OPTIMUM LINEAR MEGAWATT-FREQUENCY CONTROL
OF INTERCONNECTED ELECTRIC ENERGY SYSTEMS

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Progress is carried out by a small number of individuals with the raw materials supplied by the works of many.

Jean Sibelius
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NOMENCLATURE

Electric Energy System

Parameters

\( f^* \) - nominal system frequency Hz

\( \delta^* \) - nominal area power angle (with respect to some chosen voltage reference) rad

\( W_{\text{kin}} \) - kinetic energy MW sec

\( H \) - inertia constant sec

\( D \) - load frequency constant pu MW/Hz

\( T^* \) - tie-line power flow constant pu MW

\( R \) - self regulation of generator Hz/pu MW

\( T_t \) - turbine time constant sec

\( T_gv \) - governor time constant sec

\( K_I \) - area frequency controller integrator gain sec\(^{-1}\)

\( B \) - frequency bias constant pu MW/Hz

Variables

\( \Delta f \) - incremental frequency deviation Hz

\( \Delta \delta \) - incremental power angle deviation rad

\( \Delta P_{\text{tie}} \) - incremental tie-line power deviation pu MW

\( \Delta P_g \) - incremental generator power deviation pu MW

\( \Delta P_d \) - incremental demand power deviation pu MW
Variables

$\Delta P_c$ - incremental speed changer command deviation pu MW

$\Delta X_{gv}$ - incremental governor valve position deviation pu MW

Optimal Control Theory

$x$ - $n \times 1$ state vector

$u$ - $m \times 1$ control vector

$p$ - $n \times 1$ costate vector

$F$ - $n \times n$ state distribution matrix

$G$ - $n \times m$ control distribution matrix

$\Gamma$ - $n \times k$ disturbance distribution matrix

$K$ - $m \times n$ optimal gain matrix

$Q$ - $n \times n$ positive semidefinite state cost weighting matrix

$R$ - $m \times m$ positive definite control cost weighting matrix

$P$ - $n \times n$ positive definite matrix Riccati equation solution

$C$ - scalar cost

Special Symbols

$\dot{x}$ - time derivative of variable $x$ \(\frac{dx}{dt}\)

$\hat{\ }$ - defined as

' - transpose of matrix or vector

* - nominal value

$\Delta$ - first order perturbation

ss - steady state value of variables

pu - per unit value of variable or parameter on system basis
Abstract of Dissertation Presented to the Graduate Council in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

OPTIMUM LINEAR MEGAWATT-FREQUENCY CONTROL OF INTERCONNECTED ELECTRIC ENERGY SYSTEMS

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The interconnection of contiguous electric energy systems into large coordinated groups has made a reappraisal of existing megawatt-frequency control strategies imperative. These control strategies are based on years of operating experience, intuitive reasoning and steady state analysis. Earlier electric energy systems had interconnections which extended over limited areas and total system generation was low. Existing controllers allowed these systems to operate rather efficiently. However, interconnected electric energy systems are growing, not only in generating capability but also in area. Serious system stability problems are beginning to occur. A need to reexamine megawatt-frequency control strategies exists. New methods of megawatt-frequency control must be suggested, methods
which will allow more efficient operation of interconnected electric energy systems.

A thorough dynamic and steady state analysis of the classical megawatt-frequency control problem of multi-area electric energy systems is presented. Classical optimization techniques are applied to find best values of control parameters which minimize the Integral Square Error criterion. The results of the analysis raise serious questions about the present-day megawatt-frequency control philosophy.

To develop new methods of megawatt-frequency control, techniques of optimal control theory are used. The result of modeling the multi-area electric energy system in state variable form and stating the system specifications in terms of an integral quadratic cost is an optimal controller which is radically different from that in use today. Through study of the optimal controller structure, new feasible control methods can be suggested. These methods could give improved dynamic response and wider stability margins to present megawatt-frequency control systems.
CHAPTER I

INTRODUCTION

Definition of Megawatt-Frequency Control

An electric energy system must be maintained at a nominal operating point, characterized by specified frequency, voltage profile and load flow. It is kept in the nominal state by close control over the real and reactive power sources of the system. The total real and reactive power demands throughout the system vary widely over a 24-hour period, but for time intervals of seconds to even minutes they can be considered constant with superimposed first order perturbations \( \Delta P_d \) and \( \Delta Q_d \) respectively.

The perturbations in demand power must be matched by perturbations in generated power for exact nominal state to be maintained. However, due to the statistical nature of the loads, this can never be achieved. The best that can be hoped for is to keep the system within sufficiently close tolerance of the nominal state.

By means of sensitivity analysis,\(^1\) two very important characteristics of electric energy systems can be proven to hold for percentage-wise small, or in a
mathematical sense, first order changes in real and reactive demand powers:

1. The system frequency is sensitive only to mismatch in real power.

2. The system voltage is sensitive only to mismatch in reactive power.

Thus when demand power changes are first order, the nominal steady state control problem of electric energy systems becomes two decoupled subproblems:

1. The "megawatt-frequency" control problem or "P-f" control problem for short. By means of a frequency sensor, the frequency error, the most sensitive indicator of real power imbalance, is detected. If a frequency error exists, commands are sent to the prime movers, resulting in a change in generated real power in a direction to reduce frequency error.

2. The "megavar-voltage" control problem or "Q-V" control problem for short. By means of a voltage sensor, a voltage deviation, the most sensitive indicator of reactive power imbalance, is sensed. If a voltage deviation occurs, commands are sent to the reactive sources of the system, resulting in a change in generated reactive power in a direction to reduce the voltage deviation.

During major fault conditions the two subproblems, "P-f" control and "Q-V" control, are no longer decoupled and cannot be considered separate. In this study the first subproblem, the megawatt-frequency control problem of electric energy systems, is considered.
Interconnected Electric Energy Systems

Individual electric energy areas operating together as fully coordinated groups via interconnections called tie-lines form a multi-area electric energy pool or interconnected electric energy system. Two phrases, "electric energy area" and "fully coordinated", must be defined as they take on special meaning in referring to an interconnected electric energy system. An electric energy area is part of a company, a whole company, or a group of adjacent companies which, when viewed from an interconnection, is a single entity. This means that individual electrical connections within an electric energy area are so strong, at least in comparison to the ties between adjoining areas, that each electric energy area may be represented by a single frequency. This characteristic of an electric energy system is called coherency and is illustrated in Figure 1. To be fully coordinated, an interconnected system must be planned, designed and operated as an unit, with each part able to accept the responsibility which might be imposed upon it as a part of the system.

The responsibilities that each electric energy area has while operating as part of a multi-area electric energy pool are many. The individual areas must adjust
Control Area $i$ is characterized by system parameters $H_i, D_i, T_{gvi}, T_{ti}, R_i, P_i$ and system variables $\Delta f_i, \Delta P_{tie i}, \Delta P_{gi}, \Delta P_{ci}, \Delta P_{di}$.

Figure 1. Coherent Areas
their own generation to follow the system load changes while scheduled tie-line interchange with adjacent areas, system frequency and synchronous time are all maintained. However, by being part of an interconnected system, each area receives increased reliability and economy of operation, benefits very meaningful to an electric energy engineer.

Evolution of the Megawatt-Frequency Control Problem

The "P-f" control problem is as old as electric energy technology itself. In fact the load-frequency regulating device used today on most electric generators is a modernized version of the well-known flyball regulator invented by James Watt in the middle of the 18th Century. This was long before the first commercial electric energy system went into operation. The electric energy industry actually had its beginning with Michael Faraday, who in 1821 demonstrated the principle of operation of electric generators and motors. Two inventions gave impetus to the growth of this industry. The first invention was the incandescent light bulb invented by Thomas A. Edison in 1879. The second invention was the closed core iron transformer, developed in 1884 by Derr, Bláthy and Zipernowsky, a group of
Hungarian electrical engineers. With the electric light bulb, the public saw a practical use for electricity and demanded that it be made available. With the transformer, alternating current voltage could be handled very efficiently, allowing the industry to expand rapidly.

In 1880, the first commercial electric energy company, a hydro plant in Rochester, New York, went into operation. In 1882, the historic Pearl Street plant, the first steam electric generating plant, began producing usable power. As more individual electric energy systems appeared and grew in size and the public continued to demand more reliable service, the need for interconnection was recognized. By operating several electric energy companies together through interconnections, more diverse loads could be handled, improved reliability could be realized, and operating costs could be reduced.

To interconnect, two companies must have the same frequency, voltage level and phase on the tie-lines. As frequencies from 25 Hz to 133 Hz were common as well as a whole spectrum of voltage levels, a standard was necessary. In 1891, realizing that 25 Hz was a bad choice for an operating frequency because of light flicker, 60 Hz was chosen as a frequency standard. The standard transmission voltage level selected was 66 kilovolts.
As interconnected electric energy systems formed and total system generating capability increased, problems developed. The frequency of the interconnected systems and scheduled tie-line flows could not be held within a desired tolerance of nominal state. The only automatic load-frequency controller in operation on the system was the flyball regulator, a proportional control device which caused a change in generation in proportion to the frequency deviation. An integral controller which continues to act until the error is reduced to zero was needed. On the earliest interconnected electric energy systems this control function was performed by the load dispatchers who manually adjusted the system generation to correct frequency error. However, frequency meters in use had tolerances of ±1 cycle. Often load dispatchers in adjoining areas would be opposing each other in their efforts to reduce frequency error because of frequency meter errors.⁶ A unified plan of operation was needed.

The original system of load-frequency control of interconnected systems which was developed, consisted in giving one central member the responsibility for the frequency control. All other members of the interconnected system were only to meet their own load requirements.⁷
Theoretically the central member would only be following its own load variations.

Two objections arose to this original scheme. The first was that often the central frequency control member would be called upon to provide excessive changes in generation when several tie-lines departed from scheduled value in the same direction. The second was that continual variations in generation by the central member attempting to maintain nominal frequency caused tie-line loads to vary, making it difficult for adjoining members to maintain scheduled tie-line flows.

Due to these problems experienced by system operators in attempting to maintain the nominal state of the interconnected system, research was begun with the objective of developing supplementary load-frequency controllers, which would automatically maintain nominal frequency and scheduled tie-line flows. These supplementary controllers would aid the flyball regulator and were of two types. The "floating" or integral controller type caused corrective action to be applied at a rate proportional to the system errors, continuing action until these errors were reduced to zero. The "proportional" type applied corrective action in proportion to the system errors. The proportional type allowed some steady state error to exist.
A study made in 1932 through actual operating experience with two interconnected electric energy systems showed that better performance in maintaining system frequency and tie-line loading could be achieved by using an automatic frequency controller in each area, supplemented by a tie-line controller. Here the tie-line controller would operate in a direction to aid in maintaining nominal frequency. By this time frequency meters with accuracy of \( \pm 0.01 \) cycle were available, so the plan was entirely feasible. In 1938, this control plan of using both frequency deviation and tie-line deviation from scheduled value in the supplementary controller was again studied. It was determined that each area controller should be the floating type with the input to the controller being the Area Control Error (ACE), the sum of the area tie-line deviation and a constant times the area frequency deviation, the constant being given the name frequency Bias B. This constant B was to be determined in the following manner. Assume that the area in question is operating alone and is subject to an incremental load increase \( \Delta P_d \). The area frequency will drop a proportionate amount, the amount being determined by the area regulation and load characteristics. The load and regulation characteristic of
an area is given the name area frequency response characteristic $\beta$. If the generation in this area is commanded to increase the exact amount by which the frequency decreased, accomplished by setting $B=\beta$, steady state will be restored. If this area were part of an interconnected system and followed this same strategy, then the adjoining areas would not be burdened by load increase in the affected area.

A study in 1941 showed that in general a one percent change in the total generation of an area would result in one-tenth cycle frequency departure.\(^7\) This rule of the area frequency response characteristic of an area is adhered to today. The frequency bias constant B of each area is set equal to its average calculated area frequency response characteristic. In no case is B set less than one percent of the total area generation per one-tenth cycle.

At present five operating interconnected electric energy systems account for the whole United States.\(^{10}\) The Eastern Group follows the guidelines set forth by the Interconnected System Group (ISG) and the Western Areas follow that set forth by the North American Power Systems Interconnection Committee (NAPSIC). The standard megawatt-frequency control operating procedure for each member of an interconnected system is to use an automatic frequency control...
controller sensitive to tie-line deviation and frequency deviation, the latter being biased by the frequency bias constant B. The frequency bias constant is different for every area, being determined by the area frequency response characteristic of that area.

Research Effort

Several recent spectacular breakdowns among interconnected electric energy systems have caused renewed interest in the problem of megawatt-frequency control of interconnected electric energy systems. The method of automatic megawatt-frequency control, that of tie-line with frequency bias, was developed in the early 1940's and was based on intuition and steady-state analysis.

In this study, the "P-f" control problem of interconnected electric energy systems is presented with the realization that the problem is dynamic by nature. The classical megawatt-frequency control problem is analyzed. The Integral Square Error (ISE) criterion is used to find values of control parameters which give "best" response.

In an effort to develop a more modern control strategy, one which uses more system information than just tie-line deviation and frequency deviation, techniques
of modern optimal control theory are used. The resulting control structure allows for control methods which could considerably improve the dynamic performance of interconnected electric energy systems.
CHAPTER II
CLASSICAL MEGAWATT-FREQUENCY CONTROL

To gain a fundamental understanding of the megawatt-frequency control problem, it is necessary to study the simplest of interconnected systems, the two area interconnected system. A dynamic model is used\textsuperscript{1},\textsuperscript{12} which accurately describes the dynamics of the interconnected system experiencing first order load changes.

In an attempt to optimize system response, the ISE criterion is applied. The results of the study raise serious questions about the standard operating procedures set forth by both ISG and NAPSIC.

Dynamic System Model

The analysis that follows is based on the following assumptions:

1. The real and reactive control problems are decoupled.
2. Each electric energy area is coherent.
3. Only first order changes in system variables are assumed, second order changes being neglected.

Each electric energy area, operating at "nominal" state is characterized by nominal frequency $f^*$ and kinetic energy $W^*_{\text{kin}}$. If the area, while operating in this nominal
state, is subjected to a disturbance $\Delta P_d$, then the net power surplus will be absorbed by the system in the following three ways.

