ENHANCING HEAT TRANSPORT THROUGH OSCILLATORY FLOWS

By

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To my grandfather, John Wolsko (1911-2006)
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<td>$A$</td>
<td>Cross sectional area of channel. Applies to Chapter 1.</td>
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<td>$A$</td>
<td>Oscillation amplitude, cm.</td>
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<td>$A$</td>
<td>Constants in solutions to perturbation equations. Applies to Chapter 4.</td>
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<td>$b$</td>
<td>Width where 99% of heat transfer occurs in oscillating plate problem.</td>
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<tr>
<td>$c$</td>
<td>Coefficient for outer radius of annulus.</td>
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<td>Concentration of CSTR. Applies to Discussion Note 1 in Chapter 2.</td>
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<td>$C_{in}$</td>
<td>Inlet concentration of CSTR. Applies to Discussion Note 1 in Chapter 2.</td>
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<td>$C^*$</td>
<td>Scaled concentration of CSTR. Applies to Discussion Note 1 in Chapter 2.</td>
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<tr>
<td>$C_V$</td>
<td>Specific heat, J/(g*C).</td>
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<tr>
<td>$d$</td>
<td>Channel width. Applies to Chapter 1.</td>
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<tr>
<td>$D$</td>
<td>Molecular diffusion coefficient, cm^2/sec.</td>
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<tr>
<td>$g$</td>
<td>Gravitational constant.</td>
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<tr>
<td>$i = \sqrt{-1}$</td>
<td>Imaginary number.</td>
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<tr>
<td>$J$</td>
<td>Heat flux, W/cm^2.</td>
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<tr>
<td>$J_{COND}$</td>
<td>Conductive heat flux, W/cm^2.</td>
<td></td>
</tr>
<tr>
<td>$J_{CONV}$</td>
<td>Convective heat flux, W/cm^2.</td>
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<tr>
<td>$k$</td>
<td>Effective diffusion coefficient. Applies to Chapter 1.</td>
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<tr>
<td>$k$</td>
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\[ T = \frac{\omega \tau}{\kappa} \]

\[ \omega^2 \]
\[ Wo = A \sqrt{\frac{\omega}{2\nu}} \]  Womersley number, dimensionless. Applies to Chapter 2.

\[ Wo = \frac{\omega R^2}{v} \]  Womersley number, dimensionless. Applies to Chapter 3.

\( x \)  Coordinate in the x-direction.

\( x^* \)  Scaled length. Applies to Discussion Note 2 in Chapter 2.

\( X \)  Grouping used to simplify perturbation equations. Applies to Chapter 4.

\( y \)  Coordinate in the y-direction.

\( z \)  Coordinate in the z-direction.

\( Z \)  Grouping used to relate pressure drop to piston amplitude in annulus.

**Greek Symbols**

\[ \alpha = \frac{-i\omega}{\nu} \]  Viscous length grouping to assist in expressing solutions, cm.

\[ \alpha' = \frac{i\omega}{\nu} \]  Conjugate of \( \alpha \).

\[ \beta = \frac{-i\omega}{\kappa} \]  Diffusive length grouping to assist in expressing solutions, cm.

\( \delta \)  Size of solid gap in two compartment model, cm.

\( \varepsilon \)  Distance inner rod is displaced, cm. Applies to Chapter 4.

\( \theta \)  Angle measurement, radians.

\( \kappa \)  Thermal diffusivity, cm^2/sec.

\[ \lambda = \frac{\theta \nu}{2} \]  Dimensionless parameter. Applies to Discussion Note 2 in Chapter 2.

\( \mu \)  Dynamic viscosity, g/(cm*sec).
\( \nu \)Kinematic viscosity, cm\(^2\)/sec.

\( \zeta \) Constants in solutions to perturbation equations. Applies to Chapter 4.

\( \rho \)Density, g/cm\(^3\).

\( \tau \)Residence time, sec.

\( \omega \)Frequency, Hz.

**Superscripts**

\( ^\wedge, \sim \) Complex conjugates.

\( - \) Denotes complex conjugate (overbar)
This research project is about the use of oscillatory flows to enhance heat transport. The physical idea behind this is that if a heat source is connected to a heat sink via a fluid and the fluid is oscillated, the convective motion will bring about sharp spikes in the velocity profile which in turn will enhance the heat transport over pure conduction due to both radial and axial gradients. This research has many applications including removing heat from outer space modules, reactors, closed cabins, etc. Many parameters can be varied to maximize the convective heat transport. These parameters include pulse amplitude and frequency, pipe radius and length, as well as the transporting fluid and the geometry of the transport zone.

This problem shows from a theoretical standpoint how various pipe geometries affect the heat transfer. For example, an annular geometry is studied and it is seen that an annulus is an optimal geometry compared to a cylinder, when the flow rates through both are held fixed. This happens because the annular geometry contains an extra boundary compared to an open cylinder and the spike in the velocity is greater than the open cylindrical geometry. The comparison was taken one step further using an annulus and a two compartment model containing an inner cylinder and an outer annulus. The transport in the two compartment model proved to be higher in the case of fixed flow rates, due to the additional boundary. Lastly, an off-centered annulus
was studied, and it was shown that the net enhancement is positive for low frequencies and negative for high frequencies compared to a centered annulus.

An experiment was designed to test the various parameters listed above. The experimental results that have been obtained are in good qualitative agreement with the theoretical calculations. The experiment has shown that the heat transport is proportional to the square of the oscillation amplitude. The study also demonstrated that heat transfer increases with rising frequency. In summary, it has been shown through the experiment and the theory that oscillatory flows increase the heat transport of a system by several orders of magnitude compared to ordinary conduction.
CHAPTER 1
INTRODUCTION

This study is concerned with the investigation of heat transfer in enclosed systems. The goal is to find ways to enhance the transport of heat from a heat source to a heat sink using fluid contained connectors. These methods also need to be efficient and many times faster than molecular conduction. The application of these findings has a place in NASA’s long-term manned and un-manned missions. In a space vehicle, heat can be generated due to equipment, such as an on-board reactor. The thermal control of the environment is of utmost importance for the maintenance of crew habitat, or even for the safe operation of instruments. This heat is best conducted to external radiator sails that can then emit the heat to outer space. Getting the heat from the source to the heat sinks could be done via heat pipes and metal conductors, but the mass of such equipment is high. It is important to keep the mass on board minimal due to the immense weight during lift off and acceleration maneuvers. This then calls for a novel way to transport heat from high temperature sources in enclosed environments to sinks using fluid connectors. One such way is to use the principle of oscillatory flows to enhance heat transport. This involves the use of connecting pipes that contain a fluid which is in perpetual motion, so that the heat can be conveyed fast. To understand how the heat can be transported by simply moving fluid back and forth, a companion problem will first be presented, which involves the mass transport of species.

1.1 Physics of Mass Transport

Imagine two tanks connected by a pipe with one tank containing a species while the other tank contains a carrier. If a connecting pipe between the tanks were left open, a concentration
gradient would be formed between the two tanks in the direction of the axis of the pipe (see Figure 1.1a). This gradient would ultimately decrease and equilibrium would be achieved.

Now suppose one wants to quicken the rate at which the species move down the pipe. One way for this to occur is through the use of oscillatory flows. When a fluid is oscillated in a pipe through some driving force, it acquires a parabolic profile far away from the entrance and exit regions (Figure 1.1b). This curved shape generates a larger area for mass transfer to take place, mostly in the radial direction. The species will have a net movement down the pipe and towards the boundary. These radial concentration gradients play a large role in quickening the rate of transport. In ordinary molecular diffusion, there exist only axial concentration gradients for the species to move. However, once the oscillations begin, the particles have more ways to move toward the end of the pipe (radially and axially). Since an oscillatory flow means that the fluid must return to its original position, a reverse parabola occurs (Figure 1.1c). In the forward part of the cycle, there was a net movement of species toward the boundary. Now since there is a higher local concentration of species at the boundary compared to the core, there will be a movement of species back toward the core of the pipe. Although there is no net flow in this problem, the species will work its way down the pipe in a “zig-zag” fashion as the pulsations occur (Figure 1.1d). It can be shown that these oscillations can rapidly improve the rate at which the species is transported down the tube.

Now take the case where there are two species with a carrier fluid in one tank and pure carrier in the other, as shown in Figure 1.2. Suppose it is desired to separate the two species. If the system is left without any induced flow, the species will diffuse down the pipe at a rate that is dependent on their diffusion coefficients. Take for example two species whose diffusion coefficients are quite different from each other, helium and carbon dioxide. Carbon dioxide is
the larger molecule so it will have the slower diffusion coefficient. This means under the case of pure diffusion, the smaller helium molecules will move down the pipe quicker and can be separated out first (see Figure 1.2a). The carbon dioxide would then follow later. However, if oscillations are imposed, one will notice that something different occurs (see Figure 1.2b). As mentioned above, the pulsations will push the species toward the boundary of the tube. The species most likely to be pushed to the boundary is the faster diffuser, the helium. The carbon dioxide will remain behind in the center of the tube. When looking at the velocity profile in the tube, one will notice that the fluid speed is faster in the center compared to the boundary. This means that the carbon dioxide will actually move faster down the tube since it is in the quicker flow region. A separation is achieved with the slower diffuser being transported faster. This separation will also occur much faster compared to the case where only pure diffusion happens, since the oscillations force the two species down the pipe at faster rate.

In order for a separation to be maximized, several parameters must be optimized. In all chemical engineering problems, there exist time scales. This one is no different. In fact, multiple time constants arise that must be noted. These time constants involve the oscillation frequency, the kinematic viscosity of the carrier fluid, as well the diffusion coefficients of the species. For example, the oscillation period must be at a rate that is comparable to the time constant due to the kinematic viscosity of the fluid, so that a parabolic profile can exist. If the fluid has a low kinematic viscosity, the frequency must not be too high, or the inertial force from the pulse would overpower the viscous effects at the walls, causing the velocity profile to be much flatter, and reducing the concentration gradient in the radial direction. In fact, this suggests the existence of a dimensionless group which is a ratio of the two time constants. The analysis
will show the natural occurrence of such a group, the Womersley number. Another example is that the diffusion time constant for each species must be different. Otherwise, both species would move down the pipe at the same rate, and a distinct separation would not occur. Lastly, the viscous time scale must be greater than the diffusive time scale, since the viscous effects play a big role in producing the radial concentration gradients. This is because a more viscous fluid can generate a sharper parabolic flow profile due to the higher amount of friction at the tube walls. A longer and sharper parabola produces more radial concentration gradients which will increase the transport in the tube compared to ordinary diffusion.

1.2 Application of Mass Transport

A basic application for enhanced species transport is shown in Figure 1.3. In this example, it is important for human systems to have clean air to breathe. When humans generate carbon dioxide as they exhale, it becomes mixed in with air. This mechanism could assist in removing the carbon dioxide from the air so that it is pure when it is regenerated by plant photosynthesis. Imagine if there is a carbon dioxide and air mixture in the left hand tank, while the right hand tank consists of only water vapor. When the oscillations are induced, the slower diffusing carbon dioxide molecules will reach the right hand tank faster, and are able to be separated out first, leaving the air free from high levels of carbon dioxide.

1.3 Physics of Heat Transport

The same idea of pulsatile flows can be applied to transporting heat between a source and a sink. Take the case of a fluid in a channel. Suppose at one end, the fluid is in a hot tank, while at the other end, it is in a cold sink. Heat will be transported from the source to the sink by conduction if the fluid remains stagnant; this heat will flow down the channel at a rate proportional to the thermal diffusivity of the fluid. If the fluid is oscillated back and forth, radial
temperature gradients will form (see Figure 1.1b) and the heat movement would be enhanced down the pipe through the zig-zag movement discussed earlier. The oscillatory movement of the fluid transports the heat through convection, and when the appropriate parameters are chosen for the system, meaning the type of fluid and the geometry, the rate of convection can be significantly higher than the rate of conduction. Like in the mass transfer analog, several time scales exist. For the heat transport problem, the time scales will involve the thermal diffusivity, along with the kinematic viscosity and oscillation frequency as before.

The first time scale involves the thermal diffusivity of the fluid medium. This rate determines how effectively heat is transported through conduction. Just like in the mass transfer example, the viscous time scale (determined by the kinematic viscosity of the fluid) needs to be much larger than the diffusive time scale, so that heat can be transported in multiple directions through convection via the radial temperature gradients, as opposed to only axially, which occurs when the diffusive time scale dominates. The time constant for the oscillation frequency must again be similar to the viscous time scale in order to ensure the formation of a sharp, parabolic spike in the velocity profile.

1.4 Applications of Heat Transport

In the beginning of this chapter, the application of oscillatory flows for use in enclosed space capsules was described. One of the main advantages of this method was the limited mass required. The mass can be kept minimal since there is no net flow. The fluid to fill the channel is all that is needed, and can reused throughout. This mechanism can be run with simple fluids such as water and air. No volatile or hazardous chemicals are needed, which reduces the number of potential safety risks on board. Compare this to a heat pipe, where vaporizing fluids are sometimes used, and the pulsatile flow model seems fit for use in space capsules.
There are many relevant uses on Earth for this model as well. Many industrial processes and appliances generate excess heat that needs to be removed. These include exothermic reactions such as in polypropylene and polybutylene production, as well as in fuel cells. This would be useful for heat removal from hazardous substances such as radioactive fluids. One of the main goals of this mechanism is to remove heat quickly and efficiently so that it can be useful in many different types of processes.

1.5 Prior Research in Oscillatory Flows

The roots of this problem can be traced back to Taylor [13] and his work in dispersion. He showed through his work that a soluble substance in a long and narrow tube spreads out due to two effects: molecular diffusion and velocity gradients in the radial direction. He developed a model for dispersion that contains a diffusion coefficient which is now known as the Taylor diffusion coefficient. This “effective” diffusion value is

$$k \sim \frac{R^2 u_o^3}{D}$$  \hspace{1cm} (1.1)

Here $R$ is the radius of the tube, $u_o$ is the maximum velocity at the axis, and $D$ is the molecular diffusion coefficient. The important thing to notice is that this effective diffusion coefficient is inversely proportional to the molecular diffusion coefficient. This parallels the idea that at appropriate oscillating frequencies, there will be a reverse in the separation, where the species with the smaller molecular diffusion coefficient will have the higher effective diffusion coefficient and separate out quicker. It can also be seen from (1.1) that the best separation achievable when the fluid is oscillated is proportional to the molecular diffusion coefficient of the small molecule divided by the molecular diffusion coefficient of the large molecule.

Aris [1] later used the method of moments to see if Taylor’s result could be reproduced. Taylor had to restrict certain parameters for his conditions to be valid, such as the dimensions of
the tube and the velocity of the fluid through the pipe. Aris’ method eliminated those constraints by modeling the solute based on its moments in the direction of the flow. Through this method, Aris was able to obtain the Taylor diffusion coefficient. He also demonstrated how dispersion can be altered by changing the geometry of the cross section. He was able to show that there is less dispersion in an elliptical tube compared to a circular tube of the same area. This will be important in the modeling chapter when convection in various geometries is discussed. Bowden [2] also applied Taylor’s idea to show that this process occurs in large scales as well. In the sea for example, there is horizontal mixing due to a shearing current. This shearing current is an alternating, or oscillating, flow. He was able to show through experimental observations how the effective coefficient of horizontal diffusion is inversely proportional to the vertical eddy diffusion coefficient.

Harris and Goren [4] developed a model for oscillatory mass transfer in an open tube using an oscillating piston to drive the flow. They were able to show through this model that the enhanced species transport is proportional to the square of the piston amplitude. Their model was also developed to show how much the oscillations increase the mass transport over pure molecular diffusion. They were able to show how their model produces Aris’ result for the Taylor diffusion coefficient when the frequency of oscillations and tube radius are small compared to the kinematic viscosity and the molecular diffusion coefficient (low Womersley numbers). They also conducted an experiment to test their model using HCl. The results were in good agreement with their theory as they showed that the mass transport rate of HCl can be increased by as much as sixty times the rate of pure diffusion under the proper tuning conditions. A similar model to Harris and Goren’s is used in a later chapter to develop the heat transfer model for piston driven flow.
Watson [17] later studied how mass flux increased in a circular pipe and a two-dimensional channel when the fluid was oscillated. He concluded that the effective diffusivity was increased from the molecular diffusivity by a factor of $S$ where

$$S \sim \sqrt{\omega} \left( \frac{V}{Ad} \right)^2 \quad (1.2)$$

Here $\omega$ is the frequency of the oscillations, $V$ is the volume of the channel, $A$ is the cross-sectional area of the channel and $d$ is the width of the channel. An important point to note in this result is that the enhanced transport is proportional to the square root of the frequency. This will be seen in the derivation of the heat transfer model.

Kurzweg did extensive work in this field with Jaeger [7,8] and Zhao[9]. They did studies in both mass and heat transfer applications. Kurzweg and Jaeger performed experiments to verify Watson’s relation for the effective diffusion coefficient under oscillatory conditions. They were able to obtain a good data correlation with Watson’s theory which shows that the transport increases with the square root of the frequency. In this study, they also found that there exists an optimal frequency for a particular system (fluid medium and dimensions of tube), at which maximum transport will take place. Their results show that as frequency is increased starting from rest, there will be an increase in transport until it reaches the optimal tuning frequency, after which point it will decay. They also presented a study in high frequency oscillations in which the longitudinal dispersion coefficient was proportional to the square of the spike amplitude (as in Harris and Goren), and to the first power of frequency. They determined that the dispersion under oscillatory conditions can be more than twice as much compared to steady turbulent flow.

Kurzweg and Zhao [9] developed an experiment in heat transfer to enhance the thermal diffusivity of a fluid. They connected a hot and cold reservoir with a capillary bundle containing several small tubes. The oscillations were imposed by shaking a flexible membrane on each
tank. They were able to enhance the thermal transport in water by almost 18,000 times its molecular thermal diffusivity. It was also shown in their theoretical model that oscillating a liquid metal such as lithium can achieve a heat flux that is two orders of magnitude greater than those obtained in liquid metal heat pipes.

Others have studied the capillary and tube configurations such as Inaba et al. [5] and Peattie and Budwig [11]. They found that the tube or capillary geometry does affect the overall heat transport, and needs to be considered when designing a model. In separate studies, the input power required to oscillate a fluid in a heat transfer configuration has been examined by Nishio et al. [10] and Furukawa [3]. Nishio examined the optimum ratio of heat transport to input power and found that the ratio can be increased when there is a phase shift between the oscillations in adjacent tubes within the capillary bundle. Furukawa developed algebraic expressions for several output parameters as a function of the Womersley number, including input power, and the number of capillary tubes needed to achieve a desired transport rate.

