To my parents

A.C. Nagaraja Prasad & A.N. Kamala
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Abstract of Thesis Presented to the Graduate School of the University of Florida in Partial Fulfillment of the Requirements for the Degree of Master of Science

TWO-PORT ELECTROACOUSTIC MODEL OF A PIEZOELECTRIC COMPOSITE CIRCULAR PLATE

By

Suryanarayana A. N. Prasad

December 2002

Chairman: Dr. Mark Sheplak
Cochairman: Prof. Bhavani V. Sankar
Major Department: Department of Mechanical and Aerospace Engineering.

Accurate prediction of mode-shape and the deflection field of a circular piezoelectric composite transducer are important for the design of sensors and actuators. There is a need for a model, which lays emphasis on the physics of the problem and predicts the deflection field as a function of pressure and voltage loading. Such a theory would help in developing a non-dimensionalization procedure, which could be used to extract non-dimensional parameters for developing an optimization procedure for the design of a sensor or an actuator.

This thesis presents the development of such a theory that predicts the central deflection and mode-shape. Lumped element modeling (LEM) of the system, which is used to extract system parameters of an equivalent single degree of freedom (SDOF) system, is also discussed. The two-port network representation that is used to develop an equivalent circuit representation of the piezoelectric transduction is presented. Non-dimensionalization of the plate equations is also discussed in great detail.
CHAPTER 1
INTRODUCTION

Commonly used electroacoustic devices, such as microphones and headphones, use circular disk transducers that are piezoelectric composite plates. Recent devices, such as synthetic jet actuators, used in flow control applications, can also be driven by piezoelectric composite circular plate. Micro-fluidic pump drivers represent another relatively new application for these devices.

A design procedure, to find a set of system parameters for the optimal performance of these devices, helps to improve the performance of these devices and also widen its range of applications. Development of such a design procedure is possible only when the response of these devices to loading and change of system parameters is known. Although experimental techniques can be used to characterize piezoelectric transducers, analytical models that can predict the response of the transducers are helpful in understanding the effects of various parameters on the predicted response and also in optimizing the performance of the devices.

Determining the dynamics of a single degree of freedom (SDOF) system would be an easier task than determining the dynamic response of a corresponding complex system. Such a SDOF system would be a compact macromodel that provides physical insight and accurately captures the energy behavior and dependence on the material properties. This system is obtained by lumped element modeling (LEM) of the piezoelectric unimorph disk transducer. The lumped element parameters of the system are
obtained from the static deflection behavior of the system when subjected to pressure and voltage loading. This approximation will be valid in Fourier space and will simplify the problem of determining the complicated dynamics of the piezoelectric composite circular unimorph transducer into a static analysis of an axisymmetric piezoelectric multi-layered isotropic composite circular plate.

As the problem consists of various domains, a two-port network representation is used to model the transduction from the input port to the output port. The parameters obtained from the LEM of the plate are fed into the two-port network model of the device. This two-port model is used in developing an equivalent circuit of the device. The behavior of the device to loading is obtained from the transfer function of this equivalent circuit.

**Static displacement behavior of the piezoelectric unimorph transducer**

![Cross-sectional schematic of a clamped axisymmetric piezoelectric unimorph disk transducer](image)

Figure 1.1: Cross-sectional schematic of a clamped axisymmetric piezoelectric unimorph disk transducer

Figure 1.1 shows a cross-section of a clamped circular piezoelectric unimorph composite plate subjected to a uniform transverse pressure loading $P$ and/or a voltage $V$. A piezoceramic material of thickness $h_p$ and radius $R_i$ is bonded on top of a shim
material of outer radius \( R_2 \) and thickness \( h_y \). The loading creates a transverse displacement field \( w(r) \) and a radial displacement field \( u(r,z) \).

**Background**

Previous work in the area of piezoelectric composite circular plates focused on structures that are symmetrically layered about the neutral axis such as bimorph transducers.\(^6\) Because of midplane symmetry there is no bending-extension coupling in bimorph transducers and this simplifies the analysis. However, many of the devices cannot be manufactured in this manner (for example, most piezoelectric unimorph transducers fabricated through micro-electromechanical systems (MEMS) technology have geometry as shown in Figure 1.1). Hence there exists a need to extend the existing piezoelectric composite plate theory to the case of an axisymmetric multi-layered transversely isotropic piezoelectric composite circular plate.

Morris and Foster\(^4\) developed an optimization procedure for a piezoelectric bimorph micropump driver (the same geometry as in Figure 1.1) using finite element method (FEM) with the help of ANSYS\(^7\) software. They performed optimization of the micropump driver for both pinned and the fixed case by identifying non-dimensional \( \pi \) groups using the Buckingham theorem. They used a higher-order routine in ANSYS to accomplish this task. They have developed some empirical equations for optimal radius ratio and thickness ratio for a particular set of materials for a particular aspect ratio. They also discussed edge support effects and effect of bond layer.

Dobrucki and Pruchnicki\(^6\) formulated the problem of a piezoelectric axisymmetric bimorph and used FEM to solve the problem. They derived the equations that would determine the bending moment and extensional forces produced by the piezoelectric
material on application of an electric field. They used average elastic parameters for analyzing the composite plate. Use of the bimorph as a sensor was also discussed. They experimentally verified their results from the FEM solution. Verification of the theory with simpler geometry was also performed. They also have proved that on the rim of a clamped circular transducer the electric signal produced is zero.

Stavsky and Loewy\textsuperscript{8} solved numerically the dynamics of isotropic composite circular plates using Kirchoff’s plate theory. They found the vibrations of the composite plate to be analogous to the vibration of a homogenous shallow spherical shell. They also discussed effects due to material arrangement, radius, material and plate composition on frequency of vibration of the composite circular plate. They obtained a system of equations of the $6^{th}$ order. The solution for this system of equations can be expressed in terms of Bessel functions, the argument for which is determined from the characteristic equation of order 3. They also discussed numerical examples showing the effect that arises due to heterogeneity on vibration response of the composite to be significant.

Adelman and Stavsky\textsuperscript{9} formulated the problem of piezoelectric circular composite plates using Kirchoff’s plate theory. Static behavior and flexural-extensional vibratory response of metal-piezoceramic unimorphs and PZT-5H bimorphs possessing silver electrodes are solved numerically. Their formulation is identical to that of the formulation discussed in this thesis, except that they use variables $E_1$ and $E_2$ that relate the fictitious force/moments generated to the electric field applied instead of the comprehensive equation for fictitious forces that describe the piezoelectric transduction shown in Eq. (3.26) and Eq. (3.27). They also discussed numerical examples showing the effect of silver electrode on unimorph piezoelectric benders.
Chang and Du\textsuperscript{10} performed optimization of a unimorph disk transducer based on an electro-elastic theory assuming free boundary conditions, which is non-physical for most applications. They modified the existing Kirchoff’s plate theory by adding a term to account for the piezoelectric layer. They assumed that the electric field variation in the thickness direction could be represented as a quadratic function and the electric charge to be equal and opposite on the top and bottom electrodes of the piezoelectric layer.

Dumir et al.\textsuperscript{11} obtained a non-linear axisymmetric solution for the static and transient moderately large deflection of a laminated axisymmetric annular plate acted on by uniformly distributed ring loads by using first-order shear deformation theory. Effect of inplane inertia was neglected while the rotary inertia was considered. The material was treated to be orthotropic. They used a numeric technique called the Newmark-\(\beta\) scheme in order to solve the governing differential equations. They simplified the solution and verified the same with the solution from the classical plate theory.

**Thesis Layout**

Chapter 2 of this thesis presents a two-port, lumped-element model of an axisymmetric piezoelectric unimorph transducer with the geometry and loading described in Figure 1.1. In LEM, the individual components of a piezoelectric unimorph are modeled as elements of an equivalent electrical circuit using conjugate power variables. The synthesis of the two-port model required determination of the transverse static deflection field as a function of pressure and voltage loading.

In Chapter 3, classical laminated plate theory was used to derive the equations of equilibrium for circular laminated plates containing one or more piezoelectric layers. The equations were solved for a unimorph device wherein the diameter of the piezoelectric
layer was less than that of the shim \((R_1 < R_2)\). An exact analytical static solution of the displacement field of the axisymmetric piezoelectric unimorph is determined. The solution for annular plate obtained using the classical plate theory matches with the solution provided in the paper by Dumir et al.\(^{11}\)

Chapter 4 verifies the result obtained from Chapter 3 by theoretical means and by a Finite Element Model. Methods to estimate the model parameters are discussed and experimental verification is presented.

Chapter 5 discusses how the governing equations are used in the non-dimensionalization of the field variables, lumped element parameters and two-port network parameters. This proves to be a simpler and more comprehensive option.

The results corresponding to the work described in a Chapter are summarized at the end of the corresponding chapters. In addition, a summary of the main results is provided in the conclusions (Chapter 6). Future work and concurrent work is also discussed in Chapter 6.
CHAPTER 2
TWO-PORT NETWORK MODELING

The piezoelectric composite plate actuator represents a coupled electro-mechanical-acoustic system with frequency dependent properties determined by device dimensions and material properties. The analysis and design of such a coupled-domain transducer system is commonly performed using lumped element models.\textsuperscript{1} This is justified, because the prediction by LEM matches the actual value to within 2\%, as per Merhaut.\textsuperscript{12}

**Lumped Element Modeling**

The main assumption employed in LEM is that the characteristic length scales of the governing physical phenomena are much larger than the largest geometric dimension. For example, for the vibration of a piezoelectric plate, the bending wavelength and electromagnetic wavelength must be significantly larger than the device itself. If this assumption holds, then the temporal and spatial variations can be decoupled. This decoupling permits the governing partial differential equations of the distributed system to be “lumped” into a set of coupled ordinary differential equations through the solution of the static equations. The individual components of a piezoelectric unimorph are modeled as elements of an equivalent electrical circuit using two-port modeling.

**Two-Port Network Modeling – An Introduction**

Any linear conservative electroacoustic transduction can be modeled using the electrical analogy as a transformer or a gyrator with series and parallel impedances or
admittances on each of its ports. The transduction from one domain to another, which is a function of system parameters, is represented in terms of admittances and impedances in each domain.

Table 2.1: Conjugate power variables and corresponding dissipative and energy storage elements in various domains.

<table>
<thead>
<tr>
<th>Energy Domain</th>
<th>Effort Variable</th>
<th>Flow Variable</th>
<th>Energy Dissipater</th>
<th>Kinetic Energy Storage</th>
<th>Potential Energy Storage</th>
<th>Displacement</th>
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</thead>
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<tr>
<td>Mechanical</td>
<td>Force</td>
<td>Velocity</td>
<td>Dashpot</td>
<td>Mass</td>
<td>Spring</td>
<td>Displacement</td>
</tr>
<tr>
<td>Acoustical</td>
<td>Pressure</td>
<td>Volume Velocity</td>
<td>Vent</td>
<td>Acoustic Mass</td>
<td>Cavity</td>
<td>Volume Displacement</td>
</tr>
<tr>
<td>Electrical</td>
<td>Voltage</td>
<td>Current</td>
<td>Resistor</td>
<td>Inductor</td>
<td>Capacitor</td>
<td>Electric Charge</td>
</tr>
</tbody>
</table>

Resistors are used to represent any dissipative element. Dissipative elements in other domains are shown in the fourth column of Table 2.1. Inductors and capacitors are used to represent elements that store generalized kinetic energy and potential energy respectively. Corresponding elements in other domains are shown in fifth and sixth column of Table 2.1. The conjugate power variables, the effort and flow, are identified in each of the domains as shown in the second and third column of Table 2.1. The product of the conjugate power variables is a measure of power. In impedance analogy, elements sharing common flow are connected in series while elements sharing a common effort are connected in parallel.
Two-Port Model of Piezoelectric Transduction

A piezoelectric transducer converts electric energy into strain that is realized as a displacement in the mechanical domain. Usually a piezoelectric transduction is represented in tensor form as per IEEE standards as shown in Eq. (2.1) and Eq. (2.2).

\[ S = s^E T + dE, \]  \hspace{1cm} (2.1)

and

\[ D = dT + \varepsilon^T E, \]  \hspace{1cm} (2.2)

where \( D \) is the dielectric displacement in \([C/m^2]\), \( T \) is the stress in \([Pa]\), \( S \) is the strain, \( E \) is the electric field in \([V/m]\), \( \varepsilon \) is the permittivity in \([C/Vm]\), \( s \) is the compliance in \([1/Pa]\) and \( d \) is the piezoelectric coefficient in \([C/N]\).

A piezoelectric material responds with a strain field not only due to application of stress but also due to application of electric field. An application of stress creates a charge (due to piezoelectric transduction) in addition to the charge created due to the application of voltage across the piezoelectric (a dielectric medium).

Two-Port Model of a 1-D Piezoelectric

In case of a 1-D piezoelectric, the force \( F \) and voltage \( V \) act only in the 3-direction as shown in Figure 2.1. Application of the force not only gives rise to a deflection \( x \) but also creates a polarization represented by an electrical charge \( q \);
\[ x = C_{MS} F \quad (2.3) \]

and

\[ q = d F . \quad (2.4) \]

In the above equations, \( C_{MS} \) and \( d \) are the short-circuit mechanical compliance and effective mechanical piezoelectric coefficient (that is responsible for a strain in 3-direction due to application of electric field in the 3-direction) of the piezoelectric material respectively and are given by

\[ C_{MS} = \frac{x|_{V=0}}{F} \quad (2.5) \]

and

\[ d = \frac{x|_{P=0}}{V} . \quad (2.6) \]

Application of voltage creates a deformation \( x \) in addition to creating a polarization represented by an electric charge \( q \);

\[ x = d V \quad (2.7) \]

and

\[ q = C_{EF} V , \quad (2.8) \]

\( C_{EF} \) is the electrical free capacitance of the 1-D piezoelectric that is given by

\[ C_{EF} = \frac{k \varepsilon_0 A}{h_p} , \quad (2.9) \]

where \( k \) is the dielectric constant of the piezoelectric in the 3-direction due to application of an electric field in the 3-direction and \( \varepsilon_0 \) is permittivity of free space.

The transduction in the case of a 1-D piezoelectric in the static case, when subjected to both voltage and force load, is found by superimposing Eqs. (2.3), (2.4), (2.7) and (2.8);
Two-Port Electroacoustic Model of a Piezoelectric Composite Plate

In the case of a piezoelectric unimorph disc, application of voltage creates bending and not an extension as in the case of the 1-D piezoelectric described above. Also the focus in this thesis is oriented towards an electroacoustic model rather than an electro-mechanical model of the piezoelectric unimorph. Hence integration over the surface area of the unimorph disc needs to be performed to extend the electromechanical model of the 1-D piezoelectric to a piezoelectric unimorph disc transducer.

