The Design of a Lunar Global Navigation Satellite System

Tekilanand Persaud
University of Florida, Gainesville, Florida, 32612

An optimal Lunar navigation system was designed by finding the minimum total Delta-V required to launch and maintain a system over its entire lifetime for a design space of constellations. Considerations were made for the two largest orbital perturbations, those from third gravitational bodies, and from anisotropic Lunar gravity. The results show that the first two optimal constellations from the set of those tested are two Walker-Delta configurations with a semimajor axis of 5700 km and five orbital planes and a semimajor axis of 6700 km with three planes, both with a 75° inclination. The third was a Walker-Star configuration with a semimajor axis of 9200 km and three planes. All optimal configurations had 15 satellites.

Key words – satellite constellation design, GNSS

I. Introduction

As space exploration continues into the mid-21st century, the need for navigation systems in the vicinity of other celestial bodies arises from motivations such as servicing human operations and precisely locating unmanned systems for economic and scientific purposes. The peculiarities of the Earth-Moon system are both detrimental and advantageous to the design of a Lunar Global Navigation Satellite System (GNSS); the low mass of the Moon means that satellites in its orbit are heavily perturbed by third bodies, and mass concentrations (mascons) caused by magma upwellings in the Moon create a highly anisotropic gravity field. However, the presence of the Earth may be exploited to place spacecraft at the nearby Earth-Moon Lagrangian points to reduce the total number of spacecraft needed for a GNSS constellation.

Previous work has determined the minimum number of satellites required for total GNSS coverage of an arbitrary spherical body [1]. Some literature even addresses the particular case of the minimum satellites required for coverage of the Moon using CubeSats [2]. Still, more general studies [3] have determined the optimal constellations for a spherical body using algorithms such as symmetric/inclined-plane orbits and streets of coverage, including the time required to find the optimal solution for total coverage for each algorithm. However, these approaches are implicitly predicated on the satellite count itself contributing to the majority of the cost; over the lifetime of the constellation, other factors, such as launch Delta-V (ΔV) costs, stationkeeping, and disposal, may contribute costs high enough to warrant their exploration and minimization, especially for constellations on other celestial bodies. This paper constitutes a minimization of the first two of these costs for traditional symmetric, inclined orbits.

II. Configurations and Considerations

Regular, periodic configurations were selected using the Walker-Delta and Walker-Star configurations commonly used for contemporary navigation systems. For delta configurations, orbit planes with fixed-inclinations have their ascending nodes equally spaced about a central body. For star configurations, 90° inclination orbits are equally-spaced by their ascending node [1]. Within each plane, all satellites are equally spaced by mean anomaly in their orbit. Additional configurations were investigated using star and delta configurations supplemented with satellites in halo orbits around the Earth-Moon Lagrange points. Examples of star and delta configurations are shown in the appendix.

When choosing between star (polar) and delta (inclined) configurations, it is important to consider the primary drawbacks between each: star patterns provide strong polar coverage due to the intersection of all of the orbits above the poles. However, this makes coverage at the equator comparatively sparse. Both delta and star configurations were tested. The presence of water ice at the Moon’s poles makes them attractive sites for exploration and future Lunar bases; star configurations with redundant polar coverage would thus be more desirable [4].

The spacecraft bus was allowed to vary from masses typically seen in modern navigation systems [5] to spacecraft as small as CubeSats. The cost of the spacecraft itself and its corresponding launch cost to obtain the mission orbit are non-negligible. Since recent advances in microelectronics have allowed for the use of CubeSats for navigation [2], both CubeSats in the < 6U range and traditional spacecraft bus sizes were considered.

When designing orbits around the Moon, two major considerations must be made for the semimajor axis of the orbit: perturbations from Lunar mascons and gravitational perturbations from third bodies. The former’s strength relative to the first-order term of the Moon’s gravity increases with decreasing semimajor axis [6] and the latter’s relative strength to the Moon’s gravity increases with increasing semimajor axis, making the two conflicting parameters to be optimized. Forces from third bodies such as the Sun and Earth are included as perturbations due to the orbits being within the sphere of influence of the Moon [7].

