THE EFFECT OF THE AVAILABILITY OF THE GEOMETER'S SKETCHPAD ON LOCUS-MOTION PROBLEM-SOLVING PERFORMANCE AND STRATEGIES

BY

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THE EFFECT OF THE AVAILABILITY OF THE GEOMETER'S SKETCHPAD ON LOCUS-MOTION PROBLEM-SOLVING PERFORMANCE AND STRATEGIES

By

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Major Department: Instruction and Curriculum

The purpose of this study was to investigate the problem-solving strategies used by geometry students following instruction in a dynamic technological environment. The study explored the relationships among the students' spatial visualization ability, mathematical ability, and problem-solving strategies with and without the availability of The Geometer's Sketchpad. Solution strategies were examined for tendency to use drawings and dynamic visualization as related to the availability of the technology during a locus of points problem-solving session. Three measures of spatial visualization were taken, including ETS Card Rotations, Cube Comparisons, and Paper Folding tests. A locus-motion problem inventory was administered. Participants for the study consisted of 158 geometry students from seven classes at one high school. Following instruction using The Geometer's Sketchpad,
students were randomly assigned to two groups for the Locus-Motion Inventory: those who had the computer software available, and those who did not. Individual interviews were held with two students from each class, one from each group.

The availability of the computer was not a significant factor for performance on the LMI. Covariates of spatial ability or mathematics achievement accounted for most of the variance. Teaching the specific skills of drawings and motion on the computer resulted in those strategies being used similarly by students with and without the Sketchpad. Although this is indicated in data analyses as nonsignificant differences, the implications are positive. Results suggest that strategies learned with the technology are transferable to paper and pencil situations, and that active participation in instructional activities is important to successful performance and use of strategies. Interview data revealed that students were able to solve problems successfully on the written LMI that they had previously missed. Students were able to go beyond the known problems and solve novel questions with some discussion of the components of the question itself. Language and communication were critical to expanding the student's zone of proximal development.
CHAPTER 1
INTRODUCTION

The technological society of today demands a fundamental restructuring in the educational environment. The impact of technology provides two main issues for mathematics education: the changing perspectives on the learning of mathematics and the changing role of technology in the classroom (Mathematical Sciences Education Board [MSEB], 1990). Technology can assist in the introduction, development, and reinforcement of mathematical concepts. In the curriculum, computers and calculators offer opportunities for new conceptualizations of instruction and cognition and opportunities to learn new content, in different sequences, and with a higher degree of sophistication at every level.

The overall goal of mathematics education is for students to learn and experience the power of mathematics (National Council of Teachers of Mathematics [NCTM], 1989). Understanding mathematical concepts is the key to success and power in mathematics. However, the results of evaluations by the National Assessment of Educational Progress (NAEP) indicate that performance by students on basic skills is inadequate (Carpenter et al., 1988). The
foundations for mathematical understanding need to be strengthened. A curriculum that is active and conceptually oriented is recommended to assist students in gaining the foundations and understanding (NCTM, 1989). An environment of exploration and discovery will help develop the skills of inquiry and problem solving.

Technology may be the key to unlocking the door to a revised mathematics curriculum and learning environment. "Of all the influences that shape mathematics education, technology stands out as the one with the greatest potential for revolutionary impact" (MSEB, 1990, p. 22). The technological society is a reality and the mathematics classroom must adapt to the changing scientific workplace or fail to reach its primary goals of providing power and opportunities to the students of today in the world of tomorrow.

According to the constructivist perspective of learning, students build their own interpretative frameworks for making sense of the world, including the mathematical world (Schoenfeld, 1987a). Implementation of a dynamic technological environment that promotes visual and dynamic problem-solving strategies may influence the curriculum, the instructional techniques, and the learning perspectives in the mathematics classroom. Students construct their own knowledge by engaging in problem solving and actively reorganizing their own experiences (Cobb, Yackel, & Wood, 1991). Actively participating in "doing mathematics" aids
students through the interaction of previous knowledge with new experiences in the classroom. Providing multiple ways in which to approach mathematics learning and problem solving allows students to build knowledge from their own cognitive level. Alternative approaches and modes of thinking need to be examined for their effectiveness in facilitating understanding and concept acquisition.

The geometry curriculum with its potential for inductive and deductive reasoning provides a rich environment for problem solving and spatial thinking. However, the traditional geometry classroom, buried in proof and structure, has not been particularly conducive to an inquiry approach. A modified geometry curriculum may encourage a more versatile, integrated approach to learning. The use of computers and dynamic modeling has drastically changed the nature of mathematics education for algebra (Hershkowitz, 1990). Such a change is needed in geometry and may be promoted by combining observations and intuition with multiple representations in a dynamic environment.

**Purpose of the Study**

The purpose of this study was to investigate the problem-solving strategies used by geometry students following instruction in a dynamic technological environment. Specifically, the study explored the relationships among the students' spatial visualization ability, mathematical ability, and problem-solving
strategies with and without the availability of technology. Students were instructed with presentation and "hands on" experiences using a software product called The Geometer's Sketchpad (Jackiw, 1993). Specific lessons on locus-motion concepts provided the basis for exploring problem-solving performance and strategies. Solution strategies were examined for tendency to use drawings and dynamic visualization as related to the availability of the technology during the problem-solving session.

Rationale

To engage in mathematics is to participate in activities of problem solving. Learners have to construct their own knowledge, through a process of reflection (Davis, Maher, & Noddings, 1990; Noddings, 1990; von Glaserfeld, 1987) and in the process of purposeful activity (Krutetskii, 1976; Sowder, 1989). Metacognitive aspects of learning, such as self-monitoring, -regulation, and -evaluation, are pertinent to efficient construction of knowledge and require explicit instruction in the curriculum (Silver, 1985). Constructivism emphasizes the role of the construction process as well as the awareness of that process and how to modify it (Confrey, 1990). This study attempted to investigate the effect of a specific learning environment on the content and nature of students' learning. Investigations such as this are needed to support current
and future theories and practices (Balacheff et al., 1990; Sowder, 1989).

The Cognitive Flexibility Theory of learning, knowledge representation, and knowledge transfer acknowledges the ability to restructure one's knowledge spontaneously in many ways in response to changing situational demands (Spiro & Jehng, 1990). Revisiting the same material from multiple perspectives is the key to learning and transfer in complex domains (Spiro et al., 1991). Cognitive Flexibility combines the constructivist nature of learning and patterns of learning failure to examine dynamic cases from multiple conceptual perspectives (Spiro et al., 1991). Use of dynamic cases in this study added to the knowledge concerning students' response to multiple representations in novel situations in the complex domain of mathematics. Application of the Cognitive Flexibility Theory utilizes the random access capability of the computer to provide flexibility and pluralistic representations (Spiro & Jehng, 1990). Computer environments are an ideal tool to provide multiple representations in mathematics, allowing one to shift from one representation to another to find the form most useful (Dreyfus, 1990; Fey, 1989; Kaput, 1987, 1989; Senechal, 1990; Sowder, 1989). "Interactive technologies provide a means of intertwining multiple representations of mathematical concepts and relationships—like graphs and equations or numbers and pictorial representations of the objects the number represent" (Pea, 1987, p. 109). Several
aspects of the computer environment need to be examined, including how knowledge is affected by the environment and how students' cognitive behavior and constructs are modified (Balacheff et al., 1990).

The interactive computer environment is also revolutionizing the study of shape and visualization within the study of mathematics (Senechal, 1990; Tall & Thomas, 1989; Tillotson, 1984/1985). "Today the microcomputer is increasing the range of aids to visualization enormously, and its presence in mathematics classrooms is also stimulating a great deal of research and development in this area" (Bishop, 1989, p. 7). Spatial ability is often a controversial subject when discussing mathematical performance and achievement. The definition of spatial ability, the number of components or factors involved, the relationship of spatial ability to problem solving and mathematical performance, and the ability to learn or increase spatial skills are all topics that require continued research and investigation (Steen, 1990).

Some researchers suggest that spatial ability is a unitary trait (Johnson & Meade, 1987; Moses, 1984). Others distinguish two (Conner & Serbin, 1980; Tartre, 1990; Tillotson, 1984/1985), or three (Linn & Petersen, 1985) distinct components. Whether spatial ability is innate or subject to instruction is also an issue. Many researchers and theorists believe that spatial ability is trainable (Ben-Chaim, Lappan, & Houang, 1988; Hembree, 1992; Moses,
Furthermore, instruction is not only desirable, but necessary to improve spatial skills (Dreyfus, 1990; Krutetskii, 1976). These topics will be explored further in the literature review.

Students need explicit instruction not only in the skill of spatial visualization, but also in the ability to monitor their own endeavors in the learning process (Dreyfus, 1990; Silver, 1985; von Glaserfeld, 1987). Students need to attend to the metacognitive aspects of learning: self-monitoring, regulation, and evaluation of cognitive activity (Schoenfeld, 1989; Silver, 1985). Constructivism emphasizes that students can learn to improve their reasoning and problem-solving ability and become agents of their own learning (Paris & Byrnes, 1989; Zimmerman, 1989). This investigation explores whether instruction can influence awareness of dynamic problem-solving strategies in novel problem situations.

Improvement in, and assessment of, mathematical ability is intertwined with the mode of instruction and the activity in progress. A change in assessment that more closely correlates to instruction is needed (NCTM, 1989). It is becoming clearer that a single static level of assessment is not representative of student learning, either in terms of process or in terms of product (Campione, Brown, & Connell, 1989; Ferrara, 1987/1988; Krutetskii, 1976; Wearne & Hiebert, 1988). Assessment, like instruction, should begin
at the current knowledge level of the student, leading him or her into new mathematical territory (Campione et al., 1989; Ferrara, 1987/1988; Lampert, 1991). The difference between what a student can do alone and what he or she can do with the assistance of a teacher or more capable peer (Rohrkemper, 1989) has been termed by Vygotsky (1978) as the zone of proximal development. Placing the emphasis on the initial and possible learning states of the student, and the environment in which learning is taking place, is consistent with the constructivist perspective of cognitive change (Linn & Songer, 1991). The variety of tasks offered in this study introduced students to locus of points and extended the student’s knowledge of problem-solving processes beyond the practiced level to the next dynamic level of thinking.

Although it has been proposed that knowledge is constructed by the learner and that mathematics is a complex domain, research is needed to provide descriptions of these theories applied to specific domains (Hiebert & Wearne, 1991; Wearne & Hiebert, 1988). According to Kantowski (1981), problem-solving processes depend as much on the type of problem as on the style of the solver. Krutetskii takes this idea one step further to state that specific content ability depends on instruction. The ill-structured domain of mathematics is too broad to be investigated in global terms. A well-defined unit in geometry, such as locus of points, provides a specific domain in which to explore further the relationships of spatial visualization and the
use of dynamic technology in a constructivist learning environment. The content area of locus of points was chosen because the subject is independent of placement in the curriculum (provided that basic concepts have been introduced) and is often left out of the geometry curriculum due to time constraints.

The topic of locus of points is enhanced by a dynamic environment and the use of diagrams. The computer has the potential to provide a fruitful environment for exploring diagrams, for conjecturing and searching for patterns and generalizations (Bishop, 1989; Hershkowitz, 1990; Janvier, 1987; Lesh, 1987; Senechal, 1990; Steen, 1990). Two issues are raised by the use of calculators and computers in the classroom: finding a balance of conventional skill, understanding, and problem-solving ability that is appropriate to the new technology and discovering ways that technology opens entirely new approaches to thinking about mathematical ideas and problems (Fey, 1984). Emphasis in geometry may shift to experimentation with shapes and relations, building strong geometric intuition and a spirit of inquiry inherent in all mathematics. The instructional phase of this study explored the use of a relatively new teaching technique and medium in the geometry classroom, providing some insight into ways the curriculum might change and techniques of designing and structuring new learning activities.
Many researchers agree that computer environments provide multiple representations, important for flexibility of thought in problem solving (Dreyfus, 1990; Kaput, 1987; Sowder, 1989; Tall & Thomas, 1989). However, researchers must concentrate on determining the effect of this learning environment on what and how the students learn (Balacheff et al., 1990; Sowder, 1989). Although Sowder (1989) is referring to the study of algebra, not geometry, the same sentiment holds true: "Not much research has been done to evaluate what students learn by interacting with the programs or how the programs might be used within the context of the . . . curriculum" (p. 30).

Outline of the Study

This study was designed as a "constructivist teaching experiment" (Cobb, Wood & Yackel, 1990) to investigate the problem-solving strategies used by geometry students following instruction in a dynamic technological environment and to explore the relationships among the students' spatial visualization ability, mathematical ability, and problem-solving strategies with and without the availability of the Sketchpad. This study addressed some of the needs for research in a domain-specific content area of locus of points.

Three independent measures of spatial visualization were taken. They included tests distributed by the Educational Testing Service (ETS). The three tests were
Card Rotations, Cube Comparisons, and Paper Folding. A locus-motion problem inventory (LMI) was also administered. Participants for the study consisted of 158 geometry students from seven geometry classes at one high school. All students participated in the instruction using *The Geometer’s Sketchpad*. The instructional phase included lessons that extended previous lessons on the Sketchpad and introduced the concepts of locus of points. Students were randomly assigned to two groups for the Locus-Motion Inventory: those who would have the computer software available and those who would not. The problem-solving session followed a constructivist approach of creating a "problematic environment that would elicit the student’s adequate constructive endeavors" (Fischbein, 1990, p. 8).

Data analysis had both a quantitative and a qualitative component.

**Research Questions**

The study addressed three questions of interest:

1. Are the correct response score (CRS), tendency to use drawings (DR), and/or use of dynamic strategies (DS) that students utilize to answer locus-motion problems related to the students’ spatial visualization index (SV) or mathematics achievement (MA)?

2. Does Sketchpad availability (SA) affect correct response score (CRS), tendency to use drawings (DR), or use of dynamic strategies (DS)?
3. Are the tendency to use drawings (DR) and/or the use of dynamic strategies (DS) that students utilize when attempting to solve locus-motion problems related to the students’ correct response score (CRS)?

**Definition of Key Terms**

*Spatial visualization*, or spatial ability, is defined in this study as the ability to recognize the relationships among the elements of a given figural configuration, and to manipulate mentally one or more of those parts.

*Spatial visualization index* (SV) is operationalized as the sum of the z-scores, or standardized scores, of the three tests in the Spatial Visualization Battery.

*Mathematics achievement* (MA) is the score measured on a geometry semester examination given to all participants.

*Locus-motion inventory* (LMI) is a set of 14 problems relating to locus of points. Nine problems were used for the written portion of the LMI, and five problems were available for interviews.

*Correct response score* (CRS), or locus-motion problem-solving performance, is the number of correct responses on the written LMI.

*Locus-motion problem-solving strategies*, or methods of solutions, are scored in two ways: the tendency to use drawing (DR) and the use of dynamic problem-solving strategies (DS).
Dynamic software, or dynamic technological environment, refers to *The Geometer's Sketchpad* (Jackiw, 1993) implemented on an IBM local area network.

**Summary**

Mathematics education is in the process of change, a change, as indicated within the *Standards*, that recommends new curricula and modes of instruction and provides mathematical power to the students. As innovations are proposed, planned, and implemented, investigation into their feasibility and effectiveness is needed. Technology provides one vehicle for variations in the content addressed and the methods employed in the mathematics classroom.

This chapter presented the rationale for a study designed to examine the effect of instruction in a dynamic technological environment on performance and strategies employed in attempting locus-motion problems. The investigation included the exploration of relationships among the students' spatial visualization ability, mathematical ability, and problem-solving strategies with and without the availability of the *Sketchpad* during problem solving. The research questions pertaining to the study, as well as the key definitions, have been outlined here.

Chapter 2 contains a review of literature pertaining to the questions of interest, focusing on previous research of spatial ability, technology in education, geometry, and student cognition. A discussion of the methodology and
procedures utilized for the study is found in Chapter 3. Chapter 4 presents the results of the data analyses. The findings and their implications for instruction and research are found in Chapter 5.
CHAPTER 2
REVIEW OF RELATED LITERATURE

The theoretical premise that supports this study is that the use of a dynamic technological environment, such as The Geometer's Sketchpad, can enhance construction of knowledge by students and influence problem-solving strategies using dynamic visualization skills. The constructivist theory of learning promotes the belief that students build their own knowledge through activity and experience. Dynamic software can provide experiences in visualizing, conjecturing, and building frameworks within a specific content domain. Experience will not only influence the students' ability to perform successfully on the specific content problems in the activities, but also be useful in solving problems that are novel and perhaps more difficult.

The review of the literature presented in this chapter, therefore, focuses on studies and research pertaining to cognition and constructivist theory, spatial visualization, and curriculum and instruction in geometry. The connections between constructivism and Cognitive Flexibility Theory are explored. Within the sections on cognition and geometry, research concerning the use of technology are a point of focus.
Mathematics educators generally agree with the basic tenet of constructivist theory that learners construct knowledge within an active environment. Krutetskii states that "mathematical abilities exist only in a dynamic state, in development; they are formed and developed in mathematical activity" (1976, p. 66). The brain is not passive, but active, and engages in processes of selection, interpretation, and inference (Orton, 1987). Noddings (1990) summarizes the views of constructivist theories as follows:

1. All knowledge is constructed. Mathematical knowledge is constructed, at least in part, through a process of reflective abstraction.
2. There exist cognitive structures that are activated in the processes of construction. These structures account for the construction; that is, they explain the result of cognitive activity in roughly the way a computer program accounts for the output of a computer.
3. Cognitive structures are under continual development. Purposive activity induces transformation of existing structures. The environment presses the organism to adapt.
4. Acknowledgement of constructivism as a cognitive position leads to the adoption of methodological constructivism.
   a. Methodological constructivism in research develops methods of study consonant with the assumption of cognitive constructivism.
   b. Pedagogical constructivism suggests methods of teaching consonant with cognitive constructivism.

An example of how students build knowledge can be seen in a study by Davis in a fifth grade class (Davis & Maher, 1990). In small groups, with manipulatives available, students solved a problem involving fractional parts of a
pizza. Detailed investigation of the solution processes of two boys evinced the constructing of the solution in layers of thinking, layers of conversation. As the two students interacted with each other and with various solution approaches, the successful solution emerged from the use of concrete representation. As the students broke the problem down into parts, the tendency was to look at the parts separately, not at the whole picture. The building of concrete representations may have allowed the students to concentrate on the parts, but still be aware of the entire problem setting.

"Put into simple terms, constructivism can be described as essentially a theory about the limits of human knowledge, a belief that all knowledge is necessarily a product of our own cognitive acts" (Confrey, 1990, p. 108). Cognitive acts can be, and need to be, planned by the learner or by the instructor. Following a five day study with one teacher, supported by student interviews, Confrey developed a model of teacher's instruction that supports pedagogical constructivism. Constructivist models of teaching recommend learning environments that promote processes including acquisition of basic concepts, algorithmic skills, problem-solving heuristics, and habits of reflective thinking (Davis, Maher, & Noddings, 1990).

One of the skills that must be developed is flexibility (Davis, 1986). "Good problem solvers tend to be sufficiently flexible in their use of a variety of relevant
representational systems that they instinctively switch to the most convenient representation to emphasize at any given point in the solution process" (Lesh, Post, & Behr, 1987, p. 38). Successful problem solvers are able to build mathematical representations of problem situations and relate problems together that have a similar structure (Schoenfeld, 1987a).

Examining the skills that the successful problem solver and the gifted student possess reveals that their thinking process may be different (Dover & Shore, 1991). Dover and Shore (1991) studied 30 11-year-old students (19 of them school-identified gifted and 11 of average ability) focusing on set-breaking with the water jug problem. Results showed a three way interaction among giftedness, speed, and flexibility. The gifted students exhibited more planning, more reflection, and more flexibility.

Successful learners construct these metacognitive representations at the same time that they build the representation for the problem. Davis (1986) conducted three diverse studies that yielded similar results. One study involved a mathematics teacher solving a problem not in his area of expertise. A second study looked at a musical theme. A third study was a task-based interview of a university calculus student working a quadratic equation. In each situation two representations were built: one for the solution of the problem itself and another at the
metalevel, an observation of the method for resolving the question.

These and other metacognitive skills must be addressed directly, and students taught to recognize and apply them in the proper situations (Campione, Brown, & Connell, 1989; Hershkowitz, 1990; Janvier, 1987; Lester, 1989; Schoenfeld, 1987). Hembree (1992) found similar conclusions in a meta-analysis of 487 reports and studies in problem solving. Results of instructional methods on problem solving revealed that students who received instruction in problem solving and heuristics had higher scores than did those with no explicit training. Effects of classroom-related conditions were also examined. Computer assisted problem instruction provided better results than did paper and pencil problem solving. Teaching the specific subskill of diagram drawing also showed positive results. "Taken together, these findings suggest that the dominant factor in problem solving is less IQ than mental development" (Hembree, 1992, p. 268). However, Resnick (1989) offers a caution about metacognitive training:

This points to a fundamental problem with certain metacognitive training efforts that focus attention on knowledge about problem solving rather than on guided and constrained practices in doing problem solving. Such efforts may be more likely to produce ability to talk about processes and functions than to perform them. (p. 43)

Several studies cited by Campione et al. (1989) that investigated the inductive reasoning processes of 5-year-old and 6-year-old students indicated that students were not
given explicit instructions and did not attend to their own processes of learning. One study by Ferrara (1987/1988) included a learning phase, followed by dynamic assessment, aimed at promoting transfer. Dynamic tests, as described by Campione et al. (1989) and Ferrara (1987/1988), provided information about the ability of the student as well as the student's potential for improvement.

Assessment, like instruction, should begin at the current knowledge level of the student and lead him or her into new mathematical territory (Campione et al., 1989; Ferrara, 1987/1988; Lampert, 1991). The difference between what a student can do alone and what he or she can do with the assistance of a teacher or more capable peer (Rohrkemper, 1989) has been termed by Vygotsky (1978) as the zone of proximal development. Schoenfeld describes this process of learning: "One acquires higher order thinking skills by exercising those skills in the ZPD with the help of others and then internalizing those skills, that is, by mastering as an individual those skills for which one, at once, needed support" (1987b, p. 210). In a case study from the "Computer as Lab Partner" project, Linn and Songer (1991) referred to this difference as the range of possible cognitive changes. Students interacted with computer-based experiments, simulations, and instruction. Results indicated that, with proper guidance and active engagement of the learner, students learned to conduct investigations, made predictions and constructed principles that they were
not previously able to do. Focusing on the zone of proximal development fostered the speed of conceptual change. Salomon (1989) produced similar results in a study of reading-related metacognitive guidance with seventh grade students. Seventy-four students were randomly assigned to three treatment groups with three versions of a computer program with varying levels of guidance and varying cognitive levels of questions. Salomon reached the conclusion that "computers can serve as tools that provide guidance in a child’s zone of proximal development" (1989, p. 626).

The studies by Linn and Songer and by Salomon reflect similar cognitive constructs to those found in the Cognitive Flexibility Theory. The Cognitive Flexibility Theory of learning, knowledge representation, and knowledge transfer provides for multiple representations of the same material in rearranged instructional sequences and from different conceptual perspectives (Spiro & Jehng, 1990). The goal is to be able to apply independently the learned knowledge and processes to new situations.

An application of this theory is seen in a project described by Spiro and Jehng (1990) in which a random accessed videodisc of Citizen Kane provided the foundational instructional materials. Students engaged in theme-based or scene-based explorations. A cognitive flexibility hypertext, a computer language, allowed for texts to be constructed in multiple situations, thematically oriented.
Commentary was provided to expound upon the thematic and symbolic contrasts within the scene. The goal was not only to explore the themes and symbols within *Citizen Kane*, but also to demonstrate the complex nature of literature and provide students with active experiences with processing that complex knowledge and building new metacognitive skills.

Many of the principles of Cognitive Flexibility Theory are also the basis for constructivism (Noddings, 1990; Spiro et al., 1991). Students interpret knowledge, not absorb it. Active participation and exploration by the student is crucial. An environment of exploration allows construction of knowledge, with transfer of knowledge and skills to new situations the ultimate goal. The metacognitive aspects of constructing knowledge is important for transfer. Purposive activity with learner control and self-regulation builds new frameworks. Those activities must provide multiple representations of the concepts within the domain. Constructivist tenets outlined by Paris and Byrnes (1989) are also reflected in the research by Spiro et al. (1990, 1991). Instructional innovations that are task-oriented are needed to promote increased metacognition. Metacognitive skills are a necessary characteristic of good problem solvers, as is flexible thinking.

According to Cognitive Flexibility Theory, instruction in ill-structured domains begins with complex treatment, separated into manageable dynamic cases that provide
multiple representations of the same concept or theme. However, it is important to avoid compartmentalization of information, narrowing the focus of the idea. Revisiting the same material multiple times, in multiple ways promotes flexible thinking by the student.

