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# LIST OF ABBREVIATIONS

- $a$: Semi-major axis of elliptical contact area (m, in.); $i$ and $o$ denotes inner and outer raceway contact, respectively
- $a_1$: Life factor for reliability
- $a_2$: Life factor for manufacturing & heat treatment processes
- $a_3$: Life factor for lubrication
- $a_4$: Life factor for gradation in elastic modulus
- $a_c$: Carbide factor
- $a_{ISO}$: Integrated life adjustment factor
- $\bar{a}$: Carbide area in percent
- $b$: Semi-minor axis of elliptical contact area (m, in.); $i$ and $o$ denotes inner and outer raceway contact, respectively
- $b_i$: Polynomial coefficients for regression equation for $i \in [0, 5]$
- $c_j$: Nonlinear regression equation coefficients for $j \in [1, 3]$
- $C$: Basic dynamic capacity of bearing (N, lbf)
- $Co$: Cobalt content of the material
- CPRESS: Contact pressure from finite element analysis (Pa, psi)
- $d$: Percent (%) drop in elastic modulus from $E_{surface}$ to $E_{core}$
- $d_i$: Inner raceway diameter (m, in.)
- $d_m$: Bearing pitch diameter (m, in.)
- $d_o$: Outer raceway diameter (m, in.)
D  Ball diameter (m, in.)

$E_{ball}$  Elastic modulus of ball material (Pa, psi)

$E_{core}$  Elastic modulus of raceway/plate material at core (Pa, psi)

$E_{Effective}$  Effective elastic modulus of case hardened bearing steel (Pa, psi)

$E_{surface}$  Surface elastic modulus of raceway/plate material (Pa, psi)

$E_I$ & $E_{II}$  Elastic Modulus of body I and II (Pa, psi)

$E^*$  Effective elastic modulus of two bodies in contact (Pa, psi)

EHD  Elastohydrodynamic

$f_i$  Inner raceway groove radius to ball diameter ratio

$f_o$  Outer raceway groove radius to ball diameter ratio

$F_a$  Axial-thrust load (N, lbf)

$F_e$  Equivalent radial load (N, lbf)

$F_r$  Radial load (N, lbf)

FEA  Finite element analysis

FEM  Finite element method

HRC  Rockwell hardness

HV  Vickers hardness

k  Proportionality constant

$\kappa^*$  Diffusivity constant

K  Non-dimensional model design parameter
L  Bearing Fatigue Life, millions of revolutions or millions of stress cycles

$L_{i0}$  Bearing fatigue life in millions of revolutions corresponding to 90% survival probability

$L_i$  Fatigue life of inner raceway, millions of revolutions or millions of stress cycles

$L_o$  Fatigue life of outer raceway, millions of revolutions or millions of stress cycles

$L_s$  Bearing fatigue life in millions of revolutions corresponding to survival probability of s%

$LF$  Elastic modulus gradation life factor

$LP$  Lundberg-Palmgren

$m$  Median carbide size in micrometers

$N$  rpm

$n$  Stress-life exponent

$\bar{n}$  Total number of carbides per square centimeters

$p$  Load-life exponent

P675  Pyrowear – 675 bearing steel

$Q$  Normal load experienced by the contact (N, lbf)

$Q_c$  Basic dynamic capacity of raceways (N, lbf); i and o denotes inner and outer raceway contact, respectively

$Q_{\text{max}}$  Maximum load experienced by ball – raceway contact (N, lbf)

$Q_e$  Cubic mean equivalent radial load experienced by raceway (N, lbf); i and o denotes inner and outer raceway contact, respectively

RCF  Rolling contact fatigue

RMS/rms  Root mean square error

$R^2$  Coefficient of determination
\( R_{adj}^2 \) Adjusted coefficient of determination

\( r_i \) Inner groove radius (m, in.)

\( r_o \) Outer groove radius (m, in.)

\( \bar{S} \) Probability of survival

\( S \) Maximum compressive stress/peak contact stress (Pa, psi)

\( S_{TH} \) Maximum contact pressure with uniform elastic modulus/through-hardened raceway material (Pa, psi)

\( S_{CH} \) Maximum contact pressure for graded/case-hardened raceway (Pa, psi)

UMAT User defined material subroutine

VIM Vacuum induction melting

VAR Vacuum arc remelting

X Radial load factor

Y Axial load factor

Z Number of balls

\( \nu_I \) & \( \nu_{II} \) Poisson’s ratio of body I and II

\( \rho_{II} \) & \( \rho_{II} \) Principal curvatures of body I \( (m^{-1}, \text{in}^{-1}) \)

\( \rho_{II} \) & \( \rho_{II} \) Principal curvatures of body II \( (m^{-1}, \text{in}^{-1}) \)

\( \sum \rho \) Curvature sum of two bodies in contact \( (m^{-1}, \text{in}^{-1}) \)

\( \sum \rho_i \) Curvature sum at inner raceway contact \( (m^{-1}, \text{in}^{-1}) \)

\( \sum \rho_o \) Curvature sum at outer raceway contact \( (m^{-1}, \text{in}^{-1}) \)

\( F(\rho) \) Curvature difference of two bodies in contact
$F(\rho)_i$ Curvature difference at inner raceway contact

$F(\rho)_o$ Curvature difference at outer raceway contact

$\delta$ Case depth (m, in.)

$\mu$ Friction coefficient

$S_{\text{max} i}$ Maximum compressive hertz stress experienced by inner raceway (Pa, psi)

$S_{\text{max} o}$ Maximum compressive hertz stress experienced by outer raceway (Pa, psi)

$\alpha$ Bearing contact angle (.deg)

$\dot{\phi}_i$ Inner contact osculation

$\dot{\phi}_o$ Outer contact osculation

$a^*$ Dimensionless semi-major axis of the contact ellipse (mm, in.)

$b^*$ Dimensionless semi-minor axis of the contact ellipse (mm, in.)

ACBB Angular contact ball bearing

AFBMA Anti-Friction Bearing Manufacturers Association

AISI American Iron and Steel Institute

ANSI American National Standard Institute

CRB Cylindrical roller bearing

CVD Carbon Vacuum Degassed

DGBB Deep groove ball bearing

ECDF Empirical cumulative distribution function

ISO International Organization of Standards

LP Lundberg and Palmgren
<table>
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<td>RCF</td>
<td>Rolling contact fatigue</td>
</tr>
<tr>
<td>SAE</td>
<td>Society of Automotive Engineers</td>
</tr>
<tr>
<td>VAR</td>
<td>Vacuum arc re-melt</td>
</tr>
<tr>
<td>VIMVAR</td>
<td>Vacuum induction melt – vacuum arc re-melt</td>
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Since beginning of twentieth century, bearing manufacturers and aircraft engine manufacturers have sought to predict the fatigue endurance capabilities of rolling element bearings. The first generally accepted method to predict bearing fatigue life was published in 1940s by Lundberg and Palmgren (LP). Their work is based on Hertz theory which assumes perfectly smooth contact between two homogeneous elastic solids free from any pre-loaded condition. However, most of the modern bearing materials, such as case-hardened steels, do not satisfy these conditions. Recent, Micro/Nano indentation studies have shown inhomogeneous microstructure of these materials which leads to graded material properties in the subsurface region. Also, carburization process induces beneficial compressive residual stresses which significantly alters the subsurface stress distribution of the Hertzian contact loads. Moreover, influence of metal plasticity is not considered in rolling contact fatigue life prediction under elastic-plastic loading conditions. Therefore, bearing life predictions using LP derived fatigue life models significantly under predict endurance performance of modern bearing steels. These leads to use of oversized bearing components,
reducing the efficiency of critical mechanical and engineering systems. Hence this dissertation research is focused on correcting this under prediction of fatigue lives for modern case-hardened bearing steels. Using computational tools, in depth analysis of rolling element bearings components under different operating conditions is presented. For case-carburized steels, accurate subsurface stress fields are determined by incorporating microstructure specific material properties in the subsurface region of bearing raceways. The expected improvement in fatigue lives of case carburized steel is analyzed due to combined effect of graded material properties and residual compressive stresses. Rolling contact fatigue performance of through hardened bearing steel is investigated under elastic-plastic loading condition. Importance of metal plasticity in fatigue life prediction under these conditions is highlighted. This will help in bridging existing gap between predicted and experimentally observed fatigue lives of modern bearing steels, thereby enabling reliable bearings design.
CHAPTER 1
INTRODUCTION

Rolling Element Bearings

Discovery of rolling motion dates back to centuries when it was first understood that the amount of force required to move an object over rollers is much less than to slide the same object over the same surface (Harris (1)). Even after the invention of lubrication mechanisms which significantly reduced efforts required in sliding, rolling motion was still less difficult when it could be used. These insights led to development of bearings based on rolling motion which would eventually be used in complex mechanical systems. The term ‘bearing’ signifies their functional requirements to support load between two objects without undergoing any significant deformation. Rolling element bearings includes all types of bearings that utilize rolling motion of balls or rollers to permit constrained motion of one body relative to another with minimum friction. In majority of applications, rolling bearings are used to permit rotation of the shaft with respect to some fixed structure commonly referred as a ‘housing’. These bearings can also be used to facilitate relative linear motion or combinations of linear and rotary motions between two bodies. Hence they are used in the complex mechanical systems such as gas turbine engines and internal combustion engines to support main engine shafts. They are used in wind turbine gearboxes and some of the oil and gas services industry components such as mud rotors, mud pumps and rotating tables of drilling rigs.

During early stages of development, rolling element bearings could not compete with the endurance characteristics of hydrodynamic sliding bearings. However, this picture has changed in twentieth century with the advancement of accurate
manufacturing techniques and development of superior rolling bearing steels which can function satisfactorily for long service hours. Based on today’s technology, advantages of rolling element bearings compared to other bearing types are summarized by Harris (1) as follows:

1. Rolling element bearing operate under much less friction torque compared to conventional hydrodynamic bearings resulting into low frictional power losses
2. Difference between static friction torque and kinematic friction torque is not significant.
3. Deflection is much less sensitive to load fluctuation than that observed in conventional hydrodynamic bearings.
4. For rolling element bearings, lubrication system is less expensive as small quantity of lubricant is required for life long performance. Some rolling element bearings can also be obtained with lifelong lubricant supply.
5. They occupy shorter axial length than conventional hydrodynamic bearings
6. Within reasonable limits, variations in load, speed and operating temperatures has little effect on the performance of rolling element bearings.
7. They can support combinations of radial and thrust loads simultaneously; the range for load and speed over which they can operate satisfactorily is very wide.

Contact angle inside rolling element bearings depends on the combination of radial and thrust loads, and bearing internal geometry features.

**Rolling Bearing Types**

Rolling element bearing generally has three important components: inner, outer raceways and rolling elements (Fig. 1-1A). Depending upon the type of rolling element used bearing can be classified into four different categories:

1. Ball bearings which use spherical balls as rolling elements (Fig.1-1B)
2. Cylindrical roller bearings which use cylindrical rollers
3. Tapered roller bearings which use tapered rollers
4. Spherical roller bearings which use spherical barrels between inner and outer raceway.

For majority of bearing applications, the inner and outer raceway groove curvature radii range from 51.5 to 53% of the rolling element diameter. Ball bearings can be of deep groove (DGBB) or angular contact type (ACBB). DGBBs are generally designed for radial load where as ACBBs are designed for combined radial and thrust load. Some applications use double rows of rolling elements to increase radial load carrying capacity of the bearing. The contact angle for angular contact ball bearings generally does not exceed $40^\circ$. Thrust ball bearings has contact angle of $90^\circ$. Bearings having contact angle more than $45^\circ$ are classified as thrust bearings. They are suitable for high speed operations (Harris (1)). Compared to ball bearings, roller bearings are designed to carry much larger supporting load. They are usually stiffer and provide better endurance compared to ball bearing of similar size. Manufacturing of roller element bearing is much difficult compared to that of ball bearing. Tapered roller bearings can carry combinations of large radial and thrust load, but generally they are not used in high speed applications. Similarly spherical roller bearings are used in heavy duty applications, but they have very high friction compared to cylindrical roller bearings and are not suitable for high speed operations (Harris (1)). The load carried by each rolling element inside rolling element bearing can vary depending on its angular position with respect to external load vector and the type of the bearing. One sample case of load distribution inside radially loaded rolling element bearing is discussed in following section.
Load Distribution of Radial Rolling Bearings

Figure 1-2, shows load distribution between rolling elements of a rolling bearing under external load \( F_r \) in radial direction. The load shared by each rolling element is determined by its azimuthal angle \( \phi \), which specifies its location with respect to applied load vector. Bearing rings, shaft and housing are assumed to be rigid, except that the external applied load is supported by elastic deformations at the rolling element – raceway contacts. Assuming one of rolling element is loaded in the direction of externally applied radial load, the load distribution can be considered as symmetric as shown in Fig. 1-2. Using radial deflections at the center of each ball-raceway contact and load-deflection relations, Harris (1) has given following relation to determine rolling element loads \( Q(\phi_j) \) at the azimuth location \( \phi_j \):

\[
Q(\phi_j) = Q_{\text{max}} \left[ 1 - \frac{1}{2} \epsilon (1 - \cos \phi_j) \right]
\]  

(1-1)

where \( \phi_j = j(2\pi)/Z \); \( Z \) is the number of rolling element and \( j \) is the azimuthal position number, \( Q_{\text{max}} \) is the maximum rolling element load and \( \epsilon \) is the load distribution factor defined as:

\[
\epsilon = \frac{1}{2} (1 - \cos \phi_0)
\]

(1-2)

Generally, \( \epsilon \) is representation of the angular extent of the load zone. In Eq. (1-1), \( t \) is 1.5 and 1.1 for ball and roller bearings respectively. Sjovall proposed relationship between maximum rolling element load \( Q_{\text{max}} \) and external radial load \( F_r \) as:
\[ F_r = ZQ_{\text{max}} J_r(\varepsilon) \quad (1-3) \]

where \( J_r(\varepsilon) \) is the radial integral defined as,

\[ J_r(\varepsilon) = \frac{1}{2\pi} \int_{-\phi_o}^{\phi_o} \left(1 - \frac{1}{2\varepsilon} (1 - \cos \phi)\right) \cos \phi d\phi \quad (1-4) \]

In Eq. (1-4), \( \phi_o \) is the extent of the load zone, as shown in Fig. 1-2. During each rotation, every point on inner raceway passes through load zone as shown in Fig. 1-2. To determine cumulative damage caused by all the loads during one rotation, cubic mean equivalent radial loads are used. According to Harris (1), the cubic mean equivalent radial loads experienced by raceway can be determined as:

\[ Q_e = \left( \frac{1}{Z} \sum_{j=1}^{j=Z} Q_j^k \right)^{1/k} \quad (1-5) \]

where \( k=3 \) for rotating raceway and \( k=10/3 \) for non-rotating raceway with respect to applied load.

Bearings generally fail because of formation of spall on rolling surfaces. One such failure of inner raceway surface is shown in Fig. 1-3. The primary reason for this destruction of bearing component surfaces is rolling contact fatigue. Historically it has always been difficult to accurately formulate rolling contact fatigue. The tri-axial state of stress, non-proportional loading and out of phase stress-strain relationship makes endurance predictions very difficult. Therefore bearing industry extensively relies on empirical data of bearing fatigue lives. The current industrial standards used for bearing’s design found its origin in the Lundberg and Palmgren’s (LP) theory developed in the middle of twentieth century. LP theory uses endurance data of the bearings which were manufactured using steel and manufacturing practices available at that time.
However, over the period of past 70 years, there has been significant improvement in the quality of bearing steels and accuracy of manufacturing practices. These resulted into significant improvement in endurance capabilities of rolling element bearings and LP based life prediction methodology has proven to be inadequate to capture these advancement in steel processing. This results in use of oversized bearing with increased weight penalty on critical engineering systems. Therefore, there is need to update bearing life rating equations with parameters which are relevant to today’s materials. With this objective identified, next section describes outline of this dissertation research.

**Dissertation Outline**

Chapters in the following sections contain detailed study aimed at correcting standard mechanics and bearing design equations used for predicting fatigue lifes. In the beginning, Chapter 2 reviews some of the widely used rolling contact fatigue life prediction models. Brief history of these models along with that of bearing life rating standards is also provided. In chapter 3, typical materials used for manufacturing of rolling element bearings are discussed. Distinction between material properties of case-hardened steels and through-hardened steels is also identified. As a part of this thesis research elasticity, plasticity and numerical analysis were performed to correct for the fatigue life predictions of case-hardened and through-hardened bearing steels. Recent Micro/Nano-indentation tests indicate that for case hardened steels there exists gradient in carbide volume fraction at subsurface depth. This gradient in carbide volume fractions result in gradient in material properties, such as hardness, elastic modulus and yield strength of the case layer. Chapter 4 provides corrections to Hertz theory of contact mechanics, such that elastic stresses can be predicted for case-carburized bearing
steels which exhibit variation in elastic modulus. Chapter 5 analyzes the beneficial effect of heat treatment induced residual compressive stresses on fatigue lives of case-carburized steels with incorporation of graded material properties. Chapter 6 provides corrections for rolling contact fatigue lives of through-hardened bearing steels under elastic-plastic loading conditions. It provides details of the experimental investigation on VIMVAR M50 steel rod at 5.5GPa peak contact pressure. Influence of surface roughness and elastohydrodynamic film thickness under boundary lubrication conditions on observed fatigue lifess is also discussed. In this chapter new strain-life approach is provided to predict rolling contact fatigue lifes of through-hardened bearing steels.

Chapter 7 discusses stress-life approach to predict rolling contact fatigue life of VIMVAR M50-NiL case carburized bearing steels using median fatigue life estimates.

Figure 1-2. Radial load distribution inside rolling element bearing (Nagamoto et al. (2))
Figure 1-3. Spalled surface of the rolling element bearing inner raceway. (Photo Courtesy of Nathan Branch. Source: Branch et al. (3))
CHAPTER 2
ROLLING CONTACT FATIGUE LIFE PREDICTION MODELS

Background

Despite several advantages highlighted in previous chapter, rolling element bearing exhibit some limitations which limits their applicability and reliability in industrial applications. Experimental investigations over the past three to four decades have shown that for well lubricated rolling element bearings, ball-raceway contact generally fail due to subsurface originated fatigue commonly known as rolling contact fatigue (RCF); whereas poorly lubricated contacts inside bearings generally fail due to surface fatigue commonly observed in industrial applications. Subsurface failure of bearing material is assumed to originate from the critical locations where Hertzian contact induced shear stresses are highest. Surface failures are generally considered as manifestation of wear, scuffing and pitting mechanisms due to prolong sliding of rough surfaces. In 20th century many researchers have attempted to model rolling contact fatigue in bearing steels using empirical relations based on Weibull’s statistical theory for strength of materials. This chapter provides brief review of some of these models which are currently used by bearing manufacturers for design purpose. Discussion on surface failure mechanisms in bearings is beyond the scope of this research work.

Hertz Theory

First theory to analyze stresses at the contact of two elastic solids was developed by Hertz in early 1880s. He was concerned about the possible influence of the elastic deformation of two glass lenses in contact on Newton’s optical interference fringe patterns observed between them (Johnson (4)). Based on this, he made the hypothesis that contact area, is in general, elliptical and for the purpose of calculating local
deformations, both bodies can be approximated as elastic half space loaded over a small elliptical region of its plane surface. Based on Hertz analysis, ellipsoidal compressive stress distribution in the contact area can be given as:

\[ P = \frac{3Q}{2\pi ab} \left[ 1 - \left( \frac{x}{a} \right)^2 - \left( \frac{y}{b} \right)^2 \right]^{\frac{1}{2}} \]  

(2-1)

where, \( Q \) is the normal load experienced by two bodies in contact in X-Y plane; \( a \) and \( b \) represent semi-major and semi-minor axis of the elliptical contact area, respectively.

Let \( S \) represents maximum compressive stress experienced by two bodies in contact (i.e. at \( x=0 \) and \( y=0 \) in Eq. (2-1)). Figure 2-1 shows normalized Hertz pressure profile (Eq. (2-1)) in the contact region. The elliptical ball-raceway contacts that occur inside ball bearings under normal load are shown in Fig. 2-2.

Failure of bearings is defined as formation of the spall/fatigue cracks of measurable dimensions on the raceway surfaces (Fig. 1-3). Generally inner raceway fails first due to smaller diameter and higher rpm compared to outer raceway. For rolling elements applied stress is distributed over larger area, therefore they tend to be more durable. However, fatigue failure of ball or rollers is observed in applications where bearings are operating under heavy load (Harris (1)). But under nominal operating conditions failure of bearing inner rings is commonly observed and hence will be focus of the work presented in this article.

Experimental investigations have shown that the fatigue cracks leading to bearing ring/raceway failures originate in the subsurface region. Therefore, analysis of subsurface stresses in ball-raceway contacts is essential to predict fatigue performance of rolling element bearings. In 1930, Thomas and Hoersch (5) expanded Hertz’s original
work to determine subsurface stress distribution of two elastic solids in contact. Their work involved implementation of elliptical integrals of first and second kind to determine subsurface stress distribution along the depth direction at the center of the contact \((x=0, y=0)\). Using Thomas and Hoersch (5) solution subsurface stress distribution along the centerline of the ball-inner raceway contact inside 208 size deep groove ball bearing was determined. Based on Timken catalog ratings, bore diameter for this bearing size is 40 mm, ball diameter is 12.7 mm and axial width is 17 mm. As per standard practice, raceway groove radius to ball diameter ratio of 0.52 was assumed in this analysis. Both ball and raceway were assumed to be made up of steel with elastic modulus of 200 GPa and Poisson’s ratio of 0.3. Total normal load of \(Q=4750\) N was used which results in maximum Hertz contact pressure of \(S=3.1\) GPa with semi-major axis \(a=2.62\) mm and semi-minor axis \(b=0.28\) mm. Figure 2-3 shows variations of normalized subsurface stress components \((\frac{\sigma_x}{S}, \frac{\sigma_y}{S}, \frac{\sigma_z}{S})\) as function of normalized depth \((\frac{z}{b})\) below the center of the contact. It should be noted that due to symmetry of the Hertzian contact (Fig.s 2-1, 2-2), shear stress component along the centerline will be zero. Therefore, stress components shown in Fig. 2-3 also represent principal normal stress components at each location. The equivalent normalized stress components using Tresca and octahedral shear stress criterion are also shown. Maximum values of these principal shear stress components i.e. shear stress \((\tau_{\text{max}})\) and von-mises equivalent octahedral shear stress \((\tau_{\text{oct}})\) can be approximately expressed as a constant multiple of maximum contact pressure \(S\) as:

\[
\tau = k \times S \quad (2-2)
\]
where \( k \) is proportionality constant equal to 0.28 and 0.32 for octahedral and maximum shearing stress respectively. In point contacts, the depth at which maximum shear stress and maximum octahedral shear stress is observed is approximately same at 0.76\( b \). During the passage of loaded rolling elements over a point on the raceway surface, each point in the subsurface zone along \( Z \)-axis will be subjected to shear stress cycles which varies from 0 to \( \tau_{\text{max}} \) and 0 to \( \tau_{\text{oct}} \) at critical depth. These non-fully reversed shear stress cycles results in significant compressive mean stress which is considered to be beneficial for bearing fatigue life. Compressive mean stresses suppress propagation of fatigue cracks in the subsurface zone.

Another shear stress which needs to be considered in the analysis of ball-raceway contacts in orthogonal shear stress. If we assume elements are rolling in the direction of \( z \)-axis and \( x \)-axis is along the depth direction then shear stresses occurring in \( xz \) plane below the contact surface assume values negative to positive for values of \( x \) less than greater than zero, respectively (Harris (1)). To study variation of this shear stress for ball-raceway contact inside 208 size radial ball bearing, finite element model as shown in Fig. 2-4 was developed. Exploiting symmetry boundary conditions of Hertz contact area, only quarter sections of the ball and raceway are modelled to save computational time. Figure 2-4 shows part of the inner raceway that subtends 90 degree angle at the center of the bearing. Geometric configuration shown in Fig. 2-4 is in unloaded condition; whereas model in Fig. 2-5 shows same ball-inner raceway contact under 4750 N load applied in normal/radial direction. Raceway in this figure subtends 5 degrees angle at the center of the bearing. Both Figs. 2-4 and 2-5 are provided to show the localized nature of the Hertzian stresses in ball-raceway contact with respect to
global bearing assembly. For this configuration at critical depth, subsurface shear stress variation in xz plane is shown in Fig. 2-6. Analytical studies have shown that Eq. (2-2) can also be used to determine maximum value of $\tau_{xz}$ shear stress with $k=0.25$. In common bearing applications ratio of semi-minor axis $(b)$ to semi-major axis $(a)$ of the contact ellipse is close to 0.1. For these cases, the depth at which maximum value of $\tau_{xz}$ shear stress is observed is roughly $0.49b$ (Harris (1)) and it is located near the edge of the contact area $(x/b\approx1)$ as shown in Fig. 2-6B. Figure 2-6A highlights the node path at critical depth of 0.13 mm $(x/b\approx0.464)$ along which highest $\tau_{xz}$ shear stress is observed, as plotted in Fig. 2-6B. From these figures we can see that finite element (FE) results are in good agreement with analytical predictions. From Fig. 2-6 we can see that due to rolling motion every point at critical depth of 0.13 mm will be subjected to fully reversed stress cycle from $-(\tau_{xz})_{\text{max}}$ to $(\tau_{xz})_{\text{max}}$ subsurface shear stress in xz plane. The critical orthogonal shear stress ($\tau_0$) is defined as the maximum subsurface shear stress in xz plane i.e. $\tau_0 = (\tau_{xz})_{\text{max}}$.

Therefore, three principal shearing stresses: i.e. maximum orthogonal shear stress $\tau_0$, maximum subsurface shear stress (Tresca stress) $\tau_{\text{max}}$ and maximum octahedral shear stresses $\tau_{\text{oct}}$, can be used to predict subsurface location from where potential fatigue cracks might originate inside bearing raceways. Using these principal stresses, over past century various researchers have developed empirical relations for bearing fatigue life predictions. Following section contains review of some of these models, namely Weibull, Lundberg and Palmgren, Ioannides and Harris, Zaretsky's.
equations, which are widely accepted. Discussion regarding the premise based on which they were developed and their limitations in predicting fatigue lives of modern bearing steels is also provided.

**Weibull Equation**

While studying static failure of brittle engineering materials using statistical tools, Weibull (6) determined that ultimate strength of the material cannot be expressed as a deterministic value but a probability distribution function is required for this purpose. This led to fundamental law of Weibull’s theory:

\[
\ln(1 - F) = -\int_v n(\sigma)dv
\]

(2-3)

where \(F\) represents probability of rupture of volume \(v\) subjected to stress distribution of \(\sigma\), and \(n(\sigma)\) is function of material properties (Harris (1)). Weibull’s major contribution is realization of the fact that structural failure of material is function of the volume under stress. In applying these concepts to fatigue lives of roller bearings, Weibull (7) stated that “Due to extremely high stresses, concentrated in small volumes, large scatter in fatigue lives is an inherent property of any roller bearing. Thus, expected life of bearing for specified service conditions is a random variable which is defined by a distribution function and involves parameters of location, of scale and of shape.” These findings were corroborated by experimental investigations where nominally identical bearings subjected to nominally identical load, speed, lubrication and environmental conditions do not exhibit the same life in fatigue. Instead bearing fails according to dispersion as shown in Fig. 2-7. Slope of this curve fit (\(e\)) is called Weibull slope which is indication of dispersion in bearing fatigue lives. Statistical studies by Lundberg and Palmgren (8, 9) indicate that \(e=10/9\) for ball bearings and \(e=9/8\) for roller bearings. These values were
based on actual endurance test data of thousands of bearings fabricated from through-hardened AISI 52100 steel. Palmgren further stated that for commonly used bearing steels, \( e \) is in the range of 1.1-1.5 (Harris (1)). Because of dispersion of fatigue lives, bearing manufacturers use \( L_{10} \) life defined as the fatigue life that 90\% of the bearing population will endure. It is also termed as “basic rating life” used in bearing catalogs. The dispersion of fatigue lives for homogeneous group of bearings, such as shown in Fig. 2-7, can be represented using following Weibull’s equation:

\[
\ln \ln \frac{1}{S} = e \ln \frac{L}{L_\beta}
\]

(2-4)

where \( S \) represents probability of survival and \( L \) is the life in millions of inner-race revolutions. \( L_\beta \) is the characteristic life to be determined using available bearing endurance data. In Fig. 2-7, ordinate is \( \ln \ln (1/S) \) and abscissa is \( \ln L \). Weibull also stated that the probability of survival could be expressed as (Zaretsky et al. (10)):

\[
\ln \frac{1}{S} \sim \tau^c N^e V
\]

(2-5)

where \( V \) presents stressed volume, \( \tau \) is the critical shear stress experienced by the raceway material, \( c/e \) is the shear stress-life exponent and \( N \) is the number of stress cycles. If \( u \) is the number of stress cycles per revolution and \( L \) is the life in revolutions then \( N=ul \) (Harris (1)). Also, \( S = 1-F \), where \( F \) is probability of failure/rupture used in Eq. (2-3).

Weibull’s theory is based on the assumption that the first crack formed inside the material leads to break. This assumption may be valid for brittle materials with low fracture toughness. But in the fatigue of bearing steels, metallurgical investigations have
shown that many cracks are formed in the subsurface region which do not propagate to the surface (Harris (1)). Therefore, Weibull’s theory is not directly applicable to rolling element bearings.

