Define the two estimates for reflection coefficients as

\[ R_1 = \frac{|\eta_1^r|}{a(m)} \]  
\[ R_2 = \frac{|\eta_2^r|}{a(m)} \]

and \( R \) as the average of \( R_1 \) and \( R_2 \). Transmission coefficients at \( x_2 \) are estimated by

\[ T_1 = \frac{|\eta_{-1}^n|}{a(m)} \]  
\[ T_2 = \frac{|\eta_1^n|}{a(m)} \]

which are likewise averaged to obtain \( T \). A test of the accuracy of the solution is obtained by checking the conservation of energy requirement

\[ R^2 + T^2 \left( \frac{C_p^{n-1}k^1}{C_p^{n-1}k^n} \right) = 1 \]

By applying this model at a number of discrete frequencies and angles of incidence, prediction of the reflection characteristics for a frequency and dimensional spectrum may be built.

The results of this full numerical solution will be used to compare against the oblique and arbitrary bottom extension of non-resonant solution, and the resonant detuning solution.

### 3.4 Comparison to Existing Theories

The numerical solution to the mild slope equation offers a method to calculate the reflection coefficient valid for all values of \( 2k/\lambda \). In this section, the numerical solution will be used to compare the existing resonant and non-resonant interaction theories.

#### 3.4.1 One-dimensional Wave Field

The initial investigations of this topic concentrated on bottoms of sinusoidal form. Figure 3.1 is a plot of the three methods described previously, Mei's resonant interaction (Mei) presented in Chapter 1, the extension of the non-resonant interaction (Non-Res Extension), and the numerical solution of the mild-slope equation (Numeric). It can be