DISPERSION OF 5 MHz ZERO SOUND IN SUPERFLUID $^3$He NEAR $T_c$ IN MAGNETIC FIELDS

By

ROBERT FRANK BERG

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Abstract of Dissertation Presented to the Graduate School of the University of Florida in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

DISPERSION OF 5 MHZ ZERO SOUND IN SUPERFLUID $^3$He NEAR $T_c$ IN MAGNETIC FIELDS

By

Robert Frank Berg

December 1983

Chairman: Professor Gary G. Ihas
Major Department: Department of Physics

Measurements of the attenuation and phase velocity changes near $T_c$ of 5 MHz ultrasound have been made in the A and Al superfluid phases of $^3$He at 2.05, 9.15, 18.18, 21.53, and 31.12 bar. All measurements were performed in the low fields of either 14.3, 28.6, or 42 millitesla with the field direction parallel to the sound direction. The finite field is important in four ways. First, it enabled the anisotropic superfluid phases to be observed at arbitrarily low pressures. Second, it caused a definite orientation of the order parameter $\lambda$-vector with respect to the sound direction. Third, it allowed observation of the narrow Al phase. And fourth, the exploitation of the $^3$He NMR shift in the superfluid gave precise, in situ thermometry for the sound measurements. This is perhaps the first extensive use of superfluid $^3$He as its own thermometer.
The attenuation peak heights and the drops in the phase velocity below $T_c$ scale with pressure according to the predictions of the collisionless theory for zero sound in the A-phase, while the absolute attenuations observed are about one-quarter of that expected based on this theory. The location of the attenuation maximum corresponds to the expected clapping mode maximum at the lowest pressure but is significantly colder than that predicted at all higher pressures.
SECTION 1

INTRODUCTION

Much of the progress made in man's general understanding of the physical world has come about by conceptually separating the effects peculiar to our immediate environment from the more universal aspects of nature. The force of friction is such a "particular" effect -- realization of this was necessary before a Newton could construct a valid set of mechanical laws. Likewise, the fact that the vacuum is a more fundamental physical environment than a mixture of 80% nitrogen and 20% oxygen at 1.01 bar pressure has spurred the unceasing development of ever better vacuum pumps. Our usual environment, at a temperature of about 300 K above absolute zero, can be interpreted as an ocean of thermal excitations which affects all physical measurements made in it. Many physical phenomena are very insensitive to room temperatures but some of course are not. Indeed, many of nature's characteristics are completely drowned out at 300 K and were only discovered when the necessary low temperatures were reached. It is this ignorance of nature's hidden beauties at low temperatures that has led physicists to build ever better refrigerators.

The condensation of some form or another of matter has often served as a milestone for low temperature physics. Examples are the liquifaction of air (~80 K) in the latter part of the nineteenth century and the liquifaction of helium (4.2 K) in 1908. If one generalizes the
concept of condensation to include ordering of any kind, then the existence of "superfluidity" can be viewed as a condensation in momentum space. Such condensations of $^4\text{He}$ atoms ("helium-II") and conduction electrons ("superconductivity") were seen after the first liquifaction of helium.

In general, a superfluid system contains constituent particles (atoms or electrons) which are correlated over a distance much longer than the average interparticle distance. This "connectedness" can cause strange effects on a macroscopic scale, such as flow with absolutely zero friction. Although some macroscopic effects of superfluids can be tied together with a purely phenomenological theory (e.g. the London equations for superconductivity), a microscopic theory represents a deeper understanding. A successful superfluid microscopic theory did not exist until Bardeen, Cooper, and Schrieffer (BCS) (1957) and Bogoliubov (1958) derived the now standard "BCS" pairing picture for superconducting electrons. This theory of electrons in a metallic lattice, based on the inherently quantum mechanical nature of fermions, soon caused speculation about superfluidity in other Fermi systems. The chief such candidate was liquid $^3\text{He}$, the rare isotope of helium which has a nuclear spin of $1/2$, and the early 1960's saw theoretical predictions of superfluidity in $^3\text{He}$ (cf. Anderson and Morel, 1961; Balian and Werthamer, 1963).

The actual critical temperature $T_C$ for the transition from the normal to the superfluid state is not easy to calculate so it was not until its (accidental) discovery in 1971 (Osheroff et al., 1972a) that superfluid $^3\text{He}$ made itself known in the laboratory. The decade of the 1970's has seen an order of magnitude decrease in the routine minimum
temperatures of very low temperature laboratories. As the phase diagram of Figure 1 shows, the temperature range of .3 to 3 mK contains five different phases of $^3$He: the paramagnetic and nuclear spin-ordered solid phases as well as the "normal" and (two) superfluid liquid phases. The designations "A" and "B" of the two types of superfluid are of historical importance only. The A-phase is located in the triangle with its lower vertex at the "polycritical" point. If even a modest magnetic field is applied to these superfluid states, the zero-field picture of Figure 1 must be generalized to the three dimensional plot of Figure 2. Just above zero field, the triangle of A-phase extends itself to interpose itself between the normal and B-phase at all pressures. This region of A-phase expands until, at fields on the order of 6 kGauss, the B-phase is entirely suppressed. Finite magnetic fields also cause the appearance of the "Al" phase, nature's only magnetic superfluid. This part of the phase diagram has the shape of a very narrow wedge inserted between the A and normal phases. To see the field effects more clearly, Figure 3 takes a slice of the phase diagram at nonzero field and distorts the temperature axis to exaggerate the widths of the A and Al phases. The dotted lines show the shape at zero field. Notice how the warmest superfluid transition temperature $T_{c1}$ is raised above the zero field transition at $T_{c}$.

This thesis is a description of a series of 5 MHz ultrasound measurements made in the A and Al phases at pressures of 2.05, 9.15, 18.18, 21.53, and 31.12 bar. The application of finite magnetic fields of 14.3, 28.6 and 42 millitesla caused the slivers of A and Al phases to extend below the polycritical point thus allowing measurements spanning most of the pressure difference between zero and melting pressures. All
Figure 1

(P, log T) phase diagram for $^3$He in zero magnetic field. Superfluid phase transition lines are from "Helsinki" scale (Alvesalo et al., 1981). Ordered solid transition line is after Shigi et al. (1983).
FIGURE 1. P, log T diagram of $^3$He
FIGURE 2. P,T,B phase diagram of $^3$He.
of the measurements reported here were done at temperatures within 2% of $T_c$ since most of the important features of 5 MHz ultrasound dispersion in the superfluid occur in this narrow range. Also, for temperatures near $T_c$, the Ginzburg-Landau (e.g. see Landau and Lifshitz, 1969, ch. 14) theory which describes second-order phase transitions in terms of an "order parameter" expansion is applicable, facilitating interpretation of experimental results. The apparatus for this experiment were originally designed for an ultrasound investigation of single crystals of magnetically ordered solid $^3$He. Problems of heating and uncontrolled solid growth redirected efforts to a liquid experiment for which the apparatus were suited.

Some of the earliest investigations of $^3$He used ultrasound at 5, 15, and 25 MHz (Paulson et al., 1973) and 10 MHz (Lawson et al., 1973, 1974). The qualitative features observed in these and later measurements are a sharp peak in the attenuation just below $T_c$ accompanied by a small but swift decrease in the sound velocity. These features are now understood to result from excitation of resonant modes of the superfluid order parameter which, due to its complex tensor form, has modes of a type unique in nature.

The propagation of sound in the superfluid phase should first be placed in the perspective of sound propagation in the normal liquid phase. The velocity of sound more than doubles as the pressure is raised from zero to melting pressure (see Figure 4 constructed from data tabulated by Wheatley, 1975). As the temperature is lowered the velocity and attenuation undergo changes connected with the finite lifetime $\tau$ of the normal liquid excitations ("quasiparticles") characteristic of viscoelastic behavior (Rudnick, 1980). There is an attenuation peak
centered where the sound frequency is comparable to $\tau (\omega \tau \sim 1)$ accompanied by a small upward shift in the sound velocity, as shown in Figure 5 for 20 MHz sound at 29 bar (after Ketterson et al., 1975). The slopes on the cold and warm sides of the attenuation peak are proportional to $T^2$ and $T^{-2}$ respectively. Sound propagation in $^3$He at frequencies and (cold) temperatures such that $\omega \tau >> 1$ is termed zero sound and represents the propagation mode predicted by Landau (1957) on the basis of the normal Fermi fluid kinetic equation in the collisionless limit. The sharp changes in attenuation and velocity just below the superfluid transition temperature $T_c$ at $\sim 2.5 \text{ mK}$ can be seen in Figure 5 also.

The effects of strong fields on the attenuation of A-phase sound at multiples of 14.7 MHz at low pressure have been measured recently in France (Avenel et al., 1981; Piche et al., 1982). These measurements, chiefly concerned with nonlinear effects well below $T_c$, find that the clapping resonance occurs at the predicted temperature at low pressures, in agreement with the data reported here.

Using low fields to orient the superfluid, Ketterson et al. (1975) performed a series of 20 MHz ultrasound velocity and attenuation measurements at pressures between 17 and 28 bar which has served as a qualitative benchmark for my experiment. The measurements described here also used an orienting field but this field was strong enough and the temperature resolution fine enough to examine the A and Al phases down to 2 bar.

The sound attenuation associated with the Al phase was studied by Lawson et al. (1974) using fields up to 10 kGauss. They observed a splitting of the characteristic superfluid attenuation peak which was linear in the applied field. The Cornell group (Lawson et al., 1975)
FIGURE 4. First sound velocity $c_1(P)$
29.3 bar
20.24 MHz
after Ketterson et al. (1975)

FIGURE 5. \( \alpha(T) \) and \( c(T) \) for 20MHz sound
also has reported an observation of the velocity drop in the Al phase. There are rapid drops at both $T_{c1}$ and $T_{c2}$. To my knowledge there is no published theory explaining the dispersion of sound in the Al phase.

The choice of 5 MHz transducers represents a compromise. At temperatures near $T_c$ at 31 bar ($\sim 2.7$ mK), $\omega t \sim 3$ and the zero sound is not quite fully "developed." Near $T_c$ at 2 bar ($\sim 1.3$ mK), $\omega t \sim 10$ and the propagation mode is essentially pure zero sound. Simply using higher frequency transducers will insure pure zero sound at all pressures but the energy of a sound quantum then becomes comparable to that of a thermal fluctuation. For 5 MHz, $\hbar \omega / k = 0.24$ mK. Thus 5 and 10 MHz are reasonable frequencies for zero sound transducers if one wishes $\hbar \omega / k T_c \ll 1$.

A few words should be mentioned about the techniques used for measuring sound attenuation, sound velocity, and $^3$He temperature. The sound attenuation measurements were based on a conventional method: use of a boxcar integrator to sample and average the transient signal induced in a piezoelectric transducer by a short pulse of sound sent across the sample cell. To measure phase velocity changes, the system was configured as a phase-locked loop. This technique is less conventional than the attenuation measurement and may even be unique to this laboratory.

The discovery of large temperature-dependent nuclear magnetic resonance (NMR) frequency shifts in the superfluid phase (Osheroff et al., 1972), besides stimulating much more experimental and theoretical work in that area, also gave promise that the $^3$He sample itself could be used as a thermometer (e.g. see Richardson, 1977). To date this has not been done, perhaps because of a lack of certainty about the necessary
calibration data. A few ancillary calibrations made in the course of these experiments support the theoretical picture of the magnitude of these shifts (Leggett, 1974a) as well as corroborate most of the existing NMR shift calibrations. The net result is a basis for thermometry in the A and Al phases accurate to \(~\text{10\%}\) but with a precision achieved in magnetic fields of modest homogeneity which is comparable to LCMN thermometry. The real advantage is a truly in situ thermometer, as Figure 6 illustrates. This shows the raw sound and NMR data on a run badly disturbed by a time-dependent heat leak caused by a "touch" in one of the cryostat vibration isolation mounts. After deconvolution, the sound attenuation vs. temperature data fall on a curve similar to those of "quiet" runs but with an inhomogeneous data point distribution along that curve, where certain temperature regions were retraced due to heat spikes followed by cooling.

The results of this series of measurements are given in the last section chiefly in comparison against the collisionless theory of sound dispersion in the superfluid A-phase. The amplitudes of the peak attenuations and the velocity drops scale according to the predictions of this theory, but at the higher pressures, the temperature locations of the attenuation peaks are colder than predicted. A comparison with theory using the numerical computation scheme of Wölfle and Koch (1978) is probably required to see if a real discrepancy exists.
IN SITU THERMOMETRY

FIGURE 6. In situ $^3$He NMR thermometry
SECTION 2
PERTINENT THEORY OF $^3$He

2.1 General References

Current theoretical understanding of the superfluid phases of $^3$He rests on generalizations of the BCS theory of fermion pairing originally applied to metallic superconductivity (Bardeen, Cooper, Schrieffer, 1957). Theoretical reviews of superfluid $^3$He have been written by Leggett (1975), Anderson and Brinkman (1978), and Wölfle (1979) and experimental reviews by Wheatley (1975, 1978) and Lee and Richardson (1978). Shorter but more recent reviews can be found in the proceedings of the 1981 "Low-Temperature-16" (Physica 107-110, W.G. Clark, ed.) and 1983 "Quantum Fluids and Solids" (AIP Conference Proceedings No. 103, E.D. Adams and G.G. Ihas, eds.) conferences. What follows here is not intended as a review but only as a reminder of those theoretical aspects of $^3$He especially important for understanding the experimental results presented in this dissertation. For purposes of consistency, the notation will follow that of Wölfle's review where possible.

2.2 The Normal Fermi Liquid

Above the superfluid transition temperature of $\sim 2$ mK but significantly below the Fermi temperature of $\sim 1$ K, liquid $^3$He is well described by Landau's "Fermi liquid" theory (Landau, 1956 and 1957). The starting point for this theory is a gas of fermions of density $n$ and
particle energies $\varepsilon_{k\sigma}$ depending on momentum $p=\hbar k$ and spin projection $\sigma$. At low temperatures this gas is nearly completely condensed into its ground state: the Fermi sphere in momentum space of radius $p_F$. The Fermi energy $\varepsilon_F$ and momentum $p_F=\hbar k_F$ are related to the number density $n$ by the following relations.

$$n = \int \frac{n_{k\sigma}}{k\sigma} = \frac{k_F^3}{3\pi^2}$$

$$\varepsilon_{k\sigma} = \frac{\hbar^2 k^2}{2m^*}$$

The quantity $m^*$ is an effective mass several times greater than the bare atomic mass and is necessary for the connection to experimental results. Similarly, the energies $\varepsilon_{k\sigma}$ refer to excitations called quasiparticles, consisting of the collective motion of many $^3$He atoms but still obeying the Pauli exclusion principle.

A consequence of the nearly condensed state of quasiparticles is that the full three-dimensional momentum space picture can be reduced to the two-dimensional one of excitations near the Fermi surface. Also, the heat capacity and static magnetic susceptibility are then simply proportional to the density of states near the Fermi surface.

$$N_F = \left( \frac{\partial n}{\partial \varepsilon} \right)_{\varepsilon_F} = \frac{m^* p_F}{\pi^2 \hbar^3}$$

This is possible because the quasiparticle distribution function is that for fermions

$$n(\varepsilon) = \left\{ \exp\left[ \frac{(\varepsilon - \mu)}{kT} \right] + 1 \right\}^{-1}$$

(3)
so that at low temperatures the distribution differs from a step function centered at the chemical potential \( \mu \) only in the neighborhood of \( \mu \). Most properties of normal \(^3\)He can be described if one imagines that the quasiparticles are coupled by a weak effective interaction so that a deviation in the quasiparticle distribution from that of the ground state changes the quasiparticle energy by

\[
\delta c_{k\sigma}(r,t) = \sum_{k'\sigma'} f_{k\sigma k'\sigma'} \delta n_{k'\sigma'}(r,t)
\]  

An angular decomposition of the interaction yields the "Landau parameters" \( F_s^L, F_a^L \).

\[
f_{k\sigma k'\sigma'} = N_F^{-1} \sum_{\ell=0}^\infty \frac{P_{\ell}(|k\cdot k'|)}{(k\cdot k')} (F_s^L \delta_{\sigma\sigma'} + F_a^L \sigma\cdot\sigma')
\]  

Here, the superscripts \( s \) and \( a \) stand for symmetric and antisymmetric respectively and \( P_{\ell}(\cos\theta) \) are the Legendre polynomials. The first few Landau parameters are in principle related to simple measurements of the molar heat capacity \( C_N \), speed of (first) sound \( c_1 \), and magnetic temperature \( T^* \) (a constant derived from low temperature absolute susceptibility measurements). The pertinent relations are

\[
N_F = \frac{3 \: C_N \: n}{\pi^2 kRT}
\]

\[
F_s^0 = 3 \left( \frac{m}{m^*} \right) \frac{C_1}{V_F} - 1, \quad V_F \equiv \frac{p_F}{m^*}
\]

\[
F_s^1 = 3 \left( \frac{m^*}{m} - 1 \right)
\]
The value of $F^e$ can be obtained from spin diffusion measurements (Leggett and Rice, 1968).

2.3 **Superfluid Pairing**

If the quasiparticle interaction is the least bit attractive, at a sufficiently low temperature, the Fermi liquid picture must be supplemented to account for the inevitable instability of quasiparticles at the Fermi surface for forming bound pairs of opposite momenta. Calculation of this critical temperature $T_C$ from first principles is a difficult problem but the pair interaction can be characterized in terms of the Landau parameters. Doing so reveals that this interaction is repulsive in the s-wave orbital angular momentum state ($\lambda=0$) but attractive in the p-state ($\lambda=1$). The repulsion in the s-state might be expected from knowledge of the hard-core repulsion between $^3\text{He}$ atoms. Pairing in the p-state, the basis of the most successful models of superfluid $^3\text{He}$, necessarily must, according to the Pauli exclusion principle, be accompanied by a total spin $S=1$.

At the transition temperature, the quasiparticle energy spectrum is modified by the appearance of an energy gap and becomes

$$E_k = (\epsilon_k^2 + \Delta_k^+ \Delta_k^-)^{1/2}$$

where $\epsilon_k - \mu + \epsilon_k$ is now the quasiparticle energy measured from the chemical potential ($\mu = \epsilon_F$ at $T=0$) and $\Delta_k$ is the off-diagonal mean field acting on the Cooper pair $(k\sigma, -k\sigma')$. 

$$f_0^a = \frac{3}{p_F} kT^* m^* - 1$$ (9)
\[ \Delta_k \equiv \Delta_{k's'k} = \sum_{k'} V_{kk'} g_{k's'} \]  

(11)

Equation (11) is mathematically analogous to equation (4) except that now \( V_{kk'} \) is the attractive interaction leading to the formation of pairs and \( g_{k's'} \) is the probability amplitude of a Cooper pair \((k's',-k's')\). Here \( \Delta_{k's'} \), is a complex 2x2 matrix in spin space. The actual magnitude of the energy gap is given by

\[ |\Delta_k| = [(1/2) \text{Tr} (\Delta_k \Delta_k^+)]^{1/2} \]  

(12)

At nonzero temperatures, the probability distribution in momentum space for fermions leads to the particular form for the famous BCS self-consistency "gap equation."

\[ \Delta_{k's'} = \sum_{k'} V_{kk'} \Delta_{k's'} \frac{\tanh (E_{k'}/2kT)}{E_{k'}} \]  

(13)

The gap equation uniquely defines the gap parameter for conventional superconducting metals, where \( \ell=0 \). For \( \ell=1 \), though, several model states can be created. The fact that more than one superfluid \(^3\)He state actually exists requires the careful consideration of the various possible theoretical models. It is now widely, though not universally, believed that the two zero-field states of superfluid \(^3\)He are well described by the so-called ABM and BW models. The ABM model (cf. Anderson and Morel, 1961; Anderson and Brinkman, 1973) corresponds to the experimental A-state and the BW model (cf. Balian and Werthamer, 1963) corresponds to the experimental B-state.
In order to discuss the various model states in detail it is convenient to handle the three independent components of the 2x2 symmetric matrix \( \Delta_{k\sigma\sigma'} \), in the form of a vector \( \mathbf{d}(\mathbf{k}) \).

\[
\Delta_{k\sigma\sigma'} = \begin{bmatrix} -d_1 + id_2 & d_3 \\ d_3 & d_1 + id_2 \end{bmatrix}
\]

(14)

The square of the energy gap can be written in terms of \( \mathbf{d}(\mathbf{k}) \) as

\[
(\Delta_k \Delta_k^+)_{\sigma\sigma'} = (\mathbf{d} \cdot \mathbf{d}^* \cdot \delta_{\sigma\sigma'} + i(\mathbf{d} \cdot \mathbf{d}^*) \cdot \tau_{\sigma\sigma'}),
\]

(15)

where \( \tau \) is the "vector" of Pauli spin matrices.

The \( d_j \)'s, which deal with the spin part of the superfluid wave function, specify the amplitudes of the gap parameter for those eigenstates of the pair spin operator having eigenvalue zero. The connection to the orbital part of the wave function is made by decomposing the \( d_j \)'s along the momentum axes.

\[
d_j(\mathbf{k}) = \sum_{\alpha=1}^{3} \mathbf{d}_{j\alpha} \hat{k}_\alpha
\]

(16)

The \( l=1, s=1 \) model states of \( ^3 \)He are fully described by the tensor order parameter \( d_{j\alpha} \).

