THE EFFECT OF A MATHEMATICAL TASK FOCUSED INTERVENTION ON PROSPECTIVE ELEMENTARY SCHOOL TEACHERS BELIEFS ABOUT MATHEMATICS INSTRUCTION

By

KRISTEN APRAIZ

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2014
To Briana and Gabe
ACKNOWLEDGMENTS

My pursuit of a doctoral degree could not have been possible without the support of my professors, family, and friends.

First, I would like to thank my adviser, Dr. Tim Jacobbe, and my committee members, Dr. Kent Crippen, Dr. David Miller, and Dr. Thomasenia Adams, for their time, feedback, and expertise.

Second, I would like to thank my mom, Ronna Appleby. I am forever grateful for how you engrained in me the value of a strong work ethic, the importance of education, and the ability to persevere through life. I would like to thank my husband, Gabe, for his endless support, understanding, and patience throughout these last four years.

Finally, I would like to thank my friends and fellow graduate students for their encouragement and support. In particular, I would like to acknowledge Dr. Julie Brown, Dr. Rich Busi, Dr. Cheryl Mclaughlin, and Rhonda Williams, for always listening and providing the opportunities to bounce ideas back and forth.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>4</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>8</td>
</tr>
<tr>
<td>LIST OF ABBREVIATIONS</td>
<td>9</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>10</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>1  INTRODUCTION</td>
<td>12</td>
</tr>
<tr>
<td>Working Definitions</td>
<td>17</td>
</tr>
<tr>
<td>Theoretical Perspective</td>
<td>17</td>
</tr>
<tr>
<td>Mathematical Tasks and Cognitive Demand</td>
<td>23</td>
</tr>
<tr>
<td>Cognitive Demand</td>
<td>23</td>
</tr>
<tr>
<td>Statement of the Problem</td>
<td>30</td>
</tr>
<tr>
<td>Structure of the Dissertation</td>
<td>37</td>
</tr>
<tr>
<td>2  REVIEW OF LITERATURE</td>
<td>38</td>
</tr>
<tr>
<td>Vision for Mathematics Instruction</td>
<td>38</td>
</tr>
<tr>
<td>Mathematical Tasks</td>
<td>40</td>
</tr>
<tr>
<td>The Earlier Years of Mathematical Tasks</td>
<td>41</td>
</tr>
<tr>
<td>Teacher Education and Mathematical Tasks</td>
<td>46</td>
</tr>
<tr>
<td>Using Knowledge of Students' Mathematical Thinking</td>
<td>62</td>
</tr>
<tr>
<td>Teacher Beliefs and Teacher Change</td>
<td>65</td>
</tr>
<tr>
<td>3  METHODS</td>
<td>72</td>
</tr>
<tr>
<td>Overview</td>
<td>72</td>
</tr>
<tr>
<td>Methodology</td>
<td>75</td>
</tr>
<tr>
<td>Procedures</td>
<td>77</td>
</tr>
<tr>
<td>Participants</td>
<td>77</td>
</tr>
<tr>
<td>Identification of participants</td>
<td>77</td>
</tr>
<tr>
<td>Description of participants</td>
<td>79</td>
</tr>
<tr>
<td>Setting</td>
<td>79</td>
</tr>
<tr>
<td>Treatment</td>
<td>80</td>
</tr>
<tr>
<td>Treatment group</td>
<td>80</td>
</tr>
<tr>
<td>Control group</td>
<td>82</td>
</tr>
<tr>
<td>Data Sources</td>
<td>83</td>
</tr>
<tr>
<td>Data Collection</td>
<td>83</td>
</tr>
<tr>
<td>Beliefs instrument</td>
<td>83</td>
</tr>
<tr>
<td>Interviews</td>
<td>86</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>F</td>
<td>EXAMPLE LESSON PLAN ................................................................. 159</td>
</tr>
<tr>
<td>G</td>
<td>SAMPLE OF BELIEFS INSTRUMENT ...................................................... 161</td>
</tr>
<tr>
<td>H</td>
<td>CROSSTABULATION FOR BELIEF SURVEY DATA ..................................... 162</td>
</tr>
<tr>
<td>I</td>
<td>CODING FOR THEMATIC ANALYSIS ...................................................... 164</td>
</tr>
<tr>
<td>J</td>
<td>HEIDI AND NORA'S MATHEMATICAL TASKS ......................................... 165</td>
</tr>
<tr>
<td>K</td>
<td>INFORMED CONSENT ........................................................................ 169</td>
</tr>
<tr>
<td></td>
<td>LIST OF REFERENCES ........................................................................ 171</td>
</tr>
<tr>
<td></td>
<td>BIOGRAPHICAL SKETCH ..................................................................... 180</td>
</tr>
</tbody>
</table>
### LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-1</td>
<td>Table of Participants</td>
<td>78</td>
</tr>
<tr>
<td>4-1</td>
<td>Beliefs pretest significance values</td>
<td>96</td>
</tr>
<tr>
<td>4-2</td>
<td>Actual count for belief change score for treatment and control groups</td>
<td>96</td>
</tr>
<tr>
<td>4-3</td>
<td>Belief change score significance values.</td>
<td>97</td>
</tr>
<tr>
<td>4-4</td>
<td>Belief 5 cross-tabulation values</td>
<td>99</td>
</tr>
<tr>
<td>4-5</td>
<td>Belief 7 cross-tabulation values</td>
<td>99</td>
</tr>
<tr>
<td>4-6</td>
<td>Percentage of students in each group whose scores on Belief 5 increased 1, 2, 3, or 4 levels from presurvey to postsurvey</td>
<td>99</td>
</tr>
<tr>
<td>4-7</td>
<td>Percentage of students in each group whose scores on Belief 7 increased 1 or 2 levels from presurvey to postsurvey</td>
<td>99</td>
</tr>
<tr>
<td>4-8</td>
<td>Heidi’s and Nora’s Pre- and Post-Belief Scores</td>
<td>100</td>
</tr>
<tr>
<td>4-9</td>
<td>Descriptive Statistics on Mathematical Task Sort Scores</td>
<td>117</td>
</tr>
<tr>
<td>4-10</td>
<td>Comparison of Pre-Mathematical Task Sort Scores of Treatment and Control Groups</td>
<td>118</td>
</tr>
<tr>
<td>4-11</td>
<td>Analysis of the Task Sort Responses by Level of Cognitive Demand (n = 32 preservice teachers)</td>
<td>121</td>
</tr>
<tr>
<td>H-1</td>
<td>Belief 1 crosstabulation values</td>
<td>162</td>
</tr>
<tr>
<td>H-2</td>
<td>Belief 2 crosstabulation values</td>
<td>162</td>
</tr>
<tr>
<td>H-3</td>
<td>Belief 3 crosstabulation values</td>
<td>162</td>
</tr>
<tr>
<td>H-4</td>
<td>Belief 4 crosstabulation values</td>
<td>162</td>
</tr>
<tr>
<td>H-5</td>
<td>Belief 5 crosstabulation values</td>
<td>162</td>
</tr>
<tr>
<td>H-6</td>
<td>Belief 6 crosstabulation values</td>
<td>163</td>
</tr>
<tr>
<td>H-7</td>
<td>Belief 7 crosstabulation values</td>
<td>163</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------------</td>
<td>------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>CBMS</td>
<td>Conference Board of the Mathematical Sciences</td>
<td></td>
</tr>
<tr>
<td>CCSS</td>
<td>Common Core State Standards</td>
<td></td>
</tr>
<tr>
<td>NCTM</td>
<td>National Council of Teachers of Mathematics</td>
<td></td>
</tr>
<tr>
<td>NMAP</td>
<td>National Mathematics Advisory Panel</td>
<td></td>
</tr>
<tr>
<td>NRC</td>
<td>National Research Council</td>
<td></td>
</tr>
<tr>
<td>PST</td>
<td>Prospective Elementary School Teacher</td>
<td></td>
</tr>
</tbody>
</table>
Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

THE EFFECT OF A MATHEMATICAL TASK FOCUSED INTERVENTION ON
PROSPECTIVE ELEMENTARY SCHOOL TEACHERS BELIEFS ABOUT
MATHEMATICS INSTRUCTION

By
Kristen Apraiz

August 2014

Chair: Tim Jacobbe
Major: Curriculum and Instruction

The purpose of this study was to determine the extent to which elementary
prospective teachers’ beliefs about mathematics instruction change as a result of
participating in a 12-week intervention focused on learning about the level of cognitive
demand of mathematical tasks and writing letters to third grade elementary students. A
necessary component of implementing mathematical tasks effectively with students is to
have a foundational understanding of mathematical – task knowledge (Chapman, 2013). During the 12-week intervention the treatment group learned about features of
mathematical tasks that elicit low-level and high-level cognitive demand from students.
The prospective teachers wrote their own mathematical tasks that they included in
letters to third grade elementary students. The situative learning perspective informed
the design of the intervention. Specifically, the intervention was designed to position
participants to examine, do, and discuss mathematical tasks and participate in a letter
writing exchange with third grade students.

The study utilized mixed methods to investigate the prospective teachers’ beliefs
about mathematics instruction and knowledge of cognitive demand for mathematical
tasks. The Integrated Mathematics and Pedagogy (IMAP) Belief Survey (Philip et al., 2007) was used to collect data on beliefs. Additionally, participants of the study completed an elementary mathematical task sort on two occasions. In addition, two interviews were conducted for participants who exhibited changes in several beliefs and a change in one belief for the seven beliefs.

Results from the study indicate that the treatment group experienced significant belief changes on two of the seven beliefs. These two beliefs most closely align with the intervention. Results from the mathematical task sort indicate that participants in the treatment group were more able to accurately identify mathematical tasks as either eliciting low – or high – level cognitive demand. The outcome of the study could provide guiding principles for designing the elementary mathematics methods course where prospective teachers focus on the level of cognitive demand of mathematical tasks and have opportunities to interact with elementary school students through authentic experiences.
CHAPTER 1
INTRODUCTION

Perhaps the major challenge facing those who wish to improve the mathematics learning of U.S. students is taking seriously the fact that teaching must change.

(Hiebert, 2009, Foreword, p. ix)

Over a decade ago, the National Council of Teachers of Mathematics (NCTM) (2000) placed a strong emphasis on the need for every student to “have access to high-quality, engaging math instruction” (“A Vision for School Mathematics,” para. 3). Additionally, students need to learn in a way that creates understanding by having opportunities to discuss mathematics and engage in complex tasks while drawing upon a variety of mathematical topics (National Research Council, 2001). Students can have access to the aforementioned learning opportunities by having a teacher who has been trained to implement mathematics instruction using well-thought out mathematical tasks. Just like students, prospective and inservice teachers need opportunities to explore mathematics, collaborate with peers, and learn pedagogical strategies to help reach all students during mathematics instruction (Conference Board of the Mathematical Sciences, 2012; Stein, Smith, Hennigsen & Silver, 2009). Ideally, the teacher’s role is to help facilitate a learning environment where students are conjecturing, refining mathematical ideas, and applying mathematical knowledge to relevant, real-world tasks (National Governors Association Center for Best Practices (NGA) & Council of Chief State School Officers (CCSSO), 2010).

Establishing a classroom-learning environment that incorporates the desired student abilities takes practice, support, and time to accomplish. Prospective teachers learn about this type of teaching by reading course textbooks, watching videos, and
observing classrooms (Ball, 1990). Rarely do prospective teachers have opportunities to experiment with this type of teaching (Ball, 1990; Ebby, 2000; Hiebert, Morris & Glass, 2003; Kosko, Norton, Conn, & San Pedro, 2010). In order for students to learn mathematics at the desired level, teachers must be prepared to use mathematical tasks that will allow students to go deeply into the mathematical content (Stein et al., 2009).

Currently, with the widespread adoption of the Common Core State Standards (CCSS) (NGA & CCSSO, 2010) for mathematics across the United States, teachers have opportunities to reconsider the structure of lessons for students to learn mathematics through rigorous, cognitively demanding mathematical tasks, which emphasize justification, synthesis, and analysis of mathematics (Grossman, Reyna, & Shipton, 2011).

“Mathematics tasks are important vehicles for classroom instruction that aims to enhance students’ learning” (Shimizu, Kaur & Clarke 2010, p.1). The mathematical tasks teachers choose to implement in their classroom provide opportunities to engage students in the learning of mathematics (Chapman, 2013; National Research Council, 2001). Mathematical tasks are the problems teachers present to students. For example, a mathematical task could ask the student for one answer or an extended response that requires an explanation. Well-constructed mathematical tasks allow students to view mathematics as a connected discipline where mathematical reasoning, problem solving, and justifying the mathematics are needed to accomplish the mathematical task (National Council of Teachers of Mathematics [NCTM], 2000; National Research Council, 2001; Stein, Grover, & Henningsen, 1996). In order to use mathematical tasks effectively in mathematics instruction, both in-service and prospective teachers need
time to consider the possible outcomes mathematical tasks can elicit (National Research Council, 2001).

Prior research has revealed that teachers who are provided with opportunities to learn and focus on the cognitive demands of mathematical tasks have experienced increases in student learning and engagement during mathematics instruction (Boston & Smith, 2011; Henningsen & Stein, 1997). Teachers who participated in these studies had the opportunities to identify features of mathematical tasks that elicit low and high levels of cognitive demand, learn how to maintain the level of cognitive demand from the initial phase of choosing the task through the implementation phase in the classroom, and examine student work related to the mathematical task (Henningsen & Stein, 1997). The process of selecting a mathematical task for mathematics instruction, implementing the task with students, and discussing student results takes time for teachers to learn (Henningsen & Stein, 1997). Therefore, it is crucial for prospective teachers to have opportunities during their teacher preparation to develop and implement mathematical tasks through authentic activities, such as participating in a letter writing exchange (Kosko, et al., 2010).

The aforementioned opportunities are encompassed in classroom environments where students construct their own knowledge of mathematics through problem solving and experimentation. In order for students to experience mathematics as a genuine discipline, both prospective and practicing teachers need to experience learning mathematics the way policy documents recommend mathematics be taught (NCTM, 2000; National Mathematics Advisory Panel, 2008). The Conference Board of the Mathematical Sciences (CBMS) (2012) supports this view by recommending that
“preparation and professional development for teachers must provide opportunities to do mathematics and to develop mathematical habits of mind” (p.16).

Prospective teacher education needs to include experiences where teacher candidates can “examine and revise their assumptions about the nature of mathematics, how it should be taught, and how students learn mathematics” (NCTM, 1991, “Standard 5 – Developing as a Teacher of Mathematics”, para. 1). For instance, prospective elementary school teachers (PSTs) enter teacher education programs with about 13 years of passively watching teachers teach mathematics. These years of watching contribute to the preconceived notions PSTs have about how mathematics should be taught (Ball, 1990; Ebby, 2000, Ernest, 1989). More often than not, the mathematics instruction they believe is the way to teach is the traditional approach to teaching – where the teacher stands up in front of a room of students and lectures, while the students take notes and work on problems from textbooks (Bahr, Monroe & Shaha 2013).

During teacher education courses, PSTs experience a conflict between their own learning of mathematics and the “new” methods of teaching mathematics (Ebby, 2000; Hiebert, et al., 2003). To further complicate the PSTs understanding of mathematics teaching, they are sometimes placed in educational field experiences where the teacher is not teaching students in the same manner as preservice teachers are learning in their teacher education courses (Philipp, et al. 2007). With the implementation of the CCSS for mathematics, PSTs need opportunities where they can experiment with posing mathematical tasks to elementary students and assessing the cognitive demand of mathematical tasks in print form (CBMS, 2012).
The CCSS for mathematics place an emphasis on preparing students for future careers and colleges. The standards strive to increase the rigor and relevance of mathematics in order to ensure that students are prepared with the knowledge and skills that will lead them to having success in colleges and careers (Grossman, et al., 2011). Traditionally, individual states have set the criteria for mathematics instruction. The CCSS hold all students across the nation to the same expectations for mathematical learning. The implementation of the CCSS for mathematics will require ongoing professional development for in-service teachers and more exposure to mathematics for prospective teachers (Association of Mathematics Teacher Educators, 2011). Not only will teachers need a deep level of mathematical content knowledge, they will also need to know how to structure learning experiences for students in order to implement the Standards for Mathematical Practice (NGA & CCSSO 2010).

The purpose of this study is to determine the extent of change that can be ascertained by an intervention focused on learning about the cognitive demand of mathematical tasks. The mathematical tasks would be posed through a letter writing exchange and a review would be made of the resulting PST beliefs about mathematics instruction. The hypothesis guiding the study is that providing PSTs with opportunities to consider the cognitive demand of mathematical tasks, solve those tasks and discuss the solutions, while posing mathematical tasks through a letter writing exchange will produce significant changes in PSTs’ beliefs about mathematics instruction. The remainder of this chapter will provide justification for learning about the cognitive demand of mathematical tasks and posing mathematical tasks through a letter writing exchange.
**Working Definitions**

Mathematics education tends to use common terms throughout the literature that may be defined differently in other areas of education. In order to provide clarification for the reader, certain terms are defined.

1. **MATHEMATICAL TASK** – “defined as a classroom activity, the purpose of which is to focus students’ attention on a particular idea” (Stein, Grover, & Henningsen, 1996, p.460).

2. **PRACTICING OR INSERVICE TEACHER** – a current teacher who is working in a school.

3. **PROSPECTIVE ELEMENTARY SCHOOL TEACHER** – current postsecondary student who is enrolled in a teacher education program at a higher learning institution.

4. **STUDENTS’ MATHEMATICAL THINKING** – how students process mathematics in order to make mathematical decisions.

5. **COGNITIVE DEMAND** – the type or level of thinking produced by the student to solve a mathematical task (Stein et al., 1996).

6. **REFORM MATHEMATICS INSTRUCTION** – refers to mathematics instruction that goes beyond teaching procedures and having students recall facts; students are expected to apply knowledge, explore mathematics, and produce conjectures about how a mathematics concept is applied (NCTM, 2000).

**Theoretical Perspective**

According to Chapman (2013), there are several factors that influence the implementation of mathematical tasks: the teacher’s knowledge of content, knowledge of learners, goal for task, instructional orientation, and beliefs about mathematics instruction. Chapman claims that teachers' mathematical-task knowledge is the determining factor for how tasks are implemented in the classroom. Mathematical-task knowledge is described as “the knowledge teachers need in order to (a) select and develop tasks to promote students’ conceptual understanding of mathematics, support their development of mathematical thinking, and capture their interest and curiosity and (b) optimize the learning potential of such tasks” (Chapman, 2013, p.1). More
specifically, mathematical-task knowledge includes the teachers’ ability to select mathematical tasks that build upon students’ prior knowledge, elicit a certain level of cognitive demand, and supports students’ learning of mathematics (NCTM, 1991). Purposefully selected mathematical tasks are a key component for effective mathematics instruction (National Research Council, 2001).

Mathematical tasks are viewed as channels that provide opportunities for students to engage in problem solving and to see mathematics as a discipline with practical purposes, (i.e., one which makes sense to learn) (National Research Council, 2001). According to the National Research Council (2001) “in the classroom the teacher, the students, and the task clearly interact in a dynamic way to shape students’ learning” (p.336). A mathematical task can hold the potential to elicit a high level of cognitive demand; however it is up to the teacher and student whether the cognitive demand is maintained for the duration of the lesson. Teachers’ beliefs about mathematics instruction (i.e., how students learn) influence the implementation of the mathematical task (National Research Council, 2001).

For instance, a teacher selects a mathematical task that exhibits a high-level of cognitive demand because the task asks students to apply knowledge of a concept to a real-world application. From the time the mathematical task is first introduced through the duration of time the student attends to the task, the teacher has the ability to affect the level of cognitive demand. If the teacher believes that students should not struggle with mathematics, they may intervene in the process of making sense of the task and lower the level of cognitive demand by suggesting a possible alternative way to produce an answer. The aforementioned example is provided to show how a teacher can affect
the implementation of a mathematical task. Mathematics instruction leaves lasting impressions on students because “the mathematical tasks with which students become engaged determine not only what substance they learn but also how they come to think about, develop, use, and make sense of mathematics” (Stein et al., 1996, p.459). Preservice teachers can benefit from experimenting with posing mathematical tasks in order to learn how to use mathematical tasks effectively during instruction.

Exposure to mathematical tasks during preservice teacher education could provide opportunities for PSTs to acquire mathematical-task knowledge. Researchers, Crespo (2003) and Norton and Kastberg (2012) used the authentic activity of letter writing with PSTs. The letter writing experience provided PSTs with the opportunity to pose mathematical tasks to students, analyze the students' thinking about the mathematical task, and contemplate how to respond to students. Norton and Kastberg (2012) incorporated the levels of cognitive demand into their research study, which involved secondary preservice teachers posing mathematical tasks to high school students through letter writing, in order to demonstrate the potential cognitive demand a task could elicit from a student. Crespo (2003) had PSTs focus on the type of mathematical tasks they asked students to complete. Both studies revealed that PSTs progressed over time in their ability to pose cognitively demanding mathematical tasks.

Chapman (2013) describes mathematical-task knowledge for teaching as “multi-dimensional and thus likely to be challenging for a teacher to construct without meaningful intervention to build on her or his initial sense-making of tasks” (p.2). This dissertation focuses on building PSTs' mathematical-task knowledge, which is necessary to effectively implement mathematical tasks. Further, preservice teachers
developed select components of this knowledge by taking part in learning about the
cognitive demands of mathematical tasks and posing such tasks to elementary school
students. Another component of the study is the mathematical instructional beliefs held
by the PSTs.

Preservice teachers hold beliefs about mathematics and mathematics instruction
that date back to their years as students (Ernest, 1989). According to Philipp et al.
(2007) PSTs “hold a self-perpetuating belief that ‘If I, a college student, do not know
something, then children would not be expected to know it, and if I do know something, I
certainly don’t need to learn it again’” (p. 439). If it is true that PSTs hold such self-
perpetuating beliefs, then challenging such beliefs is a demanding task for teacher
educators to undertake. Philipp et al. (2007) posits that in order for PSTs to care about
learning mathematics for teaching, teacher educators need to recognize that PSTs care
about children first. In the study by Philipp et al. (2007), researchers used a guiding
hypothesis that if PSTs focused on understanding how children think about
mathematics, then their beliefs about mathematics instruction would change. The theory
behind the belief change is for the PSTs to connect the mathematical content
knowledge with knowledge of how to teach children to learn and understand
mathematics. Philipp et al. (2007) used the Circles of Caring model, which is an
expansion of Nel Noddings’ (1984) theory about caring, to describe the process by
which PSTs learn to care about learning mathematics for teaching.

The premise behind the Circles of Caring model is to tap into the feelings of
caring that PSTs have for children. Next, PSTs can see how children succeed in solving
a mathematical task, and they are then more inclined to learn how to incorporate more
of these experiences for children. The PSTs will be able to see how mathematical thinking plays a role in solving mathematical tasks for children. Lastly, PSTs will want to know more about mathematics to help children.

The desired outcome from the aforementioned study by Philipp et al. (2007) was for the PSTs to change their beliefs pertaining to mathematics instruction from a formulaic, procedural process to a more conceptual-based form of mathematics instruction. According to Philipp et al. (2007), the change in beliefs that the researchers were hoping to see in PSTs arrived through the act of positioning them in an environment where they engage in designed mathematical experiences (i.e., watch videos of children solving problems, read case studies, examine student work) to either act or to consider how to act with children. Due to the structure of the field experiences for the PSTs, Philipp et al. (2007) used the situative learning perspective to guide their research. The situative learning perspective is the lens through which this dissertation views the Circles of Caring theory and the acquisition of mathematical-task knowledge for PSTs.

The situative learning perspective is a part of two other perspectives – sociocultural and distributed cognition (Sawyer, 2006). The socioculturalist envisions the learner as an individual, who is part of a larger community (Lave, 1991). The activities that take place within the community help the individual acquire knowledge (Cobb & Bowers, 1999). The activity is the unit of analysis in the situative learning perspective (Rogoff, 2008). Reform documents focused on improving teacher education (e.g., NMAP [2008], NCATE [2010]), have made recommendations for preservice teachers to have opportunities which allow them to be part of the teaching community. For example,
one opportunity is a field experience where preservice teachers may observe, interact, and discuss classroom activity with peers and a teacher educator.

Interactions with others in the environment help shape learning for the individual (Vygotsky, 1978). In order to promote an optimally effective learning community, the following elements were identified by researchers as critical to facilitate learning: the role of the teacher educator, the establishment of classroom norms, and the use of tools to help preservice teachers learn about teaching using mathematical tasks (Cady, Meier & Lubinski, 2006; Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, and Perlwitz, 1991; Stein & Henningsen, 1997; Szydlik, Szydlik & Benson, 2003). For instance, in the study conducted by Szydlik et al (2003), the teacher educators facilitated a classroom learning environment, which focused on establishing social norms. The establishment of agreed upon social norms assisted the preservice teachers in working together on problem-solving tasks. During the course of the semester, the researchers were interested in how this type of learning environment affected the beliefs of preservice teachers about mathematics instruction. Several preservice teachers expressed a new understanding about mathematics as a connected discipline (Szydlik et al., 2003). For example, the preservice teachers were able to experience how several mathematical topics were needed to solve a mathematical task.

Researchers using this theory of learning place an emphasis on the setting and the activities that happen within this setting (Putnam & Borko, 2000). The tools that preservice teachers will acquire in order to learn to use mathematical tasks effectively develop through their experiences, which are situated within authentic activities. Brown, Collins and Duiguid (1989) describe such activities by stating, “Authentic activities then,
are most simply defined as the ordinary practices of the culture” (p.34). According to Putnam and Borko (2000), “authentic activities foster the kinds of thinking and problem-solving skills that are important in out-of-school settings, whether or not the activities themselves mirror what practitioners do” (p.5). Teacher preparation strives to prepare PSTs to take on leadership roles in the classroom that involve making instructional decisions on a daily basis.

The experiences provided to PSTs by teacher educators during their teacher preparation should situate learning in a setting where the activity allows PSTs to examine their thinking by constructing meaning with their peers through social interaction (Lave, 1991; Putnam & Borko, 2000). The authentic activities in this dissertation provide the context for mathematics learning to be viewed as a social process (Cobb & Bowers, 1999; Gee, 2008). The intervention for the study is designed to provide the PSTs the opportunities to work together on mathematical tasks and participate in an authentic letter writing exchange. The letter writing exchange served as an activity, which follows the Circles of Caring theory, by allowing the PST to focus on a student and learn about how the student was thinking about mathematics. During the intervention, the cognitive demand of mathematical tasks was used as a focal point for PSTs to consider the opportunities to learn mathematics provided by mathematical tasks.

**Mathematical Tasks and Cognitive Demand**

**Cognitive Demand**

For this study, the task analysis guide from Stein, Smith, Henningsen & Silver (2000) will provide a framework for analyzing the cognitive demand of mathematical tasks. The researchers define cognitive demand as “the kind and level of thinking
required of students in order to successfully engage with and solve the task” (p.11).

Stein et al. (2000) use a task analysis guide to assess the level of cognitive demand a task is capable of producing for a student. The task analysis guide consists of two categories: Lower-Level Demands and Higher-Level Demands. Within each category, there are two sub-categories.

For instance, the Lower-Level Demand category incorporates memorization tasks and procedures without connections tasks (e.g., Solve. 25 ÷5). The Higher-Level Demand category includes procedures with connections tasks and completion of mathematics tasks (e.g., think of a real-life situation that describes the following problem: 124÷12; write the problem and then solve it). Descriptions for each of the sub-categories are provided, so the teacher can analyze the mathematical task with the description to determine the level of cognitive demand a mathematical task has the potential of producing. For example, a teacher can use the task analysis guide to categorize questions on a standardized test (e.g., The Florida Comprehensive Assessment Test) in order to determine the questions’ potential cognitive demand for the student. Stein et al. (2000) stress the importance of matching the task with the learning goal for the student. The task analysis guide was developed with the intention of building on students’ prior knowledge and allowing the teacher the opportunity to scaffold student thinking, with the ultimate goal of eliciting high-level thinking from the students (Stein et al., 2000). With this in mind, the task analysis guide is a tool teachers can use to plan mathematics instruction.

