dispersion relationship, it can be seen that it is possible to calculate the phase relationships of each component as they are measured at each gage. By assuming superposition of two wave fields travelling in opposite directions, the time series will be

$$
\eta_1(t) = \sum_{n=1}^{N} A_I,n e^{-i(k_n x - \omega_n t)} + \sum_{n=1}^{N} A_R,n e^{-i(k_n (x+2z_r)+\omega_n t)}
$$

(4.9)

where $A_I$ and $A_R$ are the component amplitudes of the incident and reflected spectra, and $x_r$ is the distance from gage 1 to the point of reflection, arbitrarily set at the center of the bar field. The record at the second gage will be identical in form, except that the phases will be

$$
PH_{I,12} = k_n(x + x_{12}) - \omega_n t
$$

(4.10)

for the incident wave train, and

$$
PH_{R,12} = k(x + 2(x_r - x_{12})) - \omega_n t
$$

(4.11)

for the reflected wave train, where $x_{12}$ is the distance between gages 1 and 2. The phases will be likewise for the third gage record, with the obvious replacement of a 3 where 2 appears.

The phase lag between probes is preserved in the Fourier transform, and since it is only these that are required to complete the calculation, the initial phase, or the phase at the first gage can be factored out of each component at each gage. Thus, with phases referenced to the phase at the first gage, the spectrum at a given gage may also be described by

$$
B_{p,n} = Z_I,n e^{ik_n x_{1P}} + Z_R,n e^{-ik_n x_{1P}} + Z_{N,p,n}
$$

(4.12)

where $Z$ is the $n^{th}$ Fourier component of the wave field, $k$ is the wave number of the $n^{th}$ component, and $X_{1P}$ is the distance between the first gage and the gage in question.

It can be seen that, given only two gage spectra of known distance apart, the simultaneous equations may be solved for $Z_I$ and $Z_R$. However, to improve accuracy, additional gages may be added, and $Z_I$ and $Z_R$ solved for using a least square error approach, where