A PASSIVE WIRELESS MEMS DYNAMIC PRESSURE SENSOR FOR HARSH ENVIRONMENTS

By

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Because He Lives
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The development of sensors for use in harsh environments (e.g., high temperature, highly corrosive, radioactive, etc.) has been extremely limited due to the availability of materials that can operate in harsh conditions, maturity of fabrication for such materials, and methods for sensor interrogation. Silicon is commonly used for microelectromechanical systems (MEMS)-based sensors; however, it is not appropriate for high-temperature environments. This dissertation describes the development of a sapphire-based passive wireless MEMS dynamic pressure sensor for operation in high-temperature environments greater than 1000°C. The sensor consists of a laser-micromachined diaphragm with a passive wireless electrical resonator operating at Ku-band (i.e., 12 to 18 GHz) frequencies. Furthermore, an additional integrated temperature compensating resonator operating at Ku-band frequencies may be utilized for real-time temperature compensation. This dissertation discusses the design, fabrication, and experimental characterization of a passive wireless dynamic pressure sensor for harsh environments.
CHAPTER 1
INTRODUCTION

Sensing dynamic pressure fluctuations in high-temperature environments is a difficult problem presenting many challenges. Often, materials used for sensing devices (e.g., silicon) are not capable of operating at high temperatures since their properties (i.e., electrical, mechanical, etc.) tend to deteriorate. Additionally, sensors often require adjacent interface electronics for signal buffering, amplification, and/or temperature compensation. However, interface electronics are also compromised at elevated temperatures due to the temperature dependent electronic properties of silicon and similar semiconductors. The goal of this research is to develop a passive wireless dynamic pressure sensor that can operate in the compressor or exhaust stages of a gas turbine at temperatures up to 1000°C. The remainder of Chapter 1 introduces sensor background for dynamic pressure sensing in high-temperature environments followed by sensor requirements, research objectives, and dissertation organization.

1.1 Sensor Background

Sensors operating in harsh environments such as aeronautical environments are often subjected to high temperatures, vibration, ionizing radiation, and chemical exposure [1]. As will be discussed below, pressure is particularly an important measurand in high-temperature environments. Pressure measurements are classified as either static (i.e., time invariant) or dynamic (i.e., varies with time). A dynamic pressure sensor is capable of measuring pressure within some frequency range of interest. A microphone, for example, is a dynamic pressure sensor capable of measuring pressure fluctuations associated with acoustic phenomena. An audible
microphone measures pressure in the audible frequency range of 20 Hz to 20 kHz, whereas an aeroacoustic microphone measures pressure in the frequency range of 20 Hz to 100 kHz [2].

A gas path analysis (GPA) is commonly performed on gas turbines to determine baseline changes throughout the engine [3]. Measurement of the temperature, pressure, fuel flow, and rotor speed are important in the assessment of high-temperature gas turbines [3]. High cycle fatigue (HCF) is the largest cause of component failures in gas turbines [4]. HCF damage by aerodynamic excitation, for example, is caused by pressure fluctuations in the air flow across the turbine blades and vanes [4]. The ability to locally measure pressure fluctuations could significantly reduce HCF.

Figure 1-1. Pressure and temperature characteristics of General Electric J79 turbojet engine (adapted from [5]).
The temperature and pressure requirements for a pressure sensor vary greatly depending on the desired location in the gas turbine. Consider an industrial gas turbine (i.e., combustion turbine) such as the General Electric J79 as shown in Figure 1-1. The temperature in a gas turbine increases gradually through the compressors to the combustion chamber. The temperature further increases as the compressed air is mixed with fuel and ignited to generate a high-temperature, high-pressure air flow. As the air flows through the turbines to the exhaust the temperature then decreases. The pressure in a gas turbine follows a similar profile as the temperature. The compressor and exhaust stages of the turbine have more relaxed temperature and pressure requirements.

The exhaust temperature for a Siemens SGT-100, for example, is 545˚C [6]. The areas in a gas turbine that are at or below this temperature are the compressors and the exhaust stages. The temperature in a combustion chamber of a gas turbine can reach well above 1000˚C. Although the total pressure in a gas turbine may exceed 100 psi the dynamic pressure fluctuations in a gas turbine (e.g., 1 psi) are much smaller than the static pressure (e.g., 100 psi). As stated earlier, the goal of this research is to design a dynamic pressure sensor that can operate in the compressor or exhaust stages of a gas turbine at temperatures up to 1000˚C. Existing sensors are unable to operate in high-temperature environments due to the operability limits of existing sensor materials and their interface electronics.

Existing commercially available sensors have addressed the high-temperature issue by either water-cooling the sensing head or employing extended probe tip tubes to increase their respective measurement temperature ranges. The Kulite EWCTV312
pressure sensor [7], for example, is able to operate up to 1093°C by utilizing water-cooling to transport heat away from the sensor. An undesirable effect of water-cooling is a lowering of the local temperature of the gas being measured. The PCB Piezotronics 377B26 and Brul & Kjaer 4182 microphones [8], [9] employ probe tips to extend operating temperatures to 800°C and 700°C, respectively, by distancing the diaphragm and preamplifier from the high-temperature environment. However, there is a trade-off with probe tips between the length of the tube ℓ and the cut-off frequency f (i.e., \( f = \frac{c_o}{4\ell} \)), as well as phase-response effects [10].

<table>
<thead>
<tr>
<th>Model</th>
<th>Manufacturer</th>
<th>Max. Temp</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EWCTV312</td>
<td>Kulite</td>
<td>1093°C</td>
<td>AC</td>
<td>Water-cooled static/dynamic pressure sensor</td>
</tr>
<tr>
<td>-</td>
<td>Luna (R&amp;D)</td>
<td>1050°C</td>
<td>DC</td>
<td>Optical pressure sensor</td>
</tr>
<tr>
<td>DPT950</td>
<td>Oxsensis (R&amp;D)</td>
<td>1000°C</td>
<td>AC</td>
<td>Optical dynamic pressure sensor</td>
</tr>
<tr>
<td>377B26</td>
<td>PCB Piezotronics</td>
<td>800°C</td>
<td>AC</td>
<td>Probe tip-based microphone</td>
</tr>
<tr>
<td>4182</td>
<td>B&amp;K</td>
<td>700°C</td>
<td>AC</td>
<td>Probe tip-based microphone</td>
</tr>
<tr>
<td>-</td>
<td>NASA (R&amp;D)</td>
<td>600°C</td>
<td>DC</td>
<td>Piezoresistive (Ti/TaSi₂/Pt on SiC) pressure sensor</td>
</tr>
</tbody>
</table>

There have been significant advances in research and development (R&D) sensors as well. Luna Innovations, for example, has reported an optical sensor for pressure sensing up to 1050°C [11]. The Oxsensis DPT950 [12] is an optical-based dynamic pressure sensor for sensing up to 1000°C. Optical sensors are immune to electromagnetic interference (EMI); however, optical fibers are generally fragile and difficult to package. NASA developed a piezoresistive-based pressure sensor with Ti/TaSi₂/Pt contacts on silicon carbide (6H-SiC) for sensing up to 600°C [13]. A summary of currently existing pressure sensors and microphones for high-temperature environments is shown in Table 1-1. Mills et al. also developed an optical dynamic
pressure sensor based on sapphire capable of operating in high-temperature environments [14]; however, since the sensor has not been characterized at high temperatures it is not included in Table 1-1.

Wireless sensors are unique in that they offer the ability to operate in an environment with little to no perturbation to the environment. Wireless sensors are classified as either passive or active. Active sensors use an internal energy source (e.g., batteries) to supply energy for emitting signals. Unfortunately, batteries add significant size and weight; they are also not constructed of high-temperature compatible materials. Passive sensors, on the other hand, use an external energy source (e.g., electromagnetic radiation, vibration, etc.) to supply energy for emitting signals. Electromagnetic radio frequency (RF) energy, for example, may be utilized to determine the impedance of a passive sensor by observing the scattered energy [15].

1.2 Sensor Requirements

The requirements of a sensor are driven by the desired measurement environment and application. In order to understand the environmental requirements the performance parameters associated with the sensor must be understood first. Furthermore, the performance parameters of a dynamic pressure sensor often have trade-offs that inhibit the sensor from being broadly applicable.

1.2.1 Performance Parameters

The operation and performance of a dynamic pressure sensor are driven by several parameters of importance. The cut-on frequency $f_{ac0}$ of a dynamic pressure sensor is typically determined by geometry of the cavity-vent structure as discussed in
Chapter 3. The cut-off frequency $f_{a,o}$, on the other hand, is typically determined by the resonant frequency of the dynamic pressure sensor. The bandwidth of a dynamic pressure sensor (i.e., $\Delta f_a = f_{a,o} - f_{a,co}$) is the frequency range where the sensor has a flat band response within $+/-$ 3 dB. An audible microphone, for example, has a bandwidth of 20 Hz to 20 kHz.

The sensitivity of a dynamic pressure sensor is defined as the change in the output measurand (e.g., deflection, voltage, frequency) with respect to change in the input measurand (e.g., pressure, deflection). The acousto-mechanical sensitivity $S_{am}$, for example, relates the deflection of the diaphragm to the pressure exerted on the diaphragm [16]. The electrical sensitivity $S_e$, on the other hand, relates the output voltage to the deflection of the diaphragm. The equivalent sensitivity of a dynamic pressure sensor relates the output voltage to the pressure exerted on the transducer, in V/Pa, is

$$S = S_{am} \cdot S_e = \frac{dw}{dp} \cdot \frac{dv}{dw} = \frac{dv}{dp}.$$  \hspace{1cm} (1-1)

There is a trade-off between the sensitivity and the bandwidth of the dynamic pressure sensor given by a sensor’s sensitivity-bandwidth product as discussed in Chapter 3. The noise of a dynamic pressure sensor is some unintended electrical output in the absence of an input pressure. Noise that is truly random in the absence of an input is intrinsic, whereas noise that is due to some unwanted external source is extrinsic [17]. The minimum detectable pressure $p_{min}$ is that in which the output signal of the dynamic pressure sensor is equivalent to the noise floor. The dynamic range of a dynamic pressure sensor $DR$ is defined as the ratio of the maximum attainable pressure $p_{max}$ to the minimum detectable pressure $p_{min}$ integrated over some bandwidth or
The maximum attainable pressure of a MEMS dynamic pressure sensor is defined as the pressure at which there is a three percent departure between the ideal deflection of the diaphragm (i.e., linear deflection) and the actual deflection of the diaphragm (i.e., nonlinear deflection). The three percent departure is more well-known as the total harmonic distortion (THD). The minimum detectable pressure, on the other hand, is limited by either the thermodynamic noise of the transducer itself or the electronic noise of the interface electronics. The root-mean square pressure \( p_{rms} \) applied to a dynamic pressure sensor has standard units of Pa; however, the output is often specified in decibels of sound pressure level (i.e., dB_{SPL}) with reference to the absolute threshold of hearing \( p_o \) as

\[
p_{dB_{SPL}} = 20 \log_{10} \left| \frac{p_{rms}}{p_o} \right|.
\]  

The absolute threshold of hearing \( p_o \) is defined as the minimum sound level to which a normal human ear can perceive sound. In 1933 Fletcher et al. of Bell Laboratories decided for simplification that the absolute threshold of hearing is to have an RMS value of 20 µPa for a 1 kHz tone at 20°C [18].

A typical dynamic pressure sensor transduces a pressure change into a voltage change. An electrical resonant-based dynamic pressure sensor is capable of transducing a pressure change into an electrical resonant frequency change. The dynamic range of a resonant-based dynamic pressure sensor is given as the ratio of the maximum output frequency change \( \Delta f_o \) (i.e., electrical operating bandwidth) to the minimum resolvable frequency shift \( f_{min} \) as
\[ DR = 20 \log_{10} \left| \frac{\Delta f_o}{f_{\text{min}}} \right|. \] (1-4)

An illustration describing the sensor performance of a resonant-based dynamic pressure sensor is shown in Figure 1-2. The bandwidth and dynamic range of the pressure sensor define a window of operating space where the sensor has linear performance. As illustrated, the minimum detectable pressure \( p_{\text{min}} \) (i.e., resolution) is often limited by the noise floor of the electronics. The maximum pressure \( p_{\text{max}} \), on the other hand, is limited by the mechanics of the sensor in relation to the THD. In the region between minimum and maximum pressure there is a linear relationship between the input pressure and output frequency given by the equivalent sensitivity (i.e., \( S = \frac{df_o}{dp} \)). Equivalently, in the region between the minimum and maximum output frequency there is a linear relationship between the input pressure and output frequency.

![Diagram of sensor performance](image)

**Figure 1-2.** Operating space of a resonant-based dynamic pressure sensor.
The performance aspects of the sensor up to this point assume the sensor is operating at room temperature. The sensor’s performance will drift at elevated temperatures due to temperature-dependent material properties. Piezoresistive transducers, for example, will drift due to the temperature coefficient of resistance of the resistive elements. Piezoelectric transducers will drift due to the piezoelectric constant’s inherent temperature dependence [19]. The geometry of the sensor will also change at elevated temperatures due to the coefficient of thermal expansion (CTE) in addition to thermal stresses. A sensor must be calibrated to compensate for any material and geometric changes due to elevated temperatures. Furthermore, the sensor must maintain structural integrity (e.g., no plastic deformation) within the desired operating temperature range.

<table>
<thead>
<tr>
<th>Sensor Parameter</th>
<th>Requirement</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Range</td>
<td>172 to 5,171</td>
<td>kPa</td>
</tr>
<tr>
<td>Combined Uncertainties at RT</td>
<td>≤ 1% FS</td>
<td>V</td>
</tr>
<tr>
<td>Temperature Effect on Zero</td>
<td>± 2% FS</td>
<td>V</td>
</tr>
<tr>
<td>Temperature Effect on Sensitivity</td>
<td>0 to 2% FS</td>
<td>V</td>
</tr>
<tr>
<td>Natural Frequency</td>
<td>≥ 100</td>
<td>kHz</td>
</tr>
<tr>
<td>Acceleration Sensitivity</td>
<td>≤ 0.001% FS/g</td>
<td>V/g</td>
</tr>
<tr>
<td>Operating Temperature Range</td>
<td>up to 590</td>
<td>°C</td>
</tr>
</tbody>
</table>

1.2.2 Environmental Requirements

The Propulsion Instrumentation Working Group (PIWG) [20] is an organization of leading industrial and research participants with the common purpose of working together to advance sensor and measurement systems for gas turbine engine applications. The PIWG has defined sensor specifications addressing the pressure sensing needs in high-temperature turbines as summarized in Table 1-2. The temperature requirement is difficult for existing pressure sensors to achieve. According
to PIWG there is a current need for pressure sensors that can operate up to 590˚C. However, within the next five years there will be a need for pressure sensors that can operate up to 815˚C. As stated earlier, the temperature and pressure requirements in an industrial gas turbine vary greatly depending on the sensor location.

1.3 Research Objectives

The objective of this research is to design, fabricate, and characterize a sapphire-based passive wireless MEMS dynamic pressure sensor for harsh environments. The dynamic pressure sensor utilizes a passive wireless electrical resonator for dynamic pressure sensing. Furthermore, an additional passive wireless electrical resonator may be utilized for temperature sensing. The utilization of a passive wireless sensor allows remote sensing without the need for local active sources (e.g., batteries) or local interface electronics that are non-compatible with high-temperature environments. Additionally, the use of sapphire for the diaphragm and platinum for the electrical resonator allows the sensor to operate in high-temperature environments. The goal of this research is to develop a dynamic pressure sensor that can operate in the compressor or exhaust stages of an industrial gas turbine up to 1000˚C.

1.4 Dissertation Overview

The dissertation describing a passive wireless dynamic pressure sensor is presented in Chapters 1 through 7. Chapter 1 discussed sensor applications, requirements, and research objectives. Chapter 2 discusses general methods of transduction and wireless sensing methods followed by literature reviews of passive wireless pressure sensors and passive wireless temperature sensors. Chapter 3
discusses sensor design aspects including high-temperature material evaluation, acousto-mechanical lumped element modeling, and antenna modeling. Chapter 4 discusses sensor optimization and fabrication aspects including fabrication of a demonstration sensor followed by the high temperature sensor. Chapter 5 discusses sensor interrogation including the discussion of an integrated radar-on-a-chip for millimeter-wave frequency operation followed by the design of an electromagnetic waveguide. Chapter 6 discusses experimental characterization of the sensor including characterization of an antenna on glass and acoustic characterization of the demonstration and high temperature sensors. Chapter 7 concludes the dissertation, then discusses research contributions and future work.
Chapter 2 discusses existing transduction methods for piezoresistive, piezoelectric, electrostatic, and optical dynamic pressure sensors. Wireless sensing methods for inductive coupling, RF backscattering, and radar are then discussed followed by literature reviews of passive wireless pressure sensors and passive wireless temperature sensors. Chapter 2 ends with the proposal of a novel passive wireless dynamic pressure sensor.

2.1 Transduction Methods

As discussed earlier, a dynamic pressure sensor measures sound pressure levels (SPL) in some frequency range of interest. A dynamic pressure sensor transduces an incident fluctuating sound pressure into an electrical output. The electrical output typically requires some interface electronics for signal buffering, amplification, and/or temperature compensation. There are four transduction methods for existing dynamic pressure sensors: piezoresistive, piezoelectric, electrostatic, and optical.

2.1.1 Piezoresistive Transduction

A piezoresistive dynamic pressure sensor utilizes a thin-film resistive element to transduce a deflection in the diaphragm into a resistance change as shown in Figure 2-1. Alternatively, resistive elements may be embedded directly into the diaphragm by diffusion [21]. The resistance change due to deflection of the diaphragm is either positive (i.e., compressive strain) or negative (i.e., tensile strain). The change in resistance is attributed to a change in the length $\Delta L$, cross-sectional area $\Delta A$, and
electrical resistivity $\Delta \rho_e$ of the resistive elements as $\Delta R/R_o = \Delta L/L_o - \Delta A/A_o + \Delta \rho_e/\rho_e$.

A Wheatstone bridge is typically employed to convert the resistance change to an output voltage. The output voltage of a piezoresistive dynamic pressure sensor is dependent on the source voltage at the Wheatstone bridge electronics; therefore, the sensitivity is commonly expressed in V/Pa/V.

Figure 2-1. An illustration of a piezoresistive dynamic pressure sensor.

The sensitivity of a piezoresistive dynamic pressure sensor is directly proportional to the gauge factor, which relates the change in resistance to the strain (i.e., $\varepsilon = \Delta L/L_o$) on the diaphragm as $GF = (\Delta R/R_o)/\varepsilon$. The temperature coefficient of resistance $\alpha$ relates the change in resistance (due to change in temperature) as $\alpha \Delta T = \Delta R/R_o$. The trade-off with a high gauge factor is a high temperature coefficient of resistance. Silicon, for example, can achieve gauge factors between +100 to +170 for p-type and -140 to -100 for n-type [22]; however, the temperature coefficient of resistance for silicon is between 70 to 700 $10^{-5}/{^\circ}C$ for p-type and n-type. Piezoresistive transducers commonly employ temperature compensation electronics [23] to correct for small changes in ambient temperature; however, they are not compatible with high-temperature environments.
### 2.1.2 Piezoelectric Transduction

A piezoelectric dynamic pressure sensor utilizes a piezoelectric thin-film to transduce a deflection in the diaphragm into an electrical charge as shown in Figure 2-2. The strain $\mathbf{\hat{S}}$ in the piezoelectric film due to the applied stress $\mathbf{\hat{T}}$ on diaphragm is related to the elastic compliance constant $\mathbf{s}^E$ and the piezoelectric constant $\mathbf{d}$ of the film and the electric field component $\mathbf{E}$ as $\mathbf{\hat{S}} = \mathbf{s}^E \cdot \mathbf{\hat{T}} + \mathbf{d} \cdot \mathbf{\hat{E}}$ [24]. Furthermore, the applied stress results in a charge density in the piezoelectric film that is related to the electric permittivity $\varepsilon^T$ of the piezoelectric material as $\mathbf{D} = \mathbf{d} \cdot \mathbf{\hat{T}} + \varepsilon^T \cdot \mathbf{\hat{E}}$ [24]. The sensitivity of the pressure sensor is directly proportional to the piezoelectric constant, which relates the total charge in the film to the force applied on the diaphragm as $d = q / f$. Interface electronics including a charge amplifier [25] are typically employed due to small charges (e.g., 5 pC/N [26]) generated by the piezoelectric film.

![Figure 2-2. An illustration of a piezoelectric dynamic pressure sensor.](image)

A piezoelectric material produces an electrical polarization and corresponding electrical charge in response to an applied force. The Curie temperature $T_c$ of a piezoelectric material is the temperature where the material can no longer sustain polarization. The piezoelectric constant of a material varies as a function of the
temperature above the Curie temperature. A piezoelectric material is typically limited to operation at temperatures less than half of the Curie temperature (i.e., $T < T_c/2$). Aluminum nitride has a piezoelectric constant of 5 pC/N [26] and has shown strong interest for high-temperature environments due to its operability of up to 1150°C with piezoelectric activity [27]. Metal contacts and interface electronics inherently limit the sensor operability in high-temperature environments.

### 2.1.3 Electrostatic Transduction

An electrostatic or capacitive dynamic pressure sensor transduces a deflection in the diaphragm into a capacitance change as shown in Figure 2-3. The capacitor can be either DC- (i.e., fixed charge) or AC-biased (i.e., time varying charge). A DC-biased capacitor has an output voltage proportional to the deflection of the diaphragm (i.e., $V = q/C \propto d$). An AC-biased capacitor, on the other hand, has a modulated output voltage that is further demodulated with electronics to produce a proportional output voltage.

![Figure 2-3. An illustration of an electrostatic dynamic pressure sensor.](image)

An electret is a permanently charged thin-film commonly used on the diaphragm; however, electrets are known to not hold charge at elevated temperatures [28].
Electrostatic sensors also require local buffer amplifiers to minimize additional parasitic capacitance, which are not high-temperature compatible.

### 2.1.4 Optical Transduction

An optical dynamic pressure sensor transduces a deflection in the diaphragm to a magnitude and/or phase change of an incident light source as shown in Figure 2-4. The light source for optical dynamic pressure sensors may propagate through either free-space or utilizing fiber optics. Laser sources with lenses and mirrors are typically employed across free-space to direct collimated light. Fiber optics, on the other hand, direct light through transparent optical fibers using total internal reflection. Fiber optics are flexible, low-cost, and allow light to be re-directed easily; therefore, fiber optics are used more often than free-space optics.

![Diagram of an optical dynamic pressure sensor](image)

**Figure 2-4.** An illustration of an optical dynamic pressure sensor.

Optical dynamic pressure sensors are also considered passive (i.e., no internal energy sources) and immune to EMI since there are no local interface electronics. The intensity of light at the backend is measured using photodiodes that convert light to
current. A transimpedance amplifier is typically employed to convert the current of the photodiode to an output voltage. Optical sensors show strong promise for high-temperature environments; however, optical fibers are fragile and difficult to package.

### 2.1.5 Summary of Transduction Methods

Existing transduction methods for dynamic pressure sensors are categorized as piezoresistive, piezoelectric, electrostatic, and optical. High output impedance transducers (i.e., piezoelectric and electrostatic) require buffer amplifiers adjacent to the dynamic pressure sensor to minimize parasitic capacitance, which are not high-temperature compatible. Piezoresistive transducers require temperature compensation electronics, which are also not high-temperature compatible. Optical transducers, on the other hand, are capable of operating in high-temperature environments without electronics.

<table>
<thead>
<tr>
<th>Category</th>
<th>Wireless</th>
<th>EMI Immunity</th>
<th>High-Temp Compatibility</th>
<th>Packaging Simplicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezoresistive</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Piezoelectric</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Electrostatic</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Optical</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>RF Wireless (Proposed)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

A summary of existing transduction methods for dynamic pressure sensing is shown in Table 2-1. Piezoresistive, piezoelectric, and electrostatic transducers use wired interface electronics; therefore, they are not considered wireless. Optical transducers are considered wireless if the transducer uses free-space optics. Optical
transducers are considered to have EMI immunity and are high-temperature compatible; however, as discussed earlier optical fibers are fragile and difficult to package.

Optical sensing uses electromagnetic radiation in the infrared to ultraviolet regions of the electromagnetic spectrum. Similarly, RF wireless sensing uses electromagnetic radiation in the radio to microwave region of the electromagnetic spectrum. The electromagnetic spectrum for RF covers several kHz to hundreds of GHz. The operating band for amplitude modulation (AM) radio, for instance, is 535 to 1705 kHz, whereas the operating band for frequency modulation (FM) radio is 88 to 108 MHz. RF wireless sensing uses both near-field and far-field regions for sensing applications.

### 2.2 Passive Wireless Sensing Methods

RF passive wireless sensing is performed in either the near-field or far-field regions. Consider, for example, an electrically short antenna with length $L \leq \lambda/2$ as shown in Figure 2-5a. The near-field region is the separation distance from the transmit/receive antenna of less than one wavelength (i.e., $R < \lambda$) and is sub-divided into two regions: reactive (i.e., $R < \lambda/2\pi$) and radiative (i.e., $\lambda/2\pi \leq R \leq \lambda$). The reactive region is where either the electric (E) or magnetic (H) field dominates depending on the antenna type (e.g., magnetic H field dominate for loop antenna). The radiative region is where the fields begin to radiate outward. The transition region (i.e., $\lambda \leq R \leq 2\lambda$) between the near-field and far-field is where the fields are radiating outward and becoming planar as illustrated in the single slit optical analogy in Figure 2-5b. The
The far-field region has cross-axis E and H fields alternating 90° out of phase with the outward power decreasing as $1/R^2$.

![Diagram showing regions of near-field and far-field](image)

Figure 2-5. An illustration of near-field and far-field for (a) electrically short antenna (adapted from [29]) and (b) optical analogy.

There are three existing methods for RF passive wireless sensing: inductive coupling, RF backscattering, and radar. Inductive coupling utilizes mutual inductance to couple a transmit/receive loop antenna with associated interface electronics to an closely located adjacent loop antenna with a sensor load. Inductive coupling occurs in the near-field region at low frequencies (i.e., 100 kHz to 100 MHz) with small separation distances (i.e., < 1 cm). Alternatively, RF backscattering and radar occur in the far-field region at high frequencies (i.e, 100 MHz to 100 GHz) with large separation distances (i.e., > 10 cm). Although RF backscattering and radar both operate in the far-field region RF backscattering is primarily utilized for detecting electrical sensors, whereas radar is primarily utilized for determining physical information about an object. The three methods for passive wireless sensing are discussed in further detail below.
2.2.1 Inductive Coupling

An inductively coupled sensor typically has a closed-loop coil antenna forming an inductor. The inductance of the coil antenna is related to the number of coil turns \( N \), the magnetic flux \( \Phi \), and the current \( I \) that produces the flux as \( L = N\Phi/I \). The inductance is strongly dependent on the geometry of the coil. The inductance of a long solenoid wire, for example, with length \( \ell \) and cross-sectional area \( A \) is related to the permeability of the medium within the coil \( \mu_r \) and the number of coil turns \( N \) as \( L = \mu_r\mu_oN^2A/\ell \) [30].

The magnetic field \( H \) around a closed-loop coil and the current \( I \) passing through the coil are related by Ampere’s circuit law (i.e., magnetomotive force) as [30]

\[
NI = \int \vec{H} \cdot d\vec{\ell}.
\] (2-1)

The magnetomotive force is also related to the magnetic reluctance (i.e., resistance to magnetic flux) \( \Re \) and the magnetic flux \( \Phi \) as \( NI = \Phi(\Re_1 + \Re_2 + \Re_3) \). There are several analogous relations between magnetic and electrical circuits as shown in Figure 2-6. The magnetomotive force (i.e., \( F = NI = \Phi\Re \)) is analogous to an electrical voltage (i.e., \( V = IR \)); where the magnetic flux \( \Phi \) is analogous to an electrical current \( I \) and the magnetic reluctance \( \Re \) is analogous to an electrical resistance \( R \).

![Figure 2-6. An illustration of a ferromagnetic core with (a) equivalent magnetic circuit and (b) analogous electrical circuit (adapted from [30]).](image)
In 1831 Michael Faraday demonstrated the first electrical transformer by connecting a battery and a galvanometer (i.e., current meter) to two wires wrapped around an iron ring as shown in Figure 2-7. A magnetic field (i.e., \( \Phi = -\int v_1 dt / N_1 \)) is induced when a voltage \( v_1 \) is applied to coil 1 according to Faraday’s law of induction. The changing magnetic field \( d\Phi/dt \) further induces a voltage \( v_2 = -N_2 d\Phi/dt \) on coil 2. The ratio of the number of turns on coil 2 to the number of turns on coil 1 is \( n = N_2/N_1 \). Furthermore, the coils have a mutual magnetic flux, which relates the voltages as \( v_1/v_2 = 1/n \) [31].

The power in the two coils is equivalent by the law of conservation of energy; therefore, the ratio of the number of turns is related to the current, voltage, and inductance of the coils as [31]

\[
n = \frac{v_2}{v_1} = \frac{i_1}{i_2} = \sqrt{\frac{L_2}{L_1}}. \tag{2-2}
\]
Figure 2-8. Schematic of inductive coupling.

Consider two inductive coils placed in close proximity in free-space as shown in Figure 2-8. The two coils are coupled to one another by forward and reverse mutual inductances $M_{12} = N_2 \Phi_{12}/i_1$ and $M_{21} = N_1 \Phi_{21}/i_2$, respectively. The forward and reverse magnetic flux are equivalent; therefore, the mutual inductances are reciprocal (i.e., $M_{12} = M_{21} = M$). The self-inductances of coil 1 and coil 2 are $L_1 = N_1 \phi/i_1$ and $L_2 = N_2 \phi/i_2$, respectively. The mutual inductance is related to the product of the self-inductances as $M = \sqrt{L_1 L_2}$. A coupling coefficient $k_m$ is inserted to adjust the mutual inductance for orientation of the inductive coils such that [31]

$$M = k_m \sqrt{L_1 L_2}. \quad (2-3)$$

The inductor coils are perfectly coupled if $k_m = 1$, tightly coupled if $k_m > 0.5$, or loosely coupled if $k_m < 0.5$. The coupling coefficient is a strong function of the distance between the inductive coils [32]. The voltage on each coil is related to the mutual inductance as $v_1 = L_1 di_1/dt - M di_2/dt$ and $v_2 = L_2 di_2/dt - M di_1/dt$.

Inductive coupling occurs only in small distances due to the coupling coefficient’s dependence on distance. The distance between the inductive coils is maximized by increasing the diameters of the coils, increasing the relative permeability $\mu_r$ of the
medium in the inductive coils, or orienting the coils such that $k_m \approx 1$. The maximum frequency of an inductive coil $f_o$ is also related to the diameter of the coil $D$ as $f_o = 1/2\pi\sqrt{LC} \propto 1/D$. An inductive coil with a small diameter has a large self-resonant frequency (SRF), whereas an inductive coil with a large diameter has a small SRF.

![Diagram of resonant inductive coupling](image)

**Figure 2-9.** Schematic of resonant inductive coupling.

The addition of capacitors to an inductively coupled circuit is known as resonant inductive coupling as shown in Figure 2-9. One unique feature of resonant inductive coupling for sensors is its inherent ability to sense changes in capacitance. Resonant inductive coupling occurs at frequencies close to a common resonant frequency of coil 1 and coil 2 to maximize power transfer. The resonant frequency is related to the inductances and capacitances of the coils as

$$f_r = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L_1C_1}} = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L_2C_2}}.$$  \hspace{1cm} (2-4)

Applying Kirchhoff’s voltage law (KVL) on the primary side the voltage on coil 1 is

$$v = \left( R_1 + j \left( \omega L_1 - \frac{1}{\omega C_1} \right) \right) i_1 - j\omega M i_2.$$  \hspace{1cm} (2-5)

Applying KVL on the secondary side the voltage on coil 2 is
\[ 0 = \left( R_2 + R_L + j \left( \omega L_2 - \frac{1}{\omega C_2} \right) \right) i_2 - j \omega M i_1. \quad (2-6) \]

The impedance looking into the primary side is [33]

\[ Z_{in} = \frac{v}{i_1} = R_1 + j \left( \omega L_1 - \frac{1}{\omega C_1} \right) + \frac{\omega^2 M^2}{R_2 + R_L + j \left( \omega L_2 - \frac{1}{\omega C_2} \right)}. \quad (2-7) \]

An illustration of a resonant inductively coupled circuit simulated in LTspice with \( R_1 = R_2 = R_L = 1 \, \Omega, \, L_1 = L_2 = 100 \, \mu\text{H}, \, C_1 = C_2 = 220 \, \text{pF}, \) and \( k_m = 0.001 \) is shown in Figure 2-10. It is observed that the resonant frequency of coil 1 is matched to the resonant frequency of coil 2. Inductive coupling is commonly used for wireless power charging and wireless sensing of capacitive sensors. Inductive coupling and resonant inductive coupling occur in the near-field region. The transmitted power, however, decreases quickly with distance between the coils.

\[ \begin{array}{c}
\text{Voltage Gain, } G_v [\text{dB}] \\
\text{Frequency, } f [\text{MHz}] \\
\end{array} \]

![Figure 2-10. Illustration of resonant inductive coupling.](image)

Figure 2-10. Illustration of resonant inductive coupling.
Power transfer with 40% efficiency has been demonstrated at 9.9 MHz with a distance to coil diameter ratio of 4:1 [34]. A sensor with a 1 mm coil diameter, for example, placed 4 mm away from the interface electronics will receive 40% of the transferred power. The use of inductive coupling and resonant inductive coupling for wireless sensing is only useful for distances less than an order of magnitude higher than the diameter of the coils. To sense at larger distances without an inductive link requires operating in the far-field region.

2.2.2 RF Backscattering

The principle of operation for RF backscattering is to detect power reflected (i.e., backscattered) from a sensor in the far-field possessing an electrical impedance (i.e., load). An electromagnetic wave is first transmitted across free-space and is then incident on a sensor's antenna. The impedance mismatch between the antenna and its sensor load results in a backscatter of a portion of the incident electromagnetic wave. The impedance of the antenna has radiation resistance $R_{rad}$ and antenna reactance $X_{ant}$ elements such that $Z_{ant} = R_{rad} + jX_{ant}$. The impedance mismatch between the antenna $Z_{ant}$ and its sensor load $Z_L$ is described by the complex reflection coefficient $\Gamma_s = (Z_L - Z_{ant})/(Z_L + Z_{ant})$, which is bounded such that $-1 \leq \Gamma \leq 1$. The sensor load is a short circuit if $\Gamma = -1$, an open circuit if $\Gamma = 1$, or matched to the antenna if $\Gamma = 0$.

A schematic of a passive sensor with antenna and sensor load elements is shown in Figure 2-11. The voltage induced by the incident electromagnetic wave is represented by the open-circuit voltage $v_{oc}$. The open-circuit voltage is related to the
power received at the antenna $p_{rec}$ as

$$v_{oc} = \sqrt{p_{rec} \cdot (Z_{ant} + Z_L)} \quad [35]$$

and the voltage across the sensor load is related to the open-circuit voltage as

$$v_L = \left[ Z_L/(Z_L + Z_{ant}) \right] v_{oc}.$$ 

\[\text{Figure 2-11. Schematic of passive sensor with antenna and sensor load.}\]

Consider a co-located transmitter/receiver and sensor antenna with load oriented distance $R$ away as shown in Figure 2-12a. The power received at the sensor $p_{rec}$ is related to the directivity of the transmit and sensor antenna $D_t(\theta, \phi)$ and $D_s(\theta, \phi)$, the operating wavelength $\lambda$, the distance $R$, and the radiated power $p_{rad}$ as [35]

$$p_{rec} = D_tD_s \left( \frac{\lambda}{4\pi R} \right)^2 p_{rad}. \quad (2-8)$$

An isotropic antenna radiates power equally in all directions with a power density of $Q_i = p_{rad}/4\pi R^2$. A directive antenna generally directs most of its radiated power in a single direction with a power density of $Q_d = p_{rad}G/4\pi R^2$. The gain and directivity of a directive antenna are illustrated in Figure 2-12b. The gains of the transmitter, receiver, and sensor antennas are related to their antenna efficiencies $\eta_t$, $\eta_r$ and $\eta_s$ as $G_t = \eta_t D_t$, $G_r = \eta_r D_r$, and $G_s = \eta_s D_s$, respectively. The radiated power of the transmitter antenna is related to the source power as $p_{rad} = \eta_t P_{in}$ and the power at the sensor load is related to the received power as $p_{out} = \eta_s p_{rec}$. 

44
The one-way Friis equation describes the power at the sensor load $p_{out}$ in relation to the source power $p_{in}$ as [35]

$$p_{out} = G_t G_s \left( \frac{\lambda}{4\pi R} \right)^2 p_{in}. \quad (2-9)$$

The one-way Friis equation is expanded by including any impedance mismatches in the transmit and sensor electronic circuitry such that

$$p_{out} = G_t |1 - |\Gamma_t||^2 G_s |1 - |\Gamma_s||^2 \cdot \left( \frac{\lambda}{4\pi R} \right)^2 p_{in}. \quad (2-10)$$

The power backscattered from the antenna due to some impedance mismatch $\Gamma_s$ between the antenna and its sensor load for negligibly small impedance mismatches in the transmit circuitry is

$$p_{out}^r = G_t G_s |\Gamma_s|^2 \cdot \left( \frac{\lambda}{4\pi R} \right)^2 p_{in}. \quad (2-11)$$

The power at the receiver due to the backscattered power emitted from the antenna with sensor load is [36]
\[ p_{\text{in}}^r = G_s G_r \left( \frac{\lambda}{4\pi R} \right)^2 p_{\text{out}}^r = G_t G_r G_s^2 |\Gamma_s|^2 \left( \frac{\lambda}{4\pi R} \right)^4 p_{\text{in}}. \] (2-12)

A high impedance mismatch yields a high power at the receiver. Consider the passive sensor circuit in Figure 2-11 with a capacitive load \( Z_L = 1/j\omega C \). The input impedance looking into the passive sensor is

\[ Z_{\text{in}} = R_{\text{rad}} + j\omega L_{\text{ant}} + \frac{1}{j\omega C}. \] (2-13)

The input impedance at electrical resonance (i.e., \( f_o = 1/2\pi \sqrt{LC} \)) reduces to \( Z_{\text{in}}(f_o) = R_{\text{rad}} \). The input impedance is high for frequencies much smaller or larger than the resonant frequency (i.e., \( f \ll f_o \) or \( f \gg f_o \)). Small changes in the capacitance can be detected by observing the impedance of frequencies in close proximity to the resonant frequency. Unfortunately, the power decreases by a factor of \( 1/R^4 \) for two-way RF backscattering, which limits the maximum distance between the transmitter/receiver and the sensor. RF backscattering is commonly utilized in radio frequency identification (RFID) tag applications [37], which require remote sensing up to a few meters. Radar also operates in the far-field region and is used for determining physical information about an object.

### 2.2.3 Radar

Radar is a detection system developed using electromagnetic radio waves to determine information about an object such as its range (i.e., position) or velocity (i.e., speed). A radar system is used for searching (i.e., detecting unknown objects), tracking (i.e., measuring an objects range, velocity, angle, etc.), and imaging (i.e., extracting high resolution details) [38]. Radar is commonly used for military applications, weather
tracking, police speed detection, and automotive anti-collision systems. The most primitive radar includes a transmitter, a receiver, and associated antennas.

![Figure 2-13. Characteristics of an RF pulse.](image)

Radar systems are classified as either continuous wave or pulse wave. Continuous wave radars typically require two antennas to transmit and receive, whereas pulse wave radars use a single antenna for both transmit and receive. Pulse wave radars typically have a high peak power to detect objects at a large distance, whereas continuous wave radars operate at a low peak power. Pulse wave radars have a high susceptibility to jamming due to a low signal to noise ratio (SNR), whereas continuous wave radars have a low susceptibility due to a high SNR. The characteristics of an RF pulse are shown in Figure 2-13. The duty cycle $d_t$ of an RF pulse is the fraction of time the transmitter is turned on per period and is related to the transmit time $\tau$ and the pulse repetition interval (PRI) as

$$d_t = \frac{\tau}{PRI}.$$  \hspace{1cm} (2-14)
The listening time is the difference between the PRI and the transmit time. The average transmitted power of an RF pulse is related to the peak power $p_{pk}$ and the duty cycle $d_t$ as $p_{avg} = p_{pk} \cdot d_t$. The range resolution of an RF pulse or the minimum separation distance required to resolve two objects is related to the transmit time $\tau$ as $\Delta R = c\tau/2$ [38].

<table>
<thead>
<tr>
<th>Class</th>
<th>Features</th>
<th>Subclass</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulse</td>
<td>• Single antenna for TX/RX</td>
<td>Intrapulse</td>
<td>• Long pulse width</td>
</tr>
<tr>
<td></td>
<td>• High peak power</td>
<td></td>
<td>• Low peak power</td>
</tr>
<tr>
<td></td>
<td>• Low jamming immunity</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Low SNR</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Better for long-range objects</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Pulse</td>
<td>• Short pulse width</td>
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<td></td>
<td></td>
<td></td>
<td>• High peak power</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Measure speed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Unmodulated</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Two antennas for TX/RX</td>
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<tr>
<td></td>
<td>• Low peak power</td>
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<td></td>
<td>• High jamming immunity</td>
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<tr>
<td></td>
<td>• High SNR</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Better for short-range objects</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Modulated</td>
<td>• Measure position and speed</td>
</tr>
</tbody>
</table>

A summary of radar classifications is shown in Table 2-2. Pulse radar systems are classified as either intrapulse or pulse modulated. Intrapulse modulated radars transmit a pulse with a long pulse width, which is modulated to attain range information. Pulse modulated radars have a short pulse width for good range resolution and a high power level for increased distance. Continuous radar systems can be unmodulated or modulated. Continuous unmodulated radar systems are commonly used for measuring speed, but are not appropriate for obtaining range measurement. Continuous modulated radar systems, also referred to as frequency modulated continuous wave (FMCW) radar, are capable of determining both range and speed measurements.
A diagram of a radar system using triangle-generated FMCW is shown in Figure 2-14. A triangle-generated FMCW is implemented using a triangle voltage source fed into a voltage controlled oscillator (VCO). The VCO produces an output frequency proportional to the input voltage. An ideal VCO will output a linear sweep of frequencies with a bandwidth equivalent to the time period associated with the triangle input. The VCO output is then up-converted in frequency by an RF mixer and amplified by a power amplifier (PA). The transmitted signal then backscatters off an object and returns to the receiver where it is down-converted and compared with the triangle voltage source.