Improved student achievement can be linked to teachers’ instructional practices that are focused on the implementation of cognitively demanding tasks (Boston & Smith,
Several studies have focused on the cognitive demand of mathematical tasks in a professional development setting with teachers, and include the introduction of the task analysis guide to preservice teachers (e.g., Arbaugh & Brown, 2005; Boston & Smith, 2011; Kosko et al., 2010; Leung & Silver, 1997; Norton & Kastberg, 2011; van den Kieboom & Magiera, 2010). Each of these studies focused on building teachers’ mathematical-task knowledge by choosing mathematical tasks and determining the cognitive demand. It is important for teachers to experience mathematics, just as they want their students to experience the mathematics (CBMS, 2012). One way to do this is through working in a supportive environment where teachers feel comfortable asking questions about mathematics and explaining their thinking (Boston & Smith, 2011).

Stein and Smith (1996) investigated the use of cognitively demanding tasks with middle school teachers through their work on the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project. During the QUASAR project, middle school teachers had the opportunity to learn about the cognitive demand of mathematical tasks. As a result of this project, Stein and Smith (1996) were able to develop a Mathematical Task Framework (MTF). The framework is designed to show the stages a teacher progresses through when using tasks in mathematics instruction. For this dissertation, the treatment group focused on the beginning stage of the MTF. The beginning stage for the MTF is how a mathematical task is represented in curricular/instructional materials.

Researchers Arbaugh and Brown (2005) conducted a study which focused on the MTF. A group of geometry teachers set forth on a journey to examine the
mathematical tasks in their curriculum materials while learning about the levels of cognitive demand. They found that the time spent on analyzing the level of cognitive demand for mathematical tasks proved to be a catalyst for change in the majority of teachers’ mathematical instructional practices. According to Arbaugh and Brown (2005), “this type of intervention proved to be a non-threatening way to start teachers thinking more deeply about their practice” (p.527). Many aspects of the study by Arbaugh and Brown will be addressed with PSTs in the research design of this study.

Boston and Smith (2011) used a “task-centric approach” to professional development for secondary mathematics teachers who participated in a NSF-funded project called Enhancing Secondary Mathematics Teacher Preparation. The researchers found that the teachers who participated in the project were able to set up and enact high-level tasks significantly better than a control group of teachers. One important component of the professional development is the need for it to be “grounded in the everyday activities of teaching, thereby providing teachers with the opportunity to learn about practice from a close examination of practice” (Boston & Smith, 2011, p. 967). For instance, Darling-Hammond (2013) asserts that when teachers have the opportunity to collaborate with other teachers, they benefit from learning and sharing with each other and can directly apply their knowledge to the classroom. Another key takeaway from the study was the recognition that teachers will enter professional development with varying degrees of experience using cognitively demanding tasks. It is important to provide support for teachers as they progress, at their own pace, in recognizing and implementing these cognitively challenging instructional tasks.
The aforementioned studies that used the MTF (Stein, Grover, & Henningen 1996) focused on professional development with secondary teachers. Researchers, Kosko et al. (2010), Osana, LaCroix, Tucker, and Desrosiers (2006), Norton and Kastberg (2011), and van de Kieboom and Magiera (2010) used the framework and task analysis guide while working with PSTs on developing mathematics and mathematics teaching. In all the studies, PSTs demonstrated an understanding of how to assess mathematical tasks for cognitive demand. One aspect that the studies conducted by Kosko et al. (2010), Norton and Kastberg (2011), and van de Kieboom and Magiera (2010) encompassed was the opportunity for PSTs to implement mathematical tasks with students and then analyze the students’ mathematical thinking.

For instance, the PSTs who were part of the van de Kieboom and Magiera study were provided the opportunity to explore mathematical tasks in their mathematics content course. The exploration consisted of learning multiple ways to represent a solution to a problem, to focus on student misconceptions, and to focus on important aspects of the mathematical tasks that PSTs felt were essential to student understanding. The mathematics educators selected the mathematical tasks for the PSTs. The educators wanted the PSTs to focus on the mathematics and implementing tasks with students in order to understand how students were thinking about mathematics. A component of the mathematics content course included field experiences where PSTs were able to implement the mathematical tasks with students and reflect upon the process.

Kosko et al. (2010) and Norton and Kastberg (2011) worked with preservice secondary teachers on learning how to pose mathematical tasks through letter writing.
The idea of letter writing came from the work of Crespo (2003), who originally studied how preservice teachers posed problems to their elementary students through a letter writing exchange. Building upon Crespo’s work, Kosko et al. had the preservice teachers incorporate the NCTM (2000) Process Standards (problem solving, reasoning and proof, communication, connections, representation) and use these to build toward higher levels of cognitive demand when writing letters to high school students. Prior to sending the letter, the preservice teachers were asked to predict what processes and levels of cognitive demand the mathematical task would produce from the student. The researchers were able to measure the preservice teachers’ growth by evaluating the reflective papers written by the preservice teachers throughout the duration of the letter writing exchange. According to Kosko et al. (2010), “preservice teachers demonstrated improvements over the semester in their ability to engage students in different mathematical processes at a high level of cognitive demand” (p.211). Interestingly, the preservice teachers struggled when trying to distinguish between mathematical tasks that could be classified by the two high-level categories which were: (1) procedures with connections and (2) doing mathematics. Researchers Osana, et al. (2006) had a similar result when PSTs were asked to classify mathematical tasks by cognitive demand.

Osana et al. (2006) were interested in finding out if there was a relationship between the mathematical content knowledge of PSTs and their ability to classify mathematical tasks using the task analysis guide. The PSTs received a 45-minute lecture on cognitive demand levels of mathematical tasks during one session of their mathematics methods course. Afterwards, the preservice teachers participated in a mathematical task sort. The researchers found that preservice teachers who obtained
higher scores on the mathematical content knowledge test had a higher score for correctly sorting the mathematical tasks.

Additionally, researchers found that the PSTs lacked knowledge in understanding how elementary students’ think about mathematics. This result led Osana and colleagues (2006) to suggest “that preservice teachers’ limitations in their knowledge of children’s thinking may hinder their ability to classify accurately mathematical tasks, particular those that are specifically designed to stimulate genuine mathematical thought” (p.368). PSTs need opportunities to see how children think and respond to mathematical tasks (Osana et al., 2006). Furthermore, situating the learning for PSTs within an authentic activity such as a field experience or providing an in-class activity (i.e., viewing videos of students working on mathematical tasks) would further their understanding about the cognitive demand a particular task is capable of eliciting in students (Norton & Kastberg, 2011; Osana et al., 2006).

Learning about the potential cognitive demand a mathematical task can possess is one aspect of learning how to implement mathematical tasks. Bringing awareness of the potential cognitive demand of a mathematical task allows practicing and preservice teachers the opportunity to consider the implications for classroom instruction (Arbaugh & Brown, 2005; Norton & Kastberg, 2011; Osana et al., 2006). Each of the studies focused on strengthening practicing and preservice teachers’ mathematical-task knowledge (Chapman, 2013). PSTs need opportunities to apply their knowledge, observe students’ learning mathematics through mathematical tasks, and reflect upon their experience.
Statement of the Problem

“Tasks are central to students’ learning, shaping not only their opportunity to learn but also their view of the subject matter” (National Research Council, 2001, p.335). The National Mathematics Advisory Panel (NMAP) (2008) stresses the need for teachers to have extensive mathematical content knowledge and knowledge about how mathematical concepts connect together so mathematics is viewed as a coherent discipline. Providing students with the opportunity to learn mathematics through worthwhile mathematical tasks has been shown to increase student achievement (Boston & Smith, 2011; National Research Council, 2001; Stein et al., 1996).

Considering the desired outcome of student achievement, preservice teachers need “opportunities to examine how students are thinking about mathematical ideas, and to learn about learning paths and tasks designed to help students progress along learning paths” (CBMS, 2012, p. 32). More specifically, preservice teachers need to experience mathematics that allows them to struggle and persevere in solving a problem. This experience needs to take place in a supportive environment where preservice teachers can learn together and discuss the mathematics (Chapman, 2007). Essentially, if preservice teachers experience the process of working through a challenging problem and the feeling of accomplishment afterwards, then possibly they will encourage their students to do the same.

Teacher preparation and professional development are the necessary components for teachers to strengthen their content knowledge and pedagogical knowledge (National Research Council, 2001). Learning to teach differently from the way one was taught is a change that does not happen instantly (Hiebert, Morris, & Glass, 2003). Teacher education courses are an ideal environment for preservice
teachers to learn how to develop learning, how to pose mathematical tasks, analyze the
cognitive demand of a mathematical task, and focus on students’ mathematical thinking,
while receiving support from their peers and teacher educators.

Many aspects of planning mathematics lessons incorporate the implementation
of challenging mathematical tasks during mathematics instruction. Stein, Grover, and
Henningsen (1996) created the conceptual framework of mathematical tasks to describe
how they occur during classroom instruction. The framework consists of three phases:
1) mathematical tasks represented in curricular materials, 2) mathematical tasks as set
up by the teacher in the classroom, and 3) mathematical tasks as implemented by
students in the classroom (p. 459). There is a further dimension that features such
factors as teachers’ goals, teachers’ subject matter knowledge, class norms, and the
pedagogical knowledge of teachers, which contributes to the overall cognitive demand
of the task.

Stein et al. (1996) define the task set up “as the task that is announced by the
teacher” (p. 460). An example of task set up is explaining directions to students. The
researchers define task implementation as “the manner in which students actually work
on the task” (p.460). It is important to distinguish between these two phases in the
conceptual framework because this is where the secondary level factors influence the
mathematical task. At both the task set up and task implementation phases, the
mathematical tasks are examined for cognitive demand and the processes students
engage in while working on the task. For example, in the set-up phase, does the task
ask students to apply a memorized procedure? In the implementation phase, what are
students doing to apply the memorized procedure? The framework takes into account
the dynamic nature of the classroom where interactions between teacher and students and between students shape how the mathematical task is completed.

Stein et al. (1996) provide insight about how a mathematical task can appear to exhibit high-cognitive demand in the beginning phase of the framework; but factors may influence the outcome at the end, which sometimes leads to a decline in cognitive demand. The conceptual framework provides a concrete way of understanding the process of implementing a mathematical task and the different factors that can effect implementation. The factors in the framework that effect task set-up and task implementation are beneficial for professional developers and teacher educators when assisting preservice and practicing teachers in developing skills to effectively implement mathematical tasks into instruction.

One aspect of teaching that the mathematical task framework does not account for is the teachers' beliefs about mathematics and mathematics instruction. Raymond (1997) found a strong connection between mathematics teaching practices and teachers' mathematics beliefs. According to Raymond, “…deeply held, traditional beliefs about the nature of mathematics have the potential to perpetuate mathematics teaching that is more traditional, even when teachers hold nontraditional beliefs about mathematics pedagogy” (p. 574). Researchers Arbaugh and Brown (2005) were puzzled by a teacher who exhibited an increase in understanding about the levels of cognitive demand of mathematical tasks through a task sorting activity, but did not change his instructional practice in the classroom. The researchers were only able to speculate about the mismatch between the teachers’ progress in professional development and the lack of progress in changing his instructional practices. The
researchers inferred the possibility that the teachers’ deeply held beliefs about mathematics instruction did not allow him to change his instructional practices. Did the teacher educators take time to recognize the teachers’ beliefs? Teachers’ mathematical beliefs must be recognized and challenged in order to help them change their instructional practice (Pajares, 1992).

Other research studies (e.g., Cobb et al., 1991; Ernest, 1989) have found that teachers’ beliefs play an important role in mathematics instruction. For example, Cobb et al. (1991) wanted to initiate change in the instructional practices of second grade teachers by viewing the teachers’ current practice, which they identified as problematic. The goal of the study was to reorganize the knowledge and beliefs that teachers hold about learning and teaching. The researchers found that the teachers who participated in the project held beliefs that were more compatible to socioconstructivism. Socioconstructivism is a learning theory, which in this study describes how the teachers viewed mathematics instruction. As a result, the project teachers provided their students with mathematics instruction focused on allowing students opportunities to construct their own mathematical knowledge through engagement with mathematical tasks. The teachers had opportunities to do the mathematical tasks during a summer professional development institute. A limitation of the study was that the teachers’ beliefs were not assessed prior to the professional development. The researchers could not say whether the professional development had an effect on the teachers’ beliefs.

The NCTM’s Professional Standards for Teaching (1991) state that “selecting worthwhile mathematical tasks set the stage for learning.” The Common Core Standards for Mathematical Practice (2010) want students to learn mathematics through
the process of doing mathematics, with an end result of developing an understanding of mathematics. NCTM (1991) outlines the need for mathematical tasks to help students develop and understand mathematics, allow them to think on their own about how to arrive at a solution, and use different strategies. In order to know which type of mathematical task to use during instruction, PSTs need to consider three areas recommended by NCTM: mathematical content, the make-up of the students, and ways in which students learn mathematics.

When PSTs choose a mathematical task, they need to consider the mathematics involved. One way to assess the mathematics is do the mathematical tasks with other preservice teachers and discuss the results (Boston & Smith, 2011). Researchers Leung and Silver (1997) noticed that preservice teachers had a difficult time grasping the idea that students could have many different solutions for a mathematical task. PSTs need to think about the task they are asking students to do. For instance, is the task simply asking a student to recall procedures or does the task position students to reason about mathematics and apply that knowledge? The mathematical content involved in the tasks establishes its academic value (NCTM, 1991).

PSTs need to consider the students as learners of mathematics when selecting a mathematical task. For instance, prior knowledge students bring can provide a foundation for building to advanced mathematical concepts. Piaget’s proposition of disequilibrium is important to consider in respect to how students learn (Carter, 2008). Disequilibrium creates conflict for students when learning is taking place and creates a healthy struggle in order for students to reach a desired academic goal. Preservice teachers can choose tasks that will challenge students on different academic levels.
The act of knowing about how students learn mathematics is essential knowledge for selecting and implementing mathematical tasks. According to NCTM (1991), “well-chosen tasks allow the teacher to learn about their students’ understandings even as the tasks also press the students forward.”

The proposed study provided PSTs with the opportunity to develop an understanding of the mathematical-task knowledge needed for teaching mathematics, as well as experience with types of mathematical tasks that vary in cognitive demand (Chapman, 2013; Smith & Stein, 1998). Mathematical-task knowledge is the knowledge teachers need to implement mathematical tasks effectively during mathematics instruction (Chapman, 2013). PSTs need opportunities to experience the same type of learning they want their students to experience (Ball & Cohen, 1996). Through the course of the study, preservice teachers were exposed to learning about the task analysis guide, analyzing the cognitive demand of mathematical tasks, and created tasks that were posed to elementary students through a letter writing exchange. PSTs had the opportunity to construct a mathematical task based upon a Common Core State Standard (2010) appropriate for the content taught in the elementary classroom. The PSTs predicted the cognitive demand of the mathematical task prior to sending the letter. Upon receiving a letter back from the student pen pal, PSTs then had the opportunity to see how students responded to their mathematical tasks and re-evaluate the level of cognitive demand for that task.

Another aspect of the study was to determine which of the seven beliefs measured by the IMAP Belief Instrument (Philipp et al., 2007) showed significant change as a result of learning about the levels of cognitive demand for mathematical
tasks and participating in a letter writing exchange with third grade students. Teachers’ beliefs about mathematics have been directly linked to their instructional practices (Ernest, 1989; Raymond, 1997). Preservice teachers hold beliefs about mathematics and mathematics instruction that need to be challenged (Ernest, 1998; Pajares, 1992; Philipp et al., 2007). Teachers’ beliefs have been referred to as lenses through which teachers view mathematics and mathematics instruction (Pajares, 1992). Teacher educators would benefit from knowing how to influence a change in PSTs' beliefs by structuring experiences in a mathematics education course that could challenge beliefs. Therefore the study was interested in discovering whether learning about the levels of cognitive demand for mathematical tasks, participating in discussions about the complexity of mathematical tasks, and creating tasks that are embedded in letters to elementary school students affects preservice teachers' beliefs about mathematics instruction.

**Research Questions**

This dissertation sought to explore how a 12-week intervention impacted PSTs’ beliefs about mathematics instruction. The PSTs completed two mathematical task sorts in order to demonstrate their ability to classify mathematical tasks as having low-level or high-level cognitive demand. Additionally, PSTs constructed their own mathematical tasks implemented with third grade elementary students through a letter writing exchange. PSTs were asked to select a level of cognitive demand before the task was implemented and then to re-evaluate the task for the level of cognitive demand based on the students’ response. The study also provided PSTs with a safe environment to engage in solving mathematical tasks and participate in discussions about mathematics,
in order to strengthen mathematical-task knowledge. The following research questions guided the dissertation study:

1. To what extent did the intervention of learning about the levels of cognitive demand for mathematical tasks and implementation of mathematical tasks through letter writing with third grade students impact elementary preservice teachers' beliefs about mathematics and mathematics instruction?

2. How do elementary preservice teachers' beliefs about mathematics instruction influence their implementation of mathematical tasks?

3. Can elementary preservice teacher's identify mathematical tasks as having high-level or low-level cognitive demands and does this change after a 12-week intervention specifically focused on learning about the levels of cognitive demand and implementation of mathematical tasks in letter writing with third grade students?

**Structure of the Dissertation**

The dissertation will consist of five chapters and appendices. Chapter 1 consists of an introduction, theoretical and conceptual frameworks, and an introduction to the literature that frames this study. Chapter 2 consists of an in-depth review of literature relevant to the study. Chapter 3 describes the formal methods employed to conduct the research. Chapter 4 consists of the research results. And finally, Chapter 5 will consist of a conclusion, implications, and possible directions for future research. Appendices include a copy of the task reflection form, an example of the mathematical tasks used in the task sort, quantitative instrument samples, a schedule for the intervention, interview questions, example lesson plan, PST mathematical tasks from the letter writing, and the IRB proposal.
CHAPTER 2
REVIEW OF LITERATURE

Vision for Mathematics Instruction

The National Council of Teachers of Mathematics has a vision for school mathematics. One interpretation of this vision could resemble the following example: Imagine a classroom where a teacher and students are engaged in learning through conversations centered upon the mathematical topic of equivalent fractions. The teacher is using his/her prior knowledge of the students’ mathematical abilities to engage students in learning through the implementation of a mathematical task. The students work collaboratively to form conjectures about why two given fractions are equivalent. Students use prior knowledge and mathematical tools (e.g., fraction circles) to deepen their understanding of mathematical content. A teacher poses cognitively demanding questions to students in order to elicit higher cognitive responses such as asking students to explain how they are thinking about the mathematical task. NCTM describes this type of mathematical learning in detail in documents about reform mathematics instruction (NCTM, 2000).

When some people recall a past mathematics class they may think of a situation where they are sitting in a seat and staring at a chalkboard where the teacher is writing mathematics. The teacher may interact with students by posing questions such as, “What is nine times eight?” and the students may all speak in unison that the answer is seventy-two. The students are viewed as passive learners in this type of classroom. Over the decades, the vision of mathematics learning that NCTM (2000) proposes has been mentioned in reform documents (e.g., Curriculum and Evaluation Standards for School Mathematics, Principles and Standards for School Mathematics). Unfortunately,
this type of instruction has not become the reality of mathematics teaching and learning in the United States. A video analysis of middle school mathematics lessons from the United States revealed that teachers have not changed the way they implement mathematics instruction (Hiebert et al., 2005).

The video analysis conducted by Hiebert and others (2005) revealed that teachers in the United States repeatedly taught low-level mathematical skills, as well as procedures, without connections. When Hiebert et al. (2005) compared the instructional practices of U.S. teachers to those of teachers in other countries, using video analysis of the Trends in International Mathematics and Science Study (TIMSS) 1999, they found there was a lack of opportunity for students to discuss mathematics. The lessons from the United States revealed that teachers reinforced low-level mathematics skills such as applying procedures without justification. Analyzing the videos of the United States separately from the other countries would not have revealed the differences. However, when compared to the other countries, it was evident that the instructional practices of U.S. teachers do not challenge students to apply mathematical knowledge or make mathematical connections (Hiebert et al., 2005).

The mathematical tasks used by the U.S. teachers had the potential to elicit a high-level of cognitive demand. However, during the period of time in which the students were presented with the task and toward the end of the lesson, the teacher would lower the level of cognitive demand by providing answers or telling the students how to think about the mathematics. The researchers recommended continual support and professional development for teachers that focused on improving teachers’ practice. Perhaps a suggestion for improving the instructional methods of teachers would be to
focus on teachers’ beliefs about mathematics and mathematics instruction. Additional research could concentrate on learning about the potential level of cognitive demand of mathematical tasks.

This chapter of the dissertation will examine the relevant literature on mathematical tasks, teacher education using mathematical tasks, teachers’ beliefs, and students’ mathematical thinking.

**Mathematical Tasks**

The research studies of Doyle (1988), Hiebert and Wearne (1993), Cobb et al. (1991), Silver and Stein (1996), Stein and Henningsen (1997), and Stein et al. (2008), lay the foundation for learning about the cognitive demand of a mathematical task and how the components associated with using mathematical tasks in instruction affect student learning. Doyle (1988) categorized academic tasks as familiar and novel. Cobb et al. (1991) provided evidence that a certain type of instruction with mathematical tasks supports problem-based learning. Hiebert and Wearne (1993) examined the impact mathematical tasks and discourse had on student learning when teachers used reform-based curriculum. The work of Silver and Stein (1996) and Stein and Henningsen (1997) led to a conceptual framework for mathematical tasks. Stein et al. (2008) built upon the conceptual framework for mathematical tasks by developing five practices teachers can use to facilitate learning by using mathematical tasks. The contributions of these researchers have encouraged other researchers to delve deeper into the factors that contribute to the application of mathematical tasks. Currently, with the implementation of the Common Core State Standards (NGA & CCSSO, 2010), mathematics education is undergoing a change. This is a challenging period in
mathematics education and it is important to study how the use of mathematical tasks can lead to changes for improving teacher effectiveness and student outcomes.

**The Earlier Years of Mathematical Tasks**

The first serious discussions about how the implementation of mathematical tasks affected the learning of students started around the time NCTM was working on developing a vision for how mathematics should be taught (Ball, 1991). Early research studies focused on how mathematical tasks were used in the classroom and the environmental factors that effected the implementation of these tasks (Doyle, 1988; Marx & Walsh, 1988). An example of one of the studies is from the researcher Doyle (1988), who examined 450 junior high academic tasks across the academic areas of English, science, and mathematics. Doyle argued that knowledge is constructed based on the academic tasks presented to students during instruction. Doyle found that teachers play a critical role in the success of an academic task. Teachers affect how the students learn from the task and the work students produce from the task.

Doyle describes the concept of “task” as having four aspects of work in a classroom “(a) a goal state or end product to be achieved, (b) a problem space or set of conditions and resources available to accomplish the task, (c) the operations involved in assembling and using resources to reach the goal state or generate the product, and (d) the importance of the task in the overall work system of the class” (p.169). The observational data from this study revealed that tasks appear differently at different stages in the classroom. For instance, the task presented by the teacher may appear to elicit a high cognitive demand from the students; however, the students may interpret the task differently from the way the teacher intended and the task only elicits low cognitive demand because the student is using recall of a memorized procedure.
Based on Doyle’s examination of the tasks, he divided the academic tasks into two categories: familiar and novel. Familiar tasks focused on the memorization of facts and procedures. When students are engaged in familiar academic work, they may need to know complex knowledge; however, the outcome of the academic work is predictable – with little opportunity for students to extend their thinking. Novel tasks elicited a higher level of cognitive demand, in which students have opportunities to explore their thinking. There is not a set procedure or rule that will lead the student to the answer. The type of task presented to students affects the level of work in the classroom. For instance, Doyle found that familiar tasks were implemented seamlessly – with little disruption and distraction because students were applying procedures or recalling information. Novel tasks, on the other hand, tended to take more time and left room for unpredictability in student performance, which means teachers need to be able to address the students’ thinking about the task and even model this thinking for students.

Doyle found that math teachers tend to use familiar tasks that focus on computation and memorization of procedures. Additionally, there was no need for students to think deeply about mathematics when this occurred. Doyle hypothesized that curriculum materials might be a contributing factor explaining why familiar tasks were more prevalent in mathematics classes. Closer analysis of the mathematics curriculum revealed that the focus was on skills rather than concepts and problem solving (Doyle, 1988). The significance of Doyle’s two classifications of academic tasks is relevant to how students acquire mathematical knowledge. If teachers choose to implement tasks that focus on lower level cognitive skills, then students experience the “doing of mathematics” as recall and memorization of algorithms.
The results from the analysis of the 450 academic tasks indicate that the implementation of tasks is effected by curriculum, social order, classroom management, instruction, and learning (Doyle, 1988). Teachers use familiar work to make the class manageable and productive. Familiar tasks can lead to student misconceptions because of the lack of opportunity for them to explore, analyze, and discuss mathematics. Students working on a familiar task may just follow the procedure, but lack an understanding about how the mathematics is actually applied to the problem. Hence, Doyle (1988) is claiming that the tasks used during instruction relate directly to what students learn. If learning is designed to improve student outcomes, then further research needs to be done on effectively implementing mathematical tasks to support the student learning desired by reform policy documents.

While Doyle worked on identifying mathematical tasks, Marx and Walsh (1988) were interested in the supporting classroom structure that assisted students in higher order thinking through the implementation of academic tasks. According to Marx and Walsh, there are three elements of classroom work: “the conditions under which tasks are set, the cognitive plans students use to accomplish tasks, and the products students create as a result of their task-related efforts” (p.208). The teacher was the key to students successfully completing an academic task that matched the intended goals. For instance, when the teacher set clear goals for the academic tasks, the students were able to engage in the task at the intended academic level. On the other hand, when clear goals were not made explicit to the students, then the “tasks can have negative effects on classrooms in addition to the confusion they create in the mind of learners” (Marx & Walsh, 1988, p.215).
Marx and Walsh (1988) stress the importance of having teachers focus on students’ thinking and the cognitive plans – memory, procedure, comprehension, or opinion – needed to complete an academic task. In essence, the cognitive plans are similar to Stein and Smith (1998)’s Task Analysis Guide, which classifies mathematical tasks into four categories (e.g., memorization tasks, procedures without connections tasks, procedures with connections tasks, and doing mathematics tasks) based upon their cognitive demand. The awareness of the level of cognitive demand an academic task has is an important piece of effective instruction (Marx & Walsh, 1988). In order for teachers to have this awareness, they need to have time to work on the academic task to know the steps the student will take in completing the task. Marx and Walsh argue that “the kind of cognitive engagement students utilize …is of utmost importance rather than instructional time or time on-task per se” (p.209). This statement further illustrates the need for teacher education that is focused on doing the mathematics and learning about where students may struggle with mathematics. Next, the study conducted by Hiebert and Wearne (1993) models the principles that researchers Doyle (1988) and Marx and Walsh (1988) argue need to occur in order for mathematical tasks to have an effect on student achievement.

