PRESERVICE ELEMENTARY TEACHERS’ TWO-DIMENSIONAL VISUALIZATION AND ATTITUDE TOWARD GEOMETRY: INFLUENCES OF MANIPULATIVE FORMAT

By

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LIST OF TERMS AND ABBREVIATIONS

Attitude Toward Geometry: For this research, attitude toward geometry will consider three subcategories, enjoyment of geometry, perceived usefulness of geometry, and confidence in completing geometry problems.

Concrete Manipulatives: Manipulatives that can be touched, held, and rearranged physically.

Digital Manipulatives: “An interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer, Bolyard, and Spikell, 2002, p.373). These manipulatives are available via computer and closely resemble concrete manipulatives but can only be manipulated and moved on the screen.

Preservice Elementary Teachers: Undergraduate students within a university based teacher education program who are preparing to teach at the elementary level.

Tangrams: The set of seven pieces, one square, one parallelogram, and five triangles, two identical big triangles, one medium triangle, and two identical small triangles (see Figure 1-2).

Two-Dimensional Visualization: The act of mentally creating, recreating, and acting upon mental figures.

2D: Two-Dimensional

UGAS : Utley Geometry Attitude Scale

VST : Test of Spatial Visualization in 2D Geometry

WSAT : Wheatley Spatial Ability Test
Preservice elementary teachers prove deficient in two-dimensional visualization. Attitude toward geometry may be related to this deficiency. Preservice elementary teachers improve two-dimensional visualization using concrete or digital manipulatives and also demonstrate an interaction between two-dimensional visualization and attitude toward geometry.

Four intact groups of 74 predominantly female preservice elementary teachers enrolled in an elementary education mathematics methods course taught by four instructors at a southeastern research university completed both pretest and posttest. The quasi-experimental research design was comprised of a 2x2 matrix: the level of concrete tangram use and the level of digital tangram use. One group served as a control group; three treatment groups had access to concrete tangrams, digital tangrams, or the choice of concrete and digital tangrams to complete the intervention. Participants rotated, translated, and reflected shapes to fit within 30 tangram designs which were identical in concrete or digital format.

A repeated measures analysis of variance with alpha level of .05 showed all groups significantly increased two-dimensional visualization as measured by two tests, one test of mental rotation and the other test including mental rotation, spatial, spatio-numeric, and informal
area measurement items, $F(1, 70) = 48.97, p < .01$. Significant within group differences occurred for digital groups and nondigital groups, $F(1, 40) = 8.18, p < .01$. The digital groups mean of 83.40 was exceeded by the nondigital groups mean of 88.81. Attitude toward geometry significantly improved, $F(1,70) = 4.79, p = .03$. A relationship between two-dimensional visualization and attitude toward geometry was evidenced by the significant correlation between these two factors, correlation coefficient equals .29, $p = .01$ (N = 74); improved levels of two-dimensional visualization were correlated with more positive attitude toward geometry.

This improvement in preservice elementary teachers’ two-dimensional visualization indicates a need to research retention of two-dimensional visualization. The larger learning experienced by the nondigital groups is unexpected and demonstrates a need for the creation of a digital measure of two-dimensional visualization, isolation of the digital learning, measurement of the process of digital learning, and time allotted to adequately explore the digital environment prior to measuring digital learning.
CHAPTER 1
INTRODUCTION

Elementary students are taught mathematics from teachers who have taken a limited number of mathematics courses. Most preservice elementary teacher education programs may only require a single mathematics content course and a mathematics pedagogy course during the four year undergraduate teacher education curriculum. Because preservice elementary teacher education programs require so few mathematics courses, these courses need to deliver maximum impact in this minimal time frame. Content must be carefully selected to prepare preservice elementary teachers to become mathematically fluent, which includes questioning, reasoning, and representing information (Boaler & Humphreys, 2005). Visualization has been shown to improve mathematics performance. If preservice elementary teachers spend time practicing visualization, these abilities improve. Further, these preservice elementary teachers never see a glimmer of calculus, yet they are asked to propel their future students into careers in mathematics or science (National Research Council [NRC], 2006). In order to do this, preservice elementary teacher education programs are challenged to incorporate powerful mathematical ideas (Jones, Langrall, Thornton, & Nisbet, 2002). Promoting all students to use mathematics with competence, confidence, and enjoyment has been defined as attitude toward mathematics (Utley, 2007).

Visualization, one key to aiding preservice elementary teachers’ mathematical fluency, also yields more creative approaches in the classroom (Presmeg, 2006). Preservice elementary teachers are more likely to be, if not the mathematician, at least the stepping stone to the mathematical fluency for their future students. While mathematicians visualize, they may not talk about visualization or recognize its importance in their work. Preservice elementary teachers need to understand the importance of visualization within mathematics and in related fields.
Visualization is a skill that is situated within the geometry curriculum. Numerous studies suggest that visualization may be improved with focused practiced and instruction. Improving visualization not only improves geometry performance, but also impacts mathematics achievement (Battista, Wheatley, & Talsma, 1982) and related areas such as science, art, and engineering (Humphreys, Lubinsld, & Yao, 1993). For instance, a chemistry professor decided that his students were deficient in visualization skills and this predicted their performance in chemistry. Engineers who take a visualization test are then predicted to improve much the same as mathematics students take a math test as a predictor of future performance (Bodner & Guay, 1997).

Visualization may be downgraded in importance by mathematicians (Arcavi, 2003) instead of acclaimed for its critical importance to their work of solving problems. Students who value visualization will in turn solve problems better. Activities have been developed to improve visualization, which proves to aid geometry and mathematical studies and may prove to impact future teaching and capacity to meet future students at all levels of visualization achievement and propel them upward to increased visualization performance. Several professional organizations call for increased geometry content knowledge, particularly visualization, in preservice elementary teacher education (National Commission on Mathematics and Science Teaching for the 21st Century, 2000; National Council of Teachers of Mathematics (NCTM), 1991, 2000, 2006; NRC, 2006). Increased study and teaching of visualization and the associated spatial tasks and explorations will enable students to understand geometry (DeVault & Weaver, 1970; NCTM, 2000). Visualization instruction is proscribed for kindergarten (K) to 12th grade classrooms (NCTM, 2006; NRC, 2006), but it will not be realized easily. One of the reasons may be because preservice elementary teachers may have come through K to 12 programs that have
since been called upon for improvement. These preservice elementary teachers may be required to teach content different from the content they personally experienced in ways that may also be new to them. It is conceivable that the only exposure to geometry experienced by most preservice elementary teachers occurs during one isolated year of high school, yet these preservice elementary teachers are called upon to weave geometry concepts into the K to 12 curricula.

**Problem Statement**

Geometry, one of six content areas in mathematics, links many topics and is thus an integral part of the mathematics curriculum (NCTM, 2000, 1989). Geometry links algebra concepts of function and graphs, measurement ideas of area, perimeter, and volume, and even lays the foundation for calculus, such as preparing students to visualize the area generated when integrating a solid of revolution.

Because geometry serves to connect the mathematics curriculum, a good understanding of geometry and its practical emphasis on spatial characteristics and relationships will help students gain increased access to abstract mathematical concepts (Wheatley, 1998). Preservice elementary teachers well prepared to teach geometry will lead their students to greater success in their studies of geometry (NCTM, 2006).

Visualization, one of the four components that comprise the geometry standard (NCTM, 2000, 2006), is core to teaching the geometry curriculum (Del Grande, 1987). Visualization, the act of visualizing an object and changing the orientation of that object mentally, will aid the student working to make sense of the area of a triangle, one half base times height, when they visualize two right triangles, one rotated to fit flush with the first, and the resulting figure, namely, a rectangle with area base times height, as represented in Figure 1-1.
An important aspect in preservice elementary teacher education is visualizing mathematical relationships, and then portraying and describing this vision to support students’ learning (NCTM, 2006). Teachers must be prepared to produce a sketch of a relationship or demonstrate the manipulation of objects, such as sketching two triangles whose sum represents the area of a rectangle (see Figure 1-1). To do this, preservice elementary teachers must be expected and encouraged to construct images, which will in turn enable them to trust image constructing in solving mathematics problems (Wheatley, 1998). Manipulatives, both digital and concrete, facilitate visualization (NCTM, 1989, 2000).

Preservice elementary teachers demonstrate a deficiency in their understanding of geometry as well as in their skills to communicate geometry, particularly visualization (Hershkowitz, 1989; Owens & Outhred, 2006; Sundberg & Goodman, 2005). This deficiency is not age related, but related to experience and practice with geometry. One way to incorporate visualizing is through the use of manipulatives. Presmeg found that many students learn better with visual relationships (2006). Generalizing Presmeg’s findings would imply that many preservice elementary teachers learn better with visual relationships. Thus, augmenting preservice elementary teachers’ mathematics education with visualization techniques utilizing both digital and concrete manipulatives will aid the learning process.

This deficiency may impact preservice elementary teachers’ attitudes toward geometry. Difficulty visualizing shapes and relationships may hinder performance in geometry. However, attitude has been shown to coincide with increased performance in geometry (Usiskin, 1987). Thus, improving geometry performance, or 2D visualization in particular, may also improve attitude toward geometry. And visualization improves performance, for when visualization is improved, subsequent learning is also increased (Siegler, 2003; Wheatley, 1998).
In summary, the problem under investigation included two parts. First, preservice elementary teachers have been shown to have a deficiency in 2D visualization (Mayberry, 1983). For other populations, this deficiency decreased following increased exposure to 2D visualization activities, but this was not well established with the preservice elementary teacher population. This first part of the problem was explored using concrete manipulatives, digital manipulatives, or the choice to use concrete or digital manipulatives to influence preservice elementary teachers’ 2D visualization. The second part of the problem was the little known about preservice elementary teachers’ attitude toward geometry, its improvement after participating in 2D visualization activities and its relationship to 2D visualization. The training of preservice elementary teachers to improve both 2D visualization and attitude toward geometry must include focused practice that consists of 2D visualization activities utilizing digital or concrete manipulatives.

**Visualization**

Two-dimensional (2D) visualization is a skill used to create, recreate, and act upon mental images (Battista, Clements, Arnoff, Battista, & Borrow, 1998; Clements & Battista, 1992; Wheatley, 1998). It requires the recognition, production and interpretation of visual images that can then be mentally manipulated and explored. These visual images must be held for comparison or manipulation by visual working memory, the visual component of working memory. This allows for the creation and movement of other visual images to create new or related visual images (Stylianou, 2002). Visualization involves understanding how two or more figures relate to one another, both in similarities and differences. Further, visualization consists of constructing or deconstructing figures to arrive at more complex or simple figures, respectively. Visualization is mathematically important so that students recognize and can reproduce shapes, recognize parts of shapes, and relate figures to other figures (Arcavi, 2003;
Visualization and formal reasoning are the two primary mental abilities needed to learn and do mathematics (Battista, Wheatley, & Talsma, 1989). The term 2D visualization clarifies the focus on 2D relationships, instead of terms such as spatial arrangement (Piaget & Inhelder, 1971), spatial thinking (Bishop, 1983), spatial perception (Del Grande, 1987), spatial reasoning (Clements & Battista, 1992), imaging (Wheatley, 1998), spatial structuring (Battista et al., 1998), or spatial visualization (Clements & Battista, 1992).

Piaget and Inhelder (1971) posit that a visual image in the mind guides the creation of a spatial arrangement. Spatial thinking, as defined by Bishop (1983), consists of two aspects, namely, interpreting figural information and visual processing. The former aspect involves understanding visual representations and vocabulary, while the latter involves manipulating and transforming visual representations and images and translating abstract relationships into visual representations. Both of these components are included in Presmeg’s (1997) broader definition of visualization as constructing and transforming visual mental imagery as well as other spatial tasks involved in doing mathematics.

Spatial perception is related to the recognition and discrimination of stimuli in and from space and the relation of these stimuli with previous experiences (Del Grande, 1987). Because the concept of perception emerges from psychology, philosophy, and physics, which are ever changing, agreement upon a universally accepted definition is difficult to determine (Del Grande), the definition altered to apply to the situation.

Arcavi (2003) was the first to offer a comprehensive definition of visualization, which follows, based upon the more individualized definitions used previously by other researchers and add another dimension to the definition of visualization. Other researchers have defined visualization as “seeing the unseen” (McCormick, DeFantim, & Brown, 1987, p. 3) or organizing
data at hand and generating a solution (Fischbein, 1987). A new dimension emerges in Arcavi’s definition, “We propose that visualization can be even more than that: it can be the analytical process itself which concludes with a solution that is general and formal” (Arcavi, p. 230). This definition can be extended to include visualization not only as an important factor in the process of arriving at a solution to a mathematics problem, but as one type of foundation for the process itself. This foundation can then be used to build a solution that is both general and formal.

In defining the terms spatial reasoning, imaging, and spatial structuring, researchers note three aspects, namely, creating, recreating, and then acting upon the creation of a mental image. These definitions are presented here to demonstrate the similarities between them and the current definition of 2D visualization. Clements and Battista (1992) define spatial reasoning as the cognitive processes employed to create mental representations for constructing and manipulating spatial objects, relationships, and transformations. This idea of constructing, representing, and transforming images comprises Wheatley’s definition of imaging (1998), which closely parallels terminology used to define spatial structuring, namely, identifying, combining, and interrelating spatial components and composites (Battista et al., 1998).

Wheatley (1998) uses the terms image and imaging to avoid confusion arising from different definitions of spatial visualization and clarifies imaging as the mental activity, not as a quantitative score on a test (Wheatley & Cobb, 1990). Similar in definition but different in name, spatial structuring embodies the mental operations used to organize a set of objects prior to acting quantitatively upon those spatial objects (Battista et al., 1998). The rationale for using the term 2D visualization arises from the definition presented by Arcavi (2003) as well as from a need to clarify the focus on 2D for this research, because spatial visualization describes the process of comprehending and performing imagined movements of objects in two- and three-
dimensional space (Clements & Battista, 1992). The terms imaging and spatial structuring inform the discussion of visualization, however, 2D visualization seems to be the term that best describes the focus of this research. The definition of 2D visualization employed in this research is the creation, recreation, and action upon mental images.

**Improvement of Visualization**

Visualization is a predictor of mastering and building upon geometry skills (Clements & Battista, 1992), which include visualizing, drawing, and constructing figures, all vital elements in the geometry curriculum (Usiskin, 1987). Another example of visualizing within geometry consists of constructing a proof, where the basic components of a complex figure are indispensable and must be identified (Dreyfus & Hadas, 1987). By linking diagrams and visual images with verbal definitions and analyses, visualization builds creative thought, which is based upon spatial thinking (Clements & Battista). As with other skills, continued practice improves and maintains visualization. Similarly, preservice elementary teachers must gain access to visualization through many and varied experiences (NCTM, 2000) and practice visualization, a keenness of sight, continually so that they can gain and maintain visual skills (Presmeg, 2006).

In mathematics, visualization helps teachers in several ways, such as meeting students’ needs, improving students’ skills, and preparing students for futures in mathematics. As teachers seek for pictorial examples to clarify difficult problems, they need visualization to meet their students mentally to assess and to evaluate the pictures and ideas that students employ in their mathematical explorations (Arcavi, 2003; Clements, 2003). Preservice and inservice elementary teachers who visualize are more apt to encourage and deepen their students’ visualization skills (Presmeg, 2006), which prepares these students for careers in mathematics, science, or the arts (Clements & Battista, 1992; Humphreys et al., 1993; NRC, 2006). Thus, visualization is one tool to attract students to the study of mathematics and to promote their success in mathematics and
related fields. For instance, in an undergraduate Chemistry course, visualization was correlated with problem solving, specifically in the first stages of forming a mental representation of the problem (Bodner & Guay, 1997).

These mental representations are specifically addressed in the Representation Standard which is both processes and products of representing mathematical ideas and relationships, including a wide range of visual representations to be available to students as they solve problems (NCTM, 2000). These representations may include spoken and written language, pictures, and concrete manipulatives. An additional representation is digital manipulatives, which serve well to augment the previously mentioned representations (Shackow, 2006-2007). Digital manipulatives are computer representations of concrete manipulatives that can only be manipulated or moved on the screen.

