CALIBRATING PRESSURE SENSITIVE PAINTS USING PROPER ORTHOGONAL DECOMPOSITION

By

AHMED F. OMAR

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2006
Copyright 2006

by

Ahmed F. Omar
This dissertation is dedicated to my family.
ACKNOWLEDGMENTS

THOSE OF HIS SERVANTS ONLY WHO ARE POSSESSED OF KNOWLEDGE FEAR ALLAH; SURELY ALLAH IS MIGHTY, FORGIVING. (QURAN: SURAT FA’TIRR, VERSE 28)

THEY ASK THEE CONCERNING THE SOUL. SAY: “THE SOUL IS OF THE AFFAIR OF MY LORD: OF KNOWLEDGE IT IS ONLY A LITTLE THAT IS COMMUNICATED TO YOU, (O MEN!)”
(QURAN: SURAT AL-ISRAA’A, VERSE 85)

I express my humility and gratitude before Allah for all the blessings and successes in my life. None of these accomplishments would have been possible without the grace and mercy of Allah. I would like next to thank my parents and brother for all their sacrifices to help me in my educational pursuit. I owe them everything, and I know the world would not be enough to give to them. My fiancée is owed sincere appreciation for her support and understanding.

I would also like to thank Dr. Carroll, my advisor, for his support over the course of my Ph.D. pursuit. Special thanks are owed to Dr. Paul Hubner and Dr. Louis Cattafesta for their assistance and guidance in a time of desperate need. Further appreciation goes to the rest of my committee members, Dr. Kirk Schanze and Dr. William Lear. Finally, I would like to thank all of those who helped me through the course of my Ph.D.: friends, colleagues, and university staff.
# TABLE OF CONTENTS

LIST OF TABLES ....................................................................................................................... x

LIST OF FIGURES ................................................................................................................... xiii

NOMENCLATURE ..................................................................................................................... xxiii

ABSTRACT ................................................................................................................................. xxvi

CHAPTER

1 INTRODUCTION .................................................................................................................. 1

   Literature Review .................................................................................................................. 3
   Preface ................................................................................................................................. 3
   Historical Perspective ......................................................................................................... 4
   Photophysics of Luminescent Coatings ............................................................................ 6
   Statement of the Problem ................................................................................................. 11
   Research Efforts ................................................................................................................ 14
   Proper Orthogonal Decomposition (POD) ................................................................. 36
   Motivation and Contribution ............................................................................................ 41

2 LUMINESCENT COATINGS ............................................................................................ 46

   Overview ............................................................................................................................. 46
   The Ozone Accident .......................................................................................................... 47
   The Structure .................................................................................................................... 51
   Paint Chemistry .................................................................................................................. 52
   The Photophysics .............................................................................................................. 52
   The Temperature Dependence ....................................................................................... 55
   The Oxygen Factor .......................................................................................................... 57
   Paint Composition and Character ................................................................................... 58
   Which Luminescent Molecule? ....................................................................................... 58
   Binder Polymers ................................................................................................................. 59
   The Undercoat .................................................................................................................. 62
   Measurement System ....................................................................................................... 64
   The Excitation Source ...................................................................................................... 65
   The Detection Device ....................................................................................................... 66
   Calibration Techniques .................................................................................................... 67
# Measurement System

- **The Excitation Source**
  - Lasers ................................................................. 249
  - UV Lamps ............................................................ 250
  - Visible Light Illumination LEDs ......................... 252
- **The Detection Device**
  - CCD Cameras ..................................................... 253
  - Physical Construction ......................................... 254
  - From Light to Electrons ....................................... 255
  - Potential Wells ................................................... 256
  - Charge Transfer ................................................. 257
  - Digital Signal Processing ..................................... 259
  - CCD Chip Types .................................................. 259
    - Full-Frame CCD Chips ........................................ 259
    - Frame-Transfer CCD Chips ................................ 260
    - Back-Thinned, Back-Illuminated CCD Chips ......... 261
  - CCD Artifacts ..................................................... 261
  - Charge Transfer Efficiency .................................. 262
  - Photon (Shot) Noise ............................................. 262
  - Pixel Gain Variations .......................................... 263
  - Read-Noise ......................................................... 264
  - Saturation .......................................................... 264
  - Thermal (Dark) Current ....................................... 264
  - Camera Resolution .............................................. 266
- **Data Reduction**
  - Correcting for Non-Idealities ................................ 269
  - Dark Image Correction .......................................... 269
  - Illumination-Field Variation Correction ................. 269
  - Image-Registration correction ............................... 270
  - Flat-Field Correction .......................................... 271
- **Supplementary Topics**
  - Lifetime Approach ............................................. 272
  - Measurement Uncertainties ................................ 275
    - Measurement Systems ...................................... 276
    - Illumination source ......................................... 276
    - CCD camera .................................................... 276
    - Filtering/Spectral leakage .................................. 277
    - Physical/Chemical Properties of Coating .............. 278
    - Displacement/Deformation of Model .................... 278
  - Useful Definitions ............................................. 279
    - Binning .......................................................... 279
    - Blooming ....................................................... 280
    - Bit Depth of Camera Data .................................. 280
    - Dynamic Range .............................................. 281
    - Fill Factor ...................................................... 282
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>Response time and temperature dependency of PtTFPP/FIB with different base coatings.</td>
</tr>
<tr>
<td>1-2</td>
<td>Research effort treating PSP temperature effects.</td>
</tr>
<tr>
<td>3-1</td>
<td>Conditions and their $C$ elements.</td>
</tr>
<tr>
<td>3-2</td>
<td>Eigenvalues for case one.</td>
</tr>
<tr>
<td>3-3</td>
<td>Eigenvalues for case one-A.</td>
</tr>
<tr>
<td>3-4</td>
<td>Calibration Error case one-A.</td>
</tr>
<tr>
<td>3-5</td>
<td>Eigenvalues for case one-B.</td>
</tr>
<tr>
<td>3-6</td>
<td>Calibration Error case one-B.</td>
</tr>
<tr>
<td>3-7</td>
<td>Eigenvalues for case one-D.</td>
</tr>
<tr>
<td>3-8</td>
<td>Calibration Error case one-C.</td>
</tr>
<tr>
<td>3-9</td>
<td>Eigenvalues for case one-D.</td>
</tr>
<tr>
<td>3-10</td>
<td>Calibration Error case one-D.</td>
</tr>
<tr>
<td>3-11</td>
<td>Eigenvalues for case one-E.</td>
</tr>
<tr>
<td>3-12</td>
<td>Calibration Error case one-E.</td>
</tr>
<tr>
<td>3-13</td>
<td>Eigenvalues for case one-F.</td>
</tr>
<tr>
<td>3-14</td>
<td>Calibration Error case one-F.</td>
</tr>
<tr>
<td>3-15</td>
<td>Eigenvalues for case one-G.</td>
</tr>
<tr>
<td>3-16</td>
<td>Calibration Error case one-G.</td>
</tr>
<tr>
<td>3-17</td>
<td>Eigenvalues for case one-G.</td>
</tr>
</tbody>
</table>
3-18 Calibration Error case one-G

5-1 Coefficients of Thermal Expansion

6-1 Exposure times for the different filters

6-2 Pressure drift throughout the exposure times (case five)

6-3 Test matrix

6-4 Temperature drift throughout the exposure times (case 5)

6-5 Case one: Test-points pressure results. Actual represents pressure tap measurement and POD represents the calculated pressure via POD calibration. The precision error of the Actual readings is ± 0.006 psi.

6-6 Case three: Test-points pressure results. Actual represents pressure tap measurement and POD represents the calculated pressure via POD calibration. The precision error of the Actual readings is ± 0.006 psi.

6-7 Case five: Test-points pressure results. Actual represents pressure tap measurement and POD represents the calculated pressure via POD calibration. The precision error of the Actual readings is ± 0.006 psi.

6-8 Case seven: Test-points pressure results. Actual represents pressure tap measurement and POD represents the calculated pressure via POD calibration. The precision error of the Actual readings is ± 0.006 psi.

6-9 Comparison of different calibration techniques

6-10 Case five: Comparison between test-points pressure results for POD and intensity-ratio Actual represents pressure tap measurement and POD represents the calculated pressure via POD calibration. The precision error of the Actual readings is ± 0.006 psi.

6-11 Case seven: Comparison between test-points pressure results for POD and intensity-ratio. Actual represents pressure tap measurement and POD represents the calculated pressure via POD calibration. The precision error of the Actual readings is ± 0.006 psi.

6-12 Intensity error sources and values for Photometrics CH250A CCD camera (SI 502AB), 200 kHz, 14 bit A/D for a unity nominal gain value and a measured gain (β) of 19.1 (e^-ADU)

6-13 Shot noise uncertainty for all filters (case seven) with total precision uncertainty of 29.3cts
6-14  Intensity drift uncertainty for case seven. Reference conditions are not included in the analysis as they are acquired under isothermal conditions. The total intensity bias uncertainty due to test conditions drift is 9.7cts.

6-15  Uncertainty (95% confidence level) for key factors

A-1  Error estimates for the 16 & 14-bit CCD cameras. Camera linearity error is 0.07% and ADU speed is 200 kHz.
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>Stern-Volmer model for a range of $K_q$</td>
</tr>
<tr>
<td>1-2</td>
<td>Jablonski energy-level diagram showing the absorption and emission processes for a typical luminescent molecule</td>
</tr>
<tr>
<td>2-1</td>
<td>Coating Structure</td>
</tr>
<tr>
<td>2-2</td>
<td>Typical Absorption (left) and Emission (right) vs. Wavelength for PtTFPP in fluoroacrylic polymer binder (PtTFPP/FIB). (Bell et al. 2001)</td>
</tr>
<tr>
<td>2-3</td>
<td>Typical rate of gas permeation across a membrane (after Lu and Winnik, 2000)</td>
</tr>
<tr>
<td>2-4</td>
<td>Equipment Setup for Temperature Calibration</td>
</tr>
<tr>
<td>3-1</td>
<td>Spectral emission for two independence processes: (A) Raw spectra (B) A single raw spectrum with dependence on the higher wavelength factor and no dependence on the lower wavelength factor (C) The normalized spectra</td>
</tr>
<tr>
<td>3-2</td>
<td>P.C.A. Algorithm</td>
</tr>
<tr>
<td>3-3</td>
<td>Target transformation algorithm</td>
</tr>
<tr>
<td>3-4</td>
<td>Simulated spectral response of the luminophors: (A) Frequency domain (B) Wavelength domain</td>
</tr>
<tr>
<td>3-5</td>
<td>Shot noise as a function of intensity</td>
</tr>
<tr>
<td>3-6</td>
<td>Case one: Narrow spectrum with no overlap</td>
</tr>
<tr>
<td>3-7</td>
<td>Case one: Eigenvectors and calibration curves for pressure (left) at 79.8392° and temperature (right) at 13.6121°</td>
</tr>
<tr>
<td>3-8</td>
<td>Case one-A: Temperature Emission Amplified 10 Folds</td>
</tr>
<tr>
<td>3-9</td>
<td>Case one-A: Eigenvectors for pressure (left) at 66.5546° and temperature (right) at 0.3275°</td>
</tr>
<tr>
<td>3-10</td>
<td>Case one-B: Emission spectra</td>
</tr>
</tbody>
</table>
3-11 Case one-B: Eigenvectors for pressure (left) at 75.592° and temperature (right) at 9.365° .................................................................................................................................97

3-12 Case one-C: Emission spectra .................................................................................98

3-13 Case one-C: Eigenvectors for pressure (left) at 72.783° and temperature (right) at 6.556° .........................................................................................................................98

3-14 Case one-D: Emission spectra .................................................................................99

3-15 Case one-D: Eigenvectors for pressure (left) at 78.15° and temperature (right) at 11.923° .........................................................................................................................99

3-16 Case one-E: Emission spectra .................................................................................100

3-17 Case one-E: Reconstructed spectra .......................................................................100

3-18 Case one-E: Eigenvectors for pressure (left) at 280.1082° and temperature (right) at 166.392° ......................................................................................................................101

3-19 Case one-F: Emission spectra .................................................................................102

3-20 Case one-F: Reconstructed spectra .......................................................................102

3-21 Case one-F: Eigenvectors for pressure (left) at 100.1556° and temperature (right) at 346.3815° ......................................................................................................................103

3-22 Case one-G: Emission spectra .................................................................................103

3-23 Case one-G: Reconstructed spectra .......................................................................104

3-24 Case one-G: Eigenvectors for pressure (left) at 79.79° and temperature (right) at 13.502° .........................................................................................................................104

3-25 Case one-H: Emission spectra .................................................................................105

3-26 Case one-H: Eigenvectors for pressure (left) at 258.9° and temperature (right) at 192.68° .........................................................................................................................105

3-27 Case two: Emission spectra ....................................................................................107

3-28 Case two: Eigenvectors rotated at the appropriate angle for temperature (left) at 345.78° and pressure (right) at 106.44° ..............................................................................110

3-29 Case two: Temperature calibration .........................................................................110

3-30 Case two: Second order pressure calibration using the scorers from POD ..........111
Case two: Second order pressure calibration using the 2nd set of scorers and temperature

Case two: Third order pressure calibration using the scorers from POD at 193.46°

Case two: Pressure calibration using the scorers from POD at 193.46°

Case two: Numerically filtered emission

Case two: Numerically filtered emission eigenvectors for pressure (left) at 106.6° and temperature (right) at 346.06°

Case two: Second order pressure calibration using the 2nd set of scorers and temperature

Channel flow schematic

Channel flow: flow development and pressure distribution

Low-aspect ratio developing channel flow

Fully developed velocity profile for an incompressible laminar channel flow

Hotwire and PIV measurement schematic in the channel

Isothermal case: Centerline pressure gradient along the channel showing the deviation of the experimental results from the theoretical prediction

Non-isothermal flow schematic

Mass flow controller (AALBORG)

Channel flow schematic: Top view of channel shown

(A) Channel resting on heaters (B) Side view of channel showing the 1 inch glass plate (C) Backside of channel showing thermocouples, pressure taps, stagnation chamber and tubing connecting from the top water channel to the cooler

Close-up of the beginning of the channel showing the packaging and aluminum foam, thermocouples and pressure taps. First two test pressure taps are circled in yellow

Channel deformation due to pressure forces (modulus of elasticity for aluminum and glass is ~ 10.5 and 9.6 psi x 10⁶, respectively)

Channel deformation due to pressure forces using Plexiglas comparing the theoretical pressure profile (green line) to experimental profile (blue circles)
5-7 Calculated percentage channel height deformation (relative to no-flow conditions of 0.01”) along the centerline for 1” plate with a maximum flow rate of 100LPM, 23 psi static pressure at the beginning of the channel and 0.01” channel height. .................................................................140

5-8 Glass deformation (a comparison between theoretical and experimental results): 3/8” glass (top) and 1” glass (bottom) with percentage pressure deviation from theory in top-left corner of each plot. Flow from right to left.................................141

5-9 Experimental optical setup showing the relative position of the CCD camera and excitation sources with respect to the channel .........................................................142

5-10 Pressure response of dual-luminophor paint at 293°K (Kose 2005).................143

5-11 Temperature response of dual-luminophor paint at 14.7 psi (Kose 2005).........143

5-12 First bandpass interference filter: 550 nm, 40 ± 8 nm FWHM; dia. = 50 mm (www.mellesgriot.com)........................................................................................................144

5-13 Second bandpass interference filter: 600 nm, 40 ± 8 nm FWHM; dia. = 50 mm (www.mellesgriot.com)........................................................................................................144

5-14 Third bandpass interference filter: 650 nm, 40 ± 8 nm FWHM; dia. = 50 mm (www.mellesgriot.com)........................................................................................................145

5-15 Fourth bandpass interference filter: 700 nm, 40 ± 8 nm FWHM; dia. = 50 mm (www.mellesgriot.com)........................................................................................................145

5-16 Filters arrangement relative to paint spectral response to temperature (left) and pressure (right). ........................................................................................................146

5-17 (A) Filter wheel (B) Blue LED ISSI LM2 (excitation source) (C) CCD camera and filter wheel assembly.......................................................................................147

5-18 Effect of image registration on SNR (A) Unregistered image (B) Registered image. Bottom plots show a horizontal section of the images above. ......................148

5-19 Pixel intensity shift on the CCD array ................................................................149

5-20 Pixel shift due to thermal expansion from bottom to top of the channel...........152

6-1 Stern-Volmer relation for dual-luminophor (PtTFPP-Ruphen/PAN nanospheres in poly-t-BS-co-TFEM) at different temperature levels (Kose, 2005)...............155

6-2 Channel image showing thermocouples, calibration pressure taps and pressure test points taps. ........................................................................................................157
6-3 Effect of adding rows instead of columns to the calibration matrix. Observe the direction of the main eigenvector (black arrow) compared to the calibration eigenvector (orange arrow). .................................................................159

6-4 Intensity percent error due to shot noise vs. number of frames (14-bit CCD camera) ........................................................................................................160

6-5 Case one: Pressure profile (bottom), (a) percent pressure deviation from theory (psi), (b) temperature profile (thermocouples) over the plate (8” x 3”) ....................163

6-6 Case one: Normalized intensity (I_{run}/I_{ref}): 550nm [left] --- 600nm [right] ...........164

6-7 Case one: Normalized intensity (I_{run}/I_{ref}): 650nm [left] --- 700nm [right] ...........164

6-8 Case one: Centerline pressure response of dual-luminophor at different bandpass wavelengths .................................................................................................................165

6-9 Normalized intensity for case one (650nm) using N₂ (left): (A) Reference image acquired with no flow and exposure time of 1500ms (B) Reference image acquired with nitrogen flow and exposure time of 185ms. Exposure time for run image is 1500 ms. .................................................................................................................................166

6-10 Fluorescence image using epifluorescence microscope of Ruphen/PAN particles emission (dispersed into the PtTFPP / poly-t-BS-co-TFEM binder) at 560nm. The image shows the heterogeneous dispersion of the Ruphen/PAN particles in the poly-t-BS-co-TFEM matrix (Kose 2005) .................................................................................................................................167

6-11 Case one: Normalized intensity (I_{ref}/I_{run}). .................................................................................................................................................................................................................168

6-12 Case one: Calibration eigenvectors at 174.56° degrees (left) and eigenvalues (right) .............................................................................................................................................................169

6-13 Case one: Product of first (left) and second (right) eigenvectors and eigenvalues of calibration ........................................................................................................................................................................169

6-14 Case one: Product sum of eigenvectors and calibration eigenvalues showing 1% variation at 550nm and 15% at 650nm ........................................................................................................................................................................170

6-15 Case one: Calculated centerline-pressure profile by each eigenvector ................171

6-16 Area change due to chamfer in the glass plate at the exit ........................................172

6-17 Case one: Calibration error, absolute (left) and percentage (right) .........................172

6-18 Case one: Calculated pressure field from POD calibration ........................................173

6-19 Case one: Calculated pressure from POD calibration vs. pixel index along the channel. Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual pressure taps readings used for calibration. ..................174
6-20 Case two: Temperature profile (thermocouples) ............................................................175
6-21 Case two: Normalized intensity \( \frac{I_{\text{run}}}{I_{\text{ref}}} \): 550nm [left] --- 600nm [right].............176
6-22 Case two: Normalized intensity \( \frac{I_{\text{run}}}{I_{\text{ref}}} \): 650nm [left] --- 700nm [right].............176
6-23 Case two: Centerline temperature response of dual-luminophor coating at different bandpass wavelengths .................................................................177
6-24 Case two: Temperature linearity of dual-luminophor coating at different bandpass wavelengths ..................................................................................177
6-25 Case two: Normalized intensity \( \frac{I_{\text{run}}}{I_{\text{ref}}} \) ........................................................................178
6-26 Case two: Calibration fundamental spectra at 0.5° degrees (left) eigenvalues of calibration (right) .........................................................................................178
6-27 Case two: Product of first (left) and second (right) eigenvectors and eigenvalues of calibration ..................................................................................179
6-28 Case two: Product sum of eigenvectors and calibration eigenvalues showing 28% variation at 550nm and 18% at 650nm ......................................................180
6-29 Case two: Calculated temperature field from POD calibration (image) .................181
6-30 Case two: Calculated temperature field from POD calibration (values). Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual thermocouple readings used for calibration ..................................182
6-31 Case three: Pressure profile (bottom), (a) percent pressure deviation from theory (psi), (b) temperature profile (thermocouples) .....................................................183
6-32 Case three: Normalized intensity \( \frac{I_{\text{run}}}{I_{\text{ref}}} \): 550nm [left] --- 600nm [right].............184
6-33 Case three: Normalized intensity \( \frac{I_{\text{run}}}{I_{\text{ref}}} \): 650nm [left] --- 700nm [right].............184
6-34 Case three: Centerline pressure and temperature response of dual-luminophor at different bandpass wavelengths .........................................................185
6-35 Case three: Inverse of normalized intensity \( \frac{I_{\text{run}}}{I_{\text{ref}}} \) ......................................................186
6-36 Case three: Pressure calibration fundamental spectra at 1° degrees (left) eigenvalues of calibration (right) .................................................................187
6-37 Case three: Product of first (left) and second (right) eigenvectors and eigenvalues of pressure calibration .................................................................187
6-38 Case three: Product sum of eigenvectors and pressure calibration eigenvalues showing 18% variation at 550nm and 27% at 650nm ......................................188
6-39 Case three: Temperature calibration fundamental spectra at 0° degrees (left) eigenvalues of calibration (right) .................................................................................................................................189

6-40 Case three: Product of first (left) and second (right) eigenvectors and eigenvalues of temperature calibration ......................................................................................................................190

6-41 Case three: Product sum of eigenvectors and temperature calibration eigenvalues showing 23 % variation at 550nm and 26% at 650nm ........................................................................190

6-42 Case three: Calculated pressure field from POD calibration (image) ......................191

6-43 Case three: Calculated temperature field from POD calibration (image) ...............192

6-44 Case three: Calculated pressure field from POD calibration (values). Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual pressure taps readings used for calibration .................................................................193

6-45 Case three: Calculated temperature field from POD calibration (values). Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual thermocouple readings used for calibration .................................................................193

6-46 Case five: Temperature profile (thermocouples) ..................................................194

6-47 Case five: Pressure profile (bottom), (a) percent pressure deviation from theory (top) ..................................................................................................................................................195

6-48 Case five: Normalized intensity (I_{run}/I_{ref}): 550nm [left] --- 600nm [right] ......196

6-49 Case five: Normalized intensity (I_{run}/I_{ref}): 650nm [left] --- 700nm [right] ......196

6-50 Case five: Speckling in the 550nm image (left) and a comparison to the 700nm filter ...............................................................................................................................................197

6-51 Case five: Centerline pressure and temperature response of dual-luminophor at different bandpass wavelengths ........................................................................................................................................198

6-52 Case five: Centerline temperature profile ................................................................198

6-53 Case five: Inverse of normalized intensity (I_{run}/I_{ref}) ........................................199

6-54 Case five: Temperature calibration fundamental spectra at 89° degrees (left) eigenvalues of calibration (right) ...............................................................................................................................................200

6-55 Case five: Product of first (left) and second (right) eigenvectors and eigenvalues of calibration ...............................................................................................................................................200

6-56 Case five: Product sum of eigenvectors and calibration eigenvalues showing 9.5% variation at 550nm and 17% at 650nm ..............................................................................................................201
6-57 Case five: Pressure calibration fundamental spectra at 11° degrees (left)
eigenvalues of calibration (right) .................................................................202

6-58 Case five: Product of first (left) and second (right) eigenvectors and eigenvalues
of calibration ..................................................................................................202

6-59 Case five: Product sum of eigenvectors and pressure calibration eigenvalues
showing 4% variation at 550nm and 16% at 650nm .......................................203

6-60 Case five: Calculated temperature from POD calibration (image) ...............204

6-61 Case five: Calculated temperature from POD calibration (values). Each curve
represents a longitudinal line of pixels along the channel. Actual represents the
actual thermocouple readings used for calibration ...........................................205

6-62 Case five: Calculated pressure from POD calibration (image) ......................206

6-63 Case five: Calculated pressure from POD calibration (image). Each curve
represents a longitudinal line of pixels along the channel. Actual represents the
actual pressure taps readings used for calibration .............................................207

6-64 Case seven: Temperature profile (thermocouples) ......................................208

6-65 Case seven: Pressure profile (bottom), (a) percent pressure deviation from
theory (top) .....................................................................................................209

6-66 Case seven: Normalized intensity (I_{run}/I_{ref}): 550nm [left] --- 600nm [right] ...210

6-67 Case seven: Normalized intensity (I_{run}/I_{ref}): 650nm [left] --- 700nm [right] ....210

6-68 Case seven: Inverse of normalized intensity (I_{run}/I_{ref}) ..................................211

6-69 Case seven: Temperature calibration fundamental spectra at 93° degrees (left)
eigenvalues of calibration (right) .................................................................212

6-70 Case seven: Product of first (left) and second (right) eigenvectors and
eigenvalues of calibration ...............................................................................212

6-71 Case seven: Product sum of eigenvectors and calibration eigenvalues showing
31% variation at 550nm and 28% at 650nm ................................................213

6-72 Case seven: Pressure calibration fundamental spectra at 53° degrees (left)
eigenvalues of calibration ...............................................................................213

6-73 Case seven: Product of first (left) and second (right) eigenvectors and
eigenvalues of calibration ...............................................................................214

6-74 Case seven: Product sum of eigenvectors and calibration eigenvalues showing
8% variation at 550nm and 5% at 650nm ..................................................214
6-75 Case seven: Calculated temperature from POD calibration (image) .....................215

6-76 Case seven: Calculated pressure from POD calibration (image) .........................216

6-77 Case seven: Predicted temperature from POD calibration (values). Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual thermocouple readings used for calibration ..................................................217

6-78 Case seven: Predicted pressure from POD calibration (values). Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual pressure taps readings used for calibration ..................................................217

6-79 Temperature sensitivity (integrated intensity/°K) of the temperature and pressure luminophors at 14.7 psi (Kose 2005). ........................................................................220

6-80 Case three: Pressure calibration using intensity ratio at 650nm ..............................221

6-81 Case three: Calculated pressure using intensity ratio at 650nm ..............................221

6-82 Case three: Pressure calibration using intensity ratio (I_{550} / I_{650}) ....................222

6-83 Case three: Pressure calibration using intensity ratio (I_{600} / I_{650}) ....................223

6-84 Case three: Calculated pressure using intensity ratio at (I_{600} / I_{650}) ..................223

6-85 Case five: Pressure calibration using intensity (1 / I_{650}) .....................................224

6-86 Case five: Calculated pressure using intensity (1 / I_{650}) .....................................224

6-87 Case five: Pressure calibration using intensity (I_{600} / I_{650}) ..............................225

6-88 Case five: Calculated pressure using intensity (I_{600} / I_{650}) ..............................225

6-89 Case five: Calculated pressure using intensity (I_{600} / [I_{650}]^2) .........................226

6-90 Case five: Calculated pressure using intensity (I_{600} / [I_{650}]^{1.5}) .....................226

6-91 Case five: Calculated pressure: (A) Intensity I_{600} / [I_{650}]^{1.5} (B) POD calibration...227

6-92 Case seven: Calculated pressure using intensity (1 / I_{650}) ..............................228

6-93 Case seven: Calculated pressure using intensity (I_{600} / I_{650}) ..............................228

6-94 Case seven: Calculated pressure using intensity (I_{600} / [I_{650}]^2) .........................229

6-95 Case seven: Calculated pressure using intensity (I_{600} / [I_{650}]^{1.55}) ..................229

6-96 Case seven: Calculated pressure: (A) Intensity I_{600} / [I_{650}]^{1.55} (B) POD calibration ..........................................................230
**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(T)</td>
<td>Stern-Volmer first coefficient</td>
</tr>
<tr>
<td>B</td>
<td>Bias error</td>
</tr>
<tr>
<td>B(T)</td>
<td>Stern-Volmer second coefficient</td>
</tr>
<tr>
<td>c</td>
<td>Speed of light in vacuum (3.0 \times 10^{17} \text{ nm/s})</td>
</tr>
<tr>
<td>C</td>
<td>Scores matrix</td>
</tr>
<tr>
<td>c_p</td>
<td>Specific heats per unit mass at constant pressure</td>
</tr>
<tr>
<td>c_v</td>
<td>Specific heats per unit mass at constant volume</td>
</tr>
<tr>
<td>D</td>
<td>Data matrix, Diameter</td>
</tr>
<tr>
<td>D_p</td>
<td>Luminophor diffusion coefficients in polymer</td>
</tr>
<tr>
<td>D_q</td>
<td>Oxygen diffusion coefficients in polymer</td>
</tr>
<tr>
<td>e^-</td>
<td>Electronic</td>
</tr>
<tr>
<td>E_t</td>
<td>Energy transport rate</td>
</tr>
<tr>
<td>E_d</td>
<td>Diffusion activation energy</td>
</tr>
<tr>
<td>E_{ar}</td>
<td>Arrhenius activation energy for a non-radiative process</td>
</tr>
<tr>
<td>E_Q</td>
<td>Activation energy for oxygen diffusion in the binder</td>
</tr>
<tr>
<td>f</td>
<td>Frequency</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational force</td>
</tr>
<tr>
<td>h</td>
<td>Channel height, enthalpy, Plank’s constant (6.626176 \times 10^{-34} \text{ J·s})</td>
</tr>
<tr>
<td>I</td>
<td>Luminescence intensity</td>
</tr>
<tr>
<td>I_a</td>
<td>Absorption intensity</td>
</tr>
<tr>
<td>I_D</td>
<td>Dark charge intensity</td>
</tr>
<tr>
<td>I_{em}</td>
<td>Emission intensity</td>
</tr>
</tbody>
</table>
\( I_o \)  \hspace{1cm} \text{Luminescence intensity in the absence of oxygen}

\( I_{\text{ref}} \)  \hspace{1cm} \text{Luminescence intensity at reference conditions}

\( I_{\text{run}} \)  \hspace{1cm} \text{Luminescence intensity at run conditions}

\( k \)  \hspace{1cm} \text{Thermal conductivity (W/m·K), Boltzman’s constant (1.381 \( 10^{-23} \) J/molecule·K),}

\( k_{nr} \)  \hspace{1cm} \text{Non-radiative decay rate}

\( K_q, k_q \)  \hspace{1cm} \text{Stern-Volmer constant}

\( k_r \)  \hspace{1cm} \text{Radiative decay rate}

\( L \)  \hspace{1cm} \text{Polymer thickness (Chapter 2)}

\( \dot{m} \)  \hspace{1cm} \text{Mass flow rate}

\( N_{\text{readout}} \)  \hspace{1cm} \text{Read out noise}

\([O_2]\)  \hspace{1cm} \text{Oxygen concentration}

\( P \)  \hspace{1cm} \text{Precision error}

\( P_{O_2} \)  \hspace{1cm} \text{Partial pressure of oxygen}

\( \text{Pr} \)  \hspace{1cm} \text{Prandtl number ( Pr = \frac{\nu}{\alpha} )}

\( P_{\text{ref}} \)  \hspace{1cm} \text{Pressure at reference conditions}

\( P_{\text{run}} \)  \hspace{1cm} \text{Pressure at run conditions}

\( Q \)  \hspace{1cm} \text{Mass flow rate}

\( \cdot q \)  \hspace{1cm} \text{energy generation within object}

\( q_w \)  \hspace{1cm} \text{heat flux (W/m\(^2\))}

\( R \)  \hspace{1cm} \text{Universal gas constant (8.315 J/mol·K), Eigenvector matrix}

\( \text{Re} \)  \hspace{1cm} \text{Reynolds number ( Re = \frac{\rho m D}{\mu} = \frac{V m D}{v} )}

\( S \)  \hspace{1cm} \text{Gas solubility}

\( s(T) \)  \hspace{1cm} \text{Solubility coefficient}
$s_i$  linear solubility coefficient based on Henry’s law  
$s_{nl}$  non-linear solubility coefficient based on Langmuir’s model  
$t$  time  
$T$  Temperature, Rotation matrix  
$T_g$  Glass transition temperature  
$u$  x-component velocity  
$U$  Uncertainty  
v  y-component velocity  
$V_m$  Mean velocity  
w  z-component velocity, width  
$W_A$  Wetted perimeter  
$w_d$  Channel width  
$Z$  Squared data matrix  
$\alpha(T)$  Coefficient of linear thermal expansion, affinity coefficient  
$\beta$  Gain  
$\delta$  Viscous boundary layer thickness  
$\delta_T$  Thermal boundary layer thickness  
$\varepsilon$  Error  
$\Im$  steady-state gas flux into polymer  
$\lambda$  Wavelength, Eigenvalues  
$\mu$  Dynamic viscosity  
$\nu$  Kinematic viscosity  
$\Phi$  quantum yield of luminescence, dissipation function in energy equation  
$\rho$  Density  
$\sigma$  Standard deviation  
$\sigma_{shot}$  Shot noise  
$\tau_{em}$  Emission lifetime of an excited molecule
Pressure sensitive paints (PSP) have been gaining more popularity in experimental fluid mechanics and aerodynamic testing. A major problem with PSP pertains to its temperature dependence making the calibration process very difficult. Dual-luminophor PSP has an added temperature phosphor to provide temperature information for the calibration. Dual-luminophor systems still suffer from the inherent temperature dependence and possess added complications due to spectral cross talk and overlap. However, to date there is no successful universal calibration technique for dual-luminophor systems. Such calibration is needed for dual-luminophor PSP to become a practical experimental tool.

With this aim, a statistical technique known as proper orthogonal decomposition (POD) is used to extract pressure and temperature information from intensity data. POD works by defining and separating the main factors of any system and evaluating the proportional contribution of each factor. The calibration technique is examined by
applying the dual-luminophor PSP in a channel flow experiment. Before applying the calibration experimentally, a set of artificial data is examined using POD to provide a fundamental understanding of POD as a calibration technique. The experiment was designed to allow for different flow conditions and temperature gradients to interact, hence providing enough variation to examine the calibration technique. Seven cases are examined, with each case shedding light on a particular aspect of the calibration. The POD calibration is compared to the intensity-ratio calibration in order to emphasize the effectiveness of the technique. Finally, results are evaluated for accuracy and a detailed uncertainty analysis is performed to fully assess the POD calibration.
CHAPTER 1
INTRODUCTION

This dissertation presents novel work in the characterization and calibration of dual-luminophor pressure sensitive paints using proper orthogonal decomposition (POD). The paint is additionally implemented in a canonical flow experiment, namely a laminar incompressible channel flow, to fully characterize the paint and calibration procedure under flow-based conditions. The paint utilized in this work is PtTFPP-Ru(phen)/PAN/Poly-tBS-co-TFEM, developed by Kose (2005) and collaboratively tested under static (no flow) conditions with the author. Pressure sensitive paints (PSP) offer a promising and more effective tool for characterizing and resolving pressure fields over test-model surfaces in comparison to traditional pressure measurement techniques. Traditional techniques can offer at best a discrete representation of the pressure field. In addition, there is a rise in cost and time of experimentation, as well as restrictions on access to the entire model for pressure sensor placement. In contrast, PSP technology is relatively inexpensive and can be easily applied to any surface in very thin layers (~ 10 µm).

A typical pressure sensitive paint is composed of an oxygen sensitive luminophor embedded in an oxygen permeable polymer, which is then applied directly on the surface or on top of a primer layer to enhance light reflection. Exciting the paint with the appropriate wavelength, usually long ultraviolet (UV) to blue wavelengths, energizes the paint electronically, which is then followed by a deactivation process through different paths. The path characterizing the paint’s dependence on pressure is the oxygen quenching path. Nonetheless, other paths of deactivation compete with the oxygen
quenching path, primarily radiation. Radiation is the molecule emission of light at higher wavelength as a means of deactivation. The oxygen quenching process is directly proportional to the partial pressure of oxygen in the surrounding medium, hence the pressure of air. Accordingly, the intensity of the light emitted by the paint is inversely proportional to the pressure. However, as the luminophor is embedded in an oxygen permeable medium (polymer), the oxygen sorption and diffusion characteristics of such medium would obviously affect the quenching process. Intuitively, temperature would appreciably alter the permeability of the polymer, thus the oxygen sorption and diffusion characteristics. In addition, on the molecular level there is an inherent temperature dependence related to thermal quenching. This defines a principal concern with PSP, which is an inherent temperature dependence that necessitates a temperature compensation procedure, in combination with a typical pressure/intensity calibration, to account for temperature variations while collecting pressure information. Compensating for temperature effects is rather challenging, and while various research efforts have attempted to resolve the issue with relative success, a robust and universal procedure is yet to be born.

Dual-luminophor PSP contains an added temperature sensor, which typically emits at sufficiently different wavelengths, usually lower, such that the temperature data can be collected simultaneously to provide both a temperature compensation technique as well as mapping the temperature distribution over the surface. However, this leads to a different problem, namely, “cross talk” between the two sensors, which makes it difficult to separate the independent factors, pressure and temperature, through spectral filtration. Cross talk is an adverse effect that stems from either one luminophor absorbing the
emission of the other, a spectral overlap between the two emissions, or both. This work examines and applies POD to resolve these problems. POD identifies and separates the main factors of any process and resolves the relative magnitude, and hence importance, of each factor; in addition, it serves as a noise rejection tool.

The second part of this work focuses on examining the dual-luminophor PSP technology under flow-based conditions. Static behavior of the dual-luminophor PSP has already been quantified by the author in collaboration with Kose (2005). Flow-based measurements using the dual-luminophor PSP would allow for a thorough evaluation of the paint and the analysis/calibration process. The paint is applied in a laminar channel flow experiment with an imposed temperature gradient varying in direction from longitudinal, transverse, and oblique relative to the flow direction. The channel flow experiment is chosen as it is a canonical flow with an existing analytical solution under laminar incompressible conditions. The details of the channel flow experiment including literature review and mathematical analysis are presented in Chapter 4 and Appendix-B.

**Literature Review**

**Preface**

The aerodynamic design of any model entails fundamental fluid mechanics experiments involving the characterization of the surface pressure distribution. These measurements are usually obtained to estimate the aerodynamic forces (e.g., lift and drag) through an integration process. More crucial is the identification and understanding of specific flow phenomena that profoundly affect the design criterion, such as boundary layer separation or shock wave impingement on the surface. Occasionally, such phenomena may affect the structural aspect of the design. Additionally, computational fluid dynamics (CFD) continue to evolve and expand in application, requiring full-field
experimental techniques capable of providing more comprehensive validation.

Typically, pressure measurements are performed using traditional techniques that implement pressure taps and transducers. These devices are located on discrete points over the surface of the model. This approach has several shortcomings. The first and most obvious is the lack of a full field mapping of the pressure field, which can lead to ignorance of the flow field behavior near the surface in critical areas of the model. Further, even with prior knowledge of specific locations on the model that are potentially critical, it is not always possible to practically install such devices in the model (e.g., sharp corners, thin edges). It is rather impractical for one to try to compensate for the lack of enough spatial resolution of traditional pressure measurement techniques by adding more taps as the task is quite expensive as well as time consuming, not to mention the physical implications on the structural integrity and the inevitable compromise of the flow characteristics over the surface. To provide the reader with an idea of the cost involved in model construction, a typical pressure-instrumented aircraft wind tunnel model can cost on the order of $500,000 to $1 million to construct (McLachlan 1995). These shortcomings led to aerodynamic testing being a fairly time consuming process that adversely added to the cost and time required for any design.

**Historical Perspective**

A paper published by Dickerson and Stedman (1979) initiated research efforts in the field of flow measurements using luminescent molecules as they utilized ozone to visualize flow patterns. Ozone has excellent flow tracing characteristics due to its physical properties. A fluorescent screen with a turbulent flow passing over it was illuminated with UV, and as the ozone-air mixture passed over the screen, the ozone absorbed some of the UV, hence casting a “shadow” that is directly proportional to the
ozone concentration in the flow and the speed of the flow. By placing an additional fluorescent screen perpendicular to the flow, they were able to create a three-dimensional image of the turbulent plumes. This was the earliest recorded attempt to quantify a flow using luminescent dyes. Ironically, the whole idea was based on a mere accident as they were trying to measure the rate of photolysis of ozone in the atmosphere when some ozone leaked between a mercury lamp and a black light poster, which then cast a shadow that looked like smoke plumes. As they scrambled to find the source of the “smoke,” Dickerson realized it was the shadow of an ozone plume!

The work of Dickerson and Stedman (1979) inspired a group at the National Institute of Health (Peterson and Fitzgerald, 1980), to pioneer the concept of surface pressure characterization using luminescent coatings in the West. Nonetheless, the originality of the oxygen quenched luminescent molecules technology was first established by the German scientists Kautsky and Hirsch (1935). Peterson and Fitzgerald, (1980) described an experiment in which a surface was covered with a fluorescent dye (fluorescent yellow dye adsorbed on silica particles) that was then excited by a blue light. The dye had photoluminescence characteristics that allowed it to respond inversely to surrounding oxygen concentration. As flow over the surface was initiated, either nitrogen or oxygen was injected into the flow through a static pressure tap on the surface. In the case of nitrogen injection, a bright streak of luminescence was detected on the downstream side of the tap, while the oxygen injection resulted in a dimmer illumination downstream of the tap. The nitrogen decreased the oxygen concentration downstream of the tap and hence increased the luminescence of the paint inversely to the oxygen concentration. Even though the PSP used in this experiment was not favorable to accurate
and practical experimental implementation due to low oxygen sensitivity of the dye and the oxygen permeability of the binder as well as other problems such as poor adhesion of the dye to the surface, it undoubtedly created the potential for the use of luminescent technology in aerodynamic testing.

Concurrently, Russian scientists produced the first practical pressure sensitive coating. A year later they obtained pressure measurements with the same coating (Ardasheva et al., 1985). Around the same time a group from the Central Aero-Hydrodynamic Institute (TsAGI) in the former Soviet Union developed a new polymer-based PSP, which they applied to a cone-cylinder model at supersonic mach numbers. Their results showed that the paint suffered from very significant temperature dependence, hence establishing one of the fundamental problems with PSP that is still unresolved to this date. The Russian advancements in PSP technology was finally revealed in the West through a commercial advertisement for an “Optical Pressure and Temperature Measurement System” in the 1990 February issue of Aviation Week & Space Technology.

Over the nineties PSP technology was commercialized in the aerospace industries. Companies like McDonnell Douglas, Boeing, British Aerospace, and government research institution such as NASA, Office National d’Etudes et de Recherches Aerospatiales (ONERA) in France, to name a few, have widely implemented PSP technology into the design and testing of their products (Bell et al., 2001).

**Photophysics of Luminescent Coatings**

Pressure sensitive paints offer a potentially promising substitution to current standard pressure measurement techniques in aerodynamic testing. The PSP method allows one to obtain not only qualitative pressure images, but also quantitative absolute
pressure values at the desired locations on the model, without introducing flow-disturbing probes or affecting the structure of the model surface (Engler et al., 2000). PSP can alter the nature of the surface roughness affecting flow transition. However, PSP can be applied in very smooth layers and with careful design of both the paint and paint application such effects can be minimized. Photochemically excited molecules are embedded in an oxygen permeable binder, and once excited with the appropriate wavelength they deactivate through different mechanisms, such as a process known as oxygen quenching, hence affecting the degree of luminosity. By means of photodetectors, such as a CCD camera or through a photomultiplier tube (PMT), the emission of these molecules can be recorded and translated to yield the true pressure field. Two measurement techniques are implemented in acquiring the intensity field, the first known as the intensity based method, the other known as the lifetime method. The details and comparison of both techniques can be found in the following references (Bell et al., 2001; Hardil et al., 2002; Liu and Sullivan, 2004; Zelelow et al., 2003). This work adapts the former technique and is discussed in detail in Chapter 2.

Although PSP technology has improved, it is still in its premature stages and many problems and concerns need to be addressed in order for the technology to gain a better acceptance and implementation in aerodynamic testing. More details about the history and development of the luminescent technology, chemical formulations, decay mechanisms, paint preparation, accuracy and calibration/measurement techniques and systems are presented in Chapter 2 and can be found in even greater detail in the following references: Bell et al., 2001; Engler et al., 2000; Liu et al., 1997; Liu and Sullivan, 2004 and McLachlan et al., 1995.
Pressure sensitive paint technology is based on photoluminescence, which includes both fluorescence (emission $\sim 10^{-8} - 10^{-6}$ s) and phosphorescence (delayed emission $\sim 10^{-3}$-100 s). Probe molecules are embedded in an oxygen permeable binder when excited by incident light they deactivate through different mechanisms, namely oxygen quenching, radiation and thermal quenching. The oxygen quenching mechanism can be generally modeled by the Stern-Volmer relation (Oglesby, 1995):

$$\frac{I}{I_0} = 1 + K_q P_{O_2}$$

(1.1)

In this relation $I$ is the luminescence, $I_0$ is the luminescence in the absence of oxygen, $P_{O_2}$ is the partial pressure of oxygen, and $K_q$ is the Stern-Volmer constant. The values $I_0$ and $K_q$ are both functions of temperature. Equation (1.1) indicates an inverse relation between the partial pressure/concentration of oxygen; hence the pressure, and the intensity. The constant $K_q$ is a measure of the sensitivity of the dye to pressure. For high values of $K_q$, high accuracy of the pressure measurement is obtained at low pressure levels, while low values of $K_q$ yield higher sensitivity at high pressure levels (Oglesby, 1995). Observing Figure 1-1 it is apparent that for higher $K_q$ values, as the pressure increases the intensity ratio $\frac{I}{I_0}$ gradient becomes smaller (i.e., less sensitivity). This could result in low signal-to-noise ratio, which is most certainly the case in low speed wind tunnel testing where pressure variations are relatively small and close to atmospheric conditions, which can be accommodated by using PSP with low $K_q$. Low speed wind tunnel testing with PSP is quite challenging, and more effort is needed to
identify probes with appropriate $K_q$ values, which could perhaps be tailored to specific test conditions (Morris et al. 1993).

Figure 1-1 Stern-Volmer model for a range of $K_q$

It is not typically practical to create a “reference” condition, $I_o$, by purging the oxygen in the test cell, hence a more practical approach is to use a known condition-say atmospheric-as the reference condition as shown below in equation (1.2). Equation (1.2) reiterates the temperature dependence in the intensity, as the slope $B(T)$ and the intercept $A(T)$ are both functions of temperature. The Stern-Volmer approximation does reasonably well under two conditions: first, the range of the test pressures is adequately sensible (limited), and second, the reference condition is near the pressures of interest. Note that in equation (1.2) the reference and run intensity are under isothermal condition
to cancel out the temperature dependence; however, this approach is intrinsically impractical.

\[
\frac{I_{\text{ref}} (T_{\text{run}})}{I_{\text{run}}} = A(T_{\text{run}}) + B(T_{\text{run}}) \frac{P_{\text{run}}}{P_{\text{ref}}} \tag{1.2}
\]

Photodegradation, an intrinsic concern with luminescent molecules, is the loss of intensity due to exposure of the sensor probe to light. Molecules are much more active in their excited state than in ground state and can react to certain compounds to which they are indifferent in ground state. Addition of such compounds to a sample containing luminophor causes luminescence lifetime and intensity to decrease, a process known as “quenching of luminescence” and thus such compounds are accordingly named “luminescence quenchers.” All luminophor quenching processes are to some extent irreparable and consequently intense and continuous illumination of luminophors degrades luminophor’s illumination (Vollan et al. 1991). This is the result of the singlet oxygen molecules produced through the oxygen quenching process, which are extremely reactive. They bond with nearby molecules forming either non-luminescent molecules or causing deformations in the polymer matrix that lower measured intensities. Recent advancements in luminescent molecules and polymer sciences have nearly eliminated photodegradation of luminophors (Kose, 2005).