Researchers Hiebert and Wearne (1993) were interested in how student learning was supported through the implementation of mathematical tasks and classroom discourse. The researchers interpreted the interactions between mathematical tasks and classroom discourse in second-grade classrooms by observing the classrooms through the social-cognitive and social constructivist perspectives. Hiebert and Wearne proposed “that, in mathematics classrooms, certain kinds of instructional tasks and
discourse encourage productive ways of thinking” (p.421). For instance, classrooms utilizing mathematical problems that required students to spend more time on the problem and had more than one way to represent the answer produced higher student performance than classrooms using mathematical problems that were focused on applying a memorized procedure. Additionally, classrooms where the teachers asked more thought-provoking questions to students, such as, “How did you get your answer?” as opposed to teachers who asked questions that focused on recall of facts, created more opportunities for student achievement.

In order to test the effect different instructional approaches had on students, the researchers observed two classrooms that implemented a different way of instruction that contrasted with the traditional textbook approach. The classrooms were chosen specifically by the researchers to demonstrate the effect teacher questioning had on student outcomes. The researchers compared observations of the two classrooms with four other classrooms that focused on procedures and recall of mathematics. Overall, for all six classrooms the kind of questions that occurred the most were ones that focused on recall of facts (Hiebert & Wearne, 1993). This kind of questioning which focuses on recall of facts is prevalent during mathematics instruction across the United States (Hiebert et al., 2005).

In conclusion, Hiebert and Wearne (1993) argue there are more factors that influence the learning that takes place when implementing mathematical tasks and classroom discourse. The findings from their research study were not able to pinpoint the factors or reasons why the classrooms with higher achieving students were more influenced by the type of task and exemplified productive discourse. The researchers
were able to conclude that instructional tasks and discourse are related to the practice of teaching and learning. Based upon student performance on the assessments, the classes that focused more on procedural questions were out-performed by classes that spent more time on problems and used multiple representations. The conclusions made by Hiebert and Wearne indicate a need for further investigation into the factors that influence the implementation of academic tasks, the effect curriculum materials have on teaching, and the questioning practices of teachers.

Teacher Education and Mathematical Tasks

The NMAP (2008) recommended “that a sharp focus be placed on systematically strengthening teacher preparation, early-career mentoring and support, and ongoing professional development for teachers of mathematics at every level, with special emphasis on ways to ensure appropriate content knowledge for teaching” (p.42). Thus, in November of 2010 the National Council for Accreditation of Teacher Education (NCATE) released a “Report of the Blue Ribbon Panel on Clinical Preparation and Partnerships for Improved Student Learning.” A panel consisting of state officials, P – 12 and higher education leaders, teachers, teacher educators, union representatives, and critics of teacher education came together to take on the challenge of “addressing the gap between how teachers are prepared and what schools need” (NCATE, 2010, p. ii). The panel made it clear that a main goal was to strengthen teacher preparation by increasing the rigor and “supporting the development of complex teaching skills” to meet the needs of schools (NCATE, 2010, p. ii).

The NCATE (2010) panel recommended that teacher candidates be prepared to “ensure that all children master rigorous course content, be able to apply what they learn to think critically and solve problems, and complete high school 'college-and
workforce-ready” (p.1). This recommendation follows suit with the Common Core State Standards (NGA & CCSSO, 2010) for mathematics initiative. For example, Achieve and the U.S. Education Delivery Institute (2012) released recommendations for state and district leaders to prepare for full implementation of the CCSS. In contrast to individual state standards, the CCSS will bring an increase in the cognitive demand required by students (Achieve & U.S. Education Delivery Institute, 2012). Furthermore, the CCSS assessments (i.e., Smarter Balanced and Partnership for Assessment of Readiness for College and Careers), which are still in the developmental stages, have released assessment items showing the rigorous cognitive demands students will need to apply to complex mathematical tasks. Preservice teachers need to be prepared to meet these same challenges that their future students will face on assessments.

Specifically, NCATE (2010) recommends that preservice teacher education focus on student learning. For instance, having preservice teachers focus on student learning will prepare them to incorporate instructional strategies that use students’ prior knowledge to foster the growth of mathematical knowledge in students. Developing practices in preservice teachers where they “are innovators and problem solvers, working with colleagues constantly seeking new and different ways of teaching students who are struggling” will lead to continual growth in the teaching profession (NCATE, 2010, p.5). Next, several studies are discussed that describe how researchers contribute to teacher learning by focusing on mathematical tasks.

Arbaugh and Brown (2005) sought to embed the professional learning of a group of geometry teachers into their everyday practice by focusing on the mathematical tasks presented to students. They used a study group as a form of professional development
for the teachers. The professional development was focused on learning about the cognitive demand of mathematical tasks. Arbaugh and Brown (2005) used a task-sorting interview to measure the growth of levels of cognitive demand for teachers of mathematical tasks. Prior to the start and at the end of the professional development, teachers were asked to sort a group of high-school mathematical tasks at two different times. The categories created by the teachers were then compared using qualitative methods to draw conclusions about their understanding of the levels of cognitive demand. The results from this study reveal how teachers learned to use the mathematical task framework and became more cognizant of the tasks they chose to implement in their instruction. Additionally, teachers’ lesson plans were used to determine whether the level of cognitive demand for mathematical tasks increased because of the professional development.

Overall, the levels of cognitive demand of the mathematical tasks used in instruction did not change when considering the teachers as a group. Individually, some of the teachers showed an increase in the level of cognitive demand of the mathematical tasks in their lesson plans. Researchers Arbaugh and Brown (2005) argue that the professional development focused on mathematical task implementation and the level of cognitive demand helped to strengthen the teachers’ pedagogical content knowledge by influencing the type of mathematical tasks the teachers chose to use during mathematics instruction. Another aspect of this study that proved successful was providing teachers with the opportunity to collaborate with peers and discuss features of mathematical tasks, which produced low-level and high-level cognitive demand.
Teachers need professional opportunities to work with other teachers and discuss their work (Ball, 1993). When teachers have the opportunity to do mathematical tasks and then discuss the mathematical tasks with peers in a non-threatening professional development setting, research has indicated that their content and pedagogical knowledge are positively impacted (Chamberlin, 2005; Cobb et al., 1991; Kazemi & Franke, 2004; Prestage & Parks, 2007; Remillard & Bryans, 2004; Steele, 2005). Teachers need to understand where the students may struggle and where misconceptions can develop during the implementation of the mathematical task (CBMS, 2012; Stein et al., 2008).

In order to understand how students may struggle with mathematics, teachers need to experience the mathematics by actually doing the mathematics (Ball & Cohen, 1999; CBMS, 2012). The purpose of having teachers work on a challenging mathematical task and experience the enjoyment of completing the task as a group is a feeling that teacher educators hope that teachers want their own students to feel (CBMS, 2012). Working with other teachers in an educational setting is an ideal environment for teachers to benefit from seeing and hearing how others solve mathematical tasks (Chamberlin, 2005; Kazemi & Franke, 2004). The aforementioned opportunities rarely occur in a teacher preparation program (Ball, Thames & Phelps, 2008). In fact, most of the preparation for teachers to teach mathematics occurs in content courses (Ball et al., 2008).

Ball et al. (2008) would argue that teachers need more opportunities to develop mathematical knowledge for teaching. Recently, researchers Ball et al. (2008) theorized about the type of knowledge teachers need in order to teach mathematics. Ball et al.
(2008) have extended Shulman’s (1986) notion of pedagogical content knowledge by including the domains of knowledge of content and teaching (KCT) and knowledge of content and students (KCS) within the pedagogical content knowledge needed for teachers of mathematics. The knowledge of content and teaching is knowing how to design effective mathematical instruction and the ability to facilitate learning by deciding when to use a student’s explanation, pose a question, or extend a mathematical task (Ball et al., 2008). Knowledge of content and students encompasses the knowledge needed by the teacher to know where students will struggle in the process of learning mathematics, how to motivate students to learn mathematics by choosing interesting ways to present topics, and the ability to know if a mathematical task will be easy or hard (Ball et al., 2008).

Both domains affect the instructional decisions teachers make when selecting mathematical tasks to use during classroom instruction. According to Ball et al. (2008) “teachers need to know mathematics in ways useful for, among other things, making mathematical sense of student work and choosing powerful ways of representing the subject so that it is understandable to students” (p.404). In order to develop the recommended knowledge needed for mathematics instruction, teacher preparation programs need to look beyond stand-alone mathematical content courses and focus on experiences where preservice teachers have opportunities to develop knowledge of content and students and knowledge of content and teaching.

The Mathematical Education of Teachers (MET) II (2012) document calls for elementary teachers to experience learning similar to that by which students need to learn mathematics, (i.e., through the 8 Common Core Mathematical Practices). Many
preservice elementary teachers have the belief that mathematics is taught by following a set of rules and procedures (Ball, 1990; Kirtman, 2008; Ma, Millman, and Wells, 2008; Philipp et al., 2007). These beliefs come from their own experiences in mathematics courses. The role of the teacher educator is to guide and facilitate the learning of preservice teachers (Ball & Cohen, 1999). According to NCTM (1991), preservice teacher education needs to provide opportunities for preservice teachers “to examine and revise their assumptions about the nature of mathematics, how it should be taught, and how students learn mathematics” through the engagement in meaningful authentic activities. For example, researchers Cobb et al. (1991) used mathematical problem solving with elementary teachers during a 1-week summer institute focused on student learning that was problematic.

In the yearlong study conducted by Cobb et al. (1991), the socioconstructivist theory of knowledge was used to inform the instruction of second-grade teachers, who were focused on implementing problem-centered learning. This study incorporated reform-based mathematics instruction into experimental classrooms and compared student achievement results with the controlled classrooms. The teachers who implemented reform-based instruction received opportunities in the summer institute to work on mathematics problems that they implemented in their classrooms during the school year. A benefit of having the opportunity to work on the mathematics problems with colleagues was that they were able to talk about possible trouble areas students may experience when solving the problem. The researchers argued that professional development contributed to the success of students who had teachers who implemented the mathematical problems during instruction. This is significant because it
shows that if teachers have the opportunity to experience mathematics as learners in an environment that promotes problem solving, discussion, and experimentation, then they may be more apt to foster the same type of environment with their own students.

Additionally, findings from this study support the use of teacher and student discourse throughout instruction. The interactions between teacher and students and peer student interactions supported learning mathematics conceptually rather than procedurally, as measured by a mathematics test. For example, the researchers explained teacher and student discourse as teacher questioning and student explanations that concentrated on mathematical problems. Another key to success for the teachers who participated in the summer institute was continual support from project staff. The staff met with the teachers throughout the year, and visits consisted of sessions focused on children’s problem-solving methods centered on topics that were relevant to 2nd grade. This type of support is recommended by the MET II (2012) for quality professional development that is directly related to the instructional practices of teachers.

A reoccurring theme for the reform of teacher education is the need for teachers to have “opportunities to reconsider their current practices and to examine others, as well as to learn more about the subjects and students they teach” (Ball & Cohen, 1999, p.3). An example of a professional development project that focused on the aforementioned opportunities is the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project (Silver & Stein, 1996). The premise of the project was for a mathematics educator to provide ongoing support to teachers as they made changes in their mathematics instruction by posing cognitively demanding
mathematical tasks to students. Students had opportunities to take an active role in their learning by engaging in mathematics through mathematics instruction focused on cognitively demanding tasks.

For instance, the researchers provided a snapshot of a three-part lesson that describes how the teacher built upon students’ prior knowledge by teaching them how to convert fractions to decimals. In this snapshot, the researchers emphasized classroom norms established by both the teachers and students, the questioning teachers used while students worked collaboratively, and the explanations students shared with the class to show how they thought about the mathematical task. Silver and Stein (1996) reported that success was made possible for students when teachers had the “knowledge, skill, patience, and motivation” to facilitate the QUASAR class instruction (p.513).

Mathematical tasks from the QUASAR project were analyzed for the cognitive demand elicited throughout mathematics instruction. In the set-up phase of the mathematical task, cognitive demand was high. During the implementation phase, the cognitive demand fell. Some of the reasons for the decline in cognitive demand were attributed to the teacher telling the students what to do instead of asking questions to the students in order to engage them in thinking deeply about mathematics, not considering students’ prior knowledge, and not allowing for enough time to complete the task (Stein et al., 1996).

On the other hand, the data collected and analyzed from the QUASAR project led the researchers to discover how to maintain student engagement throughout the implementation of a mathematical task. The top five ways to maintain student
engagement are tasks that build on students’ prior knowledge, scaffolding, appropriate amount of time allotted for the mathematical task, the modeling of high level performance, and sustained encouragement from the teacher for explanations and meaning from the students (Henningsen & Stein, 1997). These methods can help teacher educators model how to maintain student engagement within preservice teacher education courses.

Next, Boston and Smith (2011) used a task-centric approach to professional development. Such professional development was part of a project titled Enhancing Secondary Teacher Preparation (ESP). Two years after the project ended, the researchers followed up with the participants to investigate whether or not the teachers were still using cognitively challenging tasks during mathematics instruction. Data sources for this study consisted of observations, instructional tasks submitted by teachers, and artifacts from the professional development. An important component to this professional development was having the participants complete a pre- and post-middle school mathematical task sort. Results from the task sort indicated that participants were able to recognize features of low-level and high-level mathematical tasks after participating in the professional development. The researchers concluded that “teachers participating in the ESP project improved their ability to select and implement cognitively challenging instructional tasks, and a subset of these teachers were shown to have sustained the improvements more than a year after the project ended” (Boston & Smith, 2011, p.970). Results from this study provide rationale for including more professional experiences for teachers that involve learning about the
cognitive demand of mathematical tasks and examining such tasks used in their everyday instruction.

Researchers Norton and Kastberg (2011) wanted to provide preservice teachers with opportunities to learn about the cognitive demand of mathematical tasks and to practice implementing mathematical tasks with students. One of the challenges of preservice teacher education, noted by the researchers, is the fact that preservice teachers do not have access to their own group of students. Teacher educators must seek out opportunities that are meaningful and impactful to the learning and beliefs of preservice teachers. Researchers have used the mathematical task framework and task analysis guide with preservice teachers, but they focused on individual parts instead of the whole piece. For instance, Norton and Kastberg (2011) and Kosko et al. (2010) introduced preservice secondary teachers to mathematical tasks through letter writing. The letter writing exchange was used as an authentic activity to engage preservice teachers in focusing on the level of cognitive demand of mathematical tasks and students’ mathematical thinking. As a result of this letter writing exchange, the “preservice teachers demonstrated improvements over the semester in their ability to engage students in different mathematical processes at a high level of cognitive demand” (Kosko et al., 2010, p.211). Letter writing has given preservice teachers the opportunity to learn how to construct mathematical tasks for students and the time to analyze how a student thinks about mathematics (Crespo, 2003; Norton & Kastberg, 2011).

Letter writing, focused on mathematical learning, was first used in a research study conducted by Crespo in 2003. Crespo (2003) found that preservice teachers, who
took part in a letter writing exchange with elementary school students, were provided with the opportunity to be the posers of problems rather than the problem-solver. This opportunity allowed preservice teachers time to reflect upon the problems they asked of their elementary school pen pals. Later, when the responses from the pen pals came, they were able to learn about the students’ mathematical thinking. Preservice teachers had the opportunity to evaluate the students’ response in their letters and construct questions that would elicit more insight into their mathematical thinking.

Initially, the preservice teachers were assigned a fourth grade elementary student. Crespo’s (2003) intent in using the letter writing activity was to provide a means for preservice teachers to experiment with posing mathematical problems to students and for preservice teachers to learn which problems provided students with opportunities to justify their mathematical thinking. Thirteen preservice teachers volunteered to participate in the study. The data collected in the study consisted of the letters sent between preservice teachers and elementary students, weekly journals from the preservice teachers, and the final case report turned in at the end of the semester. The preservice teachers used self-generated, class and course textbook problems for their letters.

Crespo (2003) identified three approaches used by the preservice teachers when they began writing: 1) make problems easy to solve; 2) pose familiar problems; 3) pose problems blindly. Further, Crespo (2003) found that the initial problems posed to the elementary students were comprised of arithmetic operations and only allowed for one answer, which is an example of low-cognitive demand. The preservice teachers wanted to simplify the problems for the students by underlining and bolding key words or by
providing hints. Crespo speculated that the choice of problems included in the letters might have been made based upon the preservice teachers own mathematical ability or lack thereof. For instance, when the problems were chosen blindly, students were more challenged by them because the problems lacked the hints seen in those problems where preservice teachers already knew the answer. Problems chosen blindly also resulted in preservice teachers realizing that elementary students had the mathematical ability to work on challenging tasks.

Over time, the preservice teachers changed the types of problems posed to elementary students. They began to ask more open-ended problems that allowed for more exploration and justification. Crespo found three themes in the changes of the types of problems: 1) trying unfamiliar problems; 2) posing problems that challenged the thinking of pupils; 3) posing problems to learn about pupil’s thinking. The changes in posing problems undertaken by the preservice teachers did not happen in isolation. In fact, preservice teachers reported that the teacher educator had an instrumental role in the changes associated with problem posing. The problems the preservice teachers engaged in during their teacher education class helped to expand their thinking about different types of mathematical problems. In essence, the experience of letter writing to elementary students while enrolled in the elementary mathematics methods course led to changes in the beliefs preservice teachers held about mathematical problems (Crespo, 2003). Crespo finds that preservice teachers must have a “mathematical understanding and their ideas for teaching” are important, “in the sense that it seems that knowing mathematics for oneself may not be a reliable predictor of good problem posing practice” (p.266).
Ball et al. (2008) would agree with Crespo and suggest that preservice teachers need more experiences, which allow them to develop pedagogical content knowledge needed to become problem posers. Furthermore, Crespo (2003) claims that a critical aspect of learning how to pose problems to students is the knowledge of students’ mathematical thinking and ways of solving problems. In conclusion, Crespo suggests the need for further research on how preservice teachers learn to pose and develop problems for students. Preservice teachers need authentic experiences where they can learn how to pose problems and reflect upon the experience to further their own knowledge of teaching mathematics.

Building upon the work of Crespo (2003), researchers Norton and Kastberg (2011) used case studies to describe the experiences of two preservice teachers, who took part in a letter writing exchange. The case studies revealed how the preservice teachers struggled with how much assistance to provide the students with when trying to encourage them to respond to the mathematical task. One preservice teacher focused on the solution he knew would give the correct answer and did not respond to the students’ mathematical thinking. Instead, the preservice teacher tried to redirect the student to follow his thinking. This incident reveals that preservice teachers may need more time and experience with solving mathematical tasks and learning there is more than one method to solve tasks.

In turn, van den Kieboom and Magiera (2010) had elementary preservice teachers work on mathematical tasks and share solutions in their methods course before implementing the mathematical tasks with elementary students. In the study, preservice teachers were enrolled in a mathematics content course, which first
introduced mathematical content and pedagogy for selected mathematical tasks involving fractions. The in-course activities consisted of preservice teachers learning from each other by showing multiple ways to represent the solution to a mathematical task, focusing on where students might develop misconceptions, and highlighting important mathematical features of the task. Next, preservice teachers had the opportunity to implement the same mathematical tasks with students. An important component to the study was the reflections the preservice teachers wrote after they evaluated their students’ mathematical thinking. The researchers, van den Kieboom and Magiera (2010), facilitated a learning environment where the focus was on learning mathematics and pedagogy together while studying students’ mathematical thinking about mathematical tasks.

Furthermore, van den Kieboom and Magiera (2010) describe the process of developing content knowledge for teaching as a cycle where the content course serves as a place for preservice teachers to learn together by doing the mathematics and reflecting on their experience implementing the same mathematical tasks with students. The research study focuses on the first step in the Mathematical Task Framework, which is to identify mathematical tasks and work out solutions to these tasks (Stein et al., 2000). The research study conducted by van den Kieboom and Magiera (2010) further illustrate the beneficial practice of having preservice teachers learn how to implement mathematical tasks in a supportive environment where they can ask questions and examine and reflect upon their practice.

First, Osana et al. (2006) investigated how elementary preservice teachers’ mathematical knowledge affected their ability to classify mathematical tasks by cognitive
demand. A teacher educator, who was familiar with the task analysis guide, instructed 26 preservice teachers on how to identify the cognitive demand of mathematical tasks. The researchers using the criteria set forth in the task analysis guide created the mathematical tasks. Additionally, the researchers were interested in knowing if the length of a task effected the classification of the task into a low-level or high-level cognitive demand category. Results from the task sorting activity indicate that preservice teachers were able to classify Level 1 tasks (memorization) with 74% accuracy and Level 2 (procedures without connections) with 56% accuracy (Osana et al., 2006). As the level of cognitive demand for mathematical tasks increased, the ability to classify a task accurately as Level 3 (procedures with connections) and Level 4 (doing mathematics) decreased.

This result directed Osana et al. (2006) to conclude “that preservice teachers’ limitations in their knowledge of children’s’ thinking may hinder their ability to classify accurately mathematical tasks, particularly those that are specifically designed to stimulate genuine mathematical thinking” (p. 368). The researchers suggest that the next direction for future research should provide preservice teachers with the opportunity to work with children and mathematical tasks. The proposed opportunity would allow preservice teachers to learn about the level of cognitive demand required by the student to effectively work on the mathematical task. Furthermore, situating the preservice teachers in a field experience where they can learn about the cognitive demand a particular mathematical task is capable of eliciting in students is an authentic activity for developing knowledge for teaching mathematics (Borko & Putnam, 2000).
In essence, preservice teachers need opportunities where they can learn about mathematical content knowledge and pedagogical content knowledge at the same time (Lampert & Ball, 1998), which contributes to mathematical-task knowledge (Chapman, 2013) and mathematical knowledge for teaching (Ball et al., 2008). Examining mathematical tasks provides an opportunity to learn about the potential cognitive demand a task can hold for potential student learning. Learning about the potential cognitive demand a mathematical task can possess is one aspect of learning how to implement such mathematical tasks. The act of bringing awareness to the potential cognitive demand of a mathematical task allows practicing and preservice teachers the opportunity to consider the possible implications for classroom instruction (Arbaugh & Brown, 2005; Boston & Smith, 2011; Kosko et al., 2010; Norton & Kastberg, 2011; Osana et al., 2006). Reform mathematics instruction posits the need for students to have opportunities to extend their thinking and apply their knowledge; meanwhile they are constructing their own understanding of mathematics (NGA & CCSSO, 2010; NRC, 2001). These learning opportunities for students only come from well-thought-out mathematical tasks from the teacher which extend beyond applying low-level mathematical procedures.

In summary, the research studies presented in this section and the research known about effective professional development and implementing mathematical tasks with preservice teachers are the guiding force behind the design of the elementary mathematics methods course for the dissertation study. Learning about mathematical tasks is related to the Standards for Mathematical Practice (NGA & CCSSO, 2010). For instance, the Standards for Mathematical Practices describe how students should learn
mathematics and also relates how teachers need to experience mathematics in order to create instructional opportunities for their students to utilize the standards to learn mathematics (CBMS, 2012). The elementary mathematics methods course was designed to provide PSTs with an opportunity to engage in solving mathematical tasks together as a group, discuss solutions, consider the potential level of cognitive demand, and provide an opportunity to create their own mathematical tasks for the letter writing exchange.

**Using Knowledge of Students’ Mathematical Thinking**

When teachers are selecting mathematical tasks to use during mathematical instruction, it is important to consider the learning needs of the students (NCTM, 1991). Teachers must use their knowledge of students’ mathematical thinking to engage the student in learning (Bransford et al., 1999). A mathematical task alone will not guarantee that the student is challenged academically; NCTM (1991) states, “the learning occurs through the discourse orchestrated by the teacher.” Researchers Stein, Engle, Smith, and Hughes (2008) recommend five practices that teachers could use to orchestrate discussions about mathematical tasks. The practices are: 1) anticipating likely student responses to cognitively demanding tasks, 2) monitoring students’ responses to the tasks during the exploration phase, 3) selecting particular students to present their mathematical responses during the discussion-and-summarizing phase, 4) purposefully sequencing the student responses that would be displayed, and 5) helping the class make mathematical connections between different students’ responses and between students’ responses and the key ideas (p.12). The knowledge the teacher possesses about students’ mathematical thinking is a crucial piece to the aforementioned five practices.
According to the Mathematical Task Framework (Stein et al., 1996) one of the factors influencing the setup of a mathematical task is the teachers' knowledge of students. The teachers' knowledge of students encompasses knowing what prior knowledge students will have about the task, how to use students’ mathematical thinking to engage them in the task, and where they might struggle when working the task. When teachers consider students’ mathematical thinking, they are able to use that knowledge to form worthwhile mathematical tasks (Chamberlin, 2005).

For example, Franke et al. (2009) studied questions asked by elementary teachers to students during an algebraic reasoning lesson to determine whether the questions asked allowed students to elaborate on their mathematical thinking. The findings from this study suggest that there is no guarantee that by asking a question a student will reveal their mathematical thinking. Some types of questions produced elaborations from the students about their mathematical thinking, but the result was not always consistent. In essence, by using students’ thinking the teachers were able to further develop a mathematical concept. More research is needed to learn how to help teachers decide which questions to ask in order to further understand students’ mathematical thinking and which type of questions influence student participation and learning (Franke et al., 2009).

Even & Tirosh (2008) stress the importance of providing teachers with the opportunities to study students’ mathematics learning and thinking in teacher education. The Cognitive Guided Instruction (CGI) study (Carpenter, Fennema & Franke, 1996) positioned inservice teachers to understand students’ mathematical thinking by “using a specific research-based model of children’s mathematical thinking” (Even & Tirosh,
The CGI model emphasizes what students can do, not what they cannot do. One of the major facets of the study was positioning teachers to recognize the informal knowledge mathematics students already possess and how to use this knowledge in the development of formal mathematics (Carpententer et al., 1996). A significant finding of the study was “that CGI teachers are significantly more successful in identifying problems their students can solve and the strategies that they use to solve them than non-CGI teachers” (p. 14). Based on the significant results from the CGI study, the same principles have been used in both preservice and inservice teacher education.

Cady, Meier, and Lubinski (2006) used the same principles as the CGI study (1996) along with NCTM (1991) practices in a study involving practicing and preservice teachers. The study followed the participants over multiple years and gathered data at pre-determined points to assess the beliefs of the participants, who were initially preservice teachers and then became practicing teachers. Interestingly, this data indicated that the experiences the preservice teachers had as participants in a project during their teacher education program had an effect on their “epistemological views and learning of mathematics” (Cady et al., 2006, p.296). When these preservice teachers transitioned to novice teachers, researchers were disappointed to report that the participants “failed to meet expectations for implementing practices aligned with NCTM practices or CGI philosophy” (Cady et al., 2006, p.296). In fact, the results the researchers were hoping to see initially were not seen until six years later, when the preservice teachers became practicing teachers. Results from this study indicate that changes in instructional practices of teachers takes time to evolve into the desired
reform-based instructional practices. Once the novice teachers became comfortable in their role as teacher, they were able to slowly implement problem-solving tasks in their classroom that focused on students’ mathematical thinking (Cady et al., 2006).

According to Ball and Cohen (1999), “The knowledge of subject matter, learning, learners, and pedagogy is essential territory of teachers’ work if they are to work as reformers imagine, but such knowledge does not offer clear guidance, for teaching of the sort that reformers advocated requires that teachers respond to students’ efforts to make sense of material” (p.11). In essence, PSTs need to see how students think and make sense of the mathematics. The teachers’ knowledge of students affects the implementation of a mathematical task as well as the teachers’ beliefs about mathematics and mathematics instruction (Chapman, 2013). There are several pieces of a hypothetical puzzle, which need to assimilate in order for teachers to teach current reform mathematics.