1. By increased kinetic energy at the rate

$$\frac{d}{dt} (W_{\text{kin}}) = \frac{d}{dt} \left[ W_{\text{kin}}^* \left(\frac{f}{f^*}\right)^2 \right]$$

$$= \frac{d}{dt} \left[ W_{\text{kin}}^* \left(1 + 2 \frac{\Delta f}{f^*}\right) \right]$$

$$= 2 \frac{W_{\text{kin}}^*}{f^*} \frac{d}{dt} (\Delta f) \quad (1)$$

2. By increased load consumption. Due to the predominance of motor load on most electric energy systems, system load changes with frequency. The rate at which system load changes with frequency evaluated at nominal settings is given the name load damping $D$.

$$D = \frac{\Delta P_d}{\Delta f} \text{ MW/Hz}$$

On an electric energy system the $D$ parameter can be found empirically.

3. By increased export of power via tie-lines with the total amount $\Delta P_{\text{tie}}$ MW defined positive out from the area.

Mathematically the power equilibrium equation describing the absorption of surplus power by area $i$ is

$$\Delta P_{gi} - \Delta P_{di} = 2 \frac{W_{\text{kin}}^*}{f^*} \frac{d}{dt} (\Delta f_i) + D_i \Delta f_i + \Delta P_{\text{tie} i} \quad (2)$$
This equation has units of MW. It is more convenient to express the power equilibrium equation in per unit, that is, in terms of $P_{ri}$, the total rated power of area $i$.

Dividing Eq. (2) by $P_{ri}$ gives

$$\Delta P_{gi} - \Delta P_{di} = 2 \frac{H_i}{f^*} \frac{d}{dt} (\Delta f_i) + D_i \Delta f_i + \Delta P_{tie i}$$

where the inertia constant $H_i$ is defined as

$$H_i = \frac{W_{kin i}}{P_{ri}} \text{ MW sec/MW} = \text{sec}$$

The inertia constant $H_i$ is virtually independent of the area size and has normal numerical values ranging from 2-8 seconds.

Equation (3) is linear and time invariant and upon Laplace transforming takes on the form

$$\left[ \Delta P_{gi}(s) - \Delta P_{di}(s) - \Delta P_{tie i}(s) \right] \frac{K_{pi}}{1 + sT_{pi}} = \Delta F_i(s)$$

where the following parameters have been introduced.

$$T_{pi} = \frac{2H_i}{f^*D_i} \text{ sec}$$

$$K_{pi} = \frac{1}{D_i} \text{ Hz/pu MW}$$

By defining the area transfer function

$$G_{pi}(s) = \frac{K_{pi}}{1 + sT_{pi}}$$
an individual control area of an interconnected system can be represented by the block diagram in Figure 2.

The incremental tie-line power $\Delta P_{tie}$.—The total real power exported from area $i$, $P_{tie i}$, is the sum of all out-flowing power from area $i$ to adjoining areas, i.e.,

$$P_{tie i} = \sum_{\nu} P_{tie \ nu}$$ \hspace{1cm} (5)

The summation $\nu$ is over all lines that terminate in area $i$.

The real power flowing over a lossless line connecting area $i$ with area $\nu$ in per unit of area $i$ is

$$P_{tie \ nu} = \frac{|V_i| |V_\nu|}{X_{i \nu} P_{ri}} \sin(\delta_i - \delta_\nu)$$ \hspace{1cm} (6)

$$\leq P_{tie max \ nu} \sin(\delta_i - \delta_\nu)$$

where

$$V_i = |V_i| e^{j\delta_i}$$

$$V_\nu = |V_\nu| e^{j\delta_\nu}$$

are the terminal bus voltages of the line and $X_{i \nu}$ is its reactance. The maximum value $P_{tie max \ nu}$ gives the maximum real power (expressed in per unit of area $i$) that can be transmitted via the line. The tie-line is termed "weak" if

$$P_{tie max \ nu} \ll P_{ri}$$
Figure 2. Coherent Area Block Diagram
If the phase angles deviate from their nominal values $\delta_{i}^{*}$ and $\delta_{v}^{*}$ by amounts $\Delta \delta_{i}$ and $\Delta \delta_{v}$ respectively, then the incremental change in tie-line power flow becomes

$$\Delta P_{tie \, iv} = \frac{\partial P_{tie \, iv}}{\partial (\delta_{i}^{*} - \delta_{v}^{*})} (\Delta \delta_{i} - \Delta \delta_{v})$$

$$= \frac{|V_{i}||V_{v}|}{X_{iv}^{p} \, v_{ri}} \cos (\delta_{i}^{*} - \delta_{v}^{*}) (\Delta \delta_{i} - \Delta \delta_{v}) \quad (7)$$

To express the incremental tie-line power in terms of frequency deviation rather than power angle deviation, the following equation is used.

$$\Delta \delta_{i} = 2\pi \int \Delta f_{i} \, dt \quad (8)$$

By combining Eqs. (7) and (8), the incremental tie-line power flow becomes

$$\Delta P_{tie \, iv} = T_{iv}^{*} \left( \int \Delta f_{i} \, dt - \int \Delta f_{v} \, dt \right)$$

$$T_{iv}^{*} \triangleq 2\pi \frac{|V_{i}||V_{v}|}{X_{iv}^{p} \, v_{ri}} \cos (\delta_{i}^{*} - \delta_{v}^{*}) \quad (9)$$

The constant $T_{iv}^{*}$ is given the name synchronizing coefficient or electrical stiffness of the tie-line.

By Laplace transforming Eq. (9) and letting $v$ take on values from 1 to the number of lines terminating in
area i, the total incremental power exported from area i results:

\[ \Delta P_{\text{tie } i}(s) = \frac{1}{s} \sum_{i} T_{i}^* (\Delta F_i(s) - \Delta F_{\text{v}}(s)) \]  \hspace{1cm} (10)

Equation (10) is symbolized in block diagram form in Figure 3.

The incremental generated power \( \Delta P_g \).--The real power generated in a synchronous machine is controlled by means of the prime mover torque. This torque is controlled by opening or closing the main steam valve in the case of a steam turbine or the water gates in the case of a hydro turbine. The steam turbine arrangement is shown in Figure 4. Several different representations of this system for small signals around nominal settings are available.\textsuperscript{1,12,13} In this study the simplest representation is chosen, one that in spite of its simplification gives valid results. The theory that follows could be applied to any of the available system representations though. The system used in this study is shown in block diagram form in Figure 5. The governor-turbine-generator has been represented by two time constants, \( T_{g} \) and \( T_{t} \), the former the time constant of the governor and the latter the time constant of the turbine. The generator response is for all practical purposes considered instantaneous.
Figure 3. Incremental Tie-line Power Out of Area i
Figure 4. Typical Turbine Control Arrangement
Figure 5. Block Diagram of Turbine Control Arrangement
The constant $R \text{ Hz/pu MW}$ is a measure of the static speed droop or self regulation of the uncontrolled generator. The unit is termed "uncontrolled" when $\Delta P_c$, the input to the speed changer, is zero. If the uncontrolled generator is running at nominal speed $f^*$ Hz at no load and is called upon to produce rated power, then the generator speed will decrease to a new steady state speed $(f^* - R)$ Hz.

**Overall single area model.**—By combining the block diagrams of Figures 2, 3 and 5, the overall single area perturbation model is obtained as shown in Figure 6. This model of an individual electric energy system will be used to represent separate areas of an interconnected electric energy system.

"P-f" Control of the Two Area System

By combining two single area perturbation models, the two area perturbation model can be formed and is shown in Figure 7. (Disregard the dotted portions of Figure 7 for the time being.)

The block containing the transfer function $a_{12}$ is explained in the following way. The term $\Delta P_{\text{tie 1}}$ in Figure 7 represents the total tie-line power flowing out of area 1 and is expressed in pu MW of area 1. Clearly the total tie-line power from area 2 in MW must equal the
Figure 6. Block Diagram of Single Area Perturbation Model
Figure 7. Block Diagram of Two Area Perturbation Model
negative of the tie-line power from area 1 in MW. But since \( \Delta P_{\text{tie} 1} \) is in per unit of area 1, then \( \Delta P_{\text{tie} 2} \) in per unit of area 2 must be

\[
\Delta P_{\text{tie} 2} = -\frac{P_{r1}}{P_{r2}} \Delta P_{\text{tie} 1} = a_{12} \Delta P_{\text{tie} 1}
\]

where \( a_{12} \) is the negative ratio of the rated powers of area 1 and 2 in MW.

A useful mechanical analog.—In attempting to grasp the load-frequency dynamics of the two area system, the following mechanical analog is helpful. The analog, shown in Figure 8, consists of two engine-load assemblies representing the two electric energy areas, interconnected via a soft spring, representing a "weak" tie-line.

If the trains are traveling at nominal speed \( v^* \) and a load change is suddenly applied to one of the assemblies, the speeds of the two trains will change by amounts \( \Delta v_1 \) and \( \Delta v_2 \) and the spring force will change by amount \( \Delta F_{\text{tie}} \). The differential equations describing these dynamics of the two train-load assemblies are identical (except for a scale factor) to those presented here describing the performance of the two area interconnected electric energy system to load disturbance. The following variables are related.
Figure 8. Mechanical Analog of Two Area Interconnected System
The Uncontrolled System—Steady State and Dynamic Analysis

Before studying how to control the two area system, an analysis of the uncontrolled two area system is given. To determine the static frequency error \( \Delta f_{1ss} = \Delta f_{2ss} = \Delta f_{ss} \), use is made of Eq. (4) with \( s = 0 \). By writing out Eq. (4) for each area with

\[
\Delta P_{g1} = -\frac{1}{R_1} \Delta f_{ss}
\]

(12)

\[
\Delta P_{g2} = -\frac{1}{R_2} \Delta f_{ss}
\]

\[
\Delta P_{\text{tie}} = a_{12} \Delta P_{\text{tie}}
\]

and eliminating \( \Delta P_{\text{tie}} \) from the equations gives

\[
\Delta f_{ss} = -\frac{\Delta P_{d2} - a_{12} \Delta P_{dl}}{D_2 + 1/R_2 - a_{12} (D_1 + 1/R_1)}
\]

(13)

Likewise, by eliminating \( \Delta f_{ss} \) from the equations used previously gives

\[
\Delta P_{\text{tie}} = \frac{\Delta P_{d2} (D_1 + 1/R_1) - \Delta P_{dl} (D_2 + 1/R_2)}{D_2 + 1/R_2 - a_{12} (D_1 + 1/R_1)}
\]

(14)
For each area, the area frequency response characteristic \( \beta \) is defined.

\[
\beta_1 \triangleq D_1 + \frac{1}{R_1} \text{ pu MW/Hz}
\]

\[
\beta_2 \triangleq D_2 + \frac{1}{R_2} \text{ pu MW/Hz}
\]

The physical significance of \( \beta \) can be stated as follows.

An area operated alone will, if uncontrolled and subject to a step load change, experience a static frequency drop inversely proportional to its \( \beta \).

This follows by using Eq. (4) with \( \Delta P_{\text{tie}} = 0 \) and \( s = 0 \).

\[
\Delta f_{ss} = - \frac{\Delta P_d}{\beta}
\]

Using Eq. (15), Eqs. (13) and (14) become

\[
\Delta f_{ss} = - \frac{\Delta P_d_2 - a_{12} \Delta P_{dl}}{\beta_2 - a_{12} \beta_1}
\]

\[
\Delta P_{\text{tie 1 ss}} = \frac{\beta_1 \Delta P_d_2 - \beta_2 \Delta P_{dl}}{\beta_2 - a_{12} \beta_1}
\]

For the equal area problem, Eq. (17) becomes particularly simple but illustrates very vividly the benefits of interconnected system operation.

With

\[
D_1 = D_2 = D
\]

\[
R_1 = R_2 = R
\]
\[ \beta_1 = \beta_2 = \beta \]
\[ a_{12} = -1 \]

Equation (17) gives
\[ \Delta f_{ss} = -\frac{\Delta P_{d2} + \Delta P_{d1}}{2\beta} \]

\[ \Delta P_{tie\,1\,ss} = -\Delta P_{tie\,2\,ss} \]
\[ = \frac{\Delta P_{d2} - \Delta P_{d1}}{2} \]  

(18)

If, for example, a step load change occurs in area 1, then
\[ \Delta f_{ss} = -\frac{\Delta P_{d1}}{2\beta} \]
\[ \Delta P_{tie\,1\,ss} = -\frac{\Delta P_{d1}}{2} \]  

(19)

Equation (19) shows that:

1. The frequency drop is only half that which would be experienced if the areas were operating alone (compare Eqs. (16) and (19)).

2. Fifty percent of the added load in area 1 is picked up by power supplied by area 2 across the tie-line.

Although some discussion of the general multi-area system is presented, this study is limited in detail to the two area system for the following important reasons:
1. It is the simplest of the multi-area systems.

2. The papers that have been published on multi-area control $^{14,15,16,17}$ have limited their analysis to the two area system. This allows a comparative basis.

3. It is necessary to completely understand the operation of the two area problem before attempting to tackle large systems.

The dynamics of the two area interconnected system were studied using the analog computer with the system data listed below:

- $P_{r1} = P_{r2} = 2000 \text{ MW}$
- $H_1 = H_2 = 5 \text{ sec}$
- $D_1 = D_2 = 8.33 \times 10^{-3} \text{ pu MW/Hz}$
- $T_{T1} = T_{T2} = .3 \text{ sec}$
- $T_{G1} = T_{G2} = .08 \text{ sec}$
- $R_1 = R_2 = 2.4 \text{ Hz/pu MW}$
- $P_{tie \text{ max}} = 200 \text{ MW}$
- $\delta_1^* - \delta_2^* = 30 \text{ deg}$
- $T_{12}^* = .545 \text{ pu MW}$
- $\Delta P_{dl} = .01 \text{ pu MW}$

Figure 9 shows the resulting variations of $\Delta f_1$, $\Delta f_2$ and $\Delta P_{tie \text{ 1}}$ to a step load change in area 1. These important results are noted:
Figure 9. Uncontrolled Response of Two Area System to Step Load Increase in Area 1
1. All three variables have static nonzero errors.
2. The two frequency errors are equal in steady state.
3. The system is oscillatory but stable.

Control Specifications

The response of the uncontrolled system, illustrated in Figure 9, is unsatisfactory in several respects. Included is a set of performance requirements suggested by NAPSIC\textsuperscript{18} for operation of multi-area electric energy systems.

1. The static frequency error following a step load change must be zero.
2. The transient frequency swings should not exceed ± .02 Hz under normal conditions.
3. The static deviation in the tie-line power flow following a step load change in either area must be zero.
4. The time error should not exceed ± 3 seconds.