An illustration of the transmitted and received signals for a triangle-generated FMCW is shown in Figure 2-15. The difference between the frequencies of the transmitted and received signals at any instantaneous time is known as the echo or beat frequency given as

$$f_b = f_t - f_r.$$  \hspace{1cm} (2-15)
The round-trip time required for a signal to complete the trip from the transmitter to the receiver is related to the distance $R$ between the transmitter/receiver and the object as $t_{rt} = 2R/c$. The ratio between the transmitted bandwidth and the transmit time for triangle-generated FMCW is a constant given as

$$\frac{\Delta f_1}{\Delta t_1} = \frac{\Delta f_2}{\Delta t_2} = \frac{\Delta f}{T}. \quad (2-16)$$

The beat frequency is determined to be a function of the distance $R$ to an object as

$$f_b = \frac{f_b}{t_{rt}} \cdot t_{rt} = \frac{B}{T} \cdot t_{rt} = \frac{2R\Delta f}{cT}. \quad (2-17)$$

The distance to an object may also be determined by observing the beat frequency as $R = cTf_b/2\Delta f$. The minimum detectable displacement $\Delta R_{min}$ for a triangle-generated FMCW is determined by the minimum time to transmit $T_{min}$, the minimum detectable beat frequency $f_{b,min}$, and the maximum linear bandwidth $\Delta f_{max}$ as

$$\Delta R_{min} = \frac{cT_{min}f_{b,min}}{2\Delta f_{max}}. \quad (2-18)$$

Figure 2-15. Frequency characteristics of triangle-generated FMCW.
The minimum detectable displacement $\Delta R_{\text{min}}$ is ultimately limited by the capabilities of the radar’s electronics. The minimum time to transmit, for example, is limited by the maximum frequency of the signal generator. Furthermore, the VCO has a finite maximum linear bandwidth. The accuracy of the source is also of concern given that electronics tend to drift with time. Signal generators have electromechanical oscillators that drift with time given by the oscillator’s frequency stability. Other factors of the signal generator such as the frequency linearity (i.e., difference between measured and desired signal) will also limit the accuracy.

![Diagram](image)

Figure 2-16. Frequency characteristics of triangle-generated FMCW for receding object.

The beat frequency is constant for a stationary object. If the object is moving (i.e., approaching or receding) as shown in Figure 2-16, then the receiver will see an associated Doppler frequency. The Doppler frequency is related to the speed of the object $v$, the operating frequency $f$, and the isentropic speed of sound $c_o$ as

$$f_d = 2 \frac{v}{c_o} f.$$  \hspace{1cm} (2-19)
The frequency observed at the receiver is related to the beat and Doppler frequencies as \( f_o = f_b \pm |f_d| \). The position of the moving object is determined by taking the average of the frequencies over time or

\[
f_b = \frac{(f_b + |f_d|) + (f_b - |f_d|)}{2}.
\]  

(2-20)

The speed of the moving object is determined by taking the difference of the frequencies over time or

\[
|f_d| = \frac{(f_b + |f_d|) - (f_b - |f_d|)}{2}.
\]  

(2-21)

There are other types of FMCW including sawtooth- and sine-generated FMCW. An ideal sawtooth has zero fall-time; however, all signal generators have some finite fall-time that will limit the maximum frequency. Sawtooth FMCW is also not useful for determining speed. A sine generator is perhaps the most common type of signal generator available. The beat frequency for sine-generated FMCW continuously alternates as the difference of two sine waves. The processing to determine object position and speed for sine-generated FMCW is more complex than other FMCW techniques. The use of FMCW for position sensing is limited by the capabilities of the radar electronics. In 2003 Venot et al. developed a radar system capable of measuring displacement up to 10 cm with less than 10 \( \mu \text{m} \) of accuracy [39]. In 2013 Pohl et al. developed a radar system capable of measuring displacement up to 50 cm with less than 5 \( \mu \text{m} \) of accuracy [40]. For position sensing of maximum displacements on the order of 1 \( \mu \text{m} \) existing FMCW is unacceptable.
2.3 Literature Review of Passive Wireless Pressure Sensors

An electrical resonator is commonly used for wireless pressure measurement. The impedance looking into an electrical resonator $Z_{in}$ with inductive $L$ and capacitive $C$ elements is minimal at the resonant frequency $f_o$. A capacitive pressure sensor has a pressure varying gap that results in a capacitance change $C(p)$. The resonant frequency changes as a function of the capacitance (i.e., $f_o(p) = 1/2\pi\sqrt{LC(p)}$). A loop antenna is commonly used as the inductive element of the electrical resonator. Intraocular pressure measurement has been a leading application for the development of wireless pressure sensors.

The first inductively coupled implantable sensor for intraocular pressure measurement was reported by Collins in 1967 [42]. A resonant frequency of 120 MHz was obtained with two 40-turn inductive coils separated by a 500 $\mu$m gap. A useful sensor distance of less than 24 mm or 10 diameters was determined for the 2.4 mm diameter coils. The pressure sensor was calibrated in saline to achieve a 3 kHz/Pa sensitivity for 67 Pa to 13 kPa.

![Figure 2-17. A capacitive pressure sensor micromachined in silicon with external inductive coil (adapted from [41]).](image-url)
Rosengren et al. later demonstrated a silicon-based intraocular pressure in 1994 [41] as shown in Figure 2-17. The silicon micromachined pressure sensor has a capacitive diaphragm. An inductive coil with 6 to 12 turns and a 5 mm diameter was wrapped around a plastic fixture for the purpose of inductive coupling. A sensitivity of 7 kHz/Pa was reported for 266 Pa to 10.6 kPa.

![Figure 2-18. An LTCC-based ceramic diaphragm of pressure sensor (adapted from [43]).](image)

A resonant pressure sensor based on low-temperature co-fired ceramic (LTCC) was reported by Fonseca et al. [43]. LTCC is a mixture of glass frit, ceramic powder, and organic binders and has a low curing temperature (i.e., 900°C). A surface inductor and capacitive diaphragm were formed by screen printed silver ink on three layers of ceramic tape, which were then laminated as shown in Figure 2-18. The reported sensitivity of the pressure sensor is 0.001 kHz/Pa for up to 700 kPa. As a glass composition, LTCC becomes conductive as the temperature increases. LTCC is also known to shrink during lamination; therefore, the dimensional accuracy is an issue.

![Figure 2-19. A flexible LCP-based pressure sensor (adapted from [44]).](image)
A flexible wireless pressure sensor based on liquid crystal polymer (LCP) was later reported by Fonseca et al. [44]. A PTFE spacer layer to define a capacitive gap with two layers of LCP to define a planar spiral inductor were laminated as shown in Figure 2-19. The reported sensitivity is 0.04 kHz/Pa for 3.8 to 400 kPa. The flexible pressure sensor was implanted in a canine with a reference sensor to simulate an abdominal aortic aneurysm; the resulting artifacts matched well with the reference sensor [44]. Chen et al. also demonstrated a flexible intraocular pressure sensor [45]. The pressure sensor was made of Parylene C, which is known for its flexibility and biocompatibility. A 4 mm diameter coil with a 20 mm separation distance between sensor and reader resulted in a 1.2 kHz/Pa sensitivity for 333 Pa to 13.3 kPa.

![Figure 2-20. An intraocular SU-8 polymer-based pressure sensor (adapted from [46]).](image)

An intraocular pressure sensor developed with SU-8 was reported by Xue et al. [46]. SU-8 is a biocompatible epoxy-based photoresist commonly used in microfluidics. The inductor coil and capacitive electrodes are formed of thick-film gold (Au) embedded in seven layers of SU-8 as shown in Figure 2-20. A 2.25 mm coil external antenna was used to interact with the sensor at a 2 mm distance. The sensitivity of the pressure sensor was reported as 8.1 kHz/Pa for 133 Pa to 7.9 kPa. An assembled implantable pressure sensor with a micro-needle was reported by Chitnis et al. [47]. A 5 mm
diameter polyimide antenna coil was assembled with a MEMS capacitive pressure sensor and a 5 mm long stainless steel needle for minimal invasive intraocular pressure monitoring. The sensitivity of the pressure sensor was reported as 0.11 kHz/Pa for 133 Pa to 6.6 kPa.

A biodegradable wireless pressure sensor was demonstrated by Luo et al. [48]. An electroplated Zn/Fe bilayer was used for the inductor coil and capacitive electrodes of the diaphragm. The diaphragm and structural surrounds were constructed from poly-L-lactic acid (PLLA). A spacer layer for the diaphragm was constructed by laminating PLLA/PCL with PLLA. The resonant frequency of the pressure sensor is 31.9 MHz with a reported sensitivity of 39 kHz/kPa for up to 20 kPa. A high-temperature co-fired ceramic (HTCC) resonant pressure sensor was reported by Tan et al. [49]. HTCC is an alumina ceramic with a high curing temperature (i.e., 1600°C), high operating temperature, and high thermal dissipation. Three layers of ceramic with an inductor and capacitive electrodes formed by screen printing are laminated. The reported sensitivity of the pressure sensor is 3.5 kHz/kPa for 100 to 500 kPa.

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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Fonseca et al. [43]</td>
<td>DC</td>
<td>Ind. Coup.</td>
<td>49.3</td>
<td>26.4</td>
<td>0.001</td>
<td>-</td>
<td>400°C</td>
</tr>
<tr>
<td>Collins [42]</td>
<td>DC</td>
<td>Ind. Coup.</td>
<td>45.7</td>
<td>120</td>
<td>3</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>Fonseca et al. [44]</td>
<td>DC</td>
<td>Ind. Coup.</td>
<td>40.4</td>
<td>35.6</td>
<td>0.04</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Xue et al. [46]</td>
<td>DC</td>
<td>Ind. Coup.</td>
<td>35.5</td>
<td>250</td>
<td>8.1</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>Chitnis et al. [47]</td>
<td>DC</td>
<td>Ind. Coup.</td>
<td>33.9</td>
<td>63.7</td>
<td>0.11</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Chen et al. [45]</td>
<td>DC</td>
<td>Ind. Coup.</td>
<td>32.0</td>
<td>350</td>
<td>1.2</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Rosengren et al. [41]</td>
<td>DC</td>
<td>Ind. Coup.</td>
<td>32.0</td>
<td>-</td>
<td>0.007</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Luo et al. [48]</td>
<td>DC</td>
<td>Ind. Coup.</td>
<td>21.7</td>
<td>31.9</td>
<td>0.039</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>Tan et al. [49]</td>
<td>DC</td>
<td>Ind. Coup.</td>
<td>13.9</td>
<td>15.2</td>
<td>0.003</td>
<td>-</td>
<td>600°C</td>
</tr>
</tbody>
</table>
A summary of the aforementioned passive wireless pressure sensors is shown in Table 2-3. All of the referenced passive wireless pressure sensors use inductive coupling as the method for wireless sensing. The only passive wireless pressure sensors to be demonstrated at high temperatures are Fonseca [43] at 400°C and Tan [49] at 600°C. Inductive coupling is well-known for wireless sensing; however, the separation distance is limited to 10x the coil diameter. A 2.5 mm diameter coil, for example, has a maximum separation distance of 25 mm (i.e., 1 in.). The material properties of a sensor are often sensitive to temperature. It is not uncommon for a sensor to be more sensitive to temperature than other desired measurands.

### 2.4 Literature Review of Passive Wireless Temperature Sensors

As stated earlier, high-temperature environments such as industrial gas turbines require sensors capable of operating at temperatures in excess of 1000°C. The electrical and mechanical properties of materials are inherently functions of temperature. The electronic output of a sensor will drift as a function of temperature. As a result, it’s often easier to design a sensor to measure temperature than some other desired measurand. A number of unique approaches have been used to measure temperature wirelessly.

A capacitively loaded slot antenna was demonstrated by Scott et al. at 19 GHz [50]. An interdigitated bimorph with cantilever beams lined up along a \( \frac{\lambda}{2} \) slot determine the reflection characteristics of an incident wave. A CTE mismatch between Au and SiO\(_2\) results in the cantilever beams naturally deflecting upwards. As the temperature increases, however, the beams deflect downward, which increases the
capacitance and decreases the resonance of the slot. The slot antenna is capable of operating up to 300°C.

Another way to measure temperature is with a surface acoustic wave (SAW) sensor. An interdigitated comb structure on a piezoelectric medium is used to transduce an incident electrical wave into a propagating mechanical wave. The mechanical wave propagates across a delay line and reflects off a reflector. The mechanical wave is then transduced back into an electrical wave. The material properties of the piezoelectric medium are a function of temperature, which results in a change in the properties of the mechanical wave. Piezoelectric materials such as aluminum nitride and langasite (La$_3$Ga$_5$SiO$_{14}$) have high melting temperature and lack phase transition up to their melting points; they are considered good candidates for high-temperature sensors.

![Interrogation of a SAW sensor using FSCW radar](image)

Figure 2-21. Interrogation of a SAW sensor using FSCW radar (adapted from [51]).

A SAW sensor on langasite was demonstrated by Canabal et al. up to 750°C [51]. Langasite is a piezoelectric material with no phase transitions up to its melting point of 1470°C. A frequency-stepped continuous wave (FSCW) radar was used to interrogate the sensor as shown in Figure 2-21. FSCW is a frequency sweep in time (i.e., chirp) by discrete steps. A change in the amplitude, frequency, and/or phase due to temperature
changes is observed in the reflected signal. An array of SAW sensors on langasite was later reported by Canabal et al. up to 900°C [52]. A sensor array with six SAW sensors on an alumina substrate was interrogated with a chirp. The group delay (i.e., change in phase over change in frequency or $\tau_g = -d\phi/d\omega$) of the reflected signal from the array was observed. The sensitivity of the SAW sensor is inferred from the group delay to be as high as 60 ppm/°C at 900°C.

![Diagram of SAW sensor array](image)

Figure 2-22. Integrated temperature and pressure SAW sensors (adapted from [53]).

The independent measurement of temperature with a sensor that has a high sensitivity to temperature and low sensitivity to pressure for accurate pressure measurement in a high-temperature environment. A novel SAW sensor with multiple reflectors for simultaneous temperature and pressure measurement was reported by Lee et al. [53]. A SAW with a top reflector for measuring pressure and a bottom reflector for independently measuring temperature is shown in Figure 2-22. The temperature sensitivity was reported as 10°/°C up to 200°C and the pressure sensitivity as 2.9°/kPa up to 550 kPa.

A temperature sensor using a parallel plate capacitor was demonstrated by Birdsell et al. up to 1000°C [54]. The inductor coil and capacitive plates were screen printed with platinum on two alumina substrates, which were then laminated with a third alumina substrate at 1400°C. The resonant frequency was reported as 29.4 MHz at 200°C with a temperature sensitivity of 3 kHz/°C up to 1000°C. An electrical resonator
with a planar interdigitated capacitor and an inductive coil was also demonstrated by Birdsell et al. [54]. An 10 \( \mu \text{m} \) inductor coil was electroplated with nickel and platinum was screen printed and laser etched to define electrodes of a co-planar interdigitated capacitor as shown in Figure 2-23. A hole is laser etched through the alumina in the center of the inductor to electrically connect the inductor coil. The resonant frequency was determined as 52 MHz and the temperature sensitivity is 4.7 kHz/˚C up to 800˚C.

![Figure 2-23. An interdigitated surface capacitor and inductor pressure sensor (adapted from [54]).](image)

A dielectrically loaded cavity resonator using silicon borocarbonitride (SiBCN) was reported by Ren et al. [55]. The SiBCN resonator as shown in Figure 2-24 is centered on a coplanar waveguide (CPW) transmission line with a two-port network analyzer attached to connectors on both ends of the CPW. The transmission spectra of the SiBCN resonator was used to determine a sensitivity of 360 kHz/˚C up to 1000˚C. Cheng et al. later demonstrated a wireless temperature sensor using an integrated SiBCN cylindrical resonator-antenna up to 1300˚C [56]. A cylindrical SiBCN cavity was coated with a platinum paste, then dried and sintered. A slot antenna was then milled out of the surface of the cylindrical cavity. The resonant frequency was measured as 10.5 GHz with a temperature sensitivity of 470 kHz/˚C up to 1300˚C.
A resonator-antenna was reported by Cheng et al. for wireless temperature sensing up to 1000°C [57]. A slot resonator was fabricated by coating an alumina substrate with a platinum paste, which was then dried and sintered. A slot was then milled out of the surface. The resonant frequency was reported as 5.1 GHz with a temperature sensitivity of 410 kHz/°C up to 1000°C. A reflective patch antenna on alumina was later reported by Cheng et al. up to 1050°C [58]. A platinum paste was patterned on the front of an alumina substrate to form a reflective patch and on the back to form a ground plane as shown in Figure 2-25. The paste was then dried and sintered. The resonant frequency was reported as 5.1 GHz with a temperature sensitivity of 480 kHz/°C up to 1050°C.

A summary of the discussed passive wireless temperature sensors is shown in Table 2-4. More than half of the referenced temperature sensors use RF backscattering for sensing, while a few use inductive coupling. The maximum operating temperatures...
for existing wireless temperature sensors is reported up to 1300°C [56]. SAW sensors have been demonstrated at separation distances of 15 cm [51] and 30 cm [53]. A number of the referenced temperature sensors use some form of a $\lambda/4$ slot for resonant-based sensing.

Table 2-4. Summary of passive wireless temperature sensors.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chen et al. [56]</td>
<td>RF Backscatter</td>
<td>1300°C</td>
<td>10.5 GHz</td>
<td>2.5</td>
<td>470</td>
</tr>
<tr>
<td>Cheng et al. [58]</td>
<td>RF Backscatter</td>
<td>1050°C</td>
<td>5 GHz</td>
<td>3.0</td>
<td>480</td>
</tr>
<tr>
<td>Ren et al. [55]</td>
<td>-</td>
<td>1000°C</td>
<td>11 GHz</td>
<td>-</td>
<td>360</td>
</tr>
<tr>
<td>Cheng et al. [57]</td>
<td>RF Backscatter</td>
<td>1000°C</td>
<td>5.1 GHz</td>
<td>3.0</td>
<td>410</td>
</tr>
<tr>
<td>Birdsell et al. [54]</td>
<td>Ind. Coupling</td>
<td>1000°C</td>
<td>29 MHz</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>Canabal et al. [52]</td>
<td>RF Backscatter</td>
<td>900°C</td>
<td>200 MHz</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>Birdsell et al. [54]</td>
<td>Ind. Coupling</td>
<td>800°C</td>
<td>52 MHz</td>
<td>-</td>
<td>4.7</td>
</tr>
<tr>
<td>Canabal et al. [51]</td>
<td>RF Backscatter</td>
<td>750°C</td>
<td>227 MHz</td>
<td>15</td>
<td>-</td>
</tr>
<tr>
<td>Birdsell et al. [59]</td>
<td>Ind. Coupling</td>
<td>675°C</td>
<td>47 MHz</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Scott et al. [50]</td>
<td>RF Backscatter</td>
<td>300°C</td>
<td>19 GHz</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Lee et al. [53]</td>
<td>RF Backscatter</td>
<td>200°C</td>
<td>440 MHz</td>
<td>30</td>
<td>-</td>
</tr>
</tbody>
</table>

2.5 Summary and Proposed Sensor

Inductive coupling, RF backscattering, and radar are three methods for RF wireless sensing. Inductive coupling operates in the near-field and is limited to separation distances less than 10x the diameter of the inductor. RF backscattering operates in the far-field and has a power decrease of $1/R^2$. Radar also operates in the far-field, however, the primary difference between radar and RF backscattering is radar is determining physical information about an object. Existing radar electronics are limited to displacement accuracies of 5 $\mu$m [40], which is unacceptable for maximum displacements on the order of 1 $\mu$m for MEMS.

Existing passive wireless pressure sensors designed for high-temperature environments have been demonstrated based on LTCC at 400°C [43] and HTCC at
The electrical and mechanical properties of a material are functions of temperature such that it’s often easier to design a temperature sensor. Existing passive wireless temperature sensors have been demonstrated on alumina up to 1300°C [56]. The need to sense dynamic pressure in high-temperature environments up to 1000°C makes passive wireless sensors a viable option. A novel passive wireless sensor is proposed for dynamic pressure sensing.

Figure 2-26. An Illustration of bottom and cross-sectional views of proposed passive wireless dynamic pressure sensor.

The proposed dynamic pressure sensor includes a reflective patch antenna with a slot on a circular diaphragm as shown in Figure 2-26. An air gap separates the diaphragm from the electrical ground plane to form a capacitive element. An incident electromagnetic wave interacts with a patch antenna and capacitive element such that a portion of the incident electromagnetic wave is reflected (i.e., backscattered) to a receiver. The strength of the reflected electromagnetic wave is a function of the
capacitive gap as well as the physical area of the antenna; for this reasoning the wireless sensing can be considered a combination of RF backscattering and radar.

The proposed sensor is inherently both passive (i.e., no local internal energy sources) and wireless (i.e., wired local interface electronics). The passive wireless sensor electrically operates at Ku-band frequencies around 15 GHz (i.e., 2 cm wavelength in free-space). Two-way wireless sensing in free-space has a $1/R^4$ power loss. Furthermore, operating in a gas turbine requires electromagnetic interaction with the sensor through the turbine wall. An electromagnetic waveguide is utilized to propagate bounded electromagnetic waves for interacting with the sensor. The sensor is placed at a distance $R > 2\lambda$ down the waveguide from the radiator such that it operates in the far-field as opposed to reported pressure sensors that are inductively coupled in the near-field. The sensor has a high-temperature compatible sapphire diaphragm and platinum-based electrical resonator. The design and fabrication aspects of the sensor are further discussed in Chapter 3 and Chapter 4, respectively.
Chapter 3 discusses sensor design aspects including discussion of high-
temperature material properties (i.e., mechanical, electrical, and thermal
characteristics). The machinability of sapphire is also discussed including using
traditional wet and dry microfabrication methods as well as laser-machining. Finally,
lumped element modeling of the dynamic pressure sensor’s acousto-mechanical
elements (i.e., diaphragm, cavity, and vent) and electrical elements (i.e., inductance and
capacitance) is discussed.

3.1 Sensor Materials

The material properties (e.g., mechanical, electrical, and thermal) of a material at
elevated temperatures is of utmost importance when designing a high-temperature
sensor. There are also trade-offs between the compatibility of a material with high
temperatures and the materials machinability. Silicon is the material of choice for most
microelectronics and MEMS devices due to its high abundance (i.e., low-cost), well-
known machinability, and unique electronic properties. Silicon is not a good candidate
for high-temperature environments as its electronic band gap [60] and Young’s modulus
[61] are strong functions of temperature. The yield strength (i.e., stress at which plastic
deformation begins) of silicon varies based on the type of silicon. Extrinsic p-type silicon
(i.e., high resistivity) has a higher yield strength than intrinsic silicon. Intrinsic silicon has
a higher yield strength than n-type silicon (i.e., low resistivity) [62]. The yield strength of
silicon also varies as a function of temperature. Silicon is known to exhibit plastic
deformation at temperatures as low as 275°C [62]. The maximum electronic operating
temperature for silicon is 150°C [63]. For high-temperature environments requiring electronics alternatives to silicon have been sought.

### 3.1.1 High-Temperature Semiconductors

The demand for high-speed, high-temperature microelectronics has inarguably pushed the development of many exotic ceramics. Ceramic materials are attractive for high-temperature environments since they retain their strength and resist corrosion [64]. Ceramic materials have high moduli (e.g., Young’s, shear, bulk); however, they are brittle since they have no ductility (i.e., ability to accommodate stress by redistribution). Ceramics are classified as either electrically insulating or semiconducting. Insulating ceramics have a high electrical resistivity (i.e., $\rho > 10^5$ $\Omega$·cm), whereas semiconducting ceramics have a low-to-medium electrical resistivity (i.e., $10^{-3} < \rho < 10^5$ $\Omega$·cm) [65].

<table>
<thead>
<tr>
<th>Ceramic</th>
<th>$\rho_m$ g/cc</th>
<th>$E$ GPa</th>
<th>MHS</th>
<th>CTE ppm/°C</th>
<th>$T_m$ °C</th>
<th>$k$ W/m·K</th>
<th>$\rho_e$ $\Omega$·cm</th>
<th>$\varepsilon_r$</th>
<th>$E_g$ eV</th>
</tr>
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<tbody>
<tr>
<td>AlN [66]</td>
<td>3.33</td>
<td>320</td>
<td>7</td>
<td>4.7</td>
<td>2227</td>
<td>180</td>
<td>$10^{13}$</td>
<td>9.0</td>
<td>6.02</td>
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<tr>
<td>Sapphire</td>
<td>3.97</td>
<td>435</td>
<td>9</td>
<td>7.9</td>
<td>2053</td>
<td>40</td>
<td>$10^{16}$</td>
<td>9.3</td>
<td>9.9</td>
</tr>
<tr>
<td>(⊥ to c-axis) [67]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diamond, C [66]</td>
<td>3.51</td>
<td>1200</td>
<td>10</td>
<td>1.2</td>
<td>4027</td>
<td>2000</td>
<td>$10^{14}$</td>
<td>5.5</td>
<td>5.4</td>
</tr>
<tr>
<td>GaAs [66]</td>
<td>5.31</td>
<td>85</td>
<td>4.5</td>
<td>5.4</td>
<td>1238</td>
<td>46</td>
<td>$10^7$</td>
<td>13.2</td>
<td>1.35</td>
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<tr>
<td>Ge [66]</td>
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<td>150</td>
<td>9</td>
<td>3.1</td>
<td>1227</td>
<td>65</td>
<td>$10^6$</td>
<td>5.3</td>
<td>3.34</td>
</tr>
<tr>
<td>GaP [66]</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GaN [66]</td>
<td>5.32</td>
<td>130</td>
<td>6</td>
<td>6.1</td>
<td>937</td>
<td>64</td>
<td>$10^4$</td>
<td>6</td>
<td>0.67</td>
</tr>
<tr>
<td>InAs [66]</td>
<td>5.66</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>InP [66]</td>
<td>4.78</td>
<td>61</td>
<td>3</td>
<td>4.6</td>
<td>1057</td>
<td>80</td>
<td>$10^6$</td>
<td>12.4</td>
<td>1.27</td>
</tr>
<tr>
<td>Si $&lt;110&gt;$  [66], [68]</td>
<td>2.32</td>
<td>170</td>
<td>7</td>
<td>2.5</td>
<td>1412</td>
<td>124</td>
<td>$10^2$</td>
<td>11.8</td>
<td>1.10</td>
</tr>
<tr>
<td>6H-SiC [66], [69]</td>
<td>3.21</td>
<td>410</td>
<td>9.5</td>
<td>5.1</td>
<td>2797</td>
<td>41</td>
<td>$10^8$</td>
<td>10.2</td>
<td>2.86</td>
</tr>
<tr>
<td>SiGe (1:1) [70]</td>
<td>3.82</td>
<td>4.2</td>
<td></td>
<td>1176</td>
<td>8.3</td>
<td></td>
<td>$10^5$</td>
<td>13.9</td>
<td>0.94</td>
</tr>
<tr>
<td>Si$_3$N$_4$ [69], [70]</td>
<td>3.31</td>
<td>317</td>
<td>9</td>
<td>3.4</td>
<td>1900</td>
<td>27</td>
<td>$10^{14}$</td>
<td>7.5</td>
<td>5.0</td>
</tr>
<tr>
<td>ZnO [66]</td>
<td>5.66</td>
<td>111</td>
<td>4</td>
<td>2.9</td>
<td>1977</td>
<td>60</td>
<td>$10^8$</td>
<td>10.8</td>
<td>3.2</td>
</tr>
</tbody>
</table>

$\rho_m$ - material density, $E$ - Young’s modulus, MHS - Moh’s hardness scale, CTE - coefficient of thermal expansion, $T_m$ - melting temperature, $k$ - thermal conductivity, $\rho_e$ - intrinsic electrical resistivity, $\varepsilon_r$ - relative permittivity, $E_g$ - band gap energy
There is no known direct relationship between the electronic band gap and the intrinsic resistivity; however, generally speaking materials with higher electronic band gaps tend to be insulating, whereas materials with lower electronic band gaps tend to be semiconducting as shown in Table 3-1.

Semiconductors are unique materials in that they may be doped to significantly alter their electronic properties [71]. For high-temperature electronics silicon carbide (SiC) is considered the material of choice. SiC has a high thermal conductivity, electric field breakdown, maximum current density, melting temperature, and is highly inert (i.e., does not chemically react with other materials). The three major crystalline forms of SiC are the cubic 3C-SiC ($\beta$-SiC), the hexagonal 4H-SiC, and the hexagonal 6H-SiC ($\alpha$-SiC). NASA’s Glenn Research Center (GRC) [72] is at the forefront of research and development of high-temperature electronics based on SiC. NASA GRC, for example, has demonstrated 6H-SiC JFET logic gates operating up to 600°C [13], long-term operation of a 6H-SiC JFET operating at 500°C for up to 10,000 hrs [73], and a 10 W RF oscillator based on SiC MESFETs on an alumina substrate up to 475°C [74].

In 2001 Werner et al. reported silicon carbide as a promising material for high-temperature electronics; however, high-quality oxides, wafer defects from micropipes, and cost were highlighted as issues limiting widespread use [75]. The maximum electronic operating temperature for silicon carbide is in excess of 600°C, whereas silicon is limited to 150°C [63]. The process maturity of silicon, however, is much higher than that of silicon carbide.

Several figures of merit have been developed for selecting the best semiconducting material for high-temperature electronics. The Johnson’s figure of merit
(JFOM) [76], for example, puts an emphasis on the power handling and high-frequency capabilities. The JFOM relates the electric field breakdown \( E_B \) and the saturated drift velocity \( v_{sat} \) of a semiconductor as

\[
JFOM = \frac{E_B^2 v_{sat}^2}{4\pi^2}.
\]  (3-1)

The Keyes’ figure of merit (KFOM) [77], on the other hand, accounts for the thermal conductivity, which becomes important during high-frequency operation. The KFOM relates the thermal conductivity \( \kappa \), saturated drift velocity \( v_{sat} \), and the dielectric constant \( \varepsilon \) of a semiconductor as

\[
KFOM = \kappa \frac{\sqrt{v_{sat}}}{4\pi \varepsilon}.
\]  (3-2)

The JFOM and KFOM are not considered accurate for low-frequency power electronics. The Baliga’s figure of merit (BFOM) [78] accounts for the permittivity and permeability, which are important at low-frequency operation. The BFOM relates the dielectric constant (i.e., \( \varepsilon = \varepsilon_r \varepsilon_o \)), carrier mobility (i.e., \( \mu = \mu_r \mu_o \)), and electric field breakdown \( E_B \) of a semiconductor as

\[
BFOM = \varepsilon \mu E_B^3.
\]  (3-3)

The JFOM, KFOM, and BFOM are applied to several wide-bandgap semiconducting materials as shown in Table 3-2. The figures of merit are normalized to silicon for comparison purposes. The figures of merit should not be mistaken for signifying that one material is better than another; they are only intended for relative comparison of materials. The figures of merit all predict that diamond has the best properties for high-temperature electronics. The figures of merit, however, are only
intended for comparing the electronic properties of materials and therefore have little usefulness in determining a material with good mechanical properties.

Table 3-2. Electrical properties of common wide-bandgap semiconductors

<table>
<thead>
<tr>
<th>SC</th>
<th>$E_g$ $^1$</th>
<th>$E_b$ $^2$</th>
<th>$\mu_n$ $^3$</th>
<th>$\mu_p$ $^4$</th>
<th>$v_{sat}$ $^5$</th>
<th>$k$ $^6$</th>
<th>$\varepsilon_r$ $^7$</th>
<th>JFOM</th>
<th>KFOM</th>
<th>BFOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>1.12</td>
<td>0.41</td>
<td>1450</td>
<td>480</td>
<td>0.86</td>
<td>1.5</td>
<td>11.7</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>GaAs</td>
<td>1.42</td>
<td>0.48</td>
<td>8600</td>
<td>130</td>
<td>0.72</td>
<td>0.46</td>
<td>12.9</td>
<td>7.11</td>
<td>0.43</td>
<td>15.3</td>
</tr>
<tr>
<td>4H-SiC</td>
<td>3.26</td>
<td>2.4</td>
<td>900</td>
<td>120</td>
<td>2</td>
<td>4.9</td>
<td>10</td>
<td>178</td>
<td>4.62</td>
<td>156</td>
</tr>
<tr>
<td>6H-SiC</td>
<td>3.03</td>
<td>2.4</td>
<td>400</td>
<td>100</td>
<td>2</td>
<td>4.9</td>
<td>9.7</td>
<td>256</td>
<td>4.62</td>
<td>116</td>
</tr>
<tr>
<td>GaN</td>
<td>3.29</td>
<td>5</td>
<td>2000</td>
<td>200</td>
<td>2.5</td>
<td>1.3</td>
<td>5.3</td>
<td>1740</td>
<td>1.37</td>
<td>2900</td>
</tr>
<tr>
<td>Diamond</td>
<td>5.47</td>
<td>20</td>
<td>4500</td>
<td>3800</td>
<td>2.7</td>
<td>20</td>
<td>5.7</td>
<td>32400</td>
<td>21.9</td>
<td>44400</td>
</tr>
</tbody>
</table>

$^1$ band gap energy [eV], $^2$ maximum electric field [MV/cm], $^3$ electron mobility [cm$^2$/V.s], $^4$ hole mobility [cm$^2$/V.s], $^5$ saturation velocity [cm/s], $^6$ thermal conductivity [W/m.K], $^7$ relative permittivity

3.1.2 Mechanical Characteristics

The strength of a material is determined by its Young’s modulus, yield strength, shear modulus, etc. The Young’s modulus, for example, relates an applied tensile stress $\sigma$ to the resulting extensional strain $\varepsilon$ as

$$ E = \frac{\sigma}{\varepsilon} = \frac{F/A_0}{\Delta L/L_o} $$

(3-4)

The Young’s modulus is constant for a small applied tensile stress $\sigma$; however, the resulting extensional strain $\varepsilon$ increases at a faster rate than the applied tensile stress beyond the yield point of a material. The specific modulus of a material $E/\rho$ is useful for comparing the strength of materials of varying densities. The specific modulus as a function of melting temperature for an assortment of ceramics (both insulating and semiconducting) and metals is shown in Figure 3-1. The specific modulus is on a log scale to separate the best choice materials into the upper-right quadrant. The best choices in increasing order of strength include silicon nitride (Si$_3$N$_4$), aluminum nitride (AlN), sapphire (Al$_2$O$_3$), silicon carbide (SiC), and diamond (C).
Diamond is superior to other materials due to its strong covalent bonding of carbon in a cubic crystal structure. Diamond exhibits excellent mechanical strength, thermal conductivity, wear resistance, and resistance to corrosion. The deposition of diamond thin-films in MEMS has shown promise for high-temperature and anti-stiction applications. Diamond thin-films have also been used to develop MEMS pressure sensors and accelerometers [80]. Diamond thin-films, however, exhibit high surface roughness due to chemical vapor deposition (CVD), which has limited the development of diamond MEMS [81].

The strength of a material at elevated temperatures is useful in predicting temperature drifting behavior. The Young’s modulus for materials, for example, tends to decrease as a function of temperature. The Young’s moduli for silicon carbide, silicon nitride [82], aluminum nitride [82], sapphire [83], and silicon <100> [61] are shown in...
Figure 3-2. The Young’s modulus for sapphire, for example, decreases five percent from room temperature to 1000K (726°C). A decrease in the Young’s modulus results an increase in the compliance of a pressure sensor’s diaphragm.

Figure 3-2. Normalized Young’s modulus as a function of temperature for select semiconductors [61], [82], [83].

3.1.3 Electrical Characteristics

The electrical resistivity of a material is of importance for sensors with electrical elements (i.e., capacitors, inductors, transistors, etc.). The electrical resistivity of a material is related to the measured resistance $R$ across some defined geometry with cross-sectional area $A$ and length $L$ as

$$\rho = \frac{RA}{L} \quad (3-5)$$

A high resistivity medium is often desired for high signal integrity in electronics; however, high resistivity doped semiconductors are considerably more expensive than intrinsic semiconductors. The resistivity changes as a function of temperature by the
temperature coefficient of resistance $\alpha$ of a material as $\rho = \rho_o(1 + \alpha(T - T_o))$. The temperature coefficient of resistance $\alpha$ for semiconductors is typically negative such that the resistivity decreases as a function of temperature. The electrical resistivity as a function of temperature for diamond [84], aluminum nitride [85], sapphire [84], and silicon nitride [86] is shown in Figure 3-3. Sapphire has a superior electrical resistivity as a function of temperature with respect to the referenced semiconductors. An electrical resistivity as high as $1 \text{ M}\Omega\cdot\text{cm}$ for sapphire is predicted up to $1800\text{K}$ ($1526^\circ\text{C}$).

![Figure 3-3. Electrical resistivity as a function of temperature for semiconductors [84]–[86].](image)

3.1.4 Thermal Characteristics

The thermal properties of a material are also important in sensor design as they determine how well a material can transfer heat. The thermal resistivity (i.e., ability to spread heat) of a material, for example, is related to the thermal resistance $R$ across some defined geometry with cross-sectional area $A$ and length $L$ as
\( \kappa^{-1} = R \frac{A}{L} \)  \hspace{1cm} (3-6)

High power densities without adequate thermal relief over time lead to degraded performance or device failure in electronics. The thermal resistivity as a function of temperature for diamond [87], aluminum nitride [88], sapphire [89], and silicon nitride [90] is shown in Figure 3-4. The thermal resistivity of diamond is superior with respect to the referenced semiconductors. Sapphire, on the other hand, has a much higher electrical resistivity at elevated temperatures than diamond, which is of utmost important for a passive wireless RF sensor.

![Graph showing thermal resistivity as a function of temperature for semiconductors](image)

Figure 3-4. Thermal resistivity as a function of temperature for semiconductors [87]–[90].

Material deformation can occur due to a change in applied force and/or temperature. The spontaneous intermixing of atoms also known as diffusion is a contributor to material deformation at elevated temperatures. The diffusion coefficient (i.e., mass diffusivity) \( D \) describes the rate of intermixing. The diffusion coefficient is
related to the diffusion constant $D_0$, activation energy of the creep mechanism $Q$, Boltzmann’s constant $k_B$, and the temperature $T$ as [91]

$$D = D_0 e^{-Q/k_B T}. \quad (3-7)$$

The mean distance that an atom travels is related to the diffusion coefficient $D$ and the mixing time $t$ as $x \approx \sqrt{Dt}$. The diffusion of atoms over time yields material deformation in the form of strain. The progressive deformation of a material over time is known as power-law creep. The general creep equation, which describes the rate of strain on a material, is related to a material constant $C$, applied tensile stress $\sigma$, the grain size of the material $d$, and the diffusion coefficient $D$ as [91]

$$\frac{d\varepsilon}{dt} = C \frac{\sigma}{d^2} D = C \frac{\sigma}{d^2} D_0 e^{-Q/k_B T}. \quad (3-8)$$

![Figure 3-5. Diffusion coefficient and creep rate as a function of temperature for semiconductors [92]–[101].](image)
A lower diffusion coefficient results in a lower rate of strain for a given applied tensile stress and temperature. The diffusion coefficients for silicon [92], gallium arsenide [93], gallium nitride [94], and sapphire [95] alloyed with platinum as a function of temperature are shown in Figure 3-5. The platinum-sapphire alloy has the lowest diffusion coefficient. The creep rates for germanium [96], silicon [97], silicon carbide [98], aluminum nitride [99], silicon nitride [100], and sapphire [101] are also shown in Figure 3-5. For elevated temperatures it is inferred that sapphire has the lowest creep rate based on the line trends.

3.1.5 Machinable Characteristics

The additive and subtractive manufacturing of silicon is well-developed primarily due to the high-demand semiconductor market. Additive processes include thin-film deposition by CVD, thermal oxidation, or physical vapor deposition (PVD) [21]. Subtractive processes include wet (i.e., isotropic) and dry (i.e., anisotropic) etching. The wet etching of silicon and silicon-based thin-films is well-known [102], [103]. A highly anisotropic dry etching method for achieving high aspect ratios as high as 30:1 in silicon [104] is known as deep reactive ion etching (DRIE). DRIE etches silicon by successively passivating the surface with carbon tetrafluoride (CF₄), then etching the silicon with sulfur hexafluoride (SF₆) [105]. The reaction of SF₆ with silicon (Si) produces volatile silicon tetrafluoride (SiF₄). Alternative ceramic materials, however, do not react as well with SF₆. An alternative to wet and dry etching is laser machining.

Laser machining uses a concentrated beam of energy to vaporize material by continuous or pulsed ablation [106]. Continuous ablation removes material by continuous melting; however, yields severe thermal damage to the material due to large
heat-affected zones (HAZ). Pulsed ablation, on the other hand, removes material by applying high-energy pulses of some limited duration. The thermal and optical penetration depths determine whether a material is most effectively machined using long-pulse or short-pulse ablation. The thermal penetration depth, in m, is related to the thermal diffusivity $\alpha$ and the diffusion time $t$ (i.e., pulse duration) as

$$d = 2\sqrt{\alpha t}.$$  \hspace{1cm} (3-9)

The thermal diffusivity, in m$^2$/s, is related to the thermal conductivity $k$, the density $\rho$, and the specific heat capacity $c_p$ of the material as

$$\alpha = \frac{k}{\rho c_p}.$$  \hspace{1cm} (3-10)

The optical penetration depth, in m, on the other hand, is related to optical absorption coefficient $\alpha$ as

$$\delta = \frac{2}{\alpha}.$$  \hspace{1cm} (3-11)

The optical absorption coefficient, in m$^{-1}$, is related to the extinction coefficient $\kappa_e$ of the material and the laser’s wavelength $\lambda$ as

$$\alpha = \frac{4\pi \kappa_e}{\lambda}.$$  \hspace{1cm} (3-12)

Laser machining of sapphire, for example, is most effectively machined with short-pulse ablation since sapphire has a thermal penetration depth (i.e., 24.4 nm) less than the optical penetration depth (i.e., 72.3 $\mu$m) [106]. Long-pulse ablation removes material by melt expulsion and results in material re-solidification on the surface and has a pulse duration of $\mu$s to ms. Short-pulse ablation, on the other hand, removes material by direct vaporization and has a pulse duration of ns to ps. The etching rate for pulsed ablation is
controlled by the pulse fluence (i.e., intensity), pulse duration, repetition rate, spot size, moving speed, etc. of the laser source [107].

