MODELING AND CHARACTERIZATION OF PIEZOELECTRIC ENERGY HARVESTING SYSTEMS WITH THE PULSED RESONANT CONVERTER

By

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To my family and friends
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MODELING AND CHARACTERIZATION OF PIEZOELECTRIC ENERGY HARVESTING SYSTEMS WITH THE PULSED RESONANT CONVERTER

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For many low-powered portable and wireless electronic applications the finite energy density of chemical batteries places limits on their functional lifetime. Through the use of energy harvesting techniques, ambient vibration energy can be captured and converted into usable electricity in order to create self-powering systems which are not limited by finite battery energy. Typical energy harvesting systems are composed of two components, a transducer that converts the mechanical vibrations into electrical energy and a power converter that efficiently delivers the harvested energy to the electronic load. The practical design of energy harvesting systems must include both components and consider how coupling between the two affects overall system performance. In order to effectively design an energy harvesting system for a specific application, a model is needed that accurately characterizes the energy harvesting process.

This work focuses on the development and experimental characterization of a system-level model for a vibration energy harvesting system. The system considered in this work is comprised of a piezoelectric composite beam transducer and a pulsed resonant converter (PRC). In addition to capturing the general electromechanical behavior, the system modeling developed in this work also considers the effects of non-ideal operation of the transducer and power converter. Specifically, this work examines the effects of non-resonant frequency operation, conduction losses in the PRC, and non-ideal switch timing. Unlike previous research, which typically focuses on only the electrical domain
behavior, the effects of harvesting energy on the mechanics of the transducer are also considered.

In this work, lumped element modeling techniques are used to model the behavior of the piezoelectric transducer. Two system-level models are presented, one using a full lumped element model (LEM) of the transducer and the other using a simplified resonant transducer model. The finite losses in the PRC are included in both models. An experimental test bed is developed, which includes several piezoelectric transducers and a discrete PRC implementation. Through experimental characterization of the energy harvesting system, it is shown that the full LEM accurately captures the behavior of the system over a range of vibration frequencies, while the simplified resonant model is only valid at a single operating frequency. The effects of modeling losses in the power converter are also demonstrated. For the specific systems implemented in this work, it is shown that an ideal model with zero losses overpredicts the power delivered to the load by 30-50%.
CHAPTER 1
INTRODUCTION AND MOTIVATION

Over the past two decades, advances in low power circuit design have spawned the increased use of portable and deployable microelectronic systems in everyday life. These systems range from portable applications, like cellular phones and PDAs, to distributed sensor networks for environmental monitoring. Typically, such systems are powered by chemical batteries, which must either be replaced or recharged when they become exhausted. This is an acceptable practice for applications where recharging is simple and battery replacement is infrequent. However, this is not the case for many standalone systems where all interaction must be performed wirelessly due to access constraints. Distributed systems proposed for environmental monitoring may be comprised of several thousand separate nodes, each requiring its own power supply. Changing or recharging this many batteries is not practical. For military systems or systems deployed in harsh environments, access may be difficult or impossible. A similar case occurs for structural monitoring of bridges or large buildings, where the system may be implanted within the structure and a battery cannot be accessed.

One proposed solution to mitigate the problems associated with finite battery energy is to create self-powering systems using energy harvesting techniques. Energy harvesting, or energy scavenging, is a broad term which refers to the process of converting environmental energy into electricity. A number of sources of environmental energy have been explored, including solar, thermal, acoustic, and vibration. By constantly harvesting energy from the environment, the lifetime of many electronic systems can be extended. For rechargeable systems with large power requirements, energy harvesting can be used to lengthen the operational lifetime between charges. When the power requirements are sufficiently low it may be possible to operate the system entirely on harvested energy.

This work explores the design of vibration-based energy harvesters used to implement self-powered electronic systems. Previous studies have demonstrated that ambient
vibration present in everyday life can be harvested, and used to generate electrical energy for low-power electronics [1-13]. Since the available sources of the ambient vibration are different, both in magnitude and frequency, for each specific environment, and the power required to maintain the operation of an electronic system depends greatly on its functionality, a specialized design is typically required for each application. Additionally, it is often desirable to keep the overall size of the self-powered system to a minimum, which means that an efficient energy harvester with a small footprint is needed. In order to design an energy harvester capable of meeting these needs, an accurate model is required which captures both its electrical and mechanical behavior. This model must not only capture the general behavior of the energy harvester under ideal design conditions, but must also account for non-ideal effects, including changes in the vibration source and parasitic losses associated with the physical implementation of the system. Many previous works in energy harvesting are limited by the use of ideal models which only consider the final electrical output of the harvester under strictly controlled operating conditions [4-13]. This work focuses on the development of electromechanical modeling techniques which include the effects of both non-ideal operation and parasitics within the energy harvesting system.

This chapter begins with a brief overview of energy harvesting systems, followed by the specific development of vibration energy harvesting. While converting mechanical vibrations into electrical energy is not a new concept, the widespread implementation of such systems has remained elusive. Deficiencies of previously reported systems are described next, and a new design methodology is proposed to help in the realization of functional energy harvesters. In order to move forward with this new design methodology, accurate and robust modeling techniques for energy harvesting systems are needed that include models of the energy harvester, the power converter, and the electronic load/energy storage unit. The models must capture not only the general behavior of the energy harvesting system, but must also account for the non-idealities and finite losses
intrinsic to these systems. The specific objectives of this work are presented next, along with the contributions of this research. This chapter concludes with an overview of the dissertation organization.

1.1 Energy Harvester Systems

A typical energy harvesting system, shown as a block diagram in Figure 1.1, comprises a transducer, a power converter, and an electronic load. The function of the transducer is to harvest environmental energy and to convert this energy into electricity. The transducer is therefore a multiple energy domain device, which couples the energy domain of the source (solar, mechanical, thermal, etc...) to the electrical domain. The role of the power converter is to ensure that a maximum amount of usable power is delivered to the load. To accomplish this, the power converter must provide the necessary loading conditions for the transducer, as well as condition the electrical signal to be used by the load. Signal conditioning typically entails rectification and voltage conversion of the transducer output to a level usable by the load electronics. The load for an energy harvesting system can represented as an electronic circuit, a storage element, or both.

![Block diagram of a typical energy harvesting system.](image)

Figure 1-1. Block diagram of a typical energy harvesting system.

1.1.1 Vibration Energy Harvesting

While a number of potential sources are available for energy harvesting, the focus of this work is on converting ambient vibrations into usable power. Sources of vibration are typically divided into two categories for energy harvesting, narrowband and broadband, depending on the frequency content of the vibration energy. Narrowband sources have
all of their energy concentrated in a small range of frequencies or at a specific single frequency. Energy in a broadband source, however, is distributed over a larger range of frequencies.

For vibration energy harvesters, mechanical energy is imparted to the transducer via direct contact with the vibrating source. The transducer then converts the mechanical energy into electrical energy, through one of several possible transduction mechanisms (magnetic, electrostatic, piezoelectric, etc...). A mass-spring-damper system is typically used to model the mechanical domain of the transducer [2, 14]. This modeling technique assumes that the portion of mechanical energy transferred to the electrical domain is small compared to the total mechanical energy in the source, and models the transducer as an ideal second-order system.

Using the second-order system model, the response of the transducer to a mechanical input will be greatest when the exciting vibration frequency corresponds to the transducer’s resonant frequency. As the vibration frequency diverges from the resonant frequency, the response of the transducer quickly diminishes. The vibration frequency typically cannot be controlled, so it is therefore desirable to design the transducer such that its resonant frequency corresponds to the frequency of the vibration source. Since the response of the transducer is only significant over a narrow frequency range, centered at its resonant frequency, only narrowband source excitations are considered in this work.

1.1.2 Deficiencies of Previous Designs

The potential for energy harvesting has been demonstrated for a wide variety of environmental sources, but functional implementations of energy harvesters are few and far between. While previous work has shown progress in modeling, implementing, and characterizing the different components of an energy harvester, the majority of previous work focuses on demonstration of the concepts and not on the design of a practical system. A few key deficiencies, common to many designs, have impeded the transition of energy harvesting from a proof-of-concept to a functional technology.
One of the major deficiencies with many previous designs of energy harvesters is referred to in this work as the design for size (DFS) concept. In a DFS system, the transducer is bound by stringent size constraints, in order to keep the total size of the self-powered system to a minimum \cite{1, 15-22}. While these minimum sized devices achieve impressive power densities, the total power produced by each device is typically less than $10 \mu W$. In contrast, electronic loads have finite power requirements often larger than $10 \mu W$, and continuous operation is not possible if these requirements are not met. Furthermore, the power density reported for these tiny devices is typically overestimated by disregarding the need for packaging and spacing between the devices \cite{1, 18, 19, 22}.

Another deficiency with many previously reported energy harvesters is the lack of the required power converter circuitry. For many cases, the harvested power is calculated by connecting the transducer directly to a resistive load \cite{19, 23-25}. A rectification circuit is often included, but a resistive load is still used to calculate power \cite{18, 26-28}. Unlike a load comprised of electronic circuits, a resistive load does not place any voltage or current limitations on the harvested power. A resistor is a purely dissipative device, and indiscriminately converts any electrical energy into heat. On the other hand, most electronic loads have specific requirements for voltage and current levels, and therefore require the presence of the power converter.

When a power converter is included in the energy harvesting system, it is often designed separately from the transducer \cite{9, 10, 12, 27, 29}, and there is no system-wide optimization of the energy harvester. In this situation, either the transducer is optimized without a specific power converter in mind, or the power converter is demonstrated with an ideal transducer. In either case, the potential for improved performance exists through a coupled design of the two components.

1.1.3 Design for Power

In practical applications, the specific electronic load required \cite{30} dictates the amount of power that must be harvested. This motivates the development of the more accurate
energy harvester system model in this work which enables a new energy harvesting design methodology, referred to as design for power (DFP), that addresses the previously discussed deficiencies encountered in many energy harvester systems. In the DFP methodology, the goal is to minimize the total system size required to achieve a specific power budget for a known vibration source. This is accomplished by a simultaneous design of both the transducer and the power converter. However, before the design can begin, an accurate system level model is needed which fully characterizes the behavior of not only the transducer and the power converter, but also the coupled interactions between the two. In addition to capturing the general behavior of the system, the model must also include the non-idealities of the energy harvester components. This includes the effects of non-ideal transducer operation as well as finite losses within the power converter.

1.2 Research Objectives and Contributions

Fundamentally, in the DFP methodology, the most important design consideration is the total harvested power produced by the energy harvesting system. An insufficient amount of harvested power results in reduced performance and possible failure of the end application. From an implementation standpoint, however, the practical design of an energy harvesting system requires more than just knowledge of the final power output. To keep the overall system size small, and any overhead power requirements to a minimum, the power converter should be implemented as a single integrated circuit (IC). The choice in specific IC technology places finite limits on the maximum voltage and current levels that can be handled. It is therefore necessary to have a complete understanding of not only the generated power levels, but also the intermediate voltage and current levels occurring within the power converter.

The goal of this research is to develop and experimentally validate a system level model for a vibration energy harvesting system using piezoelectric transduction. The system chosen for this work consists of a composite beam piezoelectric transducer and a pulsed resonant converter (PRC) power converter topology. Previous works have
separately examined modeling of the transducer beam [19, 31] and implementation of the PRC [8, 12, 13, 32, 33], however the combined modeling of the two components with the intent of a coupled, system level design has never before been presented. In previous studies where the PRC has been considered, the modeling efforts have been limited to the short-circuit resonant frequency of the transducer, and the effects of finite loss within the PRC are neglected. This work improves upon the previously developed models by including the frequency dependent behavior of the transducer as well as the power converter losses in the coupled system model. The behavior of the energy harvesting system is characterized not only in terms of the power, but also with respect to the various voltage and current signals in the power converter. Furthermore, in addition to capturing the behavior of the electrical domain, the system model developed here also demonstrates the effects that harvesting electrical energy has on the mechanical behavior of the transducer. This is an important point when considering the design of a truly minimally sized system, since the transduction of vibration energy into electricity requires motion of the transducer. The overall size of the energy harvesting system, which is typically dominated by the transducer, should therefore include the full three dimensional volume needed to operate the transducer, and not simply the two dimensional footprint.

The contributions of this work include:

- An experimental demonstration of the limited ability of the simplified resonant model to accurately capture the behavior of the energy harvesting system.

- The development and experimental verification of modeling techniques for a complete energy harvesting system model which captures the effects of frequency variation and losses within the system.

- An experimental demonstration of the importance of properly modeling power converter losses for the design of energy harvesting systems.

- The experimental demonstration that the PRC based energy harvesting system harvests maximum power at the open circuit resonance.
1.3 Organization

This dissertation is divided into 9 chapters and 7 appendices. Chapter 2 presents background information on the various transduction mechanisms that can be used to convert mechanical vibration into electrical energy. A comparison of the different mechanisms is included, along with an explanation of why piezoelectric transduction is the focus of this work. In Chapter 3, the electromechanical modeling of a piezoelectric transducer is examined. Chapter 4 provides background on the power converters that have been previously implemented with piezoelectric transducers, and justification for using the PRC in this work is provided. In Chapter 5, the complete model for the energy harvesting system is developed, and the behavior predicted by this model is explained. A comparison between the theoretical performance of the developed model and the existing resonant model is also given. Chapter 6 presents the design and fabrication techniques used to implement the transducers and energy harvesting circuitry. Chapter 7 provides a detailed examination of the methodology and results used to characterize the transducers examined in this work. Chapter 8 presents the experimental characterization used to validate the energy harvesting system modeling techniques and includes the methodology, procedure and results. Finally, Chapter 9 provides conclusions from this study as well as possible future directions for this work.
CHAPTER 2
VIBRATION TRANSDUCERS FOR ENERGY HARVESTING

This chapter examines transduction methods used to convert mechanical vibrations into electrical energy in an energy harvesting system. Three types of transduction are examined in this chapter; capacitive, magnetic, and piezoelectric. The fundamental physics used to convert mechanical vibrations into electrical energy are reviewed for each method, and example transducer geometries are presented. A literature review of previously reported work is also included. This chapter concludes with a comparison of the different transduction methods, and an explanation of why piezoelectric transduction is chosen for this work.

2.1 Capacitive Transduction

The capacitive or electrostatic transduction method uses a variable capacitor to convert mechanical vibrations into electrical energy. A simple parallel plate capacitor, shown in Figure 2-1, is useful to illustrate this type of transduction. The capacitance of this structure is given by

\[ C = \frac{\varepsilon_0 \varepsilon_r A}{d}, \]  

(2–1)

where \( \varepsilon_0 \) is the permittivity of free space, \( \varepsilon_r \) is the relative dielectric permittivity, \( A \) is the area of the plate overlap (\( A = lw \)), and \( d \) is the plate spacing. The voltage, \( V \), is related to the stored charge, \( Q \), by

\[ V = \frac{Q}{C}. \]  

(2–2)

The electrical energy stored on the capacitor can be expressed as

\[ E = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}. \]  

(2–3)

Two operating modes of capacitive charge transduction exist, the constant voltage mode and the constant charge mode [20]. As the name implies, the constant voltage mode operates by keeping the voltage across the capacitor constant during transduction. The applied voltage induces an electrical field in the dielectric, and charges of \( +Q \) and \( -Q \)
are created on the two plates. As a result of the opposite charges formed on the plates of the capacitor, an electrostatic force is present between them. The harvesting of the vibration energy occurs when work is done against the electrostatic force [34]. Typically, the mechanical energy causes a change in either the plate spacing or the overlap area, which leads to a change in the capacitance. For a constant voltage of $V_0$, Equation 2–3 becomes

$$E = \frac{1}{2}CV_0^2, \quad (2-4)$$

and the stored energy increases as the capacitance increases. Control circuitry is required to provide the initial charge and maintain constant voltage during transduction. Since the transducer cannot initiate the energy harvesting process without the initial charge, an external voltage source is typically required for the constant voltage mode.

![Figure 2-1. Parallel plate capacitor used for capacitive energy harvesting.](image)

The constant charge mode operates by maintaining a fixed amount of charge on the capacitor plates during transduction. In order to implement the constant charge mode, an external source is required to induce an initial charge on the two plates when the capacitance is at a maximum. The source is then removed, and the charge stored on the capacitor cannot change. Transduction occurs when mechanical work is done on the system to pull the plates apart. As the plates are separated, the capacitance decreases, and the amount of electrical energy stored on the capacitor increases [20]. For a constant
charge of $Q_0$, the energy stored on the capacitor is

$$E = \frac{Q_0^2}{2C}. \quad (2-5)$$

The stored energy is harvested when the capacitance reaches a minimum, corresponding to maximum energy. For this mode of operation, control circuitry is needed to establish the initial charge on the plates.

In order to remove the need for an external bias source, another implementation of capacitive energy harvesting which uses an electret has been explored. An electret is a dielectric material that has a semi-permanent electric charge, $Q_{\text{electret}}$, and creates an external electric field, $E_{\text{electret}}$. For this type of transducer, the electret material is placed between two electrodes, as shown in Figure 2-2, and acts much like a dielectric in a standard capacitor. However, unlike a typical dielectric where the electric field must be induced, the electrical field from the electret is induced by its own fixed charge. Vibration causes the free electrode to move, and charge is induced on this electrode as the overlap area changes [25]. Assuming that $E_{\text{electret}}$ is a uniform electric field, the voltage, $v_{\text{electret}}$, defined as

$$v_{\text{electret}} = E_{\text{electret}}d \quad (2-6)$$

will remain constant since the gap distance, $d$, does not change. Therefore, an electret energy harvester, with geometry shown in Figure 2-2, will operate in the constant voltage mode.

One of the primary advantages of capacitive energy harvesting is its ability to be integrated into standard silicon processes [2]. Unlike the magnetic and piezoelectric harvesters, which require specialized processing and materials incompatible with CMOS, capacitive energy harvesters can be micromachined using standard MEMS techniques. This provides the possibility of integrating the energy harvesting system on the same die as the power converter and load electronics. To produce a large variable capacitance with
a MEMS device, three topologies are typically used [2]; in-plane overlap, in-plane gap, and out-of-plane gap. An example of each topology is shown in Figure 2-3.

![Figure 2-2. Electret-based capacitive energy harvester.](image)

The main disadvantages of the non-electret capacitive transduction method are its needs for an external bias and for mechanical stops. If the storage element for the harvested energy is a battery, and the required bias voltage is low, the battery voltage may be used to provide the bias. However, if the required bias is higher than the battery voltage, voltage conversion circuitry adds to the control budget. While electret designs eliminate the need for the external bias, they often produce very high voltages and have limited charge lifetimes [35, 36]. Mechanical stops are often necessary to prevent contact between comb fingers, and to avoid stiction for out-of-plane transducers [2].

### 2.2 Literature Review - Capacitive Transduction

Early work in capacitive energy harvesting was done with non-electret structures which used bias voltages to establish the electrostatic force. Meninger examined the energy conversion for both constant charge and constant voltage harvesters [20]. He determined that the constant voltage case was capable of producing more power, but required multiple external voltage sources. A hybrid design was proposed which increased the power capability of the constant charge case, and an in-plane overlap style transducer
was designed. Simulation of this design produced output power of 8 $\mu W$ at a vibration frequency of 2520 $Hz$.

Figure 2-3. Capacitive energy harvesting topologies; A) In-plane overlap, B) In-plane gap, and C) Out-of-plane gap.

Roundy performed a thorough examination of the three different variable capacitor topologies and determined that the in-plane gap closing design was the most robust with output power comparable to the other topologies [2, 3]. Through simulation, with an input acceleration of 2.25 $m/s^2$ at 120 $Hz$, Roundy calculated an optimal design could produce 110 $\mu W$ output power and be confined to 1 $cm^3$. The power estimate was made for a constant charge system, and does not take into account the power needed for the bias control circuitry.

A number of functional capacitive energy harvesters which use a bias voltage have been demonstrated. Some examples include work done by Despesse [37], Ma [38], and Miyazaki [39]. Despesse reported a tungsten based, in-plane gap varying design, which harvested 1052 $\mu W$ from a 90 $\mu m$ displacement at 50 $Hz$. This design, however, used
electrical discharge machining to create a small macro-scale device with a volume of 18 cm$^3$. Ma presented a more traditional MEMS process to design and fabricate a harvester capable of delivering 65 nW to a resistive load. The input to this device was a 5 µm displacement at 4 kHz, but the total size was not reported. Miyazaki demonstrated an out-of-plane harvester which generated 120 nW of electrical power from an acceleration of 0.08 m/s$^2$ at 45 Hz. Again, the total size of the energy harvester was not reported.

The use of electrets to remove the need for a bias voltage is a growing trend in capacitive energy harvesting. Peano [21] developed a nonlinear dynamic model for an in-plane overlap topology to be used for device optimization. For a 5 µm displacement at 911 Hz, it was shown that a device optimized for nonlinear operation could harvest 50 µW, while a linearly optimized device could only harvest 5.8 µW.

Lo et. al [40] developed a parylene HT electret material with 3.69 mC/m$^2$. Using an in-plane overlap topology, 5.6 µW of power has been demonstrated with a 2 mm$_{pp}$ displacement at 50 Hz. The device is comprised of movable brass electrodes over a glass substrate. The total device size is approximately 25 mm$^2$.

A 16 µm thick CYTOPTM electret film was presented by Sakane [36] for use in a micromachined harvester. By doping the CYTOPTM, a charge of 1.5 mC/cm$^2$ was achieved. When a 1.2 mm$_{pp}$ displacement was applied at 20 Hz, an output power of 0.585 mW was delivered to an optimal resistive load of 4 MΩ. The total volume of this device was not clearly reported.

2.3 Magnetic Transduction

Magnetic materials emanate magnetic fields, as shown for a typical bar magnet in Figure 2-4. By convention, these fields originate from the magnetic north pole and terminate at the magnetic south pole. The number of field lines passing normally through a surface within the field is defined as the magnetic flux, Φ$_B$, which has units of webers. A magnetic flux density, $B$, is defined as the flux per unit area and has units of Tesla or
weber/m². The relationship between flux and flux density is given as

\[ \Phi_B = \int_s B \cdot d\vec{s}, \]  

(2–7)

where \( d\vec{s} \) is the differential surface area.

![Bar magnet with magnetic field lines.](image)

Figure 2-4. Bar magnet with magnetic field lines.

The electrical and magnetic domains can be linked together through Faraday’s law of magnetic induction. When \( N \) loops of a closed circuit are placed into a magnetic field, a voltage, \( v_{emf} \), is induced equal to the time rate of change of the magnetic flux,

\[ v_{emf} = -N \frac{d\Phi_B}{dt}. \]  

(2–8)

Combining Equation 2–7 and Equation 2–8, the induced voltage can be expressed in terms of \( \vec{B} \) as

\[ v_{emf} = -N \frac{d}{dt} \int_s \vec{B} \cdot d\vec{s}. \]  

(2–9)

Therefore, to induce a voltage in a closed circuit, either \( \vec{B} \), \( d\vec{s} \), or both, must be functions of time.

Magnetic transducers convert vibration energy into electrical energy through magnetic induction described by Faraday’s law. Input vibration causes relative motion between a magnet and coil, which leads to a time-varying flux and induced voltage, \( v_{emf} \), defined by
Equation 2–8. The voltage induced by the changing flux causes current to flow in the coil and delivers electrical energy to an external load.

A common configuration used for magnetic transduction is shown in Figure 2-5. A permanent magnet, attached to the housing of the transducer with a mechanical spring, is suspended above an induction coil attached directly to the housing. Vibrations applied in the z-direction cause oscillations in the position of the magnet relative to the coil, which lead to a time-varying flux. A voltage is induced in the coils, and current flows to the electrical load. A similar configuration is possible where the magnet is stationary relative to the housing and the coils move in the presence of vibration.

![Figure 2-5. Magnetic transducer used for vibration energy harvesting.](image)

The main benefit of magnetic transduction is the relatively high power densities that can be achieved. A wide variety of magnetic materials exist which exhibit very high magnetic fields and provide good coupling between the mechanical and electrical energy domains. Unlike the capacitive method where an external bias is required to provide initial charge, magnetic transducers can operate without this constraint.

The main drawback of magnetic transduction is the low output voltage that it produces. The output voltages of typical magnetic transducers are on the order of 10’s or
100’s of mV’s, which is too low for many applications. While it is possible to add step-up transformers to increase the voltage signal, the added bulk and loss of the transformer is often prohibitive. Finite core and conduction losses in transformers are significant in energy harvesting systems, where the total power is only on the order of microwatts.

Another obstacle with magnetic transducers is their integration with small-scale systems. Magnetic thin films tend to possess poor properties and produce relatively weak fields. Bulk magnetic materials are often comprised of exotic alloys and standard batch processing techniques cannot be applied.

### 2.4 Literature Review - Magnetic Transduction

Some of the first published work on magnetic transduction for vibration-based energy harvesting was performed by Williams et al [14, 41]. A device similar to that shown in Figure 2-5 was fabricated using a bulk SmCo magnet and a polyimide membrane for the spring. When the device was operated at its resonant frequency of 4.4 kHz with 0.5 µm vibration amplitude, a power of 0.3 µW was delivered to a 39 Ω resistor. The volume of the device was approximated at 25 mm³.

A slightly larger device, reported by Ching et al. [26] and Lee et al. [42], was combined with power converter circuitry and integrated into a AA-battery sized package (approx. 1 cm³). A spiral spring configuration was chosen for this design, and several different materials were analyzed as potential springs. Using a laser-fabricated Cu spring, with a NdFeB magnet, a power level of 830 µW was reported with an input vibration of 200 µm at 110 Hz.

Another spiral spring configuration, developed by Pan et al., demonstrated harvesting of 100 µW from a completely microfabricated device [43]. A silicon-based microspring structure was combined with a FePt magnet using standard Si processing. With a 60 Hz excitation, 40 mV were produced from a device whose volume measured 0.45 cm³.

Several cantilevered beam configurations have also been explored for magnetic transduction. Mizuno and Chetwynd [44] patterned coils on the surface of beams placed in
a magnetic field. It was demonstrated that as the beams vibrate, the flux change through the surface coils would produce electrical energy. For 0.64 $\mu m$ vibrations at 700 $Hz$, 4 $nW$ of power was measured for each 25 $mm$ long beam. A similar configuration was used by Kulah and Najafi [45], capable of producing 4 $nW$ from a 1 $Hz$ input.

Many of the magnetic energy harvesters that produce the largest amount of power are geared towards energy harvesting from human motion. Human walking has been studied in detail as a potential source of harvestable energy [46–48]. A simple magnetic energy harvester designed for this purpose was demonstrated by Amirtharajah and Chandrakasan [49]. A mass-spring structure, similar to that shown in Figure 2-1, was fabricated using readily available off the shelf materials. The volume of this structure was approximately 24 $cm^3$. Operating at 2 $Hz$, nearly 400 $\mu W$ were produced from a 2 $cm$ vibration displacement. A larger device, 123 $cm^3$, designed by Nakano et al., used a similar structure and demonstrated a power of 18.7 $mW$ [50].

In 2007, Arnold [51] performed a comprehensive review of magnetic energy harvesting experiments conducted over the past decade, including both rotational and vibration based generators. A scaling analysis was performed and many of the design challenges and trade-offs inherent to magnetic energy harvesting were explored.

### 2.5 Piezoelectric Transduction

The piezoelectric effect refers to a coupling between strain and polarization for certain materials due to their crystalline structure. When a material with piezoelectric properties is mechanically strained, either in compression or tension, an electric potential is induced in the material [52]. This property, illustrated in Figure 2-6, is referred to as the direct piezoelectric effect. Piezoelectricity is a reciprocal property, meaning that an applied electric potential induces a mechanical strain in the material. This is referred to as the indirect piezoelectric effect and is shown in Figure 2-7. For harvesting vibration energy, the direct piezoelectric effect is utilized to convert energy from the mechanical domain to the electrical domain.
Figure 2-6. The direct piezoelectric effect is demonstrated in A) equilibrium, B) compression and C) tension for a piezoelectric material.

Figure 2-7. The indirect piezoelectric effect is demonstrated in A) equilibrium, B) compression and C) tension for a piezoelectric material.

The presence of piezoelectric behavior in a material is determined by its crystal structure. Crystalline materials have atomic structures where the atoms are arranged in a periodic lattice. The smallest arrangement of atoms that can accurately represent the lattice is referred to as a unit cell [53]. In order for a material to exhibit piezoelectricity, the crystalline structure must be noncentrosymmetric, meaning that there is no center of symmetry within a unit cell. Of the 21 known noncentrosymmetric crystal configurations, 20 have been shown to possess piezoelectric properties. When a piezoelectric crystal is mechanically deformed, the lack of symmetry leads to the formation of electric dipoles which induce an electric field in the material [54]. Electrodes placed on the surface of the material experience a voltage differential as a result of the induced field. This effect was first demonstrated in quartz by Pierre and Jacques Curie in 1880. In addition to quartz, common piezoelectric materials include, lead zirconium titanate (PZT), aluminum
nitride (AlN), zinc oxide (ZnO), and polyvinylidene fluoride (PVDF). Table 2-1 provides a comparison of materials properties typically found in piezoelectric compounds.

Table 2-1. Properties for typical piezoelectric materials.

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Symbol</th>
<th>PZT[^55]</th>
<th>AlN[^56]</th>
<th>ZnO[^57, ^58]</th>
<th>PVDF[^59]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus [GPa]</td>
<td>E</td>
<td>52</td>
<td>283</td>
<td>98.6</td>
<td>30</td>
</tr>
<tr>
<td>Density [kg/m^3]</td>
<td>( \rho )</td>
<td>7800</td>
<td>3260</td>
<td>5700</td>
<td>1780</td>
</tr>
<tr>
<td>31 Piezo Coeff. [pC/N]</td>
<td>( d_{31} )</td>
<td>-190</td>
<td>-2.6</td>
<td>-5.5</td>
<td>20</td>
</tr>
<tr>
<td>33 Piezo Coeff. [pC/N]</td>
<td>( d_{33} )</td>
<td>390</td>
<td>5.5</td>
<td>10.3</td>
<td>30</td>
</tr>
<tr>
<td>Dielectric Const.</td>
<td>( \epsilon_r )</td>
<td>1800</td>
<td>10.7</td>
<td>8.5</td>
<td>12</td>
</tr>
<tr>
<td>31 Coupling Coeff.</td>
<td>( \kappa_{31}^2 )</td>
<td>0.118</td>
<td>0.020</td>
<td>0.040</td>
<td>0.113</td>
</tr>
<tr>
<td>33 Coupling Coeff.</td>
<td>( \kappa_{33}^2 )</td>
<td>0.497</td>
<td>0.090</td>
<td>0.139</td>
<td>0.254</td>
</tr>
</tbody>
</table>

[^55]: ceramic  
[^56]: thin-film  
[^59]: polymer

Materials with piezoelectric properties are dielectrics, meaning that they are electrical insulators and hence conduct current poorly. As such, when electrodes are placed on the surface of a piezoelectric layer, the resulting device is essentially a capacitor. Applied stress induces a voltage across the piezoelectric capacitor. However, piezoelectric materials are not perfect insulators, and the induced voltage decays with time [^54].

For a linear piezoelectric material, the electromechanical coupling can be expressed quantitatively by the constitutive equations

\[
\{ S \} = [s^E] \{ T \} + [d]^{tr} \{ E \} \tag{2-10}
\]

and

\[
\{ D \} = [d] \{ T \} + [\epsilon^T] \{ E \}, \tag{2-11}
\]

where \( \{ S \} \) is the strain vector, \([s^E]\) is the compliance matrix, \( \{ T \} \) is the stress vector, \([d]\) is the piezoelectric coefficient matrix, \( \{ E \} \) is the electric field vector, \( \{ D \} \) is the electric displacement vector, and \([\epsilon^T]\) is the permittivity matrix. The superscripts \( ^E \) and \( ^T \) indicate that the compliance and permittivity matrices are calculated with constant electric field and stress conditions, respectively [^54, ^60]. The superscript \( ^{tr} \) indicates a
matrix transpose. If $[d]$ is all zero, indicating no coupling between the mechanical and
electrical domains, then Equation 2–10 and Equation 2–11 reduce to the uncoupled
equations for an elastic dielectric material [3].

When using piezoelectric materials for vibration energy harvesting, there are two
available modes of electromechanical coupling, namely the 31 and 33 modes. The numbers
are used to represent the different modes, 1, 2 and 3, which refer to the orthogonal axes of
a 3-dimensional coordinate system. By convention, the 3-direction refers to the direction
of polarization. The 31 mode therefore describes a transducer where strain is applied in
the 1-direction, and the electric potential is generated in the 3-direction. Similarly, the
33 mode is characterized by strain in the 3-direction and an electric potential also in
the 3-direction. Both modes, 31 and 33, are shown in Figure 2-8. The geometry of the
piezoelectric material and placement of the electrodes will ultimately determine which
electromechanical mode is harnessed for transduction.

![Figure 2-8. Two common operating modes for piezoelectric energy harvesting are A) 31 and B) 33.](image)

For energy harvesters using the 31 mode, simple beam geometries are typically
implemented. The beam is either clamped at one end in a cantilevered configuration, or
clamped at both ends to provide clamped-clamped boundary conditions. Transducers with
this geometry can be further divided into unimorph and bimorph types. Unimorphs have
a single piezoelectric layer, while bimorphs have top and bottom layers with opposing
polarities. More layers can be implemented, changing the output current to voltage ratio,
but if the total thickness of piezoelectric material does not change, the total harvested power is not affected [2]. A typical 31 cantilevered beam unimorph transducer is shown in Figure 2-9.

Figure 2-9. Cantilevered beam transducer operating in the 31 mode.

As the free end of the cantilever is displaced relative to the fixed end, strain is developed in the substrate and piezoelectric layer in the 1-direction. The electrodes, placed on the top and bottom of the piezoelectric material, produce an electric field in the 3-direction. For this mode, assuming a transversely isotropic material (i.e. PZT), the constitutive equation given by Equation 2–10 and Equation 2–11, reduce to

\[ S_1 = s_{11}T_1 + d_{31}E_3 \]  \hspace{1cm} (2–12)

and

\[ D_3 = d_{31}T_1 + \varepsilon_{33}E_3. \]  \hspace{1cm} (2–13)

The stress and electrical displacement in Equation 2–12 and Equation 2–13 are coupled by the \( d_{31} \) piezoelectric coefficient. Since many piezoelectric materials are comprised of brittle ceramics, the piezoelectric and electrodes are bonded to a substrate to prevent the transducer from cracking or snapping as it is vibrates. A proof mass can be included to add more mass at the tip, thus increasing the strain induced in the piezoelectric. It also provides additional degrees of freedom for tuning the resonant frequency of the transducer.
Similarly to the 31 mode, a cantilevered beam configuration is shown to demonstrate the operation of 33 mode transducers. Figure 2-10(A) and Figure 2-10(B) show the geometry of the 33 mode transducer from the side and top views, respectively. The displacement of the free end of the cantilever, relative to the fixed end, causes strain to be induced in the piezoelectric layer in the 3-direction. The two electrodes are interdigitated fingers, which allow the electrical field to be generated in the 3-direction as well. For this mode of operation, the constitutive equations reduce to

\[ S_3 = s_{33} T_3 + d_{33} E_3 \]  \hspace{1cm} (2–14)

and

\[ D_3 = d_{33} T_3 + \varepsilon_{33} E_3. \]  \hspace{1cm} (2–15)

In the 33 mode, the coupling is therefore determined by the \( d_{33} \) piezoelectric coefficient.

One main advantage of piezoelectric energy harvesting methods is the ability to produce significant output voltages. Whereas magnetic methods are typically confined to the mV range, piezoelectric transduction is capable of producing voltages in the 10 to 100 V range. This is useful in minimizing the effects of voltage drops required for typical rectification. Also, unlike the capacitive method, piezoelectric energy harvesting can operate without an external power source for transduction.

The disadvantages of the piezoelectric method for energy harvesting are related to the material properties and fabrication. In order to create materials with high \( d_{31} \) and \( d_{33} \) coefficients, bulk processing is often necessary. Single crystal piezoelectric materials have good electromechanical coupling properties, but are very expensive to fabricate. Cheaper ceramics can be used, but are brittle and have lower coupling coefficients. In order to batch fabricate piezoelectric transducers, thin films are available. However, the thin films tend to have poor piezoelectric properties, as shown in Table 2-1. PZT films, which have the highest piezoelectric coefficients, contain lead, which is not CMOS compatible.
When choosing a piezoelectric material for a specific application, the piezoelectric coefficient ($d_{31}$ or $d_{33}$) is often considered first, since it relates both deformation to applied voltage (direct piezoelectric effect) and induced charge to applied strain (indirect piezoelectric effect). However, for energy harvesting applications, it is also important to consider the electromechanical coupling coefficient $\kappa^2$, which quantifies the ratio of energy transferred from one energy domain to another [61]. Specifically, for this vibration energy harvesting, $\kappa^2$ describes the transfer of energy from the mechanical domain to the electrical domain. For a single sheet of piezoelectric material, the value of $\kappa^2$ is a material property given by

$$\kappa^2 = \frac{d^2 E}{\epsilon}.$$  \hspace{1cm} (2–16)
2.6 Literature Review - Piezoelectric Transduction

Piezoelectric energy harvesting has been explored for a wide variety of applications. Some notable research has been conducted to harvest energy from human walking [34, 47, 59], river currents [62, 63], and even wind [11, 64]. This section focuses on small (cm scale and down) piezoelectric energy harvesters and reviews work specifically designed for self powered systems.

Many of the initial piezoelectric energy harvesters were meso-scale devices fabricated with bulk materials. Roundy [2, 3] designed a bimorph beam using a commercially available piezoelectric material, PSI-5A4E, from Piezo Systems Inc. The PZT cantilevered beam was 28 mm $\times$ 3 mm with a tungsten proof mass at the tip for adjusting the resonant frequency. The beam had top and bottom electrodes and operated in the 31 mode. From a base acceleration of 2.5 m/s$^2$ at 120 Hz, 365 $\mu$W was delivered to a resistive load.

Similar commercial 31 devices were used by Ottman [10], Sodano [65], and du Toit [1], to demonstrate piezoelectric energy harvesting. Ottman used the QP20W, a PZT bimorph device from Mide Technology Corporation, with a volume of 38.8 mm$^3$. At frequencies near 60 Hz, voltages of nearly 70 V were produced and 30 mW of power were able to be harvested. The input acceleration, however, was not reported, and power electronics were used. Sodano used the QP40N, another PZT device from Mide, which produced 900 $\mu$W across a resistor from an unreported acceleration at 30 Hz. This device volume was measured to be 1.95 cm$^3$. A similar sized device from Piezo Systems Inc., the T-226-A4-503X, with a volume of 1.37 cm$^3$, was demonstrated by du Toit to deliver 590 $\mu$W to a resistive load of 11 k$\Omega$. This device was operated at 107 Hz and 2.5 m/s$^2$ base acceleration.

Badel et al. [29] compared the harvested energy from a PZT ceramic material to that of a single crystal material. A unimorph design was implemented, with a 10 mm $\times$ 7 mm $\times$ 1 mm piezoelectric attached to a 40 mm $\times$ 7 mm $\times$ 1.5 mm shim. From a 100 $\mu$m
maximum displacement at 900 Hz, the ceramic material produced 10 µW and 2.5 V. The single crystal, 0.75PMN-0.25PT, produced 200 µW and 7 V. These power and voltage numbers were measured for a rectified dc output delivered to battery. The higher power delivered from the single crystal can be attributed to its higher electromechanical coupling coefficient.

Several PZT based, MEMS-scale transducers have been demonstrated, including work by Jeon et al. [18, 66], Fang et al. [16, 17], and Kasyap [19]. Due to the limitations of the processing techniques for MEMS, these devices have all been unimorphs. The research by Jeon used standard Si processing with a sol-gel PZT. A 170 µm × 260 µm beam was constructed with surface electrodes to operate in the 33 mode. At 13.9 kHz with a 2.56 µm tip displacement, 1 µW of dc power was delivered to a 5.2 MΩ resistor. A promising voltage of 2.4 V was reported for this input.

Fang et al. designed a 31 mode cantilever beam with a 2 mm × 600 µm × 12 µm Si shim and 600 µm × 600 µm × 500 µm Ni proof mass. A sol-gel PZT layer of 1.64 µm was deposited and Ti/Pt electrodes were patterned on top. After the Si device was released, a Ni proof mass was manually attached with glue. Rectified power output of 2.16 µW was demonstrated for 608 Hz vibrations at 9.8 m/s². A maximum voltage of 898 mV was reported for this device.

The 31 device designed by Kasyap was fabricated on an SOI wafer and used the Si substrate layer to form the proof mass. This method avoided the need for manual fabrication and gluing of multiple parts. A transducer was constructed with a 1 mm × 1 mm × 12 µm Si shim with a 2.5 mm × 4 mm × 500 µm Si proof mass. A 1 µm, sol-gel PZT layer was used with Ti/Pt electrodes. When operated at a resonant frequency of 126.6 Hz with an acceleration of 0.2 m/s², this MEMS-scale device was shown to deliver 196 nW of power to a resistive load.
2.7 Transducer Comparison

The previous sections of this chapter have examined transduction mechanisms for converting vibrations into electrical energy. Table 2-2 presents a summary of previous research for capacitive, magnetic, and piezoelectric energy harvesting schemes. While design trade-offs exist for all three of the transduction mechanisms, a piezoelectric scheme is used in this work. The main reasons for choosing piezoelectric transduction for a DFP based design are its ability to generate voltages higher than a few mV and produce power levels required for typical self-powered systems.

Table 2-2. Comparison of energy harvesters.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Accel. (m/s²)</th>
<th>Disp. (µm)</th>
<th>Freq. (Hz)</th>
<th>Area (cm²)</th>
<th>Vol. (cm³)</th>
<th>Voltage (V)</th>
<th>Power (µW)</th>
<th>Power/Area (µW/cm²)</th>
<th>Power/Vol (µW/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CAPACITIVE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Menginger [20]</td>
<td>-</td>
<td>2520</td>
<td>2.25</td>
<td>0.075</td>
<td>-</td>
<td>8</td>
<td>3.55</td>
<td>107</td>
<td></td>
</tr>
<tr>
<td>Roundy [2, 3]</td>
<td>2.5</td>
<td>-</td>
<td>120</td>
<td>3</td>
<td>1</td>
<td>6</td>
<td>110</td>
<td>37</td>
<td>110</td>
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<td>Despesse [37]</td>
<td>8.8</td>
<td>90</td>
<td>50</td>
<td>18</td>
<td>18</td>
<td>120</td>
<td>1052</td>
<td>58</td>
<td>584</td>
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<td>Ma [38]</td>
<td>760</td>
<td>5</td>
<td>4000</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>0.065</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Miyazaki [39]</td>
<td>0.08</td>
<td>-</td>
<td>45</td>
<td>-</td>
<td>-</td>
<td>0.120</td>
<td>-</td>
<td>-</td>
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<td>Peano [21]</td>
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<td>5</td>
<td>911</td>
<td>-</td>
<td>-</td>
<td>50</td>
<td>-</td>
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<td>Lo [40]</td>
<td>-</td>
<td>2000</td>
<td>50</td>
<td>50</td>
<td>38</td>
<td>5.6</td>
<td>110</td>
<td>37</td>
<td>37</td>
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<td>Sakane [36]</td>
<td>-</td>
<td>1200</td>
<td>20</td>
<td>-</td>
<td>49</td>
<td>585</td>
<td>-</td>
<td>-</td>
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<td><strong>MAGNETIC</strong></td>
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<tr>
<td>Williams [41]</td>
<td>-</td>
<td>0.5</td>
<td>4000</td>
<td>-</td>
<td>0.25</td>
<td>- 0.3</td>
<td>-</td>
<td>- 1.2</td>
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<tr>
<td>Lee [42]</td>
<td>-</td>
<td>200</td>
<td>110</td>
<td>-</td>
<td>1</td>
<td>2.2</td>
<td>830</td>
<td>- 830</td>
<td></td>
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<tr>
<td>Pan [43]</td>
<td>-</td>
<td>60</td>
<td>-</td>
<td>0.45</td>
<td>0.04</td>
<td>100</td>
<td>-</td>
<td>- 222</td>
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<tr>
<td>Mizuno [44]</td>
<td>12.7</td>
<td>0.64</td>
<td>700</td>
<td>-</td>
<td>2 320 µV</td>
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<td>-</td>
<td>0.0002</td>
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<tr>
<td>Kulah [45]</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>2.3 0.006</td>
<td>0.004</td>
<td>-</td>
<td>-</td>
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<td>Amirtharajah [49]</td>
<td>2.94</td>
<td>2 cm</td>
<td>2</td>
<td>23.5</td>
<td>-</td>
<td>400</td>
<td>-</td>
<td>- 17</td>
<td></td>
</tr>
<tr>
<td>Nakano [50]</td>
<td>1.96</td>
<td>1 cm</td>
<td>2</td>
<td>- 123</td>
<td>18,700</td>
<td>-</td>
<td>- 152</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td><strong>PIEZOELECTRIC</strong></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roundy [2]</td>
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<td>-</td>
<td>120</td>
<td>0.84</td>
<td>1 6</td>
<td>365</td>
<td>435</td>
<td>365</td>
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</tr>
<tr>
<td>Ottman [10]</td>
<td>-</td>
<td>60</td>
<td>15.3</td>
<td>0.38</td>
<td>45 30,000</td>
<td>1960</td>
<td>78,947</td>
<td>-</td>
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<tr>
<td>Sodano [65]</td>
<td>-</td>
<td>30</td>
<td>25.4</td>
<td>1.95</td>
<td>3</td>
<td>900</td>
<td>35.4</td>
<td>462</td>
<td></td>
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<tr>
<td>du Toit [1]</td>
<td>2.5</td>
<td>90</td>
<td>107</td>
<td>20.0</td>
<td>1.37</td>
<td>3.5</td>
<td>590</td>
<td>29.5</td>
<td>431</td>
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<tr>
<td>Badel [29]</td>
<td>-</td>
<td>100</td>
<td>900</td>
<td>2.8</td>
<td>0.49</td>
<td>7 200</td>
<td>71.4</td>
<td>408</td>
<td></td>
</tr>
<tr>
<td>Jeon [18, 66]</td>
<td>-</td>
<td>2.56</td>
<td>13900</td>
<td>0.044mm²</td>
<td>- 2.4</td>
<td>1</td>
<td>2262</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Fang [16, 17]</td>
<td>9.8</td>
<td>608</td>
<td>0.012</td>
<td>0.0002</td>
<td>898</td>
<td>2.16</td>
<td>108</td>
<td>10800</td>
<td></td>
</tr>
<tr>
<td>Kaeyap [19]</td>
<td>0.2</td>
<td>126.6</td>
<td>0.11</td>
<td>0.0002</td>
<td>62.6</td>
<td>0.196</td>
<td>1.78</td>
<td>31.6</td>
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</tr>
</tbody>
</table>

One of the key driving forces for using capacitive energy harvesting is its ability to integrate into CMOS and MEMS processes. Therefore, typical devices are small and the total power produced per device is low. Volumetric power densities suggest that higher power levels, comparable to the other transduction schemes, can be achieved with larger areas and multiple devices. However, these power densities often only consider the volume of the transducer. The space needed for control circuitry, power electronics, and vibratory
motion is often ignored, but quickly adds up with multiple transducers. The volumetric power density, therefore, typically over predicts the actual amount of total power that can be achieved. Since this work focuses on a design for power methodology, a minimum power budget is a key design goal. With the exception of the work by Despesse [37] and Roundy [2, 3], where macro-scale devices are used, the energy harvested using capacitive methods is insufficient for most self-powered systems.

The other reason that capacitive methods are not used in this work stems from the external voltage requirement. In order to be robust, it is desirable for a self-powered system to function strictly on harvested energy. The external bias requirement limits a capacitive energy harvesting system, by requiring that some amount of energy be stored on a battery or other storage element to provide the bias. Should this storage element become depleted, perhaps by long periods of inactivity, the bias cannot be established and no energy will be harvested. While electret harvesters bypass this concern, they produce high voltages that most IC processes cannot handle and make interfacing with power electronics difficult.

Magnetic energy harvesters face the problem of low output voltage. While the power and power density of several reported magnetic transducers are comparable to the other methods, the voltages are typically in the mV range. The extraction of power from magnetic transducers is usually accomplished by a direct connection to a small (less than 100 Ω) resistive load. For practical applications, where rectification is required, the low voltage levels are insufficient to overcome the diode drops in a typical rectifier (200 - 700 mV). It is possible to use a step-up transformer to raise the voltage, but this has a finite energy loss and additional bulk to the system.

Piezoelectric energy harvesting is chosen for this work because of its ability to produce relatively high power (∼ 100’s of µW’s) and favorable voltage conditions. With typical values between 2 and 10 V, the output voltage of piezoelectric transducers is high enough to overcome diode drops in a rectifier, but still low enough to interface with typical
IC technologies. The inability to integrate certain piezoelectric materials with CMOS, such as PZT, is not critical in this work since MEMS-scale energy harvesters have only been shown to produce a few $\mu W$'s. Such power levels are insufficient to operate self-powered systems. Larger, meso-scale piezoelectric transducers will be explored as a means to provide the power budget necessary to satisfy the design for power requirements.
CHAPTER 3
TRANSDUCER MODELING

In Chapter 2, the most common transduction mechanisms for converting mechanical vibrations into electrical energy were examined. Piezoelectric transduction was chosen for this work both for its ability to generate reasonable voltage levels and its modest power density. The electromechanical transduction, however, is only part of a complete energy harvesting system. In order to design a practical system using piezoelectric transduction, power converter circuitry must also be included. The design of a complete energy harvesting system requires a system-level model which captures the interaction between the transducer and the power converter.

While the power converter operates only in the electrical domain and can be modeled with standard circuit elements, the transducer is a complex three dimensional structure operating simultaneously in both the electrical and mechanical domains. Analytical expressions comprised of partial differential equations are typically used to model this type of spatially distributed, time-varying system; however, it is difficult to directly integrate these expressions into circuit analysis. In order to model the transducer in a form that can more easily be incorporated into circuit analysis, lumped element techniques are used. Through the use of lumped element modeling, the distributed electromechanical behavior of the transducer is lumped into discrete circuit elements, provided that the wavelength of the physical phenomena is much larger than the characteristic length scale of the transducer [52]. Both the transducer and power converter can then be analyzed using standard electrical circuit methods, such as Kirchoff’s Voltage Law (KVL) and Kirchoff’s Current Law (KCL).

