A FINITE ELEMENT APPROACH TO WEAR SIMULATION USING SUBSTRUCTURING

By

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>3</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>6</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>7</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>8</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>1 INTRODUCTION</td>
<td>10</td>
</tr>
<tr>
<td>Overview</td>
<td>10</td>
</tr>
<tr>
<td>Objective</td>
<td>11</td>
</tr>
<tr>
<td>2 WEAR MODELLING</td>
<td>12</td>
</tr>
<tr>
<td>Method</td>
<td>12</td>
</tr>
<tr>
<td>Current Trends in Wear Simulation</td>
<td>14</td>
</tr>
<tr>
<td>3 SUBSTRUCTURING</td>
<td>20</td>
</tr>
<tr>
<td>Substructure</td>
<td>20</td>
</tr>
<tr>
<td>Computational Advantages</td>
<td>20</td>
</tr>
<tr>
<td>Organizational Advantages</td>
<td>21</td>
</tr>
<tr>
<td>Substructure Modelling</td>
<td>21</td>
</tr>
<tr>
<td>Numerical Formulation</td>
<td>23</td>
</tr>
<tr>
<td>Current Trends in Substructuring</td>
<td>24</td>
</tr>
<tr>
<td>4 INTEGRATION SCHEMES</td>
<td>27</td>
</tr>
<tr>
<td>Forward Euler Method</td>
<td>27</td>
</tr>
<tr>
<td>Midpoint Integration Method</td>
<td>27</td>
</tr>
<tr>
<td>5 RESULTS AND DISCUSSION</td>
<td>29</td>
</tr>
<tr>
<td>Pin on Disc Model</td>
<td>29</td>
</tr>
<tr>
<td>Pin and Pivot Model</td>
<td>32</td>
</tr>
<tr>
<td>Comparing Forward Euler and Midpoint Approximations</td>
<td>33</td>
</tr>
<tr>
<td>6 CONCLUSION</td>
<td>54</td>
</tr>
<tr>
<td>Summary</td>
<td>54</td>
</tr>
<tr>
<td>Scope of Future Work</td>
<td>55</td>
</tr>
</tbody>
</table>
LIST OF REFERENCES .................................................................................................................. 56

BIOGRAPHICAL SKETCH ......................................................................................................... 59
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-1</td>
<td>Time comparison for pin on disc model</td>
<td>31</td>
</tr>
<tr>
<td>5-2</td>
<td>Time comparison for pin on pivot model</td>
<td>33</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>2-1</td>
<td>Flowchart for wear modelling using Archard's model</td>
<td>19</td>
</tr>
<tr>
<td>5-1</td>
<td>Flowchart for wear modelling- Non Substructured</td>
<td>35</td>
</tr>
<tr>
<td>5-2</td>
<td>Pin on disc model- Non Substructured,</td>
<td>36</td>
</tr>
<tr>
<td>5-3</td>
<td>Flowchart for wear modelling- Substructured</td>
<td>37</td>
</tr>
<tr>
<td>5-4</td>
<td>Pin on Ddsc model- Substructured (A) Parts (B) Assembly</td>
<td>38</td>
</tr>
<tr>
<td>5-5</td>
<td>Final pin surface (A) Non Substructure (B) Substructure case</td>
<td>39</td>
</tr>
<tr>
<td>5-6</td>
<td>Iteration surface wear- Pin (A) Non-Substructure (B) Substructure case</td>
<td>40</td>
</tr>
<tr>
<td>5-7</td>
<td>Final disc surface (A) Non Substructure (B) Substructure case</td>
<td>41</td>
</tr>
<tr>
<td>5-8</td>
<td>Iteration surface wear- Disc(A) Non Substructure (B) Substructure case</td>
<td>42</td>
</tr>
<tr>
<td>5-9</td>
<td>Material wear- Pin (A) Non Substructure (B) Substructure case</td>
<td>43</td>
</tr>
<tr>
<td>5-10</td>
<td>Material wear- Pivot (A) Non Substructure (B) Substructure case</td>
<td>44</td>
</tr>
<tr>
<td>5-11</td>
<td>Maximum nodal wear (A) Non Substructure (B) Substructure case</td>
<td>45</td>
</tr>
<tr>
<td>5-12</td>
<td>Final surface profile (A) Non Substructure (B) Substructure case</td>
<td>46</td>
</tr>
<tr>
<td>5-13</td>
<td>Pin pivot model- Non Substructured</td>
<td>47</td>
</tr>
<tr>
<td>5-14</td>
<td>Pin pivot model- Substructured (A) Parts (B) Assembly</td>
<td>48</td>
</tr>
<tr>
<td>5-15</td>
<td>Iteration surface wear- Pin (A) Non Substructure (B) Substructure case</td>
<td>49</td>
</tr>
<tr>
<td>5-16</td>
<td>Iteration surface wear- Pivot (A) Non Substructure (B) Substructure case</td>
<td>50</td>
</tr>
<tr>
<td>5-17</td>
<td>Material wear- Pin (A) Non Substructure (B) Substructure case</td>
<td>51</td>
</tr>
<tr>
<td>5-18</td>
<td>Material wear- Pivot (A) Non Substructure (B) Substructure case</td>
<td>52</td>
</tr>
<tr>
<td>5-19</td>
<td>Midpoint and Euler approximation</td>
<td>53</td>
</tr>
<tr>
<td>5-20</td>
<td>Error plot for different step sizes</td>
<td>53</td>
</tr>
</tbody>
</table>
A FINITE ELEMENT APPROACH TO WEAR SIMULATION USING SUBSTRUCTURING

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Wear is a critical phenomenon in most of the mechanical designs which affect the component’s life in a crucial way. Currently Archard’s wear model is largely used to simulate wear using a Finite Element Method. But this method is computationally very expensive since the analysis has to be carried out over many iterations in order to simulate the entire life cycle of a component. Here, substructuring technique is adopted to model the mating parts. Results from the substructure case is verified with the normal (non-substructure) case. Percentage of time saved with the substructure case is presented. This analysis is done for two models- Pin on Disc and Pin Pivot model. Time saved for both of them is different as it depends on the complexity of model.

