THE EFFECTS OF GRAPHING CALCULATORS AND A MODEL FOR CONCEPTUAL CHANGE ON COMMUNITY COLLEGE ALGEBRA STUDENTS' CONCEPT OF FUNCTION

BY

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This dissertation is dedicated to my parents,

Pickens and Tennie Ruth Lott,

whose love and faith I was never without

and

to Larry Vanoy Adams, my mate and friend.
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Abstract of Dissertation Presented to the Graduate School of the University Florida in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

THE EFFECTS OF GRAPHING CALCULATORS AND A MODEL FOR CONCEPTUAL CHANGE ON COMMUNITY COLLEGE ALGEBRA STUDENTS' CONCEPT OF FUNCTION

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Three treatment groups and a control group were compared on two dependent variables regarding their understanding of the concept of function. During the unit of study for the concept, Treatment Group I students used graphing calculators and participated in a conceptual change assignment. Treatment Group II students used graphing calculators only during study of the unit. Treatment Group III students participated in the conceptual change assignment only during the unit. Treatment Group IV served as a control group. Regarding students' understanding and application of the function concepts, domain and range, and their understanding of the concept of scale, the results of the covariate analysis revealed a significant treatment interaction effect. The least square means procedure indicated differences between Treatment Groups I and II, between Treatment Groups I and III, and between Treatment Groups III and IV. The group mean for students who used calculators only was vi
significantly higher than the group mean for students who used calculators and participated in the assignment. The group mean for students who participated in the assignment only was significantly higher that the group means for a) students who used graphing calculators and participated in the assignment and b) students in the control group.

Regarding students' ability to identify, construct, and define function, the results of the covariate analyses revealed a significant effect regarding the factor of conceptual change assignment. The group mean for students who participated in the assignment was significantly lower than the group mean for students who did not participate in the assignment.

Exploratory analyses revealed that the students' definitions of the concept of function were dominated by the ordered pair representation of the concept. This point-wise view of functions was further emphasized through the students' images of the concept of function.

Classroom observations of the treatment and control groups revealed additional information regarding the effect of the graphing calculator on classroom discourse.
CHAPTER I
DESCRIPTION OF THE STUDY

Introduction

One objective of reform in mathematics education has been to make the mathematics curriculum stronger. Although many mathematical concepts have gone through cycles of emphasis, deemphasis, and reemphasis in mathematics education, focus on the concept of function has been constant throughout the New Mathematics movement, the introduction of technology into mathematics education, and the effort to empower learners and educators through the application of the National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). The concept of function has been, and continues to be, upheld by the mathematics education community as a fundamental and unifying principle of mathematics (Brieske, 1973; Bruckheimer & Gowar, 1969; Herscovics, 1989; Hirsch, Weinhold, & Nichols, 1991; Krause, 1984; Papakonstantinou, 1993; Reeve, 1940).

A significant purpose of education is to assist students in acquiring concepts (Leinhardt, Zaslavsky, & Stein, 1990). Thus, students' acquisition and understanding of the concept of function should be considered by mathematics educators. Understanding of this concept has always been of great practical value. Dresden (1927) proposed that functions were a part of our everyday experiences in which dependent relationships could always be found.
Moreover, the value of the concept of function in academia is exemplified by calculus (Thorpe, 1989), because the study of calculus is essentially the study of functions and graphs of functions (Fey, 1984).

The power of technology and its application in mathematics can be realized through the use of graphing calculators as tools for learning. The graphing calculator is ideal for developing the concept of function and for focusing on its graphical representation. A renewed consideration of the concept of function has resulted from the use of graphing calculators in instruction. Two main concepts related to the concept of function, domain and range, are especially important when using the graphing calculator.

The domain is one of the most important but often neglected aspects of the concept of function, although a function is never completely defined until a domain for the function is defined (Mancill & Gonzalez, 1962). The elements of the domain of the function are the only elements to which the function can be applied (Buck, 1970), and if a domain is not specified, then one can assume "that it is the largest collection of numbers for which the function has real values" (Mancill & Gonzalez, 1962, p. 180). The range of the function follows upon clarification of the domain and the rule for the function (Orton, 1971).

A person's definition of the concept of function may "include such ideas of a function as an action, an operation, a rule, a relationship, a graph, or a picture" (Confrey, 1981a, p. 8), while the mathematical community supports a definition involving the formal relationship between a given domain and a given range. The definition of the concept of function used for the purpose of this study is described as the Dirichlet-Bourbaki definition and is stated as follows: a function is a relationship or correspondence between two sets, the domain
and the range, such that any element of the domain is related to or corresponds to one and only one element of the range.

One who fully understands the concept of function will show some evidence of understanding the concepts of domain, range, and the relationship between the domain and range (Markovits, Eylon, & Bruckheimer, 1983). By neglecting to be attentive to the concepts of domain and range, learners of mathematics fail to emphasize what Ayers, Davis, Dubinsky, and Lewin (1988) described as "the key feature of function." This feature involves the transformation of elements in the domain of the function to elements in the range of the function. The nature of this transformation depends on the relationship between the domain and the range. Moreover, inclusion of the concepts of domain and range is a necessity and an asset because, "by separating the function into its elements, we tend to make it a unity" (Hamley, 1934, p. 30).

When graphing a function whether using paper and pencil or a mechanical graphing device, appropriate domain, range, and scale for the axes must be chosen in order to obtain a useful visual description of the concept. By emphasizing the importance of choosing a domain, range, and scale for the axes for graphing the function, the learner can be led to an understanding of the relationship between the domain and range of the function.

The concept of function is important in all levels of mathematics curricula, including community college mathematics. Community colleges, among other educational roles, serve as bridges between secondary schools and four-year colleges or universities. Community college faculty often assist students to overcome their educational deficiencies and prepare them for a college or
university. The associate degree received by community college graduates is representative of a students' readiness for admittance to a college or university or for employment (American Association of Community and Junior Colleges, 1984). By emphasizing the concept of function and the related concepts of domain and range in mathematics curricula at the community college, mathematics educators have the potential to greatly affect community college algebra students' concept of function.

The concept of function is part of the foundation of mathematics. The concepts of domain and range, which are directly related to the concept of function, are the foci of the current study. Emphasis on the two concepts, at all levels of mathematical learning, implies emphasis on the concept of function.

For the purpose of preparing graphs representing the concept of function, one needs to be aware of the importance of the two concepts and the role they play in choosing an appropriate set of axes for graphing functions. Otherwise an insufficient representation of the function may result. This is especially important when employing technological tools for graphing functions.

Statement of the Problem

Algebra students typically pay little attention to choosing appropriate domain, range, and scale for the axes when graphing a function. Because the graph of the function represents the relationship between the domain and range of the function, lack of consideration of a reasonable domain, range, and scale for the axes for graphing functions perpetuates students' difficulties with the concept of function. In addition, various groups of learners of mathematics have shown evidence of difficulties with the concept of function which have
impeded their understanding of the concept. Such difficulties include students’ inability to identify, construct, and define function (Vinner & Dreyfus, 1989). These difficulties are possibly the result of students’ misunderstanding of the concepts of domain and range and the functional relationship between the two concepts.

The purpose of this study was to examine the effects of using graphing calculators and a conceptual change assignment during instruction on the students’ concept of function. The emphasis was on the students’ selection of appropriate domain, range, and scale for the axes for graphing a function and their identification, construction, and definition of function. This study was designed to test the following null hypotheses as related to college algebra students in the community college:

1. Students’ use of the graphing calculator during instruction will not affect their concept of function regarding their (a) application of the concepts of domain and range, (b) selection of appropriate domain, range, and scale for the axes for graphing functions, (c) identification, construction, and definition of function.

2. Students’ participation in a conceptual change assignment will not affect their concept of function regarding their (a) application of the concepts of domain and range, (b) selection of appropriate domain, range, and scale for the axes for graphing functions, (c) identification, construction, and definition of function.

3. Students’ use of the graphing calculator during instruction and the students’ participation in a conceptual change assignment will not interact to affect their concept of function regarding their (a) application of the concepts of domain and range (b) selection of appropriate domain, range, and scale for the axes for graphing functions, (c) identification, construction, and definition of function.
Justification of the Study

The current body of research on the concept of function and students' misconceptions of the concept is significantly deficient. Researchers have focused on the concept of function held by teachers (e.g., Even, 1989, 1990, 1993; Stein, Baxter, & Leinhardt, 1990; Vinner & Dreyfus, 1989); by college and university students (e.g., Dreyfus & Eisenberg, 1983; Vinner & Dreyfus, 1989); and by middle and high school students (e.g., Dreyfus & Eisenberg, 1982, 1984; Karplus, 1979; Markovits, Eylon, & Bruckheimer, 1988; Orton, 1971; Papakonstantinou, 1993; Thomas, 1969; Wagner, 1981). Research in this area pertaining to the community college student is essentially nonexistent. Research results obtained from the other samples can not adequately be applied without some knowledge of community college students' concept of function. One major factor is the unique population of students that attend community colleges and their distinct educational needs (Cohen & Brewer, 1987; Deegan, Tillery, & Associates, 1988). Another issue of focus is the status of the concept in the community college algebra curriculum and the attention given to emphasizing the concept.

Several major studies concerning students' difficulties, misconceptions and inconsistencies regarding mathematics and science concepts were conducted in schools, colleges, and universities outside of the United States (e.g., Dreyfus & Eisenberg, 1982, 1983, 1984; Hand & Treagust, 1991; Movshovitz-Hadar & Hadass, 1990; Vinner & Dreyfus, 1989). One cannot assume that the results are completely generalizable to populations of students in the United States. There is a need to support and/or refute researchers' findings regarding students' difficulties and misconceptions and to determine whether or not
community college students exhibit misconceptions similar to those described in the literature regarding the concept of function.

There are research studies which were conducted to examine the use of technology to teach functions and graphs (Leinhardt, Zaslavsky, & Stein, 1990). NCTM (1980, 1989) advocated the use of computers to teach functions and graphs; Fey (1984) posited that this recommendation would bring functions and graphs to the forefront of the algebra curriculum. However, the graphing calculator has not been fully exploited in educational research nor in instructional processes as a tool for enhancing conceptual understanding. Used as part of instruction, the graphing calculator can provide students with an opportunity to explore the graphical representations of functions. Specifically, graphing calculators should be used in the learning of mathematics at the community college level (Demana & Waits, 1988). Since Demana and Waits (1990) have found that students with access to technology in the learning process have success in learning algebraic concepts, it is necessary to provide additional research regarding how students approach graphing (Goldenberg & Kliman, 1988), and the benefits of employing graphing technology (Fey, 1989; Ruthven, 1990). Shumway (1990) proposed that the use of “supercalculators” (i.e., scientific calculators with graphing capabilities) in research pertaining to the development of mathematical concepts is drastically needed. Educators (e.g., Wilkinson, 1984) have suggested that technological tools, such as computers and calculators, can be used to assist students in developing mathematical concepts.

The purpose of the current study was to respond to the need for an increase in attention to conceptual understanding in mathematics education (Rosnick & Clements, 1980), and the use of technology for the learning and
teaching of algebra. The next section of this chapter is a description of the theoretical framework for the present study.

**Theoretical Framework**

Research focusing on conceptual understanding of mathematics must be supported by a theory of learning which can be applied to the learners' development of mathematical concepts and encompasses principles which are directly applicable to mathematics education. In addition to a theory of learning which can provide for these two conditions, this research must be supported by a theory which can be used to acknowledge difficulties learners might have in the process of developing concepts and to indicate a framework for dealing with misconceptions that might arise throughout the process of developing concepts. Moreover, there should be some indication that the two theories can be applied simultaneously for the benefit of concept development. Thus, the theoretical framework for the current research study will be discussed with reference to Jean Piaget's theory of genetic epistemology and Leon Festinger's theory of cognitive dissonance as applied within the context of the theory of conceptual change.

**Concepts**

Skemp (1987) described a concept as an "idea" and the name of a concept as a "sound." For example, the concept of function can be identified by the written or spoken word "function". This word is just an identifier for the concept. The concept itself is an idea, something which does not have a truth value (Bourne, Dominowski, Roger, Loftus, & Healy, 1986). Some researchers (e.g., Confrey, 1981a; Larcombe, 1985) have proclaimed that mathematical
concepts exist only in the sense that they become reality in the individual's mind. Thus the term has a twofold meaning. According to Gilbert and Watts (1983), the term concept applies to one's personal knowledge concerning an idea and it also applies to the publicly accepted (sometimes formal) explanation of the idea.

Skemp (1979, 1987) and Tennyson and Park (1980) suggested that concepts for the most part always contain within themselves other concepts. Cohen (1983) and Larcombe (1985) advocated that higher order concepts were built upon these lower order concepts. These lower order concepts should not be dismissed because one needs them as reference in defining the larger concept (Buck, 1970). Each concept needs to be defined and understood individually (Wilson, 1990). For instance, the definition of the concept of function itself is composed of some words that are themselves concepts, such as domain and range. For this reason, it can be described as a lexical definition (Vinner, 1976).

**Development of Concepts**

Jean Piaget (1896-1970) described his approach to the learner's development of concepts as genetic epistemology, indicating his concern with how the learner formed knowledge and the meaning of the knowledge to the learner (Piaget, 1981). Piaget's use of "the term genetic referred to developmental growth rather than biological inheritance" (Hergenhahn, 1976, p. 269). Other researchers (e.g., Confrey, 1991) proposed that Piaget's approach indicated that concepts develop in the mind of the learner as the learner participates in instructional experiences and attempts to solve problems. Piaget (1981) asserted that a learner can acquire mathematical
knowledge from one of two types of learning experiences: the "physical" and the "logico-mathematical". The physical experience involves the learner acting on physical objects in order to obtain characteristics about the object. The logico-mathematical experience involves the gathering of information from the actions carried out by the learner and not from the physical properties of the object (Piaget, 1975). He hypothesized that mathematical knowledge, being abstract in nature, was acquired through the logico-mathematical experience (Piaget, 1981).

It is the logico-mathematical experience which has implications for research conducted regarding the concept of function. The concept, being an idea, is not representable by physical objects, yet the representations of the concept can be acted upon by the learner. Piaget (1975) proposed that "mental or intellectual operations, which intervene in the subsequent deductive reasoning processes, themselves stem from actions: they are interiorised actions and once this interiorisation . . . is sufficient, then logico-mathematical experience in the form of material actions is no longer necessary and interiorised deduction is sufficient" (p. 6). Commenting on Piagetian theory and instruction in mathematics, Penrose (1980) wrote that operations are also actions, thus operations involving the representations of the concept of function can be conceived as actions. A basic principle of Piaget's theory of learning is that when a learner knows an object, such as a mathematical concept, the learner is able to act upon it and interact with it (Renner & Phillips, 1980; Piaget, 1981), thus performing operations involving the concept. "The important Piagetian idea is activity, not necessarily physical, concrete activity. Important for learning are active engagement and commitment, not necessarily actions on
things. Piaget's theory provides the theoretical underpinnings for an active approach to education" (Ginsburg, 1992, p. 359).

The actions performed by a learner on a mathematical object should reflect not a memorization or an adoption of actions carried out by the teacher, but at some point in time, a "reinvention" of the concept by the learner (Piaget, 1975). This reinvention is not a discovery by the learner, but an indication that the learner knows the concept by performing original actions on the representation of the concept (Ginsburg, 1992). There are a variety of non-physical actions that a learner can perform during the development of a concept. For example, Bourne, Dominowski, Roger, Loftus, and Healy (1986) proposed that a concept can be obtained by associating common characteristics of examples of the concept and by forming and testing hypotheses regarding the concept. It should be kept in mind that due to student differences, students' learning of concepts may vary (Cohen, 1983; Confrey, 1981b), because each student might perform different actions while trying to acquire a concept. For instance, Confrey (1981a) proposed that a person's definition of a concept, that is the personal or private definition for understanding the concept, is often formulated as a way to make sense of personal experiences and observations. For the learner, this is a "reinvention" of the concept for application. Confrey (1981a) stressed that the goal of mathematics education has been to transform the learner's personal definition into an academically accepted definition, in such a way that the new definition represents the learner's concept. This new, academic definition might adhere to the "classical view of concept" described by Gilbert and Watts (1983) as "the view that all instances of a concept share common properties and that these properties are necessary and sufficient to define the concept" (p. 65).
Skemp (1987) suggested that in determining whether or not one had a concept, the main condition was whether or not the person behaved accordingly when presented with new examples of the concept. Piaget (1964, 1981) argued that to have acquired an object, such as a mathematical concept, was to act accordingly when given operations to perform with understanding of the operations involved. Gilbert and Watts (1983) found that these actions could be "linguistic and non-linguistic, verbal and non-verbal" (p. 69). Historically, the mastery of a concept was described as "the demonstrated ability to recognize or identify the definition, description, or illustration of a concept or the appropriate usage of the word or words naming the concept" (Butler, 1932, p. 123).

**Misconceptions**

The classical view of the development of concepts encompasses the idea of misconception, defined by Gilbert and Watts (1983) as an error in one's conceptual structure. The researchers proposed that when a learner treats a non-example as an example, then the learner has a misconception. Gilbert and Watts (1983) also characterized "the actional view of concept" as the recognition of students' misconceptions "as being natural developmental phenomena- personally viable constructive alternatives- rather than the result of some cognitive deficiency, inadequate learning, 'carelessness' or poor teaching" (p. 67). Misconceptions may be the result of the learner's intuition (actional view) surrounding the concept or some factor of formal instruction (classical view) (Leinhardt, Zaslavsky, & Stein, 1990; Vinner, 1990) or a combination of both. Gilbert and Watts (1983) proposed "that there are no (and can be no) straightforward rules to conceptualisation that work for all
situations" (p. 69). Hence, this researcher proposes that consideration of both views in instruction is necessary to attend to students' learning of mathematics.

There are researchers (e.g., Larcombe, 1985; Linn & Songer, 1991; Steffe, 1990) who believe that emphasis should not be placed on what students do not understand, but rather on what they understand. However, misconceptions cannot be dismissed as common errors (Herscovics, 1989). One major consequence of ignoring students' misconceptions in instruction is that there is not any encouragement for "the students to impose meaning on the new material by attaching it to what is already known or altering what is already known" (Leinhardt, 1988, p. 122). Nor does instruction and curricula that take misconceptions into consideration have to be negative in nature. Students' misconceptions or inconsistencies can be used to enhance learning (Hewson & Hewson, 1984; Piaget, 1973), but there is a lack of research on this in mathematics education (Tirosh, 1990). Johnson and Johnson (1979) proclaimed that if students' misconceptions are handled properly, then conceptual understanding can be enhanced.

Driver (1981) adapted the term 'alternative framework' (actional view) instead of 'misconception' or 'inconsistency' (classical view) and suggested several reasons for focusing attention on students' ideas which were not comparable to formal, acceptable ideas. He suggested that such a focus would assist one in developing curriculum materials, teaching strategies, instructional activities, and opportunities for unrestricted learning. "Indeed identifying those areas which present particular learning problems may be an important first step in developing teaching programmes" (Driver & Erickson, 1983, p. 54).
Conceptual Change Theory

Strike and Posner (1985) and Posner, Strike, Hewson, and Gertzog (1982) suggested that learning is a conceptual change process. They employed the terms “assimilation” and “accommodation” to describe the process. These terms were first used by Piaget (Boden, 1979), and were applied as follows: “Assimilation occurs whenever an organism uses something from its environment and incorporates it. The system accommodates to the object; the changes that occur in it depend on the characteristics of the object” (Phillips, 1981, pp. 13-14). These processes are inclusive of Piaget’s theory that learning occurs through physical and/or mental activity. “Piaget’s description of how knowledge is acquired and changes developmentally is explicitly placed within the individual through the complementary processes of assimilation and accommodation” (Mason et al., 1983, p. 229).

During recent decades, science education researchers have attempted to incorporate Piaget’s terms for application in the teaching of science concepts. Strike and Posner (1985) and Posner, Strike, Hewson, and Gertzog (1982), asserted that assimilation occurs when students use previous knowledge to deal with new knowledge (Hergenhahn, 1976; Kaplan, Yamamoto, & Ginsburg, 1989; Linn & Songer, 1991), during which a major conceptual change is not required. At this stage, new knowledge becomes a part of the learners conceptual structure (Piaget, 1977). Accommodation (actional view) occurs when the learner replaces old knowledge or reorganizes old knowledge to “accommodate” new knowledge. Hergenhahn (1976) posited that assimilation can be equated with recognition, while accommodation can be equated with learning. Phillips (1981) stated that “without assimilation, there can be no accommodation, and vice versa; and accommodation is a change in structure,
the function of which is to make possible the assimilation of some stimulus pattern that is not entirely familiar" (p. 19).

Research projects pertaining to problems caused by conflicts in students' knowledge of mathematical and scientific concepts have usually been implemented to focus on the theory of conceptual change (Dreyfus, Jungwirth, & Eliovitch, 1990; Driver & Scanlon, 1988; Posner, Strike, Hewson, & Gertzog, 1982), which includes applicable frameworks of cognitive dissonance, cognitive conflict, conceptual conflict, and applications of conflict teaching (Movshovitz-Hadar & Hadass, 1990; Steffe, 1990; Tirosh, 1990; Tirosh & Graeber, 1990a, 1990b; Wilson, 1990). A model of conceptual change, the theory of cognitive dissonance was introduced by Leon Festinger in 1957 (Festinger, 1957).

"Cognitive dissonance was defined as a motivational state that impels the individual to attempt to reduce and eliminate it. Because dissonance arises from inconsistent knowledge, it can be reduced by decreasing or eliminating the inconsistency" (Wicklund & Brehm, 1976, p. 1). The underlying assumption of the theory of cognitive dissonance is that the learner, once realizing that a conflict exists within his/her conceptual structure or between his/her conceptualization and reality, will proceed to eliminate the conflict. The conflict thus acts as a catalyst for conceptual change.