Thomas and Narayanan [15,16] were able to compare the differences between the various types of driving forces used to produce oscillations. They determined that pressure driven oscillations produced a greater convective mass transport compared to the case where the moving outer wall of the tube produced the pulsations. It was also found that the ratio of convective transport to input power was identical in both cases except for extremely low frequencies where the boundary driven flow had a higher ratio. They also studied “cross over” frequencies for separations in pulsatile flow, meaning frequencies at which the two species switch in being the faster and slower transporter. It was determined that as many as three cross over frequencies can arise in certain binary gaseous systems. Thomas [14] also conducted several experiments to show that these cross over frequencies existed. He used different binary
gaseous mixtures and was able to induce the pulsations through a moving boundary. The separations were measured through the use of a gas chromatograph. In addition to finding the cross over frequencies, he also validated Kurzweg et al. that an optimal frequency existed, at which the separation would be at a maximum. All of this prior work has assisted in formulating the objectives for this project, where further progress can be made in using oscillatory flows to enhance heat transport.

1.6 Project Objectives

This project has both theoretical and experimental components. From a theoretical standpoint, the oscillatory flow model is derived after showing some simpler problems which will give the reader a better understanding of the physics. In certain cases, previous theoretical results will be verified. The model is presented in three different ways since the oscillations can be driven in various methods. The model is also discussed using various geometries which may or may not enhance the transport. These different geometries include a centered annulus, an off centered annulus, and a two compartment model involving both a cylinder and an annulus. The objective here is to determine what geometry produces the optimal transport from a theoretical perspective, as well as to see if that result changes depending on what the input parameters and conditions are.

An experimental set-up was constructed to test the basic cylindrical model for heat transport. The testing was conducted by varying key input parameters such as the amplitude of the spike, the frequency of the oscillations, the dimensions of the tube, and the type of fluid. The experimental results were compared to the theoretical model to see if they are in good agreement with each other. Also, in the case in which the experimental results deviate from the model, possible reasons are discussed. This experiment showed that heat can be transported more
effectively through the use of oscillatory flows, and can therefore be applied to removing heat in
the situations described earlier, whether it is on a large or a small scale.

Figure 1.1: Physics of mass transfer using oscillatory flows; (a) pure molecular diffusion in tube,
(b) forward pulse in oscillatory flow with molecules moving toward the boundary, (c) reverse
pulse with molecules moving toward the core, (d) end result with molecules moving down at a
faster rate compared to pure diffusion.

Figure 1.2: Physics of species separation using oscillatory flows; (a) molecular diffusion with
smaller molecules moving down tube faster (b) oscillatory flow with larger molecules remaining
in the faster core region, resulting in faster transport compared to the smaller molecules
Figure 1.3: Application to species separation in the environment
 CHAPTER 2
OSCILLATORY FLOW ALONG A FLAT PLATE

In order to understand the physics of the oscillating flow in a pipe, a simple case will first be discussed involving an oscillating plate in an infinite medium of fluid. The oscillatory nature of the plate motion is mathematically expressed as a cosine wave function. Figure 2.1 illustrates this design.

2.1 Model for Oscillatory Flow Along a Flat Plate

The oscillating plate is assumed to move in the horizontal direction. “A” represents the amplitude of the oscillating stroke, and \( \omega \) is the frequency of the plate motion. The velocity profile in this case depends only on “y.” The Navier-Stokes equation in the y-direction becomes

\[
\frac{\partial V}{\partial t} = \nu \frac{\partial^2 V}{\partial y^2}
\] (2.1)

The boundary conditions are given by no slip at the plate (\( y=0 \)), meaning the fluid will have the same velocity as the oscillating plate (\( \frac{1}{2} A \omega \cos(\omega t) \)), and that at an infinite distance away from the plate, the velocity of the fluid will approach zero. The simplest way to solve this equation is to express the velocity field as a sum of two exponentials, that is

\[
V = \hat{V} e^{i\omega t} + \tilde{V} e^{-i\omega t}
\] (2.2)

Since one term is a conjugate of the other, the algebraic manipulations are simplified and the problem can be solved using either term, meaning that

\[
\frac{i \omega \hat{V}}{\nu} = \frac{\partial^2 \hat{V}}{\partial y^2}
\] (2.3)

The solution to this equation can be found to be
\[ V = C_1 e^{\frac{1}{\sqrt{\nu}} \omega} + C_2 e^{\frac{-1}{\sqrt{\nu}} \omega} \]  

(2.4)

It can easily be seen from the boundary condition far away from the wall that \( C_1 \) must be equal to zero since the velocity must be zero. The boundary condition at the wall can be rewritten using the exponential relationship for a cosine wave, which is

\[ \cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \]  

(2.5)

Therefore the new boundary condition at the wall will be

\[ \hat{V}(y = 0) = \frac{1}{4} A \omega \]  

(2.6)

\( C_2 \) can now be solved for, and the result is

\[ \hat{V} = \frac{A \omega}{4} e^{-(1+i)\sqrt{\frac{\omega}{2\nu}} y} \]  

(2.7)

The entire expression for velocity can be found by taking twice the real part of (2.7).

Therefore

\[ V = \frac{1}{2} A \omega e^{\frac{\sqrt{\omega}}{2\nu} \omega} \cos(\omega t - \sqrt{\frac{\omega}{2\nu}} y) \]  

(2.8)

This method of solving the differential equation and the result is useful in the next step of this problem. The velocity field is used in finding the temperature profile. As in the equation of motion, only the temperature change in the y-direction will be obtained. Hence

\[ \frac{\partial T}{\partial t} + \frac{\Delta T}{L} V = \kappa \frac{\partial^2 T}{\partial y^2} \]  

(2.9)

The temperature field can be rewritten as a sum of complex exponentials much like the velocity. Therefore

\[ T = \hat{T} e^{i\omega t} + \tilde{T} e^{-i\omega t} \]  

(2.10)
The temperature equation now becomes

$$\frac{i\omega}{\kappa} \hat{T} + \frac{\Delta T}{L\kappa} \hat{V} = \frac{\hat{T}_{y}}{\hat{y}}$$

The $\Delta T/L$ term is the temperature change in the horizontal direction and can be assumed to be a constant in this case ($K$), while $\kappa$ is the thermal diffusivity. The upper boundary condition is similar to the velocity field, where at an infinite distance from the plate, the heat flux tends to zero. The lower condition would be that there is no heat flux through the wall ($dT/dy=0$). The solution becomes

$$\hat{T} = C_3 e^{-(1+i)\frac{\omega}{2\kappa}y} + C_4 e^{-(1+i)\frac{\omega}{2\kappa}y} - \frac{KA\omega}{4i} \left( \frac{1}{1-\kappa/\nu} \right) e^{-(1+i)\frac{\omega}{2\nu}y}$$

Here $C_3$ is zero due to the far-field boundary condition. The wall boundary condition can be used to solve for $C_4$ yielding

$$\hat{T} = \frac{-iKA}{2(1-\kappa/\nu)} \left( \sqrt{\frac{\kappa}{\nu}} e^{-(1+i)\frac{\omega}{2\kappa}y} - e^{-(1+i)\frac{\omega}{2\nu}y} \right)$$

Equation (2.13) can be used to solve for the actual temperature profile in (2.10) giving

$$T = \frac{KA}{2(1-\kappa/\nu)} \left[ \sqrt{\frac{\kappa}{\nu}} e^{-(1+i)\frac{\omega}{2\kappa}y} \sin(\omega t) - \frac{\omega}{2\kappa} e^{-(1+i)\frac{\omega}{2\nu}y} \sin(\omega t) \right]$$

The last quantity that needs to be found is the total heat flux. This can be expressed as the sum of the diffusive flux and the convective flux. The flux that is found is averaged over time and space to give

$$J = \frac{1}{b} \int_{0}^{b} \int_{0}^{\frac{2\pi}{\omega}} (-Kk) dt dy + \frac{\rho C_v}{b} \frac{\omega}{2\pi} \int_{0}^{\frac{2\pi}{\omega}} (VT) dt dy$$

(2.15)
In this expression, $k$ represents the thermal conductivity of the fluid, $\rho$ is the density of the fluid, $C_v$ is the specific heat of the fluid, and $b$ represents a certain distance from the oscillating plate. It is determined so that 99% of the heat transfer takes place within a region of width ‘$b$’. When (2.2) and (2.10) are inserted into (2.15), and the time integral is taken, only the cross terms will remain since the other terms integrate out to zero. This will be explained later in further detail. The equation for flux becomes

$$J = -Kk + 2 \text{Re} \left[ \frac{\rho C_v}{b} \right] \int_0^b (V \tilde{T})dy]$$  \hspace{1cm} (2.16)

Using the results found in (2.7) and (2.13), performing the integration and rearrangement produces an expression for the heat flux. It is

$$J = -Kk - \frac{\rho C_v K A^2}{V^2} \frac{\kappa}{\nu} \left[ \sqrt{\frac{\omega}{2\kappa}} - \sqrt{\frac{\omega}{2\nu}} \right] \left[ (\sqrt{\frac{\omega}{2\kappa}} + \sqrt{\frac{\omega}{2\nu}})^* \right]$$

$$\sin\left(\sqrt{\frac{\omega}{2\kappa}} b - \sqrt{\frac{\omega}{2\nu}} b\right) + \left(\sqrt{\frac{\omega}{2\kappa}} - \sqrt{\frac{\omega}{2\nu}}\right) \cos\left(\sqrt{\frac{\omega}{2\kappa}} b - \sqrt{\frac{\omega}{2\nu}} b\right)$$

$$+ \left(\sqrt{\frac{\omega}{2\nu}} - \sqrt{\frac{\omega}{2\kappa}}\right)$$  \hspace{1cm} (2.17)

The flux can also be expressed in dimensionless variables. This will be beneficial in analyzing the results and understanding the role that these different time scales play in the physics of the problem.

Define the Womersley and Prandtl numbers to be

$$Wo = A \frac{\omega}{\sqrt{2\nu}}$$  \hspace{1cm} (2.18)

and
\[
\Pr = \frac{\nu}{\kappa}
\]  

(2.19)

The amplitude can be made dimensionless as well. Thus

\[
A^* = \frac{A}{b}
\]  

(2.20)

The conductive heat flux is

\[
J_{\text{COND}} = -Kk
\]  

(2.21)

Equation (2.17) can now be expressed as a ratio of convective flux to conductive flux.

This ratio is

\[
\frac{J_{\text{CONV}}}{J_{\text{COND}}} = \frac{A^*}{8 \Pr^{1/2} (1 - \Pr^{-2})} \left\{ e^{-\left(\frac{W_o \Pr^{1/2} + W_o}{A^*}\right)} \right. \\
\left. + \frac{W_o \Pr^{1/2} - W_o}{A^*} \right\} + 1
\]

(2.22)

\[
J_{\text{CONV}} = \left(\frac{A^* W_o}{8} + 1\right)
\]  

(2.23)

2.2 Analysis of Flat Plate Model

A simple analysis can be performed on (2.22) to see what the flux should be for extreme values of the dimensionless terms. In the simple case where the plate is not oscillating, the ratio equals 1, which makes physical sense as the only transport that occurs is through conduction.

Another case to examine is when the Prandtl number becomes much greater than 1. Taking the limit as Pr goes to infinity produces a result that is

This approximation can be useful in estimating the flux for fluids with high Prandtl numbers (such as oils). A physical interpretation for this case is to look at the problem when the kinematic viscosity is much larger than the thermal diffusivity (the definition for a high Prandtl
number). Looking at the equation for the velocity field (2.8) and taking the limit when \( \nu \) tends to infinity, the result is the same as the velocity of the plate. This means that every particle of fluid moves at the same velocity as the plate. In this case there won’t be some fluid elements moving with the forward stroke and others on the backward stroke; instead they will all move together. This promotes a higher rate of heat flux due to convection as opposed to conduction. This is what would be expected since the thermal diffusion coefficient is extremely low compared to the kinematic viscosity. It can also be noted from this equation that the ratio is proportional to the square of the amplitude (observe that the amplitude appears in the Womersley number as well).

The frequency of the oscillations, the amplitude of the stroke, and the physical properties of the fluid are all input variables in this case, while the convective to conductive ratio is the output result. The equation for an infinite Prandtl number fluid (2.23) can be shown to be accurate by looking at Figure 2.2, which uses (2.22) to show heat flux for various Prandtl numbers. It can be seen that for the higher Prandtl number fluid \((Pr=500)\), the heat flux approaches the graph for \(Pr\) tending to infinity. This same line is the plot of (2.23). It also makes sense since for high \(Pr\) fluids, the heat flux increases linearly with the Womersley number from (2.23). It is obvious that a linear plot results in Figure 2.2. Therefore, it can be concluded that (2.23) is an accurate representation of (2.22) for an infinite Prandtl number.

Although it can be seen in Figure 2.2 that the convective heat flux increases with increasing Prandtl number, it should be pointed out that it only increases for a given Womersley number, and that kinematic viscosity appears in the denominator of the Womersley number. When the kinematic viscosity (or Prandtl number) becomes high, the convective transport only increases if there is an increase in the amplitude or frequency of the plate to keep the Womersley number constant. This makes physical sense since a fluid with a high viscosity is very sluggish.
Although the fluid elements are more likely to move with the velocity of the plate as stated above, there is more energy required by the system to shear a very viscous fluid back and forth. This extra energy would come from increasing the size and/or frequency of the oscillations. In fact, there will be a decrease in the transport if only the kinematic viscosity is varied and nothing else. This becomes an issue during experimentation, and is discussed more in Chapter 5.

Lastly, the flux value can be found when the Prandtl number approaches 1. This will help in determining a result for those fluids with low Prandtl numbers such as gases. Taking (2.22) as a limit when Pr approaches 1, we get

$$\frac{J_{\text{conv}}}{J_{\text{cond}}} = \left( \frac{A^*}{32} \right) \left( e^{-\frac{2Wo}{A^*}} \left( \frac{2Wo^2}{A^*} + Wo \right) - Wo \right) + 1$$

(2.24)

This equation shows that for a low Prandtl number fluid, the convective to conductive ratio will be lower than the case for a very high Prandtl number fluid simply by the fact that the ratio in (2.24) roughly scales by a factor of $e^{-2/32}$ compared to $1/8$ in (2.23). This would make sense since a lower Prandtl number fluid can mean it has a high thermal diffusivity; such as in the case of air. Not only will this drive up the conductive heat transfer, but since gases are lighter and have a lower viscosity than liquids, they will feel less of the shearing effect brought about by a moving plate. This means that the molecules will be less likely to move with the speed of the plate, and instead move at a rate proportional to the thermal diffusivity of the gas. This means less transport via convection. For both of these reasons, the convective to conductive ratio will decrease.

Although simple from a physical standpoint, the mathematics of the problem can be somewhat complicated. It is important to understand not only the physics of the problem, but the mathematical techniques as well, since they will be applied in more detail to the scenario involving oscillating flow in a pipe. Before that point is reached, two other example problems
will be presented to show how the length and time scales relate to the physics of the problem. These discussion notes are very basic chemical engineering problems, however a lot can be obtained from them. A key idea to take from these problems is to see how an input function (in these cases an input oscillation) affects the behavior of a system. Also, it is important to understand how these input functions affect the system response depending on the size and type of the input signal, as well as the size of the system. The first problem presented will involve a simple CSTR with an oscillating input. It can be easily solved from a mathematical standpoint and clearly shows the relationship between input signals and system response. The second problem is a little more complicated mathematically, but still shows the same relationship. This problem is one where a solid rod is heated at one end in an oscillatory manner. Since the first example relates to mass transfer, and the second relates to heat transfer, these problems should help the reader conclude that these systems behave in a similar way, regardless of the transport medium.

2.3 Discussion Note 1: Oscillating Input in a CSTR

Figure 2.3 shows a constant stirred tank reactor that is receiving input concentration as a sinusoidal oscillation. Writing the mass balance for the CSTR, we get

$$\frac{dC(t)}{dt} = \tau \left( C_{in}(t) - C(t) \right)$$

(2.25)

where \( C \) is the concentration in the tank, and \( \tau \) is the residence time which is equivalent to the volume of the tank divided by the volumetric flow rate. \( K \) is a concentration amount that represents the magnitude at which mass enters the tank. The inlet concentration is expressed as a sinusoidal wave to give

$$K \sin(\omega t) - C(t) = \tau \frac{dC(t)}{dt}$$

(2.26)
The initial condition for this problem is that the concentration in the tank is zero when time is equal to zero \(t=0\). The problem can be scaled where

\[
\theta = \omega t
\]

\[
W = \omega \tau
\]

and

\[
C^* = \frac{C}{K}
\]

The scaled form of (2.26) is written

\[
\sin \theta - C^* = W \frac{dC^*}{d\theta}
\]  

(2.30)

The differential equation (2.30) can be easily solved using an integrating factor to obtain

\[
C^* = \frac{\sin \theta}{W^2(\frac{1}{W^2} + 1)} - \frac{\cos \theta}{W(\frac{1}{W^2} + 1)} + C_1 e^{-\frac{\theta}{W}}
\]  

(2.31)

\(C_1\) is a constant that can be found based on the scaled initial condition that \(C^*=0\) at \(\theta=0\).

The final expression for \(C^*\) can be found after some rearranging. It is

\[
C^* = \frac{\sin \theta - W \cos \theta + W e^{-\frac{\theta}{W}}}{1 + W^2}
\]  

(2.32)

This equation describes the response of the system to the input parameters. First take the case of a very small frequency, which will cause \(W\) to become very small. If the limit of (2.32) is taken as \(W\) tends to zero, the result will be that the concentration tends to \(\sin \theta\), which is the behavior of the input wave. This makes sense if the inertia is small enough; there will be little resistance in the tank to change the response in the tank. If \(W\) were increased, it can be seen from (2.32) that the response of the tank will behave as a combination of sine and cosine waves.
Now that the inertia has become large, there will be more of a resistance from the tank, causing a time lag.

Note that the volume of the tank appears in the constant $W$. When $W$ is small, the volume of the tank can be considered small. This means that there is less room for the input signal to break up into sine and cosine parts, which is the case for small $W$. For large $W$, the volume of the tank can be large, so the signal can become different by the time it permeates throughout the tank. Being able to examine the output response of a system based on setting the input parameters is important for understanding the behavior the system. This will be critical when the oscillating flow in a pipe model is presented in the next chapter. One more example will be shown to further discuss these concepts.