In order to do a step wise wear simulation, commonly Forward Euler Approximation is used as it only requires results up to the current step. Here Midpoint Approximation is adopted to accommodate a larger step size. Since this also requires results for data point ahead of the current step, surrogate extrapolation is used for that purpose. This way larger step size is adopted without affecting the accuracy of results and there by reducing total number of iterations required to simulate the same number
of cycles. Effect of increasing step size on accuracy of results is studied for both Forward Euler and Midpoint Approximation.
Overview

Wear is a complex phenomenon to understand and model. There is comparatively less work done on it. Generally, we rely on experiments to observe effects of wear on mating parts. Modelling wear is difficult due to the geometric and material nonlinearities involved. It is mainly classified as dry and lubricated wear and abrasive and sliding wear. In spite of the large number of wear models found in the literature, no model can predict wear with reasonable accuracy a priori based only on material properties and contact information.

Any part that has a surface in contact with self or other parts having relative motion will undergo wear over a period of time. The amount and rate of wear will depend on a number of factors including contact pressure, surface properties of mating parts, loading cycles, etc. Wear in turn changes the geometry of the affected part which impacts its performance. For example a robotic arm that requires precise movements at its end will see its performance depreciate in a big way due to wear in the pivot joints that hold the arm. Similarly, if there is wear on spur gears, it will change the gear profile decreasing its efficiency. This will in turn cause more wear as it now becomes a sliding contact.

Thus, wear not only decreases the performance of a system but also brings down the life expectancy. So a proper understanding of wear is very much needed in the design phase. It will allow us to predict not only the wear amount and wear rate but also the resultant geometric changes in the mating parts.
Sliding wear, which is focused in this work, is mechanistically more complex than certain other forms of wear because it not only involves the cutting and ploughing included in abrasive wear but also the adhesion of asperities, wear debris, sub-surface crack initiation and growth, the transfer of material to and from the mating surfaces, subtle changes in surface roughness during running in, tribo-chemical film formation and many other. Though many of these factors are not considered for this study.

**Objective**

The goal of this research is to further develop the current state of techniques used in wear modelling in order to make it more efficient

The main objectives of this thesis are:

- To decrease the computational cost of modelling wear implementing Finite Element Analysis (FEA)
- To implement Substructuring technique in wear problems and study its reliability.
- To implement Midpoint Approximation Method to increase the step size and compare its results with Forward Euler Method.
CHAPTER 2
WEAR MODELLING

Method

Wear is a complex phenomenon to understand and model. There is comparatively less work done on it. Generally, we rely on experiments to observe effects of wear on mating parts. Modelling wear is difficult due to the geometric and material nonlinearities involved. It is mainly classified as dry and lubricated wear and abrasive and sliding wear. In spite of the large number of wear models found in the literature, no model can predict wear with reasonable accuracy a priori based only on material properties and contact information.

Archard was the first to propose a phenomenological wear model to describe sliding wear. It is still widely used to model wear. It assumes that the critical parameters in sliding wear are the stress field in the contact and the relative sliding distance between the contacting surfaces. The constant of proportionality in Archard’s wear model, commonly termed as the Archard’s wear coefficient, has been interpreted as the ratio of the volume worn to the volume deformed.

As shown in Figure 2-1, the approach involves post-processing of the finite element results obtained from the solution of a general contact problem with a suitable wear model to compute the progress of wear for a given time interval/sliding distance [36].

The surface nodes from the surface element groups are detected by the Wear-Processor. Inward surface normal vectors at each of the surface node are computed based on whether the contact problem is two-dimensional or three-dimensional. The
contact pressure at each of the surface nodes is computed from the normal vector and the stress tensor averaged at each surface node as:

\[ t_j = \sigma_{ij} n_i \]
\[ p = t_j n_j \]

Where \( t_j \) is the traction vector, \( \sigma_{ij} \) is the stress tensor, \( n_i \) is the inward surface normal vector at the corresponding surface node, the subscripts \( i \) and \( j \) correspond to the tensor components in two-dimensional or three-dimensional space depending on the contact situation and \( p \) is the contact pressure at each surface node.

The Wear-Processor implements Archard’s wear model to calculate the linear wear at each of the surface nodes. The wear model is implemented at the local scale, which considers the current contact pressure and the node location as:

\[ \frac{V_w}{s} = k_D F_N \]  \hspace{1cm} (2-1)
\[ \frac{V_w}{s} = k_D F_N \]
\[ \frac{h}{s} = k_D p \]  \hspace{1cm} (2-2)

Where \( V_w \) is the wear volume, \( s \) is the sliding distance, \( k_D \) is the dimensional wear coefficient, \( h \) is the linear wear, \( F_N \) is the applied normal load and \( p \) is the contact pressure at each surface node.

In the Wear-Processor, the above wear law is discretized with respect to the sliding distance as

\[ \frac{dh}{ds} = k_D p \]  \hspace{1cm} (2-3)

Normally the Forward Euler integration scheme is used to integrate the wear law over the sliding distance as
\begin{align}
\Delta h_j &= \Delta h_{j-1} + \Delta h_j \\
\Delta s_j &= \Delta s_{j-1} + k \Delta p_j \Delta s_j
\end{align}

\(h_j\) is the total wear up to the \(j^{th}\) wear step, \(h_{j-1}\) is the total wear up to the \((j - 1)^{th}\) wear step and \(\Delta h_j\) is the wear amount for the \(j^{th}\) (current) wear step.