Mathematics and problem solving are considered by many researchers (and many students!) to be complex or ill-structured domains (Davis, 1986; Polya, 1957, Resnick, 1989). As a complex domain, mathematics can be approached from the Cognitive Flexibility theoretical basis of knowledge representation and transfer. Linear presentation is not sufficient for understanding; multiple representations are necessary to provide a network of interrelated ideas (Romberg, 1988; Spiro & Jehng, 1990). A single conceptual perspective is not incorrect, just inadequate (Spiro et al., 1991). "The area of the representation of dynamic situations (involving action, transformations, movements) is particularly intriguing and corresponds to domains where children encounter learning difficulties" (Dufour-Janvier, Bednarz, & Belanger, 1987, p. 122).

"The dynamic and interactive media provided by computer software make gaining an intuitive understanding (traditionally the province of the professional mathematician) of the interrelationships among graphic, equational, and pictorial representations more accessible to the software user" (Pea, 1987 p. 96). Pea (1987) states
that the dynamic display of multiple representations is valuable not only to discover the relationships, but also to learn the skills of translating from one representation to another, consonant with the constructive perspective of building, interpreting, and relating knowledge (Resnick, 1989). "The computer appears able to offer qualitatively new thinking tools mainly through its graphic facilities and the possibility of the synchronous representation of different but related processes and situations" (Balacheff et al., 1990, p. 145).

**Spatial Visualization**

Many studies have investigated whether spatial ability is a single trait or a composite of traits. While investigating gender differences in developmental patterns of spatial ability, Johnson and Meade (1987) considered spatial ability to be a unitary trait. Moses (1977/1978) investigated the composition of spatial ability and came to the same conclusion. Moses described spatial ability as the ability "to perceive the essential relationships among the elements of a given visual situation, and to mentally manipulate one or more of these elements" (1977/1978, p. 18). The purpose of the study conducted by Moses (1977/1978) was two-fold: (1) to refine the definition of spatial ability and investigate relationships between spatial ability and variables connected with the problem-solving process; and (2) to determine the effects of
instruction on spatial ability, problem-solving performance, and degree of visuality. In the study 145 fifth-grade students in four classes were given the same battery of six tests measuring spatial ability and problem solving, prior to and following the instructional phase. The spatial tests were the Punched Holes Test, Card Rotations Test, Form Board Test, Figure Rotations Test, and Cube Comparisons Test. A Problem Solving Inventory constructed by Moses was the sixth test. The instructional phase consisted of nine weeks of lessons involving two- and three-dimensional geometric perceptions and some problem-solving tasks using visual solution processes. Correlation coefficients, factor analyses, and regression analyses were performed to investigate the decomposability of spatial ability. Pearson product-moment correlation coefficients were used to analyze relationships among the variables. Analyses of covariance were performed to analyze the effects of instruction.

Conclusions were reported as follows:

1. Spatial ability is not decomposable into the ability to manipulate an entire figure and the ability to manipulate parts of the figure. Moreover, some of the tests which have been classified as spatial tests can be solved in an analytic manner while other tests are pure spatial tests.

2. Although spatial ability is a general cognitive ability, it is a good predictor of problem-solving performance.

3. An individual with high spatial ability will frequently not write down visual solution processes as part of his solution.

4. Problem-solving performance is best predicted by a factorially pure spatial test rather than a spatial test involving other cognitive activities.

5. Successful problem-solving performance on spatial problems can be predicted better by spatial ability than by degree of visuality.
6. Spatial ability is a modifiable quantity, i.e., instruction will affect the spatial ability of an individual, affecting males more than females.

7. Instruction in certain visual processes does not significantly affect the degree of visuality of an individual. Instruction aided neither males nor females on their degree of visuality scores.

8. Instruction in certain visual processes does not significantly affect the general problem-solving performance of an individual; however, it does affect, in a positive manner, success on spatial problems.

9. Instruction does not significantly affect the problem-solving performance of males nor of females. Instruction has the same amount of effect on the problem-solving performance of both high and low spatial ability individuals.

10. Instruction significantly affects success on spatial problems more than success on analytic problems. (pp. 144-154)

Other researchers have distinguished two or three distinct components (Conner & Serbin, 1980; Linn & Petersen, 1985; Tartre, 1990; Tillotson, 1984/1985). Components have been termed by researchers as spatial orientation, spatial visualization, spatial perception, mental rotation, spatial relations, and kinesthetic imagery (Conner & Serbin, 1980; Linn & Petersen, 1985; Tartre, 1990; Tillotson, 1984/1985). "Spatial visualization is distinguished from spatial orientation tasks by identifying what is to be moved; if the task suggests that all or part of a representation be mentally moved or altered, it is considered a spatial visualization task" (Tartre, 1990, p.217). Tartre continued describing spatial orientation as "those tasks that require that the subject mentally readjust her or his perspective to become consistent with a representation of an object presented visually" (1990, p. 217).
Tartre (1990) used these definitions as she explored the role of the spatial orientation component in the solution of mathematics problems with 57 tenth-grade students. The sample of students was chosen from those who scored in the top or bottom third on the Gestalt Completion Test. According to Tartre this test was chosen because "it was the best test to capture the essence of pure spatial thought. . . . That is, the tasks would be solved holistically, it appeared unlikely that verbal or analytic processes would contribute to subjects' solutions, and the items directly required the structural organization of visual information in order to make sense out of the partial pictures" (1990, p. 220). The problem-solving interview consisted of 10 mathematics problems, geometric and nongeometric, that could be solved in more than one way. Students were asked to solve the problems, talking aloud as they did. Interviews were recorded and later coded according to the following categories: Correct answer, Done like, Failure to break set, Mental movement, Misunderstood problem, Added marks, Drew picture, Drew relation, and Estimate error. Tartre concluded that "spatial orientation skill appears to be used in specific and identifiable ways . . . accurately estimating the approximate magnitude of a figure, demonstrating the flexibility to change an unproductive mind set, adding marks to show mathematical relationships, mentally moving or assessing the size and shape of part of a figure, and getting the correct answer
without help to a problem in which a visual framework was provided" (1990, p. 227).

However, like other researchers (Fennema & Tartre, 1985; Ferrini-Mundy, 1987; Kantowski, 1981; Krutetskii, 1976; Moses, 1977/1978), Tartre (1990) noted that spatial skill, and other mathematical skills, may be linked to more general thought patterns when making sense of new material and may be directly related to the specific mathematical skill, test, or activity in progress.

To discuss spatial ability in terms of the way in which the individual solves or thinks through the problem is the basis of Krutetskii’s (1976) analysis of mathematical ability in general. Krutetskii (1976) described two different modes of thought: analytic-logical and visual-pictorial. In his analysis, these modes of thinking and the corresponding abilities were interconnected with the mathematical activity in progress. Other researchers have reached a similar conclusion that the type of activity is an important factor (Fennema & Tartre, 1985; Ferrini-Mundy, 1987; Kantowski, 1981; Moses, 1977/1978; Tartre, 1990).

Thus, the issue of the definition of spatial ability and its component traits may be further confused as the activities and thought processes in which students engage become more dynamic with the use of computer technology. "In speaking of abilities, one means the psychological characteristics of the person involved in the activity; in speaking of skills (habits), one means the psychological characteristics of the
person's activity. . . . We must stress that in analyzing skills and habits as well as ability, we are analyzing an activity" (Krutetskii, 1976, p. 71).

Tartre summed up the discussion of the definition and components of spatial ability.

Attempting to understand and discuss something like spatial orientation skill, which is by definition intuitive and nonverbal, is like trying to grab smoke: The very act of reaching out to take hold of it disperses it. It could be argued that any attempt to verbalize the processes involved in spatial thinking ceases to be spatial thinking. Spatial skill use is mental activity. Any evidence about how it is manifested must be indirect, since we cannot get into people's heads and see what they see in their mind's eye. Often, the processes involved are not even understood by the people experiencing them. The resulting indirectness of the research in this area does set limits on it but should not curtail it. If spatial skills are important to mathematics, then researchers must find ways to identify and describe the specific roles that spatial skills play in doing mathematics. (1990, p. 229)

Although the relationship of spatial skills to problem solving and mathematical ability has also been questioned, many researchers have found that a correlation does exist between visualization and mathematics (Battista, Wheatley, & Talsma, 1989; Ferrini-Mundy, 1987; Tillotson, 1984/1985; Usiskin, 1987; Vinner, 1989). Those researchers who are not convinced of the direct correlation with mathematical achievement do, however, acknowledge that pursuing the relationship of spatial skill and mathematics is worthwhile (Fennema & Tartre, 1985; Lean & Clements, 1981; Tartre, 1990). Krutetskii came to the conclusion that "the ability to visualize abstract mathematical relationships and the ability for spatial geometric concepts
... did not determine the extent of mathematical giftedness, but did determine its type, or cast of mind" (1976, p. 315). Lean and Clements expressed the need for additional investigation "before relationships between spatial ability and mathematical performance can be clarified" (1981, p. 277).

Background review in the study by Tartre (1990) previously discussed evinced the beliefs by many researchers, such as Fennema and Sherman (1977), McGee (1979), and Conner and Serbin (1985), that spatial skills are related to mathematics learning and achievement.

Ferrini-Mundy based an investigation of gender differences of achievement and spatial ability of calculus students upon the premise that there is "a well-established finding of male superiority on tests of spatial ability, as well as correlational and logical support for a relationship between mathematics performance and spatial ability" (1987, p. 126). Primary questions of interest were gender differences and the effects of the training program on calculus achievement and spatial ability. The sample of students included 66 in three large groups and 34 in each of four smaller groups. Over an eight-week period, treatment groups viewed six slide-tape modules and taped commentaries with a variety of tasks and situations with spatial visualization and orientation. Control groups participated in standard algebra and trigonometry reviews. Results indicated that there was "no treatment effect on calculus
achievement or spatial visualization ability. . . . There were interaction effects for calculus achievement and the use of visualization in solving solid-of-revolution problems and a significant treatment effect for visualization of solids of revolution" (Ferrini-Mundy, 1987, p. 126). Ferrini-Mundy (1987) suggested that significant training effects might have resulted if a wider variety of spatial tests had been used to measure spatial ability, since only the Space Relations Subtest, Form T, of the Differential Aptitude Test was used. There were indications that training may be more successful for women than for men.

Calculus content and spatial visualization was also the topic of consideration in a study by Vinner (1989) of 67 college students. The course was designed to emphasize the visual aspects of every algebraic concept and theorem. Comparisons of algebraic versus visual proofs indicated that students chose algebraic proofs even when drawings provided a visual proof upon examination. Vinner concluded that students believe that algebraic proofs are more mathematically acceptable, and memorization of formulae and algorithms more successful in assessment situations. "Thus, it seems that there is no research evidence that visual thinking is not needed for success in higher mathematics" (Vinner, 1989, p. 150).

Presmeg (1986) reached analogous conclusions to those by Vinner: that mathematical instruction and assessment may emphasize the nonvisual processes, aiding the nonvisual
learner to excel in mathematical performance. An initial investigation by Presmeg in 1985 of mathematical "stars" chosen by teachers revealed that they were predominantly nonvisualisers. Presmeg researched the effect of teacher cognitive modes, attitudes, and actions upon high school students who were "visualisers." The thirteen teachers were grouped according to the visuality of their teaching. Of the 277 seniors scored for their mathematical visuality, 54 students chosen as visualisers were selected for task-based interviews. External factors, such as time constraints of school testing procedures and teaching methods and textbooks that favor the nonvisualiser, contributed to the "preponderance of nonvisualisers amongst mathematical high achievers" (Presmeg, 1986, p. 305). Presmeg also hypothesized that visual teachers could teach visual students more effectively than intermediate or nonvisual teachers.

Regardless of the disputable nature of spatial ability and its relationship to mathematical achievement, increasing the awareness and ability of spatial visualization and spatial thinking may benefit students aiming them toward the goal of mathematical power.

In view of the fact that most teachers are unaware of the difficulties associated with visual processing in mathematics, and of the fact that these difficulties may be overcome, it seems likely that increased teacher awareness of these issues could aid visualisers in entering the category of 'stars.' (Presmeg, 1986, p. 309)
Although some studies have not reached significant results on the trainability of spatial ability, Ben-Chaim, Lappan, and Houang (1988) provided data that supported the possibility that these skills can be taught, can be learned. A sample of 1000 middle school students at three sites involving 21 teachers participated in a study of gender differences, grade differences, and effect of instruction on spatial visualization ability. Immediately before and after a three week spatial visualization unit, students were administered the Middle Grades Mathematics Project (MGMP) Spatial Visualization Test, an untimed test with 10 different types of items. A retention test was given to a subsample of students 4 weeks following the posttest. The results of the training period were significant. "The most important result of this investigation was that after the instruction intervention, middle school students, regardless of sex, gained significantly from the training program in spatial visualization tasks" (Ben-Chaim et al., 1988, p. 66).

As indicated in the previous discussion of the definition of spatial ability and its relationship to mathematical problem solving and performance, Tillotson (1984/1985) described spatial ability as having at least two components, and concluded that there is a significant relationship between spatial visualization and mathematics, specifically problem solving. Tillotson came to these conclusions as a result of a dissertation study,
investigating the nature of spatial visualization, the correlation of spatial visualization to problem-solving performance, and the effect of instruction on spatial abilities. In the study 102 sixth grade students in five classes at two comparable schools were given the same battery of four tests measuring spatial ability and problem solving prior to, and following, the instructional phase. Control and experimental groups were designated by school. The spatial tests were the Punched Holes Test, Card Rotations Test, and Cube Comparisons Test. A Problem solving Inventory constructed by Tillotson (1984/1985) was the fourth test. The instructional phase consisted of ten weeks of lessons, one 45-minute lesson per week, focusing on activities designed to improve the student's perceptual skills. During the first and last weeks the four tests were given. Correlation coefficients, factor analyses, and regression analyses were performed to investigate the decomposability of spatial ability. Pearson product-moment correlation coefficients were used to analyze relationships among the variables. Analyses of covariance were performed to analyze the effects of instruction.

Conclusions were reported as follows:

1. Spatial visualization is not a single ability.
2. Spatial visualization is not a skill taught in current mathematics curriculum.
3. Spatial visualization is a good predictor of general problem solving.
4. Spatial visualization is a trainable attribute.
5. Visualization instruction does not significantly affect problem solving performance.
6. Instruction affects performance on analytic problems differently than spatial problems. (pp. 100-104)

Supporting the inclusion of spatial skills in the mathematics curriculum, Tillotson echoed the conclusions of many other researchers (Ben-Chaim, Lappan & Houang, 1988; Dreyfus, 1990; Hembree, 1992; Krutetskii, 1976; Moses, 1977/1978; Presmeg, 1986; Vinner, 1989). Furthermore, explicit instruction is not only desirable, but necessary to improve spatial skills (Dreyfus, 1990; Krutetskii, 1976). Creating the proper environment is also an important aspect. According to the constructivist, a goal of the educator would be "the creation of a problematic environment that would elicit the student's adequate constructive endeavors" (Fischbein, 1990, p. 8).

**Geometry**

The correspondence of spatial ability and geometry is agreed upon by many researchers (Battista, Wheatley, & Talsma, 1989; Ferrini-Mundy, 1987; Tillotson, 1984/1985; Usiskin, 1987; Vinner, 1989). Preservice elementary teachers were the subjects of a study that investigated the relationship between the strategies used in geometric problem solving and two abilities, spatial visualization and formal reasoning (Battista, Wheatley, & Talsma, 1989). The study investigated the following questions:

1. Is either the selection or effectiveness of problem-solving strategies that preservice elementary teachers utilize when attempting to solve geometry
problems related to the students' spatial visualization ability or formal reasoning ability?
2. Is achievement in a geometry course for preservice elementary teachers related to either the selection of effectiveness of problem-solving strategies utilized when attempting to solve geometry problems?
3. Do those elementary teachers who are successful at geometric problem solving utilize different strategies than those who are not successful? (p. 18)

Five sections of preservice elementary teachers (81 females, 2 males) were administered a modified Purdue spatial visualization test, a modified version of the Longeot test, and an investigator-constructed geometry problem-solving test. The type of strategy used and the effectiveness of the strategy was investigated. Strategies included drawing, visualization, and nonspatial. Percent use of strategy and percent effective use of strategy scores were also given.

One of the most interesting results was that although visualization was used more frequently than drawing and nonspatial strategies, the drawing strategy was used more effectively. The more useful strategies are not the ones used the most. Recognition of this discrepancy may be useful to students as they monitor their problem-solving processes.

Battista indicated that "the balance between spatial and logical ability is likely to be an important factor in geometry performance in general" (1990, p. 48). The correlation between spatial visualization and logical reasoning and geometric problem solving was further examined by Battista in a study similar to the one with preservice
teachers. Focusing on different levels of geometry achievement and on gender differences, Battista (1990) tested five intact classes of 145 high school geometry students. The variables that were tested were spatial visualization, logical reasoning, knowledge of geometry, geometric problem-solving strategies, the discrepancy between a student’s spatial score and logical reasoning score, and use of correct drawings. Tests included Sheehan’s version of the Longeot test of formal operations, Modified Purdue Spatial Visualization test, Cooperative Mathematics Test Geometry Part 1 Form B, and the Geometric Problem Solving/Strategies test constructed by Battista. Strategies were classified as drawing, visualization without drawing, nonspatial, or none of the above. Intercorrelations between variables indicated the following results (Battista, 1990):

1. Spatial visualization and logical reasoning were significantly related to both geometry achievement and geometric problem solving for males and females.

2. Spatial visualization was significantly correlated with strategy variables of drawings and nonspatial strategies for males and with drawings and correct drawing for females.

3. Discrepancy score was significantly correlated with drawings, visualization without drawings, and nonspatial strategies for males.
4. For students with a low level of geometry achievement correlations were significantly higher between spatial visualization and geometric problem solving than between logical reasoning and problem solving.

5. Gender differences on different variables were also reported.

Battista suggested that future research investigate interrelationships between "representational schemes and problem-solving strategies in geometry" (1990, p. 57). The study also "suggests that instructional variables may be critical factors in understanding interrelationships between variables, gender differences, and geometry learning" (1990, p. 59).

An experimental verification of a method and system of exercises for developing spatial imagination was conducted by Vladimirskii (1971) to determine the role of the diagram in mastering geometric material. The premise was that the basic task of geometry is to develop geometric thought, and to apply theoretical knowledge in problem solving. Typically, the study of geometry is mostly the memorization and reproduction of proofs, not the generalization of concepts. The diagram is the most often used visual aid, but not fully used enough to promote learning of the concepts.

The experiments were conducted with sixth and ninth grade students to determine the part played by the diagram and ways of using and improving the learning by the student
with the diagram. Preliminary exercises included moving cube imaginations, use of solid and dotted drawings. Resulting problem situations developed from the fact that book diagrams are too constrained, thus restricting the understanding of the general concept. Particular relationships of the book diagram were taken to be essential features of the diagram. Conclusions from the control experiments were:

1. The diagram may be a hindrance as well as a help in the reasoning process.

2. If the concept has not been defined and learned by the student, the properties cannot be transferred to new material or problems.

The two goals of the exercise were to develop spatial imagination and foster the formation of geometric concepts, the complexity of the exercises increasing with the complexity of the diagram. The types of tasks were recognition and composition of diagrams, and explanations of the geometric relationships.

The experiment concentrated on the notion of transformation: translation, rotation, and reflection, and on shifting figures mentally without use of models. Results from the sixth grade indicated that graphic material can develop spatial imagination. Conclusions were that the major goal should be to eliminate flaws in present methods of teaching geometry, and to develop spatial imagination, and to form more precise concepts.
Hershkowitz (1989) further examined acquisition of basic geometry concepts in two experimental situations. Subjects were students in grades five, six, and seven in two schools, and elementary teachers, both preservice and inservice. The process involved defining a concept previously unknown to the participants, and having them select or draw examples of the concept. The number of critical attributes influenced the accuracy of the example choices. "There is a negative correlation between the number to concept attributes and the mean success score in the task" (1989, p. 67). A single prototype shape was often envisioned by the participants. The role of visualization is a complex process, according to Hershkowitz (1989), in that one cannot form an image of a concept without visualizing its elements, but that the visualization made may constrict the correct image of the concept. The computer may be of benefit. "Visualization and visual processes have a very complex role in geometrical processes . . . that a dynamic interaction with a geometrical microworld . . . contributes to visual flexibility. More work is needed to understand better the positive and negative contributions of visual processes" (Hershkowitz, 1990 p. 94).

Diagrams themselves do provide obstacles to the learner (Yerushalmy & Chazan, 1990; Zykova, 1969). Yerushalmy and Chazan (1990) investigated the research literature concerning the obstacles that diagrams create. In addition
to research previously mentioned (Presmeg, 1986), a study by Yerushalmy in 1986 revealed that students using computer software called the Geometric Supposer used more diagrams in a generalized end of year test than did those students that did not have Supposer experience. Experience with computer images may increase the utilization and usefulness of diagrams, and remove some of the obstacles that they cause.

From 1984 to 1988 the effect of the Supposer on students and teachers in 23 high school geometry classes was studied (Yerushalmy et al., 1990). In an inquiry approach to the teaching/learning process, data was gathered from six sources: classroom observations, student Supposer work, minutes of monthly teacher meetings, teacher interviews, teacher reflections, and student interviews. Guidelines for writing clear materials and creating good inquiry problems resulted from the investigations. Inquiry teaching is important for mathematics, and presents "challenges and difficulties" (Yerushalmy et al., 1990, p. 242) to bring it into the classroom. "The formulation of inquiry problems will be important to the successful development of guided inquiry approaches using other tool-based software environments in geometry . . . and in other domains" (Yerushalmy et al., 1990, p. 242).

Another study involving the use of the computer and the Geometric Supposer indicated an improved performance in geometry achievement. McCoy (1991) investigated the effect of the software on geometry achievement with 10th grade high
school honors students. Results indicated that there did exist an effect on problem-solving skills and on geometry achievement from integrating *Supposer* activities into the curriculum every two weeks.

Bishop restates one of these obstacles, "One problem in geometry teaching is that it is impossible to draw a generalized diagram. . . . It is therefore necessary to present many diagrammatic examples of a geometric concept if the learners are not to be restricted by the specificity of the diagram" (1983, p. 180). Bishop (1989) and Stewart (1990) further reinforced the relationship of spatial visualization and geometry, and suggested that the dynamic visual images of computer graphics may aid in the development of spatial skills.

The computer has the potential to resolve many of these obstacles, not only for spatial visualization, but also for the processes of conjecturing, searching for patterns, and generalizations, that is, the processes of "doing mathematics" and constructing knowledge (Bishop, 1989; Hershkowitz, 1990; Janvier, 1987; Lesh, 1987; Senechal, 1990; Steen, 1990).

**Summary**

The review of the literature presented in this chapter focused on studies and research pertaining to cognition and constructivist theory, spatial visualization, geometry, and the utilization of technology. According to the
constructivist theory of learning, students build new knowledge from prior knowledge and experiences within an environment of active participation. The research provided a strong basis for investigations into student learning and computer environments within domain specific content areas. Spatial visualization and geometry present rich areas for examination of content, cognition, and the learning environment. The research also indicated a need for further examination of how students build knowledge, specifically within an environment of dynamic technology available to the mathematics classroom today.
CHAPTER 3
METHODOLOGY

Overview of the Study

This chapter describes the research questions, the evaluation instruments, and the participants for the study. It outlines the processes and procedures of the design and implementation, and the data analyses that were used.

The purpose of this study was to investigate the problem-solving strategies used by geometry students following instruction in a dynamic technological environment within a "constructivist teaching experiment" (Cobb, Wood & Yackel, 1990). Specifically, the study explored the relationships among the students' spatial visualization ability, mathematics achievement, and problem-solving strategies with and without the availability of software called The Geometer’s Sketchpad. Students were instructed with teacher presentation and "hands on" experience using the Sketchpad. Specific lessons on locus-motion concepts provided the basis for exploring problem-solving strategies. The problem-solving session followed the constructivist approach of creating a "problematic environment that would elicit the student's adequate constructive endeavors" (Fischbein, 1990, p. 8). Solution strategies were
investigated for the tendency to use drawings and use of dynamic strategies in relationship to the availability of the technology during the problem-solving session with the Locus-Motion Inventory (LMI).

**Research Questions**

The study addressed three questions of interest:

1. Are the correct response score (CRS), tendency to use drawings (DR), and/or use of dynamic strategies (DS) that students utilize to answer locus-motion problems related to the students' spatial visualization index (SV) or mathematics achievement (MA)?

2. Does Sketchpad availability (SA) affect correct response score (CRS), tendency to use drawings (DR), or use of dynamic strategies (DS)?

3. Are the tendency to use drawings (DR) and/or the use of dynamic strategies (DS) that students utilize when attempting to solve locus-motion problems related to the students' correct response score (CRS)?

Research questions were investigated by more detailed statistical questions:

1. Do the relationships of SV or MA with CRS, DR, or DS vary across the Sketchpad availability groups?

2. If the relationships do not vary across Sketchpad availability, what is the relationship of CRS, DR, or DS with SV and MA?
3. Does Sketchpad availability affect CRS, DR, or DS on the Locus-Motion Inventory?