2.4 Discussion Note 2: Rod Heated Through Oscillatory Temperature Gradient

A rod is heated at one end via an oscillatory heat source and kept constant at the other end in Figure 2.4. The one-dimensional energy equation is established to be

$$ \frac{dT}{dt} = \kappa \frac{d^2T}{dx^2} $$

Here $\kappa$ represents the thermal diffusivity of the rod material. The problem can be scaled in a way similar to the previous example where

$$ \theta = \omega t $$

$$ x^* = \frac{x}{L} $$

$$ T^* = \frac{T}{K} $$

and

$$ W = \frac{L^2 \omega}{\kappa} $$

(2.33)
Again $W$ can be considered to be the input, although the characteristic length now is the length of the rod. A similar term will arise in the oscillatory flow model in a tube. The energy equation can be written in terms of complex exponentials as in (2.10), that is

$$\hat{T} = \hat{T} e^{i\theta} + \bar{T} e^{-i\theta}$$

Scaling the problem, we get

$$-i\theta W \frac{\ddot{T}^*}{\bar{T}^*} = \frac{d^2 T^*}{dx^*}$$

The solution for this equation can now be found to be

$$T^* = C_1 \cos(\sqrt{i\theta W}) + C_2 \sin(\sqrt{i\theta W})$$

The boundary conditions can be found in Figure 2.4; however the condition at the right hand end of the rod can be rewritten

$$\ddot{T}^* (x^* = 1) = i$$

since

$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

The time independent part of the temperature field is now

$$T^* = \frac{i \sin(\sqrt{i\theta W} x)}{2 \sin(\sqrt{i\theta W})}$$

The complete temperature field can now be found. It is

$$T^* = \text{Re} \left[ \frac{i \sin(\sqrt{i\theta W} x)}{\sin(\sqrt{i\theta W})} e^{-i\theta} \right]$$

After taking the real part of (2.44), and some tedious algebra, the final result of the temperature field is completed. Hence
\[ T^* = \frac{1}{[\sin(\lambda)\cosh(\lambda)]^2 + [\sinh(\lambda)\cos(\lambda)]^2} \]

\[ \{[\sin(\lambda x)\cosh(\lambda x)\sin(\lambda)\cosh(\lambda) + \sinh(\lambda x)\cos(\lambda x)\sinh(\lambda)\cos(\lambda)]\sin \theta + \]

\[ [\sin(\lambda x)\cosh(\lambda x)\sinh(\lambda)\cos(\lambda) - \sinh(\lambda x)\cos(\lambda x)\sin(\lambda)\cosh(\lambda)]\cos \theta \} \]

(2.45)

where

\[ \lambda = \sqrt{\frac{\theta W}{2}} \]

(2.46)

When \( W \) is taken to be small, it can be seen from (2.45) that \( \cos \theta \) terms will disappear, leaving the temperature response in the rod to behave as \( \sin \theta \), just like the input. Obviously, as \( W \) is increased, the temperature field will behave as a mixture of sine and cosine terms. Again, this is similar to the previous discussion note. Since \( W \) contains the characteristic length \( L \), a small value of \( W \) can mean a small rod length. The shorter length keeps the input sine wave from breaking up all the way to the end of the rod. Again, a larger characteristic length will cause a break up in the response to the inertial force, resulting in a combination of response terms.

Although these problems appear simple, the length and time scales that are generated in them can tell you a lot about the physics in the problems. Hopefully these two examples have clarified the concept of input signal relating to system behavior. It should also be noted that the oscillating plate problem presented earlier in this chapter behaves in the exact same way. The Womersley number in the problem is quite similar to \( W \) in the CSTR and solid rod examples. When the Womersley number is very small, the system behaves in the same way as the input signal, as \( \cos(\omega t) \). When the Womersley number grows, the system will behave differently, thus there is a time lag that arises. When the oscillatory flow model is presented, one may notice
similarities with these examples. There are many ways to impose the oscillations in the model, and depending on what the input signals are, the output response from the other parameters will behave in a way that is consistent with these prior examples. When the signal is small, the system will respond proportionally to the signal. A larger signal (which will occur in the oscillatory flow model) will cause a mixture of output responses from the system. Now that the reader has a better understanding of these basic problems, the oscillatory flow model can be presented.

Figure 2.1: Design for oscillating flat plate problem

Figure 2.2: Heat flux at various Prandtl numbers as a function of Womersley number
Figure 2.3: CSTR with oscillating input

Figure 2.4: Rod with oscillatory heat source on one end
Consider a model of heat transfer enhancement by oscillatory flow where the fluid resides in a cylindrical tube connected between a heat source and a heat sink. In order to oscillate the fluid in the tube, one of many methods may be used. The first is to oscillate the fluid by imposing an oscillatory pressure gradient across the ends of the tube. The second way is to “push” the fluid back and forth with an external mechanical piston. A third method is to move the wall of the tube back and forth, whence the shearing of the fluid can also cause the oscillations. The models for these three methods are first discussed for a circular cylindrical geometry.

3.1 Oscillatory Flow in a Cylindrical Tube

3.1.1 Pressure Driven Flow

To model this case, write the Navier Stokes equations. Therefore

\[ \rho \left( \frac{dV}{dt} + V \cdot \nabla V \right) = \mu \nabla^2 V - \nabla P - \rho g \]  

(3.1)

Several assumptions will be made to simplify this momentum balance. Fully developed flow is assumed in the axial (z) direction only. This means that the pressure gradient is only assumed to be in the axial direction and gravitational effects are assumed to be negligible. This reduces (3.1) for a cylindrical geometry, hence

\[ \rho \frac{\partial V_z}{\partial t} = \mu \left( \frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r} \frac{\partial V_z}{\partial r} \right) - \frac{\partial P}{\partial z} \]  

(3.2)

This equation can be rewritten as

\[ \frac{1}{\nu} \frac{\partial V_z}{\partial t} = \frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r} \frac{\partial V_z}{\partial r} - \Delta P \]  

(3.3)
where

$$\Delta P = \frac{1}{\mu} \frac{\partial P}{\partial z}$$  \hspace{1cm} (3.4)

To solve for the velocity profile, it is convenient to break the velocity field into a sum of time complex exponentials from the start, that is

$$V_z = \hat{V}_z e^{i\omega t} + \tilde{V}_z e^{-i\omega t}$$  \hspace{1cm} (3.5)

Likewise, $\Delta P$ becomes

$$\Delta P = \hat{P} e^{i\omega t} + \tilde{P} e^{-i\omega t}$$  \hspace{1cm} (3.6)

Substituting (3.5) and (3.6) into (3.3) yields

$$\frac{i \omega}{\nu} \hat{V}_z = \frac{\partial^2 \hat{V}_z}{\partial r^2} + \frac{1}{r} \frac{\partial \hat{V}_z}{\partial r} - \hat{P}$$  \hspace{1cm} (3.7)

The common exponential terms are factored out. In addition, since the two terms in (3.5) and (3.6) are complex conjugates of each other respectively, only one of the two terms needs to be solved.

In the case of an oscillating pressure drop being an input, the value must be real; therefore

$$\hat{P} = \tilde{P}$$  \hspace{1cm} (3.8)

The solution for the velocity field is

$$\hat{V}_z = C_r J_0(\alpha r) + C_y Y_0(\alpha r) + \frac{\hat{P}}{\alpha^2}$$  \hspace{1cm} (3.9)

where

$$\alpha = \sqrt{-\frac{i \omega}{\nu}}$$  \hspace{1cm} (3.10)
When (3.10) is multiplied by the characteristic length, which in this case is the radius of the tube, one can obtain a dimensionless term known as the Womersley number. This happens when

\[ \alpha R = \sqrt{-\frac{i \omega R^2}{\nu}} = \sqrt{-i \omega \alpha R^2} \]  

(3.11)

This group is similar to those developed in the Discussion Notes in Chapter 2. Note here that the length scale is the radius of the tube, which is different from the case in Discussion Note 2. In that example, the inertial term was scaled with the axial direction, as the heat through conduction moved down the rod. In this model, the critical length scale is now the tube width. This is the case because the inertial effect comes from moving the molecules in the radial direction. The tube width determines the magnitude of radial temperature gradients that can arise, and these gradients are what enhance the heat transport through convection. In addition to that, the tube length is assumed to be much greater than the tube width. This means that the effects of the gradients will be more significant over the tube width compared to the effects of the axial gradients over the tube length. This is why the radius of the tube is the important length scale for this problem.

The boundary conditions arise from observing that there is zero velocity at the wall \((r = R)\) and a finite velocity at the center point of the tube \((r = 0)\). It can be seen in (3.9) that \(C_2\) is equal to zero from the second boundary condition since the Y-Bessel function is unbounded at \(r = 0\). \(C_1\) can be found by applying the condition at \(R\). The expression for the velocity field is now found to be

\[ \hat{V}_z(r) = \frac{P}{\alpha^2}[1 - \frac{J_0(\alpha r)}{J_0(\alpha R)}] \]  

(3.12)
The solution for $\tilde{V}_z$ is simply the complex conjugate of $\hat{V}_z$. This result is then used to solve the energy equation, which is

$$\frac{\partial T}{\partial t} + V_z \frac{\partial T}{\partial z} = \kappa \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right)$$

(3.13)

Here, $\kappa$ is the thermal diffusivity. The axial temperature gradient ($dT/dz$) is assumed to be constant since entrance and exit effects are neglected in this model. The viscous dissipation term will be neglected as well. The same method can be applied to express the temperature field as a sum of complex exponentials, hence

$$T = T e^{i\omega t} + \tilde{T} e^{-i\omega t}$$

(3.14)

Using the corresponding term in the velocity field, (3.13) is now

$$\frac{i \omega}{\kappa} T + \frac{V_z}{\kappa} \frac{\Delta T}{L} = \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right)$$

(3.15)

The solution is

$$\tilde{T} = C_3 J_0(\beta r) + C_4 Y_0(\beta r) + C_5 + C_6 J_0(\alpha r)$$

(3.16)

where

$$\beta = \sqrt{-\frac{i \omega}{\kappa}}$$

(3.17)

$C_3$ and $C_4$ are constants that can be found from the boundary conditions. $C_5$ and $C_6$ are constants from the particular solution. Observe that the boundary conditions derive from the fact that the temperature is finite at the center of the tube ($r = 0$) and that there is no heat flux through the walls of the tube ($dT/dr = 0$). Once the constants are found, the temperature field solution is now complete. It is
\[ T(r) = \frac{P_i}{\alpha^2 \omega} \Delta T \left[ \frac{1 - \Pr^{-1} - \frac{J_0(\beta r)}{J_0(\alpha R)} + \frac{\alpha J_1(\alpha R)}{\beta J_1(\beta R)} \frac{J_0(\alpha R)}{J_0(\beta R)} \right] \]

\[
\Pr = \frac{V}{\kappa} \quad \text{(3.19)}
\]

The Prandtl number (Pr) is important because it compares the two time scales used in this problem, the viscous time scale and the thermal diffusive time scale. The next task is to find the total heat flux which is equal to the conductive heat flux plus the convective heat flux. This is

\[
J = -k \frac{\Delta T}{L} + \rho C_v V_z T \quad \text{(3.20)}
\]

Here \( \rho \) is the density and \( C_v \) is the specific heat. In order to evaluate the total heat transfer rate, (3.20) must be integrated over the cross sectional area and time averaged over one cycle to get

\[
Q_{TOR} = \frac{\omega}{2\pi} \int_0^{2\pi} \int_0^R 2\pi (-k \frac{\Delta T}{L} + \rho C_v V_z T) r dr dt \quad \text{(3.21)}
\]

The conductive term is a simple expression and can be easily evaluated. Inserting this into (3.21) we obtain

\[
Q_{TOR} = -k \frac{\Delta T}{L} \pi R^2 + \frac{\omega}{2\pi} \rho C_v \int_0^{2\pi} \int_0^R 2\pi V_z Tr dr dt \quad \text{(3.22)}
\]

When substituting (3.5) and (3.6) for the velocity and temperature fields respectively, we get

\[
Q_{CONV} = \frac{\omega}{2\pi} \rho C_v \int_0^{2\pi} \int_0^R 2\pi (V_z e^{i\omega t} + \dot{V}_z e^{-i\omega t})(\dot{T} e^{i\omega t} + T e^{-i\omega t}) r dr dt \quad \text{(3.23)}
\]
When the time integral is evaluated, only the “cross terms” will remain since the other
two terms integrate out to zero. Therefore

\[
\int_{0}^{2\pi} e^{2i\omega t} dt = \int_{0}^{2\pi} e^{-2i\omega t} dt = 0
\]

so

\[
Q_{\text{CONV}} = \rho C_v \int_{0}^{R} 2\pi (V_z \hat{T} + \tilde{V}_z \hat{T}) r dr
\]

(3.25)

Since these terms are complex conjugates of one another, they can be combined and
twice the real part of the remaining term can be taken to get the result for the convective heat
transfer, which is

\[
Q_{\text{CONV}} = 2 \text{Re} \left[ \rho C_v \int_{0}^{R} 2\pi (V_z \hat{T}) r dr \right]
\]

(3.26)

Evaluating the integral and combining it with the conductive part results in the final
expression for the total heat transfer, and after some algebraic manipulation we get

\[
Q_{\text{TOT}} = Q_{\text{COND}} \left[ 4P \frac{\omega^2}{R} \frac{\text{Pr}^2}{(\text{Pr}^2 - 1)} \text{Re} \left( \frac{\alpha' J_1(\alpha R)J_1(\alpha' R)J_0(\beta)}{\beta \alpha^3 J_0(\alpha R)J_0(\alpha' R)J_1(\beta)} \right) \right]
\]

(3.27)

where

\[
Q_{\text{COND}} = -k \frac{\Delta T}{L} \pi R^2
\]

(3.28)

and

\[
\alpha' = \alpha
\]

(3.29)
In order to show the enhancement of heat transfer due to the oscillations, (3.27) can be expressed as a ratio of convective heat transport to conductive heat transport to yield

\[
\frac{Q_{\text{CONF}}}{Q_{\text{COND}}} = \frac{4 P}{\omega^2 R} \frac{\Pr^2}{(\Pr^2 - 1)} \operatorname{Re}\left(\frac{\alpha' J_1(\alpha R) J_1(\alpha' R) J_0(\beta)}{\beta \alpha^3 J_0(\alpha R) J_0(\alpha' R) J_1(\beta)} - \frac{\alpha' J_1(\alpha' R)}{\alpha^3 J_0(\alpha' R)}\right)
\]  

(3.30)

A key thing to note in (3.30) is how the problem scales with the square of the pressure drop. This is because the velocity and temperature fields scale with the pressure drop. Since the convective heat transport is based on the product of the velocity and temperature fields, it should come as no surprise that the convective heat transport is proportional to the square of the driving force.

Another term in (3.30) that is important is the group where the Prandtl number appears. This term gets smaller as the Prandtl number grows. This is consistent with what was discussed in Chapter 2. A large Prandtl number can imply a very viscous fluid. More energy is required to drive a more viscous fluid; therefore, less heat will be transported convectively if the input energy is held constant. One may wonder what happens when the Prandtl number is less than 1, which is usually the case for gases. This term would become negative under those circumstances. The convective to conductive ratio will still be positive since the Bessel’s functions terms will account for the sign discrepancy. When the thermal diffusivity is greater than the kinematic viscosity, the real part of the Bessel’s functions group will be negative, keeping the overall ratio positive. All of these relationships will still exist even when the driving force is altered in the next case, where an oscillating piston is used.

3.1.2 Piston Driven Flow
If a mechanical piston is used to drive the oscillations instead of an imposed pressure drop, the model does not change very much. Suppose the position of the piston behaves as a sine wave, that is

\[ x = \frac{1}{2} A \sin \omega t \]  

(3.31)

Then the speed of the piston would be

\[ \frac{dx}{dt} = \frac{1}{2} A \omega \cos \omega t \]  

(3.32)

The momentum balance of course will not change. However, in order to get (3.12) in terms of piston amplitude, another relationship needs to be used. Since the integral of the velocity profile over the cross sectional area is equal to the volumetric flow rate, we get

\[ \int_{0}^{R} 2\pi v_z r dr = \frac{1}{2} A \omega \cos \omega t \]  

(3.33)

Also (3.33) can be re-written as

\[ \int_{0}^{R} 2\pi \hat{v}_z r dr = \frac{1}{4} A \omega \]  

(3.34)

where we have used

\[ \cos \omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2} \]  

(3.35)

Using (3.34), the pressure term can now be found in terms of amplitude to yield

\[ P = \frac{A \omega \alpha^2}{[4 - \frac{8J_1(\alpha R)}{R \alpha J_0(\alpha R)}]} \]  

(3.36)

The velocity field is now in terms of the input amplitude as (3.36) is substituted into (3.12) to give
\[ V_z(r) = A\omega \left[ 1 - \frac{J_0(\alpha r)}{J_0(\alpha R)} \right] \]

\[ 4 - \frac{8J_1(\alpha R)}{\alpha RJ_0(\alpha R)} \]

The rest of the derivation is exactly the same as before with the exception of this one substitution. The temperature field is now given by

\[ \hat{T}(r) = \frac{Ai\rho C_v \Delta T}{L} \left[ \frac{1 - \Pr^{-1} - \frac{J_0(\alpha r)}{J_0(\alpha R)} + \frac{\alpha J_1(\alpha R)J_0(\beta r)}{\beta RJ_1(\beta R)J_0(\alpha R)}}{1 - \Pr^{-1}(4 - \frac{8J_1(\alpha R)}{\alpha RJ_0(\alpha R)})} \right] \]

The total heat transfer in terms of input amplitude is

\[ Q_{TOT} = Q_{COND} \left( \frac{A^2}{R^2} \frac{\Pr^2}{(\Pr^2 - 1)} \right) * \left\{ \frac{4J_0(\beta R)}{\beta RJ_1(\beta R)} - \frac{4J_0(\alpha R)}{\alpha RJ_1(\alpha R)} \right\} \]

\[ \frac{4J_0(\alpha R)}{\alpha RJ_1(\alpha R)} - \frac{8}{\alpha^2 R^2} \left[ \frac{4J_0(\alpha')R_1(\alpha')}{\alpha' RJ_1(\alpha') - \frac{8}{\alpha^2 R^2}} \right] + 1 \]

Expressed as a convective to conductive ratio, we get

\[ \frac{Q_{COND}}{Q_{CONV}} = \left( \frac{A^2}{R^2} \frac{\Pr^2}{(\Pr^2 - 1)} \right) * \left\{ \frac{4J_0(\beta R)}{\beta RJ_1(\beta R)} - \frac{4J_0(\alpha R)}{\alpha RJ_1(\alpha R)} \right\} \]

\[ \frac{4J_0(\alpha R)}{\alpha RJ_1(\alpha R)} - \frac{8}{\alpha^2 R^2} [\frac{4J_0(\alpha')R_1(\alpha')}{\alpha' RJ_1(\alpha') - \frac{8}{\alpha^2 R^2}}] \]

This is the result obtained by Harris and Goren [4] in their model for pulsating flow, the only difference being the dimensionless Prandtl number for heat transfer replaces the Schmidt number (Sc) for mass transfer in their model. Again the driving force (amplitude) appears to the second power in the final heat transfer equation. This occurs for the same reasons as in the
pressure driven flow case. The last method, where the boundary is oscillating, will simplify the initial equations somewhat, but a similar solution will still be obtained.