Incorporating manipulatives is another avenue to improve visualization. As students manipulate concrete geometric shapes they are participating in real world connections that are the basis of Euclidean geometry (Clements & Battista, 1992). This manipulation aids the development of concepts of shapes and the creation of definitions and conjectures (Fuys, Geddes, & Tischler, 1988). Manipulatives prove to be a key component of geometry instruction, yet the textbooks rarely incorporate geometrical manipulatives with enough depth to aid students (Battista, 2001). For this reason, making up for the deficiency of geometric manipulatives in the textbooks becomes the responsibility of teachers, who must understand and be prepared to utilize manipulatives in the classroom (Battista). However, manipulative use must employ informal methods and be accompanied by intention and reflection for optimal improvement to occur (Clements & McMillen, 1996).
Tangram Explorations

Tangrams are one example of concrete manipulatives which have a complementary digital manipulative format, as well. Tangrams consist of seven pieces, or tans: five triangles (two large, one medium sized, and two small), one square, and one parallelogram (see Figure 1-2). Tangrams provide a focus for visualization because students can experiment with them formally and informally to build their geometric knowledge (NCTM, 2000). Tangrams yield multiple levels of visualization activities to improve visualization.

Tangrams bring geometry out of a static subject of study to give it dimension and vitality. Tangrams offer the learner a chance to problem solve, reason, and conjecture as they seek to fit the pieces together and search for configurations that match or go beyond what the teacher presents. Tangrams used in the classroom enliven the classroom and promote engagement, which positively influences attitude toward geometry.

Solving tangram problems improves children’s geometry understanding and 2D visualization (Olkun, 2003). The finite tangram set encourages creativity as students explore many and varied combinations of tangrams that produce different shapes, figures, or model letters of the alphabet. While tangrams consist of only seven pieces, these finite pieces yield a nearly infinite variety of activities and outcomes. They can be utilized to study ratio, fraction, percent, and decimal (Naylor, 2002). Given time to explore tangrams, questions arise and concepts emerge, such as reflection, rotation, and translation (Bohning & Althouse, 1997). Explorations of symmetry, congruence, and similarity include ordering the tans smallest to largest, or according to another criteria, placing them into Venn diagrams with categories such as shapes with one, two or three lines of symmetry, organizing tans by similarity, and creating tiling patterns by tracing the similar figures over and again, possibly including translations and rotations in an effort to create a tessellation. Students will feel empowered as they tackle this
tangram challenge. They will gain confidence and strengthen their mathematics self efficacy as they work with tangrams to explore geometry ideas.

Tangrams prove useful to see relationships between symmetry, similarity, and congruence. Symmetry, similarity, and congruence each play a role in the geometry curriculum from the earliest learners to the most advanced (NCTM, 2006; NCTM, 2000). Specific activities include line and rotational symmetry (NCTM, 2000), computations of length, area, and volume of similar solids (Duke, 1998; NCTM, 1989), and rotations to demonstrate the preservation of congruence (NCTM, 2000). Examples of symmetry are present in many real world settings, such as in art and architecture (Hancock, 2007), which include examples of similarity and congruence as well. Within each of the previously mentioned tangram activities lies a kernel of understanding pertaining to symmetry, similarity, and congruence.

In summary, tangrams have the potential to engage all learners, regardless of prior background. Tangrams are available in concrete or digital formats. And tangrams, while the pieces are few in number, are myriad in application and variety to challenge and aid all learners in their quest to visualize and make sense of geometry.

**Technology: Digital Manipulatives and Digital Tangrams**

Technology access increases as schools continue to invest in computers and internet access for more classrooms (Steen, Brooks, & Lyon, 2006). Meaningful uses of technology should be incorporated across the curriculum (International Society for Technology in Education, 2000), for appropriate use of technology promotes deeper learning of mathematics (NCTM, 2000). Technology allows the consideration of more abstract topics in meaningful ways (Clements & Sarama, 2005; Moyer, Niezgoda, & Stanley, 2005; Peressini & Knuth, 2005). Mathematical explorations incorporating technology can inform and strengthen mathematical investigations. Ideas can be tested, pictures created and compared, relationships clarified.
(Clements & Battista, 1989; Clements & Sarama; NCTM). The utilization of technology leads to advantages such as the exploration of more varied shapes, relationships, and aspects of mathematics (Clements & Battista; Clements, 1999, 2003; Clements & Sarama). Technology changes the nature of the task, both in design and execution, and also provides feedback on those tasks (Laborde, Kynigos, Hollebrands, & Strasser, 2006). The feedback on technology tasks provides multiple and varied assessments (NCTM, 1995, 2000) as students work with technology.

Technology is a key component of preservice elementary teacher education because perhaps all preservice elementary teachers did not experienced technology in their elementary classrooms in the same way that they will be required to use technology to influence the mathematics that they teach (Peressini & Knuth, 2005). Moyer et al. (2005) propose that digital manipulatives are a viable technology for young children because they closely resemble concrete manipulatives and are accessible via the Internet. For this reason, digital manipulatives are a logical choice for training preservice elementary teachers, who will directly influence young children’s learning.

Digital manipulatives improve instruction as they model student thinking processes, promote mathematical discourse, and are easy to use (Clements & McMillen, 1996; Clements & Sarama, 2005). One clear advantage to digital manipulatives is the fact that when the computer is available, digital manipulatives are always ready, requiring minimal setup, and varied, often presenting more color, help, and design choices than concrete manipulatives (Moyer et al., 2005; Shackow, 2006-2007). Digital manipulatives prove to be just as or more effective to aid student understanding (Clements & McMillen) and may foster deeper conceptual thinking (Clements, 1999; Sedig & Sumner, 2006; Steffe & Wiegel, 1994).
Digital tangrams are one example of a digital manipulative. In appearance and behavior, digital tangrams mimic their concrete counterparts. Digital tangrams have the added features such as “snapping” into place when they are near the correct location (Clements & Sarama, 2005) and offering hints or scaffolding to promote the successful solution to a digital tangram problem (Clements & McMillen, 1996). Digital tangrams appeal to contemporary learners, who are technologically savvy and find technology inherently valuable and appealing. Technology often adds a dimension of novelty that appeals to preservice elementary teachers, as well.

**Significance of the Study**

This study fills a gap left in the mathematics education research regarding visualization (Presmeg, 2006). The call goes out for research that includes both the “interactional sphere of classroom teaching and learning of mathematics at all levels” (Presmeg, 2006, p. 228) as well as in mathematical problem solving. Since Presmeg’s original research and more recent report (1986, 2006), little has been done to strengthen the body of visualization research in mathematics education. Indeed, she describes the dearth of information in the research as a lacuna in this area of visualization research (Presmeg, 2006). Given the importance of geometry and visualization, this research grants attention to these subjects (Clements & Battista, 1992). It links together with other studies to show that visualization may be improved with education that incorporates digital and concrete manipulatives (Clements & Battista, 1992; Presmeg, 2006; Wheatley, 1998).

This study brings together research about preservice elementary teachers, visualization, manipulatives, and attitude. The current body of research pertaining to visualization focuses primarily on studies of elementary students. While many studies focus on preservice elementary teachers’ visualization, none explore the implications of utilizing a single manipulative to improve visualization. In this way, the current study will contribute to the body of research regarding teacher education programs, which will benefit by exploring one intervention that
improves 2D visualization in preservice elementary teachers. Additionally, this study investigates the connection between 2D visualization and attitude as influenced by manipulative format, which contributes to the prediction of mathematics achievement (Aiken, 1985). Finally, this study addresses the issue of how students feel about geometry (Battista, 2007).

There is a debate that spatial tasks can be completed with analytic processing (Guay, McDaniel, & Angelo, 1978) or different processes such as visual or verbal, attending to the whole or to parts of the stimulus, and aided by manipulatives or body movement (Clements & Battista, 1992). This study will not resolve this debate, but will contribute to understanding how manipulatives aid instruction, particularly in which manipulative format preservice elementary teachers prefer as well as which format promotes increased visualization.

Several studies focus on tangrams to aid the instruction of geometry, but again, these studies focus on elementary students. The study performed by Smith, Olkun, and Middleton (2003) utilizes concrete and digital tangrams with fourth and fifth graders, but does not consider a group utilizing both concrete and digital tangrams. In fact, these researchers call for a study including a treatment group with access to both concrete and digital tangrams. Also, the availability of both concrete and digital tangrams will provide insight about how to aid preservice elementary teachers’ in their improvement of visualization. Thus, teacher education gains a glimpse of how to better develop preservice elementary teachers’ visualization. Shifting the focus from elementary students to the preservice elementary teachers who will become their teachers has the potential to better prepare these preservice elementary teachers to teach their future students. This study will provide an avenue for preservice elementary teachers to experience concrete and digital manipulatives for their future teaching. In a practical sense, it prepares teachers for their future classrooms that may only have access to concrete
manipulatives, and others that are fortunate enough to find both concrete and digital manipulatives. It informs the debate as to which type or combination of manipulative promotes success. This will serve to fulfill the call to foster understanding and intuition by utilizing technology (NCTM, 2000). Again, preservice elementary teachers’ attitude toward geometry is another dimension to understanding visualization.

This study is in direct answer to the call to integrate curriculum and technology standards (ISTE, 2007) because it incorporates technology into geometry curriculum in a way that addresses technology, representation, and geometry standards (Lester, 2007; NCTM, 2000). It investigates the interaction between participants and mathematical activity (Zbiek, Heide, Blume, & Dick, 2007). It highlights the technologically savvy preservice elementary teachers that are currently entering the teaching field and offers a mode to instruct them in ways that are enticing and effective for them and for their future students.

**Study Overview**

This chapter includes the purpose of the study, its rationale, and its significance to the field of mathematics education. Chapter 2 presents a review of relevant literature, including research regarding spatial visualization and how it is improved by using concrete and digital manipulatives. The design and methodology of the study are contained in Chapter 3. The quantitative analysis of the study is detailed in Chapter 4. Finally, Chapter 5 includes a summary of findings, implications for preservice elementary teachers, limitations, and recommendations for future research.
Figure 1-1. Two triangles whose sum represents the area of a rectangle.

Figure 1-2. Five triangles (two large, one medium, and two small), one square, and one parallelogram arranged in a square illustrate a tangram set.
CHAPTER 2
THEORETICAL BACKGROUND AND LITERATURE REVIEW

Theoretical Background

The framework for this study employs the models of working memory (WM) and visual working memory. As students work to improve their visualization, they are limited by their capacity to hold information in WM. Items enter WM from sensory memory and then are encoded to long term memory or discarded. WM capacity seems to be seven plus or minus two bits (Miller, 1956) held from 15 to 30 seconds.

Visual working memory coupled with auditory working memory increases capacity of working memory (Mayer & Moreno, 1998). Utilization of both visual and auditory stimulus promotes increased learning and recall. Visual pictures, objects, or digital objects all serve with verbal cues to increase WM capacity (Mayer & Moreno). Manipulatives and drawings maximize working memory by employing visual working memory.

Digital or concrete manipulatives can aid WM in solving geometry problems. Digital manipulatives offer the student multiple diagrams to clarify or offer evidence to the truthfulness of a statement or proof (Jackiw & Sinclair, 2002) and can be adapted to the various skill levels of individual learners (Sarama & Clements, 2002). Visual mathematical representations promote best practice tools and may include basic or task-based features. Basic features include conversing (commands), manipulating (point with cursor), and navigating (move over or through) while task-based features include animating, chunking, fragmenting, rearranging, or searching. When designed carefully mathematical cognitive tools will be improved (Sedig & Sumner, 2006). One mathematical cognitive tool is called Building Blocks (Sarama and Clements, 2002).
Building Blocks is a project designed to facilitate geometrical knowledge for a wide range of learners. It offers a shape builder, picture composer, and two other tools that increase in difficulty. First the learner fills in the picture with given blocks and it is readily apparent where the blocks will each fit. Next, the learner can use blocks that may not fit in such an apparent way. Finally, students must use reflections, rotations, and translations of the blocks to fill the picture. In this way, students’ skills improve to fill a picture with the given blocks. This process increases their WM capacity because they move from novice picture fillers to more expert ones (Leahey & Harris, 2001). This digital manipulative serves as a platform to improve WM capacity.

Other aids to minimize the limited capacity and rapid decay of items that are the chief limitations of WM are chunking or rehearsal (Leahey & Harris, 2001). Chunking with 2D visualization activities consists of recognizing symmetry, similarity, or congruence among figures. One example includes subjects who retain the image of two triangles fitted together to form a rectangle, a parallelogram, or a larger triangle (see Figure 2-1). These two triangles now become one figure, or a chunk of information, that minimizes the load on WM. Rehearsal is the repetition of information, such as repeating several times the two triangles fitted together until this configuration is familiar, again reducing WM load.

Sweller (1988, 1994) terms this cognitive load. To minimize cognitive load, multimedia communications should couple visual and auditory cues (Mayer & Moreno, 1998). Another possible drain on cognitive load is the physical manipulation of objects, which can be minimized by placing students in an observational role (Smith et al., 2003). This was not the case when learners were placed in acting or observing roles, for it was reported that modality did not significantly change performance (Owens, n.d.). When students observe teachers as they handle manipulatives, however, instruction proved less effective than when students handled the
Van Hiele (1986) described a model of categorizing students’ geometrical level of understanding. These levels are hierarchical in nature and begin with basic visualization, the lowest van Hiele level (zero), followed by analysis (level one), informal and formal deduction (levels two and three), and rigor (level four) (Burger & Shaughnessy, 1986). Many students do not progress through the levels (Usiskin, 1982). For this reason, it is clear to see that if students struggle with visualization, their progression to higher van Hiele levels will be hindered.

If students are deemed to be at level zero, then activities and explorations must be developed to encourage them to progress to the next van Hiele level (Battista, 2001; Crowley, 1987; Swafford, Jones, & Thornton, 1997; van Hiele, 1986, 1999). However, identifying students’ van Hiele levels does nothing to clarify the nature of the misconceptions (Callingham, 2004). Recognizing and developing student’s mathematical and geometric cognitions also involves the in-depth understanding of how students construct geometric sense making so that the teacher can aid students with their personal sense making (Battista, 2001). Van Hiele levels are one tool to make this assessment.

Technological pedagogical content knowledge [TPCK] focuses on the technology in addition to the pedagogy and the content. While Shulman (1986) developed the term pedagogical content knowledge (PCK) to describe the need to understand pedagogy specific to content, such as mathematics, reading, or science, in a similar manner, there is a need to understand technology specifically with regard to both pedagogy and content. Technology in a mathematics
classroom may look and function quite differently than in a science or reading classroom. TCPK is defined by the teacher’s skill in selecting and implementing technology within the content domain to enhance and strengthen learning. Specifically, TCPK links content, pedagogy, and technology in a web of relationships that also includes the context in which they function (Koehler & Mishra, 2005). It contains pieces of technological knowledge, pedagogical knowledge, and content knowledge, hence the term, TPCK. Termed Pedagogical Technological Knowledge (PTK) by Guerrero (2005), the idea remains the same, namely, PTK embodies teacher use of technology as instructional tool to improve teaching and learning. Detailed by Guerrero (2005) are five areas of consideration when selecting and implementing technology. First, instructional issues such as student-directed discoveries call for a shift in lesson plans to include more project based activities. Second, management issues include student attitudes and engagement such as selecting technologies that are enjoyable to the student. Third, content specific pedagogical issues come into play as the teacher selects and uses the tool. Fourth, conception and use of technology call for an authentic use rather than as a sideshow to the curriculum. This could involve a problem from the real world that connects to the curriculum. Finally, depth and breadth of content implies that the students should have adequate time and freedom to explore and make their own discoveries within the technological interface (2005) which may involve student action research projects.

Literature Review

Overview

Searches using Educational Research Information Clearinghouse (ERIC), WilsonWeb, Web of Science, Web of Knowledge, Reference Universe, JSTOR, ProQuest, Academic OneFile, PsychBooks, and EBSCOHost computer databases aided the process of compiling studies presented in dissertations and peer reviewed journals. Reference lists of key articles also
provided research worthy of inclusion. Keywords for this search included preservice elementary
teachers, geometry, visualization, and manipulatives. Symmetry, similarity, and congruence were
used to narrow the search for geometry articles. Visualization was expanded to include spatial
visualization, 2D visualization, and spatial ability. The manipulatives keyword was variously
paired with descriptors including computer, concrete, digital, and virtual.

The researcher expected to find a dearth of research regarding symmetry, similarity, and
congruence and prepared to include tesselations, rotations, and flips. Additionally, digital
definitions geometry software or digital geometry environments were considered an option to expand the
search pertaining to digital manipulatives. However, an adequate number of studies were found
directly related to the current research to make searching for these lesser related topics
unnecessary. Inservice teachers and elementary and high school students were included in the
literature due to the limited number of studies that focus on preservice elementary teachers.