4. Do the relationships of DR or DS with CRS vary across the Sketchpad availability groups?

5. If the relationships do not vary across Sketchpad availability, what is the relationship of CRS with DR and DS?

Selection of Evaluation Instruments

Measures of spatial visualization and mathematics achievement were taken to examine as predictor variables, and to provide blocking or matching variables for statistical procedures, such as those used by Tall and Thomas (1989) in their study of teaching algebra with the computer. Three independent measures of spatial visualization were taken. Mathematics achievement data was taken from the first semester geometry examination. This curriculum based test given to all regular geometry students was created by Glencoe for the Merrill Geometry text. A locus-motion problem inventory was created and administered specifically for this study. A description of each instrument is contained in the next section.

The Spatial Visualization Battery

The Spatial Visualization Battery consisted of three measures included in the Educational Testing Service Kit of Factor-Referenced Cognitive Tests. Within the ETS cognitive
factor of Spatial Orientation are the Card Rotations Test and the Cube Comparisons Test. An additional test, Paper Folding, was used from the cognitive factor of Visualization which also includes the Form Board and Surface Development Tests. Since each of the three chosen tests has different scoring schemes, the results were converted to standardized scores and summed for a single factor called spatial visualization index. Standardized scores, or z-scores, are corrected for the mean and scaled by the standard deviation of the variable. They are useful when comparing or combining different scoring schemes.

Each of these tests has been used alone or in conjunction with other tests in studies involving spatial ability by Moses (1977/1978) and Tillotson (1984/1985). As indicated in previous chapters, Tartre (1990) reported that these tests have also been cited as evidence of spatial ability by Conner and Serbin (1980) and Linn and Petersen (1985). Originally ETS considered all three tests elements of the factor called spatial visualization. They were chosen for this study based upon research to represent multiple parts of the dimensions of spatial ability under consideration.

Card Rotations Test

The purpose of this set of problems is to test the student’s ability to recognize figures that have been placed in a different orientation. Tillotson stated that the objective of the test is "to measure a student’s ability to
recognize the relationships among parts of a figure in order to identify the figure when its orientation is changed" (1984/1985, p. 42). The test has been identified with the dimension called spatial visualization or rotation (Moses, 1977/1978; Tartre, 1990; Tillotson, 1984/1985).

The test is two parts of 10 problems each. Each problem consists of an initial irregular figure on the left followed by eight various representations of the figure. The students decide whether each one of the eight figures is the same as, or different from, the one at the left, and mark a box for each as S or D. A transformation by translation or rotation is considered the same. A transformation by reflection is considered different. Students were given 3 minutes for each of the two parts of the test. The score was the number of items answered correctly minus the number answered incorrectly.

**Cube Comparisons Test**

This test displays cubes with three of the six faces showing. The purpose of the test is to measure the student's ability to recognize parts of a given configuration (Tillotson, 1984/1985). Although some analyses categorize this test as a measure of spatial visualization (Tartre, 1990; Tillotson, 1984/1985), other analyses suggest that it may relate to a different factor of mathematical ability, possibly a more analytical dimension (Moses, 1977/1978).
Each problem contains drawings of two cubes that can be drawings of the same cube in a different orientation, or must be different cubes altogether. The test contains two parts with twenty-one pairs of cubes each. Students mark S or D for each pair. Students were given 3 minutes for each of the two parts of this test. The score was the number of items answered correctly minus the number of items answered incorrectly.

**Paper Folding Test**

The purpose of this test, also known as the Punched Holes Test, is to measure the student’s ability to mentally manipulate a given spatial configuration into a different one. It also has been identified with the dimension called spatial visualization or rotation (Moses, 1977/1978; Tartre, 1990; Tillotson, 1984/1995).

In each problem, two to four figures represent a square piece of paper being folded and then punched with a hole. The student’s task is to match that representation with the correct representation of the unfolded piece of paper with hole(s) punched in the proper locations. The test contains two parts with 10 problems each. Five answer choices are provided for each problem, but only one is correct. Students were given 3 minutes for each of the two parts of the test. The score was the number of items answered correctly minus one-fourth of the number of items answered incorrectly.
Search of the literature revealed several global problem-solving inventories, and many specific to geometry or spatial orientation (Krutetskii, 1976; Moses, 1977/1978; Tillotson, 1984/1985). However, the criterion to provide domain-specific content, such as locus-motion problems, caused difficulty in finding a problem-solving inventory specific to the content and applicable to dynamic problem-solving strategies. The problem-solving inventory specific to the content of locus of points was created by the researcher, incorporating locus-motion problems similar to those found in geometry textbooks. The content area of locus of points was chosen because the subject is independent of placement in the curriculum (provided that basic concepts have been introduced) and is often left out of the geometry curriculum due to time constraints. Studies by Ferrara (1987/1988), Campione (1989), and Tillotson (1984/1985) prompted the following considerations for the initial set of 30 problems: prerequisite geometry content, varying level of difficulty, and novelty of problems.

This set of problems was reviewed by six current or preservice mathematics educators, to assist in devising the final inventory of problems. Each problem was reviewed in light of the following questions: Is the problem understandable and workable for this level of geometry class? Can the problem be solved with paper and pencil? What is the difficulty level of the problem? Will spatial
visualization techniques of motion or dynamism be useful in solving the problem? Would the problem be especially interesting in an interview setting? Based upon the reviewer ratings, the set was reduced to nine problems for the written LMI. At least one problem from each difficulty level was selected. All problems were able to be solved with pencil and paper only. Five problems were also chosen to be available for students to solve in an interview setting. See Appendix A for all LMI problems.

Of interest to the researcher was not only the performance on the problems, but also the strategies employed in reaching solutions. Problem-solving strategies identified and scored were tendency to use drawings and use of dynamic strategies, scored through observation, written work and computer drawings. Another means of ascertaining some measure of the problem-solving process used by a student was to ask the student for his or her own assessment of his strategy (Battista, 1990; Battista, Wheatley, & Talsma, 1989). Each problem on the written portion consisted of two parts. The students were asked to provide the solution to the problem, and to describe the solution or thinking process involved. Data for each problem consisted of all written work, including figures and printed copies of sketches created on the computer by students using The Geometer's Sketchpad.

Correct response score, tendency to use drawings and use of dynamic strategies were the measurements on the
Locus-Motion Inventory. The score sheet for the LMI can be seen in Appendix B. Correct response score was awarded from 0 to 1 in increments of 0.25, based on amount of success. Scores for tendency to use drawings were 0 for no drawings, 1 for one, 2 for two, 3 for multiple drawings. Scores for use of dynamic strategies were 0 or 1, depending on the indication in words, or figures, of use of motion, or dynamic thought processes. Several scoring rubrics were used as models for the one created (Battista, 1990; Battista, Wheatley, & Talsma, 1989; Fennema & Tartre, 1985; Ferrara, 1987/1988; Ferrini-Mundy, 1987; Krutetskii, 1976; Moses, 1977/1978; Presmeg, 1986; Tartre, 1990). The scoring rubric also took into consideration some concerns addressed by Lean and Clements (1981) about the types of questions and strategies required, use of incorrect solution attempts, and unwritten strategies. Although coming from two very different perspectives, Spiro et al. (1991) and Senechal (1990) suggested that data include the number of different images used. Yerushalmy and Chazan (1990) noted that students taught with the Geometric Supposer used diagrams more in the thinking process, including free-hand drawings, than those that had not been taught using the Supposer.

Sixteen percent of the problems were coded by an additional rater and interrater reliability verified (Ferrara, 1987/1988; Ferrini-Mundy, 1987; Wearne & Hiebert, 1988). Correlations between the coders for CRS, DR, and DS were 0.91, 0.83, and 0.79, respectively.
Task-based interviews were conducted with 14 students, 9% of the total sample, randomly selected within each class from each SA group. Davis (1986) described the task-based interview as students solving specific problems talking aloud, and an interviewer observing those solutions with audio or video recording. Interviews have been used in many studies and recommended by many researchers to explore the problem solving and thinking processes of students (Bishop, 1983; Confrey, 1990; Davis, 1983; Davis 1986; Davis & Maher, 1990; Ginsburg et al., 1983; Krutetskii, 1976; Presmeg, 1986; Tartre, 1990; von Glaserfeld, 1987). The types of interview strategies varied. Based upon previous research the interview was designed to observe students in three situations: explaining and confirming the solutions to problems completed on the LMI (Lean & Clements, 1981), attempting problems at the next level of difficulty from the written LMI, and attempting problems available only for interview (Campione et al., 1989; Ferrara, 1987/1988; Hoffer, 1983; Lampert, 1991; Rohrkemper, 1989). The interviewer began with the student’s work from the written LMI and proceeded to problems specifically designated for the task-based interviews, depending upon the level of difficulty and success of the student, exploring the activity of the student within his or her zone of proximal development (Vygotsky, 1978). As Krutetskii (1976) and others have noted, it is important to discover what the student has learned and what he is capable of learning with
the aid of others. All work was collected for examination, as was the transcribed audiotape of the interview (Davis, 1986; Dover & Shore, 1991; Resnick, 1989). Students were instructed to talk as they worked, a technique used in several studies and promoted within a variety of assessment tools (Ginsburg et al., 1983; Huinker, 1993; Mashbits, 1975).

Selection and Description of Population Sample

Participants for the study consisted of 158 geometry students from seven geometry classes at one high school. The seven classes were taught by four mathematics teachers interested in encouraging the use of technology in the mathematics classroom, specifically in using The Geometer’s Sketchpad in the geometry curriculum. The subjects were of mixed gender, age, and racial background in grades 9 to 12. All students participated in the instruction using The Geometer’s Sketchpad. The students were randomly assigned to two groups. One group worked the LMI in the traditional paper and pencil manner. Another group had the computer and software available as they solved the problems. Two students from each class period were randomly chosen, one from each group, to participate in the interview session. A total of 14 students worked problems within an interview setting.
Prior to the study the University of Florida’s Institutional Review Board granted permission for the investigation to take place. Students were informed and had to obtain parental permission (see Appendix C) to participate in the study.

The Spatial Visualization Battery was administered on two occasions, before any exposure to the Sketchpad and following the LMI session. A script explained the purpose and directions for the three parts of the battery. First semester examination scores were provided by the teachers. The problems from the Locus-Motion Inventory were presented to the students to be worked within one 50-minute class period. The classroom teachers administered the paper and pencil session; the researcher administered the computer session. A script was used by all administrators to execute the LMI. The interviews were conducted by the researcher, with audiorecording of the interview transcribed afterward. Each interview lasted approximately 20 minutes.

Description of the Instructional Phase

The effectiveness of instruction using technology can best be explored in the context in which it will occur in reality in the classroom (Fischbein, 1990; Kulik & Kulik, 1991). The instructional phase was a continuation of introduction to the software already in use in the geometry
classes. Teachers used the software in a presentation mode for several months. Students had several lessons of "hands on" experience with the software. The researcher presented all lessons on locus of points. The lessons were an extension of previous lessons on use of the Sketchpad, and lessons that introduced the concepts of locus of points.

There were six lessons, each lasting one 50-minute class period. Each was a combination of teacher demonstration, student discovery, and student experience with problems on the computer. Initial lessons presented the concept of locus and the steps involved in solving locus problems, specifically applied to perpendicular bisectors, angle bisectors, and triangles. Points of concurrency such as incenter, circumcenter, centroid, and orthocenter were explored. Other lessons extended the application of locus to circles. Finally, the concept of locus of points was applied to the intersection of figures, and further, to the intersections of loci.

The researcher conducted all lessons to eliminate differences in results due to teacher variability, but was aware of the possibility that a threat to validity may be introduced by the researcher influencing results. This was minimized by the use of specific presentation plans, and written activities for students with the computer. Seven problem sets and worksheets used during the instructional phase can be found in Appendix D.
Portions of the lessons were tested during the spring of 1993. The pilot was a series of lessons that introduced students to *The Geometer’s Sketchpad* and explored the intersection of figures. The lessons varied from "very guided" with specific instructions to open investigations guided only by presenting problem situations and questions. The purpose of the pilot was to make an initial evaluation of the clarity of the activities and instructions. Two groups of three students each worked on the activities with no teacher assistance or intervention. Each student provided observations and journal entries that discussed the instructions, activities, and their involvement and attitude with the computer and *The Geometer’s Sketchpad*. The results of the pilot study were used to revise the activities that introduced *The Geometer’s Sketchpad* to the students.

**Statistical Procedures**

Data analysis consisted of three parts: descriptive statistics, quantitative statistical analyses, and qualitative investigations. Descriptive statistics were obtained for all variables under consideration. For each question, appropriate statistical procedures were applied. Review of studies in the literature, especially those dealing with spatial abilities and mathematics, suggested several statistical procedures applicable to a similar study design (Ferrara, 1987/1988; Ferrini-Mundy 1987; Lean & Clements, 1991; Moses, 1977/1978; Tall & Thomas, 1989;
Tartre, 1990; Tillotson, 1984/1985). Statistical procedures for this study included correlation coefficients, analyses of covariance, and multiple regression analyses. The statistical approach is often problematic when dealing with educational issues of instruction and learning (Schoenfeld, 1985). In addition to quantitative statistical procedures, a qualitative component examined the interview data describing the problem-solving processes of individual students.

Summary

This study was designed as a "constructivist teaching experiment" (Cobb, Wood & Yackel, 1990) to investigate the problem-solving strategies used by geometry students following instruction in a dynamic technological environment and to explore the relationships among the students' spatial visualization ability, mathematical ability, and problem-solving strategies with and without the technology. Three independent measures of spatial visualization were taken. They included ETS Card Rotations, Cube Comparisons, and Paper Folding tests. A locus-motion problem inventory was also administered. Participants for the study consisted of 158 geometry students from seven geometry classes at one high school. All students participated in the instruction using The Geometer's Sketchpad. The instructional phase included six lessons that were an extension of previous lessons on use of the Sketchpad and lessons that introduced the concepts of locus of points. Students were randomly
assigned to two groups: those who would have computer software available during the LMI, and those who would not.

Data analysis had both a quantitative and a qualitative component. Statistical procedures, including descriptive statistics, correlations, analyses of covariance, and multiple regression analyses, were used to explore the questions of interest in the study. The results of data analyses are presented in Chapter 4 and discussed in Chapter 5.
CHAPTER 4
ANALYSIS OF THE DATA

This chapter contains the descriptive statistics for the variables under investigation and results of the data analyses pertinent to the questions of this study. It also includes the results of the qualitative investigations by observation and interview.

This study was designed as a "constructivist teaching experiment" (Cobb, Wood & Yackel, 1990) to investigate the problem-solving strategies used by geometry students following instruction in a dynamic technological environment and to explore the relationships among the students' spatial visualization index, mathematics ability, correct response scores, tendency to use drawings, and use of dynamic strategies with and without the Sketchpad available.

Three independent measures of spatial visualization were taken at two different occasions. The spatial visualization index included the ETS Card Rotations, Cube Comparisons, and Paper Folding tests. A locus-motion problem inventory (LMI) was also administered. Participants for the study consisted of 158 geometry students from seven geometry classes at one high school. All students participated in the instruction using The Geometer's
Sketchpad. The instructional phase included six lessons that were an extension of previous lessons on use of the Sketchpad and lessons that introduced the concepts of locus of points.

Students were assigned individually and at random to two groups for the LMI problem-solving session: those who would have the Sketchpad available, and those who would not. The problem-solving session lasted one class period of 50 minutes. Individual interviews were held on the following day with two students from each class, one from each group. Interviews lasted approximately 20 minutes during which students worked locus-motion problems aloud.

**Research Questions**

The study addressed three questions of interest:

1. Are the correct response score (CRS), tendency to use drawings (DR), and/or use of dynamic strategies (DS) that students utilize to answer locus-motion problems related to the students' spatial visualization index (SV) or mathematics achievement (MA)?

2. Does Sketchpad availability (SA) affect correct response score (CRS), tendency to use drawings (DR), or use of dynamic strategies (DS)?

3. Are the tendency to use drawings (DR) and/or the use of dynamic strategies (DS) that students utilize when attempting to solve locus-motion problems related to the students' correct response score (CRS)?
Results of Data Analysis

Descriptive statistics are presented in Table 4-1, which includes the number (N), means (M), and standard deviations (S) by Sketchpad availability (SA) for the following variables: correct response score (CRS), tendency to use drawings (DR), use of dynamic strategies (DS), spatial visualization index (SV), and mathematics achievement (MA).

MA was measured by first semester geometry examination scores. A total of 60 was possible, but the highest achieved was 56. CRS, DR and DS were the sums of the measurements of the individual problems on the locus-motion inventory (LMI). Problem scores for CRS were awarded from 0 to 1 in increments of 0.25, based on amount of success. Problem scores for DR were 0 for no drawings, 1 for one, 2 for two, 3 for multiple drawings. Problem scores for DS were 0 or 1, depending on the indication in words, or figures, of use of motion, or dynamic thought processes. SV was based on the composite z-scores for the three measures taken at the time of the LMI: Card Rotations, Cube Comparisons, and Paper Folding Tests. The correlations between the two occurrences of the spatial visualization tests were high; therefore, the measures taken closest in time to the LMI were used in analysis. Correlations between occurrences were as follows: CR = 0.74, CC = 0.64, PF = 0.72, and overall SV = 0.83.
Table 4-1. Descriptive Statistics With and Without Sketchpad

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sketchpad</th>
<th>Number</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRS</td>
<td>With</td>
<td>78</td>
<td>1.82</td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>Without</td>
<td>81</td>
<td>1.99</td>
<td>1.62</td>
</tr>
<tr>
<td>DR</td>
<td>With</td>
<td>78</td>
<td>14.08</td>
<td>6.32</td>
</tr>
<tr>
<td></td>
<td>Without</td>
<td>81</td>
<td>15.92</td>
<td>6.12</td>
</tr>
<tr>
<td>DS</td>
<td>With</td>
<td>78</td>
<td>3.45</td>
<td>2.02</td>
</tr>
<tr>
<td></td>
<td>Without</td>
<td>81</td>
<td>3.1</td>
<td>2.08</td>
</tr>
<tr>
<td>SV</td>
<td>With</td>
<td>75</td>
<td>-0.07</td>
<td>2.30</td>
</tr>
<tr>
<td></td>
<td>Without</td>
<td>90</td>
<td>0.28</td>
<td>2.37</td>
</tr>
<tr>
<td>MA</td>
<td>With</td>
<td>75</td>
<td>42.38</td>
<td>7.97</td>
</tr>
<tr>
<td></td>
<td>Without</td>
<td>94</td>
<td>42.86</td>
<td>7.71</td>
</tr>
</tbody>
</table>

Although students were randomly assigned, for the group without the Sketchpad available for the LMI, the means were somewhat higher on matching variables of SV and MA. The means were also higher for that group on the LMI for CRS and DR. Only the mean for DS was higher for the group with the Sketchpad available. Due to the scoring rubric for CRS allowing scores in increments of 0.25 the mean score is low. Students did not achieve full credit in many instances.

Research questions were investigated by more detailed statistical questions. The results of these analyses are
reported in the following paragraphs. An alpha level of 0.05 was used for all tests.

1. Do the relationships of SV or MA with CRS, DR, or DS vary across the Sketchpad availability groups?

The slope of the relationships of CRS, DR and DS with SV and MA were compared across Sketchpad availability groups by using multiple regression. The results are reported in Table 4-2 and indicate no variation over groups in the six relationships.

**Table 4-2. Results Comparing Relationships Across Sketchpad Availability Groups**

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Independent variable</th>
<th>MSE</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRS</td>
<td>SV</td>
<td>2.10</td>
<td>2.36</td>
<td>.1265</td>
</tr>
<tr>
<td></td>
<td>MA</td>
<td>2.14</td>
<td>0.80</td>
<td>.3712</td>
</tr>
<tr>
<td>DR</td>
<td>SV</td>
<td>36.52</td>
<td>0.68</td>
<td>.4123</td>
</tr>
<tr>
<td></td>
<td>MA</td>
<td>35.21</td>
<td>1.54</td>
<td>.2165</td>
</tr>
<tr>
<td>DS</td>
<td>SV</td>
<td>3.54</td>
<td>0.76</td>
<td>.3894</td>
</tr>
<tr>
<td></td>
<td>MA</td>
<td>3.78</td>
<td>0.00</td>
<td>.9898</td>
</tr>
</tbody>
</table>

2. If the relationships do not vary across Sketchpad availability, what is the relationship of CRS, DR, or DS with SV and MA?

The analyses relevant to question 1 indicated the relationship of SV and CRS, SV and DR, and SV and DS did not vary across Sketchpad availability. Similarly the
relationship of MA and CRS, MA and DR, and MA and DS did not vary across groups. Consequently the pooled relationships of SV with each of CRS, DR, and DS as well as the pooled relationship between MA and each of CRS, DR and DS were investigated using multiple regression. Results are shown in Table 4-3. For each of CRS, DR, and DS there is a significant relationship with SV and MA, \( p < .01 \).

Table 4-3. Results of Pooled Relationships of SV and MA with CRS, DR, and DS

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Independent variable</th>
<th>( r )</th>
<th>MSE</th>
<th>F</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRS</td>
<td>SV</td>
<td>.38</td>
<td>2.11</td>
<td>23.74</td>
<td>.0000</td>
</tr>
<tr>
<td></td>
<td>MA</td>
<td>.37</td>
<td>2.14</td>
<td>22.22</td>
<td>.0000</td>
</tr>
<tr>
<td>DR</td>
<td>SV</td>
<td>.30</td>
<td>36.44</td>
<td>11.17</td>
<td>.0011</td>
</tr>
<tr>
<td></td>
<td>MA</td>
<td>.34</td>
<td>35.34</td>
<td>15.05</td>
<td>.0002</td>
</tr>
<tr>
<td>DS</td>
<td>SV</td>
<td>.38</td>
<td>3.54</td>
<td>24.02</td>
<td>.0000</td>
</tr>
<tr>
<td></td>
<td>MA</td>
<td>.28</td>
<td>3.75</td>
<td>12.27</td>
<td>.0006</td>
</tr>
</tbody>
</table>

3. Does Sketchpad availability affect CRS, DR, or DS on the Locus-Motion Inventory?

To investigate the question, for each of CRS, DR and DS an analysis of covariance (ANCOVA) was conducted with SV and MA as covariates. Results are reported in Table 4-4 and indicate that Sketchpad availability does not significantly affect CRS, DR, or DS.
Table 4-4. Summary ANOVA Table for CRS, DR, and DS

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRS</td>
<td>SA</td>
<td>1</td>
<td>1.64</td>
<td>1.64</td>
<td>0.82</td>
<td>0.3673</td>
</tr>
<tr>
<td>Error</td>
<td>139</td>
<td></td>
<td>279.36</td>
<td>2.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DR</td>
<td>SA</td>
<td>1</td>
<td>109.46</td>
<td>109.46</td>
<td>3.12</td>
<td>0.0797</td>
</tr>
<tr>
<td>Error</td>
<td>139</td>
<td></td>
<td>4883.9</td>
<td>35.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DS</td>
<td>SA</td>
<td>1</td>
<td>2.89</td>
<td>2.89</td>
<td>0.85</td>
<td>0.3595</td>
</tr>
<tr>
<td>Error</td>
<td>139</td>
<td></td>
<td>474.51</td>
<td>3.41</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Do the relationships of DR or DS with CRS vary across the Sketchpad availability of groups?

The slope of the relationships of CRS with DR and DS were compared across Sketchpad availability groups by using multiple regression. The results are reported in Table 4-5 and indicate no variation across groups in the relationships.

Table 4-5. Results Comparing Relationships Across Sketchpad Availability Groups.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Independent variable</th>
<th>MSE</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRS</td>
<td>DR</td>
<td>1.15</td>
<td>1.60</td>
<td>.2077</td>
</tr>
<tr>
<td></td>
<td>DS</td>
<td>0.95</td>
<td>0.86</td>
<td>.3555</td>
</tr>
</tbody>
</table>
5. If the relationships do not vary across Sketchpad availability, what is the relationship of CRS and DR and DS?

The analyses relevant to question 4 indicated the relationships of CRS with DR and CRS with DS did not vary across Sketchpad availability groups. Consequently the pooled relationship between DR and CRS as well as the pooled relationship between DS and CRS were investigated using multiple regression. Results are shown in Table 4-6. For CRS there is a significant relationship with DR and DS, at the p < .01 level.