Piaget's models of assimilation and accommodation were also accompanied by the act of equilibration. According to Hergenhahn (1976), "since there is an innate need for harmony (equilibrium), the organism’s mental structures change in order to incorporate these unique aspects of the experience, thus causing the sought-after cognitive balance" (p. 271).

Conceptual change can be the result of new knowledge conflicting with the old; Festinger (1957) found that exposure to information was one of the
most common ways dissonance was created. He proposed that the most direct way to reduce dissonance was by "focusing on and directly dealing with dissonant relations" (Wicklund & Brehm, 1976, p. 98). Making the learner aware of misconceptions has been proposed as an important step in any strategy involving cognitive dissonance (Nussbaum & Novick, 1981).

A major goal of research has been to provide a link between theory and practice. If positive change is to occur, acknowledgement of the misconceptions that students have pertaining to the concept of function must be coupled with possible strategies to alleviate the problems, that is, there are advantages for considering the classical and actional view of concept. Hewson and Hewson (1984) adopted a model for conceptual change, which emphasized that learning is an interaction between previous and existing knowledge with the outcome depending on the interaction (actional). Accordingly, there are four criteria for conceptual change to occur when based on conflicts in students' knowledge. First, both ideas (in conflict) must be understood by the learner. Next, the student must compare both ideas and find them to be in conflict. According to Johnson and Johnson (1979), when the student has two conflicting ideas such as previous knowledge in mind which may conflict with new information, the student experiences conceptual conflict. There is almost no way to avoid this because "students will filter new input through their existing conceptions (for good or ill)" (Brophy, 1986, p. 324).

Because it is possible for the two conflicting ideas to exist in the same mind, students may need help in identifying the misconception (Fischbein, 1990). At the time that the student realizes that an alteration regarding the concept in question is needed, then cognitive conflict results (Dreyfus, 1990; Dreyfus, Jungwirth, & Eliovitch, 1990).
The third criteria involves the act of resolution. Resolution occurs either by compartmentalization or assimilation. Scwharzenberger (1982) suggested that compartmentalization meant that students have different, conflicting ideas in different mental compartments. Most people will want to resolve the conflict as soon as possible (Movshovitz-Hadar & Hadass, 1990). If the learner cannot avoid the conflict, then the learner must take action (Skemp, 1979).

Finally, there are several teaching strategies which can be used to assist students to alleviate their misconceptions. For example, the strategy of diagnosis should always be carried out as a prerequisite to instruction (Hewson & Hewson, 1984). "Once the existing, mistaken concepts are recognized then an appropriate teaching strategy can be developed, for example by producing empirical evidence which contradicts the students' beliefs" (Head, 1982, p. 636). The goal is to create conflict that the student will have to resolve in order to promote accommodation of the given concept.

Nussbaum and Novick (1981, 1982) proposed a similar model which supports a learners accommodation of concepts. The researchers suggested that accommodation is a process which can be prepared for, but not scheduled or guaranteed. They proposed that all "one can do is to try to characterize it and to look for instructional strategies that may facilitate its occurrence" (Nussbaum & Novick, 1982, p. 186). The model they suggested for facilitating cognitive accommodation involves three major objectives. The first objective is to create a learning situation that encourages learners to examine his/her conceptions prior to instruction of a topic, express these conceptions orally and written. The teacher's role involves assisting the learner in expressing
conceptions in a definitive manner and promoting an environment where learners can debate on the conceptions.

Secondly, one should attempt to create a situation where conflict is born between the students' (mis)conceptions and some academic truth or reality. "The information must be presented in such a way as to challenge or stimulate the student, for it is through this process of conflict that he integrates the new material" (McMillan, 1973, p. 36). "Piaget's theory is that cognitive development is promoted when there is a moderated degree of discrepancy between the child's cognitive structure and the new event which he encounters" (Ginsburg, 1992, p. 360).

The last objective requires one to support the learners' accommodation of the concept. "The teacher's job is not to reinforce correct answers, but to strengthen the learner's own process of reasoning" (Mason et al., 1983, p. 238). "According to Piaget, failure of previous knowledge for assimilation of an experience causes accommodation or new learning" (Hergenhahn, 1976, p. 277).

Berlyne (1965) suggested that "different forms of conceptual conflict are readily applicable to different educational subject matters" (p. 78). Thus strategies offered by science educators may need adjusting before applying them to topics in mathematics. For example, Berlyne explained that "surprise" tactics may be better for science subjects where students can be surprised by occurring phenomena that they thought impossible. (This is what Nussbaum and Novick (1982) described as a "discrepant event.") On the other hand, Berlyne suggested that "doubt" may be better for some topics in mathematics.

Cognitive dissonance has been characterized as a focus on changing a person's attitude (Berlyne, 1965; Nussbaum & Novick, 1982) rather than the
person's conceptions. Some researchers (e.g., Hewson & Hewson, 1984) have chosen to describe the phenomenon as conceptual conflict rather than cognitive dissonance or cognitive conflict. Nussbaum and Novick (1982) suggested that conceptual conflict should be used when referring to academia. Conceptual conflict was described by Berlyne (1965) as "conflict due to discrepant thoughts, beliefs, or attitudes" (p. 77).

Cognitive dissonance, cognitive conflict, and conceptual conflict are all concerned with the need of a person to resolve dissonance or conflict between two conceptions (Nussbaum & Novick, 1982). "In general, all of the theories utilize the same dynamic: if a person encounters information, facts, or experiences which makes him feel uncertain, which creates a sense of imbalance, incongruity or conflict, then that person will change his behavior or "knowledge" to integrate the information or experience into already existing norms" (McMillan, 1973, p. 35). Thus in the forthcoming review of the literature, the term(s) that the various researchers used will be reported without preference.

Burton (1984) contended that there were two ways of approaching cognitive conflict: aggressively and passively. In the aggressive mode, the student will actively seek methods to overcome the conflict, often by confronting it directly. In the passive mode, the student might accept failure and neglect to solve the conflict.
Significance of the Study

This study was supported by the need in mathematics education to provide research data pertaining to students' understanding of the concept of function, more specifically, their understanding of the concepts of domain and range, and their ability to select appropriate domain, range, and scales for the axes when graphing functions. The use of the graphing calculator in instruction requires that students using the tool consider how they select the domain, range, and scale for the axes when graphing functions. One must be aware of the effects that these elements have on the visual representation of the concept. In addition to this concern, the conceptions that the students have prior to instruction with the graphing calculator may be in conflict with new knowledge presented to them, and if so this conflict should be capitalized to enhance students' understanding of the concept. Moreover, this study included an attempt to validate and extend previous research regarding students' identification, construction, and definition of function.

There is an increased need to examine the uses of the graphing calculator in mathematics instruction and to provide for the visibility of the graphing calculator in the research literature as a tool for conceptual understanding. It is evident from the literature that research regarding conceptual understanding involving the graphing calculator is scarce.

Because very little research in mathematics education is conducted on the community college level, it is imperative that research results are not quickly applied to levels of instruction where the research is rarely conducted. This portion of the study was not simply a replication, but it was an extension of previous research. This extension included the use of a community college
sample, the use of a homogeneous sample (community college algebra students), and the factors of graphing calculator use and concept assignment participation regarding the concept of function and specifically the concepts of domain and range.

**Organization of the Study**

A review of relevant literature regarding the concept of function, the use of technology in teaching the concept of function and other related concepts, and the application of conceptual change theory in mathematics education is presented in Chapter II. The design and methodology of the study is reported in Chapter III. In Chapter IV, results of the analyses and limitations of the study are provided. A summary of the results, implications, and recommendations for future research are presented in Chapter V.
CHAPTER II
REVIEW OF THE LITERATURE

Overview

A review of the relevant literature is presented in this chapter. The areas under discussion are development of the concept of function, representations of the concept of function, difficulties students have with the concept of function, computers and calculators, graphing calculators, and the application of conceptual change theory in mathematics education research studies.

Development of the Concept of Function

The concept itself was in use long before the word "function" entered into the language of mathematics (Boyer, 1946; Miller, 1928). History has not credited one person, place, or time with creating the concept of function (Thomas, 1969). Because it is one of the oldest mathematical concepts, records of its history are not always reliable (Miller, 1928). Mention of some of the major contributors to the concept of function since the 17th century follows.

It was Descartes (1596-1650), through his emphasis on the graph of a geometric curve as a representation of an algebraic function, who influenced the connection between algebra and geometry (Sidhu, 1981; Sobel & Maletsky, 1988). This connection was a catalyst for the development of the concept of function (Confrey, 1981b; Kleiner, 1989). Dines (1919) suggested that this union of disciplines encouraged an increase of progress in the
individual fields. The presence of the concept of function in mathematics has made possible an integration between algebra and geometry and an understanding of trigonometry (NCTM 1989). The work of Fermat (1601-1665) and Descartes (1596-1650), regarding their interest in the roots of equations in two unknowns, provided the way for "variable and function of a variable" to surface in mathematics (Boyer, 1946).

Leibniz (1646-1716) introduced the term "function" into mathematics in 1673 (Ponte, 1992). He was attempting to describe the characteristics of a curve such as points on the curve and slope of the curve (Hamley, 1934; Kinney, 1922). Bernoulli (1667-1748) and Euler (1707-1783) developed similar definitions for function, including Bernoulli's definition of function as an expression composed of variables and constants (Kleiner, 1989). Euler labeled an equation or formula consisting of variables and constants as a function (Eves, 1981; Reeves, 1969). At the beginning of the twentieth century a function was commonly defined as an expression or formula (Hight, 1968). Euler went further by classifying functions as algebraic or transcendental (Miller, 1928), single-valued or multivalued, and implicit or explicit (Kleiner, 1989). Following the Eulerian concept of function (Boyer, 1946), Fourier (1768-1830) was the first mathematician to develop new areas with his investigations of heat flow (Eves, 1976, 1981; Ponte, 1992). For instance, he formed a general equation for the motion of heat in a conductor (Bell, 1937). All of these events helped to increase the strength of the concept. In some cases, the concept became more constrained by certain parameters in its definition, but in other cases the definition broadened the concept as did Dirichlet's (1805-1859) definition.
Nicholas (1966) suggested that some agreement should take place regarding the definition of function. There were many variations of Dirichlet's definition for the concept of function. More important than the statement of his definition of the concept were the doors which his definition opened in mathematics. His definition did not rely on the existence of an algebraic expression (Reeves, 1969). “A function, then, became a correspondence between two variables so that any value of the independent variable, there is associated one and only one value of the dependent variable” (Ponte, 1992). His meaning for the term included discontinuous functions (Kleiner, 1989), Bernoulli’s formula concept, Euler's equation concept, and Descartes’ geometric curve concept. Furthermore, this definition ignored the restriction that the variables had to represent real numbers (Dines, 1919). Dirichlet’s definition of the modern concept of function has dominated the mathematics curricula of today (Dreyfus, 1990). The two components of this definition of the concept of function were “arbitrariness” and “univalence” (Even, 1989, 1990, 1993). The arbitrary nature of functions eliminates the restriction that functions have to be defined by one expression, graph, or defined on specific sets of elements. The latter component, univalence, requires the restriction that each element of the domain of the function corresponds to one and only one element of the range. “The development of advanced analysis created the need to deal with differentials of orders higher than one and, therefore, to distinguish independent from dependent variables. In such a case, it became too difficult to work with multivalued symbols, and the univalence requirement was added to the definition of a function” (Even, 1993, p. 96). This correspondence between the domain and the range can either be a one-to-one or a many-to-one relationship.
Set theorists continued to expand the concept of function by defining it as a relationship between any two arbitrary sets of elements (Bennett, 1956; Johnson & Cohen, 1970; Thomas, 1969). These sets could be numerical or non-numerical (Ponte, 1992). Bourbaki gave the definition of function in terms of a set of ordered pairs, that is, as a subset of a given Cartesian product (Kleiner, 1989). In some instances, the concept has continued to be presented in this way (Janvier, 1987). The definition of the modern concept of function has often been described as the Dirichlet-Bourbaki definition.

The concept of function has a history marked by efforts to define and redefine the concept and its definition (Bennett, 1956; Eves, 1981; Hight, 1968). This growing and changing of the concept has been but one example of the conceptual change theory discussed by Confrey (1981b). She proposed that this view "portrays mathematics as having competing theories which are the results of attempts to solve outstanding problems" (p. 248). One particular problem that encouraged conceptual change was the Vibrating String problem (Miller, 1928), which influenced the evolution of the concept of function (Dines, 1919). The Vibrating String problem was used to implement an extension of the concept to include functions not previously included (Ponte, 1992). Examples of these functions would be piecewise functions and functions drawn freely by hand and not given by an algebraic expression (Kleiner, 1989). The Vibrating String problem can be examined by fixing the ends of a string at two arbitrary points on the Cartesian coordinate system then releasing the string to vibrate. Any still shot of the vibration represents the graph of a function (Confrey, 1981b). The concept of function was a result of growth, change, and conflicting ideas and theories among mathematicians.
The notion that the concept of function was important enough to be included in the mathematics curricula of the United States can be attributed to Felix Klein (Georges, 1926; Hamley, 1934), then a professor of mathematics at the University of Gottingen (Reeves, 1969). He proposed this in an 1893 speech for the International Congress of Mathematicians in Chicago (Hamley, 1934). The first people to reinforce this proposal were David Eugene Smith and E. R. Hedrick (Reeves, 1969), often publishing articles in the *Mathematics Teacher* to show their support for the concept (Hedrick, 1922; Smith, 1928). The National Committee on the Reorganization of Mathematics in Secondary Education also took on the concept of function as a major focus (Longley, 1933).

Later emphasis on the concept of function was often dominated by the idea of "functional thinking" (Booher, 1926; Georges, 1929; Lennes, 1932). According to Breslich (1932) and Hamley (1934), a person who is able to think functionally should be able to recognize relationships between variables, determine the underlying theme of the relationships, express relationships algebraically, and recognize the effects of changing variables. The educators believed that these activities characterized the application of the concept of function. The concept of function has entered into all realms of academic disciplines, especially mathematics (Breslich, 1928), and in any other context where relationships have arisen (Boyer, 1946; Carver, 1927). The principle of functionality was described as the relationship of dependence (Breslich, 1932; Kinney, 1922). Hedrick was one of the first proponents of functionality. He proposed that presenting a definition for the concept of function was not in itself adequate, but what was needed was attention to relationships (Hedrick, 1922).
Functional thinking involved thinking in terms of quantitative relationships (Georges, 1926; Hamley, 1934), described as "quantitative reasoning" by Fey (1990). The identification, analysis, and usefulness of relationships were the foci of functional thinking (Breslich, 1940), because understanding relationships was considered important to informed thinking (Booher, 1926). Fey (1990) suggested that "quantitatively literate young people need a flexible ability to identify critical relations, ... to express these relations in effective symbolic form, to use computing tools to process information and to interpret the results" (p. 65). It is the acquisition of the concept of function which would provide a basis from which learners could begin to reason quantitatively.

**Representations of the Concept of Function**

Janvier (1983) proposed that a representation of a concept is composed of three units: written symbols, real objects, and mental images. The link between the three elements is completed by verbal and non-verbal language processes. The researcher presented the concept of function as an example of a concept which does not have a real object representation, but yet remains a powerful concept.

The concept of function can be represented in multiple ways, such as by graphs, equations, tables, and arrow diagrams (Dreyfus & Eisenberg, 1982; 1984; Even, 1990; Langer, 1957; Lennes, 1932; Markovits, Eylon, & Bruckheimer, 1988; Schwarz, Dreyfus & Bruckheimer, 1990; Smith, 1972; Stein, Baxter, & Leinhardt, 1990; Thorpe, 1989). Ponte (1992) proposed that the most important representations of the concept were the numerical (tables and computations), graphical (Cartesian), and algebraic (equations)
representations. Dufour-Janvier, Bednarz, and Belange (1987) developed several reasons for emphasizing multiple representations of a concept, all of which have implications for the concept of function. Some concepts, such as the concept of function, are closely related to their representations. Each representation presents a different view of the function (Friedlander, Markovits, & Bruckheimer, 1988), but the nature of the function is explicit in its representation, if it is clearly understood.

The representations are of the same concept. In the case of function, for example, an algebraic expression, a table of values, and a graph can represent the same function. Together these representations can be used to reinforce the concept (Goldenberg, 1988).

Emphasis can be placed on trouble spots with certain representations. Tirosh (1990) found that student may be able to successfully complete a task using one representation of the concept of function, but when given the same task, but a different representation to use, the student may be unsuccessful. For example, it may be possible for a student to examine the roots of a function more closely by the graph of the function rather than by the equation of the function. The domain of a function may be easily determined by the function given as a set of ordered pairs, a table, a mapping, an algebraic equation, or by the graph of the function.

Multiple representations can be used to break up the monotony of working in the same setting for a concept. Hence, different problems can be developed concerning the different representations.

These reasons supports Confrey's (1992) theory of an “epistemology of multiple representations”. She suggested “that it is through the interweaving of
our actions and representations that we construct mathematical meaning” (p. 149). Thus, she also lends support to Piaget’s theory of genetic epistemology.

The emergence of graphing calculators in education has encouraged educators to consider the advantages of emphasizing the graphical representation of function. Yerushalmy (1991) suggested that “only one topic in a traditional algebra course utilizes visual-graphic representation in addition to the symbolic one: investigating functions” (p. 42). Functions and graphs should be central topics in algebra, one reason is because they are at the heart of elementary calculus (Fey, 1984; Hamley, 1934). Graphs and graphing should be encountered by and introduced to students as early as possible in the mathematics curriculum (Booher, 1926; Kinney, 1921; Kinney & Purdy, 1952). Georges (1926), Lennes (1932), and Buck (1970) suggested that compared to other representations of the concept of function, the graphical form of functions was most useful for students. From information obtained by observing the graph, the learner can depict various characteristics of the relationship, such as the possible values of the independent variable, that may not be as obvious through another representation. From the graph, the learner may also be able to determine the domain and range of the function. The learner can use the graphing calculator to examine the effects of changing the domain of a function or to examine the behavior of a function by restricting its domain. The graph has the potential of enhancing the concept of function (Clement, 1989; Lennes, 1932; Reeves, 1969), and is an accessible medium with which this can be done.

Learners using the graphing calculator will find it necessary to select appropriate domain, range, and scale for the axes in order to provide a useful graph of a function. Thus the students’ concept of function is enhanced through
realization that the domain dictates the resulting range and that the scale of the axes dictates the visual properties of the graph of the function. Researchers (e.g., Ayers, Davis, Dubinsky, & Lewin, 1988; Demana & Waits, 1991; Fey, 1990) are now beginning to emphasize the importance of scaling for providing appropriate graphs of functions. The consideration of the scales of the axes on which the graph appears reflects the importance one puts on viewing an appropriate graph of a function. If one is not careful to choose a reasonable scale for the two axes, "critical features" of the graph can be overlooked (Goldenberg & Kliman, 1988).

Students should not only think functionally, but they should also think graphically (Hamley, 1934) and realize that the graph is not the end of the concept of function (Breslich, 1928; Fischbein, 1987). The purpose of the graph is to increase the power of the concept of function and the function which it symbolizes (Buck, 1970; Hamley, 1934; Kinney, 1922). "The graph representing a function is also an intuitive model of that function and the function in turn, is the abstract model of a real phenomenon" (Fischbein, 1987, p. 121). The graph of a function is a representation of the quantitative relationship between variables (Blank, 1929; Fey, 1990; Linn, Layman, & Nachmias, 1987; Sidhu, 1981). Although a graph of a function is a pictorial representation of the algebraic representation of the function (Georges, 1929; Stein, Baxter, & Leinhardt, 1990), the resulting graph is controlled by the design of the coordinate system on which the learner displays the graph.

The desired outcomes of focusing on graphs of functions is to increase students' ability to interpret graphs (Clement, 1989), produce graphs, and to identify properties of functions from their graphs (Kissane, 1989). Using the graphing calculator, the graph of a function can be produced in the viewing
rectangle by inputting the algebraic representation of the function and appropriate domain, range, and scale values. The choice of the values are completely controlled by the learner (Waits & Demana, 1988). The learner has the option of altering the values at any time that the graph of the function is not a sufficient representation of the function. With pencil and paper, this would mean erasing and/or starting over on a new set of axes, causing students to become frustrated and discouraged while experimenting with different domains, ranges, and scales of axes for graphing functions. Yerushalmy (1991) posited that the lack of an efficient method of changing the scales of axes for graphing functions has caused students to only relate "prototypical graphs" to their algebraic equations.

Sidhu (1981) suggested that "a graphic representation appeals to the aesthetic sense" (p. 271), and this may help students understand the abstract concept of function (Leitzel, 1984). Graphs provide opportunities of visualization in algebra, which is very important, and in particular should be a part of the course content (Vinner, 1989). In this sense, the focus is on the "noun" of visualization, characterized by Bishop (1989) as being "the product, the object, the 'what' of visualization, the visual images" (p. 7).

Dreyfus (1990) proposed that "direct visual processing help students form more complete concept images of functions" (p. 122). When students are able to view images of functions, they become more active in the learning process (Demana & Waits, 1991; Waits & Demana, 1988), and gain more conceptual understanding of functions and their graphs (Dion, 1990). With the graphing calculator, "the power of visualization can be exploited" (Demana & Waits, 1990, p. 28). Yet this cannot happen if students are not aware of the implication of choosing appropriate domain, range, and scale for the axes for graphing
functions. This objective is one that is currently omitted form the traditional mathematics curricula (Burrill, 1992). The current emphasis on pencil and paper graphing does not provide students the opportunity to explore the implications of choosing various domains, ranges, and scales for the axes for graphing functions. In particular, teachers of traditional mathematics curricula encourage students to only consider graphing functions with "nice" domains and ranges (x and y from -10 to 10) and scales of axes equal (Hector, 1992). Unfortunately, the fact that “students learn about functional behavior as they search for an appropriate domain and range to give a full view of the function” (Hector, 1992, p. 132) has been neglected.