The first and second requirements represent the desire to keep the frequency as smooth as possible. A frequency deviation is the most sensitive indicator of a system disturbance. By keeping system frequency constant under normal operation, a system fault can be detected as soon as it occurs.

Requirement 3 represents the basic rule of power pool operation. That is, each area should, in steady state, carry its own load.
Requirement 4 represents the need to limit time error in synchronous clocks operated from system frequency. As a result of stringent frequency deviation specified in requirement 2, a time error of ± 3 seconds would take

\[3\left(\frac{1}{0.02}\right)^60 = 9000 \text{ sec}\]

or about 3 hours to accumulate. To maintain time error within limits the normal procedure is to let one member of the pool keep track of the time error by comparing "system" time with standard time. At various intervals this member commands all pool members to reduce time error to zero by a unison effort.

**Tie-Line with Frequency Bias Control Strategy**

The standard megawatt-frequency control strategy in use today is one which causes the speed changer position \(\Delta P_c\) to change at a rate proportional to tie-line deviation with frequency deviation biased. Mathematically this control strategy is of the form

\[
\Delta P_{c1} \triangleq - K_{I1} \int \left(\Delta P_{tie 1} + B_1 \Delta f_1\right) dt
\]

\[
\Delta P_{c2} \triangleq - K_{I2} \int \left(\Delta P_{tie 2} + B_2 \Delta f_2\right) dt
\]

(20)

The constants \(K_{I1}\) and \(K_{I2}\) are integrator gains and the constants \(B_1\) and \(B_2\) are the frequency bias parameters.
This control strategy is indicated by the dotted lines of Figure 7.

**Static Closed Loop System Response**

To show that the control strategy of Eq. (20) satisfies requirements 1 and 3, assume the system is at a static equilibrium point and a step load occurs in one of the two areas. If a new static equilibrium exists, then it can be achieved only after the speed changer commands $\Delta P_{c1}$ and $\Delta P_{c2}$ reach constant values. This requires that both integrands of Eq. (20) be zero.

\[ \Delta P_{\text{tie 1 ss}} + B_1 \Delta f_{\text{ss}} = 0 \]  \hspace{1cm} (21)
\[ \Delta P_{\text{tie 2 ss}} + B_2 \Delta f_{\text{ss}} = 0 \]

In view of Eq. (11), the only solution to this set of linear algebraic equations is the trivial one, i.e.,

\[ \Delta f_{\text{ss}} = \Delta P_{\text{tie 1 ss}} = \Delta P_{\text{tie 2 ss}} = 0 \]  \hspace{1cm} (22)

*This result is independent of the $B_1$ and $B_2$ values. In fact, one bias parameter (but not both) can be zero and still Eq. (22) must be true, satisfying requirements 1 and 3.*

**Dynamic Closed Loop Response**

Since, provided a static equilibrium exists to a system load change, requirements 1 and 3 are satisfied for
any frequency bias parameter value, the dynamic response as a function of the control parameters must be studied. In general, for the two area problem, there are four control parameters, but for the equal area problem

\[ K_{11} = K_{12} = K_1, \quad B_1 = B_2 = B. \]

To determine the dynamic system response as a function of control parameters \( K_1 \) and \( B \) analytically would be very difficult indeed, for the closed loop system is of 9th order. Therefore, the system was simulated on the analog computer.

Figure 10 shows the response of the two area interconnected system for fixed value of \( K_1 \) and variable \( B \), expressed in percent of \( \beta \). This figure is just one of many indicating the effect of \( K_1 \) and \( B \) on system performance.

The closed loop system becomes unstable if either \( K_1 \) or \( B \) is chosen sufficiently large. In fact Figure 10 clearly illustrates the deterioration of system stability with increasing \( B \).

Optimum Control Parameters

In order to determine the values of \( K_1 \) and \( B \) which give "best" response, it is necessary to give a definition of what is meant by "best." The well-known Integral Square Error criterion\(^9\) is chosen, and that combination of parameter values \( K_1 \) and \( B \) are desired which minimize the ISE. The ISE is defined as
Figure 10. Closed Loop Response of Two Area System to Step Load Increase in Area 1. $K_I$ Fixed
The ISE criterion is used for the following reasons:

1. It is a well-known criterion, used often in parameter optimization problems.

2. It penalizes large errors and requires the steady state value of variables to be zero.

3. It is readily adaptable to the specifications placed on the system being considered.

4. Since it is quadratic in nature, a smooth minimum is generated.

Since $\Delta f_1$ and $\Delta f_2$ closely resemble each other, only $\Delta f_1$ is penalized. The parameter $a$ in Eq. (23) is a weighting factor that determines the relative penalty between tie-line deviation and frequency deviation.

Use is made of the ISE criterion in the following manner. For a particular value of $K_I$ and $B$, the response $\Delta P_{\text{tie}}$ and $\Delta f_1$, and the integral $C$ of Eq. (23) are determined. This procedure is repeated for many different combinations of the $K_I - B$ parameter pair. Then $C$ is plotted versus $K_I$ and $B$ in a 3-dimensional figure. A bowl-shaped "cost surface" is obtained, the lowest point corresponding to the optimum parameter values $K_{I \text{ opt}} - B_{\text{ opt}}$. This is illustrated in Figure 11.
Figure 11. "Cost Surface" Showing Minimum Value of C at $K_{I\text{, opt}} \cdot B_{\text{opt}}$
Computer Results

To evaluate the system transient response for a particular $K_T - B$ parameter pair and the ISE criterion $C$ of Eq. (23), the analog computer was employed. The evaluation of $C$ required two multipliers and one summer-integrator.

The results of the study were recorded on 2-dimensional figures, the 3-dimensional Figure 11 being impractical to plot. In Figures 12, 13 and 14 are plotted "equi-B" contours which correspond to sections of the cost bowl of Figure 11, taken by looking into the B axis.

Figure 12 corresponds to the C criterion which applies one hundred percent penalty to tie-line deviation ($\alpha=0$). In Figure 13 the C criterion equally penalizes frequency deviation and tie-line deviation ($\alpha=.065$). In Figure 14 the C criterion applies all penalty to frequency deviations ($\alpha=\infty$).

Summary of Results

This study of the classical megawatt-frequency control problem was undertaken to gain an understanding of the multi-area electric energy system dynamics and the importance of control parameters on system response. Presently all U. S. utilities as part of an interconnected system use the control strategy presented in this chapter. That is, the generator speed changer position changes at a
Figure 12. Cost Function $C$ Which Penalizes Only $\Delta P_{tie 1}(t)$
Figure 13. Cost Function C Which Penalizes Both $\Delta P_{tie1}(t)$ and $\Delta f_1(t)$
Figure 14. Cost Function $C$ Which Penalizes Only $\Delta f_1(t)$
rate proportional to the ACE, the sum of tie-line deviation with frequency deviation biased. The frequency bias constant $B$ on the area frequency controllers is set equal to the area frequency response characteristic $\beta$. The reason why this is so is as follows. When a load increase occurs in area 2, the steady state power flowing across the tie-line from area 1 to area 2 equals $-\beta \Delta f_{ss}$, if the two area system is uncontrolled. This is shown by using Eq. (12) in Eq. (3) with $\Delta P_{dl} = 0$. Hence, by setting $B = \beta$, only area 2 controller will take action, the ACE of area 1 being zero. It is concluded that with this setting not only will nominal state be restored, but minimum tie-line deviations will occur. 

In this chapter both static and dynamic analysis of the multi-area electric energy system were presented. Static analysis showed that interconnected areas, when using tie-line with frequency bias type controllers, would return to nominal state after disturbance independent of frequency bias settings, provided the system remained stable. Since static analysis gave no indication of what value of frequency bias to use, dynamic analysis was applied to the problem.

To determine what system response would be preferred, the ISE criterion was used. For the two area system that was studied, using the system data given in the chapter, the best value of frequency bias to use was
approximately one-half the area frequency response characteristic. This result raises serious questions about the operating strategy in use by industry today. The value selected for the integral gain parameter $K_I$ is also very important. The electric utilities make no mention of what value of $K_I$ to use. Figure 11 shows that for fixed $B$ and high enough $K_I$, the "cost bowl" will have vertical walls. The vertical walls indicate system instability which cannot be tolerated. In fact too high a value for either integral gain $K_I$ or frequency bias $B$ could well be the cause of instability problems experienced on interconnected systems in operation today.

The study has shown that the ISE criterion which penalizes only frequency deviation is not a satisfactory criterion. Figure 14 indicates that by holding the product of $K_I$ and $B$ constant, "best" response will be obtained. To show why this is so, Eq. (20) is rewritten.

\[ \Delta P_{c1} = - \int (K_I B \Delta f_1 + K_I \Delta P_{tie 1}) dt \]

\[ \Delta P_{c2} = - \int (K_I B \Delta f_2 + K_I \Delta P_{tie 2}) dt \]

Equation (24) Since the ISE criterion requires only frequency deviation to be zero, the speed changer need only change position at a rate proportional to frequency deviation. Hence as $K_I$
decrease, B must increase so that the product $K_I B$ remains fairly constant.

The values of parameters $K_I$ and $B$ which minimize tie-line deviation are

$$B \approx 0.217$$
$$K_I \approx 0.925$$

Using these parameter values, response of the two area system to load disturbance was obtained and is shown in Figure 15. The system response is highly oscillatory, which is typical of interconnected systems in operation today. If the control strategy takes into account total information of the state of the system, rather than just tie-line deviation and frequency deviation, more damped response might be achieved. This is investigated when techniques of optimal control theory are applied in an attempt to develop a "supreme" control strategy.
Figure 15. Optimal System Response Using Classical Control Strategy
In this chapter, the techniques of optimal control theory that will be applied to the megawatt-frequency control problem are discussed. The objective of the megawatt-frequency control problem is to keep the frequency deviation and tie-line deviation within specified limits by close control over the controllable real sources of the system. In the last chapter this objective was accomplished by use of the control strategy presently in operation. To develop new type control strategies, the structure of the controller is not fixed.

This control problem of interconnected electric energy systems is readily adaptable to the well-known infinite time linear state regulator problem of optimal control theory. The state regulator problem objective is to control the system so that the system states are kept small. The solution of this problem leads to an optimal controller which is a linear function of the states of the system. This controller keeps the states near zero without excessive control effort.
The Infinite Time Linear Regulator Problem Formulation

The system state $x$.—It is assumed that the system can be represented by the linear time-invariant differential equation

$$\dot{x}(t) = Fx(t) + Gu(t)$$  \hspace{0.5cm} (25)

where

$x(t)$ - $n \times 1$ state vector
$u(t)$ - $m \times 1$ control vector
$F$ - $n \times n$ state distribution matrix
$G$ - $n \times m$ control distribution matrix

Also it is assumed that $0 < m \leq n$ and that the control vector $u(t)$ is unconstrained.

The state of the system, represented by the state vector $x$, is a very important concept. The state variables $x_1, x_2, \ldots, x_n$ are the components of the state vector $x$. These state variables are the minimum number of variables which contain sufficient information about the past history of the system to allow for computing the future of the system, assuming the control input is known. The state variables are not merely mathematical artifices, but have true physical meaning, as will be shown.

The system cost $C$.—The problem objective as stated previously is to keep the system state near zero
without excessive control effort. The transformation of this objective into a particular mathematical functional is left to the discretion of the engineer. The type functional, called the cost functional or system cost, used in this formulation is quadratic in nature. To be specific, the system cost \( C \), a scalar, is of the form

\[
C = \frac{1}{2} \int_0^\infty [x'(t)Qx(t) + u'(t)Ru(t)]dt \tag{26}
\]

where

- \( Q - n \times n \) positive semidefinite symmetric state cost weighting matrix
- \( R - m \times m \) positive definite symmetric control cost weighting matrix

To show that this cost functional represents a reasonable mathematical translation of the specifications placed on the physical system, each term of \( C \) is considered.

The term \( x'(t)Qx(t) \) with \( Q \) positive semidefinite is nonnegative for all \( x(t) \) and is zero when \( x(t) \) is zero. Since \( x'(t)Qx(t) \) is quadratic in \( x(t) \), this term of \( C \) penalizes the system much more severely for large values of the states than for small values.

The term \( u'(t)Ru(t) \) with \( R \) positive definite is positive for all \( u(t) \neq 0 \) (not all components of the vector \( u \) are zero). This term weighs the cost of the control and
penalizes the system much more severely for large control effort than for small control effort.

**Variational Approach to the Linear Regulator Problem**

To solve for the optimal controller which drives the system and minimizes the system cost $C$, the variational approach is used. The Hamiltonian $H$ for the system (Eq. (25)) and cost $C$ (Eq. (26)) is

$$H = \frac{1}{2} \{ x'(t)Qx(t) + u'(t)Ru(t) \} + p'(t)(Fx(t) + Gu(t))$$

(27)

where $p(t)$ is a $n \times 1$ costate vector.

The results of the variational approach are: Let $u^*(t)$ be a control which drives the system from an initial state $x(0)$. In order that $u^*(t)$ be optimal, it is necessary that there exist a function $p^*(t)$ such that

- **a.** $p^*(t)$ corresponds to $u^*(t)$ and $x^*(t)$ so that $p^*(t)$ and $x^*(t)$ are solutions of the canonical system

$$\dot{x}^*(t) = \frac{\partial H}{\partial p} [x^*(t), p^*(t), u^*(t)]$$

$$\dot{p}^*(t) = -\frac{\partial H}{\partial x} [x^*(t), p^*(t), u^*(t)]$$

satisfying boundary conditions

$$x(0) = x_0, \quad p(\infty) = 0$$

(28)

(29)
b. the function $H[x^*(t), p^*(t), u(t)]$ has an absolute minimum as a function of $u(t)$ at $u(t) = u^*(t)$.

The Optimal Control $u^*$

To determine the optimal control $u^*(t)$, condition (b) of the variational approach is used. Condition (b) says that for an optimal control, if $u$ is unconstrained,

$$\frac{\partial H}{\partial u} = 0 \quad (30)$$

which implies that

$$\frac{\partial H}{\partial u} = Ru(t) + G'p(t) = 0 \quad (31)$$

$$u^*(t) = -R^{-1}G'p(t)$$

Since with $u(t) = u^*(t)$, the first gradient of $H$ with respect to $u$ is zero, the second gradient of $H$ with respect to $u$

$$\frac{\partial^2 H}{\partial u^2} = R \quad (32)$$

is positive definite and all high order gradients of $H$ with respect to $u$ are zero, it follows that $H$ does have an absolute minimum at $u^*(t)$.