**Lundberg-Palmgren Equation**

Lundberg and Palmgren (8, 9) extended Weibull’s work on statistical strength of materials and proposed that probability of occurrence of fatigue crack in bearing steels should also be a function of the depth \( z_0 \) at which most severe shear stress occurs. Moreover, as explained in section 2.2, each point in the subsurface location is subjected to fully reversed shear stress cycle in the xz plane. Therefore, the total maximum subsurface shear stress, also termed as orthogonal shear stress, variation at critical depth is \( 2\tau_0 \), which is greater than either maximum Tresca stress variation from 0 to \( \tau_{max} \) or maximum octahedral shear stress variation from 0 to \( \tau_{oct} \). Hence Lundberg and Palmgren (8, 9) assumed that maximum orthogonal shear stress is the most damaging stress which leads to fatigue crack initiation in the subsurface region of the bearing inner rings. Therefore, the Lundberg-Palmgren (LP) model is defined as follow:

\[
\ln \frac{1}{S} \sim \frac{\tau_0^c N^e}{z_0^h} V \quad (2-6)
\]

where \( h \) is depth exponent and remaining terms has the same meaning as that in Eq.s (2-4, 2-5). For determination of subsurface stressed volume (\( V \), Lundberg-Palmgren provided following relation:

\[
V = a l z_0 \quad (2-7)
\]
where $a$ is the semi-major axis of the elliptical point contact area, $l$ is circumference of bearing inner ring and $z_0$ is the critical depth at which highest magnitude of orthogonal shear stress is observed. Substituting Eq. (2-7) in Eq. (2-6), we get

$$\ln \left( \frac{1}{S} \right) \sim \frac{\tau_0^c N^e a l}{z_0^{h-1}}$$

(2-8)

Substituting Eq.s (2-1, 2-2) in Eq. (2-8) and after significant mathematical manipulation and rearrangement Lundberg and Palmgren developed following relation to predict the fatigue life of ball-raceway contact (Harris (1)):

$$L_{10} = \left( \frac{Q}{Q_c} \right)^p$$

(2-9)

where $L_{10}$ is the fatigue life of ball-raceway contact in millions of revolutions, $Q_c$ is the basic dynamic capacity of the ball-raceway contact defined as the load which this contact can withstand for one million revolutions with 90% probability of survival; $p$ is load-life exponent determined as follows:

$$p = \frac{c - h + 2}{3e}$$

(2-10)

Evaluating the endurance test data of approximately 1500 bearings, Lundberg-Palmgren determined, for point contact inside ball bearings: $c=31/3$ and $h=7/3$ (Harris (1)). Substituting these values in Eq. (2-10), along with $e=10/9$ yields $p=3$ for point contact. Similar analysis for line contact problem shows $p=4$.

At any instance, rolling element bearing has plurality of contacts (as shown in Fig. 1-2). Therefore, during each rotation, point on raceway is subjected to varying load magnitudes depending upon its azimuthal (angular) location. Therefore, load history
needs to be considered to determine damage accumulation during one rotation. To solve this problem, Lundberg-Palmgren considered load distribution integrals and cubic mean equivalent loads to determine effective load experienced by rotating raceway during one rotation (section 1.3). Harris (1) provides detailed discussion regarding these concepts. Lundberg and Palmgren determined following relation to predict fatigue life of rotating raceway under plurality of contacts:

\[
L_{10} = \left[ \frac{C}{F_e} \right]^\nu
\]  

(2-11)

where \( C \) is the dynamic capacity of the entire bearing or the raceway, defined as the load which bearing’s rolling elements - raceway assembly can survive for one million revolutions with 90% probability of survival; \( F_e \) is the equivalent radial load experienced by the bearing and in the standard method of life calculation is given by

\[
F_e = XF_r + YF_a
\]  

(2-12)

Where \( X \) and \( Y \) are radial and axial load factors, and \( F_r \) and \( F_a \) are the applied radial and axial loads, respectively. Load factor values are dependent on the nominal contact angle of the bearings.

American National Standard Institute (ANSI)/Anti-Friction Bearing Manufacturers Association (AFBMA), and the International Organization for Standardization (ISO) had adopted Eq. (2-11) as a standard for rolling element bearing fatigue life prediction until 1970-80s. Bearing fatigue life prediction using LP Eq. (2-11) worked reasonably well during middle of the twentieth century. But as the manufacturing methods improved, resulting in high quality steels, as well as adoption of better lubrication practices for ball-raceway contacts led to improved endurance performance of bearings and thus Eq. (2-11)
was found to significantly underpredict fatigue lives of modern bearing steels. One of the primary reasons for the inadequacy of the LP model to predict fatigue life of modern bearing steels is the choice of failure stress as orthogonal shear stress which doesn’t account for effect of beneficial residual stresses present/developed during RCF loading. Moreover, most of the parameters used in LP life theory, including dynamic capacity, were empirically determined using endurance test data of bearing steels developed using manufacturing methods and techniques available at that time. During twentieth century, major advances in steel processing occurred with development of vacuum-melting techniques. Vacuum processing reduces or eliminates the amount of non-metallic inclusions, entrapped gases, and trace elements in structural alloys which acts as stress risers (Zaretsky (11)). Especially, implementation of combined vacuum induction melting (VIM) and vacuum arc re-melting (VAR) practices led to much cleaner variant of the steel. Therefore, these vacuum processing techniques are known to increase bearing fatigue lives by as much as much factor of 6. Advances in lubrication technologies have also contributed to improved fatigue life of rolling element bearings. Concentrated contacts inside bearing in presence of lubricant are known to generate elastohydrodynamic lubricant film, which separates the two contacting surfaces. Around 1960s, researchers found that bearing life and wear is highly dependent on the elastohydrodynamic (EHD) film thickness between rolling element and raceway surfaces (Zaretsky (12)). This EHD film thickness is dependent on lubricant rheology, surface finish and operating conditions such as temperature, load and speed. Development of tools and techniques to properly select/control these parameters has enabled rolling element bearing to operate at higher temperatures and for longer times.
In 1992, Zaretsky (13) modified original LP Eq. (2-11) with life improvement factors to account for influence of improved manufacturing methods, lubrication technologies and different reliabilities in bearing fatigue life prediction models. The bearing life rating standard after incorporation of these life factors can be written as:

\[ L_s = a_1 a_2 a_3 \left( \frac{C}{F_s} \right)^\nu \]  

(2-13)

where, \( a_1 \) accounts for different reliabilities, \( a_2 \) accounts for manufacturing and heat treatment processes of bearing steels, and \( a_3 \) is a function of ratio of central lubricant film thickness to composite rms roughness of two bodies in contact. Despite these modifications to original LP theory based life prediction methodology, experience has shown that Eq. (2-13) continue to under predict fatigue lives of bearings manufactured using latest available technologies. By 1994, actual service life of the bearings was found to be 14 times greater than that predicted by Eq. (2-13) (14). Also, based on extensive endurance testing, many bearing manufacturers had reported inter-dependence of life modification factors \( a_2 \) and \( a_3 \) used in Eq. (2-13). Therefore, to address these issues, Ioannides and Harris (15) proposed new life prediction methodology in 1985. Theory behind its development and implementation is the topic of discussion for next section.

**Ioannides and Harris Equation**

The proportionality between probabilities of survival of subsurface volume subjected to concentrated stress for \( N \) repeated cycles, such as in Eq. (2-6), is valid only when contacting surfaces are geometrically perfect and a simple normal stress occurs between them. In presence of significant surface shear stresses (due to friction),
bearing components are known to fail due to cracks originated in the surface region rather than in subsurface location (Harris (1)). In the development of their theory, Lundberg and Palmgren did not considered chances of occurrence of surface initiated fatigue in predicting fatigue life of rolling element bearings. Moreover, influence of bearing operating temperatures and surface roughness on material’s RCF performance was not considered. Microstructural investigations by Voskamp (16) and Voskamp and Mittemeijer (17) has shown significant alterations of the material microstructure in the subsurface region accompanied by development of residual stresses in circumferential, axial and radial directions of the endurance tested bearing steels. None of these effects can be considered in bearing life analysis using traditional LP model. Therefore, to account for these deficiencies in LP theory, Ioannides and Harris (IH) (15) proposed new life prediction methodology which also established linkage between structural fatigue and rolling contact fatigue. They continued the use of Weibull’s weakest link theory to predict fatigue of bearings. Based on IH model, the probability of survival of volume element $\Delta V_i$, sufficiently large to contain any defects, can be expressed as:

$$\ln \frac{1}{\Delta S_i} = F(N, \sigma_i - \sigma_{ui}) \Delta V_i$$  \hspace{1cm} (2-14)

where $\sigma_i$ is stress related fatigue criterion and $\sigma_{ui}$ is a threshold value of the criterion, i.e. fatigue limit, below which volume element will not fail (Ioannides and Harris (15)).

Similar to LP model, IH models also approximates function $F(N, \sigma_i - \sigma_{ui})$ by power law such that probability of survival of volume element $\Delta V_i$ can be expressed as:
\[ \ln \frac{1}{\Delta S_i} = A_i N^c H(\sigma_i - \sigma_{ui})(\sigma_i - \sigma_{ui})^c \Delta V_i \]  

(2-15)

where \( H(x) \) is step function, such that its value is 1 when stress \( \sigma_i \) in the element exceeds local fatigue limit \( \sigma_{ui} \), and 0 otherwise. Both \( A_i \) and \( \sigma_{ui} \) are assumed to be independent random variables. Furthermore, IH model continues the LP life philosophy that cracks formed at the critical depth, where the material is stressed most, leads to fatigue failure of the component. Therefore, survival probability of each volume element, as shown in Eq. (2-15), were weighted with exponential power of stress weighted average depth i.e. \( z_i^{-h} \). If \( n \) elements of volume \( \Delta V_i, i \in (1, n) \) make the entire volume \( V \), then probability of survival \( S \) of this entire volume for \( N \) cycles can be obtained by multiplying individual probabilities of survival of each volume element \( i \) i.e.

\[ \overline{S} = \Delta S_1 \Delta S_2 \Delta S_3 \ldots \Delta S_n \]  

(2-16)

Substituting Eq. (2-15) into Eq. (2-16), assuming \( A_i = A + \delta A_i \), where \( A \) is the average value and applying limits for integration, probability of survival of volume \( V \) can be determined as:

\[ \ln \frac{1}{\overline{S}} = AN^c \int_V H(\sigma - \sigma_u) \frac{(\sigma - \sigma_u)^c}{z^h} dV \]  

(2-17)

It should be noted that the term \( \delta A_i \) is assumed to be very small therefore, its contribution is neglected as per first approximation. Let \( I \) denote the integration term in Eq. (2-17). Because of fatigue limit, \( I \) is also random variable. Using joint probability theories and McLaurin series, Ioannides and Harris (15) rearranged Eq. (2-17) as follows:
\[
\ln \frac{1}{S} = AN^c \bar{I} \tag{2-18}
\]

where \( \bar{I} \) represents moments of \( I \), defined as:

\[
\bar{I} = \int \int \int_{V} H(\sigma - \sigma_u) \frac{(\sigma - \sigma_u)^c}{z^h} g(\sigma_u) d\sigma_u dV \tag{2-19}
\]

where \( g(\sigma_u) \) represents statistical distribution of fatigue limit \( \sigma_u \) in the stressed volume \( V \). Equation (2-18) represents probability of survival of stressed volume \( V \), which is determined using joint probability of two independent events i.e. probability of occurrence of fatigue limit \( \sigma_u \), i.e. \( g(\sigma_u) \), in this volume element and probability of its survival as given by Eq. (2-17). For very clean steels, Ioannides and Harris (15) assumed that defects of infinitesimal size in each volume element will only affect fatigue limit in that volume element, i.e. auto-correlation of fatigue limit \( \sigma_u \) with neighboring elements is assumed to be zero. Moreover, for homogeneous materials they reasoned that variation of fatigue limit \( \sigma_u \) can be neglected and assumed it has a constant value in the entire subsurface region. Therefore, for high quality homogeneous steels Eq. (2-18) can be written as:

\[
\ln \frac{1}{S} \approx AN^c \int_{V^R} \frac{(\sigma - \sigma_u)^c}{z^h} dV \tag{2-20}
\]

where \( V^R \) represents part of the volume \( V \) where \( \sigma > \sigma_u \). Equation (2-20) is flexible in terms of choice of stress criterion, i.e. any of the Tresca, orthogonal, octahedral or von-Mises equivalent stresses can be used with corresponding fatigue limit values. The stress term in Eq. (2-20) can also accommodate effect of hoop's stresses and residual
stresses, if any, present in the material, along with Hertzian contact stresses. Therefore, the stress term in Eq. (2-20) can be expanded as:

$$\sigma - \sigma_u = \sigma + \sigma_h - (\sigma_u - \sigma_r)$$  \hspace{1cm} (2-21)

where $\sigma_h, \sigma_r$ represent hoop and residual stresses in the material, respectively. At high rpm, bearing steels are known to generate significant hoop’s stresses in tangential direction. Also, residual stresses are developed due to case-carburization in both circumferential and axial directions. It should be noted that residual stresses are generally compressive in nature, therefore their contribution in Eq. (2-21), is beneficial for fatigue life.

Using extensive rolling element bearing endurance data reported by aircraft engine manufacturers and bearing manufacturers, Harris and McCool (18) has shown applicability of Eq. (2-20) for bearing fatigue life prediction. In this work they proposed use of octahedral shear stress ($\tau_{oct}$) as the most critical subsurface stress responsible for fatigue crack initiation. The perturbation in Hertzian stress field, such as due to frictional forces at the surface or stress concentration due to contaminant, can be perfectly captured by the stress term used in Eq. (2-20). Therefore, by accounting for generalized stress fields commonly observed in industrial rolling element bearing applications, IH model provides significant improvement over traditional LP model based fatigue life prediction methodology. Finally, Harris and Barnsby (19) expanded original data sets of Harris and McCool (18) to show the applicability of IH model (i.e. Eq. (2-20)) for bearing fatigue life prediction for wide variety of test cases. However, in this study they have used maximum von-mises stress for bearing fatigue life prediction. It should be noted that von-mises stress and octahedral shear stress are directly
proportional to each other. Also, Eq. (2-20) is solved by discretizing subsurface volume into ‘n’ different volume elements and numerical integration is performed using Simpson’s rule as follows:

\[
\ln \frac{1}{S} \propto \frac{N^e \pi ab^{1-h}}{9n^2} d_r \sum_{j=1}^{j=n} c_j \sum_{k=1}^{k=n} c_k \left[ \frac{(\sigma_{VM, jk} - \sigma_{VM, \text{lim}})}{r_k^h} \right]^c \tag{2-22}
\]

where, \( d_r \) represents diameter of the raceway; \( a \) and \( b \) are the semi-major and semi-minor axis of the elliptical contact area, respectively; \( c \) values are Simpson’s rule coefficients for numerical integration, while \( r \) is dimensionless distance i.e. \( z/b \) from the contact surface to the subsurface stress location \((x, y, z)\). It should be noted that for near surface volume elements, small but finite values must be used for dimensionless depth \( r \). In this research work, Eq. (2-20) will be solved using finite element (FE) generated stress fields. Discussion regarding procedure used will be provided in subsequent chapters. Harris and Barnsby (19) simplified Eq. (2-22) to determine stress-life factor \( a_{SL} \) which compares generalized stress-field with the traditionally assumed Hertzian stresses in the LP model. According to 2003 design guide, by Barnsby et al. (20), for the application of International Standard ISO 281/2, to predict life, the summation term in Eq. (2-22) can be represented as:

\[
I = \left[ \frac{\pi ab^{1-h}}{9n^2} d_r \sum_{j=1}^{j=n} c_j \sum_{k=1}^{k=n} c_k \left( \frac{(\sigma_{VM, jk} - \sigma_{VM, \text{lim}})}{r_k^h} \right)^c \right]^{-\frac{1}{e}} \tag{2-23}
\]

for 90% probability for survival. Equation (2-23) may also be evaluated according to LP assumed conditions, i.e. simple applied Hertz stress and zero limit stress \( \sigma_{VM, \text{lim}} = 0 \).
Let this be represented as $I_{LP}$. Then according to ASME design guide (Barnsby et al. (20)), $a_{SL}$ can be defined as:

$$a_{SL} = \frac{I}{I_{LP}}$$

(2-24)

Note when $I = I_{LP}$, $a_{SL} = 1$. With the development of this new stress-life factor, bearing life rating Eq. (2-11), was modified as follows:

$$L_{wo} = a_t a_{SL} \left[ \frac{C}{F_c} \right]^p$$

(2-25)

It should be noted that stress-life factor $a_{SL}$, is interdependent stress life factor. It can be considered as combination of life improvement factors $a_2$ and $a_3$ from Eq. (2-13), as it is developed to address the interdependent effects of contact stress, lubrication conditions, surface traction shear stresses, residual stresses, hoop stress and contamination induced stress into an integrated model which addresses both surface-initiated and subsurface initiated fatigue failures. Using IH model, ISO launched its new standard ISO 281 in 2007 (Ref. (14)):

$$L_{wo} = a_t a_{iso} \left[ \frac{C}{F_c} \right]^p$$

(2-26)

where, $a_{iso} = a_{SL}$. Equation (2-26) is the latest standard used for bearing fatigue life prediction. ISO 281: 2007 standard is been widely accepted by manufacturers and institutes for bearing designs e.g. this standard is mandatory for certification of wind turbine gearboxes (Ref. (14)). However, American National Standards Institute (ANSI) has not adopted this standard and especially Zaretsky (21) objected to use of fatigue
stress limit ($\sigma_u$) and interdependence of life adjustment factors. Moreover, ductile materials are known to fail due to shear stresses, therefore the choice of von-mises stress criterion which is not a shear stress, is also questionable for bearing fatigue life predictions. Some of these issues were addressed by Zaretsky’s equation which will be briefly discussed in following section.

**Zaretsky Equation**

ASME design guide (Barnsby et al. (20)) mentions that maximum shear stress amplitude is somewhat more accurate damage parameter for bearing steels. But generally it is not used in bearing life rating standards due to complexity of calculations involved in determining its magnitude. However, Zaretsky (10) proposed a bearing life prediction model using maximum shear (Tresca) stress criterion which can be written as:

$$\ln \frac{1}{S} = \alpha \tau_{\text{max}} c/e N V$$

(2-27)

where, $S$ represents survival probability of stressed volume $V$ subjected to maximum Tresca stress of $\tau_{\text{max}}$ for $N$ cycles of loading. In both Weibull Eq. (2-5) and Lundberg-Palmgren Eq. (2-6), for constant survival probability and volume element, the relationship between shear stress and life has exponent of $c/e$. This means that shear stress-life exponent depends on bearing life scatter/dispersion. Zaretsky et al. (10) argued that for most of the materials in non-rolling element fatigue the stress life exponent is independent of the scatter in bearing fatigue lives. This case should be applicable in rolling contact fatigue as well. Therefore, in the Zaretsky Eq. (2-27), we can see that for constant probability of survival and stressed volume, the relationship
between stress and fatigue life is exponential with exponent of $c$; additionally, the depth
term $z_o$ at which most severe shear stress occurs is also not used in Eq. (2-27). Similar
to IH model, Zaretsky’s model can also be used to integrate Eq. (2-27) over the entire
RCF affected region to determine complete life from the elemental stressed volumes as:

$$\ln \frac{1}{S} \propto N^c \int_{V} \tau^e dV$$  \hspace{1cm} (2-28)

Zaretsky’s model (i.e. Eq. (2-28)), does not use local element fatigue limit for life
calculation as used in IH model. However, his theory doesn’t exclude concept of fatigue
limit either. In Eq. (2-28), only those elements are used in which $\tau > \tau_u$, i.e. elements
with $\tau < \tau_u$ are considered to have infinite life. Also the fatigue life and survival
probability of the element depends on the stress $\tau$ instead of $\tau - \tau_u$. Zaretsky’s model
also differs the way in which life of the entire rolling element bearing assembly is
determined. Let us assume that $L_{10i}$, $L_{10o}$ and $L_{10or}$ represents fatigue life of inner
raceway, outer raceway and rolling element respectively. According to LP life prediction
methodology, $L_{10}$ fatigue life of the entire assembly can be determined using following
relation:

$$\left( \frac{1}{L_{10}} \right)^e = \left( \frac{1}{L_{10i}} \right)^e + \left( \frac{1}{L_{10o}} \right)^e$$  \hspace{1cm} (2-29)

where $e$ is the Weibull slope of the fatigue life scatter. Zaretsky’s model differ from Eq.
(2-29) in that it considers life of the rolling element also to determine life of the entire
assembly. Therefore, in Zaretsky’s model, $L_{10}$ life of the entire assembly can be obtained by following equation (Zaretsky et al. (22)):

$$\left( \frac{1}{L_{10}} \right)^e = \left( \frac{1}{L_{10i}} \right)^e + \left( \frac{1}{L_{10o}} \right)^e + \left( \frac{1}{L_{10or}} \right)^e$$

(2-30)

For radially loaded ball and roller bearings, life of rolling element is greater than equal to life of the outer raceway. Therefore, generally it is assumed that $L_{10or} = L_{10o}$; whereas for thrust loaded ball and roller bearings, life of rolling element is generally greater than equal to life of inner raceway, but less than life of outer raceway, therefore, for calculation purposes it is assumed that $L_{10or} = L_{10i}$ (Zaretsky et al. (22)). This approach still gives conservative estimates for fatigue life.

**Summary**

Over past entire century various researchers have attempted to develop reliable life rating models for bearing steels under rolling contact fatigue loading conditions. Three principal shearing stresses are generally used for bearing life analysis: the orthogonal shearing stress, $\tau_0$ (based on LP model); the Von-Mises equivalent octahedral shearing stress, $\tau_{oct}$ (based on Ioannides and Harris Model) and the maximum shearing stress, $\tau_{max}$ (based on Zaretsky Model). All these shearing stresses varies in proportion with the maximum Hertz pressure $S$ experienced by two bodies in contact. The first empirical relation for bearing fatigue life prediction was developed by Lundberg and Palmgren (LP) based on Weibull’s weakest link theory and utilizing Hertzian stress fields generated due to contact between two elastic solids. LP model worked reasonably well for old generation bearing steels which contain relatively higher
number of impurities in the subsurface region. However, towards the end of twentieth century, as the steel manufacturing techniques and melting practices improved, LP model seem to be inadequate for fatigue life prediction of modern bearing steels. Life prediction using LP model results in significant under prediction of fatigue performance and use of oversized bearing component in industrial applications. Primary reasons for these limitations in LP model is due to continued reliance on Hertz load generated subsurface orthogonal shear stress criterion which cannot account for surface shear stresses/lubricant effects, compressive residual stresses which are known to be beneficial for bearing fatigue lives. To correct for these under prediction of fatigue lives, in 1992, STLE Tribology committee modified original LP life rating equation with three life enhancement factors which accounted for reliability, improved bearing steel quality and better lubrication conditions, respectively. But still this modified bearing life rating formula continued to under-predict fatigue performance of modern bearing steels.

Finally, Ioannides and Harris modified the original Lundberg-Palmgren based life prediction methodology. They proposed discretization of the entire RCF affected region into different volume elements and integrated stress-life approach which takes into account survival probability of each volume element in predicting chances of fatigue in the entire subsurface region. IH model incorporates generalized stress field as opposed to Hertzian stresses used in LP equation. This makes it feasible to account for interdependent effects of lubrication conditions, hoops stresses, residual stresses on both subsurface and surface initiated fatigue of the bearing steels. IH model also establishes the concept of fatigue limit for bearings under RCF loading conditions. According to this concept, bearings operating under low load with well lubricated
contacts without any contamination effects can attain infinite lives without any chances of fatigue failures. Current ISO 281 standard for bearing fatigue life prediction is based on IH model which is been widely accepted by manufacturers and institutes for bearings designs e.g. this standard is mandatory for certification of wind turbine gearboxes. However, American National Standards Institute (ANSI) has not adopted this standard and especially notable researchers such as Erwin Zaretsky has opposed to the idea of fatigue stress limit and interdependence of life adjustment factors for bearing steels under RCF loading conditions. For the work presented in subsequent chapters, all of the three life prediction models by Lundberg-Palmgren, Ioannides-Harris and Zaretsky are used for bearing fatigue life predictions.

Figure 2-1. Normalized Hertz Pressure profile in the contact region, geometries are assumed to be in contact in XY plane.
Figure 2-2. Elliptical Ball-raceway contact inside ball bearings
Figure 2-3. Normalized subsurface stress components and equivalent stresses in depth direction at the center of the point contact with $\frac{a}{b} = 9.4$. 
Figure 2-4. Sample ball-inner raceway contact inside 208 Size (40 mm bore diameter) ball bearing
Figure 2-5. Ball-inner raceway contact of 208 size bearing under 4750 N radial load
Figure 2-6. Normalized shear stress variation in xz plane at critical depth of $0.464b$ for ball-raceway contact shown in Fig. 2-5. A) Shear stress contours in raceway material. B) Normalized $\tau_{xz}$ shear stresses parallel to the minor axis of the contact ellipse at critical depth of 0.13mm.
Figure 2-7. Weibull distribution of bearing fatigue lives
CHAPTER 3
ROLLING ELEMENT BEARING MATERIALS AND THEIR CHARACTERISTICS

Background

Discovery of rolling element bearings is dated back to 1734, when British researcher Jacob Rowe filed for the first patent on such bearings. The rolling element bearings used in Jacob Rowe’s era were made of wood, bronze and iron (Zaretsky (11)). For almost over a century, things didn’t change until last quarter of nineteenth century when Bessemer process and open-hearth melting processes were invented to improve the quality and metallurgy of the steel used for industrial applications. The first reported use of this material was by bicycle bearing manufacturers, while majority of other bearing industry probably relied on unhardened steels.

In the late 19th century demand for bearings capable of supporting heavy loads increased which led to development of carbon and chromium steels for bearing manufacturing. One of the steels reported in American Iron and Steel Institute specification (AISI), AISI 52100, was stated to be harden throughout enabling it to be reliable in applications where durability and long life are wanted. Since then it is the most widely used bearing steel today. More than 90% of the bearings manufactured today contain AISI 52100 steel. During the middle of twentieth century, carburizing variants of steel such as AISI 4320 and AISI 9310 low alloy steel were also very popular. For high temperature applications, tool steels such as AISI M-1 and AISI M-10 steels were developed (Zaretsky (11)). Thrust bearings are one of the critical components of the turbo-machinery such as aircraft gas turbine engines. Bearing materials used in these applications are expected to withstand high temperature and rotational speeds without catastrophic failures (Philip (23)). These operational
requirements led to development of modern steel processing techniques such as vacuum induction melting (VIM) and vacuum arc remelting (VAR). These methods when used in combination are known to produce cleaner variants of steels which are free from any non-metallic inclusions, entrapped gases and traced elements in structural alloys. These led to significant improvement in bearing fatigue life for aero-engine applications. VIM VAR M50 and VIM VAR M50-NiL are the two popular choices for bearing materials in jet engine applications. Towards the end of twentieth century, new corrosion resistant bearing material Pyrowear 675 (P-675) was developed by Carpenter Technology Corp. Endurance tests conducted report that its rolling contact fatigue life is improved by up to five times compared to conventional M50 bearing steel (Wells et al. (24)). Some of the recent development includes duplex hardening of steels in which thin nitride layer is applied to bearing component which have already been given a conventional hardening treatment (Ooi and Bhadeshia (25)). Duplex hardening results in increased surface hardness and possibly, compressive residual stresses in the near surface region, which gives superior tri-biological resistance to bearing components.

Rolling contact fatigue performance of bearing steels is highly dependent on the subsurface micro-structure and the presence of compressive residual stresses. Variation in material microstructure results in variation in mechanical properties in the subsurface region. These variation in mechanical properties can significantly affect subsurface Hertzian stress distribution of the two bodies in contact. Therefore it is important to quantify possible variation in these material properties to reliably predict their influence on endurance performance in rolling contact fatigue. Hence this chapter is dedicated to identifying possible spatial variation of mechanical properties of some of
the widely used bearing alloy steels, as discussed above. Resulting limitations in the applications of these materials are also highlighted.

**AISI 52100 Steel**

As discussed in the introduction, AISI 52100 steel is the most widely used through-hardened bearing steel. Generally, in machine tool spindle applications, they are air melted/vacuum degassed and tempered at 200° C. Its near surface hardness is close to 60 HRC. It has elastic modulus of 200 GPa, ultimate strength of 2.48 GPa and yield strength at 0.2% plastic strain offset is 2.14 GPa (Braza et al. (26)). Through-hardened material behavior results from uniformly distributed carbides in the subsurface microstructure. Its material properties such as hardness, yield strength and elastic modulus approximately remains constant as a function of depth in the subsurface region. The approximate chemical composition of 52100 bearing steel is shown in Table 3-1.

Voskamp (16) has extensively analyzed the performance of AISI 52100 deep groove ball bearings under rolling contact fatigue loading. He observed three stages of material response under RCF loading: Stage I – material shakedown stage – characterized by micro-plastic deformation, work hardening and buildup of residual stresses in first 1000 revolutions of operation; Stage II - Steady State – if the external load is below the shakedown limit of the material then microstructure enters steady state after stage I. In this stage bearing material resists further micro-plastic deformation. Voskamp (16) reported that bearing spends most of its life during this stage and it can span up to $10^9$ revolutions; Stage III – Instability – Plastically hardened microstructure in Stage I, gradually loosens it ability to carry the load elastically. Therefore, third stage is characterized by material softening, micro-plastic deformation,
phase transformations, changes in residual stresses (Voskamp and Mittemeijer (17))
and texture formation (Voskamp (16)). Based on experimental investigations, Voskamp
concluded that, for bearings to fail they must enter this third and final stage of
microstructure alterations. He also proposed that the changing amount of retained
austenite in the subsurface material volume can be regarded as a parameter monitoring
the three-stage development.

Studies by Wilcock and Booser has shown that AISI 52100 steel tend to lose its
hardness at temperatures above 177°C (Zaretsky (11)), making it unreliable for high
temperature applications such as aircraft gas turbine engines. Generally, in these
applications, operating hot hardness is required to be greater than Rockwell C 58. Also,
experimental investigations by Parker and Zaretsky (27), Zaretsky et al. (28) using Five-
Ball fatigue tester, indicate that low alloy content of bearing steels result in higher
fatigue lives. Therefore, for bearing operating at temperatures less than 149°C, AISI
52100 steel with the lowest alloying content, has longer fatigue life and is the most
widely used material by the bearing manufacturers.