**Teacher Beliefs and Teacher Change**

According to Philipp (2007) “the construct belief is of great interest to those attempting to understand mathematics teaching and learning” (p. 265). Beliefs about teaching and learning are important to determine how a teacher can implement reform mathematics instruction (Sztajn, 2003). For instance, a teacher may only decide to implement the mathematical content in reform mathematics standards and not use the recommended practices. Raymond (1997) defines beliefs “as personal judgments about mathematics formulated from experiences in mathematics, including beliefs about the nature of mathematics, learning mathematics, and teaching mathematics” (p.552). Understanding the nature of teachers’ beliefs about mathematics is complex because of the way these beliefs were formed (Pajares, 1992).
Preservice teachers have long formed their beliefs – defined as “psychologically held understandings, premises, or propositions about the world that are thought to be true” (Philipp, 2007, p.259) – about mathematics instruction, even before they decided to become teachers (Ball, 1990; CBMS, 2012; Chapman, 2007; Ebby, 2000; Raymond, 1997; Szydlik et al., 2003). Philipp (2007) stated, “Beliefs might be thought of as lenses through which one looks when interpreting the world” (p. 258). Through the preservice teachers’ own K–12 schooling experiences, they have formed their beliefs about mathematics instruction. During that time, they experienced mathematics for the most part differently than the method by which their preservice teacher education courses and policy documents (e.g., CCSS, NCTM Principles and Standards) envisioned mathematics would be taught. In fact, Ebby (2000) argues that preservice teacher education is a weak intervention for trying to change preservice teachers’ beliefs. Preservice teachers may experience a slight belief change during their teacher preparation, but once they become teachers, they tend to adapt to the culture at the school (Cady et al., 2006; Manouchehri, 1997).

How do the deeply held beliefs of elementary preservice teachers change? Sowder (2007) states, “many of teachers’ core beliefs need to be challenged before change can occur” (p.169). One way to challenge PSTs’ beliefs is to allow them to experience mathematics instruction differently from the way they were taught and to provide an opportunity to try mathematics instruction with students (Crespo, 2003). Sowder (2007) suggests having teachers attend to students thinking as a driving force for elementary teachers to want to change their practice and learn more mathematics.
Several studies are discussed to show how researchers structured opportunities to challenge the PSTs beliefs about mathematics and mathematics instruction.

First, Raymond (1997) examined the beliefs of elementary teachers through case-study methodology to learn how teachers’ beliefs effected their instructional practice. She found that beliefs and practice had a reciprocal relationship. Additionally, Raymond proposed a model to show the connections between teacher beliefs' and practice. Furthermore, the teachers’ beliefs about mathematics instruction did not match the actual instruction Raymond observed in the classroom. Raymond explained that the cause of the mismatch between beliefs and practice is caused by the constraints from the classroom environment. Teacher educators planning a professional development should bring awareness to the relationship between beliefs and practice through reflective discussions with the teachers (Borko, 2004).

When teachers have the opportunity to view an instructional practice different than their own, they then need to reflect upon the experience. The teacher educator needs to understand how the teachers’ beliefs about mathematics instruction play a role in how they interpret the instruction. Philipp (2007) suggests that teachers need to change their instructional practices. Then when they see results in their students, their beliefs will change. Although this may be true, Pajares (1992) claims changes in teachers’ beliefs about mathematics instruction are influenced by results, which indicate improvements in student achievement. A more complex description of belief change is advanced by Pajares (1992): “Assimilation is the process whereby new information is incorporated into existing beliefs in the ecology; accommodation takes place when new information is such that it cannot be assimilated and existing beliefs must be replaced or
reorganized. Both result in belief change, but accommodation requires a more radical alteration” (p.320). Therefore, teacher educators need to structure opportunities that challenge preservice teachers’ instructional practices in order for beliefs to be changed.

For instance, researchers Vacc and Bright (1999), Szydlik, Szydlik, and Benson (2003), and Philipp et al. (2007) studied the belief changes among elementary preservice teachers who participated in specific interventions in their teacher preparation courses. Vacc and Bright (1999) measured preservice teachers’ beliefs on four occasions throughout their teacher preparation. The researchers found the greatest change in beliefs occurred during the elementary mathematics methods course. In the course, the teacher educator implemented CGI instruction for five weeks with preservice teachers. According to Vacc and Bright (1999), “using the taxonomy of problem types and solution strategies as a guide for planning instruction, listening to how children solve problems, and exploring children’s geometrical thinking may have provided the preservice teachers in this study with the reinforcement needed to support their beliefs” (p.108). Carefully structured learning opportunities can provide occasions for preservice teachers to explore their pre-existing beliefs (Ernest, 1989).

Subsequently, researchers Szydlik and colleagues (2003) focused on the classroom environment where preservice teachers negotiated social norms while learning mathematics through problem solving. The researchers situated the learning of preservice teachers within a community of learners that focused on discussing mathematics. During the mathematics content course, the teacher educator served as a facilitator for learning and did not provide assistance to the preservice teachers as they worked together to solve mathematical problems. The instrument used to collect data
on preservice teachers’ beliefs proved to be unreliable, as researchers found conflicting statements between the preservice teacher’s responses and their reflective journal responses. Therefore, the researchers were left to draw conclusions about the study from the qualitative data. The preservice teachers expressed a feeling of mathematics confidence as a result of the experience of working together to solve problems. Additionally, several preservice teachers expressed the opinion that they were able to see mathematics as a sense-making discipline. An important aspect of this study was the supportive environment that was formed by the teacher educator, which allowed preservice teachers to participate in doing mathematics through an authentic experience.

Researchers Philipp et al. (2007) were also in search of an authentic experience that challenged preservice teachers’ beliefs about mathematics and mathematics instruction. The researchers argued that preservice teachers hold the belief that “If I, as a college student, do not know something, then children would not be expected to know it, and if I do know something, I certainly don’t need to learn it again” (Philipp et al., 2007, p.439). In order to challenge the aforementioned belief, teacher educators need to carefully construct field experiences that will have an impact on preservice teachers’ belief. For this study, preservice teachers were randomly assigned to either one of the four treatment groups or the control group. There were two treatment groups focused on students’ thinking. Both of these two treatment groups watched and analyzed videos of students participating in problem-solving. The one difference between the groups was that one group had the opportunity to engage children in problem-solving. The two other treatment groups observed and reflected on their visits to elementary schools. For this
treatment, one group of preservice teachers was assigned to teachers who were identified as reform-oriented. The other group of preservice teachers was assigned to teachers who were at schools that were convenient to the university.

Philipp et al. (2007) were interested in developing a belief instrument that would capture the complexities associated with teachers’ beliefs related to mathematics and mathematics instruction. Traditional belief inventories that utilized a Likert scale could only provide one dimension about teachers’ beliefs (Philipp et al., 2007). In order to capture the beliefs of preservice teachers, the belief instrument was constructed to collect authentic responses to the following tasks in various domains: “the domain of whole number, two [are] in the domain of fractions, and one [is] a general teaching segment” (Philipp et al., 2007, p.451). The belief instrument measures seven beliefs about mathematics and mathematics instruction. Rubrics were developed to analyze the responses from the preservice teachers. Results from the belief instrument data revealed that the two treatment groups which focused on children’s mathematical thinking “developed more sophisticated beliefs about mathematics and mathematics instruction” (Philipp et al., 2007, p.458). The treatment group of preservice teachers who were assigned to conveniently located classrooms had the highest percentage of participants with no increase in beliefs out of all the group. These results emphasize the importance of structuring a field experience that intentionally focuses on reform instruction and examining children’s mathematical thinking. In essence, if preservice teacher education is going to have an impact on preservice teachers’ beliefs, then teacher educators need to find experiences for them that will challenge their beliefs. The
act of placing preservice teachers haphazardly in convenient classrooms was shown to dramatically effect their beliefs about mathematics and mathematics instruction.

According to the above literature there is a need to understand the beliefs preservice teachers bring with them to their mathematics education courses. These beliefs serve as a lens through which they view mathematics instruction (Philipp, 2007). Teacher educators need to provide preservice teachers with opportunities to challenge their beliefs. This study explored how learning about the level of cognitive demand and developing mathematical tasks for elementary students had an impact on preservice teachers’ beliefs about mathematics and mathematics instruction. Preservice teachers need an environment where they can practice constructing mathematical tasks and evaluate the effectiveness of such tasks. In addition, preservice teachers need experiences interacting with children to learn how students think about mathematics and respond to mathematical tasks. Changing preservice teachers’ beliefs about mathematics is a challenging task, but providing them with experiences that might impact their beliefs has proved to be successful. This study builds upon research about cognitively demanding tasks and changing the beliefs of preservice teachers.
The purpose of this study was to determine the extent to which elementary preservice teachers’ beliefs change as a result of participating in a 12-week intervention focused on learning about the level of cognitive demand of mathematical tasks and writing letters to third grade elementary students. A necessary component of implementing mathematical tasks effectively with students is to have a foundational understanding of mathematical – task knowledge (Chapman, 2013). Mathematical task knowledge is “the knowledge teachers need in order to (a) select and develop tasks to promote students’ conceptual understanding of mathematics, support their development of mathematical thinking, and capture their interest and curiosity and (b) optimize the learning potential of such tasks” (Chapman, 2013, p.1). During the 12-week intervention the treatment group learned about features of mathematical tasks that elicit low-level and high-level cognitive demand from students. The preservice teachers constructed their own mathematical tasks that they included in letters to third grade elementary students.

Teachers’ beliefs about mathematics and mathematics teaching have been found to affect the mathematical tasks that teachers use for mathematics instruction (Pajares, 1992; Philipp, 2007; Raymond, 1997). Therefore, an important component of this study was to determine whether or not the 12-week intervention had an effect on the beliefs about mathematics and mathematics instruction for the treatment group. The Integrated Mathematics and Pedagogy (IMAP) Belief Survey (Philip et al., 2007) was administered at the beginning of the elementary mathematics methods course for both the treatment and control groups, and then the survey was administered again during Week 11 of the
study. Additionally, participants of the study completed an elementary mathematical task sort on Week 2 and Week 12 of the study. The purpose of the elementary mathematical task sort was to see if participants could identify a mathematical task as having low-level or high-level cognitive demand. In addition, two interviews were conducted for participants who exhibited a high belief change and low belief change for the seven beliefs. The interviews provided insight about the participants’ beliefs and how the intervention effected the participants’ experience of designing and posing mathematical tasks for third grade elementary students. The following questions guided the research study:

1. To what extent did the intervention of learning about the levels of cognitive demand for mathematical tasks and implementation of mathematical tasks through letter writing with third grade students impact elementary preservice teachers' beliefs about mathematics and mathematics instruction?

2. How do elementary preservice teachers' beliefs about mathematics instruction influence their implementation of mathematical tasks?

3. Can elementary preservice teacher’s identify mathematical tasks as having high-level or low-level cognitive demands and does this change after a 12-week intervention specifically focused on learning about the levels of cognitive demand and implementation of mathematical tasks in letter writing with third grade students?

A quasi-experimental research design was used for this study (Plano Clark & Creswell, 2010). The research design was chosen because the sample for the study is comprised of two groups of preservice teachers that were not randomly assigned. The study was conducted through the lens of the situated learning perspective (Putnam & Borko, 2000). The goal of the research was to provide a better understanding about how elementary preservice teachers’ beliefs change as a result of participating in a 12-week intervention that focused on the level of cognitive demand of mathematical tasks and writing letters to third grade students. The outcome of the study could provide guiding
principles for designing an elementary mathematics methods course where preservice teachers focus on the level of cognitive demand of mathematical tasks and have opportunities to interact with elementary school students through authentic experiences.

Participants were recruited to take part in the research through their elementary mathematics methods course. There were two sections of the same course offered in the Fall 2013 semester. One section was the control group and the other was the treatment group. Preservice teachers were not randomly assigned to the groups. However, the instructors were randomly assigned to the groups of preservice teachers. The researcher for this study was the instructor for the treatment group. A third-year graduate student in the mathematics education doctoral program was the instructor for the control group. Both instructors for the control and treatment groups used the same textbooks (i.e., Van de Walle & Lovin, 2006; Sowder, Sowder & Nickerson, 2010) and covered the same mathematical content. The differences between the two groups were the opportunities that the treatment group had to learn about the levels of cognitive demand for mathematical tasks and to participate in a letter writing exchange with third grade elementary students.

Data was collected through the use of several instruments: 1) the IMAP beliefs survey (Philipp et al., 2007); 2) general interview guide approach with two participants (Turner, 2010); and 3) elementary mathematical task sort (Arbaugh & Brown, 2005; Osana et al., 2006). The data from the beliefs survey was analyzed using a Chi-Square for one-way designs (Question 1). A thematic analysis method described by Braun and Clarke (2006) (Question 2) was used for the two participant interviews. A Wilcoxon-
ranked signed test and Whitney-Mann U test were used to analyze the elementary mathematical task sort (Question 3).

**Methodology**

This dissertation was a mixed methods study that aimed to examine whether the 12-week intervention had an effect on elementary preservice teachers’ beliefs about mathematics and mathematics instruction and how they identify mathematical tasks as low-level or high-level cognitive demand. The research was viewed through a situated learning lens (Putnam & Borko, 2000). The situative perspective focuses on how individuals participate in a group or in a classroom practice (Borko, 2004). Knowledge is situated in the activity in which it occurs (Putnam & Borko, 2000). Gee (2008) states the “situative perspective looks at knowledge and learning in terms of a relationship between an individual with both a mind and body and an environment in which the individual thinks, feels, acts, and interacts” (p.81).

Based upon the theoretical principles of the situative perspective, preservice teachers need to participate in a setting which allows them to learn from each other while acquiring the tools necessary to identify the cognitive demand of mathematical tasks. Brown, Collins, and Duguid (1989) describe the process of acquiring “tools” to aid the learner. For example, knowledge of the level of cognitive demand of mathematical tasks could be a tool preservice teachers use to analyze or construct mathematical tasks. A preservice teacher could interact with this tool in a class setting, but not be able to fully apply it to a real-life teaching situation. Therefore, this study provided PSTs with the opportunity to design mathematical tasks and pose the tasks with elementary students through a letter writing exchange on three occasions.
The intervention focused on learning about the level of cognitive demand for mathematical tasks. Learning about the conceptual mathematical task framework and task analysis guide has been shown to provide opportunities that allow preservice and inservice teachers time to explore mathematics and mathematics instruction in a supportive and safe learning environment (e.g., Arbaugh & Brown, 2005; Boston & Smith; Kosko et al., 2010; Osana et al., 2006; Silver & Stein, 1996). PSTs worked in pairs to create a mathematical task for their elementary pen pal. The letter writing exchange paired an elementary student with one or two PSTs. There were more PSTs than elementary students, therefore some PSTs had to share an elementary student. The PSTs were matched up randomly with the elementary students. The pen pals exchanged letters on three occasions during the study. Prior to sending out the mathematical task, the PSTs determined the cognitive demand of their own mathematical task and provided a rationale for the selection of cognitive demand. Afterwards, when the PSTs received their mathematical task back from their pen pal, they reevaluated the cognitive demand for their own mathematical task and provided rationale for their rating. The PSTs recorded their observations on the elementary task analysis form modified from Kosko et al. (2010) (Appendix A).

This mixed methodology was selected, as it was necessary to answer the questions posed for this study. Questions 1 and 3 allowed for quantitative measurement of preservice teachers’ beliefs about mathematics, mathematics instruction, and scoring of the mathematical task sort. In addition, Question 2 utilized qualitative methods for the participant interviews. Both sets of data were analyzed separately and then the results were compared. Finally, an interpretation of the results was made and then discussed.
(Plano & Clark, 2010). The Chi-Square test was used to compare the pre- and posttest data collected from the groups on the beliefs survey. For the mathematical task sort, a pre- and post- task sort score was determined for each participant. A Wilcoxon Signed – Ranked test and Mann-Whitney U-test for two independent groups were used to analyze the pre- and post-task sort data.

The format for the mathematical task sort activity was derived from researchers Arbaugh and Brown (2005) who implemented a pre- and post-task sort with practicing geometry teachers. The same elementary mathematical tasks were used for each task-sort activity (see Appendix B). At the conclusion of the study two participant interviews were conducted. The interviewer asked questions about the intervention, specifically about the experiences the preservice teachers had during the course and the letter writing exchange. A thematic analysis was conducted on data collected from the participant interviews (Braun & Clarke, 2006).

**Procedures**

**Participants**

**Identification of participants**

The participants for this study were elementary preservice teachers enrolled in an undergraduate elementary mathematics methods course during the Fall 2013 semester, at a large research university in the southeastern United States. The beliefs data were collected from PSTs in the control (n = 33) and treatment (n = 30) groups. The interviews were conducted with two PSTs from the treatment group. The task sort data was collected from PSTs in the control group (n = 34) and treatment group (n = 32). Since the data collection for the beliefs instrument and mathematical task sort happened on different days, the numbers of participants who completed the pre- and post-
assessments differ because some PSTs were not present on that day. Table 3-1 displays the participants for each method of data collection. The participants were not randomly assigned to these groups; however instructors were randomly assigned to sections of the course. A graduate student teaching instructor taught the control group and the researcher, who is also a graduate student teaching instructor, taught the treatment group.

Table 3-1. Table of Participants

<table>
<thead>
<tr>
<th></th>
<th>Control Group</th>
<th>Treatment Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beliefs Survey</td>
<td>n = 33</td>
<td>n = 30</td>
</tr>
<tr>
<td>Interview</td>
<td>n = 0</td>
<td>n = 2</td>
</tr>
<tr>
<td>Mathematical-task Sort</td>
<td>n = 34</td>
<td>n = 32</td>
</tr>
</tbody>
</table>

The general interview data was collected from two participants in the treatment group (Turner, 2010). The researcher chose two participants based on changes in their belief scores. These participants were chosen through purposive sampling because of the intensity in their belief change. (Flick, 2009). One participant had significant changes in beliefs for four out of the seven beliefs and the other participant had a belief change for one of the seven beliefs. Raymond (1997) created two of the questions and the researcher wrote the other eleven. The purpose of the questions was to learn more about how preservice teachers’ beliefs effected their implementation of mathematical tasks (see Appendix C). According to Turner (2010), this type of interview works well for researchers who have already established a rapport with participants, and the interview guide provides structure to the conversations.

All PSTs enrolled in the elementary mathematics methods course were invited to participate in the study. All participants were asked to fill out a consent form. The participants asked to participate in the general interviews were provided with an
additional consent form. For the treatment group, three PSTs chose not to participate, and for the control group, four PSTs chose not to participate.

**Description of participants**

All eligible participants (4 male, 73 females) were undergraduate students who were enrolled in their third semester of a teacher preparation program for elementary school teachers. This program offers both a Bachelor of Arts in Education and Master of Education. At the end of the program, preservice teachers can seek teacher certification for grades K – 6. This program requires 124 undergraduate credit hours and 36 graduate credit hours. Prior to entering this study, each participant completed the first elementary mathematics course, as well as a mathematics course at or above the college algebra level. The course used for this study is the second out of four required mathematics education courses at the university for the elementary education major.

**Setting**

The intervention took place during scheduled course meetings throughout the first twelve weeks of the Fall 2013 semester. A twelve-week timeline was based on timelines of similar studies involving mathematical tasks (e.g., Kosko et al., 2010 [12 weeks]; Norton & Kastberg, 2011 [10 weeks]; Osana et al., 2006 [1 day]; Vacc & Bright, 1999 [5 weeks]). The course met once per week for a three-hour session. The treatment and control groups met on different days. The classroom for both the control and treatment groups was set up to provide preservice teachers with the opportunity to work together in small groups. An overview of the topics for the intervention is in Appendix D.

Data from the beliefs instrument were collected during the first class meeting of the semester (pretest) and on the eleventh class meeting (posttest). The mathematical task-sort activity took place during the second class meeting (pretest) and twelfth class
meeting (posttest). Additionally, participants in the treatment group created a mathematical task for their elementary student pen pal on three occasions (sixth, ninth, and twelfth week). The elementary task analysis form was completed prior to the elementary students receiving the task, and then afterwards when the PSTs received the responses from the elementary students. This form also served as a reflection tool for PSTs as they looked back upon their letter writing exchange.

The interviews took place during the fourteenth week of class to allow for the data from the beliefs instrument and mathematical task-sort to be analyzed. The interviews took place in a conference room adjacent to the room where the course was taught in the college of education during a convenient time of day for both the researcher and the participants.

Treatment

Treatment group

The PSTs who were part of the treatment group for the study were introduced to the levels of cognitive demand for mathematical tasks, Mathematical Task Framework (Stein et al., 1996), and task analysis guide (Stein & Smith, 1998). The intention for the selected activities was to have PSTs examine, perform, and then discuss mathematical tasks that were in the curriculum and in other resources and to determine the cognitive demand. The PSTs also considered how students would approach a mathematical task and discussed possible challenges students might have with the task. During class sessions, PSTs discussed ways to help students persevere through challenging aspects of mathematical tasks without lowering the cognitive demand of the mathematical tasks. Each of the classes was structured so PSTs had an opportunity to watch videos, view student work, and engage in discussions.
During Week 3, pairs of PSTs were randomly assigned to a third grade elementary student pen pal from a school in Texas. The elementary third grade teacher made the letter writing exchange possible. She contacted the mathematics education department at the University of Florida at the time that the researcher for this study was seeking a school to exchange letters with. Each classroom at the elementary school was assigned a university to study for the whole year. The premise behind having each class study a university was to motivate the students to try to succeed academically and to go to college. The teacher described the school as a rural Kindergarten through fifth grade elementary school. The majority of the students spoke a language other than English and most students were from low socioeconomic homes. After hurricane Katrina in 2005, many displaced families moved to the town where the elementary school is located. The third grade class consisted of twenty-two students (11 boys and 11 girls).

The third grade students initiated the first correspondence of the letter writing exchange. The third grade students sent a pre-made survey consisting of favorite things (e.g., color, candy, movie, etc.) to the PSTs. In pairs, the PSTs wrote a letter introducing themselves, completed the same pre-made survey, and included a mathematical task based upon a pre-selected Common Core State Standard (2010). During the study, PSTs were able to correspond with their pen pals on three occasions. An overview of the timeline for the pen pal letters with the corresponding standard is located in Appendix E.

According to Gee (2008) “knowledge is produced through the activities and tools participants interact with” (p.81). Throughout the twelve-week intervention, PSTs were exposed to a variety of mathematical tasks. An example lesson plan is included in Appendix F. The PSTs had the opportunity to learn from each other as they solved the
mathematical tasks presented in class. In addition, they had the opportunity to create mathematical tasks for their pen pals. The PSTs used features described in the task analysis guide (Stein et al., 2000) to create their tasks for their pen pal.

**Control group**

The control group for the study followed the typical format for the elementary mathematics methods course at the university. This typical format consists of discussing previously assigned homework problems, introduction to the new content through a student-centered activity, followed by a discussion, and then a recap of the content and how it relates to teaching mathematics. The class activities were typically from the course textbook (e.g., Van de Walle & Lovin, 2006). Participants in both the control and treatment groups were taught the same mathematical content on fractions. The control group did not participate in assignments that were focused on the cognitive demand of mathematical tasks and the letter writing exchange with elementary school students.

However, the control and treatment shared a common practicum experience. Each PST in both groups was assigned to an elementary school classroom in the same school district for the whole semester. The PSTs would go to their assigned classroom once a week. Throughout the course of the semester, the PSTs were asked to complete assignments in their practicum classrooms. The assignments consisted of creating and implementing reading and mathematics lessons, completing a case study on one English-for-speakers-of other-languages student, and conducting a technology inquiry. The PSTs experiences in their practicum classrooms varied. For instance, some of the PSTs were able to create their own lessons, while other PSTs were given pre-made lessons to teach. The teaching experiences ranged from whole group to small group.
The practicum experience was a required component of the elementary education program for all third semester students.

**Data Sources**

The data sources for this study consisted of: the beliefs survey (Philipp et al., 2007), two audio-taped preservice teacher interviews, and mathematical task-sort (Smith et al., 2004). There were also written artifacts from the preservice teacher interviews, written field notes, and video-taped class sessions. The written field notes and videotaped class sessions provided a resource for monitoring the implementation of the treatment. Specifically, the researcher viewed the video-taped class sessions to ensure the content covered in the control group was consistent with the treatment group. The instructors did not share teaching resources during the study. The video-taped class sessions for the treatment group provided a resource for fidelity of implementation of the treatment for the study. The beliefs survey was taken in the form of pencil and paper responses. The two participant interviews were audio-recorded and transcribed. For the mathematical task sort, participants used a paper with pre-made columns titled “level of cognitive demand.” and rows with letters identifying the mathematical task.

**Data Collection**

**Beliefs instrument**

A pretest and posttest were used to capture any effects the treatment had on PSTs’ beliefs. The beliefs instrument was created through funding provided by the Interagency Educational Research Initiative, National Science Foundation grant to researchers at San Diego State University. The researchers were interested in creating an instrument that could measure change in PSTs’ beliefs about mathematics and mathematics instructions. They hypothesized that Likert-type surveys were not capable
of measuring such change and were determined to develop a beliefs instrument that could provide a better understanding about PSTs beliefs. It took two years to develop the instrument (Ambrose, Philipp, Chauvot, & Clement, 2003). The goal of the researchers was to "create a suitable beliefs survey by identifying characteristics of beliefs that account for the critical role they play in teaching and learning" (Philipp et al., 2007, p. 450).

Only one version of the instrument was produced in a Web-based format. A researcher modified the original Web-based format to a paper and pencil format. The original layout, design, directions, and questions remain the same. Originally each question appeared on one screen, and so each question would appear on a separate sheet of paper. There was no time limit for the survey. A selection from this instrument is provided in Appendix G.

The beliefs instrument contained sixteen questions that included complex situations and scenarios that required respondents to make teaching decisions (Philipp et al., 2007). The instrument was intended to capture beliefs that focus on implementing reform-based teaching (Philipp et al., 2007). Originally, the researchers created eight such beliefs but could only provide reliability and validity measure for seven beliefs. The seven beliefs included in the instrument are:

**Beliefs About Mathematics:**

1. Mathematics is a web of interrelated concepts and procedures (and school mathematics should be too).

**Beliefs About Learning or Knowing Mathematics, or Both:**

2. One’s knowledge of how to apply mathematical procedures does not necessarily correspond with understanding of the underlying concepts.
3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.

4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely to learn the concepts.

**Beliefs About Ways Children (Students) Learn and Perform Mathematics:**

5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect.

6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children’s initial thinking, whereas symbols do not.

7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

The researchers engaged in a rigorous process to develop the instrument. Pilot work with in-service teachers provided positive results for capturing a range of belief scores. Due to the fact that this survey was not in a typical Likert Scale format, the usual tests used to determine validity and reliability were not appropriate (Ambrose et al., 2003). Therefore, to ensure consistency of the survey, it was administered to 18 PSTs, which included follow-up interviews with the PSTs (Ambrose et al., 2003). Additionally, five mathematics education researchers, with experience in teaching and researching PSTs’ beliefs, were administered the survey and also engaged in follow-up discussions (Philips et al., 2007). The result of dispensing the survey to the aforementioned parties was confirmation of the reliability and validity of the survey. Hence, the survey measured what the researchers intended for it to measure (participants’ beliefs about mathematics instruction) and these results were found each time the survey was administered. Reliability information for the sixteen questions on the beliefs instrument was never published.
The majority of the construction time spent on the instrument was with the rubrics. Seventeen rubrics for the instrument were constructed and each one took 72 person-hours. The coders for all 17 rubrics calculated an 84% inter-rater reliability.

For this study, the instrument was administered to the participants at the beginning of the first scheduled class meeting for the Fall 2013 semester. The researcher administered the instrument by following the directions in the manual created by the authors (with the exception of the paper and pencil modification). The participants recorded their answers directly on the paper, either by selecting an answer to a multiple-choice response or filling in the short answer responses. There was a video component to the instrument, which was shown to the whole group at the same time. The participants were only identified by their study identification number, which they recorded on the instrument.