In the case where the visual representation is most useful, it is impractical to suggest that all of the work should or could be done with pencil and paper. The graphing calculator could easily be used to provide instruction which focuses on the graphical representation of function. Bishop (1989) suggested that visualization is both a 'personal' and individual' activity and the personalization characteristics (i.e., portable, hand-held) of the graphing calculator could increase the effectiveness of these factors. However, use of the graphing calculator calls for emphasis on producing sufficient graphs of functions. This in turn calls for a need to consider the limitations of the viewing rectangle of the graphing calculator. Dick (1992) proposed that lack of choosing sufficient domains, ranges, and scales can cause the “graphical behavior” of the function to be hidden.
Difficulties with the Concept of Function

The concept of function, which should be presented to students as early as possible (Eves, 1976, 1981; Goals for School Mathematics, 1963), has perhaps been the most difficult mathematical concept for students to master (Dreyfus, 1990). Many teachers have had experiences where students were simply not acquiring a mathematical concept presented to them (Malone & Dekkers, 1984). Students have often experienced difficulties in their attempts to acquire the concept of function (Dreyfus & Eisenberg, 1984; Henderson, 1970). An awareness of these difficulties and students' misconceptions are important (Confrey, 1981a; Tirosh & Graeber, 1990a, 1990b).

Many students have been hindered by a lack of intuitive understanding of the concept of function caused by misconceptions and inconsistencies (Dreyfus & Eisenberg, 1984; Herscovics, 1989; Leinhardt, Zaslavsky, & Stein, 1990; Markovits, Eylon, & Bruckheimer, 1988; Wagner, 1981). This realization was a result of research which examined difficulties students had with the concept of function. Topics on the concept of function, conflicts, and inconsistencies regarding students' learning of mathematics have been addressed by the International Group for the Psychology of Mathematics Education (IGPME) (Nesher & Kilpatrick, 1990). Students have exhibited a variety of difficulties with the concept of function which have highlighted inconsistencies in their knowledge, in regards to the concept. More research is needed regarding the concept of function (Even, 1989; Fischbein, 1990), and researchers have begun to make inquiries regarding the concept in order to aid mathematics curricula and instruction.

Stein, Baxter, and Leinhardt (1990) conducted an extensive study to examine teacher subject matter knowledge of function and graphing to
determine how one's subject matter knowledge would be a factor during instruction in the elementary grades. A portion of the study was reported as a one-subject study, which focused on the teacher's knowledge of function and graphing and instruction for fifth grade students. The measures, interview and card sorting tasks, were designed by the researchers and field-tested for the study. Reliability measures were not reported for either. The interview consisted of open-ended questions. The card sorting task has been used previously in mathematics education research. Collis (1971) employed the card sorting task in a study designed to follow concept formation in mathematics for a sample of eighth grade students in a convent school. The results from the task were used to determine what the subjects chose as "fundamental categories" by allowing the subjects to sort the cards freely. Likewise, the volunteer subject in Stein, Baxter, and Leinhardt's (1990) study was instructed to categorize the cards which depicted graphical and algebraic representations of functions. The task was validated by a group of mathematics educators. Grouping or sorting possibilities included function representation, mathematical relationships, functions versus nonfunctions, and a combination of the aforementioned.

Upon analyzing the subject's sorted cards and noting that the subject did not group together the different representations of the same function, the researchers concluded that the subject did not have mastery over the different representations of the same function. In particular, the subject in the study was lacking the connection between the graphical and the algebraic representation of function. This finding has been corroborated by the findings of many other researchers (e.g., Ayers, Davis, Dubinsky, & Lewin, 1988; Dreyfus & Eisenberg, 1983, 1984; Dunham & Osborne, 1991; Even, 1989, 1990; Fey, 1984; Graham
& Ferrini-Mundy, 1990; Hector, 1992; Kreimer & Taizi, 1983; Leinhardt, Zaslavsky, & Stein, 1990; Markovits, Eylon, & Bruckheimer, 1990; Schwarz, Dreyfus, & Bruckheimer, 1990). Because the development of the concept of function was nurtured with the connection of algebra and geometry, one might want to assume that establishing the relationship between the algebraic and graphic representations of the concept would not be difficult for some students. However, Even (1989) and Even, Lappan, and Fitzgerald (1988) observed that students who could recognize a given function in its algebraic representation could not always recognize the same function in its graphical representation.

Stein, Baxter, and Leinhardt (1990) considered the fact that many teachers in the elementary grades are not required to obtain training in mastering or instructing the concept of function and graphing. The subject was an 18-year veteran of education and described by administrators at his school as an excellent teacher, but there was no indication of how those conclusions were reached, nor of his overall mathematical education or ability. Because the concept of function has begun to be a focus in some elementary grades across the nation (Even, 1989), it might be informative to examine the novice teacher's subject matter of the concept of function and graphing. The researchers acknowledged this as well. An important implication of this study and other studies involving teachers and future teachers as subjects is that when teachers do not exhibit a strong ownership of the concept of function, then one should not expect that the students under instruction will develop a strong concept of function (Cronbach, 1942).

Misconceptions surrounding the concept of function of a sample of ninth and tenth grade students were analyzed by Markovits, Eylon, and Bruckheimer (1988). The instrument used to measure the students' performance consisted of
what the researchers considered to be sufficient elements of the concept of function. They determined that students were more successful when working with the graphical representation rather than with the algebraic representation of function. The researchers did not suggest that the algebraic representation should be dismissed from instruction, rather it was recommended that more work should be done with the graphical representation early in instruction. They believed that this might help to increase students' success with the abstract, algebraic representation. Dunham and Osborne (1991) reported on the response of 400 precalculus students to two questions of algebraic inequality between two different sets of functions. The students were asked to solve the inequalities algebraically and graphically. The researchers found that the students had the most difficulty with the graphical representation. One explanation for this occurrence offered by the researchers involved the students' "limited and superficial" experience with typical functions.

Questionnaires and individual interviews have been used by many researchers (e.g., Even, 1989, 1990, 1993; Markovits, Eylon, & Bruckheimer, 1988; Thomas, 1969) to study students' concept of function. In a study regarding subject matter knowledge and the teaching of function, Even (1989, 1990, 1993) administered questionnaires to 162 subjects, all of whom were prospective secondary teachers, and conducted interviews with ten of the participants. The instrument was developed for the study; both questionnaire and interview components were pilot tested and validated by an expert panel of educators. Neither instrument was designed to measure performance. The questionnaire involved the subjects' response to function concepts and comments on students' work. The interview was implemented to probe for further information from the subjects. One difficulty which the subjects
exhibited was a rejection of function as a function because of an incorrect assumption that functions should always be represented by formulae, as was also found by Tall and Vinner (1981), Dreyfus and Eisenberg (1983), and Vinner and Dreyfus (1989). These students also held the misconception that functions should have precise, predictable graphs. In other studies, it is reported that students have difficulties with functions that did not have the kinds of graphs that they found acceptable, such as constant and discontinuous functions (Dreyfus & Eisenberg, 1982; Hornsby & Cole, 1986; Maurer, 1974), and circular functions (Herman, 1988). Even (1990, 1993) found that students who could produce the graph of a discontinuous function were still uncertain about the graph as representative of a function because of the visible holes in the graph. In addition, she reported that the subjects did not understand the vertical line test, nor the property of univalence. The graphing calculator could play a role in giving students the opportunity to work with functions graphically. Students need the opportunity to develop individual images of a concept (Bishop, 1989), and the graphing calculator can be exploited by students who need the opportunities to examine various functions. Students can not only work with functions provided by instruction, but they are free to graph randomly chosen functions.

Markovits, Eylon, and Bruckheimer (1983) found after a series of studies that students are restricted by an overbearing linear image of function. From a larger study involving ninth grade students, the researchers were able to determine that even after changing the context of a problem involving function, the students were still guided by the linearity of functions. The researchers suggested that this phenomenon is due to the fact that the linear function is the simplest function and it is emphasized greatly in school, causing the students'
images of functions to be restricted. Along with the linear image, Papakonstantinou (1993) found that students also tend to present a quadratic image of function. Tall (1989) suggested that “by presenting mathematics to a learner in a simplified context, we inadvertently present simplified regularities which become part of the individual concept image. Later these deeply ingrained cognitive structures can cause serious cognitive conflict and act as obstacles to learning” (p. 37).

Other students have also shown the acceptance of the misconception that all functions must be linear. For a sample of high school students and for a sample of 84 college students, Karplus (1979) and Dreyfus and Eisenberg (1983), respectively, found that the property of linearity is one which students strongly depend on when completing graphs of functions or when producing graphs of functions. In their study of college students' concept of function regarding properties of linearity, smoothness, and periodicity, Dreyfus and Eisenberg (1983) administered a questionnaire to college students enrolled in a mathematics course at two Israeli universities to examine the students' concept of function. Other important difficulties noted by the researchers included difficulties by students in determining if a given relation was or was not a function and in using the concept of function to solve problems. Regarding their discussion of the analysis of the results, the researchers used such expressions as "students seem to feel" and "some students feel"; since these are subjective observations, it is questionable as to whether the conclusions based on the observations are replicable.

Tall and Vinner (1981) defined concept definition as the collection of words or phrases used to denote the concept. Tennyson and Park (1980) suggested that a concept definition should contain all the critical attributes of
the concept in question. Tall and Vinner (1981) and Tall (1990) also defined concept image as the image accompanying the concept, including mental images and characteristics of the concept. The students' idea of what a function looks like (concept image) is determined by the image created by examples from experiences and not necessarily by an instructional definition (concept definition) (Friedlander, Markovits, & Bruckheimer 1988; Tall 1981). The idea of concept image is comparable to Skemp's (1979) use of the term "object-concept". He posited that the object-concept is the most basic image of a concept which allows one to recognize whether or not a given object represents the concept. Regarding the graph of a function, the most basic image would most likely be different for each person, because the concept image the person would have in mind at any given time would be based on his/her personal definition and/or image of the concept and not necessarily the mathematically accepted definition (Tall & Vinner, 1981).

A serious error in students' thinking is a conflict between concept definition and concept image (Tall, 1986). This reflects a major inconsistency in the students' conceptual system (Scwharzenberger, 1982; Tirosh, 1990). Students may or may not be able to produce a concept definition; they may indeed have acquired the concept, but yet be unable to provide evidence of this acquisition (Henderson, 1970; Van Engen, 1953). For instance, they may not be able to give a verbal definition (Skemp, 1979, 1987), and in any case, the concept image may not be comparable (Even, 1990; Tall & Vinner, 1981; Tirosh, 1990; Wilson, 1990). In regards to the conflict between two cognitive elements, the cognitive dissonance theorist implied that conflict "can be eliminated by changing one of those elements" (Festinger, 1957, p. 18). In application of Festinger's recommendation for changing the element by adding a new
cognitive element to "reconcile" the conflicting elements, the graphing calculator as a cognitive tool can become the new element which can assist students in developing images for the concept of function. The number of images of the concept of function that the student can create with the graphing calculator is infinite, thus insuring that a variety of examples are available for examination to help the student develop sufficient images of the concept. However, the number of images of the concept of function that the student can create is not as important as the variety and quality of the images.

The subject in Stein, Baxter and Leinhardt's (1990) study dismissed the critical characteristic of function that one and only one range element can be assigned to each domain element. Wilson (1990) found that even for those students who are successful at a sorting task, it is possible that they may not be able to provide a definition of function which describes their conditions for sorting. Likewise, Papakonstantinou (1993) and Vinner and Dreyfus (1989) found that even when students could provide the Dirichlet-Bourbaki definition of function, they were in some cases unable to use this definition to identify relations which were functions. Also, students may master formal mathematical knowledge such as definitions (Cronbach, 1942), but yet be unable to apply this knowledge in a problem situation (Fischbein, 1987). Dreyfus (1990) found this to be one of the major conflicts students have regarding the concept of function. Vinner and Dreyfus (1989), Vinner (1990), and Linn and Songer (1991) would describe this phenomena as a compartmentalization of the students' knowledge. When students can provide a definition of the concept of function, they are using information which satisfies the condition of providing a sufficient definition. The same students can possibly not rely on that information as a means of identifying functions. Instead, they may rely on their images of the
concept. Therefore, asking a learner to produce a definition of the concept of function does not guarantee that the learner is able to apply the definition. This difficulty may be attributed to the fact that different students may have different mental images of function and may construct these images in different ways, depending on the images that are emphasized in instruction (Hershkowitz, Arcavi, & Eisenberg, 1987).

Results by Vinner and Dreyfus (1989) were reported from a seven-item questionnaire administered to a sample of 271 first year Israeli college students and 36 Israeli junior high school teachers. There were no reliability or validity estimates provided by the researchers for the questionnaire. In analyzing the questions the researchers consulted a panel of mathematical experts and used trained assistants to help organize the data. The subjects, students of various academic majors, were divided into four groups depending on the level of mathematics course needed for their academic major. Given this basis for division, similar groups of students at other universities could have been divided differently based on those universities' requirements.

The first four questions were identification questions, the fifth and sixth questions were construction problems, and and the seventh question was a request for a definition of function. Students were asked to explain all answers. After randomly choosing and analyzing 50 questionnaires of correct definitions, they were able to determine that the students' definitions could be sorted into six categories, which unmistakenly were parallel to some of the definitions found in the historical development of the concept of function. For example, one of the categories was function as defined as an algebraic expression or equation, which parallels Bernoulli's and Euler's conceptions of the definition. Of the various results, they found that the students supplying a Dirichlet-
Bourbaki definition increased with mathematics course level and these students gave more and better justifications for their responses to identification and construction exercises regarding the concept of function.

Papakonstantinou (1993) administered a six item questionnaire to 552 geometry, algebra, precalculus, and calculus high school students. The students were enrolled in regular and honors classes in two urban schools. The purpose of the study was to examine the students' knowledge of the concept of function and the relationship between this knowledge and the students' ability to provide mathematical and non-mathematical examples of the concept of function. The researcher found that the students had difficulty defining the concept, providing sufficient examples, and providing justifications for their responses. There was a relationship between the students' ability to define the concept and their ability to provide examples of the concept. Moreover, she concluded that the students' understanding of the concept was stronger from a visual or graphical representation approach that from any other approach.

Dreyfus (1990) reported two other "interrelated" difficulties which were difficulty in visualizing properties of functions graphically and difficulty in allowing functions to be conceived as mathematical objects. Both observations were also reported Ayers, Davis, Dubinsky, and Lewin (1988) and Schwarz, Dreyfus, and Bruckheimer (1990). This is not a new phenomenon. Van Engen (1953) suggested that this deficiency in acquisition of a concept was possibly due to the fact that there was a lack of visual aids that provided students with an opportunity to see the concept of function in action. The employment of graphing calculators would ease this problem for classroom instruction today.
Markovits, Eylon, and Bruckheimer (1988) investigated the concept of function of a sample of ninth and tenth grade students. No other information was given regarding the sample. The researchers were able to determine, by way of a questionnaire, at least seven difficulties the students had with the concept. There was not an indication of the statistical strength of the questionnaire or of the procedures for administration. The framework adopted by the researchers included two basic facts: The first was the adoption of the Dirichlet-Bourbaki definition of function and the second was the realization that multiple representations of the concept of function exist.

The difficulties uncovered by Markovits, Eylon, and Bruckheimer (1988) which have not been previously addressed will now be discussed. These researchers found that many students were deficient in understanding mathematical vocabulary. Vinner and Dreyfus (1989) also reported that many students' difficulties were due to the fact that they did not have ample mastery of terms which related to the concept of function, such as image and preimage. Confrey (1981a) reported that the terms which accompany a concept are important in the sense that they help communicate the characteristics of the concept. Dunham and Osborne (1991), Dreyfus and Eisenberg (1982), and Rosnick and Clement (1980) were able to add students' lack of understanding of variables and functional notation.

Markovits, Eylon, and Bruckheimer (1988) also observed that students often dismissed related concepts of the concept of function, such as domain and range. The researchers suggested that such related concepts should be fully understood in all representations of the concept of function or should be deemphasized in instruction. Of course, the concepts of domain and range are essential to understanding the concept of function, and emphasis on them
should not be decreased, thus leaving educators the responsibility of focusing on the concepts in instruction. Orton (1971) found that students misunderstood the concepts of domain and range as related to the graphical representation of functions. Given the graph of a function, many of the children in his study were not able to denote the domain and range of a function.

Regarding students' choices of domain and range for graphing a function, Laughbaum (1989) suggested that if students think that the graph of the function stops with the "edges" of the medium on which the graph is viewed and hence implying that the domain and range of the function end at those points, then the mathematics educator should encourage the students to view of the function using various domains and ranges. With access to a graphing calculator, this becomes a small task to facilitate.

Other ideas related to the concept of function are variable and single-valuedness (Reeves, 1969), images, preimages, zeros, and extremum (Dreyfus & Eisenberg, 1982, 1984), inverse, many-to-one, and one-to-one (Orton, 1971). Finally, the researchers discovered that students had difficulty with multi-step problems and rational number operations. In these instances, we cannot expect students to succeed with mathematics until they are able to work comfortably with integers and rational numbers (Leitzel, 1984), and are fully prepared to use skills to work problems with multi-steps.

Particularly for graphing functions, students have shown to have a lack of awareness of the importance of the scales of the axes. Dunham and Osborne (1991) reported that students often ignore the scales of the axes, and they have the tendency to assume that the scales of both axes of the Cartesian coordinate system are identical and marked in intervals of one. The researchers found that when students were encouraged to focus on the scales of the axes, the
students showed improvement in working with points on the graph, scaling, and using graphing space efficiently. They suggested that teachers should require students to examine the effect on the graph of the function when the scales of the axes are changed using graphing devices. Demana and Waits (1988) found that lack of choosing appropriate scales of axes for graphing functions could cause one to be a victim of the "pitfalls of graphing". They suggested that inappropriate scales could cause one to draw incorrect conclusions from the graph of a function. The researchers proposed that one solution to the problem was to require that students graph the same function on different sets of axes to review the effects that the scales may have on the production of the graph.

Yerushalmy (1991) reported on a study involving 35 secondary school students using the Function Analyzer (Schwartz & Yerushalmy, 1988). Of current interest are the responses of the students to tasks involving an understanding of scale. One such tasks requested that the subjects match an algebraic representation of function with its graphical representation. The task involved four linear functions graphed on sets of axes with different scale units. There were "two different pictures of the same function and two identical pictures of different functions" (Yerushalmy, 1991, p. 48). The results of the responses showed that the students were relatively successful with the two identical pictures of different functions (91% and 94%) and were not misled by the scale of the axes. However, they were less successful with the two different pictures of the same function (61% and 70%).

The researchers also presented the students with the graph of a function for which the axes were not labeled. The students were asked to make a choice between matching the graph with an algebraic equation or denoting that such an equation could not be determined from the information provided. A
majority of the students (28 out of 30) were correct in responding that the equation could not be determined. Of the 28 students, 71% had an argument involving the lack of labels and units on the axes and 29% suggested that they needed the graph of another function to make comparisons or a different coordinate system.

Educators should find a better or best method for teaching the concept of function (Cohen, 1983). Henderson (1967, 1970) outlined three ways of teaching concepts in mathematics, including the concept of function. The three ways were as follows: In the "connotative" mode, one would use the term "function" to discuss the properties and characteristics of functions which would cause one to apply the name to a given mathematical object. In the "denotative" mode, one would use the term "function" to distinguish between relations which were and were not functions and produce examples of functions. "Implicatively", the definition of the concept of function would be presented to the learner. In the case of the last method which is the most passive of the three, Vinner and Dreyfus (1989) recommended that formal definitions of concepts should only be employed to finalize the concepts through instruction once it is clear that the students have acquired the concept. In agreement, Tall (1986) stated that "formal definitions . . . are totally inadequate starting points for an unsophisticated learner lacking the cognitive structure to make sense of them" (p. 23). Likewise, it was proposed by Confrey (1981a) that "a precise definition fails to communicate the importance of the concept of function within the conceptual framework of . . . algebra" (p. 9). Other researchers (e.g., Henderson, 1967, 1970; Skemp, 1987) have proposed that the definition would help students acquire the concept. Wilson (1990)
suggested that definitions, along with examples and nonexamples of a concept work together to help students acquire concepts.

Regardless of the instructional methods used, students must become active in learning process (Kaput, 1979). In 1971, Collis reported that “the attainment of concepts in mathematics is the result, in the main, of deliberate formal teaching” (p. 12). While, Fischbein (1987), through his research on students' intuition, found that formal teaching does not provide for all of the students' academic needs. By accepting the notion that a formal presentation of definitions and concepts alone does not promote conceptual understanding, the present research study is conducted with an effort to actively involve the learner in the instructional process. Studies aimed at increasing the instructional component of the concept of function are rare compared to the work being done on students' understanding of the concept and its components (Stein, Baxter, & Leinhardt, 1990). Teachers should be prepared to organize and present concepts in a manner which would afford the students greater access to the concept (Leitzel, 1984). Cohen (1983) and Cornelius (1982) suggested that concepts should be instructionally developed so that students can understand and acquire them. One way to encourage this would be for educators to present concepts in a way such that the new concepts are related to concepts students have already acquired (Tirosh, 1990). By examining cognitive conflict in students' mathematical knowledge, we may be able to avoid compartmentalization (Steffe, 1990). Regarding the concept of function, Confrey (1981a) proposed that the aim of mathematics education was to attempt to encourage students to succeed at accommodation of the concept. Given that “assimilation is typically the way that abstract formally defined concepts are acquired” (Thomas, 1969, p. 52), the task of the current study was
to examine an attempt to assist students in avoiding compartmentalization regarding the concept of function either by assimilation, and to a greater degree, accommodation.