2.4 Comparison of the ABM and BW States

Specification of a particular form for the complex tensor \( d_{j\alpha} \) leads to a state which can be compared with other model states by means of their free energies described as expansions in the order parameter. This Ginzburg-Landau expansion near \( T_C \) must involve
only combinations of $d_{ja}$ which are gauge invariant and invariant with respect to rotations in spin and position (orbital) space. There are only six such combinations: one second-order and five fourth order. To write down these free energy invariants it is useful to first define an order parameter normalized to the rms angular average of the energy gap over the Fermi surface, $\Delta(T)$.

$$\hat{d}_{ja} \equiv \frac{d_{ja}}{\sqrt{3\Delta(T)}} \quad (17)$$

with

$$\sum_{ja} \hat{d}_{ja} \hat{d}_{ja}^* = 1 \quad (18)$$

and

$$\Delta(T) = \left[ \left( \frac{1}{4\pi} \int d\omega (1/2) Tr(\Delta^+_{k} \Delta^+_{k}) \right) \right]^{1/2} \quad (19)$$

In terms of the $d_{ja}$, the invariants are (using Einstein summation, cf. Mermin and Stare, 1973; Brinkman and Anderson, 1973)

$$I_0 = \hat{d}_{ja} \hat{d}_{ja}^* = 1 \quad (20)$$

$$I_1 = |\hat{d}_{ja} \hat{d}_{ja}|^2 \quad (21)$$

$$I_2 = \hat{d}_{i\alpha} \hat{d}_{i\alpha}^* \hat{d}_{j\beta} \hat{d}_{j\beta}^* = 1 \quad (22)$$

$$I_3 = \hat{d}_{i\alpha} \hat{d}_{j\alpha} \hat{d}_{i\beta} \hat{d}_{j\beta} \quad (23)$$
The free energy difference between the normal and superfluid states can now be written as

\[ F = F_s - F_n \]
\[ = -N F/2 (1-T/T_c) \Delta^2(T) I_0 \]
\[ + (21/80) \xi(3) N F(\pi k T_c)^{-2} \Delta^4(T) \sum_{i=1}^5 \beta_i I_i \]

In the BCS weak coupling limit, the values of the \( \beta_i \) are

\[ \beta_2 = \beta_3 = \beta_4 = -\beta_5 = -2\beta_1 = 1 \]

A class of states discovered by Balian and Werthamer (1963) minimizes the free energy for the weak coupling values in Equation (26). The BW states are those unitary states given by

\[ d_{j\alpha} = \Delta(T) e^{i\phi} R_{j\alpha} \]

where \( \phi \) is an arbitrary phase and \( R_{j\alpha} \) is an arbitrary real orthogonal (i.e. rotation) matrix. The BW state is the only \( \Delta \neq 0 \) state with an isotropic energy gap.

\[ |\Delta_k(T)| = \Delta(T) \]
Since the BW state is the weak coupling minimum energy state, the identification of the experimentally highly anisotropic A-state with the ABM model state (Anderson and Brinkman, 1973) has thus been purchased at the cost of abandoning the simple weak coupling picture of $^3$He, at least in those regions of the phase diagram where the A-state is more stable than the B-state. Deferring this problem for a moment, the ABM state can be characterized by the following unitary order parameter:

$$d_{j\alpha} = (3/2)^{1/2} \Delta(T) \hat{d}_j (\hat{n}_\alpha + i \hat{m}_\alpha)$$

(30)

where $\hat{d}$ is an arbitrary unit vector in spin space and $\hat{n}$ and $\hat{m}$ are arbitrary unit vectors in position space constrained only by $\hat{n} \cdot \hat{m} = 0$. These latter two vectors define the "angular momentum" vector by their cross-product

$$\hat{l} \equiv \hat{n} \times \hat{m}$$

(31)

which lies along the axis of the anisotropic energy gap

$$|\Delta_k(T)| = (3/2)^{1/2} \Delta(T) |\sin \Theta|$$

(32)

where $\cos \Theta = \hat{k} \cdot \hat{l}$.

2.5 Strong Coupling and the Finite Field Phase Diagram

The connection made by Anderson and Brinkman (1973) between the experimental A-phase and the theoretical ABM model state used the idea of "spin fluctuation feedback." As Leggett succinctly states, "the basic physical idea of the Anderson-Brinkman theory is that the forma-
tion of the superfluid state modifies the pairing interaction between quasiparticles, and that the precise nature of the modification depends on the particular kind of superfluid state formed" (Leggett, 1975, p. 373). Spin fluctuation exchange, though perhaps the most important feedback mechanism, is only one of several "strong coupling" effects which might modify the pairing interaction. Quantitative spin-fluctuation or "paramagnon" calculations have been performed by Brinkman et al. (1974), Kuroda (1975), Tewordt et al. (1979a,b), and Tewordt et al. (1975, 1978). Instead of using the paramagnon approach, Rainer and Serene (1976), Serene and Rainer (1979), and Sauls and Serene (1981) have expanded the corrections to the $\beta_1$ in powers of $(kT_c/\epsilon_F)$ and used the "s-p" approximation for quasiparticle scattering for their calculations. Levin and Valls (1979) have combined both the paramagnon and s-p scattering approaches. The net result of these various calculations is a modification of the $\beta_1$ from their weak coupling ($kT_c/\epsilon_F \to 0$) BCS values.

Comparison of the strong coupling calculations just mentioned with experiment is obviously important but is complicated by the fact that experimental measurements yield only combinations of the $\beta_1$. Even though experiment contradicts the earlier predictions for some of the $\beta_1$ combinations (Halperin et al., 1976), enough measurements to define all five $\beta_1$ at any single pressure have not yet been performed. Most of the useful measurements which can be performed toward this end exploit the existence of the $A_1$ phase in a finite magnetic field. As shown by Figure 3, the normal to $A$-phase transition is split by a magnetic field into two transitions at $T_{c1}$ and $T_{c2}$. This splitting, a natural consequence of the fact that $^3$He forms a spin-1 superfluid, can be viewed as
a relative shift of the Fermi energies of the "spin-up" and "spin-down" components of the quasiparticles (cf. Ambegaokar and Mermin, 1973). The size of the interposed $A_1$ phase is proportional to the dependence of the pair coupling constant $\lambda_1$ on $\varepsilon_F$ and the applied field $H$.

$$\frac{T_{c1} - T_{c2}}{T_c} = \frac{d\lambda_1}{d\varepsilon_F} \frac{\mu_0 H}{(1 + F_0^a)}$$

(33)

Measurements involving the $A_1$ phase can best be explained by first considering the Ginzburg-Landau form for the free energy difference of a system containing only paired parallel spins (no $\uparrow\downarrow$ component) as written in Wheatley's (1975) exposition of Takagi's (1974) work.

$$\Delta_{\uparrow}^2 + (\sigma + \eta)\Delta_{\downarrow}^2$$

(34)

This equation is similar to Equation (26) except that the energy gaps for the two spin populations are now separately considered and the effect of an applied field is included. The reduced temperature $t$ and field $h$ in Equation (34) are defined by

$$t \equiv (T - T_c)/T_c$$

$$h \equiv \gamma \tilde{H} H/(2kT_c)$$

(35)

where $\gamma$ is the gyromagnetic ratio for $^3$He. The parameters $\eta$, $\beta$ and $\delta$ are to be determined experimentally but with the restrictions $\beta > 0$ and $\delta < 1$. The two parameters $\beta$ and $\delta$ can be related to the $\beta_1$ (Leggett, 1975) by
\[ \beta = \left(7\zeta(3)/16\pi^2\right)(\beta_2 + \beta_4) \quad (36) \]

\[ \delta = \frac{-(\beta_2 + \beta_4 + 2\beta_5)}{(\beta_2 + \beta_4)} \quad (37) \]

where the value of the Riemann zeta function is \( \zeta(3) = 1.202 \). Experimental measurements of the characteristic BCS heat capacity jump at \( T_c \) give information on the \( \beta_1 \). The three such independent measurements are the following:

\[ \frac{C_A - C_N}{C_N} = \left[ \frac{24}{14} \zeta(3) \right] \frac{1}{\beta_2 + \beta_4 + \beta_5} = \frac{3}{2\pi^2 \beta(1-\delta)} \quad (38) \]

\[ \frac{C_{Al} - C_N}{C_N} = \left[ \frac{24}{14} \zeta(3) \right] \frac{1}{2(\beta_2 + \beta_4)} = \frac{3}{4\pi^2 \beta} \quad (39) \]

\[ \frac{C_B - C_N}{C_N} = \left[ \frac{24}{14} \zeta(3) \right] \frac{3}{3(\beta_1 + \beta_2) + (\beta_3 + \beta_4 + \beta_5)} \quad (40) \]

Other experimental quantities yield additional information. There is a small magnetic susceptibility increase in the ABM state of about 1%.

\[ \frac{\chi_A - \chi_N}{\chi_N} = \frac{n_0^2(1+F^a)}{2\beta(1+\delta)} \quad (41) \]

The transition temperatures are given by

\[ t_1 = \frac{\pi h}{k} \quad (42) \]

\[ t_2 = \frac{\pi h(1-\delta)}{(1+\delta)} \quad (43) \]

so that
\[ t_1 - t_2 = \frac{2nh}{(1+\delta)} \]  

(44)

By defining an additional parameter to measure the substate with spin projection zero, Equation (31) can be generalized to include the BW state. Levin and Valls (1977) did this to obtain the transition temperature between the A and B phases in a finite field below the polycritical point. Their rather complicated expression for \( t_{AB} \), which is proportional to \( h^2 \), will not be quoted here but it can be used to give additional information on the \( \beta_1 \) since the extra parameter characterizing the B-phase can be estimated by measuring the slope of the B-phase susceptibility as a function of temperature.

Finally, the perpendicular NMR shifts in the \( A_1 \)-phase and A-phase well below \( t_2 \) give an independent measure of \( \delta \).

\[ \frac{(d\nu^2_L/dt)_{A_1}}{(d\nu^2_L/dt)_A} = \frac{1-\delta}{4} \]  

(45)

This will be discussed further in the next section.

2.6 NMR in the A and Al Phases

The magnetic characteristics of superfluid \(^3\)He differ dramatically from those of the normal liquid. The susceptibility of the normal liquid is an essentially temperature-independent constant equal to that of a noninteracting Fermi system renormalized by the Fermi liquid parameter \( F_0^a \)

\[ \chi_N = \frac{\gamma^2 h^2 N_F / 4}{1 + F_0^a} \]  

(46)
The nuclear magnetic resonant frequency \( \omega \) is simply proportional to the magnetic field \( H \), a la Larmour:

\[
\omega_L = \gamma H \quad (47)
\]

This picture changes below the superfluid transition at \( T_c \), where the NMR A-phase resonance is seen to shift away from its normal liquid value according to the relation

\[
\omega^2 = (\gamma H)^2 + \Omega_A^2, \quad \Omega_A^2 \propto (1 - T/T_c). \quad (48)
\]

Such a shift in the resonant frequency is possible only if non-spin-conserving forces are invoked. In order to explain the A-phase frequency shifts first seen by Osheroff et al. (1972), Leggett (1973, 1974a) modified the precession equations for the nuclear spins \( S \) by the addition of a dipolar torque \( R_D \).

\[
\dot{S} = \gamma S \times H + R_D \quad (49)
\]

The magnitude of the nuclear dipolar coupling between two adjacent \(^3\text{He}\) atoms in the liquid is extremely weak. In temperature units it is \(~10^{-7} K\) and could be readily neglected at typical experimental temperatures, which are some \(10^4\) times higher, except for the correlations intrinsic to the superfluid state. This causes the effective dipolar coupling energy to be multiplied by the number of condensed Cooper pairs in the system. The tendency for the pair wave function to seek its most energetically favorable orientation is thus not "drowned out" by thermal fluctuations.
For the ABM state, the equilibrium orientation caused by $R_D$ aligns the $d$ vector along the (orbital) angular momentum vector $\vec{L}$. More generally, on sufficiently short time scales typical of NMR experiments, the motion of the $d$ vector is given by

$$\dot{d} = \gamma d \times (H - \gamma S/\chi)$$  \hspace{1cm} (50)

Calculation of the dipolar torque $R_D$ and the resulting shift in the NMR frequency has been done for the various unitary phases of superfluid $^3$He (Leggett, 1974a). In general the "longitudinal" frequency (so called because of parallel-ringing magnetic resonance experiments) is a product of a function of pressure $f(P)$ and $(1 - T/T_C)$.

$$\omega_A^2 = (2\pi)^2 \nu_A^2 = (2\pi)^2 f(P)(1 - T/T_C), \hspace{0.5cm} T < T_C$$  \hspace{1cm} (51)

where, for the ABM state,

$$f(P) = (\pi/10)<R^2>\gamma^2 N_F (1 + F_0^2) [2n(1.14 \varepsilon_c/kT_c)]^2 (kT_c)^2 (\Delta C/C_N)$$  \hspace{1cm} (52)

In Equation (52) $\varepsilon_c$ is an energy integral cutoff, estimated to be about 0.7 K by Leggett (1972). Most strong coupling effects are absorbed into the experimentally measured ratio of the jump in the heat capacity at $T_C$ to the normal heat capacity $(\Delta C/C_N)$. Here $<R^2>$ is an angular average of the square of the quasiparticle renormalization factor and is expected to be on the order of unity. The quantity $R_k$, which is averaged represents a modification to the spin current carried by particles of a free gas to give that of the Fermi liquid (Leggett, 1965; Leggett and Rice, 1968).
\[ \frac{p_\sigma}{m} + R_k \frac{p_\sigma}{m} \quad (53) \]

The nuclear magnetic resonance frequency also exhibits a shift in the Al phase which can be easily understood in terms of the Ginzburg-Landau equation (34). Takagi (1975) and Osheroff and Anderson (1974) have discussed how the shift frequency \( \nu_A \) depends on temperature by starting with the behavior of the two (different) order parameters for the up and down-spin components for the liquid. It will be useful to define a new temperature scale linear in \( T \) but with its zero at \( T_{c2} \) and its "degree" equal to the width of the Al phase.

\[ U \equiv \frac{T_{c2} - T}{T_{c1} - T_{c2}} \quad (54) \]

Minimizing the free energy of Equation (34) results in the following behavior of the gap magnitudes as a function of temperature.

\[ \Delta_+^2 = \Delta_-^2 = 0, \quad U < -1 \quad \text{(normal liquid)} \quad (55) \]
\[ \Delta_+^2 = 0, \quad \Delta_-^2 = A(1 + U), \quad -1 < U < 0 \quad \text{(Al-phase)} \quad (56) \]
\[ \Delta_+^2 = BU, \quad \Delta_-^2 = A + BU, \quad 0 < U \quad \text{(A-phase)} \quad (57) \]

Here, it is arbitrarily assumed that \( \eta > 0 \), thus favoring an up-spin Al phase. The constants \( A \) and \( B \) are related to the free energy parameters \( \beta \) and \( \delta \) by

\[ A = (\tau_1 - \tau_2)(kT) \quad (58) \]
\[ B = A/(1 - \delta). \quad (59) \]
TRANSVERSE NMR SHIFTS NEAR $T_c$

FIGURE 7. NMR shifts in the A and Al phases
A third constant $C$ relates the perpendicular resonance shift to the average of the two gaps.

\[ \nu_A^2 = C[\frac{\Delta + \Delta^*}{2}]^2 \]  

The resulting shifts in the Al and A phases are then

\[ \eta_A^2 = (c/4)A(1 = U) \quad , \quad -1 < U < 0 \]  
\[ \nu_A^2 = (C/4)[A + 2BU + 2(A + BU)^{1/2}(BU)^{1/2}] \quad , \quad 0 < U . \]  

"Deep" into the A phase the shift can be well approximated by a linear form

\[ \nu_A^2 = (C/4)[2A + 4BU] \quad , \quad U > 1 \]

The above theory gave an excellent fit to the NMR measurements of Osheroff and Anderson (1974) done at the melting pressure in fields of .49 and .74 Tesla. By taking the ratio of the slopes of the NMR shift vs. temperature in the Al and "deep" A phases, they obtained a value of $\delta = 0.25 \pm 5\%$. Figure 7 is a line drawing of this NMR shift plotted in units of $AC$ against the doubly reduced temperature $U$. The particular slope ratio chosen for this drawing is 5.33 corresponding to $\delta = 0.25$.

2.7 Sound Propagation in the Normal Fermi Liquid and the ABM-State

One manifestation of the rich behavior of superfluid $^3$He due to its complex tensor order parameter is in how the liquid disperses
sound. Many of the collective modes possible in the superfluid phases can be directly excited by sound waves of the appropriate frequencies. In order to give a qualitative explanation of sound experiments in the A-phase I will, for the most part, quote the simpler calculations expounded by Wölfle (1978) in his review on sound propagation in superfluid $^3$He.

At temperatures high enough or frequencies $\omega$ low enough sound propagation obeys the classical connections to compressibility $\kappa$, density $\rho$, and viscosity $\eta$. The velocity $c_1$ of such a hydrodynamic mode, ordinary or "first" sound, is given by

$$\frac{c_1^2}{\rho} = \frac{1}{\kappa} \tag{64}$$

and the attenuation by

$$\alpha_1 = \frac{2\omega^2}{3c_1^3}\eta. \tag{65}$$

The Fermi liquid compressibility is

$$\kappa = \frac{N_F m^2}{\rho^2 (1 + F_0^s)}, \tag{66}$$

independent of temperature while the viscosity has a $T^{-2}$ temperature dependence.

$$\eta \propto \frac{1}{5} \rho v_F^2 \propto T^{-2} \tag{67}$$
The velocity of first sound in $^3$He, taken from Wheatley (1975), is plotted in Figure 4.

In general, transport properties such as viscosity can be calculated only if one uses the general form of Landau's Fermi liquid theory which includes the effects of quasiparticle collisions. For the normal Fermi liquid, the mean quasiparticle lifetime between collisions $\tau$ is proportional to $T^{-2}$. The dynamic properties of the normal Fermi liquid can be placed in a context which can be generalized to the superfluid phases by writing down the kinetic equation for the distribution function of the quasiparticles. Starting with the Fermi distribution for static thermal equilibrium

$$f^0_k = \left[ \exp(\epsilon^0_k/kT) + 1 \right]^{-1}$$  \hspace{1cm} (68)

the linear deviation caused by the perturbing (sound) field of wave vector $q$ and frequency $\omega$, $\delta f_k(r,t)$, can be written in terms of its Fourier components

$$\delta f_k(q,\omega) = \int d^3rdt[f^0_k - f^0_k]\exp[i(q \cdot r - \omega t)]$$  \hspace{1cm} (69)

The evolution of these Fourier components is governed by the kinetic equation

$$\hbar \omega \delta f_k - \epsilon^0_+ \delta f_k + \delta f_k \epsilon^0_+ + \delta f_k \epsilon^0_- \delta f_k - \delta \epsilon_k f^0_k = -iI_k(\delta f^\prime)$$  \hspace{1cm} (70)

This important equation uses the following notation: "0" indicates equilibrium quantities, "±" means $k \pm q$, and $\delta \epsilon_k$ is the change in the
quasiparticle energy caused by both the direct gain in the external field \( \delta \varepsilon^\text{ext} \) and the shift in the distribution function \( \delta f_k \), namely

\[
\delta \varepsilon_k = \delta \varepsilon_k^\text{ext} + \text{Tr}_\sigma \sum_{k'} \int f_{k'k'} \delta f_{k'}
\]

The term on the right hand side of Equation (70) is the collision integral which operates on the deviation of the distribution function from local equilibrium.

\[
\delta f'_{k} = \delta f_{k} - \left( \frac{\partial f_0}{\partial \varepsilon^k} \right) \delta \varepsilon_k
\]

What happens if the (sound) excitation period is much shorter than the quasiparticle lifetime, i.e. \( \omega \tau \gg 1 \)? This is the "collisionless" limit, equivalent to setting the dissipation term of Equation (70) to zero. Landau (1957) showed that new propagation modes ("zero sound") may then exist, corresponding to anisotropic oscillations of the Fermi sphere (cf. Figure 8), with a separate mode for each \( k \)-component of the quasi-particle interaction. He is such that there is only one propagation mode not strongly damped (longitudinal zero sound), corresponding to a nearly spherical perturbation of the Fermi surface.

Equation (70) can be solved for its resonances by using the conservation laws for the number and current densities \( \delta n \) and \( j \), which are defined by

\[
\delta n(q, \omega) = \sum_{k \sigma} \delta f^\sigma_k (q, \omega)
\]

\[
j(q, \omega) = \sum_{k \sigma} (\hbar k/m) \delta f^\sigma_k (q, \omega).
\]
FIRST SOUND

MOMENTUM PERTURBATIONS

UNPERTURBED FERMI LEVEL

ZERO SOUND

FIGURE 8. Momentum perturbations of $c_0$ and $c_1$
These conservation laws are the respective continuity equations

\[ \omega \delta n = q \cdot j \]  \hspace{1cm} (75)

\[ \omega j = \sum_{k} \left( \frac{k}{m} \right) (v_F k \cdot q) \delta f_k \]  \hspace{1cm} (76)

which are obtained from the kinetic equation by multiplying it by \( l \) or \( k \) respectively and then summing over all \( k \).

In the collisionless, zero-sound limit the velocity and attenuation are

\[ \frac{c_0 - c_1}{c_0} = \frac{2(1 + \frac{1}{5} F_s^2)}{5(1 + F_0^3)} = \frac{2(m^*)}{15} \left( \frac{V_F}{c_1} \right)^2 , \quad (F_s^2 \equiv 0) \]  \hspace{1cm} (77)

\[ \alpha_0 = \frac{2}{15} \left( \frac{m^*}{m} \right) \left( \frac{V_F}{c_1} \right)^2 \frac{1}{c_1^\tau} . \]  \hspace{1cm} (78)

For \(^3\)He, the first and zero sound velocities differ by only \( \sim 1\% \). This is so because the hard core repulsion leads to a large value of \( F_0^s \), especially at high pressure. The zero sound attenuation has a \( T^2 \) dependence, opposite that for first sound.

The general dispersion relation for sound in normal liquid \(^3\)He including the transition region \( \omega \tau \sim 1 \) has been calculated by various means (Rudnick, 1980; Wölfle, 1976b) and requires handling of the dissipative term in Equation (70). Wölfle's approach is to construct an approximate collision integral using the \( k \)-dependent quasiparticle relaxation rates. His scheme allows generalization of the kinetic equation to the superfluid states. Figure 5 from Wölfle (1973) includes the result of such a normal fluid calculation fitted to the data of
Ketterson et al. (1975). The data points, which are well represented by the theoretical fit, are not shown.

The complicated nature of the superfluid $^3$He model states, as compared to the Fermi liquid theory, can not allow a description of superfluid sound propagation based solely on the scalar kinetic equation (70). This was first verified experimentally (Lawson et al., 1973; Paulson et al., 1973) by the dramatic changes seen in the velocity and attenuation of sound just below $T_c$ for both the A and B phases. The general features are a sharp peak in the attenuation with a width $\sim 1\% \ T_c$ and a steep decrease in the velocity starting at $T_c$. There is also some theoretical and experimental evidence for order parameter induced attenuation changes just above $T_c$ (Emery, 1975, 1976; Paulson and Wheatley 1978; Samalam and Serene, 1978; Pal and Bhattacharyya, 1979). This small effect will be neglected here.