**Interviews**

Two weeks after the study, two participant interviews were conducted using the General Interview Guide Approach (Turner, 2010). At the conclusion of the study, the researcher chose two participants from the treatment group. The participants were chosen based upon their belief change scores. Specifically, the range for the change scores were analyzed and the two participants were chosen for their growth or lack thereof out of the seven beliefs. One participant had belief changes on five of the seven beliefs and the other participant only had a belief change on one of the seven beliefs. Both participants were in third grade practicum classrooms at the same school. This characteristic is significant to the study because third grade students were also the pen pals for the letter writing exchange.
The intent of the interviews was to gain insight into how the preservice teachers interpreted their experience in the intervention. Pajares (1992) recommended that multiple data sources be used to understand the beliefs of teachers. The interviews took place in a conference room adjacent to the classroom where the course was held at the College of Education. There was no time limit on the interview and each participant was asked the same questions. Each interview was audio recorded and transcribed verbatim by the researcher. The participants already had an established rapport with the researcher, which helped the interview flow similar to a conversation (Turner, 2010).

**Mathematical Task Sort**

The design, use and analysis of a mathematical task sort was informed by research established by Arbaugh and Brown (2005), Boston and Smith (2011), Osana et al. (2006), and Smith et al. (2004). The mathematical task sort provided an assessment of pre- and post-knowledge of the cognitive demands of mathematical tasks. In order to complete the task sort, participants were provided with sixteen elementary mathematical tasks on sheets of 8.5 x 11 paper. These sixteen tasks were created through the QUASAR research study (Stein et al., 2000). Each task was assigned a letter (e.g., A through P). The participants were provided with a table that consisted of rows labeled A through P and columns labeled as memorization, procedures without connections, procedures with connections, doing mathematics, and a rationale for classification of the mathematical task. The participants were asked to sort the mathematical tasks into four categories (e.g., memorization, procedures without connections, procedures with connections, doing mathematics). The instructions were as follows:

- The papers in the plastic sleeves contain 16 elementary-school level mathematics tasks.
• Please read them through, and then sort them into one of the four categories provided in the table.

• Please provide rationale for your category selection in the last column.

The mathematical task sort was designed as a tool to be used with teachers in professional development settings (Smith et al., 2004). The purpose of the mathematical task sort was for participants to know the potential a mathematical task has to meet the student learning goals and bring awareness to features of mathematical tasks (Stein et al., 2004). All participants could engage with the mathematical task sort at some level.

The mathematical task sort was scored using criteria established by Boston (2006), who created a scoring matrix for the task sort. A score of either a 1 or a zero was assigned to a participant identifying each individual task as either high or low cognitive demand. There was also a rationale column provided for the decision of the cognitive demand. Responses in this column were also assigned a score of 1 or zero. However, only a few participants in each group completed the rationale column. Therefore, the researcher did not include this information in the pre- and post-task sort.

**Elementary Mathematical Task Form**

Preservice teachers in the treatment group focused on constructing mathematical tasks for the pen pal letters. After the mathematical task was constructed, the PSTs chose a level of cognitive demand for the mathematical task and provided a rationale for this decision. They chose the level of cognitive demand based upon the features of the task explained in the Task Analysis Guide (Stein et al., 2000). After the elementary student responded to the mathematical task, the PST re-evaluated the level of cognitive demand of the task and provided a reason for the choice in level of that cognitive demand. Kosko et al. (2010) found that the forms provided evidence of the PSTs’
abilities to predict levels of cognitive demand and indicated improvement over the period of the study. This form served as a tool for PSTs to consider the cognitive demand for the task they created for the pen pal letter. The PSTs were able to reflect upon the cognitive demand of the task after the elementary student had the opportunity to try the task.

**Data Analysis**

In order to answer Research Question 1, the data collected from the beliefs instrument was analyzed using a chi-square test for one-way designs (Shavelson, 1996). The beliefs instrument collects data for an independent variable that has two or more levels. Each of the seven beliefs that the instrument measured was treated independently. The chi-square test was performed using the Statistics Package for the Social Sciences (SPSS) 18.0. The five assumptions for this test were considered. The assumptions are as follows (Shavelson, 1996, p.558):

1. Each observation must fall in one and only one category.
2. The observations in the sample are independent of one another.
3. The observations in the sample are measured by frequencies.
4. The expected frequency for each category is less than 5 for df ≥ 2 and not less than 10 for df = 1.
5. The observed values of $X^2$ with 1 degree of freedom must be corrected for continuity in order to use the table of values of $X^2_{critical}$.

To address Research Question 2, the transcribed interview data from the two participant interviews were analyzed using thematic analysis (Braun & Clarke, 2006). Braun and Clarke (2006) describe the process of completing a thematic analysis in six phases. It is important to note that these phases do not represent a linear process; they denote a process that can take the form needed by the researcher to create themes.
Hence, the researcher can return to any phase at any point in the process of the thematic analysis (Braun & Clarke, 2006). These phases are as follows:

- Phase 1: familiarizing yourself with the data
- Phase 2: generating initial codes
- Phase 3: searching for themes
- Phase 4: reviewing themes
- Phase 5: defining and naming themes
- Phase 6: producing the report

The identified themes provided insight into the participants’ beliefs about mathematics and mathematics instruction. Throughout the analysis the researcher made note of any theme that was not consistent with the beliefs about mathematics instruction. The purpose of the interviews was to provide follow-up information about the preservice teachers’ beliefs about mathematics and mathematics instruction.

Additionally, the researcher sought assistance from a fellow graduate student researcher to read the transcripts from the interviews and code the data in order to validate the first researcher’s findings. The second researcher verified the codes and themes from the data created by the first researcher. Both researchers had a conversation about the themes that were identified in the data and found that the majority of the themes coincided with one another.

In order to address research Question 3, the mathematical task sorts were coded to produce a numeric score that represented the preservice teachers’ knowledge of the cognitive demand of mathematical tasks. The treatment group scores on the pre- and post- task sort were compared using one-tailed Wilcoxon Signed-Rank test for nonparametric data. This test was chosen to determine whether preservice teachers’ knowledge of the levels of cognitive demand increased after the 12-week intervention. The pre- and post- task sort data for both the treatment and control group was compared
using a Mann-Whitney U test for two independent groups (Shavelson, 1996). This test was chosen to examine if there were differences in the distributions of two groups or differences in the medians of two groups (Lund & Lund, 2013).

The assumptions for the Mann-Whitney U Test were as follows (Shavelson, 1996):

1. You have one dependent variable that is measured at the continuous or ordinal level.

2. You have one independent variable that consists of two categorical, independent groups (i.e. a dichotomous variable).

3. You should have independence of observations, which means that there is no relationship between the observations in each group of the independent variable or between the groups themselves.

4. You must determine whether the distribution of scores for both groups of your independent variable have the same shape or a different shape. This will determine how you interpret the results of the Mann-Whitney U Test.

The analyses for these three research questions were utilized to provide an understanding of the changes that occurred during the course of the study. In the next section the potential limitations are identified by the researcher.

**Limitations of the Study**

There were several limitations to the study. First, the sample population for the study was not randomly selected into groups. The groups of the participants were already intact prior to the study. Upon acceptance into the teacher education program, participants have been assigned a cohort with whom they take almost every course together. The participants are in close proximity to each other on a daily basis. Additionally, both groups participated in a practicum experience in the same school district one day a week. The participants were assigned to one classroom in grades Kindergarten through fifth grade. During the practicum experience the participants...
observed mathematics instruction and implemented mathematics lessons. This could pose a potential threat to the internal validity of the study. The sample for the study is a convenience sample of preservice teachers enrolled in an elementary mathematics methods course as part of a required course in their program of study. Due to this fact, this may limit the generalizations that can be made to other populations of preservice teachers.

The beliefs instrument also provides threats to internal validity of the study. For instance, there was only one copy of the instrument and it was administered twice in twelve weeks (i.e., before and after the treatment). Furthermore, the original instrument was validated as a web-based survey. Another researcher transcribed the instrument to a paper and pencil format, keeping the original formatting and following the instructions of the beliefs instrument manual. There is no validation information for the new paper and pencil format of the instrument.

The mixed methods research design was chosen to provide validity to the quantitative and qualitative data collected for the study. The design provided the opportunity for the researcher to analyze the data separately and then compare the results.

Finally, the researcher posed a threat to the validity of the study because the researcher taught the treatment group. The participants could have become aware of the researcher’s intentions for the treatment when they read the consent form. The researcher tried to not influence the participants in any way during the study.

Conclusion

This dissertation study builds upon the research conducted by Arbaugh and Brown (2005), Boston and Smith (2011), Crespo (2003), Kosko et al. (2010), Norton and
Kastberg (2011), and Osana et al. (2006) by using the Mathematical Task Framework with elementary teachers and incorporating letter writing into the elementary mathematics course. The goal of the study was to build upon prior research studies which focused on providing PSTs with opportunities to learn about the level of cognitive demand of mathematical tasks and provide a better understanding about how PSTs’ beliefs change as a result of participating in a 12-week intervention that focused on the level of cognitive demand of mathematical tasks and writing letters to third grade students. Furthermore, the researcher was interested in whether or not the treatment had an impact on the PSTs’ beliefs.

The PSTs had an opportunity to learn about the cognitive demand of mathematical tasks, as well as, analyze curriculum materials with the intention of identifying the cognitive demand of potential mathematical tasks for elementary grades, participate in doing mathematics during the course, discuss mathematics and mathematics instruction as a group, and correspond with a third grade pen pal. Overall, the dissertation study provided insight into the beliefs preservice elementary teachers hold about mathematics and mathematics and instruction. The study was intended to add to the body of literature on mathematical tasks, PSTs’ beliefs about mathematics instruction, and authentic experiences for PSTs in teacher education.
CHAPTER 4
RESULTS

The results of the data analysis are presented in this chapter, organized into three sections that correspond to the three original research questions presented in Chapter 1. The first section describes the beliefs of the elementary preservice teachers, including the results of the pre- and post-belief survey. The second section provides closer analysis of how the beliefs of two participants were impacted by the intervention. This includes the qualitative analysis of two participant interviews. The third section explains the knowledge of preservice teachers about the cognitive demands of mathematical tasks. This includes the pre- and post-mathematical task analysis for the treatment group, a comparison of mathematical task sort scores for the treatment and control group, and an analysis of the mathematical tasks classified by the treatment group.

Elementary Preservice Teachers’ Beliefs about Mathematics and Mathematics Instruction

Research Question 1: To what extent did the intervention of learning about the levels of cognitive demand for mathematical tasks and implementation of mathematical tasks through letter writing with third grade students impact elementary preservice teachers’ beliefs about mathematics and mathematics instruction? In order to answer this research question, a Chi-Square analysis was used to analyze the data collected by the beliefs survey for both the treatment and control groups (n=63). The results of the comparisons between the treatment and control group are presented in the remainder of this section.

Pre- and Post-Belief Survey

The pre- and post-belief survey change scores serve as an indicator of the treatment and control groups’ beliefs about mathematics and mathematics instruction.
after 12-weeks. Originally, the pre-belief survey was administered to seventy-three PSTs. A total of sixty-three PSTs completed both the pre- and post-belief survey. Due to attrition from the study, the scores of ten PSTs were not included in the data analysis. There were three PSTs in the treatment group and four PSTs in the control group who chose not to be part of the study. Additionally, one PST from the treatment group withdrew from the course halfway through the intervention and two PSTs did not complete both the pre- and post-belief surveys.

The researcher administered the pre- and post-belief surveys to both groups using the same directions on all four occasions. The researcher assigned a numerical score to each pre- and post-belief survey following the rubric for each of the seven beliefs. To ensure reliability when assigning a score to each of the surveys, the researcher calibrated by using the practice sets in the training manual, prior to scoring the surveys. After all of the pre- and post-surveys were assigned raw belief scores, the researcher assigned a change score based upon the following criteria: no increase or a negative value in belief score received a 0, increased one level received a 1, and increased two or more levels received a 2.

First, a Chi-Square analysis was conducted to determine if there was a difference between the pretest belief scores for the treatment and control groups (see Table 4-1). These results show that the pretest scores were not significantly different between the treatment and control groups for all seven beliefs. More specifically, the pretest scores for both groups were very similar and hence did not indicate one group was higher than the other at a certain belief change score.
Next, the actual counts for each change score level for all seven beliefs for both groups is represented in Table 4-2. The descriptive statistics revealed a significant difference between the treatment (n=30) and control group (n=33) on two out of the seven beliefs (see Table 4-3). The two beliefs are: Belief 5 (children can solve problems in novel ways before being taught how to solve such problems) \(\chi^2(2) = 8.497, p \leq .05\) and Belief 7 (the teacher should allow the children to do as much of the thinking as possible) \(\chi^2(2) = 11.004, p \leq .01\). The Chi-Square test revealed a change in beliefs for more participants in the treatment group than in the control group for Beliefs 5 and 7.

The control group participants did experience changes on beliefs. However, these changes are not significant when comparing the actual counts versus the expected count for the two groups.

<table>
<thead>
<tr>
<th>Table 4-1. Beliefs pretest significance values.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belief</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4-2. Actual count for belief change score for treatment and control groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belief</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

* T represents treatment group and C represents control group
Table 4-3. Belief change score significance values.

<table>
<thead>
<tr>
<th>Belief</th>
<th>Pearson Chi-Square Value</th>
<th>Degrees of Freedom</th>
<th>p-value (2-sided)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.859</td>
<td>2</td>
<td>0.651</td>
</tr>
<tr>
<td>2</td>
<td>1.254</td>
<td>2</td>
<td>0.534</td>
</tr>
<tr>
<td>3</td>
<td>1.280</td>
<td>2</td>
<td>0.527</td>
</tr>
<tr>
<td>4</td>
<td>0.904</td>
<td>2</td>
<td>0.636</td>
</tr>
<tr>
<td>5</td>
<td>8.497</td>
<td>2</td>
<td>0.014</td>
</tr>
<tr>
<td>6</td>
<td>0.692</td>
<td>2</td>
<td>0.708</td>
</tr>
<tr>
<td>7</td>
<td>11.004</td>
<td>2</td>
<td>0.004</td>
</tr>
</tbody>
</table>

The cross-tabulation in Table 4-4 and 4-5 below provide a breakdown of results for Belief 5 and Belief 7. For Belief 5, the treatment group had 23 participants experience at least one level of change in beliefs compared to the control group that only at least one level of belief change for 14 participants. More specifically, the actual count reveals that 12 participants in the treatment group had at least two levels of changes in beliefs, while the control group had 19 participants with no change in beliefs for Belief 5. Belief 5 involves presenting problems to children and allowing them to solve the problems before being taught how to solve them. Additionally, Table 4-6 and Table 4-7 provide the percentages for the each category of belief change by group.

The letter writing exchange may have facilitated the change in this belief. The participants in the treatment group created problems for their pen pals and were only able to provide directions. The participants were not able to provide instruction and thus had to wait to see how their pen pal responded to their problem. There is evidence for this conclusion from one of the interview participants who first analyzed the pen pal response and thought it was incorrect. Upon further inspection of the solution and the students’ explanation, a realization occurred to the participant that the elementary school student was actually correct in her thinking and the solution was correct. The participant was anticipating that the pen pal student would solve the problem like she had intended.
it to be solved, and was surprised when the pen pal student came up with an alternative solution to her preconceived one. The control group did not have the opportunity to interact with students in this manner and this may be the reason why 19 of the participants experienced no change in level in their beliefs for Belief 5.

Belief 7 is focused on allowing students to do as much thinking about mathematics as possible without teacher interference. Belief 7 for the treatment group had actual counts of 16 participants with a belief change of at least one level or higher. The control group had actual counts of 6 participants with one level change, while none had two levels of belief change. There were 14 participants with no belief change in the treatment group compared to 27 in the control group.

An explanation for the slightly higher belief change for the treatment group as opposed to the control group could be due to the way the course was structured for the treatment participants. Beliefs 5, 6, and 7 are encompassed in the category, beliefs about ways children (students) learn and perform mathematics. Belief 6 was not significant in this study and this could have been due to a missing component in the intervention which would have allowed PSTs the opportunity to see that children think differently than how adults would expect them to think about mathematics. However, the treatment group experienced mathematics as a problem-solving discipline. There were several occasions when participants were asked to work together on a task and share their solution process. The participants were also asked to consider the cognitive demand of the mathematical tasks and how to differentiate the task to accommodate different levels of cognitive demand. The remaining cross-tabulations for the five beliefs are included in the Appendix H.
Table 4-4. Belief 5 cross-tabulation values.

<table>
<thead>
<tr>
<th>Group</th>
<th>Change score 0, expected count</th>
<th>Change score 0, actual count</th>
<th>Change score 1, expected count</th>
<th>Change score 1, actual count</th>
<th>Change score 2, expected count</th>
<th>Change score 2, actual count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>12.4</td>
<td>7</td>
<td>9.5</td>
<td>11</td>
<td>8.1</td>
<td>12</td>
</tr>
<tr>
<td>Control</td>
<td>13.6</td>
<td>19</td>
<td>10.5</td>
<td>9</td>
<td>8.9</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 4-5. Belief 7 cross-tabulation values.

<table>
<thead>
<tr>
<th>Group</th>
<th>Change score 0, expected count</th>
<th>Change score 0, actual count</th>
<th>Change score 1, expected count</th>
<th>Change score 1, actual count</th>
<th>Change score 2, expected count</th>
<th>Change score 2, actual count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>19.5</td>
<td>14</td>
<td>7.6</td>
<td>10</td>
<td>2.9</td>
<td>6</td>
</tr>
<tr>
<td>Control</td>
<td>21.5</td>
<td>27</td>
<td>8.4</td>
<td>6</td>
<td>3.1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4-6. Percentage of students in each group whose scores on Belief 5 increased 1, 2, 3, or 4 levels from presurvey to postsurvey

<table>
<thead>
<tr>
<th></th>
<th>1 – level increase</th>
<th>2 – level increase</th>
<th>3 – level increase</th>
<th>4 – level increase</th>
<th>Increased ≥ 1 level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment (n = 30)</td>
<td>37%</td>
<td>27%</td>
<td>10%</td>
<td>3%</td>
<td>77%</td>
</tr>
<tr>
<td>Control (n = 33)</td>
<td>27%</td>
<td>9%</td>
<td>3%</td>
<td>0%</td>
<td>39%</td>
</tr>
</tbody>
</table>

Table 4-7. Percentage of students in each group whose scores on Belief 7 increased 1 or 2 levels from presurvey to postsurvey

<table>
<thead>
<tr>
<th></th>
<th>1 – level increase</th>
<th>2 – level increase</th>
<th>Increased ≥ 1 level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment (n = 30)</td>
<td>33%</td>
<td>20%</td>
<td>53%</td>
</tr>
<tr>
<td>Control (n = 33)</td>
<td>18%</td>
<td>0%</td>
<td>18%</td>
</tr>
</tbody>
</table>

The Beliefs of Heidi and Nora

Research Question 2: How do elementary preservice teachers' beliefs about mathematics instruction influence their implementation of mathematical tasks? In order to answer this question the researcher chose two preservice teachers from the treatment group that exhibited either a significant belief change or a minimal belief change across all seven beliefs measured by the survey.

The participants were chosen from the treatment group through purposive sampling (Flick, 2009). One participant was chosen because she had few belief changes
and one participant was chosen because she had many changes. For clarification purposes the preservice teacher with a significant belief change on five out of seven beliefs will be referred to as Heidi and the preservice teacher with one belief change will be referred to as Nora. Heidi’s and Nora’s respective scores on the beliefs surveys are presented in Table 4-8. Heidi had significant increases on five out of the seven beliefs and Nora had a belief change on one of the seven beliefs. Heidi had the highest score change for all seven beliefs. For example, on Belief 6, Heidi went from a pre-survey score of a 0 to a post-survey score of a 4. Nora had the lowest score change for all seven beliefs. For example, on Belief 7, Nora had a pre-survey score of 2 and a post-survey score of 0. Both participants were in their third semester in the elementary education program, placed at the same school in different third grade classes for their third semester practicum, and have taken the same courses together since entering the elementary education program.

<table>
<thead>
<tr>
<th>Beliefs</th>
<th>Pre-Belief Score</th>
<th>Post-Belief Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heidi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belief 1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Belief 2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Belief 3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Belief 4</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Belief 5</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Belief 6</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Belief 7</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Nora</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belief 1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Belief 2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Belief 3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Belief 4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Belief 5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Belief 6</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Belief 7</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>
Background information about the experience the PSTs had in their third semester is provided to understand the premise of the interview questions. During the third semester the PSTs were assigned to a practicum placement at an elementary school for one day a week. The PSTs were placed in pairs in elementary classrooms. The practicum experience was designed for PSTs to have an opportunity to work with English Language Learners and teach two lessons for mathematics and three lessons for reading.

In the beginning of the 12-week intervention, the preservice teachers were asked to read two articles about mathematical tasks outside of the course. One article focused on making instructional decisions and planning mathematical tasks that would elicit high cognitive demand (e.g., Breyfogle & Williams, 2008). The other article focused on implementing high-level mathematical tasks (e.g., Smith, Bill & Hughes, 2008). For the practicum experience, the preservice teachers were asked to develop two mathematical lessons. The lessons were supposed to have a mathematical task that would maintain a level of cognitive demand. After the lesson, preservice teachers were asked to evaluate their success or lack of success implementing the mathematical task and maintaining the level of cognitive demand through a lesson reflection form. The interview questions began by asking about how the preservice teachers planned their lessons for the practicum, followed by their beliefs about mathematics instruction, and finally about the pen pal experience.

The interviews were analyzed through thematic analysis (Braun & Clarke, 2006). Both interviews were completed on the same day, after the last class of the semester. The researcher had pre-selected thirteen questions for the interviews. Two of the
questions were from an interview conducted by researcher Raymond (1997), the other eleven questions were created by the researcher for this study. Each participant was asked the same thirteen questions. Throughout the interview, follow-up questions were asked to clarify responses. Each participant interview was audio-recorded and transcribed immediately following the interview. Additionally, the researcher took observation notes throughout both interviews. After the interviews were transcribed, the researcher became familiar with the data by reading the data through one time without taking notes. Then the data was reread and initial codes were generated. Next, the researcher completed another reading of the transcripts and sorted the data into potential themes (see Appendix I). Once the initial themes were established, another reading of the data took place and themes were named and defined (Braun & Clarke, 2006).

The goal of the interviews was to better understand the mathematical beliefs of two purposefully selected preservice teachers. The following themes were identified in the data:

1. Past experiences and present beliefs about mathematics
2. Authentic and inquiry-based learning
3. The role of the teacher in making instructional decisions
   a. Cognitive aspects of student understanding
   b. Social and emotional aspects of teaching

In these sections each theme is defined and evidence from the participants is provided to show how participants' beliefs are an important construct that influences the development and implementation of lessons and mathematical tasks.
Past experiences and present beliefs about mathematics instruction

The preservice teachers’ current beliefs are shaped by their past mathematical experiences. The participants try to present mathematics in the opposite way than the way they were taught mathematics. Both participants were asked to explain the best way to learn mathematics. In each of their explanations, they included an example of their prior learning experience with mathematics. Heidi and Nora had different experiences with learning mathematics. First Nora’s experience is shared, followed by Heidi’s experience. A comparison of the experiences is done through the support of the data.

Nora had a negative experience with learning mathematics. This negative experience made an impact on how she viewed mathematics and how she taught mathematics to students in practicum. When she taught her first lesson on place value, she was very nervous at the beginning and later relieved when the students did not ask any questions. These feelings reveal she was not comfortable with teaching mathematics during the first lesson. As the semester progressed Nora contributed experiences in the course, such as demonstrating how to solve mathematical tasks together as a group, thereby shaping how she envisioned mathematics should be taught. Nora wanted her students to experience success with mathematics and continually described how she used easy mathematics problems with the students.

In one of her lessons she had students create mathematics problems for her and she went through the classroom answering the problems. She wanted students to see that she could answer the problems quickly and this was something that she wanted her students to aspire toward when learning mathematics. Additionally, Nora wanted to make sure her students knew she believed in them and that she was learning along with them.
Nora describes her mathematics experience as “horrible…we basically did worksheets. I got frustrated with them and, like I wish it would have been more hands on and real life and applying it to real life is definitely a way that children should learn math.” Throughout the 12-week intervention, the course consisted of mathematical tasks that incorporated the use of manipulatives. Nora valued the use of manipulatives because she felt she was able to understand the mathematics through using them. She states, “I didn’t see a point to it when I was in elementary school. I was frustrated and there was no point for me.” It is possible that Nora was trying to change her belief about mathematics instruction, but there was a constant struggle for her to reclaim her own confidence in mathematics.

Heidi had a contrasting experience with learning mathematics. She did not struggle to learn mathematics. Heidi remembers learning procedures for easy topics such as mean, median, and mode. Her teachers used hands-on activities when students had difficulties understanding the mathematics. In order to find out how Heidi felt about this type of learning, the researcher asked her if she felt the activities should come first when learning a new mathematical topic. Heidi thought activities should indeed come first to help students understand the mathematical concept. She describes an experience in her practicum where she saw how the students did not understand the concept of multiplication and the procedure was taught to the students first. She felt the concept should have been taught before-hand so students understood why to multiply. This example is the opposite from how Heidi was taught mathematics. She was trying to change her prior experience with mathematics instruction for her students.
Heidi felt an important characteristic of good mathematics teaching is “authentic hands-on experiences.” Evidence of this characteristic is exhibited in the design of her lesson for creating bar graphs. Heidi felt strongly that students needed to collect their own data and create bar graphs based upon this data, instead of the alternative which was completing worksheets where the data was already collected or made up. She felt students need to be motivated by fun mathematics lessons which allow students to figure out problems on their own, provide an explanation to demonstrate their understanding of the concept, and not become overwhelmed with the information.

**Mathematics instruction should include authentic and inquiry-based learning**

Both participants want students to have an authentic learning experience. Their interpretation of an authentic learning experience means having students do worthwhile mathematics activities where the students are applying knowledge and see the relevance of mathematics. Nora felt it was important to connect mathematics to real life. Heidi feels the same way as Nora. However she approaches learning through inquiry-based instruction, where she wants students to figure out the mathematical tasks on their own. Both participants felt that instruction should be guided and include scaffolds for student learning. This section explains how each participant describes what they value for mathematics instruction and how it relates to their beliefs about mathematics instruction.

Nora’s view of authentic learning relates to representing mathematics in the real world. For example, she states, “I would like to show my kids how it really does relate to real life. Like multiplication is something that we use a lot. Division is something that we use a lot…to read a newspaper or to read anything they need to know, like, what does that mean. You know. A lot of people, even adults, don’t understand statistics.” She goes
Statistics is very real life. Every situation very real life. And that’s how I think they should learn it. I think they should learn it based on real life and figuring out how it is going to relate to their future.” Nora is adamant about mathematics instruction relating to real life; however when she describes the lessons she taught in practicum, she focuses more on the student outcomes than she does the connections between real life and mathematics. Later in the interview she comments about the lack of connections between mathematical topics and how she felt she was never taught how mathematics applied to real life.

Heidi views mathematics instruction through inquiry-based learning. She values the importance of having students find out the solutions to mathematical tasks on their own and for the students to explain their answer. One example to show how Heidi encouraged students to explain their work and present their solutions in small groups is shown here:

I could tell by how they explained by the words they used, did they know the correct vocabulary? And stuff like that to the students who couldn’t really explain. Because I noticed that some of the students would be just copying their neighbor. And so when those students tried to talk they couldn’t explain how they did it. They couldn’t tell me why there were three crayons in three groups. They didn’t know, like, how to explain that to me in words. The other students were doing really great. They could explain exactly what they were doing step-by-step.