This chapter presents the lumped element model used by Kasyap [19] for a cantilevered composite beam operating in the 31 mode. While a complete derivation of the model is beyond the scope of this work, a general understanding of the model and its limitations is important. An overview of lumped element modeling and conjugate power variables
for electromechanical systems is discussed first. This is followed by an explanation of
the LEM used to represent the piezoelectric transducer. The chapter concludes with the
derivation of a simplified resonant model commonly used to characterize power converters
for piezoelectric energy harvesting.

3.1 Lumped Element Modeling Overview

Lumped element modeling allows the complex, spatially distributed behavior of
multiple energy domain systems to be represented with equivalent circuit elements. A
LEM is created by localizing the distributed system into discrete elements [52]. This
technique is considered valid when the dimensions of the system are small compared to the
characteristic wavelength [61]. Since energy is domain independent, the total energy and
its components, both potential and kinetic, can be examined in any energy domain of the
system.

A pair of conjugate power variables, generalized as effort and flow, is used to represent
energy in each domain [52]. For example, in the electrical domain, the conjugate power
variables are voltage and current. The voltage represents the effort and current represents
the flow. In the mechanical domain, force is effort and velocity is flow. For both cases,
the product of the effort and flow variables is power, which is simply the energy per unit
time. By working in terms of energy and conjugate power variables, parallels can be
drawn between the discrete elements of different domains based on their interactions with
energy. For instance, in the electrical domain, potential energy is stored on a capacitor,
while in the mechanical domain potential energy is stored in the compliance of a spring.
Similar analogies exist for the storage of kinetic energy and energy dissipation. Using these
parallels, mechanical mass can be modeled with an inductor, mechanical compliance can
be modeled with a capacitor, and mechanical dissipation can be modeled with a resistor.
Table 3.1 summarizes the modeling of the lumped mechanical domain parameters as
circuit elements.
Table 3-1. Energy domain analogs.

<table>
<thead>
<tr>
<th>Conjugate Power Variable</th>
<th>Mechanical Domain</th>
<th>Electrical Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effort</td>
<td>Force</td>
<td>Voltage</td>
</tr>
<tr>
<td>Flow</td>
<td>Velocity</td>
<td>Current</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lumped Elements</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stored kinetic energy</td>
<td>Mass</td>
<td>Inductor</td>
</tr>
<tr>
<td>Stored potential energy</td>
<td>Compliance</td>
<td>Capacitor</td>
</tr>
<tr>
<td>Dissipated energy</td>
<td>Damper</td>
<td>Resistor</td>
</tr>
</tbody>
</table>

3.2 LEM for a Cantilever Beam Piezoelectric Transducer

The schematic view of a typical piezoelectric energy harvesting transducer is shown in Figure 3-1. The basic structure is that of a composite beam, which consists of a shim, piezoelectric patch and proof mass. The beam is in a cantilevered configuration, with the fixed end clamped to the vibrating surface and the free end allowed to move. For the remainder of this work, the clamped and free ends of the beam will be referred to as the base and tip, respectively. As the surface vibrates, the base moves with it, placing the entire beam in an accelerating reference frame. A distributed inertial force is developed along the transducer, causing the beam to deflect and strain to be developed in the piezoelectric material. The strain induces a voltage in the piezoelectric, thus converting the mechanical energy of the vibrating surface into electrical energy.

![Figure 3-1. Cantilevered composite beam.](image-url)
A LEM presented by Kasyap [19] that describes the behavior of the piezoelectric transducer beam is shown in Figure 3-2. The model contains elements in both the mechanical and electrical domains, as well as the transduction between the two. The notation shown here follows that of the original work, where the first subscript denotes the domain of an element (m for mechanical and e for electrical), and the second denotes the condition under which it is measured (s for short (electrical) and b for blocked (mechanical)). The mechanical domain is comprised of three components, mechanical mass, $M_m$, compliance, $C_{ms}$, and damping, $R_m$, while the electrical domain includes the piezoelectric capacitance, $C_{eb}$, and the dielectric loss, $R_e$. The conjugate power variables are the force, $F_m$, and tip velocity $U_m$, in the mechanical domain, and the voltage, $V$, and current, $I$, in the electrical domain. The electromechanical transduction is modeled with an ideal transformer whose turns ratio is $\phi$.

Figure 3-2. LEM of a composite cantilevered beam transducer.

The mechanical domain of the LEM shown in Figure 3-2 is based on modeling the cantilevered composite beam as a single degree of freedom (SDOF) mass-spring-damper system. This modeling technique assumes that the kinetic and potential energy of the distributed system are lumped at the tip of the beam and that the dynamics of the beam tip can be modeled by an effective mechanical mass, short-circuit compliance, and mechanical damping. A schematic diagram of the SDOF system is shown in Figure 3-3,
where \( x \) is the absolute displacement of the beam tip, and \( y \) is the absolute displacement of the the base. When an acceleration input is applied to the base of the system, the motion at the tip of the beam is found by summing the forces

\[
\sum F = ma. \tag{3-1}
\]

Applying the free body diagram for this system, also shown in Figure 3-3, to Equation 3–1 gives

\[
-(x - y) \frac{1}{C_{ms}} - (\dot{x} - \dot{y}) R_m = M_m \ddot{x}. \tag{3-2}
\]

By defining a relative tip displacement, \( z(t) = x(t) - y(t) \), Equation 3–2 can be re-written as

\[
M_m \ddot{z} + R_m \dot{z} + \frac{1}{C_{ms}} z = -M_m \ddot{y}. \tag{3-3}
\]

The use of Equation 3–3 allows the SDOF system shown in Figure 3-3 to be modeled in a different inertial reference frame, where the dynamic behavior of the system is now defined in terms of the relative displacement between the tip and the base, \( z \). A schematic of the SDOF system using the relative displacement is shown in Figure 3-4A. The right hand side of Equation 3–3, \(-M_m \ddot{y}\), is the product of the effective mechanical mass and the base acceleration. This term is modeled as an effective inertial tip force, \( F_m \), and is the mechanical effort variable used in the transducer LEM. The flow variable in the mechanical domain is the relative tip velocity, and is modeled as \( U_m = \dot{z} \).

![Figure 3-3](image)

Figure 3-3. SDOF mass-spring-damper system showing absolute displacement.
A comparison between Equation 3-1 and the governing equation for the RLC electrical circuit shown in Figure 3-4B,

\[ L\ddot{q} + R\dot{q} + \frac{1}{C}q = V, \]  

(3-4)

shows the mathematical analog between the lumped mechanical parameters and circuit elements. Equating \( z \) in the mechanical domain to \( q \) in the electrical domain, masses are modeled with inductors, damping is modeled with resistors, and compliance is modeled with capacitors, as described by Table 3.1. Since the behavior of second order systems is the same for both the mechanical and electrical domains, the underlying physics of the system remain the same with either representation.

![Figure 3-4. LEM of the mechanical domain represented as a A) mass-spring-damper system and B) RLC system.](image)

In a study by Erturk and Inman [67, 68] the validity of the SDOF mass-spring-damper assumption was explored for cantilevered composite beams in reference to energy harvesting systems. In this study, the response of the SDOF system was mathematically compared to that of the dynamic Euler-Bernoulli beam equation for the first resonant mode. For the comparison of the two models, the electromechanical transduction caused by the piezoelectric effect was ignored, and only the mechanical effects of the composite beam were considered. It was shown that the SDOF assumption is not valid for beams where the proof mass is small compared to the mass of the composite shim, due primarily to the effects of the distributed mass of the beam. It was further shown that a correction
factor, $\mu$, is needed to accurately model the tip displacement. In fact, without the correction factor, the SDOF assumption was shown in some cases to under predict the tip displacement by more than 35%.

The correction factor proposed by Erturk and Inman modifies the governing equation (Equation 3-3) to become

$$M_m \ddot{z} + R_m \dot{z} + \frac{1}{C_{ms}} z = \mu F_m.$$  \hspace{1cm} (3–5)

This modified governing equation is of the same form as that used to derive the transducer LEM in 3-2, with the only exception being the multiplicative factor of $\mu$ associated with the applied base force, $F_m$. Therefore, the only change of the LEM required to account for the correction factor is to multiply $F_m$ by $\mu$. An empirical approximation for the value of $\mu$ is given as

$$\mu = \frac{(M_t/M_{Beam})^2 + 0.603 (M_t/M_{Beam}) + 0.08955}{(M_t/M_{Beam})^2 + 0.4637 (M_t/M_{Beam}) + 0.05718},$$  \hspace{1cm} (3–6)

where $M_t$ is the mass at the tip and $M_{Beam}$ is the beam mass. If there is no tip mass ($M_t = 0$) the value of $\mu$ is approximately 1.566. On the other hand, if the tip mass is large compared to the mass of the beam, Equation 3–6 approaches unity, and the correction factor is not needed.

The electrical domain of the transducer LEM captures the behavior of the piezoelectric patch. Since the piezoelectric portion of the transducer is a dielectric material sandwiched between two electrodes, it is modeled with the capacitor $C_{eb}$. Here the subscript $b$ indicates that the transducer is mechanically blocked. This means that there is zero velocity in the mechanical domain, and therefore no transduction into the electrical domain. The loss due to the non-ideality of the dielectric material is modeled with the resistor $R_e$. This dielectric loss term can be related to the loss tangent $\tan \delta$ [69]

$$R_e = \frac{1}{\tan \delta (2\pi f_n C_{eb})},$$  \hspace{1cm} (3–7)

where $\tan \delta$ is a material parameter of the piezoelectric representing the ratio of power losted to power stored. The loss tangent is defined as the ratio of the resistive and reactive
parts of the dielectric impedance. The value of the loss tangent is often difficult to predict from first principles, and is typically modeled based on empirical results.

In the LEM shown in Figure 3-2, the electromechanical transduction is modeled with a ideal transformer with a turns ratio of \( \phi \). The transformer couples the conjugate power variables and allows the elements in one domain to be reflected into the other using standard circuit techniques [61]. The turns ratio, \( \phi \), is defined as

\[
\phi = -\frac{d_{eff}}{C_{ms}},
\]

where \( d_{eff} \) is the effective piezoelectric coefficient. The effective piezoelectric coefficient is defined as

\[
d_{eff} = \frac{w_{tip}}{V},
\]

where \( w_{tip} \) is the relative tip displacement of the transducer.

An intuitive examination of the model and its underlying assumptions provides a useful sanity check. If the effects from the electrical domain are removed, corresponding to the short-circuit condition where \( V = 0 \), there is no transduction and the beam is simply a mechanical structure. In this case, the mechanical domain simplifies to a second order, SDOF mass-spring-damper system as expected. In a similar fashion, if the beam is mechanically blocked, there can be no velocity and therefore \( U_m = 0 \). For the blocked case, there is no vibration energy in the mechanical domain, and the transducer reduces to a simple piezoelectric capacitor with dielectric loss.

The basic assumption that allows for the modeling of the mechanical domain as a SDOF system is that the transducer obeys linear Euler-Bernoulli beam mechanics and that it operates near to or below its first resonant frequency. This assumption places some limits on the operating range for which the LEM remains valid. For instance, Euler-Bernoulli beam theory assumes that the beam is long (length >> width) and slender (length >> thickness) and that the displacement of the beam is on the order of its
thickness [70, 71]. Therefore the model is not valid for structures that violate the geometry assumptions and base accelerations that cause large tip displacements.

3.3 Simplified Electrical Domain Model

In the previous section, a LEM comprised of circuit elements was presented to model the electromechanical behavior of piezoelectric transducers in order to more easily integrate the transducer model with the electrical domain behavior of the power converter. While the lumped model shown in Figure 3-2 is much less complex than a fully distributed electromechanical model, an even further simplified resonant model, shown in Figure 3-5, is often used for energy harvesting research [9–11, 13, 18, 31, 72–74]. In this section the derivation of this simplified resonant model from the complete LEM in Figure 3-2 will be presented and the fundamental differences between the two models will be addressed.

Figure 3-5. Simplified electrical domain LEM at resonance.

The first step in the simplification of the LEM in Figure 3-2 is to remove the ideal transformer by reflecting the mechanical domain components into the electrical domain. For the generic transformer circuit shown in Figure 3-6, the rules used for reflecting elements from the primary side to the secondary side are [75]:

- Multiply the primary impedance by \( n^2 \)
- Multiply the primary voltage by \( n \)
- Divide the primary current by \( n \)

By applying the transformer reflection rules to the LEM circuit in Figure 3-2, the reflected LEM circuit shown in Figure 3-7 is produced. The voltage source in the electrical domain of the model has been removed, and will be replaced with a power converter in
Chapter 4. The reflected variables are defined as

\[ F_m^* = \frac{F_m}{\phi}, \quad (3-10) \]

\[ U_m^* = U_m\phi, \quad (3-11) \]

\[ M_m^* = \frac{M_m}{\phi^2}, \quad (3-12) \]

\[ C_m^* = C_m\phi^2, \quad (3-13) \]

and

\[ R_m^* = \frac{R_m}{\phi^2}. \quad (3-14) \]

The addition of the * indicates that the mechanical parameter has been reflected into the electrical domain and will be continued throughout this work.

Figure 3-6. Generic transformer circuit.

Figure 3-7. LEM reflected into the electrical domain.

The next simplification made to the model is to assume that the transducer is being excited at its resonant frequency, specifically the short-circuit mechanical resonant
frequency. The impedance of the inductor (reflected mechanical mass) is given by

\[ Z_{M_m^*} = j\omega M_m^*, \tag{3–15} \]

and the impedance of the capacitor (reflected mechanical compliance) is given by

\[ Z_{C_{ms}^*} = -\frac{j}{\omega C_{ms}^*}, \tag{3–16} \]

where \( \omega = 2\pi f \). At the short-circuit mechanical resonant frequency, the impedance contributions of the effective mass and compliance sum to zero and the LEM can be represented by Figure 3-8.

Figure 3-8. Electrical domain LEM at short-circuit resonance.

The final simplification to be made to the circuit model is to combine the resistive elements \( R_m^* \) and \( R_e \). The voltage source and series resistance, \( F_m^* \) and \( R_m^* \), can be represented with an equivalent current source and parallel resistance by using a Norton transformation \[75\]. The results of this transformation shown in Figure 3-9, where \( I_{piezo} \) is defined as

\[ I_{piezo} = \frac{F_m^*}{R_m^*}. \tag{3–17} \]

The parallel resistors, \( R_m^* \) and \( R_e \), can now be combined into an equivalent resistance, \( R_{piezo} \), which is defined as

\[ R_{piezo} = \frac{R_m^*R_e}{R_m^* + R_e}. \tag{3–18} \]

Figure 3-5 shows the simplified circuit with \( R_{piezo} \). Also, the blocked electrical capacitance, \( C_{eb} \), is equivalent to \( C_{piezo} \) in Figure 3-5.
Figure 3-9. Electrical domain LEM at resonance with a Norton transformation.

While these simplifications have reduced the complexity of the LEM, it has been at the expense of model robustness. For example, making the assumption of resonant operation removes the inductor and capacitor from the LEM. However, the resonant model is only technically valid for a single specific frequency and there is no way to determine an effective bandwidth or analyze the non-resonant behavior of the transducer. From a practical point of view, trying to design a transducer to have an exact resonant frequency is not trivial. Another detriment of the simplified resonant model is that any information about the velocity (and therefore also the relative tip displacement), is lost when $R_m$ and $R_e$ are combined into $R_{piezo}$. 

\[ I_{piezo} \] 
\[ R_m \] 
\[ C_{eb} \] 
\[ R_e \]
CHAPTER 4
POWER CONVERTERS FOR PIEZOELECTRIC TRANSDUCERS

In Chapter 3, a lumped element model was presented which captures the electromechanical behavior of the piezoelectric transducer. In order to implement a practical energy harvesting system, the transducer must be combined with a power converter. The role of the power converter is to maximize the amount of energy delivered to the load electronics. In order to be effective, the power converter must condition the electrical energy generated by the transducer into a form that is compatible with the load electronics. For the piezoelectric beam transducer, modeled in Chapter 3, the generated voltage is an ac signal. However, the electronic circuits that comprise the load are designed to operate with a specific dc voltage. Deviations from the specified value can lead to poor operation or failure of the circuit. The power converter must therefore be able to rectify the ac voltage signal generated by the transducer, and often regulate the voltage level delivered to the load.

This chapter examines the various power converter topologies that have been implemented for piezoelectric-based, vibration energy harvesting. A brief discussion of different methods for modeling the load is presented first. The simple passive rectifier-capacitor circuit is presented next, followed by a more robust active configuration. Two more active schemes involving synchronized switch timing, the synchronized switch harvesting on inductor (SSHI) method and the synchronous charge extraction (SCE) method, are also examined. For simplicity and uniformity, the operation for all of the power converter configurations presented in this chapter are described using the simplified resonant model presented in Chapter 3, where the input vibration frequency is assumed to be the short-circuit resonant frequency of the transducer. The chapter concludes with a comparison of the various power converter topologies and provides justification for using the pulsed resonant converter in this work.
4.1 Load Modeling

The electronic load for an energy harvesting system is typically modeled as either an equivalent resistor [3, 11, 23, 27, 28, 63, 76] or a constant voltage source [9, 10, 29, 77]. The equivalent resistor representation is strictly dissipative, which only models energy being removed from the harvesting system, and does not take into account the behavior of storage elements. Storage elements, such as rechargeable chemical batteries, store excess harvested energy and maintain relatively stable voltage levels for the load electronics. When a battery is included in the system, the load is typically modeled as a constant voltage source. This model assumes an ideal battery, where the voltage is assumed to be independent of its stored energy. While most batteries chemistries do show at least slight variation in voltage as they are charged and discharged, the ideal situation is useful for comparing the performance of power converters. For the remainder of this chapter, the load will be modeled with a constant voltage source to represent an ideal battery.

4.2 Standard Rectifier-Capacitor Circuit

The standard rectifier-capacitor circuit is a passive topology that provides rectification and smoothing of an input signal in order to provide a stable dc output [78]. A piezoelectric transducer connected to a standard rectifier-capacitor circuit is modeled in Figure 4-1. The full-wave bridge rectifier, composed of diodes $D_1 - D_4$, rectifies the voltage $v_{piezo}(t)$, and the large capacitor, $C_{rect}$, filters out any non-dc components. Here it is assumed that the output of the piezoelectric transducer is sinusoidal with a peak amplitude, $v_{piezo}(t)$, and frequency, $\omega$. In order for this circuit to function correctly, $C_{rect}$ should be chosen such that $C_{rect} >> C_{piezo}$ [8].

For the configuration shown in Figure 4-1, Ottman et al. [9] demonstrated that the power delivered to the load could be expressed as

$$P_{rectcap} = \frac{2V_{rect}}{\pi} (I_{Piezo} - V_{rect}\omega C_{Piezo}) , \quad (4-1)$$
where $V_{\text{rect}}$ is the rectified voltage. Equation 4–1 indicates that the power delivered to the load is a function of $V_{\text{rect}}$. A maximum amount of power, $P_{\text{rectcap, max}}$, given by

$$P_{\text{rectcap, max}} = \frac{I_{\text{Piezo}}^2}{2\pi \omega C_{\text{Piezo}}},$$

(4–2)

can be achieved when $V_{\text{rect}}$ is at an optimal voltage. The optimal voltage, $(V_{\text{rect}})_{\text{opt}}$, is expressed as

$$(V_{\text{rect}})_{\text{opt}} = \frac{I_{\text{Piezo}}}{2\omega C_{\text{Piezo}}}.\quad (4–3)$$

![Passive rectifier-capacitor circuit](image)

Figure 4-1. Passive rectifier-capacitor circuit.

### 4.3 Active Rectifier-Capacitor Circuit

For the topology shown in Figure 4-1, the value of $V_{\text{rect}}$ is essentially clamped at $V_{\text{Load}}$ by the battery voltage, and can only achieve maximum power transfer if $V_{\text{Load}}$ happens to be equal to $(V_{\text{rect}})_{\text{opt}}$. While it may be possible to choose a battery whose voltage meets this requirement, this is not a very robust design technique.

In order to achieve better matching to the load, Ottman et al. [9, 10] designed the active rectifier-capacitor topology, shown in Figure 4-2. In this circuit, a DC/DC converter decouples the rectifier-capacitor from the battery load. A high-efficiency pulse width modulated (PWM) switching converter controls the value of $V_{\text{rect}}$ by varying the duty cycle of the PWM signal. An active feedback controller ensures that $V_{\text{rect}}$ is tuned to $(V_{\text{rect}})_{\text{opt}}$ for maximum power transfer. The efficiency of the switching converter demonstrated was on the order of 80%, so most of the harvested energy was delivered to the battery. A power increase of 400% over the standard rectifier-capacitor circuit was
reported [9], but this power increase will depend highly on how close $V_{Load}$ initially is to $(V_{rect})_{opt}$.

![Circuit Diagram]

Figure 4-2. Active rectifier-capacitor circuit with battery load.

### 4.4 Synchronized Switch Harvesting on Inductor (SSHI)

Another active power converter topology based on the rectifier-capacitor circuit is referred to as synchronized switch harvesting on inductor (SSHI) \[7, 27, 29, 74, 79, 80\]. The basic topology is the same as the rectifier-capacitor with the addition of an inductor and a switch. Timing of the SSHI circuit is based on the mass-spring-damper model of the piezoelectric transducer and is synchronized with the mass displacement, $w_{tip}(t)$ \[27\]. Since the effective mass is lumped at the tip, tip displacement can be used to synchronize timing \[27, 29\]. Two different SSHI configurations have been developed by Badel et al. \[29\], parallel and series, and the specific operation of each is presented in the following sections.

#### 4.4.1 Parallel SSHI Configuration

The parallel SSHI circuit, shown in Figure 4-3, places a switch and inductor in parallel with the piezoelectric element. For the majority of each vibration period, the switch, $SW_{SSHI}$, is open and the circuit operates similarly to the standard rectifier-capacitor circuit. When the displacement reaches a maximum, $SW_{SSHI}$ is closed and an LC resonant circuit is formed between $C_{piezo}$ and $L_{SSHI}$. Once the switch is closed, it remain in this mode for one half of an electrical oscillation period, $\tau_{LC}$, during which time the voltage across the piezoelectric is inverted. The waveforms demonstrating the
SSHI voltage inversion are shown in Figure 4-4. The converter is designed such that the electrical period is instantaneous compared to the mechanical oscillation period, $\tau_{osc}$,

$$\tau_{LC} = 2\pi \sqrt{\frac{C_{piezo} L_{SSH1}}{C_{piezo}}} < \tau_{osc}.$$  \hspace{1cm} (4-4)

![Parallel SSHI schematic.](image)

The voltage inversion is not perfect due to finite losses of the switch. The imperfect inversion is described by [29]

$$V_{piezo}(t_{sw} + \tau_{LC}) = V_{piezo}(t_{sw}) e^{-\frac{Q_I}{2}},$$  \hspace{1cm} (4-5)

where $t_{sw}$ is the switching instant and $Q_I$ is the quality factor of the LC circuit. The quality factor, $Q_I$, can be defined as

$$Q_I = \frac{1}{R_{SW}} \sqrt{\frac{L_{SSH1}}{C_{piezo}}},$$  \hspace{1cm} (4-6)

where $R_{SW}$ is the finite resistance of $SW_{SSH1}$ and the coil resistance of the inductor.

![Parallel SSHI waveforms.](image)
The power harvested with the parallel SSHI configuration is a function of the rectified voltage, $V_{\text{rect}}$, given by

$$P_{\text{parallel,SSHII}} = \frac{V_{\text{rect}}}{\pi} \left(2I_{\text{Piezo}} - \omega C_{\text{Piezo}} V_{\text{rect}} \left(1 - e^{\frac{\pi}{\omega t}}\right)\right).$$

(4–7)

The optimal rectified voltage, $(V_{\text{rect}})_{\text{opt}}$, which yields maximum power is found to be

$$(V_{\text{rect}})_{\text{opt}} = \frac{I_{\text{Piezo}}}{\omega C_{\text{Piezo}} \left(1 - e^{\frac{\pi}{\omega t}}\right)}.\quad (4–8)$$

In order to ensure that the rectified voltage is set to $(V_{\text{rect}})_{\text{opt}}$, a DC/DC power converter is implemented in a similar fashion to the active rectifier-capacitor circuit. When the rectified voltage is constrained to $(V_{\text{rect}})_{\text{opt}}$, the maximum power for the parallel SSHI becomes

$$P_{\text{max}} = \frac{I_{\text{Piezo}}^2}{\pi \omega C_{\text{Piezo}} \left(1 - e^{\frac{\pi}{\omega t}}\right)}.\quad (4–9)$$

### 4.4.2 Series SSHI Configuration

The series configuration of the SSHI circuit places the inductor and switch between the transducer and diode bridge in a series configuration, as shown in Figure 4-5. The switching behavior of $SW_{\text{SSHII}}$ is identical to the parallel case, where the switch is open for the majority of the vibration cycle and closes briefly when the displacement is at its maximum value.

![Series SSHI schematic](image)

Figure 4-5. Series SSHI schematic.
The piezoelectric voltage and displacement waveforms are shown in Figure 4-6. When $SW_{SSH1}$ closes, energy is transferred to the load via $L_{SSH1}$. The power delivered to the load is again a function of $V_{rect}$, and is defined as

$$P_{series,SSH1} = \frac{2V_{rect}}{\pi} \frac{1 + e^{\frac{-\pi I_{piezo}}{\omega C_{piezo}}}}{1 - e^{\frac{-\pi I_{piezo}}{\omega C_{piezo}}}} (I_{piezo} - \omega C_{piezo} V_{rect}). \quad (4-10)$$

The optimal rectified voltage is given by

$$V_{rect, opt}^{(V)} = \frac{I_{piezo}}{2\omega C_{piezo}}, \quad (4-11)$$

which results in a maximum power of

$$P_{max} = \frac{I_{piezo}^{2}}{2\pi \omega C_{piezo}} \frac{1 + e^{\frac{-\pi I_{piezo}}{\omega C_{piezo}}}}{1 - e^{\frac{-\pi I_{piezo}}{\omega C_{piezo}}}}. \quad (4-12)$$

A DC/DC power converter is again used to set the rectified voltage to $(V_{rect})_{opt}$ [29].

![Figure 4-6. Series SSHI waveforms.](image)

**4.5 Synchronous Charge Extraction**

Synchronous charge extraction (SCE) is a power conversion process that harvests electrical energy when charge accumulated on the intrinsic capacitance of the piezoelectric reaches a maximum [29]. In a synchronous charge extraction circuit, the piezoelectric element is effectively disconnected from the power converter circuitry and allowed to operate in an open-circuit configuration, shown in Figure 4-7A. Since zero current flows to the load, the generated charge is stored on the capacitance, $C_{piezo}$. The amount of energy
stored on this capacitance increases with charge, $Q_{\text{piezo}}$, and can be expressed as

$$E = \frac{1}{2} Q_{\text{piezo}}^2 C_{\text{piezo}}.$$  \hfill (4–13)

When the charge is maximized, corresponding to maximum energy, the piezoelectric element is connected to the power converter circuitry, shown in Figure 4-7B, and the stored energy is transferred to the load. Since the charge on a capacitor is directly proportional to its voltage,

$$Q_{\text{piezo}}(t) = C_{\text{piezo}} v_{\text{piezo}}(t),$$  \hfill (4–14)

the transfer of energy from the piezoelectric capacitance to the power converter circuit can be triggered from a voltage maximum.

Figure 4-7. Synchronous charge extraction circuit when A) accumulating charge, and B) transferring charge.

Several circuit configurations have been developed to implement synchronous charge extraction. The topologies, operation, and waveforms for two SCE circuits are presented in the following sections.

4.5.1 Transformer Topology

The transformer based circuit for SCE, shown in Figure 4-8, was developed by Badel et al. [29]. Operation of this circuit can be split into three repeating phases, 1-3, as shown in Figure 4-9. During phase 1, the N_{\text{switch}} (named for the type of MOSFET used to implement it) is open, and current cannot flow into the converter. As a result, charge accumulates on $C_{\text{piezo}}$, and $v_{\text{piezo}}(t)$ increases in a sinusoidal manner. When the voltage
reaches a maximum, phase 2 begins and the N\textsubscript{Switch} closes. An LC resonator is now formed between $C_{\text{piezo}}$ and the primary side of the transformer, $L_p$. As energy stored on $C_{\text{piezo}}$ is transferred to $L_p$, the current in the primary, $i_{LP}(t)$, begins to increase and $v_{\text{piezo}}(t)$ begins to decrease. The blocking diode, $D_5$, prevents the flow of current in the secondary winding, and energy is stored on the transformer. The windings of the transformer are designed such that the electrical period of this LC resonator, $\tau_{LC}$, is much smaller than the period of the mechanical vibration, $\tau_{osc}$,

$$\tau_{LC} \ll \tau_{osc}. \quad (4-15)$$

This design technique is used to ensure that the transfer of energy from $C_{\text{piezo}}$ to the transformer is almost instantaneous with respect to the vibration frequency. When $v_{\text{piezo}}(t)$ reaches zero, corresponding to a complete transfer of energy, phase 3 begins and the N\textsubscript{Switch} opens. During phase 3, energy stored on the transformer is delivered to the load via the secondary current, $i_{LS}(t)$. When all of the energy has been delivered to the load, the N\textsubscript{Switch} remains open and operation resets to phase 1.

![SCE transformer topology](image)

Figure 4-8. SCE transformer topology.

For an ideal power converter, where parasitic losses are neglected, the power harvested using this SCE topology is given as \cite{8}

$$P_{XFR} = \frac{I_{Piezo}^2 R_{Piezo}}{2\pi} \frac{\omega \tau}{1 + \omega^2 \tau^2} \left(1 + e^{-\frac{\pi}{\omega \tau}}\right)^2, \quad (4-16)$$

where $\tau = C_{\text{piezo}} R_{\text{piezo}}$. 

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4.5.2 Pulsed Resonant Converter Topology

The topology of the pulsed resonant converter, presented by Xu et al. [12], is similar to that of the transformer-based design presented in the previous section. Unlike the transformer implementation, which uses two coupled inductors (the transformer) and a single switch, the PRC uses two switches and a single inductor. The PRC is shown schematically in Figure 4-10, and the operational waveforms are shown Figure 4-11.

The operation of the PRC is split into three phases, 1-3. During phase 1, both the N_{Switch} and the P_{Switch} are open, and \( v_{\text{piezo}}(t) \) ramps up sinusoidally as charge accumulates on \( C_{\text{piezo}} \). When \( v_{\text{piezo}} \) reaches a maximum, phase 2 begins, the N_{Switch} closes, and an LC resonant circuit is formed between \( C_{\text{piezo}} \) and \( L_{PRC} \). Energy is transferred from \( C_{\text{piezo}} \)
to $L_{PRC}$ via $i_{LPRC}(t)$, which continues until $v_{piezo}(t)$ reaches zero. The zero crossing of $v_{piezo}(t)$ initiates phase 3, where $N_{Switch}$ is open and the $P_{Switch}$ is closed. During phase 3, energy stored on the inductor is transferred to the load. Phase 3 begins when the energy transfer to the load is complete, and $i_{LPRC}(t)$ is zero.

![SCE PRC waveforms](image)

Figure 4-11. SCE PRC waveforms.

The power harvested by an ideal PRC is given as [12]

$$P_{PRC} = \frac{I_{Piezo}^2 R_{Piezo}}{2\pi} \frac{\omega \tau}{1 + \omega^2 \tau^2} \left(1 + e^{\omega \tau} \right)^2,$$  \hspace{1cm} (4–17)

which is the same as that harvested by the ideal transformer topology. The result in Equation 4–17 was initially presented by Ngo et al. [8], and a complete derivation is presented in Appendix A.

### 4.6 Summary of Power Converters

In the preceding sections of this chapter, the topology and operation of power converters used for piezoelectric-based energy harvesting were presented. A summary of the power converters discussed in this chapter is shown in Table 4-1. In order to choose a power converter for this work, it is necessary to examine the intricacies of each topology and assess its suitability for a design for power system.
Table 4-1. Comparison of energy harvesters.

<table>
<thead>
<tr>
<th>Converter</th>
<th>Maximum Power</th>
<th>Switching Control</th>
<th>Self Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active Rect-Cap</td>
<td>$P_{\text{max}} = \frac{I_{\text{P} \text{iezo}}^2}{2\pi \omega C_{\text{P} \text{iezo}}}$</td>
<td>N/A</td>
<td>No</td>
</tr>
<tr>
<td>SSHI-Parallel</td>
<td>$P_{\text{max}} = \frac{I_{\text{P} \text{iezo}}^2}{\pi \omega C_{\text{P} \text{iezo}} \left(1-e^{-\frac{\pi T}{Q I}}\right)}$</td>
<td>Displacement</td>
<td>No</td>
</tr>
<tr>
<td>SSHI-Series</td>
<td>$P_{\text{max}} = \frac{I_{\text{P} \text{iezo}}^2}{2\pi \omega C_{\text{P} \text{iezo}} \left(1-e^{-\frac{\pi T}{Q I}}\right)}$</td>
<td>Displacement/Voltage</td>
<td>No</td>
</tr>
<tr>
<td>SCE-Transformer</td>
<td>$P_{\text{PRC}} = \frac{I_{\text{P} \text{iezo}}^2 R_{\text{P} \text{iezo}}}{2\pi} \frac{\omega T}{1+\omega^2 \tau^2} \left(1+e^{-\frac{\omega T}{\tau T}}\right)^2$</td>
<td>Voltage</td>
<td>Yes</td>
</tr>
<tr>
<td>SCE-PRC</td>
<td>$P_{\text{PRC}} = \frac{I_{\text{P} \text{iezo}}^2 R_{\text{P} \text{iezo}}}{2\pi} \frac{\omega T}{1+\omega^2 \tau^2} \left(1+e^{-\frac{\omega T}{\tau T}}\right)^2$</td>
<td>Voltage</td>
<td>Yes</td>
</tr>
</tbody>
</table>

4.6.1 Design Considerations for DFP Systems

A number of factors must be considered when choosing a power converter for an energy harvesting system. When employing a DFP methodology, where the importance of absolute size is secondary to the generation of a specific power target, the most important factor is the total amount of power delivered to the load electronics. To provide a fair comparison of the power converters presented in this chapter, it is assumed that each converter will deliver the maximum amount of power possible for its topology. In addition to maximum power, a practical system implementation must consider the overhead power required to operate the control electronics. The sensing and switching required for active switching power converters requires a finite amount of energy. Additionally, DC/DC switching converters used in several of the presented implementations must generate control signals and have feedback for stable PWM operation.

Size of the power converter is another important design consideration for energy harvesting systems. While many of the electronic components used for sensing and control can be fabricated on chip, the integration of magnetic and large capacitive components is not trivial, and often comes at the cost of performance. Typical designs for DC/DC
converters include large filter capacitors and inductors which should not be neglected in the design process. The number and size of such components should be kept to a minimum in order to keep total system size down.

4.6.2 Power Converter Comparison

The active rectifier-capacitor configuration is the most basic active power converter, consisting of only a diode bridge, capacitor, and DC/DC switching converter. Since there is no synchronized switching in this circuit, the active rectifier-capacitor has little sensing and control circuitry. The only waveforms that must be generated are the control signals for the DC/DC converter. While this design requires low overhead power, the total power produced by the active rectifier-capacitor circuit is typically lower than the other converters. For the SCE topologies, when $1 << \omega \tau$, the maximum power delivered to the load by both the transformer and PRC configurations is 4 times larger than the active rectifier-capacitor circuit. For the SSHI topologies, with quality factors of 5.5, the maximum power delivered is 9 times greater than the active rectifier-capacitor circuit [29].

The SSHI configurations are very similar to the active rectifier-capacitor circuit and are capable of delivering much greater power. However, the addition of the inductor in the SSHI circuits adds several design issues. As stated previously, magnetic components do not integrate well, and the inductor adds unwanted size to the system. Also, finding an inductor with a high quality factor is not trivial. Since the power output of the circuit is a function of quality factor, selection of the inductor is critical. Small inductors with small wire windings tend have larger resistance. According to Equation 4–6, the quality factor decreases with increasing resistance. Finding an inductor with high quality factor often requires larger magnetic components, which ultimately increases system size. Another issue with the SSHI configurations involves the switch timing. Since these circuits are timed with displacement extremes, the power converter must have some way of measuring displacement. Typically, the switch timing for SSHI applications is done external to the energy harvesting system using laser-based or magnetic displacement sensing techniques.
This is a major hindrance for implementing SSHI power converters into self powered systems.

Unlike the active rectifier-capacitor and SSHI topologies, the SCE configurations do not rely on DC/DC converters nor do they have optimal rectified voltages. For this reason, SCE circuits are described as self-optimized. Theoretically, a SCE circuit will deliver the same amount of power regardless of the battery voltage. While a DC/DC converter is not required for these configurations, both the transformer-based and PRC topologies require magnetic components and synchronization. The transformer based circuit has two inductors, but only requires one signal for switching. While this cuts down on the required power budget to operate the converter, the two inductors add size to the system. On the other hand, the single inductor of the PRC requires less space, but it has two switches to control. The precise timing of these switches requires more overhead power for both sensing and switching operations.

In order to implement a system based on the DFP principle, the PRC topology is attractive for several reasons. Unlike the SSHI circuits which require that the tip displacement be monitored, control of the PRC can be accomplished strictly in the electrical domain. In terms of power delivery, the PRC outperforms the active rectifier-capacitor circuit when the factor $\omega T$ is sufficiently large. While the transformer based SCE circuit requires less control circuitry and overhead power than the PRC, the size of the two inductor windings is prohibitive. Using low power IC design techniques, the power required to control the second switch of the PRC can be minimized to offset any design advantages of the transformer topology.

The remainder of this work will focus on the modeling and characterization of an energy harvesting system which implements the piezoelectric composite transducer beam presented in Chapter 3 and the PRC described here. In the next chapter, a new system level model is developed which describes the simultaneous operation of the transducer and power converter. Unlike the model shown in Figure 4-10, where the transducer is
represented by its simplified resonant form, the new model will employ the full LEM from Chapter 3 and be valid over a larger range of frequencies. In addition, losses and non-idealities associated with the implementation of the PRC will also be added to the model.
CHAPTER 5
MODELING OF PRC-BASED ENERGY HARVESTING SYSTEMS

In Chapter 4 various power converter topologies for piezoelectric energy harvesters were presented and justification for using the PRC was provided. The operation of the PRC described in Chapter 4, however, assumed ideal behavior of the power converter, with lossless switches and ideal switch timing. Furthermore, the expression presented for total harvested power (Equation 4–17) was derived from the resonant transducer model, which assumes a very specific operating condition. For the design of practical energy harvesting systems an ideal power converter model should not be assumed. The components of the PRC, including the rectification circuitry, switches, and inductor, all have some finite amount of loss which prevents a complete transfer of energy from the transducer to the load. Switch timing is also not ideal, and in the case of the PRC can reduce the amount of energy transferred from one phase to the next. Additionally, the use of the resonant transducer model places very strict operating limits on the energy harvesting system. The system behavior predicted by this model can only be applied if the short-circuit resonant mechanical frequency of the transducer and the frequency of the vibration source exactly match. Should these frequencies differ, or if the frequency of the vibration source should change, the underlying assumptions of the resonant model are no longer valid.

For the practical design of energy harvesting systems, the fact that the simplified resonant model is only valid at a single operating frequency places severe limitations on its usefulness as a design tool. A more robust model is therefore needed which captures the behavior of the energy harvesting system over a range of operating frequencies. In this chapter, an electromechanical system level model is developed which uses the full transducer LEM presented in Chapter 3, without any simplifying assumptions. In addition, the finite losses associated with the PRC are included in this model, in order to determine how the non-ideal behavior of the power converter affects the overall system...
behavior. For the sake of comparison, a second system-level model is also developed that uses the simplified resonant transducer model and includes the effects of PRC losses. By directly comparing both models with equal PRC loss, it can be determined if the behavior of the simplified model is sufficient as a tool for design approximations.

The system modeling in this chapter is divided into five sections. The first section discusses the sources of parasitic loss in the PRC, which limit the energy harvesting capabilities of the circuit. In the next section, two electromechanical system models are developed, one using the full transducer LEM and one using the simplified resonant LEM. The model development is followed by a comparison of the two cases, where several of the waveforms produced by each are directly compared. In the course of this comparison, previously unreported PRC behavior, in the form of alternating peak heights of the rectified voltage, is predicted by the models. The next section of this chapter provides an explanation for the source of the alternating peak height behavior. In the final section of the chapter, a summary of the system-level modeling is presented.

5.1 Losses in the PRC

To accurately model the physical implementation of a PRC circuit, parasitic losses should not be neglected. This is especially true for energy harvesting applications where output power levels are often on the order of $\mu$Ws. A schematic view of the full transducer LEM and non-ideal PRC circuit, shown in Figure 5-1, illustrates that every component has some finite loss.

The rectifier circuitry in Figure 5-1 is represented by diodes $D_1 - D_4$. The parasitic effect of each diode is modeled as a series combination of the forward voltage drop, $V_{\text{diode}}$, and the conduction loss, $R_{\text{diode}}$. For a standard p/n semiconductor diode the forward voltage drop is on the order of approximately 0.7 V, and can be reduced to around 0.2 V by using a Schottky diode. To further minimize the losses from the forward voltage drops, the diodes can be replaced with an active MOSFET rectifier. An active rectifier is a circuit that uses transistor switches to control the flow of current and rectify the voltage
signal. When active rectification is used, the value of becomes approximately zero and can be ignored. The resistive term remains, however, to account for the finite conduction loss of each transistor. The major disadvantage of active rectification is the external power required to control the timing of the MOSFET gates. For designers, the choice of rectification circuitry should be examined on a case by case basis in order to determine which loss mechanisms (forward voltage drop or control overhead) will have the least impact on system performance. For the model derived in this work, the diode drops are included and can simply be set to zero should synchronous rectification be used.

Figure 5-1. Schematic view of the full LEM and non-ideal PRC.

The other significant sources of parasitic loss in the PRC are the conduction losses of the switches and inductor. These are shown in Figure 5-1 as $R_{NSwitch}$, $R_{PSwitch}$, and $R_{LPRC}$. For the two switches, the resistance values represent the nominal channel resistance at the operating temperature of interest. The inductor loss term, $R_{LPRC}$, combines the conduction losses and core losses of the inductor. The effects of the parasitic losses in the PRC are different during each phase of operation. As a point of notation, a subscript is added to each of the three phases in this work to in order to better explain the flow of energy within the converter. Specifically, ($\rightarrow_x$) is added to each phase, where $x$ indicates the destination of the energy during that particular phase. During
phase1 (→capacitor), when both the N\textsubscript{Switch} and the P\textsubscript{Switch} are open, no current flows into the PRC and the conduction losses do not have any effect. For phase2 (→inductor), the N\textsubscript{Switch} is closed and the flow of energy is from the C\textsubscript{eb} to L\textsubscript{PRC}. A schematic diagram of the energy harvesting system during phase2 (→inductor) is shown in Figure 5-2A. This circuit assumes that the diodes D\textsubscript{1} and D\textsubscript{4} are conducting. The same circuit model occurs when D\textsubscript{2} and D\textsubscript{3} conduct. Since all of the resistive losses occur in series, they are combined into an effective loss element for this phase, \( R_{\text{Phase2}} \), where

\[
R_{\text{Phase2}} = 2R_{\text{diode}} + R_{\text{LPRC}} + R_{\text{N.SW}}. \tag{5–1}
\]

Similarly, the forward voltage drops of the diodes can be combined into an effective diode drop of \( V_{\text{d,eff}} \), where

\[
V_{\text{d,eff}} = 2V_{\text{diode}}. \tag{5–2}
\]

A simplified circuit diagram using the effective values is shown in Figure 5-2B.

Figure 5-2. Schematic diagram of the energy harvester during phase2 (→inductor) operation modeled with A) all loss terms and B) the effective PRC loss, \( R_{\text{Phase2}} \).
When the PRC is operating in phase3 (battery) the N Switch is open and the P Switch is closed as energy is transferred from \( L_{PRC} \) to the load. A schematic diagram of the losses during phase3 (battery) is shown in Figure 5-3A. Similarly to phase2 (inductor), an effective phase3 (battery) resistance, \( R_{Phase3} \) can be found by combining the series resistances where

\[
R_{Phase3} = 2R_{\text{diode}} + R_{LPRC} + R_{P\text{Switch}}. \tag{5-3}
\]

The forward voltage drops can again be combined into an effective drop, \( V_{d,eff} \), whose value is twice \( V_{\text{diode}} \), as was given in Equation 5-2. A simplified circuit diagram for phase3 (battery) using the effective values is shown in Figure 5-3B.

\[\text{Figure 5-3. Schematic diagram of the energy harvester during phase3 (battery) operation modeled with A) all loss terms and B) the effective PRC loss, } R_{Phase3}.\]

### 5.2 Derivation of the Electromechanical Behavior

This section presents the development of two electromechanical models for a PRC-based energy harvesting system. The first model uses the full transducer LEM from Chapter 3, while the second model uses the simplified resonant LEM. A general overview of the procedure used for modeling both systems is shown in Figure 5-4.
Figure 5-4. Flow diagram of the procedure used to model the energy harvesting system.

The model development begins by assuming that the PRC is off and the transducer is allowed to reach steady-state operation. The system waveforms \( v_{\text{rect}} (t), u_{m^*} (t), v_{Cms} (t), i_{LPRC} (t), \text{ etc. . . } \) describing the steady-state behavior are found using Fourier analysis. At some point in time the PRC is turned on, and steady-state operation continues until \( v_{\text{piezo}} (t) \) reaches a peak, which initiates phase2\((\rightarrow \text{inductor})\). The switching from one phase to another is assumed to be instantaneous and the rise times and fall times of the switches is not considered here. This assumption is reasonable if the rise and fall times of the switches are much shorter than the lengths of the individual phases.

The system waveforms during phase2\((\rightarrow \text{inductor})\) are found using time domain analysis instead of Fourier analysis, since steady-state operation can no longer be assumed as a result of the non-linear switching. Time domain analysis produces differential
equations that require initial conditions to solve. The initial conditions at the start of phase2\(_{\text{inductor}}\) are equal to the final conditions of the circuit at the end of steady-state operation. The waveforms for phase3\(_{\text{battery}}\) are found using a similar procedure. Time domain analysis is applied to the phase3\(_{\text{battery}}\) circuit, and initial conditions are found by analyzing the final conditions of phase2\(_{\text{inductor}}\).

After completing phase3\(_{\text{battery}}\) for the first time, the circuit model does not go back to steady-state, but instead enters phase1\(_{\text{capacitor}}\) operation. Again time domain methods are used, and initial conditions are calculated from the end of phase3\(_{\text{battery}}\). The circuit model then proceeds through a cyclical pattern of stepping through the phases (phase2\(_{\text{inductor}}\), phase3\(_{\text{battery}}\), phase1\(_{\text{capacitor}}\)...). The remainder of Section 5.2 is divided into three parts. In the first part, the model for the the model for the energy harvesting system using the full LEM of the transducer with the non-ideal PRC is derived. The simplified resonant LEM with the non-ideal PRC is developed next, followed by a comparison of the two cases.

### 5.2.1 System Modeling Using the Full LEM

A schematic representation of the full electromechanical system with PRC losses is shown in Figure 5-5, where the mechanical components have been reflected into the electrical domain. The resistors \(R_{\text{Phase2}}\) and \(R_{\text{Phase3}}\) represent the total losses of the PRC during phase2\(_{\text{inductor}}\) and phase3\(_{\text{battery}}\), respectively. The results of the model derivation for the individual phases are presented in the following sub-sections. The more rigorous and complete derivation for this model is presented in Appendix B.

![Figure 5-5. Schematic representation of the full LEM and PRC.](image)
5.2.1.1 Steady-state operation

During steady-state operation, the PRC is off and both the \( N_{\text{Switch}} \) and \( P_{\text{Switch}} \) are open. The driving function for this system is the reflected mechanical force applied to the base

\[
F_m^* (t) = F_m^* \sin (\omega t), \tag{5–4}
\]

where \( F_m^* = \frac{M_m a_m}{\phi} \) is the magnitude of the reflected inertial force and \( \omega \) is the radian frequency of oscillation. The term \( a_m \) is the acceleration applied to the base in the mechanical domain and has units of m/s\(^2\). The driving function is assumed to have a phase shift of zero, meaning that the phase shifts of all the other signals are referenced to it.

In order to find the other operational waveforms during steady-state, Fourier analysis is applied to the transducer circuit. The waveforms of the reactive components (currents through inductors and voltages across capacitors) during steady-state are given by

\[
v_{\text{piezo}} (t) = V_{\text{piezo}} \sin (\omega t + \theta_{v\text{piezo}}), \tag{5–5}
\]

\[
u_m^* (t) = U_m^* \sin (\omega t + \theta_{u_m^*}), \tag{5–6}
\]

\[
u_{\text{Cms}} (t) = -V_{\text{Cms}} \sin (\omega t + \theta_{v\text{Cms}}), \tag{5–7}
\]

where the waveform magnitudes are

\[
V_{\text{piezo}} = \frac{\omega R_e C_{ms}^*}{\sqrt{(1 - \omega^2 A_2)^2 + (\omega A_3 - \omega^3 A_1)^2}} F_m^*, \tag{5–8}
\]

\[
U_m^* = \frac{\sqrt{\left(\omega^2 R_e C_{eb} C_{ms}^*\right)^2 + \left(\omega C_{ms}^*\right)^2}}{\sqrt{(1 - \omega^2 A_2)^2 + (\omega A_3 - \omega^3 A_1)^2}} F_m^*, \tag{5–9}
\]

\[
V_{\text{Cms}} = \frac{\sqrt{1 + (\omega R_e C_{eb})^2}}{\sqrt{(1 - \omega^2 A_2)^2 + (\omega A_3 - \omega^3 A_1)^2}} F_m^*, \tag{5–10}
\]
the phase shifts are

\[ \theta_{vpiezo} = \frac{\pi}{2} - \tan^{-1}\left( \frac{\omega A_3 - \omega^3 A_1}{1 - \omega^2 A_2} \right), \] (5–11)

\[ \theta_{um} = \tan^{-1}\left( \frac{1}{-\omega R_e C_{eb}} \right) - \tan^{-1}\left( \frac{\omega A_3 - \omega^3 A_1}{1 - \omega^2 A_2} \right), \] (5–12)

\[ \theta_{V_{Cms}}(j\omega) = \tan^{-1}(\omega R_e C_{eb}) - \tan^{-1}\left( \frac{\omega A_3 - \omega^3 A_1}{1 - \omega^2 A_2} \right), \] (5–13)

and the constants \( A_1 \) - \( A_3 \) are defined as

\[ A_1 = R_e M_m C_{eb} C_{ms} \] (5–14)

\[ A_2 = R_e R_m C_{eb} C_{ms} + M_m C_{ms} \] (5–15)

\[ A_3 = R_e C_{eb} + R_m C_{ms} + R_e C_{ms} \] (5–16)

The waveforms for the reactive components are important because they provide the initial conditions needed to solve the differential equations for phase2 (\( \rightarrow \) inductor).