In the acoustic domain, a volume displacement is created in a piezoelectric transducer not only due to application of pressure but also due to application of voltage. Application of pressure creates a charge separation across the piezoelectric layer (due to piezoelectric transduction) in addition to the charge separation created due to the application of voltage. Hence the transduction of an axisymmetric piezoelectric unimorph disk in static case is expressed as

\[
\begin{bmatrix} \Delta \text{Vol} \\ q \end{bmatrix} = \begin{bmatrix} C_{AS} & d_A \\ d_A & C_{EF} \end{bmatrix} \begin{bmatrix} P \\ V \end{bmatrix},
\]

(2.11)

where \(\Delta \text{Vol}\) is the volume displaced by the plate due to the application of pressure \(P\) and voltage \(V\);

\[
\Delta \text{Vol} = \int_0^{R_o} 2\pi r w(r) dr,
\]

(2.12)

and \(C_{AS}\) is the short-circuit acoustic compliance of the plate. Expression for the acoustic compliance is extracted from the equivalent SDOF system by equating the strain energy of the actual system to the potential energy of the SDOF system in the following manner:
\[ C_{AS} = \frac{\Delta Vol}{P} \bigg|_{V=0} = \frac{\int_{0}^{R} w(r) dr}{P}. \] (2.13)

In Eq. (2.11), \( d_A \) is the effective acoustic piezoelectric coefficient, which is given by

\[ d_A = \frac{\Delta Vol}{V} \bigg|_{P=0} = \frac{\int_{0}^{R} w(r) dr}{V}. \] (2.14)

**Equivalent Circuit Representation and Parameter Extraction**

Assuming time harmonic function and differentiating both sides of Eq. (2.11) with respect to time yields the expression for conjugate power variable at low frequencies;

\[
\begin{bmatrix} Q \\ i \end{bmatrix} = \begin{bmatrix} j\omega C_{AS} & j\omega d_A \\ j\omega d_A & j\omega C_{EF} \end{bmatrix} \begin{bmatrix} P \\ V \end{bmatrix},
\] (2.15)

where \( \omega \) is the radian frequency. The equivalent circuit of the piezoelectric unimorph at low frequencies takes the form shown in Figure 2.2.

![Figure 2.2: Equivalent two-port circuit representation of the piezoelectric unimorph at low frequencies.](image)

In Figure 2.2, \( Q \) is the volume velocity and \( i \) is the current. The parameter \( \phi_A \) appearing as the transformer turns ratio in the equivalent circuit representation is the
electroacoustic transduction coefficient, which is given by the ratio of the effective acoustic piezoelectric coefficient and the short-circuit acoustic compliance of the plate:

$$\phi_A = \frac{d_A}{C_{AS}}. \quad (2.16)$$

When unloaded, the equivalent circuit representation in Figure 2.2 is represented as shown in Figure 2.3. The energy stored in the two circuit elements shown in Figure 2.3 should equal the energy stored in the circuit element shown in Figure 2.4. Hence the electroacoustic energy-coupling factor, which is a quantity that describes the fraction of energy converted from electric domain to the acoustic domain, is

$$K^2 = \frac{\text{P.E. stored in } \phi_A^2 C_{AS}}{\text{P.E. stored in } C_{EF}} = \frac{1}{2} \frac{\phi_A^2 C_{AS} V^2}{C_{EF}} = \frac{\phi_A^2 C_{AS}}{C_{EF} C_{EF}} = \frac{d_A^2}{C_{AS} C_{EF}}. \quad (2.17)$$

![Figure 2.3](image1.png)

Figure 2.3: Equivalent circuit representation in the electric domain of the piezoelectric unimorph.

![Figure 2.4](image2.png)

Figure 2.4: Equivalent circuit representation in the electric domain of the piezoelectric unimorph when decoupled from the acoustic domain.
The blocked electrical capacitance at the electrical side is obtained by equating the energy storage in Figure 2.3 and Figure 2.4:

\[ \frac{1}{2} C_{EB} V^2 + \frac{1}{2} \phi_A^2 C_{AS} V^2 = \frac{1}{2} C_{EF} V^2. \]  

(2.18)

Substituting Eq. (2.17) in Eq. (2.18), we obtain

\[ \frac{1}{2} C_{EB} V^2 + \frac{1}{2} C_{EF} V^2 K^2 = \frac{1}{2} C_{EF} V^2. \]  

(2.19)

which simplifies to

\[ C_{EB} = C_{EF} (1 - K^2). \]  

(2.20)

Figure 2.5: Another equivalent two-port circuit representation of the piezoelectric unimorph at low frequencies.

Transduction mechanism in the case of a piezoelectric unimorph at low frequencies has another equivalent circuit representation as shown in Figure 2.5. The parameter \( \phi_A^i \) appearing as the transformer turns ratio in the equivalent circuit representation shown in Figure 2.5 is the acousto-electric transduction coefficient, which is given by the ratio of the effective acoustic piezoelectric coefficient of the plate and free electrical capacitance of the piezoelectric layer;
\[ \phi'_A = \frac{d_A}{C_{EF}}. \]  

(2.21)

When the electrical port in the equivalent circuit representation shown in Figure 2.5 is shorted, the equivalent circuit simplifies to the circuit shown in Figure 2.6. The energy stored in the two circuit elements shown in Figure 2.6 should equal the energy stored in the circuit element shown in Figure 2.7. Hence the energy-coupling factor is derived as

\[ K^2 = \frac{\text{P.E. stored in } \phi'^2_C E_F}{\text{P.E. stored in } C_{AS}} = \frac{1}{2} \frac{\phi'^2_C E_F P^2}{C_{AS}^2} = \frac{\phi'^2_C E_F}{C_{AS} C_{EF}} = \frac{d_A^2}{C_{AS} C_{EF}}. \]  

(2.22)

![Figure 2.6: Equivalent circuit representation in the acoustic domain of the piezoelectric unimorph.](image)

![Figure 2.7: Equivalent circuit representation in the acoustic domain of the piezoelectric unimorph when electric behavior is decoupled.](image)

The open acoustic compliance at the acoustic port is obtained by equating the energy storage in Figure 2.6 and Figure 2.7;
\[ \frac{1}{2} C_{EF} P^2 \frac{\phi_A^2}{2} + \frac{1}{2} C_{AO} P^2 = \frac{1}{2} C_{AS} P^2. \]  
(2.23)

Substituting Eq. (2.22) in Eq. (2.23), we obtain

\[ \frac{1}{2} C_{AS} P^2 K^2 + \frac{1}{2} C_{AO} P^2 = \frac{1}{2} C_{AS} P^2, \]  
(2.24)

which simplifies to

\[ C_{AO} = C_{AS} \left(1 - K^2\right). \]  
(2.25)

For higher frequencies the mass of the disk becomes important and leads to resonance. The circuit in Figure 2.8 will describe the system at higher frequencies, up to the fundamental natural frequency.

![Equivalent two-port circuit representation of axisymmetric piezoelectric unimorph disk at frequencies comparable to that of the primary resonance](image)

The acoustic mass \( M_A \) of the equivalent SDOF system is extracted from the deflection response of the system by equating the kinetic energy stored in the SDOF system to that of the actual system in the following manner:

\[ M_A = \frac{2\pi}{(\Delta Vol)^2} \int_0^{R_1} \rho_A w(r)^2 rdr, \]  
(2.26)

where \( \rho_A \) is the areal density of the piezoelectric composite plate. For the geometry described in Figure 1.1, the value of the areal density remains a constant from zero-radius
to the end of the inner composite region. There is a change in value of the areal density at the interface and then remains a constant until the clamped edge. The volume displaced due to application of a load remains a constant independent of the region.

The short-circuit natural frequency \( f_s \) in Hz of the system is given by

\[
 f_s = \frac{1}{2\pi \sqrt{C_{AS}M_A}}, \quad (2.27)
\]

while the open-circuit resonant frequency \( f_o \) is given by

\[
 f_o = \frac{1}{2\pi \sqrt{C_{AO}M_A}}. \quad (2.28)
\]

By substituting Eqs. (2.25) and (2.27) in Eq. (2.28), we obtain the relationship between the resonant frequencies as

\[
 f_o = \frac{1}{2\pi \sqrt{C_{AS} \left(1-K^2\right)M_A}} = \frac{f_s}{\sqrt{\left(1-K^2\right)}}, \quad (2.29)
\]

which on simplification yields

\[
 f_o \sqrt{\left(1-K^2\right)} = f_s, \quad (2.30)
\]

The deflection field of the piezoelectric composite plate, due to unit-applied pressure and due to unit-applied voltage, is required to determine \( C_{AS}, d_A, \) and \( K \). The deflection field is obtained by analyzing the mechanical behavior of the piezoelectric unimorph, which is modeled and discussed in Chapter 3 of the thesis.
CHAPTER 3
MECHANICAL BEHAVIOR OF THE PIEZOELECTRIC COMPOSITE PLATE

For developing a two-port electroacoustic model of the piezoelectric composite plate, the quantities defined in Eq. (2.13), (2.14) and Eq. (2.26), need to be determined. These quantities depend on the vertical deflection that is determined by analysis of the mechanical behavior of the axisymmetric piezoelectric composite plate.

Problem Formulation

Problem formulation is based on the classical Kirchoff’s plate theory. The fact that one of the layers is a piezoelectric is accounted for in the constitutive relation by adding an additional strain term due to the piezoelectric layer.

Assumptions

The assumptions made in developing a linear small deflection plate theory with piezoelectric effect are as follows:

- The plate (shown in Figure 3.1), is assumed to be in a state of plane stress normal to the z-axis. In other words, the normal stress $\sigma_{zz}$ and the shear stress $\tau_{rz}$ are approximately equal to zero.
Figure 3.1: An axisymmetric multi-layered transversely isotropic circular plate with pressure load, radial load and a moment. All loads shown are considered positive.

- The shear stresses $\tau_{\theta\theta}$ and $\tau_{r\theta}$ also vanishes due to the assumed axisymmetric nature of the problem. Stress measures, corresponding to the plane stress case, (i.e.) $\sigma_{rr}$ and $\sigma_{\theta\theta}$ exist. Only, shear stress in the transverse direction exists (i.e. $\tau_{rz} \neq 0$). Figure 3.2 delineates the active stress resultants on a multi-layered composite circular plate acted on by a pressure load $P(r)$.

Figure 3.2: Cross-sectional view of the plate shown in Figure 3.1, showing the sign conventions and labels used.
The active stresses on the circular plate are shown in Figure 3.3. Figure 3.4 delineates the force, moment and shear force resultants acting on an infinitesimal element of a multi-layered isotropic composite circular plate.

Figure 3.3: Top view of the plate shown in Figure 3.1

Figure 3.4: Enlarged isometric view of the element shown in Figure 3.3 and Figure 3.2 with generalized forces acting on it.
In Figure 3.4 and Figure 3.5, $N_r$ and $N_\theta$ are the force resultants in radial and circumferential directions respectively:

$$N_r = \int_{z_1}^{z_2} \sigma_{rr} \, dz$$  \hspace{1cm} (3.1)$$

and

$$N_\theta = \int_{z_1}^{z_2} \sigma_{\theta\theta} \, dz .$$  \hspace{1cm} (3.2)$$

The moment resultants in radial and circumferential directions $M_r$ and $M_\theta$ are given by

$$M_r = \int_{z_1}^{z_2} \sigma_{rr} r \, dz$$  \hspace{1cm} (3.3)$$

and

$$M_\theta = \int_{z_1}^{z_2} \sigma_{\theta\theta} r \, dz .$$  \hspace{1cm} (3.4)$$

$Q_r$ is the shear force resultant given by
\[ Q_r = \int_{z_1}^{z_2} \tau_{r,z} \, dz \, . \quad (3.5) \]

**Equilibrium Equations**

For the case of the axisymmetric multi-layered composite, the field variables \( u_0 \) and \( \theta \) are functions of radius alone (due to axisymmetry). Therefore, the partial differentials equal the corresponding total differentials. Taking balance of forces in the radial direction, of the projection of the element (shown in Figure 3.4) in the \( r - \theta \) plane, we obtain

\[ \frac{dN_r}{dr} + \frac{1}{r} (N_r - N_\theta) = 0 \, . \quad (3.6) \]

Taking moment about the line tangent to \( r + dr \) of the element shown in Figure 3.4, we obtain

\[ \frac{dM_r}{dr} - Q_r + \frac{1}{r} (M_r - M_\theta) = 0 \, . \quad (3.7) \]

Taking balance of the forces shown in Figure 3.5 in the vertical direction, we obtain

\[ \frac{dQ_r}{dr} + P(r) + \frac{Q_r}{r} = 0 \, . \quad (3.8) \]

The moment resultants are expressed in matrix form as

\[ \begin{bmatrix} M_r \\ M_\theta \end{bmatrix} = \int_{z_1}^{z_2} \begin{bmatrix} \sigma_{r} \\ \sigma_{\theta} \end{bmatrix} \, dz \, . \quad (3.9) \]

where moments/moment resultants causing tension in top surface are considered positive.

The force resultants are expressed in matrix form as

\[ \begin{bmatrix} N_r \\ N_\theta \end{bmatrix} = \int_{z_1}^{z_2} \begin{bmatrix} \sigma_{r} \\ \sigma_{\theta} \end{bmatrix} \, dz \, . \quad (3.10) \]

where radial forces/force resultants causing tension are considered positive.
Equations (3.6) - (3.8) are the equations of equilibrium\textsuperscript{14,15,16} of an axisymmetric multi-layered composite plate. The pressure load acting on the bottom surface is uniform. Therefore, these equations are suitably modified to yield the equilibrium equations of an axisymmetric piezoelectric unimorph. The resulting expressions are given by

\[
\frac{dN_r}{dr} + \frac{(N_r - N_0)}{r} = 0, \tag{3.11}
\]

\[
\frac{dM_r}{dr} + \frac{(M_r - M_0)}{r} = -Q_r, \tag{3.12}
\]

and

\[
\frac{dQ_r}{dr} + \frac{Q_r}{r} + P = 0. \tag{3.13}
\]

**Strain–Displacement Relationships**

The field variables involved in the problem are the radial deflection in the reference plane \( u_0 \) and slope \( \theta \) respectively.

![Figure 3.6: Undeformed and deformed shape of an element of circumferential width \( \phi \) and length \( dr \) at a radial distance \( r \) from the center in the reference plane.](image)

The increment shown in Figure 3.6 contains a total derivative because of the fact that the reference plane displacements are just a function of radius due to axisymmetry,
(i.e.) \( u_0 = u_0(r) \) and \( \theta = \theta(r) \). The strains are obtained from the field variables in the following manner.

The radial strain measure for Kirchoff’s plate theory is given by

\[
\varepsilon_{rr} = \varepsilon_{rr}^0 + \kappa_r,
\]

where \( \varepsilon_{rr}^0 \) and \( \kappa_r \) are the radial strain in the reference plane and curvature in the radial direction, respectively;

\[
\varepsilon_{rr}^0 = \frac{u_0 + du_0 - u_0}{r + dr - r} = \frac{du_0}{dr}
\]

and

\[
\kappa_r = -\frac{d^2w}{dr^2} = -\frac{d\theta}{dr}.
\]

The circumferential strain measure for Kirchoff’s plate theory is given by

\[
\varepsilon_{\theta\theta} = \varepsilon_{\theta\theta}^0 + \kappa_\theta,
\]

where \( \varepsilon_{\theta\theta}^0 \) and \( \kappa_\theta \) are the circumferential strain in the reference plane and curvature in the circumferential direction respectively and are obtained by taking balance of forces acting on the element shown in Figure 3.6;

\[
\varepsilon_{\theta\theta}^0 = \frac{(r + u_0)\theta - r\theta}{r\theta} = \frac{u_0}{r}
\]

and

\[
\kappa_\theta = -\frac{1}{r} \frac{dw}{dr} = -\frac{\theta}{r}.
\]

Constitutive Equations

For an elastic plate, the stress-strain relations\(^{14,15}\) are given by
\[
\begin{align*}
\begin{bmatrix}
\sigma_{rr} \\
\sigma_{00}
\end{bmatrix} &= 
\begin{bmatrix} Q_{11} & Q_{12} \\
Q_{21} & Q_{11}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{rr} \\
\varepsilon_{00}
\end{bmatrix}, \\
\end{align*}
\tag{3.20}
\]

where

\[
[Q] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu \\
\nu & 1 \end{bmatrix}, \\
\tag{3.21}
\]

\(E\) is the Young’s modulus and \(\nu\) is the Poisson’s ratio of the layer.