Several researchers have attempted to design methods that will assist teachers in helping students acquire the concept of function and alleviate misconceptions. Many of these same researchers have studied students' misconceptions and inconsistencies regarding the concept of function. Although "there is no proven optimal entry to functions and graphs" (Leinhardt, Zaslavsky, & Stein, 1990, p. 6), the Function Block introduced by Dreyfus and Eisenberg (1981, 1982, 1984) was one such attempt to provide dialogue concerning the presentation of function in the mathematics curriculum. The main theme of this research has been that students' intuition of the concept of function should be considered, especially when the topic is first presented. According to some researchers (e.g., Dreyfus & Eisenberg, 1982, 1983, 1984; Vinner & Dreyfus, 1989), students' intuition should be assessed before the topic of function is encountered, because intuition exists before the onset of formal instruction (Leinhardt, Zaslavsky, & Stein, 1990). Fischbein (1987) advocated that one's intuition goes beyond the obvious facts one is given. Confrey (1981a) presented one characterization of a concept as its acquisition which can allow the learner to go beyond the given information. By examining students' intuition, the researcher can determine whether or not the learner can use the concept for generalization. Dreyfus and Eisenberg (1981, 1982, 1983) suggested that the concept of function be introduced into the instruction by three interrelated properties of function. The three dimensional block model displays the function concept on three axes. The x-axis represents the various settings (representations) in which functions can be presented, which allows for
lateral transfer from one setting to another. The y-axis represents concepts related to the concept of function, which involves new learning, and the z-axis represents the level of abstraction or generalization, such as problem type and number of variables, as a representation of vertical transfer. The researchers did not address the definition of the concept of function as an element of the Function Block (Kolb, 1985), nor was the idea that a function represents a quantitative relationship or functionality emphasized in the model.

According to the designers of the model, the Function Block can be used to encourage decisions regarding the sequencing and presentation of the concept in instruction and curriculum by choosing various aspects of the different axes to emphasize. One has to be careful not to overwhelm the students by mis-sequencing (Tirosh, 1990) the concept, if the Function Block is used for curriculum development.

Dreyfus and Eisenberg (1981, 1982) administered a questionnaire to 443 Israeli students of grades six through nine in 24 classes. The questionnaire was designed to examine differences, if they existed, between students' intuition of the concept of function regarding the independent variables of ability/social level, grade, sex, and setting of the concept of function. The ability/social level variable was questionable. The classification was based on social level as being dominant. Hence a student with high ability but low social economic status would be classified as "low". Garofalo (1983) pointed this out as one of the weaknesses of the variable. He also noted students then are not compared as a group by using any standardized instrument.

The representations they used in the questionnaire were arrow diagram, graph, and table. Each representation involved a function given concretely and a function given abstractly. Concepts related to the concept of function and
addressed in the instrument were image, preimage, growth, extrema, and slope. Reliability coefficients were estimated by Kuder Richardson formula 20 (KR-20) and reported as .91 for the full test and .86 and .81 respectively, for the subtests. The researchers used a four-way analysis of variance for the 1982 report of the study, but the model does not match their hypotheses. Huber (1983) noted that interactions were included in the model, but were not included in their hypotheses. He went on to highlight the fact that there were significant interactions, but the researchers only discussed the main effects.

They found that ability/social level and grade each had a strong effect on the total mean scores. The diagram representation caused students the most difficulty, and it was concluded that students with high ability/social level were most likely to perform better in the graphical representation than in the table representation. Students with low ability/social level performed better in the table representation than in the graph representation. In a study of middle and high school students, Wagner (1981) found that the sample found it easier to interpret functions presented in chart form, table, or ordered pair. The implications of these findings propose to the educator the task of determining which representations are most appropriate for the population of students in individual classes.

In a study reported in 1984, Dreyfus and Eisenberg examined a sample of seventh and eighth grade Israeli junior high school students regarding the usefulness of the Function Block. Each third of the sample was given an arrow diagram test booklet, a graph test booklet, and a table test booklet, respectively. Each booklet had a concrete situation and an abstract situation. The decision to include both a concrete situation and an abstract situation was based on the
z-axis of the Function Block, however, Even (1989) suggested that this axis was not precisely clarified and may itself have axes or branches of its own.

**Computers and Calculators**

Several researchers (e.g., Ayers, Davis, Dubinsky, & Lewin, 1988; Confrey, 1992; Schwarz & Bruckheimer, 1990; Schwarz, Dreyfus, & Bruckheimer, 1990) have designed computer programs to deal with the concept of function in instruction. A Triple Representation Model (TRM) for the concept of function was developed in Israel (Dick, 1992) and introduced into mathematics education by Schwarz and Bruckheimer (1990) and Schwarz, Dreyfus, and Bruckheimer (1990). This model for instruction of the concept of function was designed to address students' misconceptions about and difficulties with the concept of function. There are several main characteristics of the TRM. The model can be used to provide the learner with a path between the algebraic, graphical, and tabular representation of function. Instruction which relates these three representations of functions assist students in developing a deeper understanding of functions, and may help "decompartmentalize" students' concept of function (Dick, 1992).

The model is useful for providing for transfer between the three representations. Smith's (1972) research report on transfer between representations of function with grade nine students provided evidence that when students are not taught with the representation which they are asked to transfer to, then the transfer may not be completely successful. The sample of this study consisted of students participating in the Secondary School Mathematics Curriculum Improvement Study (SSMCIS). Thomas (1969, 1971) reported on a study of 201 seventh and eighth grade students participating in
the same project and using an experimental text. At that time, the mapping approach to function was emphasized, with some use of the ordered pair approach. Of the students who were competent in working with function given by arrow diagram and algebraic rule, only 85% of them were successful with the graph and ordered pair representation. As found by Even (1989, 1990), because a student understands the concept in one representation, does not guarantee that this understanding will transfer to another, newly introduced representation.

The transfer between the representations is completely automatic. The learner has the option of using the computer to operate within the transfer of the concept of function from one representation to another. However, the learner may still be hindered by the inability to conceptually apply transfer to the concept. For example, Yerushalmy (1991), in a study employing the Function Analyzer (Schwartz & Yerushalmy, 1988) a computer program involving three representations of functions, found "that learning within the linked multiple representation environment does not necessarily motivate a linked performance. Although the students presented a wide repertoire of visual and other arguments, they did not tend to use these arguments for cross-checking purposes" (p. 54).

Work within any representation is operational. This provides for activity in each representation, which will give the student the opportunity to view each representation in action. In addition, the computer environment is the focus of the curriculum.

The curriculum for function suggested by the researchers will help students to understand the concept of function through experiences with the three representations, transfer between them, and problem solving. The researchers'
observations of students using the TRM have shown significant success with the activities in the TRM. The researchers allowed an introductory unit on functions to be taught to ninth grade students for ten weeks for a total of 20 hours of instruction. Students were paired together at each computer with an enlarged screen for class demonstrations and discussions. They suggested that the model should not be used at large until more research is conducted regarding the management of computers as tools of instruction. Although research on teaching and learning algebra with computers is not new (Kieran, 1990), caution of the use of computers is still recommended by IGPME (Nesher & Kilpatrick, 1990).

Several of the previous researchers have provided answers to questions posed by Dufour-Janvier, Bednarz, and Belanger (1987) regarding the use of multiple representations of mathematical concepts:

1. Which representations should be retained? This question is attended to whenever researchers draw conclusions concerning the usefulness of certain representations and difficulties that students may have with them. One can safely assume that the representations of the concept which are evident to us at this time must be used appropriately. Perhaps all representations (e.g., graph, arrow diagram, equation, table, sets of ordered pairs) should be retained with the condition that they are called upon at appropriate times during instruction. In Confrey’s (1992) “Function Probe” software tool, “the function concept is viewed as evolving from an interweaving of a variety of representations, each of which provides a different way of constructing the idea and experimenting with it. Contrasts and commonalities among the representations provide the basis for understanding the concept” (p. 169).
2. Are there representations that are more appropriate than others for developing a concept? Thorpe (1989) suggested that "the definition of a function as a set of ordered pairs is not only too abstract for an initial introduction; it is inconsistent with the way functions are viewed and used by professionals" (p. 13). However, discussion of the concept using the ordered pair representation could be beneficial for characterizing one-to-one, many-to-one functions and inverses of functions. The graphical representation has been advocated as one of the most useful settings in which students may develop the concept of function. Orton (1971) analyzed interviews with 72 students (ages 12-17) who were either average or above-average mathematically, and he concluded that in regards to the ordered pair representation of function, "some children who could recognize functions were confused when they were asked to explain the difference between a relation and a function, and to relate their responses to sets of ordered pairs if they could" (p. 46).

3. How should the representations be used? In which context? The researchers employing the Function Block addressed the issue of presenting functions in a concrete and abstract environment and employing various representations. The context in which the representations are used is ultimately decided upon by the educator. Proper judgment must be used in determining whether abstract and/or concrete situations are best for approaching and discussing the topic of function.

4. What are the difficulties and the children's conceptions that need to be taken into account when a representation is used? The preceding portion of this review has been evidence that students have exhibited various difficulties concerning the concept of function.
5. Are there representations that are more appropriate to the level of development of the child and to where he is in regard to the learning of mathematics? Based on some of the previous research results, students of different mathematical ability may prefer different representations of the concept of function. The learner's general mathematical abilities might be factors in determining how well the learner can deal with such things as the algebra of functional equations.

Friedlander, Rosen, and Bruckheimer (1982) described a parallel coordinate axes much like Arcavi and Nachmias (1990) and Nachmias and Arcavi (1990) who designed a computer environment, called the Parallels Axes Representation, as an attempt to help students in working with the symbolic and graphical representation of function. The model consisted of two vertical, parallel number lines: the left for the domain and the right for the range. Mapping lines represent the correspondence between the two, and can be used to determine characteristics of the function they represent. For example "slope is reflected as the inclination of the line with respect to the left axes" (Friedlander, Rosen, & Bruckheimer, 1982, p. 81). This is not meant to replace the Cartesian or rectangular coordinate system, but the researchers suggested that students need to see functions represented by different systems.

The microcomputer has proven to be beneficial when used to assist learners with their acquisition the concept of function (Waits & Demana, 1988). In recognizing that even college students have difficulty with the concept of function, Ayers, Davis, Dubinsky, and Lewin (1988) employed the computer as a method for instruction. The goal was to give students an environment in which they could experience multiple examples of function. They found that the students in the computer group were more effective in dealing with concepts
and composition of function than students in the paper and pencil group. The researchers did not employ any qualitative measures to further examine the differences between the two groups. Fischbein (1990) has proposed that research techniques of "pure statistics" should be accompanied by such methods as observations and interviews, in order to get a more complete picture of the situation. Another limitation of this study involved a lack of generalizability to a target population. Their sample was unique in that every student enrolled at the university was required to own a computer, and the students were already divided into groups by the university registrar's office. Statistics were not provided for the pretest nor for the posttest, which were designed for the study.

Tall and Thomas (1988) reported on employing computers and the Socratic Method of instruction to examine students' concepts in algebra. Their "generic organizers" (Tall, 1986, 1990) were microworlds (computer environments) designed to allow students to work with examples and concepts or systems of concepts. They conducted three experiments to determine if the combination worked. Tall and Thomas (1986) used the topic of algebraic variable and found that the experimental group performed conceptually better than the control group. They administered another posttest after one year and found that the students using computers still fared better conceptually than those who had not. Research involving whole class treatment is representative of what is needed in mathematics education, because whole class instruction is a reality in education, and quality of education is more important than quantity in education (Brophy, 1986).

In regard to conceptual learning in mathematics, researchers have begun to experiment with computer algebra systems to determine their value
regarding the enhancement of concepts in algebra and calculus. Palmiter (1991) employed the computer algebra system MACSYMA in a study to examine conceptual and computational differences between students using the system and students without access to the system. Her study included 40 students in the experimental group and 41 students in the control group. The students who were enrolled in a calculus course at a university, were randomly assigned to the two groups. Both groups studied the same topics, which included the fundamental theorem of calculus, inverse of functions, and integration during the course of the experiment.

The study lasted 5 weeks for the MACSYMA group and 10 weeks for the control group. At the end of the sessions, a conceptual and computational exam were administered to both groups. The instruments were designed by the course lecturers. There was no indication in the research report of the statistical strength of the instruments. However, the lecturers agreed that the content of the exams reflected the content and difficulty of the course material. The researcher reported means and standard deviations as an indication of the differences between the two groups. Only the conceptual exam results will be an issue in this review.

An example of a problem on the conceptual exam was an analysis of a graph of a function for a given interval. It was determined that the students using the computer algebra system scored significantly higher on the conceptual exam than the students in the control group using paper and pencil only. Palmiter (1991) admitted to several weaknesses in this conclusion. The students using the computer algebra system were fully aware that they were participating in an experiment, and thus this knowledge may have led to a "Hawthorne Effect". Secondly, the results could be reflective of the individual
instructor and not the treatment itself. Lastly, the experimental group’s sessions only lasted five weeks, while the control group sessions lasted ten weeks. In this time, the control group had to deal with computational and conceptual material, while the experimental group used the computer for computational work. Hence the control group had to work manually on more material and for a greater period of time. An examination of the students using the computer algebra system may be more beneficial than examining a comparison group.

The Function Analyzer (Schwartz & Yerushalmy, 1988) was the technological tool used by Yerushalmy (1991) to examine its effects on the learning of function and graphs for a sample of 35 students in the eighth grade. The main characteristic of the computer program is the manipulation of functions in three representations: graphical, symbolic, and numerical. From observations and examinations of five tasks over the 3-month period of the study, the researcher found that the benefit of visual representation was hindered by students’ lack of “formal algebraic knowledge”. In classifying functions, “students were [more] concerned with [other] factors in the description of the function, such as symmetry, inflection, location on the grid, or visual discontinuity” (Yerushalmy, 1991, p. 50). This suggested that stressing the visual alone by providing a computer screen is not sufficient. The learner will still be inhibited if an understanding of the concept of function is not obtained.

Lynch, Fischer, and Green (1989) reported on a project concerning the use of computers in elementary algebra that would enhance students’ concepts in algebra. The concept of function was one focus of the project. The representations available by computer program were tables, graphs, and
equations. Homework, classwork, and exams were completed with emphasis on the computer and on the algebraic concepts introduced by the activities. The project directors emphasized the need to reorganize if technology will be a part of the educational process. Students became more involved in the learning process and used the tools available to explore and conjecture. Like the computer, the graphing calculator is powerful enough to be used to enhance algebraic concepts, such as the concept of function.

Roberts (1980) reviewed the impact of calculator usage provided by 34 studies of mathematics achievement and attitude. Most researchers have been only concerned with these two variables (Bell, Costello, & Kuchemann, 1983). The levels of the studies were elementary, secondary, and college. There were ten articles and 24 dissertations included in the research study. The research design for most of the studies were pretest-treatment-posttest, with the experimental group receiving instruction with a calculator (treatment), and a control group receiving instruction without a calculator.

The two areas of interest were the review for the college level studies and report of conceptual effects of calculator usage. Of the ten studies reported for the college level, only three had any concern with the conceptual effect of calculator usage, and of these, only two showed support for concept development. In the review of the ten studies, Roberts (1980) did not mention which of the ten studies involved concept development. The conclusion is weakened by the omittance of this information.

In the review by effect, Roberts (1980) stated that “few studies made any real attempt to carefully integrate calculator use into the curriculum that would illustrate how calculators can facilitate concept learning” (p. 84). This indicates the need for more researchers to conduct studies which represent an emphasis
on calculators for concept development. The researcher noted several weaknesses with the studies reviewed. First, he noted that because of the difficulty of assigning students randomly to treatment and control groups, researchers assigned groups at random, but still performed statistical analysis on the individual student, rather than the class as the unit of study and used the ANCOVA to correct for the use of nonrandom groups. In addition, most of the studies were conducted with the control and treatment groups in the same schools and with the same grade levels, thus there was a greater opportunity for exchange of information during the study. Roberts also criticized the researchers for using the same teachers for both control and treatment groups and for not allowing the use of calculators on posttests.

In the most complete study conducted regarding the use of calculators (Dick, 1988), Hembree and Dessart (1986, 1992) conducted a meta-analysis. The method of meta-analysis was developed by Gene Glass, Barry McGraw, and Mary Smith (Borg & Gall, 1989), and is used to convert the statistical findings from individual studies to an effect size. "The mean of the effect sizes for all studies included in the research review is the calculated estimate of the typical effect of the phenomenon under study" (Borg & Gall, 1989, p. 173), in this case, calculator effects for students in grades K-12. If other researchers used the same studies and statistics, the results should be the same. (Hittleman & Simon, 1992). A total of 79 studies (not referenced in the research report) were used which were composed of journal articles, published and unpublished reports, and dissertations. Because one of the weaknesses of meta-analysis is the inclusion of poorly conducted studies (Borg & Gall, 1989), the researchers indicated that they took extra precautions, such as contacting researchers, to clarify content in questionable articles. They did not indicate in
this particular report the characteristics of the weak or strong studies. The studies differed in such aspects as sample size, grade level, instrumentation, and length of treatment. In each study, there were a treatment group using calculators and a control group compared by average scores to measure effects of calculators in instruction; in most cases the students were not allowed to use the device for testing. Branca, Breedlove, and King (1992) and Wheatley and Shumway (1992) found that when students are allowed access to calculators for evaluation then their teachers can concentrate on evaluating students' conceptual growth, rather than their paper and pencil performance.

There were 17 independent variables by which the 79 studies were classified. The dependent variables were achievement and attitude. Concept development with the calculator was evident in 13 studies, but "regarding the achievement of concepts, a nonsignificant effect was found across all grade and ability levels" (Hembree & Dessart, 1986, p. 94).

Koop (1982) explored the effect of the calculator at the community college level, for the dependent variables of attitude, course completion, retention, achievement, and effects on different populations of students. Practically no research has been conducted at this level concerning concept development. The 150 subjects were randomly assigned to classes. There were three classes with calculators and three classes without calculators. They could use calculators for all classwork, homework, and exams. There were three instructors who had one of each type of class. The independent variables were treatment, sex, instructor, ethnicity, and age.

An analysis of variance was performed first with pretest data to determine whether or not this covariate could be relied on more to equate the groups or to increase the power of the analyses. When the researcher determined that the
pretest could be used to increase the power, he conducted an analysis of covariance, using the pretest and a posttest. Of interest, was the result that the older students (those over 29 years of age) confounded the statistics for the main effect of the calculator. When this group of students were excluded from the analysis, the effect was much more profound. The researcher did not suggest that this should be done, but only wanted to point out the situation. Although ethnicity was listed as a independent variable, it was not discussed in the research report.

**Graphing Calculators**

The scarceness of research projects examining the use of calculators for teaching and learning mathematics emphasizes the need for research aimed at using the graphing calculator in instruction to alleviate misconceptions in students' knowledge (Tirosh, 1990), and encourage conceptual change. Students require an atmosphere for learning that encourages trial and error (Mathematical Sciences Education Board (MSEB) & National Research Council (NRC), 1990); exploration, and conjecturing (Burrill, 1992; Demana & Waits, 1989; NCTM, 1990; Ruthven, 1992; Vonder Embse, 1990). The graphing calculator makes teaching and learning functions an active rather than a passive process.

The graphing calculator is very similar to the microcomputer (Willoughby, 1990) and in fact, has been described as a hand-held computer (Cooney, 1989; Dick, 1992; Shumway, 1989), a mini-computer (Debower & Debower, 1990), a programmable computer (Trotter, 1991), and a pocket computer (Demana, Dick, Harvey, Kenelly, Musser, & Waits, 1990; Demana & Waits, 1988, 1989, 1990). There is still dialogue regarding the use of calculators in
the undergraduate mathematics curriculum (Dion, 1990), although they can be used to have a great impact on the learning and teaching of mathematics (Leitzel, 1989). Because graphing calculators are programmable, they are equally capable of graphing as the computer (Demana & Waits, 1990). On a more practical level, graphing calculators are less expensive than computers (Demana & Waits, 1990), and because of their size, they can be physically handled by the students as a hand-held tool for learning. The graphing calculator is proving to be most beneficial for graphing and analyzing functions (Barrett & Goebel, 1991; Hector, 1992) and providing students the opportunity to enhance their understanding and intuition regarding the concept of function (Demana & Waits, 1991). It has been proposed that the graphing calculator will be the technological tool that will have an “immediate” effect on the high school mathematics curriculum (Demana & Waits, 1990), and according to Leitzel (1989) this effect will be greater than that produced by the use of computers.

NCTM (1980, 1989) and MSEB and NRC (1990) suggested the availability and use of calculators in mathematics at all levels of instruction, from the elementary level (Harvey, 1991) to the community college level (Koop, 1982). The availability of the graphing calculator will increase its impact on learning and teaching mathematics far beyond the computer, that is, until perhaps computers are more accessible than graphing calculators (Shumway, 1990). The 1992 NCTM Yearbook, Calculators in Mathematics Education (NCTM, 1992) was compiled of articles written exclusively about the use of calculators in mathematics education. In his chapter of the Yearbook, Hector (1992) suggested that graphing calculators affect the way functions are taught in the mathematics curriculum. NCTM (1989) proposed that students should not
graph functions by first constructing a table of ordered pairs by hand, one reason being that this action emphasizes a point-wise characteristic of the graph of functions (Even, 1990). The graphing calculator can not only be used to decrease the time needed for producing graphs of functions, but it can be employed to refocus attention to concepts and not skills of graphing by hand (Hirsch, Weinhold, & Nichols, 1991). Without graphing calculators students do not have an alternative (Hector, 1992), and teachers may be left without a way of introducing and focusing on the graphical representations of certain functions, if chalk and blackboard are the only resources (Morris, 1982).

Graphing calculators are programmed to allow analysis of functions. Students can analyze the graph section by section by applying the zoom-in feature to view the graph more closely or the zoom-out feature to view a broader portion of the graph (Dion, 1990; Hector, 1992; Shumway, 1989). With the trace feature, the learner can analyze more closely point on the graph (Burrill, 1992). In this sense, the function becomes a mathematical object to be visualized and studied. As a visual aid, the graphing calculator can help clarify meaning of concepts, as visual aids are capable of doing (Van Engen, 1953). By examining multiple examples of functions, students are able to compare functions (Dion, 1990), and gain a better understanding of the concept (Barrett & Goebel, 1991; Demana & Waits, 1988).