### 5.2.1.2 Phase2 Operation

After operating in steady-state mode for some time, the PRC is turned on and waits for a peak in the piezoelectric voltage, \( v_{piezo}(t) \). When the peak is detected the \( N_{Switch} \) closes and phase2 (\( \rightarrow \) inductor) begins. Since the circuit is no longer operating in steady-state, Fourier analysis can no longer be applied, and a time domain analysis is necessary. The differential equation describing the behavior of this system is given by

\[ F_m \omega \cos(\omega t) = C_{1p2} \frac{d^4 i_{LPRC}(t)}{dt^4} + C_{2p2} \frac{d^3 i_{LPRC}(t)}{dt^3} + C_{3p2} \frac{d^2 i_{LPRC}(t)}{dt^2} + C_{4p2} \frac{d i_{LPRC}(t)}{dt} + C_{5p2} i_{LPRC}(t) + \frac{V_{DcEFLJp2}^2}{C_m R_e}, \] (5–17)

where the five constants for phase2 (\( \rightarrow \) inductor), \( C_{1p2} \) - \( C_{5p2} \), are given by

\[ C_{1p2} = L_{PRC} M_m C_{eb}, \] (5–18)

\[ C_{2p2} = \frac{M_m L_{PRC}}{R_e} + L_{PRC} R_m C_{eb} + M_m C_{eb} R_{Phase2}, \] (5–19)

\[ C_{3p2} = \frac{L_{PRC} C_{eb}}{C_{ms}} + \frac{L_{PRC} R_m}{R_e} + L_{PRC} + M_m + \frac{M_m R_{Phase2}}{R_e} + R_m C_{eb} R_{Phase2}, \] (5–20)
\[ C_{4p2} = \frac{L_{PRC}}{R_e C_{ms}^*} + R_m^* + \frac{C_{eb} R_{Phase2}}{C_{ms}^*} + \frac{R_m^* R_{Phase2}}{R_e} + R_{Phase2}, \]  
\[ C_{5p2} = \frac{1}{C_{ms}} + R_{Phase2} C_{ms}^* R_e. \]

Since the governing differential equation (Equation 5–17) is fourth order, four initial conditions are needed to find a solution. These initial conditions are

\[ IC1 \equiv i_{LPRC} (t_{init,p2}) = 0, \]  
\[ IC2 \equiv \frac{di_{LPRC} (t_{init,p2})}{dt} = \frac{v_{piezo} (t_{init,p2})}{L_{PRC}} - \frac{V_{Deff,p2}}{L_{PRC}}, \]  
\[ IC3 \equiv \frac{d^2i_{LPRC} (t_{init,p2})}{dt^2} = \frac{u_m^* (t_{init,p2})}{C_{eb} L_{PRC}} - \frac{R_{Phase2} dt_{LPRC} (t_{init,p2})}{L_{PRC}} \]  
\[ \quad - \frac{v_{piezo} (t_{init,p2})}{C_{eb} L_{PRC} R_e}, \]  
\[ IC4 \equiv \frac{d^3i_{LPRC} (t_{init,p2})}{dt^3} = \frac{F_m^* \sin (\omega t_{init,p2})}{M_m^* C_{eb} L_{PRC}} \]  
\[ \quad - \left( \frac{R_{Phase2}}{L_{PRC}} + \frac{1}{R_e C_{eb}} \right) \frac{d^2i_{LPRC} (t_{init,p2})}{dt^2} \]  
\[ \quad - \left( \frac{R_{Phase2}}{R_e C_{eb} L_{PRC}} + \frac{1}{C_{eb} L_{PRC}} \right) \frac{di_{LPRC} (t_{init,p2})}{dt} \]  
\[ \quad - \frac{v_{Cms}^* (t_{init,p2})}{L_{PRC} M_m^* C_{eb}} - \frac{u_m^* (t_{init,p2})}{L_{PRC} M_m^* C_{eb}} - \frac{v_{piezo} (t_{init,p2})}{L_{PRC} M_m^* C_{eb}}, \]

where \( t_{init,p2} \) is the time when phase2 (\( \text{--inductor} \)) begins. The values of the reactive components at \( t_{init,p2} \) are continuous from end of the previous phase (either steady-state or phase1 (\( \text{--capacitor} \))), because capacitor voltages and inductor currents cannot change instantaneously.

Using the initial conditions (Equation 5–23 through Equation 5–26), the governing equation (Equation 5–17) is solved for \( i_{LPRC} (t) \). All of the other system waveforms are found using the value of \( i_{LPRC} (t) \) and its derivatives. The applied mechanical force is assumed to be unchanged by the switching and remains

\[ F_m^* (t) = F_m^* \sin (\omega t). \]
The waveforms of the reactive components during phase\textsubscript{2} (\textarrow{inductor}) are

\begin{align*}
v_{\text{piezo}} (t) &= L_{\text{PRC}} \frac{d^2 i_{\text{PRC}} (t)}{dt^2} + R_{\text{Phase2}} C_{eb} \frac{di_{\text{PRC}} (t)}{dt}, \\
u_m^* (t) &= i_{\text{LPRC}} (t) + v_{\text{piezo}} (t) \frac{1}{R_e} + C_{eb} L_{\text{PRC}} \frac{d^2 i_{\text{LPRC}} (t)}{dt^2} + R_{\text{Phase2}} C_{eb} \frac{di_{\text{LPRC}} (t)}{dt},
\end{align*}

and

\begin{align*}
v_{\text{Cms}} (t) &= F^* (t) - M_m \frac{du_m^* (t)}{dt} - v_{\text{Rm}} (t) - v_{\text{piezo}} (t),
\end{align*}

where the derivative of the reflected velocity term is given by

\begin{align*}
\frac{du_m^* (t)}{dt} &= \left( 1 + \frac{R_{\text{Phase2}}}{R_e} \right) \frac{di_{\text{LPRC}} (t)}{dt} + \left( \frac{L_{\text{PRC}}}{R_e} + R_{\text{Phase2}} C_{eb} \right) \frac{d^2 i_{\text{LPRC}} (t)}{dt^2} \\
&+ (C_{eb} L_{\text{PRC}}) \frac{d^3 i_{\text{PRC}} (t)}{dt^3}.
\end{align*}

5.2.1.3 Phase\textsubscript{3} Operation

During phase\textsubscript{2} (\textarrow{inductor}) the inductor current, $i_{\text{LPRC}} (t)$, increases as energy is transferred from the piezoelectric capacitance to the inductor. When $i_{\text{LPRC}} (t)$ reaches its maximum value, phase\textsubscript{3} (\textarrow{battery}) begins, where the N\textsubscript{Switch} is open and the P\textsubscript{Switch} is closed. As in phase\textsubscript{2} (\textarrow{inductor}), Fourier analysis is not applicable and time domain methods are used. The differential equation describing the behavior of the energy harvesting system during phase\textsubscript{3} (\textarrow{battery}) is

\begin{align*}
F_{m\omega} \cos (\omega t) &= C_{1p3} \frac{d^4 i_{\text{LPRC}} (t)}{dt^4} + C_{2p3} \frac{d^3 i_{\text{LPRC}} (t)}{dt^3} + C_{3p3} \frac{d^2 i_{\text{LPRC}} (t)}{dt^2} \\
&+ C_{4p3} \frac{di_{\text{LPRC}} (t)}{dt} + C_{5p3} i_{\text{LPRC}} (t),
\end{align*}

where the constants $C_{1p3}$ - $C_{5p3}$ are given as

\begin{align*}
C_{1p3} &= L_{\text{PRC}} M_m C_{eb}, \\
C_{2p3} &= \frac{M_m L_{\text{PRC}}}{R_e} + L_{\text{PRC}} M_m C_{eb} + M_m C_{eb} R_{\text{Phase3}}, \\
C_{3p3} &= \frac{L_{\text{PRC}} C_{eb}}{C_{ms}} + \frac{L_{\text{PRC}} R_{m} C_{eb}}{R_e} + L_{\text{PRC}} + M_m + \frac{M_m R_{\text{Phase3}}}{R_e} + R_m C_{eb} R_{\text{Phase3}},
\end{align*}
\[ C4_{p3} = \frac{L_{PRC}}{R_eC_{ms}*} + R_m^* + \frac{C_{eb}R_{Phase3}}{C_{ms}*} + \frac{R_m^*R_{Phase3}}{R_e} + R_{Phase3}, \quad (5-36) \]

and

\[ C5_{p3} = \frac{1}{C_{ms}*} + \frac{R_{Phase3}}{C_{ms}*R_e}. \quad (5-37) \]

As during phase2→inductor, Equation 5–32 is a fourth order differential equation and four initial conditions are needed. The initial conditions are

\[ IC1 \equiv i_{LPRC} (t_{init,p3}) = i_{LPRC} (t_{end,p2}), \quad (5–38) \]

\[ IC2 \equiv \frac{d}{dt}i_{LPRC} (t_{init,p3}) = \frac{v_{piezo} (t_{init,p3})}{L_{PRC}} - \frac{R_{Phase3}}{L_{PRC}} \frac{d}{dt}i_{LPRC} (t_{init,p3}) - \left( V_{battery} + V_{D_{eff},p3} \right) \frac{1}{L_{PRC}}, \quad (5–39) \]

\[ IC3 = \frac{d^2}{dt^2}i_{LPRC} (t_{init,p3}) = \frac{u_m^* (t_{init,p3})}{C_{eb}L_{PRC}} - \frac{R_{Phase3}}{L_{PRC}} \frac{d}{dt}i_{LPRC} (t_{init,p3}) - \left( V_{battery} + V_{D_{eff},p3} \right) \frac{1}{C_{eb}L_{PRC}}i_{LPRC} (t_{init,p3}) - \frac{v_{piezo} (t_{init,p3})}{C_{eb}L_{PRC}R_e}, \quad (5–40) \]

and

\[ IC4 \equiv \frac{d^3}{dt^3}i_{LPRC} (t_{init,p3}) = \frac{F_m^* \sin(\omega t_{init,p3})}{M_m^*C_{eb}L_{PRC}} \frac{d^2}{dt^2}i_{LPRC} (t_{init,p3}) \]

\[ - \left( \frac{R_{Phase3}}{L_{PRC}} + \frac{1}{R_eC_{eb}} \right) \frac{d^2}{dt^2}i_{LPRC} (t_{init,p3}) \]

\[ - \left( \frac{R_{Phase3}}{R_eC_{eb}L_{PRC}} + \frac{1}{C_{eb}L_{PRC}} \right) \frac{d}{dt}i_{LPRC} (t_{init,p3}) \]

\[ - \frac{v_{Cms}^* (t_{init,p3})}{L_{PRC}M_m^*C_{eb}} - \frac{u_m^* (t_{init,p3})}{L_{PRC}M_m^*C_{eb}} - \frac{v_{piezo} (t_{init,p3})}{L_{PRC}M_m^*C_{eb}}, \quad (5–41) \]

where \( t_{end,p2} \) is the end time of phase2→inductor, and is the same as the start time of phase3→battery, \( t_{init,p3} \), under the assumption of instantaneous switching. The initial conditions are found using the continuity of the voltages and currents of the reactive components.

Using the initial conditions (Equation 5–38 through Equation 5–41), the governing equation (Equation 5–32) is solved for \( i_{LPRC} (t) \). Equations describing the other waveforms in the system are found using the value of \( i_{LPRC} (t) \) and its derivatives. The applied
mechanical force is assumed to be unchanged by the switching and remains

\[ F_m^* (t) = F_m^* \sin (\omega t). \]  

(5–42)

The waveforms of the reactive components during phase3(\textarrowright\text{battery}) are

\[ v_{\text{piezo}} (t) = L_{PRC} \frac{di_{PRC} (t)}{dt} + R_{\text{Phase3}} i_{LPRC} (t) + V_{\text{battery}}, \]  

(5–43)

\[ u_m^* (t) = i_{LPRC} (t) + \frac{v_{\text{piezo}} (t)}{R_e} + \left( L_{PRC} C_{eb} \frac{d^2 i_{LPRC} (t)}{dt^2} + R_{\text{Phase3}} C_{eb} \frac{di_{LPRC} (t)}{dt} \right), \]  

(5–44)

and

\[ v_{\text{Cms}} (t) = F_m^* (t) - M_m^* \frac{du_m^* (t)}{dt} - v_{Rm} (t) - v_{\text{piezo}} (t). \]  

(5–45)

where the derivative of the reflected velocity term is given by

\[ \frac{du_m^* (t)}{dt} = \left( 1 + \frac{R_{\text{Phase3}}}{R_e} \right) \frac{di_{LPRC} (t)}{dt} + \left( \frac{L_{PRC}}{R_e} + R_{\text{Phase3}} C_{eb} \right) \frac{d^2 i_{LPRC} (t)}{dt^2} + (C_{eb} L_{PRC}) \frac{d^3 i_{LPRC} (t)}{dt^3}. \]  

(5–46)

### 5.2.1.4 Phase1 Operation

Phase3(\textarrowright\text{battery}) operation of the circuit ends when the current flowing through \( L_{PRC} \) reaches zero. Phase1(\textarrowright\text{capacitor}) operation then begins and both switches are open. Unlike phase2(\textarrowright\text{inductor}) and phase3(\textarrowright\text{battery}) operation where current was present in \( L_{PRC} \), the inductor is disconnected from the transducer during phase1(\textarrowright\text{capacitor}) and there is zero current flowing through it. Instead, to model the waveform behavior occurring during phase1(\textarrowright\text{capacitor}), the piezoelectric voltage, \( v_{\text{piezo}} (t) \) is used. The differential equation describing the behavior of the circuit is given as

\[ F_m^* \omega \cos (\omega t) = C_{1p1} \frac{d^3 v_{\text{piezo}} (t)}{dt^3} + C_{2p1} \frac{d^2 v_{\text{piezo}} (t)}{dt^2} + C_{3p1} \frac{dv_{\text{piezo}} (t)}{dt} + C_{4p1} v_{\text{piezo}} (t), \]  

(5–47)

where the constants \( C_{1p1} - C_{4p1} \) are given as

\[ C_{1p1} = M_m^* C_{eb}, \]  

(5–48)
\[ C_{2p1} = \frac{M_m^*}{R_e} + \frac{R_m^*}{C_{eb}}, \quad (5-49) \]
\[ C_{3p1} = \frac{C_{eb}}{C_{ms}^*} + \frac{R_m^*}{R_e} + 1, \quad (5-50) \]

and
\[ C_{4p1} = \frac{1}{R_e C_{ms}^*}. \quad (5-51) \]

Since Equation 5–47 is a third order differential equation, three initial conditions are needed to find the solution. The initial conditions are

\[ IC1 \equiv v_{\text{piezo}}(t_{\text{init},p1}) = v_{\text{piezo}}(t_{\text{end},p3}), \quad (5–52) \]
\[ IC2 \equiv \frac{dv_{\text{piezo}}(t_{\text{init},p1})}{dt} = \frac{u_m^*(t_{\text{init},p1})}{C_{eb}} - \frac{v_{\text{piezo}}(t_{\text{init},p1})}{C_{eb} R_e}, \quad (5–53) \]

and
\[ IC3 \equiv \frac{d^2v_{\text{piezo}}(t_{\text{init},p1})}{dt^2} = \frac{F_m^* \sin(\omega t_{\text{init},p1})}{M_m^* C_{eb}} - \left( \frac{1}{R_e C_{eb}} + \frac{R_m^*}{M_m^*} \right) \frac{dv_{\text{piezo}}(t_{\text{init},p1})}{dt} \]
\[ - \left( \frac{1}{M_m^* C_{eb}} + \frac{R_m^*}{M_m^* R_e C_{eb}} \right) v_{\text{piezo}}(t_{\text{init},p1}) \quad (5–54) \]

where \( t_{\text{end},p3} \) is the end time of phase 3 (battery), and is the same as the start time of phase 1 (capacitor), \( t_{\text{init},p1} \), under the assumption of instantaneous switching. Using the initial conditions (Equation 5–51 through Equation 5–53), the governing equation (Equation 5–47) is solved for \( v_{\text{piezo}}(t) \). Equations describing the remaining waveforms in the system are found using the value of \( v_{\text{piezo}}(t) \) and its derivatives. The applied mechanical force is assumed to be unchanged by the switching and remains
\[ F_m^*(t) = F_m^* \sin(\omega t). \quad (5–55) \]

The waveforms of the reactive components during phase 1 (capacitor) are
\[ u_m^*(t) = \frac{v_{\text{piezo}}(t)}{R_e} + C_{eb} \frac{dv_{\text{piezo}}(t)}{dt}, \quad (5–56) \]
and

\[ v_{C_{ms}}(t) = F_m^*(t) - M_m^* \frac{du_{m}^*(t)}{dt} - v_{Rm}(t) - v_{piezo}(t), \quad (5-57) \]

where the derivative of the reflected velocity term is given by

\[ \frac{du_{m}^*(t)}{dt} = \frac{1}{R_e} \frac{dv_{piezo}(t)}{dt} + C_{eb} \frac{d^2i_{PRC}(t)}{dt^2}. \quad (5-58) \]

### 5.2.2 System Modeling Using the Simplified Resonant LEM

In Chapter 3, the transformation of the full LEM transducer model into a more compact simplified resonant model was presented. The transformation process involved first reflecting the mechanical domain elements into the electrical domain, and then assuming a short-circuit mechanical resonant frequency of operation. Under this assumption, the impedance of the reflected mechanical mass and compliance elements were removed from the circuit, resulting in the simplified model shown in Figure 5-6A. The final simplification to the circuit model was to perform a Norton source transformation and combine the resistors, \( R_m^* \) and \( R_e \) to obtain the circuit shown in Figure 5-6B.

![Circuit schematics of the simplified resonant (A) voltage source and (B) current source models.](image)

Figure 5-6. Circuit schematics of the simplified resonant A) voltage source and B) current source models.

While the Norton transformation and resistor combination in the simplification of the circuit is useful for reducing the complexity of the model, valuable information regarding the mechanical operation of the transducer is lost. Specifically, by combining \( R_m^* \) with \( R_e \), the tip velocity of the transducer can no longer be determined, since there is no way determine what portion of the current through \( R_{piezo} \) was originally from the mechanical domain and what portion originated in the electrical domain.
For this work, instead of using the more simplified Norton equivalent circuit in Figure 5-6B, the voltage source model in Figure 5-6A is used in order to maintain information about the mechanical behavior of the transducer. The general behavior of the two circuit models is the same. Both assume that the driving mechanical acceleration occurs at the short-circuit mechanical resonant frequency, which removes the impedance effects of $C_{ms}$ and $M_m$. Also, the electrical operation, in terms of the piezoelectric capacitance, is the same because of the equivalency of Norton and Thevenin circuit models. The only disadvantage of using the voltage source model is that the differential equations used to describe the behavior of the circuit are slightly more complex than the current source model due to the presence of an extra circuit element.

A schematic representation of the electromechanical system comprised of the simplified resonant (voltage source) transducer LEM and the PRC with losses is shown in Figure 5-7. The resistors $R_{Phase2}$ and $R_{Phase3}$ represent the total losses of the PRC during phase2 (→inductor) and phase3 (→battery), respectively. The results of the model derivation for the individual phases are presented in the following sub-sections. A more rigorous and complete derivation for this model is presented in Appendix C.

![Figure 5-7. Schematic representation of the simplified resonant LEM and PRC.](image)

5.2.2.1 Steady-state operation

During steady-state operation, the PRC is off and both the $N_{Switch}$ and $P_{Switch}$ are open-circuited. The driving function for this system is the reflected mechanical force

$$F_m^*(t) = F_m^* \sin(\omega t),$$  \hspace{1cm} (5–59)
where $F_m^*$ is the magnitude of the inertial force and $\omega$ is the radian frequency of oscillation. The driving function is assumed to have a phase shift of zero, meaning that the phase shifts of all the other signals are referenced to it.

In order to find the other operational waveforms during steady-state, Fourier analysis is applied to the transducer circuit. The waveforms of the reactive components (currents through inductors and voltages across capacitors) during steady-state are given by

$$v_{\text{piezo}}(t) = V_{\text{piezo}} \sin(\omega t + \theta_{\text{vpiezo}}),$$  
(5–60)

and

$$u_m^*(t) = U_m^* \sin(\omega t + \theta_{um}),$$  
(5–61)

where the magnitudes are

$$V_{\text{piezo}} = \frac{R_e}{\sqrt{(R_m^* + R_e)^2 + (\omega C_{eb} R_e R_m^*)^2}} F_m^*,$$  
(5–62)

and

$$U_m^* = \frac{\sqrt{1 + (\omega C_{eb} R_e)^2}}{\sqrt{(R_m^* + R_e)^2 + (\omega R_m^* R_e)^2}} F_m^*,$$  
(5–63)

and the phase shifts are

$$\theta_{\text{vpiezo}} = -\tan^{-1}\left(\frac{\omega C_{eb} R_e R_m^*}{R_m^* + R_e}\right),$$  
(5–64)

and

$$\theta_{um} = \tan^{-1}(\omega R_e C_{eb}) - \tan^{-1}\left(\frac{\omega C_{eb} R_e R_m^*}{R_m^* + R_e}\right).$$  
(5–65)

The waveforms for the reactive components are important because they provide the initial conditions needed to solve the differential equations for phase2(→inductor).

### 5.2.2.2 Phase2 Operation

After operating in steady-state mode for some time, the PRC is turned on and waits for a peak in the piezoelectric voltage, $v_{\text{piezo}}(t)$. When the peak is detected the $N_{\text{Switch}}$ closes and phase2(→inductor) begins. Since the circuit is no longer operating in steady-state,
Fourier analysis can no longer be applied, and a time domain analysis is necessary. The differential equation describing the behavior of this system is given by

\[ F_m \sin(\omega t) = C1_{p2} \frac{d^2 i_{LPRC}(t)}{dt^2} + C2_{p2} \frac{di_{LPRC}(t)}{dt} + C3_{p2} i_{LPRC}(t), \]  

(5–66)

where the 3 constants for phase 2, \( C1_{p2} - C3_{p2}, \) are given by

\[ C1_{p2} = L_{PRC} R_m^* C_{eb}, \]  

(5–67)

\[ C2_{p2} = \frac{R_m L_{PRC}}{R_e} + C_{eb} R_m R_{Phase2} + L_{PRC}, \]  

(5–68)

and

\[ C3_{p2} = \frac{R_{Phase2} R_m^*}{R_e} + R_m^* + R_{Phase2}. \]  

(5–69)

Since the differential equation in Equation 5–66 is second order, two initial conditions are needed to find a solution. These initial conditions are

\[ i_{LPRC}(t_{init,p2}) = 0, \]  

(5–70)

and

\[ \frac{di_{LPRC}(t_{init,p2})}{dt} = \frac{v_{piezo}(t_{init,p2})}{L_{PRC}}, \]  

(5–71)

where \( t_{init,p2} \) is the time when phase 2 begins. The values of the reactive components at \( t_{init,p2} \) are continuous from end of the previous phase (either steady-state or phase 1).

Using the initial conditions (Equation 5–70 and Equation 5–71), the governing equation (Equation 5–66) is solved for \( i_{LPRC}(t) \). Equations describing the other waveforms in the system are found using the value of \( i_{LPRC}(t) \) and its derivatives. The applied mechanical force is assumed to be unchanged by the switching and remains

\[ F_m^* (t) = F_m^* \sin(\omega t). \]  

(5–72)
The waveforms of the reactive components during phase2 (inductor) are

\[ v_{\text{piezo}} (t) = L_{\text{PRC}} \frac{di_{\text{LPRC}} (t)}{dt} + R_{\text{Phase2}} i_{\text{LPRC}} (t), \quad (5-73) \]

and

\[ u_m^* (t) = \frac{F_m^* (t) - v_{\text{piezo}} (t)}{R_m^*}. \quad (5-74) \]

5.2.2.3 Phase3 Operation

During phase2 (inductor) the current, \( i_{\text{LPRC}} (t) \), increases as energy is transferred from the piezoelectric capacitance to the inductor. When \( i_{\text{LPRC}} (t) \) reaches its maximum value phase3 (battery) begins, where the N\text{Switch} is open and the P\text{Switch} is closed. As in phase2 (inductor), Fourier analysis is not applicable and time domain methods are used. The differential equation describing the behavior of the energy harvesting system during phase3 (battery) is

\[ F_m^* \sin (\omega t) = C_{1p3} \frac{d^2i_{\text{LPRC}} (t)}{dt^2} + C_{2p3} \frac{di_{\text{LPRC}} (t)}{dt} + C_{3p3} i_{\text{LPRC}} (t) + C_{4p3} V_{\text{battery}}, \quad (5-75) \]

where the constants \( C_{1p3} - C_{4p3} \) are given as

\[ C_{1p3} = L_{\text{PRC}} R_m^* C_{eb}, \quad (5-76) \]

\[ C_{2p3} = \frac{R_m L_{\text{PRC}}}{R_e} + C_{eb} R_m R_{\text{Phase3}} + L_{\text{PRC}}, \quad (5-77) \]

\[ C_{3p3} = \frac{R_{\text{Phase3}} R_m^*}{R_e} + R_m^* + R_{\text{Phase3}}, \quad (5-78) \]

and

\[ C_{4p3} = 1 + \frac{R_m^*}{R_e}. \quad (5-79) \]

As during phase2 (inductor), the governing equation (Equation 5-75) is a second order differential equation and two initial conditions are needed. The initial conditions are

\[ i_{\text{LPRC}} (t_{\text{init,p3}}) = i_{\text{LPRC}} (t_{\text{end,p2}}), \quad (5-80) \]
and
\[
\frac{di_{LPRC} (t_{init,p3})}{dt} = \frac{v piezo (t_{init,p3})}{L_{PRC}} - \frac{V_{battery}}{L_{PRC}} - \frac{R_{Phase3}}{L_{PRC}} i_{LPRC} (t_{init,p3}), \tag{5-81}
\]
where \(t_{end,p2}\) is the end time of phase2 (\(\rightarrow\)inductor), and is the same as the start time of phase3 (\(\rightarrow\)battery), \(t_{init,p3}\), under the assumption of instantaneous switching. The initial conditions are found using the continuity of the voltages and currents of the reactive components.

Using the initial conditions (Equation 5-80 and Equation 5-81), the governing equation (Equation 5-75) is solved for \(i_{LPRC} (t)\). Equations describing the other waveforms in the system are found using the value of \(i_{LPRC} (t)\) and its derivatives. The applied mechanical force is assumed to be unchanged by the switching and remains
\[
F_m^* (t) = F_m^* \sin (\omega t). \tag{5-82}
\]

The waveforms of the reactive components during phase3 (\(\rightarrow\)battery) are
\[
v piezo (t) = L_{PRC} \frac{di_{LPRC} (t)}{dt} + R_{Phase3} i_{LPRC} (t) + V_{battery}, \tag{5-83}
\]
and
\[
u m^* (t) = \frac{F_m^* (t) - v piezo (t)}{R_m^*}. \tag{5-84}
\]

5.2.2.4 Phase1 Operation

Phase3 (\(\rightarrow\)battery) operation of the circuit ends when the current flowing through \(L_{PRC}\) reaches zero. Phase1 (\(\rightarrow\)capacitor) operation then begins and both switches are open. Unlike phase2 (\(\rightarrow\)inductor) and phase3 (\(\rightarrow\)battery) operation where current was present in \(L_{PRC}\), the inductor is disconnected from the transducer during phase1 (\(\rightarrow\)capacitor) and there is zero current flowing through it. Instead, to model the waveforms occurring during phase1 (\(\rightarrow\)capacitor), the piezoelectric voltage, \(v piezo (t)\) is used. The differential equation
describing the behavior of the circuit is given as

\[
\frac{dv_{\text{piezo}}(t)}{dt} = \frac{F_m^* \omega \sin(\omega t)}{R_m^* C_{eb}} - \left( \frac{1}{R_e} + \frac{1}{R_m^* C_{eb}} \right) v_{\text{piezo}}(t),
\]  

(5–85)

Since Equation 5–85 is a first order differential equation, only one initial condition is needed to find the solution. The initial conditions is

\[
v_{\text{piezo}}(t_{\text{init},p1}) = v_{\text{piezo}}(t_{\text{end},p3}),
\]

(5–86)

where \( t_{\text{end},p3} \) is the time when phase3(→battery) ends and, which is equal to the time that phase1(→capacitor) begins under the assumption of instantaneous switching. Using the initial condition (Equation 5–86), the governing equation (Equation 5–85) is solved for \( v_{\text{piezo}}(t) \). Equations describing the remaining waveforms in the system are found using the value of \( v_{\text{piezo}}(t) \) and its derivatives. The applied mechanical force is assumed to be unchanged by the switching and remains

\[
F_m^*(t) = F_m^* \sin(\omega t).
\]

(5–87)

The reflected mechanical velocity term during phase3(→battery) is given by

\[
u_m^*(t) = \frac{F_m^*(t) - v_{\text{piezo}}(t)}{R_m^*}.
\]

(5–88)

## 5.3 Comparison of the Full and Simplified Models

Following the development of the full LEM and simplified resonant LEM system models, a test case was examined in order to compare the behavior predicted by both. A simulation code was written in MATLAB for both the full LEM model and the simplified resonant model. The basic algorithm for the code follows that described in Figure 5-4, and the two codes themselves are presented in Appendix D. The sample energy harvesting system is based on a transducer beam that was developed for preliminary data collection. The simulation parameters used for the two systems are shown in 5-1. The reflected input force applied to the system was 3.4 V.
Table 5-1. Simulation parameters used for comparing the full LEM model to the simplified resonant model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical Mass (Reflected)</td>
<td>$M_m^*$</td>
<td>29.77 kHz</td>
</tr>
<tr>
<td>Mechanical Compliance (Reflected)</td>
<td>$C_{ms}^*$</td>
<td>0.26 nF</td>
</tr>
<tr>
<td>Mechanical Damping (Reflected)</td>
<td>$R_m^*$</td>
<td>147.47 kΩ</td>
</tr>
<tr>
<td>Dielectric Loss</td>
<td>$R_e$</td>
<td>12 MΩ</td>
</tr>
<tr>
<td>Piezoelectric Capacitance</td>
<td>$C_{eb}$</td>
<td>16 nF</td>
</tr>
<tr>
<td>PRC Inductance</td>
<td>$L_{PRC}$</td>
<td>270 µH</td>
</tr>
</tbody>
</table>

Simulations of the sample energy harvesting system were run for both the full LEM model and the simplified resonant circuit. In order to examine the behavior of the simplified resonant model at a valid operating point, these simulations were run at the short-circuit mechanical resonant frequency. The behaviors of the two systems are compared by analyzing $v_{rect}(t)$, $i_{L_{PRC}}(t)$, and $u_m^*(t)$ in the sub-sections that follow.

5.3.1 Comparison of the Rectified Voltage

The rectified voltage, $v_{rect}(t)$, for the full and simplified resonant LEM models are shown in Figure 5-8 and Figure 5-9, respectively. For both cases, it can be seen that the energy harvesting system is first allowed to reach an open circuit steady-state where the PRC is off, and that this steady-state behavior is the same for both cases. However, once the PRC is turned on and energy harvesting is initiated, the behavior of the two models begins to differ.

For the full LEM model shown in Figure 5-8, there is a settling transient, where the peak value of $v_{rect}(t)$ initially shoots up and eventually settles down to a new steady-state value. For the simplified resonant case shown in Figure 5-9, $v_{rect}(t)$ does not exhibit this large transient period and settles quickly to a steady-state value with the PRC on. Another discrepancy between the two models is present in the peak heights of $v_{rect}(t)$. The peak values of $v_{rect}(t)$ for the full model are approximately 3 V, while the peak values for the simplified resonant model are clearly less than 3 V.
Figure 5-8. Simulated waveform of $v_{rect}(t)$ using the full LEM.

Figure 5-9. Simulated waveforms of $v_{rect}(t)$ using the simplified resonant LEM.
In order to more closely compare the behavior of $v_{\text{rect}}(t)$ for the two system models, a zoomed in view of the steady-state behavior for both models is plotted on the same time axis in Figure 5-10. For the full LEM model, the peak value of $v_{\text{rect}}(t)$ shows a bistable behavior, alternating between a high value, $v_{\text{rectPeak}}(\text{big})$, and a low value, $v_{\text{rectPeak}}(\text{small})$, while the peak value of $v_{\text{rect}}(t)$ for the simplified model remains at a constant value. For this particular case, both peak values, $v_{\text{rectPeak}}(\text{big})$ and $v_{\text{rectPeak}}(\text{small})$, predicted by the full model are higher than the single peak value predicted by the simplified resonant model. This is an important point, when considering that the energy delivered to the load during each harvesting cycle is proportional to the square of the peak value of $v_{\text{rect}}(t)$.

![Figure 5-10. Simulated waveforms of $v_{\text{rect}}(t)$ using the full and simplified resonant models.](image)

### 5.3.2 Comparison of the Inductor Current

The inductor currents, $i_{LPRC}(t)$, for the full and simplified resonant models are shown in Figure 5-11 and Figure 5-12, respectively. Once again, for the full LEM model there
is a transient period after the PRC is turned on where the peak current shoots up and eventually settles back to the energy harvesting steady-state value. This transient behavior is not seen for the simplified resonant model. The peak heights of \( i_{LPRC}(t) \) during the steady-state operation when the PRC is on are also different for the two models. For the full LEM model, the peak value is approximately 20 mA, and for the simplified resonant model the value is closer to 18 mA.

Figure 5-11. Simulated waveforms of \( i_{LPRC}(t) \) using the full LEM.

A zoomed in view of the \( i_{LPRC}(t) \) waveforms during steady-state operation is shown in Figure 5-13, for the two models. Due to the relative timescale of the figure, the current waveforms simply appear as pulses. For the full LEM model, there are again two distinct alternating peak heights for the inductor current, \( i_{LP_{RCPeak}(big)} \) and \( i_{PRCPeak(small)} \). Similarly to the case of \( v_{rect}(t) \), the simplified resonant model only appear to predict a single peak value. As will be shown Chapter 8, the simplified resonant model actually does
predict two alternating peak heights for $i_{\text{LPRC}}(t)$, however, the difference between these two peak heights is much less than that predicted by the full LEM model.

Figure 5-12. Simulated waveforms of $i_{\text{LPRC}}(t)$ using the simplified resonant LEM.

Considering the manner in which the PRC transfers energy from the transducer to the load, the fact that $v_{\text{rect}}(t)$ and $i_{\text{LPRC}}(t)$ exhibit similar behavior, in terms of the presence or absence of different height peaks, should not be surprising. A higher peak height for $v_{\text{rect}}(t)$ corresponds to a larger amount of energy stored on the piezoelectric capacitance. When this larger amount of energy is transferred to the inductor during phase2(→inductor), it will result in a correspondingly larger peak value for the inductor current. The same principle applies to the smaller peak heights. When the amount of energy stored on the piezoelectric is smaller, the peak value of the rectified voltage at the start of phase2(→inductor) will be smaller and so will the corresponding peak value of the inductor current at the end of phase2(→inductor).
Figure 5-13. Simulated waveforms of $i_{LPRC}(t)$ using both the full and simplified LEMs.

5.3.3 Comparison of the Tip Velocity

The predicted behavior of the reflected velocity, which is directly proportional to the mechanical domain velocity, is shown in Figure 5-14 and Figure 5-15 for the full and simplified resonant models, respectively. The velocity of the full LEM model shows a transient period, as in the other waveforms, that eventually settles out to a steady-state value. The velocity of the simplified resonant model, however, does not exhibit this transient behavior, but instead quickly reaches steady-state operation.

Figure 5-16 shows a close-up view of the reflected velocity waveforms for both models. While the velocity behavior predicted by the full LEM is a continuous sinusoid, the simplified resonant model has sharp, non-linear switching transitions. By comparing the instances of time where these transitions occur to the predicted $v_{rect}(t)$ and $i_{LPRC}(t)$ waveforms, the non-linear behavior is found to occur at the points in time when energy is harvested from the system.
Figure 5-14. Simulated waveforms of the tip velocity, $u_m^\ast (t)$, using the full LEM.

Figure 5-15. Simulated waveforms of the reflected tip velocity, $u_m^\ast (t)$, using the simplified resonant LEM.
Figure 5-16. Simulated comparison of $u_m^*(t)$ for the full and simplified models.

The differences in the predicted velocity behavior of the two models are a result of the simplifications made to derive the simplified resonant model from the full LEM. Assuming that the transducer is excited at its short-circuit mechanical resonant frequency, the components which capture the effects of the mechanical mass, $M_m^*$ and the mechanical compliance, $C_{ms}^*$, were removed from the model because their combined impedance contribution at this frequency is zero. This approach is mathematically valid for the steady-state analysis of linear harmonic systems [75], however, the behavior of the PRC is highly non-linear due to the presence of the rectification circuitry and switches. In addition to negating their impedance contributions to the model, the complete removal of $M_m^*$ and $C_{ms}^*$ from the full LEM also changed the continuity behavior of the system. Considering the lumped mechanical parameters in terms of their circuit component equivalents, discontinuities in the current and voltage are not physically possible for $M_m^*$ and $C_{ms}^*$, respectively. In terms of the beam mechanics, where current and velocity are
synonymous, the velocity of the simplified resonant model is unable to correctly capture the effects of momentum since there is no longer any inertial mass in the system. For the full LEM model, where the mechanical mass remains in the system, there are no discontinuities seen in the velocity waveform.

5.4 Explanation of the Two Peak Values of the Rectified Voltage

In the previous section, it was observed that the energy harvesting system level model using the full LEM predicts that the operation of the PRC will produce a brief startup transient and then settle to a steady-state value. In Figure 5-10, the steady-state behavior of the \( v_{\text{rect}}(t) \) shows that the peak heights, where energy harvesting occurs, alternate between two different values, \( v_{\text{rectPeak}}(\text{big}) \) and \( v_{\text{rectPeak}}(\text{small}) \). As a result of the two peak values in \( v_{\text{rect}}(t) \), alternating values in the peak height of \( i_{\text{LPRC}}(t) \), \( i_{\text{LPRCPeak}}(\text{big}) \) and \( i_{\text{PRCPeak}}(\text{small}) \), are also predicted. The purpose of this section is to describe the physical mechanisms which cause the alternating peak heights to occur.

A schematic view of the energy harvesting system used to describe the two peak height behavior is shown in Figure 5-17. The system is comprised of the full transducer LEM and the PRC, where the losses in the PRC have been omitted for simplicity.

![Energy harvesting system model](image)

Figure 5-17. Energy harvesting system model comprised of the full transducer LEM and PRC.

The simulated waveforms for \( v_{\text{rect}}(t) \) and the corresponding tip displacement, \( w_{\text{tip}}(t) \), are shown in Figure 5-18 for an energy harvesting system that is assumed to have been on long enough to reach steady-state operation. The timing of the peak voltages, \( v_{\text{rectPeak}}(\text{big}) \) and \( v_{\text{rectPeak}}(\text{small}) \), correspond to the maximum and minimum values of
$w_{\text{tip}}(t)$, respectively. This behavior is reasonable considering that the extrema of $w_{\text{tip}}(t)$ corresponds to a maximum amount of strain, and therefore a maximum amount value of $v_{\text{rect}}(t)$.

At point X, $v_{\text{rect}}(t)$ is at a maximum, and the energy harvesting process is initiated. Due to the large difference in the relative time scales of the input vibration and the energy harvesting process, a zoomed in version of phase2$_{\text{inductor}}$ and phase3$_{\text{battery}}$ at point X are shown in Figure 5-18B. During phase2$_{\text{inductor}}$, $v_{\text{rect}}(t)$ slopes down to 0 V and $i_{\text{LPRC}}(t)$ increases to its peak value, $i_{\text{LPRCpeak}}$ in this case, as energy is transferred from $C_{eb}$ to $L_{\text{PRC}}$. The value of $i_{\text{LPRC}}(t)$ then ramps down to zero during phase3$_{\text{battery}}$, as $L_{\text{PRC}}$ is discharged and the harvested energy is transferred to the load. Since a continuous current path must exist during phase3$_{\text{battery}}$, $C_{eb}$ conducts with a current $i_{C_{eb}}(t) \approx |i_{\text{LPRC}}(t)|$, as shown in 5-19. The reflected mechanical branch of the circuit does not participate in the phase3$_{\text{battery}}$ current path because the large inductor does not allow abrupt changes to occur in the reflected (velocity) current term $u_{m^*}(t)$. The current $i_{C_{eb}}(t)$ flowing through the capacitor causes a change in the piezoelectric voltage of the form

$$\frac{dv_{\text{piezo}}(t)}{dt} = \frac{i_{C_{eb}}(t)}{C_{eb}} \quad (5-89)$$

The value of $v_{\text{rect}}(t)$ is a rectified version of $v_{\text{piezo}}(t)$, and therefore, regardless of the slope of $v_{\text{piezo}}(t)$ given in Equation 5–52, $v_{\text{rect}}(t)$ will remain positive, as shown in Figure 5-18B.

As $L_{\text{PRC}}$ is discharged, phase3$_{\text{battery}}$ ends and phase1$_{\text{capacitor}}$ begins. During phase1$_{\text{capacitor}}$, which proceeds from point X to point Y in Figure 5-18A, both the $N_{\text{Switch}}$ and $P_{\text{Switch}}$ are open, and value of $w_{\text{tip}}(t)$ inverts from maximum positive displacement to minimum negative displacement. The change in tip displacement causes a reverse in the strain field, and the value of $v_{\text{rect}}(t)$ begins to slope down. Due to the presence of the rectifier, the value of $v_{\text{rect}}(t)$ will only continue to fall until it reaches zero volts, at which point the it will be rectified and ramp back up as shown Figure 5-18A. At point Y, the value of $v_{\text{rect}}(t)$ reaches a maximum, corresponding to $v_{\text{rectPeak}}(\text{small})$, and
the energy stored on $C_{eb}$ is harvested. The harvesting process for this peak is shown in Figure 5-18C.

Figure 5-18. Waveforms describing the two peak operation of the PRC.
During phase2 (→inductor), the value of \( v_{rect}(t) \) again falls to zero as energy is transferred from \( C_{eb} \) to \( L_{PRC} \). Once all of the energy has been transferred to \( L_{PRC} \), phase3 (→battery) begins and the stored energy is transferred to the load. A continuous current path is again required and a non-zero current \( i_{Ceb}(t) \) flows through \( C_{eb} \). The flow of current through the capacitor causes a change in \( v_{piezo}(t) \) according to Equation 5–52, which translates to the positive change seen in the value of \( v_{rect}(t) \). In comparison to the phase3 (→battery) behavior previously shown in Figure 5-18B, the \( v_{rect}(t) \) waveform shown in Figure 5-18C does not ramp up to the same height. This reduced height is a result of less energy being harvested from the piezoelectric, and therefore a smaller amount of current flowing to the load. Since the change in \( v_{rect}(t) \) during phase3 (→battery) is proportional to the current flowing through it, according to Equation 5–52, a smaller current results in a smaller change in \( v_{rect}(t) \).

Figure 5-19. Phase3 (→battery) operation of the PRC.

After the energy transfer shown in Figure 5-18C is complete, the circuit again enters phase1 (→capacitor), where the value of \( w_{tip}(t) \) is reversed during the time between Y and Z. As the value of \( w_{tip}(t) \) goes from negative to positive the value of \( v_{rect}(t) \) increases. Unlike the rectified voltage between X and Y, where the \( v_{rect}(t) \) waveform sloped down from a positive offset, the \( v_{rect}(t) \) waveform for this case slopes up from a positive offset. The net result of this is a higher peak height at point Z, \( v_{rectPeak(big)} \), as shown in Figure 5-18A. Since the peak height at point Z has more energy than that at point Y, a larger discharge current will occur during phase3 (→battery), resulting in a larger offset at the start of phase1 (→capacitor). The subsequent inversion of \( w_{tip}(t) \) will cause \( v_{rect}(t) \) to slope down.
to zero, be rectified, and produce a smaller voltage peak with a value of $v_{\text{rectPeak}}(\text{small})$. The process of alternating peak heights continues indefinitely while the system remains in steady-state operation.

### 5.5 Modeling Summary

In this chapter, the various sources of conduction loss in the PRC have been presented and two electromechanical models were developed to capture the behavior of the energy harvesting system. The first model consisted of the full transducer LEM, standard PRC components, and the effective loss terms, $R_{\text{Phase2}}$ and $R_{\text{Phase3}}$, which capture the effects of conduction loss in the PRC. A second model, consisting of the simplified resonant transducer LEM and the PRC with conduction losses was also developed. The advantage of using the full transducer LEM is in the ability of the model to predict the behavior of the system over a range of frequencies. While the simplified resonant model is less complex, it is only valid at the short-circuit mechanical resonant frequency.

A comparison of the two developed system models was performed by examining the behavior of the fundamental system waveforms predicted by each. At the short-circuit mechanical resonant frequency, it was observed that the full LEM model predicted alternating peak heights for the $v_{\text{rect}}(t)$ and $i_{\text{LPRC}}(t)$ waveforms, while the simplified resonant model predicted nearly identical peak heights for the respective signals. A detailed examination of the behavior causing the alternating peak heights was presented in Section 5.4. In terms of the tip velocity, the full LEM model predicted a smooth and continuous sinusoidal waveform. The simplified resonant model, on the other hand, showed large discontinuities in the tip velocity occurring when energy was removed from the system. The presence of these discontinuities is non-physical and is a result of removing the lumped mechanical mass and compliance terms in the simplification from the full LEM to the simplified resonant model.

In order to verify that the system operation predicted by the full LEM electromechanical model is accurate, it is necessary to compare the predicted behavior to experimental
measurements. Additionally, while the simplified resonant LEM model is technically only valid at a specific frequency, it is useful to determine its validity in approximating system behavior at other frequencies. In the following chapters, the implementation of an experimental test setup is described and the results of experimental measurements are presented.
Two theoretical models of the PRC-based energy harvesting system were developed in the previous chapter. In order to verify the accuracy and robustness of these models, it is necessary to demonstrate experimentally that the energy harvesting system behaves as the models predict under a wide array of operating conditions. To this end, an experimental test bed is required which includes multiple transducers and is capable of emulating different PRC operating conditions.

This chapter presents the design and fabrication of the experimental test bed (transducers and PRC circuitry) needed to validate the electromechanical models. The PRC circuitry is developed first because of certain limitations that this places on the design of the transducer. The design and fabrication procedures for the transducers are presented next, followed by a summary of the transducer geometries used for model validation.

### 6.1 Discrete PRC Implementation

While previous works have presented the design of pulsed resonant converters using ASICs (application specific integrated circuits) [12, 13, 32, 33], the PRC used for model validation in this work is implemented at the board level using discrete components. The primary advantage of this implementation is the ease with which components and component values can be changed on a discrete board. For instance, it is much less complex (and more cost effective) to replace or change diodes from a discrete board in order to demonstrate how the forward voltage drop affects circuit operation than it is to fabricate a different IC for each data point. It is expected that the discrete PRC will most likely have greater parasitic losses and a higher power overhead than an ASIC implementation, however the board level implementation is more practical for studying the behavior of energy harvesting systems.
As discussed in the previous two chapters, the operation of the PRC can be divided into three sequential phases which represent the accumulation and transfer of energy to the load. These three phases correspond to different configurations of the PRC switches, and the transition from one phase to another is initiated by monitoring voltage and current signals within the power converter, specifically the rectified voltage, $v_{\text{rect}}(t)$, and the inductor current, $i_{\text{LPRC}}(t)$. The underlying control theory for the PRC can be summarized as:

- Remain in phase1(→capacitor) until $v_{\text{rect}}(t)$ reaches a peak → start phase2(→inductor)
- Remain in phase2(→inductor) until $i_{\text{LPRC}}(t)$ reaches a peak (or $i_{\text{LPRC}}(t)$ crosses zero) → start phase3(→battery)
- Remain in phase3(→battery) until $i_{\text{LPRC}}(t)$ crosses zero → start phase1(capacitor)

Therefore, the control circuitry needed to properly implement PRC operation only requires the ability to detect peaks and zero crossings of these two signals.

This section examines the operation and implementation of the discrete PRC and is divided into two parts. An operational overview of the discrete PRC circuitry is first presented, followed by a detailed description of the specific circuitry and design principles used to implement the power converter.

### 6.1.1 Overview of Discrete PRC Operation

A schematic view of the PRC circuitry used in this work is shown in Figure 6-1. The PRC is essentially made up of two functional blocks, the power converter and the control circuitry. The power converter represents the portion of the circuit that is responsible for physically transferring the harvested energy from the transducer to the load and includes the rectifier circuitry, MOSFET switches, and inductor. The role of the control circuitry is to create the switching signals, $N_{\text{Gate}}(t)$ and $P_{\text{Gate}}(t)$, which drive the MOSFET switches of the converter.

Several buffer amplifiers are included to the PRC in Figure 6-1. The use of buffers in the circuit is necessary to prevent loading effects of the piezoelectric transducer by both
the control circuitry and measurement instruments. Characterization of some preliminary transducers showed piezoelectric capacitances on the order of $10 \, \mu F$ and resistances on the order of $10 \, M\Omega$. Measurement instruments, such as oscilloscopes, typically have input impedances of $1 \, M\Omega$ in parallel with $18 \, pF$. While the parallel capacitance will not significantly load the transducer, the $1 \, M\Omega$ parallel resistance will dominate and may alter the electromechanical response of the transducer.

![Functional diagram of the discrete power converter circuit.](image)

Figure 6-1. Functional diagram of the discrete power converter circuit.

The control circuitry for the power converter comprises a voltage comparator and a series of cascaded monostable multivibrators, commonly referred to as one-shots, which generate the timing signals for the PRC. A one-shot is an electronic circuit that outputs a fixed-length pulse when triggered by the edge (either rising or falling) of another switching signal. For the majority of discrete one-shot circuits, the length of the pulse is set by an external RC time constant. Since the lengths of the PRC switching pulses, $t_{NGate}$ and $t_{PGate}$, are functions of the LEM and the PRC parameters (as shown in Chapter 4),
their values will be different for each transducer. By externally controlling each RC
time constant with a potentiometer, the lengths of the PRC switching signals can be
individually tuned to achieve the proper timing for PRC operation. Additionally, by
having external control of the individual switching signals, the effects of non-ideal switch
timing can be isolated and characterized.

![Diagram](image)

Figure 6-2. Operational waveform of the discrete PRC control circuit.