Table 4-6. Results of Relationships of CRS with DR and DS.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Independent variable</th>
<th>r</th>
<th>MSE</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRS</td>
<td>DR</td>
<td>.73</td>
<td>1.15</td>
<td>176.6</td>
<td>.0000</td>
</tr>
<tr>
<td></td>
<td>DS</td>
<td>.78</td>
<td>0.95</td>
<td>234.6</td>
<td>.0000</td>
</tr>
</tbody>
</table>

Other Findings

For the group with the Sketchpad available, the number of Sketchpad drawings (NSD) was also scored. In instances in which figures were not saved nor mentioned, a score of zero was awarded. Data indicates that students did use the Sketchpad when it was available. The mean number of figures created was 4.3 with a standard deviation of 2.4. Of the 78 students that had the computer available, 73 or 94% created computer drawings for one or more problems. Results of a
regression analysis with NSD as independent variable and CRS as dependent variable indicate that the relationship was not significant, $F(1,74) = 2.11, p = .1443$.

Further analysis of the tests within the spatial visualization index offered some additional findings. The three tests that comprised the spatial visualization index were given on two occasions, before any exposure to the Sketchpad and following the LMI session. Although there was a high correlation between the scores, there were differences in the means that should be noted. Mean scores on the Card Rotations test (CR) increased from 98.65 to 115.79, an improvement of 17.1 points. On the Cube Comparison test (CC), mean scores increased from 12.55 to 16.65, an improvement of 4.1 points. The means scores on the Paper Folding (PF) test improved 2.5 points, from 7.99 to 10.46.

Additional analyses examined the relationship of the individual tests, measured at the time of the LMI, to the LMI scores to determine if any test had a more significant relationship to CRS, DR, or DS. Since there was no influence of SA with SV on either of these variables, multiple regressions with CRS, DR, and DS respectively were implemented, each time with CR-post, CC-post, and PF-post as independent variables. Results are presented in Table 4-7.
Table 4-7. Effect of Spatial Visualization Subtests on CRS, DR, and DS.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Source</th>
<th>r</th>
<th>MSE</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRS</td>
<td>CR-post</td>
<td>.19</td>
<td>2.45</td>
<td>4.28</td>
<td>.0402</td>
</tr>
<tr>
<td></td>
<td>CC-post</td>
<td>.37</td>
<td>2.18</td>
<td>22.94</td>
<td>.0000</td>
</tr>
<tr>
<td></td>
<td>PF-post</td>
<td>.32</td>
<td>2.21</td>
<td>17.59</td>
<td>.0000</td>
</tr>
<tr>
<td>DR</td>
<td>CR-post</td>
<td>.19</td>
<td>39.58</td>
<td>2.69</td>
<td>.1034</td>
</tr>
<tr>
<td></td>
<td>CC-post</td>
<td>.26</td>
<td>38.24</td>
<td>8.03</td>
<td>.0052</td>
</tr>
<tr>
<td></td>
<td>PF-post</td>
<td>.31</td>
<td>36.09</td>
<td>13.24</td>
<td>.0004</td>
</tr>
<tr>
<td>DS</td>
<td>CR-post</td>
<td>.23</td>
<td>4.14</td>
<td>7.10</td>
<td>.0085</td>
</tr>
<tr>
<td></td>
<td>CC-post</td>
<td>.35</td>
<td>3.83</td>
<td>19.47</td>
<td>.0000</td>
</tr>
<tr>
<td></td>
<td>PF-post</td>
<td>.30</td>
<td>3.88</td>
<td>14.51</td>
<td>.0002</td>
</tr>
</tbody>
</table>

Results of the multiple regression analyses indicate that CC-post had the greatest influence on CRS and DS, both at p < .01. PF-post had the greatest influence on DR also at p < .01. CR-post had the least influence on CRS, DR, and DS.

Interview Results

This study also had a qualitative component. Two students from each class period, randomly chosen, one from each group, participated in an interview session that lasted approximately 20 minutes. For each student the scores on
the written LMI had been determined prior to the interview, and three or four problems had been selected for discussion during the interview. In most cases two of the problems were from the written LMI and one chosen at the next difficulty level from a set of problems not seen by the student before. Students solved the problems aloud, with varying assistance from the researcher.

Results of analyses of variance comparing the means of the interviewees on CRS, DR, and DS with those students that took only the written LMI indicated that there were no significant differences in the means. The students being interviewed were not different from those not interviewed for the research questions.

A main purpose of the interview component of the study was to explore the ability of the student within his or her zone of proximal development (Vygotsky, 1978) and to discover what the student has learned, and what he is capable of learning with the aid of the researcher (Krutetskii, 1976). Students worked two problems from the written LMI again with varying degrees of assistance. In all cases, the students were able to solve problems successfully on the written LMI that they had previously missed. Ten of the interview students were also able to work at least one problem that they had not seen before which had a difficulty level higher than one missed on the written LMI. Sketchpad availability did not make a difference in the solutions to interview problems.
Patterns, or causes, of failure were also of interest, since they suggest ways to expand a student's "zone" (Spiro & Jehng, 1990). In many cases, students indicated that understanding the problem caused the biggest obstacle.

S1: . . . when you read the problem--it is hard to understand, but when you went over it, it way easy to comprehend.
R: So the words were the first stumbling block?
S1: Yes.

S2: I didn’t know really what it was asking for.

S3: I didn’t know how to do number three.
S3: What I did . . . see I read the problem wrong and I was trying to make them 60 feet from the race mark instead of sixty feet apart.
R: So these aren’t terrible are they?
S3: No. I just have a hard time getting the question out of the words.

S4: I knew what I had to draw but I didn’t know how to explain the locus.

S6: I didn’t understand. . . . Is that asking what will make it consist of one point?

S9: I didn’t understand this one very well at all.

S10: I really didn’t understand the question.

S12: I’m not sure what they are asking.

At times the words within the problems caused the difficulty. Meanings of terms, such as plane M, tangent, isosceles, a circle that contains two given points, were often not known. Students would ask specifically, "what does this mean?"

S8: Plane M confused me. I haven’t learned much about planes.

R: What was hard about the question? What gave you trouble to start with?
S10: The plane.
Another obstacle was the use of variable measurements, such as distance \( r \), or radius \( s \). Students commented on the lack of real numbers or specific directions to make the question understandable.

S3: Can I solve this if I don't know the radius?
S3: You don't have any numbers so you don't know what to do.

S6: This is hard to understand 'cause it doesn't say where they are.

S14: Point P is anywhere?

Although one problem-solving heuristic is to think of a similar problem, students often related problems exactly to those they had previously experienced and did not take note of differences in the question or conditions. This student made use of this heuristic in a positive manner:

S4: Matter of fact it is the very same figure as the first one with the isosceles triangle.

Remembering a similar problem, but not exactly like the one on the LMI, was a hindrance to another student. On the written test, the student indicated on two occasions that the problem was like one already solved, or that he remembered the solution. In both cases, there were distinct conditions that changed the question and the solution.

It was interesting to note the extent to which students were aware of what caused obstacles with the problems. Students were often aware that they did not understand the question. As can be seen in the quotations above, they commented on the fact that they did not understand the question or did not know the definitions of terms. They
were less aware that they had not noted differences in problem situations when similar to those already experienced.

The need for understandable materials and problems was obvious from the investigations. In many cases, students did not interpret the problem correctly. When the conditions were clarified, or the correct figure drawn, the student was able to proceed to a correct solution. The wording of the problems was critical to the correct figure and the correct solution. In three instances students made valid interpretations of the question different from the interpretation of the researcher. In two of the three instances the students produced a correct solution, but the way it was expressed was not clear to the researcher. Communication was critical from the students' and the researcher's perspective. An emphasis in the mathematics classroom on communication is important to understanding, instruction, and assessment (NCTM, 1989). That was clearly evident in these interviews.

Another purpose of the interview was to explore the use of the computer, and its potential for use. Students were asked a series of survey questions at the beginning of the problem-solving session (see Appendix E). The purpose was twofold: to put the student at ease with the interviewer, and to explore the student's attitude and beliefs about mathematics and computers. In all instances the students responded that computers did not make learning mathematics
more mechanical and boring. In two instances, students noted the technical aspects of the technology as an obstacle. In all instances, students indicated that everyone should learn something about computers.

Stated benefits of the computer included accuracy of figures and measurements, precision, easier calculations, faster drawings, and ability to move the figures while seeing changes in the figures and measurements simultaneously. Students that did not have the Sketchpad available for the written or interview sessions indicated that they would have used the computer had it been available, and perhaps would have done better. Those that had the computer available appreciated its capabilities.

S3: Well, I took the test in the classroom. I could have done better if it had been on the computer.
S3: Because I would have visualized it better. It can do much better than my drawings. I could have put a point and the locus, a midpoint . . . 'cause it would be more to scale than mine.
S3: I probably could have used it for all of them if I knew how to read the question correctly.

R: If you had had the computer, would you have used it?
S5: Yeah.
R: Would it have helped?
S5: You can move stuff around, like the circle.
R: Do you think it helped you learn it?
S5: I think.

S6: It may have helped since it's more accurate and easier to see. You can see it's an angle bisector not just a line you draw yourself and that helps. It's kind of a bother to say this is what I want and highlight everything.
R: So the technical part may not be worth the effort.
S6: Yeah.
R: Do you think the computer helped you learn?
S6: Yeah. Just 'cause it was more accurate, and the measurements. The measurements were right there and when they changed you see and that was a help.
R: Do you think the computer helped you in any way?
S9: When we were learning, it did make things clearer to see, 'cause sometimes if you sketched it out then you started seeing things and that made other things make sense.
R: Did it change the way you might solve?
S9: It might have. I think it did. If you learn on your own then you can put things on it and ask. So if you do it yourself, it makes it a lot easier.

R: You used the computer on almost every problem, right?
S13: Yeah.
R: Could you have done these if you hadn’t had the computer?
S13: It would have been a lot longer. I just can’t draw.

As assessment should be, the interview sessions themselves were a learning experience. Students were able to solve problems with assistance. Students were able to share their opinions and knowledge about the content area, the instruction, and the technology.

S1: I actually am glad that you went over it.
R: Did you learn something?
S1: Yeah. I just never have been able to learn math very well. If I really listen, then I can understand it. But not if I try to read it myself.
R: You knew some of the answers, but said you were afraid.
S1: I had ideas. That always happens to me though I won’t put it down. I won’t want to embarrass myself or something.

S3: I was making it harder than it was.

R: Are these hard?
S10: Yeah.
R: Why are they hard?
S10: They make you think.

Attitudes and beliefs, as well and knowledge and success on the content, were improved with communication between the learner and the instructor.
Summary

This study investigated the problem solving strategies used by geometry students following instruction in a dynamic technological environment and explored the relationships between the students' spatial visualization index, mathematics ability, correct response scores, tendency to use drawings, and use of dynamic strategies, with and without the Sketchpad available.

The study addressed three questions of interest:

1. Are the correct response score (CRS), tendency to use drawings (DR), and/or use of dynamic strategies (DS) that students utilize to answer locus-motion problems related to the students' spatial visualization index (SV) or mathematics achievement (MA)?

2. Does Sketchpad availability (SA) affect correct response score (CRS), tendency to use drawings (DR), or use of dynamic strategies (DS)?

3. Are the tendency to use drawings (DR) and/or the use of dynamic strategies (DS) that students utilize when attempting to solve locus-motion problems related to the students' correct response score (CRS)?

Research questions were investigated by more detailed statistical questions outlined below.

1. Do the relationships of SV or MA with CRS, DR, or DS vary across the Sketchpad availability groups?
2. If the relationships do not vary across Sketchpad availability, what is the relationship of CRS, DR, or DS with SV and MA?
3. Does Sketchpad availability affect CRS, DR, or DS on the Locus-Motion Inventory?
4. Do the relationships of DR or DS with CRS vary across the Sketchpad availability of groups?
5. If the relationships do not vary across Sketchpad availability, what is the relationship of CRS and DR and DS?

Results of both the quantitative and qualitative components of the study have been described in detail in this chapter and are summarized in this section.

Initial analyses indicated that none of the relationships of spatial visualization index with correct response score, tendency to use drawings, or use of dynamic strategies varied across Sketchpad availability. Similarly, none of the relationships of mathematics achievement with correct response score, tendency to use drawings, or use of dynamic strategies varied across groups. Results of multiple regression analyses indicated that each of the variables of correct response score, tendency to use drawings, and use of dynamic strategies had a significant relationship with spatial visualization index and mathematics achievement.

To investigate whether Sketchpad availability affects correct response score, tendency to use drawings, and use of dynamic strategies, an analysis of covariance (ANCOVA) was
conducted for each variable with spatial visualization index and mathematics achievement as covariates. Results indicated that Sketchpad availability does not significantly affect correct response score, tendency to use drawings, or use of dynamic strategies.

The analyses concerning the relationships of correct response score with tendency to use drawings and with use of dynamic strategies indicated that none of the relationships varied across Sketchpad availability groups. Results of multiple regression analyses indicated that for correct response score there is a significant relationship with both tendency to use drawings and use of dynamic strategies.

For the students with the Sketchpad available, there was no significant relationship between the number of computer-created drawings and correct response score. Students did use the Sketchpad when it was available. Subjects that had the computer available during the problem solving session created a mean of 4.3 figures on the computer. Of the 78 students that had the computer available, 73 or 94% created computer drawings for one or more problems.

Further analysis of the tests within the spatial visualization index offered some additional findings. The three tests that comprised the spatial visualization index were given on two occasions, before the students had any exposure to the Sketchpad and immediately following the written LMI session. Although there was a high
correlation between the scores, there were differences in the means.

Additional analyses examined the relationships of the individual tests to the LMI scores to determine if any test had a more significant relationship to correct response score, tendency to use drawings, or use of dynamic strategies. Results of the multiple regression analyses indicate that Cube Comparisons test had the greatest influence on correct response score and use of dynamic strategies. Paper Folding test had the greatest influence on tendency to use drawings. Card Rotations test had the least influence on correct response score, tendency to use drawings, and use of dynamic strategies.

Analysis of the interview data revealed that in all cases the students were able to solve problems successfully on the written LMI that they had previously missed. Sketchpad availability did not make a difference in the solutions to interview problems. In many cases, students indicated that understanding the problem caused the biggest obstacle. At times the words within the problems caused the difficulty. Another obstacle was the use of variable measurements, such as distance r, or radius s. Students commented on the lack of real numbers or specific directions to make the question understandable.

Although one problem-solving heuristic is to think of a similar problem, students often related problems exactly to
those they had previously experienced and did not take note of differences in the question or conditions.

Students were aware of what caused obstacles with the problems. Students were often aware that they did not understand the question. They commented on the fact that they did not understand the question or did not know the definitions of terms. They were less aware that they had not noted differences in problem situations when similar to those already experienced.

The need for understandable materials and inquiry problems was obvious from the investigations. In many cases, students did not interpret the problem correctly. When the conditions were clarified, or the correct figure drawn, the student was able to proceed to a correct solution. The wording of the problems was critical to the correct figure and the correct solution.

Another purpose of the interview was to explore the use of the computer and its potential for use. Stated benefits of the computer included accuracy of figures and measurements, precision, easier calculations, faster drawings, and ability to move the figures while seeing changes in the figures and measurements simultaneously. Students that did not have the Sketchpad available for the written or interview sessions indicated that they would have used the computer had it been available, and perhaps would have done better.
The interview sessions themselves were a learning experience. Students were able to solve problems with assistance. Students were able to share their opinions and knowledge about the content area, the instruction, and the technology. Attitudes and beliefs, as well as performance, were improved with communication between the learner and the instructor.
 overview of the study

The purpose of this study was to investigate the problem-solving strategies used by geometry students following instruction in a dynamic technological environment. Specifically, the study explored the relationships among the students' spatial visualization ability, mathematical ability, and problem-solving strategies with and without the technology. Students were instructed with teacher presentation and "hands on" experience using a software product called *The Geometer's Sketchpad*. Specific lessons on locus-motion concepts provided the basis for exploring problem-solving strategies. Methods of solutions were examined for the level of drawing and dynamic visualization used in relationship to the availability of the technology during the problem-solving session.

Three independent measures of spatial visualization were taken at two different occasions. The spatial visualization index included ETS Card Rotations, Cube Comparisons, and Paper Folding tests. A locus-motion problem inventory (LMI) was also administered. Participants
for the study consisted of 158 geometry students from seven geometry classes at one high school. All students participated in the instruction using *The Geometer’s Sketchpad*. The instructional phase included lessons that were an extension of previous lessons on use of the *Sketchpad* and lessons that introduced the concepts of locus of points.

Students were assigned individually and at random to two groups for the problem-solving session: those who did have the *Sketchpad* available, and those who did not. The problem-solving session session lasted one class period of 50 minutes. Individual interviews were held on the following day with two students from each class, one from each group. Interviews lasted approximately 20 minutes, in which students worked locus-motion problems aloud. The problem-solving session followed the constructivist approach of creating a "problematic environment that would elicit the student’s adequate constructive endeavors" (Fischbein, 1990, p. 8). Data analysis had both a quantitative and a qualitative component.

**Results and Discussion**

The theoretical premise that supports this study is that the use of a dynamic technological environment, such as *The Geometer’s Sketchpad*, can enhance construction of knowledge by students and influence problem-solving strategies using dynamic visualization skills. The
constructivist theory of learning promotes the belief that students build knowledge through activity and experience. Dynamic software can provide experiences in visualizing, conjecturing, and building frameworks within a specific content domain.

The study addressed three questions of interest:

1. Are the correct response score (CRS), tendency to use drawings (DR), and/or use of dynamic strategies (DS) that students utilize to answer locus-motion problems related to the students' spatial visualization index (SV) or mathematics achievement (MA)?

2. Does Sketchpad availability (SA) affect correct response score (CRS), tendency to use drawings (DR), or use of dynamic strategies (DS)?

3. Are the tendency to use drawings (DR) and/or the use of dynamic strategies (DS) that students utilize when attempting to solve locus-motion problems related to the students' correct response score (CRS)?

Research questions were investigated by more detailed statistical questions outlined below.

1. Do the relationships of SV or MA with CRS, DR, or DS vary across the Sketchpad availability groups?

2. If the relationships do not vary across Sketchpad availability, what is the relationship of CRS, DR, or DS with SV and MA?

3. Does Sketchpad availability affect CRS, DR, or DS on the Locus-Motion Inventory?
4. Do the relationships of DR or DS with CRS vary across the Sketchpad availability of groups?

5. If the relationships do not vary across Sketchpad availability, what is the relationship of CRS and DR and DS?

Initial analyses indicated that none of the relationships of spatial visualization index with correct response score, tendency to use drawings, or use of dynamic strategies varied across Sketchpad availability. Similarly, none of the relationships of mathematics achievement with correct response score, tendency to use drawings, or use of dynamic strategies varied across groups. Results of multiple regression analyses indicated that each of the variables of correct response score, tendency to use drawings, and use of dynamic strategies had a significant relationship with spatial visualization index and mathematics achievement.

To investigate whether Sketchpad availability affects correct response score, tendency to use drawings, and use of dynamic strategies, an analysis of covariance (ANCOVA) was conducted for each variable with spatial visualization index and mathematics achievement as covariates. Results indicated that Sketchpad availability does not significantly affect correct response score, tendency to use drawings, or use of dynamic strategies.

The availability of the computer during the problem-solving session was not a significant factor for performance on the LMI, as measured by correct responses. In each case,
the covariate of spatial ability or mathematics achievement accounted for most of the variance. Although specific to the geometric content of locus of points, these findings supplement those by Moses (1977/1978) and Presmeg (1986) that spatial ability is a good predictor of problem-solving performance.

Moses (1977/1978) found that students with high spatial ability often do not write down visual solution processes. However, in this study, students were encouraged to write down solutions, make drawings, and describe visual solution processes. Both spatial visualization and mathematics achievement were significantly related to tendency to use drawings.

Additional findings included trends seen in the subtests of the spatial visualization index. Although the study did not specifically address the components of spatial ability nor whether it is a modifiable quantity, descriptive statistics provide some interesting information that warrants comment here and, perhaps, further research. The three tests that comprised spatial visualization index were given on two occasions, before any exposure to the Sketchpad and following the LMI session. Although there was a high correlation between the scores, there were differences in the means that should be noted. Mean scores on the Card Rotations test (CR) increased from 98.65 to 115.79, an improvement of 17.1 points. On the Cube Comparison test (CC), mean scores increased from 12.55 to 16.65, an
improvement of 4.1 points. The means scores on the Paper Folding (PF) test improved 2.5 points, from 7.99 to 10.46.

Although there were differences in the means of the subtests on the two occasions, without a control group that did not use the Sketchpad at all, there can be no direct comparison. However, the fact that the mean scores on all subtests and the overall index score increased suggests that spatial ability is a modifiable quantity, as indicated in the search of the literature (Ben-Chaim, Lappan & Houang, 1988; Hembree, 1992; Moses, 1977/1978; Tillotson, 1984/1985; Vinner, 1989).

Additional analyses examined the relationship of the individual tests, measured at the time of the LMI, to the LMI scores to determine if any test had a more significant relationship to correct response scores, tendency to use drawings, or use of dynamic strategies. Results of the multiple regression analyses indicated that Cube Comparisons test had the greatest influence on correct response score ($r = .37$) and use of dynamic strategies ($r = .35$). Paper Folding test had the greatest influence on tendency to use drawings ($r = .31$). Card Rotations test had the least influence on correct response score, tendency to use drawings, and use of dynamic strategies. Of the three tests, Paper Folding and Cube Comparisons involve three dimensional figures or motion. Whereas, the Card Rotation test requires only two-dimensional movement. ETS considers the Card Rotation and Cube Comparison tests as measures of
Spatial Orientation, and the Paper Folding test as one of the measures of Visualization.

The relationships among the variables of the LMI were also investigated. The analyses concerning the relationships of correct response score with tendency to use drawings, and with use of dynamic strategies, indicated that neither of the relationships varied across Sketchpad availability groups. Results of multiple regression analyses indicated that for correct response score there is a significant relationship with both tendency to use drawings and use of dynamic strategies. Students extensively used drawings and dynamic strategies. Both variables were highly correlated to correct response scores, indicating that they were effective strategies for locus-motion problem solving.

As reflected in research, flexible thinking is related to successful problem solving (Davis, 1986; Dover & Shore, 1991; Schoenfeld, 1987; Spiro & Jehng, 1990). Flexibility is indicated by the tendency to use drawings. Based on the scoring rubric, on the average each student drew more than one figure for each problem. Research and experience indicate that lack of multiple drawings is often a limitation to success in mathematics (Vladimirskii, 1971). Results suggest that experience with computer images may increase the utilization and usefulness of diagrams. The motion capability of the computer may reduce the obstacles

Students did use the Sketchpad when it was available. Subjects that had the computer available during the problem-solving session created a mean of 4.3 figures on the computer. Of the 78 students that had the computer available, 73 or 94% created computer drawings for one or more problems.

Having learned the content with the Sketchpad available, the students imitated its capabilities even when they did not have it available for the problem-solving session. Students used such language as, "I traced the locus," "I imagined the motion," etc. that indicated that they envisioned motion as they would have on the computer. Students also commented that they would have been more accurate with their drawings, and more correct in their solutions if they had had the Sketchpad available.

Since the results indicated that both groups had similar correct response scores, it appears that the drawings and dynamic strategies produced similar results. In an environment in which assessment may be different from instruction, these results suggest that problem-solving strategies learned in one environment may transfer to others. In this study, the strategies employed with the computer carried over to the paper and pencil problems. The low mean on the correct response score may be due to the incremental scoring for each problem or to the difficulty
levels of the problems. Although the difficulty levels of the problems chosen for the LMI were agreed upon by the reviewers, the ability of the students may have been overestimated.

Although there was a correlation between spatial visualization and correct response score \( (r = .38) \), the correlation between the strategies utilized and correct response score was significantly higher \( (r = .73 \text{ for DR}, .78 \text{ for DS}) \). The findings agree with those of other researchers that instruction and practice in a specific content area are more dominant factors for problem solving than prior knowledge and skill \( \text{(Hembree, 1992; Kantowski, 1981; Krutetskii, 1976)} \).

Interview sessions provided additional findings concerning the use of drawings and the availability of the computer that supports prior research conclusions that the computer has the potential to resolve many of the obstacles of conjecturing, searching for patterns, and generalizations \( \text{(Bishop, 1989; Hershkowitz, 1990; Janvier, 1987; Lesh, 1987; Senechal, 1990; Steen, 1990)} \). Students that did not have the Sketchpad available for the written or interview sessions indicated that they would have used the computer had it been available, and perhaps would have done better. Those that had the computer available appreciated its capabilities. Benefits of the computer included accuracy of figures and measurements, precision, easier calculations, faster drawings, and ability to move the figures, seeing
changes in figures and measurements simultaneously. By providing students with "hands on" experiences with these characteristics, the development of spatial orientation skill as described by Tartre (1990) may be accelerated. Tartre concluded that "spatial orientation skill appears to be used in specific and identifiable ways . . . accurately estimating the approximate magnitude of a figure, demonstrating the flexibility to change an unproductive mind set, adding marks to show mathematical relationships, mentally moving or assessing the size and shape of part of a figure, and getting the correct answer without help to a problem in which a visual framework was provided" (1990, p. 227).