Lunar gravity models have been developed by multiple gravity mapping missions. In particular, the Gravity Recovery and Interior Laboratory (GRAIL) mission provides the most recent and comprehensive Lunar gravity model. GRAIL has determined that the Lunar surface gravity field varies by as much as 1.913 Gals (1.913 × 10⁻³ g) [6]. This corresponds to a value within 2% of the perturbation force strength from Earth.
Table 1 - Parameters for Constellations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$ (inclination, degrees)</td>
<td>15 : 15 : 90 90</td>
</tr>
<tr>
<td>$a$ (semimajor axis, km)</td>
<td>3500 : 100 : 10000 ($R_{Moon} + 200) : 50 : (R_{Moon} + 2000)$</td>
</tr>
<tr>
<td>$p$ (orbit planes)</td>
<td>2 : 1 : 10 2 : 1 : 10</td>
</tr>
<tr>
<td>$n$ (satellite count)</td>
<td>10 : 1 : 50 10 : 1 : 1000</td>
</tr>
<tr>
<td>$L$ (number of libration point satellites)</td>
<td>0 : 4 : 4 0 : 4 : 4</td>
</tr>
</tbody>
</table>

The format of the parameter values is given in (initial value): (step)/(final value). Permutation sets of these five variables were taken using the respective line from each variable (i.e. permutation set one consists of the first line of $a$, $i$, $p$, $n$, and $L$). Inclinations only 90 degrees correspond to star configurations. A set of low-altitude star configurations with many satellites was tested. $R_{Moon} = 1738$ km is the equatorial radius of the Moon. These extremely low altitude orbits were tested at an inclination of 90° because of previous literature which found stable orbits within 2° of this inclination [22]. Three additional constellations corresponding to the Moon-radius scaled versions of GPS, GLONASS, and Galileo were also tested.

at a Lunar altitude of 200 km, and decays to less than 1.0% the perturbative strength of Earth’s gravity at a semimajor axis of 5500 km.

Perturbations from non-gravitational sources such as solar radiation pressure (SRP), infrared emission, and Lunar albedo pressure are finite but small enough to ignore, using data from GRAIL as a reference. SRP was the largest non-gravitational perturbation for this mission and introduced an error on the order of 1% of the range of gravitational forces on the Lunar surface [6]. For CubeSats, this error is expected to be proportionally smaller based on the spacecraft’s surface area. Although further from the Lunar surface, these forces become more significant due to the decreased strength of variations felt due to anisotropies in Lunar gravity, they remain the same relative to Sun and Earth gravitational perturbations, which are on the order of Lunar gravity field variations at the Lunar surface. Thus, these non-gravitational perturbations may be neglected without significant discrepancies between the real and computed total optimal $\Delta V$ for stationkeeping.

Ground segment optimization and placement was not considered in computation due to the current uncertainty in the practicalities and details of such an establishment. For the same reasons, considerations for the end-of-life disposal of all spacecraft were withheld.

III. Computation

A. Approximations

Simplifying assumptions were made to expedite the computation process:

1. The orbits of the Earth around the Sun and the Moon around the Earth are stable, known and approximated accurately enough to use Kepler’s equation instead of numerical integration of forces on the respective celestial bodies.

2. Perturbations will be symmetric over large time periods; $\Delta V$ need only be computed for a single orbit of a regular, periodic constellation to obtain the stationkeeping requirements for an entire constellation since the Moon will rotate under the satellites and the Earth vector will rotate at the same rate as this rotation. Over a ten-year propagation, the Sun vector will similarly rotate around the selenocentric frame ten times.

