THREE ESSAYS ON REGULATION, PUBLIC FINANCE, AND GAME THEORY

By

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By

Huseyin Yildirim
To my fiance (Elif), my parents (Ayse and Mehmet), my sister (Huriye), and my brother

(Mustafa)
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THREE ESSAYS ON REGULATION, PUBLIC FINANCE, AND GAME THEORY

By

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This dissertation addresses three issues. The first one deals with whether regulation hinders innovation in regulated industries like telecommunications and electricity. With experience, regulated monopolists learn to employ cost reducing innovations. We characterize the optimal regulation of an innovating monopolist with unknown costs. Regulatory policy is designed to minimize current costs of service, while encouraging development of cost saving innovations. Following practice, regulated prices change periodically as the observed operating conditions of the monopolist vary. We find under optimal regulation (a) innovation is encouraged by light handed regulation allowing the monopolist to earn greater information rents while providing greater service, (b) innovation occurs in the absence of long term agreements when private information is persistent, and (c) innovation is more rapid in a durable franchise, and the regulator prefers durable franchises for exploiting learning economies.

The second issue is why charities or fundraisers commonly announce donations as they accrue. Doing so induces donors to play a sequential-move rather than simultaneous-move game. We examine the conditions under which a charity prefers such sequential play. It is known that if donors only value
contributions through their effect on the total provision of a public good, then the charity will not announce contributions sequentially. However, with more general utility functions that include additional effects such as warm-glow or snob appeal, the charity may benefit from announcing contributions.

In the last part, we characterize the equilibrium outcomes of such games with two distinct features: (1) Agents have multiple opportunities to respond to each other before the payoffs are received, and (2) they can do so only by accumulating their strategy variables over time. Our characterization depends only on agents’ reaction functions, one-shot Cournot-Nash and Stackelberg outcomes in the textbook sense. We show that having more than two opportunities to respond would not change the equilibrium outcomes and provide conditions for which equilibrium outcomes would be the same as the one-shot Cournot-Nash outcome.
CHAPTER 1
THREE ESSAYS ON REGULATION, PUBLIC FINANCE, AND GAME THEORY

This dissertation consists of three essays on regulation, public finance, and game theory.

The first essay examines the optimal regulation of a monopolist with unknown costs who learns to employ cost reducing innovations through experience. The analysis extends previous studies of incentive regulation by Baron and Myerson (1982), and Laffont and Tirole (1993) to account for learning and innovation in production. The formal model is cast in an infinitely repeated Markov game setting. Both the regulator and monopolist select strategies based on the payoff relevant history of play. History is completely summarized by the existing technology. The technology evolves stochastically through time, determined by the previous sequence of regulatory agreements. Following practice, we assume that the regulator lacks commitment power and thus offers a new contract in each period upon observing the existing technology. Dynamic programming arguments are employed to simplify this complicated dynamic regulation game. In characterizing the unique Markov perfect equilibrium, we find that (a) innovation is encouraged by light-handed regulation allowing the monopolist to earn greater information rents while providing greater service, (b) innovation occurs in the absence of long term agreements when private information is persistent, (c) innovation is more rapid in a durable franchise, and the regulator prefers durable franchises for exploiting learning economies. Within the same framework, we also consider the dynamic regulation when the monopolist incurs increasing costs that arise from its reliance on exhaustible resources to supply service. We find that the results listed above are reversed for this setting. In particular, the regulator prefers a less durable franchise.
The second essay investigates the role of announcement of contributions in fund-raising activities. By announcing contributions as they arise, the fund-raiser induces sequential play by donors in the contribution game. We then compare the total of equilibrium contributions to that in the simultaneous game without announcements. Our analysis assumes a general utility function for donors that has the literature's pure-altruism and warm-glow specifications as special cases, while allowing other motives like prestige. It is known that when donors are purely altruistic, it is in the charity's best interest not to announce contributions. However, we find that announcing contributions might yield a higher total in equilibrium when donors possess additional motives such as warm-glow. This finding rationalizes the common practice of announcing contributions as they are made. We also show the central results hold in the endogenous timing in a game where donors choose not only how much to contribute but also when to contribute.

The last essay finds the set of equilibrium outcomes for two-player games with two distinctive features: (a) before payoffs are received, players have multiple opportunities to respond to their rival, (b) players can only choose to accumulate their strategic variables over time. Examples are rent seeking activities by lobbyists to influence a policy decision, voluntary contributions by agents to a public good, and quantity competition by duopolists before the market opens. Ten qualitatively different cases arise as determined by the slopes of standard reaction functions and how players value their rival's strategic variable. Our analysis provides a unified approach to these games that shows the equilibrium set depends on the comparison between agents' one-shot Cournot-Nash and Stackelberg leader choices. The equilibrium set can be a singleton, equivalent to the standard Cournot-Nash outcome, or to one Stackelberg outcome, consist of both Stackelberg equivalents, or consist of a continuum. We show further that the equilibrium sets are invariant to the number of opportunities agents have to respond to each other. The Cournot-Nash equivalent occurs, for example, when agents have the usual free-riding
incentives in public-good contribution games. Our analytical approach can also be applied to the decumulation setting where agents can only choose to decrease their strategic choices over time. Several examples are analyzed including lobbying and duopoly competition with differentiated products.

We provide concluding remarks in chapter 5.
CHAPTER 2
REGULATING AN INNOVATING MONOPOLIST

2.1 Introduction

Beginning with the seminal paper by Baron and Myerson (1982) an elegant theory of incentive regulation has developed which outlines how a regulator optimally directs a service provider, who is privately informed about the cost of service. This theory generates a rich set of regulatory prescriptions for a wide variety of procurement and franchise environments. However the extant theory mostly pertains to stationary settings where technology and the expected cost and value of service does not change systematically with time. An important omission of current theory is that one expects technology improvements to alter systematically the cost and value of service in regulated industries like telecommunications and electricity. Further, current and future regulations affect the pace of innovation. Therefore we expect regulators to enact current policy anticipating its impact on the rate of innovation and the future expected cost and value of service.

This paper examines the optimal regulation of a privately informed service provider in a setting where the expected cost of future service depends on the level of previous service.

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1 The theory of incentive regulation and a comprehensive review of the literature is nicely summarized in Laffont and Tirole (1993).

2 See Laffont and Tirole, Chapters 9 and 10 and the references cited therein for an overview of this theory.

3 In electricity markets the development of alternating current, improvements in controlling generation and transmission, exploiting economies of scale in generation, and the introduction of load control and conservation measures have significantly reduced the cost of service. In telecommunications markets, the development of advanced wire and wireless technologies, the improvement of switches, and advances in data and information processing have created higher quality and lower cost service.
provided. The volume of previous service supplied is a measure of the supplier’s “experience” or “knowledge.” More experienced producers may reduce their future cost of service by employing superior technologies and better procedures to generate, coordinate, and deliver service to their customers. The regulated firm enjoys learning economies enabling it to reduce the future cost of service to consumers. The primary focus of our analysis is to study how incentive regulation is designed for encouraging suppliers to develop and adopt cost saving technologies.

The setting we analyze consists of a monopoly service supplier who is regulated by a commission acting on behalf of consumers. The monopolist’s total unit cost of service is comprised of an intrinsic cost determined by the current technology and a temporal cost which varies each period according to economic and market conditions. We assume the intrinsic cost is publicly known since one can readily observe the firm’s technology for generating service. However, the current temporal cost reflecting existing supply conditions is known privately by the supplier. Periodic changes in intrinsic costs arising from the implementation of a new technology occur with a probability that is increasing in the service level previously offered.

In practice, long term regulatory agreements are infeasible. This reflects the inability of regulatory bodies to legally commit to multi year franchise arrangements. Consequently the

---

4 Evidence of productivity gains in regulated industries is reflected in the productivity offsets required under price cap regulation. Productivity offsets reduce rates to correspond with productivity gains the firm is expected to achieve each year. Berndt (1991) contains a nice survey of empirical evidence and techniques for documenting the importance of experience in reducing production costs

5 For instance it seems reasonable to assume regulators can observe a new technology for generating electricity installed by the monopoly supplier. However, current supply conditions affecting the availability and price of fuel is likely to be privately known by the regulated monopolist.

6 The inability of regulatory commissions to commit future commissions to a regulatory policy is well documented. For instance Levy and Spiller (1996) provide an enlightening summary of the various ways in which regulators abuse their administrative discretion. Nonetheless, regulators do attempt to establish intermediate term commitments to follow pricing policies, such as multi year price cap agreements. Despite these attempts, regulators often can not resist pressure to modify such agreements if utility profits become too large.
regulator offers short term agreements that are renegotiated periodically. To most simply model
the regulator's limited commitment ability we assume regulatory agreements are renegotiated
each period. Aside from being realistic this assumption permits us to examine the stability of
franchise arrangements and to discover how investments in innovation may exist without binding
agreements.7

Section 2.2 of the paper presents the formal model. We assume the regulator and
monopolist select strategies based on the payoff relevant history of play. History is completely
summarized by the existing supply technology. Technology evolves stochastically through time,
determined by the previous sequence of regulatory agreements. We employ dynamic
programming arguments to analyze the parties' selection of strategies to maximize their
immediate expected return plus future expected continuation value of the franchise. This enables
us to generate the players' dynamic decision rules and to characterize the unique Markov Perfect
Equilibrium (MPE) of this regulation game.

Section 2.3 investigates how franchise agreements are optimally tailored to promote
innovation. We find the supplier and regulator both make concessions to hasten the arrival of cost
reducing technology. Innovation is more rapid when current service is increased enabling the
supplier to accelerate his rate of learning. To facilitate learning, the supplier agrees to provide
service at lower cost, and the regulator offers greater compensation for service provided. It is
noteworthy that concessions arise in equilibrium without long term agreements for dividing the

7 If long term regulatory agreements were feasible, it would be possible "in theory" for the
regulator to achieve the first best, full information optimum for our setting (see d'Aspremont and
Gerad-Varet (1979) and Rogerson (1992)). This would require the firm to sign a long term
contract stipulating all payments and supplies of service to be delivered currently and in the
future conditional upon future supply conditions. The contract would be negotiated before the
firm was privately informed about its costs of production. We note, that although such optimal
contracts exist, they require strong (and unrealistic) conditions to be satisfied. These conditions
require that contracts not be renegotiated, that the service to be supplied be precisely described at
the beginning, and that neither party be allowed to renege even if they expect large financial
losses in the future.
surplus arising from innovation between the firm and consumers. Absent long term agreements, each party faces a potential "holdup" problem. However their mutual interest in developing the cost cutting technology greatly alleviates this problem.

Section 2.3 also illustrates regulated firms will invest to become more productive even when the regulator anticipates the productivity gains. As expected, the regulator does lower the firm's payment to coincide with its reduction in supply costs. However, this ratcheting down of service payments doesn't eliminate all the firm's incentives to invest. The firm's private information about current supply conditions enables it to earn greater information rents when supply costs fall and the regulator demands more service. This is a noteworthy finding as it contrasts with several earlier studies suggesting that opportunistic behavior on the part of the regulator may preclude utilities from investing. This finding also has implications for the durability of franchise agreements. The regulator will prefer dealing long term with a single supplier rather than periodically shifting between suppliers. This is because investment incentives are greatest for long-lived suppliers as provided for in a durable franchise.

Settings may also occur where supply costs rise with previous production. For instance future costs of supplying electricity may increase when inexpensive and environmentally benign fuel inputs are used up from previous production. In section 2.4, we consider this setting and we find the observations reported for the declining service cost case are reversed here. Specifically, we illustrate how the firm and the regulator become "tougher" negotiators with current service levels decreasing in anticipation of higher future supply costs. Further, the regulator prefers less durable franchises, where service suppliers are replaced with great frequency.

Studies by Kolbe and Tye (1991), Lyon (1991, 1992, 1995), Mayo and Flynn (1988), and Tiesberg (1993) argue that utilities will be reluctant to invest if they anticipate regulators will disallow these expenditures from the utility's rate base. Such behavior prevents the utility from earning a competitive return on its investment. In contrast to this, we show investments in learning are guaranteed an appropriate rate of return in equilibrium, at least in the setting we analyze.
Section 2.5 summarizes our findings and discusses some procurement and contracting issues relating to the insights developed in the paper. Proofs of all formal results appear in the appendices. Our findings pertain to several previous papers on dynamic regulation, price cap regulation, holdup problems in contracting, learning by doing and the durability of bilateral relationships. We will relate our findings to these studies in the process of discussing our results.

2.2 A Model of Dynamic Regulation

Our model is a straightforward extension of Baron and Myerson (1982) to allow for cost reducing innovation. The parties consist of a risk neutral utility and a regulator representing risk neutral consumers. Each period the monopolist provides service, \( x \geq 0 \), yielding a flow of utility \( v(x) \) to consumers, where \( v \) is strictly increasing and concave with \( v(0) = 0 \). The firm incurs a unit supply cost \( D(c,T) = d(T) + c \) consisting of two parts. The first component, \( d(T) \), is the intrinsic cost depending on \( T \), the current technology or resources available to the firm. The state variable \( T \) assumes positive integer values and its evolution from one state to another is governed by a stochastic process described below. In settings where the firm learns from previous production, \( d(T) \) is decreasing as the utility employs better production techniques acquired through experience. Alternatively, \( d(T) \) is increasing if \( T \) measures the degree of resource depletion resulting from previous production. The second cost component, \( c \), is a transitory cost. It is known privately by the firm, and it depends on the monopolist’s current operating conditions, and access to input. Transitory cost, \( c \), is independently and identically distributed each period by the density \( f(c) > 0 \) for \( c \in [c_l, c_h] \) with \( h'(c) > 0 \) where \( h(c) = c + F(c)/f(c) \).

A noteworthy feature of our model is the persistence of private information. Since there is no temporal correlation between different realizations of private costs, \( c \), and the current state \( T \)

\[ h'(c) > 0 \]

The expression \( h(c) \) is called the virtual cost of supply by Baron and Myerson (1982) as it includes the cost of information rents earned by the privately informed firm. The requirement, \( h'(c) > 0 \) is a standard condition employed in hidden information problems to insure satisfaction of second order conditions and the existence of separating equilibria.
is public knowledge, the regulator does not learn about the efficiency of the firm from observing its past behavior. Strategic learning and signaling are significant aspects of most previous analyses of dynamic regulation. Here, however, we abstract from these issues to focus on the role of learning by doing and declining resources in regulation.

The evolution of $T$ is governed by a simple Poisson process,

$$\lambda(T+1; x, T) = ax, \ a > 0$$

(A1)

where $\lambda(\ )$ is the probability the state moves from $T$ to $T+1$ next period, given the current output level is $x \geq 0$. When learning occurs (A1) implies the firm is more likely to inherit a superior technology next period, the more it produces currently. The possibility of the monopolist reducing cost through direct investment is not analyzed here, though we briefly discuss this possibility in section 2.5.

The dynamic relationship between the regulator and the utility consists of a series of short term contracts. In practice, legal and administrative constraints prohibit a regulator from negotiating long term service agreements with utilities. Firms are unable to credibly commit to an agreement for the future supply of service without knowing their costs of production. Negotiations between the firm and regulator begin each period with both parties observing the

---

10 This is in contrast to several earlier papers that examine repeated contracting and regulation in settings where the value of exchange or the cost of providing service remains constant and the contracting parties learn about each other’s private information over time. See for example, Baron and Besanko (1984,1987), Hart and Tirole(1988), Laffont and Tirole(1987, 1988, 1990), Lewis and Sappington (1997), and Meyer (1991).

11 The constant $a$ is bounded above to insure the probability of a transition from technology $T$ to $T+1$ does not exceed one. In practice the rate of innovation might also depend on the current state of technology and on the cumulative production to date. We abstract from these features for simplicity.

12 Our model relates to a large literature on learning by doing and industrial structure. Two papers most closely related to our formal analysis are by Cabral and Riordan (1994) and Habermeier (1992). These papers investigate the impact of learning by doing on oligopoly competition. In contrast, our paper focuses on how regulated monopolies are induced to exploit leaning economies in the absence of market competition.
current state $T$. Only the firm observes its private cost $c$. Next, the regulator offers the firm a single period contract $\{P(c,T), x(c,T)\}$. $P(\cdot)$ is the firm's payment, and $x(\cdot)$ is the firm's required supply of service. Both quantities are conditioned on the firm's report of its cost, $c$, and the state, $T$. The firm follows by either accepting or rejecting the contract. If the firm accepts the contract it delivers $x(\cdot)$ and receives $P(\cdot)$. If the contract is rejected, the parties suspend negotiations until next period. This is an ongoing process beginning again next period with realizations of the state $T$, and private cost $c$, and the offering of a new contract by the regulator.

We model the ongoing relationship between the regulator and firm as a dynamic game, In each period the regulator selects a contract to maximize the expected present value of consumer surplus. The utility selects a level of service to maximize its expected present value of profits. We focus on Markov strategies where the parties condition their actions solely on the current state, $T$ and on their private information (in the case of the utility). The focus on Markov behavior has intuitive appeal. In settings like ours where there is no long term legal institution governing the franchise, it seems reasonable for the parties to predicate their behavior each period on the current state as it directly affects their payoffs. Our approach to modeling strategic behavior differs from previous analysis by Gilbert and Newberry (1994) and Salant and Woroch (1991). These analyses investigate reputational equilibria, whereby the regulator compensates the utility for its investment to maintain a reputation for honoring the regulatory compact. Although reputational concerns don't enter into our analysis, we find investment behavior is nonetheless supported in equilibrium.

**Characterizing Equilibrium**

The characterization of this complicated dynamic interaction between the regulator and utility is simplified using dynamic programming arguments. Denote by $V(T)$ and $W(T)$ the expected value of beginning the current period in state $T$ for the regulator and firm respectively. Since the horizon is infinite and we restrict attention to Markov strategies, the value functions are
independent of calendar time. The change arising in the regulator’s and firm’s value functions as the state moves from T to T+1 is \( \Delta V(T) \equiv V(T+1) - V(T) \), and \( \Delta W(T) \equiv W(T+1) - W(T) \) respectively.

The optimal strategy for the firm with private cost \( c \) in state T is to select a cost report \( c' \) to

\[
\max_{c'} \left\{ P(c', T) - x(c', T)(d(T) + c) + \delta(W(T) + \lambda(T+1, x(c', T))\Delta W(T)) \right\} \quad (2.1)
\]

when offered the menu of contracts \( \{P(c', T), x(c', T)\} \) by the regulator. The expected profit for the regulated firm in state T is

\[
W(T) = E_c W(T, c) \quad (2.2)
\]

where \( W(T, c) \) is the maximized value of firm profit in (2.1) and \( E_c \) is the expectation with respect to \( c \). Notice \( W(T, c) \) consists of current period payments minus production costs, plus the discounted (by \( \delta \in (0,1) \)) stream of future profits. Future expected profits consist of the returns from beginning next period in the same state T, \( W(T) \), plus the probability the state increases next period to \( T+1 \) multiplied by the corresponding increase in profits \( \Delta W(T) \).

The regulator’s problem, designated by \([R]\) is to offer a menu of contracts \( \{P(c, T), x(c, T)\} \) to maximize V(T) defined by

\[
V(T) = E_c \left\{ v(x(c, T)) - P(c, T) + \delta\{V(T) + \lambda(T+1, x(c, T))\Delta V(T)\} \right\} \quad [R]
\]

subject to (2.1) and

\[
W(T, c) \geq \delta W(T) \text{ for all } c \in [c_l, c_H] \quad (IR)
\]

According to \([R]\) the regulator maximizes the sum of expected current period surplus and expected discounted (by \( \delta \in (0,1) \)) future surplus. Future surplus equals \( V(T) \) plus the probability of transitioning to state \( T+1 \) multiplied by the corresponding increase in continuation value \( \Delta V(T) \). This maximization is subject to (2.1) indicating how the firm optimally reacts to the regulator’s contract menu. The maximization is also subject to the individual rationality constraint, (IR) indicating the minimum surplus the firm expects from contracting with the
regulator. The firm can guarantee itself \( \delta W(T) \), the discounted value of beginning next period in state \( T \), by refusing to contract in the current period.

Combining the firm's optimal reporting strategy embodied in (2.1), with the regulator's optimal contracting strategy characterized by the solution to [R] enables us to establish necessary and sufficient conditions for a Markov perfect equilibrium (MPE). Informally, a MPE consists of a pair of strategies for the firm and regulator constituting a perfect equilibrium for all payoff relevant histories described by the state variable \( T \). For technical reasons, we assume there exists some state \( T_H > 0 \) such that \( d(T) \) is constant for all \( T \geq T_H \). Given this assumption we can establish the following.

**Proposition 1:** There exists a unique MPE to the dynamic regulation game. The optimal contract \( \{x(c,T), P(c,T)\} \) supporting the MPE satisfies the following: \( x(c,T) \) is strictly decreasing in \( c \) for \( x(c,T) > 0 \) and,

\[
v'(x(c,T)) = -\left( \frac{F(c)}{f(c)} + d(T) + c \right) + \alpha \delta \left[ \Delta W(T) + \Delta V(T) \right] \leq 0 \quad (= \text{if } x(c,T) > 0) \tag{a}
\]

\[
P(c,T) = [d(T) + c]x(c,T) + \int_c^{\infty} x(c',T)dc' - \alpha x(c,T)\delta \Delta W(T) \tag{b}
\]

\[
W(T) = E_c \left\{ \frac{x(c,T)F(c)}{1 - \delta} \right\} \tag{c}
\]

\[
V(T) = E_c \left\{ \frac{x(c,T)F(c)R(c)}{1 - \delta} \right\} \tag{d}
\]

**Proof:** The proofs of all propositions are contained in the appendices.

---

13 Restricting \( d(T) \) to be constant for \( T \) sufficiently large significantly simplifies our formal analysis. It permits us to solve for the equilibrium by backward induction. We define \( T_H \) to be the smallest \( T \) for which \( d(T') \) is constant when \( T' > T \).