Table 3-3. Reported etching of selected ceramic materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Method</th>
<th>Etchant</th>
<th>Etch Rate</th>
<th>AR</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlN [108], [109]</td>
<td>Laser</td>
<td>KrF (λ = 248 nm)</td>
<td>150 nm/pulse</td>
<td>12:1</td>
</tr>
<tr>
<td>Al₂O₃ [110], [111]</td>
<td>Laser</td>
<td>ND:YAG, λ = 355 nm</td>
<td>200 nm/pulse</td>
<td>11:1</td>
</tr>
<tr>
<td>Al₂O₃ [110], [111]</td>
<td>ICP</td>
<td>BCl₃/CH₂Cl₂</td>
<td>100 nm/min</td>
<td>8:1</td>
</tr>
<tr>
<td>GaAs [112]</td>
<td>RIE</td>
<td>BCl₃/Cl₂/SiCl₄</td>
<td>800 nm/min</td>
<td>20:1</td>
</tr>
<tr>
<td>GaN [109], [113]</td>
<td>Laser</td>
<td>ND:YVO₄, λ = 266 nm</td>
<td>2000 nm/pulse</td>
<td>11:1</td>
</tr>
<tr>
<td>GaN [109], [113]</td>
<td>ECR</td>
<td>Cl₂/H₂/CH₄/Ar</td>
<td>160 nm/min</td>
<td>5:1</td>
</tr>
<tr>
<td>InN [109]</td>
<td>ECR</td>
<td>Cl₂/H₂/CH₄/Ar</td>
<td>100 nm/min</td>
<td>4:1</td>
</tr>
<tr>
<td>InP [114], [115]</td>
<td>Laser</td>
<td>ND:YVO₄, λ = 355 nm</td>
<td>500 nm/pulse</td>
<td>High</td>
</tr>
<tr>
<td>InP [114], [115]</td>
<td>ECR</td>
<td>Cl₂/Ar</td>
<td>2700 nm/min</td>
<td>14:1</td>
</tr>
<tr>
<td>Si [114], [116]</td>
<td>Laser</td>
<td>ND:YVO₄, λ = 355 nm</td>
<td>1200 nm/pulse</td>
<td>High</td>
</tr>
<tr>
<td>Si [114], [116]</td>
<td>RIE</td>
<td>SF₆</td>
<td>4000 nm/min</td>
<td>20:1</td>
</tr>
<tr>
<td>SiC [117], [118]</td>
<td>Laser</td>
<td>ND:YAG, λ = 355 nm</td>
<td>6 nm/pulse</td>
<td>20:1</td>
</tr>
<tr>
<td>SiC [117], [118]</td>
<td>RIE</td>
<td>SF₆</td>
<td>170 nm/min</td>
<td>13:1</td>
</tr>
<tr>
<td>Si₃N₄ [119]</td>
<td>Laser</td>
<td>ND:YAG, λ = 1064 nm</td>
<td>175 µm/pulse</td>
<td>4:1</td>
</tr>
<tr>
<td>Ti [120]</td>
<td>ICP</td>
<td>CHF₃</td>
<td>500 nm/min</td>
<td>20:1</td>
</tr>
</tbody>
</table>

The laser ablation threshold (i.e., fluency level required to etch a material) is high for ceramic materials with high optical transparencies including wide-bandgap materials. The etch rates and aspect ratios (AR) reported using dry etching and laser machining for a number of ceramics are shown in Table 3-3. Chen et al., for example, laser machined sapphire at a rate of 200 nm/pulse and achieved an aspect ratio of 11:1 using a 355 nm laser [110]. Blood et al. performed significant work in characterizing the laser machining of sapphire using an Oxford Lasers J-355 picosecond laser micromachining station with a 355 nm wavelength Coherent Talisker laser by Coherent [106], [121], [122]. Blood et al. attained a sidewall angle of 4.8° (i.e., aspect ratio of 12:1) by increasing in laser fluence to 22 J/cm² and also observed the aspect ratio generally
increases by increasing the pulse area overlap, laser fluence, and number of passes [106].

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>9.6</td>
<td>1.27</td>
<td>40.0</td>
<td>59.0</td>
<td>4.28</td>
<td>13.3</td>
</tr>
<tr>
<td>40</td>
<td>9.6</td>
<td>1.27</td>
<td>30.0</td>
<td>48.0</td>
<td>4.05</td>
<td>14.1</td>
</tr>
<tr>
<td>30</td>
<td>9.6</td>
<td>1.27</td>
<td>20.5</td>
<td>38.5</td>
<td>4.05</td>
<td>14.1</td>
</tr>
<tr>
<td>20</td>
<td>9.6</td>
<td>1.27</td>
<td>11.0</td>
<td>28.0</td>
<td>3.83</td>
<td>14.9</td>
</tr>
<tr>
<td>50</td>
<td>7.7</td>
<td>1.27</td>
<td>39.5</td>
<td>60.5</td>
<td>4.73</td>
<td>12.0</td>
</tr>
<tr>
<td>50</td>
<td>5.8</td>
<td>1.27</td>
<td>41.0</td>
<td>63.5</td>
<td>5.06</td>
<td>11.3</td>
</tr>
<tr>
<td>50</td>
<td>3.9</td>
<td>1.27</td>
<td>40.5</td>
<td>69.5</td>
<td>6.51</td>
<td>8.7</td>
</tr>
<tr>
<td>50</td>
<td>1.9</td>
<td>1.27</td>
<td>41.5</td>
<td>82.5</td>
<td>9.17</td>
<td>6.2</td>
</tr>
<tr>
<td>50</td>
<td>9.6</td>
<td>2.5</td>
<td>39.5</td>
<td>60.0</td>
<td>4.61</td>
<td>12.4</td>
</tr>
<tr>
<td>50</td>
<td>9.6</td>
<td>5.0</td>
<td>40.0</td>
<td>76.5</td>
<td>8.18</td>
<td>6.9</td>
</tr>
<tr>
<td>50</td>
<td>9.6</td>
<td>7.5</td>
<td>40.5</td>
<td>92.0</td>
<td>11.46</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Mills et al. performed follow-up work in characterizing the laser machining of sapphire tethers [107], [123]. Tethers of 1 mm length and varying widths were laser machined in a 127 µm sapphire. In addition to varying widths, the laser fluence and step down were varied to understand their effects on tether width as shown in Table 3-4. The results show that laser fluency and step down distance have a significant impact on the sidewall angle. Mills et al. also observed that cracks are a concern; however, they may be minimized by reducing laser frequency and scan speed, reducing the number of passes, and switching the spiral path to minimize heat on the outer edges of the features.

### 3.1.6 Material Selection

As discussed earlier, a sensor capable of operating in high-temperature environments must be constructed of materials capable of operating at high
temperatures. Electrically insulating portions of the sensor must also remain as electrical insulators at high temperatures. Sapphire is an excellent candidate for operating in high-temperature environments. The melting temperature of sapphire exceeds 2000°C. The electrical resistivity of sapphire is greater than 1 MΩ.cm up to 1800K (1526°C). Sapphire appears in nature, but is also produced synthetically.

Synthetic sapphire is available in both wafer and die form for processing.

The production of synthetic sapphire was enabled early on using the Verneuli process [124], which was developed for synthesis of gems. The need for high-quality single-crystal sapphire for optical applications, however, led to production of sapphire using the Czochralski process [124]; also commonly used in industry for producing high-quality silicon wafers. A rotating seed holder is first inserted into a crucible of molten alumina, then the seed holder is slowly lifted out of the molten alumina to grow a boule of crystalline sapphire.

Figure 3-6. Sapphire crystal orientation and c-plane view [89].
Sapphire is a hexagonal crystal structure with four primary plane orientations: c-plane (0001), a-plane (11-20), m-plane (10-10), and r-plane (1-102) as shown in Figure 3-6. The A-plane is perpendicular to the C-plane. The M-plane has a 30° angle with the A-plane and is also perpendicular to the C-plane. The R-plane has a 57.6° angle with the C-plane. Sapphire is anisotropic; therefore, the electrical and mechanical properties may vary significantly depending on the crystal orientation. The physical properties of C-plane and A-plane sapphire have predominantly been studied in literature. A literature review of the electrical and mechanical properties of sapphire is shown in Table 3-5.

<table>
<thead>
<tr>
<th>Source</th>
<th>Orientation</th>
<th>( \rho_m ) g/cc</th>
<th>E GPa</th>
<th>( \nu )</th>
<th>( \rho_e ) ( \Omega \cdot \text{cm} )</th>
<th>( \varepsilon_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Almaz Optics [125]</td>
<td>( \perp ) to c-axis</td>
<td>3.98</td>
<td>335</td>
<td>0.25</td>
<td>( 10^{16} )</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>( \parallel ) to c-axis</td>
<td>-</td>
<td></td>
<td></td>
<td>11.5</td>
<td></td>
</tr>
<tr>
<td>Ferro Ceramic [126]</td>
<td>( \perp ) to c-axis</td>
<td>3.97</td>
<td>250-400</td>
<td>0.29</td>
<td>( 10^{17} )</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>( \parallel ) to c-axis</td>
<td>-</td>
<td>468</td>
<td>0.28</td>
<td>( 10^{14} )</td>
<td>9.3</td>
</tr>
<tr>
<td>Insaco [127]</td>
<td>( \perp ) to c-axis</td>
<td>3.97</td>
<td>345</td>
<td>0.25-0.30</td>
<td>( 10^{16} )</td>
<td>9.3</td>
</tr>
<tr>
<td>MolTech GmbH [128]</td>
<td>( \perp ) to c-axis</td>
<td>3.98</td>
<td>345 (25°C)</td>
<td>0.27-0.30</td>
<td>( 10^{16} )</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>( \parallel ) to c-axis</td>
<td>386 (1000°C)</td>
<td></td>
<td></td>
<td>11.5</td>
<td></td>
</tr>
<tr>
<td>Saint-Gobain [67]</td>
<td>( \perp ) to c-axis</td>
<td>3.97</td>
<td>435 (25°C)</td>
<td>0.27-0.30</td>
<td>( 10^{16} )</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>( \parallel ) to c-axis</td>
<td>-</td>
<td>386 (1000°C)</td>
<td>0.27-0.30</td>
<td>11.5</td>
<td></td>
</tr>
<tr>
<td>Valley Design [129]</td>
<td>( \perp ) to c-axis</td>
<td>3.98</td>
<td>345</td>
<td>0.29</td>
<td>( 10^{14} )</td>
<td>9.4</td>
</tr>
<tr>
<td></td>
<td>( \parallel ) to c-axis</td>
<td>-</td>
<td></td>
<td></td>
<td>11.5</td>
<td></td>
</tr>
</tbody>
</table>

E - Young’s modulus, \( \nu \) - Poisson’s ratio, \( \rho_e \) - electrical resistivity, \( \varepsilon_r \) - relative permittivity

The Young’s modulus of sapphire is determined to be between 250 to 468 GPa. Fischer et al. performed strength tests on single-crystal sapphire, which yielded Young’s moduli of 384 GPa perpendicular to the C-plane (i.e., \( \parallel \) to the c-axis) or in the A- or M-planes and 436 GPa in the C-plane (i.e., \( \perp \) to the c-axis) [130]. The high Young’s modulus of sapphire makes it a good choice for high-pressure (e.g., 100 psi) static pressure sensors. The high Young’s modulus of sapphire, on the other hand, makes it
difficult to design low-pressure (e.g., 1 psi) dynamic pressure sensors. The Young’s modulus of R-plane sapphire has not been extensively studied. Salem, for example, reported a Young’s modulus of 440 GPa in the R-plane [131], whereas Vodenitcharova et al. reported a Young’s modulus of 386 GPa [132].

### 3.1.7 RF Characteristics of Sapphire at High Temperature

It is challenging to perform RF characterization in a high-temperature environment. A typical network analyzer, for example, is designed to operate up to 40°C [133]; therefore, the network analyzer must be thermally isolated from the high-temperature environment. Additionally, the material properties of the cables and RF probes are functions of temperature such that the electrical characteristics will change with any elevation in temperature. Schwartz et al. demonstrated a high-temperature RF probe station for on-wafer characterization up to 500°C and 50 GHz [134].

The probe station [134] includes a Boralectric heating element residing on an insulating block of NASA shuttle tile as illustrated in Figure 3-7. RF probes from GGB Industries have a copper heatsink affixed to the micro-coax leading to tungsten tips and reflective tape added to the probe body. A thermocouple is attached to the device under test (DUT) for localized temperature measurement and monitored using a data acquisition unit (DAQ). The phase shift associated with an elevation in temperature of the cables and probes is minimized in the measured phase of the DUT using a correction factor.
The relative permittivity of a dielectric material is the ratio of the material's absolute permittivity $\epsilon(f)$ to the permittivity of a vacuum $\epsilon_0$ or $\epsilon_r(f) = \epsilon(f)/\epsilon_0$. Another way to think of the relative permittivity is how well a material is able to hold charge with reference to a vacuum or free-space. A material with a relative permittivity of 3, for example, is able to hold 3 times the amount of charge as free-space. The loss tangent of a dielectric material is the power loss or dissipation through the material and is related to the quality factor of the material $Q$ as $\tan \delta = 1/Q$. 

Figure 3-7. A high-temperature probe station (adapted from [134]).

Figure 3-8. Au CPW quarter-wave resonator on sapphire and A-A cross-section.
A number of methods have been developed for measuring the relative permittivity and loss tangent of a dielectric material including quarter-wave resonators [135], ring resonators [136], shunt capacitors [137], and ground planes [138]. The quarter-wave and ring resonator methods are well-known and commonly employed in literature.

A quarter-wave resonator has a transmission line element that is a quarter-wave in length (i.e., \( L = \lambda / 4 \)) in an open-circuit configuration with the coplanar ground as shown in Figure 3-8. A half-wave resonator is also possible by short-circuiting the transmission line to the coplanar ground. The electric field lines wrap from the top and bottom surfaces of the transmission line to the adjacent ground plane. The field lines are not contained in a single medium (i.e., air above and dielectric below the transmission line); therefore, the measured permittivity is an effective permittivity rather than the relative permittivity. The effective permittivity \( \varepsilon_{\text{eff}} \) is related to the order of the resonance (i.e., \( n = 1, 3, 5, ... \)), resonant frequency \( f \), and length of the quarter-wave transmission line \( L \) as [135]

\[
\varepsilon_{\text{eff}} = \left( \frac{nc}{4fL} \right)^2.
\]  

(3-13)

The equivalent impedance for a quarter-wave transmission line is related to the series combination of the series resistance \( R_{\text{ESR}} \), inductance \( L \) and capacitance \( C \) of the transmission line as \( Z(\omega) = R_{\text{ESR}} + j\omega L + 1/j\omega C \). The equivalent impedance at the resonant frequency (i.e., \( \omega_o = 1/\sqrt{LC} \)) is reduced to \( Z(\omega_o) = R_{\text{ESR}} \). The loss tangent of the dielectric medium is related to the series resistance \( R_{\text{ESR}} \) and capacitance \( C \) as [35]

\[
\tan \delta = \frac{1}{Q} = \omega R_{\text{ESR}} C.
\]  

(3-14)
The skin depth is the depth from the surface of a conductor where most of the AC current resides. The skin depth $\delta$ is related to the operating frequency $f$, permeability of the conductor $\mu = \mu_r \mu_0$, and conductivity of the conductor $\sigma$ as \[\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}.\] (3-15)

The skin resistance $R_{skin}$ (i.e., AC resistance) for a planar conductor is related to the width of the conductor $W$, conductivity of the conductor $\sigma$, skin depth $\delta$, and thickness of the conductor $t$ as $R_{skin} = 1/W \sigma \delta (1 - e^{-t/\delta})$ [139]. The attenuation due to conductor loss is related to a geometry dependent parameter $P$, resistivity of the conductor $\rho$, skin depth $\delta$, and thickness of the conductor $t$ as \[\alpha = P \left[ \frac{\rho/\delta}{1 - e^{-t/\delta}} \right].\] (3-16)

Ponchak et al. studied the attenuation and relative permittivity of sapphire using CPW structures [141]. The CPW structures have 25 nm of Ti and 1.4 $\mu$m of Au on a 430 $\mu$m thick sapphire substrate. The CPW has a trace width of 130 $\mu$m and a 60 $\mu$m spacing from the coplanar ground. The attenuation and relative permittivity are determined using multiple delay lines. The attenuation as a function of frequency up to 50 GHz for temperatures from 25°C to 400°C is shown in Figure 3-9. Ponchak et al. observed that the attenuation increases linearly with temperature. The attenuation for 5 GHz and 25 GHz, for example, increases at a rate of 0.0021 dB/cm/°C. It can further be inferred that the attenuation due to conductor loss increases by $\sqrt{f}$. 

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The relative permittivity as a function of frequency up to 50 GHz for temperatures from 25°C to 400°C is shown in Figure 3-10. Ponchak et al. observed that the relative permittivity increases linearly with temperature. The relative permittivity for 25 GHz, for example, increases at a rate of 0.0024 /°C. Ponchak et al. also demonstrated CPW structures on alumina up to 50 GHz from 25°C to 400°C with attenuation and relative permittivity results similar to that of sapphire [140]. The results in Figure 3-9 and Figure 3-10 are useful in predicting the RF behavior of sapphire for temperatures up to 400°C. Furthermore, by utilizing a reference temperature sensor the change in the resonant frequency as a result of temperature can be compensated for. Further studies need to be performed to understand the RF behavior of sapphire at temperatures up to 1000°C.
3.2 Acousto-Mechanical Lumped Element Modeling

The proposed passive wireless dynamic pressure sensor transduces acoustic fluctuations into mechanical vibrations of a diaphragm. The mechanical vibrations are further transduced to a frequency shift of an electrical resonator. The acousto-mechanical and electrical portions of the dynamic pressure sensor are represented as equivalent lumped elements with analogous impedances. The impedance in the energy domain relating the effort \(e\) and flow \(f\) conjugate power variables is given as \(Z = e/f\).

The current (i.e., flow) in the electrical domain is defined as the time variant flow of charge due to an applied voltage (i.e., \(i = dq/dt\)) and the voltage (i.e., effort) is the time variant change in applied electrical work due to charge flow (i.e., \(v = dW/dq\)) [142]. The velocity (i.e., flow) in the mechanical domain is similarly defined as the time variant displacement due to an applied force (i.e., \(u = dx/dt\)) and force (i.e., effort) is the time variant change in applied mechanical work due to displacement (i.e., \(f = dW/dx\)) [142].
The power in an energy domain is the product of the effort and flow (i.e., \( p = fe \)). A summary of the electrical, mechanical, and acoustic equivalent lumped elements is shown in Table 3-6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Electrical</th>
<th>Mechanical</th>
<th>Acoustic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement (( q ))</td>
<td>( q )</td>
<td>( x )</td>
<td>( \Delta V )</td>
</tr>
<tr>
<td>Effort (( e ))</td>
<td>( v )</td>
<td>( f )</td>
<td>( p )</td>
</tr>
<tr>
<td>Flow (( f ))</td>
<td>( i = \frac{dq}{dt} )</td>
<td>( u = \frac{dx}{dt} )</td>
<td>( q = \frac{d\Delta V}{dt} )</td>
</tr>
<tr>
<td>Compliance</td>
<td>( C )</td>
<td>( \frac{1}{k} = C_m )</td>
<td>( C_a )</td>
</tr>
<tr>
<td>Force</td>
<td>( v = \frac{q}{C} )</td>
<td>( f = kx )</td>
<td>( p = \frac{\Delta V}{C_a} )</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>( S_e = \frac{dv}{dq} = C )</td>
<td>( S_m = \frac{df}{dx} = \frac{1}{k} = C_m )</td>
<td>( S_a = \frac{dp}{d\Delta V} = C_a )</td>
</tr>
<tr>
<td>Power (( p = fe ))</td>
<td>( p_e = iv )</td>
<td>( p_m = uf )</td>
<td>( p_a = qp )</td>
</tr>
<tr>
<td>Impedance (( Z = e/f ))</td>
<td>( Z_e = \frac{v}{i} )</td>
<td>( Z_m = \frac{f}{u} )</td>
<td>( Z_a = \frac{p}{q} )</td>
</tr>
</tbody>
</table>

The dynamic pressure sensor shown in Figure 2-26 may be modeled by equivalent lumped elements under the condition that the diameter of the diaphragm (i.e., \( 2a \)) is much less than the wavelength of the (i.e., \( \lambda = c_o/f \)) of the acoustic fluctuation (i.e., \( 2a \ll \lambda \)). A lumped element model assumes there is no relationship between spatial variations (e.g., voltage changes across the diaphragm) and temporal variations (e.g., time). In the electrical and mechanical domain elements are treated as if there is a point load at a single location. In the acoustic domain elements are treated as an integrated load across some area. An electrical circuit with lumped elements describing the acousto-mechanical, RF, and electrical domains of the dynamic pressure sensor is shown in Figure 3-11.

The dynamic pressure sensor acousto-mechanically transduces a pressure \( p \) applied to the circular diaphragm into a displacement field \( w(r) \). The displacement field
of the diaphragm causes the capacitance $C(w)$ of the integrated capacitor in the diaphragm to change. The pressure sensor resides at the backend of an electromagnetic waveguide. An electromagnetic wave with power $p_t$ at some Ku-band frequency in the vicinity of 15 GHz is transmitted through the frontend of an electromagnetic waveguide with a power loss $\eta_1$ where it interacts with the sensor at the backend. The power loss $\eta_1$ is related to the attenuation of the electromagnetic waveguide $\alpha$, in dB/m, and the length of the electromagnetic waveguide $\ell$ as $\eta_1 = e^{-2\alpha\ell}$.

![Figure 3-11. Schematic of electrical equivalent for dynamic pressure sensor.](image)

Furthermore, the resonant frequency of the antenna on the diaphragm shifts as a function of the applied pressure as $f_o(p) = 1/2\pi\sqrt{LC(p)}$. The incident electromagnetic wave $E^i(f)$ is then electrically filtered by the resonating antenna with a representative antenna reflection coefficient $\Gamma_a(f)$. The magnitude of the antenna reflection coefficient is bounded such that $0 \leq |\Gamma_a(f)| \leq 1$. The physical area of the antenna also contributes to the scattered power. The structural reflection coefficient is related to numerically determined resistances $R_A$ and $R_B$ such that $\Gamma_s(f) = R_B/(R_A + R_B) \propto \sqrt{\sigma_s}$. The reflected
electromagnetic wave $E'(f)$ travels back through the electromagnetic waveguide with a power loss $\eta_2$ to a receiver. The power loss $\eta_2$ is related to the attenuation of the electromagnetic waveguide $\alpha$, in dB/m, and the length of the electromagnetic waveguide $\ell$ as $\eta_2 = e^{-2\alpha \ell}$. The received power $p_r$ at the frontend of the electromagnetic waveguide is related to the transmitted power $p_t$, the power losses $\eta_1$ and $\eta_2$, and the structural and antenna reflection coefficients $\Gamma_s$ and $\Gamma_a$, respectively, as

$$p_r = \eta_1 \eta_2 \Gamma_s^2 \Gamma_a^2 p_t.$$  

(3-17)

The insertion loss, in dB, looking into an electromagnetic waveguide at the frontend with the pressure sensor at the backend is equivalent to the ratio of the received to transmitted power or $IL = 10 \log_{10} |p_r/p_t|$. The applied pressure incident on the pressure sensor can thus be determined by observing changes in the magnitude and/or phase of frequencies in the vicinity of the resonant frequency. Methods for deriving the applied pressure at the sensor will be further discussed in Chapter 6.

The dynamic pressure sensor has a diaphragm suspended over a cavity and vent structure to provide high-pass transfer characteristics, while rejecting DC pressure equilibration. The acoustic mass of the cavity $m_{ca}$ is not included in the acousto-mechanical domain since the cavity is short in length as will be further discussed. The equivalent impedance of the diaphragm is related to the acoustic resistance of the diaphragm $R_{da}$, the radiation mass $m_{ra}$, the acoustic mass of the diaphragm $m_{da}$ and the acoustic compliance of the diaphragm $C_{da}$ as $Z_d = R_{da} + j \omega (m_{ra} + m_{da}) + 1/j \omega C_{da}$. The equivalent impedance of the cavity is related to the acoustic compliance of the cavity $C_{ca}$ as $Z_c = 1/j \omega C_{ca}$. The equivalent impedance of the vent is related to the acoustic resistance of the vent $R_{va}$ and the neglected acoustic mass of the vent $m_{va}$ as
\( Z_v = R_{va} + j\omega m_{va} \). The applied pressure \( p \) is across both the diaphragm and vent elements. Furthermore, the total volume flow rate (i.e., volume velocity) \( q = q_d + q_v \) is divided between the diaphragm and vent impedance elements as \( q_d \) and \( q_v \), respectively. The total volume flow rate as a function of applied pressure \( p \) is described using KVL as

\[
q = \frac{p}{Z_{d} \parallel Z_v + Z_c} = \frac{p}{\frac{Z_d Z_v}{Z_d + Z_v} + Z_c}.
\] (3-18)

The acoustic sensitivity of the dynamic pressure sensor is related to the volumetric displacement of the diaphragm related to the flow rate of the diaphragm \( q_d \) and the applied pressure \( p \) as

\[
S_a = \frac{1}{j\omega} \frac{q_d}{p} = \frac{1}{j\omega} \frac{q}{p} \frac{Z_v}{Z_d + Z_v} = \frac{1}{j\omega} \frac{1}{Z_d Z_v + (Z_d + Z_c)Z_c}.
\] (3-19)

The dynamic pressure sensor operates between the cut-on frequency \( f_{a,co} \) and the cut-off frequency \( f_{a,o} \). The acoustic sensitivity of the dynamic pressure sensor as a function of frequency is predicted in Table 3-7. The acoustic sensitivity for operating at frequencies between the cut-on and cut-off frequencies is

\[
S_a = \frac{j\omega C_{ca} C_{da} R_{va}}{1 + j\omega R_{va} (C_{ca} + C_{da})}.
\] (3-20)

The time constant for the dynamic pressure sensor is given by \( \tau = R_{va} (C_{ca} + C_{da}) \). If the time constant is large such that \( \tau \gg 1/\omega \), then the acoustic sensitivity reduces to the parallel combination of the acoustic compliances of the cavity and diaphragm or

\[
S_a \approx C_{ca} \parallel C_{da} = \frac{C_{ca} C_{da}}{C_{ca} + C_{da}}.
\] (3-21)

The acoustic compliance of the cavity described in Eq. 3-45 is inherently low for MEMS due to small volumes that results in an effect known as cavity stiffening. The
effect of cavity stiffening is sensitivity loss in a pressure sensor. If, however, the
acoustic compliance of the diaphragm is designed to be much lower than the acoustic
compliance of the cavity, then the acoustic sensitivity is equivalent to the mechanical
sensitivity (i.e., \( S_a \approx S_m = C_{da} \)). A lower acoustic compliance of the diaphragm, on the
other hand, yields a lower mechanical sensitivity.

<table>
<thead>
<tr>
<th>Table 3-7. Summary of dynamic pressure sensor’s frequency characteristics.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
</tr>
<tr>
<td>( \omega = 0 )</td>
</tr>
<tr>
<td>( \omega &lt; \omega_{a,co} )</td>
</tr>
<tr>
<td>( \omega_{a,co} &lt; \omega &lt; \omega_{a,o} )</td>
</tr>
<tr>
<td>( \omega = \omega_{a,o} )</td>
</tr>
<tr>
<td>( \omega &gt; \omega_{a,o} )</td>
</tr>
<tr>
<td>( \omega = \infty )</td>
</tr>
</tbody>
</table>

The resonant frequency of the diaphragm is related to the radiation mass \( m_{ra} \),
acoustic mass \( m_{da} \), and acoustic compliance \( C_{da} \) of the diaphragm and the acoustic
compliance of the cavity \( C_{ca} \) as

\[
 f_{a,o} = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{(m_{ra} + m_{da}) \left( \frac{C_{ca}C_{da}}{C_{ca} + C_{da}} \right)}}. \tag{3-22}
\]

The acousto-mechanical sensitivity of the dynamic pressure sensor is related to
the displacement of the diaphragm \( dw \) in response to a change in applied pressure to
the diaphragm \( dp \). The displacement of the diaphragm \( w(p) \), however, is reduced by a
sensitivity loss \( C_{ca}/C_{da} \) due to cavity stiffening. The acousto-mechanical sensitivity is
related to the displacement of the diaphragm \( dw \) in response to a change in applied
pressure to the diaphragm \( dp \) as
\[ S_{am} = \frac{dw}{dp} = \frac{d}{dp} \left( w(p) \cdot \frac{C_{ca}}{C_{da}} \right). \]  

The electrical sensitivity of the dynamic pressure sensor is related to the change in the resonant frequency \( df_o \) as a function of a change in the nonlinear displacement of the diaphragm \( dw \) or

\[ S_e = \frac{df_o}{dw} = \frac{d}{dw} \left( \frac{1}{2\pi \sqrt{LC(w)}} \right). \]  

The equivalent sensitivity of the dynamic pressure sensor that relates a change in the resonant frequency \( df_o \) to a change in the applied pressure to the diaphragm \( dp \) is

\[ S = S_{am} \cdot S_e = \frac{df_o}{dp}. \]  

The effective quality factor of a source resonator is related to the resonant frequency \( f_o \) and the group delay \( d\phi/df \) at the resonant frequency [144]. Similarly, the minimum resolvable frequency shift of the dynamic pressure sensor is related to the resonant frequency \( f_o \), the minimum resolvable phase \( \Delta\phi \), and the electrical quality factor \( Q_e \) as [145]

\[ f_{min} = \frac{f_o \Delta\phi}{2Q_e} = \frac{\Delta f_{3dB}}{2} \cdot \Delta\phi. \]  

The minimum resolvable phase is typically limited by the measurement electronics. The phase uncertainty for an Agilent E5071C network analyzer, for example, is less than 2° (i.e., 0.035 rad) up to 20 GHz [133]. The minimum detectable pressure is related to the minimum resolvable frequency shift \( f_{min} \) and the equivalent sensitivity \( S \) of the dynamic pressure sensor as

\[ p_{min} = \frac{f_{min}}{S}. \]
The output frequency of the sensor drifts over time due to noise. The frequency drift is measured by monitoring the change in resonant frequency over time under ambient conditions (i.e., no intended applied pressure or elevation in temperature). The equivalent pressure drift of the sensor is related to the frequency drift $f_{drift}$ and the equivalent sensitivity $S$ of the passive wireless dynamic pressure sensor as

$$p_{drift} = \frac{f_{drift}}{S}. \quad (3-28)$$

If the pressure drift exceeds the minimum detectable pressure (i.e., $p_{drift} > p_{\text{min}}$), then the dynamic range of the sensor decreases. The equivalent sensitivity, which is related to the mechanical and electrical sensitivities, describes how well the dynamic pressure sensor converts an applied pressure to a change in the resonant frequency. A higher equivalent sensitivity yields a lower minimum detectable pressure. A higher electrical quality factor also yields a lower minimum detectable pressure. The diaphragm, cavity, and vent elements play a direct role in determining the acousto-mechanical sensitivity.

### 3.2.1 Diaphragm

The diaphragm is responsible for deflecting under an applied pressure. The cross-section of an unloaded dynamic pressure sensor with a circular diaphragm is shown in Figure 3-12. If a load is uniformly distributed across the circular diaphragm, then the displacement of the diaphragm as a function of the radial distance $r$ from the center is non-uniform with a maximum displacement at the center (i.e., $r = 0$). The displacement of the uniformly loaded circular diaphragm at some radial distance $r$ for small
displacements is related to the applied pressure to the diaphragm \( p \), the radius of the diaphragm \( a \), and the flexural rigidity of the diaphragm \( D \) as \[146\]

\[
w(p,r) = \frac{pa^4}{64D} \left[ 1 - \left( \frac{r}{a} \right)^2 \right]^2.
\] (3-29)

\[\text{Figure 3-12. Cross-sectional view of an unloaded dynamic pressure sensor with geometrical parameters.}\]

The maximum deflection is found at the center of the diaphragm (i.e., \( r = 0 \)) as \( w_{pk} = \frac{pa^4}{64D} \). The flexural rigidity of the circular diaphragm is related to the Young’s modulus \( E \), thickness \( h \), and Poisson’s ratio \( \nu \) of the diaphragm as \( D = \frac{Eh^3}{12(1-\nu^2)} \).

The displaced volume of the circular diaphragm as a result of uniform loading is \[147\]

\[
\Delta V = \int_0^a 2\pi rw(r)dr = \frac{\pi a^6 p}{192D}.
\] (3-30)

The acoustic compliance of the circular diaphragm is equivalent to the ratio of the displaced volume of the diaphragm \( \Delta V \) to the applied pressure \( p \) on the diaphragm or \[147\]

\[
C_{da} = \frac{\Delta V}{p} = \frac{\pi a^6}{192D}.
\] (3-31)

The mechanical sensitivity of the diaphragm in the acousto-mechanical domain is equivalent to the acoustic compliance of the diaphragm or
\[ S_m = C_{da}. \]  

(3-32)

The volume flow rate is related to the integrated velocity across the diaphragm as [147]

\[ q = \frac{d(\Delta V)}{dt} = \int_0^a 2\pi r \frac{dw(r)}{dt} dr. \]  

(3-33)

The acoustic mass of the circular diaphragm is determined by equating the lumped kinetic energy of the diaphragm to the total kinetic energy to obtain [147]

\[ m_{da} = 2\pi \int_0^a \rho_A \left( \frac{w(r)}{\Delta V} \right)^2 r dr = \frac{9\rho_A}{5\pi a^2}. \]  

(3-34)

The area density \( \rho_A \) is the mass of the diaphragm per unit area or \( \rho_A = \int_{w_1}^{w_2} \rho dw \) [147]. The radiation mass is an additional term caused by fluid particles that oscillate with the diaphragm related to the density of the fluidic medium \( \rho_o \) (e.g., air) and the radius of the diaphragm \( a \) is approximated as a piston in an infinite baffle as [10]

\[ m_{ra} = \frac{8\rho_o}{3\pi^2 a}. \]  

(3-35)

The quality factor of the diaphragm is expressed as a combination of the air damping \( Q_{air} \), thermoelastic damping \( Q_{therm} \), support losses \( Q_{sup} \), and surface losses \( Q_{sur} \) as \( Q^{-1} = Q_{air}^{-1} + Q_{therm}^{-1} + Q_{sup}^{-1} + Q_{sur}^{-1} \) [148]. The damping ratio \( \zeta = 1/2Q \) is experimentally determined from similar devices since it’s difficult to isolate the many forms of damping. The acoustic resistance associated with dampening of the circular diaphragm is related to the damping ratio \( \zeta \) as

\[ R_{da} = 2\zeta \sqrt{\frac{m_{ra} + m_{da}}{C_{da}}}. \]  

(3-36)
The aforementioned equations are utilized to describe the diaphragm’s small displacement response to an applied pressure $p$. If the applied pressure is sufficiently large enough such that the diaphragm exhibits large displacement, then the displacement of the diaphragm is related to the pressure as a third-order model. The displacement at the center of the diaphragm for large displacements using an energy-based method is related to the applied pressure to the diaphragm $p$, the radius of the diaphragm $a$, the Young’s modulus of the diaphragm $E$, the thickness of the diaphragm $h$, and the Poisson’s ratio of the diaphragm $\nu$ as [146]

$$\frac{pa^4}{EH^4} = \frac{16}{3(1-\nu^2)} \left(\frac{w_{pk,nl}}{h}\right) + \frac{3.11}{(1-\nu^2)} \left(\frac{w_{pk,nl}}{h}\right)^3. \quad (3-37)$$

For convenience the small displacement at the center of the diaphragm is $w_{pk}$ and the large displacement at the center of the diaphragm is $w_{pk,nl}$. The displacement error, in %, is determined by

$$w_{error} = \left|\frac{w_{pk} - w_{pk,nl}}{w_{pk}}\right| \cdot 100\%. \quad (3-38)$$

Sheplak et al. used a numerical approach to determine the large deflection characteristics of a circular plate with in-plane stress [149]. The maximum pressure of a silicon-based circular diaphragm with no in-plane stress at a five percent departure between the small and large displacement is determined by the Young’s modulus $E$, the thickness of the diaphragm $h$, and the radius of the diaphragm $a$ as [149]

$$p_{max} = 2.3E \left(\frac{h}{a}\right)^4. \quad (3-39)$$

The large displacement corresponding to a five percent departure from the small displacement at the maximum pressure is determined as
\[ w_{pk,nl} = 0.95 \frac{p_{max} a^4}{64D} = 0.41h(1 - v^2). \]  

(3-40)

The maximum radial stress induced on the circular diaphragm by uniform loading occurs along at the edge of the diaphragm as [146]

\[ \sigma_{r,max} = \frac{3}{4} p \left( \frac{a}{h} \right)^2. \]  

(3-41)

The yield strength is the maximum stress that is induced by the circular diaphragm before the diaphragm exhibits plastic deformation. The yield strength of silicon, for example, is 7 GPa [150]. The burst pressure (i.e., maximum applied pressure before plastic deformation) is determined by inserting the yield strength into Eq. 3-41 and solving for the pressure. For brittle materials such as silicon and sapphire the yield strength should be backed down a factor of 3 to 4 to account for the brittleness.

Another consideration is the ratio of the thickness of the diaphragm to that of the handle (i.e., \( h/m \)). The ratio of the displacement of the handle to that of the diaphragm is given as

\[ F = \frac{w_{pk,m}}{w_{pk,h}} = \frac{E_n h^3 (1 - v_m^2)}{E_m m^3 (1 - v_h^2)}. \]  

(3-42)

If the same material is used for both the diaphragm and handle, then the ratio of the displacement of the handle to that of the diaphragm reduces to \( F = (h/m)^3 \). A low handle thickness is desirable for high electrical performance (i.e., high electrical Q and high electrical sensitivity) as described in 3.3.5 and 4.1.4. A lower handle thickness, however, yields a higher compliance of the handle. A high compliance of the handle will result in a lower capacitance change between the antenna and ground plane. A trade-off exists between the achievable electrical performance and the desired electro-mechanical performance (i.e., high capacitance change).
3.2.2 Cavity

The cavity represents the cylindrical space defined by the area of the diaphragm and the cavity gap below the diaphragm. The input impedance in the cavity is related to the characteristic impedance (i.e., \( Z_o = \rho_o c_o \)), the wave number (i.e., \( k = \omega / c_o \)), and the distance from the ground plane \( z \) as \( Z_{in} = -jZ_o \cot(kz) \) [10]. Furthermore, the acoustic impedance is related to the input impedance by the area of the diaphragm (i.e., \( S = \pi a^2 \)) as \( Z_{ac} = Z_{in} / S \). The acoustic impedance in a short cavity is approximated as

\[
Z_{ca} \approx \lim_{kz \to 0} \frac{Z_{in} S}{S} = \frac{Z_o}{j\pi a^2} \left( \frac{1}{kz} - \frac{kz}{3} - \frac{(kz)^3}{45} - \cdots \right). \tag{3-43}
\]

If \( kz < 0.3 \), then the acoustic impedance in a short cavity can further be approximated by the first term as

\[
Z_{ca} = \frac{Z_o}{j\pi a^2 k z} = \frac{1}{j\omega C_{ca}}. \tag{3-44}
\]

The acoustic compliance of the cavity \( C_{ca} \) is related to the volume of the cavity \( V = \pi a^2 z \), the density of the fluidic medium \( \rho_o \) (e.g., air), and the isentropic speed of sound \( c_o \) as [10]

\[
C_{ca} = \frac{V}{\rho_o c_o z^2}. \tag{3-45}
\]

If the cavity is longer such that \( kz < 0.3 \) does not hold, then the second term in the acoustic impedance approximation must also be considered. The second term represents an acoustic mass of the cavity given as

\[
m_{ca} = \frac{\rho_o z}{3\pi a^2}. \tag{3-46}
\]
3.2.3 Vent

The vent is responsible for determining the cut-on frequency of the dynamic pressure sensor. A higher vent resistance yields a lower cut-on frequency, whereas a lower vent resistance yields a higher cut-on frequency. The volume flow rate of the diaphragm $q_d$ is related to the impedance of the vent $Z_v$, the impedance of the diaphragm $Z_d$, and the total volume flow rate $q = q_d + q_v$ as

$$q_d = \frac{Z_v}{Z_d + Z_v} q.$$  \hfill (3-47)

A high vent impedance $Z_v$ yields a high volume flow rate of the diaphragm $q_d$. The acoustic sensitivity of the dynamic pressure sensor is related to the volume flow rate of the diaphragm $q_d$ and the applied pressure $p$ as

$$S_a = \frac{1}{j\omega} \cdot \frac{q_d}{p}.$$  \hfill (3-48)

A high vent impedance $Z_v$ will also increase the acoustic sensitivity of the dynamic pressure sensor $S_a$. The cut-on frequency of the dynamic pressure sensor is related to the acoustic resistance of the vent $R_{va}$, the acoustic compliance of the cavity $C_{ca}$, and the acoustic compliance of the diaphragm $C_{da}$ as

$$f_{a,co} = \frac{1}{2\pi} \cdot \frac{1}{R_{va}(C_{ca} + C_{da})}.$$  \hfill (3-49)

A higher vent impedance will result in a lower cut-on frequency for the dynamic pressure sensor; as a result, a lower cut-on frequency has a higher bandwidth. The vent channel will have a rectangular cross-section due to the planar nature of microfabrication processing. A serpentine design is chosen with $n$ turns to increase vent resistance. The hydraulic diameter $D_h$ is related to the cross-sectional area $A = w_v d_v$, perimeter $P = 2(w_v + d_v)$, depth $d_v$, and width $w_v$ of the vent channel as [151]
\[ D_h = \frac{4A}{P} = \frac{2w_v d_v}{w_v + d_v} = \frac{2d_v}{1 + \frac{d_v}{w_v}}. \] 

(3-50)

The effective length of the vent \( L_{eff} \) is the physical length \( L \) in addition to a correction factor \( 60nD_h \) that accounts for the \( n \) turns of \( 180^\circ \) as [151]

\[ L_{eff} = L + 60nD_h. \] 

(3-51)

The acoustic resistance of the vent \( R_{va} \) is related to the dynamic viscosity of the fluidic medium \( \mu \), effective length of the vent \( L_{eff} \), and hydraulic diameter \( D_h \) as [142]

\[ R_{va} = \frac{12\mu L_{eff}}{w_v d_v^3}. \] 

(3-52)

### 3.2.4 Thermomechanical Noise

A thermomechanical noise model for the dynamic pressure sensor is shown in Figure 3-13a. The uncorrelated noise source for the diaphragm is related to the acoustic resistance of the diaphragm \( R_{da} \) as \( E_d^2 = 4k_BT R_{da}\Delta f \) [152]. Similarly, the uncorrelated noise source for the vent is related to the acoustic resistance of the vent \( R_{va} \) as \( E_v^2 = 4k_BT R_{va}\Delta f \). The noise model in Figure 3-13a is simplified using equivalent impedances \( Z_d, Z_c, \) and \( Z_v \) for the diaphragm, cavity, and vent, respectively, as shown in Figure 3-13b. The noise source across the diaphragm impedance is determined by short-circuiting the noise source for the vent (i.e., \( E_v = 0 \)) then using voltage division as [152]

\[ E_{11} = \frac{Z_d}{Z_d + \frac{Z_v Z_c}{Z_v + Z_c}} E_d = \frac{Z_d(Z_v + Z_c)}{Z_d Z_v + Z_d Z_c + Z_v Z_c} E_d. \] 

(3-53)
Similarly, the noise source across the cavity and vent impedances is determined by short-circuiting the noise source for the diaphragm (i.e., $E_d = 0$) then using voltage division as [152]

$$E_{12} = \frac{Z_d Z_c}{Z_d + Z_c} = \frac{Z_d Z_c}{Z_d Z_v + Z_d Z_c + Z_v Z_c} E_v. \quad (3-54)$$

The noise voltage associated with the diaphragm is related to the noise sources $E_{11}$ and $E_{12}$ and the impedance of the diaphragm $Z_d$ as [152]

$$E_d^2 = E_{11}^2 + E_{12}^2 = \frac{(Z_c + Z_v)^2 E_d^2 + Z_v^2 E_v^2}{(Z_c Z_d + Z_v Z_d + Z_c Z_v)^2} Z_d^2. \quad (3-55)$$

![Diagram](a) and (b)

Figure 3-13. A thermomechanical noise model for a dynamic pressure sensor (a) electrical circuit with (b) simplified impedance circuit.