Typical operation of the power converter circuitry is shown in Figure 6-2. During
phase1(→capacitor) operation all switches are open, and the rectified piezoelectric voltage,
$v_{rect}(t)$, rises sinusoidally. The comparator circuit compares the value of $v_{rect}(t)$ to $V_{TH}$,
where $V_{TH}$ is an externally applied DC voltage. While $v_{rect}(t)$ is lower than $V_{TH}$, the
comparator outputs a low signal. When \( v_{\text{rect}}(t) \) rises above \( V_{TH} \), the comparator output changes to high and produces a rising edge which triggers the generation of \( v_{\text{delay}}(t) \). The \( v_{\text{delay}}(t) \) pulse is tuned to terminate (transition from high to low) at the peak value of \( v_{\text{rect}}(t) \) and provides a falling edge to trigger the remaining signals. The combination of the comparator and the one-shot used to generate \( v_{\text{delay}}(t) \) effectively create a tunable peak detector for \( v_{\text{rect}}(t) \), where \( V_{TH} \) can be adjusted for course tuning and the length of \( v_{\text{delay}}(t) \) can be adjusted via the one-shot for more fine tuning.

The falling edge created by \( v_{\text{delay}}(t) \) is triggers the \( N_{\text{Gate}}(t) \) signal, and functions to synchronize the peak value of \( v_{\text{rect}}(t) \) with the start of phase2\((\rightarrow\text{inductor})\) as shown in Figure 6-2. As the value of \( N_{\text{Gate}}(t) \) becomes high, NSwitch closes, and the value of \( v_{\text{rect}}(t) \) decreases as charge is removed from the piezoelectric capacitance, \( C_{eb} \). The length of \( N_{\text{Gate}}(t) \) is adjusted to remain high until \( v_{\text{rect}}(t) \) reaches zero volts, which corresponds to the length of phase2\((\rightarrow\text{inductor})\). The falling edge produced by the transition of \( N_{\text{Gate}}(t) \) from high to low triggers the generation of the \( P_{\text{Gate}}(t) \) signal. Unlike the other switching signals produced by the control circuitry which control NMOS switches, the \( P_{\text{Gate}}(t) \) control signal is inverted to control a PMOS switch. The length of \( P_{\text{Gate}}(t) \) is tuned to correspond to the length of phase3\((\rightarrow\text{battery})\), and should remain low until \( i_{LPRC}(t) \) reaches zero.

### 6.1.2 Implementation of the Discrete PRC

The PRC circuitry was implemented on a two-sided copper circuit board which was fabricated at the University of Florida using an LPKF S100 milling machine. The power converter portion of the circuit was designed to allow for the discrete components (MOSFET switches, inductor, and rectifier) to be easily interchanged for debugging purposes. Instead of permanently attaching the discrete components directly to the circuit board, sockets were soldered in their place. A small, separate board with the mating hardware needed to plug into these sockets was milled for each discrete component. Additionally, in order to measure current waveforms through the inductor and switches,
small wire loops were included in the power converter layout to accommodate a Tektronix TCP312 magnetic current probe. It was assumed that any stray inductance from these wire loops was small compared to $L_{PRC}$, and that any resulting parasitic effects could be ignored.

The buffers used in the circuit were realized with AD711 operational amplifiers. A schematic diagram of the buffer circuitry is shown in Figure 6-3. These BiFET amplifiers were chosen because their high input impedance ($3 \, T\Omega/5.5 \, pF$) MOSFET input stage minimizes any loading affects on the piezoelectric transducer. Ceramic filter capacitors, shown as $C_F$ in Figure 6-3, were used to reduce any noise on the power supply lines.

![Schematic diagram of the AD711 buffer amplifier.](image)

The control circuitry for the discrete PRC, shown in Figure 6-4, was implemented using three ICs. The voltage comparator used to generate $v_{comp}(t)$ was realized with an LM311 voltage comparator biased with $\pm \, 5 \, V$. In order to prevent any loading effects on the PRC, the $v_{rect}(t)$ input to the comparator used the buffered $v_{rect}(t)$ signal. The $V_{TH}$ signal, which served strictly as a course timing adjustment, was produced by an external DC power supply. The four cascaded one-shots blocks were implemented with two CD74HC221E monostable multivibrators, where each IC has two multivibrators. These ICs used a single sided voltage bias of $+5 \, V$. The variable resistors, $R_{adj}$, attached between $V_{CC}$ and the $C_x R_x$ pins were used to adjust the pulse length produced by the one-shots. This was accomplished by varying the RC time constant formed by $R_{adj}$ and the capacitor placed between $C_x$ and $C_x R_x$. In order to have fine, medium, and course
adjustment capabilities of the pulse lengths, each instance of $R_{adj}$ on the schematic was realized with the series combination of three potentiometers, 1 $k\Omega$, 10 $k\Omega$, and 50 $k\Omega$.

Figure 6-4. Schematic diagram of the cascaded PRC controller.

The transistors used to implement the NSwitch and PSwitch for this work are the On Semiconductor NTS4001 and Infineon BSS223P, respectively. The DO3316T-104MLB (100 $\mu$H) and DO3316T-274MBL (270 $\mu$H) surface mount power inductors from Coilcraft are interchanged for different tests in Chapter 8. A photograph of the implemented PRC and control circuitry used to realize the discrete PRC is shown in Figure 6-5.
Figure 6-5. Photograph of the fabricated PRC circuitry.

The rectification circuitry, which is represented in Figure 6-1 by the four diodes, $D_1 - D_4$, is implemented using an active rectification circuit developed by Xu [13]. Unlike a passive rectifier where diodes are used to control the direction of current flow, in an active rectifier the direction of current flow is controlled by transistor switches. By replacing the diodes with an active rectifier the forward voltage drops associated with discrete diodes are removed. Therefore, the only remaining loss from the rectifier is the relatively small conduction loss from the transistors. The design trade-off for an active rectifier, however, is the overhead power needed to control the transistor switches. The active rectifier designed by Xu was originally fabricated as part of a complete ASIC PRC and controller system. Due to some of the limitations of the ASIC design, only the active rectifier circuit portion of the ASIC could be leveraged for this work.

The active rectifier portion of the ASIC designed by Xu implemented a full-wave bridge rectifier through the use of two separate circuit blocks, a high bridge and low bridge. The high bridge topology, shown in Figure 6-6A, conducted when the value of $u_{\text{piezo}}(t)$ was above the threshold voltage of the transistors. The low bridge circuit, shown in Figure 6-6B, is added to supplement the high bridge, and is optimized for rectification
below the transistor threshold voltage. Unlike the high bridge circuit, where the MOSFET gates are set by \( v_{\text{piezo}}(t) \), the gates in the low bridge require external control, namely \( v_{C_1} \) and \( v_{C_2} \). Depending on the polarity of \( v_{\text{piezo}}(t) \), the control circuitry for the low bridge sets either \( v_{C_1} \) or \( v_{C_2} \) to \( V_{dd} \), while the other is set to ground.

![Functional blocks of the active rectifier comprised A) high bridge and B) low bridge.](image)

**6.2 Transducer Design and Fabrication**

This section presents the design and fabrication techniques used to create the transducer beams. Design considerations required to implement transducer beams capable of operating with the discrete PRC are presented first, followed by the general transducer fabrication procedure. This section concludes with a summary of the transducer beam geometries specifically used in this work.

**6.2.1 Design Considerations**

In order to validate the system models of the energy harvesting system presented in Chapter 5 it is necessary to implement transducer beams capable of properly functioning with the discrete PRC. While the models presented in Chapter 5 are generally valid for any LEM parameters, non-idealities of the discrete PRC circuitry place some limits on the design of the transducers.
A fundamental issue arising from the PRC circuitry that affects the transducer design is the minimum length of the pulse widths created by the one-shot ICs. This is important because it places a lower limit on the lengths of phase2\(_{\text{inductor}}\) and phase3\(_{\text{battery}}\). With a 5 V power supply, preliminary experiments found that the smallest sustainable pulse was approximately 40 ns. However, since the rise and fall times of this signal was on the order of 15 ns, most of this pulse was spent in transition. It was found that pulse lengths of approximately 100 ns or more created much cleaner signals. Using this lower limit of 100 ns and a few simplifying assumptions, it is possible to gain some insight into the minimum area of the piezoelectric patch needed to operate with the discrete PRC.

To estimate the minimum area of the piezoelectric patch, it is assumed that the power converter operates ideally, with perfect timing and zero loss. Under this assumption the time required for the circuit to remain in phase2\(_{\text{inductor}}\), \(t_{\text{NGate}}\), is equal to the time needed for \(v_{\text{rect}}(t)\) to reach zero volts, or one fourth of the resonant period of \(L_{\text{PRC}}\) and \(C_{eb}\) [12]:

\[
t_{\text{NGate}} = \frac{\pi}{2} \sqrt{\frac{L_{\text{PRC}}}{C_{eb}}}.
\]  

(6–1)

Assuming further that the blocked capacitance and free capacitance of the piezoelectric patch are approximately equal (meaning that the electromechanical coupling factor is small)

\[
C_{eb} \approx C_{ef} = \frac{\varepsilon_r \varepsilon_0 A}{d},
\]  

(6–2)

where \(\varepsilon_0\) is the permittivity of free space, \(\varepsilon_r\) is the relative permittivity of the dielectric, \(A\) is the area of the parallel plate, and \(d\) is the thickness of the piezoelectric. Combining Equation 6–1 and Equation 6–2, and solving for \(A\) gives

\[
A = \frac{d}{\varepsilon_r \varepsilon_0 L_{\text{PRC}}} \left( \frac{2t_{\text{NGate}}}{\pi} \right)^2.
\]  

(6–3)

Choosing a value for \(L_{\text{PRC}}\) of 100 \(\mu H\), and using the specified parameters of the piezoelectric material (\(d = 0.267 mm\), \(\varepsilon_r = 3800\), \(\varepsilon_0 = 8.85 \times 10^{-12} F/m\)) and the
minimum pulse length of 100 ns, the minimum area becomes

\[ A_{\text{min, phase2}} = 0.32 \text{mm}^2. \]  

(6-4)

A similar analysis can be performed for operation during phase\(_3\)\(_{\rightarrow}\)battery, where under ideal conditions the length of phase\(_3\)\(_{\rightarrow}\)battery is \([12] \]

\[ t_{PGate} = \sqrt{L_{PRC} C_{eb} V_{\text{rectPeak}} / V_{\text{battery}}}, \]  

(6-5)

where \(v_{\text{piezoPeak}}\) is the peak value of the rectified voltage at the start of phase\(_2\)\(_{\rightarrow}\)inductor. Note that since this Equation 6-5 is based on the simplified resonant mode, only a single value of \(v_{\text{piezoPeak}}\) occurs. Again assuming the validity of Equation 6-2, the minimum area needed for the control circuit to operate during phase\(_3\)\(_{\rightarrow}\)battery can be approximated by combining Equation 6-2 and Equation 6-5

\[ A = \frac{d}{\varepsilon_r \varepsilon_0 L_{PRC}} \left( \frac{V_{\text{battery}} t_{PGate}}{v_{\text{piezoPeak}}} \right)^2. \]  

(6-6)

Making the assumption that \(v_{\text{rectPeak}}\) will always be at least one tenth of the battery voltage (which is nominally 5 V for this work), a minimum area for phase\(_3\)\(_{\rightarrow}\)battery operation is

\[ A_{\text{min, phase3}} = 79.39 \text{ mm}^2. \]  

(6-7)

Comparing Equation 6-4 and Equation 6-7, the minimum area of the piezoelectric patch should be at least 80 mm\(^2\). To account for the parasitic effects in the discrete PRC, which shorten the length of phase\(_2\)\(_{\rightarrow}\)inductor and phase\(_3\)\(_{\rightarrow}\)battery, an additional safety factor of 3 is included to ensure that these phases remain longer than 100 ns. This means that the area of the piezoelectric patch should be on the order of 200 - 300 mm\(^2\) (2-3 cm\(^2\)). Had a larger coupling coefficient been assumed, meaning \(C_{eb} \neq C_{ef}\), a piezoelectric patch with a larger area would be needed.
6.2.2 Fabrication Procedure

The basic structure of the transducer beams used in this work is shown in Figure 6-7. Each beam comprises an aluminum shim, piezoelectric patch, two thin layers of copper tape, and small lead wires to connect to the PRC. The piezoelectric material used for this work is PSI - 5H4E from Piezo Systems Inc., which is composed of lead zirconate titanate (PZT). This material is commercially available in three thicknesses, 0.127 mm, 0.191 mm, and 0.267 mm, and the PSI - 5H4E sheets are fabricated with a 100 nm thick nickel electrode is on both sides. Electrical contact to the top electrode of the piezoelectric patch is made directly, while the bottom electrode is accessed through the aluminum shim. Since both nickel and aluminum are difficult to solder to, the copper tape is included in the design to provide a surface for soldering lead wires.

![Figure 6-7. Structure of the piezoelectric beam transducer.](image)

A common fabrication procedure was adopted for all of the transducers presented in this work. The first step in this procedure was to fabricate the aluminum shim and clamp pieces. A clamp for a single transducer comprises two plates where each plate contains five holes for clamp tightening screws. A schematic of the dimensions for a single plate of the clamp structure is shown in Figure 6-8. The holes in the bottom plate were hand tapped to fit a 0-80 machine screw. An LPKF S100 milling machine operated in the 2.5D mode was employed to drill the holes and cut out the aluminum pieces for both the shims and the clamps. The shims and clamps were cut from 1 mm and 2 mm aluminum stock, respectively.
The next step in the fabrication process was to cut the piezoelectric material to size. In this work, the width of the PZT patch, wPZT, was always sized to match the shim, and the length, lPZT, was varied to achieve different LEM parameters. The PSI-5H4E is a brittle ceramic and is shipped in 2" × 2" sheets. To cut the PZT to the proper dimensions, a carbide tipped engraving pen and straight edge were used in a manner similar to cutting glass. The straight edge was placed a distance $w_{PZT}$ from the edge of the sheet and the engraving pen was run along the straight edge to score the PZT. The PZT was then carefully laid on top of the straight edge, with the score line slightly overhanging the corner, and a slight force was applied to the overhanging portion, as shown in Figure 6-9. The resulting strip (of width wPZT) was then cut to length in a similar manner.

![Figure 6-8. Dimensions for a single plate of the clamp structure.](image)

After cutting the aluminum shim and piezoelectric patch, the two layers were bonded together using low temperature silver epoxy. The epoxy used to fabricate the beams was MG Chemicals - Silver Conductive Epoxy (Cat. No 8331-14G), which was chosen simply for its availability. The two parts of the epoxy were mixed together on a glass slide and then spread along one side of the piezoelectric patch in an even layer. The two layers were then pressed firmly together by hand and allowed to set overnight.
After the epoxy was allowed to set, the composite beam was placed into the clamp as shown in Figure 6-7. The beam was positioned with a small space between the clamp and the piezoelectric patch to avoid any risk of contacting the top electrode to the clamp. Before the clamp was fully tightened, a small piece of aluminum was inserted on the opposite side from the beam to prevent any pivoting as the clamp plates were drawn together.

![Figure 6-9. Procedure for cutting PZT patch.](image)

The final step in the fabrication process was to attach the copper tape and the lead wires to the beam. Two small pieces of copper tape were cut, using scissors, and attached to the beam as shown in Figure 6-7 with the adhesive on the back side of the tape. One piece of tape was attached to the top electrode of the piezoelectric patch as close as possible to the clamped end. The other piece of tape was then attached to the bottom of the aluminum shim directly below the first piece. Lead wires were then soldered to the copper tape as close to the clamped end of the beam as possible in order to minimize loading effects on the beam. The wires were then run along the side of the clamp and secured in place with masking tape to minimize any rattling from loose wires.

### 6.2.3 Beam Geometries

Using the design guidelines and the fabrication procedure presented in the previous sections of this chapter, three different transducer beams were constructed. The
dimensions of the transducers are listed in Table 6-1. The choice of these dimensions
was chosen primarily to satisfy the minimum piezoelectric area requirements necessary
to ensure proper operation of the discrete PRC using Equation 6–7. Additionally, these
dimensions provided resonant frequencies less than 200 Hz, which is in the frequency
range of many ambient vibration sources. The length, width, and thickness of the shim
are represented by $L_s$, $W_s$, and $T_s$, respectively. Similarly, the length, width, and thickness
of the piezoelectric patch are given by $L_p$, $W_p$ and $T_p$. The next chapter presents the
experimental characterization of the three transducer beams. The techniques used to
extract the LEM parameters are discussed along with the results of the LEM extraction
experiments.

Table 6-1. Dimensions of fabricated beams.

<table>
<thead>
<tr>
<th>Beam #</th>
<th>$L_s$ [mm]</th>
<th>$W_s$ [mm]</th>
<th>$T_s$ [mm]</th>
<th>$L_p$ [mm]</th>
<th>$W_p$ [mm]</th>
<th>$T_p$ [mm]</th>
<th>$A_p$ [mm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>122.67</td>
<td>6.30</td>
<td>0.93</td>
<td>23.77</td>
<td>6.30</td>
<td>0.27</td>
<td>149.75</td>
</tr>
<tr>
<td>2</td>
<td>123.21</td>
<td>8.90</td>
<td>1.03</td>
<td>33.75</td>
<td>8.90</td>
<td>0.27</td>
<td>300.38</td>
</tr>
<tr>
<td>3</td>
<td>88.89</td>
<td>7.11</td>
<td>1.06</td>
<td>39.30</td>
<td>7.11</td>
<td>0.27</td>
<td>279.42</td>
</tr>
</tbody>
</table>
CHAPTER 7
TRANSDUCER CHARACTERIZATION

In Chapter 6, the design and fabrication of the transducer beams and PRC circuitry needed to verify the energy harvesting system models was presented. The next step in the model verification process for the energy harvesting system is to characterize the transducer beams by extracting the LEM parameters, shown in Figure 7-1. Using the extracted parameters, the modeled behavior can be compared to experimental measurements in order to confirm its validity.

Figure 7-1. Schematic of the transducer LEM.

This chapter presents both the methodology and results of the LEM parameter extraction for the three experimental transducer beams. The characterization of the transducers is divided into three parts, mechanical characterization, electrical characterization, and coupled electromechanical characterization, which correspond to the first three sections of this chapter. The final section of the chapter presents a summary of the results.

7.1 Mechanical Characterization

The mechanical characterization of the transducer beams focuses on the extraction of the lumped mechanical parameters, \( M_m, C_{ms}, \) and \( R_m \). In order to isolate the mechanical domain from the electrical domain, the piezoelectric patch is short-circuited by physically connecting the electrical leads. The resulting circuit model is shown in Figure 7-2. By short-circuiting the electrical domain, the effects of the electromechanical coupling are
negated, and the transformer can be effectively removed from the circuit. This can be also be conceptualized as an effective short circuit in the mechanical domain. It should be noted that while the short-circuit condition has removed the electromechanical coupling properties of the piezoelectric, the mechanical properties of the piezoelectric patch (stiffness, mass, etc.) will still influence the lumped mechanical parameters of the transducer.

![Schematic of the transducer LEM with the piezoelectric short-circuited.](image)

Figure 7-2. Schematic of the transducer LEM with the piezoelectric short-circuited.

After isolating the mechanical domain by short-circuiting the piezoelectric, two characterization experiments are used to find the lumped mechanical parameters. The compliance, $C_{ms}$, is first extracted using a static experiment. This is followed by a dynamic parameter extraction technique to find the values of $M_m$ and $R_m$ by curve fitting to a second order mass-spring-damper system.

### 7.1.1 Static Compliance Extraction

The short-circuit mechanical compliance is found by shorting the two lead wires of the piezoelectric patch together and applying different static forces to the tip of the beam. By measuring the resulting tip displacement, the effective compliance of the beam can be found using Hooke’s Law

$$F_{app} = \frac{w_{tip}}{C_{ms}}$$  \hspace{1cm} \text{(7-1)}

where $F_{app}$ is the force applied to the tip of the beam and $w_{tip}$ is the tip displacement.
The experimental configuration used to extract $C_{ms}$ from the transducers is shown in Figure 7-3. In this setup, force is applied to the beam by adding known masses to a scale pan suspended from the tip, and tip displacement is measured using a laser displacement sensor. For this work, coins of different denomination were used to provide the applied tip force. A simple scale pan was fashioned using folded printer paper and monofilament wire (fishing line). In order to attach the scale pan the transducer beams, a small hole was drilled into each aluminum shim, close to the free end, and a piece of monofilament wire with a hook at the end was tied through the hole. Loops were formed in the free ends of the wire used for the scale pan, and these loops were then placed around the hook to suspend the entire structure. The tip displacement was monitored using a Keyence LK G-32 laser displacement sensor. Precise positioning of the laser at the tip of the beam was accomplished using a 3-axis traverse.

The first step in the extraction of $C_{ms}$ was to measure the mass of the coins. This was done using an Ohaus EP214DC Explorer Pro digital balance. After the masses of the coins were determined the empty scale pan was attached to the beam and the laser displacement sensor was aligned over the beam tip and zeroed. The laser displacement sensor outputs a voltage which is proportional to the displacement and features an adjustable sensitivity. For the $C_{ms}$ extraction, the sensitivity was set to 1 V/mm. Coins were added to the scale pan and the displacement was recorded using the LK-Navigator software provided by Keyence. Weight was added until the magnitude of the displacement was no longer within the range of the sensor (approximately ± 5 mm). The coins and washers were then removed and the experiment was repeated two additional times using different ordering and combinations of weight.

To extract the short-circuit mechanical compliance from each of the three tests, the force from each mass combination was first calculated using

$$F_{app} = ma_m$$  \hfill (7-2)
where $a_m$ is the acceleration due to gravity ($9.8 \, m/s^2$) and $m$ is the total mass in the scale pan. A plot of the displacement vs. force was then generated for each test and a linear best fit line was applied, where the slope of the best fit line is equal to the compliance. The result of a single static compliance test for beam 3 is shown in Figure 7-4, where the compliance was found to be $5.2 \times 10^{-3} \, m/N$. The best fit regression line shown in Figure 7-4 is representative of the other compliance measurements, in that the $R^2$ value was higher than 0.99 (where $R^2 = 1$ indicates a perfect fit). It should be noted that this method of extracting $C_{ms}$ assumes that the beam behaves like a linear spring and that the tip displacement is small so as not to violate the linear Euler-Bernoulli assumption.

Figure 7-3. Experimental setup used to extract the short circuit compliance of the transducers.
7.1.2 Dynamic Mechanical Extraction

The remaining two lumped mechanical parameters, $M_m$ and $R_m$, are found by performing the dynamic mechanical characterization described by Kasyap [19], which experimentally finds the transfer function (TF) between the tip displacement and base acceleration. The experimental TF is then compared to the theoretical TF for a short-circuited transducer and curve fitting techniques are used to extract the best fit parameters for $M_m$ and $R_m$.

To find the theoretical TF for the short-circuited transducer, Kirchoff’s voltage law (KVL) is applied to the circuit model shown in Figure 7-2.

$$F_m = 1.566 M_m a_m = \left( j \omega M_m + \frac{1}{j \omega C_m} + R_m \right) U_m$$  \hspace{1cm} (7–3)

The factor of 1.566 is the correction factor, $\mu$, for a beam where the mass of the proof mass is zero [67, 68]. Substituting $U_m = j \omega w_{tip}$ into Equation 7–3 and rearranging leads to
the theoretical TF between the relative tip displacement and base acceleration

\[ TF = \frac{w_{tip}}{a_m} = \frac{M_m}{j\omega C_m} + \frac{1}{j\omega M_m + R_m j\omega} \frac{1}{1.566} \]  \hspace{1cm} (7-4)

The setup used to experimentally find the TF of the transducer is shown in Figure 7-5. The SR 780 spectrum analyzer outputs a chirp signal, which is amplified by the Bruel and Kjær (B&K) Type 2718 amp to drive the (B&K) mechanical shaker. A B&K Type 8001 impedance head is mounted between the shaker and the transducer in order to monitor the base acceleration. Since the Type 8001 impedance head uses a piezoelectric accelerometer, a PCB 422E03 charge amplifier and Series 481 signal conditioner are also needed to correctly measure acceleration.

![Figure 7-5. Experimental setup used to extract \(M_m\) and \(R_m\).](image)

The LK-G32 laser displacement sensor is again used to measure the tip displacement of the beam and is aligned using the 3-axis traverse. The output signals from both the laser displacement sensor and the accelerometer (via the signal conditioner) are fed into the inputs of the spectrum analyzer in order to measure the TF (Range: 0 - 100/200 Hz; 800 FFT lines; 30 averages; uniform windowing). It should be noted that the tip
displacement measured in this experiment is the absolute displacement, and not the relative displacement between the base and the tip used to derive the TF in Equation 7–4. However, since the displacement of the base was typically small (<5µm) compared to that of the tip (∼100µm), the relative and absolute displacement values were reasonably close. Therefore, the absolute displacement measurement was used to reduce experimental complexity. The use of the absolute tip displacement was continued for the other dynamic measurements presented in this work.

For the mechanical characterization experiments performed in this work, the spectrum analyzer was configured to capture the magnitude of the TF between tip displacement and base acceleration on a linear scale. As an example, the TF for beam 3 is shown in Figure 7-6. The experimental TF was then imported into MATLAB where the Curve Fitting Tool was used to find $M_m$ and $R_m$ by curve fitting the experimental data to the theoretical TF.

![Graph showing experimental short-circuit TF with best-fit line for beam 3.](image)

Figure 7-6. Experimental short-circuit TF with best-fit line for beam 3.
The magnitude of the theoretical TF is found from Equation 7–4 is given by

\[ |TF| = \frac{M_mC_{ms}}{\sqrt{(2\pi f R_mC_{ms})^2 + \left(1 - (2\pi f)^2 M_mC_{ms}\right)^2}} 1.566. \]  

(7–5)

The model was entered into the Curve Fitting Tool as a custom equation, and a non-linear least squares fit was performed using a trust-region algorithm. The fitted curve, LEM parameters with 95% confidence bounds, and fitting statistic are shown in Figure 7-6. The fitting statistics for beam 3, specifically \( R^2 > 0.99 \), are representative of the other two transducers. The results of the TF curve fits for beams 1 and 2 are presented in Appendix E.

### 7.2 Electrical Characterization

The electrical characterization of the piezoelectric transducer is performed to find the values of the lumped electrical domain components \( C_{eb} \) and \( R_e \). For the mechanical characterization presented in Section 7.1, the effects of the electromechanical coupling were removed (or at least effectively minimized) by short-circuiting the piezoelectric patch, in order to isolate the mechanical domain of the transducer. In a similar manner, the electrical domain is isolated by providing a blocked condition to the mechanical domain. When the beam is mechanically blocked, there is no tip displacement, and therefore no tip velocity. Figure 7-7 illustrates how the mechanical blocked condition affects the LEM of the transducer. Since the mechanical domain is open-circuited there can be no electromechanical coupling and the effects that any input voltage or current has on the transducer are isolated to the electrical domain.

The experimental setup used for the electrical characterization is shown in Figure 7-8. Since the tip displacement in the LEM, \( z \), was previously defined in Chapter 2 as the relative displacement between the base and the tip of the transducer, it is necessary to ensure that neither the base nor the tip of the beam moves in order to provide a blocked mechanical condition. The base of the beam is secured by tightly locking the clamp into an optical table post, and the tip of the beam is secured between two metal blocks. While
it is not possible to completely prevent any motion of the beam, the use of this blocking procedure should minimize the coupling between the mechanical and electrical domains. The HP4294A impedance analyzer is used to measure the parallel combination of $C_p$ and $R_p$ in the frequency range of interest for each blocked transducer.

\[ U_m = 0, \quad F_m = M_m, \quad C_{ms}, \quad R_m, \quad d_{ef}(=\Phi):1 \]

Mechanical Domain \hspace{2cm} Electrical Domain

**Figure 7-7.** Schematic of the mechanically blocked piezoelectric transducer LEM.

**Figure 7-8.** Experimental setup used for electrical characterization.

For this work, the sweep range for the impedance analyzer was set to overlap the short-circuited, mechanical resonant frequency of each device. Specifically, for beams 1 and 2, whose resonant frequencies were $\sim 60 \ Hz$, the sweep range was set between 40 $Hz$ (the lower limit of the impedance analyzer) and 100 $Hz$. For beam 3, whose resonant

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frequency was $\sim 120 \text{ Hz}$, the sweep range was set between 40 Hz and 200 Hz. For all three beams, the bandwidth setting of the impedance analyzer was set to 5 for maximum frequency resolution, and 25 averages were taken for each measurement. The results of the impedance measurement for beam 3 are shown in Figure 7-9, where $C_p = 24.5 \text{ nF}$ and $R_p = 3.2 \text{ M}\Omega$. Assuming that the LEM in Figure 7-7 accurately models the impedance of the transducer, the values of $C_{eb}$ and $R_e$ are equal to $C_p$ and $R_p$, respectively. Since the variation in $C_{eb}$ and $R_e$ is relatively small over the sweep range, the values at the short-circuit resonant frequency are assumed accurate over frequencies near the resonance. The impedance measurements for beams 1 and 2 are included in Appendix E.

7.3 Electromechanical Characterization

The electromechanical characterization experiment is used to find the effective electromechanical coupling coefficient, $d_{eff}$, which is needed for the calculation of the turns ratio, $\Phi$. The quantity $d_{eff}$ is defined as the relative tip displacement of the beam per
applied volt

\[ d_{eff} = \frac{w_{tip}}{V_{app}} \hspace{1cm} (7-6) \]

where \( w_{tip} \) is the magnitude of the tip displacement and \( V_{app} \) is magnitude of the applied voltage.

The experimental setup to perform the electromechanical characterization and extract \( d_{eff} \) is shown in Figure 7-10. In this experiment the clamp is secured to prevent motion of the base, and the transducer is excited using an Agilent 33120A function generator. The output of the function generator, \( V_{app} \), is monitored using a Tektronix 5104B oscilloscope after first passing through a buffer stage to prevent any electrical loading of the beam. The buffer for this work was realized using the AD711 configuration presented in Chapter 6.1.2 (Figure 6-3). Using the three axis traverse from the previous extraction experiments, the LK-G32 laser displacement sensor is aligned over the tip of the beam to monitor displacement.

![Figure 7-10. Experimental setup used for electromechanical characterization.](image-url)
For the beams characterized in this work, $V_{app}$ was a low frequency sine wave, between 10 and 15 Hz, whose amplitude was varied over the range of the function generator (0 - 10 V). The reason for using a low frequency sine wave was to minimize any of the resonant effects of the transducer. Had this characterization been performed at or near a resonant transducer frequency, the measured tip displacement for a given voltage would have been higher, leading to an incorrect overestimate of $d_{eff}$. By measuring $d_{eff}$ at a frequency well below resonance, a flat band response of the transducer can be assumed where the resonant effects of the device do not influence the measurement of $d_{eff}$. To extract the value of $d_{eff}$ from these measurements, displacement was plotted against $V_{app}$, and a linear best fit line was computed. The slope of this best fit line was used for $d_{eff}$, and the effective turns ratio was found using

$$\phi = \frac{d_{eff}}{C_{ms}} \quad (7-7)$$

The results of electromechanical extraction for beam 3 are shown in Figure 7-11, where the slope of the best fit line indicates an effective electromechanical coupling coefficient of $d_{eff} = 2.057 \times 10^{-6} \text{ m/N}$. The results shown here for beam 3 are representative of the other two beams, whose $R^2$ values are also higher than 0.99. The measured results of the electromechanical extraction experiments for beams 1 and 2 are presented in Appendix E.
7.4 Summary of Results

The previous three sections of this chapter have described the theory and methodology used to extract the LEM parameters for the piezoelectric transducer beams. The parameter extraction procedure included an isolated mechanical characterization, an isolated electrical characterization, and coupled electromechanical characterization of the transducer. In addition to the theory, an example measurement for each of the characterization experiments was presented in the chapter and a complete set of experimental measurements is included in Appendix E. A summary of the experimentally extracted LEM parameters for the three transducer beams used in this work is given in Table 7-1.

Table 7-1. Extracted LEM parameters for the three transducer beams.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Beam 1</th>
<th>Beam 2</th>
<th>Beam 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{ms} ) [m/N]</td>
<td>( 1.610 \times 10^{-2} )</td>
<td>( 1.089 \times 10^{-2} )</td>
<td>( 5.105 \times 10^{-3} )</td>
</tr>
<tr>
<td>(95% Con. Bds.)</td>
<td>( \pm 1.680 \times 10^{-3} )</td>
<td>( \pm 1.350 \times 10^{-3} )</td>
<td>( \pm 9.000 \times 10^{-3} )</td>
</tr>
<tr>
<td>( M_m ) [kg]</td>
<td>( 4.499 \times 10^{-2} )</td>
<td>( 5.752 \times 10^{-2} )</td>
<td>( 3.264 \times 10^{-2} )</td>
</tr>
<tr>
<td>(95% Con. Bds.)</td>
<td>( \pm 4.690 \times 10^{-5} )</td>
<td>( \pm 7.130 \times 10^{-5} )</td>
<td>( \pm 5.760 \times 10^{-3} )</td>
</tr>
<tr>
<td>( R_m ) [N/(m/s)]</td>
<td>( 1.943 \times 10^{-3} )</td>
<td>( 2.631 \times 10^{-3} )</td>
<td>( 4.134 \times 10^{-3} )</td>
</tr>
<tr>
<td>(95% Con. Bds.)</td>
<td>( \pm 0.191 \times 10^{-4} )</td>
<td>( \pm 3.120 \times 10^{-4} )</td>
<td>( \pm 7.010 \times 10^{-4} )</td>
</tr>
<tr>
<td>( C_{eb} ) [nF]</td>
<td>15.0</td>
<td>29.5</td>
<td>24.5</td>
</tr>
<tr>
<td>( R_e ) [MΩ]</td>
<td>10.0</td>
<td>4.0</td>
<td>3.2</td>
</tr>
<tr>
<td>( d_{eff} ) [m/V]</td>
<td>( 1.646 \times 10^{-6} )</td>
<td>( 2.795 \times 10^{-6} )</td>
<td>( 2.057 \times 10^{-6} )</td>
</tr>
</tbody>
</table>

The next step in the validation process for the energy harvesting system model is to compare the mathematical model with experimental results. By plugging the extracted transducer LEM parameters from Table 7-1 in the mathematical model developed in Chapter 5, the behavior of the theoretical and physical systems can be accurately compared.
EXPERIMENTAL CHARACTERIZATION OF THE ENERGY HARVESTING SYSTEM

The ultimate goal of this work is to develop and validate a model of the PRC-based energy harvesting system which accurately captures both the general behavior and the non-idealities of the system. In order to verify the accuracy of any model, it is necessary to compare the theoretical behavior with experimental measurements. In the preceding chapters modeling (Chapter 5), fabrication (Chapter 6), and transducer characterization (Chapter 7) have been presented. The final step in the process is to validate the model against experimental data.

This chapter presents the experimental characterization of the PRC-based energy harvesting system and is divided into two sections. The first section presents a first generation of experiments which were used with beam 1 to characterize the operation of the system. While the results from the first generation experiments were successful, it was determined that a few simple changes could be implemented to improve the experimental procedure and provide more accurate data. The second section of this chapter presents the modified second generation of experiments which incorporate the improvements made to the first generation. Measurement from the first and second generation experiments are compared to the theoretical models for both the full LEM and simplified resonant cases. It is demonstrated that the system model using the full transducer LEM accurately predicts the behavior of the system over a range of frequencies while the simplified resonant model does not. Furthermore, it is shown the techniques used to incorporate the non-ideal PRC effects into the model yield accurate predictions of the experimental measurements, and that including these non-idealities in the model is critical to the design process.

8.1 First Generation Experimental Characterization

The first generation characterization represents a series of experimental measurements that were performed on beam 1 to determine how the behavior of the energy harvesting system is affected by the presence of non-ideal operating conditions. Specifically for this
work, the effects of non-resonant frequency operation, losses in the power converter, and non-ideal switch timing were considered.

This section is divided into four parts. The first part presents the experimental setup used to characterize the behavior of the energy harvesting system. This is followed by the characterization of the sources of non-ideal PRC behavior and explains how the existing theoretical models are modified to include these effects. The third part presents the experimental measurements and compares these to the simulations of both the full LEM and simplified resonant models. This section concludes with a discussion of the shortcomings of this first generation characterization and provides motivation for the second generation.

8.1.1 Experimental Setup

The first generation experimental setup used to characterize the operation of the PRC-based energy harvesting system is shown in Figure 8-2. In this setup, the transducer is actuated mechanically by an applied base acceleration and the electrical waveforms of interest are simultaneously recorded. The transducer and clamp assembly, presented in Chapter 6, is mounted to a B&K Type 4810 mini-shaker which provides the mechanical input. The mini-shaker is driven by an HP 33120A function generator which is amplified by a B&K Type 2718 power amplifier. The base acceleration of the transducer is monitored using the LK-G32 laser displacement sensor set to 1 mm/V. Assuming that the base acceleration is sinusoidal, the acceleration amplitude can be related to the displacement amplitude by

$$|a_m| = \omega^2 |d_m|,$$

where $|d_m|$ is the amplitude of the sinusoidal displacement. The tip displacement is monitored using an oscilloscope triggered by the sync signal from the function generator. It was determined experimentally that the displacement output by the laser sensor was very noisy and that it was necessary to take a time average with the oscilloscope to determine the displacement amplitude.
A schematic view of the PRC is shown in Figure 8-2 which demonstrates how the electrical domain signals are monitored. The circled V’s represent voltage measurement points and the A’s represent current measurement points. All of the voltage measurements that could load the piezoelectric transducer are buffered (see Chapter 6.1.1), which allows these voltages to be directly measured using an oscilloscope. There are two different types of current measurements shown in Figure 8-2. The A with the double circle represents a measurement made by a Tektronix TCPA300 contactless current probe, and the A with a single circle represents a measurement made using a Keithly 480 picoammeter. The contactless current probe was used to examine time varying currents, while the picoammeter was used to find the average current being delivered to the load. All of the time varying signals, both voltage and current, were measured using a Tektronix TDS5104B digital oscilloscope.
8.1.2 Modeling Losses in the PRC

There are two primary sources of loss associated with the PRC implementation used for this work. The first source is the finite conduction loss associated with each of the circuit elements in the power converter. Conduction loss, which can be attributed to the transistors in the active rectifier, the inductor, and the switches, has previously been included in the model developed in Chapter 5 as $R_{\text{Phase}2}$ and $R_{\text{Phase}3}$. The other source of loss in the PRC is a delay between the $N_{\text{Switch}}$ and $P_{\text{Switch}}$ which is a result of the non-ideal behavior of the circuitry used to control the transition from phase2($\rightarrow$inductor) to phase3($\rightarrow$battery).

For the model developed in Chapter 5, the individual conduction losses were summed together to find an effective conduction loss for each phase of operation where

$$R_{\text{Phase}2} = 2R_{\text{diode}} + R_{LPRC} + R_{N\text{.SW}}, \quad (8-2)$$

and

$$R_{\text{Phase}3} = 2R_{\text{diode}} + R_{LPRC} + R_{P\text{.SW}}. \quad (8-3)$$
For the PRC implantation used in this work, the value of is the conduction loss from the active rectifier, which was given by Xu as approximately 1 Ω [13], and the values $N_{\text{Switch}}$ (NTS4001) and $P_{\text{Switch}}$ (BSS223P) are both approximately 1 Ω. The determination of $R_{LPRC}$ is somewhat more complex. The resistive loss of an inductor is frequency dependant as shown in Figure 8-3A. The current passing through the inductor during PRC operation, however, is a shark fin waveform as shown in Figure 8-3B. It is therefore not composed of a single discrete frequency, but rather a superposition of many frequencies.

In order to approximate an effective value for the inductor resistance, the power spectral density (PSD) of the shark fin waveform was computed and the frequency range with the largest signal power was determined. A plot of the PSD for the 100 $\mu$H inductor used in the first generation characterization is shown in Figure 8-4, which shows most of the signal power occurring around 150 kHz. An effective value approximately 3 Ω was then approximated using the data in Figure 8-3A. This same approximation technique was used to find an effective resistance for the other inductors used in this work. Using a value 3 Ω for $R_{LPRC}$, the effective resistances become $R_{\text{Phase 2}} = R_{\text{Phase 3}} \approx 5\Omega$.

![Figure 8-3. Plots showing A) the resistance of a 100 $\mu$H inductor vs. frequency, and B) the shark-fin PRC current waveform.](image)

The other source of loss in the PRC is related to the lag time between phase2 (→inductor) and phase3 (→battery) which is caused by the cascaded control scheme described in
Chapter 6. For the ideal PRC with zero lag time, shown in Figure 8-5A, the N\textsubscript{Switch} is closed and the P\textsubscript{Switch} is open during phase2\textsubscript{($\rightarrow$inductor)} as energy is transferred from the piezoelectric patch to the inductor. When the inductor current reaches its maximum value, the N\textsubscript{Switch} is open and the P\textsubscript{Switch} immediately closes, initiating phase3\textsubscript{($\rightarrow$battery)}. The value of $i_{battery}(t)$ at the start of phase3\textsubscript{($\rightarrow$battery)} is equal to the value of $i_{LPRC}(t)$ at the end of phase1\textsubscript{($\rightarrow$capacitor)}.

![Figure 8-4. PSD of a 100 $\mu$H inductor used for the first generation characterization.](#)

The behavior of a non-ideal PRC with finite lag time is shown in Figure 8-5B. Once again, during phase2\textsubscript{($\rightarrow$inductor)}, the N\textsubscript{Switch} is closed and the P\textsubscript{Switch} is open as energy is transferred from the piezoelectric capacitance to the inductor. When the inductor current reaches a maximum value the N\textsubscript{Switch} opens, but due to the finite delay of the one-shot that controls the P\textsubscript{Switch} there is a lag time, $t_{lag}$, before the P\textsubscript{Switch} closes and starts to conduct. Since $i_{LPRC}(t)$ does not have a conduction path during this lag time, a portion of the energy stored on $L_{PRC}$ is dissipated through parasitic channels. When the P\textsubscript{Switch} finally closes, the value of $i_{battery}(t)$ at the start of phase3\textsubscript{($\rightarrow$battery)} is smaller than the value of $i_{LPRC}(t)$ at the end of phase2\textsubscript{($\rightarrow$inductor)}, resulting in a reduction in the amount of power.
delivered to the load. For the PRC control circuit implemented in this work, a lag time of approximately 200 ns was observed.

\[
\begin{align*}
&i_{\text{battery}}(t) \\
&v_{\text{rect}}(t) \\
&N_{\text{Gate}}(t) \\
&P_{\text{Gate}}(t)
\end{align*}
\]

Figure 8-5. PRC waveforms current and voltage waveforms showing A) zero switching lag, and B) finite switching lag.

In order to improve the accuracy of the model, it is necessary to include the effects of the finite lag time in the model simulations. Directly incorporating the 200 ns delay into the simulations is not trivial for the model developed in Chapter 5 because the lag time creates a fourth phase where both switches are open and energy is stored on the inductor. Instead, an effective lag time, based upon empirical data, is implemented to capture the dissipation associated with this switching non-ideality. The effective lag is defined as

\[
t_{\text{lag, effective}} = t_{P_{\text{Switch}}} \left( 1 - \frac{i_{\text{battery, max}}}{i_{LPRC, max}} \right),
\]

where \( t_{P_{\text{Switch}}} \) is the length of time that the \( P_{\text{Switch}} \) remains closed during phase3\(_{-\text{battery}}\); \( i_{\text{battery,Peak}} \) is the maximum value of \( i_{\text{battery}}(t) \), and \( i_{LPRC, Peak} \) is the maximum value of
derivation of this effective lag time and a detailed explanation of how it is integrated with the model is presented in Appendix G.

8.1.3 Experimental Results

For the first generation experimental characterization of the energy harvesting system, the effects of non-resonant frequency operation, losses in the power converter, and non-ideal switch timing were examined. The presentation of the experimental results is divided into three parts. The first part demonstrates how the operation of the energy harvesting system is affected by variations in the mechanical input acceleration frequency. This is followed by an examination of how losses in the PRC affect the total power delivered to the load, where the effects of losses occurring during phase2 (→inductor) and phase3 (→battery) are individually considered. Finally, the effects of non-ideal switch timing are examined.

For beam 1, experimental data was recorded at three different acceleration levels, 0.4 m/s², 0.5 m/s², and 0.6 m/s². These acceleration levels were chosen empirically, because they provided reasonable signals levels without the risk of damaging any of the circuitry. Specifically, since the active rectifier circuitry was implemented on an ASIC originally designed to be operated around 2 V, the chosen acceleration values provided voltages that minimized the risk of damaging the synchronous rectifier. Furthermore, since the power supply rails of the control circuitry were only ±5 V, it was necessary to excite the transducer at an acceleration that did not produce higher open circuit voltages. In this section, the experimental results for an applied base acceleration of 0.5 m/s² are presented. The results for the other two acceleration levels can be found in Appendix F.

8.1.3.1 Frequency Variation

For the frequency variation experiment, a single frequency sinusoidal vibration was applied to the base for each data point. The amplitude of the driving signals was adjusted to achieve the desired acceleration by monitoring the base displacement with the laser.
The control circuitry of the PRC was then tuned to achieve the "normal" operation of the PRC as described in Chapter 4. Normal operation refers to the following conditions:

- $N_{\text{Switch}}$ closes when $v_{\text{rect}}(t)$ reaches its maximum value
- $N_{\text{Switch}}$ opens and $P_{\text{Switch}}$ closes when $i_{LPRC}(t)$ reaches its maximum value
- $P_{\text{Switch}}$ closes when $i_{LPRC}(t)$ reaches zero

The tuning of the PRC control circuitry was performed manually by monitoring $v_{\text{rect}}(t)$ and $i_{LPRC}(t)$ on the oscilloscope and adjusting the potentiometers of the appropriate one-shot. Once the circuit was properly tuned, the current and voltage values needed to characterize the behavior of the system were measured. For the first generation frequency variation experiment, the following quantities were recorded in order to be compared with the model predictions:

- Power delivered to the load
- $v_{\text{rectPeak}}(\text{big})$ and $v_{\text{rectPeak}}(\text{small})$
- $i_{LPRC\text{Peak}}(\text{big})$ and $i_{LPRC\text{Peak}}(\text{small})$
- $i_{\text{batteryPeak}}(\text{big})$ and $i_{\text{batteryPeak}}(\text{small})$
- $t_{NGate}$ and $t_{PGate}$

The power delivered to the load was chosen for a point of comparison as it is a key metric for any energy harvesting system, especially one using the DFP methodology. The peak values of the voltage and currents were chosen since they correspond to the switching instances which define the operation of the PRC. Additionally, the ability to accurately predict the peak values of voltage and current in a system is important if the system models are going to be used for an IC implementation. Finally, the lengths of the two gate signals were captured to determine the models’ ability to predict the strict timing behavior necessary for the PRC to function properly. The predicted behavior for the full LEM and simplified resonant model along with the experimental measurements for these system quantities are presented in the following sections.
Power

A comparison of the total harvested power as a function of frequency is shown in Figure 8-6 for the experimental data and several different cases of both the simplified resonant and full LEM models. An examination of the experimental data in shows that the power follows a resonant behavior, where the peak level occurs at a specific resonant frequency and falls off quickly as the frequency moves away from resonance. The resonant frequency of the experimental data occurs at approximately 59 Hz for beam 1. An effective lag of 135 ns was calculated from the experimental data and included in the model simulations.

![Power vs. frequency for beam 1; $a_m = 0.5 \ m/s^2$; $L_{PRC} = 100 \ \mu H$.](image)

The three curves which use the full LEM model in Figure 8-6 represent the cases of a lossless PRC without lag (green), a lossless PRC with lag (red), and a PRC where both conduction losses and lag are included (blue). The shaded regions of these three curves correspond to uncertainty in the measurement of the lumped damping parameter, $R_m$. For each respective curve, the power generated using the nominal value of $R_m$ is plotted.
with a solid line, and the 95% confidence interval for $R_m$ is used to create the upper low
bounds of the shaded region. The shaded regions in other plots follow the same format
with respect to $R_m$. Three corresponding curves using the simplified resonant model are
also included in Figure 8-6, and implement the same color scheme as the full LEM model.
For these curves, only the nominal values of $R_m$ is plotted. The simplified resonant model
is technically only valid at the short-circuit mechanical resonance of the transducer, $f_{sc}$,
which is 59.14 Hz using the nominal values of $M_m$ and $C_{ms}$ of beam 1. For the sake of
comparison, the simplified resonant model was also applied over the entire frequency range
in order to determine its potential usefulness as a reasonable approximation of the system
behavior.

The three curves for both the simplified resonant and full models are included to
quantitatively show how the non-ideal behavior of the PRC affects the harvested power.
The curves shown in green represent the case of an ideal PRC, where there are zero
conduction losses and no switching lag. This represents a best case design, and can be
used to set an upper limit on the amount of power to be expected. The data shown by
the red curves are used to illustrate the effect of the switching lag. For the full model data
shown in Figure 8-6, the switching lag reduces the peak harvested power by close to 33%,
from 7.5 $\mu W$ to 5.0 $\mu W$. The blue curves include the effects of both the switching lag and
the conduction losses of the PRC, and most accurately describe the implemented system
behavior. In this experiment, the full model shows an additional peak power reduction of
approximately 20%, from 5 $\mu W$ to 4 $\mu W$, when the PRC conduction losses are included in
the model.

Directly comparing the experimental measurements to the two models shows that
the full LEM model captures the resonant behavior of the system within approximixately
0.5 Hz for all three of its cases. The resonant frequency predicted by the full LEM model
is very close to the transducer open-circuit frequency, $f_{oc}$ of 59.47 Hz. The simplified
resonant model, on the other hand, does not capture any of the resonant behavior seen in
the experimental measurements. This is not completely unexpected, considering that the simplified resonant model is only valid at a single frequency. It is therefore more useful to compare the two models with the experimental data at $f_{sc}$, where the simplified model is applicable. For this comparison, the full and simplified resonant models, which include the effects of losses and switching lag (blue), are considered because they offer the best approximation of the experimental system. At 59.14 $Hz$, the value of the power predicted by both models are almost equal, and are both close to the experimental measurement. From Figure 8-6, it is therefore difficult to determine which model is more accurate at the short-circuit resonant frequency.

$v_{\text{rect}}$

In Chapter 5, one of the major differences presented between the simplified resonant model and the full LEM model was the presence of two different values of the peak rectified voltage, $v_{\text{rectPeak (big)}}$ and $v_{\text{rectPeak (small)}}$, seen in the full LEM model. A comparison between the experimental measurements and the values of the peak rectified voltages predicted by the theoretical models is shown in Figure 8-7. Again the full LEM model is presented with the 95% confidence bounds of $R_m$, while the nominal value of $R_m$ is used for the simplified resonant model. All of the simulated curves shown here assume both conduction losses and an effective switching lag of 135 $ns$.

