LARGE DEFLECTION BEHAVIOR EFFECT IN REINFORCED CONCRETE COLUMN UNDER SEVERE DYNAMIC SHORT DURATION LOAD

By

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A THESIS PRESENTED TO THE GRADUATE SCHOOL OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

UNIVERSITY OF FLORIDA

2010
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To my girlfriend, family and friends
ACKNOWLEDGMENTS

I would like to express my sincere gratitude to Dr. Theodor Krauthammer, my research advisor, for his precious advice and direction during the completion of this research. I would also like to thank Dr. Serdar Astarlioğlu for his constructive advice and support which greatly facilitated the completion of this research. I am thankful for Canadian Armed Forces, more particularly 1 ESU (1st Engineer Support Unit) for giving me the opportunity to complete graduate studies in structural engineering. My final gratitude goes to my family, friends and loved one who supported me during the completion of this study.
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<tr>
<td>$A_c$</td>
<td>Area of concrete</td>
</tr>
<tr>
<td>$A_{cl}$</td>
<td>Area of concrete at each layer</td>
</tr>
<tr>
<td>$A_g$</td>
<td>Area of gross section</td>
</tr>
<tr>
<td>$A_{si}$</td>
<td>Area of steel at each layer</td>
</tr>
<tr>
<td>$c$</td>
<td>Neutral axis depth</td>
</tr>
<tr>
<td>$C$</td>
<td>Damping coefficient</td>
</tr>
<tr>
<td>$C_c$</td>
<td>Compressive concrete force at mid-span</td>
</tr>
<tr>
<td>$C_c'$</td>
<td>Compressive concrete force at support</td>
</tr>
<tr>
<td>$c_m$</td>
<td>Section neutral axis depth at mid-span</td>
</tr>
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<td>$c_s$</td>
<td>Section neutral axis depth at support</td>
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<tr>
<td>$C_s$</td>
<td>Compressive steel force at mid-span</td>
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<td>$C_s'$</td>
<td>Compressive steel force at support</td>
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<tr>
<td>$D'$</td>
<td>Nominal diameter of longitudinal compression reinforcement (inches)</td>
</tr>
<tr>
<td>$D''$</td>
<td>Nominal diameter of hoops (inches)</td>
</tr>
<tr>
<td>$d_{si}$</td>
<td>Depth of steel layer</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Elastic modulus of concrete (psi)</td>
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<tr>
<td>$Enh_c$</td>
<td>Concrete enhancement factor in compression</td>
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<tr>
<td>$Enh_{ct}$</td>
<td>Concrete enhancement factor in tension</td>
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<td>$Enh_s$</td>
<td>Steel enhancement factor</td>
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<td>Stress of concrete layer</td>
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<td>$F_n$</td>
<td>Function</td>
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<td>$f_s$</td>
<td>Steel stress</td>
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<tr>
<td>$f_{si}$</td>
<td>Stress of steel layer</td>
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<td>Steel layer forces</td>
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<tr>
<td>$F_t$</td>
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</tr>
<tr>
<td>$f_u$</td>
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<tr>
<td>$f_y$</td>
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<td>$f_y^*$</td>
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<td>$h$</td>
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<td>$H''$</td>
<td>Average core dimensions of the confined concrete compression zone measure to the outside of stirrups (inches)</td>
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<td>$i$</td>
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<tr>
<td>$ILF$</td>
<td>Inertia load factor</td>
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<tr>
<td>$i_s$</td>
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<td>$K_m$</td>
<td>Mass factor</td>
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I  Length
I_u  Unsupported length
m  Mass
M_1  Column end moment at support 1
M_2  Column end moment at support 2
m_{ci}  Concrete layer moment
M_e  Equivalent mass
M_{fi}  Ultimate moment capacity due only to flexure
M_{nm}  Moment at mid-span
M_{ns}  Moment at support
m_{si}  Steel layer moment
M_t  Total mass
M_u  Ultimate moment capacity due only to shear
N  Membrane axial force
N_{ext}  External axial load
P  Pressure/axial load
P_r  Resistance function load
P_{max}  Maximum pressure
P_{so}  Incident pressure
P_o  Atmospheric Temperature
t_a  Time of arrival
t_o  Time of positive Phase
t_{o^-}  Time of negative phase
q Transverse load

$Q_{1i}$ Static reaction at support 1 at time step i

$Q_2$ Static reaction at support 2 at time step i

$Q_i$ Load at time step i

R Distance from a reference explosion

r Radius of gyration

$R_m$ Resistance function

s Spacing of hoops (inches)

S Surrounding support stiffness

SRF Shear reduction factor

t Section lateral displacement at support

TF Trust factor

$T_s$ Tensile steel force at mid-span

$T_{s'}$ Tensile steel force at support

u Displacement

$\dot{u}$ Velocity

$\ddot{u}$ Acceleration

$u_e$ Equivalent displacement

$\dot{u}_e$ Equivalent velocity

$\ddot{u}_e$ Equivalent acceleration

W Equivalent TNT weight

w Deflection

$WE_b$ External Work in beam
\[ \text{\( W_{E_e} \)} \quad \text{External Work of equivalent system} \]

\[ x \quad \text{Position} \]

\[ Z \quad \text{Scaled distance} \]

\[ z_i \quad \text{Depth of concrete layer} \]

\[ \beta \quad \text{Newmark-Beta constant} \]

\[ \Delta \quad \text{Increment space} \]

\[ \delta \quad \text{Deflection} \]

\[ \varepsilon \quad \text{Total Strain} \]

\[ \varepsilon_a \quad \text{Axial strain} \]

\[ \varepsilon_{h/2} \quad \text{Reinforced concrete strain at } h/2 \]

\[ \varepsilon_p \quad \text{Strain due to creep and shrinkage} \]

\[ \varepsilon_s \quad \text{Steel strain} \]

\[ \varepsilon_{sh} \quad \text{Hardening steel strain} \]

\[ \varepsilon_{su} \quad \text{Ultimate steel strain} \]

\[ \infty \quad \text{Infinity} \]

\[ \rho \quad \text{Steel ratio} \]

\[ \rho_r \quad \text{Ratio of the confining steel volume to the confined concrete core volume per unit length of the element in compression zone} \]

\[ \rho' \quad \text{Longitudinal compression reinforcement ratio} \]

\[ \rho^* \quad \text{Web reinforcement ratio} \]

\[ \Phi \quad \text{Curvature} \]

\[ \sigma \quad \text{Stress} \]

\[ \gamma' \quad \text{Inertia proportionality factor} \]
LARGE DEFLECTION BEHAVIOR EFFECT IN REINFORCED CONCRETE COLUMN UNDER SEVERE DYNAMIC SHORT DURATION LOAD

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May 2010

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Reinforced concrete columns may undergo large deformations when subjected to blast load. In addition to static loads that impose flexural, axial and shear effects, a combination of blast-induced transient transverse and axial loads due to blast loads may induce large deformations and lead to geometric instabilities. Furthermore, if the flexural deformation becomes very large, the structural member might shift to a tension membrane response. One needs to address all these phenomena when conducting a dynamic analysis. Advanced explicit nonlinear dynamic finite element codes may perform such analysis but require large numerical resources and very long execution times. Consequently, advanced simplified and accurate computational tools are required to address these complicated behaviors with much more limited computational resources, in support of design activities and/or rapid incident and/or damage assessments. Therefore, a new numerical computational capability was developed for the computer code DSAS to address those needs. The new approach can account for transverse and axial responses, secondary moments when undergoing large deformations, geometric instabilities, and the transition into a tension membrane. The
present thesis explains this new approach developed, explains how it was validated and presents numerical results for reinforced concrete columns undergoing explosion-induced large deformations. Conclusions and recommendations are also included.
CHAPTER 1
INTRODUCTION

Problem Statement

In recent years, many studies have been conducted by governmental and non-governmental organizations across the world such as the Engineer Research and Development Center U.S. Army, Defense Research Development Canada and the Center of Infrastructure Protection and Physical Security University of Florida, in an attempt to better understand the effect of blast, fragmentation and impact loads on buildings in order to better design against specific threats. Publications such as Unified Facilities Criteria 3-340-02 (U.S Department of Defense, 2008), ASCE Structural Design for Physical Security (ASCE, 1999), and ASCE Guideline for Blast Resistance Buildings in Petrochemical Facilities (ASCE, 1997) are few example of publications produced in light of the development accomplishments in this field and are available to engineers to better design both military and civilian structures. In spite of these guidelines, much more work still remains to be done to better understand structural behavior of a variety of different structures, structural components and structural materials when subjected to short-duration severe dynamic loads. Examples of such required works include reinforced concrete or steel connections, composite sections and reinforced concrete columns undergoing large deformation.

Of all components in a structure, columns may be the most critical ones as they carry the most amount of the structural component. In the past, they were generally not designed to sustain large lateral dynamic loads such as blast loading and, for that reason, are very susceptible to terrorist attacks. As an example, in 1995, part of the Alfred P. Murrah Federal Building in Oklahoma City collapsed after three of the four
north face columns were destroyed by a blast, resulting in 168 casualties and several injured citizens (Bangash et al. 2006). For this reason, the need to better understand column behavior quickly became clear in the field of progressive collapse. At the design stages, a finite element analysis may be the most widely used approach to verify a design for a specific threat as it offers great capabilities, but it often requires advanced knowledge of finite elements and copious amounts of time. On the other end, design guidelines may be expedient to design against specific threats but also come with a cost as they should not be applied to all cases and may not yield the most cost efficient design as conservative measures may be built into them. When it comes to a real-time possible threat on a building and counter measures that are immediately needed, both approaches are usually of no use to first respondent engineers. In such scenarios, a unified approach yielding accurate results in a restrained timeframe is desired and has now been under development for some years.

In the case of reinforced concrete elements, research and development has made great progress in recent years for the behavior of several cases such as reinforced concrete slabs, beams and columns, i.e. Ross (1983), Krauthammer et al. (1988), Shanaa (1991), Krauthammer, T., Schoedel, R., Shanaa, H. (2002), Krauthammer et al. (2004), Tran (2009), and many more. Algorithms have been developed to quickly analyze structural components using advanced single degree-of-freedom system (SDOF) approach coupled with nonlinear material behavior models and different modes of failure such as flexural and direct shear. Improvement in the analysis of the capacity of reinforced concrete columns subjected to both lateral forces and axial forces were recently accomplished by Tran et al. (2009).
In his research, Tran et al. (2009) developed an algorithm to analyze reinforced concrete column structures as seen in Figure 1-1 subjected to both a variable lateral and variable axial load. By decoupling the column and the beam elements of the structure into two distinct single degree of freedom systems, it was possible to analyze both elements at each time step and take into account variable load interaction on each element, changing with time. Parametric studies conducted during the research with similar models using finite element software and DSAS lead to the conclusion that improvement is still required for the analysis of reinforced concrete columns when undergoing large deformation. As an example, tension membrane effects in columns have yet to be implemented in such algorithm although studies show it would improve significantly the section capacity prediction. In addition, second order moments, also known as P-Delta effects, need to be incorporated in such analysis as they increase the flexural stress of the columns.

**Objective and Scope**

The primary objective of this research was to develop an expedient and accurate numerical algorithm to analyze the response of reinforced concrete columns subjected to severe short dynamic loads such as blast and impact loads. The following are the steps of this research.

- Develop a flexural resistance function calculation algorithm for columns, including tension membrane behavior, P-Delta effects and Euler buckling, and adapt the algorithm for variable axial load when present.

- Validate algorithm using finite element numerical software ABAQUS by comparing similar analysis between DSAS and ABAQUS.

- Conduct parametric studies using DSAS to fully understand the effect of large deformation and tension membrane action in reinforced concrete columns under severe short dynamic loads.
Although protective structures are designed against blast and fragmentation, this study only focused on the behavior of reinforced concrete columns under severe, short-duration dynamic loads, or, more precisely, blast loads. The effect of fragmentation load was dismissed due to the need for more research and development in this field of study to fully capture its effect on the structure and properly model its behavior using adequate numerical software.

**Research Significance**

The algorithm developed in this study was incorporated into the computer code DSAS developed at the Center for Infrastructure Protection and Physical Security at the University of Florida and will provide the means to generate more realistic reinforced concrete column resistance functions. As a result of this research, the computer code DSAS has now the capability to account for large deformations, geometric instability and tension membrane effects in addition to the other phenomena already taken into account by the program when conducting dynamic analysis of reinforced concrete section. The research is discussed in detail in the next chapters.
Figure 1-1. Blast load combination on internal structural components. [Reprinted with permission from Tran T.P. 2009. Effect of short duration high impulse variable axial and transverse loads on reinforced concrete column. M.S. dissertation (Page 48, Figure 3-1). University of Florida, Gainesville, Florida.]
CHAPTER 2
LITERATURE REVIEW

Introduction

Chapter 2 provides a review of recent studies and developments in the field of protective structures based on a number of technical reports, books and publications related to the scope of this research. The first topic discussed is an overview of the Dynamic Structural Analysis Suite (DSAS) software which will be intensively used for this research. Blast load calculations for simple and complex structures are then discussed followed by structural analysis theoretical models relevant to this research, to conclude with discussion on load-impulse diagrams.

Dynamic Structural Analysis Suite (DSAS)

DSAS has been under continuous development under the direction of Dr. Theodore Krauthammer since the early 1980s. The program is designed to perform a static and dynamic analysis of a wide range of structural elements such as reinforced concrete beams/column/slabs, masonry walls, steel components, and others, for point load, uniform load, single degree of freedom load, blast load and impact. The program has the ability to generate moment curvature for beams and columns as well as resistance functions and pressure/load impulse diagrams of all elements it analyzes. The program uses advanced single degree of freedom (SDOF) and non-linear material behavior allowing for fast analysis, making it efficient and expedient.

As this chapter reviews theoretical concepts, references will be made to the software to ensure good theoretical background comprehension of the program for subjects related to this research and how the concepts are incorporated into DSAS.
One may refer to the DSAS user manual (Astarlioglu, 2008) for a complete description of the software.

**Blast Load Calculation**

**Blast Load Calculations for Simple Structures**

A shock wave is caused by a sudden violent release of energy as a result of an explosion. There are many different sources of explosions such as chemical and nuclear, but the principle sources of interest for this research are the chemical explosive materials available to terrorism, such as TNT, capable of damaging or destroying civilian and military structures. Upon arrival of a blast wave on a structure, many factors may affect the load generated on the structure due to the shock wave propagation such as the shape of the explosive, obstacles encounter during the wave propagation phase, and the distance of the explosion to the structure. For ease of load calculation and research purposes, some assumptions are made for the calculation of the loads. First, the shape of the explosive is considered perfectly spherical, the explosion occurs far enough to assume uniformly distributed load on the structure, and no obstacles are encountered during wave propagation before time of arrival. Figure 2-1 shows a typical blast wave path to a structure.

The calculation of the load on the structure may be performed using a scaling approach represented by Equation 2-1. The explosive charge needs to be transformed into an equivalent TNT charge in the case when TNT is not the primary explosive used. UFC 3-340-02, (Department of Defense, 2008) contains charts that have been developed to simplify the calculation.

\[
Z = \frac{R}{W^{1/3}} \tag{2-1}
\]
where

W represents the equivalent TNT Weight (lbs/Kg)
R represents the radial distance from centre of explosion to target (ft/m)
Z represents the scaled distance

The free field pressure time variation presented in Figure 2-2 may be used to generate the load on a structure and one may use appropriate charts provided by UFC-3-340-02 to obtain the different parameters. The modified Friedlander equation, Equation 2-2, may then be used to compute the pressure on the structure as a function of time.

\[ P(t) = P_{\text{max}} \left[ 1 - \left( \frac{t}{T_{\text{pos}}} \right) \right] e^{-(t/\alpha)} \]  

(2-2)

where \( \alpha \) represents the decay coefficient. The impulse may then be calculated using Equation 2-3 by integrating the area under the curve.

\[ i_s = \int_0^{t_{\text{max}}} P(t) dt \]  

(2-3)

In literature, equations more complex than the Friedlander equation having better agreement with experimental results do exist, but the extra amount of computational time required to perform these calculations does not justify the small increase in accuracy and therefore are not considered for this research.

**Blast Load Calculations for Complex Structures**

In reality, when a blast wave hits a structure, the load will act on the structural elements in various ways depending on the type of structure, the layout of its structural elements, its material properties, and its architectural composition. The blast wave will
penetrate through doors, windows and various openings. Vertical structural elements will be affected by a transverse pressure while horizontal components will be affected by a vertical and transverse pressure. Figure 1-1 shows a simple example of how blast load may act on the internal members. The same approach discussed in the previous section with the appropriate chart may be used to combine all types of loads a structural element is subjected to as a function of time.

**Structural Analysis**

To properly analyze a reinforced concrete structure under dynamic load, proper constitutive models must be used. Over the past century, reinforced concrete has been studied very intensively and many book, papers and technical reports covering a wide range of the subject matter are available i.e. Ross (1983), Krauthammer et al. (1988), Shanaa (1991), Krauthammer, T., Schoedel, R., Shanaa, H. (2002), Krauthammer et al. (2004), Park and Gamble (2000), Park (1975) etc. The following section discusses the structural analysis approach use for this research and reviews the recent developments in the field of protective structures with regards to reinforced concrete analysis relevant to this research. First, a review of dynamic analysis is presented which covers the notion of equivalent single degree of freedom system, followed by the notion of a resistance function used for non-linear material behavior in a dynamic analysis. Flexural behavior is then discussed along with constitutive material models of interest for reinforced concrete columns. Different failure mechanism for reinforced concrete is then discussed, including shear and axial behavior to conclude with a review of large deformation and phenomenon such as membrane behavior which may be present during large deformation of reinforced concrete elements.
Dynamic Analysis

Dynamic analysis may be conducted for one degree of freedom problems which require simple computational effort or for multiple degree of freedom problems requiring more computational effort. The general equation of motion remains the same in both cases and is represented by Equation 2-4.

\[ m \cdot \ddot{u} + c \cdot \dot{u} + R(u) = F(t) \]  \hspace{1cm} (2-4)

where \( m \) represents the mass which generates the inertia force, \( c \) represents the damping coefficient used to calculate the damping force and \( R \) represents the resistance force or stiffness of the structural element. Usually, the more degrees of freedom used, the more accurate the representation of the actual parameters. However, for a specific location of interest, it is possible to use an equivalent single degree of freedom system approach.