### 3.3 Electrical Lumped Element Modeling

The proposed passive wireless dynamic pressure sensor has an electrical resonator represented by a patch antenna with a slot and an air gap (i.e., cavity-backed). Microstrip patch antennas of various geometries including the addition of slots have been studied [153]. The gain-bandwidth product of a patch antenna is proportional to the volume of the antenna $V$ and some constant $k$ as [154]
\[ G \cdot \Delta f = k \cdot V. \]  \hspace{1cm} (3.56)

Sievenpiper et al. validated the gain-bandwidth product’s relationship to the size of an antenna by studying the performance of 110 small antennas [155]. Patch antennas inherently have low gain-bandwidth products due to their small volumes. Patch antennas with cavities, however, have shown promise for higher gain-bandwidth products [156]. Feed-trace rectangular patch antennas with cavities in the K-band [156], V-band [157], W-band [158] and feed-probe circular patch antennas with cavities in the L-band [159] have been studied. A review of fundamental antenna theory is first necessary to better understand antenna behavior before moving on to modeling.

**3.3.1 Antenna Fundamentals**

Antennas are radiating elements widely used in wireless communications for power transmission and reception. The geometrical and material parameters of an antenna almost exclusively determine the performance of an antenna. Passive antennas have equivalent performance as both transmitters and receivers described by the Lorentz reciprocity theorem as

\[ \int \mathbf{J}_1 \cdot \mathbf{E}_2 \, dV = \int \mathbf{E}_1 \cdot \mathbf{J}_2 \, dV. \]  \hspace{1cm} (3.57)

Consider a radiating antenna (i.e., object 1) and loaded scattering antenna (i.e., object 2) in free-space separated some far-field distance as shown in Figure 3-14a. The equivalent two-port network shown in Figure 3-14b fully describing the two antennas and their interaction in free-space has an associated impedance matrix given as [160]

\[
\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.
\]  \hspace{1cm} (3.58)
The voltage across the loaded scattering antenna is related to the impedance of the load $Z_L$ as $V_2 = -I_2Z_L$ [160]. The trans-impedance parameters are considered equivalent (i.e., $Z_{12} = Z_{21}$) since the antenna is reciprocal. The voltage across the radiating antenna is related to the input impedance $Z_{in}$ as [160]

$$V_1 = Z_{in}I_1 = \left( Z_{11} - \frac{Z_{12}^2}{Z_{22} + Z_L} \right) I_1.$$  

(3-59)

Figure 3-14. A radiating antenna with loaded scattering antenna in free-space (a) electrical circuit and (b) equivalent two-port network.

The backscattered voltage is defined as the difference between the voltage across the radiating antenna $V_1$ when a load $Z_L$ is present across the scattering antenna and the voltage across the radiating antenna $V_{10}$ when then scattering antenna is unloaded (i.e., $Z_L = \infty$) or $\Delta V = V_1 - V_{10}$ [160]. The backscattered voltage is a function of the impedance parameters $[Z]$ and the load impedance $Z_L$ as [160]

$$\Delta V = Z_{in}I_1 = \left[ (Z_{11} - Z_1) - \frac{Z_{12}^2}{Z_{22} + Z_L} \right] I_1.$$  

(3-60)

The reflected (i.e., backscattered) electric field is related to the backscattered voltage $\Delta V$ and the effective length of the antenna $\ell$ as $E^r = \Delta V / \ell$. An electromagnetic wave in the far-field is a traversing wave (i.e., moves perpendicular to direction of
energy transfer) with electric $\vec{E}$ and magnetic $\vec{H}$ fields perpendicular to one another related by the impedance of free-space $\eta_0$ as [35]

$$\vec{H} = \frac{1}{\eta_0} \hat{a}_r \times \vec{E}. \quad (3-61)$$

The electric and magnetic fields for a linearly polarized electromagnetic wave in the far-field, for example, are described by vectors $\vec{E}(z,t) = \hat{x}E_o \sin(\omega t - kz)$ and $\vec{H}(z,t) = \hat{y}E_o \sin(\omega t - kz)/\eta_o$, respectively. The power density of an electromagnetic wave, in W/m², is described by the Poynting vector as the cross-product of the electric and magnetic fields or $\vec{S} = \vec{E} \times \vec{H}$ [35]. The radiated power, in W, of an electromagnetic wave is the time-averaged power density integrated over a closed spherical surface $\vec{S}$ or [35]

$$p_{rad} = \frac{1}{2} \int \text{Re}[\vec{E} \times \vec{H}^*] \cdot d\vec{S}. \quad (3-62)$$

![Figure 3-15](image)

Figure 3-15. An illustration of electromagnetic waves traversing in z-direction including (a) linear, (b) elliptical, and (c) circular polarizations.

Electromagnetic waves can travel in linear, circular, or elliptical patterns as shown in Figure 3-15. Electromagnetic waves are generally elliptical and described as a
composite electric field such as \( \vec{E}(z,t) = \hat{x}E_{xo} \sin(\omega t - kz) + \hat{y}E_{yo} \sin(\omega t - kz + \delta) \). The electric field can also be written in phasor form as \( \vec{E}(z,t) = \hat{x}E_{xo} e^{j\omega t} + \hat{y}E_{yo} e^{j(\omega t + \delta)} \).

The phase difference between the two electric fields is \( \delta \). The axial ratio is used to describe the polarization of an antenna as the ratio of the major axis \( a \) (i.e., the maximum electric field) to the minor axis \( b \) (i.e., the minimum electric field) as [161]

\[
A_R = \frac{a}{b} \quad (3-63)
\]

A linearly polarized electromagnetic wave has an axial ratio of \( A_R = \infty \), whereas a circularly polarized electromagnetic wave has an axial ratio of \( A_R = 1 \). An elliptically polarized electromagnetic wave has an axial ratio of \( 1 < A_R < \infty \). A summary of polarization types and their corresponding parameters is shown in Table 3-8.

<table>
<thead>
<tr>
<th>Polarization</th>
<th>Direction</th>
<th>( E_{xo}/E )</th>
<th>( E_{yo}/E )</th>
<th>( \delta )</th>
<th>( A_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>X</td>
<td>1</td>
<td>0</td>
<td>-</td>
<td>\infty</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>0</td>
<td>1</td>
<td>-</td>
<td>\infty</td>
</tr>
<tr>
<td></td>
<td>45(^\circ)</td>
<td>( 1/\sqrt{2} )</td>
<td>( 1/\sqrt{2} )</td>
<td>0</td>
<td>\infty</td>
</tr>
<tr>
<td>Circular</td>
<td>LHCP</td>
<td>( 1/\sqrt{2} )</td>
<td>( 1/\sqrt{2} )</td>
<td>+90(^\circ)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>RHCP</td>
<td>( 1/\sqrt{2} )</td>
<td>( 1/\sqrt{2} )</td>
<td>-90(^\circ)</td>
<td>1</td>
</tr>
<tr>
<td>Elliptical</td>
<td>-</td>
<td>( 1 &lt; A_R &lt; \infty )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The polarization vector describes the amplitudes in each axis normalized to the magnitude of the electromagnetic wave \( E_m = \sqrt{E_{xo}^2 + E_{yo}^2} \) as [162]

\[
\vec{\rho} = \frac{\vec{E}}{E_m} = \frac{E_{xo}}{E_m} \hat{x} + \frac{E_{yo}}{E_m} e^{j\delta} \hat{y}. \quad (3-64)
\]

A linearly polarized electromagnetic field in the vertical direction (i.e., \( E_{yo} = 0 \)) yields a polarization vector with a magnitude of \( |\vec{\rho}_x| = 1 \) in the vertical direction and \( |\vec{\rho}_y| = 0 \) in the horizontal direction. A circularly polarized electromagnetic field, on the
other hand, has equivalent polarization vectors in both the vertical and horizontal
directions given as $|\tilde{\rho}_x| = |\tilde{\rho}_y| = 1/\sqrt{2}$. The polarization ratio is the ratio of the
polarization component in the $y$-axis to that of the $x$-axis or [162]

$$
\tilde{r} = \frac{E_{yo}}{E_{xo}} = \frac{E_{yo}}{E_{xo}} e^{i\delta}.
$$

(3-65)

If the major axis of an elliptical polarizer lies along the $x$-axis, then the polarization
ratio is bounded such that $0 \leq |\tilde{r}| \leq 1$. A linearly polarized electromagnetic field, for
example, has a polarization ratio of $|\tilde{r}| = 0$. A circularly polarized electromagnetic field,
on the other hand, has a polarization ratio of $|\tilde{r}| = 1$. A patch antenna has an ability to
reflect electromagnetic waves that are incident to its surface as shown in Figure 3-16.

![Figure 3-16. Interaction of an incident electromagnetic wave with some arbitrary object.](image)

The ratio of the reflected electromagnetic wave $E^{r}_i$ to the incident electromagnetic
wave $E^{i}_j$ on an antenna is described by the scattering parameter [163]

$$
a_{ij} = \frac{E^{r}_i}{E^{i}_j}.
$$

(3-66)
The scattering parameters are complex and commonly expressed in polar form as
\[ a_{ij} = |a_{ij}|e^{j\theta_{ij}} \] [163]. The radar cross-section (RCS) is determined in the far-field (i.e., \( R \to \infty \)) where the electromagnetic waves are planar as [163]

\[ \sigma_{ij} = \lim_{R \to \infty} 4\pi R^2 |a_{ij}|^2 = \lim_{R \to \infty} 4\pi R^2 \left| \frac{E^r_i}{E^r_j} \right|^2. \] (3-67)

The scattering parameters in vertical (i.e., v) and horizontal (i.e., h) directions are related to the incident and reflected electromagnetic fields as [163]

\[ \begin{bmatrix} E^r_v \\ E^r_h \end{bmatrix} = \begin{bmatrix} a_{vv} & a_{vh} \\ a_{hv} & a_{hh} \end{bmatrix} \begin{bmatrix} E^i_v \\ E^i_h \end{bmatrix}. \] (3-68)

For a monostatic radar the cross-polarization scattering parameters are equivalent (i.e., \( a_{vh} = a_{hv} \)). A circularly polarized antenna (i.e., \( A_R = 1 \)) has equivalent scattering parameters in both axes (i.e., \( a_{vv} = a_{hh} \) or \( a_{vh} = a_{hv} \)) for a linearly polarized incident electromagnetic wave. An electrical antenna such as a dipole antenna directs an electromagnetic wave along a single axis very similarly to a linear polarizer.

![Figure 3-17. An optical illustration of a source passing through a set of co-polarized and cross-polarized linear polarizers.](image)
The Hittite HMC6450/6451 transceiver discussed in Chapter 5 has a daughterboard with a transmitter chip and a receiver chip placed adjacent to one another. The transmitter and receiver chips have linearly polarized antennas, which are cross-polarized to minimize transmitter to receiver leakage. An illustration of linear polarization of electrical antennas using an optical analogy is shown in Figure 3-17.

In the two scenarios a set of linearly polarizers representing transmit and receive antennas are either co-polarized or cross-polarized. If the linear polarizers are cross-polarized, then the linearly polarized source is filtered out at the output. However, if the linear polarizers are co-polarized, then a portion of the linearly polarized source is directed to the output. To direct an electromagnetic wave in the cross-polarized case requires an intermediate circular or elliptical polarizer as shown in Figure 3-18.

![Diagram of linear and circular polarizers](image)

**Figure 3-18.** An optical illustration of a set of cross-polarized linear polarizers with an intermediate circular polarizer.

In the optical domain (i.e., f > 300 GHz) a quarter-wave plate is commonly used to convert a linearly polarized electromagnetic wave into a circularly polarized electromagnetic wave. A laminar diffraction grating pattern can also be used as a circular polarizer. The diffraction grating is analogous to a dielectric slab with some effective height $h_{\text{eff}} = h/n_{\text{eff}}$ related to the effective refractive index $n_{\text{eff}}$ [164]. The
effective refractive index of the laminar diffraction grating pattern is related to the grating width \( a \), grating period \( d \), and wavelength \( \lambda \) given in [164]. The reflection coefficient for normal incidence is related to the refractive index of the surrounding medium \( n_o \) and the effective refractive index \( n_{eff} \) of the laminar grating pattern as [165]

\[
\Gamma = \frac{n_o - n_{eff}}{n_o + n_{eff}}
\]  

(3-69)

The result of an incident electromagnetic wave at the surface of a laminar diffraction grating pattern is both a reflected intensity (i.e., \( I^r = \Gamma^2 I^i \)) and an absorbed intensity (i.e., \( I^a = (1 - \Gamma^2) I^i \)). If the laminar grating pattern is oriented some offset angle \( \theta \) with respect to a linearly polarized incident electromagnetic wave in the vertical direction, then the intensity of the reflected electromagnetic wave is a function of the offset angle \( \theta \) as described by Malus’ law as \( I^r = \Gamma^2 I^i \cos^2 \theta \). An electrical antenna can also be designed to take advantage of Malus’ law. A patch antenna, for example, is intrinsically linearly polarized. A patch antenna, however, can be configured for circular polarization.

![Figure 3-19](image-url)

Figure 3-19. A feed-trace square patch antenna with (a) power divider and (b) slot (adapted from [153]).
A feed-trace square patch antenna driven by two signals that are 90° out of phase is shown in Figure 3-19a [153]. A power divider splits the feed signal into two paths: one path feeding directly to the antenna and another path with a quarter-wave transmission line to delay the signal by 90°. A maximum electric field in the vertical direction is produced when the current is maximum in the vertical direction and zero in the horizontal direction. A maximum electric field in the horizontal direction is produced when the current is maximum in the horizontal direction and zero in the vertical direction. The electric field maxima and minima are continuously alternating or rotating between the vertical and horizontal directions resulting in circular polarization. The addition of a slot to a feed-trace patch antenna can also be used to achieve circular polarization as shown in Figure 3-19b [153].

![Current Distribution on a Patch Antenna](image)

Figure 3-20. An illustration of current distribution on a patch antenna (a) without slot and (b) with slot.

A slot in a patch antenna is able to redistribute the current across the patch antenna in a way that mimics a patch antenna driven by two signals 90° out of phase. A patch antenna with no slot as shown in Figure 3-20a has a current distribution aligned in the same direction as the incident electromagnetic wave. The addition of a slot to the
patch antenna as shown in Figure 3-20b allows for redistribution of the current along both the vertical and horizontal directions for circular polarization. A patch antenna will reflect a portion of the incident electromagnetic wave at the patch’s surface. The reflected electric field $\vec{E}^r$ is related to the incident electric field $\vec{E}^i$, the surface current density $J_s$, and the surface impedance $Z_s$ as [166]

$$\hat{z} \times [\vec{E}^r + \vec{E}^i] = Z_s [\hat{z} \times \vec{J}_s].$$  \hspace{1cm} (3-70)

The reflected electric field in the far-field for a rectangular patch antenna is also described as the sum of the electric fields contributed by the surface current, the electric polarization current, and the magnetic polarization current or [167]

$$E^r(r, \theta, \phi) = E^s(r, \theta, \phi) + E^e(r, \theta, \phi) + E^m(r, \theta, \phi).$$ \hspace{1cm} (3-71)

The reflected electric field contributed by the surface current is determined using an electric-current model [167] as

$$E^s(r, \theta, \phi) = E'(r, \theta, \phi) \int_{-W/2}^{W/2} \int_{-L/2}^{L/2} J_{sx}(x', y') e^{i(k_x x' + k_y y')} dx' dy'. \hspace{1cm} (3-72)$$

The electric field contributed by the surface current is reduced by integrating to yield [167]

$$E^s(r, \theta, \phi) = E'(r, \theta, \phi) \left( \frac{\pi WL}{2} \right) \left[ \frac{\sin \left( \frac{k_y W}{2} \right)}{k_y W} \right] \left[ \frac{\cos \left( \frac{k_x L}{2} \right)}{\left( \frac{\pi}{Z} \right)^2 - \left( \frac{k_x L}{2} \right)^2} \right]. \hspace{1cm} (3-73)$$

The electric fields contributed by the electric polarization current and the magnetic polarization current are functions of $\varepsilon_r$ and $\mu_r$ of the dielectric medium for the rectangular patch antenna given in [167]. For a non-magnetic medium (i.e., $\mu_r = 1$) the magnetic polarization current is neglected. The electric field contributed by the electric polarization current has a similar form to Eq. 3-72 with an additional integration across
the height of the substrate. The electric field contributed by the electric polarization current after integrating yields [167]

\[
E^e(r, \theta, \phi) = 2E_o(WLh)(\omega\varepsilon_o)(\varepsilon_r - 1)M_o e^{-jk_o h \cos \theta} \\
\cdot \sin \theta \left[ \frac{\sin \left(\frac{k_y W}{2}\right)}{k_y W} \right] \left[ \frac{\sin(k_o h \cos \theta)}{k_o h \cos \theta} \right] \left[ \frac{(k_x L)}{2} \right] \left[ \frac{\cos \left(\frac{k_x L}{2}\right)}{\left(\frac{\pi}{2}\right)^2 - \left(\frac{k_x L}{2}\right)^2} \right].
\]

(3-74)

The reflected electric field (in the far-field) of primary interest is that field which is perpendicular to the surface of the antenna (i.e., \(\theta = 0^\circ\) and \(\phi = 0^\circ\)). The electric field contributed by the electric polarization current at \(\theta = 0^\circ\) is zero.

### 3.3.2 Structural Mode Radar Cross-Section

An object such as a flat plate or antenna can reflect (i.e., backscatter) electromagnetic waves due to two phenomena: the interaction between an incident electromagnetic wave with the physical area of the object (i.e., the structural mode) or interaction between an incident electromagnetic wave with the electrical nature of the object (i.e., the antenna mode). The RCS due to the structural mode describes the reflection due to the physical area of the antenna and is described as

\[
\sigma_s = \lim_{R \to \infty} 4\pi R^2 \left| \frac{E_s^r}{E^i} \right|^2.
\]

(3-75)

Similarly, the RCS due to the antenna mode describes the reflection due to the electrical portion of the antenna and is described as [168]

\[
\sigma_a = \lim_{R \to \infty} 4\pi R^2 \left| \frac{E_a^r}{E^i} \right|^2.
\]

(3-76)
The reflection due to the physical area of an object is dependent on the physical size (i.e., effective length) of the object $L$ and the operating wavelength $\lambda$. There are three scattering regions: low frequency (i.e., Rayleigh) (i.e., $L \ll \lambda$), resonant (i.e., Mie) (i.e., $L \approx \lambda$), or high frequency (i.e., optical) (i.e., $L \gg \lambda$) [38]. In the Rayleigh region there is little phase variation of the incident electromagnetic wave over the surface of the antenna at any instantaneous time (i.e., surface sees same electromagnetic wave). In the Mie region, however, there is significant phase variation of the incident electromagnetic wave over the surface of the antenna. In the optical region the scattering is primarily due to local independent scattering mechanisms (e.g., surface imperfections such as gaps and cracks).

\[
\begin{align*}
E_i' & \quad \text{(a)} \\
E_s' & \quad \text{(b)} \\
Z_A = R_A + jX_A \\
Z_L = 0
\end{align*}
\]

Figure 3-21. Interaction of an incident electromagnetic wave with a flat plate resulting in (a) structural mode and (b) antenna mode electrical circuit.

The structural mode gain of a flat plate in the optical region (i.e., $L \gg \lambda$) is related to the effective area of the antenna (i.e., $A_e = \eta_{ant}A$) and the operating wavelength $\lambda$ as
\[ G_s = \frac{4\pi A_e}{\lambda^2} \] [38]. The structural mode RCS for a flat plate is related to the structural mode gain of the patch antenna \( G_s \) and the effective area \( A_e \) of the antenna as [163]

\[ \sigma_s = G_s A_e = \frac{4\pi A_e^2}{\lambda^2}. \] (3-77)

Consider a flat plate with an incident electromagnetic wave in the far-field as shown in Figure 3-21. The incident electromagnetic wave interacts with the physical area of the antenna as illustrated. A portion of the incident electromagnetic wave is reflected due to interaction with the physical area of the plate. The flat plate electrically has some resistance \( R_A \) and reactance \( X_A \) and appears as a short circuit (i.e., \( Z_L = 0 \)); therefore, there is no additional reflection due to the electrical nature of the plate.

![Figure 3-22. Normalized RCS of rectangular and circuit flat plates with FEM (solid) and analytical (dotted) expressions.](image)

The normalized RCS \( \sigma/\lambda^2 \) as a function of the normalized plate width \( L/\lambda \), where the plate width for a rectangular plate is \( L = a \) and the plate width for a circular plate is \( L = \pi b/1.841 \), is shown in Figure 3-22. The solid lines represent the finite element
method (FEM) expressions obtained using Ansoft's High Frequency Structural Simulator (HFSS). The FEM model of the rectangular plate matches well with models predicted by Rao [169] and Rahmat-Samii [170]. The dotted lines represent the analytical expressions obtained using the RCS approximation of a flat plate in the optical region (i.e., \( \sigma = 4\pi A^2/\lambda^2 \)). The RCS for a rectangular flat plate, for example, is related to the width of the flat plate \( a \) and the operating wavelength \( \lambda \) as \( \sigma = 4\pi a^4/\lambda^2 \). The RCS for a circular flat plate is related to the radius of the flat plate \( b \) and the operating wavelength \( \lambda \) as \( \sigma = 4\pi^3 b^4/\lambda^2 \). It is observed in Figure 3-22 that the analytical expressions for the rectangular and circular plates are close approximations to the FEM in the Mie region (i.e., \( L \approx \lambda \)).

![Figure 3-23. Correction factor for rectangular and circular flat plates as a function of the normalized plate width \( L/\lambda \).](image)

A correction factor defined as the ratio of the FEM RCS to the analytical RCS for the flat plate model is shown in Figure 3-23. The correction factor simplifies the
analytical model by using the simplified flat plate model with an applied correction factor for the desired operating wavelength. The correction factor for a rectangular plate with a width of \( L = \frac{\lambda}{2} \) is 2.81 and the correction factor for a circular plate is 2.49.

\[
\text{Figure 3-24. Normalized RCS as a function of normalized length for dipole antenna.}
\]

Dipole antennas are commonly used in wireless communication systems. Consider an incident electromagnetic wave on a dipole antenna with length \( L \) and diameter \( D \) such that \( L \gg D \). The normalized RCS \( \frac{\sigma}{\lambda^2} \) as a function of the normalized dipole length \( \frac{L}{\lambda} \) for a dipole antenna is shown in Figure 3-24. The RCS predicted by FEM matches well with measurements by Green [171] and Dybdal et al. [172]. Green notes that the composite RCS is related to the RCS components in the phi and theta directions \( \sigma_\phi \) and \( \sigma_\theta \), respectively, as [171]

\[
\sigma = \sqrt{\sigma_\phi^2 + \sigma_\theta^2}.
\]

(3-78)
The RCS predicted in the theta direction for the dipole antenna is orders of magnitude smaller than the RCS in the phi direction; therefore, the predicted RCS in the phi direction is approximately equivalent to the composite RCS. As discussed earlier, the addition of a slot to a flat plate will redistribute the current in such a way that a portion of the incident electromagnetic wave will be reflected in both the phi and theta directions.

![Figure 3-25. Ratio of RCS in theta direction to RCS in phi direction as a function of normalized slot length \( L/\lambda \) for rectangular and circular flat plates.](image)

Consider a flat plate with a slot of length \( L \) at a 45° angle centered in the flat plate. The width of the rectangular flat plate is \( \lambda \) and the radius of the circular flat plate is \( 1.841 \lambda/2\pi \). The ratio of the RCS in the theta direction to the RCS in the phi direction as a function of the normalized length of the slot \( L/\lambda \) is shown in Figure 3-25. It is observed that as the length of the slot increases towards \( \lambda/2 \) the ratio of the RCS increases toward unity, which is indicative of circular polarization. The result of an incident
electromagnetic wave with an RCS ratio of unity is a reflected electromagnetic wave with equal power in both the vertical and horizontal directions.

### 3.3.3 Antenna Mode Radar Cross-Section

Consider an incident electromagnetic wave on two parallel flat plates in close proximity as shown in Figure 3-26a. The two plates are in close proximity such that they have an equivalent inductance $L$ and capacitance $C$ such that they electrically resonate at a frequency $f_o = 1/2\pi\sqrt{LC}$. An incident electromagnetic wave on the plates yields a reflected electromagnetic wave due to the physical area of the antenna in addition to the electrical nature of the antenna. The electrical representation of the antenna includes real and reactive impedance elements of the antenna (i.e., $Z_A = R_A + jX_A$) and the load (i.e., $Z_L = R_L + jX_L$) as shown in Figure 3-26b.

Figure 3-26. Interaction of an incident electromagnetic wave with two flats resulting in (a) structural mode and (b) antenna mode electrical circuit.
The real portion of the antenna impedance can further be represented with radiation resistance $R_{rad}$ and dissipation resistance $R_d$ as $R_A = R_{rad} + R_d$. The reactive portion of the antenna impedance is inherently inductive; therefore, the reactance is described as $X_A = \omega L$. The current in the antenna is related to the induced open-circuit voltage $V_{oc}$, the impedance of the antenna $Z_A$, and the impedance of the load $Z_L$ as $I = V_{oc} / (Z_A + Z_L)$. The real power delivered to the load is related to the current $I$ and the real impedance of the load $R_L$ as [173]

$$P_L = I^2 R_L = \frac{V_{oc}^2 R_L}{|Z_A + Z_L|^2}. \quad (3-79)$$

The effective area of power absorption for the impedance load is related to the real power delivered to the load $P_L$ and the incident power density $P^i = (E^i)^2 / \eta_o$ as [173]

$$A_L = \frac{P_L}{P^i} = \frac{\eta_o V_{oc}^2 R_L}{(E^i)^2 |Z_A + Z_L|^2}. \quad (3-80)$$

The antenna mode RCS is related to the gain of the antenna $G_A$ and the effective area of power absorption for the impedance load $A_L$ as

$$\sigma_a = G_A A_L. \quad (3-81)$$

The normalized RCS $\sigma / \lambda^2$ in the theta direction as a function of the normalized plate width for two circular plates $L / \lambda = \pi b / 1.841 \lambda$ with a $\lambda / 2$ slot and varying separation distances $d$ in free-space is shown in Figure 3-27. The normalized RCS in the theta direction increases as the plate width approaches $L = \lambda / 2$. A lower plate separation distance results in a higher RCS quality factor.

The composite RCS of an antenna due to both the structural and antenna modes $\sigma_s$ and $\sigma_a$, respectively, is described as [173]
\[ \sigma = \left| \sqrt{\sigma_s} - (1 - \Gamma_a)\sqrt{\sigma_a}e^{j\phi} \right|^2. \]  

(3-82)

The relative phase \( \phi \) describes the phase difference between the structural and antenna modes. The complex antenna reflection coefficient describes the mismatch between the antenna impedance \( Z_a \) and the load impedance \( Z_L \) of the antenna as \( \Gamma_a = (Z_A - Z_L)/(Z_A + Z_L) \) [173]. The antenna mode RCS of an antenna is determined experimentally by measuring the composite RCS \( \sigma \) and the structural mode RCS \( \sigma_s \) as

\[ (1 - \Gamma_a)\sqrt{\sigma_a}e^{j\phi} = \sqrt{\sigma_s} - \sqrt{\sigma}. \]

The structural mode RCS \( \sigma_s \) is determined by measuring the composite RCS \( \sigma \) under a short-circuit case.

Figure 3-27. Normalized RCS in theta direction as a function of normalized plate width for two circular flat plates with a \( \lambda/2 \) slot and varying separation distances \( d \).

The ratio of the RCS in the theta direction to the RCS in the phi direction for two circular flat plates with a \( \lambda/2 \) slot and varying separation distances is shown in Figure 3-28. An electrical resonance in the vicinity of \( L = \lambda/2 \) means the flat plate antenna is reflecting a portion of the incident energy in the phi direction to the theta direction. The
strength of the reflected energy is related to both the antenna and structural modes. Although more energy is being coupled from the phi to the theta directions for increasing separation distances it is seen in Figure 3-27 that the quality factor in the theta direction actually decreases with increasing separation distances.

Figure 3-28. Ratio of RCS in theta direction to RCS in phi direction as a function of normalized plate width for two circular flat plates with a $\lambda/2$ slot and varying separation distances $d$.

The reflected electric field due to an incident electric field $E^i$ on a patch antenna is related to the composite RCS $\sigma$ as

$$E^r \propto \sqrt{\sigma} E^i = |\sqrt{\sigma_s} - (1 - \Gamma_a)\sqrt{\sigma_a}e^{j\phi}|E^i. \quad (3-83)$$

A patch antenna has reflected electric field components in the vertical and horizontal directions $E^r_v$ and $E^r_h$, respectively. The reflected electric field can also be described with polarization factors to represent the division in the vertical and horizontal directions as
\[ E^r = E_v^r + E_h^r = |\rho_v|E^r + |\rho_h|E^r. \]  

(3-84)

The polarization factors have magnitudes bounded such that \( 0 \leq \rho \leq 1 \). The polarization factors for circular polarization, for example, are \( 1/\sqrt{2} \). The expressions in Eq. 3-83 and Eq. 3-84 are combined to fully describe the reflected electric fields in the vertical and horizontal directions as a function of the structural and antenna mode RCS of the patch antenna as

\[ E_v^r \propto |\rho_v| \cdot |\sqrt{\sigma_s - (1 - \Gamma_a)} \sqrt{\sigma_a} e^{j\phi}|E^i \]  

(3-85)

and

\[ E_h^r \propto |\rho_h| \cdot |\sqrt{\sigma_s - (1 - \Gamma_a)} \sqrt{\sigma_a} e^{j\phi}|E^i. \]  

(3-86)

The expressions in Eq. 3-85 and Eq. 3-86 fully describe the electric fields reflected by a patch antenna due to an incident electric field. The polarization factors are determined experimentally as they are difficult to model.

### 3.3.4 Existing Patch Antenna Models

There are two analytical models commonly used to analyze rectangular microstrip antennas: the transmission line model and the cavity model [174]. The transmission line model is a distributed element based equivalent, whereas the cavity model is a lumped element based equivalent as shown in Figure 3-29. The transmission line model assumes the patch antenna to appear as an open-circuited transmission line. The impedance of the patch antenna as a function of the position \( x \) along the patch is [174]

\[ Z(x) = \frac{V(x)}{I(x)} = Z_0 \cot \frac{\pi x}{L}. \]  

(3-87)

The cavity model assumes the patch antenna is a narrow-band resonator (i.e., lossy cavity). The cavity model of a patch antenna treats the patch as a cavity with a
magnetic wall boundary placed at the perimeter of the patch antenna [175]. The first resonant mode of the patch antenna appears as a parallel RLC circuit with an input impedance given as [175]

\[ Z_{in} = R_{in} + jX_{in} = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}}. \tag{3-88} \]

The lumped element capacitance is determined by the length \( L \), width \( W \), and height \( h \) of the patch and the feed position \( x \) as [175]

\[ C = \frac{\varepsilon_o \varepsilon_r LW}{2h} \cos^2 \frac{\pi x}{L}. \tag{3-89} \]

The lumped element inductance is determined by the capacitance \( C \) in Eq. 3-89 and the resonant frequency \( \omega_o \) as \( L = 1/C \omega_o^2 \). The lumped element resistance is also related to the capacitance \( C \) in Eq. 3-89 as well as the quality factor \( Q = 1/\tan \delta \) and the resonant frequency \( \omega_o \) as \( R = Q/\omega_o C \). The cavity model is an appropriate model for describing the scattering behavior of a patch antenna and will be used for modeling of the circular patch antenna.

![Figure 3-29. A rectangular patch antenna with lumped element equivalent (adapted from [174]).](image)

### 3.3.5 Electrical Antenna Model

The two most common patch antennas are the rectangular and circular patch antennas as shown in Figure 3-30. The width of a rectangular patch antenna is related
to the operating wavelength $\lambda$ as $2b = \lambda/2$ [175]. The radius of a circular patch antenna is related to the operating wavelength $\lambda$ as $b = 1.841\lambda/2\pi$ [175]. Furthermore, the area of a circular patch antenna with radius $b$ is given as $A = \pi b^2$.

The operating frequency of a circular patch antenna is related to the speed of light in free-space $c$, the radius of the circular patch antenna $b$, and the effective permittivity $\varepsilon_{eff}$ as [176]

$$f_o = \frac{1.841c}{2\pi b \sqrt{\varepsilon_{eff}}}$$

(3-90)

The area of a circular patch antenna is described as an integration in polar coordinates as $A = \int_0^{2\pi} \int_0^b rdrd\theta$. The fill factor is described as the area of the patch antenna with respect to the area of the circular diaphragm (i.e., $F = A_{ant}/A_d$). A trade-off exists between the size of the diaphragm and the operating frequency that will be discussed further in the Chapter 4. A circular patch antenna will be the main focus of this research as it is able to approach a fill factor of 1, whereas a rectangular patch is limited to a fill factor of $2/\pi$ or 0.64.
The dynamic pressure sensor has an antenna and ground plane forming an electrical capacitor as shown in Figure 3-31a. The nominal gap distance between the electrodes is given as \( d = w_o + m \). The nominal capacitance (i.e., unloaded diaphragm) between the antenna and ground plane is the parallel combination of the capacitance of the handle (i.e., \( C_1 = \varepsilon_{r1}\varepsilon_o A/m \)) and that of the air gap (i.e., \( C_2 = \varepsilon_o A/w_o \)) or

\[
C_o = \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_o A}{d} \left[ \frac{\varepsilon_{r1}}{1 + \varepsilon_{r1} \frac{w_o}{m}} \cdot \left( 1 + \frac{w_o}{m} \right) \right] = \frac{\varepsilon_{eff}\varepsilon_o A}{d}. \tag{3-91}
\]

The area of the capacitor (i.e., \( A = \pi b^2 \)) does not consider the fringe fields along the periphery of the circular plates. The fringe fields are significant and must be accounted for by utilizing an effective radius \( b_e \) in place of the radius \( b \) as [176]

\[
b_e = b \sqrt{1 + \frac{2d}{\pi b\varepsilon_{eff}} \left( \ln \frac{\pi b}{2d} + 1.7726 \right)}. \tag{3-92}
\]

The effective permittivity between the antenna and ground plane for an unloaded diaphragm is related to the permittivity of the handle \( \varepsilon_{r1} \), the nominal air gap \( w_o \), and the handle thickness \( m \) as [159]
\[
\varepsilon_{\text{eff}} = \frac{\varepsilon_r}{1 + \varepsilon_r \frac{w_o}{m}} \left(1 + \frac{w_o}{m}\right). \tag{3-93}
\]

In the case of \(m \gg w_o\) the effective permittivity is approximated as \(\varepsilon_{\text{eff}} \approx \varepsilon_r / \left(1 + \varepsilon_r \frac{w_o}{m}\right)\). The patch antenna electrically resonates at \(f_o = 1/2\pi \sqrt{L_o C_o}\) for an unloaded diaphragm. The resonant frequency is also related to the wavelength \(\lambda\) as \(f_o = c/\lambda \sqrt{\varepsilon_{\text{eff}}}\). The equivalent inductance for the patch antenna is related to the nominal distance between the antenna and ground plane \(d = w_o + m\), the permeability of free-space \(\mu_o\), and the area of the patch antenna \(A = \pi b^2\) as [176]

\[
L_o = \frac{\lambda^2 \varepsilon_{\text{eff}}}{4\pi^2 c^2 C_o} = \frac{\lambda^2 d \mu_o}{4\pi^2 A} = \frac{\mu_o d}{\pi (1.841)^2}. \tag{3-94}
\]

The nominal capacitance may also be written in an alternate form as

\[
C_o = \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_r \varepsilon_o A}{m} \left[\frac{1}{1 + \varepsilon_r \frac{w_o}{m}}\right] = C_1 \cdot \frac{1}{1 + \varepsilon_r \frac{w_o}{m}}. \tag{3-95}
\]

It is observed that the nominal capacitance \(C_o\) is bounded for all \(w_o/m\) such that \(0 \leq C_o \leq C_1\). Under a uniformly applied pressure the diaphragm will deflect in such a way that the peak deflection is at the center of the diaphragm. It is convenient to describe the average deflection of the diaphragm for simplification in solving for the capacitance of a diaphragm under a uniformly applied pressure. The average deflection of a diaphragm is determined by integrating over the area of the antenna then dividing by the area of the antenna \(A = \pi b^2\) or

\[
\bar{w} = \frac{1}{\pi b^2} \int_0^b 2\pi rw(p,r) dr = w_{pk} \left[1 - \left(\frac{b}{a}\right)^2 + \frac{1}{3} \left(\frac{b}{a}\right)^4\right]. \tag{3-96}
\]

A small antenna radius (with respect to the diaphragm radius) has a high average deflection. A high average deflection results in a high dynamic range \(DR\). A high
dynamic range may also be achieved using a high permittivity material for the handle. The trade-off with a small antenna-to-diaphragm radius is a high operating frequency.

The capacitance between a circular patch antenna and ground plane with a diaphragm under a uniformly applied pressure is related to the air gap \( g(p, r) = w_o - w(p, r) \) as

\[
C(p) = \frac{\varepsilon_r \varepsilon_0}{m} \int_0^{2\pi} \int_0^b \frac{r}{1 + \varepsilon_r \frac{g(p, r)}{m}} drd\theta. \tag{3-97}
\]

The effective permittivity between the circular patch antenna and the ground plane for a diaphragm under a uniformly applied pressure is

\[
\varepsilon_{eff}(p) = \frac{\varepsilon_r}{A} \int_0^{2\pi} \int_0^b \frac{r}{1 + \varepsilon_r \frac{g}{m}} \cdot \left(1 + \frac{g}{m}\right) drd\theta. \tag{3-98}
\]

For a given pressure the effective permittivity varies as a function of the radius \( r \) of the diaphragm. To illustrate consider a diaphragm under a uniform load with maximum deflection at the center (i.e., \( w = w_{pk} \)). The effective permittivity \( \varepsilon_{eff} \) as a function of the normalized radius \( r/a \) is shown in Figure 3-32. The normalized displacement of the diaphragm \( w/w_{pk} \) is also shown. A high diaphragm-to-handle thickness ratio (i.e., \( h/m \)) results in a high permittivity variation (or change) under the diaphragm. A high permittivity variation yields a high electrical sensitivity (i.e., \( S_e \propto \frac{d}{dw} \left[ \frac{1}{\sqrt{\varepsilon_{eff}}} \right] \)), which results in a high dynamic range \( DR \).
Figure 3-32. Effective permittivity of pressure sensor as a function of the normalized radius of the diaphragm with a sapphire handle.

The capacitance can also be solved for directly with a Taylor series expansion by substituting \( g(p) = w_o - w \) into Eq. 3-97 and re-arranging to obtain

\[
C(p) = \frac{\varepsilon_r \varepsilon_o}{m + \varepsilon_r w_o} \int_0^b \int_0^{2\pi} \frac{r}{1 - \frac{\varepsilon_r}{m + \varepsilon_r w_o} \cdot w(p, r)} \, dr \, d\theta.
\] (3-99)

A dimensionless parameter \( \gamma = \varepsilon_r w_{pk} / (m + \varepsilon_r w_o) \) is used to replace \( w_{pk} \) and the integration variable \( r \) is replace with the dimensionless variable \( x = \sqrt{\gamma} \left[ 1 - \left(\frac{r}{a}\right)^2 \right] \)

such that the capacitance is reduced to [177]

\[
C(p) = C_o \cdot \frac{1}{\sqrt{\gamma}} \int_0^{\sqrt{\gamma}} \frac{dx}{1 - x^2} = C_o \cdot \frac{1}{\sqrt{\gamma}} \tanh^{-1} \sqrt{\gamma}.
\] (3-100)

A Taylor series expansion is utilized to expand the hyperbolic tangent function such that the capacitance is equivalent to [177]
\[ C(p) = C_o \cdot \left( 1 + \frac{\gamma}{3} + \frac{\gamma^2}{5} + \cdots \right). \]  
\( (3-101) \)

The electrical quality factor is related to the loaded quality factor of the antenna \( Q_l \), and the quality factors associated with surface waves \( Q_{sw} \), dielectric loss \( Q_d \), and conductor loss \( Q_c \) [153] as

\[
\frac{1}{Q_e} = \frac{1}{Q_l} + \frac{1}{Q_{sw}} + \frac{1}{Q_d} + \frac{1}{Q_c}. \hspace{1cm} (3-102)
\]

The quality factor associated with surface waves \( Q_{sw} \) for thin substrates is ignored [153]. The quality factor due to dielectric loss is related to the loss tangent \( \tan \delta \) of the dielectric material as \( Q_d = 1/\tan \delta \). The quality factor due to conductor loss is related to the thickness of the conductor \( t \) and the skin depth \( \delta = 1/\sqrt{\pi \sigma f} \) [35] as \( Q_c = t/\delta = t\sqrt{\pi \sigma f} \). The aforementioned quality factors are generally much larger than the loaded quality factor contributed by the antenna such that the equivalent quality factor is approximated as \( Q_e \approx Q_l \). The loaded quality factor of the patch antenna is related to the radius of the antenna \( b \), the radiation resistance \( R_{rad} \), the parallel resistance \( R_p \), the effective permittivity of the antenna \( \varepsilon_{eff} \), the distance between the antenna and ground plane \( d = w_o + m \), and the impedance of free-space \( \eta_o = \sqrt{\mu_o/\varepsilon_o} \) as

\[
Q_l = R_{rad} \parallel R_p \cdot \frac{C}{L} = \frac{1.841 \pi b R_{rad} R_p \sqrt{\varepsilon_{eff}}}{(R_{rad} + R_p)d \eta_o}. \hspace{1cm} (3-103)
\]

Balanis assessed the relationship between the permittivity and bandwidth of a microstrip patch antenna using a volumetric relationship [178]. The quality factor of a circular patch antenna using a volumetric relationship is related to the effective permittivity \( \varepsilon_{eff} \) as
\[ Q = \frac{f_o}{\Delta f} \propto \frac{1}{b^2 h} \propto \left(\sqrt{\varepsilon_{eff}}\right)^2 \cdot \frac{1}{\sqrt{\varepsilon_{eff}}} = \sqrt{\varepsilon_{eff}}. \]  

(3-104)

The volumetric relationship by Balanis is in agreement with the loaded quality factor of the circular patch antenna. The parallel resistance of the antenna \( R_p \) is generally much higher than the radiation resistance of the antenna \( R_{rad} \) (i.e., \( R_p \gg R_{rad} \)) such that the equivalent electrical quality factor is determined as

\[ Q_e = Q_l \approx \frac{1.841 \pi b R_{rad} \sqrt{\varepsilon_{eff}}}{\eta_0}. \]  

(3-105)

The radiation resistance \( R_{rad} \) is difficult to model; however, if the dielectric thickness is neglected the radiation resistance is approximated by a closed-form solution given as [175]

\[ R_{rad} = \frac{V_o^2}{2 P_r} = \frac{\lambda^2 \eta_0}{\pi^3 b^2 \left[ \frac{4}{3} - \frac{8}{15} (k_o b)^2 + \cdots \right]}. \]  

(3-106)

The radiation resistance, however, is typically determined experimentally. The radiation resistances of a circular microstrip antenna with dielectric permittivities of 2.32 and 9.8, for example, are measured as 430 \( \Omega \) and 1200 \( \Omega \) (for frequencies less than 400 MHz), respectively [175]. The radiation resistance of the antenna is important as it determines the electrical quality factor, which further determines minimum resolvable frequency \( f_{\text{min}} \) and the dynamic range \( DR \).