Following are the most related studies in terms of content and design. Also included are
studies that relate closely in one particular aspect or are closely related in methodology. Studies
are organized into three sections including visualization, geometry, attitude toward geometry,
and technology. Within each section, studies incorporating concrete or digital manipulatives and
preservice elementary teachers are presented. Some details of studies including elementary or
high school students are also included because they inform the discussion of 2D visualization
utilizing digital and concrete manipulatives. Symmetry, similarity, and congruence, while the
focus of only a few studies, are included throughout the literature review where they are most
related to the larger topic presented, namely, visualization, geometry, attitude toward geometry,
or technology.
Visualization

Visual teachers demonstrate traits associated with creativity in teaching and make connections across the curriculum, to other subjects in school, to other areas in students’ lives, and to the real world (Presmeg, 1986). Visual learners tend to exhibit characteristics associated with creativity such as playfulness, humor, self-awareness and adaptability (Presmeg, 1991). Visualization efficacy may be limited or at times aided by concrete imagery and increased by pattern imagery with its fluid nature and basis on pure relationships (Presmeg, 1986). Students’ difficulty understanding 2D visualization concepts propel them into searching for descriptions that utilize three-dimensional (3D) concepts, but this may be a symptom of limited 2D visualization proficiency (Callingham, 2004). Conversely, students struggling to make sense of 3D arrays of cubes call upon their understanding of 2D arrays of squares. Researchers bent upon discovering 3D conceptualizations of students revisited their research agenda and decided to simplify the study from 3D arrays of cubes to 2D arrays of squares to analyze students’ spatial structuring (Battista et al., 1998; Battista & Clements, 1998).

Arcavi (2003) introduces several strengths of visual arguments to illustrate and clarify mathematics in three cases. The first case is that of purely symbolic representations, such as lattice points to represent fractions. The second case is algebraic or counterintuitive proofs, such as the difference between the length of a rope measuring the circumference of the earth at the equator and the length of rope measuring the circumference of the earth on posts six feet above the equator (38 feet longer). The third case is reconnecting with the meaning behind the mathematics, as in slope equaling an intercept problem.

This author illustrates the power of visual arguments in mathematics. These visual arguments aid geometry instruction, particularly visualization instruction within a geometry context. While this was not a study designed to improve visualization, it does characterize and
point out the key role of visualization within mathematics. It is interesting to note the statement that drives this author’s comments, namely, “visualization offers a method of seeing the unseen” (McCormick et al., 1987, p. 3)

Diagrams and visual tools aid teaching and comprise valuable heuristics, but often are undervalued in mathematics theory and practice. Students don’t always see what the teacher or researcher sees (graphs of lines $y = 2x+1$, $3x+1$, $4x+1$, predict $5x+1$; didn’t predict going through $0,1$). Mathematicians visualize not only to illustrate ideas, but also to reason, problem solve, and prove. Both perceptual and conceptual reasoning are aided by visualization (Arcavi, 2003).

Battista (2001) suggests a series of steps students must explore informally in order to gain an understanding of formal structuring, or visualization, in mathematics. These steps include spatial structuring, geometric structuring, and axiomatic structuring (See Figures 2-2, 2-3, and 2-4). The first and most basic step involves the construction of a mental organization or form for an object or spatial environment. This is the basic building block, followed by adding geometric concepts to analyze that structure and finally by analyzing this construction into a system through logical deduction. For example, the author gives the example of row by column organization of a shape to determine area of the shape. This is an example of spatial structuring (see Figure 2-2), while geometric structuring may include the rotation of a shape around a particular axis (see Figure 2-3). Axiomatic structuring could include the tiling of space or a shape with triangles, which requires the decomposition of shapes into triangles (see Figure 2-4).

**Studies of Visualization**

Positive correlations between spatial ability and mathematics achievement emerge in the literature across the grade levels (Battista, 1990; Fennema & Sherman, 1976, 1977; Guay & McDaniel, 1978; Wheatley & Reynolds, 1996). Particularly for preservice elementary teachers, visualization is related to problem solving, with more visualization than non-visualization.
strategies used (Battista et al., 1989). Visualization improves in digital environments among both high and low achieving students (Smith et al., 2003). Dixon (1995a) suggested that many and varied visualization opportunities will not only predict and improve achievement, but also could close the achievement gap for students for whom English is not their first language. Olkun, Altun, and Smith (2005) have shown this to be true for students from varied socio-economic status (Olkun et al., 2005). These same students indicated ownership of a computer at home. Those who had no access to computers at home performed at a lower level than their peers with computers at home. This difference in scores was minimized and improved visualization resulted when these fourth and fifth graders were given access to computers at school (Olkun et al., 2005). These studies illustrate methods to narrow the achievement gap that exists in mathematics education.

Geometry concepts hold the potential to improve visualization, particularly visual perception skills, including eye-motor coordination, figure-ground perception, constancy in shape and size, position in space, spatial relationships, visual discrimination, and visual memory; clearly understood visual perceptual skills lead to design of geometry programs and mathematical activities that will improve visual perception (Clements & Battista, 1992). While a standard high school geometry course will not yield improved visualization, an informal geometry course for preservice elementary teachers will yield improved visualization (Clements & Battista).

Another avenue to improve visualization is the use of manipulatives (Clements & Battista, 1992). Studies of concrete, digital and both concrete and digital manipulatives are included herein. Six studies focusing on concrete manipulatives (Battista et al., 1998; Bishop, 1980; Fuys et al., 1988; Jackson, 1999; Martin, Lukong, & Reaves, 2007; Putney & Cass, 1998)
will be followed by eight studies employing digital manipulatives (Dixon, 1995a, 1995b; Olkun et al., 2005; Sarama, Clements, Swaminathan, McMillen, & Gomez, 2003; Sedig, 2007; Smith et al., 2003; Steen, 2002; Steen et al., 2006; Wyeth, 2002). Finally, six visualization studies pertaining to concrete and digital manipulatives, reported subsequently, show that access to digital manipulatives improves performance more than access to concrete manipulatives (Gerretson, 1998; Olkun, 2003; Olkun et al., 2005; Smith et al.; Taylor, 2001; Zacchi & Amato, 1999). And one case study shows no significant differences between visualization performance using concrete or digital manipulatives (Fung, 2005).

**Visualization Studies Employing Concrete Manipulatives**

Battista and Clements (1998) found that students struggle to develop strategies for enumerating cubes in cube buildings. They need instructional tasks to develop mental models and meaningful strategies because difficulties arise from a lack of coordination between views of cube buildings and the mental models of cube buildings (Battista & Clements). Battista et al. (1998) report these findings again, namely, students struggle to construct 3D arrays of cubes, and so they revised their research questions and began a new study to discover students’ 2D structuring of arrays of squares. Twelve second graders were interviewed 2 to 3 times for 45 minutes each session. Interviewers presented various rectangles varying in size from 3 rows and 4 columns to 5 rows and 6 columns to be covered with squares. As an aid, tick marks were drawn along the sides and some interior spaces of the rectangles. Students were asked for an original prediction and a drawing prediction before tiling the figure to determine the number of squares needed to cover the rectangle. From these responses, researchers detailed three levels of spatial structuring and concluded that students’ organizing actions promote a spatial structuring of arrays when students use a set of squares to aid both motion and perception. Coordinating actions move students from local structuring to global structuring. Also, geometric and visual
thinking are founded upon structuring 2D and 3D space. Additionally, arrays, coordinate systems, shapes, and geometric transformations serve to structure space. Finally, “studying the processes by which students structure space offers us a new and powerful perspective on investigating children’s construction of geometric and spatial ideas” (p. 531). Also noted is the need to understand 2D arrays of squares prior to introducing an area model for multiplication. This seminal work lays the foundation for future research to investigate students’ spatial structuring.

Key to aiding students’ conceptual development of geometry includes social constructing familiar spatial environments (Davis & Hyun, 2005), knowing which van Hiele level best describes their current performance, offering tasks and activities to propel them upward toward the next van Hiele level, and allowing them to struggle with problems with appropriate and well-timed scaffolding (Battista, 2001). One researcher limited their research by determining students’ van Hiele level in order to understand their understanding of tessellations (Callingham, 2004). Then, as the author studied students’ descriptions of tessellation patterns, it was noted that students tended to describe tessellations in terms of 2D or 3D shapes; some students saw movement of the shapes, and students at higher van Hiele levels describe tessellations as comprised of familiar shapes. The author postulated that students’ understanding of 2D representational conventions may be limited even during the later years of primary school. Further, the author proposes that students who see movement in the tessellation patterns may be more adequately prepared to recognize both line and rotational symmetry. And the author pointed to the limitation of identifying van Hiele levels because it omits the nature of the misconceptions (Callingham, 2004).
Also interesting to note is a doctoral dissertation designed to explore the difference between concrete, digital, or both concrete and digital manipulatives with a focus on probability (Taylor, 2001). Subjects (N=83) were partitioned into four groups, one control and three treatment. Treatment groups received similar lessons but had access to concrete, digital, or both concrete and digital manipulatives. In this case, a two by two matrix with level of concrete or digital manipulative revealed that students in the digital group, those with access to either digital or both concrete and digital manipulatives, were the only ones to achieve a significant difference in their probability performance (Taylor, 2001). This supports other literature that suggests technology improve mathematics instruction.

**Geometry Studies Using Concrete Manipulatives**

Seven studies focused on the use of concrete manipulatives in a geometry setting show the value of using manipulatives to teach geometry. These studies conclude that students adapt more readily to their environment (Martin et al., 2007), improve geometry performance (Bishop, 1980; Fuys et al., 1988; Jackson, 1999; Raphael & Wahlstrom, 1989), promote organizing actions to structure 2D arrays (Battista et al., 1998) and influence preservice elementary teachers to use manipulatives in their future teaching (Putney & Cass, 1998). While the Battista et al. study is detailed above under visualization studies employing concrete manipulatives, the other five studies are discussed in more detail presently.

Manipulative tools when used appropriately significantly improve the performance of children on spatial tasks and geometry (Bishop, 1980; Fuys et al., 1988; Raphael & Wahlstrom, 1989). Manipulatives make the most impact and promote more advanced reasoning when they are handled by the children and not just observed as teachers handle the manipulatives (Clements & McMillen, 1996; Jackson, 1999; Martin et al., 2007). While it may be more time intensive to include manipulatives, lessons that utilize manipulatives positively impact students and their
mathematical achievement; after observing sixteen classrooms, Jackson concludes that the majority of classrooms openly displayed (87%), included in lessons (82%), and allowed students to directly use (85%) manipulatives. Additionally, K to 2nd grade students (N=26, K N=8, 1st N=8, 2nd N=10) show more advanced reasoning given the opportunity to adapt their environment using manipulatives, change non-triangles into triangles in more advanced ways with manipulatives (pipe cleaners) and interact with and adapt to their environment given the manipulatives (Martin et al.). Martin et al. studied eleven subjects using manipulatives while fifteen subjects using pictures to first identify triangles from a given set of shapes and then to change the non-triangles into triangles from that same set of shapes. The students worked with shapes constructed of pipe cleaners for the manipulative group (N=11) or pictured on a worksheet for the picture group (N=15). These findings demonstrate the usefulness of manipulatives to aid geometry instruction.

**Geometry Studies Using Tangrams**

Researchers utilizing tangrams with first and second grade students noted that to place the tans, some students saw the diagram not only as divided into triangular regions, but also could correctly place the triangles and other tans mentally; still other students used trial and error with the concrete tangram pieces to determine the placement of the tans (Wheatley & Cobb, 1990). Students were given five tangram pieces (two small triangles, a medium triangle, a square, and a parallelogram) and asked to construct a square (See Figure 2-5). As often as needed, students were shown a picture of the square with the three triangles correctly placed. This highlights the visual processes promoted by tangram activities.

Wheatley (1998) reports research employing a tangram task, consisting of giving the students a figure (square) and briefly showing the pattern for completing the figure and showing the pattern again, if needed highlights students’ capability to from mental images. Students may
place some pieces, then stare off into space or ask to see the pattern again to complete their image, but the quality of number of placement of tans and sophistication of number constructions is in direct correspondence. Wheatley notes that tangrams are among the activities that encourage visualization as students form mental images; tangrams have also been demonstrated to promote “mathematics meaningful making” (p. 76).

Tangrams were used to investigate the influence of digital and concrete manipulatives on student achievement in visualization. Olkun (2003) considered the effect of and training with manipulatives (digital and concrete) as well as effects due to gender or grade. Fourth and fifth graders (N = 99) completed the pretest at the end of September and were assigned to three groups with 31 students in each group based on pretest scores to produce no difference between groups. (Six participants dropped out of the study, so 93 students comprise the sample.) One control group and two treatment groups completed the pretest and posttests, which are identical and consisted of 24 items (5 spatial, 7 spatio-numeric, 7 mental rotation, and 5 informal area measurement items). Piloted with 120 fourth to sixth graders, a reliability coefficient of alpha = 0.768 was reported for the pretest. The treatment consisted of 30 tangram designs, ranging from easy to more complex in number of pieces and transformations, and were completed during one session in mid October (80 to 120 minutes) digitally or using concrete tangrams, respectively, by the two treatment groups. Immediately following the treatment, the posttest was administered. Results reported include significant differences between experimental and control groups, but no significant difference between digital and concrete treatment groups. Higher scores in the experimental groups point to a definite benefit to using manipulatives for geometry instruction, but lead to no clear preference for digital or concrete manipulatives in this case. Increased gains using concrete manipulatives by fourth graders and similarly using digital manipulatives by fifth
graders suggests those types of manipulatives may be most effective for those age categories, but alternatively, this difference may also be due to increased computer experience of fifth graders (Olkun, 2003). Perhaps computers should simply be introduced earlier for all age categories. If students in fourth grade had been given access to computers at an earlier age, then their results may more closely parallel those from the fifth graders.

Findings that concrete and digital manipulatives improved visual thinking (Olkun, 2003) are supported by another study utilizing digital and concrete tangram designs. These designs proved to increase geometry scores and yielded a positive impact on students’ 2D geometric shape reasoning (Olkun et al., 2005). A sample of 100 Turkish fourth graders was selected from a larger sample of 279 students from 5 school sites in rural, urban, and suburban locations. These 100 subjects, selected according to pretest scores to form five homogenous groups, included three treatment groups and two control groups. The three treatment groups completed one session of 80 to 120 minutes of digital tangram designs, rested for five minute, and then took the posttest. The pretest was reported to have a reliability coefficient of alpha = 0.78 (n=279, number of items = 29) while the posttest reliability coefficient was alpha = 0.76 (n=100, number of items = 29). The treatment consisted of 40 digital tangram designs that required increasingly more pieces and more transformations as students progressed through the designs. Posttest scores showed that initial differences due to owning a computer or not were minimized by the digital tangram intervention, which points to appropriate interventions to increase the likelihood of students’ disadvantages being recovered (Olkun et al., 2005). Researchers also conclude that as students combine two equilateral triangles to form a square, parallelogram or larger triangle, they improve their capability to reason geometrically. Gender and socioeconomic status were considered as added comparisons in the data analysis in order to show that tangrams
not only increase performance, but also reduce the gap between male and female subjects as well as across socioeconomic status levels (Smith et al., 2003).

**Preservice Elementary Teachers’ Attitude Toward Geometry**

Attitude towards mathematics includes cognitive, affective, and performance components (Aiken, 1985). Affective measures include confidence, motivation, and engagement, which have been shown to be high in undergraduates electing to take mathematics classes (Galbraith & Haines, 1998). In 1972, Aiken’s measure of attitude included an enjoyment scale, while in 1974, Aiken augmented the previous scale with a value of mathematics scale. Aiken’s 1972 scale was used in the following study. Concrete manipulative materials as part of an instructional approach for 150 preservice elementary teachers were utilized to determine if preservice elementary teachers’ attitudes changed during the course of 40 hrs of work over a 10 week quarter. To improve the sample size, data was gathered from three smaller samples over three semesters. By administering the Mathematics Attitude Scale (Aiken, 1972) as both pretest and posttest and computing the gain score (gain score = pretest – posttest), a t-test revealed that attitude improved, both in negative attitudes diminishing and positive attitudes increasing. This highlights the effectiveness of concrete manipulatives to help improve preservice elementary teachers’ attitude toward mathematics (Putney & Cass, 1998).