**AISI M50 Steel**

As discussed in section 3.1, AISI M50 steel is one of the popular choices for the
aircraft gas turbine engine mainshaft bearings. It maintains its hardness at higher
temperatures which makes it highly desirable for Aeroengine applications where
bearings must tolerate vibratory stresses, bending moments and high rotation speeds at
high temperatures and aggressive lubrication conditions. M50 bearings are generally
tempered three times at 540°C. This secondary hardening process, also termed as
tempering, results in greater hardness, mainly due to precipitation of stable carbides.
Such a heat treatment also results in removal of retained austenite which affects dimensional stability of the component (Ooi and Bhadeshia (25)).

‘M’ in M50 indicate that primary alloying element in this steel is molybdenum. Table 3-1 shows the approximate chemical composition of this steel by weight percent. This through-hardened variant of steel has approximate yield strength of 2.8 GPa at 0.2% plastic strain offset (Klecka et al. (29)). Typically, it has hardness of 60-66 HRC (Ooi and Bhadeshia (25)). Its elastic modulus is in the range of 200-230GPa depending upon carbide volume fraction in the material. These carbide volume fractions of through hardened steels, such as AISI M50 and AISI 52100 steels, approximately remain constant as function of depth in the subsurface material. Therefore material properties such as elastic modulus, yield strength and hardness doesn’t vary as function depth.

Increase in gas turbine efficiency can be achieved by increasing temperatures or by increasing operational speed. The most practical alternative is increasing operational speed. Therefore, in these applications bearings are expected to operate at higher DN values which are calculated by multiplying rpm by bore diameter for bearing’s inner ring (mm). Through hardened steels such as M50, typically have low fracture toughness (approximately $K_{IC} \leq 24$ MPa m$^{1/2}$ (Zaretsky (11)). Hence, bearings made with M50 material fail catastrophically at DN values greater than 2.4 million (Philip (23)). At high operational speeds, tensile stresses in hoop’s direction lead to inner ring fracture and subsequent failure. In such applications where higher toughness is desired, carburized-grade steel variants are used.

Braza et al. (26) analyzed the microstructural alterations in M50 bearing inner rings operating under 3.62GPa maximum Hertzian stress, at 3000 rpm with minimum
lubricant film thickness to composite surface roughness ratio of 5.5. The inner rings were ran for longer lives between 200 to 300 million of cycles. Braza reported formation of butterflies around larger carbides which acts as a stress risers in subsurface microstructure. No significant softening was observed in the stressed zone unlike the one observed by Voskamp (16) in 52100 steel. Therefore, for low to medium speed applications with high operating temperatures AISI M50 steel is the most widely used material in aerospace applications.

**AISI M50-NiL**

As discussed above, through-hardened materials fails catastrophically at DN values greater than 2.4 million. To prevent such system failures in high speed applications, materials with high fracture toughness are desired. Fracture toughness is a material property defined as the stress required to initiate rapid fracture in presence of local defect/crack. It is inversely proportional to carbon content and hardness of the steel. Therefore, when carbon content is low the fracture toughness of the steel is high. This can be achieved by carburizing-grade steels which have low carbon content so that heat treatment results in moderate surface hardness and high core toughness. Carburizing process is used prior to heat treatment to diffuse carbon into near surface layers so that surface hardness is equivalent to that of through-hardened steels; whereas lower carbon concentration in core region results in higher fracture toughness (Zaretsky (11)). Fracture toughness can also be improved by adding nickel without affecting steel’s hardness. Carburized steels are generally heat treated to a temperature above 875°C. At this temperature, nickel in presence of high chromium and low carbon environment causes steel to become fully austenitic (Philip (23)). This results in steel microstructure with uniformly distributed fine carbides which significantly improves
rolling contact fatigue life of bearing steels. These insights led to modification of chemistry of AISI M50 steel by decreasing the amount of carbon and increasing the amount of nickel. This new variant of M50 was termed as M50-NiL; where ‘Ni’ stands for increased nickel and ‘L’ stands for low carbon content. Table 3-1 shows chemical composition of M50-NiL in weight percentage. Similar to M50, M50-NiL is generally tempered three times for temperatures up to 525° C (Braza et al. (26)). It is been reported that both M50 and M50-NiL can withstand temperatures of up to 600° C (e.g. in gas turbine applications) without any significant microstructural changes (Philip (23)).

M50-NiL steel is case carburized so that up to an approximate depth of 0.75 mm its hardness matches with that of through hardened M50 steel at about 60-64 HRC (740-840 HV) (Zaretsky (11)). It has core fracture toughness of over $60 \text{ MPa m}^{1/2}$ ($50 \text{ ksi in}^{1/2}$) compared to through-hardened AISI M50 steel which has fracture toughness of $29 \text{ MPa m}^{1/2}$ ($20 \text{ ksi in}^{1/2}$). The M50-NiL core hardness is Rockwell C 43 to 45 (Zaretsky (11)). Case-carburization also develops favorable internal compressive residual stresses of up to 400 MPa in both circumferential and axial directions (Forster et al. (30)). These stresses are known to significantly improve rolling contact fatigue life of case-carburized steels. These issues will be analyzed independently in subsequent chapters.

Recent micro/nano indentation studies by Klecka et al. (29) indicate that there exists gradient in carbide concentration in the microstructure of subsurface region of case-carburized steels such as M50-NiL. This gradation in carbide concentration results in gradation in material properties such as hardness, yield strength and elastic modulus. Klecka et al. (29), reported that elastic modulus for M50-NiL can vary from 228 GPa in
the surface region to 202 GPa in the core region, about 13.4% variation over 2mm case depth. For the same sample, the yield strength was reported to vary from 2.8 GPa (@0.2% plastic strain offset) in the surface region to 1.3 GPa (@0.2% plastic strain offset) in the core region. The Vickers hardness was reported to vary from 825 kg/mm\(^2\) in surface region to 500 kg/mm\(^2\) in the core region. The variation in elastic modulus will alter the subsurface stress distribution in the ball-raceway Hertzian contact. One of the goals of this research was to analyze these perturbations in Hertzian stresses and resulting influence on rolling contact fatigue life. Techniques used and the results observed are discussed in subsequent chapters.

Similar to 52100 and M50 steel, Braza et al. (26) also studied the microstructural alterations in the M50-NiL steel under rolling contact fatigue loading. Braza reported formation of white etched zone consisting of many bands some at a steep angle and some parallel to the surface. Since the carbides in M50-NiL steel are less than 1 micron in size no evidence of butterflies was found under prolonged RCF loading up to 305 million cycles. Similar to M50 steel, no softening of the material was reported. However, under peak contact pressure of 5.5 GPa, Bhattacharya et al. (31) has reported significant hardening of the material (about 15% in kg/mm\(^2\)) over 246 million cycles of RCF loading. However, under nominal loading conditions this much hardening of the subsurface material is not expected. Therefore, because of its high durability under extreme temperature and speed conditions, M50-NiL is widely used in Aeroengine applications.

Pyrowear–675

Most often turbo-machinery components require materials with superior corrosion resistance that will protect them from degradation while awaiting insertion in to drivetrain
systems as well as in adverse operating conditions. To satisfy these customer needs, Pyrowear 675 (P-675) was first developed by Carpenter Technology Corp (Wells et al. (24)). Its chemical composition by weight percentage is given in Table 3-1. Alloy steels with chromium content greater than 12% are considered corrosion resistant (Zaretsky (11)). Pyrowear-675 is case-carburized steel with high surface hardness and wear resistance, good internal impact toughness, ductility and fracture toughness with inherent corrosion and heat resistance. Its high hot hardness makes it suitable choice for high temperature applications where M50 and M50-NiL steels are currently being used. Depending upon heat treatment used its surface hardness can range from HRC 59.3 to 64.2. Typical core material hardness is HRC 50 (Wells et al. (24)). Ball rod test conducted by Wells et al. (24) at 5.42 GPa peak Hertzian stress showed fivefold increase in rolling contact fatigue life of P-675 rods compared to M50 rods manufactured from the same heat treatment. Similar to M50-NiL, P-675 is also case-carburized steel. Therefore, it is expected to have significant residual stresses in the subsurface region/case layer which will improve its rolling contact fatigue performance. However, data regarding residual stress measurements in P-675 steel is not available in the published literature.

Micro/Nano indentation studies by Klecka et al. (29) indicate that gradation in carbon concentration over the case layer results in gradation in material properties for P – 675 steel. Similar to M50-NiL steel, its elastic modulus varies from 224 GPa to 202 GPa, about 11% variation, over 1mm case depth. For the same sample, yield strength is reported to vary from 3 GPa in surface region to 1.3 GPa (@0.2% plastic strain offset) in the core region. As highlighted in previous section, these gradation in material
properties will significantly alter the subsurface stress distributions in ball-raceway contact. And one of the objective of this research was to quantify these variations in the subsurface stresses to determine the improved performance of this bearing material under rolling contact fatigue loading conditions.

**Ceramic Bearings**

All the materials discussed from sections 3.2 till 3.5, corresponds to inner and outer raceways of the rolling element bearings. Balls and rollers used in these bearings are often manufactured from steels. These combination of steels balls/rollers rolling over steel raceways classifies them as steel bearings. Some of the applications in aerospace industry, which demand higher power transmission to weight ratio, use silicon nitride balls on steel raceway. These are called as hybrid bearings or ceramic bearings. Ceramic balls are lighter than steel balls, hence force on the outer raceway is reduced. This in-turn reduces friction and rolling resistance. As well, ceramic balls are harder, smoother and have better thermal properties compared to steel balls. These factors lead to 10 times better fatigue life for ceramic bearings compared to steel bearings.

**Summary**

Aeroengine bearing materials are expected to withstand vibratory stresses, bending moments and high rotation speeds at elevated temperatures and aggressive lubrication conditions. Therefore bearing steels are generally designed to resist rolling contact fatigue, wear, scuffing, pitting failure mechanisms and to maintain structural integrity. Most failures in bearings originate at surface location rather than at subsurface locations. To prevent such random occurrences of these events, bearing steels are generally heat treated to surface hardness of HRC 60-64. AISI 52100 steel is the most
widely used through-hardened bearing material. Its uniform carbide composition results in uniform material properties in the subsurface region. However, due to steel chemistry, AISI 52100 steel is known to lose its hot hardness at high temperatures, making it unreliable for gas turbine engine applications. To address this issue, AISI M50 through-hardened steel was developed for high temperature applications. It can maintain its hot hardness for temperatures up to $600^0$ C. Similar to AISI 52100 steel, its material properties remain constant in the subsurface region. Due to high carbon content, M50 steels have low fracture toughness, which leads to catastrophic failures of bearing inner rings in high speed applications. His problem was solved with the development of carburizing-grade steels such as M50-NiL and P-675. Case-hardening results in higher carbon concentration near the surface region which almost linearly decreases to lower concentration in the core region. Therefore, this process enables materials to achieve high fracture toughness in the core region and high surface hardness. Case-hardened steels can be used in applications with DN values up to 3 million. Another characteristic of case-carburization is linear gradation in material properties such as elastic modulus, yield strength and hardness in the subsurface region. This variation in material properties will alter the subsurface stress distribution of two bodies in contact. Bearings normally operate under loads which are much below the yield limit. Therefore, there is need to quantify influence of variation in elastic modulus on the observed fatigue lifes of case-hardened steels under RCF loading. These issues will be addressed using analytical and numerical techniques such as finite element method in subsequent chapters.
Table 3-1. Approximate chemical compositions of bearings steels in wt% (Wells et al. (24), Ooi and Bhadeshia (25))

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Si</th>
<th>Cr</th>
<th>Mo</th>
<th>V</th>
<th>Ni</th>
<th>Mn</th>
<th>Co</th>
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<td>52100</td>
<td>1</td>
<td>0.2</td>
<td>1.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td>M50</td>
<td>0.83</td>
<td>0.25</td>
<td>4.1</td>
<td>4.25</td>
<td>1</td>
<td></td>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td>M50-NiL</td>
<td>0.13</td>
<td>0.18</td>
<td>4.1</td>
<td>4.25</td>
<td>1.2</td>
<td>3.5</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Pyrowear-675</td>
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<td>0.4</td>
<td>13</td>
<td>2.6</td>
<td>0.6</td>
<td>2.6</td>
<td>0.65</td>
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</table>
CHAPTER 4
EXTENDED HERTZ THEORY OF CONTACT MECHANICS FOR CASE HARDENED STEELS WITH IMPLICATIONS FOR BEARING FATIGUE LIFE

Background

A number of applications in tribology, geology, optoelectronics, biomechanics, fracture mechanics, and nanotechnology involve components with compositionally graded materials (Giannakopoulos and Suresh (32)). Transmission components such as rolling element bearings, gears etc., are heat treated using carburization and nitriding processes to enhance resistance to damage and deformation under repeated contact. These surface treatment processes result in gradient in carbide and nitride concentration in the subsurface region, leading to spatial variation of mechanical properties such as elastic modulus, yield strength, and hardness (Klecka et al. (29)). The mechanical response of materials with spatial gradients in composition and structure has been a subject of many experimental and analytical studies by number of researchers (32-36). Giannakopoulos and Suresh (32, 34) analyzed axisymmetric graded half space problem for a point load, and for conical, spherical and flat indenters. In their study, frictionless contact, constant Poisson’s ratio and Young’s Modulus variation with depth either as simple power law \( E(z) = E_0 z^{k'} \), \( 0 \leq k' < 1 \) or exponential variation \( E(z) = E_0 e^{az} \) were assumed. Liu et al. (35) solved axisymmetric frictionless contact problem of functionally graded materials (FGMs) using transfer matrix method and Hankel integral transform technique. With the help of numerical techniques they obtained solutions for contact pressure and contact region for indenters of various geometries. Guler and Erdogan (36) analyzed contact problem of graded metal/ceramic coatings for planar elasticity (plane stress and plane strain) conditions. They considered continuously varying thermo-mechanical properties to analyze influence of material
inhomogeneity constant, the coefficient of friction and various length scale parameters on the critical stresses that determine fatigue and fracture of coating. All the solutions from previous studies are applicable only to limited cases such as axisymmetric and planar contact problems. However, in most of the industrial applications, especially for ball bearings, elliptical contact exists between ball and raceway. These conditions require contact analysis of elastically graded materials under three-dimensional loading. Moreover, previous studies primarily rely on extension of Boussinesq’s method whereas practices in bearing industry have evolved to use of Hertz equations for stress, deformation and fatigue life calculations (Harris (1), Zaretsky (13)). With these limitations identified, in this chapter goal was to study contact problem of case-hardened ball bearing raceways using Hertz equations under three dimensional as well as axisymmetric loading conditions.

Traditional Hertzian contact solution for determining subsurface elastic stress fields for two bodies in contact is applicable only for homogeneous materials (Harris (1)). Bearing $L_{10}$ life, defined as number of revolutions for which 90% of bearings survive at a given load, is a strong function of peak Hertzian contact pressure and the peak orthogonal subsurface shear stress (Zaretsky et al. (22)). Variations in contact pressure and subsurface stress field due to elastic modulus variation in the case layer of a case-hardened steel is expected to introduce a significant correction to bearing life prediction. Therefore, the primary objective of this work is to analyze the influence of elastic modulus gradation on the peak Hertzian contact pressure and subsurface stress-field experienced by the ball-raceway contact. For this, detailed mesh density/convergence study of the 3D finite element models (FEM) of the ball-raceway contact with elastic
modulus gradient is performed. Discussion regarding procedures adopted for the
development of these models along with the challenges encountered in FEM
simulations of ball-bearing contacts are also presented.

**Hertz Theory of Contact Mechanics**

As discussed in Chapter 2, stresses developed at the contact of two elastic solids
were first analyzed by Hertz in early 1880s. He proposed that contact area is, in
general, elliptical and for the purpose of calculating local deformations, both bodies can
be approximated as elastic half spaces loaded over a small elliptical region of their
surfaces (Johnson (4)). Based on Hertz analysis, ellipsoidal compressive stress
distribution in the contact area can be written as:

\[
\sigma = \frac{3Q}{2\pi ab} \left[ 1 - \left( \frac{x}{a} \right)^2 - \left( \frac{y}{b} \right)^2 \right]^{1/2}
\]

where, \( Q \) is the normal load experienced by two bodies in contact in X-Y plane; \( a \) and \( b \)
represent semi-major and semi-minor axes of the elliptical contact area. \( S \) will be used
to represent maximum compressive stresses experienced by two bodies in contact.

Hertz also provided expressions for semi-major axis \( a \) and semi-minor axis \( b \) as
(Johnson (4)):

\[
a = \sqrt[3]{\frac{3Q}{2\pi \rho} \left[ \frac{(1 - \xi_I^2)}{E_I} + \frac{(1 - \xi_{II}^2)}{E_{II}} \right]^{1/3}}
\]

\[
b = \sqrt[3]{\frac{3Q}{2\pi \rho} \left[ \frac{(1 - \xi_{II}^2)}{E_I} + \frac{(1 - \xi_I^2)}{E_{II}} \right]^{1/3}}
\]
where $\zeta$ and $E$ are Poisson’s ratio and elastic modulus of bodies I and II respectively; 

$\sum_p$ is the summation of principal curvatures of two bodies in contact; $a^*$ and $b^*$ are non-dimensional parameters defined by curvature difference of two bodies (Johnson (4)).

It is well known that bearings generally fail due to rolling contact fatigue arising from continuously varying subsurface shear stresses (Harris (1)). Three principal shearing stresses are generally used for bearing life analysis: the orthogonal shearing stress, $\tau_0$ (based on LP model Eq. (2-6)); the Von-Mises equivalent octahedral shearing stress, $\tau_{oct}$ (based on Ioannides and Harris model Eq. (2-20)) and the maximum shearing stress, $\tau_{max}$ (based on Zaretsky’s model Eq. (2-27)). All these shearing stresses vary in proportion with the maximum Hertz pressure $S$ experienced by two bodies as:

$$\tau = k \times S$$

(4-4)

where the proportionality constant $k$ is equal to 0.25, 0.28 and 0.32 for the orthogonal, octahedral and the maximum shearing stress, respectively, for commonly observed contact conditions (i.e. $b/a \approx 0.1064$). For roller bearings these values are 0.25, 0.29 and 0.30 respectively. For ball bearings (point contact), the subsurface depths where maximum orthogonal, octahedral and subsurface shear stress observed are 0.5$b$, 0.72$b$, 0.76$b$ respectively; Corresponding depths for roller bearings are 0.5$b$, 0.79$b$, 0.79$b$ respectively. Analytical expressions Eq. ((4-1)-(4-4)) were derived based on the assumption that subsurface properties such as elastic modulus and hardness remain constant (Thomas and Hoersch (5)). However, for case-hardened bearing steels such as M50-NiL and P-675, a linear gradation of carbide volume fraction with depth, and
consequently a linear variation in elastic modulus with depth have been observed. Due to this gradient, traditional Hertz solution cannot be used for determining the peak contact pressure and subsurface stress fields. Therefore, 3D finite element model (FEM) of the ball-raceway contact with modulus gradient built into the raceway is used in present study.

**Sensitivity of Bearing Fatigue Life to Elastic Modulus Variations**

It is well known that the bearing fatigue life, $L$, is inversely proportional to equivalent dynamic load $F_e$ and also maximum Hertzian contact pressure $S$ as (Zaretsky et al. (22)):

$$L \propto \frac{1}{[F_e]^p} \propto \frac{1}{S^n} \quad (4-5)$$

where $p$ and $n$ represent load-life and stress-life exponents, respectively. For point contact, stress-life exponent is three times the load-life exponent ($n = 3p$). Based on bearing endurance data available in 1950s, Lundberg and Palmgren (8) evaluated load-life exponent $p = 3$ and therefore $n = 9$. The study by Parker and Zaretsky (37) indicates that for vacuum-processed steels $p = 4$ and $n = 3p = 12$. These results are based on fatigue data collected from five-ball fatigue tester with maximum Hertzian stress in the range of 4.5GPa to 6GPa. However, reevaluation studies by Londhe et al. (38), based on actual bearing fatigue lives reported by bearing and aircraft engine manufacturers (Harris and McCool (18)), indicate that load life exponent $p$ for ball bearings is 4.1 and the corresponding stress-life exponent is then $n = 3*p = 12.3$. These results are based on Bayesian statistics approach rather than traditional minimization of root-mean-square approach used for determining unknown parameters. As seen in Eq. (4-5), because
fatigue life is proportional to the contact pressure raised to large exponent (9, 12 or 12.3), life estimates are sensitive to even small changes in contact pressure. Even subtle reductions in peak contact pressure due to elastic modulus gradient can significantly improve fatigue life prediction of ball bearings.

Analytical study presented in the Appendix shows that peak Hertz contact pressure experienced by ball bearings is fairly sensitive to variations in elastic modulus of raceway material (Londhe (39)). For example, a 10% reduction in elastic modulus for raceway material results in a 3.6% drop in the peak contact pressure, due to reduced contact stiffness. This results in a 38.7% correction in bearing fatigue life prediction with stress-life exponent \( n \) of 9. These numerical results are based on analysis of 209 single row radial deep groove ball bearings with inner and outer raceway diameters of 52.291 mm (2.059 inch) and 77.706 mm (3.059 inch). Groove radii for both raceways and ball diameter were 6.6 mm (0.26 inch) and 12.7 mm (0.5 inch), respectively. Bearing was loaded radially with 8900 N (2000.8 lbs) force on 9 rolling elements.

Similarly, with stress-life exponents of 12, and 12.3 the expected correction in fatigue life is 54.7% and 56.4%, respectively. Therefore, for accurate prediction of fatigue life of case-carburized bearing steels, Hertz equations must be corrected to account for varying elastic modulus from the surface to core region of the carburized raceway material.

At the micro-structural level, case hardened steels contain carbide precipitates surrounded by steel matrix. These carbides are usually stiffer than steel matrix. Therefore, composite elastic modulus is used to represent equivalent elastic modulus of carbide inclusions and steel matrix at each case depth. As discussed in Chapter 2, for
M50-NiL steel, composite elastic modulus decreases linearly from about 228 GPa to 202 GPa, 11.4% variation over 2mm case depth. Similar data for P-675 indicates that the composite elastic modulus varies linearly from 224 GPa to 202 GPa, about 9.82% variation, over 1mm case depth. In this analysis, finite element models were constructed to study the dependency of peak Hertzian stress on the linearly varying elastic modulus of a raceway material for different geometries under circular and elliptical contact loads. A wide range of variations in elastic moduli and case depths were considered. Results from FEA analyses are presented in following sections. Simulations include steel-on-steel and ceramic-on-steel contact of varying ball sizes.

**Finite Element Analysis**

ABAQUS/Standard software was used for this nonlinear FE contact analysis. The type of contact models that were developed for this study are shown in Fig. 4-1. Three dimensional ball-on-raceway contact, as shown in Fig. 4-1A, was used to simulate elliptical contact area observed inside 6309 deep groove ball bearing (DGBB). Geometrical dimensions for the model are specified in Table 4-1. Simulations included contact between a silicon nitride ball on steel raceway and steel ball on steel raceway using one-quarter symmetry. Symmetric boundary conditions were used along planes perpendicular to elliptical contact area and passing through semi-major and semi-minor axis. Similarly, a ball on plate model with a circular contact was also developed using axi-symmetric boundary conditions as shown in Fig. 4-1B. This model was simulated for silicon nitride and steel balls of 12.7 mm (0.5 inch) and 25.4 mm (1 inch) diameter on a steel substrate. Details about the geometries used in each simulation are summarized in Tables 4-1. A sample three dimensional ball-inside-channel model (as per Zaretsky et al. (10)) with infinite bore diameter was also considered in this analysis. As shown in
Fig. 4-1C, in this model symmetric boundary conditions were used as well to save computational time. In all of the models, bottom edge/surface of the plate, raceway and channel was fixed.

It should be noted that both through-hardened and case-hardened variants of steel were used in both 3D and Axi-symmetric models. For through-hardened steels uniform elastic modulus of 200 GPa was used for ball, raceway, plate and channel material. Since, in open literature, elastic modulus variation data is only available for M50-NiL and P-675 case-hardened steels, majority of simulations were performed for these materials. However hypothetical samples of case-carburized steels were also considered in some of the simulations to generalize the results. These steels can be considered to be processed by: pack carburizing, gas carburizing, liquid carburizing and vacuum/LPC carburizing methods. These carburization techniques are commonly used by bearing manufacturers (Schneider and Chatterjee (40)). Typical case depths attainable from these manufacturing methods can vary between $50 \mu m - 1.5 \text{mm}$. The case hardness from these processes is reported in the range of 50-63 HRC (about 20% variation). Corresponding concentrations of carbide volume fraction and elastic moduli variations of the case layer are approximated to follow a similar trend (Klecka et al. (29)). Generally, for aerospace main shaft bearings, a case depth of 2mm is common with 10-20% linear decrease in case layer elastic modulus.

To study the influence of these graded properties of raceway material on the peak contact pressure under elliptical contact loading conditions, a wide range of operating conditions, as detailed in Table 4-3, were considered in the analyses. The 3D 6309 DGBB and ball-channel models, and Axi-symmetric ball-plate model (shown in
Fig. 4-1) were simulated for nominal loads ranging from 100 N (22.48 lbs) to 13500 N (3034.92 lbs). At each load, simulations were performed for silicon nitride and steel balls in contact with M50-NiL and P-675 steel raceway/plate material. The resulting peak contact pressures were in the range of 1.5 GPa (217.56 ksi) – 3.1 GPa (449.62 ksi), which is the typical operating range for majority of industrial ball bearing applications. Additionally, case-depths of 0.5 mm and 1.5mm were used in some of the simulations (model #s 29-33 in Table 4-3) with different possible gradations in elastic modulus from surface to core region. Let $E_{\text{surface}}$ and $E_{\text{core}}$ denote surface and core elastic moduli, respectively, of the raceway/plate/channel materials. Let ‘δ’ be the case depth and ‘d’ be the fraction of the elastic modulus variation from $E_{\text{surface}}$ to $E_{\text{core}}$ over this case depth defined as:

\[
d = \left( \frac{E_{\text{surface}} - E_{\text{core}}}{E_{\text{surface}}} \right)
\]  

Figure 4-2 depicts elastic modulus variation for M50-NiL material in comparison to through-hardened steel (denoted as TH) as a function of subsurface depth.

While the above gradations were considered only for raceway/plate/channel material, the elastic moduli of the balls were considered to have no gradation i.e., 320GPa for silicon nitride balls and 200GPa for steel balls in all of these models. The spatial variation of elastic modulus in the desired domain of raceway/plate/channel was implemented in ABAQUS via a user defined material subroutine ‘UMAT’, by assigning numerical values to each material point of the discretization element. In some of the simulations, gradient in elastic modulus was setup using temperature dependent material properties and gradation in temperature over the case-layer of
raceway/plate/channel material. Details about the discretization element and mesh techniques used are discussed in the following section.

Computational modelling of nonconformal Hertzian contact between the ball and raceway (or two elastic bodies) is challenging despite significant advances in computational resources. Hence to obtain a balance between computational cost and accuracy of the FE simulations, mesh discretization scheme, as shown in Fig. 4-1, was developed for all of the models. For the ball-raceway model, fine mesh was used near the center of elliptical contact area. In this region, smallest dimensions of 3D solid element for raceway geometry were 145.4 µm x 7.2 µm x 3.07 µm (for semi-major axis, semi-minor axis and subsurface depth, respectively). For quarter section of ball geometry, near the center of contact area smallest dimension of 3D solid element were 61.79 µm x 53.73 µm x 24.35 µm. For the raceway, because of steeper gradient, very fine mesh elements were used along semi-minor axis ‘b’. This level of mesh refinement was necessary to determine accurate contact pressure, contact dimensions and subsurface shear stresses for geometries with uniform elastic modulus of raceway material. Since contact pressure is highly sensitive to the curvature of geometries, quadratic elements were used instead of linear elements (Kim (41)). Structured meshing technique along with hexahedral elements (C3D20) was used for the entire FE analysis of ball-raceway and ball-channel models. With this mesh refinement, approximate time for one 3D ball-raceway contact simulation was 48 hours on 32GB RAM fast processor computer.

The ball-plate model shown in Fig.4-1B used very refined axisymmetric mesh. At the center of the contact area, smallest dimension of the 2D element for plate geometry
were 11.11 µm X 6.86 µm and for ball geometry they were 39.90 µm X 34.14 µm.

Similar to ball-raceway model, structured meshing technique with quadratic quadrilateral elements (CAX8) were used.

The FEA results were validated against analytical solutions available in Harris (1) and Thomas and Hoersch (5), for the case of through hardened or uniform elastic modulus materials, for test configurations shown in Table 4-2. The goal of this analysis was to determine the mesh refinement level that would result in acceptable convergence of contact pressure and subsurface stresses for cases where analytical solutions were known. Finite element analysis results shown in Table 4-2 for maximum contact pressure ($S$) are within 1% of the analytical solution. Table 4-2 also indicates maximum values of critical subsurface shear stresses and the corresponding depths at which they are observed. For axisymmetric ball-plate model, the errors between analytical and FEA solutions for critical subsurface stresses are less than 2%. The errors in the predicted contact pressure from analytical and computational results were under 3% for all the simulations.