The electrical sensitivity of the dynamic pressure sensor is related to the resonant frequency \( f_o = c/\lambda\sqrt{\varepsilon_{eff}} \) as

\[ S_e(p) = \frac{df_o}{dw} = \frac{d}{dw} \left( \frac{c}{\lambda\sqrt{\varepsilon_{eff}(p)}} \right). \]  

(3-107)
The antenna reflection coefficient describes a band-pass transfer response of the RCS. The antenna reflection coefficient is related to the parallel resistance $R_p$, the radiation resistance $R_{rad}$, the inductance $L$ of the antenna, and the capacitance $C$ between the antenna and ground plane as

$$\Gamma_a = \frac{E^r}{E^i} = \frac{R_p \parallel Z_L \parallel Z_C}{R_{rad} + R_p \parallel Z_L \parallel Z_C} = \frac{R_p / R_{rad}}{1 + \frac{R_p}{R_{rad}} + j\omega R_p C + \frac{R_p}{j\omega L}}$$

(3-108)

The antenna reflection coefficient at electrical resonance (i.e., $f_o = 1/2\pi\sqrt{LC}$) reduces to

$$\Gamma_a(f_o) = \frac{E^r}{E^i} = \frac{R_p}{R_{rad} + R_p}.$$  

(3-109)

### 3.3.6 Temperature Effects

The radius of the slot antenna changes as a function of temperature. The radius of the slot antenna is related to the CTE of the antenna $\alpha$ and change in temperature $\Delta T = T - T_o$ as

$$b(T) = b(T_o)(1 + \alpha\Delta T).$$  

(3-110)

The CTE $\alpha$ of the antenna determines the amount of expansion for elevated temperatures $T > T_o$. The CTE of aluminum, for example, is 27 ppm/°C up to 500°C [66], whereas the CTE of platinum is 9 ppm/°C [66]. The permittivity of the handle is also a function of temperature (i.e., $\varepsilon_r(T)$). The permittivity of the handle is related to the thermal coefficient of dielectric constant (TCDk) $\gamma$ and the change in temperature $\Delta T' = T - T_o$ as

$$\varepsilon_r(T) = \varepsilon_r(T_o)(1 + \gamma\Delta T').$$  

(3-111)
The TCDk of the handle determines the change in the dielectric constant for elevated temperatures \( T > T_o \). The TCDk of Borofloat® 33, for example, is +12705 ppm/°C up to 450°C [179], whereas the TCDk of sapphire is +1550 ppm/°C up to 800°C [89]. The permittivity of air is a function of the pressure, temperature, and humidity [180]. The permittivity of dry air at room temperature is 1.00054 with a TCDk of -2 ppm/°C [180]. The change in the effective permittivity at elevated temperatures is driven by the TCDk of the handle.

The resonant frequency of the slot antenna at elevated temperatures drifts as a function of the expanding radius \( b(T) \) and the effective permittivity \( \varepsilon_{eff}(T) \) as \( f_o(T) = 1.841c / 2\pi b(T) \sqrt{\varepsilon_{eff}(T)} \). The fractional change in the resonant frequency is related to the CTE of the slot antenna \( \alpha \) and the TCDk of handle \( \gamma \) as

\[
\frac{f_o(T)}{f_o} = \frac{b}{b(T)} \frac{\varepsilon_{eff}}{\varepsilon_{eff}(T)} = \frac{1}{1 + \alpha \Delta T} \frac{1}{\sqrt{1 + \gamma \Delta T}}. \tag{3-112}
\]

The CTE of platinum is much smaller than the TCDk of sapphire (i.e., \( \alpha \ll \gamma \)) such that for the high temperature sensor the fractional change in the resonant frequency for the slot antenna is reduced to

\[
\frac{f_o(T)}{f_o} \approx \frac{1}{\sqrt{1 + \gamma \Delta T}}. \tag{3-113}
\]

### 3.3.7 Antenna Scaling

Suppose the radius of a circular patch antenna is scaled by a factor \( S \) such that the scaled radius is \( b' = b / S \). The area of the circular patch antenna is related to the radius \( b \) as \( A = \pi b^2 \). The scaled area is related to the scaling factor \( S \) as
\[ A' = \pi (b')^2 = \frac{A}{S^2}. \]  

The operating wavelength of a circular patch antenna is related to the radius \( b \) as \( \lambda = 2\pi b/1.841 \). The scaled operating wavelength is related to the scaling factor \( S \) as
\[ \lambda' = \frac{2\pi b'}{1.841} = \frac{\lambda}{S}. \]  

The frequency of a circular patch antenna is related to the operating wavelength \( \lambda \) as \( f = c/\lambda \sqrt{\varepsilon_{\text{eff}}} \). The scaled frequency is related to the scaling factor \( S \) as
\[ f' = \frac{c}{\lambda' \sqrt{\varepsilon_{\text{eff}}}} = S f. \]  

The scaled time period associated the frequency of the circular patch antenna is related to the scaling factor \( S \) as
\[ T' = \frac{1}{f'} = \frac{T}{S}. \]  

The structural RCS of a circular patch antenna is related to the physical area of the antenna \( A \) and the operating wavelength \( \lambda \) as \( \sigma = 4\pi A^2/\lambda^2 \). The scaled RCS is related to the scaling factor \( S \) as
\[ \sigma' = \frac{4\pi \cdot (A')^2}{\lambda'} = \frac{\sigma}{S^2}. \]  

The effective permittivity \( \varepsilon_{\text{eff}} \) of the circular patch antenna is not a function of the scaling factor \( S \). The capacitance of a circular patch antenna is related to the physical area of the antenna \( A \) as \( C = \varepsilon_{\text{eff}} \varepsilon_o A/d \). The scaled capacitance is related to the scaling factor \( S \) as
\[ C' = \frac{\varepsilon_{\text{eff}} \varepsilon_o A'}{d} = \frac{C}{S^2}. \]
The scaled frequency is related to the scaled capacitance as \( f' = \frac{1}{2\pi}\sqrt{L'C'} \), which can be used to validate Eq. 3-116.

**Table 3-9. Summary of scaling relationships for circular patch antenna parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Full scale</th>
<th>Sub-scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>( b )</td>
<td>( b/S )</td>
</tr>
<tr>
<td>Area</td>
<td>( A )</td>
<td>( A/S^2 )</td>
</tr>
<tr>
<td>Wavelength</td>
<td>( \lambda )</td>
<td>( \lambda/S )</td>
</tr>
<tr>
<td>Frequency</td>
<td>( f )</td>
<td>( Sf )</td>
</tr>
<tr>
<td>Time Period</td>
<td>( T )</td>
<td>( T/S )</td>
</tr>
<tr>
<td>RCS</td>
<td>( \sigma )</td>
<td>( \sigma/S^2 )</td>
</tr>
<tr>
<td>Capacitance</td>
<td>( C )</td>
<td>( C/S^2 )</td>
</tr>
</tbody>
</table>

A summary of the described parameters and their relationship to the scaling factor \( S \) is shown in Table 3-9. The relationships in Table 3-9 give insight into some trade-offs with scaling of the circular patch antenna. One particularly important trade-off is that between the frequency and RCS. A circular patch antenna designed to operate at a frequency that scales with \( S \) will have an RCS that scales with \( 1/S^2 \). The two-way path loss described in Chapter 5 for the power loss through the electromagnetic waveguide also scales as \( S^4 \). A lower frequency is desired for lower power loss and higher RCS; however, a lower frequency results in a higher antenna area. The size of the sensor constrains the size of the antenna. A 10 mm x 10 mm die, for example, can have a maximum antenna diameter of 8 mm.

### 3.4 Summary

The sensor design aspects were discussed in Chapter 3. First, the properties of materials (i.e., mechanical, electrical, and thermal) at elevated temperatures was discussed. Sapphire in particular has excellent mechanical, electrical, and thermal characteristics at elevated temperatures including greater than 1 M\( \Omega \).cm of electrical
resistivity up to 1800K (1526°C). The machinability of high-temperature materials was also discussed. Unfortunately, ceramics such as sapphire tend to be difficult to machine; however, previous work by Blood and Mills provides the groundwork for effective laser machining of sapphire. Finally, lumped element modeling of the sensor including acousto-mechanical, thermomechanical noise, and electrical modeling of the antenna was discussed.
Chapter 4 discusses sensor optimization and fabrication aspects associated with the passive wireless sensor. An optimization is first performed on the design parameters associated with the demonstration sensor. Next, the fabrication of a proof-of-concept sensor with a silicon-based diaphragm is discussed to validate the sensor design followed by the fabrication of a high-temperature sensor with a sapphire-based diaphragm to yield a high-temperature sensor. Finally, packaging of the fabricated sensors is discussed for experimental characterization.

4.1 Optimization

The design of a dynamic pressure sensor requires careful selection of design parameters such that an optimum performance is achieved. The design parameters of a pressure sensor have trade-offs that must be considered. Sensitivity, for example, has a trade-off with the bandwidth defined by the sensitivity-bandwidth product. A higher sensitivity, for instance, results in a lower bandwidth, whereas a higher bandwidth results in a lower sensitivity. Each design parameter typically has bounds to which the parameter can fall within. The optimum performance is achieved by selecting a parameter to be minimized then performing a routine which finds the set of parameters to satisfy the minimization condition.

The goal for the dynamic pressure sensor is to maximize the mechanical sensitivity for a given maximum pressure. The mechanical sensitivity of the sensor is related to the stiffness of the diaphragm as $S_m = 1/k_{da}$. Suppose the diaphragm thickness $h$ is fixed such that the mechanical sensitivity is a function of the diaphragm
radius $a$ as $S_m \propto a^6$. Furthermore, suppose the maximum attainable pressure is primarily a function of the diaphragm radius $a$ as $p_{\text{max}} \propto 1/a^4$. The mechanical sensitivity as a function of the maximum attainable pressure is shown in Figure 4-1.

![Figure 4-1. An illustration of Pareto front for mechanical sensitivity of a dynamic pressure sensor.](image)

The maximum mechanical sensitivity for a given maximum pressure lies on the Pareto front [181]. The feasible region below the Pareto front represents all attainable mechanical sensitivities for a given maximum pressure. By performing an optimization the set of parameters that yield the highest mechanical sensitivity are found for a given maximum pressure. The equivalent sensitivity is proportional to the mechanical sensitivity; therefore, a higher mechanical sensitivity yields a higher equivalent sensitivity.
4.1.1 Design Parameters

There are three categories of parameters associated with designing the dynamic pressure sensor: geometric, material, and performance. The diaphragm’s geometric parameters include the diaphragm radius $a$ and thickness $h$. The diaphragm’s material parameters include the Young’s modulus $E$ and electrical resistivity $\rho$. Each design parameter can further be classified as either fixed or variable. The fixed parameters include the material properties of the diaphragm as well as the thickness of the diaphragm. The radius of the diaphragm is a variable parameter. The microfabrication processing capabilities introduce upper and lower bounds on possible geometric parameters. A list of associated parameters for the dynamic pressure sensor are shown in Table 4-1.

The handle thickness is determined by the wafer thickness of 300 $\mu$m for Borofloat® 33 (for the demonstration sensor) and the die thickness of 200 $\mu$m for sapphire (for the high temperature sensor). A higher antenna thickness yields a higher signal reflection on the antenna; however, thicknesses of 500 nm (for the demonstration sensor) and 150 nm (for the high temperature sensor) were selected for cost purposes. The nominal air gap was chosen to yield the desired maximum pressure (i.e., 3 kPa for demonstration sensor and 5 kPa for high temperature sensor) within the bounds of the diaphragm radius (i.e., 2 to 4 mm). The vent length and width were determined to yield a cut-on frequency of $<20$ Hz.

As discussed earlier, the material parameters including the Young’s modulus of the diaphragm, the electrical resistivity of the antenna, and the permittivity of the handle are fixed. The maximum pressure $p_{max}$ is pre-determined for the specific application.
The variable geometric parameters include the diaphragm thickness and radius, the antenna radius, and the antenna’s slot length and width. The antenna slot length and width are determined using an electromagnetic FEM solver. The variable performance parameters include the electrical operating frequency and the equivalent sensitivity.

Table 4.1. Fixed and variable parameters of the dynamic pressure sensor [68], [130], [179].

<table>
<thead>
<tr>
<th>Type</th>
<th>Var.</th>
<th>Units</th>
<th>Value</th>
<th>Alt.</th>
<th>Description</th>
</tr>
</thead>
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<td>μm</td>
<td>300</td>
<td>200</td>
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<td>150</td>
</tr>
<tr>
<td></td>
<td></td>
<td>w_o</td>
<td>μm</td>
<td>15</td>
<td>20</td>
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<td>μm</td>
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<td>100</td>
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<td></td>
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<td></td>
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<td>Performance</td>
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<td>s</td>
<td>kHz/Pa</td>
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</table>

1 silicon, 2 sapphire, 3 aluminum, 4 platinum, 5 Borofloat® 33

4.1.2 Objective Function

As discussed earlier, the mechanical sensitivity of the dynamic pressure sensitivity is related to the stiffness of the diaphragm as \( S_m = 1/k_{da} \). The objective of the
optimization is to minimize the stiffness of the diaphragm $k_{da}$ within the bounds of the associated design parameters $X$ such that

$$\min_X f_{obj}(X) = k_{da}. \quad (4-1)$$

The associated design parameters $X$ are described by

$$X = \{a, h, f, b\}. \quad (4-2)$$

### 4.1.3 Constraints

The design parameters $X$ are subject to bounds given by $LB \leq X \leq UB$. The lower and upper bounds $LB$ and $UB$, respectively, for each variable design parameter are found in Table 4-1. There are linear constraints on design parameters $a$, $h$, $f$, and $b$. For instance, the minimum antenna radius $b_{\text{min}}$ is determined by the maximum operating frequency $f_{\text{max}}$ as $b_{\text{min}} = 1.84c/2\pi f_{\text{max}}\sqrt{\varepsilon_{\text{eff}}}$. Similarly, the maximum antenna radius $b_{\text{max}}$ is determined by the minimum operating frequency $f_{\text{min}}$ as $b_{\text{max}} = 1.84c/2\pi f_{\text{min}}\sqrt{\varepsilon_{\text{eff}}}$. The design antenna radius must also be between the minimum and maximum antenna radii or $b_{\text{min}} \leq b \leq b_{\text{max}}$. The radius of the diaphragm $a$ is related to the radius of the antenna $b$ as

$$b \leq a. \quad (4-3)$$

For a pre-determined maximum pressure $p_{\text{max}}$ the radius of the diaphragm $a$ is related to the thickness of the diaphragm $h$ as

$$a \left(\frac{p_{\text{max}}}{2.3E}\right)^{1/4} - h = 0. \quad (4-4)$$
The maximum displacement of the diaphragm $w_{pk,nl}$ is bounded to be less than or equal to the nominal air gap $w_o$ such that the thickness of the diaphragm $h$ is determined as

$$h \leq \frac{w_o}{0.41(1 - \nu^2)}.$$  \hspace{1cm} (4-5)

The radius of the diaphragm $a$ is bounded between 2 and 4 mm such that it is large enough to have an inclusive antenna operating in the Ku-band, while also being small enough such that there’s at least 1 mm distance between the edge of the diaphragm and the edge of the 10 mm x 10 mm sensor die. The thickness of the diaphragm $h$ for the demonstration sensor is bounded between 15 and 50 $\mu$m such that a nominal air gap $w_o$ of 15 $\mu$m is met. The operating frequency $f$ is bounded between 14 and 16 GHz to allow operation near the center of the operating frequency band of the electromagnetic waveguide (i.e., 12 to 18 GHz). The radius of the antenna $b$ is selected such that the antenna will have a resonant frequency around 15 GHz.

The optimization of the aforementioned design parameters is attained using the fmincon function in MATLAB. The slot length $l_s$ and width $w_s$ of the antenna are optimized using Ansoft’s HFSS. HFSS is an FEM solver commonly used for simulating electromagnetic structures such as transmission lines, antennas, filters, and packages.

### 4.1.4 Results

The optimization of the diaphragm is performed using the fmincon function in MATLAB. The written code used for optimizing the demonstration and high temperature sensors is found in Appendix A. The demonstration sensor is designed such that two diaphragm sizes (i.e., 7.6 mm and 6.4 mm) satisfy corresponding pressure
requirements (i.e., 3 kPa and 6 kPa) for a pre-determined maximum displacement of 13.5 μm. The optimized design parameters for the demonstration sensor are given in Table 4-2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>$P_{\text{max}} = 3 \text{ kPa}$</th>
<th>$P_{\text{max}} = 6 \text{ kPa}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diaphragm Diameter</td>
<td>$2a$</td>
<td>7.6 mm</td>
<td>6.4 mm</td>
</tr>
<tr>
<td>Diaphragm Thickness</td>
<td>$h$</td>
<td>35 μm</td>
<td>35 μm</td>
</tr>
<tr>
<td>Cavity Depth</td>
<td>$w_o$</td>
<td>15 μm</td>
<td>15 μm</td>
</tr>
<tr>
<td>Maximum Displacement</td>
<td>$w_{pk,nl}$</td>
<td>13.5 μm</td>
<td>13.5 μm</td>
</tr>
<tr>
<td>Antenna Diameter</td>
<td>$2b$</td>
<td>5.8 mm</td>
<td>5.8 mm</td>
</tr>
<tr>
<td>Nom. Operating Frequency</td>
<td>$f_o$</td>
<td>14.9 GHz</td>
<td>14.9 GHz</td>
</tr>
</tbody>
</table>

The optimization of the antenna’s slot length $l_s$ and width $w_s$ are performed with HFSS. HFSS solves for the scattering parameters of a structure by utilizing an adaptive mesh at one or more solution frequencies. The adaptive mesh iteratively decreases the mesh element size (i.e., increases number of elements) until successive scattering parameters converge below some desired scattering parameter error. HFSS also has an optimization feature for automated sweeping of one or more design parameters. An antenna in the far-field is solved for using the HFSS Integral Equation (IE) solver that utilizes the method of moments (MoM) technique [166] to solve for the currents on the surface of the antenna in free-space.

HFSS IE requires a discrete sweep of frequency to obtain frequency-related information about the antenna. The uses of a discrete sweep significantly increases the required computational time for HFSS. A linearly polarized incident plane wave with a 1 V/m electric field is utilized to represent the plane wave incident on the antenna. The plane wave is polarized in an axis that is normal to the surface of the antenna. As discussed earlier, the circular patch antenna has a slot with the purpose of converting
the linear incident plane wave into a circularly polarized plane wave. HFSS IE computes the RCS in both the co-axis and cross-axis.

Figure 4-2. Resonant frequency as a function of antenna air gap for a 5.7 mm circular patch antenna.

First, a circular patch antenna with a 50 \( \mu \text{m} \) silicon device layer and a 270 \( \mu \text{m} \) Borofloat® 33 handle layer is simulated with a nominal air gap of 14 \( \mu \text{m} \). The slot length and slot width of the patch antenna are 800 \( \mu \text{m} \) and 100 \( \mu \text{m} \), respectively. The resonant frequency as a function of the air gap for a circular patch antenna with a 5.7 mm diameter antenna is shown in Figure 4-2. As expected a lower air gap yields a lower resonant frequency. The electrical sensitivity of the 5.7 mm circular patch antenna is approximated as 71 kHz/nm in the full air gap range of 14 \( \mu \text{m} \).
Next, the slot width of a circular patch antenna is varied between 60 to 200 μm. The slot length and air gap are held at 800 μm and 9 μm, respectively. The cross-axis RCS and the quality factor as a function of the slot width for a circular patch antenna with a 5.7 mm diameter antenna are shown in Figure 4-3. It is observed that a lower slot width results in a lower RCS. The quality factor does not appear to be a function of the slot width. Furthermore, the RCS does not vary much for widths of 100 μm or more. The resonant frequency of the circular patch antenna decreases slightly at a rate of approximately 142 kHz/μm for slot lengths from 60 to 200 μm.
Next, the slot length of a circular patch antenna is varied between 400 μm and 1200 μm. The slot width and air gap are held at 100 μm and 9 μm, respectively. The cross-axis RCS and the quality factor as a function of the slot length for a circular patch antenna with a 5.7 mm diameter antenna are shown in Figure 4-4. It is observed that a lower slot length results in a lower RCS. Furthermore, a lower slot length results in a higher quality factor. The quality factor does not vary much for slot lengths less than 800 μm and decreases for slot lengths greater than 800 μm. The resonant frequency of the circular patch antenna decreases slightly at a rate of approximately 125 kHz/μm for slot lengths from 400 to 1200 μm.

The effects of intentional (e.g., by design) or unintentional (e.g., due to packaging) rotational misalignment of the antenna are also important to understand. The cross-axis RCS as a function of the offset angle θ for a circular patch antenna with a 5.7 mm
diameter antenna is shown in Figure 4-5. The offset angle is defined as an offset from the slot’s designed angle of 45°. The slot length, slot width, and air gap are 800 μm, 100 μm, and 9 μm, respectively. The cross-axis RCS has little change for offset angles of less than 15°, then gradually decreases for offset angles greater than 15°. The FEM suggests that for small rotational misalignments there is minimal effect on the RCS of the circular patch antenna.

![Graph](image)

Figure 4-5. RCS as a function of offset angle for 5.7 mm circular patch antenna.

The thickness of the handle is should also be studied to determine its effects on the performance of the circular patch antenna. The slot length, slot width, and air gap are 800 μm, 100 μm, and 9 μm, respectively. The handle thickness is varied between 100 and 500 μm. The RCS and quality factor as a function of the handle thickness for a circular patch antenna with a 5.7 mm diameter antenna is shown in Figure 4-6. It is observed that a lower handle thickness results in a higher RCS. Furthermore, a lower
handle thickness results in a higher quality factor. A lower handle thickness also results in a higher compliance of the handle, which in turn results in a lower capacitance change between the antenna and ground plane. The result of a lower capacitance change is a decrease in the electrical sensitivity of the circular patch antenna.

Figure 4-6. RCS and quality factor as a function of handle thickness for 5.7 mm circular patch antenna.

The resonant frequency of a circular patch antenna is a strong function of the handle thickness as shown in Figure 4-7. A low tolerance for a thin handle can result in a high change in the nominal resonant frequency of a circular patch antenna. A circular patch antenna with a 5.7 mm diameter antenna and a handle thickness of 150 µm with a +/- 10% tolerance, for example, can have a nominal resonant frequency with a +/- 300 MHz deviation.
Figure 4-7. Resonant frequency as a function of handle thickness for 5.7 mm circular patch antenna.

4.2 Fabrication of Demonstration Sensor

A demonstration sensor is first fabricated to validate the ability to sense dynamic pressure using RF backscattering. The demonstration sensor will also validate the sensor’s performance as predicted by HFSS. The demonstration sensor is fabricated in silicon, which is very abundant (i.e., low-cost), has mature machinability, and has good mechanical properties for developing MEMS [150]. The demonstration sensor uses common microfabrication processes such as wet etching and DRIE. The sensor is fabricated in a class 100 clean room environment at the University of Florida’s Nanoscale Research Facility. The demonstration sensor has a silicon diaphragm with an affixed Borofloat® 33 handle on the back side of the sensor. Borofloat® 33 is selected as the handle for the demonstration sensor due to its low-cost and compatibility with microfabrication processes. A comparison of some common
mechanical and electrical properties of silicon, Borofloat® 33, and sapphire are shown in Table 4-3.

Table 4-3. Summary of mechanical and electrical properties for sensor materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho_m ) g/cc</th>
<th>( E ) GPa</th>
<th>( \nu )</th>
<th>( \rho_e ) ( \Omega ).cm</th>
<th>( \varepsilon_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon &lt;110&gt; [68]</td>
<td>2.32</td>
<td>170</td>
<td>0.28</td>
<td>( 10^{-2} )</td>
<td>11.8</td>
</tr>
<tr>
<td>Borofloat® 33 [179]</td>
<td>2.20</td>
<td>64</td>
<td>0.20</td>
<td>( 10^{12} )</td>
<td>4.6</td>
</tr>
<tr>
<td>A-Plane Sapphire [130]</td>
<td>3.97</td>
<td>384</td>
<td>0.27</td>
<td>( 10^{16} )</td>
<td>11.5</td>
</tr>
</tbody>
</table>

E - Young’s modulus, \( \nu \) - Poisson’s ratio, \( \rho_e \) - electrical resistivity, \( \varepsilon_r \) - relative permittivity

A 100 mm silicon-on-insulator (SOI) wafer with a 475 +/- 10 \( \mu \)m handle layer, 50 +/- 1 \( \mu \)m device layer, and 0.5 +/- 0.025 \( \mu \)m oxide layer from Ultrasil Corporation [182] is used in the fabrication of the demonstration sensor. The SOI wafer has a resistivity of 1 to 20 \( \Omega \).cm in the handle layer and 0.001 to 0.002 \( \Omega \).cm in the device layer. The SOI wafer is cleaned using a standard RCA cleaning process consisting of an organic SC-1 clean, followed by an oxide clean, then an ion SC-2 clean; the details of the cleanings are found in Appendix C.

Figure 4-8. Fabrication process for demonstration dynamic pressure sensor.
The general fabrication process for the silicon-based demonstration sensor is shown in Figure 4-8. The first step of the fabrication process is to DRIE 15 μm into the silicon device layer of the SOI wafer to define a cavity for the air gap and a trench (i.e., long, narrow opening) for the vent channel. The DRIE successively uses 130 sccm of SF₆ + 13 sccm of O₂ to etch the silicon and 85 sccm of C₄F₈ to passivate the silicon. The second step is to blanket deposit aluminum on the SOI wafer, then pattern and wet etch to define the ground plane in the cavity of the silicon device layer. To coat the surface of the silicon evenly with photoresist it’s important that the thickness of the photoresist be at least 1/3 thicker than any subtractive features in the silicon (i.e., 15 μm deep cavities); as such, 20 μm of photoresist is coated on the silicon.

![Fabricated SOI wafer with aluminum deposited cavity and vent hole](image)

Figure 4-9. Fabricated SOI wafer with aluminum deposited cavity and vent hole (photo courtesy of John Rogers).

The third step is to etch a vent hole in the silicon device layer of the SOI wafer by DRIE as shown in Figure 4-9. In addition to the vent hole for each die, 100 μm wide die
streets are also etched such that a separate die release step is not needed. The fourth step is to remove the backside silicon of the SOI wafer by DRIE. The oxide is subsequently removed by an RIE etch using 30 sccm of CF$_4$, 20 sccm of CHF$_3$, and 50 sccm of Ar. In addition to the backside silicon being removed, die streets are also etched such that the die are released after the oxide etch. The released die of the demonstration sensor are shown in Figure 4-10.

![Fabricated demonstration dynamic pressure sensor die](image)

**Figure 4-10.** Fabricated demonstration dynamic pressure sensor die (photo courtesy of John Rogers).

The fifth step is to attach the demonstration sensor to a Borofloat® 33 handle using an adhesive. The silicon and glass die are aligned by hand, then another piece of glass is laid on top of the silicon die at a 45° angle to protect the diaphragm. A micrometer is then used to hold the die together and a Loctite® Super Glue ULTRA Gel Control [183] is applied around the perimeter of the silicon and glass die. The adhesive
is applied to the die using an dispensing tip by Nordson [184]. The demonstration sensor with a Borofloat® 33 handle is shown in Figure 4-11.

![Figure 4-11. 10 mm x 10 mm demonstration dynamic pressure sensor die with Borofloat® 33 handle (photo courtesy of John Rogers).]

### 4.3 Fabrication of High-Temperature Sensor

The materials used to construct a high-temperature dynamic pressure sensor must be able to operate at high temperatures. Sapphire is an excellent material for operating at high temperatures in terms of its mechanical, electrical, and thermal properties. Ceramic materials like sapphire, on the other hand, are difficult to machine. Cracking is also a common issue with laser machining of ceramics. Mills observed cracking at the tether bases of a floating element shear stress sensor machined in sapphire [107]. The parameters of the laser source may be adjusted to minimize cracking. The general fabrication process for the high-temperature dynamic pressure sensor is shown in Figure 4-12.
1. Pattern metal on diaphragm and handle
2. Laser etch die
3. Attach die together

Figure 4-12. Fabrication process for high-temperature dynamic pressure sensor.

### 4.3.1 Laser Machined Sensor

The high-temperature sensor is constructed of a 10 mm x 10 mm x 50 \( \mu \text{m} \) A-plane sapphire die for the diaphragm and a 10 mm x 10 mm x 200 \( \mu \text{m} \) A-plane sapphire die for the handle from Valley Design [185]. The first step is to deposit and pattern metal on the sapphire die. A 300 nm layer of chrome is deposited and patterned on the two sapphire die to promote lift-off during the lift-off process. A 10 nm layer of titanium followed by 150 nm of platinum are then deposited. The antenna and ground planes are formed by performing a lift-off process. The second step is to send the die to Oxford Lasers [186] where thru holes are machined in both die for bonding purposes and a 20 \( \mu \text{m} \) deep pocket for the cavity and a vent channel in the handle die are machined. The third step is to align the two die by hand, then a piece of glass is laid on top of the diaphragm die at a 45° angle to protect the diaphragm. A micrometer is then used to hold the die together and a Loctite® Super Glue ULTRA Gel Control [183] is applied at
the corner tips of the sapphire die. The Loctite® adhesive is not high-temperature compatible and is only applied for temporarily holding the die together. The die are then released and a Cotronics 989F alumina adhesive [187] is applied around the perimeter of the die. Cotronics 989F is a high-temperature compatible adhesive for operating up to 1900°F (i.e., 1030°C). A Master-Mite 10008 heat gun [188] is used to decrease the cure time of the adhesive to just a few minutes.

4.3.2 Alternative Sensor

The laser machining of sapphire can be cost prohibitive as well as time consuming. An alternative to subtractive machining of the cavity and vent channel is to perform an additive step. A cavity is formed by the addition of an inner spacer layer between the diaphragm and handle sapphire die. The spacer layer also functionally serves as the vent channel. A high-temperature metal such as titanium is deposited at a controlled rate for precise definition of the cavity and vent channel. The high resistivity of the sapphire isolates the antenna from the ground plane; therefore, the low resistivity of the metal does not interfere with the electrical performance of the sensor. The alternative process for the fabrication of a high-temperature sensor without laser machining is shown in Figure 4-13. It should be noted that the alternative sensor may be limited at high temperatures due to the stress induced by CTE expansion of the titanium in the lateral direction. In this regard, although the laser machined sensor is more expensive it may also be more robust at high temperatures.
1. Pattern metal on diaphragm and handle

2. Pattern spacer layer on die

3. Attach die together

Figure 4-13. Alternative fabrication process for high-temperature dynamic pressure sensor.

The high-temperature sensor is constructed of a 10 mm x 10 mm x 50 μm A-plane sapphire die for the diaphragm and a 10 mm x 10 mm x 200 μm A-plane sapphire die for the handle from Valley Design [185]. An antenna and ground plane are formed on the two sapphire die using a lift-off process as described in 4.3.1. A 2.5 μm layer of titanium is then deposited and patterned on the die to form the cavity and vent channel. Hydrofluoric acid (HF), known to be an effective etchant of titanium [102], is used for patterning the titanium. The diaphragm and handle die are then attached using the same process described in 4.3.1. The fabricated high-temperature sapphire sensor using an alternative additive fabrication process is shown in Figure 4-14. The antenna diameter for the high-temperature sapphire sensor is 3.8 mm compared to an antenna diameter of 5.7 mm for the demonstration sensor due to a higher permittivity for sapphire.
Figure 4-14. 10 mm x 10 mm high-temperature sapphire-based dynamic pressure sensor die (photo courtesy of John Rogers).

The demonstration and high-temperature pressure sensors are inspected with an optical microscope to obtain the lateral dimensions and with thickness measuring tools to obtain the geometric parameters of the sensors as shown in Table 4-4. The nominal air gap and ground/antenna thicknesses are measured in the clean room using a Dektak 150 profilometer. The nominal air gap is determined by the etching rate and time of the DRIE as well as the thickness of the ground plane. The antenna and ground plane thicknesses are determined by the deposition rate and time of the sputterer. The thickness of the Borofloat® 33 handle is measured by a micrometer with a 1 μm resolution. The manufacturer of the Borofloat® 33 glass wafers specified a nominal thickness of 300 μm with a 10% tolerance. An average of 267 μm was measured across the Borofloat® 33 wafer.
Table 4-4. Summary of measured geometries for demonstration and high-temperature pressure sensors.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Units</th>
<th>Demonstration Design</th>
<th>Measured</th>
<th>High Temp Design</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna Diameter</td>
<td>$2b$</td>
<td>mm</td>
<td>5.60</td>
<td>5.60</td>
<td>3.80</td>
<td>3.79</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5.70</td>
<td>5.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5.80</td>
<td>5.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slot Length</td>
<td>$l_s$</td>
<td>μm</td>
<td>800</td>
<td>800</td>
<td>400</td>
<td>397</td>
</tr>
<tr>
<td>Slot Width</td>
<td>$w_s$</td>
<td>μm</td>
<td>100</td>
<td>100</td>
<td>80</td>
<td>76</td>
</tr>
<tr>
<td>Ground Diameter</td>
<td>-</td>
<td>mm</td>
<td>7.50</td>
<td>7.45</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Diaphragm Diameter</td>
<td>$2a$</td>
<td>mm</td>
<td>7.60</td>
<td>7.60</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Diaphragm Thickness</td>
<td>$h$</td>
<td>μm</td>
<td>35.0</td>
<td>35.8</td>
<td>50</td>
<td>-</td>
</tr>
<tr>
<td>Handle Thickness</td>
<td>$m$</td>
<td>μm</td>
<td>300</td>
<td>267</td>
<td>200</td>
<td>-</td>
</tr>
<tr>
<td>Nominal Air Gap</td>
<td>$w_o$</td>
<td>μm</td>
<td>14.0</td>
<td>13.3</td>
<td>4.85</td>
<td>4.62</td>
</tr>
<tr>
<td>Ground Thickness</td>
<td>-</td>
<td>nm</td>
<td>1000</td>
<td>820</td>
<td>150</td>
<td>175</td>
</tr>
<tr>
<td>Antenna Thickness</td>
<td>$t$</td>
<td>nm</td>
<td>500</td>
<td>500</td>
<td>150</td>
<td>175</td>
</tr>
</tbody>
</table>

4.4 Sensor Packaging

The packaging of the passive wireless dynamic pressure sensor consists of two conductive plates; referred to as plate 1 and plate 2 with detailed drawings in Appendix F. The conductive plates are fabricated in aluminum as shown in Figure 4-15. The plates are used for both securing the sensor for incident pressure and interaction with electromagnetic waves. Plate 1 has a recess for allowing the sensor to sit flush with the surface of the plate and also has an opening incorporated in the recess to expose the antenna to the electromagnetic waveguide. Plate 2 has an opening with an area smaller than the sensor for the purpose of holding the sensor in place.

The thickness of plate 1 is 1/8 in. (i.e., 3.17 mm) and the thickness of plate 2 is 1/4 in. (i.e., 6.35 mm). Both plates have two openings for number 6 (i.e., 0.140 in. diameter) screws to secure the plates to the electromagnetic waveguide. Plate 2 also has four openings for number 14 (i.e., 0.250 in. diameter) screws to secure the plate to a plane wave tube.
4.5 Summary

The sensor optimization and fabrication aspects were discussed in Chapter 4. The sensor was optimized to minimize the stiffness of the diaphragm, which yields a higher mechanical sensitivity. The fabrication of a demonstration sensor for validating the sensor design and performance was also discussed followed by the fabrication of a high-temperature sensor using laser machined sapphire for the sensor diaphragm and platinum for the electrical antenna. Finally, the sensor packaging consisting of two conductive plates for securing the sensor and interacting with electromagnetic waves was discussed. The sensor packaging for the passive wireless dynamic pressure sensor is simplistic. The plates were designed as proof of concept for demonstrating the sensor, but can be optimized for field applications.
5.1 Radar Fundamentals

Harald Friis of Bell Labs was an early pioneer in radio propagation and radar. Friis derived an analytical expression relating the one-way power transmitted by one antenna (i.e., transmitter antenna) to the power received by another antenna (i.e., receiver antenna). The power at the receiver antenna is related to the radiated power $p_t G_t$, the transmitter and receiver gains $G_t$ and $G_r$, the operating wavelength $\lambda$, and the separation distance $R$ between the antennas as [38]

$$p_r = \frac{p_t G_t G_r \lambda^2}{(4\pi R)^2}. \quad (5-1)$$
The Friis expression is useful in describing a one-way radio system as shown in Figure 5-1. The power loss due to free-space, also known as free-space path loss, is proportional to $1/R^2$. A radar system is similar to a one-way radio system in that power is transmitted over free-space from a transmitter antenna. The receiver for a radar, however, is an object with a RCS. A part of the incident power at the object is reflected (i.e., backscattered) based on the area of the object (i.e., RCS). The reflected power is then transmitted over free-space to a receiver antenna. A monostatic radar system has a single antenna for the transmitter and receiver. A bistatic radar system has separate transmitter and receiver antennas.

![Figure 5-1. Illustration of operation of a one-way radio and two-way radar.](image)

The power at the receiver antenna for a monostatic radar system is related to the radiated power $p_t G_t$, the transmitter and receiver gains $G_t$ and $G_r$, the operating wavelength $\lambda$, the separation distance $R$ between the antennas, and the RCS $\sigma$ of the object as [38]

$$p_r = \frac{p_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4}. \quad (5-2)$$
The minimum receivable power, also known as the sensitivity, for a monostatic radar system is related to the maximum separation distance $R_{\text{max}}$ between the antenna and the object as

$$S_{\text{min}} = \frac{p_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R_{\text{max}}^4}. \quad (5-3)$$

The noise power of the receiver due to thermal noise is related to Boltzmann’s constant $k_B$, the noise factor $F$, the operating temperature $T_o$, and the receiver bandwidth $\Delta f$ as $p_n = k_B F T_o \Delta f$. The noise power for a 1 Hz bandwidth at room temperature, for example, is -174 dBm. The noise factor defined as the ratio of the SNR looking into the receiver to the SNR looking out of the receiver is bounded such that $F \geq 1$. The noise figure is used to specify the noise of a receiver in decibels as $NF = 10 \log_{10} |F|$ [189]. The signal-to-noise ratio is defined as the ratio of the receiver power to the noise power of the receiver or

$$\text{SNR} = \frac{p_r}{p_n} = \frac{p_t G_t G_r \sigma \lambda^2}{k_B F T_o \Delta f (4\pi)^3 R^4}. \quad (5-4)$$

A low noise figure $NF$ and a high signal-to-noise ratio $\text{SNR}$ are desirable performance characteristics of a receiver. The minimum signal-to-noise ratio of a receiver can be determined from Eq. 5-3 and Eq. 5-4 as

$$\text{SNR}_{\text{min}} = \frac{S_{\text{min}}}{p_n} = \frac{S_{\text{min}}}{k_B F T_o \Delta f}. \quad (5-5)$$

The minimum signal-to-noise ratio is another performance characteristic of a receiver proportional to the receiver sensitivity. A low receiver sensitivity, for example, has a low minimum signal-to-noise ratio.
The minimum required transmitter power can be determined by the receiver sensitivity using a link budget [36]. A link budget adds or subtracts the decibel gains or losses throughout a system to determine the net power gain or loss at the receiver. Consider a monostatic radar system as an example. The transmitter power given as
\[ P_{t, dB} = 10 \log_{10}|p_t| \]
first goes through the transmitter antenna with gain \( G_{t, dB} \). The transmitted power radiates outward with a free-space path loss of \( FSPL = 20 \log_{10}|4\pi R/\lambda| \). The power radiated then reflects off an object with a RCS \( \sigma_{dB} = 10 \log_{10}|\sigma| \). The reflected power then radiates outward with a free-space path loss \( FSPL \). The reflected power at the receiver antenna goes through the receiver antenna with a gain \( G_{r, dB} \). The received power is then determined by the sum of the gains and losses as
\[ P_{r, dB} = P_{t, dB} + G_{t, dB} + FSPL + \sigma_{dB} + FSPL + G_{r, dB}. \]

The receiver power must be greater than or equal to the receiver sensitivity (i.e., \( p_r \geq S_{min} \)) for the object to be detected by the monostatic radar system. The RCS of the object described in 3.3.2 is related to its physical characteristics (i.e., physical area \( A \)) and electrical characteristics (i.e., electrical resistivity \( \rho \), directivity \( D \), reflection coefficient \( \Gamma \)). The directivity \( D(\theta, \phi) \) is a function of the azimuth \( \phi \) and elevation \( \theta \) angles corresponding to where the object is in space with relation to the antenna. There has been much effort to make physically large objects appear electrically small to avoid radar detectability. The F-117 Nighthawk, for example, has angled surfaces to deflect power in other directions as well as radar absorptive materials.
5.2 Radar-on-a-Chip Electronics

A radar system has electronics on the frontend behind the transmitter antenna and on the backend behind the receiver antenna. The capabilities of the electronics determine how well the radar system is able to detect objects. A radar system operating at millimeter-wave frequencies has short wavelengths such that an entire radar system can be developed on a single chip; also known as radar-on-a-chip. The operating frequency for a passive inductor or capacitor scales up as the length and width of the passive scales down by $S$ (i.e. $f' = f/S$). The operating frequency of a transistor, which is inversely proportional to the time delay $\tau$ (i.e., $f \propto 1/\tau$), scales up as the length and width scale down by $S$ (i.e., $f' \propto 1/\tau' = S/\tau$) [71]. The building blocks of a radar-on-a-chip include an oscillator, filtering elements, frequency mixers, and amplifiers.

The transmitter and receiver antennas may also be integrated with a radar-on-a-chip. A monostatic radar system with a single antenna for transmitting and receiving can operate as half-duplex or full-duplex. A half-duplex communication system either transmits or receives at any given time. A full-duplex communication system is able to simultaneously transmit and receive at the same time.

5.2.1 Full-Duplex Antenna Architectures

A full-duplex communication system is hard to achieve due to the high isolation needed to prevent cross-talk between the transmitter and receiver electronics. An RF circulator is a three-port device used for separating the transmitter from the receiver [190]. A circulator is constructed of magnetic ferrite materials that allow power to be transferred to a clockwise adjacent port. A circulator does not allow power to be transferred to a counter-clockwise port, which means a circulator is non-reciprocal. The
insertion loss due to the power transfer of a circulator from port $i$ to port $j$ in decibels is 
$$IL_{ji} = 20 \log_{10} |S_{ji}|.$$ The return loss or power reflected back to port $i$ in decibels is 
$$RL_i = 20 \log_{10} |S_{ii}|.$$ The isolation describes how well the circulator rejects power transfer from port $j$ to a counter-closewise adjacent port $i$ in decibels as 
$$I_{ij} = 20 \log_{10} |S_{ij}|.$$

![Figure 5-2. A single-junction circulator (a) block diagram and (b) example performance.](image)

A single circulator (i.e., single-junction circulator) for a full-duplex communication system is shown in Figure 5-2a. An ideal circulator has infinite isolation; however, in a practical circulator has some finite isolation such that a part of the transmitted signal will leak through the circulator to the receiver. Additionally, some finite power will be reflected to receiver from the antenna due to an impedance mismatch between the circulator and antenna. Additional reflections may also be seen at the receiver due to multi-path objects beyond the antenna.