3. Non-gravitational perturbation forces are negligible at all altitudes. These include SRP, albedo pressure, infrared emission, and the Lunar atmosphere. The perturbative strength of harmonic terms is reduced below 1.0% of Earth’s perturbative strength at semimajor axes greater than 5500 km and may be neglected beyond this point to improve computation times.

4. Effects such as apsidal precession, ejection, libration, precession of the line of nodes, and other perturbations to the Moon’s orbit are negligible due to their small magnitude relative to their mean value [8].

5. Line of sight is enough to provide coverage to a point. No considerations were made towards limitations on the cone of contact for instrumentation and antennae. Furthermore, points of contact on the Lunar surface were assumed to have no elevation mask limitations due to irregular terrain.

6. Halo orbits around Earth-Moon libration points will have small enough deviations from the libration point and have periods short enough that satellites at these points may be modelled as being at the libration point itself. This removes the need to solve for the halo orbits in the circular restricted three-body problem.

7. The differences in $\Delta V$ needed to cause commercial rocket booster stages carrying the satellites to reach the Moon at different altitudes in the same planes (i.e. an equatorial orbit from which a plane change may be made into the correct final orbit) is negligible.

8. Regular stationkeeping will be conducted to maintain the satellites’ orbits. This stationkeeping will be conducted using electric propulsion (EP) with an exhaust velocity of 30 km/s, corresponding to modern high-performance EP, whose specific impulse typically exceeds 3000 s [5].
B. Methods

Calculations for orbit propagation and continuous coverage were made in Matlab and parallelized in Open Multi-Processing (OpenMP) for C++. OpenMP offers an open-source method of performing the same computations across multi-threaded processors. Hybrid versions of OpenMP and Message-Passing Interface are available to parallelize code across multiple cores for supercomputing applications across extremely large trade spaces. Verification of results was conducted in Analytical Graphics’ Systems Tool Kit.

IV. Segments

A. Coverage

Coverage for a constellation of artificial satellites in regular, periodic, circular orbits about a spherical body may be computed using a purely geometrical argument. However, inclusion of auxiliary satellites at the libration points precludes this method. Instead, coverage was computed numerically with the satellites in a stationary position, since the satellites were presumed to conduct regular stationkeeping to preserve their original orbits.

In a geodetic system, the flattening term provides a measure of an object’s oblateness; for the Moon, this value is $f = 1/870$, [9], [10]. Despite this value, which is relatively low to that of Earth’s, the Moon was simulated as an oblate sphere when performing coverage calculations.

Coverage was determined using an iteration over a discretized Lunar surface consisting of 50 latitudinal and longitudinal divisions instead of using a purely geometrical argument, which would have been computationally cheaper [1]. This method enables irregular constellation configurations to be tested, such as those with auxiliary spacecraft at the Earth-Moon Lagrangian points. The computational algorithm for determining whether a perfect ellipsoid has complete GNSS coverage was performed in the following steps:

1. Discretize the surface of the Moon into a three-dimensional array of points.
2. Determine the positions of the satellites.
3. Iterate across each point on the Lunar surface, and at each point, iterate across every satellite, verifying that the vector between the point and a satellite does not pass through the Moon for at least four satellites. This was done by determining the angle made between the surface-to-satellite vector and the vector normal to the Moon’s surface at each point.

B. Orbit Propagation

Orbits were propagated using oblate spherical harmonic Lunar gravity with the first 50 terms of the GRAIL mission [11], the $j_2$ Lunar oblateness term [12] and third body perturbations from the Earth and Sun with a timestep of 30 s and a fourth-order Runge-Kutta numerical integrator.