### 3.1.3 Boundary Driven Flow

Suppose now that the oscillations are generated through the outer wall moving back and forth in the axial direction with a velocity of $1/2^*A_0\omega \cos(\omega t)$. The momentum balance will be the same as (3.3), however there will not be a driving force term from the pressure drop as the motion is driven by the boundaries. The solution is solved in the same way as before, except there is only a homogeneous solution, hence

$$\hat{V}_z = C_1J_0(\alpha r) + C_2Y_0(\alpha r)$$  \hspace{1cm} (3.41)

The boundary condition at the center of the tube is the same as before, meaning $C_2$ is equal to zero, and using (3.35), the velocity at the wall is equal to

$$\hat{V}_z(r = R) = \frac{1}{4} A \omega$$  \hspace{1cm} (3.42)

$C_1$ can now be solved for and the velocity profile is

$$\hat{V}_z(r) = \frac{A \omega J_0(\alpha r)}{4 J_0(\alpha R)}$$  \hspace{1cm} (3.43)

The temperature profile can be solved as before. The boundary conditions remain the same as in the other cases; this means that we have finite temperature at the center and no heat flux through the wall.

$$\hat{T}(r) = \frac{Ai \Delta T}{4 \Delta (1 - Pr)} \left[ J_0(\alpha r) - \frac{\alpha J_1(\alpha R)J_0(\beta r)}{\beta J_1(\beta R)J_0(\alpha R)} \right]$$  \hspace{1cm} (3.44)

The total heat transfer is determined in the same way as before to get
\[ Q_{TOT} = Q_{DIFF} \left( \frac{A^2 \alpha \beta}{16R(1 - Pr^{-1})} \right) \]  
\[ \left[ \frac{\alpha' J_0(\beta R)J_1(\alpha' R) - \beta J_1(\beta R)J_0(\alpha' R)}{(\beta^2 - \alpha'^2)J_1(\beta R)J_0(\alpha' R)J_0(\alpha R)} \right] + 1 \]

and the ratio which is

\[ \frac{Q_{CONV}}{Q_{COND}} = \frac{A^2 \alpha \beta}{16R(1 - Pr^{-1})} \left[ \frac{\alpha' J_0(\beta R)J_1(\alpha' R) - \beta J_1(\beta R)J_0(\alpha' R)}{(\beta^2 - \alpha'^2)J_1(\beta R)J_0(\alpha' R)J_0(\alpha R)} \right] \]

Although these three methods of oscillating have a different driving force, the one thing that appears in all three models is that the overall heat transfer is proportional to the square of the driving force. For example, if the amplitude is doubled, then the heat transfer will be quadrupled. This relationship was tested experimentally and will be discussed in Chapter 5.

The relationship of input signal to output response can be seen in each of these cases as well. It was shown in the Discussion Notes in Chapter 2 that the output response depends on the size of the input signal and the size of the system. In looking at the final expression for heat transfer in each of these examples, it is evident that the system response is a combination of terms (Bessel’s functions) unlike the input signals. For example, when the wall oscillates as a cosine function, the velocity and temperature profiles will behave as a combination of sine and cosine functions (expressed as Bessel’s functions for a cylindrical geometry). Just like in the last chapter, the break up of the input signal will occur when the Womersley number reaches a certain limit. The specific value is not relevant here, but it is important to see that the oscillatory flow model has a simple input signal, and increasing that signal will result in a different output response, no matter what the means of oscillation are.

The open cylinder is a common configuration for a connecting tube. However, one might imagine that potential improvements can be made by increasing the number of boundaries, which would in turn increase the amount of radial temperature gradients, by generating more velocity
spikes. A simple configuration that comes to mind that meets this criterion is that of an annular tube case to which we now turn.

3.2 Oscillatory Flow in an Annulus

Notice from Figure 3.4 that the inner boundary generates an additional spike when viewing this cross section. The method for the annular flow model is exactly the same as the cylindrical flow model with the exception of the inner boundary condition. Although this is the only difference, this will make the result much more complicated. The three variations presented above will also be shown here.

3.2.1 Pressure Driven Flow

The solution for the momentum equation is identical to (3.9). However, with the new boundary condition at the inner wall, $C_2$ is no longer zero. The boundary condition at the inner wall ($r = R$) and the outer wall ($r = cR, c>1$) are now the same, that is the velocity equals zero. The solved velocity field is now

\[
\hat{V}_z(r) = \frac{P}{\alpha} \left\{ 1 - \frac{J_0(\alpha r)[Y_0(\alpha c R) - Y_0(\alpha R)]}{J_0(\alpha R)Y_0(\alpha c R) - J_0(\alpha c R)Y_0(\alpha R)} \right\} \frac{Y_0(\alpha r)[J_0(\alpha R) - J_0(\alpha c R)]}{J_0(\alpha R)Y_0(\alpha c R) - J_0(\alpha c R)Y_0(\alpha R)}
\]

As in the case of the velocity field, the inner and outer wall boundary conditions on temperature are the same, meaning there is no heat flux through either wall (therefore $dT/dr = 0$ at these walls). The solution for the temperature field is similar to (3.16), with an additional particular solution that is proportional to the Y-Bessel’s function. Therefore

\[
\hat{T} = C_1 J_0(\beta r) + C_2 Y_0(\beta r) + C_3 + C_4 J_0(\alpha r) + C_5 Y_0(\alpha r)
\]

$C_3, C_4, \text{ and } C_5$ are found from solving for the particular solution, while $C_1$ and $C_2$ are determined from the boundary conditions all yielding
The expression for the total heat transfer is equal to

\[ Q_{\text{tot}} = -k \frac{\Delta T}{L} \pi R^2 (e^2 - 1) + 2 \text{Re} \int_r^{cR} 2\pi V_\tilde{z} T rdr \]  

(3.54)

\( \tilde{V}_z \) is the conjugate of \( V_z \). Since the expression is now quite complicated, MAPLE® is used to further evaluate (3.54). The next case for annular flow, the oscillating piston problem, is
quite similar to the oscillating pressure drop case. The same method to relate the pressure drop model to the piston model for a cylinder will be used now, with a change in the integration limits to account for the geometry difference.

3.2.2 Piston Driven Flow

The solution for the piston driven problem is the same as the pressure driven problem with the exception of one substitution. By inserting (3.47) into (3.34) and solving for the scaled pressure, we get

$$\hat{P} = \frac{A\omega\alpha^2 R^2(c^2 - 1)}{8Z}$$  \hfill (3.55)

where

$$Z = \left\{ \frac{R^2(c^2 - 1)}{2} - \frac{R[cJ_i(\alpha R) - J_i(\alpha R)][Y_0(\alpha R) - Y_0(\alpha R)]}{\alpha[J_0(\alpha R)Y_0(\alpha R) - J_0(\alpha R)Y_0(\alpha R)]} \right\}$$

(3.47) through (3.53) can now be found in terms of piston amplitude, and the final result can be determined using (3.54).

3.2.3 Boundary Driven Flow

For the problem where the outer wall is oscillating, the boundary condition for the momentum equation will be the same as (3.42). The velocity will be zero at the inner wall. Using these to solve for the velocity profile, we get

$$V_z(r) = \frac{A\omega}{4} \frac{[J_0(\alpha R)Y_0(\alpha R) - Y_0(\alpha R)J_0(\alpha R)]}{[Y_0(\alpha R)J_0(\alpha R) - J_0(\alpha R)Y_0(\alpha R)]}$$

(3.57)

There is no heat flux through either wall, so the temperature solution is

$$\hat{T}(r) = C_1J_0(\beta r) + C_2Y_0(\beta r) + C_3J_0(\alpha r) + C_4Y_0(\alpha r)$$

(3.58)

where
\[ C_1 = \frac{\alpha}{\beta} \left\{ \frac{C_3 [J_1(\alpha R)Y_1(\beta c R) - J_1(\alpha c R)Y_1(\beta R)]}{[J_1(\beta c R)Y_1(\beta R) - J_1(\beta R)Y_1(\beta c R)]} \right\} + \]

\[ C_2 = \frac{-C_1 \beta J_1(\beta R) - \alpha C_4 Y_1(\alpha R)}{\beta Y_1(\beta R)} \]

\[ C_3 = \frac{-A\Delta Ti}{4L(1 - Pr^{-1})} \frac{Y_0(\alpha R)}{Y_0(\alpha R)J_0(\alpha R) - Y_0(\alpha R)J_0(\alpha R)} \]

\[ C_4 = \frac{-A\Delta Ti}{4L(1 - Pr^{-1})} \frac{J_0(\alpha R)}{Y_0(\alpha R)J_0(\alpha R) - Y_0(\alpha R)J_0(\alpha R)} \]

The overall heat transfer can be found by solving equation (3.54). In summary, the input parameters for this model are the amplitude and frequency of the driving force (either the pressure drop, the piston, or moving wall), the length and radius of the connecting tube, the fluid used, the geometry of the tube (cylindrical or annular), and the axial temperature drop. Once all these parameters are inserted into the appropriate model, the conductive, convective, and total heat transport can be calculated. Now that the cylindrical and annular models have been presented, it is important to determine which geometry is more optimal, and under what conditions.

### 3.3 Comparison of Annular and Cylindrical Geometries

Four cases are discussed to determine whether the cylindrical or annular geometry is preferable for enhanced heat transfer. In the first case, the heat transport in an open tube is compared to one in an annular geometry whose outer radius is held equal to the radius of the open tube. As the inner rod’s radius is decreased, one expectedly recovers the results for an open tube. The comparison between the two geometries is made holding the pressure drop per unit
length fixed. In a companion calculation, the volumetric flow rate in a half cycle is held fixed instead of the pressure drop. While an idea regarding the effect of the walls can be obtained, the comparison between the two geometries is not entirely fair because the cross-sectional areas of the two geometries are not kept equal. Thus two other calculations that mirror the first two cases are examined with the only difference being that the cross-sectional area of the cylinder and annulus are now held fixed. The results are telling.

The Prandtl number used in the calculations corresponds to water and is set to a value of 7. The physical properties of water were used when evaluating the density, viscosity, thermal conductivity, and the specific heat. The other parameter values held constant in all the calculations are the following: tube length of 10 cm, a frequency of 0.1 Hz, and an axial temperature difference of 10 deg C. These values are of little concern as the qualitative nature is what is important.

Figure 3.5 depicts the ratio of the transport in the annulus to the transport in the cylinder while the inner radius of the annulus is varied. A value of 1 cm for the outer radius was used in the first two sets of calculations. It is observed that as the inner radius is decreased in the annulus, the values of the heat transport come close to the open tube result. One is led to the conclusion that the open tube cylindrical geometry is a more optimum design compared to an annulus. The reason for this conclusion is that the available region for heat transfer decreases as the inner radius increases. This means that for a fixed pressure drop, as the size of the inner rod increases, the resistance to flow increases, and the sharpness in velocity spikes decrease, leading to a lower heat transport. The transport decreases from the open tube even when the radius of the inner rod is small. This is expected as the resistance to flow increases dramatically when a thin rod is introduced leading to a decrease in heat transfer. Likewise, as the inner rod diameter
gets larger, the annular geometry asymptotically becomes a thin gap, and the heat transport slowly approaches zero.

In the first case, the pressure drop was held fixed, but if the flow displacement in one half-cycle was held fixed, and the comparison was made between the two geometries, something different may occur and that is indeed the case. As there is now a smaller cross sectional area in the annular region than the open cylinder, the pressure drop will have to increase in the annulus so that the flow rates can be the same in both cases. The results of the calculations are depicted in Figure 3.6.

The calculations depicted in the figure now show that the heat transport is enhanced with an annular geometry compared to an open tube provided that the flow rate per half-cycle is assumed equal in both geometries. The flow rate was calculated by integrating the velocity over the flow area for one half-cycle to give

$$\text{Flowrate} = \frac{\pi}{\pi} \int \int 2\pi V_z r \, dr \, dt$$

The pressure drop (which is contained in the solution for the velocity field) in the annulus was then changed so that the flow rates would be equal. On calculating the pressure drop increase in the annulus as the inner radius increases, it is evident that this is a reason why the transport increases in the annular case. Another contributing factor to the increase in the pressure drop is due to the resistance of the inner rod. The wall friction can lower the flow rate, so the pressure drop must increase to overcome that resistance and make the flow rate the same as the cylinder. For the cases where the inner radius of the annulus is high, the transport in the annulus becomes roughly 15 times higher than the corresponding cylindrical case. The pressure drop is related to the amplitude of the spike, meaning that a higher pressure drop in the tube will
result in a greater amplitude of oscillation, a higher velocity spike, and hence, greater transport. This is why the annulus is the more optimal method in the case of fixed flow rates.

In the first two cases, the area changes as the radius of the inner rod was changed. This raises the question as to whether the comparison between a cylindrical and an annular geometry is a fair one. To make a fair comparison, consider the case where the cross-sectional areas and the pressure drop are held equal for both annular and open tubes. This means that as the inner radius of the annular geometry changes, the cross-sectional area of the companion open tube must also change so that both configurations will have the same amount of area available for heat transport. The results for this comparison are graphically depicted in Figure 3.7 for a definite frequency. The calculations show that the heat transport is again greater in the open cylinder, just like the first case. It appears to have an advantage over the annular case by 1-2 orders of magnitude for smaller cross-sectional areas. Again, if the flow rate is held fixed, for the same cross-sectional area, we see that the heat transfer favors the annular configuration.

The issue with the case of fixed flow rates, when the areas were not held equal to each other, was that the lower cross-sectional area in the annulus drives up the pressure drop needed in order to have the flow the same as the cylindrical case, and consequently leads to larger spikes in the flow profile. However, in the last set of calculations the cross-sectional areas are the same for the open tube and annulus, and therefore this factor is now not an issue. The question then is: why does the annular geometry still give a higher transport? The reason is that in the last case, as before, the pressure drop increases because the inner rod offers resistance on account of wall friction. This factor continues to remain important even where the cross-sectional area is fixed and made equal. One will notice in looking at Figure 3.8, that the ratio at high inner radii is still large (about 8 times), although not as large as before (15 times). This makes physical sense
since the cross-sectional area discrepancy is now eliminated in Figure 3.8, leaving only the inner rod inhibitor to play a role in driving up the pressure drop.

From the calculations it appears that the open tube geometry is best when the pressure drop is held fixed, but the annular geometry is better when the volumetric flow rate is held fixed. The ultimate message is that the optimal case depends on what one holds fixed, pressure drop or flow rate.

Another factor in determining which geometry is best is to see what system gives a higher heat transport output per unit of mechanical power input. The power only takes into account the amount needed to cause the oscillatory fluid movement. The power needed to move an oscillating piston of a given mass for example is not incorporated as this is subject to the materials used. The mechanical power is calculated using the following relationship, which is

\[
Power = \frac{\omega}{2\pi} \int_{0}^{a} \int_{R}^{c} 2\pi V_z (\text{pressure}) r dr dt \quad (3.64)
\]

For the case of a cylinder, the lower limit of integration in \( r \) will be zero. In order to obtain the pressure in terms of \( P \) from (3.64), it must be converted to units of pressure. Therefore

\[
Power = \frac{\omega}{2\pi} \int_{0}^{a} \int_{R}^{c} 2\pi V_z P \mu_l r dr dt \quad (3.65)
\]

(3.5) and (3.6) can be inserted into (3.65) giving

\[
Power = \frac{\omega}{2\pi} \int_{0}^{a} \int_{R}^{c} 2\pi V_z \left( e^{i\omega t} + V_z e^{-i\omega t} \right) \left( \hat{P} e^{i\omega t} + \hat{P} e^{-i\omega t} \right) \mu_l r dr dt \quad (3.66)
\]

The integration with respect to time can be easily done (see (3.24)) leaving a final expression for power which is
To demonstrate a comparison we will first take the case of fixed cross sectional area. The case where the pressure drops are held fixed produce Figure 3.9. There is no need to consider the case of fixed flow rates because the results are identical to Figure 3.9. These sets of calculations show that the ratio of transport/input power is higher for the open tube case. Again, the annular case results approach the cylindrical values when the inner rod gets small. The reasoning for the input power being greater for the annular case is because there are two boundaries in the annulus compared to just one for the open tube. The extra boundary means there is more viscous effects. This means more energy is required to oscillate the fluid since it now must overcome the viscous effects at two walls as opposed to just one. This must be considered when determining the geometry for the most efficient heat transport. If the amount of input power is a concern, then that needs to be factored into the decision.

In summary, four methods have been worked out to compare the heat transport in an open cylinder versus an annulus. Although the results can differ depending on the method of choice, the results of each method make physical sense. It can be concluded from this study that the annular model would be optimal if the system is able to handle a large pressure drop, and if input power is not a concern. If not, then the open cylinder would be the geometry of choice.

3.4 Comparison of Two Compartment Flow versus One Compartment Flow

3.4.1 Heat Transfer

It is possible to make another comparison involving cylindrical and annular geometries. Ranger [12] proposed a model for steady Poiseuille flow through concentric pipes with various areas of cross sections. He determined that it was possible to have two different geometric
models which have the same cross section and the same flow rate, although each model contains a different number of “compartments.” For example, the one compartment model consists of a large annulus, while the two compartment model has an annulus and a cylinder. Note that the outer radii of both models ($R_4$) are set equal to each other. In order to make the cross sectional area of both models equal, a constraint needs to be used. It is

$$R^2 = R_3^2 - R_2^2$$  \hspace{1cm} (3.68)

In the case of the one compartment model, the flow rate is

$$Q = \frac{\pi \Delta P}{8 \mu} [R_2^4 - R_3^4 + R_4^2] [R_2^2 + R_3^2 - R_2^2 - \frac{(R_4^2 - R_3^2 + R_2^2)}{\ln R_4 - \frac{1}{2} \ln (R_3^2 - R_2^2)}]$$  \hspace{1cm} (3.69)

Note that (3.68) was substituted for $R$ where appropriate. For the two compartment model, the flow rate is

$$Q = \frac{\pi \Delta P}{8 \mu} [R_2^4 + R_3^4 - R_4^4 - \frac{(R_4^2 - R_3^2)^2}{\ln(R_4 / R_3)}]$$  \hspace{1cm} (3.70)

The derivation for (3.68) through (3.70) was done by Ranger for Poiseulle flow and can be found in [12]. A ratio can now be established relating the two flow rates as

$$S = \frac{Q}{Q_S}$$  \hspace{1cm} (3.71)

When $S=1$, the flow rates become equal. This means that for a given value of $R_4$, $R_3$ and $R_2$ can be adjusted so that the flow rates are equal. Pressure drop and viscosity will not play a role since they factor out of (3.71). There exists a value of $R_2$ where $S=1$ for a set $R_3$ value (again assuming $R_4$ is held fixed). In order to examine the behavior of the one compartment system versus the two compartment system, certain relationships need to be developed for use in the oscillatory flow model. For this study, assume that the piston driven model is used in both
The one and two compartment models. It is important for this problem that the pistons are external to the channels. See Figure 3.11 for a visual depiction.

The shaded areas represent the flow regions. The external pistons in both cases are assumed to oscillate with the exact same amplitude and frequency, and they both have the same cross sectional area since the outer radius \( R_4 \) of the one and two compartment models are assumed to be the same. This means that the volumetric flow rate going into the one compartment system will be equal to that going into the two compartment system. The only issue that now arises is the “break-up” of the flow into each section of the two compartment model. Since the models were derived earlier for the cylinder and the annulus, the amplitude needs to be known for each case so that the transport can be found in both sections separately. The transport in each section is simply added together to obtain the total transport for the two compartment model.