The surface nodes are then shifted inwards to accommodate for the wear. The element size limits the inward shift which in turn limits sliding distance. To address this problem, the mesh size is increased, but this will affect accuracy of results. So, in order to maintain accuracy for long sliding distance, the sliding distance is divided into small steps and the model is remeshed at the end of each step.

**Current Trends in Wear Simulation**

In the recent past, wear simulation implementing Archard’s wear model through FEA have been conducted which are in close agreement with the experimental results.

Simulation of 2D wear model of pin on disk configuration was done for measurement of wear profile [1]. Development of inclination was observed due to more severe wear at leading edge than the trailing one. Inclination reached a steady state after which the nodal wear is constant which was verified through experimental results.

Similarly, pin on disc contact configuration test [2] were done to calculate stress and contact pressure field distributions which were in accordance with analytical formulations for cylindrical flat punch contact but the mass losses calculated from numerical model were larger than experimental ones.

By using adaptive method for the time step [3] between the geometry updates, simulation time was decreased. A greater time step was used in the early steps of simulation while a smaller one was used in the terminating steps.
A model was formulated within a Lagrangian framework capable of accounting for large plastic deformations and history-dependent material behavior [4]. Continuous adaptive remeshing was used as a means of eliminating deformation induced element distortion, and resolving fine features of the wear process such as contact boundary layers.

Boundary element formulation for sliding wear was proposed, for modelling three-dimensional wear problems [5]. Since it only models and remeshes boundary of the bodies in contact, as they lose material, making it relatively easy and robust, saving computational resource and allowing large scale simulations.

Influence of the surface topography on the wear in sliding contacts was estimated in [6]. It simulated wear by discretizing the wear law with respect both to time and space and using the Euler integration scheme. Even after applying that scheme. It was concluded the main disadvantages of FEA to be its high computing time and computer disk space consumption.

A simple simulation time step optimization routine was developed [7], evaluating the integration step duration for every solution step individually on the basis of the fixed maximum wear increment. Assuming the linear wear law to be valid, the FEA wear simulation results for a given contact geometry and a given load were treated based on wear coefficient sliding distance change equivalence.

Linear increment was obtained in sequential wear simulation results. It was observed experimentally that there is a linear growth in wear depth as a result of using constant wear coefficient [8]. The wear coefficient between materials changed for repeated contacts. By using constant wear coefficient, initially high wear rate was
covered for the rest of the contact. Thus, in the simulation for the same number of
contacts, deeper wear values were calculated and safer results were reached.

Global Incremental Wear Model (GIWM) [9], an incremental implementation of
Archard’s wear model on the global scale for modeling sliding and rolling/sliding wear
was developed specifically for the pin-on-disc and twin-disc setup. The wear model
identification and its verification was done using the GIWM, and the transient wear
simulation was done on representative gear tooth flank of a planetary gear train using
the Wear Processor.

High temperature pin-on-disc tests were carried out to simulate the tribological
behavior of oxide scale in the role bite during hot rolling [10]. The evolution of the friction
coefficient during the pin-on-disc test was classified into three stages based on the
oxide film and its growth. Large cracks and pores were found inside the oxide scale,
indicating that the severe ‘banding’ phenomena could happen when the oxide scale
reaches a critical value. A large amount of wear debris observed on the pin wear track
confirming that wear mechanism at that stage was a mixture of adhesive, abrasion and
oxidation.

A procedure was adopted to simulate the rail profile evolution due to uniform
wear [11]. It considered a combination of the coupling dynamics of the freight vehicle
and track, non-Hertzian contact model and a material wear model. The proposed rail
wear model was verified by a case study. The shapes of the simulated worn rail profile
agreed with the field measurements. A contact model considering plasticity was needed
to determine effect of plastic deformation on the prediction of gauge corner wear of high
rails.
A computational mechanics-based approach was proposed to predict the sliding wear caused by a loaded spherical pin contacting a rotating disc [12]. The central region of the wear track was considered to experience plane strain conditions and a ratchetting wear model was applied to estimate the rate of wear caused by the relative sliding between the pin and the disc. The approach relied on the 3D plastic deformation. It predicted wear rates obtained experimentally in pin-on-disc tests on an Al–Si coating, and the trend was consistent with the experimental data.

Experiments performed have confirmed that, under the conditions analyzed, the temperature effect on the wear rate is more critical than that of pressure. As it was observed in previous investigations with other aluminum matrix composites [13], the role of the reinforcement particles is to increase the transition temperature from the mild to the severe wear regimes. These tendencies can also have been observed from the numerical model, although quantitative agreement with the experiments is not achieved.

The sliding part of gear tooth contact was simulated in pin-on-disc experiments [14]. The friction properties, wear resistance, and damage mechanisms of steel gear materials were investigated and compared. Dependence of coefficient of friction on the speed of disc rotation, porosity in the material and combinations of pin-disc material was established.

Expressions were developed to determine the maximum contact temperature at the pin/disk interface during pin-on-disk tribotests [15]. The predictions matched with thermal FEA of pin/disk contacts. The equations were also applied to study maximum contact temperature for pin-on-disk testing of metallic, ceramic, and polymeric materials.
As per the uncertainty guidelines ISO and NIST, friction and the wear rate of thin films by pin-on-disc were measured and analyzed different parts of standard uncertainties, such as the effect of pin holder misalignment or the role of operator in estimation of the wear track cross-section area [16]. Calculations were given to show why variance should be computed from the difference between maximum and minimum measured value instead of standard deviation. The procedure to estimate the uncertainties for low number of measurement were shown.