14 The equilibrium reported in Proposition 1 is for the case where some subset of the higher cost types are shut down each period, so \( x(c,T) = 0 \) for \( c \) sufficiently large. This arises whenever the range of possible costs types is sufficiently large. For consistency and simplicity we focus on this case throughout the paper.
According to part (a) of Proposition 1, in equilibrium the output assigned to a firm of type \( \{c,T\} \) maximizes the net benefit from service by setting marginal net benefit to zero when output is strictly positive. Net marginal benefit includes the marginal benefit from current consumption, \( v'(x) \), minus the marginal cost of production, \( (d(T) + c) \) and the firm's information rents \( F(c)/f(c) \), plus the discounted expected change in future surplus from an increase in output, given by \( a\delta[\Delta W(T) + V(T)] \). Part (b) indicates the firm receives a payment equal to its direct cost of production \( [d(T) + c]|(c,T) \) plus the rent accruing from its private information \( \int_{c}^{W} x(c',T)dc' \), minus, \( ax(c,T)\delta W(T) \), the expected increase in future revenue resulting from an improvement in technology. Notice the regulator taxes away the firm's additional expected surplus, \( ax(c,T)\delta W(T) \), from a technology improvement by lowering the firm's payment by that amount. Parts (c) and (d) of Proposition 1 provide a convenient closed form expression for the expected surplus earned by the firm and regulator respectively.

### 2.3 Dynamic Regulation: Learning by Doing

This section characterizes optimal regulation when the monopolist learns to reduce service cost with experience. To capture the possibility of service costs declining with previous production, we introduce

**ASSUMPTION 1**: \( d(T) \) is weakly decreasing in \( T \) for \( T \leq T^H \) and \( d(T^H-1) > d(T^H) \)

Assumption 1 implies intrinsic cost, \( d(T) \) never increases and it strictly decreases as \( T \) transitions from \( T^H-1 \) to \( T^H \). Thus \( d( T^H ) \) is the minimal intrinsic cost the firm can achieve. Since \( d(T) \) is only weakly decreasing, several improvements in technology may be required before service costs strictly decline.

In settings where supply costs decrease with experience, it seems likely both parties benefit as the franchise matures. This conjecture is confirmed in

**PROPOSITION 2**: If Assumption 1 is satisfied the equilibrium value functions for the firm and the regulator, \( W(T) \) and \( V(T) \) are strictly increasing in \( T \) for \( T < T^H \).
Proposition 2 indicates transitioning to a higher technology *strictly* increases both the firm’s flow of future profits and the future flow of consumer surplus even when the new technology doesn’t strictly reduce service costs. Apparently each technology transition moves the firm closer to an eventual cost reducing state, thus increasing the future flow of profits and consumer surplus. In addition, although the regulator can observe the firm’s adoption of cost reducing technology she fails to tax away all the increase in surplus. The ratcheting down of compensation payments is incomplete as the firm retains a share of the additional surplus for herself. We explore these two factors in greater detail in the next two subsections.

2.3.1 Regulation to exploit learning economies

How is regulation tailored to exploit learning economies? To answer this question it is instructive to review the properties of the single period or static regulatory equilibrium as first analyzed by Baron and Myerson (1982). In a static environment without learning, technology remains at its initial level, \( T \), throughout time. In this case the optimal regulated service schedule denoted by, \( x^0(T,c) \), satisfies

\[
v'(x^0(c,T)) - \left( d(T) + c + \frac{F(c)}{f(c)} \right) \leq 0 \quad (= \text{if } x^0(c,T) > 0) \tag{2.3}
\]

According to (2.3) the regulator selects service to maximize only current net benefits. Current service does not affect future benefits, in contrast to settings in which learning arises. A comparison of the service levels induced in settings where learning does and does not occur and the impact of learning on induced service is summarized in,

**PROPOSITION 3**: If Assumption 1 hold, then for \( T \leq T^u \),

\( (a) \) \( x(c,T) \geq x^0(c,T) \), with \( (>) \) for \( x(c,T) > 0 \)

\( (b) \) \( x(c,T) \) is strictly increasing in \( T \), for \( x(c,T) > 0 \)
Part (a) of Proposition 3 indicates the regulator optimally induces greater service from the firm when learning economies exist. The regulator induces the firm to provide greater service to expedite the adoption of new cost reducing technology. The arrival of advanced technologies benefits both the firm and consumers. These benefits are manifested in two ways both leading to the firm increasing its service level. First, the marginal cost of inducing the firm to supply more service is reduced by the amount, \(a \Delta W(T) > 0\), when learning is possible. This amount is the expected increase in the firm’s future profits resulting from an increase in present service. The firm supplies service at a lower price expecting to earn greater profits in the future and since service costs are reduced the regulator optimally induces more output from the firm.

The second factor leading to greater service is that marginal benefit to consumers from service increases by the amount \(a \Delta V(T) > 0\) when learning is possible. This amount measures the additional future expected consumer surplus from a marginal increase in current service. As a consequence of this added marginal benefit, the regulator induces the firm to provide greater service compared to the static, no learning case.

The regulator induces greater service anticipating returns from learning in the future. This anticipatory behavior may also improve current regulatory performance. In a static setting the regulator induces too little service to limit the monopolist’s information rents. When the regulator induces greater service to encourage innovation, she may bring current production closer to static efficiency levels, thus improving current performance.

Part (b) of Proposition 3 implies the firm supplies greater service when operating an advanced technology for \(T \leq T^H\). This arises for two reasons. First the adoption of a new technology may reduce service cost leading the regulator to demand more service. This factor leads to greater service whether or not learning economies exists. However, service increases even when the advanced technology does not immediately reduce cost so that \(d(T+1) = d(T)\) for some \(T\). In this case the future returns from current service increase because an improvement in
technology brings the firm closer to an eventual cost reduction. Thus the regulator calls for
greater service to hasten the arrival of this cost reducing technology.

2.3.2 Incentives to innovate and the distribution of gains from learning

Proposition 2 indicates learning economics resulting in an improved technology strictly
benefit the firm and consumers. This occurs even though the regulator observes the new
technology and the resulting reduction in intrinsic cost. Despite her power to set contract terms,
the regulator is unable to fully "ratchet down" (Freixas et al., 1985, Laffont and Tirole, 1987 and
Weitzman, 1980) the service price to tax away all the firm's gains from a cost reduction. To see
this, recall from part (b) of Proposition 1, that the monopolist's payment is given by

\[ P(c, T) = [d(T) + c]x(c, T) + \int_{c'}^u x(c', T)dc' - ax(c, T)\delta AW(T) \]

This formula, indicates payment is reduced by \( ax(c, T)\delta AW(T) \), the monopolist's expected
increase in future surplus. This reduction in payment is similar to the productivity offset
employed to adjust service rates under price cap regulation.\(^{15}\) This payment reduction does not
capture all of the increase in monopoly profit from a cost saving innovation. As technology
improves, the regulator induces greater levels of service, (by part (b), Proposition 3) affording the
monopolist greater information rents. This increase is reflected by the term \( \int_{c'}^u x(c', T)dc' \), in the
payment formula.

The monopolist's ability to capture some share of the increased surplus from a cost
reducing innovation explains why the firm offers service at lower prices to promote innovation
without long term regulatory agreements. Accepting lower payment for service is a relationship
specific investment the firm makes to increase the future surplus generated by the franchise. In
this instance a hold up problem exists (Williamson 1985, Hart and Moore 1988) because the

\(^{15}\) See the Rand Journal Symposium on Price Cap Regulation (1989) and Sappington and
Weisman (1996), Chapter 6 for more information on price cap regulation.
monopolist is not guaranteed a return from his investment absent a long-term regulatory agreement. Having a persistent source of private information that does not erode with experience permits the monopolist to earn a sufficient return on his investment to partially ameliorate the hold up problem. Notice also that investment arises in our setting even without an implicit "regulatory compact" supported by reputational concerns as modeled in Gilbert and Newberry (1994) and Salant and Woroch (1991). Our findings suggest earlier analyses of multi period regulation which presume the monopolist’s private knowledge is eventually revealed may overstate the difficulties arising from limited regulatory commitment.

To examine the distribution of learning gains between the firm and consumers we compute \( \Delta W(T) / (\Delta W(T) + \Delta V(T)) \) to measure the percentage of technology gains accruing to the firm. From parts (c) and (d) of Proposition 1 this ratio is given by

\[
\frac{\Delta W(T)}{\Delta W(T) + \Delta V(T)} = \frac{E_c((x(c,T+1) - x(c,T))F(c))}{E_c((x(c,T+1) - x(c,T))F(c)(1 + h'(c)))}
\] (2.4)

An easy interpretation of (2.4) arises for the case where \( F(c) = \left(\frac{c-c_L}{c_H-c_L}\right)^\eta \) with \( \eta > 0 \). This is an isoelastic generalization of the uniform distribution with \( \eta = 1 \) corresponding to the uniform density. For this case, \( h'(c) = 1 + 1/\eta \), and (3.2) simplifies to

\[
\frac{\Delta W(T)}{\Delta W(T) + \Delta V(T)} = \frac{1}{2 + 1/\eta}
\] (2.5)

Inspecting (2.5) it is clear when \( \eta \) is small consumers capture most of the additional surplus from a technology advance. When \( \eta \) is small most of the probability mass is concentrated on small values of \( c \) knowing the monopolist’s costs are likely to be small, the regulator can safely offer the firm a demanding contract requiring it to supply significant service with small compensation. Conversely when \( \eta \) is large there is a substantial probability the firm’s private cost of service is high. Consequently the regulator must offer the firm more generous compensation to supply service. This enabling the firm to capture a greater share of the surplus generated from the
technology improvement. In the limiting case where \( \eta \) becomes large the firm shares equally with consumers in the gains from learning.

2.3.3 Durability of the franchise relationship

Until now we have assumed the same firm continually services consumers. However, conceivably the firm may go out of business or leave to service other customers. This section analyzes the regulator's preference for dealing with short lived and long lived firms. Suppose there is some exogenous survival probability \( \mu \in [0,1] \) the firm remains to serve consumers for another period. This probability depends on factors not explicitly modeled here that include the firm's outside opportunities and consumers' inclination to switch suppliers. Assume, with probability \( 1-\mu \) the firm leaves the franchise, whereupon it is immediately replaced with another supplier. It is costless for consumers to switch suppliers, and the new supplier is able to adopt the incumbent's existing technology. This would arise for instance, if consumers own the equipment that firms utilize to provide service as often occurs with municipal utilities. Assuming the firm is costlessly replaced without loss of technological know how is unrealistic, but it serves to reinforce the findings reported in

**PROPOSITION 4:** Given Assumption 1 the regulator prefers the most durable franchise with \( \mu = 1 \).

The intuition for Proposition 4 follows from our earlier discussion of how one optimally regulates to exploit learning economies. Learning is best cultivated in a durable relationship. A long lived firm willingly provides service at reduced compensation in anticipation of increased future profits from learning. The value of learning is reduced when the firm's tenure is limited. Consequently, the firm requires larger payment for current service if it does not expect to share in the returns from learning. The regulator reacts by decreasing the level of service demanded in current periods, thus retarding the rate of innovation. All of this conspires to reduce the future surplus from learning generated under the franchise. This further suggests the turnover of
regulated firms should be small in settings where learning economies are important and that the rate of innovation is likely to be greater in durable relationships.\textsuperscript{16}

2.4 Dynamic Regulation with Increasing Costs

In some settings supply costs may rise with previous production. For instance the cost of electricity may increase from previous production if the utility exhausts the supply of inexpensive fuels.\textsuperscript{17} We formally describe the setting where supply costs rise with previous production by the following:

\textit{ASSUMPTION 2}: \( d(T) \) is weakly increasing in \( T \) for \( T \leq T^H \) and \( d(T^{H-1}) < d(T^H) \).

According to the assumption production costs are never decreasing and they are strictly increasing as \( T \) transitions from \( T^{H-1} \) to \( T^H \).

Under Assumption 2, the beneficial effects on regulation arising from the possibility of future cost reduction are no longer present. In fact, there is an adverse side to dynamic regulation. Current service is reduced below the level that would prevail in a static setting, and expected profits and consumer surplus fall as the regulatory relationship matures. We record these findings in the following

\textit{PROPOSITION 5}: If Assumption 2 is satisfied,

\( (a) \) The equilibrium expected firm profits, \( W(T) \), and expected consumer surplus \( V(T) \) are strictly decreasing in \( T \), for \( T < T^H \).

\( (b) \) Service, \( x(c,T) \), is strictly decreasing in \( T \) for \( x(c,T) > 0 \).

\( (c) \) \( x(c,T) \leq x^0(c,T) \), with \( (<) \) for \( x^0(c,T) > 0 \)

\( (d) \) The regulator prefers short-lived franchises, with \( \mu \) as small as possible.

\textsuperscript{16} This prediction is consistent with McCabe (1996) who reports suppliers learned to reduce design and construction costs of electric power plants more when they worked for one rather than several customers over time.

\textsuperscript{17} The cost of fuel might increase because it is more difficult to obtain, or because it is dirtier and requires more clean up to meet environmental emission standards.
When production costs are increasing, both parties realize an increase in current service will cause expected costs to rise in the future. This increase in costs reduces the expected surplus both parties expect as reported in part (a) of proposition 5. Greater future costs cause the regulator to induce smaller service levels as access to inputs deteriorates as reported in part (b). The firm requires greater compensation in current periods to supply service as compensation for the reduction in profits arising in the future. Similarly, the regulator receives smaller benefits from current production service, since future expected surplus decreases as current service increases. These two effects lead to the finding in part c that the regulator reduces service below the level occurring in a static setting.

Part (d) of the proposition examines the regulator’s preference for franchise durability in this setting. When parties anticipate their relationship surplus will decline with experience, they will seek less durable relationships. The advantage of a short term franchise is the firm is willing to supply current service at a lower cost, when his future stake in the franchise is reduced An increase in current service doesn’t diminish the firm’s future profits if he is unlikely to be serving the same consumers again. The regulator prefers to engage firms with a short time horizon because they are “softer” bargainers, willing to accept smaller payment to provide current service.

Combining part (d) of Proposition 5 with Proposition 4 suggests the regulator’s preference for durable franchises depend on the potential for the relationship to grow. In growing environments where technological advance is likely to reduce future service costs, consumers will prefer maintaining a stable relationship by retaining a single supplier throughout the lifetime of the franchise. This naturally assumes that only a single firm may supply consumers at one time. The possibility of several suppliers serving customers simultaneously is mentioned briefly in the
concluding section. Conversely, when exchange surplus decreases with previous production, 
regulators prefer short term franchises with frequent turnover of service suppliers.

2.5 Conclusion

Technological progress resulting in less costly service is important in regulated 
industries, especially telecommunications and electricity. This paper extends the incentive 
regulation literature by characterizing the optimal regulation of an innovating monopolist with 
private and changing supply costs. Our principle findings are that (1) innovation is encouraged by 
light handed regulation allowing the monopolist to earn greater information rents while providing 
greater service, (2) both the monopolist and consumers strictly benefit from cost reducing 
innovation, (3) innovation occurs in the absence of long term agreements when private 
information is persistent, and (4) innovation is more rapid in a durable franchise, and the 
regulator prefers durable franchises for exploiting learning economies.

This paper has focused on cost reducing innovation. It seems clear the results reported 
here would also apply when experience permits the monopolist to improve the quality of 
customer service. Besides learning by doing, other vehicles like R&D may exist for discovering 
new technologies In these instances it would be interesting to analyze how the regulator induces 
the appropriate mixture of innovation activities.

Another application for the techniques developed here is to study of optimal procurement 
policy. Cost reductions from learning are important in repeated procurements of factories, ships, 
aircraft and weapons. The optimal procurement procedure would trade off the advantages of 
purchasing from a single supplier to exploit learning economies against the benefits of dealing 
with several suppliers to maintain competition.

18 The importance of learning economies is documented in Fox (1988) and Gansler (1989).
CHAPTER 3
WHY CHARITIES ANNOUNCE DONATIONS: A POSITIVE PERSPECTIVE

3.1. Introduction

Fund raising by charities and other nonprofit institutions is commonly characterized by announcement of receipts as they accrue. Telethons continuously update their receipts. United Way campaigns post signs telling what fraction of their total goal has been reached. Universities frequently announce large contributions at the official commencement of their capital campaigns, and regularly provide updates. What is the role of announcement in the fund-raising process?

Most of the literature on voluntary provision of public goods presumes simultaneous moves in the contribution game. With announcements, however, donors would play sequentially. This suggests a potential explanation for announcement in fund raising: Sequential equilibria might produce a higher total than results in the simultaneous-move alternative. In fact, Varian (1994) shows that the sequential-contribution game provides less of the public good in the "standard" model in which agents care only about the total supply of the public good.¹

Why then, do fund-raisers commonly facilitate sequential play in the contribution game? One possibility is that the standard utility specification is wrong and that donors have additional motives such as warm-glow effects. The warm-glow utility specification introduces an agent's own contribution into the utility function, so that an agent gets utility not only from the total provision but also from his own contribution. The warm-glow specification has reconciled two inconsistencies between the theoretical predictions of the standard model of voluntary contributions and empirical evidence on contributions. The standard model predicts small

¹ While not the focus of this paper, we provide here generalizations of Varian's analysis to the n-agent case and to the case of endogenous timing of contributions.
(arbitrary) income redistributions among contributors will be neutral, i.e., not change utility levels or total contributions. Second, in sufficiently large economies, only the rich contribute to the provision of public goods. These striking predictions, however, conflict with the empirical evidence on private charities. White (1989), for example, indicates that nine out of ten Americans donate to charities. Steinberg (1989) provides empirical evidence that neutrality fails. With the warm-glow utility specification, income redistributions are not neutral and even in large economies poorer people with such preferences will make contributions. Other recent models departing from the standard one suggest that enhancement of prestige and reputation may also be relevant in making charitable contributions.

To consider whether these types of effects can make sequential-contribution equilibria better for the charity, we consider a two-agent model with a general utility function that includes the standard and warm-glow effect models as special cases, as well as allowing other motives for contributing. For the general utility function, sequential equilibria can produce a higher supply of the public good than can the simultaneous-move alternative. This can occur, for example, under the warm-glow specification. If a charity has the power to choose the order of agents' contributions or if an order arises exogenously, the charity will sometimes prefer to announce contributions as they arise. We also show how the result generalizes to the n-player case.

The environment with announcements may better be modeled as allowing the order of contributions to arise endogenously. Here agents decide when they will contribute in addition to

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2 See Warr (1983), Bergstrom, Blume, and Varian (1986), and Bernheim (1986).


5 See Glazer and Kondrad (1996) and Harbaugh (1998a, 1998b). These papers are discussed further in the next footnote.
the amount that they will contribute. We also show that making a commitment to announce contributions may also be preferred in several versions of the endogenous-timing game.

Our findings provide additional support for utility specifications that admit effects like warm glow. They also point toward the organizational role that a charity can play in the provision of public goods.\(^6\)

Andreoni (1998) and Vesterlund (1998) also offer explanations for announcements of initial donations in voluntary contribution equilibria. Andreoni presumes expenditure must exceed a threshold for the public good to be of value, implying no contributions will frequently be a simultaneous-move equilibrium even when an equilibrium with positive provision also exists. By enticing one or more donors to contribute first and then announcing that total, the zero-contribution equilibrium can be eliminated. Vesterlund presumes incomplete information among donors about the quality of the charity, which can be resolved by a donor's costly inspection. By committing to announce the first contribution, a high-quality charity can sometimes induce an inspection and subsequent signaling of quality via the initial contribution. Our analysis complements these alternative explanations. For example, after the threshold has been met in a model like Andreoni's, our analysis rationalizes continued announcement of

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\(^6\) In addition to the papers discussed next in the text, Glazer and Konrad (1996), Harbaugh (1998a, 1998b) and Bilodeau and Slivinski (1997) also analyze the behavior of charitable organizations. Konrad and Glazer characterize a charity as providing a means to signal wealth, which requires public disclosure of donations. Harbaugh demonstrates the value of categorical reporting of donations in raising funds, when a motivation for giving is prestige. Because donors' utility functions in Konrad and Glazer's and Harbaugh's models do not depend on other donors' contributions, there is no role for sequential announcements. Bilodeau and Slivinski show the value of specialization of charities when there are multiple public goods, as this allows donors better control of use of their contributions. They do not consider the possibility of sequential announcements. In a non-game theoretic model where a charity's administrator does not necessarily share the same ideology as donors, Rose-Ackerman (1981) demonstrates that government grants may increase private donations. This can occur, for example, if grants are given conditional on charity's adopting an ideology closer to donors', or grants give donors better information about the charity. In her 1987 paper, using a similar model, she also shows that a charity might change its ideology closer to donors' to rely more on private donations if government grants are cut back.
contributions. While our analysis relies on motivations for giving like warm glow that these alternatives do not require, our results hold in the otherwise simplest setting with complete information and a concave provision technology.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 presents the main results. Section 4 considers extensions to the endogenous-timing game. Section 5 provides some examples. Section 6 draws some conclusions. An appendix contains most of the more technical analysis.

3.2 The Model

Let there be two agents with utility functions:

$$U^i = U^i(x_i, Y, h^i(y_i, y_j)), \quad i, j = 1, 2, i, j;$$

where $y_i$ denotes agent $i$'s monetary contribution to the public good, $Y = y_i + y_j$ denotes the total provision of the public good, and $x_i$ is (monetary) private-good consumption.\(^7\) Assume that $U^i$ is increasing strictly in $x_i$ and $Y$, and weakly in $h^i$; twice continuously differentiable; and strictly quasi-concave in $(x_i, Y^i, h^i)$ on the interior of agent $i$'s constraint set (presented below). To insure interior solutions, we adopt the Inada conditions: $\lim_{x_i \to 0} U^i_i(.) = \infty$ and $\lim_{x_i \to \bar{x}_i} U^i_i(.) = 0$, where $\bar{x}_i$ is endowed income and subscripts stand for partial derivatives.\(^8\) We also assume $h^i$ is continuous, twice differentiable, and weakly monotonic in both arguments. At times, it will be useful to assume further that $h^i$ is concave.

This form for the utility function is more general than the common specifications in the literature, and incorporates them as special cases. If $h^i(.)$ is a constant, then the present model

\(^7\) Prices are held fixed throughout.

