$Z_N$ is the error spectrum for a particular gage. Following directly from Funke and Mansard (1980), define

\[
\beta_n = \frac{k_n}{x_{12}}
\]

(4.13)

\[
\gamma_n = \frac{k_n}{x_{13}}
\]

(4.14)

Equation 4.12 may be restated for all three gages as

\[
e_{1,n} = Z_{I,n} + Z_{R,n} - B_{1,n}
\]

(4.15)

\[
e_{2,n} = Z_{I,n}e^{i\beta_n} + Z_{R,n}e^{-i\beta_n} - B_{2,n}
\]

(4.16)

\[
e_{3,n} = Z_{I,n}e^{i\gamma_n} + Z_{R,n}e^{-i\gamma_n} - B_{3,n}
\]

(4.17)

where

\[
\epsilon_{p,k} = -Z_{N,p,n} + f_e(Z_{I,n}, Z_{R,n})
\]

(4.18)

where $f_e$ is an expression for the error associated with the entire domain, thus common to all three gages.

Now a least squares fit may be used to find those values of $Z_R$ and $Z_I$ for which the sum of the squares of $\epsilon_{p,n}$, for all values of $p$ is a minimum. This will occur at

\[
f_e(Z_{I,n}, Z_{R,n}) = 0.
\]

(4.19)

Therefore, it is required that the sum of the squared error over each gage

\[
\sum_{p=1}^{3} (\epsilon_{p,n})^2 = \sum_{p=1}^{3} (Z_{I,n}e^{i\psi_{p,n}} + Z_{R,n}e^{-i\psi_{p,n}} - B_{p,n})^2
\]

(4.20)

be minimized, where $\psi_{p,n}$ is either $\beta$ or $\gamma$.

It is assumed that a minimum will be reached when both partial derivatives are zero. Differentiating 4.20 with respect to $Z_I$ and $Z_R$ results in,

\[
\sum_{p=1}^{3} (Z_{I,n}e^{i\psi_{p,n}} + Z_{R,n}e^{-i\psi_{p,n}} - B_{p,n})e^{i\psi_{p,n}} = 0
\]

(4.21)

and

\[
\sum_{p=1}^{3} (Z_{I,n}e^{i\psi_{p,n}} + Z_{R,n}e^{-i\psi_{p,n}} - B_{p,n})e^{-i\psi_{p,n}} = 0
\]

(4.22)