After determining reasonable mesh refinement level, next step is to identify design parameters of interest. As per Hertz theory, these parameters are principal curvatures of two bodies, their material properties and the normal load acting on the contact. For linearly graded materials such as case-hardened bearing steels, three more design parameters: surface elastic modulus $E_{surface}$, fractional variation in elastic modulus from surface to core i.e. $d$ (as per Eq. (4-6)) and case-depth $\delta$, were defined. Comprehensive FE study was performed by varying each of these design parameters to analyze peak contact pressure for elastically graded materials. Table 4-3 shows details
of the 33 different simulated test cases used in this study. Out of these, 10 correspond to ball-raceway contact inside 6309 DGBB, 21 correspond to ball-plate model and 2 correspond to ball-channel model. Poisson’s ratio of 0.3 was kept constant for all the materials. Since a case depth of 2mm is most frequently used in majority of aerospace bearing applications, most of the test cases were run for a case layer of up to 2mm depth.

**Results and Discussion**

The maximum contact stress ($S$) experienced between the ball and raceway/plate/channel in each of the 33 test configurations are shown in Table 4-3. For the case-hardened raceway: it is denoted as $S_{CH}$ and that experienced by through-hardened raceway under identical ball material, geometry and loading conditions is denoted as $S_{TH}$. M50-NiL and P-675 were the two representative case-carburized bearing steels used because their elastic modulus variation over case-depth is reported in the published literature. However, test cases 29-33 include different possible variants of case-carburized steels. These were considered to generalize the analysis results. From the results presented in Table 4-3, it can be concluded that peak contact pressure experienced by case-hardened raceway will always be different than that experienced by through-hardened raceway even if other design parameters are kept constant. For example, for 6309 DGBB with steel ball rolling over steel raceway, under 1279.1 N load (i.e. test case # 2) the peak contact pressure varies by up to 3.61%. Using Eq. (4-5), this 3.61% variation in peak pressure will correspond to 37.57% correction in life with $n=9$. With $n=12$ or 12.3, the corrections in the life will be even higher i.e. 53% and 54.64% respectively. Therefore, this underlines the necessity of the fact that gradations
in the elastic modulus of case-hardened steel must be considered for fatigue life predictions of bearing raceways. Comparing the contact pressures experienced by case-hardened steel and through-hardened steels, it may appear that through-hardened bearing steels will have higher life than case-hardened bearing steels (as per Eq. (4-5)). However, in reality case-carburized bearing steels are known to outperform their through-hardened counterparts under identical operating conditions (Harris et al. (42), Errichello et al. (43)). This improved RCF performance of case-carburized bearing steels is mainly attributed to presence of residual compressive stresses in circumferential and axial directions of bearing inner rings and fine carbide microstructure in the subsurface region. Analysis on the influence of residual compressive stresses on Hertzian contact stresses in the presence of graded material properties will be reported in a separate chapter. The main goal of the present analysis is to analyze peak contact pressure for elastically graded bearing raceways and provide simple correction to Hertz equations. For this, non-linear regression analysis was performed as explained in the later sections of this chapter. For all the test cases presented in Table 4-3, maximum orthogonal, Tresca and von-Mises stresses were analyzed in the subsurface region of raceway material. Regression estimates for proportionality constant $k$ (in Eq. (4-4)) were 0.247, 0.316 and 0.59, respectively, for ball-raceway contact inside 6309 DGBB bearing. These estimates of $k$ for graded material are almost identical to that of uniform elastic modulus/through-hardened material. Therefore, no significant difference was observed in proportionality constant $k$ for case-hardened and through-hardened bearing steels.
Based on finite element analysis it was observed that peak contact pressure experienced by case-hardened steels is significantly different than that experienced by through-hardened steels under identical operating conditions. Due to high stress-life exponents of 9 (LP model), 12 (Zaretsky’s model) and 12.3 (Londhe et al. (38)), account of even smaller perturbations in peak contact pressure will result in significant correction in bearing fatigue life prediction. Therefore, effects due to gradation of concentration of carbide particles and resulting variations in elastic modulus must be considered for accurate stress calculations and fatigue life predictions for case-hardened bearing raceways.

**Mesh Convergence Study**

As discussed in previous section, reasonable mesh size was determined for three dimensional and axi-symmetric models to minimize the computational cost of finite element simulations. A mesh convergence study was undertaken to check the influence of mesh size on contact pressure variations for 6309 DGBB ball-raceway model presented in Table 4-2. A total of 180980 second order 3D solid elements were used, out of which 16432 were used to discretize the ball geometry and 164548 were used to discretize the raceway geometry. For convergence study total mesh size was reduced by an approximate factor of 2 in each step. Therefore, total number of second order 3D solids elements in four mesh discretization schemes were 180980, 112849, 65144 and 16365 respectively. Models with through-hardened and M50-NiL steel properties for raceway were simulated using this four mesh discretization schemes. Results obtained for mesh convergence study are shown in Fig. 4-3, which shows that the variations in peak contact pressure from this four mesh discretization schemes are insignificant. The variation in peak contact pressure for model with 65144 no. of elements and model with
180980 no. of elements is less than 0.8% for both the materials. For all the 12 test cases corresponding to the 3D analysis, errors in maximum contact pressures from FE simulations and analytical solutions were less than 3%. Therefore, these confirm that contact pressures have converged satisfactorily. In the present analysis, total of 180980 no. of elements were used for the 3D models shown in Table 4-3. Similar analysis was performed for half-inch axisymmetric ball-plate model. Total number of CAX8 elements selected were 9231, 20334, 45400 and 77475. Each mesh discretization scheme was used to simulated contact of steel ball on steel plate and steel ball on M50-NiL plate. Peak contact pressures observed are plotted in Fig. 4-4 as function of total number of elements. This plot confirms that peak contact pressures have converged for axisymmetric models.

**Effective Elastic Modulus of Case Hardened Bearing Steels**

The composite elastic modulus of case layer decreases linearly from surface to core material. In order to use Hertz equations for such graded material, effective elastic modulus of the case layer must be determined. This effective elastic modulus can also be used in bearing fatigue life prediction equations which are mainly applicable for through-hardened steels/homogeneous materials. To determine this modulus, an equivalent contact pressure approach will be used with traditional Hertz equations. Using Eq. (4-1), peak contact pressure can be expressed as:

\[
S = \frac{3Q}{2\pi ab}
\]  

(4-7)

Substituting expressions for \(a\) and \(b\) (i.e. Eq. (4-2) and Eq. (4-3)) in Eq. (4-7), we get
For case carburized raceway, all the terms, except elastic modulus, are constant in Eq. (4-8). Let $E_{ball} = E_I$, be the elastic modulus of ball and $E_{TH} = E_H$ be the elastic modulus of through-hardened steel raceway. $S_{TH}$ represents the peak contact pressure for the raceway with uniform elastic modulus of $E_{TH}$. Let $S_{CH}$ represent peak contact pressure for graded case-hardened raceway material. In each of the 33 test cases, contact pressures for case-hardened and through-hardened raceway under similar operating conditions were determined and are included in Table 4-3. Let $E_{Effective}$ represent the effective elastic modulus of the case layer which predicts peak contact pressure of $S_{CH}$ from traditional Hertz Eq. (4-8). Therefore, using Eq. (4-8), for identical operating conditions, these terms can be related as,

$$\frac{S_{TH}}{S_{CH}} = \left[\frac{1}{E_{ball} + \frac{1}{E_{Effective}}} + \frac{1}{E_{ball} + \frac{1}{E_{TH}}}\right]^{-2/3}$$

Equation (4-9) can be used to determine the effective elastic modulus such that peak Hertz contact pressure for through hardened raceway material with elastic modulus of $E_{Effective}$ is $S_{CH}$. Thus with $E_{Effective}$ the analytical Hertzian solution can be used for determining peak contact pressures, subsurface stress fields and contact dimensions for case hardened steels such as M50-NiL and P-675. Using Eq. (4-9), effective elastic modulus of case-carburized steels was determined for each of the 33 test cases. These
values are presented in Table 4-4 which reveal that effective elastic modulus of case-hardened steel is a function of the design parameters of ball-raceway contact area. Especially, geometries with the same material exhibit different effective elastic modulus of case-layer under different loads, as can be seen from test cases 2 and 8. For larger dimensions of the ball-raceway contact area, the effective elastic modulus of case-layer is significantly lower than the surface elastic modulus. This is because Hertzian stresses are distributed over larger subsurface volume hence effect of gradations in elastic modulus is felt more at the surface. From Table 4-4, it can also be seen that sharper gradients in elastic modulus, i.e. smaller case-depths δ or higher gradation d, results in significantly lower effective elastic modulus for case-hardened steel than its value at the surface (test cases 29-33). Therefore, these results indicate that effective elastic modulus of case-hardened bearing steel is not only dependent on gradation parameters but is also a function of operating conditions of the bearing raceways. Next step is to determine reasonable surrogate to predict effective elastic modulus of case-hardened steel depending design parameters of interest, to enable the use of Hertz equations for stress-fatigue life calculations.

**Statistical Analysis**

In this section, the aim is to determine mathematical relationship between effective elastic modulus of case-layer, and geometry, material, elastic modulus gradation and load parameters of ball-raceway contact. Let \( f \) be the function which represents relationship between these parameters, which can be represented as:

\[
E_{\text{Effective}} = f(E_{\text{ball}}, E_{\text{Surface}}, d, \delta, Q, \xi_I, \xi_H, \sum \rho, F(\rho))
\]  

(4-10)
Surrogate/meta models were constructed to determine an approximate functional form for \( f \) in order to establish relationship between design parameters and effective elastic modulus of carburized steel. FE simulation results are numerical approximations with some inherent errors commonly termed as noise. Regression and kriging are two popular choices for surrogate construction, but for noisy data points regression surrogate is better choice because it can predict approximate global trend in the region that is far away from the current design space. Kriging surrogates are generally preferred for noise free data without any numerical errors. Therefore, regression techniques were used to determine surrogate for 'f. Certain non-dimensional parameters common to Hertzian contacts are used in this analysis. \( E^* \) is the effective elastic modulus of two bodies in contact defined as,

\[
\frac{1}{E^*} = \frac{(1 - \xi_I^2)}{E_{ball}} + \frac{(1 - \xi_{II}^2)}{E_{Surface}}
\]  

(4-11)

The non-dimensional model design parameter \( K \) is defined as,

\[
K = \frac{1}{\delta \left( \frac{2Q}{E^* \sum \rho} \right)^{\frac{1}{2}}}
\]

(4-12)

Curvature difference of two bodies in contact is non-dimensional term therefore it is not considered in the definition of \( K \). Moreover, its affect was found to be captured by regression coefficients. MATLAB software program was used for the regression analysis to determine curve fit equation based on the 33 training data points presented in Tables 4-3 and 4-4. Values of the non-dimensional model design parameter \( K \) were determined for each test case. It was found that linear regression techniques do not provide satisfactory surrogate based on these 33 training data points. However, it was
observed that non-linear regression techniques provide a reasonable surrogate to predict \( E_{\text{Effective}} \) as a function of \( E_{\text{Surface}}, \ E_{\text{Core}}, \ d \) and \( K \). Using the simplest nonlinear form, this surrogate can be represented as

\[
E_{\text{Effective}} = \frac{c_1 E_{\text{Surface}} + c_2 E_{\text{Core}}}{1 + c_3 dK}
\]  

(4-13)

where \( c_1, c_2 \) and \( c_3 \) are the constants determined using least-squares fit methods. It should be noted that in Eq. (4-13), core elastic modulus can be represented as \( E_{\text{Core}} = (1-d)E_{\text{Surface}} \). The nonlinear regression predicts coefficient of determination i.e. \( R^2 = 0.992 \) and \( R^2_{\text{adj}} = 0.992 \) for this model. The estimates of coefficients in Eq. (4-13) are \( c_1 = 0.95023, \ c_2 = 0.050402 \) and \( c_3 = 0.49188 \). The P-values for the corresponding estimates are close to 0, indicating that they are statistically significant. The estimated RMS error based on Eq. (4-13) and the 33 data points presented in Tables 4-3 and 4-4 is 1.01 GPa. Due to very high coefficient of determination of close to 1, this semi-numerical solution can be considered as correction to Hertz analytical equations for linear elastically graded materials such as case-hardened bearing steels. Next section represents validation of this method to predict peak contact pressure for case-carburized bearing steels.

**Validation Study**

To validate this approach of equivalent peak contact pressures, FEA simulations were performed again with effective elastic modulus for raceway/plate/channel material for all the 33 sample test cases presented in Table 4-3. The results obtained for this cross validation of peak contact pressures are also presented in Table 4-4, which shows that this approach works very well for up to 10% and 20% gradations in elastic modulus.
over 2mm and 1mm case depth. For these cases, the errors in actual peak contact pressures experienced by case-hardened steels and the ones obtained using effective elastic modulus of raceway materials are less than 1%. For Axisymmetric ball-plate models, all the errors are less than 0.2%, even for up to 9.1% gradation in elastic modulus over 0.5mm case-depth and 16.67% gradation over 1.5mm case-depth (as per test cases # 29-31). Comparing the results for models 2 and 8, we can see that for identical geometry and material properties (i.e. 6309 DGBB ball bearing with M50-NiL raceway) the effective elastic modulus of case-hardened steel is lower at higher loads. For 1279.1N load it is 223.1GPa whereas for 2800N load it is 221.74GPa. Therefore, account of gradations in elastic modulus of carburized steels is particularly useful for ball-raceway contact area with larger dimensions and at higher loads. Results for test case #33 indicate that effective elastic modulus is significantly lower than surface elastic modulus for extreme gradations in elastic modulus i.e. up to 13.04% gradation over 1.5mm case-depth. For 1 inch ceramic ball – steel channel contact, the effective elastic modulus is just 215.66GPa compared to 230GPa - 200GPa variation in elastic modulus over 1.5mm case-layer used in this model. Even for such an extreme design case, validation error is still less than 0.3%. Figure 4-5 shows plot of 33 data points where peak contact pressures obtained using effective elastic modulus are plotted against peak contact pressures obtained using elastically graded raceway/plate/channel material. As expected, the trend line has an approximate slope of 1 with coefficient of determination: $R^2 = 1$, which confirms validity of this method.
Summary

Many modern bearing steels such as M50-NiL and P-675 are case-hardened, to develop a high fracture toughness core with lower carbon content and hardened surface region with higher carbon content. Due to this gradation in carbide microstructure from surface to core, there exists gradient in material properties such as elastic modulus, hardness and yield strength over the case-layer. Due to high stress-life exponents of 9, 12 or 12.3, bearing fatigue life is very sensitive to variations of peak Hertz pressure experienced by ball-raceway contact. Therefore, determination of accurate Hertz contact pressure experienced by ball-raceway contact is necessary for accurate fatigue life prediction and reliable bearings design. Hertz theory is mainly applicable for homogeneous materials such as through-hardened steels. It can't be directly used for inhomogeneous materials such as case-carburized bearing steels. In this study, comprehensive finite element analyses were performed to study peak Hertz pressure for elastically graded materials for different variations of ball-raceway contact design parameters. Because of the inverse elastic gradient, the peak contact pressure experienced by case-hardened steel is different than that experienced by through-hardened steel, under identical geometry and loading. The concept of effective elastic modulus is introduced which may enable the use of Hertz equations for case-carburized bearing steels. Results from a wide range of simulations are used to determine an accurate regression equation for an effective elastic modulus of case-hardened steel. Using this surrogate, effective elastic modulus can be determined as weighted sum of surface and core elastic moduli of case-hardened bearing steel and geometrical and loading parameters. Even though in present work, gradations in elastic modulus were assumed to be from 230 to 200GPa or 240 to 200GPa, this analysis can be adopted for
any possible gradations in elastic modulus over different possible case-depths. This approach can significantly simplify stress-life analysis for case-carburized bearing steels, as complex, time consuming 3D finite element simulations can be avoided.
Figure 4-1. Von-Mises stress contours. A) For 3D Ball-raceway contact inside radially loaded 6309 deep groove ball bearing. B) For Axi-symmetric ball-plate contact. C) 3D Ball-inside-channel model.
Figure 4-2. Elastic modulus variation as function of depth for M50NiL and through-hardened bearing steel (TH)
Figure 4-3. Mesh convergence study for 3D ball-raceway model of contact inside 6309 DGBB
Figure 4-4. Mesh convergence study for Axi-symmetric ball-plate model
Figure 4-5. Comparison of actual peak contact pressure for case-hardened bearing steel and through-hardened bearing steel with elastic modulus of $E_{\text{Effective}}$.
Table 4-1. Geometrical Properties of Three-dimensional and Axisymmetric finite element models

<table>
<thead>
<tr>
<th>Model Type</th>
<th>FE Model Dimensions</th>
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<tbody>
<tr>
<td></td>
<td>6309 DGBB</td>
</tr>
<tr>
<td></td>
<td>1/2 inch Ball-Plate</td>
</tr>
<tr>
<td></td>
<td>1 inch Ball-Plate</td>
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<tr>
<td></td>
<td>Ball-Channel</td>
</tr>
<tr>
<td>Dimensionality</td>
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<tr>
<td>Bore diameter (mm)</td>
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<tr>
<td>O.D. (mm)</td>
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<tr>
<td>Width (mm)</td>
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<tr>
<td>Ball diameter (D, mm)</td>
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<tr>
<td>Raceway groove curvature radius/D</td>
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</tr>
<tr>
<td></td>
<td>Axisymmetric</td>
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<td></td>
<td>Infinite</td>
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<td>Infinite</td>
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<tr>
<td></td>
<td>0.52</td>
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<td>12.7</td>
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Table 4-2. Comparison between FEA and Analytical solutions for through-hardened bearing steels

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<td>FEA Solution</td>
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<td>Semi-minor axis: 'b' (mm)</td>
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<td>Normalized Maximum Shear Stress: $\tau_{max}/S$</td>
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<td>Normalized depth: $z_0/b$</td>
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Table 4-3. Material gradation parameters, normal load and peak contact pressure observed for case-hardened and through-hardened bearing steels for each FEA simulation test case

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<th>Maximum Contact Pressure</th>
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Table 4-4. Validation of equivalent contact pressure approach and effective elastic modulus

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<th>Percent Gradation in Modulus from surface to core : d%</th>
<th>Case-depth δ (mm)</th>
<th>Effective Elastic Modulus (MPa)</th>
<th>Actual Peak Pressure for graded material S (GPa)</th>
<th>Peak Pressure using Effective Elastic Modulus</th>
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<td>Case-depth ( \delta ) (mm)</td>
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Table 4-5. Bearing properties used for sensitivity study

| Inner Raceway diameter ($d_i$) | 52.291 mm |
| Outer Raceway diameter ($d_o$) | 77.706 mm |
| Ball Diameter ($D$) | 12.7 mm |
| Number of Balls ($Z$) | 9 |
| Inner groove radius ($r_i$) | 6.6 mm |
| Outer groove radius ($r_o$) | 6.6 mm |
| Radial Load Applied ($F_r$) | 8900 N |
| rpm ($N$) | 1800 |
| Contact angle ($\alpha$) | 0 deg. |
| Bearing pitch diameter ($d_m$) | 65 mm |

Quantities | Case A | Case B |
--- | --- | --- |
Young's Modulus ($E_i$) and Poisson's ratio ($\nu_i$) of raceway | $E_i = 200$ GPa, $\nu_i = 0.3$ | $E_i = 180$ GPa, $\nu_i = 0.3$ |
Young's Modulus ($E_{II}$) and Poisson's ratio ($\nu_{II}$) of balls | $E_{II} = 200$ GPa; $\nu_{II} = 0.3$ |
CHAPTER 5
INFLUENCE OF RESIDUAL STRESSES AND GRADED MATERIAL PROPERTIES ON
FATIGUE LIFE PREDICTIONS UNDER ELASTIC LOADING

Background

In 1960s, researchers at General Motors Research Laboratories extensively analyzed influence of residual stresses on the fatigue performance of rotating steel components including that of rolling element bearings (Zaretsky (11)). These stresses can be induced through either microscopic/macroscopic surface deformations from mechanical processing or by transformations of the microstructure of the steel by heat treating. They can be either tensile or compressive in nature. The magnitude and the distribution of these processing induced residual stresses is dependent on the metallurgical character of the steel and the severity of the operation (Scott et al. (44)). Typical operations which can be used to induce residual stresses are heat treatment, rolling, shot peening, diamond burnishing and sever grinding. Except heat treatment all other operations can be classified as mechanical processes, which induce compressive residual stresses no deeper than 0.01 in. to 0.012 in or even shallower depth. Scott et al. (40) showed adverse effects of these mechanically disturbed surface layers on fatigue performance of the specimen. Their work showed that electrolytic removal of shallow deformed surface layer, which accompanies introduction of residual stresses by mechanical processing methods, results in improved fatigue life of the specimen. However, compressive residual stresses induced due to heat treatment generally extend up to a much deeper region up to a depth of 765 microns or even higher (Forster et al. (30) and Philip (23)). Several experimental studies ((Zaretsky (11), Braza et al. (26)) have shown that they improve rolling contact fatigue performance of bearing steels. Load induced tensile stresses must overcome these compressive residual
stresses to do any damage. Gentile and Martin (45) reported that ball bearing lives were almost doubled when metallurgical induced compressive residual stresses were present in inner races (Zaretsky (11)).

Experimental investigations have shown that case-hardened steel such as M50-NiL has compressive residual stresses in pre-fatigue condition. Therefore, there is need to accurately quantify the influence of these beneficial residual stresses on the expected fatigue performance of this case-carburized steels. With this objective identified, following chapter is dedicated to analyzing influence of compressive residual stresses on fatigue life predictions of case-hardened steels. To determine exact stress state, gradations in elastic modulus is also considered.

**Residual Stress Profile from Carburization Process**

As highlighted in previous section, carburization process induces residual stresses in most of the case-carburized bearing steels. Forster et al. (30) determined the pre-fatigue residual stresses in M50-NiL steel in circumferential directions as function of depth. Approximate plot of their measurements is shown in Fig. 5-1. For reference, residual stress measurement in M50 steel is also shown. These measurements corresponds to 208-size, 40 mm bore bearings with raceways manufactured from M50-NiL and M50 steels. The M50-NiL and M50 bearings were made to standards used for aviation engine bearings, i.e. both materials were vacuum induction melted – vacuum arc remelted and tempered three times at 525\(^0\) C and 540\(^0\) C, respectively. Residual stress measurements were reported to be made using X-ray diffraction analysis.

Numerous studies have shown beneficial effect of these Residual stresses on the fatigue lives of the case-carburized steels (Zaretsky et al. (46), Broszeit and Zwirlein
These stresses are shown to alter critical subsurface shear stresses induced due to Hertzian loading of ball-raceway contacts. For light to moderately loaded bearings (i.e. $\tau_{\text{max}} \approx 414 \text{ MPa, } S \approx 1-1.5 \text{ GPa}$) the potential improvement in bearing fatigue lives can be up to 12 times; whereas for heavily loaded bearings (i.e. $\tau_{\text{max}} \approx 724 \text{ MPa, } S \approx 2-2.5 \text{ GPa}$) the expected improvement in life can be up to 4 times (Zaretsky (11)). These analytical results are corroborated by experimental endurance performance of M50-NiL bearing steels compared to M50 steels. To prove relative fatigue resistance of M50-NiL and M50 steels in aircraft engine applications, Harris et al. (42) conducted endurance tests on 6309 deep groove ball bearings with inner rings manufactured from these steels. These tests were conducted on 15 R2 endurance test rigs with heavy interference fit between bearing bore and shaft to generate tensile hoop stresses up to 181 MPa, commonly observed in high speed applications. Marginal lubrication conditions were used with 3µm filter and Exxon Turbo 2389 lubricant. All of these test conditions were used to minimize the testing time. Bearing inner rings were manufactured from VIM VAR M50 and M50-NiL steels; whereas outer rings and balls were manufactured from martensitic, stabilized AISI 52100 steel. For each material 40 samples of bearings were tested. For through-hardened M50 steel inner rings, 31 inner rings failed because of spalling and 3 tests were stopped because of failure of auxiliary components such as balls or cages. After 500 million cycles, 6 tests were suspended because no failure was observed. Most of the 31 failures of inner rings observed were before 100 million cycles. Bearings with case-carburized M50-NiL steel inner rings were tested under similar conditions. In these tests 12 auxiliary component failures were reported with 28 suspensions after 500 million cycles of revolution. These endurance
tests conducted by Harris et al. (42) clearly shows improved endurance capabilities of case-hardened M50-NiL steel compared to through-hardened M50 steel. They attributed this high endurance of M50-NiL steel to residual stresses of 175 MPa which were reported in those samples and high fracture toughness of the core region.

**Combined Effect of Gradation in Elastic Modulus and Residual Stresses**

As discussed in section 4.5, due to gradation in elastic modulus from 228GPa to 202GPa over 2mm case depth, the peak contact pressure experienced by M50-NiL material will always be higher than that experienced by through hardened steel under similar load. This higher contact pressure is mainly attributed to smaller deformation of the contact area. Therefore, due to smaller contact dimensions RCF affected volume is smaller for case-carburized bearing steels (as per Eq. (4-7)). From Eq. (2-6) and (2-27), it can be seen that life is inversely proportional to the RCF affected volume. Therefore, gradation in elastic modulus is expected to introduce small improvement in fatigue lifes of case-carburized bearing steels due to smaller RCF affected volume. Also, the residual compressive stresses due to case-carburization (as shown in Fig. 5-1) will minimize the magnitude of principal subsurface shear stresses. Studies by Zaretsky (11) and Voskamp and Mittemeijer (17) have shown that residual compressive stresses from case carburization can be linearly superimposed on the Hertzian stresses generated due to contact between ball and raceway. Therefore, due to residual stress profile shown in Fig. 5-1, the subsurface principal normal stress components in circumferential and axial direction will be reduced by 400MPa in the entire case layer up to 0.76mm. Combined effect of gradations in elastic modulus and compressive residual stresses one sample theoretical problem is analyzed as follows:
For the 6309 DGBB bearing studied in chapter 4, the raceway groove radius is 9.08 mm and ball diameter is 8.73 mm. The inner raceway radius at the contact is 27.51 mm. For the contact between ceramic ball (E=320GPa) and through hardened steel (E=200GPa) raceway under 1279.1N load the maximum Hertz contact pressure is determined to be 1.89GPa and lengths of semi-major and semi-minor axis are 1.7621 mm and 0.1832 mm. Corresponding highest magnitude of Tresca shear stress in subsurface region is 595.39MPa at a depth of 0.141mm. Now for the same configuration through hardened steel raceway is substituted with M50-NiL raceway with modulus gradation from 228GPa to 202GPa over 2mm case depth. Based on analysis presented in chapter 4, effective elastic modulus of the case-carburized steel was determined to be 223.803GPa. Using Hertz theory, corresponding contact pressure was determined to be 1.979GPa and contact dimensions are 1.7228mm and 0.1791mm, respectively. The maximum value of subsurface Tresca shear stress is 622.871MPa at a depth of 0.1378mm. By using principal of superposition this magnitude of subsurface shear stress will reduced by a stress of 200MPa due to residual stresses. It should be noted that principal normal stress in radial direction is not affected due to residual stresses. Therefore, debit in Tresca shear stress will be 200MPa only at a depth of 0.1378mm. Using Zaretsky Eq. (2-27), corresponding life improvement factor for M50-NiL steel compared to through hardened steel (TH) will be:

\[
LF = \frac{L_{M50NiL}}{L_{TH}} = \left[ \frac{595.39}{622.871 - 200} \right]^{3m} X \frac{V_{TH}}{V_{M50NiL}}
\]

Defining RCF affected volume as per Eq. (2-7) with critical depth for Tresca shear stress and c=31/3 and m=1.1, life improvement factor is determined to be LF=35.75.

From this calculations, it can be inferred that fatigue life of 6309 DGBB bearing with
M50-NiL case carburized raceway is 35.75 times higher than the fatigue life of the same bearing with through hardened raceway. This explains the superior endurance performance of case carburized bearings reported by Harris et al. (42). So it can be seen that due to gradations in elastic modulus and residual compressive stresses M50-NiL bearings are guaranteed to outperform through hardened bearings under identical loading conditions.

**Summary**

Influence of residual stresses on fatigue endurance of case-carburized steels is very well studied, but its effect in presence of graded material properties is not completely understood. Hence using effective elastic modulus surrogates developed in Chapter 4 and linear superposition principal, combined effect of graded elastic modulus and residual compressive stresses was analyzed for one sample contact inside 6309 DGBB bearing with ceramic balls and M50-NiL raceway. It was determined that case-carburization can improve fatigue life of bearing by as much as 35.75 times under 1.89GPa peak contact pressure. Lower loads will predict even higher improvement in fatigue life for case-carburized bearings. The analytical predictions are corroborated by experimental findings on endurance performance of case-carburized bearings. Therefore, case-carburized bearings should be preferred in applications where high reliability, long service life is desired and costs of bearings is not major concern.
Figure 5-1. Residual stress profiles in circumferential direction for M50-NiL and M50 steels in pre-fatigue (untested) condition.
CHAPTER 6
ROLLING CONTACT FATIGUE LIFE PREDICTION UNDER ELASTIC-PLASTIC LOADING

Background

During normal service life, bearing materials are subjected to multibillion contact stress cycles of non-proportional Hertzian loading between rolling elements and raceways. For raceway, maximum shear stresses are known to occur in the subsurface region. As very small volume of raceway material is subjected released tri-axial compression loading cycles, subsurface cracks can initiate at material inhomogeneities or stress concentration sites and propagate towards surface leading to spall formation. Therefore, rolling contact fatigue life of bearing steel is considered as random variable which is function of inhomogeneity in the material microstructure. Hence bearing industry uses probabilistic design equations to predict rolling contact fatigue life of bearings, based on its operating conditions. As discussed in Chapter 2, probabilistic design equations for rolling contact fatigue life predictions of bearing steels are proposed by Lundberg-Palmgren, Ioannides-Harris and Zaretsky. Therefore, in the present research, RCF tests are conducted on VIMVAR M50 steel rod using three-ball-rod accelerated RCF test bench, ceramic balls and Royco 500 synthetic Turbine oil, to check accuracy of each of these models in predicting fatigue lifes of modern bearing materials. Lundberg-Palmgren, Ioannides-Harris and Zaretsky’s empirical life prediction models are calibrated using experimental estimates of $L_{10}$ fatigue life. These calibrated models are then validated using experimental data available in the published literature. Vickers micro-indentation studies and three – dimensional finite element analysis is performed to determine accurate stresses in the subsurface region of the bearing steel rod. Life modification factors corresponding to materials, elastohydrodynamic lubrication
mechanism, friction and lubricant contamination for this test setup are determined as per STLE’s 1994 (Zaretsky (13)) design guide.