An examination of the frequency dependant operation of the experimental data shows a resonant behavior similar to that of the power shown in Figure 8-6, where the maximum voltage values occur around 59 $Hz$. As expected, the full model does a reasonable job of capturing the frequency behavior and again appears to be shifted by approximately 0.5 $Hz$ and corresponds to $f_{oc}$.

In terms of the magnitudes of $v_{\text{rectPeak (big)}}$ and $v_{\text{rectPeak (small)}}$, neither the full LEM model nor the simplified resonant model provides a good prediction. The full LEM model predicts that the two voltage peak values will be similar away from resonance, and will diverge as the frequency approaches $f_{oc}$. The experimental results, however, show that
the values of the two peak voltages have very little separation, and that the magnitude is similar to the value predicted by $v_{rectPeak}(big)$. For the simplified resonant model only a single peak voltage value is predicted. However, even at $f_{sc}$, were the simplified resonant model should be most accurate, the predicted values are almost half of the experimental results.

![Graph showing $v_{rectPeak}$ vs. frequency](image)

Figure 8-7. $v_{rectPeak}$ vs. frequency for beam 1; $a_m = 0.5 \, m/s^2$; $L_{PRC} = 100 \, \mu H$

$i_{LPRC}$

As was the case for the rectified piezoelectric voltage, the full LEM model of the energy harvesting system developed in Chapter 5 predicts two different values for the peak inductor current, $i_{LPRCpeak}(big)$ and $i_{LPRCpeak}(small)$, while the simplified resonant model does not. Figure 8-8 presents a comparison between the experimental results and the two simulated models for the peak values of $i_{LPRC}(t)$ which occur at the end of phase\textsubscript{2}($\rightarrow$inductor).
Similarly to previous results, the experimental data displays a resonant behavior which is captured using the full LEM model. The frequency separation between the measured and modeled curves is once again 0.5 Hz. The differences in the magnitudes of \( i_{LPRC\text{Peak}}(\text{big}) \) and \( i_{LPRC\text{Peak}}(\text{small}) \) are once again not captured well by either of the models. The full LEM model predicts a significant separation of the peak current values around the resonant frequency, while the experimental results show only a small separation. The simplified resonant model, on the other hand, predicts a small separation of the peak current values, but greatly under predicts their magnitude.

![Graph showing \( i_{LPRC\text{Peak}} \) vs. frequency for beam 1; \( a_m = 0.5 \text{ m/s}^2 \); \( L_{PRC} = 100 \mu \text{H} \).](image)

Figure 8-8. \( i_{LPRC\text{Peak}} \) vs. frequency for beam 1; \( a_m = 0.5 \text{ m/s}^2 \); \( L_{PRC} = 100 \mu \text{H} \).

While the experimental measurements in Figure 8-8 do not show the level of separation between \( i_{LPRC\text{Peak}}(\text{big}) \) and \( i_{LPRC\text{Peak}}(\text{small}) \) predicted by the full LEM model, the experimental results are at least consistent when the observed behavior of \( v_{\text{rect}}(t) \) from Figure 8-7 is considered. At the beginning of phase 2\(_{\text{inductor}}\), the value of the energy stored on the capacitor is proportional the peak value of the rectified
voltage, $v_{\text{rect}}(t)$. At the end of phase $2_{\text{inductor}}$, when all of the energy has been transferred to the inductor, the value of the stored energy is proportional to the peak value of the inductor current, $i_{LPRC}(t)$. If subsequent peak values of $v_{\text{rect}}(t)$ are close in magnitude, as shown in Figure 8-7, the corresponding peak values of $i_{LPRC}(t)$ will also exhibit similar magnitudes, as shown in Figure 8-8.

$i_{\text{battery}}$

Both the experimental and modeled values of the peak battery currents, $i_{\text{batteryPeak (big)}}$ and $i_{\text{batteryPeak (small)}}$, are shown in Figure 8-9. A comparison between the full LEM case and the experimental data follows the same trends as previous results, where the full LEM model predicts the frequency behavior seen in the experimental data, with a difference of $\sim 0.5$ Hz. While the full LEM model predicts two different peak heights, whose separation is largest at the open circuit resonant frequency, the experimental measurements are once again close together. The simplified resonant model predicts two closely spaced peak heights at $f_{sc}$, and comes reasonably close to the magnitude of the experimental data.

![Graph](image-url)

Figure 8-9. $i_{\text{battery}}$ vs. frequency for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 100 \mu \text{H}$. 

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Figure 8-10 shows a comparison between experimental results and the behavior predicted by the two simulated models for the value of $t_{NGate}$ as a function of frequency. For both the full and simplified resonant models, the value of $t_{NGate}$ is constant, regardless of the operating frequency. The values predicted by both models is 1.89 $\mu$s and the waveforms overlap in Figure 8-10. Similar behavior occurs for the experimental data, where the mean value over the frequency range is 1.61 $\mu$s, with a standard deviation of 46.20 ns. One potential source for the random deviations in the experimental values can be attributed to the fact that the $N_{Gate}(t)$ signal was manually adjusted for each frequency to end on the peak value of $i_{LPRC}(t)$. Determination of the exact peak within a few nanosecond was not trivial due to noise in the $i_{LPRC}(t)$ signal.

![Figure 8-10](image)

Figure 8-10. $t_{NGate}$ vs. frequency for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 100 \mu H$.

The constant value for $t_{NGate}$ as a function of frequency is explained by considering the phase2($\rightarrow$inductor) operation of the PRC shown in Figure 8-11. During phase2($\rightarrow$inductor),
energy is transferred from $C_{eb}$ to $L_{PRC}$. Assuming that the other LEM components and the parasitic effects of the system do not greatly affect behavior of the energy transfer, the process reduces to a simple LC oscillation. The amount of time required to transfer all of the stored energy from the piezoelectric capacitance to the inductor, which is the length of $N_{Gate}(t)$, is one quarter of the LC oscillation period [13]

$$t_{NGate} = \frac{\pi \sqrt{C_{eb}L_{PRC}}}{2}.$$  (8–5)

Since this value is only dependant on $C_{eb}$ and $L_{PRC}$, the frequency of the applied acceleration does not change the value of $t_{NGate}$. As a point of reference, the value of $t_{NGate}$ predicted from Equation 8–5 is also shown in Figure 8-10.

![Figure 8-11. Phase2(→inductor) operation of the PRC approximated as an LC oscillator.](image)

The effects of frequency on the value of $t_{PGate}$ for the experimental measurements and simulated results are compared in Figure 8-12. As a result of the experimental implementation used to realized the discrete PRC in Chapter 6, the control circuitry only allowed for a single value of $t_{PGate}$. For both models, however, two alternating values of $t_{PGate}$, $t_{PGate}(big)$ and $t_{PGate}(small)$ were predicted. Unlike the case for $t_{NGate}$, which did not show any frequency dependence, the experimental data for $t_{PGate}$ exhibits similar resonant behavior to many of the previously presented measurements.

The full LEM model shows similar frequency behavior to the experimental data, with a 0.5 Hz frequency shift in peak values. In terms of the magnitude, most of the
experimental values for the length of $t_{PGate}$ are between $t_{PGate}(big)$ and $t_{PGate}(small)$ predicted by the full model. While the simplified resonant model again does not capture any of the frequency behavior, at $f_{sc}$ it provides a reasonably accurate estimate for the experimentally measured value of $t_{PGate}$.

![Diagram](image)

Figure 8-12. $t_{PGate}$ vs. frequency for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 100 \mu H$.

### 8.1.3.2 Variation of PRC Conduction Losses

Two separate experiments were performed to characterize the effects of the conduction losses in the power converter from $R_{Phase2}$ and $R_{Phase3}$. The first experiment measured how variations of $R_{Phase2}$ affect the power delivered to the load for a constant value of $R_{Phase3}$, and in the second $R_{Phase3}$ was varied while $R_{Phase2}$ was held constant. For both cases, the respective resistance value was varied by placing discrete resistors in the conduction paths. The resistors used to vary $R_{Phase2}$ were inserted between the $N_{Switch}$ and ground, and those used to vary $R_{Phase3}$ were inserted between the load and ground as shown in Figure 8-13.
The data collection procedure used for both of these experiments was similar to that of the frequency variation case. A discrete resistor was first placed in the appropriate conduction path and then a sinusoidal base vibration was applied at 59 Hz. This frequency was used for all of the conduction loss experiments presented for beam 1. The base acceleration value was set using the laser displacement sensor. The control circuitry of the PRC was again manually tuned to achieve the normal operation described in the frequency variation experiment. A measurement of the power was then taken for each resistance data point.

Figure 8-13. Schematic of the PRC demonstrating the position of $R_{\text{Phase2}}$ and $R_{\text{Phase3}}$.

**Power vs. $R_{\text{Phase2}}$**

The effects of $R_{\text{Phase2}}$ on the power delivered to the load for beam 1 are shown in Figure 8-14. Both the modeled data and the experimental measurements indicate a reduction in the harvested power as the conduction loss of the PRC increases. This is behavior is logical, considering that any losses in the PRC would reduce the amount of power which is transferred from the piezoelectric to the load. The experimental data shows good agreement with both the LEM and simplified resonant model with losses.
The agreement between these two models should not be surprising considering that both models predict similar levels of harvested power for this frequency in Figure 8-6. As a reference, the simplified resonant model with zero PRC loss is also included to demonstrate the importance of modeling the non-ideal behavior of the energy harvesting system. The power for the simplified resonant model with zero loss was presented in Chapter 4 (given by Equation 4–17) and over predicts the measured power by a factor of almost 3.

Figure 8-14. Power vs. $R_{\text{Phase}2}$ for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 100 \, \mu\text{H}$; $f = 59 \, \text{Hz}$.

**Power vs. $R_{\text{Phase}3}$**

The manner in which power varies with $R_{\text{Phase}3}$ for beam 1 is presented in Figure 8-15. As was the case with $R_{\text{Phase}2}$, the total harvest power decreases with increasing PRC conduction losses. An examination of the experimental data shows good agreement with both the full LEM and simplified resonant models which include PRC loss. The slope of the data from the simulated models is slightly greater than the experimental case, but is
within experimental uncertainty for most data points. The resonant model with a zero loss PRC is included in this figure to again demonstrate the importance of including losses in the modeling procedure of energy harvesting systems.

![Figure 8-15. Power vs. $R_{Phase3}$ for beam 1; $a_m = 0.5 \, m/s^2$; $L_{PRC} = 100 \, \mu H$; $f = 59 \, Hz$.](image)

### 8.1.3.3 Effects of Timing Offsets

The goal of this characterization was to determine the effects that non-ideal switch timing has on the operation of the energy harvesting system. However, due to the topology used to implement the discrete PRC, this characterization focuses solely on the behavior of the $N_{Switch}$. The effects of timing variation on the $P_{Switch}$ could not be examined because the implemented circuitry could not stop the flow of current through the $P_{Switch}$ until the energy stored on $L_{PRC}$ had completely discharged into the battery. The exact mechanism responsible for preventing the closure of the $P_{Switch}$ was not thoroughly examined and is left for future work.

To characterize the effects of non-ideal timing of the $N_{Switch}$, the length of the $N_{Gate}(t)$ signal was manually varied by changing the potentiometer of the one-shot...
controlling it. Data was collected in a manner similar to that for the previous experiments. The value of $t_{NGate}$, was set using an oscilloscope, and the acceleration was then adjusted by monitoring the laser displacement sensor. Once the system was properly tuned, a power measurement was taken and the process was repeated for all of the data points.

The results of varying $t_{NGate}$ on the harvested power are shown in Figure 8-16 along with the behavior predicted by the full LEM and simplified resonant models. Both the experimental data and the full LEM model show a peak in the power around 1.2 $\mu$s, where the magnitude of the results is over-predicted by the simulation. The shape of the power curve predicted by the simplified resonant model shows similar behavior to the full model, with a peak power value occurring when $t_{NGate}$ is around 1.6 $\mu$s.

Examining the behavior of the full LEM model shown in Figure 8-16, it can be seen that maximum power does not occur at the value of $t_{NGate} = 1.89$ $\mu$s, as predicted by the model in Figure 8-10, but rather occurs at the smaller value of $t_{NGate} = 1.2$ $\mu$s. The value of 1.89 $\mu$s for $t_{NGate}$ corresponds to the case where phase2 (→inductor) is tuned to...
end when the value of $i_{LPRC}(t)$ reaches a maximum. If $t_{NGate}$ is less than 1.89 $\mu$s, the switching will occur before the maximum value is reached. Similar behavior is shown for the experimental data. The mean value of $t_{NGate}$ from Figure 8-10, which was found by manually tuning the end of phase2$_{inductor}$ to coincide with a peak value of $i_{LPRC}(t)$, is approximately 1.6 $\mu$s, while the maximum power occurs when $t_{NGate}$ is 1.2 $\mu$s. In terms of the circuit operation, this means that it may be possible to increase the total harvested power by ending phase2$_{inductor}$ before the value of $i_{LPRC}(t)$ reaches a peak.

8.1.4 Deficiencies of the First Generation Experiment

While capturing and processing data for the experimental characterization of beam 1, several minor deficiencies in the experimental process were observed. For instance, in the operational waveforms where the frequency was varied, there is a small shift between the resonant frequencies of the model simulations and the experimental measurements of approximately 0.5 $Hz$. One possible explanation for this discrepancy in resonant frequency can be attributed to a poorly clamped beam. In some preliminary experiments with beam 1, masking tape was wrapped around the clamped end of the beam in an attempt to electrically isolate it from the clamp. The tape was still present in the first generation experimental characterization, and following the measurements it was discovered that the presence of the tape allowed the beam to slide slightly within the clamp. It is possible that a small displacement of the beam within the clamp altered the effective LEM parameters from the extracted values used in the model simulations.

Another issue in the first generation experimental setup is the use of the LK-G32 laser displacement sensor to monitor the acceleration of the base. According to the specification sheet for the laser, the minimum detectable displacement is on the order of $\pm 5 \mu m$. For the accelerations and frequencies of interest, Equation 8–1 indicates that displacement amplitude also on the order of $\pm 5 \mu m$ must be monitored by the laser. Operating the laser near its minimum detectable displacement gives rise to some concerns about the accuracy of its measurements.
While it is believed that these deficiencies do not invalidate the data from the first generation experimental characterization, they are certainly not desirable. In the next section of this chapter, an improved experimental setup is presented which was used to address the aforementioned issues with the measurement and characterization.

8.2 Second Generation Experiment

The second generation characterization builds on the first generation by improving upon some of the experimental issues which were discusses in Section 8.1.4. As in the first generation characterization, a number of experiments are performed to characterize both the general behavior of the energy harvesting system as well as the effects of system non-idealities.

This section is divided into two parts. In the first part, the modifications made to the first generation experimental procedure are discussed in detail. This is followed by a presentation of the experimental measurements which were made using the second generation setup, and compares the experimental behavior to both the full LEM and simplified resonant models.

8.2.1 Modifications to First Generation Experiment

The primary deficiencies with the first generation experiment involved the method implemented to clamp the transducer beam and the use of the laser displacement sensor to monitor the base acceleration. A poor clamping condition is suspected of causing the 0.5 Hz frequency shift between the experimental data and the simulations using the full model. The levels of displacement monitored with the laser displacement sensor were near its minimum detectable signal, and the accuracy of the acceleration values derived using the laser are therefore questionable. Several modifications were made to the first generation experimental setup in order to correct these deficiencies.

To improve the clamping of the transducer beam structures, the masking tape used for electrical isolation of beam 1 was not used in the fabrication of the subsequent beams.
Additionally, nuts were added to the clamp structure in order to allow for the clamping screws to be tightened more securely, as shown in Figure 8-17.

Figure 8-17. Improved clamp structure for second generation characterization.

The issues associated with the laser displacement sensor were addressed by modifying the experimental setup, as shown in Figure 8-18. Instead of using the laser displacement sensor to calculate the base acceleration from a displacement measurement, the applied base acceleration was calculated using a Polytec MSV 300 scanning laser vibrometer. The laser vibrometer (LV) employs optical interferometry techniques to measure the velocity of a surface. For single frequency, sinusoidal excitations, the magnitude of the velocity, $|v_m|$, can be related to the acceleration amplitude by

$$|a_m| = |v_m| \omega.$$  \hspace{1cm} (8–6)

The primary motivation for using the LV system to monitor the base acceleration was the improved resolution over the laser displacement sensor. For the frequency range interest, the LV has a resolution of 0.3 $\mu m/s$, which translates to an acceleration of 113 $\mu m/s^2$. This is almost four orders of magnitude improvement over the laser displacement sensor. Additionally, by using the LV to monitor the base acceleration, the laser displacement sensor can be used to monitor the tip displacement of the transducer beam. Since the magnitude of the tip displacement is much larger than that of the base, there are no
issues with the laser displacement sensor operating near its noise floor. The rest of the experimental setup used for the second generation characterization experiments is the same as that used for the first generation.

![Diagram of experimental setup](image)

Figure 8-18. Modified second generation experimental setup.

### 8.2.2 Experimental Results

For the second generation experimental characterization of the energy harvesting system, the same series of tests performed for the first generation on beam 1 were applied to beams 2 and 3 for several different acceleration levels. In this section, the results for beam 2 are presented for an acceleration of $0.5 \ m/s^2$, and were chosen as representative examples for the complete dataset. The presentation of the experimental results is again divided into three parts. The effects of non-resonant frequency operation are presented first, followed by the effects of losses in the power converter. Finally, the effects of non-ideal switch timing are examined. Complete results for both beams 2 and 3 are presented in Appendix F. The experimental results for each of the three tests are compared to the full LEM and simplified resonant system level models.
8.2.2.1 Frequency Variation

The procedure for the second generation frequency variation experiment is nearly identical to that presented in Section 8.1.3.1. The primary differences between this experiment and the previous one are the use of the LV to monitor the base acceleration and the use of laser displacement sensor to monitor tip displacement.

Power

The total harvested power as a function of frequency is shown in Figure 8-19 for the experimental data and several cases of the simplified resonant and full LEM models. Using the experimental results, an effective lag value of 52.4 ns was calculated for beam 2. A comparison of the experimental measurements and the full model with both conduction losses and lag (blue) shows good agreement in both the magnitude and the frequency behavior. Peak power for both curves occurs around the open-circuit resonant frequency of 64.4 Hz. Since PRC operation forces the transducer to operate in an open circuit configuration for the majority of the time, it is reasonable that the energy harvesting system follows the open-circuit resonant behavior. Examining the simplified model at the short-circuit resonant frequency of 63.6 Hz, there is also good agreement between the experimental data and the model. It is interesting to note that while simplified resonant model accurately predicts the harvested power at the short-circuit mechanical frequency, maximum power actually occurs at the open-circuit frequency. For the data shown in Figure 8-19, the difference between the power at open and short circuit frequencies is more than 30%.

The red and green curves in Figure 8-19 are included to again illustrate the effects of loss and switch lag in the PRC. For the full model, the switching lag reduces the total harvested power by approximately 10%, and the conduction losses of the PRC account for another 20%.

Comparing the results of the power vs. frequency for beam 1 (presented in Section 8.1.3.1) and beam 2, the simulation results and experimental measurements are in better
agreement for beam 2. This is most likely due to the changes made to the experimental setup from the first to second generation. The 0.5 $Hz$ shift in resonant frequency between the simulated and measured data is no longer present in the 2nd generation, where nuts were added to improve the boundary conditions. In addition to the improved agreement in the frequency behavior, the magnitude of the power predicted in the 2nd generation also shows better agreement with experiment. Since the LV gives much better resolution in the base acceleration measurement than the laser displacement sensor, it is reasonable to assume that the simulated and experimental acceleration amplitudes more closely match each other in the 2nd generation characterization.

![Power vs. frequency for beam 2; $a_m = 0.5 \, m/s^2$, $L_{PRC} = 270 \, \mu H$.](image)

$v_{rect}$

A comparison between the experimental measurements and the values of the peak rectified voltages predicted by the theoretical models is presented in Figure 8-20. As with the power data, there is good agreement between the frequency behavior of the
experimental data and the full model, both of which seem to have resonant frequencies around the open circuit resonance of 64.4 Hz.

As was the case for beam 1, neither model seems to accurately capture the magnitude of $v_{\text{rectPeak}}(\text{big})$ and $v_{\text{rectPeak}}(\text{small})$. Both the full LEM model, and the simplified resonant model at short-circuit resonance, under predict both the big and small values of the rectified voltage peaks. Additionally, the full model over predicts the separation between the big and small peak values for all but a few cases.

![Figure 8-20. $v_{\text{rectPeak}}$ vs. frequency for beam 2; $a_m = 0.5 \text{ m/s}^2$, $L_{PRC} = 270 \mu\text{H}$.](image)

A comparison between the experimental results and the two simulated models for the peak values of $i_{\text{LPRC}}(t)$, $i_{\text{LPRCPeak big}}$ and $i_{\text{LPRCPeak small}}$, is shown in Figure 8-21. The experimental data and the full model show good agreement in their respective frequency behavior. Away from the open-circuit resonant frequency, where the peak values of $i_{\text{LPRC}}(t)$ are close together, the full model provides an accurate prediction of
experimental results. However, near the resonant frequency where the model predicts the peak values will diverge, the experimental data does not show the expected separation. At the short-circuit resonant frequency, the simplified model again under predicts the value of the current peaks.

Similarly to beam 1, while the level of separation between $i_{LPRCPeak}(big)$ and $i_{LPRCPeak}(small)$ predicted by the full model is lower than the measured data, the experimental results are consistent with the behavior of $v_{rect}(t)$ in Figure 8-20. The small amount of separation between $v_{rectPeak}(big)$ and $v_{rectPeak}(small)$, translates into a small amount of separation for $i_{LPRCPeak}(big)$ and $i_{LPRCPeak}(small)$.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>$i_{LPRCPeak}$ [mA]</th>
<th>$i_{LPRCPeak}(small)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>62</td>
<td></td>
<td></td>
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<tr>
<td>63</td>
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<tr>
<td>64</td>
<td></td>
<td></td>
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<tr>
<td>65</td>
<td></td>
<td></td>
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<tr>
<td>66</td>
<td></td>
<td></td>
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<tr>
<td>67</td>
<td></td>
<td></td>
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<tr>
<td>68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>69</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$F_{sc} = 63.6 \text{ Hz}$

$F_{oc} = 64.4 \text{ Hz}$

Figure 8-21. $i_{LPRC}$ vs. frequency for beam 2; $a_m = 0.5 \text{ m/s}^2$, $L_{PRC} = 270 \mu\text{H}$.

$i_{battery}$

The experimentally captured and modeled values of the peak battery current, $i_{batteryPeak}(big)$ and $i_{batteryPeak}(small)$, are shown in Figure 8-22. Comparing the full model to the experimental data shows that both follow the same open-circuit resonant
behavior as the other signals tested in the frequency variation experiment. Similarly to the behavior of $v_{rect}(t)$ and $i_{LPRC}(t)$, the full model predicts the battery current well at frequencies away from the open circuit resonant frequency, where $i_{batteryPeak\,(big)}$ and $i_{batteryPeak\,(small)}$ converge. Near the open-circuit resonance, however, the experimental data does not show the large separation predicted by the full model. At the short-circuit mechanical resonant frequency, the simplified resonant model gives a close approximation of the peak values of $i_{battery}(t)$. This is consistent with the power measurement for beam 2, where the simplified resonant model also provides an accurate prediction of the experimental measurement.

![Graph showing $i_{batteryPeak\,(big)}$ and $i_{batteryPeak\,(small)}$ vs. frequency]

Figure 8-22. $i_{battery}$ vs. frequency for beam 2; $a_m = 0.5 \text{ m/s}^2$, $L_{PRC} = 270 \mu H$.

$N_{Gate}$

A comparison between the experimental results and the two simulated models for the value of $t_{N_{Gate}}$ is shown in Figure 8-23. As with beam 1, the frequency behavior predicted by both the full and simplified resonant models predicts a constant value
which is independent of frequency. The length of the NGate signal using the full and simplified resonant models are 4.32 $\mu s$ and 4.36 $\mu s$, respectively. The experimental results show similar behavior, where the mean is 4.23 $\mu s$ with a standard deviation of 85.50 $ns$. Again, the slight differences in the experimental measurements are most likely attributed to the manually tuning procedure used to realize the switching lengths. The value of $t_{NGate}$ predicted by Equation 8–5 is also presented in Figure 8–23 in order to compare the behavior of both the experimental data and the models to an ideal LC oscillation. While the ideal value is slightly over predicts the measured values, it provides a simple expression to approximate $t_{NGate}$.

![Figure 8-23. $t_{NGate}$ vs. frequency for beam 2; $a_m = 0.5 \ m/s^2$, $L_{PRC} = 270 \ \mu H$.](image)

$P_{Gate}$

Figure 8-24 shows the effects of frequency on the value of $t_{PGate}$ for the experimental measurements and simulated results. Due to the implementation of the discrete PRC circuitry, there is only a single value of $t_{PGate}$ at each frequency point, while the models
both predict two values, \( t_{PGate}(big) \) and \( t_{PGate}(small) \). For both the experimental measurements and the full LEM model, the maximum values of \( t_{PGate} \) occur near the open-circuit resonant frequency. For the simplified model at \( f_{sc} \), the value of \( t_{PGate} \) is underestimated when compared to the experimental measurements.

Figure 8-24. \( t_{Gate} \) vs. frequency for beam 2; \( a_m = 0.5 \, m/s^2 \), \( L_{PRC} = 270 \, \mu H \).

**Tip Displacement**

For the second generation characterization experiments, the laser displacement sensor was implemented to monitor the tip displacement of the transducer beam. By monitoring the beam tip while energy is being harvested, it is possible to determine how well the developed models capture the effects that energy harvesting operation has on the beam mechanics.

A comparison of the experimental data and model simulations for the amplitude of tip displacement as a function of frequency is shown in Figure 8-25. Both the experimental measurements and the full model show that the tip displacement reaches its maximum
value at the open circuit resonant frequency. The magnitude of the experimental results and the full model show good agreement over the displayed frequency range.

\[ f_{sc} = 63.6 \, \text{Hz} \]
\[ f_{oc} = 64.4 \, \text{Hz} \]

Figure 8-25. Tip displacement vs. frequency for beam 2; \( a_m = 0.5 \, \text{m/s}^2 \), \( L_{PRC} = 270 \, \mu\text{H} \).

The results from the simplified resonant model require some additional explanation. For the sinusoidal motion of the transducer tip, the amplitude of the displacement can be related to the amplitude of the velocity by Equation 8–6. In Chapter 5, however, it was shown that the velocity waveform exhibited large discontinuities as a consequence of removing the inductor from the LEM. The velocity waveform at \( f_{sc} \), 63.6 \( \text{Hz} \), is shown in Figure 8-26 for this test. Since the discontinuities are nonphysical, and simply a result of the model, they were ignored in the calculation of the tip displacement, and the velocity waveform in Figure 8-26 was approximated as a sinusoid. Even ignoring the high amplitude discontinuities, the simplified resonant model over predicts the tip displacement by a factor of approximately 2 in Figure 8-25. If these discontinuities are not ignored, the simplified resonant model only moves further away from the experimental results.
8.2.2.2 Variation of PRC Conduction Losses

In order to examine the effects of the PRC conduction losses in the second generation experimental characterization, the experimental procedure used in Section 8.1.3.2 was employed. Separate experiments for individually measuring the effects of $R_{\text{Phase}2}$ and $R_{\text{Phase}3}$ were performed using the setup presented in Figure 8-13. The effects of varying the conduction loss on the total harvested power are discussed in this section.

**Power vs. $R_{\text{Phase}2}$**

The effects of the $R_{\text{Phase}2}$ on the total harvested power at are shown in Figure 8-27 for the modeled and experimental systems when operated at 63.5 Hz. As the level of conduction loss during phase2(→inductor) increases, the power decreases for both the experimental and simulated cases. The simplified resonant model shows slightly better agreement with the experimental measurements than the full model, but the general trend of the curves is similar. The presence of parasitic losses which were not modeled, may
explain the over predictions by the simulated and experimental data. Some discrepancy between the full and simplified models is expected, since the operating frequency for this test is not exactly the short-circuit mechanical resonant frequency required by the simplified model.

Figure 8-27. Power vs. \( R_{\text{Phase2}} \) for beam 2; \( a_m = 0.5 \, \text{m/s}^2 \); \( L_{PRC} = 270 \, \mu\text{H} \); \( f = 63.5 \, \text{Hz} \).

**Power vs. \( R_{\text{Phase3}} \)**

Figure 8-28 demonstrates the effects that phase3\(_{-\text{battery}}\) conduction losses have on total harvested power. Unlike the previous experiments, whose effective lags were found to be 52.4 ns, the effective lag for this experiment was calculated to be 106.0 ns. Similarly to the case for \( R_{\text{Phase2}} \), as the value of \( R_{\text{Phase3}} \) is increased, the power delivered to the load is reduced. A comparison between the two modeled cases and the experimental data indicates that both models over-predict the results measured in the experimental circuit. The slopes of the modeled cases and experimental measurements, however, are in good agreement, indicating perhaps some bias error in the experiment. Again, the presence of parasitic effects not included in the model could be the cause of this discrepancy.
A comparison of the effects of $R_{\text{Phase}2}$ and $R_{\text{Phase}3}$ on the harvested power for beam 2 indicates that the conductive losses which occur during phase2($\rightarrow$inductor) have a greater impact on the power reaching the load than those losses which occur during phase3($\rightarrow$battery). Since the length of phase2($\rightarrow$inductor) is greater than the length of phase3($\rightarrow$battery) for this particular case, the conduction losses which make up $R_{\text{Phase}2}$ will waste more energy than the losses of $R_{\text{Phase}3}$. Should this be reversed, and phase3($\rightarrow$battery) is longer than phase2($\rightarrow$inductor), the opposite behavior would be expected.

![Figure 8-28. Power vs. $R_{\text{Phase}3}$ for beam 2; $a_m = 0.5 \, m/s^2$; $L_{PRC} = 270 \, \mu H$; $f = 63.5 \, Hz$.](image)

### 8.2.2.3 Effects of Timing Offsets

The effects of variation in the timing signal $N_{\text{Gate}}(t)$ on the total harvested power were characterized using the experimental approach previously discussed in Section 8.1.3.3. The experimental results and model simulations for power as a function of $t_{NGate}$ is shown are Figure 8-29. As was the case in the first generation characterization, both the full and simplified resonant models predict the general shape of the experimental data. In terms of the magnitude, there is a slight over-prediction of the experimental measurements by
both models. Since the frequency of operation used for these measurements is close to
the short-circuit mechanical resonant frequency, the magnitude of the simplified model
provides a reasonable prediction for the experimental data. Had a different operating
frequency been chosen, the agreement between the experimental measurements and the
simplified resonant model would most likely not occur.

Figure 8-29. Power vs. $t_{NGate}$ for beam 2; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$; $f = 63.5 \text{ Hz}$.

As discussed in the first generation characterization, the experimental results and
model simulations shown in Figure 8-29 both indicate that an increased amount of
power is harvested when $t_{NGate}$ is smaller than the value used in the frequency variation
experiments. In the frequency variations experiments, $t_{NGate}$ was tuned to force the
circuit to remain in phase2\text{\rightarrow inductor} until $i_{LPRC}(t)$ reached a peak. The experimental
values of $t_{NGate}$ required to achieve this were 4.23 $\mu s$ and 4.26 $\mu s$ for the experimental
measurements and full LEM, respectively. Figure 8-29 indicates that an increased amount
of power could be harvested if a value of $t_{NGate} = 3.69 \mu s$ and 3.05 $\mu s$ are used for the
experimental and full LEM cases.
8.3 Summary of Experimental Results

In the preceding two sections of this chapter, the behaviors predicted by both the full LEM and simplified resonant system models were compared to the experimental measurements from the implemented energy harvesting system. In order to examine the ability of the models to accurately predict the non-ideal behavior of the system, three separate experiments were performed, which captured the effects of frequency variation, conduction losses, and variations in the timing of the switching signals. In this section, a summary of the results from the first and second generation characterizations is presented, and general conclusions are made about the importance of an accurate system level model with respect to the design of energy harvesting systems.

For the frequency variation experiments presented in the previous sections, it was shown that the full LEM model is capable of accurately predicting the resonant behavior of the various signals in the energy harvesting system over the range of tested frequencies. Considering the case of beam 2 in the second generation experiment, where the transducer was rigidly clamped, the frequency where maximum power was harvested occurred at the open-circuit resonant frequency of the transducer. Since the PRC operates with the transducer in an open-circuit configuration for the majority of each energy harvesting cycle, it is reasonable that the harvested power should have a maximum value at the open-circuit resonant frequency. In addition to the power, the maximum values of the other frequency-dependant PRC signals tested also occurred at the open-circuit resonant frequency. This can be further justified by considering that the values of these PRC signals ($v_{rect}(t)$, $i_{LPRC}(t)$, $i_{battery}(t)$ and $t_{PGate}$) are all proportional to the total harvested power, and the frequency behavior between these signals and the power should be the same. Unlike the full LEM model, the simplified resonant model is only applicable at the short-circuit mechanical resonant frequency. When the simplified resonant model was applied at other frequency values, it did not predict the resonant behavior observed in the experimental measurements. Therefore, in a practical design, where the frequency of
the vibration source and the designed resonant frequency of the transducer are typically not in perfect agreement, the simplified resonant model is not suitable as a design tool for approximating the behavior for a PRC-based energy harvesting system. Assuming that it were possible to design the short-circuit mechanical resonant frequency of the transducer to perfectly match the frequency of the source vibration, the results from beam 2 show that the power levels at $f_{sc}$ are lower than those achieved at $f_{oc}$. Therefore, for a PRC-based energy harvesting system, the design goal should be to match the open-circuit resonant frequency of the transducer to the frequency of the vibration source.

While the full LEM model accurately predicted the resonant behavior presented in the frequency variation experiments, there was some discrepancy observed between the predicted values and experimental measurements in terms of the alternating peak behavior of $v_{rect}(t)$, $i_{LPRC}(t)$, $i_{battery}(t)$ and $t_{PGate}$. For these four signals, the full LEM model predicts alternating peak heights, where the difference in the height between consecutive peaks is function of frequency. When the vibration frequency is equal to the open-circuit resonant frequency the difference is the peak heights reaches its maximum value. As the vibration frequency moves away from $f_{oc}$, the difference between the peak heights is reduced and the height of consecutive peaks approach a single value. Comparing the model predictions to the experimental data for beam 2, the largest difference between peak heights is observed at the resonant frequency, and this difference is reduced as the input frequency moves away from resonance. The magnitude of the separation between consecutive peaks for the experimental measurements, however, is over predicted by the full LEM model. This over prediction is most likely caused by the circuit implementation used to realize the discrete PRC switching control. Since only a single value for $t_{NGate}$ and $t_{PGate}$ are generated in the discrete PRC, the lengths of phase2(→inductor) and phase3(→battery) are fixed. If these values are tuned to accommodate a large peak height, instability of the system occurs when a much smaller peak height is
encountered. Similarly, if the circuit is tuned to accommodate a small peak height and encounters a peak height that is too much larger, the system again becomes unstable. In order to gather experimental data, a stable system was needed, and as a result of tuning the system for stable operation, the difference in heights of the alternating peaks was reduced. If instead of using fixed values for the lengths of phase2 (→inductor) and phase3 (→battery), a more sophisticated control circuit was implemented which featured individualized timing for each of the alternating peak values, it is expected that the behavior of the modified system would more closely match that predicted by the full LEM model.

The tip displacement of the transducer beam was measured in the second generation frequency variation experiment in order to characterize the effects that removing energy from the electrical domain of the transducer have on its mechanical operation. The results predicted by the full LEM model show good agreement with the experimental measurements of the tip displacement. For the simplified resonant model, however, the discontinuous behavior resulting from the removal of $M_m$ and $C_{ms}$ from the model lead to an over-prediction of the tip displacement. The ability of the full LEM model to predict the tip displacement allows for a design which captures the full volumetric space requirements of the operating energy harvesting system, and not just the two-dimensional area of the transducer footprint.

The importance of accurately modeling the losses and non-ideal behavior of the PRC in the energy harvesting system model was demonstrated in Figure 8-6 and Figure 8-19, which plot power as a function of frequency. Considering the full LEM model case, these two figures show that the power predicted using an ideal PRC model with zero conduction losses and zero switch lag (green) over predicts the power by 30-50% compared with the model which includes conduction losses and switch lag (blue). In a system implemented using the DFP methodology, a difference of this magnitude could severely affect the system performance.
The specific effects of the conduction losses of the PRC during phase2 (→inductor) and phase3 (→battery) on the total harvested power were examined by sweeping the values of \( R_{Phase2} \) and \( R_{Phase3} \). As expected, the amount of total harvested power decreased with increasing conduction losses. This behavior was predicted by both models and confirmed by the experimental measurements. For this study, the measured data indicated that value of \( R_{Phase2} \) had a greater effect on the total harvested power than the value of \( R_{Phase3} \). This is most likely due to the fact that the length of phase2 (→inductor), and therefore the conduction time of \( R_{Phase2} \), was several times longer than the length of phase3 (→battery).

The effect of timing variations in the \( N_{Gate}(t) \) signal predicted by both models showed good agreement with the experimental measurements. Since the frequency chosen to perform these tests was near the short-circuit mechanical resonance of both transducer beams 1 and 2, the difference between the full LEM and simplified resonant models is expected to be small. Had a different vibration frequency been chosen, the simplified resonant model would not exhibit the same level of agreement. An examination of the results for both beams showed that the maximum amount of power was not harvested for the value of \( t_{NGate} \) shown to produce normal PRC operation, but rather at a smaller value of \( t_{NGate} \) which would result in a non-complete transfer of energy during phase2 (→inductor).

While the exact mechanism which causes this behavior is not fully understood, the presence of this higher order behavior in experimental measurements helps to provide validity to the modeling techniques presented in this work.

As a final point in this chapter, the electromechanical coupling, \( \kappa^2 \), is presented for each of the three transducer beams in Table 8-1. In Chapter 2, the \( \kappa^2 \) values for isolated piezoelectric materials (not part of a transducer) were presented and shown to simply be material properties of the given piezoelectric. For the composite beams used in this work, the overall \( \kappa^2 \) for each transducer is found from the short- and open-circuit resonant frequencies

\[
\kappa^2 = \frac{\omega^2_o - \omega^2_s}{\omega^2_o},
\]  

\( (8-7) \)
where $\omega_s$ and $\omega_o$ are the short-circuit and open-circuit radian frequencies, respectively. For all three of the transducer beams, the electromechanical coupling coefficient is smaller for the transducer (shim, clamp, and piezoelectric patch) than for the isolated piezoelectric patch used in its fabrication. This means that the addition of the shim and clamp structure effectively reduced the electromechanical coupling of the piezoelectric. However, for a practical design, the inclusions of a shim or substrate is often needed for structural integrity and tuning the transducer to the proper operating frequency.

Table 8-1. Properties for typical piezoelectric materials.

<table>
<thead>
<tr>
<th>Beam #</th>
<th>$\omega_s$ ($\pi$ Hz)</th>
<th>$\omega_o$ ($\pi$ Hz)$^\dagger$</th>
<th>$\kappa^2$ (material)</th>
<th>$\kappa^2$ (transducer)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2\pi$ (59.1 Hz)</td>
<td>$2\pi$ (59.5 Hz)$^\dagger$</td>
<td>0.118</td>
<td>0.011</td>
</tr>
<tr>
<td>2</td>
<td>$2\pi$ (63.6 Hz)</td>
<td>$2\pi$ (64.4 Hz)</td>
<td>0.118</td>
<td>0.025</td>
</tr>
<tr>
<td>3</td>
<td>$2\pi$ (123.3 Hz)</td>
<td>$2\pi$ (125.3 Hz)</td>
<td>0.118</td>
<td>0.032</td>
</tr>
</tbody>
</table>

$^\dagger$ Estimated using LEM
Harvesting energy from environmental vibrations allows for the implementation of self-powered electronic systems, capable of functional lifetimes beyond that provided by the finite energy density of a battery. The design of an efficient energy harvesting system, including both a transducer and power converter, requires an accurate model that fully captures the coupled electromechanical behavior of the system. For practical designs implemented with non-ideal circuitry and capable of operating outside of controlled laboratory settings, the system-level model of the energy harvester must also be able to capture the effects of non-idealities in system operation. When a DFP methodology is applied, the overall size of the system becomes an important design factor as well. This work has presented the theoretical development and experimental validation of a system-level electromechanical model for a PRC-based, piezoelectric energy harvesting system that captures the effect of non-resonant frequency and parasitic losses in the power converter and piezoelectric energy harvester. In this chapter, a summary of this project is first presented, followed by a discussion of the future direction for this research.

9.1 Research Summary

In Chapter 3, the full lumped element model of a composite-beam piezoelectric transducer was first presented. By assuming that the transducer was operated at its short-circuit mechanical resonant frequency, the simplified resonant transducer model may be derived. While this simplified model can be found in a large portion of the research on piezoelectric energy harvesting, the requirement of a specific operating frequency places severe limitations on its usefulness as a practical design tool. The behavior of an energy harvesting system modeled using the simplified resonant model is technically invalid if the frequency of the vibration source does not exactly match the short-circuit resonant frequency. Nevertheless, the simplified resonant model was still considered in this work
in order to determine its potential usefulness in approximating the behavior of the energy harvesting system driven at frequencies near to the short-circuit resonance.

In Chapter 4 and Chapter 5, the operation of the PRC was presented and the finite sources of parasitic loss were discussed. For a typical PRC implementation, the primary source of loss are the conduction losses occurring during phase2 (inductor) and phase3 (battery) as energy is transferred from the transducer to the load. The conduction losses during these two phases were shown to be comprised of conduction losses from the rectification circuitry, inductor, and switches. Two separate system-level models were then developed using the full LEM and simplified resonant transducer models, and included the effects of conduction loss. A sample energy harvester system was then used to compare the two models at the short-circuit resonant frequency to ensure the validity of the simplified resonant case. The results of this comparison showed that the behaviors predicted by each of the models were quite different. For the full LEM model, the peak values of the rectified voltage and inductor current alternated between two values. This behavior corresponds to alternating amounts of energy being transferred to the load for consecutive energy harvesting cycles. In the case where the simplified resonant model was used, the values of the consecutive peaks heights were nearly identical. More importantly, both of the two alternating values of the peak height predicted by the full LEM model were higher than the peak values predicted by the simplified resonant model.

The differing behavior of the full LEM and simplified resonant models is a result of the simplifying assumptions made in the derivation of the simplified case. By assuming short-circuit mechanical resonant operation, the net impedance produced by the effective lumped mass and compliance terms in the full LEM becomes zero, and in the derivation of the simplified model these two terms were simply removed. Removing these terms is mathematically valid for linear analysis of the circuit model at steady-state, however the non-linear switching behavior of the PRC invalidates their removal. In addition to impedance effects, these two components capture the inertial and compliance behavior.
of the mechanical system, which prevent abrupt discontinuities in the motion of the transducer. For the simplified resonant model, where these components are absent, large discontinuities in the mechanical velocity were incorrectly predicted during phase2($\rightarrow$inductor) when large currents were generated in the PRC.

In order to validate the energy harvesting system behavior predicted by the developed models, an experimental test bed consisting of several transducers and a discrete PRC circuit was implemented. Chapter 6 and Chapter 7 presented the design of the experimental test bed and the experimental characterization of the transducer beams, respectively. The LEM parameters extracted from the experimental characterization were used in the full LEM and simplified resonant models to predict the behavior of the implemented energy harvesting system.

Chapter 8 presented the setup and results used to experimentally validate the operation of the energy harvesting system. For each of the three transducers used in this work, three characterization experiments were performed to determine the models’ ability to predict the non-resonant and non-ideal behavior of the system. These three experiments included varying the source frequency, varying the conduction losses in the PRC, and varying the timing of the $N_{\text{Switch}}$. This third experiment was performed to determine how premature and delayed switching behavior affects the operation of the system.

The results of the experimental characterization showed good agreement with the full LEM model. In terms of frequency behavior, both the modeled results and experimental measurements demonstrated a resonant peak occurring at the open-circuit resonant frequency of the transducer. Considering that the PRC is disconnected from the transducer for more than 99% of its operation, this open-circuit frequency behavior is not at all unexpected. For the simplified resonant model, good agreement was observed at the short-circuit resonant frequency, but the model does not capture the experimentally observed resonant behavior and predicts an essentially constant value over the frequency range of interest. Applying the experimental results of the frequency behavior to the
design of practical energy harvesting systems, two conclusions can be drawn. **First, the simplified resonant model should not be used as a tool for design approximations, because of its poor ability to predict the system behavior away from the short-circuit resonance.** Second, and perhaps more important, when designing a system to harvest energy from a specific known vibration source, the open-circuit resonant frequency of the transducer should be designed to correspond to the frequency of the source.

The importance of modeling the PRC losses was also demonstrated in the experimental characterization of the energy harvesting system. An ideal system model with zero PRC loss over-predicts by 30-50% the total harvested power at the open-circuit resonant frequency. When the losses were included in the simulated model, the difference was within experimental uncertainty.

The specific effects of the conduction losses during phase2\(_{\text{inductor}}\) and phase3\(_{\text{battery}}\) was demonstrated by manually changing the resistances in the respective energy transfer conduction paths. As expected, the increase in the conduction path resistances resulted in a reduced amount of power reaching the load. In addition to be experimentally measured, this behavior was also predicted by the full LEM model. For this work, the effect of increased resistance in the phase2\(_{\text{inductor}}\) conduction path was larger than for the phase3\(_{\text{battery}}\) path, most likely due the fact that phase2\(_{\text{inductor}}\) was longer than phase3\(_{\text{battery}}\). The relative lengths of these two phases will vary depending on the specific energy harvesting system. From a design perspective, it is important to understand how the specific resistance values will affect the final power, especially in the choice of components. In these experiments, the dominant source of conduction loss was the inductor. To reduce the conduction loss from the inductor, a larger gauge wire could be used, but at the expense of increased inductor size. A similar issue arises for the design of the switches used in the PRC. A switch with a larger gate width will generally have lower conduction loss, but at the cost of increased gate area, and therefore increased
overhead power required to for switching. For the design of an energy harvester using the DFP methodology, an understanding of the trade-offs between size and power is critical.

The general effects of offsets in the timing of the $N_{\text{Switch}}$ on the total harvested power observed in the experimental measurements were also captured by the model. While the initial goal of this experiment was to determine to what extent improper timing of the $N_{\text{Switch}}$ would reduce the total harvested power, it was discovered that it might be possible to increase the total harvested power by closing the $N_{\text{Switch}}$ before all of the energy stored on the piezoelectric capacitance is transferred to the inductor. The convenience of switching at the moment of complete energy transfer is that the inductor current is at a peak, and a peak detector can be implemented to trigger the switching event. Finding a signal to accurately trigger the $N_{\text{Switch}}$ before the peak occurs is an interesting challenge and a possible method to increase the energy harvested using a PRC.

In addition to capturing the behavior of the electrical domain of the energy harvesting system, the full LEM model was also used to examine the effects that harvesting energy from the transducer has on its mechanical operation. The tip displacement was experimentally measured and showed good agreement with the values predicted by the model. In the design of an energy harvester where DFP is employed, this knowledge is important in order to determine the full three-dimensional volume that the harvester will occupy.

### 9.2 Future Direction

The ultimate goal of this research is to enable the design of minimally-sized, energy harvesting systems using the DFP methodology. In order to optimally design an energy harvester for a specific application, an accurate model of the system was first needed. The modeling and characterization efforts presented in this work therefore represents an important initial step in the overall design process. Before any PRC-based energy harvesting systems can be implemented on a wider scale, however, additional research is needed in several areas.
Building upon the coupled electromechanical modeling of the transducer and power converter, an important next step is to determine the sensitivity of the energy harvesting system performance to the various design parameters, including both the transducer geometry and PRC waveforms. Since the largest response from the transducers occurs at a single specific frequency, which was shown for the PRC to be the open-circuit resonant frequency, the design of a highly efficient energy harvester requires that the resonant frequency of the transducer occurs at (or very near to) the frequency of the vibration to be harvested. The resonant frequency of the transducer is a function of the LEM parameters, which are based upon the device geometry, and any variation in these parameters from their designed values will ultimately effect the resonant frequency. It is reasonable to assume that there will be some finite difference between the designed and fabricated transducer geometry, and it is therefore important to know how these differences will affect the performance of the system, especially in terms of the total harvested power. Similarly, for the PRC waveforms, which are critical in the timing of the PRC and transfer of power to the load, it is important to understand how sensitive the voltage and current signals are to slight variations in both transducer and PRC parameter values.

In this work, the characterization of energy harvesting operation yielded some new and previously undocumented behavior for PRC-based energy harvesting systems. In Chapter 5 and Chapter 8, it was shown that the non-linear switching of the PRC caused the rectified piezoelectric voltage to exhibit two different peak heights, both in simulation and in an implemented system. Also in Chapter 8, simulations showed that it might be possible to increase the amount of harvested power by changing the timing of the \( N_{\text{Switch}} \). While this work provided a brief explanation for the occurrence of the two peak heights, a more detailed examination is required to understand how the \( N_{\text{Switch}} \) timing can affect the total harvested power. With a more detailed understanding on the origins of these two phenomena, it may be possible to leverage them in order to design a more efficient energy
harvesting system. Additional research will then also be needed to determine the best implementation of the control circuitry needed to reap any benefits which may arise.

This research focused on the modeling and characterization of PRC-based energy harvesting systems implemented with constant-amplitude, single-frequency vibration sources. However, many viable sources of environmental vibration energy do not meet these criteria. An interesting future direction for this research is to adapt the modeling techniques developed here to systems with more complex vibration spectra, such as asynchronous impulses. Sources of vibration impulses could include anything from a human foot-strike to the slamming of a door. When these impulses are applied to the piezoelectric cantilevered beam transducer, the resulting piezoelectric voltage waveform is a decaying sinusoid vibrating at its damped natural frequency. One of the greatest advantages to an impulse-based energy harvester is that it does not require careful matching of its resonant frequency to the source frequency. This greatly simplifies the design process. Since the PRC is timed on the heights of the voltage peaks and zero crossings, the control for the energy harvesting process could remain unchanged.
APPENDIX A
DERIVATION OF LOSSLESS PRC OPERATION USING THE SIMPLIFIED RESONANT MODEL

In previous work by Ngo et al. [8], the power generated by a lossless, PRC-based energy harvesting system is presented using the resonant transducer model discussed in Chapter 5. A schematic of this energy harvesting system is shown in Figure A-1. In the work by Ngo, a closed-form approximation of the power delivered to the load for a lossless system is given as

\[
P_{PRC-res} = \frac{4\pi}{\pi} \frac{\omega \tau}{1 + \omega^2 \tau^2} \left(1 + e^{-\frac{\pi}{\omega \tau}}\right)^2,
\]

(A–1)

where \( \tau = C_{piezo} R_{piezo} \). A full derivation, however, is not provided. The purpose of this appendix is to present a complete derivation of Equation A–1 using standard circuit analysis.