In the case of a column having a distributed mass, distributed varying forces, and a varying stiffness, a complete dynamic analysis may become cumbersome computationally since the column can be divided in an infinitesimal number of degrees of freedom. For a simple problem such as evaluating the localized deflection of a structural element, an equivalent system may be used to accurately calculate the structural response at a desired location using a single degree of freedom system (SDOF) having an equivalent stiffness \( K_e \), mass \( M_e \) and load \( F_e \). The equation of motion may then be expressed by Equation 2-5 for a linear elastic system

\[ M_e \cdot \ddot{u} + c \cdot \dot{u} + K_e \cdot u = F_e(t) \]  \hspace{1cm} (2-5)

and Equation 2-6 for nonlinear system

\[ M_e \cdot \ddot{u} + c \cdot \dot{u} + R_e = F_e(t) \]  \hspace{1cm} (2-6)
The equivalent system parameters are calculated such that the equivalent SDOF system displacement at the degree of freedom location is equal to the real structure at the same point in time and both systems assume the same displaced shape. As Biggs (1964) pointed out, stresses and forces in the equivalent system must be based on displaced shapes, as they are not directly equivalent to the same quantities in the real structure. Since we are dealing with reinforced concrete material having non linear material properties in the elastic and non-elastic domain, the general academic stiffness term $K$ will be replaced by the notion of resistance function discussed later in this section.

The following procedure to obtain the equivalent SDOF parameters was presented by Biggs (1964) for an elastic or perfectly plastic domain and did not account for the transition phase in between. Furthermore, it assumed ideal boundary conditions. Krauthammer et al. (1988) improved this concept by developing a procedure based on Biggs (1964) to account for the transition between elastic behaviors to inelastic behaviors valid for all boundary conditions and is discussed in this section.

**Shape function**

As shown in Figure 2-4, when an element of a structure undergoes deformation it assumes a deformation profile that can mathematically be represented by a shape function $\Phi(x)$. The shape function can then be used to derive a mathematical model using laws of physics to calculate the equivalent SDOF terms required for the equivalent system. It may also be used to calculate the reaction at the support.

**Equivalent mass calculation**

The equivalent mass may be calculated using an energy balance solution by balancing the kinetic energy in both systems.
The total kinetic energy of the beam is calculated using Equation 2-7

\[ KE_b = \frac{1}{2} \int_0^L m(x) \cdot \dot{u}(x, t)^2 \, dx \]  

(2-7)

Where the displacement velocity is calculated using Equation 2-8

\[ u'(x, t) = \phi(x) \cdot \dot{u}(t) \]  

(2-8)

The kinetic energy in the equivalent system may then be obtained using Equation 2-9

\[ KE_e = \frac{1}{2} \cdot M_e \cdot \left[ \dot{u}_e(t) \right]^2 \]  

(2-9)

Then \( M_e \) is solved for by equating Equation 2-7 and Equation 2-9

\[ M_e = \frac{\int_0^L m(x) \cdot \dot{u}(x, t)^2 \, dx}{\dot{u}_e(t)^2} \]  

(2-10)

One may then use Equation 2-11

\[ \dot{u}_e(t) = \dot{u}(t) \]  

(2-11)

and substituting Equation 2-11 into Equation 2-10 to obtained a new equivalent mass relationship expressed by Equation 2-12

\[ M_e = \int_0^L m(x) \cdot \phi(x)^2 \, dx \]  

(2-12)

The mass factor \( K_m \), representing the ratio of the equivalent mass to total mass, can then be calculated by Equation 2-13

\[ K_m = \frac{M_e}{M_t} \]  

(2-13)

In a dynamic analysis, the equivalent mass and equivalent mass factor will be affected at each time step by the change in displacement profile. To account for this, the
linear interpolation presented by Equation 2-14 was used by Krauthammer et al. (1988) to calculate the equivalent mass factor at each time step.

\[
K_m = K_{mi} + \frac{K_{m(i+1)} - K_{mi}}{\Delta_{(i+1)} - \Delta_i} \cdot (\Delta - \Delta_i) \tag{2-14}
\]

Where \( \Delta_i \leq \Delta \leq \Delta_{(i+1)} \)

Once hinge formation starts to occur, the equivalent mass will remain constant, since the inelastic deformed shape function is assumed to be effected only minorly (Krauthammer et al. 1988).

DSAS uses a simplified approach where it generates an equivalent mass function curve based on displacement \( M_e(\Delta) \), which reduces computational time.

**Equivalent load calculation**

Following the same approach as that of the equivalent mass, the equivalent load \( F_e \) is then derived by balancing the external work done on the real system to the equivalent system. The work done on the real system is calculated using Equation 2-15

\[
WE_p = \int_0^L w(x,t) \cdot u(x,t) dx + \sum_{i=1}^n F(t)_i \cdot u(t)_i \tag{2-15}
\]

Where the displacement \( u(x) \) can be substituted by Equation 2-16

\[
u(x,t) = \phi(x) \cdot u(t) \tag{2-16}
\]

To obtain Equation 2-17

\[
WE_p = \int_0^L w(x,t) \cdot (\phi(x) \cdot u(t)) dx + \sum_i \left[ F(x_i, t)_i \cdot (\phi(x_i)_i \cdot u(t)_i) \right] \tag{2-17}
\]

The work done on the equivalent system can be expressed by Equation 2-18

\[
WE_e = F_e \cdot u_e(t) \tag{2-18}
\]
Where $F_e$ can be obtained by balancing the work done on both systems to be equal as shown in Equation 2-19

$$F_e \cdot u_e(t) = \int_0^L w(x,t) \cdot (\phi(x) \cdot u(t)) \, dx + \sum_i \left[ F(x_i,t) \cdot (\phi(x_i)) \cdot u(t) \right]$$

(2-19)

To finally obtain Equation 2-20

$$F_e = \frac{\int_0^L w(x,t) \cdot (\phi(x) \cdot u(t)) \, dx + \sum_i \left[ F(x_i,t) \cdot (\phi(x_i)) \cdot u(t) \right]}{u_e(t)}$$

(2-20)

And since

$$u_e(t) = u(t)$$

(2-21)

Equation 2-20 may be simplified to Equation 2-21

$$F_e = \int_0^L w(x,t) \cdot \phi(x) \, dx + \sum_i \left[ F(x_i,t) \cdot (\phi(x_i)) \right]$$

(2-22)

The load factor $K_L$ representing the ratio of the equivalent load to total load may be calculated with Equation 2-23

$$K_L = \frac{F_e}{F_i}$$

(2-23)

Krauthammer et al. (1988) used the same approach as was used for the equivalent mass factor to compute the equivalent load factor at every time step during the dynamic analysis using Equation 2-24

$$K_L = K_{Li} + \frac{K_{Li(i+1)} - K_{Li}}{u_{(i+1)} - u_i} \cdot (u - u_i)$$

(2-24)

Where $u_i \leq u \leq u_{(i+1)}$

Again, a simplified approach was implemented in DSAS where it generates a load factor function curve based on displacement $R_e(u)$, which reduces computational time.
Mass factor and load factor calculation: DSAS approach

As mentioned in the previous section, DSAS uses a modified approach based on the previous theory to evaluate the load factor and the mass factor at every time step of the analysis and does so by using finite beam elements. Based on material properties, section properties and the curvature relationship, DSAS will generate an equivalent resistance function for a given section (displacement versus load) using Equation 2-25

\[ F_e^i = \sum_{i} f_j^i \cdot \frac{d_j^i}{d_{mid}^i} = R_e^i \]  

(2-25)

where \( j \) represents the node location and \( i \) the load increment. Using the same approach, it will generate the equivalent mass using Equation 2-26

\[ M_e^i = \sum_{i} M_j^i \cdot \left( \frac{d_j^i}{d_{mid}^i} \right)^2 \]  

(2-26)

Finally, it will generate the loading function using Equation 2-27

\[ F_e(u, t) = \frac{F_e(u)}{w(u)} \cdot w(t) \]  

(2-27)

where \( w \) represents the distributed static load applied on the element that would cause the control displacement, \( u \).

Numerical integration of equivalent SDOF

Various approaches may be used to numerically integrate the equation of motion. The method is usually chosen to minimize computational time and provide better accuracy for a given type of problem. The method used for the scope of this research is a special case of the Newmark-Beta Method. This method is an implicit method meaning the equation of motion is satisfied at time \( t + \Delta t \) unlike an explicit method, where the equation of motion is satisfied at time \( t \). For multi degree of freedom system,
an implicit method may have been computationally inefficient requiring a full stiffness matrix inversion which would be very inefficient for a non-linear resistance function. However, this method has proven to be very efficient and stable for a single degree of freedom system. The method uses Equation 2-28 and Equation 2-29 to compute velocities and displacement:

\[
\dot{u}_{t+\Delta t} = \dot{u}_t + \frac{\Delta t}{2} \left( \dddot{u}_t + \dddot{u}_{t+\Delta t} \right) \tag{2-28}
\]

\[
u_{t+\Delta t} = u_t + \dot{u}_t \cdot \Delta t + \left( \frac{1}{2} - \beta \right) \cdot \dddot{u}_t \cdot \Delta t^2 + \beta \cdot \dddot{u}_{t+\Delta t} \cdot \Delta t^2 \tag{2-29}
\]

Where \(\beta\) is taken as 1/6. The following are the steps the method uses to solve the equation of motion.

- Compute \(\dddot{u}_t\) using the equation of motion and known \(u_t\) and \(\dot{u}_t\).
- Estimate \(\dddot{u}_{t+\Delta t}\), usually chosen as \(\dddot{u}_t\) or zero for first time step.
- Compute \(\dddot{u}_{t+\Delta t}\) and \(u_{t+\Delta t}\) using Equation 2-28 and Equation 2-29
- Compute \(\dddot{u}_{t+\Delta t}\) using the equation of motion and previously computed \(\dddot{u}_{t+\Delta t}\) and \(u_{t+\Delta t}\).
- Iterate the process using newly computed \(\dddot{u}_{t+\Delta t}\) until \(\dddot{u}_{t+\Delta t}\) satisfy the convergence tolerance.
- Once convergence is satisfied, continue to next time step and repeat process.

The value of time step must also be carefully chosen to ensure accuracy and stability. For the Newmark-Beta method, typical values when dealing with short impulsive load are one tenth of the natural period or one twelfth of the pulse duration. This takes into account the effect of plasticity on the system period. The main criterion of importance is the time step must be small enough to properly capture the loading
time history since blast occurs in a very short duration of time. Therefore, it is not uncommon to use a time step of 0.0001 second in the case of blast loading. To computationally improve the calculation time, one can use a time step of 0.0001 second during loading phase and then use a bigger time step during free vibration phase.

**Dynamic reactions**

The following procedure to compute reactions at the supports were developed by Krauthammer et al. (1988) to overcome the limitations of the elastic – perfectly plastic material model previously used in literature such as Biggs (1964), which are also only applicable to a limited number of load cases (Krauthammer et al. 1988). The distribution of the inertia force is assumed to be equal to the deformed shape function as seen in Figure 2-6.

- For each load step in the load-deflection relationship, compute the reaction and proportionality factor corresponding to each support using Equation 2-30 and Equation 2-31.

\[
\gamma_{1i} = \frac{Q_{1i}}{Q_i} \quad (2-30)
\]

\[
\gamma_{2i} = \frac{Q_{2i}}{Q_i} \quad (2-31)
\]

Where

- \(Q_{1i}\) and \(Q_{2i}\) represent the static reaction at load time step \(i\)
- \(Q_i\) represent the load at time step \(i\)
- \(\gamma_{1i}\) and \(\gamma_{2i}\) represent the load proportionality factors.

- Compute the inertial load factor ILF at every load step \(i\) using the following relationship.
\[
ILF_i = \frac{\int_0^L \phi(x_i) \, dx}{L}
\] (2-32)

- By using the principle of linear beam theory, inertia proportionality factors \(Y'_{1i}\) and \(Y'_{2i}\) are approximated for every load time step. At this point, the inertia proportionality factor can only be approximated due to the fact that the magnitude of the inertia load is unknown, therefore the required iterative procedure to evaluate these factors precisely cannot be performed.

- Using Equation 2-33 and Equation 2-34, compute the end reactions of the element

\[
V_{1i} = (\gamma_{1i} \cdot Q(t)) + (ILF \cdot Y'_{1i} \cdot M_i \cdot \ddot{u})
\] (2-33)

\[
V_{2i} = (\gamma_{2i} \cdot Q(t)) + (ILF \cdot Y'_{2i} \cdot M_i \cdot \ddot{u})
\] (2-34)

Where

- \(Q(t)\) represents the forcing function
- \(M\) represents the mass of the element
- \(\ddot{u}\) represents the acceleration

Equation 2-35 is used to calculate \(\gamma_{1i}, \gamma_{2i}, \gamma'_{1i}\) and \(\gamma'_{2i}\) at every time step

\[
\gamma = \gamma_i + \left(\frac{\gamma_{i+1} - \gamma_i}{u_{i+1} - u_i}\right) \cdot (u - u_i)
\] (2-35)

Where \(u_i \leq u \leq u_{i+1}\) and \(u\) represents the dynamic displacement for a specific time step and \(\gamma\) is simply the generic name for load and inertia proportionality factors.

**Resistance Function**

The resistance function is by definition the restoring force \(R_m\) exercised by an element to regain its initial condition when subjected to a load. It is important to note that the resistance function of an element will be different for each different type of deformation. For example, the resistance function for an axial deformation on a
concrete column will not be the same as for a flexural deformation on the same column. Also, in the case of reinforced concrete, the resistance function due to axial load will not be the same in tension and in compression. Figure 2-7 shows an example of a resistance function obtained using the software DSAS V3.0 for a reinforced concrete section. A detailed discussion on the software DSAS may be found in a later section.

Resistance function \( R(u) \) allows for the dynamic equilibrium equation for a SDOF system to be written as presented by Equation 2-36

\[
m \cdot \dddot{u} + c \cdot \ddot{u} + R(u) = F(t)
\]  

(2-36)

The maximum resistance is the total load having the given distribution which the element could support statically. The stiffness is numerically equal to the total load of the same distribution which would cause a unit deflection at the point where the deflection is equal to that of the equivalent system. (Biggs, 1964)

Using Biggs definitions quoted above, the equivalent SDOF equation of motion may be expressed with Equation 2-37

\[
M_e \cdot \dddot{u} + C \cdot \ddot{u} + K_L R(t) = F_e(t)
\]  

(2-37)

where

\[
F_e(t)
\]

is the equivalent force as a function of time

\[
M_e
\]

is the equivalent mass

\[
M_e(u')
\]

is the equivalent mass as a function of displacement

\[
C
\]

is the damping coefficient parameter

\[
K_L
\]

is the load factor

Using the previously describe definition by (Biggs, 1964), one may assume Equation 2-38

\[
K_R = K_L
\]  

(2-38)
where $K_r$ and $K_L$ represent the resistance factor and load factor.

In the case of dynamic analysis, one must also consider the effect of load reversal into the resistance function model. Many models may be found in the literature but the model used in this research was developed by Krauthammer et al. (1990) based on Sozen (1974) models and is seen in Figure 2-8. Line $A'$-$A$ represents the loading and unloading in the linear domain. For instance, when the element is loaded, displacement will start going from $O$ to $A$. If $A$ is not reached, the unloading will occur elastically following line $A$ to $A'$ and will oscillate until damping brings the structure to rest at $O$. If yielding is reached, the beam is said to have reached yielding and plastic deformation will start occurring. If point $C$ is reached, the beam is said to have failed in flexure. Otherwise, the beam will unload following points $B$, $D$, $E$, $F$, $G$, $D'$ and $E'$. The beam will then come to rest with a plastic deformation when all energy has been dissipated through damping.

**Flexural Behavior**

Flexural behavior of reinforced concrete is probably the subject mostly documented in the literature since most designs are controlled by flexure, (Park and Pauley, 1975, MacGregor, 2009, Park and Gamble, 2000, McCormack and Brown, 2009). The concept of a moment curvature relationship has been extensively used to represent the flexural behavior of reinforced concrete. The moment curvature function is generated using strain compatibility and equilibrium. The following assumptions have been used to develop the numerical algorithm embedded into DSAS. First, multi-axial stresses are ignored and therefore, only uni-axial stresses are considered in the concrete beam. Second, plane sections remain plane before and after bending based on Bernoulli’s principle. Finally, stress-strain curves for concrete and steel are known.
Procedure may be found in literature where tension is ignored since the concrete under tension stress quickly loses its effect when cracks are formed in the tensile region. In this case, the tension is not ignored since strain rate may have a significant effect on the tension strength of the concrete when subjected to blast or other loading cases. A more detailed discussion on strain rate effect is presented in a later section.

Figure 2-9 shows a stress and strain diagram of a reinforced concrete cross section under flexural stress. The confined concrete is separated from the unconfined concrete and the cross section is divided into layers. By using equilibrium and compatibility, the stress strain curve may be obtained by incrementing the curvature $\Phi$ from 0 to failure of the section using the following steps:

- Increment $\theta$
- Assume strain distribution
- Calculate strain $\varepsilon_i$ for each layer of the section
- Calculate each stress $\sigma_i$ as a function strain $\varepsilon_i$ using material's constitutive model
- Calculate moment and forces
- Iterate the process until equilibrium, Equation 2-39, is satisfied

$$\sum F_x - P = 0$$  \hspace{1cm} (2-39)

- Repeat process for new value of $\theta$ until ultimate strain has been reach.

The constitutive material models used to generate the moment curvature diagram for confined and unconfined concrete under high intensity load and steel have been developed by Krauthammer et al. (1988) and are presented in Figure 2-10, Figure 2-11 and Figure 2-12.
For the unconfined concrete stress strain model the model is divided into three stages represented by Equation 2-40, Equation 2-41 and Equation 2-42 for different strain values:

From \( 0 \leq \varepsilon_c \leq 0.002 \)

\[
f_c = f_c' \left[ \frac{2 \cdot \varepsilon_c}{0.002} - \left( \frac{\varepsilon_c}{0.002} \right)^2 \right]
\] (2-40)

From \( 0.002 \leq \varepsilon_c \leq 0.004 \)

\[
f_c = f_c' \left[ 1 - Z'(\varepsilon_c - 0.002) \right]
\] (2-41)

Where \( Z' \) represents the line slope

From \( \varepsilon_c \geq 0.004 \)

\[f_c = 0\] (2-42)

For the confined concrete stress strain model, Figure 2-11, the model is also divided into three stages for different values of strain and represented by Equation 2-43, Equation 2-44 and Equation 2-48:

From \( 0 \leq \varepsilon_c \leq \varepsilon_o \)

\[
f_c = f_c' \left[ \frac{E_c \cdot \varepsilon_o}{f''_c} \cdot \frac{\varepsilon}{\varepsilon_o} - K \cdot \frac{\varepsilon}{\varepsilon_o} \right] \\
1 + \left( \frac{E_c \cdot \varepsilon_o}{K \cdot f''_c} - 2 \right) \cdot \frac{\varepsilon}{\varepsilon_0}
\] (2-43)

From \( \varepsilon_o \leq \varepsilon_c \leq \varepsilon_{0.3k} \)

\[
f_c = K \cdot f_c' \left[ 1 - 0.8 \cdot Z \cdot \varepsilon_o \left( \frac{\varepsilon}{\varepsilon_o} - 1 \right) \right]
\] (2-44)

Where
\[ \varepsilon_o = 0.0024 + 0.005 \left( 1 - \left( \frac{0.734}{h''} \right) \cdot s \right) \cdot \left( \frac{\rho_r \cdot f''_y}{\sqrt{f'_c}} \right) \]  

(2-45)

\[ K = 1 + 0.0091 \cdot \left( 1 - 0.245 \cdot \frac{s}{h''} \right) \cdot \left( \rho_r + \frac{D''}{D'} \cdot \rho' \right) \cdot \frac{f''_y}{\sqrt{f'_c}} \]  

(2-46)

\[ Z = \frac{0.5}{\frac{3}{4} \cdot \rho_r \cdot \sqrt{\frac{h''}{s}} + \left( \frac{3 + 0.002 \cdot f''_y}{f'_c - 1000} \right) - 0.002} \]  

(2-47)

From \( \varepsilon_{0.3k} \leq \varepsilon_c \)

\[ f'_c = 0.3 \cdot K \cdot f''_c \]  

(2-48)

And,

- \( E_c \) is the elastic modulus of concrete (psi)
- \( H'' \) is the average core dimension of the confined concrete compression zone, measured to the outside of stirrups (inches)
- \( \rho_r \) is the ratio of the confining steel volume to the confined concrete core volume per unit length of the element in compression zone.
- \( \rho' \) is the longitudinal compression reinforcement ratio
- \( f''_y \) is the yield stress of the hoops
- \( s \) is the spacing of the hoops (inches)
- \( D'' \) is the nominal diameter of the hoops (inches)
- \( D' \) is the nominal diameter of the longitudinal compression reinforcement (inches)

It is important to note the parameters \( K, Z \) and \( \varepsilon_o \) will vary as the location of the neutral axis changes and therefore, as DSAS increments the section curvature, a new confined stress strain curve is generated for the calculation of the moment curvature.
Furthermore, different cross section will have different stress strain curves unique to them.