Coating the surface of a model with paint can potentially alter the airflow characteristics over the model, making the paint intrusive. This intrusion can be in the form of actual physical alteration such as adding thickness to the surface of the model or an alteration to the effective shape of the model, e.g., altering the boundary layer development (Schairer et al., 1998). Physical alteration does not necessarily imply a change of the roughness of the surface; on the other hand, effective alteration is a direct
result of roughness change of the surface. For example, a rougher surface due to the paint would lead to premature transition of the boundary layer, or it could have the opposite effect if the paint forms a smoother contact-surface with the flow relative to the actual surface. Further, paint could affect readings from the calibration pressure taps by rounding square edges or forming a protuberance (disturbance) around the opening. Schairer et al. (1998) assert that the latter was not observed and if it was to be a concern then it would be detected as a consistent offset between the paint-on and paint-off pressure readings at every location and condition. The issue is further addressed by Vanhoutte et al. (2000), as they examined the effect of the paint under low speed, low Reynolds number and high subsonic conditions for a swept wing. Low speed examination showed that an increased surface roughness due to the paint yielded higher drag forces due to the prevention of the occurrence of a bursting laminar separation bubble. High subsonic speeds testing indicated the increased roughness induced significant drag increase, while increased thickness showed lesser, but considerable, drag increase. The reader interested in more of the details and mechanics of PSP intrusion is encouraged to read Vanhoutte et al. (2000) and Amer et al. (2001), as the subject is merely addressed here for completion.

**Statement of the Problem**

A principal concern with pressure sensitive paints is their inherent temperature dependence, an issue that remains unresolved in the literature. Other issues include: accuracy, feasible environmental range of application, time response, invasiveness and cost-effectiveness (Weiss, 2002). The accuracy of PSP measurements is affected by several factors that include model deformation, paint self-illumination, excitation source stability, image registration, and photobleaching. Nevertheless, temperature dependence
continues to be the primary source of error in PSP measurements as innovations in the
fields of CCD fabrication, digitizers and data processors have greatly reduced related
errors associated with measurements. Liu et al. (2001) investigated the different sources
of error in PSP measurements and concluded that temperature effects dominate PSP
uncertainty. Therefore, a calibration technique that is robust and accurate is essential for
PSP applications. Schanze et al. (1997) examined the different sources of the temperature
dependence in an attempt to pave the way for identifying the appropriate solutions. The
polymer they employed is a typical PSP formulation comprising a luminescent
polypyridine Ru (II) complex dispersed within a poly-dimethylsiloxane (PDMS) binder.
The dye is excited with blue light (\(\lambda_{ex} \approx 450\) nm) and luminesces strongly in the red
wavelengths (\(\lambda_{max} \approx 620\) nm). The formulation was tested over the 0-14 psi range and
yielded a Stern-Volmer quenching constant \(K_{SV}\) of 0.70 psi\(^{-1}\). Mathematically the
intensity, \(I_{em}\), and natural emission decay lifetime, \(\tau_{em}\), can be expressed as follows (Bell
et al., 2001).

\[
I_{em} = \frac{C k_r}{k_r + k_{nr} + k_q [O_2]} \quad (1.3)
\]

\[
\frac{1}{\tau_{em}} = k_r + k_{nr} + k_q [O_2] \quad (1.4)
\]

where \(C\) is a constant that linearly relates the intensity to the quantum efficiency;
\(k_r, k_{nr}\) and \(k_q\) are the radiative, non-radiative and oxygen quenching first-order rate
constants, respectively. The deactivation process yields light only through radiative
decay, while the other two decay mechanisms yield temperature.
As illustrated by Hager et al. (1975a, 1975b), the radiative decay is typically temperature independent. Further, Van Houten and Watts (1976) characterized the general temperature dependence of the non-radiative decay rate behavior with an Arrhenius function. They attempted to physically separate these different effects through the experiment to identify the main source of luminescence and its relative magnitude. They eliminated the oxygen quenching path by reducing the pressure to vacuum or simply deoxygenating the paint with an inert gas, i.e., argon. In such a case the ratio of intensity-lifetime is simply dependent on the nonradiative (temperature) decay rate $k_{nr}$.

Conversely, oxygen diffusivity in the binder, which is directly related to $k_q$, was the main
source of temperature dependence under oxygen rich conditions. Schanze et al. (1997) tested the dye embedded in two different mediums, namely ethanol and PDMS. They concluded that the intrinsic temperature dependence is not strongly influenced by the medium under vacuum or degassed conditions. However, in the presence of air the lifetime temperature dependence is substantial for a PSP sensor dispersed in PDMS, while it is almost non-existent when ethanol is the medium. This is because the oxygen quenching pathway is characterized by much lower activation energy for ethanol in contrast to PDMS, which has typical activation energies on the order of 11 kJ/mol. Activation energy is often used to denote the minimum energy needed for a specific chemical reaction to occur. Further, in the presence of oxygen, they found that oxygen quenching is the predominant decay mechanism (> 90% of the total decay). Finally, by comparing the lifetime emission of the polypyridine Ru(II)/PDMS in vacuum and in oxygen rich environment, they showed that oxygen quenching is the main factor influencing the temperature dependence due to the larger $\tau/\Delta\text{temp.}$ gradient, further suggesting that in the absence of oxygen the lifetime temperature dependence is solely due to the nonradiative temperature dependence. These results indicate that in order to minimize the temperature dependence, the medium (binder) used must possess the lowest possible activation energy for oxygen diffusivity.

**Research Efforts**

A paper by Puklin et al. (2000) examined a PSP probe, PtTFPP, embedded in copolymer of heptafluoro-$n$-butyl methacrylate and hexafluoroisopropyl methacrylate (FIB), possessing an activation energy of 1.5 kJ/mol, which satisfies the criterion set forth in the previous work by Schanze et al. (1997) for low temperature dependence in PSP
(i.e., low activation energy). They introduced the concept of an “ideal” paint, which they define as one for which the Stern-Volmer relation (intensity-ratio) under isothermal conditions for the run and reference condition is approximately independent of temperature, and the intensity ratio under isobaric conditions is approximately pressure independent. In a simpler sense, any paint can be described as ideal if all the Stern-Volmer plots for different temperatures collapse to a single curve under isothermal conditions. Hence, an ideal PSP can be more conveniently corrected for temperature dependency; however, this definition of ideality is applicable only for a limited temperature range (Puklin et al., 2000). Ideality provides the ease of determining the pressure value if both the temperature data and the intensity ratio under isobaric conditions are known, thus decoupling temperature data from any pressure dependency. Using a lifetime measurement approach under static conditions, they examined the dye for different pressures between vacuum and atmospheric pressure under three temperature conditions (10°C, 30°C, & 50°C). Their results showed -0.6%/°C temperature dependence in the PSP. In a follow-up paper, Gouin (2000b) investigated the feasibility of achieving an ideal PSP by annealing the polymer. As they noted, annealing the paint (PtTFPP) made the paint behave ideally with very small temperature dependence (-0.52%/°C) with an almost identical slope under vacuum and atmospheric pressure conditions. They also found that non-annealed samples exhibit higher temperature dependence with a varying slope between vacuum and atmospheric conditions (-0.54%/°C and -0.80%/°C respectively). The annealing process has to take place above the glass transition temperature, \( T_g \), of the polymer to achieve this ideal behavior. Heating a polymer network above \( T_g \) has the effect of relaxing the chains, loosening the
entanglements in the network, and allowing the system to achieve thermodynamic stability. This will ease oxygen diffusion through the polymer as the relaxation of the network implies fewer entanglements leading to easier diffusion and lower activation energy.

A third publication by the same group at the University of Washington (Gouin, 2000c) examined the effect of using a second base coat under the PtTFPP/FIB paint. Such combination typically results in an increase in the response time and the temperature dependence of the PSP. The accuracy of the PSP measurements depends on many factors, one of which is the uniformity of the model surface, as the light emitted by the PSP is not totally captured by the photodetector (CCD, PMT, etc.). A determinant of how much light is captured by the photodetector is the intrinsic reflectivity of the model surface. Reflectivity is defined as the ratio of reflected light intensity to incident light intensity at the surface of a material that is considerably thick such that the reflectance is independent of the thickness; thus commonly described as the intrinsic reflectance of the surface. Surface reflectance is Lambertian (diffuse), Phong (specular) or a combination of both. Ideal Lambertian surface apparent reflectance is independent of the observer; i.e., the object brightness is independent of the angle between the observer and the surface. This contrasts with glossy surfaces, as the apparent brightness is highest when the observing angle is equal to the source angle. Most real objects have some mixture of Lambertian and Phong qualities. As models typically incorporate different materials on the surface, ranging from the actual material of the model to wax and Bondo material, variation of the reflectivity of the surface induces significant error accordingly. For that reason, a uniform base, known as a base coat, is typically applied to the surface to create
a homogeneous canvas. A base coat is generally a polymer which is embedded with a white substrate (pigments), e.g., titanium dioxide, which possesses a high index of refraction and good dispersion in the polymer (high hiding power), but can sometimes photoxidize organic compounds when irradiated with UV (< 390nm). Unfortunately, base coats have been shown to influence PSP functionality. This is partially attributed to the change of the dynamic oxygen equilibrium of the PSP layer as it interacts mutually with the surrounding air as well as the base coat. The combination eventually reaches equilibrium after a certain response time. The equilibrium process is directly tied to the oxygen diffusion in the polymers. This effect is only imperative when performing high frequency or transient measurements, as response time is important. The more prominent effect is the apparent influence of the base coat on the temperature dependency of the PSP layer, which in turn can potentially yield adverse affects to the desirable temperature dependency characteristics, mentioned previously, of an ideal paint. They examined five different base coats: FIB, polymethylmethacrylate (PMMA), polyacrylonitrile (PAN), polyvinyl acetate (PVA) and a silicone (SR-900). Gouin et al., (2000c) found that oxygen diffusion in the polymer is the main factor influencing the temperature dependency as vacuum results were almost identical. Further, the oxygen concentration gradient that exists between the base coat and the PSP layer is the reason behind the direct influence of the base coat on the temperature dependency of the PSP, as different base coats would hold different concentrations. Comparing permeable polymers (FIB and silicon base coat) versus impermeable polymers (PMMA, PAN and PVA) showed significant increase in both response time and temperature dependency at atmospheric conditions for the latter polymers except for PAN; the results are tabulated below for convenience. Their results
assert the importance of choice of a specific base coat for each particular PSP.

Table 1-1 Response time and temperature dependency of PtTFPP/FIB with different base coatings

<table>
<thead>
<tr>
<th>Base-Coat Polymer</th>
<th>Response time (sec)</th>
<th>Temperature dependence@ atm. (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No base coat</td>
<td>0.6</td>
<td>-0.52%</td>
</tr>
<tr>
<td>FIB</td>
<td>0.8</td>
<td>-0.56%</td>
</tr>
<tr>
<td>PMMA</td>
<td>15</td>
<td>-1.06%</td>
</tr>
<tr>
<td>PVA</td>
<td>7</td>
<td>-0.75% (-2% above 35°C)</td>
</tr>
<tr>
<td>PAN</td>
<td>0.9</td>
<td>-0.7%</td>
</tr>
<tr>
<td>Silicone (SR-900) (PVC)</td>
<td>1.1</td>
<td>-0.69%</td>
</tr>
</tbody>
</table>

In Table 1-1 Pigment Volume Concentration (PVC) is defined as Gouin et al. (2000c).

\[
PVC = \frac{\left(\frac{\text{mass of pigment}}{\text{density of pigment}}\right)}{\left(\frac{\text{mass of pigment}}{\text{density of pigment}}\right) + \left(\frac{\text{mass of binder}}{\text{density of binder}}\right)}
\]  

Their last publication in the series, (Gouin, 2000d), demonstrates how non-ideal paints can be idealized through the addition of inorganic pigments. They show that the diffusion of oxygen is affected by the addition of pigments to the polymer and that the decay rate behavior of the paint is affected as well; however, not all the quenching rate constants were affected. They further noticed that a specific concentration of pigments made the paint act ideally. Using fine grade (1-5 µm) aluminum oxide (Al₂O₃) with pigment volume concentration of 31% they were able to closely match the temperature dependence of PtTFPP at vacuum, yielding an ideal paint. Again, it is important to note that such ideality is pertinent to limited temperature ranges and isothermal conditions.

Woodmansee et al. (1997) investigated the issue of temperature dependence as they examined three different PSPs and two TSPs formulations as well as four different temperature compensation methods. The paint was applied to a transverse-jet-in-
crossflow experiment. As the behavior of the specific paint is not of relative importance to this work and is well documented in their paper, reduction methods will be discussed herein and only related paint behavior if needed. They compared four different reduction methods: isothermal, in-situ, K-fit and temperature corrected pressure. The isothermal approach, as implied by its name, ignores temperature variations by assuming that the temperatures at run conditions are identical to the calibration run. The calibration coefficients are obtained through a static calibration of a specimen and then fitting the data in a least squares sense. This approach under-predicts the pressure as it ignores the temperature drop due to the operation of the tunnel. On average, the isothermal calibration had an 8.0%f.s. drop relative to the pressure taps measurements and 2.8%f.s. root mean square (rms) deviation. Nonetheless, the predicted pressures mapped a similar trend as the pressure taps, which makes such an approach good for qualitative purposes only. However, this approach may be useful quantitatively if the run images have a steady and identical temperature distribution, in which case the reference image must be acquired immediately after the termination of the each run of the experiments.

The second approach, in-situ, accounts for the temperature variation by placing pressure taps on the painted surface and using that pressure information to evaluate the calibration coefficients. However, pressure taps need to be positioned in predetermined locations to ensure the coverage of the entire domain of the pressure test range (i.e., lowest and highest pressure areas). To ensure this, a run test prior to installation of the pressure taps with the surface painted may be necessary to obtain a qualitative picture of the pressure field. Further, adequate numbers of pressure taps need to be positioned through the intermediate range to secure acceptable accuracy in the results. The overall
results for this approach showed that the *in-situ* method had the smallest error of all methods with 1.0%f.s. mean and 0.55%f.s. RMS differences.

A hybrid technique between the earlier techniques is the K-fit approach. It is typically implemented when the model lacks enough pressure taps to cover a wide enough pressure range. It relies on the fact that the pressure vs. intensity curves collapse if the run and reference temperatures are both identical (i.e., isothermal). This of course limits the experiments allowed by many constraints, such as air-flow speed, geometry of the model, steady conditions, etc. Nevertheless, as mentioned previously, if a reference (wind-off) image is acquired immediately after the run image, temperature variations should vanish in the normalization process, unless temperature gradients are severe and change significantly between the two runs. The K-fit approach empirically scales the reference image to emulate a reference image at a different temperature condition. This approach is better than the isothermal technique with a 4.7%f.s. mean and 2.7%f.s. RMS differences, yet it is still inferior to the *in-situ* calibration approach.

The last examined technique, temperature-corrected pressure calibration, utilizes a temperature sensitive paint (TSP) to calibrate the PSP measurements. The TSP is applied to the model after PSP experiments and then the two images are aligned to sub-pixel accuracy. Knowing the intensity ratio and the temperature at each pixel, a 2-D surface is constructed with the pressure as a function of both intensity ratio and temperature. This method eliminates the need for an *in-situ* pressure tap data. Unfortunately, the authors did not have great success with this approach due to several predicted problems such as photodegradation, shelf-life degradation and calibration surface effects. The authors conclude by recommending a multi-step *in-situ* calibration.
Hubner et al. (1997b) suggested a temperature compensation model for PSP that was rather successful. Their work was motivated by the lack of a generic model of luminescent intensity in terms of pressure and temperature. They based their model on a Stern-Volmer with an Arrhenius-type temperature dependency. In general, oxygen sorption in an ideal and homogenous liquid is directly related to the partial pressure of the oxygen as described by Henry’s law, assuming equilibrium between the oxygen within the binder and the oxygen above it:

\[ [O_2] = s(T) P_{O_2} \]  

where \( s(T) \) is the solubility coefficient. However, as described by Hubner et al. (1997a), over expanded pressure ranges the solubility coefficient depends on pressure as well,

\[ [O_2] = s(T, P_{O_2}) P_{O_2} \]  

This leads to the redefining of the intensity-pressure calibration from equation (1.2) to extend it to higher order polynomial expansion,

\[
\frac{I_{\text{ref}}(T_{\text{run}})}{I_{\text{run}}} = A(T) + B(T) \left( \frac{P_{\text{run}}}{P_{\text{ref}}} \right) + C(T) \left( \frac{P_{\text{run}}}{P_{\text{ref}}} \right)^2 + \ldots
\]  

(1.8)

For limited pressure ranges, a linear solubility model can be used to estimate the pressure dependence. Large pressure ranges may be conveniently modeled with higher order polynomials; however, polynomials do not always adequately model the intensity-pressure relationship. This is a result of the uneven dispersion of luminophors in binders, which leads to heterogeneous distribution of the luminophor within the binder causing molecular aggregation and consequently self-quenching of excited probes and multiple exponential decays (as opposed to single exponential decays for luminophors dissolved in fluid solvent). Hubner et al. (1997a) proposed a model based on sorption theory to
compensate for these effects. For a given isotherm, the sorption model expands Henry’s law by adding a non-linear term based on Langmuir’s model of penetrant immobilization,

\[
O_2 = s_l(T)P_{O_2} + s_n(T)\frac{\alpha(T)P_{O_2}}{1+\alpha(T)P_{O_2}}
\]  

(1.9)

where \(s_l\) is the linear solubility coefficient based on Henry’s law, \(s_n\) is the non-linear solubility coefficient based on Langmuir’s model and \(\alpha(T)\) is the affinity coefficient \((\text{absorption})/\text{desorption}\) of penetrants. Taking the limit of equation (1.9) as the partial pressure of oxygen goes to zero and infinity reveals a dual sorption (DS) model-equations

\[
\lim_{P_{O_2} \to 0} \frac{\partial [O_2]}{\partial P_{O_2}} = s_l + s_n\alpha(T)
\]  

(1.10)

\[
\lim_{P_{O_2} \to \infty} \frac{\partial [O_2]}{\partial P_{O_2}} = s_l
\]  

(1.11)

Application of the DS model to equation (1.8) yields the proposed model by Hubner et al. (1997a),

\[
\frac{I_{\text{ref}}}{I_{\text{run}}} = A + B\left(\frac{P}{P_{\text{ref}}}\right) + C\frac{D\left(P/P_{\text{ref}}\right)}{1+D\left(P/P_{\text{ref}}\right)}
\]  

(1.12)

The coefficients \((A, B, C \& D)\) in this model are functions of both temperature and reference conditions. The model was applied to two PSP coatings and their results showed the superiority of their model relative to a second and even a third order polynomial, with a pressure ratio root mean square error (rmse) of 0.0004 and 0.001 for in-range and off-range data, respectively (PSP-A), which was also comparable to their second dye (PSP-B). The concern with this model is the difficulty in decoupling the
temperature dependency of the sensitivity coefficients as a direct result of the added nonlinear pressure terms, Hubner et al. (1997b). This created the motivation behind the subsequent work of Hubner et al. (1997b), as they proposed yet another model based on a Stern-Volmer with an Arrhenius-type temperature dependency. This model, unlike the DS model, has coefficients that depend only on temperature formulated from the radiative, non-radiative and quenching decay processes of the luminescence. They used a quadratic to model the solubility expansion coefficient (i.e., PSP sorption). As they describe in their paper, the radiative decay is almost temperature independent and hence usually regarded as constant. On the other hand, the non-radiative decay is significantly temperature dependent and is characterized by an exponential decay process in addition to an offset that is temperature independent.

\[
k_{nr} = k_{nr,0} + D_{nr} e^{(-E_{nr}/RT)}
\]  

(1.13)

The oxygen quenching process is governed by the sorption and the diffusion of the oxygen in the binder matrix, with Arrhenius type decay approximation over small temperature ranges. This implies composite temperature dependence in Henry’s law, namely the product of the oxygen concentration in the binder, \([O_2]\), and the Stern-Volmer constant, \(k_q\), hence the total activation energy is the sum of the activation energies of each process. Earlier in this review the work of Schanze et al. (1997) modeled the oxygen process as a simple exponential mainly dependent on oxygen diffusivity and established that oxygen quenching accounts for more than 90% of the total decay of the luminescent probe they used. The final form of Hubner et al. (1997b)’s model is as follows:
\[
\frac{I_{\text{ref}}}{I} = A + B \left( \frac{P}{P_{\text{ref}}} \right) + C \left( \frac{P}{P_{\text{ref}}} \right)^2
\]  

(1.14)

where,

\[
A(T, T_{\text{ref}}, P_{\text{ref}}) = \frac{K_{r,o} + K_{nr,o} + D_w e^{\left( \frac{-E_{q,1}}{RT} \right)}}{\text{den}} \]

(1.15)

\[
B(T, T_{\text{ref}}, P_{\text{ref}}) = D_{q,1} e^{\left( \frac{-E_{q,1}}{RT} \right)} \frac{1}{\text{den}} \]

(1.16)

\[
C(T, T_{\text{ref}}, P_{\text{ref}}) = D_{q,2} e^{\left( \frac{-E_{q,2}}{RT} \right)} \frac{1}{\text{den}} \]

(1.17)

\[
den = k_{r,o} + k_{nr,o} + D_w e^{\left( \frac{-E_{q,1}}{RT_{\text{ref}}} \right)} + D_{q,1} e^{\left( \frac{-E_{q,1}}{RT_{\text{ref}}} \right)} P_{\text{ref}} + D_{q,2} e^{\left( \frac{-E_{q,2}}{RT_{\text{ref}}} \right)} P_{\text{ref}}^2 \]

(1.18)

The first coefficient, \( A \), represents the radiative and non radiative effects, while the second and third coefficients, \( B \) and \( C \), characterize the oxygen quenching process. The coefficients are clearly a function of the run as well as the reference temperatures. The activation energies, \( E_{q,1} \), \( E_{q,1} \), \& \( E_{q,2} \), the rate constants \( k \), and the factors premultiplying the exponential decay terms are to be determined by performing a standard emission-lifetime calibration. These coefficients are functions of the specific luminophor properties and binder characteristics. Nonetheless, as stated already, run temperature information is still needed to evaluate the coefficients. Plotting these coefficients as a function of the run temperature it was clear that the coefficient, \( A \), was relatively temperature independent, while the other coefficients are noticeably temperature dependent. The authors appeal to the need for “dual-luminophor” paint, a paint containing two probes that measure pressure and temperature simultaneously and independently, which would provide the temperature information needed for calibration. Their model predicted the pressure with
rmse of 0.011 and absolute pressure uncertainty of 0.2 psi. Such high uncertainties in the measurements may not be tolerable in many applications, moreover; their results were based on 70 isotherms, underlining the difficulty in obtaining temperature corrections for PSP measurements.

Given the various limitations and requirements for selecting a practical combination of an oxygen sensor and a suitable binder, another proposed solution included adding non-oxygen quenched temperature-dependent phosphor to the paint to map out the temperature field and correct for temperature dependence. Coyle and Gouterman (1999) attempted to correct for lifetime measurement using such an approach. They defined a criterion for ideal TSP that can be co-embedded with PSP in a dual-luminophor formulation. The TSP criterions are:

1. Excite in the same spectral region as the PSP
2. Emit at a sufficiently different spectral region
3. Exhibit strong temperature dependence
4. Exhibit no pressure dependence

The PSP sensor of their choice was platinum meso-tetra (pentafluorophenyl) porphyrin (PtTFPP), which emits around 650nm, as for the temperature sensor they utilized the phosphor La$_2$O$_2$S:Eu$^{3+}$, which emits strongly at 514nm (Struck, 1970), both embedded in a (FIB) polymer. Both sensors are excited in the UV range, 337nm and 392nm for La$_2$O$_2$S:Eu$^{3+}$ and PtTFPP respectively. The two dyes were prepared separately and the temperature dye was sprayed first and allowed to cure overnight and then the PtTFPP/FIB was sprayed on top of the existing film. Inspecting the emission spectra of both dyes separately one can observe that the emission of the La$_2$O$_2$S:Eu$^{3+}$ overlaps with the excitation and emission of the PtTFPP, which could potentially lead to spectral interference. Maintaining a constant pressure field across the sample, but varying the
pressure between each run (0-1atm.), the temperature emission was recorded and it was completely independent of the pressure between the range of 10-50ºC with a lifetime decay-to-temperature slope of 1.9%/°C and they were able to fit the emission to an Arrhenius function, as predicted by the theory. Their temperature calibration yielded a root-mean-square error of 1.11ºC for 20 different data points. PtTFPP showed a lifetime decay temperature dependence of -0.3%/°C under vacuum conditions. To calculate the pressure they used the following 2nd order relation:

\[
P = a\left(\frac{\tau_{\text{vac}}}{\tau}\right)^2 + b\left(\frac{\tau_{\text{vac}}}{\tau}\right) + c
\] (1.19)

Implementing the previous equation as the calibration function to determine the pressure and temperature for fifteen different environmental conditions, five pressures and three temperatures, their results yielded a maximum absolute error of 3.0ºC, with the error in the predicted temperature increasing with the increase of temperature. The corrected pressure measurements had a maximum error of 4% and an average error of about 1%, while the non-corrected data had a maximum error of 16%. The authors argue that the difference between the emissions of PtTFPP and the combined dual-paint is not due to the interference of the 615nm emission of the La2O2S:Eu3+ with the PtTFPP emission, as they have observed La2O2S:Eu3+ emission at 625nm on the order of 400 µs, while their observation of PtTFPP decay curves fit to a double exponential model did not have a 400 µs component, yet they fall short of offering an alternative explanation. Research at the University of Florida unquestionably indicates a change in the intensity emission behavior of the PSP sensor with the addition of the TSP sensor; however, as our results are based on intensity measurements compared to lifetime measurements approach
by Coyle and Gouterman (1999), the author desists from disputing their rationale.

In a paper presented by Ji et al. (2000) a temperature independent PSP based on a bichromophoric luminophor was developed and tested. In contrast to prior work that typically utilized metalloporphyrins or polypyridine rythenium (II) complexes as the luminescent probes in formulating PSP, they used supramolecular assemblies. Supramolecular assemblies are two or more luminescent chromophores that are chemically bonded together (Balzani et al., 1991), they often posses photophysical properties that are distinct from those of the isolated units. They used Ru-pyrene as the luminescent probe utilizing its very long-lived exited state (Simon et al., 1997). As illustrated by Hager et al. (1975a and 1975b), under vacuum conditions the radiative decay for PSP complexes is typically temperature independent. Further, Van Houten and Watts (1976) successfully characterized the general temperature dependence of the non-radiative decay rate behavior for Ruthenium complexes with an Arrhenius function. Under oxygen rich conditions, close to atmospheric levels, the radiative and thermal (non-radiative) decay rates contribute insignificantly towards the overall decay rate relative to the oxygen quenching. As described earlier in this review by Schanze et al. (1997) this implies that under these conditions, the temperature dependence of the paint is mainly characterized by the temperature dependence of the oxygen quenching decay rate (i.e., oxygen diffusivity in the binder), which is also commonly characterized by an Arrhenius type decay. Intermediate conditions between vacuum and atmosphere are characterized by a composite PSP temperature dependence behavior of the two previously mentioned states (i.e., vacuum and atmosphere). The probe’s excited state lifetime has moderate temperature dependence; nonetheless a nearly temperature-
independent Stern-Volmer calibration is coincidentally feasible under isothermal conditions between the run and reference images (i.e., ideal paint). The Stern-Volmer relation is expressed as:

\[
\frac{I_o (T_{\text{run}}, p = 0)}{I (T_{\text{run}}, p)} = 1 + K_q (T_{\text{run}}) p
\]  

(1.20)

Differentiating this expression with respect to temperature and equating it to zero would set the condition needed to achieve a non-temperature dependent Stern-Volmer constant \( K_q \). Through some mathematical manipulation and assuming that \( K_{nr} \gg K_r \) the condition yields \( \frac{E_d}{E_{nr}} \approx 1 \), where \( E \) is the activation energy and subscripts \( d \) and \( nr \) refer to oxygen diffusion and non-radiative decay, respectively. In a simpler sense, if the temperature dependence of the oxygen diffusivity in the binder and the temperature dependence of the non-radiative decay are nearly identical (i.e., slope), then the paint can be described as an ideal paint.

They attempted to incorporate the sensor in PDMS as a binder, but the sensor had a poor emission, consequently they synthesized MPP acrylate polymer binder, which is relatively polar, hence able to dissolve polar transition metal salts contrary to PDMS. Subsequent to establishing the calibration curves and plotting the results, the Stern-Volmer plots, for temperatures of (25-55ºC) over a pressure range of 0.005 atm to 1 atm, coincided almost exactly, hence indicating that the Ru-pyrene/MPP Stern-Volmer calibration is temperature independent over the specified range of conditions. They reiterated the same examination using a different sensor, namely PtTFPP, with the same binder MPP and the results showed that the temperature-independence Stern-Volmer behavior is unique to the Ru-pyrene/MPP system. Finally, they examined the lifetime
emission (luminescent decay rate) to determine the activation energies for each sensor under close to vacuum conditions and at 1 atm in order to separate the oxygen quenching and non-radiative decay channels. The Ru-pyrene/MPP system had an equivalent decay rate, which is synonymous to equivalent activation energies, thus verifying their temperature-independence criterion stated in advance. On the other hand the PtTFPP/MPP system had different activation energies. This work showed that ideal paint formulations greatly hinge on the appropriate selection and matching of both the luminophor and the polymer.

Another approach suggested in the literature is adding an environmental-conditions-independent sensor that is embedded within the formulation of a dual-luminophor PSP to provide an internal reference, thus replacing the wind-off image, and using an intensity based approach to resolve the pressure and temperature. The feasibility of such a concept was investigated by Subramanian et al. (2000 and 2001) where they implemented a non-pressure sensitive paint (NPSP) sensor to utilize as a reference sensor. As the paper points out, it is nearly always the case that when two sensors are mixed together in the same paint matrix, spectral interference is inevitable. The sensor emitting in the lower part of the spectrum could influence the excitation and thus the emission of the other sensor emitting at higher wavelengths. This interference is very hard to resolve and could lead to more error in the data in addition to the already existing temperature dependence error. Theoretically, any added sensors to the PSP sensor should be excited in the same spectral domain and emit at the same wavelength, in order to eliminate multiple excitation sources and the need for a filter wheel on the camera, hence limiting the available choices of sensors. This can be accomplished by means of one of
the following two methodologies. The same pressure sensor can be utilized except that it
would be embedded in an oxygen impermeable binder. This approach is impractical due
to the infeasibility of creating a completely oxygen impermeable binder, knowing that
any oxygen diffusion in the binder would certainly lead to serious errors. The second
approach is to utilize a sensor that is not quenched by oxygen and is temperature
insensitive. This necessitates that such a sensor emits at a different wavelength than any
other sensor embedded in the same paint matrix in order for it to be entirely
distinguishable from any other emission in the spectrum. The emission from this sensor
would then serve as the reference for each individual run and thus eliminating wind-off
referencing. Further advantages to such an approach include: the potential of using the
NPSP for correcting temperature dependence if it was possible to match the temperature
effects of both PSP and NPSP, and the NPSP can be used as a target-marker for model
deformation determination during wind-tunnel testing.

Subramanian et al. (2001) avoided mixing the two sensors together, instead they
applied the paints separately on different parts of their test specimen (disk) with a dark
ring separating both dyes, conceivably to avoid spectral interference; nonetheless, such a
compromise may not be always tolerable. They used six different combinations of PSPs,
PSP binders, NPSPs and NPSP binders. The environmental range in which they
conducted their experiments ranged between 0.001-2.7 atm and from 15-35°C. Their
results conclude that at constant temperature the NPSP was reasonably invariable with
the change of pressure and that the PSP had an order of magnitude higher rate of change
with pressure. However, upon imposing a temperature gradient on the specimen, there
was an obvious variation in the calibration curves, due to the inhomogeneity of the
temperature dependence between PSP and NPSP sensors. The PSP showed temperature dependency even under isothermal conditions for the run and reference intensities. This fact forced them to concede the infeasibility of their approach to correct for temperature dependence. They further investigated whether a spectral cross-talk exists between the two luminophors by eliminating the dark ring and it was not evident in some formulations, but it was significant in other formulations; nevertheless it was minimized when a dark ring separated the two luminophors. They observed spectral leakage when the NPSP and the PSP had the same luminescent probe embedded in a different binder, non-oxygen and oxygen permeable binder, respectively. However, they affirmed that none of the binders were absolutely oxygen impermeable.

The department of chemistry at the University of Washington, Khalil et al. (2004), has collaborated with Innovative Scientific Solution Inc. (ISSI) and presented a paper in which they did similar work to that of Subramanian et al. (2001). Khalil et al. (2004) point out in their paper that internal reference methods avoid several problems that arise with intensity measurements such as light source changes, index of refraction, optical geometry variations and fluctuations in film thickness and dye concentration. They based their experiment on a technique called the yardstick, which is in essence similar to having two sensors, one of which is affected by a specific factor, e.g. pressure, while the other is independent of that factor. The latter then serves as a reference for corrections. For that they experimented with a near infrared porphyrin sensor, namely platinum tetra (pentafluorophenyl) porpholactone PtTFPL. The sensor emits around a distant 733nm, allowing for another reference dye to be added without the concern of spectral interference. The reference dye of their choice was magnesium tetra (pentafluorophenyl)
porphine (MgTFPP), which luminesces at 650nm. The two sensors were dispersed in a FIB polymer, taking advantage of its relatively low temperature dependence effects and its consistent temperature dependence at vacuum and atmospheric conditions. In their experimental setup, both dyes are excited at 460nm ± 10nm, with two CCD cameras equipped with the appropriate filters capturing the intensity emission. Their environmental operational range spanned temperatures between 5 and 50ºC (the ideal range in which the PSP/FIB behave ideally) and a pressure ranging from 1 to 21 psi. Their results show that the pressure sensor and the reference dye exhibit temperature dependency. Nonetheless, by taking the ratio of the reference sensor to the pressure sensor they were able to reduce the temperature dependency to -0.1%/°C, which is a significant reduction from 0.6%/°C for the PtTFPP embedded alone in the FIB polymer. As they concede in their work, these results would be close to unattainable if there was any spectral interference between the two sensors, thus limiting the pool of sensor/reference/binder candidates available. Furthermore, these results are limited to a specific temperature range which imposes further limitations on testing conditions. In a subsequent work, Zelelow (2003), they replaced the reference sensor with a temperature sensor, namely tri (β-diketonate) phenanthroline europium (Eu chelate complex), which is virtually an oxygen independent sensor that emits around 615 nm. The results closely matched their previous work with the temperature sensor providing the potential to resolve temperature gradients as small as 0.001ºC. However, the luminescence intensity temperature coefficient for PtTFPL increased to 0.33%/°C. It is further noted the cost of the material used in synthesizing both of their coatings is rather expensive relative to
other available systems.\textsuperscript{1} They applied their formulation to an automotive model, Gouterman et al. (2004), and their results yielded an rms of 0.06 psi for the in-situ calibration. As the TSP has -0.25%/°C temperature sensitivity, they were able to reduce the temperature sensitivity of the PSP from -0.32%/°C to -0.07%/°C. However, they assess that the corresponding 33% percent improvement in temperature correction (reduction in rms) is below expectation for such reduction in the temperature sensitivity. They did not provide actual quantitative estimates of their error, nonetheless, their $C_p$ figure shows certain discrepancies between predicted and actual measurements. Their formulation suffered from considerable photodegradation (0.5%/hour at 1 bar and 40°C). Their approach shows potential for dual-luminophor PSP to resolve the temperature dependency issue, and a more robust data reduction and analysis technique would optimize results.

Similar work presented by Mitsuo et al. (2003) utilizes a temperature sensitive dye for temperature compensation in dual-luminophor PSP. The temperature probe of their choice was Rhodamine B (RhB), which absorbs in the UV range and has a broad emission starting at 550 nm and extending to 750 nm, with peak emission around 580 nm. The PtTFPP peak absorbance overlaps with the tail emission of RhB, which couples the PtTFPP emission with the RhB emission. This phenomenon is known as cross talk, which makes decoupling pressure and temperature information from the spectral emission fairly complicated. They modeled the temperature by a fourth order polynomial and the pressure by a third order polynomial. By first compensating for the temperature effects in the emission intensity of the PtTFPP using the wind on temperature information

\textsuperscript{1} Personal communication with Kose
and information from the PtTFPP calibration curves, they estimated pressure calibration coefficients that are temperature independent. The pressure standard deviation was around 6‰f.s., while the temperature estimates incurred a 1.5 K error. Once more, resolving spectral interference using simple calibration procedure has proven to be insufficient for accurate results, and the need for a more profound mathematical approach is inevitable.

Compensating for the temperature sensitivity in PSP using a co-embedded TSP falls under two categories, compensation using absolute temperature measurements, or by matching the temperature sensitivity of the PSP (Goss et al., 2005). The later is obviously more convenient; however, it can only be applied to ideal paint measurements as ideality indicates that the paint possesses constant temperature sensitivity. Non-ideal paints have variable temperature sensitivity; hence absolute temperature information is needed for practical measurements. PtTFPP embedded in FIB, as mentioned previously, is classified as an ideal paint, thus most functional dual-luminophor systems employ it as the pressure probe.

A question that is recurrently raised when using luminescent coatings is whether to use intensity based systems or a lifetime decay approach. Hardil et al. (2002) asserts that intensity based systems are quite complicated due to the added excitation sources, wind-off image and filtering system. They offer an alternative approach in which they have formulated a new oxygen-permeable sol-gel-based paint, containing both temperature and pressure luminophors. The fluorescence decay times of the two luminophors are separated by several orders of magnitude, which allows for the luminescent decay measurement to be separated in the time domain. In addition the two luminophors have
been chosen such that they have similar excitation and emission spectral regions to avoid multiple filtering regions and multiple excitation sources; hence image registration issues are completely avoided. Gouterman et al. (1997) explains the difference between the life time approach, adapted by Hardil et al. (2002) and intensity based approach, more commonly implemented and is the adapted technique in this work. The lifetime is an intrinsic property of the luminophor, which, unlike intensity, is virtually independent of external perturbations; hence the requirement for a wind-off image is eliminated. Nonetheless, lifetime is still dependent on temperature and hence a correction is still required. Gartenburg et al. (1991), attempted to solve the problem by implementing an additional infrared camera to provide a temperature correction for the pressure. Others (Alaruri et al., 1999 and Coyle and Gouterman, 1999) added an additional temperature sensitive luminophor to provide the temperature data for correction, which still required added cameras, excitation sources and filters. Hardil et al. (2002) approach follows previous work done by Bedwell et al. (1998) with the exception that they implement an array of LEDs rather than a laser for excitation. In addition, the work is specific for sol-gel-paints, [Ru (dpp) 3]2+. It is worth mentioning that the intensity ratio, defined in equations (1.1) and (1.20), is equal to the lifetime ratio, except that the absolute value of the reference and run lifetimes are independent of external perturbations. Hardil et al. (2002) lists three conditions for an ideal temperature luminophor:

1. It has the same excitation wavelength as the PSP luminophor
2. It is completely insensitive to oxygen
3. Has a significantly different decay time compared to the PSP luminophor
Most two and three-component systems that have been developed satisfy the first two conditions, while the third condition is a disputed topic. The question then is how do the two measurement systems compare to each other? Unlike the intensity based method, lifetime approach requires that both the excitation source and CCD camera be triggered with a delay shift between them, otherwise the CCD camera may incorporate extraneous light from the excitation source. This introduces complications into the control system, especially when decay times could be on the order of nanoseconds, and error could be easily introduced in the system if extraneous light is leaked into the measurement, which could be hard to detect as well. Not to mention the degree of sophistication required in the CCD camera in order to be able to handle such short exposure times. Furthermore, for each condition, different exposure time for different triggering cycles, usually on the order of 300 cycles, is required and need to be integrated in the same image frame. Other complications rise from the fact that in such a process, it is always assumed that the luminescence from the excitation source is absolutely constant and that the PSP luminophor decays to zero during the excitation sequence. The latter is not an issue for the intensity based system, while the effect of the drifting of the excitation source is not as nearly as significant compared to the lifetime system. As reported by Hardil et al. (2002), due to this drifting effect, their SNR was low enough to induce a temperature error of ± 3°C.

**Proper Orthogonal Decomposition (POD)**

POD, also known as the Karhunen-Loève expansion after Kari Karhunen and Michel Loève, is a classical statistical technique that is often implemented to simplify/decompose a dataset. First introduced by Kosambi (1943), POD is a method of identifying patterns in data, and expressing the data in a certain least squares optimal
sense. Data patterns are often hard to identify, especially in dataset of high dimensionality, where graphical representation is unfeasible. It makes possible the reduction of multidimensional systems to lower-dimensional approximate descriptions (Chatterjee, 2000). Implementations of POD are mainly in dynamic testing, image processing, signal processing, and general data compression. Practical applications include identifying coherent structures in turbulence, vibration analysis, astronomy, combustion analysis, and chemical substance identification, to name a few.

POD entails a mathematical procedure that transforms a number of potentially correlated variables into a smaller number of uncorrelated variables known as principal components. The first principal component forms an axis that accounts for the largest variance in the data, and each succeeding component accounts for as much of the remaining variability as possible. From a purely mathematical perspective, it is a linear transformation that chooses a new coordinate system for the data set such that the greatest variance by any projection of the data set comes to lie on the first axis, the second greatest variance on the second axis, and so on. In simpler terms, it provides a spatial basis for the modal decomposition of a system of functions. It enables the extraction of basis functions commonly called eigenfunctions. Quintessentially, it is an efficient procedure that captures dominant components of multidimensional systems and then reconstructing the system to the desired precision by using the relevant set of modes, effectively reducing the order of the system. POD entails two main objectives: the reduction, sometimes even characterization of the dimensionality of the dataset and recognition of fundamental factors (e.g. coherent structures in turbulence, exhaust gaseous compositions, etc.).
A key advantage of POD is its ability to generate an “abstract” modal decomposition that is data dependent without any prior understanding of the process yielding the data. This enables the extraction of the principal factors where a priori knowledge of the driving potential is insufficient to guide the basis function selection (Rathinam and Petzold, 1993). Moreover, unlike traditional linear transforms, POD does not have a fixed set of basis vectors; rather its basis vectors depend on the data set. This means that POD can be used for reducing dimensionality of the dataset without altering the key characteristics of the dataset that contribute most to its variance by eliminating the relatively less significant components.

POD is often confused with one of its sub-processes, namely Principal Component Analysis (PCA), which identifies the process by which raw eigenfunctions are established. The details are provided in Chapter 3, nonetheless it is sufficient at this point to note that POD involves more than the basic extraction of the modal factors, it further works to transform this abstract solution to physically significant solution; however, this transformation then involves an intuition or “feel” for the expected reduced modes.

A paper published by Carroll et al. (1999) explored the idea of using POD to separate the pressure and temperature information as well as for temperature compensation of the pressure calibration functions of measurements from dual-luminophor coatings. As stated previously, two main problems emerge with dual-luminophor systems: spectral interference between the two probes and cross talk between the two probes (i.e., emission overlap and absorption of emission of the lower wavelength probe by the higher wavelength probe, respectively). As described previously, any spectrum contains different variations embedded into it; nonetheless, one should be able
to account for a finite number of independent parameters that constitute the spectrum (Carroll et al., 2002). These independent parameters would vary in the magnitude and range at which they contribute to the spectrum, hence allowing us to extract the main parameters that form the spectrum and easily separating each parameter and identifying it with a physical parameter that we are interested in. Once these components are obtained a new spectrum is constructed which contains the direct contribution of such parameters of interest omitting extra factors that may well be of no interest or inherently unwanted contributions such as noise. The process simply hinges upon finding the eigenvectors and eigenvalues of the data matrix. Once these are established, the eigenvalues are compared and the largest $n$ number of eigenvalues and their corresponding eigenvectors are used only to reconstruct the spectrum. The number of eigenvectors selected depends on two aspects: how many factors one expects to be the main contributors and the relative expected ratio between the smallest eigenvalue, the factor with the least contribution and effect, to the largest eigenvalue. These criterions depend on the application and the degree of accuracy desired in the reproduced spectrum. A thorough description of the mathematical analysis is presented in Chapter 3 and the reader is encouraged to read through Malinowski et al. (2002) for a full and comprehensive mathematical derivation and explanation of POD.

Carroll et al. (1999) selected two luminophors such that one is purely temperature dependent while the other is both pressure and temperature dependent. The full spectrum was collected through a spectrophotometer for constant temperature and varying pressure and vice versa, to ensure the behavior of each dye. They analyzed the full spectrum using the POD and yielded the need for three factors to accurately reconstruct the raw spectra.
and to compensate for mutual interactions between the two luminophors. They managed to restrain the average percent error in the retrieved environmental conditions to within 2%. However, the work did not perform the analysis on a discrete set of data (using CCD camera), and they performed basic orthogonal rotation. The work presented initial characterization of POD, however, it stopped short of fully evaluating POD application to dual-luminophor paints by applying it to various coatings with different compositions and implementing a real case experiment where more involved data are obtained. In a subsequent paper Carroll et al. (2001), reiterated their application of the POD analysis to the same dye (Ruphen/Coumarin-7) they experimented with before and a new dye/polymer coating, namely PtTFPP/Diethyloxadicarbocyanine iodide (DOCI). They concluded that unlike the Ru system, the PtTFPP/DOCI data can be adequately represented with only two factors. However, their calibration curves for the DOCI showed some temperature dependence and they limited their calibration functions to a one-dimensional function. Further, they did not explore non-orthogonal rotation or surface rotation. Their pressure calibration yielded a percentage error between 0.32% and 1.3%. Nonetheless, they established the potential for POD to serve as data reduction technique for dual-luminophor PSP paints.

Kose (2005) developed a dual-luminophor system (PtTFPP-Ru(phen)/PAN/Poly-tBS-co-TFEM) which was successful in providing accurate pressure and temperature information implementing POD to resolve the data. Mathematical characterization and implementation of the POD was developed by the author and the calibration was collaboratively performed yielding very low error estimates of pressure and temperature. The main objective of the work was to formulate a successful dual-luminophor system
and to perform static testing to assess potential success in wind tunnel testing.

