THE ROLE OF DIFFERENTIATION AND STANDARDS-BASED ASSESSMENT IN THE MATHEMATICS LEARNING OF STRUGGLING AND ADVANCED LEARNERS IN A DETRACKED HIGH SCHOOL GEOMETRY CLASSROOM

By

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To Trent, Briana, Patryk, Mom, and Dad
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THE ROLE OF DIFFERENTIATION AND STANDARDS-BASED ASSESSMENT IN THE MATHEMATICS LEARNING OF STRUGGLING AND ADVANCED LEARNERS IN A DETRACKED HIGH SCHOOL GEOMETRY CLASSROOM

By

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Chair: Nancy Fichtman Dana
Major: Curriculum and Instruction

To gain insight into how using differentiated instruction and standards-based assessment supported my students’ learning in a detracked, honors geometry classroom, I employed the methodology of practitioner research to examine and reflect on the development and implementation of a standards-based differentiated instructional unit based on the Pythagorean Theorem. Data collected and analyzed included field notes and audiotaped conversations during classroom activities; student artifacts from classroom activities and assessments; verbatim transcripts from audiotaped student interviews; teacher journal entries chronicling significant events and actions taken during the development and implementation of this unit; and notes from peer debriefing during and following the unit’s implementation.

As I reviewed, analyzed, and reflected upon the data, my findings fell into two categories: “how” and “why” using differentiated instruction and standards-based assessment supported my struggling learners and challenged my advanced learners during the implementation of the Pythagorean Theorem unit. Within the first category, “how,” three themes emerged. Planning and implementing a differentiated and
standards-based unit of instruction supported my struggling learners and challenged my advanced learners by: (1) using homogeneous grouping when introducing new content; (2) using heterogeneous grouping when reviewing content; and, (3) allowing for both teacher and student choice for grouping throughout the unit’s implementation. Within the second category, “why”, three themes also emerged. Planning and implementing a differentiated and standards-based unit of instruction supported my struggling learners and challenged my advanced learners because: (1) differentiation and standards-based learning was framed using scaffolded notes, manipulative materials, podcasts, and problem-solving activities; (2) a high degree of structure was used when struggling students and advanced students worked together in heterogeneous groups; and, (3) a holistic grading procedure used for both formative and summative assessments provided insights for both me and my students into their meeting a level of proficiency on the unit’s learning targets. These findings indicate how planning for and implementing a differentiated, standards-based instructional unit can support the learning needs of both struggling and advanced learners in a detracked honors geometry classroom.
CHAPTER 1
INTRODUCTION AND BACKGROUND

Introduction and Purpose Statement

The achievement gap, usually based on standardized test scores and graduation rates, highlights and emphasizes the differences between struggling and advanced students. This achievement gap can also be described as the ‘opportunity gap,’ as the inequities that exist both racially and economically in society mirror the same gap in achievement found in our schools (Carter, 2013; Welner & Carter, 2013). All students deserve fair and equal access to academic opportunities, but the opportunity gap that persists in schools, on both personal and organizational levels, highlights the differences between the systemic advantages some students possess compared to the systemic disadvantages that other students have, resulting in different learning opportunities for students (Darling-Hammond, 2013; Krownapple, Kosi, & Keeny, 2010; Welner & Carter, 2013). Schools have to be culturally responsive in meeting their students’ unique academic and social needs by acknowledging their strengths and providing the opportunities necessary to allow every student a fair chance for academic success (Welner & Carter, 2013). This principle of fairness and equity in schools dates back nearly two centuries to when Horace Mann (1849) described education as “the great equalizer of the conditions of men – the balance-wheel of the social machinery” (p. 59). Many educators unwittingly participate in maintaining the disparities between their advantaged and disadvantaged students, because students’ achievement and opportunity relate to their own perceptions of teachers’ expectations, the relationships they have with teachers, and their chances for participation in school. All of these factors contribute to and allow the opportunity gap between advantaged and
disadvantaged students to be further broken down into three components related to school climate and teachers’ beliefs and biases: the expectations gap, the relationships gap, and the participation gap (Krownapple et al., 2010; Quaglia, Fox, & Corso, 2010).

The expectations gap can exist from both the teachers’ and students’ viewpoints. Teachers form impressions of their students’ individual capacities for learning, and the academic level and type of course in which the students are enrolled can influence these preconceived notions. These impressions can create an artificial ceiling in the minds of teachers beyond which, whether due to intelligence or effort, their students are incapable of going and thereby limiting their potential for learning (Harris, 2012; Powell, 2011a; Quaglia et al., 2010; Tyson, 2013). The students’ perspective of the expectations gap stems from their own impressions of their teachers’ opinions of their (the students’) ability to learn. Struggling students tend to disconnect emotionally, behaviorally, and intellectually from the learning process if they believe their teachers do not believe in them, stunting their academic potential and further increasing the opportunity gap (Krownapple et al., 2010; Ladson-Billings, 2013; Quaglia et al., 2010; Welner & Carter, 2013).

The relationships gap denotes the parallel between the quality of students’ relationships with their teachers as well as the strength of their academic efforts (Quaglia et al., 2010). This relationship can have either a positive or a negative impact on students’ social and academic growth (Powell, 2011a). The interactions between a teacher and student strongly influence that student’s self-esteem and self-concept in addition to their academic success in the classroom (Tutwiler, 2007). Students seek genuine, caring relationships with their teachers, and they want to be treated with
respect and know that their efforts to learn are valued (Krownapple et al., 2010; Powell, 2011a; Rightmyer, 2011). Students need to feel their teachers are interested in their welfare in and out of the classroom and will not give up on them (Bondy, Ross, Hambacher, & Acosta, 2012). Teachers who strive to achieve these relationships can strengthen their students’ engagement with school, which can lead to an increase in their learning (Ladson-Billings, 2013; Quaglia et al., 2010).

The participation gap encompasses the differences between the majority of students who tend to be less involved in all aspects of the school’s learning and social environment and the smaller percentage of students who have the opportunity to take full advantage of experiences the educational process has to offer (Quaglia et al., 2010). The social components of learning, such as teacher-student dialogue, making connections with content, and developing deep thinking skills related to content, are critical to the learning process (Carter, 2013; Krownapple et al., 2010; Powell, 2011a). Equally important are the student-to-student interactions through discussion and shared discovery in order to foster critical thinking and provide access to varying points of view (Carter, 2013; Darling-Hammond, 2013; Watanabe, 2012). Schools need to provide settings where students feel safe asking questions and sharing their thoughts in order to allow them to fully engage in their own education (Powell, 2011b; Quaglia et al., 2010).

To close the opportunity gap for all students, educators need to work on eliminating the expectations, relationships, and participation gaps. By doing this, all students, especially the struggling students, will benefit from an equitable educational experience and have opportunities to achieve (Quaglia et al., 2010). Teachers can take steps to reduce the opportunity gap by creating detracked, homogeneous learning
environments so that all students can benefit from instruction from high-quality teachers in rigorous courses.

Part of the expectations gap that affects students in mathematics classes is ability tracking. In most schools, ability grouping for mathematics is a common practice, as many math teachers believe they can teach more effectively by narrowing the range and specificity of instruction for students whose knowledge is at the same level (Boaler, 2006). The labeling of students by ability level (low, regular, and advanced) creates the artificial barriers within the learning environment which feed directly into the expectations gap. Students placed in lower tracks are faced with learning disparities that can affect their academic self-images and their motivation to excel. These struggling students tend to have low self-esteem and believe they are incapable of learning with their realization of their teachers’ low expectations and knowledge that they are not afforded the same opportunities as their peers in advanced classes. Struggling students disengage in the learning process when they become aware of their teachers’ opinions, and this withdrawal further widens the participation gap. This, in turn, leads to these students meeting their teachers’ low expectations, thereby making it nearly impossible to create the solid, trusting relationships that are needed to close the relationship gap between teachers and students (Bernhardt, 2014; Burris & Garrity, 2008; Chmielewski, Dumont, & Trautwein, 2013; Powell, 2011a; Quaglia et al., 2010; Trautwein, Lüdtke, Marsh, Köller, & Baumert, 2006; Watanabe, 2012; Werblow, Urick, & Duesbery, 2013).

Schools where teachers have the discretion to make decisions about the implementation and foci of their mathematics curricula and use specific structures in the
classroom to assess their students’ progress in mastering concepts have met with recent success in detracking mathematics (Horn, 2006). As teachers detrack mathematics classrooms, it becomes important for them to find ways to differentiate their instruction and ensure assessments are focused on demonstrating learning target proficiency and mastery in order to effectively implement equity-based instruction in mathematics (Dueck, 2014; Guskey, 2007; Marzano, 2007; O’Connor, 2011; Stiggins & Chappuis, 2005). A critical aspect of successful detracking is teachers’ honest self-assessment of their beliefs about their students’ ability to learn in order to avoid unconsciously perpetuating the disparities in opportunity they are striving to eliminate within their classrooms (Bernhardt, 2014; Cox, 2011; Harris, 2012; Krownapple et al., 2010; McGee, Wang, & Polly, 2013).

In my local context, I am concerned that many high school students homogeneously tracked into ability-grouped mathematics classes do not learn mathematics in equitable classroom environments or have access to identically rigorous curricula. Also, if students are placed into heterogeneously grouped mathematics classrooms, the curriculum and teaching strategies might not meet the needs of struggling students in their current learning environment. Furthermore, teachers’ preexisting beliefs about struggling students might interfere with their ability to achieve academic success, as best practices and differentiating instruction alone are not sufficient to ensure high levels of achievement for all students (Burris & Garrity, 2008; Burris, Wiley, Welner, & Murphy, 2008; Powell, 2011a, 2011b).

With this concern, during the 2013-2014 academic year, I detracked the geometry courses I teach so that all students taking geometry were enrolled in honors
geometry. The data from the Geometry End Of Course (EOC) exam, a standardized test administered to all geometry students in my state each year, indicate that the detracking process was effective with a 96% passing rate for my geometry students. However, after examining the data more closely, I noticed that students who performed at lower achievement levels were either black males or students who came from families with lower socioeconomic status. These data indicate that even with success for the majority of my students, the achievement, or opportunity gap, is still an issue within my detracked classroom. Moreover, although my advanced students meet the minimum level of proficiency on the mathematics standards for geometry, I felt as though I did not provide opportunities for a deeper understanding and application of their knowledge.

Within my detracked geometry classroom, I want to provide additional supports for my students who struggle to learn mathematics, as well as challenge my students who have advanced understandings of mathematics concepts. Differentiated instruction and standards-based assessment offer promise to achieve this goal. For successful implementation of differentiated instruction and standards-based assessment in the classroom, teachers must create units of study that teach the same high-level rigorous curriculum using differentiated instructional methods to reach all students. These methods should especially enable struggling students to grasp concepts more effectively, motivate them to participate and engage in meaningful dialogue, and keep them from falling behind their peers (Burris & Garrity, 2008; Ferlazzo, 2013; Rollins, 2014). At the same time, teachers should develop instructional materials that push advanced learners to increase and develop their knowledge and skills in areas explicitly
connected to the standards (Chapman & King, 2014). Included in delivering differentiated units of study is the process of using standards-based grading, a student-centered grading process based on student mastery of specific content standards that engages students in learning and makes grades a more accurate reflection of that learning (Miller, 2013; Tierney, Simon, & Charland, 2011; Welsh, D’Agostino, & Kaniskan, 2013). Using differentiated instructional methods to deliver content that builds on students’ mathematics thinking and problem-solving skills and elicits student discussions and explanations of solutions, while holding each student to the same high expectations by using a grading system based on student proficiency of content, increases student achievement in mathematics classrooms (Schoen, Cebulla, Finn, & Fi, 2003). Hence, I wished to incorporate differentiated instruction and standards-based assessment into my teaching in my detracked geometry classroom.

**Purpose and Research Questions**

The purpose of my study was to understand the ways differentiated instruction and standards-based assessment supported the mathematics learning of my struggling students and advanced learners in a detracked honors geometry classroom. Combining these teaching and grading strategies with the creation of a classroom environment where the teacher cares and respects students has the potential to significantly narrow the opportunity gap by shrinking the expectations, relationships, and participation gaps (Quaglia et al., 2010).

As I implemented a differentiated instructional unit using standards-based grading, the research question that guided my study was:

- In what ways do differentiated instruction and standards-based assessments support struggling students and challenge advanced learners in a detracked honors geometry classroom?
Methodology

To gain insight into the use of differentiated instruction and standards-based assessment in my classroom, I employed the methodology of practitioner research to examine and reflect on my practice. Recall that practitioner research, or inquiry, is a methodical and deliberate means of studying one’s own practice and initiating critical discussion based on the breakdown and examination of collected data (Campbell, 2013; Cochran-Smith & Lytle, 1993, 2009; Dana, Thomas, & Boynton, 2011; Dana & Yendol-Hoppey, 2008, 2009; Lytle, 1996). Practitioner inquirers purposefully collect and analyze data to make determinations about their own practice in order to improve their students’ academic success (Campbell, 2013). This type of inquiry is distinguished from the way evidence-based practices are researched in the field, where research design, quality of research, quantity of research, and magnitude of effect of supporting studies are fundamental components in determining the meaningfulness of the study that extends beyond the context of one classroom to numerous classrooms, schools, and communities. Research from studying evidence-based practices support the entire field of education by explaining processes or procedures that have the potential to improve student outcomes across a broad field of settings, whereas practitioner research has a much narrower focus, providing insight to a single or small group of teachers in order to make change within their own personal context (Cook & Cook, 2011; Dana & Yendol-Hoppey, 2014).

I developed and implemented a standards-based differentiated unit of study and accompanying standards-based assessments and chose scoring structures for students in my geometry class that included different levels of instruction so that all students would meet proficiency based on the adopted geometry standards. As I developed and
implemented this unit, I systematically and intentionally collected data to determine in what ways differentiated instruction and standards-based assessment supported the mathematics learning of my struggling students and advanced learners. These data were collected in four ways.

First, I gathered documents and artifacts from each class meeting, including student artifacts showing differentiation and achievement toward mastery/proficiency of learning targets. Second, I wrote down my individual teacher observations during class time with the students in the form of field notes. I also periodically audio-recorded student conversations during class time and transcribed those recordings into my field notes. Next, I audio-recorded students’ reflections to gain insight about their needs while implementing differentiated classroom instruction and activities and standards-based assessments. Finally, as a practitioner researcher, I systematically and intentionally studied my way of work through personal reflection in a journal to better understand and meet the needs of my struggling students and advanced learners.

**Significance of the Study**

During this study, I focused on developing and implementing a differentiated instructional unit and standards-based assessments for my geometry students within my classroom. Practitioner inquiry is a self-reflexive process that allows teachers to provide an insider’s “perspective that makes visible the ways that students and teachers together construct knowledge and curriculum” (Cochran-Smith & Lytle, 1993, p. 43). I intentionally built reflection into my research process that included collecting, analyzing, and using data to make decisions about my practice and my students’ learning. In my classroom, *all* students’ learning has the potential to be increased through this process,
as the changes I made in my practice were based on their needs as determined through reflection on the data I collected.

Since practitioner inquiry allows teachers to disseminate their discoveries by sharing their research-based knowledge with their students, colleagues, administration, community, and profession, the results of this study have the potential to impact other teachers at a local, state, and national level (Campbell, 2013). This study’s results have the potential to influence and improve differentiated instructional teaching and standards-based assessment practices within my department and school as we move towards implementing standards-based grading school-wide within the next two years.

Although there are a plethora of books and guides for educators about differentiating instruction and standards-based assessment and grading, the research on how to implement these practices successfully into high school mathematics classes is limited. Therefore, the results of this study provide insights into how to improve teaching practices to support struggling students’ academic achievement in high school mathematics classes. This study also contributes to research and provides insights into how to narrow the opportunity gap by closing the expectations, relationships, and participation gap (Quaglia et al., 2010).

**Summary and Statement of Collaboration**

Chapter 1 presented how students’ achievement and opportunity connect to their perceptions of teacher expectations, relationships with their teachers, and the chances for participation they encounter in the classroom (Quaglia et al., 2010). I shared how teachers’ differentiating instruction and using standards-based assessments can narrow the opportunity gap for struggling students, as well as increase the complexity of learning to challenge the more advanced learners. I also explained how I examined my
work by creating a differentiated unit of instruction and using standards-based assessments within that unit. Finally, I discussed how the results of this study contribute to improving my own instruction and the possible learning of others on a local, state, and national level.

As I embarked on this study, I was fortunate to have two colleagues who worked on the same grade-level team as I do, share a passion for closing the achievement/opportunity gap, and were fellow cohort members in the professional practice doctoral program for which I completed this dissertation research for my culminating experience in the program. Mickey MacDonald teaches high school biology at my school. As I engaged in my study to investigate the ways differentiated instruction and standards-based assessment supported the mathematics learning of my struggling students and advanced learners in a detracked honors geometry classroom, Mickey was embarking on a parallel study to investigate the ways differentiated instruction and standards-based assessment supported the science learning of her struggling students and advanced learners in a detracked Honors Biology classroom. Melanie Harris teaches music at my school, and she studied the ways music and academic teachers support one another in closing the opportunity gap at our school (Harris, 2015). Hence, as is common practice in the conduct of practitioner inquiry (Dana & Yendol-Hoppey, 2014) as well as in the dissertation work of students obtaining a professional practice doctorate (Browne-Ferrigno & Jensen, 2012), we collaborated and supported one another in the conduct of our independent but related studies. Because our studies were so closely related to one another, they share the same foundational literature base. For this reason, we collaboratively wrote portions of Chapter 2 of this
dissertation, with relevant portions of Chapter 1 for our own independent studies appearing in all of our completed dissertation works.

**Overview of Dissertation**

In order to situate this study, I will review the literature about differentiated instruction and standards-based assessment and examine how they both relate to narrowing the opportunity gap for students in Chapter 2. Chapter 3 details the methodology for this study, including a description of the context, the data collection and participants, and the data analysis of the study. Chapters 4 through 6 present the results of this study. Chapter 4 details my planning of the standards-based differentiated unit, Chapter 5 describes its implementation and the ways data informed my instructional decisions as I was teaching the unit, and Chapter 6 details what I learned from this study based on analysis of my entire data set after I had completed teaching the unit. I conclude this dissertation in Chapter 7 by stating the implications of my learning for my practice, as well as the field of education.
CHAPTER 2
REVIEW OF LITERATURE

The purpose of this study was to determine the ways differentiated instruction and standards-based assessment supported the mathematics learning of my struggling students and advanced learners in a detracked honors geometry classroom. As such, there are several bodies of literature that provide the foundation for this investigation, including literature on the achievement/opportunity gap, tracking and detracking practices, the provision of differentiated instruction, as well as the use of standards-based assessment. In Chapter 2, I provide a review of the literature in each of these areas to situate my study within the field.

Dana (2013) notes the importance of using the literature to inform the study of your own practice as a teacher researcher sharing that “when teachers inquire, their work is situated within a large, rich, preexisting knowledge base that is captured in such things as books, journal articles, newspaper articles, conference papers, and websites” (p. 33). Hence, teacher researchers need to analyze what has been written and published related to their topics of study and draw “relationships between the knowledge and the theory produced by others” and the knowledge teacher researchers are “generating locally from practice” (Dana, 2013, p. 34). To accomplish this goal, I cite several types of literature in Chapter 2, including empirical studies published in reputable peer reviewed journals, conceptual/theoretical pieces published by scholars in the field, and practical pieces published both in practitioner journals and book to guide practitioners in the process of translating research and theory into their practice. I so doing, I demonstrate the ways, as a teacher researcher, I became “well-informed on what current knowledge exists in the field related to (my) topic” (Dana, 2013, p. 34).
Gaps in Educational Opportunity

In the United States, the achievement gap refers to the disparity that exists in academic achievement between minoritized groups, primarily African Americans, Hispanics, and American Indians, and the dominant group, primarily Whites, as measured by standardized test scores (Darling-Hammond, 2013; J. V. Clark, 2013). This gap also exists in the differential achievement of students based on socioeconomic status. Students from low-income families are three times less likely to reach proficiency on standardized tests in math and science than their more wealthy peers are across every grade level (J. V. Clark, 2013). The achievement gap between African American students and White students begins before they enter kindergarten and is maintained, and often increases, as they matriculate through elementary, middle, and high school (Fryer & Levitt, 2004). Unfortunately, the achievement gap is considered to be a direct consequence of the inequalities that exist within our society and is better represented by the term opportunity gap (Boykin & Noguera, 2011; Ladson-Billings, 2007; Milner, 2010).

The Opportunity Gap

The opportunity gap encompasses systemic disparities in teacher quality, curricula, school funding, healthcare, wealth, education, affordable housing, and quality childcare (Boykin & Noguera, 2011; Welner & Carter, 2013). In an interview with Richard Rothstein, Research Associate at the Economic Policy Institute, and Kati Haycock, founder of The Education Trust, Holland (2007) found opposing perspectives on the impact educators can have on the achievement gap because of the larger opportunity gap. Rothstein believed that the opportunity gap is too driven by inequalities within society for schools to significantly impact the achievement gap. Conversely,
Haycock believed that with high quality instruction, rigorous curricula, and continuous support through professional learning, teachers can significantly impact the achievement gap. Rothstein (2006), however, does believe that increasing services such as early child education, summer enrichment programs, community-based health care, and stable housing for low-income and minority students could significantly reduce the achievement gap as such services address many of the gaps that limit opportunity. Rothstein (2006) estimated that such an increase in services would require a $12,500 annual increase in per pupil expenditure to reduce the effect of the opportunity gap in order to eliminate the achievement gap.

**The Expectations, Relationships, and Participation Gaps**

Although Rothstein (2006) questioned the influence that teachers have in reducing the achievement gap because of the larger opportunity gap, other educational researchers believe that schools can effectively address the achievement gap by dividing the opportunity gap into three elements in which educators have some control: the expectations gap, the relationships gap, and the participation gap (Quaglia, Fox, & Corso, 2010). Similar to Haycock’s affirmation that educators can reduce the achievement gap in spite of societal inequalities that students bring with them into school, Quaglia, Fox, and Corso (2010) espoused that educators can reduce the achievement gap by maintaining high expectations for all students, building respectful and caring relationships between adults and students, and engaging all students in the academic and social aspects of school life.

**The expectations gap.** The first gap in which teachers have control is the expectations gap which can be defined from both the teachers’ perspective and the students’ perspective. Teachers’ expectations for and assumptions about their
individual students’ potential to succeed creates the first expectations gap (Harris, 2012; Quaglia et al., 2010). If a student is not enrolled in advanced or college-track courses, teachers might doubt that student’s academic ability and assume he or she is not intelligent, not able to learn, or lacks the capability to meet high academic standards (Harris, 2012; Tyson, 2013). Teachers’ deficit beliefs about their students’ capacity and motivation to learn compromise student access to high-quality educational experiences and rigorous curricula, decreasing their opportunities for learning (Harris, 2012; Powell, 2011a; Quaglia et al., 2010; Tyson, 2013).

Quaglia et al. (2010) describes the second expectations gap as “involv(ing) the difference between students’ expectations of themselves and what they perceive to be teachers’ opinion of their potential” (para. 4). If students suspect that teachers have negative beliefs about their ability to learn, they lack self-motivation to engage in the classroom or take advanced courses (Krownapple et al., 2010; Quaglia et al., 2010). Striving for good grades and passing standardized tests are unimportant to students who think their teachers do not believe in them. Students are less engaged intellectually, emotionally, and behaviorally in school, which tends to further widen the opportunity gap as these children never reach their full potential (Krownapple et al., 2010; Ladson-Billings, 2013; Quaglia et al., 2010; Welner & Carter, 2013).

**The relationships gap.** The second gap, the *relationships gap*, can affect student achievement and refers to the connection between students’ effort in their academic work and the quality of the relationships they have with their teachers (Quaglia et al., 2010). The relationships that students develop with their teachers are crucial, as they can either support or hinder students’ social growth and quality of
learning in the classroom (Powell, 2011a). The ways in which teachers relate and interact with students influence their academic achievement as well as their self-concept and self-esteem (Tutwiler, 2007). Students want to have caring, personal, and authentic relationships with their teachers (Powell, 2011a; Rightmyer, 2011). They want their teachers to treat them as learners, demonstrating respect, understanding, and a commitment to teaching, and showing them they are valued, worthy, accepted, and appreciated (Krownapple et al., 2010; Powell, 2011a; Rightmyer, 2011). Students need to sense that their teachers care about their well-being and have concern about them in and out of the classroom. They want their teachers to refuse to give up on their success and to display a “no excuses” approach to teaching in which they truly believe that their students “can, will, and must succeed” (Bondy et al., 2012, p. 423).

Culturally responsive teachers eliminate deficit thinking and previous beliefs about their students, their families, and their cultures, and thus validate their students as learners (Powell, 2011a; Rightmyer, 2011). By incorporating students’ language, social identities, and cultural resources into their curriculum, teachers connect with their students, and their families, creating learning environments where everyone values human differences (Powell, 2011a, 2011b; Rightmyer, 2011). Deliberately fostering relationships and forming partnerships with students’ families can influence students’ academic success. When teachers increase family participation and allow parents to be part of the decision-making processes, students’ achievement, attitudes, and behaviors have been found to improve, “regardless of the family structure, socioeconomic status, race, level of parent education, size of the family or age of the child” (Seitz, 2011, p. 62). According to Adkins-Coleman (2010), teachers who build solid, trusting relationships
with their students and their families, and also blend academic demands with compassion, are better able to foster their students’ connectedness with school and increase student investment in learning. Such relationship building reduces the relationships gap which may lead to a reduction in the achievement gap (Krownapple et al., 2010; Quaglia et al., 2010).

The participation gap. The final gap within the control of educators is the participation gap. The participation gap describes the disparities in opportunity and advantage between a small number of students who actively engage in classes and school activities and a larger number of students who do not fully participate in the educational process (Quaglia et al., 2010). Learning is a social activity where teachers guide students to make connections and enter into dialogue with each other to support the meaning-making process and to think critically (Carter, 2013; Krownapple et al., 2010; Powell, 2011a). Students need opportunities to interact with each other to increase their academic involvement, encourage critical thinking through participation and conversation, and gain access to various perspectives to increase their learning (Carter, 2013; Darling-Hammond, 2013; Watanabe, 2012). According to Powell (2011b) and Quaglia et al. (2010), schools that created learning environments that fostered meaningful learning opportunities and encouraged students to participate fully in their educational pursuits by striving for goals, asking questions, and taking risks effectively reduced the participation gap.

In addition, in some impoverished cultures, students and their families may believe that schooling does not make a difference in their futures, because they have not experienced any sense of purpose, achievement, optimism, or connectedness to
education. Tileston and Darling (2008) indicated that these beliefs influence how some students from low SES backgrounds avoid participation in the academic life of school, which may result in chronic absenteeism, sleeping in class, or not completing assignments. Additionally, students of color and from poverty, who have experienced repeated failure in school, also tend to avoid participating in class because they do not want to risk sharing incorrect answers and experiencing embarrassment in front of their peers (Rollins, 2014). This behavior results in the inability of teachers to assess their students' content knowledge because of the avoidance of possible public humiliation (Rollins, 2014; Tileston & Darling, 2008).

According to Powell (2011b), some students of color and those living in poverty may limit their own academic success by developing oppositional identities through the avoidance of speaking in class, completing their homework, and getting good grades, because they do not want to be “perceived or interpreted as (acting) White” (p. 40). Teachers’ encouragement and high expectations set in a caring, learning community where all students work together, practice respect, and support each other’s personal goals was shown to collectively increase students’ participation in class (Powell, 2011b).

To offset these behaviors, Ferlazzo (2013) and Tileston and Darling (2008) indicated that teachers who differentiated their instruction and helped their students see the relevance of content to their lives increased student engagement and participation in class. Several scholars also found that teachers who work with students as a team using formative assessments and specific feedback to engage in conversations about learning increase motivation and participation so all students can take more ownership of their learning (Davies, 2007; Guskey, 2007; Stiggins, 2007).
All students deserve fair and equal access to academic opportunities, but the opportunity gap that persists in schools, on both personal and organizational levels, highlights the differences between the systemic advantages some students possess compared to the systemic disadvantages that other students have, resulting in differential learning opportunities for students (Darling-Hammond, 2013; Krownapple et al., 2010; Welner & Carter, 2013). One such organizational structure that limits learning opportunities for students and is also deeply rooted in individual belief systems about teaching and learning is the practice of tracking (Oakes, 2005; Tutwiler, 2007).

**Tracking**

The term tracking refers to the process by which students are grouped into different classes based on a combination of one or more of the following criteria: standardized test scores, perceived ability, work ethic, prior classroom achievement, teacher recommendations, IQ scores, and motivation (Burris & Garrity, 2008; NASSP, 2006; Oakes, 2005; Tyson, 2013). Students are assigned either to different courses with more or less rigorous curricula or to different levels of the same course within a grade level (Burris & Garrity, 2008; Heubert & Hauser, 1999). Tracking is different from ability grouping, with the former occurring in high schools and some middle schools and the latter occurring within elementary schools (Loveless, 2013). In elementary schools, students are typically clustered into ability groups for reading and math instruction within the same class (Loveless, 1998; Loveless, 2013) while in secondary schools students are typically tracked into different courses based on perceived and/or real differences in prior learning and/or ability (Burris & Garrity, 2008; Loveless, 1998; Loveless, 2013). According to the 2013 Brown Center Report on American Education: How Well are American Students Learning, Loveless (2013) reported that tracking has increased over
the past two decades, particularly in math at the 8th grade level, and continues to persist in all academic subjects in high schools following a decrease in the mid-1990s which coincided with a national debate on the merits of tracking. Although tracking debates have fluctuated in recent history, the practice of tracking in high schools has its roots in changes that were taking place in America at the turn of the 20th century.

**Birth of Tracking**

Most educators, parents, and students accept tracking as an integral component of the culture of American schooling (Ansalone & Biafora, 2010; Burris & Garrity, 2008) never questioning its origins or its merit. School tracking was set in place to address significant changes that were occurring near the end of the 19th century in America. Oakes (2005) and Rury (2005) detailed three significant changes during this historical time period that precipitated the creation of the comprehensive high school replete with differentiated tracks.

First, Oakes (2005) described how America was flooded with poor, uneducated immigrants who needed to be “Americanized” at the turn of the 20th century. Education was viewed as an appropriate way to assimilate large immigrant populations into American ways of acting and thinking, ways in which middle- and upper-class northern, White, European Americans acted and thought.

Second, Jeynes (2011) pointed out that scientific Darwinism led to social Darwinism, a belief that evolution should be used to improve society and when applied to schooling, only the “fittest”, most intellectual individuals should be educated. Such a belief about who was fit and who was unfit for education provided scientific evidence for the dominant group (northern, White, European Americans) to attribute class differences to differences in innate intelligence which from the dominant group’s view
was fixed and unchanging (Oakes, 2005). Thus, to serve the dominant group, schooling needed at least two tracks: one that prepared middle- and upper-class students for college and one that taught low-class immigrants the skills needed to work in factories (Loveless, 1998; Oakes, 2005; Rury, 2005).

The final change that impacted the birth of tracking in secondary schools in the early 20th century was the factory model of mass production which had revolutionized industry and was then applied to schooling (Oakes, 2005; Rury, 2005). The assembly line model of learning was viewed as a cost-effective and efficient way to mass produce educated adults prepared for either movement into a factory job or into advanced studies (Rury, 2005). This model led to the development of the college preparatory track and the vocational track in the comprehensive high schools established in the early to mid-1900s, a model of education that has changed little to this day (Mirel, 2006; Rury, 2005). Through differential tracking, the social and cultural stratification that exists in society and is currently maintained in schools (Oakes, 2005) was born out of a response to the social and historical changes that faced America during the late 1800s and early 1900s. Many assumptions about tracking by educators and parents help to sustain its prevalent use in high schools (Oakes, 2005).

Assumptions that Support Tracking

Despite the predominance of literature which questions the efficacy of tracking (Burris et al., 2008; Oakes, 2005; Watanabe, 2012), this practice has increased over the past two decades (Loveless, 2013). Because academic tracking is so ingrained in the ways in which schools operate, particularly high schools, many people view tracking as natural, never questioning its merit, and even going so far as to tout the benefits of tracking based on assumptions which lack a research basis. Oakes (2005) argued
against several of the assumptions that supporters of tracking believe are reasons to maintain such a system.

**Homogeneous grouping is better for students.** One such assumption that educators who support tracking hold is homogeneous grouping is academically favorable for all students (Oakes, 2005). Educators demonstrate this assumption in their beliefs about the learning environment best-suited for high-achieving and low achieving students. For both groups of students, the assumption is that mixed ability grouping hinders their achievement. Burris (2006) found that heterogeneous grouping in middle school mathematics actually increased the likelihood of all student groups taking advanced math classes, improved achievement scores for low-SES and minority students, and showed no statistically significant differences in achievement between high-achieving (gifted) students enrolled in homogeneous math classes compared to these same students enrolled in heterogeneous math classes. This finding is especially important since lower math achievement limits opportunities for students as early as third grade (Stinson, 2004) and becomes a gatekeeper for college (Spielhagen, 2006; Watanabe, 2012). In a meta-analysis of research on the effects of over four decades of detracking, Rui (2009) also found that detracking significantly improves the achievement of traditionally low-tracked students while having no effect on the achievement of traditionally high-tracked students. Conversely, the effect of academic tracking on low track students leads to low self-esteem (Van Houtte, Demanet, & Stephens, 2012), widening of the achievement gap (J. V. Clark, 2013), and an increase in dropout rates (Werblow, Urick, & Duesbury, 2013).
**Process for placement into tracks.** A second assumption that many educators and parents alike believe about tracking is that placement into academic tracks is based on real differences in student ability and achievement and the processes for such placements are appropriate and fair. Three types of criteria are used for such placements: standardized test scores, teacher recommendations, and student and parent choice with the first two criteria encountered in most high school track placements (Oakes, 2005).