Observation were made through electron microscope of the warn surface [17] for pin-on-disk experiment for micro-grooving or micro-cutting, adhesion and micro-delamination or micro-flaking. Through the trend, it was seen that the instantaneous wear ratio is approximately constant also, were rate increased with the increase in the hardness difference between pin and disc.

Calculations for flash temperature and mean surface temperature at the apparent contact area were proposed [18]. The increase in bulk temperatures of the rubbing bodies decreased the flow stresses of the rubbing materials to a certain extent, which resulted in an increase in the plastic zone size in the sub- surfaces of the rubbing bodies. Consequently, the friction coefficient as well as wear rate increased with increasing sliding speed.

A numerical approach to simulate the progressive accumulation of wear in oscillating metal on metal reciprocating pin-on-disk tribometer was used to measure wear rate and the results from the finite element analysis were compared with the block-on-ring experimental results [31]. An extrapolation scheme was used to reduce computational costs of the simulation.
In order to increase the step size to decrease the computational time, kriging was used to fit the available history of the stress intensity factor and crack growth direction [32]. The surrogate allowed extrapolation ahead of the current data point, which enabled the use of the midpoint approximation method for both stress intensity factor and crack growth direction. An algorithm for adjusting the step size was introduced which produced comparable accuracy with a significantly reduced number of function evaluations.

Figure 2-1. Flowchart for wear modelling using Archard’s model
CHAPTER 3
SUBSTRUCTURING

Substructure

Substructure is a collection of elements grouped together with all the degree of freedom except the retained ones eliminated [34]. It is connected to rest of the model by retained degree of freedom at retained nodes. Retained nodes and degrees of freedom are those that will be recognized externally at the usage level (when the substructure is used in an analysis), and they are defined during generation of the substructure. It can also contain a set of internal load cases and boundary conditions (not specified at the retained nodes) that can be activated and scaled [33]. It has linear response but allows for large rotations and translations. If the eigen modes are retained, it can allow dynamic response. For the rest of the model, it is like an optional mass, stiffness, damping or a set of scalable load vectors. A single substructure can be used for identical pieces in a structure (such as the teeth of a gear)

Computational Advantages

System matrices (stiffness, mass) are smaller. Due to the creation of the substructure, only the retained degrees of freedom and the associated reduced stiffness (and mass) matrix are used in the analysis until it is necessary to recover the solution internal to the substructure.

Efficiency is improved when the same substructure is used multiple times. Once the stiffness calculation and substructure reduction is done, the substructure itself can be used multiple times, resulting in a significant savings in computational effort.

Substructuring can isolate a large part of the model to save time during reanalysis. Those parts that will remain unchanged can be modelled as substructure.
These parts can be isolated to save the computational effort involved in forming the
stiffness of those parts of the structure.

In a problem with local nonlinearities, such as a model that includes interfaces
with possible separation or contact, the iterations to resolve these local nonlinearities
can be made on a very much reduced number of degrees of freedom if the substructure
capability is used to condense the model down to just those degrees of freedom
involved in the local nonlinearity.

**Organizational Advantages**

Substructuring provides a systematic approach to complex analyses. The design
process often begins with independent analyses of naturally occurring substructures.
Therefore, it is efficient to perform the final design analysis with the use of substructure
data obtained during these independent analyses.

Substructure libraries in commercial CAE software allow analysts to share
substructures. In large design projects, large groups of engineers must often conduct
analyses using the same structures. Substructure libraries provide a clean and simple
way of sharing structural information.

Many practical structures are so large and complex that a finite element model of
the complete structure places excessive demands on available computational
resources. Such a large linear problem can be solved by building the model,
substructure by substructure, and stacking these level by level until the whole structure
is complete and then recovering the displacements and stresses locally, as required.

**Substructure Modelling**

Substructuring modelling is division of original structure into components,
planned at outset and usually adopted as a way to manage a large finite element
projector as a way to fit analysis into limited computer resources [33]. With
substructuring, separate substructures interact. A time independent analysis of
substructure proceeds as follows:

1. Divide finite element model into two or more parts (substructure). Division should
   be made where parts have few interconnections so that reduced substructure
   matrix will not be large.

2. For each substructure, create a finite element model and generate its global
   equations \( \{K_s\}\{D_s\} = \{R_s\} \). Reduce this equation (by Gauss elimination) until only
   attached DOF \( \{D_a\} \) remains. So the equation comes down to \( \{K_a\}\{D_a\} = \{R_a\} \).

3. Assemble the reduced equation sets of all substructure to obtain global
   equations \( \{K_A\}\{D_A\} = \{R_A\} \). Here \( \{D_A\} \) contains all attachment DOF \( \{D_a\} \).

4. Solve equations \( \{K_A\}\{D_A\} = \{R_A\} \) for \( \{D_A\} \). Hence attachment DOF \( \{D_a\} \) are known
   for exact substructure. Return to substructure equations \( \{K_s\}\{D_s\} = \{R_s\} \) that was
   partially solved in step 2. Now solve for remaining DOF in \( \{D_S\} \) by back
   substitution. Finally, post process to obtain gradients (stresses) in elements.

Static condensation is done prior to assembly. The substructure is regarded as a
single element has many internal DOF. Sometimes attachment DOF are referred as
‘masters’ and internal as ‘slaves’. Generally, in order to attach the substructure to the
structure, attachment d.o.f on mating boundaries must match in number, location, type
and orientation. However with more developed methods recently, it can be done even if
these parameters do not match [33].