From Chapter 2, it can be understood that Eq. (3.22) represents the inverse constitutive relation of a piezoelectric material as per IEEE standards.

\[
S = s E T + dE \\
\tag{3.22}
\]

Thus for an axisymmetric piezoelectric composite plate discussed above, the constitutive relations are given by

\[
\begin{align*}
\begin{bmatrix}
\sigma_{rr} \\
\sigma_{00}
\end{bmatrix} &= [Q]
\begin{bmatrix}
\varepsilon_{rr} \\
\varepsilon_{00}
\end{bmatrix} + z
\begin{bmatrix} \kappa_r \\
\kappa_0 \end{bmatrix} - E_{33}
\begin{bmatrix} d_{31} \\
d_{31} \end{bmatrix}, \\
\end{align*}
\tag{3.23}
\]

where \(E_{33}\) is the uniform electric field across the thickness of the piezoelectric layer and \(d_{31}\) is the piezoelectric coefficient in the plane perpendicular to the direction of application of the electric field. In the above expression, number 3 represents the \(z\)-direction and number 1 represents the radial direction.

The plate constitutive equations are obtained by integrating the constitutive equations of the individual layers through the thickness and are given by

\[
\begin{align*}
\begin{bmatrix}
N_r \\
N_0
\end{bmatrix} &= [A][\varepsilon^0] + [B][\kappa] - \begin{bmatrix} N^p_r \\
N^p_0
\end{bmatrix}, \\
\end{align*}
\tag{3.24}
\]

and
\[
\begin{bmatrix}
M_r \\
M_0
\end{bmatrix} = [B][\varepsilon^0] + [D][\kappa] - \begin{bmatrix}
M_r^p \\
M_0^p
\end{bmatrix},
\]

(3.25)

where \( [A] = \int_{z_1}^{z_2} [Q] dz \) is the extensional rigidity matrix, \( [B] = \int_{z_1}^{z_2} [Q] z dz \) is the flexural-extensional coupling matrix and \( [D] = \int_{z_1}^{z_2} [Q] z^2 dz \) is the flexural rigidity matrix. The flexural-extensional coupling matrix is zero, if the material layers are symmetrical about the reference plane (For example, a piezoelectric bimorph). The force resultants, \( N_r^p \) and \( N_0^p \), generated internally due to the application of \( E_{33} \) across the piezoelectric layer are given by

\[
\begin{bmatrix}
N_r^p \\
N_0^p
\end{bmatrix} = \int_{z_1}^{z_2} E_{33} \begin{bmatrix}
Q_{11} & Q_{12} \\
Q_{12} & Q_{11}
\end{bmatrix} \begin{bmatrix} d_{31} \\
\\
\end{bmatrix} dz.
\]

(3.26)

The moment resultants, \( M_r^p \) and \( M_0^p \), generated internally due to the application of \( E_{33} \) across the piezoelectric layer are given by

\[
\begin{bmatrix}
M_r^p \\
M_0^p
\end{bmatrix} = \int_{z_1}^{z_2} E_{33} \begin{bmatrix}
Q_{11} & Q_{12} \\
Q_{12} & Q_{11}
\end{bmatrix} \begin{bmatrix} d_{31} \\
\\
\end{bmatrix} zdz.
\]

(3.27)

From the form of Eq. (3.26) and Eq. (3.27), we can infer that the material is transversely isotropic;

\[
N_r^p = N_0^p = N^p
\]

(3.28)

and

\[
M_r^p = M_0^p = M^p.
\]

(3.29)
Substituting the strain-displacement relationships and curvature-displacement relationships shown in Eqs. (3.14) – (3.19) in the plate constitutive relationships shown in Eqs. (3.24) and (3.25), we obtain

\[
\begin{bmatrix}
N_r
\end{bmatrix} = \begin{bmatrix}
A_{11} \frac{du_0(r)}{dr} + A_{12} \frac{u_0(r)}{r} - B_{11} \frac{d\theta(r)}{dr} - B_{12} \frac{\theta(r)}{r} \\
A_{12} \frac{du_0(r)}{dr} + A_{11} \frac{u_0(r)}{r} - B_{12} \frac{d\theta(r)}{dr} - B_{11} \frac{\theta(r)}{r}
\end{bmatrix} \begin{bmatrix}
N_r^p
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
M_r
\end{bmatrix} = \begin{bmatrix}
B_{11} \frac{du_0(r)}{dr} + B_{12} \frac{u_0(r)}{r} - D_{11} \frac{d\theta(r)}{dr} - D_{12} \frac{\theta(r)}{r} \\
B_{12} \frac{du_0(r)}{dr} + B_{11} \frac{u_0(r)}{r} - D_{12} \frac{d\theta(r)}{dr} - D_{11} \frac{\theta(r)}{r}
\end{bmatrix} \begin{bmatrix}
M_r^p
\end{bmatrix}
\]

**Governing Differential Equations**

Equations (3.30) and (3.31) are substituted in the plate equations of equilibrium shown in Eqs. (3.11) – (3.13). The field variables in the resulting expression, are decoupled (see Appendix A) by introducing the variables \(\alpha\) and \(D_{11}'\) defined as

\[
\alpha = \frac{B_{11}}{A_{11}}
\]

and

\[
D_{11}' = \left(D_{11} - \frac{B_{11}^2}{A_{11}}\right)
\]

Assuming that the electric field is uniform and the material is transversely isotropic is used to show that the internal force and moment resultants, generated by the piezo, has a constant value all along the reference plane. The resulting expression is simplified to yield the governing differential equations, which are
\[
\frac{d^2 \theta (r)}{dr^2} + \frac{1}{r} \frac{d \theta (r)}{dr} - \frac{\theta (r)}{r^2} = \frac{Pr}{2D_1},
\]  
(3.34)

and

\[
\frac{d^2 u_0(r)}{dr^2} + \frac{1}{r} \frac{d u_0(r)}{dr} - \frac{u_0(r)}{r^2} = \frac{Pr \alpha}{2D_1}.
\]  
(3.35)

**General Solution**

The general solution to the above set of differential equations is determined as

\[
u_0 = a_1 r + \frac{a_2}{r} - \frac{\alpha}{D_{11}} \left( \frac{Pr^3}{16} \right),
\]  
(3.36)

and

\[
\theta = b_1 r + \frac{b_2}{r} - \frac{1}{D_{11}^*} \left( \frac{Pr^3}{16} \right),
\]  
(3.37)

where \(a_1, a_2, b_1\) and \(b_2\) are constants that are determined by applying the boundary conditions. A detailed derivation of this solution is shown in Appendix A.

**The Problem of Piezoelectric Unimorph Disk Transducer**

The problem of circular piezoelectric unimorph disk transducer shown in Figure 1.1 is solved by combining the solution for deflection field of an annular plate of inner radius \(R_1\) and outer radius \(R_2\), with the solution for deflection field of the inner composite plate of radius \(R_1\). To differentiate between the constants involved in the solutions in the central composite region and outer annular region, symbols \(^{(1)}\) and \(^{(2)}\) are used to denote inner and outer regions respectively.

**Central Composite Plate**

In the central composite plate the central deflections of the plate is bounded:
and

\[ u_0(0) < \infty \] (3.38)

and

\[ \theta(0) < \infty. \] (3.39)

Substituting Eq. (3.38) and Eq. (3.39) in the general solution, we obtain

\[ a_2^{(1)} = 0 \] (3.40)

and

\[ b_2^{(1)} = 0. \] (3.41)

Therefore the deflections are given by

\[ u_0^{(1)}(r) = a_1^{(1)} r - \frac{\alpha}{D_{11}^{(1)}} \left( \frac{Pr^3}{16} \right) \] (3.42)

and

\[ \theta^{(1)}(r) = b_1^{(1)} r - \frac{1}{D_{11}^{(1)}} \left( \frac{Pr^3}{16} \right). \] (3.43)

In matrix form, Eq. (3.42) and Eq. (3.43) is written as

\[
\begin{bmatrix}
  u_0^{(1)}(r) \\
  \theta^{(1)}(r)
\end{bmatrix}
= r
\begin{bmatrix}
  a_1^{(1)} \\
  b_1^{(1)}
\end{bmatrix}
- \frac{Pr^3}{16D_{11}^{(1)}} \begin{bmatrix}
  \alpha \\
  1
\end{bmatrix}.
\] (3.44)

From Eq. (3.44), we obtain

\[
\begin{bmatrix}
  \frac{du_0^{(1)}(r)}{dr} \\
  \frac{d\theta^{(1)}(r)}{dr}
\end{bmatrix}
= \begin{bmatrix}
  a_1^{(1)} \\
  b_1^{(1)}
\end{bmatrix}
- \frac{3Pr^2}{16D_{11}^{(1)}} \begin{bmatrix}
  \alpha \\
  1
\end{bmatrix}.
\] (3.45)

Substituting Eqs. (3.44) - (3.45) in Eqs. (3.30) - (3.31), we obtain the following expression for generalized force resultants in the radial direction:
\[
\begin{bmatrix}
N_r^{(i)}(r) \\
M_r^{(i)}(r)
\end{bmatrix} = \begin{bmatrix}
A_{11}^{(i)} + A_{12}^{(i)} & -B_{11}^{(i)} - B_{12}^{(i)} \\
B_{11}^{(i)} + B_{12}^{(i)} & -D_{11}^{(i)} - D_{12}^{(i)}
\end{bmatrix}\begin{bmatrix}
a_1^{(i)} \\
b_1^{(i)}
\end{bmatrix} + \frac{Pr^2}{16D_{11}^{(i)}} \begin{bmatrix}
3B_{11}^{(i)} + B_{12}^{(i)} - 3A_{1}^{(i)} \alpha - A_{12}^{(i)} \alpha \\
3D_{11}^{(i)} + D_{12}^{(i)} - 3\alpha B_{11}^{(i)} - \alpha B_{12}^{(i)}
\end{bmatrix} - \begin{bmatrix}
N^p \\
M^p
\end{bmatrix}.
\]
(3.46)

Substituting Eqs. (3.32) - (3.33) in Eq. (3.46), we obtain

\[
\begin{bmatrix}
N_r^{(i)}(r) \\
M_r^{(i)}(r)
\end{bmatrix} = \begin{bmatrix}
(A_{11}^{(i)} + A_{12}^{(i)}) & -(B_{11}^{(i)} + B_{12}^{(i)}) \\
(B_{11}^{(i)} + B_{12}^{(i)}) & -(D_{11}^{(i)} + D_{12}^{(i)})
\end{bmatrix}\begin{bmatrix}
a_1^{(i)} \\
b_1^{(i)}
\end{bmatrix} + \frac{Pr^2}{16D_{11}^{(i)}} \begin{bmatrix}
B_{12}^{(i)} - A_{12}^{(i)} \alpha \\
-\alpha B_{12}^{(i)} + 3D_{11}^{(i)} + D_{12}^{(i)}
\end{bmatrix} - \begin{bmatrix}
N^p \\
M^p
\end{bmatrix}.
\]
(3.47)

At the interface, the deflections are given by

\[
\begin{bmatrix}
u_0^{(i)}(R_i) \\
\theta^{(i)}(R_i)
\end{bmatrix} = R_i\begin{bmatrix}
a_1^{(i)} \\
b_1^{(i)}
\end{bmatrix} - \frac{PR_1^3}{16D_{11}^{(i)}}\begin{bmatrix}
\alpha
\end{bmatrix}
\]
(3.48)

and the generalized force resultants are given by

\[
\begin{bmatrix}
N_r^{(i)}(R_1) \\
M_r^{(i)}(R_1)
\end{bmatrix} = \begin{bmatrix}
(A_{11}^{(i)} + A_{12}^{(i)}) & -(B_{11}^{(i)} + B_{12}^{(i)}) \\
(B_{11}^{(i)} + B_{12}^{(i)}) & -(D_{11}^{(i)} + D_{12}^{(i)})
\end{bmatrix}\begin{bmatrix}
a_1^{(i)} \\
b_1^{(i)}
\end{bmatrix} + \frac{PR_1^2}{16D_{11}^{(i)}} \begin{bmatrix}
B_{12}^{(i)} - A_{12}^{(i)} \alpha \\
-\alpha B_{12}^{(i)} + 3D_{11}^{(i)} + D_{12}^{(i)}
\end{bmatrix} - \begin{bmatrix}
N^p \\
M^p
\end{bmatrix}.
\]
(3.49)

**Outer Annular Plate**

For the outer annular plate, \([B]\) vanishes because of the symmetry about its neutral axis, which in turn implies that \(D_{11}^{(2)} = D_{12}^{(2)}\), and \(\alpha = 0\). As the outer annular plate is isotropic \(D_{12}^{(2)} = \nu D_{11}^{(2)}\) and \(A_{12}^{(2)} = \nu A_{11}^{(2)}\).

The solution in the annular region is obtained from the general solution by applying fixed condition at the clamped edge and interface matching condition at the inner edge. The fixed condition at the clamped edge is applied by making

\[
u_0(R_2) = 0
\]
(3.50)

and

\[
\theta(R_2) = 0.
\]
(3.51)
Substituting Eqs. (3.50) and (3.51) in the general solution, we obtain

\[ a_2^{(2)} = -a_1^{(2)} R_2^2 \]  

and

\[ b_2^{(2)} = -b_1^{(2)} R_2^2 - \frac{PR_2^4}{16D_{11}^{(2)}}. \]  

Substituting back Eqs. (3.52) and (3.53) into the general solution, we obtain

\[
\begin{bmatrix}
  u_0^{(2)}(r) \\
  \theta^{(2)}(r)
\end{bmatrix} = \begin{bmatrix} r = R_2^2 \\ r \end{bmatrix} \begin{bmatrix} a_1^{(2)} \\ b_1^{(2)} \end{bmatrix} - \frac{P}{16D_{11}^{(2)}} \begin{bmatrix} 0 \\ r^3 - \frac{R_2^4}{r} \end{bmatrix}.
\]  

(3.54)

Taking the first derivative of Eq. (3.54) with respect to the radius, we obtain

\[
\begin{bmatrix}
  \frac{d}{dr} u_0^{(2)}(r) \\
  \frac{d}{dr} \theta^{(2)}(r)
\end{bmatrix} = \begin{bmatrix} 1 + \frac{R_2^2}{r^2} \\ \frac{a_1^{(2)}}{b_1^{(2)}} \end{bmatrix} - \frac{P}{16D_{11}^{(2)}} \begin{bmatrix} 0 \\ 3r^2 + \frac{R_2^4}{r^2} \end{bmatrix}.
\]  

(3.55)

Substituting \( r = R_1 \) in Eq. (3.54), we obtain the deflections at the interface, which are given by

\[
\begin{bmatrix}
  u_0^{(2)}(R_1) \\
  \theta^{(2)}(R_1)
\end{bmatrix} = \begin{bmatrix} R_1 - \frac{R_2^2}{R_1} \\ \frac{R_1}{R_1} \end{bmatrix} \begin{bmatrix} a_1^{(2)} \\ b_1^{(2)} \end{bmatrix} - \frac{P}{16D_{11}^{(2)}} \begin{bmatrix} 0 \\ R_1^3 - \frac{R_2^4}{R_1} \end{bmatrix}.
\]  

(3.56)

The generalized force resultants in the outer annular region are given by

\[
\begin{bmatrix}
  N_r^{(2)}(r) \\
  M_r^{(2)}(r)
\end{bmatrix} = \begin{bmatrix}
  A_1^{(2)} \frac{du_0^{(2)}(r)}{dr} + A_2^{(2)} \frac{u_0^{(2)}(r)}{r} \\
  -D_1^{(2)} \frac{d\theta^{(2)}(r)}{dr} - D_2^{(2)} \frac{1}{r} \theta^{(2)}(r)
\end{bmatrix}.
\]  

(3.57)
Substituting Eq. (3.54) and Eq. (3.55) in Eq. (3.57), we obtain a relationship for generalized force resultants in the radial direction only in terms of the constants involved in the analytical solution for deflection:

\[
\begin{bmatrix}
N_r^{(2)}(r) \\
M_r^{(2)}(r)
\end{bmatrix} = \left(1 + \nu_s + \frac{R^2}{r^2} (1 - \nu_s) \right) \begin{bmatrix}
A^{(2)}_{ii} a^{(2)}_i \\
-D^{(2)}_{ii} b^{(2)}_i
\end{bmatrix} + \frac{Pr^2}{16} \left\{ \begin{array}{c}
0 \\
(3 + \nu_s + \frac{R^4}{r^4} (1 - \nu_s))
\end{array} \right\}.
\] (3.58)

The generalized force resultants at the interface are given by

\[
\begin{bmatrix}
N_r^{(2)}(R_i) \\
M_r^{(2)}(R_i)
\end{bmatrix} = \left(1 + \nu_s + \frac{R^2}{R_i^2} (1 - \nu_s) \right) \begin{bmatrix}
A^{(2)}_{ii} a^{(2)}_i \\
-D^{(2)}_{ii} b^{(2)}_i
\end{bmatrix} + \frac{PR_i^2}{16} \left\{ \begin{array}{c}
0 \\
(3 + \nu_s + \frac{R_i^4}{R_i^4} (1 - \nu_s))
\end{array} \right\}.
\] (3.59)

**Interface Compatibility Conditions**

At the interface, deflections and the resultants must match the following conditions:

\[
u_0^{(1)}(R_i) = u_0^{(2)}(R_i),
\] (3.60)

\[
\theta^{(1)}(R_i) = \theta^{(2)}(R_i),
\] (3.61)

\[
N_r^{(1)}(R_i) = N_r^{(2)}(R_i)
\] (3.62)

and

\[
M_r^{(1)}(R_i) = M_r^{(2)}(R_i).
\] (3.63)

From the above derivation, it can be noted that the piezoelectric effect does not appear explicitly in the general solution but is introduced by the interface matching conditions of the generalized force resultants shown in Eq. (3.62) and (3.63).