Simply by knowing the characteristic times of $^3$He, one might estimate when changes in the sound dispersion will take place as the frequency is varied. (Varying temperature changes the characteristic times, also leading to dispersion.) Figure 9 shows the various regimes defined by these (temperature-dependent) characteristic times, with $1/\tau$ dividing the hydrodynamic and collisionless regimes. The frequency defined by the superfluid energy gap defines an additional region. For a sufficiently warm temperature ($T_1$ on Figure 9), the gap frequency is less than $1/\tau$ and the intervening regime can be called "gapless"; the energy gap is not well defined due to quasiparticle collisions. At colder temperatures ($T_2$), the region above $1/\tau$ but well below $\Delta(T_c)/\hbar$ is called by Wölfle (1978) "macroscopic," signifying that the order parameter is in a local equilibrium state not significantly perturbed by the sound frequency.
HYDRODYNAMIC (FIRST SOUND) → COLLISIONLESS (ZERO SOUND)

\[ \frac{\Delta(T_1)}{\hbar}, \quad \frac{1}{\tau}, \quad \frac{\Delta(T_2)}{\hbar} \]

← GAPLESS → MACROSCOPIC → ω

FREQUENCY REGIMES IN $^3$HE

FIGURE 9. Frequency regimes for sound in $^3$He
Calculations of the dispersion of pure zero sound (no quasiparticle collisions) in the ABM state have been done by Wölfle (1973, 1975a,b, 1976a,b,c), Ebisawa and Maki (1974), and Serene (1974). Wölfle's approach, originally due to Betbeder-Matibet and Nozières (1969), starts with the kinetic equation (70), drops the collision term, and generalizes the distribution function to a matrix form appropriate for a superfluid description. This results in expressions for the perturbations of the order parameter which are induced by the sound which in turn can be related to the expected sound dispersion.

Cooper pairing of quasiparticles of opposite momenta in the superfluid state causes a new type of long range order in the system. The quasiparticle (diagonal) distribution function

$$f_{k\sigma \sigma', (r,t)} = \int \frac{d^3 q}{(2\pi)^3} \exp(iq \cdot r) \langle a^+_k \sigma a_{k+q/2} \sigma' \rangle$$

is then joined by an off-diagonal distribution proportional to the expectation value of the pairing

$$g_{k\sigma \sigma', (r,t)} = \int \frac{d^3 q}{(2\pi)^3} \exp(iq \cdot r) \langle a_{k-\sigma} a_{k+q/2} \sigma' \rangle$$

where $a^+_k (a_k)$ is a creation (annihilation) operator acting on a state of momentum $k$ and $k_{\pm} = k \pm q/2$.

The quantity $g_k$ is zero in the normal state and cannot even be observed directly in the superfluid. Rather, changes in $g_k$ are seen by their coupling to $f_k$. By defining a matrix distribution function in particle-hole space
\[ \vec{n}_k(r,t) = \begin{bmatrix} f_k & g_k \\ + & 1 - f_{-k} \end{bmatrix} \]  

(81)

and an analogous energy matrix

\[ \vec{\varepsilon}_k(r,t) = \begin{bmatrix} \varepsilon_k & \Delta_k \\ \Delta^+ & -\varepsilon_{-k} \end{bmatrix} \]  

(82)

the linearized response to a sound wave \((q,\omega)\) in the collisionless limit can be written as the matrix analogue of Equation (70).

\[ \delta \vec{n}_k - \varepsilon_k^0 \delta \vec{n}_{k+} + \delta \vec{n}_{k-} \varepsilon_{k-}^0 - \delta \varepsilon_{k+} \varepsilon_{k-}^0 + \varepsilon_{k-}^0 \delta \varepsilon_{k-} = 0 \]  

(83)

The quantity \(\delta \vec{n}_k = \vec{n}_k - \vec{n}_k^0\) is the linear deviation of the distribution function from its equilibrium value, which is

\[ \vec{n}_k^0 = \frac{1}{2} \left[ \overline{I} + \frac{\varepsilon_k^0}{2E_k} \tanh \left( \frac{E_k}{2kT} \right) \right]. \]  

(84)

As before, energies are measured from the Fermi surface

\[ \varepsilon_k^0 = \frac{k^2}{2m^*} - \mu \]  

(85)

and the superfluid energy spectrum includes the effect of the energy gap

\[ E_k = (\varepsilon_k^2 + \Delta_k^+ \Delta_k^-)^{1/2} \]  

(86)

The perturbations in the components of the energy matrix \(\vec{\varepsilon}_k\) are related to the perturbations of the two distribution functions.
\[ \delta \varepsilon_k = \sum_{k'} f_{kk'} \delta f_{k'} + \delta \varepsilon_{k}^{ext} \]  
\[ (87) \]

\[ \delta \Delta_{k \sigma \sigma'} = \sum_{k'} g_{kk'} \delta g_{k'} \sigma \sigma' \]  
\[ (88) \]

In analogy to the normal Fermi liquid equation (4), the equilibrium energy gap is related to the off-diagonal distribution function by the self consistency equation

\[ \Delta_{k \sigma \sigma'} = \sum_{k'} g_{kk'} \sigma \sigma' \delta g_{k'} \sigma \sigma' \]  
\[ (89) \]

thus defining \( g_{kk'} \sigma \sigma' \).

The matrix kinetic equation (83) is a system of linear integral equations which can be solved for the case where the interactions \( f_{kk'} \) and \( g_{kk'} \) are assumed to be independent of the magnitude of \( k \) \((|k| = k_F)\). \( \text{Wölfle's (1976b) approach is to rewrite Equation (83) by defining "vectors" composed of combinations of } \delta f_{k}, \delta g_{k}, \delta \varepsilon_{k}, \text{and } \delta \Delta_{k} \text{ and multiply them by appropriate matrices which contain only equilibrium quantities for their entries. Solving the resulting matrix equation is then more straightforward, leading to explicit expressions for } \delta \varepsilon_{k} \text{ and } \delta \Delta_{k}. \)

\[ \delta \varepsilon_{k} - \delta \varepsilon_{k}^{ext} = \int \frac{d\omega}{4\pi} f_{kk'} \left( \frac{n'}{\hbar \omega - n'} \right) (1 - \lambda_{k}) \delta \varepsilon_{k} - \frac{\lambda_{k}}{2} (\delta \varepsilon_{k} + \delta \varepsilon_{-k'}) \]

\[ + \frac{(\hbar \omega + \hbar^2 n')}{4 |\Delta_{k}|^2} \lambda_{k} \left( \delta \Delta_{k}, \delta \Delta_{k}^{+} - \Delta_{k}, \delta \Delta_{k}^{+} \right) \]  
\[ (90) \]

\[ \delta \Delta_{k} + \sum_{k'} g_{kk'} \theta_{k'} \delta \Delta_{k'} = \frac{N_F}{2} \int \frac{d\omega}{4\pi} g_{kk'} \left( \frac{\lambda_{k'}}{|\Delta_{k}|^2} \right) \left( \lambda_{k} \left( \omega + \hbar n' \right) \delta \varepsilon_{k}, \right. \]

\[ - \frac{1}{2} \left( \hbar^2 \omega^2 - \hbar^4 n'^2 \right) \delta \Delta_{k}, + \Delta_{k}, \Delta_{k}^{+}, \delta \Delta_{k}^{+}, + \Delta_{k}, \delta \Delta_{k}^{+}, \delta \Delta_{k}^{+}, \Delta_{k'} \} \]  
\[ (91) \]
For the above two equations, the following abbreviations were used:

\[ \eta \equiv \hbar^2 k \cdot q / m^* \]  

(92)

\[ \Theta_k \equiv (1/2E_k) \tanh(E_k/2kT) \]  

(93)

\[ \lambda_k(q, \omega, T) \equiv -4|\Delta_k|^2 \int_{-\infty}^{\infty} \frac{d\varepsilon_k}{\varepsilon_k} \left[ (\hbar \omega)^2 \Theta_k + \eta^2 \varepsilon_k \frac{d\Theta_k}{d\varepsilon_k} \right] \]  

(94)

\[ D_k \equiv (\hbar \omega)^2 \left[ (\hbar \omega)^2 - 4E_k^2 \right] - \eta^2 \left[ (\hbar \omega)^2 - 4\varepsilon_k^2 \right] \]  

(95)

Equations (90) and (91) were arrived at assuming that the gap parameter \( \Delta_{\sigma \sigma} \), is unitary but otherwise are not specific to any model state. Specialization to the ABM state requires the particular form of the ABM gap, namely Equation (30) inserted into Equation (14).

\[ \Delta_{k \sigma \sigma'} = i\sqrt{3} \Delta(T) \left[ (\hat{n} + i\hat{m}) \cdot \hat{k} \right] \left[ d \cdot (\tau_2)_{\sigma \sigma'} \right] \]  

(96)

Now, by a particular choice of coordinates, the perturbation of the gap parameter can be written in terms of the azimuthal components of the first spherical harmonic \( Y_{lm} \).

\[ \delta \Delta_{k \sigma \sigma'} = i\sqrt{6} \frac{\Delta(T)}{\hbar \omega} \sum_{m=-1}^{+1} \sum_{j=1}^{3} \left( \frac{8\pi}{3} \right)^{1/2} Y_{lm}(^k) \delta_m \delta_m \]  

(97)

Using these forms for the gap and its perturbation induced by the sound in the general equations (90) and (91) results in an expression for each of the three expansion parameters \( \delta d_1, \delta d_0, \delta d_{-1} \). Collective modes (resonant oscillatory distortions) of the order parameter show up as poles of the \( \delta d_m \) in the complex frequency plane.
The equilibrium (no sound) order parameter is proportional to $Y_{11} (k)$ as can be seen by choosing coordinates such that $\hat{n} = \hat{x}$ and $\hat{m} = \hat{y}$ in Equation (96). It turns out that there is no pole associated with $\hat{d}_1$.

There is one pole associated with $\hat{d}_{-1}$; excitation of this mode amounts to a distortion of the equilibrium gap parameter by an amount proportional to $Y_{1-1} (k)$. In the limit of $T$ near $T_c$, the temperature dependence of this mode's frequency is (Wölfle, 1975)

$$\hbar \omega_{c1} = [(4/5)(2\sqrt{6} - 3)]^{1/2} \sqrt{3/2} \Delta(T)$$
$$= 1.2326 \Delta_0(T)$$

(98)

where $\Delta_0(T) = \sqrt{3/2} \Delta(T)$ is the maximum magnitude of the ABM gap.

The $\hat{d}_{-1}$ mode is often referred to as the "clapping" mode as the $Y_{1-1} (k)$ oscillations results in a movement of the $\hat{n}$ and $\hat{m}$ vectors towards and away from each other as Figure 10 shows.

The equation for $\hat{d}_0$ yields two modes. These modes are sometimes called "normal flapping" and "super flapping." Figure 10 illustrates normal flapping: $\hat{n}$ and $\hat{m}$ "flap" up and down, thus rocking the $\hat{z}$-vector back and forth away from its equilibrium position. Near $T_c$, the temperature dependences of the two flapping frequencies are given by

$$\hbar \omega_{nf1} = \sqrt{4/5} \Delta_0(T) \left[1 - \left(28/3\pi^4\right) \zeta(3)(\Delta_0(T)/kT_c)\right]$$

(99)

$$\hbar \omega_{sf1} = 2 \Delta_0(T) \left[1 - \left(28/5\pi^4\right)\zeta(3)(\Delta_0(T)/kT_c)\right]$$

(100)
FIGURE 10. Clapping and flapping modes
The above gap resonance features, as well as those due to breaking of Cooper pairs ($\omega = 2\Delta$), show up in the general expressions for the sound attenuation and velocity. The function $\xi^0$ containing this dispersion information can be written in a form emphasizing the importance of the relative orientation of the sound vector $\mathbf{q}$ and the gap axis $\mathbf{l}$ (Serene, 1974):

$$\xi^0 = \xi^0_\parallel \cos^4(\beta) + \xi^0_c 2\sin^2(\beta)\cos^2(\beta) + \xi^0_\perp \sin^4(\beta)$$  \hspace{1cm} (101)

where $\cos\beta \equiv \mathbf{q} \cdot \mathbf{l}$ and $F^S_2 \equiv 0$. The component functions are

$$\xi^0_\parallel = (45/4) \left[ \langle \lambda \cos^4 \theta \rangle - \langle \lambda \cos^2 \theta \rangle^2 / \langle \lambda \rangle \right]$$  \hspace{1cm} (102)

$$\xi^0_c = (45/8) \left[ 3 \langle \lambda \sin^2 \theta \cos^2 \theta \rangle - \langle \lambda \cos^2 \theta \rangle \langle \lambda \sin^2 \theta \rangle / \langle \lambda \rangle \
- \langle \lambda \cos^2 \theta \rangle^2 / [ \langle \lambda \cos^2 \theta / \sin^2 \theta \rangle - 2(\Delta_0 / \omega)^2 \langle \lambda + \hat{\varepsilon} \rangle \cos^2 \theta ] \right]$$  \hspace{1cm} (103)

$$\xi^0_\perp = (45/16) \left[ (3/2) \langle \lambda \sin^4 \theta \rangle - \langle \lambda \sin^2 \theta \rangle^2 / \langle \lambda \rangle \
- (1/4) \langle \lambda \sin^2 \theta \rangle^2 / [ \langle \lambda \rangle - 2(\Delta_0 / \omega)^2 \langle \lambda \sin^2 \theta \rangle ] \right]$$  \hspace{1cm} (104)

Here $\langle A \rangle \equiv (1/4\pi) \int d\Omega A$ is the angular average and

$$\cos \theta \equiv \mathbf{k} \cdot \mathbf{l}$$

$$\hat{\varepsilon} \equiv \int_{-\infty}^{\infty} d\varepsilon_k \frac{\Delta_k^2}{E^2_k} \frac{d}{dE^2_k}(\theta_k)$$  \hspace{1cm} (105)

$$\lambda \equiv \int_{-\infty}^{\infty} d\varepsilon_k \frac{\tanh(E_k/2kT)}{[E^2_k - (\omega/2)^2]} \frac{\Delta_k^2}{2E^2_k}$$  \hspace{1cm} (106)
For $T$ near $T_c$, 

$$
\lambda = i \pi \left( \frac{\Delta_0}{\hbar \omega} \right) \tanh \left( \frac{\hbar \omega}{4kT} \right) \frac{\sin^2 \theta}{\left( \frac{\hbar \omega}{2 \Delta_0} - \sin^2 \theta \right)^{1/2}} + O \left( \frac{\Delta_0}{kT_c} \right)^2 \quad (107)
$$

For zero sound of low enough frequency, $\lambda$ simplifies to $(\hbar \omega/kT)$ times a function of $(\Delta_0/\hbar \omega)$ only. This makes it possible to express the sound attenuation and velocity in terms of a universal function defined by

$$
\chi = \left( \frac{2kT}{5\hbar \omega} \right) \xi^0 \quad (108)
$$

The sound attenuation is then

$$
\alpha = \frac{\hbar \omega^2}{c_0 F_0^s kT} \text{Im}(\chi) \quad (109)
$$

and the shift in the sound velocity from its zero sound value $c_0$ is

$$
\frac{c - c_0}{c_0} = - \frac{\hbar \omega}{F_0^s kT} \text{Re}(\chi) \quad (110)
$$

Figures 11 and 12 from Wolfle (1973) show what the imaginary and real parts of the components of the universal function $\chi(\Delta_0/\hbar \omega)$ look like. Although pairbreaking at $\Delta_0/\hbar \omega = 1/2$ is implicitly included, the dominant features are the various gap resonance modes. In particular, $\chi_\perp$ shows a strong, sharp peak in the attenuation with a corresponding feature in the velocity shift. The overall anisotropy of dispersion is of course due to the anisotropy of the order parameter itself. The maximum value of the attenuation for any orientation is proportional to the prefactor of Equation (109).
So far, the effects of quasiparticle collisions have been ignored. A good accounting of their effects, which should include the background $T^2$ attenuation behavior and a broadening of the collective modes peaks, might start by adding a dissipative term to the right hand side of the matrix kinetic equation (83). This is a generalization of the scalar collision integral. Wölfle and Koch (1977) were able to construct an approximate form of this dissipative term that was numerically tractable. Their calculations, involving some 30 double integrals on angle and energy, give fair agreement with the attenuation data of Paulson et al. (1977) and the attenuation and velocity data of Ketterson et al. (1975). In particular, the clapping attenuation peak is broadened and the corresponding "derivative-like" structure in the velocity shift is no longer present.

The presentation of the preceding pages, chiefly concerned with the theory of collisionless sound in superfluid $^3$He, is intended to serve as a basis for comparing the experimental results to be described later in this thesis. In our experiment the magnetic field was parallel to the sound vector, thus orienting the $\lambda$-vector of the superfluid perpendicularly to the sound (except for a small fraction next to the container boundaries). Thus only $\chi_\perp$ of Equations (101) through (111) will be considered for comparing the sound attenuation and velocity data.
$$\text{Im}\{\chi\}$$

Scaled attenuation

$$\hbar \omega \ll kT$$
$$T = T_c$$

FIGURE 11. Theoretical attenuation, Im($\chi$)
FIGURE 12. Theoretical velocity shift, $\text{Re}(\chi)$
SECTION 3
APPARATUS

The apparatus for performing these experiments necessarily include the refrigerator, constructed before my tenure at the University of Florida. Since construction details have not been reported previously, I will do so here. Much of the general discussion of refrigeration principles follows Lounasmaa (1974).

3.1 Large-Scale Features

The cryostat and most electronic systems rest in a 24 m³ copper screen room. Electronic filtering for the 125 VAC lines includes, besides the usual low pass network, provision for elimination of the occasional bursts at 3510 Hz used for campus clock synchronization. All vacuum pumps and the dilution refrigerator gas handling board are outside the screen room as well as the computer and signal averager for data acquisition. The signal averager has analog electrical connections to the screen room electronics via three 1-MHz low-pass filters.

Figure 13 indicates the gross dimensions of the cryostat and dewar. The "super-insulated" dewar (Cryogenics Associates, 62 liters) and cryostat are suspended from a triangular aluminum plate which is supported at its three corners by optical-bench air mounts. For demagnetization of copper nuclei, a custom-built, American Magnetics, niobium-titanium superconducting 8 tesla magnet is used. This magnet, with its compensating coils, is mounted directly on the 10-liter vacuum
FIGURE 13. Cryostat and dewar
Figure 14

8 Tesla superconducting solenoid and some of the contents of the vacuum jacket.

A 1 K pot
B still
C continuous concentric heat exchanger
D discrete heat exchangers
E mixing chamber
F flexible heat link
G squeeze connection to bundle flange
H copper demagnetization bundle
I thermal shields
J vacuum jacket wall
K indium heat switch
L vertical Helmholtz coil pair
M compensation windings
N 8 tesla solenoid
FIGURE 14. Solenoid and vacuum jacket
jacket and the entire assembly hangs from the pumping line and baffle assembly in the neck. In order to conduct up to 78 amperes between the magnet and its external current supply, a two-level system of current leads is used. The lower portion is a "sandwich" of normal and superconducting metal strips which connects to the upper, normal vapor-cooled portion near the top of the dewar belly. These vapor-cooled leads, developed especially for low-duty cycle use (Berg and Ihas, 1983), lowered the overall helium boiloff rate by ~30% when they replaced the leads supplied by the magnet manufacturer. The average consumption of liquid helium (from a two month period including five magnetization cycles) is now 15.5 liters per day, including transfer losses.

Figure 14 shows the outlines of the contents of the vacuum jacket, consisting of three chief portions: the dilution refrigerator, the copper bundle, and the experimental volume.

3.2 Dilution Refrigerator

Over the last two decades, dilution refrigeration has emerged from nonexistence to become the method of choice for cooling scientific experiments to temperatures significantly below 1 K. The power of this technique can be briefly explained by comparison with the older techniques of $^3$He evaporative cooling. Both $^3$He evaporative and dilution cooling use the binding energy for extracting single atoms from the bulk concentrated phase. Both can be operated in a continuous, circulating mode for an indefinite length of time and are essentially unaffected by the presence of magnetic fields. The evaporative cooling power of a single atom is its liquid binding energy $L$ and the rate of $^3$He atom extraction is proportional to the vapor pressure, which is exponential
in temperature. Thus, the overall cooling power goes as

\[ Q(\text{evaporation}) \sim L \exp\left(-\frac{L}{kT}\right) \]  

(112)

and in practice the highest possible pumping speeds give a minimum temperature of about 0.3 K for a pure \(^3\)He cryostat.

Dilution refrigeration works because of the nonzero solubility of \(^3\)He in \(^4\)He at arbitrarily low temperatures, about 6.4\% below 40 mK. Cooling, obtained by transferring \(^3\)He atoms from the pure into the dilute phase in a "mixing chamber," is driven by an osmotic gradient, which in turn is produced by pumping \(^3\)He vapor from a "still" which is at a temperature high enough to give a reasonable circulation rate. Even though the cooling power per \(^3\)He atom decreases with temperature as \(T^2\), the circulation rate, \(\dot{n}\), is independent of mixing chamber temperature so that the overall power for dilution cooling is

\[ Q(\text{dilution}) \sim \dot{n}_3 T^2. \]  

(113)

If careful account is taken of factors such as thermal isolation and viscous heating, the practical limit of a dilution refrigerator is determined by the quality of the heat exchangers between the incoming and outgoing liquid streams. Within the past few years, temperatures below 3 mK have been achieved by groups using fine silver powder heat exchangers (e.g. Frossati, 1978; Oda et al., 1983).

Our dilution refrigerator is of conventional (pre-1978) design. The 1 K pot, of 270 cm\(^3\) volume, liquifies the incoming \(^3\)He. A "pickup" capillary, with an impedance dictated by the heat load on the
pot, replenishes the $^4$He supply from the 4.2 K bath. A manually operated valve allows the pot to be filled in about 10 minutes at the time of first cooldown or if the pickup impedance becomes blocked. Such blocking did occur during this quite extended experiment and daily use of the manual fill valve was required. Careful use of the valve resulted in negligible entropy increase of the experiment, even during the 1 mK runs.

The $^3$He still (100 cm$^3$) contains a "film burner" designed for a $^3$He purifier (Kirk and Adams, 1974) and reduces the $^4$He content of the vapor removed to about 1%.