Previous studies of attitude yielded improved attitude toward mathematics when knowledgeable teachers employed concrete materials (Sowell, 1989). Future studies of attitude could include control groups for comparison to the treatment groups, following up with the preservice elementary teachers who learned with manipulatives to determine if they will in turn teach with manipulatives, and investigating attitude over time to see if the positive increase in attitude toward mathematics is sustained (Putney & Cass, 1998). Only the first suggestion will be incorporated into the proposed research study.
Steen et al. (2006) found that during a two week geometry unit the treatment teacher’s daily journal entries about student beliefs, attitudes, and activities revealed that less engaged, less attentive, less motivated learners increased in engagement, attentiveness, and motivation as they worked with digital manipulatives. This treatment teacher felt digital manipulatives should be used early on and often in mathematics instruction to increase their apparent effectiveness (Steen et al.). This improvement in attitude toward mathematics has been noted by Sowell (1989), as well.

A call for research was issued by Clements and Battista (1992) to discover the specific cognitive constructions students use when using manipulatives, the types of computer environments that foster geometric learning, and how teachers can become “theoretically cognizant” (p. 458) as they utilize innovative materials and environments and knowledge about students’ learning. One aspect of students’ cognitive constructions includes their attitude, which consists of such factors as enjoyment, confidence, and usefulness (Utley, 2007) and has been shown to improve with the implementation of digital manipulatives (Sedig, 2007).

**Studies of Technology Implementation in Geometry Instruction**

Technology studies specific to geometry instruction tend to focus on the technology implementation rather than upon the steps that led to the selection of that technology (Fung, 2005; Moyer et al., 2005; Olkun, 2003; Olkun et al., 2005; Smith et al., 2003; Steen et al., 2006; Zacchi & Amato, 1999). In one case study, the subject worked on the mathematics with access to his own computer and the sole attention of the researcher (Fung). Another study was designed to give students access to their own computer by completing the intervention during the normally scheduled computer lab time of Turkish fourth and fifth graders (Olkun et al.). The single teacher moved around the computer lab monitoring student work. Alternately, students who worked together at one computer showed similar gains when working directly or watching a peer work.
on the computer (Smith et al.). Another study supports this stance, namely, students given access
to or the opportunity to observe technology performed at similar levels (Owens, n.d.). This
indicates that it is beneficial to pair students as they work in digital environments.

Often, it is the case that the researcher or teacher is responsible for multiple students, who
may be sharing one computer as a pair or trio of students (Moyer et al., 2005). Grouping subjects
about one computer does not seem to diminish the effects of the technology implementation.

**Studies incorporating digital manipulatives**

As schools increase the number and power of computers available to students in schools,
teachers need to deepen their knowledge of how best to implement technology. Implementation
of technology, such as digital manipulatives, will require good pedagogy, people, and
performance (Ferdig, 2006).

Nine studies exploring the use of digital manipulatives in geometry settings conclude that
technology increases geometry performance (Clements & Battista, 1989; Dixon, 1995a, 1995b;
Olkun et al., 2005; Sedig, 2007; Steen, 2002; Steen et al., 2006; Smith et al., 2003), promotes
exploration and discovery (Wyeth, 2002), and facilitates awareness of geometric properties
(Sarama et al., 2003). Six of these studies appeared in refereed journals within the last five years
and two studies are unpublished doctoral dissertations focused on geometry and digital
manipulatives (Dixon, 1995a; Steen, 2002).

The design of technology promotes technology implementation (Laborde, 2001; Sedig,
2007; Steffe & Olive, 2002; Wyeth, 2002). Child-initiated play, open-ended and discovery-
oriented, able to account for multiple achievement levels, minimal entry level knowledge, and
multiple solutions are some of the design issues called for (Bredekamp & Copple, 1997; Resnick,
1998). Flow, or easy transitions and a purpose to continue through the technology, is another
goal to achieve in technology educational designs (Sedig, 2007).
As reported in the previous section, Olkun et al. (2005) uncovered the impact computers produce on Turkish fourth and fifth graders’ 2D geometry reasoning. An intervention of digital tangram designs, described as a playful discovery of mathematics, provided an effective integration of technology and mathematics content (Olkun et al.). While initially those subjects with no computer at home scored lower, that difference was minimized with digital tangram designs (Olkun et al.). Steen (2002) studied the impact of digital manipulatives on 31 first grade students found that text, digital manipulatives, and written instruction promoted significant gains in the treatment group, while gains, though not significant, were made in the control group, who had access to text and written exercises only (Steen). These 31 first grade students were two intact classes taught by two teachers of similar educational and experience. A coin toss determined the treatment group that would use digital manipulatives to complete practice activities on wireless internet capable laptops. Both groups received pretest and posttests, instruction from the same text, and even the same practice when digital manipulatives were not available for that topic. Pretest and posttests consisted of the first and second grade geometry tests in the text. Pilot testing and reliability were not reported for these tests, perhaps because they are relatively standard tests. Significant gains in treatment group, and a gain, though not significant, in control group lead the researcher to conclude that these first graders are ready for third grade instruction in the second grade (Steen).

Sarama et al. (2003) found that digital manipulatives facilitate awareness of geometric properties. Investigating students’ conceptual development of 2D space, including grid or coordinate systems and rectangles, the researchers sought to describe the role of representation, including computer representations. From 4 fourth grade classes (N=80) in urban and suburban settings, case studies were compiled for 3 average students and their partners, who were given
pretest and posttest (paper and pencil) and pre and post interviews. These case studies provided the data for this research. The whole class worked for 13 sessions on sunken ships and grid patterns units. Students were paired to work on digital manipulative activities. Researchers observed, interviewed, and gathered work from the children. Digital manipulatives provided rich assessment situations and promoted creativity. Additionally, researchers found that students benefit from the incorporation of real-world examples that are then gradually faded out as the abstract concept becomes clear (Sarama et al.).

**Studies comparing concrete and digital manipulatives**

Five studies comparing concrete and digital manipulatives conclude that the digital manipulatives promoted more creative solutions, increased knowledge transfer, and communication of mathematical ideas (Moyer et al., 2005). Participants using digital manipulatives outperform those who work with concrete manipulatives (Gerretson, 1998). Reported above are the results that fourth and fifth graders utilizing digital and concrete tangram designs outperform those in the control group; manipulatives are beneficial to 2D geometry reasoning, but which type of manipulative to use may depend on age (Olkun, 2003).

**Value added features of digital manipulatives**

Of note is the idea that digital manipulatives must contain a value added feature, something that gives them value beyond that of their concrete counterparts, to improve performance or justify their adoption. Digital manipulatives include but are not limited to digital objects that enable learners to gain access to a wider variety of colors, shapes, and configurations of physical objects. In the case of educational software paired with concrete manipulatives, the extra explanations and lines that were designed to be a value added feature of the software were instead a distraction to students. The concrete manipulatives clarified the very points that were meant to be clarified by the extra explanation in the software (Zacchi & Amato, 1999). This
supports findings by Mayer and Moreno (1998) that digital information should have a clean design utilizing auditory cues rather than including extra written or visual cues.

The digital setting has the value added of enticing students to the mathematics because of the draw technology has on students today (Sarama & Clements, 2002). It helps students work more efficiently because the setup and clean up are quieter than concrete manipulatives. Digital manipulatives are increasingly varied and available on the Internet, while concrete manipulatives are limited to supplies on hand or time available to create new ones (Shackow, 2006-2007). And the time spent with digital manipulatives may be longer than the time spent with concrete manipulatives because learners may get caught up in the technology, lose track of time, and work longer than they had planned or anticipated (Sarama & Clements, 2002).

In some instances, concrete manipulatives could be substituted quite easily for digital manipulatives. However, middle school students have been shown to think concrete manipulatives childish; they may prefer digital manipulatives and see them as more desirable than concrete manipulatives (Shackow, 2006-2007). This argument may prove true for preservice elementary teachers and not only for middle school students. One researcher investigated performance on similarity tasks utilizing a dynamic geometry learning environment (Gerretson, 1998). Preservice elementary teachers (N=52) from 3 sections of an elementary education mathematics methods course were randomly assigned within each section to treatment and control groups. Each group completed a pretest, three sessions, and a posttest. Treatment group sessions were completed in a computer lab using The Geometer’s Sketchpad (Jackiw, 1995), while the control group worked in a mathematics laboratory with access to concrete manipulatives, protractor, and ruler. Controlling for initial differences using the pretest and standardized achievement tests, an analysis of covariance shows that the treatment groups
outperformed the control group. The digital treatment group performed above the concrete treatment group (Gerretson). This study points to the effectiveness of digital manipulatives over concrete manipulatives.

Similar gains have been shown to occur for not only concrete manipulatives, but also for digital manipulatives. Students given the option to utilize both concrete and digital manipulatives surprised researchers when they arranged the concrete manipulatives not on the desk next to the computer but instead held up the concrete manipulatives right on the computer screen (Zacchi & Amato, 1999). They placed tangram shapes on the screen in an effort to find the correct angle of rotation, used a reflection mirror to discover the correct axis for reflection, and placed Popsicle sticks on the computer screen as they solved their problems. Researchers altered their set of concrete manipulatives to fit this behavior during the second session, creating simpler manipulatives that could be more easily utilized on the computer screen. They exchanged a geometric grid used for rotation to simple Popsicle sticks connected at one end so that students could find the angle of rotation and see it instead of the angle being hidden beneath the grid. Also, they added tracing paper to save the students from contorting their bodies to see the angle of reflection for their shape. In addition to the mirror, the students could trace the shape on the computer screen and then fold the tracing paper to find the correct angle (Zacchi & Amato). Students’ comments pointed out that the concrete manipulatives helped them understand and gave them something to work with when they couldn’t visualize it, were 3D, and were something to hold in hand (Zacchi & Amato). Concrete manipulatives used on the screen merged operations in concrete and digital worlds and blurred distinctions between the concrete and abstract world (Zacchi & Amato). Thus the concrete manipulatives aided visualization when paired with digital manipulative explorations.
Another study employed both concrete and digital manipulatives to investigate children’s use of digital manipulatives and how they impacted or enhanced children’s understanding of mathematical concepts and skills. An action research project was designed to include two classrooms of 18 kindergarteners and 19 second graders and their two teachers (Moyer et al., 2005). By analyzing observation data and student work, researchers conclude that the 18 kindergarteners produce more patterns with more creativity using digital pattern blocks than when using wooden pattern blocks or drawing patterns (Moyer et al.).

The 18 kindergarteners participated in three days of patterning lessons using first paper, second digital pattern blocks, and finally concrete pattern blocks. Researchers compared the number of patterns, the number of blocks in pattern stem, the number of blocks per pattern and the average number of blocks per pattern (Moyer et al., 2005). Digital pattern blocks produced greater numbers and variety of patterns. Likewise, the 19 second graders use place value and transfer place value knowledge to non-pictoral addition after using digital place value rods. Digital tens rods seemed to encourage students’ written work to simulate the computer, for they circled ten ones and drew an arrow to show they were being traded for a ten rod, as in the lasso feature of the digital ten rods. Also, ten ones were lined up next to a ten rod to show ten ones equal one ten rod (Moyer et al.). These second graders participated in two days of place value instruction, using digital place value rods on the first day and the second day using four pieces of paper, where they recorded both the solutions and written explanations. Again, work with the digital place rods points to an increased understanding of place value and a greater variety of strategies used to solve the problems. Indeed, researchers found that for kindergartners and second graders, digital manipulatives allowed communication of mathematical ideas, such as
reasoning, problem solving, justification of solutions, increased variety of solutions expressed, and in greater understanding of patterns and place value.

**Summary**

Students’ 2D visualization varies widely from student to student and surprisingly, across age categories (Callingham, 2004; Clements, 2003). Studies to date include those of van Hiele levels (Burger, 1986; Burger & Shaughnessy, 1986; Callingham, 2004; Mayberry, 1983; Usiskin, 1982;), spatial ability or spatial visualization (Battista et al., 1998; Clements & Battista, 1992; Dixon, 1995a, 1995b; Fung, 2005; Gerretson, 1998; Hershkowitz, 1989; Olkun, 2003; Olkun et al., 2005; Robinson, 1994; Smith et al., 2003), and dynamic geometry software (DGS) (Clements & Battista, 1989; Dixon; Jackiw & Sinclair, 2002; Marriotti, 2002; Papert, 2002; Sarama & Clements, 2002). Studies also consider concrete or digital manipulatives as a means to improve visualization (Battista et al., 1998; Fung, 2005; Moyer et al., 2005; Olkun et al., 2005; Steen, 2002; Steen et al., 2006). Findings show that visualization can be improved and manipulatives, particularly digital manipulatives, are a viable intervention. Incorporation of concrete or digital manipulatives plays a role in improving preservice elementary teachers’ attitude toward geometry.

Visualization studies include six studies focused on concrete manipulatives within the visualization setting (Battista et al., 1998; Bishop, 1980; Fuys et al., 1988; Jackson, 1999; Martin et al., 2007; Putney & Cass, 1998). Geometry is shown to improve, particularly visualization, with interventions such as digital manipulatives (Dixon, 1995a; Olkun, 2003; Sedig, 2007; Steen, 2002; Steen et al., 2006; Taylor, 2001), concrete manipulatives (Battista et al., 1998; Jackson; Martin et al., 2007), and a combination of digital and concrete manipulatives (Moyer et al., 2005; Olkun et al., 2005; Wyeth, 2002; Zacchi & Amato, 1999). These interventions prove to narrow
the achievement gap (Olkun et al., 2005; Sedig, 2007) and improve visualization (Sedig, 2007; Smith et al., 2003).

Studies to date show the importance of selecting appropriate technologies, such as digital manipulatives, Super Tangrams (Sedig, 2007), and tangram designs. Nine studies of digital manipulatives, including both digital manipulatives and digital tangrams, yield increased performance levels (Clements & Battista, 1989; Dixon, 1995a, 1995b; Olkun et al., 2005; Sarama et al., 2003; Sedig; Smith et al., 2003; Steen, 2002; Wheatley, 1998; Wyeth, 2002). The effective implementation of technology yields increased variety and output in student work, as well as improving student’s skills to reason and solve problems, then justify those solutions (Moyer et al., 2005). One method of technology implementation includes augmenting paper and pencil practice with technology practice, demonstrating that when geometry instruction includes lessons about visualization utilizing technology, students’ achievement improves (Clements & Battista; Dixon).

Finally, four studies comparing both concrete and digital manipulatives show increased gains for digital manipulatives (Gerretson, 1998; Moyer et al., 2005; Steen, 2002; Taylor, 2001), while one study (Olkun, 2003) and a case study (Fung, 2005) show no significant differences on measures of visualization between concrete and digital interventions. A related study granted access to both concrete and digital manipulatives to subjects and found that this diminished distinctions between abstract and concrete world (Zacchi & Amato, 1999).

Preservice elementary teachers’ 2D visualization may be influenced by the study of digital and concrete tangrams to perform symmetry, similarity, and congruence tasks. The literature yields much in the way of tessellations, transformations, and rotations with a focus on manipulatives or dynamic geometry software such as Geometer’s Sketchpad (Jackiw, 1995) and
less on symmetry, similarity (Gerretson, 1998), and congruence. While tangrams both digital and concrete are mentioned in the literature (Olkun et al., 2005; Sedig, 2007; Smith et al., 2003; Wheatley, 1998; Wheatley & Cobb, 1990), digital and concrete manipulatives of various types are addressed more frequently in the literature. Studies also focus on elementary or middle school students (Callingham, 2004) leaving a dearth of research regarding preservice elementary teachers’ use of tangrams. However, the research that compares students in grades 5 to 8 with preservice and inservice elementary teachers yields findings that categorize all of these learners with similar behavior patterns on visual tasks (Hershkowitz, 1989). This gives credence to comparing literature from students and teachers, both preservice and inservice.

Preservice elementary teacher education continues to receive focus. Training for these teachers must be carefully designed and implemented to prepare them for their future classrooms (Presmeg, 2006). Part of this training is attention to their attitude toward geometry (Utley, 2004). Creativity and reasoning lead to improved attitudes toward mathematics (Putney & Cass, 1998; Steen, 2002). Improving attitude will be part of a solid foundation preparing teachers for future teaching (Putney & Cass). One key to improved attitude toward geometry is the incorporation of manipulatives, concrete (Sowell, 1989) or digital (Sedig, 2007). Digital and concrete manipulatives promote visualization within the geometry content, which content is ever important for teacher training (Clements, 2003; Presmeg; Shulman, 1986; Taylor, 2001). As preservice elementary teacher content knowledge improves, their attitude also improves (Sedig, 2007; Usiskin, 1987).