**Motivation and Contribution**

Pressure sensitive paints have been gaining acceptance in the practical market of aerodynamic testing; however, the technology is still short of being easily and universally applicable due to several difficulties that require resolution for the technology to reach its full potential. The full scale and practical implementation of dual-luminophor PSP technology is hindered to a great extent by the inherent temperature dependence of the paint. Aerodynamic testing can involve unavoidable and considerable temperature variations over the surface of the model and successful PSP implementation hinges on eliminating temperature effects that induces error in pressure field measurements. Various efforts have been addressing the issue in pursuit of satisfactory resolution including: PSP coatings with low temperature dependence (Gouterman et al., 1997), various mathematical error reduction and calibration models (e.g. isothermal and K-fit) (Hubner et al., 1997a; McLachlan et al., 1993; Woodmansee et al., 1997), simultaneous TSP imaging to provide a pixel-by-pixel temperature compensation (Woodmansee et al., 1997), adding an environmentally independent sensor to replace wind-off reference (Subramanian et al., 2000 and 2001) and most promisingly a TSP luminophor that is co-embedded in the same binder with the PSP in a dual-luminophor system.(Coyle and Gouterman, 1999; Khalil et al., 2004; Mitsuo et al., 2003 ; Zelelow et al., 2003).
<table>
<thead>
<tr>
<th>Calibration technique</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
</table>
| *In situ* isothermal calibration: Acquire wind-off images at atmospheric conditions | • The simplest approach  
• Suitable for isothermal conditions | • Temperature effects can not be accounted for  
• Calibration R.M.S. error on the order of 1.25 psi for PtTFPP/FIB (Bell et al, 2001), with pressure range of 29 – 72.5psi.  
• Calibration r.m.s. error on the order of 0.2 psi for ODU PSP (Woodmansee et al., 1998), with pressure range of 1 – 15.4psi. |
| *In situ* calibration: Acquire wind-off images immediately after wind-on conditions | • Easy technique  
• Temperature effects absorbed in calibration coefficients | • Need to ensure temperature stability  
• Wind tunnel must be turned off  
• Limitation on number of images that can be acquired for reference condition, especially for long exposure times, due to temperature drift from wind-on conditions.  
• Significant temperature variation between different parts of model require local calibration and pressure taps |
| *A priori* calibration | • Few pressure taps scattered over the model are needed  
• More controlled environment for calibration  
• More efficient for practical applications (less time spent in the wind tunnel) | • Separate experiment for calibration in required  
• Need to have a prior knowledge of expected pressure and temperature levels in the experiment to optimize the calibration process  
• Pressure taps need to encompass the pressure range, and hence any sharp pressure gradients could be unobservable  
• Temperature information must be obtained on the model using TSP (symmetric models with symmetric flow conditions). Asymmetric models or flow conditions can be mapped for temperature using dual-luminophor systems  
• Same batch of paint must be used for both calibration and experiment  
• Calibration functions are typically biquadratic in pressure and |
<table>
<thead>
<tr>
<th>Calibration technique</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>temperature (Bell et al., 2001), necessitating a minimum of 9 points of calibration points with both pressure and temperature information</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calibration r.m.s. error on the order of 0.021psi for OPTROD PSP (Bell et al., 2001), with pressure range of 0.75 - 14.7psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ideal paints: Hybrid technique (K-fit calibration)</td>
<td>Simpler than \textit{a priori} calibration (only one temperature condition is calibrated for \textit{a priori})</td>
<td>Inferior to \textit{in-situ} calibration</td>
</tr>
<tr>
<td>Useful for ideal paints when extrapolation beyond pressure taps range is needed</td>
<td>In some experiments, the $K$ factor is pressure dependent, yielding a more complicated calibration</td>
<td></td>
</tr>
<tr>
<td>Superior to \textit{in-situ} isothermal calibration</td>
<td>Calibration r.m.s. error on the order of 0.99 psi for ODU PSP (Woodmansee et al., 1998), with pressure range of 1 – 15.4psi.</td>
<td></td>
</tr>
<tr>
<td>Separate temperature measurement using IR camera</td>
<td>Non-intrusive</td>
<td>Ineffective for Mach number less than ~ 0.75 (Kammeyer et al., 2002)</td>
</tr>
<tr>
<td>Reasonably accurate (standard uncertainty of 0.202 psi) given high temperature gradients</td>
<td>Multiple cameras required to cover all surfaces</td>
<td>Increased processing time</td>
</tr>
<tr>
<td>Separate temperature measurement using TSP</td>
<td>Calibration procedure is simple</td>
<td>Added uncertainty due to the IR camera system</td>
</tr>
<tr>
<td>Only two filters are needed</td>
<td>Limited to symmetric models and conditions</td>
<td></td>
</tr>
<tr>
<td>\textit{In situ} calibration, hence less cost</td>
<td>Thickness and roughness of PSP and TSP are difficult to match, hence surface flow conditions are typically different.</td>
<td></td>
</tr>
<tr>
<td>No chemical interaction between P/TSP and no spectral interference</td>
<td>PSP uncertainty limited by TSP uncertainty</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Thermocouples needed for TSP calibration</td>
<td></td>
</tr>
</tbody>
</table>
Dual-luminophor systems have suffered from spectral interference and cross-talk problem that yielded high errors in measurements and difficulties in the calibration process. POD technique, first implemented to dual-luminophor systems by Carroll et al. (1999), showed the potential for resolving such issues mathematically. Nonetheless, full characterization and application of POD to paint systems is yet to be carried out that would present a better understanding of the mathematical process and relate it to the physical properties of the paints, hence allowing for optimal optimization and resolution of the data. Furthermore, a real-case test application of the paint would serve to validate both the paint formulation and the analysis process and to further address the various issues associated with acquiring measurement with dual-luminophor paints under flow-based conditions.

A successful dual luminophor system has been developed by the Schanze group at the University of Florida (Kose, 2005), using the formulation PtTFPP-Ru(phen)/PAN/Poly-tBS-co-TFEM. The paint has been calibrated and tested in collaboration with the author under static condition (vacuum chamber), and POD method has been applied to correct for temperature effectively. POD was capable of separating the pressure and temperature information with pressure and temperature 95% confidence error estimates of 0.1 psi and 0.4 K, respectively. Full characterization and application of POD to paint systems is essential in order to completely understand the mathematical process and relate it to the physical properties of the paints, hence allowing for optimal optimization and resolution of the data. Most importantly, full assessment of POD as a calibration technique is needed and a comprehensive analysis including uncertainty estimates would establish the feasibility of POD a calibration technique.
A channel flow experiment is utilized to evaluate the paint. The channel flow experiment provides a well controlled environment with existing analytical solutions/approximations. A temperature gradient will be imposed both along and/or across flow direction to simulate several cases of boundary conditions. The overall accuracy and uncertainty analysis will be assessed to evaluate POD as a calibration technique. Further, other calibration techniques are discussed and compared, when possible, to examine the effectiveness of POD relative to existing calibration techniques.
CHAPTER 2
LUMINESCENT COATINGS

This chapter presents a fundamental description of pressure and temperature sensitive paints and the process of acquiring measurements and reducing and processing intensity data. The information presented herein is intended for readers acquainted with the subject. More detailed discussion and information is provided in Appendix-A.

Overview

Resolving the pressure and temperature fields is a typical requisite in nearly all experimentation in the field of fluid mechanics and aerodynamic testing. Surface pressure information is imperative in characterizing various flow phenomena from boundary layer separation/reattachment and shear layers to shock wave impingement on surfaces. It is further employed in computational fluid dynamics (CFD) validation and perhaps more commonly in calculating the various aerodynamic loads such as lift, drag, wing torsion, etc. Temperature measurement is vital in supersonic and hypersonic flows and in flows involving significant heat transfer. Typically, holes are drilled in the surface to allow for pressure taps and thermocouples in order to measure pressure and temperature, respectively. Such practice could compromise the structure of the model, the flow field, the accuracy of the measurements or the overall practicality of the experiment. Furthermore, the resolved field is only a discrete representation of the full field true distribution and hence an interpolation, or yet worse an extrapolation, procedure is required to resolve the full field leading to added inaccuracies in the data.
The Ozone Accident

While working on an experiment intended to measure the rate of photolysis of ozone in the atmosphere at the University of Michigan chemistry laboratory R.R. Dickerson stumbled on an idea that was destined to inspire a new field for flow measurement (Dickerson and Stedman, 1979). As they were in the midst of running the experiment, some ozone seeped out and streamed over a black light poster (UV sensitive fluorescent screen) that was illuminated by mercury lamp (i.e., UV radiation). The streaming ozone induced an illusion of smoke plumes, which left the poor chemists scrambling for the source of the fire puffing the smoke. A short-lived thrill is broken by the realization of Dickerson that the plumes are simply the “shadow” of the ozone over the black poster. The ozone absorbed some of the UV radiation as it passed over the black poster and hence lesser light intensity illuminated the black poster, thus lowering its emission intensity. At the time, helium bubble flow visualization techniques seemed fascinating, and smoke and density change techniques were nothing short of a scientific marvel. Compared to these techniques, ozone was superior in many aspects, from its excellent tracing properties to its relative inexpensiveness to its character as a non-intrusive technique. There is just only problem with ozone; it is totally invisible to the human eye, which might raise some eyebrows of skeptical experimental aerodynamicist. Dickerson’s observation surely provided a resolution to the issue, but more importantly, it inspired two biomedical engineers at the National Institutes of Health, Maryland to develop a new flow visualization technique based on oxygen quenching of fluorescence (Peterson and Fitzgerald, 1980).

John Peterson and Raphael Fitzgerald founded the field of flow measurement
techniques using luminescent coatings. The diminution of phosphorescence of luminescent dyes in the presence of oxygen was first discovered by Kautsky and Hirsch (1935). Luminescent molecules that are strongly quenched by oxygen are typically simple aromatic compounds and are easily exited in the longer wavelengths of UV range. The concern is their emission range and intensity. Luminescence in the visible range of the spectrum is desired for practicality and well distinguishable emission from excitation is required for accurate measurements. Peterson & Fitzgerald used Fluorescent Yellow dye adsorbed on silica particles for their measurements, which has an excitation peak at 466 nm and emits strongly at 519 nm. Their technique measured flow patterns qualitatively and was rather simple; nonetheless, it established the potential for the technology.

Few years after the publication of Peterson and Fitzgerald (1980), several research groups started developing paint systems and research advancements increased steadily. Among the most successful groups is the Gouterman group at the University of Washington (UW). They developed an oxygen-quenching porphyrin probe for oxygen concentration detection in blood that was rather successful. They managed to implement the technology qualitatively but effectively in pressure measurements on aerodynamic models by the late 1980s. The first quantitative measurements took place in the summer of 1989 at the National Aeronautics and Space Administration (NASA) Ames Research Center using the UW coating (Kavandi et al., 1990; McLachlan et al., 1993; Gouterman et al., 1997).

On the other side of the northern hemisphere, Soviet scientists were concurrently carrying out similar research. As early as the late 1970s, Soviet researchers at the Central
Aero-Hydrodynamic Institute (TsAGI) explored the possibility of using oxygen-quenched luminophors for pressure measurements in wind tunnel testing. Theoretical work initiated by Zakharov in the 1960s inspired two brilliant researches, Pervushin and Nevsky, to develop the first PSP formulation by 1981, which they successfully used to obtain initial experimental results merely a year later (Ardasheva et al., 1985). By the mid 1980s another group at TsAGI developed an original polymer-based PSP formulation and tested it in supersonic flows using a lifetime approach (see following sections). High temperature sensitivity of the PSP was the main drawback of their new system. Research continued with a shift towards intensity-based approach (see following sections), which led to a more practical system that was later commercialized by the Italian company INTECO (Volan and Alati, 1991). After a long period of ignorance and misrecognition by the Western research community, the early 1990s revealed the Soviet advancement, then former USSR, through a commercial advertisement in the 1990 February issue of *Aviation Week & Space Technology*.

The work of TsAGI was demonstrated experimentally by Deutsche Forschungsanstalt fur Luft- und Raumfahrt e.V. (DLR) in Germany in January of 1991. Using intensity-based method, pressure field measurements using the TsAGI coating in the high speed wind tunnel facility in Göttingen were acquired on a cropped delta wing model. The results were validated by pressure taps and oil flow visualization techniques in addition to temperature variations on the model surface (Engler et al., 1991). The paint was calibrated in a *priori* method and error estimated for the paint from temperature dependence was reported to be less than 0.3% per degree by TsAGI. The paint layer experienced local temperature gradients less than 3°C, while the overall temperature
increased by as much as 12°C. The temperature was acquired through an infrared camera and just three thermocouples. They observed an increase in the pressure standard deviation following the application of the paint. They experienced significant errors in their pressure estimates due to curvature effects that caused uneven reflection from the surface to the photodetector, which could have been accounted for by taking a reference image. However, the paint results were provided by INTECO and it appears that calibration and data processing techniques were still premature. Volan and Alati (1991) further elaborated on the TsAGI system and compared calibration techniques (see calibration section). At this point it is sufficient to state that a calibration procedure is necessary for the paint involving the establishment of a relation between the property under investigation (e.g. pressure) and the captured illumination of the paint, accommodating different variables affecting the calibration, such as temperature dependence and illumination degradation. Moreover, error estimates greatly depend on the calibration technique, which in turn depends on the experimental setting (i.e., temperature variations/local gradients between run conditions, model movement, paint composition, etc.); hence, the degree of accuracy desired in the data is what eventually dictates the extent and complexity of the calibration procedure.

The 1990s brought prosperity to luminescent luminophor technology in wind tunnel testing. Major companies and organization competed to implement and further develop the technology to their advantage. Industrial institutions such as NASA, Boeing Seattle, Boeing St. Louis (formerly McDonnell Douglas), British Aerospace in the United Kingdom, Office National d’Etudes et de Recherches Aerospatiales (ONERA) in France, National Aerospace Laboratory (NAL) in Japan, Deutsche Forschungsanstalt fur Luft-
und Raumfahrt e.V. (DLR) in Germany are among the main industrial implementers of the technology. Academic research is encouragingly growing and has increasingly spread to many educational institutions around the globe. The University of Florida (UF), Purdue University and UW are among the leaders in current research advancements.

**The Structure**

Luminescent coatings contain a sensor molecule that is embedded in a transparent oxygen-permeable polymer binder, as in the case of Pressure Sensitive Paint (PSP), or oxygen-impermeable polymer binder, as in the case of Temperature Sensitive Paint (TSP), both of which can be dissolved in a vaporizable solvent and then sprayed on the surface, with a diffusely reflecting base underneath (i.e., primer) as shown in Figure 2-1.

![Figure 2-1 Coating Structure](image)

These coating luminesces proportionally with pressure and temperature levels when excited by electromagnetic radiation (light) of appropriate energy (i.e., wavelength). More accurately stated, the illumination of the coating is rather quenched by pressure and/or temperature. The illumination is then captured by a photodetection device, such as a charge-coupled device (CCD) camera or a photomultiplier tube (PMT) after passing the emission through appropriate filter(s) to separate the various regions of emission in the
spectrum. A typical absorption and emission spectrum for PtTFPP/FIB is shown in Figure 2-2.

![Graph showing typical absorption and emission spectra for PtTFPP/FIB.](image)

**Figure 2-2. Typical Absorption (left) and Emission (right) vs. Wavelength for PtTFPP in fluoroacrylic polymer binder (PtTFPP/FIB). (Bell et al. 2001).**

**Paint Chemistry**

This section details the photophysical processes driving the luminescence of luminophors. Fundamental issues and concerns that arise and unfavorably affect the luminophor emission are identified and addressed. Further, this section states the selection characteristics for each component of the paint system and the selected components for the dual-luminophor system implemented in this work.

**The Photophysics**

According to quantum theory, the electromagnetic energy is transmitted in discrete amounts (i.e., in units or packets) called quanta. A quantum of electromagnetic energy is called a photon. The energy carried by each photon is proportional to its frequency. An atom or molecule of a substance usually does not emit energy; it is then said to be in a low-energy or ground state. When an atom or molecule in the ground state absorbs a
photon, it is raised to a higher energy state, and is said to be excited. The substance spontaneously returns to a lower energy state by emitting a photon with a frequency proportional to the energy difference between the excited state and the lower state. In the simplest case, the substance will return directly to the ground state, emitting a single photon with the same frequency as the absorbed photon.

Luminescence is often used to describe the cold emission of electromagnetic radiation at a different wavelength than that at which it is absorbed. Luminescence is triggered by the movement of electrons within the substance from higher energy levels to lower energy levels (i.e., deactivation). Luminescence can initiate through various paths, from simple oxidation of the substance to complex processes such as chemiluminescence carried out by living organisms (e.g. Fireflies) in which chemical reactions induce luminescence. In the scope of pressure/temperature sensitive paints, luminescence is more accurately referred to as photoluminescence. Photoluminescence is the instantaneous emission of light from a substance under the influence of optical excitation (Gfroerer, 2000). As a luminescent molecule absorbs a photon of light a molecular photoluminescence (fluorescence and phosphorescence) radiative phenomenon follows. Fluorescence is a process that occurs when an electron in a molecule transitions from the lowest singlet state to the singlet ground state. While phosphorescence occurs when the electron transitions from the lowest excited triplet state to the singlet ground state. Unlike fluorescence, phosphorescence is a delayed emission with a longer lifetime and wavelengths (Liu et al., 1997). Upon excitation an electron transition to a higher energy level, however, there are only two permitted states: The singlet state and the triplet state. At the excited singlet state an electron would possess more energy relative to an excited
triplet state. Furthermore, it is more probable for an electron at the singlet state to transit to another singlet state than to a triplet state (intersystem transition). Hence, after a molecule that is initially in the singlet ground state absorbs a photon it will most likely convert to an excited singlet state. Within picoseconds, the excited state relaxes to the lowest singlet state, without emission of radiation, through internal conversion. The remaining energy in the lowest singlet state may then be dissipated via radiation or through various radiationless mechanisms, such as thermal and oxygen quenching.

In view of that, a luminescent molecule of an environmental sensor absorbs light and becomes electronically excited to an elevated energy state, which is followed by an energy emission to return to ground state. The absorption process takes place initially under thermal equilibrium state of the molecule then following excitation the molecule migrates to various vibrational levels of a new electronic state. As a result, there exists an energy deficit between emission and excitation energies, thus radiative emission occurs at longer wavelengths relative to absorption, a phenomenon known as the Stokes shift (Lakowicz, 1999). In general, the energy emission (deactivation) is predominantly categorized under two groups: radiative-decay mechanisms in which energy is released as light and non-radiative-decay mechanisms in which energy is transferred to the surrounding medium (e.g. heat transfer). The pressure sensor molecules used in PSPs have an added feature that allows them to return to ground state by colliding with oxygen molecules, a process known as oxygen quenching Kautsky and Hirsch (1935). Therefore, as the ambient pressure increases the partial pressure of oxygen in the air increases and thru sorption and diffusion, the oxygen concentration within the binder increases. This leads to an increased quenching effect on the sensor leading to a lower intensity. TSP
coatings are typically embedded in oxygen impermeable binders and function on the basis of the sensitivity of the luminescent molecules (luminophor) to their thermal environment. The molecules reach an excited state by absorption of a photon and then deactivates through the emission of a photon. A rise in temperature of the luminescent molecule will increase the probability that the molecule will return to the ground state by a radiationless process, which is known as thermal quenching. However, TSP shows some apparent pressure, more accurately oxygen concentration, dependence, which is unavoidable as an absolutely oxygen impermeable binder is yet to exist. Nonetheless, the pressure dependence in most TSP systems is rather insignificant relative to thermal quenching effects.

**The Temperature Dependence**

Unfortunately, PSP usually exhibits undesired temperature dependence, which necessitates a correction to the calibration process. This dependability stems primarily from the fact that oxygen diffusion and solubility in the polymer depend on temperature, while the inherent temperature sensitivity of the luminescent molecules contributes to this dependability on a secondary basis (Schanze et al. (1997)). Recalling equations 1.3 and 1.4, the decay rate constants are temperature dependent except the radiative decay rate $k_r$.

\[
I_{em} = \frac{Ck_r}{k_r + k_{nr} + k_q [O_2]} \quad (1.3)
\]

\[
\frac{1}{\tau_{em}} = k_r + k_{nr} + k_q [O_2] \quad (1.4)
\]

Henry’s law states that for quasi-steady pressure variations, the gas concentration in a medium is proportional to the gas concentration contiguous to the medium. Properties of the binder affect oxygen sorption and diffusion as well. These properties
are: the solubility and diffusion of the gas in the binder, the density of the luminophor in
the binder, the quenching efficiency, the luminophor(s) embedded within, the electronic
state of the luminophor (triplet or singlet), and lastly the permeability of the binder (Bell
et al., 2001). It should be noted that recalling the discussion in the literature review of the
work of Gouin (2000b), these properties can be appreciably altered by adjoining layers
(i.e., primer). Temperature affects some of these properties, specifically, the solubility
and diffusion of the gas in the binder and the permeability of the binder itself. Following
Smoluchowski’s model, the quenching second-order rate constant can be described as
(Szmacinski and Lakowicz, 1995)

\[ k_q = 4\pi N p \left( D_p + D_q \right) \]  \hspace{1cm} (2.1)

where \( D_p \) and \( D_q \) are the diffusion coefficients of the luminophor and the oxygen in the
polymer, \( N \) is the number of luminophor molecules per millimole in the binder and \( p \) is a
factor characterizing the quenching mechanisms. The diffusion coefficients are functions
of temperature, and can be related to temperature in an Arrhenius form over limited
temperature ranges.

\[ D_q \propto e^{\frac{-E_Q}{RT}} \]  \hspace{1cm} (2.2)

where \( E_Q \) is the activation energy for oxygen diffusion in the binder, \( R \) is the universal
gas constant (8.315 J/mol.K) and \( T \) is the average thermodynamic temperature in °K of
the binder.

The non-radiative first-order decay rate constant represents the intrinsic
temperature dependence of the luminophor. It is further decoupled into two parts, a
temperature-independent component and a thermally activated intersystem crossing
function. The thermally activated component is usually modeled with an Arrhenius
functions in a similar fashion to the diffusion coefficient in equation (2.2). The overall all
temperature dependence of the paint is a composite function of both the intrinsic
temperature dependence and the oxygen quenching rate constant. Typically, the oxygen
quenching path is the more dominant; however, some formulations show opposite
dependence (Liu et al. 1997).

**The Oxygen Factor**

Oxygen is a diradical molecule, that is, it possesses a pair of equal energy
molecular orbits and two unpaired electrons. Electrons spin in orbits as they rotate about
an axis passing through the electron. Molecules whose outermost pair of electrons have
parallel spins are in the “triplet state;” molecules whose outermost pair of electrons have
anti-parallel spins are in the “singlet state.” Ground-state oxygen is in the triplet state its
two unpaired electrons have parallel spins, a characteristic that, according to rules of
physical chemistry, does not allow them to react with most molecules. Thus, ground-state
or triplet oxygen is not very reactive. However, triplet oxygen can be activated by the
addition of energy, and transformed into reactive oxygen species such as singlet oxygen.
Singlet oxygen is produced as a result of the absorption of light energy. When triplet
oxygen absorbs sufficient energy to reverse the spin of one of its unpaired electrons, it
forms the singlet state. Singlet oxygen though not a free radical it is highly reactive.

When an excited luminescent molecule collides with triplet oxygen, energy transfer
occurs and singlet oxygen is produced. As energy requirement for triplet oxygen
transformation is rather low (1.0 eV), triplet oxygen is recognized as an excellent
quencher. This is typically followed by a radiation deactivation process around 1240 nm
and an intersystem crossing leading to vibrational relaxation. They issue here is the
lifetime of the singlet oxygen and its concentration relative to the triplet oxygen within
the binder as it might react irreversibly with the components of the paint given enough time. The phenomenon is commonly as photobleaching and is highly dependent on the surrounding environment and hence differs from one paint composition to the other.

**Paint Composition and Character**

As stated previously in this chapter, paint system contains three major components, the luminophor, the binder and the primer. The binder accommodates the luminophor and regulates the interaction between the luminophor and the surrounding medium, whereas the primer provides a homogenous base coat for the paint layer, which is advantageous for the overall performance of the paint.

**Which Luminescent Molecule?**

Luminophors are selected to have characteristic such as: high quantum yield (defined later in this chapter), long emission lifetime, photostability under extended excitation and a distinguishable Stokes shift (Bedlek-Anslow, 2000). Platinum porphyrins and ruthenium complexes are among the most utilized sensors for PSP due to their high oxygen sensitivity and long lifetimes. When combined with a temperature probe in a dual luminophor system, self-quenching behavior is observed with increasing rate as the luminophor concentrations are increased, i.e., ~ 10nm molecular spatial separation distance, (Bell et al., 2001). Polymer binders must be robust enough to sustain skin friction and all other forces on the surface. In addition, they should be easy to apply in order to achieve a thin smooth film on the surface (~ 10 µm), thus ensuring that they would not change the aerodynamic or structural properties of the model and allow for a potentially fast dynamic response.

The molecular pressure probe used in this work is PtTFPP, a fluorinated tetraphenyl porphyrin derivative which is extremely photostable (i.e., retains physical
properties under exposure to light), possess a large phosphorescence quantum yield (90% in 3-methylpentane) and has a long lived triplet lifetime (120 µs) (Zelelow et al., 2003). Its characteristic phosphorescence long lifetime allows enough time for oxygen quenching within the decay lifetime of luminophor in proper mediums. Absorption bands are observed at the long wavelength UV range (390 nm), and in the low and high green wavelengths (506 nm and 540 nm). The green wavelengths absorption bands create a problem for dual-luminophor systems as TSP phosphors emits at that range. This leads to cross talk between the luminophors and an induced variation in the PtTFPP emission that is temperature dependent. Main emission of PtTFPP is in the red wavelengths spectral domain (651 nm) with a less significant emission at the 712 nm wavelength, far enough from typical TSP emission. Spectral separation of the PSP and TSP emissions is essential in dual-luminophor systems that implement intensity based measurement systems. The TSP phosphor is tris-(1,10-phenanthroline)ruthenium(II) dichloride (Ruphen). Strong absorbance is evident near the 450 nm range while emission is near the 580 nm wavelength. Ruphen exhibits slight oxygen quenching properties due to its relatively long lived life time (0.6 µs). To eliminate this dependence the Ruphen is encapsulated polymer-based nanospheres comprised of polyacrylonitrile (PAN), which has extremely low gas permeability. Both the PSP and the TSP (encapsulated) are dispersed into poly-t-BS-co-TFEM polymer.

**Binder Polymers**

Binders play the important role of mediating between the surrounding oxygen and the luminophor molecules (i.e., diffuse oxygen within). The characteristics of the polymer are then crucial to the overall performance of the paint. The diffusivity of a gas in a polymer medium is quantified by imposing a specie gradient across the polymer and
then measuring the gas flux across the polymer as a function of time, a procedure known as gas permeation across a membrane (Lu and Winnik, 2000). Initial gas diffusion is typically slow and exponential like, then following some time lag a steady-state gas flux is manifested through the membrane (Figure 2-3).

Figure 2-3 Typical rate of gas permeation across a membrane (after Lu and Winnik, 2000)

The steady-state flux, $\mathcal{J}$, is linearly proportional to the gas diffusion coefficient in the membrane, $D$, the gas solubility in the membrane under equilibrium state, $S$, and the pressure gradient across the membrane, $\Delta p$, and is inversely proportional to the membrane thickness, $L$.

$$\mathcal{J} = \frac{DS\Delta p}{L} \quad (2.3)$$
The diffusion of the gas in the polymer is permitted through the molecular-size voids into which the gas molecules can travel. As described by the free-volume model, formation of packets of free volume via thermal activation opens paths for molecular transportation within the solid polymer, in contrast to diffusion through liquids which is driven by mere translational displacement. Intrinsically, temperature will have the affect of increasing the volume of this free space leading to enhanced diffusion. This diffusion enhancement is often characterized by an Arrhenius behavior. Temperature has a multifaceted effect on solid polymers. As the polymer bypasses the glass transition temperature, $T_g$, thermal expansion becomes more significant and the free volume of the system allows for large-amplitude molecular motion of the polymer backbone, leading to more complex temperature dependence in gas diffusion. This is contrary to the temperature dependence of gas diffusion characterized by a simple activation energy model under the glass transition temperature. Often, additives such as plasticizers are incorporated in polymers to lower their glass transition temperature.

An ideal polymer will be completely temperature independent and possess excellent oxygen sorption and diffusion characteristics. The first is yet to exist; however, Schanze et al. (1997) developed a realistic criterion for the selection of polymers in order to minimize temperature dependency. Their results indicate that in order to minimize the temperature dependence, the medium (binder) used must posses the lowest possible activation energy for oxygen diffusivity. In other words, the lower the intrinsic “resistance” of the binder to oxygen diffusion, the lesser temperature effects will be to the overall emission. Puklin et al. (2000) developed a binder that satisfies the criterion set by Schanze et al. (1997). The group developed the FIB polymer, which exists in the glassy
state at room temperature and holds a glass transition temperature of 70°C. It has high pressure sensitivity relative to typical silicone resins, with robust and smooth application to surfaces and low temperature sensitivity (-0.6%/°C).

**The Undercoat**

An undercoating, i.e., primer, coating is typically applied first on the surface before applying the paint. The primer coating contains white pigments embedded in a polymer, usually the same polymer used for the paint to minimize chemical alteration of the paint when applied on top of the primer, as the paint will still be in aqueous form dissolved in the solvent which in turn once applied will dissolve and mix some of the primer layer with the paint. Having identical polymers for the paint and the primer layers further ensures that oxygen sorption and diffusion are similar and hence avoiding oxygen gradient across the paint that could potentially change the paint characteristics (Gouin et al., 2000c). Primer layers are advantageous for various reasons. First, they possess high index of refraction and high hiding power. Index of refraction is a parameter used to describe the interaction of electromagnetic radiation with matter. It indicates how much the light is slowed down while traveling through a specific medium relative to vacuum. Whereas the speed of all electromagnetic radiation in vacuum is the same, it is a function of frequency in any medium. This index of refraction is the ratio of the speed of sound to the phase velocity (the velocity at which the phase of any one frequency component of the wave will propagate) of the radiation wave, not to be confused with the envelop velocity (i.e., the velocity with which the overall shape of the wave’s amplitude propagates and is what dictates the rate at which information and energy may be transmitted by the wave). Thus index of refraction is typically bigger than one in proportion to the density of the material, that is the denser the material, the more the light
is slowed down. That is advantageous as it provides more reflected light both back into the paint layer after penetrating it to the primer and from molecular emission back to the camera. Hiding power is defined as the ability of paint to obscure the surface over which it has been applied, which is strongly influenced by the degree of dispersion of pigments in the binder media. The more the pigment particles agglomerate, the worse the hiding power becomes.

Another advantage of primer coats is that they eliminate any intrinsic reflection of the surface due to various materials that compose the surface. The uniformity of the surface guarantees uniform reflection, which eliminates errors due to reflection variation that could be significant depending on the surface material composition, and which can be further manifested as an error source if model movement occurs. However, primer coats, even with identical polymer as the paint, induce some undesirable effects as they interact with the paint layer. As shown by Puklin et al. (2000) and Gouin et al. (2000c), primers can adversely alter the response time of the paint as well as the temperature dependence of the paint. Oxygen equilibrium across the paint layer is a deterministic factor for the quenching process, and a primer that is oxygen permeable, especially one with a different permeability, induces an oxygen gradient across the interface with the paint layer that affects the PSP characteristics. Binders with low oxygen permeability provide a resolution to such problem, such as polyacrylonitrile PAN (Kose, 2005). A complete elimination of primer effects is not feasible because the white pigments (e.g. titanium dioxide) are known to affect the oxygen diffusion in the polymer, (Schappacher et al., 2003), in addition to other undesirable effects such as photoxidation.
Measurement System

A schematic of the experimental setup for intensity based static measurement system is provided in Figure 2-4 showing the different components and their arrangement. In this setup the specimen is placed inside a vacuum chamber, where the pressure can be precisely monitored and controlled, wedged between two rectangular aluminum blocks. The top block is heated, while the bottom is cooled imposing a quasi-one-dimensional temperature gradient along the length of the specimen, if desired. The specimen is excited with the proper light source (UV lamp, LED, laser, etc.) and the photonic emission is then passed through the appropriate filter(s) then to the cooled CCD camera, which converts the photonic energy into electrical current. The Electronic Unit (E/U), which encloses an analog-to-digital converter, amplifies and processes the current produced and passes it to the frame grabber card installed in the PC for final processing by the test personnel. The pressure of the chamber is measured via a pressure transducer and the temperature is measured through five equally-spaced thermocouples fitted on the back surface of the specimen.
The Excitation Source

Excitation source plays a key role influencing the accuracy and quality of data. Luminophors typically have multiple absorption peaks with moderate bandwidths, a desired character of the probe to avoid broad absorption and spectral overlap and interference. Therefore, the excitation source must provide sufficient and uniform illumination at these absorption bands to produce an output luminescence signal capable of saturating the detector in relatively short exposure time, thus taking advantage of detector’s signal-to-noise (SNR) potential. However, the illumination should not be bright enough to cause photodegradation of the luminophor or overwhelm the detector’s well depth (see useful definitions in Appendix-A). Further, the excitation source must have absolutely no emission at the luminophor spectral emission range. A spatially
uniform illumination avoids the formation of local regions where the signal is low relative to the noise level with respect to the rest of the imaged field, inducing an inconsistent SNR field. If continuous illumination is desired, then a main character is the stability of the illumination output by the source, while excellent pulsed excitation depends on repeatable excitation signal levels.

Some of these induced variations can be accounted for in the calibration process such as inhomogeneous illumination fields (see calibration section), however, variations in the overall output signal level (source drifting) is not easily accounted for. A drift in the excitation signal will have the effect of reducing the strength of the paint emission, falsely indicating higher pressure/temperature levels. Constant monitoring of the output excitation signal, though possible, is quite impractical, especially when using multiple excitation sources. A suggested approach in the literature is to embed an environmental-insensitive probe in the paint (a multicomponent system) that depends only on the excitation signal. This approach provides an effective correction technique that eliminates excitation field variations; nonetheless, it is fairly challenging to find a probe that will absorb in the same spectral range as the other probes and yet emits in an empty range of the spectrum not occupied by other luminophors, which becomes even more intricate in systems containing more than one probe (e.g. pressure and temperature dual-luminophor systems.)

**The Detection Device**

The detection device plays a deterministic role in how accurate PSP measurements are and the degree of resolution obtainable. This could further impinge on the feasibility of certain experiments, such as low speed wind tunnel PSP measurements, where pressure variations are small and the corresponding intensity variations are fairly
diminutive. Accordingly, an understanding of the basic principles of operation of
detection sources is fundamental for the comprehension of the setup and acquisition
processes as well as error sources and uncertainty analysis.

**Calibration Techniques**

A calibration procedure is needed to translate intensity information to the
corresponding pressure and temperature. However, to establish the calibration relations a
prior knowledge of some environmental condition is required. This can be accomplished
through an *in situ* or a priori approach or a hybrid approach. Each approach is defined in
the following sections.

**A Priori Calibration**

This approach is based on static (no flow) calibration procedure carried
independently in a calibration chamber where the environmental conditions are well
controlled and monitored. A coupon is coated with the paint and then imaged inside a
pressure controlled environment, such as a vacuum chamber, and the intensity of the
specimen is recorded as the pressure inside the chamber is varied, hence creating a
relation between the two variables. In applications where temperature variations are
expected, a temperature gradient is imposed on the coupon and the pressure is varied
while maintaining the temperature gradient and a calibration surface is produced where
the pressure is a function of both the intensity and the temperature. The calibration
relations are then utilized to retrieve pressure and temperature information from the
actual test data, such as in a wind tunnel. The general form of the priori calibration is:

\[
p = \sum_{l=0}^{L} \sum_{k=0}^{K} a_{lj} T^l \left( \frac{I_{ref}}{I} \right)^L
\]

(2.4)
It is usually sufficient to represent the pressure with a biquadratic function in equation (2.4) (i.e., $L=K=2$). The higher the order of the pressure calibration function the more points needed to solve for all the coefficients, in this example, the biquadratic functions has nine coefficients and thus nine intensity and temperature measurements are needed. The range of the pressure and temperature calibration conditions depends on the range of these conditions in the application as well as the accuracy needed. Extrapolation usually yield high errors, hence, the environmental envelop should cover all expected test conditions. The general behavior of the intensity within envelop should be known through spectroscopy analysis to determine the resolution of the measurements desirable in each different region of the pressure and temperature ranges.

A reference image is still needed to account for spatial variation (i.e., paint thickness, illumination field, etc.); however, the reference condition in this case may or may not be identical to the reference condition in the actual testing environment (wind-off). If the two conditions are not identical then the test normalized ratios are multiplied by the reference-to-wind-off intensity ratio before using equation (2.4).

Priori calibration offers simplicity and convenience by eliminating the need for installing pressure taps and thermocouples in the model. Further, it allows for the full coverage of the environmental test range. This would be rather difficult if one attempts to installs pressure taps on the model surface, as no prior knowledge exists to provide information for optimal taps location or the physical infeasibility of installing the pressure tap at these locations. In the former case, the paint could be applied and a qualitative estimation of the pressure field is obtained to determine the best locations for installing pressure taps. Unfortunately, this approach may involve higher errors because
of different camera and illumination source positions (Liu et al., 1997). Even though the same batch of paint is used to paint both the calibration coupon and the test model, variations still occur due to different application of the paint, environmental/surface containments, etc. Nonetheless, more often, this calibration technique is accurate enough for typical test environments with considerable practicality. The results are exceptionally accurate if the calibration is carried in a wind tunnel where the static pressure and temperature can be controlled under wind-off conditions (i.e., pressurized tunnels).

**In situ Calibration**

In this approach the calibration is carried in a wind tunnel under flow conditions. Pressure taps are installed in the model and the pressure variation over the model are recorded simultaneously with intensity values. The pressure function is typically a polynomial of the second-order, with a first-order representation sufficing many applications.

\[
P = \sum_{h=0}^{H} b_h \left( \frac{I_{\text{wind-off}}}{I} \right)^h
\]  

(2.5)

The temperature effects are absorbed in the coefficients, which unless isothermal conditions are present, increases the standard deviation of the calibration data and yields, in most cases, double-valve points that correspond to different conditions. To compensate for this, thermocouples can be installed to record the temperature and the calibration curve is transformed to a calibration surface. The *in situ* approach may seem self-defeating as it still requires pressure taps and thermocouples to be installed in the model, however, the number of calibration taps is significantly less than the typical number of pressure taps required to map out the pressure field.
**Hybrid Calibration**

This technique combines the convenience of a priori approach with the accuracy of the *in situ* method. The pressure calibration is established first in a static cell, and then the calibration is corrected for temperature variation between static and flow conditions via a factor $K$. The approach was explained earlier in the Literature Review Chapter. The technique relies on the very little temperature variations on the order of few degrees and a PSP coating exhibiting ideal behavior. Thus, this approach fails when significant temperature variation occur during testing and with non-ideal paint/polymer systems.

**PSP Calibration**

In the presence of an oxygen-quencher a quantity known as the quantum yield of luminescence $\Phi$, more commonly known as the quantum efficiency, is modeled under oxygen rich conditions as:

$$\Phi = \frac{\text{rate of luminescence emission}}{\text{rate of excitation}} = \frac{I}{I_a} = \frac{k_r}{k_r + k_{nr} + k_q[O_2]} = k_r \tau$$

(2.6)

where $I$ is the luminescent intensity, $I_a$ is the absorption intensity and $\tau$ is the lifetime of an excited molecule. Under vacuum conditions, equation (2.6) becomes:

$$\Phi_0 = \frac{k_r}{k_r + k_{nr}} = k_r \tau_0$$

(2.7)

where $\Phi_0$ and $\tau_0$ are the quantum efficiency and lifetime of the probe under vacuum conditions, respectively. Dividing the quantum efficiency under vacuum by that under oxygen rich yields:

$$\frac{\Phi_0}{\Phi} = \frac{I_a}{I} = \frac{k_r + k_{nr} + k_q[O_2]}{k_r + k_{nr}} = \frac{k_q[O_2]}{k_r + k_{nr}}$$

$$\frac{\Phi_0}{\Phi} = \frac{I_a}{I} = 1 + k_q[O_2] \tau_0$$

(2.8)
This form is known as the Stern-Volmer equation. It is not practical to pull a vacuum in order to estimate the vacuum lifetime of the sensor molecule, thus a known condition, such as atmosphere, is utilized instead as the reference condition. Henry’s Law relates the oxygen concentration to the partial pressure of oxygen as shown in equation (1.6). The intensity ratio with respect to an atmospheric reference condition is hence represented as:

\[
\frac{I_{\text{ref \ atm}}}{I} = \left \{ \frac{k_r + k_{nr} + k_q [O_2]}{k_r + k_{nr} + k_q [O_2]} \right \}_{\text{ref}} = \frac{k_r + k_{nr} + k_q P_{O_2}}{k_r + k_{nr} + k_q (1)} = A(T) + B(T) P_{O_2}
\]

(2.9)

\[A(T) = \frac{k_r + k_{nr}}{k_r + k_{nr} + k_q} ; \quad B(T) = \frac{k_q}{k_r + k_{nr} + k_q} ; \quad P_{O_2} = \frac{P}{P_{\text{ref}}}
\]

The general form of the Stern-Volmer equation is expressed as:

\[
\frac{I_{\text{ref}}}{I} = \sum_{n=0}^{N} A_n(T) \left( \frac{P}{P_{\text{ref}}} \right)^n
\]

(2.10)

The coefficients \(A_n(T)\) are functions of temperature, as expected due to the temperature dependence in \(k_D\) and \(k_Q\), and are to be determined through the calibration process, in which pressure and intensity data are acquired and then a least-square fit procedure is performed to determine the coefficients. Typically a second order polynomial \((N = 2)\) is sufficient to accurately fit the experimental calibration data. The process of taking the ratio between the luminescence intensity and some reference intensity is essential in order to eliminate illumination spatial non-uniformities, coating thickness variation and luminophor uneven dispersion in the binder.
Temperature Compensation Models

As described in Chapter 1, PSP exhibits temperature dependence primarily due to the dependence of the polymer binder permeability on temperature, which in turn effects the oxygen diffusion and sorption in the binder. On a secondary level, under oxygen rich environments, the inherent temperature quenching rate constant $k_D$ plays an insignificant role but becoming more significant as the oxygen concentration decreases until it constitutes the only temperature dependence at vacuum conditions. The temperature dependence can be modeled by a single exponentially decaying function at vacuum and a multi-exponential decaying function around atmospheric conditions (Schanze et al., 1997). The proportionality constants and activation energies depend on the sensor probe and the polymer; however, up to the date of this work, there has been no successful effort to globally model this dependence for a wide range of pressures and temperatures.

Calibration provides a mean to compensate for the temperature dependence. In situ techniques have no explicit temperature dependence; rather they absorb these effects in the calibration coefficients as the temperature spatial variations are averaged out among all points included in the calibration (Bell et al., 2001). This means that each set of coefficients correspond to a unique pressure and temperature condition. This approach is useful when temperature variations are small enough (~10°-20° C depending on the PSP composition) to avoid double value points on the calibration surface that correspond to different pressure/temperature values. Further, extrapolating for points outside the calibration envelop entails high uncertainty. In wind tunnel experiments, the run image precedes the reference image as the reference image is acquired immediately after the termination of the run image to ensure identical temperature distribution throughout both images. This prohibits practical execution of long experiments as the tunnel must be
stopped after each run to allow for the reference image. In the presence of significant
temperature gradients on the model surface, a localized calibration is needed with enough
pressure taps for each region.

A priori calibration provides a more global calibration as the pressure is calibrated
as a function of both normalized intensity and temperature. Nonetheless, this necessitates
that temperature field estimation on the model surface be performed. As this calibration
approach offers a pixel-by-pixel temperature compensation, the only reasonable
temperature measurement technique would have to be a TSP system. Early attempts
suggested imposing a TSP layer either beneath or on top of a PSP layer in order to
simultaneously map out pressure and temperature fields (Harris and Gouterman, 1995;
Oglesby et al., 1995). Latter attempts incorporated the two sensors in the same binder in a
dual-luminophor system (Carroll et al., 1999). In either approach the convenience of
having a second sensor to map-out the temperature field comes with a highly undesirable
consequence, namely spectral interference. Spectral interference occurs whenever two
luminophors are co-embedded in the same binder. The emission of the lower wavelength
sensor probe, i.e., TSP, usually overlaps with part of the excitation region of the higher
wavelength sensor probe, i.e., PSP. This adds to the temperature dependence complexity
as the PSP emission is partially dependent on the intensity of the TSP emission. This
makes the decoupling of the two effects rather intricate using typical calibration and
compensation techniques. Secondary detrimental effects include: spectral overlap due to
broad emission of the lower wavelength luminophor, chemical interaction, particles
coagulation resulting in an uneven dispersion of the two luminophors in the binder and
increased uncertainty in the temperature estimates as it is embedded in an oxygen
permeable binder (TSPs possess finite oxygen sensitivity).

This work utilizes a dual-luminophor system (Kose, 2005) and a mathematical statistical technique known as POD to provide the calibration functions. POD is capable of separating the different factors constituting any dataset; further, it provides a physical insight into the various processes involved. A comprehensive discussion of the approach and calibration procedure is presented in Chapter 3.

TSP Calibration

TSP sensors are typically dispersed in an oxygen-impermeable polymer binder, thus the last term in the denominator of equation (2.6) can be eliminated reducing the equation to:

$$
\Phi = \frac{I}{I_a} = \frac{k_r}{k_r^* + k_{nr}} = k_r r_0
$$

(2.11)

The temperature rate constant $k_D$ is modeled as an Arrhenius function of the form (Liu et al., 1997):

$$
k_{nr} \propto \exp\left(-\frac{E}{RT}\right)
$$

(2.12)

where $E$ is the Arrhenius activation energy and $R$ is the universal gas constant. The Arrhenius behavior can be incorporated in the inverse of equation (2.11) to yield:

$$
\frac{I_a}{I} = \frac{k_r + k_{nr}}{k_r} = 1 + \frac{k_{nr}}{k_r} = 1 + \frac{A \exp\left(-\frac{E}{RT}\right)}{k_r}
$$

(2.13)

where $A$ is a proportionality constant. Subtracting two different temperature condition, with one of them chosen at absolute zero ($T = 0$) eliminates the unity constant.
Dividing equation (2.14) by a reference temperature condition, an approximate intensity ratio is realized as:

\[
\ln \left( \frac{I(T)}{I(T_{ref})} \right) = \frac{E}{R} \left( \frac{1}{T} - \frac{1}{T_{ref}} \right)
\]

(2.15)

In theory the plot of equation (2.15) is a straight line with slope of \(\frac{E}{R}\). However, from experimental observation this relation fits experimental data but only over certain temperature range which varies with different coating formulations. Therefore, a more general form is required and can be expressed in the form:

\[
\frac{I(T)}{I(T_{ref})} = F \left( \frac{T}{T_{ref}} \right)
\]

(2.16)

where \(F\) is a general function that could vary from a polynomial, exponential or some other function that fits the experimental data over a certain temperature range.
CHAPTER 3
PROPER ORTHOGONAL DECOMPOSITION

This chapter presents the proper orthogonal decomposition (POD) analysis. It details all the involved steps and mathematical as well as physical implications. Artificial data are then examined and analyzed via POD to characterize the POD and gain a more comprehensive understanding of the analysis process.

Overview

The idea of characterizing the pressure and temperature fields using specialized luminescent coatings (i.e., TSP or PSP separately) is presented in the previous chapters; however, the aforementioned approach is only capable of characterizing each field autonomously. The concept of formulating a coating containing both pressure and temperature sensors embedded in the same binder presented a rather fascinating supposition that is well worth investigating. Two obvious questions emerge through such concept: Are the two sensors (luminophors) going to interact and affect each other? Is it possible to identify and separate the emission from each sensor and consequently be able to accurately determine the pressure and temperature? Even though the two questions are fundamentally tied together, the first is more of a chemist subject, while the second is more of a question for a mathematician, hence my dilemma as an engineer!