Heidi values student explanations. On another occasion, Heidi describes a situation when she assumed her pen pal got the incorrect answer based upon the students’ illustration of how to divide a circle in half two different ways. Heidi’s problem with the student work can be seen in the appendix (see Appendix J). When she read the pen pal’s written explanation she realized the student actually answered the problem correctly. Additionally, Heidi made it a point to include directions for her pen pal to
explain and show her work. This suggests that Heidi wanted to know how the student was thinking about the mathematics and whether her pen pal understood the mathematical task.

Heidi’s beliefs about mathematics instruction align with how she implements mathematical tasks. Nora struggles with what she believes should happen with learning mathematics and how she implements mathematics instruction. Both participants believe learning mathematics should be fun for the students. They both feel guided learning and scaffolding instruction through hands-on activities and manipulatives are a means to make mathematics instruction fun and engaging for students. This is in contrast to how they were taught mathematics in elementary school.

**The role of the teacher in making instructional decisions**

This theme was derived from the data through two sources: the cognitive aspects of student understanding and the social and emotional aspects of teaching. Both participants mentioned student understanding consistently throughout the interview; however they referred to it in different ways. Hence, the researcher felt it necessary to separate the theme into two sub-themes to explain how the participants were describing the role of the teacher. First, the cognitive aspects of student understanding are discussed, followed by the social and emotional aspects of teaching.

**Cognitive aspects of student understanding**

The participants focused on different aspects of mathematics instruction related to cognitive demand in both their lessons and pen pal letters. These aspects illustrate a picture of the way in which preservice teachers are learning to teach mathematics. Heidi focused on cognitive aspects of student learning, while Nora focused on the emotional
aspects of student learning. A conclusion provides a description of similarities found between the approaches the participants used to teach mathematics.

Heidi focused on the students’ mathematical ability when planning and implementing her lessons and writing her pen pal letters. One example is in her response to an interview question, which asked her if she focused on the cognitive demand of mathematical tasks or mathematical questions when developing her lessons. She stated:

I think I definitely did but like I said according to student ability. So I figured not just all around the problem is high cognitive demand I feel while I was teaching it I feel like I was manipulating things on the spot. Like make it higher for some students.

This example shows Heidi predetermined the students’ abilities before implementing the lesson and then, in the moment of teaching, she would change the mathematical problem. For instance, if students were struggling with the problem, she would change the problem to an easier one, or, if they were able to do the original problem, provide an extension to the problem. Heidi’s method of teaching could be described as differentiating instruction for her students.

Another example of how Heidi used the students’ mathematics ability to design a mathematical task was in the letter writing exchange. Heidi explains:

I kind of like umm…saw her math ability with the very first problem and I kind of gave her an easier one and saw how she did with that and then based the other problems. She did very well with the first task. So then, like me and Krista made the next one more challenging.

The pen pal letter writing exchange began with the elementary students sending a filled-out questionnaire about their interests (e.g., favorite color, favorite candy) to the preservice teachers. The preservice teachers had no prior knowledge of their pen pals’ mathematical ability. It was up to the preservice teacher to determine how to write a
mathematical task that was based upon a Common Core State Standard and elicit a preselected level of cognitive demand. Heidi explained that she started with an easy mathematical task, and then, when the student completed the task correctly she began to pose more challenging mathematical tasks. This is an example of how Heidi used scaffolding techniques to write mathematical tasks for her pen pal.

Heidi felt it was important to start with easy problems and work toward more challenging ones. Another example comes from a worksheet she created for students on the topic of multiplication. Heidi used pre-made groups for students to place objects in to illustrate the concept of grouping for multiplication. She chose this representation for the topic because she felt it illustrated the concept of multiplication. As the worksheet progressed the pre-made groups disappeared, allowing the students to make their own groups that illustrated the multiplication problem.

Heidi was also concerned that students understood mathematics conceptually. Heidi explains, “I think it goes back to the conceptual, like we wanted to make sure that she was understanding. Like she could explain why she worked it out the way she did.” In the pen pal letters Heidi sent, she always asked her pen pal to provide a different way to solve the problem. Prior to the intervention Heidi describes her instruction as:

Like just give them the problem and not ask them to explain it. Not ask them to critique someone else’s work and tell them why it’s wrong. Just give them the problem and they would just do the procedure. Rather now I know that to understand how they are thinking and why they answered that way is probably the most important part about solving math problems.

Her response indicates she is open-minded to having students share their solution strategies, and mathematics should be taught in a non-procedural manner. This response also indicates that Heidi values a students’ explanation about how they solved a problem, more than just seeing a numerical response.
On the contrary, Nora was concerned with student outcomes. She taught her lessons with the goal of having the students know what they needed for the test. When asked if she focuses on the cognitive demand for mathematical tasks or mathematical questions when developing her lessons she replied, “No not really. I don’t focus on the cognitive demand. No, that’s not something I really thought about.” This statement reveals that the cognitive demand was not considered for her lessons or the pen pal letters. Nora made problems simple for her students and pen pal so they would get them correct. She wanted her students to feel successful with mathematics, and this meant answering mathematics problems correctly. Nora based instructional decisions on making the mathematics easy for the students.

Another example comes from her pen pal letters. She first wrote her mathematical tasks for her pen pal by only using words. Her pen pal would not respond and she assumed the problem was too hard for him. Instead of using words she drew pictures to illustrate a mathematics problem that she described as easy and was able to get a response from her pen pal.

Later in the interview Nora describes how she knows when students understand the mathematics. Nora states:

I would throw things in and I knew that they really understood it when they could answer those questions that came to my mind right away…because that means that they have a deeper understanding of just what I am saying or just what I explained.

Nora thought students demonstrated their understanding of mathematics when they could quickly respond to a question. She made up questions on the spot to keep students responding quickly. Nora interrupts students’ quick responses as a way of demonstrating understanding of the mathematical material.
Both participants were concerned with challenging the students academically. Heidi interpreted challenging as progressively creating mathematical tasks that increased cognitive demand. Nora interpreted challenging students as creating questions in the moment and students were able to answer them right away. For example, Nora states:

Because they were answering the questions that I already planned to ask. Like they have already got the concept that I planned to teach them and I wanted to extend it a little more so I would ask, like, oh what if I did this? What if I did that? And they were able to answer those questions too. So I was like they really get this. They are not just understanding this at a surface level. They get what I’m saying.

The example also reveals that Nora believes her students have a deeper level of understanding and that she is successful with her teaching methods. Heidi also created questions in the moment but she considered the individual students’ needs, unlike Nora who would ask questions to the whole group of students she was teaching. Interestingly, neither participant planned in advance to challenge students during their lessons. They had to think quickly about how to engage the students by posing questions, which they felt challenged the students. The pen pal letter writing exchange provided Heidi with the opportunity to think, plan, and consider how to pose a challenging mathematical task to her pen pal.

In conclusion, both participants indicated they want students to experience success with easier mathematical tasks before they move toward more cognitively demanding tasks. They felt their role, as the teacher, was to scaffold the students’ learning by building upon easy skills through less cognitively demanding mathematical tasks.
Social and emotional aspects of teaching

On several occasions the participants mentioned the need to know students and motivate them to learn mathematics. The pen pal letter writing exchange was an opportunity for the participants to know their pen pal without physically meeting them. This opportunity is just one instance of both participants taking into account student interests when designing mathematical tasks. Nora and Heidi both indicated that they cared about students as learners. In this section, the social and emotional aspects of teaching with a focus on student understanding are described to illustrate how the participants’ beliefs are implemented in their lessons and mathematical tasks.

Nora felt students need to have a teacher who supports and believes in them. When Nora was asked by the researcher, “What are the three most important characteristics of good mathematics teaching?” Her first response was “believing in your students because I wish somebody would have believed in me.” Nora tries to show students through her teaching and in her letters to her pen pal that she believes in them and cares, in order not to perpetuate the feelings she had when learning mathematics. For instance, she feels the students need to know the teacher is there to support them as they solve problems. Nora stresses the need to work as a team in the classroom and to solve problems together. In essence, Nora is trying to change the way she experienced mathematics instruction.

Nora also wanted her pen pal to feel like she was learning alongside him. For example, in this section, she describes the letter writing exchange where she and her pen pal are exchanging mathematics problems:

I loved it. I was glad that he was feeling like he was almost like – I’m going to give her some. Like he was feeling like it was equal. I would give him one and he would give me one. Which is the way I wanted him to feel, like
it was equal and I’m not just sending you math problems for you to solve because I am a teacher but you can give me some, too, because I still need the practice too.

It was very important to Nora that the students felt she was part of the community of learning. She wanted them to feel comfortable with her and with mathematics. This is another example of how she wanted to provide students with a different learning experience than the way she experienced mathematics.

Heidi believed she motivated her pen pal to respond to the mathematical tasks by including the student’s name and interests in the mathematical tasks she created for the letters. Heidi describes how she made a connection with her pen pal in the following response:

We tried to base on her interests too so that would make her excited to do it. She was very excited and then she would get the problems. After she would get the problem she would write underneath it and color so it seemed like she was excited because we would, like, use her name in it. I think making them interested is like definitely a big thing for us.

In essence, Heidi and her classmates were excited and motivated to write the letters when the pen pals included something personal in the letters such as drawings or sentences about their life. The motivation for the letter writing exchange was evident in the participants and pen pal students’ responses to each other. Heidi tried hard to find ways to motivate her pen pal and she valued this aspect of teaching. In another example from the interview, Heidi describes how a student from her practicum classroom took pride and ownership of her work.

On this occasion, Heidi explained how she knew she had a successful mathematics lesson. Following the lesson Heidi collected the student work for a class assignment. All of the students wanted their work back to show their parents. Heidi described the reaction of one student: “she was so excited to, like, take it home and she
was proud of her work so I feel like that is when you know when you have a successful lesson.” Heidi’s interpretation of success is different from Nora. Heidi interprets successful mathematics teaching as when students are excited and proud of their work. Nora interprets success as students arriving at the correct answer.

Heidi and Nora both agreed that motivating students and making learning fun through hands-on activities and manipulatives is important for teaching mathematics. Heidi considers students when planning lessons. For example, Heidi describes how she structures a lesson:

You are presenting them with a lot of information because that’s when you have to kind of teach the new, like, concepts. So I made sure to make that very explicit and like, not just ramble on because I know they would get overwhelmed. And definitely like modeling and think-a-louds. I think that was, like, the most important part of the structure because that really shows them, like, you are solving the problem and they get to see how they need to think about when they are solving it.

This excerpt indicates that Heidi thinks it is important to model the mathematical behavior she expects of her students. Manipulative use was recognized by both participants as a way to have students engaged and motivated to learn mathematics. Nora wishes she were taught mathematics by using manipulatives and seeks out opportunities to use them with her students in her practicum classroom. For instance, Nora felt the students had to know how to use the manipulatives in order to show the concept of place value. Nora and Heidi implemented their beliefs into mathematical tasks through hands-on activities and using manipulatives in lessons.

In conclusion, both participants took the time to create problems they felt their pen pals would be motivated to respond to. For example, Heidi would incorporate her pen pal’s name and her interests into the mathematical problem. Nora wanted her pen pal to feel he could send her mathematics problems too. Heidi and Nora wanted students to be
excited about mathematics and therefore cared about the way the students learn mathematics.

All in all, the past experiences with mathematics instruction that Heidi and Nora experienced has led them to want to change how they teach mathematics. Both participants want students to feel successful with solving mathematics problems. They did not want to have students struggle with mathematics. However, they both went about relieving the struggle in different ways. Both participants believed students should be taught mathematics through authentic and hands-on activities. The themes illustrate how deeply intertwined mathematical beliefs are when the participants present mathematics to students.

Heidi experienced significant belief changes in four of the seven beliefs. Evidence of her belief changes has been is seen in her interactions with her pen pal and designing her lesson plans. Nora’s prior experience with mathematics has led her to focus on motivating students to see mathematics differently. For instance, she wants students to see mathematics as a discipline that makes sense and can be applied to a student’s life. Overall, experiences in the intervention and practicum classroom have impacted how Heidi and Nora interrupt mathematics and mathematics instruction.

Preservice Teachers’ Knowledge of the Cognitive Demands of Mathematical Tasks

Research Question #3: Can elementary preservice teacher’s identify mathematical tasks as having high-level or low-level cognitive demands, and does this change after a 12-week intervention specifically focused on learning about the levels of cognitive demand and implementation of mathematical tasks in letter writing with third grade students? To answer this question, comparisons were made between the
treatment groups’ pre- and post-mathematical task sort scores and between the mathematical task sort scores of the treatment group and control group. The results of these comparisons are presented in the remainder of this section.

**Pre- and Post- Intervention Mathematical Task Sort**

The pre- and post-intervention mathematical task sort scores provide evidence of preservice teachers’ knowledge of cognitive demands prior to and following their participation in a 12-week intervention. Thirty-two preservice teachers participated in the pre- and post-intervention mathematical task sort. For each of the 16 tasks in the mathematical task sort, preservice teachers received 1 point for correctly classifying the task as high-level or low-level according to the Task Analysis Guide (Stein et al., 2000). The mathematical tasks classified as high-level include “doing mathematics” and “procedures with connections.” The mathematical tasks classified as low-level include “memorization” and “procedures without connections.” The highest possible score on the mathematical task sort is 16 points.

Scores on the pre-mathematical task sort ranged from 6 to 14, with a mean score of 10.22. Post-mathematical task sort scores ranged from 7 to 14, with a mean score of 11.66. Results of the Wilcoxon Signed-Rank tests for non-parametric, paired data indicate that the increase of 1.44 between the means of the pre- and post-mathematical task sorts was significant (z = -2.694; p = 0.007 [two-tailed]). These results suggest that for the treatment group of preservice teachers, their knowledge of cognitive demands of mathematical tasks increased following the 12-week intervention (see Table 4-9). The effect of the increase will be described later in this section, and the analysis of the intervention will identify events that might have provided opportunities for this learning to occur.
Comparing the Treatment Group to the Control Group

The treatment groups’ pre- and post-mathematical task sort scores were compared to the task sort scores of the control group. The results indicate whether the treatment group had a greater knowledge of the cognitive demands of mathematical tasks at the end of the study than the control group who did not participate in the intervention.

The pre-mathematical task sort scores from the 34 control group preservice teachers ranged from 5 to 15 points, with a mean of 9.71. The post-mathematical task sort scores from the 34 control group preservice teachers ranged from 5 to 14, with a mean of 10.22. Results of the Mann-Whitney U test comparing the mathematical task sort scores of the treatment group and the control group are listed in Table 4-10. The Mann-Whitney U test was conducted to determine if there were differences in pre-mathematical task sort scores, post-mathematical task sort scores, and change scores between the treatment and control groups. Distributions of the pre-mathematical task sort scores, post-mathematical task sort scores, and change scores for the treatment and control group were similar, as assessed by visual inspection.

The median pre-mathematical task sort scores were not statistically significantly different between the treatment (Mdn = 10) and control (Mdn = 10), U = 485.5, z = -.760, p = .447. The median change scores for the mathematical task sorts were not statistically significantly different between the treatment (Mdn = 0) and control (Mdn =2),
U = 450.5, z = -1.207, p = 0.228. The median post-mathematical task sort scores were statistically significantly different between the treatment (Mdn = 12) and control (Mdn = 11), U = 353, z = -2.481, p = .013.

The results indicate that both groups were relatively equivalent on the pre-mathematical task sort, indicating both groups had prior knowledge of the cognitive demand of mathematical tasks. Additionally, when change scores were used as the dependent variable, there was no significant difference between the groups. However, the results indicate that the post-mathematical task sort scores for the treatment group were significantly higher than the control group. The intervention, which focused on learning about the cognitive demand of mathematical tasks, appears to have provided preservice teachers with knowledge of mathematical tasks.

Table 4-10. Comparison of Pre-Mathematical Task Sort Scores of Treatment and Control Groups

<table>
<thead>
<tr>
<th></th>
<th>Mean (SD)</th>
<th>Mean Difference vs. Control Group</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-Mathematical Task</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sort:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment Group (n = 32)</td>
<td>10.22 (1.896)</td>
<td>0.51</td>
</tr>
<tr>
<td>Control Group (n = 34)</td>
<td>9.71 (2.223)</td>
<td>NA</td>
</tr>
<tr>
<td><strong>Post-Mathematical Task</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sort:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment Group (n = 32)</td>
<td>11.66 (1.977)</td>
<td>1.22*</td>
</tr>
<tr>
<td>Control Group (n = 34)</td>
<td>10.44 (2.205)</td>
<td></td>
</tr>
</tbody>
</table>

*Results are significant at p < .05 [two-tailed]

**Descriptive Data on Preservice Teachers’ Task Sort Responses**

A closer analysis is provided to understand the significance in the pre- and post-mathematical task sort scores for the treatment group. Specifically, information is provided on whether gains in pre- and post-mathematical task sort scores could be attributed to an improvement in participants’ ability to identify high-level and low-level
mathematical tasks. Table 4-11 provides data to illustrate the nature of changes in participants’ mathematical task sort responses over time.

Participants struggled with identification of high-level tasks. The category with the most incorrect responses was “Doing Mathematics” (DM). Participants had the same percentage of incorrect classifications for both the pre- and post-mathematical task sort. Of the 160 instances in which DM tasks were classified on the task sort (i.e., 5 DM per participant times 32 participants), 118 incorrect classifications (74%) occurred on both the pre- and post-mathematical task sort. As shown in Table 4-11, all of the participants classified at least one DM task incorrectly.

The results for the “Procedures with Connections” (PWC) tasks were slightly better than the DM tasks. There were five PWC tasks on the task sort creating 160 instances. On the pre-mathematical task sort, participants incorrectly classified 98 times out of the 160 instances (61%) and 84 times out of the 160 instances (53%) for the post-mathematical task sort. This result indicates that participants held prior knowledge of PWC tasks. It is possible that the intervention helped the participants recognize features of PWC tasks. Similar to the DM task, all of the participants classified at least one PWC task incorrectly. Overall, the higher-level tasks were more difficult for participants to classify. Participants incorrectly classified 63% of the higher-level tasks for the post-mathematical task sort. Hence, the participants’ ability to correctly identify high-level mathematical tasks did not contribute to the increase in the treatment group participants’ task sort scores. Participants had difficulty correctly classifying DM tasks and only showed a slight improvement with classifying PWC tasks.
Table 4-11 illustrates that the treatment group participants had better success classifying low-level tasks. On the post-mathematical task sort, only 48% of the low-level tasks were classified incorrectly. There were 128 occurrences of the “Procedures without Connections” (PWOC) tasks (i.e., 4 tasks times 32 participants). On the pre-mathematical task sort PWOC tasks were incorrectly classified in 101 of the 128 occurrences (79%). On the post-mathematical task sort, 83 tasks (65%) were incorrectly classified, which indicates an improvement in the participants’ ability to classify PWOC tasks. Table 4-11 shows that only one teacher was correctly able to classify all PWOC on the post-mathematical task sort. Two “Memorization” (MEM) tasks were on the task sort creating 64 instances (i.e., 2 tasks times 32 participants) where MEM tasks were classified. Participants incorrectly classified 22 instances out of 64 (34%) on the pre-mathematical task sort and 10 times out of 64 (16%) on the post-mathematical task sort. Twelve participants incorrectly classified at least one MEM task on the pre-mathematical task sort, and eight participants did so on the post-mathematical task sort. These results suggest that the participants exhibited a slightly improved ability to classify low-level tasks on the post-mathematical task sort.

The descriptive data provide support to validate the increase in the treatment groups’ post-mathematical task sort scores were not effected by repeated measures (i.e., the scores did not improve simply because the participants were completing the mathematical task sort for the second time) and the participants were not learning the “correct answers” for the mathematical task sort in the intervention. The treatment group improved in their ability to correctly classify mathematical tasks with low-level cognitive demand.
In summary, participants in the treatment group had a significant change in Beliefs 5 and 7, compared to the control group. The control group did not have a significant change in any of the seven beliefs, compared to the treatment group. However, all participants in the study had a belief change for at least one of the seven beliefs. The interview data from Heidi and Nora revealed that they want to teach mathematics differently from how they were taught. There are similarities and differences between the instructional approaches utilized by Heidi and Nora. The analyses of the mathematical task sort suggest that PSTs struggle with identifying high-level cognitively demanding tasks. Chapter 5 will discuss the conclusions that can be made from the results of the statistical analyses and interviews presented in this chapter.

<table>
<thead>
<tr>
<th>Level of Cognitive Demand</th>
<th># of Tasks</th>
<th>Total # of classifications</th>
<th># of incorrect classifications</th>
<th># of teachers incorrectly classifying a task at that level</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-Level:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doing Mathematics</td>
<td>5</td>
<td>160</td>
<td>118</td>
<td>32</td>
</tr>
<tr>
<td>Procedures with Connections</td>
<td>5</td>
<td>160</td>
<td>98</td>
<td>32</td>
</tr>
<tr>
<td>Low-Level:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Procedures without Connections</td>
<td>4</td>
<td>128</td>
<td>101</td>
<td>32</td>
</tr>
<tr>
<td>Memorization</td>
<td>2</td>
<td>64</td>
<td>22</td>
<td>12</td>
</tr>
</tbody>
</table>

*Total number of classifications is determined by multiplying the number of tasks at that level by 32 (the number of preservice teachers)*
CHAPTER 5
DISCUSSION

Importance of this study

The results from this study are important to PST education, specifically the elementary mathematics methods course, because in the treatment group there appears to be a change in beliefs that are aligned with reform-based mathematics instruction and an increase of knowledge about features of mathematical tasks. The treatment group had significant changes on two beliefs that are included in the broad category titled “Beliefs About Children’s (Students’) Learning and Doing Mathematics.” Specifically, these beliefs include Belief 5: Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect. They also include Belief 7: During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible. Additionally, the results for the study indicate that the treatment group PSTs could accurately identify low-level mathematical tasks after the 12-week intervention and had a significant increase in overall ability to identify mathematical tasks by low- and high-level cognitive demand.

This study contributes to research on elementary preservice teacher mathematics education by finding evidence through a belief survey, mathematical task sort, and two interviews, which provide indication of change in mathematics instruction beliefs and knowledge of the cognitive demand of mathematical tasks following participation in a 12-week intervention. The researcher designed and facilitated a series of learning experiences that allowed PSTs to have the opportunity to do mathematics,
analyze mathematical tasks, create mathematical tasks, and pose mathematical tasks through a letter writing exchange, in order to challenge PSTs’ beliefs about mathematics instruction and develop aspects of mathematical-task knowledge (Crespo, 2003).

In Chapter 1, the argument was presented that in order for students to learn mathematics at the desired level, teachers must be prepared to use mathematical tasks that will allow students to go deeply into the mathematical content (Stein et al., 2009). Research has shown that when teachers are provided with opportunities to learn and focus on the cognitive demands of mathematical tasks, they experienced increases in student learning and engagement during mathematics instruction (Boston & Smith, 2011; Henningsen & Stein, 1997). Therefore, PST education serves as the stage where PSTs have opportunities to develop and experiment with posing mathematical tasks through authentic activities with students (Crespo, 2003).

The purpose of this study was to determine the extent of such development, and included a 12-week intervention which focused on learning about the cognitive demand of mathematical tasks and posing mathematical tasks through a letter writing exchange. The study evaluated the effects of the levels of cognitive demand for mathematical tasks on PST’s and their beliefs about mathematics instruction. In order to portray this phenomena, the study utilized separate pre- and post-assessments for the knowledge of levels of cognitive demand for mathematical tasks and the beliefs about mathematics instruction, along with two participant interviews from the treatment group.

The results from this study provide evidence that following the 12-week intervention the treatment group had significant changes in 2 of the 7 beliefs. Additionally, the treatment group showed growth in their knowledge about levels of
cognitive demand of mathematical tasks. At the end of the intervention, the treatment group significantly increased: 1) their knowledge of the cognitive demands of mathematical tasks (i.e., their task sort scores); 2) their knowledge of Belief 5: Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect (i.e., the change score for the belief); and 3) their knowledge about Belief 7: During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible (i.e., the change score for the belief).

The change scores for Beliefs 1, 2, 3, 4, and 6 did not increase significantly when comparing the treatment and the control groups’ actual versus predicted counts. However, when comparing the individual belief changes for each group, the treatment group had more participants who experienced belief changes for at least three of the seven beliefs than the control group. The task sort scores for the treatment group were significant when comparing pre- and post-scores. A comparison of the control and treatment group revealed a significant difference in the post-mathematical task sort scores. Therefore, the treatment groups’ knowledge of mathematical tasks and two beliefs changed significantly compared to the control group, due to the participation in the 12-week intervention focused on mathematical tasks.

According to the National Research Council (2001), approaches to teaching have little effect on student learning. Instead, “successful interaction among three elements: teachers’ knowledge and the use of mathematical content, teachers’ attention to and handling of the students, and students’ engagement in and use of mathematical tasks”
are the keys to successful student learning (National Research Council, 2001, p.111).
Although this study did not measure student achievement or teachers' mathematical content knowledge, the growth in PSTs’ knowledge for the level of cognitive demand of mathematical tasks is the initial step in the MTF. This growth is an important first step in recognizing the potential a mathematical task has in eliciting thinking about mathematics from a student. Research studies that have focused on engaging both preservice and inservice teachers in learning about the level of cognitive demand have indicated growth in the type of mathematical tasks posed by participants (Arbaugh & Brown, 2005; Boston & Smith, 2011; Kosko et al., 2010; Silver & Stein, 1996). Researchers have also found that there are cases where a teacher elects to use a mathematical task with features of high cognitive demand and in the course of implementing the mathematical task, pedagogical decisions are made by the teacher which lower the overall cognitive demand of the mathematical task (Arbaugh & Brown, 2005; Silver & Stein, 1996). The instructional decisions teachers make effect the success of implementing high cognitive demand mathematical tasks. Teachers’ beliefs about mathematics instruction play an important role in the presentation of mathematics (Remillard & Bryans, 2004). According to Ernest (1989), teachers' beliefs about mathematics instruction depend upon the successful implementation of reform-based mathematics instruction. Therefore, based on evidence from prior research about the positive effects learning about the cognitive demand of mathematical tasks has on mathematics instruction, the results from this study show the potential to help create opportunities for learning in elementary preservice teacher mathematics education courses.
The intervention appears to have provided PSTs with the opportunity to participate in a learning environment where they solved and discussed mathematical tasks together and acquired the tools necessary to identify the cognitive demand of mathematical tasks (Brown et al., 1989, Gee, 2008). The following section will provide an explanation for the effectiveness of the intervention in contributing to PSTs’ belief changes and knowledge of the level of cognitive demand of mathematical tasks as identified by the results of this investigation.

**Explanations for Results**

This section offers explanations for the results obtained in the study. How did the intervention affect PSTs' beliefs about mathematics instruction? How did the two PSTs selected for interviews use mathematical tasks? What types of mathematical tasks did the PSTs have a difficult time identifying? Potential explanations for these questions are presented in the remainder of this section.

**The Intervention Provided PSTs with the Opportunity to Do Mathematics**

The intervention for the study was designed to provide the PSTs with opportunities to work together on mathematical tasks and participate in an authentic letter writing exchange. During the intervention, the cognitive demand of mathematical tasks was used as a focal point for PSTs to consider the prospects mathematical tasks provide students to learn mathematics. Throughout the 12-week intervention, PST’s were presented with a variety of opportunities to engage with mathematical tasks. Aspects of the intervention provided PSTs with a chance to develop mathematical-task knowledge (Chapman, 2013) and challenge their beliefs about mathematics instruction.