Equation (28) gives the conditions $x^*(t)$ and $p^*(t)$ must satisfy
\[
\dot{x}^*(t) = \frac{\partial H}{\partial p} = Fx^*(t) - GR^{-1}G'p^*(t) \tag{33}
\]

\[
\dot{p}^*(t) = -\frac{\partial H}{\partial x} = -Qx^*(t) - F'p^*(t)
\]

These system canonical equations can be written in the form

\[
\begin{bmatrix}
\dot{x}^*(t) \\
\dot{p}^*(t)
\end{bmatrix} = M \begin{bmatrix}
x^*(t) \\
p^*(t)
\end{bmatrix}
\tag{34}
\]

where \(M\), a \(2n \times 2n\) system canonical matrix, is defined as

\[
M \triangleq \begin{bmatrix}
F & -GR^{-1}G' \\
-Q & -F'
\end{bmatrix}
\tag{35}
\]

To determine \(u^*(t)\) as a feedback control law, a transformation between \(p(t)\), the costate vector, and \(x(t)\), the state vector, is needed. It is assumed that such a transformation does exist and is given by

\[
p^*(t) = P(t)x^*(t) \tag{36}
\]

where \(P(t)\), a \(n \times n\) positive definite symmetric matrix, is a solution to the matrix Riccati equation.

By using Eq. (36) in Eq. (33), the first order nonlinear matrix differential Riccati equation results
\[ \dot{P}(t) = -PF - F'P + PGR^{-1}G'P - Q \] (37)

\[ P(t = \infty) = 0 \]

Equation (37) is solved for its positive definite steady state solution with

\[ \dot{P}(t) = 0 \] (38)

Then, using Eq. (36) in Eq. (31) gives

\[ u^*(t) = -R^{-1}G'Px^*(t) \triangleq Kx^*(t) \] (39)

where \( K \), of dimension \( m \times n \), is a constant optimal gain matrix and \( u^*(t) \), the optimal controller, is in the feedback form desired. Substituting Eq. (39) into the first of Eqs. (33) gives

\[ x^*(t) = Fx^*(t) + GKx^*(t) \triangleq Fcx^*(t) \] (40)

The response of the optimal system is the solution of Eq. (40).

The System Conditions—Controllability and Observability

What conditions must the system satisfy for the matrix Riccati equation (Eq. (37)) to have a positive definite symmetric steady state solution? To answer this question, the concepts of controllability and observability must be considered.
When a system in the form of Eq. (25) is said to be controllable, it means that by suitable choice of the elements of the gain matrix $K$ of Eq. (39), the closed loop poles of the system may be specified. Stated mathematically, the system of Eq. (25) is controllable if and only if the rank of the $n \times nm$ partitioned matrix $A$, where

$$A \triangleq [G, FG, \ldots, F^{n-1}G]$$

is equal to $n$ or equivalently if and only if the matrix $A$ has $n$ linearly independent column vectors.

To discuss the concept of observability, the matrix $D$ of dimension $\ell \times n$ ($\ell \leq n$), and the vector $y$ of dimension $\ell \times 1$ are defined as

$$Q = D'D$$

$$y(t) = Dx(t)$$

The components of $y$ are a linear combination of the state variables of the system. The system described by Eq. (25) is said to be observable from $y$ if, given the information contained in $y$, complete information about the state $x$ may be reconstructed. Stated mathematically the system is observable from $y$ if and only if the rank of the $n \times n\ell$ partitioned matrix $B$, where
B = \left[ D', F'D', F'^2D', \ldots, F'^{n-1}D' \right] \quad (43)

is equal to n, or equivalently if and only if there is a set of n linearly independent column vectors of B.

The two conditions the system represented by Eq. (25) with system cost C of Eq. (26) must satisfy for the Eq. (37) to have a unique positive definite steady state solution are:

a. The system must be observable from \( y = Q^{1/2}x = Dx \)
b. The states of the system which are unstable must be controllable

A proof of these necessary conditions can be found elsewhere.\(^2^2\)

The Eigenvector Matrix Riccati Equation Solution

The positive definite solution \( P \) of Eq. (37) can be obtained in several ways. One method would be to use some numerical differential equation solver routine. Since the boundary condition on \( P(t) \) is at the final time, Eq. (37) would be solved backwards in time until \( P(t) \) reached a steady state solution. Another method would be to use Eq. (38) and solve the \( n(n + 1)/2 \) nonlinear algebraic equations for the elements of the positive definite \( P \). The method employed\(^2^3\) however, is one which uses the eigenvectors of the stable eigenvalues of the system canonical matrix \( M \) of Eq. (35).
If the system conditions stated previously are satisfied, then all \( n \) eigenvalues of \( F_c \) of Eq. (40) will have negative real parts. A similarity transformation is performed on \( F_c \) with the \( n \times n \) matrix \( S \).

\[
S^{-1}F_cS = J
\]

(44)

If it is assumed that the first \( k \) eigenvalues of \( F_c \) are real \( (\lambda_k = \sigma_k) \) and the remaining \( (n - k)/2 \) pairs are complex \( (\lambda_{k+1} = \sigma_{k+1} + j\omega_{k+1}) \), then

\[
J = \text{diag} \{\lambda_1, \lambda_2, \ldots, \lambda_k, \begin{bmatrix} \sigma_{k+2} & \omega_{k+2} \\ \omega_{k+2} & \sigma_{k+2} \end{bmatrix}, \ldots, \begin{bmatrix} \sigma_n & \omega_n \\ -\omega_n & \sigma_n \end{bmatrix} \}
\]

(45)

By definition let

\[
L \triangleq PS
\]

(46)

Then, using Eqs. (37) and (44),

\[
LJ = -F'L - QS
\]

(47)

and

\[
\begin{bmatrix} S \\ \hline L \end{bmatrix} J = \begin{bmatrix} F & -GR^{-1}G' \\ \hline -Q & -F' \end{bmatrix} \begin{bmatrix} S \\ \hline L \end{bmatrix}
\]

(48)
The $2n \times 2n$ matrix \( \begin{bmatrix} S \\ L \end{bmatrix} \) is the "real modal" matrix of $M$ corresponding to the $n$ stable eigenvalues of $M$, or identically the eigenvalues of the optimal system Eq. (40), and is formed in the following way.

If $v_\eta$ is a real eigenvector associated with real eigenvalue $\lambda_\eta$ and $v_{R\nu} + jv_{I\nu}$ is the complex eigenvector associated with complex eigenvalue $\lambda_\nu$, then

\[
\begin{bmatrix} S \\ L \end{bmatrix} \triangleq \begin{bmatrix} v_1, \ldots, v_k, v_{Rk+1}, v_{Ik+1}, v_{Rk+3}, v_{Ik+3}, \ldots \end{bmatrix}
\] (49)

From Eq. (46), the solution to the steady state Riccati equation is

\[
P = LS^{-1}
\] (50)

and since $P$ is symmetric

\[
P = (S^{-1})'L'
\] (51)

**Digital Computer Application**

A digital computer program was written in Fortran IV (a flow chart and program listing are included in the Appendix), which, when given the system matrices $F$ and $G$, and the cost matrices $Q$ and $R$, checks for controllability and observability of the system and computes the optimal feedback gain matrix $K$. 
Controllability of the pair \((F, G)\) was examined in the following way. The partitioned matrix \(A\) of Eq. (41) was calculated, normalizing the columns of \(A\) to 1 at each step. After \(A\) was computed, each row of \(A\) was normalized to 1 and the determinant of \(AA'\) was evaluated. If the determinant is nonzero, the matrix \(A\) has rank \(n\).

Observability of the pair \((F, Q)\) was evaluated in the same manner using the partitioned matrix \(B\) of Eq. (43).

The measure of convergence of the steady state solution \(P\) of Eq. (37) was determined by calculating the norm\(^2\) of \(\dot{P}\), defined as

\[
\|\dot{P}\|^2 \triangleq \sum_{i,j} \max (|\dot{P}_{ij}|)
\]

where \(\dot{P}_{ij}\) is the \(i\)-\(j\)th element of \(\dot{P}\). \(\dot{P}\) was calculated from Eq. (37) once \(P\) was determined. This norm was always less than \(1.0 \times 10^{-3}\) in all cases that were studied.

The eigenvalue-eigenvector solver used was IGVEC5—CHARD.\(^5\)
CHAPTER IV

MODERN MEGAWATT-FREQUENCY CONTROL

The optimal control theory discussed in the last chapter is applied to the megawatt-frequency control problem to develop a "supreme" control strategy. The interconnected electric energy system is modeled in state variable form. The performance specifications the system must satisfy are stated mathematically in terms of an integral quadratic system cost. The optimal controller which drives the system and minimizes the system cost $C$ is a linear function of the system states. The optimal feedback gains are determined for four different system costs. System response, obtained from the analog computer, is much improved compared to system response which results when using classical control strategies discussed in Chapter II.

For each of the four system costs, the optimal controllers for each area are noninteracting. Each area controller depends only on the states of its respective area. The control strategies suggested in this chapter could be implemented on existing interconnected systems, giving greatly improved system dynamic response.
The Dynamic System in State Variable Form

The dynamic model that was developed in Chapter II will be used again. However, since the dynamic system equations are to be put in state variable form, they are repeated.

The power equilibrium equation in per unit of area \( i \) is

\[
\frac{2H_i}{f^*} \frac{d}{dt} \Delta f_i + D_i \Delta f_i + \Delta P_{\text{tie} \ i} = \Delta P_{g_i} - \Delta P_{d_i} \tag{53}
\]

The incremental tie-line power flowing out of area \( i \) in per unit of area \( i \) is

\[
\Delta P_{\text{tie} \ i} = \sum_v \left[ T_{i,v}^* \int \Delta f_i dt - \int \Delta f_v dt \right] \tag{54}
\]

In Chapter II the governor-turbine-generator arrangement was represented by two time constants. The block diagram is shown in Figure 5. In this chapter the governor-turbine-generator arrangement is represented by two differential equations associated with the time constants \( T_{gv} \) and \( T \). A new variable \( \Delta X_{gv} \), governor valve position in pu MW is introduced (see Figure 4). The incremental change in generation to incremental change in frequency deviation or governor valve position is described by the following differential equations in pu MW.
\[
\frac{d}{dt} \Delta P_g = -\frac{1}{T_t} \Delta P_g + \frac{1}{T_t} \Delta X_{gv} \\
\frac{d}{dt} \Delta X_{gv} = -\frac{1}{T_{gv}} \Delta X_{gv} - \frac{1}{T_{gv}^R} \Delta f + \frac{1}{T_{gv}} \Delta P_c
\]  

Combining Eqs. (53) and (54) along with Eq. (55) gives a set of three differential equations describing the dynamic performance of area i to incremental load changes in that area.

\[
\begin{align*}
\frac{2H_i}{f^*} \frac{d}{dt} \Delta f_i &+ D_i \Delta f_i + \sum_{i=1}^n T_{vi} \left[ \int \Delta f_i dt - \int \Delta f_v dt \right] = \Delta P_{gi} - \Delta P_{di} \\
\frac{d}{dt} \Delta P_{gi} &+ \frac{1}{T_{ti}} \Delta P_{gi} + \frac{1}{T_{ti}} \Delta X_{gvi} \\
\frac{d}{dt} \Delta X_{gvi} &+ \frac{1}{T_{gvi}} \Delta X_{gvi} - \frac{1}{T_{gvi}^R} \Delta f_i + \frac{1}{T_{gvi}} \Delta P_{ci}
\end{align*}
\]

The state and control variables.—Equation (56), which describes a single area of the multi-area system, must be rewritten in terms of the state and control variables. The state and control variables are defined in the following way.

\[
\begin{align*}
\frac{2H_i}{f^*} \frac{d}{dt} \Delta f_i &+ D_i \Delta f_i + \sum_{i=1}^n T_{vi} \left[ \int \Delta f_i dt - \int \Delta f_v dt \right] = \Delta P_{gi} - \Delta P_{di} \\
\frac{d}{dt} \Delta P_{gi} &+ \frac{1}{T_{ti}} \Delta P_{gi} + \frac{1}{T_{ti}} \Delta X_{gvi} \\
\frac{d}{dt} \Delta X_{gvi} &+ \frac{1}{T_{gvi}} \Delta X_{gvi} - \frac{1}{T_{gvi}^R} \Delta f_i + \frac{1}{T_{gvi}} \Delta P_{ci}
\end{align*}
\]

\[
\begin{align*}
\frac{2H_i}{f^*} \frac{d}{dt} \Delta f_i &+ D_i \Delta f_i + \sum_{i=1}^n T_{vi} \left[ \int \Delta f_i dt - \int \Delta f_v dt \right] = \Delta P_{gi} - \Delta P_{di} \\
\frac{d}{dt} \Delta P_{gi} &+ \frac{1}{T_{ti}} \Delta P_{gi} + \frac{1}{T_{ti}} \Delta X_{gvi} \\
\frac{d}{dt} \Delta X_{gvi} &+ \frac{1}{T_{gvi}} \Delta X_{gvi} - \frac{1}{T_{gvi}^R} \Delta f_i + \frac{1}{T_{gvi}} \Delta P_{ci}
\end{align*}
\]

\[
\begin{align*}
\begin{align*}
x_1 &\triangleq \int \Delta P_{tie} dt \\
x_2 &\triangleq \Delta P_{g1} \\
x_3 &\triangleq \Delta f_i \\
x_4 &\triangleq \Delta P_{cl} \\
x_5 &\triangleq \Delta X_{gvi}
\end{align*}
\end{align*}
\]

\[
\begin{align*}
\begin{align*}
x_1 &\triangleq \int \Delta P_{tie} dt \\
x_2 &\triangleq \Delta P_{g1} \\
x_3 &\triangleq \Delta f_i \\
x_4 &\triangleq \Delta P_{cl} \\
x_5 &\triangleq \Delta X_{gvi}
\end{align*}
\end{align*}
\]
To maintain frequency deviation in each area within tolerance levels, close control is kept over the real power sources in each area. This is accomplished by varying the speed changer position. The control force in each area is the incremental speed changer position deviation.

The variables of an area which change as a result of surplus power in that area are the governor valve position deviation, generator output deviation, frequency deviation and tie-line power deviation. These variables are defined as states. The integral of tie-line deviation must also be defined as a state. One of the performance specifications placed on the system is that tie-line deviation should be zero in steady state following a step load disturbance. Tie-line deviation will be zero only when the integral of tie-line deviation reaches a steady state value. To guarantee this, information about the integral of tie-line deviation must be made available.