The isolation at the transmitter can be improved by using a dual-junction circulator [190] as shown in Figure 5-3a. The performance characteristics of the two circulators is assumed to be equivalent for comparison. The insertion loss from the transmitter to the antenna, in decibels, is twice the insertion loss of a single circulator (i.e., $I_{\text{Ant,Tx}} =$
The isolation between the receiver and the antenna is the same as a single circulator (i.e., $I_{\text{Ant,Rx}} = I_{12}$). The isolation between the transmitter and the receiver is also the same as a single circulator (i.e., $I_{\text{Rx,Tx}} = I_{12}$). The isolation between the antenna and transmitter, on the other hand, is twice the isolation of a single circulator (i.e., $I_{\text{Tx,Ant}} = 2I_{12}$). The dual-junction circulator has improved antenna to transmitter isolation; however, there is no improvement in the isolation between the transmitter and receiver. The isolation between the transmitter and receiver is important for reducing cross-talk between the transmitter and receiver in simultaneous transmit/receive operation.

![Diagram](image)

Figure 5-3. A dual-junction circulator (a) block diagram and (b) example performance.

The transmitter to receiver isolation can be improved by using a triple-junction circulator [190] as shown in Figure 5-4a. The performance characteristics of the three circulators is assumed to be equivalent for comparison. The insertion loss from the transmitter to the antenna, in decibels, is twice the insertion loss of a single circulator (i.e., $I_{\text{Ant,Tx}} = 2I_{21}$). The isolation between the receiver and the antenna is the twice the isolation of a single circulator (i.e., $I_{\text{Ant,Rx}} = 2I_{12}$). The isolation from the transmitter to the receiver is the same as a single circulator (i.e., $I_{\text{Rx,Tx}} = I_{12}$); however, the isolation
from the receiver to the transmitter is twice the isolation as a single circulator (i.e., \( I_{Tx,Rx} = 2I_{12} \)). Additionally, the isolation between the antenna and the transmitter is twice the isolation of a single circulator (i.e., \( I_{Tx,Ant} = 2I_{12} \)).

Figure 5-4. A triple-junction circulator (a) block diagram and (b) example performance.

In summary, single-, dual, and triple-junction circulators can be used for full-duplex communications. A single-junction circulator has the lowest insertion losses for power transfer between the transmitter, receiver, and antenna. The isolation between transmitter and receiver, however, is limited to the isolation characteristics of the circulator. The isolation between the antenna and transmitter for a dual-junction circulator is twice the isolation of a single circulator. The isolation from the receiver to the transmitter for a triple-junction circulator is twice the isolation of a single circulator. The transmitter to receiver isolation, however, for all circulator configurations is the same as a single circulator. Another device with an isolation feature is the directional coupler.

A directional coupler is a four-port device used for power dividing as shown in Figure 5-5. The directional coupler has an isolation port (not shown) with minimal power transfer. Directional couplers are constructed with two adjacent transmission lines (i.e.,
coupled line coupler) or with $\lambda/4$ transmission line segments forming a branch-line (i.e., quadrature hybrid coupler) [191]. The insertion loss of a directional coupler, in decibels, is $IL = 20 \log_{10}|S_{21}|$. The isolation of a directional coupler, or power transferred to the isolation port, is $I = 20 \log_{10}|S_{41}|$. The forward and reverse coupling for a directional coupler relate power transferred to the coupled port as $C_{\text{fwd}} = 20 \log_{10}|S_{31}|$ and power leaked from the transfer port to the isolation port as $C_{\text{rev}} = 20 \log_{10}|S_{42}|$, respectively. The coupler’s ability to direct power is given as $D = 20 \log_{10}|S_{31}/S_{41}|$.

![Figure 5-5](image)

Figure 5-5. A directional coupler (a) block diagram and (b) example performance.

As discussed earlier, there is cross-talk between the transmitter and receiver when using a circular for full-duplex communications. Y.K. Chan et al. proposed a cancellation network for reducing the cross-talk [192] as shown in Figure 5-6. The cancellation network has a circulator with two quadrature hybrid couplers and a tunable antenna. The transmitter power is divided between the antenna and the tunable attenuator. The power at the attenuator is adjusted, then combined at the receiver with a 180° phase shift. The power leaked through the circulator is combined with the power through the attenuator and effectively cancel one another out. The cable length between devices may need adjusting to compensate for phase differences in the couplers.
Knox reported an alternative way to cancel the cross-talk using a balanced feed network [193] as shown in Figure 5-7. A balanced feed network has two quadrature hybrid couplers and two circulators. The transmitted power follows a path through a circulator to a split feed patch antenna where the antenna reflects the power to the receiver with a 180° phase shift. The transmitter power also follows a path through another circulator to a 90° port of the split feed patch antenna where the antenna reflects the power to the receiver with no phase shift. The power at the receiver due to cross-talk is effectively cancelled out due to the incoming power signals being nearly equal in amplitude and opposite in phase.
5.2.2 Phase-Locked Loops

An oscillator is important to the functionality of a radar system. The oscillator provides a reference signal for the electronics as well as is used in the frequency mixing stages. A basic feedback oscillator shown in Figure 5-8 is useful for understanding oscillators. The transfer function for a closed-loop oscillator with positive feedback is [71]

\[
\frac{v_o}{v_i} = \frac{A}{1 - A\beta}
\]  

(5-6)

The transfer function (i.e., system) is unstable as the closed loop gain approaches unity (i.e., \(A\beta \rightarrow 1\)). An unstable (i.e., oscillating) system is the result of the magnitude of the closed loop being unity (i.e., \(|A\beta| = 1\)) and the phase shift around the loop being zero (i.e., \(\angle A\beta = 2\pi n\)); also known as the Barkhausen criterion.

![Diagram of basic feedback oscillator.](image)

An oscillator may be constructed with an operational amplifier and passives. The upper operating frequency of operational amplifiers, however, is on the order of 1 MHz due to limited bandwidth and slew-rates. An oscillator constructed with transistors and reactive elements can operate at frequencies well past 100 MHz [71].
The phase noise of an oscillator is the frequency domain representation of time domain instabilities; also known as jitter. A sine wave with amplitude and phase noise shown in Figure 5-9 is represented as \( v(t) = (v_p + v_n(t)) \cos \left( 2\pi f_c \left( t + \frac{\theta_n(t)}{2\pi f_c} \right) \right) [194]. \)

The sine wave voltage can further be approximated assuming the phase noise is much less than 90° (i.e., \( \theta_n(t) \ll \pi/2 \)) as

\[
v(t) \approx (v_p + v_n(t)) \cos(2\pi f_c t) - (v_p + v_n(t)) \theta_n(t) \sin(2\pi f_c t).
\] (5-7)

The spectrum of the sine wave voltage in the frequency domain is

\[
S_v(f) = \frac{v_p + v_n(t)}{2} \left[ \delta(f - f_c) + \delta(f + f_c) \right] \\
+ \frac{v_p + v_n(t)}{2} [S_\theta(f - f_c) - S_\theta(f + f_c)].
\] (5-8)

The spectrum of phase noise is represented through Leeson’s equation as

\[
\mathcal{L}(f + f_c) = 10 \log_{10} \left| \frac{S_v(f)}{S_v(f_c)} \right| = 10 \log_{10} |S_\theta(f + f_c)|.
\] (5-9)

The phase noise spectrum for an oscillator is given by Leeson’s equation as [195]
\[
\mathcal{L}(f + f_c) = 10 \log_{10} \left\{ \frac{k_B T F}{2p_{av}} \left[ \frac{f_o}{2Q} \right]^2 \cdot \frac{f_c}{(f + f_c)^3} + \frac{f_o}{(f + f_c)^2} + \frac{1}{f + f_c} \right\} + 1 \right\}.
\]

(5-10)

Figure 5-10. Phase noise for an oscillator.

The phase noise as a function of the offset frequency from an oscillator is shown in Figure 5-10. The phase noise close to the oscillator is dominated by device flicker noise (i.e., \(1/f^3\) noise) that causes random frequency modulation. The white noise that causes frequency modulation (i.e., \(1/f^2\) noise) is the dominate phase noise further away from the oscillator. The flicker noise (i.e., \(1/f\) noise) is typically dominated by the flat-band white noise far away from the oscillator. The overall noise floor increases as an effect of phase noise. The effective signal-to-noise ratio of the oscillator is \(SNR = 20 \log_{10} |1/2\pi f_{rms}|\) [196]. An oscillator with a higher quality factor results in a lower phase noise. The phase noise can also be reduced by using a phase-locked loop [191].
A phase-locked loop is a feedback system for comparing the phase generator by a VCO to a reference input [197] as shown in Figure 5-11. The difference in the frequency between the reference and the feedback divider is fed into the VCO. The closed-loop transfer function describing a phase-locked loop is

\[
\frac{f_0}{f_{\text{ref}}} = \frac{K_{pd}Z(s)K_v}{s} \frac{1}{1 + K_{pd}Z(s)K_v/s \times \frac{1}{N}} = \frac{1}{1 + \left(\frac{K_{pd}Z(s)K_v}{s}\right) \frac{1}{N}}.
\]

(5-11)

If the open loop gain is much greater than unity (i.e., \(K_{pd}Z(s)K_v/s \gg 1\)), then the closed-loop transfer function is reduced to \(f_0/f_{\text{ref}} = N\). It is observed that in addition to tracking a reference input a phase-locked loop is also useful for generating a frequency that scales to the reference frequency.

### 5.2.3 Transformers

A center-tap transformer as shown in Figure 5-12 is commonly used in power supplies for driving half- and full-bridge rectifiers. A center-tap transformer is also useful for developing single- and double-balanced mixers. The voltages on the secondary coils are related to the number of windings on the primary and secondary coils as \(v_2 =\)
\((N_2/N_1)v_1\) and \(v_3 = (N_3/N_1)v_1\). The operating frequency of a transformer scales up accordingly as the radius of the coils scale down by \(S\) (i.e., \(f' = f/S\)).

![Center-tap transformer diagram](image)

**Figure 5-12.** A center-tap transformer.

Additive processing has been used to fabricate thin-film transformers on silicon. A variety of single layer transformers including parallel conductive winding [198], interwound winding [199], and concentric spiral winding [200] have been demonstrated. Also, multi-layer transformers have been demonstrated with dielectric films for an overlay winding [201].

### 5.2.4 Frequency Mixers

A frequency mixer is a nonlinear device used for producing the sum or difference of two frequencies. A frequency mixer is used in communication systems for mixing up to a higher frequency (i.e., up-converting) or mixing down to a lower frequency (i.e., down-converting). An open-close switch is fundamentally a mixer [202] as illustrated in Figure 5-13. The input alternates on and off at a switching rate of \(f_1 = 1/2T_1\). The switch is open and closed at a switching rate of \(f_2 = 1/2T_2\). The output of the switch results in an on value when both the switch is open and the input is on and the output is off otherwise. The time periods illustrated in Figure 5-13 show the nonlinear mixing
behavior of an ideal mixer producing the fundamentals (i.e., $f_1, f_2$), harmonics (i.e., $3f_1, 3f_2$, etc.) and intermodulation products (i.e., $f_1 + f_2, f_1 - f_2, 2f_1 + f_2, 2f_1 - f_2$, etc.).

![Diagram of switching behavior of a mixer](image)

**Figure 5-13.** Analogous switching behavior of a mixer.

A frequency mixer has a local oscillator (LO) in place of the switch. An up-converting mixer has an intermediate frequency (IF) in place of the input and an RF frequency in place of the output. A single diode is the simplest practical frequency mixer [203]. An ideal diode is a switch with infinite resistance (i.e., $R = \infty$) for zero voltage across the diode (i.e., $v_D = 0$) and zero resistance (i.e., $R = 0$) for positive voltage across the diode (i.e., $v_D > 0$). The current through the diode is also infinite for a positive voltage across the diode. The finite current in a real diode is related to the voltage across the diode $V_D$, ideality factor $n$, thermal voltage $V_T$, and saturation current $I_S$ as [71]

$$I = I_S(e^{V_D/nV_T} - 1). \quad \text{(5-12)}$$

The output voltage of the diode using a Taylor series expansion with a small signal approximation of the exponential term (i.e., $e^x - 1 = \sum_{n=1}^{\infty} x^n / n! \approx x + x^2 / 2$) is related to the voltage across the diode $v_D \approx v_1 \sin \omega_1 t + v_2 \sin \omega_2 t$ as
\[ v_o \propto (v_1 \sin \omega_1 t + v_2 \sin \omega_2 t) + \frac{1}{2}(v_1 \sin \omega_1 t + v_2 \sin \omega_2 t)^2 + \ldots \]  

The second term of the Taylor series expansion is expanded as

\[ v_1^2 \sin^2 \omega_1 t + 2v_1v_2 \sin \omega_1 t \sin \omega_2 t + v_2^2 \sin^2 \omega_2 t. \]  

The \( 2v_1v_2 \sin \omega_1 t \sin \omega_2 t \) term is further expanded using trigonometric identities as

\[ \frac{v_1^2}{2}(1 - \cos 2\omega_1 t) + v_1v_2[\cos(\omega_1 - \omega_2)t - \cos(\omega_1 + \omega_2)t] \]

\[ + \frac{v_2^2}{2}(1 - \cos 2\omega_2 t). \]  

The output voltage of the diode has fundamentals (i.e., \( \omega_1, \omega_2 \)), sum and difference terms (i.e., \( \omega_1 + \omega_2, \omega_1 - \omega_2 \)), harmonics (i.e., \( 2\omega_1, 2\omega_2, \ldots \)), and intermodulation products. The intermodulation products for a mixer are often specified in a table such that the power levels for possible products can be accounted for within a desired operating frequency band. The output of a mixer often has a band-pass filter to minimize unwanted harmonics and intermodulation products. The performance of a frequency mixer (e.g., isolation between ports) can be improved by using single-balanced or double-balanced mixers, which have been demonstrated with planar transformers [204] and coupled line couplers [205].

Figure 5-14. A single-balanced half ring mixer (a) schematic and (b) equivalent circuit.
A single-balanced half ring mixer has two diodes with a center-tap transformer as shown in Figure 5-14. An input RF voltage is applied across the primary coil of the transformer. The magnitude of the LO voltage is assumed to be much larger than the magnitude of the RF voltage (i.e., \( v_{LO} \gg v_{RF} \)) to turn the diode on. The output IF voltage is the sum of the two input voltages (i.e., \( v_{IF} = v_{LO} + v_{RF} \)) for a positive LO (i.e., \( v_{LO} > 0 \)) and the difference of the two input voltages (i.e., \( v_{IF} = v_{LO} - v_{RF} \)) for a negative LO (i.e., \( v_{LO} < 0 \)). The single-balanced mixer has RF to LO isolation; however, there is no isolation between the LO and IF.

![Diagram of single-balanced half ring mixer](image)

**Figure 5-15.** A double-balanced half ring mixer (a) schematic and (b) equivalent circuit.

A double-balanced half ring mixer has two diodes and two center tap transformers as shown in Figure 5-15. An input RF voltage is applied across the primary coil of one of the transformers. The configuration of the double-balanced mixer is similar to that of a single-balanced mixer with the addition of a second transformer coil to the output for isolation between the LO and IF. The output IF voltage is equivalent to the input RF voltage (i.e., \( v_{IF} = v_{RF} \)) for a positive LO (i.e., \( v_{LO} > 0 \)) and zero (i.e., \( v_{IF} = 0 \)) for a negative LO (i.e., \( v_{LO} < 0 \)). The double-balanced half ring mixer has isolation between
all ports. The diodes and transformers of a double-balanced half ring mixer can be fabricated on silicon using standard microfabrication processes.

5.2.5 Transistor Amplifiers

The transistor is the basic building blocks for active devices (i.e., oscillators, frequency mixers, amplifiers, tunable filters, etc.). A transistor can be used as either a switch or an amplifier. An npn bipolar junction transistor (i.e., npn BJT) is created by diffusing an n-type island in a p-type semiconductor then diffusing a p-type sub-island into the n-type island as shown in Figure 5-16. Similarly, a pnp bipolar junction transistor (i.e. pnp BJT) is created by diffusing a p-type island in a n-type semiconductor then diffusing an n-type sub-island into the p-type island. An npn BJT has two p-n junctions which form a base-emitter junction and a base-collector junction as shown in Figure 5-16. The current flowing through the emitter is related to the collector $I_C$ and base $I_B$ currents as $I_E = I_C + I_B$. The current gain relates the collector current of a BJT to the base current as $\beta = I_C/I_B$.

![Figure 5-16. An npn BJT transistor with (a) schematic symbol and (b) I-V curves.](image-url)
An npn BJT is in the active region when the collector-to-emitter voltage \( V_{CE} \) is greater than the base-to-emitter voltage \( V_{BE} \), which must also be positive (i.e., \( V_{CE} > V_{BE} > 0 \)). The collector current in the active region is related to the saturation current \( I_S \), the base-to-emitter voltage \( V_{BE} \), the thermal voltage \( V_T = k_B T/q \), the collector-to-emitter voltage \( V_{CE} \), and the Early voltage \( V_A \) as [65]

\[
I_C = I_S e^{V_{BE}/V_T} \left( 1 + \frac{V_{CE}}{V_A} \right) \approx I_S e^{V_{BE}/V_T}.
\] (5-16)

The Early voltage \( V_A \) is typically large (i.e., \( V_{CE} \ll V_A \)) such that it has little effect on the collector current. The small-signal transconductance of a BJT is related to the collector current \( I_C \) and the base-to-emitter voltage \( V_{BE} \) as

\[
g_m = \frac{\partial I_C}{\partial V_{BE}}.
\] (5-17)

An amplifier is created by integrating electrical passives (i.e., resistors and capacitors) with the BJT transistor. Three well-known amplifier configurations are the common emitter, common base, and common collector amplifiers [71].

![Diagram of an n-channel FET transistor](a)

An n-channel field effect transistor (i.e., n-channel FET) is created by diffusing p-type islands in a n-type semiconductor as shown in Figure 5-17. Similarly, a p-channel
field effect transistor (i.e., p-channel FET) is created by diffusing n-type islands in a p-type semiconductor. The current flowing through the gate is negligible (i.e., $I_G \approx 0$) due to the large gate-to-source capacitance; therefore, the current flowing through the drain is equivalent to the current flowing through the source (i.e., $I_D = I_S$).

An n-channel FET is in the saturation region when the drain-to-source voltage $V_{DS}$ is greater than the difference between the gate-to-source voltage and the threshold voltage (i.e., $V_{GS} - V_{TN}$); which also must be positive (i.e., $V_{DS} > V_{GS} - V_{TN} > 0$). The drain current in the saturation region is related to the transconductance parameter $k_n = \mu_n C_{ox} (W/L)$, the gate-to-source voltage $V_{GS}$, the threshold voltage $V_{TN}$, the channel-length modulation $\lambda$, and the drain-to-source voltage $V_{DS}$ as [71]

$$I_D = \frac{k_n}{2} (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS}) \approx \frac{k_n}{2} (V_{GS} - V_{TN})^2.$$  \hfill (5-18)

The channel-length modulation $\lambda$ is generally small (i.e., $V_{DS} \ll 1/\lambda$) such that it has little effect on the drain current. The small-signal transconductance of a FET is related to the drain current $I_D$ and the gate-to-source voltage $V_{GS}$ as

$$g_m = \frac{\partial I_D}{\partial V_{GS}}.$$  \hfill (5-19)

The integration of electrical passives with a FET transistor creates well-known amplifier configurations such as the common source, common gate, and common drain amplifiers [71].

### 5.3 A Commercial Radar-on-a-Chip

There is high one-way atmospheric attenuation near 60 GHz due to high oxygen absorption, which results in a one-way attenuation of nearly 15 dB/km [38]. A two-way
radar system would see a 30 dB/km loss, which makes operating near 60 GHz ideal for covert one-way communications. The 60 GHz operating band is also unlicensed by the Federal Communications Commission (FCC) [206], which further makes it attractive for industrial and commercial radar applications. IBM [207] demonstrated first generation transmitter and receiver chips operating from 59 to 64 GHz using silicon germanium bipolar complementary metal oxide semiconductor (SiGe BiCMOS) process technology [208]. IBM later demonstrated second generation chips with additional features to the transmitter and receiver [209]. The receiver, for example, had baseband variable-gain amplifiers (VGA) to improve the receiver gain to 72 dB.

Hittite Microwave [210] later licensed millimeter-wave integrated circuits from IBM [211] and released HMC6000 [212] transmitter and HMC6001 [213] receiver chips. Hittite also released a HMC6450/6451 60 GHz transceiver development kit [214] for prototype development using the transmitter and receiver chips. The performance for the different generations by IBM and Hittite Microwave is shown in Table 5-1.

<table>
<thead>
<tr>
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<th></th>
<th></th>
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<tbody>
<tr>
<td>Frequency Range</td>
<td>59 to 64 GHz</td>
<td>57 to 64 GHz</td>
<td>57 to 64 GHz</td>
</tr>
<tr>
<td>Tx Saturated Output Power</td>
<td>16 to 17 dBm</td>
<td>15 to 16 dBm</td>
<td>16 dBm</td>
</tr>
<tr>
<td>Tx 1 dB Output Power</td>
<td>10 to 12 dBm</td>
<td>10 to 12 dBm</td>
<td>11 dBm</td>
</tr>
<tr>
<td>Tx Conversion Gain</td>
<td>34 to 37 dB</td>
<td>30 to 33 dB</td>
<td>38 dB</td>
</tr>
<tr>
<td>Tx Power Consumption</td>
<td>513 mW</td>
<td>822 mW</td>
<td>800 mW</td>
</tr>
<tr>
<td>Rx 1 dB Input Power</td>
<td>-36 dBm</td>
<td>-36 dBm</td>
<td>-36 dBm</td>
</tr>
<tr>
<td>Rx LNA Gain</td>
<td>20 dB</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Rx Noise Figure</td>
<td>5 to 6.7 dB</td>
<td>5.5 to 6.5 dB</td>
<td>7 dB</td>
</tr>
<tr>
<td>Rx Conversion Gain</td>
<td>38 to 40 dB</td>
<td>72 dB</td>
<td>2 to 67 dB</td>
</tr>
<tr>
<td>Rx Power Consumption</td>
<td>526 mW</td>
<td>547 mW</td>
<td>610 mW</td>
</tr>
</tbody>
</table>

The commercially available HMC6450/6451 transceiver development kits have the ability to transmit and receive up to 1.8 GHz of bandwidth in the frequency range of 57
to 64 GHz. The transmitter and receiver chips also have antennas co-packaged with each chip. The input and output signal routing to the transmitter and receiver chips has differential in-phase (I) and quadrature-phase (Q) interfaces. The differential I and Q of the transmitter up-converts signals up to 2 GHz (i.e., baseband) to millimeter-wave frequencies near 60 GHz. The receiver down-converts millimeter-wave frequencies down to baseband. The transmitter and receiver chips allow interfacing with frontend and backend electronics capable of operating up to 2 GHz.

Figure 5-18. A 60 GHz transmitter (adapted from [208]).

A diagram describing the transmitter is shown in Figure 5-18. The transmitter chip has an integrated frequency synthesizer with a low phase noise LO for operating between 16.3 and 18.3 GHz. The input I and Q baseband signals are modulated with the LO such that the IF signal is between 8 and 9.1 GHz. The IF signal is then filtered and amplified by a VGA. The amplified signal is then up-converted to an operating frequency between 57 and 64 GHz. The RF signal is amplified with a pre-driver and PA then transmitted to the antenna.
A diagram describing the receiver is shown in Figure 5-19. The receiver chip also has an integrated frequency synthesizer with a low phase noise LO for operating between 16.3 and 18.3 GHz. The RF signal from the antenna is amplified by a low noise amplifier (LNA) then down-converted to an operating frequency between 8 and 9.1 GHz. The IF signal is then filtered and amplified by a VGA. The amplified IF signal is modulated with an LO to baseband I and Q signals. Finally, the baseband signals are then amplified by a VGA. The output power of the receiver chip as a function of the input power to the transmitter chip is shown in Figure 5-20 for varying TX attenuation. The gain with no applied internal attenuation is approximately 37 dB for an input power less than -45 dBm. The gain decreases for an input power greater than -45 dBm; which indicates the PA of the transmitter is saturating. The gain decreases proportionally to increases in the transmitter IF attenuation for an input power less than -45 dBm indicating linearity.
Figure 5-20. Output power of HMC6450 as a function of input power with varying TX attenuation.

The output power of the receiver chip as a function of the input power to the transmitter chip is shown in Figure 5-21 for varying RX attenuation. The gain decreases proportionally to increases in the receiver IF attenuation for an input power less than -45 dBm indicating linearity at the receiver. The transmitter and receiver chips each have co-packaged linearly polarized folded dipole antennas with 7 dBi of gain. The folded dipole antennas for the transmitter and receiver chips are oriented perpendicular to one another to minimize cross-talk between adjacent chips on the same board. The output power of the receiver to input power of the transmitter characteristics for a HMC6450 transceiver shown in Figure 5-21, however, indicates there is not sufficient isolation between the transmitter and receiver antennas for full-duplex communications.
Figure 5-21. Output power of HMC6450 as a function of input power with varying RX attenuation.

The cross-talk between the on-board transmitter and receiver chips is estimated by experimental data using a link budget as shown in Figure 5-22. The output power from the receiver is measured as -8 dBm for an input power to the transmitter of -45 dBm. The transmitter has a conversion gain of 38 dB for an input power of -45 dBm [208]. The antenna of the transmitter and receiver chips each have 7 dBi of gain. The receiver has a conversion gain of 65 dB [213]. The cross-talk between the on-board transmitter and receiver chips is determined as 80 dB. The use of a single-board transceiver as a radar system requires that the cross-talk be greater than the sum of the two-way free-space path loss and the RCS of an object. The RCS of a circular patch antenna operating at 60 GHz is approximately -68 dBsm. The separation distance for detecting the circular patch antenna is determined as less than 0.8 mm, which is not practical.
The HMC6450 is determined to not be practical for wireless sensing; however, the HMC6451 has the same capabilities as the HMC6450 without the co-packaged antennas. The HMC6450 can interface to an electromagnetic waveguide using coax-to-waveguide adapters. The electromagnetic waves propagate down the waveguide and interact with the sensor. The main advantages of operating at millimeter-wave frequencies is the scaling down of the antenna size and scaling down of the electromagnetic waveguide. For demonstration purposes, the design of an electromagnetic waveguide operating in the Ku-band (i.e., 12 to 18 GHz) is discussed.

5.4 Electromagnetic Waveguide

An electromagnetic waveguide is used for propagating electromagnetic waves; it is a hollow tube constructed of a conductive material (e.g., aluminum). A rectangular waveguide shown in Figure 5-23 is the most commonly used geometry. An electromagnetic wave can effectively propagate through a waveguide in one of two
possible modes: TE mode (i.e., electric field) or TM mode (i.e., magnetic field) [216].

The dimensions of the hollow cross-sectional area determine which mode the waveguide will allow to propagate. There is a minimum operating frequency (i.e., cut-off frequency) for each mode.

![Figure 5-23. A rectangular electromagnetic waveguide.](image)

The cut-off frequency of a rectangular waveguide is related to the permeability \( \mu = \mu_r \mu_o \) and permittivity \( \varepsilon = \varepsilon_r \varepsilon_o \) of the medium (e.g., air), the number \( m \) of half-wavelengths across the width \( a \), and the number \( n \) of half-wavelengths across the height \( b \) as [216]

\[
f_c^{mn} = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}.
\] (5-20)

The first mode is the TE\(_{10}\) mode with a cut-off frequency given as \( f_c^{10} = 1/2a\sqrt{\mu\varepsilon} \). The next mode after the TE\(_{10}\) mode is the TE\(_{20}\) mode with a cut-off frequency given as \( f_c^{20} = 1/a\sqrt{\mu\varepsilon} = 2f_c^{10} \). The bandwidth of electromagnetic waves propagating in the TE\(_{10}\) mode is \( \Delta f = f_c^{10} = 1/2a\sqrt{\mu\varepsilon} \).
The Electronic Industries Alliance developed a set of standards for dimensions of rectangular waveguides and their corresponding operating frequencies. A WR-62 waveguide, for example, has a width of 0.622 in (i.e., 15.8 mm) and a height of 0.311 in (i.e., 7.9 mm). The WR-62 waveguide operates in the Ku-band from 12.4 to 18 GHz.

The average power delivered to the backend of the electromagnetic waveguide for a TE\textsuperscript{10} mode is related to the attenuation of the electromagnetic waveguide $\alpha$ and the length of the electromagnetic waveguide $\ell$ as \[ p_{10} = \frac{1}{2} \int Re[\vec{E} \times \vec{H}^*] \cdot d\vec{S} = p_o e^{-2\alpha \ell}. \] (5-21)

The attenuation in the electromagnetic waveguide is related to the conductor $\alpha_c$ and dielectric $\alpha_d$ attenuation as $\alpha = \alpha_c + \alpha_d$. The conductor attenuation for a TE\textsuperscript{10} mode is a function of the width $a$ and height $b$ of the electromagnetic waveguide as well as the resistivity of the walls of the electromagnetic waveguide [216].

### 5.4.1 Electromagnetic Interference

The environment around an electromagnetic waveguide has both natural (e.g., solar radiation) and man-made (power lines, electromagnetic radiation) EMI. The EMI from power lines commonly couples into electronic circuits through electric fields or magnetic induction [217]. Interface electronics associated with electrostatic, piezoelectric, and piezoresistive dynamic pressure sensors are susceptible to EMI. The passive wireless sensor, however, does not have local electronics. The sensor is also contained in an electromagnetic waveguide with EMI isolation from the environment.

The waveguide is constructed of aluminum with a skin depth of 0.67 $\mu$m at 15 GHz. The
minimum thickness necessary to attenuate any environmental EMI is three skin depths or 2.01 \( \mu \)m. The thickness of the proof of concept waveguide is 11 mm.

### 5.4.2 Waveguide Design

An electromagnetic waveguide with two perpendicular channels is designed to interact with the dynamic pressure sensor as shown in Figure 5-24. The two channels have cross-sectional dimensions of 15.8 mm x 7.9 mm for operating at Ku-band frequencies. The frontend of the electromagnetic waveguide is designed to be compatible with WR-62 coax-to-waveguide adapters. The backend of the electromagnetic waveguide has an opening for interfacing a sensor placed between two conductive plates as discussed in Chapter 4. The electromagnetic waveguide is constructed if two identical, but mirrored sections as shown in Figure 5-24.

![Figure 5-24. A two channel electromagnetic waveguide with cross-sectional view.](image)

The dimensions for the electromagnetic sections are found in Appendix F. The outer dimensions of the electromagnetic waveguide are a width of 3 in. (i.e., 76.2 mm),
a height of 1.5 in. (i.e., 38.1 mm), and a length of 195.2 mm. The opening at the backend of the electromagnetic waveguide is 26.7 mm x 15.8 mm. The conductive plates are designed such that the sensor is centered about a 3 mm wide ground plane between the two channels. The electromagnetic waveguide fabricated with aluminum is shown in Figure 5-25. The two aluminum sections each have four holes for alignment dowels and ten threaded holes for number 10 (i.e., 0.187 in. diameter) screws. There are also two threaded holes at the backend of the electromagnetic waveguide for number 6 (i.e., 0.140 in. diameter) screws to secure the plates.

![Figure 5-25. Fabricated aluminum electromagnetic waveguide for passive wireless dynamic pressure sensor (photo courtesy of John Rogers).](image)

### 5.5 Summary

The design of a system for producing electromagnetic waves to interact with the passive wireless pressure sensor was discussed in Chapter 5. The fundamentals of
radar were first discussed to understand a background on radar. The antenna architectures for simultaneously transmitting and receiving were then discussed. The electronics of a radar-on-a-chip were then discussed including oscillators, frequency mixers, and amplifiers. A commercially available radar-on-a-chip from Hittite was then discussed for wireless interaction with a pressure sensor at millimeter-wave frequencies. The HMC6450 was then determined to not be practical for operating with a pressure sensor due to its cross-talk. An electromagnetic waveguide with the ability to propagate Ku-band frequencies to the pressure sensor at the backend of the waveguide was then discussed. The waveguide allows the interaction of electromagnetic waves with the pressure sensor, while simultaneously being located in a harsh environment.
Chapter 6 discusses experimental characterization of the dynamic pressure sensor fabricated in Chapter 4. Characterization of an antenna on glass is first discussed to demonstrate the interaction of an antenna in the electromagnetic waveguide. The fundamentals of plane wave tubes are then discussed as a precursor to acoustic characterization. The acoustic characterization of silicon-based AC and DC demonstration pressure sensors is then discussed followed by a sapphire-based AC high temperature pressure sensor.

### 6.1 Characterization of an Antenna on Glass

To validate the interaction of an antenna-based sensor in an electromagnetic waveguide an antenna on glass is constructed. The antenna on glass shown in Figure 6-1 is fabricated using standard microfabrication processes at the University of Florida’s Nanoscale Research Facility. The antenna on glass has 500 nm of aluminum patterned on the front side of a 270 μm Borofloat® 33 glass die to form a circular patch antenna with a slot. An additional 500 nm of aluminum is patterned on the back side to form a ground plane for the circular patch antenna. The details of the fabrication of the antenna on glass are in Appendix B. Antennas with diameters of 5.5, 5.6, 5.7, and 5.8 mm are fabricated to demonstrate operation of circular patch antennas at electrical frequencies in the vicinity of 15 GHz. The slot for the antennas has a length of 800 μm and a width of 100 μm.
The antenna on glass is affixed to the backend of the electromagnetic waveguide with the two conductive plates discussed in Chapter 4. The frontend of the waveguide is attached to two PE9803 coax-to-waveguide adapters [218] from Pasternack. The coax-to-waveguide adapters are attached to coax cables that connect to an Agilent E5071C network analyzer. The network analyzer is swept from 12 to 18 GHz in steps of 4 MHz (i.e., 1501 points). The measurement error in the RF cables is neglected by aligning the coax-to-waveguide adapters together then performing a divide math function on the E5071C to subtract out the error due to the cables and coax-to-waveguide adapters. Ideally, a two-port waveguide calibration is performed on the coax-to-waveguide adapters; however, the calibration standards for waveguides are cost prohibitive and not necessary for accurately determining the insertion loss. After calibration the coax-to-waveguide adapters are then connected to the frontend of the electromagnetic waveguide.

Figure 6-1. Fabricated 5.6 mm antenna on glass (photo courtesy of John Rogers).
Figure 6-2. Measured and FEM insertion loss for 5.6 mm antenna on glass with measured resonant peak at 14.288 GHz.

The insertion loss as a function of frequency for a 5.6 mm antenna on glass is shown in Figure 6-2. The measured results are shown in the solid line, while the predicted results using an FEM model are shown in the dotted line. There are error bars in the measured results indicating a measured magnitude uncertainty of 0.4 dB, which matches the uncertainty specified for an Agilent E5071C network analyzer up to 20 GHz [133]. There is a strong peak around 13.3 GHz, which was predicted to be more than three orders of magnitude lower by the FEM model. The peak is likely attributed to the geometries at the backend of the electromagnetic waveguide.

The antenna on glass has a measured resonant frequency of 14.288 GHz, which matches the FEM model within 0.85%. The measured electrical quality factor is 83, which is slightly lower than the predicted electrical quality factor of 98. The variation in the quality factor is likely attributed to the assumption of an infinite electrical resistivity in
the FEM model. The measured results for 5.5, 5.6, 5.7, and 5.8 mm antennas is shown in Table 6-1. The measured uncertainty for the antennas is determined as 660 kHz, which is negligible in comparison to the resonant frequencies.

<table>
<thead>
<tr>
<th>Ant. Diameter</th>
<th>Resonant Freq. (MHz)</th>
<th>Error</th>
<th>Electrical Q Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5 mm</td>
<td>14504 +/- 0.660</td>
<td>0.89%</td>
<td>Meas. 88 FEM 100</td>
</tr>
<tr>
<td>5.6 mm</td>
<td>14228 +/- 0.660</td>
<td>0.85%</td>
<td>Meas. 83 FEM 98</td>
</tr>
<tr>
<td>5.7 mm</td>
<td>14080 +/- 0.660</td>
<td>1.21%</td>
<td>Meas. 76 FEM 128</td>
</tr>
<tr>
<td>5.8 mm</td>
<td>13888 +/- 0.660</td>
<td>1.94%</td>
<td>Meas. 89 FEM 79</td>
</tr>
</tbody>
</table>

To better predict the performance of an antenna the material properties of the glass can be extracted using test structures (i.e., half-wave resonators) fabricated on glass. The relative permittivity of Borofloat® 33 is given by Schott as 4.6 [179]; however, accounting for measurement variations in the material can result a more accurate prediction. The effective permittivity of a material is related to the length of the half-wave shorted resonator $\ell$ and the resonant frequency $f$ as $\varepsilon_{\text{eff}} = \left(\frac{c}{2f \ell}\right)^2$. The relative permittivity for a grounded coplanar waveguide is related to the effective permittivity as $\varepsilon_r \approx 2\varepsilon_r - 1$. The fabricated half-wave resonators on glass are shown in Figure 6-3.

![Figure 6-3. Fabricated half-wave resonator test structures on glass (photo courtesy of John Rogers).](image)
The resonators have designed lengths of 8.96, 7.47, 6.38, 5.58, and 4.96 mm; which correspond to resonant frequencies at 10, 12, 14, 16, and 18 GHz. The test structures are measured using 150 μm pitch coplanar ground-signal-ground (GSG) probes [219] from GGB Industries. The Agilent E5071C network analyzer is swept from 8 to 20 GHz in steps of 10 MHz (i.e., 1201 points). The error due to the cables and probes is neglected by performing a SOLT calibration using a CS-5 alumina calibration substrate [220] from GGB Industries. The effective permittivity of Borofloat® 33 measured by the test structures is shown in Table 6-2. The length of the resonators is determined by optical observation under a microscope; therefore, there is some inherent uncertainty of the measurement determined to be 0.01 mm. The measured effective permittivity is within 10% of the referenced value of 4.6.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8.96 mm</td>
<td>8.95 mm +/- 0.01</td>
<td>10 GHz</td>
<td>9.74 GHz</td>
<td>4.92 +/- 0.01</td>
</tr>
<tr>
<td>7.47 mm</td>
<td>7.46 mm +/- 0.01</td>
<td>12 GHz</td>
<td>11.55 GHz</td>
<td>5.06 +/- 0.01</td>
</tr>
<tr>
<td>6.38 mm</td>
<td>6.37 mm +/- 0.01</td>
<td>14 GHz</td>
<td>13.71 GHz</td>
<td>4.88 +/- 0.02</td>
</tr>
<tr>
<td>5.58 mm</td>
<td>5.57 mm +/- 0.01</td>
<td>16 GHz</td>
<td>15.67 GHz</td>
<td>4.88 +/- 0.02</td>
</tr>
<tr>
<td>4.96 mm</td>
<td>4.96 mm +/- 0.01</td>
<td>18 GHz</td>
<td>17.64 GHz</td>
<td>4.87 +/- 0.02</td>
</tr>
</tbody>
</table>

### 6.2 Plane Wave Tube Fundamentals

A plane wave tube is used to acoustically excite a sensor with a shear stress and/or pressure field. A plane wave tube is a long rectangular tube typically constructed of a conductive material with a hollow rectangular inner cross-section where acoustic waves propagate. A typical plane wave tube has an acoustic source on the frontend and an acoustic termination such as an open- or close-ended backend [10] as shown in Figure 6-4.
The pressure traversing along the plane wave tube is related to the incident pressure $P^+$, reflected pressure $P^-$, wave number $k = \omega/c_o$, and position along the tube $z$ as [10]

$$p(z, t) = P^+ e^{-j kz} + P^- e^{+j kz}. \quad (6-1)$$

Similarly, the particle velocity along the plane wave tube is related to the incident and reflected particle velocities $U^+ = P^+/Z_o$ and $U^- = -P^-/Z_o$, respectively, as [10]

$$u(z, t) = U^+ e^{-j kz} + U^- e^{+j kz}. \quad (6-2)$$

The characteristic acoustic impedance of the fluid in the plane wave tube (e.g., air) is the ratio of the incident pressure to the incident particle velocity or [10]

$$Z_o = \frac{P^+}{U^+} = -\frac{P^-}{U^-}. \quad (6-3)$$

The complex reflection coefficient $R$ at the backend of the tube (i.e., $z = 0$) is the ratio of the pressure reflected back from the termination impedance $Z_n$ to the pressure incident on the termination or [10]

$$R = \frac{P^-}{P^+} = \frac{Z_n - Z_o}{Z_n + Z_o}. \quad (6-4)$$
The reflection coefficient for a tube perfectly matched to the termination is zero; however, in general the reflection coefficient at the termination is bounded such that \(-1 \leq R \leq 1\). The reflection coefficient at any point along the tube beginning from the termination is \(R(z) = Re^{j2kz}\). The pressure standing wave ratio (SWR) is the ratio of the maximum pressure to the minimum pressure along the tube (i.e., \(SWR = |P_{max}|/|P_{min}| = |P^+ + P^-|/|P^+ - P^-| = (1 + |R|)/(1 - |R|)\)). The impedance at any point along the tube is [10]

\[
Z(z) = \frac{P(z)}{U(z)} = Z_o \frac{e^{-j kz} + Re^{j kz}}{e^{-j kz} - Re^{j kz}}
\]

(6-5)

Figure 6-5. Standing wave magnitude and impedance characteristics for closed backend tube.
The reflection coefficient for a plane wave tube with a closed backend (i.e., \( Z_n = \infty \)) is \( R = 1 \). The pressure and particle velocity for a closed backend plane wave tube are \( p(z, t) = P^+ (e^{-jkz} + e^{+jkz}) \) and \( u(z, t) = U^+ (e^{-jkz} + e^{+jkz}) \), respectively [10]. The pressure and particle velocity at the backend (i.e., \( z = 0 \)) are \( p = 2P^+ \) and \( u = 0 \), respectively, as shown in Figure 6-5. The impedance along the closed tube is [10]

\[
Z(z) = -jZ_o \cot(kz).
\] (6-6)

A plane wave tube developed at the University of Florida as shown in Figure 6-6 is used for acoustic characterization of the passive wireless pressure sensors. The plane wave tube is constructed of aluminum and is 38 in. in length. The tube has an inner cross-section of 1 in. x 1 in and a wall thickness of 1 3/4 in. The plane wave tube allows acoustic plane waves to propagate through the tube with up to the cutoff frequency. The cutoff frequency of a rectangular plane wave tube where propagating acoustic waves become evanescent (i.e., decay) is related to the isentropic speed of sound \( c_o \), the number \( m \) of half-wavelengths across the width \( a \), and the number \( n \) of half-wavelengths across the height \( b \) as [10]

\[
f^m_n = \frac{1}{2c_o} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}.
\] (6-7)

The first cutoff frequency occurs at \( f^1_0 = c_o/2a \). The cutoff frequency for the acoustic plane wave tube at the University of Florida with a 1 in. x 1 in. rectangular cross-section is 6.69 kHz.
6.3 Acoustic Characterization

A dynamic pressure sensor is a transducer capable of converting an applied pressure to a diaphragm into an electrical output. An electrical resonant-based dynamic pressure sensor transduces an applied pressure to a diaphragm into a change in the electrical resonant frequency of the sensor. To further illustrate the transduction of an electrical resonant-based dynamic pressure sensor consider the pressure at the end of a plane wave tube as shown in Figure 6-7. The electrical output frequency of the sensor is nominal when the pressure at the termination of the plane wave tube is zero (i.e., \( f(p = 0) = f_0 \)). The pressure at the termination increases/decreases as a function of time at a rate proportional to the sine of the acoustic frequency (i.e., \( p(t) = p_{max} \sin(2\pi f_a t) \)).
An accurate measurement of the pressure in real-time requires sampling at a rate such that \( t_s \leq 1/100 \, f_a \). The sampling time required for an acoustic frequency of 1 kHz, for example, is 10 \( \mu \)s. The Agilent E5071C network analyzer has a cycle time of 2 ms for a 51 point sweep for a 200 MHz bandwidth [133]. A lower cycle time can be achieved with a lower number of points and a higher IF bandwidth. The accuracy of the data, however, becomes compromised at some level. The use of a network analyzer for real-time pressure measurement is not practical. For calibration purposes, however, an alternative method to measuring data in real-time is to observe the maximum change in the magnitude (or phase) over frequencies in the vicinity of the nominal resonance of the sensor.