For a sample of 67 college calculus students, Vinner (1989) applied a treatment of visualizing algebraic notions in the course and providing opportunities for students to choose between algebraic and visual proofs. He administered several questionnaires from which he was able to determine that the students were more likely to choose algebraic over visual. However, "reliable access to graphic calculators is likely to encourage both students and
teachers to make more use of graphic approaches to solving problems" (Ruthven, 1990, p. 447). Like the computer the graphing calculator can be used to provide visual images (Bloom, Comber, & Cross, 1987).

Ruthven (1990) compared secondary school mathematics students who were classified into two groups: those using the graphing calculator as a tool in the classroom and those not having regular access to graphing calculators, on a questionnaire and 12 graphing items. The independent variables were treatment, grade, school, and sex. The researcher did not give a reason for including the variable of sex, nor was a reason given for including an interaction of sex and treatment in the statistical model.

The students were from classrooms in England schools which were participating in the Graphics Calculators in Mathematics Project, in which six teachers had at least one class with permanent access to graphing calculators. The researcher used four schools with parallel classes. The statistical strength of the questionnaire was not reported. He did not report on the entire test of graphing items, but only on a subtest of symbolization items. He found that the graphing calculators encouraged relationships between the symbolic and graphic representation of function. Describing the graphing calculator as a "cognitive tool", he was convinced that it aids conceptual understanding, as are many other researchers (e.g., Branca, Breedlove, & King, 1992; Dance, Jeffers, Nelson, & Reithaler, 1992; Rubenstein, 1992; Wheatley & Shumway, 1992).

In a study designed to examine the impact of graphing calculators on high school students' acquisition of function concepts and instructional processes, Rich (1990) used the t-test to compare the means of an achievement test and an attitude survey of treatment and control groups. The treatment groups consisted of in-tact classes with access to graphing calculators and a
precalculus textbook which was written with an emphasis on a graphical and technological approach to function. The control group consisted of in-tact classes with access to scientific calculators without graphing capabilities and a traditional precalculus textbook. One of the main differences between the two textbooks was the emphasis on graphing and the use of graphing calculators. The researcher did not explore the effect that the use of different textbooks might have had on the results of the analyses.

In regards to achievement, the treatment group had higher mean than the control group, but the t-test analysis did not indicate any significant difference between the two means. Further analysis of a formal interview, which dealt with misconceptions regarding the concept of function, was conducted with high, average, and low ability students. The interviews revealed the following when comparing the treatment and control groups.

1. The treatment groups were better able to graph functions.
2. The students in the treatment groups were better able to work with functions algebraically and graphically.
3. The treatment groups showed a better understanding of the relationship between algebraic equations and graphs of functions.
4. The treatment groups understood the concept of inverse of a function and absolute value functions better.
5. Students in the treatment groups were better prepared to respond to questions.
6. The treatment groups had a better understanding of functional relationships.
7. Both groups were capable of dealing with variables and functional notation.

8. The treatment groups had more insight on initial images of graphs.

**Conceptual Change Theory in Mathematics Education**

Because accommodation is difficult to accomplish and is a more radical action than assimilation, Strike and Posner (1985) and Posner, Strike, Hewson, and Gertzog (1982), found that either compartmentalization will occur or an attempt at assimilation will be made. Schwarz, Dreyfus, and Bruckheimer (1990) described compartmentalization as the act of restricting one's knowledge to a particular context. By this description, the researchers implied that students either willfully or unconsciously employ certain pieces of information depending on the situation they are involved in.

Students' misconceptions are real concerns for educators, and they need to be aware of them (Wagner, 1981). Before curricula and instruction can be altered to deal with students' misconceptions, attention must be give to examining the nature of particular misconceptions (Nussbaum & Novick, 1981, 1982). Much of this work is taking place in science education (Dreyfus, Jungwirth, & Eliovitch, 1990; Nussbaum & Novick, 1981, 1982; Pines & West, 1986). Brophy (1986) suggested that by developing curriculum and instruction that addresses students' misconceptions, we may be able to significantly reduce these misconceptions. As the theory of conceptual change has been applied in mathematics education via the introduction of cognitive dissonance, cognitive conflict, conceptual conflict, and the use of the conflict teaching approach, encouraging assimilation and accommodation, it has become evident through limitations, warnings and suggestions reported in several
studies (e.g., Movshovitz-Hadar & Hadass, 1990; Tirosh, 1990; Tirosh & Graeber, 1990a; Williams, 1991; Wilson, 1990) that some modification to the intervention of cognitive conflict will be necessary.

The application of the theory of conceptual change, although not widely used in mathematics education, has provided for success in the teaching of science concepts. Stavy and Berkovitz (1980) used a "conflict-producing technique" to increase science students' development of the concept of temperature. Upon discovering that the children had conflicting representations for temperature, the researchers intervened with worksheets designed to induce cognitive conflict through activities. For a sample of 77 students, the researchers administered pretests and post-tests to two treatment groups and one control group. One treatment group received a revised curriculum and whole group instruction to induce cognitive conflict; the other treatment group received individual instruction to induce cognitive conflict, and the control group received neither. The cognitive conflict was induced by worksheet activities. Results supported the conclusion that the conflict inducement produced positive results regarding the students' acquisition and understanding of the concept of temperature. In applying the theory of conceptual change in science education, Stavy and Berkovitz (1980) have recommended that curricula and instructional methods be designed to induce cognitive conflict, with the goal of assisting students in acquiring concepts. Hand and Treagust (1991) provided a model for designing curriculum materials to encourage cognitive conflict. Their model consisting of 15 lessons involved seven worksheets which addressed students' misconceptions through practical activities and challenging questions. The misconceptions identified in interviews were the basis for the worksheets.
Sierpinska (1987) and Williams (1991) examined students' concept of mathematical limit. In his study examining college calculus students' concept of limit, Williams (1991) applied treatment in which he attempted to encourage conceptual change in the students by inducing cognitive conflict through a conflict teaching approach. The researcher's aim was to present characterizations of limit which would be in conflict with the students' concepts of limit. A survey was administered to 341 university calculus students. The objective of the survey was to determine the students' initial concept of limit. Eventually, 10 students were chosen to participate in the treatment portion of the study.

The 10 students met as a group with the researcher on five occasions during a 7-week period, during which time clarification of a definition for limit was performed, three characterizations of limit were presented as inducers of conflict, and students were asked to explain whether or not their view of limit was altered by the previous information. Williams (1991) concluded that the students were unwilling to integrate a formal view and formal definition into their knowledge structure. Although the researcher acknowledged failure in the treatment based on his goal of having the students alter their concept of limit, he concluded that a closer examination of the students' knowledge of function was needed to determine the underlying assumptions which caused them to have misconceptions about the concept of limit. The researcher did not suggest that technological advances may have been beneficial in the study. It is possible that the students were inhibited by the static representation of functions that the researcher made available. The graphing calculator would have been an excellent tool for the students to use to examine the functions as dynamic models of mathematics. With graphing calculators, the subjects would
have had the opportunity to examine a great variety of functions. The lack of employment of any technological tool in the study perhaps was a major weakness considering the topic which was of focus. Similarly, Sierpinska (1987) found that the induced conflict did not encourage major changes in the students' concept of limit. However the researcher was encouraged that discussion of the "mental conflict" would produce some change in the future.

In Festinger's (1957) original thesis, he proposed that the goal of inducing cognitive dissonance was to provide motivation for a student to attempt to reduce the dissonance or conflict; this goal was attained in Williams' (1991) and Sierpinski's (1987) study, because they were able to create situations where the students could examine their concepts and question the validity of their concepts. These studies represent one of the obstacles which science educators have found in attempting to induce cognitive conflict: the learner may not encounter conflict which insures learning (Dreyfus, Jungwirth, & Eliovitch, 1990; Driver & Erickson, 1983). For example, Rosnick and Clement (1980) obtained similar results from a study involving the testing and interviewing of university students enrolled in precalculus and calculus courses. The researchers asked students to translate information given in a table, graph, or picture into an algebraic equation. They found that the students had serious misconceptions regarding variable and equation. Through tutoring sessions, they found that "although students's behavior for the most part was changed, . . . their conceptual understanding of equation and variable remained, for the most part, unchanged" (p.18).

Tirosh and Graeber (1990a) employed the conflict teaching approach in an attempt to examine preservice teachers' concept of division. Treatment consisted of individual interviews with the 21 subjects. The researchers
conducted the interviews with the goal of assisting the students in becoming aware of and reflecting upon their misconceptions and inconsistencies. One type of conflict induced involved encouraging the subjects to review their conceptions regarding division and their calculations of division. The subjects in the study exhibited improvement in translating information given in word problems and in translating written statements of division into mathematical language. The results provided evidence that if the approach is used properly, it is possible to have positive outcomes. This approach does have its limitations. The student may not even recognize that he has a conflict (Wicklund & Brehm, 1976; Wilson, 1990), and if so, may not respond positively. In addition, the learner's self-concept as a learner of mathematics may be negatively affected.

Cognitive conflict was addressed by Wilson (1990) in a study designed to examine students' discrepancies between definitions and examples. Tirosh, Graeber, and Wilson (1990) proposed two types of inconsistencies. The first type of inconsistency involved discrepancies between students' mathematical constructs and "conventional" mathematical constructs. An example of this inconsistency regarding the concept of function would be a discrepancy between the learner's definition of the concept and the definition which is proposed by the mathematics community. The second type was an inconsistency within the students' repertoire of mathematical constructs. Wilson's (1990) study concentrated on the second type of inconsistency. She warned that although students may acknowledge inconsistencies, they may still resist conceptual change. (This resistance is a major component of Festinger's theory (Festinger, 1957; Wicklund & Brehm, 1976)). However, she encouraged
the use of inconsistencies in instruction, which stresses definitions, examples, and "logical reasoning".

The academic level of students has been a major factor in several studies regarding inconsistencies. Vinner and Dreyfus (1989) found that subjects from higher levels (academically) were able to provide more and better justifications for their responses than their lower level counterparts. Differences between low and high ability students also surfaced in Markovits, Eylon, and Bruckheimer's (1988) study of students' concept of function. They found that students of lower ability had more success with mathematical activities which were in "story" form than with mathematical activities which were essentially algebraic in nature. The highest ability students did not have this difficulty.

Regarding the issue of scale of axes for graphing, Goldenberg and Kliman (1988) conducted interviews with high school juniors and seniors at the precalculus level and eighth graders enrolled in first year algebra. All of the students were characterized "bright and articulate" and chosen for this reason to insure that the researchers would obtain usable data. The purpose of the study was to accumulate, through video and personal observation, a set of metaphors that the students would use for examining graphs regarding the scales of the axes. For example, one such metaphor involved changing scale as a way of "magnifying" the graph. More important than the metaphors which the researchers collected, was the method which they used to encourage students to react in the way that the researchers wanted them to react.

To insure themselves that the students would be very responsive, the researchers selected problems which they predicted would cause conflict and discrepancies in the students' knowledge of graphing. For instance, the researchers asked the students to provide information regarding the graphs of
parabolas, and the responses from the students were typical of what students "expect" the graphs of quadratic equations to look like: a curve with symmetry with respect to the line through its vertex. However, the researchers presented the students with a quadratic equation and the corresponding graph of the parabola on a set of axes scaled in such a way as to cause the graph to resemble a vertical line. This action caused conflict within the students' conceptual structure of quadratic equations and graphs of parabolas. The students were encouraged, during the interview, to examine their conceptions and make suggestions as to why such an event would occur. Some students, having determined that the choice of scale of the axes affected the graph of the function, still did not believe that the graph was valid. However, encouraging the students to examine their conceptions and to consider that perhaps their conceptions regarding the graphing of quadratic functions and functions in general were inadequate provided an environment for active learning.

**Summary**

Numerous misconceptions regarding the concept of function were found to exist with learners by examining their intuitions about the concept. Acknowledgement of this was profound; what was missing were examples of curricula and instructional methods to help alleviate these inconsistencies in students' mathematical knowledge. Yet this assistance would have to be such that it is accessible to students and teachers. At least three models were suggested: the Function Block, the Triple Representation Model, and the Parallel Axes Representation model. They are all to be commended, but they are not yet well known or even accessible and have not been proven to be useful on a global scale.
It is apparent from the review that little attention has been given to domain and range, two very important subconcepts of function. In order for each and every function to be defined, it must have a defined domain. Changing the domain of the function, changes the function itself and may do so to the degree that the mathematical object is no longer a function. These two concepts make up the foundation of the concept of function. It is proposed that by building up students' understanding of these two concepts, educators will be closer to insuring that students are on an upward path to acquiring the concept of function.

Besides the difficulty students showed in denoting the domain and range of functions as stated in the literature, at least two of the other misconceptions could be addressed by focusing on the subconcepts of domain and range. The first was the students' inability to make a connection between the graphical and algebraic representation of the same function. If the students' were aware of the role of the domain in defining functions, the pairing of the different representations of the concept might be less difficult for students. For example, a function that is defined only in the domain of values greater than or equal to zero would be expected to have a graph that appears to the right of the y-axis. However, the learner would have to be aware of the importance of knowing the domain of a function.

Secondly, some misconceptions focused on the fact that students expected graphs of functions to be acceptable, precise, and predictable. Having knowledge that the domain has a direct effect on the graph of the function would alert students to the fact that graphs of functions are sometimes "out of the ordinary", but with value placed on the role of the domain, "out of the ordinary" can become ordinary. The idea of an "acceptable" graph is also
related to students lack of consideration of the scale of axes on which the graph is produced. The omittance of such an important detail causes students to hold on to their conceptions regarding what certain graphs should look like. When students ignore the domain, range, and/or scales of axes for the graph of a function, they may be overlooking important details of the function represented by the graph. The graph of the function represents the functional relationship between the domain and range of the function, without sufficient encouragement to consider these three main aspects of the graph of a function, students are limited in their understanding of the concept of function. Furthermore, to assure that students will be appropriately guided in "reading" graphs correctly, the effects that the domain, range, and scales of axes has on the graph should be examined.

By adopting the theory of conceptual change, we can make these misconceptions beneficial in the learning process by inducing cognitive conflict through activities which will encourage students to deal with their misconceptions. Activities allowing for an examinations of misconceptions will assist students in dealing with their misconceptions. Secondly, graphing calculators are viable educational tools and are as powerful as the microcomputer. The graphing calculator would seem to be the most appropriate tool for dealing with misconceptions surrounding the concept of function. Yet these devices, although they are quite accessible, are rarely employed as a tool for conceptual change. Use of the graphing calculator forces one to examine appropriate domains, ranges, and scales of axes for graphing functions. In fact, the graphing calculator is perhaps the most feasible medium for conducting such activities. One should consider that one reason why the topic has been omitted in traditional algebra courses because an
efficient medium for examining appropriate domains, ranges, and scales of axes for graphing functions has not been readily accessible. Yet the use of the graphing calculator for this purpose has not been exploited in the teaching and learning of algebra.

Several trends surfaced from an analysis of the research reported. For the most part, the subjects participating in the various studies were not chosen randomly, but instead were volunteer or opportunity samples. Many of the samples were classes of students participating in special education projects, rather than "normal" classroom settings. This condition makes it almost impossible to generalize the research findings to a target population, given the special situations under which the studies were conducted. However, the constraints placed on researchers using school samples are often unavoidable. When students are assigned to classes by the school or students in colleges and universities register for classes by choice, it is difficult to have a random sample. In some cases, it may be possible to choose groups randomly to be assigned to treatment and control but few researchers denoted having this particular option. These studies cannot be dismissed without consideration for the information that the researcher has provided for the mathematical community. It is still important to consider the individual research results of studies that are limited in their generalizability. Such information can serve as a foundation for future research findings.

In most studies, questionnaires were administered to obtain information rather than to make comparisons between groups or test performance. Several of the researchers did not acknowledge the need to collect and/or report reliability and validity evidence for their instruments. In many cases, the instruments were used to make inferences regarding students' intuition,
misconceptions, and inconsistencies regarding the concept of function, yet the reader could not be assured that the instruments were in their best condition for completing the given tasks. Likewise, reliability estimates were not given for many of the measurements, thus there is no indication of the replicability of the study with the same or a comparable sample using the same instrument. In cases where the instrument was used for comparison, lack of adequate pretesting measures and use of nonrandom sampling techniques may have caused the power of statistical treatment to be weakened.
CHAPTER III
RESEARCH DESIGN AND METHODOLOGY

Research Objective

The purpose of this study was to investigate community college algebra students' concept of function regarding their understanding of the concepts of domain and range, their selection of appropriate domain, range, and scale for the axes for graphing polynomial functions, and their identification, construction, and definition of function. Of interest were the effects of employing the graphing calculator and a conceptual change assignment on the above factors. The study was designed to test the following null hypotheses:

1. Students' use of the graphing calculator during instruction will not affect their concept of function regarding their (a) application of the concepts of domain and range, (b) selection of appropriate domain, range, and scale for the axes for graphing functions, and (c) identification, construction, and definition of function.

2. Students' participation in a conceptual change assignment will not affect their concept of function regarding their (a) application of the concepts of domain and range, (b) selection of appropriate domain, range, and scale for the axes for graphing functions, and (c) identification, construction, and definition of function.

3. Students' use of the graphing calculator during instruction and the students' participation in a conceptual change assignment will not interact to affect their concept of function regarding their (a) application of the concepts of domain and range (b) selection of appropriate domain, range, and scale for the axes for graphing functions, and (c) identification, construction, and definition of function.
Conceptual Change Assignment

The Conceptual Change Assignment (see Appendix A) was designed by the researcher. The assignment was modeled after the accommodation model given in Nussbaum and Novick (1981, 1982). The assignment was implemented immediately before the instructor formally presented the topic of function and the concepts of domain and range. Part I of the assignment was designed to stimulate a learning environment which would encourage the learners to examine their conceptions before being presented with formal instruction regarding the concepts of domain, range, and scales of axes for graphing functions. By the time the students reached the topic of function including the concepts of domain and range, they had covered sections regarding graphing linear and quadratic equations. Part I of the assignment represented an opportunity for students to review these exercises, although they were unaware that they were formally dealing with the concepts of domain and range and scaling axes for graphing functions.

Part II of the assignment was designed to create conflict between the students’ conceptions regarding the concepts of domain, range, and selection of domains, ranges, and scales of axes for graphing functions. The exercises in Part I were typical exercises in algebra regarding the domain and range of linear, quadratic, and cubic functions and typical scales of axes for graphing functions. However, the exercises in Part II were designed to encourage conflict between what the learner already knew and what was being presented as mathematical possibilities. The inducement of conflict was planned to arise when the learner realized that the domains, ranges, and scales of axes did not
have to be typical, that the domains, ranges, and scales of axes used for the equations in Part I did not work for equations in Part II, and that the domain and range of the function dictated the design of the Cartesian coordinate system.

Part III of the assignment was designed to encourage and support the learner’s accommodation of the concepts of domain, range, and scales of axes for graphing functions. To support the fact that domains and ranges did not have to be typical, the learner was encouraged to view functions with different domains and ranges. To support the learner’s accommodation of the fact that the scales of axes affect how the graph will look, the learner was encouraged to view functions with different scales of axes.

**Domain / Range / Scale Instrument**

The Domain / Range / Scale Instrument (see Appendix B) was designed by the researcher and administered as a pretest and posttest. The objectives of the instrument and the relationship between the objectives and the hypotheses of the study are given in Table 1.

**Identification / Construction / Definition Instrument**

The instrument denoted by the researcher as the Identification / Construction / Definition Instrument was adapted from Vinner and Dreyfus (1989) (see Appendix C). The objective of administering the instrument was to determine students’ concept image and concept definition of the concept of function. One purpose of the instrument was to examine the students’ image of the concept and the students’ definition of the concept. The instrument consisted of seven items. Items one through four were identification items; items five and six were construction items. These first six items addressed the
<table>
<thead>
<tr>
<th>Items</th>
<th>Hypothesis</th>
<th>Objective: To test the student's ability to...</th>
</tr>
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<tbody>
<tr>
<td>1 - 4</td>
<td>(1-3)a</td>
<td>denote the domain and range of functions given algebraically.</td>
</tr>
<tr>
<td>5 - 7</td>
<td>(1-3)a</td>
<td>denote the domain and range of functions given graphically.</td>
</tr>
<tr>
<td>8 - 10</td>
<td>(1-3)a</td>
<td>identify specified function values for functions given algebraically.</td>
</tr>
<tr>
<td>11 - 13</td>
<td>(1-3)a</td>
<td>identify specified function values for functions given graphically.</td>
</tr>
<tr>
<td>14 - 15</td>
<td>(1-3)b</td>
<td>graph functions given algebraically with domain restrictions.</td>
</tr>
<tr>
<td>16</td>
<td>(1-3)b</td>
<td>graph functions given algebraically without domain restrictions.</td>
</tr>
<tr>
<td>17*</td>
<td>(1-3)b</td>
<td>identify functions which satisfy specified domain and range restrictions.</td>
</tr>
<tr>
<td>18 - 22**</td>
<td>(1-3)a,b</td>
<td>distinguish between the properties of the function and the properties of its graph.</td>
</tr>
<tr>
<td>23 - 26</td>
<td>(1-3) a,b</td>
<td>choose appropriate domain and range restrictions and reasonable scales to provide complete graphs of functions.</td>
</tr>
<tr>
<td>27</td>
<td>(1-3)a,b</td>
<td>recognize the effect that a domain restriction and scale of the axes may have on the graph of a function.</td>
</tr>
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</table>

Note: (1-3)a,b refers to hypotheses 1a, 1b, 2a, 2b, 3a, and 3b.

*This item appears in Yerushalmy (1991).

**These items appear in Markovits, Eylon, & Bruckheimer (1988).
students' image of the concept. The items required identification of functions and nonfunctions and the creation of a functional equation based on a written description of a functional relationship. The last item was a request to the students to provide a definition of the concept. All of these items addressed hypotheses 1c, 2c, and 3c. The students were asked to explain all of their response.