\(^8\) Bergstrom, Blume, and Varian (1986) extensively analyze the impacts of income redistributions on the private provision of a public good without assuming interior solutions in a standard model. Our focus is different from theirs, thus analysis around boundaries is less interesting and would unduly complicate our paper.
reduces to the standard model. If $h'(.) = y_i$, then it reduces to the warm-glow model. The general form assumed here admits other possibilities in which agent $i$ may be affected by the other agent's contribution not only through $Y$, but also through some other private interests captured in $h'(.)$. For instance, each agent may be concerned about his contribution relative to the other agent in as much as this effects social prestige and reputation. Suppose, for example, that two rival businessmen are invited to donate to support a public school. They may compete for potential customers' goodwill by means of their relative contributions. In this case, $h'_2 < 0$ due to a *snob effect*. In other cases, $h'_2 > 0$ is plausible due to a *bandwagon effect*. For football fans, their team's victory is a public good. One fan's cheering at the stadium may increase others' utilities both by increasing total support, which may help the team win, and also by directly making it more fun to attend and cheer.

Agent $i$ allocates his endowment, $I_i$, between $x_i$ and $y_i$. As is common in the literature, suppose that both private good and individual contributions are normal in income (but not luxuries), i.e., $0 < \frac{\partial x_i^d}{\partial I_i} < 1$ and $0 < \frac{\partial y_i^d}{\partial I_i} < 1$, where $x_i^d(I_i,y_j)$ and $y_i^d(I_i,y_j)$ are the ordinary demand functions.  

Throughout the analysis, we assume preferences and incomes are common knowledge among the agents. Assume, for now, that the charity shares the same information. Moreover, for now, also let the charity have the power to ask for contributions in a particular order and assume that each agent can contribute only once. We examine more limited control settings below.

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9 Normality of demand for individual contributions can be related to an assumption of normality of demand for total contributions $Y$. Substitute into $U(.)$ the constraint $y = Y-y_j$, and define $U^*(x,Y;y_j) = U^*(x,Y,y_j(Y-y_j,y_j))$. In our model, the natural definition of the ordinary demand for Y, $Y^d(I,y_j)$, is the Y that maximizes $U^*(x,Y;y_j)$ over $(x,Y)$ subject to $x+Y = I$. The demand for individual contributions (i.e., the solution to [P1] below) is given by: $y_i^d = Y_i^d(I+y_j,y_j)-y_j$, where we use that the solution $y_i^d$ is interior. Hence, $\frac{\partial y_i^d}{\partial I_i} = \frac{\partial Y_i^d}{\partial I_i}$, so the normality restrictions are the same.
including with multiple contributions. In a fund-raiser, a charity may ask for contributions in a particular order and reveal the contributed amounts as they arise, thereby inducing agents to play a sequential game. Alternatively, the charity may induce simultaneous play by not revealing contributions until all have been made regardless of whether they arrive sequentially. The timing of the contribution game is exogenous from the agents' perspective. Our general results also can be applied to compare cases in which the "natural play" of the contribution game is sequential or simultaneous-move. Varian (1994) provides interesting examples of such games.\textsuperscript{10}

To keep the analysis simple, we assume that both the sequential- and simultaneous-move games have a unique and interior (subgame-perfect) pure-strategy Nash equilibrium.\textsuperscript{11} Let $G_0$ and $G_k$, $k=1,2$, denote, respectively, the simultaneous- and sequential-move games where agent $k$ moves first in the latter cases. Let $Y(G_k)$ and $y_i(G_k)$ denote, respectively, the equilibrium total provision of the public good and the equilibrium contribution of agent $i$ under game $G_k$, $k = 0,1,2$.

Solve [P1] below to find agent $i$'s best-reply function: $y_i=f'(y_i)$.

Max $U'(x_i,Y,h'(y_i,y_j))$ \quad [P1]

\begin{align*}
x_iy_i \\
\text{s.t.} & \quad x_i + y_i = I_i \\
& \quad y_i + y_j = Y \\
& \quad x_i, y_i \geq 0
\end{align*}

Substitute for $x$, and $Y$, and rewrite [P1]:

Max $U'(I_i - y_i, y_i + y_j, h'(y_i,y_j))$ \quad s.t. $0 \leq y_i \leq I_i$

\textsuperscript{10} For example, Varian considers a variation on the Samaritan's dilemma game played between the older and the younger generations. This game is obviously inherently sequential.

\textsuperscript{11} Sufficient conditions for these results are most readily and meaningfully established in the particular applications. For the sequential model, concavity of $h'(y_i,f(y_i))$ in $y_i$, where $f$ is agent $j$'s best-reply function, along with our other assumptions, constitutes one set of sufficient conditions. In the simultaneous case, uniqueness is the key requirement (given our other assumptions), and, of course, assumptions that insure a unique intersection of best-reply functions are required. The examples in Section 5 demonstrate the ease in applying our results.
Using that the solution is interior, the first-order condition is:

$$-U_1' + U_2' + U_3' h_1' = 0. \quad (3.1)$$

Our assumptions above imply that second-order condition is satisfied\(^\text{12}\), i.e. that

$$K_i = U_{11} + U_{22} - 2U_{12} + 2h_1'(-U_{13} + U_{23} + U_{33} h_1' / 2) + U_3' h_{11} \leq 0. \quad (3.2)$$

From (1), the slope of agent i’s best-reply function is:

$$\frac{\partial f^i}{\partial y_j} = \frac{U_{12} - U_{22} - U_{32} h_1' + U_{13} h_2' - U_{23} h_2' - U_{33} h_1 h_2' - U_3' h_{12}'}{K_i}. \quad (3.3)$$

The "total effect" of agent j’s contribution on i’s utility is:

$$\frac{dU^i}{dy_j} = U_{12} + U_{32} h_1'. \quad (3.4)$$

Interpretations of (3.3) and (3.4) will be provided in the next section.

The simultaneous-move equilibrium of \(G_0\) occurs at the intersection of \(y_i = f(y_j)\) and \(y_j = f^i(y_j)\). The subgame-perfect equilibrium of the sequential-move game \(G_i\) occurs at the value of \(y_i\) on \(y_j = f(y_j)\) that maximizes \(U_i^*\). The latter is found by solving the program:

$$\text{Max} \quad U'(I-y_i,y_i+f(y_i),h(y_i,f(y_i))). \quad [P2]$$

$$0 \leq y_i \leq I_i$$

We assume that \([P2]\) is a quasi-concave programming problem. Given our other assumptions, one set of sufficient conditions is that \(f(y_i)\) is concave (convex) in \(y_i\) for \(U(x,y_i+y_j,h(y_i,y_j))\) increasing (decreasing) in \(y_j\). These conditions are satisfied, for example, in the examples in Section 5.

### 3.3 Results

Proposition 1 contains the main results of this paper.

\(^{12}\) Using concavity of \(h(y_i,y_j)\), it is straightforward to write \([P1]\) as a quasi-concave programming problem.
PROPOSITION 3.1 The signs of \( \frac{dU^i}{dy_j} \frac{\partial f^j}{\partial y_i} \) and \( (y_i(G_j) - y_i(G_0)) \) are the same. Furthermore, \( (\frac{dU^i}{dy_j} \frac{\partial f^j}{\partial y_i}) \) and \( (Y(G_j) - Y(G_0)) \) have the same (opposite) sign whenever \( (1 + \frac{\partial f^j}{\partial y_i}) \) is positive (negative) for \( y_j \) between and including \( y_j(G_0) \) and \( y_j(G_1) \).\(^{13}\)

The proof is presented in the appendix. Here we provide an intuitive discussion. Figures 1-4 illustrate several possibilities regarding the slopes of best-reply functions and the total effect of agent i's contribution on agent j's utility. In Figure 3.1 for example, both agent 1's best-reply function and utility are increasing in agent 2's contribution. The latter and quasi-concavity of utility in \((y_1, y_2)\) imply that agent 1's indifference curves open upward with utility increasing as \( y_2 \) increases. Agent 2 obtains utility from agent 1's contribution, but his best-reply function decreases in \( y_1 \). The two Stackelberg equilibria and the simultaneous-move equilibrium are shown in each figure. We refer to these figures and rationalize the cases at various points below.

Whether sequential or simultaneous play yields a larger contribution by agent i depends on whether \( \frac{dU^i}{dy_j} \) and \( \frac{\partial f^j}{\partial y_i} \) have the same or opposite signs. By switching i and j in (3) and comparing (3) with (4), one can see that any pattern of signs of these expressions is possible since each expression has terms not in the other. Consider \( \frac{dU^i}{dy_j} \). If agent 2 increases his contribution by one dollar, agent 1 gets extra utility \( U^i_{2} \) due to the one dollar increase in total supply of the public good and extra utility (or disutility) \( U^i_{3} h^i_{2} \) from the other private interests discussed above. Although \( U^i_{2} \) and \( U^i_{3} \) are always non-negative, the sign of \( h^i_{2} \) can vary across plausible settings.

\(^{13}\) If \( \frac{dU^i}{dy_j} = 0 \), then "whenever \( (1 + \frac{\partial f^j}{\partial y_i}) \) is not needed. If \( \frac{\partial f^j}{\partial y_i} = -1 \) over the given range, then \( Y(G_1) = Y(G_0) \).
When the effect of an increase in \( y_2 \) on the private interests of agent 1 is negative, \( \frac{dU^j}{dy_2} \) indicates the degree of relative importance between this and the positive effect on the total supply of the public good. One might say that a dominating public-good effect implies that the net effect is positive.

Now consider the example in Figure 3.1 where \( \frac{dU^j}{dy_2} > 0 \) and \( \frac{\partial f^2}{\partial y_1} < 0 \). If agent 1 moves first, he knows he can engender the simultaneous-move outcome by committing to \( y_1(G_0) \). However, if agent 1 contributes less, agent 2 would respond to this by increasing her contribution, which, in turn, increases agent 1’s utility. Agent 1’s incentive at the margin is then to decrease his contribution. This incentive carries over globally under our concavity assumptions. Whether total provision of the public good increases relative to the simultaneous case depends on whether agent 2 raises her contribution more than agent 1 decreases his, which depends on the sign of \( (1 + \frac{\partial f^2}{\partial y_1}) \). If the latter expression is negative between \( y_1(G_0) \) and \( y_1(G_1) \), then \( Y(G_1) > Y(G_0) \).

In general, normality assumptions about private good consumption and contributions have some relation to the slopes of the best-reply functions. To see this, use (1) to calculate

\[
\frac{\partial y_i^d}{\partial I_i} = \frac{U_{i1} - U_{i2} - U_{i3} + U_{i4} + U_{i5} h_i + U_{i6} h_i + U_{i7} (h_i)^2 + U_{i8} h_i K_i}{K_i} \tag{3.6}
\]

and

\[
\frac{\partial x_i^d}{\partial I_i} = 1 - \frac{\partial y_i^d}{\partial I_i} = \frac{-U_{i4} + U_{i2} + U_{i3} h_i + U_{i4} h_i + U_{i5} h_i + U_{i6} (h_i)^2 + U_{i7} h_i}{K_i} \tag{3.7}
\]

Substituting (3.7) into (3.3) yields

\[
\frac{\partial f^j}{\partial y_j} = \frac{\partial x_i^d}{\partial I_i} \left( h_i - h_{i2} \right) \left( -U_{i2} + U_{i3} h_i + U_{i4} h_i + U_{i5} h_i + U_{i6} (h_i)^2 + U_{i7} h_i \right) + \frac{\partial y_i^d}{\partial I_i} \left( h_i - h_{i2} \right) U_i \tag{3.8}
\]
Equation (3.8) shows how agent $i$ would respond to a change in agent $j$'s contribution. For example, suppose agent $j$ increases his contribution by one dollar. This affects $i$'s behavior through several channels. To distill these effects, it is useful to think of agent $i$ as initially reducing his contribution by one dollar. Then agent $j$'s increased contribution corresponds to an increase in agent $i$'s income of one dollar. Due to the normality assumptions on demands, agent $i$ will spend a fraction of the one dollar increase on the private good. This implies a net decrease in contribution to the public good. This negative income effect is reflected in the first term of (8). If agents' preferences obey the standard model (i.e., $h'$ is constant), then this income effect is the sole determinant of the slope of the best-reply function, and this is why the slope lies between $-1$ and $0$ in the standard model. As we will see, this is the main intuition behind the first part of the Corollary below.

With general utility functions, however, there are other effects reflected in the remaining terms in (8). To clarify these terms, it is useful to note that $\frac{dU^i_3}{dy_i} = -U^i_{13} + U^i_{23} + U^i_{33}h^i_1$, which is the total effect of agent $i$'s own contribution on his marginal utility from the additional motives. For example, in the warm-glow model which has $h^i_1 = 1$ and $h^i_2 = h^i_{11} = h^i_{12} = 0$, the slope simplifies to $\frac{dF^i}{dy_j} = -\frac{\partial x^i_d}{\partial I_i} + \frac{1}{K_i} \frac{dU^i_3}{dy_i}$, and $\frac{dU^i_3}{dy_i}$ derives solely from the warm-glow effect. Agent $i$'s increased expenditure on the private good and associated reduction in own contribution affects his marginal value of making contributions, as captured by this term. If, for example, $U^i_{33}$ is negative and dominates the other terms in $\frac{dU^i_3}{dy_i}$, then $\frac{\partial F^i}{\partial y_j} > -\frac{\partial x^i_d}{\partial I_i}$ (using $K_i \leq 0$). In any warm-glow specification having $dU^i_3/dy_i < 0$, agent $i$ will either not decrease his contribution as much as dictated by the income effect or will increase his contribution. An example of the latter is analyzed in Section 5. Yet another interpretation of slopes for the warm-glow model will be
given in the second part of the Corollary below. In the model with general \( h' \), the more direct effects of the change in \( y_j \) will come into play as well as effects operating through concavity properties of \( h' \).

To get further insight, it is useful to consider the implications of Proposition 1 in the special cases, i.e., the standard and warm-glow models.

**COROLLARY 3.1**

A) (Varian) If donors care only about the total supply of the public good, then the simultaneous-move game yields the highest amount of the public good, and agent \( i \) contributes less under \( G_i \) than under \( G_0 \).

B) In the warm-glow utility model in which \( h'(y_j) = y_j \), the signs of \( \frac{\partial f'_i}{\partial y_i} \) and \( (y_i(G_i) - y_i(G_0)) \) are the same. Moreover, \( \frac{\partial f'_i}{\partial y_i} \) and \( (Y(G_i) - Y(G_0)) \) have the same (opposite) sign whenever \( (1 + \frac{\partial f'_i}{\partial y_i}) \) is positive (negative).\(^{14}\)

Again, proof is in the appendix. Part A of the corollary coincides with Varian’s (1994) result. The important point is that in the standard model, the highest provision of a public good is secured under the simultaneous-move game. If a charity has the power to arrange the timing of agents' contributions and the standard model holds, then the charity wants to induce donors to contribute simultaneously. The charity may ensure simultaneous contributions simply by not revealing any information about contributed amounts as they arise. However, this does not rationalize the common observation that some charities do reveal information about contributed amounts.

\(^{14}\) If \( \frac{\partial f'_i}{\partial y_i} = -1 \), then \( Y(G_i) = Y(G_0) \).
Part B of the corollary indicates that when agents have warm-glow preferences, it may be possible to engender higher total contributions when donors move sequentially than when they move simultaneously. In addition, the ranking of total contributions is determined completely by the slope of the second mover's best-reply function, since the first mover always prefers a higher contribution by the second mover. A sufficient condition here for a higher total under sequential play is that the second mover's best-reply function is upward sloping. Equilibria in this case are illustrated in Figure 3.2. There could be a case with warm-glow agents with downward sloping best-reply functions where the sequential equilibria might still have a higher total. Vesterlund observes that a property of sequential equilibrium in the standard model of utility is that the first mover would contribute more if given the chance. Hence, contributing twice must not be feasible if we are to take the sequential equilibria seriously. Note that this caveat is unnecessary when warm-glow agents have best-reply functions that are upward sloping. In the sequential equilibrium here, the leader makes a choice above his best-reply function so would not want to contribute more if given the opportunity. In Section 4 we analyze such games.

Best-reply functions in the warm-glow model have some relatively simple interpretations.

Again, substituting for \( h_1' = 1 \), \( h_2' = h_{12} = 0 \) and utilizing equation (3), one can see that \( \frac{\partial f^i}{\partial y_j} \) is positive if and only if \( \frac{dU_2^i}{dy_1} = -U_{21} + U_{22} + U_{23} \) is positive. Expression \( \frac{dU_2^i}{dy_1} \) is the total effect of agent i's own contribution on his marginal utility of public good. If utility from the warm-glow effect is sufficiently complementary with total provision, i.e., if \( U_{23} \) is sufficiently positive, then best-reply functions will be upward sloping. Manzoor, Gronberg, and Hwang (1997) empirically analyzed contributions to public-radio stations in the U.S. and found upward sloping best-reply

15 The agents in Figure 3.1 have asymmetric preferences but could both be warm-glow types. The sequential equilibrium where agent 2 moves first clearly has a higher total than in the simultaneous equilibrium. As discussed above, the total if agent 1 moves first may be higher.
functions thus supporting a model with effects like warm glow. Our theoretical study complements these empirical findings.

With warm-glow utility functions and upward-sloping best replies, announcement by a charity leads to a Pareto improvement over the simultaneous-contribution equilibrium (see Figure 3.2). Ideally, a charitable organization would dictate a particular order of moves. In the radio-station example, granting the station such knowledge and control may be unrealistic. However, our results imply gains from announcing donations as they arise in any exogenously determined order as long as both warm-glow players have upward sloping best-reply functions.

So far we have shown gains might result with announcement only for the case of two players. We show in the appendix that, with n warm-glow players that have upward-sloping, best-reply functions, strict gains arise simply from announcing the first donation. We also provide slightly stronger conditions sufficient for higher contributions as the frequency of announcement rises, in the subset of games where announcement begins after $m < n$ contributions have been made and then continues after every subsequent contribution. That is, we show total contributions rise as $m$ declines. This implies, for example, always announcing is better than never announcing when the conditions are met.

In the next section we show how our results apply if the timing of contributions is determined endogenously. In Section 5, we examine an example of a warm-glow specification, and also an example with a snob effect.

3.4 Endogenous Timing

The ideal charity would have the power and information about agents that permit it to set the order of contributions that raises the highest total. Gathering this information and engendering such an outcome might be a key function of fund-raisers. In this section, we relax the assumption that the charity has the power to choose a particular order of contributions while maintaining the assumption of complete information. It may be difficult or inefficient for even a
fully informed charity to induce a particular order of moves. The charity may have difficulty contacting donors in the preferred order. With many small donors, it may be relatively efficient to solicit donations from everyone at once, as in the public radio and TV examples. Hence, we assume a period of time over which donations may be made and let the agents choose the timing and amounts of their contributions. Again, the question is whether the charity can make use of being able to announce the contributed amounts as they arise in this setting. Endogenous-timing games have been analyzed by Hamilton and Slutsky (1990, 1993) and van Damme and Hurkens (1996), and the results can be applied to our problem.

Specifically, assume a charity collects contributions from two agents over two periods. We assume first that agents may contribute only once, but in the period they choose. It may be impractical, i.e., prohibitively costly, to contribute twice. Allowing contributions in both periods is also analyzed below, and our main points continue to hold. We assume the charity reports the magnitude of any donations that arise in period 1, before period 2 donations if relevant. We ask whether the charity gains from such reporting. For simplicity, we abstract from discounting as is realistic if the periods are short. Let $y_i(t)$, $t = 1, 2$, denote agent $i$’s contribution in period $t$. The pure strategy set of agent $i$ consists of: (a) all strategies $y_i(1) \in (0, 1]$ and $y_i(2) = 0$; and (b) all strategies $y_i(1) = 0$ and $y_i(2) = \gamma^2(y_i(1))$, with $\gamma^2(\cdot)$ any function from $[0, 1] \to [0, 1]$. We require subgame perfection in equilibrium. This game corresponds to the action-commitment game of Hamilton and Slutsky (1990). Proposition 3.2 reports properties of the equilibrium set.

**Proposition 3.2** (Hamilton and Slutsky) In the endogenous-timing contribution game, exactly three subgame-perfect Nash equilibria exist: an equilibrium where both donors contribute the one-period, simultaneous-move equilibrium amounts in period 1; and both Stackelberg
equilibria, with the leader contributing the one-period leader's amount in the first period and the follower contributing the one-period follower's amount in the second period.\textsuperscript{16}

The arguments can be easily summarized.\textsuperscript{17} If donor $i$ expects $j$ to make his Stackelberg-leadership contribution in the first period, then $i$ can do no better than to wait and respond accordingly in the second period. In turn, $j$'s leadership strategy is optimal given $i$ will wait to contribute. If both donors expect the other to make their one-period, simultaneous-move equilibrium contribution in the first period, then each can do no better than to contribute in the first period. That is, there is nothing to be gained by waiting, given the equilibrium expectation. Both donors waiting to contribute is not an equilibrium because either donor would be better off first playing his Stackelberg leadership contribution in the first period given the expectation that the other donor will wait.

The implications for the desirability of announcement are somewhat obscured by the presence of the multiplicity of equilibria.\textsuperscript{18} Suppose that both Stackelberg equilibria yield greater total contributions than the simultaneous-move equilibrium as with warm-glow players and upward-sloping best-reply functions. If the agents can coordinate on an equilibrium under announcements, then the charity has nothing to lose from announcing and gains if the resulting equilibrium is one of the Stackelberg equilibria. Note, too, that a failure of the agents to coordinate on an equilibrium would never lead to a lower total in this case. That is, in any case where the agents make first-period choices consistent with alternative equilibria, the total will

\textsuperscript{16} Van Damme and Hurkens (1996) show that the "action-commitment game" we are studying has no mixed-strategy equilibria unless no player prefers his Stackelberg leadership payoff to the payoff in a mixed-strategy equilibrium of the one-period game. Under our assumptions, no mixed-strategy equilibria exist in the one-period game, so no mixed-strategy equilibria exist in the endogenous-timing game.

\textsuperscript{17} For proof, see Hamilton and Slutsky (1990, theorem VII).

\textsuperscript{18} Van Damme and Hurkens (1996) show that further refinements of equilibrium fail to eliminate any of the equilibria so long as either player strictly prefers leadership to simultaneous moves, which will be so except in contrived cases of our problem.
never be below the simultaneous-move total. If either plays their Stackelberg leadership amount in the first period, then the total will be higher. If they both wait attempting to be followers, then they will end up playing the simultaneous-move total in the second period.

