This form of the governing equation was then solved analytically. Kirby (1989) develops essentially the same form but for intermediate depth, dispersive waves. Also, the form of the governing equation is extended to accommodate obliquely incident waves. Since the derivatives of \( \cosh kh \) in the extended mild slope equation (Kirby, 1986) given in Chapter 2 are of \( O(\epsilon) \) and using the same variable definitions, it may be rewritten as

\[
\nabla \cdot (f \nabla \eta) + k^2 p \eta = 0
\]

where

\[
p = CC_g, \quad f = p - \frac{g \delta}{\cosh^2 kh}
\]

Introducing the variable transformation

\[
\eta = f^{-1/2} W
\]

6.4 becomes

\[
\nabla^2 W + [k^2 + A(k^2 \delta + \frac{\nabla^2 \delta}{2})] W = 0
\]

where

\[
A = \frac{g}{CC_g \cosh^2 kh} = \frac{4k}{2kh + \sinh 2kh} = \alpha'
\]

as in Chapter 2.

If \((x, y)\) is the horizontal plane and \(h = h(x), \delta = \delta(x), \partial \partial y \equiv 0\), equation 6.7 becomes

\[
W_{xx} + [k^2 + A(k^2 \delta + \frac{\delta_{xx}}{2})] W = 0
\]

Allowing oblique incidence, let

\[
m = k \sin \theta = \text{constant}
\]

6.10

equation 6.9 becomes

\[
\dot{W}_{xx} + [(k^2 - m^2) + A(k^2 \delta + \frac{\delta_{xx}}{2})] \dot{W} = 0 \quad W = \dot{W} e^{imy}
\]

6.11