There is a need for research that seeks to improve 2D visualization through the use of manipulatives, both digital and concrete, and to aid preservice elementary teachers in their
preparation to teach. In addition to improved 2D visualization, research should explore how preservice elementary teachers’ attitude toward geometry improves.
Figure 2-1. Two triangles form a square, a parallelogram, and a larger triangle.

Figure 2-2. Example of spatial structuring; without gridlines drawn in, students struggle to visualize tiling a rectangle with squares.

Figure 2-3. Example of geometric structuring; right triangle rotated about the right angle vertex 90, 180, and 270 degrees.

Figure 2-4. Example of axiomatic structuring; decomposition of rectangle into triangles.

Figure 2-5. Two small triangles, a medium triangle, a square, and a parallelogram form a square.
CHAPTER 3  
METHODOLOGY

The purpose of this study was to investigate the impact of digital manipulatives, concrete manipulatives or the choice of digital and concrete manipulatives on preservice elementary teachers’ 2D visualization skills within the geometry content. Of secondary interest to the researcher was preservice elementary teachers’ attitudes toward geometry. Accordingly, the researcher sought to investigate changes in attitude toward geometry as a result of the intervention as well as relationship of attitude toward geometry to performance on 2D visualization measures.

This study was an extension of a study of Turkish fourth and fifth graders which demonstrated that concrete tangrams improved 2D visualization. The population was altered to study preservice elementary teachers, who perform at levels similar to fourth and fifth graders (Fuys et al., 1988; Mayberry, 1983). The addition of a group with access to both digital and concrete manipulatives as suggested by Olkun (2003) was included in the current study. And an attitude toward geometry measure was employed to measure not only any influence upon attitude but also any correlation between attitude toward geometry and 2D visualization. These gains in 2D visualization supported literature suggesting that practice can improve 2D visualization (Olkun; Olkun et al., 2005; Smith et al., 2003; Wheatley, 1997, 1998).

Research Questions

The following research questions were employed for this study.

- **Research question 1:** Does use of digital tangrams influence preservice elementary teachers’ 2D visualization on tangram designs?

- **Research question 2:** Does use of concrete tangrams influence preservice elementary teachers’ 2D visualization on tangram designs?

- **Research question 3:** Does the choice to use concrete or digital tangrams influence preservice elementary teachers’ 2D visualization on tangram designs?
• **Research question 4:** Does the use of concrete, digital, or the choice to use concrete or digital tangrams to complete tangram designs influence preservice elementary teachers’ attitudes toward geometry?

• **Research question 5:** Does a relationship between preservice elementary teachers’ 2D visualization and attitude toward geometry exist?

**Selection of Evaluation Instruments**

Both 2D visualization and attitude toward geometry were measured in a pretest and posttest design. To evaluate 2D visualization, two measures were used. First, Wheatley (1996) added Form B to the Wheatley Spatial Ability Test (WSAT) that was first developed in 1978. Second, in 2005 Olkun et al. added five items to the Test of Spatial Visualization in 2D Geometry (VST). These updated versions of the WSAT and VST were employed to measure 2D visualization for this study; attitude toward geometry was measured as well. In 2007, Utley published reliability and validity for the Utley Geometry Attitude Scales (UGAS). A description of each test as well as reliability and validity associated with each test follow.

The WSAT focused specifically on 2D visualization and mental rotations. Tangram shapes were not involved, but the mental rotations necessary for the tangram intervention were similar to the mental rotations needed to complete the WSAT. The WSAT consisted of 100 multiple choice items that were to be completed within an eight minute time limit. The items were grouped in sets of five. One shape was pictured and subjects were asked to mark yes or no on five related shapes, yes if the shape can be obtained by rotating the original shape. All together, to explain the test, distribute and collect papers, the test took approximately 25 minutes to administer. Norms were reported for third through fifth grades (Wheatley, 1996). The publisher reported an internal consistency (K-R20 = .92) and suggested that it was a valid and reliable test. Other studies agreed with this assertion and showed that the WSAT predicted

The Purdue Visualization of Rotation test (Guay, 1977), the Shephard-Metzler Rotations test (Shephard & Metzler, 1971; Vandenberg, 1975), The Minnesota Paper Folding Test and the Revised Minnesota Paper Foam Board tests were considered inadequate measures for this research. The first two measure 3D visualization (Bodner & Guay, 1997; Vandenberg, 1975) and the latter two have been shown to be analytic processing instead of gestalt processing, or processing mental representations of figures as organized wholes (Bodner & Guay; Guay et al., 1978). Guay et al. studied spatial ability tests and found that the former two tests are least likely and the latter two most likely to be confounded by analytic processing.

Tangram pieces were incorporated in the VST (Olkun et al., 2005) (see Appendix A). This test was developed by researchers to measure 2D visualization in conjunction with the tangram intervention selected for the current research. Twenty-four items (Olkun, 2003) were augmented by five more items. The augmented measure was tested for reliability by pretesting fourth and fifth grade students (N = 279), and then posttesting a sampling of those same students (N = 100). Reliability was reported as alpha = .78 on the pretest (N=279) and alpha = .76 on the posttest (N=100). Olkun et al. noted validity is evidenced by significant differences on the pretest between the fourth graders (N = 224) and fifth graders (N = 55). This measure contained four subsections, namely, spatial, spatio-numeric, mental rotation, and informal area measurement items with eight, eight, eight and five items, respectively (Olkun et al., 2005). In this way, it incorporated elements of the WSAT and the added dimension of spatial and measurement items. For this reason, it was deemed a good complement to use in tandem with the WSAT to document 2D visualization prior to and following the treatment.
Additionally, participants completed the UGAS (Utley, 2007) to measure preservice elementary teacher attitudes toward geometry (see Appendix B). This test was specific to geometry and consisted of three scales, similar to the Mathematics Attitude Scale (Aiken, 1972, 1974) and consisted of 32 items, each item is a 5-point Likert-scale survey question, with 17 items worded positively and 15 items worded negatively (Utley). Using a sample of 264 undergraduate students, internal consistency reliability analysis of all 32 items yielded Cronbach’s alpha = 0.96 (Utley). Reliability on the three subscales, confidence, usefulness, and enjoyment were, respectively, alpha = 0.95, alpha = 0.93 and alpha = 0.92 (Utley). While Utley suggested that the UGAS may be useful in examining the relationship between achievement in geometry and students’ attitudes toward geometry (2007), she also devised geometry activities to improve attitude (Utley, 2004). This test was administered in a pretest and posttest design to detect changes in preservice elementary teachers’ attitude toward geometry in an effort to contribute to evidence for using the UGAS to detect change in attitude.

**Selection of Treatment**

Among treatments to improve visualization were whole courses, daily activities such as those in *Quick Draw* (Wheatley, 2007), and ideas relating to tangrams. Books devoted to tangrams include educational books, such as Tangramath (Seymore, 1971). Multiple software and internet resources existed for tangrams, as well. A treatment consisting of 30 tangram designs (see Appendix C) were specifically selected to proceed from simple to complex in terms of number of pieces and rotations to complete the problems. While these designs were originally used with fourth and fifth grade students to improve their 2D visualization, preservice elementary teachers’ 2D visualization has been shown to be similar to fourth and fifth graders (Mayberry, 1983). These designs promoted improved 2D visualization and future research including a third treatment group who had the choice of using digital or concrete tangrams was
suggested. Thus, it was deemed useful to pursue this study employing these 30 designs giving students a focus as well as practice in concretely representing their answers on their solution page (see Appendix D). The designs engaged participants in covering the designs with geometric shapes, which proved beneficial to mathematical development (Wheatley & Reynolds, 1996).

**Pilot Study**

The pilot study was completed in four phases to determine that the WSAT and VST instruments and both concrete and digital interventions were suitable for the population of preservice elementary teachers as well as to the population of fourth and fifth graders used by previous researchers (Spencer, n.d.). The WSAT was piloted on 52 preservice elementary teachers, the VST on 27 preservice elementary teachers, the concrete intervention with 27 preservice elementary teachers and the digital intervention with 24 preservice elementary teachers. Reliability for the WSAT was .94 (N = 26, number of items = 100). Reliability for the VST was .70 (N = 22, number of items = 24). These participants noticed three details that aided the implementation of the VST including wording of the items, writing directly on the test, and defining the word cover. These three details helped to clarify what instructions the researcher needed to provide to the study participants.

These preservice elementary teachers were in their junior year one semester prior to elementary education mathematics methods enrolled in three separate sections of a course titled teaching reading in primary grades taught by three different instructors. It was helpful to see their reaction to the instruments and interventions. Their suggestions and comments were incorporated into the study. From this pilot, it appears that the WSAT and VST as well as the concrete and digital interventions, though developed for fourth and fifth graders, are appropriate for preservice elementary teachers.
To pilot the interventions, 27 preservice elementary teachers completed the 30 tangram designs using concrete tangram sets and 24 preservice elementary teachers completed digital tangram designs. The purpose of this portion of the pilot study was to gather data to support the use of the intervention for this population (see Appendix D). Originally, this intervention, 30 tangram designs ranging from simple to complex in number of tangram pieces and number of transformations to complete each design, was employed for a sample of fourth and fifth graders (Olkun et al., 2005). Tangrams are typically included within an elementary education mathematics methods course to demonstrate area or other geometry concepts (Van de Walle, 2004). Within 2D geometry, tangrams and pattern blocks are the two most well known activities to explore the creation of larger shapes from smaller shapes or to fit together small shapes to form larger shapes (Van de Walle, 2004). Data collected during this portion of the pilot study provided evidence to determine the validity of utilizing this intervention with preservice elementary teachers. Also, the duration for the intervention was settled at 30 to 45 minutes.

Preservice elementary teachers worked in pairs or trios to complete the intervention. Pairs and trios promoted discourse, competition, scaffolding, positive feedback, and engagement. Preservice elementary teachers in an elementary education mathematics methods course typically work in pairs or small groups to complete activities that prepare them for their future classrooms. In this way, treatment and control groups matched. Participants in the pilot study communicated about the ease or difficulty of solutions, the challenge to find a solution, and the method to record those solutions. Competition between groups promoted a playful atmosphere and encouraged students to avail themselves to find solutions. Scaffolding occurred naturally. One pair worked as if one person with four hands, moving tangram pieces in and out of the workspace fluidly. Another participant noted that while watching her partner struggle to place
tangram pieces, she saw where one piece needed to be, placed it there, and watched as the solution came together. This led to encouragement from each partner and engagement as they worked together to finish the designs. It was found that pairs provide a vital component of the intervention as students work together to solve the problems and to keep the orientation of the activity playful (Steffe & Weigel, 1994).

The four phases of the pilot study provided evidence of the suitability of both measures and interventions for the population of preservice elementary teachers, yielded reliability data for the measures for this population, and suggested the timing appropriate for the current study. Also, the instructional phase was enriched with added explanations and definitions to make the administration of the study uniform for all groups.

**Selection and Description of Sample**

The study sample consisted of 74 preservice elementary teachers attending a southeastern research university. These participants were predominantly female of various ability levels and enrolled in four sections of elementary education mathematics methods course taught by four instructors. A quasi-experimental design was used to measure four groups as intact classes that formed three treatment groups and one control group. The treatment groups received additional practice in 2D visualization by completing 30 tangram designs and recording their solutions. The treatment groups completed the same 30 tangram designs using concrete or digital tangrams. The first group had access to concrete tangrams, the second group had access to digital tangrams, and the third group had access to both concrete and digital tangrams. The control group continued with the regular course outline, which includes one 3-hour session focusing on geometry. All groups completed pretesting and posttesting prior to the regularly scheduled geometry instruction for the course.
Historically, these preservice elementary teachers are observed to be deficient in geometry (Fuys et al., 1988; Mayberry, 1983). They may struggle to recall geometry learned from one class in high school and have little or no experience with geometry manipulatives. Preservice elementary teachers’ deficiencies in geometry may impact their attitude toward geometry (Usiskin, 1987). Alternately, attitude toward geometry may impact preservice elementary teachers’ 2D visualization. Technology is also inherently appealing to today’s learners. This sample did not participate in a geometry test, but demonstrated various ability levels on the 2D visualization tests. They also experienced difficulty recalling definitions for symmetry, similarity, and congruence. While some subjects were familiar with tangrams, other subjects noted never seeing tangrams before. Participants also were observed to work diligently with a generally positive attitude, though attitudes varied overall. And participants were noted to be familiar and comfortable working with technology, demonstrating a preference for technology by electing to work solely with technology when both concrete and digital tangrams were available.

**Procedures**

Prior to the study, the researcher obtained permission from the University Institutional Review Board for the investigation to take place (see Appendix E). The preservice elementary teachers were informed and asked to sign a consent form to participate in the study (see Appendix F). The researcher administered the pretest, intervention, and posttest in class. A schedule for the procedures for the two 90-minute sessions included a first session to pretest, define 2D visualization, tangrams, symmetry, similarity, and congruence and complete five simple tangram problems and a second session to complete 30 tangram designs and posttest (see Table 3-1).
Treatment Design

Practice during the course includes a three-hour geometry lesson focused on 2D and 3D geometry content. The activities, discussions, and assessment focus on preparing the preservice elementary teachers to teach elementary students the basic skills and relationships of geometry. Preservice elementary teachers work in pairs or small groups to complete these activities and assessments.

Preservice elementary teachers rely on the practice they receive in class to prepare for their future students. For this reason, this study will focus on tangram activities that are readily transferable to the elementary classroom. These activities have been shown to further the 2D visualization of Turkish fourth and fifth grade students (Olkun, 2003). While preservice elementary teachers in age are further ahead than fourth and fifth grade students, in van Hiele level, they often perform at a similar level to that of the fourth and fifth grade students (Mayberry, 1983). Van Hiele levels are not age dependent, but dependent upon experience and exposure to geometry principles and content. Including focused geometry experiences into the preservice elementary teachers’ geometry training is designed to influence their 2D visualization.

Manipulatives may prove to produce an effect not only for 2D visualization but also for attitude toward geometry. Concrete or digital manipulatives can then become a tool to aid instruction of preservice elementary teachers’ future students in geometry. Tangrams are a non-threatening manipulative because teachers may have seen them before or view them for their entertainment value.

All sections received one 3-hour class session focused on geometry as part of the regular schedule for the course. Preservice elementary teachers worked in pairs or small groups during this instruction to explore activities that will enliven their future classrooms. This regular geometry lesson occurred after the intervention to avoid contamination of the data. The three
treatment groups worked to complete 30 tangram designs for the first 90 minutes of two 3-hour class sessions. In this way, they completed the pretest, intervention, and posttest prior to the regularly scheduled geometry instruction.

The treatment groups had access to concrete tangrams and digital tangrams. The study was performed in a quasi-experimental pretest-posttest design with the WSAT (Wheatley, 1996), VST (Olkun et al., 2005) and UGAS (Utley, 2007) tests administered as both pretest and posttest. All preservice elementary teachers participated in the sessions in during their scheduled classtime.

The preservice elementary teachers participating in the concrete tangram group had access to a set of concrete tangrams. The digital group met in a computer lab to access the digital tangrams. Finally, those in the digital and concrete group also met in the computer lab to access both the concrete tangrams as well as the computers for the digital tangrams. It was anticipated that the latter group would either use a combination of concrete and digital tangrams or show preference for a single type of tangram format.

**Treatment: Tangram Designs**

The description of the visualization activities are comprised of a definition of 2D visualization, and introduction to tangrams, the solution of five simple tangram designs, and 30 tangram designs that range in difficulty. These tangram designs promoted exploration and creativity to encourage preservice elementary teachers to explore tangram problem solutions. The first tangram designs require fewer tangram pieces and fewer transformations, while the final tangram designs utilize all seven tangram pieces and require more transformations to complete (Olkun et al., 2005).

If 2D visualization concepts are studied early in geometry, it will enliven the geometry curriculum (Niven, 1987). For this reason, the treatment was implemented early during the 2007
fall semester. As preservice elementary teachers worked to cover the tangram designs with the tangram pieces, they explored concepts of 2D visualization, which arise by combining two small triangles to form a square, bigger triangle, or parallelogram in an attempt to cover the tangram designs (Dana, 1987; Farrell, 1987). These covering actions comprise informal exploratory activities to teach 2D visualization, which includes recognition and creation of similar shapes (Friedlander & Lappan, 1987). By arranging tangram pieces within the tangram outlines, preservice elementary teachers engage in a matching activity that aids the identification of similar shapes (Friedlander & Lappan). Various tangram designs presented shapes from nature (Usiskin, 1987).