The intervention was designed and implemented in ways consistent with the situative learning theory (Brown et al., 1989; Cobb & Bowers, 1999; Gee, 2008; Lave,
For instance, the situative learning theory places an emphasis on the activities that take place within the community to help the individual acquire knowledge (Cobb & Bowers, 1999). Based upon the situative learning theory, in order for learning to occur the following components need to be included in the design of the learning environment: establish classroom norms, identify the role of the teacher educator as the facilitator of the group, provide opportunities to socially construct the meaning of mathematics with peers, and engage in activities which positioned PSTs to acquire the tools to learn about the cognitive demand of mathematical tasks. These four components were included in the intervention.

First, to begin building a classroom-learning environment the instructor and participants created course expectations. The initial activity in Session 1 of the intervention had the participants work together in groups of four to create expectations for the course, each other, and the instructor. Each group had the opportunity to share their expectations with the class. The instructor also shared the expectations she had for the participants. During Session 2, the instructor facilitated a discussion about what it means to teach student-centered mathematics. Many participants shared their own experiences with learning mathematics. For example one participant shared, “I would also find myself giving up halfway through the problem and waiting for the teacher to tell me how to figure it out the ‘right’ way (which he or she was going to do no matter what).”

Following the discussion, the participants were asked how to convert an improper fraction to a mixed number. In small groups, the participants were asked to share their methods for converting. The objective for this activity was to find out how the participants were converting the improper fraction to a mixed number. Were they
recalling a procedure or did they represent an improper fraction with pictures? The instructor wanted the participants to consider how they were taught mathematics. Next, the participants watched a video clip of a child struggling to remember the procedure for converting a mixed number to an improper fraction. The clip was intended to challenge or confirm their beliefs about mathematics teaching and learning. In this same session, the participants were introduced to the Standards for Mathematical Practice. This introduction included background knowledge about the Standards for Mathematical Practice and a video where students used the Standards for Mathematical Practice in their learning of mathematics. This sequence of activities assisted in the creation of classroom norms.

From the start of the intervention, the instructor made it clear to the participants that her role was to facilitate learning and the role of the participants was to engage in learning about mathematics through mathematical tasks (Szydlik et al., 2003). Previous studies (e.g., Szydlik et al., 2003; van den Kieboom & Magiera, 2010) made the role of the instructor explicit in the establishment of creating a learning environment where preservice teachers engaged in problem-solving and generating mathematical explanations. The opportunities to participate in solving mathematical tasks in groups and discussing solution strategies has been recommended as an effective practice to help preservice and inservice teachers become comfortable using mathematical tasks during instruction (Remilard & Bryans, 2004). The mathematical tasks used in the intervention were chosen based upon mathematical content and certain features of cognitive demand.
An explanation about the mathematical tasks and the ways in which the tasks were used in the intervention is provided to form an understanding of what events may have contributed to the results in the study. In Session 3, participants were first introduced to mathematical tasks that involve fractions. The participants were asked to work in small groups with four fraction mathematical tasks that were taken from a National Assessment of Education Progress 2009 fourth grade test. After the participants worked on the tasks, they were asked to consider three questions: 1) What mathematical practices did you use to solve these tasks? 2) How are these two tasks similar? How are they different?; 3) What prior knowledge did students need to know in order to answer the questions? The participants were given the opportunity to discuss the results of each task and address the questions. Following this activity the participants were asked to work on a Brownie Problem (Flores & Klein, 2005), which can be classified as a fair-sharing task. The participants solved the task as a group and then shared their results with the group. This activity provided the participants with the opportunity to see how their peers represented the solution to the problem in different ways.

For Session 4 and 9, participants were asked to work on mathematical tasks for the Inside Mathematics (Noyce Foundation, 2012). The tasks selected included a fifth grade fraction task that asked students to consider where to place fractions on a number line and compare fractions using fraction benchmarks, and a decimal task. Both tasks were given to participants to solve and discuss on respective occasions. Following the discussion, participants were given student work that showed how students interpreted the mathematical task. First the participants were asked to use a rubric to
score the students work. Second, the participants were asked questions that positioned them to analyze the student work responses and group them in categories. For example, one of the questions asked was: “What does this tell you about students’ understanding of fractions?” Discussions arose from the PSTs about how students would respond to the tasks and what decisions teachers need to make to help the student understand the mathematics needed to answer the task.

In Session 6 a multiplication fraction task was presented to PSTs. This task posed a dilemma for some participants because they were only familiar with the standard algorithm to multiply fractions. The task asked the participants to use pattern blocks to represent the multiplication and provide a reason for why the standard algorithm for fraction multiplication works.

For Session 10, the participants were placed in groups and asked to work on a measurement task that involved finding measurements of Barbie and then were assigned to construct a real-life model of Barbie. This task allowed the participants to decide how to act in order to develop a real-life model of Barbie. Several design features for the intervention focused on developing mathematical-task knowledge through doing the mathematic tasks, discussing results, analyzing student work from the mathematical task, and discussing features for the mathematical task that would elicit cognitive demand.

The participants were formally introduced to the levels of cognitive demand during the second part of Session 4. The first activity was to complete two fraction tasks, selected by the instructor, which had features of low- and high-level cognitive demand. The participants were asked to identify the strategies they could use to solve
the two tasks, identify the mathematics concepts, and make a list of similarities and differences between the tasks. The instructor provided sheets of poster paper for the participant responses for strategies, mathematics concepts, and similarities and differences. The poster papers were all combined into one list. Afterwards, the participants were asked to classify the two tasks as low- or high-cognitive demand and develop reasons for their classification. This activity was the participant’s first experience in the intervention with low- and high-level cognitive demand. In preparation for Session 5, participants were asked to review the third grade sample questions for the Partnership for Assessment of Readiness for College and Careers assessment items (e.g., Flower Gardens, Fractions on the Number Line, and Mariana’s Fractions) by evaluating the Mathematical Practices used to solve the task and the level of cognitive demand for the task. During class, the participants engaged in a discussion based upon their findings for mathematical tasks. A lot of questions arose from the participants about how to adequately prepare students for mathematical tasks. Many of the participants felt that if they were not able to answer the mathematical tasks on their own, their students would be unable to answer them as well. Following this discussion the preservice teachers completed a middle school mathematical task sort. The participants were asked to sort the middle school tasks into categories of their choice and to describe the categories on their paper. Then the participants discussed with their group the categories they created. Next, the instructor passed out a table with the four categories that represented the level of cognitive demand. The participants were asked to sort the tasks into one of the four categories. Afterwards, participants were asked to share one task from each of the categories, identify features that allowed for
classification to that category, and describe what adaptations could be made to raise the cognitive demand (if the task was classified as low-cognitive demand).

The conclusion for the middle school task sort activity ended with a discussion about the features of mathematical tasks and the classification of cognitive demand for the tasks. For instance, will a task with a real life application always be classified as high-level cognitive demand? The goal for the discussion was for participants to realize that the cognitive demand in written form had the potential to complete the mathematical task. When the mathematical task was used in context, the potential cognitive demand of the mathematical task may or may not be maintained based upon factors in the classroom. A limitation to the mathematical task sort is the lack of an opportunity to see how students interact with the mathematical task.

Preservice teacher education faces the challenge provided to PSTs when they teach their own students (Norton & Kastberg, 2012). Therefore, this study sought to provide PSTs with an authentic opportunity, which was participating in a letter writing exchange. The letter writing exchange facilitated the development of mathematical tasks and allowed the PSTs to pose the tasks in the pen pal letters. The letter writing covered “three important aspects of mathematics teaching practice: posing tasks, analyzing pupils’ work, and responding to pupils’ ideas” (Crespo, 2003, p.246). On three occasions, the participants had the opportunity to write letters to third grade students. These opportunities served as a time for participants to experiment with posing mathematical tasks. The participants had the opportunity to write about anything they wanted to in the letters. For instance, some of the participants chose to share their experience in college, such as attending sporting events; others included information
about where they grew up. The only requirement was to include a mathematical task based upon a pre-determined Common Core State Standard. The instructor selected the standard to reflect the content that the students were learning at their school.

The participants had the opportunity to write their first pen pal letter during Session 6. For the letter they were asked to create a mathematical task using a second grade Common Core State Standard for partitioning circles and rectangles. The participants had the freedom to create the mathematical task. The instructor suggested the use of the Task Analysis Guide to identify features of the mathematical tasks that exhibited the cognitive demand they desired from the task. Participants had the opportunity to write their second pen pal letter during Session 9. The mathematical task focused on a second grade standard that dealt with place value. For instance, if you have the number 897, explain what the 8, 9, and 7 represent.

The third pen pal letter was written in Session 11 and focused on a second grade standard for the foundations of multiplication. One of the intentions of the letter writing exchange was to make a connection between the university coursework and the actual practice of teaching (Kagan, 1992). Prior research studies recommend structured experiences that allow PSTs the opportunity to see how students’ think about mathematics (Kagan, 1992; Philipp et al., 2002; Philipp et al., 2007).

All in all, the intervention provided learning experiences that focused on creating classroom norms, establishing roles of the instructor and participants, structuring activities where mathematical task knowledge was developed through social interactions with peers, and acquiring the tools to learn about the cognitive demand of mathematical tasks. The activities, which occurred during the intervention, are the unit
of analysis for this study (Rogoff, 2008). Situating the PSTs in a classroom environment, which focused on doing mathematical tasks, considering the cognitive demand a particular task elicited in students, and posing mathematical tasks to the student, culminates in an authentic activity which develops knowledge for teaching mathematics (Borko & Putnam, 2000; Crespo, 2003). Increases in participants’ ability to identify the cognitive demand of mathematical tasks and changes in mathematics instruction beliefs suggest that participants benefited from the opportunities presented to them in the intervention. Exposure to mathematical tasks in the intervention also contributed to changes in the treatment group PSTs’ beliefs about mathematics instruction.

Belief Changes

According to Ernest (1989), teachers’ instructional beliefs about mathematics need to be recognized in order for mathematical tasks to be properly implemented. With this in mind, the intervention was focused on providing opportunities, which challenged PSTs beliefs about mathematics instruction by using the situative perspective to inspire the design for the intervention and provide an authentic letter writing exchange experience. The treatment group had experiences in the intervention that positioned them to solve mathematical tasks together, discuss mathematics, and share solutions (Szydlik et al., 2003). These experiences allowed participants to experience mathematics as a sense-making discipline (Szydlik et al., 2003; van den Kieboom & Magiera, 2010).

The quantitative data for beliefs indicate that more PSTs in the treatment group increased their belief scores on at least 3 beliefs, compared to the control group, who experienced less change in beliefs. One possible explanation for the score increase can be attributed to the intervention and specifically the letter writing exchange. PSTs in the
treatment group had the opportunity to experiment with posing mathematical tasks through a letter writing exchange. This opportunity could have ignited a change in Belief 5 because PSTs had the chance to build a relationship with a pen pal, write mathematical tasks for the student, and respond to the students’ answer (Bahr et al., 2013). The letter writing exchange served as an important tool in promoting change in PSTs’ beliefs. Philipp et al. (2002) found significant belief changes for PSTs who worked exclusively for 3 sessions with a third grade student on mathematical tasks related to place value. All of the participants had the opportunity to work with students during the study, due to the practicum placement, which was a component of the third semester in the elementary teacher preparation program.

The control and treatment group both shared a common field experience during the 12-week study. Both groups were assigned to a K-5 classroom at the same school for one day a week. All participants had the same assignment of creating and implementing two mathematics lessons to a group of students. The difference between the two groups who contributed to a significant belief change for Beliefs 5 and 7 was the letter writing exchange that the treatment group took part in. The letter writing exchange contributed to a significant belief change for Belief 5 due to the nature of the authentic activity. Researchers Philip et al. (2007), reported similar findings for the “Children’s Mathematical Thinking Experiences – Live” (CMTE – L) group of PSTs. The researchers intentionally focused the CMTE – L participants to see that “children understood mathematics they have not been formally taught” by providing PSTs with preselected mathematical tasks to implement with an individual child (p. 446). The letter
writing exchange incorporated the construction of a mathematical task and provided PSTs with the opportunity to see how the student responded to the task.

The PSTs in the treatment group did not have an opportunity to instruct students, nor did they have prior knowledge of the students' mathematical ability. The letter writing exchange was an opportunity for PSTs to experiment with posing mathematical tasks (Crespo, 2003). In essence, the letter writing exchange supported the change in Belief 5. Philip et al. (2007) found that the PSTs who were in the CMTE treatments experienced a larger increase in change of beliefs for both Belief 5 and 7 than the PSTs who were in the “Mathematical Observation and Reflection Experiences” (MORE) treatments. The MORE treatments had PSTs observe teachers who were either reform-oriented or located in a classroom convenient to the college campus. Results from both studies indicate that experiences where PSTs implement mathematical tasks with children and have the opportunity to consider students’ mathematical thinking are more likely to experience changes in Beliefs 5 and 7 than PSTs who are only observing mathematics instruction. Similar results for Beliefs 5 and 7 were recorded by Bahr et al. (2013). In their study, PSTs who were simultaneously enrolled in an elementary mathematics methods course and attended a field experience at a local public school experienced significant changes in Beliefs 5 and 7 due to the direct application of knowledge they were taught in their university course. Consequently, both studies, Philipp et al. (2007) and Bahr et al. (2013), experienced similar results for PSTs who had opportunities to work with children and engage children in learning mathematics. These results align with findings from this study in which PSTs had an opportunity to see how students learn.
The qualitative data provides evidence of the sophistication of beliefs held by Heidi and Nora. For instance, Heidi describes her desire to make mathematics meaningful to students through real-life mathematical tasks. She was able to teach a group of students during her practicum experience and then use this same knowledge about teaching and apply it to constructing a mathematical task for her pen pal. According to Ernest (1989), knowledge of organization and management of teaching mathematics is acquired through experimentation. Heidi had several opportunities to experiment with teaching and posing mathematical tasks, which contributed to her development of pedagogical content knowledge.

PSTs experimented with writing and posing mathematical tasks. They wanted to get a response from their pen pal and the task set-up did not always provide the response they were expecting. Therefore, they had to structure the task differently the next time they wrote the letter. The tasks from Heidi and Nora are shown in Appendix J. Learning how to present tasks to students was a challenge because the PSTs did not have the opportunity to respond to students in the moment and did not have prior knowledge of the students’ mathematical ability. On the other hand, this challenge provided time for PSTs to consider how to respond to their pen pal, which is not always possible in a live classroom setting (Crespo, 2003). Additionally, PSTs had to experiment with methods to learn more about their pen pals’ mathematical ability. The practicum and letter writing exchange experiences provided different contexts for PSTs to experience how students learn mathematics.

In the interviews there was conflicting evidence about what it meant for a student to be challenged during mathematics instruction. For instance, on one hand, when Nora
described how she structured a lesson, she stated that students needed to be challenged and she was coming up with questions to do this. On the other hand, she wanted students to experience success and therefore she made it easy for them by giving them something she knew they would get so the students would be set up for success. A possible explanation for this occurrence can be explained by Philipp et al. (2007), “Because many PSTs have little mathematical experience with children, they are initially able to project only their own, too often negative, mathematical experiences onto those of children, with the result that they avoid placing children in challenging situations (e.g., never asking children to solve a problem before they have been shown how to do so)” (p. 441). It is promising that Nora recognizes the need to challenge students. However, her past experiences with learning mathematics strongly effect the experiences she provides her students with, and she does not want her students to feel the way she did when learning mathematics (e.g., frustrated, upset). Heidi provided similar explanations with regards to challenging students and having them feel successful.

One difference between the way Nora and Heidi presented mathematics is that Heidi used scaffolding in her mathematics instruction by first using easy mathematical problems and then progressing to challenging problems, depending on the students’ mathematical ability. She also did the same thing when constructing and posing the mathematical tasks in the letter writing exchange. Crespo (2003) found that the types of problems created and posed by PSTs in a letter writing exchange and related questions for the problem changed over time. The participants in her study contributed these changes to their exposure to mathematical tasks in their university course. Similarity,
Heidi felt the intervention contributed to her learning more about mathematics concepts because of all the hands-on activities. Heidi had significant changes on 4 of 7 beliefs measured in this study. These reported changes for Heidi are significant as we consider the difficulty researchers have reported in changing PSTs’ beliefs about mathematics instruction (Crespo, 2003; Philipp et al., 2007; Vacc & Bright, 1999).

A finding from the interviews that was underlying the way both participants interacted with their students (both practicum and pen pal) was an element of caring. This study recognized the fact that PSTs care about students (Philipp et al., 2007). The notion of caring about students was present in the mathematics instruction presented by PSTs to students. For example, Nora cared about the students’ feelings about mathematics and did not want them to be embarrassed if they did not know the answer. She also wanted the students to see that mathematics was fun. Nora was interested in the emotional outcome of learning mathematics and did not focus on the mathematical content.

In contrast, Heidi was concerned with not overwhelming students with new information and therefore implemented mathematical tasks in a learning progression. Heidi felt the gradual increase in cognitive demand was the best way to reach all students. The Circles of Caring model (Philipp et al., 2007) was used to hook PSTs into learning more about mathematics. In Heidi’s case, there is evidence that it worked. However, for Nora her own beliefs’ about mathematics contributed to her lack of pedagogical development. Ernest (1989) suggests “The argument is that such conceptions have a powerful impact on teaching through such processes as the selection of content and emphasis, styles of teaching, and modes of learning” (p.20).
Nora would probably benefit from another semester of a similar mathematics course paired with a practicum experience, in order for her to continue to teach differently than the way she herself was taught (Nicol, 1999). Hence, this intervention might have more of an impact if it was introduced earlier in the elementary education program.

A combination of teaching mathematics lessons and focusing on one students’ mathematical thinking during the letter writing exchange supported PSTs views of mathematics instruction. An analysis of the two PSTs’ interviews provides evidence that both PSTs were trying to teach mathematics in a way that was opposite from how they experienced mathematics instruction. This fact goes against the literature that argues teachers are more likely to teach mathematics the way they have been taught, and perpetuates the belief that mathematics is a set of rules and procedures with no connections (Ball, 1988; Cady et al., 2006; Kagan, 1992). Although it is clear that their beliefs about mathematics instruction influence the experiences they provide to their practicum students and pen pal, there is hope that these two PSTs are developing a vision for reform-based mathematics instruction (Nicol, 1999). Explanations for the increases in participants’ task sort scores will be offered in the section that follows.

**Increased awareness of level of cognitive demand for mathematical tasks**

One explanation for the significant increases in the treatment groups’ task sort scores is their experience with analyzing features for mathematical tasks presented during the course of the intervention. Additionally, the treatment group had the opportunity to write, pose, and receive a response from a student for mathematical tasks. The treatment groups’ task scores increased from the pre- and post-mathematical task sort, and were significantly higher following their participation in the intervention than the scores of the control group who did not participate in the
intervention. Data from pre- and post-mathematical task sort scores by categories of cognitive demand reveal that participants in the treatment group had the highest accuracy on tasks, which had the potential for eliciting low levels of cognitive demand. Osana et al. (2006) studied a similar population of elementary preservice teachers and had the same result for the classification of low-level cognitive demand mathematical tasks.

Another significant finding was, the accuracy to classify a task as eliciting high levels (e.g., procedures with connections, doing mathematics) of cognitive demand decreased for participants in the treatment group. A similar result was found in the literature (Osana et al., 2006; Kosko et al., 2010). Osana and colleagues (2006) theorized that the result of inaccurate high-level classifications was due to the participants not having opportunities to work with children because they lacked knowledge of children’s’ mathematical thinking. This study did have that component and the participants experienced the same result. Perhaps an explanation for the inaccuracy of classification for high-level tasks is due to lack of exposure to high-level tasks in the PSTs K-12 academic schooling (Crespo, 2003).

The letter writing exchange was intended to support the PSTs in developing their understanding of the levels of cognitive demand. For instance, each PST completed a table that allowed them to hypothesize about the level of cognitive demand for the mathematical task they created and provide evidence for this decision prior to sending the pen pal letter (Norton & Kastberg, 2011). After the task came back, they were asked to re-evaluate the level of cognitive demand. An analysis of whether or not the mathematical task created by the PST was correctly identified by cognitive demand was
not completed since this form served as a reflective tool. However, the researcher did look over the tables after every letter-writing occasion to see how the PSTs classified the mathematical tasks. The two participants who participated in interviews, both stated that they posed challenging tasks to their pen pals. The tasks from all three occasions for each participant can be seen in Appendix J. An extension of this study would be to analyze the mathematical tasks for features representative of the levels of cognitive demand and compare the classifications made by PSTs.

PSTs in the treatment group were exposed to a variety of mathematical tasks during the 12-week intervention. The comparison of task sort scores for the two groups reveals that the treatment group scores were significant after the 12-week intervention. Hence, exposing PSTs to mathematical tasks, having them discuss the mathematical task, do the mathematical task, and create their own mathematical tasks led to increases in knowledge for identifying mathematical tasks by low- and high-levels of cognitive demand.

**Effectiveness of the intervention**

The effectiveness of the intervention in producing changes in PSTs knowledge of cognitive demand for mathematical tasks and changes in Belief 5 and 7 is notable given the short amount of time in which the intervention occurred. The focus of mathematical tasks and the letter writing exchange in the intervention can be credited with contributing to those changes. The intervention provided exposure to the level of cognitive demand of mathematical tasks (Arbaugh & Brown, 2005; Boston & Smith, 2011; Silver & Stein, 1996), group participation in performing mathematics tasks (Crespo, 2003; Nicol, 1999; Szydlik et al., 2003), and experimentation with writing and posing mathematical tasks (Crespo, 2003; Norton & Kastberg, 2012). Crespo (2003)
suggests a lot of attention has been given to the ability of PSTs to solve mathematical
tasks; however little attention has been given to their ability to construct and pose
mathematical tasks to students. Based upon the research about mathematical tasks,
the researcher felt it was important for the PSTs to consider the different features of
mathematical tasks by examining mathematical tasks in curriculum materials and ones
that serve as assessment items, and then discussing the aspects of mathematical
tasks.

This study demonstrates that PSTs developed aspects of mathematical task
knowledge as described by Chapman (2013). This knowledge is related to the
mathematical knowledge needed for teaching as defined by Hill et al. (2008). Hill et al.
(2008) who used an egg metaphor to represent the knowledge a teacher needs to teach
mathematics. Within the egg there exist two sides, which are divided into sections. The
side that relates to this study is Pedagogical Content Knowledge. Within Pedagogical
Content Knowledge, the section that closely relates to the knowledge participants
acquired through the intervention is the Knowledge of Content and Students (KCS).
According to Hill et al. (2008), KCS is defined as “content knowledge intertwined with
knowledge of how students think about, know, or learn this particular content” (p. 375).
On a small scale, the letter writing exchange positioned PSTs to create a mathematical
task based upon a mathematical standard, to consider how the student would think
about the mathematical task, and afterwards to examine how the student responded to
the task. There were several tools which PSTs utilized to learn about mathematical
tasks.
The Mathematical Tasks Framework and Task Analysis Guide were the tools, which influenced the development of PSTs’ knowledge. The Task Analysis Guide was utilized by PSTs while designing mathematical tasks for the pen pal letters. Additionally, the Task Analysis Guide was used when analyzing and discussing mathematical tasks during intervention sessions. The Mathematical Tasks Framework was used to show the progression of how a mathematical task is enacted within mathematics instruction. Through consistent focus on the cognitive demand of mathematical tasks, the tools provided to the treatment PSTs were useful in constructing mathematical tasks for their pen pal letters and identifying features of mathematical tasks.

The design of the intervention contributed to the significance for 2 of 7 beliefs about mathematics instruction measured by the beliefs survey. For the treatment group, the shared experience of having a pen pal contributed to PSTs’ beliefs about mathematics instruction and supported overall PST learning (Putnam & Borko, 2000). Nicol (1999) recommended that PSTs have opportunities to work with students in order to produce a change in their beliefs about mathematics instruction. The letter writing experience was an authentic activity that presented opportunities to PSTs that might not normally occur in an elementary classroom. The letter writing exchange helped maintain a sense of community because the PSTs cared about their pen pal students and spent time crafting mathematical tasks for their letters.

In preparation for the future, the intervention could be improved in ways that would further influence the beliefs held by PSTs and challenge their ability to create cognitively challenging tasks. For instance, PSTs would benefit from more interactions with students through letter writing. Many of the participants provided verbal feedback
for the pen pal activity, and the most frequent response was that they wished they had more opportunities to correspond with their pen pals. During the 12-week intervention, the participants received only three letters. Additionally, PST could have benefitted from opportunities to collaborate on analyzing the mathematical tasks prior to sending them to the pen pal students and after the letters came back to the PSTs. These opportunities would have allowed for PSTs to discuss how the pen pal students were approaching and solving the problems. These interactions could have led to a change in Belief 6, which was not significant in this study. Further, there are a couple of questions that were ignited from this study and worth investigating in the future: Could more frequent interactions between PSTs and pen pals contribute to increased belief changes? and Would the mathematical tasks posed by the PSTs increase in cognitive demand? These are examples of questions that would be worthy of investigation in future studies.

The following section will describe how the intervention and methodology utilized in this study builds on prior PST research and can inform future elementary preservice teacher mathematics courses.

**Contributions of this Investigation**

This study contributes to the growing body of research on elementary mathematics teacher preparation. The results of this study provide evidence of the effectiveness for applying the situative learning perspective to PST learning, as utilized in several other preservice teacher education studies (i.e., Crespo, 2003; Nicol, 1999; Philipp et al., 2008; Szydlik et al., 2003). In this study, a situative learning approach to the design and facilitation of the intervention appeared to be successful in supporting development of knowledge and beliefs about mathematics instruction for PSTs. The PSTs had opportunities to engage in mathematics by actually doing mathematical tasks
and contributed to learning within the intervention. Specifically, as suggested by prior studies (i.e., Crespo, 2003; Norton & Kastberg, 2012; Szydlik et al., 2003), engaging PSTs in solving mathematical tasks provided them with an opportunity to construct their own tasks, and experiments with posing the task was a valuable tool for learning about the level of cognitive demand and influencing their beliefs about mathematics instruction. The PST interviews provide evidence to support this claim. In order to offer meaningful experiences to PSTs, it is highly recommended by researchers (i.e., Bahr et al., 2013; Nicol, 1999; Norton & Kastberg, 2012; Philipp et al., 2007) to include authentic learning opportunities that focus on how students think and learn about mathematics.

Teacher educators are faced with a dilemma about the best ways to prepare PSTs (MET, 2012). These dilemmas arise when teacher educators are met with resistance from the PSTs to learn mathematics. Many PSTs hold a preconceived notion that their mathematical knowledge is sufficient for teaching elementary school students and therefore do not see the value of learning more mathematics (MET, 2012; Philipp et al., 2007). Teacher educators face the test of structuring experiences for PSTs that challenge their beliefs about mathematics and provide opportunities to engage students in learning mathematics.

This study also contributes to research on effective field experiences for elementary preservice teachers (Bahr et al., 2013; Philipp et al., 2007). The current study focused on providing PSTs with opportunities to learn about the levels of cognitive demand and to participate in a letter writing exchange. This study built on the success of teacher educators who used the Task Analysis Guide and mathematical task sort.
with practicing teachers in professional development settings (i.e., Arbaugh & Brown, 2005; Boston & Smith, 2011, Silver & Stein, 1996). In professional development studies, the participants had access to students and were able to use mathematical tasks in their mathematics instruction. The participants in this study did not have their own students. Therefore the letter writing exchange provided the authentic experience of writing and posing mathematical tasks to students (Crespo, 2003; Norton & Kastberg, 2012).

In the following section, the conclusions, limitations, and suggestions for future research are presented.