The oblateness and harmonic terms were implemented as a potential in (1) with reference radius $R_e$, oblate coefficients $J$, normalized Stokes coefficients $C_n^m$ and $S_n^m$, and normalized Legendre Polynomial $P_l^m$. Here, $r, \phi,$ and $\theta$ are spherical coordinates (radius, longitude, and latitude) [7], [13]. $M$ and $N$ are the number of terms chosen. $G$ and $M$ are the familiar terms of gravitational constant and mass of the Moon. Data with the normalized Stokes coefficients and their uncertainties are made available by NASA [13].

$$V = \frac{GM}{r^3} \left[ 1 - \sum_{k=2}^{M} \frac{R_e^k}{r^k} \right] P_k \cos(\phi) + \sum_{l=1}^{N} \sum_{m=0}^{N} \left( \frac{R_e}{r} \right)^l \tilde{P}_{lm} \sin(\theta) \left( \tilde{C}_{lm} \cos(m\phi) + \tilde{S}_{lm} \sin(m\phi) \right) \right]$$

Third-body perturbations were computed using the easily-implemented Cowell’s Method [8]. Here, $a$ is the acceleration, $ps$ denotes the vector from the perturbing body to the satellite, and $pm$ denotes the vector from the perturbing body to the Moon. $M_p$ is the mass of the perturbing body.

$$a = -\frac{GM}{r^3} \hat{r} - GM_p \left( \frac{\hat{r}_{ps}}{r_{ps}} - \frac{\hat{r}_{pm}}{r_{pm}} \right)$$

Despite the Lunar sphere of influence extending to a radius of 66,000 km, perturbations of eccentricity and inclination were observed at over 0.02 and 5°, respectively at 35,000 km circular selenocentric orbits after ten orbits, suggesting the need for lower-radius orbits in an initial analysis. This allowed the trade space of parameters to be iterated across in Table 1 for the full study to be narrowed to decrease computation time.

C. Launch Costs and Total $\Delta V$

Launch to Low Earth Orbit (LEO) was readily calculated from the velocity required for a circular, geocentric staging orbit of 320 km [14].

Trans-Lunar Injections (TLI) between LEO and the Moon were calculated using a minimization of $\Delta V$ with an elliptical transfer orbit. This results in a maximum time of transfer, which is an insignificant burden to the mission at approximately 120 hours. The transfer velocity is 10.8 km/s from an initial LEO of 320 km altitude and 0° flight path angle [8]. The same maneuver can be performed from a highly-inclined LEO to facilitate more efficient distribution of star-type constellations through changes of ascending node once at the Moon. The overall $\Delta V$ is computed from an approximation of a Hohmann Transfer from the parking orbit to a circular orbit of the same semimajor axis as the Moon. It is possible to insert to the Moon’s orbit using low-energy transfers and three-body dynamics in the Earth-Moon system and systematically construct transfers similar to the low-energy trajectory taken by the Hiten probe [15]. These trajectories may provide a 20% saving on $\Delta V$ over that required for a traditional Hohmann transfer to the Moon. However, for simplicity of formulation, Hohmann transfers shall be used to compute transfer costs to the Moon.

Once in Lunar orbit, the $\Delta V$ for an inclination change into the correct final Lunar orbit was calculated. For star configurations, where all satellites are in a polar orbit, it is cheaper to launch directly into a high Earth inclination and make the TLI in an inclined plane; changing each plane then requires the minimum inclination change from a Lunar polar orbit. From each plane,
the $\Delta V$ for phasing maneuvers were calculated to create equal separation for satellites in each orbital plane.

The $\Delta V$ for stationkeeping was calculated over a ten-year period. Since the perturbations in orbital elements were expected to be the same across all planes, only the $\Delta V$ required to correct eccentricity, inclination, and semimajor axis was computed. Stationkeeping was simulated on a monthly basis and the orbit re-propagated therefrom on the osculating orbit in the simulation.