The stress-strain model used for steel is also taken from Krauthammer et al. (1988) and is shown in Figure 2-12 and is separated in 3 stages: elastic using Equation 2-49, perfectly plastic using Equation 2-50 and hardening using Equation 2-51.

From A to B: $\varepsilon_s \leq \varepsilon_y$

$$f_s = E_s \cdot \varepsilon_y$$  \hspace{1cm} (2-49)

From B to C: $\varepsilon_y \leq \varepsilon_s \leq \varepsilon_{sh}$

$$f_s = f_y$$  \hspace{1cm} (2-50)

From C to D: $\varepsilon_s \geq \varepsilon_{sh}$

$$f_s = f_y \cdot \left[ \frac{m \cdot (\varepsilon_s - \varepsilon_{sh}) + 2 \cdot (\varepsilon_s - \varepsilon_{sh}) \cdot (60 - m)}{60 \cdot (\varepsilon_s - \varepsilon_{sh}) + 2 \cdot (30 + r + 1)^2} \right]$$  \hspace{1cm} (2-51)

where

$$m = \left( \frac{f_s}{f_y} \right) \cdot \frac{(30 \cdot r + 1)^2 - 60 \cdot r - 1}{15 \cdot r^2}$$  \hspace{1cm} (2-52)

$Y$ is the difference from Ultimate strain to strain hardening

$E_s$ is the steel modulus of elasticity

$\varepsilon_s$ is the steel strain

$\varepsilon_{sh}$ is the strain where steel hardening begins

$\varepsilon_{su}$ is the ultimate strain of steel

$f_s$ is the steel stress

$f_u$ is the ultimate steel stress
$f_y$ is the steel yield stress

It is important to note that when a reinforced concrete section fails in flexure, it is a combination of both flexural behavior and diagonal shear behavior and not only a flexure failure. Adjustments are required to account for diagonal shear behavior in the dynamic analysis and are discussed next.

**Diagonal Shear Behavior**

Diagonal shear failure occurs on reinforced concrete sections where flexural stress and shear stress are acting together with a significant amount of stress creating cracks perpendicular to the principle tensile stresses along the member causing brittle failure, thus the need to provide web reinforcement.

For the case of this study, a shear reduction factor will be used based on Krauthammer et al. (1988) findings that diagonal shear must not be neglected when computing the deflection of beams influenced by shear, especially for deep beams and slender beams.

The shear reduction concept was first developed by Krauthammer et al. (1979) and later modified by Krauthammer et al. (1988) to better account for the effect of diagonal shear in deep and slender beams. Previous research conducted by Kani (1966) and Leoanhardt (1964) on beams without web reinforcement indicated the shear span to effective depth ratio $a/d$ was governing the strength of simply supported rectangular reinforced concrete beams, where $a$ is the distance from the load to the support and $d$ is the distance from the compression external fiber to the first layer of steel in tension. Two different type of failure were observed based on $a/d$,

- Shear compression failure: $1.0 \leq a/d \leq 2.5$
- Diagonal failure: $2.5 \leq a/d \leq 7$
Kani (1966) proposed a model for the influence of shear on beams having no web reinforcement as shown in Figure 2-13. Lines P1 and P3 represent the limit where there is no need for reduction of moment capacity and line P2 represent the minimum moment capacity. Krauthammer et al. (1979) represented the influence of shear on the section capacity for flexure using Equation 2-53

\[ SRF = \frac{M_u}{M_{fl}} \]  

(2-53)

\( M_u \) being the ultimate moment capacity due to shear and flexure and \( M_{fl} \) the ultimate moment capacity due only to flexure.

To obtain the \( SRF \), Krauthammer et al. (1979) developed the relationship presented by Equation 2-54, Equation 2-55 and Equation 2-56 based on Kani (1966) experimental results to describe the minimum SRF ratio as a function of \( \rho \).

\[ 0 < \rho < 0.65\%: \left( \frac{M_u}{M_{fl}} \right)_m = 1.0 \]  

(2-54)

\[ 0.65\% < \rho < 1.88\%: \left( \frac{M_u}{M_{fl}} \right)_m = 1.0 - 36.6 \cdot (\rho - 0.0065) \]  

(2-55)

\[ 1.88\% < \rho < 2.88\%: \left( \frac{M_u}{M_{fl}} \right)_m = 0.6 \]  

(2-56)

Using Equation 2-54, Equation 2-55 and Equation 2-5, Krauthammer et al. (1988) developed mathematical models to account for shear influence on reinforced concrete moment capacity of a given section with and without web reinforcement and for both under-reinforced and over-reinforced sections. Figure 2-14 shows the model developed by Krauthammer et al. (1988).
For the case of this study, the important factor to take from the Krauthammer et al. (1988) investigation is the modification of the model when web reinforcements are present. Krauthammer et al. (1988) developed the relationship presented by Equation 2-57 to compute the minimum moment capacity ratio, point P$_2$ on Figure 2-13.

\[
\left(\frac{M_{u}}{M_{\beta}}\right)_m = \left(\frac{M_{u}}{M_{\beta}}\right)_m + \left[1.0 - \left(\frac{M_{u}}{M_{\beta}}\right)_m\right] \cdot \tan(\alpha)
\]  

(2-57)

Where \(\left(\frac{M_{u}}{M_{\beta}}\right)_m\) is the modified minimum moment capacity ratio accounting for the effect of transverse reinforcement and \(\alpha\) is taken as the angle of diagonal compression strut at ultimate and is calculated using Equation 2-58 and Equation 2-59 for different type of beams.

Deep rectangular beam:

\[
\alpha = 2.72 \cdot \rho^* \cdot (a / d) + 4.08
\]  

(2-58)

Slender rectangular beam:

\[
\alpha = 3.06 \cdot (\rho^* \cdot \sqrt{a / d}) + 7.22
\]  

(2-59)

\(\rho^*\) is the web reinforcement ratio, calculated using the following relationship

\[
\rho^* = \frac{\rho'' \cdot f''_y}{f'_c}
\]  

(2-60)

\(\rho''\) is the web reinforcement ratio

\(f''_y\) is the yield stress of the stirrup

\(f'_c\) is the compressive strength of concrete

This approach was embedded into DSAS. The computed moment is multiplied by SRF and the curvature is divided by SRF.
Direct Shear

Direct shear failure occurs at a location of high concentration of shear forces such as support and point load locations. In this case, cracks form almost perpendicular to the beam. (Krauthammer et al. 2002).

In the case of direct shear failure, Krauthammer et al. (1986) developed a shear resistance function model based on an empirical model develop by Mattock and Hawkins (1972) and Murtha and Holland (1982) to include the effect of load reversal and is represented in Figure 2-15.

To account for direct shear failure in a dynamic column analysis one may use Krauthammer et al. (1988) approach to verify both direct shear failure and flexural failure at each time step of the analysis. The method uses two single-degree-of-freedom systems running simultaneously, one for direct shear analysis and the other for flexural analysis. This method is already implemented in DSAS and has been intensively used in previous research conducted at the Center for Infrastructure Protection and Physical Security. A similar approach is also used to evaluate variable axial load on a column from adjacent components and is discussed in the next section.

Axial Behavior and Analysis

When axial forces are acting on a concrete beam or column, their effect may be taken directly into account in the moment curvature relationship since all forces acting on the section must be in equilibrium. Typically, the presence of compressive axial load increases the ultimate moment of the section; however it also reduces its ductility. If the axial force is less than the balanced axial force, the moment capacity will be increased, and if the axial force in compression exceeds the balance point, the moment capacity will be reduced. For most cases, compressive axial forces will also improve the shear resistance capacity of the section.
Holmquist and Krauthammer (1984) proposed a model to account for shear capacity enhancement due to compressive axial force. They proposed the thrust factor $TF$, calculated using Equation 2-61,

$$TF = 1 + 0.0005 \cdot (P/A_g)$$  \hspace{1cm} (2-61)

which multiplies the stress axis in the force slip model, when compressive axial loads are present.

When a variable axial force is acting on a column, it will result in a variable resistance function. Figure 2-16 from Tran (2009) shows the influence different values of axial forces may have on a section when subjected to both lateral and axial forces.

To evaluate the resulting axial load on the column due to upward and downward pressures acting on the horizontal element attached to the column as seen in Figure 1-1, one may use a SDOF system to analyze the response of the horizontal element and transfer the calculated reaction at the support to the column. It is then possible to use a SDOF to evaluate the column response to the load combination. Such analyses must be conducted simultaneously at each time step as both member reactions to the load affect one another, especially if one member is to fail. Also, within one element, it may be required to have more than one DOF to evaluate each possible failure mode.

In order to account for the variable axial forces on the column and to solve for the dynamic SDOF equation equilibrium, one may generate a series of resistance functions within an upper and lower bound envelop for the flexural resistance function as seen in Figure 2-17. Theses curves may then be used to interpolate the resistance value at a given deflection for the variable axial forces. The upper bound would represent the resistance function for the maximum compressive axial forces generated by the loading
function and the lower bound would represent the maximum axial tension forces or minimum axial compressive force. A search algorithm may then be used at each time step to solve for the resistance function point within the envelope during the dynamic analysis. The aim of generating a set of curves to interpolate from is to minimize the computational time by serving as a starting point to solve for the resistance function during the flexural dynamic analysis. At the present time, DSAS is regenerating a complete resistance curve every time the axial forces varies which becomes very time consuming when the axial load varies continuously during the analysis.

**Rate Effect**

Researchers such as Evan (1942), Wastein (1953) and Soroushian et al. (1986) have reported in literature an increase in strength material for material subjected to high loading rates (Shanna, 1991). Shanna (1991) reported two techniques used in analysis, One of these is a dynamic enhancement factor based on straining rate to increase material properties used in the derivation of moment-curvature relationships, diagonal shear and direct shear relationships. Another technique used is to directly apply the enhancement factor to the resistance function by multiplying the shear and flexure capacity of the section by the enhancement factor (Shaana, 1991).

One may use one of the following rate to develop the enhancement factor as they are correlated, the loading rate, the stress rate or the strain rate. Since the strain is the controlling parameter in the computation of the moment curvature relationship, it is much easier computationally to use the strain rate as the independent parameter. Shanna (1991) and Krauthammer et al. (2002) offer a more detailed discussion on rate effect and proposed the use of the Soroushian and Obaseki (1986) model for the enhancement factor.
For steel material, the enhancement factor presented by Equation 2-62 may be used.

\[
Enh_s = \frac{f_y (\text{dynamic})}{f_y (\text{static})} = \frac{f_u (\text{dynamic})}{f_u (\text{static})}
\]

\[
Enh_s = \left[ 3.1 + 1.2 \cdot f_y + (0.65 + 0.05 \cdot f_y) \cdot \log_{10} (\partial \varepsilon / \partial t) \right] / f_y
\]  

(2-63)

And for concrete materials, the enhancement factor presented by Equation 2-64 may be used.

\[
Enh_c = \frac{f' (\text{dynamic})}{f' (\text{static})}
\]

(2-64)

For \( \frac{\partial \varepsilon}{\partial t} < 1.6E^{-5} \text{inch/inch/sec} \)

\[
Enh_c = 1.14 + 0.03 \cdot \log_{10} (\partial \varepsilon / \partial t)
\]

(2-65)

For \( \frac{\partial \varepsilon}{\partial t} > 1.6E^{-5} \text{inch/inch/sec} \)

\[
Enh_c = 1.38 + 0.03 \cdot \log_{10} (\partial \varepsilon / \partial t)
\]

(2-66)

Finally for concrete in tension, Equation 2-67 may be used

\[
Enh_{ct} = \exp(0.0164 \cdot E^{2.086})
\]

(2-67)

Where,

\[
E = 7.0 + \log_{10} (\partial \varepsilon / \partial t)
\]

(2-68)

**Large Deformation**

The behavior of structural elements under large deformation is well known in literature and is often referred to as second order analysis. In such analyses, the effect of P-Delta is taken into account and the Euler-Buckling is verified at each time step of
the analysis. The following review the notion of P-Delta effect and a finite element method that implicitly accounts for its effect.

**P-Delta effect**

Columns may be classified into two categories: short columns and slender columns. The difference lies in the influence of axial force combined with lateral deflection on section strength capacity. Short columns are classified as such as the strength relies solely on strength of material and section geometry and would fail before enough lateral deflection occurs for a P-Delta effect to be significant on the section capacity. In the case of a slender column, the eccentricity of an axial load may create a significant moment amplification that cannot be ignored which results in a reduction of column strength capacity.

There are two types of second order moments that are required to be taken into account in a P-Δ analysis. The first type of moment is induced by a deflection to the central cords causing a load eccentricity as seen in Figure 2-18. The second type occurs when the supports move away from the cord, creating an extra eccentricity for the applied load to the central cord. For the case of this study, only the first type will be considered as supports are not expected to move significantly compared to the center of the column when subjected to a blast load.

MacGregor (2009) in its slender column section discussed in details the different paths of failure a column may take using a reinforced concrete interaction diagram (Figure 12-8 of the reference). He demonstrated a linear correlation for a short column and a second order correlation for a slender column. The linear correlation of the short column is due to the fact that it fails before P-Δ moment has a significant influence on the section. The second order correlation is due precisely to the second moment
introduced as the slender columns deflect. ACI 318-08 provide slenderness ratios for braced and un-braced members where slenderness effects may be neglected for ease of design.

Non braced members:

\[
\frac{k \cdot l_u}{r} \leq 22
\]  

(2-69)

Braced members:

\[
\frac{k \cdot l_u}{r} \leq 34 - 12 \cdot (M_1 / M_2) \leq 40
\]  

(2-70)

Where

- \( K \) is the effective length factor
- \( l_u \) is the unsupported length
- \( r \) is the radius of gyration of cross section
- \( M_1 / M_2 \) is the ration of end moment and \( M_1 \leq M_2 \).

Those slenderness ratios were developed for static analysis and may not apply for the case of severe short duration dynamic loads. Therefore, one shall take into consideration P-Delta effect for all type of columns regardless of their slenderness ratio when dealing with blast loading.

Two types of failure may occur for slender column: material failure and stability failure. Stability failure occurs when

\[
\frac{\partial M}{\partial P} \rightarrow \infty
\]  

(2-71)

Therefore, in a dynamic analysis, stability failure shall be verified at each time step of the dynamic analysis.
In order to design for second moment effect, the moment magnifier method presented by MacGregor (2009) combined with a first-order analysis approach may be used. For the case of this study an exact approach solution using finite element application and numerical procedure to develop the resistance curve in DSAS was used based on the following discussion and further discussed in Chapter 3.

**Development of resistance function curve in DSAS**

To generate the resistance curve one may use a finite beam element approach that implicitly accounts for the axial forces in its formulation. The following is a short review of the DSAS finite beam element formulation procedure used to generate the resistance function of an element followed by a discussion of a modified finite beam element formulation.

**DSAS procedure in steps:**

- Generate the moment curvature of the reinforced concrete cross section.
- Divide the column into multiple beam elements with 2 degrees of freedom per node as seen in Figure 2-20.
- Using a displacement control approach implicitly through the cylindrical arc-length method developed by Crisfield M.A (1981) and equivalent parameters, increment displacement at mid span and obtain rotation and displacement of all nodes due to mid-span displacement.
- Obtain Curvature at each node using a strain-displacement matrix and Equation 2-72 and Equation 2-73.

\[
\sigma = E \cdot \varepsilon = E \cdot \{B\} \cdot \{U\} \quad (2-72)
\]

\[
\varepsilon = \{B\} \cdot \{U\} = \phi \quad (2-73)
\]

- Recover Forces at each node using Equation 2-74 and obtain the stiffness term from the moment curvature diagram for each node.

\[
\{f\} = [K] \cdot \{U\} \quad (2-74)
\]

- Repeat steps by incrementing displacement until failure
The cylindrical arc-length method mentioned above is an iterative solution developed by Crisfield M.A (1981) to capture snap-through buckling in a finite element analysis. When a load control analysis is performed, the load deflection path will follow O-A-C in Figure 2-19, creating a dynamic snap-through which is not truly representative of the element behavior. In order to truly capture the element behavior, a deflection control analysis is required. Path O-A-B-C of Figure 2-19 shows the true load deflection path of an element. To properly capture this path using finite elements, a solution allowing tracing negative slope is required. To do so, Crisfield (1981) used a cylindrical arc-length iterative approach where the finite governing differential equation is checked along the arc until convergence occurs. A more detailed discussion of this method may be found in Crisfield (1981).