A charity may also be able to facilitate sequential play in an endogenous-timing environment. If contributors attempt to give simultaneously, the charity acts to delay one contribution. If the phones in a telethon ring simultaneously, one contribution is taken first, while the phone operator stalls the contribution of the other, of course reporting the first contribution when completed. If hands of contributors are raised simultaneously at a live fund raiser, then the M.C., perhaps ceremoniously, flips a coin to see who gets to contribute first.

To analyze this, modify the above endogenous-timing game so that, if both attempt to contribute in the same period, then agent i is selected with probability \( \frac{1}{2} \) to contribute first, i's contribution is announced, and then j is able to contribute. Selecting the leader with probability \( \frac{1}{2} \) is sensible if the charity cannot distinguish the agents (but knows the distribution of types). The agents understand the process. A player's strategies must specify all feasible choices taking account of the possibilities of Stackelberg subgames if both attempt to contribute simultaneously. The subset of player i's strategies where player i attempts to contribute in the first period specifies: a contribution \( y_i \in (0,1] \) if j waits; a contribution from the same set if j also attempts to contribute in the first period and i is selected to lead; and a response function \( y_j(y_i) \) (with obvious restrictions) if j is selected to lead. The subset of strategies for player i if he waits until the second period to contribute is analogously augmented for the possibility that j also waits. For this modified endogenous timing contribution game, we have:

**Proposition 3.3**

A) There exists a subgame-perfect Nash equilibrium which is unique with both players contributing in the first (second) period and each one-period, Stackelberg-equilibrium outcome
arising with probability \( \frac{1}{2} \), if both players prefer leading (following) in the one-period Stackelberg game.

B) Subgame-perfect Nash equilibrium is unique with player i contributing his one-period, Stackelberg-leadership amount in period 1 and player j contributing his one-period, Stackelberg-follower amount in the second period, if player i (j) prefers leading (following) in the one-period Stackelberg game.

These results are easy to confirm. The extensive form of the game and subgame perfection imply that the outcome must be equivalent to one of the two Stackelberg equilibria. Then, for example, in the case of mutual preference for leading in part A, by contributing in the first period with probability one, a player maximizes his probability of the Stackelberg-leadership outcome for any strategy of the other agent that satisfies subgame perfection. Similar arguments establish the other possibilities. Note that any of these cases are possible as illustrated in Figures 3.1-3.3. Moreover, a higher total can result in all relevant Stackelberg equilibria in any of these cases. It is also possible that when both Stackelberg outcomes are equilibrium possibilities, one has a higher total and the other a lower total than the simultaneous-move game. This could occur, for example, in a case with asymmetric agents. If the informational structure is such that a charity can do no better than select the leader with probability \( \frac{1}{2} \) when both attempt to contribute simultaneously (as we have assumed), then we must examine the specifics, including the charity's payoff function, to determine what is best for the charity.

A final variation of endogenous timing we analyze permits agents to contribute more than once. More specifically, suppose agent i selects a non-negative contribution \( y^t_i, t = 1,2 \), in each of two periods (again with no discounting). Agent i's aggregate contribution continues to be denoted by \( y_i = y^1_i + y^2_i \), and the agent totals \((y_1, y_2)\) continue to enter the utility functions. With
no announcements after first-period contributions, it is easy to see that equilibrium has
\[ y_i = y_i(G_0). \]

If the charity announces first-period contributions, then agent i’s strategy set consists of:
\[ y_i^1 \in [0,1] \text{ and any function } y_i^2(y_i^1, y_j^1) \text{ from } [0,1] \times [0,1] \rightarrow [0,1] - y_i^1 \].

The sets of subgame perfect equilibria for the three cases depicted in Figures 3.1-3.4 are reported in proposition 3.4.
We say that equilibrium is equivalent to a Stackelberg equilibrium if both agents’ aggregate contributions conform to the Stackelberg levels, and likewise about the simultaneous-move equilibrium.

**PROPOSITION 3.4**

A. If both agents have upward sloping best-reply functions and utilities that increase in the other agent’s contribution (e.g., see Figure 3.2), then the equilibrium set consists of one or the two Stackelberg equivalents.

B. If agent i has an upward sloping best-reply function, agent j has a downward sloping best reply function, and both agents' utilities increase in the others' contribution (e.g., see Figure 3.1), then equilibrium must be equivalent to the Stackelberg equilibrium in which agent i leads (e.g., G2 in Figure 3.1).

C. If both agents have downward sloping best-reply functions with utilities that increase in the other agent’s contribution, then equilibrium must be equivalent to the simultaneous-move equilibrium.

D. (Saloner) If both agents have downward sloping best-reply functions with utilities that decrease in the other’s contribution (e.g., see Figure 3.3), then the set of equilibria consist of all contribution pairs on the outer envelope of the best-reply functions between and including the Stackelberg equilibria.

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19 Any \( y_i^1 \leq y_i(G_0) \) and \( y_i^2 = y_i(G_0) - y_i^1 \) is an equilibrium strategy for agent i. This multiplicity is irrelevant to aggregate contributions and to payoffs.
The proof and more detail about the equilibrium strategies is provided in Appendix B. While the results are somewhat different than in the endogenous-timing game where agents can contribute only once, the main point that announcement leads to equilibria with a greater total holds in important cases. In case A of Proposition 4.4 that we have been emphasizing, for example, both equilibria have a greater total. In fact the only difference in this case is that the set of equilibria now does not include the simultaneous-move equivalent. Equilibrium has unique totals in Cases B and C of Proposition 4, in the former case with a greater total than with no announcement, and with the same total in the latter case. We provide an example of Case D in the next section, which is analytically equivalent to a standard Nash-Cournot output game. Saloner (1987) has analyzed the two-period output game, and has shown the continuum of equilibria described in Proposition 4D. We have restricted attention here to these four cases to save space and because we feel they are adequate to convey the results.20

3.5 Two Examples

The first example has symmetric warm-glow agents with upward sloping best-reply functions. Let there be two agents, i = 1,2, with utility functions:

\[ U^i(x_i, Y, y_i) = x_i^\alpha + (1 + k y_i) Y + y_i^\alpha, \]  

with \( 0 < \alpha < 1 \) and \( k > 0 \).

Further assume that agent i is endowed with an income, \( I_i \), such that \( I_i \leq \left( \frac{\alpha(1-\alpha)}{2k} \right)^{1/2} \). It is straightforward to establish that \( U^i(I_i-y_iy_i+y_iy_i) \) is strictly quasi-concave in \((y_i, y_i)\), and the best-reply functions \( f(y_i) \) are increasing, concave, and satisfy \( f(0) > 0 \). Hence, equilibria are interior and unique, and agents contribute more individually and in total in the Stackelberg equilibria.

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20 Ten distinct cases can be identified as delineated by the slopes of best-reply functions and how utility depends on the other agent’s contribution. The techniques employed in Appendix B can be used to analyze each case.

21 This assumption is made to meet the normality assumptions on demands.
The key here is that $U_{23}^i$ is sufficiently positive.\footnote{As we indicated in the corollary, the warm-glow model does not necessarily guarantee that sequential contributions yields more of the public good than does simultaneous contributions. For example, suppose agent i has the above utility function whereas agent j has utility function $U_j^i(x_j,Y,y_j) = \log x_j + \log Y + \log y_j$. Agent j then has a best-reply function with slope between -1 and 0, and the sequential equilibrium where agent i moves first yields less of the public good than would simultaneous contributions.} The second example posits utility functions for the two agents of the form:

$$U'(x_i, Y, h'(y_i, y_j)) = x_i Y \frac{1}{y_i}$$

This specification embodies a snob effect or reputation motive. Holding $Y$ constant, i's utility declines as j's contribution rises, associated with the implied decline in i's relative contribution. Best-reply functions are given by:

$$f'(y_i) = \max\{\frac{1 - y_j}{2}, 0\}$$

as depicted in Figure 3.3. In Figure 3.3, we assume $I_1 > I_2$, but that the incomes are close, which leads to somewhat different Stackelberg outcomes.

Without announcement, i.e., in the simultaneous-move equilibrium, it is interesting that the outcome is precisely as if the agents had standard Cobb-Douglas preferences: $U^i = x_i Y$. By failing to announce, the charity would not allow the snob effect to come into play.

The sequential equilibria are depicted in Figure 3.3, and the total is higher in each case than in the simultaneous-move equilibrium. Relative to the simultaneous-move choices, the incentive of the leader is to increase his contribution to induce a reduction in the other agent's contribution. Because the follower is sluggish to reduce her contribution, the total rises. If the lower-income agent leads, then the Stackelberg equilibrium is interior ($G_3$). If the higher-income agent leads, then the Stackelberg equilibrium is at the "corner" where the follower just ceases to
In both cases, there is here a preference for leading over following, as leading permits one to win the prestige game. The increase in total contributions in the Stackelberg games is in stark contrast to the contribution game with standard utility function (depending only on the total contribution). In the standard case, the leader prefers higher contribution by the follower (the indifference curves open away from a player's contribution axis), and the leader reduces his own contribution to get the follower to contribute more.

3.6 Conclusion

Charities frequently announce donations as they accrue. In this paper, we view these announcements as a means of inducing a sequential game among donors as an alternative to having them contribute simultaneously. Such a rationalization for announcement by charities fails if agents have the standard utility function since then contributions would decline. We have shown that with more general utility functions including the warm-glow one, however, it is possible that a charity increases total contributions by announcing the magnitudes of realized contributions. This provides another argument in favor of more general utility functions when analyzing voluntary contributions. Moreover, it supports the idea that adopting a simultaneous-move assumption in such analyses should not be taken for granted. This paper also points to the proactive role that a charity might play in collecting donations which is increasingly being recognized in the literature.

Perhaps the strongest assumption maintained in our analysis is complete information about preferences among donors. Investigating the incentives to announce contributions when all parties lack information about donor preferences is a topic for more study. This research also

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23 This example does not conform perfectly to all the assumptions we made in the general analysis (e.g., \( U'(I_i-y_i,y_i+y_j,h(y_i,y_j)) \) is not everywhere quasi-concave). But it is well behaved enough that the results apply. If one is bothered by the infinite utility that agent 1 obtains in \( G_1 \), then modify utility to have \( y_j + k \) in the denominator of the middle expression of (10). For \( k \) small, the results are virtually the same. We have not done this in the text just to keep the analysis simpler.
further motivates empirical efforts to understand what drives donors to contribute. We have shown that the linkage between donor preferences and the game form that maximizes contributions admits a variety of possibilities. We know, for example, that if agents have warm-glow utility functions and upward-sloping best-reply functions, then Stackelberg equilibria produce higher contributions than in the simultaneous-move equilibrium. But it is not enough to know only that best-reply functions are upward sloping. Donors could have upward-sloping best-reply functions due purely to competition for prestige, e.g., if \( U' = x(k+y, -y) \). Then the Stackelberg equilibria are associated with a lower total. Hence, understanding well the form of the utility function can be important to making basic predictions about the nature of contribution equilibria.
CHAPTER 4
TWO-AGENT MULTI-PERIOD GAMES OF ACCUMULATION

4.1 Introduction

In many interesting settings, agents repeatedly interact before their final payoffs accrue. Leading examples include private contributions of donors to a public good, rent-seeking activities of lobbies to influence a policy decision, policy choices of political candidates before the election, and quantity competition of duopolists before the market clears. The main features of such game settings are twofold: (i) Before payoffs are received, agents have multiple opportunities to respond to their opponents, and (ii) agents can only choose to accumulate their strategy variables over time. For instance, in the rent seeking game, opposing lobbies exert efforts to influence the policy decision in favor of their own interests. Since this is a time-consuming process and lobbies may observe the opponents' efforts before the final decision, they may choose to put more effort later. The possibility of responding later will certainly introduce a dynamic perspective to the lobbying game, and thus may affect the total efforts exerted in equilibrium. However, most studies of such games alluded above have been centered on the assumption that agents make their choices once and usually simultaneously. Our objective in this paper is to examine the consequences of multiple response possibility to the equilibrium outcomes and agents' payoffs.

A brief preview of our main findings and the organization of the paper are as follows. We present the model in section 2. In the model, there are two agents whose preferences and strategy spaces are common knowledge. Agents have a finite number of periods (T) until final payoffs are received. In each period, they observe the history of the game, and then simultaneously choose whether or not to increase their previous strategy variables.
In section 3, we fully characterize the set of equilibrium outcomes when \( T = 2 \). The noteworthy feature of this characterization is that it describes the set in terms of agents' reaction functions, the one-shot Cournot, and the Stackelberg outcomes in the textbook sense. Thus, it gives a convenient recipe to see the equilibrium outcomes in different scenarios. Next we investigate the necessary and sufficient conditions under which the Cournot equilibrium is the unique outcome of our dynamic setting. The condition requires that each agent’s Stackelberg leader choice be less than his corresponding Cournot amount. This finding certainly has intuitive appeal. For instance, in the standard model of private contributions to a public good, agents like to free ride on others. Thus, each agent’s Stackelberg leader amount is less than his Cournot amount. However, if agents were to contribute again, then the leader would find her contribution too little, and contribute more later. Interestingly though, this intuition holds for other possible outcomes thereby ruling out all but Cournot equilibrium. We also find that considering an arbitrary but finite number of periods has absolutely no effect on the equilibrium outcomes. That is, an equilibrium outcome for an arbitrary \( T \) can be engendered as one for \( T = 2 \), and vice versa. This is because in most settings actions occur only in the first period, and therefore adding one more period would not change this behavior. This simple yet startling result implies for example that the equilibrium total contributions in the public good game, or total efforts exerted in the rent seeking game would be the same even if agents could have more opportunities to respond to each other.

Settings may arise where agents can only reduce their first period choices over time. For instance, duopolists producing differentiated products and competing in price may find themselves bounded to the first period prices, and may only cut back later to keep customers’ goodwill, or political candidates announcing tax rates may prefer reducing the rate rather than increasing later to keep voters’ goodwill. It turns out that our techniques for the accumulation case goes through for this decumulation case and results change with slight modifications as we
note in section 4. Finally, section 5 draws some conclusions and provides future research avenues.

Before proceeding, it is useful to relate and distinguish our paper from previous work. In Romano and Yildirim (forthcoming), using specific utility functions we consider a two-period contribution game to a public good to see the role of announcements in fund-raising activities. Here we provide more general results for a two-period accumulation setting, extend to more than two periods, and also consider the decumulation possibility. Hamilton and Slutsky (1990, 1993), and Van Damme and Hurkens (1996), among others, analyze the endogenous timing games where firms choose not only how much to produce but also when to produce. Our analysis extends and complements this literature by considering multiple production possibilities for each firm as opposed to once.\(^1\) Admati and Perry (1991) investigate the conditions under which a socially desirable joint project can be completed when parties take turns to make contributions towards the completion of the project. Our model can be viewed in a similar context except now parties can contribute in every period and the public good is continuous instead of being a discrete one.\(^2\)

4.2. The Model

There are two agents who have a finite number (T) of periods before their payoffs accrue. Agents have bounded action spaces \(F_i = [0, I_i] \) for \( i = 1, 2 \). In period \( t \in \{1, ..., T\} \), they simultaneously choose \( y_i^t \in [0, I_i - Y_i^{t-1}] \) upon observing the history of the game, where we assume \( y_i^0 = 0 \) and define the cumulative outcome \( Y_i^t \) at the end of period \( t \) as:

---

1 Saloner (1987) examines a two-period version of our model for homogenous good duopolists. We will link some of his results to ours below.

2 Varian (1994) also considers sequential contributions to a continuous public good where agents contribute only once and the Stackelberg leader has commitment power. Our model complements his by highlighting the difficulty of this commitment.
\[ Y_i^n = \sum_{n=1}^{i} Y_i^n \]  

(4.1)

To lighten notation, we use \( Y_i = Y_i^T \) for the final outcomes.

Agents enjoy the payoffs at the end of period T, and thus have the following utility functions:

\[ U^i = U^i(Y_i, Y_j), i, j = 1, 2, i \neq j \]  

(4.2)

where \( U^i() \) is continuous, twice differentiable, and strictly quasi-concave in \((Y_i, Y_j)\). Both utility functions and the action spaces are common knowledge, and Subgame Perfect Nash Equilibrium is our equilibrium concept in the ensuing analysis.

Our analytical technique is based on the reaction functions, the one-shot Cournot-Nash, and the Stackelberg outcomes in the textbook sense. Thus, we define the reaction function of agent \( i \) as:

\[ f^i(Y_j) = \arg\max_{Y_i} U^i(Y_i, Y_j) \]  

(4.3)

The above assumptions on \( U^i() \) guarantee that \( f^i() \) is indeed a function. We focus here on the sufficiently well-behaved games, and thus also assume that the reaction functions are monotonic. We note however that once the monotonic case is understood, it is easy to modify our results as we demonstrate in the applications below.

The one-shot Cournot-Nash equilibrium \((Y_i(G_0), Y_j(G_0))\) occurs at the intersection of the reaction functions. We assume that it is unique, and interior. Also, the Stackelberg equilibrium \((Y_i(G_i), Y_i(G_i))\) with \( i \) as the leader occurs where

\[ Y_i(G_i) = \arg\max_{Y_i} U^i(Y_i, f^j(Y_i)), \text{ and } Y_j(G_i) = f^j(Y_i(G_i)) \]  

(4.4)

To insure the uniqueness of the Stackelberg outcome, we assume \( U^i(Y_i, f^j(Y_i)) \) is strictly quasi-concave in \( Y_i \). This assumption also implies that the Stackelberg leader’s payoff increases
monotonically as we move along the follower's reaction function towards the Stackelberg equilibrium.

Before proceeding to the main analysis, we record the following preliminary finding which compares the Cournot-Nash and Stackelberg outcomes.

**PROPOSITION 4.1**: In equilibrium, agent i's Stackelberg leader amount is greater (less) than his Cournot-Nash amount if and only if \( \left( \frac{dU_i}{dY_i} \frac{\partial f_i}{\partial Y_i} \right) \) is positive (negative).

Proof: All proofs are contained in the appendices. Q.E.D.

### 4.3 The Accumulation Game

In this section, we fully characterize the set of equilibrium outcomes when agents can only accumulate their strategy variables over time. To do so, we first start with the simpler setting where \( T = 2 \), and then consider the case with an arbitrary but finite \( T \) to compare the resulting outcomes.

#### 4.3.1 \( T = 2 \) Case

We assume here that agents have only two periods until the payoffs are received and define the following sets.

\[
S_1 = \{(Y_i, Y_j) \in F_i \times F_j \text{ such that } Y_i \geq f'(Y_i)\} \\
S_2 = \{(Y_i, Y_j) \in F_i \times F_j \text{ such that } Y_i = f'(Y_i) \text{ for at least one agent}\} \\
S_3 = \{(Y_i, Y_j) \in F_i \times F_j \text{ such that } Y_i \leq Y_i(G_i) \text{ whenever } Y_i \neq f'(Y_i)\} \\
S_4 = \{(Y_i, Y_j) \in F_i \times F_j \text{ such that if } Y_i = f'(Y_i), \text{ then either } Y_j \geq Y_j(G_i) \text{ or } Y_i \geq Y_i(G_i)\}
\]

We also define \( S = S_1 \cap S_2 \cap S_3 \cap S_4 \). Before proceeding, we record the following second period equilibrium strategies that hold for any setting.
LEMMA 4.1: The following strategies constitute an equilibrium in the second period. For \( i, j = 1,2, \) and \( i \neq j \),

\[
y^2_i(y^i_j, y^j_j) = \begin{cases} 
0, & \text{if } y^i_i \geq f^i(y^i_j) \text{ and } y^j_j \geq f^j(y^j_i) \\
Y_i(G_0) - y^i_i, & \text{if } y^i_i \leq Y_i(G_0) \text{ and } y^j_j \leq Y_j(G_0) \\
0, & \text{if } y^i_i \geq Y_i(G_0) \text{ and } y^j_j \leq f^j(y^j_i) \\
f^i(y^j_j) - y^i_i, & \text{if } y^j_j \geq Y_j(G_0) \text{ and } y^j_j \leq f^j(y^j_i) 
\end{cases}
\]

These strategies have been originally exploited by Saloner (1987) when he analyzes homogenous good Cournot duopolists with two production periods. Interestingly though, they are still equilibrium strategies for other applications regardless of reaction functions being upward or downward sloping. Together with Lemma 1, the following result provides a convenient recipe for equilibrium outcomes in any setting.

PROPOSITION 4.2: For \( T = 2 \), a \((Y_0, Y_1)\) pair in the feasible set is an equilibrium outcome if and only if it is in \( S \).

This finding fully characterizes the set of equilibrium outcomes for different applications in a unified manner and reveals that the conditions given in sets \( S_i(i = 1, \ldots, 4) \) above are indeed necessary and sufficient for a feasible pair to be an equilibrium. It should be clear that these conditions are based only on the usual definitions of reaction functions, the one-shot Cournot-Nash, and the Stackelberg outcomes and thus easy to apply. Since the proof is by construction, one can also follow the first period strategies together with second period strategies in Lemma 1 supporting any particular outcome in specific applications. In general, as the game carries over the characteristics of both the standard Cournot and Stackelberg models, the outcomes will be "mixtures" of the outcomes of those in the Cournot and Stackelberg models. Even so, it is easy to observe from the proof of Proposition 1 that all actions occur in the first period for all equilibrium outcomes possibly including the Stackelberg ones unless Stackelberg outcomes are the only ones in a setting. This is because in such cases second period strategies provide sufficient threat for
both agents and the intuition will be clearer when we discuss different application throughout. As a result, observing agents taking actions early on and doing nothing later should not mean that they play the one-shot Cournot game. In fact, the following finding records the exact conditions under which the Cournot-Nash outcome is the unique equilibrium of our dynamic setting.

**Proposition 4.3:** For \( T = 2 \), the one-shot Cournot-Nash outcome, \((Y_i(G_0), Y_j(G_0))\), is the unique equilibrium outcome if and only if each agent's Stackelberg leader amount is less than his corresponding Cournot-Nash amount, i.e., \( Y_i(G_i) < Y_i(G_0) \) for \( i = 1, 2 \).