Standardized test scores and teacher/counselor recommendations add to the meritocratic legitimacy of such placements implying that students have ‘earned’ their track placement and “deserve to be there” (Oakes, 2005, p. 9). However, these placements are predicated on results of standardized tests that include questions intentionally designed to sort students (Dalal & Gunderman, 2011; Oakes, 2005) and often include cultural bias (Grodsky, Warren, & Felts, 2014). Questions that cannot show differences in students’ ability are eliminated from the test even when they measure student mastery at a particular grade level (Oakes, 2005). Additionally, teacher recommendations are often based on student work ethic and student behaviors rather than student ability (Demanet & Van Houtte, 2012; Van Houtte, Demanet, & Stevens, 2013).

When students are not placed in advanced academic tracks, parents of students who have higher socioeconomic status use their networking resources to negotiate changes in track placement for their child (Werum, Davis, & Cheng, 2011). Sil (2007) affirmed that “…fixed policies [placement within academic tracks] become negotiable when advantaged parents lobby for better placement for their children” (p. 122).
Students whose parents do not have such networking resources have little recourse for questioning such placement (Sil, 2007).

Collectively, the use of standardized test results from tests which do not necessarily measure grade level mastery, teacher recommendations which include factors not associated with ability, and parental negotiations for better track placements of students from higher socioeconomic status contribute to the inequitable placement of White students into advanced tracks and minority students into general tracks.

**Ease of teaching homogeneous groups.** A third assumption that supporters of tracking offer is that teaching is easier when students are homogenously grouped. Grant (2011) described the ease with which high school teachers who only teach honors level and Advanced Placement (AP) students find teaching. Worthy (2010) provided an alternative view of teaching for teachers assigned a group of low-track students. Unlike the homogeneously grouped honors and AP classes, homogeneously grouped low-track classes are the most difficult to teach and hardest to manage (Worthy, 2010).

These three assumptions, used as reasons why educators and parents alike support the continued practice of academic tracking, have masked a century’s worth of social reproduction that continues to pervade our schools (Oakes, 2005; Tyson, 2013).

**Problems Inherent with Tracking**

The inequitable placement of students into tracks is considered a form of institutional racism (Sensoy & DiAngelo, 2012) that serves to reproduce the social and cultural stratification that exists in society (Biafora & Ansalone, 2008; Burris, 2014; Chmielewski, 2014; Fiel, 2013; Oakes, 2005; Tyson, 2013). Much literature questions the efficacy of tracking practices for both high-achieving and low-achieving students.
alike. Such practices result in three significant issues related to tracking: differences in teacher quality and instruction between higher academic and lower academic tracks (Burris et al., 2008; Darling-Hammond, 2013; Tienken & Zhao, 2013; Tyson, 2013; Watanabe, 2012; Welner & Carter, 2013); the perpetuation of racial and class-based inequalities (Carter, 2013; Darling-Hammond, 2013; Powell, 2011b; Powers, 2011; Rothstein, 2013; Tyson, 2013; Watanabe, 2012), and the questionable nature with which students are assigned to particular tracks (Carter, 2013; Tienken & Zhao, 2013; Tyson, 2013; Watanabe, 2012).

**Teacher and instructional quality.** The first issue resulting from tracking involves differences in teacher and instructional quality. Research indicated that students placed in lower-tracked classes are not provided the same quality of instruction as students placed in advanced classes (Darling-Hammond, 2013; Tienken & Zhao, 2013; Tyson, 2013; Watanabe, 2012; Welner & Carter, 2013). One of the most significant factors related to improving student achievement is teacher quality (USDOE, 2002). Teacher quality and subsequently instructional quality have been consistently identified as the most important school-based factors affecting student achievement (Darling-Hammond, 1999; Rockoff, 2004). Since measuring teacher and instructional quality is so complex, Kennedy (2008) argued that instead of using the term ‘teacher quality,’ educators should explore ‘teacher qualities’ when examining the multiple factors that teachers possess that impact student achievement.

Teacher qualities can be grouped broadly into three categories: personal resources, performance qualities, and teacher effectiveness (Kennedy, 2008). Personal resources are qualities that teachers have prior to beginning teaching such as
credentials, subject-area and cultural knowledge, and beliefs about teaching and learning. Performance qualities relate to the day-to-day work that happens within the classroom such as classroom management, lesson preparation, collaboration with colleagues, and communication with parents and students. Teacher effectiveness is directly related to the impact that a teacher has on student achievement. Research on the effects of tracking indicated that students in lower-tracked classes have less-experienced teachers who do not have the qualifications to teach rigorous curriculum that challenges their students to think critically (Berry, 2013; Burris & Garrity, 2008; Carter & Welner, 2013; Heubert & Hauser, 1999; Watanabe, 2012). Studies showed that teachers’ experience, certification status, preparation for teaching, and academic background all contribute significantly to their students’ academic achievement, because teachers with more expertise and experience have the ability to make good curriculum decisions and bring more continuity to their instructional practice (Carter &Welner, 2013; Darling-Hammond, 2010, 2013). Researchers found a trend in the way school administrators assign more experienced teachers to higher academic tracks as a way of rewarding those teachers (Kalogrides, Loeb, & Béteille, 2013). Granting the instruction of advanced courses as a reward for powerful, successful, more experienced teachers, and assigning remedial classes as a punishment to teachers who are weak, undeserving, or have less experience and training, effectively creates a hierarchy of teaching assignments that further disenfranchises students in lower academic tracks who are predominately students of color and students who come from high poverty (Burris & Garrity, 2008; Carter, 2013; Darling-Hammond, 2010, 2013; Heubert & Hauser, 1999; Kalogrides & Loeb, 2013; Tyson, 2013).
Race and class-based inequality. Minority students and students living in poverty tend to score lower on standardized tests than White students. Students of color and/or low socioeconomic status (SES) are predominantly tracked into low-level classes where failure rates tend to be higher and student motivation is likely to be lower (Heubert & Hauser, 1999; Tyson, 2013). In fact, research indicated that in racially diverse schools, honors or advanced courses are likely to have twice as many White and Asian students than the courses designed for low-ability and non-college-bound students that are disproportionately filled with students of color or from a low socioeconomic status family (Burris & Garrity, 2008; Carter, 2013; Darling-Hammond, 2010, 2013; Heubert & Hauser, 1999; Tyson, 2013). In these racially mixed schools, White students are more likely to be placed in honors or advanced courses before their equally capable Black counterparts, resulting in de facto segregation by tracking (Darling-Hammond, 2010; Dixson & Rousseau, 2005; Tyson, 2013).

Some schools claim to have increased the number of minority students in courses with more rigorous curricula after the Nation at Risk report was published in 1983, but the gains in the proportion of these students taking courses that are more rigorous are much greater than what is reflected in their standardized test scores (Barton & Coley, 2010). The National Assessment of Educational Progress (NAEP) studied high school graduates’ transcripts from 1990 to 2005 to determine why the large increases in the proportion of high school students taking more advanced courses did not result in higher NAEP scores. Barton and Coley (2010) discussed how the analysis of these data led to wonderings about whether schools really increased their curricula’s rigor or possibly just changed the name of the courses taken by minority students.
Another wondering was whether the gaps between students’ enrollment and test scores were the result of unprepared teachers teaching the advanced courses for these groups of students. Relabeling courses and failing to provide equitable resources for all students does not give every child a fair chance at being successful and inevitably separates students by characteristics that correlate statistically with race, ethnicity, and class (Barton & Coley, 2010; Loveless, 2013; Welner & Carter, 2013). In fact, schools that continue to separate their students yet insist they allocate equal curricular materials and competent teachers “[fail] to address the problem that tracking and ability grouping constitute(s) not merely differentiation but stratification – that is, unequal distribution of status” (Heubert & Hauser, 1999, pp. 102-103).

The 2013 Brown Center Report on American Education shows that there has been an increase in tracking since the detracking debate made national headlines between 1994 and 1998, and it again has become a commonplace activity in many schools (Loveless, 2013). A reason for the resurgence of tracking in schools is attributed to the accountability systems put in place in many states that was then reinforced by the accountability provisions of the No Child Left Behind Act of 2002. Because educators must focus their attention on students showing proficiency on state standards by passing standardized achievement tests, some educators feel that grouping students who are struggling into homogeneous remedial classes is justified. Therefore, tracking and ability grouping may continue to thrive due to educators’ fear of meeting adequate yearly progress (AYP) perpetually rising (Loveless, 2013).

**Assignment to tracks.** A significant reason that tracking persists in schools is that many teachers believe they can teach homogeneous groups more effectively,
narrowing the range and specificity of instruction for students whose knowledge is at the same level (Boaler, 2006; Oakes, 2005). Moreover according to Loveless (2013), many teachers believe that teaching heterogeneous classes is problematic, because their students’ learning ability is so diverse. Unfortunately, the inequities that result in course tracking can negatively affect the academic self-concept and achievement of students placed in the lower tracks (Chmielewski et al., 2013; Trautwein, Lüdtke, Marsh, Köller, & Baumert, 2006). Teachers label their students as “low” kids, “regular” kids, and “advanced” kids, indicating their biases about students through these labels (Burris & Garrity, 2008). This belief system sustains tracking practices in schools and limits learning opportunities for struggling students. When these struggling students, separated from the elite group enrolled in advanced courses, realize that their teachers have low expectations of them, it results in them having low self-esteem, low expectations, and identifying themselves as incapable of learning, thereby increasing their chances to disengage from the learning process (Bernhardt, 2014; Burris & Garrity, 2008; Carter, 2013; Darling-Hammond, 2013; Powell, 2011a; Watanabe, 2012; Werblow, Urick, & Duesbery, 2013) and increasing non-academic behaviors (Demanet & Van Houtte, 2012; Van Houtte, Demanet, & Stevens, 2013).

Furthermore, the literature reveals that some teachers stereotype their low-income students and students of color as automatically being low achievers, because they do not conform to ‘typical’ behaviors or norms that educators define for academic success (Carter, 2013; Demanet & Van Houtte, 2012; Van Houtte, Demanet, & Stevens, 2013). Bernhardt (2014), Givens (2007) and Yosso (2005) all indicated that even though teachers may say they treat all students the same way, it is not always true,
because adults bring their own experiences and opinions about cultural and social capital to the classroom. Teachers may believe that White students have more value than Black students, because they view White students as hard-working, coming from more influential families, and possessing cultural and social class differences that influence their achievement (Carter, 2013; Rothstein, 2004, 2013; Rousseau & Dixson, 2006; Tyson, 2013). Placing students into tracks using cultural definitions of intelligence and standards of behavior systematically disadvantages students of color, causing them shame and humiliation, and reinforces the inequities between Whites and Blacks (Dixson & Rousseau, 2005; Ladson-Billings, 2006; Rousseau & Dixson, 2006; Sensoy & DiAngelo, 2012; Welner & Carter, 2013). According to several scholars, this kind of deficit thinking maintained the disparities between Black and White students, the hegemony of the privileged White community, and social and racial injustice within our educational institutions (Carter, 2013; Hinchey, 2010; Rousseau & Dixson, 2006; Yosso, 2005).

By the time students are in high school, students have institutional constraints that limit what classes they take, including prior placement, course scheduling, prerequisites, and sequencing (Loveless, 2013; Tyson, 2013). Students are grouped by ability in elementary school, as early as Kindergarten or first grade, and students in these homogeneous groups eventually become classmates in tracked courses in middle and high school (Darling-Hammond, 2010; Loveless, 2013). School policies such as prerequisites and sequencing impede students from switching from the lower to higher-tracked classes, making it almost impossible for students who are ‘late bloomers’ to ever take advanced courses as a senior if they started in remedial classes as a
freshman (Tyson, 2013; Watanabe, 2012). Teachers and counselors assume that low-track students will not be able to get into college and reinforce struggling students’ lack of choice in courses by recommending they look at their futures ‘realistically’, thereby negatively influencing students’ beliefs about success and increasing the expectations gap with their lack of support and stereotypical assumptions (Darling-Hammond, 2010; Quaglia et al., 2010).

In addition, several scholars have noted that parental intervention tends to widen the expectations gap by preserving the differentiation of curriculum in schools between White, middle-class students and poor students or students of color, as administrators and counselors are influenced by the greater clout of predominantly White parents from a higher socioeconomic status (Burris & Garrity, 2008; Darling-Hammond, 2010; Heubert & Hauser, 1999). Parents of advanced or gifted students believe that their children’s education will be compromised if they are not provided challenging curriculum in separate, homogeneous, high-tracked classrooms. These opinions influence schools to continue using tracking as a social stratification system (Burris & Garrity, 2008; Darling-Hammond, 2010; Heubert & Hauser, 1999; Tyson, 2013).

**Elimination of Tracking through Differentiation and Standards-Based Classrooms**

One possible way to begin to dismantle the social stratification that exists in high schools is to eliminate tracking (Burris & Garrity, 2008; Oakes, 2005). Simply eliminating tracking by randomly grouping all students into heterogeneous classrooms, however, does not solve the problems that are inherent when tracking is in place. The elimination of tracking, also called detracking, does not guarantee that all students will receive challenging curricula, the best teachers, or high quality instruction, nor does detracking guarantee that racial and class inequalities will be reduced. Moving to a detracked
classroom does, however, guarantee an increase in the diversity of learners within a classroom (Burris & Garrity, 2008; Watanabe, 2012).

To address this increased diversity, educators consider how to meet the learning needs of two groups of students in their mixed-ability classrooms: students who have a history of struggling in school and students who have consistently been very successful academically. Schools that have successfully eliminated tracking have embedded practices that support learners who traditionally struggle in school, particularly students of color living in poverty, while continuing to challenge advanced learners by providing the curriculum that has been customarily reserved for the highest tracked students as the curriculum that all students receive (Burris & Garrity, 2008; Oakes, 2005; Watanabe, 2012).

**Best Practices in Schools that have Eliminated Tracking**

Examples of schools that have successfully eliminated tracking include the Preuss School in San Diego, California (Tyson, 2013; Watanabe, 2012) and South Side High School in Long Island, New York (Burris & Garrity, 2008; Oakes, 2005). Tyson (2013) described The Preuss School, a charter school associated with the University of California, San Diego, as a school in which all students are enrolled in the one academic track the school offers, which is the college preparatory track. The school serves 800 low-income students in grades 6-12 who come from families whose parents have not attended college. The racial composition of Preuss School is 59% Hispanic, 23% Asian, and 12% African American. Ranked the top, transformative high school in America for three consecutive years by *Newsweek* (C. Clark, 2013), students who struggle academically are identified early and the school establishes support structures so they can successfully meet the rigorous college preparatory curriculum (Alvarez &
Mehan, 2006). Such structures include tutoring after-school, on Saturdays, and within advisory classes that reinforce and remediate content understanding. Additionally, teachers receive continuous professional learning in research-based practices on how students learn and how to teach for understanding. Because of the early identification of struggling learners followed by intensive interventions and ongoing professional learning of faculty, Alvarez and Mehan (2006) noted that the numbers of students who are at risk academically decreased as students matriculate from 6th grade to 12th grade.

Carol Corbett Burris (2014), Principal of South Side High School in the Rockville Center School District in Long Island, New York, shared her school’s story into detracking, with an emphasis on the changes that occurred in South Side High School beginning in the latter half of the 1990s. Prior to the elimination of tracking, only 32% of African American and Hispanic students earned a Regent’s diploma, while 88% of White and Asian students earned this diploma. During the nine year time period in which tracking was eliminated at South Side High School, graduating with a Regent’s diploma designation indicated that a student was prepared for a four-year post-secondary institution. Nine years into the detracking reform, 95% of African American and Hispanic students earned a Regents diploma and 100% of White and Asian students earned this diploma. The elimination of tracking has impacted Regent’s diploma graduation rates, and has nearly eradicated the achievement gap among the two groups of students at this high school. Similarly to The Preuss School, Burris (2014) and Tyson (2013) attribute the achievement success of South Side High School students to the equal learning opportunities and resources that all students receive as
well as the support structures that are in place for students who struggle to meet the demands of the more rigorous academic expectations.

When tracking is eliminated in schools such as Pruess School in California and Southside High School in New York, the diversity in initial achievement levels of students can be higher than in homogeneously-grouped classrooms (Staples, 2008). Student background knowledge within a mixed-ability classroom substantially increases as does cultural diversity, especially in schools with diverse student populations. One way that teachers can meet the diverse learning needs of all students in mixed-ability classrooms is through differentiation (Tomlinson, 2014).

**Differentiated Instruction**

The literature reveals that when teachers differentiate their instruction to meet the needs of all students in their classrooms and focus on creating standards-based assessments for students to demonstrate mastery and proficiency of learning targets, they can effectively eliminate tracking and meet the learning needs of diverse learners (Dueck, 2014; Guskey, 2007; Marzano, 2007; O’Connor, 2007, 2011; Stiggins & Chappuis, 2005). According to Tomlinson (1999, 2000, 2014), teachers in heterogeneous classrooms created units of study that taught the same rigorous curriculum using differentiated instructional methods to reach all students with varied skills and abilities. Additionally, teachers provided high-quality curriculum and instruction that maximized their students’ capacity and potential for learning, especially enabling struggling students to grasp concepts more effectively, motivating them to participate and engage in meaningful dialogue, and keeping them from falling behind their peers (Burris & Garrity, 2008; Morgan, 2014; Tomlinson, 1999, 2000, 2008). At the same time, teachers challenged their advanced students to explore and push beyond only
mastering the minimum content standards (Burris & Garrity, 2008; Ferlazzo, 2013; Morgan, 2014; Rollins, 2014; Tomlinson, 1999, 2000, 2008).

Teachers use differentiated instruction to adjust the content, process, or product of teaching and/or learning to maximize their students' ability to learn and apply knowledge. These processes ensure that what students learn, how they learn it, and how they demonstrate mastery of that knowledge matches their individual readiness levels, interests, and preferred modes of learning (Anderson, 2007; Lewis & Batts, 2005; Rock, Gregg, Ellis & Gable, 2008; Tomlinson, 2004; Watts-Taffe, Laster, Broach, Marinak, McDonald Connor & Walker-Dalhouse, 2012; Wormeli, 2005). A common misconception about differentiation is that it is synonymous with individualized instruction. This misconception is misleading. Tomlinson (2014) established that differentiated instruction does not mean that within a unit of study a classroom of students will be engaged in twenty-five different activities, one activity tailored to the needs of each student. Rather, within a unit of study, the teacher provides a modification in the curriculum only when there is a student need and only if such a modification will likely result in an increase in meeting the learning goal or skill that cannot be achieved through non-differentiated instruction. Such differentiation occurs either through the content in which students engage, the process through which students engage with content around a learning goal, or the product that the student produces to show his or her understanding of the learning goal (Anderson, 2007; Lewis & Batts, 2005; Rock et al., 2008; Tomlinson, 2014).

**Differentiating the Content**

Teachers may differentiate the content of their lessons by adjusting what they plan to teach or altering how their students acquire the required knowledge,
understanding, and skills (Anderson, 2007; Lewis & Batts, 2005). It is important for
teachers to familiarize themselves with their content standards or prescribed curriculum
and identify the key concepts or big ideas for which their students must demonstrate
mastery. Using clear statements of intended learning allows teachers to identify the
concepts that need to be taught in depth so that students have time to make sense of
the important ideas and master critical content (McTighe & Brown, 2005; Rock et al.,

By identifying learning targets, teachers avoid varying the objectives or
performance expectations that serve as a foundation for all of their students, and they
can clearly define the outcomes students should accomplish, which provide coherence
to their learning experiences (Anderson, 2007; McTighe & Brown, 2005; Rock et al.,
2008; Watts-Taffe et al., 2012). At the same time, teachers can respond to the diverse
needs of both their struggling and advanced learners by differentiating the depth of
content that students explore related to the identified learning targets (McTighe &
Brown, 2005).

**Differentiating the Process**

Teachers may also differentiate the process of teaching and learning by shifting
from content-centered classrooms to student-centered classrooms. Teachers analyze
their students’ varying methods for understanding and assimilating facts, concepts, or
skills, and they use this information to plan the activities they use in their classrooms
based on their students’ readiness to learn, interests, preferences, strengths, and needs
(Anderson, 2007; Lewis & Batts, 2005; Rock et al., 2008; Watts-Taffe et al., 2012).
examining the spread and distribution of their students, teachers could determine what
materials and tasks would challenge each student, making connections between lessons and students’ lives, and using differentiated instruction to find a balance between instruction, remediation, and enrichment for their students.

In addition, teachers can vary the processes by which students learn by allowing students to work at different paces, giving extra time to struggling students or giving advanced students options for working ahead and exploring topics related to the content standards being taught (Lawrence-Brown, 2004; Rock et al., 2008). Modifying the ways in which students work together is an important aspect of differentiated instruction, as students have more opportunities for success if given choices to work alone, in pairs, or collaborative small groups (Anderson, 2007; Lewis & Batts, 2005; Rock et al., 2008). Teachers also have the ability to tailor instruction and provide support through modeling or coaching for small groups or individuals when students have access to work in learning environments that are conducive to their unique learning preferences (Anderson, 2007; Lewis & Batts, 2005; Morgan, 2014; Rock et al., 2008; Tomlinson, 1999, 2000, 2008). Teachers can scaffold their lessons and provide additional support or enrichment activities using technology as a tool for differentiating instruction (Lawrence-Brown, 2004; Morgan, 2014; Tomlinson, 2000; Watts-Taffe et al., 2012).

**Differentiating the Product**

Because assessment is an essential part of differentiated instruction, teachers must decide in what ways their students will demonstrate mastery of the content standards in their classrooms. As part of their planning routine, teachers should use effective assessments to drive their instruction and provide learning targets for their students to reach (Anderson, 2007; Rock et al., 2008; Stiggins et al., 2006; Wormeli, 2006). Effective informal and formal assessments are valid, reliable, and instructionally
useful in evaluating what students know, what students do not know, and helping teachers and students make action plans for future learning (Stiggins et al., 2006; Watts-Taffe et al., 2012). Teachers can use formative assessments and student self-assessments during instruction to evaluate students’ understanding of the content and adapt or modify their lessons or activities based on students’ needs or instructional situations (Parsons, Dodman & Cohen Burrowbridge, 2013; Rock et al., 2008; Stiggins et al., 2006; Tomlinson, 2008; Watts-Taffe et al., 2012).

Using a variety of formative and student self-assessments gives teachers and students opportunities to examine the students’ learning needs, as they may vary from lesson to lesson, even for individual students (Lawrence-Brown, 2004; Stiggins et al., 2006). Lawrence-Brown (2004) and Lewis and Batts (2005) suggest that teachers should avoid creating fixed groups with which to work, especially if they are based on ability, as doing so might result in creating tracking within their classrooms and creating problems associated with having lower expectations for struggling students.

Furthermore, Anderson (2007) stated that teachers can also differentiate the product of teaching and learning by allowing students to select from a variety of formats to demonstrate what they know at the end of a unit of instruction. When students have choices about how they can provide evidence of their learning in ways that represent their unique learning preferences, interests, and strengths, they are more engaged in the learning process, they have greater self-efficacy, and they take ownership of their learning (Anderson, 2007; Tomlinson, 2008). Teachers can measure students’ in-depth knowledge using different types of summative assessments such as specified rubrics, tests, projects, portfolios, independent learning contracts or studies, or other
standardized measures of achievement (Lewis & Batts, 2005; Rock et al., 2008; Stiggins et al., 2006; Tomlinson, 2008; Watts-Taffe et al., 2012).

**Standards-Based Grading**

The purpose of standards-based grading is to “compare student performance to established levels of proficiency in knowledge, understanding, and skills” (McMillan, 2009, p. 108) using a system that is based on specific learning targets and performance standards known to all teachers, students, parents, and other stakeholders (O’Connor, 2007; Tierney, Simon, & Charland, 2011). Standards-based grading differs from traditional, or norm-referenced, grading, which is the practice of summarizing student performance on various assignments by calculating the mean of scores and reporting that average as a single, cumulative number or letter grade used to compare and rank the students to each other (Guskey & Jung, 2013; McMillan, 2009; O’Connor, 2011; Reeves, 2008; Tierney et al., 2011). By connecting student performance only to academic achievement based on learning expectations and standards and excluding nonacademic factors, such as effort or participation, students’ grades are a more accurate reflection of their learning and are less likely to be inconsistent, unfair, invalid, or unreliable (Miller, 2013; Reeves, 2006; Tierney et al., 2011; Welsh, D’Agostino, & Kaniskan, 2013).

The intention of standards-based grading is to provide information about students’ academic achievement. Therefore, the grades should be a reflection of students’ mastery of the curriculum objectives and learning standards for that course (Tierney et al., 2011). Guskey and Bailey (2001) and Stiggins et al. (2006) suggested that educators should first identify the major learning targets or standards that they expect their students to learn in their courses. Most teachers use their state’s
performance standards to identify their prescribed subject-specific goals (Harris, 2012). When teachers know the goals and objectives on which they will assess their students, they use them as a guide when planning differentiated units of study that help their students explore the concepts more deeply (Harris, 2012; Welsh et al., 2013). Guskey and Jung (2013) and McMillan (2009) stressed that the standards that drive teachers’ instruction should describe the specific knowledge, skills, abilities, and dispositions that they desire their students to know and demonstrate through interactions and instructional activities in classroom learning environments with their teachers and peers.

While planning for instruction based on specific standards for their students, teachers define the graduated levels of performance on which their students are assessed for each learning target (Guskey & Bailey, 2001; Welsh et al., 2013). These levels of performance are often illustrated and explained to students in the form of rubrics (McMillan, 2009; Reeves, 2011). These rubrics serve as a means for teachers to provide specific feedback to their students about their progress and achievement toward mastering the established learning goals and standards and to guide collaborative conversations between teachers, students, and parents for improving students’ learning (Guskey & Jung, 2013; Miller, 2013; Reeves, 2011).

Teacher feedback influences their students’ mastery of the learning targets when it is communicated to their students often and in numerous ways, both orally and in written format, and specifically identifies areas where further learning and improvement is needed (McMillan, 2009; Reeves, 2011). Research indicated that when teachers provide students with timely feedback that evaluated their performance and provided suggestions for improvement, students became more intrinsically motivated to succeed,
had a deeper understanding of the material they were learning, and demonstrated stronger self-efficacy (McMillan, 2009). As students self-evaluate and identify the areas in which they succeed and struggle, they develop a sense of ownership and responsibility toward learning (Guskey & Jung, 2013; McMillan, 2009).

**Summary and Conclusions**

In Chapter 2, I provided an overview of four bodies of literature that provide the foundation for this study: (1) literature on the achievement/opportunity gap, (2) literature on tracking/detracking, (3) literature on differentiation, and (4) literature on standards-based grading. These four bodies of literature converge to situate my study in the field of education. In Chapter 3 of this dissertation, I describe the methodology I used to investigate the ways differentiated instruction and standards-based assessment supported the mathematics learning of my struggling students and advanced learners in a detracked honors geometry classroom.
CHAPTER 3
METHODOLOGY

In order to design and teach a unit of instruction using differentiated instruction and standards-based assessment to support my honors geometry students’ mastery of mathematics content standards, I engaged in practitioner research to better understand and meet the needs of my struggling and advanced students. Teacher inquiry is a systematic and intentional research approach used to study my own practice and foster intellectual professional discourse with my colleagues through the collection, analysis, and interpretation of data (Campbell, 2013; Cochran-Smith & Lytle, 1993, 2009; Dana & Yendol-Hoppey, 2008, 2009; Lytle, 1996). As a practitioner inquirer, I intentionally collected and used data to make decisions about my practice to increase my students’ learning and mastery of mathematics content (Campbell, 2013). This study documented my teaching decisions and actions, as well as my students’ outcomes, that occurred through my work as a practitioner researcher.

In Chapter 3, I define practitioner research as the method for my study. I also provide information regarding the context of the study, introduce the students I focused on throughout the study of my own practice (my participants), and explain the data collection and analysis methods employed. Finally, I describe my background, perspectives, and trustworthiness as a researcher.

Practitioner Research

Reflective teachers strive to learn about their own instruction in order to improve and become better practitioners (Herr & Anderson, 2005), and then, through those reflections, increase their own professional growth and make changes to improve their own teaching and their students’ learning in their classrooms and schools (Dana &
Yendol-Hoppey, 2009). According to Cochran-Smith and Lytle (2009), being a practitioner researcher allows a teacher to take on a “duality of roles,” (p. 41) enabling her to simultaneously be a classroom instructor while also being a researcher participating in the inquiry process from the inside. While engaged in practitioner inquiry, teachers record details of their students’ learning and their own classroom practice at the same time they “systematically document from the inside perspective their own questions, interpretive frameworks, changes in views over time, dilemmas, and recurring themes” (Cochran-Smith & Lytle, 2009, p. 44). The accumulation of teachers’ data collected during cycles of inquiry, when shared with their colleagues in their schools, local communities, state institutions, or to national audiences, have the ability to influence local and public school policies and reform, increasing the individual teacher’s scope of influence beyond her classroom walls (Cochran-Smith & Lytle, 1993; Herr & Anderson, 2005). I chose practitioner research as the methodology for my study because it allowed me to examine my own practice within the context of my classroom. Practitioner research enabled me to concurrently improve my practice, increase my knowledge of differentiating classroom instruction and using standards-based assessments within my detracked honors geometry classroom, and share my findings with other teachers locally and publicly.

The state of Florida adopted the Mathematics Florida Standards (MAFS) in March, 2014, which were adapted from the Common Core State Standards for Mathematics (Common Core State Standards Initiative, n.d.), and subsequent changes were made to the Geometry End-of-Course (EOC) Test (Florida Department of Education, n.d.). Due to these changes, geometry teachers are attempting to find
effective ways to help their students learn the required content. Because the changes made to the state’s mathematics standards are so recent, there is very little research related to the successful implementation of instructional practices or student learning to guide teachers in helping their students master the material. Using practitioner research as a structure to examine their work within the classroom has the potential to help teachers understand the ways in which differentiated instruction and standards-based assessment can improve their students’ understanding of these standards. Practitioner research allowed me to intentionally observe, examine, and question, through an organized systematic data collection process, how changes to my classroom instruction and learning activities in my classroom increased my knowledge and understanding regarding my practice and students’ learning, especially for my struggling students and my advanced learners (Lassonde, Ritchie, & Fox, 2008). By engaging in practitioner inquiry, the evidence from the data collected during the study allows me to make informed and systematic decisions about my future practice and advocate for instructional changes to a larger audience (Dana & Yendol-Hoppey, 2009).

**Context for Study**

I teach at P. K. Yonge Developmental Research School, a K-12 laboratory school associated with the University of Florida. P. K. Yonge is designed as a special school district under Florida Department of Education funding. Established in 1934, the school serves 1150 students, whose demographic population approximates the demographic composition of the state of Florida. The school’s mission is to “develop innovative solutions to educational concerns in the state and to disseminate successful instructional programs to other school districts” (“About P. K. Yonge,” 2014), so having
a student population that is representative of the state of Florida’s student population is critical for transferability of best practices in teaching and learning to other educators.

Because I am the only geometry teacher at the school where I teach, I am responsible for every student enrolled in geometry to master the course standards and pass the state’s Geometry EOC Test each year. In previous years, there were two levels of geometry courses offered to students, honors geometry for students performing on grade-level and more advanced students, and ‘regular’ geometry for struggling students. Research indicates that many high school students homogenously tracked into ability-grouped mathematics classes do not learn mathematics when placed in equitable classroom environments or have access to identically rigorous curricula (Heubert & Hauser, 1999). Also, if students are placed into heterogeneously grouped mathematics classrooms, the curriculum and teaching strategies might not meet the needs of struggling or advanced students within their current learning environment (Burris & Garrity, 2008). This research influenced me to detrack the geometry courses during the 2013-2014 academic year so that all students taking geometry were enrolled in honors geometry. The data from the 2013-2014 Geometry EOC Test indicate that the detracking process was effective with a 96% passing rate for our school’s students. However, after examining the data more closely, I noticed that some of my struggling students performed at lower achievement levels than desired, and some of my more advanced students did not perform as high as expected. In addition, with the changes to the MAFS and new Geometry EOC Test, there was a need to make changes to my instruction and assessment to meet the needs of all of the learners in my classroom,
especially the struggling and more advanced students, so they can master or possibly expand upon the standards and meet the expectations of passing the test.

To gain insights into the ways differentiated instruction and standards-based assessments support struggling students and challenge advanced learners in a detracked honors geometry classroom, I focused my teacher research on a particular unit of study that historically had been challenging for struggling learners and difficult for me to provide extension activities that encourage higher-level thinking for my advanced learners. “Investigation of the Pythagorean Theorem” was a two-week unit that began in February and ended in March during the 2014-2015 academic year. The detailed accounting of the planning and implementation of this unit of study appears in chapters 4 and 5 of this dissertation and was an important component of my practitioner research.