To account for the axial forces, Smith and Griffiths (1998) presented the following modification to the Euler-Bernoulli beam element. The governing differential equation is modified and is presented by Equation 2-75

\[ E \cdot I \cdot \frac{\partial^4 w}{\partial x^4} + P \cdot \frac{\partial^2 w}{\partial x^2} = q \]  \hspace{1cm} (2-75)

Where

- \( E \) is the elastic modulus of the section
- \( I \) is the moment of inertia of the section
- \( P \) is the axial load (compression > 0)
- \( w \) is the deflection
- \( x \) is the position on the beam
- \( q \) is the transverse load per unit length carry by the element.
The Galerkin method and finite element discretisation was used to solve the matrix Equation 2-76 for the axial force contribution in the system equilibrium.

\[
\pm P \int_0^L \frac{\partial N_i}{\partial x} \cdot \frac{\partial N_j}{\partial x} \, dx \cdot \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix} \quad (2-76)
\]

Where \( i,j = 1,2,3,4 \)

And lead to the stiffness matrix Equation 2-77

\[
+ P \begin{bmatrix}
\frac{36}{L} & 3 & -\frac{36}{L} & 3 \\
\frac{L}{3} & 4 \cdot L & -3 & -3 \\
-\frac{36}{L} & -3 & \frac{36}{L} & -3 \\
\frac{L}{3} & -L & -3 & 4 \cdot L
\end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix} \quad (2-77)
\]

Then the equilibrium equation becomes Equation 2-78.

\[
\begin{bmatrix}
\frac{12}{L^3} & \frac{6}{L^2} & -\frac{12}{L^3} & \frac{6}{L^2} \\
\frac{L^3}{6} & \frac{L^2}{4} & -\frac{6}{L^2} & \frac{L^2}{2} \\
\frac{12}{L^3} & \frac{L^2}{6} & \frac{12}{L^3} & \frac{L^2}{6} \\
\frac{L^3}{6} & \frac{L^2}{2} & -\frac{6}{L^2} & \frac{4}{L}
\end{bmatrix} \cdot E \cdot I \cdot \begin{bmatrix}
\frac{36}{L^3} & 3 & -\frac{36}{L^3} & 3 \\
\frac{L^3}{3} & 4 \cdot L & -3 & -3 \\
-\frac{36}{L^3} & -3 & \frac{36}{L^3} & -3 \\
\frac{L^3}{3} & -L & -3 & 4 \cdot L
\end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix} = f \quad (2-78)
\]

For a displacement control approach, this method may be modify in step 5 in the presented procedure to account for the extra curvature created by the P-Delta effect using Equation 2-79 and Equation 2-80.

\[
\phi_1 \rightarrow \phi_1 + P_1 \cdot \frac{u_1}{[E \cdot I]_1} \quad (2-79)
\]

\[
\phi_2 \rightarrow \phi_2 + P_2 \cdot \frac{u_2}{[E \cdot I]_2} \quad (2-80)
\]
Where the stiffness $E \cdot I$ may be approximated using the moment curvature diagram for
the unmodified curvature and implying a sufficient number of elements are present.

It is important to note there have been many different finite element models
developed over the years better suited for large deformation problems than the one
presented here. For example, Bathe et al. (2005) presented two formulations for large
strain and large deformation, the total formulation and the rate formulation.

The total formulations are based on kinematic quantities defined with
respect to an adopted reference configuration and on incremental solution
procedure and the rate formulations are based on integration of constitutive
relations involving stress and strain rate. (Bathe et al. 2005).

The modified Euler-Bernoulli beam element is an approximate method based on
the theory of small rotation and deflection that may be used for reinforced concrete
elements as they are expected to fall into tension membrane behavior while being in the
small deflection/deformation range. Krauthammer et al. (1988) proved that small
deflection theory and large deflection theory hold true for deflection up to 15% of the
beam length in the case of simply supported beams.

Furthermore, it was also shown by Krauthammer et al. (1984), Krauthammer et al.
(1986), and Park and Pauley (1975) that compressive membrane effects for slabs start
developing at a deflection of about 0.5*h where h is the slab thickness. It was also
shown that tensile membranes fully develop in between a deflection of h and 2*h.
Krauthammer et al. (2003) also showed tension membrane effects in deep beams,
point C in Figure 2-22, start occurring at about 0.17*h. In the case of columns, recent
studies conducted by Tran et al. (2009) demonstrated that an increase of axial forces on
columns resulted in a smaller deflection at failure which indicated transition into a
tension membrane behavior would occur at a smaller deflection than without axial
forces.

All the above demonstrated that the use of large deformation finite elements for
reinforced concrete columns is not necessary since by the time the central deflection
reaches 15%, slender columns and even short columns will have already transited into
tension membrane. Therefore, the a modified Euler-Bernoulli beam element is expected
to provide excellent results in the case of reinforced concrete sections.

**Euler Buckling**

The Euler Buckling represents a failure mode of a structural element subjected to
a compressive stress exceeding its capacity. The theory was developed by Euler
Bernoulli using its beam theory formulation and goes as follows.

Using Equation 2-81 for a simply supported column with only an axial load applied
on it

\[ E \cdot I \cdot \frac{du}{dx} = M \]  \hspace{1cm} (2-81)

where the moment may be calculated using Equation 2-82

\[ M = -P \cdot u \] \hspace{1cm} (2-82)

the solution for displacement, \( u \), will take the form presented by Equation 2-83

\[ u = C_1 \cdot \sin(\lambda \cdot x) + C_2 \cdot \cos(\lambda \cdot x) \] \hspace{1cm} (2-83)

where

\[ \lambda = \sqrt{\frac{P}{E \cdot I}} \] \hspace{1cm} (2-84)

The constants \( C_1 \) and \( C_2 \) may then be solved using boundary conditions. For this case
one may show that \( C_2 \) is 0 for \( x =0 \) and \( C_1 \) is not zero for \( x = L \). Therefore if \( C_1 \) is not
zero but the boundary condition states the displacement must be zero, it implies that the variable in the $\sin(var)$ must be zero.

Thus Equation 2-85

$$\lambda \cdot L = n \cdot \pi$$

(2-85)

where $n$ represents the mode shape of the deflection function leading minimum $P$ for $n = 1$.

Therefore using Equation 2-84 and Equation 2-85 one may solve Equation 2-86 for the critical load $P_{cr}$

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I}{L^2}$$

(2-86)

Many reference books provide the Euler buckling equation for different possible cases. However, it is important to note this formulation is based on the assumption of small angle deformation, linear elastic material, no yielding before buckling and the presence of only pure axial load. This is, however, not true in reality but in most cases, calculating the Euler buckling represents safety verification in design and an upper limit for analysis.

For the case of reinforced concrete, there are two different types of Euler buckling that may be verified. The first one is the global buckling where the entire section is in compressive buckling and the second one is where buckling occurs in the longitudinal reinforcement between stirrups. The following section discusses the two different types during a dynamic analysis.

**Dynamic analysis: Reinforce concrete global buckling**

When a column is subjected to a blast loading or any lateral load, the deflection of the column will be due to the combined effects of both axial and lateral load. Therefore
during a dynamic analysis, it is possible to approximate the Euler buckling load at each
time step by using the effective stiffness of the column. This approach may be facilitated
by using a finite element code since curvature may be obtained at each node and the
stiffness may then be obtained for each node using the moment curvature diagram of
the reinforced concrete section. Therefore, for a one degree of freedom system, the
effective stiffness is simply the average of all stiffness.

Another method that has been used in literature is the energy balanced method.
To use this approach in a single degree of freedom dynamic analysis, the axial load
must also be expressed in terms of equivalent axial load using an energy balance
solution and a shape function. In the equation of motion, the axial load only affects the
stiffness and when combined with the stiffness for the flexural stiffness, a new natural
frequency is formed at each time step as any lateral displacement or axial load variation
will have an effect on the system stiffness.

If one considers  $K_f$ to be the equivalent flexural stiffness and $K_a$ to be the
equivalent axial stiffness, the natural frequency of the system may then be expressed
using Equation 2-87

$$\omega_e = \sqrt{\frac{K_f - K_a}{M_c}}$$

(2-87)

when the axial load acts in compression, the natural frequency would be decreased.

From Equation 2-87, it is possible to solve for the value of $P_{cr}$ using Equation 2-88

$$K_f - K_a = 0$$

(2-88)

to yield Equation 2-89
\[
P_{cr} = \frac{\int k(x)\varphi^2(x)dx}{\int [\varphi(x)]^2 dx}
\]

(2-89)

A more detailed discussion on the derivation of the previous equation may be found in Tedesco et al. (1999). By using Equation 2-89, it is possible to form a graph of \( P_{cr} \) versus central displacement that may be used by advanced dynamic analysis software such as DSAS to verify the buckling load at each time step of the analysis.

**Dynamic analysis: Reinforce concrete compression longitudinal rebar buckling**

Buckling may occur in elements subjected to compressive forces within a composite section such as reinforced concrete when conditions for such phenomenon are present. Test experiments conducted by Yamashiro and Siess (1962) on reinforced concrete beam members subjected to bending, shear and axial load have revealed the presence of local buckling of the compression longitudinal steel reinforcement on several test specimens. There were two type of compression failure observed, steel compression failure and concrete compression failure. It was, however, observed that when a beam failed due to concrete compression failure, the longitudinal steel had buckled as well. It was also observed that concrete compression failure was occurring in lightly reinforced concrete specimens. In the case of reinforced concrete columns, the amount of longitudinal steel reinforcement is generally much larger compare to reinforced concrete beam and one may expect a steel compression failure to occur before concrete compression failure.

When using a finite element approach such has the one used by DSAS, concrete compression failure is easily captured using the moment curvature diagram and finite elements as concrete will failed in layers. To capture the steel compression failure,
Yamashiro and Siess (1962) proposed using the Euler Buckling formulation for longitudinal steel in compression. The unsupported length is then taken as the stirrups spacing and the formulation is solve for a fixed-fixed case in view of the fact that the test specimen demonstrated that all rebar buckling occurred between two adjacent ties having the shape of a fixed-fixed buckling case. Furthermore, it was also observed that the buckling of the steel never occurred downward or upward but rather horizontally.

Equation 2-90 is the relationship used by Yamashiro and Siess (1962) to account for the possibility of rebar buckling in between two ties in terms of buckling stress as the strain in the compression steel and the stress strain curve were known.

\[ f_{cr} = \frac{\pi^2 \cdot E_t}{(s / r)^2} \]

(2-90)

where

- \( E_t \) is the tangent modulus of the steel
- \( s \) is the spacing of the ties
- \( r \) represents the radius of compression bars

At the time, the computational results did not quite match the experiment results as some beams continue to provide flexural strength after the buckling of longitudinal reinforcement. The assumption by Yamashiro and Siess (1962) was that the beam would fully fail if the longitudinal steel buckles. The reason why the beam continued to provide flexural strength may be explained by the fact the stress in the beam were redistributed to the remaining of the section after the compression longitudinal rebar buckled. This limitation may however be overcome by the use of a finite element code and the moment curvature diagram for a given section.
For a dynamic analysis using a finite element code, it is possible to modify the steel stress strain curve relationship in the compression range to account for the Euler buckling. One may solve for the stress or the strain at which Euler buckling occurs using the following procedure

- Increment stress (or increment strain)
- solve for the secant stiffness
- calculate the Euler buckling stress corresponding to the secant stiffness
- compare the calculated Euler buckling stress to the actual incremental stress
- Increment until Euler buckling stress is equal to the incremental stress

Using this approach, it is possible to capture buckling in longitudinal steel element and continue the analysis has the stress distribution in the failed region of the cross section would simply be redistributed to the un-failed region.

**Membrane Behavior**

When a reinforced concrete section goes into large deformation, the section may experience an increase in flexural capacity due to compressive membrane action which may be followed by tension membrane action caused by catenary action of steel reinforcement when conditions for such effects are present. In that case, the membrane behavior start occurring once reinforced concrete stops behaving in an elastic manner and falls into a non-linear material behavior. Principle cracks form at the centre of the element and continue to increase as the beam/column deflects. Once the beam is fully cracked, only the steel continues to provide strength until all steel layers have failed. Figure 2-21 shows a simplified transition progression into compressive membrane followed by tension membrane.
For general design purposes, the membrane effect is generally not taken into account as the serviceability issue does not allow for large deformation to occur. The effect was although studied in literature for limited cases such as slabs under large deformations (Park 1964, Park and Gamble, 2000 and Guice et al. 1989) and resistance slabs under fire (Bailey, 2000). The subject is also of interest in the field of progressive collapse as it may provide sufficient time for a building to be evacuated before complete collapse of the building occurs or in some cases may actually prevent the collapse of the building.

Figure 2-22 shows a general load-deflection/resistance-function at mid-span of a fully fixed two way rectangular reinforced concrete slab. The membrane effect development of a slab section or a beam/column section is based on the same principles. The steel reinforcement must be continuous along the entire length of the element and be well anchored into the boundary to allow for compression and tensile membranes to develop. A continuous section would behave the same as long as the steel is not discontinued and is long enough to have proper length development for the steel beyond the continuous supports. To develop a compressive membrane, the boundaries have to be stiffed enough to allow for outward reaction as the section deforms as seen in Figure 2-23. Only then would the compressive membrane have a significant impact on the section flexural strength capacity. Park and Gamble (2000) showed that compressive membrane forces in slabs may improve by 1.5 to 2 times the ultimate capacity of both one-way and two-way slabs.

The section behaves in compressive membrane up to point A of Figure 2-22. As the section continues to deflect, transition into tensile membrane occurs, as seen in
segment A-B of Figure 2-22. In that region, the beam is nearly fully cracked and concrete no longer provides sufficient strength for the section flexural capacity to continue to increase. Segment B-C of Figure 2-22 represents the tensile membrane region, which shows an increase in section capacity as the deflection increases. At that point, the section is fully cracked and only the steel provides flexural strength to the section. The steel will continue to carry the load until it fails. This phase is characterized by tensile membrane.

**Compressive membrane calculation**

Compressive membrane calculation theory has been developed for slabs by various authors. The following is a review of one way slab compressive membrane calculation based on Guice et al. (1989) and Park (1964). One may also use this theory to approximate compressive membrane behavior of column, as long as good judgment is used, as the deformation profile for one-way slabs used to develop the theory differ from the deformation profile of columns.

Using geometry of deformation as seen in Figure 2-24, one may derive Equation 2-91

\[
\cos(\phi) = \frac{x + t}{x + (h - c_s) \cdot \tan(\phi) - c_m \cdot \tan(\phi) - \varepsilon \cdot x}
\]

(2-91)

where

\[\varepsilon\] represents the total strain, shrinkage and creep strains

\[t\] represents the lateral movement of one support

\[c_m\] represents the neutral axis depth at mid-span

\[c_s\] represents the neutral axis depth at support

\[x\] represents the horizontal projected length of deformed element
Guice et al. (1989) re-wrote Equation 2-91 using trigonometric identities and small angle deformation to obtain Equation 2-92

\[ c_s + c_m = h - \frac{\delta}{2} - \frac{\varepsilon \cdot x^2}{\delta} - \frac{x \cdot t}{\delta} \]  

(2-92)

Equation 2-92 may then be rearranged to yield Equation 2-93

\[ c_s + c_m = h - \frac{\delta}{2} - \frac{x^2}{\delta} \left( \varepsilon + \frac{t}{x} \right) \]  

(2-93)

Park and Gamble (2000) developed another expression to solve for \( c_s \) and \( c_m \) using Figure 2-23 using Equation 2-94.

\[ \left( \beta \cdot l + 0.5 \cdot \varepsilon \cdot (1 - 2 \cdot \beta) \cdot l + t \right) \cdot \sec(\phi) = \left( h - c_s \right) \cdot \tan(\phi) + (1 - \varepsilon) \cdot \beta \cdot l - c_m \cdot \tan(\phi) \]  

(2-94)

which may be simplified to Equation 2-95

\[ h - c_s - c_c = \frac{2 \cdot \beta \cdot l \cdot \sin\left(\frac{\phi}{2}\right) + \varepsilon \cdot \beta \cdot l \cdot \cos(\phi) + 0.5 \cdot \varepsilon \cdot (1 - 2 \cdot \beta) \cdot l + t}{\sin(\phi)} \]  

(2-95)

to yield Equation 2-96

\[ c_s + c_m = h - \frac{\delta}{2} - \frac{\beta \cdot l^2}{2 \cdot \delta} \left( \varepsilon + \frac{2 \cdot t}{l} \right) \]  

(2-96)

using small angle approximation and the trigonometry identities presented by Equation 2-97 and 2-98

\[ \sin(\phi) = 2 \cdot \sin\left(\frac{\phi}{2}\right) = \frac{\delta}{\beta \cdot l} \]  

(2-97)

and

\[ \cos(\phi) = 1 \]  

(2-98)

For one-way slabs, \( \beta \) may be taken as 1 and the variable \( l \) may be substituted for \( x \) to obtain a similar expression to Guice et al. (1989) presented by Equation 2-99.
\[ c_x + c_m = h - \frac{\delta}{2} - \frac{x^2}{2 \cdot \delta} \left( \varepsilon + \frac{2 \cdot t}{x} \right) \] (2-99)

The difference between both expression lies in small angle approximation and trigonometry identities where Guice et al. (1989) and Park and Gamble (2000) relied on different simplification approaches. By comparing Equation 2-99 and Equation 2-92, one can see that the difference relies on the strain \( \varepsilon \) where the Park and Gamble (2000) equation uses half the strain displacement used by Guice et al. (1989). Experimental investigation of Guice et al. (1989) theoretical model to calculate the compressive membrane capacity of several slabs reported an excellent correlation with the experimental ultimate deflection. A mean value of analysis predictions over experimental result of 0.96 with standard deviation of 0.14 and a variance of 1.9% were reported (Guice et al. 1989).

In order to solve for a unique solution more equations are required to solve for the unknown terms introduced by Equation 2-92. By using horizontal equilibrium of each force components in the reinforced concrete section in Figure 2-25, one may use

Equation 2-100

\[ C_c' + C_s' - T_s' = C_c + C_s + T \] (2-100)

or simply use Equation 2-101

\[ \sum F_{sup\ port} = \sum F_{midspan} \] (2-101)

It is now possible to use an algorithm as discussed in the flexural behavior section to solve for each forces \( C_s, C_c, T_s, C_s', C_c' \) and \( T_s' \) in a more accurate manner by dividing the section in multiple layer as seen in Figure 2-9. The forces may then be expressed using Equation 2-102, Equation 2-103, Equation 2-104 and Equation 2-105.
\[ F_{si} = f_{si} \cdot A_{si} \]  \hfill (2-102)

\[ F_{ci} = f_{ci} \cdot A_{ci} \]  \hfill (2-103)

\[ m_{si} = F_{si} \cdot \left( \frac{h}{2} - d_{si} \right) \]  \hfill (2-104)

\[ m_{ci} = F_{ci} \cdot \left( \frac{h}{2} - z_{i} \right) \]  \hfill (2-105)

where

- \( F_{si} \) represents the steel layer forces
- \( F_{ci} \) represents the concrete layer forces
- \( A_{si} \) represents the area of steel at each layer
- \( A_{ci} \) represents the area of concrete at each layer
- \( d_{si} \) represents the depth of each steel layer
- \( z_{i} \) represents the depth of each concrete layer
- \( m_{si} \) represents the steel layer moment
- \( m_{ci} \) represents the concrete layer moment
- \( f_{si} \) represents the steel stress
- \( f_{ci} \) represents the concrete stress

Using a stress-strain relationship with previously defined material models \( f_{si} \) and \( f_{ci} \) may be expressed as a function of neutral axis depth \( c_{s} \) or \( c_{m} \), Equation 2-106, depending on side of interest.

\[ f_{si} = F(\varepsilon_{si}) \]  \hfill (2-106)

where \( \varepsilon_{si} \) may be expressed using Equation 2-107

\[ \varepsilon_{si} = \frac{\varepsilon_{cu}}{c} \cdot (d_{i} - c) \]  \hfill (2-107)
Using the same approach, one may obtain the stress of concrete as a function of central axis depth using Equation 2-108

\[ f_{ci} = F_i \left[ \frac{\varepsilon_{cu}}{c} \left( z_i - c \right) \right] \]  

(2-108)

The total section forces and moment may then be expressed using Equation 2-109 and Equation 2-110

\[ M_{nm} = \sum_i m_{si} + \sum_i m_{ci} \]  

(2-109)

\[ N = \sum_i F_{si} + \sum_i F_{ci} \]  

(2-110)

By looking at the external forces equilibrium of the reinforced concrete element, one may derive Equation 2-111 for the sum of external moment on the element as seen in Guice et al. (1989) and using Figure 2-26

\[ M_{ns} + M_{nm} - N \cdot \delta = F_{c} \cdot \frac{x}{2} \]  

(2-111)

Where \( M_{ns} \) may be calculated using the section forces equilibrium at the support using the same approach as for \( M_{nm} \).