**Solution Techniques**

The expressions continued to increase in complexity. Analytical expression for the deflection field in terms of basic parameters is huge and does not provide any
physical insight and is hard to evaluate or code. Hence two different techniques are
employed to obtain the solution.

Simple Analytical Solution

The first technique is to obtain a simple analytical solution by making valid
assumptions based on reality. The solution is simplified by introducing constants, which
are functions of interface deflections. Such a solution is useful for the purpose of non-
dimensionalization. Such an effort to obtain an analytical expression is discussed in
Appendix B. The MATLAB code implementing the same is attached in Appendix C.

Numerical Method to Obtain Constants

The second technique is to code up the above equations in matrix form in
MATLAB. The equivalent stiffness matrix of each of the regions is evaluated. The
matrices are assembled to form an equivalent global stiffness matrix. The deflections and
generalized force resultants at the interface are found by inverting the stiffness matrix.
The constants and hence the deflection field is found from the evaluated interface
deflections. The MATLAB code is attached in Appendix C of the thesis.
CHAPTER 4
MODEL VERIFICATION

Verification of the analytical form of the solution obtained in Chapter 3 is required before using it to develop any design procedure. The following section deals with the verification of the solution obtained from Chapter 3 by theoretical and experimental means in addition to verifying it with a finite element model.

Theoretical Verification

To verify the theory, the solution for a classical plate obtained from the piezoelectric composite plate theory should match exactly the classical plate solution for a homogenous plate. The solution obtained from the theory will be equal to that of the classical plate solution if the equivalent flexural rigidity \( D_{11}^* \) defined in Chapter 3 is same as the flexural rigidity \( D \) of the classical plate. The flexural rigidity of a classical circular plate is given by

\[
D = \frac{Eh^3}{12(1-\nu^2)}. \tag{4.1}
\]

The equivalent flexural rigidity \( D_{11}^* \) of the classical plate is given by

\[
D_{11}^* = D_{11} - \frac{B_{11}^2}{A_{11}}. \tag{4.2}
\]

Substituting the expression for flexural rigidity in Eq. (4.2) for a homogenous plate, we obtain
\[ D'_{11} = \int_{z}^{z+h} \frac{E}{1-v^2} z^2 dz - \frac{\left( \int_{z}^{z+h} \frac{E}{1-v^2} z dz \right)^2}{\int_{z}^{z+h} \frac{E}{1-v^2} dz} . \quad (4.3) \]

Pulling the constants out of the integral in the above expression, we obtain

\[ D'_{11} = \frac{E}{1-v^2} \left( \int_{z}^{z+h} z^2 dz - \frac{\left( \int_{z}^{z+h} zdz \right)^2}{\int_{z}^{z+h} dz} \right) . \quad (4.4) \]

Computing the definite integrals in the above expression, we obtain

\[ D'_{11} = \frac{E}{1-v^2} \left( \frac{z^3}{3} \left[ \frac{z^{1+h}}{1+h} \right]^2 - \frac{\left( \frac{z^2}{2} \right)^2}{\frac{z^{1+h}}{1+h}} \right) . \quad (4.5) \]

which on further simplification yields

\[ D'_{11} = \frac{E}{1-v^2} \left( \frac{(z+h)^3}{3} - \frac{(z^2)^2}{2} \frac{2}{h} \right) . \quad (4.6) \]

Using elementary arithmetic identities, the above expression is simplified to yield

\[ D'_{11} = \frac{E}{1-v^2} \left( \frac{3z^2h + 3zh^2 + h^3}{3} - \frac{(2z + h)^2}{4} \right) . \quad (4.7) \]

which is simplified further to yield

\[ D'_{11} = \frac{E}{1-v^2} \left( \frac{h^3}{3} + z^2h + zh^2 - \frac{4z^2h + 4zh^2 + h^3}{4} \right) . \quad (4.8) \]

The above expression on simplification yields
\[ D_{ii}^* = \frac{E}{1-\nu^2} \left( \frac{h^3}{3} + z^2 h + z h^2 - z^2 h - z h^2 - \frac{h^3}{4} \right) = \frac{E h^3}{12(1-\nu^2)} = D, \quad (4.9) \]

which implies that the equivalent flexural rigidity appearing in the piezoelectric circular composite plate theory reduces to flexural rigidity of a classical circular plate for a homogenous plate.

**Finite Element Model**

The finite element model of the piezoelectric unimorph disk was made for several extents of piezoelectric patch in ABAQUS CAE,\(^{17}\) both for pressure loading and voltage loading. The geometry and the material properties used corresponding to Brass/PZT bender (Shim = Brass, Piezo = PZT) are tabulated in Table 4.1. A sample input file for each of the following cases of applied loading is attached in Appendix D of the thesis.

**Pressure Loading**

A short-circuit across the PZT was modeled by applying an equipotential boundary condition on the top and bottom surfaces of the piezoelectric patch. A pressure load equivalent of 1000 \( Pa \) was applied on the bottom surface of the shim layer. The shim was meshed with 8-noded linear axisymmetric brick elements while the piezoelectric layer was meshed with similar brick elements with piezoelectric stresses. The geometry was scaled by a factor of 1000 (this is done in order to avoid numerical truncation errors in the solver) and the pressure loading was diminished by a factor of 1000 in order that the output deflections are directly in meters. The maximum static deflection \( w_0(0) \) for each case was determined both from the analytical solution (obtained from the MATLAB code attached in Appendix C) and the finite element
model. The results were plotted as a function of normalized radius of the piezoelectric patch used as shown in Figure 4.1.

Table 4.1: Properties of the piezoelectric unimorph disk used in the finite element model

<table>
<thead>
<tr>
<th>Geometrical properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer radius (radius of the mount)</td>
<td>500 µm</td>
</tr>
<tr>
<td>Radius of the piezoelectric layer</td>
<td>0-500 µm</td>
</tr>
<tr>
<td>Thickness of the shim</td>
<td>5 µm</td>
</tr>
<tr>
<td>Thickness of the piezoelectric layer</td>
<td>2 µm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mechanical properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus of the shim</td>
<td>90 Gpa</td>
</tr>
<tr>
<td>Poisson’s ratio of the shim</td>
<td>0.3</td>
</tr>
<tr>
<td>Density of the shim</td>
<td>8700 kg/m³</td>
</tr>
<tr>
<td>Young’s modulus of the piezoelectric layer</td>
<td>30 Gpa</td>
</tr>
<tr>
<td>Poisson’s ratio of the piezoelectric layer</td>
<td>0.3</td>
</tr>
<tr>
<td>Density of the piezoelectric layer</td>
<td>7500 kg/m³</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Electric and Dielectric properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative permittivity of the piezoelectric layer</td>
<td>1000</td>
</tr>
<tr>
<td>Piezoelectric constant responsible for an extension in 1-direction due to application of field in the 3-direction (d_{31})</td>
<td>-50 pC/N</td>
</tr>
</tbody>
</table>

Plot of the center deflection of a circular piezoelectric unimorph obtained from the analytical solution, described in Eqs. (B.31) and (B.37) matches the solution obtained from the finite element model to within 1%. Furthermore, mesh refinement studies
indicated that the finer the mesh, the lesser the deviation of the solution obtained from the finite element model to that obtained from the theory. (The case shown in Figure 4.1 corresponds to the case where spacing between the nodes was 0.001 mm.)

![Graph showing comparison of maximum deflection for different radii of the piezoelectric material as predicted by the analytical solution and finite element model for pressure application.](image)

**Figure 4.1:** Comparison of maximum deflection for different radii of the piezoelectric material as predicted by the analytical solution and finite element model for pressure application.

**Voltage Loading**

The finite element model made for pressure loading was modified to yield a finite element model of the piezoelectric unimorph disk subjected to voltage loading. In this case, the pressure loading was reduced to zero. A potential boundary condition of unit strength was applied on the top surface of the piezoelectric layer while the bottom surface
was retained at zero potential. Other parameters were retained at the same value as in the previous case.

Figure 4.2 shows that the plot of the center deflection of a circular piezoelectric unimorph obtained from the analytical solution, described in Eqs. (B.31) and (B.37) matches the solution obtained from the finite element model made for voltage loading to within 1%. Mesh refinement studies indicated lesser deviation with a finer mesh similar to the case with pressure loading. (The case shown in Figure 4.2 corresponds to the case where spacing between the nodes was 0.001 mm.)

![Figure 4.2: Comparison of maximum deflection for different radii of the piezoelectric material as predicted by the analytical solution and finite element model for a unit voltage loading.](image-url)
Experimental Verification

In order to further validate the model, experiments were conducted in the Dynamics and Control Laboratory at the University of Florida.

A periodic chirp signal of 5 $V$ amplitude with frequency ranging from $\sim 100$ $Hz$ to 4000 $Hz$ was applied across the circular piezoelectric unimorph (APC International Ltd. Model APC850) to determine its natural frequency and mode shape. A Polytech PI laser scanning laser vibrometer (MSV200), shown in Figure 4.3, was used to measure the transverse deflection of the clamped piezoelectric composite plate due to the application of voltage across the piezoelectric material. The geometry and material properties of the piezoelectric unimorph bender used for experiment are shown in Table 4.2.
Table 4.2: Properties of the piezoelectric bender APC 850

<table>
<thead>
<tr>
<th>Geometric properties</th>
<th>Mechanical properties</th>
<th>Electric and Dielectric properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_2 )</td>
<td>( E_s )</td>
<td>( \varepsilon_r )</td>
</tr>
<tr>
<td>11.7 mm</td>
<td>89.6 GPa</td>
<td>1750</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>( v_s )</td>
<td>( d_{31} )</td>
</tr>
<tr>
<td>10.0 mm</td>
<td>0.324</td>
<td>-175 pC/N</td>
</tr>
<tr>
<td>( h_s )</td>
<td>( \rho_s )</td>
<td></td>
</tr>
<tr>
<td>0.221 mm</td>
<td>8700 kg/m(^3)</td>
<td></td>
</tr>
<tr>
<td>( h_p )</td>
<td>( E_p )</td>
<td></td>
</tr>
<tr>
<td>0.234 mm</td>
<td>63 GPa</td>
<td></td>
</tr>
<tr>
<td>( v_p )</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td>( \rho_p )</td>
<td>7700 kg/m(^3)</td>
<td></td>
</tr>
</tbody>
</table>

Experiments were then performed by applying a voltage of 5 VAC at 100 Hz via conductive copper tape attached to the two sides of the composite plate and the laser was scanned across the surface.

Figure 4.4: Measured displacement frequency response function obtained by converting velocity measurements using \( 1/j\omega \) integrating factor.
The 100 Hz test frequency was very small compared to the measured natural frequency of approximately 3360 Hz, as shown in the frequency response of the piezoelectric unimorph disk transducer found in Figure 4.4. As the experiment was conducted at such a low frequency, mass effect was neglected and the mode shape obtained should approximate the static mode shape.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Theory</th>
<th>Experiment</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>$142 , Pa/V$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_{as}$</td>
<td>$1.40e-013 , m^4/s^2/kg$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_A$</td>
<td>$13800 , kg/m^4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_A$</td>
<td>$1.98e-011 , m^3/V$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_{EF}$</td>
<td>$20.8 , nF$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_{EB}$</td>
<td>$18.0 , nF$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>$0.37$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_s$</td>
<td>$3620 , Hz$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>$3360 , Hz$</td>
<td>$7.6 - 15.9$</td>
<td></td>
</tr>
<tr>
<td>$f_o$</td>
<td>$3890 , Hz$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_0(0)$</td>
<td>$0.107 , \mu m/V$</td>
<td>$0.0923 , \mu m/V$</td>
<td>$15.9$</td>
</tr>
</tbody>
</table>

The results from the experiments were compared with the corresponding analytical solutions. They show matching of the mode shape to a considerable extent. However the deflection is off by 15.9%. The natural frequency, obtained from the
experiment (around 3360 Hz) does not lie between short-circuit and open-circuit resonant frequencies, obtained from the theory (see extracted lumped and two-port network parameter table shown in Table 4.3). It is expected that, the natural frequency lie within this range. This is because, in the case of a piezoelectric with a short (potential difference across the terminals is zero), the natural frequency will correspond to short-circuit resonant frequency and a piezoelectric in static case (zero current) would correspond to the open-circuit case. Since the experiment is performed in a dynamic condition rather than static and with a value of voltage applied across the terminals, the resonant frequency is expected to lie in between the open-circuit and the short-circuit value. Table 4.3 shows the value of the lumped element and two-port network parameters obtained from the theory. It is found that, the values obtained deviates from the theory to a considerable extent.

**Causes for Deviation of the Experimental Results from Theory**

It should be noted that the theory neglects the bond layer between the piezoceramic patch and the brass shim and thus assumes that a perfect bond exists between the piezoceramic and the shim. In this case, the estimated bond layer thickness is 25 \( \mu m \) (1 mil), which must be accounted for.

In addition, the axisymmetric assumption implies that the circular piezoceramic patch is bonded in the center of the shim. Commercial unimorphs exhibit some non-uniformity in this regard.

Furthermore, the piezoceramic patch has a thin metal electrode layer (e.g., silver) of a different radius. Fringing field effects are not modeled in the present calculations. Silver electrode also effects in mechanically stiffening the plate, which is not accounted
for in this calculation. An accurate analysis would involve three sections viz. a three-layered inner composite disc, a two-layered composite annular plate and an outer annular plate.

Finally, it should be emphasized that an ideal clamped boundary is difficult to achieve in practice. In the current experimental setup, thick clamp plates are used in conjunction with several bolts uniformly spaced around the circumference.
The analytical expressions for the transverse deflections derived in Chapter 3 of the thesis, are too complicated to obtain any sort of a physical insight into scaling. To facilitate design of a piezoelectric unimorph using the analytical solution, a non-dimensional representation of the transverse deflection is necessary. Such a representation of the transverse deflection would prove to be a good design tool.