The heat exchanger system between the still and the mixing chamber consists of a continuous exchanger followed by six step exchangers. The continuous exchanger, 1.5 m in length, is simply a thin-walled copper-nickel tube of 1.8 mm inside diameter which contains the dilute stream and an inner capillary of 0.51 mm inside diameter for the concentrated stream. Each of the six step exchangers is a pair of copper tubes silver-soldered together and packed with copper powder (~200 mesh, C-110, U.S. Bronze). Bulk flow is through a central longitudinal tube created by a "greened" steel wire during the sintering at 900°C. The volume available to the helium ranges from 2 cm$^3$ to 4 cm$^3$ in each of the six exchangers.

The 30 cm$^3$ copper mixing chamber contains copper powder (~400 mesh) sintered in place at 850°C.

Figure 15 characterizes the performance of our dilution refrigerator by plotting its cooling power as function of mixing chamber temperature for three different circulation rates. The circulation rates of 17, 25, and 36 micromoles per second correspond to still heater
DILUTION REFRIGERATOR PERFORMANCE

\[ \nabla = 17 \text{ micromoles/second} \\
\circ = 25 \\
\Delta = 36 \]

COOLING POWER (\(\mu W\))

FIGURE 15. Dilution refrigerator performance
powers of 130, 360, and 700 microwatts respectively. Film burner power was 140 microwatts for each curve. The minimum mixing chamber temperature in the unloaded state is about 14 mK although temperatures as low as 10 mK lasting for a few hours have been observed. These latter incidents have always been associated with the lowering of the magnetic field at the experiment or the copper bundle and are attributed to demagnetization of the copper nuclei in the mixing chamber by the fringing fields of these magnets. This effect is verified by the comparable heating seen at the mixing chamber when one of these fields is raised at the same speed used when mixing chamber cooling was observed.

3.3 Nuclear Cooling Stage

Achieving refrigeration sufficient to study the superfluid phases or the magnetically ordered solid phase of $^3$He usually requires adiabatic nuclear demagnetization cooling. Although cooling to these temperatures is possible with Pomeranchuk compression or demagnetization of the electronic paramagnetic salt cerous magnesium nitrate (CMN), the former is limited to melting curve pressure (34.3 bar at 3 mK) and both lose cooling power just below the ordering temperatures of the refrigerants, about 1 mK for both $^3$He and CMN.

Nuclear cooling can be simply understood by considering the thermodynamics of a noninteracting nuclear dipole system placed in a magnetic field, B. The Zeeman interaction energy for each dipole is

$$\epsilon_m = -\mu g m B$$ (114)

where $m$ is the magnetic quantum number, $g$ is the nuclear gyromagnetic ratio, and $\mu = e\hbar/2Mc$ is the nuclear magneton. For $n$ moles, the partition function is
where

\begin{align}
R &= \text{ideal gas constant} \\
\mu_0 &= \text{free space permeability} \\
\Lambda &= N_o I(I+1) \mu_0 \mu g^2/3k = \text{molar nuclear Curie constant} \\
b^2 &= \text{dipolar interaction}
\end{align}

Under ideal conditions of no heat leaks or nonnuclear heat capacities, the final temperature, \( T_f \), after reducing the field from \( B_f \) to \( B_1 \), is related to the initial temperature by

\[ T_f = \frac{B_f^2 + b^2}{B_1^2 + b^2} \frac{1}{2} T_i \tag{119} \]

and the heat capacity is unchanged.
The construction of the nuclear cooling stage for our cryostat is similar to that described by Muething (1979). The two major differences between the Ohio State University nuclear stage and ours at the University of Florida are the wire diameter and the $^3$He heat exchanger; these differences will be discussed in more detail. As was done at OSU, copper was chosen for the demagnetization material. Copper gives the advantages of high thermal conductivity, no superconducting transition, ready availability in wire form, and low cost. Its disadvantages are its low Curie constant ($\Lambda = 4.04 \times 10^{-12} K/\text{mole}$) and its relatively large Korringa constant of 1.1 sec·K.

3.4 **Bundle Construction**

Since the construction details of our bundle differ somewhat from those of the Ohio State University bundle, the building process will be described here. The gross features of the final product are a close-packed cylindrical copper wire bundle 5.6 cm in diameter and 40 cm long welded to an 11 cm diameter copper plate or flange. This flange has an array of holes for clamping devices, such as the $^3$He heat exchanger, to the bundle.

The wire chosen for the bundle was a commercial grade, coated magnet wire (Essex Corporation) whose special, high temperature insulation (Allex) allowed the bundle to withstand temperatures of 300°C for extended periods of time. Thus, the wires were protected during the welding operation and the slight flowing of the Allex during annealing could be used to bond the bundle together without the use of epoxy,
suspected as the cause of large time dependent leaks (Konter et al., 1977). The wire chosen was (#24 AWG, 0.51 mm diameter) based on the measured 4.2 K resistivity of the wire material, \( \rho = 5 \times 10^{-11} \text{ ohm}\cdot\text{m} \). The net eddy current heating power in a changing magnetic field, \( B \), is

\[
P = \frac{\pi N^2 r^4 B^2}{8\rho}
\]

(121)

where \( N \) is the number of wires of length \( l \) and radius \( r \). For a typical "cold" rate of \( 3 \times 10^{-4} \text{ Tesla/sec} \) this gives \( P = 10 \text{ nW} \), an acceptable level for this process.

Preparation of the wire for bundle construction commenced by winding some 200 turns onto a 27 cm diameter light aluminum drum. Two cuts through the wires along the axis of the drum gave two tresses of wires 42 cm long. About forty such tresses were made. Allex is resistant to most chemical stripping agents so, in order to remove 4 cm of insulation from one end of each tress, the tresses were dipped in a bath of molten sodium hydroxide (NaOH) contained in a stainless steel beaker. Immediately afterwards, the wires were bathed in a weak solution of acetic acid to neutralize any residue of the strong base. Finally, they were rinsed twice with distilled water in an ultrasonic cleaner.

Collecting the wires together for welding was begun by straightening the tresses and carefully laying them together in a trough. A hose clamp then drew the bare ends into a close-packed array 5.1 cm in diameter. A 5.1 cm i.d., 5.6 cm o.d. cylindrical OPHC copper collar with an interior bevel on one end was forced around the wires by alternately pounding on the collar and sliding the hose clamp down
towards the center of the wires. The protruding 3 mm of base wire provided the fill metal during welding. The bundle of wires was immediately clamped into a support jig, designed so that most of the bundle could be lowered into a glass nitrogen dewar (see Figure 16). The dewar was filled with liquid nitrogen until only the OFHC collar and the stripped ends were not immersed. Then the tops of the wires and collar were housed in a glove box continuously purged by helium gas drawn from evaporating liquid helium. An arc welder operating at five kilowatts in the D.C. mode with an argon gas shielded tip fused the wire tops together. After correcting minor nonuniformities of this first weld by filing, the bundle flange, an 11 cm diameter plate with a central hole to mate to the bundle, was slid over the fused wire tops and came to rest on the collar. The two pieces were then welded together using the glove box and other procedures as before, after which the entire assembly was ready for wrapping.

The bundle flange was now fitted with a bearing race so that, by inserting the bearing balls, the flange with its protruding wires was left free to turn with respect to this support. The next step was to attach two #24 Allex insulated copper wires at the top of the bundle, which, as it rotated, would be wrapped along its length by these wires. The two wires were fed from opposite sides by a spring and brake assembly, which used feedback to give a very constant and balanced tension. As the wrap progressed, the bundle was struck sharply and simultaneously with two hammers in an opposing fashion all around the perimeter of the bundle. This compressed the wires of the bundle together so that, when the wires were eventually cut and tied off, the final diameter was < 5.6 cm and quite uniform.
The wrapped bundle and flange structure was removed from the rotating support and inserted into a clamp designed to straighten the bundle and preserve its alignment with respect to the flange. Heating for five days at 275°C in a slightly flowing $^4$He atmosphere annealed the copper and caused the Allex insulation to flow slightly and thus bond the individual wires together. It was possible to avoid completely using epoxy and still have a quite rigid structure.

The last step was to trim the bundle to the appropriate length using a "glass wheel" (as used for cutting stainless steel tubing). The cut end showed only a few dislocations in the hexagonal close-packed array of 9000 wires.

3.5 Thermal Isolation and Heat Leaks

Effort expended on refrigeration for a millikelvin cryostat must be complemented with measures to prevent the flow of heat into or generation of heat within the final cooling stage. All material connections between stages of different temperatures were constructed to minimize undesired heat flow along these connections. The load bearing supports between stages are 1 cm diameter AGOT graphite rods. All electrical connections and helium carrying capillaries were heat sunk at the 1 K pot, the still, the mixing chamber, and the copper bundle. Electrical heat sinks usually relied on a thin layer of cigarette paper soaked with Stycast 2850FT (Emerson and Cuming) as an electrically insulating thermal conductor. Capillaries were thermally anchored by soldering 10 to 30 cm of the line around a metal post and bolting the post to the refrigerator. Silver solder was used at the lower stages to avoid problems with soft solder superconductivity. The lowest temperature capillary connections were made with 300 micron (12 mil) o.d.
copper-nickel tubes. Most electrical connections were made with copper-nickel wire except for the connections to the lowest temperature stage, which were made with Nb-Ti superconducting wires (the copper cladding was removed from all but the end connections of each length of wire).

Two nested thermal shields anchored at the still and mixing chamber reduce the influx of heat due to blackbody radiation. The upper portion of these shields is of welded sheet copper and the lower "tail" section is a cylinder of longitudinally aligned copper wires ("coil foil") coated with a hardened slurry of epoxy (Stycast 1266, Emerson and Cuming) and finely ground coconut charcoal. This design (Tanner et al., 1977) has a high infrared absorption coefficient while minimizing the heat load due to eddy current heating during changes of the copper bundle field. The bottoms of the shields are aligned with each other, the vacuum jacket, and the copper bundle by a series of teflon "bicycle wheels" with nylon mono-filament spokes. The influx of infrared radiation to the lower temperature stages was also reduced by placing tabs of copper foil at the vacuum jacket pumping port and over unused feed-through holes at the various refrigerator stages.

The heat link between the bundle and the mixing chamber consists of the heat switch, the flexible link, and the squeeze connectors. The squeeze connectors allow quick mounting and removal of the bundle from the cryostat, thus allowing the mounting of the experiment to be chiefly a table-top procedure. To establish the squeeze connection, a gold-plated copper collar, welded to the flexible link, is pressed over a matching gold-plated copper post welded to the bundle flange. A thick nylon ring is then pressed around this assembly. As the cryostat cools, contraction of this nylon ring provides the strong
squeezing force necessary (Muething et al., 1977). The flexible link is two bundles of copper wires each about 1 cm$^2$ in cross section. At its upper termination is a heat switch of conventional design which consists of two copper posts of semicircular cross section separated by Vespel (DuPont) rods and surrounded by a small persistable superconducting solenoid. The superconducting link is a strip of indium about 1 cm long and about 10 mm$^2$ in cross-section (March and Symko, 1965). Indium is readily available in pure form, is easily fabricated and attached to the refrigerator by virtue of its low melting point, and has a relatively low critical field of 28 millitesla.

Despite these various thermal isolation measures, the heat leak to the bundle was frustratingly high at 50 to 100 nanowatts for the first several weeks of the run during which the data for this thesis were taken. Tests showed that no nearby electrical equipment and, with the possible exception of the dilution refrigerator mechanical pump, no nearby mechanical equipment were implicated. The heat leak due to electrical and capillary connections to the bundle was expected to be $< 1$ nanowatt. As the data on Figure 17 show, the heat leak to the bundle eventually dropped to about 3 nanowatts or about 50 picowatts per mole of refrigerant. Since we knew of no changes external to the cryostat which could have caused this drop in the heat leak, we surmised that the depletion of some energy reservoir in the bundle had been taking place during this time (see Pobell, 1982).

Our primary suspect for the time-dependent heat leak is ortho-para conversion of hydrogen molecules. The associated energy, transition rate, and equilibrium concentrations of this process are well known for the bulk solid (Silvera, 1980). Such a heat leak as a function of time would be
FIGURE 17. Time-dependent heat leak to bundle
\[ Q = \frac{3.8 \times 10^{-3} n}{(Kt+1/X(0))^2} \]  

where \( t \) is the time in hours of cooldown from 300 K (essentially infinite temperature as far as the ortho concentration is concerned) to "0 K" and \( n \) is the number of moles of hydrogen involved. In the denominator \( X(0) \) is the high-temperature limit of ortho concentration \((3/4)\) and \( K \) is the rate constant \((0.019 \text{ hour}^{-1})\). The curve plotted onto the data of Figure 17 is Equation (122) for \(3.5 \times 10^{-4} \) mole of hydrogen. Ignoring the residual heat leak of about 3 nanowatts, the fit is rather good. About \(5 \times 10^{-3} \) mole of hydrogen gas was used as an exchange gas for cooling the cryostat down to 10 K and, although it should have ended up in solid form at the bottom end of the vacuum jacket wall, we cannot be certain of this. Also, a recent report by (Mueller et al., 1983) cites the discovery of pockets of hydrogen gas in copper used for construction of a refrigerator. Without some form of microscopic analysis to prove otherwise, such a possibility for the bundle wires or the body of the \( ^3\text{He} \) heat exchanger is only speculative.

### 3.6 Superfluid-Handling Apparatus

At the heart of the cryostat is the experimental region outlined schematically by Figure 18. The "anchor" to the bundle flange is the \( ^3\text{He} \) heat exchanger and on this structure the compressor, strain gauge, and cell body sit. Demountable struts raise the body of the heat exchanger to center the cell body in the magnetic field. These two OFHC copper pieces bolt to both the bundle flange and the heat exchanger; the mating surfaces are gold-plated. Also bolted to the bundle flange are a carbon resistance thermometer, a resistance heater, a heat sink for the
Heat exchanger plus mounted containers

FIGURE 18. Heat exchanger plus mounted containers
12 nongrounded connections to the experimental electrical devices, and a capillary heat sink for the \(^4\)He line to the compressor. The six ground connections needed were made by soldering these wires to a metal tab bolted to the bundle flange which in turn was electrically connected to the upper part of the cryostat through both Cu-Ni capillaries and a superconducting wire.

The advantage given by this nuclear cooling stage design which allows the bundle, heat exchanger, and cryostat to be connected to one another by simple bolt and squeeze connections is that quick "turn-around" times are possible. While one experiment is being run, another can be constructed and tested on a separate heat exchanger. The practical realization of this advantage naturally requires the existence of at least two heat exchangers; for this reason, the exchanger described below was designed and built.

3.7 Sinter Cell Tests for the \(^3\)He Heat Exchanger

Various materials and preparation schemes have been employed to build millikelvin heat exchangers. Although these schemes were nicely reviewed by Harrison (1979) in the more general context of the Kapitza resistance problem, I decided to make a direct experimental comparison of several types using consistent methods of thermometry, heat input, and cell design. Recently, mechanical and electrical measurements of submicron copper and silver powder sinters were performed to further understand their effectiveness as heat exchangers (Robertson, et al., 1983).

Until recently, copper has been the most commonly used material for low-temperature heat exchangers due to its high bulk thermal conductivity and its availability in the forms of foils, wires, flakes,
and powders. The copper powder we used (Vacuum Metallurgical) in these tests was nominally 0.03 micron in particle diameter. However, electron micrographs showed a characteristic diameter closer to 0.07 micron. The black appearance of this powder is apparently due to the presence of a large (60-80%) copper oxide volume fraction. This estimate is based on the 16% mass loss that occurred upon heating a sample to 450°C in a hydrogen atmosphere.

In order to build an oxide-free copper exchanger, we tried two approaches. The first was to reduce the powder at low temperatures until a 15% mass loss was achieved. The resulting powder, which had suffered a little particle size increase (.08 to .14 micron diameters) was then "cold-pressed" (at room temperature) into a copper cell. The second approach was to first press the raw powder into the cell and then heat it in a reducing atmosphere to obtain a 15% mass loss. A third approach, untried by us, is to "presinter" the powder before pressing it into the cell for sintering. The "presintering" might more appropriately be called "oxide reduction."

In recent years, due to its low nuclear heat capacity and relative cleanliness, silver powder, especially the .07 micron "Japanese powder", (Vacuum Metallurgical) has been the most popular material for millikelvin heat exchangers. Since a sample of our silver powder suffered less than 1% mass loss upon heating to 300°C, we decided to cold-press the raw powder into a silver-plated copper cell with no heat treatment whatsoever.

Palladium, with its high paramagnetism, showed promise as a heat exchanger material for $^3$He samples, as verified by foil measurements of Avenel et al. (1973). However, to our knowledge, only two
palladium powder exchangers have ever been built. Our palladium test cell was sintered according to the recipe devised at Ohio State University, (see Muething, 1979) which employs carbon monoxide and hydrogen as reducing agents, and helium to flush out the hydrogen before cooldown to prevent its absorption by the palladium.

A total of five cells, whose particulars are described in Table 1, were built. A cross-section of a sinter test cell is shown in Figure 19. The chamber in the oxygen-free copper body was cut out by spark etching and abrasive cleaning was necessary to remove the resulting residue on the walls. Final cleaning consisted of an acid bath dip and heating in a hydrogen atmosphere to remove surface oxides.

For economical thermometry, 0.5 watt, 220 ohm "old" Speer carbon resistors were ground to a 1.6 mm thickness and attached to leads with silver paint. These thermometers, calibrated against the $^3$He melting curve, worked well down to 5 mK when immersed directly in the liquid $^3$He.

Thermal time constants of the sinter cells were measured by applying a current pulse to the Cu-Ni heater sufficient to cause an initial 5 to 10% temperature rise over the outside temperature, which was that of the temperature-regulated mixing chamber. After the passage of a small signal transient, attributed to the response of the thermometer bridge electronics to the heater pulse and the finite response time of the carbon thermometer, the $^3$He temperature decayed approximately exponentially to that of the mixing chamber. The thermal time constants obtained were converted to thermal boundary resistances using the interpolated and extrapolated heat capacity data of Greywall and Busch (1982). For comparison purposes, these resistances are plotted
normalized to sinter volume in Figure 20. Although a normalization with respect to sinter surface area would be more interesting theoretically, the sinters were too small to allow accurate area measurements. The multiplication of the resistances by the temperature in Figure 20 demonstrates the $T^{-1}$ dependence of boundary resistance often seen before in this regime.

Although the tests for all of the cells were not at the same pressure, data taken in the silver powder cell at 4.6, 14.6, and 27.6 bar gave resistances within ±25% of their averaged values. Thus a direct comparison using Figure 20 should be valid to at least this uncertainty.

The best results were obtained with the copper powder reduced and sintered after packing (cell #5) and with cold-pressed silver powder (#1). The palladium (#4), which did not do as well, is interesting, since if surface area is estimated by average particle diameter, it is actually about three times better than cells #1 and #5. Cold-pressed fine copper powder whether oxidized or not (cells #2 and #3), does very poorly below 50 mK.

By far, the best exchanger "per unit difficulty" is the cold-pressed silver powder cell. For this reason I built a full-scale $^3$He heat exchanger of this type.
SINTER TEST CELL

FIGURE 19. Sinter test cell
FIGURE 20. Sinter test cell results

SINTER TEST RESULTS

R = thermal resistance
T = temperature
V = volume
Table 1: Descriptions of sinter test cells

<table>
<thead>
<tr>
<th>#</th>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>0.07 micron Ag powder (Vacuum Metallurgical) cold-pressed at 850 bar into Ag-plated cell. 43% packing fraction. 4.6 bar $^3$He pressure.</td>
<td></td>
</tr>
<tr>
<td>#2</td>
<td>0.03 micron Cu powder (Vacuum Metallurgical) was first reduced (165-180°C in He+H$_2$ atmosphere for 120 minutes; this caused 15.3% mass loss) then cold-pressed at 1110 bar. 43% packing fraction. 14.6 bar $^3$He pressure.</td>
<td></td>
</tr>
<tr>
<td>#3</td>
<td>Untreated 0.03 micron Cu powder (Vacuum Metallurgical) cold-pressed at 1200 bar. 34% packing fraction. 14.6 bar $^3$He pressure.</td>
<td></td>
</tr>
<tr>
<td>#4</td>
<td>1 micron Pd powder (Leico) packed at 1110 bar then sintered according to OSU recipe (Muething, 1979) using CO (810-860°C for 2 hours) and H$_2$ (570-600°C for 5 hours) and He &quot;flushouts&quot; while cooling. 52% packing fraction. 3.2 bar $^3$He pressure.</td>
<td></td>
</tr>
<tr>
<td>#5</td>
<td>0.03 micron Cu powder (Vacuum Metallurgical) packed at 855 bar then sintered at 200-220°C for 230 minutes in H$_2$ atmosphere. This caused 13% axial shrinkage and 15.5% mass loss. 34% packing fraction. 3.2 bar $^3$He pressure.</td>
<td></td>
</tr>
</tbody>
</table>

3.8 $^3$He Heat Exchanger Construction and Performance

The gross dimensions of the OFHC copper body of the heat exchanger and the locations of the cell mounting holes (see Figure 21) were chosen to duplicate those of the first exchanger for this cryostat, which used sintered palladium powder. Three independent wells, each divided into two or three annular sections by 0.8 mm thick cylindrical copper walls, were cut into the copper body using a spark etch milling technique.
FIGURE 21. Heat exchanger details
machine (EDM). The dividing walls insured that no portion of the sinter would be more than 1.6 mm from bulk copper. For sealing vessels to the heat exchanger wells with gold o-rings, a sealing surface much harder than annealed copper is required. To this end, and also to provide hard material for tapping the bolt circles for cell attachment, six flat brass rings were inlaid into grooves cut into the exchanger body at the upper and lower entrances of the three wells. The best means of doing this turned out to be first coating the appropriate exchanger and brass ring surfaces separately with silver solder (Silvaloy 355, Eutectic), removing all traces of flux, and finally brazing the rings into place in a hydrogen atmosphere at 700°C using greened stainless steel weights to prevent "floating" of the rings. This final brazing simultaneously annealed the copper body. A final, light EDM cut was made to align the inside diameters of the brass inlays with that of their respective wells. The protruding inlays were then machined to nearly the level of the copper body and drilled and tapped with appropriate bolt circles (#2-56).