In summary, tangram designs promoted the exploration of 2D visualization. Patterns from nature, squares, and other shapes encouraged preservice elementary teachers to engage in transformations of tangram pieces to complete tangram designs.

**Description of the Instructional Phase**

Preservice elementary teachers participated in the intervention during two 90-minute sessions. These sessions were taught by the researcher because each group had a different instructor, which would introduce instructor variability into the study. All sessions occurred during the regularly scheduled elementary education mathematics methods courses during 2007 fall semester. These two sessions occurred in two consecutive weeks, with the pretest administered during the first session and the post test completed during the second and final session. The control group completed the informed consent form, which briefly described the study, and the pretests and posttests.

Whole class discussion set the stage for the tangram problem exploration. Visualization was defined: Visualization, the act of creating, recreating, and acting upon mental figures, can be improved with practice and intention, deteriorates over time if not practiced with intention, and
aids problem solving. Because visualization is one of the four geometry content areas pinpointed by NCTM (2000), at the beginning of the study, participants were told the impact that this can make not only on their geometry teaching but also upon their future students’ geometry success (Presmeg, 2006). Numerous studies suggest that visualization may be improved with focused practiced and instruction. Improving visualization not only improves geometry performance, but also impacts mathematics achievement (Battista, Wheatley, & Talsma, 1982) and related areas such as science, art, and engineering. For instance, a chemistry professor decided that his students were deficient in visualization skills and this predicted their performance in chemistry (Bodner & Guay, 1997). Engineers who took a visualization test were then predicted to improve (Humphreys et al., 1993) much the same as mathematics students who took a mathematics test as a predictor of future performance. Visualization may be downgraded in importance by mathematicians (Arcavi, 2003) instead of acclaimed for its critical importance to their work of solving problems. Students who value visualization will in turn solve problems better (Arcavi). Tangrams are one tool to improve visualization.

The whole class discussion continued with a description of tangrams. It was suggested that preservice teachers use tangrams as a cultural element because of their Chinese origin. The link tangrams provide between mathematics and literature by incorporating Grandfather Tang’s Story (Tompert, 1990) or other books about tangrams was also mentioned. Next, the diversity of tangram designs, including geometrical, animal, or alphabet shapes was noted. Finally, the simplicity of deriving a set of tangrams from a square was discussed.

A few preliminary problems familiarized students with placing the tangram pieces. They were asked to place four tangram pieces in a rectangle, three tangram pieces in a square, and five tangram pieces in a square (see Figure 3-1). These problems gave students a sense of how to
combine the tangram pieces to form larger images as well as provide simple examples for use in their future classrooms. Suggestions of finding the 13 convex shapes that can be formed with all seven tangram pieces or forming a square, triangle, rectangle, parallelogram or trapezoid from one, two, three, four, five, six, or seven tangram pieces were provided to give participants a sense of how to adapt tangram activities for different learners. This also served to demonstrate the many and varied uses of tangrams in the classroom.

With definitions clarified, the students were paired. They were told that the designs progress from easy to challenging and that if a particular design was difficult, that design may be set aside to solve later. Then, participants were given the answer sheet. A sample problem was solved to demonstrate how to record a solution. Finally, the set of tangram designs on paper accompanied with concrete tangram sets were distributed, except for the digital tangram group, who viewed the tangram sets and designs only on the computer. In the case of the digital group and the digital and concrete group, the researcher logged in to the computers and had the Tangram software and file open ready for students to begin.

Students were given 45 minutes to complete the tangram designs. The simple tangram designs at the beginning of the intervention enticed participants to challenge themselves with the more complex subsequent tangram designs. And this increasing difficulty also improved working memory capacity, for as participants engaged in the intervention, their skill chunking two or more tangram pieces into a larger shape improved, as noted in the description of the pilot study (Spencer, n.d.). At the conclusion of the intervention, the posttests were administered.

**Statistical Procedures**

A quasi-experimental design was used for the study. Four groups with one intact class in each group made up three treatment and one control group. The concrete group utilized concrete tangrams. The digital group used computers to access digital tangrams. And the digital and
concrete group had access to both concrete and digital tangrams. The control group simply completed pretest and posttests. The quasi-experimental research design was comprised of a 2x2 matrix: the level of concrete tangram use and the level of digital tangram use. The objective was to determine the effect of the independent variables, individually and interactively, upon the dependent variables. The dependent variable was 2D visualization. The independent variable was tangram format, concrete, digital, or both concrete and digital.

To analyze the data collected during the study, a 2x2 repeated measures analysis of variance (ANOVA) was conducted. Two factors, concrete and digital tangrams, with two levels each were considered (see Figure 3-2). The WSAT (Wheatley, 1996) and VST (Olkun et al., 2005) served to control for 2D visual skill level and showed that there were no between-group differences in the statistical analysis of the first four research questions. All four groups completed both the pretest and posttests.

The UGAS (Utley, 2007) demonstrated the relationship between attitude toward geometry and 2D visualization based upon a correlation of UGAS and first WSAT and then VST. A 2x2 repeated measures ANOVA with the WSAT and VST as dependent variables and group as the fixed factor was conducted. The UGAS was employed as both pretest and posttest to explore changes in attitude toward geometry upon the completion of the 30 tangram designs. No between-group differences appeared in the analysis of UGAS with WSAT and VST making the repeated measures ANOVA sufficient for analysis of the data.

**Summary**

In an effort to discover differences between preservice elementary teachers’ 2D visualization following tangram explorations using concrete, digital, and concrete and digital tangrams, the design of the study followed a quasi-experimental pretest and posttest control group design. Four sections of an elementary education mathematics methods course provided
the sample of 74 preservice elementary teachers, with each section being assigned as one of the three treatment or one control groups. The timeline consisted of two 90 minute sessions, with the pretest given at the start of the first session and the posttest administered at the conclusion of the second session. Sessions for all groups were taught by the researcher during the regular classtime. The concrete tangram group utilized concrete tangrams; the digital tangram group used computers to access digital tangrams; and the digital and concrete group used both concrete tangrams and computers with access to digital tangrams.
Table 3-1. Schedule for procedures.

<table>
<thead>
<tr>
<th>Group</th>
<th>Pretest, definition of terms, 5 designs</th>
<th>Complete 30 tangram designs and posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>Oct 15</td>
<td>Oct 29</td>
</tr>
<tr>
<td>Concrete</td>
<td>Sept 13</td>
<td>Sept 20</td>
</tr>
<tr>
<td>Digital</td>
<td>Oct 11</td>
<td>Oct 18</td>
</tr>
<tr>
<td>Digital and Concrete</td>
<td>Oct 22</td>
<td>Oct 29</td>
</tr>
</tbody>
</table>
Figure 3-1. Four tangram pieces form a rectangle; three and five tangram pieces form a square.

Figure 3-2. Repeated measures ANOVA design with two levels.
CHAPTER 4
RESULTS

This chapter summarizes the results pertaining to 2D visualization in regards to the five research questions of the study. The first three research questions examine 2D visualization in connection with the format of the tangram intervention, concrete, digital, or both concrete and digital. The last two research questions address attitude toward geometry, its improvement upon completion of the intervention and its correlation to 2D visualization. These research questions will be explored in sequence followed by a discussion of findings.

Results

A brief synopsis of the analysis of the data will be followed by the results related to each of the research questions. This synopsis includes assumptions of the statistical model, the associated statistics, and the relation of these statistics to the research questions. The results of the ANOVA with repeated measures for two group factors (concrete and digital) and one within factor (occasion of the pretest or posttest) for each test are included in this chapter.

Analysis of the Data

Three ANOVAs with repeated measures were used for the tests, two 2D visualization tests (WSAT and VST) and the attitude toward geometry test (UGAS). Each ANOVA had two non-repeated factors, concrete and digital, with two levels: concrete groups had access to concrete tangrams and nonconcrete groups did not have access to concrete tangrams; digital groups had access to digital tangrams and nondigital groups did not have access to digital tangrams. Each ANOVA had a repeated factor, occasion, with two levels: pretest and posttest. The complete table of raw data values is in Appendix F. An alpha level of .05 was used for all statistical tests.
The ANOVA for the WSAT indicated no significant difference between the concrete and the nonconcrete group as well as no significant difference between the digital and the nondigital groups. A significant effect was found for the WSAT pretest and posttest averaged across the four groups, $F(1, 70) = 48.97), p < .01$. Within the occasions of the WSAT pretest and posttests, the digital factor also showed a significant interaction with occasion, $F(1, 70) = 8.18, p < .01$ (see Table 4-1). The significant interaction is shown in Figure 4-1. The means for nondigital and digital groups on the WSAT pretest were 74.94 and 77.50, respectively; the means for nondigital and digital groups on the WSAT posttest were 88.81 and 83.40, respectively.

Similarly, the ANOVA for the VST indicated no significant difference between the concrete and the nonconcrete groups as well as no significant difference between the digital and the nondigital groups. No significant effect was found for the VST pretest and posttest averaged across the four groups. A significant interaction was found within the occasions of the VST pretest and posttests for the digital factor, $F(1, 70) = 5.67, p = .02$ (see Table 4-2). The significant interaction is shown in Figure 4-2. The means for nondigital and digital groups on the VST pretest were 25.56 and 26.50, respectively; the means for nondigital and digital groups on the VST posttest were 26.38 and 25.90, respectively.

As with the results of 2D visualization, the ANOVA for the UGAS indicated no significant difference between the concrete and the nonconcrete groups as well as no significant difference between the digital and the nondigital groups. A significant effect was found for occasion of the UGAS pretest and posttest, $F(1, 70) = 4.79, p = .03$ (see Table 4-3). No significant effect was found within the concrete and nonconcrete groups and the occasion of the UGAS pretest and posttest. Additionally, no significant effect was found within the digital and nondigital groups and occasion.
To address the fifth research question, two correlations were conducted, first, a correlation of the UGAS posttest and WSAT posttest and second, a correlation of the UGAS posttest and VST posttest. Correlation was not significant between UGAS and WSAT, \( r(74) = .21, p = .08 \). Correlation was significant between the UGAS and the VST, \( r(74) = .29, p = .01 \). This demonstrated the positive relationship between attitude toward geometry and 2D visualization. Increased levels of 2D visualization corresponded with increased attitude toward geometry.

This concludes the analysis of the data. Statistical values and their interpretations will now be presented in connection to each of the five research questions.

**Research Questions, Associated Statistical Values, and Interpretations**

The first three research questions are presented together because they each address the question of how various formats of tangrams influence 2D visualization. The last two research questions seek to explain variations in attitude toward geometry in connection to 2D visualization.

**The Influence of Tangram Format upon Two-Dimensional Visualization**

The first research question addressed the influence of digital tangrams upon 2D visualization and is stated as follows: Does use of digital tangrams influence preservice elementary teachers’ 2D visualization on tangram designs? Results demonstrate that all groups significantly improved performance from pretest to posttest on the WSAT. The statistically significant difference within-groups occurred on both the WSAT and VST. These results are detailed in Tables 4-1 and 4-2.

Within the digital and nondigital groups, the nondigital group (N = 34) experienced larger learning than the digital groups (N = 40). The interaction between digital and nondigital groups with test is illustrated in Figures 4-1 and 4-2 for the WSAT and VST, respectively. The means
for nondigital and digital groups on the WSAT pretest were 74.94 and 77.50, respectively; the means for nondigital and digital groups on the WSAT posttest were 88.81 and 83.40, respectively. The means for nondigital and digital groups on the VST pretest were 25.56 and 26.50, respectively; the means for nondigital and digital groups on the VST posttest were 26.38 and 25.90, respectively.

The second research question considered the connection between 2D visualization and concrete tangrams and is stated as follows: Does use of concrete tangrams influence preservice elementary teachers’ 2D visualization on tangram designs? Participants with access to concrete tangrams (N = 32) completed 30 tangram designs and showed significantly increased performance levels from pretest to posttest. However, access to concrete tangrams did not produce a significantly different performance from the non-concrete participants (N = 42) on either the WSAT or the VST, indicating that while concrete manipulatives improve performance, when seeking to improve 2D visualization, concrete manipulatives may not serve to promote a significantly different performance from participants who do not have access to concrete manipulatives.

The third research question considers the interaction of digital and concrete tangrams and is stated as follows: Does the choice to use concrete or digital tangrams influence preservice elementary teachers’ 2D visualization on tangram designs? Results indicate that the digital tangrams yielded a significant effect when compared to nondigital treatment groups. In this instance, all groups made significant gains from pretest to posttest, but the nondigital groups, experienced larger learning. Additionally, there is no interaction between digital and concrete, indicating that digital, concrete, or a combination of digital and concrete tangrams aid 2D
visualization of preservice elementary teachers, but no statistically significant difference exists between the digital and digital and concrete groups.

In summary, all groups made significant gains from pretest to posttest on the WSAT. The nondigital groups experienced the largest learning as measured by the WSAT and the digital groups displayed the higher average of group means from pretest to posttest on the VST. This concludes the results of the first three research questions and will be followed by the results of the final two research questions.

**Attitude Toward Geometry**

The previous three research questions addressed the influence of digital or concrete tangrams on preservice elementary teachers’ 2D visualization. The final two research questions evaluate the influence of attitude toward geometry on preservice elementary teachers’ 2D visualization, both the influence of 2D visualization activities upon attitude toward geometry as well as the relationship between attitude toward geometry and 2D visualization.

The fourth research question was designed to detect an influence of 2D visualization activities upon attitude toward geometry and was stated as follows: Does the use of concrete, digital, or the choice to use concrete or digital tangrams to complete tangram problems influence preservice elementary teachers’ attitude toward geometry? Participants completed the UGAS prior to and upon completion of the treatment to determine change in attitude toward geometry. A repeated measures ANOVA was conducted to detect significant differences within groups upon attitude toward geometry at the completion of the intervention. The results showed a significant difference within groups from pretest to posttest before and after the intervention in preservice elementary teachers’ attitude toward geometry. This provides evidence that as participants 2D visualization skills improved, their attitudes toward geometry significantly improved, as well.
The fifth and final research question was proposed to determine the correlation that may exist between 2D visualization and attitude toward geometry. It is stated as follows: Does a relationship between preservice elementary teachers’ 2D visualization and attitude toward geometry exist? While results showed that attitude toward geometry was significantly correlated at the alpha level of .05 with preservice elementary teachers’ performance on the VST, this was not true for performance on the WSAT. The correlation coefficient was .29 from a correlation of UGAS and VST with a significance of .01. The correlation of UGAS and WSAT yielded a correlation coefficient of .21 and a significance of .08, which did not reach criterion. This result was somewhat contradictory. While UGAS scores were not significantly correlated to WSAT scores, UGAS scores were significantly correlated with VST scores. This indicated the relationship between attitude toward geometry and 2D visualization. Students who maintained a positive attitude toward geometry performed at increased levels on measures of 2D visualization.
Table 4-1. Repeated measures analysis of variance WSAT source table.

<table>
<thead>
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<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test</td>
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<td>1</td>
<td>3457.77</td>
<td>48.97**</td>
</tr>
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<td>Test*Digital</td>
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<td>577.53</td>
<td>8.18**</td>
</tr>
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<td>Test*Concrete</td>
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<td>15.97</td>
<td>.27</td>
</tr>
<tr>
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<td>4.50</td>
<td>.06</td>
</tr>
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<td></td>
<td></td>
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</tr>
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<td>.34</td>
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<td>.59</td>
</tr>
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</tr>
<tr>
<td>Error</td>
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</tr>
</tbody>
</table>

** significant at the alpha = .05 level

Table 4-2. Repeated measures analysis of variance VST source table.