From a chemical prospective, the goal is find the proper combination of pressure and temperature luminophor such that they will mix conveniently and disperse evenly in the binder without considerably changing the chemical composition or the luminescent characteristics of each other. In addition they should possess a comparable wavelength
excitation-region but distinguishable emission. Finally, any cross-talk between the two luminophor should be limited to minimal.

Mathematically, the simplest approach is to ratio the emission of each luminophor to a fixed reference, such as a third luminophor that has a constant emission with respect to the environmental conditions but dependent on the excitation intensity level and paint thickness, or take the ratio of one emission to the other. The latter approach does not always yield accurate results and may perhaps be somewhat simplistic, especially when cross-talk is influential. A different mathematical approach, which is the approach adapted in this work, is to implement a mathematical technique known as proper orthogonal decomposition.

**POD Analysis**

The principal mathematical difficulty of the calibration process lies in three aspects: separating the pressure and temperature information, since a single field maps both parameters at the same time, accounting for the inherent temperature dependence in the pressure sensor and cross-talk between the two sensors, and most importantly establishing a calibration curve/surface for each parameter.

**Introduction**

In general, a spectrum encompasses scores of diverse variations that derive from the quantities under investigation, intersystem relations and dependence, instrument variations (i.e., noise) and environmental conditions. Nonetheless, such complex and correlated variations can usually be reduced to a set of finite number of independent variations (factors) shaping the spectral data. The criterion governing the possibility of such process of factorization is that the data matrix can be expressed as a linear sum of product terms (a.k.a. variation spectra). If it were feasible to calculate such variation
spectra, then this data could be used instead of the raw spectral data for building the calibration model. These variation-spectra, often called eigenvectors or factors, are then used to reconstruct the original spectrum by multiplying each one by a different constant scaling factor, commonly known as eigenvalues or scores, and adding the results together until the new spectrum closely matches the original spectrum. This method of breaking down a set of spectroscopic data into its most basic variations is a least-square technique known as Principal Components Analysis (PCA). PCA yields a form of a solution that is known as an abstract solution. Such solution has no physical significance or implication by itself, thus further mathematical manipulation is required in order to quantify the final solution as meaningful. Such process is known as Target Transformation (TA) and it involves rotating the newly constructed spectrum until a physically meaningful solution is realized. This rotation can be as simple as a two-dimensional orthogonal (single-axis) rotation or as complicated as an \( n \)-dimensional non-orthogonal rotation.

**Mathematical Formulation**

A full discussion of the mathematical aspect of the POD analysis is essential in the understanding of the process. Further, key aspects of the analysis are determined based on the process under investigation, and hence a comprehensive discussion of the mathematical procedure is essential.

**Principle component analysis**

As mentioned previously, for POD to yield successful results, the data matrix must be expressible as a linear sum of product terms (eigenvalues and eigenvectors). This dictates that the data matrix must be square. The different noise sources and the character of each noise were discussed in the previous chapter. It was further noted that some errors increase proportionally with the magnitude of the signal such as shot noise, hence the
need for a statistical normalization process. In order to give each column in the data matrix an equal statistical weight, each element is normalized by the square root of the sum of the squares of all the elements in its column. However, normalizing the data has an irrecoverable effect that may not be practical in certain processes. Luminescence application in fluid mechanics is one application that yields adverse effects when data is normalized. The source of the problem stems from the fact the normalization process causes a mutual dependence between all regions of the spectrum, hence imposing an additional complication to the analysis. Consider the simple case where a spectrum is a function of two independent factors that emit at two distinct wavelengths with no overlap as shown in Figure 3-1.

Figure 3-1 Spectral emission for two independence processes: (A) Raw spectra (B) A single raw spectrum with dependence on the higher wavelength factor and no dependence on the lower wavelength factor (C) The normalized spectra
Each spectrum varies with only one factor at only one of the two emission regions, while remaining constant in the other region (i.e., invariant with the other factor). Upon normalization, the spectra show a new mutual dependence between the two factors. The decomposition of the normalized spectra would still yield only two eigenvectors, however, the new dependence can not be easily resolved to retrieve the original spectrum. Therefore, the calibration functions are much more complicated and involved, which defeats the purpose of the POD as a model reducing technique. The important question is then whether the analysis can be carried out without the normalization process while yielding accurate results. The answer to this question determines the potential for the POD analysis as a calibrator to a process. In the particular case of luminescent coatings, this calls for a thorough understanding of the error sources and their effect and scaling throughout the spectral range. Noise is shot noise limited for high intensity levels (> few hundred electrons) and is preamplifier or dark current limited for low intensity levels and long exposure times, respectively.

For most steady luminescent coatings applications in wind tunnel testing high intensity levels are present. Dynamic applications and low pressure applications, such as acoustics, do not enjoy such advantage. The experiment under investigation in this work fortunately belongs to the former applications; hence noise is shot noise limited. A typical scientific grade CCD chip has a FWC of about 330,000 e⁻, which yields shot noise of 0.17%, 0.25%, and 2.5% for FW, half-FW and 2,500 e⁻ intensity levels, respectively. This shows that noise variation between different regions of the spectrum can be effectively eliminated by using appropriate, but usually different, exposure times to utilize the FWC for each region of the spectrum.
The next step in the analysis is to square the data matrix by obtaining the covariance matrix, $Z$, which is obtained by pre-multiplying the data matrix, $D$, by its transpose. Define the normalized data matrix as:

$$D_{m,n} = \sum_{j=1}^{n} r_{m,j} c_{j,n} = [R][C]$$

(3.1)

here $m$ corresponds to the number of wavelengths spanning the emission and $n$ is the number of environmental conditions. Then the covariance matrix is given by:

$$Z = D^T D = Z_{n,n}$$

(3.2)

The next step is to extract the eigenvalues, $\lambda$, and their corresponding eigenvectors, $Q$, from the covariance matrix. Utilizing a math solver such as MATLAB, the eigenvalues are extracted and arranged in a descending order with respect to their magnitude. The magnitude of these eigenvalues serves as a gauge for the importance of their corresponding eigenvectors. Accordingly, a criterion, such as a cutoff ratio of the eigenvalue to the largest eigenvalue, has to be set to determine the minimum acceptable eigenvector. The principal eigenvectors constitute an optimized, mutually orthogonal coordinate system. Each successive eigenvector accounts for the next maximum possible variance in the data. In the factor space, the largest eigenvector is oriented as to account in a least-square sense for the greatest possible variance in the data. Further, it passes through the greatest concentration of data points and defines the best one-factor model for the data. While the first two factors define a plane passing through the greatest concentration of data points, with the second factor pointing in the direction that accounts for as much as possible of the variance not accounted for by the first factor. The eigenvalue extraction process is continued until all the significant eigenvectors are
extracted. The remaining eigenvectors characterize noise and insignificant factors embedded in the data and are consequently omitted. Now the new data matrix, $D_w$, can be reconstructed using the deduced eigenvalues and eigenvectors. The mathematical sequence of this process is presented in equations (3.3) through (3.7).

$$[Q]^{-1}[Z][Q] = \begin{bmatrix} \lambda_j & \delta_{jk} \end{bmatrix} = \hat{\lambda}$$  \hspace{1cm} (3.3)

$$[Z]Q_j = \hat{\lambda}_j Q_j$$  \hspace{1cm} (3.4)

$$[Q]^{-1} = [Q]^T$$  \hspace{1cm} (3.5)

$$[D] = [U][Q]^T$$  \hspace{1cm} (3.6)

where,  \hspace{1cm} $[U] = [D][Q] = [R]$

$$[Q]^T = [C]$$

$$\therefore [D_w] = [R][C]$$  \hspace{1cm} (3.7)

In equation (3.7), $R$ is called the row matrix with an $m \times N$ dimension, where $N$ is the number of factors, and $C$ is called the column matrix with $N \times n$ dimension.

**Target transformation**

After extracting the principal factors of the data set, a rotation is usually required in order to transform the abstract solution to a physically meaningful solution. We expect to have at least two principal components representing the two main variants in the data, namely pressure and temperature, nevertheless, a third factor representing cross-talk and/or mutual dependence could be notably influential. Hence, a discussion of three-dimensional non-orthogonal rotation is essential. The following set of matrices express a three-dimensional non-orthogonal rotation applied to axis $x$, $y$, and $z$ respectively.
The coefficients multiplying the trigonometric functions, \( a_l, b_l \) and \( c_l \), with \( l = x, y \) and \( z \), are called the orthogonality coefficients. Theses coefficients determine the degree of skewness imposed on each axes, with unit value imposing orthogonality. To carry out the transformation, the right hand side of equation (3.7) is multiplied by \([T]\) and \([T]^{-1}\) in the following manner:

\[
[T] = \begin{bmatrix}
1 & 0 & 0 \\
0 & a_x \cos(\theta_x) & -b_x \sin(\theta_x) \\
0 & c_x \sin(\theta_x) & d_x \cos(\theta_x)
\end{bmatrix}
\]
\(\text{(3.8)}\)

\[
[T_y] = \begin{bmatrix}
a_y \cos(\theta_y) & 0 & -b_y \sin(\theta_y) \\
0 & 1 & 0 \\
c_y \sin(\theta_y) & 0 & d_y \cos(\theta_y)
\end{bmatrix}
\]
\(\text{(3.9)}\)

\[
[T_z] = \begin{bmatrix}
a_z \cos(\theta_z) & -b_z \sin(\theta_z) & 0 \\
c_z \sin(\theta_z) & d_z \cos(\theta_z) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
\(\text{(3.10)}\)

The coefficients multiplying the trigonometric functions, \( a_l, b_l \) and \( c_l \), with \( l = x, y \) and \( z \), are called the orthogonality coefficients. Theses coefficients determine the degree of skewness imposed on each axes, with unit value imposing orthogonality. To carry out the transformation, the right hand side of equation (3.7) is multiplied by \([T]\) and \([T]^{-1}\) in the following manner:

\[
[D_w] = \left( [R] \begin{bmatrix} [T]^T \end{bmatrix} \right) \begin{bmatrix} [T]^{-1} \end{bmatrix} [C]
\]
\[
\therefore [D_w] = [R]_{\text{transformed}} [C]_{\text{transformed}}
\]
\(\text{(3.11)}\)

If the transformed solution has a physical significance, then a real solution has been established so that:

\[
[D_{\text{real}}] = [R]_{\text{real}} [C]_{\text{real}}
\]
\(\text{(3.12)}\)

A physical understanding of the process under investigation is necessary to interpret the rotated data and determine a physically meaning solution. For instance, in luminescent coating application, a pre-knowledge of the emission spectrum of the luminophor will aid in identifying the rotation process.
POD Behavior

The behavior of the POD as a model reduction technique and a calibrator for luminescent data is examined in this section. Artificial data is utilized to provide as much insight in the original process being examined and hence the behavior of the POD analysis.

Introduction

In order to methodically understand and comprehend the POD analysis, a theoretical set of data will be created representing a dual-luminophor emission. The data will have different distinct characteristics to evaluate the effect of each variation on the POD analysis. Two different cases of data will be analyzed:

1. The first set of data represents an ideal and simple spectroscopic emission, in which there is no mutual dependence between temperature and pressure.
2. The second set of data has the pressure emission depending on temperature (spectroscopic).

In addition, the following subcases will be examined as well:

- The filtered and integrated emission.
- Shot noise will be imposed on the signal.
- Spectral overlap between factors (i.e. cross-talk)

Each set of data is utilized to accentuate a particular feature of the POD. The eventual goal is to examine whether the POD analysis is capable of extracting the main modes of the data even if other variations, such as noise, overlap, cross talk, etc., are present in the spectra. Furthermore, the analysis will assess the accuracy of the extracted modes as a function of SNR, degree of cross talk and overlap.
Algorithm

The following chart details the P.C.A. algorithm and the various steps involved.

![Figure 3-2 P.C.A. Algorithm](image-url)
Target transformation

Equation (3.11) is used to perform a non-orthogonal rotation on the reconstructed data. The main focus of the rotation is on the $C$ matrix. The elements in the $C$ (eigenvalues) matrix are directly related to the environmental conditions. For example, if we hold the temperature constant for five spectra and vary only the pressure, after performing the POD we find a single eigenvector representing the general pressure variation, and each of the five elements in the $C$ matrix scale that eigenvector according to the corresponding pressure level. Target transformation may not be needed at all in such case, as the variables are directly related. If the two parameters do not vary independently, then the rotation would be more involving to apply due to the mutual dependence between them. In such case we would have to solve two simultaneous equations with two unknowns, temperature and pressure. Each equation states one of the environmental parameters as a function of the eigenvalues. If only two factors are needed to constitute the data, then these equations represent surfaces in the eigenvalues plane. However, if three factors are needed to reconstruct the data, then the equation lies in a four dimensional space. Figure 3-3 details the target transformation process.
Reconstruct the data matrix from the extracted eigenvalues and eigenvectors

Loop 360° for the rotation angle

Loop for the orthogonality coefficients

Rotate the data and reconstruct the rotated data matrix

Fit the pressure and temperature to the eigenvalues in a least square sense and calculate the residual error

Search for the minimum residual error and the corresponding rotation angle and orthogonality coefficients

Terminate the coefficient loop

Terminate the rotation loop

Figure 3-3 Target transformation algorithm
**Data generation**

To generate a theoretical data for dual-luminophor emission equations (3.13) through (3.16) are implemented to calculate the peak values of each spectrum (Liu et al, 2001). In the following expressions $I_{ref}$ and $P_{ref}$ are the reference luminescent intensity and pressure at a known temperature, respectively.

\[
\left( \frac{I}{I_{\text{press}}} \right) = A(T) + B(T) \frac{P}{P_{\text{ref}}} \quad (3.13)
\]

\[
A(T) \approx A(T_{\text{ref}}) \left[ 1 + \frac{E_{nr}}{RT_{\text{ref}}} \left( \frac{T - T_{\text{ref}}}{T_{\text{ref}}} \right) \right] \quad (3.14)
\]

\[
B(T) \approx B(T_{\text{ref}}) \left[ 1 + \frac{E_{p}}{RT_{\text{ref}}} \left( \frac{T - T_{\text{ref}}}{T_{\text{ref}}} \right) \right] \quad (3.15)
\]

The pressure is represented by equation (3.13), the Stern-Volmer relation, which is widely used as operational calibration relation for PSP measurements. The coefficients $A(T)$ and $B(T)$ are temperature-dependent, $E_{nr}$ is the Arrhenius activation energy for a non-radiative process, $E_p$ is the activation energy for oxygen diffusion, and $R$ is the universal gas constant. Typical values of these coefficients for (Bath Ruth + silica gel in GE RTV 118) are: $A(T_{\text{ref}}) = 0.13$, $B(T_{\text{ref}}) = 0.87$, $\frac{E_{nr}}{RT_{\text{ref}}} = 2.82$, and $\frac{E_p}{RT_{\text{ref}}} = 4.32$ over a temperature range from 293K to 333K, with the reference temperature is $T_{\text{ref}} = 298$K.

The temperature is characterized by the Arrhenius functions as follows:

\[
\left( \frac{I}{I_{\text{ref}}} \right)_{\text{temp}} = e^{- \frac{E_{nr}}{R} \left( \frac{1}{T} - \frac{1}{T_{\text{ref}}} \right)} \quad (3.16)
\]
The emission of the luminophors follows a Gaussian distribution in the frequency domain:\(^2\)

\[
I(f) = \frac{A I}{\sigma \sqrt{2\pi}} e^{-\frac{(f - \mu_f)^2}{2\sigma^2}}
\]  

(3.17)

where \(AI\) is the amplitude-control constant of the intensity, \(\sigma\) is the standard deviation of the spectrum, \(f\) and \(\mu_f\) are the frequency and center frequency in Hz, respectively.

This distribution is symmetric in the frequency domain, however, upon transforming the spectrum to the wavelength domain it shows the familiar luminophor behavior of asymmetric spread. The transformation is guided by observing that energy is related to the wavelength by the following relation:

\[
E = h f = \frac{h c}{\lambda}
\]  

(3.18)

here \(h\) is Plank’s constant \((6.626176 \times 10^{-34} \text{ J} \cdot \text{s})\) and \(c\) is the speed of light in vacuum \((3.0 \times 10^{17} \text{ nm/s})\). The spectrum can then be expressed in the wavelengths \(\lambda\) domain as:

\[
I(\lambda) = \frac{AI}{\sigma \sqrt{2\pi}} e^{-\frac{(\frac{c}{\lambda} - \mu_f)^2}{2\sigma^2}}
\]  

(3.19)

This relationship will produce a spectrum that will tail off at the higher wavelengths and in consequences replicates the effects spectral overlap between the two emissions (i.e., TSP & PSP).

\(^2\) Personal communication with Dr. Schanze
There are two types of noise that will be introduced in the spectra, white noise and shot noise. White noise is a type of noise that spans all different frequencies. The adjective white is used to describe this type of noise because of the way white light works. White light is made up of all of the different colors (frequencies) of light combined together, in the same way; white noise is a combination of all of the different frequencies. At the CCD output, the signal from the image sensor is converted from the charge domain to the voltage domain by means of a sense capacitor and a source-follower amplifier; this amplifier has a resistance that causes thermal noise. The effective resistance in this case is the output impedance of the source follower. This type of thermal noise is sometimes called Johnson noise, or as better recognized, white noise, since its magnitude is independent of frequency.
The noise in electrons can be described as:

$$\sigma_{white} = \sqrt{\frac{4kTBR_{out}}{\Delta V \Delta N}} A_y$$ (3.20)

where $k$ is Boltzman’s constant (J/K), $T$ is the temperature (K), $B$ is the noise power bandwidth (Hz) and $R$ is the effective channel resistance ($\Omega$). To simulate this type of noise the “rand” function in MATLAB will be used to generate the random noise.

A fundamental characteristic of white noise is its randomness. In performing the POD analysis, this change in different spectra, due to the different noise in each spectrum, necessitates a unique and different rotation for each different set of data. This is due to the fact that the POD analysis does not eliminate noise from data regardless of its magnitude; rather it significantly damps the noise out. The magnitude of the change in the new rotation is related to the relative difference between the noises. Therefore, a different rotation is essential in order to obtain the most accurate calibration curves.

Nonetheless, since the comparative change in noise in each set of data is relatively small, given that the magnitude of the noise itself is usually small, the new rotation would be slightly differ from the original rotation, sparing us the hassle of repeating the 360° search. High-level noise in the data might on the other hand result in an irreversible distortion to the data that is unrecoverable by the POD.

Shot Noise is the noise associated with the random arrival of photons at any detector, as described in Chapter 2. Since the time between photon arrivals is governed by Poisson statistics, the uncertainty in the number of photons collected during a given period of time is:

$$\sigma_{\text{shot}} = \sqrt{I}$$ (3.21)
where $\sigma_{\text{shot}}$ is the shot noise and $I$ is the intensity, both expressed in electrons. So a 10,000-electron exposure will have a shot noise of 100 electrons. This implies that the best signal-to-noise ratio possible for a 10,000-electron signal is $10,000/100 = 100$, accordingly, the maximum noise level is 1% of the signal.

Figure 3-5 Shot noise as a function of intensity
Analysis

In order to automate the process of finding the new rotation, one needs to perform a least square analysis on the surfaces relating the environmental conditions to the $C$ matrix. Looking at Table 3-1, the temperature is varied for the first ten runs, while the pressure is held constant. The pressure is then increased and the same temperature variation is applied, resulting in a total of ninety conditions.

<table>
<thead>
<tr>
<th>Environmental Conditions</th>
<th>$C^T$ matrix elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ $P$</td>
<td>$c_{1,1}$ $c_{2,1}$ $c_{3,1}$</td>
</tr>
<tr>
<td>300 2</td>
<td>$c_{1,2}$ $c_{2,2}$ $c_{3,2}$</td>
</tr>
<tr>
<td>307 3.5</td>
<td>$c_{1,3}$ $c_{2,3}$ $c_{3,3}$</td>
</tr>
<tr>
<td>314 5</td>
<td>$c_{1,4}$ $c_{2,4}$ $c_{3,4}$</td>
</tr>
<tr>
<td>321 6.5</td>
<td>$c_{1,5}$ $c_{2,5}$ $c_{3,5}$</td>
</tr>
<tr>
<td>328 8</td>
<td>$c_{1,6}$ $c_{2,6}$ $c_{3,6}$</td>
</tr>
<tr>
<td>335 9.5</td>
<td>$c_{1,7}$ $c_{2,7}$ $c_{3,7}$</td>
</tr>
<tr>
<td>342 11</td>
<td>$c_{1,8}$ $c_{2,8}$ $c_{3,8}$</td>
</tr>
<tr>
<td>349 12.5</td>
<td>$c_{1,9}$ $c_{2,9}$ $c_{3,9}$</td>
</tr>
<tr>
<td>356 14</td>
<td>$c_{1,n}$ $c_{2,n}$ $c_{3,n}$</td>
</tr>
</tbody>
</table>

The number of eigenvectors extracted for each case will determine the number of independent variables. We expect only two main independent factors and perhaps a third factor that accounts for combined minor effects, such as noise and spectral interference.
Case One (Independent Emission)

This is the first examined case with no mutual dependence between the two emission regions.

The eigenvalues (Table 3-2) show that only the first two factors shape the spectra while the third is 14 orders of magnitude smaller, and hence has no significance.

![Figure 3-6 Case one: Narrow spectrum with no overlap](image_url)

The eigenvalues (Table 3-2) show that only the first two factors shape the spectra while the third is 14 orders of magnitude smaller, and hence has no significance.

<table>
<thead>
<tr>
<th>Table 3-2 Eigenvalues for case one</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenvalue</td>
</tr>
<tr>
<td>$\lambda_1$</td>
</tr>
<tr>
<td>$\lambda_2$</td>
</tr>
<tr>
<td>$\lambda_3$</td>
</tr>
</tbody>
</table>
Figure 3-7 Case one: Eigenvectors and calibration curves for pressure (left) at 79.8392° and temperature (right) at 13.6121°

Pressure calibration is a linear fit while temperature was fitted via a quadratic expression. Absolute calibration error for the pressure is less than 2e-06 psi and for the temperature it is less than 0.1K. The two factors have a relative rotational angle of 66.2°. Next, seven sub-cases are examined to shed light on specific issues pertinent to the understanding of POD with the results summarized at the end of these cases.
Case One-A (Temperature Emission Amplified by 10)

The temperature emission is amplified by 10.

Figure 3-8 Case one-A: Temperature Emission Amplified 10 Folds

Figure 3-9 Case one-A: Eigenvectors for pressure (left) at 66.5546° and temperature (right) at 0.3275°

<table>
<thead>
<tr>
<th>Table 3-3 Eigenvalues for case one-A</th>
<th>Table 3-4 Calibration Error case one-A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenvalue</td>
<td>Magnitude</td>
</tr>
<tr>
<td>λ₁</td>
<td>1.24e+005</td>
</tr>
<tr>
<td>λ₂</td>
<td>3.07e+002</td>
</tr>
<tr>
<td>λ₃</td>
<td>-5.89 e-011</td>
</tr>
</tbody>
</table>
Case One-B (Temperature Emission Broadened to Overlap with Pressure)

The temperature emission is expanded to slightly overlap the pressure emission.

![Intensity vs Wavelength Graph](#)

Figure 3-10 Case one-B: Emission spectra

![Eigenvalues Matrix](#)

Figure 3-11 Case one-B: Eigenvectors for pressure (left) at 75.592° and temperature (right) at 9.365°

### Table 3-5 Eigenvalues for case one-B

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>4.81e+003</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>1.89e+002</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>2.46 e-012</td>
</tr>
</tbody>
</table>

### Table 3-6 Calibration Error case one-B

<table>
<thead>
<tr>
<th>Error Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. press. error</td>
<td>5.49e-005</td>
</tr>
<tr>
<td>RMS press. error</td>
<td>2.88e-004</td>
</tr>
<tr>
<td>Max. temp. error</td>
<td>0.095</td>
</tr>
<tr>
<td>RMS temp. error</td>
<td>0.595</td>
</tr>
</tbody>
</table>
Case One-C (Temperature Emission Greatly Broadened to Overlap with Pressure)

The temperature emission is expanded to greatly overlap the pressure emission.

Figure 3-12 Case one-C: Emission spectra

Figure 3-13 Case one-C: Eigenvectors for pressure (left) at 72.783° and temperature (right) at 6.556°

<table>
<thead>
<tr>
<th>Table 3-7 Eigenvalues for case one-D</th>
<th>Table 3-8 Calibration Error case one-C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Eigenvalue</strong></td>
<td><strong>Magnitude</strong></td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>$1.17e+004$</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>$1.61e+002$</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>$-8.6e-012$</td>
</tr>
</tbody>
</table>
Case One-D (Close Emission: Temperature at 600 nm and Pressure at 650 nm)

The center frequencies of the two emissions are close.

Table 3-9 Eigenvalues for case one-D

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>4.97e+003</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>1.03e+002</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>-5.04e-012</td>
</tr>
</tbody>
</table>

Table 3-10 Calibration Error case one-D

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. press. error</td>
<td>6.08e-006</td>
</tr>
<tr>
<td>RMS press. error</td>
<td>3.19e-005</td>
</tr>
<tr>
<td>Max. temp. error</td>
<td>0.095</td>
</tr>
<tr>
<td>RMS temp. error</td>
<td>0.595</td>
</tr>
</tbody>
</table>
Case One-E (Shot Noise Imposed on the Spectra)

Shot noise randomized by multiplying it by a normalized random error (randn in MATLAB).

Figure 3-16 Case one-E: Emission spectra

Figure 3-17 Case one-E: Reconstructed spectra
Figure 3-18 Case one-E: Eigenvectors for pressure (left) at 280.1082° and temperature (right) at 166.392°

Table 3-11 Eigenvalues for case one-E

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>4.13e+003</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>1.79e+002</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>4.25e-003</td>
</tr>
</tbody>
</table>

Table 3-12 Calibration Error case one-E

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. press. error</td>
<td>0.02</td>
</tr>
<tr>
<td>RMS press. error</td>
<td>0.089</td>
</tr>
<tr>
<td>Max. temp. error</td>
<td>0.277</td>
</tr>
<tr>
<td>RMS temp. error</td>
<td>0.925</td>
</tr>
</tbody>
</table>
Case One-F (Shot Noise Amplified by 10)

Shot noise amplified by 10.

Figure 3-19 Case one-F: Emission spectra

Figure 3-20 Case one-F: Reconstructed spectra
Figure 3-21 Case one-F: Eigenvectors for pressure (left) at 100.1556° and temperature (right) at 346.3815°

Table 3-13 Eigenvalues for case one-F

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>1.25e+005</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>5.43e+003</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>4.04e-001</td>
</tr>
</tbody>
</table>

Table 3-14 Calibration Error case one-F

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. press. error</td>
<td>0.028</td>
</tr>
<tr>
<td>RMS press. error</td>
<td>0.104</td>
</tr>
<tr>
<td>Max. temp. error</td>
<td>0.423</td>
</tr>
<tr>
<td>RMS temp. error</td>
<td>1.601</td>
</tr>
</tbody>
</table>

Case One-G (Filtered Spectra with Shot Noise Amplified by 10)

Shot noise amplified by 10.

Figure 3-22 Case one-G: Emission spectra
Figure 3-23 Case one-G: Reconstructed spectra

Table 3-15 Eigenvalues for case one-G

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>8.87e+004</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>3.84e+003</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>2.35e-002</td>
</tr>
</tbody>
</table>

Table 3-16 Calibration Error case one-G

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. press. error</td>
<td>0.034</td>
</tr>
<tr>
<td>RMS press. error</td>
<td>0.116</td>
</tr>
<tr>
<td>Max. temp. error</td>
<td>0.487</td>
</tr>
<tr>
<td>RMS temp. error</td>
<td>1.540</td>
</tr>
</tbody>
</table>

Figure 3-24 Case one-G: Eigenvectors for pressure (left) at 79.79° and temperature (right) at 13.502°
Case One-H (Filtered Noisy Spectra Using Only Two Filters)

This case is intended to demonstrate the impact of using only two filters on the POD analysis. The ability to use less number of filters means more costly efficient experiment, less time required for data acquisition and analysis, and less time for test conditions to vary and hence improved accuracy.

Figure 3-25 Case one-H: Emission spectra

Figure 3-26 Case one-H: Eigenvectors for pressure (left) at 258.9° and temperature (right) at 192.68°
Table 3-17 Eigenvalues for case one-G

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>8.19e+04</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>3.62e+03</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>-0.09e-09</td>
</tr>
</tbody>
</table>

Table 3-18 Calibration Error case one-G

<table>
<thead>
<tr>
<th></th>
<th>Max. press. error</th>
<th>RMS press. error</th>
<th>Max. temp. error</th>
<th>RMS temp. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. press. error</td>
<td>0.0328</td>
<td>0.1269</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMS press. error</td>
<td>0.4171</td>
<td>1.3858</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Summary

1. Normalizing the data produces a third eigenvector that is 4 orders of magnitude lower than the first two, while in the raw data the third eigenvector was 12 orders of magnitude lower.

2. Increasing the accuracy of the rotation angle increases the accuracy of the calibration (1 order of magnitude / decimal)

3. If the center frequencies of the two peaks and their widths are exactly matched (regardless of the relative magnitude) only one eigenvector emerges. Target transformation is not possible as there is only vector. Varying the width will produce a second factor, in practice it is almost impossible to have two emissions with the same exact emission frequency and broadness. Varying the center frequency by 1 nm will produce two eigenvectors. Error in all cases is not adversely affected.

4. Filtering nearly eliminates the noise from the filtered signal as the integration process acts as an averaging process and the results are comparable to spectroscopic results

5. Fitting the temperature data to an exponential fit doesn’t produce accurate results as the data range is small and the fitting process fits within that domain, hence a bigger temperature range will improve the fitting process. Instead a quadratic calibration function suffices with good accuracy for small temperature ranges.

6. Non-orthogonal rotation produces rotation angles for pressure and temperature that are significantly close to each other. Orthogonal rotation produces better results (less error), however the rotation angles are significantly apart.

7. POD managed to separate the pressure and temperature data effectively in all cases, except the case of identical center frequencies and spectral emission width.

8. Adding noise to the signal is identified by POD as a third factor that is still relatively insignificant in magnitude compared to the first and second eigenvalues, but significantly higher than the case of no noise. Increasing the magnitude of the noise increases the magnitude of the third eigenvalue.
9. In all presented cases, only two factors were used. Using only two filters produced comparable results to using four filters. This significant result shows that the number of extracted factors determines the number of filters needed for calibration.

**Case Two (Temperature Dependent Pressure Emission)**

In this case the effect of temperature on pressure is examined. The temperature dependence is based on equations (3.13), (3.14) and (3.15). The same analysis performed on case one is carried out and yields similar results. Only the noise infected data, both spectroscopic and filtered, are presented. A summary at the end of this section addresses omitted results and provides discussion.

![Figure 3-27 Case two: Emission spectra](image)

**Calibration Surface Fitting**

The intensity is calculated in the artificial data using

\[
\frac{I}{I_{ref_{kemp}}} = e^{\frac{E_{nr}(1 - \frac{1}{T_{ref}})}{R}}
\]

The temperature can thus be retracted by the following equation.

\[
\frac{1}{T} = \frac{R}{E_{nr}} \left[ \ln \left( \frac{I}{I_{ref_{kemp}}} \right) \right] + \frac{1}{T_{ref}}
\]

(3.22)
Using $C_{1,n}$ to represent $\frac{I}{I_{ref \_temp}}$ in the above equation the temperature is accurately calculated with a maximum absolute error of $4 \times 10^{-5}$ K. In order to account for the temperature effects in the pressure calibration, a surface fit is needed with the pressure as a function of the two sets of scores $C_{1,n}$ and $C_{2,n}$. The first set $C_{1,n}$ accounts directly for the temperature, in fact, the calibration below will show that they are interchangeable.

Recalling the theoretical function for generating the pressure intensity data

$$\left(\frac{I_{ref}}{I_{press}}\right) = A(T) + B(T) \frac{P}{P_{ref}}$$

which is expressed in terms of the pressure as:

$$\frac{P}{P_{ref}} = \left[\left(\frac{I_{ref}}{I}\right) \frac{1}{B(T)}\right] - \left(\frac{A(T)}{B(T)}\right)$$  \hspace{1cm} (3.23)

The first term is a function of both pressure and temperature, while the second term is purely temperature dependent. These coefficients can be expressed in terms of the temperature via simple algebraic manipulations of equations (3.14) and (3.15) (Appendix-b) to yield the following:

$$\frac{A(T)}{B(T)} \approx 0.15 \left(\frac{0.01T}{1 + 0.01T}\right)$$  \hspace{1cm} (3.24)

$$\frac{1}{B(T)} = \frac{1}{0.012612T - 2.8884}$$  \hspace{1cm} (3.25)

The ratio of the constants $\frac{A(T)}{B(T)}$ is an order of magnitude smaller than the constant multiplying the intensity ratio $\frac{1}{B(T)}$. The first term in equation (3.23) is a product of the intensity ratio and some function of temperature. Hence a product of the two sets of
scores is appropriate to represent the first term. The second term can thus be represented in the calibration function by the temperature scores $C_{i,n}$. These two representations are not necessarily linear. A second order calibration function for the pressure can then be represented by the following equation.

\[
\begin{pmatrix}
C_{11} & C_{11}C_{21} & C_{11}^2 & C_{11}^2C_{21} & 1 \\
C_{12} & C_{12}C_{22} & C_{12}^2 & C_{12}^2C_{22} & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
C_{1(n-1)} & C_{1(n-1)}C_{2(n-1)} & C_{1(n-1)}^2 & C_{1(n-1)}^2C_{2(n-1)} & 1 \\
C_{1n} & C_{1n}C_{2n} & C_{1n}^2 & C_{1n}^2C_{2n} & 1
\end{pmatrix}
\begin{pmatrix}
a \\
b \\
c \\
d \\
e
\end{pmatrix}
= 
\begin{pmatrix}
P_1 \\
P_2 \\
\vdots \\
P_{n-1} \\
P_n
\end{pmatrix}
\]

Solving for the coefficients yields a calibration surface for the pressure as a function of both sets of scores.
Figure 3-28 Case two: Eigenvectors rotated at the appropriate angle for temperature (left) at 345.78° and pressure (right) at 106.44°

The temperature calibration based on equation (3.22) yielded a STD in the test points of 0.06°K and a maximum absolute error of 0.17°K. The pressure calibration is expressed by the following equation.

\[
P = 43.755 C_1 + 519.74 C_1 C_2 + 1088.4 C_1^2 - 160.15 C_1^2 C_2 + 0.44783 C_2 + 0.44783\]  \hspace{1cm} (3.27)
Figure 3-30 Case two: Second order pressure calibration using the scorers from POD

Figure 3-31 Case two: Second order pressure calibration using the 2nd set of scorers and temperature
The third order calibration function is expressed by the following equation.

\[
P = 38.483 C_1 + 516.79 C_1 C_2 + 1036.8 C_1^2 - 93.442 C_2^2 - 163.64 C_1^3 - 11038 C_1^2 C_2 + 0.2689
\]  
(3.28)

Figure 3-32 Case two: Third order pressure calibration using the scorers from POD at 193.46°

Figure 3-33 Case two: Pressure calibration using the scorers from POD at 193.46°
A third order calibration slightly improves the results from a STD of 0.02 for the second order fit to a STD of 0.012. POD was capable of separating the two environmental conditions and retrieve the character of the original functions in the calibration functions. The filtered data exhibits the same results as shown below.

![Graph showing intensity vs wavelength for two cases.](image1)

**Figure 3-34** Case two: Numerically filtered emission

![Graph showing intensity vs wavelength for two cases.](image2)

**Figure 3-35** Case two: Numerically filtered emission eigenvectors for pressure (left) at 106.6° and temperature (right) at 346.06°
Figure 3-36 Case two: Second order pressure calibration using the 2nd set of scorers and temperature

Summary

1. Remarks made for case one are applicable for case two

2. A calibration surface was required for the pressure. This is, however, should not be taken as universal for all paint formulations. The artificial data was generated based on a particular paint characteristics and other formulations may should different characteristics that would allow for one dimensional calibration functions. Factors controlling this calibration are spectral cross-talk, spectral overlap, PSP temperature sensitivity, and its mathematical character (i.e., linear, exponential, etc.)

3. Accuracy of the calculated pressure using POD is a function of degree of cross-talk between the luminophors, degree of temperature dependence in the PSP probe, and the magnitude of noise in the signal. Spectral overlap is easily resolved by the POD as long as the higher wavelength emission does not absorb lower wavelength emission. If absorption occurs, then the problem is classified as cross-talk rather than simple spectral overlap.
CHAPTER 4
LAMINAR CHANNEL FLOW

This chapter presents theoretical discussion of the channel flow experiment. The chapter starts with the simple isothermal pressure driven flow and then examines the non-isothermal solution. Only the isothermal case theoretical solution is presented. The non-isothermal cases are discussed and the governing equations for the solution are presented without solution. The non-isothermal solution is presented as an aid for future work.

Preface

A channel flow experiment is employed in this work because it provides a well controlled environment allowing the calibration technique to be fully assessed. By controlling the boundary temperatures and the mass flow rate inside the channel, hence the pressure gradient, the pressure and temperature fields can be predicted by theory. Fluid flows can be generally categorized as internal or external. Internal flows are entirely bounded by solid surfaces, while external flows are unbounded such as the flow over an airfoil. More precisely, in external flows boundary layer growth on a surface is not externally constricted (i.e., by a solid boundary), while such growth is eventually constrained in internal flows. An internal flow experiment, namely a channel flow, is utilized in this work as an application to demonstrate the feasibility of dual-luminophor PSP in dynamic testing. Internal flows can be classified as either laminar or turbulent depending on the Reynolds number (Re).

\[
\text{Re} = \frac{\rho V D}{\mu} = \frac{V D}{\nu} = \frac{\text{inertia forces}}{\text{viscous forces}}
\]  

(4.1)
where \( \rho \), \( \mu \) and \( \nu \) are the fluid density, dynamic and kinematic viscosities, respectively, \( V \) is the characteristic velocity of the flow, and \( D \) is the hydraulic diameter. Analytical solutions exist for various laminar internal flows including some classical flows that are fundamental in any fluid mechanics text. Turbulent flows do not enjoy such simplicity, hence analytical solutions are not possible and empirical data and semi-empirical theories constitute our main understanding of such flows. Laminar and turbulent flows can be further subcategorized into compressible and incompressible.

A channel flow can be often assumed to be two-dimensional by having a sufficiently large aspect ratio \( \frac{w_d}{h} \gg 1 \); where \( w_d \) is the channel width and \( h \) is the channel height, see Figure 4-2 below. The flow can be driven by a moving wall, gravity, or a longitudinal (flow direction) differential pressure with stationary walls or a combination of two or more driving potentials. The experiment utilized in this work is purely pressure driven, a classical flow known as Poiseuille flow in a rectangle channel.

![Figure 4-1 Channel flow schematic.](image-url)
The following discussion is after (Fox and McDonald, 1998; Panton, 1996; Schlichting, 1979; White, 1974). Typically, the received flow entering the channel is modeled as a slug flow (i.e., uniform parallel velocity) free of vorticity, which is rather difficult to establish in actual applications. In the experiment carried out in this work, the main difficulty lies in assuring a uniform channel height. The channel height is 0.25 mm, therefore, paint thickness variation and channel deformation are possible sources of channel height variation. Further, paint roughness could alter the surface characteristics causing early transition in the channel. In channel flow, the center flow particles are accelerated by the pressure forces, and hence the pressure must have a negative gradient.
along the flow direction while fluid particles adjacent to the solid boundaries experience viscous effects (i.e., friction) that cause a velocity deficit in the regions near the walls known as the boundary layers. These layers grow as the flow advances downstream deforming the initial uniform velocity profile. As the flow velocity is brought to zero at the wall by the no-slip condition and is gradually increased away from the wall, this velocity deficit dictates an increase in the centerline velocities for an incompressible flow to maintain a constant mass flow rate. Further, the development of this boundary layer causes a longitudinal reduction of the inviscid core of the flow. After a specific length known as the hydrodynamic entrance length the two layers (top and bottom) merge forming the famous parabolic velocity profile. At this point, the flow has become completely viscous and has reached a fully developed stage. Velocity gradients in the axial direction of the flow vanish and the velocity reaches its maximum value at the centerline. Further, the pressure gradient is a constant in the axial direction as it balances the viscous forces in the developed region of the channel.

This analysis is a reasonable approximation under the condition that the aspect ratio is large enough. If the height of the channel is comparable to the channel width then four boundary layers, instead of two, interact and the hydrodynamic entrance length may be longer and is determined by the longer dimension as shown in Figure 4-3. Furthermore, the axially fully developed flow would possess a lateral gradient and the maximum velocity would vary across the flow. In the case of the high aspect ratio channel the viscous effects would need a very long channel length for the lateral boundary layers to diffuse and merge and hence relatively short channels can be modeled accurately neglecting end-wall effects.
A key dimensionless parameter of the flow is the Reynolds number. Experiments have shown that for typical engineering applications such a flow would transition from laminar to turbulent around $\text{Re}_h \approx 1400\ (2300$ for smooth circular channels). However, much higher transition values are attainable by careful fabrication of the channel surface to ensure the lowest friction coefficient possible and more importantly through meticulous attention to the incoming flow to prevent any disturbances in the upstream approaching flow. Further, the velocity profile in the fully developed region is subject to the flow condition (i.e., laminar or turbulent). Turbulent flows have “fuller” velocity profiles due to the mixing effect, which is the case for most practical channel flows except highly viscous flows. The hydrodynamic length is hence obviously a function of the Reynolds number and for laminar flow is (Shah and London, 1978):
This empirical relation shows that for laminar flows entrance lengths can be about 100 \( h \). Turbulent flows, however, are independent of the Reynolds number (Incropera, 1981) and show much shorter entrance length on the order of 25-40 \( h \) (Fox and McDonald, 1998), once more, due to the enhanced mixing.

Although this classical problem may strike the reader as a simple fluid mechanics problem, research efforts are still ongoing to formulate a more accurate solution that accounts for entrance effects and turbulence as well as heat transfer issues. For example, in the design of flow-intake devices, accurate pressure loss analysis in the entrance region is detrimental. A more recent application that relies on accurate channel flow analysis is microchannels, where the length of the channel is shorter than the entrance length (Kohl et al., 2005; Muzychka and Yovanovich, 1998). Validating the dual-luminophor paint in the fully developed region will enable the extension of the paint application to the entrance lengths as well to more complicated flows/geometry.

**Isothermal Flow Solution Mathematical Derivation**

The governing equations for a channel flow in rectangular coordinates are:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0
\]  \( (4.3) \)

Navier-Stokes equations for a Newtonian fluid with constant viscosity:

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] - \frac{\partial P}{\partial x} + \rho g_x
\]  \( (4.4) \)
\[ y - \text{momentum} \]
\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right] - \frac{\partial P}{\partial y} + \rho g_y \tag{4.5}
\]

\[ z - \text{momentum} \]
\[
\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = \mu \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] - \frac{\partial P}{\partial z} + \rho g_z \tag{4.6}
\]

**Assumptions/Justifications**

1. **Two-dimensional flow with zero cross flow/gradients** \( w = 0, \frac{\partial ( )}{\partial z} = 0 \): The aspect ratio of the channel \( \frac{w}{h} \gg 1 \) is large enough such that the flow is only in the x-direction and the channel length is relatively short, thus preventing viscous effects, due to the sidewalls, to laterally diffuse into the channel, except very near the sidewalls.

2. **Parallel laminar incoming flow**: The low velocity approaching flow is treated through a series of turbulence damping layers (aluminum and packaging foam) to even and damp out any fluctuations in the velocity and turbulence induced by the flow path. Further, the mass flow rate is investigated experimentally (Chapter 5) showing a linear relation with respect to the pressure drop validating laminar conditions for the entire channel (Panton, 1996).

3. **Steady laminar incompressible flow** \( \frac{DP}{Dt} = 0 \): Mass flow rate well controlled and measurements acquired under steady conditions (i.e., no transient measurements). Mass flow rate (as described in Chapter 5) is low enough that the Mach number is below 0.3 (incompressible) and the Reynolds number is below 1400, ensuring laminar conditions. This further implies that the velocity divergence is zero (i.e., continuity equation).

4. **Body forces are only due to gravity and act in the negative z-direction.**

Hence, the reduced system of equations is:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4.7}
\]

\[
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \frac{\partial p}{\partial x} \tag{4.8}
\]
\[ \rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] - \frac{\partial p}{\partial y} \quad (4.9) \]

\[ 0 = -\frac{\partial p}{\partial z} + \rho g_z \quad (4.10) \]

**Boundary Conditions**

The boundary conditions for a pressure driven flow between two rigid infinitely wide parallel plates are:

\[ u = v = 0 \quad @ \quad y = 0 \& h \quad (4.11) \]

The boundary conditions are the no-slip for the axial velocity component and no velocity penetration at the wall, i.e., zero vertical velocity component.

**The Fully Developed Region (Poiseuille Flow)**

In the fully developed region further simplification allow for an exact solution for laminar flows. As the term “fully-developed” implies, the velocity profile ceases to vary in the flow directions. Only important equations/solutions are presented in this chapter.

The details of the solution can be found in Panton (1996), White (1991), and Schlichting (1979) and is provided in Appendix-B. The velocity solution is:

\[ u(y) = -\frac{h^2}{2\mu} \frac{\partial P}{\partial x} \left[ \frac{y}{h} - \left( \frac{y}{h} \right)^2 \right] \quad (4.12) \]

This is the famous Poiseuille parabolic profile with a maximum velocity value occurring at the centerline \( y = \frac{h}{2} \).

\[ u_{\text{max}} = -\frac{h^2}{8\mu} \frac{\partial P}{\partial x} \quad (4.13) \]

It is often convenient to non-dimensionalize the velocity by the maximum or mean velocity in order to have a universal representation of the velocity profile.
\[
\frac{u}{u_{\text{max}}} = 4 \left[ \frac{y}{h} - \left( \frac{y}{h} \right)^2 \right]
\] (4.14)

Figure 4-4 Fully developed velocity profile for an incompressible laminar channel flow

The channel height is 0.01” making any velocity measurement inside the channel to verify the velocity profile rather difficult. Anemometry probes (e.g. hotwire) would not be suitable due to flow interference. Although probes with wire diameter on the order of micrometers exist; the probe itself is much larger in dimension and hence will impose on the flow. Particle Image Velocimetry (PIV) measurements would also be close to impossible. Seeding particles on the order of micrometers are available allowing for PIV measurement without having concerns about the relative size of the seeding particles relative to the channel height. Condensation on the glass is, however, a concern. Nonetheless, incense has been used successfully in the literature to evade seeding particles problems (Holman, 2006). The main difficulty with PIV measurements in the channel is the optical alignment of the light sheet. The measured section of the channel is
relatively long (10.18”) and hence any slight misalignment in the light sheet would result in an incorrect measurement of the flow field. As shown in Figure 4-5, if the light sheet is misaligned, the measured flow field would not correspond to the flow field at a certain channel elevation, \( u(y = \text{constant}) \), rather the flow field inside the channel at different elevation, \( u(y \neq \text{constant}) \).