At this stage leak tests uncovered two passages between screw holes tapped into the brass inlays and their respective wells due to gaps in the solder apparently caused by flux residue. These were successfully sealed by injections of Stycast 2850FT (Emerson and Cuming).

Preparation of the exchanger for packing with silver powder including light scrubbing to remove the EDM residue, followed by an acid bath dip to remove a partial brass surface on the copper body caused by diffusion of zinc from the brass inlays during annealing. All surfaces of the heat exchanger body were then plated with silver using a commercial jeweler's electroplating solution and fine silver anode (Jadow).
Although the depth of the wells probably attenuated the electric field near the well bottoms, this was apparently compensated by the electronegativity difference of silver and copper and the end result was at least an optically thick coating of silver on all exchanger surfaces.

After silver plating, the brass sealing surfaces were lapped to the same level as the copper body and then polished by wet sanding with fine emery paper. Final cleaning was done in an ultrasonic bath.

Packing the annular sections of the exchanger wells was done in layers one to three mm thick per pressing in order to assure uniformity of volume packing fraction throughout. The nominally 0.07 micron diameter "Japanese" silver powder was packed using a press constructed from an inexpensive 1.5 ton automobile hydraulic jack. Pressures applied to the silver varied from 370 to 410 bar. The final pressing left the 487 gram copper exchanger body holding some 61 grams of pressed silver powder. The volume packing fraction of the three wells varied from 46% to 48%.

Figure 21 shows the arrangement of the two sizes of holes which were drilled into the packed sinter. The two large holes, 2.4 mm in diameter, anticipate possible future use of the exchanger in flow experiments and greatly enhance hydrodynamic heat flow through the helium between the top and bottom of each well. The array of small holes, 1.0 mm in diameter, decreases the average distance between bulk liquid helium and the sinter interior.

Addition of the array of small holes improved the thermal time constant of the $^3$He exchanger at 3 mK from about 30 to about 6 minutes. The required density of these holes should be comparable to the characteristic length of the sinter, $l^*$, defined by (see Muething, 1979)
\[(\xi^*)^2 = \frac{k^*}{h_K} \frac{V}{A}\]  

(123)

where

\[h_K = \text{Kapitza conductance per unit area}\]

\[k^* = \frac{k_3 k_s}{k_3 + k_s}\]

\[k_3 T = ^3\text{He conductance in the sinter}\]

\[V, A = \text{sinter volume, area}\]

Using \(h_K\) derived from a measurement of Ahonen et al. (1978) and \(k_s\) found by normalizing the bulk value by the sinter to bulk ratio for palladium found by Muething and setting \(V/A\) to that of a single spherical particle of 0.07 micron diameter gives \(\xi^* = 2\) mm.

While the final version of this cold-pressed silver powder \(^3\text{He}\) heat exchanger has not been completely characterized as a function of helium temperature and pressure its performance has been entirely satisfactory. Below 10 mK, the thermal time constant for heat flow between 1/2 mole of helium and the copper bundle is about six minutes except at temperatures much below three millikelvin where the \(\tau_1\) of the copper nuclei becomes the important thermal "resistance" (\(\tau_1\) is 1.1 sec\cdot K for copper).

3.9 Compressor

Only two of the three heat exchanger wells were used in this experiment. Most of the liquid \(^3\text{He}\) sample was contained in these two wells and in the compressor. This compressor, whose outline is shown by
Figure 22, allows the $^3$He pressure to be precisely controlled, especially at pressures above 29.3 bar, where solid $^3$He begins to form in the warmer regions of the sample fill line.

This arrangement of concentric bellows is not the usual one for $^3$He compressors. The more common figuration is that used in Pomeranchuk (1950) compressors where the $^3$He bellows is everted so that a compression consists of an expansion of the $^3$He bellows, thus avoiding inadvertent compression of the solid helium which might form in the bellows' folds. Since solid growth in the compressor was not desired in the first place, the present design saves valuable space in the experimental region.

The two bellows are made of single-ply beryllium copper (Robert Shaw). The top and bottom beryllium-copper pieces act as caps for the $^3$He container walls of 321 stainless steel. The post inside the $^3$He bellows acts as a stop and conserves $^3$He volume while still providing channels sufficient for keeping the $^3$He in thermal equilibrium with the rest of the sample. All metal-to-metal seals were made with Stycast 2850FT (Emerson and Cuming) to avoid the use of superconducting solder; these seals held to pressures of at least 37 and 17 bar on the $^3$He and $^4$He sides respectively. Movement of the bellows could be detected by measuring the capacitance between electrodes glued to the outside of the $^4$He brass cap and the inside of the stainless wall.

The position of this compressor determines the total experimental $^3$He volume. The maximum stroke of 8.6 mm causes a volume change of 1.0 cm$^3$. Thus, when the $^4$He volume in the compressor is increased from its minimum of 7.6 cm$^3$ to its maximum of 11.0 cm$^3$, the total experimental $^3$He volume decreases from 11.4 cm$^3$ to 10.4 cm$^3$. 
\[ ^3 \text{He} \text{ Compressor} \]

**FIGURE 22.** \(^3\text{He}\) compressor
As mentioned in the introduction, attempts to grow single crystals of solid $^3$He in the sound transducer region failed because of solid nucleation elsewhere. My chief suspicion is that this solid grew inside the compressor due to a heat leak associated with "fast" (~1 bar/hour) compression. Irreproducibility and minor nonmonotonic behavior of the volume transducer signal suggests that the outer electrode may have become partially unglued from the stainless steel. Friction due to this defective transducer or to "scraping" of other parts may be the source of trouble. As part of a pressure regulation loop there have been no problems with the compressor, however.

3.10 Pressure Measurement and Control

A capacitive strain gauge measured the sample pressure in situ. The design used is similar to the original one first used for $^3$He by Straty and Adams (1969). Figure 23 shows a schematic diagram of the electronics used for the pressure and volume measurements. Although both transducers required a three-terminal capacity measurement, a total of only five cryostat wires was used by putting both signals on a common line at different frequencies. In both systems, a ratio transformer primary is driven at the reference frequency of either one or five kilohertz. The secondary is then adjusted so that the voltage at the sum point is nulled. The reference capacitors (20 pF) are anchored to the LHe pot.

Submicron position changes could be seen by the volume transducer but it had non-monotonic and irreproducible behavior attributed to a loose electrode. The sensitivity of the pressure transducer was about $10^{-4}$ bar and worked well up to 36 bar. The settings of the ratio transformer for the strain gauge were related to sample pressure through a
calibration made against a 1500 psi (103 bar) Heise Bourdon pressure
gauge. 22 points between 4 and 36 bar were recorded. This procedure
took several hours as each point required usually more than 10 minutes
to change the pressure and wait for pressure equilibrium. The resulting
data were fit to a second-order polynomial with an rms deviation of less
than 0.1 bar which is more than the estimated inaccuracy of the Heise
gauge obtained from $^3$He melting curve comparisons at the nuclear order-
ing temperature (1.0 mK) and at the melting curve minimum pressure (318
mK).

The pressure measurement system described above was used to
close the sample pressure by the feedback scheme outlined in Figure
24. The error signal of the $^3$He pressure lock-in amplifier drives a
heater in a $^4$He "bomb" suspended just beneath, and weakly coupled ther-
ically to, the top of the vacuum jacket. By appropriately fine tuning
the $^4$He and $^3$He amounts the bomb can be caused to operate near the
liquid-vapor critical point at 5.2 K where the expansivity of the
$^4$He is large. Gross adjustments to the sample volume and density are
made by adding or removing gas through the $^4$He and $^3$He fill capillar-
ies. The two control valves are always left shut except during these
adjustments.

The dependence of sound velocity on pressure imposes a re-
quirement on the stability of the pressure during a single run where
changes in phase velocity are measured. Referring to Figure 4 shows
that a velocity precision of $\Delta c/c \sim 10^{-5}$ requires $\Delta P \sim 10^{-4}$ bar in the
region where $(dc/dP)/c$ is greatest. The stability of pressure regula-
tion, as estimated by observation of the volume transducer output,
indicated that this requirement was met.
FIGURE 24. Pressure control loop
3.11 Sound Cell Contents

The cylindrical beryllium-copper cell slung underneath the middle well of the $^3$He heat exchanger contains an assembly of parts bathed in $^3$He and shown by Figure 25. Nine separate pieces fit inside the outer sleeve machined from Stycast 1266 (Emerson and Cuming). This sleeve is a hollow cylinder with four longitudinal slots allowing good $^3$He thermal conduction and absolute alignment of the radiofrequency (RF) coils with respect to externally applied fields.

The fraction of the total $^3$He sample actually probed is only about 1%: that portion contained between the two sound transducers separated by the cylindrical spacer. Sound pulses are transmitted and received by the two 9.53 mm diameter, 5 MHz, gold-plated X-cut quartz piezoelectric transducers. Ground connections to the transducers' inner faces are made via silver paint coating the ends of the spacer. Electrical connections to the opposite faces of the transducers are made by light, nonmagnetic springs, which also hold the transducers in place against the spacer. A small amount of indium solder holds a connecting wire (not shown) to the end of each spring.

The machined epoxy spacer defines the sound path length (6.25 ± .05 mm) and diameter (5.0 mm). These estimates allow for the thermal contraction of Stycast 1266 (Swift and Packard, 1979). Some 23 mm$^2$ of radial holes, including those created by the "crenelations" at the spacer ends, provide thermal conduction paths for the helium inside.

In order to perform NMR measurements on the same sample probed by the sound, a flattened "saddle-shaped" RF coil was wound on grooves cut into the spacer. The location of this coil is the reason that the more usual choice of metal or quartz for a spacer material was not
FIGURE 25. Sound cell contents
made. The coil was constructed by winding half of some 100 to 150 turns of #44 (45 micron) formvar-insulated copper wire on each side of the "saddle". At 930 kHz, its inductance was 95 microhenries and its Q was 15. The coil's orientation placed the field it produced perpendicular to the plane of rotation of the static field.

One interesting note is that a naive calculation of the eddy current heating induced in the gold transducer coating can give a value as high as many nanowatts at the RF levels and frequencies routinely used in this experiment. Apparently the details of the RF field configuration and its intersection with the gold film prevent such a disaster from occurring. The heating seen at RF currents ten times that ordinarily used was less than or on the order of one nanowatt.

Not shown in Figure 25 is a miniature heater for nucleation of solid growth for magnetically ordered solid $^3$He experiments. This heater consisted of a dab (~0.1 mm$^3$) of Stycast 1266 (Emerson and Cuming) containing 0.07 micron silver powder (Vacuum Metallurgical) near the percolation limit. This slurry was hardened across the cut faces of a twisted pair of 45 micron formvar-insulated copper wires. Heaters of this type had resistances of several hundred to several thousand ohms which were stable at low temperatures. Their small size enabled the location of nucleated solid $^3$He to be well controlled. Unfortunately, unpredictable "burnouts" limited their reliability. For the purposes of this (liquid) experiment it is sufficient to know that the unused heater extends radially inward to the cell center in the midplane of the spacer and takes up only 2% of the sound path area.

A second RF coil consisting of 220 turns of #44 (45 micron) formvar-insulated copper wire was wound in a "double solenoid" config-
uration around a machined epoxy tube, which was packed with 155 mg of 10 micron platinum powder (Leico). Cigarette paper seals at the tube's ends allowed $^3$He inside and defined the platinum packing fraction of 16%. This RF coil has an inductance of 130 microhenries and a Q of 15 at 250 kHz. This arrangement is for platinum NMR thermometry.

The other machined epoxy pieces are a spring backing plate and annular cylinders designed to reduce the $^3$He cell volume.

After arranging all ten connecting wires in place and gently tying the stack of epoxy parts into the slotted sleeve to tension the springs, the assembly was fitted into the metal sound cell body already bolted to the exchanger and fixed in orientation with respect to the exchanger. A beryllium-copper cap containing the wire feedthroughs was bolted and sealed to the sound cell body using a single gold o-ring. The two electrical feedthroughs consisted of two copper-nickel capillaries ($\sim$1 mm diameter) sealed by Stycast 2850FT (Emerson and Cuming).

### 3.12 Magnetic Fields

Magnetic fields applied to this experiment were created by a system of superconducting coils contained within the vacuum jacket and attached to the mixing chamber. This system, constructed by Gregory Spencer, consists of a vertical and horizontal "quasi-Helmholtz" solenoid pair, each having separate gradient and sweep capabilities. The location of the main vertical coil inside the vacuum jacket is shown in Figure 14.

Only the vertical quasi-Helmholtz pair was used in this experiment. The prefix "quasi" refers to the fact that the pair is analyzed numerically as a notched solenoid whose notch length is chosen for the best compromise between central field homogeneity and the desired size of the "homogeneous" region.
The field manipulating system pertinent to this experiment, outlined by Figure 26, consists of the main vertical coil, its associated flux transformer, the horizontal gradient coil, and the room-temperature driving electronics. Fields of 14, 28, and 42 mT were persisted in the vertical coil pair. With the experiment at 2 mK, changing the vertical field at the rate of $2 \times 10^{-5}$ Tesla/sec (0.2 G/sec) produced only a small increase in the bundle temperature. The interaction of the horizontal coil's field with the bundle and exchanger was such that it caused an order of magnitude more eddy current heating at comparable sweep rates.

Built into the circuit of the vertical coil is the secondary winding of a flux transformer. This field sweeping system (Spencer et al., 1982) gives the advantages of higher precision and less ambiguity due to flux exclusion by the main coil compared to the more usual method of sweeping with an independent coil. A power operational amplifier (BOP 36-5M, Kepco) configured for voltage-controlled current output drives the primary winding of the flux transformer. Its output amplitude is determined by the relative values of resistors $R_4$ ($1\Omega$, 100W power resistor), and $R_1$ (4000 $\Omega$ inside BOP). The voltage, $V_3$, across $R_3$ (1 m$\Omega$ precision shunt) is proportional to the output current. The 12-bit digital ramp constructed by James Robinson drives the BOP with a triangular sweep pattern and provides the trigger pulse for starting data acquisition by the digital oscilloscope. Driving the swept field by a current instead of a voltage minimizes problems due to changes in the lead resistances ($R_5$) and inductance-induced voltage lags at the waveform "corners."
Figure 27 indicates the field homogeneity over the sample seen by the $^3$He NMR coil. The sweep of $4.1 \times 10^{-5}$ Tesla superposed on a steady 14.3 mT shows that the majority of the affected $^3$He is in a field of homogeneity $\Delta B/B = 4 \times 10^{-4}$.

The horizontal gradient coil was used to enhance the tip of the NMR peak. Its effect on the overall field homogeneity or central field value was not important.

Field perturbations caused by the bundle demagnetization solenoid and large steel objects outside the cryostat caused some minor problems. In spite of its compensation winding, the bundle solenoid contributed a field to the sample region amounting to some 0.02% of the solenoid central field. Thus, even small demagnetizations produced a measurable shift of the NMR resonance. In addition, the bundle solenoid exhibited a relaxation of its fringing field comparable to the magnitude of the shift with a characteristic time of one hour after small ($\sim 3$ mT) demagnetizations.

A drift of $\sim 1 \times 10^{-11}$ Tesla/sec superposed on most NMR measurements was attributed to decay of either the persisted Helmholtz pair or bundle solenoid, equivalent to magnet decay times of order 10 or 0.1 years respectively.

A number of spontaneous jumps in the observed NMR frequency corresponding to field changes of $\sim 10^{-9}$ Tesla were eventually attributed to changes in the static field caused by changes in the positions of nearby large steel objects such as gas bottles or nitrogen dewars. Figure 28 shows the effect produced by various positions of a 0.1 m$^3$ hollow steel dewar. The observed field shift has very little hysteresis and is in approximate quantitative agreement with the effect expected
FIGURE 26. Static field control
Figure 27. NMR lineshape
Figure 28. Static field perturbation test
from the dipole field induced in a permeable object in the earth's field.

3.13 $^3$He NMR Electronics

The scheme I used to measure the $^3$He nuclear magnetic resonance (NMR) absorption signal can be described as that of Rollin's (1949) with three modifications. Rollin's method measures the voltage across a parallel LC "tank" circuit tuned to the NMR frequency and driven by a constant current. The amplitude and phase of this RF voltage is affected by the sample placed in the inductor at the resonance, which is found by sweeping the frequency.

The first modification to this scheme is to use field sweeping instead of frequency sweeping and thereby avoid complications caused by frequency-dependent properties of the electronic components. If the amplitude of the swept field is small, as was the case here ($\Delta B/B < 10^{-3}$), there are no problems associated with field-dependent properties of the sample.

Figure 29 should be referred to for an explanation of the second modification, namely cancellation of the RF "carrier" signal. The idea, which is the motivation for the various examples cited by Andrew (1956), is to use a sensitive means of detecting the "out-of-balance" NMR signal without saturating the detector.

The particulars of the "bridge" used are as follows. After attenuation to the desired voltage level, an RF synthesizer (General Radio 1163-A) drives the primary of a center-tapped RF transformer (Mini-Circuits MCL T4-1). The resulting two outputs, differing by 180°, drive approximately symmetric LC tuned circuits through the large resistances $R_1$ and $R_1'$ ($\sim$100 kΩ). The net voltage is coupled by $C_3$ and
$C_3'$ (22 pF) to the low-noise preamplifier (Analog Modules LNVA-V) and then detected. The resistance $R_3$ (120 $\mu$) is necessary to kill parasitic oscillations of the preamp and $L_3$ (1 microhenry) blocks 60 Hz variations of the preamp output. Tuning is accomplished by first setting $C_2$ to give the Larmour resonant frequency in the cryostat coil and then adjusting the "dummy circuit" component (primed) values to null the bridge output. While extinction ratios as small as $10^{-4}$ were possible, tuning stability typically limited this value to $\sim 10^{-3}$.

Ideally, the noise level of a measurement involving a string of amplifiers such as this should come entirely from the preamplifier. However, without the above carrier cancellation scheme, the noise of the high frequency lock-in detector is dominant. This is because the preamp output must be attenuated to avoid saturating the lock-in. The signal (and preamp noise) are thus reduced while the lock-in noise remains the same. Use of the bridge enhances the signal to noise ratio for $^3$He NMR under typical conditions by a factor of six.

The high frequency lock-in mentioned above (Princeton Applied Research 5202) represents the third modification to the Rollin scheme. Although not quantitatively characterized here, the use of phase-sensitive detection greatly enhances the sensitivity of the spectrometer. The vector sum of the in-phase and quadrature signals is the final analog output of the spectrometer.

3.14 Ultrasound Electronics

The electronic system used for sound measurements is illustrated by Figure 30. It consists of means for sending a short sound burst through the liquid helium and then detecting, amplifying, and processing this burst to get either attenuation or phase velocity change information about the medium of propagation.
ULTRASOUND ELECTRONICS

FIGURE 30. Ultrasound electronics
A five megahertz synthesizer's output (Hewlett-Packard 3325A) is gated and amplified by a Matec 310HP, which is triggered at 30 Hz synchronously with the 125VAC line. The resulting output, 10 cycle bursts of 5 MHz RF coherent with the pulse envelope, drives the transmitting quartz crystal after attenuation to a peak-to-peak amplitude of 0.60 volt. The signal induced in the receiving transducer is amplified first by a ham radio tuned preamplifier (Palomar P-308) and then by a 200 MHz wideband amplifier (Matec 625).

Different arrangements are then used to measure either attenuation or velocity. For measurements of attenuation, the amplified received pulse is detected and filtered at 500 kHz. A boxcar integrator (Princeton Applied Research CW-1) using a gate width of about 2 microseconds and gate delay of 20 to 30 microseconds gives the average voltage of the received waveform in the vicinity of the waveform maximum. From this amplitude the attenuation of the $^3$He relative to that at a reference temperature can be ascertained.

Changes in the phase velocity of the $^3$He can be measured if the phase of the received RF pulse is compared against a stable reference (here, a PTS 200 synthesizer tied to the HP synthesizer). Simply summing the reference and the received signal will give an output which goes as the cosine of the angle between the two inputs. Unfortunately, for changes in the input phase difference of more than a few degrees, this cosine relation between output voltage and input phase is a nonlinear one and even gives zero sensitivity at the 0 and 180-degree points. However, by using feedback, the phase of the transmitted signal can be continuously adjusted to keep it near the most favorable point (in terms of phase sensitivity) in relation to the reference phase.
Closing the two open switches in Figure 26 adds the reference signal to the received pulse before detection and allows the boxcar output to become the error signal for the phase controlling input of the synthesizer.

The analysis of the resulting phase-locked loop is discussed in Appendix A. From this analysis, one can see that the sensitivity of the loop output to changes in the phase of the received signal is (from (A10))

\[
\frac{\partial V}{\partial \phi} \cong \frac{1}{a}
\]  

(124)

where \(a\) is the phase-programming co-efficient of the helium signal synthesizer. For the Hewlett-Packard 3325A, \(a = 2.97\) radians/volt.
4.1 Refrigeration

Demagnetization cooling is inherently a cyclic process unlike dilution cooling. The period of this cycle for this experiment lasted from several days to two weeks and consisted of the steps outlined below. Magnetization of the 8T solenoid was typically done in about one hour using a rate of increase of the central field of 2.7 mT/second. The heat of magnetization of the copper nuclei typically resulted in bundle temperatures of 60 to 80 mK. After about three days the dilution refrigerator had reduced the bundle temperature to 28 to 30 mK resulting in a heat capacity of 7 to 8 J/K. The heat switch between the mixing chamber and the bundle was then opened and the bundle field was reduced in three steps. First, the central field was reduced relatively rapidly at 2.7 mT/sec from its initial value at 8.2 to 8.5 T down to 3.3 T. This rapid decrease conserves liquid helium by reducing the integrated heating in the upper portion of the magnet leads but does not appreciably raise the bundle entropy as the resulting eddy current heating occurs while the bundle is still relatively "warm" (>10 mK). Second, the field was further reduced from 3.3 T down to the neighborhood associated with the $^3$He superfluid transition (.15 to .50 T) at the slower rate of 0.27 mT/sec. Finally, the solenoid field was carefully adjusted to place the $^3$He temperature just below $T_c$. 
Over the week or two following the gross demagnetization, minute step decreases in the solenoid field were made one to four times per day to keep the nuclear stage in the vicinity of $T_C$. Although the temperature remained within 3% of $T_C$ during this entire time, the bundle entropy was of course continuously increasing, with a resulting loss in heat capacity. When the temperature drift rate had risen to an unacceptably high level (or some other experimental contingency arose) the bundle was remagnetized to begin another cycle.