The Poisson’s ratio of the shim materials and piezoelectric materials are close to each other with a value around 0.3. Hence, an assumption that the Poisson’s ratio of the shim and piezoelectric material are the same, would simplify the problem. Furthermore, the governing equations contain terms that are either ratios of the Poisson’s ratio or ratios of the difference of unity and square of Poisson’s ratio. These quantities are still closer to unity. Therefore, in order to simplify the non-dimensionalization procedure, the effect of Poisson’s ratio is ignored. Morris and Foster\textsuperscript{4} have also neglected the effect of Poisson’s ratio. Non-dimensionalization is carried out using the Buckingham $\pi$ Theorem.\textsuperscript{18}

**Buckingham $\pi$ Theorem**

The independent parameters involved in this simplified problem are $E_p$, $E_s$, $h_p$, $h_s$, $R_1$, $R_2$, $d_{31}$, $V$, $P$, $w_0$ and $u_0$, which are 11 independent variables. The three dimensions involved in the problem are that of length $L$, force $F$ and voltage $V$. Therefore there should exist eight (11-3 = 8) independent non-dimensional variables.
To design the best piezoelectric disc for a particular shim, it is better to non-dimensionalize the variables with respect to the shim variables. This leaves $E_s$ to be the basic dimension to non-dimensionalize variables with units of pressure and leaves $h_s$ or $R_2$ to be the basic dimension to non-dimensionalize variables with units of length.

Choosing $R_2$ for normalizing the length scale, we obtain

$$\frac{w_0}{R_2}, \frac{h_0}{R_2}, \frac{d_{31}V}{R_2}, \frac{w}{R_2}, \frac{h}{R_2}, \frac{d_{31}V}{R_2}$$

$w_0$ and $h_0$ to be the 8 non-dimensional variables.

The basic field variables can be expressed in non-dimensional form as

$$\nu_0^* = \frac{w_0}{R_2} = f_1 \left( \frac{E_p}{E_s}, \frac{P}{E_s}, \frac{R_1}{R_2}, \frac{h_1}{R_2}, \frac{h_p}{R_2}, \frac{d_{31}V}{R_2} \right)$$

and

$$u_0^* = \frac{u_0}{R_2} = f_2 \left( \frac{E_p}{E_s}, \frac{P}{E_s}, \frac{R_1}{R_2}, \frac{h_1}{R_2}, \frac{h_p}{R_2}, \frac{d_{31}V}{R_2} \right)$$

However, such a representation will not provide any sort of physical insight that would facilitate design. Hence a non-dimensional form of the primary variables, which could be expressed in the best possible form, is required.

Morris and Foster$^4$ represented the optimal radius ratio $(r^*)_{opt}$ and thickness ratio $(t^*)_{opt}$ (i.e. their field variables) as functions of aspect ratio $\alpha$, Young’s modulus ratio $\beta$ and a ratio $D^*$ defined in Eq. (5.3);

$$\alpha (r^*)_{opt} = g_1 (\alpha, \beta),$$

$$(t^*)_{opt} = g_2 (\alpha, \beta)$$

and
\[
\frac{\left( t^* \right)_{eq}}{\alpha} = g_3(D^*),
\]

where

\[
r^* = \frac{R_1}{R_2}, \quad t^* = \frac{t_p}{t_s} \quad \text{and} \quad D^* = \frac{D_{11}^{(2)}}{E_p R_2^3}.
\]  

(5.3)

They numerically found the functional dependence of their field variables on the fore-
mentioned ratios and developed some empirical relationships. Since the problem
discussed in this thesis, is not directly concerned with the optimization of a particular
device, the field variables are chosen to be \( u_0 \) and \( w_0 \).

A non-dimensional representation of these field variables, are conventionally
obtained by non-dimensionalizing the plate equations. However, non-dimensionalizing
the plate equations is also a complex task. Therefore, governing equations that provide
physical insight are used for the purpose of non-dimensionalization.

The response of the central composite plate, when subjected to a pressure \( P \) and
voltage \( V \), is represented by

\[
u_0^{(1)}(r) = a_1^{(1)} r,
\]  

(5.4)

\[\theta^{(1)}(r) = a_2^{(1)} r - \frac{Pr^3}{16D_{11}^{(1)}},\]

(5.5)

\[N_r^{(1)}(r) = A_{11}^{(1)} (1+\nu) a_1^{(1)} - N^p \]

(5.6)

and

\[M_r^{(1)}(r) = -D_{11}^{(1)} (1+\nu) a_2^{(1)} + \frac{Pre^{3+\nu}}{16} - M^p,\]

(5.7)
where the number (1) in superscript indicates deflections and generalized forces of the central composite plate along an axis where coupling matrix vanishes.

For the outer annular plate, \([B]\) vanishes because of the symmetry about its neutral axis. Hence the response of the outer annular plate is given by

\[
u_0^{(2)}(r) = a_1^{(2)} r + \frac{b_1^{(2)}}{r},
\]

\[
\theta^{(2)}(r) = a_2^{(2)} r + \frac{b_2^{(2)}}{r} + \frac{Pr^3}{16D_{11}^{(2)}},
\]

\[
N_r^{(2)}(r) = A_1^{(2)} (1 + \nu) a_1^{(2)} - A_1^{(2)} (1 - \nu) \frac{b_1^{(2)}}{r^2},
\]

and

\[
M_r^{(2)}(r) = D_{11}^{(2)} (1 + \nu) a_2^{(2)} - D_{11}^{(2)} (1 - \nu) \frac{b_2^{(2)}}{r^2} + \frac{Pr^2 (3 + \nu)}{16},
\]

where the number (2) in superscript indicates deflections and generalized forces of the outer annular plate.

**Non-Dimensional Deflection for Pressure Loading**

The non-dimensional vertical deflection \(w_0^*(\bar{r})\) is obtained from the expression shown in Eq. (5.12). The symbol * in the superscript indicates a non-dimensional quantity.

\[
w_0(\bar{r}) = \frac{\kappa}{R_2^3} \int_0^{\bar{r}} \theta^{(1)}(r) dr + \int_{\bar{r}}^1 \theta^{(2)}(r) dr.
\]

Substituting \(\bar{r} = \frac{r}{R_2}\) in Eq. (5.12), we obtain
The non-dimensional form of the slope is obtained by dividing the slope with the forcing term in Eq. (5.9) to yield

\[
\theta^{(1)} = \frac{\theta^{(1)}}{PR_2^3} \quad \text{and} \quad \theta^{(2)} = \frac{\theta^{(2)}}{PR_2^3}.
\]  

The letter P in the superscript is used to represent non-dimensional parameters that are obtained when the piezoelectric unimorph is subjected to pressure loading alone. Even though the slope by itself is non-dimensional, the above form is more useful because it does not vary with change in loading and the overall dimensions.

Substituting Eq.(5.14) in Eq. (5.13), we obtain

\[
\int_0^\frac{n}{R_2} \theta^{(1)} - \int_0^\frac{n}{R_2} \theta^{(2)} d\bar{r}. \tag{5.15}
\]

Eq. (5.15) implies that the non-dimensionalized vertical deflection is given by

\[
w_0^* (\bar{r}) = \frac{w_0 (r)}{PR_2^3}.
\]  

The non-dimensional vertical displacement obtained in Eq. (5.16) is not dependent on the aspect ratio of the shim (see Figure 5.4 and Figure 5.6). It is dependent on the ratios of the radius, thickness and Young’s modulus.

\[
w_0^* (\bar{r}) = \frac{E_p}{E_y} \frac{h_p}{h_1} \frac{R_1}{R_2}. \tag{5.17}
\]

The non-dimensional form of the field variable shown in Eq. (5.17) is much simpler than the form indicated in Eq. (5.1). The best way to represent the whole set of deflections of a piezoelectric unimorph in a compact form is to plot the non-dimensional
center deflection against the radius ratio \( R_1/R_2 \) for different values of \( h_p/h_s \) for a particular \( E_p/E_s \) as shown in Figure 5.1 - Figure 5.5. The value of \( h_s/R_2 \) used in the first five plots of each of the non-dimensional variables discussed in the section is 0.01. Commerially available disc benders manufactured by APC International limited have the Young’s modulus ratio varying between 0.6 and 0.8. The Young’s modulus of PZT deposited in MEMS level device can be as low as 30 GPa. The Young’s modulus of the shim layer (Silicon) is around 150 GPa resulting in a Young’s modulus ratio of 0.2. Since, Silicon is a moderately anisotropic material with an anisotropic coefficient\(^{19}\) of 1.57 (which is close to unity. For an isotropic material, the value of anisotropic coefficient is unity), the Silicon layer is treated as transversely isotropic in this analysis. Hence values of 0.02, 0.2, 0.4, 0.6 and 0.8 were selected for the ratio of the Young’s Modulii. The Young’s modulus ratio value was taken to as small as 0.02 to accommodate PVDF – Aluminum benders. In order to show that these non-dimensional variables do not vary with change in aspect ratio \( R_2/h_s \), a plot of the non-dimensional variable with value of Young’s modulus ratio at 0.6 and the value of \( h_s/R_2 \) at 0.02. The 4\(^{th}\) and 6\(^{th}\) plots of each of the non-dimensional variables discussed in the following section are exactly the same even though the aspect ratios are different. This proves the non-dependent nature of the non-dimensional variables on aspect ratio.

The denominator is multiplied with a factor of 0.25, to make its value equal to that of the central deflection of a clamped classical circular plate acted upon by a pressure load;
The expression for non-dimensional vertical deflection is represented in terms of ratios (mentioned in the beginning of this chapter) as

\[
W_0^{ps}(\tilde{r}) = \frac{W_0(r)}{PR_2^4} \frac{Eh_1^3}{16(1-V^2)}
\]

which on simplification yields

\[
W_0^{ps}(\tilde{r}) = \frac{W_0(r)}{R_2} \frac{3}{4}(1-V^2) \frac{P}{E_s} \left( \frac{R_2}{h_s} \right)^3.
\]

Figure 5.1: Non-dimensional plot of the center deflection of a piezoelectric unimorph disc with \(E_p/E_s = 0.02\) subjected only to a pressure load.
Figure 5.2: Non-dimensional plot of the center deflection of a piezoelectric unimorph disc with $E_p/E_s = 0.2$ subjected only to a pressure load.

Figure 5.3: Non-dimensional plot of the center deflection of a piezoelectric unimorph disc with $E_p/E_s = 0.4$ subjected only to a pressure load.
Figure 5.4: Non-dimensional plot of the center deflection of a piezoelectric unimorph disc with $E_p/E_s = 0.6$ subjected only to a pressure load.

Figure 5.5: Non-dimensional plot of the center deflection of a piezoelectric unimorph disc with $E_p/E_s = 0.8$ subjected only to a pressure load.
Figure 5.6: Non-dimensional plot of the center deflection of a piezoelectric unimorph disc with $E_p/E_s = 0.6$ and $h_i/R_z = 0.02$, subjected only to a pressure load.

**Non-Dimensionalization for Voltage Loading**

Equations (5.4) – (5.11) are modified for the case when voltage alone is applied in the following manner:

\[ u_0^{(1)}(r) = a_1^{(1)} r, \quad (5.21) \]

\[ \theta^{(1)}(r) = a_2^{(1)} r, \quad (5.22) \]

\[ N_r^{(1)}(r) = A_{11}^{(1)} (1 + \nu) a_1^{(1)} - N_r^p, \quad (5.23) \]

\[ M_r^{(1)}(r) = -D_{11}^{(1)} (1 + \nu) a_2^{(1)} - M_r^p, \quad (5.24) \]

\[ u_0^{(2)}(r) = a_1^{(2)} r + \frac{b_1^{(2)}}{r}, \quad (5.25) \]
\[ \theta^{(2)}(r) = a_2^{(2)} r + \frac{b_2^{(2)}}{r}, \]  
(5.26)

\[ N_r^{(2)}(r) = A_1^{(2)} (1+v) a_1^{(2)} - A_1^{(2)} (1-v) \frac{b_1^{(2)}}{r^2}. \]  
(5.27)

and

\[ M_r^{(2)}(r) = D_1^{(2)} (1+v) a_2^{(2)} - D_1^{(2)} (1-v) \frac{b_2^{(2)}}{r^2}. \]  
(5.28)

The expression for non-dimensional vertical deflection \( w_0^v(\tilde{r}) \) is obtained from the expression

\[ w_0(\tilde{r}) = \int_0^{\tilde{r}} \theta^{(1)} dr + \int_{\tilde{r}}^{r} \theta^{(2)} dr. \]  
(5.29)

Substituting \( \tilde{r} = \frac{r}{R_2} \) in Eq. (5.29), we obtain

\[ w_0(\tilde{r}) = R_2 \int_0^{\eta} \theta^{(1)} d\tilde{r} + \int_{\eta}^{\tilde{r}} \theta^{(2)} d\tilde{r}. \]  
(5.30)

The non-dimensional form of the slope when subjected to voltage loading alone is obtained by dividing the slope with an equivalent of the forcing term found in Eq. (5.24) to yield

\[ \theta^{(1)v} = \frac{\theta^{(1)}}{\left(\frac{M_p R_2}{D_1^{(2)}}\right)} \quad \text{and} \quad \theta^{(2)v} = \frac{\theta^{(2)}}{\left(\frac{M_p R_2}{D_1^{(2)}}\right)}. \]  
(5.31)

The letter V in the superscript is used to represent non-dimensional parameters that are obtained when the piezoelectric unimorph is subjected to voltage loading alone. Even though the slope by itself is non-dimensional, the above form is more useful because it
does not vary with change in loading, piezoelectric constant, relative permitivity of the piezoelectric and the overall dimensions.

Substituting Eq. (5.31) in Eq. (5.30), we obtain

\[
\frac{w_0(r)}{R_2} \left( \frac{M^2 R_2}{D^{(2)}_{11}} \right) = \int_0^n \left( \theta^{(i)}(r) \right) d\tilde{r} + \int_0^1 \left( \theta^{(2)}(r) \right) d\tilde{r}. \tag{5.32}
\]

Equation (5.32) implies that the non-dimensional vertical deflection is given by

\[
w_0^{\star}(\tilde{r}) = \frac{w_0(r)}{\left( \frac{M^2 R_2}{D^{(2)}_{11}} \right)}. \tag{5.33}
\]

The non-dimensional vertical displacement obtained in Eq. (5.33) is also independent of aspect ratio and is only a function of the radius ratio, thickness ratio and Young’s modulus ratio, which is much simpler to represent than the form indicated in Eq. (5.1);

\[
w_0^{\star}(\tilde{r}) = f \left( \frac{E_p}{E_\varepsilon}, \frac{h_p}{h_\varepsilon}, \frac{R_1}{R_2} \right). \tag{5.34}
\]

The best way to represent the whole set of deflections is to plot it in the same manner as in the case of pressure loading. The denominator was multiplied by a factor of \( \frac{1}{1+\nu} \) to make its value equal to that of the deflection obtained for a classical circular plate acted on by a moment equal to piezoelectric couple shown in Eq. (3.27). The revised expression is given by

\[
w_0^{\star}(\tilde{r}) = \frac{w_0(r)}{\left( \frac{M^2 R_2}{D^{(2)}_{11}(1+\nu)} \right)}. \tag{5.35}
\]

The expression for non-dimensional vertical deflection is represented in terms of ratios (mentioned in the beginning of this chapter) in the following manner:
\[ w_0^v(\bar{r}) = \frac{w_0(r)}{ \left( \frac{E_p}{1-v} d_{31} V \left( h_p + h_s \right) \left( \frac{E_s h_s}{E_s h_s + E_p h_p} \right) R_2^2 } \right) \left( \frac{E_s h_s^3}{12(1-v^2)(1+v)} \right). \] (5.36)

\[ (i.e.) \]

\[ w_0^v(\bar{r}) = \frac{w_0(r)}{ \left[ \frac{1}{R_2} \frac{E_p h_p}{E_s h_s} \frac{1 + \frac{E_p}{E_s} \left( \frac{h_p}{h_s} \right)^2}{12} \left( \frac{d_{31} V}{R_2} \right) \right]. \] (5.37)

The variation of the center deflection of a unimorph subjected only to a voltage load, with variation in non-dimensional ratios is delineated in Figure 5.7 - Figure 5.12.