![Figure 4-5 Hotwire and PIV measurement schematic in the channel](image)

Even though the velocity could have not been measured experimentally, it can be estimated from the mass flow rate, which is a controlled variable in this experiment. Further, once the velocity profile is known other quantities can be obtained via simple relations. The volume flow rate \( Q \) information is essential in the characterization of any fluid process. It is easily computed by integrating the velocity over the cross sectional area.

\[
Q = \int_{0}^{w} \int_{0}^{h} u \, dy \, dz = \frac{wh^3}{12\mu} \frac{\partial P}{\partial x} \tag{4.15}
\]
This relation will be used to validate laminar flow conditions in the channel flow and estimate the Mach and Reynolds numbers. As seen in the equation, the volume flow rate is linearly related to the pressure drop in the fully developed region for laminar flows. This linear behavior is not valid for turbulent flows (Panton, 1996). Once the volume flow rate is known, the mean velocity is readily obtained.

\[
\frac{Q}{wh} = \frac{h^2}{12\mu} \frac{\partial P}{\partial x} \quad (4.16)
\]

Note that the average velocity is two-thirds the maximum velocity for a rectangular channel; however, the average velocity is only one-half the maximum velocity for circular cross sections. This is due to higher frictional forces as result of the larger effective wetted perimeter. Equations (4.15) and (4.16) are utilized to verify flow conditions. The latter provides a mean velocity and hence the Mach and Reynolds numbers. At the maximum flow rate of 100LPM (0.0017 m³/sec) and standard conditions, the centerline pressure gradient in the fully developed region (9.18") was measured experimentally to be 7.3 psi (50318 Pa), which yields an average velocity of:

\[
u_m = \frac{h^2}{12\mu} \frac{\partial P}{\partial x} = \frac{(0.00025)^2}{12(1.82e-05)} \left( \frac{50318}{0.23317} \right) \\
\approx 62 \left( \frac{m}{\text{sec}} \right) \quad (4.17)
\]

Using the flow rate information:

\[
u_m = \frac{Q}{wh} = \frac{0.0017}{(0.1016)(0.00025)} \\
\approx 65 \left( \frac{m}{\text{sec}} \right) \quad (4.18)
\]
The discrepancy in the calculated average velocity based on the measured mass flow rate and the centerline pressure gradient indicates that the channel is deforming (see Chapter 5), hence the pressure gradient is not constant across the channel.

![Isothermal case: Centerline pressure gradient along the channel showing the deviation of the experimental results from the theoretical prediction.](image)

Using equation (4.15), the calculated flow rate from the measured centerline pressure gradient is:

$$ Q = \frac{wh^3}{12\mu} \frac{\partial P}{\partial x} = \frac{(0.1016)(0.00025)^3}{12(1.82e-05)} \left( \frac{50318}{0.23317} \right) $$

$$ Q = 0.00157 \left( \frac{m}{\text{sec}} \right) $$

This value is 6% lower than the experimental measurement from the mass flow controller. There was some insignificant leaks between the aluminum and glass plates, however, the discrepancy is mainly due to channel deformation. Increasing the channel
height from 0.25 mm to 0.25527 mm yields identical results for the flow rate calculated from equation (4.19) and the experimental value, and hence the two calculated mean velocity values from equations (4.17) and (4.18) are matched. The modified channel height represents an average channel height and is ~2% (5 µm) higher relative to the no-flow channel height. One possible way to accurately estimate the channel deformation is to install an array of pressure taps at the beginning and end of the channel. The pressure taps will provide transverse pressure profile, which in turn will indicate any height variation. The calculated mean velocity based on the modified channel height is:

\[ \mu_m \bigg|_{h = 0.25527 \text{ mm}} = 64.38 \left( \frac{m}{\text{sec}} \right) \]  

(4.20)

Thus, the nondimensional numbers are computed to yield:

\[ M \approx 0.2 \]  

(4.21)

\[ \text{Re} \approx 1100 \]  

(4.22)

These values confirm initial assumptions of incompressible and laminar flow, respectively. Further, the pressure gradient behavior was observed experimentally and showed a linear relation with respect to mass flow rate.

**Non-Isothermal Fully Developed Flow**

In the previous analysis isothermal conditions where assumed, but more importantly viscosity was presumed constant allowing for the decoupling of the momentum and energy equation (neglecting free convection). The following analysis is based on Incropera (1981). The term “fully-developed thermal region” for flows with convection heat transfer is based on a non-dimensional temperature difference as opposed to the actual temperature. When this dimensionless temperature is independent of \( x \) the
flow is termed fully developed. Hence the fully developed thermal condition is:

\[
\frac{\partial}{\partial x} \left[ \frac{T_w(x) - T(x,y)}{T_{w}(x) - T_{m}(x)} \right] = 0 \tag{4.23}
\]

where \( T_w \) is the wall temperature and \( T_m \) is the flow mean temperature. If the boundary condition is a constant temperature along the channel \( \frac{\partial T_w}{\partial x} = 0 \), then the fully developed condition reduces to:

\[
\frac{\partial T}{\partial x} = \frac{(T_w - T)}{(T_w - T_m)} \frac{\partial T_m}{\partial x} \tag{4.24}
\]

while for a constant heat flux we obtain:

\[
\frac{\partial T_w}{\partial x} = \frac{\partial T_m}{\partial x} \tag{4.25}
\]

or alternatively:

\[
\frac{\partial T}{\partial x} = \frac{\partial T_m}{\partial x} \tag{4.26}
\]

The foregoing results clearly show that knowing the mean temperature is essential in describing the temperature distribution throughout the fully developed region of the channel. The temperature distribution at the surface \( \frac{\partial T}{\partial x} \) is the quantity sought after in this work as a mean of validating the measured temperature profiles by the paint. However, the surface temperature is not constant due to forced convection effects and there is no heat flux at the surface, rather a constant heat flux at \( x = 0 \) and \( L \) and at \( z = \pm \frac{W_d}{2} \). The boundary conditions are clearly more involving than the simple cases of constant heat flux and constant temperature. A simple energy balance would yield the following expression for the mean temperature in terms of convection.
\[ \frac{dT_m}{dx} = \frac{W}{m c_p} h(T_w - T_m) \] (4.27)

Obviously, this still calls for a known temperature distribution or heat flux over the surface, which is neither available nor simple. In order to estimate the surface and mean temperatures, the temperature profile \( T(x, y) \) must be then determined by solving the energy equation. The energy equation for a Newtonian fluid in rectangular coordinates is:

\[ \rho \frac{De}{Dt} + p \nabla \cdot V = \nabla \left( k \cdot \nabla T \right) + \mu \Phi \] (4.28)

where \( e \) is the internal energy per unit mass, where \( \Phi \) is customarily called the dissipation function and is always positive definite as viscosity dissipates energy from the system.

For the fully developed viscous region in the channel flow problem, using the same assumptions used in the velocity derivation, the energy equation reduces to:

\[ \rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right) = \left( u \frac{\partial p}{\partial x} \right) + k \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right\} + \mu \left( \frac{\partial u}{\partial y} \right)^2 \] (4.29)

The temperature boundary and initial conditions are (Figure 4-7):

\[
\begin{align*}
T(x, y) &= T_0 \quad @ x = 0, y \\
T(x, y) &= T_w(x) \quad @ x \geq 0, y = 0 \\
h_{\text{glass}} \frac{\partial T(x, y)}{\partial y} &= h_{\text{air}} \left( T_{w,up} - T \right) \quad @ x \geq 0, y = h \\
h_{\text{metal}} \frac{\partial T(x, y)}{\partial y} &= h_{\text{air}} \left( T_{w,bottom} - T \right) \quad @ x \geq 0, y = 0 \\
\frac{\partial T(x, y)}{\partial t} &= T_i - \left( \frac{T_1 - T_2}{L} \right) x \quad @ t = 0; \quad x, y = 0
\end{align*}
\] (4.30)
These boundary conditions represent a balance between forced convection and conduction at the upper and lower walls, a uniform temperature profile at the beginning of the channel, while the lower wall having an interface boundary condition with the metal. The problem is not easily realized because the bottom boundary condition \((v = 0)\) is an interface boundary. This means that convection due to the flow and conduction in the metal are interfacing and constantly updating each other, hence the two problems must be numerically solved simultaneously until steady conditions are realized. The conduction problem for the metal plate is governed by the general heat equation shown below.
\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \left( \frac{\rho c_p}{\kappa} \right) \frac{\partial T}{\partial t}
\] (4.31)

where \( q \) represents energy generation within the plate. Conduction and radiation on the outside boundaries can be neglected as forced convection in the channel is more significant. The two problems interface the terms \( \frac{\partial T}{\partial x} \) and \( \frac{\partial^2 T}{\partial x^2} \). For each iteration equation (4.31) is solved first with the temperature profile at the interface updated by the solution for one time step, then equation (4.29) is solved with the updated boundary and its solution updates the interface boundary. The numerical solution is reiterated until convergence occurs. The governing equations represent the 2-dinemsional solution in \( x \) and \( y \) coordinates. Realistically, the temperature is also varying in the \( z \)-direction, hence the problem is 3-dimansional. Besides the obvious intricacy of the problem, it is further beyond the scope of this work and thus is not further pursued. Nonetheless, the theory can be numerically solved to validate the experimental results for future work.
CHAPTER 5
EXPERIMENTAL SETUP

This chapter describes the experimental setup and the data acquisition process, including hardware specifications and design. It starts by describing the hardware and the different devices utilized. The latter part presents the optical setup and key issues involved with the imaging system, such as image registration due to thermal expansion.

Hardware Description

Channel

The channel is comprised of two rectangular plates (14” x 6” x 1"): one aluminum and the other is window (float)-glass for optical access. The two plates are clamped together with a U-shape hard gasket sandwiched in between them creating the channel cavity. The thickness of the gasket determines the channel height; however, C-clamps are used to attach the aluminum and glass plates which would affect the channel height as well due to localized clamping profile instead of a uniform profile. The gasket used in this experiment is 0.01” (0.25 mm) thick, which then constitutes the channel height. The channel height was chosen to maximize the pressure gradient along the channel.

Compressed air is supplied through two large tanks which are charged by a 300HP rotary screw compressor (Quincy QSI-1000) capable of supplying 950 SCFM at 200 psig. The compressor capacity well exceeds the required stagnation pressure of ~ 50 psi. Moisture in the compressed air is removed via dual-desiccant dryers (Hydronix). This is a crucial requirement as moist air could lead to condensation on the glass, especially when heating and cooling are applied. The air channels into the lab through a series of 3” and
1” lines that lead to the mass flow controller MFC (AALBORG – GF C471S). The mass flow controller plays a central role in the success of the experimental work since maintaining constant flow rates, and consequently pressure levels, is vital for accurate calibration. The MFC has a maximum flow rate of 3.531 SCFM or 100 SLM (Standard Liter per Minute). An excitation voltage of 12V is supplied to the MFC, while controlling the flow rate is accomplished via an analog output signal from a National Instrument (NI) 12-bit 100KS/sec data acquisition card (DAQPad- 6020E) attached to an NI BNC 2120 board. The MFC has an input voltage range of 0-5V and a calibration constant of 0.05 V/SLM. A circuit attached to the MFC supplies an output voltage providing feedback to monitor the stability of the flow rate.

![Mass Flow Controller (AALBORG)](image)

Flow then enters a cylindrical PVC stagnation chamber (2” D x 4” L) through the side where the incoming flow velocity is significantly reduced on the opposite wall of the cylinder before diffusing into the chamber. The chamber is mounted perpendicularly with respect to the channel and hence the flow at the exit of the chamber is forced to turn 90°
degrees. The chamber-to-exit area ratio is ~ 79, ensuring stagnation conditions in the chamber. A static and stagnation pressure ports are inserted into the chamber to ensure stagnation conditions and provide stagnation back pressure information. At the exit of the stagnation chamber a ½ inch thick circular piece of packaging foam lays flushed with the surface of the aluminum plate to abate any turbulence generation due to the 90° degrees turn. A rectangular strip of aluminum foam is embedded in a recessed area downstream of the packaging foam at the beginning of the channel to damp out any left over turbulence.

Figure 5-2 Channel flow schematic: Top view of channel shown.
Two cylindrical straits ahead of the foam house two 120V 600W electrical heaters (Omega CHROMALOX CIR-2066) controlled by a variable voltage controller (POWERSTAT). Two water channels serve to circulate cold antifreeze providing temperature boundary control. One channel is longitudinally positioned at the top of the channel (relative to the Figure 5-2), while the other is perpendicularly laid at the very end of the channel. Anti-freeze is circulated in the water channels via a cooler (Haake G/D3). The entire assembly rests on three hot-plates (Thermolyne Cimarec 375 W) providing heat generation from the bottom side of the channel. The aluminum plate has twenty T-type thermocouples (Omega) and seventeen 0.02” diameter pressure taps arranged on the surface as shown in Figure 5-2. All thermocouples and pressure taps are flushed with the surface and are free of burr. Thirteen pressure taps line up the centerline of the channel and provide pressure information for calibration. Four additional pressure taps are utilized to examine the accuracy of the calibration and the two-dimensionality of the flow. The surface of the aluminum plate has a mirror-surface polish (600 grit) in order to prevent premature transition of the flow to turbulence. The paint is applied on top of the aluminum using an OPTIMA 5000 gravity spray-gun with a 0.25” nozzle head. No primer was used in the experiment to avoid more thickness uncertainty. After the paint is applied and allowed enough time to cure a Terry cloth is used to smooth the surface. The surface is further treated with compressed air to remove any residual paint or primer loose particles. The paint is manually applied in a systematic and steady motion to minimize any surface thickness variation. Furthermore, the amount of paint applied is optimized by applying just enough paint layers to yield enough intensity emission upon excitation without having an average thickness that would impact the overall channel
height. The last two steps (i.e., uniform and optimal thickness) are important for ensuring the most agreement upon comparing the theoretical and measured pressure distributions.

Figure 5-3 (A) Channel resting on heaters (B) Side view of channel showing the 1 inch glass plate (C) Backside of channel showing thermocouples, pressure taps, stagnation chamber and tubing connecting from the top water channel to the cooler.
The above figure shows a close-up of the beginning of the channel. As seen in the figure, the greatly decelerated flow from the stagnation chamber exists through a layer of packaging foam. The flow impinges on the glass plate and is then forced to enter the channel through the aluminum foam strip. The flow is then accelerated inside the channel under differential pressure forces meeting ambient conditions at the exit. The stagnation pressure is a function of the mass flow rate, channel height, and viscous forces inside the channel.

**Channel Deformation**

In this experiment large pressure gradients were sought to ensure significant intensity variation due to the paint’s shallow gradient near and above atmospheric conditions. The pressure forces due to the flow inside the channel caused the glass and aluminum plate to deform altering the effective cross sectional area of the channel.
Several attempts were made before choosing 1 inch thick glass as the material for the optically accessible plate. A 3/8 inches window-glass piece was fitted in an aluminum frame and bonded to the frame by epoxy. The glass cracked upon clamping the aluminum frame to the other metal plate under flow-on conditions. It is conceivable that the local stress by the C-clamps combined with the pressure inside the channel lead to glass cracking. The second attempt utilized Plexiglas, as it a possesses higher modulus of elasticity. Plexiglas, however, is more prone to scratching and is often coated with chemicals that luminesces upon excitation with blue/UV radiation. The Plexiglas proved durable and did not crack; nonetheless, Plexiglas has relatively high thickness tolerance (± 0.05 mm) and therefore created an uneven channel height which was clearly observable in the pressure distribution along the flow direction (Figure 5-6). Plexiglas would be the optimal choice if the channel height is large enough for such small surface
variations to be insignificant. For that reason (i.e., surface flatness) a 3/8 inches glass plate was reused without inserting it into any frame, but rather a full size plate was directly clamped to the metal plate.

Figure 5-6 Channel deformation due to pressure forces using Plexiglas comparing the theoretical pressure profile (green line) to experimental profile (blue circles).

The 3/8” glass deformed almost twice as much as the 1” glass as shown in Figure 5-8. This confirms that pressure deviation from theory is due to glass deformation. Using simple mechanics of equations (Beer and Johnston, 1981) one can deduce deflection profile as shown in Figure 5-7. The plate is modeled as a wide beam fixed at one end and free at the other. This simplification is undertaken for two reasons: first, the deformation of the beam would be higher than that of a plate clamped at three sides, hence this estimate would represent a worst case scenario. Second, modeling the deformation of a plate is rather involved and requires finite element calculations. The load (i.e., pressure) is modeled according to the theoretical Poiseuille flow solution. For a 6” wide beam the
deflection is maximum at the beginning of the channel, which is consistent with experimental observations.

Figure 5-7 Calculated percentage channel height deformation (relative to no-flow conditions of 0.01”) along the centerline for 1” plate with a maximum flow rate of 100LPM, 23 psi static pressure at the beginning of the channel and 0.01” channel height.

Bernoulli’s equation predicts that the pressure scales as the velocity square and continuity predicts a linear relation between velocity and area (i.e., channel height for a fixed width). Therefore the maximum variation from the figure above is ~ 3.2%. Figure 5-8 shows 4% maximum deviation in the pressure.
Figure 5-8 Glass deformation (a comparison between theoretical and experimental results): 3/8” glass (top) and 1” glass (bottom) with percentage pressure deviation from theory in top-left corner of each plot. Flow from right to left
Optical Setup

A schematic of the optical setup for the experiment is shown in Figure 5-9. The CCD camera utilized is a Photometrics CH250A coupled with a 14-bit 200 kHz A/D converter (CE200A) for pixel voltage conversion. The CCD chip (SITe502CB/AR-X) is 512 x 512 pixels thinned scientific grade array with a FWC of 329,000 e⁻. The camera head is thermoelectrically cooled to minimize thermal noise in the CCD chip output. In order to capture the entire channel with the CCD camera, a Nikon NIKKOR 50 mm lens is attached to the front of the camera. Small focal lengths produce a “fish-eye” effect in the image observed as warping of the image biased towards the center of the image.

Figure 5-9 Experimental optical setup showing the relative position of the CCD camera and excitation sources with respect to the channel
A filter wheel is positioned in front of the lens to resolve the spectral range. In order to select the appropriate filters a spectroscopic analysis of the paint emission is needed. The following two figures show the paint spectral response due to pressure and temperature.

Figure 5-10 Pressure response of dual-luminophor paint at 293°K (Kose 2005)

Figure 5-11 Temperature response of dual-luminophor at 14.7 psi (Kose 2005)
Based on the spectral data above, four bandpass interference filters (Melles Griot) are selected. As shown in the following four figures, the four filters have a center frequencies of 550nm (03 FIV 044), 600nm (03 FIV 046), 650 nm (03 FIV 048), and 700nm (03 FIV 058).

Figure 5-12 First bandpass interference filter: 550 nm, 40 ± 8 nm FWHM; dia. = 50 mm (www.mellesgriot.com)

Figure 5-13 Second bandpass interference filter: 600 nm, 40 ± 8 nm FWHM; dia. = 50 mm (www.mellesgriot.com)
Observing both the emission spectra and the bandpass filters plots, the first filter (550 nm) should only capture the temperature emission. The second filter overlaps with the beginning of the pressure emission and hence is expected to observe some of pressure emission. The third and fourth filters (650 and 700 nm, respectively) capture the pressure emission due to pressure and temperature effects as well. The former overlaps with the
main pressure emission, while the latter observes only the tail of the emission. This intensity variation between the different filters would necessitate different exposure times to maintain similar SNR throughout the spectra.

Figure 5-16 Filters arrangement relative to paint spectral response to temperature (left) and pressure (right).

The excitation sources are two blue super-bright LED’s by ISSI that is approximately centered at 465nm. The LED’s are cooled with an internal fan to maintain lamp stability. LED’s were positioned far enough from the channel to create a more uniform illumination field and avoid sharp deviations in the intensity ensuring optimal SNR over the entire image. Care should be practiced to also avoid having the reflection of the LED’s on the image. This is simply accomplished by locating the LED’s far enough laterally. All optical surfaces are cleaned with an appropriate solvent to remove any oil residue or dust particles.

Data was collected using a Pentium III computer and NI LabView software. The software interfaced with Photometrics software V++ to acquire the images. A NI SCXI
1000 frame accessorized with a NI SCXI-1303 module was used to read all thermocouple data and a Mensor digital pressure gauge (Model 15000) collected pressure readings (f.s.: 25 psia and accuracy: 0.025% f.s.). A mechanical pressure scanner was used to switch between the appropriate pressure channels.

Figure 5-17 (A) Filter wheel (B) Blue LED ISSI LM2 (excitation source) (C) CCD camera and filter wheel assembly
**Image Registration**

Image registration is the process of aligning two images to sub-pixel accuracy often utilized in PSP application in wind tunnels. In real experiments model movement is inevitable due to aerodynamic loads, tunnel vibration, and camera movement. PSP luminophors disperse inhomogenously in binders and tend to aggregate when embedded in nano-spheres in dual-luminophor systems (Kose, 2005). This creates sharp spatial gradients due to this uneven dispersion. As optical distortion occurs, e.g. due to model movement, these variations create a “rough” image effect in the normalized image (run/ref), as shown in Figure 5-18.

![Figure 5-18 Effect of image registration on SNR (A) Unregistered image (B) Registered image. Bottom plots show a horizontal section of the images above.](image-url)
When an image moves, the intensity value of each pixel changes according to the following relation (ignoring pixel-to-pixel variations of the CCD array):

\[
I_{i,j\mid m} = a_1 I_{i,j} + a_2 I_{i\pm1,j\pm1}
\]

(5.1)

where \( I_{i,j\mid m} \) is the modified intensity value, \( a_1 \) and \( a_2 \) are the confidents of translation.

The translation coefficients depend on the amount of shift from the reference image. For a non-integer pixel shift \((n < 1)\) both coefficients are non-zero and scale linearly with pixel shift, hence a linear interpolation is possible. For example, assume that an image moves 20% in the \( x \)-direction and 60% in the \( y \)-direction, Figure 5-19, with pixel intensities of 10, 100, 30, and 70 counts for pixels 1, 2, 3, and 4, respectively.

Figure 5-19 Pixel intensity shift on the CCD array
The modified intensity of pixel 3 is a combination of the intensity values of pixels 1, 2, 3, and 4. Assuming homogenous electron flux at each pixel surface, the relative contribution of each pixel to the shifted pixel depends on the relative area of each pixel that overlaps with the shifted pixel multiplied by its intensity value. Therefore, for our example the modified intensity of pixel 3 would be:

\[ I_{3m} = 10(0.48) + 100(0.12) + 30(0.32) + 70(0.08) = 32 \]  \hspace{1cm} (5.2)

Model deformation is not limited to pure translation; models warp, rotate, and even stretch/shrink. For such deformations more complicated image registration routines are needed, such as projective and polynomial transformation. To enable initial prediction of type(s) of transformation needed, models are fitted with reference points that map the frame of the model. Following image acquisition, image pairs are compared utilizing these reference points and appropriate transformation can be determined.

**Thermal Expansion**

Materials undergo thermal expansion as their temperature increases. Experiments show that for small temperature changes (< 100°C) the change in length \( \Delta L \) is directly proportional to the change of temperature \( \Delta T \) (Young, 1992). Further, the length of the object relates directly to the amount of thermal expansion. The overall relation may be expressed as:

\[ \Delta L = \alpha L_0 \Delta T \]  \hspace{1cm} (5.3)

where \( \alpha \) is the coefficient of linear thermal expansion (1/°C) and \( L_0 \) is the initial length of the object. It is important to note that this relation is an approximation valid only for small temperature variations, while for higher temperature changes \( \alpha \) would depend on the initial temperature of the material and the relation ceases to hold its linearity.
The channel is mated between aluminum and window-glass plates of equal thickness. The following table lists the values of $\alpha$ for both materials, notice that $\alpha$ for aluminum is an order of magnitude higher than for glass.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\alpha \left( 10^{-6}/^\circ C \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>24</td>
</tr>
<tr>
<td>Glass</td>
<td>4.9</td>
</tr>
</tbody>
</table>

The material of the channel experiences relatively small temperatures changes (~50 °C), however, this change is sufficient to induce an unfavorable increase in both the length and width of the channel requiring image registration. The channel rests vertically on a series of hot-plates, and hence any increase in width will yield a spatial translation in the positive y-direction. Aluminum possess higher thermal expansion coefficients, we calculate the vertical shift in the image to be:

$$\Delta L_y = \frac{24}{10^6} (152.4)(50) = 0.183 \text{mm}$$

and similarly for the longitudinal length we obtain:

$$\Delta L_x = \frac{24}{10^6} (355.6)(50) = 0.426 \text{mm}$$

The CCD camera produces a 512 x 512 pixels image and the imaged longitudinal portion of the channel is 10.2 inches, which yields a spatial resolution of:

$$\frac{512 \text{ pixels}}{259.08 \text{ mm}} = 1.976 \left( \frac{\text{pixels}}{\text{mm}} \right)$$

which translates to a maximum pixel shift of:

$$1.976 \left( \frac{\text{pixel}}{\text{mm}} \right) \times 0.183 \text{(mm)} = 0.362 \text{(y-pixel)}$$

$$1.976 \left( \frac{\text{pixel}}{\text{mm}} \right) \times 0.426 \text{(mm)} = 0.842 \text{(x-pixel)}$$
Thermal expansion for free objects is zero at the geometric center and maximum at the peripheries. For our channel, each mm is equivalent to 0.988 pixels, and hence the bottom row of intensities corresponding to the bottom side of the channel would not move, while the top row would move by a total of 0.362 pixels with a linear translation in between the top and bottom row. Therefore a linear transformation is required and a constant spatial shift is not sufficient. Image registration effects are more pronounced for the temperature filters (i.e., 550nm and 600nm) due to the nature of the TSP probe and its dispersion in the binder.

Figure 5-20 Pixel shift due to thermal expansion from bottom to top of the channel
CHAPTER 6
RESULTS AND ANALYSIS

This chapter presents the results of the experimental work and provides discussion and analysis of the results. The analysis presented herein is based on the POD analysis presented in chapter 3. The data is analyzed using the MATLAB codes provided in Appendix-A.

Procedure and Test Cases

The following sub-sections address key aspects of the experimental procedure, data acquisition and the various test cases.

Exposure Times

In order to maintain similar SNR throughout the spectrum, different exposure times are needed to utilize the camera FWC at each wavelength. The following table shows the distinct variation between exposure times, note that the boundaries of the spectrum require the longest exposure times and hence possess the lowest intensity emission levels.

Table 6-1 Exposure times for the different filters

<table>
<thead>
<tr>
<th>Filter</th>
<th>550nm</th>
<th>600nm</th>
<th>650nm</th>
<th>700nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exposure time (ms)</td>
<td>3400</td>
<td>560</td>
<td>1500</td>
<td>3500</td>
</tr>
</tbody>
</table>

Sixteen frames were acquired per image to ensure low shot noise. Given the relatively long aggregate exposure times, it is essential that the environmental conditions do not drift during the acquisition process. Each case was allowed enough time to reach steady state conditions and the mass flow controller was able to maintain steady pressure levels throughout the experiment. In addition, the pre and post-run pressure and temperature values are recorded and are compared to ensure stability of conditions for
each run. The pressure deviation is shown in Table 6-2 through an entire set of measurement. Further, the first and last filter conditions are compared to determine any variation, which would be accounted for in the uncertainty analysis.

Table 6-2 Pressure drift throughout the exposure times (case five)

<table>
<thead>
<tr>
<th>Quantity/Filter</th>
<th>550nm</th>
<th>600nm</th>
<th>650nm</th>
<th>700nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum absolute error (psi)</td>
<td>0</td>
<td>0.0063</td>
<td>0.012</td>
<td>0.016</td>
</tr>
<tr>
<td>Mean Error (psi)</td>
<td>0</td>
<td>-0.0028</td>
<td>-0.0076</td>
<td>-0.012</td>
</tr>
<tr>
<td>STD (psi)</td>
<td>0</td>
<td>0.0033</td>
<td>0.0044</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Data Matrix

Seven cases were examined to fully characterize the paint and calibration process. The experiments have three variables: pressure (i.e., flow), temperature, and the direction of the temperature gradient with respect to the flow direction.

Table 6-3 Test matrix

<table>
<thead>
<tr>
<th>Temperature Profile</th>
<th>Flow OFF</th>
<th>Flow ON</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>Reference</td>
<td>Case 1</td>
</tr>
<tr>
<td>Longitudinal (flow direction)</td>
<td>Case 2</td>
<td>Case 3</td>
</tr>
<tr>
<td>Transverse (perpendicular to flow)</td>
<td>Case 4*</td>
<td>Case 5</td>
</tr>
<tr>
<td>Oblique</td>
<td>Case 6*</td>
<td>Case 7</td>
</tr>
</tbody>
</table>

* Results not shown to avoid redundancy

The response of the pressure probe PtTFPP, i.e., Stern-Volmer relation (SV), Figure 6-1, shows a very shallow gradient of ~ 3 %-psi⁻¹ near atmospheric conditions, and
exhibits lower gradients when embedded in the dual-luminophor system (Kose, 2005). Kose (2005) did not characterize the response at higher than atmospheric pressure levels due to hardware limitations; however, the trend of the SV predicts an even shallower gradient. In order to ensure significant intensity variations the pressure gradient along the channel is preset and maximized by the mass flow controller to about $6.4263 \pm 0.0087$ psi (95% confidence level).

![Figure 6-1 Stern-Volmer relation for dual-luminophor (PtTFPP-Ruphen/PAN nanospheres in poly-t-BS-co-TFEM) at different temperature levels (Kose, 2005)](image)

Hardware and flow conditions imposed a limitation on the maximum obtainable temperature gradient. The cooler pump is not capable of circulating water fast enough to maximize heat transfer, in particular, in the case of temperature gradients perpendicular or oblique to the flow field. From Fourier law of conduction (Equation 4.24) for constant heat flux and thermal conductivity coefficient, as the length of the object along which
heat is conducted increases, the temperature difference increases, and vice versa. This limited the maximum temperature gradient perpendicular to the flow at any given cross section to ~ 10° C for cases 4-7.

**POD Analysis**

The details of the POD analysis are presented in chapter 3. Only key elements of the analysis are discussed below for convenience. The analysis of the artificial data showed that in a spectral dataset depending on two independent factors (i.e., pressure and temperature) a base factor shapes the spectrum and a second “complementary” factor scales the different regions of the spectrum. The scores associated with the latter are thus used for calibrating the independent factors. The calibration functions are distinguished and realized by applying a different rotation to the complementary factor and hence the scores. The complementary factor is extracted from a dataset that contains both intensity and corresponding environmental conditions, hence two calibration data sets are possible. Upon decomposing and rotating the calibration dataset the coefficients for the calibration function are extracted and a calibration set of scores are determined. This set of scores is then implemented as the guiding calibrator for unknown conditions. The intensity information for unknown environmental conditions is added to the calibration data and POD is applied to the entire data matrix. Only the first data columns containing calibration information are used to determine the appropriate rotation angle. As the unknown conditions differ from the calibration conditions a different rotation angle is obtained by searching the data space in a 360° hunt.

**Calibration Matrix vs. Test Matrix**

The approach adapted by the author in establishing the calibration function via POD is based on an *in-situ* approach. Specific points on the channel surface contain
pressure taps and thermocouples (Figure 6-2). The normalized intensity values and environmental values from these calibration points are utilized to estimate the calibration functions. The calibration matrix contains both the intensity values and the environmental values arranged as four rows corresponding to the four filters and $n$ columns corresponding to the number of calibration points. The test matrix contains intensity values (normalized) and its columns are added one full set (4 rows x 501 columns) at a time to the right of the calibration matrix columns.

Figure 6-2 Channel image showing thermocouples, calibration pressure taps and pressure test points taps.

The entire matrix is processed by the POD analysis with curve/surface fitting reevaluated for each new combined matrix throughout the full rotation space ($360^\circ$) to optimize the calibration. The calibration and test matrix is shown below in equation (6.1).
The test set has to be collected along the same direction as the calibration points. This means if there is a direction in which the calibration points are collected (i.e., longitudinal or lateral) then the test points to be evaluated must be collected along that direction. Since the pressure calibration points span the longitudinal direction of the channel in order to maximize the calibration envelope, test sets are chosen as the intensity values of the longitudinal lines along the channel. This is important as the test set is added to the calibration matrix and prior to decomposing the combined matrix, it is squared by post-multiplying the matrix by its transpose. This can be thought of as inducing “more weight” around a specific portion of the calibration data causing a false bias on the calibration, and consequently its orientation (i.e., the calibration eigenvector orientation). Therefore, if data points are concentrated around a specific value on the calibration curve, they will cause a shift in the orientation of the calibration eigenvector.
and hence the calibration scores. This rule is not a limitation on POD, rather it serves to optimize computational time only.

Figure 6-3 Effect of adding rows instead of columns to the calibration matrix. Observe the direction of the main eigenvector (black arrow) compared to the calibration eigenvector (orange arrow).

Consider case 3 with temperature and pressure gradients along the flow direction. Lines perpendicular to the flow direction have almost constant temperature and pressure values and hence the eigenvector will significantly deviate from its calibration position if these data lines are inserted in the matrix (instead of line parallel to the flow). This becomes less of a problem as the environmental condition field encompasses significant multidirectional gradients (e.g. cases 6 and 7).
Image Filtering

Shot noise is the main contributor to noise associated with the imaging system. Sixteen frames were acquired per filter to minimize shot noise. For our 14-bit CCD camera, readout noise is 13.3 e⁻/frame (R.M.S.) at 200 kHz. The total readout noise is then equal to 13.3 x 16 = 212.8 e⁻. Averaging 16 frames at an average intensity of 191,000 e⁻ (10,000 counts) yields shot noise, i.e., uncertainty, of 1748 e⁻ (91.5 counts).

![Figure 6-4 Intensity percent error due to shot noise vs. number of frames (14-bit CCD camera)](image)

Read out and shot noise for the CCD camera are comparable at average intensity levels of 57300 e⁻ (3,000 counts). Lower intensity levels yield read-out-noise limited intensity and higher intensity levels are shot noise limited. For this work, sufficient intensities, averaging 191,000 e⁻ (10,000 counts), make the data shot noise limited. The
relative uncertainty for 16 frames is 0.057 %, compared to 0.08 % and 0.23 % for 8 and 1 frames, respectively. Therefore, very modest image filtering was required to smooth pixel variation after image registration. Unless otherwise stated, the averaging square size utilized is 3 x 3 pixels.

**Results**

The following subsections detail the results for the different cases. The results are presented in more details in earlier cases and any repeated results or conclusions are briefly referenced back to previous cases. Each case represents a unique condition that examines a particular aspect of the POD calibration process. A general uncertainty analysis is presented at the end of this chapter. Temperature effects influence the accuracy of the pressure calibration, thus, the uncertainty in the pressure differs between cases due to the different temperature gradients orientation and magnitude. The pressure is examined at four discrete points on the surface of the channels containing pressure taps. The results are tabulated and compared to the theoretically predicted uncertainty at the end of the uncertainty section. Temperature uncertainty is not emphasized for each case as it remained constant for the various cases and it has been the observation of the author that typical uncertainty reported in the literature for TSP paints are similar to the presently examined dual-luminophor system. Observed calibration temperature uncertainty (95% confidence) through the experimental work is 0.6°C for cases 2 and 3, and 1.5°C for cases 5 and 7. The thermocouples have an error of ±0.5°C or 0.4% f.s., whichever is greater, and a STD of 1°C or 0.75% f.s., whichever is greater. This yields temperature uncertainty due to thermocouples of 2°C. The error in temperature was independent of the pressure levels of each case, but varied with the orientation of the temperature gradient relative to pressure. Further, temperature accuracy of a degree or so,
which is typical of TSP (Cattafesta et al, 1998), is more than sufficient for most practical applications. However, dual-luminophor systems have shown higher errors (~2-3°C) in the temperature estimates (Hardill et al, 2002, Mitsuo et al, 2003.) In addition to the previous temperature uncertainties, temperature deviation during image acquisition (Table 6-4) added to the total uncertainty.

<table>
<thead>
<tr>
<th>Quantity/Filter</th>
<th>550nm</th>
<th>600nm</th>
<th>650nm</th>
<th>700nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum absolute error (psi)</td>
<td>0</td>
<td>0.52</td>
<td>0.49</td>
<td>0.59</td>
</tr>
<tr>
<td>Mean Error (psi)</td>
<td>0</td>
<td>-0.30</td>
<td>-0.34</td>
<td>-0.39</td>
</tr>
<tr>
<td>STD (psi)</td>
<td>0</td>
<td>0.16</td>
<td>0.17</td>
<td>0.23</td>
</tr>
</tbody>
</table>

**Case One (Isothermal Longitudinal Pressure Gradient)**

The objective of this case is to isolate the pressure response of the paint in the absence of temperature gradients (Table 6-3). This will identify the spectral response of both luminophors (P/TSP) due to pressure and more importantly expose any pressure dependence in the temperature probe. Further, this will help guide the calibration process of the pressure in the presence of temperature gradients by identifying the main character (i.e., functional form of calibration) of the pressure response.
Figure 6-5 Case one: Pressure profile (bottom), (a) percent pressure deviation from theory (psi), (b) temperature profile (thermocouples) over the plate (8” x 3”)

(a) Percent deviation from Theory (psi)

(b) °C Temperature profile (thermocouples)
Figure 6-6 Case one: Normalized intensity ($I_{run}/I_{ref}$): 550nm [left] --- 600nm [right]

Figure 6-7 Case one: Normalized intensity ($I_{run}/I_{ref}$): 650nm [left] --- 700nm [right]
Figure 6-5b shows a maximum temperature variation of 0.5°C over the channel, which is within the thermocouple 95% confidence uncertainty estimate of 2°C, ensuring isothermal conditions. The pressure profile has a maximum deviation of ~ 2% from the Poiseuille-flow theoretical solution. The deviation in the pressure is attributed to the bowing of the glass plate due to pressure forces. As detailed in chapter 5, this was evident from the pressure profile using a 3/8 inches thick glass plate compared to the 1 inch plate used in the experiments. The deviation is still tolerable in terms of pressure variation. Nonetheless, a more important effect is observed in the intensity images due to the thickness of the glass and its deformation. As the glass plate deforms it creates different light transmission characteristics between the paint and the CCD camera due to light refraction as the incident angle is non-zero for the run images. The reference images suffer no such deformation and hence upon normalizing the images “false” effects are observed, especially at the ends (inlet and exit) of the channel where the incident angle is highest. These effects are shown in Figure 6-8.

Figure 6-8 Case one: Centerline pressure response of dual-luminophor at different bandpass wavelengths
The intensity level at the beginning of the channel does not decrease with the pressure increase; rather it is nearly flat at the higher wavelength and curves up at lower wavelengths. This observation was suspected to be due the SV behavior, however upon investigating lower pressures the effect persisted. An attempt was made to acquire reference conditions using nitrogen to replicate the glass deformation without imposing a pressure field observable by the paint (the paint responds to oxygen concentration not pressure as described in chapter 2). This attempt was not successful because the normalized image was terribly noisy. This noise was not due to image registration or high shot noise in the signal. The results, Figure 6-9, show significant increase in the level of “noise” in the normalized image for which the reason is not apparently clear.

Figure 6-9 Normalized intensity for case one (650nm) using N₂ (left): (A) Reference image acquired with no flow and exposure time of 1500ms (B) Reference image acquired with nitrogen flow and exposure time of 185ms. Exposure time for run image is 1500 ms.
The image suffers from a “spottiness” effect rather than a noisy grainy effect observed when image registration is an issue. Figure 6-10 shows the microscopic aggregation of the Ruphen/PAN particles in the PtTFPP / poly-t-BS-co-TFEM binder and the observed inhomogeneous dispersion. Kose (2005) noted that this variation was barely obvious on the macroscopic scale. In the channel flow experiment this was slightly observed in the normalized image of the 550nm filter (Figure 6-6). It should be noted that the intensity false color resolution was refined to show the inhomogeneity in the image. Further, an intensity variation of -1.6% is observed in the image due to convection. This intensity variation corresponds to ~1°C (irresolvable by thermocouples) according to the reported sensitivity value of -1.45%/°C. The heterogeneous nature of oxygen/nitrogen diffusion in the binder matrix combined with small temperature variations are a plausible cause of the spottiness in the normalized image. It is rather impossible to remove all of the oxygen in the binder (Weaver et al, 1999), and hence any left over oxygen in the binder would yield spurious results.

Figure 6-10 Fluorescence image using epifluorescence microscope of Ruphen/PAN particles emission (dispersed into the PtTFPP / poly-t-BS-co-TFEM binder) at 560nm. The image shows the heterogeneous dispersion of the Ruphen/PAN particles in the poly-t-BS-co-TFEM matrix (Kose 2005).
Figure 6-8 shows less than 0.5% variation in the centerline temperature intensity (550nm) with pressure, while the 600nm filter observes some of the pressure variation. The 650 and 700nm filters, with more emphasis in the former (~12% decrease in intensity), predominantly capture pressure information. This agrees with theoretical prediction of insignificant oxygen quenching effects for the temperature phosphor.

**POD Calibration**

Intensity information is analyzed through POD yielding the following results:

![Figure 6-11 Case one: Normalized intensity (I_ref/I_run)](image)

POD extracted two eigenvectors as shown in the following two figures. The spectrum in Figure 6-11 shows a very small intensity variation of about 1.5% at 550nm compared to 16.5% 650nm. POD produces linear sum of factors, hence two factors are needed, one to account for the main variant in the spectrum and another to shape and scale the overall spectrum throughout the spectral range. However, in the absence of temperature effects, both vectors can be used for calibration.
Figure 6-12 Case one: Calibration eigenvectors at 174.56° degrees (left) and eigenvalues (right)

Figure 6-13 Case one: Product of first (left) and second (right) eigenvectors and eigenvalues of calibration

The two are identical in shape; however, the corresponding eigenvalues shape them differently. As seen in Figure 6-13, the first product (1st eigenvector) maps the main variant, i.e., pressure, while the second adds to it to reconstruct the original spectrum scaling the different regions of the spectrum as shown in the following figure. The second eigenvector is thus the calibration vector.
Figure 6-14 Case one: Product sum of eigenvectors and calibration eigenvalues showing 1% variation at 550nm and 15% at 650nm

The pressure calibration is a linear fit of the intensity data, equation (6.2), with the coefficients extracted at the appropriate rotation angle. The calibration eigenvalues were recalibrated each time a run intensity column (501 data points) was inserted into the matrix to optimize the calibration. Higher-order even functions yielded erroneous results while odd powers indistinctly improved the calibration.

\[
P = a + bC^2 = 53.492 + 789.99C^2
\]  

(6.2)
Figure 6-15 Case one: Calculated centerline-pressure profile by each eigenvector

Figure 6-15 shows the predicted centerline pressure by both eigenvectors at the same rotation angle. The second eigenvector over-predicts the pressure below calibration range and slightly under-predicts the pressure above the calibration range. The intensity flat behavior near the inlet of the channel is evident in the calibration. The calibration adheres to low errors relative to the taps readings except near the entrance. The rise in the predicted pressure near the exit of the channel is expected as the flow experiences a positive area change forcing the flow to plunge below atmospheric pressure to recover atmospheric conditions at the exit.
Figure 6-16 Area change due to chamfer in the glass plate at the exit

The calibration has an average error of 0.0638 psi (0.325 %) and a maximum error of 0.276 psi at the maximum pressure (22.022 psi), with the next highest absolute error of 0.114 psi and an R.M.S. of 0.0072 psi. The uncertainty of the calibration is ±0.19 psi.

Figure 6-17 Case one: Calibration error, absolute (left) and percentage (right)

Table 6-5 Case one: Test-points pressure results. Actual represents pressure tap measurement and POD represents the calculated pressure via POD calibration. The precision error of the Actual readings is ± 0.006 psi.

<table>
<thead>
<tr>
<th>Actual (psi)</th>
<th>POD (psi)</th>
<th>Absolute error (psi) /Percent error</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.03</td>
<td>21.71</td>
<td>0.32 (1.45 %)</td>
</tr>
<tr>
<td>21.35</td>
<td>21.25</td>
<td>0.1 (0.47 %)</td>
</tr>
<tr>
<td>19.22</td>
<td>19.15</td>
<td>0.07 (0.36 %)</td>
</tr>
<tr>
<td>16.77</td>
<td>16.61</td>
<td>0.16 (0.95 %)</td>
</tr>
</tbody>
</table>

The predicted pressure field is shown in Figure 6-18. Four test points are compared to the predicted values and the results show a maximum deviation of 0.351 psi from the actual values and an average error less than 0.72% as shown in Table 6-5.
Figure 6-18 Case one: Calculated pressure field from POD calibration
Figure 6-19 Case one: Calculated pressure from POD calibration vs. pixel index along the channel. Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual pressure taps readings used for calibration.

It is worthy of noting that columns that pass through high intensity points exhibit slightly higher errors. This is because the direction of the calibration eigenvector is slightly altered because of the addition of this high intensity pixel value. These high-intensity pixels correspond to points on the surface of the channel with contaminants that luminesces highly relative to the paint (e.g. large primer particles lodged inside the spray-gun) or points where paint particles have coagulated and formed a high concentration of luminescent particles. These high-intensity points are not eliminated during the normalization process. In the reference image these points reach saturation but they do not in the run images, hence upon normalization they exhibit a different trend relative to surroundings pixels. Nonetheless, these points can be digitally filtered to resolve their adverse contribution.
Case Two (Longitudinal Temperature Gradient)

The objective of this case is to isolate the temperature response of the paint in the absence of pressure gradients. This will further identify the spectral response of the both luminophors (P/TSP) and more importantly expose any temperature dependence in the pressure probe, an expected outcome. Additionally, this will help guide the calibration process of the pressure in the presence of temperature gradients by identifying the temperature dependence character in the pressure response.

Figure 6-20 Case two: Temperature profile (thermocouples)
Figure 6-21 Case two: Normalized intensity ($I_{run}/I_{ref}$): 550nm [left] --- 600nm [right]

Figure 6-22 Case two: Normalized intensity ($I_{run}/I_{ref}$): 650nm [left] --- 700nm [right]
The temperature response of the paint shows a 26% percent variation relative to reference conditions in the intensity (550nm) with temperature, while the 600nm filter observes 24% variation. The 650 and 700nm filters show a strong temperature dependence of 17% and 13.5%, respectively, as expected. The temperature effect on the intensity is linear in all spectral regions.
POD Calibration

Intensity information is analyzed through POD yielding the following results:

Figure 6-25 Case two: Normalized intensity ($I_{\text{run}}/I_{\text{ref}}$)

POD extracted two eigenvectors as shown in the following two figures. The spectrum is shown in Figure 6-25. The two eigenvectors account for the temperature variation and the indirect temperature effect in the pressure spectral range.

Figure 6-26 Case two: Calibration fundamental spectra at 0.5° degrees (left) eigenvalues of calibration (right)
Similar to case 1, the two vectors are identical in shape; however, the corresponding eigenvalues scale them differently. As seen in Figure 6-27, the first product (1st eigenvector) maps the main variant, i.e., temperature, while the second scales the data in the pressure range inversely with respect to the temperature range. The second eigenvector serves by enhancing the temperature effect at 550nm and to a lesser extent at the 600nm range, while reducing the temperature effect at the 650 and 700nm wavelengths, with more damping in the latter. The addition of the two eigenvectors retrieves the calibration spectrum shown below.
The Temperature calibration is a linear fit of the intensity data, equation (6.3), with the coefficient extracted at the appropriate rotation angle. The calibration eigenvalues were recalibrated each time a run intensity column was inserted into the matrix to optimize the calibration. Once more, higher order calibration functions with even powers yielded erroneous results while odd powers insignificantly improved the calibration.

\[ T = a_T + b_T C_1 \]  

(6.3)

The linear character of the calibration function suggests a strong relation between the nature of the temperature field and the calibration character. The predicted temperature field is shown in Figure 6-29, with an average calibration error of 0.3 °C.
Figure 6-29 Case two: Calculated temperature field from POD calibration (image)
Figure 6-30 Case two: Calculated temperature field from POD calibration (values). Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual thermocouple readings used for calibration.