A main purpose of the interview component of the study was to explore the ability of the student within his or her zone of proximal development (Vygotsky, 1978) and to discover what the student has learned and what he is capable of learning with the aid of the researcher (Krutetskii, 1976). In the interview session, all students were successful on the problems with varying degrees of assistance from the investigator. Students were able to go beyond the known problems and solve those novel questions with some discussion of the components of the question itself. Once students understood the question, they demonstrated the ability to change an unproductive mind set (Dover & Shore, 1991; Tartre, 1990). They were able to recognize the new conditions and proceed to successful
solutions. Sketchpad availability did not make a difference in the solutions to interview problems.

Patterns, or causes, of failure were also of interest, since they suggest ways to expand a student’s zone. Student comments indicated that the first phase of problem solving is indeed the most important, the most critical (Polya, 1957). Understanding the problem was the biggest hindrance to successful solutions. In many cases, students indicated that understanding the problem caused the biggest obstacle.

Understanding of the problem may have been curtailed by lack of an initial figure. Once students understood and were able to draw the figure, the solution was often well within their grasps. Assisting students in building that initial visual framework is indeed a critical step in the problem-solving process (Polya, 1957). Language and communication were most important to expanding the student’s zone of proximal development.

Students were aware of what caused obstacles with the problems. Students were often aware that they did not understand the question. As can be seen in the quotations in Chapter 4, students commented on the fact that they did not understand the question or did not know the definitions of terms.

Anecdotal evidence is found in students’ descriptions of how they solved the problems that indicates that students often related the current problem to those previously experienced. Failure to recognize differences in the
previous problems and the new ones was often a hindrance. Students are accustomed to repeating on assessments the same kind of problems experienced during instruction and practice.

The need for comprehensible materials and problems was obvious from the investigations. In many cases, students did not interpret the problem correctly. When the conditions were clarified, or the correct figure drawn, the student was able to proceed to a correct solution. The wording of the problems was critical to the correct figure and the correct solution. Communication was critical from the students’ and the researcher’s perspective. An emphasis in the mathematics classroom on communication is important to understanding, instruction, and assessment (NCTM, 1989).

Limitations

This study contained limitations that affect the generalizability of the results. Participants consisted of high school geometry students in one high school with four teachers. The domain was content-specific to locus of points. A limitation of scoring the LMI lies in the use of self-report documentation of how students solved problems, and drawing conclusions from the figures and written solutions provided. Since there were no common standardized tests that could be used for matching, common teacher/textbook tests were used to match mathematics achievement. Since the study lasted over a period of
several days, variability of attendance was a problem. Students were absent due to sickness, personal reasons, and school related activities. Absences were taken into account for the testing days, but were not available for the instructional period.

The researcher conducted all lessons to eliminate differences in results due to teacher variability, but was aware of the possibility that a threat to validity may be introduced, by the researcher influencing results. This was minimized by the use of specific presentation plans, and written activities for students with the computer. Teacher variability was possible in the amount of orientation the students had to the Sketchpad prior to the study. All teachers planned a similar amount of exposure to the software, but there was no firm record of utilization.

The findings would have been strengthened had there been a control group that did not use the computer for instruction or assessment in locus-motion problems. Future research should provide that comparison. However, the same caveat will exist in the future as it did in this study, that teachers already believe in the impact of the dynamic technology and want to utilize it. The control group might need to be located in another situation, where the technology was not available, which introduces other limitations into the study.
The results of this study are generalizable only to situations involving similar participants, content, and classes of variables.

**Implications and Recommendations**

Mathematics education is in the process of change, a change as indicated within the *Standards* that recommends new curricula and modes of instruction and assessment. As changes are proposed, planned, and implemented, investigation into their feasibility and effectiveness are needed. This study added to the body of knowledge that already exists as technology makes its impact upon education. Future investigations can build upon this research and continue to expand the knowledge base for educational practices in a technological environment.

Future research should strive to provide control information for direct comparison. The findings would have been strengthened had there been a control group that did not use the computer for instruction or assessment in locus-motion problems. Because of the desire of all the participating teachers to have use of the *Sketchpad*, and ethically not wanting to deprive any teacher or student the advantage of the computer, there was no control group. However, as mentioned above, some care will be needed in the design of that research.

Addition of a control group would allow investigation of gains in spatial visualization scores, as a whole and
with individual components. Differences in the means of the subtests on the two occasions suggests that spatial ability is a modifiable quantity. Direct comparisons would provide more conclusive information. Although the use of drawings was very prevalent regardless of the Sketchpad availability, comparison with a control group would be instructive. Data on the number of problems attempted would provide more informative data. Due to the time constraints, students did not attempt all problems.

Computers indeed are useful in broadening the mathematics curricula, with locus of points a prime example. Changes in instruction must redirect the emphasis and create an environment of inquiry, conjecture, discovery, and active involvement. Other studies might use geometry content that occurs earlier in the curriculum while the range of students' knowledge is not as variable as it might be at the end of the year. Content that changes the basic approach to the geometry curriculum, such as transformational geometry, might also be explored.

Design of instructional materials, activities, and problems are critical to the success of any changes in instruction, curricula, or assessment. The design and structure of learning activities with technology is difficult. Models are formative; materials are developmental. Teachers and researchers alike must strive to create an environment of inquiry based upon well-planned lessons, activities, and questions. The teachers
participating in the study gained valuable experience with technology based lessons, using a variety of instructional techniques such as teacher-presentation, "hands on" experience by the students, "show and go" lessons in which the instructor demonstrates and then the students explore and practice, and lessons in which the students read instructions only and attempt to follow. Further investigations should examine the correct response scores and whether the scoring rubric or level of difficulty was problematic. Low performance scores may have obscured meaningful results.

Language and communication play an important part in the teaching/learning process. Future research should emphasize the importance of communication and explore the socialization processes in the mathematics classroom. The need for comprehensible materials and problems was obvious from the investigations. In many cases, students did not interpret the problem correctly. When the conditions were clarified, or the correct figure determined, the student was able to proceed to a correct solution. The wording of the problems was critical to the correct figure and the correct solution.

In addition to the knowledge gained about teaching and learning, the participating teachers and the researcher gained experience with alternative forms of assessment. The information gained in the interview sessions gave insight to the disadvantages of paper and pencil assessment. Teachers
were excited to see that students can be encouraged to be visualizers and that they respond to that encouragement. Several of the teachers modified the end of the year examination to measure the use of figures as part of assessment. Using the computer in assessment was also being incorporated into the evaluation program.

Future investigations of this technological environment and the cognitive constructs of students might utilize the measurement and construction capability of the Sketchpad more completely. This would explore the influence of multiple representations in figures, measurements, and words to a fuller extent. Exploring the means to capture motion as well as figures on the computer would provide useful data. Students indicated that the ability to construct specific figures and properties, such as midpoints, was helpful.

Mathematical communication and socialization of the student in the problem-solving process would also be of interest. Another set of written problems, presented after the interview sessions with the communication and socialization interactions, would indicate if performance results and strategies were improved.

There is also a need to continue the investigation of gender differences, age or grade differences, and attitudes and beliefs of students and teachers in a technological environment.
Summary

This study was designed to investigate the problem-solving strategies used by geometry students following instruction in a dynamic technological environment. Specifically, it explored the relationships among the students' spatial visualization index, mathematics ability, correct response scores, tendency to use drawings, and use of dynamic strategies with and without the Sketchpad available.

Due to the design of the study, the lack of significant differences does not indicate lack of findings. That there were no significant differences in the groups with and without the Sketchpad available suggests that strategies learned with the technology available are transferable to paper and pencil problem-solving situations. Students can be encouraged to use a different learning style and to expand their problem-solving strategies. The major limitation was that all of the students had access to the Sketchpad during instruction. However, this very aspect of the design was what made it important to the teachers and students participating in the study.

Technology can assist in the introduction, development, and reinforcement of mathematical concepts, in developing intuition and insight. Technology may be the key to unlocking the door to a constructivist curriculum that is active and conceptually-oriented.
The results reinforce the constructivist belief that students build knowledge representations and improve problem-solving strategies based upon experience and active involvement in exploratory activities. In agreement with other research, this study suggests that active participation in instructional activities is a better predictor of successful problem-solving performance and use of strategies than is spatial visualization or mathematics achievement. Instruction and practice in specific content areas are more dominant factors for problem solving than prior knowledge and skill.

Communication and socialization in mathematics education is important to understanding, instruction, and assessment. Language and communication were critical to expanding the student's zone of proximal development.

The technological society is a reality and the mathematics classroom must adapt to the changing scientific workplace or fail to reach its primary goals of providing power and opportunities to the students of today in the world of tomorrow. Students are capable of learning, of obtaining mathematical power. They need to be challenged with content and instructional techniques that will release the potential for learning that they possess. Continued collaboration between educational researchers and practitioners will ensure the development of mathematical communities and learning environments that will promote
mathematical literacy in the technological society in which the students of today will participate.
APPENDIX A
LOCUS-MOTION INVENTORY PROBLEMS

1. Plane M contains segment XY. What is the locus of points in the plane that are the vertices of all isosceles triangles having segment XY as the base?

2. Two points do not determine a circle because there are an infinite number of circles that contain both of them.
   a) Mark two points on your paper that are about an inch apart and then construct (draw) several circles that contain both of the points.
   b) What relationship do the centers of these circles have to the line segment that joins the two points?

3. Given a circle O and a point P on the circle. Describe the locus of the midpoints of all chords of the circle with P as one endpoint.

4. Given any triangle ABC, find the locus of the circumcenter as vertex C moves along a line parallel to side AB. (The circumcenter is the point of concurrency of the perpendicular bisectors of the sides of the triangle.)

5. What is the locus of points in a plane that a distance $d$ from a point A and also are a distance $t$ from a given line not containing A?

6. A physical education teacher places two students 60 feet apart. The students are supposed to race to a finish
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marker. Describe all the possible placements of the marker that will make the race fair.

7. Describe the locus of points make by a point on the outermost part of the wheel of a bicycle as it travels down the road.

8. Make a sketch and describe the set of all points in a plane which are centers of circles of a given radius \( r \) that contain a given point A.

9. Consider the set of all points in a plane equidistant from two given points and equidistant from two given parallel lines. Depending on the situation, the set may consist of:
   a) one point
   b) one line
   c) the empty set (there is no point that satisfies the conditions)
   d) All of the above

Explain your answer.

10. What is the locus of the centers of all circles that are tangent to each of two parallel lines?

11. Given circle P with radius \( r \). A second circle, with radius \( s \) \((s < r)\) moves so that it is always tangent to circle P. Describe the locus of the center of the second circle.

12. The orthocenter of a triangle is the point of concurrency of the altitudes. Given equilateral triangle ABC with the orthocenter. What is the path of the
orthocenter as angle A gets larger? approaches 90 degrees?
What happens to the altitudes as angle A approaches (and reaches) 180 degrees? What does that mean for the orthocenter as angle A approaches 180 degrees?

13. Given point P in the interior of angle ABC. Find the locus of points that are in the interior of angle ABC, are equidistant from the sides of angle ABC, and are a given distance r from point P.

14. ABCD is any convex quadrilateral.
   a) Explain how to find a point which is equidistant from line AD and line AB and also equidistant from D and C.
   b) Explain how to find a point which is equidistant from lines AB, AD and DC.
   c) Do the points of part a) and part b) coincide? sometimes? always? never? Explain your answer.
APPENDIX B
SCORE SHEET FOR LOCUS-MOTION INVENTORY

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APPENDIX C
CONSENT FORMS
Parental Consent Form

Your child’s class has been selected to participate in my Ph.D dissertation research project on geometry. I am a mathematics teacher on leave from GHS completing my degree in mathematics education at the University of Florida. I have been assisting the teachers at GHS in writing lessons for geometry using the mathematics IBM lab and software called The Geometer’s Sketchpad. During the year your child’s teacher has included those lessons as part of the regular curriculum. Students are currently learning to use the software in the laboratory environment. Lessons with the Sketchpad will continue as part of the curriculum after the study period is over.

The information gathered for this study will be used to investigate the relationships between spatial visualization, problem-solving strategies, performance and assessment in geometry; and how this relationship might be affected by the use of current technology, The Geometer’s Sketchpad software. Students will be asked to take a battery of ETS Spatial Visualization Tests that will take approximately 20 minutes of class time. The study will follow 3-4 lessons with The Geometer’s Sketchpad on locus of points. Students will spend one class period working problems from a Locus-Motion Inventory. The software will be available to half of the students, not available to half. Students for each group will be chosen randomly, as will several students to work some of the problems in an interview setting. All problems can be worked with paper and pencil only.

The visualization test will be evaluated and rated to investigate the effect of the technology on spatial visualization abilities. The written and oral problem-solving inventory questions will be evaluated and rated for correctness and strategy used. Scores from the teacher grades will be used for matching purposes statistically. All individual student scores will be kept confidential. Each student will be identified only by a number. There is no risk involved for your child, nor will there be monetary compensation. Participation or non-participation in the problem sessions will not affect your child’s grade in class. Students may skip any question that they do not wish to answer. Students who do not wish to participate in the project will be given assignments for the class periods in which the geometry and spatial visualization problem solving will take place. You or your child has the right to withdraw permission for your child’s data to be used without any penalty or prejudice.

If you would like further information concerning this study, feel free to contact me at the University or at home. Please sign the form on the space provided indicating whether your child may participate, and have your child return the signed portion to his or her geometry teacher.
Also inform your child whether he or she has permission to participate in this study.

Thank you for your interest and support,

Stephanie Robinson  
Graduate Student in Mathematics Education, University of Florida  
392-0761 Ext. 263 (work)  
372-0700 (home)

RETURN BOTTOM PORTION

I have read and I understand the procedure described above. I will _____ / will not _____ allow my child, _______________________, to participate in Stephanie Robinson’s geometry study. I have retained the top portion of this letter.

_________________________  _______________________
Parent/Guardian Signature  date

_________________________  _______________________
2nd Parent/Witness Signature  date
Teacher Consent Form

Thank you for agreeing to participate in my Ph.D. dissertation research project on geometry. I am a mathematics teacher on leave from GHS completing my degree in mathematics education at the University of Florida. The information gathered for this study will be used to investigate the relationships between spatial visualization, problem-solving strategies, performance and assessment in geometry; and how this relationship might be affected by the use of current technology, The Geometer’s Sketchpad software. This is a software package available for teachers and students in the Mathematics IBM Computer Lab at GHS.

Your involvement in this research will consist of assisting in the administration of ETS Spatial Visualization tests which take approximately 20 minutes, and administration of the written Locus-Motion Inventory for one class period at the end of the study. You will assist with the lesson that introduce your classes to The Geometer’s Sketchpad and the lessons on locus of points. I will prepare these lessons. You will provide scores for matching and comparison purposes from your geometry diagnostic test and semester tests for students participating in the study. All individual student scores will be kept confidential. Each student will be identified only by a number.

There is no risk involved for you. Participation or non-participation in this study will not affect you or your job. Using the technology available could enhance the learning and teaching of mathematics in all content areas. You have the right to withdraw permission for involvement without any penalty or prejudice.

If you would like further information concerning this research, feel free to contact me at the University or at home. Please sign the form on the space provided and return the bottom portion to me.

Thank you for your interest and support,

Stephanie Robinson
Graduate Student in Mathematics Education, University of Florida
392-0761 Ext. 263 (work) 372-0700 (home)

RETURN BOTTOM PORTION

I have read and I understand the procedure described above. I agree to participate in Stephanie Robinson’s geometry study. I have retained the top portion of this letter.

Signature: __________________________ date __________________________

Teacher

Approved for use through March 1995.
Child Assent Script

During the semester this class will be participating in a research project concerned with how technology impacts learning geometry and spatial visualization. You will be using the Mathematics IBM Lab and software called *The Geometer’s Sketchpad* to explore certain geometry lessons. You have seen the software used by your teacher in class, and are now learning to use the software yourself. An extension of the introductory lessons will include 3–4 lessons using the software. Then you will work problems involving the content just learned. The software will be available to some students during the problem-solving session. Others will work the problems traditionally with paper and pencil only. All problems can be worked with paper and pencil only. Some of the problems will be strictly written, others will be worked in an interview setting. Your solution methods will be used to analyze the different ways that students solve geometry problems. In addition you will take a battery of ETS Spatial Visualization test, which will take approximately 20 minutes of class time.

Individual student scores will be kept confidential. Each student will be identified only by a number. There is no risk involved for you. Participation or non-participation in the testing specific to the study will not affect your grade in class but could serve as a valuable learning experience. You may skip any question that you do not wish to answer. If you object to participation in this study, you do not have to take part. Your teacher will provide assignments for you during the two class periods that the geometry and spatial visualization problem solving will take place. You may decide to withdraw from the project at any time.
APPENDIX D
INSTRUCTIONAL ACTIVITY SHEETS
Problem Set 1

Name

Open LM1P1.

1. Describe the set of all points, in a plane, which are 1 cm from a given segment 6 cm long and also are 2 cm from the midpoint of the segment.

![Diagram of segment AB with midpoint C and distances](image)

Distance(B to A) = 6.0 cm

2. A segment intersects the sides of angle DEF in points J and K. Locate a point of segment GH which is equidistant from ray ED and ray EF.

![Diagram of triangle DEFG with points J, K, and H](image)
3. Describe the set of all points in a plane which are equidistant from two intersecting lines.

4. Describe the set of all points in a plane which are equidistant from two intersecting lines and are 2 cm from the point of intersection.
Problem Set 2

Name ________________________________
Open L2P1.

For each of the following, draw a figure showing the locus of points in a plane. Then describe the locus.

1. All points that are 1 cm from a circle with radius of 3 cm.

2. All centers of all circles passing through two distinct points.
3. All midpoints of all chords formed by secants (lines that intersect the circle in two points) drawn to a circle from a point outside the circle.

4. Given two concentric circles with radii of 1 cm and 3 cm. What is the locus of points in the plane of the two circles and equidistant from them?
1. Construct a triangle and all three perpendicular bisectors of the sides of that triangle. The perpendicular bisectors will meet at one point, the point of concurrency, called the circumcenter. Sketch the figure on your screen on your paper.

2. Construct a triangle and all three angle bisectors of the angles of that triangle. The bisectors will meet at one point, the point of concurrency, called the incenter. Sketch the figure on your screen on your paper.

3. There are two more important points of concurrency concerning a triangle. One is the point of concurrency of the medians, called the centroid. The other is the point of concurrency of the altitudes, called the orthocenter. Sketch both situations on your paper.
1. Open LMS4, LMS5, LMS6. For each of the figures, change the triangle to be acute, then right, then obtuse and for each type triangle tell where the point of concurrency lies, and how its path moves as the triangle changes.

2. Figure A. Point of Concurrency __________________________________________
   Where? Acute ___________________________ Right ___________________________
          Obtuse __________________________________________
   How does the path of the point move? ______________________________________

3. Figure B. Point of Concurrency __________________________________________
   Where? Acute ___________________________ Right ___________________________
          Obtuse __________________________________________
   How does the path of the point move? ______________________________________

4. Figure C. Point of Concurrency __________________________________________
   Where? Acute ___________________________ Right ___________________________
          Obtuse __________________________________________
   How does the path of the point move? ______________________________________

5. Figure D. Point of Concurrency __________________________________________
   Where? Acute ___________________________ Right ___________________________
          Obtuse __________________________________________
   How does the path of the point move? ______________________________________
Using New Sketch, draw and describe on your paper all possible ways the given figures can intersect.

1. two lines
2. a circle and a line
3. two circles
4. a circle and two parallel lines
5. two concentric circles and a line
6. two parallel lines and a third line

Draw a diagram to find the locus of points that satisfy the conditions. Then describe the locus.

1. all points that are equidistant from the points of intersection of two given circles.
2. all centers of all circles tangent to two intersecting lines.
3. all the points in the plane that are 1.5 cm from a given line and 2 cm from a given points on the line.
4. all points in the plane that are equidistant from the rays of an angle and equidistant from two points on one of the sides of the angle.
5. all the points in the plane on or inside of a given angle that are equidistant from the sides of the angle and 2 cm from the vertex of the angle.
6. Given line m parallel to line n, points A and B on line m, and the distance between m and n is less than 1/2 (AB):

a. How many points are there in the locus of all points P on line n such that triangle PAB is an isosceles triangle?

b. How many points are there in the locus of all points P on line n such that triangle PAB is a right triangle?

c. How many points are there in the locus of all points P on line n such that triangle PAB is an equilateral triangle?
LOCUS-MOTION

WORKSHEET 4

NAME ________________________________

For each of the following problems, construct the figure and describe the locus.

1. Given two points R and S in the plane of angle ABC. Find the locus of points that are in the interior of angle ABC, are equidistant from the sides of angle ABC, and are equidistant from R and S.

2. Describe the set of all points which are at a given distance from a given point P, and equidistant from P and another point Q.

3. Describe the set of all points in a plane at a given distance from a given point and equidistant from two given parallel lines.

4. Given two parallel lines m and n and a third line j all in a plane. Find the locus of points in the plane that are equidistant from m and n, and are a given distance from j.

5. Describe the set of all points in a plane at a given distance from a point and at a given distance from a given line.
A quadrilateral is a polygon with four sides. Some quadrilaterals are parallelograms, trapezoids, rectangles, rhombi, and squares. Some are convex, some are not.

1. Do the diagonals of a quadrilateral always intersect each other?

2. ABCD is a trapezoid with bases AB and CD. Find the set of all points equidistant from A and B, the set of all points equidistant from B and C, those from C and D, and those from D and A. Under what conditions will there exist a point P, in the plane of the trapezoid, equidistant from A, B, C, and D? (When will all of the loci intersect?)

3. Given a segment XY. Find the locus of the points of intersection of the diagonals of all possible rhombi which have XY as a side.
APPENDIX E
INTERVIEW QUESTIONS

Survey questions for interview students were taken from an in-school survey regarding the mathematics laboratory. Students indicated whether they agreed or disagreed with the statements.

1. I almost never have gotten shook up during a math test.
2. My mind goes blank and I am unable to think clearly when working math problems.
3. I usually have been at ease during math tests.
4. Mathematics makes me feel uneasy and confused.
5. Using computers makes learning mathematics more mechanical and boring.
6. Everyone should learn something about computers.
7. Computers solve problems better than people do.
LIST OF REFERENCES


Stephanie Osgood Robinson was born in Tallahassee, Florida. As a child she lived with her family in Tallahassee, Atlanta, GA, and Jacksonville, FL. She graduated from Gainesville High School, Gainesville, FL. Following two years at Florida State University, she was elected to Phi Beta Kappa and received her undergraduate degree with honors in mathematics from the University of Florida. She pursued a career in data processing in Orlando and Jacksonville, prior to returning to the University of Florida to complete a master’s degree in education.

She taught mathematics at Gainesville High School for eight years, full time and then part time while beginning her doctoral studies. As a research assistant, she worked with Dr. Mary Grace Kantowski and with Dr. Elroy Bolduc on the Mathematics Leadership Project and High School Competency Training Project for the State of Florida. She also worked as a teaching assistant for the Department of Instruction and Curriculum.

She is married to Robert Robinson and has one daughter, Kalena Carmen. In the fall of 1994 Stephanie will be an assistant professor in the College of Education at the University of Tennessee, Knoxville.
I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Mary Grace Kantowski, Chair
Professor of Instruction and Curriculum

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Elroy J. Bolduc
Professor of Instruction and Curriculum

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

James J. Algina
Professor of Foundations of Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

John W. Gregory
Professor of Instruction and Curriculum
I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Charles W. Nelson
Professor of Mathematics

This dissertation was submitted to the Graduate Faculty of the College of Education and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

December, 1994

Dean, College of Education

Dean, Graduate School
EFFECTS OF CONTINGENT AND NON-CONTINGENT REINFORCEMENT ON CHILDREN'S ACADEMIC PERFORMANCE

By

CARLOS M. ALVAREZ

A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

1974
ACKNOWLEDGMENTS

I wish to express my most sincere gratitude to Dr. James C. Dixon, Chairman, advisor and friend, whose patience, guidance and continuous support not only through the course of this investigation but also throughout my graduate career have been invaluable and greatly appreciated. I also gratefully acknowledge Dr. William D. Wolking, Co-Chairman of my doctoral committee for his help in the organization and critique of this manuscript.

Special gratitude is extended to Dr. Henry S. Pennypacker for the invaluable amount of teaching which served as an inspiration for the present investigation. His interest, advice and creative suggestions are greatly appreciated. I am further indebted to the other members of this committee, Dr. Ted Landsman and Dr. Donald Avila for their assistance and criticism.

Special thanks to my good friends José Valle and Alejo Vada for the long hours of hard work spent with me in the preparation and implementation of the pilot work that preceded this investigation. Their continuous encouragement and sincere testimony of friendship will always be remembered.