![Resonant Transducer Model](image1)

![Pulsed Resonant Converter](image2)

Figure A-1. Simplified resonant transducer LEM and PRC power converter.

For the energy harvesting system presented in this derivation it is assumed that no energy is dissipated in the PRC and that the switch timing is instantaneous and ideal. Under these assumptions, all of the energy stored on \( C_{piezo} \) during phase1 (\( \rightarrow \) capacitor) will be transferred to the load during phase2 (\( \rightarrow \) inductor) and phase3 (\( \rightarrow \) battery). Therefore, the amount of power delivered to the load can be approximated using only the energy stored during
phase1(\rightarrow\text{capacitor}) and the energy transfer frequency

\[ P_{PRC} = E_{\text{phase1}} f_{\text{transfer}}, \quad (A-2) \]

where \( f_{\text{transfer}} = 2f_{\text{vibration}} \) due to the rectifier of the voltage signal. The energy stored on the piezoelectric capacitance during phase1(\rightarrow\text{capacitor}) is given by

\[ E_{\text{phase}} = \frac{1}{2} C_{\text{piezo}} v_{\text{piezo}}^2, \quad (A-3) \]

where \( v_{\text{piezo}} \) is the piezoelectric voltage. Combining Equations A–2 and A–2, the power delivered to the load becomes

\[ P_{PRC} = C_{\text{piezo}} v_{\text{piezo}}^2 f_{\text{vibration}}. \quad (A-4) \]

Since \( C_{\text{piezo}} \) and \( f_{\text{vibration}} \) are known for a specific transducer geometry and input vibration, only the piezoelectric voltage at the end of phase1(\rightarrow\text{capacitor}) is needed to find the power.

During phase1(\rightarrow\text{capacitor}) both the N_{\text{Switch}} and P_{\text{Switch}} are open and the transducer operates in an open circuit condition. Applying KCL to the transducer provides

\[ i_{\text{piezo}}(t) = i_{R_{\text{piezo}}}(t) + i_{C_{\text{piezo}}}(t). \quad (A-5) \]

The current source, which represents the mechanical domain input, is sinusoidal and of the form

\[ i_{\text{piezo}}(t) = I_{\text{piezo}} \sin(\omega t), \quad (A-6) \]

where \( I_{\text{piezo}} \) is magnitude of the source, and \( \omega_{\text{vibration}} = 2\pi f_{\text{vibration}} \) is the radian frequency.

The currents \( i_{R_{\text{piezo}}} \) and \( i_{C_{\text{piezo}}} \) can be expressed as

\[ i_{R_{\text{piezo}}}(t) = \frac{v_{\text{piezo}}(t)}{R_{\text{piezo}}}, \quad (A-7) \]

and

\[ i_{C_{\text{piezo}}}(t) = C_{\text{piezo}} \frac{\partial v_{\text{piezo}}(t)}{\partial t}, \quad (A-8) \]
Substituting Equations A–6, A–7, and A–8 into Equation A–5 gives

\[ I_{\text{piezo}} \sin (\omega t) = \frac{v_{\text{piezo}}(t)}{R_{\text{piezo}}} + C_{\text{piezo}} \frac{\partial v_{\text{piezo}}(t)}{\partial t}, \quad (A-9) \]

which can be rearranged into the more standard form

\[ \frac{\partial v_{\text{piezo}}(t)}{\partial t} + \frac{v_{\text{piezo}}(t)}{\tau} = \frac{I_{\text{piezo}}}{C_{\text{piezo}}} \sin (\omega t). \quad (A-10) \]

The solution to the differential equation given as Equation A–10 is the sum of the homogenous and particular solutions.

The homogenous solution is found by setting the differential equation to zero

\[ \frac{\partial v_{\text{piezo}}(t)}{\partial t} + \frac{v_{\text{piezo}}(t)}{\tau} = 0, \quad (A-11) \]

and has the form

\[ v_{\text{piezo}}(t)_{\text{homogenous}} = ke^{\frac{t}{\tau}}, \quad (A-12) \]

where \( k \) is a constant determined by boundary conditions.

The particular solution requires solving Equation A–10 with the sinusoidal term, and has the standard solution given by

\[ v_{\text{piezo}}(t)_{\text{particular}} = A \cos (\omega t) + B \sin (\omega t), \quad (A-13) \]

where \( A \) and \( B \) are constants. The derivative of Equation A–13 is also needed to solve Equation A–10 and is given by

\[ \frac{\partial v_{\text{piezo}}(t)_{\text{particular}}}{\partial t} = -A\omega \sin (\omega t) + B\omega \cos (\omega t). \quad (A-14) \]

Substituting Equations A–13 and A–14 into Equation A–10 gives

\[ -A\omega \sin (\omega t) + B\omega \cos (\omega t) + \frac{A \cos (\omega t) + B \sin (\omega t)}{\tau} = \frac{I_{\text{piezo}}}{C_{\text{piezo}}} \sin (\omega t). \quad (A-15) \]
Equation A–15 can then be rearranged to
\[
\sin (\omega t) [-A\omega \tau + B] + \cos (\omega t) [A + B\omega \tau] = \frac{I_{\text{piezo}} \tau}{C_{\text{piezo}}} \sin (\omega t). \tag{A–16}
\]

To find the values of the coefficients the sine and cosine terms are compared. Equating the sine terms in Equation A–16 provides
\[
-A\omega \tau + B = \frac{I_{\text{piezo}} \tau}{C_{\text{piezo}}}. \tag{A–17}
\]
Solving Equation A–17 for the coefficient \(B\) gives
\[
B = \frac{I_{\text{piezo}} \tau}{C_{\text{piezo}}} + A\omega \tau. \tag{A–18}
\]
Equating the cosine terms from Equation A–16 gives
\[
A + B\omega \tau = 0. \tag{A–19}
\]
Substituting Equation A–18 into A–19 and solving for the coefficient \(A\) provides
\[
A = -\frac{I_{\text{piezo}} \omega \tau^2}{C_{\text{piezo}} (1 + \omega^2 \tau^2)}. \tag{A–20}
\]
Finally, substituting Equation A–20 into Equation A–19 and solving for the coefficient \(B\) gives
\[
B = \frac{I_{\text{piezo}} \tau}{C_{\text{piezo}} (1 + \omega^2 \tau^2)}. \tag{A–21}
\]
The total solution, which is the sum of the homogenous and particular solutions, is
\[
v_{\text{piezo}} (t) = A\omega \cos (\omega t) + B \sin (\omega t) + ke^{-\frac{t}{\tau}}. \tag{A–22}
\]
Substituting Equations A–20 and A–21 into Equation A–22 provides
\[
v_{\text{piezo}} (t) = \frac{I_{\text{piezo}} \tau}{C_{\text{piezo}} (1 + \omega^2 \tau^2)} [-\omega \tau \cos (\omega t) + \sin (\omega t)] + ke^{-\frac{t}{\tau}}. \tag{A–23}
\]
In order to find the value of \(k\) a boundary condition for the energy harvesting system is needed. To set the boundary condition it is first necessary to define a the instant that
the N\text{Switch} closes as \( t_{sw} \). At this instant the voltage across the capacitor falls to zero, and assuming that the voltage is transferred almost instantaneously from the \( C_{\text{piezo}} \) to \( L_{\text{PRC}} \), the following boundary condition can be used
\[
y_{\text{piezo}}(t_{sw}) \approx y_{\text{piezo}}(t_{sw}^+) = 0. \tag{A–24}
\]

Applying this boundary condition to Equation A–23 gives
\[
y_{\text{piezo}}(t) = 0 = \frac{I_{\text{piezo}}}{C_{\text{piezo}}(1 + \omega^2 \tau^2)} \left[ -\omega \tau \cos(\omega t_{sw}) + \sin(\omega t_{sw}) \right] + ke^{-\frac{t_{sw}}{\tau}} \tag{A–25}
\]

Rearranging Equation A–25 for and solving for \( k \) gives
\[
k = \frac{I_{\text{piezo}} \tau}{C_{\text{piezo}}(1 + \omega^2 \tau^2)} \left[ \omega \tau \cos(\omega t_{sw}) - \sin(\omega t_{sw}) \right] e^{\frac{t_{sw}}{\tau}} \tag{A–26}
\]

Substituting the value of \( k \) from Equation A–26 into Equation A–23 provides a solution for the piezoelectric voltage in terms of the transducer parameters and the time, \( t \)
\[
y_{\text{piezo}}(t) = \frac{I_{\text{piezo}}}{C_{\text{piezo}}(1 + \omega^2 \tau^2)} \times \left[ -\omega \tau \cos(\omega t) + \sin(\omega t) + \left( \omega \tau \cos(\omega t_{sw}) - \sin(\omega t_{sw}) \right) e^{\frac{t_{sw}}{\tau}} \right] \tag{A–27}
\]

As previously mentioned, the operating frequency of the PRC is twice that of the vibration due to the presence of the rectifier. In other words, the PRC is periodic with a period of \( \frac{T_{\text{vibration}}}{2} \). This means that it takes a time of \( \frac{T_{\text{vibration}}}{2} \) for the piezoelectric voltage to reach its maximum after \( t_{sw} \). The maximum voltage across the piezoelectric capacitance can therefore be defined as
\[
V_{sw}(t_{sw}) = y_{\text{piezo}} \left( t_{sw} + \frac{T_{\text{vibration}}}{2} \right). \tag{A–28}
\]

Noting that
\[
\frac{T_{\text{vibration}}}{2} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}. \tag{A–29}
\]
Equation A–28 can be expressed as

\[ V_{sw}(t_{sw}) = v_{piezo} (t_{sw} + \frac{\pi}{\omega}). \tag{A–30} \]

Substituting Equation A–30 into Equation A–27 and simplifying gives

\[ V_{sw}(t_{sw}) = \frac{I_{piezo}}{C_{piezo}} \times \left[ \frac{\omega \tau}{\sqrt{1 + \omega^2 \tau^2}} \cos \left( \frac{\omega t_{sw}}{\tau} \right) - \sin \left( \frac{\omega t_{sw}}{\tau} \right) + \left( \omega \tau \cos \left( \frac{\omega t_{sw}}{\tau} \right) - \sin \left( \frac{\omega t_{sw}}{\tau} \right) \right) e^{-\frac{\omega t_{sw}}{\tau}} \right]. \tag{A–31} \]

Further rearranging of Equation A–31 leads to a simplified form

\[ V_{sw}(t_{sw}) = \frac{I_{piezo}}{C_{piezo}} \times \left[ \frac{\omega \tau}{\sqrt{1 + \omega^2 \tau^2}} \left( \cos \left( \frac{\omega t_{sw}}{\tau} \right) - \sin \left( \frac{\omega t_{sw}}{\tau} \right) \right) \right]. \tag{A–32} \]

It is next necessary to find the value of \( t_{sw} \) which will provide the largest value of the switching voltage, \( V_{sw} \). This done by taking the derivative of Equation A–32 with respect to \( t_{sw} \) and setting it equal to zero

\[ \frac{\partial V_{sw}(t_{sw})}{\partial t_{sw}} = 0 = \frac{I_{piezo}}{C_{piezo}} \left( \frac{\omega \tau}{\sqrt{1 + \omega^2 \tau^2}} \left( -\omega^2 \tau \sin \left( \frac{\omega t_{sw}}{\tau} \right) - \omega \cos \left( \frac{\omega t_{sw}}{\tau} \right) \right) \right). \tag{A–33} \]

Dividing through by the constant terms and rearranging provides

\[ \omega^2 \tau \sin \left( \frac{\omega t_{sw}}{\tau} \right) = -\omega \cos \left( \frac{\omega t_{sw}}{\tau} \right). \tag{A–34} \]

Using a trigonometric identity for tangent gives

\[ \tan \left( \frac{\omega t_{sw}}{\tau} \right) = \frac{-1}{\omega \tau}. \tag{A–35} \]

Since \( \tan \left( \frac{\omega t_{sw}}{\tau} \right) \) is a negative value, the angle must be in either quadrant II or IV as shown in Figure A-2.

Initially assuming the angle is in quadrant IV (quadrant II will be examined shortly),

\[ \cos \left( \frac{\omega t_{sw}}{\tau} \right) = \frac{\omega \tau}{\sqrt{1 + \omega^2 \tau^2}}. \tag{A–36} \]
and
\[ \sin(\omega t_{sw}) = -\frac{1}{\sqrt{1 + \omega^2 \tau^2}}. \] (A–37)

Substituting the trigonometric identities from Equation A–36 and Equation A–37 into Equation A–32 gives
\[ V_{sw\ max} = \frac{I_{\text{piezo}\tau}}{C_{\text{piezo}} (1 + \omega^2 \tau^2)} \left( 1 + e^{\frac{-\pi}{\omega\tau}} \right) \left( \omega \tau \frac{\omega \tau}{\sqrt{1 + \omega^2 \tau^2}} + \frac{1}{\sqrt{1 + \omega^2 \tau^2}} \right). \] (A–38)

Simplifying and rearranging Equation A–38
\[ V_{sw\ max} = \frac{I_{\text{piezo}\tau}}{C_{\text{piezo}} \sqrt{1 + \omega^2 \tau^2}} \left( 1 + e^{\frac{-\pi}{\omega\tau}} \right). \] (A–39)

Had the assumption been made that the angle was in quadrant II, a negative value of Equation A–39 would have resulted, corresponding the negative part of the rectified voltage. Since the voltage is squared in the determination of the harvested power, the choice in quadrants does not ultimately affect the outcome.

Figure A-2. Illustration of the angles described by Equation A–35

The power can now be calculated from Equation A–4 by substituting in the piezoelectric voltage from A–39 and using
\[ f_{\text{vibration}} = \frac{\omega}{2\pi}. \] (A–40)
Combining terms and simplifying yields the total harvested power from the PRC using the resonant transducer model

\[ P_{PRC} = \frac{I_{piezo}^2 R_{piezo}}{2\pi} \frac{\omega_T}{1 + \omega^2 \tau^2} \left( 1 + e^{-\frac{\tau}{\omega_T}} \right)^2. \]  

(A–41)
APPENDIX B
DERIVATION OF THE FULL LEM WITH LOSSES

In Chapter 5, the operation of the energy harvesting system comprised of the full LEM was modeled using differential equations to describe the behavior of the system during each of the three phases. Here, a full derivation of the governing differential equations and initial conditions is presented for each phase.

**Phase 1**

During phase 1 (→capacitor), both switches are open, and the energy harvesting system model can be reduced to the open-circuit transducer model, shown schematically in Figure B-1.

Applying KVL to Loop 1 results in

\[ F_m^* \sin(\omega t) = u_m^*(t) + v_{Mm^*}(t) + v_{Cms^*}(t) + v_{Rm^*}(t) + v_{piezo}(t). \] \( \text{(B–1)} \)

Substituting in the voltage values of the reflected mechanical domain components, Equation B–1 becomes

\[ F_m^* \sin(\omega t) = M_m^* \frac{du_m^*(t)}{dt} + \frac{1}{C_{ms^*}} \int_{-\infty}^{t} u_m^*(t) \, dt + R_m u_m^*(t) + v_{piezo}(t). \] \( \text{(B–2)} \)

The integral term is removed by taking the time derivative of both sides of Equation B–2,

\[ F_m^* \omega \cos(\omega t) = M_m^* \frac{d^2 u_m^*(t)}{dt^2} + \frac{1}{C_{ms^*}} u_m^*(t) + R_m \frac{du_m^*(t)}{dt} + \frac{dv_{piezo}(t)}{dt}. \] \( \text{(B–3)} \)
Next the value of \( u_m^* (t) \) is found by applying KCL to Node 1

\[
u_m^* (t) = i_{Ceb} (t) + i_{Re} (t), \tag{B-4}
\]

where the values of the currents \( i_{Ceb} (t) \) and \( i_{Re} (t) \) are given by

\[
i_{Ceb} (t) = C_{eb} \frac{dv_{piezo} (t)}{dt}, \tag{B-5}
\]

and

\[
i_{Re} (t) = \frac{v_{piezo} (t)}{R_e}. \tag{B-6}
\]

Substituting in the values from Equations B–4 through B–6 into Equation B–3 yields

\[
F_m^* \omega \cos (\omega t) = M_m^* C_{eb} \frac{d^3 v_{piezo} (t)}{dt^3} + \left( \frac{M_m^*}{R_e} + R_m^* C_{eb} \right) \frac{d^2 v_{piezo} (t)}{dt^2} + \left( \frac{C_{eb}}{C_{ms}^*} + \frac{R_m^*}{R_e} + 1 \right) \frac{dv_{piezo} (t)}{dt} + \frac{1}{C_{ms}^* R_e} v_{piezo} (t). \tag{B-7}
\]

Equation B–7 can be further simplified to

\[
F_m^* \omega \cos (\omega t) = C_{1p1} \frac{d^3 v_{piezo} (t)}{dt^3} + C_{2p1} \frac{d^2 v_{piezo} (t)}{dt^2} + C_{3p1} \frac{dv_{piezo} (t)}{dt} + C_{4p1} v_{piezo} (t), \tag{B-8}
\]

where \( C_{1p1} \) through \( C_{4p1} \) are coefficient values based on the transducer LEM. The coefficient values are

\[
C_{1p1} = M_m^* C_{eb}, \tag{B-9}
\]

\[
C_{2p1} = \frac{M_m^*}{R_e} + \frac{R_m^*}{C_{eb}}, \tag{B-10}
\]

\[
C_{3p1} = \frac{C_{eb}}{C_{ms}^*} + \frac{R_m^*}{R_e} + 1, \tag{B-11}
\]

and

\[
C_{4p1} = \frac{1}{R_e C_{ms}^*}. \tag{B-12}
\]

In order to solve Equation B–8 for \( v_{piezo} (t) \), the initial conditions at the start of phase 1 (\( \rightarrow \) capacitor) are used. Since Equation B–8 is a 3rd order ordinary differential
equation, three initial conditions are required. The first initial condition is that the voltage across the piezoelectric capacitance, $C_{eb}$, cannot change instantaneously. Therefore, the piezoelectric voltage at the start of phase1 (→capacitor) must be equal to the piezoelectric voltage at the end of phase3 (→battery)

$$IC1 \equiv v_{piezo}(t_{init,p1}) = v_{piezo}(t_{end,p3})$$  \hspace{1cm} (B-13)

where $t_{end,p3} = t_{init,p1}$ is the instant when the circuit switches from phase3 (→battery) to phase1 (→capacitor).

The second initial condition is found from the current flowing through $C_{eb}$. Applying KCL to Node1 gave the expression in Equation B–4. Substituting in the values for $i_{Ceb}(t)$ and $i_{Re}(t)$ from Equations B–5 and B–6 gives

$$u_m^*(t) = C_{eb} \frac{dv_{piezo}(t)}{dt} + \frac{v_{piezo}(t)}{R_e},$$  \hspace{1cm} (B-14)

which can be rearranged and solved for $t = t_{init,p1}$ to give

$$IC2 \equiv \frac{dv_{piezo}(t_{init,p1})}{dt} = \frac{u_m^*(t_{init,p1})}{C_{eb}} - \frac{v_{piezo}(t_{init,p1})}{C_{eb}R_e}$$  \hspace{1cm} (B-15)

The third initial condition is found by applying KVL to Loop1, which leads to the expression given by Equation B–1. Substituting the values of $v_{Mm^*}(t)$, where

$$v_{Mm^*}(t) = M_m \frac{d}{dt} u_m^*(t),$$  \hspace{1cm} (B-16)

and $v_{Rm^*}(t)$ into Equation B–1 gives

$$F_m^* \sin(\omega t) = M_m^* \frac{d}{dt} u_m^*(t) + v_{Cms^*}(t) + R_m^* u_m^*(t) + v_{piezo}(t)$$  \hspace{1cm} (B-17)

Next, substituting in the value of $u_m^*(t)$ from Equation B–14 into Equation B–17 and rearranging gives
\[ F_m \sin(\omega t) = M_m^* C_{eb} \frac{d^2 v_{\text{piezo}}(t)}{dt^2} + M_m^* \frac{dv_{\text{piezo}}(t)}{dt} + M_m^* \frac{di_{\text{LPRC}}(t)}{dt} \]

\[ + v_{\text{CMS}}(t) + R_m^* u_m^*(t) + v_{\text{piezo}}(t). \]

Rearranging Equation B–18 gives the third initial condition

\[ IC3 \equiv \frac{d^2 v_{\text{piezo}}(t_{\text{init,p1}})}{dt^2} = \frac{F_m^* \sin(\omega t_{\text{init,p1}})}{M_m^* C_{eb}} - \left( \frac{1}{R_e C_{eb}} + \frac{R_m^*}{M_m^* R_e C_{eb}} \right) \frac{dv_{\text{piezo}}(t_{\text{init,p1}})}{dt} - \left( \frac{1}{M_m^* C_{eb}} + \frac{R_m^*}{M_m^* R_e C_{eb}} \right) v_{\text{CMS}}(t_{\text{init,p1}}) - \frac{v_{\text{CMS}}(t_{\text{init,p1}})}{M_m^* C_{eb}}. \]

### Phase 2

During phase 2 (\(\rightarrow\) inductor), the energy harvesting system model can be reduced to the schematic shown in Figure B-2.

![Figure B-2](image-url)

**Figure B-2.** Schematic of the energy harvesting system during phase 2 (\(\rightarrow\) inductor).

Applying KVL to Loop 1 in Figure B-2 results in

\[ F_m^* \sin(\omega t) = v_{Mm^*}(t) + v_{\text{CMS}^*}(t) + v_{Rm^*}(t) + v_{\text{piezo}}(t). \]

(B–20)

Substituting in the voltages values of the reflected mechanical domain components, Equation B–20 becomes

\[ F_m^* \sin(\omega t) = M_m^* \frac{du_m^*(t)}{dt} + \frac{1}{C_{ms}^*} \int_{-\infty}^{t} u_m^*(t) \, dt + R_m u_m^*(t) + v_{\text{piezo}}(t). \]

(B–21)
The integral term is removed by taking the time derivative of both sides of Equation B–21,

\[ F_m^* \omega \cos (\omega t) = M_m^* \frac{d^2 u_m^* (t)}{dt^2} + \frac{1}{C_{ms}^*} u_m^* (t) + R_m^* \frac{du_m^* (t)}{dt} + \frac{dv_{piezo} (t)}{dt}. \] (B–22)

Next the value of \( u_m^* (t) \) is found by applying KCL to Node 1

\[ u_m^* (t) = i_{Cb} (t) + i_{Re} (t) + i_{LPRC} (t) \], (B–23)

where the values of the currents \( i_{Cb} (t) \) and \( i_{Re} (t) \) were given by Equation B–5 and Equation B–6. Substituting in the values from Equation B–23 into Equation B–22 yields

\begin{align*}
F_m^* \omega \cos (\omega t) &= M_m^* C_{cb} \frac{d^3 v_{piezo} (t)}{dt^3} + \left( \frac{M_m^*}{R_e} + R_m^* C_{cb} \right) \frac{d^2 v_{piezo} (t)}{dt^2} \\
&\quad + L_m^* \frac{d^2 i_{LPRC} (t)}{dt^2} + \left( \frac{C_{cb}}{C_{ms}^*} + \frac{R_m^*}{R_e} + 1 \right) \frac{dv_{piezo} (t)}{dt} \\
&\quad + R_m^* \frac{di_{LPRC} (t)}{dt} + \frac{1}{C_{ms}^* R_e} v_{piezo} (t) + \frac{1}{C_{ms}^*} i_{LPRC} (t) \, .
\end{align*}

(B–24)

Equation B–24 is a function of both \( v_{piezo} (t) \) and \( i_{LPRC} (t) \). These two terms can be related by examining the inductor, \( L_{PRC} \). The voltage-current relationship for \( L_{PRC} \) is

\[ v_{LPRC} (t) = L_{PRC} \frac{di_{LPRC} (t)}{dt} = v_{piezo} (t) - i_{LPRC} (t) R_{Phase2} - V_{eff,piezo} \]. (B–25)

Solving Equation B–25 for \( v_{piezo} (t) \) and its derivatives gives:

\begin{align*}
\frac{dv_{piezo} (t)}{dt} &= L_{PRC} \frac{d^2 i_{LPRC} (t)}{dt^2} + R_{Phase2} \frac{di_{LPRC} (t)}{dt} \, , \quad (B–26) \\
\frac{d^2 v_{piezo} (t)}{dt^2} &= L_{PRC} \frac{d^3 i_{LPRC} (t)}{dt^3} + R_{Phase2} \frac{d^2 i_{LPRC} (t)}{dt^2} \, , \quad (B–27) \\
\frac{d^3 v_{piezo} (t)}{dt^3} &= L_{PRC} \frac{d^4 i_{LPRC} (t)}{dt^4} + R_{Phase2} \frac{d^3 i_{LPRC} (t)}{dt^3} \, , \quad (B–28)
\end{align*}

and

\begin{align*}
\frac{d^4 v_{piezo} (t)}{dt^4} &= L_{PRC} \frac{d^5 i_{LPRC} (t)}{dt^5} + R_{Phase2} \frac{d^4 i_{LPRC} (t)}{dt^4} \, . \quad (B–29)
\end{align*}
Substituting the values from Equations B–26 through B–29 into B–24 simplifies into an equation of only $i_{LPRC}(t)$

$$F_m \omega \cos(\omega t) = M_m C_{eb} L_{PRC} \frac{d^4i_{LPRC}(t)}{dt^4}$$

\[+ \left( M_m C_{eb} R_{Phase2} + \frac{M_m L_{PRC}}{R_e} + R_m C_{eb} L_{PRC} \right) \frac{d^3i_{LPRC}(t)}{dt^3} \]

\[+ \left( M_m + \frac{M_m R_{Phase2}}{R_e} + R_m C_{eb} R_{Phase2} \right) \frac{d^2i_{LPRC}(t)}{dt^2} \]

\[+ \left( \frac{C_{eb} L_{PRC}}{C_{ms}^*} + \frac{R_m L_{PRC}}{R_e} + L_{PRC} \right) \frac{di_{LPRC}(t)}{dt} \]

\[+ (R_{Phase2}) \frac{di_{LPRC}(t)}{dt} + \left( \frac{1}{C_{ms}^*} + \frac{R_{Phase2}}{C_{ms}^* R_e} \right) i_{LPRC}(t) \]

\[+ \frac{V_{D_{eff} q2}}{C_{ms}^* R_e}. \quad \text{(B–30)} \]

Equation B–30 is further simplified to

$$F_m \omega \cos(\omega t) = \left( C_{1p2} \frac{d^4i_{LPRC}(t)}{dt^4} + C_{2p2} \frac{d^3i_{LPRC}(t)}{dt^3} + C_{3p2} \frac{d^2i_{LPRC}(t)}{dt^2} \right)$$

\[+ C_{4p2} \frac{di_{LPRC}(t)}{dt} + C_{5p2} i_{LPRC}(t) + \frac{V_{D_{eff} q2}}{C_{ms}^* R_e} \quad \text{(B–31)} \]

where $C_{1p2}$ through $C_{5p2}$ are coefficient values based on the transducer LEM and the value of $L_{PRC}$. The coefficient values are

$$C_{1p2} = L_{PRC} M_m C_{eb}, \quad \text{(B–32)}$$

$$C_{2p2} = \frac{M_m L_{PRC}}{R_e} + L_{PRC} R_m C_{eb} + M_m C_{eb} R_{Phase2}, \quad \text{(B–33)}$$

$$C_{3p2} = \frac{L_{PRC} C_{eb}}{C_{ms}^*} + L_{PRC} R_m + L_{PRC} + M_m + \frac{M_m R_{Phase2}}{R_e} + R_m C_{eb} R_{Phase2}, \quad \text{(B–34)}$$

$$C_{4p2} = \frac{L_{PRC}}{R_e C_{ms}^*} + R_m + \frac{C_{eb} R_{Phase2}}{C_{ms}^*} + \frac{R_m R_{Phase2}}{R_e} + R_{Phase2}, \quad \text{(B–35)}$$

and

$$C_{5p2} = \frac{1}{C_{ms}^*} + \frac{R_{Phase2}}{C_{ms}^* R_e}. \quad \text{(B–36)}$$

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In order to solve Equation B–31 for \( i_{LPRC} (t) \), the initial conditions at the start of phase2(\( \rightarrow \)inductor) are used. Since Equation B–31 is a 4th order ordinary differential equation, four initial conditions are required. The first initial condition is that the current flowing through \( L_{PRC} \) is zero at the start of phase2(\( \rightarrow \)inductor) given as

\[
IC1 \equiv i_{LPRC} (t_{init,p2}) = 0.
\] (B–37)

where \( t_{init,p2} = t_{end,p1} \) is the instant when the PRC switches from phase1(\( \rightarrow \)capacitor) to phase2(\( \rightarrow \)inductor).

The second initial conditions is found from the voltage across \( L_{PRC} \). Applying KVL to Loop2 yields

\[
v_{piezo} (t) = L_{PRC} \frac{di_{LPRC} (t)}{dt} + R_{Phase2} i_{LPRC} (t) + V_{D_{eff,p2}},
\] (B–38)

which can be rearranged and solved for \( t = t_{init,p2} \) to give

\[
IC2 \equiv \frac{di_{LPRC} (t_{init,p2})}{dt} = \frac{v_{piezo} (t_{init,p2})}{L_{PRC}} - \frac{R_{Phase2}}{L_{PRC}} i_{LPRC} (t_{init,p2}) - \frac{V_{D_{eff,p2}}}{L_{PRC}}.
\] (B–39)

Applying the result of Equation B–37 to Equation B–39, the second initial condition reduces to

\[
IC2 \equiv \frac{di_{LPRC} (t_{init,p2})}{dt} = \frac{v_{piezo} (t_{init,p2})}{L_{PRC}} - \frac{V_{D_{eff,p2}}}{L_{PRC}}.
\] (B–40)

The third initial condition is found by applying KCL at Node1 in Figure B-2, where

\[
u_m^\ast (t) = i_{Ce_b} (t) + i_{Re} (t) + i_{LPRC} (t).
\] (B–41)

Substituting the values of \( i_{Ce_b} (t) \) and \( i_{Re} (t) \) into Equation B–41 gives

\[
u_m^\ast (t) = C_{eb} \frac{dv_{piezo} (t)}{dt} + \frac{v_{piezo} (t)}{R_e} + i_{LPRC} (t).
\] (B–42)

Next, substituting the value of \( v_{piezo} (t) \) and its derivative, which were given as Equations B–26 and B–27, into Equation B–42 and solving at \( t = t_{init,p2} \), the third initial condition
can be found

\[ \text{IC} 3 \equiv \frac{d^2 i_{\text{LPRC}}(t_{\text{init}, 2})}{dt^2} = \frac{u_m^*(t_{\text{init}, 2})}{C_{eb} L_{\text{PRC}}} - \frac{R_{\text{Phase}2}}{L_{\text{PRC}}} \frac{d i_{\text{LPRC}}(t_{\text{init}, 2})}{dt} - \frac{1}{C_{eb} L_{\text{PRC}}} i_{\text{LPRC}}(t_{\text{init}, 2}) - \frac{v_{\text{piezo}}(t_{\text{init}, 2})}{C_{eb} L_{\text{PRC}} R_e}. \] (B–43)

Applying the result of Equation B–37 to Equation B–43, the third initial condition reduces to

\[ \text{IC} 3 \equiv \frac{d^2 i_{\text{LPRC}}(t_{\text{init}, 2})}{dt^2} = \frac{u_m^*(t_{\text{init}, 2})}{C_{eb} L_{\text{PRC}}} - \frac{R_{\text{Phase}2}}{L_{\text{PRC}}} \frac{d i_{\text{LPRC}}(t_{\text{init}, 2})}{dt} - \frac{v_{\text{piezo}}(t_{\text{init}, 2})}{C_{eb} L_{\text{PRC}} R_e}. \] (B–44)

Finally, the fourth initial condition is found by taking the KVL of Loop 1, given in Equation B–20. Substituting the values of \( v_{Mm^*}(t) \) and \( v_{Rm^*}(t) \) into Equation B–20 gives

\[ F_m^* \sin(\omega t) = M_m^* \frac{d}{dt} u_m^*(t) + v_{Cms^*}(t) + R_m u_m(t) + v_{\text{piezo}}(t). \] (B–45)

Next, substituting in the value of \( u_m^*(t) \) from Equation B–41 into Equation B–45 and rearranging gives

\[ F_m^* \sin(\omega t) = M_m^* C_{eb} \frac{d^2 v_{\text{piezo}}(t)}{dt^2} + \frac{M_m^*}{R_e} \frac{dv_{\text{piezo}}(t)}{dt} + M_m^* \frac{d i_{\text{LPRC}}(t)}{dt} \]

\[ + v_{Cms^*}(t) + R_m^* i_{Lm^*}(t) + v_{\text{piezo}}(t) \] (B–46)

The value of \( v_{\text{piezo}}(t) \) from Equations B–27 and B–28 is then substituted into the first two terms of Equation B–46. Evaluating the results and setting \( t = t_{\text{init}, 2} \) provides the fourth initial condition.

\[ \text{IC} 4 \equiv \frac{d^3 i_{\text{LPRC}}(t_{\text{init}, 2})}{dt^3} = \frac{F_m^* \sin(\omega t_{\text{init}, 2})}{M_m^* C_{eb} L_{\text{PRC}}} \]

\[- \left( \frac{R_{\text{Phase}2}}{L_{\text{PRC}}} + \frac{1}{R_e C_{eb}} \right) \frac{d^2 i_{\text{LPRC}}(t_{\text{init}, 2})}{dt^2} \]

\[- \left( \frac{R_{\text{Phase}2}}{R_e C_{eb} L_{\text{PRC}}} + \frac{1}{C_{eb} L_{\text{PRC}}} \right) \frac{d i_{\text{LPRC}}(t_{\text{init}, 2})}{dt} \]

\[- \frac{v_{Cms^*}(t_{\text{init}, 2})}{L_{\text{PRC}} M_m^* C_{eb}} - \frac{u_m^*(t_{\text{init}, 2})}{L_{\text{PRC}} M_m^* C_{eb}} - \frac{v_{\text{piezo}}(t_{\text{init}, 2})}{L_{\text{PRC}} M_m^* C_{eb}}. \] (B–47)
Phase3

During phase3, the energy harvesting system model can be reduced to the schematic shown in Figure B-3.

![Figure B-3. Schematic of the energy harvesting system during phase3](image)

Applying KVL to Loop 1 in Figure B-3 results in

\[ F_m \sin(\omega t) = v_{Mm^*}(t) + v_{Cms^*}(t) + v_{Rm^*}(t) + v_{piezo}(t). \]  \(\text{(B–48)}\)

Substituting in the voltages values of the reflected mechanical domain components, Equation B–48 becomes

\[ F_m \sin(\omega t) = M_m \frac{du_m^*(t)}{dt} + \frac{1}{C_{ms^*}} \int_{-\infty}^{t} u_m^*(t) \, dt + R_m u_m^*(t) + v_{piezo}(t). \]  \(\text{(B–49)}\)

The integral term is removed by taking the time derivative of both sides of Equation B–49

\[ F_m \omega \cos(\omega t) = M_m \frac{d^2 u_m^*(t)}{dt^2} + \frac{1}{C_{ms^*}} u_m^*(t) + R_m \frac{du_m^*(t)}{dt} + \frac{dv_{piezo}(t)}{dt}. \]  \(\text{(B–50)}\)

Next the value of \(u_m^*(t)\) is found by applying KCL to Node 1

\[ u_m^*(t) = i_{Ceb}(t) + i_{Re}(t) + i_{LPRC}(t), \]  \(\text{(B–51)}\)

where the values of the currents \(i_{Ceb}(t)\) and \(i_{Re}(t)\) were given by Equation B–5 and Equation B–6. Substituting in the values from Equations B–5, B–6, and B–51 into Equation B–50 yields

\[ \text{(204)}\]
\[ F_m^* \omega \cos(\omega t) = M_m^* C_{eb} \frac{d^3 v_{\text{piezo}}(t)}{dt^3} + \left( \frac{M_m^*}{R_e} + R_m^* C_{eb} \right) \frac{d^2 v_{\text{piezo}}(t)}{dt^2} \]
\[ + L_m^* \frac{d^2 i_{\text{LPRC}}(t)}{dt^2} + \left( \frac{C_{eb}}{C_{ms}} + \frac{R_m^*}{R_e} + 1 \right) \frac{dv_{\text{piezo}}(t)}{dt} \]
\[ + R_m \frac{di_{\text{LPRC}}(t)}{dt} + \frac{1}{C_{ms} R_e} v_{\text{piezo}}(t) + \frac{1}{C_{ms}} i_{\text{LPRC}}(t). \quad (B-52) \]

Equation B–52 is a function of both \( v_{\text{piezo}}(t) \) and \( i_{\text{LPRC}}(t) \). These two terms can be related by examining the inductor, \( L_{\text{PRC}} \). The voltage-current relationship for \( L_{\text{PRC}} \) is

\[ v_{\text{LPRC}}(t) = L_{\text{PRC}} \frac{di_{\text{LPRC}}(t)}{dt} = v_{\text{piezo}}(t) - i_{\text{LPRC}}(t) R_{\text{Phase3}} - V_{D_{eff,3}} - V_{\text{battery}}. \quad (B-53) \]

Solving for \( v_{\text{piezo}}(t) \) and its derivatives gives:

\[ v_{\text{piezo}}(t) = L_{\text{PRC}} \frac{di_{\text{LPRC}}(t)}{dt} + i_{\text{LPRC}}(t) R_{\text{Phase3}} + V_{D_{eff,3}} + V_{\text{battery}}, \quad (B-54) \]
\[ \frac{dv_{\text{piezo}}(t)}{dt} = L_{\text{PRC}} \frac{d^2 i_{\text{LPRC}}(t)}{dt^2} + R_{\text{Phase3}} \frac{di_{\text{LPRC}}(t)}{dt}, \quad (B-55) \]
\[ \frac{d^2 v_{\text{piezo}}(t)}{dt^2} = L_{\text{PRC}} \frac{d^3 i_{\text{LPRC}}(t)}{dt^3} + R_{\text{Phase3}} \frac{d^2 i_{\text{LPRC}}(t)}{dt^2}, \quad (B-56) \]
and
\[ \frac{d^3 v_{\text{piezo}}(t)}{dt^3} = L_{\text{PRC}} \frac{d^4 i_{\text{LPRC}}(t)}{dt^4} + R_{\text{Phase3}} \frac{d^3 i_{\text{LPRC}}(t)}{dt^3}. \quad (B-57) \]

Substituting the values from Equations B–54 through B–57 into B–52 simplifies into an equation of only \( i_{\text{LPRC}}(t) \)
\[ F_m \omega \cos (\omega t) = M_m^* C_{eb} L_{PRC} \frac{d^4 i_{LPRC}(t)}{dt^4} + \left( M_m^* C_{eb} R_{Phase3} + \frac{M_m^* L_{PRC}}{R_e} + R_m^* C_{eb} L_{PRC} \right) \frac{d^3 i_{LPRC}(t)}{dt^3} + \left( M_m^* + \frac{M_m^* R_{Phase3}}{R_e} + R_m^* C_{eb} R_{Phase3} \right) \frac{d^2 i_{LPRC}(t)}{dt^2} + \left( \frac{R_m^* L_{PRC}}{R_e} + L_{PRC} \right) \frac{d^2 i_{LPRC}(t)}{dt^2} + \left( \frac{R_m^* R_{Phase3}}{R_e} + \frac{C_{eb} R_{Phase3}}{C_{ms}^*} \right) \frac{di_{LPRC}(t)}{dt} + \left( \frac{L_{PRC}}{C_{ms}^* R_e} + R_{Phase3} \right) \frac{di_{LPRC}(t)}{dt} + \left( \frac{1}{C_{ms}^*} + \frac{R_{Phase3}}{C_{ms}^* R_e} \right) i_{LPRC}(t) + \frac{(V_{D_{eff,p3}} + V_{battery})}{C_{ms}^* R_e}. \] 

Equation B–58 simplifies further to become

\[ F_m^* \omega \cos (\omega t) = C_{1p3} \frac{d^4 i_{LPRC}(t)}{dt^4} + C_{2p3} \frac{d^3 i_{LPRC}(t)}{dt^3} + C_{3p3} \frac{d^2 i_{LPRC}(t)}{dt^2} + C_{4p3} + C_{5p3} i_{LPRC}(t) + \frac{(V_{D_{eff,p3}} + V_{battery})}{C_{ms}^* R_e}. \] 

where \( C_{1p3} \) through \( C_{5p3} \) are coefficient values based on the transducer LEM and the value of \( L_{PRC} \). The coefficient values are

\[ C_{1p3} = L_{PRC} M_m^* C_{eb}, \]  

\[ C_{2p3} = \frac{M_m^* L_{PRC}}{R_e} + L_{PRC} R_m^* C_{eb} + M_m^* C_{eb} R_{Phase3}, \]  

\[ C_{3p3} = \frac{L_{PRC} C_{eb}}{C_{ms}^*} + \frac{L_{PRC} R_m^*}{R_e} + L_{PRC} + M_m^* + \frac{M_m^* R_{Phase3}}{R_e} + R_m^* C_{eb} R_{Phase3}, \]  

\[ C_{4p3} = \frac{L_{PRC}}{R_e C_{ms}^*} + R_m^* + \frac{C_{eb} R_{Phase3}}{C_{ms}^*} + \frac{R_m^* R_{Phase3}}{R_e} + R_{Phase3}, \]  

and

\[ C_{5p3} = \frac{1}{C_{ms}^*} + \frac{R_{Phase3}}{C_{ms}^* R_e}. \]

In order to solve Equation B–59 for \( i_{LPRC}(t) \) the initial conditions at the start of phase3(\( \rightarrow \)battery) are used. Since Equation B–59 is a 4th order ordinary differential
equation, four initial conditions are required. The first initial condition is found by recalling that the current through the inductor, \( L_{PRC} \), cannot change instantaneously. Therefore, the current at start of phase3 (→battery) must be equal to the current at the end of phase2 (→inductor),

\[
IC1 \equiv i_{LPRC} (t_{init_p3}) = i_{LPRC} (t_{end_p2}).
\]  

(B–65)

where \( t_{end_p2} = t_{init_p3} \) is the instant when the circuit switches from phase2 (→inductor) to phase3 (→battery).

The second initial conditions is found from the voltage across \( L_{PRC} \). Applying KVL to Loop2 yields

\[
v_{piezo} (t) = L_{PRC} \frac{di_{LPRC} (t)}{dt} + R_{Phase3} i_{LPRC} (t) + V_{battery} + V_{D_{eff_p3}}.
\]  

(B–66)

which can be rearranged and solved for \( t = t_{init_p3} \) to give

\[
IC2 \equiv \frac{di_{LPRC} (t_{init_p3})}{dt} = \frac{v_{piezo} (t_{init_p3})}{L_{PRC}} - \frac{R_{Phase3} i_{LPRC} (t_{init_p3})}{L_{PRC}} - \frac{(V_{battery} + V_{D_{eff_p3}})}{L_{PRC}}.
\]  

(B–67)

The third initial condition is found by applying KCL at Node1 in Figure B-3, where

\[
u_{m}^* (t) = i_{Ceb} (t) + i_{Re} (t) + i_{LPRC} (t).
\]  

(B–68)

Substituting the value of \( i_{Ceb} (t) \) and \( i_{Re} (t) \) into Equation B–68 gives

\[
u_{m}^* (t) = C_{eb} \frac{dv_{piezo} (t)}{dt} + \frac{v_{piezo} (t)}{R_{e}} + i_{LPRC} (t).
\]  

(B–69)

The derivative of \( v_{piezo} (t) \) is found from Equation B–66 and is given by

\[
\frac{dv_{piezo} (t)}{dt} = L_{PRC} \frac{d^2 i_{LPRC} (t)}{dt^2} + R_{Phase3} \frac{di_{LPRC} (t)}{dt}.
\]  

(B–70)

Substituting Equations B–66 and B–70 into Equation B–69 and solving at \( t = t_{init_p3} \), the third initial condition can be found
Finally, the fourth initial condition is found by taking the KVL of Loop 1, given as Equation B–48. Substituting the values of $v_{Mm^*}(t)$ and $v_{Rm^*}(t)$ into Equation B–48 gives

$$F_m^* \sin(\omega t) = M_m^* \frac{d}{dt} u_m^*(t) + v_{Cms^*}(t) + R_m^* u_m^*(t) + v_{piezo}(t).$$  \hspace{1cm} (B–72)

Next, substituting in the value of Equation B–69 into Equation B–72 and rearranging gives

$$F_m^* \sin(\omega t) = M_m^* C_{eb} \frac{d^2 v_{piezo}(t)}{dt^2} + \frac{M_m^*}{R_e} \frac{dv_{piezo}(t)}{dt} + M_m^* \frac{di_{LPRC}(t)}{dt}$$

$$+ v_{Cms^*}(t) + R_m^* i_{Lm^*}(t) + v_{piezo}(t).$$  \hspace{1cm} (B–73)

The value of $v_{piezo}(t)$ from Equation B–66 is then substituted into the first two terms of Equation B–73. Evaluating the results and setting $t = t_{\text{init,}3}$ provides the fourth initial condition.

$$IC^4 \equiv \frac{d^3 i_{LPRC}(t_{\text{init,}3})}{dt^3} = \frac{F_m^* \sin(\omega t_{\text{init,}3})}{M_m^* C_{eb} L_{PRC}}$$

$$- \left( \frac{R_{\text{Phase}3}}{L_{PRC}} + \frac{1}{R_e C_{eb}} \right) \frac{d^2 i_{LPRC}(t_{\text{init,}3})}{dt^2}$$

$$- \left( \frac{R_{\text{Phase}3}}{R_e C_{eb} L_{PRC}} + \frac{1}{C_{eb} L_{PRC}} \right) \frac{di_{LPRC}(t_{\text{init,}3})}{dt}$$

$$- \frac{v_{Cms^*}(t_{\text{init,}3})}{L_{PRC} M_m^* C_{eb}} - \frac{u_m^*(t_{\text{init,}3})}{L_{PRC} M_m^* C_{eb}} - \frac{v_{piezo}(t_{\text{init,}3})}{L_{PRC} M_m^* C_{eb}}.$$  \hspace{1cm} (B–74)
APPENDIX C
DERIVATION OF THE SIMPLIFIED RESONANT LEM WITH LOSSES

In Chapter 5, the operation of the energy harvesting system comprised of the simplified resonant LEM was modeled using differential equations to describe the behavior of the system during each of the three phases. Here, a full derivation of the governing differential equations and initial conditions is presented for each phase.

Phase 1

During phase1\(_{(-\text{capacitor})}\), both switches are open, and the energy harvesting system model can be reduced to the open-circuit transducer model, shown schematically in Figure C-1.

![Figure C-1. Schematic of the energy harvesting system during phase1\(_{(-\text{capacitor})}\).](image)

Applying KVL to Loop\(_1\) results in

\[
F_m^* \sin(\omega t) = u_m^*(t) R_m^* + v_{\text{piezo}}(t) .
\]

The value of \(u_m^*(t)\) next found by applying KCL to Node\(_1\)

\[
u_m^*(t) = i_{\text{Ceb}}(t) + i_{\text{Re}}(t),
\]

where the values of the currents \(i_{\text{Ceb}}(t)\) and \(i_{\text{Re}}(t)\) are given by

\[
i_{\text{Ceb}}(t) = C_{\text{eb}} \frac{dv_{\text{piezo}}(t)}{dt},
\]

and

\[
i_{\text{Re}}(t) = \frac{v_{\text{piezo}}(t)}{R_e}.
\]
Substituting in the values from Equations C–2 through C–4 into Equation C–1 yields

\[ F_m \sin(\omega t) = R_m C_{eb} \frac{dv_{\text{piezo}}(t)}{dt} + \left( \frac{R_m}{R_e} + 1 \right) v_{\text{piezo}}(t). \] (C–5)

Equation C–5 is further simplified to

\[ F_m \sin(\omega t) = C_{1p1} \frac{dv_{\text{piezo}}(t)}{dt} + C_{2p1} v_{\text{piezo}}(t). \] (C–6)

where \( C_{1p1} \) and \( C_{2p1} \) are coefficient values based on the transducer LEM.

\[ C_{1p1} = R_m C_{eb}, \] (C–7)

and

\[ C_{2p1} = \frac{R_m}{R_e} + 1. \] (C–8)

In order to solve Equation C–6 for \( v_{\text{piezo}}(t) \), the initial conditions at the start of phase1\((\rightarrow\text{capacitor})\) are used. Since Equation C–6 is a first order differential equation, a single initial condition is required. The initial condition is found by considering that the voltage across \( C_{\text{piezo}} \) cannot change instantaneously between phase3\((\rightarrow\text{battery})\) and phase1\((\rightarrow\text{capacitor})\)

\[ IC1 \equiv v_{\text{piezo}}(t_{\text{init,p1}}) = v_{\text{piezo}}(t_{\text{end,p3}}). \] (C–9)

where \( t_{\text{end,p3}} = t_{\text{init,p1}} \) is the instant when the circuit switches from phase3\((\rightarrow\text{battery})\) to phase1\((\rightarrow\text{capacitor})\).

**Phase2**

During phase2\((\rightarrow\text{inductor})\), the energy harvesting system model can be reduced to the schematic shown in Figure C–2.

Applying KVL to Loop1 in Figure C–2 results in

\[ F_m \sin(\omega t) = u_m(t) R_m + v_{\text{piezo}}(t). \] (C–10)

The value of \( u_m(t) \) next found by applying KCL to Node1
\[ u_m^* (t) = i_{Ceb} (t) + i_{Re} (t) + i_{LPRC} (t), \quad (C-11) \]

where the values of the currents \( i_{Ceb} (t) \) and \( i_{Re} (t) \) were given by Equation C–3 and Equation C–4. Substituting in the values from Equation C–11 into Equation C–10 yields

\[ F_m^* \sin (\omega t) = R_m^* C_{eb} \frac{dv_{piezo} (t)}{dt} + \left( \frac{R_m^*}{R_c} + 1 \right) v_{piezo} (t) + R_m^* i_{LPRC} (t). \quad (C-12) \]

Figure C-2. Schematic of the energy harvesting system during phase 2 (\( \rightarrow \) inductor).