As anticipated, the temperature calibration accuracy is superior due to the linear nature of the temperature field and the absence of significant noise interference. These result will guide the calibration for the following cases keeping in mind to explore a more suitable calibration function to the temperature field.
Case Three (Simultaneous Longitudinal Pressure and Temperature Gradients)

The objective of this case is to study the effect of a parallel pressure and temperature flow fields on the calibration. The pressure and temperature are decreasing longitudinally and hence have an aggregate lessening effect on the intensity rather than an opposing effect.

Figure 6-31 Case three: Pressure profile (bottom), (a) percent pressure deviation from theory (psi), (b) temperature profile (thermocouples)
Figure 6-32 Case three: Normalized intensity ($I_{run}/I_{ref}$): 550nm [left] --- 600nm [right]

Figure 6-33 Case three: Normalized intensity ($I_{run}/I_{ref}$): 650nm [left] --- 700nm [right]
The temperature field in case 2 was exposed to forced convection by the flow yielding case 3. The temperature field was shrunk from 18°C to 11°C difference along the flow direction. The corresponding decrease in the intensity is linear with the decrease in temperature in the 550nm range with a slope of ~1.45%/°C. The intensity at 650nm decreased by 22 % compared to 12 % for the adiabatic case, which translates to -1.1%/°C, which is the value (at 14.7 psi) reported by Kose (2005). As the paint characteristics were not characterized at higher than atmospheric pressures by Kose (2005), there is no current reference for comparison, however, the values are in agreement with the trend of increasing temperature dependence with increasing pressure as reported by Kose (2005). Further, Kose (2005) used spectroscopic results, while this work uses filtered (integrated) intensity data. The pressure profile still shows the bowing behavior but the temperature field has increased the thermodynamic pressure, increasing the pressure deviation from the adiabatic theoretical solution.
POD Calibration

Intensity information is analyzed through POD yielding the following results:

Figure 6-35 Case three: Inverse of normalized intensity ($I_{\text{run}}/I_{\text{ref}}$)

POD extracted two eigenvectors, however, unlike the previous two cases; two different rotation angles are needed to extract the pressure and temperature information. The rotation angles are 0° degrees and 1° degree for the temperature and pressure, respectively. This is a significant result as it manifests the fact that the pressure and temperature fields are parallel. The spectrum in Figure 6-35 shows comparable intensity variation at 550nm and 650nm.
As seen in Figure 6-36, the second eigenvector has a near zero value at 650nm, hence the scores associated with the first eigenvector mainly scale the pressure. From Figure 6-37, the contribution of the second product at the 650nm wavelength is only 0.7% relative to the contribution of the first product. This is because the temperature and pressure gradients are in the same direction and hence the temperature effect on the pressure in this case can be thought of as a bias, particularly since temperature effects were observed from case two to be nearly linear in the pressure region. It is interesting to
observe that the effect of the second eigenvector is the smallest at the 650nm, while it is comparable at the other wavelength, including the 700nm wavelength. This emphasizes the fact that the pressure main emission is at the 650nm, while the 700nm captures only the tail of this emission. Further, it suggests a reemergence and extension of temperature probe emission at the higher wavelengths.

Figure 6-38 Case three: Product sum of eigenvectors and pressure calibration eigenvalues showing 18% variation at 550nm and 27% at 650nm

The pressure calibration is a linear fit of the intensity data, similar to equation (6.2), with the coefficient extracted at the appropriate rotation angle \( a = -6.1312, b = 562.6 \). The pressure calibration has an average error of 0.0295 psi and a maximum error of 0.129 psi at the maximum pressure (21.851 psi), with the next highest absolute error at 0.0646 psi. The test points exhibit a maximum error of 0.241 psi (1.45%) and average error of
0.153 psi (0.826 %). The predicted pressure error has not increased compared to case 1, further, high pressure (>21 psi) accuracy has improved.

Table 6-6 Case three: Test-points pressure results. Actual represents pressure tap measurement and POD represents the calculated pressure via POD calibration. The precision error of the Actual readings is ± 0.006 psi.

<table>
<thead>
<tr>
<th>Actual</th>
<th>POD</th>
<th>Absolute error (psi) /Percent error</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.85</td>
<td>21.77</td>
<td>0.08 (0.37 %)</td>
</tr>
<tr>
<td>21.17</td>
<td>21.09</td>
<td>0.08 (0.38 %)</td>
</tr>
<tr>
<td>19.03</td>
<td>18.82</td>
<td>0.21 (1.10 %)</td>
</tr>
<tr>
<td>16.62</td>
<td>16.37</td>
<td>0.25 (1.50 %)</td>
</tr>
</tbody>
</table>

The Temperature calibration is acquired in a similar manner to pressure with a slightly different rotation angle. In this case the second eigenvector has insignificant contribution at the 600nm wavelength. Further, only the scores associated with the first eigenvector collapse to single line as shown below. The temperature error is similar to case 2. The eigenvector mapping the pressure had to be slightly rotated by a degree to point towards the other variant, i.e., temperature.

Figure 6-39 Case three: Temperature calibration fundamental spectra at 0° degrees (left) eigenvalues of calibration (right)
Figure 6-40 Case three: Product of first (left) and second (right) eigenvectors and eigenvalues of temperature calibration

The reconstructed spectra is shown below.

Figure 6-41 Case three: Product sum of eigenvectors and temperature calibration eigenvalues showing 23% variation at 550nm and 26% at 650nm
Figure 6-42 Case three: Calculated pressure field from POD calibration (image)
Figure 6-43 Case three: Calculated temperature field from POD calibration (image)
Figure 6-44 Case three: Calculated pressure field from POD calibration (values). Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual pressure taps readings used for calibration.

Figure 6-45 Case three: Calculated temperature field from POD calibration (values). Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual thermocouple readings used for calibration.
Case Five (Perpendicular Temperature Gradient with Longitudinal Pressure Gradient)

In this case the temperature gradient is imposed perpendicular to the flow direction. The temperature profile is nearly constant along each lateral station. This will allow the investigation of POD’s ability to distinguish the relative angle between the pressure and temperature gradient. Further, the accuracy of the results would assess the ability of POD to fully separate the variables. Unlike case 3, the intensity variation would not be constant across the channel, while the pressure is assumed constant laterally, due to the temperature gradient.

Figure 6-46 Case five: Temperature profile (thermocouples)
Figure 6-47 Case five: Pressure profile (bottom), (a) percent pressure deviation from theory (top)
Figure 6-48 Case five: Normalized intensity ($I_{\text{run}}/I_{\text{ref}}$): 550nm [left] --- 600nm [right]

Figure 6-49 Case five: Normalized intensity ($I_{\text{run}}/I_{\text{ref}}$): 650nm [left] --- 700nm [right]
It is observed that image registration is more important in this case compared to previous cases. This is due to the nature of the pressure and temperature fields and thermal expansion as discussed in chapter 5. A constant image translation is not sufficient and a translation gradient is required. The metal expansion is small enough that a linear gradient suffices. This is confirmed by the “noise” level in the different filtered images. The noise is highest at the 550nm wavelength and lowest at the 750nm filter. The TSP particles are more pronounced (i.e., observed in the spectral images) at the lower wavelength and hence a lateral temperature gradient effects on image registration is most observed at these wavelengths. The following figure shows this effect, notice the speckles the 550nm image causing sharp gradients between neighboring pixels, thus making image misalignment more pronounced.

Figure 6-50 Case five: Speckling in the 550nm image (left) and a comparison to the 700nm filter
The centerline temperature profile is evident across the spectral range. The 550nm filter reveals the extent of invariability in the temperature profile along the centerline station. Although the temperature profile is not exactly constant along each lateral station of the channel, it varies insignificantly enough (< 4°C) for the character of the POD to be fully distinguished.
POD Calibration

Intensity information is analyzed through POD yielding the following results:

Figure 6-53 Case five: Inverse of normalized intensity ($I_{\text{run}}/I_{\text{ref}}$)

POD extracted two eigenvectors form the temperature calibration matrix, shown below, similar to case 2. The eigenvectors represent the pressure and temperature as was the case before. At the minimum-error rotation angle the scores of the second eigenvector scale the temperature. The scores associated with the first eigenvector can also be used at the same rotation angle to scale the temperature, however, producing nosier calibration. This is because this set of scores represent the intensity variation in the pressure spectral range due to temperature change.
Figure 6-54 Case five: Temperature calibration fundamental spectra at 89° degrees (left) eigenvalues of calibration (right)

Figure 6-55 Case five: Product of first (left) and second (right) eigenvectors and eigenvalues of calibration

The optimum rotation angle for calibration is 89° degrees and the mean calibration error is consistently 0.3°C. The reconstructed temperature calibration spectrum is shown below.
The temperature calibration is a quadratic fit of the intensity data, equation (6.4), with the coefficient extracted at the appropriate rotation angle. A linear fit produced less accurate results. Further, the normalized image vectors must be taken in the direction of the temperature gradients (i.e., rows not columns) for the calibration procedure to yield correct values.

\[ T = aC_z^2 + bC_z + c \]  \hspace{1cm} (6.4)

The pressure calibration yielded two eigenvectors with the calibration utilizing the first eigenvector. The calibration mean error was 0.061 psi with a maximum error of 0.24 psi.
Figure 6-57 Case five: Pressure calibration fundamental spectra at 11° degrees (left) eigenvalues of calibration (right)

Figure 6-58 Case five: Product of first (left) and second (right) eigenvectors and eigenvalues of calibration
The pressure calibration is a linear fit of the intensity data similar to previous cases. The results for the test points show a slight increase in the predicted error relative to case 3 (Table 6-7). The robustness of the POD analysis is evident in the results. The relative angle between the two calibration eigenvectors is 78° degrees showing the ability of POD to identify the relative orientation. The relative angle is less than 90° degrees because the temperature field was not constant along lateral stations as discussed above.

Table 6-7 Case five: Test-points pressure results. Actual represents pressure tap measurement and POD represents the calculated pressure via POD calibration. The precision error of the Actual readings is ± 0.006 psi.

<table>
<thead>
<tr>
<th>Actual</th>
<th>POD</th>
<th>Absolute error (psi) /Percent error</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.48</td>
<td>22.34</td>
<td>0.14 (0.63)</td>
</tr>
<tr>
<td>21.81</td>
<td>22.05</td>
<td>0.24 (1.10)</td>
</tr>
<tr>
<td>19.62</td>
<td>19.40</td>
<td>0.22 (1.12)</td>
</tr>
<tr>
<td>17.04</td>
<td>16.89</td>
<td>0.14 (0.88)</td>
</tr>
</tbody>
</table>
Figure 6-60 Case five: Calculated temperature from POD calibration (image)
Figure 6-61 Case five: Calculated temperature from POD calibration (values). Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual thermocouple readings used for calibration.

The temperature field in Figure 6-60 shows the angularity (i.e., relative angle between) of the temperature with respect to the flow direction. Observing the figure above, it is evident that the temperature calibration was not able to accurately observe the sharp gradient between pixel indices 165 and 265; however, the values are off by about 1.2°C. The sharp gradient is caused by a 0.5” gap between the hot-plates. A temperature gradient of 2°C is possible within the 2” distance separating the two thermocouples.

From Fourier law, a temperature gradient of 2°C over a distance of 2” with thermal conductivity of ~200 W/m.K for aluminum, and a cross sectional area of 2 inch², the wattage needed is about 10W. This shows that such gradient is possible as...
Figure 6-62 Case five: Calculated pressure from POD calibration (image)
Figure 6-63 Case five: Calculated pressure from POD calibration (image). Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual pressure taps readings used for calibration.

Observing the figure above, the pressure calibration was able to observe the lateral increase in the pressure due to temperature increase. Ideal gas law predicts roughly 0.5 psi difference laterally due to thermodynamic effects. However, the calibration seems to slightly over predict the pressure close to the channel exit. Unfortunately, no pressure validation was available for exit conditions; nonetheless, the calibration shows consistent accuracy within the calibration domain.
Case Seven (Oblique Temperature Gradient with Longitudinal Pressure Gradient)

In this case the temperature gradient is oblique relative to the flow direction. The temperature profile varies along each lateral station. This will further confirm the POD’s ability to distinguish the relative angle between the pressure and temperature gradient. Further, the accuracy of the results would assess the ability of POD to fully separate the variable. This is the most challenging case for the POD calibration and would set the benchmark for its success.

Figure 6-64 Case seven: Temperature profile (thermocouples)
Figure 6-65 Case seven: Pressure profile (bottom), (a) percent pressure deviation from theory (top)
Figure 6-66 Case seven: Normalized intensity ($I_{\text{run}}/I_{\text{ref}}$): 550nm [left] --- 600nm [right]

Figure 6-67 Case seven: Normalized intensity ($I_{\text{run}}/I_{\text{ref}}$): 650nm [left] --- 700nm [right]
POD Calibration

Intensity information is analyzed through POD yielding the following results:

![Graph showing normalized intensity vs. wavelength.](image)

Figure 6-68 Case seven: Inverse of normalized intensity ($I_{run}/I_{ref}$)

POD results are in complete agreement with previous results and the extracted eigenvectors and eigenvalues are shown below. The calibration functions are once more linear for the pressure and quadratic for the temperature with similar calibration accuracy compared to case 5.
Figure 6-69 Case seven: Temperature calibration fundamental spectra at 93° degrees (left) eigenvalues of calibration (right)

Figure 6-70 Case seven: Product of first (left) and second (right) eigenvectors and eigenvalues of calibration
Figure 6-71 Case seven: Product sum of eigenvectors and calibration eigenvalues showing 31% variation at 550nm and 28% at 650nm

Figure 6-72 Case seven: Pressure calibration fundamental spectra at 53° degrees (left) eigenvalues of calibration (right)
Figure 6-73 Case seven: Product of first (left) and second (right) eigenvectors and eigenvalues of calibration

Figure 6-74 Case seven: Product sum of eigenvectors and calibration eigenvalues showing 8% variation at 550nm and 5% at 650nm

POD successfully recognized the change of orientation between the pressure and temperature from 78° degrees in case five to 40° degrees in case seven. Further, the calibration accuracy is consistent with previous cases.
Table 6-8 Case seven: Test-points pressure results. Actual represents pressure tap measurement and POD represents the calculated pressure via POD calibration. The precision error of the Actual readings is ± 0.006 psi.

<table>
<thead>
<tr>
<th>Actual</th>
<th>POD</th>
<th>Absolute error (psi) /Percent error</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.28</td>
<td>23.42</td>
<td>0.14 (0.60)</td>
</tr>
<tr>
<td>22.51</td>
<td>22.54</td>
<td>0.03 (0.13)</td>
</tr>
<tr>
<td>20.01</td>
<td>19.76</td>
<td>0.25 (1.25)</td>
</tr>
<tr>
<td>17.10</td>
<td>16.88</td>
<td>0.22 (1.29)</td>
</tr>
</tbody>
</table>

Figure 6-75 Case seven: Calculated temperature from POD calibration (image)
Figure 6-76 Case seven: Calculated pressure from POD calibration (image)
Figure 6-77 Case seven: Predicted temperature from POD calibration (values). Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual thermocouple readings used for calibration.

Figure 6-78 Case seven: Predicted pressure from POD calibration (values). Each curve represents a longitudinal line of pixels along the channel. Actual represents the actual pressure taps readings used for calibration.
The Case for POD

A fundamental question that is inherently tied with the principle of using POD to calibrate the paint is why POD? What advantage does POD offer compared to simpler techniques? Such techniques could be as simple as *a priori* calibration that encompasses a wide range of temperatures expected during the experiment. Other techniques suggested in the literature (Bell et al 2001, Woodmansee et al 1998) are presented again in Table 6-9. To assess the robustness and advantage/disadvantage of using POD, two different calibration techniques are utilized. The first technique is a simple curve fit of the intensity data at the pressure spectral emission region (650nm). The second technique uses an intensity ratio of the intensities from the temperature (550nm) and pressure spectral regions.

<table>
<thead>
<tr>
<th>Calibration technique</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
</table>
| *In situ* isothermal calibration: Acquire wind-off images at atmospheric conditions | ▪ The simplest approach  
▪ Suitable for isothermal conditions | ▪ Temperature effects can not be accounted for  
▪ Calibration R.M.S. error on the order of 1.25 psi for PtTFPP/FIB (Bell et al, 2001), with pressure range of 29 – 72.5psi.  
▪ Calibration r.m.s. error on the order of 0.2 psi for ODU PSP (Woodmansee et al 1998), with pressure range of 1 – 15.4psi. |
| *In situ* calibration: Acquire wind-off images immediately after wind-on conditions | ▪ Easy technique  
▪ Temperature effects absorbed in calibration coefficients | ▪ Need to ensure temperature stability  
▪ Wind tunnel must be turned off.  
▪ Limitation on number of images that can be acquired for reference condition, especially for long exposure times, due to temperature drift from wind-on conditions.  
▪ Significant temperature variation between different parts of model |
<table>
<thead>
<tr>
<th>Calibration technique</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
</table>
| **A priori** calibration | ▪ Few pressure taps scattered over the model are needed  
▪ More controlled environment for calibration  
▪ More efficient for practical applications (less time spent in the wind tunnel) | ▪ Separate experiment for calibration in required  
▪ Need to have a prior knowledge of expected pressure and temperature levels in the experiment to optimize the calibration process  
▪ Pressure taps need to encompass the pressure range, and hence any sharp pressure gradients could be unobservable  
▪ Temperature information must be obtained on the model using TSP (symmetric models with symmetric flow conditions). Asymmetric models or flow conditions can be mapped for temperature using dual-luminophor systems  
▪ Same batch of paint must be used for both calibration and experiment  
▪ Calibration functions are typically biquadratic in pressure and temperature (Bell et al 2001), necessitating a minimum of 9 points of calibration points with both pressure and temperature information  
▪ Calibration r.m.s. error on the order of 0.021psi for OPTROD PSP (Bell et al 2001), with pressure range of 0.75 - 14.7psi |
| **Hybrid technique (K-fit calibration)** | ▪ Simpler than *a priori* calibration (only one temperature condition is calibrated for *a priori*)  
▪ Useful for ideal paints when extrapolation beyond pressure taps range is needed  
▪ Superior to *in-situ* isothermal calibration | ▪ Inferior to *in-situ* calibration  
▪ In some experiments, the *K* factor is pressure dependent, yielding a more complicated calibration  
▪ Calibration r.m.s. error on the order of 0.99 psi for ODU PSP (Woodmansee et al 1998), with pressure range of 1 – 15.4psi. |
Curve Fit of the 650nm Intensity Data and Intensity Ratios

A simple approach to account for temperature effects in pressure measurements is to normalize the pressure emission by that of the temperature. This approach would effectively eliminate the temperature dependence under the postulation that both pressure and temperature luminophors show identical temperature sensitivity (i.e., percent intensity variation/°C). The dual-luminophor system used in this work (PtTFPP-Ruphen/PAN / poly-t-BS-co-TFEM) shows a temperature sensitivity of 1.4%/°C and 1.1%/°C for Ruphen (temperature) and PtTFPP (pressure), respectively.

Figure 6-79 Temperature sensitivity (integrated intensity/°K) of the temperature and pressure luminophors at 14.7 psi (Kose 2005).

The proximity of the temperature sensitivity of the two luminophors makes the intensity-ratio approach a reasonable approximation for run conditions for which temperature variations are small (~3-5°C) relative to reference conditions. However, this approach is not suitable for higher temperature variations from the reference condition.
Case Three

First we attempt fitting pressure intensity data from the 650nm range to pressure tap data. This approach is an *in-situ* isothermal approach for which temperature effects are not accounted for in the calibration.

![Graph of pressure calibration using intensity ratio at 650nm](image1)

**Figure 6-80 Case three: Pressure calibration using intensity ratio at 650nm**

![Graph of calculated pressure using intensity ratio at 650nm](image2)

**Figure 6-81 Case three: Calculated pressure using intensity ratio at 650nm**
For case three, this approach yielded comparable results to POD calibration results, but only within the calibration range. This is because the temperature gradient is decreasing linearly along the flow direction, thus the overall outcome is a DC offset to the intensity field. Next we try normalizing the 650nm data by the other two temperature wavelengths (i.e., 550nm and 600nm).

![Normalized Intensity (I550nm / I650nm) vs Pressure (psi)](image)

Figure 6-82 Case three: Pressure calibration using intensity ratio ($I_{550}$ / $I_{650}$)

The ratio ($I_{550}$/$I_{650}$) exhibits a double-value behavior for higher pressures and hence is not valid as a calibration scheme. On the other hand, the ratio ($I_{600}$/$I_{650}$) exhibits no such behavior and could be possibly valid for calibration. Results of this calibration are shown in Figure 6-84 and show the inaccuracy of the calibration, especially at high pressure levels and outside the calibration envelope.
Figure 6-83 Case three: Pressure calibration using intensity ratio ($I_{600} / I_{650}$)

Figure 6-84 Case three: Calculated pressure using intensity ratio at ($I_{600} / I_{650}$)
Case Five

Figure 6-85 Case five: Pressure calibration using intensity ($1 / I_{650}$)

Figure 6-86 Case five: Calculated pressure using intensity ($1 / I_{650}$)
Figure 6-87 Case five: Pressure calibration using intensity (I_{600} / I_{650})

Figure 6-88 Case five: Calculated pressure using intensity (I_{600} / I_{650})
Figure 6-89 Case five: Calculated pressure using intensity \((I_{600}/[I_{650}]^2)\)

Figure 6-88 and Figure 6-89 show an interesting trend of improved accuracy by changing the power of the denominator \((I_{650})\).

Figure 6-90 Case five: Calculated pressure using intensity \((I_{600}/[I_{650}]^{1.5})\)
Figure 6-91 Case five: Calculated pressure: (A) Intensity $I_{600}/[I_{650}]^{1.5}$ (B) POD calibration.

Table 6-11 Case five: Comparison between test-points pressure results for POD and intensity-ratio Actual represents pressure tap measurement and POD represents the calculated pressure via POD calibration. The precision error of the Actual readings is ± 0.006 psi.

<table>
<thead>
<tr>
<th>Actual</th>
<th>POD</th>
<th>$I_{600}/[I_{650}]^{1.5}$</th>
<th>Absolute error (psi)</th>
<th>Percent error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>POD</td>
<td></td>
</tr>
<tr>
<td>22.48</td>
<td>22.34</td>
<td>21.80</td>
<td>0.14 (0.63)</td>
<td></td>
</tr>
<tr>
<td>21.81</td>
<td>22.05</td>
<td>21.67</td>
<td>0.24 (1.10)</td>
<td></td>
</tr>
<tr>
<td>19.62</td>
<td>19.40</td>
<td>19.37</td>
<td>0.22 (1.12)</td>
<td></td>
</tr>
<tr>
<td>17.04</td>
<td>16.89</td>
<td>17.39</td>
<td>0.14 (0.88)</td>
<td></td>
</tr>
</tbody>
</table>

The results show a maximum absolute error of 0.68 psi, which is much higher than the POD calibration. Evidently, the simple intensity-ratio technique is not capable of accounting and compensating for temperature effects in the pressure data. The only case for which the technique yielded comparable results was case three. Nonetheless, even in
such simple interaction between pressure and temperature (case three), the intensity-ratio technique yields notably inaccurate results outside the calibration range.

**Case seven**

Figure 6-92 Case seven: Calculated pressure using intensity ($1 / I_{650}$)

Figure 6-93 Case seven: Calculated pressure using intensity ($I_{600} / I_{650}$)
Figure 6-94 Case seven: Calculated pressure using intensity \((I_{600} / [I_{650}]^2)\)

Figure 6-95 Case seven: Calculated pressure using intensity \((I_{600} / [I_{650}]^{1.55})\)
Figure 6-96 Case seven: Calculated pressure: (A) Intensity $I_{600}/[I_{650}]^{1.55}$ (B) POD calibration.

Table 6-12 Case seven: Comparison between test-points pressure results for POD and intensity-ratio. Actual represents pressure tap measurement and POD represents the calculated pressure via POD calibration. The precision error of the Actual readings is ± 0.006 psi.

<table>
<thead>
<tr>
<th>Actual</th>
<th>POD</th>
<th>$I_{600}/[I_{650}]^{1.55}$</th>
<th>Absolute error (psi) /Percent error</th>
<th>POD $I_{600}/[I_{650}]^{1.55}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.28</td>
<td>23.42</td>
<td>22.35</td>
<td>0.14 (0.60)</td>
<td>0.93 (3.99)</td>
</tr>
<tr>
<td>22.51</td>
<td>22.54</td>
<td>22.22</td>
<td>0.03 (0.13)</td>
<td>0.29 (1.28)</td>
</tr>
<tr>
<td>20.01</td>
<td>19.76</td>
<td>19.48</td>
<td>0.25 (1.25)</td>
<td>0.53 (2.65)</td>
</tr>
<tr>
<td>17.10</td>
<td>16.88</td>
<td>17.74</td>
<td>0.22 (1.29)</td>
<td>0.64 (3.74)</td>
</tr>
</tbody>
</table>

The results of case seven are comparable to the previous case (case five). This shows the robustness and effectiveness of the POD approach in calibrating dual-luminophor paints. The calibration approach is universal and is thus applicable to any flow conditions without any fundamental restrictions. However, the POD is more involved, both mathematically and physically, and must be well understood to yield
accurate results. The main issues pertinent to the success of the POD approach were discussed in previous sections. Next we turn our attention to the uncertainty analysis to complete the analysis and assess the accuracy of the POD approach. Although POD results showed high potential for the approach, without identifying the accuracy of the calibration and the results the overall success of the POD would not be complete.

Error Sources and Uncertainty

The first question facing any experimental calibration technique is: how accurate are the results? A calibration approach such as POD is rather involving and hence may contain many sources of errors. Combined, these errors could yield any calibration technique futile. We start by defining error, its sources and then conclude by presenting the uncertainty analysis. The term “error” refers to the magnitude of deviation in the measured value from the true value, or more specifically the estimated value of the true value. Of course, one can never estimate the error in any measurement without knowing the true measured value. On the other hand, the true value is something that we can never know without measuring it, which is inherent some scale of error due to equipment, the experiment, or the experimentalist. One can conclude then that we can never actually measure the true value, and hence we can never know the exact error, either. This makes the entire argument seem circular! However, we can usually estimate the likelihood that a measuring error exceeds some specific value (Beckwith 1993). Experimentalists rely on tools such as statistics to quantify such likelihood within a specific confidence level, say 95%. This means that 95% of the time, we can claim the measurement error of that instrument to be bounded by some limit, while 5% of the time the error exceeds those limits. The error bounds on the measurement are commonly called the uncertainty of that measurement. Error is defined as:
\[ Error = \varepsilon = x_m - x_{\text{true}} \]  

(6.5)

where \( x_m \) and \( x_{\text{true}} \) are the measured and true value of the measurement, respectively.

The uncertainty bounds to within a certain confidence level are thus:

\[ -U_x \leq \varepsilon \leq U_x \]  

(6.6)

where \( U_x \) is the uncertainty in the measurement. Equivalently, the uncertainty can be expressed as:

\[ x_m - U_x \leq x_{\text{true}} \leq x_m + U_x \]  

(6.7)

The first step in quantifying the uncertainty is to identify the various sources of errors. Despite the diversity of error sources, most errors can be placed into two general classes: bias and precision errors. Bias errors are systematic by nature, meaning they repeat themselves with the same magnitude and in the same direction every time a measurement is recorded. Therefore, they can be usually accounted for easily by subtracting this offset from any reading. However, to recognize bias error, a more accurate measuring instrument is needed to quantify its value in the instrument under investigation. Precision errors are inherently random and differ for each successive measurement in both magnitude and direction. Fortunately, this type of error typically averages to zero if enough measurements are recorded. The readings will cluster around a central (i.e., mean) value and spread to some limit around this value creating a distribution. Enough readings are needed to define such distribution. Upon defining the distribution of the error, statistical analysis is employed to estimate the bounds of such error.
Figure 6-97 Bias and precision error

Classification of Error

In general, error categories may overlap and are at times ambiguous. Some errors behave as precision in one case and as bias in others; further, some do not clearly fit into either category. However, we can generally classify errors as follows:

1. Bias errors (systematic)
   - Calibration errors
   - Consistent human errors or defective equipment
   - System resolution limitations
   - Loading errors
2. Precision errors (random)
   - Fluctuating experimental conditions (either inherent to measured quantity or due to external factors)
   - Insufficient system sensitivity
3. Alternating errors (sometimes bias and sometimes random)
   - Instrument backlash, friction, hysteresis
   - Calibration and or test environmental conditions drift
   - Variations in experimental procedure or definitions
We know shed more light on some of these errors that pertain to our experiment starting with bias errors. Calibration errors are an obvious source and are evidently important to this work. A zero-offset is typically required to “tare” the equipment. For instance, the pressure transducer used in measuring the various pressures throughout the experiment is validated for calibration bias against a more accurate instrument (DRUCK), which has some bias itself, but rather much smaller. If the calibration offset is not accurate, then a bias value will be added to all measured values. Moreover, if the instrument is not checked for such bias regularly, such bias error could slip unnoticed. On the other hand, thermocouple readings were simply validated against each other. The channel has 20 thermocouples, and if under no-flow isothermal conditions a thermocouple reading is off by a specific value relative to the remaining thermocouples, then such error is recorded. However, thermocouples usually have a significant STD, and hence deviations within this precision uncertainty fall in the ambiguous category as they may be sometimes random and other times systematic. Resolution limitation can induce a bias error equal to \( \pm \frac{1}{2} \) the minimum possible resolution. In our experiment, the imaging system was superior to pressure and temperature instruments, with the temperature system possessing the poorest resolution. An important sub-category of bias error is the loading error. Loading error results from equipment interference with the measured value. For instance, paint, pressure taps, thermocouples, and the glass plate used in this experiment influenced, to some extent, the flow characteristics. Nonetheless, in our experiment the actual flow was not under investigation, rather, the paint and the calibration procedure. Therefore, such errors can be tolerated in this case, but most definitely not in actual experiments.
Precision errors vary in their sources, but exhibit a different trait than bias errors. Fluctuating experimental conditions can cause such error. CCD camera systems are typically cooled to maintain steady output and low noise. If the camera system is turned on and not allowed enough time to reach steady state conditions, such as the A/D unit which needs to reach the operating temperature of ~ -25°C, precision errors will be introduced into the measured signal. The same argument goes for test cases were a temperature gradient was imposed on the channel. For that reason, environmental conditions were monitored throughout the experiment. It is important to note that, strictly speaking, drifting test conditions are not errors, rather a limitation of the design or an inherent feature of the phenomenon under investigation. However, the same statistical techniques can be applied and such variations can be treated as errors.

The most difficult error type is alternating error due to the ambiguity of the error source and hence the error processing procedure (i.e., subtract a bias, average large enough sample, etc.). In our experiment, the main alternating error was due to temperature effects on the imaging system and the test conditions. Relatively long exposure times were used (~ 8 minutes) for which temperature variation occurred and affected the measurement to some extent. The imaging system had to be calibrated for every case to estimate dark current noise by taking an image with the excitation sources off. The heated flow exhausted into an enclosure that included the CCD camera system and the channel itself. The increased temperature could have influenced the dark current reading and thus the bias calibration. Conversely, the variation of the temperature during such long acquisition time would induce a random error on the measurement. Only the latter was quantified, as its outcome is much higher with respect to dark noise error.
Uncertainty Analysis

As with any experiment, the accuracy of the predicted environmental conditions are limited by the uncertainty in those measurements. The total uncertainty defined in an r.m.s. fashion is:

\[ U_x = \sqrt{B_x^2 + P_x^2} \]  

(6.8)

\[ P_x = \frac{1.96 \sigma_x}{\sqrt{m}} \]  

(95%) for \( m \geq 30 \)  

(6.9)

where \( B \) is the bias error, \( P \) is the precision error roughly estimated as twice the standard deviation \( \sigma_x \) and \( m \) is the number of samples. For a linear function \( y \) of \( n \) independent variables \( x_i \) the total uncertainty is then defined as:

\[ U_y = \sqrt{\left(\frac{\partial y}{\partial x_1} U_{x_1}\right)^2 + \left(\frac{\partial y}{\partial x_2} U_{x_2}\right)^2 + \cdots + \left(\frac{\partial y}{\partial x_n} U_{x_n}\right)^2} \]  

(6.10)

Sajben (1993) discussed predicted pressure measurements via PSP and concluded seven variables (from the Stern-Volmer relation) contributing to the uncertainty in the pressure.

\[ U_p = f(I, I_{ref}, I_a, I_{a,ref}, T, T_{ref}, P_{ref}) \]  

(6.11)

where \( U_p \) is the total pressure uncertainty, \( I \) is the run intensity, the subscripts \( \text{ref} \) and \( a \) denote reference and excitation conditions, respectively, and \( T \) is the temperature. Each variable may have two types of noise embedded within, bias or random. Observing the variables, they can be lumped into three variables, intensity, temperature, and pressure.

We start by examining the uncertainty in the intensity. Intensity uncertainty stems mainly inherent sources: shot noise, dark current, digitizer bias, and readout noise and external sources that are due to signal drift pertinent to test conditions drift. Other minor
sources not addressed in this analysis include paint photodegradation, digitizer resolution, pixel-gain variation, and image registration. All main error sources are precision errors except for dark current noise. Dark current and readout noise are estimated from the camera specification, while digitizer bias is accounted for by subtracting a dark image from both the reference and run images.

Table 6-13 Intensity error sources and values for Photometrics CH250A CCD camera (SI 502AB), 200 kHz, 14 bit A/D for a unity nominal gain value and a measured gain ($\beta$) of 19.1 (e-/ADU)

<table>
<thead>
<tr>
<th>Error sources</th>
<th>Average Value</th>
<th>Error Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dark current (e^-/pixel/sec @ -25°C)</td>
<td>6</td>
<td>P</td>
</tr>
<tr>
<td>Digitizer bias (cts)</td>
<td>~110</td>
<td>B</td>
</tr>
<tr>
<td>Readout noise (e-/RMS)</td>
<td>13.3</td>
<td>P</td>
</tr>
<tr>
<td>Shot noise (cts for m number of images)</td>
<td>$\sqrt{I}$</td>
<td>P</td>
</tr>
</tbody>
</table>

The total dark current uncertainty $U_d$ is exposure time dependent and hence is computed as:

$$
U_d = \frac{1}{\beta} \sqrt{\left( \frac{1.96 \times 6 \times 3.4}{\sqrt{16}} \right)^2 \left( \frac{1.96 \times 6 \times 0.56}{\sqrt{16}} \right)^2 \left( \frac{1.96 \times 6 \times 1.5}{\sqrt{16}} \right)^2 + \left( \frac{1.96 \times 6 \times 3.5}{\sqrt{16}} \right)^2}
$$

$$
U_d = 0.79 \text{ cts}
$$

Shot noise follows Poisson statistics and is effectively reduced by averaging multiple images. The uncertainty due to shot noise for a single frame is then characterized as the square root of the collected electrons, equation (6.13).

$$
U_e = \sqrt{e^-} = \sqrt{\beta I}
$$

(6.13)
The uncertainty in the intensity is thus:

\[ U_I = \frac{U_e}{\beta} = \sqrt{\frac{I}{\beta}} \]  

(6.14)

For multiple frames, the precision uncertainty (95%) is thus expressed as (Mendoza, 1997):

\[ P_I = 1.96 \frac{U_I}{\sqrt{m}} = 1.96 \sqrt{\frac{I}{m\beta}} \]  

(6.15)

where \( \beta \) is the ADU gain (=19.1 e⁻/ADU for our camera). Sixteen frames were acquired to minimize the uncertainty in the intensity measurement. As each case has a different intensity level due to different environmental conditions, case 7 was chosen for uncertainty calculation as it possesses the lowest run intensity levels, hence the highest uncertainty. The following table summarizes the intensity statistics for case 7.

<table>
<thead>
<tr>
<th>Intensity Data</th>
<th>550nm</th>
<th>600nm</th>
<th>650nm</th>
<th>700nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (cts)</td>
<td>5256.1</td>
<td>5782</td>
<td>7017.8</td>
<td>6444.1</td>
</tr>
<tr>
<td>Average shot noise (cts)</td>
<td>72.5</td>
<td>76</td>
<td>83.8</td>
<td>80.3</td>
</tr>
<tr>
<td>( P_I ) (cts)</td>
<td>8.1</td>
<td>8.5</td>
<td>9.4</td>
<td>9</td>
</tr>
<tr>
<td>Mean</td>
<td>10147</td>
<td>11129</td>
<td>12162</td>
<td>10252</td>
</tr>
<tr>
<td>Average shot noise (cts)</td>
<td>100.7</td>
<td>105.5</td>
<td>110.3</td>
<td>101.2</td>
</tr>
<tr>
<td>( P_I ) (cts)</td>
<td>11.3</td>
<td>11.8</td>
<td>12.4</td>
<td>11.4</td>
</tr>
</tbody>
</table>

All precision errors in Table 6-14 are then summed in an r.m.s. approach yielding a total precision uncertainty due to shot noise ±29.3cts. Ignoring excitation source and photodegradation intensity drift, test condition intensity drift is only considered. The longest exposure time for an image is 3.5 seconds, and as environmental conditions (i.e.,
pressure and temperature) were monitored, pressure drift was smaller than the precision of the pressure transducer of 0.00625psi. Temperature variation was more difficult to assess because temperature drift (~ 0.08°C) was smaller that its uncertainty. This falls into the ambiguous error category were it could be due to manufacturer specification error limits (i.e., bias) of 0.5°C or the STD limit of 1°C. In the worst case it would contribute as a bias error, and hence it will be treated as such. For the 550nm emission, a 1°C change caused a decrease in intensity of about 76cts, thus 0.08°C error would yield a precision error of ~6 cts. The intensity drift for all filters are shown in the table below.

Table 6-15 Intensity drift uncertainty for case seven. Reference conditions are not included in the analysis as they are acquired under isothermal conditions. The total intensity bias uncertainty due to test conditions drift is 9.7cts

<table>
<thead>
<tr>
<th>Intensity Data</th>
<th>550nm</th>
<th>600nm</th>
<th>650nm</th>
<th>700nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature drift (°C)</td>
<td>0.08</td>
<td>0.04</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>Temperature sensitivity (cts/°C)*</td>
<td>76.2</td>
<td>80.9</td>
<td>77.2</td>
<td>46.4</td>
</tr>
<tr>
<td>$P_I$ (cts)</td>
<td>6.1</td>
<td>3.2</td>
<td>6.2</td>
<td>2.8</td>
</tr>
</tbody>
</table>

*Based on average intensities from Table 6-14

Additional contributor to total error originated from image registration due to CCD pixel-to-pixel gain variations. To correct for this bias error a flat field image is needed to normalize the registered image. Flat field images are acquired using a uniform illumination source. As described by Cattafesta et al (1998), a uniform source is assembled by using an integrating sphere in conjunction with a bandpass filter identical in its optical characteristics to the bandpass filter used for luminescence detection. Cattafesta et al (1998) reported ±1% variation due to this effect. The uniform excitation source was not available for this experiment; consequently a value of 1% will be added to the total intensity uncertainty to accommodate this effect.
The total uncertainty in the measured intensity is thus given by:

\[ U_I = \sqrt{B_I^2 + P_I^2} \]  

(6.16)

The total bias error is:

\[ B_I = \sqrt{N_{id}^2 + (N_{db})^2} = \sqrt{0 + 9.7^2} \]

\[ B_I = 9.7 \text{ cts} \]  

(6.17)

The total precision error is computed by the r.m.s. of all precision errors, noting that dark current is multiplied by a factor of two to account for both run and reference images, and the readout noise is multiplied eight times to account for all filters in both run and reference conditions:

\[ P_I = \sqrt{N_{sh}^2 + 2(N_d^2) + 8(N_{rd})^2} \]

\[ P_I = \sqrt{(29.3)^2 + 2(0.79^2) + 8(1.15)^2} \]  

\[ P_I = 29.5 \text{ cts} \]  

(6.18)

In the above expressions, \( N \) is the noise and the subscripts \( d, rd, db, \) and \( id \) denote dark current, readout, digitizer bias, and intensity drift, respectively. The digitizer bias noise was omitted as it was subtracted from both run and reference images. The total uncertainty in the intensity is thus given by:

\[ U_I = \sqrt{(9.7)^2 + (29.5)^2} \]

\[ U_I = 31.1 \text{ cts} \]  

(6.19)

It is often more convenient to express such uncertainty as a percentage with the mean intensity of all filters for both run and reference images as follows:
The uncertainty in the calculated pressure from POD calibration has a fundamental difference from a simple Stern-Volmer relation. The coefficients of the linear fit produced by the POD for pressure calibration are temperature independent. This is a significant and important result for two reasons. First, it shows that POD successfully fully compensated for the temperature effects in the pressure. Second, as a result, run and reference temperature uncertainties, other than intensity drift, do not contribute to the pressure uncertainty. Further, both the STD and the pressure drift for the run pressure are an order of magnitude smaller than the pressure transducer accuracy (0.006psi), thus only the latter value need to be added to the total uncertainty of the pressure. The pressure calibration throughout all the cases was a linear function of a set of scores from the POD calibration, which carries the same uncertainty of the intensity ratio \( \frac{I_{\text{ref}}}{I} \), thus the overall uncertainty with 95% confidence level is then characterized as:

\[
U_P = \sqrt{\left( \frac{\partial P}{\partial I} \frac{U_{I_{\text{ref}}}}{I} \right)^2 + \left( B_P \right)^2}
\]

Excitation source errors are omitted in this analysis. The excitation sources used have been characterized and observed to be very stable with time and were always left on, but covered between runs to avoid photobleaching effects, to avoid the initial drift in the source. Therefore, it will be assumed as an insignificant contributor to the total
uncertainty. The total pressure uncertainty from POD calibration is then:

\[
U_p = \sqrt{U_I^2 + (B_p)^2}
\]

\[
U_p = \sqrt{(0.36)^2 + (0.025)^2}
\]

\[
U_p = \pm 0.361\% \quad (95\%)
\]  \quad (6.23)

Similarly, the temperature uncertainty is computed and is tabulated below. The uncertainty analysis corroborates the robustness and accuracy of POD calibration compared to other techniques reported in the literature. Mitsuo et al (2003) reported a percent uncertainty of 5-8\% (at \(\sim 4-6\)psi) in the calculated pressure using PtTFPP/Rhodamine B(RhB)/Poly-IBM-co-TFEM dual-luminophor system. They used a simple Stern-Volmer calibration with temperature independent coefficients correcting for temperature effects only into the reference image. POD calibration has been shown to produce excellent results for dual-luminophor systems. Further, POD provides extends beyond the mathematical envelop providing physical insight into the process under investigation. This concludes the analysis chapter. A summary of the results, final thoughts and future work are presented in the following chapter.
Table 6-16 Uncertainty (95% confidence level) for key factors

<table>
<thead>
<tr>
<th>Variable</th>
<th>Average Value</th>
<th>Bias Error $^3$</th>
<th>Random Error (STD) $^3$</th>
<th>Uncertainty 95% confidence limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$ $^1$</td>
<td>5256 cts</td>
<td>DiB=0.0 cts $^2$</td>
<td>RoN=0.7 cts $^4$</td>
<td>±8.4 cts $^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>ShN=8.3 cts $^5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DkN=1.07 cts $^6$</td>
<td></td>
</tr>
<tr>
<td>$I_{ref}$ $^1$</td>
<td>10147 cts</td>
<td>DiB=0.0 cts $^2$</td>
<td>RoN=0.7 cts $^4$</td>
<td>±11.6 cts $^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>ShN=11.5 cts $^5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DkN=1.07 cts $^6$</td>
<td></td>
</tr>
<tr>
<td>$T_{thermocouple}$</td>
<td>30-70°C</td>
<td>0.5°C</td>
<td>1°C</td>
<td>±0.55°C $^7$</td>
</tr>
<tr>
<td>$P_{tap}$</td>
<td>23-16 psi</td>
<td>0.00625 psi</td>
<td>0.0003 psi</td>
<td>±0.00626 psi</td>
</tr>
<tr>
<td>$P_{ref}$</td>
<td>14.72 psi</td>
<td>0.00625 psi</td>
<td>0.0003 psi</td>
<td>±0.00626 psi</td>
</tr>
<tr>
<td>$T_{psp}$</td>
<td>30-70°C</td>
<td>0.808°C</td>
<td>1.193°C</td>
<td>±0.86°C</td>
</tr>
<tr>
<td>$P_{psp}$</td>
<td>24-14.5 psi</td>
<td>0.025% psi</td>
<td>0.36% psi</td>
<td>±0.361% psi</td>
</tr>
</tbody>
</table>

$^1$ Case 7, filter 550nm

$^2$ After subtracting the dark image

$^3$ RoN=Readout noise, ShN=Shot noise, DkN=Dark noise, DiB=Digitizer bias

$^4$ A/D frequency = 200 kHz

$^5$ Gain = 19.1 e/ADU

$^6$ Dark current = 6 e/pixel/second

$^7$ Number of samples = 64
CHAPTER 7
CONCLUSIONS AND FUTURE WORK

This chapter provides a summary of the work presented in this dissertation. It also provides direction for future work and possible research paths.

Conclusions

Pressure sensitive paints have been steadily advancing as a measurement technique in various engineering applications. The technology has been hindered since its emergence by several drawbacks. One of the main difficulties with PSP is the calibration process. PSP formulations have always suffered from an inherent temperature dependence causing spurious pressure measurements. Different calibration techniques are presented in the literature which attempt to compensate for the temperature dependency of PSPs. These techniques lack the universality and offer modest accuracy, particularly when temperature gradients are significant. Several research efforts investigated the idea of combining two luminophors that measure pressure and temperature simultaneously, hence providing temperature information for pressure calibration. These dual-luminophor systems exhibited undesirable behaviors such as cross-talk and spectral interference making the extraction of pressure and temperature information more complicated. This work presented a calibration technique using POD which was capable of separating the pressure and temperature information from raw intensity data and fully compensate for temperature dependence in the pressure emission. Furthermore, the technique extends beyond the mathematical envelope of calibration and provides physical insight into the process under investigation. POD calibration is not limited to isothermal conditions
between run and reference conditions, small/moderate temperature gradients, similar
temperature sensitivity of both luminophors or spectrally independent emission. Further,
it adds as a noise reduction technique and a modal analysis tool.

A channel flow experiment provided several test conditions to evaluate POD as a
calibration technique. The first and second cases showed the paint response to pressure
and temperature independently. The rest of the cases examined different mixed
conditions between pressure and temperature. The analysis revealed the various
advantages and provided a measure for the accuracy of POD as a calibration technique.

The following is a summary of the main advantages of POD.

1. **POD fully separates pressure and temperature information**: This was evident in
   the pressure calibration functions. All pressure calibration functions were linear
   and the calibration coefficients were constants (i.e., temperature independent).
   Temperature calibration was a quadratic fit.

