By factoring $e^{i2\pi n N_b}$ out of the bracketed term, thereby forcing it into a sine form with an argument of $2\pi n N_b$ where $n N_b$ are integer, it can easily be shown that the final form for the case of $2l = n\lambda$ is

$$I = \frac{N_b L}{2} = \frac{N_b \pi}{\lambda}$$  \hfill (2.55)

2.5.2 The Non-Resonant Case

In the case where the $x$ component of the wave is away from resonance or

$$2l - n\lambda \neq 0$$  \hfill (2.56)

$I$ will change considerably. The integral portion is stated

$$I = \frac{1}{2i\lambda} \left[ \frac{e^{iN_b 2\pi N_b} e^{i\left(\frac{3}{2}\right) 2\pi n N_b} - 1}{2l + n\lambda} - \frac{e^{-iN_b 2\pi N_b} e^{i\left(\frac{3}{2}\right) 2\pi n N_b} - 1}{2l - n\lambda} \right]$$  \hfill (2.57)

Letting

$$\gamma = \frac{2l}{\lambda}$$  \hfill (2.58)

2.58 becomes

$$I = \frac{-2\gamma}{\lambda(n^2 - \gamma^2)} e^{i\pi N_b} \left[ \frac{e^{i\pi N_b} - e^{-i\pi N_b}}{2i} \right]$$  \hfill (2.59)

Applying trigonometric identities and rearranging, gives

$$I = \frac{\gamma^2}{l(\gamma^2 - n^2)} e^{iN_b L} \sin l N_b L$$  \hfill (2.60)

2.5.3 Full Solution

For a bottom with the positive branch of a cosine curve imposed on a flat bottom, the reflection coefficient for a given wavenumber component of the wave field, $R$, would be calculated by summing the effect of all non-resonant Fourier components of the bottom plus the effect of the resonant component. The full solution is given by

$$R = \sum_{n \neq \frac{2l}{\lambda}}^\infty a_n \frac{1}{l(\gamma^2 - n^2)} e^{iN_b L} \sin l N_b L + a_n \frac{N_b \pi}{\lambda} \bigg|_{n = \frac{2l}{\lambda}}$$  \hfill (2.61)

where $n$ is the $n^{th}$ Fourier component of the bottom perturbation, $\lambda$ is $2\pi/L$ with $L$ being the bar spacing, $N_b$ is the number of bars in the field, and $l = k \cos \theta$. 