The final equations required to obtain a unique solution need to account for the axial thrust in order to have a complete picture of membrane behavior in the section equilibrium model. The magnitude of the thrust may be affected by various factors such as support movement, axial strain deformation and creep. To account for these, Guice and al (1989) developed a relationship which accounts for the strain due to axial deformation, the elastic shortening strain and the creep and shrinkage strain. Equation 2-112 neglects the effect of longitudinal reinforcement on the axial stiffness.

\[ \varepsilon = \frac{N}{E_c \cdot Ac} + \varepsilon_p \]  

(2-112)
where

\( \varepsilon_a \) represents the axial strain

\( E_c \) represents the concrete elastic modulus

\( A_c \) represents the gross section area

\( \varepsilon_p \) represents the creep and shrinkage strain

If lateral displacement occurs at the column support it may be taken into account using Equation 2-113 and assuming the displacement is elastic.

\[
t = \frac{N}{S}
\]  \hspace{1cm} (2-113)

Where

\( t \) represents lateral displacement at support

\( N \) represents the compressive axial forces at support

\( S \) represents the surrounding support stiffness.

then Equation 2-99 may be written as

\[
c_s + c_m = h \left( -\frac{\delta}{2} - \frac{x^2}{\delta} \left( \frac{N}{E_c A_c} + \varepsilon_p + \frac{N}{S \cdot x} \right) \right)
\]  \hspace{1cm} (2-114)

Creep and shrinkage prior to the loading of the section may be ignored since data on the matter are often not available in practice and may also be ignored during blast loading due to short load duration. Therefore Equation 2-114 may be written as

Equation 2-115

\[
c_s + c_m = h \left( -\frac{\delta}{2} - \frac{x^2}{\delta} \left( \frac{N}{E_c A_c} + \frac{N}{S \cdot x} \right) \right)
\]  \hspace{1cm} (2-115)

The formulation may be further modified to account for external axial forces as shown in Equation 2-116
\[ c_s + c_m = h - \frac{\delta}{2} - \frac{x^2}{\delta} \left( \frac{N}{E_c \cdot Ac} + \frac{N - N_{ext}}{S \cdot x} \right) \quad (2-116) \]

where \( N_{ext} \) represents the external axial forces taken as negative for compression and positive for tension.

In the case of most columns, no lateral displacement or separation is expected to occur at the support since columns support large loads, making the connection very stiff in that regard. Therefore, one may consider the connection stiffness to be infinite and rewrite Equation 2-116 to obtain Equation 2-117

\[ c_s + c_m = h - \frac{\delta}{2} - \frac{x^2}{\delta} \left( \frac{N}{E_c \cdot Ac} \right) \quad (2-117) \]

Although the external axial term disappears from Equation 2-116, it remains part of the equilibrium of the external forces and Equation 2-111 may be adjusted to account for the P-Delta effect created by the extra axial load to obtain Equation 2-118.

\[ M_{ns} + M_{nm} - (N + N_{ext}) \cdot \delta = F_c \cdot \frac{x}{2} \quad (2-118) \]

\( c_s \) and \( c_m \) may now be computed using Equation 2-114 or Equation 2-116 depending on the cases for any displacement \( \delta \). With \( c_s \) and \( c_m \) known, one may calculate all internal forces in the concrete section element and finally, one may use Equation 2-117 to solve the load carried by the section.

**Tension membrane calculation**

The tension membrane forces may be computed by applying the general cable theory to the steel reinforcement. The following is a review of this approach and only considers the case of a uniformly distributed load on the section. Under uniformly
distributed load, the section deflection profile is expected to take the form of a parabola as seen in Figure 2-27 where the deflected curve may be described by Equation 2-119

\[ y = a \cdot x^2 \]  

(2-119)

To solve for the parameter \( a \) in the parabola equation, one may use a truss approach where the cable is divided into an infinite number of truss link together supporting the distributed load as seen in Figure 2-28. At any location on the curve, the axial force \( d_s \) may be represented into vertical and horizontal component force \( v \) and \( t \).

Equation 2-120 is then derived using similar triangle.

\[ \frac{t}{v} = \frac{dx}{dy} \]  

(2-120)

The shear force at any point may then be expressed by Equation 2-121 and Equation 2-122

\[ v = t \cdot \frac{dy}{dx} \]  

(2-121)

and

\[ dv = t \cdot \frac{d^2 y}{dx^2} = w \]  

(2-122)

where \( w \) is taken as the load per unit length. By integrating Equation 2-122, one obtain Equation 2-123

\[ t \cdot \frac{dy}{dx} = w \cdot x + C \]  

(2-123)

and since at \( x = 0 \) the deflection is at a maximum, the constant \( C \) become 0.

Integrating once more, one obtain Equation 2-124

\[ t \cdot y = \frac{w \cdot x^2}{2} + C \]  

(2-124)
and at $x = L/2$, $y = u$, one obtained Equation 2-125 for $C$

$$C = t \cdot u - w \cdot \frac{L^2}{8} \quad (2-125)$$

then Equation 2-124 becomes Equation 2-126

$$t \cdot y = \frac{w \cdot x^2}{2} + t \cdot u - w \cdot \frac{L^2}{8} \quad (2-126)$$

One may sum the moments at one end of the parabola for the equilibrium of half the parabola to obtained Equation 2-127

$$t \cdot u = w \cdot \frac{L^2}{8} \quad (2-127)$$

where in this case $t$ is the horizontal force at $x = 0$ where there is no vertical force.

Then substituting Equation 2-127 into Equation 2-126 one obtain Equation 2-128

$$t \cdot y = \frac{w \cdot x^2}{2} + w \cdot \frac{L^2}{8} - w \cdot \frac{L^2}{8} \quad (2-128)$$

and solve for Equation 2-129

$$y = \frac{w \cdot x^2}{2 \cdot t} \quad (2-129)$$

The term $a$ in the parabola Equation 2-119 may be taken expressed using Equation 2-130

$$a = \frac{w}{2 \cdot t} \quad (2-130)$$

and using Equation 2-127, one may substitute $w$ in Equation 2-130 to obtain Equation 2-131

$$a = \frac{4 \cdot u}{L^2} \quad (2-131)$$
The steel strain may then be calculated using the arc-length formula to solve for the change in length. Therefore, using symmetry and solving one half of the parabola, one obtain Equation 2-132

\[ L_{arc} = \int_{0}^{L/2} \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \]  

one may recall from Equation 2-119

\[ \frac{dy}{dx} = 2 \cdot a \cdot x \]  

Then solving the integral equation to obtain Equation 2-134

\[ L_{arc} = \frac{\ln(\sqrt{1 + a^2 \cdot L^2} + L \cdot a)}{4 \cdot a} + \frac{L}{4} \cdot \sqrt{1 + a^2 \cdot L^2} \]  

The total strain may then be computed Equation 2-135 and Equation 2-136

\[ \varepsilon = \frac{2 \cdot L_{arc} - L}{L} = \frac{2 \cdot L_{arc}}{L} - 1 \]  

\[ \varepsilon = \frac{\ln(\sqrt{1 + a^2 \cdot L^2} + L \cdot a)}{2 \cdot a \cdot L} + \frac{\sqrt{1 + a^2 \cdot L^2}}{2} - 1 \]  

By defining

\[ \alpha = a \cdot L \]  

and

\[ \beta = \sqrt{1 + \alpha^2} \]

Equation 2-136 becomes Equation 2-139

\[ \varepsilon = \frac{1}{2} \left[ \frac{\ln(\beta + \alpha)}{\alpha} + \beta \right] - 1 \]
The strain is then expressed as a function of central deflection $u$. Using stress strain compatibility, one may solve the load deflection curve for any steel configuration in a reinforced concrete section.

Park and Gamble (2000) compared theoretical model calculations of tension membrane load deflection curve versus experimental data for different slabs. The results demonstrated a need to adjust the theoretical calculation of the tension membrane to the experimental tension membrane. The theoretical membrane curve needs to be shifted up and merged to the end of the load deflection curve of the compressive membrane theoretical curve for appropriate results.

Krauthammer (1986) developed the Equation 2-140 for tension membrane behavior into one way-slabs that account for such shift.

$$w(u) = \frac{8 \cdot (M_n + T \cdot u)}{L^2} \tag{2-140}$$

where

$w$ represents the applied distributed load

$M_n$ represents the nominal section moment

$T$ represents the tension in the steel

$u$ represents the mid-span deflection.

Since columns and beams are literally one-way slabs, one may use Equation 2-140 to adjust the pure tension membrane calculated using the cable theory. The tension membrane behavior would then begin at a pressure value calculated using Equation 2-141

$$\frac{8 \cdot M_n}{L^2} \tag{2-141}$$
and the shift would be done following a perpendicular path to the pure membrane behavior as shown in Figure 2-29.

As a column deflects under a severe short dynamic load, it is possible for the column to stop behaving as a beam/column and behave as a tension membrane if the axial load is taken over by other elements of the structure. This is possible for the cases where the column would fail in flexure or due to a stability issue yielding to sudden buckling of the section. In the case of buckling a sudden change into tension membrane behavior would occur. If such a scenario occurs, the resistance function of the section must be modified to account for the tension membrane behavior. Figure 2-30 shows an example of such transition into tension membrane and Figure 2-31 shows a column resistance function generated using the new version of DSAS with shifted tension membrane resistance function. It is important to note the maximum resistance function in tension membrane may be limited by an upper limit base on steel development length or pulling capacity of the connection.

Load-Impulse Diagrams

The Pressure-Impulse (P-I) diagrams or more accurately referred to as Load-Impulse Diagrams, were developed to determine levels of damage on a structure and have also been used in the past to evaluate human response to shock wave generated by an explosion. After World War II, they became widely used in the field of protective structure engineering.

As seen in Figure 2-32, the P-I diagram may be divided into three different regimes. The impulsive regime is characterized by short load duration where the maximal structural response is not reached before the load duration is over. The dynamic regime is characterized by the maximum response being reached close to the
end of the loading regime. Lastly, the quasi–static regime is characterized by a structure having reached its maximum response before the applied load is removed.

Figure 2-32 also shows one of the advantages of using the P-I diagram to better differentiate the impulsive regime and quasi-static regime by mean of vertical and horizontal asymptotes. The points on the P-I curve represent the combination of pressure or load and impulse that would cause failure based on a predefine damage or failure criteria. If a combination of load and impulse is located on the left and below the curve, the structure will not exceed the predetermined acceptable damage level or failure criteria. The predetermined acceptable damage level or failure criteria will only be exceed for a combination of load and impulse located above and on the right side of the curve.

For simple problems, the P-I diagram may be generated using closed form solutions. For other cases, the energy balanced method may be used to establish the quasi-static and impulsive asymptote and the dynamic range may be approximated by fitting a curve based on available data. More details on these methods may be found in literature such as Krauthammer (2008). For more complex loading scenario, a numerical solution must be employed to generate the P-I curve. Blasko et al. (2007) developed a numerical solution that was incorporated into DSAS and will be used in the scope of this research and. A detailed description of his method may be found in Blasko et al. (2007).

**Summary**

This chapter presented various theoretical models from various fields of study that is used in Chapter 3 to develop a new computational tool to analyze reinforced concrete column subjected to blast load and undergoing large deformation. An introduction to
explosive load calculation was presented followed by a review of dynamic analysis procedures for single degree of freedom system. A review of different concepts used in the field of protective structure for reinforced concrete material was also conducted before discussing large deformation behavior of reinforced concrete and load-impulse diagram.
Figure 2-1. Blast environment from airburst. [Adapted from Krauthammer, T. 2008. Modern Protective Structure (Page 72, Figure 3-5). CRC Press, Boca Raton, Florida.]

Figure 2-2. Free field pressure-time variation. $P_{so}$ = Incident over pressure, $P_o =$ Atmospheric pressure, $t_a =$ Time of arrival, $t_o =$ Positive phase duration, $t_o^-$ = Negative phase duration, $i_s =$ Positive phase impulse, $i_s^- =$ Negative phase impulse. [Adapted from Krauthammer, T. 2008. Modern Protective Structure (Page 68, Figure 3-1). CRC Press, Boca Raton, Florida.]
Figure 2-3. One degree of freedom system representation of a real structural element.

Figure 2-4. Deformation profile.
Figure 2-5. Equivalent load diagram.

Figure 2-6. Determination of dynamic reaction for a beam with arbitrary boundary conditions. [Adapted from Krauthammer, T., Shahriar, S. 1988. A Computational Method for Evaluating Modular Prefabricated Structural Element for Rapid Construction of Facilities, Barriers, and Revetments to Resist Modern Conventional Weapons Effects. Rep. No. ESL-TR-87-60 (Page 122, Figure 44). Engineering & Services Laboratory Air Force Engineering & Services Center, Tyndall Air Force Base.]
Figure 2-7. Resistance functions example for a reinforced concrete material with fixed boundaries conditions obtain with DSAS V3.0.
Figure 2-10. Unconfined concrete stress strain curve. [Adapted from Krauthammer, T., Shahriar, S. 1988. A Computational Method for Evaluating Modular Prefabricated Structural Element for Rapid Construction of Facilities, Barriers, and Revetments to Resist Modern Conventional Weapons Effects. Rep. No. ESL-TR-87-60 (Page 9, Figure 2). Engineering & Services Laboratory Air Force Engineering & Services Center, Tyndall Air Force Base.]
Figure 2-12. Steel Stress-strain curve model. [Adapted from Krauthammer, T., Shahriar, S. 1988. A Computational Method for Evaluating Modular Prefabricated Structural Element for Rapid Construction of Facilities, Barriers, and Revetments to Resist Modern Conventional Weapons Effects. Rep. No. ESL-TR-87-60 (Page 11, Figure 3). Engineering & Services Laboratory Air Force Engineering & Services Center, Tyndall Air Force Base.]
Figure 2-13. General model for shear influence on beams without web reinforcement. [Adapted from Krauthammer, T. 2008. Modern Protective Structure (Page 229, Figure 5-26). CRC Press, Boca Raton, Florida.]
Figure 2-15. Direct shear resistance envelop and reversal loads. [Adapted from Krauthammer, T., Shahriar, S. 1988. A Computational Method for Evaluating Modular Prefabricated Structural Element for Rapid Construction of Facilities, Barriers, and Revetments to Resist Modern Conventional Weapons Effects. Rep. No. ESL-TR-87-60 (Page 126, Figure 46). Engineering & Services Laboratory Air Force Engineering & Services Center, Tyndall Air Force Base.]
Figure 2-16. Influence of axial force on section response. [Reprinted with permission from Tran T.P. 2009. Effect of short duration high impulse variable axial and transverse loads on reinforced concrete column. M.S. dissertation (Page 91, Figure 5-7). University of Florida, Gainesville, Florida.]
Figure 2-17. Flexural resistance function envelope.
Figure 2-18. P-Delta effect on pinned column.
Figure 2-19. Snap-Through buckling.

Figure 2-20. Beam element.
Figure 2-21. Membrane behavior transition beam/column.
Figure 2-22. General flexural resistance functions including compression membrane and tension membrane behavior.

Figure 2-23. Free body diagram of deformed slab strip. [Adapted from Park, R., Gamble, W.L. 2000. Reinforced Concrete Slabs. (Page 642, Figure 12.4 ) John Wiley & Sons, Inc. New-York, New-York.]
Figure 2-24. Compressive membrane geometric model. [Adapted from Guice, L.K., Slawson, R., Rhomberg, E.J. 1989. Membrane analysis of flat plate slabs. (Page 88, Figure 5) ACI Structural Journal, Vol 86(1), 83-92.]

Figure 2-25. Compressive membrane section. [Adapted from Guice, L.K., Slawson, R., Rhomberg, E.J. 1989. Membrane analysis of flat plate slabs. (Page 88, Figure 6) ACI Structural Journal, Vol 86(1), 83-92.]
Figure 2-26. Compressive membrane external section forces. [Adapted from Guice, L.K., Slawson, R., Rhomberg, E.J. 1989. Membrane analysis of flat plate slabs. (Page 88, Figure 5) ACI Structural Journal, Vol 86(1), 83-92.]

Figure 2-27. Tension membrane geometric deformation.

Figure 2-28. Cable/truss deformation.
Figure 2-29. Pure tension membrane modification.
Figure 2-30. Resistance function modification path for stability failure.
Figure 2-31. DSAS combined resistance function example with shifted tension membrane.
Figure 2-32. Typical response functions. [Reprinted with permission from Tran T.P. 2009. Effect of short duration high impulse variable axial and transverse loads on reinforced concrete column. M.S. dissertation (Page 44, Figure 2-18). University of Florida, Gainesville, Florida.]
CHAPTER 3
METHODOLOGY

Introduction

This chapter presents the enhanced development of RC column analysis implemented into the non-linear SDOF dynamic analysis procedure in DSAS to account for the different large deformation behavior phenomenon that may occur when a RC column is subjected to blast loads. A structural overview of the problem is presented to familiarize the reader with the analytical problem addressed in this research. A discussion on the different improvements made to the dynamic analysis solution is also presented. Simplified algorithms are provided, as well as example calculations from both, previous and improved, analysis methods. Finally, the chapter concludes with the overall general analysis algorithm that incorporates all discussed dynamic analysis theory and procedures.

Structure Overview

The following discussion sets the problem parameters and presents the assumptions made in this study. The structures of interest are reinforced concrete columns subjected to blast loads. The support conditions for the columns are considered to be fixed at both ends. It is assumed that all longitudinal reinforcements are continuous through the entire span of the column, and well anchored into the supports such that no pullout of steel reinforcement could occur allowing for tension membrane behavior to be developed. It is also assumed that once flexural failure occurs, the axial load on the column will be taken by other elements of the structure allowing for tension membrane behavior to occur. Figure 3-1 illustrates the structural problem that satisfies the above-stated conditions.
When a blast load hits the structure, it will generate a time varying pressure/load that may be characterized by \( F(t) \). When conditions are present for the blast wave to pursue its way into the building through various openings, upward and downward loads, characterized by \( P_u(t) \) and \( P_d(t) \), may occur on horizontal structural elements transferring their loads to the reinforced concrete columns. Under such conditions, various phenomena may occur, and need to be addressed during the analysis:

- The first phenomenon of interest is the possibility that the RC columns undergo large deformation and transit into a tension membrane behavior. This case would only occur if the axial load acting on the columns is taken by other structural elements once the column fails in flexure.