4.2 Thermometry

Carbon resistance thermometers which had been previously calibrated against a germanium resistance thermometer (in turn calibrated against the $^3$He melting curve) were bolted to the $^3$He sample heat exchanger body, the mixing chamber, and various other, higher temperature places in the cryostat. In addition, thermometers calibrated against the exchanger body resistor were bolted to the bundle flange and one of the heat exchanger struts. The two-lead resistances of these thermometers were measured with a commercial AC-bridge (Linear Research) and were used as general indicators for the timing of bundle magnetizations and demagnetizations and the general "health" of the dilution refrigerator.

We determined $^3$He temperatures relative to $T_C$ by two methods: NMR of platinum nuclei and NMR of the sample itself. $^3$He NMR was used for all of the sound vs. temperature measurements while platinum NMR was used only for ancillary calibrations.

The susceptibility of platinum nuclei, which apparently follows the Curie law $\chi \propto 1/T$, (see Lounasmaa, 1974) can be measured by a pulsed resonance method to yield a relative measure of temperature. If
the platinum metal is finely divided to give a large surface area to volume ratio the coupling of the nuclei to the surrounding medium ($^3\text{He}$) is fairly rapid owing to its relatively short nuclei-lattice relaxation time $\tau_1$ ($\tau_1 T = 0.03 \text{ sec} \cdot \text{K}$). Our platinum thermometer is located close to the region probed by the sound (see Figure 25) so that it can measure the temperature of the central sample and also enjoy the magnetic field homogeneity of that volume. The driving, amplifying, and detecting electronics are contained in a commercial spectrometer (Instruments for Technology) whose tuned circuits limit temperature measurement to fields of 14, 28, and 56 mT. Typically, the spectrometer is configured to drive the RF coil surrounding the platinum so as to give a "tipping" pulse of $< 10^\circ$ from the direction (vertical) of the static field. This tipping angle (actually a classical view of the particular distribution of quantum mechanical spins with either "up" or "down" projections) decays back to zero according to $\tau_1$. However, the current induced in the RF coil after the tipping pulse is caused by the (classical) precession at the Larmour frequency of these tipped spins. The length of this signal is governed by the effective nuclei-nuclei dephasing time $\tau_2$, limited by our static field homogeneity to several hundred microseconds. This signal, after amplification, detection, integration, and subtraction of a small instrument-induced background, yields a number whose magnitude is inversely proportional to the temperature of the platinum nuclei.

The perpendicular NMR shift $\nu_A^2$ seen in $^3\text{He}$ below the transition temperature $T_C$ is very sensitive to temperature but apparently has not been relied upon until now as a thermometer due in part to questions of accuracy of the calibration information. Examination of the equa-
tions of Section 2.6 shows that, at any particular pressure, two calibrations are required: the products CA and CB. Here I will argue that data already existing in the literature coupled with a few additional measurements give a foundation for reasonably accurate thermometry in the A and Al phases near $T_c$ for all pressures.

Leggett's (1974) prediction for the characteristic A-phase shift frequency $\nu_A$ is explicitly given by Equation (51) but this expression includes the quantities $\varepsilon_c$ and $\langle R^2 \rangle$, which are not directly measurable. If these two quantities are only weakly dependent on pressure then, to an excellent approximation, the expression for the pressure dependence of the slope of $\nu_A^2$ vs. temperature can be used with $\varepsilon_c/k$ defined as 0.7 K and $\langle R^2 \rangle$ equal to a constant fit to experimental data. Figure 31 justifies this procedure by showing various groups' experimental determinations of $f(P)$ (also listed in Table 2) as well as points calculated from the theoretical relation (51) with $\varepsilon_c/k = 0.7$ K and $\langle R^2 \rangle = 0.70$. This particular value of $\langle R^2 \rangle$ was chosen in order to cause the calculated values of $f(P)$ to fall near most of the experimental points. The data of Webb et al. (1975) may differ from the others because of differences in temperature scales. As these problems of temperature scaled have not been completely resolved (and may never be as far as old data is concerned) the LaJolla data were ignored for making the choice of $\langle R^2 \rangle$. 
TABLE 2

\[ f(P) \text{ determinations, where } v_A^2 = f(P)(1-T/T_C) \]

<table>
<thead>
<tr>
<th>Pressure (bar)</th>
<th>( f(P)(10^{10}\text{Hz}^2) )</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>34.3</td>
<td>5.04 ± 0.02</td>
<td>Osheroff and Anderson (1974) + Halperin et al. (1975)</td>
</tr>
<tr>
<td>21.5</td>
<td>3.8</td>
<td>Webb et al. (1975)</td>
</tr>
<tr>
<td>25.8</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td>29.4</td>
<td>5.3</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>21.1</td>
<td>2.87 ± 0.20</td>
<td>Ahonen et al. (1976)</td>
</tr>
<tr>
<td>25.4</td>
<td>3.55 ± 0.10</td>
<td></td>
</tr>
<tr>
<td>29.0</td>
<td>4.28 ± 0.20</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>4.78 ± 0.20</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>3.17 ± 0.20</td>
<td>Giannetta (1980)</td>
</tr>
<tr>
<td>27</td>
<td>3.63 ± 0.20</td>
<td></td>
</tr>
<tr>
<td>2.05</td>
<td>0.57 ± 0.06</td>
<td>This work</td>
</tr>
</tbody>
</table>

Questions of temperature scale also enter into the calculation of the theoretical \( f(P) \) from experimental data. Values of \( T_C \) and \((\Delta C/C_N)\) were interpolated from the data of Alvesalo et al. (1981) and values of \( N_F \) and \( F_0^a \) were taken from Appendix B of Wheatley (1975) but scaled according to the more recent heat capacity measurements at Bell Labs (Greywall, 1983). See Appendix B for details.

Although the experimental determinations of the slope of \( v_A^2 \) are sufficiently dense between 20 and 34 bar to calibrate the \(^3\)He NMR thermometry there, extrapolation to significantly lower pressures is risky without at least one low pressure determination of the slope of \( v_A^2 \) vs. temperature. Doing so at 2.05 bar in the 6 \( \mu \)k "window" of A-phase created by a 28 mT field was accomplished by relying on the fact that our residual heat leak is time-independent. In the refrigeration mode where small demagnetization steps are used to keep the \(^3\)He and
copper bundle temperature very near $T_c$ and for solenoid fields $B$ such that the $^3$He heat capacity is negligible compared to that of the magnetized bundle, the temperature drift rate $\dot{T}$ is simply related to the solenoid current $I$ and its average rate of change $\dot{I}$ necessary to keep $T$ near $T_c$.

$$\frac{\dot{T}}{T_c} = \frac{\dot{B}}{B} = \frac{\dot{I}}{I}$$

(125)

Knowledge of $\dot{I}/I$ combined with a measurement of $d\nu_A^2/dt$ then yields $f(P)$. For NMR thermometry in or near the Al-phase using $\nu_A^2$ one must know the Ginzburg-Landau parameter $\delta$ which measures the coupling of the up and down-spin gaps. The ratio of the temperature derivatives of $\nu_A^2$ in the A and Al phase is

$$\frac{(d \nu_A^2/dT)_A}{(d \nu_A^2/dT)_{Al}} = \frac{CA}{4CB} = \frac{(1 - \delta)}{4}$$

(126)

At the melting pressure, Osheroff and Anderson (1974) found $\delta = 0.25 \pm 5\%$. To my knowledge there has been no determination of $\delta$ at lower pressures although one would expect $\delta$ to decrease at lower pressures as it is a "strong coupling" parameter of sorts. Although I had hoped to obtain low pressure measurements of $\delta$, the narrow width of the Al phase in our low fields allowed determinations of limited precision, as the data of Figure 32 show. Nevertheless, this plot does strongly suggest that $0 < \delta < 0.25$ is a good estimate at all $^3$He pressures. For purposes of data reduction I chose $\delta = 0.25$ for all pressures. If $\delta$ should indeed be as low as zero at the lower pressures, this would result in the maximum possible Al phase temperature measurement error of 25%.
Figure 31

Various determinations of the slope $f(P)$ of $v^2$ vs. temperature near $T_c$. The points at multiples of 3 bar are calculated from the theoretical relation (Equation (51)) using the input of Appendix B and with $\langle R^x \rangle = 0.70$. 
FIGURE 31. Slope of $\nu_A^2(T)$, $f(P)$
4.3 NMR in $^3$He

In typical operation the field $B_1$ produced by the $^3$He NMR coil was $\sim 3 \times 10^{-5}$ tesla ($\sim 0.3$ Gauss). Changing the static field and thus the NMR frequency involved the following procedure (see Figure 29). The persistence of the vertical field coil was broken and its current was ramped toward the new field at $\sim 2 \times 10^{-5}$ tesla/sec. Meanwhile, the frequency synthesizer, cryostat tank circuit, and "dummy" tank circuit were tuned to the new frequency with the latter adjustment made last to achieve an extinction ratio of $< 10^{-3}$. When the vertical field reached the desired value it was persisted. From then on, frequency and not field adjustments were made to maintain the Larmour resonance condition as the "background" field changed due to drift and effects of the bundle solenoid fringing field.

Some fine tuning of the static field was done by applying a weak horizontal field gradient. This did not significantly improve the overall field homogeneity but did have a beneficial effect on the peak structure. This structure, as seen in Figure 33, is caused by the homogeneity of the static field $H(r)$, weighted by the local $^3$He density $n(r)$ (which can be zero due to displacement of epoxy parts, etc.) and $H_1(r)$, the field of the RF coil. The signal at a particular frequency $\omega$ is

$$S(\omega) \propto \int d^3r \, H_1(r)n(r) \left[ \frac{\gamma H(r) \tau_2}{1 + (\gamma H(r) - \omega)^2 \tau_2^2} \right] \tag{127}$$

The horizontal gradient chosen was that which enhanced the "spike" appearance of the central part of the lineshape, improving the precision of the measured frequency shift, which was determined by fitting a
FIGURE 33. Field tuning of NMR peak
parabola to the tip of this spike. Figure 33 shows a series of line-shapes obtained at different gradient currents. In this example −350 mA gives the best shape for "tracking" purposes.

4.4 Ultrasound Signals

The excitation voltage amplitude (0.60 volt peak-to-peak) and duty cycle (2.0 μsec every 1/30 sec) were chosen as a compromise between heating and available output signal. Pulsing the transmitting transducer at a 6 Hz rate but at 15.1 volts peak-to-peak caused some 30 to 60 nW heating at ~1.5 mK, 20.5 bar (normal liquid). Assuming that this heating is proportional to the input electronic power, the local heat leak due to the transducers during data collection was then some 0.2 to 0.5 nW. How much of this is directly due to the viscosity of $^3$He is not known though this should be calculable.

Figure 34 shows an example of an averaged detected pulse illustrating the boxcar window position in time relative to the received signal. The structure following the "zeroth" echo is not quantitatively understood but is apparently due to the coherent summation of successive echoes enabled by the ringing of the quartz transducers. The boxcar window was always placed in the vicinity of the first maximum to avoid effects due to following echoes and also to avoid the spurious feedthrough signal as much as possible. This feedthrough signal, visible as the portion existing before the first helium signal at 28 microseconds, is apparently a sum of excitations caused by ringing of the tuned preamp and the receiving transducer caused by capacitive pickup of the electronic transmitter pulse. Its magnitude relative to the "real" helium signal is large for this example (2.05 bar) as the normal liquid attenuation is largest for low pressures. The amplitude of the feedthrough
signal was always only a few percent of the helium signal except near the superfluid attenuation maximum at 2.05 bar. The amplitude dependence of the phase-locked loop for velocity measurement (see Appendix A) then became an overwhelming effect and denied the possibility of performing A-phase velocity measurements at that pressure.

The relative magnitude of the feedthrough-induced signal was larger when the crystals were driven at their next odd harmonic, 15 MHz. Because of uncertainty in the resulting data corrections which might be required, no 15 MHz data will be presented in this thesis.

To indicate the linearity of the entire receiving chain, Figure 35 shows a plot of signal output vs. input relative to the amplitude at just above $T_c$ at 21.53 bar. Nonlinearity is much less than 10% until below a relative amplitude of about 0.3, again affecting only the 2.05 bar runs.

A small, periodic modulation seen in some of the velocity data was found to be related to the air temperature of the screen room, with the periodicity caused by cycling of the supplemental air conditioner directed into the screen room. This coupling of the velocity data to room temperature apparently occurred in the phase shifting circuit of the signal synthesizer (Hewlett-Packard 3325A), which has a rated error of ±1° phase/1°C. The amplitude of these room temperature-induced oscillations was damped by a factor of $\sim 3$ by a passive low-pass thermal "filter" added to the air blower input of the synthesizer.

As a final note, no effects (due to the nature of $^3$He or otherwise) caused by coupling of the NMR and the sound were ever noticed. Tewordt and Schopohl (1979) have studied this possibility theoretically.
FIGURE 34. Detected sound pulse appearance
FIGURE 35. Linearity of attenuation measurements
4.5 Data Acquisition

Recording the information contained in the $^3$He NMR and ultrasound signals included conversion of the analog signals to digital form and some "real time" data reduction. The entire process followed a two minute period timed by the digital ramp sweeping the static NMR field. The various states of the system during this two minute period are illustrated by Figure 36. The output of the digital ramp is shown at the top of the figure. At the start of an up sweep, a trigger signal sent by the ramp causes the 2-channel signal averager (Nicolet 1170, actually only used for digital to analog conversion) to begin recording the NMR and ultrasound signals (MEASURE mode). At the end of the 30-second sweep, the Nicolet sends this information to the computer (Hewlett-Packard 9845A). This "READOUT" mode lasts 80 seconds due to the slow speed of the BCD serial data transfer. After receiving the data, the computer finds the highest NMR point and fits a parabola to the neighboring points. The single number representing the fitted peak location is recorded. The sound data, simultaneously accumulated during the field sweep, are fit to linear function of time $S(t)$. The value of $S(t)$ at the time represented by the NMR peak location is recorded. Data files stored on magnetic cassette tapes thus consisted of up to 200 two-component "points" at two minutes per point.

Data taking commenced after demagnetizing the bundle sufficiently to cool the $^3$He to .99 $T_c$. After an equilibration time of 30 minutes, the $^3$He temperature drifted linearly up toward $T_c$ over the next several hours. Points were accumulated above $T_c$ to establish data baselines before stopping data acquisition. Calibrations of the sound signal were performed at least once at each pressure at just above $T_c$ to
obtain data reduction parameters. Appendix C is the BASIC program executed on the 9845A computer during data acquisition.
FIGURE 36. Data acquisition cycle
5.1 **Data Examination**

A total of about 210 files of raw sound and NMR data were stored on magnetic cassette tapes for these measurements, representing approximately 3 weeks of actual data acquisition spread out over four months. Of the 5 MHz runs (the large majority), about one third were rejected upon examination of the data plots due to problems caused by a high level of noise in the NMR data or an insufficient bundle demagnetization. A smaller fraction of the remaining runs were rejected after data reduction when it became evident that these runs had been done at incorrect instrument settings.

For data reduction, each data file was examined and the point nearest $T_{c1}$ as identified by its signature in the sound or NMR data was defined. This point plus a second, warmer point, typically defined an hour later in the data, established a region of normal fluid data for baseline purposes. Reduction of the NMR and sound data then proceeded separately as described below and the sets of reduced data were afterward combined to give plots of sound dispersion vs. temperature. The BASIC program used for data reduction is listed in Appendix D.

5.2 **Temperature**

Before deconvoluting the temperature of the $^3$He sample from the measured NMR shifts, some corrections for systematic errors had to
be made. The first correction dealt with the uniform change in time of the NMR frequency caused by decay of one or both of the persisted solenoids in the cryostat. A baseline defined by a linear fit to the defined normal region was subtracted from all of the NMR data. The effects of a comparable relaxation change seen shortly after persisting the bundle solenoid were avoided by ignoring any data taken in a period of about an hour following the persistence of this solenoid.

The second correction dealt with discontinuities in the NMR data apparently caused by changes in the positions of large iron objects in the neighborhood of the cryostat. The data were corrected for these "jumps" simply by requiring continuity at the dislocation and adding an appropriate constant to all of the points to the right of the dislocation. Such a "healing" process, required an average of about once every 10 hours of data, was usually extremely successful as judged by the smooth appearance of the data afterwards.

After performing these systematic corrections, the magnitude of $v^2_A$ was calculated and from this the temperature was deduced. The relation between $v^2_A$ and the actual data, namely signal averager addresses, can be derived for field sweeping as

$$\frac{v^2_A}{A} = 2\nu\gamma H_2(n-n_0)/N + \left[\gamma H_2(n-n_0)/N\right]^2 \quad (128)$$

Here, the signal averager records the NMR amplitude as it steps through $N$ addresses. During this time the static field is modulated by the total amount $H_2$. $(n-n_0)$ is the address difference of the shifted peak from its location at $T_c$ and $\nu$ is the constant frequency of observation. For this experiment, the second term of Equation (128) was
\( \sim 3 \times 10^{-4} \) times the size of the linear term and was ignored for the data analysis.

Knowing \( \nu_A^2 \), the reduced temperatures are obtained by inverting Equations (61) and (62).

\[
U = \frac{4\nu_A^2}{AC} - 1, \quad 0 < \nu_A^2 < AC/4 \quad (129)
\]

\[
U = \frac{(4\nu_A^2 - AC)^2}{16BC \nu_A^2}, \quad AC/4 < \nu_A^2 \quad (130)
\]

\[
\frac{T}{T_c} = 1 - \frac{[(1-\delta)/2 + U](T_{c2} - T_{c1})}{T_c} \quad (131)
\]

The values of \( \delta \) and \( \nu \) (cf. Equations (42, 43, 44)) used for thermometry deconvolution are the melting curve values obtained by Osheroff and Anderson (1974). As each file's data was reduced, an estimate of the location of the \( T_{c2} \) point was made as a check against these values. Within the scatter of the data (\( \sim 20\% \)) the ratio of this independent estimate of the Al-phase width to that expected was always unity.

The values of the A-phase shift coefficient \( BC \) were obtained by interpolation from Figure 31 (\( \langle R^2 \rangle = 0.70 \)). These parameters are listed in Appendix B.

As an example, Figure 37 shows the NMR data for a single short run. It contains the unusually high number of three defects (at 94, 140 and 208 minutes). Important features are points taken as the sample cooled immediately after demagnetization (0-40 minutes), points in the Al-phase (230-298 minutes), and normal points showing the background drift (300-400 minutes). After subtracting the background drift,
FIGURE 37. Deconvolution of temperature from NMR
healing the discontinuities, and dropping the first hour of data, the reduced temperature vs. time is very nearly linear up to $T_{c1}$ ($U = -1$) where of course the thermometry no longer works. The greater scatter of points in the Al phase ($U = -1$ to 0) is caused by the smaller slope of the NMR shift there as shown by Figure 7.

Within the scatter of the data, on each run the temperature drift $\dot{T}$ in the A and Al phases was always the same with no discontinuity. This is at first surprising in light of the fact that the heat capacity jump at $T_c$ is split into approximately equal parts at $T_{c1}$ and $T_{c2}$ by a magnetic field (Halperin et al., 1976). Apparently the heat leak which caused $\dot{T}$ entered the nuclear stage via the $^3$He instead of vice versa. This would be expected since $\tau_1$ for the copper nuclei at these temperatures is long enough to dominate the thermal relaxation of the $^3$He.

5.3 **Sound Amplitude and Velocity**

Sound amplitude data were subtracted from a baseline defined as the average of the raw data in the same normal region span used to define the NMR baseline. Subsequent reduction of the data depended on whether velocity or attenuation data was handled.

The calibrations of the voltages representing attenuation were made by making two measurements at $T > T_c$ that were 3dB apart and assuming linearity. The exception to this procedure was for the 2 bar calibration, where the attenuation was high enough that the nonlinearity of the measuring system became important. Here, the calibration was defined as the difference between the normal operating value and that obtained with no transducer excitation whatsoever. This definition increased the accuracy of the high attenuation points at the expense of
the lower attenuation points. Since the absolute attenuation was not measured, all measurements are referred to the attenuation at $T_c$.

Calibrations of the sound velocity data were made in the limit of small sound velocity changes $\Delta c$, where the relative sound velocity change in a cell of length $L$ is proportional to the shift in the received phase of the sound after crossing the cell.

$$\frac{\Delta c}{c} = \frac{-c}{\omega L} \Delta \phi$$  \hspace{1cm} (131)

The calibration between phase shifts and the output voltage of the phase-locked loop was simply a matter of changing the signal synthesizer phase by a known amount relative to the reference and recording the change in the phase-locked loop voltage.

A third calibration defined the undesired effect of the signal amplitude on the output of the phase measurement system (see Appendix A). This effect was assumed to be linear in the amplitude of the signal. The correction was a maximum of $\sim 10\%$ at the lowest pressure and significantly less at higher pressures.

The deconvoluted data plots typically exhibit a knot of points at $T_{c1}$. This represents the normal liquid points which, due to the absence of a real NMR shift above $T_{c1}$, were compressed into the region near $T_{c1}$. 
SECTION 6
RESULTS AND INTERPRETATIONS

6.1 Assumptions

Figures 38 through 49 display the coldest temperature run for each of the different pressure and field combinations. For all of the runs, the applied field \( H \) was parallel to the sound vector \( q \). Ideally this should limit the axis of the order parameter \( \hat{\mathbf{\xi}} \) to lie in the plane perpendicular to \( H \) and \( q \). Since the sound dispersion should depend only on \( \hat{\mathbf{\xi}} \cdot q \), the theoretical results calculated for the perpendicular components of the attenuation and velocity shift should apply.