From an Earth parking orbit of 500 km altitude to a halo orbit with point of closest approach to Earth-Moon Lagrangian Point 1 (EM-L1) of 185 km, the $\Delta V$ is approximately 3.2 km/s [16], [17]. To EM-L2, the cost is 4.8 km/s for a near-rectilinear halo orbit [18]. Transfer velocities to EM-L4 and EM-L5 were found using a Hohmann Transfer to the Moon’s orbit conducted at a time such that the satellite will rendezvous with the Lagrangian point. A 6U CubeSat was used as the spacecraft size for the Lagrangian points. This choice minimizes the $\Delta V$ penalty, since solar sails may be used for Lagrangian point and interplanetary missions [19], [20].

Propellant mass for stationkeeping was estimated from the spacecraft dry mass and the $\Delta V$ required for stationkeeping using the rocket equation [21]. Due to the limited $\Delta V$ capability of CubeSat propulsion systems, the satellite masses were changed to include greater propellant mass based on the $\Delta V$ required for stationkeeping.

The primary drawback of only minimizing total $\Delta V$ is that real costs, such as those associated with larger spacecraft sizes and having to pack spacecraft into discrete commercial rocket boosters were neglected. For example, if a commercial rocket has enough $\Delta V$ capability to launch satellites into one configuration of 15 satellites or into another with 18 satellites, both providing complete Lunar coverage, the benefit of manufacturing three fewer satellites while purchasing the same launch provider contract may remain unseen without examination of the constellation parameters. However, the advantage of the $\Delta V$ method is that the benefits of having fewer, lighter satellites were captured by the fact that many large satellites will rapidly inflate $\Delta V$ costs, especially due to the exponential nature of the rocket equation.

V. Results

Although a binary check was not performed with spacecraft positions with time, the $\Delta V$ required to correct ten randomly-selected orbits for monthly stationkeeping was within 1% in the simulation versus Systems Tool Kit. Similarly, for 100 orbits evenly spaced across inclination and semimajor axis, the position error after one month of propagation was less than 1 kilometer.

Analysis in literature enables whole families of orbits to be eliminated. For example, the orbits tested at low altitudes had an inclination which minimized or completely removed the $\Delta V$ required to keep them in a stable orbit considering the Moon’s uneven gravity field [22]. Realistically, perturbations from third bodies mean that no orbit of the Moon will be completely stable, much like how the presence of the Sun destabilizes EM-L4 and EM-L5.

The parameters of the constellations tested are given in Table 1. The winning constellations are three sets of constellations. The first is a delta-type constellation with $\alpha = 5700$ km, $i = 75^\circ$, and five planes. The second has $\alpha = 6700$ km, $i = 75^\circ$, and three planes. The third is a star-type ($90^\circ$ inclination) three-plane constellation with a semimajor axis of 9200 km. All winning constellations had 15 satellites with no auxiliary satellites at the libration points. The most efficient constellation at $\alpha = 5700$ km has a total $\Delta V$ required for complete constellation insertion and maintenance at 209 km/s. Other configurations follow the same trends, having below 220 km/s. This trend holds for the top 24 configurations, which have the same parameters as the top three, but semimajor axes that vary as much as 500 km from the nominal value. However, when total impulse is used instead of $\Delta V$, the third configuration is the most efficient. This is because the overall stationkeeping requirements of this configuration are more than five times smaller than the other two configurations. The plane-change $\Delta V$ of the first two configurations make them overall a lower $\Delta V$ than the third.

Furthermore, the next sets of winning constellations have $\Delta V$ values greater than 240 km/s but no more than 18 satellites. The uncertainty in savings provided by efficient techniques such as low-energy Lunar transfers means that more detailed computation is required to differentiate between the 0.2% most optimal constellations from the set of those tested.

The same result holds for traditional GNSS spacecraft dry masses, including those of GPS Block II, GLONASS, and Galileo. This is likely due to the small stationkeeping requirements of the winning constellations relative to the remainder of the design space and high efficiency of the EPS system assumed in use.

Figures with trends in total $\Delta V$ versus parameters such as semimajor axis, inclination, plane count, and satellite count for constellations providing complete coverage are shown in the appendix. The number of free parameters makes visualization difficult, even with a shaded contour plot, but when the plots are viewed together, the overall trend is seen; the lowest $\Delta V$ points in each plot correspond to the parameters of the winning constellations.