For each area there are a set of five state variables and one control input. However, in the two area problem which is considered, the tie-line deviation in the first area is proportional to the tie-line deviation in the second by a constant. This constant was originally defined in Eq. (11) of Chapter II.
\[ \Delta P_{\text{tie} 2} = -\frac{P_{r1}}{P_{r2}} \Delta P_{\text{tie} 1} = a_{12} \Delta P_{\text{tie} 1} \quad (58) \]

In this case, an additional state for the integral of tie-line deviation in area 2 is not needed.

The two area problem—the matrices F and G.—For the two area problem there are nine states and two control inputs.

\[
\begin{align*}
\dot{x}_1 & \triangleq \int \Delta P_{\text{tie} 1} \, dt \\
\dot{x}_2 & \triangleq \int \Delta f_1 \, dt \\
\dot{x}_3 & \triangleq \Delta f_1 \\
\dot{x}_4 & \triangleq \Delta P_{g1} \\
\dot{x}_5 & \triangleq \Delta X_{gvl} \\
\dot{x}_6 & \triangleq \int \Delta f_2 \, dt \\
\dot{x}_7 & \triangleq \Delta f_2 \\
\dot{x}_8 & \triangleq \Delta P_{g2} \\
\dot{x}_9 & \triangleq \Delta X_{gv2} \\
\end{align*}
\]

(59)

By substituting the definition of the state and control variables into the six differential equations which define the two area problem, the system equations can be written in the vector matrix form

\[ \dot{x}(t) = Fx(t) + Gu(t) + P_d \quad (60) \]

where
\[
\begin{bmatrix}
0 & T^{*}_{12} & 0 & 0 & 0 & 0 & -T^{*}_{12} & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{f^{*}T^{*}_{12}}{2H_{1}} & -\frac{f^{*}D_{1}}{2H_{1}} & \frac{f^{*}}{2H_{1}} & 0 & \frac{f^{*}T^{*}_{12}}{2H_{1}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{T_{t1}} & \frac{1}{T_{t1}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{a_{12}T^{*}_{21}}{2H_{2}} & 0 & 0 & 0 & a_{12}T^{*}_{21} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{t2}} \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{t2}} & \frac{1}{T_{t2}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{g2}R^{2}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{g2}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{f^{*}}{2H_{1}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{f^{*}}{2H_{2}} & 0 & 0 \\
\end{bmatrix}
\]

\[F = \begin{bmatrix}
0 & 0 & 0 & \frac{1}{T_{g2}}R^{2} & 0 & -\frac{1}{T_{g2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}\]

\[G' = \begin{bmatrix}
0 & 0 & 0 & 0 & \frac{1}{T_{g2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}\]

\[\Gamma' = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}\]

\[\text{(61)}\]
The matrix \( \Gamma \) of dimension \( 9 \times 2 \) is called the disturbance distribution matrix.

The megawatt-frequency control system model represented by Eq. (60) is not in the desired form for the following reasons:

1. In the optimal control theory presented in Chapter III, a \( \Gamma \) matrix was not presented in the system model.

2. The objective of the state regulator problem is to keep the system states near zero. However, because of the step load disturbance, requiring zero steady state value for all system states is not consistent with performance specifications placed on the system. For example, to maintain zero frequency deviation in steady state, increased demand power must be met by increased generation.

\[
\Delta P_g = \Delta P_d
\]  

Some of the other states will also have nonzero steady state values.

To clear up this apparent dilemma, a new state vector \( x^1(t) \) is defined as

\[
x^1(t) \triangleq x(t) - x_{ss}
\]  

where \( x_{ss} \) is the steady state of the system. The control \( u_{ss} \) needed to maintain the system in steady state is defined by Eq. (60) with \( \dot{x}(t) = 0 \).

\[
0 = Fx_{ss} + Gu_{ss} + \Gamma P_d
\]

\[
u_{ss} = - (G'G)^{-1}G'Fx_{ss} - (G'G)^{-1}G'TP_d
\]
A new control vector \( u^1(t) \) is defined as
\[
u^1(t) = u(t) - u_{ss} \tag{65}\]

The equation for the new state \( x^1(t) \) becomes
\[
\dot{x}^1(t) = Fx^1(t) + Gu^1(t) + Fx_{ss} + Gu_{ss} + TP_d \tag{66}
\]
which in light of Eq. (64) reduces to
\[
\dot{x}^1(t) = Fx^1(t) + Gu^1(t) \tag{67}
\]
\[x^1(0) = -x_{ss}\]

This change of system and control variables simply shifted the reference position of the system. The desired state is now the origin with zero control. The superscript 1 is dropped to prevent unnecessary notation problems. The matrices \( F \) and \( G \) remain unchanged.

**The System Specifications—the \( Q \) and \( R \) Matrices**

To define the \( Q \) and \( R \) matrices, a set of performance specifications the system is to satisfy must be stated. These are the same as presented in Chapter II, but will be repeated here.

1. The static frequency deviation following a step load change must be zero.
2. The static change in tie-line power following a step load change must be zero.
3. The transient frequency deviation should not exceed \( \pm 0.02 \) Hz under normal conditions.
4. The time error represented by the integral of frequency deviation should not exceed ± 3 seconds.

To define these specifications mathematically requires the sum of the following terms.

From condition 1

\[(\Delta f_1)^2 + (\Delta f_2)^2\]

From condition 2

\[(\Delta P_{tie 1})^2 = (T_i^* \left[ \int \Delta f_1 dt - \int \Delta f_2 dt \right])^2\]

From conditions 3 and 4

\[\left( \int \Delta f_1 dt \right)^2 + \left( \int \Delta f_2 dt \right)^2\]

Defining these variables in terms of their respective states and putting the products in matrix form \(x'Qx\) specifies \(Q\) to be
Large control effort is penalized by adding the terms $u_1^2 + u_2^2$ which requires $R$ to be

$$ R = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} $$

\hspace{1cm} (69)

**Computer Results**

Four different combinations of $Q$ and $R$ are presented to show their effect on system response. The $F$ and $G$ matrices did not change. The equal area problem was studied using the following system data.

$P_{r1} = P_{r2} = 2000$ MW

$H_1 = H_2 = 5$ sec

$D_1 = D_2 = 8.33 \times 10^{-3}$ pu MW/Hz

$T_{t1} = T_{t2} = .3$ sec
T_{gv1} = T_{gv2} = 0.08 \text{ sec}

R_1 = R_2 = 2.4 \text{ Hz/pu MW}

P_{tie \ max} = 200 \text{ MW}

\delta_1^* - \delta_2^* = 30 \text{ deg}

T_1^* = 0.545 \text{ pu MW}

\Delta P_{d1} = 0.01 \text{ pu MW}

Case 1 Q and R as defined in Eqs. (68) and (69)

Case 2 Q, 100R

Case 3 10Q, R

Case 4 10Q, 100R

Shown in Table 1 are the optimal feedback gains for each case. Once the optimal feedback gains were determined, the optimally regulated system was simulated on the analog computer.

Figure 16 is a graph of the system dynamic response using the optimal feedback gains of case 1. A comparison of this figure with the response of the optimum classical megawatt-frequency controller (Figure 15) very vividly illustrates the improvement in system response achieved by using total information of the system in the controller.

Figure 17 shows the dynamic response of the system for case 2 when control effort is penalized more severely
<table>
<thead>
<tr>
<th>Case 1 (Q, R as Defined)</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
<th>X6</th>
<th>X7</th>
<th>X8</th>
<th>X9</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1</td>
<td>-.707</td>
<td>-.299</td>
<td>-.932</td>
<td>-1.28</td>
<td>-.296</td>
<td>-.701</td>
<td>-.064</td>
<td>-.030</td>
<td>-.006</td>
</tr>
<tr>
<td>u_2</td>
<td>.707</td>
<td>-.701</td>
<td>-.064</td>
<td>-0.030</td>
<td>-.006</td>
<td>-.299</td>
<td>-.932</td>
<td>-1.28</td>
<td>-2.296</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2 (Q,100R)</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
<th>X6</th>
<th>X7</th>
<th>X8</th>
<th>X9</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1</td>
<td>-.071</td>
<td>-.046</td>
<td>-.039</td>
<td>-.074</td>
<td>-.020</td>
<td>-.054</td>
<td>-.013</td>
<td>-.020</td>
<td>-.005</td>
</tr>
<tr>
<td>u_2</td>
<td>.071</td>
<td>-.054</td>
<td>-.013</td>
<td>-.020</td>
<td>-.005</td>
<td>-.046</td>
<td>-.039</td>
<td>-.074</td>
<td>-.020</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 3 (10Q, R)</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
<th>X6</th>
<th>X7</th>
<th>X8</th>
<th>X9</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1</td>
<td>-2.24</td>
<td>-1.93</td>
<td>-3.48</td>
<td>-3.41</td>
<td>-.678</td>
<td>-1.24</td>
<td>-.003</td>
<td>.035</td>
<td>.006</td>
</tr>
<tr>
<td>u_2</td>
<td>2.24</td>
<td>-1.24</td>
<td>-.003</td>
<td>-.035</td>
<td>.006</td>
<td>-1.93</td>
<td>-3.48</td>
<td>-3.41</td>
<td>-.678</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 4 (10Q, 100R)</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
<th>X6</th>
<th>X7</th>
<th>X8</th>
<th>X9</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1</td>
<td>-.224</td>
<td>-.080</td>
<td>-.196</td>
<td>-.350</td>
<td>-.089</td>
<td>-.236</td>
<td>-.038</td>
<td>-.038</td>
<td>-.009</td>
</tr>
<tr>
<td>u_2</td>
<td>.224</td>
<td>-.236</td>
<td>-.038</td>
<td>-.038</td>
<td>-.009</td>
<td>-.080</td>
<td>-.196</td>
<td>-.350</td>
<td>-.089</td>
</tr>
</tbody>
</table>
Figure 16. System Dynamic Response Case 1
Figure 17. System Dynamic Response Case 2
than in case 1. The system response is oscillatory and has a long settling time.

The system response for case 3 is shown in Figure 18. Increased penalty is now placed on state deviations rather than control effort. The system response is heavily damped and has short settling time.

Increased penalty is placed on both state deviations and control effort for case 4. System response is shown in Figure 19. The system is oscillatory but state deviations are not as great and settling time is not as long as those of the system response of case 2.

Controller Structure

The controller for each area is a linear function of all nine states of the system. This is shown by writing out the optimal controller for case 1.

\[
\begin{align*}
    u_1 &= -0.707x_1 - 0.300x_2 - 0.932x_3 - 1.28x_4 - 0.296x_5 - 0.701x_6 \\
     &\quad - 0.064x_7 - 0.030x_8 - 0.006x_9 \\
    u_2 &= 0.707x_1 - 0.702x_2 - 0.064x_3 - 0.030x_4 - 0.006x_5 - 0.300x_6 \\
     &\quad - 0.932x_7 - 1.28x_8 - 0.296x_9
\end{align*}
\]

(70)

It appears that in this form each area controller depends on information not only from its own area but also from the opposite area. However, by replacing each state
Figure 18. System Dynamic Response Case 3
Figure 19. System Dynamic Response Case 4
variable of Eq. (70) by its physical variable and by using the following equalities between the states and tie-line deviations

\[ \Delta P_{\text{tie} 1} = T_{12}^* (x_2 - x_6) \]
\[ \frac{d\Delta P_{\text{tie} 1}}{dt} = T_{12}^* (x_3 - x_7) \] (71)
\[ \Delta P_{\text{tie} 2} = - \Delta P_{\text{tie} 1} \]
\[ \frac{d\Delta P_{\text{tie} 2}}{dt} = - \frac{d\Delta P_{\text{tie} 1}}{dt} \]

the controllers for each area become essentially non-interacting.

This is shown in Eq. (72).

\[ u_1 = -0.707 \left[ \Delta P_{\text{tie} 1} + 1.41 \Delta f_1 \right] dt \]
\[ + 1.28 \Delta P_{\text{tie} 1} + 0.117 \frac{d\Delta P_{\text{tie} 1}}{dt} \]
\[ - 0.996 \Delta f_1 - 1.28 \Delta P_{g1} - 0.296 \Delta X_{gv1} \]
\[ - 0.030 \Delta P_{g2} - 0.006 \Delta X_{gv2} \] (72)

\[ u_2 = -0.707 \left[ \Delta P_{\text{tie} 2} + 1.41 \Delta f_2 \right] dt \]
\[ + 1.28 \Delta P_{\text{tie} 2} + 0.117 \frac{d\Delta P_{\text{tie} 2}}{dt} - 0.996 \Delta f_2 \]
\[ - 1.28 \Delta P_{g2} - 0.296 \Delta X_{gv2} - 0.030 \Delta P_{g1} - 0.006 \Delta X_{gv1} \]
By writing the controller in this form it is revealed that area 1 controller depends weakly on information from area 2 and vice-versa for controller 2 and area 1. To illustrate this point, the coefficients of $\Delta P_{g2}$ and $\Delta X_{gv2}$ in $u_1$ and the coefficients of $\Delta P_{g1}$ and $\Delta X_{gv1}$ in $u_2$ are set equal to zero and this suboptimal system simulated. The system response is shown in Figure 20. In Table 2 the optimal feedback gains are shown when the controller is in the form of Eq. (72). In all four cases, the system controllers are essentially non-interacting.