In time-independent analysis, substructuring does not introduce any additional
approximation into the finite element model. The dynamic equivalent of substructuring is
‘component mode synthesis’ which does introduce additional approximation.

If attachment d.o.f \( \{D_A\} \) are not affected much by design changes, analysis of
individual substructure can be proceeded independently, occasionally assembling to
update \( \{D_A\} \). Different groups, subcontractors can be assigned individual substructure.
Iterative solution is required in case of nonlinearity, it can be expensive if the entire structure is involved in calculation. If nonlinearity is confined to a small part, the rest can be modelled as substructure which can greatly reduce the computational cost. Substructuring does increase bookkeeping as large number of files are generated.

**Numerical Formulation**

The reduced stiffness matrix is easily derived when only static response is considered [38]. Since the response of a substructure is entirely linear, its contribution to the virtual work equation for the model of which it is a part is

$$\delta W = \begin{bmatrix} \delta u^R & \delta u^E \end{bmatrix} \left( \begin{bmatrix} \Delta P^R \\ \Delta PE \end{bmatrix} - \begin{bmatrix} K^{RR} & K^{RE} \\ K^{ER} & K^{EE} \end{bmatrix} \begin{bmatrix} \Delta u^R \\ \Delta u^E \end{bmatrix} \right)$$  \hspace{1cm} (3-1)

Where \(\{\Delta P^R\}\) and \(\{\Delta P^E\}\) are consistent nodal forces on retained and internal nodes applied to the substructure during its loading as a substructure and tangent stiffness matrix is:

$$[K] = \begin{bmatrix} K^{RR} & K^{RE} \\ K^{ER} & K^{EE} \end{bmatrix}$$  \hspace{1cm} (3-2)

Since the internal degrees of freedom in the substructure, \(\{u^E\}\), appear only within the substructure, the equilibrium equations conjugate to \(\{\delta u^E\}\) in the contribution to the virtual work equation given above are complete within the substructure, so that

\(\{\Delta P^E\} - [K^{ER}]\{\Delta u^R\} - [K^{EE}]\{\Delta u^E\} = 0\)

These equations can be rewritten to define \(\Delta u^E\) as

$$\{\Delta u^E\} = [K^{EE}]^{-1} \left( \{\Delta P^E\} - [K^{ER}]\{\Delta u^R\} \right)$$  \hspace{1cm} (3-3)

The substructure's contribution to the static equilibrium equations is, therefore,

$$\delta W = \begin{bmatrix} \delta u^R \end{bmatrix} \left( (\{\Delta P^R\} - [K^{RE}][K^{EE}]^{-1} \{\Delta P^E\}) - ([K^{RR}] - [K^{RE}][K^{EE}]^{-1}[K^{ER}]) \{\Delta u^R\} \right)$$
Thus, for static analysis the substructure’s reduced stiffness is

\[
\overline{K} = [K^{RR}] - [K^{RE}] [K^{EE}]^{-1} [K^{ER}]
\]  

(3-4)

A reduced mass matrix is calculated by projecting the global mass matrix to the subspace of the substructure modes by a technique known as Guyan reduction but this study does not require a reduced mass matrix.

**Current Trends in Substructuring**

When the computational FEA was in a developing stage, [19] A number of substructuring aspects like multilevel sub-structuring algorithms, use of hypermatrix and other sparse matrix schemes, use of substructuring in automated design systems and substructuring in elastoplastic problems were considered. Also, ways to enhance computational efficiency of substructuring analysis were suggested.

A methodology was presented to allow for a separate analysis of uncoupled, readily identifiable substructures with the coupling effect accounted for through the interaction forces that develop at the interfaces between the substructures [20]. It was extended to include material nonlinearities and was efficient for problems with localized nonlinearities, where an interface can be placed.

The use of the substructuring technique for the solution of two-dimensional non-linear problems of dynamic response with the direct time integration method was examined [21]. It introduced some schemes that considerably reduced the computational expense in analyses of locally and fully non-linear dynamic problems as compared with a traditional analysis. Numerical examples were presented where the time reduction was obtained.
An efficient condensation technique for the proper use of available computer power and avoid any detrimental effects on computer system was presented that incorporated sub-structures in shells of revolution [22]. The technique has the ability of using fixed-size arrays in the computer program irrespective of the size of the sub-structure, thus allowing the optimum use of computer memory.

Substructuring technique was applied to obtain the detailed stress distributions in some regions of the Railcar Underframe Bolster (welded joints) in order to assess the fatigue life of the component [23].

A procedure that removes the dynamic effects of a flexible fixture from a subcomponent using modal substructuring was presented [24]. The approach improved the modal basis of the substructure, so that it could be used to more accurately estimate the modal parameters of the built-up system. New types of constraints were also presented, which constrain the modal degrees of freedom of the substructures, avoiding the need to estimate the connection point displacements and rotations.

A method to incorporate advanced substructuring features into “classic” linear dynamics within Abaqus was provided [25]. Usage of advanced features together with scalable solvers to realize more detailed physics than is currently available.

Ways of estimating the uncertainty in modal substructure models, providing the experimentalist with an approach that could be used to evaluate the fidelity of a substructure model were explored [26]. It allowed one to detect cases where the substructuring problem was very sensitive to uncertainty, so a remedy can be sought, and even a solution could be provided for the measure of expected scatter in the predictions.
The use of superelement for the stress and deflection analysis of a typical fighter wing structure was carried out [27]. Three methods of analyses were carried out and compared: practical/theoretical analysis; finite element analysis with the conventional element modeling approach; and finite element analysis with the superelement modeling. Result showed a good agreement between the three methods.

A validation test was proposed to give confidence or reveal flaws in the experimentally derived substructure model [28]. The validation was done evaluate the effect of measurement errors in noise and to simulate substructuring of indeterminate connections.