**Conclusions, Limitations, and Future Research**

The results of this study are important for several reasons. First, the intervention contributed to a significant increase in Belief 5 and 7. Both of these beliefs represent beliefs about students’ learning and doing mathematics, which was one of the focal points of the study. However, a limitation of the study was the fact that Belief 6 was not significant. In the future the intervention can include the component of having the PSTs share their letters with one another and analyze the mathematical tasks prior to sending the letters by solving the mathematical task and discussing how the pen pal student would solve the task. Upon the letters arrival of the letters, the PSTs can analyze and discuss to discuss how the pen pal student solved the mathematical task. This initial component to intervention could potentially provide the opportunity for Belief 6 to change significantly.

The significant increase in 2 of the 7 beliefs is important because it has been documented in the literature that trying to change PSTs beliefs is difficult due to years of prior experience as students themselves. Based upon the interview responses from the two participants who had either a high or low change in beliefs, it is difficult to determine
which aspects of the intervention had a significant overall effect on PSTs beliefs. Taking this into account, the intervention effected the PSTs in different ways depending on their prior beliefs about mathematics instruction. For instance, Nora states, “I like that it (the university course) is not so focused on math. It is more our discovery of how to do things.” Nora was able to participate in the program and did not feel threatened by the elementary mathematics course because it did not feel like the mathematics instruction she experienced in this past. Taking this into consideration, the intervention did have an effect on Nora’s beliefs about mathematics instruction. Heidi presents a different perspective about the intervention in this statement,

I feel like that everything we talked about all year about the conceptual understanding that definitely – because beforehand if you would have told me to write a problem I would have done it how I learned. Like just give them the problem and not ask them to explain it, not ask them to critique someone else’s work and tell them why it’s wrong. Just give them the problem and they would just do the procedure. Rather now I know that to understand how they are thinking and why they answered that way is probably the most important part about solving math problems.”

Heidi and Nora were selected based on their contrasting results from the belief survey. It would have been helpful if a participant were chosen that had average belief changes to gain their perspective about the intervention. Although a conclusion from two participant responses is hard to draw, it can be stated that the intervention did effect PSTs beliefs about mathematics instruction in different positive ways. One question that is important for the future is, will the intervention have a lasting effect on PSTs’ beliefs about mathematics and mathematics instruction once they make the transition to a practicing teacher?

Second, exposure to different mathematical tasks, doing mathematics as a community, writing mathematical tasks, and posing mathematical tasks through letter
writing contributed to the participants’ ability to recognize features of mathematical tasks and categorize the tasks by high- and low-level cognitive demand. A more in-depth look at the results from the mathematical task sort reveals need for PSTs to have experience with high-level cognitive demand tasks. The participants were most successful identifying the low-level cognitive demand tasks. A suggestion to improve the knowledge of high-level tasks could be to take low-level tasks and change certain features of the task to potentially elicit a higher level of cognitive demand. Also, would the results have been different if the intervention was started earlier in the elementary education program as opposed to waiting until the third semester in the program. The researcher found that several PSTs in the treatment group struggled with their own mathematical content knowledge of fractions. Due to the lack of knowledge, they struggled with the fraction tasks and sometimes became frustrated when other PSTs were able to explain the task and they did not understand. The researcher assumed that the PSTs were already prepared to take the course with the required prerequisite content knowledge. The PSTs lack of knowledge limited some of their interactions with the mathematical tasks. An additional construct to measure for future studies involving mathematical tasks and beliefs is mathematical content knowledge.

Third, this study utilized both quantitative and qualitative methods to analyze PSTs beliefs and mathematical task knowledge. Reflection forms for mathematical tasks, pen pal letters, and videotaped class sessions supported the identified changes in the study. Future research studies would seek a larger sample size, a different instrument to collect data for mathematical-task knowledge, a longer time frame for data collection, and an additional instrument to measure mathematical content knowledge.
The sample for this study was a convenience sample and the groups were already intact. Prior to the beginning of this study, the majority of the participants had been in their group for one year. The participants were familiar with one another and took all of their courses together every semester. The two groups could have interacted with one another and discussed what was occurring in the different classes resulting in a diffusion threat to internal validity.

Another threat to internal validity that this study faced was attrition. There were participants in both groups who chose not to participate in the study. Additionally, both groups had participants who did not complete the pre- and post-assessments for either beliefs or mathematical task sort. The data for these participants (e.g., the ones who chose not to participate and the ones with incomplete assessments) were removed from the data analysis, which poses another threat to internal validity.

The beliefs instrument caused a threat to internal validity. First, there was only one form for the survey that was used for both the pre- and post-test. The participants could have become familiar with the survey questions and therefore rushed through the post-test by filling in the same information. Second, the original version of the survey was designed and implemented for use on the web. Therefore, it was only validated as a web-based survey. A researcher took careful time and precision to produce an identical paper and pencil form of the beliefs survey. The paper and pencil form of the beliefs survey was not validated and could pose a threat to the study.

Next, the mathematical task sort was designed for use in professional development settings with practicing teachers. The mathematical task sort does not have validity or reliability information because it is a tool used to bring awareness to
features of mathematical tasks based upon the cognitive demand. Other researchers, who drew conclusions about teachers’ knowledge of cognitive demand, have used this tool. The researcher asked the participants in both groups to give reasons why they chose the classification category for the mathematical task. Less than half of the participants followed the researchers’ directions for both the pre- and post-mathematical task sort. Arbaugh and Brown (2005) and Boston and Smith (2011) used the reasons as qualitative data and incorporated it into the overall score for the mathematical task sort. Unfortunately, this data was not available for this study due to lack of compliance from the participants.

Finally, there exists the possibility of researcher bias. The researcher was the instructor for the treatment group. Participants in the treatment group were aware that the researcher was completing this study for part of the degree requirements and may have felt obligated either to participate or give perceived desirable responses on the beliefs survey. The researcher made a conscious effort to stay true to the design structure of the intervention and communicate openly with participants about the study. Notes were taken after each class session for the treatment group, and both the treatment and control group were videotaped. The video recordings were used to monitor the study. However, the implementation of the intervention could still have posed a threat to the internal validity.

In summary, the researcher attempted to conduct a study following rigorous guidelines for education research. The results from this study are multifaceted and serve to inform the design of future elementary mathematics education courses. It would be beneficial to continue to use mathematical tasks with preservice teachers by
providing them with a classroom environment where they can discuss features and solutions of the mathematical tasks. Additionally, PSTs can extend their pedagogical knowledge by learning how to pose questions that engage students to further learn about the mathematical task. Aspects of learning how to ask questions arose from the data in both PST interviews. Another avenue worth exploring is the connection between the mathematical content included in the mathematical task for the letter writing and the Common Core State Standards. Are PSTs correctly interpreting the Common Core State Standards? Lastly, it would be interesting to measure the PSTs beliefs at the beginning of the first semester of the elementary education program and then again at the end of the third semester, after the intervention is implemented in the first semester. Vacc and Bright (1999) measured PSTs’ beliefs at the beginning and ending of each semester during the elementary education program to determine which events contributed to a significant belief change. Following a similar timeline, it would be beneficial to measure PSTs beliefs the first semester in the program and again in their third semester to determine whether the intervention in the third semester has a significant impact on PSTs’ beliefs. Measuring PSTs’ beliefs for a longer duration would provide more data about the effects of the intervention. Future research studies can learn how to support PSTs as they experiment with writing and posing mathematical tasks for students.
APPENDIX A
ELEMENTARY MATHEMATICAL TASK ANALYSIS FORM

Name: _______________________
Date: _______________________

<table>
<thead>
<tr>
<th>Level of Cognitive Demand</th>
<th>Predicted</th>
<th>Evidence for Prediction, or Reasons</th>
<th>Observed</th>
<th>Evidence for Observation, or Reasons Why Expected Level of Cognitive Demand Were not Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memorization</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Procedures without connections</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Procedures with connections</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doing mathematics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use the student response to your task to reflect on and analyze ways in which you can improve future task design.

**Form adapted from Kosko et al. (2010)**
APPENDIX B
TASKS FROM THE QUASAR ELEMENTARY SCHOOL TASK-SORTING ACTIVITY

TASK A

Manipulatives or Tools Available: One triangle pattern block

Using the edge of a triangle pattern block as the unit of measure, determine the perimeter of the following pattern-block trains.

![Triangle Pattern Blocks]

TASK B

Manipulatives or Tools Available: Calculator

<table>
<thead>
<tr>
<th>Product</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \times 2$</td>
<td>4</td>
</tr>
<tr>
<td>$2 \times 2 \times 2$</td>
<td>8</td>
</tr>
<tr>
<td>$2 \times 2 \times 2 \times 2$</td>
<td>16</td>
</tr>
<tr>
<td>$2 \times 2 \times 2 \times 2 \times 2$</td>
<td>32</td>
</tr>
</tbody>
</table>

If the pattern shown continues, could 375 be one of the products in this pattern? Explain why or why not.

APPENDIX C
EXAMPLE OF QUESTIONS FOR GENERAL INTERVIEW GUIDE APPROACH

• How did you develop your math lessons for your practicum experience?

• When you developed your lessons did you focus on the cognitive demand of the mathematical task(s) or mathematical questions?

• Describe how you implemented mathematical tasks in your teaching of mathematics.

• How did you know when students understood the mathematical tasks?

• What do you think is the best way for students to learn math?

• What are the three most important characteristics of good mathematics teaching? (Raymond, 1997)

• How do you know when you’ve had a successful mathematics lesson? (Raymond, 1997)

• For the pen pal assignment, what did you do to make a mathematical task for your student?

• Did your student provide you with insight about how they were doing the mathematical task?

• Describe your overall thoughts about the pen pal assignment.

• Did any experience in the course help prepare you to construct mathematical tasks?
APPENDIX D
COURSE OVERVIEW FOR 12 WEEKS OF THE STUDY
<table>
<thead>
<tr>
<th>Session 1:</th>
<th>Session 2:</th>
<th>Session 3:</th>
<th>Session 4:</th>
<th>Session 5:</th>
<th>Session 6:</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 21&lt;sup&gt;st&lt;/sup&gt;, 2013</td>
<td>August 28&lt;sup&gt;th&lt;/sup&gt;, 2013</td>
<td>September 4&lt;sup&gt;th&lt;/sup&gt;, 2013</td>
<td>September 11&lt;sup&gt;th&lt;/sup&gt;, 2013</td>
<td>September 18&lt;sup&gt;th&lt;/sup&gt;, 2013</td>
<td>September 25&lt;sup&gt;th&lt;/sup&gt;, 2013</td>
</tr>
<tr>
<td>Introductions &amp; Data Collection</td>
<td>Data Collection for pre-mathematical task sort</td>
<td>Introduction to the Task Analysis Guide</td>
<td>Middle School Mathematical Task Sort and discussion</td>
<td>Multiplication and Division Task</td>
<td></td>
</tr>
<tr>
<td>Instrument</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assignment:</td>
<td>Assignment:</td>
<td>Assignment:</td>
<td>Assignment:</td>
<td>Assignment:</td>
<td></td>
</tr>
<tr>
<td>Complete four fraction tasks</td>
<td>Compare features of fraction tasks</td>
<td>Partition Task</td>
<td>Fractions Task on ordering fractions with student work</td>
<td>Adding and Subtracting Fraction Task</td>
<td>First pen pal letter with mathematical task focused on partitioning shapes</td>
</tr>
<tr>
<td>Part-to-Whole Tasks</td>
<td>Assignment: Read article – Designing and Implementing Worthwhile Tasks</td>
<td>Assignment: Analyze 3&lt;sup&gt;rd&lt;/sup&gt; grade fraction tasks for the PARCC with the Task Analysis Guide</td>
<td>Assignment: Read article – Thinking through a lesson: Successfully implementing high-level tasks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematical Practices Gallery</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Walk</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Session 7:</td>
<td>Session 8:</td>
<td>Session 9:</td>
<td>Session 10:</td>
<td>Session 11:</td>
<td>Session 12:</td>
</tr>
<tr>
<td>October 2&lt;sup&gt;nd&lt;/sup&gt;, 2013</td>
<td>October 9&lt;sup&gt;th&lt;/sup&gt;, 2013</td>
<td>October 16&lt;sup&gt;th&lt;/sup&gt;, 2013</td>
<td>October 23&lt;sup&gt;rd&lt;/sup&gt;, 2013</td>
<td>October 30&lt;sup&gt;th&lt;/sup&gt;, 2013</td>
<td>November 6&lt;sup&gt;th&lt;/sup&gt;, 2013</td>
</tr>
<tr>
<td>Dividing fraction task</td>
<td>Decimal Task</td>
<td>Fraction, Decimal, and Percent Task</td>
<td>Measurement Task</td>
<td>Beliefs Posttest</td>
<td>Post – Mathematical Task Sort</td>
</tr>
<tr>
<td>Second Pen Pal Letter with</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mathematical task</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>focused on place value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Third Pen Pal letter with</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mathematical task</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>focused on algebraic thinking</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# APPENDIX E
OVERVIEW OF PEN PAL LETTER WRITING

<table>
<thead>
<tr>
<th>Session</th>
<th>Date</th>
<th>Common Core State Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>September 25(^{th}), 2013</td>
<td><strong>CCSS.Math.Content.2.G.A.3</strong> Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</td>
</tr>
<tr>
<td>9</td>
<td>October 16(^{th}), 2013</td>
<td><strong>CCSS.Math.Content.2.NBT.A.1</strong>– Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g. 706 equals 7 hundreds, 0 tens, and 6 ones.</td>
</tr>
<tr>
<td>12</td>
<td>November 6(^{th}), 2013</td>
<td><strong>CCSS.Math.Content.2.OA.C.4</strong> – Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.</td>
</tr>
</tbody>
</table>
Session 5

Goals:
- Participants will learn about features of mathematical tasks.
- Participants will recognize that mathematical tasks vary in level of cognitive demand.
- Participants will learn about the Task Analysis Guide.
- Participants will discuss characteristics of mathematical tasks based upon the level of cognitive demand.

Activity 1: Comparing Tasks (15 minutes):
- Participants will work on two tasks called Necklace Task and the Musical Fractions Task

Necklace Task
Karen is stringing a necklace with beads. She puts green beads on $\frac{1}{2}$ of the string and purple beads on $\frac{3}{10}$ of the string. How much of the string does Karen cover with beads?

Musical Fractions Task
The fraction assigned to a musical note represents its value. The following are common values:
$\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$
How many combinations of these notes can you find that equal the value of a whole note?

- Participants will compare the features of the two tasks in dyads.
- They will make a list of similarities and differences.
- Next, they will discuss which task would provide the most information about students’ mathematical thinking.
- Afterwards, results from the discussions will be shared. A list of qualities for both tasks will be made and compared. The participants will develop criteria for the two tasks.

Activity 2: Mathematical Task Sort (60 minutes):
- Participants will sort a set of middle school mathematical tasks. A handout will be provided where they can place the task into a category (memorization, procedures without connections, procedures with connections, doing mathematics).
- Participants will complete the middle school mathematics task sort, first individually and then with a partner.
The results for the categories will be shared with the whole group and each task will be discussed.

The participants will develop a set of criteria for each category.

Next, the teacher educator will introduce the participants to the Task Analysis Guide.

The participants will compare their categories to the Task Analysis Guide.

A discussion will be held about the features of each task.

**Overview of Levels of Cognitive Demand (15 minutes):**

- A discussion about the levels of cognitive demand and how they relate to student achievement in mathematics.

**Activity 3: Adding and Subtracting Task (45 minutes):**

- In groups, participants will complete the adding and subtracting task using pattern blocks.
- First, the participants are asked to complete the task on their own. Next, they are asked to share the results with the group.
- Lastly, participants are asked to create their own task using the pattern blocks.
- Closure for the activity consists of a whole group discussion about the mathematical task. The teacher educator asks the participants to discuss the mathematical knowledge needed to complete the task, where students might struggle, and the potential to be a cognitively demanding task.

**Wrap-up for the session (35 minutes):**

- Discussion about homework problems from the previous class session.
- Discussion about the course reading from the previous class session and how it relates to this week’s course topic.
3. Teachers often ask children to share their strategies for solving problems with the class. Read the following student answers and indicate whether each makes sense to you. Then, click on the button at the bottom of the page to continue.

<table>
<thead>
<tr>
<th>Student</th>
<th>Problem</th>
<th>Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carlos</td>
<td>149 + 286</td>
<td>Written on paper&lt;br&gt;11&lt;br&gt;149&lt;br&gt;+286&lt;br&gt;435</td>
</tr>
<tr>
<td>Henry</td>
<td>149 + 286</td>
<td>Henry says, “I know that 40 and 80 is 120, and one hundred and two hundred makes 300, and 120 and 300 is 420, and 9 and 6 is 14, so 420 and 10 is 430, and 4 more is 434.”</td>
</tr>
<tr>
<td>Elliott</td>
<td>149 + 286</td>
<td>Written on paper&lt;br&gt;149&lt;br&gt;+286&lt;br&gt;300&lt;br&gt;120&lt;br&gt;15&lt;br&gt;435</td>
</tr>
<tr>
<td>Sarah</td>
<td>149 + 286</td>
<td>Sarah says, “Well, 149 is only 1 away from 150, so 150 and 200 is 350, and 80 more is 430, and 6 more is 436. Then I have to subtract the 1, so it is 435.”</td>
</tr>
</tbody>
</table>

Does Carlos’s reasoning make sense to you?  
- [ ] Yes  
- [ ] No

Does Henry’s reasoning make sense to you?  
- [ ] Yes  
- [ ] No

Does Elliott’s reasoning make sense to you?  
- [ ] Yes  
- [ ] No

Would you like to see a further explanation?  
- [ ] Yes  
- [ ] No

Click here to see a further explanation.
APPENDIX H
CROSSTABULATION FOR BELIEF SURVEY DATA

Table H-1. Belief 1 crosstabulation values.

<table>
<thead>
<tr>
<th>Group</th>
<th>Change score 0, expected count</th>
<th>Change score 0, actual count</th>
<th>Change score 1, expected count</th>
<th>Change score 1, actual count</th>
<th>Change score 2, expected count</th>
<th>Change score 2, actual count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>19</td>
<td>20</td>
<td>6.7</td>
<td>7</td>
<td>4.3</td>
<td>3</td>
</tr>
<tr>
<td>Control</td>
<td>21</td>
<td>20</td>
<td>7.3</td>
<td>7</td>
<td>4.7</td>
<td>6</td>
</tr>
</tbody>
</table>

Table H-2. Belief 2 crosstabulation values.

<table>
<thead>
<tr>
<th>Group</th>
<th>Change score 0, expected count</th>
<th>Change score 0, actual count</th>
<th>Change score 1, expected count</th>
<th>Change score 1, actual count</th>
<th>Change score 2, expected count</th>
<th>Change score 2, actual count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>21.9</td>
<td>20</td>
<td>3.8</td>
<td>5</td>
<td>4.3</td>
<td>5</td>
</tr>
<tr>
<td>Control</td>
<td>24.1</td>
<td>26</td>
<td>4.2</td>
<td>3</td>
<td>4.7</td>
<td>4</td>
</tr>
</tbody>
</table>

Table H-3. Belief 3 crosstabulation values.

<table>
<thead>
<tr>
<th>Group</th>
<th>Change score 0, expected count</th>
<th>Change score 0, actual count</th>
<th>Change score 1, expected count</th>
<th>Change score 1, actual count</th>
<th>Change score 2, expected count</th>
<th>Change score 2, actual count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>14.8</td>
<td>14</td>
<td>8.1</td>
<td>10</td>
<td>7.1</td>
<td>6</td>
</tr>
<tr>
<td>Control</td>
<td>16.2</td>
<td>17</td>
<td>8.9</td>
<td>7</td>
<td>7.9</td>
<td>9</td>
</tr>
</tbody>
</table>

Table H-4. Belief 4 crosstabulation values.

<table>
<thead>
<tr>
<th>Group</th>
<th>Change score 0, expected count</th>
<th>Change score 0, actual count</th>
<th>Change score 1, expected count</th>
<th>Change score 1, actual count</th>
<th>Change score 2, expected count</th>
<th>Change score 2, actual count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>17.1</td>
<td>17</td>
<td>7.1</td>
<td>6</td>
<td>5.7</td>
<td>7</td>
</tr>
<tr>
<td>Control</td>
<td>18.9</td>
<td>19</td>
<td>7.9</td>
<td>9</td>
<td>6.3</td>
<td>5</td>
</tr>
</tbody>
</table>

Table H-5. Belief 5 crosstabulation values.

<table>
<thead>
<tr>
<th>Group</th>
<th>Change score 0, expected count</th>
<th>Change score 0, actual count</th>
<th>Change score 1, expected count</th>
<th>Change score 1, actual count</th>
<th>Change score 2, expected count</th>
<th>Change score 2, actual count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>12.4</td>
<td>7</td>
<td>9.5</td>
<td>11</td>
<td>8.1</td>
<td>12</td>
</tr>
<tr>
<td>Control</td>
<td>13.6</td>
<td>19</td>
<td>10.5</td>
<td>9</td>
<td>8.9</td>
<td>5</td>
</tr>
</tbody>
</table>
### Table H-6. Belief 6 crosstabulation values.

<table>
<thead>
<tr>
<th>Group</th>
<th>Change score 0, expected count</th>
<th>Change score 0, actual count</th>
<th>Change score 1, expected count</th>
<th>Change score 1, actual count</th>
<th>Change score 2, expected count</th>
<th>Change score 2, actual count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>17.6</td>
<td>16</td>
<td>7.1</td>
<td>8</td>
<td>5.2</td>
<td>6</td>
</tr>
<tr>
<td>Control</td>
<td>19.4</td>
<td>21</td>
<td>7.9</td>
<td>7</td>
<td>5.8</td>
<td>5</td>
</tr>
</tbody>
</table>

### Table H-7. Belief 7 crosstabulation values.

<table>
<thead>
<tr>
<th>Group</th>
<th>Change score 0, expected count</th>
<th>Change score 0, actual count</th>
<th>Change score 1, expected count</th>
<th>Change score 1, actual count</th>
<th>Change score 2, expected count</th>
<th>Change score 2, actual count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>19.5</td>
<td>14</td>
<td>7.6</td>
<td>10</td>
<td>2.9</td>
<td>6</td>
</tr>
<tr>
<td>Control</td>
<td>21.5</td>
<td>27</td>
<td>8.4</td>
<td>6</td>
<td>3.1</td>
<td>0</td>
</tr>
</tbody>
</table>
APPENDIX I
CODING FOR THEMATIC ANALYSIS
Heidi’s Mathematical Tasks

Task 1:

Samantha has her friend over for pizza. If shared fairly, how much of the pizza will each friend get? Show 2 ways.

Task 2:

Pencils are packed 10 in a box.

Samantha needs 130 pencils. How many boxes does she need? Show your work.
Task 3:

Samantha won first place at her school's Math Competition! She can pick from two prizes:

Choice 1: You can take $15 home with you today.

Choice 2: You can take $3 for the next 10 days.

1) Which choice (1 or 2) gives you more money? How much more?

2) Which choice did you pick? Why?
Nora:
Task 1:

Look at the pictures and respond to the questions below.

1. Which shape(s) show one half of the shape shaded? (Write the letter(s) of the shape and explain.)

2. Which shape(s) show one third of the shape shaded? (Write the letter(s) of the shape and explain.)
Task 2:

John had 2 hundred dollar bills:

![Image of two $100 bills]

and he wants to buy shirts that cost 10 dollars each. How many shirts can he buy with the money that he has?

Task 3:

You have 10 pieces of candy. You want to share them with a friend so that you each have the same amount of candy. How many pieces of candy will each of you get?
Dear Potential Participant:

The Unified Elementary ProTeach program is very interested in your experience and learning in the program. The faculty want to know how the various experiences you have in the program affect your knowledge and beliefs about teaching as well as your teaching practices and effectiveness as a teacher. The purpose of this letter is to secure your consent for participation in a study of your development as a teacher from the point of entry into the program through your first year of teaching. The following types of information may be collected:

- **Survey:** A variety of instruments may be used to document your perspectives and practices.
- **Artifacts:** A variety of artifacts may be collected to document your perspectives and practices. These may include but are not limited to lesson plans, reflective journal, papers, tests, and projects completed to meet course requirements, and samples of children’s work with names removed.
- **Online archives:** Some UEP classes and field experiences use technology to facilitate student learning. Posting may be archived.
- **Observations:** Observation data in the form of field notes or as recorded with an observation instrument may be collected in college classrooms and in elementary classrooms to document your teaching practice. Observations may include video recordings.
- **Interviews:** Interviews lasting approximately 30-60 minutes may be conducted to gain insight into your perspectives and practices. Interviews will be audio or video recorded and transcribed. Interviews will focus on the following kinds of questions
  - Describe what you are expected to do in this course or field experience.
  - What has been challenging for you? Please explain.
  - What have you learned as a result of your participation?
  - How has your participation made a difference in the way you think about teaching or in your development as a teacher?
  - How has your participation made a difference in the way you teach or intent to teach?
  - How has your participation affected your students’ learning (in elementary classrooms).

Approved by
University of Florida
Institutional Review Board 02
Protocol # 2011-1-0673
For Use Through 09/22/2014

The Foundation for The Gator Nation
An Equal Opportunity Institution

2403 Norman Hall
PO Box 117048
Gainesville, FL 32611-7048
352-392-4215
http://education.ufl.edu/school
• Quantitative academic data: Data such as GPA, grades, and standardized test data such as SAT, GRE, or Florida Teacher Certification Examination performance data.

Your participation is strictly voluntary. Non-participation or denied consent to collect some or all of the data listed above will not affect your grades or your status as a UEP student. In addition, you may request at any time that your data not be included. The data that are collected will be analyzed by a team of researchers consisting of university faculty and/or doctoral students who work with UEP students. Your identity will be protected through use of pseudonyms, and your confidentiality will be protected to the full extent provided by law. I do not perceive that there are any risks for your participation in the study. In fact, UEP students generally enjoy the opportunity to reflect on their own learning and have a voice in shaping the UEP program.

Please sign and return to me this copy of the letter. A second copy is for your records. If you have any questions about this study of the procedures for data collection, please contact me (colvin@coe.ufl.edu, 273-4218). If you have questions about the rights of research participants, you can contact the University of Florida Institutional Review Board Office, P.O. Box 112250, University of Florida Gainesville, FL 32611.

Sincerely,

Suzanne M. Colvin
Principal Investigator
Director, Unified Elementary Proteach Program

I have read the procedure described above for the study of student learning in the Unified Elementary Proteach Program. I agree to participate, and I have received a copy of this description.

__________________________  _________________________
Signature of Participant     Date
LIST OF REFERENCES


Flick, U. (2009). *An Introduction to Qualitative Research*. SAGE.


BIOGRAPHICAL SKETCH

Kristen Apraiz graduated from Florida State University in December 2003 with a Bachelors of Science in secondary mathematics education. After graduation, Kristen extended her studies by pursuing a Master of Science in mathematics education from Florida State University and graduated in August 2004. Immediately following graduation, Kristen started her first teaching position at New Smyrna Beach High School in New Smyrna Beach, Florida. She taught high school mathematics, coached the boys and girls swim teams, and was the adviser for the student government association. Kristen was also the teacher leader for the Algebra professional learning community.

In 2008, Kristen accepted a faculty position at Daytona State College in the College of Adult Education. At Daytona State College, she taught high school mathematics courses and college preparatory mathematics courses. In 2010, Kristen enrolled at the University of Florida to begin working on a Doctor in Philosophy in curriculum and instruction with an emphasis on mathematics education. During her third year in the doctoral program she accepted a teaching position at Ivy Hawn Charter School for the Arts in Lake Helen, Florida. She served as the middle school mathematics teacher and elementary mathematics coach. Kristen graduated in August 2014 and is a clinical assistant professor in the School of Teaching and Learning in the College of Education at the University of Florida. She continues to research elementary preservice teacher education with a focus on the cognitive demand of mathematical tasks.