Participants

For this study, I chose to focus my gaze as a researcher on one section of a heterogeneous, detracked honors geometry course that contained students of varying achievement levels, including struggling and advanced learners. I selected my sixth period class to study as this class contained more struggling and more advanced learners than any of my other classes. I provide a brief background for each student in my study in the following paragraphs. All names are pseudonyms.

Struggling Learners

I identified five students in this class period that often struggled with understanding content and solving problems during previously taught units in honors geometry. Using differentiated instruction and standards-based assessment, I wanted
to help these struggling students comprehend and apply the Pythagorean Theorem successfully to reach at least the minimum level of proficiency.

**Beth.** Beth was a 14-year-old White female 9th-grader from a divorced family. She lived with her remarried mother, her stepfather, and her 6-year-old stepsister in a middle-income neighborhood. Beth participated occasionally, preferring to daydream, doodle, or socialize during class time if not continuously reminded to participate. She became quite anxious before assessments, and she often shared that she did not have a great deal of confidence in her past mathematics classes. Beth maintained a C/D average in honors geometry.

**Frank.** Frank, an articulate 16-year-old White male 10th-grader from an upper-income family, regularly participated in class discussions, but disliked solving problems that required multiple steps or writing. He was always polite and cooperative, but he usually let his peers take the lead in activities that required more than talking. Frank’s unwillingness to put forth effort into practicing mathematics outside of the classroom led to his maintaining a D average in honors geometry due to his low performance on assessments.

**Nancy.** Nancy was a quiet 16-year-old multiracial female 10th-grader whose parents were divorced. She divided her time between the two lower-income households, spending the school week with her single White mother and the weekends with her single African-American father. Nancy completed most of her practice assignments on time, but she lacked confidence in her abilities to do math and seldom spoke in class. She shared that she did not feel comfortable asking for help, even when
she did not understand. Nancy did not perform well on her assessments, but managed to maintain a C average in honors geometry.

**Susan.** Susan was a boisterous 15-year-old African-American female 10th-grade student who lived with her single mother in a lower-income neighborhood. She was highly social and loved to be the center of attention in class. Despite Susan’s high level of participation in class and her desire to have fun while learning, she frequently made careless mistakes or did not fully grasp geometric concepts, which resulted in poor performances on her assessments and a C/D average in honors geometry.

**Ursa.** Ursa was an athletic and talkative 14-year-old African-American female 9th-grader, and she lived with her single mother in a middle-class neighborhood. Ursa loved reading and writing, but she disliked mathematics intensely. She occasionally asked questions in class, but she avoided doing anything related to mathematics after she left the classroom. Ursa liked to socialize, but she let her peers take the lead during class activities and completed the minimum amount of work possible. She often talked about how she was unable to understand anything related to mathematics if she was not in the classroom. Due to her avoidance behaviors in and out of the classroom, Ursa had a D/F average in honors geometry.

**Advanced Learners**

Within this diverse class period, there were also four advanced learners that excelled in my honors geometry course, understanding and mastering the content standards quickly and efficiently, as well as scoring very high on all of their assessments. I wanted to find ways to challenge these students so that they could expand their basic understanding of the Pythagorean Theorem.
Jeff. Jeff was an inquisitive 15-year-old White male 9th-grader who lived with both of his parents in an upper-income neighborhood. He was not socially outgoing, but he was friendly with everyone and worked cooperatively during group activities. Jeff’s love of learning was evident, as he would frequently stay after class to ask questions, discuss alternate problem-solving methods, or share information he found on the internet that related to the content presented in class. He demonstrated mastery of all of the learning targets for each unit and maintained an A average in honors geometry.

Lisa. Lisa, a conscientious 14-year-old Hispanic female 9th-grader, was the middle of three daughters to a single mother living in a low-income community. Although quiet in nature, Lisa was quick to step in and help other students that needed assistance during class when she saw they were struggling without them first requesting her help. She rarely struggled with the content and performed well on her assessments, and routinely held an A/B average in honors geometry.

Tammy. Adopted into a middle-income White family as an infant, Tammy was a friendly 14-year-old Hispanic female 9th-grader who enjoyed learning mathematics. She was diligent in her daily routines, kept a meticulous notebook, and had a tendency to speak aloud during activities and solving practice problems. Before assessments, Tammy second-guessed herself and had to be reassured that she knew the content. Even though she would lose her confidence momentarily, Tammy consistently revealed her mastery of the learning targets on her assessments and always carried an A average in honors geometry.

Victor. Victor was a fiery, redheaded 14-year-old White male 9th-grader, and was an only child from a middle-income family. Victor had a short temper and had little
tolerance for students who did not put forth effort into understanding the learning targets they were responsible for mastering. Even though he frequently did not complete all of his assigned practice problems, he showed his understanding of mathematics by performing well on his assessments regularly and had an A average in honors geometry.

Data Collection

Practitioner inquiry becomes a natural part of teachers’ daily work and interactions with their students, so data are collected in a variety of forms, most coming directly from the classroom (Cochran-Smith & Lytle, 2009; Dana & Yendol-Hoppey, 2009). To gain insights into how differentiated instruction and standards-based assessments supported Beth, Frank, Nancy, Susan, and Ursa, and challenged Jeff, Lisa, Tammy, and Victor, in a detracked honors geometry classroom during my teaching of a unit on the Pythagorean Theorem, I gathered data from a variety of sources. In this study, I collected data in four ways.

First, I collected student artifacts and work produced as a natural and normal part of the activities and assessments that took place in the classroom as part of the Pythagorean Theorem unit. As I systematically collected and analyzed student work each day, these data revealed student understanding of the identified state standards and learning targets associated with the Pythagorean Theorem and provided information for determining necessary shifts in my instruction.

Second, I gathered field notes in the classroom during each day of implementing the Pythagorean Theorem unit. Field notes allow a teacher to document the action that is occurring within her classroom (Creswell, 2013; Dana & Yendol-Hoppey, 2009). I kept a notebook near me at all times during each class period and wrote down my
observations of the students’ behavior. There were also occasions where I wanted to capture students’ conversations during instruction, so I used my cell phone’s voice recording device to capture this data. I then transferred the data into my field notes after each class period.

The third type of data I collected were from student interviews. Interviewing students to gain insights into their thinking provided rich sources of data during my study to help me determine how differentiated instruction and standards-based assessments influenced my students’ progression toward mastery of learning targets related to the Pythagorean Theorem (Dana & Yendol-Hoppey, 2009). According to Dana and Yendol-Hoppey (2014):

“Interviewing” as a form of data collection can be as simple as circulating during instruction and asking questions to students about their learning. What makes the instructional activity of asking students questions an “interview” is that this act relates directing to the (research question a teacher is) exploring (p. 105).

Such was the case in my study. I asked my students questions as I circulated around the room during classroom activities. As I circulated and questioned students about their learning, I kept a record of these conversations in the notebook where I recorded my field notes, subsequently using these notes to help me determine if I needed to adjust the activities or assessments during the implementation of the unit.

In addition to informal interviews, I scheduled one-on-one interviews with selected struggling and advanced students, and recorded their responses. These one-on-one interviews took place before or after school, and I asked the students questions to ascertain their views of the differentiated activities and standards-based assessments used during the Pythagorean Theorem unit. These questions were:
What teaching methods that I do help you understand/extend what you’re learning in geometry about the Pythagorean Theorem?

What might I do to help you understand/extend the learning targets?

What might you do to help you understand/extend the learning targets?

How are you demonstrating mastery/advanced understanding of the learning targets?

What are your (and my) next steps toward helping you demonstrate proficiency/advanced understanding of the learning targets?

I recorded the interviews and conversations without commenting or making judgments, as that might have interfered with the interpretation of the students’ data (Creswell, 2013). During the data analysis process, I returned to these recordings and took notes on the students’ responses to each question.

Finally, the fourth way I collected data was to keep a practitioner researcher journal where I recorded my personal insights about how the implementation of differentiated instruction and standards-based assessments influenced my students’ learning of the Pythagorean Theorem. Each day, I spent 10-15 minutes writing in my journal to reflect on my teaching practices, the actions and words of my students, and the students’ work produced in class as a result of their participation in the planned activities and assessments. Each day I used the same questions to guide my journal reflections and writing:

- What is/are the most critical incident(s) I observed today related to my differentiation of instruction and use of standards-based assessments within the “Investigation of the Pythagorean Theorem” unit?

- I believe this was a critical incident because …

- What actions (if any) might I take in my teaching based on my observation of this/these critical incident(s)?
I had the opportunity to immediately reflect upon the day’s work by following a purposeful daily routine and then plan for my next steps for instruction and assessment.

Data Analysis

My data analysis was both formative and summative in nature. Because data collection and analysis are iterative processes, using the data for formative assessment is a critical step in my research study (Dana & Yendol-Hoppey, 2009). According to Dana (2013),

Formative data analysis takes place throughout the inquiry study. The processes of data collection and data analysis do not exist independently of one another and proceed in a chronological lockstep manner. Rather, these processes are iterative in nature. As teacher researchers collect data throughout a study, they seek to understand what those data mean and use these understandings to make decisions about instruction and the next steps in the inquiry journey (p. 50).

After each instructional day, I formatively assessed what my students and I did through reflecting in my journal. I also used informal student observations, interviews, and conversations from the day as a type of formative assessment. I used the information from these formative assessments to determine any instructional or assessment changes I made before the next day with the students. Any changes made based on my reflections or student observations, interviews, and conversations were noted within my practitioner researcher journal.

The work and assessments that my students produced were analyzed to determine if mastery of the learning targets related to the Pythagorean Theorem progressed at an acceptable rate, and to determine what interventions needed to take place for my struggling students or extended activities given to my advanced learners. Each differentiated activity was examined separately to determine if specific activities were beneficial in helping my students learn the standards. I studied each standards-
based assessment individually to determine if students preferred or performed better on specific types of assessments. I also analyzed which activities and assessments worked best for my struggling students and my advanced students.

Summative data analysis occurred after I collected all of the data at the conclusion of the Pythagorean Theorem unit. In order to gain insight from the entire set of data, I assembled all of the data generated during the study in one location. My field notes, student interviews, and teacher journal were organized electronically and protected by password, and my student artifacts were kept in a secure location. I read through all of the data in its entirety several times to understand it as a whole before breaking it into parts (Agar, 1980). I sorted the data by student and placed them on posters I created for each struggling and advanced student that I had identified at the beginning of my study (Figure 3-1). I highlighted key concepts or ideas that emerged from the data and wrote notes and memos to help me interpret what the data meant. During this data analysis process, I was able to find research stories that related back to my study’s purpose (Creswell, 2013; Dana & Yendol-Hoppey, 2009).

As engaging in practitioner inquiry requires critical reflection from both theory and practice, my data analysis included interpreting the stories found in my data and connecting that information to previous research and literature (Cochran-Smith & Lytle, 2009). Finally, I used questions from The Reflective Educator’s Guide to Classroom Research (Dana & Yendol-Hoppey, 2014, p. 181) to reflect on and interpret what I learned from my data:

- What have I learned about myself as a teacher?
- What have I learned about children?
• What have I learned about the larger context of schools and schooling?

Figure 3-1. Student posters for data analysis notes and memos

• What are the implications of what I have learned on my teaching?
• What changes might I make in my practice?
• What new wonderings do I have?

My answers to these questions appear in Chapter 7 of this dissertation.
Credibility and Trustworthiness of Study

Lincoln and Guba (1985) suggest that the work of qualitative researchers is strengthened when the traditional notions of reliability and validity are reframed in terms of trustworthiness. To ensure the credibility and trustworthiness of my work as a practitioner researcher, I used triangulation to verify and provide validity to my data findings. Triangulation is a process of examining evidence and looking for similarities and differences from multiple sources of data for specific themes (Creswell, 2013; Dana & Yendol-Hoppey, 2009). I used student artifacts and work, field notes, student interviews, and a practitioner researcher journal to provide evidence for and enhance my learning, as well as the credibility of my study.

Another way I ensured trustworthiness in my study was to conduct peer debriefing. Peer debriefing, or discussing the research process with a trusted colleague, allowed me to step outside the study and engage in collaborative analyses of my data (Lincoln & Guba, 1985). As previously stated in Chapter 1, Mickey MacDonald, the biology teacher on my grade-level team, was completing a parallel practitioner research study in her detracked, Honors Biology classroom where she implemented a unit of study using differentiated instruction and standards-based assessment practices within a similar time frame as my own study. To establish the credibility and trustworthiness of my study, Mickey served as a peer debriefer and provided a second perspective that helped me understand the data that I gathered as well as my own position within the study (Creswell, 2013). I discussed my observations from my initial read of my data with her, and she shared what she was seeing in her own data, noticing both areas of overlap and divergence in our observations. I captured what we discussed using sticky notes and added them to the posters that contained my student
data and used them as part of the data analysis process suggested by Dana and Yendol-Hoppey (2009).

A final technique I used to increase trustworthiness of my study was to provide rich, thick descriptions of my data. When the practitioner researcher writes thorough descriptions that include many details, it helps readers feel as though they were present during data collection (Lincoln & Guba, 1985). I provide rich, thick descriptions of my planning and teaching of the Pythagorean Theorem unit in Chapters 4 and 5.

### Researcher Background, Biases, Values, and Experiences

Being a practitioner researcher required me to acknowledge and be cognizant of my personal biases, prejudices, and unexamined impressions and assumptions that affect my teaching practice and may have influenced my research study (Creswell, 2013; Dana & Yendol-Hoppey, 2009; Herr & Anderson, 2005). Therefore, it was critical that I was aware of any predispositions, values, and experiences that I might have brought to this study. To accomplish this goal, in this section of Chapter 3, I include a brief reflection on who I am as both a teacher and a practitioner researcher.

I have been a secondary mathematics teacher for twenty-four years. I taught seventh and eighth grade for the first seventeen years, I taught high school Algebra for one year, and I have been teaching high school geometry for seven years. During the one year that I taught high school Algebra, I was also the District Mathematics Coordinator for grades K-12 half time, but I realized I loved teaching more than being in an administrative role, so I returned to the classroom and my students. I hold National Board Certification in Early Adolescence/Mathematics, and going through that process of certification led to me being more reflective about my teaching and finding ways to help my students succeed and enjoy mathematics.
I am the first person in my family to graduate from college, but it seemed like my family always expected me to do so. I graduated from Florida State University in April, 1991, with a Bachelor’s of Science degree in Secondary Science/Math Teaching from the College of Arts and Sciences. Getting an advanced degree was a personal goal of mine, and my father influenced me to pursue my Master’s degree. He was a retired Marine Corps drill sergeant, and he inspired me to always become better and reach for the next step in my career. Because I was teaching, and I became pregnant with my daughter during my time in school, it took me four years to complete my Master’s degree in Educational Leadership from the University of Florida in December, 2000. My father passed away in 1998, but I could always hear him coaching me and pushing me to do my best in my head. My work in this doctoral program is dedicated to him.

I have always been strong in mathematics, and my desire to become a teacher came at an early age. I did not want to teach middle school, and I find it amusing that I have spent most of my career teaching and enjoying young adolescents. Moving to high school mathematics was a difficult decision for me, but I needed to grow and challenge myself professionally. Geometry was my weakest content area, so taking on that subject has been a struggle for me personally, but it also influenced my practitioner inquiry over the past six years as I focused on using various forms of assessment (formative, summative, and student self-assessment) to increase student understanding and achievement in my classroom (MacDonald, Weller & Miller, 2014; Weller, 2012a, 2012b, 2012c).

My passion for helping students achieve has increased each year based on new strategies and wonderings that continue to evolve with each cycle of teacher inquiry in
which I engage. During the 2009-2010 academic year, I incorporated the use of formative assessments in my geometry class to determine how my assessment of students’ knowledge could change and improve how I teach my students and what I could do to help them learn. Using “assessment for learning rather than assessment of learning” (Leahy, Lyon, Thompson & Wiliam, 2005, p. 19) allowed me to identify my students’ needs, design my future instruction, and provide feedback to my students so they could improve their work (Weller, 2012a).

I focused on helping my students become more self-aware of what they had mastered and what they needed to do to master the material that they had not mastered as part of my practitioner inquiry conducted during the 2011-2012 academic year. I incorporated what I was already doing in my classroom using learning targets, and I developed an inquiry project examining my current use of Assessment for Learning and helping my students become more self-aware of what standards (learning targets) they had mastered. I found that the more the students were involved in the process of self-assessment, the more they had a sense of control of their studying. They were then more focused on what they needed to do to improve the outcome of their summative assessments (Weller, 2012b).

Because some of my students were still not performing as well as I would have liked, especially some of my African-American students, I questioned whether changing my classroom practices with the use of formative assessment and interactive learning targets was sufficient to increase all of my students’ achievement. To help me answer this question, I engaged in practitioner inquiry during the first semester of 2012. I examined how I could scaffold the use of self-assessment of learning targets and
formative assessment data to teach four struggling students how to reflect on what they knew, reflect on what they did not know, and develop action steps to fill in the gaps before they took their summative assessment. My data analysis indicated that creating and completing a plan for mastering learning targets not yet mastered was difficult for students to self-manage without significant teacher assistance and intervention (Weller, 2012c). Although engaging in self-assessment and goal setting gave my students more ownership in their learning, the self-assessments, when not teacher-made, were inaccurate, and the goals the students set were unrealistic or ineffective (Chappuis, 2009).

My grade-level team colleagues, Mickey MacDonald and Cody Miller, a biology teacher and English teacher respectively, and I decided to collaborate during the 2013-2014 academic year to investigate additional ways to increase student motivation, engagement, and find success for all of our students. Our inquiry focused on how we could effectively eliminate tracking in the 9th grade core subjects (biology, geometry, and language arts) and continue to meet the learning needs of all students. By enrolling students in honors sections of these core subjects and modifying our instruction to include both online and face-to-face instruction, we effectively detracked our classes (MacDonald, Weller & Miller, 2014). However, although the data indicated that I was meeting the needs of most of my students in my geometry class, my struggling students were still not meeting proficiency of the standards, nor were my advanced students challenged beyond the mastery of the learning targets. Therefore, for my dissertation study, I decided to build on my previous work in this area and engaged in practitioner inquiry to examine how differentiated instruction and standards-based assessment can
meet the needs of all of the learners in my classroom, including my struggling and advanced learners.

**Conclusion**

In Chapter 3, I defined practitioner research as a way to engage in critical examination of teaching and learning practices. I described the context of this study and introduced the students I focused on, as well as explained the ways I collected and analyzed data. Finally, I described my background, perspective, and trustworthiness as a teacher researcher.

In Chapters 4, 5, and 6, I share the results of my study. In Chapter 4, I describe how I planned a differentiated and standards-based mathematics unit focused on the Pythagorean Theorem to support my struggling and advanced learners. Then, in Chapter 5, I describe the implementation of that unit providing a thorough description of the unit as it unfolded that includes many details to help readers feel as though they were present during data collection. After providing a detailed accounting of the unit as it unfolded in Chapter 5, in Chapter 6 I share what I learned as a result of engaging in summative analysis of my data after I had completed the teaching of this unit.
CHAPTER 4
PLANNING A DIFFERENTIATED AND STANDARDS-BASED MATHEMATICS UNIT:
THE PYTHAGOREAN THEOREM

Chapter Overview

The purpose of this study was to understand the ways differentiated instruction and standards-based assessment supported the mathematics learning of my struggling students and advanced learners in a detracked honors geometry classroom. The first step in my work as a practitioner researcher to address the question, “In what ways do differentiated instruction and standards-based assessments support struggling students and challenge advanced learners in a detracked honors geometry classroom?” was to plan a differentiated and standards-based unit. As discussed in Chapter 3, the unit I chose to develop and study was based on the Pythagorean Theorem. In Chapter 4, I provide a chronological accounting of my planning of this standards-based differentiated unit of instruction.

To contextualize the planning of this unit, I begin with a detailed description of my school and classroom context for developing the standards-based differentiated unit focused on the Pythagorean Theorem. Next, I discuss how I developed student-friendly learning targets from two Florida standards related to the Pythagorean Theorem. Then I describe the construction of the unit’s summative assessment and chosen scoring rubric aligned to the learning targets. I explain how I planned the unit’s daily schedule and instructional activities. Finally, I conclude Chapter 4 with a description of developing formative assessments related to the Pythagorean Theorem. By providing this rich, thick description of the unit’s development, I will set the stage for Chapter 5 where I will discuss the implementation of this standards-based differentiated unit of
instruction focused on the Pythagorean Theorem in a detracked honors geometry classroom.

**The Detracked Honors Geometry Classroom in a Developmental Research School**

As previously stated, I work at P.K. Yonge Developmental Research School, a K-12 lab school that serves approximately 1150 students and is a department within the College of Education at the University of Florida. P.K. Yonge admits students using a lottery system so that demographics of the student population match the state of Florida’s demographics for race, gender, and socioeconomic level. 48% of students that attend P.K. Yonge are White and 52% are minority (23% Black, 17% Hispanic, 6% Multiracial, 4% Asian/Pacific-Islander, and 2% other). 50% of P.K. Yonge’s students fall below Florida’s median income, and 24% of the students are on free and reduced lunch. 12% of the students have an individualized education plan or a 504 plan, and each day students travel from over 37 different cities to attend classes at P.K. Yonge (National Center for Education Statistics, n.d.).

P.K. Yonge operates as its own state school district, and its mission is to develop, evaluate and disseminate exemplary K-12 educational programs. Within the school, there are elementary (K-5) and secondary (6-12) divisions, and the secondary division separates further into a middle school (6-8) and high school (9-12) that shares the same physical space. Middle school classes are heterogeneously mixed, with each student enrolling in the same level of core subject-area classes (English/Language Arts, Science, Social Studies, and Mathematics) while taking two electives each year. Upon entering high school, students have the choice of taking different levels of core courses and electives (regular, honors, or Advanced Placement).
The secondary school uses a modified block scheduling, with students attending each class for 250 minutes per week. Students follow a traditional 50-minute period schedule on Mondays, and on Tuesdays through Fridays, they attend three classes per day for 100-minute periods. They have periods 1, 2, and 3 on Tuesdays and Thursdays, and periods 4, 5, and 6 on Wednesdays and Fridays. On Tuesdays through Fridays, high school students have a split period during the third block of the day (periods 3 and 6), attending the first half of class for 50 minutes, breaking for lunch for 30 minutes, and then returning for the second 50 minutes of class after the lunch period.

The new Mathematics Florida Standards (MAFS) were approved by the Florida State Board of Education in early 2014. With the adoption of these new standards came the expectation that students enrolled in geometry will use critical thinking, problem solving, and communication skills to demonstrate their knowledge of the standards, showing what they know and are able to do, on the newly developed Geometry End-of-Course Test (Florida Department of Education, n.d.). The adoption of the new standards and subsequent test created the necessity for resources and activities to help students engage with and communicate about the content in order to truly understand the content they were to master.

As previously discussed, prior to 2013, standard geometry and honors geometry courses were offered at P.K. Yonge. Students enrolled in the standard geometry course typically struggled in understanding the mathematical concepts. These students also represented the lower socioeconomic and minority population in comparison to the students enrolled in honors geometry. Comparing the state course standards indicated only two differences between the standard geometry and honors geometry courses.
Also, all students, regardless of the course in which they were enrolled, were expected to take the same Geometry End-of-Course Test. At the start of the 2013-2014 academic year, I decided to “detrack” my standard geometry and honors geometry classes to provide an equitable learning experience for all of my students (MacDonald, Weller, & Miller, 2014). Since 2013, every student enrolls in honors geometry at P.K. Yonge, and I teach every section of the course.

The range of mathematical understanding varies greatly in my classes. I have students who still struggle with their basic multiplication facts and students who are ready to move on to more challenging mathematics courses within the same classroom period. This dichotomy of learners presents a challenge for planning and teaching units that require students to explore and learn both concrete and abstract mathematical concepts. It is necessary for me to create a learning environment where all students are successful at mastering learning targets on unit summative assessments and, eventually, passing the Geometry End-of-Course Test.

One of the areas of study in geometry where some of my students have historically struggled in the past focuses on the Pythagorean Theorem. Most students enrolled in my honors geometry course make the connections between what they learned in the mathematics course taught in the academic year previous to taking the geometry course, expanding their prior knowledge about the Pythagorean Theorem and mastering the standards. However, although they are introduced to the Pythagorean Theorem in Algebra 1, not every student within my heterogeneously grouped classroom comes to me with the same prerequisite knowledge. Some students struggle with understanding how the formula \(a^2 + b^2 = c^2\) is derived, and in previous academic
years these students have resorted to learning the minimum amount of information to pass a unit test and memorizing the formula to solve basic problems. Other advanced students go beyond understanding and using the basic formula, and they quickly grasp the many other mathematical concepts that root themselves in the Pythagorean Theorem. Instead of completing more rote problems for practice, these students want to have the opportunity to push themselves and learn something new and more challenging. With these considerations in mind, I chose to design a unit based on the standards which focus on the Pythagorean Theorem that my students are required to master for passing the Geometry EOC Test that incorporated differentiated activities that met the needs of my diverse students.

**Identifying the State Standards and Learning Targets**

The first step to planning the unit was identifying the Florida Standards that focus on the Pythagorean Theorem.

- **MAFS.912.G-SRT.3.8**: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

- **MAFS.912.G-GPE.1.1**: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

I teach how to use trigonometric ratios to solve right triangles in applied problems, later in the honors geometry course as part of a unit that focuses specifically on the concept of similarity. Therefore, I based the current unit on using the Pythagorean Theorem to solve right triangles in applied problems from the first standard, as well as the second standard (CPALMS, 2014-2015a; CPALMS, 2014-2015b).

After identifying the state standards for this particular unit, I rewrote them as learning goals in student-friendly language so that the students would be able to
I understand what I expected them to master during this unit, and, subsequently, self-assess their own learning (O’Connor, K., 2011; Stiggins, Arter, Chappuis, & Chappuis, 2006). I then divided the learning goals into learning targets to match the order of the lessons presented in the textbook that I use as a resource in my classes, Discovering Geometry, An Investigative Approach (Serra, 2008), breaking down the content into smaller chunks so that the information would be easier for my students to process and use (Marzano & Brown, 2009). Student-friendly learning targets, also called shared learning targets, break down larger learning goals into smaller pieces that students can use to determine what they need to learn during a lessons or unit (Brookheart, Long, and Moss, 2011; Leahy et al., 2005). Because some of the standards required the students to have prerequisite knowledge and skills, I included other learning targets in the unit. For example, one of the standards requires the students to derive the equation of a circle. It was logical for the students to understand the relationship between the Pythagorean Theorem and the distance formula before using that information to derive the equation of a circle. Therefore, I identified and included other skills not specified in the state standards that my students would need to know as they learned the Pythagorean Theorem. Table 4-1 summarizes the student learning targets and specific skills I developed for this unit based on the Florida standards.

**Developing the Unit Summative Assessment**

After examining the state standards and rewriting them into student-friendly learning targets, the next step in planning the unit was to write the summative assessment and determine the scoring rubric for that assessment. Creating the summative assessment and choosing the scoring rubric prior to instruction guided my decisions for how I would teach the unit, including the design of different activities that
would engage my students in the learning process. I worked with Dr. Timothy Jacobbe, an Associate Professor and Mathematics Educator Coordinator at the University of Florida’s School of Teaching and Learning within the College of Education, to create the summative assessment and scoring rubric to measure my students’ mastery of the identified learning targets for the Pythagorean Theorem.

Table 4-1. Pythagorean Theorem learning targets by learning goal

<table>
<thead>
<tr>
<th>Florida Standard: MAFS.912.G-SRT.3.8: Use trigonometric properties and the Pythagorean Theorem to solve right triangles in applied problems.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can derive, understand, and use the relationship between the length of the legs and the length of the hypotenuse of a right triangle to solve problems.</td>
</tr>
</tbody>
</table>

LT 1  I can define the Pythagorean Theorem.
LT 2  I can find missing lengths of sides of triangles using the Pythagorean Theorem.
LT 3  I understand and can use the converse of the Pythagorean Theorem to solve problems.
LT 4  I can use Pythagorean triples to determine if a triangle is a right triangle.
LT 5  I understand the relationship between the length of the legs and the length of the hypotenuse in a 45°-45°-90° triangle, and I can use this relationship to find unknown lengths of sides of isosceles right triangles.
LT 6  I understand the relationship between the lengths of the shorter and longer legs and the length of the hypotenuse in a 30°-60°-90° triangle, and I can use this relationship to find unknown lengths of sides of right triangles with angle measures of 30°, 60°, and 90°.
LT 7  I can apply the Pythagorean Theorem and its converse to solve word problems and to solve application problems in 3-D.

Florida Standard: MAFS.912.G-GPE.1.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

I can derive, understand, and use the Pythagorean relationship on a coordinate plane.

<table>
<thead>
<tr>
<th>Florida Standard: MAFS.912.G-GPE.1.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can derive, understand, and use the Pythagorean relationship on a coordinate plane.</td>
</tr>
</tbody>
</table>

LT 8  I can derive and use the distance formula to solve problems.
LT 9  I can derive the equation of a circle and use it to solve problems.

My objective when writing the summative assessment for the Pythagorean Theorem unit was to include questions that demanded the students have a deeper
understanding of the learning targets and apply that understanding to solving problems instead of having questions that only entailed students memorizing and recalling geometric vocabulary and conjectures. Requiring them to make connections between what they learned to the application problems involving the Pythagorean Theorem would allow the students to have a better understanding of what they might encounter later on the Geometry EOC Test.

Dr. Jacobbe and I reviewed numerous test items related to the Pythagorean Theorem, as we did not want to have problems that were too similar or required the students to answer the same kind of problems repeatedly. We also did not want to have an excessive amount of problems, as the students might grow fatigued from having a test that was overly long. Dr. Jacobbe and I eventually chose eleven problems for the summative assessment that would measure my students’ understanding of the learning targets for the Pythagorean Theorem unit (refer to Appendix A).

The problems at the beginning of the summative assessment were less challenging, providing students an opportunity to show that they could find the missing lengths of sides of triangles using the Pythagorean Theorem, use Pythagorean triples to determine if a triangle is a right triangle, and use the converse of the Pythagorean Theorem to solve problems. Moving further into the test, the questions became progressively difficult, requiring my students to synthesize information from different parts of the unit or previous units to solve them. These questions entailed students having to apply the Pythagorean Theorem and its converse to solve word problems or application problems in 3-D. There were also problems that required students to understand the relationships between the lengths of the legs and hypotenuse in
isosceles right triangles and 30°-60°-90° triangles to solve problems to find unknown lengths of sides of triangles, find areas of figures, and calculate the circumference of a circle. Finally, students had the opportunity to show their mastery of solving problems using the distance formula and demonstrate their understanding of circle equations.

After writing the summative assessment and model solutions, I chose to apply a scoring rubric previously developed by Dr. Jacobbe (Jacobbe, 2014). The problems were divided into groups by page (page 1: problems 1-3; page 2: problems 4-6; page 3: problems 7 and 8; page 4: problems 9 and 10; page 5: problem 11). Each group received a score out of 4 points. Each part (problem) in each group would be scored as Essentially Correct (E), Partially Correct (P) or Incorrect (I) using the model solutions as a guide, and the point assignment was described for each part.

A student’s problem would be scored as Essentially Correct (E) if it met the following criteria:

- **Mathematical Understanding**: Completely addresses all parts of a problem, solutions are complete and correct
- **Strategies and Reasoning**: Uses appropriate, efficient, and sophisticated strategy that leads to a solution; evaluates reasonableness
- **Communication**: Provides a clear, organized, and complete explanation detailing how problems are solved; easy for reader to follow

A student’s problem would be scored as Partially Correct (P) if it met the following criteria:

- **Mathematical Understanding**: Solution is mainly complete and correct; may have minor errors, but answer is still reasonable
- **Strategies and Reasoning**: Uses appropriate strategy that leads to a solution; may have minor gaps
- **Communication**: Provides a clear, but not perfectly organized explanation; mainly easy for reader to follow, but problems may remain
A student’s problem would be scored as Incorrect (I) if it met the following criteria:

- **Mathematical Understanding**: No solution or the solution is not correct
- **Strategies and Reasoning**: Uses strategies that do not help solve the problem; or no clear use of a strategy
- **Communication**: Minimal, unclear, or no explanation provided

We decided that there might be times where particular responses would be scored as E+, E-, P+, P-, or I+, and this information would be useful for assigning the final grade for the assessment. Using this “EPI” format for scoring would allow me to examine my students’ work more holistically, examining the quality and effort each student put into his work instead of only grading for the correctness of the answer.

After determining that the “EPI” format would work for scoring each problem, Dr. Jacobbe described his method for determining a descriptive and numerical grade to represent the overall score on the summative assessment (Table 4-2). He linked descriptors to a numerical score, combining the scores for each group of problems to make the total score out of a possible 20 points. The points were then converted to a percentage ranging from 50% to 100%.

The lowest score a student could make on the assessment was a 50%, which was the same lowest score my students could make on previous unit assessments in my class. However, instead of rounding a score up to a 50% as I had in previous units, students were unable to score below a 50% using the new scoring method using the point system. This method complemented the use of the “EPI” grades for each problem, as assigning the final score also used a holistic approach to grading the summative assessment by considering the quality of the students’ effort and work...
provided on the assessment instead of assessing only whether students’ answers were correct or incorrect.