A finite element model updating by combining the substructure model updating method with the response surface model updating method was proposed [29] to update the finite element model of a certain combined cable-stayed suspension bridge. It was observed that no additional approximation was introduced due to substructuring.

When working with movable substructures, the order reduction capability has been limited by the large amount of connection degrees of freedom which must be kept in the reduced representation to cover all possible positions of substructures. A sequential assembly of substructure solved this issue, where these degrees of freedom were eliminated from the reduced model [30]. Results indicated that the proposed method predicted correctly the position-dependent dynamic behavior of a machine.
CHAPTER 4
INTEGRATION SCHEMES

Approximation schemes are used to project the solution to forward data points from the present available solutions. There are different types of Integration schemes with varying level of accuracy. Increasing the step size decreases time as it requires less steps to simulate the same number of cycles but it also decreases the accuracy of the results. So a tradeoff has to be achieved to get an optimum step size for a given accuracy margin.

Forward Euler Method
Euler approximation scheme has widely been used for most of the iterative simulations including wear modelling. Since the method only requires slope of the function up to the current step which we already have in order to get the solution for next step. With this approach, by increasing the step size, there is a significant decrease in accuracy

\[ y_{n+1} = y_n + h \cdot f(x_n, y_n) \]  (4-1)

Comparing Equation 4-1 with Equation 2-5 it is seen that \( y_{n+1} \) in our case is \( h_{n+1} \) which is the wear height for the next step. \( y_n \) is wear height for current step \( h_n \). \( h \) is the step size and \( f(x_n, y_n) \) is the wear amount \( K_p \Delta s \) for the current step which we get by running simulation on the current geometry. Hence it is just cumulative addition of wear heights for each step.

Midpoint Integration Method
Midpoint approximation has higher accuracy than that of Euler approximation [35]. It requires function evaluation at two points to get to the next step. One function
evaluation is required at the current step and the other at midpoint of current step and the next step.

\[ y_{n+1} = y_n + h \left( f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right) \right) \quad (4-2) \]

Comparing with Equation 2-5 the Equation 4-2 is little different in the sense that it requires two function evaluation. The first evaluation is \( f(x_n, y_n) \) which is same as that in forward Euler method but here this result is not used to get to \((n+1)\)th step. Instead it is used to get to \((n+1/2)\)th step which is \((x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n))\). Than second function evaluation is performed at this midpoint to get to \((n+1)\)th step. Hence it will require two simulations to be carried out for the same step size as compared to Euler’s method. To overcome this, surrogate extrapolation is used. By fitting a linear regression surrogate to the history of past points and extrapolating it to \((n+1/2)\)th step, the need of one simulation is eliminated.

In this way to simulate \(n\) steps, instead of doing \(2n\) simulation, we do \(n\) extrapolations and \(n\) simulations. Hence the number of simulations required for \(n\) steps is same as that of Euler approximation. But since the accuracy of midpoint method is higher compared to Euler’s approximation, step size can be increased without affecting accuracy of results and thus reducing the simulation time further. This method is discussed in more detail in Chapter 5.
CHAPTER 5
RESULTS AND DISCUSSION

**Pin on Disc Model**

As shown in Figure 5-2, the first model is Pin on Disc one where the pin is of 3mm diameter and 10mm height. The Disc section has a width of 10mm and a height of 5mm. Material for both Pin and pivot is AISI 4140 alloy steel that has 200 Gpa Modulus of Elasticity, 0.3 Poisson’s Ratio and wear coefficient k of $1 \times 10^{-5}$ mm$^3$/Nm. Disc is held fixed at the bottom edge. A pressure of 20Mpa is applied on the top surface of pin and it is displaced by 5mm in +X direction. Bottom edge of the Pin and top edge of Disc are defined as frictionless contact surface. Two small springs are attached to top of the Pin to make sure that it does not undergo large displacement in pressure application step. Top node of spring is fixed in y direction only. It has an elastic modulus of only 20Mpa which is negligible compared to that of Pin hence attaching springs does not affect the solution.

The steps for wear analysis is shown in Figure 5-1, Simulation is performed on the model which has first step as the pressure step and the second one as Sliding step. Sliding is done in 20 steps so the total distance of 5mm is divided into 20 segments of 0.25mm each. Once this is done, from the output database contact pressure at all nodes on contact surface is obtained. As there are 20 steps, we have a set of 20 pressure values for each node. Archard’s wear model is applied to each node for each of the 20 pressure values to get corresponding wear amounts. Then these 20 wear amounts are added to get total wear for that node. This procedure is done for each node on top and bottom contact surface. Each nodal wear amount is multiplied with an extrapolation factor of 100 to represent 100 cycles. Then for a given node, the
coordinates of that node are shifted in the direction of surface normal at that node by an amount equal to the total wear amount. This is repeated for each node. Thus, we get an updated model. Now the simulation is performed on this updated model to get another updated model by repeating the entire process. At the end of 10 iterations which represent 10X100=1000 wear cycles, the resultant profile as seen from Figure 5-6 has stabilized. Hence the net wear amount at the end of 10 iterations is multiplied with another extrapolation factor of 100 so total cycles simulated becomes100,000. This is similar to the extrapolation scheme adopted by Mukras [37]

As per the analytical calculation, converting all the data in mm we get:

\[ h = k p \Delta s \text{ (total cycles)} \]

\[ h = \left(10^{-5} /1000\right) \times 20 \times 5 \times 100,000 \]

\[ h = 0.1 \text{ mm} \]

From Figure 5-5 the simulation results match with analytical solution. As seen from Figure 5-9 and Figure 5-10 the total material wear for each iteration stabilizes and remains constant. It is same for both Pin and pivot since they are of the same material.