- The second phenomenon of interest is the possibility of second order moment to arise, also referred to as P-Delta effect.

- The third phenomenon is the possibility of buckling to occur due to the weakening of the section stiffness as the column deflects coupling with an axial load that may increase due to the dynamic axial load.

Chapter 2 reviewed different structural behavior analysis theories that were used to develop different theoretical models and algorithm procedures that were incorporated into DSAS to account for the different modes of behavior and are presented in this section.

**Dynamic Analysis**

This section lays down the different algorithms and dynamic analysis approaches used to develop the overall improved dynamic analysis algorithm procedure to address the different phenomena discussed in the last section.

**Tension Membrane Analysis**

The dynamic analysis of the reinforced concrete column subjected to blasts loads is conducted using the advanced non-linear SDOF dynamic analysis software DSAS and uses a variety of the concepts discussed in Chapter 2. Since the analysis is time
dependant and used resistance functions to calculate the effective SDOF equivalent
stiffness of the column at each time step for various deflection, it is possible to develop
a resistance function for the tension membrane behavior to replace the resistance
function of the flexural behavior once flexural failure occurs and continue the dynamic
analysis until tension membrane failure occurs or dynamic behavior stops. Figure 3-2
illustrates the main steps in the algorithm developed and incorporated into DSAS to
create the tension membrane resistance function based on the cable theory presented
in Chapter 2.

Figure 3-3 and Table 3-1 set the parameters, as an example, for the pure tension
membrane resistance function illustrated in Figure 3-4. The large displacement is mainly
due to the fact the maximum strain considered to be possible for the steel is 15% and
since the length of the steel reinforcement is long in this example, it allows for a large
central deflection before failure occurs.

During the dynamic analysis, the flexural resistance function is first generated and
the maximum moment capacity of the section is obtained from the moment curvature
diagram. It is then possible to apply Equation 2-141 to shift the tension membrane base
on the discussion of Chapter 2 to obtain the resistance function shown in Figure 3-5.

It is important to note here both the flexural and tension resistance functions have
been combined together to show a combination of both. In the dynamic analysis
however, once transition occurs into tension membrane behavior, there is no going back
into a flexural behavior mode as the concrete no longer provides strength to the section.
Therefore, the hysteresis loop will be conducted on the tension membrane resistance
function only. One may realize the total displacement on the shifted curve is less than
the previously generated tension membrane resistance curve. The reason is the maximum allowable stress for the steel cannot increase when the membrane is shifted up and therefore has to fail at an earlier central displacement.

The equivalent parameters in the tension membrane range must be evaluated to complete the equivalent SDOF equation of motion. To do so, the same approach discussed in Chapter 2 is used. The equivalent mass and equivalent load are computed using the Equation 3-1, Equation 3-2, Equation 3-3 and Equation 3-4

\[ M_e^j = \sum_{i} M_j \left( \frac{d_j}{d_{mid}} \right)^2 \] (3-2)

\[ F_e^i = \sum_{i} f_i \cdot \frac{d_j}{d_{mid}} \] (3-4)

Since the membrane deflection take the form of a parabola, the load factor and mass factor remain constant throughout the analysis and the solution yield Equation 3-5 and 3-6.

\[ M_e = \frac{8}{15} \cdot L \cdot M_{Total} \] (3-5)

\[ F_e = \frac{2}{3} \cdot L \cdot w(x) \] (3-6)

**P-Delta Effect**

To account for P-Delta effect, the finite element beam formulation was simply modified using Equation 2-79 and Equation 2-80 to account for the axial effect when
computing the nodes rotations. This method formulation may be used for both slender and short columns with and without axial forces.

**Buckling**

As discussed earlier, there are two types of buckling that may occur in a reinforced concrete section, the first of which being the overall buckling of the column and second being buckling of the compressive longitudinal reinforcement. The following discusses both types and how they were handled in the dynamic analysis.

**Column buckling**

The Euler Buckling theory is introduced in the dynamic analysis as a check allowing the user using the program DSAS to be aware of the possibility of such stability failures to occur. When the Euler buckling is included in the dynamic analysis and such failure occurs, transition into tension membrane will suddenly occur. The theory presented in Chapter 2 to calculate the Euler buckling during a single degree of freedom system was adapted and incorporated into the Software DSAS to generate a function of the critical buckling load versus the central displacement that may be used at each time step to check for buckling. Figure 3-6 represents the algorithm that was incorporated in DSAS to create such function. The incorporation of this algorithm was quite simple as the program used finite beam elements that provide displacement and rotation at each node. An example of the resultant Euler-Buckling function is presented in Figure 3-7 for a 12 ft reinforced concrete column that will be introduced in more details in the next chapter.

**Buckling of compressive longitudinal reinforcement**

In the case of local buckling, the steel stress strain curve was modified in the compressive range to have failure occur once the Euler Buckling stress was reached in
between rebars. This modified the moment curvature diagram of the section and therefore modified the flexural resistance function of the section. Since the resistance function is generated by means of finite elements, a localized failure in a rebar element does not imply total failure of the structural element and therefore the analysis is continued until overall failure of the section occurs. Figure 3-8 shows flexural resistance functions examples of a given section with and without compressive longitudinal reinforcement buckling.

**Flexural Failure and Transition to Tension Membrane**

Different conditions were incorporated into the dynamic analysis program for the transition into tension membrane. Those conditions are based on geometric behavior and theoretical models.

**Column buckling transition to tension membrane behavior**

Different scenarios may trigger a stability failure as the column central deflection increase such as an increase in axial load, a diminution of the section effective stiffness under a given axial load or a combination of both. During the dynamic analysis, if the axial load increases, the flexural resistance function is re-generated to account for the stiffness changes due to the axial load. Figure 3-9 is an example of two different flexural resistance functions for two different axial loads. These resistance functions also include the effect of local buckling as discussed previously and the shear reduction factor. An increase in the axial load results into push of the flexural resistance function to the left allowing less deflection capacity and transverse load carrying capacity. Figure 3-10 is the resistance function for the case of 800 kip axial load as shown previously but it includes the tension membrane resistance curve. Note this section is heavily reinforced with # 11 rebar having an ultimate strain of 15%
As one may observe by looking at Figure 3-10, if a stability failure is to occur and the column is to transit into tension membrane behavior, the only capacity the section would have is the steel carrying capacity. Therefore, as the section loses its flexural strength capacity, the resistance functions suddenly falls onto the tension membrane resistance function and continues to deflect following the tension membrane resistance function path. Figure 3-11 represents the general case for such transition.

**Flexural failure transition into tension membrane**

The most obvious transition into tension membrane is due to flexural failure. The program DSAS already takes into account the different modes of flexural failure such as material failure and diagonal shear failure. The only concern here is the transition into tension membrane behavior. As one may have observed from Figure 3-10, transition into tension membrane shall occur at the end of the flexural resistance curve, which corresponds to the deflection where the concrete no longer provides resistance to the section. Therefore, as the flexural resistance suddenly drops to zero, the tension membrane resistance function curve is intercepted and the program continues the dynamic analysis using the tension membrane resistance curve. To capture the interception point, a simple algorithm was programmed that simply compares both curves.

**Transition into tension membrane due to excessive deformation**

For any given element acting as a beam, column or slab, if the central deflection is greater than the section depth, the entire section will then fall into tension. This type of behavior, however, may not be fully captured by the current finite element procedure as it is using beam element. Figure 3-12 is an example of DSAS V3.0 limitations as the resistance function goes to about twice the section depth which, for this case, was 16
inches. Note that this resistance function does not account for local buckling and has no axial load on it.

Obviously, a column carrying no axial load does not exist in reality. This is more or less an extreme case. However, to ensure transition occurs due to the geometric limitations of the section, a simple condition was introduced in the algorithm that simply transitions into tension membrane if the central deflection becomes greater than the section depth.

**DSAS Overall Dynamic Analysis Algorithm**

Figure 3-13, Figure 3-14, and Figure 3-15, are the simplified program flowchart of the improved DSAS procedure for the analysis of reinforced concrete columns subjected to blast loading.

**Summary**

The dynamic analysis procedure and algorithms for the analysis of reinforced concrete columns subjected to blast loads were described in this Chapter. First, the structural components of interest were discussed along with the assumptions used for the problem analysis. Various improvements were discussed to capture different possible behaviors reinforced concrete columns with fully fixed-fixed supports could undertake. Those improvements of the currently used dynamic analysis procedure make for a more complete algorithm/program that provides a strong analysis capacity of reinforced concrete columns undergoing large deformation. In the following chapter, the developed program will be validated using finite element software and a past study of reinforced concrete behavior subjected to blast loads.
Figure 3-1. Reinforced concrete column model.
Figure 3-2. Tension membrane algorithm.

3-3. RC column tension membrane calculation example.
Figure 3-4. Pure tension membrane resistance function calculation example.
Figure 3-5. DSAS combined resistance function example.
Figure 3-6. Buckling check algorithm.
Figure 3-7. Euler-buckling function obtained from DSAS DPLOT linked software.
Figure 3-8. Flexural resistance functions only.
Figure 3-9. Resistance functions for different axial load on same column.
Figure 3-10. Combined resistance function for constant 800 kips axial load.
Figure 3-11. Resistance function transition into tensile membrane behavior.
Figure 3-12. Flexural resistance function of RC column with no axial load.
Figure 3-13. DSAS simplified column analysis flowchart chart 1.
Figure 3-14. DSAS simplified column analysis flowchart chart 2.
Figure 3-15. DSAS simplified column analysis flowchart chart 3.
Table 3-1. Reinforced concrete column parameters.

<table>
<thead>
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<td>$f_u$ (psi)</td>
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<td>$f_f$ (psi)</td>
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CHAPTER 4
ANALYSIS

Introduction

The objectives validation of the research and the confirmation of the methodology presented in Chapter 3 were conducted using the finite element software ABAQUS/Explicit V6.8 (Dassault Systemes, 2008). The first step was to validate the material model using experimental data obtained from Feldman and Siess (1958) experiment on reinforced concrete beams subjected to impact loads. The second step was to apply the material model to both DSAS V3.0 and ABAQUS V6.8 and finally use the developed model to validate the tension membrane analysis algorithm by analyzing a reinforced concrete column under different loading conditions.

Material Model Validation

The reinforced concrete model used for reinforced concrete was developed using experimental data from Feldman and Siess (1958) on reinforced concrete beams subjected to impact load. For ABAQUS V6.8, the concrete damaged plasticity (CDP) model was used for the concrete material as it is suitable for concrete subjected to dynamic and cyclic loading such as blast loading. The following citation was directly taken from ABAQUS V6.8 theory manual to describe the model.

The concrete damaged plasticity model is primarily intended to provide a general capability for the analysis of concrete structures under cyclic and/or dynamic loading. The model is also suitable for the analysis of other quasi-brittle materials, such as rock, mortar and ceramics. Under low confining pressures, concrete behaves in a brittle manner; the main failure mechanisms are cracking in tension and crushing in compression. The brittle behavior of concrete disappears when the confining pressure is sufficiently large to prevent crack propagation. In these circumstances failure is driven by the consolidation and collapse of the concrete microporous microstructure, leading to a macroscopic response that resembles that of a ductile material with work hardening. (AB AQUS V6.8 User Theory Manual 2008)
The model parameters used for analysis were derived by Tan Loo Yong at CIPPS as part of his M.S. thesis titled Characterizing a Reinforced Concrete Connection for Progressive Collapse Assessment, to be published in May 2010. He validated his concrete model by analyzing the different beams from the experiment conducted by Feldman and Siess (1958). Dynamic increase factors for both concrete and steel were applied on the materials stress-strain curve to properly capture the dynamic effect onto the material behavior. Table 4-1 presents typical values used for such factors.

Table 4-2 presents the CDP parameters developed to model beam C from Feldman and Siess (1958) experiment. The steel stress strain material model was based on the experiment data but was transformed into true stress and logarithmic strain as required by ABAQUS V6.8 explicit. Table 4-3 presents the different steel material parameters for Beam C.

Figure 4-1 shows the developed model in ABAQUS V6.8. The element types used were cubic 8 node reduced integration with hourglass control (C3D8R) for the concrete element and 2D Timoshenko beam element for the steel elements (B31). Finally, gravity loads were applied along with a point load to simulate the impact load located at the center of the beam having the load time history presented in Figure 4-2.

Figure 4-3 illustrates the displacement time history of Beam C experiment data versus ABAQUS V6.8 results. The maximum displacement obtained from ABAQUS V6.8 is 3.03 inches versus 3.01 inches from the experiment, which represents a difference of 0.66%.

The results obtained from ABAQUS V6.8 proved to be representative of the experiment for the peak displacement value. However, the results obtained from
ABAQUS V6.8 past the peak displacement varies from the experiment as ABAQUS V6.8 lacks the ability to properly capture the hysteresis behavior of reinforced concrete models in free vibration mode. This limitation has been reported in previous studies conducted using the ABAQUS software for impact and blast loads and will be disregarded for the remainder of the validation. However, since structural damages are related to the peak support rotation, one can compare the peak displacements.

When pushing the Timoshenko beam element into large deformation under higher load, it was found that the Timoshenko formulation was unable to handle large axial deformation. A Timoshenko hybrid element formulation to handle such case is available in ABAQUS/Standard V6.8 but unfortunately not in ABAQUS/Explicit V6.8. Therefore, the Timoshenko beam elements were replaced by truss elements and the material properties for the steel were modified to reflect proper dynamic increase factor for truss elements. Figure 4-4 shows the results obtained for Beam C using truss elements compared with Timoshenko beam elements, DSAS V3.0 and the experiment results. The maximum displacement obtained using truss elements is 3.0 and represents a difference of 0.33% compared to experimental data.

The same beam was then subjected to a triangular blast load having a peak reflected pressure of 1181 psi and load duration of 0.00132375 sec. The results are shown in Figure 4-5 for DSAS V3.0 and ABAQUS V6.8. The peak displacement value obtained from ABAQUS V6.8 is 1.72 inches versus 1.66 inches obtained from DSAS 3.0 for a difference of 3.33%. The same exercise was conducted with an increased triangular load function having a peak reflected pressure of 1777 psi and a load duration of 1.28 msec. The results are shown in Figure 4-6. The results obtained for this case
demonstrated a difference of 1.60% for a maximum displacement of 3.36 inches for DSAS V3.0 and 3.41 inches for ABAQUS V6.8.

The material model developed for ABAQUS V6.8 and DSAS V3.0 based on the Feldman and Siess (1958) experiment proved to yield accurate results for beam cases.

**Column Model Validation**

The model was further validated against a reinforced concrete column having no axial load subjected to various combinations of triangular blast load. The column design was chosen based on previous research conducted by Thien Tran (2008) on reinforced columns subjected to blast. The column cross section was 16X16 inches and the column free span length was 12 ft. 8 # 11 longitudinal rebar was placed at 1.5 inches, 8 inches and 14.5 inches deep leaving 1.5 inches of cover all around. The longitudinal reinforcements were closed with stirrups spaced at 12 inches. The column continued six inches at each end to properly model boundary conditions. A steel plate was emplaced at the top and bottom of the column to properly distribute the axial point load onto the surface of the column. The supports were fixed in all directions and the longitudinal reinforcement ends were also fixed in the vertical direction to ensure full development capacity of the reinforcement. Figure 4-7 shows a general view of ABAQUS V6.8 model and Table 4-4 shows the peak displacement results obtained for all different loading cases from DSAS V3.0 and includes the percent difference from ABQUS V6.8 for both beam and truss elements models.

Overall, the average percent difference is 13.37% for truss element cases versus 17.92% from Timoshenko beam element cases. However, when ignoring the smaller load case, the average percentage difference for truss and beam elements is respectively 6.26% and 17.89%. The truss element model proved to yield more
accurate results than the beam element model and also proved to be capable of handling large deformation, and as a result, higher loading cases.

**Modified DSAS Validation**

To validate the modification done to the software DSAS, the previously discussed cases were tested using the modified version of DSAS to ensure the program would yield the same results as DSAS V3.0 for cases where modification done to improved the program were not expected to be engaged during an analysis. Table 4-5 represents the results obtained from both programs.

Under low impulsive load, the results are exactly the same as expected as neither compressive longitudinal reinforcement buckling nor tension membrane behavior influence the response. However, under the higher impulsive load of 1875 psi-msec, the tension membrane was engaged because the compression longitudinal reinforcement buckling was not taken into account, yielding a higher tension membrane shift due to a higher nominal moment on the moment curvature diagram. Finally, as the impulsive load was increased, direct shear failure occurred. Although tension membrane is captured, it has no significance as the column has already failed. This will not be the case under the presence of axial load since the axial load increases the direct shear capacity of the section while reducing the maximum lateral deflection in flexure. This case was more or less a beam case with the configuration of a column used for validation purposes only.

Figure 4-8 is an example of flexural time history analysis to compare the modified DSAS 3.0 response versus ABAQUS 6.8 using truss element.
Both programs yield very close peak displacements; however, one may see the peak displacements occurring at different times. This is mainly due to the fact both programs used different approaches to calculate the stiffness of the column during the dynamic analysis which yielded different response periods. However, both analyses yield acceptable results from a design standpoint as the main interests are the peak displacement and reactions.

**Tension Membrane Validation**

To validate the modified version of DSAS in tension membrane range, the previous column was subjected to several series of blast loads while being subjected to a constant axial load. For each series constant axial loads were chosen to be located below balance point, at balance point and above balance point based on the interaction diagram of the column presented in Figure 4-9.

Two analytical steps were used in an attempt to capture the effect of the axial load on the column behavior during the ABAQUS 6.8 analysis. The first step consisted of capturing the displacement time history of the support under only axial loading. The second step was to reproduce the effect of the axial load on the column by imposing a support displacement time history as of step one. This allowed the stress due to the axial load to develop into the column prior of being subjected to a blast load and also respects the assumption that the supports are fixed when the column is subjected to transverse load and undergoing tension membrane behavior.

The first axial load imposed on the first series of lateral loads is 500 kip. Because the analysis is conducted using ABAQUS Explicit, the axial load was gradually applied from 0 to 0.1 sec in order to reduce the analysis time for the support to come to rest. It was found that a total time of 0.15 sec was sufficient to properly capture the effect of the
axial load on the column. Figure 4-10 represents the support displacement time history for the three different axial loads imposed on the column.