This result certainly has intuitive appeal. Note first that Proposition 1 above lists which settings result in \( Y_i(G_i) < Y_i(G_0) \) for agent \( i \), and thus yield \((Y_i(G_0), Y_i(G_0))\) as the unique outcome. Consider for example the standard model of private contributions to a public good\(^3\), where agents have the following utility functions:

\[
U^i = U^i(x_i, Y_i + Y_j) \tag{4.5}
\]

Here \( x_i \) represents \( i \)'s private good consumption. Under mild assumptions, this model implies that agents have downward sloping reaction functions. Since each agent likes the other's contribution, i.e., \( \frac{dU^i}{dY_j} > 0 \), Proposition 1 reveals that \( Y_i(G_i) < Y_i(G_0) \). That is, agents like to free ride on others whenever they get the chance. Suppose indeed that agent \( i \) commit to his Stackelberg leader amount, \( Y_i(G_i) \), in the first period and contributing nothing later. Then the best agent \( j \) can do is to choose his corresponding follower amount. However, in such a case agent \( i \) would find his contribution too little and have an incentive to give more later, which eliminates the Stackelberg outcome as an equilibrium. Note though that this argument rules out only the Stackelberg outcome. There may be other candidates. However, if there were another equilibrium such that agent \( i \) were not on his reaction function, then from Theorem 1, we would have

\(^3\) See Bergstrom, Blume, and Varian (1986).
\( Y_i \leq Y_i(G_i) \) for the possible equilibrium. Since agent i already finds \( Y_i(G_i) \) too little and cannot commit to this amount, he would not certainly be able to do so for any other amount lower. Thus, both agents must be on their reaction functions in equilibrium implying the Cournot-Nash outcome as the unique one in our dynamic setting. Besides providing partial insight to Proposition 3, this argument also validates the simultaneous-move assumption made in most analyses of the above model to give a sharp prediction of equilibrium even when the actual play has a dynamic element.

In the same spirit, one can imagine other interesting cases that satisfy Proposition 2\textsuperscript{4}. For instance, if agents, say two businessmen, are to contribute to a public good, and concerned mainly about giving more than others to gain, perhaps, their customers’ goodwill, then we can postulate the following utility functions:

\[
U^i = x_i(k + Y_i - Y_j)
\]

where \( k \) is just a positive constant. In such a case, it is easy to verify that each agent has an upward sloping reaction function, and dislikes the other’s contribution, i.e., \( \frac{dU^i}{dY_j} < 0 \). Proposition 1 then implies that \( Y_i(G_i) < Y_i(G_0) \) for agent i. That is, if agent i were to be the Stackelberg leader and commit to his contribution, then he would contribute less than the corresponding Cournot-Nash amount to soften the “competition” in the contribution game. However, he would find his Stackelberg leader amount too low and certainly have an incentive to give more in the second period. Similar intuition goes through for the other possible outcomes, yielding the Cournot-Nash outcome.

Although two-period setting seems to provide most of the intuition in our dynamic setting, in reality agents may have more opportunities to respond to each other before the final

\textsuperscript{4} See Romano and Yildirim (forthcoming) for different examples in public good game context.
payoffs are received. To analyze this, we relax the assumption $T = 2$ and let $T$ be an arbitrary but finite number in the following subsection.

4.3.2 $T > 2$ Case

Suppose agents now have $T (>2)$ periods to reach the final outcome. In such a case, we are interested in knowing the set of equilibrium outcomes, and more importantly if it is any different from that of when $T = 2$. That is, does allowing agents to have more opportunities to respond matter in equilibrium at all? The following finding answers this question.

**PROPOSITION 4.4:** In the accumulation game,

a) For $T > 2$, a $(Y_b, Y_f)$ pair in the feasible set is an equilibrium outcome if and only if it is in $S$.

b) A $(Y_b, Y_f)$ pair is an equilibrium outcome for $T > 2$ if and only if it is an equilibrium outcome for $T = 2$.

This is a striking result. Part (a) fully characterizes the equilibrium set and reveals that it coincides with the one for $T = 2$ setting. Furthermore, any equilibrium outcome for $T > 2$ can be engendered as one for $T = 2$, and vice versa as recorded in Part (b). Thus, extending periods has absolutely no effect on the equilibrium outcomes. Consider for example a $(Y_b, Y_f)$ equilibrium pair for $T = 2$ where $y_b^2 = y_f^2 = 0$. Then, an opportunity to play once again would clearly be irrelevant in equilibrium. This is the case for all equilibrium pairs except when the Stackelberg outcomes are the only ones in a setting. However, in such a setting, once the Stackelberg leader commits to his leader amount in the first period, whether the follower reaches the corresponding follower amount at once, or distributes over periods would not affect the final outcome.

This simple result has strong implications in different applications. For instance, the equilibrium total contributions in the public good game, or total efforts exerted in the rent seeking game would not change if the agents had more than two opportunities to revise their decisions. Also, Saloner’s (1987) Cournot duopolists would come up with the same equilibrium production
if they had more chances to respond before the market clears. In what follows, we examine the equilibrium outcomes of these games in details.

4.3.3 Applications

Here we provide interesting applications to demonstrate our findings.

Application 1: Differentiated Product Duopolists

We consider the following model proposed first by Dixit (1979) and also analyzed by Singh and Vives (1984).

Suppose there are two firms in a sector where each firm produces a differentiated product. Firm i has a demand function of the form:

\[ p_i = \alpha_i - \beta_i q_i - \gamma q_j, \quad i, j = 1, 2, \text{ and } i \neq j \]  \hspace{1cm} (4.7)

where \( \alpha_i, \beta_i, (\beta_i \beta_2 - \gamma^2) \), and \((\alpha_i \beta_j - \alpha_j \gamma)\) are all assumed to be positive. Products are substitutes (complements) if \( \gamma \) is positive (negative). Firm i has a constant marginal cost, \( m_i \), with no fixed costs or capacity constraint, and thus its profit functions is given by \( \Pi_i = (p_i - m_i)q_i \).

Firms engage in quantity competition. It is easy to verify that i’s reaction function is

\[ f^i(q_j) = \frac{\alpha_i - m_i}{2\beta_i} - \frac{\gamma}{2\beta_i} q_j \]  \hspace{1cm} (4.8)

where \( \alpha_i - m_i > 0 \). This model yields a unique one-shot Cournot-Nash, \((q_i(G_0), q_j(G_0))\), and unique Stackelberg outcomes, \((q_i(G_i), q_j(G_j))\) where i leads, satisfying our assumptions.

Consider first the case where products are substitutes, i.e., \( \gamma > 0 \), and assume that production is time-consuming. There are two periods before the market clears. In the first period, firms simultaneously choose their first period outputs. The chosen outputs become common knowledge, and then, in the second period, the firms simultaneously decide to add, if any, to their previous production. This case coincides with Saloner’s (1987) model and a special case of our
accumulation game with $T = 2$. Applying Proposition 2, the set of equilibrium outcomes is

$$S = \{(q_i, q_j) \text{ such that } q_i = f'(q_i) \text{ and } q_i(G_s) \leq q_i \leq q_i(G_t), i, j = 1, 2, i \neq j\}.$$ 

Thus, $S$ includes a continuum of outcomes including the one-shot Cournot-Nash, and Stackelberg outcomes. As Saloner also points out, all equilibrium production occurs in the first period even for the Stackelberg outcomes.

Now consider the case where products are complements, i.e., $\gamma < 0$. Then the equilibrium set contains only the two Stackelberg outcomes. Proposition 4 above extends these findings and reveals that the equilibrium sets would remain unchanged even if firms interact more than twice before the market clears.

Application 2: A Rent-seeking Model

Consider the following stylized rent-seeking model developed first by Tullock (1980).

There are two risk-neutral parties trying to influence the decision of a policy-maker in favor of their own interests. Examples of such decisions include awarding monopoly rights, government contracts, or favorable legislation. Rent-seeking activities might take the form of political lobbying, bribes, or campaign contributions to political candidates. Party $i$ attaches a positive value of $v_i$ if the final decision is in its favor, and zero otherwise. The likelihood of $i$ winning however is given by

$$P_i(x_1, x_2) = \frac{x_i}{x_1 + x_2}, \quad \text{and } P_i(0, 0) = 1/2 \quad (4.9)$$

where $x_i \geq 0$ and represents $i$'s rent-seeking expenditures, or effort. Thus party $i$ has a payoff function:

---

5 Basu (1992) demonstrates the collusive behavior of Cournot duopolists in a finitely repeated game where the stage game is Saloner's two period setting. Our finding reinforces this result by implying that it would remain unchanged even if the stage game included more than two periods.

6 See Buchanan (1980) for an introduction to this problem.
\[ U^i(x_1, x_2) = v_i \frac{x_i}{x_1 + x_2} - x_i \]  
\hspace{2cm} (4.10)

From here, \( i \)'s reaction function can be found as:\(^7\)

\[ f^i(x_j) = \begin{cases} (v_i x_j)^{1/2} - x_j, & \text{if } x_j \in (0, v_i] \\ 0, & \text{if } x_j > v_i \end{cases} \]  
\hspace{2cm} (4.11)

Although most analyses of this model have used the simultaneous-move assumption and focused on the Cournot-Nash equilibrium, Linster (1993) examines the Stackelberg rent-seeking extension and compares the resulting outcomes. Here we borrow from Linster's findings.

We first note that \( f(x_j) \) is increasing for \( x_j \in (0, v_i/4) \) and decreasing for \( x_j \in (v_i/4, v_i) \), thus admitting its maximum when \( x_j = v_i/4 \). This means that \( f(\cdot) \) is non-monotonic globally and violates our monotonicity assumption. Even so, we will argue below that our results hold. The Cournot-Nash and Stackelberg equilibriums all exist and unique in the model and are as follows:

\[ x_i(G_0) = \frac{v_i^2 v_j}{(v_1 + v_2)^2} \]  
\hspace{2cm} (4.12)

\[ x_i(G_1) = \frac{v_i^2}{4v_j}, \quad \text{and} \quad x_j(G_1) = \frac{v_i}{2} - \frac{v_i^2}{4v_j} \]  
\hspace{2cm} (4.13)

When \( v_1 = v_2 \), Cournot-Nash and Stackelberg equilibriums coincide. Thus we consider the more realistic and interesting case where \( v_1 \neq v_2 \), say \( v_1 > v_2 \). Although it would not harm our analysis, we avoid corner Stackelberg outcomes by assuming \( v_2 > v_i/2 \). Before proceeding, we note the following ordering:

\[ v_2/4 < x_1(G_0) < x_1(G_1) \text{ and } x_2(G_2) < x_2(G_0) < v_i/4. \]  
\hspace{2cm} (4.14)

\(^7\) \( f(0) \) is not defined for obvious reasons.
To motivate our analysis, we note that rent-seeking might be a time-consuming process. Until the final decision is made, once parties put efforts, they may still have time to respond and further their efforts upon observing the rival’s. However, even the Stackelberg analysis will fail to capture this multiple response possibility. Thus, our following analysis complements the previous studies on the subject.

Consider the accumulation game for $T = 2$ within the rent-seeking context. Despite the reaction functions are non-monotonic violating our assumption, it is not hard to see that the second period strategies in Lemma 1 will still hold here as well, and so will Proposition 2 and 4 above. Thus, the equilibrium set is

$$S = \{(x_1, x_2) \text{ is such that } x_2 = f^2(x_1) \text{ and } x_1(G_0) \leq x_1 \leq x_1(G_1)\}$$

First, note that the Stackelberg outcome where party 2 with the lower valuation leads cannot be sustained as an equilibrium. This is because even when party 2 commits to its leader amount, and indeed party 1 follows, in the second round party 2 would find its effort too low and have an incentive to increase. Second, in equilibrium, since $x_2^* = f^2(x_1^*)$, total rent-seeking activities $x_1^* + x_2^* = (v_2^* x_1^*)^{1/2}$, which is increasing in $x_1^*$. In addition, since, in equilibrium, $x_1(G_0) \leq x_1^* \leq x_1(G_1)$, the Cournot-Nash, and Stackelberg outcome where high valuation party leads yield a lower and an upper bound for the total equilibrium activities, respectively. How tight the interval is depends on how close parties’ valuations are.

Finally, these conclusions would remain intact even if the parties’ had more chances to respond than twice as Theorem 2 implies.

**4.4 The Decumulation Game**

In some settings, agents’ first period decisions may bind them in future periods and “decumulating” later may be the only strategic option. For example, two competing political candidates who are to announce tax rates may find themselves bound to the first period choices.
Thus, if they were to respond upon observing the opponent’s decision, they would perhaps revise their first period choices and announce only tax cuts to keep voters’ goodwill. Yet another conceivable example is that two differentiated good duopolists competing in prices may be quite reluctant to raise their prices if they are to revise their first period choices later. It turns out however that these decumulation cases can be readily analyzed within our framework with slight modifications. Formally now agent $i$ chooses $y^1_i \in [0,1]$ and $y^1_i \in [-Y^{t-1}_i,0]$ for $t = 2, \ldots, T$. All other assumptions remain unchanged.

Similar to the previous section, we first define the following sets:

- $S'_1 = \{(Y_i, Y_j) \in F_i \times F_j \text{ such that } Y_i \leq f^i(Y_j)\}$

- $S'_2 = \{(Y_i, Y_j) \in F_i \times F_j \text{ such that } Y_i = f^i(Y_j) \text{ for at least one agent}\}$

- $S'_3 = \{(Y_i, Y_j) \in F_i \times F_j \text{ such that } Y_i \geq Y_i(G_i) \text{ whenever } Y_i \neq f^i(Y_j)\}$

- $S'_4 = \{(Y_i, Y_j) \in F_i \times F_j \text{ such that if } Y_i = f^i(Y_j), \text{ then either } Y_j \leq Y_j(G_j) \text{ or } Y_i \leq Y_i(G_i)\}$

We also define $S' = S'_1 \cap S'_2 \cap S'_3 \cap S'_4$ and record the following

**LEMMA 4.2:** For $T = 2$, the following strategies constitute an equilibrium in the second period in the decumulation game. For $i, j = 1,2, \text{ and } i \neq j$,

$$y^2_i(y^1_i, y^1_j) = \begin{cases} 
0, & \text{if } y^1_i \leq f^i(y^j_i) \text{ and } y^1_j \leq f^j(y^j_i) \\
y^j_i(G_0) - y^1_i, & \text{if } y^1_i \geq Y_i(G_0) \text{ and } y^1_j \geq Y_j(G_0) \\
0, & \text{if } y^1_i \leq Y_j(G_0) \text{ and } y^1_j \geq f^j(y^j_i) \\
f^j(y^j_i) - y^1_i, & \text{if } y^1_j \leq Y_j(G_0) \text{ and } y^1_i \geq f^i(y^j_i) 
\end{cases}$$

With these strategies in mind, the following result records the main findings for the decumulation game.
PROPOSITION 4.5: In the decumulation game,

a) For \( T = 2 \), a \((Y_i, Y_j)\) pair in the feasible set is an equilibrium outcome if and only if it is in \( S' \).

b) For \( T = 2 \), the one-shot Cournot-Nash outcome is the unique outcome if and only if each agent's Stackelberg leader amount is greater than his corresponding Cournot-Nash amount.

c) A \((Y_i, Y_j)\) is an equilibrium outcome for \( T > 2 \) if and only if it is an equilibrium outcome for \( T = 2 \).

To provide intuition behind the result and show the ease of applying it, we continue considering the differentiated duopoly model analyzed in section 4.3.3 above. However, this time we assume firms engage in price competition, and write the demand functions by inverting (4.7) as

\[
q_i = a_i - b_i p_i + c p_j, \quad i, j = 1, 2, \text{and } i \neq j \quad (4.15)
\]

where we let \( \delta = \beta_1 \beta_2 - \gamma^2 \), \( a_i = (\alpha_i \beta_i - \alpha_i \gamma) / \delta \), \( b_i = \beta_i / \delta \), and \( c = \gamma / \delta \). Note that \( a_i \) and \( b_i \) are positive due to the assumptions made in section 4.3.3. Firm \( i \)'s profit function continues to be

\[
\Pi^i = (p_i - m_i)q_i. \quad \text{It is easy to verify in this case that firm } i \text{'s reaction function is}
\]

\[
f^i(p_j) = \frac{a_i + b_i m_i}{2b_i} + \frac{c}{2b_i} p_j \quad (4.16)
\]

This model also yields unique Cournot-Nash, i.e. one-shot simultaneous-move, unique Stackelberg equilibriums, satisfying our assumptions. Furthermore, since \( \frac{d\Pi^i}{dp_j} = c(p_i - m_i) \), proposition 4.1 above reveals that firm \( i \)'s equilibrium price is greater when it is the Stackelberg leader than its corresponding Cournot-Nash choice. Interestingly, this is so whether products are substitutes or complements, i.e., independent of the sign of \( c \).

Suppose now that firms have a pre-play stage where before finally selling their products, they simultaneously choose their prices and announce them. These prices are observed by both, and then they have one more chance to revise their decisions before selling. It is conceivable to
assume that they will only make price-cuts in the revision stage to keep their customers’ goodwill. This refers to our decumulation game when $T = 2$. It is easy to follow from Part (a) of Theorem 4.3 above that the firms will end up offering their one-shot Cournot-Nash prices in the market. The uniqueness of this equilibrium is also due to Part (b) of the same result. The intuition is that if there were another equilibrium such that firm $i$ is not on its reaction function, then it would be offering a price greater than or equal to its Stackelberg choice revealed by set $S_3'$. However, in this case firm $i$ would find this price too high and have an incentive to cut back in the second round. Thus, both have to be on their reaction functions in equilibrium, implying the unique Cournot-Nash outcome. It is interesting to note that this holds regardless of products being substitutes or complements. As a result, simultaneous-move analysis of the differentiated duopoly model gives sharp equilibrium predictions even if the actual play involves a pre-play game like ours. In fact, this result is robust in that if the firms had more opportunities to respond in the pre-play stage, the equilibrium outcome would not change at all.

4.5 Conclusion

Our analysis provides a convenient recipe to find the equilibrium outcomes of games the with two distinct features: (1) Agents have multiple opportunities to respond to each other until their payoffs accrue, (2) agents can only choose to accumulate their strategic variables over time. This recipe depends only on agents’ standard reaction functions, one shot Cournot-Nash and Stackelberg outcomes. We find that equilibrium outcomes would not change even if agents had more than two opportunities to respond. We provide conditions under which the outcomes would be the same as in one shot Cournot-Nash game. In the present analysis, we consider the cases where today’s actions are equally costly as tomorrow’s actions, i.e. discount factor is unity for both agents. However, it would be interesting to see the robustness for different discount factors.\(^8\)

\(^8\) Pal (1991) does such a comparative static analysis based on Saloner’s (1987) model.
CHAPTER 5
CONCLUDING REMARKS

This dissertation sheds light on three different issues on regulation under incomplete information, the organizational role of charities and other nonprofit institutions, and the effects of multiple response possibilities on agents’ equilibrium behavior in a dynamic environment.

The first chapter extends the incentive regulation literature by characterizing the optimal regulation of an innovating monopolist with private and changing supply costs. Our principle findings are (1) innovation is encouraged by light handed regulation allowing the monopolist to earn greater information rents while providing greater service, (2) both the monopolist and consumers strictly benefit from cost reducing innovation, (3) innovation occurs in the absence of long term agreements when private information is persistent, and (4) innovation is more rapid in a durable franchise, and the regulator prefers durable franchises for exploiting learning economics.

The second chapter rationalizes a common observation that charities frequently announce donations as they accrue. We view these announcements as a means of inducing a sequential game among donors as an alternative to having them contribute simultaneously. Such a rationalization for announcement by charities fails if agents have the standard utility function since then contributions would decline. We have shown that with more general utility functions including the warm-glow one, however, it is possible that a charity increases total contributions by announcing the magnitudes of realized contributions. This provides another argument in favor of more general utility functions when analyzing voluntary contributions. Moreover, it supports the idea that adopting a simultaneous-move assumption in such analyses should not be taken for
The third chapter provides a convenient recipe to find the equilibrium outcomes of games the with two distinct features: (1) Agents have multiple opportunities to respond to each other until their payoffs accrue, (2) agents can only choose to accumulate their strategic variables over time. This recipe depends only on agents’ standard reaction functions, one shot Cournot-Nash and Stackelberg outcomes. We find that equilibrium outcomes would not change even if agents had more than two opportunities to respond. We provide conditions under which the outcomes would be the same as in one shot Cournot-Nash game. In the present analysis, we consider the cases where today’s actions are equally costly as tomorrow’s actions, i.e. discount factor is unity for both agents. However, it would be interesting to see the robustness of results for different discount factors.
APPENDIX A
PROOFS OF CHAPTER 2

Proofs of Propositions 2.1-2.3:

For convenience we presume Assumption 1 holds, although the proof of Proposition 1 doesn't require this. In the text, we establish that any MPE is characterized by a menu of contracts \{P(c,T), x(c,T)\} that maximize

\[ V(T) = \max \ E_c \{v(x(c,T)) - P(c,T) + \delta[V(T) + \lambda(T + 1; x(c,T), T)\Delta V(T)] \} \]  

subject to:

\[ W(T) = E_c \max_{c'(c)} \{P(c'(c),T) - x(c'(c),T)[d(T) + c] \] 

\[ + \delta[W(T) + \lambda(T + 1; x(c'(c),T), T)\Delta W(T)] \} \]  

\[ = E_c W(T, c) \]  

\[ W(T, c) \geq \delta W(T) \quad \forall c \in [c_L, c_H]. \]  

Without loss of generality, we require \( c'(c) = c \) for all \( c \) by the Revelation Principle. By employing usual arguments for characterizing mechanisms satisfying [IR] and [IC] (see Fudenberg and Tirole (1991, pp.253–268.), we require that

\[ W(T, c) = \delta W(T) + \int_{c_L}^{c_H} x(c', T) \, dc' \]  

(a 1)

\[ x(c,c) \text{ is nonincreasing in } c \]  

(a.2)

Substituting for (a.1) into the expression for \( W(T,c) \) in [IC] establishes part (b) of Proposition 1.