Equation C–12 is a function of both \( v_{piezo} (t) \) and \( i_{LPRC} (t) \). These two terms can be related by examining the inductor, \( L_{PRC} \). The voltage-current relationship for \( L_{PRC} \) is given by

\[ v_{LPRC} (t) = L_{PRC} \frac{di_{LPRC} (t)}{dt} = v_{piezo} (t) - i_{LPRC} (t) R_{Phase2} - V_{D_{eff,p2}}. \quad (C-13) \]

Solving Equation C–13 for \( v_{piezo} (t) \) and its derivative gives:

\[ v_{piezo} (t) = L_{PRC} \frac{di_{LPRC} (t)}{dt} + i_{LPRC} (t) R_{Phase2} + V_{D_{eff,p2}}, \quad (C-14) \]

and

\[ \frac{dv_{piezo} (t)}{dt} = L_{PRC} \frac{d^2 i_{LPRC} (t)}{dt^2} + R_{Phase2} \frac{di_{LPRC} (t)}{dt}. \quad (C-15) \]

Substituting the values from Equations C–14 and C–15 into C–12 simplifies into an equation of only \( i_{LPRC} (t) \)
\[ F_m \sin(\omega t) = (L_{PRC}R_m^*C_{eb}) \frac{d^2i_{LPRC}(t)}{dt^2} \]
\[ + \left( \frac{R_mL_{PRC}}{R_e} + C_{eb}R_mR_{Phase2} + L_{PRC} \right) \frac{di_{LPRC}(t)}{dt} \]
\[ + \left( \frac{R_{Phase2}R_m^*}{R_e} + R_m^* + R_{Phase2} \right) i_{LPRC}(t) \]
\[ + \left( \frac{R_m^*}{R_e} + 1 \right) V_{D, eff, p2} \]

Equation C–16 is further simplified to

\[
F_m \sin(\omega t) = C1_{p2} \frac{d^2i_{LPRC}(t)}{dt^2} + C2_{p2} \frac{di_{LPRC}(t)}{dt} + C3_{p2}i_{LPRC}(t)
\]
\[ + \left( \frac{R_m^*}{R_e} + 1 \right) V_{D, eff, p2}, \]

where the 3 constants for phase2 (inductor), \( C1_{p2} - C3_{p2}, \) are given by

\[ C1_{p2} = L_{PRC}R_m^*C_{eb}, \] (C–18)
\[ C2_{p2} = \frac{R_mL_{PRC}}{R_e} + C_{eb}R_mR_{Phase2} + L_{PRC}, \] (C–19)

and

\[ C3_{p2} = \frac{R_{Phase2}R_m^*}{R_e} + R_m^* + R_{Phase2}. \] (C–20)

In order to solve Equation C–17 for \( i_{LPRC}(t) \), the initial conditions at the start of phase2 (inductor), \( t = t_{init, p2} \) are used. Since Equation C–17 is a second order differential equation, two initial conditions are required. The first initial condition is that the current flowing through \( L_{PRC} \) is zero at the start of phase2 (inductor) given as

\[ IC1 \equiv i_{LPRC}(t_{init, p2}) = 0. \] (C–21)

The second initial conditions is found from the voltage across \( L_{PRC} \). Applying KVL to Loop2 yields

\[ v_{piezo}(t) = L_{PRC} \frac{di_{LPRC}(t)}{dt} + R_{Phase2}i_{LPRC}(t) + V_{D, eff, p2}, \] (C–22)
which can be rearranged and solved for \( t = t_{\text{init},2} \) to give

\[
IC_2 \equiv \frac{d i_{\text{LPRC}}(t_{\text{init},2})}{dt} = \frac{v_{\text{piezo}}(t_{\text{init},2})}{L_{\text{PRC}}} - \frac{R_{\text{Phase}2} i_{\text{LPRC}}(t_{\text{init},2})}{L_{\text{PRC}}} - \frac{V_{\text{D,eff},2}}{L_{\text{PRC}}}. \tag{C-23}
\]

Applying the result of Equation C–21 to Equation C–23, the second initial condition reduces to

\[
IC_2 \equiv \frac{d i_{\text{LPRC}}(t_{\text{init},2})}{dt} = \frac{v_{\text{piezo}}(t_{\text{init},2})}{L_{\text{PRC}}} - \frac{V_{\text{D,eff},2}}{L_{\text{PRC}}}. \tag{C-24}
\]

**Phase 3**

During phase 3 (\( \rightarrow \text{battery} \)), the energy harvesting system model can be reduced to the schematic shown in Figure C-3.

![Figure C-3. Schematic of the energy harvesting system during phase 3 (\( \rightarrow \text{battery} \)).](image)

Applying KVL to Loop 1 in Figure C-3 results in

\[
F_m \star \sin(\omega t) = u_m(t) R_m + v_{\text{piezo}}(t). \tag{C-25}
\]

The value of \( u_m(t) \) next found by applying KCL to Node 1

\[
u_m(t) = i_{\text{Ceb}}(t) + i_{\text{Re}}(t) + i_{\text{LPRC}}(t), \tag{C-26}
\]

where the values of the currents \( i_{\text{Ceb}}(t) \) and \( i_{\text{Re}}(t) \) were given by Equation C–3 and Equation C–4. Substituting in the values from Equation C–26 into Equation C–25 yields

\[
F_m \star \sin(\omega t) = R_m C_{eb} \frac{dv_{\text{piezo}}(t)}{dt} + \left( \frac{R_m^*}{R_e} + 1 \right) v_{\text{piezo}}(t) + R_m^* i_{\text{LPRC}}(t). \tag{C-27}
\]
Equation C–27 is a function of both \( v_{piezo}(t) \) and \( i_{LPRC}(t) \). These two terms can be related by examining the inductor, \( L_{PRC} \). The voltage-current relationship for \( L_{PRC} \) is given by

\[
v_{LPRC}(t) = L_{PRC} \frac{di_{LPRC}(t)}{dt} = v_{piezo}(t) - i_{LPRC}(t) R_{Phase3} - V_{D, ef f, 3} - V_{battery}.
\] (C–28)

Solving Equation C–28 for \( v_{piezo}(t) \) and its derivative gives:

\[
v_{piezo}(t) = L_{PRC} \frac{di_{LPRC}(t)}{dt} + i_{LPRC}(t) R_{Phase3} + V_{D, ef f, 3} + V_{battery},
\] (C–29)

and

\[
\frac{dv_{piezo}(t)}{dt} = L_{PRC} \frac{d^2i_{LPRC}(t)}{dt^2} + R_{Phase3} \frac{di_{LPRC}(t)}{dt}.
\] (C–30)

Substituting the values from Equations C–29 and C–30 into C–27 simplifies into an equation of only \( i_{LPRC}(t) \)

\[
F_m^* \sin(\omega t) = (L_{PRC} R_m^* C_{eb}) \frac{d^2i_{LPRC}(t)}{dt^2} + \left( \frac{R_m L_{PRC}}{R_e} + C_{eb} R_m R_{Phase3} + L_{PRC} \right) \frac{di_{LPRC}(t)}{dt} + \left( \frac{R_{Phase3} R_m^*}{R_e} + R_m^* + R_{Phase3} \right) i_{LPRC}(t) + \left( \frac{R_m^*}{R_e} + 1 \right) V_{D, ef f, 3} + \left( \frac{R_m^*}{R_e} + 1 \right) V_{battery}.
\] (C–31)

Equation C–31 is further simplified to

\[
F_m^* \sin(\omega t) = C_{1p3} \frac{d^2i_{LPRC}(t)}{dt^2} + C_{2p3} \frac{di_{LPRC}(t)}{dt} + C_{3p3} i_{LPRC}(t) + \left( \frac{R_m^*}{R_e} + 1 \right) (V_{D, ef f, 3} + V_{battery}),
\] (C–32)

where the 3 constants for phase3(\(\rightarrow\)battery), \(C_{1p3} - C_{3p3}\), are given by

\[
C_{1p3} = L_{PRC} R_m^* C_{eb},
\] (C–33)

\[
C_{2p3} = \frac{R_m L_{PRC}}{R_e} + C_{eb} R_m R_{Phase3} + L_{PRC},
\] (C–34)
and
\[ C3_{p3} = \frac{R_{Phase3}R_m^*}{R_e} + R_m^* + R_{Phase3}. \] (C–35)

In order to solve Equation C–32 for \( i_{LPRC}(t) \), the initial conditions at the start of phase3(battery), \( t = t_{init,p3} \) are used. Since Equation C–32 is a second order differential equation, two initial conditions are required. The first initial condition is found by recalling that the current through the inductor, \( L_{PRC} \), cannot change instantaneously. Therefore, the current at start of phase3(battery) must be equal to the current at the end of phase2(inductor),
\[ IC1 \equiv i_{LPRC} (t_{init,p3}) = i_{LPRC} (t_{end,p2}). \] (C–36)
where \( t_{end,p2} = t_{init,p3} \) is the instant when the circuit switches from phase2(inductor) to phase3(battery).

The second initial conditions is found from the voltage across \( L_{PRC} \). Applying KVL to Loop2 yields
\[ v_{piezo}(t) = L_{PRC} \frac{di_{LPRC}(t)}{dt} + R_{Phase3}i_{LPRC}(t) + V_{D_{eff,p3}} + V_{battery}, \] (C–37)
which can be rearranged and solved for \( t = t_{init,p2} \) to give
\[ IC2 \equiv \frac{di_{LPRC}(t_{init,p3})}{dt} = \frac{v_{piezo}(t_{init,p3})}{L_{PRC}} - \frac{R_{Phase3}}{L_{PRC}} \frac{i_{LPRC}(t_{init,p3})}{L_{PRC}} - \frac{V_{D_{eff,p3}} + V_{battery}}{L_{PRC}}. \] (C–38)
Applying the result of Equation C–36 to Equation C–38, the second initial condition reduces to
\[ IC2 \equiv \frac{di_{LPRC}(t_{init,p3})}{dt} = \frac{v_{piezo}(t_{init,p3})}{L_{PRC}} - \frac{V_{D_{eff,p3}} + V_{battery}}{L_{PRC}}. \] (C–39)
APPENDIX D
MATLAB CODE

phase1_waveforms.m

% PHASE1_WAVEFORMS calculates the key waveforms during phase 1. First the
differential equation for vpiezo(t) during phase 1 is solved using ode45
on phase1_ode. Next the waveforms are calculated. The waveforms are
returned as a single array [waveforms].

% USAGE:
% [waveforms] = phase1_waveforms(IC,tstart)
% where IC are the initial conditions of the reactive components at the
% start of phase 1, and tstart is the starting time of phase 1.

function [waveforms] = phase1_waveforms(IC,tstart)

% Progress tracking
~~~~~~~~~~~~~~~~~~~~~~~~~~~~
% Used for debugging
% status = 'Finding phase1 waveforms'
~~~~~~~~~~~~~~~~~~~~~~~~~~~~
% Declaration of global variables
global Lm_ref Cm_ref Rm_ref Ceb Re Fmag_ref fn refine
omega = 2*pi*fn;

% ODE Solution
% ODE integration limit - The end time, tend, is equal to the start time
% plus ∆t, where ∆t SHOULD overestimate the time that it takes
% for vpiezo(t) to reach its maximum value.
Δt = 1/2/fn*1.2;
% Calculate the end time
tend = tstart + Δt;
% ODE solver options
% NOTE: Maxstep can be decreased if the program is not catching the
% zero crossings of iLPRC(t) and vpiezo(t)
options = odeset('refine',refine,'MaxStep',0.005*abs(Δt));
% Command to solve the ode defined @phase3_ode
[t,vpiezo] = ode45(@phase1_ode,[tstart tend],IC,options);
% Find the the point where the absolute value of the piezoelectric
% voltage is at a maximum. Y is the value of the maximum voltage and I is
% the index. The index, I, is used to determine where how much of the
% differential equation solution is needed to complete phase 1.
% Find the length of vpiezo. vpiezo is an array, where the number of rows
\texttt{\% is equal to the length, and each column represents vpiezo, dvpiezo, and d2vpiezo, respectively,}
\texttt{\[length\_vpiezo\ \text{width}\_vpiezo\}=\text{size}(vpiezo);}
\texttt{\% Find the halfway point of vpiezo. Floor is used to make sure this value}
\texttt{\% is an integer.}
\texttt{half\_length\_vpiezo = floor(length\_vpiezo/2);}
\texttt{\% Find the point where the second derivative is minimum. Absolute value is}
\texttt{\% chosen in case there is not an exact zero value of the second derivative,}
\texttt{\% and a negative value might be closest to zero.}
\texttt{[Y I] = min(abs(vpiezo(half\_length\_vpiezo:length\_vpiezo,2)));}
\texttt{\% Since the index, I, was found for a vector starting halfway through the}
\texttt{\% total vpiezo vector, half\_length\_vpiezo must be added to I to find the}
\texttt{\% index with respect to the original vpiezo. This may be off by one point,}
\texttt{\% but should be accurate enough assuming a good spread of time points.}
\texttt{I = I + half\_length\_vpiezo - 1;}
\texttt{\% Eliminate all of the times after iLPRC is max. The vector time}
\texttt{\% only represents the time during phase1, while the vector t is not}
\texttt{\% limited to the length of phase1, but instead continues on.}
\texttt{time\_p1 = t(1:I);}
\texttt{\% LEM/PRC waveforms during phase 1}
\texttt{iLPRC\_p1 = time\_p1*0; \% Inductor current}
\texttt{vpiezo\_p1 = vpiezo((1:I),1); \% Piezoelectric voltage}
\texttt{iLm\_p1 = vpiezo\_p1/Re + Ceb\_vpiezo\_p1(1:I),2); \% Velocity}
\texttt{F\_p1 = Fmag\_ref*sin(omega*time\_p1); \% Force from source}
\texttt{vRm\_p1 = iLm\_p1*Rm\_ref; \% Force across mechanical damping}
\texttt{\% The derivative of iLm\_phase1. This value is needed to find the voltage}
\texttt{\% across vCm(t). The value is found by using KCL and simply taking the}
\texttt{\% derivative.}
\texttt{d\_dt\_iLm\_p1 = vpiezo\_p1(1:I),2)/Re + vpiezo\_p1(1:I),3)*Ceb;}
\texttt{\% The voltage/force across the lumped mechanical compliance. The can be}
\texttt{\% related to the mechanical displacement qCm\_phase1.}
\texttt{vCm\_p1 = F\_p1 - Lm\_ref*d\_dt\_iLm\_p1 - vRm\_p1 - vpiezo\_p1;}
\texttt{\% Mechanical displacement during phase 1}
\texttt{qCm\_p1 = Cm\_ref*vCm\_p1;}
\texttt{\% Results vector}
\texttt{waveforms = [time\_p1 F\_p1 iLm\_p1 vCm\_p1 qCm\_p1 abs(vpiezo\_p1) abs(iLPRC\_p1)];}

\texttt{phase2\_waveforms.m}

\texttt{\% PHASE2\_WAVEFORMS calculates the key waveforms during phase 2. First the}
% differential equation for iLPRC(t) during phase 3 is solved using ode45
% on phase2.ode. Next the waveforms are calculated. The waveforms are
% returned as a single array [waveforms].
% USAGE:
% [waveforms] = phase2_waveforms(IC,tstart)
% where IC are the initial conditions of the reactive components at the
% start of phase 2, and tstart is the starting time of phase 2.
function [waveforms] = phase2_waveforms(IC,tstart)

%% Progress tracking

%%%%%%

%% Used for debugging

%% Declaration of global variables

global Lm_ref Cm_ref Rm_ref Ceb Re LPRC Fmag_ref fn R2 N_points NGate_offset

omega = 2*pi*fn;

%% ODE Solution

%% ODE integration limit - This is set to 1.2 times the 0.25 of the period
%% to make sure and capture the zero crossing of vpiezo(t) and the maximum
%% value of iLPRC(t).

Delta_t = 1.2*(pi/2*sqrt(LPRC*Ceb));

%% Calculate the end time

tend = tstart + Delta_t;

%% In order for the answer of the ODE to be evenly spaced in time, it is
%% necessary to define a time vector, TSPAN, where the points of TSPAN are
%% evenly spaced.

dt = (tend-tstart)/N_points;

tspan = [tstart:dt:tend];

%% ODE solver options

options = odeset('refine',refine,'MaxStep',0.0005*abs(Delta_t));

%% Command to solve the ode defined @phase2_ode

[t,iLPRC] = ode45(@phase2_ode,tspan,IC,options);

%% Find the maximum current. Y is the value of the maximum current and I is
%% the index. The index, I, is used to determine where how much of the
%% differential equation solution is needed before phase2 ends and phase3
%% starts.

[Y I] = max(abs(iLPRC(:,1)));
Eliminate all of the times after iLPRC is max. The vector time_phase2 only represents the time during phase2, while the vector t_phase2 is not limited to the length of phase2, but instead continues on.

time_p2 = t(1:I);

ILEM/PRC waveforms during phase2

iLPRC_p2 = iLPRC((1:I),1);  % Inductor current
diLPRC_p2 = iLPRC((1:I),2);  % 1st derivative of inductor current
d2iLPRC_p2 = iLPRC((1:I),3);  % 2nd derivative of inductor current
d3iLPRC_p2 = iLPRC((1:I),4);  % 3rd derivative of inductor current

vpiezo_p2 = d_iLPRC_p2*LPRC + R2 *iLPRC_p2;  % Piezoelectric voltage
iCeb_p2 = Ceb *LPRC*d2iLPRC_p2 + R2 *Ceb*d_iLPRC_p2;  % Current through Ceb
iRe_p2 = vpiezo_p2/Re;  % Current through Re

iLm_p2 = iLPRC_p2 + iRe_p2 + iCeb_p2;  % Velocity - in the electrical domain
F_p2 = Fmag_ref*sin(omega*time_p2);  % Force from source - in electrical domain

vRm_p2 = iLm_p2*Rm_ref;  % The derivative of iLm_p2. This value is needed to find the voltage
across vCm. The value is found by using KCL and simply taking the
% derivative.

d_dt_iLm_p2 = (1+R2/Re)*diLPRC_p2 + ... 
(LPRC/Re+R2*Ceb)*d2iLPRC_p2 + (Ceb+LPRC)*d3iLPRC_p2;

vCm_p2 = F_p2 - Lm_ref*d_dt_iLm_p2 - vRm_p2 - vpiezo_p2;
% Mechanical displacement during phase 2

qCm_p2 = Cm_ref*vCm_p2;

Results vector

waveforms = [time_p2 F_p2 iLm_p2 vCm_p2 qCm_p2 abs(vpiezo_p2) abs(iLPRC_p2)];

default waveforms.m

% PHASE3_WAVEFORMS calculates the key waveforms during phase 3. First the
% differential equation for iLPRC(t) during phase 3 is solved using ode45
% on phase3_ode. Next the waveforms are calculated. The waveforms are
% returned as a sigle array [waveforms].
% USAGE:
% [waveforms] = phase3_waveforms(IC,tstart)
% where IC are the initial conditions of the reactive components at the
% start of phase 3, and tstart is the starting time of phase 3.
function [waveforms] = phase3_waveforms(IC,tstart)
%% Progress tracking
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Used for debugging
% status = 'Finding phase3 waveforms'
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Declaration of global variables

global Lm_ref Cm_ref Rm_ref Ceb Re LPRC Fmag_ref fn Vbatt R3

global N_points refine Vdiode3 switching_delay

omega = 2*pi*fn;

%% ODE Solution
% ODE integration limit - The end time, tend, is equal to the start time
% plus Δt = LPRC*ΔiLPRC(t)/Vbatt, where Δt SHOULD
% overestimate the time that it takes for iLPRC(t) to fall to zero.
Δt = 1.1*abs(LPRC*IC(1)/Vbatt);
% The end time of the integration
tend = tstart + Δt;
% In order for the answer of the ODE to be evenly spaced in time, it is
% necessary to define a time vector, TSPAN, where the points of TSPAN are
% evenly spaced.
dt = (tend-tstart)/N_points;
% 10000 can be changed if more/less points are needed
tspan = tstart:dt:tend;
% ODE solver options
% NOTE: Maxstep can be decreased if the program is not catching the
% zero crossings of iLPRC(t) and vpi(eo)(t)
%options = odeset('refine',refine,'MaxStep',0.0005*abs(Δt));
% Command to solve the ode defined @phase3_ode
[t,iLPRC] = ode45(@phase3_ode,tspan,IC);%,options);
% Find the maximum current. Y is the value of the maximum current and I is
% the index. The index, I, is used to determine where how much of the
% differential equation solution is needed before phase2 ends and phase3
% starts.
[Y I] = min(abs(iLPRC(:,1)));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Testing
%I = I + 1000
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Eliminate all of the times after iLPRC is max. The vector time_p3
% only represents the time during phase3, while the vector t is not
% limited to the length of phase3, but instead continues on.
time_p3 = t(I:I);
%% LEM/FRC waveforms during phase3
% Numerical solutions to the differential equation
iLPRC_p3 = iLPRC((1:I),1);  % Inductor current
d_iLPRC_p3 = iLPRC((1:I),2);  % 1st derivative of inductor current
d2_iLPRC_p3 = iLPRC((1:I),3);  % 2nd derivative of inductor current
d3_iLPRC_p3 = iLPRC((1:I),4);  % 3rd derivative of inductor current

% Calculation of the system waveforms for phase 3
vpiezo_p3 = LPRC*d_iLPRC_p3+R3*iLPRC_p3+Vbatt+Vdiode3;  % Piezoelectric voltage
iCeb_p3 = LPRC*Ceb*d2_iLPRC_p3+R3*Ceb*d_iLPRC_p3;  % Current through Ceb
iRe_p3 = vpiezo_p3/Re;  % Current through Re
iLm_p3 = iCeb_p3 + iRe_p3 + iLPRC_p3;  % Velocity
F_p3 = Fmag_ref*sin(omega*time_p3);  % Force from source
vRm_p3 = iLm_p3*Rm_ref;  % Force across mechanical damping
% The derivative of iLm_phase3. This value is needed to find the voltage
% across vCm. The value is found by using KCL and simply taking the
% derivative.
d_dt_iLm_p3 = (1+R3/Re)*d_iLPRC_p3 + ...
(LPRC/Re*R3*Ceb)*d2_iLPRC_p3 + (Ceb*LPRC)*d3_iLPRC_p3;
% The voltage/force across the lumped mechanical compliance. The can be
% related to the mechanical displacement qCm_phase3.
vCm_p3 = F_p3 - Lm_ref*d_dt_iLm_p3 - vRm_p3 - vpiezo_p3;
% Mechanical displacement during phase 3
qCm_p3 = Cm_ref*vCm_p3;
% Computing the power delivered to the battery
dt_phase3 = dt;
I;
% I_delay = 1;
I_delay = floor(switching_delay/dt_phase3);
% This takes care of the problems when switching_delay = 0
if I_delay == 0
    I_delay = 1
end
iLPRC_p3_delay = zeros(size(iLPRC_p3));
iLPRC_p3_delay(I_delay:length(iLPRC_p3),1) = iLPRC_p3(I_delay:length(iLPRC_p3),1);
%difference = max(iLPRC_p3) -max(iLPRC_p3_delay)
%size(iLPRC_p3_delay)
%size(iLPRC_p3)
%size(iLm_p3)
%plot(time_p3,iLPRC_p3,time_p3,iLPRC_p3_delay,'r')
%pause
% Results vector
waveforms = [time_p3 F_p3 iLm_p3 vCm_p3
qCm_p3 abs(vpiezo_p3) abs(iLPRC_p3) iLPRC_p3_delay];

IC_phase1.m

% Find the initial conditions for phase 1
% The differential equation during phase 1 is a 3th order ODE of vpiezo(t),
% therefore there are 3 initial conditions.
function [IC] = IC_phase1(vpiezo_0,iLm_0,vCm_0,iLPRC_0,t_init)
% Declaration of global variables
% These are only the global variables needed in this function.
global Lm_ref Rm_ref Ceb Re Fmag_ref fn
% Calculate the constants needed in this function
omega = fn*2*pi;
% This section calculates the intial/boundary conditions
% IC1 = vpiezo(t_init)
IC1 = vpiezo_0;
% IC2 = d/dt(vpiezo(t_init))
IC2 = iLm_0/Ceb-vpiezo_0/(Ceb*Re);
% IC3 = d2/dt2(vpiezo(t_init))
IC3 = Fmag_ref/(Lm_ref*Ceb)*sin(omega*t_init)-
     (1/(Re*Ceb)+Rm_ref/Lm_ref)*((iLm_0/Ceb-vpiezo_0/(Ceb*Re))-
     vCm_0/(Lm_ref*Ceb)-(1/(Lm_ref*Ceb)+Rm_ref/{Lm_ref*Re*Ceb}))*vpiezo_0;
% Vector of initial conditions
IC = [IC1;IC2;IC3];

IC_phase2.m

% Find the initial conditions for phase 2.
% The differential equation during phase 2 is a 4th order ODE of iLPRC(t),
% therefore there are 4 initial conditions.
function [IC] = IC_phase2(vpiezo_0,iLm_0,vCm_0,iLPRC_0,t_init)
% Declaration of global variables
% These are only the global variables needed in this function.
global Lm_ref Rm_ref Ceb Re LPRC Fmag_ref fn R2 Vdiode2
% Calculate the constants needed in this function
omega = fn*2*pi;
% This section calculates the intial/boundary conditions
% The start of phase 2 is at time, t = t_init.
% IC1 = iLPRC(t_init)
IC1 = 0;
% IC2 = d/dt(iLPRC(t_init))
IC2 = (vpiezo_0-Vdiode2)/LPRC;
% IC3 = d2/dt2(iLPRC(t_init))
IC3 = iLm_0/(LPRC*Ceb)-vpiezo_0/(LPRC*Ceb*Re)-R2/LPRC*IC2;
% IC4 = d3/dt3(iLPRC(t_init))
IC4 = Fmag_ref*sin(omega*t_init)/(Lm_ref*Ceb*LPRC) - (R2/LPRC + 1/(Re*Ceb))*IC3 - ...
(R2/(Re*Ceb*LPRC)+1/(Ceb*LPRC))*IC2 - ...
vCm_0/(Lm_ref*Ceb*LPRC) - iLm_0*Rm_ref/(Lm_ref*Ceb*LPRC) ...
- vpiezo_0/(Lm_ref*Ceb*LPRC);
% Vector of initial conditions
IC = [IC1;IC2;IC3;IC4];
phase1_ode.m

% phase1_ode
% This program solves for vpiezo(t) during phase 1 of PRC operation.
function dydt = phase1_ode(t,y)
% Declaration of global variables
global Lm_ref Cm_ref Rm_ref Ceb Re Fmag_ref fn
% Short-circuit radian frequency
omega = 2*pi*fn;
% Some constants from the differential equation
AA = Lm_ref*Ceb;
BB = Lm_ref/Re+Rm_ref*Ceb;
CC = Ceb/Cm_ref+Rm_ref/Re+1;
DD = 1/(Re*Cm_ref);
% Solution of the differential equation
dydt = [y(2);y(3);(Fmag_ref*omega*cos(omega*t)-BB*y(3)-CC*y(2)-DD*y(1))/AA];

phase2_ode.m

% phase2_ode
% This program solves for iLPRC during phase 2 of PRC operation.
function dydt = phase2_ode(t,y)
% Declaration of global variables
global Lm_ref Cm_ref Rm_ref Ceb Re LPRC Fmag_ref fn R2 Vdiode2
% Short-circuit radian frequency
omega = 2*pi*fn;
% Some constants from the differential equation
% Given as C1_p2 - C5_p2 in em_coupling_p2_derivation.doc
AA = LPRC*Lm_ref*Ceb;
BB = Lm_ref*LPRC/Re+LPRC*Rm_ref*Ceb + Lm_ref*Ceb*R2;
CC = LPRC*Ceb/Cm_ref+LPRC*Rm_ref/Re+LPRC*Lm_ref + Lm_ref*R2/Re + Rm_ref*Ceb*R2;
DD = LPRC/(Re*Cm_ref)+Rm_ref + Ceb*R2/Cm_ref + Rm_ref*R2/Re + R2;
EE = 1/Cm_ref + R2/(Cm_ref*Re);
FF = Vdiode2/(Cm_ref*Re);
% Solution of the differential equation
dydt = [y(2);y(3);y(4);(Fmag_ref*omega*cos(omega*t)...-BB*y(4)-CC*y(3)-DD*y(2)-EE*y(1)-FF)/AA];

phase3_ode.m
This program solves for iLPRC during phase 3 of PRC operation.

```matlab
function dydt = phase3_ode(t,y)

% Declaration of global variables
global Lm_ref Cm_ref Rm_ref Ceb Re LPRC Fmag_ref fn Vbatt R3 Vdiode3

% Short-circuit radian frequency
omega = 2*pi*fn;

% Some constants from the differential equation
% Given as Cl_p3 - C5_p3 in em_coupling_p3_derivation.doc
AA = Lm_ref*Ceb*LPRC;
BB = Lm_ref*Ceb*R3 + Lm_ref*LPRC/Re + Rm_ref*Ceb*LPRC;
CC = Lm_ref + Lm_ref*R3/Re + Rm_ref*Ceb*R3 + ...
    Rm_ref*LPRC/Re + Ceb*LPRC/Cm_ref + LPRC;
DD = Rm_ref + Rm_ref*R3/Re + Ceb*R3/Cm_ref + LPRC/(Re*Cm_ref) + R3;
EE = 1/Cm_ref + R3/(Cm_ref*Re);
FF = Vdiode3/(Re*Cm_ref) + Vbatt/(Re*Cm_ref);

% Solution of the differential equation
dydt = [y(2);y(3);y(4);(Fmag_ref*omega*cos(omega*t) ...
      - BB*y(4)-CC*y(3)-DD*y(2)-EE*y(1)-FF)/AA];
```

const_calc.m

% CONST_CALC calculates several constants that are used in much of the
% derivation. There is no input. The value of the EH system parameters
% are read in from the .txt file and then the constants are calculated.
%
% USAGE:
% [omega,tau,T,A1,A2,A3] = const_calc()

```matlab
function [omega,tau,T,A1,A2,A3] = const_calc()

% Declaration of global variables

% Calculate the constants
A1 = Re*Lm_ref*Ceb*Cm_ref;
A2 = Re*Rm_ref*Ceb*Cm_ref + Lm_ref*Cm_ref;
A3 = Re*Ceb + Rm_ref*Cm_ref + Re*Cm_ref;
omega = 2*pi*fn;
tau = Ceb*Re;
T = 1/fn;
```

energy_calc.m
% This function calculates the energy delivered to the battery during phase 3
% for each energy harvesting cycle.
function [energy] = energy_calc(waveforms)
% Declaration of global variables
global Vbatt
% Calculation of the energy delivered to the load during phase 3
% Energy is calculated by integrating the current, iLPRC(t), during phase 3
% and multiplying it by the time step (dt) and the battery voltage (Vbatt).
% It is necessary to multiply by dt because trapz integrates assuming unit
% spacing, not spacing of dt.
% Since the waveforms are calculated with even spacing, the spacing in
% time can be found by finding the difference between any two consecutive
% time points.
dt = waveforms(2,1) - waveforms(1,1);
% Energy is calculated as E = V*I*dt
energy = trapz(waveforms(:,8)) * dt * Vbatt;

PRC_off_steady_state.m

% PRC_OFF_STEADY_STATE calculates the steady state waveforms of the EH
% system when the PRC is off. This value is calculated for a user
% specified number of periods at steady-state up to the time when
% vpiezo reaches its maximum value.
% USAGE:
% [waveforms] = PRC_off_steady_state(filename, n_periods)
% where n_periods is the number of periods to include in the waveforms.
% The returned value, waveforms, is an array of the waveforms in the
% following order: waveforms = [t' F' iLm' vCm' qCm' vpiezo' iLPRC']
function [waveforms] = PRC_off_steady_state(n_periods)
% Declaration of global variables
global Cm_ref Ceb Re Fmag_ref
% System constants needed for waveforms
[omega, tau, T, A1, A2, A3] = const_calc();
% Steady-state magnitudes and phases
% Magnitudes
vpiezo_mag = Fmag_ref * omega * Re * Cm_ref / sqrt((1 - omega^2 * A2)^2 + ...
           (omega * A3 - omega^3 * A1)^2);
iLm_mag = sqrt((omega^2 * Re * Ceb * Cm_ref)^2 + ...
               (omega * Cm_ref)^2) / sqrt((1 - omega^2 * A2)^2 + ...
\( + (\omega \cdot A_2 - \omega^3 \cdot A_1)^2 \cdot F_{\text{mag}} \cdot \text{ref}; \)
\( v_{\text{Cm}} = \sqrt{1 + (\omega \cdot \tau)^2} / \sqrt{(1 - \omega^2 \cdot A_2)^2 + \ldots} \)
\( (\omega \cdot A_3 - \omega^3 \cdot A_1)^2 \cdot F_{\text{mag}} \cdot \text{ref}; \)

\% Phases
\( \phi_{\text{vpiezo}} = -\pi/2 - \tan((\omega \cdot A_3 - \omega^3 \cdot A_1)/(1 - \omega^2 \cdot A_2)); \)
\( \phi_{\text{iLm}} = \tan(-1/(\omega \cdot \tau)) - \tan((\omega \cdot A_3 - \omega^3 \cdot A_1)/(1 - \omega^2 \cdot A_2)); \)
\( \phi_{\text{vCm}} = \tan((\omega \cdot A_3 - \omega^3 \cdot A_1)/(1 - \omega^2 \cdot A_2)); \)

\% Generating the steady-state waveforms
\% Time when \( \text{vpiezo} \) is at its max + extra periods
\( t_{\text{vpiezo\_max}} = \pi/2/(\omega \cdot \text{vpiezo}) + n \cdot \text{periods} \cdot \tau; \)
\% Time vector
\( t = \text{linspace}(0, t_{\text{vpiezo\_max}}, 10000); \)
\( dt = t(400) - t(399); \)
\% Waveforms
\( F = F_{\text{mag}} \cdot \text{ref} \cdot \sin(\omega \cdot t); \quad \% \text{Force} \)
\( \text{vpiezo} = v_{\text{piezo}} \cdot \text{mag} \cdot \sin(\omega \cdot t + \phi_{\text{vpiezo}}); \quad \% \text{Piezoelectric voltage} \)
\( i_{\text{Lm}} = i_{\text{Lm\_mag}} \cdot \sin(\omega \cdot t + \phi_{\text{iLm}}); \quad \% \text{Mechanical current - reflected velocity} \)
\( v_{\text{Cm}} = -v_{\text{Cm\_mag}} \cdot \sin(\omega \cdot t + \phi_{\text{vCm}}); \quad \% \text{Mechanical compliance voltage} \)
\( q_{\text{Cm}} = C_{\text{m\_ref}} \cdot v_{\text{Cm}}; \quad \% \text{Mechanical displacement} \)
\( i_{\text{LPRC}} = 0 \cdot t; \quad \% \text{Inductor current} \)
\% Results vector
\( \text{waveforms} = \left[ t' \ F' \ i_{\text{Lm}}' \ v_{\text{Cm}}' \ q_{\text{Cm}}' \ \text{vpiezo}' \ i_{\text{LPRC}}' \right]; \)

\begin{verbatim}
parameter_sweep.m

%',
% This program simulates PRC operation for a user-set number of cycles
% and allows for a parametric sweep of any variable.
% Program Initialization
clear all; close all; clc
% Declaration of Global Variables
global Lm_ref Cm_ref Rm_ref Ceb Re LPRC Fmag_ref fn Vbatt R3
global R2 N_points refine Vdiode2 Vdiode3 NGate_offset switching_delay
refine = 4;
% Define the LEM/PRC Parameters
% Input the system parameters

dt = t(400) - t(399); 
\end{verbatim}
Lm = 4.499e-4;  % [kg]
Cm = 1.610e-2;  % [m/N]
Rm = 2.134e-3;  
Ceb = 15.0e-9;  % [F]
Re = 10e6;  % [Ohms]
LPRC = 100e-6;  % [H]
deff = 1.646e-6;  % [m/V]
fn = 59;  % [Hz]
am = 0.7;  % [m/s^2]
Vbatt = 5;  % [V]

% Parasitics in the PRC
R2 = 0;  % [Ohms]
R3 = 0;  % [Ohms]
Vdiode2 = 0;  % [V]
Vdiode3 = 0;  % [V]
Ngate_offset = 0;  

% This is the delay between the switches of phase 2 and phase 3
switching_delay = 0;  % [s]

%% Calculate Remaining LEM/PRC Parameters
% Calculation LEM parameters based off other inputs
Fmag = Lm*am*1.566;
phi = deff/Cm;

% Calculate the reflected LEM parameters
Lm_ref = Lm/phi^2;
Cm_ref = Cm*phi^2;
Rm_ref = Rm/phi^2;
Fmag_ref = Fmag/phi;

%% Define the Sweep Range
% First parameter value
start = 55;
% Parameter step value
step = 0.125;
% Last parameter value
stop = 65;
% Sweep range
range = start:step:stop;

%% Define Sweep Variables
% Number of EH cycles for each parametric point.
EH_cycles = 200;
% Number of integration points used to solve the phase 2 and phase 3 ODEs.
N_points = 2500;
To increase speed, the outputs for the parametric sweeps are pre-allocated to be zero vectors of the appropriate size.

Calculate the number of swept parametric points:
\[
preset = \frac{(\text{stop}-\text{start})}{\text{step}} + 1;
\]

Pre-allocate each output variable:
- \( vrect_{\text{big}} = \text{zeros}(\text{presize},1) \)
- \( vrect_{\text{small}} = \text{zeros}(\text{presize},1) \)
- \( iLPRC_{\text{big}} = \text{zeros}(\text{presize},1) \)
- \( iLPRC_{\text{small}} = \text{zeros}(\text{presize},1) \)
- \( ibattery_{\text{big}} = \text{zeros}(\text{presize},1) \)
- \( ibattery_{\text{small}} = \text{zeros}(\text{presize},1) \)
- \( ngate_{\text{big}} = \text{zeros}(\text{presize},1) \)
- \( ngate_{\text{small}} = \text{zeros}(\text{presize},1) \)
- \( pgate_{\text{big}} = \text{zeros}(\text{presize},1) \)
- \( pgate_{\text{small}} = \text{zeros}(\text{presize},1) \)
- \( \text{power} = \text{zeros}(\text{presize},1) \)
- \( \text{vss} = \text{zeros}(\text{presize},1) \)
- \( \text{max}_\text{qCm} = \text{zeros}(1,3) \)
- \( \text{w}_\text{tip} = \text{zeros}(\text{presize},1) \)

% Parametric Sweep
% Initialize the parametric progress (shows the percentage of parametric points that have been swept)
\[
g = \text{waitbar}(0, '\text{Parametric Sweep Progress}', '\text{Position}', [345 350.1250 270 56.2500]);
\]
% Initialize a parameter counter
\[
p_\text{counter} = 0;
\]
\[
\text{for} \ \text{parameter} = 62
\]
\[
p_\text{counter} = p_\text{counter} + 1;
\]
\[
\text{fn} = \text{parameter};
\]
\[
\text{h} = \text{waitbar}(0, '\text{Individual Parameter Progress}');
\]
% Initialize variables that start over for each parametric sweep. \( ngate \) and \( pgate \) are the lengths of phase 2 and phase 3, respectively.
% \( iLPRC_{\text{peak}} \) is the value of \( iLPRC(t) \) at the end of phase 2 (when it is at its maximum value).
\[
ngate = \text{zeros}(\text{EH_cycles},1);
\]
\[
pgate = \text{zeros}(\text{EH_cycles},1);
\]
\[
iLPRC_{\text{peak}} = \text{zeros}(\text{EH_cycles},1);
\]
\[
ibattery_{\text{peak}} = \text{zeros}(\text{EH_cycles},1);
\]
% Calculate some parameter specific constants
[\omega, \tau, T, A1, A2, A3] = const_calc();

% STEADY-STATE OPERATION

% This section calculates the steady-state operation of the EH system,
% before the PRC is turned on.

N_ss = 2;

% Calculate the steady-state waveforms

[ss_waveforms] = PRC_off_steady_state(N_ss);

% Create waveform vectors for the different voltages, currents, etc.

time = ss_waveforms(:,1);
force = ss_waveforms(:,2);
velocity = ss_waveforms(:,3);
vCm = ss_waveforms(:,4);
qCm = ss_waveforms(:,5);
vpiezo = ss_waveforms(:,6);
iLPRC = ss_waveforms(:,7);
ibattery = zeros(size(iLPRC));
vss(p_counter) = max(vpiezo);

% Calculate the maximum value of qCm (used to find w_tip)
max_qCm(1,1) = max(qCm);

% PHASE 2 - INITIAL CONDITIONS

% This section calculates the initial conditions for phase 2. Here the
% initial conditions are calculated analytically, while later
% iterations will simply use the conditions at the end of the previous
% phase.

% Phase shift of vpiezo relative to Fmag_ref, under steady-state

phi_vpiezo = -pi/2 - atan((\omega *A3 - \omega^3 *A1)/(1-\omega^2*A2));

% Time when phase2 begins

tinit_p2 = pi/2/\omega - phi_vpiezo/\omega + N_ss*T;

% Find the values of the reactive components at the end of steady-state
% Vpiezo

vpiezo_0_p2 = \omega *Re*Cm_ref/sqrt((1-\omega^2*A2)^2 + ...
                     (\omega*A3-\omega^3*A1)^2)*Fmag_ref;

% Velocity = iLm

iLm_0_p2 = sqrt((\omega^2*Re*Ceb*Cm_ref)^2+...
\[(\omega \cdot \text{Cm}_\text{ref})^2 / \sqrt{((1 - \omega^2 \cdot A2)^2 + \ldots)} \]
\[(\omega \cdot \text{A3} - \omega^3 \cdot \text{A1})^2 / \sqrt{1 + (\omega \cdot \tau)^2} \cdot \text{Fmag}_\text{ref};\]

\% Voltage across compliance = vCm
\vCm_\text{0.p2} = \omega \cdot \tau \cdot \text{Fmag}_\text{ref} / \sqrt{((1 - \omega^2 \cdot A2)^2 + \ldots)}
\vCm_\text{0.p2} = \omega \cdot \tau \cdot \text{Fmag}_\text{ref} / \sqrt{((1 - \omega^2 \cdot A2)^2 + \ldots)};

\% Inductor current = iLPRC
\iLPRC_\text{0.p2} = 0;

\% Calculate the initial conditions vector for phase 2
\text{IC}_\text{p2} = \text{IC}_\text{phase2} (\text{vpiezo}_\text{0.p2}, \text{iLm}_\text{0.p2}, \vCm_\text{0.p2}, \iLPRC_\text{0.p2}, \text{tinit}_\text{p2});

\% Track the maximum piezoelectric voltage
\text{vpiezo}_\text{peak} = \text{vpiezo}_\text{0.p2};

\% PHASE 2 - CALCULATION
%

\% This section uses the phase 2 initial conditions and the phase 2 ode
\% to solve for the phase 2 waveforms.

\text{time} = [\text{time}; \text{p2 \_ waveforms}(\ldots,1)];
\text{force} = [\text{force}; \text{p2 \_ waveforms}(\ldots,2)];
\text{velocity} = [\text{velocity}; \text{p2 \_ waveforms}(\ldots,3)];
\vCm = [\vCm; \text{p2 \_ waveforms}(\ldots,4)];
\text{qCm} = [\text{qCm}; \text{p2 \_ waveforms}(\ldots,5)];
\text{vpiezo} = [\text{vpiezo}; \text{p2 \_ waveforms}(\ldots,6)];
\iLPRC = [\iLPRC; \text{p2 \_ waveforms}(\ldots,7)];
\text{ibattery} = [\text{ibattery}; \text{zeros(size(p2 \_ waveforms)(\ldots,1)))];

\% Calculate ngate length for this EH cycle
\text{ngate}(1) = \text{p2 \_ waveforms}(\text{length(p2 \_ waveforms)(\ldots,1))) - \text{p2 \_ waveforms}(1,1);

\% Calculate the maximum value of qCm (used to find w\_tip)
\text{max} \cdot \text{qCm}(1,2) = \text{max} (\text{p2 \_ waveforms}(\ldots,5));

\% PHASE 3 - INITIAL CONDITIONS
%

\% This section calculates the initial conditions for phase 3, which are
\% simply equal to the final conditions of phase 2.

\% Time when phase 3 begins
\text{tinit}_\text{p3} = \text{p2 \_ waveforms}(\text{length(p2 \_ waveforms)},1);

\% Find the values of the reactive components at the end of phase 2
% Vpiezo
vpiezo_0.p3 = p2_waveforms(length(p2_waveforms),6);
% Velocity = iLm
iLm_0.p3 = p2_waveforms(length(p2_waveforms),3);
% Force across compliance = vCm
vCm_0.p3 = p2_waveforms(length(p2_waveforms),4);
% Inductor current = iLPRC
iLPRC_0.p3 = p2_waveforms(length(p2_waveforms),7);
% Calculate the initial conditions vector for phase 3
IC_p3 = IC_phase3(vpiezo_0.p3,iLm_0.p3,vCm_0.p3,iLPRC_0.p3,tinit_p3);
% Calculate maximum inductor current = iLPRC
iLPRC_peak(1) = iLPRC_0.p3;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%% % PHASE 3 - CALCULATION % %
% This section uses the phase 3 initial conditions and the phase 3 ode to solve for the phase 3 waveforms. % % Find the waveforms for phase 3
[p3_waveforms] = phase3_waveforms(IC_p3,tinit_p3);
% Add new waveform information to waveform vectors
time = [time; p3_waveforms(:,1)];
force = [force; p3_waveforms(:,2)];
velocity = [velocity; p3_waveforms(:,3)];
vCm = [vCm; p3_waveforms(:,4)];
qCm = [qCm; p3_waveforms(:,5)];
vpiezo = [vpiezo; p3_waveforms(:,6)];
iLPRC = [iLPRC; p3_waveforms(:,7)];
ibattery = [ibattery; p3_waveforms(:,8)];
% Calculate pgate length for this EH cycle
pgate(1) = p3_waveforms(length(p3_waveforms(:,1))) - p3_waveforms(1,1);
% Calculate maximum battery current = ibattery
ibattery_peak(1) = max(p3_waveforms(:,8));
% Calculate the energy delivered to the battery during phase 3
energy = energy_calc(p3_waveforms);
% Calculate the time where phase 3 starts in order to fine EH frequency.
time_p3_peak = [tinit_p3];
% Calculate the maximum value of qCm (used to find w_tip)
max_qCm(1,3) = max(p3_waveforms(:,5));
% EH LOOP

% This section repeats phases 1-3 for a user entered number of times.
% This allows the transient effects to die out so that all of the
% important parameters can be calculated at "PRC-on steady-state".

for N = 1:(EH_cycles-1)

% Increment the EH_cycle progress bar (shows the progress of each
% parametric point).
waitbar(N/(EH_cycles-1),h);

% PHASE 1 - INITIAL CONDITIONS

% This section calculates the phase 1 initial conditions from the
% final conditions at the end of phase 3.

% Time when phase 1 begins
 tinit_p1 = p3_waveforms(length(p3_waveforms),1);

% Find the values of the reactive components at the end of phase 3
% Piezoelectric voltage = Vpiezo
 vpiezo_0_p1 = p3_waveforms(length(p3_waveforms),6);
% Velocity/inductor current = iLm
 iLm_0_p1 = p3_waveforms(length(p3_waveforms),3);
% Force across compliance = vCm
 vCm_0_p1 = p3_waveforms(length(p3_waveforms),4);
% Inductor current = iLPRC
 iLPRC_0_p1 = p3_waveforms(length(p3_waveforms),7);

% Calculate the initial conditions vector for phase 1
 IC_p1 = IC_phase1(vpiezo_0_p1,iLm_0_p1,vCm_0_p1,iLPRC_0_p1,tinit_p1);

% PHASE 1 - CALCULATION

% This section uses the phase 1 initial conditions and the phase 1
% ODE to solve for the phase 1 waveforms.

% Find the waveforms for phase1
[p1_waveforms] = phase1_waveforms(IC_p1,tinit_p1);

% Add new waveform information to waveform vectors
 time = [time;p1_waveforms(:,1)];
 force = [force; p1_waveforms(:,2)];
 velocity = [velocity; p1_waveforms(:,3)];
vCm = [vCm; p1_waveforms(:,4)];
qCm = [qCm; p1_waveforms(:,5)];
vpiezo = [vpiezo; p1_waveforms(:,6)];
iLPRC = [iLPRC; p1_waveforms(:,7)];
ibattery = [ibattery; zeros(size(p1_waveforms(:,1)))];

% Calculate the maximum value of qCm (used to find \( w_{\text{tip}} \))
max_qCm(1,1) = max(p1_waveforms(:,5));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% PHASE 2 - INITIAL CONDITIONS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% This section calculates the phase 2 initial conditions from the
% final conditions at the end of phase 1.

% Time when phase 2 begins

% Find the values of the reactive components at the end of phase 1
% Piezoelectric voltage = Vpiezo

% Velocity/inductor current = iLm

% Force across compliance = vCm

% Inductor current = iLPRC

% Calculate the initial conditions vector for phase 2

% Track the maximum piezoelectric voltage

% PHASE 2 - CALCULATION

% This section uses the phase 2 initial conditions and the phase 2
% ODE to solve for the phase 2 waveforms.

% Find the waveforms for phase 2

% Add new waveform information to waveform vectors

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```matlab
qCm = [qCm; p2_waveforms(:,5)];
vpiezo = [vpiezo; p2_waveforms(:,6)];
iLPRC = [iLPRC; p2_waveforms(:,7)];
ibattery = [ibattery; zeros(size(p2_waveforms(:,1))));

% Calculate gate length for this EH cycle
ngate(N+1) = p2_waveforms(length(p2_waveforms(:,1))) - p2_waveforms(1,1);

% Calculate the maximum value of qCm (used to find \( w_{tip} \))
max_qCm(1,2) = max(p2_waveforms(:,5));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% PHASE 3 - INITIAL CONDITIONS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% This section calculates the initial conditions for phase 3, which
% are simply equal to the final conditions of phase 2.

% Time when phase 3 begins
tinit_p3 = p2_waveforms(length(p2_waveforms),1);

% Find the values of the reactive components at the end of phase3
% Piezoelectric voltage = Vpiezo
vpiezo_0_p3 = p2_waveforms(length(p2_waveforms),6);
% Velocity/inductor current = iLm
iLm_0_p3 = p2_waveforms(length(p2_waveforms),3);
% Force across compliance = vCm
vCm_0_p3 = p2_waveforms(length(p2_waveforms),4);
% Inductor current = iLPRC
iLPRC_0_p3 = p2_waveforms(length(p2_waveforms),7);

% Calculate the initial conditions vector for phase 3
IC_p3 = IC_phase3(vpiezo_0_p3,iLm_0_p3,vCm_0_p3,iLPRC_0_p3,tinit_p3);

% Calculate maximum inductor current = iLPRC
iLPRC_peak(N+1) = iLPRC_0_p3;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% PHASE 3 - CALCULATION
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% This section uses the phase 3 initial conditions and the phase 3 ode
% to solve for the phase 3 waveforms.