The oscillation amplitude and frequency of the external piston is assumed to be set. The volume displacement equation is used to obtain the oscillation amplitude in the annular channel of the one compartment model. Thus

\[
A_{ex} R_4^2 = A_{1a} (R_4^2 - R^2)
\]  

(3.72)

\( A_{ex} \) is the amplitude of the external piston and \( A_{1a} \) is the amplitude inside the annulus (the \( \pi \) on each side of the equation cancel). \( A_{1a} \) can be solved for since everything else in (3.72) is known. The transport in the annulus for the one compartment model can now be obtained by using the equations for piston driven flow in an annulus described earlier. In order to obtain the amplitude in the cylinder and annulus for the two compartment system, two relationships are needed to solve for the two unknown amplitudes. The first says that the total flow rate is equal to the sum of the flow rates in the cylinder and the annulus. Therefore
\[
A_{2c} R_4^2 = A_{2a} (R_4^2 - R_3^2) + A_{2c} R_2^2
\]  

(3.73)

\(A_{2c}\) and \(A_{2a}\) are the oscillation amplitudes inside the cylinder and annulus of the two compartment model respectively. Since the volumetric flow rates are being used in this equation, note that the oscillation frequency and the constant \(\pi\) factor out of each term in (3.73).

The second relationship involves the resulting pressure drops generated by the moving piston in each compartment. Since the piston moves externally, the resulting pressure drop in each compartment must be the same. This makes physical sense since there will be only one pressure drop from the beginning of the pipe to the end of the pipe. This means that the right hand sides of (3.36) and (3.55) can be set equal to one another, yielding

\[
\frac{A_{2c} (R_4^2 - R_3^2)}{8Z} = \frac{A_{2c}}{\left[4 - \frac{8 J_1(\alpha R_2)}{R_2 \alpha J_0(\alpha R_2)}\right]}
\]  

(3.74)

The expression for \(Z\) can be found in (3.56) where the outer radius is equal to \(R_4\) and the inner radius is equal to \(R_3\) instead of \(c R\) and \(R\), respectively. There are now two equations for these two unknown amplitudes (\(A_{2a}\) and \(A_{2c}\)), meaning they can now be solved. Comparisons were made at various frequencies for several different values of a parameter \(\delta\). This is the width of the annular solid (no flow) region in the two compartment model (equal to \(R_3\) minus \(R_2\)).

Since the cross sectional areas are the same for each model for the flow regions, the regions where there is no flow is also identical between the one and two compartment systems, because \(R_4\) is the same for both cases. This means a lower \(\delta\) corresponds to more available room for flow. The problem was solved using a numerical solver, with water as the fluid medium, a pipe length of 50 cm, a temperature difference of 10 C, an outer radius (\(R_4\)) of 1 cm, and an external piston length of 8 cm. The results are given as a ratio of the heat transfer in the one compartment.
model to the heat transfer in the two compartment model for various sizes of $\delta$ (0.1, 0.3, and 0.5 cm).

In all three instances, the heat transport in the two compartment model is greater than the one compartment model. This makes sense by understanding what is going on with the physics of the problem. In the two compartment model, there are a total of three boundaries in the cross section (the outer edge of the annulus, the inner edge of the annulus, and the outer edge of the cylinder). There are only two boundaries in the one compartment model (the outer and inner edge of the annulus). Keep in mind that the total flow rates must be the same through each model. Since there is no flow at the boundaries, and very little flow right next to the boundaries (the parabolic spike in the velocity profile approaches zero near the walls), there will be an extra region of very little flow in the two compartment model since it has one more boundary. This will cause a local decrease in the flow rate in these regions. To account for the extra region of slow flow, the amplitude of the spike must grow proportionately, so as to keep the overall flow rates the same for each model. It has been shown earlier in this chapter that increasing the amplitude in the flow profile drives up the heat transport by the square of the amplitude increase. This is why the two compartment model is more favorable, since the extra boundary will produce larger spikes in the flow profile.

The trend for the curves in Figures 3.12, 3.13, and 3.14 is for the one compartment model to approach the transport of the two compartment model, then tail off as frequency increases. This is due to the sharp spike forming in the wide annular region of the one compartment model as frequency grows, then the profile begins to flatten and the transport begins to plateau off as the frequency becomes large. The cylindrical region will maintain a sharper velocity profile for
a longer period of time since there is no inhibiting rod in the middle. This will cause the ratio to
(decrease at high frequency.

As the $\delta$ value increases, it can be seen that the maximum ratio approaches 1, which is
when the transport in both models are equal, but it never reaches 1. Note that the maximum
points on the curves seem to approach 1 asymptotically, so when $\delta$ becomes 1 cm, the models
will have the same transport. This makes sense because if the solid (no flow) region
encompasses the entire area enclosed by the outer wall, both models will have the same amount
of heat transport going through it, which is zero.

A comparison has been made between two geometries to determine which transports heat
better through it, which in this case is the two compartment model. These models can also be
studied to figure out which provides a better separation between two species.

3.4.2 Mass Transfer and Separation of Species

The same models can be compared to determine which transports mass more efficiently.
Since the mathematics of the models for heat and mass transport are virtually the same, the
obvious conclusion can be reached that the two compartment model is better since it transports
heat at a higher rate, which is indeed true. Let us take this one step farther to see how effectively
each model can separate out a mixture of two species in a carrier fluid.

This problem was first presented in the Chapter 1. For this study, helium and carbon
dioxide will be separated in a nitrogen carrier. Starting from rest, the helium molecules will
separate out faster than the carbon dioxide since helium has a larger diffusion coefficient. When
the oscillation frequency becomes high enough, the carbon dioxide will emerge first as the small
helium molecules get trapped near the boundaries in the slow flow regions. Eventually, there
will be an optimal frequency for which the separation ratio is at a maximum. This is consistent
with the results of Kurzweg and Jaeger [8]. The same parameters were used for this calculation as in the heat transfer calculation discussed above, with the exception of the fluid medium. The results are again presented for different values of $\delta$. The separation ratio in the figures represents the mass flow rate of carbon dioxide divided by the mass flow rate of helium. Note in Figure 3.16 that the dotted line at the end of the curve represents an estimate due to inaccuracy of the machine performing the calculation. Since this region occurs well after the maximum is achieved, it is not significant.

The maximum separation ratio that is achieved in the one and two compartment models is about the same in all three cases. It can also be seen how the size of the peaks grow when $\delta$ increases, from about 2.5 for small $\delta$, to about 3.8 for larger $\delta$. This is because the amount of solid (no flow) space grows with $\delta$. Since the external piston amplitude remains the same, the amplitudes of the spikes in the tubes must grow to satisfy (3.72) and (3.73).

Even though the maximum ratios are similar in each case, one big difference is at what frequency the maximums occur. In each case, the highest ratio occurs at a much lower frequency in the two compartment model compared to the one compartment model. Roughly 3 to 4 times the optimal frequency of the two compartment system is needed to achieve the optimal frequency for the one compartment system. The reasoning for this is very similar to the heat transfer analog. Since there are more boundaries in the two compartment model, there are more slow flow regions. This means that the helium molecules can move quickly into the slow part of the flow near the walls, since there is more available space. The separation will occur faster since the helium molecules become quickly trapped, which means the larger carbon dioxide molecules will have more room in the faster moving core region, and therefore transport quickly down the tube.
Achieving the separation at a lower frequency is beneficial since it limits the amount of power needed to drive the system. The input power needed to oscillate the fluid increases with increasing frequency. Therefore the same separation can be produced with either model, but much less energy is being used in the two compartment system. The conclusion can be reached that the two compartment model is the more optimal geometry, whether heat or mass is being transported, or if a separation is desired. One last consideration will be made in the next chapter regarding the geometry of the system, that being what happens when the inner rod of an annulus is moved to an off centered position.

Figure 3.1: Oscillations driven by pressure drop

Figure 3.2: Oscillations driven by a piston
Figure 3.3: Oscillations driven by moving boundary

Figure 3.4: Oscillations in an annular geometry

Figure 3.5: Heat transport in annular versus cylindrical geometry with the outer radius and pressure drop held fixed
Figure 3.6: Heat transport in annular versus cylindrical geometry with the outer radius and flow rate held fixed

Figure 3.7: Heat transport in annular versus cylindrical geometry with the cross sectional area and pressure drop held fixed
Figure 3.8: Heat transport in annular versus cylindrical geometry with the cross sectional area and flow rate held fixed

Figure 3.9: Output/Input Ratio with Fixed Pressure Drop and Cross Sectional Area
Figure 3.10: Diagrams for one and two compartment models respectively (flow regions shaded)

Figure 3.11: External piston oscillating one compartment model (top) and two compartment model (bottom).
Figure 3.12: Heat transport in the one vs. two compartment model at various frequencies at small $\delta$

Figure 3.13: Heat transport in the one vs. two compartment model at various frequencies at medium $\delta$
Figure 3.14: Heat transport in the one vs. two compartment model at various frequencies at large $\delta$

Figure 3.15: Separation ratio for various frequencies for the one compartment model (dashed line) and two compartment model (solid line) at small $\delta$
Figure 3.16: Separation ratio for various frequencies for the one compartment model (dashed line) and two compartment model (solid line) at medium $\delta$.

Figure 3.17: Separation ratio for various frequencies for the one compartment model (dashed line) and two compartment model (solid line) at large $\delta$. 
In the previous chapter we showed that oscillatory flow in an annular geometry leads to an enhancement of transport over an open geometry under certain conditions. Another case to examine is when the inner cylinder in the annular geometry is moved to a slightly off-centered position (see Figure 4.1). A model of this problem will first be presented, followed by a calculation to see what option is better, and under what circumstances.

4.1 Modeling an Off Centered Annulus

Let the deviation in distance of the inner rod from the center be small and be denoted as $\varepsilon$. To get the relation between the radial distance in the distorted geometry with the radius in the reference geometry, it is seen that

$$[X - \varepsilon]^2 + Y^2 = R_0^2$$  \hspace{1cm} (4.1)

Converting this into polar coordinates we get

$$R^2 - \varepsilon^2 R \cos \theta = R_0^2 - \varepsilon^2$$  \hspace{1cm} (4.2)

$R$ can be expressed in terms of $\varepsilon$ to get the mapping from the reference to the current configurations. To get a better understanding of the mapping equations, see Johns and Narayanan [6]. Thus

$$R(\theta, \varepsilon) = R_0 + \varepsilon R_1(\theta_0) + \frac{1}{2} \varepsilon^2 R_2(\theta_0)$$  \hspace{1cm} (4.3)

Here $R_1$ and $R_2$ are determined from their definitions yielding

$$R_1 = \frac{dR}{d\varepsilon}(\varepsilon = 0) = \cos \theta_0$$  \hspace{1cm} (4.4)
\[ R_2 = \frac{d^2 R}{d\varepsilon^2} (\varepsilon = 0) = -\frac{\sin^2 \theta_0}{R_0} \]  \hfill (4.5)

These results will be useful in obtaining the boundary conditions at the displaced inner surface. For the distorted geometry, let the driving force come from the inner and outer walls oscillating in the axial direction, both moving in phase, and whose speed is expressed as \(1/2*\omega_0 \cos(\omega t)\). Using the mapping of the inner boundary, the velocity and temperature profiles can be obtained at the various orders. The expression for the lowest order (i.e., \(\varepsilon^{(0)}\)) is obtained in a manner as done earlier. Since the zeroth order problem is for the case of the centered inner rod, this simply becomes the boundary driven problem in an annulus shown in Chapter 3, with the exception that the boundary condition at the inner wall is now equal to the condition at the outer wall. The velocity field is now (the solved constants are listed in the Appendix)

\[ V_{z0} = A_{0,1} + B_{0,1} \cos(\theta) \]  \hfill (4.6)

Note that the same method is used here to express velocity as a sum of time complex exponentials. Solving the first order problem will be different from the base case, since velocity will become a function of \(\theta\), in addition to the radial position. The first order equation is

\[ \frac{d^2 \hat{V}_{z1}}{dr_o^2} + \frac{1}{r_0} \frac{d \hat{V}_{z1}}{dr_o} + \alpha^2 \frac{\hat{V}_{z1}}{r_0^2} + \frac{1}{r_0^2} \frac{d^2 \hat{V}_{z1}}{d\theta_0^2} = 0 \]  \hfill (4.7)

At first order, the boundary condition at the outer wall yields the first order velocity field to be zero. This is true because there is no displacement at the outer surface, so even though the outer wall is moving at fixed amplitude, there are no first order \(\varepsilon\) terms, implying that the velocity is zero at the outer boundary to first order or any order higher than zero. For the inner boundary, the surface is displaced, so a mapping must be used yielding
\[ V_z = V_{z0}(R_0, \theta_0) + \varepsilon [V_{z1}(R_0, \theta_0) + R_1(\theta_0) \frac{dV_{z0}}{dr_0}(R_0)] + \frac{1}{2} \varepsilon^2 [V_{z2}(R_0, \theta_0) + 2R_1(\theta_0) \frac{dV_{z1}}{dr_0}(R_0) + R_1^2(\theta_0) \frac{d^2V_{z0}}{dr_0^2}(R_0) + R_2(\theta_0) \frac{dV_{z0}}{dr_0}(R_0)] + \ldots \] (4.8)

In order to obtain the boundary conditions at the inner wall for each order greater than zero, set the terms of the desired order in (4.8) equal to zero and obtain a condition for the velocity at the wall for that particular order. Note that these boundary conditions stay the same when the velocity is expressed as a sum of exponentials since there are no time dependent terms in the boundary conditions.

At first order, the inner wall boundary condition is

\[ \hat{V}_{z1}(R_0, \theta_0) = -R_1 \left. \frac{d\hat{V}_{z1}}{dr_0} \right|_{R_0} = -\cos \theta_0 \left. \frac{d\hat{V}_{z0}}{dr_0} \right|_{R_0} \] (4.9)

The \( \theta \) dependence is seen to be proportional to \( \cos \theta \) in order to satisfy differential equation (4.7) and the inner wall boundary condition (4.9). The solution would then be

\[ \hat{V}_{z1} = (A_{1}\psi_1(\alpha r_0) + B_{1}\psi_1(\alpha r_0)) \cos \theta \] (4.10)

The differential equation for \( O(\varepsilon^2) \) is the same as (4.7) with the exception of \( V_{z2} \) replacing \( V_{z1} \) in the notation. The second order boundary condition is

\[ \hat{V}_{z2}(R_0, \theta_0) = -2\cos \theta_0 \left. \frac{d\hat{V}_{z1}}{dr_0} \right|_{R_0} - \cos^2 \theta_0 \left. \frac{d^2\hat{V}_{z0}}{dr_0^2} \right|_{R_0} + \sin^2 \theta_0 \left. \frac{d\hat{V}_{z0}}{dr_0} \right|_{R_0} \] (4.11)

The complete result for the second order velocity field is

\[ \hat{V}_{z2} = [A_{2}\psi_1(\alpha r) + B_{2}\psi_1(\alpha r)] + [C\psi_1(\alpha r) + DJ_0(\alpha r)] \cos 2\theta \] (4.12)

This form of the solution makes sense since all three terms in (4.11) can be broken up into \( \theta \) dependent and \( \theta \) independent terms through the use of the double angle formulae. For this
problem, only the $\theta$ independent terms are taken into account. The reasoning for this will be explained later. This completes the calculation of the velocity field in the problem. These expressions are used to solve the temperature field equations. The method for solving the temperature field equations is the same as before. Each order in $\epsilon$ of the temperature profile is solved using the corresponding velocity profile (i.e., the first order temperature field uses the first order velocity solution). The temperature is broken up into a sum of two exponential terms just like the velocity field case. The boundary conditions for the base case is that there is no flux through the walls ($dT/dr=0$). For the base case, the temperature field is identical to (3.56).

Therefore

$$
\hat{T}_0 = A_{0r} Y_0(\beta r) + B_{0r} J_0(\beta r) + \xi_{y0} Y_0(\alpha r) + \xi_{y0} J_0(\alpha r)
$$

The first order problem for the temperature field has the same form as (3.15) plus a $\theta$ dependent term. It is

$$
\frac{d^2 \hat{T}_1}{dr_0^2} + \frac{1}{r_0} \frac{d \hat{T}_1}{dr_0} + \beta \hat{T}_1 + \frac{1}{r_0^2} \frac{d^2 \hat{T}_1}{d\theta_0^2} = \frac{\Delta T}{L\kappa} V_{z1}
$$

In a manner used for treating the boundary conditions for the velocity field, we have the temperature field mapping for the inner rod, which is

$$
\frac{dT}{dr_0} = \frac{dT_0}{dr_0}(R_0, \theta_0) + \varepsilon \left[ \frac{dT_1}{dr_0}(R_0, \theta_0) + R_1(\theta_0) \frac{d^2 T_0}{dr_0^2}(R_0) \right] + \frac{1}{2} \varepsilon^2 \left[ \frac{d^2 T_2}{dr_0^2}(R_0, \theta_0) + 2R_1(\theta_0) \frac{d^2 T_1}{dr_0^2}(R_0, \theta_0) \right] + R_1^2(\theta_0) \frac{d^3 T_0}{dr_0^3}(R_0) + R_2(\theta_0) \frac{d^2 T_0}{dr_0^2}(R_0) \right] + ...
$$
The flux is equal to zero at the outer wall and is zero for all subsequent orders. The solution to this first order problem has a homogenous and particular solution multiplied by a \( \cos \theta \) term, just like the velocity field. It is

\[
\hat{T}_1 = [A_{1T}Y_1(\beta r) + \xi_{11}Y_1(\alpha r) + \xi_{1J}J_1(\alpha r)] \cos \theta
\]  

As in the case of the velocity field, the boundary condition for the temperature field at the inner wall is found by taking terms of the appropriate \( \varepsilon \) order in (4.15), and solving for the respective order.

For the second order problem, the differential equation for temperature is the same form as (4.14) with \( T_2 \) and \( V_{z2} \) replacing \( T_1 \) and \( V_{z1} \), respectively. As in the second order velocity problem, the solution is broken up into two parts to satisfy the \( \theta \) dependent and \( \theta \) independent boundary condition, and again, only the \( \theta \) independent part of the solution will be useful. We get

\[
\hat{T}_2 = A_{2T}Y_0(\beta r) + B_{2J}J_0(\beta r) + \xi_{2y}Y_0(\alpha r) + \xi_{2J}J_0(\alpha r)
\]  

The cosine and sine functions can be broken up as in the second order velocity problem and only the \( \theta \) independent terms are needed.