Second case of the same model is modelled with substructures. As seen from Figure 5-4, Except for the lower part, rest of the Pin is made into substructure. Similarly, except for the top contacting region, rest of the Disc is substructured.

Substructure generation happens only once at the start, before the first iteration where its stiffness matrix is reduced only to the retained nodes. Retained Nodes are the nodes which would be required to connect the substructure with the main structure for further analysis. For each wear iteration, we only deal with this reduced stiffness matrix which contain DOF of the retained nodes.
These two substructures are modelled in separate files, nodes on top and bottom edge are selected as retaining nodes. Static condensation of stiffness matrix is done to reduce it to only the retained nodes during Substructure generation step. These reduced matrices are imported in the main model as shown in Figure 5-4 and the assembly is done. After assembly, rest of the problem is solved in the same way as in previous non-substructure case. The procedure is given in Figure 5-3.

Results of both Non-Substructure and Substructure case is compared. As seen in Figure 5-5, Figure 5-6 and Figure 5-12, the development of wear profile on the disc is same for both the cases. Figure 5-11 shows maximum nodal wear for each iteration. Also, the disc profile is identical as seen in Figure 5-7 and Figure 5-8. The material wear in each iteration shown in Figure 5-9 and Figure 5-10 is also same as that of non-substructure case. Hence it concluded that solution to this wear problem is identical for both the cases.

The run time is summarized in Table 5-1. The total time taken for analysis in the Non-Substructure case 1705.5s. For the substructure case, we need to add the time required for both substructure generation to the total run time for the second case which comes out to be 968.35 s. Hence we get a 43.2% time saving in the Substructure case compared to the non-Substructure case. All simulations were run on a machine with 6GB ram and Intel Core i5 processor.

<table>
<thead>
<tr>
<th></th>
<th>SubSt. Pin</th>
<th>SubSt. Disc</th>
<th>Job time</th>
<th>Total Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non SubSt.</td>
<td>21.23</td>
<td>23.94</td>
<td>923.18</td>
<td>968.35</td>
</tr>
<tr>
<td>% saving</td>
<td>0</td>
<td>0</td>
<td>1707.55</td>
<td>1705.55</td>
</tr>
</tbody>
</table>

43.22%
Pin and Pivot Model

The second model analyzed is Pin Pivot model as seen in Figure 5-13. Here the pin is rotating inside of a fixed pivot as in a pivot joint or a sleeve bearing. The outer diameter of pin is in contact with the inner diameter of pivot. Hence these two contact surfaces will undergo wear. Pin diameter is 10mm, pivot inner diameter is 10.2mm and outer diameter is 18mm. Both Pin and Pivot are made of AISI 4140 alloy steel that has 200 Gpa Modulus of Elasticity, 0.3 Poisson’s Ratio and wear coefficient k of $1 \times 10^{-5}$ mm$^3$/Nm. As in Figure 5-13 the bottom edge of pivot is held fixed. A force of 40N is applied in +Y direction to the Pin in the force application step. Pin is rotated by 0.5 radian in the rotation step. Center of pin is constrained in X direction. Nodes on inner diameter of pivot and those on outer diameter of pin are selected as contact nodes.

The wear analysis is done in the same way as that of Pin and Disc model. In this case, the rotation step is divided in to 20 steps of equal length. Once we get the wear amount for each node as it was done in Pin and Disc model, each node is shifted in the direction of normal to the surface which in this case is radial. The contact surface profile update at the end of each cycle is shown in Figure 5-15 and Figure 5-16 for the Pin and Pivot respectively. Also the material wear in each iteration is plotted in Figure 5-17 and Figure 5-18. After first three iterations, the wear amount stabilizes and remains constant for the rest of iterations. The material wear for each iteration for Pin and pivot is same since the material is same.

This model was also modeled using substructuring as shown in Figure 5-14. In this case only the parts that have contact surfaces are not modeled as substructure. For pin, the inner part is modelled as substructure while the outer ring is not since it has the contact surface. The center node and the nodes on circumference are retained nodes.
Similarly for the pivot, except for the inner ring, rest of the part is modelled as substructure. Nodes on bottom edge and those on inner circumference of the pivot substructure are retained nodes.

Wear simulation was carried out for the substructure case similar to non-substructure case and the resulting wear surface of pin and pivot can be seen in Figure 5-15 and Figure 5-16. Comparing this results with that of non-substructure model, it can be seen that we are getting the same results.

The run time is summarized in Table 5-2. The total time taken for analysis in the Non-Substructure case is 4348 s. For the substructure case, we need to add the time required for both substructure generation to the total run time for the second case which comes out to be 1314s. Hence we get a 69.7% time saving in the Substructure case compared to the non-Substructure case. Again, all simulations were run on a machine with 6GB RAM and Intel Core i5 processor.

<table>
<thead>
<tr>
<th></th>
<th>SubSt. Pin</th>
<th>SubSt. Pivot</th>
<th>Job time</th>
<th>Total Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>SubSt.</td>
<td>23.74</td>
<td>32.29</td>
<td>1258.0</td>
<td>1314.03</td>
</tr>
<tr>
<td>Non SubSt.</td>
<td>0</td>
<td>0</td>
<td>4348.0</td>
<td>4348.0</td>
</tr>
<tr>
<td>% saving</td>
<td></td>
<td></td>
<td></td>
<td>69.77%</td>
</tr>
</tbody>
</table>

**Comparing Forward Euler and Midpoint Approximations**

Euler integration scheme as discussed in chapter 4 is most commonly used as it only requires function evaluation up to the current step. That is, in order to get to \((n+1)^{th}\) step, function evaluation of \(n^{th}\) step is required.
The maximum nodal wear for each iteration is plotted in Figure 5-19 for the Pin on Disc model. It is seen that the wear plot stabilizes and becomes linear after first three iterations.