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<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
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<td>Test</td>
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<td>.13</td>
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<td>18.64</td>
<td>5.67**</td>
</tr>
<tr>
<td>Test*Concrete</td>
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<td>1</td>
<td>.06</td>
<td>.02</td>
</tr>
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<td>Test<em>Digital</em>Concrete</td>
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<td>1</td>
<td>.18</td>
<td>.05</td>
</tr>
<tr>
<td>Error</td>
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<td>1.76</td>
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<tr>
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<td>1</td>
<td>1.06</td>
<td>.07</td>
</tr>
<tr>
<td>Error</td>
<td>996.83</td>
<td>70</td>
<td>14.24564.13</td>
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</tr>
</tbody>
</table>

** significant at the alpha = .05 level
Table 4-3. Repeated measures analysis of variance UGAS source table.

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<td><strong>Within-Subjects Effects</strong></td>
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</tr>
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<td>Test</td>
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<td>1</td>
<td>.13</td>
<td>.00</td>
</tr>
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<td>Test*Concrete</td>
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<td>1</td>
<td>31.17</td>
<td>.92</td>
</tr>
<tr>
<td>Test<em>Digital</em>Concrete</td>
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<td>59.57</td>
<td>1.75</td>
</tr>
<tr>
<td>Error</td>
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<td>1101.95</td>
<td>1.62</td>
</tr>
<tr>
<td>Concrete</td>
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<td>1</td>
<td>12.44</td>
<td>.02</td>
</tr>
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<td>Error</td>
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<td>680.79</td>
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</tr>
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</table>

** significant at the alpha = .05 level
Figure 4-1. Digital and nondigital interaction with WSAT test.

Figure 4-2. Digital and nondigital interaction with VST test.
CHAPTER 5
CONCLUSION

This chapter includes a summary of findings and explores the conclusions, implications, limitations and recommendations for future research to be drawn from the results discussed in the previous chapter. Conclusions are organized into two categories, improving 2D visualization and attitude toward geometry. These conclusions are followed by implications for preservice elementary mathematics teacher education. Limitations and recommendations for future research conclude this chapter.

Summary

Preservice elementary teachers prove deficient in two-dimensional visualization. Attitude toward geometry may be related to this deficiency. Preservice elementary teachers improve two-dimensional visualization using concrete or digital manipulatives and also demonstrate an interaction between two-dimensional visualization and attitude toward geometry.

Four intact groups of 74 predominantly female preservice elementary teachers enrolled in an elementary education mathematics methods course taught by four instructors at a southeastern research university completed both pretest and posttest. The quasi-experimental research design was comprised of a 2x2 matrix: the level of concrete tangram use and the level of digital tangram use. One group served as a control group; three treatment groups had access to concrete tangrams, digital tangrams, or the choice of concrete and digital tangrams to complete the intervention. Participants rotated, translated, and reflected shapes to fit within 30 tangram designs, which were identical in concrete or digital format.

A repeated measures analysis of variance with alpha level of .05 showed all groups significantly increased two-dimensional visualization as measured by two tests, one test of mental rotation and the other test including mental rotation, spatial, spatio-numeric, and informal
area measurement items, $F(1, 70) = 48.97, p < .01$. Significant within group differences occurred for digital groups and nondigital groups, $F(1, 40) = 8.18, p < .01$. The digital groups mean of 83.40 was exceeded by the nondigital groups mean of 88.81. Attitude toward geometry significantly improved, $F(1,70) = 4.79, p = .03$. A relationship between two-dimensional visualization and attitude toward geometry was evidenced by the significant correlation between these two factors, correlation coefficient equals .29, $p = .01$ ($N = 74$); improved levels of two-dimensional visualization were correlated with more positive attitude toward geometry.

This improvement in preservice elementary teachers’ two-dimensional visualization indicates a need to research retention of two-dimensional visualization. The larger learning experienced by the nondigital groups is unexpected and demonstrates a need for the creation of a digital measure of two-dimensional visualization, isolation of the digital learning, measurement of the process of digital learning, and time allotted to adequately explore the digital environment prior to measuring digital learning.

**Improving Two-Dimensional Visualization**

This study supports other studies that demonstrate the improvement in elementary students’ visualization (Battista et al., 1998; Clements & Battista, 1992; Dixon, 1995a; Fung, 2005; Gerretson, 1998; Hershkowitz, 1989; Moyer et al., 2005; Olkun, 2003; Olkun et al.; 2005; Robinson, 1994; Smith et al., 2003; Steen, 2002; Wheatley, 1998; Wheatley & Reynolds, 1996). Participants in all groups made significant improvements in 2D visualization. This aligns with other literature that suggests that attention to visualization improves visualization (Wheatley, 1998; Wheatley & Cobb, 1990). Visualization improved with minimal attention to this aspect of mathematics.

This current study was an extension of a study of Turkish fourth and fifth graders, incorporating a different population, an additional treatment group, an additional measure of
visualization, and a measure of attitude toward geometry. The population was altered to study
preservice elementary teachers, who perform at levels similar to fourth and fifth graders
(Mayberry, 1983). The addition of a treatment group with access to concrete and digital
manipulatives as suggested by Olkun (2003) was included in the current study. The WSAT was a
second measure of 2D visualization. The UGAS was employed to measure not only any
influence of 2D visualization upon attitude but also any correlation between 2D visualization and
attitude toward geometry.

The researcher conjectured about why students in the digital and concrete group elected
to work with digital tangrams as well as why this interesting difference between digital and
nondigital groups occurred. The clear preference for digital tangrams is contrary to what
occurred in the pilot study, for these participants reported a preference for concrete tangrams
because they could try multiple placements of various tangram pieces simultaneous to their
partner’s explorations. Participants of a concrete pilot group noted a distinct dislike for the
digital tangram software because the software limited the placement of tangram pieces; digital
group participants with access to concrete tangrams noted this limiting factor as a reason for
electing to work with the digital tangrams. Digital tangrams allow for the placement of only one
tangram piece at a time. Participants particularly mentioned that when they worked with the
concrete tangram set, there were too many options as far as placing, rotating, and reflecting
tangram pieces. Additionally, participants noted that once a tangram piece was placed on the
computer, they could note its placement on their answer sheet, while concrete tangram pieces
had too many varieties of placement for students to do this. This supports research of Japanese
and American teaching; while Japanese teachers described the ideal lesson plan as including time
to explore the underlying concept with minimal teacher direction, American teachers incorporate
gradual assistance from the teacher for students to discover new concepts (Jacobs & Morita, 2002). Further, one reason for preference of digital tangrams may stem from the standard formation of the digital tangram pieces at the start of each of the 30 designs. Unlike the digital tangrams, the concrete tangram pieces were arranged in a variety of ways at the start of each design. This has been described by Clements (1999) as an advantage when students are given less choice or “freedom” (p. 55) with digital manipulatives.

One explanation for why nondigital groups performed at higher levels than digital groups may stem from the paper and pencil test for all groups (Eid, 2004/2005). Additionally, at the conclusion of the intervention immediately prior to posttesting, participants in the nondigital groups practiced concrete representations while participants in the digital groups practiced visual representations. Concrete group participants who completed the designs more quickly than their peers increased their practice of concrete representation as they thoughtfully recorded written solutions to the tangram designs. In contrast, participants in the digital groups spent less time with these concrete representations and found the tangram designs within the tangram software, completing a few more tangram designs.

**Attitude Toward Geometry**

The value of improving 2D visualization, while important, has new importance when taken in conjunction with attitude toward geometry. This study demonstrates that a significant difference occurred for attitude toward geometry (Utley; 2004). This result suggests attitude toward geometry is malleable. Attitude toward geometry also served as an indicator of 2D visualization, based upon participants’ UGAS scores being significantly correlated with 2D visualization on the VST. Increased levels on 2D visualization were correlated with increased levels of attitude toward geometry.
As preservice elementary teachers’ 2D visualization improved, their attitude toward geometry improved as well. This supported Aiken’s claim that attitude toward mathematics and mathematics ability were dynamically and interactively related (1985) and augmented studies of visualization and geometry abilities (Olkun, 2003; Wheatley, 1998). This study demonstrated that both concrete and digital manipulatives improved attitude toward geometry, adding to Sowell’s (1989) findings that knowledgeable teachers who incorporate concrete teaching aids facilitate improved attitudes toward mathematics.

**Implications for Preservice Elementary Teacher Education**

As mathematics teacher educators seek to design programs of study that will prepare mathematics teachers for their future classrooms, the issues of content and pedagogy always emerge. Content must be designed to fit within the framework and time schedule of the mathematics teacher education courses. Pedagogy should prepare preservice elementary mathematics teachers for their future classroom.

This intervention proved to benefit preservice elementary teacher education in two ways. First, this study demonstrated that incorporating this intervention within the context of an elementary education mathematics methods course resulted in a significant difference in preservice elementary teachers’ 2D visualization with a minimal amount of time sufficient to complete the intervention. Second, because 2D visualization was correlated with attitude toward geometry, this intervention demonstrated the potential to improve not only 2D visualization but also attitude toward geometry. The fact that this intervention was included prior to the regular geometry instruction that occurs during an elementary education mathematics methods course led to the conclusion that this intervention was appropriate within the context of another elementary education course, such as science, technology or art, which include visualization components. It was important to the research to complete the geometry component of the
syllabus after the completion of the intervention and testing to avoid contamination of data by traditional geometry instruction. The researcher asked the instructors to postpone geometry instruction for this reason. This leads to a future research direction to include this intervention or a variation thereof at multiple points in the mathematics, science, technology, or art curricula.

One issue that is evident from the results points to the need to incorporate manipulative use with reflection for increased understanding. The control group outperformed all others, suggesting that manipulative use alone is not sufficient to promote significant improvements (Moyer, 2001) but must be accompanied by attention to mental capacity, such as reflection upon the actions to build mathematical meaning. The concrete group engaged in a short period of reflection following the intervention. This time for reflection occurred between the conclusion of the 30 tangram designs and the administration of the posttests for participants that completed the intervention more quickly than their peers. Participants reflected during this time about which designs were most challenging and what strategies were most helpful. In contrast, the digital groups continued to explore tangram designs within the software rather than capturing this moment for reflection. It is possible that the subsequent posttest results demonstrate that a period of reflection promotes increased learning.

This study supports other work using pairs to promote discourse and understanding (Olkun, 2003; Utley, 2004). If preservice elementary teachers note this, they may be encouraged to incorporate pairs during class time in their future classrooms to promote discourse. Additionally, research of small group learning and its influence upon geometry thinking, knowledge, and instruction could be further addressed (Battista, 2007). The researcher recorded data about the interaction of students during the intervention. It was noted that as the participants worked together in pairs, ideas and solutions emerged. Pairs provided scaffolding to one another
as well as encouragement (Sarama & Clements, 2002). Conversation was supportive as each partner worked toward solutions. When one student struggled to complete the design, the other student used fingers to point on the screen where a particular tangram piece should fit. Some pairs began by assigning roles, but these roles shifted back and forth as they worked in tandem. All pairs recorded their solutions on the answer sheet, engaging in discourse as they sought for and recorded solutions. Individuals working alone to complete this intervention may not experience similar results.

No information regarding preservice elementary teachers’ computer experience was collected for the current study, but these participants are deemed nondigital natives. Their future students will be digital natives. This indicates that preservice teachers most likely perform at different levels than the students that they will teach. For this reason, research should be conducted to measure how digital non-natives teach digital natives. Additionally, creating content or lessons for future students rather than completing tangram designs may be more appropriate to influence preservice elementary teachers’ 2D visualization.

**Limitations**

The design of this study evolved from circumstances and settings that were partly beyond the control of the researcher. Research bias, selection and assignment to groups, ceiling effect and test-retest effect were some of the lurking variables that were anticipated in the design of this study. Careful consideration to these threats was given to minimize their influence upon the data.

Bias due to researcher collecting all pretest and posttest data as well as implementing the intervention, was seen as less of a lurking variable than the variability that would have been introduced by four elementary education mathematics methods instructors implementing the treatment. Preplanned lessons and strictly defined activities for participants in all classes minimized this threat. Intact and nonrandomly assigned classes were used for treatment and
control groups, which is not uncommon for studies conducted in educational settings. Another limitation of this study may be described as a ceiling effect. Examination of the plots shows a marginal ceiling effect present in the data for the WSAT and VST measures. Finally, test-retest effect was minimized by including a control group.

**Recommendations for Future Research**

Future research should explore and develop several aspects of the current work. Studying the impact of 2D visualization and attitude toward geometry over time and on teaching is one future research direction. Additionally, the creation of a digital measure and effective implementation of technology are further extensions of this study.

**Two-dimensional visualization and attitude over time**

Perhaps the most apparent extension to this study is the measurement of not only 2D visualization but also of geometry achievement. This would support research noting the connection between increased visualization and geometry achievement (Seigler, 2003; Wheatley, 1998). This connection between visualization and geometry could also lead to improved mathematics achievement. Additionally, research of small group learning and its influence upon geometry thinking, knowledge, and instruction could be further addressed (Battista, 2007).

Finally, to test retention of 2D visualization and stability of attitude toward geometry participants could be posttested again at the end of the semester or year to support or refute the proposition that the effects of digital tangrams on 2D visualization are not temporary. (Ben-Chaim, Lappan, and Houang, 1988; Putney & Cass, 1998).

**Impact of two-dimensional visualization and attitude on teaching**

Longitudinal research that follows preservice elementary teachers into their future classrooms would ascertain the effect of this intervention upon 2D visualization and future teaching, particularly if the preservice teachers who learned with manipulatives teach with
manipulatives (Putney & Cass, 1998). It is suggested that teachers who visualize in turn deepen their students’ tendency to visualize (Presmeg, 2006). Qualitative data could be gathered through observation or teacher journaling to add further evidence to this claim (Steen et al., 2006). Further study of participants as teachers could determine if this intervention led to visualization instruction or visually responsive instruction in their classrooms. Additionally, future research focused on constructs connected to technology will inform classroom instruction (Zbiek, et al., 2007). Also, measures of attitude could be further studied to determine their influence on preservice elementary teachers’ future classrooms and teaching.

**A new measure: digital and mental rotations**

Testing in this study included only paper-based tests; future studies could explore digital tests, which have been noted to promote increased scores (Eid, 2004-2005). A measure designed to measure digital learning is currently needed to promote research of 2D visualization in digital environments to account for encoding specificity.

Attitude toward mathematics has been shown to be correlated with achievement (Aiken, 1985). Preservice elementary teachers’ attitude toward geometry as measured by the UGAS was significantly correlated with 2D visualization when the latter is measured by the VST. One reason for the significant correlation on the VST may be due to the various types of items presented in that measure, namely, spatial (8 items), spatio-numeric (8 items), mental rotation (8 items), and informal area measurement problems (5 items). In contrast, a significant difference in attitude or correlation of attitude with 2D visualization does not result when 2D visualization was measured by the WSAT, which may be related to the singular focus of the WSAT, mental rotations, or the number of items on each measure, 100 items on the WSAT as opposed to 29 items on the VST. The VST covered a broader range of topics, which may have promoted improvements in attitude toward geometry. Finally, the WSAT is timed to promote holistic
processing rather than analytic processing; the VST is not timed and therefore may measure analytic rather than holistic processing.

**Effective implementation of technology**

As technology continues to be called for in today’s classrooms, effective uses of technology are needed, which include measuring digital learning, isolating the technology, measuring the process, and providing time to explore both digital and concrete formats prior to selecting one format. The development of a digital measure was mentioned previously. Including a treatment group with access to concrete and digital manipulatives was a detail added to the current study because of a call for this by Olkun (2003). The researcher noted that participants in this digital and concrete group elected to work exclusively with the digital tangrams, with the exception of one participant. This indicates a need to allow participants time to explore both digital and concrete manipulatives prior to offering a choice of manipulative format (Salomon & Gardner, 1986). Further exploration should include interviews of participants’ to understand prior experience with and selection of manipulative format.
APPENDIX A
TEST OF SPATIAL VISUALIZATION IN TWO-DIMENSIONAL GEOMETRY

TEST OF SPATIAL VISUALIZATION IN 2D GEOMETRY

ID #:  
Date:  

Please read instructions carefully for each item & mark one letter. You may skip items.

1. Which of the gray shapes on the right cannot cover the shape on the left?

<table>
<thead>
<tr>
<th>A) 1 &amp; 2</th>
<th>B) 1 &amp; 3</th>
<th>C) 3 &amp; 4</th>
<th>D) 1 &amp; 4</th>
</tr>
</thead>
</table>

2. Which of the gray shapes we should use to cover the shape on the left?

<table>
<thead>
<tr>
<th>A) 1 &amp; 2</th>
<th>B) 1 &amp; 3</th>
<th>C) 1, 2 &amp; 4</th>
<th>D) 3 &amp; 4</th>
</tr>
</thead>
</table>

3. Which of the gray shapes we should use to cover the shape on the left?

<table>
<thead>
<tr>
<th>A) 1, 3 &amp; 4</th>
<th>B) 1 &amp; 3</th>
<th>C) 1 &amp; 2</th>
<th>D) 3 &amp; 4</th>
</tr>
</thead>
</table>

4. Which of the gray shapes we should use to cover the shape on the left?

<table>
<thead>
<tr>
<th>A) 1 &amp; 2</th>
<th>B) 1, 2 &amp; 3</th>
<th>C) 1, 2 &amp; 4</th>
<th>D) 3 &amp; 4</th>
</tr>
</thead>
</table>
5. How many of the gray triangle we should use to cover the blank spaces of the square placed on the left?