This fact that area controllers are non-interacting is very important to the electric utilities. Presently individual power areas of interconnected groups measure area frequency deviation, area tie-line deviation, area generation deviation and integral of frequency deviation (time error). However, the area megawatt-frequency controller is only a function of the integral of the first two variables, frequency and tie-line deviation. The megawatt-frequency controllers of each area could be redesigned to be linear functions of all the area system variables with nominal expense. This new controller would maintain the system within very close tolerance of the desired nominal state.
Figure 20. System Suboptimal Response
<table>
<thead>
<tr>
<th>Case</th>
<th>$u_1$</th>
<th>$f \Delta P_{\text{tie} 1}$</th>
<th>$f \Delta f_1$</th>
<th>$\Delta P_{\text{tie} 1}$</th>
<th>$d \Delta P_{\text{tie} 1}$</th>
<th>$\Delta f_1$</th>
<th>$\Delta P_{g 1}$</th>
<th>$\Delta X_{g v 1}$</th>
<th>$\Delta P_{g 2}$</th>
<th>$\Delta X_{g v 2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>- .707</td>
<td>- 1.00</td>
<td>1.28</td>
<td>.117</td>
<td>- .996</td>
<td>- 1.28</td>
<td>- .296</td>
<td>- .030</td>
<td>- .006</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>- .071</td>
<td>- .146</td>
<td>.100</td>
<td>.024</td>
<td>- .052</td>
<td>- .074</td>
<td>- .020</td>
<td>- .020</td>
<td>- .005</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>- .224</td>
<td>- .316</td>
<td>.433</td>
<td>.700</td>
<td>- 2.33</td>
<td>- .350</td>
<td>- .090</td>
<td>- .038</td>
<td>- .009</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>$u_2$</th>
<th>$f \Delta P_{\text{tie} 2}$</th>
<th>$f \Delta f_2$</th>
<th>$\Delta P_{\text{tie} 2}$</th>
<th>$d \Delta P_{\text{tie} 2}$</th>
<th>$\Delta f_2$</th>
<th>$\Delta P_{g 2}$</th>
<th>$\Delta X_{g v 2}$</th>
<th>$\Delta P_{g 1}$</th>
<th>$\Delta X_{g v 1}$</th>
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<tr>
<td>1</td>
<td>- .707</td>
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<td>- .090</td>
<td>- .038</td>
<td>- .009</td>
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</tr>
</tbody>
</table>
Summary of Results

In this chapter the megawatt-frequency control problem of interconnected electric energy systems was formulated as a linear state regulator problem of optimal control theory. The system dynamics were described by a vector matrix differential equation. The performance specifications were stated mathematically in terms of an integral quadratic cost. An optimal controller which significantly improves the transient response of the interconnected system was obtained.

The structure of the optimal controller has shown that:

1. By using all information about the system in the area controller, significant improvements in system response can be achieved.

2. The area controllers are essentially noninteracting.

3. Area megawatt-frequency controllers should not be restricted to functions of only integral of tie-line and frequency deviation as is presently being done. They should also be linear functions of tie-line deviation, rate of tie-line deviation, frequency deviation, governor valve position deviation and area generation deviation of their respective areas.
CHAPTER V

CONCLUSIONS

A study of the megawatt-frequency control problem of interconnected electric energy systems has been presented. To help in an understanding of the problem, a history of megawatt-frequency control was introduced. In the introduction the evolution of the present megawatt-frequency control strategy was discussed. This control strategy is based on both intuition and steady state analysis.

A detailed two area electric energy system model was used to study the dynamics of the interconnected system. The system must satisfy a set of dynamic and static performance criteria, which were stated in the text. The uncontrolled interconnected system did not satisfy the criteria. The controlled system, using a control strategy similar to that in operation today, satisfied the static performance criteria, independent of control parameter values, provided the system was stable. This result disagrees with past analysis.

To determine values of control parameters which would best satisfy dynamic performance criteria, parameter
optimization techniques were applied. Values of control parameters which minimized the Integral Square Error criterion gave best response. The result of applying the ISE criterion to the classical megawatt-frequency control problem, using system data enumerated in the text, indicated that the "optimum" frequency bias setting should be about one-half that in use today. This difference between control gains suggested in this study and those in use on operating systems could be the cause of serious system instability problems that have actually been experienced. Simulation of the interconnected system with the "optimum" control parameter values revealed that the system response was oscillatory.

Techniques of optimal control theory were applied to the megawatt-frequency control problem to develop new type control strategies. The megawatt-frequency control problem was formulated as an infinite time linear regulator. The optimal controller which results when using this formulation is a linear function of the present state of the system. Essentially non-interacting feedback controllers for each area were obtained. These new controllers considerably improved system response.

The results of the analysis of the megawatt-frequency control problem cast serious doubts for the first time on
the standard operating procedure set forth by interconnected system groups. Past conclusions drawn using static analysis to judge interconnected system response have been proven incorrect. A parameter optimization study suggested using control parameter settings different from those in use for best response. To improve system response, the optimal control theory study suggested using new type control strategies.

Future work should include study of the megawatt-frequency control problem using more sophisticated system models. Models which consider interaction between the "P-f" and "Q-V" channels of operation should be used. This model would more accurately represent the system. The optimal control theory techniques used in this study could be applied to systems of larger dimensions with ease. Some work in this area is presently being done.26

A study of the effect of system size on the degree of decoupling of controllers between interconnected areas should be made. If one area's size is much larger than another's, does this mean that one area should be dependent on the other for its control action?

As interconnected electric energy systems continue to grow, design problems become almost insurmountable.
To tackle these enormous problems, the only route to follow for solutions is one which uses optimal control theory and digital and analog computers. The results already look promising.
FLOW CHART

Start

Read System Identification

Problem:

Old

Read System Dimensions NP,NU

Read Matrices F,G
Read Matrices Q,R

Print Out Header and Matrix F

Problem:

Old

New

Compute and Print Out Eigenvalues of F

Print Out Matrices G, Q, R

Determine Controllability and Observability Properties of System

Form the System Canonical Matrix M
Compute the Eigenvalues of the Matrix M

Compute the Eigenvectors of M Corresponding to the Negative Eigenvalues of M

Solve for P, the Positive Definite Riccati Equation Solution

Compute and Print Out the Norm of P

Compute and Print Out Optimal Feedback Gains

Compute and Print Out the Closed Loop System Matrix $F_c$ and Its Eigenvalues

To Start
THIS FORTRAN IV PROGRAM CALCULATES THE OPTIMAL FEEDBACK GAINS OF
THE INFINITE TIME LINEAR REGULATOR PROBLEM

DIMENSION F(10,10), G(10,10), P(10,10), Q(10,10), GT(10,10), R(10,10),
1 MGRIGT(10,10), A(10,10), B(10,10), C(10,10), M(20,20), RSR(20), RSI(20),
2 ID(20)

COMMON A, B, C
DOUBLE PRECISION DA(10,10), DB(10,10), DC(20,20), DET
REAL MGR, GT, M
INTEGER S, 'S'

READ IN COMMENT CARD. IF THE SYSTEM IS THE SAME SKIP READIN OF F + G
99 READ (5, 113, END=999) ID
READ (5, 100) NPROB
IF (NPROB .EQ. 5) GO TO 2

READ IN SYSTEM DIMENSIONS
READ (5, 102) NP, NU

READ IN SYSTEM DISTRIBUTION MATRIX
CALL MXIN (F, NP, NP)

READ IN CONTROL DISTRIBUTION MATRIX
CALL MXIN (G, NP, NU)

READ IN STATE COST FUNCTION WEIGHTING MATRIX
CALL MXIN(Q, NP, NP)

READ IN CONTROL COST FUNCTION WEIGHTING MATRIX
CALL MXIN(R, NU, NU)

WRITE OUT HEADER AND INPUT DATA
WRITE (6, 107) NP, NU, ID
WRITE (6, 109)
CALL MXOUT (F, NP, NP)
IF (NPROB .EQ. 5) GO TO 12
WRITE (6, 119)

FIND THE EIGENVALUES OF F
CALL MEGVAL (F, NP)
12 WRITE (6, 110)
CALL MXOUT (G, NP, NU)
WRITE (6, 111)
CALL MXOUT (Q,NP,NP)
WRITE (6,160)
CALL MXOUT (R,NU,NU)

C DETERMINE IF SYSTEM IS CONTROLLABLE AND OBSERVABLE FROM THE COST
C FUNCTION
CALL CONOBS (NP,NU,NP,F,G,Q)
IF (NU .GT. 1) GO TO 7
R(1,1) = -1./R(1,1)
GO TO 9

7 DO 8 I = 1,NU
     DO 8 J = 1,NU
     R(I,J) = -R(I,J)
     CALL MXINV (R,NU,DET,IER)
     C FORM THE SYSTEM CANONICAL MATRIX M

9 CALL MTRANS(NP,NU,G,GT)
     CALL MMUL (NP,NU,NU,G,R,P)
     CALL MMUL (NP,NU,NP,P,GT,MGR)
     DO 4 I = 1,NP
         DO 4 J = 1,NP
         M(I,J) = F(I,J)
         M(I,J+NP) = MGR(I,J)
         M(I+NP,J) = -Q(I,J)

4 M(I+NP,J+NP) = -F(J,I)
     N = NP*2

C FIND THE EIGENVALUES OF M
DO 15 I = 1,N
     DO 15 J = 1,N
     CC(I,J) = M(I,J)
     CALL CHARD (DC,N,RSR,RSI,0.0,0.20)

15 CALL SORT (M,N,2C,RSR,RSI,DC,IER)
     IF (IER .EQ. 1) GO TO 99
     DO 70 I = 1,NU
     DO 70 J = 1,NU

C FIND THE EIGENVECTORS OF THE EIGENVALUES WITH NEGATIVE REAL PARTS
     CALL CHARD (DC,N,RSR,RSI,0.0,0.20)
     CALL SORT (M,N,2C,RSR,RSI,DC,IER)
     IF (IER .EQ. 1) GO TO 99
     DO 70 I = 1,NU
     DO 70 J = 1,NU

C
DA(I,J) = DC(J,I)
70 DB(I,J) = DC(J+NP,I)
C SOLVE FOR P - THE MATRIX RICCATI SOLUTION
CALL DINVER (DA,NP,10,DB,NP,10,DET,IER)
DO95I=1,NP
DO95J=1,NP
95 P(I,J) = DB(I,J)
C WRITE OUT P
WRITE (6,180)
WRITE (6,180)
CALL MXOUT (P,NP,NP)
C FIND THE NORM OF P - DOT
CALL MMUL (NP,NP,NP,P,MGRIGT,A)
CALL MMUL (NP,NP,NP,P,A,B)
CALL MMUL (NP,NP,NP,P,F,A)
CALL MADD (NP,NP,A,B,A)
CALL MADD (NP,NP,A,C,A)
CALL MTRANS (NP,NP,F,B)
CALL MMUL (NP,NP,NP,B,P,C)
CALL MADD (NP,NP,A,C,A)
CALL MNORM (NP,NP,A,W)
WRITE (6,129) W
C CALCULATE THE STEADY STATE OPTIMAL GAINS
CALL MMUL (NU,NU,NP,R,GT,Q)
CALL MMUL (NU,NU,NP,Q,P,B)
WRITE (6,170)
CALL MXOUT (B,NU,NP)
C FIND THE CLOSED LOOP SYSTEM DISTRIBUTION MATRIX
CALL MMUL (NP,NP,NP,MGRIGT,P,C)
DO 1 I = 1,NP
DO 1 J = 1,NP
1 G(I,J) = F(I,J) + C(I,J)
C WRITE OUT F - CLOSED LOOP
WRITE (6,130)
CALL MXOUT (C,NP,NP)
C FIND THE EIGENVALUES OF F - CLOSED LOOP
WRITE (6,190)
J = 0
DO 17 I = 1,N
IF (RSR(I) .GT. 0.0) GO TO 17
   J = J + 1
   WRITE (6,125) J,RSR(I),RSI(I)
CONTINUE
17 CONTINUE

C START THE NEXT PROBLEM
GO TO 99

999 WRITE (6,998)
   CALL EXIT
100 FORMAT (A1)
102 FORMAT (2I3)
107 FORMAT (1H1/T2,'THIS LINEAR QUADRATIC LOSS PROBLEM IS OF ORDER ',I13/T2,'AND HAS ',I3,' CONTROL INPUT(S).'//1H ,20A4)
109 FORMAT (T2,'SYSTEM DISTRIBUTION MATRIX')
110 FORMAT (T2,'CONTROL DISTRIBUTION MATRIX')
111 FORMAT (T2,'STATE COST FUNCTION WEIGHTING MATRIX')
113 FORMAT (20A4)
119 FORMAT (T2,'EIGENVALUES OF THE OPEN LOOP SYSTEM')
125 FORMAT (T3,I4,4X,1PE16.5,4X,1PE16.5)
129 FORMAT (T2,'THE NORM OF P - DOT = ',1PE15.5)
130 FORMAT (T2,'CLOSED LOOP SYSTEM DISTRIBUTION MATRIX')
160 FORMAT (T2,'CONTROL COST FUNCTION WEIGHTING MATRIX')
170 FORMAT (T2,'STEADY STATE OPTIMAL GAINS')
180 FORMAT (T2,'STEADY STATE MATRIX RICATTI EQUATION SOLUTION')
190 FORMAT (T2,'EIGENVALUES OF CLOSED LOOP SYSTEM DISTRIBUTION MATRIX 1'/T20,'REAL',15X,'IMAG')
998 FORMAT (1H1)
STOP
END
SUBROUTINE CHARC(A,N,RR,RI,Crit,IPRNT,NVAR)

CHARD VERSION OF MARCH 8, 1967 - J.C. BIDWELL

THIS SUBROUTINE COMPUTES THE EIGENVALUES OF A REAL MATRIX

SYMmetric OR NONSYMmetric

THE INPUT MATRIX IS TRANSFORMED BY SIMILARITY TRANSFORMATIONS

INTO ONE OF THE FROBENIUS FORMS WHERE ROW 1 CONTAINS ALL BUT

THE LEADING COEFFICIENT OF THE CHARACTERISTIC EQUATION - THE

LEADING COEFF. IS OF COURSE 1.0

ACCURACY IS INCREASED BY MAXIMIZING DIVISOR BY INTERCHANGING

ROWS AND COLS.

THE ROOTS OF THE CHARACTERISTIC EQ. ARE SOLVED USING

A D-ALEMBERT LEMMA TECHNIQUE

WHERE ALL VALUES IN A ROW TO THE LEFT OF THE DIAGONAL ARE LESS

THAN INPUT CRITERION (CRIT) PROGRAM SUBDIVIDES PROBLEM USING

RF TO OPERATE ON TWO OR MORE LOWER ORDER POLYNOMIALS.