**Rolling Contact Fatigue Testing**

Accelerated rolling contact fatigue tests were conducted using three ball-rod test bench. Schematic of this experimental test setup is shown in Fig. 6-1 (Bhattacharyya et al. (48)). The 9.5 mm (3/8 in.) diameter \(d\) and 127 mm (5 in.) long cylindrical test specimen is held in vertical position by collet as shown in Fig. 6-1A. This assembly of specimen and collet is mounted on direct drive electric motor, which is line with the specimen below the table, and rotated at 3600 rpm. The three 12.7 mm (1/2 in.) diameter \(D\) balls are used to apply radial load on the cylindrical test specimen, as shown in Fig. 6-1B. These balls are equally spaced by bronze retainer (Fig. 6-1C) and are thrust loaded by two tapered roller bearing cups. Thrust loading is applied on tapered cups using three set of springs each of 9.98 \(N/mm\) (57 lbs/in) stiffness. These springs are calibrated with dead weight of 830.26 N (186.65 lbs), which corresponds to 1181.71 N (265.66 lbs) radial load between each ball and cylindrical test specimen. After each test, rod is advanced in axial direction to conduct next run. Therefore this test bench provides most efficient way of evaluating bearing materials under RCF loading, as multiple experiments can be conducted on single test specimen. During each run lubrication is applied by drip feed from the top side and in the beginning of the tests total immersion of rotating components is used to prevent wear at startup. An accelerometer and shutdown device monitors the vibration of the tapered cup housings. When a preset level of vibrations is exceeded, indicating presence of fatigue spall or surface crack, the
motor is automatically shut down (Glover (49)). Total test time is recorded by hour meter which is electrically connected to the motor.

In the current investigation, M50 steel cylindrical test specimen was tested till failure. It was produced by vacuum induction melting followed by vacuum arc remelting (VIMVAR) process. Balls were made of ceramic (Si$_3$N$_4$) material. For each test, separate set of three balls were used and care was taken to ensure that all ceramic balls used in current investigation came from the same heat treatment. Before and after every test: M50 steel specimen, ceramic balls, brass retainer and collet were cleaned with soap and water solution in ultra-sonic cleaner for 10 mins to remove any form of metal debris and foreign particles that might influence failure of rod. Before installing these components they were dried using air blower. Tapered cups were also cleaned with cloth to wipe out any metal debris. All of the tests were conducted using ROYCO 500 Synthetic Turbine Engine oil. Its specifications are same as MIL-L-23699C ester based synthetic oil which is used by U.S. military in aviation turbine engines and helicopter gear boxes. It is known to provide adequate protection against thermal and oxidative degradation mechanisms at higher temperatures (Beane et al. (50)). During all tests, drip feed rate of the lubricating oil was monitored every 2 hours and was maintained at 8-10 drops per minute. After each test the used oil was stored in separate container and not reused for subsequent testing, as per guidelines provided by Vlcek and Zaretsky (51). All of tests in current investigation were conducted at room temperature. Next Vickers micro-indentation was performed to determine Hardness of the Virgin VIMVAR M50 steel rod sample.
Vickers Micro-indentation Hardness Measurements

To determine hardness of the VIMVAR M50 steel rod, a 4.7625 mm (\(\frac{3}{16}\) in.) thick sample from end portion of virgin rod was cut along radial section using cutting saw. This sample was mounted on acrylic for polishing purpose. Conventional metallographic polishing techniques were used to prepare sample for hardness measurements. Multiple polishing steps were performed in automatic polisher with pad grid sizes ranging from 9µm to 1µm diamond. Quasi-static Vickers indentation experiments were conducted using Wilson® Instruments Tukon™ 2100B indentation tester. Using 100g load, indents were made on polished face of the virgin sample from surface up to 1558.27µm depth. Indents were made at 10 different depths with average separation of 153.39µm, as shown in Fig. 6-2A. First indent was made at 177.79µm depth from the surface. At each depth, 5 indents were made with average separation of 152.44µm. Optical micrograph of one sample pyramid shape Vickers indent is shown in Fig. 6-2B. At each depth average value of Hardness measurements from 5 idents is used. Observed variation of Vickers hardness measurements is shown in Fig. 6-2C. It can be seen that hardness values are nearly constant up to depth of 1558.27µm, which is expected for through-hardened steels. Average value of Vickers Hardness measurements from 50 indents is 854.84 kg/mm². This average value of Vickers hardness is used for determining life improvement factor for VIMVAR M50 steel, as per STLE life factors. Average diagonal length for pyramid shape Vickers idents is found to be 14.73µm. After hardness measurements, accelerated RCF tests were conducted using three ball-rod test bench. Observed results are discussed in following sections.
Experimental Results

As discussed earlier, VIMVAR M50 steel rod was life tested using Silicon nitride balls and Royco 500 Turbine Engine oil lubricant on three ball-rod test bench and 830.26N dead weight. Total 9 such tests were conducted out of which 7 tests were stopped due to formation of spall on the rod. Of the remaining two tests, one of test was suspended due to formation of spall on silicon nitride ball after 32.2 hours of testing and other test was suspended after 125.1 hours of testing due to shortage of turbine engine oil (this is the 9th test). Weibull probability plot of the test times for 7 failures is shown in Fig. 6-3. This plot is prepared using Weibull++ software. It can be seen that under nominally identical operating conditions, life of the VIMVAR M50 steel rod can vary from 22.3 hours to 202.2 hours, which is approximately an order of magnitude variation.

Median ranks of the spalled test times are adjusted for suspended test times. Using Eq. (2-4), straight line is fit through 7 data points, as shown in Fig. 6-3. The Weibull slope ‘e’ is estimated to be 1.33 with 90% confidence bound interval as [0.82, 2.14]. The estimated $L_{10}$ life at 90% probability of survival is 17.97 hours with 90% confidence bound as [5.25, 41.1] hours. These estimates are based on rank regression on Y, median ranks and standard ranking method for determining failure order number. The confidence bounds are determined using likelihood ratio. The rank regression method is used to determine Weibull slope because maximum likelihood method gives biased estimates of parameters (Harris (1), Vlcek and Zaretsky (51)). Using this method, the Weibull slope estimates of 1.33 is in good agreement with the $e=10/9$ which was estimated by Lundberg and Palmgren (8) based on fatigue data of thousands of bearings. Figure 6-3 also shows 90% confidence bounds around straight line curve fit. Optical micrograph (from Olympus BX51 microscope) of the spall observed on one of
the 7 tracks is shown in Fig. 6-4. Its approximate dimensions are 671.43µm in length and 589.03µm in width direction. Here length is assumed to be perpendicular to rolling direction and width is parallel to the rolling direction. In three ball-rod test bench, one revolution of rod corresponds to 2.389 stress cycles between ceramic balls and VIMVAR M50 steel rod (Glover (49)). Therefore, $L_{10}$ fatigue life of 17.97 hours corresponds to 9.273 millions of contact stress cycles. These estimate of fatigue life for VIMVAR M50 steel rod is in good agreement with the life test data reported by Glover (49) using similar materials and test setup. In his work, $L_{10}$ fatigue life is reported to vary between 4.0 to 5.6 millions of contact stress cycles with Steel balls and 1083.98 N load. Glover (49) also studied influence of two different lubricants: Exxon’s Terresic 100, an ISO viscosity grade 100 mineral oil and MIL-L-23699 Synthetic turbine engine oil. In present study, experimental results from current investigation are used to calibrate empirical life prediction equations proposed by Lundberg-Palmgren (Eq. (2-6)), Ioannides – Harris (Eq. (2-20)) and Zaretsky (Eq. (2-27)). And fatigue life data reported by Glover (49) for VIMVAR M50 steel using similar test bench will be used for validation study. Experimental results of both the studies are summarized in Table 6-1. The $L_{10}$ fatigue life is reported as total number of contact stress cycles experienced by the rod. Failure index represents total number of failures observed out of total number of tests conducted. Before calibrating LP, IH and Zaretsky’s life prediction equations, influence of operating conditions must be considered using life factors $a_1, a_2$ and $a_3$, as discussed in Eq. (2-13). How these factors were determined is discussed in following section.
STLE Life Factors for Operating Conditions

In Table 6-1, all of the fatigue lifes are reported for 90% probability of survival. Therefore, reliability factor $a_i$ is 1 for all the calibration and validation data sets. First life factors for calibration data set no. 1 were determined as follows:

Life Factor $a_2$: As per STLE design guide, life factor for AISI M50 steel is 2. Also for vacuum induction melting and vacuum arc re-melting practice the life improvement factor is 6. As discussed earlier, average value of Vickers indentation hardness is 854.84 $\text{kg/mm}^2$ for the VIMVAR M50 steel rod tested in calibration data set no. 1. This approximately corresponds to 65.75 HRC Rockwell C hardness. In general, it is known that higher the hardness of bearing steel at given operating temperature, longer will be the fatigue life. Therefore, following formula is used, as per STLE design guide, to determine life improvement factor corresponding to 65.75 HRC Rockwell C hardness:

$$LF = \exp\left\{m[(RC)_T - 60]\right\}$$

(6-1)

where $(RC)_T$ is hardness of the bearing steel at operating temperature and $m=0.1$. Therefore, life improvement factor corresponding to Rockwell C hardness of 65.75 HRC is 1.78. Through hardened steel do not contain any residual stresses therefore their effect is not considered. Therefore, for VIMVAR M50 steel rod with 65.75 HRC hardness, materials and processing life factor $a_2 = 6 \times 2 \times 1.78 = 21.36$.

Lubrication life factor $a_3$: It is well established that due to hydrodynamic action, contacting surfaces are generally separated by thin film of lubricant. Due to high pressures in the contact region, the lubrication process is accompanied by elastic deformation of the contacting surfaces. This process is referred as elastohydrodynamic
(EHD) lubrication mechanism. In 1949, Grubin was the first to identify presence of EHD films in rolling contact machine elements (Zaretsky (52)) and provided expression for calculating minimum lubricant film thickness. Since then numerous equations are proposed to determine EHD film thickness, by different researchers notably by Dowson and Higginson (1966), Archard and Cowking (1965-66) and Hamrock and Dowson (1977) (Zaretsky (52)). However, Coy and Zaretsky (53) determined that variations in the EHD film thicknesses calculated from these equations are less than variations between different experimental data sets used to verify these theories. Therefore, in present analysis minimum lubricant film thickness formula provided by Hamrock and Dawson (54) is used. Based on their theory, minimum lubricant film thickness in non-dimensional form for point contact can be obtained using following equation:

\[ H^o = \frac{3.63U^{0.68}G^{0.49}(1-e^{-0.68k_i})}{Q_z^{0.073}} \]

where \( h^o = \frac{h^o}{\mathcal{R}} \); \( h^o \) is minimum lubricant film thickness between two contacting bodies and \( \mathcal{R} \) is equivalent radius in rolling direction. In Eq. (6-2), ellipticity ratio \( k_i = \frac{a}{b} \), where \( a \) and \( b \) are semi-major and semi-minor axis of elliptical contact area between two bodies; \( U \), \( G \) and \( Q_z \) are dimensional less speed, material and load parameters, respectively. They can be determined using following set of equations (Harris (1)):

\[ U = \frac{\eta_o u}{2E \mathcal{R}} \]  \hspace{1cm} (6-3)

\[ G = \lambda E' \]  \hspace{1cm} (6-4)

\[ Q_z = \frac{Q}{E \mathcal{R}^2} \]  \hspace{1cm} (6-5)
In Eq. (6-3), \( \eta_o \) is lubricant viscosity at atmospheric pressure and entrainment velocity \( u = u_1 + u_2 \), where \( u_1 \) and \( u_2 \) are surface velocities of the two bodies in contact. In Eq. (6-4), \( \lambda \) represents pressure coefficient of viscosity. In Eq. (6-5), \( Q \) is the normal load acting on the contact. In Eq.s (6-5), \( E \) represents reduced modulus of elasticity for two bodies, which is obtained using following equation:

\[
\frac{1}{E} = \frac{1}{2} \left( \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right)
\]

where \( E_1, E_2 \) are elastic moduli and \( v_1, v_2 \) are Poisson’s ratios of body 1 and 2, respectively. In above equations, equivalent radius in rolling direction \( R \) is obtained using following equation:

\[
\frac{1}{R} = \frac{2}{D + \frac{2}{d}}
\]

where \( D \) and \( d \) are diameters of ceramic ball and VIMVAR M50 steel rod, respectively.

As discussed earlier in calibration data set, ROYCO 500 synthetic turbine oil was used. Based on reference manual its pressure coefficient of viscosity (\( \lambda \)) = 0.0155

\[
\text{mm}^2/\text{N} \quad (1.07 \times 10^{-4} \text{ in}^2/\text{lb})
\]

and kinematic viscosity at 40\(^\circ\)C, \( \eta_b = 25.7 \) cSt (55). Using density of 1 \( \text{g/cm}^3 \), absolute viscosity of ROYCO 500 oil was determined to be \( \eta_o = 2.57 \times 10^{-8} \text{ N s/mm}^2 \). At 3600 rpm, the entrainment velocity of lubricant in ball rod contact area is \( U = 3223.59 \text{ mm/s} \) (Glover (49)). In this analysis, for VIMVAR M50 steel rod elastic modulus of 205GPa and poisson’s ratio of 0.3 were used as per Bhattacharya et al. (56). Also for Ceramic balls elastic modulus of 310GPa and posision’s ratio of 0.27 are used as per Bhattacharya et al. (31). Substituting all of the above lubricant and
material parameters for two bodies, in Eq.s ((6-3)-(6-7)), with normal load of 1181.71 N, non-dimensional speed, material and load parameters were determined as: \( U = 5.65 \times 10^{-11} \), \( G = 4175.92 \) and \( Q_z = 5.93 \times 10^{-4} \), respectively. From these parameters and Eq. (6-2), minimum lubricant film thickness was determined as \( h^\circ = 0.076 \mu m \) \((2.99 \times 10^{-6} \text{ in.})\). Harris (1) has given following approximate relation to determine film thickness at center of the contact \( (h_c) \) from minimum lubricant film thickness:

\[
h_c = \frac{4}{3} h^\circ
\]

(6-8)

Therefore, for calibration data set 1, the lubricant film thickness at the center of contact is \( h_c = 0.101 \mu m \) \((3.99 \times 10^{-6} \text{ in.})\). According to Zaretsky (52), at higher speeds, lubricant starvation effects needs to be considered when lubricant flow number \( (G \times U) \geq 2 \times 10^{-7} \). For calibration data set 1, lubricant flow number \( G \times U = 2.36 \times 10^{-7} \), which indicates beginning of starvation of lubricant condition in the ball-rod contact area. Therefore, lubricant film reduction factor of 0.9 was determined as per STLE design guide. The measure of effectiveness of the lubricant film thickness in contact area is \( \Lambda \) ratio defined as ratio of central film thickness and composite surface roughness of the rolling element surfaces (i.e. \( \frac{h_c}{\Lambda} \)) (Zaretsky (13)). It should be noted that \( h_c \) includes effects of film thickness reduction due to lubricant starvation in contact area. If root mean square (rms) surface roughness of bodies 1 and 2 are \( \Omega_1 \) and \( \Omega_2 \) then composite surface roughness is defined as:

\[
\Omega = \left( \Omega_1^2 + \Omega_2^2 \right)^{\frac{1}{2}}
\]

(6-9)
For the VIMVAR M50 steel rod tested in calibration data set 1, centerless ground surface finish is 0.0762µm (3µin.) arithmetic average (AA). Surface finish of ceramic balls is 0.0254µm (1µin.) arithmetic average (AA). As per STLE design guide, arithmetic average surface finish (AA) and rms surface finish, for statistically random surface are related as:

$$\Omega_{AA} = 0.8\Omega_{rms}$$  \hspace{1cm} (6-10)

Using Eq.s (6-9) and (6-10), composite surface roughness for VIMVAR M50 steel rod and Ceramic balls is determined to be 0.1µm (3.95µin.). Therefore, the lubricant film parameter (Λ) for this configuration is 0.91. As per STLE design guide, corresponding life modification factor \(a_i\) is determined to be 0.36. However, it should be noted that above film thickness calculations were based on elastohydrodynamic lubrication assumptions. According to Rahnejat (57), lubrication regimes are characterized as: 5 < \(\Lambda < 100\): Hydrodynamic lubrication; 3 < \(\Lambda < 10\): Elastohydrodynamic lubrication; 1 < \(\Lambda < 5\): Partial/mixed lubrication; \(\Lambda < 1\) boundary lubrication. Therefore, \(\Lambda=0.91\) corresponds to boundary lubrication regime. Recently, Zhu and Wang (58) proposed that for mixed lubrication \(\Lambda\) ratio roughly spans from 0.01-0.05 to 0.6-1.2. However, in present work, \(\Lambda < 1\) is used as boundary lubrication regime because it provides conservative estimates of life modification factor corresponding to friction between ball and rod surfaces. This will be discussed in later part of this section. In boundary lubrication, there is not complete separation of rolling surfaces due to EHL film and surface asperities on both bodies will force direct solid to solid contact. Influence of surface roughness/asperities in EHL contacts is studied by various researchers including notably those by Greenwood and Morales-Espejel (59), Wang et al. (60), Xu and Sadeghi (61) and Wang et al. (62).
Results from these studies are application specific with individual EHL running conditions and specific surface roughness profiles (Rahnejat (57)). General empirical relation similar to that of Hamrock and Dowson Eq. (6-2) is not available. Also, detailed information of surface roughness profiles of VIMVAR M50 steel rod and Ceramic balls tested in calibration data set 1 is not available. Additionally in this work, scope of EHL film thickness calculations is limited to determination of lubrication life modification factor as per STLE design guide. In boundary lubrication regime (i.e. Λ < 1), variation in lubrication life modification factors is not as significant as variation in experimental fatigue lifes. Therefore, for calibration data set 1, lubrication life modification factor of 0.36 based on Λ = 0.91 is used.

For calibration data set 1, slide to roll ratio is roughly 0.23. For determining slide to roll ratio, sliding due to gyroscopic motions of the ceramic balls is not considered because of the heavy normal load used in this application (Harris (1)). For such a gross sliding of balls over rod surface, effect of friction on observed fatigue life needs to be considered. Based on Stribeck-Hersey curve given in STLE design guide, coefficient of friction for boundary lubrication is 0.15. Coefficient of friction in mixed lubrication regime is usually less than 0.1 (Rahnejat (57). Life modification factor due to frictional effects is inversely related to coefficient of traction. Hence, assuming Λ < 1 as boundary lubrication regime gives conservative estimate of life modification factor due to frictional effects. Therefore, life modification factor of 0.71 was determined corresponding to 0.15 coefficient of friction between ball and rod surfaces.

Life Modification Factor for Levels of Contamination: In addition to above life modification factors, life adjustment factor for level of contamination must be applied.
Absolute filter rating for three-ball rod test bench used in this work is 3µm. Based on STLE design guide, corresponding life modification factor is 1.4 for point contact. As discussed earlier, all the tests in calibration data set 1 were conducted at room temperature. Therefore, issue of water corrosion of rolling surfaces due to moisture in the air needs to be addressed. As per Ref. (55) and (63), Royco 500, MIL-L-23699 Synthetic Turbine oils and Exxon Terresstic 100 Mineral oil contains modern additive packages which resists rust and corrosion due to water environments. Therefore, life modification factor due to corrosion from water was not considered in this analysis.

After determining life modification factors for reliability ($a_1$), materials manufacturing and processing ($a_2$), lubrication and friction ($a_3$), and contamination levels ($a_4$), basic LP life is obtained using experimental fatigue lifes as follows:

$$L_{10,L_P} = \frac{L_{10,Exp}}{a_1a_2a_3a_4} \quad (6-11)$$

For calibration data set 1, using $a_1=1$, $a_2=21.36$, $a_3=0.26$, $a_4=1.4$, basic LP life is determined to be $L_{10,L_P} = 1.19$ millions of contact stress cycles. Using similar procedure basic LP lifes were determined for all the validation data sets 1-6. For these calculations, kinematic viscosity ($\eta_b$) for Exxon Terresic 100 mineral oil at 40°C is assumed to be 100cSt (63). The pressure coefficient of viscosity was determined to be $0.025 \frac{mm^2}{N} (1.724 \times 10^{-4} \frac{in^2}{lb})$ as per following relation from Harris (1) for mineral oils:

$$\lambda = 0.1122 \left( \frac{\eta_b}{10^4} \right)^{0.163} \quad (6-12)$$
Density for Terresic 100 mineral oil is $0.887 \frac{g}{cm^3}$ (63). Surface roughness for AISI 52100 Grade-24 balls is 0.089µm (3.5µin.) arithmetic average AA (Glover (49)). The VIMVAR M50 steel rods used in validation data sets 1-6 were processed using standard hardening heat treatment used for M50 steel. Therefore, in this analysis hardness values of 63 HRC were used for these rods. The corresponding life modification factor of 1.35 for Rockwell C hardness of 63 was determined using Eq. (6-1). The calculated basic LP lifes ($L_{10,LP}$) for these validation data sets are also given in Table 6-1. The lubrication film thickness parameter ($\Lambda$) for all the test cases are summarized in Table 6-2. It should be noted that mineral oil (Exxon Terresic 100) has viscosity of 100cSt, which is significantly higher than viscosity of synthetic oils (Royco 500, MIL-L-23699) of 25.7cSt, at 40°C. But still for nearly identical configuration, lubricant $\Lambda$ ratio for mineral oil is less than 1, as shown in Table 6-2. This is because contact lubricant flow number ($G \times U$) for mineral oil is $1.31 \times 10^{-6}$, which is significantly higher than $2.36 \times 10^{-7}$ determined for synthetic oils. This predicts significant starvation of lubricant condition in the ball-rod contact area. The elastohydrodynamic film reduction factor is reduced by 58% to 0.38 due to higher contact lubricant flow number. Therefore, in present analysis it is observed that increase in EHL film thickness due to higher viscosity is compensated by lubricant starvation effect in the ball-rod contact area. From Table 6-1, it can be seen that experimental $L_{10}$ fatigue life of rod in calibration data set is higher than lifes observed in all of the validation data sets, even if higher load is used. This can be explained by higher hardness of 65.75 HRC of the VIMVAR M50 steel rod used in calibration data set 1 over the conventional 63 HRC hardness of the VIMVAR M50 steel rod used in validation data sets 1-6. Moreover, additional improvement in life comes
from higher lubricant film parameter Λ as shown in Table 6-2. For Λ < 1, surface smearing or deformation accompanied by wear might be the possible reasons for the formation of the spall on the rod as shown in Fig. 6-4. This fact is also corroborated by higher slide to roll ratio. Ceramic balls used in calibration data set 1 are smoother than AISI 52100 (Grade -24) balls used in Validation data sets 1-6. It should be noted that subsurface stresses in VIMVAR M50 steel rod will be higher in calibration data set 1 than those for same rod in validation data sets 1-6, due to higher stiffness of the ceramic balls over AISI 52100 steel balls which leads to smaller contact area and higher contact pressure. This issue will addressed in subsequent sections. Therefore based on analysis in this section, it can be seen that observed experimental life of rolling contact machine elements can be significantly influenced by material hardness, lubricant (EHL) film thickness and starvation effects, surface roughness and frictional effects due to sliding of rolling surfaces and contamination levels of the lubricating oil. After analyzing influence of all these operating conditions next step is to analyze subsurface stresses in ball-rod contact.

**Subsurface Stresses**

Using Hertz theory of contact mechanics, subsurface stresses can be determined for single ball-rod contact shown in Fig. 6-1B. For these, as mentioned earlier, elastic modulus and Poisson’s ratio of steel rod is assumed to be 205GPa and 0.3, respectively. These parameters for silicon nitride balls were assumed to be 310GPa and 0.27, respectively. The geometrical and loading parameters were used as discussed in the experimental method section. The results based on elastic Hertz calculations are summarized in Table 6-3. These calculations were performed using MATLAB subroutine, to solve for elliptical integrals to determine contact dimensions. Maximum
subsurface von-mises, Tresca, and orthogonal shear stresses in rod and corresponding critical depths are also provided. All of the values are within 2-3% of the values provided in non-dimensional charts from Harris (1). Therefore, based on elastic calculations, 1181.71 N load between ceramic ball and steel rod results in 6.34GPa peak Hertz contact pressure and 3.87GPa peak von-mises stress in the subsurface region of the rod. Yield strength of high strength bearing steels is typically in the range of 2.5 – 3.5GPa. Using Tabor’s rule (64), yield strength of the material can be determined based on its hardness measurements. It should be noted that in Eq. (6-1), life improvement factor for Rockwell C hardness of 65.75 is determined with respect to Rockwell C hardness of 60. Therefore, for determining yield strength of steel rod, hardness of HRC 60 (≈ 698 kg/mm² Vickers hardness) is used in present analysis. The Tabor relationship between material hardness (H) and yield strength (σ_y) can be given as, \( H = C \sigma_y \)

where \( C \) is constraint factor. In recent studies Klecka et al. (29), Bhattacharyya et al. (56), Allison et al. (65) reported 2.4, 2.63, 2.66, 2.8 and 2.7 values for constrain factor \( C \) for through-hardened high strength bearing steels. Therefore in this study, average value of 2.638 is used for constraint factor \( C \). Using Tabor’s relationship, constraint factor of 2.638 and Vickers hardness of 698 kg/mm² yield strength of the bearing steel rod was determined to be 2.59GPa. It can be seen that maximum von-mises stress of 3.87GPa (based on elastic assumptions) for single ball-rod contact is significantly higher than the yield strength of 2.59GPa. Therefore, significant yielding of the subsurface rod material is expected at global level in the first loading cycle. Under elastic-plastic loading conditions, Hertz equations are not valid for determining contact stresses.
Hence, finite element model, as shown in Fig. 6-5, was developed for this experimental setup to determine contact stresses between ceramic ball and steel rod under elastic-plastic loading condition. To minimize computational time, only quarter geometries of the ball and rod were simulated, exploiting symmetric boundary conditions of the contact area. Very fine mesh was used near the center of the contact and at the boundary. Both the geometries were meshed using three dimensional second order hexahedra elements (C3D20T). Total of 152090 elements were used to discretize both the ball and rod geometries, to ensure accurate estimation of contact stresses under elastic-plastic loading conditions. This simulation was performed using ABAQUS 6.13-2/Standard software. Total time for one simulation is 135 hours on 32GB RAM multi-processor fast computer. This much computational cost for this simulation is expected due to nonlinear geometric boundary conditions and material properties i.e. contact and metal plasticity, respectively. Non-linear hardening behavior was used to simulate metal plasticity of the bearing steel rod. Klecka (66) reported power law hardening behavior for through hardened bearing steels based compression test results and indentation studies. Based on this rule, flow stress ($\sigma$) and flow strain ($\varepsilon$) can be related as:

$$\sigma = K\varepsilon^n,$$

where $K$ is strength coefficient and $n$ is strain hardening exponent which was assumed to be 0.05 based on data reported by Klecka (66) for samples from same heat treatment. Strength coefficient is determined to be $K=3.54\text{GPa}$ based on yield strength of 2.59GPa at plastic strain offset of 0.2%. The true stress-true strain curve, used in this analysis, based on these material properties is shown in Fig. 6-6. It should be noted that true strain values contain 0.2% plastic strain offset for initial yielding of the material. True stress – true strain curve from Fig. 6-6 indicate that actual stress state under
elastic-plastic loading is significantly different than that under elastic assumptions. This non-linear hardening behavior was assigned to steel rod in Abaqus model using J2 plasticity behavior. Ceramic materials generally have higher hardness/yield strength hence elastic-plastic material properties were not assigned to silicon nitride balls. Results from FE simulation are also summarized in Table 6-3 under elastic-plastic loading. It can be seen that under elastic-plastic loading maximum subsurface von-mises stress in steel rod is just 2.81GPa which is about 1.06GPa (about 27%) lower than 3.87GPa determined under elasticity assumptions. Table 6-3 also compares percent (%) variation in different contact parameters under elastic and elastic-plastic loading assumptions. For similar configuration, it can be seen that actual contact pressure drops by 13.14% from 6.34GPa to 5.51GPa, just by accounting for metal plasticity. The contact dimensions: semi-major axis (a) and semi-minor axis (b) also increases by 7.32% and 4.44%, respectively. It is interesting to note that ellipticity ratio \( k = \frac{a}{b} \), is not significantly different for two loading conditions. For elastic loading, \( k_1 = 1.74 \) and for elastic-plastic loading \( k_1 = 1.79 \). Therefore, changes due to variation in this parameter in Hamrock and Dawson Eq. (6-2) are insignificant. Also, it can be seen that due to low strain hardening exponent of 0.05, actual maximum von-mises stress to peak contact pressure ratio is significantly lower for elastic-plastic loading condition compared to that for elastic loading condition. It decreases by 16.68%. Similar, results were observed for maximum subsurface shear (Tresca) stress to peak contact pressure ratio. However, surprisingly it was observed that maximum orthogonal shear stress to peak contact pressure ratio almost remain constant, indicating that subsurface orthogonal shear stress is not affected due to metal plasticity. Figure 6-7 shows
equivalent plastic strain (PEEQ) in the subsurface region of the steel rod after first loading cycle. It can be seen that plastic strain is observed only in subsurface region whereas strains near the surface region are only elastic in nature. Therefore, EHL film thickness calculations using Hamrock and Dawson Eq. (6-2) were assumed to be remain valid under these conditions. Moreover, no theories are available in published literature which can predict lubricant EHL film thicknesses under gross metal plasticity of the contacting bodies.