Midpoint approximation was also implemented in the same model. As discussed in Chapter 4, to move from n to n+1 step it requires function evaluation of n\(^{th}\) step as well as (n+1/2)\(^{th}\) step. Here surrogate extrapolation is used. Linear regression surrogate of first order is used because the equation of wear if of first order and also the plot in Figure 5-19 is linear. Hence first four step evaluation is done with Euler approximation as in the previous case. After the fourth step, midpoint approximation is adopted. So a linear surrogate of first order is fitted to the first four points and the next midpoint which is 4.5\(^{th}\) step is found by extrapolating that surrogate. Hence for this evaluation, simulation is replaced with surrogate extrapolation. Once we have the 4.5\(^{th}\) step, using Equation 4-2, wear for fifth step is calculated. This way, to go from fourth to fifth step, one simulation and one extrapolation is required. Now for the sixth step, surrogate extrapolation is done based on history up to fifth step. This process of curve fitting and extrapolation is required for each node in each step.

Results with different step sizes are compared in Figure 5-20. The maximum nodal wear at the end of 1000 simulated cycles is taken here. Simulations were done for a step size of 1, 1.2, 1.5 and 2 for both Euler and Midpoint approximations. The result from simulation with step size 1 was considered as base for both the approximations. Results from other three step sizes were compared against that base and percentage error in these results were plotted. It can be seen that with Euler approximation, increase in percentage error is large compared to that of Midpoint approximation. To an
extent that error with step size of 2 in Midpoint approximation is less than the error of Euler approximation with step size 1.2.

Figure 5-1. Flowchart for wear modelling- Non Substructured
Figure 5-2. Pin on disc model- Non Substructured,
Figure 5-3. Flowchart for wear modelling- Substructured
Figure 5-4. Pin on Ddsc model- Substructured (A) Parts (B) Assembly
Figure 5-5. Final pin surface (A) Non Substructure (B) Substructure case
Figure 5-6. Iteration surface wear - Pin (A) Non-Substructure (B) Substructure case
Figure 5-7. Final disc surface (A) Non Substructure (B) Substructure case
Figure 5-8. Iteration surface wear- Disc(A) Non Substructure (B) Substructure case
Figure 5-9. Material wear- Pin (A) Non Substructure (B) Substructure case
Figure 5-10. Material wear- Pivot (A) Non Substructure (B) Substructure case
Figure 5-11. Maximum nodal wear (A) Non Substructure (B) Substructure case
Figure 5-12. Final surface profile (A) Non Substructure (B) Substructure case
Figure 5-13. Pin pivot model- Non Substructured
Figure 5-14. Pin pivot model- Substructured (A) Parts (B) Assembly
Figure 5-15. Iteration surface wear- Pin (A) Non Substructure (B) Substructure case
Figure 5-16. Iteration surface wear- Pivot (A) Non Substructure (B) Substructure case
Figure 5-17. Material wear- Pin (A) Non Substructure (B) Substructure case
Figure 5.18. Material wear- Pivot (A) Non Substructure (B) Substructure case
Figure 5-19. Midpoint and Euler approximation

Figure 5-20. Error plot for different step sizes
CHAPTER 6
CONCLUSION

Summary

This thesis focused on ways to decrease the computational cost associated with wear simulations. First method explored for this purpose was substructuring while the other was to implement use on Midpoint approximation scheme.

After a close analysis of the results, it can be concluded that implementing substructuring to model parts of the entire geometry does not affect the results of simulation. This is seen in both Pin on Disc and the Pin Pivot model that surface profile of substructure case was identical with that of the non-substructure case. Results being same, comparing total run time, Pin on Disc model saved 43.2% time with substructuring while Pin Pivot model being a more complex one, saved 69.7% of time. This is large time saving given the accuracy of results.

Compared to Euler approximation, the combination of Midpoint approximation and Surrogate extrapolation gave more accurate results with large step size. Hence Midpoint approximation allows for larger step size and in turn saving simulation time without much compromise in accuracy.

Hence when applied to even larger and more complex problems, this technique can result in saving simulation time in terms of hours and days. Since substructuring is very cheap method in terms of computational space, the requirement for high performance computing platform can be eliminated for many wear simulation problems. This will not only make simulation process faster but can also make it possible to simulate larger problems which are currently not possible with present state of art computing.
Scope of Future Work

To make the technique of substructuring more widely accepted for wear simulation, it needs to be validated with more wear models present in the literature. More complex and nonlinear problems need to be simulated to test the results and time saving. Also for any given wear model, up to what extent can parts be substructured has to be studied so that there can be some standardization of how close to the contact surface can a part be substructured. More complex geometries with uneven surfaces, high contact forces and thermal stresses need to be modelled with substructures to validate this technique. Also models which undergo high wear amount resulting in significant geometric changes need to be simulated with substructuring. Methods other than Euler approximation and Midpoint approximation need to be tested for their accuracy. Also for a given model, how to optimize the step size to decrease the time keeping accuracy within a certain range is a matter of study.
LIST OF REFERENCES


BIOGRAPHICAL SKETCH

Hardik Acharya was born in Anand in the State of Gujarat, India. He graduated with Bachelor of Engineering degree in Mechanical Engineering from Gujarat Technological University, 2014. After graduation, he was a Research Fellow at Indian Institute of Technology, Gandhinagar. He enrolled in Master of Engineering program at Mechanical & Aerospace Engineering Department at University of Florida. His areas of interest include Solid Mechanics, FEA and Design.