The results obtained from ABAQUS V6.8 and DSAS for a reinforced concrete column under 500 kips axial load subjected to different loading cases are shown in Table 4-6. The results obtained from the modified program DSAS for the case of 500 kips axial load demonstrated the column would not fail in flexure and would not undergo tension membrane behavior under an impulsive load of 1134 and 1354 psi-msec. For the case of 1875 psi-msec, the modified DSAS program demonstrated the column would undergo flexural failure followed by tension membrane behavior. ABAQUS V6.8 however demonstrated all cases would undergo tension membrane behavior based on the strain in the compression longitudinal reinforcement at mid-span going from compression to tension. The reason is that the effect of the axial load quickly disappears as the column deflects under a transverse load due to the boundary conditions remaining fixed. Under real conditions, the data obtained for the 500 Kips axial loads would have been closer to the modified DSAS results as axial load taken below balanced point increase the flexural capacity of the column. It is interesting to note DSAS V3.0 analysis of the impulse case of 1875 psi-msec did not yield a flexural failure. However, when including the local buckling effect and ignoring the possibility of tension membrane, the modified version of DSAS yield a flexural failure. This is demonstrated in Figure 4-11 which represents the flexural resistance function of both cases. In the case where longitudinal reinforcement buckling is included, the maximum permissible displacement is 3.99 inches and therefore demonstrated the column would fail in flexure and undergo tension membrane behavior.
In the case of an axial load of 800 kip and 1500 kip, Table 4-7 and Table 4-8, all cases demonstrated flexural failure and tension membrane behavior in the modified DSAS program. ABAQUS V6.8, under the prescribed analysis condition, also demonstrated tension membrane behavior for all cases. However, these results are somewhat biased, as discussed earlier, by the fact the axial load does not remain constant up to flexural failure during the analysis onto the longitudinal reinforcement. Under higher lateral loads, the effect of the axial load becomes less important and this may be observed by looking at the percent difference from all the previous cases, as the central deflection increases, the percent difference from both programs improves. This point is further enforced when comparing the results obtained from ABAQUS V6.8 versus Modified DSAS where no axial load is present as demonstrated in Table 4-9 where the percent difference for all cases are within 15% and mostly within 10%.

The last discussion only proved that the approach used in ABAQUS V6.8 lacks the capacity to properly capture the effect of the axial load up to flexural failure but this limitation could be overcome by increasing the axial load. Therefore, the column was subjected to higher lateral load where the axial load is expected to have less effect on the overall analysis and the results are presented in the Table 4-10 and Table 4-11 for the case of 800 kips and 1500 kips. The 500 kips cases are ignored as the column would failed in direct shear failure prior to reaching those central deflection levels.

The results obtained from the last analysis proved to be much more conclusive to evaluate the tension membrane behavior. In the case of 800 kips axial load, although the percent difference was in the range of 16%, the results proved to be consistent under different load cases. For the case of 1500 kips axial load, the percent difference
varies but are overall is within 10%. The suspected reason for percent difference variation in the case of 1500 kip is the need to better understand the shifting condition of the tension membrane resistance function. The results obtained are overall remarkable when considering one program uses over fifty thousand elements while the other only uses a few and also considering the complexity of ABAQUS versus DSAS.

Direct shear failure was observed for the 800 kips axial load during the modified DSAS program run. Even though DSAS detects a direct shear failure, it performs a full flexural analysis. The results obtained may still be used for comparison. It was however not the intent to prove DSAS capability of handling direct shear failure and no further discussion on the subject will be made as part of the results analysis section of this dissertation. One thing to note, however, an increase in axial load increases the direct shear capacity of a section which explains why direct shear failure was not detected during the 1500 kips axial load analysis under the same loading conditions as the 800 kips axial load.

Figure 4-12 shows the deflected shape of the columns for the last reported cases. The maximum principal stress are reported but are not representative of the maximum stress in the steel rebar as truss elements are being used which may only report S11 (axial) stress. In this case, the maximum principal stresses are located at the interface of the reinforcement and the steel plate on top of the column is used to uniformly distribute the axial stress due to axial load. However, Figure 4-13 shows the axial stress time history of steel rebar at mid-span. It demonstrates compression reinforcement goes from compression stress to tension stress and the maximum tensile stresses are below ultimate capacity. The transition into tension membrane behavior of the compressive
reinforcement occurs at 0.1535 sec of the analysis corresponding to a displacement of 2.82 in. This represents a transition displacement of 17.63% of the section depth (h) which is comparable to transition point of deep beam as expected based on Krauthammer et al. (2003) where 17% was observed in deep beam. Finally, Figure 4-14 shows the minimum principal stress where the concrete is shown to be crushed and in tension.

To conclude on the validation of the 12ft column, Figure 4-15, Figure 4-16 and Figure 4-17 are example of various time responses of central displacement obtained from both program for different cases.

**Slender Column**

To confirm the validation of the SDOF algorithm ability to handle large deformation behavior into a reinforced concrete column with a fixed boundary condition, the previous column was expended 12 feet, making it fall into a slender column classification. All other properties remained the same. The column was subjected to an axial load of 1200 kips and the same approach discussed earlier was used in ABAQUS V6.8 to mimic the effect of the axial load. Table 4-12 are the results obtained from both programs.

The results obtained are in good agreement. Under higher load, the percent difference increases a little above 10%. However, after increasing the load until direct shear failure occurs, the percent difference is 14.6% which is an acceptable range.

Figure 4-18 and Figure 4-19 shows the deformed shape of the column and the S11 (axial) stress at the maximum displacement step for the last two cases analyzed. It demonstrates both cases have remaining capacity in the steel to develop tension membrane behavior.
Summary

This chapter presented the different steps and approaches used to validate the modification made to the dynamic analysis procedure embedded into the software DSAS for the analysis of reinforced concrete columns having fixed boundary conditions and the capacity to undergo tension membrane behavior under large deformation. First, a material model was developed for both DSAS V3.1 and ABAQUS V6.8 based on past experimental data on reinforced concrete beams subjected to impact load. The material model was then used to model a reinforced concrete column in ABAQUS V6.8 and DSAS 3.1 and again tested and validated under severe short dynamic transverse load. Finally, the modification made to the program to account for large deformation and tension membrane behavior were validated. The results obtained were conclusive and both ABAQUS and the modified DSAS yielded results within 15% difference which at this point may be considered acceptable. However, it is important to understand limitations and assumptions made for the validation and will be discussed in the next chapter as part of the discussion and recommendations.
Figure 4-1. Beam C meshing.
Figure 4-2. Beam C load time history.
Figure 4-3. Displacement time history beam C ABAQUS vs. experiment.
Figure 4-4. Beam C experiment time history comparison.
Figure 4-5. DSAS V3.0 vs. ABAQUS V6.8 analysis with triangular blast load $Pr = 1183$ psi and $t = 1.32$ msec.
Figure 4-6. DSAS V3.0 vs ABAQUS V6.8 analysis with triangular blast load $Pr = 1777$ psi and $t = 1.28$ msec.
Figure 4-7. Reinforced concrete column model in ABAQUS V6.8. A) General model. B) Steel reinforcement cage.
Figure 4-8. Flexural time history analysis comparison modified DSAS vs. ABAQUS V6.8 for triangular blast load $Pr=1777$ psi and $t = 0.00127630$. 
Figure 4-9. Interaction diagram for 12 ft long column model obtained with Modified DSAS DPlot linked software.
Figure 4-10. Support displacement time history for different constant axial load cases.
Figure 4-11. Flexural resistance function comparison of local buckling with a constant axial load of 500 kips.
Figure 4-12. Principal maximum stress, case 1500 kip axial load, \( Pr=5262 \), \( t = 0.00142721 \).
Figure 4-13. Steel reinforcement axial stress at mid-span, case 1500 kip axial load, $Pr = 5262$ psi, $t = 0.00142721$ sec.
Figure 4-14. Principal minimum stress, case 1500 kip axial load, Pr=5262 psi, t = 0.00142721 sec.
Figure 4-15. Flexural time history example case with 800 kips constant axial load, triangular blast load with Pr = 2922 psi and t = 1.28 msec.
Figure 4-16. Flexural time history example case with 1500 kips constant axial load, triangular blast load with Pr = 2922 psi and t = 1.28 msec.
Figure 4-17. Flexural time history example case with 1500 kips constant axial load, triangular blast load with $P_r = 4757$ psi and $t = 1.39$ msec.
Figure 4-18. Twenty-Four foot column maximum general principal stress results case: Pr = 2922 psi, t = 0.00128337 sec.
Figure 4-19. Twenty-Four foot column maximum general principal stress results case: Pr = 3400 psi, t = 0.00128337 sec, direct shear failure reported by DSAS.
### Table 4-1. Reinforced concrete design dynamic increase factors.

<table>
<thead>
<tr>
<th>Type of Stress</th>
<th>Far design range</th>
<th>Close design range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steel reinforcement</td>
<td>Concrete</td>
</tr>
<tr>
<td></td>
<td>fdy/fy</td>
<td>fdu/fu</td>
</tr>
<tr>
<td>Bending</td>
<td>1.17</td>
<td>1.05</td>
</tr>
<tr>
<td>Diagonal tension</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Direct shear</td>
<td>1.10</td>
<td>1.00</td>
</tr>
<tr>
<td>Bond</td>
<td>1.17</td>
<td>1.05</td>
</tr>
<tr>
<td>Compression</td>
<td>1.10</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Adapted from Krauthammer, T. Hinman, E. Rittenhouse, T. Structural system- Behavior and design philosophy. (Page 3-24, Figure 3.1) Structural Design for Physical Security- State of the practice rep, ASCE, Reston, Virginia.

### Table 4-2. Concrete damaged plasticity model parameters.

<table>
<thead>
<tr>
<th>Mass density</th>
<th>2.21E-07</th>
<th>Lbf*s²/in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young modulus</td>
<td>4816</td>
<td>ksi</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

**Concrete damaged plasticity parameters**

<table>
<thead>
<tr>
<th>Dilatation angle</th>
<th>Eccentricity</th>
<th>f₀₀₀/f₂₀₀</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.1</td>
<td>1.16</td>
<td>0.666</td>
</tr>
</tbody>
</table>

**Compressive behavior**

<table>
<thead>
<tr>
<th>Yield stress (ksi)</th>
<th>Inelastic strain</th>
<th>Damage parameter</th>
<th>Inelastic strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.355</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6.24636</td>
<td>0.0007031</td>
<td>0</td>
<td>0.000703</td>
</tr>
<tr>
<td>6.426</td>
<td>0.0010674</td>
<td>0</td>
<td>0.001067</td>
</tr>
<tr>
<td>5.88124</td>
<td>0.0018797</td>
<td>0.084774</td>
<td>0.00188</td>
</tr>
<tr>
<td>4.24697</td>
<td>0.0029182</td>
<td>0.339097</td>
<td>0.002918</td>
</tr>
</tbody>
</table>

**Tensile behavior**

<table>
<thead>
<tr>
<th>Yield stress (ksi)</th>
<th>Cracking strain</th>
<th>Damage parameter</th>
<th>Cracking strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.872</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.437</td>
<td>0.0009054</td>
<td>0.499427</td>
<td>0.000905</td>
</tr>
<tr>
<td>0.001</td>
<td>0.0018108</td>
<td>0.99</td>
<td>0.001811</td>
</tr>
</tbody>
</table>
Table 4-3. Steel properties of beam C for ABAQUS, US unit with stress in ksi.

<table>
<thead>
<tr>
<th>Steel general properties</th>
<th>Mass density</th>
<th>7.3386E-07 Lbf*s²/in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass density</td>
<td>7.3386E-07</td>
<td>Lbf*s²/in</td>
</tr>
<tr>
<td>Young's modulus</td>
<td>29520</td>
<td>ksi</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tension steel</th>
<th>Increased stress</th>
<th>Nominal strain</th>
<th>True stress (ksi)</th>
<th>True strain</th>
<th>Plastic strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>51.414</td>
<td>0.00174</td>
<td>51.504</td>
<td>0.00174</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>0.15000</td>
<td>86.250</td>
<td>0.13976</td>
<td>0.137</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compression steel</th>
<th>Increased stress</th>
<th>Nominal strain</th>
<th>True stress (ksi)</th>
<th>True strain</th>
<th>Plastic strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>52.106</td>
<td>0.00177</td>
<td>52.198</td>
<td>0.00176</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>0.15000</td>
<td>86.250</td>
<td>0.13976</td>
<td>0.137</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stirrups</th>
<th>Increased stress</th>
<th>Nominal strain</th>
<th>True stress (ksi)</th>
<th>True strain</th>
<th>Plastic strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>47.129</td>
<td>0.00160</td>
<td>47.204</td>
<td>0.00160</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>0.15000</td>
<td>86.250</td>
<td>0.13976</td>
<td>0.137</td>
<td></td>
</tr>
</tbody>
</table>

Table 4-4. Reinforced concrete column under no axial load results.

<table>
<thead>
<tr>
<th>Triangular load parameters</th>
<th>DSAS V3.0 max disp. (in)</th>
<th>FEA results ABAQUS V6.8</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflected pressure (psi)</td>
<td>Incident reflected impulse (psi-msec)</td>
<td>Time duration (msec)</td>
<td>No SRF</td>
</tr>
<tr>
<td>1183</td>
<td>783</td>
<td>1.324</td>
<td>0.60</td>
</tr>
<tr>
<td>1777</td>
<td>1134</td>
<td>1.276</td>
<td>1.08</td>
</tr>
<tr>
<td>2134</td>
<td>1354</td>
<td>1.269</td>
<td>1.47</td>
</tr>
<tr>
<td>2922</td>
<td>1875</td>
<td>1.283</td>
<td>2.57</td>
</tr>
<tr>
<td>3602</td>
<td>2369</td>
<td>1.315</td>
<td>3.85*</td>
</tr>
</tbody>
</table>

*Direct shear failure.
Table 4-5. Reinforced concrete column under no axial load.

<table>
<thead>
<tr>
<th>Triangular load parameters</th>
<th>DSAS V3.0 max disp. (in)</th>
<th>Modified DSAS Including tension membrane behavior max disp. (in)</th>
<th>FEA Results ABAQUS V6.8 mid-span disp. (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflecte d pressure (psi)</td>
<td>Reflecte d impulse (psi-msec)</td>
<td>Time duration (msec)</td>
<td>No SRF</td>
</tr>
<tr>
<td>1183</td>
<td>783</td>
<td>1.324</td>
<td>0.60</td>
</tr>
<tr>
<td>1777</td>
<td>1134</td>
<td>1.276</td>
<td>1.08</td>
</tr>
<tr>
<td>2134</td>
<td>1354</td>
<td>1.269</td>
<td>1.47</td>
</tr>
<tr>
<td>2922</td>
<td>1875</td>
<td>1.283</td>
<td>2.57</td>
</tr>
<tr>
<td>3602</td>
<td>2369</td>
<td>1.315</td>
<td>3.85</td>
</tr>
</tbody>
</table>

# Tension membrane engaged, # Direct shear failure.

Table 4-6. Twelve foot reinforced concrete column under 500 kip axial load.

<table>
<thead>
<tr>
<th>Triangular load parameters</th>
<th>DSAS results</th>
<th>FEA results</th>
<th>Results comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflected pressure (psi)</td>
<td>Incident reflected impulse (psi-msec)</td>
<td>Time duration (msec)</td>
<td>DSAS V3.0 max disp. (in)</td>
</tr>
<tr>
<td>1777</td>
<td>1134</td>
<td>1.276</td>
<td>1.41</td>
</tr>
<tr>
<td>2134</td>
<td>1354</td>
<td>1.269</td>
<td>2.05</td>
</tr>
<tr>
<td>2922</td>
<td>1875</td>
<td>1.283</td>
<td>4.05</td>
</tr>
</tbody>
</table>

# Tension membrane engaged.

Table 4-7. Twelve foot reinforced concrete column under 800 kip axial load.

<table>
<thead>
<tr>
<th>Triangular load parameters</th>
<th>DSAS results</th>
<th>FEA results</th>
<th>Results comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflected pressure (psi)</td>
<td>Incident reflected impulse (psi-msec)</td>
<td>Time duration (msec)</td>
<td>DSAS V3.0 max disp. (in)</td>
</tr>
<tr>
<td>1777</td>
<td>1134</td>
<td>1.276</td>
<td>1.33 #</td>
</tr>
<tr>
<td>2134</td>
<td>1354</td>
<td>1.269</td>
<td>1.33 #</td>
</tr>
<tr>
<td>2922</td>
<td>1875</td>
<td>1.283</td>
<td>1.33 #</td>
</tr>
</tbody>
</table>

# Tension membrane engaged, # Direct shear failure, # Flexural failure.
Table 4-8. Twelve foot reinforced concrete column under 1500 kip axial load.

<table>
<thead>
<tr>
<th>Triangular load parameters</th>
<th>DSAS results</th>
<th>FEA results</th>
<th>Results comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflected pressure (psi)</td>
<td>Incident reflected impulse (psi-msec)</td>
<td>Time duration (msec)</td>
<td>DSAS V3.0 max disp. (in)</td>
</tr>
<tr>
<td>1777</td>
<td>1134</td>
<td>1.276</td>
<td>0.78³</td>
</tr>
<tr>
<td>2134</td>
<td>1354</td>
<td>1.269</td>
<td>0.78³</td>
</tr>
<tr>
<td>2922</td>
<td>1875</td>
<td>1.283</td>
<td>0.78³</td>
</tr>
</tbody>
</table>

¹Tension membrane engaged, ³Direct shear failure, ³Flexural failure.

Table 4-9. Comparison of ABAQUS results with DSAS V3.0 without axial load.

<table>
<thead>
<tr>
<th>Reflected pressure (psi)</th>
<th>ABAQUS V6.8 with 500 kip axial load max disp. (in)</th>
<th>ABAQUS V6.8 with 800 kip axial load max disp. (in)</th>
<th>ABAQUS V6.8 with 1500 kip axial load max disp. (in)</th>
<th>Modified DSAS V3.0 (with SRF &amp; rebar buckling) no axial load</th>
<th>% difference 500 kip case</th>
<th>% difference 800 kip case</th>
<th>% difference 1500 kip case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1777</td>
<td>1.72</td>
<td>1.90</td>
<td>2.04</td>
<td>1.77</td>
<td>2.37</td>
<td>7.30</td>
<td>13.67</td>
</tr>
<tr>
<td>2134</td>
<td>2.52</td>
<td>2.56</td>
<td>2.55</td>
<td>2.39</td>
<td>5.25</td>
<td>6.47</td>
<td>6.41</td>
</tr>
<tr>
<td>2922</td>
<td>4.69</td>
<td>4.68</td>
<td>4.86</td>
<td>4.18</td>
<td>10.76</td>
<td>10.60</td>
<td>14.00</td>
</tr>
</tbody>
</table>

Table 4-10. Twelve foot reinforced concrete column under 800 kip axial load con't.