As it can be seen from Table 6-3, maximum contact pressure, maximum von-mises and Tresca shear stresses, and contact dimensions are significantly different under elastic-plastic loading conditions compared to that under elastic loading assumptions. Therefore, influence of metal plasticity cannot be ignored in fatigue life prediction Eq.s (2-6), (2-20) and (2-27). Equations (2-9) and (2-13) were determined based on elastic Hertz contact stresses hence they are not used in present study. Equations (2-6), (2-20) and (2-27) are the only available models and being empirical in nature they were used to predict fatigue lifes under elastic-plastic loading conditions. Following section discusses calibration and validation study of these models using experimental data in given in Table 6-1.

**Calibration of Fatigue Life Models**

The Lundberg-Palmgren, Ioannides and Harris, and Zaretsky Eq.s (2-6), (2-20) and (2-27), respectively were calibrated using basic LP life determined for calibration data set 1 from Table 6-1. The critical subsurface stresses and corresponding depths that were used are given in Table 6-3. The estimated values of the proportionality constants for various models are summarized in Table 6-4. In present study, exponents $e=10/9$, $c=31/3$ and $h=7/3$ (as determined by LP) were used in all of the models to
ensure consistency. In Table 6-4, $K_{LP,\text{elastic}}$ denotes proportionality constant for LP model, i.e. Eq. (2-6), with elastic Hertz stresses, contact dimensions as per Table 6-3 and RCF affected volume as defined by Eq. (2-7). $K_{LP,\text{elastic-plastic}}$ denotes proportionality constant for LP model (Eq. (2-6)) with elastic-plastic stresses and contact dimensions as determined using FE simulations (Table 6-3). Under elastic-plastic loading, definition of RCF affected volume is assumed to be same that of LP model i.e. Eq. (2-7), except contact dimensions and subsurface critical depth are as per FE simulation results. For determination of proportionality constant of Ioannides and Harris model (Eq. (2-20)), same elastic-plastic stresses were used. Moreover, two different variations of IH theory were used. In first case, fatigue limit $\sigma_u$ was assumed to remain constant (i.e. $\sigma_u = C$ ) in the subsurface region over all the finite elements. In present analysis, fatigue limit of $\sigma_u = 684\text{MPa}$ was used for through hardened bearing steels, as per Barnsby et al. (20). In second case, fatigue limit of 684MPa was assumed to be linearly decreasing to 0 for elastic-plastic loading beyond yield strength up to ultimate strength of the material, as discussed by Ioannides and Harris (15). Moreover, ultimate strength to yield strength ratio of 1.2 is used as it is reasonable for hard bearing steels (Bhattacharyya et al. (31)). The integral in Eq. (2-20) was solved by discretizing subsurface material of the rod into 900 finite elements and adding up contribution of individual volume elements as shown below:

$$ I = \int_{V_R} \left(\sigma - \sigma_u\right)^c \frac{1}{z^h} dV = \sum_{i=1}^{900} \left(\sigma - \sigma_u\right)^c \frac{1}{z^h_i} \Delta V_i $$

where volume elements $\Delta V_i, i \in [1, 900]$, make up subsurface volume $v_R$ in the radial plane passing through semi-major axis. The scale factor along circumference direction
was not considered in determining volume for both calibration and validation data sets. Equation (6-13) was solved by integrating MATLAB and Abaqus softwares. The calibrated value of proportionality constant \( K_{IH} \) using both the approaches i.e. constant fatigue limit \( (\sigma_u = C) \) and linearly decreasing fatigue limit \( (\sigma_u \neq C) \) are given in Table 6-4. Similar to Eq.s (2-6) and (2-20), Zaretsky Eq. (2-27) was also solved using elastic-plastic stresses, contact dimensions and critical subsurface shear stress depths determined from FE simulations. The corresponding value of proportionality constant \( K_z \) is provided in Table 6-4. Calibrated Eq.s (2-6), (2-20) and (2-27) were validated using fatigue life data (from Glover (49)) as summarized in Table 6-1. Details of the validation study are provided in following section.

**Validation Study**

Similar to steel rod – ceramic ball contact, finite element model was developed to simulate steel rod – steel ball contact for validation data sets (1-6). Elastic-plastic material properties were assigned to both ball and the rod, as 1083.98N radial load results in plastic deformation of material in the subsurface region of both the bodies. Rockwell C hardness of 63 was used for the ball material. For elastic-plastic loading between steel rod and steel ball at 1083.98N load, maximum contact pressure is observed to be 5.06GPa. The ratio of peak von-mises stress, peak Tresca shear stress and peak orthogonal shear stress to maximum contact pressure was found to be 0.54, 0.29 and 0.24, respectively. Peak equivalent plastic strain (PEEQ) in the rod after first cycle is \( 5.9 \times 10^{-3} \) whereas that in the steel ball is \( 2.33 \times 10^{-3} \). It can be seen that peak mises stress to maximum contact pressure ratio is decreased from 0.54 at 1083.98N load for steel-steel contact to 0.51 at 1181.71N load for ceramic-steel contact (Table 6-
3), due to non-linear hardening of the material. Similar, argument can be made for the peak Tresca shear stress to maximum contact pressure ratio, as it decreases from 0.29 to 0.27. However, from both the models it was observed that ratio of peak orthogonal shear stress to maximum contact pressure is almost same in presence of metal plasticity and even under only elastic loading conditions. This indicate that peak orthogonal shear stress in the subsurface region of the material is not affected due to plastic deformation of the material, whereas ratios of peak von-mises and Tresca shear stress to maximum contact pressure continues to decrease with increased loading.

Using elastic and elastic-plastic stresses, contact dimensions and subsurface critical shear stress depths from FEA, calibrated Eq.s (2-6), (2-20) and (2-27) were used to predict fatigue lifes for validation data sets (1-6). Figure 6-8 shows plot of logarithm of the ratio of experimental LP fatigue life (from Table 6-1) to the life predicted using calibrated models. Symbols used to denote LP model with elastic loading is (LP(E)), LP model using elastic-plastic loading is (LP(EP)), Zaretsky model with elastic-plastic loading is (Zaretsky(EP)), Ioannides and Harris model with elastic-plastic loading and constant fatigue limit in the subsurface region is (IH(EP, $\sigma_u = c$)), Ioannides and Harris model with elastic-plastic loading and linearly decreasing fatigue limit is (IH(EP, $\sigma_u \neq c$)).

This logarithms of the life ratios for all the 6 validation data sets are plotted in this figure. Ideally, predicted life should be same as actual experimental life. Therefore, $\log_{10} = 0$ is the target as shown in the figure. From this plot it can be seen that, LP model with elastic stresses results in highest error in rolling contact fatigue life prediction. All the 6 validation data points corresponding to LP(E) are farthest from target of $\log_{10}$=0. It can also be seen that LP model with elastic-plastic loading condition provides some
improvement over LP model with elastic loading conditions. However, rolling contact fatigue life prediction using Zaretsky (Eq. (2-27)) and Ioannides-Harris model (Eq. (2-20)) with elastic-plastic loading condition and $\sigma_u = c$ are even better, as all the 6 validation data points lie close to the target of $\log 1=0$. It was found that IH model with constant fatigue limit provides better fatigue life prediction than IH model with linearly decreasing fatigue limit as a function of yielding of the material. For all the data sets, logarithm of the ratio of actual life to the predicted life are less than zero because the observed basic LP life for validation and calibration data sets are comparable (Table 6-1). Calibrated Eqs. (2-6), (2-20) and (2-27) predict higher basic LP life for validation data sets than that for calibration data set, owing to the lower operating stresses from lower loads under elastic-plastic loading condition. Also, it should be noted that Zaretsky’s model (Eq. (2-27)) with Tresca shear stress criterion provide better fatigue life prediction than both versions of Ioannides and Harris model (Eq. (2-20)) under elastic-plastic loading condition. This result is consistent with several metallurgical investigations using transmission electron microscope, which reported significant plastic alterations in the subsurface region of the material at depth which closely matches with maximum Tresca shear stress location (Harris (1)). The total RCF affected volume, as defined by Eq. (2-7) is comparable for calibration and validation data sets. The IH model uses von-mises stress in each element subtracted by fatigue limit, therefore the ratio of integrals (i.e. Eq. (6-13)) for two cases get amplified due to reduced magnitude of stresses. Hence life prediction using IH models deviate more from target than that from Zaretsky model. However, both Zaretsky and IH models are better for these loading condition, as maximum Tresca shear stress and von-mises stress magnitude are influenced in
presence of metal plasticity. Therefore, life prediction using these models along with STLE life modification factors can reasonably predict fatigue performance of VIMVAR M50 steel under elastic-plastic loading condition.

**Dynamic Capacity and Metal Plasticity**

From the data presented in Table 6-1, it can be seen that even after yielding in first cycle VIMVAR M50 steel rod has $L_{10}$ fatigue life in the range of 4 to 9.273 millions of contact stress cycles. And even after accounting for all the STLE life modification factors as per Zaretsky (13), basic LP life for all the data sets is roughly 1 million contact stress cycles. This life is observed when plastic strain after first loading cycle is 0.59% for steel rod – steel ball contact at 1083.98N load and 1.04% for steel rod – ceramic ball contact at 1181.71N load, at global scale. As per LP model, dynamic capacity is defined as the load for which contact will survive one million stress cycles with 90% probability of survival. Bearing steels developed during the time when LP model was developed were relatively of poor quality compared to today’s manufacturing practices. Hence, amount of load required to fail the material in one million stress cycles was in elastic domain. However, due to superior quality of today’s bearing steels, significant plastic deformation is required in first cycle to fail the material in one million contact stress cycles, as observed in current experimental investigation. Therefore, expressions for dynamic capacity need to account of influence of metal plasticity, as the subsurface stress field under elastic-plastic loading significantly deviate from that predicted by Hertzian theory (Table 6-3). Accounting for these effects will lead to realistic mechanics based rolling contact fatigue life prediction approaches which will show better agreement between predicted life and experimental life as shown in Fig. 6-8, leading to reliable mechanical and engineering systems.
Strain Life Equation

As it can be seen from Fig. 6-7, significant plastic deformation is generated after first loading cycle. Generally, in fatigue theory under non-zero plastic strain, common practice is to use strain-life equation instead of stress-life equations. Therefore, this section discusses development of new strain-life equation to predict rolling contact fatigue life of bearing steels under elastic-plastic loading condition.

In rolling contact fatigue, it is difficult to conduct stress controlled or strain controlled tests due to nonlinearity of the contact mechanics. Experimental investigation in current work is based on load controlled tests. Recently, Bhattacharyya et al. (31) investigated performance of VIMVAR M50-NiL case-carburized steel rod under similar testing conditions. In their study, significant increase in subsurface hardness and contact track width under elastic-plastic loading of VIMVAR M50-NiL case-carburized steel rod is observed. The track width is reported to have increased by 56.25% over 246 millions of contact stress cycles and the subsurface hardness at critical depth is found to have increased by 13.51% over the entire life of the specimen. However, neither there is any reported increase in track width nor any noticeable hardening was observed over initial 43000 loading cycles; indicating that at global scale material is under elastic-shakedown after yielding in first cycle. Bearing steels contain many alloying elements which forms hard carbides in steel matrix. These carbides are known to act as stress risers, therefore locally material continues to accumulate plastic strain via ratchetting mechanism even if at global scale it is in elastic-shakedown stage. This accumulated plastic strain manifests as increase in hardness after millions of contact stress cycles, as was observed in study by Bhattacharyya et al. (31). However, influence of microstructural attributes and ratchetting mechanism in overall fatigue life prediction is
usually not considered because of the variability in manufacturing conditions which leads to non-uniform/variable subsurface microstructure of nominally identical bearing steels. Study by Kotzalas (67) has shown that even though changes in residual stress over time are function of the materials response to fatigue, instantaneous values of material constants can’t be determined. Hence the only viable option is to use pre-fatigue stress field and initial loading conditions to predict life of bearing steels under rolling contact fatigue loading. Using this approach, Bhattacharyya et al. (48) recently proposed stress-life equation, using traditional S-N diagram to predict fatigue life of VIMVAR M50-NiL case-carburized bearing steels. In their work both volume average stresses and point stresses were used to construct S-N diagram. In present analysis, this approach is corrected using strain-life equation, as appropriate damage parameter for any material beyond its elastic limit stress is plastic strain. Typical loading cycles for VIMVAR M50 steel rod in three ball rod test setup at 1181.71N load is shown in Figure 6-9. During first loading cycle, the material follows the path O-A-B-C. In remaining cycles, material is under elastic shakedown at global scale, therefore it follows path C-B-C-B. However, as discussed earlier, loading cycles at local scale around carbide particles can be significantly different depending upon microstructural attributes. It should also be noted that loading shown in Fig. 6-9 is actually compressive in nature. Equivalent von-mises stress is used to represent three dimensional loading cycle. From this figure, it can be seen that maximum stress ($\sigma_{\text{max}}$) in any loading cycle is 0 and minimum stress ($\sigma_{\text{min}}$) in first loading cycle is -2.81GPa. This classify as released compression or zero-compression loading cycles where R, defined as ratio of minimum and maximum stresses (Collins (68)), is $\infty$. Therefore, under these non-fully reversed
loading condition effect compressive mean stress needs to be considered. It should be
noted that it is because of these compressive mean stress material survives for millions
of stress cycles even if it is yielded in first cycle. Mean strain effects are dominant only
when fatigue life is few hundreds of stress cycles and plastic strain range component
dominaates over elastic strain range component. In past various researchers have
proposed strain-life models accounting for mean stress effects. The most widely used
models are Morrow's strain life model, Manson and Halford's model which accounts for
the effect of mean stress on both elastic and plastic strain components, and Smith-
Watson-Topper model which assumes product of maximum stress and strain amplitude
remains constant for different combinations of stress and strain at given life (Ince and
Glinka (69)). Smith Watson Topper model is not useful for RCF loading condition
because maximum stress is 0. Therefore, it predicts infinite life under compression. In
present study Morrow's strain life model with compressive mean stress corrections is
used. It includes Morrow's mean stress correction in Morrow's strain life equation. With
mean stress corrections Morrow's strain-life equation can be given as:

\[
\frac{\Delta \varepsilon}{2} = \frac{\sigma_f - \sigma_m}{E} (2N_f)^b + \varepsilon_f (2N_f)^c
\]  

(6-14)

In Eq. (6-14), \(\frac{\Delta \varepsilon}{2}\) is total strain amplitude, which is summation of elastic and plastic
strain amplitudes; \(N_f\) is total number of loading cycles to failures and hence
\(2N_f\) represents total number of reversals; \(b\) is Basquin slope/fatigue strength exponent
and \(c\) is fatigue ductility exponent. These exponents are defined as slope of elastic and
plastic strain amplitudes versus reversals to failure curve on log-log plot. \(\varepsilon_f\) represents
fatigue ductility coefficient, defined as strain intercept at one load reversal i.e. \(2N_f = 1\)
that leads to failure. Similarly, $\sigma_f$ is fatigue strength coefficient which defines stress amplitude for fully reversed loading leading to fail in first cycle. E is elastic modulus of the material. In Eq. (6-14), $\Delta \varepsilon/2$ is assumed to be strain amplitude from stabilized hysteresis loop of cyclic stress-strain loading. But as discussed earlier, due to continuously evolving nature of RCF loading and insufficient material property data, $\Delta \varepsilon/2$ is considered as strain-amplitude in initial loading cycle. Equation (6-14) is valid under the assumptions that material is experiencing strain range of $\Delta \varepsilon/2$ for $2N_r$ number of reversals. However, from Fig. 6-9 it can be seen that material loading is composed of two different total strain ranges at global scale. In first cycle (i.e. O-A-B-C), total strain range = $\varepsilon_b - \varepsilon_o$, consists of elastic and plastic strain ranges and in remaining cycles (i.e. C-B-C-B..), total strain range = $\varepsilon_b - \varepsilon_c$, which is only elastic in nature due to elastic shakedown of the material. For such a varying amplitude ranges, cumulative fatigue damage theory proposed by Palmgren-Miner will be used. It is also known as linear damage rule which can be defined as follows:

If component is subjected to stress amplitude $S_i$ for $n_i$ number of cycles and if $N_i$ is defined as total life of the component under $S_i$ stress amplitude from S-N diagram, then damage fraction ($D_i$) over $n_i$ cycles due of stress amplitude $S_i$ is defined as follows:

$$D_i = \frac{n_i}{N_i} \quad (6-15)$$
Therefore, Palmgren-Miner’s hypothesis proposes that damage fraction at any stress level $S_i$ is linearly proportional to the ratio of the number of cycles of the operation to the total number of cycles that will result in failure at same stress level (Collins (68)).

If $S_i, i \in [1, n]$, represents different spectrum of stress amplitudes to which component is subjected, then failure is predicted to occur if:

$$D_1 + D_2 + D_3 + \ldots + D_n \geq 1$$

(6-16)

In fatigue loading when cycles to failure are unknown, conservative approach is to use following form of Palmgren-Miner’s hypothesis:

$$\sum_{i=1}^{n} \frac{n_i}{N_i} = 1$$

(6-17)

In present analysis, two different spectrum of loadings are considered for the three-ball rod test setup. From Fig. 6-9, it can be seen that at global level material undergoes loading along path O-A-B-C i.e. initial yielding cycle and C-B-C-B i.e. elastic shakedown stage. Path O-A-B-C will be denoted as spectrum 1 loading and path C-B-C-B denoted as spectrum 2 loading. For the calibration data set 1, basic LP fatigue life is 1.19 millions of contact stress cycles. This life will be used to calibrate Morrow’s strain life equation with compressive mean stress corrections (i.e. Eq. (6-14)) to determine unknown material parameters. This life of 1.19 millions of contact stress cycles corresponds to combined spectrum of loading cycles i.e. path O-A-B-C and path C-B-C-B. Path O-A-B-C is active for first cycle whereas path C-B-C-B is active for remaining cycles. Therefore, $n_1 = 1$ and $n_2 = 1.19 \times 10^9 - 1$. Total cycles to failure i.e. $N_1$ and $N_2$, if each of these loading paths were active separately are unknown. Using material properties i.e. elastic modulus of 205GPa and strain hardening exponent of 0.05,
corresponding elastic and plastic strain amplitude components can be determined for each spectrum of loading. Cyclic stress and strain amplitudes and their elastic, plastic components are related as follows:

\[
\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2} = \frac{\Delta \sigma}{2E} + \left[ \frac{\Delta \sigma}{2K} \right]^{2n}
\]  \hspace{1cm} (6-18)

where \( \frac{\Delta \varepsilon_e}{2} \) and \( \frac{\Delta \varepsilon_p}{2} \) are elastic and plastic strain amplitudes and \( \frac{\Delta \sigma}{2} \) is stress amplitude; \( n \) and \( K \) are cyclic strain hardening exponent and cyclic strength coefficient, respectively. Typically, for VIMVAR M50 bearing steel parameters \( n \) and \( K \) are unknown. Moreover, to apply Eq. (6-18) in just first loading cycle, they can be assumed to be same as static strength hardening exponent \( (n) \) and strength coefficient \( (K) \).

Therefore, for loading path O-A-B-C, stress amplitude \( \frac{\Delta \sigma}{2} = 1.405 \text{ GPa} \). Corresponding elastic strain amplitude \( \frac{\Delta \varepsilon_e}{2} = 6.854 \times 10^{-3} \) and plastic strain amplitude \( \frac{\Delta \varepsilon_p}{2} = 4.94 \times 10^{-3} \). Therefore, total strain amplitude \( \frac{\Delta \varepsilon}{2} = 0.01179 \) for loading path O-A-B-C. Equation (6-14) is solved using two different approaches: In first approach highest magnitude of stresses at critical depth are used and in second approach volume average stresses and strain are used. Second approach is necessary because failure due to rolling contact fatigue is strong function of the volume of the material loaded. Hence, effect volume can be considered in Eq. (6-14) by using volume average stresses/strains. For determining, volume average stress and strain, RCF affected volume is assumed to be same as volume defined by LP model i.e. critical depth \( X \) semi-major axis of contact ellipse \( X \) circumference of the raceway. For calibration data set 1, elements comprising this volume are highlighted in Fig. 6-10. Over this region,
average stress was determined to be 2.305GPa. Similarly, volume average elastic and plastic strain amplitudes in first loading cycle (i.e. path O-A-B-C) were determined to be 0.005622 and 0.00135, respectively. Therefore, volume average total strain amplitude in first cycle was determined to be $6.972 \times 10^{-3}$. Similar calculations were performed for loading path C-B-C-B. Total strain amplitude is $6.854 \times 10^{-3}$ and volume averaged total strain amplitude is $5.622 \times 10^{-3}$. These estimates of total strain amplitude and volume averaged total strain amplitude can be substituted in Eq. (6-14) to determine unknown material parameters for bearing steel. This procedure is demonstrated using critical point strains as follows:

Substituting total strain amplitude for path O-A-B-C in Eq. (6-14), we get:

$$0.011794 = \frac{\sigma_f}{E} - \frac{\sigma_m}{E} (2N_1)^b + \varepsilon_f (2N_1)^c$$  \hspace{1cm} (6-19)

Similarly, substituting total strain amplitude for path C-B-C-B in Eq. (6-14), we get:

$$6.854 \times 10^{-3} = \frac{\sigma_f}{E} - \frac{\sigma_m}{E} (2N_2)^b$$ \hspace{1cm} (6-20)

As discussed earlier, in Eq.s (6-19) and (6-20), $\sigma_f$ is fatigue strength coefficient which represents stress amplitude for the material to fail in first cycle. This value is not reported for bearing steels in published literature. Therefore, in present study it is assumed to be same as ultimate strength of the material. Moreover, ratio of ultimate strength to yield strength is assumed to be 1.2, which is common for hard bearing steels (Bhattacharyya et al. (31)). Hence, ultimate strength of the bearing steel was determined to be 3.116GPa. Similarly, $\varepsilon_f$ which represents fatigue ductility coefficient/plastic strain amplitude which results in failure in first cycle, is defined at plastic strain
corresponding to ultimate strength of the material. Therefore, \( \varepsilon_f = 0.07796 \) is used in present study. It should be noted that \( N_1 \) and \( N_2 \) in Eq.s (6-19) and (6-20) are related by Eq. (6-17) as:

\[
\frac{1}{N_1} + \frac{1.19 \times 10^6}{N_2} - 1 = 1
\]

(6-21)
as per Palmgren-Miners cumulative fatigue damage rule. Both \( N_1 \) and \( N_2 \) are unknown. Also, exponents \( b \) and \( c \) in Eq.s (6-19) and (6-20) needs to be determined. To determine these unknowns, relationship between exponents \( b \) and \( c \), and cyclic strain hardening exponent \( n' \) is used. As per Collins (68), based on energy arguments they are related as follows:

\[
b = \frac{-n'}{1 + 5n'}
\]

(6-22)

\[
c = \frac{-1}{1 + 5n}
\]

(6-23)

Therefore, Eq.s (6-19)-(6-23) with 5 unknowns \( b, c, n', N_1 \) and \( N_2 \) were solved simultaneously using fmincon optimization solver from MATLAB. Their values were determined to be \( b = -0.0757 \), \( c = -0.6216 \), \( n' = 0.1217 \), \( N_1 = 1.6376 \times 10^3 \) and \( N_2 = 1.1907 \times 10^6 \) cycles. This procedure of determining material parameters was again performed with volume average elastic and plastic strain amplitudes. The estimated values of these parameters for both the approaches are summarized in Table 6-5. Typically, values of \( b \) ranges from -0.05 to -0.15 and \( c \) ranges from -0.5 to -0.8. From Table 6-5, it can be seen that for both the approaches values of exponents \( b \) and \( c \) lies in this range. This confirms suitability of strain-life approach to predict life of bearing steels under rolling contact fatigue loading.
Most of the bearings in service are operating below peak contact stress of 2GPa. This loading condition lies purely in elastic domain at global scale. As the plastic strain amplitude $=0$, the Morrow’s strain life Eq. (6-14) with mean stress corrections reduces to following form for these operating conditions:

$$\frac{\Delta \varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E} (2N_f)^b$$

(6-24)

For example, at 2GPa peak contact pressure, basic LP life of bearing steel rod with these dimensions will be 22.55 billions of contact stress cycles with $b=-0.0757$.

Multiplying it with STLE life factors of $a_1=1$, $a_2=21.36$, $a_3=0.8$ and $a_4=1.4$, total life is determined to be 539.47 billions of contact stress cycles. These life corresponds to the $\frac{3}{8}$ in. diameter VIMVAR M50 steel rod with HRC 65.75 hardness, 3µin. average surface roughness (AA), in contact with $\frac{1}{2}$ in. diameter silicon nitride ball with 1µin. average surface roughness (AA) under the load of 2GPa peak contact pressure and ROYCO 500 synthetic turbine oil lubricant. Therefore, this approach predicts significantly higher life for modern high strength through hardened bearing steels under normal loads, as observed in service.

**Summary**

Aerospace bearing steels are often tested under accelerated rolling contact fatigue test conditions. Using three ball-rod test setup, accelerated RCF tests were conducted on VIMVAR M50 steel rod with HRC 65.75 hardness. Ceramic balls were used. Springs were calibrated to apply radial load of 1181.71N between individual ball-rod contacts. This load resulted in significant plastic deformation in the subsurface region of the rod. Total 9 such tests were conducted using ROYCO 500 synthetic
turbine oil lubricant. Out of these 9 tests, 7 tests showed spalls on the rod and 2 tests were suspended due to ball spallation and inadequate lubricant. Using standard Weibull Statistics $L_{10}$ fatigue life for VIMVAR M50 steel rod was estimated. STLE life modification factors were determined for this test setup. Due to high load and surface roughness, the characteristic $\Lambda$ lubricant parameter for this testing condition was found to be less than 1, which indicate boundary lubrication regime. Similar analysis was performed on the fatigue data available in published literature from identical test setup. It was observed that increased hardness and higher lubricant parameter $\Lambda$ results in higher fatigue life for VIMVAR M50 steel rod even if it is operating under higher loads/stresses. Basic LP lifes were determined using experimental life estimates and STLE life modification factors. To determine elastic-plastic stresses in the subsurface region of rod, three dimensional finite element simulations were performed. It was observed that due to non-linear hardening of the material subsurface elastic-plastic stress field is significantly different than Hertzian elastic field. Actual peak contact pressure in the steel rod drops by as much as 13.14% due to plastic deformation of the material. Ratios of maximum von-mises and Tresca shear stresses to peak contact pressure are reduced by up to 17%. However, ratio of maximum orthogonal shear stress to peak contact pressure is not affected in presence of metal plasticity. Total 7 estimates of basic LP $L_{10}$ fatigue life, based on 89 experimental results, were used for calibration and validation study of empirical rolling contact fatigue life models proposed by Lundberg-Palmgren, Ioannides-Harris and Zaretsky. It was found that Zaretsky and Ioannides–Harris model with elastic-plastic stresses show better agreement between predicted and observed fatigue life. However, life prediction using LP model with elastic
stresses significantly deviate from observed fatigue life. Hence, under these loading conditions, elastic-plastic stresses must be considered in rolling contact fatigue life prediction. Moreover, it was observed that even after significant plastic deformation in first cycle, bearing steel still survive for millions of contact stress cycles. Therefore, the load required to fail modern high strength bearing steels in one million contact stress cycles lies in elastic-plastic domain of the material loading. Therefore, plastic deformation of the material needs to be accounted for in basic dynamic capacity expressions.