Pilot Study

The researcher conducted a pilot study during the Fall 1992 term using two community college algebra classes in a north-central Florida community college. The classes participated in activities involving graphing calculators and a conceptual change assignment regarding the concept of function.

After the implementation of the conceptual change assignment in one class, several changes were made regarding the content of the assignment before it was administered to the other class. First, the length of the assignment was decreased to allow time for whole group discussion at the end of the assignment. To facilitate this, two graphing activities which were repetitive were eliminated. However, an additional graphing activity was added to increase the diversity of the types of equations used in the assignment.

Furthermore, the researcher observed that the success of the assignment for students using the graphing calculator was enhanced by the students' previous experience with the graphing calculator. Because the students were already familiar with the operation of graphing calculators, they were able to concentrate on the assignment and not on the operation of the calculator. There were only a few questions asked regarding the operation of the calculator. All other questions and comments posed by the students were concerned with the activity. Observation of the students' responses to the
assignment revealed that they were able to relate their algebraic responses to questions regarding the domain and range of functions to the graph of the functions. Furthermore, the induction of conflict was successful given that the students reached a point of conflict during the activity and indicated that certain events did not correspond with what they "knew" to be true. After examining the situation, the students began to seek solutions for the problem. At the completion of the assignment, the students were able to discuss their written and graphical responses with knowledge of what they had learned by using the graphing calculator to complete the assignment. Prior use of the graphing calculator did not negatively effect the objective of the assignment.

Two instruments were administered to the two classes of students to examine the students' concept regarding their understanding of domain and range, their selection of appropriate domain, range and scale for the axes for graphing functions, and their identification, construction, and definition of function. To make changes to the instruments and gather evidence of validity, the instruments were not submitted to the classes simultaneously. The length of the Domain / Range / Scale Instrument was decreased because of time constraints of the class periods. To facilitate this, several exercises from the various objectives of the instrument were eliminated. Several questions on the Identification / Construction / Definition Instrument were determined to be too difficult for the students. They were replaced with comparable, but less difficult exercises. The objective of the exercises remained the same.

To insure that the content of the instruments reflected content domain of college algebra regarding the concept of function, the content validity of instruments was provided by using the literature as a guide, adoption of several items from other instruments which were related to the current topic and
an analysis of the content of the instruments by a mathematician and a mathematics educator. The researcher submitted the instruments to the two experts for their review and later met with them to discuss the content and objectives of the instruments. The two experts provided their professional assessment of the instruments which the researcher incorporated into the design of the instruments.

Reliability estimates for the final version of the Domain / Range/ Scale Instrument and the final version of the Identification / Construction / Definition Instrument were estimated by Kuder Richardson (KR-20) and were given as .84 and .77, respectively. The difficulty and discrimination indices for the individual items of the two research instruments are reported in Tables 2 and 3. The difficulty index for each item was given as the proportion of students who answered the item correctly. The discrimination index for each item was calculated by subtracting the proportion of students in the lower half of the group who answered the item correctly from the proportion of students in the upper half of the group who answered the item correctly.

Research Population and Sample

The population for this study consisted of students enrolled in college algebra at public community colleges. Community colleges, although rightfully deserving of a place in America’s educational system, are often omitted from the network of educational research, at least more so than elementary, middle, and high schools, 4-year colleges and universities. Wattenbarger (1989) reported that there were 1,300 community colleges in the United States and that 38% of all college and university students seeking bachelors degrees were graduates of a community college. Furthermore, 47% of all undergraduate
<table>
<thead>
<tr>
<th>Item</th>
<th>Difficulty</th>
<th>Discrimination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.231</td>
<td>.190</td>
</tr>
<tr>
<td>2</td>
<td>.154</td>
<td>.024</td>
</tr>
<tr>
<td>3</td>
<td>.615</td>
<td>.714</td>
</tr>
<tr>
<td>4</td>
<td>.154</td>
<td>.024</td>
</tr>
<tr>
<td>5</td>
<td>.154</td>
<td>.333</td>
</tr>
<tr>
<td>6</td>
<td>.308</td>
<td>.357</td>
</tr>
<tr>
<td>7</td>
<td>.462</td>
<td>.690</td>
</tr>
<tr>
<td>8</td>
<td>.692</td>
<td>.262</td>
</tr>
<tr>
<td>9</td>
<td>.769</td>
<td>.429</td>
</tr>
<tr>
<td>10</td>
<td>.846</td>
<td>.286</td>
</tr>
<tr>
<td>11</td>
<td>.308</td>
<td>.047</td>
</tr>
<tr>
<td>12</td>
<td>.154</td>
<td>.333</td>
</tr>
<tr>
<td>13</td>
<td>.462</td>
<td>.381</td>
</tr>
<tr>
<td>14</td>
<td>.231</td>
<td>.500</td>
</tr>
<tr>
<td>15</td>
<td>.308</td>
<td>.357</td>
</tr>
<tr>
<td>16</td>
<td>.462</td>
<td>.071</td>
</tr>
<tr>
<td>17</td>
<td>.538</td>
<td>.547</td>
</tr>
<tr>
<td>18</td>
<td>.154</td>
<td>.333</td>
</tr>
<tr>
<td>19</td>
<td>.615</td>
<td>.404</td>
</tr>
<tr>
<td>20</td>
<td>.926</td>
<td>.143</td>
</tr>
<tr>
<td>21</td>
<td>.615</td>
<td>.096</td>
</tr>
<tr>
<td>22</td>
<td>.385</td>
<td>.524</td>
</tr>
<tr>
<td>23</td>
<td>.077</td>
<td>.167</td>
</tr>
<tr>
<td>24</td>
<td>.154</td>
<td>.024</td>
</tr>
<tr>
<td>25</td>
<td>.154</td>
<td>.024</td>
</tr>
<tr>
<td>26</td>
<td>.308</td>
<td>.357</td>
</tr>
<tr>
<td>27</td>
<td>.385</td>
<td>.833</td>
</tr>
</tbody>
</table>
Table 3
Difficulty and Discrimination Indices
Identification / Construction / Definition Instrument

<table>
<thead>
<tr>
<th>Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difficulty</td>
<td>.615</td>
<td>.538</td>
<td>.615</td>
<td>.615</td>
<td>.615</td>
<td>.154</td>
<td>.385</td>
</tr>
<tr>
<td>Discrimination</td>
<td>.675</td>
<td>.875</td>
<td>.675</td>
<td>.350</td>
<td>.350</td>
<td>.250</td>
<td>.625</td>
</tr>
</tbody>
</table>

minority students are enrolled in community colleges (Koltai & Wilding, 1991). These statistics can be used to support the need to include the community college system in the mathematics education research base of undergraduate institutions. If this is not done, we as educators are taking the risk of omitting many learners from our research sample. These facts provide reason for an increased effort to include community colleges in more educational research.

More specifically, as society requires more persons who can work efficiently in a quantitative and technological environment, it will be necessary to focus attention on curricula and instruction which provides students the opportunity to develop a strong concept of function. One recommendation made in the report, “The Status of Science, Engineering, and Mathematics Education in Community, Technical, and Junior Colleges” is that community college faculty should attempt to maintain learning environments which enhances students’ learning of science, engineering, and mathematics (Koltai & Wilding, 1991). The graphing calculator actually could be used to play a part in all three disciplines, but in particular, this recommendation supports the need to include the use of technology in the learning and teaching of algebra. In this
report, the authors also reminded the reader that the community college is an active educational institution involved in the advancement of science and mathematics.

The research sample consisted of 128 college algebra students enrolled in a north-central Florida community college. There were eight classes of students: six treatment classes and two control classes. This community college was selected because the researcher had previously established a professional relationship with administrators and instructors. The researcher had also taught algebra and general mathematics courses at the college, and thus was familiar with the atmosphere of the learning environment, departmental policies, and general characteristics of instructors and students.

All classes were intact, thus random assignment of students to the classes was not possible. The students in the classes represented characteristics of the population of students enrolled in this particular community college and community colleges in general. The researcher chose to use day classes because the night classes are typically smaller classes and consist of students who are employed full-time during the day. Otherwise, there was no indication that the students in the sample were significantly different from all other students enrolled in college algebra. To aid with the description of the demographics and other descriptors of the students within the sample, each student in the study was requested to complete an information sheet (see Appendix D). The chi-square and one-way analysis of variance (ANOVA) procedures were employed to determine if there were significant differences among the groups in the sample regarding gender, age, race, enrollment status number of secondary and postsecondary mathematics courses, and grade point average.
There were more females participating in the study, however the arrangement of the students by gender in the four groups was nonsignificant (see Table 4).

### Table 4
Frequency and Percentage of Gender by Group

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator &amp; Assignment</td>
<td>30</td>
<td>10 (33%)</td>
<td>20 (67%)</td>
</tr>
<tr>
<td>Calculator</td>
<td>26</td>
<td>11 (42%)</td>
<td>15 (58%)</td>
</tr>
<tr>
<td>Assignment</td>
<td>33</td>
<td>11 (33%)</td>
<td>22 (67%)</td>
</tr>
<tr>
<td>Control</td>
<td>39</td>
<td>19 (49%)</td>
<td>20 (51%)</td>
</tr>
<tr>
<td>Total</td>
<td>128</td>
<td>51 (40%)</td>
<td>77 (60%)</td>
</tr>
</tbody>
</table>

Note: The chi-square value of 2.46 for 3 degrees of freedom with \( p = .482 \) was nonsignificant.

The average ages for the four groups in the study are presented in Table 5. The results from the ANOVA procedure revealed that age is significant for both the calculator factor and the assignment factor (see Table 6). The students who used calculators during the study were significantly younger than students who did not use calculators during the study. Furthermore, students who participated in the conceptual change assignment were significantly younger than students who did not participate in the conceptual change assignment.

The categorization of the students by race is presented in Table 7. Because of the number of categories with less than five students, employing the chi-square procedure was not appropriate. However, the information in the
Table revealed that a significant number of the students participating in the study were Caucasian/White for all four groups.

### Table 5
**Average Age by Group**

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Average Age in Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator &amp; Assignment</td>
<td>30</td>
<td>19</td>
</tr>
<tr>
<td>Calculator</td>
<td>26</td>
<td>21</td>
</tr>
<tr>
<td>Assignment</td>
<td>33</td>
<td>20</td>
</tr>
<tr>
<td>Control</td>
<td>39</td>
<td>25</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>128</td>
<td>21</td>
</tr>
</tbody>
</table>

### Table 6
**Analysis of Variance: Age**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III Sum of Squares</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator</td>
<td>1</td>
<td>179.49</td>
<td>6.94*</td>
<td>.0095</td>
</tr>
<tr>
<td>Assignment</td>
<td>1</td>
<td>259.76</td>
<td>10.04*</td>
<td>.0019</td>
</tr>
<tr>
<td>Calculator * Assignment</td>
<td>1</td>
<td>87.32</td>
<td>3.37</td>
<td>.0686</td>
</tr>
<tr>
<td>Model</td>
<td>3</td>
<td>607.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>124</td>
<td>3209.01</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *significant for p = .05
Table 7
Frequency and Percentage of Race by Group

<table>
<thead>
<tr>
<th>Race</th>
<th>Group</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>IV</td>
<td>Total</td>
<td></td>
</tr>
<tr>
<td>African American</td>
<td>1 (3%)</td>
<td>0</td>
<td>4 (12%)</td>
<td>3 (7%)</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Asian/Pacific Islander</td>
<td>1 (3%)</td>
<td>1 (4%)</td>
<td>2 (6%)</td>
<td>1 (3%)</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Caucasian/White</td>
<td>26 (88%)</td>
<td>23 (88%)</td>
<td>25 (76%)</td>
<td>34 (87%)</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>Hispanic</td>
<td>1 (3%)</td>
<td>2 (8%)</td>
<td>1 (3%)</td>
<td>1 (3%)</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>American Indian</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>1 (3%)</td>
<td>0</td>
<td>1 (3%)</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Note: I--Calculator & Assignment Group (n=30)  III--Assignment Group (n=33)
II--Calculator Group (n=26)                     IV--Control Group (n=39)

The enrollment status of the students by group is reported in Table 8. According to the results from the chi-square procedure, there were no significant differences between the groups regarding the number of students in the groups who attended college full-time and the number of students who attended college part-time.

The average grade point average of the students by group are reported in Table 9. The averages are based on the grade point averages reported by the students. Due to some circumstances, such as first-time enrollment, some students did not have grade point averages at the time of the study. For the reported grade point averages, the ANOVA results revealed that grade point average was significant for the factor of calculator (see Table 10). Students who used the calculator during the study had a significantly lower grade point average than students who did not use calculators during the study.
Table 8
Frequency and Percentage of Course Load by Group

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Full-Time</th>
<th>Part-Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator &amp; Assignment</td>
<td>30</td>
<td>28 (93%)</td>
<td>2 (7%)</td>
</tr>
<tr>
<td>Calculator</td>
<td>26</td>
<td>22 (85%)</td>
<td>4 (15%)</td>
</tr>
<tr>
<td>Assignment</td>
<td>33</td>
<td>31 (94%)</td>
<td>2 (6%)</td>
</tr>
<tr>
<td>Control Group</td>
<td>39</td>
<td>31 (79%)</td>
<td>8 (21%)</td>
</tr>
<tr>
<td>Total</td>
<td>128</td>
<td>112 (88%)</td>
<td>16 (12%)</td>
</tr>
</tbody>
</table>

Note: The chi-square value of 4.67 for 3 degrees of freedom with \( p = 0.197 \) was nonsignificant.

Table 9
Average Grade Point Average by Group

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Grade Point Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator &amp; Assignment</td>
<td>25</td>
<td>2.86</td>
</tr>
<tr>
<td>Calculator</td>
<td>20</td>
<td>2.85</td>
</tr>
<tr>
<td>Assignment</td>
<td>27</td>
<td>3.10</td>
</tr>
<tr>
<td>Control</td>
<td>37</td>
<td>3.12</td>
</tr>
<tr>
<td>Total</td>
<td>109</td>
<td>2.98</td>
</tr>
</tbody>
</table>

Note: Grade point average was based on a 4.0 scale.

The students were also asked to report the number of mathematics courses taken in high school and in college. The results of the tabulation of this information is given in Table 11. There were no significant differences between
the four groups regarding the number of mathematics courses taken by the students.

Table 10
Analysis of Variance: Grade Point Average

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III Sum of Squares</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator</td>
<td>1</td>
<td>1.59</td>
<td>5.18*</td>
<td>.0249</td>
</tr>
<tr>
<td>Assignment</td>
<td>1</td>
<td>.03</td>
<td>.10</td>
<td>.7535</td>
</tr>
<tr>
<td>Calculator * Assignment</td>
<td>1</td>
<td>.12</td>
<td>.42</td>
<td>.5198</td>
</tr>
<tr>
<td>Model</td>
<td>3</td>
<td>1.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>105</td>
<td>32.26</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *significant for p = .05

Table 11
Frequency of Mathematics Courses Taken by Group

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>High School</td>
</tr>
<tr>
<td>Calculator &amp; Assignment</td>
<td>30</td>
<td>98</td>
</tr>
<tr>
<td>Calculator</td>
<td>26</td>
<td>96</td>
</tr>
<tr>
<td>Assignment</td>
<td>33</td>
<td>108</td>
</tr>
<tr>
<td>Control</td>
<td>39</td>
<td>104</td>
</tr>
<tr>
<td>Total</td>
<td>128</td>
<td>406</td>
</tr>
</tbody>
</table>

Note: The chi-square value of 7.20 for 3 degrees of freedom with p = .066 was nonsignificant.
Instructional Materials

All college algebra students in the study used the same college algebra textbook: *Algebra for College Students* (Lial, Miller, & Hornsby, 1992), and all instructors used the same topical outline according to departmental policy. Before reaching the topic of function, the students studied linear and quadratic equations, during which time, they were instructed in the processes of paper and pencil graphing due to departmental policy. Sections of the text covered for the unit on functions during the study are given in Table 12.

<table>
<thead>
<tr>
<th>Text Section</th>
<th>Section Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.6</td>
<td>Introduction to Functions; Linear Functions</td>
</tr>
<tr>
<td>8.1</td>
<td>Algebra of Functions</td>
</tr>
<tr>
<td>8.2</td>
<td>Quadratic Functions; Parabolas</td>
</tr>
<tr>
<td>8.3</td>
<td>More About Parabolas and Their Applications</td>
</tr>
</tbody>
</table>

Each section listed objectives for the topic, followed by examples. Three definitions for the concept of function presented in the text were as follows:

1. A function is a relation for which each value of the first component of the ordered pair, there is one and only one value of the second component.

2. A function is a set of ordered pairs for which the first component is unique.

3. A function is a rule or correspondence such that each domain value is assigned to one and only one range value.
Graphing with technology in the above sections was not emphasized except in the graphing calculator sections at the end of chapter eight. All instructors were at liberty to decide on the method of presentation of the topics in the course, but were required to cover the course topics given by the departmental outline.

A Casio 7000G graphing calculator was made available during class to each student in the treatment classes using graphing calculators. They were taught how to operate the calculator by their instructors. A description of the calculator's capabilities as related to the study is given in Appendix E.

Instructors

Letters were sent to all but one instructor of college algebra to request their participation in the study. One instructor was omitted from the invitation due to her involvement in her own dissertation research. Six instructors who responded positively to the invitation agreed to participate in the study. To describe the characteristics of the instructors, they were requested to complete a profile information sheet (see Appendix F).

All six instructors had at least attained a master's degree in either mathematics or mathematics education. The average number of years of experience that each instructor had at the community college level was 18 years. In addition, on average, each instructor had taught college algebra 26 times throughout their years of teaching. All but one of the instructors were lacking experience using the graphing calculator to teach mathematics.

The instructors assigned to use graphing calculators in the study were given a graphing calculator during the term prior to the study and received individual instruction from the researcher in the use of graphing calculators as
required for the study. The instructors were instructed not to collaborate on issues regarding the research study. The researcher was available throughout the duration of the study to meet with these instructors to discuss issues regarding the use of the graphing calculator and answer questions regarding the capabilities of the graphing calculator. The instructors assigned to use the conceptual change assignment in the study met with the researcher to discuss the theory which was the basis of the assignment and the implementation of the assignment.

**Design of the Study**

A nonequivalent control group design was used to collect the data. Each group consisted of two classes of students. Treatment Group I was allowed to use graphing calculators on in-class assignments and participated in the conceptual change assignment. Treatment Group II had access to graphing calculators for in-class assignments, and Treatment Group III participated in the conceptual change assignment only. The Control Group was not provided with graphing calculators and did not participate in the conceptual assignment. This factorial (quasi-) experiment involved a 2X2 design: two levels of graphing calculator use, and two levels of in-class assignment. The objective was to determine the effect of the two independent variables, individually and interactively, on the dependent variable (posttest scores).

The first factor in the experiment was students' access and use of the graphing calculator during instruction. Interest in this factor developed from the need to determine if students could engage in concept development better with the use of the graphing calculator than without use of the graphing calculator. An implication is that if students perform better on the function concept
instruments with the use of the graphing calculator than without its use, then attention needs to be given to integrating this tool in mathematics education instruction.

The second factor in the experiment was participation in a concept development assignment focusing on students' (mis)conceptions. Interest in this factor resulted from the overwhelming existence of difficulties which students had with the concept of function. An implication is that if students perform better on the function concept instruments after participating in such an assignment, then attention should be given to focusing on students' (mis)conceptions before formally presenting concepts.

**Procedures**

All instruments were administered in class by the individual instructor. A schedule for the procedures is provided in Table 13.

<table>
<thead>
<tr>
<th>Group</th>
<th>Pretests</th>
<th>Treatment</th>
<th>Posttests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>Feb. 1993</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Treatment consisted of the use of graphing calculators during in-class assignments, explorations, and discussions and/or participation in a conceptual change assignment. Students were encouraged to use the graphing calculators freely, with and without prompting from the instructor. The goal was
to make the graphing calculator a normal part of the learning environment. The instructors were not provided with specific guidelines for incorporating the graphing calculator into instruction.

The conceptual change assignment was designed to focus on the concepts of domain and range and choosing of appropriate domains, ranges, and scales of axes for graphing functions. The students worked on the assignment prior to formal instruction on the topic.

To control for internal validity, the pretests were administered near the beginning of the semester and before the students were presented with the unit on function. The posttests were administered at the completion of the first half of the unit on functions by the instructor.

The analysis of covariance (ANCOVA) was used to examine mean differences between the four groups in the study. The Domain / Range / Scale Instrument and the Identification / Construction / Definition Instrument were used as pretests (covariates) and as posttests. With ANCOVA one can examine the individual scores as they vary around a regression line, rather than as they vary around the mean of the scores. An assumption of ANCOVA is that the regression lines for each group are parallel, that is, they have equal slopes. The slope of the regression line represents the relationship between the covariate and the dependent variable. ANCOVA uses the covariate as the independent variable to make adjustments to the dependent variable. The effectiveness of the covariate as an adjustment factor is determined by the variation in the relationship between the covariate and the dependent variable. The advantage of applying ANCOVA is to control for any pretreatment differences that might exist between the groups and to determine whether or not the treatment effect is significant.
In addition, the researcher was interested in the students' definitions and images of the concept of function. Distinct difficulties with the concept of function which the four groups exhibited in their responses to the instruments were also examined.

To complement the quantitative analyses of the instruments, the researcher also observed each group of classes. Of interest were the students' reactions to the use of the graphing calculator. The researcher also noted any other differences between the groups which were pertinent to the outcome of the study.
CHAPTER IV
RESULTS

Analysis for Domain / Range / Scale Instrument

Descriptive statistics for the pretest and posttest results for the Domain / Range / Scale Instrument are given in Table 14. At the institution where the study was conducted, a score of 70% is the lowest score that is considered as a sufficient score for a student to “pass” an examination. The overall posttest mean of 22.22% indicates that the groups were not successful with the instrument in regards to the percentage of items answered correctly.