WHERE -

A IS A DOUBLE PREC. NVAR BY NVAR DIMENSIONED MATRIX

N IS ORDER OF MATRIX USED

RR,RI STORAGE ARRAYS FOR NROROTS

CRIT IS DIVISOR CRITERIA (NORMAL 0)

IPRNT - IF NOT ZERO ROUTINE PRINTS ROOTS

NVAR IS DIM OF MATRIX(MAX)

CHARD USES POLYRF,LEMBRT,POLYEV Routines AND COEFER COMMON DATA

DIMENSION A(NVAR,NVAR),RR(1),RI(1)

DOUBLE PRECISION COEF(21)

DIMENSION XX(20),YY(20)

DOUBLE PRECISION SUM,DIV,ROW(20),COL(20)

DOUBLE PRECISION A,X

CALL OVERFL(JACK)

THE CODING USING THE 3000 NUMBERS HAVE TO DO WITH A CUSTOM

MATRIX NORMALIZATION FOR A SPECIAL CLASS OF PROBLEMS

IF N GE 20 DIVIDE ALL MATRIX ELEMENTS BY 10.0

IF (N.LT.20) GO TO 3100

DO 3050 I = 1,N
DO 3050 J = 1,N
3050 A(I,J) = A(I,J)/10.0
3100 CONTINUE
JACK=0
M=N
NR=0
1 L=M
2 K=L-1
BIG=CRIT
JJ=0
C FIND LARGEST ROW ELEMENT TO LEFT OF DIAGONAL
CO 10 J=1,K
AA=0ABS(A(L,J))
IF (AA.LE.BIG) GO TO 10
BIG = AA
JJ=J
10 CONTINUE
C IF ALL ELEMENTS LEFT OF DIAGONAL ARE LE CRITERIA GO TO COMPUTE
C EIGENVALUES OF REDUCED MATRIX
IF (JJ.EQ.0) GO TO 70
C SHIFT ROWS AND COLS IF NECESSARY
IF (JJ.EQ.K) GO TO 40
DO 20 J=1,M
X = A(JJ,J)
A(JJ,J) = A(K,J)
20 A(K,J)=X
DO 30 I=1,L
X= A(I,JJ)
A(I,JJ)=A(I,K)
30 A(I,K)=X
40 CONTINUE
C MAKE SIMILARITY TRANSFORMATION ON MATRIX
DIV=A(L,K)
C ROW IN EFFECT IS THE LEFT OR INVERSE SIMILARITY MATRIX
SIMILARITY MATRIX

C COL IN EFFECT IS THE RIGHT
DO 42 J=1,M
ROW(J)=A(L,J)
42 COL(J)=-ROW(J)/DIV
COL(K)=1.0/DIV

C (ROW + I) * A WHERE ROW IS KTH ROW, I THE IDENTITY MA
DO 50 J=1,M
SUM=0.0DO
DO 45 I=1,M
45 SUM=SUM+A(I,J)*ROW(I)
50 A(K,J)=SUM

C A * (COL + I) WHERE COL IS KTH ROW, I THE IDENTITY MA
C FIRST K ROWS LESS KTH COL.
DO 60 I=1,K
DO 60 J=1,M
IF (J.EQ.K) GO TO 60
A(I,J)=A(I,J)+A(I,K)*COL(J)
60 CONTINUE
C LTH ROW
DO 65 J=1,M
65 A(L,J)=0.0DO
C KTH COL
A(L,K)=1.0DO
DO 68 I=1,K
68 A(I,K)=A(I,K)*COL(K)
L=L-1
IF (L.EQ.1) GO TO 70
GO TO 2
C SET UP TO COMPUTE ROOTS OF REDUCED OR FULL MATRIX
70 CONTINUE
IF (L.EQ.M) GO TO 200
CCEP(1) = 1.0
J=1
DO 80 I=L,M
J=J+1
CCEP(J)=-A(L,I)
80 CONTINUE
C J BECOMES DEGREE OF POLYNOMIAL
J=J-1
CALL OVERFL(JACK)
IF (JACK.EQ.1) WRITE (6,1082)
1082 FORMAT(1HO,15X,17HOVERFLOW IN CHARD)
CALL POLYRF(COEPL,J,XX,YY,IERR)
IF (IERR.NE.0) WRITE (6,1085) IERR
1085 FORMAT(1HO,10X,13HPOLRF IERR =,10)
C STORE J RCOTS
DO 90 I=1,J
NR=NR+1
RR(NR)=XX(I)
90 RI(NR)=YY(I)
IF (NR.GE.N) GO TO 500
M=N-NR
IF (M.EQ.1) GO TO 220
GO TO 1
C ONE EIGENVALUE IS A DIAGONAL ELEMENT
200 NR=NR+1
RR(NR)=A(L,L)
RI(NR)=0.0
210 IF(NR.EQ.N) GO TO 500
IF(L.EQ.2) GO TO 220
M=N-NR
GO TO 1
220 NR=NR+1
RR(NR)=A(1,1)
RI(NR)=0.0
GO TO 210
500 CALL OVERFL(JACK)
IF (JACK.EQ.1) WRITE (6,1510)
1510 FORMAT(1H0,15X,15HOVERFLOW IN RF )
C IF N GE 20 MULT. ALL ROOTS BY 10.0
IF (N.LT.20) GO TO 3200
DO 3150 I = 1,N
RR(I) =10.0*RR(I)
3150 RI(I) =10.0*RI(I)
3200 CONTINUE
IF (IPRNT.EQ.0) RETURN
C PRINT OUT N ROOTS IF CALLED FOR
WRITE (6,1520)
1520 FORMAT (T20,'REAL',15X,'IMAG')
DO 540 I=1,N
540 WRITE (6,1540) I,RR(I),RI(I)
1540 FORMAT(1H ,2X,14,4X,1PE16.6,4X,1PE16.6)
RETURN
END
SUBROUTINE CONOBS (NP,NU,NY,F,G,H)

C THIS SUBROUTINE TAKES MATRICES F NP X NP, G NP X NU, H NY X NP
C AND CHECKS FOR CONTROLLABILITY OF THE PAIR (F,G) AND OBSERVABILITY
C OF THE PAIR (F,H). THE DETERMINANT OF THE PARTITIONED
C CONTROLLABILITY OR OBSERVABILITY MATRIX POSTMULTIPLIED BY ITS
C TRANSPOSE IS EVALUATED

DIMENSION F(10,10),G(10,10),H(10,10),B(10,100),S1(10,10),W(10,10),
1S(10,10)

DOUBLE PRECISION DET
COMMON S1,S,W
LO=NU
DO 32 I=1,NP
DO 32 J=1,NU
32 S(I,J)=G(I,J)
DO 33 I=1,NP
DO 33 J=1,NP
33 S1(I,J)=F(I,J)
DO 85 ITES=1,2
IF (LO .EQ. 0) GO TO 90
CALL NORML(S,NP,LO)
DO 35 I = 1,NP
DO 35 J = 1,LO
35 B(I,J) = S(I,J)
L=1
MOU=NP-1
DO 40 IT=1,MOU
CALL MMUL (NP,NP,LO,S1,S,W)
CALL NORML(W,NP,LO)
DO 20 I=1,NP
DO 20 J=1,LO
J1=(J+L*LO)
B(I,J1)=W(I,J)
20 S(I,J)=W(I,J)
40 L=L+1
NDIM=L*LD
CALL NORML(B,NP,NDIM)
   DO 1000 JJ = 1,NP
      SUM = 0.0
      DO 1000 KK = 1,NDIM
         CG 1001 SUM = SUM + B(JJ,KK)*B(JJ,KK)
1001   W(JJ,JJ) = SUM
CALL MXINV(W,NP,DET,IER)
IF (DET .GT. 1) GO TO 87
IF (IER) 60,70,80
60 WRITE (6,200)
200 FORMAT (25H OVERFLOW ANSWER IN DOUBT )
   GO TO 90
70 WRITE (6,300) DET
300 FORMAT (T2,'CONTROLLABLE DET = ',1PE14.4)
   GO TO 90
80 WRITE (6,400) DET
400 FORMAT (T2,'NOT CONTROLLABLE DET = ',1PE14.4)
90 LO=NY
   DO 93 J=1,NY
   DO 93 I=1,NP
93   S(I,J)=H(J,I)
   CALL MTRANS(NP,NP,F,S1)
85 IF (IER) 160,170,180
160 WRITE (6,200)
   GO TO 91
170 WRITE (6,901) DET
901 FORMAT (T2,'OBSERVABLE DET = ',1PE14.4)
   GO TO 91
180 WRITE (6,902) DET
902 FORMAT (T2,'NOT OBSERVABLE DET = ',1PE14.4)
RETURN
END
SUBROUTINE DINNER ( A, NA, NAD, B, NB, NBD, DET, IERROR )
C THIS SUBROUTINE IS A MODIFICATION OF THE UNIVERSITY OF FLORIDA C
C COMPUTER CENTER'S INVERT. IT USES DOUBLE PRECISION AND HAS BEEN C
C RENAMED DINNER. C FOSHA 2-69 C
DOUBLE PRECISION A,B,BD,SAVE,PIVOT,DET
DIMENSION A(NAD,NA),B(NBD,NB),BD(10),INDEX(10)
CALL OVERFL(INX)
DET = 1.000
IERROR = 0
DO 130 I = 1, NA
PIVOT = 0.000
C SEARCH FOR PIVOTAL ELEMENT
DO 60 J = I, NA
IF (DABS(A(J,I)) .LE. DABS(PIVOT)) GO TO 60
PIVOT = A(J,I)
INDEX(I) = J
60 CONTINUE
IF (DABS(PIVOT) .LT. 1.0D-6) GO TO 250
IF (INDEX(I) .EQ. I) GO TO 90
DET = -DET
C INTERCHANGE ROWS TO PUT PIVOTAL ELEMENT ON DIAGONAL
DO 80 L = 1, NA
SAVE = A(I,L)
A(I,L) = A(INDEX(I), L)
80 A(INDEX(I), L) = SAVE
90 DET = DET*PIVOT
A(I,I) = 1.000
DO 91 KK=1,NA
91 A(I,KK) = A(I,KK)/PIVOT
C REDUCE NON-PIVOTAL ROWS
DO 130 LJ = 1, NA
IF (LJ .EQ. I) GO TO 130
SAVE = A(LJ, I)
A(LJ, I) = 0.000
DO 120 K = 1, NA
120 A(LJ,K) = A(LJ,K) - SAVE * A(I,K)
130 CONTINUE
C INTERCHANGE COLUMNS
NA1 = NA + 1
DO 160 KKK = 1, NA
K = NA1 - KKK
IF (INDEX(K) .EQ. K) GO TO 160
DO 98 L = 1, NA
SAVE = A(L,K)
A(L,K) = A(L, INDEX(K))
98 A(L, INDEX(K)) = SAVE
160 CONTINUE
C A INVERSE IS NOW STORED IN A
C FIND SOLUTION VECTORS FOR ALL CONSTANT VECTORS INPUT
IF (NB .LE. 0) GO TO 210
DO 190 K = 1, NB
DD(I) = 0.000
180 DD(I) = DD(I) + A(I,J) * B(J,K)
190 B(I,K) = DD(I)
C SOLUTION VECTORS NOW IN B
C CHECK FOR OVERFLOW CONDITION AND SET ERROR SIGNAL
210 CALL OVERFL(INX)
IF (INX .EQ. 2) RETURN
DET = 0.000
IERROR = -1
RETURN
C IF CONTROL REACHES 250, MATRIX IS SINGULAR
250 IERROR = +1
DET = 0.000
RETURN
SUBROUTINE IGVEC (H, RR, RI, N, NDIM, VR, VI)

THIS SUBROUTINE WAS ORIGINALLY CALLED IGVEC5 AS WRITTEN BY J. VANNNESS BUT WAS MODIFIED BY C FOSHA 13 NOVEMBER 1968 AND RENAMED IGVEC.

THIS IS A GENERAL EIGENVECTOR SOLVER WHERE H IS A REAL MATRIX AND THE EIGENVALUE IS REAL OR COMPLEX. IT SOLVES FOR V IN HV = LV WHERE L IS THE INPUT EIGENVALUE

INPUTS

H - N ORDER SINGLE PREC. MATRIX DIMENSIONED NDIM X NDIM
RR,RI - REAL AND IMAG. PARTS OF EIGENVALUES. DOUBLE PREC.
N - ORDER OF MATRIX AND VECTORS
NDIM - FIXED DIMENSION LIMITS OF H, VR, AND VI.

OUTPUTS

VR, VI - REAL AND IMAG. EIGENVECTOR ARRAYS - DOUBLE PREC.

DIMENSION H(NDIM,NDIM)
DOUBLE PRECISION A(20,20)
DOUBLE PRECISION VR(NDIM), VI(NDIM), RR, RI, TEMP
DIMENSION NCOL(20)
T = RI
IF (T.EQ.0.0) GO TO 35

A = H**2

98 A(I,J) = A(I,J) + H(I,K)*H(K,J)

A = H**2 - 2.0*RR*H(I,J) + RR**2 + RI**2 FOR RI NOT 0.
DO 20 I = 1,N
DO 20 J = 1,N
20  A(I,J) = A(I,J) - 2.0*RR*H(I,J)
DO 30 I = 1,N
30  A(I,I) = A(I,I) + RR*RR + RI*RI
GO TO 55

C
C
A = H - RR FOR RI = 0.0
C
C
35 DO 40 I = 1,N
DO 40 J = 1,N
40  A(I,J) = H(I,J)
DO 50 I = 1,N
50  A(I,I) = A(I,I) - RR

C
C
NORMALIZE MATRIX BY MAKING MAX. ELEMENT 1.0
C
C
55 BIG = 10.0E-25
IBIG = 0
JBIG = 0
DO 70 I = 1,N
DO 70 J = 1,N
X = DABS(A(I,J))
IF (X.LT.BIG) GO TO 70
BIG = X
IBIG = I
JBIG = J
CONTINUE
70  TEMP = A(IBIG,JBIG)
DO 80 I = 1,N
DO 80 J = 1,N
80  A(I,J) = A(I,J)/TEMP
DO 90 I = 1,N
90  NGUL(I) = I
SOLVE FOR X USING CROUT METHOD MAXIMIZING ALONG DIAGONAL
STOPPING WHEN DIAGONAL ELEMENTS REMAINING BECOME SMALL

N1 = N - 1
ICOLX = 0
DO 200 K = 1, N
K1 = K - 1
BIG = 10.0E-26
IBIG = 0
DO 115 I = K, N
VI(I) = A(I, I)
IF (K.EQ.1) GO TO 115
DO 110 L = 1, K1
110 VI(I) = VI(I) - A(I, L)*A(L, I)
115 CONTINUE
IF (K.EQ.N) GO TO 220
IF (R.EQ.0.0) GO TO 118
IF (K.LT.0.1) GO TO 118
X = VI(N) - VI(N1)
X = ABS(X)
IF (X.LT.10.E-30) GO TO 185
118 DO 120 I = K, N
X = CABS(VI(I))
IF (X.LT.10.E-30) GO TO 120
IBIG = I
BIG = X
120 CONTINUE
IF (IBIG.EQ.C) GO TO 185
IF (IBIG.EQ.K) GO TO 140