2. **The calibration functions are simple**: Kose (2005) used POD to calibrate the paint
   in a priori approach. The calibration function were nonlinear second order
   surfaces with no obvious relation to the known pressure and temperature behavior
   as a function of intensity data. This work examined the behavior of POD as a
   calibrator using artificial data, which provided a fundamental understanding of the
   analysis. The calibration function were linear and quadratic for pressure and
   temperature, respectively. These calibration functions agrees with the theoretical
   prediction of the paint behavior.

3. **POD provides physical insight about the pressure and temperature field**: The
   eigenvectors are extracted at an appropriate rotation angles. These angles are
   related to the orientation of the pressure and temperature fields relative to each
   other. For instance, in case three, the pressure and temperature gradients were
   longitudinal, and the relative angle between the two eigenvectors was 1° degree.
   For case five the temperature gradient was mainly in the transverse direction, the
   relative angle between the calibration eigenvectors was 78° degrees. The last case,
   case seven, had an oblique temperature gradient relative to the pressure gradient
   and the relative angle was 40° degrees.

4. **Accurate results**: The results of the test points and the uncertainty analysis
   demonstrated the accuracy of the POD calibration. Absolutes errors averages ~0.2
   psi for the test points with an uncertainty of ±0.36% psi. This is far superior to
   results for similar dual-luminophor systems with uncertainty estimates of ±5-7%
   psi, Mitsuo et al (2003). Furthermore, since POD calibration produced
temperature independent calibration coefficients, pressure uncertainty is
temperature independent, yielding improved pressure accuracy. The results were
also compared to relatively rudimentary calibration techniques using direct
intensity-ratios of two different wavelengths spectral data. The ratio technique
had an average absolute error of 0.6 psi for case seven, with maximum error of ~
1 psi near the calibration limits. The intensity-ratio error increases significantly as
the temperature gradient increases. This is evident by comparing the results for
case five and case seven. On the other hand, POD error is steady and independent
of the temperature field. Furthermore, POD calibration was accurate even outside
the calibration envelope. The intensity ratio approach fails significantly even
within the calibration envelope at high pressures.

5. POD calibration eases the constraints on paint formulations: The capability of
POD to spectrally separate pressure and temperature information allows for larger
pool-of-candidates of dual-luminophor systems. Several research papers in the
literature investigated various pressure and temperature luminophors in an attempt
to find the best combination with minimal cross talk and spectral interference.
This limits the number of viable dual-luminophor systems. POD opens the door
for more luminophors to be used and more importantly, perhaps, for the
possibility of tailoring the formulation to different applications.

Future Work

The potential for dual-luminophor PSP in experimental applications is yet to be
fully realized due to the difficulty of the calibration process. The work presented herein
presented a successful technique for calibrating dual-luminophor PSP. The next step
would be to apply the paint and calibration in more involving applications, such as wind
tunnel testing. Several wind tunnel applications exist in the literature which are well
established and can be utilized to further validate the paint/calibration system. Airfoil
applications for different flow conditions would be of great importance to the validation
process. In particular, an application with high pressure and temperature gradients would
serve to substantiate the capability of POD to separate pressure and temperature effects
and provide accurate results. In addition, a low speed case with small pressure gradients
would emphasize the accuracy aspect of the POD calibration. Several issues need to be
addressed in either case. First, the number of calibration points needed as a function of
the environmental conditions range. Second, the extent of the calibration accuracy beyond the calibration envelope is a key factor that needs to be further addressed. Third, different paint formulations should be used to fully assess the success of POD as a calibration technique. An interesting case to investigate would involve a non-directional temperature gradient, such as that from a heat source in the middle of the channel. This will identify a key aspect of POD behavior, which is whether POD would still be successful under non-obvious gradient orientation conditions.

PSP is relatively and significantly less intrusive compared to traditional pressure measuring techniques. There are many intricate applications in which dual-luminophor PSP can be implemented and proven superior compared to traditional techniques. A case that is currently under investigation at the University of Florida examines the flow over porous media used as injector face plates inside rocket engines combustion chambers. Such application would truly establish dual-luminophor technology in both the academic and commercial sectors and provide more potential for the technology. A second application would be micro-channel flows. Such experiment would examine key aspects of the paint and calibration as well. Several issues can be investigated using a micro-channel experiment: the intrusiveness of the paint would be greatly emphasized, the number of calibration points needed, and the accuracy and sensitivity of both the paint and the calibration process. More practical applications involve rotating airfoils, such as turbine or compressor blades. Dual-luminophor systems would prove superior in such application, following success of PSP, with an added bonus of mapping temperature variations as well. There is also considerable demand for high-temperature (>400 K) dual-luminophor systems. The development of such system is currently ongoing at the
University of Florida (Schanze group). Dual-luminophor would be a very robust and successful tool for CFD validation. Instead of typical discrete information to validate the numerical simulation, leaving many areas in the flow invalidated; dual-luminophor PSP would provide for a full field validation.
APPENDIX A
IMAGING SYSTEM

This appendix provides comprehensive details about the different component of imaging system. The information is primarily directed to readers with little to no background in imaging system.

Measurement System

Various components come together to create an imaging system including excitation source, imaging device, and supporting components. The following sections discuss each component and various issues pertinent to its application in luminescence imaging.

The Excitation Source

Excitation source plays a key role influencing the accuracy and quality of data. Luminophors typically have multiple absorption peaks with moderate bandwidths, a desired character of the probe to avoid broad absorption and spectral overlap and interference. Therefore, the excitation source must provide sufficient and uniform illumination at these absorption bands to produce an output luminescence signal capable of saturating the detector in relatively short exposure time, thus taking advantage of detector’s signal-to-noise (SNR) potential. However, the illumination should not be bright enough to cause photodegradation of the luminophor or overwhelm the detector’s well depth (see definitions in the supplementary topics section). Further, the excitation source must have absolutely no emission at the luminophor spectral emission range. A spatially uniform illumination avoids the formation of local regions where the signal is
low relative to the noise level with respect to the rest of the imaged field, inducing an inconsistent SNR field. If continuous illumination is desired, then a main character is the stability of the illumination output by the source, while excellent pulsed excitation depends on repeatable excitation signal levels.

Some of these induced variations can be accounted for in the calibration process such as inhomogeneous illumination fields (see calibration section), however, variations in the overall output signal level (source drifting) is not easily accounted for. A drift in the excitation signal will have the effect of reducing the strength of the paint emission, falsely indicating higher pressure/temperature levels. Constant monitoring of the output excitation signal, though possible, is quite impractical, especially when using multiple excitation sources. A suggested approach in the literature is to embed an environmental-insensitive probe in the paint (a multicomponent system) that depends only on the excitation signal. This approach provides an effective correction technique that eliminates excitation field variations; nonetheless, it is fairly challenging to find a probe that will absorb in the same spectral range as the other probes and yet emits in an empty range of the spectrum not occupied by other luminophors, which becomes even more intricate in systems containing more than one probe (e.g. pressure and temperature dual-luminophor systems.)

**Lasers**

Laser (Light Amplification by the Stimulated Emission of Radiation) is an optical oscillator, which is made out of a solid, liquid or gas that emits uniform and coherent light. Unlike ordinary light sources, coherent light produced by a laser is made up of waves of the same wavelength and in phase. Lasers have the capability of delivering pure light in an almost-perfectly parallel beam (collimated), of one or more discrete
frequencies. Laser emission can be continuous or pulsed and is used in a myriad of applications. A laser is an optical oscillator, which is made out of a solid, liquid or gas with mirrors at both ends. Lasers also implement luminescence principles in their functionality. The lasing material is excited with appropriate energy, thus electronically energizing the atoms, causing them to jump to higher orbits, creating a “population inversion.” When a photon with a frequency corresponding to the energy difference between its excited state and its ground state strikes an excited atom, the excited atom is stimulated to emit a second photon of the same frequency and in phase with the striking photon. The striking photon and the emitted photon may then each strike other excited atoms, stimulating further emissions of photons, all of the same frequency and all in phase. The light is then passed through a laser medium between two mirrors. The mirrors continue to reflect the light back and forth; this process increases the intensity of the light waves and induces a chain reaction. The combination of spontaneous emission first, which is followed by stimulated emission, causes the laser to “lase,” which means it generates a coherent beam of light of a single frequency.

Lasers are of great advantage in certain PSP applications. Their narrow-band emission can be easily tailored for each specific luminophor, though not practical, and is easily distinguished in the spectrum. They are most advantageous in applications where optical access is difficult, such as imaging turbine blades inside the engine. Using fiber optics, laser beams can be funneled into complex structures to provide illumination for the paint. Moreover, lasers, as already stated, can be pulsed with very short illumination windows (~ 10 ns), making them ideal for unsteady measurements and rotating machinery. On the other hand, it requires cautious and continual surveillance of the
spectrum and energy output intensity to avoid excitation induced errors. Lasers are, however, unsuitable for applications requiring uniform illumination fields and always pose a safety concern as coherent light sources due to high energy beams.

**UV Lamps**

Ultraviolet electromagnetic radiation occupies wavelengths shorter than that of the visible region on the spectrum. It can be subdivided into near UV (380–200 nm wavelength) and vacuum UV (200–10 nm). While invisible to the human eye; illuminating certain materials (i.e., luminescent molecules) with UV radiation prompts the visible emission of fluorescence and phosphorescence. Gas discharge lamps, in particular mercury vapor discharge lamps, are among the most common source of continuous ultraviolet source. The proportion of ultraviolet emitted by a mercury vapor lamp varies considerably with current density, but the operating pressure of the lamp mainly governs the spectral quality. Low-pressure mercury vapor lamps possess a very sharp and discrete emission at 254nm, which is significantly below typical PSP excitation range. High-pressure lamps are the typical choice for luminescent probes applications as they emit well within the excitation range of most practical probes with a characteristic peak output at 365nm and secondary peaks at 334nm and 313nm.

Excitation via mercury-lamps is principally at discrete wavelengths, narrowing the potential pool of compatible probes. UV sources with more continuous emission spectra include xenon arc lamps (commonly used as sunlight simulators), deuterium arc lamps, mercury-xenon arc lamps, metal-halide arc lamps, and tungsten-halogen incandescent lamps. UV lamps require considerable time to warm up and produce significant heat emission, which might induce errors in the measurement if the lamp is close enough to the test model. It is of an utmost importance for researchers using UV radiation to
practice safety by shielding themselves, especially under elongated exposure times, as UV exposure of the skin or eyes is fairly hazardous.

**Visible Light Illumination LEDs**

Some luminophors have excitation absorption bands in the blue range of the spectrum (e.g. ruthenium complexes). Light Emitting Diodes (LEDs) are used in many electrical and electronic product on the market, from a tiny on/off light to digital readouts, flashlights, traffic lights and perimeter lighting. LEDs are more efficient and require considerably less power compared to incandescent bulbs. Assembled out of arrays of semiconductor diodes, high-quality LEDs are capable of emission at a single wavelength of light when charged with electricity. Lesser quality LEDs have lower purity compounds, hence possess a broader emission, yet are still practical for most engineering applications. LEDs have very long durable life and are available in various colors depending on the material used for the tips of the probes. Red and yellow LEDs utilize aluminum indium gallium phosphide (AlInGaP), green and blue utilize indium gallium nitride (InGaN) and with the addition of phosphor, white LEDs are produced. Blue LEDs are the most widely used excitation source in luminescent probes applications due to the excitation range of most luminophors that lies in the blue range of the spectrum. They are relatively expensive to buy (~ $ 2,500), but are rather practical over a wide range of applications, possess low-drifting intensity characteristics and emit low temperature relative to incandescent bulbs.

**The Detection Device**

The detection device plays a deterministic role in how accurate PSP measurements are and the degree of resolution obtainable. This could further impinge on the feasibility of certain experiments, such as low speed wind tunnel PSP measurements, where
pressure variations are small and the corresponding intensity variations are fairly diminutive. Accordingly, an understanding of the basic principles of operation of detection sources is fundamental for the comprehension of the setup and acquisition processes as well as error sources and uncertainty analysis.

**CCD Cameras**

A charge-coupled device (CCD) is a sensor for recording images, consisting of an integrated circuit containing an array of linked/coupled, capacitors. Controlled through an external circuit, each capacitor can transfer its electric charge to one or other of its neighbors producing a stream of charges that is then passed to other components for further processing.

![Figure A-1 Schematic of CCD imaging system](image)

When a lens on the CCD array projects an image, it results in an accumulated electric charge on the capacitors that is proportional to the local light intensity. Once the array has been electronically charged, a control circuit causes each capacitor to shift its contents to its neighbor in a systematic way. The last capacitor in the array unloads its charge into an amplifier that converts the charge into a voltage. The process continues...
until the entire contents of the array are mapped as voltage field. The electric voltage is then sampled, digitized and stored in memory for processing. CCDs contain grids of pixels and widely used in digital cameras, optical scanners and video cameras as light-sensing devices. They typically have a light response of 70% to incident light making them by far superior and more efficient than photographic film, which have typical light response of 2%.

**Physical Construction**

A standard CCD device is composed of metal oxide semi-conductor (MOS) capacitor, a composite structure of doped semi-conducting substrate and combination of aluminum or heavily doped polysilicon fused with an electrical connector (commonly known as an etched electronic gate) that is insulated in the middle by a layer of silicon dioxide (SiO2), and various supporting electronics. Combined they work to collect, store and then digitize the analog information for computer processing.

The MOS capacitor (pixel) constitutes the building block of the CCD. The CCD array consists of two areas: the sensing area, the region of pixels exposed to light, and the serial register, a segment of pixels coated with an opaque material. As a voltage bias is applied to each pixel gate, it creates a potential well, thus triggering these pixels to accept...
and collect the electrons generated via the incoming electromagnetic radiation in direct proportion. These collected charge packets are then stored and transferred from the sensing area to the serial register, which in turn shifts each charge packet orderly for subsequent readout by the analog-to-digital (A/D) electronic unit.

**From Light to Electrons**

Silicon crystalline enjoys favorable electronic properties under exposure to electromagnetic radiation in the visible spectrum. Through photoelectric effects and thermal agitation of the silicon crystal lattice, output electric charge packets are generated in proportion to the incoming electromagnetic radiation. The latter effect is actually an undesirable source of error known as thermal noise or more famously dark current. As explained earlier, an electron can be elevated to a higher energy level as a photon strikes the atom, occasionally causing the freeing of the electron from the atom. Similarly, as light strikes the CCD surface, it frees electrons from the covalent bonds between neighboring atoms to move around, which accumulate in the capacitors. Several factors influence the probability of a photon collision causing an atom to release an electron. The two central factors are: blockage by circuits on the CCD surface prohibiting light from entering and the wavelength of the incident light (longer wavelengths photons can infiltrate to certain depths of the CCD array without colliding with atoms, while some shorter wavelengths may simply reflect off the surface.)

**Potential Wells**

Quantifying the number of photons that fall on the photoreactive surface resulting in a release of an electron is a true representation of the CCD’s sensitivity. The ratio of incident photon to collision-induced electron release is known as the quantum efficiency and is often reported as a percentage. The process of quantifying the number of electrons
takes place in potential wells. A potential well is a thin layer of silicon dioxide, encrusted with a conductive gate structure, grown on a silicon layer. As a positive electrical potential is applied to these gates, a depletion region is created providing a “well” to store the photon-induced free electrons.

![Potential Well Structure](image)

Figure A-3 Potential Well Structure

The well collects the electronic charges until it reaches its full capacity, which is typically on the order of a million electrons. The well is indiscriminate to electrons regardless of their source, hence electrons generated from thermal agitation and high energy particles add to the total stored charge.

**Charge Transfer**

The CCD array is configured into multiple vertical shift registers and usually one horizontal shift register, both requiring different clock patterns. After the electrons are collected by the pixels in the potential well, they are then shifted along the CCD array by the action of regular electronic pulses. The register collects one line at a time and transports the pixel charges in a serial manner to the on-chip output stage. A circuit that deposits the electrons from each pixel into a sense capacitor counts the electrons.

Farad’s law states that the potential difference across a capacitor is equal to the
electric charge accumulated inside the capacitor divided by its capacitance; accordingly a charge will develop a voltage across the sense capacitor proportional to the incident light intensity collected by the corresponding pixel. The output voltage is a series of stepped DC voltages. The circuitry measures this voltage and amplifies it then drains the capacitor. This procedure provides an effective grayscale image of how much light has fallen on each individual pixel in the CCD array.

1. Charges accumulate on the parallel-register due to light exposure
2. Charges are shifted from the parallel register (1st column) to the serial register
3. Charges in the first block of the serial register are shifted into the output node
4. Charges at the output node are collected for signal processing
5. Steps 3 & 4 are repeated till the entire serial register is read
6. Steps 1 through 4 are repeated till all the parallel registers are read

Figure A-4 CCD readout sequence

Each pixel retains three different voltage levels: the reset feed-through, the reference level, and the pixel level. The opaque pixels have the voltage level representing black that is utilized as reference voltage to adjust the signal offset. The reset feed-through voltage is the potential applied to set the sense capacitor to the initial reference voltage. The reference voltage can be relatively high, up to 12V but after the decay of this feed-through potential the capacitor will reflect the reference voltage level. Once the capacitor has been reset, a switch opens and the pixel charge is transferred to the capacitor, altering its voltage.
**Digital Signal Processing**

The voltage output of the CCD array is amplified and passed to the analog-to-digital converter (A/D). The A/D converter divides the electronic charges of the CCD pixels that are readout from the chip by a certain factor (i.e., the conversion factor) thus delivering a digital signal measured in counts (or gray levels). The maximum possible conversion factor would be the full-well capacity divided by the bit depth (i.e., maximum possible intensity range.) However, quite often the conversion factor is lower than this theoretical value. The conversion factor provides a mean of controlling the digitization process. For example, for low light level experiments a lower conversion factor is desired, which implies a higher bit A/D. This allows for faster saturation as fewer electrons are needed, while the light-to-gray scale digitization is finer (i.e., fewer electrons per count) and hence the resolution of the image is improved allowing for the detection of smaller variations.

An example would better illustrate the concept. The CCD camera used in this work has a full-well capacity of 330,000 electrons and is coupled with a 14-bit A/D converter. This means that the well can collect up to 330,000 electrons before reaching its capacity, and the A/D converter has a saturation limit of 16,384 counts, which in turn implies that at the CCD saturation point the smallest resolvable variation by the A/D converter, ignoring noise, is on the order of 20 electrons. A 16-bit A/D converter would ideally be able to resolve variations of only 5 electrons for the same CCD camera. Noise undesirably reduces the dynamic range and thus the resolution of the image.

**CCD Chip Types**

**Full-Frame CCD Chips**

This type is has a lower sensitivity in the spectral range and has the simplest
architecture, hence less expensive and advantageous in applications where high
sensitivity measurements is not influential. Full-frame CCD chips consist of a high-
density array of photodiodes that convert the incoming photons into electrical potentials
according to the process described previously. Their advantage lies in their very high fill
factor, values of 97-99 % are typical, which means that the entire surface is functional
with very little space between the photodiodes that is not functional in radiation detection
and hence lesser photons are lost. This CCD type requires a mechanical shutter or
synchronized illumination to avoid image smearing artifacts that occur as a result of the
continuous illumination during parallel register readout.

**Frame-Transfer CCD Chips**

This type compensates for the full-frame CCDs inability to simultaneously detect
light and perform the readout process. This is accomplished by using a two-part chip in
which one half is exposed to light and collects photons while the other serves as a
temporary data storage only and is masked to protect it from incident photons. During the
exposure of an image, the data of the previous image are readout from the storage array
via the serial shift register through an output amplifier and the A/D converter. As soon as
exposure and readout of the old image are completed the newly accumulated charges are
very rapidly moved across the CCD array; this process is thus termed "frame transfer."
The process of transferring data between the two parts of the CCD is relatively shorter
compared to the readout process. This allows for faster image processing as each cycle
yields an acquired image and as well as a simultaneously readout immediately preceding
image, compared to only one acquired and read image per cycle. Unlike the full-frame
CCD chips, a frame-transfer CCD chip can operate continuously without a shutter at a
high rate.
**Back-Thinned, Back-Illuminated CCD Chips**

A rather expensive and delicate type of chips for high-end scientific-grade CCD cameras, this type is superior due to its high quantum efficiency exceeding 80% between 450 and 650 nm, compared to only 65% for regular CCD chips. Polysilicon layers, as mentioned previously, compose the gates for the parallel registers and are relatively transparent at long wavelengths but becomes opaque at wavelengths shorter than 400 nm, thus attenuating light at the latter wavelengths. As shown in Figure A.3, light enters the parallel register through these gates in standard CCDs, however, in back-thinned CCDs the back is thinned by acid etching down to about 10 µm so that it becomes transparent allowing for light to enter from the back (silicon side) where there is no gate structure. These sensors have a substantially improved sensitivity extending from the soft x-ray to the near infrared range as compared to standard CCD chips. A substantial downside of this chip type is a readout noise that is usually considerably higher than that of standard chips even at slow digitization speed. Higher dark noise is typical for these CCDs as well.

**CCD Artifacts**

Inherently, CCD imaging incorporates noise (artifacts) as a result of the manufacturing and operation of the CCD device and the intrinsic nature of electromagnetic radiation. The four principal noise levels are the signal photon shot noise, dark current noise, dark bias noise and read noise. Artifacts superimposed on data obtained with CCDs are either multiplicative or additive. Multiplicative artifacts are corrected for via a division process, while additive artifacts are accounted for through subtraction. CCD Artifacts include:

1. Charge Transfer Efficiency
2. Photon (Shot) noise
3. Pixel gain variations (flat field)
4. Read-noise
5. Saturation
6. Thermal (Dark) Current

**Charge Transfer Efficiency**

CCD imagers rely on efficient charge transfer from one pixel to the neighboring pixel during the readout process. As charges from distant pixels relative to the output amplifier undergo many transfers, on the order of hundreds or even thousands depending on the CCD size, the efficiency of this process is vital for the overall performance of the CCD operation. Charge transfer efficiency (CTE) is defined as the fraction of electrons that are successfully passed onto the next position during the horizontal and vertical shifting associated with the readout process. Typical CCDs exhibit a CTE of 0.99999; still, this can leave a significant portion of the charge in downstream pixels. For example, a CCD chip with a FWC of 330,000 electrons, a CTE of 0.99999 and a chip size of 512x512 would lose 1,685 electrons during the transfer for the furthest pixel. This is only about 0.5% of the total charge; however, if the chip size is expanded to 2048x2048, then the lost charge is 6689 electrons or 2% of the total charge. A CTE of 0.99995 would yield a charge loss of 32,120 electrons or 10% of the total charge for a 2048x2048 chip size. Lost charges are thus a function of location and the further away from the output amplifier the higher the relative loss. Relatively low CTE values result in minor smearing effect with the smears pointing away from the readout edge of the CCD.

**Photon (Shot) Noise**

Shot noise derives from the stochastic nature of the photon flux, thus is a fundamental property of the quantum nature of electromagnetic radiation. A steady
illumination source emits photons according to a Poisson distribution over any time interval. Accordingly, the charge collected by the CCD chip exhibits the same Poisson distribution with a characteristic noise equal to the square root of the intensity. Thus, in order to reduce shot noise, higher signal is required. For instance, a signal of 100 photons has a shot noise of 10 photons or 10%, while a signal of 10,000 photons has a shot noise of 100 electrons or 1%. Since this noise is caused by the natural statistical variation of the light, it is independent of the camera hardware or design. Combining intensities of adjacent pixels, on the expense of lower spatial resolution, during the readout process, a procedure termed “binning,” reduces the shot noise. This is especially beneficial in low intensity applications where a sacrifice in the spatial resolution is tolerable.

In general, the term shot noise is applied to any noise component reflecting a Poisson statistical variation, or uncertainty, in measurements of the number of photons collected during a given time interval.

**Pixel Gain Variations**

This is multiplicative noise source that stems from various causes. It is a gain variation from pixel to pixel the can be seen as a non-uniform canvas known as a flat field image. Causes include: quantum efficiency variation between pixels due to wavelength dependence, fringing problems, which are wavelength dependent interference creating global interference patterns, dust particles in the optical path that cause visible and repeatable patterns on the data frame, and quantum efficiency hysteresis that is a function of exposure history. In order to account for gain variations a calibration flat field frame is captured to scale the gain in each pixel to the average gain of the CCD via a normalization process.
Read-Noise

Read noise, also known as preamplifier noise, is a direct result of the process of measuring the amount of charge in a given pixel by time integrating the measured current. Quoted in number of electrons, read-noise is considered a mean quantity for the CCD array. Read noise can be effectively reduced by averaging multiple frames and the attenuation goes as the square root of the number of frames.

Saturation

Saturation is reached when either the potential wells are filled or the A/D converter reaches the maximum digitization value equal to $2^{\text{(number of bits)}} - 1$. When saturation is reached information at that pixel is lost, and even worse as the release of electrons continues by incident photons, extra charge spills over to neighboring pixels (usually above and below as the potential barriers are smaller in the vertical direction.) Most systems are designed so that the A/D converter saturates before the CCD wells saturate, in other words the CCD full well is much larger than the A/D converter maximum count capacity. This results in the added benefit that one remains within the linear response range of the CCD until A/D saturation occurs. Another problem with saturation is emphasized when the illumination field is significantly inconsistent or surface geometry is such that light reflection is greatly divergent (e.g. substantial surface curvature.) This creates localized regions where pixels are saturated while other pixels have very low intensity near noise levels. Increasing exposure time to fill up low intensity pixels would only lead to more saturated pixels.

Thermal (Dark) Current

Breaking covalent bonds to free electrons can be initiated by thermal effects in addition to photonic collisions. This adds a “false” charge to the overall charge
accumulated by the potential wells; hence thermal current is an additive noise. Dark noise arises from statistical variation in the number of electrons thermally generated within the silicon structure of the CCD, which is independent of photon-induced signal, but highly dependent on device temperature. It arises mainly from two sources: general background heat from supporting electronics and locally hot pixels. The former induces quite low noise contribution (few electrons per hour), while the latter displays vastly higher rates of thermal electrons generation. For both, the accumulation rate of electrons is proportional to exposure time and sensitive to temperature with a decaying exponential pattern. If a pixel reaches saturation on the time scale of the read-out process, known as “Super-hot” pixels, it creates a hot column, as pixel values downstream in that column pass through the hot pixel during the read process. Theoretically, dark current is linear with time and is accounted for by acquiring data frames with the shutter closed with integration time equivalent to run frames.

Dark noise follows a Poisson relationship to dark current, and is equivalent to the square-root of the number of thermal electrons generated within the image exposure time. The theoretical dark current follows the following relationship,

\[ n = AT^{\frac{3}{2}} e^{-\left(\frac{E_g}{2kT}\right)} \]  

(A.1)

where \( n \) is the dark current, \( A \) is proportionality constant, \( T \) is the temperature in degrees Kelvin, \( k \) is Boltzmann’s constant and \( E_g \) is the silicon band gap energy (about 1.2 eV.)

Cooling the CCD reduces the dark current dramatically, and in practice, high-performance cameras are usually cooled to a temperature at which dark current is negligible over a typical exposure interval. Roughly, an 8-10 °C decrease in temperature
reduces the dark current by about half.

**Camera Resolution**

Camera resolution is an important parameter for any CCD camera. Digital images are composed of pixels representing the charge level of the miniature photodiodes on the CCD chip. Each photodiode integrates the intensity of a tiny area of the image and hence the size of the photodiode relates directly to the image resolution. The overall size of the CCD chip and the size of the focused image on the CCD chip also contribute to the overall resolution. It should be noted, however, that even though small CCD pixels improve the resolution, they also reduce the dynamic range of the CCD because the full-well capacity is dependent on the photodiodes size. Furthermore, increased resolution generally equals higher readout speed, which in turn dictates the requirements for faster and more elaborate subsequent electronics.

For scientific grade CCDs, pixel size could be as small as few micrometers to 48 \( \mu \text{m} \), with total imaging area in the range of 1 to 24 cm². In order to avoid signal acquisition defects, such as aliasing, moiré, or heat frequencies, each resolution element in the image plane must be covered by at least two pixels on the CCD.

**Photomultiplier Tubes (PMTs)**

A PMT is an extremely sensitive electronic detector composed of a vacuum tube that converts incident light into electrical current and amplifies it. Photomultiplier tubes are advantageous in some applications because they are more sensitive to light than standard CCD elements with a wider range of frequencies as well (i.e., ultraviolet, visible and near infrared spectrum.) PMTs are capable of multiplying the signal produced from the incident light to such an extent that detection of single photons is possible. The main construction of a PMT is a glass tube which houses a series of dynodes and an anode
under vacuum conditions (Figure A 5). Vacuum is necessary to facilitate movement of electrons through the tube and prevent electron scattering by air particles.

When photons strike the photocathode material, a thin deposit on the entry window of the device, electrons are freed due to the photoelectric effect, in a manner similar to that of CCD chips. These photons are directed by the focusing electrode towards the electron multiplier (i.e., dynode), where electron multiplication occurs. A dynode is an electrode arranged in a series assembly with each dynode possessing a more positive charge than its predecessor. Electron collision instigates secondary emission at the surface of each dynode. Such an arrangement is able to amplify the infinitesimal current emitted by the photocathode by (typically) many hundreds of millions. Electrons past the photocathode are accelerated towards the first dynode due to the potential difference causing a bombardment of electrons on the dynode surface. This releases more electrons from the dynode surface that are then accelerated to the next dynode. The geometry of the dynode chain is such that a cascade occurs with an ever-increasing number of electrons being produced at each stage. The whole process is repeated with more and more electrons being generated. A 12 stage dynode PMT will typically generate a gain of incoming charge on the order of 10 million. Eventually, the spawned electrons arrive at the anode and induce a surge of voltage pulse corresponding to incident photon on the photocathode.
The combination of high gain, low noise, high frequency response and large area of collection makes PMTs ideal for low intensity radiation applications. PMTs operation is based on two principles: the particle(s) to be detected have to be converted to electrons by the photocathode before the amplification process initiates and that the amplification is caused by a cascade of acceleration electrodes (i.e., dynodes) capable of accelerating the electrons to enough speeds allowing for the generation of more than one new electron upon impact of the dynode.

**Data Reduction**

Using intensity or lifetime measurement system, intensity information is recorded in the form of image planes. The intensity information needs to go three processes to finally yield pressure and/or temperature information in the spatial domain. The data reduction processes are:

1. Correcting for non-idealities in the data originating during data acquisition
2. Creating calibration curves or surfaces
3. Transforming the 2-D calibrated intensity information in the calibrated image to the actual 3-D model
Correcting for Non-Idealities

Dark Image Correction

As explained earlier, heat in the camera system induces a dark current that adds to the total collected signal. In addition, charge drainage from the CCD and the amplifier offset is typically imperfect, hence adding to the measured signal. The existence of ambient light will also add to the noise total. A “dark” image is typically acquired to compensate for such problems. An image with the shutter closed with an identical exposure time as run images indicates the offset charge imposed on the data, and through a simple subtraction of this offset charge associated noise is significantly reduced. In cases where considerable ambient light is present, the shutter should be opened, with the excitation source off, to better account for the noise.

Illumination-Field Variation Correction

Illumination-field variation stems from two sources: variations due to the uneven distribution of light on the model and model movement that changes the illumination field character. The former is easily compensated for by acquiring a “reference” image and normalizing the run images by it. A reference image is acquired under no-flow (a.k.a. wind-off) condition, such that all the intensity variations are due to the physical setup. The latter source, model movement, is much harder to compensate for. The most successful effort implemented a reference sensor incorporated in the paint composition. The reference sensor is only responsive to illumination intensity and thus independent of environmental changes. It usually is excited in the same spectral range as the other luminophors but emits at lower wavelengths (≈ 510 nm). The emission of the reference sensor is utilized to normalize the raw intensities. It further have the advantage of
eliminating the need for a wind-off image as well as significantly reduce model movement error, especially if the detection device is equipped with a filter screen that allows for simultaneous acquisition of different wavelengths. A potential for these referencing probes is their implementation in temperature compensation for PSP sensors. It is hypothesized that if the reference probe and the pressure probe can be matched for their temperature dependency, then normalizing the pressure emission by the reference emission would eliminate the temperature error. Unfortunately, finding such a probe is yet to be accomplished and further problems emerge whenever a probe is embedded in the binder such as spectral cross-talk.

**Image-Registration correction**

In wind-tunnel testing models move due to the aerodynamic loads acting on them. Such movement/warping deformation leads to significant errors in the necessary normalization process. Spatially, samples exhibit sharp gradients due to the nature of the paint mixture and the application process. These gradients could be sharp enough to devastate the normalized images. A simple way to reduce these spatial variations is to either slightly defocus the image or implement some filtering/smoothing algorithm to the raw images prior to normalization. This is useful only when spatial variations are modest, however, significant spatial gradients require image registration algorithm.

Image registration is a process in which two images are matched via translation, rotation and resizing/warping. The objective of the process is such that when the two images are laid on top of each other pixels will exactly match. The registration process could be a simply rigid-body transformation (i.e., projective) involving only translation and rotation, or it could incorporate physical deformation corrections (polynomial). Bell & McLachlan (1996) presented the transformation models from the (x,y) coordinates to
the \((x',y')\) as:

Projective:
\[
\begin{align*}
x &= \frac{a_1x' + a_2y' + a_3}{c_1x' + c_2y' + 1}, \\
y &= \frac{b_1x' + b_2y' + b_3}{c_1x' + c_2y' + 1}
\end{align*}
\tag{A.2}
\]

Polynomial:
\[
\begin{align*}
x &= \sum_{m=0}^{M} \sum_{n=0}^{N} a_{nm} x'^{n} y'^{m}, \\
y &= \sum_{m=0}^{M} \sum_{n=0}^{N} b_{nm} x'^{n} y'^{m}
\end{align*}
\tag{A.3}
\]

These coefficients are determined by measuring the centroid coordinates of some target points on both images. The target points should be well defined and easy to identify and contain few pixels to suppress deviation of the centroid. They should also be distributed over the surface of the model strategically to ensure proper transformation.

Rigid-body transformations require at least four points, while polynomial transformations require as little as three points for a first-order polynomial and ten points for third-order transformations. The degree of accuracy needed increases with the decrease in pressure and temperature gradients, fortunately, model movement decreases as well.

**Flat-Field Correction**

Camera lenses generate an inhomogeneous light field as they deposit more light near the focal point and lesser light near the periphery of the image. Additionally, the pixel gain variation, discussed in the CCD artifacts section, adds to the non-uniformity of the image. This correction is needed when model movement and/or deformation occurs; otherwise these variations vanish in the normalization process. As local pixel variations could be quite significant, an image registration procedure is first needed. The process involves acquiring a “flat-field” image and normalizing the run images by this image after subtracting the dark image from both images. The flat-field image is collected via imaging the inside of an integrating sphere or capturing any image through several layers
of diffusing “opal” glass (Bell et al. 2001). Unsurprisingly, an image added to the overall correction process means an increase in the total shot noise contribution. The increase in shot noise is quantified by Mendoza (1997) as:

$$\sqrt{1 + \left( \frac{\text{no. summed data images}}{\text{no. summed flat-field images}} \right)}$$

(A.4)

This dictates the acquisition of five times as many flat-field images as the number of raw images to reduce the shot noise to 10% of the theoretical minimum uncertainty. This relation goes by a factor of ten, that is to reduce the shot noise to 1%, 50 flat-field images are required.

**Supplementary Topics**

Further issues concerning data measurement and reduction are discussed and detailed in the following subsections.

**Lifetime Approach**

The lifetime of luminescence is defined as the time required for the luminescence intensity to decay from some initial value to 1/e of that initial intensity. As lifetime is an intrinsic character of each specific molecule, the emission is independent of illumination field variations, molecule concentration, photodegradation, paint thickness or any other non-intrinsic character of the molecule. However, temperature dependence is an intrinsic behavior of luminescent molecules, and hence temperature effects still persist in lifetime measurements. Lifetime measurements employ very short integration times on the order of few microseconds (2~5 $\mu$s) as the entire decay process of the emission is typically less than 20 $\mu$s. Mechanical shutters are incapacitated under these demanding conditions, hence special imaging systems such as PMTs, phase-sensitive cameras and intensified CCD cameras with pulsed excitation sources. Lifetimes can be measured by phase
fluorimetry (phosphorimetry). Measurements of the phase shift between a harmonic, say, a sinusoidally modulated excitation signal and the emitted signal, luminescence intensity time decay (lifetime) of the sensor molecule can be quantified as follows (Liu et al. 1997):

\[
\frac{dl}{dt} = -\frac{l}{\tau} + A \exp(i2\pi ft) \tag{A.5}
\]

where \( A \) is absorption amplitude of the emission and \( f \) is the frequency of the excitation signal. The solution to this relation is:

\[
I = \tau A \exp \left[ i \left( \frac{\phi}{2\pi ft} \right) \right] \tag{A.6}
\]

If the modulation frequency is fixed, the lifetime can be expressed as:

\[
\tau = \frac{\tan(\phi)}{2\pi f} \tag{A.7}
\]

where \( \phi \) is the phase shift. The emission has a phase shift delay relative to the excitation signal that increases with increased emission. Phase-sensitive cameras or lock-in amplifiers are used to capture the excitation and emission signals during two gated intervals. The first gate is phase locked with the excitation signal and the second gate captures the emission after excitation with a delaying phase shift. The process is better illustrated in the following diagram:
Another approach is to measure the lifetime decay in the time domain. The specimen is excited by radiation, at which time the shutter of the camera is closed. The molecules absorb the radiation and reach a maximum intensity peak, then the excitation is turned off and the emission of the molecules starts to decay. The first time gate opens as soon as the excitation source is turned off and the first decaying emission is captured. Excitation is initiated again but with a time delay in the exposure cycle relative to the end of the excitation cycle under identical exposure times. This is the second exposure gate, and the process is repeated with an incremental constant delay between the excitation and the exposure cycles. Typically, two or three gates suffice to determine the lifetime of the molecule.
The intensity of the emission follows an exponentially decaying behavior, thus lifetime can be related to the integrated intensity and the exposure times as follows:

\[
\tau = \frac{t_{\text{exp}}}{\ln\left(\frac{I_1}{I_2}\right)} \quad \text{(A.8)}
\]

Even though the lifetime approach provides certain advantages compared to the intensity approach, it still lacks the simplicity and practicality of the latter approach. Sophisticated equipments are needed and noise is more influential in lifetime measurements, rendering it not favorable for practical use.

**Measurement Uncertainties**

Uncertainties stem from three main sources: measurement systems, Physical and Chemical properties of coating, and displacement/deformation of the test model.
Measurement Systems

This includes the illumination source, detectors and filtering/spectral leakage.

Illumination source

In order to avoid introducing noise to the measurement the illumination source must produce uniform and steady illumination at the proper excitation wavelengths for the luminophor. It should also be adequately bright to produce a luminescence signal that could saturate the detector, hence utilizing the full capacity of the detector’s signal to noise ratio (SNR) potential. However, it should not be overly bright such that it will cause utter saturation or lead to excessive photodegradation of the luminophor.

CCD camera

The four principal noise sources are shot noise, dark current noise, dark bias noise and readout noise. Shot noise is a statistical variation that stems from the nature of the uncertainty of electromagnetic radiation, causing the number of photons per unit time collected by the CCD to conform to Poisson statistical distribution. The rms of the shot noise is then characterized as , hence the SNR is also , and SNR is highest when the full-well capacity of the CCD is exploited through longer exposure times or higher intensity illumination source. Such noise can be substantially reduced by averaging multiple images. Dark current is a thermally induced charge resulting in an accumulation of electrons in the detector. It has an rms of , where is the dark charge intensity level. Conveniently, it can be noticeably reduced by simply cooling the CCD head. Dark bias noise is simply a zero-offset bias in the digitizer that can be easily corrected for by subtracting a dark image (image with the shutter closed with the same exposure time as the run images) from all the run images. Readout noise is a function of the data transfer frequency, the higher the data transfer frequency the higher the noise, however through
careful design of the readout electronics of the detector it can be minimized. These four noise sources result in either random error, which can be reduced via acquiring numerous readings, or bias error (fixed and systematic error). Thus, the total uncertainty $U_{\text{total}}$ with 95% confidence level is expressed as (Mendoza 1997):

$$
(u_{\text{total}})^2 = (u_{\text{bias}})^2 + \left( \frac{2u_{\text{random}}}{\sqrt{m}} \right)^2
$$

(A.9)

where $m$ is the number of samples (i.e., images). The bias error includes dark bias and readout noise, and random error includes shot noise and dark current noise.

Table A-1 Error estimates for the 16 & 14-bit CCD cameras. Camera linearity error is 0.07% and ADU speed is 200 kHz

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Photometrix Camera (16 bit)</th>
<th>Photometrix Camera (14 bit)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CH 350A/S1502</td>
<td>CH 250A/S1502AB</td>
</tr>
<tr>
<td>Shot noise $\sqrt{I @ FWC}$ SITE502 CCD, ADU cts/frame</td>
<td>256</td>
<td>128</td>
</tr>
<tr>
<td>Dark current noise $e^-$/pixel/sec @ 25°C</td>
<td>2.8</td>
<td>6</td>
</tr>
<tr>
<td>Dark bias noise, ADU cts</td>
<td>130</td>
<td>110</td>
</tr>
<tr>
<td>Gain $e^-$/ADU (gain= 1 / 2)</td>
<td>5.2 / 1.3</td>
<td>19.1 / 4.8</td>
</tr>
<tr>
<td>Readout noise @ 200kHz $e^-$/RMS (gain=1 / 2)</td>
<td>17.5 / 13.3</td>
<td>13.3 / 10.3</td>
</tr>
</tbody>
</table>

Filtering/Spectral leakage

Filters must be utilized to separate the different regions of the spectrum and to separate the illumination source from the illumination signal of the coatings. Adequate absorption filter(s) should be installed in front of the light source to cutoff the excitation light after the end of the absorption region and before the emission region. Interference filter(s) are installed in front of the camera to precisely resolve the various regions in the spectrum and to stop leakage from the light source at the lower wavelengths. Allowing
light leakage from the excitation source would have the effect of reducing the resolution available, since the CCD camera is blind to wavelengths, it captures the reflected light regardless of its spectral origin, and thus a reflected light from the excitation source would be added to the total signal acting as a bias error.

**Physical/Chemical Properties of Coating**

One of the main problems with luminescent coatings is photodegradation which could lead to a drifting signal leading to incorrect intensity levels and the need for recalibration. Depending on the luminophor and the binder used the luminescence intensity of the coating will decay at a certain rate. Therefore, care should be practiced in both choosing a good luminophor/binder combination and acquiring data over elongated periods of time. Cross talk between different luminophors embedded in the same binder is prompted when a luminophor quenches the other by absorbing the emission from the lower-emitting luminophor, hence inducing error in the emission intensity of the quenched luminophor. Other error sources include temperature dependence in PSP coatings, variation in paint thickness, uneven dispersion of the luminophor(s) in the binder and pressure and temperature hysteresis, a process that leads to a change in the calibration curve due to the transition of the polymer from a hardened state to a rubbery state when it is first heated above its glass temperature leading to a change in the thermal quenching rate.

**Displacement/Deformation of Model**

Subjected to aerodynamic loads, especially at high speeds (higher dynamic pressures), models can deform or even displace from their original position. To correct for this bias a process known as image registration is required. Image registration is the process of aligning the run image with the reference (wind-off) image, before
normalizing them to even out all the physical non-uniformities in the coating, such that all the corresponding pixels are in perfect alignment. The extent of how perfectly the alignment should be performed depends on the magnitude of illumination gradients and the desired pressure and temperature accuracy. If the model displaces in a nonlinear manner then a third-order polynomial coordinate transformation is unavoidable. An alignment procedure consists of allocating physically fixed points on the model to use a reference, usually called registration dots; these dots should cover few pixels on the image. The centroid of these registration dots is located and matched, via shifting/warping of the image, between all the images to sub-pixel accuracy.

**Useful Definitions**

**Binning**

Binning, also called on-chip integration, is a process intended to improve the signal-to-noise ratio by primarily reducing shot noise and secondarily readout noise through a process of combining charges of neighboring CCD pixels and during readout. The result is an increased signal and thus an improved sensitivity and an improved signal-to-noise ratio. Binning is achieved first by the addition of charges from neighboring wells of the serial register into a super-pixel (serial binning) followed by subsequent superposition of rows of data in the serial shift register (parallel binning). This allows for shorter exposure time, thus less photodegradation of the specimen, less smearing in applications involving vibration and better dynamic response. This is accomplished at the expense of spatial as information is lost between pixels.

The binning factor is modifiable in both the global and local levels. It can be varied across the CCD width or more globally for the entire chip. Pixels are always aggregated in a square pattern to avoid image distortion. Readout time is shortened as one over the
square root of the binning factor. Time saved in the serial register is limited to the serial binning only which takes place after the data have been readout into the output node prior to being passed on to the output amplifier, while parallel binning occurs on the actual CCD chip. Binning can also be done after image acquisition in the computer image memory, however, this approach does not improve neither readout time nor noise.

**Blooming**

Overexposed pixels with saturated potential wells observed in bright regions of the image can spill their excess charge in neighboring pixels. This imposes an artificial additive intensity value for the receiving pixels that are virtually impossible to be accounted for. This condition is termed blooming. Many chip designs incorporate overflow drain structures, also known as potential barriers, to prevent spilling from saturated pixels. These barriers are typically less effective in the column direction relative to the row direction. Even though it might be obvious that lowering exposure time or illumination intensity would resolve the problem, it is quite common to have “bad” pixels that exhibit high intensity levels that would prevent adequate exposure durations.

**Bit Depth of Camera Data**

The bit depth is a reflection of the binary range of the possible grayscale values available for the image. It is the process in which the analog signal (voltage) output is digitized by the A/D converter of the camera. It is similar to the dynamic range of the CCD chip, which represents the exploitable capacity of the potential wells. Higher bit depth provides a finer scaling of the produced image during the digitization process. Bit depths range from 8 to 16 bit, that is, an 8-bit A/D converter has a maximum deviations of $2^8 = 256$ available grayscales, while the 16-bit A/D converter would offer $2^{16} = 65,536$ possible grayscale levels for the same signal. The theoretical pixel-per-grayscale
value is determined by dividing the full-well capacity by the maximum A/D converter deviations. For instance, a CCD chip with a full-well capacity of 330,000 electrons when coupled with a 16-bit A/D converter would yield 330,000/65,536 = 5.035 electrons per grayscale. In simpler terms, the A/D converter would be able to distinguish intensity gradient as small as 5 electrons in the image, but lesser variations would go undetectable. Sometimes the binary range is given in decibels with one bit equaling 6 dB (i.e., 8-bit = 48 dB, 12 bit = 72 dB, etc.)

**Dynamic Range**

Potential wells collect charges induced through electromagnetic radiation; nonetheless, other charges are induced via other sources (e.g. thermal agitation) that are also collected in the potential well. These false charges “eat” into the dynamic range of the CCD chip by imposing a dc offset. It is quantified as the FWC divided by the additive noise. The higher the dynamic range the more reliable is the quantification of differences between the dimmest and the brightest intensities of an image taken. The dynamic range (Dr) is sometimes measured in decibels: Dr = 20 x log (FWC/Noise).