Figure 5.7: Non-dimensional plot of the center deflection of a piezoelectric unimorph disc with \( E_p/E_s = 0.02 \) subjected only to a voltage load.
Figure 5.8 Non-dimensional plot of the center deflection of a piezoelectric unimorph disc with $\frac{E_p}{E_s} = 0.2$ subjected only to a voltage load.

Figure 5.9: Non-dimensional plot of the center deflection of a piezoelectric unimorph disc with $\frac{E_p}{E_s} = 0.4$ subjected only to a voltage load.
Figure 5.10: Non-dimensional plot of the center deflection of a piezoelectric unimorph disc with $E_p/E_s = 0.6$ subjected only to a voltage load.

Figure 5.11: Non-dimensional plot of the center deflection of a piezoelectric unimorph disc with $E_p/E_s = 0.8$ subjected only to a voltage load.
Figure 5.12: Non-dimensional plot of the center deflection of a piezoelectric unimorph disc with $E_p/E_s = 0.6$ and $h_p/h_s = 0.00001$, subjected only to a voltage load.

To further enable design and optimization of a circular piezoelectric unimorph, non-dimensional lumped element and two-port network parameters are needed.

**Non-Dimensionalization of Lumped Element and Two-Port Network Parameters**

Some of the lumped element and two-port network parameters are obtained from the transverse deflection by expressions described in Chapter 2 of this thesis. The non-dimensional parameters of these variables, has the same dependence on the corresponding dimensional variables, as the transverse deflection. In other words, the equivalent non-dimensional parameter is obtained from the corresponding dimensional parameter by dividing it with the value of the parameter for a classical circular plate.

The non-dimensional short-circuit acoustic compliance is given by
\[ C'_{AS} = \frac{C_{AS}}{(C_{AS})_{Shim}} \]  

(5.38)

where \((C_{AS})_{Shim}\) represents the short-circuit acoustic compliance of the shim alone:

\[ (C_{AS})_{Shim} = \frac{\pi R_s^6 (1 - \nu_s^2)}{16 E_s h_s^3}. \]  

(5.39)

Above expression for \((C_{AS})_{Shim}\) in terms of non-dimensional ratios is given by

\[ (C_{AS})_{Shim} = \frac{1}{16} \left(1 - \nu_s^2 \right) \left( \frac{P}{E_s} \right) \left( \frac{R_s}{h_s} \right)^4 \left( \frac{\pi R_s^2 h_s}{P} \right). \]  

(5.40)

The non-dimensional acoustic compliance is also a function of the Young’s modulus ratio, thickness ratio and radius ratio. The best way to represent these variables will be similar to that of the deflections.

Figure 5.13 - Figure 5.18 delineates such plots of short-circuit acoustic compliance of the piezoelectric unimorph.

![Figure 5.13: Non-dimensional short-circuit acoustic compliance plots for \(E_p/E_s = 0.02\).](image)

Figure 5.14: Non-dimensional short-circuit acoustic compliance plots for $E_p/E_s = 0.2$.

Figure 5.15: Non-dimensional short-circuit acoustic compliance plots for $E_p/E_s = 0.4$. 
Figure 5.16: Non-dimensional short-circuit acoustic compliance plots for $E_p/E_s = 0.6$.

Figure 5.17: Non-dimensional short-circuit acoustic compliance plots for $E_p/E_s = 0.8$. 
Figure 5.18: Non-dimensional short-circuit acoustic compliance plots for $E_p/E_s = 0.6$ and $h_t/R_2 = 0.02$.

The non-dimensional acoustic mass is given by

$$M_A^* = \frac{M_A}{(M_A)_{Shim}},$$  \hspace{1cm} (5.41)

where $(M_A)_{Shim}$ represents the acoustic mass of the shim alone;

$$M_A = \frac{1.8 (\rho_A)_{Shim}}{\pi R^2},$$ \hspace{1cm} (5.42)

where $(\rho_A)_{Shim}$ represents the areal density of the shim layer.

The non-dimensional mass like the deflections are a function of the usual three ratios. In addition it is also a function of density ratio. Density ratio, like Young’s Modulus ratio, is unique for a certain set of material. Aluminum/PVDF benders have
Young’s modulus ratio of 0.02 - 0.05. Therefore the density ratio of Aluminum/PVDF benders (i.e. 1760/2700) is chosen for the plots shown in Figure 5.19 - Figure 5.20. The Silicon/PZT benders have their Young’s Modulus ratio varying from 0.2 - 0.6. Therefore the density ratio of Silicon/PZT benders (i.e. 7500/2300) is chosen for the plots shown in Figure 5.21 - Figure 5.23 and Figure 5.26. The Brass/PZT benders have their Young’s Modulus ratio varying from 0.6 - 0.8. Therefore the density ratio of Brass/PZT (i.e. 7500/8700) benders produced by APC International Ltd. is chosen for the plots shown in Figure 5.24 - Figure 5.25. Figure 5.26, like the plots of previous non-dimensional ratios, has all ratios similar to Figure 5.23 except the aspect ratio. The plots appear the same indicating that the non-dimensional acoustic mass does not vary with change in aspect ratio.

Figure 5.19: Non-dimensional acoustic mass plots for $\frac{E_p}{E_s} = 0.02$ (Aluminum/PVDF).
Figure 5.20: Non-dimensional acoustic mass plots for $E_p/E_s = 0.05$ (Aluminum/PVDF).

Figure 5.21: Non-dimensional acoustic mass plots for $E_p/E_s = 0.2$ (Silicon/PZT).
Figure 5.22: Non-dimensional acoustic mass plots for $E_p/E_s = 0.4$ (Silicon/PZT).

Figure 5.23: Non-dimensional acoustic mass plots for $E_p/E_s = 0.6$ (Silicon/PZT).
Figure 5.24: Non-dimensional acoustic mass plots for $E_p/E_s = 0.6$ (Brass/PZT).

Figure 5.25: Non-dimensional acoustic mass plots for $E_p/E_s = 0.8$ (Brass/PZT).
Figure 5.26: Non-dimensional acoustic mass plots for $E_p/E_s = 0.6$ and $h_s/R_z = 0.02$ (Silcon/PZT).

Unlike the parameters that were non-dimensionalized by quantities possessing a physical significance, some of the two-port network parameters do not have quantities for the shim alone described. We can obtain the non-dimensional form of the two-port network parameters by finding the equivalent parameters for the shim alone from first principles.

The effective acoustic piezoelectric coefficient as defined in Chapter 2 is obtained by computing the volume displaced by a unit application of voltage due to the vertical deflection field, analytical expression for which is obtained in Chapter 3. Therefore, the non-dimensional form is derived by evaluating the volume displaced due to a displacement field obtained by the application of a moment equal to the piezoelectric couple generated on the shim of radius $R_z$. 

\[ d_A^* = \frac{d_A}{(d_A)_{Shim}}, \]  

(5.43)

where \((d_A)_{Shim}\) represents the effective acoustic piezoelectric coefficient of the piezoelectric with dimensions of the shim that is obtained from the following expression:

\[
(d_A)_{Shim} = \frac{\int_0^{R_2} M_p \left( R_2^2 - r^2 \right) D^{(2)}_{11} (1+\nu) 2\pi rdr}{V}. \]  

(5.44)

Substituting for the terms appearing in the above expression, we obtain

\[
(d_A)_{Shim} = \frac{\int_0^{R_2} \frac{E_p}{1-\nu} d_{31} V h_s \left( R_2^2 - r^2 \right) 2\pi rdr}{\frac{E_s h_s^3}{12(1-\nu^2)(1+\nu)}} \frac{1}{V}. \]  

(5.45)

Pulling the constants out of the integral in the above expression, we obtain

\[
(d_A)_{Shim} = 24\pi \frac{E_p}{E_s} \frac{d_{31} h_s}{h_s} \int_0^{R_2} \left( R_2^2 - r^2 \right) rdr. \]  

(5.46)

The above expression is simplified as

\[
(d_A)_{Shim} = 6\pi R_2^2 \left( \frac{E_p}{E_s} \right) d_{31} \left( \frac{R_2}{h_s} \right)^2. \]  

(5.47)

which can be written as

\[
(d_A)_{Shim} = 6 \left( \frac{d_{31} V}{R_2} \right) \left( \frac{E_p}{E_s} \right) \left( \frac{R_2}{h_s} \right)^3 \left\{ \frac{\pi R_2^2 h_s}{V} \right\}. \]  

(5.48)
Figure 5.27: Non-dimensional effective acoustic piezoelectric coefficient plots for $E_p/E_s = 0.02$.

Figure 5.28: Non-dimensional effective acoustic piezoelectric coefficient plots for $E_p/E_s = 0.2$. 
Figure 5.29: Non-dimensional effective acoustic piezoelectric coefficient plots for $E_p/E_s = 0.4$.

Figure 5.30: Non-dimensional effective acoustic piezoelectric coefficient plots for $E_p/E_s = 0.6$. 
Figure 5.31: Non-dimensional effective acoustic piezoelectric coefficient plots for $E_p/E_s = 0.8$.

Figure 5.32: Non-dimensional effective acoustic piezoelectric coefficient plots for $E_p/E_s = 0.6$ and $h_p/h_s = 0.00001$. 
The best way to represent this non-dimensional parameter is the same as that of the previously obtained parameters (see Figure 5.27 - Figure 5.32). It is found that $d_A$ is directly proportional to the square of the aspect ratio.

Non-dimensional electrical free capacitance was defined in the same manner as the ratio of the electrical free capacitance of the piezoelectric composite circular plate to capacitance of a piezoelectric disc with the dimensions of the shim. (i.e.)

\[
C_{EF}^* = \frac{C_{EF}}{(C_{EF})_{Shim}},
\]

where

\[
(C_{EF})_{Shim} = \frac{\varepsilon \pi R_s^2}{h_s}.
\]

(i.e.)

\[
C_{EF}^* = \frac{\varepsilon \pi R_2^2}{h_p} = \left(\frac{R_1}{R_2}\right)^2 \frac{\varepsilon \pi R_2^2}{h_s} = \left(\frac{R_1}{R_2}\right)^2 \left(\frac{h_p}{h_s}\right).
\]

Now that the non-dimensional form of all the independent two-port parameters and lumped parameters are known, non-dimensional representation of other parameters can be found directly by replacing the form of variables visible in the expressions with the corresponding non-dimensional variables.

Hence the non-dimensional representation for $\phi_A$ is given by

\[
\phi_A^* = \frac{d_A^*}{C_{AS}^*}.
\]

Eq. (5.52) implies that
Substituting Eqs. (5.40) and (5.47) in Eq.(5.53), we obtain

$$\phi_A^* = \phi_A \left( \frac{C_{As}}{d_{as}} \right)_{shim}, \tag{5.53}$$

which on simplification, gives

$$\phi_A^* = \phi_A \left( \frac{1}{16} \left( 1 - v^2 \right) \left( \frac{P}{E_s} \right) \left( \frac{R_2}{h_s} \right)^4 \left( \frac{\pi R_2^2 h_s}{P} \right) \right) \left( \frac{d_{s1} V}{R_2} \right) \left( \frac{E_p}{E_s} \right) \left( \frac{R_2}{h_s} \right)^3 \left( \frac{\pi R_2^2 h_s}{V} \right), \tag{5.54}$$

which on simplification, gives

$$\phi_A = \phi_A^* \left( \frac{d_{s1} V}{R_2} \right) \left( \frac{E_p}{E_s} \right) \left( \frac{R_2}{h_s} \right) \left( \frac{P}{E_s} \right) \left( \frac{R_2}{h_s} \right)^3 \frac{1}{96 (1 - v^2)} \left( \frac{P}{V} \right). \tag{5.55}$$

The plots of non-dimensional $\phi_A$ are shown in Figure 5.33 - Figure 5.38.

![Figure 5.33: Non-dimensional $\phi_A$ for $E_p/E_s = 0.02$.](image-url)
Figure 5.34: Non-dimensional $\phi_A$ for $E_p/E_s = 0.2$.

Figure 5.35: Non-dimensional $\phi_A$ for $E_p/E_s = 0.4$.
Figure 5.36: Non-dimensional $\phi_A$ for $E_p/E_s = 0.6$.

Figure 5.37: Non-dimensional $\phi_A$ for $E_p/E_s = 0.8$. 
The non-dimensional representation for $\phi_A'$ is obtained in the same manner as that of $\phi_A$ and is given by

$$\phi_A' = \frac{d_A^*}{C_{EF}^*}.$$  \hspace{1cm} \text{(5.56)}

Substituting Eqs. (5.43) and (5.49) in Eq. (5.56), we obtain

$$\phi_A' = \frac{d_A}{(d_A)_{shim}} \frac{C_{EF}^*}{C_{EF}^*},$$ \hspace{1cm} \text{(5.57)}

which on simplification, gives

$$\phi_A' = \phi_A' \frac{(C_{EF}^*)_{shim}}{(d_A)_{shim}}.$$ \hspace{1cm} \text{(5.58)}

Substituting Eqs. (5.48) and (5.50) in Eq.(5.58), we obtain
Multiplying both sides of the above equation with the denominator of the right-hand side of Eq. (5.59), we obtain

$$
\phi'_A \phi''_A = \frac{\epsilon \pi R_2^2}{h_x} \frac{1}{R_2} \left( \frac{d_{31} V}{R_2} \right) \left( \frac{E_p}{E_s} \right) \left( \frac{R_2}{h_x} \right)^3 \left( \frac{\pi R_2^2 h_x}{V} \right) \left( \frac{h_x^2}{\epsilon V} \right),
$$

(5.59)

which on simplification, gives

$$
\phi'_A = \phi''_A \left( \frac{d_{31} V}{R_2} \right) \left( \frac{E_p}{E_s} \right) \left( \frac{R_2}{h_x} \right)^3 \left( \frac{h_x^2}{\epsilon V} \right) \left( \frac{6d_{31}^2 E_s}{\epsilon} \right) \left( \frac{V}{P} \right),
$$

(5.60)

Figure 5.39 - Figure 5.44 depicts the variation of non-dimensional $\phi_A$ with variation in non-dimensional ratios.

![Graph](image-url)
Figure 5.40: Non-dimensional $\phi'_A$ for $E_p/E_s = 0.2$.

Figure 5.41: Non-dimensional $\phi'_A$ for $E_p/E_s = 0.4$. 

Figure 5.42: Non-dimensional $\Phi_A'$ for $E_p/E_s = 0.6$.

Figure 5.43: Non-dimensional $\Phi_A'$ for $E_p/E_s = 0.8$. 

The coupling coefficient represents the ideal fraction of energy transduced to the other domain and by definition is non-dimensional. The coefficient provides an indication of the electroacoustic energy conversion for the unimorph, but does not yield the actual value because it does not account for electrical and mechanical losses as well as electrical and mechanical loads. Therefore the equivalent representation for $K$ will be referred to as the universal representation for the coupling coefficient, since this representation is independent of the electric and dielectric properties of the piezoelectric layer. It is denoted by $K^*$ and is given by

$$K^* = \sqrt{\frac{(d_A^*)^2}{C_{AS}^*C_{EF}^*}},$$

which on simplification yields
Substituting Eqs. (5.40), (5.48) and (5.50) in Eq. (5.63), we obtain

\[ K^* = \frac{d^*_a}{\sqrt{C_{AS}^* C_{EF}^*}}. \]

It is noted that \( K \) is directly proportional to \( d_{31}/\sqrt{\varepsilon} \). Even though Young’s modulus ratio appears in the relationship, nothing can be said about the dependency of \( K \) on Young’s modulus ratio.