Substituting the expression for \( P(c'(c), T) \) into [R], where \( c'(c) = c \), and integrating by parts one obtains

\[ V(T) = \max_{x(c,T) \geq 0} \frac{1}{1 - \delta} E_c \left\{ v(x(c,T)) - [d(T) + c + \frac{F(c)}{f(c)}]x(c,T) - a x(c,T) \frac{\delta}{\Delta W(T) + \Delta V(T)} \right\} \]  

(a.3)

The maximization in (a.3) requires that part (a) of Proposition 2.1 be satisfied. Further totally differentiating the expression in part (a) of Proposition 2.1 with respect to \( c \) confirms \( x(c,T) \) is strictly decreasing in \( c \) for \( x(c,T) > 0 \) as stated in the Proposition.
To obtain part (c) of Proposition 2.1, substitute from (a.1) for \( W(T, c) \) to obtain

\[
W(T) = E_c \left[ \delta W(T) + \int_c^c x(c', T) \, dc' \right] \quad \Leftrightarrow \quad W(T) = E_c \left[ \frac{x(c, T) F(c)}{1 - \delta} \right] \tag{a.4}
\]

where the second part of (a.4) follows from the first by integrating by parts.

Part (d) of Proposition 2.1 is obtained from (a.3) by integrating by parts and employing part (a) of Proposition 2.1.

The proof of Proposition 1 is completed by showing the value functions \( V(T) \) and \( W(T) \) exist and are unique. The proof is by induction, and in the course of this proof we also establish Propositions 2.2 and 2.3.

For \( T \geq T^H \), \( d(T) \) is constant. Therefore since the instantaneous payoff functions for both players are invariant with \( T \) for \( T \geq T^H \) all states \( T \) are payoff equivalent implying \( V(T) = V(T^H) \) and \( W(T) = W(T^H) \) for \( T \geq T^H \). In this case, part (c) combined with the fact \( \Delta W(T) = \Delta V(T) = 0 \) uniquely defines \( x(c, T^H) = x(c, T) \) for all \( T \geq T^H \) implicitly by

\[
v'(x(c, T^H) - [d(T^H) + h(c)]) \leq 0 \quad (= 0 \text{ if } x(c, T^H) > 0). \tag{a.5}
\]

Combining (a.5) with parts (c) and (d), we have

\[
W(T^H) = E_c \left[ \frac{x(c, T^H) F(c)}{1 - \delta} \right] \tag{a.6}
\]

\[
V(T^H) = E_c \left[ \frac{x(c, T^H) F(c) h'(c)}{1 - \delta} \right] \tag{a.7}
\]

For \( T = T^H - 1 \): Define \( Z(T) = W(T) + V(T) \). By parts (c) and (d) of Proposition 2.1,

\[
Z(T) = E_c \left[ x(c, T)(1 + h'(c)) F(c) \right] \tag{a.8}
\]

\[
\Delta Z(T^H - 1) = E_c \left[ (x(c, T^H) - x(c, T^H - 1))(1 + h'(c)) F(c) \right] \tag{a.9}
\]

Also part (a) implies \( x(c, T^H - 1) \) is defined by

\[
v'(x(c, T^H - 1)) = [d(T^H - 1) + h(c)] + a \Delta Z(T^H - 1) \leq 0 (= 0 \text{ if } x(c, T^H - 1) > 0) \quad \tag{a.10}
\]

Equations (a.9) and (a.10) together define a mapping \( G(\cdot) \) which maps values of \( \Delta Z(T^H - 1) \) into \( \Delta Z(T^H - 1) \). A fixed point of that mapping exists if

\[
\Delta \tilde{Z}(T^H - 1) = G(\Delta \tilde{Z}(T^H - 1)) = E_c \left[ [x(c, T^H) - x(c, T^H - 1, \Delta \tilde{Z}(T^H - 1)) [1 + h'(c)] F(c) \right]
\]

\[
\tag{a.11}
\]

where \( x(c, T^H - 1, \Delta \tilde{Z}(T^H - 1)) \) is given by equation (a.9) where \( \Delta Z(T^H - 1) = \Delta \tilde{Z}(T^H - 1) \).
Note $G(\Delta Z(T^H - 1))$ is continuously decreasing in $\Delta Z(T^H - 1)$, $\Delta Z(T^H - 1) = 0 < G(0)$, and $\Delta Z(T^H - 1) = (d(T^H) - d(T^H - 1)) / a\delta > G((d(T^H) - d(T^H - 1)) / a\delta) = 0$ implying there exists a unique fixed point

$$\Delta \tilde{Z}(T^H - 1) \in (0, (d(T^H) - d(T^H - 1)) / a\delta)$$

(a.12)

This fixed point uniquely determines $V(T^H - 1)$ and $W(T^H - 1)$ as $Z(T^H) = W(T^H) + V(T^H)$ is known and,

$$Z(T^H - 1) = Z(T^H) + \Delta \tilde{Z}(T^H - 1)$$

(a.13)

$$W(T^H - 1) = E_e \left\{ \frac{1}{1 + h'(c)} \right\} Z(T^H - 1)$$

(a.14)

$$V(T^H - 1) = E_e \left\{ \frac{h'(c)}{1 + h'(c)} \right\} Z(T^H - 1)$$

(a.15)

Note also that equations (a.12) – (a.14) imply $\Delta W(T^H - 1)$ and $\Delta V(T^H - 1)$ are strictly positive. And part (c) of Proposition 2.1 therefore implies $x(c, T^H) \geq x(c, T^H - 1)$, (with > for $x(c, T^H) > 0$), thus establishing Proposition 2.2 and 2.3 for $T = T^H - 1$.

To complete the induction argument, suppose $W(T)$ and $V(T)$ exist and are unique for all $T' \geq T$ and $\Delta Z(T) > 0$ for all $T \leq T' \leq T^H - 1$. Then to show existence and uniqueness of $W(T - 1)$ and $V(T - 1)$, we search for a fixed point, $\Delta \tilde{Z}(T - 1)$ satisfying

$$\Delta \tilde{Z}(T - 1) = G(\Delta \tilde{Z}(T - 1)) = E_e \left\{ [x(c, T) - x(c, T - 1; \Delta \tilde{Z}(T - 1))] [1 + h'(c)] F(c) \right\}$$

(a.16)

where $x(c, T - 1; \Delta \tilde{Z}(T - 1))$ is determined by

$$v'(x(c, T - 1; \Delta \tilde{Z}(T - 1)) - [d(T - 1) + h(c)] + a\delta [\Delta \tilde{Z}(T - 1)] \leq 0$$

(a.17)

(= 0 if $x(c, T - 1; \Delta \tilde{Z}(T - 1)) > 0$)

Note $G(\Delta Z(T - 1))$ is continuously decreasing in $\Delta Z(T - 1)$, $\Delta \tilde{Z}(T - 1) = 0 < G(0)$ and $\Delta \tilde{Z}(T - 1) = \Delta Z(T) + [d(T) - d(T - 1)] / a\delta > G(\Delta Z(T) + (d(T) - d(T - 1)) / a\delta) = 0$, implying there exists a unique fixed point satisfying

$$\Delta \tilde{Z}(T - 1) \in (0, \Delta Z(T) + (d(T) - d(T - 1)) / a\delta)$$

(a.18)

This fixed point uniquely determines $V(T - 1)$ and $W(T - 1)$ as $Z(T) = W(T) + V(T)$ is known and

$$Z(T - 1) = Z(T) + \Delta \tilde{Z}(T - 1)$$

(a.19)
\[ W(T-1) = E_e \left[ \frac{1}{1 + h'(c)} \right] Z(T-1) \]  
(a.20)

\[ V(T-1) = E_e \left[ \frac{h'(c)}{1 + h'(c)} \right] Z(T-1) \]  
(a.21)

Note also that equations (a.18) – (a.21) imply \( \Delta W(T-1) \) and \( \Delta V(T-1) \) are strictly positive. And part (a) of Proposition 2.1 therefore reveals \( x(c, T) \geq x(c, T - 1) \), (with \( > \) for \( x(c, T) > 0 \), thus establishing Propositions 2.2 and 2.3 for \( T < T^{11} - 1 \).

Proof of Proposition 2.4:

We let \( \rho = \mu \delta \) denote the firm’s effective discount factor and use the subscript \( \rho \) to indicate the dependence of the equilibrium strategies and payoffs on \( \rho \). Arguing as in the proof of Proposition 1 MPE is characterized by the solution to the following problem:

\[ V(T) = \max_{x_p(c, T)} \left\{ \frac{1}{1-\delta} E_e \left\{ \begin{array}{c} v(x_p(c, T)) - \left[ d(T) + h(c) \right] x_p(c, T) + \delta V(T) \\ + a x_p(c, T) \left[ \delta \Delta V_p(T) + \rho \Delta W_p(T) \right] \end{array} \right\} \right\} \]  
(a.22)

The maximization in equation (a.22) implies

\[ v'(x_p(c, T)) - \left[ d(T) + h(c) \right] + a \Delta Z_p(T) \leq 0 \quad (=0 \text{ if } x_p(c, T) > 0) \]  
(a.23)

where \( \Delta Z_p(T) \equiv \delta \Delta V_p(T) + \rho \Delta W_p(T) \). The corresponding value function for the firm is given by

\[ W_p(T) = E_e \left\{ \rho W_p(T) + (\delta - \rho) \overline{W} + \int_{c'}^{c} x_p(c', T) dc' \right\} \]  
(a.24)

where \( \overline{W} \) is the firm’s expected continuation value if it leaves the franchise for another market. The expressions in (a.22) and (a.24) can be integrated and rewritten to yield:

\[ V_p(T) = E_e \left[ \frac{x_p(c, T) h'(c) F(c)}{1 - \delta} \right] \]  
(a.25)

\[ W_p(T) = E_e \left[ \frac{x_p(c, T) F(c)}{1 - \rho} \right] + \frac{\delta - \rho}{1 - \rho} \overline{W} \]  
(a.26)

Employing arguments used to prove Propositions 2.1 – 2.3, we know a unique MPE exists for each value of \( \rho \) and that \( \Delta W_p(T) \) and \( \Delta V_p(T) \) are strictly increasing for \( T < T^{11} \) under Assumption 1.
Consider different discounted survival probabilities $\rho_1 > \rho_2$. We wish to show that $V_{\rho_1}(T) > V_{\rho_2}(T)$ for all $T < T^H$. We argue by induction. Beginning with $T \geq T^H$, it is clear from (b.4) $V_{\rho_1}(T) = V_{\rho_2}(T)$ since $\Delta V_{\rho}(T) = \Delta W_{\rho}(T) = 0$ and therefore $x_{\rho}(c, T)$ is independent of $\rho$.

Consider $T = T^H - 1$. Suppose $\Delta Z_{\rho_1}(T) \leq \Delta Z_{\rho_2}(T)$. By (a.23) that implies $x_{\rho_1}(c, T) \geq x_{\rho_2}(c, T)$. Since (a.25) and (a.26) imply

$$\Delta V_{\rho}(T) = E_c \left[ \frac{(x_{\rho}(c, T + 1) - x_{\rho}(c, T))F(c)h'(c)}{1 - \delta} \right] \quad (a.27)$$

$$\Delta W_{\rho}(T) = E_c \left[ \frac{(x_{\rho}(c, T + 1) - x_{\rho}(c, T))F(c)}{1 - \rho} \right] \quad (a.28)$$

and $x_{\rho}(c, T + 1) = x_{\rho_2}(c, T)$, it follows that $\Delta V_{\rho_1}(T) \geq \Delta V_{\rho_2}(T)$, $\Delta W_{\rho_1}(T) \geq \Delta W_{\rho_2}(T)$ thus contradicting our original supposition. Consequently,

$$\Delta Z_{\rho_1}(T) < \Delta Z_{\rho_2} \quad (a.29)$$

Since

$$\Delta V_{\rho}(T) = \frac{1}{1 - \delta} \frac{E_c h'(c)}{\delta} \Delta Z_{\rho}(T) \quad (a.30)$$

(a.29), (a.30), and the fact that $\rho_1 > \rho_2$ imply

$$\Delta V_{\rho_1}(T) < \Delta V_{\rho_2}(T) \quad (a.31)$$

Since $V_{\rho_1}(T + 1) = V_{\rho_2}(T + 1)$, (b.14) implies $V_{\rho_1}(T) > V_{\rho_2}(T)$.

To complete our proof, suppose $V_{\rho_1}(T') > V_{\rho_2}(T')$ for all $T < T' < T^H - 1$. If we assume $\Delta Z_{\rho_1}(T) \leq \Delta Z_{\rho_2}(T)$, we can employ the same argument by contradiction used above for $T^H - 1$ case to show that $\Delta V_{\rho_1}(T) < \Delta V_{\rho_2}(T)$. From this, it follows that $V_{\rho_1}(T) = V_{\rho_1}(T + 1) - \Delta V_{\rho_1}(T) > V_{\rho_2}(T) = V_{\rho_2}(T + 1) - \Delta V_{\rho_2}(T)$ since $V_{\rho_1}(T + 1) > V_{\rho_2}(T + 1)$. This completes our proof.

Proof of Proposition 2.5:

Proposition 2.5 is proved with the same exact arguments employed in the proofs of Proposition 2.1-2.4, only under the conditions set out in Assumption 2 rather than Assumption 1.
APPENDIX B
PROOFS FOR CHAPTER 3

Proof of Proposition 3.1: Assume \( \frac{dU_i}{dy_j} \frac{\partial f^j}{\partial y_i} > 0 \). In the equilibrium for \( G_0 \), both agents are on their best-reply functions so that (3.1) holds for both of them. In the sequential game \( G_n \), \( j \) is on his best-reply function so that (3.1) still holds for \( j \). Here, agent \( i \) satisfies the following first-order condition, derived from [P2]:

\[
[-U_i' + U_2' + U_3' h_i'] + \frac{dU_i}{dy_j} \frac{\partial f^j}{\partial y_i} = 0. \tag{A1}
\]

If we evaluate (A1) at \( G_0 \), then the bracketed term vanishes by (3.1). Hence, quasi-concavity of \( U_i(I_i, y_i, h(y_i)) \) in \( y_i \) and \( \frac{dU_i}{dy_j} \frac{\partial f^j}{\partial y_i} > 0 \) imply \( y_i(G_i) > y_i(G_0) \).

Now consider total provision of the public good. \( Y = y_i + y_j = y_i + f(y_i) \) in both equilibria. Then, \( \frac{\partial Y}{\partial y_i} = 1 + \frac{\partial f^j}{\partial y_i} \). Thus, if \( \frac{\partial f^j}{\partial y_i} > -1 \), then \( \frac{\partial Y}{\partial y_i} > 0 \). Since \( y_i(G_i) > y_i(G_0) \), \( Y(G_i) > Y(G_0) \).

Analogously, if \( \frac{\partial f^j}{\partial y_i} < -1 \), then \( \frac{\partial Y}{\partial y_i} < 0 \). In this case, \( y_i(G_i) > y_i(G_0) \) implies \( Y(G_i) < Y(G_0) \). Of course, if \( \frac{\partial f^j}{\partial y_i} = -1 \), then \( Y(G_i) = Y(G_0) \) independent of the assumption made at the beginning of the proof.

By using a similar argument to that above, the results for \( \frac{dU_i}{dy_j} \frac{\partial f^j}{\partial y_i} < 0 \) easily follow.

Finally, assuming \( \frac{dU_i}{dy_j} \frac{\partial f^j}{\partial y_i} = 0 \) and comparing (3.1) and (A1) cover the final possibility, completing the proof. Q.E.D.

Proof of Corollary 3.1:
A) Under the hypothesis of the Corollary, \( h_1^2 = h_2 = h_1^i = 0 \). From (3.8), \( \frac{\partial f_j^i}{\partial y_i} = -\frac{\partial x_j^i}{\partial y_i} \), which implies \(-1 < \frac{\partial f_j^i}{\partial y_i} < 0\) due to the normality assumptions on demands. From (3.4), \( \frac{dU_i^j}{dy_j} = U_2^i > 0 \). Thus, \( \frac{dU_i^j}{dy_i} \frac{\partial f_j^i}{\partial y_i} < 0 \). Utilizing Proposition 3.1, \( y_i(G_i) < y_i(G_0) \) and \( Y(G_i) < Y(G_0) \) easily follow.

B) By assumption, \( \frac{dU_i^j}{dy_j} = U_2^i > 0 \). By applying Proposition 3.1, the results follow. Q.E.D.

Results with n Warm-Glow Players and Upward Sloping Best-Reply Functions: Assume there are \( n \) warm-glow players. Let \( y_i^R(Y_i) \) denote agent \( i \)'s best-reply function, \( i = 1, 2, ..., n \), where \( Y_i = \sum_{j \neq i} y_j \), denotes other agents' total contribution. Observe that, for warm-glow utility, the generalization of the total effect in (4) satisfies:

\[
\frac{dU_i^j}{dy_i} \left( \frac{\partial f_j^i}{\partial y_i} \right) > 0.
\]

Assume:

\( y_i^R(Y_i) \) is increasing, continuous, and twice differentiable; \( (a-1) \)

and

Simultaneous-move equilibrium among any subset of the \( n \) agents (with other agents' \( y_i \)'s exogenous) exists and is unique \( (a-2) \)

Refer to the quasi-sequential game where agent \( i \) publicly selects \( y_i \) first, followed by simultaneous choices of the remaining agents as the "i-leader game."

Proposition B-1: Total contributions in the i-leader game exceed those in the n-agent, simultaneous-move game for any \( i \).

Proof: Let \( R_i \) denote the collective best-reply function of the \( n-1 \) agents other than \( i \). It satisfies

\[
R_i = \sum_{j \neq i} y_j^e(y_j);
\]

by definition:

where \( y_j^e(y_j) \) is agent \( j \)'s equilibrium choice of \( y_j \) in the simultaneous-move stage of the game.

It must be that:
Differentiate \( e^{-l,0} \), hence, as \( y_j \) is the best reply to \( R_i \cdot y_j^R + y_i \). Differentiate (A3) and rearrange:

\[
y_j^\sigma = \frac{(R_{-i}^R + 1)y_j^{R'}}{1 + y_j^{R'}}
\]  

(A4)

Now, (A2), (A4), and (a-1) imply \( R_i \in [-1,0] \) is impossible. If \( R_i \in (-1,0] \), then (A4) and (a-1) imply \( y_j^\sigma > 0 \), and (A2) implies \( R_i > 0 \), a contradiction. If \( R_i = -1 \), then (A4) implies \( y_j^\sigma = 0 \), again contradicting (A2). Hence, the collective best-reply function has slope everywhere positive or everywhere less than minus one. (This holds everywhere or \( R_i \) would have to jump, violating (a-1).)

The result follows then by application of Proposition 3.1. This follows since the i-leader game and n-agent simultaneous-move game are equivalent to the two-agent games of Proposition 1 with "one agent" described by the best-reply function \( R_i(y_i) \). Q.E.D.

Remark 1: The proof admits the possibility that the collective best-reply function of the -i agents is downward sloping, while their individual best-reply functions are upward sloping. Although this may be impossible, it is easy to deal with such a case as we have anyway.

Remark 2: A very similar argument can be made that establishes, for the standard model of utility, that any i-leader Stackelberg equilibrium has lower total contributions than in the simultaneous-move equilibrium. To show this, show that \( R_i(y_i) \in (-1,0) \) given \( y_i^R \in (-1,0) \). Using [A2] and [A4] a contradiction to any \( R_i(y_i) \in (-1,0) \) is easily generated. Then apply Proposition 1. Hence, Varian's result can be so generalized.

We now develop a proposition that implies among other things that announcing after every contribution yields a higher total than in the n-player, simultaneous-move equilibrium. We continue to assume agents have warm-glow utility functions, and each has upward-sloping best-reply function (i.e., a-1 applies). Assume further that agents will make their contributions consecutively in an exogenously specified order, and number the agents 1 through n in the order that they will contribute. An n-tuple vector consisting of A's for "announcement," and N's for "no announcement," denotes a commitment by the charity to follow a policy of announcing the cumulative contributions following the m\(^{th}\) contribution if and only if the m\(^{th}\) element of the vector is an A. Assume the preferences of agents, their order of contributions, and the charity's committed announcement strategy are common knowledge. Then, since announcement after the last contribution is irrelevant, the charity has \( 2^{n-1} \) distinct strategies, and there are this many corresponding contribution games. Our result concerns the subset of n games corresponding to
announcement strategies of the form \((N,N,...,N,A,A,...,A)\), i.e., consisting of \(m \in \{0,1,...,n-1\}\) \(N\)'s followed by \(n-m\) \(A\)'s. (Hence, we assume announcement always after the \(n\)th contribution, though, again, not announcing here would be equivalent since everyone can deduce the total.) Hence, the games we consider has a subset of \(k=\text{m+1}\) agents first make simultaneous choices, followed by the remaining \(n-k\) players playing sequentially. Let \(G^k\), \(k=1,2,...,n\), denote the game of this subset where the initial announcement follows the \(k\)th contribution. Of course, \(G^1\) is the \(n\)-agent sequential game and \(G^n\) is the \(n\)-agent simultaneous-move game.

Let \(y_i^e(G^k)\) denote the equilibrium contribution of agent \(i\) in game \(G^k\), and assume it is positive and unique for all \(i,k=1,2,...,n\). Let \(Y^e(G^k) = \sum_{i=1}^{n} y_i^e(G^k)\). we provide conditions such that \(Y^e(G^{k-1}) > Y(G^k), k=2,3,...,n\). For any game \(G^k\), \(i > k\) and \(k \leq h < i\), let:

\[
y^B_{ih}(X), X = \sum_{i=1}^{h} y_i
\]

(A5)

denote agent \(i\)'s quasi-equilibrium contribution conditional on agents 1 through \(h\) collectively contributing \(X\) (and, otherwise, the game being played according to the announcement strategy of \(G^k\)). That is, \(y^B_{ih}\) is agent \(i\)'s choice in the subgame where the first \(h < i\) agents have chosen collectively \(X\), and agents \(h+1, h+2, \text{etc.}\) play sequentially, agent \(i\) a member of the latter group. We assume \(y^B_{ih}\) is unique for all \(X\) and feasible \(k, i,\) and \(h\), i.e., that all such subgames of \(G^k\) have a unique equilibrium. \(y^B_{ih}(X)\) is a generalization of a best-reply function. Observe that \(y^R_{n,k}(X) = y^B_{n,k}(X)\) and \(y^B_{ih}(\sum_{j=1}^{h} y_j^e(G^k)) = y^e_i(G^k)\).