<table>
<thead>
<tr>
<th>Triangular load parameters</th>
<th>DSAS results</th>
<th>FEA results</th>
<th>Results comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflected pressure (psi)</td>
<td>Incident reflected impulse (psi-msec)</td>
<td>Time duration (msec)</td>
<td>DSAS V3.0 max disp. (in)</td>
</tr>
<tr>
<td>4208</td>
<td>2844</td>
<td>1.352</td>
<td>1.32³</td>
</tr>
<tr>
<td>4757</td>
<td>3305</td>
<td>1.390</td>
<td>1.32³</td>
</tr>
<tr>
<td>5262</td>
<td>3755</td>
<td>1.427</td>
<td>1.32³</td>
</tr>
</tbody>
</table>

¹Tension membrane engaged, ³Direct shear failure, ³Flexural failure.
Table 4-11. Twelve foot reinforced concrete column under 1500 kip axial load con't.

<table>
<thead>
<tr>
<th>Reflected pressure (psi)</th>
<th>Incident reflected impulse (psi-msec)</th>
<th>Time duration (msec)</th>
<th>DSAS V3.0 max disp. (in)</th>
<th>Modified DSAS V3.0 max disp. (in)</th>
<th>ABAQUS V6.8 average mid-span disp. (in)</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>4208</td>
<td>2844</td>
<td>1.352</td>
<td>0.77(^3)</td>
<td>8.60(^1)</td>
<td>9.10</td>
<td>5.51</td>
</tr>
<tr>
<td>4757</td>
<td>3305</td>
<td>1.390</td>
<td>0.78(^3)</td>
<td>10.48(^1)</td>
<td>10.33</td>
<td>1.47</td>
</tr>
<tr>
<td>5262</td>
<td>3755</td>
<td>1.427</td>
<td>0.78(^3)</td>
<td>12.26(^1)</td>
<td>13.48</td>
<td>9.03</td>
</tr>
</tbody>
</table>

\(^1\)Tension membrane engaged, \(^2\)Direct shear failure, \(^3\)Flexural failure.

Table 4-12. Twenty-four foot reinforced concrete column results.

<table>
<thead>
<tr>
<th>Reflected pressure (psi)</th>
<th>Incident reflected impulse (psi-msec)</th>
<th>Time duration (msec)</th>
<th>DSAS V3.0 max disp. (in)</th>
<th>Modified DSAS V3.0 max disp. (in)</th>
<th>ABAQUS V6.8 Average mid-span disp. (in)</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1777</td>
<td>1134</td>
<td>1.276</td>
<td>0.53(^3)</td>
<td>6.44(^1)</td>
<td>6.74</td>
<td>4.44</td>
</tr>
<tr>
<td>2134</td>
<td>1354</td>
<td>1.269</td>
<td>0.52(^3)</td>
<td>8.23(^1)</td>
<td>9.02</td>
<td>8.80</td>
</tr>
<tr>
<td>2922</td>
<td>1875</td>
<td>1.283</td>
<td>0.49(^3)</td>
<td>12.73(^1)</td>
<td>14.76</td>
<td>13.75</td>
</tr>
<tr>
<td>3400</td>
<td>2182</td>
<td>1.284</td>
<td>0.49(^3)</td>
<td>15.56(^2)</td>
<td>18.22</td>
<td>14.59</td>
</tr>
</tbody>
</table>

\(^1\)Tension membrane engaged, \(^2\)Direct shear failure, \(^3\)Flexural failure.
CHAPTER 5
DISCUSSION AND RECOMMENDATIONS

Introduction

In this chapter, the observations made as part of the validation of the tension membrane are further discussed. First, a discussion on the limitation of the analysis procedure based on results obtained is conducted. Second, a parametric study of the modified DSAS program is carried out to identify key issues that may help someone design a RC column capable of undergoing tension membrane behavior. Finally, recommendations are made for future development.

Limitations

Various limitations were encountered during the validation procedures that were only discussed briefly in the previous chapter; more specifically these limitations are regarding the effect of axial load on the column. The first limitation discussed here is the lack of ability to properly capture the effect of axial load on the overall column during the dynamic analysis in the software ABAQUS V6.8. The main purpose of columns is obviously to carry large axial forces and as such, they are usually not designed to sustain large lateral loads. On the other hand, when buildings are designed to be able to sustain the loss of a column, the adjacent components are not designed to take over the axial load on the lost column before flexural failure occurs. The reality is, if the structure is to overtake the axial load onto the column prior to flexural failure, no one can predict accurately the exact time and the exact vertical support displacement at flexural failure using a finite element code or any other analysis tool available today to conduct such analysis. Furthermore, the axial load time history on the column going from fully loaded to zero would also be impossible to predict and yet very important to be taken into
account. The fact is, for any given design able to sustain the loss of a column, flexural failure would occur before significant axial load variation would have an effect on the column response and the problem needs to be treated as such. This was however the limitation encountered with the software ABAQUS V6.8 where it was not possible to model both a constant axial load onto the column and going into tension membrane at flexural failure. An induced support displacement approach was then used in an attempt to mimic the effect of a constant axial load onto the column but was very limited as the effect of the axial load was quickly dissipated as lateral deflection occurred. It was however found that axial load effect became less important as the transverse load increased and the model was then validated under higher load.

In the same matter, DSAS also lacks the ability to properly capture the effect of support displacement during an analysis. It assumes fixed boundary conditions at the support and the axial effect if taken into account when DSAS numerically developed the moment curvature relationship prior to conducting the dynamic analysis. One may argue a validation approach using ABAQUS V6.8 would have been to first conduct an analysis of the column under both axial and transverse load, capture the time displacement time history of the support until flexural failure occur and mimic the effect of the axial load by imposing the same displacement time history at the support. This would have been ideally the best route to take. However, because DSAS does not have the ability to capture the effect of support displacement for the analysis of column, this approach was not preferable as the results from ABAQUS would have yielded much higher central displacement and there would have been many difficulties capturing the exact time of flexural failure. Note that in this case, the displacement time history of the support due
to both axial and transverse load would have been much higher than the displacement time history due only to axial load which was not significant enough to be taken into account during the validation in Chapter 4. So not only was the validation limited by the uses of ABAQUS, it was also limited by the capacity of DSAS. Nonetheless, DSAS proved to have the capability to yield very good results under the conditions it was design for and in the case of a significant support displacement, DSAS had the ability to predict within an acceptable range if the column would actually undergo tension membrane or fail in direct shear failure.

When conducting a dynamic analysis, one must take into account the physical limitation of the overall structure and not only the capacity of the column to undergo tension membrane behavior. If a column is subjected to an explosion strong enough to bring the column into a tension membrane state, one must consider not only the damage done to the column but also to the area surrounding the column. For example, while the concrete of the column is likely to be crushed into pieces, concrete within the steel reinforcement may remain trapped inside the steel cage or not based on the stirrup spacing and overall reinforcement configuration. This will have an effect on the central displacement of the column as the loss of mass leads to the loss of inertia force. This problem may be overcome by minimizing the stirrup spacing or through the use of FRP to wrap the column. Another important factor is the capacity of the connection to resist blast load. One must considered the damage that may occur at the fixed connection itself and how it may affect the column capacity to develop tension membrane behavior. For example, the longitudinal rebar nearest to the blast may be completely cut due to fragments while the rebar farthest from the blast load remains
intact. In that case tension membrane could developed but with less resistance capacity.

Finally, the SDOF approach itself has its own assumptions and limitations that need to be considered. One of these is the assumed shape function of the tension membrane to take the form of a parabola. This shape function represents the ideal case where once the column undergoes tension membrane, the concrete does not provide movement restriction to the steel rebar. However, this may not be the case as concrete is likely to remain caged in the rebar and change the shape of the deflection. Also, the failure criteria of the steel is uni-axial, and therefore does not consider the stress from lateral load on the steel. On the other end, once concrete is crushed, it is impossible to predict the exact lateral stress on the rebar. However, as seen in the validation and further discussed in the next section, direct shear failure would occur before full tension membrane capacity may be developed.

Overall, the improvements made to the SDOF procedure as part of this thesis provide an excellent tool for analysis within certain limitations. Those limitations could be better addressed by conducting experiments and will be discussed with more depth in a later section. Nonetheless, good agreement was obtained from both programs during the validation which permits a further investigation of the benefit tension membrane behavior may bring to a column under blast loads. Therefore, the improved software is put to work in the next section to identify key benefits and better understand the tension membrane behavioral mode with different columns configurations.

**Parametric study**

The following section conducts parametric studies to better understand the behavioral mode of reinforced concrete columns undergoing large deformation. First a
parametric study to better understand the effect of direct shear failure in the overall analysis is conducted for the 12ft column used in the previous section having different longitudinal rebar sizes. Second, pressure-impulse diagram of the 12 ft columns are presented to demonstrate the benefit of tension membrane capabilities.

**Column Parametric Study**

The first key issue for a column to be able to undergo tension membrane behavior under blast load is to have the ability to resist direct shear failure. Different factors will affect the direct shear capacity of a section but the main one to be considered in the case where tension membrane behavior is desired is the axial load and the size of longitudinal reinforcement. Therefore the 12 foot column designed as part of the validation was modified to compare the effect of using different longitudinal reinforcement rebars sizes. The columns were subjected to various loading scenarios and the results are presented in Table 5-1.

The different resistance functions for each columns analyzed under a 500 kips axial load are presented in Figure 5-1 represents. Note the resistances functions are combined for demonstration purposes, once tension membranes occur, only tension membrane resistance functions remain for the dynamic analysis. The results highlighted in orange in Table 5-1 represent the central displacements obtained under tension membrane behavior. The results obtained clearly demonstrate the tension membrane capacity is limited by the direct shear failure capacity of the section under blast. In other words, direct shear failure is expected to occur before a column may reach its full capacity in tension membrane and fails as such. As the rebar size decreases, the direct shear capacity decreases as well as the tension membrane capacity. However, as the
axial load is increased, the direct shear capacity is increased which in some cases provides extra strength to the column to develop higher tension membrane capacity.

When comparing the extra strength provided by the tension membrane, the results are impressive. For the worst case loading of 5262 psi reflected pressure under an axial load of 1500 kip and #11 reinforcement, the results show an increased transverse load capacity of 715% when compared with the 735.9 psi reflected pressure case that did not develop tension membrane behavior. Figure 5-2 represents the interaction diagram of the column analyzed as part of the parametric study. Note that no design reduction factors are applied.

**P-I Diagram**

In this section, the load-impulse diagram of the 12 ft RC column is investigated for a constant axial load of 800 kip and 1500 kip. Figure 5-3 and 5-4 are the load-impulse diagram obtained including direct shear and flexural behavior from both modified and DSAS V3.0.

Both cases, 800 kip and 1500 kip axial load, demonstrated the benefit of the tension membrane action of the RC column. The modified DSAS yield a flexural load-impulse curve much higher than DSAS V3.0 which only includes flexural behavior. The results obtained also demonstrated in the dynamic response range of high explosive, the direct shear failure dominated the failure mechanism up to impulsive load response 7285 psi-msec and 7253 psi-msec respectively for 800 kip and 1500 kip axial load cases. To bring the RC column to fail in tension membrane without failing in direct shear, higher impulsive load are required. This may be of interest for a building to resist enhanced blast weapons such as thermobaric explosives.
Future Development/Recommendations

The analytical procedure and algorithm developed in this research to conduct a single degree of freedom analysis of RC column under blast induced transient and axial loads demonstrated considerable progress in the capability to conduct large deformation analysis. The following discussion identifies key development issues that shall be considered in the future to further develop the capacity of capturing the tension membrane behavior of reinforced concrete columns as well as better capturing large deformation behavior during the analysis.

The first recommendation is to conduct field experiments to validate the approach presented in this research. During the experiment, key issues other than tension membrane behavior shall be considered. The first issue of great interest is to capture and study the spalling and the capacity of the steel reinforcement cage to retain concrete from flying out and becoming hazards. This would provide better insight into what percentages of loads are actually transferred onto the steel once the concrete crushed and spalled and if they occur before or after transiting to tension membrane. The second would be to capture the damage occurring at the support to evaluate the capacity of the steel to carry tension membrane behavior and develop a reduction factor for the resistance function or steel capacity based on the findings. The third one would be to evaluate local damages that may occur to the steel rebar and develop a reduction factor if needed. Finally, a recommendation is made to capture the effect of support displacement versus fully fixed while undergoing tension membrane behavior in order to develop a more realistic model for DSAS.

The second recommendation is regarding software application of ABAQUS. It would be of great interest to develop a reinforced concrete model in ABAQUS/Explicit fit
for blast that actually models the spalling of concrete. Such a model would better account for the change of inertia forces due to loss of concrete mass.

The final recommendation is regarding the software applications of DSAS. Obviously, based on the first recommendations, the findings should be incorporated into DSAS as part of future studies conducted at CIPPS. However, one aspect that was not discussed is the capacity of capturing the effect of compression membrane behavior that was discussed in Chapter 2 that may have a significant effect on the column response. This effect was not observed during this research as it was not of interest but previous studies on reinforced concrete slabs demonstrated compression membranes may have significant effect on the section capacity and shall considered. A study on Timoshenko beam elements is currently underway at CIPPS that should allow capturing such effect in DSAS.

During this research, only distributed loads were investigated. However, the development of tension membrane behavior is also possible under different type of load such as point load. Jun and Hai, (2010) investigated experimentally the effect of compressive membrane behavior and tension membrane behavior in RC Beam subjected to point load. Figure 5-5 and Figure 5-6 are pictures obtained from Jun and Hai (2010) that demonstrate the experiment set-up. The experimental resistance function obtained and shown in Figure 5-7 clearly demonstrated the capacity of the RC beam to fully develop the tension membrane behavior. The maximum central displacement obtained is about 24 inches which is in the displacement range modified DSAS would yield for comparable section length. The failure mechanism at the end support observed during the experiment also corresponds to the failure mechanism of
tension membrane behavior embedded into DSAS based on the cable theory. The displacement profile of the entire beam at failure is not conclusive with the shape of the cable theory as parts of the RC beam/column remain un-cracked. This may be due to the fact this was more or less a static test and no spalling occurred as it is expected to occur when subjected to blast load. Therefore more investigation of RC beam/column subjected to impact loads is required to properly understand the displacement profile of the RC beam in the tension membrane range in order to develop a tension membrane resistance function algorithm to handle point loads due to impact.

The pictures shown in Figure 5-8 to Figure 5-11, were obtained from previous tests conducted by the Defense Threat Reduction Agency on RC columns. They show a reinforced concrete columns undergoing tension membrane behavior when subjected to a blast load. These pictures are good visual examples of fully developed catenary action and further justify the recommendations made in this section.

**Conclusion**

A new computational capability was developed for DSAS to address geometric instability and the possible transition into the tension membrane behavioral mode. This significantly extends the capability of DSAS to analyze reinforced concrete columns subjected to explosive load. However, further research and experiment on the matter is required. For example, it is assumed the structure would free the column of its axial load once it falls into tension membrane. However, this generally implies that a vertical displacement of the column supports would occur which needs to be capture in the dynamic analysis.

When hardened structure are to be designed to sustain large loading capacity and survive the loss of a column component, one shall consider the design of such columns.
as to obtain extra lateral capacity by developing a tension membrane behavior. This study demonstrated the significant lateral strength one may gain due to tension membrane behavior. Such lateral support, although the element may not carry axial load, continues to provide obstacle fragment, to secondary blast and secondary fragment as well as continuing to support any lateral load a column may be designed to support. However, the cost will quickly increase as larger diameter of steel rebar is going to be required and it is likely to make for an over designed reinforced concrete column and contradict the design code. However, because it is designed for tension membrane behavior and so the remaining of the structure may take on the axial load, the issue of sudden failure is not an issue in this case.

The effect of direct shear failures also plays a significant role as it was demonstrated during the validation and parametric study that the column is likely to fail in direct shear prior to reach full tension membrane capacity. However, the study showed important increase in flexural capacity due to tension membrane that proved to provide a significant flexural capacity compared with a column having no tension membrane capacity.

Finally, the overall study provided important improvements to rapidly and efficiently analyze reinforced concrete columns subjected to blast while yielding accurate values.
Figure 5-1. Twelve foot reinforced concrete column combined resistance function comparison.
Figure 5-2. Interaction diagram 12 ft RC column with different rebar sizes.
Figure 5-3. Load-Impulse diagram 12 ft RC column under 800 kips axial load.
Figure 5-4. Load-Impulse diagram 12 ft RC column under 1500 kips axial load.

Figure 5-5. Experimental set-up. [Reprinted with permission from Jun, Y., Hai, T.K. 2010. Progressive Collapse Resistance of RC Beam-Column Sub-assemblages. (Page 3, Figure 3). Proc. 3rd Int. Conf. on Design and Analysis of Protective Structure (DAPS2010), Singapore, 10-12 May, 2010.]
Figure 5-6. Failure mode of the section. [Reprinted with permission from Jun, Y., Hai, T.K. 2010. Progressive Collapse Resistance of RC Beam-Column Sub-assemblages. (Page 3, Figure 4). Proc. 3rd Int. Conf. on Design and Analysis of Protective Structure (DAPS2010), Singapore, 10-12 May, 2010.]

Figure 5-7. Experimental resistance function. [Reprinted with permission from Jun, Y., Hai, T.K. 2010. Progressive Collapse Resistance of RC Beam-Column Sub-assemblages. (Page 4, Figure 5). Proc. 3rd Int. Conf. on Design and Analysis of Protective Structure (DAPS2010), Singapore, 10-12 May, 2010.]
Figure 5-8. Reinforced concrete column undergoing tension membrane behavior picture 1. [Picture provided by the Defense Threat Reduction Agency]
Figure 5-9. Reinforced concrete column undergoing tension membrane behavior picture 2. [Picture provided by the Defense Threat Reduction Agency]
Figure 5-10. Reinforced concrete column undergoing tension membrane behavior picture 3. [Picture provided by the Defense Threat Reduction Agency]
Figure 5-11. Reinforced concrete column undergoing tension membrane behavior picture 4. [Picture provided by the Defense Threat Reduction Agency]
Table 5-1. Twelve foot reinforced concrete column analyzed with various longitudinal reinforcement size rebar.

<table>
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<th>Triangular Load Parameters</th>
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<th>#9 Longitudinal Reinforcement</th>
<th>#11 Longitudinal Reinforcement</th>
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<td>Incident reflected impulse (psi-msec)</td>
<td>Time duration (msec)</td>
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1 Tension membrane engaged 2Direct shear failure.
LIST OF REFERENCES

ACI 318-08, (2008). *Building Code Requirement for Structural Concrete (ACI 318-08) and Commentary (ACI 318R-08)*, American Concrete Institute, Farmington Hills, M.I.


BIOGRAPHICAL SKETCH

In June 2000, Dave Morency enrolled in the Canadian Regular Officer Training Plan (ROTP) where he attended the Royal Military College of Canada and graduated with a bachelor degree in Civil Engineering in 2005. He then served as a combat engineer officer and completed one tour of duty in Afghanistan. In 2008, he was selected by the Canadian army to return to school and complete graduate studies with a specialization in Force Protection. He was then accepted to attend the University of Florida where he studied under the direction of Professor Dr. Theodor Krauthammer as a structural engineer graduate student.