New Strain-life approach is proposed to predict rolling contact fatigue life under elastic-plastic loading condition. Morrow’s strain life equation with mean stress corrections is used for this case. Palmgren-Miner’s cumulative fatigue damage rule is used to account for damage fraction contributions from different loading paths. Unknown material parameters were determined using both point strain amplitudes at critical depth and volume average strain amplitudes over RCF affected area, and basic LP lifes of calibration data set 1. The estimates of Basquin slope and fatigue ductility exponent, from both the approaches, lies in the range which is normally reported for high strength bearing steels. New strain-life equation along with STLE life modification factors can also be used to predict RCF lifes of bearing steels, under service loads, which are commonly observed in industrial and aerospace applications.
Figure 6-1. Assembly of experimental test setup. A) RCF test rig set up. B) Schematic of three-ball-on-rod test setup. The test rod is placed in a brass retainer and spring loaded with three silicon nitride balls. C) Brass retainer with silicon nitride balls. (Photo Courtesy of author)
Figure 6-2. Micro-Vickers hardness measurements. A) Optical micrograph of the Vickers indent on polished face of the VIMVAR M50 virgin sample. B) Optical micrograph of the sample pyramid shape Vickers indent. C) Average Vickers hardness measurements as function of depth from the surface of the polished face. (Photo Courtesy of author)
Figure 6-3. Weibull plot of observed times till failure
Figure 6-4. Optical micrograph of the one of the 7 tracks containing spall. (Photo Courtesy of author)
Figure 6-5. Finite Element Model for Ball-Rod Contact with 1/4\textsuperscript{th} geometries to exploit symmetric boundary conditions.
Figure 6-6. True Stress - True Strain response of bearing steel under elastic and elastic-plastic material behavior
Figure 6-7. Equivalent Plastic strain (PEEQ) in the subsurface region of the steel rod under elastic-plastic loading condition.
Figure 6-8. Logarithm of the ratio of actual life to predicted life for the validation data sets (1-6), using LP model with elastic loading (E), LP model with elastic-plastic loading (EP), Zaretsky model with elastic-plastic loading (EP), Ioannides and Harris model using elastic-plastic loading (EP) and constant fatigue limit ($\sigma_u = c$), Ioannides and Harris model using elastic-plastic loading (EP) and linearly decreasing fatigue limit ($\sigma_u \neq c$).
Figure 6-9. Cyclic loading cycles at critical depth for bearing steel rod in ball-rod test setup
Figure 6-10. RCF affected volume used for determining volume average stresses and strains
Table 6-1. Summary of Experimental data sets selected in present study

<table>
<thead>
<tr>
<th>Data Set No.</th>
<th>Material</th>
<th>Radial Load between ball and rod (N)</th>
<th>rpm</th>
<th>Lubricant</th>
<th>Failure Index</th>
<th>Experimental ( L_{10} ) life of rod (millions of cycles)</th>
<th>Basic LP life (millions of cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>VIMVAR M50 Silicon Nitride</td>
<td>1181.71</td>
<td>3600</td>
<td>Royco 500</td>
<td>7/9</td>
<td>9.273</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>Calibration Data Set</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>VIMVAR M50 AISI 52100 (Grade - 24)</td>
<td>1083.98</td>
<td>3600</td>
<td>Terresic 100</td>
<td>24/24</td>
<td>5.3</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>Validation Data Sets (Glover (49))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>VIMVAR M50 AISI 52100 (Grade - 24)</td>
<td>1083.98</td>
<td>3600</td>
<td>Terresic 100</td>
<td>24/24</td>
<td>4.3</td>
<td>0.84</td>
</tr>
<tr>
<td>3</td>
<td>VIMVAR M50 AISI 52100 (Grade - 24)</td>
<td>1083.98</td>
<td>3600</td>
<td>Terresic 100</td>
<td>8/8</td>
<td>5</td>
<td>0.97</td>
</tr>
<tr>
<td>4</td>
<td>VIMVAR M50 AISI 52100 (Grade - 24)</td>
<td>1083.98</td>
<td>3600</td>
<td>Terresic 100</td>
<td>8/8</td>
<td>5.1</td>
<td>0.99</td>
</tr>
<tr>
<td>5</td>
<td>VIMVAR M50 AISI 52100 (Grade - 24)</td>
<td>1083.98</td>
<td>3600</td>
<td>MIL-L-23699</td>
<td>8/8</td>
<td>4</td>
<td>1.13</td>
</tr>
<tr>
<td>6</td>
<td>VIMVAR M50 AISI 52100 (Grade - 24)</td>
<td>1083.98</td>
<td>3600</td>
<td>MIL-L-23699</td>
<td>8/8</td>
<td>5.6</td>
<td>1.58</td>
</tr>
</tbody>
</table>
Table 6-2. Lubricant \( \Lambda \) ratio for all the data sets given in Table 6-1

<table>
<thead>
<tr>
<th>Data Set No.</th>
<th>( \Lambda ) ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration Data Set 1</td>
<td>0.91</td>
</tr>
<tr>
<td>Validation Data Set 1</td>
<td>0.79</td>
</tr>
<tr>
<td>Validation Data Set 2</td>
<td>0.79</td>
</tr>
<tr>
<td>Validation Data Set 3</td>
<td>0.79</td>
</tr>
<tr>
<td>Validation Data Set 4</td>
<td>0.79</td>
</tr>
<tr>
<td>Validation Data Set 5</td>
<td>0.64</td>
</tr>
<tr>
<td>Validation Data Set 6</td>
<td>0.64</td>
</tr>
</tbody>
</table>
Table 6-3. Contact dimensions, stresses and critical depth under elastic and elastic-plastic loading conditions between ball and rod.

<table>
<thead>
<tr>
<th>Contact Parameters</th>
<th>Elastic Loading</th>
<th>Elastic-Plastic Loading</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact Pressure S (GPa)</td>
<td>6.34</td>
<td>5.51</td>
<td>13.14</td>
</tr>
<tr>
<td>Semi-major axis a (mm)</td>
<td>0.40</td>
<td>0.43</td>
<td>7.32</td>
</tr>
<tr>
<td>Semi-minor axis b (mm)</td>
<td>0.23</td>
<td>0.24</td>
<td>4.44</td>
</tr>
<tr>
<td>Maximum von-mises stress / S</td>
<td>0.61</td>
<td>0.51</td>
<td>16.68</td>
</tr>
<tr>
<td>Maximum mises depth z/b</td>
<td>0.58</td>
<td>0.62</td>
<td>7.53</td>
</tr>
<tr>
<td>Maximum Tresca stress/S</td>
<td>0.32</td>
<td>0.27</td>
<td>16.49</td>
</tr>
<tr>
<td>Maximum Tresca stress depth z/b</td>
<td>0.62</td>
<td>0.62</td>
<td>0.15</td>
</tr>
<tr>
<td>Maximum orthogonal shear stress/S</td>
<td>0.23</td>
<td>0.24</td>
<td>4.24</td>
</tr>
<tr>
<td>Maximum orthogonal shear stress depth z/b</td>
<td>0.42</td>
<td>0.43</td>
<td>0.78</td>
</tr>
</tbody>
</table>
Table 6-4. Calibration Results

<table>
<thead>
<tr>
<th>Proportionality Constant</th>
<th>Estimated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\text{LP, elastic}}$</td>
<td>$1.41 \times 10^{-12} \text{ mm}^{2/3} \text{ GPa}^{-3/3}$</td>
</tr>
<tr>
<td>$K_{\text{LP, elastic–plastic}}$</td>
<td>$3.93 \times 10^{-12} \text{ mm}^{2/3} \text{ GPa}^{-3/3}$</td>
</tr>
<tr>
<td>$K_{\text{IH, elastic–plastic}}$</td>
<td>($\sigma_u = C$) $3.56 \times 10^{-50} \text{ mm}^{2/3} \text{ GPa}^{-3/3}$</td>
</tr>
<tr>
<td>$K_{\text{IH, elastic–plastic}}$</td>
<td>($\sigma_u \neq C$) $1.6 \times 10^{-50} \text{ mm}^{2/3} \text{ GPa}^{-3/3}$</td>
</tr>
<tr>
<td>$K_{Z, elastic–plastic}$</td>
<td>$1.03 \times 10^{-10} \text{ mm}^{-3} \text{ GPa}^{-310/27}$</td>
</tr>
</tbody>
</table>
Table 6-5. Estimates of unknown parameters for Morrow’s Strain-Life equation with mean stress corrections

<table>
<thead>
<tr>
<th>Bearing steel material parameters</th>
<th>Point Strains</th>
<th>Volume average strains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basquin slope (b)</td>
<td>-0.0757</td>
<td>-0.0892</td>
</tr>
<tr>
<td>Fatigue ductility exponent (c)</td>
<td>-0.622</td>
<td>-0.554</td>
</tr>
<tr>
<td>Cyclic strain hardening exponent (n)</td>
<td>0.122</td>
<td>0.161</td>
</tr>
<tr>
<td>Number of cycles to failure (N1)</td>
<td>1637.61</td>
<td>121406.4</td>
</tr>
<tr>
<td>for loading path O-A-B-C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of cycles to failure (N2)</td>
<td>1190726</td>
<td>1190009</td>
</tr>
<tr>
<td>for loading path C-B-C-B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 7
A NEW APPROACH TOWARDS LIFE PREDICTION OF CASE HARDENED BEARING STEELS SUBJECT TO ROLLING CONTACT FATIGUE

Background

Hardened and tempered steels are excellent choices for bearings as their microstructures are designed to maximize the resistance to plastic deformation during intense rolling contact fatigue loading. Carburizing, nitriding and shot peening are the processes commonly used to introduce compressive residual stresses in the near-surface and subsurface regions of bearing steels to reduce the effective fatigue stress amplitude and hence increase the RCF life of bearings. Studies by Ooi and Bhadeshia (25), Braza et al. (26) and Bhattacharyya et al. (56) have shown that these surface-hardened steels exhibit greater resistance to RCF deformation than their through-hardened counterparts. Surface topography of a component also plays a crucial role in fatigue life of a component. Smooth surfaces and absence of surface shear favor subsurface originated spalling. This mechanism is the dominant mode of failure in rolling element bearings that have smooth surfaces and operate under Elastohydrodynamic lubrication (EHL) conditions. Surface originated pitting occurs in cases where lubrication breaks down in presence of surface asperities, dents, and scratches and the bearings operate under mixed lubrication. A numerical RCF model of machine components with mix lubrication condition presented by Epstein et al. (70) predicted shorter fatigue life for components with rougher surfaces.

During RCF loading, the bearing steels pass through a series of microstructural changes which are manifested as dark etching regions (DERs), light etching regions (LERs), white etching bands (WEBs) and white etching areas (WEAs). These microstructural alterations take place along with plastic strain accumulation and these
changes depend on the magnitude of contact stress, temperature, and number of cycles. These microstructural changes are markers of RCF deformation and may qualitatively represent various stages of material degradation. However, a quantitative estimation of RCF life of the bearings require life prediction models which might be either probabilistic or deterministic in nature. As discussed in earlier chapters, Lundberg and Palmgren (8, 9) were the first to provide probabilistic life prediction model that was later adopted into ISO standard for load rating and life of rolling element bearings. Although the LP model worked reasonably well for older generation “dirty” bearing steels ($L_{10}$ life < 2000 hrs), it significantly under predicts the RCF life of modern ultra-clean VIMVAR bearing steels. This under prediction of fatigue life for modern bearing steels occurs due to use of maximum orthogonal shear stress ($\tau_0$) in LP model and the calculation of $\tau_0$ based on elastic Hertzian contact theory which overestimates the stress under elastoplastic loading. In order to overcome some of the limitations of LP model, Ioannides and Harris (15) (IH) proposed a new life prediction model. They divided the RCF affected stressed volume into small discrete regions and then integrated the probability of survival of each region to determine the overall probability of survival of entire volume. A fatigue limit stress ($\sigma_f$) was introduced in their model below which failure is not possible. However, this model also suffers from the limitation of using elastic Hertzian stresses which continues to result in under prediction of RCF life in bearings. Also, the fatigue limit stress ($\sigma_f$) for bearing steels subjected to RCF loading is unknown. In this chapter, experimental and numerical methodology is presented which attempts to correct these limitations to obtain a better RCF life prediction for bearings.
S-N diagrams are frequently used to determine the fatigue limit of a material. A critical parameter to construct the S-N diagram is the stress amplitude at which the material fails due to fatigue loading. Although S-N diagrams are available for steels loaded under uniaxial fatigue loading, no mechanistic method for determination of fully reversed stress amplitude and construction of S-N diagram for bearing steels under RCF loading is available in literature. The cyclic stress–strain response, which represents the evolution of stress under applied strain amplitude in the subsurface under RCF is considerably different from uniaxial high cycle fatigue (HCF) response, due to the presence of triaxial stress state and non-proportional loading with changing planes of maximum shear. Also, the mean compressive stress present in the RCF stress cycle plays crucial role in determining the effective strain amplitude in the material. The plastic strain amplitude during nominal RCF loading being small (\(0 < \Delta \varepsilon_p < 0.003\)) and therefore, the cyclic stress-strain diagrams and S-N diagrams based on uniaxial fatigue loading are not representative of the localized multiaxial cyclic response due to RCF. A methodology to construct cyclic stress-strain response of M50-NiL case-hardened bearing steel is already presented in a recent study by Bhattacharyya et al. (31). However, to construct an effective S-N diagram of bearing steels under RCF, the following critical questions should be considered: (a) what is the appropriate definition of stress amplitude for S-N diagram construction for RCF loading? (b) Is volume average stress amplitude a better representation of RCF than the maximum stress amplitude within RCF affected region? To address this questions, in following sections previously developed experimental methodology is described which allows one to evaluate the elasto-plastic von Mises stresses within the RCF affected
subsurface regions in terms of micro-hardness values (Bhattacharyya et al. (31)). Use of actual elasto-plastic stress instead of Hertzian elastic contact stress will provide a link to material-specific life prediction. By hardness measurements, it is possible to determine the size of the RCF affected zone after millions of cycles which is otherwise impossible to determine. This work discusses results from experimental testing of M50-NiL steel rod, using three-ball-on-rod test setup over several hundred million cycles, to examine the evolution of subsurface elasto-plastic stresses as a function of RCF cycles. Micro-indentation hardness measurements were performed within the plastically deformed subsurface regions to measure the changes in material hardness as a function of RCF cycles. These hardness numbers are converted to elasto-plastic stresses and provide critical information about the size and intensity of the RCF affected zone. Next, by using the 3D elastoplastic stress fields from a finite element model the S-N diagram is constructed for case-carburized M50-NiL steel using maximum von Mises stress amplitude as well as volume average stress amplitude. The fatigue limit stress of M50-NiL based on both the stress amplitudes is determined. The effect of elastic modulus and yield strength variation over the steel case depth on the $L_{10}$ life is also discussed in detail. This approach is expected to provide insight into the design of bearing material microstructure for advanced aerospace applications.

**Material and Microstructure**

The material used is an M50-NiL case carburized steel with 2 mm deep case region. The carbon concentration is 0.8 wt% at the case region and decreases to 0.1 wt.% at the core. The composition of this material is provided in Bhattacharyya et al. (31). The virgin material microstructure is shown along with the hardness distribution in
Fig. 7-1. Note from Fig. 7-1A that the carbide concentration is higher in the case region and gradually decreases in the core region. Such variation in carbide content over the case depth causes the Vickers hardness to decrease from 736±5 HV (kg/mm²) at the depth of 75 μm from the surface to 450±5 HV (kg/mm²) at the depth of 2 mm (core) (Fig. 7-1B). The carbide volume fraction over the case depth as measured by Klecka et al. (29) is provided in Fig. 7-1C along with the hardness numbers at those depths. The microstructure is typical of a hardened and tempered martensitic steel and consists of tempered martensitic matrix with uniformly distributed carbides. The carbides are roughly spherical in shape and are 1 μm in diameter. A typical microstructure of the core region is shown in Fig. 7-1D.

**Experimental Testing**

Ball-on-rod RCF tests are conducted on a 12.7 cm (5 in.) long and 9.525 mm (0.375 in.) diameter M50-NiL case-carburized steel rod using three silicon nitride (Si₃N₄) balls. A schematic of the three-ball-on-rod test is shown in Fig. 6-1. The rod is spring loaded radially by three 12.7 mm (0.5 in.) diameter Si₃N₄ balls. A constant calibrated radial load of 831.4 N (186.9 lbf) is applied between the ball and the rod. A maximum contact pressure 5.5 GPa at the ball-rod contact is computed using an elastic-plastic finite element model. Elastic Hertzian maximum contact pressure is calculated to be 5.79 GPa at the same load. This contact pressure is significantly greater than nominal stress level (2-3 GPa) at which the bearings are typically operated, as these are accelerated RCF tests. The rod is rotated at 3600 rpm, which results in accumulation of 8600 contact cycles per minute. Each test was conducted for either a specified number of cycles or until failure (spallation), which is detected by an accelerometer that stops the test. Once a test is completed, the rod can be advanced and a new test can be
conducted at another location on the rod for different numbers of cycles. Therefore, at the end of the experiments, the rod consists of several RCF tracks on its surface where each track corresponds to a specific number of RCF cycles. In current experiment, load was kept constant and number of cycles were varied between 4300 to 246 million as provided in Table 7-1. Only a single test was conducted for track nos. 1-6 and 10-11 and two tests were conducted for each case between track numbers 7-9. Three tracks spalled after 171.8 million, 228 million and 246 million cycles.

Each RCF track was sectioned longitudinally (along the length of the rod) and radially (or circumferentially) along AA’ and BB’, respectively. Typical optical micrographs of the RCF affected regions in the longitudinal and radial section are shown in Figs. 7-2A and 7-2B, respectively. Note that in longitudinal section, the RCF affected regions are semicircular in shape. In radial section, the RCF affected zone is the shape of a single circular ring. Each RCF track was sectioned, polished and etched with 3% Nital (3 vol % nitric acid+97 vol % ethyl alcohol) to reveal the subsurface regions.

**Micro-indentation Hardness Measurements**

Micro-Vickers indentations were conducted within the RCF affected regions in the longitudinal section of each RCF track as shown schematically in Fig. 7-3A. The indentation load was 100 g and the average diagonal length of the resulting indent in the case region was 15 µm. Therefore, sufficient number of carbides are present beneath the indent and an average hardness over the indentation area is obtained. The first indent was created at 75 µm depth from the surface, and the distance between two indents were 75 µm to eliminate any interaction between indents and free surface. An optical micrograph of the etched longitudinal section revealing semicircular RCF
affected region is shown in Fig. 7-3B. As the etching process creates uneven surface, the micro-indentations are not conducted on etched surfaces. Instead, the etching is removed by polishing and the indentation experiments were conducted on un-etched but polished surface as shown in Fig. 7-3C. Two longitudinal sections were obtained from each track and were used for the subsequent hardness measurements. The hardness numbers are the average values based on the measurements from two longitudinal sections, and hence accounts for the variability of microstructure, fatigue damage formation and evolution.

**Numerical Method**

A numerical model is developed to obtain von Mises stresses at the subsurface material points after the first static contact between the virgin ball and the rod material. While, the measured hardness values after millions of cycles determine the evolved elastoplastic von Mises stress and enables us to determine the size of the RCF affected region, the stresses developed at the first contact or first cycle are considered as initial far-field applied stresses. This model is necessary as it is not practical to conduct an experiment for a single RCF cycle in the current machine which rotates at 3600 rpm.

A 3-dimensional elastic-plastic finite element (FE) contact model of the ball-on-rod RCF test setup was developed to determine subsurface von Mises stress distribution for the initial static loading condition. Quarter symmetry of the ball-on-rod contact is used in the 3D contact simulations, as shown in Fig. 7-4A. Near the center of the contact, a fine mesh discretization is used so that 831.4 N normal load results in peak pressure \( P_{\text{max}} \) of 5.5GPa. Figure 7-4B also reveals the elliptical contact area at the interface whose semi-major axis and semi-minor axis dimensions are 366 μm and
197 μm, respectively. The resulting subsurface von-mises stress distribution for ball-on-rod contact is shown in Fig. 7-4C. The generated maximum von Mises stress is 3006 MPa and occurs at the depth of 114 μm below the rod surface. The maximum orthogonal shear stress is 1331 MPa.

The total strain ($\varepsilon$) has two components elastic strain ($\varepsilon_e$) and plastic strain ($\varepsilon_p$) such that

$$\varepsilon = \varepsilon_e + \varepsilon_p$$  \hspace{1cm} (7-1)

The elastic and plastic behavior is modeled by linear elastic and simple power-law models, respectively as per

$$\sigma = \begin{cases} E\varepsilon & \sigma \leq \sigma_y \\ K\varepsilon^n & \sigma \geq \sigma_y \end{cases}$$  \hspace{1cm} (7-2)

where $\sigma$ is flow stress, $\varepsilon$ is the true strain, $\sigma_y$ is the yield strength, $E$ is elastic modulus, $K$ is strength coefficient, and $n$ is strain-hardening exponent.

The variations of elastic-plastic material properties, $\sigma_y$, $K$ and $E$ as a function of case depth were determined from monotonic compressive stress-strain responses developed by Branch et al. (71). Monotonic stress-strain curve follows power-law hardening behavior (Eq. (7-2)) and are plotted for various case depths in Fig. 7-4D. The strain-hardening exponent $n = 0.056$ remains constant over the entire case layer of M50-NiL steel. Due to gradation in carbon concentration over case layer there exists gradient in material properties, such as elastic modulus and yield strength for M50-NiL steel. Figure 7-4D indicates elastic modulus variation from 228 GPa in the surface region ($E_{Surface}$) to 200 GPa in the core region ($E_{Core}$) over 2 mm case depth. The strength coefficient at each case depth were determined using corresponding variations
in the yield strength from surface to the core region of the M50-NiL steel. In FE model, material properties were defined as a function of temperature. Therefore, depth dependent material properties variation was achieved using predefined linearly varying temperature fields in the case-layer of the rod material. These temperature fields were kept constant during entire analysis as no heat interaction was defined. The elastic modulus of the silicon nitride ball was kept constant at 310 GPa. A total of 152090 quadratic hexahedral elements of the type C3D20T were used to discretize the quarter symmetric ball-on-rod model. The von Mises yield criterion with isotropic hardening behavior is used. This FE simulation was performed using ABAQUS 6.13-2/Standard software on a multiprocessor computer with simulation time of approximately 34 hrs.

Results

The gradation in carbide content over the 2-mm deep case region in M50-NiL steel results in a variation in the hardness over the case depth, as shown in Fig. 7-1B. This is the virgin material hardness before RCF testing. The RCF loading results in the formation of plastically deformed regions at the subsurface, as shown schematically in Fig. 7-2A and 7-2B. No such RCF affected region was observed under optical microscopy until 43000 cycles of contact, and no change in subsurface material hardness could be measured. RCF affected region could only be observed after 13.5 million cycles and first change in hardness was also measured.

The hardness distribution within the RCF affected region formed within the subsurface of the longitudinal section after 13.5 million cycles is shown in Fig. 7-5A. This corresponds to the smallest RCF affected region that was observed among all the tracks in Table 7-1. At each case depth, up to 300 µm from the surface, the hardness is highest at the center of the plastic zone and decreases with increasing distance from
the center and eventually matches the virgin material hardness outside the RCF affected zone. Similar hardness distribution is observed at other case depths, although the extent of hardening decreases with depth. Therefore, maximum hardness at each case-depth is always measured at the centerline of the RCF affected zone (Fig. 7-5A). The maximum hardness within the entire RCF affected region was measured at the depth of 75 µm from the surface. At 375 µm depth beneath the surface, only a small variation (±5 kg/mm²) from the virgin material hardness at that depth was measured. This hardness is considered approximately same as the virgin material hardness at every material point along the entire width of the RCF affected region. Therefore, 375 µm depth from the surface defines the end of the hardened region. Interestingly, a narrow softened region is observed just beneath the hardened region, and also just outside the contact patch (hardened region), at a lower depth from the surface. A detailed description of the hardness profiles and their evolution as a function of RCF cycles is provided in Bhattacharyya et al. (31).

The changes in hardness values from the virgin material hardness are plotted as hardness contours in Fig. 7-5B for 13.5 million cycles. Clearly, the RCF affected region consists of hardened and softened regions. Hardness contour plots showing the formation and growth of the hardened and softened regions within RCF affected regions as a function of fatigue cycles are provided in detail in (Bhattacharyya et al. (72)). It is evident from Fig. 7-7A that, the hardened and softened regions grow bigger in size and the severity of hardening increases with cycles. At sufficiently higher number of cycles, (Fig. 7-7A) the softened zone forms a network around the hardened region. The failure
might occur when the softer regions join and reach surface to form spall on the RCF track.

**Discussion**

The RCF affected plastically deformed region does not nucleate at the surface, but initiates at the subsurface region according to Hertzian contact theory. Plastic strain accumulates due to the action of subsurface stresses generated due to Hertzian contact and results in hardening and softening within the RCF affected region as shown in Fig. 7-5.

The reasons for hardening and softening have been rationalized based on Hertzian contact theory. It was proposed that the triaxial released compression (compression-zero) fatigue causes hardening of material and uniaxial fatigue with evolved tension causes softening of material. Therefore, within hardened region all three principal stress components are compressive during loading, i.e., $\sigma_1 < 0, \sigma_2 < 0$ and $\sigma_3 < 0$. But the principal stresses relax to zero during unloading. Therefore, the region which exhibits significant hardening is subjected to *triaxial released compression* and therefore has compressive mean stress component in the stress cycle. The effective stress at various material points within the hardened region can be determined as equivalent von Mises stress ($S_{VM}$) which is same as the stress range ($\Delta S_{VM}$) and will be used in subsequent sections to construct S-N diagram. This von Mises stress is compressive in nature and has a compressive mean stress ($S_m = \Delta S_{VM} / 2$). Therefore, equivalent fully-reversed alternating stress amplitude is determined using Goodman’s relation as,
\[ \Delta S_{VM}^{FR} / 2 = S_a / \left(1 - \frac{S_m}{S_u}\right) \]  

(7-3)

where, \( \Delta S_{VM}^{FR} / 2 \) is the equivalent fully reversed alternating stress amplitude after mean stress correction (i.e. at zero mean stress), and \( S_m \) is the mean stress of a fatigue stress cycle given by average of maximum and minimum stress in that cycle

\[ = S_{\text{max}} / 2 = \Delta S_{VM} / 2 \] for compression-zero fatigue cycle. \( S_a \) is the alternating stress with respect to the mean stress = \( S_{\text{max}} / 2 = \Delta S_{VM} / 2 = S_m \) for compression-zero fatigue cycle, and \( S_u \) is the ultimate strength of the material. In this study, the ultimate strength to yield strength ratio has been assumed to be 1.21 which is reasonable for hard bearing steels. Therefore, the ultimate strength \( S_u = 3500 \text{ MPa} \) which corresponds to 8% monotonic true strain based on Fig. 7-4D.

**Global Stress-life (\( S_u - N_f \)) Approach for M50-NiL:**

\( S_u - N_f \) diagrams are commonly used to estimate the fatigue limits of structural alloys under uniaxial fatigue loading condition, where \( S_u \) is the alternating stress and \( N_f \) is the cycles to failure. In this section, two different approaches are introduced to construct S-N diagram of M50-NiL bearing steel subjected to RCF. The first approach is based on the maximum von Mises stress amplitude within the RCF affected region and the second approach considers the volume average von Mises stress within the RCF affected region to determine the volume average stress amplitude.

**Stress-Life (\( S_u - N_f \))** diagram based on maximum von Mises stress amplitude:

Due to the complex multiaxial stress state experienced by a highly localized subsurface material, effective \( S_u - N_f \) diagrams for RCF loading has only been constructed
recently (Bhattacharyya et al. (31)) based on the maximum von Mises stress amplitude. According to the 3D FE model, the far-field applied strain amplitude at the location of maximum von Mises stress is predominantly elastic, \((\Delta \varepsilon_p = 0.012, \Delta \varepsilon_e \approx 0.005)\).

Therefore, stress-life \((S_a - N_f)\) approach may serve as an appropriate description of global RCF life of a bearing steel.

It is assumed that the material fails in the first-half cycle when the \(\Delta S_{VM}/2\) reaches ultimate strength of the material. The maximum hardness was measured at the central point at 75 µm depth. However, the highest von Mises stress is expected at a depth of 114 µm as observed from the FEA shown in Fig. 7-4C. This discrepancy is due to the fact that the hardness numbers were measured at an interval of 75 µm which did not provide spatial resolution for picking the exact location of the point of maximum hardening from microhardness measurements. The maximum hardness can occur at a depth between 75 µm and 150 µm. Hardness measurements at a smaller interval (less than 75 µm) will provide more spatial resolution and will reveal more accurate location of the highest hardened material point within the RCF affected region. However, in this analysis it is considered that the material point at 114 µm depth as most critical point since the von Mises stress level is highest. From the FE model, the computed maximum von Mises stress at 114 µm depth is \(S_{VM} = 3006\) MPa. The equivalent fully-reversed far-field alternating stress amplitude \((\Delta S_{VM}^{FR}/2)\) is 1051 MPa at this point at \(P_{max} = 5.5\) GPa. The magnitude of fully reversed von Mises stress amplitude \((\Delta S_{VM}^{FR}/2 = 0.19\ P_{max})\) is smaller than the maximum orthogonal shear stress amplitude \((\Delta \tau_\alpha/2 = 0.24\ P_{max})\), which is determined from the 3D FE model to be 1331 MPa. Experimental results in Table 7-1
indicate that three RCF tracks of M50-NiL steel spalled after 171.8, 228 and 246 million cycles when loaded at $P_{\text{max}} = 5.5$ GPa. Further note that the material fails in the first-half cycle, when the $\Delta S_{\text{VM}}^{\text{max}} / 2$ reaches the ultimate strength (3500 MPa) of the material. Based on these four data points, the $S_\alpha - N_f$ diagram was constructed and is shown in Fig. 7-6. Fitting a power-law equation through the data points yields the stress-life equation for M50-NiL steel as

$$\frac{\Delta S_{\text{VM}}}{2} = 3500(2N_f)^{-0.061}$$

(7-4)

The exponent in Eq. (7-4) is known as Basquin slope, $b$ and its magnitude is similar to hard steels. The corresponding stress-life exponent, is defined as $n = 1/b$ is 16.39, which is significantly higher than the stress-life exponent of 9 proposed by Lundberg and Palmgren (8, 9) using orthogonal shear stress and more recent stress life exponent of 12 as proposed by Parker and Zaretsky (37) for vacuum processed post 1960 bearing steels. A higher stress exponent suggests a greater fatigue life of a bearing. Therefore, the S-N diagram based on equivalent fully reversed von Mises stress predicts higher life of a bearing than LP theory. The lower magnitude of $\Delta S_{\text{VM}}^{\text{FR}} / 2$ than the orthogonal shear stress is responsible for the higher life predicted by von Mises criteria and reflects the beneficial effect of compressive mean stress on the RCF life of bearing steels. Due to high sensitivity of fatigue life to applied stresses, even subtle variations in stress-field can lead to significant alterations in bearing material's fatigue performance. Equation (7-4) and Fig. 7-6 can be used to predict median fatigue life of M50-NiL bearing steel under different alternating stress amplitudes. For example, for a nominally loaded typical aerospace bearing subjected to a peak Hertzian stress of $P_{\text{max}} =$
2.8 GPa (~ 400 ksi), the resulting far-field amplitude of the released-compression stress cycle at the critical point can be obtained from elastic contact stress theory (Harris (1)) to be \( S_{VM} = 0.614 P_{\max} \) and the corresponding mean stress is, \( S_m = 0.307 P_{\max} \). Based on these values, the equivalent fully-reversed \( \Delta S_{VM}/2 \) at that material point loaded at \( P_{\max} = 2.8 \) GPa was determined from Eq. (7-3) to be 690 MPa. The corresponding \( N_f \) is obtained from Eq. (7-4) to be \( 3.6 \times 10^{11} \) (360 billion) cycles. Therefore, at nominal Hertzian stress, this material is able to survive very high number of RCF cycles.