I would also like to express my appreciation to Mr. Phillip Alvers, Principal of Moseley Elementary School, who facilitated the space and students who participated in the study; also, to David Smolen for programming assistance.
Finally, special thanks to my wife Arminda, not only for her direct participation in this research as co-experimenter, but also for her encouragement and continuous support throughout my graduate career.
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EFFECTS OF CONTINGENT AND NON-CONTINGENT REINFORCEMENT ON CHILDREN'S ACADEMIC PERFORMANCE

By

Carlos M. Alvarez

August, 1974

Chairman: Dr. James C. Dixon
Co-Chairman: Dr. William D. Wolking
Major Department: Psychology

In the present investigation children's performance in an academic task was studied under two types of environmental arrangements: one in which consequences were conditionally related to their performance and another in which the environmental outcomes were administered independently of their performance.

Two experiments were conducted. In Experiment I eight first-grade children were individually exposed on a daily basis to two different experimental conditions. In one of them, each S was given tokens contingent upon his correct performance on an arithmetic task. In a second condition each S received tokens which were given independent of his performance on the same task. The quality of performance was measured by accuracy of responses to the arithmetic problems and rate of improvement. Productivity, operationally defined as the frequency of arithmetic problems attempted per session regardless of accuracy, was also measured in terms of rates of problems completed per
session and rate of change in frequency over time. Tokens obtained during the performance could be later exchanged for a variety of toys and candies.

In Experiment II seven first-grade children were individually exposed on a daily basis to two different experimental conditions. In one of them, tokens were delivered contingent upon the S's correct responses on the arithmetic task, but withdrawal of tokens also occurred everytime the S made an incorrect response. In a second experimental condition, delivery and withdrawal of tokens occurred on a non-contingent basis, that is, independently of the correctness of responses. As in Experiment I, tokens obtained during the performance could be later exchanged for a variety of toys and candies.

Results showed no significant difference between Ss' average level of accuracy under the contingent and non-contingent administration of tokens. Both experimental conditions produced acceleration or a trend towards improvement in accuracy in all Ss participating in both experiments with the exceptions of one S who showed deceleration in the level of accuracy over time under the non-contingent condition.

These findings then, failed to show the dramatic difference previously reported in the literature, regarding the differential effects produced by contingent and non-contingent reinforcement. It was argued, however, that generalization across experimental conditions probably accounts largely for the non-differential effects found in this study.

In terms of productivity, Ss surprisingly showed higher rates of improvement in work output over time when tokens were delivered and
withdrawn non-contingent upon their performance. The possibility of explaining these results in terms of superstitious responding was explored. It was argued, however, that future replication of this phenomenon is necessary before making any generalizations.
CHAPTER I
INTRODUCTION

One of the basic characteristics of the work conducted with instrumental responses, beginning with Thorndike and continuing with Skinner and his colleagues, has been their emphasis on the notion of contingency. For many years contingency has been considered necessary for gaining control over voluntary behavior. Accordingly, contingency became a fixture in experimental designs for operant conditioning research, and has been a basic characteristic of the investigation of schedules of reinforcement conducted by Skinner and his colleagues.

Contingency has been generally understood as a set of conditional probabilities, that is, the probability that given one event another event will occur. In instrumental conditioning, it is usually interpreted as the probability that given a response, reinforcement will occur. For example, control over voluntary responses is obtained by arranging contingencies between pre-selected responses and environmental consequences. Implicit in the notion of contingency, therefore, is the prior emission of a response in order for reinforcement to occur.

This notion of contingency, however, does not seem to explain all the possible relations between response and outcomes to which organisms are sensitive. Environmental events can occur when no specific response has been emitted and they can also affect on-going behavior. In this investigation an attempt was made to study this latter dimension of the
relationships between response and environmental outcomes. That is, an investigation was conducted to study some of the possible effects produced by experimental arrangements which are independent of performance, and also to compare the effects produced by arranging a contingency between performance and consequences.

In this investigation, children's performance in an academic task was studied under two types of environmental arrangements: one in which consequences were conditionally related to their performance and another in which the environmental outcomes were administered independently of their performance.

**Review of the Literature**

Until Skinner's work on schedules of intermittent reinforcement, it was common in most reinforcement studies for every response to be followed by reinforcement, that is, the response was a necessary and sufficient condition for the delivery of reinforcement. Intermittent reinforcement dropped the sufficiency condition but retained the necessity rule according to which the occurrence of the response was still required for procurement of reinforcement. In other words, whatever intermittent reinforcement schedule was in effect, based on either number of responses or temporal parameters, reinforcement would not occur without the necessary antecedent response. Intermittent schedules of reinforcement are usually considered the most frequently occurring schedule in the natural world. That is, while continuous reinforcement can be provided quite easily in the laboratory, it is unlikely that most behaviors could be reinforced all the time outside the laboratory.
In non-laboratory situations behaviors are not only reinforced on intermittent schedules, but also, there seem to be a number of outcomes in the natural environment that appear to occur independently of the organism's responding. In some instances, for example, non-responding which usually may fail to produce reinforcement, sometimes produces it. On such occasions there may be environmental arrangements in operation that are quite different from the typical schedules of reinforcement discussed by Skinner, in which responding is a necessary criterion for reinforcement to occur.

At this point, it appears relevant to mention Seligman, Maier and Solomon's (1971) and Maier, Seligman and Solomon's (1969) argument regarding the independence between environmental events and responses and its subsequent effects. According to these authors, traditional learning theorists have particularly emphasized the organism's learning of two relations between response and reinforcement, either that a certain response produces reinforcement or that the response no longer produces the rewarding consequence. However, a third relation can also be learned by the organism, that there is no relationship between responding and outcome. For Seligman, Maier and Solomon (1971) learning that reinforcement is independent of the presence or absence of a response is also an active way of learning, which in fact may interfere with learning new relationships in subsequent situations. This is the basis for their theory of "learned helplessness" that attempts to explain the phenomenon of passivity observed in some animals when facing trauma.

Skinner also addressed himself to the issue of what appears to be a lack of relationship or independence between reinforcement delivery and
the organism's responding. Skinner (1948) conducted a study in which clock-scheduled reinforcements were delivered to pigeons without any reference to the on-going behavior at the time of reinforcement delivery. Most of the birds in the study developed a pronounced stereotyped pattern of behavior, the particular patterns varying with individual animals. Skinner labelled this pattern of behavior as "superstitious behavior" since, according to the experimenter, this kind of behavior had occurred as a consequence of "accidental conditioning." Skinner reasoned that although no experimental "contingency" had been established beforehand, an effective "contingency" could be inferred from the observed stereotyped pattern of behavior. Skinner describes the process of "accidental conditioning" in the following manner:

The conditioning process is usually obvious. The bird happens to be executing some response as the hopper appears; as a result it tends to repeat this response. If the interval before the next presentation is not so great that extinction takes place, a second "contingency" is probable. This strengthens the response still further and subsequent reinforcement becomes more probable. It is true that some responses go unreinforced and some reinforcements appear when the response has not just been made, but the net result is the development of a considerable state of strength. (pp. 168-169)

However, according to Lachter, Cole and Schoenfeld (1971) it was possible for Skinner, in his description of the process of accidental conditioning, to salvage the concept of contingency by appealing to "(a) the fact that the behavior stream is continuous; (b) to the inference that a reinforcer even when applied without pre-selection of a response must be contingent upon some response; (c) to the presumption that whatever response is in the proper temporal relation to the reinforcer takes the impact of the 'contingency' and emerges as the conditioned response. To rescue contingency in this way, however, is to rob it of at least part
of its meaning, because every reinforcement schedule must then be asserted as being contingent, or conversely, that no schedule can be said to be non-contingent [p. 233]."

To Schoenfeld, Cole, Lang and Mankoff (1973): "Regardless of how the 'superstition' demonstration is viewed it made two things clear: first, the 'contingency' conventionally described as 'procurement' and 'production' could be adequately stated in temporal terms alone; and second, that operant conditioning was possible without it, that is, without an experimentally specified dependence of the temporal distribution of $S^r$ (reinforcement) upon the temporal distribution of $R$ (response) [p. 156]."

The review of the literature that follows includes another aspect of contingencies that may be relevant to this study. Contingency has traditionally implied temporal contiguity between response and reinforcing event, temporal contiguity meaning the relatively immediate occurrence of reinforcement after the organism's response. This notion that immediate reinforcement or temporal contiguity is a critical condition for changing behavior has been set out explicitly by Skinner (1953) and supported by many other behaviorists who explain the way subjects learn when they are classically or instrumentally conditioned by the principle of "association by contiguity." The rat learns the connection between lever-press and food pellet. The dog learns the connection between the bell and food.

According to the principle of contiguity we ought to find a 1 : 1 correspondence between each response and reinforcement; that is, each instrumental response learned by any organism must be reinforced somehow, and reinforcement must follow each particular response very closely in time. This strict interpretation of the principle
of contiguity appears more difficult to apply in the case of schedules of intermittent reinforcement. However, Mowrer and Jones (1945) offered the "response-unit hypothesis" that seems to reconcile them. According to Mowrer and Jones, some studies in the past have shown that the reinforcing effects of reward applied not only to the immediately preceding behavior, but also, in a lesser degree, to behaviors that are somewhat more remote from the reinforcement. Therefore, in the response-unit hypothesis it is argued that "response" must be re-defined to include the whole sequence of behaviors leading to reinforcement.

It is only recently that the principle of temporal contiguity has been challenged (Baum, 1973; Bloomfield, 1972; Rachlin, 1970) as a universal description of the relations between environmental stimuli, responses and reinforcements. For example, Rachlin (1970) suggests the possibility that in instrumental conditioning organisms learn a set of correlations between rates of response and reinforcements. He gives an illustration of the organism's learning correlations: "Let us consider the rat that is in a chamber with the bar. Sometimes the bar is pressed at a slow rate, food comes at a slow rate. When the bar is pressed at a rapid rate, food comes at a rapid rate. We notice that in situations of this kind, rats tend to press the bar rapidly. It is possible that the critical feature of the relationship between response and reinforcement is their correlation in rate, [p. 158]."

Rachlin also makes a distinction between connections and correlations. He argues that correlation between response and reinforcement must not be confused with the pre-arranged schedules of reinforcement discussed by Ferster and Skinner (1957) where certain responses produce
reinforcement immediately. When responses are only correlated with reinforcements, reinforcing events may be received between responses, as well as preceding or immediately after responses. For example, the organism's learning of a positive correlation between its responses and reinforcement will be reflected in an increased rate of responding produced by increased reinforcement and vice versa when learning a negative correlation.

A study by Herrnstein and Hineline (1966) appears to show results that favor the correlation hypothesis. This experiment is also discussed by Rachlin as testing whether correlations can be learned by organisms when not accompanied by direct connections between responses and reinforcements. Herrnstein and Hineline found that rats learned bar-pressing responses when not followed by any other consequence than a reduction in the frequency of electric shock received. In this experiment the experimental design was arranged in such a way that there was not a 1:1 connection between responses and shocks, or between responses and periods of no-shock. In the experiment, rapid rates of bar pressing responses were correlated with a slow rate of shocks; and when rats pressed the bar at a slow rate, shocks came at a rapid rate. The authors found that under these conditions, rats learned to press the bar rapidly.

Results obtained in other studies are also interpreted by Rachlin under the light of the "correlation hypothesis." For example, in a series of investigations conducted by Maier, Seligman and Solomon (1969) and Seligman, Maier and Solomon (1971) some dogs were exposed to electric shock in a harness, delivered independently of their responses. The dogs could do nothing to escape or avoid the shock in those circumstances.
According to the "correlation hypothesis," responses emitted by the dogs in the harness were completely uncorrelated with the rate of shock received. On the next day, the same dogs were put into a shuttle box where signalled shocks were delivered and the dogs given the possibility of escaping and avoiding them by jumping over a barrier. Most of the dogs previously exposed to independent shock did not learn to escape. These dogs appeared to have given up and passively accepted the electric shock. This group of dogs differed significantly in their reaction to the shuttle box from the dogs who had been previously exposed to escapable shock. Some of the passive dogs eventually learned to escape shock, but not until after being dragged across the shuttle several times by the experimenters. Obviously, prior exposure to inescapable shock or shock delivered independently of their behavior produced a strong interference effect on the dog's subsequent behavior in the shuttle box. Previous experience severely retarded and interfered with their learning abilities to escape and avoid shock. Rachlin, as mentioned before, interprets these results in terms of the correlation hypothesis as follows: The dogs learned in the initial condition a "zero correlation" between responding and environmental events. Learning this zero correlation, however, interfered with their learning of the new positive correlation present in the shuttle box.

Rachlin explains further: "The evidence provided by the Seligman, Maier and Solomon experiment is that correlations, even when they are zero correlations, are learned (in the sense that they have potent effect on behavior). It would be difficult to explain this experiment in terms of connections, because the important feature of the first stage of the
experiment when shocks and responses were uncorrelated was that there were no connections established. If there were no connections, what could have been learned? In terms of correlations, however, the experiment is easy to explain, because zero correlations exist on the same continuum as positive correlations. There is not reason why they cannot be learned just as easily as positive correlations. Then, the zero correlation, already learned, interferes with the effect of the positive correlation imposed later [p. 165]."

The review of the literature that follows leaves the theoretical aspects of contingencies and concentrates on reviewing empirical evidence collected in several areas of research regarding the differential effects of administering positive contingent and non-contingent outcomes. In most of the following studies contingency vs. non-contingency were operationally defined in the way proposed by Schoenfeld and Farmer, 1970: contingency means "that the distribution in time of R [response] determines the distribution in time of reinforcements" while, on the other hand "non-contingency means that the temporal distribution of reinforcements is not determined by the temporal distribution of responses [p. 221]."

First, the empirical evidence collected so far regarding organisms' preference for either type of reinforcement appears fairly conclusive at the present time. Organisms in general seem to prefer response-contingent reward over response-independent reward. For example, Jensen (1963) found that when 200 rats were given a choice between eating pellets from a dish placed on the floor of a Skinner box or pressing a bar to earn pellets, all but one of the rats left the dish with "free pellets"
and resumed pressing the bar to obtain food during the 40 minute choice period. Neuringer (1969) and Carder and Berkowitz (1970) also found that animals seem to prefer response-contingent food over food delivered on a non-contingent basis. Singh (1970) found that not only rats, but also children, prefer to work to obtain reward even when having the possibility of obtaining it free. And, more recently, Tarte and Collier (1972) conducted three experiments with pre-school and elementary school children to investigate behavioral preferences when allowed the choice between obtaining candies or pennies by taking them from a dish or by pulling a lever. Results of the study confirmed previous findings. They showed children's high preference (77%) for earned rewards.

With regard to the effects of non-contingent positive outcomes, studies from different areas of research have reported investigating such phenomena. For example, in the area of Internal vs. External Control developed by Rotter (1966) we find studies investigating the effects of "random or non-contingent reinforcement" upon Ss' expectancies for future reinforcement. According to Rotter's (1954) social learning theory the probability of any behavior to occur in a particular situation is a function of two important variables: first, the person's expectancy that the given behavior will be instrumental in obtaining the available reinforcement, and second, the value of the reinforcement for the individual behaving. Thus, in Rotter's theory, reinforcement's main function is to strengthen Ss' expectancy that a particular behavior will be followed by that reinforcement in the future.

Experimenters in the area of Internal vs. External control have exposed Ss to what they have called "skill vs. chance" conditions. In
the first condition Ss are usually instructed that reward in the tasks
to be performed is determined by their own skill, in other words, con-
ingent upon their performance. In the second condition, Ss are instructed
that obtaining reward is outside their control, that is dependent upon
chance, fate or powerful others. The basic hypothesis underlying these
experiments is that if a person perceives a reinforcement as being con-
ingent upon his own behavior, the occurrence of positive or negative
reinforcement will strengthen or weaken the probability for that behavior
to occur again in the same or similar condition. If S sees reinforce-
ment as being out of his control or non-contingent, then the behavior
preceding reinforcement will have a lower probability to re-occur.

To test the above hypothesis some studies were carried out comparing
verbal expectancies for future reinforcement under chance and skill con-
ditions. One of the first studies was that undertaken by Phares (1957) who
used an ambiguous color matching task and instructed half of the Ss that
the task was so difficult that success would be a matter of luck and
another half that success would be a matter of skill. Expectancy was
measured by the number of chips a S would be willing to bet on his
probability of being correct on the succeeding trial. Despite the fact
that both groups received the same number and sequence of reinforcements,
the skill conditions had greater effect on raising or lowering expectancies
for future reinforcements than chance conditions.

In a second study reported one year later, James and Rotter (1958)
investigated the effects of partial vs. 100% reinforcement on the extinction
of expectancies developed under skill and chance conditions. According to
James and Rotter (1958, p. 398), "On the basis of social learning theory,
it was reasoned that under chance conditions of 100% reinforcement, S received no cues of non-reinforcement during the training series. When extinction begins he utilizes the first non-reinforcements as cues that the situation has changed. Since control of the situation is perceived as external there is a sudden shift in expectancy and extinction is rapid. With partial reinforcement under chance conditions S receives cues of non-reinforcement as well as reinforcement. When extinction is begun non-reinforcement does not produce a new cue which he can use to discriminate a change in situations. Consequently, extinction is more gradual and more trials are required before the situation is recategorized." It is interesting to note that Sheffield (1949) had already offered a similar interpretation of this phenomenon. However, hers was primarily based upon stimulus-response concepts while James and Rotter introduced the expectancy variable.

To investigate the validity of Rotter's social learning theory hypothesis, James and Rotter (1958) used a simple but ambiguous card-guessing task in which success was totally controlled by the experimenter, although it did not appear so to the Ss. Instructions were then used to create skill vs. chance conditions. Ss in the skill condition were instructed that success in the task depended primarily upon their own skills, while Ss in the chance condition were told that success in such a task was entirely a matter of luck. To summarize, results of the study seemed to be in agreement with the hypothesis postulated by the authors of the study. Under the skill condition, the 50% reinforcement schedule produced more rapid extinction of expectancies than the 100% reinforcement schedule, while the chance condition showed the results typical of
previous studies using partial reinforcement: the 100% reinforcement schedule produced faster extinction than the 50% or partial reinforcement schedule. Expectancies of success were measured by Ss' own rating on an 11-point scale. Extinction was considered to occur when the verbalized expectancy fell to either 0 or 1 on three consecutive trials.

Another experiment reported by Rotter, Liverant and Crowne (1961) replicated James and Rotter's findings without using different instructions to create chance vs. skill conditions. In this study, Ss were exposed to two different tasks which would be regarded as skill- and chance-controlled on the basis of the Ss' previous cultural experience. The skill-determined task involved a motor-skill apparatus (Vertical Aspiration Board) while the chance-determined task involved the card-guessing procedure used by James and Rotter. Results of this study indicate that verbal expectancies showed less increments and decrements under the situation where reinforcement was perceived by Ss as being non-contingent upon their behavior (chance condition). Ss showed much greater fluctuations of verbal expectancies under the skill condition. This study also replicated James and Rotter's (1958) findings regarding the resistance to extinction offered by verbal expectancies from Ss exposed to 100% vs. partial reinforcement under chance vs. skill conditions.

Other studies have also tested, at least indirectly, for any differential effects produced by contingent vs. non-contingent positive outcomes. For example, Bandura and Perloff (1967) conducted a study with 7- to 10-year-old children in which the authors utilized a self-monitored reward procedure in one of the experimental conditions. That is, one
group of children was exposed to an experimental condition in which Ss had to set their own standards of behavior required to attain reinforcement, and also had to administer the rewards to themselves when reaching their self-prescribed standard of performance (contingent reward). A second group of children was exposed to another experimental condition that consisted of an externally controlled reward system. In this condition reward was administered on a non-contingent basis, that is, each child "inherited" a number of tokens before playing the game. A third group of children was treated as a control group. Results of the study showed that both the self-contingently monitored (contingent reward) and the externally controlled (non-contingent) reward conditions were equally efficacious in inducing behavioral productivity as compared to the control condition.

A second study also using the self-monitored system in one of the conditions, tested for the differential effect upon behavioral productivity of contingently administered vs. non-contingently imposed reward. In this study, Liebert, Spiegler and Hall (1970) required Ss to set their own standards and to turn a crank to achieve them. Ss in the contingent-reward condition self-administered tokens whenever the self-selected standards were attained. Ss in the non-contingent reward condition received the tokens at equal time intervals while playing, being yoked to Ss in the contingent condition with respect to density of reinforcement and intervals of time for delivery of reinforcement. The exchange value of tokens (none, low and high) was also an experimental variable manipulated by the experimenters. Results of this study showed that the productivity of children receiving exchangeable tokens exceeded that of
the control group, who received no tokens. The number of responses (cranks) emitted per session was directly related to the value of the tokens, in both experimental groups. However, an interesting result was obtained despite the fact that Ss in the externally-controlled reward system were explicitly told that they were being rewarded on a non-contingent basis, they, too, responded differentially to the different values assigned to the tokens. Even more significant, Ss in the non-contingent condition changed their standards significantly more than Ss in the contingent-reward condition. Liebert, Spiegler and Hall (1970) interpreted these results as follows: "Thus, it appears that non-contingently rewarded Ss may well have changed their standard frequently in an effort to "understand" or maximize the contingency, a phenomenon that may have numerous counterparts in life situations where the criteria for productivity are ambiguous [p. 246]."

One of the most typical methods of comparing the effects of contingent vs. non-contingent positive events has been to shift, within the same individual, from a functioning contingent schedule to a non-contingent schedule, and vice versa. The effects on the rate of responding produced by the shift are then carefully observed and measured. For example, the results most often reported in the animal and human literature when using this method has been a decrease in the rate of responding when organisms are exposed to non-contingent positive events as compared to the rate maintained during the stage of contingent reinforcement. A typical study in the human literature is that conducted by Hart, Reynolds, Baer, Brawley and Harris (1968). In this study, the experimenters used the reversal design incorporating the two different contingencies of reinforcement
upon cooperative play responses of a 5-year-old girl in a pre-school setting. A baseline phase was recorded, then non-contingent social reinforcement from teachers was introduced. After several sessions of receiving non-contingent social attention and approval, teachers began to administer the social reinforcement only contingent upon cooperative play or approximations to it. Following evidence of changes in the rate of cooperative play behaviors, each condition was discontinued and returned to a prior condition for the purpose of demonstrating experimental control over the behavioral changes. Hart et al. (1968) reported data showing that non-contingent social attention and approval did not appreciably develop cooperative play in the girl. However, significant rate changes were observed everytime the social reinforcement was made contingent upon cooperative play behavior.

Schoenfeld, Cole, Land and Mankoff (1973) argue, however, that shifts from contingent to non-contingent schedules of reinforcement usually produce an increased variability of responding since no constraint is placed on the topography of the S's behaviors when exposed to non-contingent reinforcement as compared to contingent reinforcement. This greater variability in behavior, they believe, possibly accounts for the greater reduction in rates of responding.

They also claim that a lower rate of responding under non-contingent reward is not a universal finding. Other occasional reports have been made of indefinite maintenance of ongoing rates of responding during prolonged exposure to what the authors called non-contingent schedules of reinforcement. For example, a study by Fenner (1969) reports that key pecking acquired by two pigeons on an autotraining schedule was
maintained by a non-contingent fixed interval 6-sec. schedule of reinforcement. Neuringer (1970) reinforced with grain the first three pecks emitted by pigeons on a response-key. Neuringer found later that pigeons continued to peck the response-key "superstitiously" throughout 50 experimental sessions in which grain was presented independent of the subjects' behaviors.

The following studies are especially relevant to the present investigation because they are concerned with children's academic performance, although they deal only with the effects of contingent reinforcement. For example, Kirby and Shields (1972) reported increases in the rate of Ss' responses in arithmetic tasks when using a fixed ratio schedule of praise and immediate correctness feedback contingent upon Ss' performance. They also reported a collateral increase in percentage of time in attending behavior when the above procedure was used. Removal of such treatment produced decreases in both rate of responses and attending behavior. Hopkins, Schutte and Garton (1971) using an ABAB experimental design also found greater effectiveness of using the procedure of allowing access to playground contingent upon children's completion of their academic assignments as compared to a baseline condition. These authors reported the superiority of using reinforcement contingent upon children's performance in producing increases in work rates and trends towards fewer errors. Lahey, McNees and Brown (1973) have also reported significant improvements in children's reading comprehension with the utilization of praise and pennies made contingent upon correct answers. Chadwick and Day (1971) measured the effects of contingent tangible reinforcers (points) and social reinforcements on
children's academic performance. Three dependent variables were measured by the authors of this investigation: percentage of time at work, work output per minute and accuracy of performance. All three dependent variables showed higher increases when tangible and social reinforcements were administered contingent upon Ss' performances as compared to a baseline condition. Other studies which have reported similar findings when using contingent reinforcement conditions include Knapczyk and Livingston (1973) and Brigham, Graubard and Stans (1972).