Now that the velocity and temperature equations have been determined up to second order in \( \varepsilon \), the heat transport can now be obtained. The expression for the transport is the same as before, which is

\[
Q = 2 \rho C_v \Re \int_0^{2\pi R_e} \int \hat{V}_z T rdrd\theta
\]  

where

\[
\hat{V}_z = \text{conjugate}(V_z)
\]
Note that the time dependence has already been factored out leaving only the cross terms (cf. Chapter 3). In the case of a perturbed surface, the transport rate $Q$ will have to be expressed as a perturbation series as well, so

$$Q = Q_0 + \varepsilon Q_1 + \frac{1}{2} \varepsilon^2 Q_2 + \ldots$$  \hspace{1cm} (4.20)$$

where

$$Q_1 = \frac{dQ}{d\varepsilon}$$  \hspace{1cm} (4.21)$$

and

$$Q_2 = \frac{d^2Q}{d\varepsilon^2}$$  \hspace{1cm} (4.22)$$

Since $Q$ is expressed as an integral, the derivative of $Q$ can be determined using Leibnitz’s rule for differentiating integrals. Note that the velocity and temperature terms can be expressed as derivatives, which will simplify the algebra somewhat. Therefore

$$V_{z_1} = V_{z_0} + \varepsilon V_{z_1} + \frac{1}{2} \varepsilon^2 V_{z_2} + \ldots$$  \hspace{1cm} (4.23)$$

where

$$V_{z_1} = \frac{dV_z}{d\varepsilon}$$  \hspace{1cm} (4.24)$$

and

$$V_{z_2} = \frac{d^2V_z}{d\varepsilon^2}$$  \hspace{1cm} (4.25)$$

The same expansion can be done for the temperature. Using Leibnitz’s rule to find $Q_1$, it is found that

$$Q_1 = \rho c_v \int \int [\bar{V}_{z_0} \hat{T}_1 + \hat{T}_0 \bar{V}_{z_1}] r dr d\theta - \int \cos \theta (\bar{V}_{z_0} \hat{T}_0) R d\theta$$  \hspace{1cm} (4.26)$$
Looking at (4.26), it can be concluded that $Q_1$ is equal to zero. Both terms in the double integral contain a $\cos \theta$ term from the first order expression and integration over $2\pi$ makes the entire integral equal to zero. The same thing occurs in the single integral with the $\cos \theta$ term, making that integral equal to zero as well.

The expression for $Q_2$ can be found, and after some cancellations, the result is

$$Q_2 = \rho C_v \int \left[ \tilde{V}_{z0} \dot{T}_2 + 2\tilde{V}_{z1} \dot{T}_1 + \tilde{V}_{z2} \dot{T}_0 \right] \rho r dr \, \theta \, \theta +$$

$$\int R_0 R_1 \frac{1}{2} T_0 (R_0) \frac{d V_{z0}}{d \theta} (R_0) d \theta$$

From this expression it can be understood why the $\theta$ dependent terms were neglected in the second order velocity and temperature expressions. If the expressions for the perturbed velocity and temperature field were inserted into (4.27) including all the $\theta$ dependent parts, several terms would result that either contained a $\cos \theta$, or a $\cos 2\theta$ term. Since the integration is carried out over $2\pi$ radians in the $\theta$ direction, all of these terms will integrate out to zero. This is why the $\theta$ dependent terms were discarded in the second order perturbation, since they would just contribute to the terms that would eventually integrate out to zero. A final expression can now be obtained for $Q_2$ after further evaluating the term in the single integral. It is

$$Q_2 = 4\pi \rho C_v \text{Re} \left\{ \int_{R_0}^{R_1} \left[ \tilde{V}_{z0} \dot{T}_2 + \tilde{V}_{z1} \dot{T}_1 + \tilde{V}_{z2} \dot{T}_0 \right] \rho r dr + \frac{\dot{T}_0 (R_0)}{2} \frac{d V_{z0}}{d \theta} (R_0) \right\}$$

Since $Q_1$ is equal to zero, (4.28) must be added to $Q_0$ to get the final equation for the total heat transport in the off-centered annular geometry to give
\[ Q = 4\pi p C_v \text{Re} \left\{ \int_{R_0}^{\infty} [V_{z0} \hat{T}_0] r dr + \right\} \]

\[ \frac{1}{2} e^{2} \left[ \int_{R_0}^{\infty} [V_{z0} \hat{T}_0 + V_{z1} \hat{T}_1 + V_{z2} \hat{T}_2] r dr + \frac{T_0(R_0)}{2} \frac{d}{dr} \hat{V}_{z0}^2 (R_0) \right] \]  \hspace{1cm} (4.29)

4.2 Analysis of Off Centered Model

One of the important results that need to be obtained from these equations is a solution for \( Q \) at various frequencies. This will determine if the off centered annulus is a beneficial geometry, or a detrimental one, since \( Q \) is equal to the change in transport due to the perturbation. The results for a hypothetical fluid with a Prandtl number of 10, a kinematic viscosity of 0.1 cm\(^2\)/s, and amplitude of 10 cm are presented in Figure 4.2. Like the comparison between the cylinder and the annulus presented in Chapter 3, the qualitative nature is what is important here. MAPLE\textsuperscript{®} was used to solve (4.29) at various frequencies. It is seen here that the change in transport is positive for small frequencies, and quickly becomes negative for higher frequencies. Although this has been shown from a mathematical standpoint, some explanation needs to be given why this makes physical sense.

When the annulus is positioned in the center, the only gradients that exist during the oscillations are in the radial direction. If the annulus were moved to an off centered position, there will be gradients forming in the \( \theta \) direction in addition to the radial direction. In order to visualize this better, imagine that the off-centered position of the annulus creates a “thin” region and a “thick” region (see Figure 4.3). The thin region consists of radial lengths less than \( R_0 \), and the thick region contains the area of fluid that has radial lengths greater than \( R_0 \). The total cross sectional area of the thick region turns out to be greater than the thin region. As stated before, an azimuthal gradient is formed. The heat will then flow from the thin region to the thick region.
Since the thick region is the freer flowing region, it will be easier for the heat to transport down the tube, leading to a positive change. This argument was proven in Chapter 3 when the heat transport decreases when the region of flow becomes more constricted. This was the case of an inner rod being stuck inside an open tube to form an annulus. The positive gain in transport from the thick region is higher than the loss from the thin region, causing the overall net change to be positive for low frequencies.

When the frequency increases, something different happens. Since the walls are oscillating, the velocity near the walls will be higher than in the center of the tube. This is similar to the oscillating flat plate problem described in Chapter 2, when the velocity is at its highest at the wall and decreases as one moves away from it. This is especially true when the frequency is high, because there will be a sharper velocity gradient going from the wall to the core. Since the thick region has more available area farther away from the walls, the heat is essentially moving from the fast to the slow part of the flow (from the thin region to the thick region). This means that heat will take a longer period time to move down the tube. This phenomenon will become more and more dominant as the frequency is increased. That is why the net transport becomes negative, since the effect of fast moving walls dominates over the effect of the free flowing region, and continues to be more and more negative as frequency is increased. Now that the models have been presented with various geometric comparisons, experiments can be designed to test the accuracy of these models.
Figure 4.1: Difference between centered annulus (clear inner rod) and off-centered annulus (shaded inner rod)

Figure 4.2: Net change in transport due to off centered inner rod at various frequencies
Figure 4.3: Thin and thick regions that result from an annulus that is off-centered
CHAPTER 5
THE EXPERIMENT AND RESULTS FROM THE EXPERIMENTATION

The theory presented thus far shows how the enhancement of heat transfer depends on frequency of oscillation, the amplitude of the priming motion, and the geometry. An experimental apparatus was designed to test the validity of the theoretical model. This experiment was meant to verify the case of piston driven flow in a cylindrical tube. The apparatus is described below as well as the method that was used to obtain the data. The reader will learn about the difficulties that arose when designing the testing procedure, as well as the logic behind the alterations that were made to improve the performance of the experiment. The possible experimental errors are also discussed, and finally the results are given with the discussion as to why the data agrees or disagrees with certain aspects of the theory.

5.1 Experimental Set-Up

The experimental design that was used in this study is pictured in Figure 5.1. A sketch is shown in Figure 5.2a and 5.2b. The top cylindrical tank was used as the heat source, while the bottom tank was the heat sink, with a connecting tube between them. The top tank was approximately 6 L in volume, and the bottom tank had a volume of 4.5 L. The length of the pipe in the figure was 30 cm, although that length could be varied. The fluid medium filled the bottom tank, the connecting pipe, and most of the upper tank. The orientation of the heat source above the heat sink was optimal since a lighter (warmer) fluid was on top of a heavier (colder) fluid. This minimized the effect of natural convection arising from gravitational effects. The oscillations were imposed by a mechanical piston that was attached to a bull wheel which was rotated by a power drill. The amplitude of the oscillations was changed by adjusting the position of the hinge on the bull wheel, while the frequency was changed by varying the speed of the
power drill. It was important to use a material for the piston that could handle a reasonable amount of torque (from the drill). In the early exploratory stage of experimentation, several pistons broke because of this issue. After a while, the acrylic piston was exchanged for a piston made of PVC (polyvinyl chloride). This piston had a higher resistance to torque, and when properly lubricated with oil, it was able to last the entire experimentation process. The material selected for the tanks and the connecting pipe was plexiglass. This was chosen since its thermal conductivity is low (0.18 W/m.K) compared to the fluids used during experimentation. These fluids were water (conductivity of 0.60 W/m.K) and a 60% glycerol-water mixture (0.38 W/m.K). It was important to have a tank made of material that had a low conductivity since the model assumed that no heat was lost through the walls of the tank. A material with a higher thermal conductivity could transport heat out of the pipe in the radial direction, which would provide a source of error.

An important part of running this experiment effectively was to control the temperature of the heat source and the heat sink. This kept the axial temperature difference constant throughout the experiment and consistent with the model. This was accomplished with the assistance of controlled temperature baths that circulated constant temperature water through both the heat source and the heat sink.

In the heat source, in addition to having the controlled temperature bath circulate the water in the tank through a copper coil, a heater was placed in the tank that was controlled by a PID controller. A mixing rod attached to a motor was also present in the heat source which assisted in keeping the temperature uniform in the bath. Copper was chosen as the coil material due to its high thermal conductivity. When the set point for the heater in the tank was established, the heater turned on until it reached the set point since the set point was greater than
room temperature. However, as there was an overshoot as the system was trying to reach steady state, the temperature bath had a set point which was slightly below the set point of the heater. This assisted in removing the excess heat that the heater emitted as it tried to maintain constant temperature in the upper tank. This was necessary since heat emitted by the heater in overshooting the set point in theory really had nowhere to escape. Since the diameter of the connecting tube was extremely small compared to the tank, excess heat would have taken an inordinate amount of time to move itself down the tube and out of the tank. This “cooling loop” expedited the process.

In the lower tank there was also a cooling loop with a temperature bath; however, this loop took up a very large amount of the volume of the tank. Larger amounts of coils were needed in the sink since in addition to keeping the sink at constant temperature; there was heat that was transported down the tube that needed to be removed once the oscillations began. Now that the apparatus has been described, we discuss the method that was developed to gather data.

5.2 Experimental Method

When both tanks reached steady state, the heater in the heat source was on at a certain rate to maintain the set point temperature. The controller measured this rate and it was recorded. This was important to do since this rate took into account the heat loss through the coils as well as losses to the ambient. Although the assumption of no heat loss through the walls of the tanks and walls of the pipe was made, no amount of insulation could have made this assumption totally accurate. Therefore heat was still lost through these walls, although the heat source and connecting pipe were covered with multiple layers of foam insulation. This rate of heat loss needed to be established before the oscillations began, since it had to be factored out of the total rate. Another way that the heat leakages from the top tank were minimized was by the set point
of the heat source. The set point used for these experiments was always between 24 and 26 degrees Celsius, which were slightly above room temperature. This made the temperature gradient between the hot tank and the ambient rather small, which aided in reducing heat loss out of the heat source. The set point of the bottom tank was slightly below room temperature, between 16 and 18 degrees Celsius. Again, this minimized the temperature gradient between the heat sink and the surroundings. Although the axial temperature difference between the source and the sink was only between 6 and 10 deg C for the experimental testing, the theory still showed that a significant amount of heat transfer would occur.

When the oscillations began for a given amplitude and frequency, heat was transported quickly down the pipe. This drove the temperature of the top tank down, since heat was being lost to the sink. In order to compensate for this, additional heat was added to the source to drive the temperature back up to the set point. The heat that was put back into the top tank to maintain the set point was equal to the heat that was transported down the tube to the heat sink. Again, to get the actual rate of heat that was transported down the tube, one must subtract the rate of heat loss to the ambient and coils, which were determined before the oscillations started. In the bottom tank, the temperature tried to increase as heat entered it from the pipe. This heat was removed via the copper coils so that a constant temperature could still be maintained in the heat sink. In both tanks, the coils removed the heat, and this heated water went into a chiller, where it was cooled down. This chilled water then was sent back to the temperature bath, where it was used to export constant temperature water back to the tanks. This cyclic process occurred throughout the experiment.

In order to check the accuracy of the experimental results against the theory, several different parameters were tested. One theoretical conclusion that was desired to be tested was to
see if the oscillations improved the heat transport by multiple orders of magnitude as predicted by the model. However, in order to do that, the pure conductive case needed to be evaluated as well. Unfortunately, the time required for the linear temperature profile to form in the tube via conduction was very high when water was used as the transporting fluid. Since the model assumed that this linear temperature profile existed, this must form first before the conductive heat transfer is determined. The time constant for the conductive case scaled as the pipe length squared over the thermal diffusivity. Remember that there were no oscillations in the conductive case, so the heat transport occurred strictly in the axial direction, making the characteristic length the length of the tube and not its radius. Even with a relatively small pipe length of 30 cm in this experiment, it would still take on the order of one week for the steady linear profile to form. It would be possible for this time to decrease if the pipe length was really small. However, the model also neglected the entrance and exit effects as discussed before, meaning that the oscillation amplitude used would have to decrease substantially as well. This small oscillation amplitude would have enhanced the transport in a very minimal way, and would not have been very useful.

Another possible way to avoid this problem would be to use a fluid with a much higher thermal diffusivity, since this would have lowered the amount of time needed for the steady state profile to form down the tube. Air would have been a possible substitute since its thermal diffusivity is over 100 times that of water. However, a problem arose when air was put into the tank. Mixing air in the tank proved to be very difficult as it was observed that there were severe temperature fluctuations in the heat source. Since the model assumed a well mixed heat source, another mixing method had to be used instead of the stirring rod. A few small fans were inserted into the tank to help keep the air constantly circulating. However, it turned out that heat was
being generated from the fan motors, which in the end would skew the results since not all the heat being added to the tank would have come from the heater, some would have come from the fan motors. Since only the output of the heater got measured, this would have become a large source of error. In the end, it was determined that air wasn’t a feasible test fluid, and water became the fluid of choice again.

It was then decided to compare different convective cases since the time needed for a convective steady state to form was on the order of a few minutes under the influence of oscillations. Different parameters in the apparatus were varied to see if indeed the experimental results followed the theoretical model. Before that occurred, the sources of error in the experiment needed to be evaluated to determine how much they would affect the results.

5.3 Error Analysis

In the experimental set-up, there were several sources of error that arose. The first issue was related to the physical properties of the fluid. There are several that come into play when calculating the heat transported through the system. These include the thermal conductivity, the kinematic viscosity, and the density of the fluid. These properties are functions of temperature, so their values varied when the temperature changed. The property values obtained for both water and the 60% glycerol-water mixture were those at room temperature (20 C). Since the set point of the upper tank was a few degrees higher than room temperature (24-26 C), and the set point of the lower tank was a few degrees below room temperature (16-18 C), it was assumed that the average temperature in the tank was roughly that of the room temperature. However, if the upper temperature limits of the tank were used, the average temperature in the pipe would be 22 C instead of 20 C. It was also possible for the room temperature to have been greater than 20 C, which might have influenced the average temperature in the pipe. However the pipe was well
insulated and it was unlikely that the room temperature excursion would have had much of an effect. Therefore the physical properties of water and glycerol needed to be examined between 20 and 22 C. A range of values was obtained (see Tables 5.1 and 5.2).

These data show that even with a two degree temperature difference, there is a difference in the Prandtl number of the fluid. The contributing factor to this is the kinematic viscosity, which appears in the Prandtl number. It can be seen that for water there can be as much as a 2.3% deviation from the average Prandtl number and a corresponding deviation of 3.5% for the glycerol-water mixture.

Other errors arose from deviations in the dimensions of the experiment from design values. The pipe had a total design length of 30 cm. It was divided into two pieces, each having a tolerance of 0.0127 cm. The tolerance in the radius of the pipe was also 0.0127 cm. Two different radii were used in the experiment, whose sizes were 3/16 in. and 1/8 in. The error then in the pipe length was 0.085%, while the error in radius was 2.68% for the wide tube, and 4.00% for the narrow tube.

The amplitude measurement was taken by determining the length of the piston hinge from the center of the bull wheel. This was equivalent to the actual length the piston was moving. The maximum possible amplitude that the piston could have generated was 7.14 cm, and minimum amplitude of 2.73 cm. Note that this wasn’t the actual amplitude of the flow in the pipe since the tube in which the piston oscillated had a different radius then that in the actual pipe. The amplitude in the pipe can be easily determined by using the following formula which is that of equal volume displacement. It is

$$A_{piston} (R_{piston})^2 = A_{pipe} (R_{pipe})^2 \quad (5.1)$$
whence the amplitude in the pipe is calculated. The pipe amplitude not only varied when the piston amplitude was changed, but also with the radius of the pipe, as different sizes were used during experimentation. The piston amplitude was measured using calipers. Although the calipers were quite accurate, an eyeball measurement needed to be taken to place the caliper ends with the center of the hinge and the center of the bull wheel. The error in the measurement was assumed to be about 0.5 mm. This means that for the smallest piston amplitude, the maximum error that could exist is 1.83%. A larger piston amplitude would produce a smaller percentage error.

The temperature drop between the heat source and heat sink was dependent on the accuracy of the controllers in the respective tanks. The controller produced very good results as the temperature range was always within 0.05 °C of the set point in both tanks at steady state. Temperature measurements were taken every 5 seconds, meaning that 240 measurements were taken over the 20 minute trial period to obtain an average temperature reading. It was unlikely that the error in the average temperature was anything over 0.01 °C for each tank. The error in an 8 °C temperature gradient (the typical temperature difference) would have been about 1/4%.

Another source of error in the experiment occurred when measuring the frequency. Since the bull wheel was attached to a drill with a motor, there would be surges or decreases in the voltage to the motor. This would cause a change in the frequency measurement for short periods of time. When this occurred, the drill was adjusted quickly to bring the frequency back to the original set point. It was determined that the maximum change that was brought about in the frequency by these voltage changes was no more than 0.2 Hz. This change usually lasted no more than a minute before it was noticed and rectified. The amount of time for a trial period was 20 minutes, meaning that the difference in the average frequency was 0.01 Hz per voltage
change. The number of voltage changes usually never exceeded 3 in a trial period, which made the total deviation in the average frequency 0.03 Hz at most. The frequencies used during the entire experimentation process ranged between 0.9 and 1.7 Hz. The maximum error for small frequencies would have been roughly 3.3% and 1.8% for large frequencies.

So far, several input parameters have been discussed in regard to the error in their measurements. These include kinematic viscosity, thermal diffusivity, thermal conductivity, temperature gradient, pipe length, pipe width, oscillation amplitude, and oscillation frequency. Note that all these parameters appear in (3.40), which was the model equation for piston driven flow in a cylinder. Since the overall heat transport was proportional to all of these parameters in different ways, the maximum and minimum values that arose from errors were substituted into (3.40) for each term to get the maximum and minimum possible value for the overall heat transport. There was another source of error in the experiment that doesn’t appear in (3.40). When the actual amount of heat transported in the experiment was determined, the time that the heater was on was counted by a timer in the computer that controls the heater. This amount of time had an error associated with it. The timer was rounded off to the nearest hundredth of a second. When a start time and stop time were determined, there was a rounding error associated with each time in addition to the start and stop time for when the heat loss to the ambient was calculated. Since there was a maximum of 0.005 s of rounding error with each timing, a total of 0.02 s in error could result when 4 timings are done. Considering the heater was on anywhere from 3 to 100 s depending on the trial, the error associated with this could range from 0.02% to 0.67%. In the case where a ratio of two trials was taken, 8 timings were done, and the maximum total error doubled to 0.04 s, resulting in a percentage error ranging from 0.04% to 1.33%. This additional error was added into the maximum and minimum values for the overall heat transfer.
to obtain the total experimental error. This total error is expressed on a trial to trial basis with 
error bars shown on the plots in the results section.