A 218 Angular contact ball bearing rotating at 15,000 rpm with 16 rolling elements, inner raceway contact angle of 51 deg., bearing pitch diameter of 125.26 mm and thrust load of 10,000 lb is typically operated for 100,000 hours. The methodology to calculate the number of stress cycles corresponding to this service life is provided by Harris (1). Based on these calculations, the corresponding number of stress cycles over this service life are \( N_f = 8 \times 10^{11} \) cycles at the inner raceway contact. The corresponding \( \Delta S_{VM}/2 \) obtained from Eq. (7-4) is 630.4 MPa. In this work, RCF limit of a bearing steel is defined at \( 10^{12} \) cycles, which is close to the life calculated for 218 Angular contact ball bearing. The corresponding \( \Delta S_{VM}/2 \) obtained from Eq. (7-4) is 622 MPa. Thus, the effective von Mises stress amplitude of 622 MPa can be considered to represent the RCF fatigue strength or limit \( (S_f) \) of M50-NiL, based on maximum von Mises stress criteria. The corresponding maximum Hertzian stress is, \( P_{\max} \approx 2.46 \) GPa (357 ksi). It must be noted here that loading will be globally elastic at this \( P_{\max} \).

Stress-Life \( (S_{\sigma} - N_f) \) Diagram based on volume-averaged von Mises stress amplitude: Bearings fail due to spall formed over considerable volume of the material,
rather than the localized failure assumed in $S_a - N_f$ diagram. Therefore, a stress estimate representative of the entire RCF affected region may be used to construct $S_a - N_f$ diagram for RCF loading. Note that the RCF affected region is small in size (several hundred microns in width and depth) and stress state is multi-axial unlike uniaxial high-cycle fatigue test specimens. The von Mises stress being different at different material points within RCF affected region, a volume average von Mises stress is used for construction of $S_a - N_f$ diagram in this case.

The plastically deformed zone at the first cycle (from the 3D FE model) is not the true representation of RCF affected region for determining far-field (applied) volume average von Mises stress. The stressed region formed after the first contact cycle grows over millions of RCF cycles until failure occurs. Therefore, the size of the RCF affected region is larger and is assumed to be same as the size of the fully grown plastic zones formed after 171.8, 228 and 246 million cycles. Then, the magnitude of the volume average von Mises stress within this larger stressed volume will be determined from the 3D FE model generated von Mises stresses shown in Fig. 7-4C.

To determine the size of RCF affected volume for S-N diagram construction, only the hardened region within the fully grown plastically deformed zone formed within a spalled track is considered. The subsurface hardness contour plot for spalled RCF track after 246 million cycle is shown in Fig. 7-7A. Note that both the hardened and softened regions are bigger than those developed after 13.5 million cycles (Fig. 7-5B). The softened regions after 246 million cycles form surrounding the hardened zone in the form a network. The hardened region formed after 246 million cycles is 800 μm wide and 435 μm deep within the subsurface. This $800 \mu m \times 435 \mu m$ section
is used to obtain volume average von Mises stress. The 3D FE model generated section which is used to compute the volume average von Mises stress is shown in Fig. 7-7B. The volume average far-field von Mises stress is computed by dividing the entire hardened region into \( N \) number of discrete elements and integrating the von Mises stress contributions from each element as shown in Fig. 7-7B and then averaging them over the entire RCF affected volume according to,

\[
\sigma_{\text{VM}}^{\text{av}} = \frac{\sum_{k=1}^{N} V_k \times (\sigma_{\text{VM}})_k}{\sum_{k=1}^{N} V_k}
\]

(7-5)

where \((\sigma_{\text{VM}})_k\) is the von Mises stress at the centroid of each discrete volume element with corresponding volume \(V_k\). The entire RCF affected region is comprised of volume elements \(k = 1\) to \(N\).

Based on Eq. (7-5), the volume average von Mises stress is 1910 MPa which is considerably smaller than the maximum von Mises stress which is 3006 MPa. Note that, the volume average von Mises stress is even smaller than the average initial (virgin) yield strength of the 435 μm deep case layer which is 2798 MPa. This average virgin material yield strength was determined by averaging the experimentally measured virgin material hardness \((H)\) values (Fig. 7-1B) and then by converting them to yield stress \((\sigma_y)\) by using Tabor’s rule expressed as \(\sigma_y = H/C\). Here \(C\) is the constraint factor of a material and its magnitude for M50-NiL steel was determined by Klecka et al. (29) to be 2.5. Therefore, although the subsurface case-material is subjected to a volume average stress amplitude smaller than its average yield strength, plastic strain accumulates within the subsurface after several hundred million cycles of RCF loading.
The volume average ultimate strength of the 800 μm × 435 μm RCF affected (hardened) region was determined assuming the volume average ultimate strength to average virgin material yield strength (2798 MPa) ratio of 1.21. This ratio can be assumed to remain constant because strain-hardening exponent is constant for the entire case-layer (Fig. 7-4D). The resulting volume average ultimate strength = 3388 MPa. The fully reversed stress amplitude (ΔS_{VM}/2) based on volume average von Mises stress was calculated from Eq. (7-3) to be 745 MPa. Furthermore, the material fails in first half cycle when the fully reversed volume average stress becomes equal to the average ultimate strength of the entire RCF affected region. Therefore, the corresponding power-law equation becomes,

\[ \frac{\Delta S_{VM}}{2} = 3388 (2N_f)^{-0.076} \]  

(7-6)

The S_{a} - N_ f diagrams based on the volume average von Mises stress as well as the maximum von Mises stress are plotted in Fig. 7-6. Note that the magnitude of Basquin slope (exponent of the power law equation) for volume averaged S_{a} - N_ f curve is greater than the maximum von Mises curve. Therefore, the S_{a} - N_ f curve based on maximum von Mises stress predicts higher fatigue strength longer fatigue life of the material than the volume averaged S_{a} - N_ f curve. The stress-life exponent from Eq. (7-6) is \(1/0.076 = 13.16\), which is greater than the stress life exponent of 9 (LP model) but is consistent with the value of 13.5 recently reported by Londhe et al. (38) for modern case-hardened bearing steels. Based on Eq. (7-6), the fatigue strength corresponding to \(N_f = 10^{12}\) cycles result in \(S_f = 394\) MPa which is lower than the fatigue strength based on maximum von Mises stress criteria. The lower magnitude of volume average fatigue
limit suggests that, the RCF crack not necessarily initiate at the highest stressed material point. Crack initiation may occur at any material point within the RCF affected region and hence accounts for the variability in the microstructure and statistical variation in strength due to the presence of defects such as inclusions, inhomogeneities, carbides etc. Clearly, the $S_a - N_f$ curve based on maximum von Mises stress overestimates the RCF strength and life of bearing steels.

**Effect of Gradation in Elastic Modulus and Hardness on RCF life of Carburized Bearing Steels**

To study the influence of gradation in material properties on expected RCF life of case-hardened steels and to compare that with a through-hardened steel, an additional ball-on-rod FE model was developed with uniform homogeneous material properties for the rod in the subsurface region. The ball-on-rod contact was simulated at the same load as in the case-hardened model. This homogeneous material assumption for the rod is equivalent to that of through-hardened steel, in which a uniform elastic modulus of 228 GPa and a uniform yield strength of 2.94 GPa were used. This yield strength is the surface yield strength of the M50-NiL used in the current study. Similar elastic modulus and yield strength are observed in M50 steel variants and are same as the elastic modulus of the surface region of M50-NiL case-carburized steel.

Based on this homogeneous model, the peak contact pressure of the through-hardened material is 5.58 GPa which is about 1.42% higher than 5.5 GPa obtained from the case-hardened model (Fig. 7-4). The maximum von Mises stress occurs at the depth of 114 μm and the magnitude is 3085 MPa, as compared to 3006 MPa for the case-hardened M50-NiL. The maximum orthogonal shear stress ($S_{xy}$) occurs at a depth of 115 μm and the magnitude is 1348 MPa for through-hardened steel as compared to
1331 MPa for case-hardened steels. The FE contours of the orthogonal shear stress distribution for both the models are shown in Fig. 7-8. Furthermore, for through-hardened steel, the vertical (Y-direction) deformation at the center of the contact is 9.345 µm, which is 2.76% smaller compared to 9.603 µm observed for the case-hardened material. This larger deformation at the center of the contact, explains the 1.42% decrease in peak contact pressure experienced by the case-hardened steels. Therefore, the appropriate design of microstructure by case-hardening can decrease the magnitude of Hertzian contact stress and von Mises stress within the RCF affected material volume and improve bearing’s RCF life.

In the following section, $L_{10}$ life of case-hardened and through bearings is compared using peak contact pressure and maximum orthogonal shear stress. First, the peak contact pressures obtained for through-hardened steel and case-hardened steel are used to determine expected improvement in fatigue life for graded materials using following approach,

The peak contact pressure, $P_{\text{max}}$, experienced by the ball-rod contact under normal load $Q$, are related according to the Hertz theory as:

$$P_{\text{max}} = \alpha (Q)^{1/3} \tag{7-7}$$

Using Lundberg-Palmgren (LP) theory (8, 9), $L_{10}$ fatigue life of the contact can be related to normal load $(Q)$ experienced by the contact as:

$$L_{10} = \alpha \left(\frac{1}{Q}\right)^p \tag{7-8}$$

where $p$ is the load-life exponent, whose value can be 3 and 4 based on Lundberg-Palmgren model and Zaretsky’s model, respectively for point contact loading. Recently,
based on Harris and McCool’s extensive RCF data, Londhe et al. (38) determined the load-life exponent to be 4.1 for point contact loading. Combining Eqs. (7-7) and (7-8), we can determine relationship between peak contact stress \( P_{\text{max}} \) and the \( L_{10} \) fatigue life of the contact as:

\[
L_{10} \propto \left( \frac{1}{P_{\text{max}}} \right)^n
\]  

(7-9)

where, \( n = 3p \) is stress-life exponent, which can be 9, 12 or 12.3 based on \( p=3,4 \) or 4.1, respectively. The Eq. (7-9) can be used to compare peak contact pressures observed in ball-on-rod model (Fig. 7-4) for case-hardened and through-hardened rod materials, to predict improved fatigue performance of graded materials. Let \( P_{\text{max}}^{CH} \) and \( P_{\text{max}}^{TH} \) be the peak contact pressure for case-hardened and through-hardened materials, respectively. Therefore, ratios of fatigue lives of case-hardened steel \( (L_{10})_{CH} \) to that of through hardened steel \( (L_{10})_{TH} \) can be determined using Eq. (7-9) as:

\[
\frac{(L_{10})_{CH}}{(L_{10})_{TH}} = \left( \frac{P_{\text{max}}^{TH}}{P_{\text{max}}^{CH}} \right)^n
\]  

(7-10)

As discussed earlier, under constant load of 831.4 N, about 1.42% increase in peak contact pressure was observed for through-hardened material \( (S_{CH} = 5.58 \text{GPa}) \) as compared to case-hardened M50-NiL \( (S_{CH} = 5.5 \text{GPa}) \). Due to this drop in peak contact pressure from \( P_{\text{max}}^{TH} \) (5.58 GPa) to \( P_{\text{max}}^{CH} \) (5.5 GPa), the expected improvement in fatigue life of case-hardened steel compared to through hardened counterpart is 13.88% from Eq. (7-10) with \( p=3 \), i.e. \( n=9 \). It can be noted that higher stress-life exponent of 12 and
12.3 will predict even higher i.e. 18.92% and 19.43% improvement in fatigue lives for carburized steels, respectively.

A similar analysis can be conducted for both through-hardened and case-hardened steels based on the variation in maximum orthogonal shear stress \( S_{xy} \) within the RCF affected region. Based on LP model, the probability of survival \( \hat{S} \) of a subsurface volume \( V \), subjected to orthogonal shear stress \( S_{xy} \) at a depth of \( z_o \), for \( N \) number of cycles can be give as:

\[
\ln \frac{1}{\hat{S}} \propto S_{xy}^c \frac{N^e}{z_o^h} V \tag{7-11}
\]

where, \( c, e \) and \( h \) are stress-life, Weibull slope and critical-depth exponents, respectively. For a constant probability of survival, Eq. (7-11) can be rearranged as:

\[
L = N \propto \left( \frac{1}{S_{xy}} \right)^{\frac{1}{e}} \left( \frac{1}{V} \right)^{\frac{1}{e}} \left( z_o \right)^{\frac{h}{e}} \tag{7-12}
\]

In LP model (Eq. (7-11)), the subsurface volume subjected to rolling contact fatigue was expressed as:

\[
V = a\zeta z_o \tag{7-13}
\]

where, \( a \) is semi-major axis of the ellipsoidal contact area, \( \zeta \) circumference of the rod and \( z_o \) is the subsurface depth at which critical orthogonal shear stress \( S_{xy} \) is observed. Equation (7-12) can also be used to compare fatigue lives of case-hardened steel and through-hardened steels. As can be seen from above results, the variations in contact dimensions and subsurface depth at which critical orthogonal shear stress is observed are negligible in case-hardened and through-hardened material, hence they
can be assumed to remain constant. Therefore, Eq. (7-12) can be used to compare fatigue lives of case-hardened (CH) steel and through-hardened (TH) steel as:

\[
\frac{L_{10\text{CH}}}{L_{10\text{TH}}} = \left( \frac{S_{XY\text{TH}}}{S_{XY\text{CH}}} \right)^c
\]

The values of exponent \(c\) and \(h\) for M50-NiL and M50 steels are not available in the published literature. Therefore, in this work \(c = 31/3\) and \(e = 10/9\) as proposed by Lundberg and Palmgren (8, 9) will be used for fatigue life comparison of case-hardened and through-hardened steels. Substituting, \((S_{XY\text{CH}}) = 1331\) MPa and \((S_{XY\text{TH}}) = 1348\) MPa in Eq. (7-14), case-hardened steels are expected to have 12.53% more fatigue life than through-hardened counter parts under identical geometry and loading conditions. Based on peak contact pressure approach the expected improvement in bearing fatigue life of case-hardened steels can be up to 19.43%. Therefore, this analysis confirms the fact that LP model under predicts the improved rolling contact fatigue performance of case-hardened steels compared to through-hardened steels.

A similar comparison of \(L_{10}\) life prediction can be conducted based on von Mises stress theory. Ioannides and Harris divided the RCF affected volume into discrete regions and integrated the contribution from each region to predict the RCF life. Their model can be improved by using elastoplastic von Mises stresses (obtained from micro-indentation hardness measurements).

It must be finally mentioned that, in this configuration, if we assume similar surface properties for through-hardened and case-hardened steels, the expected improvement in bearing fatigue life due to gradation in material properties is only up to 19% because of small dimensions of the ball-on-rod contact area arising due to non-
conformal geometry of ball and rod contact. It is shown by Londhe et al. (73) that the gradation effects are felt more at the surface for ball bearings with larger contact patch dimensions. Due to conforming raceway groove (concave surface), the length of semi-major axis for typical ball-raceway contact, under nominal loading conditions, can be up to 1 mm to 2 mm. For such wider contact patch, the variation in peak contact pressure and subsurface stresses due to gradation in material properties in the subsurface region is significantly higher. Also, a steeper gradation in elastic modulus and yield strength along with the presence of compressive residual stress results in even higher improvement in fatigue lives for case-carburized steels under identical geometrical and loading conditions. Therefore, case-carburized steels are expected to outperform through-hardened steels under similar rolling contact fatigue loading conditions.

**Summary**

Three ball-on-rod tests were conducted for several hundred million cycles at 5.5 GPa Hertzian contact stress to monitor the progressive evolution of plasticity due to RCF. Based on micro-hardness data and 3D FE model a methodology is provided to construct stress-life ($S_a - N_f$) diagram of M50-NiL case hardened bearing steel subjected to RCF loading. Micro-indentation hardness measurements reveal significant increase in material hardness within the plastically deformed subsurface region. The $S_a - N_f$ diagram for M50-NiL steel is constructed for the first time based on both the maximum von Mises stress amplitude as well as the volume average stress amplitude within the RCF affected volume. The fatigue limit of M50-NiL is estimated at $10^{12}$ cycles to be 622 MPa and 394 MPa based on the maximum von Mises stress and volume average von Mises stress, respectively. The gradation in material properties and
presence of residual compressive stresses can improve the RCF life of a material. A case-carburized material (such as M50-NiL, P-675 steel etc.) with gradually decreasing elastic modulus and yield strength from case to core region exhibits greater life than the uniform through-hardened material (such as M50, AISI 52100 steel etc.). It was shown that LP model provides conservative prediction of RCF life. The degree of life improvement depends significantly on the contact geometry and the magnitude of gradation in material properties. The obtained results help to develop a robust mechanistic life prediction model for bearing steels subjected to RCF.
Figure 7-1. Microstructural attributes of M50-NiL case-hardened steels. A) The alloy carbides are shown in the inset as white phase. AFM image of the white carbides in the inset reveals their spherical shape. B) Hardness distribution as a function of case-depth. Note that the case depth is around 2 mm. C) The variation of carbide volume fraction and hardness as a function of case-depth. D) Optical image of microstructure of the core region of M50-NiL steel. (Photo Courtesy of Abir Bhattacharyya. Source: Bhattacharyya et al. (48)).
Figure 7-2. Micrographs of RCF affected region. A) RCF-affected region in the longitudinal section. B) RCF-affected region in the radial section. Note that the RCF-affected regions form within subsurface. (Photo Courtesy of Abir Bhattacharyya. Source: Bhattacharyya et al. (48)).
Figure 7-3. Indentation map for hardness measurements. A) Schematic of RCF affected region in the longitudinal section and grid of Vickers micro-indents within the RCF affected region. B) An optical micrograph of RCF affected zone formed in the longitudinal section after 13.5 million cycles. C) Optical micrograph of micro-indents within the RCF affected region in the longitudinal section. The indents provide the hardness values at various material points which are representative of von Mises stresses at those material points. (Photo Courtesy of Abir Bhattacharyya. Source: Bhattacharyya et al. (48)).
Figure 7-4. Results from numerical study. A) 3D FE contact model of a quarter section of an M50-NiL rod and Silicon Nitride (Si₃N₄) ball. B) The contact pressure distribution on the surface of the rod. C) Subsurface von-mises stress distribution in XY and YZ planes of the rod. D) The monotonic stress-strain graphs at various case-depths and the core region of the M50-NiL steel rod. Note: the strain hardening exponent at all case-depths is 0.056. Linear gradation of elastic modulus from surface to core region across M50-NiL rod cross section is also shown.
Figure 7-5. Micro-indentation hardness measurements. A) Hardness distribution within RCF affected region after 13.5 million cycles of RCF. Hardened and softened regions are observed. Dashed vertical line shows the centerline of the RCF affected region. Within hardened region, at each depth, the hardness is highest at the centerline and reduces to virgin material hardness with increasing distance from the centerline. B) Contour plots showing changes in hardness from the virgin material hardness for 13.5 million cycles Bhattacharyya et al. (48).
Figure 7-6. S-N diagrams of M50 NiL steel. The dashed line represents the S-N response based on maximum von Mises stress amplitude within the RCF affected region. The solid line is the S-N response based on volume average stress amplitude of RCF affected region. RCF limit is defined at $N_f = 10^{12}$ cycles Bhattacharyya et al. (48).
Figure 7-7. Volume average stress and strain determination. A) Fully grown RCF affected region after 246 million cycles. The dashed region is the hardened zone of dimension 800 μm (width) × 435 μm (depth). B) Selected elements in RCF affected region for volume average stress and strains determination. Note that only half of the width is shown due to quarter model. Entire hardened region is divided into multiple discrete elements shown by the red grid lines. The volume average von Mises strain and volume average plastic strain over the cross section is 1910 MPa and 0.00047 Bhattacharyya et al. (48).
Figure 7-8. Subsurface orthogonal shear stress $S_{xy}$ distribution in Ball-on-rod model. A) For case-hardened steel. B) For through-hardened steel.
Table 7-1. Number of test cycles for each RCF track. M stands for millions. Note that tracks 9, 10 and 11 spalled due to RCF

<table>
<thead>
<tr>
<th>Track #</th>
<th>RCF loading cycles</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4300</td>
<td>Did not spall</td>
</tr>
<tr>
<td>2</td>
<td>8600</td>
<td>Did not spall</td>
</tr>
<tr>
<td>3</td>
<td>17200</td>
<td>Did not spall</td>
</tr>
<tr>
<td>4</td>
<td>43000</td>
<td>Did not spall</td>
</tr>
<tr>
<td>5</td>
<td>13.5 M</td>
<td>Did not spall</td>
</tr>
<tr>
<td>6</td>
<td>27.5 M</td>
<td>Did not spall</td>
</tr>
<tr>
<td>7</td>
<td>38.7 M</td>
<td>Did not spall</td>
</tr>
<tr>
<td>8</td>
<td>158.4 M</td>
<td>Did not spall</td>
</tr>
<tr>
<td>9</td>
<td>171.8 M</td>
<td>Spalled</td>
</tr>
<tr>
<td>10</td>
<td>228 M</td>
<td>Spalled</td>
</tr>
<tr>
<td>11</td>
<td>246 M</td>
<td>Spalled</td>
</tr>
</tbody>
</table>
In current investigation, detailed study of rolling contact fatigue performance of case hardened and through hardened bearing steels is performed. Influence of variations in material properties on Hertz contact stress fields is analyzed for carburized bearing steels. Using three dimensional and axisymmetric finite element models, contact stress fields are studied for different gradations in elastic modulus over different case depths and under different loads. Data generated from 33 different simulations was used to train nonlinear surrogates to predict effective elastic modulus as function of design parameters of ball raceway contact. This effective elastic modulus can be used in Hertz equations to predict contact stresses for elastically graded case carburized steels. Influence of residual compressive stresses in presence of graded material properties is also analyzed. It is estimated that case carburized steels will have significantly higher fatigue lives than through hardened bearing steels under identical operating conditions.

Using three ball rod test bench, accelerated rolling contact fatigue tests were conducted on VIMVAR M50 steel rod. It was observed that even after significant plastic deformation in first cycle, material still survives for more than 1 million contact stress cycles. Therefore, expressions for dynamic capacity needs to be updated to account for influence of metal plasticity and elastic-plastic stresses. Influence of operating conditions such as material hardness, lubricant film thickness and surface roughness on observed fatigue life of steel rod is analyzed. Calibration and validation study of traditional rolling contact fatigue life models is performed using current experimental data and results available in published literature. From finite element simulations, it was
determined that orthogonal shear stress is not affected in presence of metal plasticity. Hence, Zaretsky model with Tresca shear stress criterion and Ioannides – Harris model with von-mises stress criterion are more accurate in predicting observed fatigue lifes compared to Lundberg – Palmgren model. Moreover, life predictions using elastic stresses only, results in significant deviation between predicted and observed life. New strain life equation is proposed for rolling contact life prediction using Morrow’s mean stress corrections. The estimated values of Basquin slope and fatigue ductility exponent for bearing steels are in good agreement with the values commonly reported for metals. This strain-life equation with life modification factors as per STLE design guide can be used to predict rolling contact fatigue lives even under elastic loading conditions.
APPENDIX
SENSITIVITY ANALYSIS

To demonstrate the influence of elastic modulus variations on the bearing fatigue lives a 209 single row deep groove ball bearing example from Londhe (39) will be used. Detailed information about its component geometries and material properties is provided in Table 4-5. Using standard bearing macro geometry relations we can determine the osculation values for the inner and outer raceways as:

\[
\begin{align*}
 f_i &= \frac{r_i}{D} = 0.52; \quad \phi_i = \frac{1}{2f_i} = 0.962; \\
 f_o &= \frac{r_o}{D} = 0.52; \quad \phi_o = \frac{1}{2f_o} = 0.962
\end{align*}
\]

For inner raceway – ball contact, curvature sum and curvature difference were calculated using principal radii of curvatures for two bodies \( \rho_{II} = \rho_{2I} = \frac{2}{D} \),

\[
\begin{align*}
 \rho_{III} &= \frac{2}{d_i} \text{ and } \rho_{2II} = \frac{1}{f_i D} \text{ respectively:} \\
 \sum \rho_i &= \rho_{II} + \rho_{2I} + \rho_{1II} + \rho_{2II} = 0.2018 \ m m^{-1} \\
 F(\rho)_i &= \frac{(\rho_{II} - \rho_{2I}) + (\rho_{1II} - \rho_{2II})}{\sum \rho} = 0.93997
\end{align*}
\]

Similarly for outer raceway - ball contact, curvature sum and curvature difference can be calculated using Eq. (A-1) as: \( \sum \rho_o = 0.1378 \ m m^{-1}; \ F(\rho)_o = 0.91209. \)

After determining the geometrical properties of the bearing, contact stresses and deformations can be determined.

**Case A: Elastic Modulus of Raceway Material = 200 GPa**

To determine ball - raceway contact dimensions, MATLAB subroutines were developed to solve for complete elliptical integrals of first and second kind, and load distribution integrals (Harris (1)). Maximum load experienced by ball – raceway contact
was found to be \( Q_{\text{max}} = 4527.88 \text{ N} \). At this load inner and outer raceway contact dimensions were determined to be
\[
a_i = 2.591 \text{ mm}, \quad b_i = 0.2772 \text{ mm}, \quad a_o = 2.5054 \text{ mm}, \quad b_o = 0.3424 \text{ mm}
\]
Using traditional Hertz solutions, peak compressive stress experienced by inner and outer raceways were determined as \( S_{\text{max}i} = 3011.59 \text{ MPa} \) and \( S_{\text{max}o} = 2521.42 \text{ MPa} \).

Basic Dynamic capacity for inner and outer raceways were determined using following equations (Harris (1)):
\[
C = 93.2 \left[ \frac{2f}{2f - 1} \right]^{0.41} \left( \frac{1 + \gamma}{1 - \gamma} \right)^{1.39} \left[ \frac{\gamma}{\cos \alpha} \right]^{0.3} D^{1.8} \left( \frac{Z}{1} \right)^{-\frac{1}{3}}
\]
where, \( \gamma = \frac{D \cos \alpha}{d_m} \). Using Eq. (A-2), basic dynamic capacity for inner and outer raceway contact was found to be \( Q_{ci} = 7054.95 \text{ N} \) and \( Q_{co} = 13958.14 \text{ N} \), respectively. Cubic mean equivalent radial load (Eq. (1-5)) experienced by rotating inner and outer raceways is \( Q_{ei} = 2489.76 \text{ N} \) and \( Q_{eo} = 2604.82 \text{ N} \), respectively. In this analysis LP model is used to compare fatigue lives obtained from elastic modulus variations.

\[
L = \left[ \frac{Q}{Q_e} \right]^3
\]

Therefore, based on dynamic capacity and equivalent radial load we can calculate fatigue lives for both inner and outer raceway using Eq. (A-3) as,
\[
L_i = \left[ \frac{Q_{ci}}{Q_{ei}} \right]^3 = 22.75 \text{ million cycles}; \quad L_o = \left[ \frac{Q_{co}}{Q_{eo}} \right]^3 = 153.87 \text{ million cycles}
\]

Combined life of bearing can be obtained from the following equation (i.e. Eq. (2-29))
\[ L_{10} = (L_i^{-1.11} + L_o^{-1.11})^{-0.9} = 20.485 \text{ million cycles} = 189.672 \text{ hrs} \]  

(A-4)

**Case B: Elastic Modulus of Raceway Material = 180 GPa**

Now let us assume that elastic modulus of the bearing raceway material is decreased by 10% from 200GPa to 180GPa. Also it will be assumed that Poisson’s ratio of the material and elastic modulus of the ball remains constant. Using MATLAB subroutine, maximum load \( (Q_{\text{max}}) \) experienced by ball – raceway contact was determined to be 4521.44 N. At this load, inner and outer contact dimensions were found to be:

- \( a_i = 2.6364 \text{ mm}, \ b_i = 0.2821 \text{ mm}, \ a_o = 2.5497 \text{ mm}, \ b_o = 0.3485 \text{ mm} \)

Peak compressive Hertz stress experienced by inner and outer raceways were \( S_{\text{maxi}} = 2904.18 \text{ MPa} \) and \( S_{\text{maxo}} = 2430.79 \text{ MPa} \), respectively. Using Eq. (4-2) and stress life exponent of 9, we can relate fatigue lives with peak hertz stresses experienced by the raceways in cases A and B as,

\[
\frac{L_B}{L_A} = \left[ \frac{S_{\text{maxA}}}{S_{\text{maxB}}} \right]^9
\]

(A-5)

Solving Eq. (A-5), we get fatigue life of the inner and outer raceways of the bearing in Case B as, \( L_i = 31.55 \text{ million cycles} \) and \( L_o = 213.92 \text{ million cycles} \) respectively. From Eq. (A-4), combined fatigue life of the bearing in case B is 28.404 million cycles. Therefore, if we decrease elastic modulus of the raceway material by 10% then, the combined bearing fatigue life is improved by 38.66% from Case A to Case B. Hence this analysis confirms that bearing fatigue lives are highly sensitive to the variations in elastic modulus of raceways and there is need to incorporate life modification factors.
corresponding to gradient in elastic modulus for accurate prediction of fatigue lives of case hardened bearing steels.
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BIOGRAPHICAL SKETCH

Nikhil Londhe was born in Maharashtra state of India. He obtained his master's degree in mathematics and bachelor’s degree in mechanical engineering from Birla Institute of Technology and Science, Pilani. He joined University of Florida, in fall 2012 to pursue Master of Science degree in Department of Mechanical and Aerospace Engineering with specialization in solid mechanics, design and manufacturing. He completed his master’s studies with thesis specializing in durability of rolling element bearings. Later on, he continued his PhD dissertation research on same topic. For this research, he worked with Dr. N. K. Arakere in Computational Solid Mechanics Lab. Nikhil is recipient of Graduate School Doctoral Dissertation Award from University of Florida. He also won prestigious Walter D. Hodson Award for 2017 from Society of Tribologists and Lubrication Engineers in recognition of the best paper published by society from lead author under the age of 35.