Table 4-2. Summative assessment descriptors, total scores, and final grades

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>Total Score</th>
<th>Final Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exceptional</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Secure</td>
<td>18-19</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>16-17</td>
<td>90</td>
</tr>
<tr>
<td>Developing</td>
<td>14-15</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>12-13</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>10-11</td>
<td>75</td>
</tr>
<tr>
<td>Beginning</td>
<td>8-9</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>6-7</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>60</td>
</tr>
<tr>
<td>Minimal</td>
<td>4</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>0-3</td>
<td>50</td>
</tr>
</tbody>
</table>

Planning the Unit’s Daily Schedule and Instructional Activities

With the summative assessment and rubric completed, the next step to planning the unit was to develop each day’s face-to-face teaching and learning activities, being mindful about differentiating my instruction to meet the needs of all my students. This included making lesson plans for the direct instruction of the learning targets for the whole class each day as well as designing small group activities for re-teaching important concepts related to those learning targets to my struggling learners and challenging my advanced learners.

Unit’s Daily Schedule

I matched the unit’s learning targets with the sections from my textbook to create an outline for the order of the lessons. I found that the first five sections of the textbook’s chapter provided the geometric definitions, conjectures, and practice
problems for the identified learning targets. I then made a calendar to determine the
number of days needed to teach each section, provide a thorough review, and
administer the summative assessment (Table 4-3).

Table 4-3. Planning calendar for Pythagorean Theorem unit

<table>
<thead>
<tr>
<th>Day</th>
<th>Time</th>
<th>Textbook lesson</th>
<th>Learning Targets</th>
</tr>
</thead>
</table>
| Day 1 | 50 min| 9.1 – The Theorem of Pythagoras  
9.2 – The Converse of the Pythagorean Theorem | • I can define the Pythagorean Theorem.  
• I can find missing lengths of sides of triangles using the Pythagorean Theorem.  
• I understand and can use the converse of the Pythagorean Theorem to solve problems.  
• I can use Pythagorean triples to determine if a triangle is a right triangle. |
| Day 2 | 100 min| 9.3 – Two Special Right Triangles  
9.4 – Story Problems | • I understand the relationship between the length of the legs and the length of the hypotenuse in a 45°-45°-90° triangle, and I can use this relationship to find unknown lengths of sides of isosceles right triangles.  
• I understand the relationship between the lengths of the shorter and longer legs and the length of the hypotenuse in a 30°-60°-90° triangle, and I can use this relationship to find unknown lengths of sides of right triangles with angle measures of 30°, 60°, and 90°.  
• I can apply the Pythagorean Theorem and its converse to solve word problems and to solve application problems in 3-D. |
| Day 3 | 100 min| 9.5 – Distance in Coordinate Geometry | • I can derive and use the distance formula to solve problems.  
• I can derive the equation of a circle and use it to solve problems. |
| Day 4 | 50 min| Review |                  |
| Day 5 | 100 min| Review |                  |
| Day 6 | 100 min| Summative Assessment |                  |

**Scaffolded note pages.** After determining the timeline of the unit, I made scaffolded note pages for my students to use during the direct instruction of each section of the unit. The note pages provide a detailed outline, or scaffold, that the students follow and fill in during direct instruction of the lesson. Using the supplemental
materials provided by the textbook, I combined the learning targets I had written from the state standards related to the Pythagorean Theorem with the key topics, definitions, conjectures, and examples from the textbook sections 9.1 through 9.5 to create the note pages using a word processing program. These note pages helped students stay focused during direct instruction and provided an organizational structure for students who lack executive functioning skills (refer to Appendix B).

Included in the note pages was a self-assessment for students to examine and track their understanding of the learning targets for each lesson. Before instruction began, I asked the students to rate their understanding of each learning target by putting a star on the proficiency scale that ranges from 0 to 4 (Table 4-4).

Table 4-4. Student self-assessment rating scale

<table>
<thead>
<tr>
<th>Numerical rating</th>
<th>Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 = No Idea!</td>
<td>I have never heard of this.</td>
</tr>
<tr>
<td>1 = Emerging</td>
<td>I need help with all of the concepts and problems.</td>
</tr>
<tr>
<td>2 = Partially Proficient</td>
<td>I know the simple concepts and problems.</td>
</tr>
<tr>
<td>3 = Proficient</td>
<td>I know all of the simple and complex concepts and problems.</td>
</tr>
<tr>
<td>4 = Advanced</td>
<td>I can go beyond what is taught in class and use the concepts for other problems.</td>
</tr>
</tbody>
</table>

After teaching each lesson, I asked the students to rate their understanding of the learning targets again using the same proficiency scale but marking with a checkmark during the second rating. The students could then see growth in their understanding of the learning targets after participating in the lesson, and they could identify the learning targets with which they still needed help. The students revisited these learning targets later in the unit when we reviewed for the summative assessment as a class, using their proficiency scales to track their progress toward meeting or exceeding the learning targets that were derived from the state standards (Marzano,
My goal was to have every student eventually self-assess at the proficient level (level 3) on every learning target before the summative assessment.

**Instructional classroom activities.** Once I completed writing the scaffolded notes, I made or found different instructional activities designed to reinforce my students’ understanding of the learning targets for each lesson. I wanted activities to help my struggling students comprehend the geometric concepts that lead to using the Pythagorean Theorem to solve problems beyond just memorizing and using the formula $a^2 + b^2 = c^2$. I also wanted to include activities that required my advanced learners to push their thinking and extend their understanding of the learning targets. For lesson 9.4, which focused on solving word problems, I created two levels of problems related to the same learning targets for the students to solve in homogeneous small groups. I made a sheet of basic, more concrete problems for students who might struggle with applying the Pythagorean Theorem. I also created a sheet of challenging problems that would involve my advanced learners to extend their thinking beyond the basic application of the Pythagorean Theorem.

**Hands-on manipulatives for visualization.** For some of the activities, I needed ways for my strugglers to visualize the math by using hands-on manipulatives. For example, in order to understand how the lengths of the sides of the legs and hypotenuse in a right triangle relate to the areas of squares, I designed an activity for students to model this relationship using right triangles made from poster board (with side lengths of 3-4-5, 6-8-10, and 5-12-13 inches) and 1-inch color tiles. I also found 3-dimensional cubes and pencils to use for modeling a specific problem with which students have difficulty visualizing. Additionally, I cut out paper circles with different
radii lengths for the activity that explores circle equations to help students envision and understand how to derive the equation of a circle on a Cartesian plane (Schwartz, n.d.).

Although I did not make any hands-on manipulatives for my advanced students, I considered their needs as I was planning the unit. For example, I chose more abstract problems related to the learning targets for them to solve in small groups to push these students beyond solving problems requiring only rudimentary use of the Pythagorean Theorem. I also determined that using compasses would be more challenging than having them use prefabricated circles during the circle activity.

Because differentiated instruction centers on the needs of the students as they learn, I knew that I would have to plan other instructional activities to re-teach concepts to my struggling students and challenge my advanced learners as we moved through the day-to-day lessons for the Pythagorean Theorem.

**Online podcasts for practice problems.** After I completed developing the instructional classroom activities, I proceeded to create podcasts that related to the unit that my students could access at school or at home. An integral daily activity in this unit was for students to engage in problem solving through independent practice. Students were required to complete practice problems that correlate to the learning targets/state standards for each lesson that I taught. During this practice, the students applied the geometric concepts, definitions, and conjectures to solve problems of varying degrees of difficulty. Based on my experience teaching other units, I realized that many times students might need to finish this practice outside of class time, as there may not be enough time in class to complete the assignments.
Some students require extra help to complete these problems, but their parents and/or guardians may not have the sufficient background knowledge to assist them with these tasks. Using the ShowMe® app on my iPad®, I created podcasts of every practice problem for the unit to allow students to access my help via computer while they were at home or if I was unavailable for face-to-face teaching or tutoring. My students referred to these podcasts as “Virtual Weller.”

**Developing the Unit Formative Assessments**

Having completed writing the scaffolded notes, creating differentiated activities that included accompanying hands-on manipulatives, and creating podcasts of every assigned practice problem for the unit, I returned to creating assessments for the unit. With the summative assessment completed, I needed to decide what kinds of formative assessments I wanted to use to determine my students’ comprehension of the learning targets related to the Pythagorean Theorem. These formative assessments would help me determine which of my students were struggling with understanding the material and which were ready for more challenging work related to the concepts taught. I intended to use various forms of formative assessments during the two-week unit, including my observations and conversations during individual and group work, students’ ability to complete assignments in and out of class with accuracy, and individual paper-and-pencil assessments. I decided to include three individual paper-and-pencil formative assessments in this unit.

I focused on the following learning targets for this state standard for the first two formative assessments:

**MAFS.912.G-SRT.3.8**: Use trigonometric properties and the **Pythagorean Theorem** to solve right triangles in applied problems.
I can derive, understand, and use the relationship between the length of the legs and the length of the hypotenuse of a right triangle to solve problems.

- I can define the Pythagorean Theorem.
- I can find missing lengths of sides of triangles using the Pythagorean Theorem.
- I understand and can use the converse of the Pythagorean Theorem to solve problems.
- I can use Pythagorean triples to determine if a triangle is a right triangle.

I wrote a pre-assessment to administer on Day 1 of the unit so that I could ascertain my students’ prior knowledge of the Pythagorean Theorem. I then wrote a formative assessment for Day 2 to measure my students’ understanding of these same learning targets.

I decided to use a scoring rubric for the second formative assessment using the “EPI” format that Dr. Jacobbe had developed for scoring the summative assessment. My intention was to introduce my students to this type of rubric prior to using it for the summative assessment so that they would understand how I would be grading their work. It also gave me an opportunity to practice and become more comfortable using the scoring rubric to examine my students’ work more holistically.

For the third paper-and-pencil formative assessment, I focused on the following learning targets for both state standards related to this unit:

MAFS.912.G-SRT.3.8: Use trigonometric properties and the Pythagorean Theorem to solve right triangles in applied problems.

I can derive, understand, and use the relationship between the length of the legs and the length of the hypotenuse of a right triangle to solve problems.

- I understand the relationship between the length of the legs and the length of the hypotenuse in a 45°-45°-90° triangle, and I can use this relationship to find unknown lengths of sides of isosceles right triangles.
- I understand the relationship between the lengths of the shorter and longer legs and the length of the hypotenuse in a 30°-60°-90° triangle, and
I can use this relationship to find unknown lengths of sides of right triangles with angle measures of 30°, 60°, and 90°.

**MAFS.912.G-GPE.1.1:** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

*I can derive, understand, and use the Pythagorean relationship on a coordinate plane.*

- I can derive and use the distance formula to solve problems.
- I can derive the equation of a circle and use it to solve problems.

I chose not to use a scoring rubric for the third paper-and-pencil formative assessment, because I wanted to use the results of that assessment immediately during the class period to group the students for an activity.

**Conclusion**

In Chapter 4, I provided a chronological accounting of my planning of a standards-based differentiated unit of instruction focused on the Pythagorean Theorem within the context of a detracked honors geometry classroom at a university’s developmental research school. I first provided a description of the school and classroom context in which the unit would take place. Next, I discussed how I developed student-friendly learning targets from identified Florida standards related to the Pythagorean Theorem as well as the summative assessment for the unit and how I chose to apply the scoring rubric Dr. Jacobbe had developed. I explained how I planned the unit’s daily schedule and instructional activities, detailing how I generated a daily calendar, created scaffolded note pages, organized classroom activities, and produced online podcasts for practice problems. Finally, I described how I developed individual paper-and-pencil formative assessments to use during the unit. By first writing learning targets based on the state standards, creating the assessments, and
designing the instructional activities, I was able to design a detailed instructional unit aligned with specific goals for what I wanted my students to know and be able to do related to the Pythagorean Theorem (Stiggins et al., 2006; Tomlinson & Moon, 2013; Wiggins & McTighe, 2005). In Chapter 5, I will provide a thorough description of the daily happenings as I implemented this standards-based differentiated unit of study based on the Pythagorean Theorem.
CHAPTER 5
IMPLEMENTING A DIFFERENTIATED AND STANDARDS-BASED MATHEMATICS UNIT: THE PYTHAGOREAN THEOREM

Chapter Overview

Once the differentiated and standards-based mathematics unit was planned, it was ready for implementation. The teaching of this unit was five days in length and took place from Monday, February 23, 2015 through Wednesday, March 4, 2015, followed by the administration of the summative assessment on Friday, March 6, 2015. In Chapter 5, I provide a day-to-day chronological accounting of my implementation of the unit.

The accounting of each day of the implementation of the unit and the actions I took presented in Chapter 5 were carefully reconstructed using my data – field notes, student artifacts, student interviews, and entries made in my journal each day. Reconstructing the day-by-day implementation of the unit is an important part of my work as a teacher researcher, as the chronological descriptions of each day provide insights into the role formative data analysis played in my work. According to Dana (2013), formative data analysis is an important component of practitioner inquiry as teacher researchers use data to “make decisions about instruction” (p. 50). Hence, Chapter 5 illuminates the ways I used data to adjust student groupings and student activities to support the learning of my struggling and advanced students. By providing this rich, thick description of the unit’s implementation and the ways I used data to make instructional decisions throughout the teaching of this unit, I reveal the details associated with teaching a differentiated and standards-based unit.

Chapter 5 is organized by each day of instruction. I begin each day-by-day accounting with an overview of time structure, state standards, and learning targets taught during the class period. Following the overview, I describe the events of the
class period starting with the opening activities, continuing with the classroom instruction and activities, and ending with how I wrapped up instruction. Each day ends with student reflections on the instruction of the day captured through interviews. After the description of the five instructional days, the final section of Chapter 5 details the administration and results of the summative assessment and summative re-assessment.

**Implementing the Pythagorean Theorem Unit Day 1: Monday, February 23, 2015**

**Overview**

The first day of the unit took place during a 50-minute class period and was based on the textbook’s lessons 9.1 (The Theorem of Pythagoras) and 9.2 (The Converse of the Pythagorean Theorem). I had identified the state standard and written the learning targets for these lessons while planning the unit.

**MAFS.912.G-SRT.3.8**: Use trigonometric properties and the Pythagorean Theorem to solve right triangles in applied problems.

*I can derive, understand, and use the relationship between the length of the legs and the length of the hypotenuse of a right triangle to solve problems.*

- I can define the Pythagorean Theorem.
- I can find missing lengths of sides of triangles using the Pythagorean Theorem.
- I understand and can use the converse of the Pythagorean Theorem to solve problems.
- I can use Pythagorean triples to determine if a triangle is a right triangle.

**Opening Activities**

**Pre-assessment.** I wanted to determine which students had already mastered the learning targets for these two sections before beginning direct instruction, because the concepts I was to cover were essentially a review of the Pythagorean Theorem taught in Algebra 1, the mathematics course in which my students enrolled in the year
prior to taking honors geometry. To accomplish this, I administered an individual pencil-and-paper formative pre-assessment to my students at the start of the class period. My plan was to use the results to group the students based on their background knowledge and ability to solve problems involving the Pythagorean Theorem.

The pre-assessment contained five problems from the textbook’s teaching resources package, *Discovering Geometry: An Investigative Approach, Assessment Resources*, asking the students to solve problems using the Pythagorean Theorem based on the learning targets for lessons 9.1 and 9.2 (Figure 5-1) (DeCarli, 2008, p. 129). The students could use calculators during the pre-assessment if they wished, as they would be able to use calculators during the Geometry End-of-Course Test.

![Figure 5-1. Student pre-assessment for Pythagorean Theorem Unit (DeCarli, 2008, p. 129)](image-url)
Students completed their pre-assessments in 5-10 minutes, and as the students turned in their papers, I quickly graded them (FN, February 23, 2015, p. 1). My struggling learners performed as I had expected on the pre-assessment, with all of them solving 0, 1, or 2 problems out of 5 total problems correctly. However, only three of my advanced learners performed as I had predicted with scores of 4 or 5, as Victor had answered just 1 problem correctly (Table 5-1). I used the students’ scores from their pre-assessments to create groups for the classroom instruction and activities.

<table>
<thead>
<tr>
<th>Pseudonym</th>
<th>Type of Learner</th>
<th>Score (out of 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beth</td>
<td>Struggling</td>
<td>0</td>
</tr>
<tr>
<td>Frank</td>
<td>Struggling</td>
<td>0</td>
</tr>
<tr>
<td>Jeff</td>
<td>Advanced</td>
<td>5</td>
</tr>
<tr>
<td>Lisa</td>
<td>Advanced</td>
<td>4</td>
</tr>
<tr>
<td>Nancy</td>
<td>Struggling</td>
<td>2</td>
</tr>
<tr>
<td>Susan</td>
<td>Struggling</td>
<td>1</td>
</tr>
<tr>
<td>Tammy</td>
<td>Advanced</td>
<td>4</td>
</tr>
<tr>
<td>Ursa</td>
<td>Struggling</td>
<td>1</td>
</tr>
<tr>
<td>Victor</td>
<td>Advanced</td>
<td>1</td>
</tr>
</tbody>
</table>

Creating homogeneous groups. Students were assigned to one of two groups after the pre-assessment. I felt that students who answered 3, 4, or 5 questions correctly on the pre-assessment had sufficient background knowledge of the lesson’s learning targets from what they had learned about the Pythagorean Theorem during their previous Algebra 1 course. Therefore, I excused these students from the direct instruction of lessons 9.1 and 9.2 to begin the practice problems for these sections immediately. The students in this group also had access to computers in the classroom if they wished to view the online podcasts of the practice problems that I had made for these lessons if they needed more assistance. Jeff, Lisa, and Tammy were part of the
Students who answered 0, 1, or 2 questions correctly on the pre-assessment moved to the front of the classroom with me for direct instruction on lessons 9.1 and 9.2 using the scaffolded notes I created. I hoped that my struggling students (Beth, Frank, Nancy, Susan, and Ursa) would be more comfortable participating and asking questions during direct instruction if they were part of a smaller group. I was also curious about why Victor, one of my advanced learners, had not performed as I had expected on the pre-assessment.

Class Instruction and Activities

Direct instruction: Introducing the Pythagorean Theorem. I began the direct instruction process by distributing the scaffolded notes for lessons 9.1 and 9.2 and asking the students to rate themselves by marking where they were on the self-assessment scale with a star as I read each learning target aloud. As I glanced at their papers, I saw that Beth, Frank, Nancy, Susan, and Ursa rated themselves at a 1, and Victor rated himself at a 3 (FN, February 23, 2015, p. 1).

After the students completed the self-assessment, I defined the parts of a right triangle and discussed the special relationship between the lengths of the legs and the length of the hypotenuse, and how that relationship is known as the Pythagorean Theorem.

To help my students visualize the Pythagorean Theorem, I gave them poster board triangles and 1-inch color tiles to model the relationship between the areas of squares built on the sides of right triangles. Using the tiles in their small groups, the
students were able to see how the sum of the areas of the squares built on the legs of a right triangle is equal to the area of the square built on the hypotenuse (Figure 5-2).

Figure 5-2. Color tiles activity – student examples of right triangles

There were a few “ah-hah” moments among my struggling students as they made connections between what they had learned in previous math courses with what they had just modeled in their groups using the manipulatives.

Nancy: That’s what that is? I remember learning about $a^2 + b^2 = c^2$, but I didn’t know that’s what that meant. There are actually squares there.

Susan: That makes a lot more sense to me now.

Frank: I never knew that. Does it work for only right triangles? (FN, February 23, 2015, p. 1)

Frank’s comment led to a discussion among all of the students in the small direct instruction group about whether or not the Pythagorean Theorem worked for acute and obtuse triangles. I sketched an acute triangle (with side lengths 6, 7, and 8, and all three angles less than 90°) and an obtuse triangle (with side lengths 4, 10, and 12, with one angle greater than 90°) on the board. I asked the students to use the 1-inch color tiles to model one of the triangles by forming squares on each side as they had done with the right triangle examples.
The students were engaged in a lively discussion while making their models, but I noticed that Victor was quiet, looked flushed in the face, and seemed frustrated. When I asked him if he was okay, he said, “I know all of these learning targets. I just messed up on my quiz” (FN, February 23, 2015, p. 2). I encouraged him to show me what he knew and asked him to help other students as they completed the activity. I assured him that the groups were not permanent and that he would be able to be in a different group later based on his performance on the next formative assessment. Victor seemed to accept this, and he started working and urged others in his group to participate more.

The students completed building their acute and obtuse triangles. They concluded that the Pythagorean Theorem did not apply to triangles that were not right triangles, as the sum of the areas of the squares built on the shorter sides of an acute or obtuse triangle does not equal the area of the square built on the longest side (Figure 5-3).

Figure 5-3. Color tiles activity – student examples of acute and obtuse triangles

I asked if anyone would volunteer to explain if they had found any kind of relationship between the sum of the squares of the two shorter sides and the square of longest side. Victor raised his hand and explained his thinking to the group.
Victor: It seems that, in the acute triangle, when you square the two shorter sides and add them up, it’s greater than the longest side when it’s squared. But when you do that with the obtuse triangle, when you square the two shorter sides and add them up then it’s less than the big side squared (TJ, February 23, 2015, p. 2).

I told Victor that his comparison of the acute and obtuse triangles was correct, and I wrote a summary of his conclusion on the board: If the sum is less, it is an obtuse triangle, and if the sum is greater, it is an acute triangle. Frank and Susan seemed to follow Victor’s explanation, but Beth, Nancy, and Ursa still looked confused. I decided to move along and revisit this concept later, as I needed to move on with the note pages and finish before the class period ended. However, after my conversation with Victor and his insightful explanation of acute and obtuse triangles, I believe that he had a better understanding of the Pythagorean Theorem than what his pre-assessment had indicated (TJ, February 23, 2015, p. 3).

I reiterated that during the color tiles activity, the groups determined that the Pythagorean Theorem worked for right triangles but not acute or obtuse triangles. Therefore, if three positive integers were substituted into the Pythagorean Theorem and satisfied the equation, then the triangle must be a right triangle. I defined and showed them examples of these Pythagorean Triples, concluding the discussion by explaining that they had actually been discussing the converse of the Pythagorean Theorem during their color tiles activity.

I wanted the students to be able to apply what they learned to solving problems, and I had three examples on the scaffolded notes for them to solve. Because the students had been able to see the relationships between the sides of a right triangle during the colored tiles activity, I believed that they would have success solving the
example problems. However, I found that a few of the students, including Beth, Nancy, and Ursa, did not make immediate connections between the activity and the examples due to the triangles being labeled with different variables other than a, b, or c, not reading the questions carefully, or having difficulties when a picture was not provided. After encouraging them to draw pictures or look for right triangles, the three girls were able to understand how to solve the example problems. Frank and Susan solved the problems and articulated their answers with confidence (FN, February 23, 2015, p. 2).

After completing the examples, I asked the students to go back and complete the self-assessment again, this time marking where they were on the scale with a check mark. Victor rated himself as a 4; Frank and Susan rated themselves as a 3; and, Beth, Nancy, and Ursa rated themselves at a 2.

**Wrapping up instruction.** The class period ended soon after the groups finished the examples, and I told the students to complete their assignments on their own. I reminded them to visit Virtual Weller if they needed help with doing the problems on their own.

**Student Reflections**

I interviewed several students from both groups after the events of Day 1, asking them what teaching methods and structures I used in class helped them understand or extend what they were learning about the Pythagorean Theorem. The students who participated in the direct instruction and activities revealed how they felt about being part of that group:

Nancy: I think that breaking us into groups helped me feel more comfortable with asking questions. Sometimes I get nervous asking questions when the people I’m with already know what they’re doing. The people in my group had questions like me so I didn’t feel stupid asking them (SI 1, February 24, 2015, p. 1).
Ursa: It makes it easier for me not to have the whole class do the notes together. I liked that we did the pre-test, and then the ones that understood started their homework right away, and then we had a smaller section taking the notes and asking questions. It made me feel better knowing that more people didn’t know as much like I did, and we did it together, and it was easier for me. Other people in the groups up front had questions like I did (SI 1, February 24, 2015, p. 2).

Frank: You helped us understand the Pythagorean Theorem better when you split us into groups. The people that knew it didn’t need extra help, so they were doing more of their own thing because they were at a higher understanding level. For the people who didn’t know it, you focused more of your attention to us and it was more individualized and more specific to what we needed. I like how you split us into groups because I was able to ask more questions. I felt more comfortable asking the questions (SI 1, February 24, 2015, p. 2).

Victor: I was in the group in the front of the room with you because I messed up on the pre-test. After the first couple of minutes, I figured out what I did wrong and wanted to move because I was bored (SI 1, February 24, 2015, p. 2).

My struggling learners, Beth, Nancy, Ursa, and Frank, also discussed their understanding of the learning targets after completing the color tiles activity to model the Pythagorean Theorem:

Beth: I liked how we used the tiles to find the areas and how that matched up on the triangle sides for the Pythagorean Theorem (SI 1, February 24, 2015, p. 1).

Nancy: When we did the tiles so that we could add up the squares to see how the problems worked, it made more sense to me (SI 1, February 24, 2015, p. 1).

Ursa: It was easier to understand where \( a^2 + b^2 = c^2 \) came from when we looked at the areas of the three squares (SI 1, February 24, 2015, p. 2).

Frank: I liked the presentation and the pictures we made. You were explaining the Pythagorean Theorem in more depth to us so that we could understand it. I could visualize it a lot better after we made the areas with the tiles (SI 1, February 24, 2015, p. 2).
My advanced students, Jeff and Tammy, talked about how they liked having the opportunity to skip the direct instruction of the lessons and immediately begin working on their practice problems independently:

Jeff: I like that you did a pre-test, because if we knew how to use the Pythagorean Theorem, we didn't have to sit through a long lesson forever and we could actually do the work and practice. I would definitely prefer that than sitting through a lesson on material that I already know. But if I'm not sure that I know the stuff that we are learning, I would rather sit through it just to make sure I'm not missing anything that I need to know

Tammy: Since I already knew the Pythagorean Theorem, I was able to figure out the questions on the pre-assessment that I did know and then try the ones that I didn't know. But I tried to use what I know to figure them out and I guess I did okay. I was good with you not making me sit there and learn something I already knew. If I didn't know what I was doing, I would want to be in the other group. But for the stuff from these practice problems, I just went on Virtual Weller to help me figure out the practice problems that I didn't know

The students talked about what they would want to do in future class periods to help them become more proficient in their understanding of the learning targets. Jeff and Tammy shared that they preferred to have opportunities to challenge their thinking and extend their knowledge of the learning targets:

Jeff: I would want to do something that pushes me to think more and learn more about the topic. But not just busy work

Tammy: I like to do harder kinds of problems about the stuff I already know, but I don't like to do it alone. I want to work with others. I would rather learn something new and push myself than be bored with stuff I already know

My struggling learners talked about wanting help from others in understanding the content:

Nancy: I like it when you go over practice problems that I don't understand so I can see what I'm supposed to do
Ursa: I liked being in the small group yesterday, but I think that I want to be with other people who know more about what is going on sometimes. Like when we have to work out the problems, I want to work with people who can tell me how to work the stuff out (SI 1, February 24, 2015, p. 2).

Frank: I like working with other people instead of by myself. It’s easier to talk about what to do to solve the problems than figure it out on my own (SI 1, February 24, 2015, p. 2).

Nancy and Frank also discussed what they needed to do on their own to increase their understanding of the learning targets:

Nancy: I need to do the homework and understand how I got the answers and what I need to do on the problems. I need to study the notes more and ask more questions if I don’t understand something instead of just watching you do them. I guess I should participate more in class (SI 1, February 24, 2015, p. 1).

Frank: I’m going to be reviewing problems, the notes, quick look at what we do in class. I will probably try to do some of the homework. I usually don’t do my homework, but I might this time. I don’t like to do the homework (SI 1, February 24, 2015, p. 2).

Implementing the Pythagorean Theorem Unit Day 2: Wednesday, February 25, 2015

Overview

Day 2 of the Pythagorean Theorem unit was a 100-minute class period split into two 50-minute sections separated by a 30-minute lunch period. The lessons used in the textbook this day were 9.3 (Two Special Right Triangles) and 9.4 (Story Problems), aligning with one of the state standards and the learning targets I wrote to correlate with the standard.

MAFS.912.G-SRT.3.8: Use trigonometric properties and the Pythagorean Theorem to solve right triangles in applied problems.

I can derive, understand, and use the relationship between the length of the legs and the length of the hypotenuse of a right triangle to solve problems.
• I understand the relationship between the length of the legs and the length of the hypotenuse in a 45°-45°-90° triangle, and I can use this relationship to find unknown lengths of sides of isosceles right triangles.

• I understand the relationship between the lengths of the shorter and longer legs and the length of the hypotenuse in a 30°-60°-90° triangle, and I can use this relationship to find unknown lengths of sides of right triangles with angle measures of 30°, 60°, and 90°.

• I can apply the Pythagorean Theorem and its converse to solve word problems and to solve application problems in 3-D.

Opening Activities

Creating heterogeneous groups. I decided to group the students for reviewing the previous class period’s assignment (from lessons 9.1 and 9.2) using the results of the pre-assessments that they took on Day 1. I created heterogeneous groups, combining 2-3 students from the group that had worked independently on Day 1 with 2-3 students from the group who had participated in the direct instruction of the lesson during that class period. Ursa’s statement from the previous day about how she liked to work with other students who could explain “how to work the [problems]” influenced my choice to create heterogeneous groups for discussing the practice problems (SI 1, February 24, 2015, p. 2). After interviewing and formatively assessing Victor on Day 1 during direct instruction, I determined that his prior knowledge of the Pythagorean Theorem was more extensive than his pre-assessment score indicated, and I categorized him with students who had worked independently when I created the heterogeneous student groups for the review activity (TJ, February 23, 2015, p. 3-4).

Review activity. I wanted my students to review some of the practice problems from the assignment before administering the paper-and-pencil formative assessment on learning targets from Day 1. I asked the students to write down the practice problems from the assignment with which they had difficulties on sticky notes.
individually and then compare their sticky notes and choose the three problems that appeared most often in their group. I provided three pieces of blank paper for each group on which the students, writing one problem per sheet, solved the problems together. I handed the blank sheets of paper to students who scored high on the pre-assessments and had worked independently on Day 1, asking them to be the scribes for their groups. Jeff, Lisa, and Tammy were amongst the students chosen as scribes. The scribes in each group wrote down one of their chosen practice problems on each sheet of paper, and then began to work them out.

After a few minutes, I noticed that the scribes and other students who worked in the independent group from Day 1 were taking the lead, talking through and solving each problem without much input from the students who had participated in the direct instruction group during the previous class (FN, February 25, 2015, p. 1). I decided to change up the procedures in each group to engage all of the students in the activity. I told the students that they would pass their papers every 40 seconds, and they would each take turns being the scribe for their groups with everyone still contributing to explaining the problem-solving process. I explained that I would keep time and circulate through the room to eavesdrop on their conversations. I also made clear that I expected to hear everyone contribute and would use a random name generator to choose students to explain more details if I did not notice active participation from each member of the group. I also said that each group had a “teacher save” if they were unable to solve a problem and needed a hint, but it had to be a unanimous decision to get my help (TJ, February 25, 2015, p. 1).
The change in student participation was noticeable, because all of the students had to pay attention to what each member of the group did before they had to pass the paper. The 40-second time limit and possibility of random questions from me caused students to increase their engagement, resulting in discussions that were animated and focused on solving the practice problems (TJ, February 25, p. 1). Group 2 was the only group to use their “teacher save,” as the students in that group wanted to verify they were explaining a word problem correctly to a student that had been absent on Day 1. All of the groups finished their problems within 10 minutes, and I concluded the activity by asking if there were any other questions for me before the individual formative assessment (FN, February 25, 2015, p. 1). Since the students did not ask any questions, I collected the groups’ papers and asked the students to separate for the paper-and-pencil formative assessment.

**Formative assessment.** The paper-and-pencil formative assessment consisted of two questions (see Figure 5-4). The first question asked students to identify three numbers that could be the side lengths of a right triangle. The second question had two parts, the first requiring the students to determine if a given triangle was a right triangle given the lengths of the three sides. For the second part of the question, students were asked to find the lengths of the two legs of a right triangle (whole numbers) given the length of the hypotenuse (whole number). I explained that I would grade the quizzes after class and would be sharing the results during the next class meeting to give me the opportunity to determine how well the students were mastering the learning targets. I also wanted to have the opportunity to show the students the new EPI scoring rubric with their graded quizzes so that they would be able to use the standards-based grading
tool to help them understand what they needed to work on to improve their understanding of the learning targets (see Figure 5-5). The students took between 5-10 minutes to finish. After the students submitted their completed quizzes to me, they previewed their scaffolded note pages for lessons 9.3 and 9.4 and rated themselves on the self-assessment scale by marking them with a star (FN, February 25, 2015, p. 2)

Figure 5-4. Day 2: Student formative assessment
Classroom Instruction and Activities

**Direct instruction: Special right triangles.** I moved into the direct instruction of the lessons about solving problems with special right triangles (45°-45°-90° and 30°-60°-90° triangles), keeping the students in the same groups from the previous activity. I read the learning targets aloud, asking them to think about where they made a star on
the self-assessment scale for each one so that they could keep in mind what they needed to learn that day. I walked around as I read the learning targets, glancing at the students’ papers to get an idea of where they rated themselves. The ratings ranged between 0 and 2, and I did not see any marks of 3 or 4 (FN, February 25, 2015, p. 2).