A few studies have reported the differential effects of contingent as well as non-contingent reward upon academic performance. Glynn (1970), for example, utilized four different groups of children to compare the effects produced by self-determined and experimenter-determined token reinforcement (contingent conditions), chance-determined token reinforcement (non-contingent condition), and no token treatment (control condition). Results of this study showed that the non-contingent condition produced a level of accuracy in Ss' performance that was not only lower than those produced by the two contingent conditions but also lower than the level of accuracy maintained by the control group. In another study, Brigham, Finfrock, Breunig and Bushell (1972) exposed six school kindergarten children to three different reinforcement conditions: a baseline condition in which no tokens were administered, another condition in which tokens were administered on Ss' correct responses and finally a condition in which tokens were delivered on a non-contingent basis. They found that children were more accurate when the tokens were delivered contingent upon their correct responses and that non-contingent delivery of tokens reduced accuracy below baseline levels. On the other
hand, these authors reported that an analysis of the absolute rate of responses—correct and incorrect responses—also showed that children had nearly doubled their productivity during the non-contingent phase as compared to the contingent phase.

In sum, then, studies measuring the differential effects produced on academic and non-academic behaviors, by the contingent and non-contingent administration of rewards have reported evidence of significant increases in rates of responding under contingent reinforcement conditions. When both conditions of reinforcement were compared, contingent reinforcement typically showed superiority over non-contingent reinforcement in producing positive effects. Furthermore, some studies reported decreases or deterioration of Ss' rates of responding under the non-contingent condition. Only one study reported that although contingent reinforcement produced higher accuracy in Ss' academic performance, an analysis of their absolute rate of responding showed that Ss nearly doubled their work output under the non-contingent phase. All studies reviewed utilized either reversal or group-statistical designs in their comparison of the effects of contingent and non-contingent reinforcement.

**Statement of the Problem**

It was the purpose of this investigation to test for any differential effects on children's academic performances produced by the exposure of each child to two different experimental conditions: one in which external consequences were arranged to be administered contingent upon their responding on the academic task and another one in which outcomes were arranged on a non-contingent basis. Again, for the purpose of this study, contingency and non-contingency were operationally defined along
the lines proposed by Schoenfeld and Farmer (1970). That is, in the contingent condition, the temporal distribution of the experimentally pre-specified responses determined the distribution in time of the external consequences, while in the non-contingent condition the temporal distribution of outcomes was not determined by the temporal distribution of responses.
CHAPTER II

METHOD

Two experiments were conducted. In Experiment I a group of first-grade children were individually exposed on a daily basis to two different experimental conditions. In one of them, each $S$ was given tokens contingent upon his correct performance on an arithmetic task. In a second condition each $S$ received tokens which were given independent of his performance on the same task. The quality of performance was measured by accuracy of responses to arithmetic problems and rate of improvement. Productivity, operationally defined as the frequency of arithmetic problems attempted per session regardless of accuracy, was also measured in terms of rates of problems completed per session and rate of change in that frequency over time. Tokens obtained during the performance could be later exchanged for a variety of toys and candies.

In Experiment II another group of first-grade children was exposed on a daily basis to two different experimental conditions. In one of them, tokens were delivered contingent upon the $S$'s correct responses on the arithmetic task, but withdrawal of tokens also occurred every time the $S$ made an incorrect response. In a second experimental condition, delivery and withdrawal of tokens occurred on a non-contingent basis, that is, independently of the correctness of responses. As in Experiment I, tokens obtained during the performance could be later exchanged for a variety of toys and candies.
In this investigation a multiple schedule design (Ferster and Skinner, 1957) was adopted. The multiple schedule design is generally considered more sensitive to individual differences than the ABAB design. Using a multiple schedule, findings of differential responding are quite powerful experimental results. In this particular type of schedule, differential rates of responding are more clearly seen as being a product of the specific control produced by the different stimuli and schedules of reinforcement used in each component. Two main disadvantages are generally attributed to the multiple schedule design, however. First, it is usually quite difficult, especially when using human Ss, to employ two or more experimental conditions per day on the same subject, as compared to employing a single condition for several days. Second, there is always the possibility of generalization from one experimental condition to another.

During the pilot work conducted previous to this investigation, an interesting phenomenon was observed. Children generally did not show any evidence of differential performance when exposed to both contingent and non-contingent reward conditions. They seemed not to discriminate between the experimental conditions and furthermore, they appeared to work as hard under both types of arrangements.

Several environmental stimuli were then systematically manipulated throughout the pilot work and later introduced in this investigation, especially those that appeared to facilitate the process of discrimination. Environmental stimuli were made quite different for both conditions: different experimenters, different rooms and different color sheets associated with each experimental condition. All of these
environmental variables were, of course, counterbalanced among Ss to control for their possible direct effects. Finally, each S had a prolonged exposure to both experimental conditions that was also expected to facilitate discrimination.

These procedures were used in Experiments I and II; however, to facilitate the process of discrimination even further, another variable was introduced in Experiment II: withdrawal of rewards. Introduction of the withdrawal of reinforcement procedure was also made in response to the findings of the pilot work that the administration of contingent vs. non-contingent positive events alone did not seem to produce any significant difference in the way children performed under both experimental conditions.

The withdrawal of reinforcement procedure commonly known as "response cost" has been extensively used in laboratory research with human Ss (Weiner, 1962; 1963; 1964; 1965a; 1965b; 1969). This procedure has also proved to be a powerful one in controlling behavior when used contingent upon Ss' behaviors outside the laboratory and in therapeutic situations (e.g., Boren and Colman, 1970; Kazdin, 1971; Winkler, 1970).

Phillips, Phillips, Fixen and Wolf (1971) for example, contrasted the relative efficacy of token reinforcement, response cost and both procedures combined with predelinquent boys. These experimenters found that the administration of positive reinforcement plus a response cost procedure resulted in a greater percentage of correct response on a quiz administered after watching a TV news program. In another study, Schmauk (1970) reports that response cost seemed to make Ss more aware of the contingencies when compared to other punishment procedures.
Experiment I

Subjects

Eight students (4 boys and 4 girls) selected by the teacher from a first grade class at Moseley Elementary School, Palatka, Florida, served as subjects. The Ss were between 6 and 7 years old (see Appendix A for further description of Ss).

Materials

Two distinctively different rooms at Moseley Elementary School were used for the experiment. In each room there was an armchair where the S could sit on one side of a wooden screen. On the other side of the screen there was a table containing a carousel projector, a whistle and a bell that were used by the experimenter (E). The screen did not allow S to fully see E on the other side of the screen but E could observe the S's written performance from the other side (see Appendix B). The carousel projector contained slides with numbers 0 through 20, equivalent to the number of tokens to be collected by S. The slides were projected on a screen placed in front of S. Next to the S's chair a box was placed containing a large number of tokens that could be picked up by S at the end of his performance, depending on the last number projected on the screen. Also, on the floor next to the S's chair was a pile of children's comics.

Two sets of sheets of paper containing simple counting tasks were used. Sheets of one set were blue and the other set was yellow. On top of the blue sheets, a circle appeared at each corner. Below each of the circles were 5 different arithmetic problems. Each problem consisted of circles and squares alternated randomly. On the right hand side of each problem four numbers appeared in column. One of the numbers represented the correct number of circles contained in that specific problem. Two
different blue sheets containing a total of 20 problems were presented to every child during each session (see Appendix C).

The yellow sheets were identical in format except that a square appeared at each corner instead of a circle. On the right hand side of each problem four numbers appeared in column. One of the numbers represented the correct number of squares contained in that specific problem. Two different yellow sheets containing a total of 20 problems were presented to every child during each session (see Appendix D).

To control for the level of difficulty of problems on both yellow and blue sheets, problems appearing on the two different color sheets contained equivalent numbers of circles and squares. Also, the alternative answers were the same for each type of sheet.

**Procedure**

**Preparatory stage:**

For three consecutive days, each child was brought to the rooms by each experimenter to get acquainted with the task, tokens and prizes. Each child was primarily familiarized with the way in which numbers were projected on the screen in front of him. The procedure to follow was explained to him by $E$ in the following manner: Everytime he came to the room he would have to sit in the armchair and at the sound of the whistle he would begin to work on the problems placed in front of him. The procedure of counting the number of circles or squares in each problem, depending on the sheet presented, was also explained to him. $E$ explained that it would be his task to choose the number of circles or squares from four alternative answers on the right. First, he would have to complete the left hand column of the sheet before continuing with the
problems on the right hand column. In case of completing one sheet, he could continue working on the second sheet available on his desk. During the three days of the trial period, each child was shown the procedure of obtaining tokens that could be exchanged later for toys or candies. E explained to S that during his performance, numbers on the screen would change--the noise of the carousel projector plus the bell ringing would indicate to him that a new and higher number of tokens was projected on the screen. Numbers projected on the screen would represent the number of tokens he was getting. E explained to S that at the end of 2 minutes he would hear the whistle again and at that time he would have to stop working. Then, he would pick up a number of tokens equivalent to the last number shown on the screen. This procedure was practiced several times during the three days of preparation for the experiment. During this trial period, E let the child exchange tokens for a variety of toys and candies of different token-prices, attempting to establish the tokens as conditioned reinforcers for the child.

E also explained to S the purpose of the comics on the floor next to him, that if at any time he felt tired, he could pick the comics up and read them as he wished.

Experimental procedure:

Every day each S was conducted first to one of the rooms to have a 2-minute session with one of the Es. At the end of his first session with one of the Es, and after collecting the tokens, S went to the second room to work with a second E for two more minutes. A period of time ranging between 5 and 10 minutes usually elapsed between one session and the other.
In one of the rooms, one of the Es administered the contingent condition. That is, after the sound of the whistle E added one point on the screen paired with the sound of the bell ringing at the end of every correct choice made to a problem. At the end of 2 minutes E sounded the whistle again indicating that the session was over.

In another room, a second E administered the non-contingent condition. That is, after the sound of a whistle E began adding one point on the screen paired with the bell ring, according to a time schedule prepared in advance of the session. This schedule was independent of the S's performance on this task. The number of points projected on the screen, to be exchanged later for tokens, was presented according to the number of tokens obtained by S during the alternate session of contingent delivery of tokens.

Both experimental conditions were presented daily and their order of presentation alternated, to control for any effects produced by the order in which they were presented to S. Also, Es and environmental stimuli such as rooms and their corresponding set of task sheets (blue or yellow) were counterbalanced among all Ss to control for their possible effects.

At the end of the two daily sessions S was allowed to exchange the tokens for his choice of toys and candies. The number of days exposed to the experimental conditions varied with each child. Although the study was conducted for a total of 21 school days, children were absent from school some days during this period.
Experiment II

Subjects

Eight students (4 boys and 4 girls) were originally selected by their teacher and assigned to this experiment. However, one of the girls was ill 14 days out of the 21 days in which the experiment was conducted and, consequently, the data on her performance were later discarded from the final results. Ss for this experiment were students from a first grade class at Moseley Elementary School, Palatka, Florida. The Ss' ages ranged between 6 and 7 years old. Ss in Experiment II were different from those Ss participating in Experiment I (see Appendix A for further description of Ss).

Materials

The same two rooms and equipment used in Experiment I were also used for this experiment. The same two Es were involved in this experiment, too. Only a new instrument was introduced: a "cricket" was utilized during the sessions to announce the losing of points projected on the screen.

Two sets of sheets, identical to those used in Experiment I, were also used in this experiment.

Procedure

Preparatory stage:

The first three days of the preparatory stage were conducted in the same way as in Experiment I, except that in this experiment Ss were also acquainted with the procedure used by E to communicate to them the loss of a point or token. Loss of a token was indicated by a change in the number projected on the screen (to a lower number) paired with the sound of the "cricket." Gaining one point would be indicated as in Experiment I, by
a change in the number projected on the screen (to a higher number) and the sound of the bell.

Experimental procedure

Every day each S was conducted to one of the rooms, where one of the experimental conditions was administered. At the end of the first session in that room S was conducted to a second room for the administration of a second experimental condition. A period of time ranging between 5 and 10 minutes usually elapsed between one session and the other.

In one of the rooms, one of the Es administered the contingent condition. After signaling the beginning of the session by using the sound of the whistle the experimenter began adding one point to the screen paired with the sound of the bell at the end of every choice by S. In this experiment, however, E also took away one point (projected on the screen and paired with the sound of a cricket) every time an incorrect choice occurred. At the end of the 2-minute session the E sounded the whistle indicating that the session was over. The same type of sheets were used for each S throughout his exposure to this experimental condition. At the end of the session S was allowed to pick up the number of tokens equivalent to the points projected on the screen.

When conducted to the other room, a second E administered the non-contingent condition. That is, after the initial signal of the whistle E began adding one point paired with the sound of the bell or taking away one point paired with the sound of the cricket, according to an independent and previously elaborated schedule. At the end of two minutes after the
initial signal, $E$ whistled again indicating the end of the session. The same type of sheet was used on a daily basis in this condition. At the end of this session, $S$ was allowed again to collect tokens equivalent to the last number projected on the screen. At the end of the two daily sessions $S$ was allowed to exchange the tokens obtained in both conditions for toys or candies.

Both experimental conditions were presented daily as in Experiment I, and their order of presentation alternated. Also, $Es$ and environmental stimuli were counterbalanced among $S$s.
CHAPTER III

RESULTS

The differential effects produced by the experimental conditions were measured on two dependent variables: accuracy and productivity of the S's daily performance on the arithmetic task. The basic measure of accuracy utilized in this investigation was the Accuracy Ratio. The Accuracy Ratio is defined as the ratio between frequency of correct responses and frequency of incorrect responses (Pennypacker, Koenig and Lindsley, 1972). The productivity of S was measured by the daily frequency of arithmetic problems attempted per session, regardless of the accuracy of his performance.

Since information regarding proportional changes of a behavior is valuable, the accuracy ratios and frequencies of problems completed per session were plotted on Standard Behavior Charts (Pennypacker et al., 1972). Figures 1 through 30 (Appendix E) show graphically the daily accuracy ratios for each S under each experimental condition. Figures 31 through 60 (Appendix F) present graphically the daily frequency of problems completed by each S during each experimental session. By plotting behavior frequency data on a logarithmic scale a linear graphic representation of proportional rather than absolute changes in behavior frequencies can be obtained (Koenig, 1972).

In order to facilitate an understanding of the results presented in this chapter, it may be helpful to present a brief description of the
Standard Behavior Chart as well as some of the procedures used in the analysis of the data.

Standard Behavior Chart

The Standard Behavior Chart is actually a semi-logarithmic chart, whose vertical scale includes six logarithmic cycles. This vertical dimension of the Standard Behavior Chart represents the scale of frequencies or rates of responses. The units of measurement are movements or responses per minute. The horizontal scale represents the time scale that runs for 140 consecutive days or a total of 20 calendar weeks.

Celeration

This is a measure of behavior change over time. Frequencies or ratios displayed on the Standard Behavior Chart are usually either accelerating (x) or decelerating (:) as time passes. The term celeration is used to describe the acceleration or deceleration relationships. Celeration is actually a measure of change in frequency of responding that occurs within a week's period of time. The celeration coefficient is the ratio value that describes the slope of the line of best fit calculated for a series of frequencies. The line of best fit is obtained by using the least squares method of regression. One of the most important characteristics of celeration is that it provides a measure of behavior change which is independent of the initial rate of behavior, so that it gives a measure of improvement without reference to initial accuracy or speed.

Accuracy Ratio Celeration

This is a measure of the change in accuracy that takes place over time. The accuracy ratio celeration provides the opportunity to compare
across Ss the celerating effects on accuracy ratios produced by different experimental arrangements. Measuring celerating effects rather than the absolute values of changes in performance facilitates comparison among Ss allowing for individual differences in levels of performance. Figures 61 through 75 (Appendix G) graphically show on the left side of each figure the accuracy ratio celerations for each S under both experimental conditions in Experiments I and II.

Productivity Celeration

This is a measure of change in productivity or frequency of problems attempted that occurs over time. Figures 61 through 75 (Appendix G) show graphically on the right side of each figure the productivity celerations for each S under the two experimental conditions in Experiments I and II.

Analysis of the Accuracy of Ss' Performance

Table 1 shows the geometric means* and standard deviations of the accuracy ratios per experimental condition for each S in Experiment I. A matched t-test computed on the difference between the geometric means of Ss in the two experimental conditions shows a non-significant difference (t = .31). Delivery of tokens contingent upon the S's correct responses did not seem to affect significantly the average performance of Ss as compared to the condition in which tokens were delivered on a non-contingent basis.

*The geometric mean is considered to be the proper average for any ratio measures. Since the measures used in this investigation--frequency of responses, accuracy ratio and celerations--are ratios, the geometric mean was chosen as the appropriate measure of central tendency.
**TABLE 1**

GEOMETRIC MEANS AND STANDARD DEVIATIONS OF THE ACCURACY RATIOS PER CONDITION FOR EACH S - EXPERIMENT I

GEOMETRIC MEANS:

<table>
<thead>
<tr>
<th>Subjects</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contingent</td>
<td>4.79</td>
<td>.79</td>
<td>4.67</td>
<td>1.93</td>
<td>3.04</td>
<td>.56</td>
<td>3.75</td>
<td>6.41</td>
</tr>
<tr>
<td>Non-Contingent</td>
<td>6.02</td>
<td>.82</td>
<td>5.90</td>
<td>1.78</td>
<td>3.37</td>
<td>.49</td>
<td>4.99</td>
<td>3.70</td>
</tr>
</tbody>
</table>

STANDARD DEVIATIONS:

<table>
<thead>
<tr>
<th>Subjects</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contingent</td>
<td>2.21</td>
<td>1.42</td>
<td>1.50</td>
<td>1.74</td>
<td>1.91</td>
<td>1.52</td>
<td>2.39</td>
<td>1.27</td>
</tr>
<tr>
<td>Non-Contingent</td>
<td>1.80</td>
<td>1.65</td>
<td>1.34</td>
<td>1.85</td>
<td>1.76</td>
<td>1.70</td>
<td>1.85</td>
<td>1.81</td>
</tr>
</tbody>
</table>

\[ t = .31; \text{n.s.}; \text{d.f.} = 7.\]
Table 2 shows the geometric means and standard deviations of the accuracy ratios per experimental condition for each S in Experiment II. A matched $t$-test shows a non-significant difference between the geometric means of the accuracy ratios produced by the condition in which delivery and withdrawal of tokens were made contingent upon Ss' performance and those produced by the other experimental condition in which delivery and withdrawal were made non-contingent upon Ss' behavior ($t = .42$).

A matched $t$-test computed for all Ss from both experiments also showed a non-significant difference ($t = .05$). Results appear to indicate that administration of tokens on a contingent and on a non-contingent basis did not seem to affect differently the average accuracy performance of Ss.

Finally, a matched $t$-test was computed on the difference between the over-all average levels of accuracy obtained by Ss participating in Experiment I and those obtained by Ss in Experiment II. The $t$-test showed a statistically significant difference between both groups ($t = 2.75; p < .02$, two tailed; $d.f. = 28$). That is, Ss in Experiment I showed on the average a significantly higher level of accuracy in their performance than Ss in Experiment II.

Table 3 shows the Accuracy Ratio Celerations for each S under each experimental condition in Experiment I. A matched $t$-test computed on the log of the celerations produced by the two experimental conditions showed a non-significant difference ($t = 2.22$). That is, the delivery of tokens on a contingent and on a non-contingent basis failed to produce significantly different effects on the rates of accuracy changes of Ss'
TABLE 2
GEOMETRIC MEANS AND STANDARD DEVIATIONS OF THE ACCURACY RATIOS PER CONDITION FOR EACH S - EXPERIMENT II

GEOMETRIC MEANS:

<table>
<thead>
<tr>
<th>Subjects</th>
<th>9</th>
<th>10</th>
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<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contingent</td>
<td>1.61</td>
<td>.292</td>
<td>.54</td>
<td>2.78</td>
<td>4.62</td>
<td>.53</td>
<td>.85</td>
</tr>
<tr>
<td>Non-Contingent</td>
<td>2.82</td>
<td>.298</td>
<td>.94</td>
<td>1.91</td>
<td>3.32</td>
<td>.47</td>
<td>.55</td>
</tr>
</tbody>
</table>

STANDARD DEVIATIONS:

<table>
<thead>
<tr>
<th>Subjects</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contingent</td>
<td>2.09</td>
<td>1.78</td>
<td>1.60</td>
<td>1.86</td>
<td>2.02</td>
<td>1.78</td>
<td>1.94</td>
</tr>
<tr>
<td>Non-Contingent</td>
<td>2.35</td>
<td>1.89</td>
<td>1.58</td>
<td>1.99</td>
<td>2.12</td>
<td>1.79</td>
<td>1.92</td>
</tr>
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</table>

\[ t = .42; \text{n.s.}; \ d.f. = 6 \]
<table>
<thead>
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<tbody>
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<td>XI.35</td>
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<td>2</td>
<td>XI.10</td>
<td>XI.23</td>
</tr>
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<td>3</td>
<td>XI.18</td>
<td>XI.16</td>
</tr>
<tr>
<td>4</td>
<td>X2.02</td>
<td>X1.53</td>
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<tr>
<td>5</td>
<td>XI.36</td>
<td>X1.16</td>
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<td>6</td>
<td>XI.60</td>
<td>X1.16</td>
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<tr>
<td>7</td>
<td>XI.59</td>
<td>X1.37</td>
</tr>
<tr>
<td>8</td>
<td>XI.15</td>
<td>XI.35</td>
</tr>
</tbody>
</table>

\( t = 2.22; \text{n.s.; } d.f. = 7 \)
performances. It is also important to note that accelerating effects occurred in all cases under the contingent and non-contingent conditions.

Table 4 shows the Accuracy Ratio Celerations for each S under the two experimental conditions in Experiment II. This Table shows that accelerating effects were found in all cases, except for S #10 who showed a deceleration in accuracy under the non-contingent condition. Six out of seven Ss showed greater celeration on accuracy under the contingent condition. However, a matched t-test computed on the log of the celerations under both experimental conditions showed that the difference in celerations was statistically non-significant (t = 1.91). That is, the delivery and withdrawal of tokens contingent upon Ss' correct and incorrect responses to the arithmetic task did not produce any significantly greater celerating effect on the accuracy of Ss' performance when compared to that produced by the delivery and withdrawal of tokens on a non-contingent basis. Also, in Experiment II, most Ss (with one exception) showed acceleration in the accuracy of their performances under both experimental conditions.

However, when a matched t-test was conducted on the log of the accuracy ratio celerations produced by contingent vs. non-contingent administration of tokens, for all Ss from Experiments I and II combined, a statistically significant difference was obtained (t = 2.72; p < .02, two tailed; d.f. = 14) in favor of the contingent conditions.

Analysis of Ss' Productivity

Table 5 shows the geometric means and standard deviations of the frequency of problems attempted daily by each S regardless of accuracy under each experimental condition in Experiment I. A matched t-test
### TABLE 4

ACCURACY RATIO Celerations per Condition for each S - Experiment II

<table>
<thead>
<tr>
<th>Subjects</th>
<th>Contingent</th>
<th>Non-Contingent</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>X2.02</td>
<td>X1.38</td>
</tr>
<tr>
<td>10</td>
<td>X1.31</td>
<td>÷1.16</td>
</tr>
<tr>
<td>11</td>
<td>X1.15</td>
<td>X1.43</td>
</tr>
<tr>
<td>12</td>
<td>X1.33</td>
<td>X1.23</td>
</tr>
<tr>
<td>13</td>
<td>X1.50</td>
<td>X1.47</td>
</tr>
<tr>
<td>14</td>
<td>X1.26</td>
<td>X1.04</td>
</tr>
<tr>
<td>15</td>
<td>X1.35</td>
<td>X1.06</td>
</tr>
</tbody>
</table>

$t = 1.91; \text{n.s.}; d.f. = 6$
<table>
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</tr>
</thead>
<tbody>
<tr>
<td>Contingent</td>
<td>3.72</td>
<td>2.65</td>
<td>2.77</td>
<td>2.38</td>
<td>2.94</td>
<td>3.08</td>
<td>4.10</td>
<td>2.94</td>
</tr>
<tr>
<td>Non-Contingent</td>
<td>3.56</td>
<td>2.35</td>
<td>3.17</td>
<td>2.29</td>
<td>2.81</td>
<td>2.72</td>
<td>4.48</td>
<td>2.89</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Subjects</th>
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<th>5</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Contingent</td>
<td>1.24</td>
<td>1.33</td>
<td>1.16</td>
<td>1.42</td>
<td>1.27</td>
<td>1.58</td>
<td>1.22</td>
<td>1.24</td>
</tr>
<tr>
<td>Non-Contingent</td>
<td>1.40</td>
<td>1.28</td>
<td>1.13</td>
<td>1.49</td>
<td>1.18</td>
<td>1.55</td>
<td>1.34</td>
<td>1.34</td>
</tr>
</tbody>
</table>

\[ t = .39; \text{n.s.}; d.f. = 7 \]
computed on the geometric means of problems attempted under the experimental condition in which delivery of tokens was made contingent upon S's performance and that obtained for the condition in which tokens were delivered non-contingently, was non-significant ($t = .39$). That is, the delivery of tokens contingent upon S's accuracy of performance vs. the non-contingent delivery of tokens did not seem to differentially affect the average level of S's productivity.