Since these parameters depend on the fore-mentioned parameters, the best possible representation of these non-dimensional parameters is in no way different from the representation of those basic parameters. Figure 5.45 - Figure 5.50 depict the variation of \( K^* \) with variation in non-dimensional ratios.
Figure 5.46: $K'$ for $E_p/E_s = 0.2$.

Figure 5.47: $K'$ for $E_p/E_s = 0.4$. 
Figure 5.48: $K'$ for $E_p/E_s = 0.6$.

Figure 5.49: $K'$ for $E_p/E_s = 0.8$. 
Figure 5.50: $K^*$ for $E_p/E_s = 0.6$ and $h_s/R_2 = 0.02$.

The non-dimensional variables obtained in this chapter are dependent only on the thickness ratio, Young’s modulus ratio and the ratio of the radii of the shim and the piezoelectric material. The plots shown in Figure 5.1 - Figure 5.50 depict variation of the different non-dimensional parameters with respect to these non-dimensional ratios. Therefore these charts can be used in creating a procedure for design of the best piezoelectric unimorph disc for a chosen device.
CHAPTER 6
CONCLUSIONS AND FUTURE WORK

Two-port electroacoustic model of a piezoelectric composite circular plate has been developed in this thesis. The analysis developed for the mechanical behavior of an axisymmetric piezoelectric multi-layered composite circular plate in this thesis is for any axisymmetric circular piezoelectric transducer.

The solution converges to already existing classical plate solutions upon simplification. FEM indicates that the center deflection matches with the theory to within 1% as predicted by Merhaut. The experiments with the Brass-PZT benders using laser vibrometry indicate that the modeshape is predicted to a considerable level of accuracy. However the natural frequency and center deflection are off by 15.9%. This is attributed to the problem with clamping, bond layer thickness and presence of a silver electrode. The experimentally measured frequency lies close to the predicted natural frequency range indicating the validity of the lumped assumption.

A set of non-dimensional parameters was identified for the problem of piezoelectric axisymmetric composite disc transducers. These parameters are useful in representing the deflection field of the composite plate in a complete manner for a wide range of system parameters. These parameters also have been useful in determining non-dimensional two-port network variables, some of which are directly the system parameters that need to be optimized for maximum performance of the piezoelectric transducer for the device of our choice.
Some of the non-dimensional two-port network parameters do not vary even with change in material properties in addition to not varying with change in the overall dimension. Based on the device requirement and design constraints the parameter that needs to be extremized can be pulled out of the non-dimensional charts discussed in Chapter 5. The value of the non-dimensional parameter, the corresponding radius ratio, thickness ratio and Young’s modulus ratio can be read from the charts in Chapter 5. The non-dimensional parameters so read can be converted to their actual form by using the relationship between the corresponding dimensional and non-dimensional forms in Chapter 5. Thus the device specifications can be determined. If the shim is designed, the charts can be used to find the best piezoelectric patch that would make the device function optimally.

The non-dimensional parameters also facilitate a possibility of development of a higher order design procedure for determining an optimal geometry of the piezoelectric material for maximum actuation/ sensing/ energy reclamration/ impedance tuning.

A synthetic jet using PZT-Brass disc as the actuating element and a MEMS based PZT microphone are being designed at the Interdisciplinary Microsystems Group at the University of Florida using a similar procedure. Future work will involve extending the theory for plates with large initial in-plane stresses. A large deflection analysis of the circular piezoelectric composite plate will extend the usage to high-displacement actuators like the THUNDER\textsuperscript{20} and RAINBOW\textsuperscript{20}. 
APPENDIX A
DETAILED DERIVATION OF THE GENERAL SOLUTION FROM PLATE CONSTITUTIVE EQUATIONS

Rewriting Eq. (3.30) from Chapter 3, we have
\[
\begin{bmatrix}
N_r \\
N_\theta
\end{bmatrix} = 
\begin{bmatrix}
A_{11} \frac{du_0(r)}{dr} + A_{12} \frac{u_0(r)}{r} - B_{11} \frac{d\theta(r)}{dr} - B_{12} \frac{\theta(r)}{r} \\
A_{21} \frac{du_0(r)}{dr} + A_{22} \frac{u_0(r)}{r} - B_{21} \frac{d\theta(r)}{dr} - B_{22} \frac{\theta(r)}{r}
\end{bmatrix} \begin{bmatrix}
N_r^p \\
N_\theta^p
\end{bmatrix}. \tag{A.1}
\]

Rewriting Eq. (3.31) from Chapter 3, we have
\[
\begin{bmatrix}
M_r \\
M_\theta
\end{bmatrix} = 
\begin{bmatrix}
B_{11} \frac{du_0(r)}{dr} + B_{12} \frac{u_0(r)}{r} - D_{11} \frac{d\theta(r)}{dr} - D_{12} \frac{\theta(r)}{r} \\
B_{21} \frac{du_0(r)}{dr} + B_{22} \frac{u_0(r)}{r} - D_{21} \frac{d\theta(r)}{dr} - D_{22} \frac{\theta(r)}{r}
\end{bmatrix} \begin{bmatrix}
M_r^p \\
M_\theta^p
\end{bmatrix}. \tag{A.2}
\]

Restating equations of equilibrium from Chapter 3, we have
\[
\frac{dM_r}{dr} + \frac{(M_r - M_\theta)}{r} = -Q_r, \tag{A.3}
\]
\[
\frac{dN_r}{dr} + \frac{(N_r - N_\theta)}{r} = 0, \tag{A.4}
\]
and
\[
\frac{dQ_r}{dr} + \frac{Q_r}{r} + P(r) = 0. \tag{A.5}
\]

For the case described in Figure 1.1, Eq. (A.5) becomes
\[
\frac{dQ_r}{dr} + \frac{Q_r}{r} = -P(r) = -P. \tag{A.6}
\]
Multiplying both sides by \( r \) (integration factor) and integrating once with respect to the radius \( r \), we obtain

\[
rQ_r = -\int P(r) r dr = -\int P r dr = \frac{-P r^2}{2} + c. \tag{A.7}
\]

Applying the boundary condition that,

\[
Q_r(r) = \frac{-P r}{2} + \frac{c}{r} \quad \text{at} \quad r = R, \tag{A.8}
\]

we obtain

\[
Q_r(R) = \frac{-P \pi R^2}{2 \pi R} = \frac{-PR}{2}, \tag{A.9}
\]

which implies \( c = 0 \).

Hence

\[
Q_r(r) = \frac{-P r}{2}. \tag{A.10}
\]

Substituting back Eq. (A.10) in Eq. (A.3), we obtain

\[
\frac{dM_r}{dr} - \frac{Pr}{2} + \frac{1}{r} (M_\theta - M_\theta) = 0. \tag{A.11}
\]

Substituting Eq. (A.1) in Eq. (A.4), we obtain

\[
\left\{ \frac{d}{dr} \left( A_{11} \frac{du_0(r)}{dr} + A_{12} \frac{u_0(r)}{r} - B_{11} \frac{d\theta(r)}{dr} - B_{12} \frac{1}{r} \theta(r) - N_r^p \right) + \right. \\
\left. \frac{1}{r} \left( A_{21} \frac{du_0(r)}{dr} + A_{22} \frac{u_0(r)}{r} - B_{21} \frac{d\theta(r)}{dr} - B_{22} \frac{1}{r} \theta(r) - N_\theta^p \right) \right\} = 0, \tag{A.12}
\]
which is simplified as

\[
\begin{align*}
\left( A_{11} \left[ \frac{d^2 u_0(r)}{dr^2} + 1 \frac{du_0(r)}{r \, dr} + \frac{u_0(r)}{r^2} \right] - B_{11} \left[ \frac{d^2 \theta(r)}{dr^2} + 1 \frac{d\theta(r)}{r \, dr} + \frac{\theta(r)}{r^2} \right] \right) &= 0 \quad (A.13)
\end{align*}
\]

This simplification occurs because the piezoelectric properties and the field are uniform on the \( r-\theta \) plane making \( \frac{d}{dr} \left( N^p_r \right) + \frac{1}{r} \left( N^p_r - N^p_\theta \right) \) go to zero.

Similarly substituting Eq. (A.2) in Eq. (A.11), we obtain

\[
\begin{align*}
B_{11} \left[ \frac{d^2 u_0(r)}{dr^2} + 1 \frac{du_0(r)}{r \, dr} + \frac{u_0(r)}{r^2} \right] - D_{11} \left[ \frac{d^2 \theta(r)}{dr^2} + 1 \frac{d\theta(r)}{r \, dr} + \frac{\theta(r)}{r^2} \right] &= \frac{Pr}{2} \quad (A.14)
\end{align*}
\]

Substituting \( r = e^s \) in Eq. (A.13) and Eq. (A.14), we obtain

\[
\begin{align*}
\left( D^2 - 1 \right) \left[ A_{11} u_0(s) - B_{11} \theta(s) \right] &= 0, \quad (A.15)
\end{align*}
\]

and

\[
\begin{align*}
\left( D^2 - 1 \right) \left[ B_{11} u_0(s) - D_{11} \theta(s) \right] &= \frac{Pe^{3s}}{2}. \quad (A.16)
\end{align*}
\]

Eq. (A.15) and Eq. (A.16) can be written as:

\[
\begin{align*}
\left( D^2 - 1 \right) [y_1] &= 0, \quad (A.17)
\end{align*}
\]

and

\[
\begin{align*}
\left( D^2 - 1 \right) [y_2] &= \frac{Pe^{3s}}{2}, \quad (A.18)
\end{align*}
\]

where,

\[
y_1 = A_{11} u_0(s) - B_{11} \theta(s), \quad (A.19)
\]

and

\[
y_2 = B_{11} u_0(s) - D_{11} \theta(s). \quad (A.20)
\]
We know that, general solution to the above equations are given by

\[ y_1 = ae^t + be^{-s}, \quad (A.21) \]

and

\[ y_2 = ce^t + de^{-s} + y_p. \quad (A.22) \]

(i.e.)

\[ y_1 = ar + \frac{b}{r}, \quad (A.23) \]

and

\[ y_2 = cr + \frac{d}{r} + y_p. \quad (A.24) \]

If \( y_1 \) and \( y_2 \) (which are linear combinations of \( u_0 \) and \( \Theta \)) were just functions of \( r \) and \( \frac{1}{r} \) then \( u_0 \) and \( \Theta \), will also be in the same form as \( y_1 \) and \( y_2 \), which results in the following expression of the primary variables:

\[ u_0 = a_1 r + \frac{a_2}{r} + \text{particular solution}, \quad (A.25) \]

and

\[ \Theta = b_1 r + \frac{b_2}{r} + \text{particular solution}. \quad (A.26) \]

The particular solution to the equation

\[ \left( D^2 - 1 \right) [y_2] = \frac{Pe^{3s}}{2}, \quad (A.27) \]

is given by
\[ y_p = \frac{P e^{3x}}{2 (3^2 - 1)} = \frac{P e^{3x}}{16} = \frac{P r^3}{16}. \]  

(A.28)

This correction for particular solution is found by inverting the relationship between \( y_1, y_2 \) and \( u_0, \theta \) in the following manner:

\[
\begin{pmatrix}
  u_0 \\
  \theta
\end{pmatrix} = \frac{1}{(A_{11} D_{11} - B_{11})} \begin{pmatrix}
  D_{11} & -B_{11} \\
  B_{11} & -A_{11}
\end{pmatrix} \begin{pmatrix}
  0 \\
  Pr^3
\end{pmatrix}.
\]

(A.29)

The quantities

\[
D_{11} - \frac{B_{11}^2}{A_{11}} = D_{11}^*,
\]

(A.30)

and

\[
\frac{B_{11}}{A_{11}} = \alpha.
\]

(A.31)

are defined to simplify the solution. The resulting expression for solution is given by

\[
u_0 = a_t r + \frac{a_2}{r} \frac{\alpha}{D_{11}} \left( \frac{Pr^3}{16} \right),
\]

(A.32)

and

\[
\theta = b_t r + \frac{b_2}{r} - \frac{1}{D_{11}^*} \left( \frac{Pr^3}{16} \right),
\]

(A.33)

which is same as the solutions shown in Eq. (3.36) and Eq. (3.37) of the equation of Chapter 3.
APPENDIX B
ANALYTICAL SOLUTION

An analytical solution is obtained in terms of variables that are functions of primary variables in the following manner.

**Analytical Expression for Radial Defection \( u_0 \) and Slope \( \theta \)**

Equations (3.44), (3.47), (3.56) and (3.58) is simplified by defining the following constants:

\[
A_i^{eq} = \left( A_{i1}^{(1)} + A_{i2}^{(1)} \right), \quad B_i^{eq} = \left( B_{i1}^{(1)} + B_{i2}^{(1)} \right), \quad D_i^{eq} = \left( D_{i1}^{(1)} + D_{i2}^{(1)} \right),
\]

\[
f_1 = \frac{A_{11}^{(1)}}{D_{11}^{(1)}}, \quad f_2 = \frac{B_{11}^{(1)}}{D_{11}^{(1)}}, \quad f_3 = \frac{D_{11}^{(1)}}{D_{11}^{(1)}},
\]

\[
P^* = \frac{PR_1^2}{16}, \quad f_4 = (1 + v_s) + \frac{R_2^2}{R_1^2} (1 - v_s),
\]

and

\[
f_5 = (3 + v_s) + \frac{R_2^4}{R_1^4} (1 - v_s).
\]

(B.1)

In the inner composite region, generalized forces at the interface described in Eq. (3.49) simplifies to the form shown in Eq. (B.2) by using the above constants:

\[
\begin{bmatrix}
N_i^{(1)}(R_1) \\
M_i^{(1)}(R_1)
\end{bmatrix}
= 
\begin{bmatrix}
A_i^{eq} & -B_i^{eq} \\
B_i^{eq} & -D_i^{eq}
\end{bmatrix}
\begin{bmatrix}
a_1^{(1)} \\
b_1^{(1)}
\end{bmatrix}
+ P^*
\begin{bmatrix}
f_2 - f_1 \alpha \\
-\alpha f_2 - 3 + f_3
\end{bmatrix}
- \begin{bmatrix}
N^p \\
M^p
\end{bmatrix}.
\]

(B.2)

Rewriting Eq. (3.48), we have

\[
\begin{bmatrix}
u_0^{(1)}(R_1) \\
\theta^{(1)}(R_1)
\end{bmatrix}
= R_1
\begin{bmatrix}
a_1^{(1)} \\
b_1^{(1)}
\end{bmatrix}
- \frac{PR_1^3}{16D_{11}^{(1)}}
\begin{bmatrix}
\alpha \\
1
\end{bmatrix}.
\]
In the outer annular region, generalized forces at the interface described in Eq. (3.59) on substituting the constants described in Eq. (B.1) reduces to

\[
\begin{bmatrix}
N^{(2)}_r (R_i) \\
M^{(2)}_r (R_i)
\end{bmatrix} = \begin{bmatrix}
A^{(2)}_i \\
0
\end{bmatrix} \begin{bmatrix}
a^{(2)}_i \\
b^{(2)}_i
\end{bmatrix} + P^* \begin{bmatrix}
0 \\
f_5
\end{bmatrix}.
\] (B.3)