Before beginning the lesson 9.3 scaffolded notes, I asked the students if they remembered how to simplify radicals, as this was a prerequisite skill for solving problems using the relationships between the legs and hypotenuses in $45^\circ$-$45^\circ$-$90^\circ$ triangles and $30^\circ$-$60^\circ$-$90^\circ$ triangles. Many students, including Beth, Frank, Nancy, Susan, Tammy, and Ursa, raised their hands indicating that they did not remember how to simplify radicals, although they had learned the procedures in their Algebra 1 course (FN, February 25, 2015, p. 3). I decided to teach the students a method for simplifying radicals that I learned when I took Algebra 1. I thought this method might help my students, especially those who needed a visual representation, to understand how to simplify radicals (TJ, February 25, 2015, p. 1).

I explained to my students that my teacher told me to picture the radical sign as a clothes dryer. The number under the radical sign needed to be factored into prime numbers, and each prime number represented a sock in the dryer. It was my job to then remove the socks and fold them into pairs. Each pair of socks then became one folded pair (an individual prime number) outside the radical/dryer. The socks that did not have a match went back under the radical/into the dryer. The individual pairs of socks outside were then multiplied together, and the socks under the radical/in the dryer were multiplied back together. I used matched and unmatched socks during my story to illustrate the process of simplifying a radical. All of the students watched the procedure,
and many laughed at the story. I wrote five problems on the board that were similar to
the problem I had solved during my story and asked the students to solve them in their
groups, using the same procedure we had used while reviewing the practice problems
from Day 1. The students used the “sock story” to simplify the radicals, and all of the
groups were successful. Beth, Frank, Susan, and Tammy appeared comfortable with
solving the problems with the members of their groups, referring to the sock example to
explain how they found the answers. However, I noted that both Nancy and Ursa were
hesitant when they had to begin a problem on their own and needed more
encouragement from the students in their groups whenever it was their turn to be their
groups’ scribe (FN, February 25, 2015, p. 3).

After reviewing how to simplify radicals, I continued to the first investigation
focused on isosceles right (45°-45°-90°) triangles on the scaffolded notes for lesson 9.3.
I led the class as a whole group through the first step of the investigation using the
Pythagorean Theorem to solve for the missing hypotenuse in each triangle and writing
the answers in simplified radical form. The students then completed the second and
third steps with their groups. To encourage the students to participate, I circulated
among the groups, selecting individual students to explain their problem-solving
processes using my random name generator. Frank was willing to explain his answers,
and Susan shared her answers after more coaxing from me (FN, February 25, 2015, p.
3-4).

After bringing the class back together as a whole group to complete the Isosceles
Right Triangle Conjecture, I continued to use the scaffolded notes to guide the direct
instruction for the lesson. The students completed the patterns related to the 30°-60°-
90° triangles and filled in the table, as they were quick to see the similarities to the previous investigation about isosceles right triangles. I led the whole group in summarizing the pattern they had discovered during the investigation in their groups and completing the 30°-60°-90° Triangle Conjecture as the bell rang for the students to go to lunch (FN, February 25, 2015, p. 5).

During lunch, I copied the Isosceles Right Triangle Conjecture and the 30°-60°-90° Triangle Conjecture on chart paper and hung them on the wall of my classroom. When the students returned from lunch, I asked them to solve the example problems on the last page of the lesson 9.3 scaffolded notes individually. Many of the students completed the problems in less than one minute, but Beth, Nancy, Susan, and Ursa took about 30 more seconds to complete their problems. Nancy and Susan kept examining the chart paper on the wall as they completed their work, using the pictures as a guide to help them solve the problems. After all of the students were finished, I explained each problem on the board, asking the students to give me a thumbs-up if they got the problem correct. Every student gave me a thumbs-up on the first two problems, but Ursa indicated she had an incorrect answer for the third problem. After completing the examples, I asked the students to rate themselves on their self-assessments and mark where they were on the scale with a check mark. As I circulated through the room, I saw mostly ratings of 3 and 4, with Beth, Nancy, and Ursa rating themselves at a 2 (FN, February 25, 2015, p. 5).

**Direct instruction: Story problems.** I started lesson 9.4 by having the students complete the self-assessment for the one learning target. Jeff and Victor rated themselves at 3, and all of the other students rated themselves at 1 or 2 (FN, February
I read the word problem example from the scaffolded notes aloud to the class. I then explained that there were two separate right triangles within the problem that they needed to locate. To help the students visualize the problem, I distributed 3-dimensional clear rectangular prisms with pencils cut out to represent the diagonals in the figure shown in the example problem to students who had rated themselves at 1 or 2 (see Figure 5-6). I was curious to know if these students would hand off the cubes to other students in their group or if they would examine the cube and try to solve the problem on their own first. Susan and Beth both kept their cubes and started to look for the triangles with their group members. In another group, a student immediately handed his cube to Frank, and Frank kept the cube and identified the triangles formed by the pencils, showing his group where the triangles were located without any assistance (FN, February 25, 2015, p. 6).

Figure 5-6. 3-dimensional clear rectangular prism with pencils

I asked the students to sketch the two triangles separately on their papers and determine how to use both of them to solve the problem. Within the groups, I noticed that many students seemed confident with solving the problem. Tammy was among the students in her group that completed their sketches quickly and then took turns talking through each step. Ursa was very confused and needed members of her group to
explain what to do two different times. Frank, who had confidently identified the triangles in the cube, also had trouble labeling the triangles and finding how they overlapped for solving the problem. Lisa, Susan, and Victor animatedly debated the process together while other members of their group watched and copied from their papers quietly. Nancy and Beth had been in a group together, but Nancy had left school during lunch. Two students tried to help Beth solve the problem, but she just copied down what they had, saying, "I don't think I get this" repeatedly (FN, February 25, 2015, p. 6).

After verifying that all of the groups solved the example correctly, I asked the students to rate themselves on their level of comfort with solving word problems using the Pythagorean Theorem on the self-assessment at the top of the lesson 9.4 scaffolded note page.

Since the students worked in heterogeneous groups since the start of the period, my plan was to have them in homogeneous groups for the word problem activity. To differentiate the instruction, students who rated themselves at a 3 or 4 were given word problems that were more challenging and required multiple steps to solve. Students who rated themselves at a 1 or 2 worked on word problems that necessitated fewer steps but still met the requirements of the learning targets for the lesson. I allowed the students to choose their own groups for this activity, and I circulated through the room to observe and informally assess the students' understanding and ability to solve word problems (TJ, February 25, 2015, p. 2-3).

**Differentiated activities for homogeneous groups.** There were several students who rated themselves at a 3 or 4 on their self-assessments. Jeff and Victor
chose to work together with another student on the challenging problems, and they had a great rapport with each other. They all contributed to solving the problems, arguing and teasing each other about how they wanted to set up the problems to solve them. They had difficulties solving one of the problems, but they persevered in trying to solve it (FN, February 25, 2015, p. 7).

Susan also worked together in a group with two of her friends on the challenging problems. I was surprised that Susan rated herself as a 3 on her self-assessment, because she had rated herself lower on Day 1. I had noticed that Susan seemed to be more confident after the group activities today, and I hoped that that confidence would continue when solving the more challenging word problems. I observed that she asked her other group members a lot of questions and needed a great deal of assistance from them with sketching and labeling pictures for solving the problems. However, Susan did not give up and she participated the entire class period (FN, February 25, 2015, p. 7).

Finally, Lisa and Tammy formed a group with two other students to work on the more challenging problems. Tammy worked through the problems methodically, occasionally describing her problem-solving process aloud to the group. Lisa took on the role of answering the other two students’ questions and helping them as they made their way through the problems, staying focused and involved in the process (FN, February 25, 2015, p. 8).

When the students working on the more challenging problems showed signs of great frustration, I reassured them that it was okay if they did not understand how to proceed with solving them immediately. I encouraged them to persevere and help each other. After telling them this, some of the students moved between the groups to ask
questions or provide assistance to each other. None of the groups finished the
problems before the end of the class period. I noticed that Jeff and Victor were
particularly frustrated that they could not solve one of the problems, and they asked if I
could show them how to set up the problem during lunch the next day (TJ, February 25,
2015, p. 3).

Students who had rated themselves at a 1 or 2 on the self-assessment worked
on problems that were more basic, required fewer steps to solve, yet met the conditions
of reaching the proficiency level on the learning targets for the lesson. Beth and Ursa
chose to work together in a group with two other students, and they worked diligently to
solve the problems. They had their note pages on their desks and referred back to the
example often as they worked on solving the problems. They reminded each other to
start by sketching pictures, asked each other for assistance or if they were labeling their
figures correctly, and were totally engaged in the problem-solving process. When I
asked this group if they needed assistance, the only questions I received were whether
they were setting up the problems correctly. This was the only group to complete all of
the problems that I assigned for the activity. When they finished, they started on the
practice problems on the daily assignment together. They only completed one practice
problem before the end of the period, but the group seemed pleased with that
accomplishment (TJ, February 25, 2015, p. 3).

In contrast, the other group working on the more basic word problems needed
much more guidance and encouragement from me to stay engaged with the activity.
The members of this group, including Frank, just stared at their papers without
attempting to begin the first problem. I read the problem aloud and asked them where
they thought they should start, but I received silence and blank looks from the students. I asked them to look back at their notes to see what steps we took as a class to solve the example problem. The boys stared at the notes silently, so I gave them a hint for starting by asking a question.

Mrs. Weller: Do you think drawing a picture would help?

Frank: Oh, yeah, okay (FN, February 25, 2015, p. 7).

Frank picked up his pencil and sketched a right triangle for the first problem, and the other members of the group followed his lead. I walked away to check on the other groups, and when I returned, the boys had not proceeded past sketching the triangle. I had to prompt this group to do each step, asking, “What do you do next?” each time I walked by. I spent more time with this group than with any other during this activity, and they only completed the first problem. I was surprised that they did not complete more than this, as Frank had been more engaged and successful when he was with his heterogeneous group earlier in the class period (TJ, February 25, 2015, p. 3).

Wrapping up instruction. As the class period ended, I collected the students’ papers so that I could use them as a formative assessment. I announced that the students needed to complete the practice problems for an out-of-class assignment, reminding them to use the online podcasts if they needed help from Virtual Weller with setting up and solving the practice problems (TJ, February 25, 2015, p. 4).

Student Reflections

The students reflected on working within different groups during the class period. My struggling students, Frank, Beth, Ursa, and Susan, discussed working in the heterogeneous groups for review and homogeneous groups for understanding the new learning targets after direct instruction:
Frank: I liked being in the different groups. For some things it’s better to work with different groups. Like if you need to ask questions or get them to explain. And sometimes you can learn something from other people that you didn’t know, different techniques, and get a different perspective. But when you’re learning a brand new thing or way too confused, it’s better to be separated so that you can have more help. I like how you did the different ways to look at the special right triangles with the tables and pictures to know where the patterns come from. That was cool. And I liked it when you used the cube to help us see what the problem was supposed to look like when we took notes (SI 2, February 26, 2015, p. 2).

Beth: I’m glad we worked in groups, because I panic when I’m on my own, especially when I see a word problem (SI 2, February 26, 2015, p. 2).

Ursa: Talking about the homework problems with the people who were in the other group last class helped me. I couldn’t figure out what to do next when I got the paper sometimes, but (other students) helped me do it. The pencils and cube thing helped me see how to figure out that cube problem on the notes. I’m more of a picture person. I’m glad that I was with people working on the word problems that were on the same level as me. We could take turns talking and feel relaxed about working them out. I was able to talk more and not feel stupid if I didn’t know what to do next, because we all were pretty much on the same level (SI 2, February 26, 2015, p. 2).

Susan: I felt that I understood what we had done in the last class and doing the problems in our groups helped me, too. I wanted to work with the other group, because I thought I knew what I was doing after the notes. It was really hard, though. I probably should have moved back to the other group, but I didn’t want to. I don’t think I can do those problems if I had to do them by myself (SI 2, February 26, 2015, p. 1).

Victor and Jeff, two of my advanced students, also talked about being in heterogeneous and homogeneous groups during the class period:

Victor: The group I was in going over the homework was okay. I liked being in the other group when we worked on the word problems. It was less boring than the first day, because if I already understand something I get bored doing it over and over. We got stuck on the one problem, but because we kept fighting about it and trying to figure it out, I was more interested than the other day (SI 2, February 26, 2015, p. 1).

Jeff: Going over the homework with the group you put me in at the beginning was all right, but not everyone had their homework. You gave us something to do that made sure everyone did something even if they
didn’t have it, so at least they didn’t just sit there and do nothing. The pattern thing for the special right triangles was helpful. I had looked in the book a little after doing the homework, and I saw those triangles, so I looked them up online before coming to class. What you did on the notes helped them make more sense to me. I liked getting the challenging problems instead of the ones that I knew that I could do already. Except for that one problem. We argued a lot on that one and we still didn’t figure it out. I liked working with people who understand the math the way I do. It’s less frustrating for me when we all work on trying to figure out the math (SI 2, February 26, 2015, p. 1).

Victor and Ursa both commented on how using the hands-on manipulatives helped them with visualization during problem solving:

Victor: Using the cube in class helped us think about what to do for solving the problem (SI 2, February 26, 2015, p. 1.)

Ursa: I like it when you draw pictures or when we get to use 3-D things to help us understand (SI 2, February 26, 2015, p. 2).

My students reflected on their understanding of the learning targets and their next steps in their learning.

Victor: Those word problems definitely pushed us. They were hard, but I think it’s better to further my knowledge doing those than the easier problems. Since we got a lot of them, you can see we’re working and learning more (SI 2, February 26, 2015, p. 1).

Susan: I’ve been doing the problems and talking more about them in class. That helps me when I’m with you, but I don’t know what to do when I’m at home (SI 2, February 26, 2015, p. 1).

Frank: During the problem solving part of class I did more than everyone, and then I stopped to see what everyone else was going to do because I didn’t want to do the wrong thing. I need to remember more stuff from the notes and problems we do in class (SI 2, February 26, 2015, p. 2).

Beth: I feel better when I’m working with the group. I can do it then. But when I do the problems on my own, I forget it all. Like on the assessment we did. I don’t think I did very well. I need more practice (SI 2, February 26, 2015, p. 2).
Implementing the Pythagorean Theorem Unit Day 3: Friday, February 27, 2015

Overview

The third day of the Pythagorean Theorem unit focused on the textbook lesson 9.5 (Distance in Coordinate Geometry and Circle Equations). The class period was 100 minutes in length, split into two 50-minute sections separated by a 30-minute lunch period. I had identified the state standard and written the learning targets for these lessons while planning the unit.

MAFS.912.G-GPE.1.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

I can derive, understand, and use the Pythagorean relationship on a coordinate plane.

- I can derive and use the distance formula to solve problems.
- I can derive the equation of a circle and use it to solve problems.

Opening Activities

Creating heterogeneous groups. Because I was concerned that there were going to be more questions from my struggling students from the assignment from lessons 9.3 and 9.4 about special right triangles and word problems, I decided to use the students’ self-assessment ratings after taking the Day 2 notes as a guide for creating their heterogeneous groups. Beth, Frank, Susan, and Ursa had shared that they preferred reviewing problems with other students who had greater understanding of the concepts, so grouping the students heterogeneously gave them this opportunity. Before reviewing the practice problems, I decided to give the students a warm-up problem that linked to the day’s lesson about the distance formula.
**Warm-up problem.** As the students walked into class, I handed them a warm-up problem on a half-sheet of paper. The problem assessed skills taught to the students earlier in the year. It required the students to use a coordinate grid to sketch the location of the three vertices of a triangle, identify cardinal directions (North and East), and identify that the triangle was a right triangle. The students then had to use the Pythagorean Theorem to find the distance between two points on the coordinate grid, which were two vertices of the right triangle. This warm-up had dual purposes of reviewing previously learned skills and concepts (plotting points on a coordinate grid, identifying a right triangle, using the Pythagorean Theorem to solve for a missing side length) and previewing the day’s lesson of deriving and using the distance formula.

Each of the groups worked together and they were all successful in solving the problem. I walked around the room and checked with each group as they completed their work. I noticed that Ursa asked Tammy to help her set up the problem, and I was pleased that Ursa showed more confidence and enthusiasm with using the Pythagorean Theorem than on Day 1 or Day 2. After all of the groups found the correct answer, we continued with the day’s agenda, moving to the review activity to check their homework assignment (FN, February 27, 2015, p. 1).

**Review activity.** Like on Day 2, I gave my students sticky notes and requested that each individually write down the numbers of the practice problems from the assignment that they found difficult to solve. During his reflection about Day 2, Jeff had shared his appreciation for my having every group member accountable for participating in the previous class’s review activity. To engage all of the students with reviewing how to solve story problems, I decided to have the students complete a small-group activity.
to examine the assigned practice problems from that lesson (FN, February 27, 2015, p. 1).

I asked each group to get one piece of chart paper and each group member to take a different colored marker. I assigned a different story problem from the assignment to each group, with two groups having the same problem (there were three problems assigned from lesson 9.4). I asked the groups to solve and show each step in the problem-solving process. I shared my expectations that every person in the group had to contribute and help each other understand the procedure for solving these story problems. Since each member of the group had a different colored marker, I was able to assess every person’s contribution to the problem-solving process from looking at the chart paper. As the students completed their posters, I circulated through the room to eavesdrop on their conversations (TJ, February 27, 2015, p. 1).

Students who had solved the challenging word problems during the previous class took leadership roles in their groups with organizing how to solve the problems and who needed to write down the different steps. Since they knew that they were accountable as a group, I noticed that students kept reminding their each other that they needed to make sure that the posters represented each person’s participation with a different color (FN, February 27, 2015, p. 2).

Although all of the students shared in the creation of the posters, it was clear that some of the struggling students were unsure of what steps to take and how to write down the information. I noticed that Nancy asked others in her group to repeat specific steps and give more explanation about how to solve the problem since she had been absent from class during lesson 9.4. Beth was very hesitant to write anything down on
her paper without Victor conveying to her exactly what to write. In contrast, Frank and Susan jumped in to work together on using the Pythagorean Theorem to calculate answers after asking other students in their group for help with setting up the problem. Susan seemed more confident after figuring out what the problem should look like, and Frank showed greater enthusiasm with solving the problems than he had been during the previous class period (FN, February 27, 2015, p. 2).

All of the groups hung their posters with the correct solutions on the walls when they were finished. The students then moved from poster to poster to examine each other’s work. Students who had attempted the assignment made corrections on their papers, and those who had not completed the practice problems made notes so that they could try to solve them on their own, or with the online podcasts, later (FN, February 27, 2015, p. 2).

While the students completed their posters in the group activity, I went through the individual sticky notes to determine which problems from the lesson 9.3 assignment the students wrote down most often so that I could review those with the whole group after we completed the story problems activity. I chose the five problems that the students indicated were the most difficult for them on the board for the whole group. I used a random name generator to call on students when I wanted to prompt them to give me the next step in the problem-solving process. If the chosen student did not know the next step, I allowed them to choose a friend in their group to help them figure out what to do next. Susan, who had worked on the more challenging problems after rating herself at a 3 on Day 2, needed help to answer questions. She had shared with me that she had struggled with solving the more difficult problems during her reflection...
and had indicated that she was not sure she would be able to solve story problems on her own (FN, February 27, 2015, p. 3). We concluded the review activity and moved to the direct instruction of lesson 9.5.

Classroom Instruction and Activities

Direct instruction: Distance formula. I distributed the scaffolded note sheets for the direct instruction of lesson 9.5, asking the students to rate themselves on the learning targets with a star as I read them aloud. I noticed that the ratings were lower than in previous lessons in the unit, with only Jeff and Victor rating themselves at a 1 and all of the other students rating themselves at a 0 (FN, February 27, 2015, p. 3). I recalled that while I was planning the unit, I had predicted that this lesson was likely to be challenging for my students, as most of them would not have much background knowledge or prior experience with the concepts and mathematics associated with this state standard (TJ, February 27, 2015, p. 2).

For the first part of the investigation for lesson 9.5, students used given segments on coordinate grids as hypotenuses of right triangles. They had to draw the legs of each right triangle along the grid lines and then find the length of each segment using the Pythagorean Theorem. I referred back to the warm-up that the students solved at the start of the class period, as the problem-solving method for these examples was almost identical to those they had used during the warm-up. The next part of the investigation required the students to graph a pair of points and find the distance between them. Some of the students were unable to make a connection between the previous examples and these new problems, and some of the struggling students did not attempt to begin (TJ, February 27, 2015, p. 2).
To engage the students, I used the random name generator to choose students as characters in the example problems. After choosing a name, the student became a “superhero” character who had to fly from one point to another. Many students, including my struggling and advanced learners, became animated, laughing and calling out next steps after I inserted the name of one of the students in the class (Quincy) into the example.

Mrs. Weller: “Superhero Quincy” has to travel from point (-1, 2) to point (11, -7). We need to find the shortest distance that “Superhero Quincy” could fly from one point to the other. How can we do this?

Quincy: Woo hoo!

When Quincy yelled out, the class laughed, indicating their engagement. I asked them how I should represent where he was going to fly on their coordinate grid. Jeff, Mike (another student in the class), and Susan explained the process.

Jeff: Plot the points and connect them with a segment.

Mike: Isn’t that just like the problems we just did, then?

Susan: Hey, you’re right. Now I know what we’re supposed to do.

I modeled plotting the points and connecting them with a segment on the board while the students completed the process on their own papers.

Susan: Now draw the legs in and make it a right triangle.

I drew in the legs and was about to ask for the next step when Quincy interrupted.

Quincy: Use the Pythagorean Theorem to find out how far I’m going to fly (FN, February 27, 2015, p. 4; TJ, February 27, 2015, p. 3).

The students had all completed the problem, so I asked them to compare their answers in their groups and give me a thumbs-up if they had the same answers or thumbs-down if there were any differences. All of the students gave me a thumbs-up,
so I displayed my answer on the board, and verified that the students had the correct answer. Susan wiggled in her seat, excited that she had solved the problem (FN, February 27, 2015, p. 4).

The next example in the investigation included points that were further apart and it was unreasonable to plot them on a coordinate grid. To keep the students engaged, I again used the random name generator to create a “superhero” for the problem. I asked them how we could find how far “Superhero Victor” had to fly if we did not use a coordinate grid. I reminded the students that the distance that “Superhero Victor” had to fly was still the hypotenuse of a right triangle, referring to the figure I included with the example (TJ, February 27, 2015, p. 4).

A student asked if they needed to find out the length of the legs of the triangle. I answered affirmatively, and I reminded them that we had already calculated horizontal and vertical distances in a previous unit that year when we reviewed slope (which they had learned in Algebra 1). When I asked them how we could find the lengths of the legs in the right triangle for “Superhero Victor”, Victor responded.

Victor: Figure out how far you go horizontally by subtracting the two x’s, and then the two y’s for the vertical leg (FN, February 27, 2015, p. 5).

I asked the students to do that in their groups and then tell me what values they found for the lengths of the legs. Nancy, Ursa, and Beth looked confused and watched the other students in their groups do the work first. I noticed that Jeff, Tammy, and Lisa automatically explained to students in their groups what they had done to find the leg lengths (TJ, February 27, 2015, p. 4).

After verifying that all of the groups had found the horizontal and vertical distances, I reminded them to use the Pythagorean Theorem to find how far “Superhero
Victor” flew. Nancy said, “That’s the part I can do” (FN, February 27, 2015, p. 5). I asked the groups to give me a thumbs-up when they had their final answer. When all of the students had given me the signal, I revealed the answer, and all of the students indicated that they had the correct answer (TJ, February 27, 2015, p. 4; FN, February 27, 2015, p. 5).

I wrote the Distance Formula \( distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \) on the board, and I asked the students to copy it on their lesson 9.5 scaffolded notes. I then explained that they used this formula to solve the “Superhero” problems. \( x_2 - x_1 \) was the horizontal distance between the given points, so its value was the length of one of the legs of the right triangle, and \( y_2 - y_1 \) was the vertical distance between the given points, so its value was the length of the other leg of the right triangle. I asked the students to find the answer for Example A on their note sheets, which did not have a picture. I reminded them to think about how the “superhero” still needed to fly between the points and that they still had a right triangle (TJ, February 27, 2015, p. 5).

I noticed that some students instinctively helped the struggling students within the groups. I heard a student tell Beth, “Remember to subtract the \( x \)’s and then the \( y \)’s to find the legs.” I saw Beth nod her head and glance at the other student’s paper to verify what she had done. Lisa and Tammy automatically checked in with students in their groups, giving advice and support. Jeff and Victor verified answers, but they were not as quick to offer assistance until the other students in their groups had attempted to solve the problem (FN, February 27, 2015, p. 6).

As the groups were finishing the example, I circulated through the room to check their answers. After I confirmed that they had all solved the problem correctly, I asked
them to rate themselves on the first learning target on the self-assessment at the top of their lesson 9.5 scaffolded note page since the lunch period was about to begin. No students rated themselves at a 0 or 1, Nancy rated herself at a 2, and all of the other students rated themselves at a 3 or 4 (FN, February 27, 2015, p. 6). I felt that this part of the lesson was very successful, and the students left the room to go to lunch (TJ, February 27, 2015, p. 5).

**Direct instruction: Circle equations.** When the students returned from lunch, I reminded the students that they had been very successful with using the distance formula to solve problems. As I began the next part of my direct instruction for lesson 9.5, I thought to myself that transferring the same concept with writing a circle equation using the center and the length of the radius would be easy for the students. Since I was using coordinate grids and showing them where the right triangles were located within the circles, the students were essentially using the same formula as they had when they solved their “Superhero” problems. However, some students were confused almost immediately as soon as a circle appeared. Tammy was noticeably agitated, and this was surprising as she was one of my more advanced students who had been part of the group that solved the challenging problems on Day 2 (TJ, February 27, 2015, p. 6).

I decided to add some examples to the direct instruction part of the lesson by making up a story about a pony ride. I drew a stick figure pony attached to a stake by a rope on the board, and I showed the students that the pony traveled in a circle during his ride. I asked the students to identify the part of the circle that the rope represented. Jeff correctly identified the radius, so I then added coordinates to the picture, asking
them to write the equation of the circle if the stake was located on a map that resembled
a coordinate plane with the stake located at point (3, 2), and the length of the rope was
8 units. I instructed the students to look at the examples they had on their note pages
to help them write the equation (FN, February 27, 2015, p. 7; TJ, February 27, 2015, p.
6).

I noticed that my struggling learners did not understand how to write the equation
of a circle. Jeff, Lisa, and Victor were all successful in writing the equation, but Beth,
Frank, Nancy, Susan, and Ursa all relied on members of their groups to help explain
what to do. I saw that Tammy was still looking stressed, as she was not grasping the
concept as quickly as she usually did. Although more students in the class seemed to
understand how to solve the problem, it was clear that my struggling learners, and
Tammy, still did not comprehend what they were doing (TJ, February 27, 2015, p. 6).

To extend the problem, I then asked if the length of the rope would change if I
changed the location of the stake. A few of the students said no, and some of them
shook their heads in agreement. I changed the coordinate to (-4, 6) and asked the
groups to write the equation. The same students explained how the equation changed
within their groups, but I saw a few students begin to show more understanding. I saw
Tammy sit up straighter in her seat as though she was beginning to comprehend what
she was doing, and I noticed that Frank and Susan both nodded their heads as
someone in their group explained what he did to change the equation (FN, February 27,
2015, p. 8).

I then gave the students a question that required them to find a circle’s new
equation after translating and dilating it. I incorporated previously learned material
based on learning targets from a previous unit with the material that students were responsible for mastering in the current unit. I knew that items on the Geometry End-of-Course test could combine several state standards into a single question, and I wanted to expose my students to these types of questions. I had also hoped to challenge my more advanced students by pushing them to solve and explain how to combine what they had learned to others in their groups (TJ, February 27, 2015, p. 6).

I explained how to do the problem step-by-step. To give the students more practice in their groups, I wrote a similar problem on the board and asked the groups work together to solve it. Jeff, Victor, and Lisa all took the lead with starting the problem in their individual groups (FN, February 27, 2015, p. 8). I noticed that several of the struggling students looked confused, and they listened and watched the more advanced students solve and talk through the problems. I explained how to solve the problem to the whole group after all of the small groups had found their solutions (TJ, February 27, 2015, p. 7).

I finished the direct instruction of lesson 9.5 by having the students complete the self-assessment by marking where they were on the scale with a checkmark.

**Differentiated activity for homogeneous groups.** I wanted to move the students into homogeneous groups to complete an activity related to finding the equation of a circle (Schwartz, n.d.). The activity required that the students write equations for circles with varying sizes of radii, and required them to move the circles to different locations, incorporating in the standards related to translating and dilating circles. This activity described a dog whose leash is tied to a stake, and the area where the dog could roam in its yard. The area the dog could roam was actually the area of a
circle, and the location of the stake was its center. The dog’s leash length varied through the problem. Students had to determine where in the yard the dog could be located and write circle equations to represent that location and the size of the circle. I read the introduction of the activity aloud to the whole group. Before breaking into small groups to complete the problem, I reminded the students that the dog’s leash was similar to the pony’s rope in the examples that we did together. I also reiterated the directions, asking the groups to identify the center of the circle (coordinates) and using the length of the leash as the radius when writing their circle equations.

I allowed the students that rated themselves at a 3 or 4 form their own groups near the back of the room, informing them that compasses were available to use to draw circles on their activity sheets. Jeff, Lisa, and Victor all chose to work in the same group for the activity (FN, February 27, 2015, p. 9). The groups started to work immediately, and I was able to turn my attention to the students who were struggling with understanding the learning target.

I asked the students who rated themselves at a 1 or 2 (there were no ratings of 0) to form groups near the front of the room. Tammy and Ursa had chosen to work in the same group, Nancy and Susan were together in another group, and Beth and Frank were in a third group (FN, February 27, 2015, p. 9).

Instead of using compasses as the students who had rated themselves with a 3 or 4 were using, I gave the struggling students hands-on manipulatives to aid with visualizing the problem. The manipulatives were blue circles that represented the different areas of the circles with the specific radii lengths created by a dog’s leash. While the students worked together, using the manipulatives to determine where the
dog could be located on the coordinate grid, I moved between the three groups, asking specific questions to get the students to verbalize the process of writing circle equations (TJ, February 27, 2015, p. 7).

- What numbers do you use in the equation?
- Why did you put the numbers in the equation in that order?
- Why did you use that number after the equal sign in the equation?
- When the circle moved to a different location, what changed and what stayed the same, and why? (FN, February 27, 2015, p. 9).

Tammy rated herself at a 2 for lesson 9.5 (FN, February 27, 2015, p. 9). This was different from both Day 1 and Day 2, because she had been part of the independent group (IND) and had rated herself at a 3 or 4 on lessons 9.3 and 9.4 on Day 2 (TJ, February 23, 2015, p. 1; TJ, February 25, 2015, p. 2-3). I noticed that she struggled when we started the investigation for writing circle equations, but I had thought she understood the concepts by the end of direct instruction. However, I could see that having the manipulatives made Tammy more comfortable because she could actually visualize how the location of the circles determined how to write the equations. Tammy talked her way through each problem, saying every part of how she was solving the problem aloud. All of the students in her group silently listened and watched her place the circles on the worksheet (FN, February 27, 2015, p. 10). Since I could hear Tammy’s thought processes and could verify that she was beginning to master the learning targets, I made sure I asked my questions to the other students in the group so that I knew they understood how to write circle equations (TJ, February 27, 2015, p. 9).

In the second group, both Nancy and Susan were trying to work on the problems individually, not wanting to work with others in the group. Susan was able to answer my
questions correctly, but Nancy struggled, giving incorrect answers worded as questions back to me. I eventually was able to lead her to a couple of correct answers, but I could see that she was not internalizing the process for finding the equations of circles (FN, February 27, 2015, p. 10).

Beth and Frank worked well together in the third group, taking a methodical approach to their work, checking every step with each other and agreeing on their circle equations before moving on to the next part of the problem (FN, February 27, 2015, p. 10-11).

I checked in with the other groups in the back of the class occasionally, but they were all doing well and staying on task. Jeff and Victor were quite animated and loud, arguing about where to place their largest circle on their coordinate grid. I wondered why they were not using compasses to construct their circles, as having them would make it easier to determine where to locate the largest circle. Since they were engaged in the activity, I decided to let them push each other to find the answer on their own (FN, February 27, 2015, p. 11).

**Wrapping up instruction.** 10 minutes before the end of the period, I asked the students to hand in their activity papers so that I could look over them before our next class meeting. I distributed the students’ graded formative assessments and the EPI scoring rubric I used to grade them. I explained that the standards-based grading tool indicated what they needed to work on to improve their understanding of the learning targets. I offered to answer any questions they had, and said I would be available later if they wanted to look at their work and the rubric in more detail.
As the students packed up their belongings at the end of class, I reminded them to use the online podcasts if they needed assistance from Virtual Weller to complete the practice problems on their assignment. I also announced that if the students wanted copies of the solutions from the challenging story problems from the activity on Day 2, they could get them from me before they left. Jeff, Lisa, and Victor all obtained copies of the solutions before leaving for the day (TJ, February 27, 2015, p. 11).