   A) 2  
   B) 4  
   C) 6  
   D) 8

6. How many of the gray triangle we should use to cover the shape placed on the left?

   A) 3  
   B) 4  
   C) 5  
   D) 6

7. How many of triangle B we should use to cover the triangle A placed on the left?

   A) 2  
   B) 3  
   C) 4  
   D) 5

8. How many of the gray triangle we should use to cover the shape placed on the left?

   A) 2  
   B) 3  
   C) 4  
   D) 5
9. How many of the gray triangle we should use to cover the shape placed on the left?

A) 3
B) 4
C) 5
D) 6

10. I have a geometric shape. I use 6 of the shape A to cover that shape. How many of the shape B would I need to cover the same shape?

A) 1
B) 3
C) 5
D) 7

11. I have a geometric shape. I use 2 of the shape A to cover that shape. How many of the shape B would I need to cover the same shape?

A) 3
B) 4
C) 5
D) 6

12. I have a geometric shape. I use 6 of the shape A to cover that shape. How many of the shape B would I need to cover the same shape?

A) 2
B) 3
C) 4
D) 8
13. I have a geometric shape. I use 3 of the shape A to cover that shape. How many of the shape B would I need to cover the same shape?

A) 4  
B) 5  
C) 6  
D) 7

14. How many of the gray triangle we should use to cover the shape placed on the left?

A) 7  
B) 6  
C) 5  
D) 4

15. How many of the gray triangle should we use to cover the shape placed on the left?

A) 3  
B) 4  
C) 5  
D) 6

16. How many of the gray triangle we should use to cover the shape placed on the left?

A) 3  
B) 4  
C) 5  
D) 6
17. Which one of the shapes can be obtained by rotating the figure on the left in clockwise direction?

18. Which one of the shapes can be obtained by rotating the figure on the left in clockwise direction?

19. Which one of the shapes can be obtained by rotating the figure on the left in clockwise direction?

20. Which one of the shapes can be obtained by rotating the figure on the left in clockwise direction?
21. Which one of the shapes can be obtained by rotating the figure on the left in clockwise direction?

[Images of four shapes, labeled 1, 2, 3, 4]

A) 1
B) 2
C) 3
D) 4

22. Which one of the shapes can be obtained by rotating the figure on the left in clockwise direction?

[Images of four shapes, labeled 1, 2, 3, 4]

A) 1
B) 2
C) 3
D) 4

23. Which one of the shapes can be obtained by rotating the figure on the left in clockwise direction?

[Images of four shapes, labeled 1, 2, 3, 4]

A) 1
B) 2
C) 3
D) 4

24. Which one of the shapes can be obtained by rotating the figure on the left in clockwise direction?

[Images of four shapes, labeled 1, 2, 3, 4]

A) 1
B) 2
C) 3
D) 4
This is a unit (small) square. How many unit (small) square are there in each of the shapes below.

25. a) 7  
    b) 8  
    c) 9  
    d) 10

26. a) 9  
    b) 10  
    c) 11  
    d) 12

27. a) 10  
    b) 11  
    c) 12  
    d) 13

28. a) 20  
    b) 18  
    c) 16  
    d) 14

29. a) 16  
    b) 15  
    c) 14  
    d) 13
APPENDIX B
UTLEY GEOMETRY ATTITUDE SCALES

ID# ______________________  Date: _______________

For the following statements, circle your level of agreement with each of the following statements.

SD - if you strongly disagree
D - if you disagree
N - if your feeling is neutral
A - if you agree
SA - if you strongly agree

1. I am sure that I can learn geometry concepts.     SD     D     N     A     SA
2. I believe that I will need geometry for my future.  SD     D     N     A     SA
3. Geometry problems are boring.                     SD     D     N     A     SA
4. When I leave class with a geometry question unanswered, I continue to think about it.  SD     D     N     A     SA
5. I often have trouble solving geometry problems.    SD     D     N     A     SA
6. When I start solving a geometry problem, I find it hard to stop working on it.       SD     D     N     A     SA
7. Time drags during geometry class.                SD     D     N     A     SA
8. I am confident I can get good grades in geometry.  SD     D     N     A     SA
9. When I can’t figure out a geometry problem, I feel as though I am lost and can’t find my way out. SD     D     N     A     SA
10. Geometry has no relevance in my life.             SD     D     N     A     SA
11. I lack confidence in my ability to solve geometry problems.  SD     D     N     A     SA
12. Geometry is not a practical subject to study.     SD     D     N     A     SA
13. I feel sure of myself when doing geometry problems. SD     D     N     A     SA
14. Geometry is fun.                                SD     D     N     A     SA
15. I just try to get my homework done for geometry class in order to get a grade.        SD     D     N     A     SA
16. Geometry is an interesting subject to study.  
17. I can see ways of using geometry concepts to solve everyday problems.  
18. For some reason even though I study, geometry seems unusually hard for me.  
19. Geometry is not worthwhile to study.  
20. I often see geometry in everyday things.  
21. Geometry problems often scare me  
22. I am confident that if I work long enough on a geometry problem, I will be able to solve it.  
23. Solving geometry problems is enjoyable.  
24. I will need a firm understanding of geometry in my future work.  
25. Working out geometry problems does not appeal to me.  
26. I do not expect to use geometry when I get out of school.  
27. Geometry tests usually seem difficult.  
28. I will not need geometry for my future.  
29. I can usually make sense of geometry concepts.  
30. Geometry has many interesting topics to study.  
31. Geometry is a practical subject to study.  
32. I have a lot of confidence when it comes to studying geometry.  


*Permission is granted by Juliana Utley, August 2007, for Kristin Spencer to place a version of this instrument in the appendix of her dissertation.
APPENDIX D
TANGRAM DESIGN ANSWER SHEET
Pilot Study: The Appropriateness of Two-Dimensional Visualization Intervention and Measures with a Preservice Teacher Sample

The pilot study was completed in four phases to determine that the WSAT and VST instruments and both concrete and digital interventions were suitable for the population of preservice elementary teachers as well as to the population of fourth and fifth graders used by previous researchers. The WSAT was piloted on 52 preservice elementary teachers, the VST on 27 preservice elementary teachers, the concrete intervention with 27 preservice elementary teachers and the digital interventions with 24 preservice elementary teachers. These preservice elementary teachers were in their junior year one semester prior to elementary education mathematics methods course enrolled in three separate sections of a course titled teaching reading in primary grades taught by three different instructors. It was helpful to see their reaction to the instruments and interventions. Their suggestions and comments were incorporated into the study. From this pilot, it appears that the WSAT and VST as well as the concrete and digital interventions, though developed for fourth and fifth graders, are appropriate for preservice elementary teachers.

The researcher conducted the first phase of the pilot study using 26 preservice elementary teachers attending a southeastern research university. These preservice elementary teachers were given the WSAT as a means to establish the validity of using this measure with preservice elementary teachers, who have been reported to perform at a fourth or fifth grade level (Mayberry, 1983). Reliability for the WSAT was .94 (N=26, number of items = 100). The pilot yielded a mean of 79.23 with a standard deviation of 14.68, median of 84.5, mode of 85.5, and a high of 100 and low score of 45. These scores are similar to the grade five norms reported for the
WSAT, which are reported at 100 = 98th percentile, 69 = 51st percentile, and 20 = 1st percentile (Wheatley, 1996). This phase of the pilot study also gave the researcher practice in administering the WSAT and led to the acquisition of an accurate timing device.

To complete the first and second phases of the pilot study, 22 preservice elementary teachers completed the WSAT and the VST. Reliability for the VST was .70 (N = 22, number of items = 24) with a mean of 19.36 and standard deviation of 3.30, high of 28 and low of 16. On the spatial, spatial-numeric, mental rotation, and informal area measurement subscales, reliability was, respectively, .06 (number of items = 4), .68, .63, and .22 (number of items = 4) with means, 3.38 (sd = 0.74), 6.90 (sd = 1.51), 6.10 (sd = 1.76), and 3.38 (sd = 0.80), respectively. The mean for the WSAT was 80.69 with a standard deviation of 17.16. With a high test score of 100 and low of 49.5, this sample performed close to the normative data published for fifth graders (Wheatley, 1996). Mean score for the VST was 24.5 with a standard deviation of 3.19. These participants noticed three details that aided the implementation of the VST. First, items two through nine and items fourteen through sixteen ask a question about covering a shape and are worded “we should use” instead of “should we use.” This detail distracted pilot participants, so the researcher made a point to clarify that point during the study. Secondly, the researcher asked the pilot participants to record their answers on a separate answer sheet instead of writing directly on the test. Several participants mentioned that they would have liked to sketch in their solutions, so the researcher made it clear to study participants that they were welcome to do so. Finally, the word “cover” confused participants in the pilot study. They were uncertain of this term’s definition. The researcher defined this term for participants prior to distributing the pretest and posttests during the study. Namely, a cover is a combination of shapes that leaves no spaces uncovered but does not extend beyond the shape to be covered. It must cover the shape
completely with no overlap. These three details surfaced during the pilot study and helped to clarify what instructions the researcher needed to provide to the study participants. In scoring these tests, the researcher also noted two items that were coded incorrectly on the answer key. The researcher verified this with Olkun (personal communication, 9/21/07) and corrected the answer key.

To pilot the interventions, 27 preservice elementary teachers completed the 30 Tangram designs using concrete Tangram sets and 24 preservice elementary teachers completed digital Tangram designs. The purpose of this portion of the pilot study was to gather data to support the use of the intervention for this population (see Appendix D). Originally, this intervention, 30 Tangram designs ranging from simple to complex in number of tangram pieces and number of transformations to complete each design, was employed for a sample of fourth and fifth graders (Olkun et al., 2005). Tangrams are typically included within an elementary education mathematics methods course to demonstrate area or other geometry concepts. Within 2D geometry, tangrams and pattern blocks are the two most well known activities to explore the creation of larger shapes from smaller shapes or to fit together small shapes to form larger shapes (Van de Walle, 2004). Data collected during this portion of the pilot study provided evidence to determine the validity of utilizing this intervention with preservice elementary teachers.

For the third phase of the pilot study, 27 preservice elementary teachers worked in pairs or trios completing Tangram designs utilizing concrete Tangrams. While fourth and fifth graders needed 80 to 120 minutes to complete the Tangram designs, preservice elementary teachers were only given twenty-five minutes to work. Fifteen of the preservice elementary teachers completed all 30, two completed 26, three complete 23, two completed 21, and three completed 20 designs.
Two participants chose not to record their design solutions, so it is unknown how many designs they completed. Thus, the duration for the intervention was settled at 30 to 45 minutes.

Pairs and trios promoted discourse, competition, scaffolding, positive feedback, and engagement. Preservice elementary teachers in an elementary education mathematics methods course typically work in pairs or small groups to complete activities that will prepare them for their future classrooms. In this way, treatment and control groups will match. Participants in the pilot study communicated about the ease or difficulty of solutions, the challenge to find a solution, and the method to record those solutions. Competition between groups promoted a playful atmosphere, encouraged students to avail themselves to find solutions, and even led two groups to complete all the designs first and then go back to record their solutions later. The researcher wondered if these groups would remember solutions. In fact, these students noticed the ease with which they could solve the designs on the second time through, even the designs they’d skipped because they were too difficult the first time around. Scaffolding occurred naturally. One pair worked as if one person with four hands, moving tangram pieces in and out of the workspace fluidly. Another subject noted that while watching her partner struggle to place pieces, she saw where one shape needed to be, placed it there, and watched the solution come together. This led to encouragement from each partner and engagement as they worked together to finish the designs.

The preservice elementary teachers included comments about their response to the intervention. They reported enjoying the Tangram designs. They noted the most difficult designs. And, as mentioned above, they reported finding those difficult designs easier upon return. This improved capacity excited subjects. Engagement and activity were high during the treatment. Participants who completed the designs first and revisited the designs to record their answers
noticed their improved skills and suggested this as a method for implementation during the study. The researcher concurred.

Five preservice elementary teachers completed the 30 Tangrams with a few moments remaining. So, while the other participants continued with the 30 Tangrams, these five participants worked on Macintosh laptops from the department laptop cart. These laptops were logged into by the researcher in anticipation of a few participants reviewing Macintosh-compatible software. As mentioned above, it was determined that the Macintosh laptops made rotating the digital Tangram pieces cumbersome. Additionally, participants noted that after working with concrete Tangram sets, the digital Tangrams seemed to stifle the number of combinations of Tangram pieces available. They mentioned that while concrete Tangram sets allow both participants to simultaneously explore combinations of various Tangram pieces, the digital Tangrams did not. They mentioned that the Macintosh Tangram software did not allow them to simultaneously explore combinations of various digital Tangram pieces as they could with the concrete Tangram set.

The fourth phase of the pilot study explored the digital Tangram intervention. The sample of 22 preservice elementary teachers who took the WSAT and VST during phases one and two of the pilot study were scheduled to complete the 30 digital tangram designs on computers and take the WSAT and VST as posttests. These participants worked for twenty-five minutes, completing designs. While the computers were functioning properly, the file containing the 30 designs was protected and accessible only to the administrator. Instead, participants worked on designs within the Tangram software. Only after the participants had completed several tangram designs within the software did the researcher realize that this file was on a USB key that could have given the participants access to the file of 30 tangram designs. Because of the computer file
access being denied, timing for the digital tangram designs could only be estimated. Also, because these participants worked within the software but not with the 30 specific tangram designs that comprise the intervention, they were not posttested. In any case, participants noted that the tangram pieces must be precisely placed to snap into place. They quickly figured out how to rotate pieces. Two pairs noticed that the parallelogram cannot be reflected or flipped within this software. This technicality is accounted for in the file of 30 tangram designs, because they can be solved with rotations and no reflections. Pairs worked together taking turns with the mouse, except for four pairs who chose for only one partner to use the mouse while the other partner advised, pointed to the screen, and suggested solutions. One participant chose to work solo instead of joining a pair to create a trio. Three pairs even found one design (design 4 of 25 very hard designs) that registers solved when in fact a small triangular region remains uncovered. Again, it was found that pairs provide a vital component of the intervention as students work together to solve the problems and to keep the orientation of the activity playful (Steffe & Weigel, 1994).

These four phases of the pilot study provide evidence of the suitability of both measures and interventions for the population of preservice elementary teachers, yield reliability data for the measures for this population, and suggest the timing appropriate for the current study. Also, the instructional phase was enriched with added explanations and definitions to make the administration of the study uniform for all groups.
## APPENDIX F
### DATA SET

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<th>Concrete ID#</th>
<th>WSATpre</th>
<th>VSTpre</th>
<th>UGASpre</th>
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Figure F-2: Digital data set.
REFERENCES


BIOGRAPHICAL SKETCH

I graduated Mountain View High School in Orem, Utah May 1990. I earned both a BS in mathematics with a minor in English in December 1995 and an MS in mathematics with a minor in statistics in April 1998 from Brigham Young University in Provo, Utah. I completed the doctoral degree in curriculum and instruction with an emphasis in mathematics education in May 2008 at the University of Florida. I married Aaron Tull Spencer June 2000 and welcomed four children, Nathan, Isaac, Jacob, and Sarah into our family.

Mr. Thompson was instrumental in setting my sights upon studying mathematics. In his algebra class, he taught me the excitement of discovering mathematics when he allowed me to study the material at my own speed, completing all the odd-numbered exercises and checking them in the back of the book.

Another defining moment in my mathematics career occurred on the Newport Beach peninsula. I was struggling to complete my assignments for Mr. Lawlor’s Pre-Calculus course, namely, Gauss-Jordan reduction of matrices. After three or four attempts, I discovered the importance of method in mathematics. Solving from top to bottom and left to right was critical to this solution. When I finally figured this out, I did cartwheels on the beach. My self efficacy improved knowing I could decompose a difficult problem to find a solution. When I selected Calculus during the time slot that precluded Modern Dance Technique and Theory, my course was set for life. I decided that I would need to struggle to learn mathematics but that I would always dance. So, I changed my undergraduate major to mathematics from modern dance. This has made all the difference.

I express gratitude to Dr. Thomasenia Adams, a miracle worker, for helping me see the completion of this one step in my eternal journey of education, intelligence, and light. Finally, my husband’s desire to pursue a graduate degree far from my home influenced my decision to
pursue this doctoral degree. Initially, I thought it might curb my homesickness to study, but in fact, it has been the fulfillment of a lifelong dream.