Table 14
Pretest and Posttest Means and Standard Deviations for Domain / Range / Scale Instrument

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Pretest Mean %</th>
<th>S.D.</th>
<th>Posttest Mean %</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator &amp; Assignment</td>
<td>30</td>
<td>11.30</td>
<td>16.10</td>
<td>17.73</td>
<td>9.06</td>
</tr>
<tr>
<td>Calculator</td>
<td>26</td>
<td>7.57</td>
<td>11.38</td>
<td>25.23</td>
<td>16.02</td>
</tr>
<tr>
<td>Assignment</td>
<td>33</td>
<td>6.94</td>
<td>8.36</td>
<td>27.55</td>
<td>16.45</td>
</tr>
<tr>
<td>Control</td>
<td>39</td>
<td>1.18</td>
<td>2.88</td>
<td>19.15</td>
<td>12.36</td>
</tr>
<tr>
<td>Total</td>
<td>128</td>
<td>6.33</td>
<td>10.92</td>
<td>22.22</td>
<td>14.14</td>
</tr>
</tbody>
</table>

99
By employing the ANCOVA procedure, differences in the pretest results among the four groups were accounted for. The ANCOVA results (see Table 15) revealed that when considering the dependent variable, posttest scores on the Domain / Range / Scale Instrument, there is a significant interaction effect between the factors of calculator and assignment. The results of the covariate analysis warrant rejection of the following hypothesis:

Students' use of the graphing calculator during instrumentation and the students' participation in a conceptual change assignment will not interact to affect their concept of function regarding their application of the concepts of domain and range and their selection of appropriate domain, range, and scale for the axes for graphing functions.

Table 15
Analysis of Covariance: Domain / Range / Scale Instrument

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III Sum of Squares</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator</td>
<td>1</td>
<td>261.77</td>
<td>1.42</td>
<td>.2358</td>
</tr>
<tr>
<td>Assignment</td>
<td>1</td>
<td>8.69</td>
<td>.05</td>
<td>.8285</td>
</tr>
<tr>
<td>Calculator x Assignment</td>
<td>1</td>
<td>1867.68</td>
<td>10.13*</td>
<td>.0019</td>
</tr>
<tr>
<td>Pretest</td>
<td>1</td>
<td>577.45</td>
<td>3.13</td>
<td>.0793</td>
</tr>
<tr>
<td>Model</td>
<td>4</td>
<td>2719.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>123</td>
<td>22686.29</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *significant for p = .05

To analyze the interaction, the least square means procedure was employed (see Table 16). There were three significant group differences. There were significant differences between the calculator and assignment group and the calculator group, between the calculator and assignment group
and the assignment group, and between the assignment group and the control group when analyzing the dependent variable of posttest scores on the Domain / Range / Scale Instrument. The calculator group had a significantly higher mean score than the calculator and assignment group. The assignment group had a significantly higher mean score than both the calculator and assignment group and the control group.

<table>
<thead>
<tr>
<th>Adjusted Means</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>16.19</td>
<td>Calculator &amp; Assignment</td>
</tr>
<tr>
<td>24.97</td>
<td>Calculator</td>
</tr>
<tr>
<td>27.41</td>
<td>Assignment</td>
</tr>
<tr>
<td>20.22</td>
<td>Control</td>
</tr>
</tbody>
</table>

Note: *significant for p = .05

Analysis for Identification / Construction / Definition Instrument

Descriptive statistics for the pretest and posttest results for the Identification / Construction / Definition Instrument are given in Table 17. At the institution where the study was conducted, a score of 70% is the lowest score that is considered as a sufficient score for a student to “pass” an examination. The overall mean of 56.08% indicates that the groups were not successful with the instrument in regards to the percentage of items answered correctly.
Table 17
Pretest and Posttest Means and Standard Deviations for Identification / Construction / Definition Instrument

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Pretest Mean %</th>
<th>S.D.</th>
<th>Posttest Mean %</th>
<th>S.D.</th>
<th>Adj. M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator &amp; Assignment</td>
<td>30</td>
<td>39.47</td>
<td>27.48</td>
<td>51.63</td>
<td>24.70</td>
<td>49.57</td>
</tr>
<tr>
<td>Calculator</td>
<td>26</td>
<td>23.04</td>
<td>31.57</td>
<td>59.85</td>
<td>17.98</td>
<td>60.41</td>
</tr>
<tr>
<td>Assignment</td>
<td>33</td>
<td>25.58</td>
<td>27.17</td>
<td>52.42</td>
<td>19.96</td>
<td>52.58</td>
</tr>
<tr>
<td>Control</td>
<td>39</td>
<td>19.82</td>
<td>22.13</td>
<td>60.08</td>
<td>18.27</td>
<td>61.16</td>
</tr>
<tr>
<td>Total</td>
<td>128</td>
<td>26.56</td>
<td>27.51</td>
<td>56.08</td>
<td>20.47</td>
<td></td>
</tr>
</tbody>
</table>

Adj. M indicates adjusted mean.

By employing the ANCOVA procedure, differences in the pretest results among the four groups were accounted for. The ANCOVA results (see Table 18) revealed that when considering the dependent variable, posttest scores on the Identification / Construction / Definition Instrument, there is a significant main effect for the factor of conceptual change assignment. The students who participated in the assignment had a significantly lower group mean than the students who did not participate in the assignment.

The results of the covariate analysis warrant rejection of the following hypothesis:

Students’ participation in a conceptual change assignment will not affect their concept of function regarding their identification, construction, and definition of function.
Table 18

Analysis of Covariance: Identification / Construction / Definition Instrument

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III Sum of Squares</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator</td>
<td>1</td>
<td>107.94</td>
<td>.27</td>
<td>.6033</td>
</tr>
<tr>
<td>Assignment</td>
<td>1</td>
<td>2829.11</td>
<td>7.11*</td>
<td>.0087</td>
</tr>
<tr>
<td>Calculator x Assignment</td>
<td>1</td>
<td>39.90</td>
<td>.10</td>
<td>.7520</td>
</tr>
<tr>
<td>Pretest</td>
<td>1</td>
<td>2282.36</td>
<td>5.74</td>
<td>.0181</td>
</tr>
<tr>
<td>Model</td>
<td>4</td>
<td>4308.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>123</td>
<td>48926.82</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *significant for p = .05

The results of the covariate analysis warrant acceptance of the following hypotheses:

Students' use of the graphing calculator during instruction will not affect their concept of function regarding their identification, construction, and definition of function.

Students' use of the graphing calculator during instruction and the students' participation in a conceptual change assignment will not interact to affect their concept of function regarding their identification, construction, and definition of function.

Tests for Interactions

Analysis of covariance (ANCOVA) was employed to analyze the posttest scores from the Domain / Range / Scale Instrument and the Identification / Construction / Definition Instrument. For both dependent variables the assumption of parallel regression lines for the ANCOVA was checked. The purpose was to determine whether or not there existed an interaction between the pretests and the treatment. Both F-values were nonsignificant (see Tables 19 and 20), which implied that there were no interactions between the pretests...
and the treatments. Thus the regression lines are parallel (implying homogeneous slopes), and the use of ANCOVA in the analyses was appropriate.

Table 19
Pretest x Treatment Groups Interaction Test for Domain / Range / Scale Instrument

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III Sum of Squares</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groups</td>
<td>3</td>
<td>616.26</td>
<td>1.110</td>
<td>0.3475</td>
</tr>
<tr>
<td>Pretest</td>
<td>1</td>
<td>208.70</td>
<td>1.130</td>
<td>0.2902</td>
</tr>
<tr>
<td>Pretest x Groups</td>
<td>3</td>
<td>498.75</td>
<td>0.90*</td>
<td>0.4439</td>
</tr>
<tr>
<td>Model</td>
<td>7</td>
<td>3218.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>120</td>
<td>22187.54</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *nonsignificant for p = .05

Table 20
Pretest x Treatment Groups Interaction Test for Identification / Construction / Definition Instrument

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III Sum of Squares</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groups</td>
<td>3</td>
<td>2682.90</td>
<td>2.260</td>
<td>0.0848</td>
</tr>
<tr>
<td>Pretest</td>
<td>1</td>
<td>2227.52</td>
<td>5.630</td>
<td>0.0192</td>
</tr>
<tr>
<td>Pretest x Groups</td>
<td>3</td>
<td>1477.15</td>
<td>1.25*</td>
<td>0.2965</td>
</tr>
<tr>
<td>Model</td>
<td>7</td>
<td>5785.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>120</td>
<td>47449.69</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *nonsignificant for p = .05
Exploratory Analyses

Open-Ended Responses: Domain, Range, Scale

On the Domain / Range / Scale Instrument, the students were presented with the following scenario:

Three students were given identical function equations along with a domain for each. The students were instructed to graph the function. The graphs generated by the students were all different, but considered correct.

The students were asked to explain the reasoning behind the results. The responses to this open-ended question revealed that although the students had difficulty answering the other items on the instrument, there was a definite amount of conceptual understanding of the concepts of domain, range and scale present in the groups. The following is a list of examples given by several students in each of the four groups.

Treatment Group I: Calculator and Assignment Group

1. Different ranges, different view of graph.
2. They used different points for the range (y values).
3. Different scales for each student.
4. Because the domains were different.

Treatment Group II: Calculator Group

1. The students used different scales.
2. The scales of the graphs were different.
3. Each student may have used different units on their graph, giving the different looking graphs.
4. Because of the different ranges the student could of used.
Treatment Group III: Assignment Group

1. A function is a set of ordered pairs. The domain is the first numbers of the ordered pair. The function was the same but the graph of the numbers started on a different x point.

2. They may have used different scales, like one could have did the x axis by fives and the other one could have used tens.

3. The students used different coordinate systems with different numbers.

Control Group

1. The domains may have been different.

2. The scales used by each were different.

3. Depended on how they numbered their scale of x and y axis.

The justifications provided by the students indicate that there was some awareness on their part regarding the effect that different domains and scales for the axes have on the graph of a function. In addition, students using the graphing calculator were more likely to discuss “the view of the graph” than students not using the graphing calculator. The students who did not use graphing calculators were aware that different domains and different scales for the axes will cause graphs of the same function to be different, but for some their responses indicated that they were thinking of the graphs as different graphs rather than different views of the same graph.

Subjects' Definitions of the Concept of Function

The number of students who gave definitions for the concept of function and the number of students who gave academically acceptable definitions for
the concept are presented in Table 21. A definition was coded as academically acceptable if the definition contained the required components of the representation in which the definition was given. (See Appendix G for an example of acceptable and unacceptable definitions.) Only 70 students (55%) in the study gave a definition of the concept of function at the time of the administration of the posttests. Of the 70 students who gave definitions, only 42 (60%) gave academically acceptable definitions.

There were no significant differences between the groups regarding the number of students giving definitions of the concept of function. However, there were significant differences between the groups regarding the number of students giving academically acceptable definitions of the concept of function. The control group gave significantly more academically acceptable definitions than any other group. Nineteen of 39 students (48%) in the control group gave academically acceptable definitions. The calculator and assignment group followed with 13 out of 30 students (43%) giving academically acceptable definitions.

Also of interest were various definitions of the concept of function which the subjects in the study would provide (see Table 22). (See Appendix H for a list of examples of definitions given by category). The various representations of the concept of function found in the literature were used to guide the categorization of the definitions given by the students. Seventy-three percent of all of the students who gave definitions gave a definition in the ordered pair representation. The control group's definitions were significantly dominated by the ordered pair representation. Of the 24 students in this group who gave definitions, 23 (96%) gave a definition in the ordered pair representation.
### Table 21
Frequency of Students Giving Definitions by Group

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Definitions</th>
<th>Academically Acceptable Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator &amp; Assignment</td>
<td>30</td>
<td>18 (60%)</td>
<td>13 (43%)</td>
</tr>
<tr>
<td>Calculator</td>
<td>26</td>
<td>12 (46%)</td>
<td>4 (15%)</td>
</tr>
<tr>
<td>Assignment</td>
<td>33</td>
<td>16 (48%)</td>
<td>6 (18%)</td>
</tr>
<tr>
<td>Control</td>
<td>39</td>
<td>24 (62%)</td>
<td>19 (79%)</td>
</tr>
<tr>
<td>Total</td>
<td>128</td>
<td>70</td>
<td>42</td>
</tr>
</tbody>
</table>

Note: The Chi-Square value of 2.38 for 3 degrees of freedom with $p = .502$ was nonsignificant. The Chi-Square value of 11.73 for 3 degrees of freedom with $p = .008$ was significant.

### Table 22
Categorization of Definitions by Group

<table>
<thead>
<tr>
<th>Category Representation</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>Percent of All Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordered Pair</td>
<td>11</td>
<td>5</td>
<td>12</td>
<td>23</td>
<td>73%</td>
</tr>
<tr>
<td>Graph</td>
<td>7</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>16%</td>
</tr>
<tr>
<td>Equation</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>8%</td>
</tr>
<tr>
<td>Problem</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3%</td>
</tr>
</tbody>
</table>

Note: I--Calculator & Assignment Group (n=30) II--Calculator Group (n=26) III--Assignment Group (n=33) IV--Control Group (n=39)
The Chi-Square value of 10.86 for 3 degrees of freedom with $p = .013$ was significant.

The graphical representation was the next dominant definition category.
The calculator and assignment group gave more definitions in the graphical
representation than any other group. Thirty-nine percent of all students in the calculator and assignment group who gave definitions gave a definition in the graphical representation. The calculator group gave more definitions in the equation representation than any other group. Forty-one percent of all student definitions given by the calculator group were in the equation representation.

**Subjects' Images of the Concept of Function**

Of interest to the researcher were the subjects' images of the concept of function. The images were denoted in the students' explanations to items on the Identification / Construction / Definition Instrument. Table 23 contains the categorization of the images. The representations of the concept of function were used to guide the categorization of the students' images of the concept of function. The students' image of the concept of function was dominated by the vertical line representation. After examining the explanations given by the

<table>
<thead>
<tr>
<th>Category</th>
<th>Group I</th>
<th>Group II</th>
<th>Group III</th>
<th>Group IV</th>
<th>Percent of All Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Line</td>
<td>21</td>
<td>10</td>
<td>15</td>
<td>12</td>
<td>58%</td>
</tr>
<tr>
<td>Ordered Pair</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>19</td>
<td>30%</td>
</tr>
<tr>
<td>Other</td>
<td>2</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>12%</td>
</tr>
</tbody>
</table>

Note: I--Calculator & Assignment Group (n=30) II--Calculator Group (n=26) III--Assignment Group (n=33) IV--Control Group (n=39)

The chi-square value of 11.20 for 3 degrees of freedom with p = .011 was significant (excluding the "other" category).
students for identifying and constructing functions, it became evident that 58% of all of the students who gave images, gave the graphical representation in form of the vertical line representation.

The number of students in the calculator and assignment group who had a vertical line image of the concept of function were significantly higher than any other group. Eighty-eight percent of all of their explanations contained the ordered pair image. In fact, the vertical line representation and the ordered pair representation are so closely related that one can conclude that 88% of all the students' images of the concept of function were point-wise images. That is, the students' images depended on what happens with individual points on a graph or in a set of ordered pairs.

**Classroom Observations**

During the months of February and March, the researcher conducted classroom observations in both the treatment and control group classes. The researcher observed the classes before and during the time which the concept of function was being presented. All of the instructors in the study were informed that the researcher would observe individual classes on a random basis. The method of observation involved the completion of an observation form (see Appendix I) and the videotaping of several classes. The purpose of the observations was to ascertain the discourse in the classrooms during the course of the study which might complement the discussion of the quantitative analyses results.
Limitations of the Study

Several limitations must be considered when interpreting the results of the study. The instructors participating in the study were volunteers, and the individual roles each person played was assigned by the researcher. Although the instructors volunteered for the study, they were not given an indication of what the individual roles would be before agreeing to participate in the study. In addition, each instructor exhibited different teaching styles which could have affected the presentation of the content.

Due to the course enrollment policy of the institution where the study took place and the nature of the recruitment of instructor participants, random assignment of the research sample was not possible. The characteristics of the students participating in the study were not unlike college algebra students not participating in the study. However, it is questionable as to whether or not the pretests were able to control for all of the background variables regarding differences between the groups. As indicated, the students using graphing calculators in the study were significantly younger and had significantly lower grade point averages than students not using the tools. In addition, the students participating in the conceptual change assignment were significantly younger than students not participating in the assignment.

Because the students' performance on the instruments did not affect the students' grades, the students' may have lacked motivation to perform to a certain standard. The students were aware of this condition before agreeing to participate in the study. Furthermore, at the time of the
study, there did not exist any items which could be used as covariates which were standardized or which were not directly related to mathematics or which would complement the conceptual framework of the study.
CHAPTER V
CONCLUSION

Summary

The purpose of this study was to investigate community college algebra students' concept of function regarding their understanding of the concepts of domain and range, their selection of appropriate domain, range, and scale for the axes for graphing polynomial functions, and their identification, construction, and definition of function. A nonequivalent control group design was used to collect the data. Each group consisted of two classes of students. Treatment Group I was allowed to use graphing calculators on in-class assignments and participated in a conceptual change assignment. Treatment Group II had access to graphing calculators only for in-class assignments, and Treatment Group III participated in the conceptual change assignment only. The Control Group was not provided with graphing calculators and did not participate in the conceptual change assignment. This factorial (quasi-) experiment involved a 2X2 design: two levels of graphing calculator use and two levels of in-class conceptual change assignment participation. The objective was to determine the effect of the two independent variables, individually and/or interactively, on the dependent variable of posttest scores on the Domain / Range / Scale Instrument and the Identification / Construction / Definition Instrument.

One hundred twenty-eight community college algebra students participated in the study. Random assignment of the students to the four groups
was not possible, because the classes were intact. The instructors of the classes volunteered to participate in the study. At the beginning of the 1993 spring term, all of the students completed the Domain / Range / Scale Instrument and the Identification / Construction / Definition Instrument as pretests. The treatment, use of graphing calculators and participation in the conceptual change assignment, was introduced with the unit of study involving the concept of function. All of the students studied the same content for the unit. Students in the control group, however, were not allowed to use graphing calculators nor participate in the conceptual change assignment during the study of the unit.

ANCOVA was employed to examine the posttest scores from the Domain / Range / Scale Instrument and the Identification / Construction / Definition Instrument. For the Domain / Range / Scale Instrument, the ANCOVA results led to the rejection of the following null hypothesis:

Students' use of the graphing calculator during instruction and the students' participation in a conceptual change assignment will not interact to affect their concept of function regarding their application of the concepts of domain and range, their selection of appropriate domain, range, and scale for the axes for graphing functions.

The significant interaction effect was further examined by employing the least square means procedure. Results of the group comparisons revealed significant differences between the calculator and assignment group and the calculator group, between the calculator group and the assignment group and the assignment group, and between the assignment group and the control group when analyzing the dependent variable of posttest scores on the Domain / Range / Scale Instrument. The calculator group had a significantly higher mean score than the calculator and assignment group. The assignment group
had a significantly higher mean score than both the calculator and assignment group and the control group.

For the Identification / Construction / Definition Instrument, the ANCOVA results led to the rejection of the following null hypothesis:

Students' participation in a conceptual change assignment will not affect their concept of function regarding their identification, construction, and definition of function.

The students who participated in the assignment had a significantly lower group mean than the students who did not participate in the assignment.

The ANCOVA results led to the acceptance of the following null hypotheses:

Students' use of the graphing calculator during instruction will not affect their concept of function regarding their identification, construction, and definition of function.

Students' participation in a conceptual change assignment will not affect their concept of function regarding their identification, construction, and definition of function.

An exploratory analyses of the Identification / Construction / Definition Instrument revealed that only 55% of the students participating in the study were able to give a definition for the concept of function at the time of the posttest. Although there were no significant differences between the groups regarding the number of students who provided a definition for the concept of function, there were significant differences between the groups regarding the number of students who provided academically acceptable definitions for the concept. The control group gave significantly more academically acceptable definitions than any other group. The calculator and assignment group gave the second most academically acceptable definitions of the concept of function.
Seventy-three percent of all students who gave a definition for the concept of function, gave a definition in the ordered pair representation. The control group's definitions were significantly dominated by the ordered pair representation. The graphical representation was the next dominant definition category. The calculator and assignment group gave more definitions in the graphical representation than any other group. The calculator group gave more definitions in the equation representation than any other group.

The students' image of the concept of function was dominated by the vertical line representation. Fifty-eight percent of all of the students who gave explanations for identification items exhibited ownership of the graphical representation of the concept in form of the vertical line representation. The number of students in the calculator and assignment group who had a vertical line image of the concept were significantly higher than any other group. Eighty-eight percent of all of the students in this group had a vertical line image. The majority of all of the students had point-wise images of the concept of function. That is, the students' images depended on what happened with individual points on a graph or points in a set of ordered points.

Discussion

The interaction of the use of graphing calculators and participation in a conceptual change assignment was found to significantly affect the student's concept of function regarding application of the concepts of domain and range, and the selection of appropriate domain, range, and scale for the axes for graphing functions. Further analyses of the interaction revealed that students employing the graphing calculators only and the students participating in the conceptual change assignment only were more affected by the two separate
treatments than the students who employed the graphing calculators and participated in the conceptual change assignment. Almost every student and five of the six instructors in the study were unfamiliar with graphing calculators before participation in the study. None of the instructors had formally applied conceptual change theory in their teaching of mathematics. Therefore the graphing calculator and the conceptual change assignment were two new introductions into the calculator and assignment group. The students and instructor for this group were required to operate under circumstances involving two new issues into the classroom, while the calculator group and the assignment group only had to deal with one new introduction into the classroom.