Let the collective best reply of all sequential agents in game \(G^k\), \(k < n\) be

\[
B^k(X) = \sum_{i=k+1}^{n} y^B_{ik}(X), X = \sum_{i=1}^{k} y_i
\]

(A6)

For the analysis below, the behavior of agent \(k+1\) in game \(G^k\), i.e., the first sequential agent, is pivotal. It is convenient to have additional notation for that agent's generalized best-reply function. Let \(y'(X,k), X = \sum_{i=1}^{k} y_i\) denote agent \((k+1)\)'s best choice conditional on \(X\) in game \(k, k=1,2,...,n-1\). \((y'(X,k)\) also equals \(y^B_{k+1,k}(X)\) in game \(k\), but using the latter notation when needed would be more awkward.) The key additional assumption is
\[ \frac{\partial y^e(X, k)}{\partial X} > 0 \text{ for all } X \text{ and } k = 1, 2, \ldots, n-1, \] (a-3)

and for any ordering of the players.

Note that (a-3) will be satisfied for \( k = n-1 \) by (a-1). Assumption (a-3) must be checked in any example to apply the proposition below.

Now consider the comparison of game \( G^{k-1} \) to game \( G^k \), \( k = 2, 3, \ldots, n \). We have:

\[ B_{k-1}(X) = y^e(X, k-1) + B^k(X + y^e(X, k-1)), \] (A7a)

\[ B_{k-1}'(X) = \frac{\partial y^e}{\partial X}[1 + B^k(X + y^e)] + B^k'(X + y^e). \] (A7b)

**Lemma 1.** \( B^k > 0 \) for all \( k = 1, 2, \ldots, n-1 \).

**Proof.** \( B^{k-1} > 0 \) by (a-1). By (a-3) and (A7b), then, the result holds by recursion. Q.E.D.

Relevant to comparing equilibria in games \( G^k \) and \( G^{k-1} \), as we will see, is the examination of incentives in game \( G^{k-1} \) assuming the first \( k-1 \) agents (i.e., the simultaneous movers) choose their equilibrium contributions in game \( G^k \). We have:

**Lemma 2.** For \( k = 2, 3, \ldots, n \), the quasi-equilibrium of game \( G^{k-1} \) where agents \( i = 1, 2, \ldots, k-1 \) choose \( y^i(G^k) \) is the same as the equilibrium of game \( G^k \).

**Proof.** Consider agent \( k \) in game \( G^{k-1} \). If all agents \( i = 1, 2, \ldots, k-1 \) choose \( y^i = y^i(G^k) \), then agent \( k \) has precisely the same incentives as in equilibrium in game \( G^k \). Hence, \( y^k_k = y^k_i(G^k) \). Since the remaining agents would choose \( y^k_{ik} = \left( \sum_{i=1}^{k} y^i(G^k) \right) \) in both games, the result holds.

**Lemma 3.** In game \( G^{k-1}, k = 2, 3, \ldots, n \), at the non-equilibrium vector of strategies for the first \( k-1 \) agents \( y^i = y^i(G^k) \), the local incentive of the latter agents \( i = 1, 2, \ldots, k-1 \) is to increase their contribution.

\[ U^i = U^i(l_i - y^i, \sum_{j=1}^{k} y^j + y^i + B^k(\sum_{j \neq i} y^j, y^i)); \]

**Proof.** In game \( G^k \), for \( i = 1, 2, \ldots, k \), agent \( i \)'s payoff function is:

and first-order condition is:

\[ -U^i_1 + U^i_2(1+B^k) + U^i_3 = 0 \] (A8)
In game $G^{k-1}$, at the non-equilibrium vector of contributions for agents $i = 1, 2, \ldots, k-1$, $y_i = y^*_i(G^k)$, and for agent $i$ has

$$\frac{dU^i}{dy_i} = -U^i_1 + U^i_2 (1 + B^{k-1} \sum_{j=1}^{k-1} y^*_j(G^k)) + U^i_3; \quad (A9)$$

where the partial derivatives of $U^i$ have the same arguments as in (A8) by Lemma 2. However, $B^{k-1}(\cdot)$ in (A9) exceeds $B^k(\cdot)$ in (A8), implying $\frac{dU^i}{dy_i}$ in (A9) is positive, i.e., that the result holds.

That $B^{k-1}$ in (A9) exceeds $B^k$ in (A8) follows from (A7b), using that the argument of $B^k$ in (A7b) equals $\sum_{i=1}^k y^*_i(G^k)$, again by Lemma 2, and applying (a-3) and Lemma 1. Q.E.D.

With these results it is easy to show:

**Proposition A-2.** Total contributions in the equilibrium of game $G^{k-1}$ exceed those in equilibrium of game $G^k$, $k = 2, 3, \ldots, n$.

**Proof.** If $\sum_{i=1}^{k-1} y^*_i(G^{k-1}) > \sum_{i=1}^{k-1} y^*_i(G^k)$, then the result holds by Lemmas 1 and 2. The proof of the latter is a corollary of Vives's (1990) stability result (Theorem 5.1, p. 313) for games with strategic complements (i.e., supermodular games). Vives shows a more general version of the following result. If players in a simultaneous-move game have upward-sloping best-reply functions, then, starting at a strategy vector below the players' best-reply functions, a Cournot tatonnement process converges monotonically upward to an equilibrium point of the game. The simultaneous-stage game among the first $k-1$ players of $G^{k-1}$ has upward-sloping, best-reply functions by (a-3). This follows since the generalized best-reply function in (a-3) of player $k$ in $G^{k-1}$ is also player $k$'s best-reply function in the simultaneous stage of game $G^k$. Then, applying Vives's theorem, by Lemma 3 and our assumption of uniqueness of equilibrium, the result follows. Q.E.D.

We first provide a set of simple results that severely restrict the set of equilibrium candidates, in some cases reducing to a singleton.

**Lemma B1:** In equilibrium, $y_i^2 = \max \{0, f^i(y_j) - y_i^1\}$.

**Proof:** If $f^i(y_j) - y_i^1 \geq 0$, then $y_i = f^i(y_j)$ is feasible and obviously best for agent $i$. If, however, $f^i(y_j) - y_i^1 < 0$, then $y_i^2 = 0$ is best for agent $i$, which follows from quasiconcavity of $U^i(I, y, \lambda, y_j, h^i(y_i, y_j))$ in $(y_i, y_j)$.
Corollary B1: In equilibrium, \( y_i \geq f'(y_j) \).

Proof: This immediately follows from Lemma B1.

Lemma B2: In equilibrium, \( y_i = f'(y_j) \) for at least one agent.

Proof: Suppose not. Then \( y_i > f'(y_j) \) for both agents and \( y_i^2 = y_j^2 = 0 \), which follow from Lemma B1 and Corollary B1. Thus \( y_i^1 > f'(y_j^1) \) for each agent. However, in the first period, given agent j’s contribution agent i would be better off by reducing his amount due to quasiconcavity of \( U'(\cdot) \) in \((y_i, y_j)\), a contradiction.

Lemma B3: If, in equilibrium, agent i has \( y_i \neq f'(y_j) \), then \( y_i \leq y_i(G_i) \).

Proof: Given \( y_i \neq f'(y_j) \), it must be that \( y_i > f'(y_j) \) from the above Corollary. Applying Lemma B1, it must also be that \( y_i^2 = 0 \) and thus \( y_i^1 = y_i \). Since \( y_i > f'(y_j) \), we have \( y_j = f'(y_i) \) from Lemma B2, implying that \( y_j^1 \in [0, f'(y_i)] \). Now we argue that if \( y_i^1 > y_i(G_i) \), then agent i could increase his utility by marginally reducing \( y_i^1 \).

If \( y_j^1 \) is such that following the marginal reduction in \( y_i^1 \), agent j can choose \( y_j^2 \) such that \( y_j = f'(y_i) \) (i.e., if the nonnegativity constraint on \( y_j^2 \) does not bind), then agent i is better off due to quasiconcavity of \( U'(\cdot) \) in \((y_i, y_j)\). If, however, \( y_j^1 \) is such that following the marginal reduction in \( y_i^1 \), agent j cannot choose \( y_j^2 \) such that \( y_j = f'(y_i) \) (i.e., the nonnegativity constraint on \( y_j^2 \) binds), then \( y_j \) would be unchanged. In the latter case, agent i is better off since \( y_i^1 > f'(y_j) \) and \( U'(\cdot) \) is quasiconcave in \((y_i, y_j)\).

Proof of Proposition 4.4:

A) Define the following sets:

\[
F_1 = \{(y_1, y_2) \in [0, 1] \times [0, 1] \mid y_1 = f'(y_2) \text{ and } y_2(G_0) \leq y_2 \leq y_2(G_2)\} \quad \text{and} \\
F_2 = \{(y_1, y_2) \in [0, 1] \times [0, 1] \mid y_2 = f'(y_1) \text{ and } y_1(G_0) \leq y_1 \leq y_1(G_1)\}.
\]

Let \( F = F_1 \cup F_2 \). Applying Lemma B1-B3 and Corollary B1, the elements of \( F \) are the only equilibrium candidates in this case (see figure 3.2).

Take any \((y_1, y_2)\) in \( F_2 \) such that \( y_1 \leq y_1(G_i) \). Since \( y_2 = f'(y_1) \), we have \( y_2^1 \leq f'(y_1) \) by definition. Now we argue that agent 1 can engender \((y_1(G_i), y_2(G_i))\) as the equilibrium outcome.
Suppose agent 1 chooses $y_1^2 = y_1(G_1)$. Then the equilibrium in period 2 has $y_1^2 = 0$ and $y_2^2 = y_2(G_1) - y_2^1$. In fact, below it is clarified that this is the only equilibrium. Since $(y_1(G_1), y_2(G_1))$ is a strictly better outcome for agent 1, $(y_1, y_2)$ cannot be an equilibrium outcome. This argument rules out all the points but $y_1 = y_1(G_1)$ in $F_2$. A similar argument reduces $F_1$ only to the point where $y_2 = y_2(G_2)$.

Here we provide a set of strategies under which point $G_1$ is an equilibrium outcome. Consider the following strategies:

$$y_1^1 = y_1(G_1), \quad y_2^1 = 0,$$

and

$$y_1^2(y_1^1, y_2^1) = \begin{cases} 
0, & \text{if } y_1^1 \geq f^i(y_1^1) \text{ and } y_1^j \geq f^j(y_1^1), \\
y_1(G_0) - y_1^1, & \text{if } y_1^1 \leq y_1(G_0) \text{ and } y_1^1 \leq y_1(G_0), \\
0, & \text{if } y_1^1 \geq y_1(G_0) \text{ and } y_1^1 \leq f^j(y_1^1), \\
f^i(y_1^1) - y_1^1, & \text{if } y_1^1 \leq y_1(G_0) \text{ and } y_1^1 \leq f^j(y_1^1),
\end{cases}$$

where $i, j = 1, 2$ and $i \neq j$.

Consider the second period strategies and suppose $y_1^1 \geq f^i(y_1^1)$ and $y_1^j \geq f^j(y_1^1)$. Given that $y_2^2 = 0$, $y_i^2 = 0$ since $U(.)$ is quasiconcave in $(y_i, y_i)$. Now suppose $y_1^1 \leq y_1(G_0)$ and $y_1^j \leq y_1(G_0)$. Further suppose $y_1^2 = y_1(G_0) - y_1^1$ is given. This implies $y_j = y_j(G_0)$. Since $y_1(G_0) - y_1^1 \geq 0$, and by definition agent i's best response to $y_i(G_0)$ is $y_i(G_0)$, we have $y_i^2 = y_i(G_0) - y_1^1$.

In the third case, suppose $y_1^1 \leq y_1(G_0)$ and $y_1^j \leq f^j(y_1^1)$. Further suppose $y_2^2 = f^i(y_1^1) - y_1^j$ is given, implying that $y_j = f^j(y_1^1)$. This reveals that $y_1^2 = 0$ due to quasiconcavity of $U(.)$ in $(y_i, y_i)$. Given $y_1^2 = 0$, obviously $y_2^2 = f^j(y_1^1) - y_1^j$ is the best response for agent j. Finally, consider the last case in the second period. Given $y_1^2 = 0$, since $y_1^j \leq f^i(y_1^1)$, $y_2^2 = f^j(y_1^1) - y_1^j$ is the best response for agent i. Now given $y_1^2 = f^i(y_1^1) - y_1^j$, i.e., $y_i = f^i(y_1^1)$, $y_1^2 = 0$ is the best-response for agent j due to quasiconcavity of $U(.)$ in $(y_i, y_i)$.

Thus the second period strategies are equilibrium strategies for agents.

Now consider the first period strategies. Suppose $y_1^1 = y_1(G_1)$ is given. In this case, the
best agent 2 can do is to reach the point $G_1$ (see figure 3.2). Given the second period strategies, agent 2 can achieve this goal by contributing nothing in the first period so that the second period strategies will dictate $y^1_2 = 0$ and $y^2_2 = y_2(G_1)$. Thus, $y^1_2 = 0$ is a best response for agent 2. Now suppose $y^1_2 = 0$ is given. Again the best agent 1 can do is to reach $G_1$. Given the second period strategies, he can do so by contributing $y_1(G_1)$ in the first period.

Thus the above strategies constitute an equilibrium at point $G_1$. The same second period strategies and $y^1_1 = 0, y^1_2 = y_2(G_2)$ along with a similar argument above proves that $G_2$ is an equilibrium outcome as well.

B) Applying Lemma B1-B3 and Corollary B1, the set $F_1$ defined in part (A) above contains the equilibrium candidates for this case (see figure 3.1). Using a similar argument to the proof of part (A), we can rule out all the points but $y_2 = y_2(G_2)$ in $F_1$. Also the same second period strategies above, and $y^1_2 = y_2(G_2), y^1_1 = 0$ along with similar arguments prove that $G_2$ is indeed an equilibrium outcome.

C) In this case, applying Lemma B1-B3 and Corollary B1, $(y_1(G_0), y_2(G_0))$ is the only equilibrium candidate. The same second period strategies in part (A), and $y^1_1 = y_1(G_0), y^1_2 = y_2(G_0)$ along with similar arguments prove that point $G_0$ is an equilibrium outcome.

D) Applying Lemma B1-B3 and Corollary B1, the set $F$ defined in part (A) contains the equilibrium candidates in this case (see figure 3.3). However, here any point can be supported as an equilibrium outcome. Take any $(y_1, y_2)$ in $F$. Let $y^1_1 = y_1, y^1_2 = y_2$ and let the second period strategies remain the same as in part (A). Similar arguments in part (A) will confirm that these are equilibrium strategies. Since this case is analytically equivalent to Saloner's (1987) Cournot duopolists with two production periods, the interested reader can refer to that paper for more details.
APPENDIX C
PROOFS FOR CHAPTER 4

Proof of Proposition 4.1:
Assume \( \frac{dU^i}{dY_j} \frac{\partial f^j}{\partial Y_i} \bigg|_{G_0} > 0 \). In the Cournot-Nash equilibrium \((G_0)\), both agents are on their reaction functions so that

\[
\frac{dU^i}{dY_i} = 0 \tag{A1}
\]

for both \( i \) and \( j \). In the Stackelberg equilibrium where \( i \) leads \((G_i)\), \( j \) is on his reaction function whereas agent \( i \) satisfies the following first-order condition:

\[
\frac{dU^i}{dY_i} + \frac{dU^i}{dY_j} \frac{\partial f^j}{\partial Y_i} = 0 \tag{A2}
\]

If we evaluate (A2) at \( G_0 \), the first term vanishes by (A1). Thus, the strict quasi-concavity of

\[
U^i(Y_i, f(Y_i)) \text{ in } Y_i \text{ and } \frac{dU^i}{dY_j} \frac{\partial f^j}{\partial Y_i} \bigg|_{G_0} > 0 \text{ imply that } Y_i(G_i) > Y_i(G_0).
\]

Using a similar argument, the results for \( \frac{dU^i}{dY_j} \frac{\partial f^j}{\partial Y_i} \bigg|_{G_0} \) being negative and zero easily follow. Q.E.D.

**Lemma A1:** Suppose there is a unique interior Cournot-Nash equilibrium in the one-shot game. Also suppose \( Y_j = f^j(Y_i) \) for a feasible \((Y_i, Y_j)\). Then \( Y_i < Y_i(G_0) \) if and only if \( Y_i < f^i(Y_j) \).
Proof of Lemma A1:

Define the functions \( f = f^i \circ f^j \) and \( F(x) = f(x) - x \). Note that the Cournot-Nash equilibrium \((Y_i(G_0), Y_j(G_0))\) is such that \( Y_i(G_0) \) is a fixed point of \( f \), and \( Y_j(G_0) = f^i(Y_j(G_0)) \).

Also note that \( F(Y_i(G_0)) = 0 \). Since \( Y_i \in [0, I_i] \), we have \( F(I_i) \leq 0 \) and \( F(0) \geq 0 \).

\((\Rightarrow)\) Suppose for some feasible \((Y_i, Y_j)\) pair we have \( Y_j = f^j(Y_i) \) and \( Y_i < Y_i(G_0) \). Suppose however that \( Y_i \geq f^i(Y_j) \). That is, \( Y_i \geq f(Y_j) \), or \( F(Y_j) < 0 \). Since \( F(0) \geq 0 \) and \( F() \) is continuous, from the Intermediate Value Theorem, there exists some \( \tilde{Y}_i \in [0, Y_i] \) such that \( F(\tilde{Y}_i) = 0 \). However, then \((\tilde{Y}_i, Y_j) \) is another Cournot-Nash equilibrium, which contradicts the uniqueness assumption.

\((\Leftarrow)\) Suppose now that for some feasible \((Y_i, Y_j)\) pair we have \( Y_j = f^j(Y_i) \) and \( Y_i < f^i(Y_j) \). However, suppose \( Y_i \geq Y_i(G_0) \). That is, \( Y_i < f(Y_j) \), or \( F(Y_j) > 0 \). Since \( F(I_i) \leq 0 \) and \( F() \) is continuous, there exists some \( \tilde{Y}_i \in (Y_i, I_i] \) such that \( F(\tilde{Y}_i) = 0 \). However, then \((\tilde{Y}_i, Y_j) \) is another equilibrium, a contradiction.

Q.E.D.

Proof of Lemma 4.1:

Consider the second period strategies and suppose \( y_i^1 \geq f^i(y_j^1) \) and \( y_j^1 \geq f^j(y_i^1) \). Given that \( y_i^2 = 0, y_j^2 = 0 \) since \( U() \) is quasi-concave in \((Y_i, Y_j)\). Now suppose \( y_i^1 \leq Y_i(G_0) \) and \( y_j^1 \leq Y_j(G_0) \). Further suppose \( y_j^2 = Y_j(G_0) - y_j^1 \) is given. This implies \( Y_j = Y_j(G_0) \). Since \( Y_i(G_0) - y_i^1 \geq 0 \), and by definition agent i’s best response to \( Y_j(G_0) \) is \( Y_i(G_0) \), we have \( y_i^2 = Y_i(G_0) - y_i^1 \).

In the third case, suppose \( y_i^1 \geq Y_i(G_0) \) and \( y_j^1 \leq f^j(y_i^1) \). Further suppose \( y_j^2 = f^j(y_i^1) - y_j^1 \) is given, implying that \( Y_j = f^j(y_i^1) \). This reveals that \( y_i^2 = 0 \) due to quasi-concavity of \( U() \) in \((Y_i, Y_j)\). Given \( y_i^2 = 0 \), obviously \( y_j^2 = f^j(y_i^1) - y_j^1 \) is best response for agent j. Finally, consider the last case in the second period. Given \( y_j^2 = 0 \), since \( y_i^1 \leq f^i(y_j^1) \), \( y_i^2 = f^i(y_j^1) - y_i^1 \) is best response for agent i. Now given \( y_i^2 = f^i(y_j^1) - y_i^1 \), i.e.,
\[ Y_i = f^i(y^1_j), \quad y^2_j = 0 \] is best response for agent \( j \) due to quasi-concavity of \( U(\cdot) \) in \( (Y_i, Y_j) \).

Q.E.D.

**CLAIM 1:** \( (Y_i(G_0), Y_j(G_0)) \) is an equilibrium outcome for \( T = 2 \) under the following cases:

Case 1: \( Y_i(G_0) > Y_i(G_j) \) and \( Y_j(G_0) > Y_j(G_j) \)

Proof: Suppose \( y^1_j = Y_j(G_0) \) is given. If \( y^1_j > Y_i(G_0) \), then \( y^2_j = 0 \). In this case, there are two possibilities. If \( y^1_j \geq f^j(y^1_j) \), then \( y^2_j = 0 \). However, \( y^1_j = Y_i(G_0) \) is better for agent \( i \) due to quasi-concavity of \( U(\cdot) \) in \( (Y^\prime Y_j) \). On the other hand, if \( y^1_j < f^j(y^1_j) \), then \( y^2_j = f^j(y^1_j) - y^1_j \). Since the resulting outcome is on agent \( j \)'s reaction function and \( Y_i > Y_i(G_0) > Y_i(G_j) \), agent \( i \) is worse off than \( y^1_j = Y_i(G_0) \) due to strict quasi-concavity of \( U(Y_i, f(Y_j)) \) in \( Y_i \). Thus, \( y^1_j = Y_i(G_0) \) is a best response.

Now consider the possibility of \( y^1_j \leq Y_i(G_0) \). Then, \( y^2_j = 0 \) and \( y^2_j = Y_i(G_0) - y^1_j \). Thus, the resulting outcome is \( Y_i = Y_i(G_0) \). As a result, given \( y^1_j = Y_j(G_0) \), \( y^1_j = Y_i(G_0) \) is a best response for agent \( i \).