To date, the only quantitatively successful attempt at fitting experimental A-phase sound data is a series of rather involved numerical calculations for 24, 26, and 33.5 bar (Wölfle and Koch, 1978). I cannot say at this time if this scheme would fit our data. Certainly, it cannot be directly applied to the Al phase as Wölfle and Koch assume zero magnetic field. Instead, I will use the collisionless theory as a guide for comparing the results between pressures. The Ginzburg-Landau result for the magnitude of the energy gap will be assumed here

\[
\left( \frac{\Delta}{kT_c} \right)^2 = \frac{2\pi^2}{3} \left( \frac{\Delta C}{C_N} \right) \left( 1 - \frac{T}{T_c} \right)
\]

(132)

where it is hoped that all strong coupling effects are accounted for by the experimentally measured heat capacity jump at \( T_c \), \( \left( \Delta C/C_N \right) \).
FIGURE 38. Data plot: 2 bar, 14 mT
FIGURE 39. Data plot: 2 bar, 42 mT
FIGURE 40. Data plots: 9 bar, 14 mT
FIGURE 41. Data plots: 9 bar, 42 mT
Reduced temperature ($T/T_c$)

FIGURE 42. Data plots: 18 bar, 14 mT
FIGURE 43. Data plots: 18 bar, 29 mT
Reduced temperature (T/Tc)

FIGURE 44. Data plots: 18 bar, 42 mT
Reduced temperature ($T/T_c$)

FIGURE 45. Data plots: 22 bar, 14 mT
FIGURE 46. Data plots: 22 bar, 42 mT
FIGURE 47. Data plots: 31 bar, 14 mT
FIGURE 48. Data plots: 31 bar, 29 mT
FIGURE 49. Data plots: 31 bar, 42 mT
6.2 Attenuation

The simplest characterization of the A-phase attenuation data is made by a list of the heights and temperatures of the maximum attenuation peaks. Since no significant systematic variation of those quantities as a function of magnetic field strength was seen, average values at each pressure can lump the 14, 29, and 42 mT data together.

The collisionless theory says that the sound attenuation should be

\[ \alpha_o = \frac{\hbar \omega^2}{c_0 F S kT_c} \text{Im}(\chi) \]  
(133)

where \( \chi \) is a universal function of \((\Delta/\hbar \omega)\) only. If one interprets the maximum attenuation peak at each pressure as being caused by the same value of \( \chi \) (e.g. \( \chi(1/1.23) \) for the clapping mode) then the heights of these peaks should scale as the prefactor of \( \chi \) in Equation 133. Figure 50 shows a plot of the peak heights divided by the appropriate factor, essentially a plot of the "experimental Im(\chi)" vs. pressure. Considering that the peak height increases by a factor of 30 from the highest to the lowest pressure, this scaling works remarkably well. The "experimental Im(\chi)" values are significantly smaller than the collisionless theory prediction. The clapping resonance of Figure 50 has its maximum at \( \text{Im}(\chi) = 1.08 \) whereas the experimental data lie in the range of 0.17 to 0.33. This discrepancy could be explained qualitatively by invoking the "smearing out" effect of quasiparticles with finite lifetimes.

Unfortunately, at the lowest pressure, one would expect only a fourth of this smearing effect as exists at the highest pressure because of the \( T^2 \) dependence of the quasiparticle lifetime. Much more cannot be said without numerical calculations to include these dissipation effects.
If the attenuation peak maximum is indeed due to the clapping resonance then $T/T_c$ for this peak should result from inserting $\gamma \omega = 1.23 \Delta$ into Equation (132). Figure 51 is a plot of this theoretical value and the experimental results vs. pressure. The agreement is good only at low pressures. From the numerical plots in Wölfle and Koch (1978), the location of the $\alpha_\perp$ peak is apparently little affected by the inclusion of dissipation. For this reason, quasiparticle lifetime effects may not explain the peak location discrepancy. Problems with input data for the thermometry calibration could be suspect but it should be noted that corrections to $(\Delta C/C_N)$ or $kT_c$ shift both the experimental and theoretical points in the same direction. Another possibility is that specular reflection in the sound cell significantly mixes in the attenuation components important for $\mathbf{\hat{z}} \cdot \mathbf{q} \neq 0$. For example, the peak in $\alpha_\perp$ occurs at a temperature 2.4 times colder than the $\alpha_\perp$ peak due to clapping, about the ratio of the discrepancy for the high pressure data of Figure 51. The data of Paulson et al. (1977) also show 5 MHz attenuation peaks which are "too cold" but the discrepancy is less, only about a factor of 1.7.

The attenuation peak at 2.05 bar shows a distinct twofold structure near the maximum attenuation. This feature was also seen in horizontal fields, for $\mathbf{\hat{q}} \cdot \mathbf{H} = 0$. It may represent structure masked at higher pressures due to quasiparticle collisions such as a contribution by $\alpha_\perp$. Another possibility is that the extra peak is an artifact caused by a combination of RF feedthrough and the rapidly changing velocity of sound in this region. Such a "beating" effect is quite possibly the origin of the attenuation satellites seen in the B-phase at low pressures (Paulson and Wheatley, 1978).
FIGURE 50. Scaled attenuation maxima
LOCATIONS OF ATT‘N MAXIMA

$\Delta = \text{COLLISIONLESS THEORY PREDICTIONS}$

$\left(1 - \frac{T}{T_c}\right) \times 10^3$

$P \text{ (bar)}$

FIGURE 51. Locations of attenuation maxima
The effects of the Al phase on the sound attenuation can be easily seen in most of the data at 42 mT. The rounded shoulder appearance is apparently a separate attenuation peak starting to separate from the main peak as suggested by the higher field data of Lawson et al. (1974).

6.3 Phase Velocity Changes

The rate at which the sound velocity drops below $T_c$ from its normal liquid value increases by roughly a factor of two on going from the highest pressure (31 bar) to the lowest pressure at which a velocity measurement was made (9 bar). The low temperature limit of this velocity drop is expected on theoretical grounds to be the first sound velocity $C_1$. However, an attempt to scale the velocity runs by $(C_0 - C_1)/C_0$ did not work very well. Turning again to the collisionless theory for superfluid sound dispersion, the velocity change near $T_c$ is seen to be proportional to the gap for $T$ near $T_c$ (cf. Equation (110)).

$$\frac{c - c_0}{c_0} \propto \frac{\Delta(T)}{F_0^s kT_c}$$

This suggests a scaling by the factor

$$\frac{\Delta}{F_0^s kT_c (1 - T/T_c)^{1/2}} \propto \frac{(\Delta C/C_N)^{1/2}}{F_0^s}$$

Figure 52 shows that such a scaling is approximately valid.

The shape of the velocity vs. temperature curves is qualitatively different from the zero sound picture, which predicts exactly zero shift in the velocity for temperatures above the pair breaking limit $\hbar \omega = 2\Delta(T)$. Although the shape of the velocity curves initially
suggests it, fitting the curves to a constant times \((1 - T/T_c)^{1/2}\) was not successful. For lower temperature data \(\hbar\omega \ll \Delta\), where the velocity is expected to go as the viscosity, such fits might be possible.

There may be a connection to viscosity in the Al phase velocity data. The shapes of the reduced viscosity plots in finite fields (Alvesalo et al., 1975) are suggestive.

The lowest pressure data at 9 bar seems to suggest little or no drop in the velocity above \(T_{c2}\).

6.4 Metastability at the AB Transition

At the lower pressures of 2 and 9 bar, the first-order character of the transition between the A and B phases of superfluid caused hysteresis in the location of the transition temperature \(T_{AB}\). Figure 53 shows a particularly clear example of supercooling and superheating at 9.15 bar in a 14.3 mT field. The upper and lower plots are raw data vs. time for the NMR and ultrasound signals respectively. The temperature as a function of the time first drops after the initial bundle demagnetization, "turns around" at 30 minutes, and then warms at 40 nanokelvin/minute, passing through \(T_{c1}\) at 212 minutes.

As the sample warms, the B-phase exists between 62 and 154 minutes. This is marked by both the NMR and sound data - the later transition occurs very near the sound attenuation maximum and the resulting amplitude jump in the data is small. These sound signatures, plus the different slope of the B-phase NMR, eliminate the possibility that the NMR jumps were caused by a perturbed static field as was the case for Figure 37.

The temperature for the B to A transition occurs in this run at about the same place for runs without A-supercooling so it is not
A-B HYSTERESIS

FIGURE 53. AB transition hysteresis
clear that the B-phase is superheated here. The degree of supercooling of the A-phase is comparable to other measurements (e.g. see Wheatley, 1975) but the apparently unprovoked A to B transition is an unusual feature. It may be significant to note that no highly magnetic solid such as $^3$He or CMN refrigerant was in direct contact with the sample. Also, the immediately previous history of the sample before this run started included about 80 minutes at $\sim$1.001 $T_c$.

6.5 Summary and Future Work

The two chief results of these experiments are a practical demonstration of $^3$He NMR as a superfluid thermometer and a systematic comparison of low-frequency zero sound behavior in superfluid $^3$He-A with the collisionless theory over a pressure range of 29 bar. In particular, the pressure scaling of the sound attenuation and velocity is in fair agreement with the collisionless predictions. The temperatures of the attenuation peaks show a major discrepancy, especially at high pressures. This discrepancy, about a factor of 2.4, is the same as that observed by Paulson et al. (1977) if their temperature scale is the one which produced their NMR shift data on Figure 31.

Other features observed in the data include a double attenuation peak at 2 bar and strong supercooling of the A-phase at low pressures. The data for the A1 phase awaits theoretical explanation.

Extensions of these measurements in the future should gather data at new pressures, magnetic fields, and sound frequencies. The pressures between 2 and 9 bar and in the vicinity of the zero-field polycritical point would be interesting; the former because of the apparently qualitative changes in the sound attenuation which take place there and the latter because of the possible "step" structure in the
data of Figure 50. Different field orientations could be used to determine the importance of the contributions of $\lambda_\parallel$ and $\chi_\perp$ (see Wheatley, 1973). Finally, attenuation measurements at the next transducer harmonic, 15 MHz, would allow comparisons with the experimental work of other groups, in addition to those made here at 5 MHz.
APPENDIX A

PHASE LOCKED LOOP ANALYSIS

Figure 54 shows the various phase shifts in the loop involving the two synthesizers, the $^3$He sample and the amplifying, summing, and detecting electronics.

The various phases are

$\phi_0 =$ reference synthesizer phase

$\phi_1 =$ signal synthesizer phase

$\phi_3 =$ phase of signal after passing through cables, amplifiers, transducers, and sample cell

The various voltages are

$A\sin(\omega t+\phi_1+\phi_3)$ = signal input to adder

$B\sin(\omega t+\phi_0)$ = reference signal input to adder

$V_1 =$ AC-coupled output of detected signal to boxcar

$V_2 =$ boxcar output which controls signal synthesizer phase

Since the duty cycle of the pulsed signal is extremely low, the AC-coupled output from the detector to the boxcar is just

$$V_1 = \sqrt{A^2+B^2+2AB\cos(\phi_1-\phi_0+\phi_3)} - B$$

(A1)

The output of the boxcar is
\[ V_2 = CV_1 \quad , C = \text{constant} \quad (A2) \]

which controls the phase of the helium signal synthesizer via the relation

\[ \phi_1 = \phi_0' + aV_2 \quad , \quad \phi_0' = \text{constant} \quad (A3) \]

Using (A2) and (A3) in (A1) gives

\[ V_2 = C \left[ A^2 + B^2 + 2AB \cos(\phi_0' - \phi_0 + aV_2) \right]^{1/2} - B \quad (A4) \]

In the ideal limit that \( B >> A \), (A4) reduces to

\[ V_2 = (CA/B)[A/2 + B \cos(\phi_0' - \phi_0 + aV_2)] \quad (A5) \]

With the loop locked, the quantity in brackets can be set near zero volts. This, plus the requirement that \( B >> A \), allows me to set

\[ \cos(\phi_0' - \phi_0 + aV_2) = \pi/2 - (\phi_0' - \phi_0 + aV_2) \quad (A6) \]

Inserting (A6) into (A5), the output of the loop is seen to be

\[ V_2 = \frac{(CA/B)}{\left(1 + aCA\right)} \left[ A/2 - B(\phi_0' - \phi_0 + aV_2 - \pi/2) \right] \quad (A7) \]

The dependence of \( V_2 \) on changes in \( \phi_3 \) (and therefore on changes in the helium phase velocity) is simply
\[ \frac{\partial V_2}{\partial \phi_3} = \frac{CA}{(1 + aCA)} \quad (A8) \]

The (unwanted) dependence of \( V_2 \) on amplitude changes in more complicated.

\[ \frac{\partial V_2}{\partial A} = \left[ \frac{1}{A} - \frac{aC}{(1 + aCA)} \right] V_2 + \frac{CA/B}{(1 + aCA)} \quad (A9) \]

Equations (A8) and (A9) can be simplified further since \( aCA \gg 1 \), i.e. the loop has a high gain.

\[ \frac{\partial V_2}{\partial \phi_3} = \frac{1}{a} \quad (A10) \]

\[ \frac{\partial V_2}{\partial A} = \frac{1}{a} \left( \frac{V_2}{CA} + \frac{1}{2B} \right) \quad (A11) \]
FIGURE 54. Phase-locked loop
### APPENDIX B

#### DATA REDUCTION PARAMETERS

<table>
<thead>
<tr>
<th>P (bar)</th>
<th>$T_c$ (mK)</th>
<th>$c_1$ (m/sec)</th>
<th>$(c_1 - c_0)$ (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.05</td>
<td>1.34</td>
<td>216</td>
<td>5.96</td>
</tr>
<tr>
<td>9.15</td>
<td>1.99</td>
<td>287</td>
<td>3.85</td>
</tr>
<tr>
<td>18.18</td>
<td>2.44</td>
<td>346</td>
<td>2.80</td>
</tr>
<tr>
<td>21.53</td>
<td>2.55</td>
<td>363</td>
<td>2.62</td>
</tr>
<tr>
<td>31.12</td>
<td>2.75</td>
<td>408</td>
<td>2.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P (bar)</th>
<th>$\frac{F_s}{F_0}$</th>
<th>$(\Delta C/C_N)$</th>
<th>$f(P)(10^{10} \text{Hz}^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.05</td>
<td>13.5</td>
<td>1.42</td>
<td>0.57</td>
</tr>
<tr>
<td>9.15</td>
<td>28.8</td>
<td>1.59</td>
<td>1.45</td>
</tr>
<tr>
<td>18.18</td>
<td>48.4</td>
<td>1.72</td>
<td>2.58</td>
</tr>
<tr>
<td>21.53</td>
<td>54.5</td>
<td>1.76</td>
<td>3.07</td>
</tr>
<tr>
<td>31.12</td>
<td>78.5</td>
<td>2.02</td>
<td>4.60</td>
</tr>
</tbody>
</table>

**Quantity**

- $T_c$: superfluid transition in zero field
- $c_1$: speed of first sound in normal $^3$He
- $\frac{\Delta C}{C_N}$: heat capacity jump at $T_c$
- $F_s/F_0$, etc.: Fermi liquid parameters
- $c_0 - c_1$: $2c_1/5(1 + F_s^0), F_2^2 \equiv 0$
- $f(P)$: pressure dependence of $\nu_A^2$

**Source**

- Alvesalo et al. (1981)
- Wheatley (1975)
- Alvesalo et al. (1981)
- Wheatley (1975); but scaled by Greywall (1983)
- calculated from above quantities
- calculated from above quantities
APPENDIX C

DATA ACQUISITION PROGRAM

10 | Last modification on 30 June 1989.
20 | PEAD is for taking simultaneous sound and HIP peak data.
30 | Data digitized by the Nicolet 1170 signal averager is collected
40 | by the subroutine "Dump". The HIP data in n1chem(1:200) is
50 | searched to find the highest peak. The "tip" of this peak is
60 | defined by a second-order fit. Also, the integral of
70 | n1chem(1:11) is calculated. This number, called moment, is linear
80 | by (m/f) which in turn is proportional to the frequency of the
90 | half of n1chem is fit linearly.
100 | Kernal position of highest HIP point
110 | Max value of highest HIP point
120 | Moment is defined above. (Moments in SHORT form)
130 | Y0 value of sound phase shift voltage at line 2 after a linear fit

200 | OPTION BASE 0
210 | DIM Coeff(4),X(200),Y(200)
220 | INTEGER n1chem, Min, Max, F, 00, 0, File, Ip, In File
230 | SHORT Chem, V0
240 | CON INTEGER n1chem(4084) HIP peak position, sound voltage
250 | CREATE File200.20
260 | ASSIGN #1 TO File200
270 | INPUT "Give the first DATA file number", 00
280 | File1 = File
290 | INPUT "Give number of files required for this run", 00
300 | File2 = File1 + 200 points: File3 = File1 + 400 min file
310 | LINPUT "Give today's date", Date
320 | O = 000
330 | 0 = 00
340 |
350 |
360 |
370 | This section sets up PEAD for multiple-file runs
380 | EXIT GRAPHICS
390 | IF O = 0 THEN STOP
400 | File1 = "DATA\VALUES\data1"
410 | O = 0
420 | IF O = 0 THEN STOP
430 | WRITE 10 11,6:13
440 | WRITE 10 11,6:13
450 | CPERTE File2, 200, 20
460 | ASSIGN #1 TO File2
470 | PRINT #1: Dumb
480 | PEAD #1, 1
490 | WRITE 10 11,6:10
500 | PRINT "Start 20 second WAIT"
510 | FOR I = 1 TO 8
520 | WAIT 10000
530 | NEXT I
540 | PRINT "End 20 second WAIT"
550 | GOTO 640
560 |
570 1
580 CREATE File, 200, 20
590 ASSIGN #1 TO File 1
600 PPRINT #1, "Dummy"
610 READ #1, 1
620 1
630 1
640 PRINTER IS 0
650 PRINT " = "
660 PPRINT File 1, "PIE-8", Date
670 PPRINT " = "
680 PRINTER IS 16
690 1
700 1
710 Pe!
720 Input: CALL Dump
730 Xmin=00
740 Xmax=-22763
750 Xmax=0
760 Moment=0
770 1
780 1
790 This section finds the HMP peak (highest peak) and calculates a first moment of sorts.
800 FOR I = 2 TO 2009 due to faulty data transmission
810 IF I = 0 THEN GOTO 200
820 IF I = 1 THEN GOTO 200
830 IF I = 2 THEN GOTO 200
840 IF I = 3 THEN GOTO 200
850 IF I = 4 THEN GOTO 200
860 IF I = 5 THEN GOTO 200
870 NEXT I
880 X0 = Moment
890 1
900 1
910 1
920 1
930 This section finds the HMP peak (highest peak) and calculates a first moment of sorts.
940 FOR I = 2 TO 2009 due to faulty data transmission
950 IF I = 0 THEN GOTO 200
960 IF I = 1 THEN GOTO 200
970 IF I = 2 THEN GOTO 200
980 IF I = 3 THEN GOTO 200
990 IF I = 4 THEN GOTO 200
1000 IF I = 5 THEN GOTO 200
1010 NEXT I
1020 I = I + 1
1030 Degree = 2
1040 CALL Polynomial(X, Y, N, Degree, Coeff(1), Reg1, Reg2, Total2, Regm, Regn, IF, Direct, Direct, Dftot, Abort)
1050 Xmax = Coeff(1) Y = Coeff(2) Xmax = Xmax
1060 CALL Plot(Ipar, Ipar, Xmax, Xmin, Ymin, Coeff(1)) Display of the fit.
1070 1
1080 1
1090 1
1100 1
1110 This section picks out sound points and fits them to a line.
1120 1 = 1
1130 FOR J = 2049 TO 4088 STEP II Use "II" to reduce analysis time.
1140 J = J + 1
1150 Y(I) = Xmax - 1
1160 I = I + 1
1170 N = 1
1180 NEXT J
1190 Degree = 1
1200 CALL Polynomial(X, Y, N, Degree, Coeff(1), Reg1, Reg2, Total2, Regm, Regn, IF, Direct, Direct, Dftot, Abort)
1210 Y(0) = Coeff(1) X = Coeff(2) Xmax = Xmax + 2048
1220 1
1230 1
1240 1
1250 1
1260 FOR I = 1 TO M
1270 PRINT "POINT #", I, X(I), Y(I)
1280 N = 1
1290 1
1300 1
1310 1
1320 1
...
1300: This section prints point values on hard copy and on tape file.
1301: P: "Point": Pmax: Pmax: Pmax: Pmax: Pmax: Pmax: Y0 = Y0
1302: P: "Point": Pmax: Pmax: Pmax: Pmax: Y0
1303: P: "Point": Pmax: Pmax: Pmax: Pmax: Y0
1304: 
1400: 
1401: 
1402: P=P+1
1403: IF P > 200 THEN 370
1404: GOTO 720
1405: END
1406: 
1407: 'Advance the point number counter
1500: '200 points per tape file
1480 !
1490 !
1500 SUE Folor: INTEGER Ip, iPart, Ys
1510 INTEGER Xp, Coeff(1), Coeff(2)
1520 INTEGER Xs, Ys, Xdif, Ydif, Xav, Yav, X1, X2, Y1, Y2
1530 INTEGER Ymax, Xmax, Xmin, Ymin
1540 INTEGER N, M, L, T
1550 PLOTTER IS 12, "GRAPHICS"
1560 GRAPHICS
1570 !
1580 LIMIT 0.250, 0.200
1590 LOCATE 20, 110, 15, 25
1600 X1 = -1ipar
1610 X2 = ipar
1620 Y1 = Coeff(0) - ABS(Coeff(1)) * iPart + Coeff(2) * iPart - ipar
1630 Y2 = Ymax
1640 Xdif = ABS(X2 - X1)
1650 Ydif = ABS(Y2 - Y1)
1660 Xav = (X1 + X2) / 2
1670 Yav = (Y1 + Y2) / 2
1680 scale X1; Y1, Y2
1690 GRID Xdif / 5, Ydif / 5, Xav, Yav, 1, 1
1700 FRAME
1710 LONG 4
1720 !
1730 !
1740 !
1750 GOTO 2010
1760 FOR Y = Y1 TO Y2 STEP Ydif 5
1770 MOVE X1, Y1 - 0.5 - Ydif
1780 LABEL USING "DDDDDDDDDD":
1790 NEXT Y
1800 MOVE Xav, Y1 - 1 - Ydif
1810 LABEL USING "I": "
1820 !
1830 !
1840 FOR Y = Y1 TO Y2 STEP Ydif 5
1850 MOVE X1 - 1 - Xdif, Yav
1860 LABEL USING "DDDDDDDDDDDD":
1870 NEXT Y
1880 DEC DEG
1890 LDIF 90
1900 MOVE X1 - 2 + Xdif, Yav
1910 LABEL USING "K": "
1920 !
1930 !
1940 LDIF 0
1950 Title = ""
1960 !
1970 INPUT "Enter title", Titles
1980 MOVE Xav, Y2 + 1 + Ydif
1990 LABEL USING "K"; Titles
2000 !
2010 !
2020 !
2030 INPUT "Give a data point symbol .", Az
2040 AS = ""
2050 FOR I = 1 TO Iparr
2060 MOVE X(I), Y(I)
2070 LABEL USING "A"; Az
2080 NEXT I
2090 !
2100 !
2110 !
2120 !
2130 !
2140 !
2150 SUBEND
SUB Polynomial(X),Y,H,Degree,Coeffs,Totals,Regs,Resms
  F,Dreg,Dfreg,Dtot,Abort)
ON ERROR GOTO Bomb
Abort=0
OPTION BASE 0
DIM Matrix(Degree,Degree),In(Degree,Degree),E(Degree)
REDIM Coeffs(Degree)
IF Degree<N/2 THEN SUBEXIT 'Check for higher degree than possible
Dfreg=Degree
Dfres=N-1-Degree
Dfot=Dfreg*Dfres
FOR K=0 TO Degree 'Set up system of equations
  Matrix(K,J)=0
  FOR I=1 TO N
    Matrix(K,J)=Matrix(K,J)+FNG(K)*FNG(J)
  NEXT I
ENDO Matrix(K,J)=Matrix(K,J)
ENDO B(K)=0
FOR I=1 TO N
  B(K)=B(K)+Y(I)
ENDO Z=2*Y(I)*Y(I)
ENDO
NEXT I
NEXT K
MAT In=INV(Matrix)
MAT Coeffs=In*B
FOR I=1 TO N
  X1=X1+X(I)
ENDO
ENDO X2=X2+X(I)*X(I)
Y1=Y1+Y(I)
Y2=Y2+Y(I)*Y(I)
ENDO Z2=Z+X(I)*Y(I)
ENDO
HETO X1/N
Y1=Y1/N
Total=2-N+Y1+Y1
GOSUB Regs
Regress=Total-Regress 'Residual sum of squares
REGS=Regress/Dfreg
RESS=Regress/Dfres
OFF ERROR
DEFULT ON
F=Regress/Resms
DEFAULT OFF
SUBEXIT
Regs=Regress=0
FOR I=1 TO N
  J=0
  FOR L=0 TO Degree
    J=J+X(L)*Coeffs(L)
  NEXT L
  Regs=Regress+(J-Y1)^2
  NEXT I
RETURN
SUBEXIT
DEF FNG(K)=X(I)^N
Bomb: Abort=1
SUBEND
Dump is a modification of Don Bakalyar's program for reading Nicolet data. Instead of explicitly telling the Nicolet to READOUT, the 9845 "listens" and the Nicolet "talks" as soon as it has done one sweep (Nicolet AUTO CYCLE mode). Occasionally, the Nicolet fails to "talk"; it's then "kicked" explicitly.