![Diagram](image)

Figure 6-7. Pressure magnitude along plane wave tube and corresponding output frequency of sensor for (a) no pressure, (b) half maximum pressure, and (c) maximum pressure at end wall of plane wave tube.
The Agilent E5071C network analyzer has a max hold function for data acquisition. The max hold function works by successively sweeping the desired frequency range of the analyzer; the data is stored in the memory, then compared to the previous sweep. If the current data point at a particular frequency has a value greater than the previously stored data point at the same frequency, then the current data point is stored in memory. The maximum frequency deviation from the nominal resonant frequency occurs when the pressure at the termination is maximum (i.e., $p = p_{\text{max}}$). The sensitivity of the sensor at an acoustic frequency $f_a$ can be determined by observing the maximum frequency deviation as a function of the applied pressure.

### 6.3.1 Reference Microphone Calibration

A reference microphone is commonly used for calibrating a pressure sensor so that an accurate determination of the pressure applied to the DUT sensor is known. Consider a first reference microphone at the end wall of a plane wave tube and a second reference microphone that is perpendicular to the first reference microphone as shown in Figure 6-8. An Agilent 33120A function generator [221] provides a sine wave at some desired acoustic frequency, which is then fed into a Crown Audio XLS1500 power amplifier [222]. The XLS1500 has a 300 W power rating for an 8 $\Omega$ speaker, which yields a peak current rating of 6.1 A (or an RMS current rating of 4.3 A). The output of the power amplifier is connected to a BMS 4950 compression driver speaker [223], which transduces the electrical sine wave into an acoustic pressure wave. The acoustic pressure wave then propagates through the plane wave tube and interacts with the reference microphones located at the end of the tube.
The reference microphones are 1/8\" Type 4138 microphones [224] with 1/4\" Type 2670 preamplifiers [225] by Bruel and Kjaer. The reference microphones are powered by a Type 2804 microphone power supply [226] by Bruel and Kjaer. The outputs from the reference microphones are also amplified by the microphone power supply and fed into a NI PXIe-4499 signal analyzer [227] by National Instruments. A Type 4231 sound calibrator [228] by Bruel and Kjaer is used to determine the sensitivity of the reference microphones. The sound calibrator emits a known 94 dB\text{SPL} sound source at 1 kHz such that the sensitivity of the reference microphones can be analytically determined. The amplitude spectral density $A(f)$, in V/Hz$^{1/2}$, is related to the observed power spectral density $S_x(f)$, in V$^2$/Hz, from the signal analyzer as $A(f)/\sqrt{2} = \sqrt{S_x(f)}$. The pressure
applied to the reference microphone is related to the amplitude spectral density $A(f)$, the sensitivity of the microphone $S$, and the absolute threshold of hearing $p_o$ as [10]

$$\text{dB}_{\text{SPL}} = 20 \log_{10}\left| \frac{A(f) \cdot S}{\sqrt{2} \cdot p_o} \right|. \quad (6-8)$$

The sensitivity of reference microphone 1 is determined as 0.507 mV/Pa and the sensitivity of reference microphone 2 is determined as 0.557 mV/Pa. To accurately measure the DUT sensor a reference microphone is ideally co-located at the end wall with the DUT sensor. The reference microphone and DUT sensor, however, will experience slightly different pressures given their different locations. One method to neglect any variation in the pressure at the microphone and DUT sensor is to orient the sensors in such a way that their locations can be swapped. The output of the reference microphone and the output of the DUT are related as $H_{12}$ [17]. The transfer function of the DUT sensor is related to the transfer function of the reference microphone $H_1$ and the geometric mean of the transfer function $H_{12}$ at an original orientation $H_{12}^0$ and a swapped orientation $H_{12}^s$ as $H_2 = H_1 \sqrt{H_{12}^0 \cdot H_{12}^s}$ [17]. The electromagnetic waveguide, however, encloses the entire end wall.

Alternatively, a second reference microphone is placed perpendicular to the end wall and calibrated with the first reference microphone such that the pressure applied at the end wall can be determined numerically. The second reference microphone tracks the first reference microphone up to 152 dB$_{\text{SPL}}$ at 1 kHz. The second reference microphone has a 0.6 dB deviation at 154 dB$_{\text{SPL}}$ as shown in Table 6-3. The pressure at the end wall without the first reference microphone can now be determined by observing the pressure of the second reference microphone.
Table 6-3. Measured reference microphone pressures at 1 kHz.

<table>
<thead>
<tr>
<th>Freq</th>
<th>Voltage</th>
<th>Amp Click</th>
<th>End Wall Mic dB_{SPL}</th>
<th>Top Mic dB_{SPL}</th>
<th>Current mA_{rms}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kHz</td>
<td>70 mV_\text{p}</td>
<td>2</td>
<td>99.7</td>
<td>99.6</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>150 mV_\text{p}</td>
<td>2</td>
<td>105.4</td>
<td>105.4</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>280 mV_\text{p}</td>
<td>2</td>
<td>110.1</td>
<td>110.1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>480 mV_\text{p}</td>
<td>2</td>
<td>115.1</td>
<td>115.0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>850 mV_\text{p}</td>
<td>2</td>
<td>120.0</td>
<td>120.0</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>560 mV_\text{p}</td>
<td>3</td>
<td>125.0</td>
<td>124.9</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>1.02 V_\text{p}</td>
<td>3</td>
<td>130.0</td>
<td>130.0</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>1.08 V_\text{p}</td>
<td>4</td>
<td>135.0</td>
<td>135.0</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>1.33 V_\text{p}</td>
<td>5</td>
<td>140.0</td>
<td>140.0</td>
<td>124</td>
</tr>
<tr>
<td></td>
<td>1.38 V_\text{p}</td>
<td>7</td>
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<td>222</td>
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<td></td>
<td>1.61 V_\text{p}</td>
<td>9</td>
<td>150.0</td>
<td>150.0</td>
<td>406</td>
</tr>
<tr>
<td></td>
<td>1.60 V_\text{p}</td>
<td>10</td>
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<td>460</td>
</tr>
<tr>
<td></td>
<td>1.62 V_\text{p}</td>
<td>11</td>
<td>152.0</td>
<td>151.9</td>
<td>516</td>
</tr>
<tr>
<td></td>
<td>1.88 V_\text{p}</td>
<td>11</td>
<td>153.0</td>
<td>152.7</td>
<td>606</td>
</tr>
<tr>
<td></td>
<td>2.08 V_\text{p}</td>
<td>12</td>
<td>154.0</td>
<td>153.4</td>
<td>803</td>
</tr>
</tbody>
</table>

6.3.2 Sensitivity Calibration of AC Demonstration Sensor

The next step after calibrating the microphones in the plane wave tube is to replace the end wall microphone (i.e., reference microphone 1) with the electromagnetic waveguide and the dynamic pressure sensor as shown in Figure 6-9. The electromagnetic waveguide is connected to an Agilent E5071C network analyzer [133]. Prior to connecting the electromagnetic waveguide to the network analyzer a SOLT calibration is performed to neglect the error in the cables using an Agilent 85052D calibration kit [229]. The cables are then connected to the waveguide adapters on the electromagnetic waveguide. The phase response is observed in the vicinity of the resonant frequency of the sensor. The max hold function is utilized such that as the applied pressure increases the maximum change in the phase crossover is observed.
Three dynamic pressure sensors with antenna diameters of 5.6, 5.7, and 5.8 mm with silicon diaphragms of 7.6 mm are characterized in the plane wave tube. The nominal resonant frequency of the dynamic pressure sensor with an antenna diameter of 5.6 mm is 15.344 GHz. The FEM model of the dynamic pressure sensor predicted a nominal resonant frequency of 15.379 GHz. The change in the resonant frequency as a function of the applied pressure for the dynamic pressure sensor with an antenna diameter of 5.6 mm is shown in Figure 6-10. The dynamic pressure sensor has a measured sensitivity is 2.25 kHz/Pa with a linear pressure range of 900 Pa. The dynamic pressure sensor was designed to operate up to 3 kPa; however, it is observed that it is able to measure pressure linearly up to 900 Pa. The setup was broken down and the sensitivity calibration repeated to yield a measured sensitivity of 1.87 kHz/Pa. The sensitivity bounds in Figure 6-10 show the upper and lower sensitivity bounds determined as +/- 0.09 kHz/Pa for a measured sensitivity of 2.25 kHz/Pa.
Figure 6-10. Change in the resonant frequency as a function of applied pressure for an AC demonstration pressure sensor with 5.6 mm antenna and repeatability data.

The change in the resonant frequency as a function of the applied pressure for the dynamic pressure sensor with an antenna diameter of 5.7 mm is shown in Figure 6-11. The nominal resonant frequency of the dynamic pressure sensor with an antenna diameter of 5.7 mm is 15.104 GHz. The FEM model of the dynamic pressure sensor predicted a nominal resonant frequency of 15.116 GHz. The dynamic pressure sensor has a measured sensitivity is 2.20 kHz/Pa with a linear pressure range of 900 Pa. The setup was broken down and the sensitivity calibration repeated to yield a measured sensitivity of 2.22 kHz/Pa. The sensitivity bounds in Figure 6-11 show the upper and lower sensitivity bounds determined as +/- 0.08 kHz/Pa for a measured sensitivity of 2.22 kHz/Pa.
Figure 6-11. Change in the resonant frequency as a function of applied pressure for an AC demonstration pressure sensor with 5.7 mm antenna and repeatability data.

The change in the resonant frequency as a function of the applied pressure for the dynamic pressure sensor with an antenna diameter of 5.8 mm is shown in Figure 6-12. The nominal resonant frequency of the dynamic pressure sensor with an antenna diameter of 5.8 mm is 15.056 GHz. The FEM model of the dynamic pressure sensor predicted a nominal resonant frequency of 14.862 GHz. The dynamic pressure sensor has a measured sensitivity is 4.81 kHz/Pa with a linear pressure range of 900 Pa. The setup was broken down and the sensitivity calibration repeated to yield a measured sensitivity of 5.10 kHz/Pa. The sensitivity bounds in Figure 6-12 show the upper and lower sensitivity bounds determined as +/- 0.16 kHz/Pa for a measured sensitivity of 5.10 kHz/Pa.
The measured resonant frequencies for demonstration pressure sensors with antenna diameters of 5.6, 5.7, and 5.8 mm matched very well with the analytical model as shown in Table 6-4. The measured sensitivities for the static and dynamic pressure sensors were 2 to 5 times smaller than the analytical model. The measured uncertainty is determined by the uncertainty of the reference microphone. The Type 4138 [224] reference microphone with Type 2670 [225] preamplifier has an uncertainty of 0.2 dB determined by the uncertainty of the Type 4231 [228] sound calibrator. A Monte Carlo analysis of the analytical model resulted in an analytical uncertainty of 0.7 kHz/Pa. The sensitivity difference is likely attributed to loose fitting of the sensor in its package (i.e., conductive plates). The repeatability data in Figure 6-10, 6-11, and 6-12 shows the sensitivity varies between setup breakdowns. The loose fitting of the sensor in the package leads to pressure leaks and decreased sensitivity.
Table 6-4. Performance characteristics for AC demonstration pressure sensors.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5.6 mm</td>
<td>15.344</td>
<td>0.23%</td>
<td>2.25 +/- 0.09</td>
</tr>
<tr>
<td>5.7 mm</td>
<td>15.104</td>
<td>0.08%</td>
<td>2.22 +/- 0.08</td>
</tr>
<tr>
<td>5.8 mm</td>
<td>15.056</td>
<td>1.31%</td>
<td>5.10 +/- 0.16</td>
</tr>
</tbody>
</table>

6.3.3 Frequency Drift of AC Demonstration Sensor

The minimum detectable pressure of the dynamic pressure sensor is determined by the drifting behavior of the sensor. Furthermore, the dynamic range of the pressure sensor is determined by the minimum detectable pressure. The frequency drift is determined by observing the resonant frequency of the dynamic pressure sensor over time with no applied pressure. As described earlier, the dynamic pressure sensor with a 7.6 mm diameter silicon diaphragm and an antenna diameter of 5.7 mm has a nominal resonant frequency around 15.104 GHz. The change in the resonant frequency is determined by observing the phase crossover over a time of 30 min. The change in the resonant frequency as a function of time for a dynamic pressure sensor with an antenna diameter of 5.7 mm is shown in Figure 6-13.

The average frequency shift over 30 min is 19.7 kHz, which yields a minimum detectable frequency of 27.8 kHz (i.e., \( f_{\text{min}} = \sqrt{2} f_{\text{avg}} \)). The minimum detectable frequency is used to determine the minimum detectable pressure as \( p_{\text{min}} = f_{\text{min}} / S \). The minimum detectable pressure for a dynamic pressure sensor with an antenna diameter of 5.8 mm and sensitivity of 5.10 kHz/Pa is 5.4 Pa. The maximum frequency shift over time is also observed to determine the measurement uncertainty in the frequency drift data. A maximum frequency change of 70 kHz was observed over 30 min for the dynamic pressure sensor.
6.3.4 Sensitivity Calibration of DC Demonstration Sensor

The sensitivity of a dynamic (i.e., AC) pressure sensor is considered constant for frequencies between the cut-on to cut-off frequencies (i.e., pass band). The sensitivity of a static (i.e., DC) pressure sensor, on the other hand, is constant for frequencies between DC and the cut-off frequency. Assuming an AC and DC sensor have equivalent geometries (with the exception of the dynamic pressure sensor’s vent) the two sensors exhibit the same sensitivity. It is easier to measure the response of a DC pressure from an instrumentation point of view as a time invariant pressure yields a time invariant output. Consider a pressure calibrator in replacement of the plane wave tube as shown in Figure 6-14.
Figure 6-14. Test setup for sensitivity calibration of DC sensor using a pressure calibrator.

The Fluke 718-30G pressure calibrator [230] has a range of -12 to +30 psi (i.e., -82.7 to 206.8 kPa) with a 0.001 psi (i.e., 6.8 Pa) resolution. The Fluke 718-30G has an internal hand-pump for applying pressure, which can then be tuned with a fine adjustment knob. The Fluke 718-30G also has an internal reference pressure sensor for precision pressure measurements. It is particularly challenging to apply and maintain a DC pressure to the sensor due to unintentional leakage paths in the environment. The potential leakage paths are minimized by following basic rules. Clamps are added to the ends of the 1/4" PVC tube where the tube interfaces with 1/4" barb to 1/8" NPT connectors. The threaded 1/8" NPT connectors are also wrapped with PTFE thread seal tape [231] to prevent any leaks around the threads. The sensor is placed in a receded opening of a conductive plate. To seal the space between the sensor and the plate Silly Putty [232] is applied around the periphery of the sensor. Silly Putty is also applied along the perimeter of the plates where the plates meet.
Despite all attempts to prevent leaks, they are still present by observation of measurement of a time constant decay in the applied DC pressure. For calibration purposes the decay is small relative to the sweep time of the network analyzer. Three static pressure sensors with antenna diameters of 5.6, 5.7, and 5.8 mm with silicon diaphragms of 7.6 mm are characterized with the pressure calibrator. The nominal resonant frequency of the static pressure sensor with an antenna diameter of 5.6 mm is 15.120 GHz. The change in the resonant frequency as a function of the applied pressure for the static pressure sensor with an antenna diameter of 5.6 mm is shown in Figure 6-15. The static pressure sensor has a measured sensitivity of 3.07 kHz/Pa. The setup was broken down and the sensitivity calibration repeated to yield a measured sensitivity of 3.42 kHz/Pa. The measured sensitivity is slightly higher than that of the dynamic pressure sensor, which is good validation of the measured results for the dynamic pressure sensor.

![Figure 6-15. Change in the resonant frequency as a function of applied pressure for a DC demonstration pressure sensor with 5.6 mm antenna and repeatability data.](image-url)
The change in the resonant frequency as a function of the applied pressure for the static pressure sensor with an antenna diameter of 5.7 mm is shown in Figure 6-16. The nominal resonant frequency of the dynamic pressure sensor with an antenna diameter of 5.7 mm is 14.784 GHz. The dynamic pressure sensor has a measured sensitivity is 3.13 kHz/Pa. The setup was broken down and the sensitivity calibration repeated to yield a measured sensitivity of 2.31 kHz/Pa.

![Figure 6-16. Change in the resonant frequency as a function of applied pressure for a DC demonstration pressure sensor with 5.7 mm antenna and repeatability data.](image)

The change in the resonant frequency as a function of the applied pressure for the static pressure sensor with an antenna diameter of 5.8 mm is shown in Figure 6-17. The nominal resonant frequency of the dynamic pressure sensor with an antenna diameter of 5.7 mm is 14.496 GHz. The dynamic pressure sensor has a measured sensitivity is 2.07 kHz/Pa. The setup was broken down and the sensitivity calibration repeated to yield a measured sensitivity of 2.52 kHz/Pa.
The three static pressure sensors are summarized in Table 6-5. The measured resonant frequencies for demonstration pressure sensors with antenna diameters of 5.6, 5.7, and 5.8 mm matched well with the analytical model. The measured sensitivities for the three pressure sensors were 3 to 5 times smaller than the analytical model. As discussed in the AC sensitivity calibration the sensitivity difference is likely attributed to the placement of the sensor between the conductive plates. The repeatability data shows the sensitivity varies between setup breakdowns. The placement of the sensor in the package with Silly Putty does not make a perfect seal. Pressure leaks still exist in the package which leads to decreased sensitivity.

Figure 6-17. Change in the resonant frequency as a function of applied pressure for a DC demonstration pressure sensor with 5.8 mm antenna and repeatability data.
Table 6-5. Performance characteristics for DC demonstration pressure sensors.

<table>
<thead>
<tr>
<th>Ant. Diameter</th>
<th>Resonant Freq. (GHz)</th>
<th>Error</th>
<th>Sensitivity (kHz/Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Meas.</td>
<td>Analytical</td>
<td>Meas.</td>
</tr>
<tr>
<td>5.6 mm</td>
<td>15.120</td>
<td>15.379</td>
<td>1.68%</td>
</tr>
<tr>
<td>5.7 mm</td>
<td>14.784</td>
<td>15.116</td>
<td>2.20%</td>
</tr>
<tr>
<td>5.8 mm</td>
<td>14.496</td>
<td>14.862</td>
<td>2.46%</td>
</tr>
</tbody>
</table>

Next, a hysteresis study of the demonstration pressure sensor is done to observe the output behavior of the pressure sensor as the pressure is increased up to the maximum pressure then decreased back to zero. The resonant frequency change as a function of applied pressure for a dynamic pressure sensor with an antenna diameter of 5.7 mm is shown in Figure 6-18. It is observed that there is an 80 kHz difference in the resonant frequency as the pressure is increased then decreased. The difference is likely attributed to the loose fitting of the sensor in the package (i.e., conductive plates).

![Figure 6-18. Hysteresis characterization of AC demonstration pressure sensor with 5.7 mm antenna.](image-url)
6.3.5 Sensitivity Calibration of AC High Temperature Sensor

A high temperature dynamic pressure sensor is now calibrated using the same setup discussed in 6.3.2. The dynamic pressure sensor with an 8 mm diameter sapphire diaphragm is characterized with a nominal resonant frequency of 14.784 GHz. The change in the resonant frequency as a function of the applied pressure for the dynamic pressure sensor is shown in Figure 6-19.

![Figure 6-19](image)

Figure 6-19. Change in the resonant frequency as a function of applied pressure for an AC high temperature pressure sensor with 3.8 mm antenna.

The dynamic pressure sensor has a measured sensitivity is 21.7 +/- 0.70 kHz/Pa. The setup was broken down and the sensitivity was measured at 10.6 +/- 0.43 kHz/Pa. The analytical model predicts a sensitivity of 23.7 kHz/Pa, which matches well with the measured sensitivity. The dynamic pressure sensor was designed to operate up to 5 kPa; however, it is observed that it is able to measure pressure linearly up to 800 Pa.
The observed resonant frequency does not change past 800 Pa. The cause for this is likely determined by the sensor’s fabrication.

A film of photoresist was deposited and patterned on a 2.5 μm layer of titanium, then wet etched to create the cavity and vent channel for the sensor. There was some residue before deposition of the photoresist that caused a portion of the resist above the cavity to not be fully developed, which resulted in the cavity not being fully etched during the wet etch. The height of the portion of titanium left in the cavity measures with a height of 2.5 μm. The feature is visibly seen in Figure 4-14.

### 6.3.6 Frequency Drift of AC High Temperature Sensor

As described earlier, the dynamic pressure sensor with an 8 mm diameter sapphire diaphragm and an antenna diameter of 3.8 mm has a nominal resonant frequency of 14.784 GHz. The change in the resonant frequency is determined by observing the phase crossover over a time of 30 min. The change in the resonant frequency as a function of time for the high temperature sensor with a 3.8 mm antenna is shown in Figure 6-20. The average frequency shift over 30 min is 92 kHz, which yields a minimum detectable frequency of 130 kHz (i.e., \( f_{\text{min}} = \sqrt{2} f_{\text{avg}} \)). The minimum detectable pressure for the dynamic pressure sensor with an antenna diameter of 3.8 mm and sensitivity of 21.7 kHz/Pa is 6.0 Pa. The maximum frequency shift over time is also observed to determine the measurement uncertainty in the frequency drift data. A maximum frequency change of 209 kHz was observed over 30 min for the dynamic pressure sensor.
6.4 Summary

The experimental characterization of a dynamic pressure sensor for harsh environments was discussed in Chapter 6. The characterization of an antenna on glass was first discussed to validate the interaction of an antenna with the electromagnetic waveguide. The fundamentals of plane wave tubes was then discussed as a precursor to experimental characterization of a dynamic pressure sensor in an acoustic plane wave tube. The acoustic characterization of silicon-based demonstration and sapphire-based high temperature sensors was then discussed for characterizing the sensitivity and frequency drift of the sensors. The silicon-based demonstration pressure sensors included both static and dynamic types with sensitivities around 2 to 6 kHz/Pa. The sapphire-based high temperature pressure sensor had a sensitivity of up to 21 kHz/Pa.

Figure 6-20. Resonant frequency drifting behavior over time for an AC high temperature pressure sensor.
CHAPTER 7
CONCLUSIONS AND FUTURE WORK

Chapter 7 concludes with a summary of the work described in the dissertation followed by research contributions and future work.

7.1 Summary

Chapter 1 discussed the sensor background including applications and sensor requirements followed by research objectives to serve as a motivation for a passive wireless dynamic pressure sensor. Chapter 2 discussed methods of transduction including piezoresistive, piezoelectric, electrostatic, and optical. Wireless sensing methods including inductive coupling, RF backscatter, and radar were also discussed followed by literature reviews of passive wireless pressure and temperature sensors. Finally, a novel passive wireless pressure sensor was proposed. Chapter 3 discussed high-temperature material selection followed by lumped element modeling of acousto-mechanical and electrical sections. Chapter 4 discussed optimization of a demonstration sensor followed by fabrication of a silicon-based demonstration sensor and sapphire-based high-temperature sensor. Chapter 5 discussed sensor interrogation including discussion of a commercially available integrated radar-on-a-chip followed by the design of an electromagnetic waveguide. Chapter 6 discussed experimental characterization of demonstration and high-temperature pressure sensors in an acoustic plane wave tube.

7.2 Research Contributions

The contributions of this research include:
Realization of the first passive wireless MEMS dynamic pressure sensor for high-temperature environments operating at Ku-band frequencies;

Demonstration of said passive wireless MEMS dynamic pressure sensor with an electromagnetic waveguide; where utilization of the waveguide allows the passive wireless MEMS dynamic pressure sensor to operate in a harsh environment such as a gas turbine.

### 7.3 Future Work

The passive wireless dynamic pressure sensor is packaged and characterized to reside at the backend of an electromagnetic waveguide. The goal of this research is to operate in a gas turbine at temperatures up to 1000°C. The requirements for a waveguide operating in harsh environments include low electrical resistivity, high thermal conductivity, and a high melting temperature. The temperature at the frontend of the electromagnetic waveguide must be low enough to keep the coax-to-waveguide adapters operational. Methods of dissipating heat (e.g., heat-sinks, lengthening of waveguide, reducing thickness of waveguide walls, etc.) should be studied to ensure the temperature at the coax-to-waveguide adapters is close to ambient.

The packaging of the passive wireless pressure sensor is conceptually simplistic. The experimental results, however, show that there is room for improvement in the package design. The sensor die needs to be sealed tightly in the conductive plates. The plates and any additional sealing mechanism should be high-temperature compatible and ideally have a CTE matched to the sensor.
The ability to operate at low frequencies is desirable for using cost effective and commercially available electronics. The advantage of interacting with the sensor at millimeter-wave frequencies is a smaller waveguide and smaller sensor. The HMC6451 60 GHz transceiver could be integrated to up-convert baseband (e.g., L-band) to millimeter-wave frequencies, then down-convert the millimeter-wave frequencies back to baseband. Alternatively, electronics could be developed to up-convert signals from baseband to a higher frequency band (e.g., Ku-band), then down-convert signals back to baseband.
APPENDIX A
DEMONSTRATION SENSOR OPTIMIZATION

optimization.m

% Dynamic Pressure Sensor Optimization
% John Rogers
%

format short; % five digit format
format compact; % suppress line feeds
clc; clear all; % clear command window and clear all data

% target performance parameters
p_max = 3e3; % target maximum pressure [kPa]
z = 15e-6; % depth of cavity [um]
w_max = 13.5e-6; % maximum nonlinear displacement of diaphragm [um]

% material parameters
ce = 3e8; % speed of light in free-space [m/s]
eo = 8.854e-12; % permittivity of free-space [F/m]
uo = 4*pi*1e-7; % permeability of free-space [H/m]
co = 340.29; % isentropic speed of sound in air [m/s]
po = 1.225; % density of air [kg/m^3]
u = 1.983e-5; % dynamic viscosity of air [Pa-s]
v_si = 0.27; % Poisson’s ratio of silicon [-]
E_si = 170e9; % Young’s modulus of <110> silicon [GPa or GN/m^2]
p_si = 2328;  % density of silicon [kg/m^3]
e_r = 4.6;  % relative permittivity of Borofloat 33 [-]

% geometric parameters (fixed)
m_t = 1e-6;  % metal thickness [um]
wo = z - m_t;  % initial air gap [um]
L = 1e-3;  % physical length of vent [mm]
n = 0;  % number of 180deg vent turns [-]
d_v = z;  % depth of vent channel [um]
m = 270e-6;  % thickness of handle [um]
bo = 2.9e-3;  % radius of antenna [mm]

% geometric parameters (variable)
a = [2 4]*1e-3;  % radius of diaphragm [mm]
h = [10 50]*1e-6;  % thickness of diaphragm [um]
w_v = [100 100]*1e-6;  % width of vent channel [um]

% electrical parameters (variable)
fe = [13 16]*1e9;  % electrical operating frequency [GHz]

% define lower and upper bound functions
lb = [a(1) h(1) w_v(1)];  % lower bounds
ub = [a(2) h(2) w_v(2)];  % upper bounds
% randomly generate initial conditions

mm = 1;

N = 5; % number of successful runs

P = 50; % maximum number of consecutive unsuccessful runs

P0 = P;

while mm <= N
    for nn = 1:length(lb)
        x0(nn) = lb(nn) + rand(1,1)*(ub(nn)-lb(nn));
    end

% define linear constraints for A and B matrices

A = [0 1 0];

b = [w_max/(0.41*(1-v_si^2))];

Aeq = [(p_max/(2.3*E_si))^(0.25) -1 0];

beq = [0];

[x,k_da,exitflag] = fmincon('objfun',x0,A,b,Aeq,beq,lb,ub,",",v_si,E_si);

a = x(1);

h = x(2);

w_v = x(3);

clc;
if exitflag < 0
    mm = mm;
    P = P-1;
elseif exitflag > 0
    y(mm) = x(1);
    mm = mm + 1;
    P = P0;
end
if P == 0
    mm = N;
end
end

% report convergence results
fprintf('Convergence Results:
')
fprintf('---------------------
')
fprintf('Diameter of Diaphragm: %5.2f [mm].
', 2*y*1e3)
fprintf('---------------------
')
fprintf('% flexural rigidity of diaphragm [Pa-m^3]
D = E_si*h^3/(12*(1-v_si^2));
')
% calculate acoustic elements of diaphragm

\[ C_{da} = \frac{1}{k_{da}}; \] % acoustic compliance [m/N]
\[ m_{da} = \frac{9(p_{si}h)}{(5\pi a^2)}; \] % acoustic mass [kg]
\[ m_{ra} = \frac{8p_{o}}{(3\pi^2 a^2)}; \] % radiation mass [kg]

% calculate acoustic elements of cavity

\[ C_{ca} = \frac{p_{o}a^2z}{(po*co^2)}; \] % acoustic compliance [m/N]

% calculate acoustic elements of vent

\[ D_h = \frac{2d_v}{(1+d_v/w_v)}; \] % hydraulic diameter [um]
\[ R_{va} = \frac{12u(L+60n*D_h)}{w_v*d_v^3}; \] % acoustic resistance [Ohms]

% calculate cut-on and cut-off frequencies

\[ f_{a_co} = \frac{1}{(2\pi R_{va}*(C_{ca}+C_{da}))}; \] % cut-on frequency [Hz]
\[ f_{a_o} = \frac{1}{(2\pi \sqrt{(m_{ra}+m_{da})*(C_{ca}C_{da}/(C_{ca}+C_{da}))})}; \] % cut-off frequency [Hz]

% define compliance ratio

\[ R = \frac{C_{ca}}{C_{da}}; \]

% call electrical function

\[ [f_{re},f_{re\_max},f_{re\_min},Se,S,f_{\_max},phase_{\_min},Qe,f_{\_min},DR,p_{\_min}] = \]
\[ \text{elec}(p_{\_max},a,D,R,bo,e_r,wo,m,ce); \]
% check that antenna radius is between representative radii for operating frequencies

if f_re_min < fe(1) || f_re_max > fe(2)
    fprintf('Antenna Operates Outside of Desired Operating Frequency Band!
    n
    n')
end

% check that diaphragm radius is greater than or equal to antenna radius

if a < bo
    fprintf('Diaphragm is too Small for Desired Antenna!
    n
    n')
end

% report results

fprintf('Optimization Results:
')

fprintf('---------------------
')

fprintf('Max. Pressure: %5.2f [Pa].
',p_max)

fprintf('Max. Diaphragm Displacement: %5.2f [um].
',w_max*1e6)

fprintf('Antenna Diameter: %5.2f [mm].
',2*bo*1e3)

fprintf('Diameter of Diaphragm: %5.2f [mm].
',2*a*1e3)

fprintf('Thickness of Diaphragm: %5.2f [um].
',h*1e6)

fprintf('Cut-on Frequency: %5.2f [Hz].
',fa_co)

fprintf('Cut-off Frequency: %5.2f [kHz].
',fa_o/1e3)

fprintf('Electrical Sensitivity: %5.2f [kHz/nm].
',Se)

fprintf('Equivalent Sensitivity: %5.2f [kHz/Pa].
',S)

fprintf('Minimum Detectable Pressure: %5.2f [Pa].
',p_min)
fprintf('Dynamic Range: %5.2f [dB].\n', DR)
% Objective Function for Dynamic Pressure Sensor
% John Rogers

function [k_da] = objfun(x,v_si,E_si)

% flexural rigidity of diaphragm [Pa-m^3]
D = E_si*(x(2))^3/(12*(1-v_si^2));

% calculate acoustic compliance of diaphragm [m/N]
C_da = pi*(x(1))^6/(192*D);

% calculate acoustic stiffness of diaphragm [N/m]
k_da = 1/C_da;
end
elec.m

% Electrical Function for Dynamic Pressure Sensor
% John Rogers

% function [f_re,f_re_max,f_re_min,Se,S,f_max,phase_min,Qe,f_min,DR,pmin] = elec(p_max,a,D,R,bo,e_r,wo,m,ce)

% plot electrical resonant frequency vs applied pressure
T = 10; % number of samples
p = p_max; % initial applied pressure [Pa]

for kk = 1:(T+1)
    % calculate effective permittivity for distributed diaphragm [-]
    syms r
    wpk = p*a^4/(64*D)*R; % calculate center displacement of diaphragm
    w_avg = double(1/(pi*bo^2)*int(2*pi*r*wpk*(1-r^2/a^2)^2,0,bo)); e_eff = e_r/(1+e_r*(wo-w_avg)/m)*(1+(wo-w_avg)/m); d = m+wo-w_avg;

    % calculate resonant frequency of antenna
    f_re(kk) = 1.841*ce/(2*pi*bo*sqrt(1+2*d/(pi*bo*e_eff)*(log(pi*bo/(2*d))+1.7726))*sqrt(e_eff)); x(kk) = p;
end
$y(kk) = (wo - w_{avg}) \times 1e6$

$p = p - p_{\text{max}}/T$

end

% plot electrical sensitivity
subplot(1,2,1)
plot(y,f_re/1e9)
grid on
title('electrical sensitivity')
xlabel('air gap, g [\text{um}]')
ylabel('electrical resonant frequency, fo [\text{GHz}]')
set(gca,'xdir','reverse')

% plot equivalent sensitivity
subplot(1,2,2)
plot(x,f_re/1e9)
grid on
title('equivalent sensitivity')
xlabel('applied pressure, p [\text{Pa}]')
ylabel('electrical resonant frequency, fo [\text{GHz}]')
% electrical sensitivity [kHz/nm] and equivalent sensitivity [kHz/Pa]

\[
f_{\text{re\_max}} = f_{\text{re}(T+1)};
\]

\[
f_{\text{re\_min}} = f_{\text{re}(1)};
\]

\[
g_{\text{max}} = y(T+1);
\]

\[
g_{\text{min}} = y(1);
\]

\[
Se = (f_{\text{re\_max}}-f_{\text{re\_min}})/(g_{\text{max}}-g_{\text{min}})/1e6;
\]

\[
S = (f_{\text{re\_max}}-f_{\text{re\_min}})/p_{\text{max}}/1e3;
\]

% dynamic range [dB]

\[
f_{\text{max}} = f_{\text{re\_max}}-f_{\text{re\_min}}; \quad \text{% maximum frequency shift [Hz]}
\]

\[
\text{phase\_min} = 0.035; \quad \text{% min phase uncertainty of Agilent E5071C [rad]}
\]

\[
Qe = 100; \quad \text{% electrical quality factor [-]}
\]

\[
f_{\text{min}} = f_{\text{re\_max}}/(2*Qe)\text{phase\_min}; \quad \text{% minimum resolvable frequency shift [Hz]}
\]

\[
DR = 20\log10(f_{\text{max}}/f_{\text{min}});
\]

% minimum detectable pressure [Pa]

\[
p_{\text{min}} = f_{\text{min}}/(S*1e3);
\]

\[
f_{\text{re}(T+1)}
\]

end
APPENDIX B
ANTENNA ON GLASS FABRICATION

1 – Create Antenna Mask

1.1 – Expose Chrome Mask
   A. Pattern PR using Heidelberg Laser Writer with 20 mm head
   B. Invert Antenna Mask
   C. Develop in AZ 400K (3:1 H₂O:400K) developer for 60 s
      a. Triple rinse in DI water
      b. N₂ dry

1.2 – Wet Etch
   A. Etchant: Chrome Etchant (Transene 1020)
   B. Time: 90 s
   C. Triple rinse in DI water
   D. N₂ dry

1.3 – Strip Photoresist
   A. Place in bath of Baker PRS-3000 at 70°C for 5 min
   B. Triple rinse in DI water
   C. N₂ dry
2 – Metalize Test Antenna on Glass

2.1 – Clean Borofloat Wafer
   A. Place in unheated piranha (16:3:1 H₂O:H₂SO₄:H₂O₂) solution for 10 min
   B. Triple rinse in DI water
   C. N₂ dry
   D. Dehydration bake in oven at 125°C for 20 min

2.2 – Deposit 500 nm of Al using KJL CMS-18 Sputterer
   A. Deposit Rate: 1.1 A/s
   B. Rotation: 20 rpm
   C. Power: 250 W (DC)
   D. Chamber Pressure: < 10⁻⁶ Torr

2.3 – Pattern Photoresist
   A. Spin 1.2 μm of AZ 1512 positive PR using SUSS MicroTec Delta 80RC
      spin coater with recipe 1512_1.2
   B. Soft bake at 112°C for 2 min
   C. Expose PR on SUSS MicroTec MA6 mask aligner (use Test Ant Mask)
      a. Contact Mode: Hard
      b. Lamp Power: 7.5 mW/cm² @ 365 nm
      c. Exposure Time: 20 s
      d. Exposure Dose: 150 mJ/cm²
D. Develop in AZ 300MIF developer for 75 s
   a. Triple rinse in DI water
   b. N\textsubscript{2} dry

E. De-scum in Anatech SCE600 barrel plasma asher
   a. Oxygen Flow Rate: 300 sccm
   b. RF Power: 300 W
   c. Time: 60 s

2.4 – Wet Etch
   A. Etchant: Al Etchant
   B. Time: ~10 min (until features fully visible)
   C. Triple rinse in DI water
   D. N\textsubscript{2} dry

2.5 – Strip Photoresist
   A. Place in bath of Baker PRS-3000 at 70°C for 5 min
   B. Triple rinse in DI water
   C. N\textsubscript{2} dry
   D. De-scum in Anatech SCE600 barrel plasma asher
      a. Oxygen Flow Rate: 600 sccm
      b. RF Power: 600 W
      c. Time: 3 min
3 – Metalize Backside of Glass

3.1 – Deposit 500 nm of Al using KJL CMS-18 Sputterer

A. Deposition Rate: 1.1 A/s
B. Rotation: 20 rpm
C. Power: 250 W (DC)
D. Chamber Pressure: < 10-6 Torr

4 – Die Separation

A. Attach wafer to dicing tape with Antenna side up
B. Dice wafer into 10 mm x 10 mm die strips using ADT 7100 series dicing saw
   a. Blade Type: resin bonded CA-008-140-030-H from Dicing Blade Technology
   b. Spindle Speed: 22k rpm
   c. Cut Speed: 1 mm/s
C. Remove die strips from dicing tape
1 – Create Chrome Masks

1.1 – Expose Chrome Mask
   A. Pattern PR using Heidelberg Laser Writer with 20 mm head
   B. Invert Ground Mask
   C. Flip Cavity, Ground, and Vent Masks in X-Axis
   C. Develop in AZ 400K (3:1 H₂O:400K) developer for 60 s
      a. Triple rinse in DI water
      b. N₂ dry

1.2 – Wet Etch
   A. Etchant: Chrome Etchant (Transene 1020)
   B. Time: 90 s
   C. Triple rinse in DI water
   D. N₂ dry

1.3 – Strip Photoresist
   A. Place in bath of Baker PRS-3000 at 70°C for 5 min
   B. Triple rinse in DI water
   C. N₂ dry
2 – Clean SOI Wafer

2.1 – Organic Clean SOI (SC1)
   A. 5:1:1 H₂O:NH₄OH:H₂O₂ at 75°C for 10 min
   B. Triple rinse in DI water
   C. N₂ dry

2.2 – Oxide Clean SOI (HF Dip)
   A. 50:1 H₂O:HF at RT for 60 s
   B. Triple rinse in DI water
   C. N₂ dry

2.3 – Ion Clean SOI (SC2)
   A. 5:1:1 H₂O:HCl:H₂O₂ at 75°C for 10 min
   B. Triple rinse in DI water
   C. N₂ dry
   D. Dehydration bake in oven at 125°C for 20 min

3 – Cavity Etch Si Layer

3.1 – Apply HDMS to Top Si
   A. Place wafer on HDMS hot plate for 3 min
   B. Dispense HDMS vapor for 30 s
   C. Wait 2 min before removing from hot plate
3.2 – Pattern Photoresist

A. Spin 1.2 μm of AZ 1512 positive PR using SUSS Microtec Delta 80RC spin coater with recipe 1512_1.2

B. Soft bake at 112°C for 2 min

C. Expose PR on SUSS MicroTec MA6 mask aligner (use Cavity Mask)
   a. Contact Mode: Hard
   b. Lamp Power: 7.5 mW/cm² @ 365 nm
   c. Exposure Time: 20 s
   d. Exposure Dose: 150 mJ/cm²

D. Develop in AZ 300MIF developer for 75 s
   a. Triple rinse in DI water
   b. N₂ dry

E. De-scum in Anatech SCE600 barrel plasma asher
   a. Oxygen Flow Rate: 300 sccm
   b. RF Power: 300 W
   c. Time: 60 s

F. Hard bake PR in oven at 125°C for 20 min

3.3 – Etch Cavity

A. STS DRIE 15 μm at 1.5 μm/min using recipe Bao2_jr
   a. RF Power: 500 W @ 13.56 MHz
   b. Platen Power: 12 W
   c. Etch Flow Rate: 130 sccm (SF₆) + 13 sccm (O₂)
d. Etch Time: 13 s

e. Passivation Flow Rate: 85 sccm (C₄F₈)

f. Passivation Time: 7 s

3.4 – Strip Photoresist

A. Place in bath of Baker PRS-3000 at 70°C for 5 min

B. Triple rinse in DI water

C. N₂ dry

D. De-scum in Anatech SCE600 barrel plasma ashers
   a. Oxygen Flow Rate: 600 sccm
   b. RF Power: 600 W
   c. Time: 3 min

4 – Metalize Ground in Cavity

4.1 – Deposit 1000 nm of Al using KJL CMS-18 Sputterer

A. Deposit Rate: 1.1 A/s

B. Rotation: 20 rpm

C. Power: 250 W (DC)

D. Chamber Pressure: < 10⁻⁶ Torr

4.2 – Pattern Photoresist

A. Spin 10 μm of AZ 9260 positive PR using SUSS Microtec Delta 80RC spin coater with recipe 9260_10
B. Soft bake at 112°C for 80 s