Suppose now that \( y^1_j = Y_i(G_0) \) is given. Since \( Y_i(G_0) = f^i(Y_j(G_0)) > f^i(Y_j(G_j)) = Y_i(G_j) \) by hypothesis, we analyze two cases regarding the monotonicity of \( f(\cdot) \). If \( f(\cdot) \) is upward sloping, then we must have \( Y_j(G_j) < Y_j(G_0) \), where \( y^1_j = Y_j(G_0) \) would be a best response for agent \( j \) from the above discussion. If, on the other hand, \( f(\cdot) \) is downward sloping, then \( Y_j(G_0) < Y_j(G_j) \). Consider \( y^1_j < Y_j(G_0) \). Then, \( y^2_j = 0 \) and \( y^2_j = Y_j(G_0) - y^1_j \) where \( Y_i = Y_i(G_0) \) and \( Y_j = Y_j(G_0) \). Thus, \( y^1_j = Y_j(G_0) \) yields the same payoff for \( j \). Now consider \( y^1_j > Y_j(G_0) \). Then \( y^1_j > f^j(y^1_j) \), implying that \( y^2_j = 0 \). Also since \( y^1_j > Y_j(G_0) = f^j(y^1_j) \), we have \( y^2_j = 0 \). However, \( y^1_j = Y_j(G_0) \) is a better response. Thus, given \( y^1_j = Y_i(G_0) \), \( y^1_j = Y_j(G_0) \) is a best response for agent \( j \).

Overall, since, for \( y^1_i = Y_i(G_0) \) and \( y^1_j = Y_j(G_0) \), second period strategies dictate \( y^2_i = y^2_j = 0 \), \( (Y_i(G_0), Y_j(G_0)) \) is supported as an equilibrium outcome.
Case 2: \( Y_f(G_0) > Y_f(G_j) \) and \( Y_f(G_0) > Y_f(G_j) \)

Proof: We have \( Y_j(G_0) = f^j(Y_i(G_0)) > f^i(Y_j(G_i)) = Y_j(G_i) \) and \( Y_i(G_0) = f^i(Y_j(G_0)) > f^i(Y_j(G_i)) = Y_i(G_j) \). If both reaction functions are upward sloping, it must be that \( Y_i(G_0) > Y_i(G_j) \) and \( Y_j(G_0) > Y_j(G_j) \). In this case, that \( y^1_i = Y_i(G_0) \) and \( y^1_j = Y_j(G_0) \) together with the second period strategies in Lemma 1 support \( (Y_i(G_0), Y_j(G_0)) \) is an equilibrium follows from a similar argument in case 1 above.

If \( f(.) \) is upward and \( f(.) \) is downward sloping, then we have \( Y_j(G_0) > Y_j(G_j) \) and \( Y_i(G_0) > Y_i(G_j) \). Again a similar argument to that above shows that \( (Y_i(G_0), Y_j(G_0)) \) is an equilibrium outcome. It is also easy to show that when both reaction functions are downward sloping, the Cournot-Nash outcome is supported as the equilibrium. Q.E.D.

Claim 2: \((Y_n, Y_j)\) is an equilibrium outcome if \( Y_i = f^i(Y_j), Y_j > f^j(Y_i), Y_i(G_0) < Y_j < Y_j(G_j) \), and \( Y_i > Y_i(G_j) \).

Proof: First note that since \( Y_j(G_j) > Y_j(G_0) \), from Proposition 1 we have \( \left( \frac{dU^j}{dY_j} \frac{\partial f^i}{\partial Y_j} \right) \) must be positive. Suppose both \( \frac{dU^j}{dY_j} \) and \( \frac{\partial f^i}{\partial Y_j} \) are positive. Then, since \( Y_j < Y_j(G_j) \), \( Y_i = f^i(Y_j) < f^i(Y_j(G_j)) = Y_i(G_j) \), a contradiction. Thus, both must be negative. This immediately implies that \( Y_i < Y_i(G_0) \). Now we show that \( y^1_i = Y_i, y^1_j = Y_j \) together with the second period strategies in Lemma 1 constitute an equilibrium.

Suppose \( y^1_i = Y_i \) is given and suppose \( y^1_j = Y_j \) is not a best response for agent j. If \( y^1_j \leq Y_j(G_0) \), then the second period strategies dictate \( y^2_j = Y_j(G_0) - Y_i \) and \( y^2_i = Y_i(G_0) - y^1_j \), which is a worse outcome for agent j than \((Y_n, Y_j)\) due to strict quasi-concavity of \( U(Y_j, f(Y_j)) \) in \( Y_j \). If, however, \( Y_j(G_0) < y^1_j < Y_j \), then \( Y_i < f^i(y^1_j) \). In this case, \( y^2_j = 0 \) and \( y^2_j = f^i(y^1_j) - Y_i \), which is again a worse outcome for agent j for the same reason.
Finally, assume $y_j^1 > Y_j$. Then, $y_j^1 > f^i(Y_i)$ and $Y_i > f^i(y_j^1)$. Thus, $y_i^2 = y_j^2 = 0$. However, agent $j$ is worse off due to quasi-concavity of $U(.)$ in $(Y_i, Y_j)$. Thus, $y_j^1 = Y_j$ is a best response. Similar arguments show that $y_i^1 = Y_i$ is a best response to $y_j^1 = Y_j$ as well. Since for these first period strategies we have $y_i^2 = y_j^2 = 0$, such $(Y_i, Y_j)$ is indeed an equilibrium. Q.E.D.

**Lemma A2:** For $T = 2$, in equilibrium, $y_i^2 = \max \{ 0, f^i(Y_j) - y_j^1 \}$.

**Proof:** If $f^i(Y_j) - y_j^1 \geq 0$, then $Y_i = f(Y_i)$ is feasible and obviously best for agent $i$. If, however, $f^i(Y_j) - y_j^1 < 0$, then $y_i^2 = 0$ is best for agent $i$, which follows from $U'(.)$ in $(Y_i, Y_j)$. Q.E.D.

**Proof of Proposition 4.2:**

$(\Rightarrow)$ Suppose $(Y_n, Y_j)$ is an equilibrium outcome. We will show that it is in $S_i$ ($i = 1, \ldots, 4$), respectively.

(S1): The equilibrium pair is in $S_1$ follows from Lemma A2.

(S2): Suppose the equilibrium pair is not in $S_2$. Then $Y_i > f(Y_i)$ for both agents and $y_i^2 = y_j^2 = 0$, which follows from Lemma A2. Thus $y_i^1 > f^i(y_j^1)$ for each agent. However, in the first period, given $j$’s contribution agent $i$ would be better off by reducing his amount, a contradiction.

(S3): Given $Y_i \neq f^i(Y_j)$, it must be that $Y_i > f(Y_i)$ since the pair is in $S_1$. From Lemma A2, it must also be that $y_i^2 = 0$ and thus $y_i^1 = Y_i$. Since $Y_i > f(Y_i)$, we have $Y_j = f(Y_j)$ from lemma A2. Now we argue that if $y_i^1 > Y_i(G_i)$, then agent $i$ could increase his utility by marginally reducing his first period choice.

If $y_i^1$ is such that following the marginal reduction in $y_i^1$, agent $j$ can choose $y_j^2$ such that $Y_j = f(Y_j)$, then agent $i$ is better off due to $U(Y_i, f(Y_j))$ in $Y_i$. If, however, $y_i^1$ is such that following the marginal reduction in $y_i^1$, agent $j$ cannot choose $y_j^2$ such that $Y_j = f^j(Y_i)$, then $Y_j$ would be unchanged. In the latter case, agent $i$ is better off since $y_i^1 > f^i(Y_j)$ and $U(.)$ in its arguments.

(S4): Suppose that $(Y_n, Y_j)$ is an equilibrium outcome with $Y_j = f(Y_j)$. Suppose, however, that $Y_i < Y_i(G_i)$ and $Y_j < Y_j(G_j)$. Since the pair is in $S_1$, we have $Y_i \geq f^i(Y_j)$. Thus from Lemma A1,
Thus Claim for \( Y_i, Y_j \) implies \( Y_j(G_i) < Y_i(G) \). Given agent i's first and second period strategies, i can do better by engendering \( (Y_i(G_i), Y_j(G_j)) \) as an equilibrium outcome.

Let \( y_i^1 = Y_i(G_i) \). Then since \( Y_i(G_i) > Y_i(G_0) \) and \( y_i^1 < f(y_i^1) = Y_j(G_i) \), agent i's second period strategy dictates \( y_i^2 = 0 \). Again since \( y_j^1 < Y_j(G_i) \) and \( Y_j(G_i) > Y_j(G_0) \),
\[
y_j^2 = f(y_j^1) - y_j^1 = Y_j(G_0) - y_j^1.
\]
Thus \( Y_j = Y_j(G_i) \).

(\( \Longleftrightarrow \)) Take any \( (Y_i, Y_j) \) in \( S \). We analyze two cases:

Case 1: \( Y_i = f(Y_j) \)

If \( Y_j = f(Y_i) \), then it means \( Y_i = Y_i(G_0) \) and \( Y_j = Y_j(G_0) \). Claim 1 shows that such a pair is indeed an equilibrium outcome if it is in \( S_4 \). Now suppose \( Y_j \neq f(Y_i) \). It must be that \( Y_j > f(Y_i) \) and \( Y_j < Y_j(G_j) \) since the pair is in \( S_1 \) and \( S_3 \), respectively. Also since \( Y_i = f(Y_j) \), being in \( S_4 \) reveals either \( Y_j \geq Y_j(G_j) \) or \( Y_j \geq Y_j(G_j) \). From Lemma A1, we have \( Y_j > Y_j(G_0) \). We analyze different possibilities.

Suppose \( Y_j < Y_j(G_j) \). Then \( Y_i > Y_i(G_i) \) due to being in \( S \). Claim 2 shows that such a pair is supported as an equilibrium outcome. Now consider \( Y_j = Y_j(G_j) \). This means \( Y_i = f(Y_j) = Y_i(G_i) \). It is straightforward to show that this pair can be supported as an equilibrium by the second period strategies together with \( y_j^1 = Y_j(G_i) \) and \( y_i^1 = 0 \).

Case 2: \( Y_i \neq f(Y_j) \)

This case is qualitatively the same as case 1 by changing subscripts. Q.E.D.

Proof of Proposition 4.3:

(\( \Rightarrow \)) Suppose \( (Y_i(G_0), Y_j(G_0)) \) is the unique equilibrium outcome. Suppose however that w.o.l.g. \( Y_i(G_i) > Y_i(G_0) \) for agent i. Since this pair is an equilibrium outcome and
\[
Y_j(G_0) = f(Y_i(G_0)),
\]
we have \( Y_j(G_0) \geq Y_j(G_0) \). However it is easy to see that such \( (Y_i(G_i), Y_j(G_j)) \) pair is in \( S \) and thus an equilibrium as well, which contradicts the uniqueness assumption.

(\( \Leftarrow \)) Suppose that \( Y_i(G_i) < Y_i(G_0) \) for both agents. From Claim 1, we know that in this case \( (Y_i(G_0), Y_j(G_0)) \) can be obtained as an equilibrium.

Now suppose that there exists a feasible \( (Y_i, Y_j) \neq (Y_i(G_0), Y_j(G_0)) \) that is also an equilibrium. Then it must be in \( S \). This implies w.o.l.g \( Y_i = f(Y_j) \) and \( Y_j \neq f(Y_i) \). This also implies that \( Y_j < Y_j(G_0) \), which follows from having \( Y_j \leq Y_j(G_j) \) and \( Y_j(G_j) < Y_j(G_0) \) by hypothesis. Since i is on his reaction function, Lemma A1 reveals \( Y_j < f(Y_j) \). However, this contradicts \( Y_j > f(Y_j) \) following from the pair being in \( S \). Q.E.D.

Now we will prove Theorem 2 by first proving the following findings.
LEMMA B1: If \((Y_n, Y_j)\) is an equilibrium outcome, then \(y^*_i = \max \{0, f(Y_j) - Y_i^{T-1}\}\), and

\[
y^*_i = \begin{cases} 
0, & \text{if } f^i(Y_j) - Y_i^{t-1} \leq 0 \\
\leq f^i(Y_j) - Y_i^{t-1}, & \text{otherwise}
\end{cases}
\]

for \(t = 2, \ldots, T - 1\).

Proof: (by backward induction)

First note that \(y^*_i = \max \{0, f(Y_j) - Y_i^{T-1}\}\) immediately follows from the two period setting.

Now consider \(t = T - 1\) and suppose \(f(Y_j) - Y_i^{T-2} \leq 0\). Since, by definition, \(Y_i^{T-2} \leq Y_i^{T-1}\), we have \(y^*_i = 0\). Given this, \(y_i^{T-1} = 0\) is best for agent \(i\) due to quasi-concavity of \(U'(.)\) in \((Y_n, Y_j)\). Suppose now that \(f(Y_j) - Y_i^{T-2} > 0\). In this case, \(Y_i = f(Y_j)\) is feasible and best for agent \(i\). He can reach \(Y_i\) either by setting \(y_i^{T-1} = Y_i - Y_i^{T-2}\), in which case \(y_i^* = 0\) follows, or by setting \(y_i^{T-1} < Y_i - Y_i^{T-2}\) which implies \(Y_i^{T-1} < f(Y_j)\) and thus reveals

\(y^*_i = f(Y_j) - Y_i^{T-1}\).

To complete the induction argument, suppose \(y^*_i\) satisfies the above functional forms for \(t = t_0, t_0 + 1, \ldots, T\) for some \(t_0 \geq 3\). Now we will show that \(y_{i}^{t-1}\) admits the same functional form.

Consider the case \(f^i(Y_j) - Y_i^{t-2} \leq 0\). Since, again by definition, \(Y_i^{t-2} \leq Y_i^{t-1} \leq \ldots \leq Y_i\), it must be that \(y_i^{t-2} = \ldots = y_i^{T} = 0\) due to the induction assumption. Given this, \(y_i^{t-2} = 0\) is best for agent \(i\). Now consider \(f^i(Y_j) - Y_i^{t-2} > 0\). Again \(Y_i = f(Y_j)\) is feasible and best for agent \(i\). He can choose to set \(y_i^{t-2} = Y_i - Y_i^{t-2}\) in which case \(y_i^{t} = \ldots = y_i^{T} = 0\) as dictated by the induction hypothesis. However, if he chooses to set \(y_i^{t-2} < Y_i - Y_i^{t-2}\), then he can achieve \(Y_i\) in the remaining periods given the functional form \(y_i^{t}\) where \(t = t_0, \ldots, T\). Q.E.D.

COROLLARY B1: If \((Y_n, Y_j)\) is an equilibrium outcome, then \(Y_i \geq f(Y_j)\).

Proof: Since, in equilibrium, \(y^*_i = \max \{0, f(Y_j) - Y_i^{T-1}\}\), \(Y_i \geq f(Y_j)\) follows. Q.E.D.

LEMMA B2: If \((Y_n, Y_j)\) is an equilibrium outcome, then \(Y_i = f(Y_j)\) for at least one agent.

Proof: Suppose not. Then \(Y_i > f(Y_j)\) for both agents from Corollary above. Lemma B1 implies for both agents \(y^*_i = 0\), \(t = 2, \ldots, T\). Thus \(y_i^* = Y_i\) for both agents. However, in the first period, given \(j\)'s amount, agent \(i\) would be better off by at least marginally reducing his amount due to quasi-concavity of \(U'(.)\) in \((Y_n, Y_j)\), a contradiction. Q.E.D.
LEMMA B3: If \((Y_i, Y_j)\) is an equilibrium outcome and \(i\) has \(Y_i \neq f(Y_j)\), then \(Y_i \leq Y_i(G_j)\).

Proof: First note that given \(Y_i \neq f(Y_j)\), we have \(Y_i > f(Y_j)\) due to Corollary above. Also from Lemma B1, it must be that for agent \(i\) \(y_i^t = 0, t = 2, \ldots, T\). Thus \(y_i^1 = Y_i\). From Lemma B2, we have \(Y_j = f(y_j^1)\). Suppose \(Y_i > Y_i(G_j)\). We will argue that agent \(i\) would be better off by marginally reducing \(y_i^1\). Given \(\{y_i^1, \ldots, y_i^T\}\), suppose \(i\) chooses \((y_i^1 - \varepsilon)\) in the first period. If \(f(y_i^1 - \varepsilon) < Y_j\), then agent \(j\) would change any \(y_j^t, t = 2, \ldots, T\). In this case, \(i\) is better off since \(y_i^1 > f(Y_j)\). If, however, \(f(y_i^1 - \varepsilon) > Y_j\), then, given \(\{y_j^2, \ldots, y_j^T\}\), \(\bar{Y}_j = f_i(y_i^1 - \varepsilon)\) is feasible and best for \(j\). In this case, since \(y_i^1 > Y_i(G_j)\), agent \(i\) is better off due to quasi-concavity of \(U'(y_i^1, f(Y_j))\) in \(Y_i\). Thus, we must have \(Y_i \leq Y_i(G_j)\). Q.E.D.

LEMMA 4: If \((Y_i, Y_j)\) is an equilibrium outcome and \(Y_i = f(Y_j)\), then either \(Y_i \geq Y_i(G_j)\) or \(Y_j \geq Y_j(G_i)\).

Proof: Similar to proving in \(T = 2\) case that the equilibrium outcome is in \(S_4\). However, in this case, given \(y_i^1 = Y_i(G_i)\), agent \(j\) can accumulate his Stackelberg follower amount in \(T\) periods. Q.E.D.

Proof of Proposition 4.4:

(a)

(⇒) Easily follows from Corollary B1, Lemma B2 – 4 above.

(⇐) Now we will show by construction that the points in \(S\) are also equilibrium outcomes for \(T > 2\).

Define \((\bar{Y}_i, \bar{Y}_j)\) as \(U^j(Y_i(G_j), Y_j(G_j)) = U^j(\bar{Y}_i, \bar{Y}_j)\) and \(\bar{Y}_j = f^j(\bar{Y}_j)\). Let

\[L_j = \{(Y_i, Y_j) \in F_i \times F_j \mid U^j(Y_i, Y_j) \leq U^j(Y_i(G_i), Y_j(G_j))\}\]

\[K_j = \{(Y_i, Y_j) \in L_j \mid \bar{Y}_j \leq Y_j \leq Y_j(G_j)\}\]

\[A_j = \{(Y_i, Y_j) \in K_j \mid Y_j > f(Y_i)\text{ and } Y_j \leq Y_j(G_j)\}\]

\[B_j = \{(Y_i, Y_j) \in K_j \mid Y_j > f^j(Y_i)\text{ and } \bar{Y}_j \leq Y_i \leq Y_i(G_i)\}\]

\[C_j = \{(Y_i, Y_j) \in F_i \times F_j \mid Y_j \leq f(Y_i)\text{ and } Y_j \leq Y_j(G_i)\}\]

Define the following strategies from I to IV:

\[y_i^t = \begin{cases} 0, & \text{if } Y_i^{t-1} > f^i(Y_j^{t-1}) \text{ and } Y_j^{t-1} \geq f^j(Y_i^{t-1}) \\ Y_i(G_0) - Y_i^{t-1}, & \text{if } Y_i^{t-1} \leq Y_i(G_0) \text{ and } Y_j^{t-1} \leq Y_j(G_0) \\ 0, & \text{if } Y_i^{t-1} \geq Y_i(G_0) \text{ and } Y_j^{t-1} \leq f^j(Y_i^{t-1}) \\ f^j(Y_j^{t-1}) - Y_i^{t-1}, & \text{if } Y_j^{t-1} \geq Y_j(G_0) \text{ and } Y_i^{t-1} \leq f^i(Y_j^{t-1}) \end{cases}\]
\[
y_i^t = \begin{cases} Y_i(G_i) - Y_i^{t-1}, & \text{if } [Y_i^{t-1} \leq Y_i(G_i) \text{ and } Y_j^{t-1} \leq Y_j] \text{ or } (Y_i^{t-1}, Y_j^{t-1}) \in K_j \\
Y_i(G_i) - Y_i^{t-1}, & \text{if } (Y_i^{t-1}, Y_j^{t-1}) \in A_i \\
0, & \text{otherwise}
\end{cases}
\]

\[
y_i^t = \begin{cases} f^i(Y_j^{t-1}) - Y_i^{t-1}, & \text{if } Y_j^{t-1} \geq Y_j \text{ and } Y_i^{t-1} \leq f^i(Y_j^{t-1}) \\
f^i(Y_j^{t-1}) - Y_i^{t-1}, & \text{if } Y_i^{t-1} \leq f^i(Y_j^{t-1}) \\
0, & \text{otherwise}
\end{cases}
\]

Case 1: \(Y_i(G_i) \leq Y_i(G_0)\) or \(Y_j(G_i) \leq Y_j(G_0)\) for both agents

In this case, the first period strategies used in \(T = 2\) case (see Claim 1 and 2 above) together with adopting the strategy I for \(t = 2, \ldots, T\) support the outcomes in \(S\) as equilibria.

Case 2: \(Y_i(G_i) \leq Y_i(G_0)\) or \(Y_j(G_i) \leq Y_j(G_0)\) for agent i and \(Y_j(G_i) \geq Y_j(G_0)\) and \(Y_i(G_i) \geq Y_i(G_0)\) for agent j

In this case, agent j adopting strategy II for \(t = 2, \ldots, T - 1\), and strategy I for \(t = T\), and agent i adopting strategy I for \(t = 2, \ldots, T\) together with the first period strategies described in Claim 2 yield the point in \(S\) as the equilibrium.

Case 3: \(Y_i(G_i) \geq Y_i(G_0)\) and \(Y_j(G_i) \geq Y_j(G_0)\) for both agents

Agent i’s adopting strategy III for \(t = 2, \ldots, T - 1\) and strategy I for the last period, and agent j’s adopting strategy IV for \(t = 2, \ldots, T - 1\) and strategy I for the last period, together with both agents’ adopting first period strategies described in Claim 2 yield the outcomes in \(S\) as equilibrium results Q.E.D.

(b) Easily follows from Theorem 1 and part (a) of Theorem 2. Q.E.D.

Proofs of Lemma 4.2 and Proposition 4.5:

The proofs follow similar lines to the proofs of Lemma 1, Proposition 2 and 4.
REFERENCES


BIOGRAPHICAL SKETCH

Huseyin Yildirim was born in Denizli, Turkey, in 1972. He received his B.S. in industrial engineering at Bilkent University, Turkey in 1993. After finishing his M.A. in economics at Bilkent, he joined University of Florida for a Ph.D. in economics in 1995. He will graduate in May 2000 and continue his career at Duke University as an assistant professor.
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