Consider a CCD with a full well capacity of 330,000 electrons, and a read noise of 9 electrons per pixel at the specified read-out rate and an average dark current of 16 electrons per pixel, the dynamic range would be 330,000/(9+16), or 13200. In order to utilize the full dynamic range of the CCD, a camera incorporating 14-bit analog-to-digital conversion is required, having the ability to detect 16,384 (2 to the 14th power) grayscale levels (or 0.805 electrons per gray-level step). If 12-bit A/D conversion is used, only 4,096 gray levels can be displayed, corresponding to 3.22 electrons per grayscale step. On the other hand, a camera having 16-bit A/D conversion, which has the capability of discriminating 65,536 gray levels, will be limited by the dynamic range (4500 electrons
per pixel) of the CCD, and will not offer improvement over the 14-bit A/D converter.

A primary goal in the manufacture of scientific-grade CCD cameras is to maximize the signal available and minimize the noise, resulting in maximum dynamic range. Cooling the CCD minimizes thermal noise, as well as optimizes clocking, sampling, and other read-out electronics, consequently, the noise associated with each read-out cycle would been reduced in some high-performance scientific grade CCD cameras to as little as 3-5 electrons per pixel at typical read-out rates of approximately 1 MHz.

**Fill Factor**

Pixels do not always possess a 100% photon-active surface where photonic energy is transferred to electrons. The ratio of the active area to the overall pixel area is known as the fill factor. In cases where the fill factor is less than ideal, pixels exhibit moiré structures as inactive areas on the pixel surface yield no information for the corresponding regions on the focused image. This becomes an issue for images with small spatial dimension. Further, as a portion of the incident light is not utilized, the quantum efficiency of the CCD array will be decreased.

**Light Sensitivity**

Light sensitivity is a quantity representing the minimum number of electrons detectable by the CCD chip within the imposed noise. It is characterized by the number of photons needed to generate enough electrons to produce a unity signal-to-noise ratio (SNR). It is mathematically defined as the ratio of the SNR to the quantum efficiency of the CCD chip. Light sensitivity is wavelength dependent as the quantum efficiency is wavelength dependent as well.

**Linearity**

A crucial characteristic of CCD camera is the time performance of the chip. The
intensity history in the time domain should exhibit a linear behavior to ensure accurate response to the measured radiation. Non-linear behavior would either lead to quick saturation of the high intensity regions leaving the lower intensity regions with very little count levels in the digitized image, or would magnify lower intensity gradients on the expense of the higher intensities gradients, which would create augmented uneven error distribution as errors such as shot noise scale with the intensity. Scientific grade CCD systems have a very linear response with nonlinearities in the range of a few tenths of a percent.

**Signal to Noise Ratio (SNR)**

High SNR is imperative for accurate CCD imaging. The main noise sources affecting the SNR are: shot noise, dark current and readout noise. Mathematically, this is expressed as:

$$SNR = \frac{I}{\sqrt{\left(\sqrt{I}\right)^2 + \left(\sqrt{I_D}\right)^2 + N_{\text{readout}}^2}} = \frac{I}{\sqrt{I + I_D + N_{\text{readout}}^2}}$$  \hspace{1cm} (A.10)

It is apparent from the above equation that at high intensity levels shot noise is the main contributor to the overall SNR since dark current noise and the readout noise are quite constant and are relatively small. Therefore, at high intensity levels the SNR is:

$$SNR \approx \frac{I}{\sqrt{I}} = \sqrt{I}$$  \hspace{1cm} (A.11)

This implies that for high SNR values, the potential wells must be exploited to their limit. The following figure shows a graphical representation of the above equation for CCD chip with a FWC of 330,000, showing that at the FWC the noise is less than 0.2% of the total signal while at 10,000 electrons the relative noise is 1%. 
In low intensity applications where long exposure times are utilized, dark noise increases, which in turn affects the dynamic range and to a much lesser extent the SNR. If short exposure times are utilized, both dark noise and readout noise become significant contributors to the total noise. It is important not to confuse the actual intensity collected by the CCD chip with the digitized signal output of the A/D converter. Using a higher bit A/D converter would not increase the intensity, rather the resolution of the processed signal.
This appendix provides the details of the derivation for the laminar channel flow solution for both the isothermal and non-isothermal cases.

**Isothermal Flow Solution Mathematical Derivation**

The governing equations for a channel flow in rectangular coordinates are:

\[\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0\]  

\[\rho \frac{\partial^2 u}{\partial x^2} + \rho \frac{\partial^2 u}{\partial y^2} + \rho \frac{\partial^2 u}{\partial z^2} = \nabla \cdot \mathbf{f} \]  

\[\rho \frac{\partial^2 v}{\partial x^2} + \rho \frac{\partial^2 v}{\partial y^2} + \rho \frac{\partial^2 v}{\partial z^2} = \nabla \cdot \mathbf{f} \]  

\[\rho \frac{\partial^2 w}{\partial x^2} + \rho \frac{\partial^2 w}{\partial y^2} + \rho \frac{\partial^2 w}{\partial z^2} = \nabla \cdot \mathbf{f} \]

Assumptions/Justifications

1. Two-dimensional flow with zero cross flow/gradients \( \frac{w_x}{w_y} \approx 1 \): The aspect ratio of the channel is large enough such that the flow is only in the x-direction and the channel length is relatively short, thus preventing viscous effects due to the sidewalls, to laterally diffuse into the channel, except very near the sidewalls.
2. Parallel laminar incoming flow: The low velocity approaching flow is treated through a series of turbulence damping layers (aluminum and packaging foam) to even and damp out any fluctuations in the velocity and turbulence induced by the flow path. Further, the mass flow rate is investigated experimentally (Chapter 5) showing a linear relation with respect to the pressure drop validating laminar conditions for the entire channel (Panton, 1996).

\[ \frac{DP}{Dt} = 0 \]

3. Steady incompressible flow: Mass flow rate well controlled and measurements acquired under steady conditions (i.e., no transient measurements). Mass flow rate (as described in Chapter 5) is low enough that the Mach number is well below 1400. This further implies that the velocity divergence is zero (i.e., continuity equation)

4. Body forces are only due to gravity and act in the negative z-direction.

Hence, the reduced system of equations is:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

\[ \rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \frac{\partial p}{\partial x} \]  

\[ \rho \left( \frac{\partial v}{\partial x} + u \frac{\partial v}{\partial y} \right) = \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] - \frac{\partial p}{\partial y} \]  

\[ 0 = -\frac{\partial p}{\partial z} + \rho g_z \]

**Boundary Conditions**

The boundary conditions for a pressure driven flow between two rigid infinitely wide parallel plates are:

\[ u = v = 0 \quad \text{at} \quad y = 0 \text{ & } h \]  

The boundary conditions are the no-slip for the axial velocity component and no velocity penetration at the wall, i.e., zero vertical velocity component.
The Fully Developed Region (Poiseuille Flow)

In the fully developed region further simplification allow for an exact solution for laminar flows. As the term “fully-developed” implies, the velocity profile ceases to vary in the flow directions, hence:

\[
\frac{\partial u}{\partial x} = 0 \quad (B.10)
\]

This assumption reduces equation (B.5) to:

\[
\frac{\partial v}{\partial y} = 0 \quad (B.11)
\]

Integrating the last expression once yields a constant vertical velocity component with respect to \( y \), and that at most it is a function of \( x \). However, the solid wall boundary condition necessitates that the \( v \) vanishes at the wall, hence \( v = 0 \) everywhere in the fully developed region. The \( y \)-momentum equation reduces to:

\[
0 = -\frac{\partial p}{\partial y} \quad (B.12)
\]

Thus,

\[
p_y = \text{const} \quad (B.13)
\]

The flow is pressure driven with the pressure gradient in the \( x \)-direction, thus the pressure is a function of both \( x \) and \( z \). Since vertical pressure variations are due to the hydrostatic forces, it follows upon integrating equation (B.8) that:

\[
p(x, z) = -\rho g_z + P(x) \quad (B.14)
\]

Notice the change in the sign of the body force term \( \rho g_z \) as the force acts in the negative direction. In the previous equation \( P(x) \) is the pressure along the bottom wall. Substituting for the pressure in the \( x \)-momentum equation (B.6):
\[
0 = \mu \left[ \frac{\partial^2 u}{\partial y^2} \right] - \frac{\partial P}{\partial x} \tag{B.15}
\]

The last expression reiterates that for pressure driven laminar incompressible flow in a rectangular channel shear stress is balanced by the pressure forces in the fully developed region. The velocity, and hence the shear stress, is solely a function of \( y \) while the pressure term is only a function of \( x \). Thus:

\[
\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \frac{\partial P}{\partial x} = \text{const} \tag{B.16}
\]

Integrating twice and applying the no slip boundary condition:

\[
u(y) = -\frac{h^2}{2\mu} \frac{\partial P}{\partial x} \left[ \frac{y}{h} - \left( \frac{y}{h} \right)^2 \right] \tag{B.17}
\]

This is the famous Poiseuille parabolic profile with a maximum velocity value occurring at the centerline \( y = \frac{h}{2} \).

\[
u_{\text{max}} = -\frac{h^2}{8\mu} \frac{\partial P}{\partial x} \tag{B.18}
\]

It is often convenient to non-dimensionalize the velocity by the maximum or mean velocity in order to have a universal representation of the velocity profile.

\[
\frac{u}{u_{\text{max}}} = 4 \left[ \left( \frac{y}{h} \right) - \left( \frac{y}{h} \right)^2 \right] \tag{B.19}
\]
Once the velocity profile is known other quantities can be obtained via simple relations. Volume flow rate $Q$ information is essential in the characterization of any fluid process. It is easily computed by integrating the velocity over the cross sectional area.

$$Q = \int_{0}^{h} \int_{0}^{w} u \, dy \, dz = \frac{wh^3}{12\mu} \frac{\partial P}{\partial x}$$ \hspace{1cm} (B.20)

This relation will be used to validate laminar flow conditions in the channel flow. As seen in the equation, the volume flow rate is linearly related to the pressure drop in the fully developed region. This linear behavior is not valid for turbulent flow (Panton, 1996). Once the volume flow rate is known, a mean velocity is readily obtained.

$$u_m = \frac{Q}{wh} = \frac{h^2}{12\mu} \frac{\partial P}{\partial x}$$ \hspace{1cm} (B.21)

Note that the average velocity is two-thirds the maximum velocity; however, the average velocity is only one-half the maximum velocity for circular cross sections. This is due to higher frictional forces as result of the larger effective wetted perimeter.
Non-Isothermal Fully Developed Flow

In the previous analysis isothermal conditions were assumed, but more importantly viscosity was presumed constant allowing for the decoupling of the momentum and energy equation (neglecting free convection). In this section, we solve the same problem under non-isothermal conditions.

Uniform Temperature/Heat Flux Boundary Condition

Consider the same channel with both lower and upper walls \( y = 0; y = h \) at some constant temperature higher than the flow mean temperature \( T_w > T \), where the subscript \( w \) indicates the lower wall, or have a constant heat flux, which is commonly defined using Fourier’s law of conduction expressed as:

\[
q_w^* = -k \frac{\partial T}{\partial x}
\]  

where \( q_w^* \) is the heat flux and \( k \) is the thermal conductivity which is a characteristic of the wall material. Inside the channel, convection heat transfer occurs and a thermal boundary layer begins to develop in a way similar to the viscous boundary layer. If either a constant temperature or constant heat flux is maintained at the wall, fully developed thermal conditions are eventually attained.
The thermal entrance length, $L_T$, for laminar flows is experimentally observed to be (Incropera and Dewitt, 1981):

$$\frac{L_T}{h} = 0.05 \text{Re}_h \text{ Pr} \quad (B.23)$$

where Pr is the Prandtl number defined as:

$$\text{Pr} = \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}} = \frac{c_p \mu}{k} = \frac{\nu}{\alpha} \quad (B.24)$$

The Prandtl number provides a measure of the relative effectiveness of momentum and energy diffusion via diffusion in the viscous and thermal boundary layers, respectively. This nondimensional number solely involves fluid properties with no dependence on the flow geometry or velocity. For gases, it is near unity, hence momentum and energy transfer by diffusion are comparable. It is a significant parameter in heat-transfer applications as it compares the rate of momentum diffusion to that of energy. It follows that the value of Pr dictates the relative growth of the viscous and thermal boundary layers and can be approximated as:
where \( n \) is a positive integer, \( \delta \) and \( \delta_T \) are the thicknesses of the viscous and thermal boundary layers, respectively. Thus, for gasses the extent of growth (i.e., thickness) of the two layers is approximately equal.

It is imperative to note that the concept of a mean temperature is not as simple as the mean velocity. First, to estimate the mean temperature, \( T_m \), we integrate the product of the mass flux \( \rho u \) and the internal energy per unit mass \( c_v T \) over the cross section of the channel to yield the thermal energy transport rate \( E_v \). That is,

\[
\dot{E}_v = \int_A \rho u c_v T dA
\]  
(B.26)

For which the general expression for the mean temperature is:

\[
T_m = \frac{\dot{E}_v}{m c_v} = \frac{\int \rho u c_v T dA}{m c_v}
\]  
(B.27)

For an incompressible flow in a rectangular duct it follows that the mean temperature is:

\[
= \frac{\int \rho u c_v T dA}{\rho u_m w_d h}
\]  
(B.28)

\[
T_m = \frac{1}{u_m h} \int_0^h u T(x, y) dy
\]  
(B.29)

In forced convection internal flows, heat transfer is continuous along the entire length of the channel and hence temperature increases/decreases longitudinally. Therefore, the temperature is always a function of both \( x \) and \( y \) and flow direction...
gradients never vanish, unlike the velocity. This is also different from the case of external flows as the temperature is constant in the flow direction.

It might then seem contradictory to use the term “fully-developed thermal region” for flows with convection heat transfer. However, in such flows the term fully developed is based on a non-dimensional temperature difference as opposed to the actual temperature. To realize the dimensionless temperature we need to introduce Newton’s law of convection, which is:

$$q'_{w} = h_{c} (T_{w} - T_{m}) \quad (B.30)$$

where $h_{c}$ is the local convection heat transfer coefficient (W/m²·K). Newton’s law states that a temperature difference must exist for heat transfer to occur, thus, it is reasonable to use a temperature difference to express the temperature. Utilizing all the known temperatures (wall, mean, and flow), we reach the dimensionless temperature:

$$\frac{T_{w} - T(x, y)}{T_{w} - T_{m}} \quad (B.31)$$

When this dimensionless temperature is independent of $x$ the flow is termed fully developed. Hence the fully developed thermal condition is:

$$\frac{\partial}{\partial x} \left[ \frac{T_{w}(x) - T(x, y)}{T_{r}(x) - T_{m}(x)} \right] = 0 \quad (B.32)$$

If a constant temperature boundary condition is imposed on the flow, then the fully developed conditions depend only on the mean and flow temperatures. We can gain further insight in the fully developed region by differentiating the temperature difference with respect to $y$ and evaluating the derivative at the wall.
At the wall, there exists no fluid motion due to the no-slip condition; the only path for heat transfer is conduction (or radiation, not included here). Substituting Fourier’s and Newton’s Laws of conduction and convection, respectively yields:

\[
\frac{h}{k} = \text{const} \tag{B.34}
\]

Thus for a constant properties flow, \( h_c \) is constant in the thermally fully developed region. The convection coefficient \( h_c \) strongly depends on the wall temperature gradient, which is in turn heavily dependent on conditions in the thermal boundary layer. Further, this implies that \( h_c \) must vary prior to fully developed conditions, reaching its maximum value at the channel entrance and \( \delta_r = 0 \), then it decays rapidly approaching a constant value in the fully developed region. If we expand fully developed condition, equation (B.32), we get:

\[
\left( 1 - \frac{T - T_m}{T_w - T_m} \right) \frac{\partial T}{\partial x} - \frac{T - T_m}{T_w - T_m} \left( \frac{T - T_m}{T_w - T_m} \right) \frac{\partial T}{\partial x} = 0
\]

\[
\left( 1 - \frac{T - T_m}{T_w - T_m} \right) \frac{\partial T}{\partial x} - \frac{T - T_m}{T_w - T_m} \left( \frac{T - T_m}{T_w - T_m} \right) \frac{\partial T}{\partial x} = 0 \tag{B.35}
\]

\[
\left( 1 - \frac{T - T_m}{T_w - T_m} \right) \frac{\partial T}{\partial x} - \frac{T - T_m}{T_w - T_m} \left( \frac{T - T_m}{T_w - T_m} \right) \frac{\partial T}{\partial x} = 0
\]
\[
\frac{\partial T}{\partial x} = \frac{\partial T_w}{\partial x} - \frac{(T_w - T)}{(T_w - T_m)} \frac{\partial (T_n - T_m)}{\partial x} \quad \text{(B.36)}
\]

If the boundary condition is a constant temperature along the channel \( \frac{\partial T_w}{\partial x} = 0 \), then the fully developed condition reduces to:

\[
\frac{\partial T}{\partial x} = \frac{(T_w - T)}{(T_w - T_m)} \frac{\partial T_m}{\partial x} \quad \text{(B.37)}
\]

and for a constant heat flux we first recall that \( h_c \) is constant and from equation (B.30) we find:

\[
\frac{\partial T_w}{\partial x} = \frac{\partial T_m}{\partial x} \quad \text{(B.38)}
\]

Substituting back in equation (B.36):

\[
\frac{\partial T}{\partial x} = \frac{\partial T_m}{\partial x} \quad \text{(B.39)}
\]

The foregoing results clearly show that knowing the mean temperature is essential in describing the temperature distribution throughout the fully developed region of the channel. The temperature distribution at the surface \( \frac{\partial T}{\partial x} \) is the quantity sought after in this work as a mean of validating the measured temperature profiles by the paint. In our experiment, the surface temperature is not constant due to forced convection effects and there is no heat flux at the surface, rather a constant heat flux at \( x = 0 \) and \( L \) and at \( z = \pm \frac{w_d}{2} \). The boundary conditions are clearly more involved than the simple cases of constant heat flux and constant temperature. A simple energy balance would yield the following expression for the mean temperature in terms of convection.
\[
\frac{dT_m}{dx} = \frac{W_A}{m c_p} h (T_w - T_m)
\]  
(B.40)

Which can be further simplified for the cases of constant heat flux and constant temperature at the surface yielding equations (B.41) and (B.42), respectively.

\[
T_m(x) = T_{m,\text{outlet}} + \left(\frac{q_w W_A}{m c_p}\right) x
\]
(B.41)

\[
\frac{T_w - T_m}{T_w - T_{m,\text{inlet}}} = \exp\left(-\frac{W_A \bar{h}}{m c_p} x\right)
\]
(B.42)

In the previous equations, \(W_A\) is the wetted perimeter area and \(\bar{h}\) is the average value of \(h\) from the channel inlet to \(x\). Obviously, this still calls for a known temperature distribution or heat flux over the surface, which is not available nor simple. In order to estimate the surface and mean temperatures, the temperature profile \(T(x, y)\) must be then determined by solving the energy equation.

The energy equation for a Newtonian fluid in rectangular coordinates is:

\[
\rho \frac{De}{Dt} + p \nabla \cdot V = \nabla \cdot (k \nabla T) + \mu \Phi
\]  
(B.43)

where \(e\) is the internal energy per unit mass, where \(\Phi\) is customarily called the dissipation function and is always positive definite as viscosity dissipates energy from the system.

Using Stokes’ hypothesis from the definition of the mechanical pressure \(\Phi\) is expressed as:

\[
\Phi = 2 \left\{ \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 \right\} + \left\{ \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z} + \frac{\partial v}{\partial z}\right)^2 \right\}
\]

\[
+ \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2
\]  
(B.44)
The total derivative \( \frac{D}{Dt} \) and the divergence operator \( \nabla \) are expressed as:

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + \langle \nabla \cdot \nabla \rangle
\]

(B.45)

\[
\nabla ( \cdot ) = \frac{\partial ( \cdot )}{\partial x} + \frac{\partial ( \cdot )}{\partial y} + \frac{\partial ( \cdot )}{\partial z}
\]

(B.46)

From thermodynamic relations and for an ideal gas:

\[
h = e + \frac{p}{\rho}
\]

(B.47)

\[
dh = c_p dT
\]

\[
de = c_v dT
\]

(B.48)

where \( h \) is the enthalpy, \( c_p \) and \( c_v \) \([J/kg \cdot K]\) are the specific heats per unit mass at constant pressure and volume, respectively. Using the above equations, employing the general continuity equation, equation (B.1), to substitute for the velocity divergence in equation (B.43) and assuming constant thermal conductivity, the energy equation can be rewritten as:

\[
\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = \left( \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} \right) + k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \mu \nabla^2 \Phi
\]

(B.49)

For the fully developed viscous region in the channel flow problem, using the same assumptions used in the velocity derivation, the energy equation reduces to:

\[
\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right) = \left( u \frac{\partial p}{\partial x} \right) + k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \left( \frac{\partial u}{\partial y} \right)^2
\]

(B.50)
The temperature boundary and initial conditions are:

\[
T(x,y) = T_0 \quad @ x = 0, y \\
T(x,y) = T_w(x) \quad @ x \geq 0, y = 0 \\
h_{\text{glass}} \frac{\partial T(x,y)}{\partial y} = h_{\text{air}}(T_{w, ap} - T) \quad @ x \geq 0, y = h \\
h_{\text{metal}} \frac{\partial T(x,y)}{\partial y} = h_{\text{air}}(T_{w, bottom} - T) \quad @ x \geq 0, y = 0 \\
\frac{\partial T(x,y)}{\partial t} = T_1 - \frac{(T_1 - T_2)}{L} x \quad @ t = 0; \ x, y = 0
\]

(B.51)

Figure B-3 Non-isothermal flow schematic
These boundary conditions represent a balance between forced convection and conduction at the upper and lower walls, a uniform temperature profile at the beginning of the channel, while the lower wall having an interface boundary condition with the metal. The problem is not easily realized because the bottom boundary condition \((y = 0)\) is an interface boundary. This means that convection due to the flow and conduction in the metal are interfacing and constantly updating each other, hence the two problems must be numerically solved simultaneously until steady conditions are realized. The conduction problem for the metal plate is governed by the general heat equation shown below.

\[
\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + q = \rho c_p \frac{\partial T}{\partial t} \tag{B.52}
\]

where \(q\) represents energy generation within the plate. Assuming constant thermal conductivity:

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \left( \frac{\rho c_p}{k} \right) \frac{\partial T}{\partial t} \tag{B.53}
\]

Conduction and radiation on the outside boundaries can be neglected as forced convection in the channel is more significant. The two problems interface the terms \(\frac{\partial T}{\partial x}\) and \(\frac{\partial^2 T}{\partial x^2}\). For each iteration equation (B.53) is solved first with the temperature profile at the interface updated by the solution for one time step, then equation (B.50) is solved with the updated boundary and its solution updates the interface boundary. The numerical solution is reiterated until convergence occurs. Besides the obvious intricacy of the problem, it is further beyond the scope of this work and thus is not further pursued.
Nonetheless, the theory can be numerically solved to validate the experimental results for future work.
APPENDIX C
MATLAB CODES FOR POD ANALYSIS

Image Registration

% Program to perform image registration and compare theoretical and
% experimental pressure values

clear all
clc

% Channel Parameters
L=9.18*2.54/100; % length
w=4*2.54/100; % width
mu=1.82*10^(-5); % dynamic viscosity
h=0.00025; % channel height
x=(1:2/3:9); % x-spacing for the centerline pressure taps

% Looping for the four filters
for i=44:2:50
  gstd=10000;
  for al=0.232:0.01:0.232
    % Reading images
    run=double(imread(['c1CHNL_','num2str(i),'_PT2.tif']));
    ref=double(imread(['c1CHNL_','num2str(i),'_ref.tif']));
    dark=double(imread(['c1CHNL_','num2str(i),'_dk.tif']));

    % Rotating images
    run2=imrotate(run,-al,'bicubic');
    run=run2;
    runn=run;

    ref2=imrotate(ref,-0.25,'bicubic');
    ref=ref2;
   reff=ref;
    [mm,nn]=size(run);

    % Image registration
    % %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % % image registration

    % %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % % image registration

  end
end
% horizontal shift
aa=1;
for hs=0.76:0.01:0.76
    bb=1;
    ref=reff;
    for j=2:nn
        ref2(:,j)=ref(:,j-1)-(ref(:,j-1)-ref(:,j))*hs;
    end
    ref2(:,nn)=ref(:,nn);
    ref4=ref2;
end
% Vertical shift
for vs=0.08:0.01:0.08
    for j=1:nn-1
        ref2(j,:)=ref4(j,:)-(ref4(j,:)-ref4(j+1,:))*vs;
    end
    ref2(:,nn)=ref(:,nn);
    ref=ref2;
    ratio1=run./ref;
    bb=bb+1;
    ratio=run./ref;
end
% computing the STD of two diagonal areas on the image to
% optimize rotation and image registration
    gg=std(std(ratio(100:150,180:230)))+std(std(ratio(400:450,280:330)));
if gg < gstd
    ratioC=ratio;
    gstd=gg;
    alpha=al;
    vsC=vs;
    hsC=hs;
end
    ref=ref4;
end
aa=aa+1;
gg
end
ratio=run./ref;

% Loading pressure and temperature information
D=load(['c1CHNL_',num2str(i),'_PT2.txt']);

Tmp=D(2:22,[1 3]); % Temperature
Tmpstd=D(2:22,[2 4]); % STD for Temperature
T=mean(Tmp,2); % Average Temperature
Tstd=mean(Tmpstd,2); % Average STD for Temperature

% Rearranging the temperature matrix for plotting
for j=4:-1:1
    T1(5:-1:1,5-j)=T(1+(j-1)*5:5+(j-1)*5,1);
end

Prss=D(23:42,[1 3]); % Pressure
Prssstd=D(23:42,[2 4]); % STD for Pressure
P=mean(Prss,2); % Average Pressure
Pstd=mean(Prssstd,2); % Average STD for Pressure

Q=mean(D(1,[1 3]),2)*20*0.0000166666666666666; % Average mass flow

dP=-Q*(12*mu)/h^3/w*L; % Theoretical Poiseuille Pressure Gradient
P0=P(20,1)*101325/14.7-dP; % Pressure offset
Ptheory=dP/L*([x 10.18]-1)*2.54/100+P0; % Theoretical Poiseuille Pressure

% Fitting the pressure to a polynomial
[P1,S] = POLYFIT(x',P(1:13,1),2);
P2=P1(1,1)*[x 10.18].^2+P1(1,2)*[x 10.18]+P1(1,3);

% Setting the background color for the images
a=min(min(ratioC(10:505,210))-0.02);
b=max(max(ratioC(10:505,210))+0.02);
c=(b-a)/3+a;

run(:,1:123)=b;
run(:,315:512)=b;
ref(:,1:123)=b;
ref(:,315:512)=b;
ratio(:,1:132)=c;
ratio(:,323:512)=c;
ratio(1:8,:) = c;
ratio(512:517,:) = c;

% Plotting the run, reference and normalized images
figure(1)
imagesc(run)
colorbar

figure(2)
imagesc(ref)
colorbar

figure(3)
imagesc(ratioC, [a b])
title(['filter ',num2str(i),' PT2 hs= ', num2str(hs),' vs= ', num2str(vs)])
colorbar

% Plotting temperature contours from thermocouple information
figure(4)
contourf(T1,256,'linestyle','none')
colorbar

% Plotting the theoretical and experimental pressure
figure(5)
clf
plot(x, P(1:13,:), 'o-')
hold on
plot([x 10.18], Ptheory*14.7/101325,'o-r')
plot([x 10.18], P2, 'm')
grid on
xlabel('x-position (inch)')
ylabel('Pressure (psi)')

% Plotting pressure percent deviation in pressure
figure(6)
plot((P(1:13,:)-(Ptheory(1,1:13)*14.7/101325))/P(1:13,:)*100,'o-')
xlabel('x-position (inch)')
ylabel('Percent deviation from theory')
grid on

pause

end
Calibration Intensity/Pressure and Temperature Extraction

% Program to extract calibration matrices for pressure and temperature
clear all
clc

% Channel Parameters
L=9.18*2.54/100; % length
w=4*2.54/100; % width
mu=1.82*10^(-5); % dynamic viscosity
h=0.00025; % channel height
x=(1:2/3:9); % x-spacing for the centerline pressure taps

% Looping for the four filters
for i=44:2:50
    gstd=10000;
    for al=0.232:0.01:0.232
        % Reading images
        run=double(imread(['c1CHNL_',num2str(i),'_PT2.tif']));
        ref=double(imread(['c1CHNL_',num2str(i),'_ref.tif']));
        dark=double(imread(['c1CHNL_',num2str(i),'_dk.tif']));

        % Rotating images
        run2=imrotate(run,-al,'bicubic');
        run=run2;
        runn=run;
        ref2=imrotate(ref,-0.25,'bicubic');
        ref=ref2;
       reff=ref;
        [mm,nn]=size(run);

        % Image registration
        for hs=0.76:0.01:0.76
            bb=1;
            ref=reff;
            for j=2:nn
                ref2(:,j)=ref(:,j-1)-(ref(:,j-1)-ref(:,j))*hs;
            end
            ref2(:,nn)=ref(:,nn);
    end
end
ref4=ref2;

% Vertical shift
for vs=0.08:0.01:0.08
    for j=1:nn-1
        ref2(j,:)=ref4(j,:)-(ref4(j,:)-ref4(j+1,:))*vs;
    end
    ref2(:,nn)=ref(:,nn);
    ref=ref2;
    ratio1=run./ref;
    bb=bb+1;
end

ratio=run./ref;

% computing the STD of two diagonal areas on the image to
% optimize rotation and image registration

    gg=std(std(ratio(100:150,180:230)))+std(std(ratio(400:450,280:330)));

if gg < gstd
    ratioC=ratio;
    gstd=gg;
    alpha=al;
    vsC=vs;
    hsC=hs;
end

ref=ref4;
end

aa=aa+1;

end

end

% ####################################################################

% Initializing the matrices averaging
runs=run;
refs=ref;

% Specifying the averaging square size
sqsize=5;
% Smoothing the run and reference images
length=(sqsize-1)/2;
for js=131:322
    for is=[sqsize:517-sqsize]
        runs(is,js)=sum(sum(run(is-length:is+length,js-length:js+length)))/(sqsize^2);
        refs(is,js)=sum(sum(ref(is-length:is+length,js-length:js+length)))/(sqsize^2);
    end
end

% ##############################################################
% Writing the test data matrix
ratio=runs./refs;
[sm,sn]=size(ratio);
sm=sm-16;
sna=131;
spb=322;
sma=9;
smb=509;
for si=1:snb-sna+1
    ratioCCD((si-1)*(smb-sma+1)+1:si*(smb-sma+1),d)=
        ratio(sma:smb,si+sna-1);
end

% Loading pressure and temperature information
D=load(['c1CHNL_',num2str(i),'_PT2.txt']);  % Environmental data
Tmp=D(2:22,[1 3]);  % Temperature
Tmpstd=D(2:22,[2 4]);  % STD for Temperature
T=mean(Tmp,2);  % Average Temperature
Tstd=mean(Tmpstd,2);  % Average STD for Temperature

% Rearranging the temperature matrix for plotting
for j=4:-1:1
    T1(5:-1:1,5-j)=T(1+(j-1)*5:5+(j-1)*5,1);
end

Prss=D(23:42,[1 3]);  % Pressure
Prssstd=D(23:42,[2 4]);  % STD for Pressure
P=mean(Prss,2);  % Average Pressure
Pstd=mean(Prssstd,2);  % Average STD for Pressure

Q=mean(D(1,[1 3]),2)*20*0.00001666666666666666;  % Average mass flow
\[ dP = -Q \cdot (12 \cdot \mu / h^3 / w \cdot L); \] % Theoretical Poiseuille Pressure Gradient
\[ P_0 = P(20,1) \cdot 101325 / 14.7 - dP; \] % Pressure offset
\[ P_{theory} = dP / L \cdot ([x \cdot 10.18] - 1) \cdot 2.54 / 100 + P_0; \] % Theoretical Poiseuille Pressure

% Fitting the pressure to a polynomial
\[ [P1, S] = \text{POLYFIT}(x', P(1:13,1), 2); \]
\[ P_2 = P1(1,1) \cdot [x \cdot 10.18] \cdot 2 + P1(1,2) \cdot [x \cdot 10.18] + P1(1,3); \]

% matrix size n x n (n=temp thermocouple size, m=pressure tap size)
n = 5;
m = 5;

% coordinates (upper=U) (lower=L) (R=row) (C=column)
UR = 74;
UC = 149;
dR = 101;
dC = 50;
dP = 33;
width = 4; % width of outer rectangle
n1 = (n-1)/2;
m1 = (m-1)/2;
nj = (n1 + width);
mi = (m1 + width);

% #-----------------#-----------------#-----------------#-----------------#  
% Temperature loop
% % % % .................. mi
% % : : : : : :  : width
% % : : : : : :  : m1
% % : : : : : :  :
% % : : : : : :  :
% % ---------------------------
%  : width  UR/UC
% %
% nj  n1
% #-----------------#-----------------#-----------------#-----------------#  
% Extracting temperature intensity calibration
b = 1; c = 1;
for j = 3*dC+(UC):-dC:UC
a=1;
for i=UR:dR:4*dR+(UR)
    Top=mean(mean(ratio(i-m1:i-m1,j-nj:j+nj)));
    Bot=mean(mean(ratio(i+m1:i+m1,j-nj:j+nj)));
    Lft=mean(mean(ratio(i-m1:i+m1,j-nj:j-n1)));
    Rgt=mean(mean(ratio(i-m1:i+m1,j+n1:j+nj)));
    InT(a,b)=(Top+Bot+Lft+Rgt)/4;
    IT(c,d)=(Top+Bot+Lft+Rgt)/4;
    a=a+1;
    c=c+1;
end
b=b+1;
end

%######################################################
% Pressure Extraction (calibration)
% Test Points
PNT1C=floor(UC+(dC/2));
PNT1R=UR+4*dR;
PNT2C=floor(UC+3*dC);
PNT2R=floor(UR+3*dR+(dR/2));
PNT3C=floor(UC+(dC/2));
PNT3R=UR+2*dR;
PNT4C=floor(UC+(dC/2));
PNT4R=floor(UR+(dR/2));
% #####################################################
% Calibration Points
a=1;
for i=4*dR+(UR):-33:UR
    IP(a,d)=mean(mean(ratio(i-mi:i+mi,floor(UC+dC*1.5)-
    mi:floor(UC+dC*1.5)+mi)));
    a=a+1;
end
IP(a,d)=mean(mean(ratio(PNT1R-mi:PNT1R+mi,PNT1C-
    mi:PNT1C+mi)));
IP(a+1,d)=mean(mean(ratio(PNT2R-mi:PNT2R+mi,PNT2C-
    mi:PNT2C+mi)));
IP(a+2,d)=mean(mean(ratio(PNT3R-mi:PNT3R+mi,PNT3C-
    mi:PNT3C+mi)));
IP(a+3,d)=mean(mean(ratio(PNT4R-mi:PNT4R+mi,PNT4C-
    mi:PNT4C+mi)));
%######################################################

d=d+1;
% Plotting Calibration Curves
figure(3)
plot(IT(1:13,:),'.-')
grid on
title('Temperature')
legend('550 nm','600 nm','650 nm','700 nm')

figure(4)
plot(IP(1:13,:),'.-')
grid on
title('Pressure')
legend('550 nm','600 nm','650 nm','700 nm')

% Calibration Parameters
% Centerline Pressure data
PCNTR=mean(P(1:13,:),2);

% X and Y coordinates of the temperature thermocouples starting from the
% top right of the plate (looking at the image from Matlab)
a=1;
for i=1:20
    Tx(i,1)=a;
    if mod(i,5) == 0
        a=a+1;
    end
end

a=1;
for i=1:20
    Ty(i,1)=a;
    a=a+2;
    if mod(a,11) == 0
        a=1;
    end
end

% Fitting temperature to a surface to calculate centerline temperature
% at the centerline along the pressure taps (TCNTR)
Tf=[Tx.^2 Ty.^2 Tx.*Ty Tx Ty ones(20,1)];
TCof=linsolve(Tf,mean(T(1:20,:),2));
% TCof=(Tf\mean(T(1:20,:),2));
Ty=x;
Tx=2.5;
TCNTR = flipud(TCof(1,1)*Tx.^2+TCof(2,1)*Ty.^2+TCof(3,1)*Tx.*Ty+TCof(4,1)*Tx+TCof(5,1)*Ty+TCof(6,1));

PCalbMatrix(1,:) = TCNTR;
PCalbMatrix(2,:) = PCNTR';
PCalbMatrix(3:6,:) = IP(1:13,:);'';

% Temperature Calibration matrix
TCalbMatrix(1,:) = mean(T(1:20,:),2);'';
TCalbMatrix(2:5,:) = IT';

% Writing data matrices and calibration matrices
dlmwrite('TCalbMatrix.txt', TCalbMatrix, 'delimiter','	','precision','%.12f');
dlmwrite('PCalbMatrix.txt', PCalbMatrix, 'delimiter','	','precision','%.12f');
dlmwrite('dataMatrix.txt', ratioCCD, 'delimiter','	','precision','%.12f');

POD Code

% Program to extract pressure and temperature using POD
clear all
cclc

% Test parameters
% (2) (1,84)
% (1) (2,32)
% (3) (2,234)
% (4) (1,353 not 86)
% 19:128:147

% Number of calibration points to use
np = 13

% Loading data
data = load('PCalbMatrix.txt');
D2 = 1./data(3:6,(13-np)+1:13);
Wavelength = (550:50:700);
T = data(1,:);
P = data(2,(13-np)+1:13);
ImgData = (flipud(load('dataMatrix.txt')));

ird = 1;
iar = 501;

% Loop to calculate pressure/temperature field (195 columns of 501
% rows data)
for ir=1:195

    D2(1:4,np+1:iar+np)=1./ImgData((ir-1)*iar+1:iar*ir,1:4)';
    [mm,nn]=size(D2);

% Initiating POD
Lambda1=0;
RR=zeros(n,n);
Z=D'*D;
CC=zeros(m,n); Rc=zeros(m,n); HH=zeros(m,n); e=1; Vec=0;
p=1;
while e > 0
    if p > 1
      Z=RR;
    end
    % calculating the eigenvectors and eigenvalues
    RR=Z-Lambda1*Vec*Vec';
    [Q,Lambda]=eig(RR);
    Lambda1=max(max(Lambda));
    if p ==1
        maxii=Lambda1;
    end

    for y=1:n
        for t=1:n
            if Lambda1 == Lambda(t,y);
                x=y;
            end
        end
    end
    Vec=Q(:,x);
    % Terminating the loop if the eigenvalue is less than 0.00001 of
    % the Maximum eigenvalue
    if Lambda1 > maxii*0.00001
        HH=D*Q;
        Rc(:,p)=HH(:,x);
        CC(p,:)=Vec';
    else
        e=0;
    end
    p=p+1;
end

% Reconstructing the R & C matrices
R=zeros(m,p-2);
C=zeros(p-2,n);
for aa=1:p-2
    R(:,aa)=Re(:,aa);
    C(aa,:)=CC(aa,:);
end
R1=R;
C1=C;

% Plotting the reconstructed spectra
figure(2)
clf
plot(Wavelength,R*C)
title('Reconstructed Spectra')
xlabel('Wavelength (nm)')
ylabel('Intensity')
grid on

PerrMn2=100000;
a=1;

% Target Transformation
for phii=0:360
    phi=phii*pi/180;
    % Rotation Matrix
    Tr=[cos(phi) -sin(phi); sin(phi) cos(phi)];
    R=R1*Tr;
    C=inv(Tr)*C1;

    % Plotting the eigenvectors and eigenvalues
    figure(3)
    plot(Wavelength,R(:,1),'o-b',Wavelength,R(:,2),'o-r')
title(['Fundemental Spectra @ theta = ',num2str(phii)])
xlabel('Wavelength (nm)')
legend('R1','R2')
grid on

    figure(4)
    plot((1:2/3:9),C(1,1:13)','o-b',(1:2/3:9),C(2,1:13)','o-r')
    legend('C1','C2')
grid on

    % Fitting the pressure calibration data to the calibration
    % function of order N
    N=1;
Pcof=polyfit(C(1,1:np),P,N);
N=N+1;
for i=1:N
    Prss1(i,:)=Pcof(1,i)*C(1,:).^(N-i);
End
Prss=sum(Prss1);

% Calculating the error parameters for the calibration
Perr(a,:)=(P-Prss(1,1:np));
PerrMn(a,1)=mean(abs(P-Prss(1,1:np)));
PerrMx(a,1)=max(abs(P-Prss(1,1:np)));

% Condition of minimum error
if PerrMn(a,1) < PerrMn2
    PP=Prss;
    PerrMn2(a,1)=PerrMn(a,1);
    PerrMx2(a,2)=phii;
    PerrMn(a,1)=PerrMn(a,1);
end

a=a+1;
pause(0.05)
end

% Writing predicted pressure data
PMatrix(ird,:)=PP(1,np+1:np+iar);

ird=ird+1;
end

% Plotting the predicted values and image and writing data
np=13;
P=data(2,(13-np)+1:13);

figure(1)
plot((73:33:481-(34*abs(np-13))),fliplr(P),'ok')
hold on
plot(flipud(PMatrix'),'g')
ylabel('Pressure (psi)')
grid on

figure(2)
imagesc(PMatrix, [min(PMatrix(:,95)) max(PMatrix(:,95))])
colorbar
dlmwrite('PImage.txt',PMatrix,'	')
APPENDIX D
DERIVATION OF SOME EQUATIONS

POD Analysis

Derivation of Pressure Calibration Coefficients

Starting with the Stern-Volmer relation and the definition of the temperature dependent coefficients:

\[
\frac{P}{P_{\text{ref}}} = \left[ \left( \frac{I_{\text{ref}}}{I} \right) \frac{1}{B(T)} \right] - \left( \frac{A(T)}{B(T)} \right) \tag{D.1}
\]

\[
A(T) \approx A(T_{\text{ref}}) \left[ 1 + \frac{E_{\text{nr}}}{RT_{\text{ref}}} \left( \frac{T - T_{\text{ref}}}{T_{\text{ref}}} \right) \right] \tag{D.2}
\]

\[
B(T) \approx B(T_{\text{ref}}) \left[ 1 + \frac{E_p}{RT_{\text{ref}}} \left( \frac{T - T_{\text{ref}}}{T_{\text{ref}}} \right) \right] \tag{D.3}
\]

Taking the ratio of the coefficients:

\[
\frac{A(T)}{B(T)} \approx \frac{A(T_{\text{ref}})}{B(T_{\text{ref}})} \left[ 1 + \frac{E_{\text{nr}}}{RT_{\text{ref}}} \left( \frac{T - T_{\text{ref}}}{T_{\text{ref}}} \right) \right] \left( \frac{A(T_{\text{ref}})}{B(T_{\text{ref}})} \right)^{-1} \left[ 1 + \frac{E_p}{RT_{\text{ref}}} \left( \frac{T - T_{\text{ref}}}{T_{\text{ref}}} \right) \right]^{-1} \tag{D.4}
\]

\[
\frac{A(T)}{B(T)} \approx \alpha \left[ 1 + a \left( \frac{T - T_{\text{ref}}}{T_{\text{ref}}} \right) \right] \left[ 1 + b \left( \frac{T - T_{\text{ref}}}{T_{\text{ref}}} \right) \right] \tag{D.5}
\]
Substituting the values for the different constants:

\[
\frac{A(T)}{B(T)} \approx \alpha \left( \frac{a_i}{b_i + b_{ref} T} + \frac{a_{ref} T}{b_i + b_{ref} T} \right) = \alpha \left( \frac{a_i}{b_i + b_{ref} T} + \frac{a_{ref}}{b_i + b_{ref} T} \right) \tag{D.6}
\]

In the ratio \( \frac{A(T)}{B(T)} \), the first ratio \( \frac{-1.82}{T - 4.32} \) is two orders of magnitude smaller compared to the second term \( \frac{0.009463}{1 + \frac{0.014496}{T}} \), therefore the constants can be replaced with a unity value \( \frac{-1}{T - 1} \) with the same results as the ratio is still preserved due to the triviality of the constant in the denominator compared to the temperature. The second term can be replaced as follows \( \frac{0.01}{1 + 0.01} \) with the same results (slightly different rotation angle). Replacing the actual values makes it easier to simulate real data as the constants are a function of the luminophor.

\[
\therefore \left( \frac{-1.82}{T - 4.32} \right) \ll \left( \frac{0.009463}{1 + \frac{0.014496}{T}} \right) \tag{D.7}
\]
\[
\frac{A(T)}{B(T)} \approx \alpha \left( \frac{0.009463}{\frac{1}{T} + 0.014496} \right) \approx \alpha \left( \frac{0.01T}{1 + 0.01T} \right) \tag{D.10}
\]

\[
\frac{A(T)}{B(T)} \approx \alpha \left( \frac{0.01T}{1 + 0.01T} \right) \tag{D.11}
\]

The second coefficient can be similarly expressed as:

\[
\frac{1}{B(T)} = \frac{1}{B(T_{ref}) \left[ 1 + \frac{E_p}{RT_{ref}} \left( \frac{T - T_{ref}}{T_{ref}} \right) \right] - \frac{1}{1 + b \left( \frac{T - T_{ref}}{T_{ref}} \right)}} = \frac{1}{B(T_{ref}) \left[ 1 + \frac{RT_{ref}}{E_p} \right] - \frac{1}{1 + b \left( \frac{T - T_{ref}}{T_{ref}} \right)}} \tag{D.12}
\]

\[
\frac{1}{B(T)} = \frac{1}{B(T_{ref}) + B(T_{ref})b \left( \frac{T - T_{ref}}{T_{ref}} \right)} - \frac{1}{B(T_{ref}) + b \left( \frac{T - T_{ref}}{T_{ref}} \right) - B(T_{ref})b} \tag{D.13}
\]

\[
\frac{1}{B(T)} = \frac{1}{B_{ref} - B_b + B_{ref}T} = \frac{1}{BB + B_{ref}T} \tag{D.14}
\]

\[
\frac{1}{B(T)} = \frac{1}{0.012612T - 2.8884} \tag{D.15}
\]

The ratio \( \frac{1}{B(T)} = \frac{1}{0.012612T - 2.8884} \) can not be approximated, as the calibration results would be less accurate because the temperature product is comparable in magnitude to the free constant in the denominator.
LIST OF REFERENCES


Gfroerer, T. H., Photoluminescence in Analysis of Surfaces and Interfaces Encyclopedia of Analytical Chemistry R.A. Meyers (Ed.) John Wiley and Sons Ltd, Chichester, 2000 pp. 9209–9231


Weiss, P.E., “Pressure Sensitive Paints in Variable Flowfield Wind Tunnel Testing, A feasibility study submitted for the course ESC 300,” University of Toronto, Fall 2002.


BIOGRAPHICAL SKETCH

The author was born on February 20th 1977, in the city of Salmiya, Kuwait. He grew up in the Persian Gulf, spending most of his teenage years in Bahrain. He graduated from high school at the age of sixteen and then joined the mechanical engineering Department at the University of Bahrain, Bahrain. He earned an associate degree in mechanical engineering and then joined Embry-Riddle Aeronautical University, Daytona Beach, Florida, earning a Bachelor of Science in aerospace engineering in the Spring of 1999. He continued his graduate school at ERAU earning a Master of Science in aerospace engineering in the Spring of 2001. In the fall of 2001, Dr. Omar moved to Gainesville, Florida, joining the Mechanical and Aerospace Engineering Department. He worked under the supervision of Dr. Bruce Carroll working on several research projects, earning his Ph.D. in the summer of 2006.