**Student Reflections**

The students discussed the classroom activities that helped them understand or extend what they were learning about the Pythagorean Theorem. My struggling students, Frank, Nancy, and Ursa, talked about using the hands-on manipulatives to help them understand circle equations:

Frank: When we were laying the blue circles out on our maps for the dog, we were able to visualize where to place the circles and then write equations. I liked how the colors popped out when we were using the circles, and I really don't like using a compass (SI 3, February 27, 2015, p. 2).

Nancy: Doing the dog activity, it made more sense being able to use the circles and move them around, especially when we had to switch how big the radius was and figure out how the equations were the same or different. I don't think I would have been able to figure out how to do the work without them (SI 3, February 27, 2015, p. 2).

Ursa: I liked using the blue circles for dog activity. The grid showed me the places the circles went. I figured out how the last number of the equation didn't change if the circle was the same size (SI 3, February 27, 2015, p. 2).

My advanced students, Jeff, Victor, and Tammy, reflected about their experiences with understanding the learning targets:

Jeff: I’m glad we were able to work in the groups we wanted to work in for the dog problem (SI 3, February 27, 2015, p. 2).

Victor: When we did the dog problem, I forgot to use a compass. I guess we would have finished quicker if I had remembered. We had a lot of fun
together, because we really didn’t have to explain anything to each other (SI 3, February 27, 2015, p. 1).

Tammy: I did not rate myself very high on the self-assessment and went to the other group than I had been in before. I didn’t feel strange doing that even though I usually work with the other group. I knew those people who are normally in the group that I’m in were going to go fast and beyond what you’re supposed to know. And if I wasn’t feeling completely comfortable with the topic yet I wasn’t going to want to do that because I would confuse myself completely and I wouldn’t grasp the basics of the concept I was trying to understand. The blue circles helped me a little. I could have done it without the circles because you just had to find where to put them and count the squares to get the equation. I really didn’t need the manipulatives – and I don’t like compasses. I guess I just needed to talk my way through the problems to understand what to do. I know the circle equations after doing the activity and am comfortable solving problems with them (SI 3, February 27, 2015, p. 1).

The students discussed how they demonstrated mastery or advanced understanding of the learning targets and the actions they were going to take next, and some commented on the format of the paper-and-pencil formative assessments from Day 2 and the scoring rubric that I used for grading it. Jeff, Tammy, and Victor, my advanced students, shared their thoughts:

Jeff: I like the new format for the assessment with the rubric. It might help with figuring out where you need more help or practice (SI 3, February 27, 2015, p. 2).

Tammy: You kept listening to me work out the problems during the dog activity, so you heard that I was finally understanding what I was doing. I’m definitely going to use Virtual Weller on the homework (SI 3, February 27, 2015, p. 1).

Victor: You know, I still don’t think I get special right triangles completely. I don’t know, I guess I’ll try to go back and go over the problems (SI 3, February 27, 2015, p. 1).

My struggling learners also reflected on the next steps in their learning:

Ursa: I will listen to Virtual Weller some more when I do my homework. I like working in groups better than working alone (SI 3, February 27, 2015, p. 2).
Frank:  For some reason I completely messed up the squared part of the formula on my quiz, so it messed up all of my calculations. When I looked over the rubric, I realized that I forgot all about the pictures we made with the squares. I guess I need to do more than glance at my notes before a quiz and practice more at home. I just don’t want to have to think about it after school. I’m okay with it when I’m in class, and I get the stuff fine when I’m there, but I want to do other things when I get home. I know I need to do the assignments so that I can do better on the test (SI 3, February 27, 2015, p. 1-2).

Nancy:  I really like the EPI grading thing that you used for the quiz. Are we using that on the test, too? I’m confused on some of the homework problems. I understand the Pythagorean Theorem, but there’s a lot of extra stuff, like with the special right triangles that’s still confusing. I think I need more help. I use Virtual Weller, but I sometimes tune out when I’m listening because I’m really tired when I get home. Are we going to do more problems with those and the circles? (SI 3, February 27, 2015, p. 2).

Beth:  I guess practicing more could help if I have time to do it. Can we do stuff where someone is there explaining it to me and everyone else in my group? (SI 3, February 27, 2015, p. 3).

Implementing the Pythagorean Theorem Unit Day 4: Monday, March 2, 2015

Overview

The class period for Day 4 of the Pythagorean Theorem unit was 50 minutes in length. The focus of this day was to review the concepts taught during the previous class focused on textbook lesson 9.5 (Distance in Coordinate Geometry and Circle Equations). The state standard and written learning targets were the same as the third day of the unit.

MAFS.912.G-GPE.1.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

I can derive, understand, and use the Pythagorean relationship on a coordinate plane.

• I can derive and use the distance formula to solve problems.
• I can derive the equation of a circle and use it to solve problems.
Classroom Instruction and Activities

Review activity for heterogeneous groups: Part 1. At the start of the class period, I noticed that eight of my students were absent due to personal reasons, academic field trips, or sports functions. I decided that I would not give the paper-and-pencil formative assessment I had written for lessons 9.3 and 9.5 since a large percentage of my students were missing. Instead, I decided to use the results of the previous day’s formative assessments to group my students for reviewing the previous class period’s assignment from lesson 9.5, postponing the paper-and-pencil formative assessment to the next class period. I created heterogeneous groups using a more random method by using a deck of cards (TJ, March 2, 2015, p. 1).

After observing my students’ leadership skills, work ethic, and problem solving abilities in the different groupings during the first three days of the unit, I chose five students to be “group captains.” Jeff, Lisa, Tammy, and Victor were among the five students chosen as group captains (FN, March 2, 2015, p. 1). Although Tammy had placed herself in a group that needed more assistance for the circle activity, I had determined that she had mastered the process for finding a circle equation through formative assessment (TJ, March 2, 2015, p. 1). She had also expressed her confidence in solving the problems during her interview with me (SI 3, February 27, 2015, p. 1). I gave each captain one card (each labeled with a number 1 through 5), asking them to each choose to sit at a different area in the room (FN, March 2, 2015, p. 1).

The remaining students, who had rated themselves from 1 through 4 during Day 3, each chose cards at random from a deck (with cards each labeled with a number 1 through 5), and they joined their group captains. Beth and Susan joined Jeff in his
group, while Victor, Nancy, Lisa, and Tammy were each located in one of the four remaining groups. Ursa and Frank were both absent from class. I still had some control with forming the groups, as I made sure that each one included a student confident with his mastery of the learning targets from the unit. I placed the rest of the students into groups randomly to avoid them feeling that I was placing them in specific locations because I thought they were struggling with understanding the material (TJ, March 2, 2015, p. 1).

I used an activity to help students review and understand the practice problems assigned at the end of Day 3. I told the whole class that they were in their “expert” groups, and I described how the activity would proceed. Each member of each “expert” group received a copy of one of the practice problems that I selected from the lesson 9.5 assignment (e.g., every member of Group 1 had the same assigned problem, every member of Group 2 had the same assigned problem, etc.). The members in each group would work together to become “experts” at solving their assigned problem. The groups had 5-7 minutes to solve their problem, making sure they all knew and could explain each step of the problem-solving process. All members of the group were responsible for helping each other understand and feel comfortable explaining how to do the problem to others. If the members of an “expert” group needed assistance, they had one “save” that they could use to get help from me, but there had to be a unanimous vote to use the “save” (signaled by all of the group members raising their hands together) (TJ, March 2, 2015, p. 2).

When I started the timer, all of the students started to work on their problems with the captains in each group beginning the conversations. I noticed that if a student did
not understand a step, they would interrupt whoever was talking and get clarification. Susan and Nancy each asked for more explanations numerous times in their groups. Group 2 used their “save,” when Victor had difficulties breaking a problem into smaller steps for another student. Group 5 also used their “save,” because Tammy (who had struggled during Day 3) wanted to verify that the circle equation that they had written was the correct answer to the problem (FN, March 2, 2015, p. 2).

Review activity for heterogeneous groups: Part 2. After all of the “expert” groups finished solving their problems, I reassigned the students to new “teaching” groups. One (or two) students from each of the “expert” groups had the responsibility of teaching the problem they had just solved to members from other “expert” groups. I gave each student a sheet that had the five problems on it so that they could all work out the problems as the “expert” explained the steps. I informed the groups that they had a “save” available to them if they wanted it, but the vote to use it still had to be unanimous. The students moved to their new “teaching” groups (TJ, March 2, 2015, p. 2).

The students took turns teaching their problems to each other within the “teaching” groups, and I circulated between the groups as the activity progressed. Every student was engaged in listening, writing, and asking clarifying questions through each presentation (FN, March 2, 2015, p. 2-3). The “teaching” part of the activity took between 12-15 minutes, as the students seemed comfortable explaining their problems to each other in an efficient manner (FN, March 2, 2015, p. 3).

Differentiated activities for homogeneous groups. After completing the problems, I asked the students to rate themselves using the self-assessment ratings
that we used from the scaffolded note pages, holding up their fingers “over their hearts”
with their numbers, so that I could see how they self-assessed without them revealing
their ratings to others. Students who rated themselves as a 3 or 4 would have the
choice to work individually or in small groups on the review assignment. Students who
rated themselves as a 1 or 2 would have the opportunity to work with me on the
assignment in a small group.

The majority of the students rated themselves as a 3 or 4. I told the students
they could choose their own groups, and I noticed that the students gathered in the
same combinations from the previous class periods. Although they were in groups, the
students did not converse with each other very much. They were quiet and on task as
they worked on the review assignment (TJ, March 2, 2015, p. 2).

Only a few students rated themselves as a 1 or 2, so the small group positioned
itself at the front of the room with me. Beth and Nancy started to work on their assigned
practice problems together, occasionally asking me for assistance. Instead of just
answering their questions, I would ask them a question to prompt them to find the next
steps on their own and think about how they were solving the problems. These
questions included:

- What do you need to find/what is the problem asking you to do?
- Did you draw a picture so that you can see what you need to find/do?
- What is the next step? Why?
- Does what you are doing make sense? Is it reasonable? Why?
- What does your answer mean? Is it reasonable? Why? (FN, March 2, 2015, p. 3)
**Wrapping up instruction.** The small group was able to complete about one-third of the assignment before the end of the period. I noticed that the students who had rated themselves higher on the self-assessment scale had completed between one-half and three-quarters of the assignment. I asked the students to complete the assignment before the next class period, reminding them to use the online podcasts for assistance (TJ, March 2, 2015, p. 3).

**Student Reflections**

The students talked about the classroom activities for reviewing the Pythagorean Theorem. Beth and Nancy, two of the students who were struggling with mastering the learning targets, shared their thoughts:

Beth: I don’t know. Sometimes I think I understand what I’m doing, but then I can’t. It’s not like when we were learning the stuff at the beginning when I could ask whatever questions I wanted. I’m supposed to know it now and not ask questions, because then I feel stupid if I do (SI 4, March 3, 2015, p. 2).

Nancy: I’m still confused on some of the problems. The questions you asked us at the end of class when we were doing the work helped some when I was setting up the problems. Can we do something in groups where people who know what they are doing can help me again? (SI 4, March 3, 2015, p. 2).

My advanced learners, Tammy and Jeff, also reflected about the review activity:

Tammy: I liked the activity we did because it gave us a chance to talk to each other about solving the problems. I like being able to explain what I’m thinking (SI 4, March 3, 2015, p. 1).

Jeff: I already understood the math we were doing in our groups today, but more review is okay. I’m not sure Beth and Susan really understand what they are doing on their own. They can do it if I help them with every step, but they are really lost if they have to do it by themselves (SI 4, March 3, 2015, p. 1).
Overview

The fifth day of the Pythagorean Theorem unit was a 100-minute class period split into two 50-minute sections separated by a 30-minute lunch period. I planned an activity for the students to review material from all of the lessons in the Pythagorean Theorem unit during this class period. Therefore, the students would review all of the state standards and learning targets I had identified while planning the unit.

MAFS.912.G-SRT.3.8: Use trigonometric properties and the Pythagorean Theorem to solve right triangles in applied problems.

I can derive, understand, and use the relationship between the length of the legs and the length of the hypotenuse of a right triangle to solve problems.

- I can define the Pythagorean Theorem.
- I can find missing lengths of sides of triangles using the Pythagorean Theorem.
- I understand and can use the converse of the Pythagorean Theorem to solve problems.
- I can use Pythagorean triples to determine if a triangle is a right triangle.

MAFS.912.G-GPE.1.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

I can derive, understand, and use the Pythagorean relationship on a coordinate plane.

- I can derive and use the distance formula to solve problems.
- I can derive the equation of a circle and use it to solve problems.

Opening Activities

Formative assessment. I did not immediately assign my students to specific groups at the start of the class period, because I wanted to use the results of the paper-and-pencil formative assessment for lessons 9.3 and 9.5 to place them in groups. I used the activity from Day 2 to formatively assess the students on the learning targets.
for lesson 9.4 (word problems) (TJ, February 25, 2015, p. 4), so that is why the paper-and-pencil formative assessment focused on determining my students’ understanding of lessons 9.3 and 9.5.

The paper-and-pencil formative assessment contained four problems based on the learning targets from lesson 9.3 (special right triangles) and 9.5 (distance formula and circle equations) (see Figure 5-7). The students could use calculators during the assessment if they wished, as they would be able to use calculators during the Geometry End-of-Course Test. However, I requested the students to leave their answers in simplified radical form (if applicable) for the first two problems.

Figure 5-7. Day 5: Student formative assessment
As the students turned in their papers, I graded them so that I could use the data from them to form groups for the review activity (Table 5-2). Lisa was absent from class, so she did not take the formative assessment (FN, March 4, 2015, p. 1).

<table>
<thead>
<tr>
<th>Pseudonym</th>
<th>Score (out of 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beth</td>
<td>0</td>
</tr>
<tr>
<td>Frank</td>
<td>0</td>
</tr>
<tr>
<td>Ursa</td>
<td>1</td>
</tr>
<tr>
<td>Nancy</td>
<td>2</td>
</tr>
<tr>
<td>Susan</td>
<td>3</td>
</tr>
<tr>
<td>Victor</td>
<td>3</td>
</tr>
<tr>
<td>Jeff</td>
<td>4</td>
</tr>
<tr>
<td>Tammy</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 5-2. Day 5 formative assessment results

Creating heterogeneous groups. I arranged the students into groups, placing 1-2 students scoring 3 or 4 on the formative assessment with 1-2 students scoring 0-2 on the formative assessment. Another consideration when forming the groups was creating an atmosphere where my struggling students felt at ease asking questions of the more advanced students if they needed help solving the review problems. I announced who was in each group for the activity, and the students began to change to their new seats (TJ, March 4, 2015, p. 1).

As the students were sitting down with their new groups, four students walked into the room late because they had been finishing an assessment in another class. Since these students did not take the formative assessment that the other students had just completed, I decided to add them to groups based on the data from formative assessments from previous class periods. Tammy and Frank were together in a group, and Jeff, Nancy, and Ursa were in another group together. Victor, Susan, and Beth were each in one of the other three remaining groups (FN, March 4, 2015, p. 1; TJ, March 4, 2015, p. 1).
Classroom Instruction and Activities

Review activity: Game show. The activity for reviewing the Pythagorean Theorem unit was a game show format where students would choose a question from one of five categories. There were five questions per category, each assigned point values from 100 to 500. The five categories were Basic Conjectures, Using the Theorem, Distance and Radicals, Special Right Triangles, and Word Problems and Circles. Each student had individual accountability during the activity, because all members of the group had to show their work on their own paper. I used the PowerPoint program to create the game board, and I designed the individual student activity paper for the activity, organizing it to match the categories and point values for each problem (Figure 5-8).

Teams were assigned a color (red, blue, green, yellow, and orange), and given a stack of small pieces of paper matching the teams’ color. I explained that teams would take turns choosing a question from the game board. Individuals in each group had to
work together and help each other understand and solve the problem. As a team, they had to come to consensus about their final answer for the problem. They would write their final answer on one of the small piece of colored paper, and a team member would bring it to me. All groups that had written the correct answer on the paper would receive the designated number of points for that problem. After each problem, groups could ask for clarification or more explanation from me if they had questions.

As the game progressed, I noticed that Nancy and Susan were putting a great deal of effort into understanding each problem, consistently asked Jeff to explain how he solved some of the problems. Tammy took time to talk through each step of each problem while the other members of her group, including Frank, listened and occasionally asked why she chose to solve a problem in the manner that she chose. In Victor’s group, some students just watched and copied what he and another student worked through together. I encouraged Ursa and Beth, who had both been struggling throughout the unit, to take a more active role and ask questions of their group members, but they both chose to passively listen and copy the work from their groupmates (FN, March 4, 2015, p. 2-3).

At the end of the game show review activity, the individual members of the teams with the highest point total each received a small box of Nerds® candy, and the individual members of the rest of the teams all received a piece of Starburst® candy. After completing the heterogeneous review activity, I used the remaining class time to review with a smaller group of students who had not rated themselves at a 3 or above (TJ, March 4, 2015, p. 2).
Differentiated activities for homogeneous groups. There were 20 minutes left in the class period, so I decided to separate the students into two homogeneous groups for a final review time. Using the formative assessment scores from the beginning of the class period as a guide, I separated the students into two groups. Students who scored a 3 or 4 on their assessment were given a worksheet from the textbook with challenging problems focused on the learning targets from lesson 9.5. Jeff, Susan, Tammy, and Victor were among these students, and I allowed them to choose the members of the groups with which they wished to work. Students who scored a 1 or 2, including Beth, Frank, Nancy, and Ursa, worked with me at the board reviewing practice problems from the textbook review assignment I had asked them to complete at the end of Day 4. I told the students who scored a 3 or 4 on the formative assessment, as well as the students who had arrived to class late, that they could choose to review problems with me if they wanted to instead of solving the problems on the worksheet (TJ, March 4, 2015, p. 3).

I asked the students in the group at the board to choose practice problems from their review assignments that they would like me to solve and explain. The questions that the students chose focused mainly on the basic use of the Pythagorean Theorem and solving problems using special right triangles (FN, March 4, 2015, p. 3). The students did not ask any questions about using the distance formula or circle equations, even though those sections were difficult for many of the students, especially my struggling learners. As I worked through the solutions for each problem, I used a “think-aloud” process, telling the students what I was thinking and the questions I was asking myself, making sure that my answers made sense in the context of each problem. I
hoped that by modeling the problem-solving process aloud, students would continue to use the methods on their own (TJ, March 4, 2015, p. 3).

**Wrapping up instruction.** Near the end of the class period, I gave students who had worked on the worksheet a copy of the solutions, telling them that it was not an assignment, but they could continue solving the problems on their own for practice. I reminded the entire class that they should use the online podcasts on Virtual Weller as a tool for review. I asked the students to choose practice problems that I had assigned for each lesson and solve them again for practice, checking their solutions online. I suggested they review special right triangles, the distance formula, and circle equations, since had asked about those topics most frequently during the review activity that class period. I invited students to stay after school for extra help if they had any other questions, but all of the students left when the period ended (TJ, March 4, 2015, p. 3-4).

**Student Reflections**

The students talked about the teaching methods and structures used in class that helped them understand or extend what they were learning about the Pythagorean Theorem, and some of them went into more detail about being in both heterogeneous and homogenous groupings during the different activities throughout the unit. Tammy and Victor, two of my advanced students shared their thoughts about being in their groups:

Tammy: I was okay with the game. Eddie and Quincy worked on all of the problems and were interested in knowing what to do and get points for our team. Frank didn’t do as much. Well, sort of. He copied down what I was doing and knew a little, but he didn’t really talk a lot and help the group. Being with people at the same level when I’m learning really helps. Like when we did the word problems. With the circle equations, I was with a different group than I was before so that I could figure out what to do. For reviewing, I’m okay being with different people, like we did for the game. I don’t get really frustrated if other people in the group need help, because I
think I review more when I have to explain what I’m doing to other people. Helping them helps me (SI 5, March 5, 2015, p. 2).

Victor: I didn’t want to be in my group for the game. I wanted to be with Jeff or Annie, because we know the math. Sometimes I get tired of having to explain to others. I know I can help, but if they don’t want my help, then I get angry and frustrated (SI 5, March 5, 2015, p. 3).

My struggling students, Frank, Nancy, and Ursa, talked about their experiences in heterogeneous and homogeneous groups:

Frank: For review, I like more diverse people in my group. Maybe another person on my level and then some people who are a little more above me so that they can help me out. For explanation purposes, not just for the game. When I’m learning new material, I want people on the same level as me because if there is someone that knows what they are doing, they will do all of the work and then I don’t do anything. I don’t need to do it, because they already did it for me. I think I know what I’m doing when people are talking about the problems. Tammy talked through every problem in the game, and it all made sense to me (SI 5, March 5, 2015, p. 1).

Nancy: I feel better about taking this test than I usually do. I feel like I’ve been able to ask more questions and have lots of people explain the problems to me. When we learn the new information, I learn better and don’t feel dumb asking questions when I know that everyone around me is in the same place. But when we review, I want people who get the stuff to help me. The different groups have sort of forced me to talk to more people that I usually do in class. Usually we stay with the same people, but changing it up every day has been kind of fun (SI 5, March 5, 2015, p. 1).

Ursa: The different groups for learning the material, you know, working with kids who are at the same level as me, has been working pretty well, I think. And then, when we review, having kids who can explain what I might be missing and helping me catch up helps. It’s not like when we’re learning it and I’m feeling lost. I know more than I did when we started, and the people who really know what they’re doing helps me fill in some of the blanks that I have and make it easier for me. The activities that you did in class where we worked together and see the problems helped, and I was more motivated to work when we had the game (SI 5, March 5, 2015, p. 2).
I also asked the students to discuss their thoughts about and plans for studying for the upcoming summative assessment. Both the advanced and struggling learners shared how they were going to prepare for the test:

Tammy: I’m going to practice problems from old assignments that I’m still confused about with Virtual Weller (SI 5, March 5, 2015, p. 2).

Victor: I think I’m going to do well on my test. I knew how to do most of the problems on the review. I still need to work on my special right triangles and make sure I don’t mix up the 30-60-90 with the 45-45-90 (SI 5, March 5, 2015, p. 3).

Frank: I will try to review for the test, maybe look through my notes or listen to Virtual Weller (SI 5, March 5, 2015, p. 1).

Ursa: I need to practice on my own. I still couldn’t get a lot of the answers by myself during the game. I’ll practice with Virtual Weller (SI 5, March 5, 2015, p. 2).

**Summative Assessment: Friday, March 6, 2015**

**Overview**

I administered the summative assessment during a 100-minute class period. I obtained permission for my students to eat during the middle school lunch period instead of the high school lunch period, because the middle school students eat lunch before the 100-minute block at the end of the day. By eating lunch at the earlier time, the students did not have to take their summative assessment in two sections separated by their lunch period, but, instead, had an uninterrupted block of time to complete the assessment. Students who were absent on the date the summative assessment was administered in class (Day 6: March 6, 2015) would take the test after school the following week (March 10, 2015).
Administration of the Summative Assessment

As the students gathered their pencils and calculators to take the individual summative assessment, I reminded the students that they would be graded using the EPI scoring rubric, referring to the summary of what EPI meant that I had written on the board before class. I requested that they show me all of their work on their papers, as I would use all of the evidence on their papers to determine what they knew about the Pythagorean Theorem unit. I distributed the summative assessment and asked them to glance through the assessment as I previewed it with them. I explained that there were 11 problems on the assessment, which was less than they had seen on their previous units’ summative assessments, but I assured the students that I would be able to determine if they had mastered the learning targets for the unit by examining their work on these 11 problems. As they began, I wished the students luck and reminded them that they had up to 90 minutes to complete the assessment (TJ, March 6, 2015, p. 1-2).

My advanced students looked relaxed and focused as they took their summative assessment. Jeff and Victor both seemed confident when they submitted their work. Victor grinned and whispered, “I got this,” as he handed it to me. Tammy told me that she checked each problem at least two times to make sure that she included everything. When Lisa gave me her assessment, she looked pale and tired, and she asked to go to the nurse because she was not feeling well (FN, March 6, 2015, p. 1-2).

My struggling learners did not seem as confident as my advanced students. Nancy seemed anxious when she gave me her paper, whispering, “I think I did okay on this.” Susan handed her paper to me and quickly walked away, avoiding making eye contact with me. Ursa, looking discouraged, explained that she forgot everything as
soon as she started the assessment, and she said, “I’m sure I bombed that whole thing” as she packed up her materials to leave (FN, March 6, 2015, p. 1-2).

Beth and Frank were absent on the day the summative assessment was administered in class, so they came after school on Tuesday, March 10, 2015, (4 days after the original administration date) to complete the summative assessment. Before distributing the assessment, I asked them if they had any questions related to the unit’s learning targets that they wanted me to answer. Beth asked me if I would explain the special right triangle rules, so I pulled out the chart papers I had made on Day 2 with the Isosceles Right Triangle Conjecture and the 30°-60°-90° Triangle Conjecture. After I explained the relationships between the legs and hypotenuse of each kind of triangle, I asked them if they needed any more information about special right triangles. They said no, and Beth added, “I understand how to use the rules now, thanks.” When I asked Frank and Beth if they had any other questions, they both indicated that they did not and were ready to take the assessment. However, when they finished, Frank turned in his paper saying, “I know I screwed up on this,” as he handed it to me, and Beth, seeming distracted, gave me her summative assessment and hurried out the door (FN, March 10, 2015, p. 1).

**Results of Summative Assessment and Opportunities to Re-Assess**

I returned the students’ summative assessments to them one week after the original test day. I reviewed the scoring rubric with the students, asking them to examine how I scored each problem on the assessment. The students took time to read the problems, comparing their papers and scores with each other. Almost all of my advanced students had scored in the Exceptional or Secure range of the scoring rubric, but Lisa scored in the Developing range of the scoring rubric (Table 5-3). In contrast,
most of my struggling students scored in the Minimal range of the scoring rubric, except for Nancy, who scored in the Developing range of the scoring rubric (Table 5-4).

Table 5-3. Summative assessment results for advanced learners

<table>
<thead>
<tr>
<th>Pseudonym</th>
<th>Descriptor</th>
<th>Total Score</th>
<th>Final Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeff</td>
<td>Exceptional</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Lisa</td>
<td>Developing</td>
<td>11</td>
<td>75</td>
</tr>
<tr>
<td>Tammy</td>
<td>Secure</td>
<td>18</td>
<td>95</td>
</tr>
<tr>
<td>Victor</td>
<td>Secure</td>
<td>17</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 5-4. Summative assessment results for struggling learners

<table>
<thead>
<tr>
<th>Pseudonym</th>
<th>Descriptor</th>
<th>Total Score</th>
<th>Final Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beth</td>
<td>Minimal</td>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>Frank</td>
<td>Minimal</td>
<td>4</td>
<td>55</td>
</tr>
<tr>
<td>Nancy</td>
<td>Developing</td>
<td>10</td>
<td>75</td>
</tr>
<tr>
<td>Susan</td>
<td>Minimal</td>
<td>4</td>
<td>55</td>
</tr>
<tr>
<td>Ursa</td>
<td>Minimal</td>
<td>2</td>
<td>50</td>
</tr>
</tbody>
</table>

I informed the whole group that there would an opportunity to take a re-assessment for the Pythagorean Theorem unit, as I wanted them to become proficient on the unit’s learning targets before taking the Geometry EOC test. I made clear that the new summative assessment would not include the same questions as the original assessment, but it would have different questions that would assess their knowledge of the same learning targets that were on the previous summative assessment. I also clarified that all students had the opportunity to re-assess regardless of their original scores (TJ, March 13, 2015, p. 1).

I then explained to the students that they were required to complete some of the practice problems from lessons 9.1 through 9.5 again before taking the re-assessment. I wanted the students to become proficient on the learning targets for the unit, and I thought that taking the time to solve practice problems from the assignments for each lesson would allow them to go back and determine where they needed more help to
understand the material. The students would then re-assess and have the opportunity to show that they had mastered the learning targets. I had created a packet that had the problems that I wanted the students to solve, and I distributed them to the students. I told the students that I would be available for face-to-face help after school two days during the week in addition to having the online podcasts for assistance. I announced that I would administer the new assessment in one week, on March 20, 2015, at the end of the school day (TJ, March 13, 2015, p. 1-2).

**Student Reflections**

The students shared their thoughts about the results of the summative assessment. Tammy and Lisa, two of my advanced students, discussed the scoring rubric and their understanding of the results of the summative assessment:

**Tammy:** The EPI grading helped me see that if I only missed a little part of a problem then it didn’t mean that I didn’t understand the whole thing. Like getting a P on #10. I knew what to do, but then I guess I lost my mind somewhere and added the area of the triangle. I’m not sure why I did that, but at least I was on the right track for solving the problem (SI 6, March 12, 2015, p. 1).

**Lisa:** I’m so glad I can do this test again. I know this stuff, but I really felt sick the day we took the test. I had such a horrible headache. I like that the EPI sheet breaks up the parts of the test so that I can see that it isn’t just all wrong. I made such stupid mistakes. I want to go over the problems I missed before I do my retake just to make sure I know what I’m doing. I think I know it, but I’d feel better talking to you before I take it (SI 6, March 12, 2015, p. 2).

My struggling students talked about their performance on the summative assessment, and some of them discussed their next steps for re-assessing:

**Ursa:** I completely blanked out when I got the test. When I have to do the math by myself, I get overwhelmed and I can’t figure out what to do. I am going to have to learn pretty much everything again (SI 6, March 12, 2015, p. 1).

**Nancy:** I did really good on this test compared to my other test scores from other units. I think the highest test score I’ve ever had in this class is a 65. I like
that we couldn’t make less than a 50 on the test. Can we keep using the EPI grading system for the rest of the year? I think I can get better grades if I'm not stressed out about getting a really low grade (SI 6, March 12, 2015, p. 1).

Susan: I don’t know what happened. I looked at the test and nothing looked familiar. It made sense in class and on Virtual Weller, but then I freaked out. I can’t believe it’s a 55. I missed so much, it should probably be a 25. That would really kill my average (SI 6, March 12, 2015, p. 2).

Frank: I was absent on the day you gave the test, and I guess I just forgot everything. I really need to work on this stuff. I just don’t like to do the work when I’m not in the classroom (SI 6, March 12, 2015, p. 3).

**Summative Re-Assessment: Friday, March 20, 2015**

**Overview**

I administered the summative re-assessment after school two weeks after the administration of the original summative assessment. Every student could choose to take the re-assessment regardless of the original summative assessment score. The summative re-assessment consisted of problems similar to those found on the original summative assessment (refer to Appendix C). I used the same scoring rubric from the original summative assessment for grading the summative re-assessment. Students had 90 minutes to take the re-assessment, which was the same amount of time given for the original summative assessment.

**Administration and Results of the Summative Re-Assessment**

Although I had personally asked the students who had scored at the Minimal, Beginning, or Developing levels to take the re-assessment, only Lisa, Susan, and Ursa chose to take the test. Nancy came by after school that afternoon to tell me she was happy with her score on the first assessment.

The three girls shared their thoughts about how they thought they performed on the re-assessment when they handed me their tests.
Lisa: I know I did better. I felt better, and I wasn’t distracted by anything (FN, March 20, 2015, p. 1).

Susan: I think I did better. I know I’m still not good with the special right triangles (FN, March 20, 2015, p. 1).

Ursa: I don’t think this was any better than the last one. I don’t know what’s wrong with me. I just can’t do math on my own no matter how much I study (FN, March 20, 2015, p. 1).

I graded the re-assessments using the scoring rubric, and all three of the girls improved their scores from the original summative assessment (Table 5-5).

### Table 5-5. Comparison of original summative assessment and re-assessment

<table>
<thead>
<tr>
<th>Pseudonym</th>
<th>Descriptor</th>
<th>Original</th>
<th>Re-Assessment</th>
<th>Total Score</th>
<th>Original</th>
<th>Re-Assessment</th>
<th>Final Grade</th>
<th>Original</th>
<th>Re-Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lisa</td>
<td>Developing</td>
<td>Secure</td>
<td>11</td>
<td>19</td>
<td>75</td>
<td>95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Susan</td>
<td>Minimal</td>
<td>Beginning</td>
<td>4</td>
<td>7</td>
<td>55</td>
<td>65</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ursa</td>
<td>Minimal</td>
<td>Minimal</td>
<td>2</td>
<td>4</td>
<td>50</td>
<td>55</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Student Reflections**

Students who did and did not take the summative re-assessment shared their thoughts about taking the summative re-assessment.

Lisa: I am so glad that I had the opportunity to take the retest. I knew the material, but I just couldn’t concentrate with the horrible headache I had (SI 7, March 24, 2015, p. 1).

Susan: I did better. But I wish I could have done more. I was okay when we did the activities in class a lot of the time. Even with the extra time to prepare, I didn’t do as well as I had wanted to on the retest (SI 7, March 24, 2015, p. 1).

Ursa: I just can’t take tests. I only understand when we do everything together (SI 7, March 24, 2015, p. 1-2).

Beth: I probably should have taken the retest, but it was a lot of work to go back and work on the stuff again (SI 7, March 24, 2015, p. 2).