% Find the waveforms for phase3
[p3_waveforms] = phase3_waveforms(IC_p3,tinit_p3);

% Add new waveform information to waveform vectors
time = [time;p3_waveforms(:,1)];
force = [force; p3_waveforms(:,2)];
velocity = [velocity; p3_waveforms(:,3)];
```

235
vCm = [vCm; p3_waveforms(:,4)];
qCm = [qCm; p3_waveforms(:,5)];
vpiezo = [vpiezo; p3_waveforms(:,6)];
ILPRC = [ILPRC; p3_waveforms(:,7)];
ibattery = [ibattery; p3_waveforms(:,8)];

% Calculate pgate length for this EH cycle
pgate(N+1) = p3_waveforms(length(p3_waveforms(:,1))) - p3_waveforms(1,1);

% Calculate maximum battery current - ibattery
ibattery_peak(N+1) = max(p3_waveforms(:,8));

% Calculate the energy delivered to the battery during phase 3
energy = [energy energy_calc(p3_waveforms)];

% Calculate the time where phase 3 starts in order to fine EH frequency.
time_p3_peak = [time_p3_peak tinit_p3];

% Calculate the maximum value of qCm (used to find \( \omega_{tip} \))
max_qCm(1,3) = max(p3_waveforms(:,5));

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CALCULATE VRECT_BIG AND VRECT_SMALL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if EH_cycles >=3
    vrect_big(p_counter) = max(vpiezo_peak((EH_cycles-1):EH_cycles));
    vrect_small(p_counter) = min(vpiezo_peak((EH_cycles-1):EH_cycles));
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CALCULATE ILPRC_BIG AND ILPRC_SMALL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if EH_cycles >=3
    ILPRC_big(p_counter) = max(ILPRC_peak((EH_cycles-1):EH_cycles));
    ILPRC_small(p_counter) = min(ILPRC_peak((EH_cycles-1):EH_cycles));
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CALCULATE IBATTERY_BIG AND IBATTERY_SMALL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if EH_cycles >=3
    ibattery_big(p_counter) = max(ibattery_peak((EH_cycles-1):EH_cycles));
    ibattery_small(p_counter) = min(ibattery_peak((EH_cycles-1):EH_cycles));
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% NGATE_BIG AND NGATE_SMALL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if EH_cycles >= 3
    ngate_big(p_counter) = max(ngate((EH_cycles-1):EH_cycles));
    ngate_small(p_counter) = min(ngate((EH_cycles-1):EH_cycles));
end

% PGATE_BIG AND NGATE_SMALL

if EH_cycles >= 3
    pgate_big(p_counter) = max(pgate((EH_cycles-1):EH_cycles));
    pgate_small(p_counter) = min(pgate((EH_cycles-1):EH_cycles));
end

% CALCULATE PEAK W_TIP VALUE (in [m])

w_tip(p_counter) = max(abs(max_qCm))/phi;

% CALCULATE POWER

if EH_cycles >= 3
    cycle_energy = energy(EH_cycles-1) + energy(EH_cycles);
    freq = 1/(time_p3_peak(EH_cycles) - time_p3_peak(EH_cycles-2));
    power(p_counter) = cycle_energy*freq;
end

% Close the EH_cycle progress bar
close(h)

% Increment the parametric progress bar (shows the percentage of
% parametric points have been swept)
waitbar((p_counter)/presize,g);
end

% Close the parametric sweep progress bar
close(g)
toc
parameter_saver
save beam6_sweepf_05_ideal_highRm.mat data

paramter_saver.m
% This program saves the simulation variables as a single variable

%% Save the LEM/PRC parameters

% Unreflected parameters
data.Lm = Lm;
data.Cm = Cm;
data.Rm = Rm;
data.Ceb = Ceb;
data.Re = Re;
data.LPRC = LPRC;
data.deff = deff;
data.fn = fn;
data.am = am;
data.Vbatt = Vbatt;

% Parasitics in the PRC
data.R2 = R2;
data.R3 = R3;
data.Vdiode2 = Vdiode2;
data.Vdiode3 = Vdiode3;

% Reflected parameters
data.Fmag = Fmag;
data.phi = phi;
data.Lm_ref = Lm_ref;
data.Cm_ref = Cm_ref;
data.Rm_ref = Rm_ref;
data.Fmag_ref = Fmag_ref;

% Sweep parameters
data.EH_cycles = EH_cycles;
data.N_points = N_points;

%% Save the waveform data
data.vrect_big = vrect_big;
data.vrect_small = vrect_small;
data.power = power;
data.range = range;
data.iLPRC_big = iLPRC_big;
data.iLPRC_small = iLPRC_small;
data.ngate_big = ngate_big;
data.ngate_small = ngate_small;
data.pgate_big = pgate_big;
data.pgate_small = pgate_small;
data.ibattery_big = ibattery_big;
data.ibattery_small = ibattery_small;
data.Rm_ref = Rm_ref;
data.w_tip = w_tip;
data.vss = vss;
APPENDIX E
COMPLETE TRANSDUCER CHARACTERIZATION RESULTS

This appendix provides a complete account for all of the transducer characterization experimental data used to extract the LEM parameters for beams 1-3. This includes the mechanical, electrical, and electromechanical characterization experiments.

Mechanical Characterization

In order to extract the mechanical domain LEM parameters, which includes $C_{ms}$, $M_m$, and $R_m$, two experiments were carried out for each beam. The first experiment was a static compliance extraction, described in Chapter 7.1.1, and the second was the dynamic mechanical characterization experiment, described in Chapter 7.1.2.

Static Compliance Extraction

For the static compliance extraction experiment, a tip force was applied to the transducer beam and the resulting tip displacement was measured. Tip displacement was plotted as a function of force, and the effective compliance was calculated as the slope of a linear best-fit line of the data. The tip force was realized by hanging a scale-pan at the tip of the beam and adding coins with known masses to the scale-pan. The masses of the coins used for this experiment is provided in Table E-1. For each of the three beams, the test was repeated several times and a mean value was taken.

Table E-1. Masses of the coins used to extract the compliance.

<table>
<thead>
<tr>
<th>Penny</th>
<th>Mass [g]</th>
<th>Dime</th>
<th>Mass [g]</th>
<th>Washer</th>
<th>Mass [g]</th>
<th>Quarter</th>
<th>Mass [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>2.508</td>
<td>D1</td>
<td>2.279</td>
<td>W1</td>
<td>0.613</td>
<td>Q1</td>
<td>5.693</td>
</tr>
<tr>
<td>P2</td>
<td>2.500</td>
<td>D2</td>
<td>2.282</td>
<td>W2</td>
<td>0.603</td>
<td>Q1</td>
<td>5.734</td>
</tr>
<tr>
<td>P3</td>
<td>2.470</td>
<td>D3</td>
<td>2.285</td>
<td>W3</td>
<td>0.567</td>
<td>Q1</td>
<td>5.721</td>
</tr>
<tr>
<td>P4</td>
<td>2.505</td>
<td>D4</td>
<td>2.282</td>
<td>W4</td>
<td>0.636</td>
<td>Q1</td>
<td>5.670</td>
</tr>
<tr>
<td>P5</td>
<td>2.465</td>
<td>D5</td>
<td>2.256</td>
<td>W5</td>
<td>0.552</td>
<td>Q1</td>
<td>5.704</td>
</tr>
</tbody>
</table>

The results of the static compliance measurements for beam 1 are presented in Table E-2 through Table E-4. A plot of tip displacement vs. force for each of the three tests is shown in Figure E-1.
Table E-2. Test 1 - Static compliance measurement of beam 1.

<table>
<thead>
<tr>
<th>Mass Objects</th>
<th>Actual Mass [g]</th>
<th>Absolute Disp. [m]</th>
<th>Force [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>P5</td>
<td>2.465</td>
<td>0.00036</td>
<td>0.024157</td>
</tr>
<tr>
<td>P4,P5</td>
<td>4.970</td>
<td>0.00074</td>
<td>0.048706</td>
</tr>
<tr>
<td>P3-P5</td>
<td>7.440</td>
<td>0.00111</td>
<td>0.072912</td>
</tr>
<tr>
<td>P2-P5</td>
<td>9.940</td>
<td>0.00149</td>
<td>0.097412</td>
</tr>
<tr>
<td>P1-P5</td>
<td>12.448</td>
<td>0.00190</td>
<td>0.121990</td>
</tr>
<tr>
<td>P1-P5,D1</td>
<td>14.727</td>
<td>0.00228</td>
<td>0.144325</td>
</tr>
<tr>
<td>P1-P5,D1,D2</td>
<td>17.009</td>
<td>0.00265</td>
<td>0.166689</td>
</tr>
<tr>
<td>P1-P5,D1-D3</td>
<td>19.294</td>
<td>0.00303</td>
<td>0.189081</td>
</tr>
<tr>
<td>P1-P5,D1-D4</td>
<td>21.576</td>
<td>0.00341</td>
<td>0.211445</td>
</tr>
<tr>
<td>P1-P5,D1-D5</td>
<td>23.832</td>
<td>0.00379</td>
<td>0.233554</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1</td>
<td>24.445</td>
<td>0.00388</td>
<td>0.239561</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1,W2</td>
<td>25.048</td>
<td>0.00399</td>
<td>0.245470</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1-W3</td>
<td>25.615</td>
<td>0.00408</td>
<td>0.251027</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1-W4</td>
<td>26.251</td>
<td>0.00420</td>
<td>0.257260</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1-W5</td>
<td>26.803</td>
<td>0.00428</td>
<td>0.262669</td>
</tr>
</tbody>
</table>

Table E-3. Test 2 - Static compliance measurement of beam 1.

<table>
<thead>
<tr>
<th>Mass Objects</th>
<th>Actual Mass [g]</th>
<th>Absolute Disp. [m]</th>
<th>Force [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>0.613</td>
<td>0.00009</td>
<td>0.006007</td>
</tr>
<tr>
<td>W1,D1</td>
<td>2.892</td>
<td>0.00042</td>
<td>0.028342</td>
</tr>
<tr>
<td>W1,D1,D2</td>
<td>5.174</td>
<td>0.00076</td>
<td>0.050705</td>
</tr>
<tr>
<td>W1,D1-D3</td>
<td>7.449</td>
<td>0.00112</td>
<td>0.073000</td>
</tr>
<tr>
<td>W1,D1,D4</td>
<td>9.731</td>
<td>0.00147</td>
<td>0.095364</td>
</tr>
<tr>
<td>W1,D1-D5</td>
<td>11.987</td>
<td>0.00183</td>
<td>0.117473</td>
</tr>
<tr>
<td>W1,D1-D5,P1</td>
<td>14.495</td>
<td>0.00224</td>
<td>0.142051</td>
</tr>
<tr>
<td>W1,D1-D5,P1,P2</td>
<td>16.995</td>
<td>0.00265</td>
<td>0.166551</td>
</tr>
<tr>
<td>W1,D1-D5,P1-P3</td>
<td>19.465</td>
<td>0.00307</td>
<td>0.190757</td>
</tr>
<tr>
<td>W1,D1-D5,P1-P4</td>
<td>21.970</td>
<td>0.00348</td>
<td>0.215306</td>
</tr>
<tr>
<td>W1,D1-D5,P1-P5</td>
<td>24.435</td>
<td>0.00387</td>
<td>0.239463</td>
</tr>
</tbody>
</table>

Table E-4. Test 3 - Static compliance measurement of beam 1.

<table>
<thead>
<tr>
<th>Mass Objects</th>
<th>Actual Mass [g]</th>
<th>Absolute Disp. [m]</th>
<th>Force [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>5.693</td>
<td>0.00085</td>
<td>0.055791</td>
</tr>
<tr>
<td>Q1,Q2</td>
<td>11.427</td>
<td>0.00174</td>
<td>0.111985</td>
</tr>
<tr>
<td>Q1-Q3</td>
<td>17.148</td>
<td>0.00269</td>
<td>0.168050</td>
</tr>
<tr>
<td>Q1-Q4</td>
<td>22.818</td>
<td>0.00360</td>
<td>0.223616</td>
</tr>
<tr>
<td>Q1-Q5</td>
<td>28.522</td>
<td>0.00455</td>
<td>0.279516</td>
</tr>
<tr>
<td>Q1-Q5,P1</td>
<td>31.030</td>
<td>0.00498</td>
<td>0.304094</td>
</tr>
<tr>
<td>Q1-Q5,P1,P2</td>
<td>33.530</td>
<td>0.00543</td>
<td>0.328594</td>
</tr>
</tbody>
</table>
The results of the static compliance measurements for beam 2 are presented in Table E-5 through E-8. A plot of tip displacement vs. force for each of the four tests is shown in Figure E-2.

Table E-5. Test 1 - Static compliance measurement of beam 2.

<table>
<thead>
<tr>
<th>Mass Objects</th>
<th>Actual Mass [g]</th>
<th>Absolute Disp. [m]</th>
<th>Force [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>P5</td>
<td>2.465</td>
<td>0.00022</td>
<td>0.024157</td>
</tr>
<tr>
<td>P4,P5</td>
<td>4.970</td>
<td>0.00045</td>
<td>0.048706</td>
</tr>
<tr>
<td>P3-P5</td>
<td>7.440</td>
<td>0.00073</td>
<td>0.072912</td>
</tr>
<tr>
<td>P2-P5</td>
<td>9.940</td>
<td>0.00096</td>
<td>0.097412</td>
</tr>
<tr>
<td>P1-P5</td>
<td>12.448</td>
<td>0.00120</td>
<td>0.121990</td>
</tr>
<tr>
<td>P1-P5,D1</td>
<td>14.727</td>
<td>0.00144</td>
<td>0.144325</td>
</tr>
<tr>
<td>P1-P5,D1,D2</td>
<td>17.009</td>
<td>0.00165</td>
<td>0.166688</td>
</tr>
<tr>
<td>P1-P5,D1-D3</td>
<td>19.294</td>
<td>0.00190</td>
<td>0.189081</td>
</tr>
<tr>
<td>P1-P5,D1-D4</td>
<td>21.576</td>
<td>0.00213</td>
<td>0.211445</td>
</tr>
<tr>
<td>P1-P5,D1-D5</td>
<td>23.832</td>
<td>0.00238</td>
<td>0.233554</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1</td>
<td>24.445</td>
<td>0.00244</td>
<td>0.239561</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1,W2</td>
<td>25.048</td>
<td>0.00250</td>
<td>0.245470</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1-W3</td>
<td>25.615</td>
<td>0.00256</td>
<td>0.251027</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1-W4</td>
<td>26.251</td>
<td>0.00263</td>
<td>0.257260</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1-W5</td>
<td>26.803</td>
<td>0.00270</td>
<td>0.262669</td>
</tr>
</tbody>
</table>
Table E-6. Test 2 - Static compliance measurement of beam 2.

<table>
<thead>
<tr>
<th>Mass Objects</th>
<th>Actual Mass [g]</th>
<th>Absolute Disp. [m]</th>
<th>Force [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>0.613</td>
<td>0.00030</td>
<td>0.006007</td>
</tr>
<tr>
<td>W1,D1</td>
<td>2.892</td>
<td>0.00022</td>
<td>0.028342</td>
</tr>
<tr>
<td>W1,D1,D2</td>
<td>5.174</td>
<td>0.00044</td>
<td>0.050705</td>
</tr>
<tr>
<td>W1,D1-D3</td>
<td>7.449</td>
<td>0.00066</td>
<td>0.073000</td>
</tr>
<tr>
<td>W1,D1,D4</td>
<td>9.731</td>
<td>0.00090</td>
<td>0.095364</td>
</tr>
<tr>
<td>W1,D1-D5</td>
<td>11.987</td>
<td>0.00112</td>
<td>0.117473</td>
</tr>
<tr>
<td>W1,D1-D5,P1</td>
<td>14.495</td>
<td>0.00137</td>
<td>0.142051</td>
</tr>
<tr>
<td>W1,D1-D5,P1,P2</td>
<td>16.995</td>
<td>0.00162</td>
<td>0.166551</td>
</tr>
<tr>
<td>W1,D1-D5,P1-P3</td>
<td>19.465</td>
<td>0.00187</td>
<td>0.190757</td>
</tr>
<tr>
<td>W1,D1-D5,P1-P4</td>
<td>21.970</td>
<td>0.00214</td>
<td>0.215306</td>
</tr>
<tr>
<td>W1,D1-D5,P1-P5</td>
<td>24.435</td>
<td>0.00238</td>
<td>0.239463</td>
</tr>
</tbody>
</table>

Table E-7. Test 3 - Static compliance measurement of beam 2.

<table>
<thead>
<tr>
<th>Mass Objects</th>
<th>Actual Mass [g]</th>
<th>Absolute Disp. [m]</th>
<th>Force [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>5.693</td>
<td>0.00045</td>
<td>0.055791</td>
</tr>
<tr>
<td>Q1,Q2</td>
<td>11.427</td>
<td>0.00102</td>
<td>0.111985</td>
</tr>
<tr>
<td>Q1-Q3</td>
<td>17.148</td>
<td>0.00158</td>
<td>0.168050</td>
</tr>
<tr>
<td>Q1-Q4</td>
<td>22.818</td>
<td>0.00216</td>
<td>0.223616</td>
</tr>
<tr>
<td>Q1-Q5</td>
<td>28.522</td>
<td>0.00275</td>
<td>0.279516</td>
</tr>
<tr>
<td>Q1-Q5,P1</td>
<td>31.030</td>
<td>0.00302</td>
<td>0.304094</td>
</tr>
<tr>
<td>Q1-Q5,P1,P2</td>
<td>33.530</td>
<td>0.00330</td>
<td>0.328594</td>
</tr>
<tr>
<td>Q1-Q5,P1-P3</td>
<td>36.000</td>
<td>0.00356</td>
<td>0.352800</td>
</tr>
<tr>
<td>Q1-Q5,P1-P4</td>
<td>38.505</td>
<td>0.00382</td>
<td>0.377349</td>
</tr>
<tr>
<td>Q1-Q5,P1-P5</td>
<td>40.970</td>
<td>0.00408</td>
<td>0.401506</td>
</tr>
<tr>
<td>Q1-Q5,P1-P5,D1</td>
<td>43.249</td>
<td>0.00433</td>
<td>0.423840</td>
</tr>
<tr>
<td>Q1-Q5,P1-P5,D1,D2</td>
<td>45.531</td>
<td>0.00455</td>
<td>0.446204</td>
</tr>
<tr>
<td>Q1-Q5,P1-P5,D1-D3</td>
<td>47.816</td>
<td>0.00482</td>
<td>0.468597</td>
</tr>
<tr>
<td>Q1-Q5,P1-P5,D1-D4</td>
<td>50.098</td>
<td>0.00505</td>
<td>0.490960</td>
</tr>
<tr>
<td>Q1-Q5,P1-P5,D1-D5</td>
<td>52.354</td>
<td>0.00342</td>
<td>0.513069</td>
</tr>
</tbody>
</table>

Table E-8. Test 4 - Static compliance measurement of beam 2.

<table>
<thead>
<tr>
<th>Mass Objects</th>
<th>Actual Mass [g]</th>
<th>Absolute Disp. [m]</th>
<th>Force [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>0.613</td>
<td>0.00006</td>
<td>0.006007</td>
</tr>
<tr>
<td>W1,W2</td>
<td>1.216</td>
<td>0.00013</td>
<td>0.011917</td>
</tr>
<tr>
<td>W1-W3</td>
<td>1.783</td>
<td>0.00018</td>
<td>0.017473</td>
</tr>
<tr>
<td>W1-W4</td>
<td>2.419</td>
<td>0.00025</td>
<td>0.023706</td>
</tr>
<tr>
<td>W1-W5</td>
<td>2.971</td>
<td>0.00032</td>
<td>0.029116</td>
</tr>
<tr>
<td>W1-W5,D1</td>
<td>4.637</td>
<td>0.00057</td>
<td>0.045443</td>
</tr>
<tr>
<td>W1-W5,D1,D2</td>
<td>6.316</td>
<td>0.00081</td>
<td>0.061897</td>
</tr>
</tbody>
</table>
The results of the static compliance measurements for beam 2 are presented in Table E-9 through Table E-11. A plot of tip displacement vs. force for each of the four tests is shown in Figure E-3.
Table E-9. Test 1 - Static compliance measurement of beam 3.

<table>
<thead>
<tr>
<th>Mass Objects</th>
<th>Actual Mass [g]</th>
<th>Absolute Disp. [m]</th>
<th>Force [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>P5</td>
<td>2.465</td>
<td>0.00013</td>
<td>0.024157</td>
</tr>
<tr>
<td>P4,P5</td>
<td>4.970</td>
<td>0.00026</td>
<td>0.048706</td>
</tr>
<tr>
<td>P3-P5</td>
<td>7.440</td>
<td>0.00038</td>
<td>0.072912</td>
</tr>
<tr>
<td>P2-P5</td>
<td>9.940</td>
<td>0.00051</td>
<td>0.097412</td>
</tr>
<tr>
<td>P1-P5</td>
<td>12.448</td>
<td>0.00063</td>
<td>0.121990</td>
</tr>
<tr>
<td>P1-P5,D1</td>
<td>14.727</td>
<td>0.00075</td>
<td>0.144325</td>
</tr>
<tr>
<td>P1-P5,D1,D2</td>
<td>17.009</td>
<td>0.00087</td>
<td>0.166688</td>
</tr>
<tr>
<td>P1-P5,D1-D3</td>
<td>19.294</td>
<td>0.00099</td>
<td>0.189081</td>
</tr>
<tr>
<td>P1-P5,D1-D4</td>
<td>21.576</td>
<td>0.00111</td>
<td>0.211445</td>
</tr>
<tr>
<td>P1-P5,D1-D5</td>
<td>23.832</td>
<td>0.00122</td>
<td>0.233554</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1</td>
<td>24.445</td>
<td>0.00125</td>
<td>0.239561</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1,W2</td>
<td>25.048</td>
<td>0.00128</td>
<td>0.245470</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1-W3</td>
<td>25.615</td>
<td>0.00132</td>
<td>0.251027</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1-W4</td>
<td>26.251</td>
<td>0.00135</td>
<td>0.257260</td>
</tr>
<tr>
<td>P1-P5,D1-D5,W1-W5</td>
<td>26.803</td>
<td>0.00138</td>
<td>0.262669</td>
</tr>
</tbody>
</table>

Table E-10. Test 2 - Static compliance measurement of beam 3.

<table>
<thead>
<tr>
<th>Mass Objects</th>
<th>Actual Mass [g]</th>
<th>Absolute Disp. [m]</th>
<th>Force [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1,D1</td>
<td>2.892</td>
<td>0.00013</td>
<td>0.028342</td>
</tr>
<tr>
<td>W1,D1,D2</td>
<td>5.174</td>
<td>0.00024</td>
<td>0.050705</td>
</tr>
<tr>
<td>W1,D1-D3</td>
<td>7.449</td>
<td>0.00035</td>
<td>0.073000</td>
</tr>
<tr>
<td>W1,D1,D4</td>
<td>9.731</td>
<td>0.00046</td>
<td>0.095364</td>
</tr>
<tr>
<td>W1,D1-D5</td>
<td>11.987</td>
<td>0.00057</td>
<td>0.117473</td>
</tr>
<tr>
<td>W1,D1-D5,P1</td>
<td>14.495</td>
<td>0.00070</td>
<td>0.142051</td>
</tr>
<tr>
<td>W1,D1-D5,P1,P2</td>
<td>16.995</td>
<td>0.00082</td>
<td>0.166551</td>
</tr>
<tr>
<td>W1,D1-D5,P1-P3</td>
<td>19.465</td>
<td>0.00096</td>
<td>0.190757</td>
</tr>
<tr>
<td>W1,D1-D5,P1-P4</td>
<td>21.970</td>
<td>0.00108</td>
<td>0.215306</td>
</tr>
<tr>
<td>W1,D1-D5,P1-P5</td>
<td>24.435</td>
<td>0.00120</td>
<td>0.239463</td>
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</tbody>
</table>

Table E-11. Test 3 - Static compliance measurement of beam 3.

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<thead>
<tr>
<th>Mass Objects</th>
<th>Actual Mass [g]</th>
<th>Absolute Disp. [m]</th>
<th>Force [N]</th>
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</thead>
<tbody>
<tr>
<td>Q1</td>
<td>5.693</td>
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<td>0.055791</td>
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<tr>
<td>Q1,Q2</td>
<td>11.427</td>
<td>0.00055</td>
<td>0.111985</td>
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<tr>
<td>Q1-Q3</td>
<td>17.148</td>
<td>0.00084</td>
<td>0.168050</td>
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<tr>
<td>Q1-Q4</td>
<td>22.818</td>
<td>0.00112</td>
<td>0.223616</td>
</tr>
<tr>
<td>Q1-Q5</td>
<td>28.522</td>
<td>0.00141</td>
<td>0.279516</td>
</tr>
<tr>
<td>Q1-Q5,P1</td>
<td>31.030</td>
<td>0.00155</td>
<td>0.304094</td>
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<tr>
<td>Q1-Q5,P1,P2</td>
<td>33.530</td>
<td>0.00168</td>
<td>0.328594</td>
</tr>
<tr>
<td>Q1-Q5,P1-P3</td>
<td>36.000</td>
<td>0.00183</td>
<td>0.352800</td>
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Table E-11. Continued

<table>
<thead>
<tr>
<th>Mass Objects</th>
<th>Actual Mass [g]</th>
<th>Absolute Disp. [m]</th>
<th>Force [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1-Q5,P1-P4</td>
<td>38.505</td>
<td>0.00196</td>
<td>0.377349</td>
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<td>Q1-Q5,P1-P5</td>
<td>40.970</td>
<td>0.00207</td>
<td>0.401506</td>
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<td>43.249</td>
<td>0.00220</td>
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<tr>
<td>Q1-Q5,P1-P5,D1,D2</td>
<td>45.531</td>
<td>0.00231</td>
<td>0.446204</td>
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<tr>
<td>Q1-Q5,P1-P5,D1-D3</td>
<td>47.816</td>
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<td>Q1-Q5,P1-P5,D1-D4</td>
<td>50.098</td>
<td>0.00257</td>
<td>0.490960</td>
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<tr>
<td>Q1-Q5,P1-P5,D1-D5</td>
<td>52.354</td>
<td>0.00270</td>
<td>0.513069</td>
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</table>

Figure E-3. Results of the three static compliance extractions for beam 3.

**Dynamic Mechanical Characterization**

The dynamic mechanical characterization was used to extract the values of $M_m$ and $R_m$. For these experiments, a mechanical excitation was applied to the base of the beam and the resulting tip displacement was measured. The measured results were used to
calculate a transfer function between the tip displacement and base acceleration. The measured transfer function was compared to the theoretical transfer function, which assumed a SDOF mechanical system, and curve fitting techniques were used to extract $M_m$ and $R_m$. Both the experimentally measured TF and fitted data for the dynamic mechanical characterization of beams 1-3 are shown in Figures E-4 through E-6. The 95% confidence bounds for each fitted parameter are included in the figures.

**Figure E-4.** Experimental short-circuit TF with best-fit line for beam 1.

**Figure E-5.** Experimental short-circuit TF with best-fit line for beam 2.
Electrical Characterization

The electrical domain LEM parameters, $C_{eb}$ and $R_e$, were extracted by mechanically blocking the motion of the transducer beam and measuring the electrical impedance with an impedance analyzer. A more detailed explanation of the electrical characterization was presented in Chapter 7.2. The results of the electrical characterization for beams 1-3 are shown in Figure E-7, Figure E-8, and Figure E-9, respectively.
Electromechanical Characterization

The effective piezoelectric coefficient, $d_{eff}$, was found for each beam using the electromechanical characterization experiment described in Chapter 7.3. In this experiment, a voltage was applied to the transducer and the resulting tip displacement was measured. The tip displacement as a function of applied voltage was then plotted and the value of
$d_{eff}$ was calculated as the slope of the resulting curve line. The experimental data used to calculate $d_{eff}$ for beams 1-3 is shown in Table E-12, and the resulting curves are shown Figures E-10 through E-12.

Table E-12. Beam 1 data from electromechanical characterization.

<table>
<thead>
<tr>
<th>Voltage [V] pk-to-pk</th>
<th>Tip Disp. [m] pk-to-pk</th>
<th>Voltage [V] pk-to-pk</th>
<th>Tip Disp. [m] pk-to-pk</th>
<th>Voltage [V] pk-to-pk</th>
<th>Tip Disp. [m] pk-to-pk</th>
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<td>2.64</td>
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<td>0.00000478</td>
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<td>2.47</td>
<td>0.00000736</td>
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<td>0.00000476</td>
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<td>3.30</td>
<td>0.00000536</td>
<td>3.28</td>
<td>0.00000972</td>
<td>3.28</td>
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<tr>
<td>3.72</td>
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<td>4.11</td>
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<td>4.12</td>
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<td>4.95</td>
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<td>4.92</td>
<td>0.00000984</td>
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<tr>
<td>4.52</td>
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<td>0.00001680</td>
<td>5.76</td>
<td>0.00001168</td>
</tr>
<tr>
<td>4.72</td>
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<td>6.57</td>
<td>0.00001960</td>
<td>6.58</td>
<td>0.00001304</td>
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<tr>
<td>4.92</td>
<td>0.00000788</td>
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<td>0.00002120</td>
<td>7.40</td>
<td>0.00001510</td>
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<td>5.36</td>
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<td>0.00001650</td>
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<td>9.08</td>
<td>0.00001870</td>
</tr>
<tr>
<td>6.16</td>
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<td>9.93</td>
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<td>9.92</td>
<td>0.00002020</td>
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<tr>
<td>6.60</td>
<td>0.00001124</td>
<td>10.74</td>
<td>0.00002920</td>
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<td>0.00002200</td>
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<tr>
<td>6.72</td>
<td>0.00001152</td>
<td>11.58</td>
<td>0.00003140</td>
<td>11.55</td>
<td>0.00002370</td>
</tr>
<tr>
<td>7.12</td>
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<td>12.42</td>
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<td>12.38</td>
<td>0.00002580</td>
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<tr>
<td>7.40</td>
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<tr>
<td>7.80</td>
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<td>14.07</td>
<td>0.00003910</td>
<td>14.04</td>
<td>0.00002890</td>
</tr>
</tbody>
</table>

$R^2 = 0.9939$

$Slope = 1.646 \times 10^{-6} \text{ [m/V]}$

Figure E-10. Electromechanical characterization results for beam 1.
Figure E-11. Electromechanical characterization results for beam 2.

Figure E-12. Electromechanical characterization results for beam 3.
APPENDIX F
COMPLETE EXPERIMENTAL DATA RESULTS

In Chapter 8, selected results were presented to demonstrate the characterization of the PRC-based energy harvesting system for two of the three transducer beams. In this appendix, a complete account of the experimental results for all three of the transducer beams is presented.

**Beam 1 - 0.4 m/s² - Frequency Variation**

![Graph showing power vs. frequency for beam 1 with labels for Full LEM/Lossless PRC (0 Lag), Resonant LEM/Lossless PRC (0 Lag), Full LEM/Lossless PRC (168 ns Lag), Resonant LEM/Lossless PRC (168 ns Lag), and Non-Ideal PRC (168 ns Lag). The figure also includes labels for f_sc = 59.14 Hz and f_oc = 59.47 Hz.]

Figure F-1. Power vs. frequency for beam 1; a_m = 0.4 m/s²; L_{PRC} = 100 μH.
Figure F-2. $v_{\text{rectPeak}}$ vs. frequency for beam 1; $a_m = 0.4 \, \text{m/s}^2$; $L_{\text{PRC}} = 100 \, \mu\text{H}$.

Figure F-3. $i_{\text{LPRCPeak}}$ vs. frequency for beam 1; $a_m = 0.4 \, \text{m/s}^2$; $L_{\text{PRC}} = 100 \, \mu\text{H}$.
Figure F-4. $i_{\text{batteryPeak}}$ vs. frequency for beam 1; $a_m = 0.4 \, m/s^2$; $L_{PRC} = 100 \, \mu H$.

Figure F-5. $t_{N\text{Gate}}$ vs. frequency for beam 1; $a_m = 0.4 \, m/s^2$; $L_{PRC} = 100 \, \mu H$. 

$\mu = 1.63 \, \mu s$ (Exp. Data Mean)

$\sigma = 48.20 \, ns$ (Exp. Data Std. Dev.)
Figure F-6. $t_{PGate}$ vs. frequency for beam 1; $a_m = 0.4 \text{ m/s}^2$; $L_{PRC} = 100 \mu H$.

Beam 1 - 0.5 $\text{ m/s}^2$ - Frequency Variation

Figure F-7. Power vs. frequency for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 100 \mu H$. 
Figure F-8. $v_{\text{rectPeak}}$ vs. frequency for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{\text{PRC}} = 100 \mu\text{H}$.

Figure F-9. $i_{\text{LPRCPeak}}$ vs. frequency for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{\text{PRC}} = 100 \mu\text{H}$.
Figure F-10. $i_{\text{batteryPeak}}$ vs. frequency for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{P RC} = 100 \mu\text{H}$.

Figure F-11. $t_{\text{NGate}}$ vs. frequency for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{P RC} = 100 \mu\text{H}$.
Figure F-12. $t_{PGate}$ vs. frequency for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 100 \text{ µH}$.

Beam 1 - 0.5 m/s² - Variation of PRC Conduction Losses

Figure F-13. Power vs. $R_{Phase2}$ for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 100 \text{ µH}$; $f = 59 \text{ Hz}$. 

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Figure F-14. Power vs. $R_{\text{Phase3}}$ for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{\text{PRC}} = 100 \text{ } \mu\text{H}$; $f = 59$ Hz.

**Beam 1 - 0.5 m/s$^2$ - Effects of Timing Offsets**

Figure F-15. Power vs. $t_{\text{NGate}}$ for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{\text{PRC}} = 100 \text{ } \mu\text{H}$; $f = 59$ Hz.
Beam 1 - 0.6 m/s² - Frequency Variation

Figure F-16. Power vs. frequency for beam 1; \( a_m = 0.6 \text{ m/s}^2 \); \( L_{PRC} = 100 \mu H \).

Figure F-17. \( v_{rectPeak} \) vs. frequency for beam 1; \( a_m = 0.6 \text{ m/s}^2 \); \( L_{PRC} = 100 \mu H \).
Figure F-18. $i_{LPRCpeak}$ vs. frequency for beam 1; $a_m = 0.6 \text{ m/s}^2$; $L_{PRC} = 100 \mu\text{H}$.

Figure F-19. $i_{batteryPeak}$ vs. frequency for beam 1; $a_m = 0.6 \text{ m/s}^2$; $L_{PRC} = 100 \mu\text{H}$. 
Figure F-20. $t_{NGate}$ vs. frequency for beam 1; $a_m = 0.6 \ m/s^2$; $L_{PRC} = 100 \ \mu H$.

Figure F-21. $t_{PGate}$ vs. frequency for beam 1; $a_m = 0.6 \ m/s^2$; $L_{PRC} = 100 \ \mu H$. 

$\mu = 1.62 \ \mu s$ (Exp. Data Mean) 
$\sigma = 49.00 \ n s$ (Exp. Data Std. Dev.) 
$f_{sc} = 59.14 Hz$ 
$f_{oc} = 59.47 Hz$
Figure F-22. Power vs. frequency for beam 2; $a_m = 0.5 \, m/s^2$; $L_{PRC} = 270 \, \mu H$.

Figure F-23. $v_{rectPeak}$ vs. frequency for beam 2; $a_m = 0.5 \, m/s^2$; $L_{PRC} = 270 \, \mu H$. 

Beam 2 - 0.5 m/$s^2$ - Frequency Variation
Figure F-24. $i_{LP\text{RCPeak}}$ vs. frequency for beam 2; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$.

Figure F-25. $i_{batteryPeak}$ vs. frequency for beam 2; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$. 
Figure F-26. $t_{NGate}$ vs. frequency for beam 2; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$.

Figure F-27. $t_{PGate}$ vs. frequency for beam 2; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$. 
Figure F-28. $w_{tip}$ vs. frequency for beam 2; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$.

**Beam 2 - 0.5 m/s$^2$ - Variation of PRC Losses**

Figure F-29. Power vs. $R_{Phase2}$ for beam 1; $a_m = 0.5 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$; $f = 63.5 \text{ Hz}$.
Figure F-30. Power vs. $R_{Phase^3}$ for beam 1; $a_m = 0.5 \, m/s^2$; $L_{PRC} = 270 \, \mu H$; $f = 63.5 \, Hz$.

**Beam 2 - 0.5 $m/s^2$ - Effects of Timing Offsets**

Figure F-31. Power vs. $t_{NGate}$ for beam 1; $a_m = 0.5 \, m/s^2$; $L_{PRC} = 270 \, \mu H$; $f = 63.5 \, Hz$. 
Beam 2 - 0.6 m/s² - Frequency Variation

Figure F-32. Power vs. frequency for beam 2; $a_m = 0.6$ m/s²; $L_{PRC} = 270 \mu H$.

Figure F-33. $v_{rectPeak}$ vs. frequency for beam 2; $a_m = 0.6$ m/s²; $L_{PRC} = 270 \mu H$. 
Figure F-34. $i_{LPRCPeak}$ vs. frequency for beam 2; $a_m = 0.6 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$.

Figure F-35. $i_{batteryPeak}$ vs. frequency for beam 2; $a_m = 0.6 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$. 

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Figure F-36. $t_{NGate}$ vs. frequency for beam 2; $a_m = 0.6 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$.

Figure F-37. $t_{PGate}$ vs. frequency for beam 2; $a_m = 0.6 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$. 
Figure F-38. \( w_{\text{tip}} \) vs. frequency for beam 2; \( a_m = 0.6 \text{ m/s}^2 \); \( L_{PRC} = 270 \text{ \mu H} \).

**Beam 2 - 0.7 m/s\(^2\) - Frequency Variation**

Figure F-39. Power vs. frequency for beam 2; \( a_m = 0.7 \text{ m/s}^2 \); \( L_{PRC} = 270 \text{ \mu H} \).
Figure F-40. $v_{rectPeak}$ vs. frequency for beam 2; $a_m = 0.7 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$.

Figure F-41. $i_{LPRCPeak}$ vs. frequency for beam 2; $a_m = 0.7 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$. 
Figure F-42. $i_{\text{batteryPeak}}$ vs. frequency for beam 2; $a_m = 0.7 \text{ m/s}^2$; $L_{\text{PRC}} = 270 \mu H$.

Figure F-43. $t_{\text{NGate}}$ vs. frequency for beam 2; $a_m = 0.7 \text{ m/s}^2$; $L_{\text{PRC}} = 270 \mu H$. 
Figure F-44. $t_{\text{PGate}}$ vs. frequency for beam 2; $a_m = 0.7 \text{ m/s}^2$; $L_{\text{PRC}} = 270 \mu\text{H}$.

Figure F-45. $w_{\text{tip}}$ vs. frequency for beam 2; $a_m = 0.7 \text{ m/s}^2$; $L_{\text{PRC}} = 270 \mu\text{H}$. 
Beam 3 - 0.9 $m/s^2$ - Frequency Variation

![Graph showing power vs frequency for beam 3; $a_m = 0.9 \, m/s^2$; $L_{PRC} = 270 \, \mu H$.](image)

Figure F-46. Power vs. frequency for beam 3; $a_m = 0.9 \, m/s^2$; $L_{PRC} = 270 \, \mu H$.

![Graph showing $v_{rectPeak}$ vs frequency for beam 3; $a_m = 0.9 \, m/s^2$; $L_{PRC} = 270 \, \mu H$.](image)

Figure F-47. $v_{rectPeak}$ vs. frequency for beam 3; $a_m = 0.9 \, m/s^2$; $L_{PRC} = 270 \, \mu H$. 

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Figure F-48. $i_{LPRC_{\text{Peak}}}$ vs. frequency for beam 3; $a_m = 0.9 \text{ m/s}^2$; $L_{PRC} = 270 \mu\text{H}$.

Figure F-49. $i_{\text{battery}_{\text{Peak}}}$ vs. frequency for beam 3; $a_m = 0.9 \text{ m/s}^2$; $L_{PRC} = 270 \mu\text{H}$.
Figure F-50. $t_{NGate}$ vs. frequency for beam 3; $a_m = 0.9 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$.

Figure F-51. $t_{PGate}$ vs. frequency for beam 3; $a_m = 0.9 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$. 
**Beam 3 - 1.4 m/s² - Frequency Variation**

Figure F-52. $w_{tip}$ vs. frequency for beam 3; $a_m = 0.9 \ m/s^2$; $L_{PRC} = 270 \ \mu H$.

Figure F-53. Power vs. frequency for beam 3; $a_m = 1.4 \ m/s^2$; $L_{PRC} = 270 \ \mu H$. 
Figure F-54. $v_{\text{rectPeak}}$ vs. frequency for beam 3; $a_m = 1.4 \text{ m/s}^2$; $L_{\text{PRC}} = 270 \text{ } \mu\text{H}$.

Figure F-55. $i_{L_{\text{PRC}}\text{Peak}}$ vs. frequency for beam 3; $a_m = 1.4 \text{ m/s}^2$; $L_{\text{PRC}} = 270 \text{ } \mu\text{H}$.
Figure F-56. $i_{\text{batteryPeak}}$ vs. frequency for beam 3; $a_m = 1.4 \, m/s^2$; $L_{PRC} = 270 \, \mu H$.

Figure F-57. $t_{\text{NGate}}$ vs. frequency for beam 3; $a_m = 1.4 \, m/s^2$; $L_{PRC} = 270 \, \mu H$. 
Figure F-58. $t_{PGate}$ vs. frequency for beam 3; $a_m = 1.4 \text{ m/s}^2$; $L_{PRC} = 270 \mu\text{H}$.

Figure F-59. $w_{tip}$ vs. frequency for beam 3; $a_m = 1.4 \text{ m/s}^2$; $L_{PRC} = 270 \mu\text{H}$.
Beam 3 - 1.4 $m/s^2$ - Variation of PRC Losses

Figure F-60. Power vs. $R_{\text{Phase2}}$ for beam 3; $a_m = 1.4 \, m/s^2$; $L_{\text{PRC}} = 270 \, \mu H$; $f = 123 \, Hz$.

Figure F-61. Power vs. $R_{\text{Phase3}}$ for beam 3; $a_m = 1.4 \, m/s^2$; $L_{\text{PRC}} = 270 \, \mu H$; $f = 123 \, Hz$. 
Beam 3 - 1.4 m/s² - Effects of Timing Offsets

Figure F-62. Power vs. \( t_{\text{NGate}} \) for beam 3; \( a_m = 1.4 \) m/s²; \( L_{\text{PRC}} = 270 \) µH; \( f = 123 \) Hz.

Beam 3 - 1.6 m/s² - Frequency Variation

Figure F-63. Power vs. frequency for beam 3; \( a_m = 1.6 \) m/s²; \( L_{\text{PRC}} = 270 \) µH.
Figure F-64. $v_{\text{rectPeak}}$ vs. frequency for beam 3; $a_m = 1.6 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$.

Figure F-65. $i_{LPRCPeak}$ vs. frequency for beam 3; $a_m = 1.6 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$. 
Figure F-66. $i_{\text{batteryPeak}}$ vs. frequency for beam 3; $a_m = 1.6 \, m/s^2$; $L_{PRC} = 270 \, \mu H$.

Figure F-67. $t_{NGate}$ vs. frequency for beam 3; $a_m = 1.6 \, m/s^2$; $L_{PRC} = 270 \, \mu H$. 

Experimental Data

$\mu = 3.87 \, \mu s$ 
(Exp. Data Mean) 

$\sigma = 48.1 \, ns$ 
(Exp. Data Std. Dev.)
Figure F-68. $t_{PGate}$ vs. frequency for beam 3; $a_m = 1.6 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$.

Figure F-69. $w_{tip}$ vs. frequency for beam 3; $a_m = 1.6 \text{ m/s}^2$; $L_{PRC} = 270 \mu H$. 
APPENDIX G
MODELING SWITCH LAG IN THE PRC

One of the results of using the cascaded control circuitry presented in Chapter 6 to synchronize the timing of the PRC switches is the creation of a lag time between phase2\(_\text{\rightarrow inductor}\) and phase3\(_{\text{\rightarrow battery}}\), as shown in Figure G-1. At the end of phase2\(_{\text{\rightarrow inductor}}\), the \(N_{\text{Gate}}(t)\) signal transitions from high to low which causes the \(N_{\text{Switch}}\) to open and triggers the generation of the \(P_{\text{Gate}}(t)\) signal. Due to the finite delay of the one-shot and the rise time of the \(P_{\text{Gate}}(t)\) signal, a period of dead time is created where neither of the switches conducts. Implementing the effects of this behavior in the model developed in Chapter 5 is difficult, primarily because the lag time creates a fourth phase where both switches are open and the inductor is energized. In order to include the effects of the finite lag time into the model, an effective lag is calculated which can be more easily incorporated into the model. The derivation for the effective lag is presented here.

Considering first the case of zero lag time, \(i_{\text{battery}}(t)\) form can be approximated as a triangle waveform, represented with the blue curve in Figure G-1, where the peak value is equal to the peak value of \(i_{\text{LPRC}}(t)\) at the end of phase2\(_{\text{\rightarrow inductor}}\), and the length of the signal is equal to the length of \(P_{\text{Gate}}(t)\). The triangular shape is a result of the fact that the inductor discharges into a constant voltage source load. For this type of discharge, the inductor voltage is given as

\[
\nu_{\text{LPRC}}(t) = V_{\text{battery}} = L_{\text{PRC}} \frac{di_{\text{battery}}(t)}{dt}, \quad (G-1)
\]

which can be rearranged to express the constant slope as

\[
\frac{di_{\text{battery}}(t)}{dt} = \frac{V_{\text{battery}}}{L_{\text{PRC}}}, \quad (G-2)
\]

To calculate the power delivered to the load in the simulated model for this case, the instantaneous battery power is integrated over the length of \(P_{\text{Gate}}(t), t_{P_{\text{Gate}}}, \) and...
multiplied by the energy harvesting frequency (which is twice the applied vibration frequency due to rectification)

\[ P = 2f_{vib} \int_{t_{P_{\text{Gate}}}}^{t_{N_{\text{Gate}}}} i_{\text{battery}}(t)V_{\text{battery}} dt. \]  

\( (G-3) \)

Figure G-1. The effect of lag on the power delivered to the load.

For the case where switch lag is present, the battery current is represented by the red curve in Figure G-1. When the \( N_{\text{Switch}} \) opens, there is no conduction path for \( i_{LPRC}(t) \) and some of the energy is dissipated during the lag period. As a result, the current stored on the inductor is decreased. When the \( P_{\text{Switch}} \) closes, the inductor is discharged to the
load via $i_{\text{battery}}(t)$. The slope of $i_{\text{battery}}(t)$ is the same as for the zero lag case, given in
Equation G–2, since the both $V_{\text{battery}}$ and $L_{\text{PRC}}$ are constant. The initial value of $i_{\text{battery}}(t)$
is reduced, and therefore so is the length of the $P_{\text{Gate}}(t)$ signal required for complete
 discharge of $L_{\text{PRC}}$ to the load. The power delivered to the load is again calculated with
Equation G–3.

To capture the effects of the switch lag without implementing major changes to
the model, the power delivered by the zero lag case is slightly modified to deliver the
same amount of power as the case with lag. This is accomplished by calculating the
power delivered to the load with a reduced integration period. Instead of calculating the
power by integrating the $i_{\text{battery}}(t)$ waveform for the entire length of phase3(\rightarrow \text{battery}), the
integration begins a time equal to $t_{\text{lag,eff}}$, after to the start of phase3(\rightarrow \text{battery})

$$P = 2f_{\text{vibration}} \int_{t_{\text{Gate, start}}+t_{\text{lag,eff}}}^{t_{\text{Gate, end}}} i_{\text{battery}}(t) V_{\text{battery}} dt$$  \hspace{1cm} (G–4)

The value of $t_{\text{lag,eff}}$ is found by comparing the two triangles that form $i_{\text{battery}}(t)$ for
the cases with and without lag shown in Figure G-1. The effective lag is defined as the
time that the zero lag case must be delayed for the two triangles to have the same area,
which is equal to the difference is the conduction times

$$t_{\text{lag, effective}} = x_1 - x_2.$$  \hspace{1cm} (G–5)

The relationship between $x_1$ and $x_2$ is found by considering that the slope of both
$i_{\text{battery}}(t)$ curves are equal. The angle at which both lines cross the time axis are therefore
equal

$$\theta_1 = \theta_2,$$  \hspace{1cm} (G–6)

and the tangent of equals angles must also be equal

$$\tan(\theta_1) = \tan(\theta_2).$$  \hspace{1cm} (G–7)
Applying the definition of \( \tan(\theta) \) to Equation G–7

\[
\frac{y_1}{x_1} = \frac{y_2}{x_2}
\]  

(G–8)

and solving for \( x_2 \)

\[
x_2 = x_1 \frac{y_2}{y_1}.
\]  

(G–9)

Substituting the value of \( x_2 \) from Equation G–9 into Equation G–5

\[
t_{\text{lag, effective}} = x_1 - x_1 \frac{y_2}{y_1} = x_1 \left( 1 - \frac{y_2}{y_1} \right).
\]  

(G–10)

Finally substituting in the values of \( x_1, y_1, x_2, \) and \( y_2 \) into Equation G–10 give the result for \( t_{\text{lag, eff}} \)

\[
t_{\text{lag, effective}} = t_{\text{PSwitch}} \left( 1 - \frac{i_{\text{battery, max}}}{i_{\text{LPRC, max}}} \right),
\]  

(G–11)

where \( i_{\text{battery, max}} \) and \( i_{\text{LPRC, max}} \) are the maximum values of \( i_{\text{battery}}(t) \) and \( i_{\text{LPRC}}(t) \).
REFERENCES


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BIOGRAPHICAL SKETCH

Alex Phipps was born in Ohio and moved to Gainesville, FL when he was four years old. He grew up in Gainesville and graduated from Buchholz High School in 2000. He attended the University of Florida where he received his bachelor’s degree in electrical engineering in 2004. Alex’s graduate work was also done at the University of Florida, where he studied power electronics and energy harvesting with Dr. Khai Ngo and Dr. Toshikazu Nishida. Alex received a master’s degree in electrical engineering in 2006 and will receive his PhD in electrical engineering in 2010. Upon graduation Alex will begin working for the Space and Naval Warfare Systems Command (SPAWAR) in San Diego, CA.