MAKE SIMILARITY TRANSFORMATION BY INTERCHANGING
COLUMNS K AND IBIG AND ROWS K AND IBIG

I = NCOL(IBIG)
NCOL(IBIG) = NCOL(K)
NCOL(K) = I
DO 125 I = 1,N
    TEMP = A(I,K)
    A(I,K) = A(I,IBIG)
125  A(I,IBIG) = TEMP
    DO 130 J = 1,N
    TEMP = A(K,J)
    A(K,J) = A(IBIG,J)
130  A(IBIG,J) = TEMP
140  IF (K.EQ.1) GO TO 165
    DO 145 I = K,N
    DO 145 L = 1,K1
145  A(I,K) = A(I,K) - A(I,L)*A(L,K)
165  KPI = K + 1
    DO 175 J = KPI,N
    IF (K.EQ.1) GO TO 175
    DO 170 L = 1,K1
170  A(K,J) = A(K,J) - A(K,L)*A(L,J)
175  A(K,J) = A(K,J)/A(K,K)
GO TO 200
185  ICCLX = K
GO TO 210
200  CONTINUE
210  IF (ICCLX.LT.N1) GO TO 900
    IF (ICCLX.GT.N1) GO TO 220
    A(N1,N1) = VI(N1)
    N2 = N-2
    DO 215 I = 1,N2
    A(N,N1) = A(N,N1) - A(N,I)*A(I,N1)
215  A(N1,N) = A(N1,N) - A(N1,I)*A(I,N)
    A(N,N) = VI(N)
    VI(N) = 1.0DO
    DO 240 K = 1,N1
I = N - K
VI(I) = 0.000
IL = I + 1
DO 240 L = IL, N
240 VI(I) = VI(I) - A(I,L)*VI(L)
DO 260 I = 1, N
J = NCOL(I)
260 VR(J) = VI(I)
C
           CALCULATE SI = -1/RH(H - RR*I)*SR
IF (T .EQ. 0.0) GO TO 335
DO 300 I = 1, N
VI(I) = -RR*VR(I)
DO 290 J = 1, N
290 VI(I) = VI(I) + H(I,J)*VR(J)
300 VI(I) = -VI(I)/RI
RETURN
335 DO 350 I = 1, N
350 VI(I) = 0.000
RETURN
900 WRITE (6,1900) ICOLX,(VI(I),I=1,N)
1900 FORMAT(1H0,//20X,22HIGVEC ERROR = ICOLX =,I10/(6X,1P10E12.4))
RETURN
END
SUBROUTINE LEMBR\_T

C THIS ROUTINE SYSTEMATICALLY FINDS A ROOT OF A POLYNOMIAL
C USING A SIMPLE CAGING SCHEME BASED ON D-AMEMBERTS LEMMA
COMMON /COEFE\_R/PR(21),M,X,Y,AP,RR,RI,IERRR
DIMENSION NFLAG(5),U(5),V(5),P(5)
DOUBLE PRECISION PR
EQUIVALENCE (P,P1),(P(2),P2),(P(3),P3),(P(4),P4),(P(5),P5)
L = 1
RR = 0.0
RI = 0.0
SIGN = 1.0
IFLAG = 0
JFLAG = 0
KFLAG = 0
DEL = 0.5
DDEL = 8.0
GO TO 5 I = 1,5
5 NFLAG(I) = 0
10 IF (IFLAG.LT.5) GO TO 25
IF (JFLAG.GT.0) GO TO 25
20 IFLAG = 0
IF (KFLAG.LT.3) GO TO 22
RR = RR + SIGN/19.0
RI = RI + SIGN/13.0
SIGN = -2.0*SIGN
21 CONTINUE
IF (ABS(RI).LE.1.0) GO TO 22
SIGN = SIGN/97.0
RR = SIGN/3.0
RI = -SIGN
GO TO 21
22 KFLAG = KFLAG + 1
DEL = DDEL*DEL
DDEL = DDEL + 1.3
24 NFLAG(L) = 0
   GO TO 30
25 IFLAG = IFLAG + 1
30 CONTINUE
   DO 40 I = 1,5
   IF (NFLAG(I) .NE. 0) GO TO 38
   X = RR
   Y = RI
   IF (I .EQ. 1) GO TO 35
   IF (I .EQ. 2) X = X + DEL
   IF (I .EQ. 3) X = X - DEL
   IF (I .EQ. 4) Y = Y + DEL
   IF (I .EQ. 5) Y = Y - DEL
35 U(I) = X
   V(I) = Y
   CALL PCLYESV
   P(I) = AP
38 NFLAG(I) = 0
40 CONTINUE
   IF (JFLAG .GT. 27) GO TO 60
   DO 45 I = 1,5
   IF (P(I) .GT. 1.0E-07) GO TO 48
45 CONTINUE
   GO TO 60
48 DIF1 = AMAX1(P1,P2,P3,P4,P5)
   DIF2 = AMIN1(P1,P2,P3,P4,P5)
   DIF = DIF1 - DIF2
   IF ((DIF .GE. 1.0) .AND. (P1 .LT. 1.0)) GO TO 55
   IF (P1 .EQ. 0.0) GO TO 60
   DIF = DIF/P1
   IF (DIF .LT. 0.001) GO TO 20
55 CONTINUE
60 CONTINUE
   DO 70 J = 1,5
I = J
IF (P(J) .EQ. 0.0) GO TO 100
70 CONTINUE
DIF2 = AMIN1(P2, P3, P4, P5)
IF (P1 .GT. DIF2) GO TO 80
IF (DEL .LT. 1.0 .OE-30) RETURN
DEL = 0.5 * DEL
XX = RR + DEL
YY = RI + DEL
IF (XX .EQ. RR) .AND. (YY .EQ. RI)) RETURN
IF ((XX .EQ. RR) .AND. (RI .EQ. 0.0)) RETURN
IF ((RR .EQ. 0.0) .AND. (RI .EQ. YY)) RETURN
IF (JFLAG .GT. 100) GO TO 220
JFLAG = JFLAG + 1
NFLAG(1) = 1
GO TO 30
80 AMINY = P2
N = 2
DO 85 I = 3, 5
IF (P(I) .GT. AMINY) GO TO 85
N = I
AMINY = P(I)
85 CONTINUE
L = 3
IF (N .EQ. 3) L = 2
IF (N .EQ. 4) L = 5
IF (N .EQ. 5) L = 4
NFLAG(1) = 1
NFLAG(L) = 1
U(L) = U(1)
U(1) = U(N)
V(L) = V(1)
V(1) = V(N)
P(L) = P1
P1 = P(N)
RR = U(1)
RI = V(1)
GO TO 10
100 RR = U(I)
   RI = V(I)
   RETURN
220 IERRR = 2
RETURN
END
SUBROUTINE MATRIX
C
THIS SUBROUTINE HANDLES ELEMENTARY MATRIX OPERATIONS
C
DIMENSION A(10,10), B(10,10), C(10,10)
C
ENTRY MADD (M,N,A,B,C)
C
A + B = C WHERE A M X N, B M X N, C M X N
C
DO 29 I=1,M
DO 29 J=1,N
29 C(I,J)=A(I,J) + B(I,J)
C
RETURN

ENTRY MEGVAL (A, NP)
C
FIND THE EIGENVALUES OF MATRIX A NP X NP
C
DOUBLE PRECISION DA(10,10)
C
DIMENSION RSR(10), RSI(10)
C
DO 45 I = 1, NP
DO 45 J = 1, NP
45 DA(I,J) = A(I,J)
C
CALL CHARD (DA, NP, RSR, RSI, 0.0, 0.0, 10)
C
RETURN

ENTRY MMUL (MP, NP, NU, A, B, C)
C
A X B = C WHERE A MP X NP, B NP X NU, C MP X NU
C
DO 11 L=1, MP
DO 11 I=1, NU
SUM = 0.0
DO 31 J=1, NP
31 SUM = SUM + A(L,J)*B(J,I)
C
C(L,I) = SUM
C
RETURN

ENTRY MNORM (M, N, A, W)
C
NORM OF MATRIX A M X N IS W
C
W = 0.0
DO 30 I = 1, M
X = 0.0
DO 20 J = 1, N
IF (ABS(A(I,J)) .GT. X) X = ABS(A(I,J))
20 CONTINUE
30 W = W+X
   RETURN
   ENTRY MTRANS (M,N,A,B)
C
   A M X N TRANSPOSED = B N X M
   DO 10 I=1,M
   DO 10 J=1,N
10  B(J,I) = A(I,J)
   RETURN
   ENTRY MXINV (A,M,N)
C
   READ MATRIX A M X N
   DO 50 I=1,M
   50 READ(5,100) (A(I,J), J=1,N)
   100 FORMAT (8E10.3)
   RETURN
   ENTRY MXOUT (A,L,M)
C
   WRITE MATRIX A L X M
   DO 103 I=1,L
   103 WRITE (6,109) (A(I,J), J=1,M)
   109 FORMAT ((1H,1PE14.4))
   RETURN
ENTRY NORM(A, NR, NC)
NORMALIZE MATRIX A NR X NC
IF (NC .GT. NR) GO TO 12
   DO 4 J = 1, NC
      SUM = 0.0
   DO 3 I = 1, NR
      SUM = SUM + A(I, J)**2
   SUM = SQRT(SUM)
   IF (SUM .EQ. 0.) GO TO 4
   DO 5 I = 1, NR
      A(I, J) = A(I, J) / SUM
5 CONTINUE
GO TO 14
12 DO 40 I = 1, NR
    SUM = 0.0
   DO 32 J = 1, NC
      SUM = SUM + A(I, J)**2
   SUM = SQRT(SUM)
   IF (SUM .EQ. 0.) GO TO 40
   DO 6 J = 1, NC
      A(I, J) = A(I, J) / SUM
6 CONTINUE
40 CONTINUE
14 END
RETURN
SUBROUTINE POLYEV
COMMON /COEFER/PR(21),M,X,Y,AP,RR,RI
DOUBLE PRECISION PR, U, V, US
U = PR(1)
V = 0.0DO
DO 20 I =2, M
US = U
U = X*U - Y*V + PR(I)
20  V = X*V + Y*US
AP = DABS(U) + DABS(V)
RETURN
END
SUBROUTINE POLYRF(P,N,X,Y,IERR)
COMMON /COEFEK/PR(21),M,A,E,AP,RR,RI,IERRR
DOUBLE PRECISION P(1)
DIMENSION X(1),Y(1)
DOUBLE PRECISION PR,D,X2,XY
IF ((N.LT.1).OR.(N.GT.20)) GO TO 200
IERR = 0
IERRR = 0
J = 1
M = N+1
DO 10 I = 1,M
PR(I) = P(I)
10 CONTINUE
IF (N.EQ.1) GO TO 100
15 CONTINUE
CALL LEMBRT
IF (RI.EQ.0.0) GO TO 40
IF (RR.EQ.0.0) GO TO 16
TEST = RI/RR
IF (ABS(TEST).LT.0.000001) GO TO 40
16 CONTINUE
X(J) = RR
Y(J) = RI
IF (IERRR.NE.0) GO TO 220
IF(J.EQ.N) RETURN
J = J + 1
X(J) = RR
Y(J) = -RI
IF(J.EQ.N) RETURN
J = J + 1
M = M - 2
X2 = 2.0*RR
XY = -(RR*RR + RI*RI)
DO 20 I = 2,M
20 CONTINUE
PR(I) = PR(I) + X2*PR(I-1)
PR(I+1) = PR(I+1) + XY*PR(I-1)
20 CONTINUE
   IF (M.EQ.2) GO TO 100
   GO TO 15
40 CONTINUE
   X(J) = RR
   Y(J) = 0.0
   IF (IERRR.NE.0) GO TO 220
   IF (J.EQ.N) RETURN
   J = J + 1
   M = M - 1
   GO TO 50
50 PR(I) = PR(I) + RR*PR(I-1)
   IF (M.EQ.2) GO TO 100
   GO TO 15
100 D = PR(1)
   X(J) = -PR(2)/D
   Y(J) = 0.0
   RETURN
200 WRITE (6,1200) N
1200 FORMAT(1H0,10X, 3HN =,I4,21HOUSTIDE LIMITS POLYRF)
   IERR=1
   RETURN
220 IERR = IERRR
   RETURN
   END
SUBROUTINE SORT (H,N,NDIM,RSR,RSI,SR,IER)

THIS SUBROUTINE SAVES THE STABLE EIGENVALUES OF THE SYSTEM.

DIMENSION H(NDIM,NDIM),RSR(NDIM),RSI(NDIM)
DOUBLE PRECISION VR(20),VI(2C),RR,RI,SR(NDIM,NDIM)
M = N/2
NIGV = 0
IER = 0
DO 1 I = 1,N
IF (RSR(I)) 1,25,30
1 CONTINUE
WRITE (6,180)
180 FORMAT (T2,'HAMILTONIAN HAS ZERO ROOT. NO SS SOLUTION EXISTS')
IER = 1
GO TO 9
30 CONTINUE
RR = RSR(I)
RI = RSI(I)
IF (RSI(I) .EQ. 0.) GO TO 4
IF (RSI(I) .LT. 0.) GO TO 1
NIGV = NIGV + 1
CALL IGVEC (H,RR,RI,N,NDIM,VR,VI)
DO 6 II = 1,N
6 SR(II,NIGV) = VR(II)
NIGV = NIGV + 1
DO 7 II = 1,N
7 SR(II,NIGV) = VI(II)
GO TO 1
4 NIGV = NIGV + 1
CALL IGVEC (H,RR,RI,N,NDIM,VR,VI)
DO 2 II = 1,N
2 SR(II,NIGV) = VR(II)
1 CONTINUE
IF (NIGV .NE. M) WRITE (6,99) NIGV,M
99 FORMAT (T2,'NIGV = ',I2,' M = ',I2)
9 RETURN
REFERENCES


BIOGRAPHICAL SKETCH

Charles Edward Fosha, Jr., was born August 25, 1942, at Pensacola, Florida. In June, 1960, he was graduated from Millington Central High School. In June, 1965, he received the degree of Bachelor of Electrical Engineering from the University of Florida. In September, 1965, he enrolled in the Graduate School of the University of Florida. He worked both as a teaching assistant and a research assistant in the Department of Electrical Engineering from January, 1966, until June, 1969. He received the degree of Master of Electrical Engineering in August, 1966. From September, 1966 until the present he has pursued his work toward the degree of Doctor of Philosophy.

Charles Edward Fosha, Jr. is married to the former Jane Jordan Ansley. He is a member of Eta Kappa Nu, Institute of Electrical and Electronic Engineers, and Simulation Councils, Inc.
This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Engineering and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

June, 1969

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