In addition, the mean for the assignment group on the Domain / Range / Scale Instrument was significantly higher than the mean for the control group. The assignment was designed to focus attention on misconceptions that the students had about the concepts of domain and range and scales for graphing functions. Its design, based on findings from the literature, was purposely developed to assist the students to logically deal with their conceptions. The students were able to participate in the assignment using paper and pencil, which can be considered natural conditions for the community college mathematics classroom.

Overall the students in the study were not successful with the Domain / Range / Scale Instrument. This applies to both the treatment groups and the control group. The instrument was particularly designed to examine the students' concepts of domain and range and the students' understanding of the scale of the axes for graphing functions. The results from the analyses of the
Domain / Range / Scale Instrument indicate that the students had difficulty with the following:

1. Denoting the domain and range of functions given algebraically
2. Denoting the domain and range of functions given graphically
3. Identifying specific function values for functions given algebraically
4. Identifying specific function values for functions given graphically
5. Graphing functions given algebraically with domain restrictions
6. Graphing functions given algebraically without domain restrictions.
7. Identifying functions which satisfy specified domain and range restrictions
8. Distinguishing between the properties of the function and the properties of its graph
9. Choosing appropriate domain and range restrictions and reasonable scales to provide complete graphs of functions
10. Recognizing the effect that a domain restriction and scale of the axes may have on the graph of a function.

Participation in the conceptual change assignment was found to significantly affect the student's concept of function regarding their identification, construction, and definition of function. The group mean for students who participated in the assignment was significantly lower than the group mean for students who did not participate in the assignment. This indicates that the assignment was not appropriate to assisting students with identification, construction, and definition of function. In fact, one can conclude that the
assignment was a hindrance for students attempting to develop these abilities. Perhaps this is an indication that encouraging students to attend to their misconceptions and inconsistent knowledge is not useful for all activities in the mathematics classroom, and the length of time necessary for development of concepts should be greatly considered.

Further analyses of the Identification / Construction / Definition Instrument revealed that the students' definition of function was dominated by the ordered pair representation, and the students' image of function was dominated by the vertical line representation. The ordered pair representation and the vertical line representation for the concept of function both adhere to functions on a point-wise basis. The students did not seem to respond to the arbitrary nature of functions discussed by Even (1989, 1990, 1993). They viewed functions as collections of points or ordered pairs. The students may have exhibited such difficulties with the concept of function, because the students' concept of function was dominated by these two representations.

The difficulties with the concept of function that the students exhibited are consistent with the findings of several researchers (e.g., Dreyfus, 1990; Papakonstantinou, 1993). The students had difficulty mastering the definition of function and applying the concepts of domain and range. The students' concept image of function was dominated by a point-wise view of function. According to Hershkowitz, Arcavi, and Eisenberg (1987), students construct mental images of function according to the images that are emphasized in instruction. Indeed, during the observations, the researcher noted that the definitions which the instructors stressed during instruction was based on a function as a relation which met the condition of the first component in each ordered pair in the set having a unique value.
Because of the results of the analyses, the researcher is led to propose that the students in the study had assimilated the concept of function, but they had not accommodated the concept. According to Strike and Posner (1985) and Posner, Strike, Hewson, and Gertzog (1982), this indicates that a major conceptual change was not required by the students. This is also in agreement with Nussbaum and Novick (1981, 1982) who suggested that accommodation is a process which can be prepared for, but not scheduled or guaranteed.

Implications

Implications for Mathematics Curricula

The results reported in this study have several implications for mathematics curricula. First and foremost is the need to truly embed concept of function and functional thinking in mathematics curricula. This includes emphasis on the concepts of domain and range and emphasis on the graphical representation of the concept. The results indicate that as an underlying concept of mathematics, the introduction or reintroduction of the concept of function and the concepts of domain and range at the postsecondary level does not provide a foundation for the concepts. The concepts of domain and range are two of the most important concepts surrounding the concept of function, and also must be thought of as underlying concepts of mathematics. Attention also needs to be given to definitions of the concept of function. Care should be taken to determine the appropriate time to introduce formal definitions to students and to determine which definitions should be used in the curricula.
It is evident from the literature review that formal definitions may not be appropriate for introducing or reinforcing the concept of function. Dominant definitions, such as the ordered pair representation, which do not assist students in understanding the concept of function should be complemented with other definitions which prove to be more useful for conceptual understanding. This indicates a need to include multiple representations of the concept of function in the algebra curriculum. In addition, a student's ability to provide a definition of the concept of function does not necessarily indicate that the student has ownership of the concept. Attention should be given to developing and/or collecting appropriate curricula materials and activities which will assist educators in assessing students' understanding of the concept of function.

During the observations of the treatment and control group classes, the researcher noted that the students and instructors in the treatment classes using graphing calculators were more involved with relating the concepts of domain and range to the concepts of function than the other four classes. This can be attributed to the fact that use of the graphing calculator requires that the user attends to the conditions of the domain and range of the function and the design of the coordinate axes in order to obtain a usable graph of the function. Furthermore, the presence of the graphing calculator in these classrooms allowed the instructor to require the students to examine more graphs of functions and to examine the graphs of more complicated functions than those addressed in the outline for the unit on functions.
Implications for Mathematics Instruction

There are several implications for mathematics education resulting from the analyses in the study. First, instruction focusing on functions without focusing on the concept of function does not truly address the concept or the idea of function. Students who do not own the concept of function can not be expected to be able to use the graphing calculator its fullest benefit. They must have a basic understanding of the concept in order to understand the reasoning behind the operation of the graphing calculator. Otherwise, the student will see the graphing calculator as a machine for doing mathematics instead of a tool for learning. This is consistent with Yershulamy (1991) who suggested that stressing the visual alone through the use of a technological tool is not sufficient. The learner will still be inhibited if an understanding of the concept of function is not obtained.

The presence of the graphing calculator in the treatment classes did have an impact on instruction which the researcher did not address quantitatively. During observations, the researcher noted that when comparing the groups, instruction varied drastically. Before the study actually began, the researcher observed the classes and took notes regarding the discourse in the classroom between the instructor and the students and between students. The mode of instruction in all of the classes was the lecture mode, the teacher as the presenter and the students as passive learners. The instructor talk dominated the class with students only speaking to ask questions of the instructor or quietly ask questions of each other. The instructor remained at the front of the classroom and address the class as a whole when questions were asked. This was prevalent in all of the classes. After the introduction of the
graphing calculators into the four treatment classes, the researcher noted that
this classroom discourse changed. The instructors using the graphing
calculators were more willing to allow students to talk to each other and work
together. The direction of the instructors' questions changed from addressing
the whole class to addressing individual students and small groups of students.
The instructor was more certain that the students were attentive and were
actively engaged in the learning process. In addition, the students were more
willing to make suggestions to the instructors rather than only asking questions.
The phrase “what if we . . . ?” was heard more after the introduction of the
graphing calculators than at any other time and was rarely heard at all in the
other four classes.

The researcher was also able to note that the students using the
graphing calculators exhibited behaviors which indicated that the students had
a positive disposition toward using the graphing calculators. Exclamation
remarks such as, “wow, look at this graph” and “this is really cool” were often
heard in the classes where students were using graphing calculators. When
the students were becoming aware of the power of the graphing calculator,
several of them stated that they wish that they had graphing calculators when
they were learning about functions “the first time”. In fact, the students were so
interested in the graphing calculators that they requested permission to use the
graphing calculators in class and on the instructors’ tests even after the study
was complete.

In regards to assessment, the instructors using the graphing calculator
were able and willing to walk around the classroom and observe the students
working and examine the work produced by the student. The previous lecture
mode style of instruction did not allow for such observations and examinations.
Furthermore, the students were comfortable with sharing the results on their graphing calculators with students who, at the particular moment, were not successful with working with their own graphing calculators. This action allowed the students to assist each other with assessing their own learning.

The use of the conceptual change assignment indicates that a planned focus on students' misconceptions regarding the concepts of function, domain, and range can aid students' in successfully dealing with their misconceptions. However, as indicated by the results of the study, caution should be taken regarding the selection of misconceptions or difficulties which are addressed.

**Recommendations**

The results from the study indicate a wide range of recommendations for future research. Future research should examine methods by which the concept of function can receive the emphasis that it needs in order to become an underlying concept for learners of mathematics. This research needs to address ways that the concept can be more readily embedded in the conceptual structure of the learner and not just the foundation of the discipline of mathematics. This issue should be focused on the use of technology to aid mathematics educators in reaching this goal.

Future studies are needed which can address the impact of graphing calculators for teaching and learning algebra over an extensive period of time. These types of studies are needed to trace patterns in teaching and learning and to examine the importance of existing patterns and different behaviors while students are learning. Questions to be asked are: How does the use of the graphing calculator during a sequence of algebra courses affect the way
students learn algebra? What changes are quantitatively and/or qualitatively evident when students are using graphing calculators to study algebra? How does the use of the graphing calculator affect the instructor's assessment of students' learning of algebra? Are graphing calculators beneficial for teaching and learning all algebra concepts?

Furthermore, issues regarding the use of graphing calculators in presecondary and secondary education should be addressed. This study points to the need to address the concept of function before the postsecondary level. The students' unfamiliarity with such an important learning tool suggests that graphing calculator use in the lower grades is warranted. However, questions regarding how graphing calculators can and should be used in these grades are warranted.

Regarding curricula materials for mathematics, research needs to be conducted which explores which, if any, curricula materials are needed which would enhance students' learning when using the graphing calculator as a tool. In addition, other studies which incorporate ideas and philosophies outside of the discipline of mathematics education are needed. There is much that can be learned from research in science education regarding the development of concepts. From research in psychology regarding the learner's thought processes, mathematics educators may be able to develop methods by which instruction can focus more on how the student learns and how one's learning style changes over time to affect one's learning of mathematics. From research in sociology, mathematics educators may be able to address the meaning of interactions in the classroom. This is especially important when technology is present in the classroom. Questions regarding how the technology affects the
social patterns of the classroom should be addressed, as this may affect the effectiveness of the use of the technology.
APPENDIX A
CONCEPTUAL CHANGE ASSIGNMENT
Conceptual Change Assignment

Part I:

1. Answer both questions for each equation:

   What are all of the possible real numbers that can be substituted for x?
   What are all of the possible real numbers that can result for y based on substituted values of x?

   a. \( y = \frac{x^4}{x - 15} \)

   b. \( y = -x^2 \)

   c. \( y = \frac{1}{3} x^3 \)

2. Graph each of the above equations on three separate rectangular coordinate systems. Record your graphs in the space below.
Part II:

3. Graph each of the equations below on three separate rectangular coordinate systems. Record your graphs in the space below.

   a. \( y = 300x \)  
   b. \( y = x^2 + x - 60 \)  
   c. \( y = x^3 - 55 \)

Part III:

4. Graph the following equations. Use the denoted descriptions for labeling the axis. Record your graphs on the back of the previous page.

   \begin{array}{ccc}
   
   \text{x-axis/scale} & \text{y-axis/scale} \\
   \hline
   \text{a. } y = -4x^2 - 5 & -100, 100/? & -4000, 4000/? \\
   \text{b. } y = -x^3 - x + 5 & -50, 50/5 & -90, 90/10 \\
   & -50, 50/5 & -90, 90/3 \\
   & -50, 50/2 & -90, 90/10 \\
   & -50, 50/? & -90, 90/? \\
   \end{array}
APPENDIX B
DOMAIN / RANGE / SCALE INSTRUMENT
Determine the domain and range of the following functions.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( y = \sqrt{x} )</td>
<td></td>
</tr>
<tr>
<td>2. ( y = \frac{x}{x^2 - 4} )</td>
<td></td>
</tr>
<tr>
<td>3. ( y = 17x^3 + x^2 - x )</td>
<td></td>
</tr>
<tr>
<td>4. ( y = \sqrt{-x} )</td>
<td></td>
</tr>
</tbody>
</table>

5. [Graph of a function]

6. [Graph of a function]

7. [Graph of a function]
Identify the specified function values for the following functions.

8. \( f(x) = -7x^2 + x + 4 \)
   \[ f(3) = \quad \]

9. \( f(x) = \frac{4}{5}x^3 - \frac{2}{5} \)
   \[ f(5) = \quad \]

10. \( f(x) = 6\sqrt{x} \)
    \[ f(9) = \quad \]

11. \( f(-8) = \quad \)

12. \( f(12) = \quad \)

13. \( f(0) = \quad \)
Graph the following functions on the back of the previous page.

14. \( f(x) = x^4 - 4 \) for \( x \leq 0 \)

15. \( f(x) = 2x^2 - 5 \) for \(-3 \leq x \leq 4\)

16. \( f(x) = \sqrt{x} \)

17. Indicate the graph that represents a function with domain \( 2 \leq x \leq 6 \) and range \(-1 \leq y \leq 4\). Choose one and explain your choice.

![Graphs]

Explanation: 

18. Here are four possible descriptions of the following line. Choose one and explain your choice.

\[ y = 5x \]
\[ y = x \]
\[ y = \frac{1}{5}x \]
\[ \text{cannot be determined} \]

![Line Graph]

Explanation: 

---

Item 17 was reprinted with permission from *The Ideas of Algebra, K-12(1988)* by the National Council of Teachers of Mathematics.
The following are four graphical descriptions of linear functions in systems with different scale units. Choose the function that each line describes. Explain your choice for each case.

19. 

![Graph of a line with scale units]

- $y = \frac{1}{2} x$
- $y = 2x$
- $y = 4x$
- $y = 10x$

Explanation:

20. 

![Graph of a line with scale units]

- $y = \frac{1}{2} x$
- $y = 2x$
- $y = 4x$
- $y = 10x$

Explanation:
21. 

\[ y = \frac{1}{2} x \]
\[ y = 2x \]
\[ y = 4x \]
\[ y = 10x \]

Explanation: 

22. 

\[ y = \frac{1}{2} x \]
\[ y = 2x \]
\[ y = 4x \]
\[ y = 10x \]

Explanation: 

Determine an appropriate set of axes on which the following functions can be completely graphed. A complete graph is one which shows all relevant information. You do not need to provide the graph.

<table>
<thead>
<tr>
<th>x-axis</th>
<th></th>
<th>y-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>maximum</td>
<td>scale</td>
</tr>
</tbody>
</table>

23. $y = 3x^2 - x + 8$
24. $f(x) = 64x - 144$
25. $f(x) = -27x^2$
26. $f(x) = -x$

27. Three students were given identical function equations along with a domain for each. The students were instructed to graph the function. The graph generated by the students were all different, but considered correct. Explain why the graphs looked different.
APPENDIX C
IDENTIFICATION / CONSTRUCTION / DEFINITION INSTRUMENT
Identification/Construction/Definition  Instrument

Does there exist a function whose graph is:

1.

\[ \begin{array}{c}
\text{Yes} \\
\text{No}
\end{array} \]

Explanation: 

2.

\[ \begin{array}{c}
\text{Yes} \\
\text{No}
\end{array} \]

Explanation:
4. Does there exist a function that assigns every number to itself?

Yes. What is the equation for this function?

No

Explaination:

5. Does there exist a function that assigns every number to its square?

Yes. What is the equation for this function?

No

Explaination:
6. Does there exist a function that assigns every number, except 0, to its square and to 0 it assigns 1?

   ____ Yes. What is the equation for this function? ____________________________

   ____ No

Explanation: _____________________________________________________________

    _______________________________________________________________________

7. What is function? _______________________________________________________

    _______________________________________________________________________

    _______________________________________________________________________

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APPENDIX D
STUDENT INFORMATION SHEET

Please answer the following questions as completely as possible.

1. What are the last four digits of your social security number? _________

2. What is your age? ___

3. What is your sex? (Check one)
   ____ Male
   ____ Female

4. What is your race? (Check one)
   ____ African American/Black
   ____ Asian/Pacific Islander
   ____ Caucasian/White
   ____ Hispanic (including Mexican, Cuban Puerto Rican, South or Central American)
   ____ Native American/American Indian
   ____ Other -- Please specify: ____________________________

5. What is your marital status? (Check one)
   ____ Married
   ____ Never Married
   ____ Divorced, Separated, Widowed

6. What is your current grade point average? ______

7. What is your academic major? ____________________________
8. Are you seeking a degree? (Check one)
   ___ Yes -- What degree are you seeking? ________________________________
   ___ No

9. Are you attending college full-time? (Check one)
   ___ Yes
   ___ No

10. What mathematics courses did you take in high school?
    (Check all that apply)
     ___ Pre-Algebra
     ___ Algebra
     ___ Geometry
     ___ Trigonometry
     ___ Calculus
     ___ Business Math
     ___ Statistics/Probability
     ___ Discrete Mathematics
     ___ Other -- Please specify: ______________________________________
11. What mathematics courses have you taken in community college or college up to the current date? (Check all that apply)
   ____ Elementary algebra
   ____ General mathematics
   ____ Statistics/Probability
   ____ Geometry
   ____ Calculus
   ____ College algebra 1
   ____ College algebra 2
   ____ Trigonometry
   ____ Other -- Please specify: ________________________________

12. Have you had previous experience using a graphing calculator?
   (Check one)
   ____ Yes
   ____ No
APPENDIX E
CASIO FX 7000G GRAPHING CALCULATOR PROFILE

(Adapted from Casio fx-7000G Owner’s Manual)

The calculator has four keys which can be used to initiate and work on graphing. They are described as follows:

1. Mode Display/Plot Key - The mode display key is used to establish confirmation of a chosen mode. The Plot Key is used to plot points on the coordinate system.

2. Graph/Trace Key - Pressing Graph prompts the calculator to prepare to accept an algebraic equation for graphing. The Trace Key allows one to control a cursor which can be directed to trace a graph and display the components of the dependent and independent variable.

3. Range/Factor Key - The Range Key is used to set or change the domain and range of the viewing rectangle. The Factor Key is used to magnify or reduce the view of the graph.

4. Graph-text/Clear Screen Key - The Graph-text Key allows the user to switch between the graph and the text. The Clear Screen Key clears the viewing rectangle of all graphs.
APPENDIX F
INSTRUCTOR INFORMATION SHEET

Please answer the following questions as completely as possible.

1. What is your name?______________________________________

2. a. What is the highest academic degree you have attained?
    _________________________________________________________
    b. What is the subject area of this degree?____________________

3. How many years of teaching experience do you have at the following levels?
   ___Elementary School
   ___Middle School
   ___High school
   ___Community College
   ___College/University

4. How many times have you taught college algebra I at this community college?___

5. What is your philosophy of teaching mathematics:______________________________
    _______________________________________________________________________
    _______________________________________________________________________
    _______________________________________________________________________

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6. Have you ever used a graphing calculator as part of instruction?
   ___ Yes. Under what conditions? ________________________________
   ________________________________
   ___ No. Why not? ________________________________
   ________________________________

7. How would you best define a mathematical function? __________
   ________________________________
   ________________________________
APPENDIX G
EXAMPLES OF ACADEMICALLY ACCEPTABLE AND UNACCEPTABLE FUNCTION DEFINITIONS

Acceptable:

1. A function is a point of x and y values when and only when there is only one y value for every x value.
2. For every independent variable x, there is only one dependent variable y.
3. A relation where there are no two x values which have the same y value.
4. A graph where it has to two same x coordinates.
5. A set of points where the x value has its own value, but the y value can be shared.

Unacceptable:

1. Any graph that a vertical line can be drawn through.
2. An invisible line that touches another invisible line once vertically.
3. When there are two or more coordinates that equal x.
4. A function is any number that does not go through the x term more than once.
5. A formula following the pattern ax + by = c, where no two points share the same x coordinate.
Appendix H
Examples of Function Definitions by Category

Set of Ordered Pairs:
1. For every value of x, there is one and only one y value.
2. A relation where there are no two x values which are the same.
3. When each y is not assigned the same x value.

Graph:
1. A graph that when tested by the vertical line test is only crossed once.
2. A function is a line that if a person used a vertical line test would only cross x only once.
3. A function is a line that you only hit once when you do the vertical line test.

Equation:
1. A formula following the pattern ax + by = c, where no two points share the same x coordinate.
2. Equation of line f(x).
3. Equation related to another equation.

Problem:
1. The way something is solved.
2. A function is any problem that deals with domain and range.

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APPENDIX I
OBSERVATION FORM

TEACHER

OBSERVATION #______

DATE_________________

TIME_______

TOPIC_________________________________

BOOK SECTION______

#STUDENTS_______

CLASS AGENDA:
1.
2.
3.

MODES OF INSTRUCTION
1.
2.
3.

TEACHER-STUDENT INTERACTION:

STUDENT-STUDENT INTERACTION:

QUESTIONS ASKED:

METHODS OF ASSESSMENT/EVALUATION:
1.
2.
3.

COMMENTS SPECIFIC TO TECHNOLOGY/GRAPHING CALCULATOR:
REFERENCES


Markovits, Z., Eylon, B., & Bruckheimer, M. (1988). Difficulties students have with the function concept. In The ideas of algebra, K-12 (pp. 43-60). Reston, VA: NCTM.


BIOGRAPHICAL SKETCH

Thomasenia Lott Adams, the youngest of eight children, was born on February 6, 1965. She attended the public schools of Saluda, South Carolina. In 1987, she graduated with honors from South Carolina State College with a Bachelor of Science degree in mathematics. Upon receiving the McKnight Doctoral Fellowship, she moved to Gainesville, Florida, and began her graduate work at the University of Florida. In 1988, she married Larry Vanoy Adams. A master's degree in instruction and curriculum from the University of Florida was completed in 1990. In 1993, she completed the doctorate in instruction and curriculum from the University of Florida. She joined the faculty of the Department of Instruction and Curriculum of the University of Florida in 1993.
I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Eleanore L. Kantowski, Chair
Professor of Instruction and Curriculum

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

M. David Miller
Associate Professor of Foundations of Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

John Gregory
Professor of Instruction and Curriculum

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Donald Bernard
Associate Professor of Instruction and Curriculum

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Li-Chien Shen
Associate Professor Mathematics
This dissertation was submitted to the Graduate Faculty of the College of Education and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August 1993

[Signature]
Dean, College of Education

[Signature]
Dean, Graduate School