5.4 Note on Viscous Dissipation

A term that was neglected in the energy balance earlier was that of viscous dissipation 
due to movement of the fluid in the tube. If this term were to appear in the energy balance, 
(3.13) would be rewritten as

$$\frac{\partial T}{\partial t} + V_z \frac{\partial T}{\partial z} = \kappa \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \frac{\mu}{\rho C_v} \left( \frac{\partial V_z}{\partial r} \right)^2 \tag{5.2}$$

We can scale (5.2) to obtain

$$Wo \Pr \left( \frac{dT^*}{dt^*} + \frac{A}{L} V_z^* \right) = \kappa \left( \frac{\partial^2 T^*}{\partial r^2} + \frac{1}{r^*} \frac{\partial T^*}{\partial r^*} \right) + Br \left( \frac{\partial V_z^*}{\partial r^*} \right)^2 \tag{5.3}$$

Where the Brinkman number is defined as

$$Br = \frac{\mu A^2 \omega^2}{k \Delta T} \tag{5.4}$$

Typical experimental values were used to determine the Brinkman number. When water 
was considered as the fluid medium, this dimensionless group turned out to be five orders of magnitude lower than the first order conductive term and four orders of magnitude lower for the glycerol-water mixture. The dimensionless groups of the time dependent and convective terms were even greater than the conductive term. Although from a scaling perspective, we have reasoning to drop the viscous dissipation term, we need to evaluate the entire term to see how much heat is really lost due to the viscosity. Therefore, a calculation was performed by integrating the viscous dissipation term over the cross sectional area and comparing it to the output heat transport. It turned out the heat transport was over 500 times the heat lost to viscous dissipation for the case of water as a fluid, and over 50 times the heat lost in the case of a
glycerol-water mixture. From these results, it can be concluded that the viscous dissipation term is insignificant, and this why it was neglected in the energy balance.

5.5 Experimental Results and Discussion

5.5.1 Varying Amplitude with Water

The first series of experiments were meant to verify the relationship that the convective heat transferred is proportional to the square of the piston amplitude. Initially, it was determined that the time needed to reach the conductive steady state for water was roughly 120 hours for the 30 cm pipe length. Through a theoretical calculation it was figured that the oscillations increase the overall transport by a factor of about 600. A rough estimate was made to determine the amount of time needed for the convective steady state to obtain. It was roughly 12 minutes. Once the oscillations were in effect for that length of time, it was assumed that the velocity profile in the tube was fully developed and heat was being transported at a constant rate.

Initially, the amount of heat input to the top tank over a 20 minute period was measured without oscillations in order to determine the amount of heat lost to the ambient. Once that measurement was completed the oscillations were started. The steady state profile was developed and another 20 minute measuring period was conducted for an input piston amplitude. In the next experiment, the amplitude was changed and there was a wait time until the steady state formed at the new amplitude. The heat transported was again measured over a span of 20 minutes. It should be noted here that the frequency was set between 1-1.5 Hz for every trial. Although there was a variation in frequency over all the trials, each individual trial had the same frequency for each of the two amplitudes tested.

Figure 5.3 shows the experimental results compared to a theoretical curve at various amplitude ratios. The amplitudes chosen for these trials could not exceed 40% of the pipe length.
since a larger amplitude would have induced entrance and exit effects- effects that were neglected in the model. This limited the amount of possible amplitude ratios available, which is why the ratios shown in the figure are between 1.1 and 1.6.

Observe from the figure that the data points are quite close to the theoretical curve for amplitude ratios between 1.2 and 1.5. For the extremely low and extremely high ratios, the experimental results were slightly higher than the model prediction. The reasoning for the difference at the high ratios could possibly be due to the entrance and exit effects. Although a 40% limit was placed on the amplitude size, this is just a rough guess as to when the entrance and exit effects become relevant.

When an oscillation pulse is generated, the temperature gradients ought to exist only in the pipe and the model assumes that these gradients are not felt in either tank. However, when a spike becomes large, temperature surges can occur up through the entire length of the pipe and into the tank. Since the heater and controller were both in the top tank, a sudden temperature drop in the tank would have obviously caused more heat to be added to the tank to keep the temperature at the set point.

Now the model only takes into account heat transported between the ends of the pipe. Because a larger portion of the tank was now cooler, not just at the spot where the pipe was connected, a greater amount of heat would have to be added to cool the tank down. These are considered the entrance and exit effects because the transport occurs outside the ends of the pipe in addition to inside the pipe. This excess heat increases the total amount of time the heater is on during an experimental run. This overshoot causes the heat transport ratio to become larger than expected (since the transport ratio is given with the higher amplitude result in the numerator) and this is consistent with what the data shows for high amplitude ratios. If higher amplitude ratios
were used than the ones shown in Figure 5.3 one would likely see the error become even greater, as the spikes generated would become ever larger, affecting the tanks more and more.

The discrepancy at lower amplitude ratios would not be for the same reason as the high end ratios, since the spikes were not as large. It is likely that the error in the data points between the amplitude ratio of 1.1 and 1.2 are due to some additional heat leakages in the experiment. These heat leakages will be discussed in a later section in this chapter. Looking at the points in the middle of the data set (ratios between 1.2 and 1.3), it can be seen that these points are in very good agreement with the theory.

5.5.2 Varying Amplitude with Glycerol-Water Mixture

The same experimental method discussed in the previous section was applied to a fluid with a higher Prandtl number, a 60% glycerol-40% water mixture which has a Prandtl number around 85. The idea behind running this part of the experiment is to see if the model is a good predictor for a more viscous fluid and also to see if a higher Prandtl number fluid would behave worse than a low Prandtl number fluid for varying amplitudes. This could possibly be so as a higher Prandtl number fluid usually decreases the convective transport in the pipe. The decrease occurs because a higher Prandtl number fluid usually is more viscous (the glycerol water-mixture is roughly 10 times the kinematic viscosity of water). The term "usually" is used here since thermal diffusivity can affect the Prandtl number as well. Two fluids can have different Prandtl numbers even when they have the same kinematic viscosity due to differing thermal diffusivity values. It has been shown from the model in Chapter 2 that the heat transport decreases with decreasing Womersley number. Since the kinematic viscosity appears in the denominator of the definition of the Womersley number (see Chapter 2), a more viscous fluid will decrease that value, assuming the oscillation frequency and tube radius are kept constant. Although ratios in
heat transported are being considered here, this could affect the accuracy of the experiment. For example, suppose at a certain amplitude, the heat per unit time transported down a water filled pipe is 2 W. If the amplitude were doubled, the transport is predicted to become 8 W according to the model. Now suppose at the same original amplitude the heat transported in a glycerol-water pipe is 1 W. If twice the amplitude was used, the theoretical transport would become 4 times that, or 4 W. For the water example, the rate of heat transport increases by 6 W when the amplitude is doubled. In the glycerol water case, the increase is only 3 W. Suppose in both cases the experiment produced an increase in transport that was 1 W higher than expected. The percent error would be 16.67% for the water case (6 W +/- 1 W) versus 33.33% for the glycerol-water mixture (3 W +/- 1 W). From this example, it could be concluded that the glycerol-water mixture is more sensitive from an percent accuracy standpoint. Using this fluid could prove to be a better test to see how accurate the experiment really is. Figure 5.4 shows the results for the glycerol-water mixture.

The results here show better agreement with the theoretical prediction compared to the trials with pure water. All the data points are within the experimental error bars. It should be noted though that the very high amplitude ratios were not taken into account in this experiment due to the larger errors obtained in the last experiment at those respective values. If large amplitude ratios were tested with the glycerol-water mixture, the errors associated with them would be even higher as a more viscous fluid generates sharper velocity spikes, causing even more temperature gradients to form in the tanks when the amplitudes are large. That is why only ratios below 1.4 were tested. This data set is a positive sign, as good agreement with the theory was obtained for a fluid medium that was less conducive to transporting heat, meaning that it
was more difficult to measure, due to smaller changes in transport when the amplitude was varied.

In these first two data sets, only the accuracy of ratios was examined. Although the results were quite good, the actual heat transported in the experiment was not determined. If the amplitude is increased by 10% from a first case to a second case, the experiment has shown that it can produce about a 20% increase in heat transport. However, does that 20% mean an increase of 1 W to 1.2 W, or 100 to 120 W? There is clearly a big difference between 0.2 and 20 W. It is important to determine the actual amount of heat transported in the tube to see how completely accurate the experiment turns out to be. This will be examined in the next section as the frequency is varied.

5.5.3 Varying Frequency

The frequency of the oscillations was altered to see how it affects the overall heat transport. The same glycerol-water mixture was used as the fluid medium during this set of experiments. The set-up of the trials was similar to the previous experimental runs in that a 20 minute period run was conducted with no oscillations to determine the heat lost to the ambient. Then the heat transfer over a 20 minute period with oscillations was measured for various frequencies. Again, enough time was allotted for the oscillations to reach steady state before each trial commenced. Frequency values ranged from 0.9 to 1.7 Hz. This was an appropriate frequency range for measurement since the oscillations were produced by a drill turning a bull wheel. Frequencies lower than these were difficult to keep constant since the minimum speed of the drill produced frequencies around 0.9 Hz. Higher frequency values were not used in order to keep the amount of torque on the bull wheel from the drill low. A safety concern was that the piston could snap if it were oscillating too fast (this occurred during preliminary trials). Also,
there was an upper limit for frequency since turbulent flow needed to be avoided in the pipe, since the model assumed laminar flow. The results for the experimental trials for varying frequency are shown in figure 5.5.

It is clear from this data that there is a noticeable deviation from the model prediction. For the larger frequencies, there is roughly an entire order of magnitude difference between the experimental results and the theoretical model. This discrepancy didn’t occur in the first two experiments as only the ratio of heat transport was compared to the theoretical predictions. It appears as though the deviation got larger as the frequency was increased.

One reason for a higher experimental heat transport was due to heat losses in the system while the oscillations were occurring. Heat losses to the ambient were measured before the experimental trials are conducted. However this really just takes into account the heat losses from the heat source to the ambient, since the heater and controller were all in the top of the tank. When there were no oscillations, the controller only recognized heat that was being lost through the tank walls, since the surface area of the tank was drastically higher than the surface area of the region where the tank and pipe were connected. Also, the location of the heater and controller was much closer to the walls of the tank compared to the pipe (see Figure 5.2a). All of this means that when the rate of heat loss to the ambient was determined, only the heat loss out of the tank was included. There was still the potential for heat leakages through the pipe walls while the oscillations were occurring. As stated before, although the pipe was insulated, no amount of insulation could totally prevent heat from being lost through the pipe walls. In addition to that, the fluid medium and pipe material which were discussed above assisted in limiting the amount of heat leakages through the walls, but again, it was not a complete fix to the problem. Also, as the radius of the tube could be varied, different “inserts” were machined.
These inserts fit into a larger size tube which is depicted in Figure 5.6. When the inserts were fit into the larger tube, there were air gaps that formed between the wall of the insert and the wall of the outer tube. The heat could possibly move through the inner wall and into the air gaps, which is not part of the fluid channel. This could contribute to part of the heat loss; however, since the movement of the fluid is periodic, some of the heat may return back to the fluid in the reverse stroke.

The entrance and exit effects could be another reason for the experimental heat transfer being high compared to the theory. The amplitude used for all of these frequency measurements was rather high. This was needed since the theoretical heat transfer would have been quite low for a small amplitude case, and it would have been difficult to obtain a measurement from the experiment to compare with the theory for such a small amount. Since the amplitude in the experimentation was high, the larger spike in the channel might have caused the entrance and exit effects to contribute in overshooting the theoretical heat transport as well.

It should be noted that Thomas [14] faced the same issue in his mass transfer experiment, where the experimental mass transfer was much higher than the model prediction. However, in his case, he didn’t have leakages of mass through the walls, unlike this heat transfer experiment. He suggested that secondary flows may have formed in his channel, causing a deviation in the parabolic profile. This could also be a problem in the heat transfer experiment, especially when the oscillation frequency grows. All of these issues could help explain why the experimental rate of heat transfer was much higher than expected.

Although there is a disagreement in the experimental results with the theory in terms of total heat transported down the pipe, it is still important to see how much increasing the frequency of the oscillations improve the heat transport, and if it is consistent with the theory.
order to do that, a normalized heat transport was developed. This was done by dividing the theoretical transport by some value at every frequency in the range to get a normalized theoretical curve. The value that was chosen for the normalization was the experimental heat transfer value at a mid-range frequency. It was selected because it is allows for a comparison with the model in the cases where the frequency is increased or decreased. Figure 5.7 shows these results.

It can be seen that the change in the heat transport is much more drastic than the model predicts. As frequency increases, there is a much larger increase in the transport. Again, this is likely due to the reasons discussed earlier. It should also be noted that only 4 frequencies were used. In an ideal situation, many more frequency values would be chosen, however the nature of the oscillating mechanism prevented this from happening. Considering a drill was being used to drive the oscillations, it was difficult to keep a constant speed going for every possible frequency within the available range. The drill setting was kept constant by placing a clamp on the trigger mechanism and loosening or tightening it to vary the speed. The frequencies selected here for the experiment were those frequencies that the drill could produce at a fairly constant rate. It was very difficult to tighten the clamp by even the slightest amount so as to only increase the frequency by say 0.05 Hz, as usually any minor change to the clamp brought about no change in the drill speed. After several minor tweaks, there would eventually be a larger surge in the speed. This increase roughly correlated to a frequency increase of 0.2 Hz. That is why there is roughly a 0.2 Hz difference between the chosen frequencies. Ideally, a mechanism should be designed to generate perfectly constant frequencies at any setting over a large range.

Several other trials were conducted to see if these results could be improved. However, the regular and normalized plots were very similar to Figures 5.5 and 5.7 for subsequent runs.
The fluid medium was also changed from the glycerol-water mixture back to pure water. Unfortunately, there was no difference in the outcome.

This experimental test did not produce the desired results except for the fact that it was demonstrated experimentally that heat transport does increase when the oscillation frequency is increased. It was shown from this trial that measuring the actual heat transported in the system (not a ratio) can be quite difficult due to heat losses that can be very tough to eliminate.

5.5.4 Varying Tube Radius

The final experimental test involved changing the pipe radius. In these trials, two different radial sizes were used, 3/16 in. and 1/8 in. Again, the heat lost to the ambient was calculated before the 20 minute trials were conducted, one at each radius length. A ratio between the two trials was developed and compared to the model. In this set of trials, amplitude was adjusted so that the theoretical heat transport was roughly the same between the two cases.

Looking at (5.1), the amplitude of the pulsation in the pipe must increase when the tube radius is decreased for the equation to still be satisfied. Adjusting the amplitude is necessary in order for the total flow rates to be roughly the same. The point of this set of experimental trials was not to see how radius affects the transport, but to see if the system can transport the same amount of heat when the radius is changed by altering the other parameters, namely the amplitude (and in some cases the frequency, although it was used minimally since past experimental results showed the accuracy was not very good compared to the amplitude variation). This test could be considered to be a culmination of the previous trials to see if multiple parameters can be adjusted to produce a desired result. Figure 5.8 shows the experimental ratio versus the theoretical ratio. The solid line has a slope of 1, which represents the case when the experiment agrees with the theory. Since it was desired to have the total amount of heat transfer be equal between the two
cases according to the theoretical model, most of the ratios that were chosen were very close to 1.

The results were reasonably satisfactory as some points were in good agreement with the theory and some were slightly outside the error bars. Keeping in mind all of the qualitative errors that were discussed involving the amplitude and frequency, these results are consistent with what was obtained in previous trials. Even though the frequency was not changed, the air gaps were still present in the tube (see Fig. 5.6). This combined with the entrance and exit effects generated from large amplitudes is a likely reason for the deviation in some of the data points from the model.

Another possible contributing factor to the error was the disturbance that occurred when changing the pipe inserts. When amplitude needed to be changed, the hinge on the bull wheel was adjusted. When the frequency needed to be changed, the clamp on the drill was adjusted. In the case where the radius size was switched, several parts of the experiment needed to be disassembled. This included draining the top tank and the connecting pipe, shutting off the mixer, the cooling loop, the temperature bath, and the controller. Once the insert was finally switched and the experiment was reassembled, several minutes had elapsed. Although it is hard to pinpoint a single error arising from this (except for maybe a change in room temperature during the transition period), a lot of experimental parts were switched out or adjusted. There was also a period of several minutes where the system had to be shut off. All of this could have contributed to the likelihood that the system didn’t reach the exact same equilibrated state once it was started up again. This could have led to a different response from the controller perhaps or the cooling loop. This is something that is hard to measure quantitatively, but these potential
small disturbances combined with the other sources of error mentioned above could explain some of the data points lying outside of the error bars.

5.6 Summary of Results

During the experimentation process, several of the input parameters were varied to see how they affected the heat transport from the heat source to the heat sink. These parameters included the type of fluid (with drastically different Prandtl numbers), the amplitude of the piston, the frequency of the oscillations, and the radius of the tube. Several plots were developed that show the performance of the experiment compared to the theoretical model. Initially, one of the important characteristics of the data was that all the points obeyed the theoretical trend. When amplitude was increased, the heat transport increased. When frequency was increased, the heat transport increased, etc. How accurately the data followed the model varied from trial to trial. In some instances, the results were quite close to the model, in others the data points laid just outside the error bars, and in a couple trials (mostly in the frequency variation) there was quite a large discrepancy between the theory and the data points. Whenever there was some deviation from the model, whether small or large, possible explanations were given as to why the errors occurred. These results are a good beginning in showing how much oscillatory flows can improve heat transport.

5.7 Potential Improvements on this Work

There are many ways to change the experiment to improve the accuracy of the results. Others who follow up on this work might consider making the following changes:

- Design a better way to adjust frequency so that there are more values for which the frequency can remain stable. One way could be to use a constant power supply to prevent voltage surges as well as using a motor with a shaft connecting to the bull wheel.
The speed of the motor could be adjusted with a voltage regulator as opposed to the current method with a clamp on a drill. This could be taken one step further and have the frequency controlled by a computer program, much like the controller in the heat source which monitors the temperature.

- Develop ways to minimize the heat leakages in the pipe. This could include designing inserts that eliminate the “air gaps” or finding a material to make the tube and inserts out of which have a significantly lower thermal conductivity than the fluid medium (the fluid medium could also be changed to promote a bigger discrepancy).