C. Spin 10 μm of AZ 9260 positive PR using SUSS Microtec Delta 80RC spin coater with recipe 9260_10

D. Soft bake at 112°C for 3 min

E. Hydration for 90 min

F. Expose PR on SUSS MicroTec MA6 mask aligner (use Ground Mask)
   a. Contact Mode: Hard
   b. Lamp Power: 7.5 mW/cm² @ 365 nm
   c. Exposure Time: 200 s
   d. Exposure Dose: 1500 mJ/cm²

G. Develop in AZ 400K (3:1 H₂O:400K) developer for 12 min
   a. Triple rinse in DI water
   b. N₂ dry

H. De-scum in Anatech SCE600 barrel plasma asher
   a. Oxygen Flow Rate: 300 sccm
   b. RF Power: 300 W
   c. Time: 60 s

4.3 – Wet Etch

A. Etchant: Al Etchant

B. Time: ~20 min (until features fully visible)

C. Triple rinse in DI water

D. N₂ dry
4.4 – Strip Photoresist
   A. Place in bath of Baker PRS-3000 at 70°C for 5 min
   B. Triple rinse in DI water
   C. N₂ dry

5 – Etch Vent Hole

5.1 – Apply HDMS to Si
   A. Place wafer on HDMS hot plate for 3 min
   B. Dispense HDMS vapor for 30 s
   C. Wait 2 min before removing from hot plate

5.2 – Pattern Photoresist
   A. Spin 10 μm of AZ 9260 positive PR using SUSS Microtec Delta 80RC spin coater with recipe 9260_10
   B. Soft bake at 112°C for 80 s
   C. Spin 10 μm of AZ 9260 positive PR using SUSS Microtec Delta 80RC spin coater with recipe 9260_10
   D. Soft bake at 112°C for 3 min
   E. Hydration for 90 min
   F. Expose PR on SUSS MicroTec MA6 mask aligner (use Vent Mask)
      a. Contact Mode: Hard
      b. Lamp Power: 7.5 mW/cm² @ 365 nm
      c. Exposure Time: 200 s
d. Exposure Dose: 1500 mJ/cm²

G. Develop in AZ 400K (3:1 H₂O:400K) developer for 12 min
   a. Triple rinse in DI water
   b. N₂ dry

H. De-scum in Anatech SCE600 barrel plasma asher
   a. Oxygen Flow Rate: 300 sccm
   b. RF Power: 300 W
   c. Time: 60 s

5.3 – Etch Vent

A. STS DRIE 50 μm at 1.5 μm/min using recipe Bao2_jr
   a. RF Power: 500 W @ 13.56 MHz
   b. Platen Power: 12 W
   c. Etch Flow Rate: 130 sccm (SF₆) + 13 sccm (O₂)
   d. Etch Time: 13 s
   e. Passivation Flow Rate: 85 sccm (C₄F₈)
   f. Passivation Time: 7 s

5.4 – Strip Photoresist

A. Place in bath of Baker PRS-3000 at 70°C for 5 min

B. Triple rinse in DI water

C. N₂ dry
6 – Backside Si Etch

6.1 – Apply HDMS to Si
   A. Place wafer on HDMS hot plate for 3 min
   B. Dispense HDMS vapor for 30 s
   C. Wait 2 min before removing from hot plate

6.2 – Pattern Photoresist (Device Side)
   A. Spin 10 \( \mu \text{m} \) of AZ 9260 positive PR using SUSS Microtec Delta 80RC spin coater with recipe 9260_10
   B. Soft bake at 112°C for 3 min
   C. Hard bake PR in oven at 125°C for 5 min

6.3 – Pattern Photoresist (Back Side)
   A. Spin 10 \( \mu \text{m} \) of AZ 9260 positive PR using SUSS Microtec Delta 80RC spin coater with recipe 9260_10
   B. Soft bake at 112°C for 80 s
   C. Spin 10 \( \mu \text{m} \) of AZ 9260 positive PR using SUSS Microtec Delta 80RC spin coater with recipe 9260_10
   D. Soft bake at 112°C for 3 min
   E. Hydration for 90 min
   F. Expose PR on EVG 620 mask aligner (use Backside Mask)
      a. Contact Mode: Hard
      b. Exposure Dose: 1000 mJ/cm\(^2\)
G. Develop in AZ 400K (3:1 H$_2$O:400K) developer for 12 min
   a. Triple rinse in DI water
   b. N$_2$ dry

H. De-scum in Anatech SCE600 barrel plasma asher
   a. Oxygen Flow Rate: 300 sccm
   b. RF Power: 300 W
   c. Time: 60 s

6.4 – Etch Backside and Oxide

A. Attach wafer to Si carrier using Revalpha double-coated 150°C heat release tape

B. STS DRIE 475 µm at 2.0 µm/min using recipe Bao2_jr
   a. RF Power: 500 W @ 13.56 MHz
   b. Platen Power: 12 W
   c. Etch Flow Rate: 130 sccm (SF$_6$) + 13 sccm (O$_2$)
   d. Etch Time: 13 s
   e. Passivation Flow Rate: 85 sccm (C$_4$F$_8$)
   f. Passivation Time: 7 s

C. Remove oxide in Trion RIE
   a. RF Power: 150 W
   b. Etch Flow Rate: 30 sccm (CF$_4$) + 20 sccm (CHF$_3$) + 50 sccm (Ar)
   c. Etch Rate: 580 A/min
D. Remove wafer from carrier by placing on hot plate

E. Remove die from tape

6.5 – Strip Photoresist

A. Place in bath of Baker PRS-3000 at 70°C for 5 min

B. Triple rinse in DI water

C. N₂ dry

7 – Bond Sensor to Glass Antenna

7.1 – Spin Photoresist/Remove Backside Al

A. Attach Glass Antenna die with antenna side up to Si carrier using Revalpha double-coated 150°C heat release tape

B. Spin 10 µm of AZ 9260 positive PR using SUSS Microtec Delta 80RC spin coater with recipe 9260_10

C. Soft bake at 112°C for 3 min

D. Hard bake PR in oven at 125°C for 5 min

E. Remove die from carrier by placing on hot plate

F. Develop in AZ 400K (3:1 H₂O:400K) developer for 12 min

   a. Triple rinse in DI water

   b. N₂ dry

7.2 – Strip Photoresist

A. Place in bath of Baker PRS-3000 at 70°C for 5 min
B. Triple rinse in DI water
C. N₂ dry

7.3 – Mount Sensor Die to Glass Antenna

A. Align sensor die to Glass Antenna with cavity down
B. Use micrometer to clamp die together
C. Apply adhesive around perimeter of die
APPENDIX D
HIGH TEMPERATURE SENSOR FABRICATION

1 – Create Chrome Masks

1.1 – Expose Chrome Mask

A. Place ink side of photomask on PR side of Cr mask; lay clear glass on photomask

B. Expose PR on SUSS MicroTec MA6 mask aligner
   a. Contact Mode: Flood
   b. Lamp Power: 7.5 mW/cm$^2$ @ 365 nm
   c. Exposure Time: 15 s
   d. Exposure Dose: 112 mJ/cm$^2$

C. Develop in AZ 400K (3:1 H$_2$O:400K) developer for 60 s
   a. Triple rinse in DI water
   b. N$_2$ dry

1.2 – Wet Etch

A. Etchant: Chrome Etchant (Transene 1020)

B. Time: 90 s

C. Triple rinse in DI water

D. N$_2$ dry
1.3 – Strip Photoresist

A. Place in bath of Baker PRS-3000 at 70°C for 5 min
B. Triple rinse in DI water
C. N₂ dry

2 – Clean Sapphire

2.1 – Clean Die

A. Place in Baker PRS-3000 at 70°C for 2 min
B. Triple rinse in DI water
C. N₂ dry

3 – Metalize Sapphire

3.1 – Deposit 300 nm of Cr using KJL CMS-18 Sputterer

A. Deposit Rate: 3.2 A/s
B. Rotation: 20 rpm
C. Power: 400 W (RF)
D. Chamber Pressure: < 10^-6 Torr

3.2 – Pattern Photoresist

A. Spin 1.2 μm of AZ 1512 positive PR using SUSS Microtec Delta 80RC spin coater with recipe S_1512_1.2
B. Soft bake in oven at 105°C for 2 min
C. Expose PR on SUSS MicroTec MA6 mask aligner
a. Contact Mode: Hard

b. Lamp Power: 7.5 mW/cm² @ 365 nm

c. Exposure Time: 20 s

d. Exposure Dose: 150 mJ/cm²

D. Develop in AZ 300MIF developer for 75 s

a. Triple rinse in DI water

b. N₂ dry

3.3 – Wet Etch

A. Etchant: Chrome Etchant (Transene 1020)

B. Time: 13 min (over-etch Cr to make undercut)

C. Triple rinse in DI water

D. N₂ dry

3.4 – Deposit 10 nm of Ti using KJL CMS-18 Sputterer

A. Deposit Rate: 1.23 A/s

B. Rotation: 20 rpm

C. Power: 400 W (RF)

D. Chamber Pressure: < 10⁻⁶ Torr

3.5 – Deposit 150 nm of Pt using KJL CMS-18 Sputterer

A. Deposit Rate: 4.26 A/s

B. Rotation: 20 rpm
C. Power: 250 W (DC)
D. Chamber Pressure: < 10^-6 Torr

3.6 – Lift-off Photoresist
   A. Place in MicroChemicals NMP at 70°C for 30 min with ultrasonic wand
   B. Triple rinse in DI water
   C. N₂ dry
   D. Place in Transene 1020 for 3 min to remove remaining Cr
   E. Triple rinse in DI water
   F. N₂ dry

4 – Laser Etch Sapphire
   4.1 – Laser Machine Sapphire at Oxford Lasers

5 – Bond Sapphire
   5.1 – Bond Sapphire Die
       A. Align sapphire die
       B. Clamp die together with micrometer
       C. Apply adhesive at corners of die
       D. Remove clamp
       E. Apply ceramic adhesive around perimeter and in holes
1 – Create Chrome Masks

1.1 – Expose Chrome Mask

A. Place ink side of photomask on PR side of Cr mask; lay clear glass on photomask

B. Expose PR on SUSS MicroTec MA6 mask aligner
   a. Contact Mode: Flood
   b. Lamp Power: 7.5 mW/cm^2 @ 365 nm
   c. Exposure Time: 15 s
   d. Exposure Dose: 112 mJ/cm^2

C. Develop in AZ 400K (3:1 H_2O:400K) developer for 60 s
   a. Triple rinse in DI water
   b. N_2 dry

1.2 – Wet Etch

A. Etchant: Chrome Etchant (Transene 1020)

B. Time: 90 s

C. Triple rinse in DI water

D. N_2 dry
1.3 – Strip Photoresist
   A. Place in bath of Baker PRS-3000 at 70°C for 5 min
   B. Triple rinse in DI water
   C. N₂ dry

2 – Clean Sapphire

2.1 – Clean Die
   A. Place in Baker PRS-3000 at 70°C for 2 min
   B. Triple rinse in DI water
   C. N₂ dry

3 – Metalize Antenna and Ground Plane

3.1 – Deposit 300 nm of Cr using KJL CMS-18 Sputterer
   A. Deposit Rate: 3.2 A/s
   B. Rotation: 20 rpm
   C. Power: 400 W (RF)
   D. Chamber Pressure: < 10⁻⁶ Torr

3.2 – Pattern Photoresist
   A. Spin 1.2 μm of AZ 1512 positive PR using SUSS Microtec Delta 80RC spin coater with recipe S_1512_1.2
   B. Soft bake in oven at 105°C for 2 min
C. Expose PR on SUSS MicroTec MA6 mask aligner
   a. Contact Mode: Hard
   b. Lamp Power: 7.5 mW/cm\(^2\) @ 365 nm
   c. Exposure Time: 20 s
   d. Exposure Dose: 150 mJ/cm\(^2\)

D. Develop in AZ 300MIF developer for 75 s
   a. Triple rinse in DI water
   b. N\(_2\) dry

3.3 – Wet Etch
   A. Etchant: Chrome Etchant (Transene 1020)
   B. Time: 13 min (over-etch Cr to make undercut)
   C. Triple rinse in DI water
   D. N\(_2\) dry

3.4 – Deposit 10 nm of Ti using KJL CMS-18 Sputterer
   A. Deposit Rate: 1.23 A/s
   B. Rotation: 20 rpm
   C. Power: 400 W (RF)
   D. Chamber Pressure: < 10-6 Torr
3.5 – Deposit 150 nm of Pt using KJL CMS-18 Sputterer

A. Deposit Rate: 4.26 A/s
B. Rotation: 20 rpm
C. Power: 250 W (DC)
D. Chamber Pressure: < 10-6 Torr

3.6 – Lift-off Photoresist

A. Place in MicroChemicals NMP at 70°C for 30 min with ultrasonic wand
B. Triple rinse in DI water
C. N₂ dry
D. Place in Transene 1020 for 3 min to remove remaining Cr
E. Triple rinse in DI water
F. N₂ dry

4 – Metalize Cavity and Vent Channel

4.1 – Deposit 2500 nm of Ti using KJL CMS-18 Sputterer

A. Deposit Rate: 1.23 A/s
B. Rotation: 20 rpm
C. Power: 400 W (RF)
D. Chamber Pressure: < 10-6 Torr
4.2 – Pattern Photoresist

A. Spin 1.2 µm of AZ 1512 positive PR using SUSS Microtec Delta 80RC spin coater with recipe S_1512_1.2

B. Soft bake in oven at 105°C for 2 min

C. Expose PR on SUSS MicroTec MA6 mask aligner
   a. Contact Mode: Hard
   b. Lamp Power: 7.5 mW/cm² @ 365 nm
   c. Exposure Time: 20 s
   d. Exposure Dose: 150 mJ/cm²

D. Develop in AZ 300MIF developer for 75 s
   a. Triple rinse in DI water
   b. N₂ dry

4.3 – Wet Etch

A. Etchant: Titanium Etchant (50:1 H₂O:HF)

B. Time: 3 min

C. Triple rinse in DI water

D. N₂ dry

4.4 – Strip Photoresist

A. Place in bath of Baker PRS-3000 at 70°C for 5 min

B. Triple rinse in DI water

C. N₂ dry
5 – Bond Sapphire

5.1 – Bond Sapphire Die

A. Align sapphire die

B. Clamp die together with micrometer

C. Apply adhesive at corners of die

D. Remove clamp

E. Apply ceramic adhesive around perimeter
APPENDIX F
WAVEGUIDE AND PLATE DRAWINGS

Cross-Section of Electromagnetic Waveguide

Aluminum Plate for Glass Antenna
Aluminum Plate for Plane Wave Tube

- Diameter of the hole: 5.7 mm
- Diameter of the inner hole: 5.7 mm
- Diameter of the outer hole: 33.25 mm
- Length: 111 mm
- Width: 70 mm

Dimensions:
- 7 mm
- 15.7 mm
- 31 mm
- 33.25 mm
- 51.5 mm
- 23.5 mm
- 31 mm
- 33.25 mm
- 6.35 mm
Aluminum Plate for DC Pressure Measurements

- Dimensions:
  - 38 mm
  - 30.2 mm
  - 10 mm
  - 10 mm
  - 15.05 mm
  - 15.05 mm
  - 76.2 mm
  - 38.1 mm
  - 76.2 mm

- Other measurements:
  - 8 mm
  - 2 mm
  - 3.57 mm
  - 10.2 mm
  - 6.35 mm
Operational amplifiers are electrical blocks consisting of anywhere from 10 to 100 transistors designed to behave in a certain way. The output voltage of an operational amplifier is related to the input difference voltage $v^+ - v^-$ by the open loop gain $A_o$ as $v_o = A_o(v^+ - v^-)$. An ideal operational amplifier has infinite open loop gain (i.e., $A_o = \infty$) and the input difference voltage is zero (i.e., $v^+ = v^-$). The output voltage for an ideal amplifier must then be $v_o = 0$. Additionally, an ideal operation amplifier is assumed to have no current flow into the input terminals (i.e., $i^+ = i^- = 0$). However, real operational amplifiers have finite open loop gain and non-zero input difference voltage yielding some non-zero output voltage. A summary of performance characteristics of operational amplifiers for an ideal amplifier compared to Linear Technology’s LT1007 [233] and LT1028 [234] low-noise amplifiers is shown in Table G-1.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Symbol</th>
<th>Ideal</th>
<th>LT1007</th>
<th>LT1028</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input offset current</td>
<td>$I_{os}$</td>
<td>0</td>
<td>7</td>
<td>12</td>
<td>nA</td>
</tr>
<tr>
<td>Input bias current</td>
<td>$I_b$</td>
<td>$\pm 10$</td>
<td>$\pm 25$</td>
<td>nA</td>
<td></td>
</tr>
<tr>
<td>Input offset voltage</td>
<td>$V_{os}$</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>µV</td>
</tr>
<tr>
<td>Input resistance</td>
<td>$R_{in}$</td>
<td>$\infty$</td>
<td>7</td>
<td>0.3</td>
<td>GΩ</td>
</tr>
<tr>
<td>Output resistance</td>
<td>$R_{out}$</td>
<td>0</td>
<td>70</td>
<td>80</td>
<td>Ω</td>
</tr>
<tr>
<td>Open-loop gain</td>
<td>$A_o$</td>
<td>$\infty$</td>
<td>20</td>
<td>30</td>
<td>V/µV</td>
</tr>
<tr>
<td>Gain-bandwidth product</td>
<td>GBW</td>
<td>$\infty$</td>
<td>8</td>
<td>75</td>
<td>MHz</td>
</tr>
<tr>
<td>Slew rate</td>
<td>SR</td>
<td>$\infty$</td>
<td>2.5</td>
<td>15</td>
<td>V/µs</td>
</tr>
<tr>
<td>Common mode rejection ratio</td>
<td>CMRR</td>
<td>$\infty$</td>
<td>130</td>
<td>126</td>
<td>dB</td>
</tr>
<tr>
<td>Power supply rejection ratio</td>
<td>PSRR</td>
<td>$\infty$</td>
<td>130</td>
<td>133</td>
<td>dB</td>
</tr>
</tbody>
</table>

An ideal operational amplifier also has infinite bandwidth. However, a real operational amplifier has finite bandwidth. The gain-bandwidth product (GBW) is the product of the open-loop gain and the bandwidth. Alternatively, the 3 dB bandwidth is
found by dividing the open loop gain by gain-bandwidth product by the open loop gain (i.e., \( f_{3dB} = \frac{GBW}{A_o} \)). For example, the LT1007 has a gain-bandwidth product of 8 MHz and open-loop gain of 20 V/\( \mu \)V (146 dB), which yields a 0.4 Hz 3 dB bandwidth. If, on the other hand, the LT1007 is being used to drive an audio signal in the audible frequency range of 20 Hz to 20 kHz, then the maximum allowable gain is 400 V/V (52 dB). The open-loop gain and closed-loop gain for the LT1007 using a non-inverting configuration with \( R_2 = 399 \, k\Omega \) and \( R_1 = 1 \, k\Omega \) is simulated using LTspice as shown in Figure G-1.

![Figure G-1. Gain-bandwidth product for LT1007.](image)

The voltage rails of an operational amplifier limit the maximum voltage swing on the output voltage. Ideally, the output voltage would be allowed to swing from \(-V_S\) to \(+V_S\). However, in real amplifiers the actual output voltage swing allowed is always less than \( \pm V_S \). To illustrate this concept a small signal at 10 kHz is placed on the input of a
non-inverting LT1007 operational amplifier with $R_2 = 399 \, k\Omega$ and $R_1 = 1 \, k\Omega$. The voltage rails for the operational amplifier are set at +10 V and -10 V. The output voltage for an input of 10 mV is 3.8 V and the output voltage for an input of 20 mV is 7.6 mV as shown in Figure G-2. However, the output voltage for an input of 25 mV is roughly 8.8 V instead of the expected 9.5 V. The output is considered soft clipping since the sine wave are just beginning to compress. However, the output voltage for an input of 30 mV is also 8.8 V. The output is considered hard clipping since the sine wave is severely compressing.

![Figure G-2. Clipping of output for LT1007.](image)

An alternative way of visualizing clipping of the output is to plot the output voltage as a function of the input voltage. The output voltage as a function of the input voltage for the non-inverting LT1007 operation amplifier is shown in Figure G-3. The ideal output voltage curve assumes the output stays linear across all input voltages. However, the actual output voltage curve will gradually deviate from the ideal curve as
the input voltage is incremented. This approach is commonly used in RF characterization of power amplifiers. However, in power amplifiers the input is a power signal instead of a voltage signal. The place where the output power deviates 1 dB from the ideal curve is known as the 1 dB compression point. To maximize output power for a transceiver it is very common to operate near or at the 1 dB compression point.

![Graph showing output compression for LT1007](image)

**Figure G-3. Output compression for LT1007.**

If the operational amplifier is driven into compression, then the output will be distorted. In the time domain the sine wave clips; however, in the frequency domain harmonics are generated. For illustration a non-inverting LT1007 operational amplifier is driven well into compression with an 80 mV small signal at 10 kHz as shown in Figure G-4. The fundamental frequency $f_o$ appears on the output with the addition of harmonics (i.e., $3f_o$, $5f_o$, $7f_o$, $9f_o$, etc.). The THD is the ratio of the total output voltage contributed to all of the harmonics to the output voltage contributed to the fundamental (i.e., $THD = \sqrt{v_2^2 + v_3^2 + v_4^2 + \cdots / v_1}$). The majority of the power is in the fundamental
frequency \( f_o \) and the third and fifth harmonics \( 3f_o \) and \( 5f_o \). The THD considering just the third and fifth harmonic for the non-inverting operational amplifier is approximately 27%.

![Graph showing output voltage vs. frequency](image)

**Figure G-4.** Harmonic generation for LT1007.

Other performance characteristics of operational amplifiers include input resistance, output resistance, and slew rate. The input resistance is the resistance looking into the operational amplifier from the input terminals. If an ideal operational amplifier is assumed to have no current flow into the input terminals, then the operational amplifier has infinite input resistance. The output resistance is the resistance looking into the operational amplifier from the output and is zero for an ideal operational amplifier. Next, consider the output signal of an operational amplifier is \( v_o = V_p \sin 2\pi f t \). The rate of change of the output is then \( dv_o(t)/dt = 2\pi f V_p \cos 2\pi f t \). The slew rate is equivalent to the maximum rate of change of the output, which occurs at the zero crossing (i.e., \( SR = 2\pi f_{max} V_p \)) [235]. As stated earlier, the GBW yields a trade-off
between bandwidth and gain. Similarly, the slew rate yields a trade-off between bandwidth and peak output voltage.

![Graph showing output of LT1007 in response to step input.]

Figure G-5. Output of LT1007 in response to step input.

The slew rate of an operational amplifier is determined by applying a voltage step to the input, then observing the output response on an oscilloscope. For illustration a unity gain buffer using the LT1007 is simulated using LTspice with a 5 V step input. The response time for the output to track the input is roughly 1.9 \( \mu s \) as shown in Figure G-5. The slew rate can then be calculated as \( SR = \frac{\Delta V}{\Delta t} \) or roughly 2.6 \( V/\mu s \) for the LT1007. The slew rate simulated by LTspice is in good agreement with the 2.5 \( V/\mu s \) given in Table G-1.
APPENDIX H
OPERATIONAL AMPLIFIER NONIDEALITIES

The open-loop gain for a real operational amplifier is not infinity as assumed for an ideal operational amplifier. One might question how a finite open-loop gain might affect a closed-loop configuration. Consider a non-inverting operational amplifier configuration with a closed-loop voltage gain of $v_o = 1 + R_2/R_1$. If the open-loop gain is not infinite, then the input difference voltage is not zero (i.e., $v^+ \neq v^-$). The voltage on the negative input terminal is then $v^- = (R_1/(R_1 + R_2))v_o = v_o/(1 + R_2/R_1)$. The output voltage is once again equivalent to $v_o = A_o(v^+ - v^-)$. The voltage on the positive input terminal is connected to the input source (i.e., $v^+ = v_i$). The closed-loop voltage gain can now be expressed

$$
\frac{v_o}{v_i} = \frac{A_o}{1 + \frac{A_o}{1 + \frac{R_2}{R_1}} \frac{1 + \frac{R_2}{R_1}}{A_o + 1}}.
$$

(H-1)

If the finite open-loop gain $A_o$ is much greater than the ideal closed-loop voltage gain (i.e., $A_o \gg 1 + R_2/R_1$), then the closed-loop voltage gain reduces to $v_o/v_i = 1 + R_2/R_1$. If, on the other hand, the finite open-loop gain $A_o$ is much less than the ideal closed-loop voltage gain (i.e., $A_o \ll 1 + R_2/R_1$), then the closed-loop voltage gain reduces to $v_o/v_i = A_o$. 

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Figure H-1. Non-inverting operational amplifier with grounded input and output source.

Next, one might question how the finite open-loop gain affects the output resistance. Consider a non-inverting operational amplifier with the positive input terminal grounded \( v^+ = 0 \) and the output connected to a voltage source \( v_x \) as shown in Figure H-1. The voltage on the output is \( v_x = (i_x + i_o)(R_1 + R_2) \). The output current of the operational amplifier is \( i_o = (A_o v_{ld} - v_x)/R_o \), where the differential input voltage is \( v_{ld} = v^+ - v^- = -v_1 \). The voltage across resistor \( R_1 \) is equivalent to \( v_1 = (R_1/(R_1 + R_2))v_x \).

The output resistance can now be expressed

\[
R_{out} = \frac{v_x}{i_x} = \frac{1 + \frac{R_2}{R_1}}{\frac{1}{R_1} + \left(1 + \frac{A_o}{1 + \frac{R_2}{R_1}}\right)\left(1 + \frac{R_2}{R_1}\right)R_o}. \tag{H-2}
\]

If the finite open-loop gain \( A_o \) is much greater than the ideal closed-loop gain (i.e., \( A_o \gg 1 + R_2/R_1 \)), then the output resistance reduces to \( R_{out} = 0 \). If, on the other hand, the finite open-loop gain \( A_o \) is much less than the ideal closed-loop gain (i.e., \( A_o \ll 1 + R_2/R_1 \)), then the output resistance reduces to \( R_{out} = R_o \parallel (R_1 + R_2) \).
Next, one might question how the finite open-loop gain affects the input resistance. Consider a non-inverting operational amplifier with the positive input terminal connected to a voltage source $v_x$ as shown in Figure H-2. The voltage on the output of the operational amplifier is $v_o = A_v v_{id} = A_v (v_x - v_1)$. The voltage $v_1$ across resistor $R_1$ is equivalent to $v_1 = i_o R_1$. The voltage $v_1$ across resistor $R_1$ can also be expressed as a voltage division $v_1 = (R_1/(R_1 + R_2))v_o = A_v (R_1/(R_1 + R_2))(v_x - v_1)$. The voltage $v_1$ is written as a function of $v_x$ as $v_1 = (A_v R_1/(R_1 + R_2 + A_v R_1))v_x$. The current on the positive input terminal is $i_x = (v_x - v_1)/R_{id}$. The input resistance can now be expressed

$$R_{in} = \frac{v_x}{i_x} = \frac{R_{id}}{1 - \frac{A_v R_1}{R_1 + R_2 + A_v R_1}} = \frac{R_{id}}{1 - \frac{R_2}{1 + \frac{R_2}{R_1} + A_v}}.$$  \hfill (H-3)

If the finite open-loop gain $A_v$ is much greater than the ideal closed-loop gain (i.e., $A_v \gg 1 + R_2/R_1$), then the input resistance reduces to $R_{in} = \infty$. If, on the other hand, the
finite open-loop gain $A_o$ is much less than the ideal closed-loop gain (i.e., $A_o \ll 1 + R_2/R_1$), then the input resistance reduces to $R_{in} = R_{id}$.

![Figure H-3. Difference operational amplifier with base and offset currents.](image)

An ideal operational amplifier has no current flow into the input terminals (i.e., $i^+ = i^- = 0$). However, a real operational amplifier has some finite amount of current on the input terminals (i.e., $i^+ \neq i^-$). The base bias current is the average of the currents on the input terminals (i.e., $I_b = (i^+ + i^-)/2$). Additionally, there exists a finite offset current between the two input terminals given as $I_{os} = i^+ - i^-$. Consider a difference operational amplifier with base bias and offset currents as shown in Figure H-3. If the input and output terminals are grounded, then the positive and negative input terminals dissolve to $v^+ = -(R_3 || R_4)I_b$ and $v^- = -(R_1 || R_2)I_b$, respectively. The voltage on the output $v_o = A_o(v^+ - v^-)$ is now offset proportionally to the magnitude of the base bias current $I_b$.

However, the output offset due to base bias current may be minimized. The desired output voltage due to bias currents is zero so assume $v^+ = v^-$. If the desired output voltage due to bias currents is zero, then it follows that $R_1 || R_2 = R_3 || R_4$. The designer now has the choice to properly choose well-matched resistors to minimize the
output voltage due to base bias current. Unfortunately, the output voltage due to offset current given as \( v_o = \pm I_{os} R_2 \) cannot be eliminated. However, the output voltage due to offset current is minimized by choosing an \( R_2 \) that is relatively small, but not too small as to saturate the operational amplifier.

The offset voltage on the input terminals associated with the offset current is also known as the differential input voltage \( v_{id} \) and is given as \( V_{os} = v_{id} = v^+ - v^- \). However, if a common-mode voltage is applied, then the voltage on the input terminals is \( v_{cm} = (v^+ + v^-)/2 \). In an ideal operational amplifier there is no common-mode voltage since the voltages on the input terminals are equivalent. However, a real operational amplifier has some finite offset voltage. The open-loop gain of an operational amplifier is \( A_o = v_o/v_{id} \). The common-mode gain of an operational amplifier is \( A_{cm} = v_o/v_{cm} \). The ratio of the open-loop gain to the common-mode gain is known as the common-mode rejection ratio (CMRR) and is given as

\[
CMRR = 20 \log_{10} \left| \frac{A_o}{A_{cm}} \right| = 20 \log_{10} \left| \frac{v_{cm}}{v_{id}} \right|. \tag{H-4}
\]

An ideal operational amplifier has a CMRR of infinity. However, real operational amplifiers typically have a CMRR of 70 to 120 dB. Another way to think of CMRR is an operational amplifier’s ability to reject common-mode voltages on the output as the output voltage for a real operational amplifier is \( v_o = A_o v_{id} + A_{cm} v_{cm} \). Additionally, a real operational amplifier has some output offset due to ripple voltage in the power supply. The power supply gain of an operational amplifier is \( A_r = v_o/v_r \). The ratio of the open-loop gain to the power supply gain is known as the power supply rejection ratio (PSRR) and is given as [236]
\[ PSRR = 20 \log_{10} \left| \frac{A_o}{A_r} \right| = 20 \log_{10} \left| \frac{v_r}{v_o} A_o \right|. \] (H-5)

An ideal operational amplifier has a PSRR of infinity. However, real operational amplifiers typically have a PSRR of 100 dB. Another way to think of PSRR is an operational amplifier’s ability to reject ripple voltage from the power supply on the output as the output voltage for a real operational amplifier is \( v_o = A_o v_{id} + A_r v_r \).
APPENDIX I
NETWORK AND SPECTRUM ANALYZER FUNDAMENTALS

A network analyzer is used to measure the electrical scattering parameters of a DUT across some frequency band of interest. The scattering parameters used to describe the characteristics of the DUT are input and output incident voltages $V_1^+$ and $V_2^+$, respectively, and input and output reflected voltages $V_1^-$ and $V_2^-$, respectively. The input and output reflection coefficients are $S_{11} = V_1^- / V_1^+$ and $S_{22} = V_2^- / V_2^+$, respectively, and the forward and reverse voltage gains are $S_{21} = V_2^- / V_1^+$ and $S_{12} = V_1^- / V_2^+$, respectively. The reflected voltages at the two ports are related to the incident voltages by the scattering matrix as [35]

$$
\begin{bmatrix}
V_1^- \\
V_2^-
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
V_1^+ \\
V_2^+
\end{bmatrix}.
$$

A network analyzer with one RF source and four receivers is shown in Figure I-1 [237]. The incident power signals are measured and used as references in calculating the scattering parameters [238]. Additionally, directional couplers are used to separate the incident and reflected power signals at the input and output of the DUT. A tuned receiver with a LO, RF mixer, and IF filter is used to down-convert an RF power signal to IF. The magnitude and phase of the IF is then measured by an analog-to-digital converter (ADC).
The IF bandwidth associated with the IF filter in a tuned receiver is adjustable. If the IF bandwidth decreases, then the noise floor decreases and the sensitivity increases. The dynamic range of a network analyzer is the difference between the maximum receiver power $p_{\text{max}}$ and the receiver noise floor $p_n$ (i.e., $DR = p_{\text{max}} - p_n$). The dynamic range also increases as the noise floor decreases. However, the trade-off with a decreased IF bandwidth is a decrease in the measurement speed. Another way to increase the dynamic range is by averaging. A network analyzer can average complex data that includes phase information to decrease the noise floor. A spectrum analyzer, on the other hand, is unable to decrease the noise floor by averaging since it does not utilize phase information. However, a network analyzer requires a calibration kit to calibrate out the losses in the cables such that the reference place for measuring is brought to the end of the cables. Alternatively, if RF probes are connected to a
network analyzer, then a calibration substrate is required to bring the reference plane to the end of the tips of the probes.

For two-port cable calibration a commonly used method is the short-open-load-thru (SOLT). For wafer-level measurements transmission lines may be used to mimic inductive shorts, capacitive opens, and thru lines at the wafer-level; however, a surface-mount 50 Ω load is required for SOLT. Alternatively, the thru-reflect-line (TRL) method uses a thru line, a highly reflective short or open, and an additional line of a different length than the thru line [239]. However, multiple lines may be required for broadband frequency coverage. California Eastern Laboratories has an application note providing general guidelines for designing TRL standards [240]. A thru-reflect-match (TRM) calibration replaces the additional line with a broadband 50 Ω load for broadband coverage.

For high-frequency wafer-level measurements GGB Industries of Naples, FL has developed coplanar ground-signal-ground (GSG) probes that operate from DC to 67 GHz with less than 1.1 dB of insertion loss [219]. Additionally, GGB Industries has alumina calibration substrates with SOLT, TRL, and TRM capabilities. The calibration substrate calibrates out all scattering parameter errors up to the tip of the probes. Any additional errors due to cable bending are naturally suppressed due to minimal movement of the cables at a probe station.

Although a calibration brings the measurement plane to the input of the DUT (i.e., end of cables or tips of probes), additional de-embedding of the transmission line may be necessary for a more accurate representation of the device [241]. The de-embedding may be performed either during measurement or after measurement using an ideal
transmission line model. A network analyzer with a port extension feature can extend the measurement plane to the DUT using either an open or short equivalent to that of the input line for real-time de-embedding. However, the port extension assumes the line is ideal with a linear phase response and constant impedance. A port extension effectively adds phase length to the scattering parameters.

Figure I-2. An illustration of a spectrum analyzer (adapted from [242]).

A network analyzer measures the incident and reflected voltages at known source frequencies. A spectrum analyzer, on the other hand, is used to measure unknown signals. A spectrum analyzer is capable of measuring harmonics, intermodulation products, sidebands, etc. A spectrum analyzer is a single-port receiver and unable to measure phase. An illustration of a typical spectrum analyzer is shown in Figure I-2. The architecture of a spectrum analyzer is similar to that of an AM radio receiver. The RF input signal from the DUT passes through an RF input attenuator and pre-selector or low-pass filter, then down-converts to an IF frequency by mixing with a LO. The IF signal then goes through an IF amplifier and IF filter where harmonics and
intermodulation products from the mixer are rejected. Next, an envelope detector allows the envelope of the RF input signal to pass through to the video filter.
The physical length of a transmission line and its associated frequency are related by \( \lambda_{\text{phy}} = c/f_{\text{phy}}\sqrt{\varepsilon_{\text{eff}}} \). In general, for a circuit to be considered lumped element the operating frequency should be much less than the frequency associated with the physical length (i.e., \( f \ll f_{\text{phy}} \)). A lumped element has voltage and current that are equal at any point along the associated physical length. Basic mesh and nodal analysis using KVL and KCL, respectively, can be applied to lumped elements [243]. A distributed element, on the other hand, has voltage and current that change as a function of the position along the physical length.

Consider a resistor of length \( \lambda_{\text{phy}} \) as shown in Figure J-1. The normalized current \( i(z, t = 0)/I_0 = \cos(-\beta z - \pi) \) is plotted against position along the resistor. The current along the resistor for operating frequencies much greater than one-eighth of the frequency associated with the resistor length (i.e., \( f \gg f_{\text{phy}}/8 \)) varies largely along the resistor. The current along the resistor for operating frequencies less than or equal to
one-eighth of the frequency associated with the resistor length (i.e., \( f \leq f_{phy}/8 \)), on the other hand, is very uniform along the resistor.

\[
\begin{align*}
& + \quad R'\Delta z \quad L'\Delta z \quad \quad + \\
& v(z, t) \quad G'\Delta z \quad C'\Delta z \quad v(z + \Delta z, t) \\
& - \quad \quad \quad \quad z \quad \quad \quad \quad z + \Delta z
\end{align*}
\]

Figure J-2. Schematic of distributed element circuit (adapted from [35]).

In the 1880s Oliver Heaviside developed a model to describe the electromagnetic behavior of transmission line segments. A transmission line has series resistance and inductance elements \( R = R'\Delta z \) and \( L = L'\Delta z \), respectively, and shunt conductance and capacitive elements \( C = C'\Delta z \) and \( G = G'\Delta z \), respectively, as shown in Figure J-2. The distributed element circuit is described using KVL, which yields [35]

\[
v(z, t) - v(z + \Delta z, t) = i(z, t)R'\Delta z + L'\Delta z \frac{\partial i(z, t)}{\partial t}.
\]

\[
\begin{align*}
& + \quad Z_s \quad \quad z \quad \quad + \\
& v_s \quad \quad \quad \quad R, L, C, G \quad v(z, t) \quad v_L \\
& - \quad \quad \quad \quad z = - \ell \quad \quad \quad \quad z = 0
\end{align*}
\]

Figure J-3. An electrical transmission line.
The telegrapher’s equations describe the voltage and current on the transmission line as a function of time and position along the transmission line as shown in Figure J-3. The voltage along the transmission line is related to the incident voltage $V_o^+$, reflected voltage $V_o^-$, propagation constant $\gamma = \alpha + j\beta$, position along the transmission line $z$, operating frequency $\omega$, and time $t$ as [35]

$$v(z, t) = V_o^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_o^- e^{+\alpha z} \cos(\omega t + \beta z). \quad (J-2)$$

The current along the transmission line is related to the incident current $I_o^+ = V_o^+ / Z_o$, and reflected current $I_o^- = -V_o^- / Z_o$ as [35]

$$i(z, t) = I_o^+ e^{-\alpha z} \cos(\omega t - \beta z) + I_o^- e^{+\alpha z} \cos(\omega t + \beta z). \quad (J-3)$$

The telegrapher’s equations are often given in phasor form as $v(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$ and $i(z) = I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z}$. The characteristic impedance $Z_o$ is a ratio of either the incident or reflected voltage to current. The characteristic impedance is not a function of the length of the line. Materials used for RF design exhibit properties where $\omega L \gg R$ and $\omega C \gg G$ such that the transmission line is approximated to be lossless. The characteristic impedance of a transmission line is [35]

$$Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \sqrt{\frac{R' \Delta z + j \omega L' \Delta z}{G' \Delta z + j \omega C' \Delta z}} = \sqrt{\frac{R' + j \omega L'}{G' + j \omega C'}} \approx \frac{L}{C}. \quad (J-4)$$

The phase constant $\beta$ is the change in phase along the transmission line as the wave propagates. For the lossless transmission line the phase constant is $\beta = \omega / u_p = 2\pi / \lambda = \omega \sqrt{LC}$. The reflection coefficient $\Gamma$ is the ratio of the voltage reflected back from the load to the voltage incident on the load. Ideally, the reflection coefficient is zero and the line is said to be perfectly matched to the load. In general, the complex reflection coefficient due to some mismatch between the load and characteristic impedance is [35]
\[
\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}.
\]  \hspace{1cm} (J-5)

The reflection coefficient at any point along the transmission line beginning from the load is \( \Gamma(z) = \Gamma e^{2\gamma z} \). The voltage standing wave ratio (VSWR) is the ratio of the maximum voltage to the minimum voltage along the transmission line (i.e., \( VSWR = |V_{max}|/|V_{min}| = |V_o^+ + V_o^-|/|V_o^+ - V_o^-| = (1 + |\Gamma|)/(1 - |\Gamma|) \)). The impedance at any point along the transmission line is

\[
Z(z) = \frac{V(z)}{I(z)} = \frac{Z_o e^{-\gamma z} + \Gamma e^{+\gamma z}}{e^{-\gamma z} - \Gamma e^{+\gamma z}}. \tag{J-6}
\]

The impedance looking into the transmission line is known as the input impedance and is a function of the length of the line \([35]\)

\[
Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta \ell}{Z_o + jZ_L \tan \beta \ell}. \tag{J-7}
\]

The input impedance for a line of length \( \ell = \lambda/4 \), for example, is \( Z_{in} = Z_o^2/Z_L \). The input impedance for a line of length \( \ell = \lambda/2 \) is \( Z_{in} = Z_L \). For a shorted transmission line with \( Z_L = 0 \) the input impedance is \( Z_{in} = jZ_o \tan \beta \ell \). An open transmission line, on the other hand, with \( Z_L = \infty \) the input impedance is \( Z_{in} = -jZ_o \cot \beta \ell \). One interesting observation is that at \( \lambda/4 \) the shorted transmission line electrically appears as an open circuit, whereas an open transmission line electrically appears as a short circuit.

Consider, for example, a short \( \lambda/2 \) transmission line as shown in Figure J-4 \([244]\). The voltage at the end of the transmission line (i.e., \( z = 0 \)) is zero. The voltage increases along the transmission line until the voltage is maximum at \( z = -\lambda/4 \) where the transmission line appears to be an open circuit. The voltage then decreases until it
is zero once again at the beginning of the transmission line (i.e., \( z = -\lambda /2 \)) where the transmission line appears to be a short circuit.

![Figure J-4](image)

Figure J-4. Distributed element short \( \lambda/2 \) transmission line with lumped element equivalents.

Similarly, consider an open \( \lambda/2 \) transmission line as shown in Figure J-5 [244]. The current at the end of the transmission line (i.e., \( z = 0 \)) is zero. The current increases along the transmission line until the current is maximum at \( z = -\lambda /4 \) where the transmission line appears to be a short circuit. The current then decreases until it is zero once again at the beginning of the transmission line (i.e., \( z = -\lambda /2 \)) where the transmission line appears to be an open circuit.
Figure J-5. Distributed element open $\lambda/2$ transmission line with lumped element equivalents.

If the frequency band of operation is narrow in relation to the operating frequency (i.e., $\Delta f \ll f_0$), then a circuit is said to have a high quality factor. Lumped element circuits typically have lower quality factors than their distributed element equivalents. Distributed element circuits can operate at much higher frequencies than lumped element circuits given their small dimensions in comparison to their operating frequencies. Additionally, distributed element circuits can be constructed on low-loss dielectrics.
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BIOGRAPHICAL SKETCH

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