COM INTEGER Nicmem(4007)

Kick counter
Looking for ERROR 20 in ENTER Nicolet... line
Next word from 9845 is control word.

This conversion is necessary. If conversion is not
made, the minus sign instead of the space is recognized
as the delimiter.

Saves USART for receiving data.

Transfer from Nicolet to 9845

This section "kicks" the Nicolet if it fails to READOUT
and gives ERROR 20.

This section frees Nicolet to READOUT (again)

Data transfer from Nicolet to 9845

Using freefield format.
APPENDIX D
DATA REDUCTION PROGRAM

10 ! Last modification on 11 October 1983.
20 ! TCE reduces data taken using PEAKX (<X>1), A-phase NMR shifts are converted
30 ! to reduced temperatures, Sound velocity phase shifts are changed
40 ! to normalized sound velocity shifts and amplitude data are
50 ! converted to normalized attenuation values.
60 !
70 !
80 ! SUB Linear: first-order fit to data.
90 ! SUB Average: zero-order fit to data.
100 ! SUB Points: displays point values near indicated point.
120 ! Hertz used instead of radians/second.
130 !
140 !
150 ! Ref.1=J.C. Wheatley, Rev. Mod. Phys. 47, 415 (1975)
200 !
210 !
220 !
230 !
240 OPTION BASE 0
250 ! Miscellaneous string variables: File$, Rs$, Z5$, Cold$
260 ! Next line holds experimental data variables
270 ! SHORT P(200), Xmax(200), Ymax, Xmax(200), Y0(200)
280 ! Next four lines hold input parameters for data reduction
290 ! SHORT Fsound, L, Gamma, Alpha, Delta
300 ! INTEGER Day, Month
310 ! COM Attn$
320 ! SHORT Haveep, Hnmr
330 ! SHORT Pbar, Tc, C0, Hnmr, Z2, Z3, Z4
340 ! Next line holds parameters derived from input parameters
350 ! SHORT Z1, A1, Bc, Ac
360 ! Next three lines hold plotting variables
370 ! COM SHORT X1(5), X2(5), Y1(5), Y2(5)
380 ! INTEGER 0, 15, 16, 17, 18
390 ! DIM Xlab$(50), Ylab$(50), Title$(50)
400 ! Next four lines hold variables for NMR analysis
410 ! SHORT Xmax0, Ftc2, Xshift(200), U(200)
420 ! INTEGER Imin, Imax, Ic1, Ic2, Ic1, In, Ip
430 ! INTEGER Iguess, I211, I212, I210, I221, I222, I200(5)
440 ! REAL C1, C2, D1, D2, Anmr, Bnmr
450 ! Next line holds extra variables for sound analysis
460 ! SHORT Fsound, 25(100)
470 ! Output numbers are the following:
480 ! SHORT T(200), Xshift(200), A1ratio, Udif
490 !
500 !
510 !
520 !
530 ! Sound frequency in hertz
540 ! Length of (cold) sound path in meters
550 ! g-factor in hertz per gauss
560 ! Width coefficient in kelvin per gauss
570 ! See Ref.3
580 ! NMR slope ratio parameter; see Refs.1, 3
1.00, T14'

![Image](image_url)
1280 'A
1290 Pbar=18.18
1300 Tc=2.44E-3
1310 Co=c342
1320 Nu2=2.58E10
1330 Z2=1220
1340 Z2=1565
1350 IF Nrun<160 THEN Z3=2933  'Boxcar sensitivity .5V (not 1V)
1360 Z3=2220
1370 RESTORE 1290
1380 MAT READ Z5(23)
1390 DATA .001,1.00,94,79,70,73,75,76,78,79,80,81,82,83,84,85,1,1,1,1,1,1
1400 GOTO 1730
1410 !
1420 !
1430 Pbar=21.53
1440 Tc=2.55E-3
1450 Co=c361
1460 Nu2=3.07E10
1470 Z2=1220
1480 Z3=4322
1490 Z4=240
1500 RESTORE 1520
1510 MAT READ Z5(23)
1520 DATA .001,1.00,97,82,84,86,88,89,91,92,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1
1530 GOTO 1730
1540 !
1550 !
1560 Pbar=31.12
1570 Tc=2.75E-3
1580 Co=c406
1590 Nu2=2.8E10
1600 Z2=1220
1610 IF Nr run<40 THEN Z2=60  'Nicolet on +1V scale
1620 Z3=2702
1630 Z4=160
1640 IF Nr run<40 THEN Z4=80  !Nicolet on +1V scale
1650 RESTORE 1670
1660 MAT READ Z5(23)
1670 DATA .001,1.00,97,91,93,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1
1680 GOTO 1730
1690 !
1700 !
1710 !
1720 !
1730 A1=Alpha=Fmnr/(Gamma*.00267)  'Width of Al phase in reduced temperature
1740 ! Use "Helsinki scale" (Ref.5)
1750 Bc=Nu2-A1
1760 Ac=Bc*(1-Delta)
1770 phi=2*Pi/4
1780 Z1=2*Fmnr*Hsweep/Gamma/2046
1790 ! Conversion factor for A-phase shift.
1800 ! This is the shift where Tc2 occurs
1810 ! Ignores the term quadratic in HB.
1820 ! 2046*(1/2) of Nicolet memory.
1830 !
1840 !
1850 IF Attn='A' THEN 1970
1860 Z5='H'
1870 ! Table of amplitude (not att'n) data used to correct the velocity.
1880 ! The entries are separated by temperature intervals of Tstep.
1890 ! INPUT "Type Y to prevent amplitude correction to velocity",Z5
1900 IF Z5='H' THEN 1970
1910 Z5(0)=1
1920 FOP I=1 TO 9  'This loop sets Z5(*)=1
1930 Z5(I)=1
1940 NEXT I
1950 !
1960 !
1970 ! This section reads in raw data.
1980 FOP I=1 TO 5
1990 Y(I)=22000  'X(I),Y(I)=minimum,maximum plot values
2000 Y(2)=22000
2010 X(1)=0
2020 X(2)=200
2030 NEXT I
2040 !
ON ERROR GOTO 2220  \Look for last data point (ERROP 59)
FOR I=1 TO 200
IF Nrun=44 THEN 2110
READ #1, P(I), Xmax(I), Xmax(I), Xmax(I), Y0(I)  \This is a serial READ
GOTO 2120
READ #1: P(I), Xmax(I), Xmax(I), Xmax(I), Y0(I)  \Switch Xmax with Xmax
Y(1)=MIN(Xmax(I), Y1(I))
Y(1)=MIN(Y(I), Y1(I))
IF Xmax(I)=Y1(I) THEN 15=1  \Xmax is the lowest NMR peak
IF Y0(I)=Y1(I) THEN 17=1  \Y0(I) is the lowest sound peak
Y2(I)=MAX(Xmax(I), Y2(I))
Y2(I)=MAX(Y(I), Y2(I))
IF Xmax(I)=Y2(I) THEN 16=1  \Xmax is the highest NMR peak
IF Y0(I)=Y2(I) THEN 18=1  \Y0(I) is the highest sound peak
Imax=1
NEXT I
OFF ERROR
2230
2240
2250
2260
2270  \GRAPHICS allows choices of I1,Itc2,Itc1,In to be made
2280 0=1
2290 !Display NMR data
2300 ! Q=counter for passes through Dplot
2310 Skip=2
2320 XIabs=`````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````n
ON ERROR GOTO 2220
FOR I=1 TO 5
I200(I)=0
NEXT I
2420
2430 RES="N"
2440 INPUT "To make piecewise NMR data corrections type Y",RF
2450 IF RF="Y" THEN 2690
2460 Nheal=Nheal+1
2470 INPUT "Give a point near defect point", Iguess
2480 CALL Points(Iguess,Xmax(I))
2490 INPUT "Give the (rightmost) point closest to the first defect", I200(Nheal)
2500 I200(I)=I200(I)
2510 I211=I200+4
2520 I212=I200-1
2530 I221=I200+1
2540 I222=I200+3
2550 !Now fit 2 straight lines to 3 points on either side of discontinuity.
2560 CALL Linear(Xmax(I), I211, I212, C1, D1, Regss, Resss, Totals, Regms, Resms, F, Dfreg , Dfres, Dftot)
2570 CALL Linear(Xmax(I), I221, I222, C2, D2, Regss, Resss, Totals, Regms, Resms, F, Dfreg , Dfres, Dftot)
2580 A=C1-C2
2590 B=D1-D2
2600 Xmax(I)=A*X+B-1200
2610 FOR I=1200 TO Imax
2620 Xmax(I)=Xmax(I)+Xmax(I)
2630 NEXT I
2640 CALL Dplot(Skip, XIabs, YIabs, Titles, 0, 1, Imax, F(+), Xmax(I))
2650 PAUSE
2660 GOTO 2430
2670
2680
2690 INPUT "Give earliest valid point", I1
2700
2710 !
2720 ! This section is for files followed by a warmer file.
2730 Cold="N"
2740 INPUT "If there is a following file Y", Cold
2750 IF Cold="Y" THEN 2840
2760 INPUT "Give NMR baseline fit parameters: Anmr(Y-intercept), Bnmr(slope)", Anmr, Bnmr
2770 INPUT "Number of points (min/2) between last point and first point of Tc run", Igap
2780 Anmr=Anmr-Bnmr*(Imax+1) !Anmr correction
2790 INPUT "Give the average sound value for Tc(Tc (sound))", Asound
2800 Imax=Imax !Use all points above 1
2810 GOTO 2900
2820 !
2830 !
2840 INPUT "Give Tc2, Tc1 points and a normal point.", Itc2, Itc1, In
2850 !
2860 !
2870 ! This section establishes the NMR data baseline.
2880 !
2890 call Average(Imax, In, Anmr, Bnmr)
2900 call Linear(Imax, Itc1, Itc2, In, Anmr, Bnmr, Regss, Resss, Totalss, Regss, Resms, F, Df
2910, Dfrs, Dflot)
2920 IF Bn<0 THEN Anmr=Anmr+Bn+1 AND (Bn=0) !Assume only positive drifts
2930 !
2940 !
2950 !
2960 !
2970 !
2980 !
2990 !
3000 !
3010 !
3020 !
3030 !
3040 !
3050 !
3060 !
3070 !
3080 !
3090 !
3100 !
3110 !
3120 !
3130 !
3140 !
3150 !
3160 !
3170 !
3180 !
3190 !
3200 !
3210 !
3220 !
3230 !
3240 !
3250 !
3260 !
3270 !
3280 !
3290 !
3300 !
3310 !
3320 !
3330 !
3340 !
3350 !
3360 !
3370 !
3380 !
3390 !
3400 !
3410 !
This section establishes the sound data baseline.

IF Cold<>"Y" THEN 3480 !Sound already by manual input
CALL A=rage(Y0(x),Itc1,In,Asound)

GOTO 3530 !No sound plots

GRAPHICS MOV E,Asound
FOR I=1 TO Imax
DRAW I,Asound
NEXT I
PAUSE
FOR I=1 TO Imax
Y01=Y0(I)+Asound

!Subtract out baseline/drift

!This section calculates the sound attenuation.
IF Attn<>"A" THEN 3650 !Skip this if dealing with phase
Cshift(1)=-.01/L-LOG(Y01/Y02)
!Units are inverse centimeters.

GOTO 3740

PAUSE

This section corrects phase data for amplitude dependence.
JW=(1.001-<T(1)>/25<0>)!Multiples of Tstep
IF J<1 THEN J=1
J0=INT(J)
CZ=C25>1=J-J0)+<Z5(J0+1)-25<J0>)!Linear interpolation between 25's
Y01=Y01+24<(1-25)

!Z5=estimated amplitude of sound signal
NEXT I

This section plots the sound data versus the reduced temperature

Skip=5
LAB="REDUCED TEMPERATURE (T/Tc)"
YLAB="SOUND VELOCITY SHIFT (C-C(Tc1)/C(Tc1) (ppm)"

IF Attn<>"A" THEN YLAB="ATTN DIFFERENCE FROM Tc1 (1 cm)"

Title=Files" 
"""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""

X(4)=T(1)
Y2(4)=T(In)
Y1(4)=MIN(Cshift(17),Cshift(18))
Y2(4)=MAX(Cshift(17),Cshift(18))
X(4)=991
X2(4)=1,001
Y2(4)=2000
Y2(4)=0

INPUT "X1,X2,Y1,Y2 for plot corners. Defaults are min/max boundaries",X(4)
,X2(4),Y1(4),Y2(4)
CALL Dplot(Skip,Xlab,Ylab,Titles,0,11,In,T(4),Cshift(+))

PAUSE

R=#="N"

INPUT "Type Y to replot",R
IF R<>"Y" THEN GOTO 3880

R=#="H"

INPUT "Type Y for hard copy.",R
IF R<>"Y" THEN 4090

PRINTEP IS 0 !Thermal printer

PRINT "Day, Month, Year",;Day;Month;Year
PRINT "Heating points":;I200;1;I200;2;I200;3;I200;4;I200;5
PRINT "11,1tc2,1tc1,In":;11;1tc2;1tc1;In
PRINT "Airatio, Udir=":;Airatio;",";Udir
PRINT "Hsweep=":;Hsweep;"Gauss"

DUHF GRAPHICS ICRT

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INPUT "Type Y to PRINT reduced data onto a tape file.", R
IF R="Y" THEN 4260
IF R="N" THEN 4150
Neufile="V"\"VALS(Nrun)\&":T15" ¥"V" for "velocity"
GOTO 4160
Neufile="R"\"VALS(Nrun)\&":T15" ¥"R" for "attenuation"
CREATE Neufile,200,8
PRINT #2,;Pbar,Hi
FOR I=I+1 TO In
  J=J+1
  IF J=200 THEN 4230
  PRINT #3,1-I+1;T(I),Cshift(I)
NEXT I
GOTO 530
END
MODEL: Y=H+E*X
FOP I=I TO In
X1=X1+I
Y1=Y1+Y(I)
C=C+1
X2=X2*I+1
Y2=Y2+Y(I)
NEXT I
X1=X1/N
Y1=Y1/N
C=C/N+1
Total sq=(C-H=N+Y1)/(X2-H*N+X1)
Subreg=(C-H=N+Y1)^2/(X2-H*N+X1)
Ress=Total sq
Regss=Total sq-Regss
Resms=Regss/(N-2)
F=Regss/Resms
Dfreg=1
Dfres=N-2
FMegms.Resms-
SUBEND
SUB Dplot(Skip,Xlab,Ylab,Title*,INTEGER Istart,Istop,SHORT X(>,Y(>)

COM Rits
COM SHORT X1(>,X2(>,Y1(>,Y2(>)
SHORT Xav(>,Yav(>,Xdif(>,Ydif(>)
PLOTTER IS 15,"GRAPHICS"
! PLOTTER IS "9872A"
OUTPUT 705:VS10
GRAPHICS IS 13,"GRAPHICS"
LIMIT 0,385,0,285
! LOCATE 20,130,15,95
! LOCATE 20,128,15,85
SCALE X(K>,X2(K>,Y(K>,Y2(K>)
! Defines plot boundaries
Xdif(Q)=-hes<Q>-Xi<.Q^>
Ydif<Q>=(Y2<Q>-Y1<Q))
Xav<Q)=((Xl<Q>+X2<Q>)>-2
Yav<Q)>(Y1<Q>+Y2<Q>)^2
GRID Xdif(Q)/-10, Ydif(Q)/10, X(K>, Y(K>
FIXES Xdif(0)/10, Ydif(Q)/10, X(>, Y(>)
FRAME
LORG
PROC
FOR X=XKG:TO X2<G> STEP Xdif(Q)/Sk
MOVE X,Y(K)-.05*Ydif(Q)
ON Q GOTO 4980,4980,4980.5000
LABEL USING "DDD";X
GOTO 5010
NEXT X
MOVE Xav(Q),Y(K)-.10*Ydif(Q)
LABEL USING "K";Xlabf
FOR Y=Y1(Q) TO Y2<Q) STEP Ydif(Q)/Sk
MOVE X1(Q)-.05-Xdif(Q),Y
ON Q GOTO 5060,5060,5060.5000
LABEL USING "DDD";Y
GOTO 5090
LABEL USING "D.DDD";Y
NEXT Y
MOVE X1(Q)-.17+Xdif(Q),Yav(Q)
DEG
LDIF 90
LABEL USING "K";Ylabf
LDIF 0
LFRAME
SUBEND

FOR I=Istart TO Istop
MOVE X(I),Y(I)
LABEL USING "A";":."
NEXT I
SUBEND

SUB Points(INTEGER Iguess,SHORT X(>)
! Points displays values of data points in the vicinity of the input point.
FOR I=Iguess-5 TO Iguess+5
PRINT I, X(I)
NEXT I
SUBEND
APPENDIX E

CITED MANUFACTURERS

American Magnetics, Inc., P.O. Box R, Oak Ridge, Tennessee 37820.

Analog Modules, Inc., 400 Sansu Street, Longwood, Florida 32750.

DuPont Company, Plastic Products and Resins Department, Wilmington, Delaware 19898.

Emerson and Cuming, Dielectric Materials Division, Canton, Massachusetts 02021.

General Radio Company, 300 Baker Avenue, West Concord, Massachusetts 01742.

Hewlett-Packard, Inc., 3495 Deer Creek Road, Palo Alto, California 94304.

Intermagnetics General Corporation, Charles Park, P.O. Box 566, Guilderland, New York 12084.


Kepco, Inc., 131-38 Sanford Avenue, Flushing, New York 11352.


Linear Research, 5231 Cushman Place, San Diego, California 92110.

Matec, Inc., 60 Montebello Road, Warwick, Rhode Island 02886.

Mini-Circuits Laboratories, 2625 East 14th Street, Brooklyn, New York 11235.

Nicolet Instrument Corporation, 5225 Verona Road, Madison, Wisconsin 53711.

Palomar Engineers, P.O. Box 455, 1520-G Industrial Avenue, Escondido, California 92025.

Princeton Applied Research Corporation, P.O. Box 2565, Princeton, New Jersey 08540.
Programmed Test Sources, Inc., 9 Beaverbrook Road, P.O. Box 617, Littleton, Massachusetts 01460.

Robertshaw Controls Company, Transportation and Bellows, Marketing Group, P.O. Box 400, Knoxville, Tennessee 37901.

Vacuum Metalurgical Company, Ltd., Tokyo, Japan.
REFERENCES


J.P. Harrison (1979) J. Low Temp Phys. 37, 467.


R.C. Richardson (1977) Physica 90B, 47.


Robert Frank Berg was born 7 August 1955 in Birmingham, Alabama. He received a public education in Huntsville, Alabama and later received the degree of B.S. in Physics with Distinction from the University of Virginia. His post-graduate education has consisted of two years at Duke University leading to an M.A. degree followed by four years at the University of Florida in pursuit of a Ph.D. degree in experimental physics. His wife of four years is Carol K. Emerson. A member of the American Physical Society, his published papers and abstracts include the following titles.

Publications


I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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Gary G. Thas, Chairman
Associate Professor of Physics

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E. Dwight Adams
Professor of Physics

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[Signature]
Charles L. Beatty
Professor of Materials Science and Engineering

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[Signature]
Pradeep Kumar
Associate Professor of Physics
I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Samuel B. Trickey
Professor of Physics

This dissertation was submitted to the Graduate Faculty of the Department of Physics in the College of Liberal Arts and Sciences and to the Graduate School, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

December 1983

Dean for Graduate Studies
and Research