Table 6 shows the geometric means and standard deviations of the productivity of Ss under each experimental condition in Experiment II. In one of the experimental conditions in Experiment II, tokens were delivered and withdrawn from Ss contingent upon the accuracy of their performance, in another condition the delivery and withdrawal of tokens was made on a non-contingent basis. A matched $t$-test on the average productivity under both experimental conditions also showed a non-significant difference ($t = .79$).

Table 7 shows the productivity celerations for each S under each experimental condition in Experiment I. A matched $t$-test on the log of the celerations of the two experimental conditions was non-significant ($t = .25$). That is, the delivery of tokens contingent upon S's correct responses did not appear to affect the S's productivity celeration differently from the delivery of tokens independent of the S's performance.

Table 8 shows the productivity celerations for each S under the two experimental conditions in Experiment II. Surprisingly, six out of seven Ss showed greater acceleration of their productivity under the condition in which the delivery and withdrawal of tokens was made independent of Ss' performance. A matched $t$-test on the logs of the
# Table 6

Geometric means and standard deviations of the productivity per condition for each S - Experiment II

<table>
<thead>
<tr>
<th>Subject</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contingent</td>
<td>3.01</td>
<td>3.51</td>
<td>2.10</td>
<td>2.91</td>
<td>3.52</td>
<td>3.14</td>
<td>3.52</td>
</tr>
<tr>
<td>Non-Contingent</td>
<td>3.17</td>
<td>5.44</td>
<td>1.87</td>
<td>2.81</td>
<td>3.52</td>
<td>3.03</td>
<td>3.46</td>
</tr>
</tbody>
</table>

Standard deviations

<table>
<thead>
<tr>
<th>Subject</th>
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<th>10</th>
<th>11</th>
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<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contingent</td>
<td>1.22</td>
<td>1.37</td>
<td>1.37</td>
<td>1.24</td>
<td>1.22</td>
<td>1.45</td>
<td>1.28</td>
</tr>
<tr>
<td>Non-Contingent</td>
<td>1.27</td>
<td>1.32</td>
<td>1.34</td>
<td>1.24</td>
<td>1.25</td>
<td>1.54</td>
<td>1.42</td>
</tr>
</tbody>
</table>

$t = .79; \text{n.s.}; d.f. = 6$
<table>
<thead>
<tr>
<th>Subjects</th>
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<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contingent</td>
<td>X1.08</td>
<td>X1.11</td>
<td>X1.03</td>
<td>X1.12</td>
<td>X1.07</td>
<td>X1.32</td>
<td>X1.10</td>
<td>X1.13</td>
</tr>
<tr>
<td>Non-Contingent</td>
<td>X1.17</td>
<td>X1.09</td>
<td>X1.04</td>
<td>X1.01</td>
<td>X1.02</td>
<td>X1.35</td>
<td>X1.14</td>
<td>X1.11</td>
</tr>
</tbody>
</table>

\[ t = .25, \text{n.s.; d.f.} = 7 \]
TABLE 8

PRODUCTIVITY CELERATIONS PER CONDITION FOR EACH S - EXPERIMENT II

<table>
<thead>
<tr>
<th>Subjects</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contingent</td>
<td>X1.05</td>
<td>X1.07</td>
<td>X1.07</td>
<td>X1.04</td>
<td>X1.06</td>
<td>X1.25</td>
<td>X1.12</td>
</tr>
<tr>
<td>Non-Contingent</td>
<td>X1.11</td>
<td>X1.16</td>
<td>X1.18</td>
<td>X1.09</td>
<td>X1.04</td>
<td>X1.32</td>
<td>X1.22</td>
</tr>
</tbody>
</table>

\[ t = 3.94; \ p < .01, \ \text{two tailed}; \ d.f. = 6 \]
celerations produced under the contingent compared to the non-contingent experimental condition showed a significant difference ($t = 3.94; p < .01$, two tailed; $d.f. = 6$). That is, delivery and withdrawal of tokens non-contingently upon Ss' performance produced a significantly higher acceleration in the productivity of Ss as compared to that produced by the delivery and withdrawal of tokens contingent upon Ss' academic performance.
CHAPTER IV
DISCUSSION

Results obtained in this investigation regarding the differential effects produced by the contingent and non-contingent administration of tokens on children's academic performance require cautious interpretation. Beginning with the effects on accuracy of performance, Table 1 shows no superiority for the contingent over the non-contingent condition. Indeed, 5 out of 8 Ss in Experiment I achieved higher (absolute) accuracy under the non-contingent condition. Table 2, which shows the same kind of data under the response cost procedure, also shows no significant difference in the average level of accuracy produced by contingent and non-contingent delivery and withdrawal of tokens. These findings failed to support most of the findings in the literature which have generally found superiority of contingent reinforcement [Chadwick and Day, 1971; Kirby and Shields (1972); Knapczyk and Livingston (1973); Hopkins, Schutte and Garton (1971); Brigham, Graubard and Stans (1972); Lahey, McNees and Brown (1973)].

Turning now from mean accuracy ratios to accuracy ratio celerations, a measure of change over time, Tables 3 and 4 show that both experimental conditions produced acceleration or trend towards improvement in accuracy in all Ss participating in both experiments, except in one case. Only S #10 showed accuracy acceleration under the contingent condition and deceleration in accuracy when exposed to non-contingent delivery and
withdrawal of tokens. Also, when comparing the rates of celeration produced by contingency and non-contingency conditions for each experiment, the differences were statistically non-significant except when comparing the differential effects produced in all Ss participating in Experiments I and II. That is, when all Ss from both experiments were combined, the contingent administration of environmental consequences produced an over all higher rate of improvement in accuracy over time as compared to that produced by non-contingent administration.

Results of this investigation then, failed to show the dramatic difference in effects produced by contingent and non-contingent reinforcement previously reported by Glynn (1970); Brigham et al. (1972); and Hart et al. (1968).

Two particular characteristics of the method employed in this investigation deserve careful evaluation especially when comparing present findings to previous ones. First of all, the task involved the presentation of the same set of problems over and over again throughout the whole experiment. It is likely that this procedure practically insured that a certain degree of improvement would take place under any conditions just as a consequence of the Ss' familiarity with the task and repeated practice with the same problems. Second, most studies comparing the differential effects of contingent and non-contingent reinforcement have either exposed different groups of Ss to the different experimental conditions, or have used the reversal procedure of shifting within the same S from a functioning contingent schedule to a non-contingent schedule, and vice versa. In contrast to previous research, this investigation used a multiple schedule design, exposing every S to
both experimental conditions on a daily basis. It is possible that, although special care was taken to make stimuli under the two experimental conditions quite different from each other, the tasks remained similar in nature, especially when contrasted with their other activities, and were presented quite close in time. It is very likely therefore, that some generalization across experimental conditions occurred.

Finally, a comparison between the overall average levels of accuracy achieved by Ss in Experiments I and II showed that the group of Ss who participated in Experiment I performed with a significantly higher level of accuracy than Ss in Experiment II. Although the difference may be attributed to the procedure of token administration, it is also possible that the difference in accuracy may reflect the initial differences in performance level of Ss in the two experiments. Ss from both groups seemed to differ in their levels of performance from the beginning of the experiment. In fact, 5 out of 8 Ss participating in Experiment I showed during their initial performance accuracy ratios equal to or above 1, as compared to only 2 out of 7 Ss participating in Experiment II. However, a t-test of the difference between the initial level of accuracy of both groups, defined as Ss' performance during the first three days, failed to achieve statistical significance (t = 1.55).

While the accuracy ratio emphasized correct responses in relation to incorrect ones, the productivity measure is a more direct measure of output. An analysis of the mean productivity shown on Tables 5 and 6 indicate that the average level of children's productivity under both experimental conditions did not differ significantly. However, looking at the rate of increase in productivity, Tables 7 and 8 show that all
SSs participating in both experiments increased their output over time under both experimental conditions. Previous studies had consistently reported increases in work output when children were reinforced contingently upon different aspects of their academic performances (e.g., Hopkins, Schutte and Garton, 1971; Brigham, Graubard and Stans, 1972). These investigations, however, did not attempt to compare productivity under contingent and non-contingent reinforcement. Part of the present findings then, seem to be consistent with previously reported ones, since an increase in productivity over time was also found when children were exposed to contingent reward.

The most interesting results are found, however, when a comparison is made between the rates of increase in productivity shown by SSs under both experimental conditions. Contrary to expectations, a test of significance showed that the non-contingent delivery and withdrawal of tokens accelerated the rate of SSs' productivity significantly more than their administration on a contingent basis. This difference was statistically significant only when the response cost procedure was used in Experiment II.

Although this last result appears difficult to interpret, it should be noted that a previous study (Brigham, Finfrock, Breunig and Bushell, 1972), primarily interested in comparing the differential effects of different reinforcement contingencies on the accuracy of children's academic performances, reports a similar finding. They report that an analysis of the absolute rate of academic responses—both correct and incorrect responses—revealed that children nearly doubled their rate of productivity during the experimental phase in
which a non-contingent procedure of reinforcement was used, compared to the phase in which a contingent procedure was used.

It is possible to interpret these results in terms of adventitious reinforcement. That is, tokens administered non-contingently on Ss' accuracy of performance may have adventitiously reinforced their rate of productivity. Liebert, Spiegler and Hall (1970) made a similar interpretation of their findings in which Ss who were non-contingently rewarded changed their standards to obtain reinforcement significantly more frequently than Ss receiving reward contingent upon the appropriate motor responses. They argued that their findings could be interpreted in terms of supersitious responding, and furthermore, using a cognitive description of the results, they interpreted Ss' behaviors in the non-contingent condition as possibly representing an effort on their part to understand or maximize contingency.

In regard to this last interpretation, it may be relevant to raise the question of the possible influence played by past history of reinforcement on the behaviors exhibited by Ss in these investigations. For example, it is the impression of the experimenters participating in this study that children generally came to the experimental sessions with the strong expectation that obtaining tokens during their performance would depend on how they did in the assigned task. Therefore, based upon the assumption that many of the children's experiences in school frequently involve getting reward according to their academic performances, it is possible to infer that such history of reinforcement may have been responsible for Ss' performance during this investigation. Furthermore, such history of reinforcement may have been an important
factor which interfered with the process of discrimination, especially since the present experiment took place in a school setting and also involved an academic task. Future research should attempt to control for the effects of past history of reinforcement of Ss on their responding to contingent and non-contingent environmental arrangements.

Certain factors obviously limit the generalization of the present findings. First, the limited number of Ss participating in both experiments suggests the need for replication. Second, the length of time that Ss were exposed to the experimental conditions may not have been sufficient to insure appropriate discrimination between contingency and non-contingency. Further research should investigate the extent to which these findings are determined by the experimental designs utilized, i.e., reversal procedure vs. multiple schedule design. Schoenfeld et al. (1973) for example, have claimed that shifts from contingent to non-contingent schedules of reinforcement (as in reversal designs) usually produce an increased variability of response since no constraint is placed on the topography of the S's behavior when exposed to non-contingent reinforcement. They argued, then, that this greater variability in behavior possibly accounts for the greater reduction in rates of responding reported by other investigations.

Future research should also be addressed to the question of generalization across conditions when each S is exposed to both contingent and non-contingent environmental arrangements. Procedures that may facilitate the S's discrimination of the experimental conditions should be incorporated into the experimental designs utilized by future investigators. For example, it may be worthwhile to place S
under concurrent schedules involving the procedures proposed by Findley (1958, 1962). Findley has proposed that switching behavior and the conditions for switching in a concurrent schedule need to be more explicit and more subject to the experimenter's control. Such procedures may not only facilitate the discriminative process but could also provide information about the S's preference for a given schedule arrangement.
CHAPTER V
SUMMARY

In the present study, it was argued that the traditional notion of contingency does not seem to explain all the possible relations between response and outcomes to which organisms are sensitive. There may also be outcomes when no specific response has been emitted and they can also affect on-going behavior. The present investigation then, aimed at studying this latter dimension of the relationships between response and outcome. That is, the effects produced by the administration of tokens independent of Ss' behaviors were compared to the effects produced by arranging a contingency between their performance and obtaining tokens. For this purpose, children's performance in an academic task was studied under two types of environmental arrangements: one in which the receipt of tokens was conditionally related to their performance, and another in which the tokens were received independent of their performance.

A review of the literature suggested the superiority of administering reward contingent upon responses over their non-contingent administration, to produce positive increases in behavior, especially in higher levels of academic performance. Some studies even reported decreases or deterioration of Ss' behaviors when exposed to non-contingent reward. Only three studies showed somewhat unexpected results. One of them reported that self-monitored-contingent reinforcement and non-contingent
externally imposed reward were equally efficacious in inducing productivity among Ss. A second study reported that Ss changed their standards to obtain reinforcement significantly more frequently when rewarded on an externally based schedule than when rewarded according to a self-monitored schedule. Finally, a third study reported that although contingent reinforcement produced higher accuracy in children's academic performance, an analysis of their absolute rates of responding showed that Ss nearly doubled their productivity during the non-contingent phase of the experiment.

The present investigation consisted of two separate experiments. In Experiment I eight first-grade children were individually exposed on a daily basis to two different experimental conditions (multiple schedule design). In one of them, each S was given tokens contingent upon his correct performance on an arithmetic task. In a second condition each S received tokens which were given independent of his performance on the same task. The quality of performance was measured by accuracy of responses to arithmetic problems and rate of improvement. Productivity, operationally defined as the frequency of arithmetic problems attempted per session regardless of accuracy, was also measured in terms of rates of problems completed per session and rate of change in frequency over time. Tokens obtained during the performance could be later exchanged for a variety of toys and candies.

In Experiment II seven first-grade children were exposed on a daily basis to two different experimental conditions. In one of them, tokens were delivered contingent upon the S's correct responses on the arithmetic task, but withdrawal of tokens also occurred every time the
S made an incorrect response. In a second experimental condition, delivery and withdrawal of tokens occurred on a non-contingent basis, that is, independently of the correctness of responses. As in Experiment I, tokens obtained during the performance could be later exchanged for a variety of toys and candies.

Results of the experiments can be summarized as follows:

1) In terms of Ss' accuracy of performance, no significant difference was found between Ss' average level of accuracy under the contingent and non-contingent condition. Also, both experimental conditions produced acceleration or a trend towards improvement in accuracy in all Ss participating in both experiments with the exception of one S who showed deceleration in the level of accuracy over time under the non-contingent condition.

These findings then, failed to show the dramatic difference previously reported in the literature regarding the differential effects produced by contingent and non-contingent conditions. Neither did the present results support most of the previous findings which have shown superiority of the contingent arrangements.

To help account for these differences in results some characteristics of the method employed in the present investigation, such as the kind of task involved and the multiple schedule design utilized, were discussed and contrasted with previous research. It was argued that generalization across experimental conditions probably accounted largely for the non-differential effects found in this study.

2) In terms of productivity, Ss surprisingly showed higher rates of improvement in work output over time when tokens were delivered and
withdrawn non-contingent upon their performance. The possibility of explaining these results in terms of superstitious responding was explored. It was argued however, that future replication of this phenomenon is necessary before making any generalizations. Finally, some suggestions for future research were offered.
APPENDIX A

DESCRIPTION OF SS AND NUMBER OF SESSIONS ATTENDED
### Experiment I

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APPENDIX B

DIAGRAM OF EXPERIMENTAL SETTING SHOWING RELATIVE POSITION OF E AND S
APPENDIX C
BLUE SHEETS CONTAINING ARITHMETIC PROBLEMS
APPENDIX D

YELLOW SHEETS CONTAINING ARITHMETIC PROBLEMS
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APPENDIX E

DAILY ACCURACY RATIOS FOR EACH S UNDER EACH EXPERIMENTAL CONDITION
Figure 1. Daily accuracy ratios obtained by S #1 under the contingent condition in Experiment I.
Figure 2. Daily accuracy ratios obtained by S #1 under the non-contingent condition in Experiment I.
Figure 3. Daily accuracy ratios obtained by S #2 under the contingent condition in Experiment I.
Figure 4. Daily accuracy ratios obtained by S #2 under the non-contingent condition in Experiment I.
Figure 5. Daily accuracy ratios obtained by S #3 under the contingent condition in Experiment I.
Figure 6. Daily accuracy ratios obtained by S #3 under the non-contingent condition in Experiment I.
Figure 7. Daily accuracy ratios obtained by S #4 under the contingent condition in Experiment I.
Figure 8. Daily accuracy ratios obtained by S #4 under the non-contingent condition in Experiment I.
Figure 9. Daily accuracy ratios obtained by S #5 under the contingent condition in Experiment I.
Figure 10. Daily accuracy ratios obtained by S #5 under the non-contingent condition in Experiment I.
Figure 11. Daily accuracy ratios obtained by S #6 under the contingent condition in Experiment I.
Figure 12. Daily accuracy ratios obtained by S #6 under the non-contingent condition in Experiment I.
Figure 13. Daily accuracy ratios obtained by S #7 under the contingent condition in Experiment I.
Figure 14. Daily accuracy ratios obtained by S #7 under the non-contingent condition in Experiment I.
Figure 15. Daily accuracy ratios obtained by S #8 under the contingent condition in Experiment I.
Figure 16. Daily accuracy ratios obtained by S #8 under the non-contingent condition in Experiment I.
Figure 17. Daily accuracy ratios obtained by S #9 under the contingent condition in Experiment II.
Figure 13. Daily accuracy ratios obtained by S #9 under the non-contingent condition in Experiment II.
Figure 19. Daily accuracy ratios obtained by S #10 under the contingent condition in Experiment II.
Figure 20. Daily accuracy ratios obtained by S #10 under the non-contingent condition in Experiment II.
Figure 21. Daily accuracy ratios obtained by S #11 under the contingent condition in Experiment II.
Figure 22. Daily accuracy ratios obtained by S #11 under the non-contingent condition in Experiment II.
Figure 23. Daily accuracy ratios obtained by S #12 under the contingent condition in Experiment II.
Figure 24. Daily accuracy ratios obtained by S #12 under the non-contingent condition in Experiment II.
Figure 25. Daily accuracy ratios obtained by S #13 under the contingent condition in Experiment II.
Figure 26. Daily accuracy ratios obtained by S #13 under the non-contingent condition in Experiment II.
Figure 27. Daily accuracy ratios obtained by S #14 under the contingent condition in Experiment II.
Figure 23. Daily accuracy ratios obtained by S #14 under the non-contingent condition in Experiment II.
Figure 29. Daily accuracy ratios obtained by S #15 under the contingent condition in Experiment II.
Figure 30. Daily accuracy ratios obtained by S #15 under the non-contingent condition in Experiment II.
APPENDIX F

DAILY RATES OF PRODUCTIVITY OBTAINED BY EACH S UNDER EACH EXPERIMENTAL CONDITION
Figure 31. Daily rates of productivity obtained by S #1 under the contingent condition in Experiment I.
Figure 32. Daily rates of productivity obtained by S #1 under the non-contingent condition in Experiment I.
Figure 33. Daily rates of productivity obtained by S #2 under the contingent condition in Experiment I.
Figure 34. Daily rates of productivity obtained by S #2 under the non-contingent condition in Experiment I.
Figure 35. Daily rates of productivity obtained by S #3 under the contingent condition in Experiment I.
Figure 36. Daily rates of productivity obtained by S #3 under the non-contingent condition in Experiment I.
Figure 37. Daily rates of productivity obtained by S #4 under the contingent condition in Experiment I.
Figure 38. Daily rates of productivity obtained by S #4 under the non-contingent condition in Experiment I.
Figure 39. Daily rates of productivity obtained by S #5 under the contingent condition in Experiment I.
Figure 40. Daily rates of productivity obtained by S #5 under the non-contingent condition in Experiment I.
Figure 41. Daily rates of productivity obtained by S #6 under the contingent condition in Experiment I.
Figure 42. Daily rates of productivity obtained by S #6 under the non-contingent condition in Experiment I.
Figure 43. Daily rates of productivity obtained by S #7 under the contingent condition in Experiment I.
Figure 44. Daily rates of productivity obtained by S #7 under the non-contingent condition in Experiment I.
Figure 45. Daily rates of productivity obtained by S #8 under the contingent condition in Experiment I.
Figure 46. Daily rates of productivity obtained by S #8 under the non-contingent condition in Experiment I.
Figure 47. Daily rates of productivity obtained by S #9 under the contingent condition in Experiment II.
Figure 48. Daily rates of productivity obtained by S #9 under the non-contingent condition in Experiment II.
Figure 49. Daily rates of productivity obtained by S #10 under the contingent condition in Experiment II.
Figure 50. Daily rates of productivity obtained by $S$ #10 under the non-contingent condition in Experiment II.
Figure 51. Daily rates of productivity obtained by S #11 under the contingent condition in Experiment II.
Figure 52. Daily rates of productivity obtained by S #11 under the non-contingent condition in Experiment II.
Figure 53. Daily rates of productivity obtained by S #12 under the contingent condition in Experiment II.
Figure 54. Daily rates of productivity obtained by S #12 under the non-contingent condition in Experiment II.
Figure 55. Daily rates of productivity obtained by S #13 under the contingent condition in Experiment II.
Figure 56. Daily rates of productivity obtained by S #13 under the non-contingent condition in Experiment II.
Figure 57. Daily rates of productivity obtained by S #14 under the contingent condition in Experiment II.
Figure 58. Daily rates of productivity obtained by S #14 under the non-contingent condition in Experiment II.
Figure 59. Daily rates of productivity obtained by S #15 under the contingent condition in Experiment II.
Figure 60. Daily rates of productivity obtained by S #15 under the non-contingent condition in Experiment II.
APPENDIX G

ACCURACY RATIO AND PRODUCTIVITY CELERATIONS FOR EACH S UNDER EACH EXPERIMENTAL CONDITION
Figure 61. Accuracy ratio and productivity celerations for S #1 under the contingent and non-contingent conditions in Experiment I.
Figure 62. Accuracy ratio and productivity celerations for S #2 under the contingent and non-contingent conditions in Experiment I.
Figure 63. Accuracy ratio and productivity celerations for S #3 under the contingent and non-contingent conditions in Experiment I.
Figure 64. Accuracy ratio and productivity celerations for S #4 under the contingent and non-contingent conditions in Experiment I.
Figure 65. Accuracy ratio and productivity celerations for S #5 under the contingent and non-contingent conditions in Experiment I.
Figure 66. Accuracy ratio and productivity celerations for S #6 under the contingent and non-contingent conditions in Experiment I.
Figure 67. Accuracy ratio and productivity celerations for S #7 under the contingent and non-contingent conditions in Experiment I.
Figure 68. Accuracy ratio and productivity celerations for S #8 under the contingent and non-contingent conditions in Experiment I.
Figure 69. Accuracy ratio and productivity celerations for S #9 under the contingent and non-contingent conditions in Experiment II.
Figure 70. Accuracy ratio and productivity celerations for S #10 under the contingent and non-contingent conditions in Experiment II.
Figure 71. Accuracy ratio and productivity celerations for S #11 under the contingent and non-contingent conditions in Experiment II.
Figure 72. Accuracy ratio and productivity celerations for #12 under the contingent and non-contingent conditions in Experiment II.
Figure 73. Accuracy ratio and productivity celerations for S #13 under the contingent and non-contingent conditions in Experiment II.
Figure 74. Accuracy ratio and productivity celerations for $S$ #14 under the contingent and non-contingent conditions in Experiment II.
Figure 75. Accuracy ratio and productivity celerations for S #15 under the contingent and non-contingent conditions in Experiment II.
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BIOGRAPHICAL SKETCH

Carlos M. Alvarez was born December 30, 1944, in Cardenas, Cuba. He attended Colegio de Belen in La Habana, Cuba, where he graduated from High School in 1961. He came to the United States in August, 1962. In March, 1970, he received his Bachelor of Arts degree in psychology at the University of Florida.

In March, 1970, he enrolled in the Graduate School of the University of Florida where until the present he has pursued his work toward degrees of Master of Arts and Doctor of Philosophy. He received his Master of Arts degree in Psychology in December, 1971, and in August, 1973, he completed his Internship in Clinical Psychology. Currently he is working as a Clinical Psychologist at Tri-County Mental Health Service in Palatka, Florida.

Carlos M. Alvarez is a member of Phi Kappa Phi, Honorary Scholastic Society, and student member of several professional organizations. He is married to the former Arminda Alvarez and is the father of two children.
I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

James C. Dixon, Chairman
Professor of Psychology

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

William D. Wolking, Co-Chairman
Professor of Special Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Henry S. Pennypacker
Professor of Psychology

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Theodore Landsman
Professor of Counselor Education
I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

[Signature]
Donald Avila
Professor of Foundations of Education

This dissertation was submitted to the Graduate Faculty of the Department of Psychology in the College of Arts and Sciences and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August, 1974

[Signature]
Dean, Graduate School