Frank: I just didn’t want to do it. I will do better on the next test (SI 7, March 24, 2015, p. 2).
Conclusion

In Chapter 5, I provided a chronological accounting of my implementing a standards-based differentiated unit of instruction focused on the Pythagorean Theorem within the context of a detracked honors geometry classroom at a university’s developmental research school. I showed how I used daily formative data analysis to adjust my instruction, using the information to create homogeneous and heterogeneous student groups and differentiate the classroom instruction and activities. I described the administration of the summative assessment and re-assessment, and shared my students’ reflections about how differentiating the instruction and providing standards-based assessments helped them learn. In Chapter 6, I will describe what I learned from this implementation about differentiation and standards-based assessment with struggling and advanced learners in a detracked honors geometry classroom.
CHAPTER 6
LEARNING FROM MY TEACHING OF A DIFFERENTIATED AND STANDARDS-BASED UNIT OF STUDY: THE PYTHAGOREAN THEOREM

Chapter Overview

In Chapter 5, I shared a day-by-day account of my teaching of the Pythagorean Theorem unit, the role differentiated instruction and standards-based assessment played in the unit’s implementation, and the ways I, as a teacher researcher, used the process of formative data analysis to make decisions about my teaching as the unit unfolded. After I completed teaching the unit in its entirety, I next engaged in the process of summative data analysis. According to Dana (2013):

While important insights are gleaned from the process of formative data analysis, as one nears the end of a cycle of inquiry, it’s critical to engage in summative data analysis as well. Summative data analysis involves stepping back at the end of one inquiry cycle and taking a look at the entire data set as a whole… New and different types of insights are gleaned from the independent looks at isolated portions of data done previously during formative data analysis (p. 53).

Chapter 6 provides a report of what I learned from engaging in the process of summative data analysis.

The new and different types of insights that I gleaned from the process of summative data analysis fell into two categories related to my research question: “In what ways do differentiated instruction and standards-based assessments support struggling students and challenge advanced learners in a detracked honors geometry classroom.” The first category relates to how differentiated instruction and standards-based assessment supported my struggling learners and challenged my advanced learners. The second category relates to why differentiated instruction and standards-based assessment supported my struggling learners and challenged my advanced
learners. In the next sections of Chapter 6, I discuss both the “how” and “why” categories and three themes that emerged within them.

**How Differentiated Instruction and Standards-Based Assessment Supported the Learning of My Struggling and Advanced Learners**

The first category of insights based on summative data analysis is how differentiated instruction and standards-based assessment supported my struggling learners and challenged my advanced learners. Within this category, three themes emerged. Planning and implementing a differentiated and standards-based unit of instruction supported my struggling learners and challenged my advanced learners by:

1. using homogeneous grouping when introducing new content,
2. using heterogeneous grouping when reviewing content, and
3. allowing for both teacher and student choice for grouping throughout the unit’s implementation.

**Using Homogeneous Grouping when Introducing New Content**

I define homogeneous grouping for the purposes of this study as the creation of two groups. The first group consists of learners with minimal to no knowledge of the learning target(s) for the lesson, while the second consists of learners who have basic knowledge of the learning target(s) for the lesson and are ready to deepen and enhance their understanding of the learning target(s).

Recall in Chapter 5 that homogeneous groups were created on Day 1 to provide direct instruction on the first two lessons of the unit that introduced the derivation of the Pythagorean Theorem and using the formula to solve for side lengths of right triangles to students who had scored 2 or less on the pre-assessment. Students who had scored 3 or above on the pre-assessment were allowed to begin their practice problems immediately. On Day 2, homogeneous groups were formed after students completed
their self-assessments following direct instruction on learning targets related to special right triangles and solving story problems. This gave students who had rated themselves at a 1 or 2 the opportunity to work on basic problems to increase their understanding of the content, while students who had rated themselves at a 3 or 4 solved challenging problems related to the learning targets. Similarly, homogeneous groups were formed on Day 3 after direct instruction focused on the distance formula and finding equations of circles. This allowed students who rated themselves at a 1 or 2 the opportunity to work with hands-on manipulatives and receive further assistance to learn the concepts related to finding circle equations, and giving students who rated themselves at a 3 or 4 the chance to work together at a quicker pace without assistance. On Days 4 and 5, homogeneous groups were created for review purposes, and the students who rated themselves at a lower level of understanding worked together in small groups with teacher assistance, while students who had indicated they were at a higher level of understanding of the learning targets worked in their own groups without any extra help.

When reviewing the data across all five of these homogeneous grouping episodes that occurred throughout the entire unit, it became clear that students who struggled benefitted as they reported feeling more comfortable asking questions in small groups with students who had a similar understanding level to their own. For example, Nancy, Ursa, and Frank mentioned this feeling of comfort multiple times in relationship to different lessons within the Pythagorean Theorem unit:

Nancy: I think that breaking us into groups helped me feel more comfortable with asking questions. Sometimes I get nervous asking questions when the people I'm with already know what they're doing. The people in my group
had questions like me so I didn’t feel stupid asking them (SI 1, February 24, 2015, p. 1).

When we learn the new information, I learn better and don’t feel dumb asking questions when I know that everyone around me is in the same place (SI 5, March 5, 2015, p. 1).

Ursa:  It made it easier for me not to have the whole class do the notes together...then we had a smaller section taking the notes and asking questions. It made me feel better knowing that more people didn’t know as much like I did, and we did it together, and it was easier for me. Other people in the groups up front had questions like I did (SI 1, February 24, 2015, p. 2).

I’m glad that I was with people working on the word problems that were on the same level as me. We could take turns talking and feel relaxed about working them out. I was able to talk more and not feel stupid if I didn’t know what to do next, because we all were pretty much on the same level (SI 2, February 26, 2015, p. 2).

Frank:  You helped us understand the Pythagorean Theorem better when you split us into groups. The people that knew it didn’t need extra help, so they were doing more of their own thing because they were at a higher understanding level. For the people who didn’t know it, you focused more of your attention to us and it was more individualized and more specific to what we needed. I like how you split us into groups because I was able to ask more questions. I felt more comfortable asking the questions (SI 1, February 24, 2015, p. 2).

When you’re learning a brand new thing or way too confused, it’s better to be separated so that you can have more help (SI 2, February 26, 2015, p. 2).

Nancy, Ursa, Frank, and all of the students who typically struggled in my sixth period geometry class were not the only students who discussed a feeling of comfort when placed in homogeneous groups. Advanced learners also expressed a feeling of comfort, although they defined it differently. Rather than comfort in asking questions to clarify understanding, my advanced learners found comfort in not being asked to sit through a lesson on material they were already proficient in understanding, and,
instead, having the time to work on problems together with others who understood the material well:

Jeff: I like that you did a pre-test, because if we knew how to use the Pythagorean Theorem, we didn’t have to sit through a long lesson forever and we could actually do the work and practice. I would definitely prefer that than sitting through a lesson on material that I already know (SI 1, February 24, 2015, p. 1).

I liked working with people who understand the math the way I do. It’s less frustrating for me when we all work on trying to figure out the math (SI 2, February 26, 2015, p. 1).

Tammy: I was good with you not making me sit there and learn something I already knew (SI 1, February 24, 2015, p. 1).

Victor: I liked being in the [advanced] group when we worked on the word problems. It was less boring...because if I already understand something I get bored doing it over and over (SI 2, February 26, 2015, p. 1).

In addition to feeling comfortable in homogeneous groups, my advanced students also experienced a higher level of enjoyment as they persevered through the problem-solving process. They liked engaging in conversations with students at the same learning level of understanding when a problem proved to be challenging for them.

Victor: We got stuck on the one problem, but because we kept fighting about it and trying to figure it out, I was more interested (SI 2, February 26, 2015, p. 1).

We had a lot of fun together (SI 3, February 27, 2015, p. 1).

Jeff: I liked getting the challenging problems...we argued a lot on that one and we still didn’t figure it out (SI 2, February 26, 2015, p. 1).

While both struggling and advanced learners reported benefits of being homogeneously grouped for instruction based on their understanding of learning targets, heterogeneous grouping also played an important role in both struggling and advanced students’ learning during the unit.
Using Heterogeneous Grouping when Reviewing Content

I define heterogeneous grouping for the purposes of this study as the creation of groups that combine learners who had minimal to no knowledge of the learning target(s) for the previous day’s lesson with learners who had basic to advanced knowledge of the learning target(s) for the previous day’s lesson.

Recall in Chapter 5 that I assigned students to heterogeneous groups on Days 2, 3, and 4 at the start of the class period to review the assigned practice problems from the previous days’ lessons. On Day 2, I used the pre-assessment data to group the students, and I used the students’ self-assessment ratings to create the heterogeneous groups on Day 3. On Day 4, I used a more random method of assignment for creating groups for the review activity, first assigning specific advanced students who had demonstrated greater understanding of the content to each group, and then using a deck of cards for assigning the rest of the students to the heterogeneous groups. Finally, I used the results from a paper-and-pencil formative assessment to create heterogeneous groups for the game show review activity on Day 5, placing some students who scored higher with other students who scored lower on the assessment in each group.

When reviewing the data across all four of these heterogeneous grouping episodes that occurred throughout the entire unit, I found that my students who struggled with understanding the content benefitted because they had the opportunity to hear and see various ways to solve problems as they engaged in conversations with their classmates. Ursa, Frank, Susan, Beth, and Nancy discussed numerous times how they preferred hearing their peers’ points of view while in the heterogeneous groups to
help them make sense of the problem-solving procedures necessary to master the
learning targets:

Ursa: I think that I want to be with other people who know more about what is
going on sometimes. Like when we have to work out the problems, I want
to work with people who can tell me how to work the stuff out (SI 1,
February 24, 2015, p. 2).

Talking about the homework problems with the people who were in the
other group last class helped me. I couldn’t figure out what to do next
when I got the paper sometimes, but (other students) helped me do it (SI
2, February 26, 2015, p. 2).

When we review, having kids who can explain what I might be missing
and help me catch up helps. It’s not like when we’re learning it and I’m
feeling lost. I know more than I did when we started, and the people who
really know what they’re doing helps me fill in some of the blanks that I
have and make it easier for me (SI 5, March 5, 2015, p. 2).

Frank: I like working with other people instead of by myself. It’s easier to talk
about what to do to solve the problems than figure it out on my own (SI 1,
February 24, p. 2).

Sometimes you can learn something from other people that you didn’t
know, different techniques, and get a different perspective (SI 2, February
26, 2015, p. 2).

For review, I like more diverse people in my group. Maybe another person
on my level and then some people who are a little more above me so that
they can help me out. For explanation purposes...Tammy talked through
every problem in the game, and it all made sense to me (SI 5, March 5,
2015, p. 1).

Susan: I’ve been doing the problems and talking more about them in class. That
helps me (SI 2, February 26, 2015, p. 1).

Beth: I feel better when I’m working with the group. I can do it then (SI 2,
February 26, 2015, p. 2).

Can we do stuff where someone is there explaining it to me and everyone
else in my group? (SI 3, February 27, 2015, p. 3).

Nancy: I feel like I’ve been able to ask more questions and have lots of people
explain the problems to me...when we review, I want people who get the
stuff to help me (SI 5, March 5, 2015, p. 1).
The advanced learners also expressed that participating in learning activities within heterogeneous groups had advantages in reinforcing their own learning:

Tammy: I liked the activity we did because it gave us a chance to talk to each other about solving the problems. I like being able to explain what I’m thinking (SI 4, March 3, 2015, p. 1).

For reviewing, I’m okay being with different people, like we did for the game. I don’t get really frustrated if other people in the group need help, because I think I review more when I have to explain what I’m doing to other people. Helping them helps me (SI 5, March 5, 2015, p. 2).

Jeff: I already understood the math we were doing in our groups today, but more review is okay (SI 4, March 3, 2015, p. 1).

Whether students were heterogeneously or homogeneously grouped for instruction based on their understandings of the learning targets, the ways that groups were formed is noteworthy of discussion.

**Allowing for Both Teacher and Student Choice for Grouping**

The third theme to emerge as I summatively analyzed my data related to how differentiated instruction and standards-based assessment supported the learning of my struggling and advanced learners was allowing for both teacher and student choice for grouping throughout the unit’s implementation. Within the description of each day’s activities in Chapter 5, I discussed how groups were created for different parts of the days’ lessons. Sometimes I created the groups using paper-and-pencil formative assessments, or I used a random method to create heterogeneous groups. Other times, students used their self-assessments to place themselves in a homogeneous group depending on their own learning needs.

When I reviewed the data, it became clear that using self-assessment provided an opportunity for both my struggling and advanced learners to be metacognitive about their own learning and determine whether they needed more direct instruction or
assistance with understanding the learning targets or were ready to explore the concepts in more depth through problem-solving activities. All of students had the power to choose whether they needed more instruction for mastering a particular learning target, and they took their learning into their own hands and became more cognizant of what they needed to be successful.

I noticed that the students who struggled with understanding the learning targets tended to rate themselves honestly, and most chose to participate in the homogeneous group that provided additional learning activities or direct instruction from me that increased that understanding. There were rare occasions when these students found themselves in situations that might have been too challenging for them, but they usually chose to stay in the groups in which they had placed themselves.

Susan: I wanted to work with the other (advanced) group, because I through I knew what I was doing after the notes. It was really hard, though. I probably should have moved back to the other group, but I didn’t want to (SI 2, February 26, 2015, p. 1).

Many students who placed themselves in the advanced or independent groups for challenging activities shared that they would be comfortable choosing to be in the group receiving more instruction if they did not understand a learning target.

Jeff: If I’m not sure that I know the stuff that we are learning, I would rather sit through it just to make sure I’m not missing anything I need to know (SI 1, February 24, 2015, p. 1).

Tammy: If I didn’t know what I was doing, I would want to be in the other group (SI 1, February 24, 2015, p. 1).

Tammy’s reflection after Day 3 described her metacognitive process in rating herself and her comfort in choosing to be in the group that received more assistance in understanding circle equations.
Tammy: I did not rate myself very high on the self-assessment and went to the other group than I had been in before. I didn't feel strange doing that even though I usually work with the other group. I knew those people who are normally in the group that I'm in were going to go fast and beyond what you're supposed to know. And if I wasn’t feeling completely comfortable with the topic yet I wasn’t going to want to do that because I would confuse myself completely and I wouldn’t grasp the basics of the concept I was trying to understand…I guess I just needed to talk my way through the problems to understand what to do. I know the circle equations after doing the activity and am comfortable solving problems with them (SI 3, February 27, 2015, p. 1).

Tammy’s ability to choose her group and subsequent differentiated learning activity allowed her to achieve a level of success with understanding and mastering the learning target.

My data also indicated that not allowing students to choose their group placement might cause frustration and disengagement in class. When I used the pre-assessment to place students into homogeneous groups on Day 1, I placed Victor in the direct-instruction group. He became very frustrated and did not want to participate.

Victor: I was in the group in the front of the room with you because I messed up on the pre-test. After the first couple of minutes, I figured out what I did wrong and wanted to move because I was bored (SI 1, February 24, 2015, p. 2).

It was only after I reassured Victor that he would not have to remain in this group for the entire unit that he started to work and encourage others to participate. Knowing that he would be able to change his placement increased his engagement in class. Hence, one reason both teacher and student choice in group placement plays a critical role in how differentiated instruction and standards-based assessment supported the learning of my struggling and advanced learners was that homogeneous groups were not defined by a student’s fixed ability and did not remain stagnant for the unit’s duration. Rather, groups were defined by students’ proficiency in meeting specific learning targets for the unit.
and were fluid and flexible, with students changing to different groups throughout the unit based on their own self-assessments and performance on formative assessments. Teacher formed and student selected groups worked in concert with one another to assure students were receiving what they perceived they needed to be successful in meeting learning targets.

**Why Differentiated Instruction and Standards-Based Assessment Supported the Learning of My Struggling and Advanced Learners**

With “the how” articulated, the second category of insights based on summative data analysis that emerged focused on why differentiated instruction and standards-based assessment supported my struggling learners and challenged my advanced learners. Like “the how” category previously described in the last section, within “the why” category, three themes also emerged. The differentiated and standards-based unit of instruction based on the Pythagorean Theorem supported my struggling learners and challenged my advanced learners because:

1. differentiation and standards-based learning was framed using scaffolded notes, manipulative materials, podcasts, and problem-solving activities,

2. a high degree of structure was used when struggling students and advanced students worked together in heterogeneous groups, and

3. a holistic grading procedure used for both formative and summative assessments provided insights for both me and my students into their meeting a level of proficiency on the unit’s learning targets.

**Framing Differentiation and Standards-Based Learning**

An important part of providing support for struggling and advanced learners is planning specific instructional activities to help them understand or extend their
knowledge of the content. Recall in Chapter 4 that I started planning my unit by first identifying the state standards that I expected my students to master and then rewriting them as learning targets in student-friendly language. I was then able to use these learning targets to create a summative assessment that would allow me to determine if my students had mastered the standards. The deliberate process of examining, rewriting, and using the standards to create a standards-based assessment allowed me to then determine where in my instructional unit based on the Pythagorean Theorem my struggling students would require the content to be broken down into smaller pieces, as well as identify areas where I could challenge my advanced learners.

When reviewing the data across the five instructional days of the implementation of the unit, I found that both my struggling and advanced learners benefitted from my using standards-based structure to frame the unit. The process guided me to create scaffolded notes, activities using hands-on manipulatives for visualization, online support with podcasts, and problem-solving activities to support and extend learning with structures in place to ensure that all students were accountable for their individual participation.

Each day during direct instruction, I gave scaffolded notes to my students to help them stay organized, focused on the learning targets, and manage their own learning. Including a self-assessment on the scaffolded notes for the students to use at the beginning and end of the lesson provided them with a tool for rating their own understanding and determining whether or not they had made progress toward mastering the learning targets. I also used the student self-assessment to create homogeneous groups for differentiation after direct instruction. The scaffolded notes
served many purposes in framing the differentiation and standards-based learning for all of my students.

During the planning of the unit, an area of need that I identified for my struggling students was being able to visualize the Pythagorean Theorem. During previous units of study that academic year, I realized that some of my struggling students had trouble understanding abstract concepts because they could not visualize the mathematics without having a concrete example. As I examined the state standards and wrote the learning targets for the unit I used in my study, I was able to identify the learning targets that would necessitate my struggling students having concrete examples during the instruction of the unit in order to understand the Pythagorean Theorem beyond memorizing and using the formula. I framed the differentiation and standards-based learning for the students who struggled with understanding the learning targets by creating or finding activities that included using hands-on manipulatives so that they could visualize the mathematics they were using.

When reviewing the data from the classroom activities where hands-on manipulatives were used, I found that being able to visualize the Pythagorean Theorem aided several students in their understanding of the learning targets. For example, the color tiles activity allowed students to model the relationship between the lengths of the sides of the legs and hypotenuse in a right triangle when they built the areas of the squares on each side of the triangle. Nancy, Ursa, and Frank reflected that using these manipulative materials aided in their understanding of the derivation of the Pythagorean Theorem equation on Day 1:
Nancy: When we did the tiles so that we could add up the squares to see how the problems worked, it made more sense to me (SI 1, February 24, 2015, p. 1).

Ursa: It was easier to understand where \( a^2 + b^2 = c^2 \) came from when we looked at the areas of the three squares (SI 1, February 24, 2015, p. 2).

Frank: I liked the ... pictures we made... I could visualize [the Pythagorean Theorem] a lot better after we made the areas with the tiles (SI 1, February 24, 2015, p. 2).

Frank, Ursa, and Victor also commented on how using three-dimensional manipulatives helped them determine how to visualize information given in word problems during the classroom activity on Day 2.

Frank: I liked it when you used the cube to help us see what the problem was supposed to look like when we took notes (SI 2, February 26, 2015, p. 2).

Ursa: The pencils and cube thing helped me see how to figure out that cube problem on the notes. I'm more of a picture person. I like it when you draw pictures or when we get to use 3-D things to help us understand (SI 2, February 26, 2015, p. 2).

Victor: Using the cube in class helped us think about what to do for solving the problem (SI 2, February 26, 2015, p. 1).

Using hands-on manipulatives on Day 3 helped Frank, Tammy, Nancy, and Ursa visualize dilations and translations as they explored the standards related to writing circle equations.

Frank: When we were laying the blue circles out on our maps for Fido, we were able to visualize where to place the circles and then write equations. I liked how the colors popped out when we were using the circle (SI 3, February 27, 2015, p. 2).

Tammy: The blue circles helped me a little (SI 3, February 27, 2015, p. 1).

Nancy: Doing the Fido activity, it made more sense being able to use the circles and move them around, especially when we had to switch how big the radius was and figure out how the equations were the same or different. I don't think I would have been able to figure out how to do the work without them (SI 3, February 27, 2015, p. 2).
Ursa: I liked using the blue circles for Fido. The grid showed me the places the circles went (SI 3, February 27, 2015, p. 2).

By using a standards-based structure to frame the differentiated unit of instruction, I was able to determine what abstract learning targets might prove difficult for my students who struggle to understand geometric concepts without having concrete visual representations. The planning structure gave me a way to identify those learning targets and create the hands-on manipulatives so that I could frame the differentiation and standards-based learning opportunities that increased my students' understanding of the Pythagorean Theorem.

Completing practice problems that correlate to the learning targets/state standards is an expectation I have for my students, but often times my students must complete these problems outside of class time without having their my input or assistance during the problem-solving process. As part of my planning process using a standards-based structure, I created podcasts of all of the practice problems I assigned to my students using the ShowMe® app on my iPad®. During the creation of the podcasts, I was able to verify whether the practice problems that I assigned to my students connected to the learning targets related to the Pythagorean Theorem.

When reviewing my data, I found that these podcasts benefitted both my struggling and advanced students' learning by giving them academic support from “Virtual Weller” via computer while they were out of the classroom:

Ursa: I will listen to Virtual Weller some more when I do my homework (SI 3, February 27, 2015, p. 2).

I'll practice with Virtual Weller (SI 5, March 5, 2015, p. 2).

Tammy: I just went on Virtual Weller to help me figure out the practice problems I didn’t know (SI 1, February 24, 2015, p. 1).
I’m definitely going to use Virtual Weller on the homework (SI 3, February 27, 2015, p. 1).

I’m going to practice problems from old assignments that I’m still confused about with Virtual Weller (SI 5, March 5, 2015, p. 2).

Susan: It made sense in class and on Virtual Weller (SI 6, March 12, 2015, p. 2).

My students benefitted from my providing support in understanding of the learning targets by framing differentiation and standards-based learning through the creation of the “Virtual Weller” podcasts.

Using a standards-based structure to frame the unit gave me the opportunity to differentiate my instruction for both my struggling and advanced learners during small-group activities. For example, in Chapter 4, I explained that during my planning of the lesson that focused on solving word problems, I created two levels of problems related to the same learning targets for the students to solve in homogeneous small groups. Students who struggled with applying the Pythagorean Theorem answered easier, more concrete problems that addressed the learning targets, while the advanced learners solved problems that challenged them to extend their thinking beyond the basic application of the Pythagorean Theorem.

In reviewing the data, I found that planning differentiated activities that stimulated critical thinking without forcing advanced students to complete basic or rote practice created a challenging learning environment for advanced learners. Jeff, Tammy, and Victor discussed their preference for completing more challenging work:

Jeff: I … want to do something that pushes me to think more and learn more about the topic. But not just busy work (SI 1, February 24, 2015, p. 1).

I liked getting the challenging problems instead of the ones that I knew that I could do already (SI 2, February 26, 2015, p. 1).
Tammy: I would rather learn something new and push myself than be bored with stuff I already know (SI 1, February 24, 2015, p. 1).

Victor: Those word problems definitely pushed us. They were hard, but I think it’s better to further my knowledge doing those than the easier problems (SI 2, February 26, 2015, p. 1).

In sum, planning this unit using a differentiation and standards-based lens led to the creation of four pedagogical tools that were used in the implementation of this unit to both support and extend learning: scaffolded notes, activities using hands-on manipulatives for visualization, online support with podcasts, and problem-solving activities. In addition to these four specific pedagogical tools, my data also indicated that the pedagogy of heterogeneous group activities was also an important ingredient to explain why differentiation and standards-based assessment supported my struggling and advanced learners.

**A High Degree of Structure for Activities in Heterogeneous Groups**

Recall in Chapter 5 that there were moments during the instructional days when some students were not engaged in the activities designed to increase or extend their understanding of the learning targets during the heterogeneous group activity. For example, on Day 2, I observed that many of the struggling students in the heterogeneous groups were not participating in the opening problem-solving activity. I decided to make changes in the planned activity to engage all of the students, adding physical movement, the addition of a time limit, and the possibility of random questioning by me. On Day 3, since many of the students did not complete the practice problems from the previous day’s assignment and were unable to contribute in their group activity, I decided to adjust the activity by having the students record their work on chart paper using different colored markers.
When reviewing the data across these episodes, it became clear that providing structures to increase individual accountability within the heterogeneous groups benefitted the students who struggled with the content by keeping them on task and talking about the mathematics while reinforcing the learning targets and strengthening their problem-solving skills. For example, Nancy and Ursa both mentioned how working in the heterogeneous groups increased their participation:

Nancy: The different groups have sort of forced me to talk to more people (SI 5, March 5, 2015, p. 1).

Ursa: The activities that you did in class where we worked together and see the problems helped, and I was more motivated to work when we had the game. (SI 5, March 5, 2015, p. 2).

The structures also released the advanced students from having the responsibility of completing all of the work in their groups by themselves, as Jeff expressed:

Jeff: Going over the homework with the group you put me in at the beginning was all right, but not everyone had their homework. You gave us something to do that made sure everyone did something even if they didn’t have it, so at least they didn’t just sit there and do nothing (SI 2, February 26, 2015, p. 1).

Making changes that provided a higher degree of structure within the heterogeneous groups during the differentiated unit of instruction increased my struggling students’ engagement in the learning activities, and my advanced students were not obligated to be accountable for the entire group’s work completion. All of the students participated and communicated more about the mathematics to increase their understanding of the learning targets. Besides providing a high degree of structure for my students during classroom activities, my data suggested that using a holistic grading procedure for both formative and summative assessments provided insights for both me and my students into their proficiency in meeting the unit’s learning targets.
A Holistic Grading Procedure Provided Insights into Students’ Mastery of the Learning Targets

I define holistic grading procedures for the purposes of this study as using a variety of methods for determining student understanding of the learning targets, including paper-and-pencil assessments, observations, and student reflections, considering the quality and effort of their work instead of only grading for the correctness of their answers.

Recall in Chapter 5 that I utilized numerous kinds of formative assessment to ascertain my students’ understanding of the daily learning targets. I used paper-and-pencil assessments, observations, and student reflections to create different heterogeneous groups for activities on Days 1, 2, 3, 4, and 5. My students used self-assessments as part of their note-taking procedures to determine their understanding of new material on Days 2 and 3, and they performed quick self-checks of their individual needs for additional review or assistance on Days 4 and 5.

As I reviewed my data, I found that it was important for me to use formative assessment to guide my day-by-day planning as the unit progressed to make essential modifications and effectively differentiate instruction to support my students’ learning. For example, during the planning of the unit, I decided to use the results of my students’ paper-and-pencil pre-assessment and create differentiated groups on Day 1. Students who answered 3, 4, or 5 questions correctly on the pre-assessment worked on practice problems independently on the first day of the unit, and students who answered 0, 1, or 2 questions correctly engaged in direct instruction of the first two lessons with me. Although this process seemed to work well, Victor seemed to be “misplaced” in the direct instruction group and was reluctant to engage in the activity. I verified this as the
activity continued when Victor shared his comparison of the sum of the squares of the
two shorter sides and the square of the longest side in both acute and obtuse triangles.
Victor also reflected about his frustration at being with struggling students for direct
instruction during a conversation after Day 1.

Victor: I was in the group in the front of the room with you because I messed up
on the pre-test. After the first couple of minutes, I figured out what I did
wrong and wanted to move because I was bored (SI 1, February 24, 2015,
p. 2).

After talking to Victor and observing his performance in class, I was able to use
formative assessment to ascertain that he had sufficient background knowledge of the
learning targets and changed his placement for grouping during the next class period’s
activities. His subsequent performance throughout the unit indicated that he had
advanced knowledge of the learning targets associated with the state standards for the
Pythagorean Theorem, and I felt that my using formative assessments to make a
change in his group placement for Day 2 was appropriate.

Another example of my using formative assessments in my day-to-day planning
occurred after Day 3. I used evidence from my field notes, teacher journal, student
reflections, and student work from the previous day’s problem-solving activity as
formative assessments to determine how I would group the students at the start of the
class period on Day 4, using my observations of the students’ leadership skills, work
ethic, and problem solving abilities to choose group captains. During the activity on
Day 3, Tammy had placed herself in a group of students that needed more assistance
for finding the equation of a circle. However, my observations of her during the activity
and the ensuing conversation with Tammy indicated that she had mastered the learning
targets associated with this lesson, and I chose her to be a group captain.
Tammy: I guess I just needed to talk my way through the problems to understand what to do. I know the circle equations after doing the activity and am comfortable solving problems with them...You kept listening to me work out the problems during the dog activity, so you heard that I was finally understanding what I was doing (SI 3, February 27, 2015, p. 1).

When I reviewed the data, I noticed that students used formative assessment to determine their own understanding of the content. When my students became metacognitive of their learning, they decided what they needed to do to increase their level of proficiency with the learning targets. After self-assessing and reflecting on their learning, Frank, Beth, Nancy, and Ursa mentioned several times that they needed to practice problems to increase their understanding the content related to the Pythagorean Theorem:

Frank: I need to remember more stuff from the notes and problems we do in class (SI 2, February 26, 2015, p. 2).

I know I need to do the assignments so that I can do better on the test (SI 3, February 27, 2015, p. 2).

I will try to review for the test, maybe look through my notes or listen to Virtual Weller (SI 5, March 5, 2015, p. 1).

Beth: I need more practice (SI 2, February 26, 2015, p. 2).

I guess practicing more could help if I have time to do it (SI 3, February 27, 2015).

Nancy: I think I need more help (SI 3, February 27, 2015, p. 2).

Ursa: I will listen to Virtual Weller some more when I do my homework (SI 3, February 27, 2015, p. 2).

I need to practice on my own... I'll practice with Virtual Weller (SI 5, March 5, 2015, p. 2).

In Chapter 4, I described how Dr. Jacobbe’s EPI scoring rubric allowed me to examine my students’ assessments more holistically. Besides giving me a tool to determine my students’ understanding of the learning targets, my students found that
the EPI scoring rubrics also helped them understand how they performed on their assessments and determine their next steps for learning.

Frank: For some reason I completely messed up the squared part of the formula on my quiz, so it messed up all of my calculations. When I looked over the rubric, I realized that I forgot all about the pictures we made with the squares. I guess I need to do more than glance at my notes before a quiz and practice more at home (SI 3, February 27, 2015, p. 1).

Jeff: I like the new format for the assessment with the rubric. It might help with figuring out where you need more help or practice (SI 3, February 27, 2015, p. 2).

Tammy: The EPI grading helped me see that if I only missed a little part of a problem then it didn’t mean that I didn’t understand the whole thing. Like getting a P on #10. I knew what to do, but then I guess I lost my mind somewhere and added the area of the triangle. I’m not sure why I did that, but at least I was on the right track for solving the problem (SI 6, March 12, 2015, p. 1).

Lisa: I like that the EPI sheet breaks up the parts of the test so that I can see that it isn’t just all wrong. I made such stupid mistakes. I want to go over the problems I missed before I do my retake just to make sure I know what I’m doing. I think I know it, but I’d feel better talking to you before I take it (SI 6, March 12, 2015, p. 2).

Using holistic grading procedures helped my students be metacognitive about their understanding of the learning targets related to the Pythagorean Theorem, and allowed them to create action plans for their own learning before re-assessing.

Summary and Conclusions

In sum, as I implemented a standards-based differentiated unit of study based on the Pythagorean Theorem, I recorded classroom observations in my field notes, reflected daily in my teacher journal, and recorded student reflections about the process during interviews. These data indicate that the learning of both my struggling and advanced students was supported by using homogeneous grouping when introducing new content, using heterogeneous grouping when reviewing content, and allowing for
both teacher and student choice for grouping throughout the implementation of the unit. Providing opportunities to participate in differentiated instructional activities in various types of group settings was advantageous for both my struggling and advanced learners. Deciding whether to use homogeneous or heterogeneous groups was based on the type and purpose of the instructional activity and how the structure of the groups influenced students' learning. Teacher and student choice for creating those groups also contributed to students being metacognitive of their learning.

My data also indicated that my ability to use differentiated instruction and standards-based assessment to support the learning of my struggling and advanced learners was aided by framing that learning using the pedagogical tools of scaffolded notes, activities using hands-on manipulatives for visualization, online support with podcasts, and problem-solving activities. Using a high degree of structure to ensure individual student accountability during heterogeneous group activities facilitated a classroom environment where all students had the opportunity to increase their understanding of the learning targets. My students and I were provided insights for meeting a level of proficiency on the unit’s learning targets through the use of a holistic grading procedure for both formative and summative assessments, providing me with information to make adjustments to the daily lessons and giving the students the opportunity to be metacognitive and create plans for their own learning.