VI. Conclusions

The results show that three optimal configurations arise. The most optimal is a five-plane, 5700 km semimajor axis 75° inclination delta configuration. The next has $p = 3$, $i = 75^\circ$, and $a = 6700$ km. The last has $p = 3$, $i = 90^\circ$, and $a = 9200$ km. All optimal constellations, including the 24 most-optimal configurations found, have 15 satellites, the minimum number required to provide total coverage.

Additional work is required to optimize the configurations’ packing into current launch vehicles; the limits in discretizing satellites, especially numerous CubeSats, into the correct amount of launch vehicles changes the overall minimization of the mission cost. Furthermore, no consideration was made to the cost of CubeSat propulsion methods, which may significantly alter the total cost. However, the minimization of $\Delta V$ required to fulfill each constellation providing total coverage provides a starting point from which these additional
factors may be taken into consideration, especially if using techniques that take advantage of the Moon’s uneven gravity field and rotation rate to design stable orbits with minimal maintenance costs. The orbits found in literature [22] proved unsatisfactory for GNSS purposes due to the high number of satellites required to provide coverage at such low altitudes. However, the work still proved useful in eliminating other orbits at similar altitudes.

Further investigation may also involve a combined propagation and access check segment with improved computational implementation; the designer may be able to entirely remove propulsion and allow the constellation to drift, or infrequently employ passive techniques such as retractable solar sails for orbital correction. In this case, perturbations may cause certain orbit planes to precess and change their ascending node faster than others between correction periods, distorting the perfect coverage of an unperturbed constellation. Computational efficiency will be needed for more detailed work, since Cowell’s Method, while simpler to formulate and implement, is as much as an order of magnitude slower than Encke’s Method for calculating perturbation strengths.

Lastly, future analysis should consider the access of the constellation with regards to the position of the ground segments of the system, either on the Moon or on Earth, and with regards to whether redundant coverage is desirable, as in contemporary navigation systems.
VII. Appendix

Acknowledgments

I would like to thank Prof. John Conklin and the Precision Space Systems Laboratory for their support and advice. My thanks also go to Prof. Riccardo Bevilacqua and Prof. Darin Acosta for their expertise. I am also grateful to the University of Florida and its excellent faculty, staff, and students for providing a most worthwhile undergraduate experience.

References

Additional Figures

Units are in kilometers for plots showing orbits. The Moon is depicted as its surface was discretized in the computation, with a 50 x 50 grid across its surface.

Figure 1 Example Walker-Delta configuration. This example scales up USA’s GPS system so the Lunar satellites are at a semimajor axis with the same multiple of the central body’s radii as GPS with $i = 56^\circ$, $p = 4$, and $n = 24$.

Figure 2 Example Walker-Star configuration with 210 satellites in 5 orbital planes and a 200 km altitude.

Figure 3 Example 30-day propagation of an orbit with $a = 10,000$ km and $i = 45^\circ$. The ascending node visibly precesses without notable changes to other orbital elements.
Figure 4 A plot of the inclination vs. ΔV; additional results in the 90° region are omitted, with the most optimal results being shown.

Figure 5 The trend in the plane count is easily recognized by the lowest ΔV in each plane count.

Figure 6 The trend in the semimajor axis is less obvious, with the ΔV of the optimal constellations in each semimajor axis progressively lowering in the low-altitude regime due to the decreasing effect of the Moon’s anisotropic gravity field. Results above 8000 km/s total ΔV are omitted.

Figure 7 The trend in the number of satellites is easily seen to favor fewer satellites. Again, higher satellite counts are omitted since the trend of increasing ΔV with greater satellites count increases.
Figure 8 The first winning constellation.

Figure 9 The second winning constellation.

Figure 10 The third winning constellation.