- Expand the range for certain variables. Allow for a greater range of amplitudes and frequencies to be tested. Improve the mixing mechanism so that lighter fluids (i.e., gases) can be temperature controlled.

The following improvements can be made which won’t necessarily improve the accuracy of the current results, but will help show why oscillatory flows can increase the heat transport in a system:

- Develop a way to measure the convective to conductive ratio, through either adjusting the parameters or redesigning the experiment to reduce the amount of time needed for the conductive steady state to exist in the pipe.

- Redesign the experiment where the transport region is a capillary bundle. Using tubes of very small radii will drastically increase the convective to conductive ratio. This is how Kurzweg and Zhao [9] were able to generate such a large ratio (~18000x).
• Test the other two methods of imposing oscillations. Allow for a pressure valve to
generate a pressure drop across the pipe, or design the experiment which would have a
moving boundary to impose the oscillations.

• Allow the experiment to consist of various geometries in the transport tube. Let the tube
be either a centered or off centered annular region, or a two compartment region with a
cylinder and annulus. It would be of interest to see if experimental results could be
generated which would support the theoretical arguments made earlier comparing
different geometries.

Figure 5.1: Experimental set-up
Figure 5.2a: Schematic diagram of heat source

Figure 5.2b: Schematic diagram of heat sink
Figure 5.3: Heat transport increase as a result of piston amplitude increase for low Prandtl number fluid (water). Theoretical curve (solid line) is shown with experimental results (circles) with error bars.

Figure 5.4: Heat transport increase as a result of piston amplitude increase for high Prandtl number fluid (glycerol-water mixture). Theoretical curve (solid line) is shown with experimental results (circles) with error bars.
Figure 5.5: Heat transport increase as a result of oscillation frequency increase for high Prandtl number fluid (glycerol-water mixture). Theoretical curve (solid line) is shown with experimental results (circles) with error bars.

Figure 5.6: Large tube with separate insert (left), insert placed in large tube (right) creating an air filled gap (shaded regions)
Figure 5.7: Heat transport increase as a result of oscillation frequency increase for high Prandtl number fluid (glycerol-water mixture). Normalized theoretical curve (solid line) is shown with experimental results (circles) with error bars.

Figure 5.8: Experimental ratio compared to theoretical ratio for changing radius. Solid line represents agreement between experiment and theory. Experimental data points (circles) are shown with error bars.
Table 5.1: Physical Properties of Water at Different Temperatures

<table>
<thead>
<tr>
<th>Physical Property</th>
<th>Water @ 20 C</th>
<th>Water @ 22 C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (g/cm³)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Thermal Conductivity (W/cm/C)</td>
<td>0.0059</td>
<td>0.0059</td>
</tr>
<tr>
<td>Kinematic Viscosity (cm²/s)</td>
<td>0.0100</td>
<td>0.0096</td>
</tr>
<tr>
<td>Specific Heat (J/g/C)</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>Prandtl Number (Pr)</td>
<td>7.1</td>
<td>6.8</td>
</tr>
</tbody>
</table>

Table 5.2: Physical Properties of a Glycerol-Water mixture at Different Temperatures

<table>
<thead>
<tr>
<th>Physical Property</th>
<th>Gly.-Water @ 20 C</th>
<th>Gly.-Water @ 22 C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (g/cm³)</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Thermal Conductivity (W/cm/C)</td>
<td>0.0038</td>
<td>0.0038</td>
</tr>
<tr>
<td>Kinematic Viscosity (cm²/s)</td>
<td>0.094</td>
<td>0.087</td>
</tr>
<tr>
<td>Specific Heat (J/g/C)</td>
<td>3.1</td>
<td>3.1</td>
</tr>
<tr>
<td>Prandtl Number (Pr)</td>
<td>89</td>
<td>83</td>
</tr>
</tbody>
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CHAPTER 6
CONCLUSIONS AND FUTURE RESEARCH

6.1 Conclusions

Several examples were presented to show how oscillations can improve heat transport in a pipe. In these examples, various parameters were examined to determine what conditions produce the best transport. These parameters included the geometry of the channel, the dimensions of the channel, the type of fluid medium, and the oscillation frequency and amplitude. In all these examples, a physical explanation was provided to bolster the argument of why the system behaved as it did.

The oscillatory flow model was presented in three different ways, using a pulsating pressure drop, an external oscillating piston, and an oscillating moving boundary, for both a cylindrical and an annular geometry. For all these scenarios, it was found that the heat transfer was proportional to the square of the driving force. It was also shown that the heat transfer decreases with increasing Prandtl number. In each case, the input signal of the oscillation mechanism was a single periodic function, and the response of the system was a combination of periodic functions with a phase lag. This was due to the size of the signal and the size of the system.

A comparison was made between the cylinder and the annulus to determine which geometry is more optimal for heat transfer. Depending on what parameters were held fixed, different conclusions could be reached. In the case where the pressure drop was held fixed, the cylinder provided a higher rate of transport. When the flow rate through the tubes was held fixed, the annulus provided the better results. These trends held whether the outer radius of the cylinder and annulus were the same or whether the cross sectional areas were held the same. The
inner rod of the annulus imposed a resistance to flow which drove the transport down when the pressure drop was held fixed. For the case of fixed flow rates, the resistance in the annulus lowered its flow rate, meaning the pressure drop had to increase to keep the flow rate the same as in a cylindrical tube. This drove the transport in the annulus up for the case of fixed flow rates. It was also determined that the cylinder required less power to drive the fluid back and forth compared to the annulus, since substantial energy must be exerted by the fluid to overcome the viscous effects of two walls in the annulus compared to just one for the cylinder.

This geometric study was taken one step further where an annulus was compared to a two compartment model where the geometry consisted of an inner cylinder and an outer annulus. The conclusion was reached that the two compartment model always produced a higher rate of heat transport assuming the cross sectional areas of both models were held fixed. The comparison was also made to determine which model produced a better separation between two species. Although the effective separation was similar between the two models in terms of magnitude, the maximum separation occurred at a much lower frequency for the two compartment model versus the one compartment model. The lower frequency correlates to a lower amount of energy needed to produce the optimal separation. The two compartment model produced the optimal transport in all cases since the flow rates were held fixed in both models. As in the comparison of the annulus versus the cylinder, there is an extra boundary in the two compartment model, which acts as a resistance to flow, so to account for that, the amplitude of the spikes in the two compartment channels must increase to keep the flow rate identical to the one compartment model. Since the two compartment model has larger spikes, the transport will be higher in that model.
The last effect of geometry to be studied was that of an off centered annulus. A perturbation study was presented using the scenario where both the inner and outer walls were oscillating in phase. This showed how the heat transfer changed when the inner rod of the annulus was moved to an off centered position. The off centered model provided a positive change in heat transport for very low frequencies, but quickly decreased as the frequency was raised. This was due to the “thick” region being larger than the “thin” region. When the walls oscillated fast enough, there was more fluid in the thick region that was farther away from the fast moving walls, compared to the thin region. This produced the net decrease in the transport at higher frequencies.

An experiment was designed in which multiple parameters were varied to compare its performance to the model. These included the oscillation amplitude, the fluid medium (water and a glycerol-water mixture were interchanged), the oscillation frequency, and the tube radius. For most of the data points that were taken through all these experiments, there was a reasonable agreement with the theoretical model within the error bars. This was especially true for when the amplitude, fluid medium, and tube radius were varied. The experimental results obeyed the relationship of the heat transport being proportional to the square of the piston amplitude, which was predicted by the model, for different Prandtl number fluids. For the case of varying frequency, the agreement was not very good with the theory. This was because the total heat transported was measured as the frequency was changed. In the other experiments, a comparison of two cases was made. The experimental accuracy was found by determining a ratio for the heat transported in the two cases, and comparing that to a theoretical ratio. Any error in the actual heat transfer would have been virtually eliminated since it would have been evident in both cases. For the trials where frequency was varied, the error in the total heat transport could
be attributed to heat leakages at different spots in the tube which were difficult to measure quantitatively. Overall, the experimental results were encouraging as the oscillatory flow model provided a good comparison with the data points.

6.2 Future Research in Oscillatory Flows

The oscillatory flow model can be put to use in many areas of research today. The separation of species example has applications beyond those mentioned in the introductory section. The first involves the detection of dangerous chemicals. This is an era when there is a deep concern regarding biological and chemical weapons. It is consequently important to be able to detect harmful species. This project can of course be employed in the separation of species, which can be a valuable tool. This method could serve as a helpful step of concentrating the harmful species before an unknown sample containing the species enters a detector. It is obvious that it is easier for a detector to find a hazardous chemical when there are fewer substances in a sample. By separating out certain species of a sample through this pulsating mechanism, samples would be made simpler for the detector. Another application would be the use of this project to separate viruses from samples. Since new and dangerous viruses threaten society all the time, it would be beneficial to create a mechanism that could separate out harmful particles. In addition, the pulsating mechanism can be used for spin-off technologies such as DNA and protein separation. Other “creative” uses for this technology could include electromagnetic fields, in which an imposed field is oscillated to move charges in a particular direction. Also, gas molecules which are entrapped in porous media could be removed through oscillatory flows. This could occur by means of an oscillating pressure drop on either side of the matrix, where the force exerted on the molecules could assist in moving them out of the porous matrix.
From a heat transfer standpoint, there was an emphasis put on this method as a way to transport heat out of enclosed space capsules. Although the heat transfer experiment that was conducted follows the theoretical model well, it has yet to be seen whether this experiment would perform well under zero gravity conditions. One might conclude that this experiment would produce better results at zero gravity since natural convection would be totally eliminated. It is also important to see if this technology would work on a larger scale, since the volume of outer space modules are several orders of magnitude greater than the volume of the experiment conducted in the laboratory. Hopefully NASA will look into testing this method on their space habitats and use oscillatory flows as a way to improve heat removal in their applications.
APPENDIX

CONSTANTS IN THE SOLUTIONS TO THE OFF CENTERED ANNULUS PROBLEM

\( A_{0v} = \frac{1}{4} A \omega [J_0(\alpha R_0) - J_0(\alpha cR_0)] \)
\( \frac{Y_0(\alpha cR_0)J_0(\alpha cR_0) - Y_0(\alpha R_0)J_0(\alpha cR_0)}{Y_0(\alpha R_0)J_0(\alpha R_0) - Y_0(\alpha R_0)J_0(\alpha R_0)} \) \hspace{1cm} (A.1)

\( B_{0v} = \frac{1}{4} A \omega [Y_0(\alpha R_0) - Y_0(\alpha cR_0)] \)
\( \frac{J_1(\alpha R_0)Y_1(\alpha R_0) - J_1(\alpha cR_0)Y_1(\alpha cR_0)}{J_1(\alpha R_0)Y_1(\alpha R_0) - J_1(\alpha R_0)Y_1(\alpha R_0)} \) \hspace{1cm} (A.2)

\( A_{1v} = \alpha J_1(\alpha R_0) \left. \frac{dV_0}{dr_0} \right|_{r_0} \)
\( \frac{J_1(\alpha cR_0)Y_1(\alpha R_0) - J_1(\alpha R_0)Y_1(\alpha cR_0)}{J_1(\alpha R_0)Y_1(\alpha R_0) - J_1(\alpha R_0)Y_1(\alpha R_0)} \) \hspace{1cm} (A.3)

\( B_{1v} = \alpha Y_1(\alpha R_0) \left. \frac{dV_0}{dr_0} \right|_{r_0} \)
\( \frac{J_1(\alpha cR_0)Y_1(\alpha R_0) - J_1(\alpha R_0)Y_1(\alpha cR_0)}{J_1(\alpha R_0)Y_1(\alpha R_0) - J_1(\alpha R_0)Y_1(\alpha R_0)} \) \hspace{1cm} (A.4)

\( A_{2v} = \frac{-J_0(\alpha R_0)\left(-\frac{dV_z}{dr_0}\right|_{r_0} - \frac{1}{2} \left. \frac{d^2V_z}{dr_0^2} \right|_{r_0} + \frac{1}{2R_0} \left. \frac{dV_z}{dr_0} \right|_{r_0}}{Y_0(\alpha R_0)Y_0(\alpha R_0) - J_0(\alpha R_0)Y_0(\alpha R_0)} \) \hspace{1cm} (A.5)

\( B_{2v} = \frac{Y_0(\alpha R_0)\left(-\frac{dV_z}{dr_0}\right|_{r_0} - \frac{1}{2} \left. \frac{d^2V_z}{dr_0^2} \right|_{r_0} + \frac{1}{2R_0} \left. \frac{dV_z}{dr_0} \right|_{r_0}}{Y_0(\alpha R_0)Y_0(\alpha R_0) - J_0(\alpha R_0)Y_0(\alpha R_0)} \) \hspace{1cm} (A.6)

\( \xi_{g'v} = \frac{iA_{g'v} \Delta T}{\omega (1 - Pr^{-1})L} \)
\( \frac{1}{\omega (1 - Pr^{-1})L} \) \hspace{1cm} (A.7)

\( \xi_{g''v} = \frac{iB_{g''v} \Delta T}{\omega (1 - Pr^{-1})L} \)
\( \frac{1}{\omega (1 - Pr^{-1})L} \) \hspace{1cm} (A.8)

Where \( g \) is the corresponding \( \varepsilon \) order of the constant (0,1,2, etc.)
\[ A_{0T} = \frac{-\beta B_{0T} J_1(\beta R_0) - \alpha \xi_{0j} Y_1(\alpha R_0) - \alpha \xi_{0j} J_1(\alpha R_0)}{\beta Y_1(\beta R_0)} \]  

(A.9)

\[ B_{0T} = \frac{\alpha}{\beta} \left( \xi_{0j} Y_1(\alpha R_0) Y_1(\beta c R_0) + \xi_{0j} J_1(\alpha R_0) J_1(\beta c R_0) \right) \]  

(A.10)

\[ \xi_{0j} Y_1(\alpha c R_0) Y_1(\beta R_0) + \xi_{0j} J_1(\alpha c R_0) J_1(\beta R_0) \]  

\[ J_1(\beta c R_0) Y_1(\beta R_0) - J_1(\beta R_0) Y_1(\beta c R_0) \]

\[ R_1 = \beta Y_0(\beta R_0) - Y_1(\beta R_0) / R_0 \]  

(A.11)

\[ R_2 = \beta I_0(\beta R_0) - J_1(\beta R_0) / R_0 \]  

(A.12)

\[ R_3 = \alpha Y_0(\alpha R_0) - Y_1(\alpha R_0) / R_0 \]  

(A.13)

\[ R_4 = \alpha I_0(\alpha R_0) - J_1(\alpha R_0) / R_0 \]  

(A.14)

\[ R_1' = \left( \frac{2}{R_0^2} - \beta^2 \right) Y_1(\beta r) - \frac{\beta}{R_0} Y_0(\beta r) \]  

(A.15)

\[ R_2' = \left( \frac{2}{R_0^2} - \beta^2 \right) J_1(\beta r) - \frac{\beta}{R_0} J_0(\beta r) \]  

(A.16)

\[ R_3' = \left( \frac{2}{R_0^2} - \alpha^2 \right) Y_1(\alpha r) - \frac{\alpha}{R_0} Y_0(\alpha r) \]  

(A.17)

\[ R_4' = \left( \frac{2}{R_0^2} - \alpha^2 \right) J_1(\alpha r) - \frac{\alpha}{R_0} J_0(\alpha r) \]  

(A.18)

\[ A_{1T} = \frac{-B_{1\epsilon} \kappa_2 - \xi_{1j} \kappa_3 - \xi_{1j} \kappa_4}{\kappa_1} \]  

(A.19)

\[ B_{1T} = \frac{c_1 \left( -\beta A_{0T} R_1 - \beta B_{0T} R_2 - \alpha \xi_{0j} R_3 - \alpha \xi_{0j} R_4 \right)}{R_2 c_1 - R_1 c_2} + \frac{c_1 \left( -\xi_{1j} R_3 - \xi_{1j} c_3 R_1 / c_1 + \xi_{1j} c_4 R_1 / c_1 \right)}{R_2 c_1 - R_1 c_2} \]  

(A.20)

\[ c_1 = \beta Y_0(\beta c R_0) - Y_1(\beta \kappa R_0) / c R_0 \]  

(A.21)

\[ c_2 = \beta I_0(\beta c R_0) - J_1(\beta c R_0) / c R_0 \]  

(A.22)
\[ c_3 = Y_0(\alpha c R_0) - Y_1(\alpha c R_0) / c R_0 \]  
(A.23)

\[ c_4 = J_0(\alpha c R_0) - J_1(\alpha c R_0) / c R_0 \]  
(A.24)

\[ A_{2T} = \frac{-\beta B_{2T} J_1(\beta c R_0) - \alpha \xi_{2y} Y_1(\alpha c R_0) - \alpha \xi_{2y} J_1(\alpha c R_0)}{\beta Y_1(\beta c R_0)} \]  
(A.25)

\[ B_{2T} = \frac{Y_1(\beta c R_0)}{\beta (J_1(\beta c R_0) Y_1(\beta R_0) - J_1(\beta R_0) Y_1(\beta c R_0))} \]  

\[ (X + \alpha \xi_{2y} Y_1(\alpha R_0) + \alpha \xi_{2y} J_1(\alpha R_0)) - \]  

\[ \frac{Y_1(\beta R_0)}{Y_1(\beta c R_0)} (\alpha \xi_{2y} Y_1(\alpha c R_0) + \alpha \xi_{2y} J_1(\alpha c R_0)) \]

\[ X = -[A_{1T} R_1^* + B_{1T} R_2^* + \xi_{1y} R_3^* + \xi_{1y} R_4^*] + \]  

\[ \frac{1}{2} (\beta A_{0T} R_1 + \beta B_{0T} R_2 + \alpha \xi_{0y} R_3 + \alpha \xi_{0y} R_4) - \]  

\[ \frac{1}{2 R_0} (\beta A_{0T} R_1 + \beta B_{0T} R_2 + \alpha \xi_{0y} R_3 + \alpha \xi_{0y} R_4) \]  
(A.27)
LIST OF REFERENCES


Mechanics, 21 (2) (1965) 83-95.


bundles of various cross sections, JSME International Journal, 43 (3) (2000)
460-467.

16-58.

flows subjected to high-frequency oscillations, Physics of Fluids, 26 (6) 1380-
1382.

[8] U.H. Kurzweg, M.J. Jeager, Tuning effect in enhanced gas dispersion under oscillatory
conditions, Physics of Fluids, 29 (4) (1986) 1324-1325.

hydrodynamic technique for achieving large effective thermal conductivities,

characteristics along liquid columns of oscillation-controlled heat transport
2457-2470.

923-934.

[12] K.B. Ranger, Research note on the steady Poiseuille flow through pipes with multiple
connected cross sections, Physics of Fluids, 6 (6) (1994) 2224-2226.


BIOGRAPHICAL SKETCH

The author was born on June 3, 1980, in Pittsburgh, Pennsylvania. His family moved to Northern Virginia in 1986, and he graduated from Centreville High School in Clifton, Virginia, in 1998. He attended the University of Notre Dame and graduated with a B.S. in chemical engineering in 2002. He will graduate with his Ph.D. in chemical engineering from the University of Florida in December 2006.