Having shared what I learned as a practitioner researcher through the process of summative data analysis in Chapter 6, I will focus on the implications of this study in Chapter 7.
CHAPTER 7
REFLECTIONS ON MY TEACHING OF A DIFFERENTIATED AND STANDARDS-BASED UNIT OF STUDY: IMPLICATIONS FOR MY WORK AS A TEACHER RESEARCHER

Chapter Overview

In this dissertation, I described in detail the capstone project that served as the culminating experience for my professional practice doctoral program at the University of Florida. This capstone experience consisted of studying my own practice through the process of teacher research as I designed, implemented, and analyzed a unit of instruction to be both differentiated and standards-based in order to address learner variability in my detracked honors geometry classroom. Up to this point in the dissertation, I have introduced this inquiry (Chapter 1), shared the literature that informed this inquiry (Chapter 2), discussed the methodology I employed to inquire into my practice (Chapter 3), and detailed my planning of this unit (Chapter 4), my implementation of this unit (Chapter 5), and what I learned about the ways differentiation and standards-based instruction can meet the needs of both struggling and advanced learners in my geometry classroom (Chapter 6).

According to Dana & Yendol-Hoppey (2014), at the end of each cycle of inquiry, it is important to reflect on one’s learning as a teacher researcher to articulate the implications of one’s work. As noted in Chapter 3, this can be accomplished by pondering a series of questions:

1. What have I learned about myself as a teacher?
2. What have I learned about children?
3. What have I learned about the larger context of schools and schooling?
4. What are the implications of what I have learned on my teaching?
5. What changes might I make in my practice?


The purpose of Chapter 7 of my dissertation is to discuss implications of my work by reflecting on the six questions named above. Before addressing each question, I first provide a brief summary of my dissertation to serve as a backdrop for my answers to these questions.

**Summary and Overview of the Dissertation**

The purpose of my study was to understand the ways differentiated instruction and standards-based assessment supported the mathematics learning of my struggling students and advanced learners in my detracked honors geometry classroom. Chapter 1 introduced this purpose by describing the reasoning behind detracking and the ways detracking, while a valuable practice to decrease the achievement gap, also increases learner variability within the same classroom. Differentiation and standards-based assessment hold promise to help teachers address this variability by simultaneously meeting the needs of students who struggle to learn mathematics and students who are advanced in their mathematics understandings. As a geometry teacher committed to creating a more equitable learning environment for all of my students in a detracked honors geometry classroom, I was interested in exploring the ways differentiation and standards-based assessment could be incorporated into my teaching. Chapter 1 concluded by articulating the significance of this study and the research question that guided the study: In what ways do differentiated instruction and standards-based assessments support struggling students and challenge advanced learners in a detracked honors geometry classroom?
Chapter 2 presented a review of four bodies of literature that provided the foundation for the study: the achievement/opportunity gap, tracking and detracking practices, the provision of differentiated instruction, and the uses of standards-based assessment. Overall, Chapter 2 focused on how these four bodies of literature converged to provide a rationale for the study and focused on exploring the reasons why differentiated instruction and standards-based assessments hold promise to increase the learning of struggling and advanced students in a detracked mathematics classroom.

Chapter 3 concentrated on the chosen method for the study, which was practitioner research. An in-depth description of practitioner research was provided, as well as a detailed description of the researcher who conducted the study. The details of the intervention were discussed, and the student participants were introduced. Chapter 3 concluded with the details of data collection and data analysis methods.

Chapter 4 provided an explanation of the planning of a standards-based differentiated unit of study based on the Pythagorean Theorem, including the creation of the learning targets, the construction of the formative and summative assessments, the choosing of the scoring rubric, and the planning of the daily schedule and classroom activities. The explanation of the unit’s planning set the stage for Chapter 5, where I discussed implementing the Pythagorean Theorem unit in my classroom.

In Chapter 5, I described the chronological implementation of a standards-based differentiated unit of study based on the Pythagorean Theorem. Using field notes, student artifacts, student interviews, and my journal entries, I provided a rich, thick description of the day-to-day implementation of the unit to provide insights into the role
formative data analysis played in my work and how I used data to make instructional decisions throughout the teaching of the unit. I also discussed the administration of the summative assessment and re-assessment, and shared my students’ reflections where they revealed how differentiated instruction and standards-based assessments helped them learn. Chapter 5 created a thorough description of the unit as it unfolded to help readers understand the details of teaching in a differentiated and standards-based way.

Chapter 6 focused on what I learned from planning and implementing the standards-based differentiated unit of study based on the Pythagorean Theorem through engaging in the process of summative data analysis after the unit had been taught. The insights I gleaned from the data fell into two categories, how and why differentiated instruction and standards-based assessment supported my struggling learners and challenged my advanced learners. In Chapter 6, I discussed how all of my students benefitted from participating in heterogeneous and homogeneous groupings during class instruction and activities, and teacher and student choice in the creation of these groups contributed to the students’ metacognition about their learning. I also shared how using a differentiated and standards-based frame for planning the unit, providing a high degree of structure for classroom activities, and using a holistic grading procedure to provide insights into my students’ understanding of the content also supported the learning of my struggling and advanced students.

Having recapped the details of this practitioner research study, I turn now to reflecting on the implications for what I learned from this inquiry using the questions suggested by Dana and Yendol-Hoppey (2014).
What Have I Learned About Myself as a Teacher?

A current, popular work that is being read by educators across the nation is Carol Dweck’s *Mindset: The New Psychology of Success*. Dweck (2006) shares that “although people may differ in every which way – in their initial talents and aptitudes, interests, or temperaments – everyone can change and grow through application and experience” (p. 7). An important lesson that I have learned about myself as a teacher from this study is that I need to continue to have a “growth mindset” in regards to my students. Some teachers, parents, and students think that some individuals do not have the ability to learn mathematics, and that those who struggle with understanding mathematics should be tracked into remedial classes. I believe *everyone* can learn mathematics if given the opportunity, the tools, and the encouragement they need to learn and be successful. This study indicates that scheduling students who are at different levels of understanding into the same detracked classroom can work if the teacher provides instruction and activities that meet the needs of the students who struggle to learn the content and challenge those who are ready to move beyond their basic understanding of the material. Integrating the processes of differentiation and standards-based assessment into a detracked mathematics classroom allows all learners, both those who struggle, and those who are advanced, to develop a “growth” mindset, rather than have a “fixed” mindset about their mathematics ability. Through differentiated and standards-based instruction, all learners can be challenged with a rigorous curriculum, yet this curriculum is delivered in ways to meet the different learning needs of the students in my classroom.

This study has renewed my commitment in working with students regardless of their perceived ability to learn mathematics. Having this mindset pushes me to work
harder for each student that I teach and strengthens my belief in “the potential of every student to work intelligently and hard in pursuit of success,” because I “embrace the responsibility of crafting curriculum and instruction that will maximize the success of each learner in the class” (Tomlinson & Imbeau, 2014, p. 4). Through this study, I have gained insights into the potential differentiated and standards-based instruction hold to push my detracked honors geometry classroom to be an even more equitable learning environment for all students. This study has inspired me to work to refine my approach to differentiation and standards-based instruction as well as to continue to pursue additional research-based practices to push my advanced and struggling students past their comfort thresholds in learning.

**What Have I Learned About Children?**

A lesson I have learned about my students from this study is that they need a safe and comfortable learning environment that includes a support system that encourages them to learn and be productive. This support system differs depending on the needs of the student. On a related note, this study reminded me that the same student could have different learning needs on different days, and, therefore, learning strategies need to be adjusted in relationship to those needs on any given day with any given lesson material. Knowing that my students have different needs gives me a better understanding of how I can help them learn and the role differentiated and standards-based assessments might play in the process.

Another lesson I have learned about children from this study is that they have to be encouraged to reflect on what they know and do not know in order to recognize what they need to increase their own understanding of a unit’s learning targets. O’Connor (2011) suggests that when teachers help their students identify their strengths and
areas needing improvement, they give them the tools for setting goals and monitoring their progress in learning. I saw that my students became more engaged in their learning, leading to more accountability and a heightened level of ownership and responsibility for their learning during the Pythagorean Theorem unit. However, I realized that my students do not automatically come to me with these metacognitive skills, and I still need to find ways to help them become better self-reflective learners so they can use that information to communicate and collaborate more effectively with me in creating and executing plans for their learning.

What Have I Learned About the Larger Context of Schools and Schooling?

Tracking for mathematics instruction continues to be a prominent practice in high schools across the nation. Although many educators view tracking as an easy solution to meeting the needs of struggling and advanced learners (Grant, 2011), tracking has been shown to negatively affect the academic achievement and self-concept of students assigned to the lower track (Chmielewski et al., 2013; Trautwein et al., 2006). My study offers insights into what the teaching of a unit of instruction in a detracked mathematics classroom might look like for other schools who endeavor to detrack their high school mathematics courses.

Through differentiation and standards-based instruction, teachers can use various types of formative assessments to diagnose which students need more assistance with mastering the learning targets and identify students that are ready for more challenging work. Teachers can also utilize the results from paper-and-pencil formative assessments, observations, and student reflections to plan the next steps in their instruction, create targeted practice or extension activities for different levels of student understanding, and offer effective feedback to aid students in creating action
plans for learning. I plan to present what I have learned from this study to educators across the nation at conferences, such as the National Council of Teachers of Mathematics (NCTM) conferences and the Association for Supervision and Curriculum Development (ASCD) conferences. In so doing, I hope to entice other mathematics educators to abandon the practice of tracking that is so prevalent within the larger context of schools and schooling in order to create more equitable learning opportunities for all students.

**What are the Implications of What I Have Learned on My Teaching?**

My study provided insights into the ways planning and implementing a differentiated and standards-based unit of study based on the Pythagorean Theorem supported both my struggling and advanced learners within my detracked classroom. One of the lessons that I learned that has implications for my teaching is that I need to take a more deliberate approach to planning all of my units of instruction in a manner similar to how I planned the Pythagorean Theorem unit so that I can meet the diverse needs of all of my students in all of the units of instruction I teach. This entails creating lessons that take into consideration the different levels of learning within my detracked classroom. Essential in planning these units is that I identify areas in the lessons where a lack of prerequisite skills might impede student understanding or places where common misconceptions might occur that confuse student comprehension. Once identified, I need to determine if I need to provide concrete representations of abstract concepts for students who struggle with the learning targets. I also must be ready with activities for my advanced students that challenge or extend their knowledge of the content. Finally, I need to remember to assess constantly, and in different ways, to gain insight into my students’ understanding of the learning targets (Laud, 2011).
Because I have been teaching for many years, I realize that I have a preference in teaching style. I spend a great deal of time planning a unit of instruction, making sure to address each detail of every activity that I want to use and the way I want it to unfold during instruction. After conducting this study, I realize that even though deliberate planning is necessary for providing a differentiated and standards-based unit of instruction, I need to be more flexible in making changes to these planned lessons that support the learning of my students. These changes need to be made daily, or even on the spot during a lesson or activity, to meet the needs of specific students. “One-size-fits-all” kinds of lesson where all of my students miraculously understand the content when I teach it the first time do not exist for detracked classrooms. Being attentive to students’ needs and having flexibility in finding ways to meet those needs is key to teaching in a detracked classroom, because the varying range of student abilities requires different teaching techniques at different times. Wormeli (2007) discusses how teachers need to establish a mind-set to be responsive to their students’ academic needs, and suggests that, as teachers, we ask ourselves, “Are we willing to teach in whatever way is necessary for students to learn best, even if that approach doesn’t match our own preferences?” (p. 8). It is difficult to change what has always seemed to work in my practice, but with detracking my classroom, I have had to come to the realization that what I prefer may not always be best for the students in my classroom. My students’ comfort must come before my own. While the processes of differentiation and standards-based assessment are not practices I had a great deal of comfort with prior to this study, through this study I have seen the ways these practices help to make
both students who struggle and students who are advanced more comfortable in their learning.

**What Changes Might I Make to My Practice?**

A significant part of conducting this study was having the opportunity to communicate with a colleague about what I was doing in my classroom to increase the learning of my struggling and advanced students. As previously mentioned, Mickey MacDonald conducted a study similar to my own in which she examined how differentiated instruction and standards-based assessment supported the struggling and advanced learners within her detracked Honors Biology classroom. Having established a strong, trusting relationship with another teacher with whom I could share my concerns and elicit support helped me stay focused on the planning and implementation of the unit of instruction, as well as gain insights into my work as a practitioner researcher. Therefore, as a result of this study, I will seek additional opportunities to connect and collaborate with colleagues in relationship to creating more equitable learning environments for all students in our school through the practices of differentiation and standards-based assessment.

The administration at P. K. Yonge Developmental Research School has chosen to shift to using standards-based assessment and grading schoolwide by the start of the 2017-2018 academic year. This type of instruction and assessment is new to many teachers at our school, and I believe that the results from my study of using differentiated instruction and standards-based assessments in my detracked honors geometry classroom could be beneficial in helping the teachers make these changes. In particular, the experience that I have gained through analyzing the data from my study has the potential to provide knowledge and insights to my colleagues as the
school moves to using standards-based assessment and grading. Therefore, a change
I will make to my practice as a result of this study is to increase my capacity as a
teacher leader at my school. While I have engaged in multiple cycles of practitioner
inquiry prior to this study, this study has given me the confidence to step up as a leader
and change agent at my school to a greater extent to what I have done in the past,
particularly as my whole school transitions to standards-based assessment.

Crowther (1997) defines teacher leadership as: An ethical stance that is
based on views of both a better world and the power of teaching to shape
meaning systems. It manifests in actions that involve the wider
community in the long term. It reaches its potential in contexts where
system and school structures are facilitative and supportive (p. 15).

I have worked, and plan to continue to work with my colleague, Mickey MacDonald, to
lead school-wide inquiry into standards-based grading practices at P. K. Yonge
Developmental Research School. According to Katzenmeyer and Moller (2001),

When given opportunities to lead, teachers can influence school reform
efforts. Waking the sleeping giant of teacher leadership has unlimited
potential in making a real difference in the pace and depth of school
change (p. 102).

This study has provided me with knowledge of practice that has awakened the sleeping
giant of teacher leadership within me. As I participate to greater extent than I have in
the past in leading school-wide initiatives, I endeavor to make a real difference in the
pace and depth of school change at P. K. Yonge.

**What New Wonderings do I Have?**

While I learned a great deal about the ways differentiation and standards-based
assessments supported the learning of both students who struggled and students who
were advanced in my class, in analyzing the data from my study, I noticed that although
my struggling students increased their metacognition about their understanding of the
learning targets during their participation in the direct instruction and differentiated classroom activities, they did not perform as well on the summative assessments as I had hoped. Even with specific feedback and numerous forms of formative assessments, I believe these students do not fully understand the connection between the learning targets and solving problems that demonstrate proficiency on the standards on which they are assessed. I also think that many of my students, especially those who struggle with understanding the content, need help to figure out what to do if they finally make this connection with creating and following through on action plans to master the learning targets.

I believe my next step in helping students make connections to the learning targets in order to become more metacognitive is to make detailed rubrics for the formative assessments that list each specific learning targets, link them back to the problems on the assessment, and indicate whether the evidence the students provide shows their mastery of the learning targets. My hope is that by doing this, the students use the information to self-assess and then make plans to master the material before they take the summative assessment. Therefore, the results of this study have led me to have the following wonderings:

- In what ways does my providing specific feedback on detailed rubrics linked to the learning targets for each lesson help my students understand what they know and do not know?

- In what ways do my students use the information from the formative assessment rubrics to help them focus on the learning targets not yet mastered to prepare for summative assessments?

In addition to helping my students become more metacognitive about the mathematics content they need to master in my classroom, I want to help my students become more metacognitive about their habits of work, specifically in the areas of their
personal work ethic, perseverance, and accountability. By increasing students’ awareness of how academic success is linked to their own study habits, such as time-management, persistence, and responsibility with completing learning tasks, I can assist them with creating doable and meaningful action plans designed to increase their self-efficacy for attaining their own learning goals (Zimmerman, Bonner, and Kovach, 1996). With this goal in mind, I have an additional wonderings to explore within the context of my classroom:

- In what ways can I communicate and help students understand the correlation between their habits of work and their academic achievement and learning?
- In what ways can I increase student self-efficacy and help them develop into self-regulated learners?

Practitioner research provides the opportunity for me to examine my practice through numerous cycles of inquiry, and I look forward to pursuing the answers to these wonderings by continuing to engage in that process in my detracked honors geometry classroom.
Summative Assessment

Investigation of the Pythagorean Theorem

Solve each problem below, showing all your work.

1. \( x = \) ______ cm

[Diagram of a right triangle with sides 18, 30, and \( x \)]

2. Is \( \triangle ABC \) a right triangle? Explain why or why not.

[Diagram of \( \triangle ABC \) with sides 9, 12, and 18]

3. Show \( \overline{AB} \) as the hypotenuse of a right triangle. Then find \( \overline{AB} \).

\[ \overline{AB} = \] ______

[Diagram of \( \overline{AB} \) on the coordinate plane with points A and B]
4. \( AB = \underline{\phantom{000}} \text{ cm} \)

5. What is the length of the longer leg of a 30°-60°-90° triangle with a hypotenuse of length \( 24/\sqrt{3} \) cm? Show all your work and draw a picture of the triangle you used to find your answer.

\[ \underline{\phantom{000}} \]

6. Find the area of an equilateral triangle with sides measuring 6 meters. Show all your work and draw a picture of the triangle you used to find your answer.

\[ \text{Area} = \underline{\phantom{000000000000}} \]
7. Two jets pass directly over each other at 10 p.m. One jet is travelling south at 960 km/h; the other is traveling east at 900 km/h. How far apart are the two jets at midnight? Draw a picture to illustrate the situation and show all your work.

8. The equation of a circle is \((x - 3)^2 + (y - 3)^2 = 4\).
   - Find the center and radius of this circle.
   - Graph the circle on the coordinate plane.

   Center: _______________  Radius: _______________

   The circle is translated 2 units to the right and 4 units up and then is dilated by a factor of 3.
   - Find the new center and radius of the circle.
   - What is the equation of the new circle?
   - Graph the circle on the same coordinate plane as the original circle.

   Center: _______________  Radius: _______________

    Equation: ____________________________
9. Find the circumference of a circle circumscribed about a square with perimeter 90 in. Show all your work and draw a picture to find your answer.

\[ \text{Circumference} = \] 

10. Find the area of the shaded region if \( AB = 6\sqrt{3} \) cm.

\[ \text{Area} = \]
11. The diagram shows a rectangle with its vertices on a circle. The circle has a radius of 5 and is centered at the origin.

![Diagram of a circle with vertices A, B, C, and D]

a. Write an equation that relates $x$ and $y$ for any point $(x, y)$ on the circle.

b. Without using a ruler, find the length of each side of inscribed triangle $ABD$. Show all your work.

\[ AB = \text{__________} \quad BD = \text{__________} \quad AD = \text{__________} \]

c. What type of triangle is $\triangle ABD$? How do you know?

*Questions came from Discovering Geometry: An Investigative Approach and Connected Mathematics 3: Looking for Pythagoras*
Lesson 9.1: The Theorem of Pythagoras
Lesson 9.2: The Converse of the Pythagorean Theorem

Learning Targets

<table>
<thead>
<tr>
<th>4: Advanced</th>
<th>3: Proficient</th>
<th>2: Partially Proficient</th>
<th>1: Emerging</th>
<th>0: No credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can derive, understand, and use the relationship between the length of the legs and the length of the hypotenuse of a right triangle to solve problems</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I can define the Pythagorean Theorem</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I can find missing lengths of sides of triangles using the Pythagorean Theorem</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I understand and can use the converse of the Pythagorean Theorem to solve problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I can use Pythagorean triples to determine if a triangle is a right triangle.</td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

In a right triangle, the side opposite the right angle is called the _________________.

The other two sides are called _________________.

In the figure at right, a and b represent the lengths of the legs, and c represents the length of the hypotenuse.

There is a special relationship between the lengths of the legs and the length of the hypotenuse. This relationship is known today as the Pythagorean Theorem.

**The Pythagorean Theorem**

In a right triangle, the sum of the squares of the lengths of the legs equals ____________.

Color Tiles Activity
If \(a, b,\) and \(c\) are the lengths of the three sides of a triangle and they satisfy the Pythagorean equation, \(a^2 + b^2 = c^2,\) then the triangle must be a right triangle.

Three positive integers that work in the Pythagorean equation are called___________________________.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
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<tr>
<td>6</td>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>16</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

**Converse of the Pythagorean Theorem**

If the lengths of the three sides of a triangle satisfy the Pythagorean equation, then the triangle ____________.

Let’s look at examples to see how you can use the Pythagorean Theorem to find the distance between two points.

How high up on the wall will a 20-foot ladder touch if the foot of the ladder is placed 5 feet from the wall?

Find the area of the rectangular rug if the width is 12 feet and the diagonal measures 20 feet.

Les wanted to build a rectangular pen for his guinea pig. When he finished, he measured the bottom of the pen. He found that one side was 54 inches long, the adjacent side was 30 inches long, and one diagonal was 63 inches long. Is the pen really rectangular?
Lesson 9.3: Two Special Right Triangles

<table>
<thead>
<tr>
<th>Learning Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Advanced</td>
</tr>
<tr>
<td>3 Proficient</td>
</tr>
<tr>
<td>2 Partially Proficient</td>
</tr>
<tr>
<td>1 Emerging</td>
</tr>
<tr>
<td>0 Novice</td>
</tr>
</tbody>
</table>

I understand the relationship between the length of the legs and the length of the hypotenuse in a 45°-45°-90° triangle, and I can use this relationship to find unknown lengths of sides of isosceles right triangles.

I understand the relationship between the lengths of the shorter and longer legs and the length of the hypotenuse in a 30°-60°-90° triangle, and I can use this relationship to find unknown lengths of sides of right triangles with angle measures of 30°, 60°, and 90°.

In this lesson you will use the Pythagorean Theorem to discover some relationships between the sides of two special right triangles.

One of these special triangles is an isosceles right triangle, also called a 45°-45°-90° triangle. Each isosceles right triangle is half a square, so these triangles show up often in mathematics and engineering.

Investigation - Isosceles Right Triangles

In this investigation you will simplify radicals to discover a relationship between the length of the legs and the length of the hypotenuse in a 45°-45°-90° triangle. To simplify a square root means to write it as a multiple of a smaller radical without using decimal approximations.

Step 1: Find the length of the hypotenuse of each isosceles right triangle at right. Simplify each square root.

Step 2: Complete this table. Draw additional triangles as needed.

<table>
<thead>
<tr>
<th>Length of each leg</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>10</th>
<th>l</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of hypotenuse</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 3: Do you see a pattern between the length of the legs and the length of the hypotenuse?
Isosceles Right Triangle Conjecture

In an isosceles right triangle, if the legs have length \( l \), then the hypotenuse has length _________.

Another special right triangle is a \( 30^\circ-60^\circ-90^\circ \) triangle, also called a \( 30^\circ-60^\circ \) right triangle, that is formed by bisecting any angle of an equilateral triangle. The \( 30^\circ-60^\circ-90^\circ \) triangle also shows up often in mathematics and engineering because it is half of an equilateral triangle.

1. Why must the angles in \( \triangle ABC \) (or \( \triangle ACD \)) be \( 30^\circ, 60^\circ \), and \( 90^\circ \)?

2. How does \( BD \) compare to \( AB \)? How does \( BD \) compare to \( BC \)?

3. In any \( 30^\circ-60^\circ-90^\circ \) triangle, how does the length of the hypotenuse compare to the length of the shorter leg?

Let’s use this relationship between the shorter leg and the hypotenuse of a \( 30^\circ-60^\circ-90^\circ \) triangle and the Pythagorean Theorem to discover another relationship.

**Investigation – \( 30^\circ-60^\circ-90^\circ \) Triangles**

In this investigation you will simplify radicals to discover a relationship between the lengths of the shorter and longer legs in a \( 30^\circ-60^\circ-90^\circ \) triangle.

**Step 1:** Use the relationship from the questions you answered above to find the length of the hypotenuse of each \( 30^\circ-60^\circ-90^\circ \) triangle at right. Then use the Pythagorean Theorem to calculate the length of the third side. Simplify each square root.

**Step 2:** Complete this table. Draw additional triangles as needed.

<table>
<thead>
<tr>
<th>Length of shorter leg</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>10</th>
<th>( a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of hypotenuse</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of longer leg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Step 3: Do you see a pattern between the length of the longer leg and the length of the shorter leg?

**30°-60°-90° Triangle Conjecture**

In a 30°-60°-90° triangle, if the shorter leg has length $a$, then the longer leg has length _______ and the hypotenuse has length _______.

**EXAMPLE**

Find the lettered side lengths. All lengths are in centimeters.

\[
\begin{align*}
a &= \quad & a &= \quad & a &= \quad, \ b &= \\
\end{align*}
\]
Lesson 9.4: Story Problems

<table>
<thead>
<tr>
<th>Learning Targets</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can apply the Pythagorean Theorem and its converse to solve word problems and to solve application problems in 3-D.</td>
<td>Advanced</td>
<td>Proficient</td>
<td>Partially Proficient</td>
<td>Emerging</td>
<td>No idea!</td>
</tr>
</tbody>
</table>

You have learned that drawing a diagram will help you to solve difficult problems. By now you know to look for many special relationships in your diagrams, such as congruent polygons, parallel lines, and right triangles.

What is the longest stick that will fit inside a 24-by-30-by-18-inch box?

![Diagram of a box with dimensions and a stick labeled with unknown length x]
Lesson 9.5: Distance in Coordinate Geometry

### Learning Targets

<table>
<thead>
<tr>
<th>4</th>
<th>Advanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can derive, understand, and use the Pythagorean relationship on a coordinate plane (the distance formula).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3</th>
<th>Proficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>I know all of the simple concepts and problems.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2</th>
<th>Partially Proficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>I know the simple concepts and problems.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>I need help with all of the concepts and problems.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0</th>
<th>No level</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have never heard of this.</td>
<td></td>
</tr>
</tbody>
</table>

- I can use the distance formula to solve problems.
- I can derive the equation of a circle.

### Investigation – The Distance Formula

Use each segment as the hypotenuse of a right triangle. Draw the legs along the grid lines. Find the length of each segment using the Pythagorean Theorem.

#### a.

![Graph a](image)

#### b.

![Graph b](image)

Graph each pair of points, then find the distances between them.

#### a. \((-1, -2), (11, -7)\)

![Graph c](image)

#### b. \((-9, -6), (3, 10)\)

![Graph d](image)
What if the points are so far apart that it’s not practical to plot them? For example, what is the distance between the points $A(15, 34)$ and $B(42, 70)$? A formula that uses the coordinates of the given points would be helpful. To find this formula, you first need to find the lengths of the legs in terms of the $x$- and $y$-coordinates. From your work with slope triangles, you know how to calculate horizontal and vertical distances.

**DISTANCE FORMULA:**

**Example A:** Find the distance between $A(8, 15)$ and $B(-7, 23)$.

**Example B:** Write an equation for the circle with center $(5, 4)$ and radius 7 units.

**Example C:** Write the equation for the circle.
RETEST - Summative Assessment
Investigation of the Pythagorean Theorem

Solve each problem below, showing all your work.

1. \( m = \) _____ cm

\[ \text{Diagram}\]

2. Is \( \triangle ABC \) a right triangle? Explain why or why not.

\[ \text{Diagram}\]

3. Find the perimeter of \( \triangle MNP \). Round your answer to the nearest tenth.

\( \text{perimeter} = \) _____ in

\[ \text{Diagram}\]
4. \( AB = \) \_\_\_\_\_\_ cm

5. What is the length of the hypotenuse of a 30°-60°-90° triangle with a longer leg of length 16 m? Show all your work and draw a picture of the triangle you used to find your answer.

6. Find the area of an equilateral triangle with sides measuring 8 feet. Show all your work and draw a picture of the triangle you used to find your answer.

\[ \text{Area} = \]
7. Two sports cars leave the same city at 9 a.m. One car heads south at 60 mph, while the other travels east at 45 mph. How far apart are they at noon? Draw a picture to illustrate the situation and show all your work.

8. The equation of a circle is $(x + 2)^2 + (y - 5)^2 = 9$.
   - Find the center and radius of this circle.
   - Graph the circle on the coordinate plane.

   Center: ________________  Radius: ________________

The circle is translated 6 units to the right and 4 units down and then is dilated by a factor of 2.
   - Find the new center and radius of the circle.
   - What is the equation of the new circle?
   - Graph the circle on the same coordinate plane as the original circle.

   Center: ________________  Radius: ________________

   Equation: ____________________________
9. Find the circumference of a circle inscribed in a square with diagonal $8\sqrt{2}$ m. Show all your work and draw a picture to find your answer.

\[ \text{Circumference} = \underline{\phantom{000}} \]

10. Find the area of the shaded region if $KL = 6\sqrt{2}$ cm.

\[ \text{Area} = \underline{\phantom{000}} \]
11. Write an equation of a circle centered at the origin with a radius of 7 meters.

12. Determine the length of each side of triangle \( ABC \) with vertices \( A(5, 4), B(7, 12), \) and \( C(13, 6) \). Show all your work.

\[ AB = \quad BC = \quad AC = \]

What type of triangle is \( \triangle ABC \)? How do you know?

*Questions came from *Discovering Geometry: An Investigative Approach* and *Connected Mathematics 3: Looking for Pythagoras*.*
Dear Parent/Guardian,

I am a doctoral student in the School of Teaching and Learning at the University of Florida conducting research on using a teaching method called differentiated instruction and standards-based assessment under the supervision of Dr. Nancy Dana. For the capstone project for my degree, I wish to document the ways these two important practices (differentiated instruction and standards-based assessment) will support the mathematics learning of students currently enrolled in Geometry. Differentiation is a way of teaching where the teacher proactively modifies the curriculum, teaching methods, resources, learning activities, and student products to meet the needs of individual students and/or small groups of students to maximize the learning opportunities for each student in the classroom. With standards-based assessment, students’ grades are based on how well they learn and achieve mastery of the mathematics standards and are monitored and reported separately from their work habits, attendance, and class participation. The results of my study may help teachers better understand how to use these teaching practices to meet the learning needs of all of their students and help them master the mathematics content standards. With your permission, I would like to ask your child to volunteer for this research.

The study will be conducted during the unit focused on the Pythagorean Theorem. “Investigation of the Pythagorean Theorem” is a two week unit that begins in February and ends in March. Participation in this study does not change the instruction your student is currently receiving in Geometry. With your permission, I will collect artifacts of your child’s work, ask them to respond to questions about their learning experiences in my classroom, and perform a review of their academic data. Your child’s identity will be kept confidential to the extent provided by law. Students’ names will be replaced with pseudonyms. When the study is completed and the data have been analyzed, all will be destroyed. No names will be used in any reports. Participation or non-participation in this study will not affect grades or placement in any programs.

You and your child have the right to withdraw consent for your child’s participation at any time without consequence. There are no known risks or immediate benefits to the participants. No compensation is offered for participation. If you have any questions about this research protocol, please contact me via email at kweller@pvu.ufl.edu, or contact my faculty supervisor, Dr. Dana, at (352) 273-4204. Questions or concerns about your child’s rights as research participant may be directed to the IRB02 office, University of Florida, Box 112250, Gainesville, FL 32611, (352) 392-0433.

Kristin Weller  
Doctoral Candidate  
School of Teaching and Learning  
College of Education  
University of Florida

I have read the procedure described above. I voluntarily give my consent for my child, ___________________________________________________________________, to participate in Mrs. Weller’s differentiated instruction/standards-based assessment study. I have received a copy of this description.

________________________________________________________________________  ___________________________________________________________________  ____________________________
Parent/Guardian  
Date

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REFERENCES


Boykin, W., & Noguera, P. (2011). *Creating the opportunity to learn: Moving from research to practice to close the achievement gap*. Alexandria, VA: ASCD.


Clark, J. V. (2013). Introduction. In J. V. Clark (Ed), *Closing the achievement gap from an international perspective: Transforming STEM for effective education* (pp. 3-6). New York: Springer.


Darling-Hammond, L. (2013). Inequality and school resources: What it will take to close the opportunity gap. In P. Carter & K. Welner (Eds.), *Closing the opportunity gap: What America must do to give every child an even chance* (pp. 77-97). New York, NY: Oxford University Press.


Mann, H. (1849). *Twelfth annual report of the board of education, together with the twelfth annual report of the secretary of the board*. Boston: Dutton and Wentworth.


Rothstein, R. (2013). Why children from lower socioeconomic classes, on average, have lower academic achievement than middle-class children. In P. Carter & K. Welner (Eds.), *Closing the opportunity gap: What America must do to give every child an even chance* (pp. 61-74). New York, NY: Oxford University Press.


BIOGRAPHICAL SKETCH

Kristin Nashan Weller received her Bachelor of Science degree in secondary science/mathematics teaching in 1991 from Florida State University, her Master of Education in educational leadership at the University of Florida in 2000, and her Ed.D. in curriculum, teaching, and teacher education from the University of Florida in the spring of 2016. She is currently a University School Associate Professor at the University of Florida’s lab school, P.K. Yonge Developmental Research School. Kristin is a 25-year veteran high school and middle school mathematics instructor, spending 17 years teaching seventh-grade math before moving to her current position as a high school geometry teacher. She enjoys sharing her classroom experiences with other educators and has done so at conferences and workshops throughout the country. Her research interests are increasing student engagement and academic achievement through teachers’ use of effective classroom practices, curriculum, and assessment. Kristin resides in Gainesville, Florida, with her husband and children.