Rewriting Eq. (3.56), we obtain

\[
\begin{bmatrix}
u^{(2)}_0 (r) \\
\theta^{(2)} (r)
\end{bmatrix} = \begin{bmatrix}
r - \frac{R^2_r}{r} \\
0
\end{bmatrix} \begin{bmatrix}
a^{(2)}_i \\
b^{(2)}_i
\end{bmatrix} - \frac{P}{16D^{(2)}_{11}} \begin{bmatrix}
0 \\
r^3 - \frac{R^4_r}{r}
\end{bmatrix}.
\] (B.4)

Substituting Eqs. (3.48) and (3.56) in interface conditions shown in Eqs. (3.60) & (3.61), we obtain

\[
\begin{bmatrix}
a^{(1)}_i \\
b^{(1)}_i
\end{bmatrix} - \frac{PR^3_i}{16D^{(1)}_{11}} \begin{bmatrix}
a^{(2)}_i \\
\theta^{(2)} (R_i)
\end{bmatrix} = \begin{bmatrix}
R_i - \frac{R^2_i}{R_i} \\
0
\end{bmatrix} \begin{bmatrix}
a^{(2)}_i \\
b^{(2)}_i
\end{bmatrix} - \frac{P}{16D^{(2)}_{11}} \begin{bmatrix}
0 \\
r^3 - \frac{R^4_i}{R_i}
\end{bmatrix}.\] (B.4)

Dividing both sides of the above equation by $R_i$ and combining terms containing $P^*$, we obtain

\[
\begin{bmatrix}
a^{(1)}_i \\
b^{(1)}_i
\end{bmatrix} = \begin{bmatrix}
1 - \frac{R^2_i}{R_i} \\
0
\end{bmatrix} \begin{bmatrix}
a^{(2)}_i \\
b^{(2)}_i
\end{bmatrix} - P^* \begin{bmatrix}
-\frac{\alpha}{D^{(2)}_{11}} \\
\frac{1}{D^{(2)}_{11}} \left(1 - \frac{R^4_i}{R_i} \right) - \frac{1}{D^{(1)}_{11}}
\end{bmatrix}.
\] (B.5)

The above expression in matrix form is simplified to yield

\[
a^{(1)}_i = \left(1 - \frac{R^2_i}{R_i} \right) a^{(2)}_i + \frac{P^* \alpha}{D^{(1)}_{11}}
\] (B.6)

and

\[
b^{(1)}_i = \left(1 - \frac{R^2_i}{R_i} \right) b^{(2)}_i - \frac{P^*}{D^{(2)}_{11}} \left(1 - \frac{R^4_i}{R_i} \right) + \frac{P^*}{D^{(1)}_{11}}.
\] (B.7)
Substituting Eqs. (B.2) - (B.3) in interface compatibility conditions shown in Eqs. (3.62) - (3.63), we obtain

\[
\begin{bmatrix}
A_1^{eq} & -B_1^{eq} \\
B_1^{eq} & -D_1^{eq}
\end{bmatrix}
\begin{bmatrix}
ad_1^{(1)} \\
b_1^{(1)}
\end{bmatrix}
+ P \begin{bmatrix}
f_2 - f_1 \alpha \\
-\alpha f_2 + 3 + f_3
\end{bmatrix}
+ \begin{bmatrix}
N^p \\
M^p
\end{bmatrix}
= \begin{bmatrix}
A_1^{(2)} a_1^{(2)} \\
-D_1^{(2)} b_1^{(2)}
\end{bmatrix} f_4 + P \begin{bmatrix}
0 \\
f_5
\end{bmatrix}.
\]

(B.8)

Rewriting the above expression, we obtain

\[
\begin{bmatrix}
A_1^{(2)} a_1^{(2)} \\
-D_1^{(2)} b_1^{(2)}
\end{bmatrix} f_4 = \begin{bmatrix}
A_1^{eq} & -B_1^{eq} \\
B_1^{eq} & -D_1^{eq}
\end{bmatrix}
\begin{bmatrix}
ad_1^{(1)} \\
b_1^{(1)}
\end{bmatrix}
+ P \begin{bmatrix}
f_2 - f_1 \alpha \\
-\alpha f_2 + 3 + f_3 - f_5
\end{bmatrix}
- \begin{bmatrix}
N^p \\
M^p
\end{bmatrix}.
\]

(B.9)

Eq.(B.9) is simplified by defining the following constants:

\[ t_1 = f_2 - f_1 \alpha \]  

(B.10)

and

\[ t_2 = -\alpha f_2 + 3 + f_3 - f_5. \]  

(B.11)

Substituting Eqs. (B.10) - (B.11) in Eq.(B.9), we obtain

\[ a_1^{(2)} = \frac{A_1^{eq} a_1^{(1)} - B_1^{eq} b_1^{(1)} + P^* t_1 - N^p}{A_1^{(2)} f_4} \]  

(B.12)

and

\[ b_1^{(2)} = \frac{D_1^{eq} b_1^{(1)} - B_1^{eq} a_1^{(1)} - P^* t_2 + M^p}{D_1^{(2)} f_4}. \]  

(B.13)

Substituting for \( a_1^{(1)} \) and \( b_1^{(1)} \) from Eqs. (B.6) - (B.7) in Eqs. (B.12) and (B.13), we obtain

\[ a_1^{(2)} = \frac{A_1^{eq} \left(1 - \frac{R_2^2}{R_1^2}\right) a_1^{(2)} + P^* \alpha\left(1 - \frac{R_2^2}{R_1^2}\right) b_1^{(1)} - B_1^{eq}\left(1 - \frac{R_2^2}{R_1^2}\right) b_1^{(2)} - \frac{P^*}{D_1^{(2)} R_1^2} \left(1 - \frac{R_4^4}{R_1^4}\right) + \frac{P^*}{D_1^{(1)} R_1^2}}{A_1^{(2)} f_4} + P^* t_1 - N^p. \]  

(B.14)

and
\[ b_1^{(2)} = \frac{D_1^{eq} \left( \left( 1 - \frac{R^2}{R_i^2} \right) b_1^{(2)} - \frac{P^*}{D_1^{eq}} \left( 1 - \frac{R^4}{R_i^4} \right) + \frac{P^*}{D_1^{eq}(1)} \right) - B_1^{eq} \left( \left( 1 - \frac{R^2}{R_i^2} \right) a_1^{(2)} + \frac{P^* \alpha}{D_1^{eq}(1)} \right) - P^* t_2 + M^p }{D_1^{(2)} f_4} \]  

(B.15)

The above expressions are simplified to yield

\[ a_1^{(2)} = \frac{-B_1^{eq} \left( \left( 1 - \frac{R^2}{R_i^2} \right) b_1^{(2)} - \frac{P^*}{D_1^{(2)}} \left( 1 - \frac{R^4}{R_i^4} \right) + \frac{P^*}{D_1^{(1)}} \right) + P^* t_1 - N^p + \frac{P^* \alpha A_1^{eq}}{D_1^{(1)}} }{A_1^{(2)} f_4 - \left( 1 - \frac{R^2}{R_i^2} \right) A_1^{eq}} \]  

(B.16)

and

\[ b_1^{(2)} = \frac{-B_1^{eq} \left( \left( 1 - \frac{R^2}{R_i^2} \right) a_1^{(2)} + \frac{P^* \alpha}{D_1^{(1)}} \right) - P^* D_1^{eq} \left( 1 - \frac{R^4}{R_i^4} \right) + \frac{P^* D_1^{eq}}{D_1^{(1)}} - P^* t_2 + M^p }{D_1^{(2)} f_4 - \left( 1 - \frac{R^2}{R_i^2} \right) D_1^{eq}} \]  

(B.17)

Eqs. (B.14) – (B.17) are simplified by defining the following constants:

\[ g_1 = A_1^{(2)} f_4 - \left( 1 - \frac{R^2}{R_i^2} \right) A_1^{eq}, \]  

(B.18)

\[ g_2 = B_1^{eq} \left( 1 - \frac{R^2}{R_i^2} \right), \]  

(B.19)

and

\[ g_3 = D_1^{(2)} f_4 - \left( 1 - \frac{R^2}{R_i^2} \right) D_1^{eq}. \]  

(B.20)

Substituting Eqs. (B.18) – (B.20) in Eqs. (B.14) – (B.17), we obtain

\[ a_1^{(2)} = \frac{-g_2 b_1^{(2)} + \frac{P^* B_1^{eq}}{D_1^{(2)}} \left( 1 - \frac{R^4}{R_i^4} \right) - \frac{P^* B_1^{eq}}{D_1^{(1)}} - P^* t_1 - N^p + \frac{P^* \alpha A_1^{eq}}{D_1^{(1)}} }{g_1} \]  

(B.21)

and
\[ b_1^{(2)} = \frac{-g_3 a_1^{(2)} - \frac{P^* B_1^{eq} \alpha}{D_1^{(1)}} - \frac{P^* D_1^{eq}}{D_1^{(2)}} \left( 1 - \frac{R_2^4}{R_1^4} \right) + \frac{P^* D_1^{eq}}{D_1^{(1)}}}{g_3} - P^* t_2 + M^p }{g_3}. \] (B.22)

Substituting Eq. (B.22) in Eq. (B.21), we obtain

\[ a_1^{(2)} = \frac{P^*}{g_3 g_1 - g_2^2 \left[ \frac{g_2}{g_1} \left( \frac{B_1^{eq} \alpha}{D_1^{(1)}} + \frac{D_1^{eq}}{D_1^{(2)}} \left( 1 - \frac{R_2^4}{R_1^4} \right) - \frac{D_1^{eq}}{D_1^{(1)}} + t_2 - \frac{M^p}{P^*} \right) \right] + \left[ \frac{g_3}{g_1} \left( \frac{B_1^{eq}}{D_1^{(1)}} \left( \frac{1}{R_1^4} - \frac{R_2^4}{R_1^4} \right) - \frac{B_1^{eq}}{R_1^4} + t_1 - \frac{N^p}{P^*} + \frac{\alpha A_1^{eq}}{D_1^{(1)}} \right) \right]. \] (B.23)

Similarly substituting Eq. (B.21) in Eq. (B.22), we obtain

\[ b_1^{(2)} = \frac{P^*}{g_3 g_1 - g_2^2 \left[ \frac{g_2}{g_1} \left( \frac{B_1^{eq} \alpha}{D_1^{(1)}} + \frac{D_1^{eq}}{D_1^{(2)}} \left( 1 - \frac{R_2^4}{R_1^4} \right) - \frac{D_1^{eq}}{D_1^{(1)}} + t_2 - \frac{M^p}{P^*} \right) \right] + \left[ \frac{g_3}{g_1} \left( \frac{B_1^{eq} \alpha}{D_1^{(1)}} \left( \frac{1}{R_1^4} - \frac{R_2^4}{R_1^4} \right) - \frac{B_1^{eq}}{R_1^4} + t_1 - \frac{N^p}{P^*} + \frac{\alpha A_1^{eq}}{D_1^{(1)}} \right) \right]. \] (B.24)

Substituting Eq. (B.24) in Eq. (B.6), we obtain

\[ a_1^{(1)} = \frac{P^*}{g_3 g_1 - g_2^2 \left[ \frac{g_2}{g_1} \left( \frac{B_1^{eq} \alpha}{D_1^{(1)}} + \frac{D_1^{eq}}{D_1^{(2)}} \left( 1 - \frac{R_2^4}{R_1^4} \right) - \frac{D_1^{eq}}{D_1^{(1)}} + t_2 - \frac{M^p}{P^*} \right) \right] + \left[ \frac{g_3}{g_1} \left( \frac{B_1^{eq} \alpha}{D_1^{(1)}} \left( \frac{1}{R_1^4} - \frac{R_2^4}{R_1^4} \right) - \frac{B_1^{eq}}{R_1^4} + t_1 - \frac{N^p}{P^*} + \frac{\alpha A_1^{eq}}{D_1^{(1)}} \right) \right]. \] (B.25)

Substituting Eq. (B.23) in Eq. (B.7), we obtain

\[ b_1^{(1)} = \frac{P^*}{g_3 g_1 - g_2^2 \left[ \frac{g_2}{g_1} \left( \frac{B_1^{eq} \alpha}{D_1^{(1)}} + \frac{D_1^{eq}}{D_1^{(2)}} \left( 1 - \frac{R_2^4}{R_1^4} \right) - \frac{D_1^{eq}}{D_1^{(1)}} + t_2 - \frac{M^p}{P^*} \right) \right] + \left[ \frac{g_3}{g_1} \left( \frac{B_1^{eq} \alpha}{D_1^{(1)}} \left( \frac{1}{R_1^4} - \frac{R_2^4}{R_1^4} \right) - \frac{B_1^{eq}}{R_1^4} + t_1 - \frac{N^p}{P^*} + \frac{\alpha A_1^{eq}}{D_1^{(1)}} \right) \right]. \] (B.26)

Equations (B.23) – (B.26) gives the expression for the constants appearing in the solution shown in Eqs. (3.44) and (3.56). Hence we have an analytical solution for the primary field variables radial deflection \( u_0 (r) \) and the slope \( \theta (r) \).
Analytical Expression for Vertical Deflection $w_0$

Vertical deflection $w_0(r)$ is calculated by integrating the slope $\theta(r)$ with respect to the radius in each of the regions and by applying the fixed boundary condition at the clamp in the outer annular region and by applying the compatibility condition that the deflection should be continuous at the interface in the central composite region. Therefore, in the outer annular region,

\[
\frac{w_0^{(2)}}{\text{const}}(r) = \int \theta^{(2)}(r) \, dr + \text{Const}, \tag{B.27}
\]

(i.e.)

\[
w_0^{(2)}(r) = b_1^{(2)} \left( \frac{r^2 - R_2^2}{2} - R_2^2 \ln \left( \frac{r}{R_2} \right) \right) + \frac{P \left( 4R_2^4 \ln \left( \frac{r}{R_2} \right) - r^4 \right)}{64D_{11}^{(2)}} + \text{const} \tag{B.28}
\]

By applying the clamped boundary condition that

\[
w_0^{(2)}(R_2) = 0, \tag{B.29}
\]

in the expression for $w_0^{(2)}(r)$ given in Eq. (B.28), we obtain the expression for vertical deflection in the outer annular region, which is given by

\[
w_0^{(2)}(r) = b_1^{(2)} \left( \frac{r^2 - R_2^2}{2} - R_2^2 \ln \left( \frac{r}{R_2} \right) \right) + \frac{P \left( 4R_2^4 \ln \left( \frac{r}{R_2} \right) - r^4 + R_2^4 \right)}{64D_{11}^{(2)}}. \tag{B.30}
\]

Hence the vertical deflection at the interface is given by

\[
w_0^{(2)}(R_1) = b_1^{(2)} \left( \frac{R_1^2 - R_2^2}{2} - R_2^2 \ln \left( \frac{R_1}{R_2} \right) \right) + \frac{P \left( 4R_2^4 \ln \left( \frac{R_1}{R_2} \right) - R_1^4 + R_2^4 \right)}{64D_{11}^{(2)}}. \tag{B.31}
\]

In the inner composite region the vertical deflection is given by

\[
w_0^{(1)}(r) = \int \theta^{(1)}(r) \, dr + \text{Const}, \tag{B.32}
\]
Thus the vertical deflection at the interface is given by

$$w_0^{(1)}(r) = b_1^{(1)} \frac{r^2}{2} - \left( \frac{P r^4}{64 D_{11}^{(1)}} \right) + \text{const}.$$  \hspace{2cm} (B.33)

By applying the interface compatibility condition that

$$w_0^{(1)}(R) = w_0^{(2)}(R),$$  \hspace{2cm} (B.35)

in Eqs. (B.34) and (B.31), we obtain

$$\text{const} = b_1^{(2)} \left( \frac{R_1^2 - R_2^2}{2} - R_2^2 \ln \left( \frac{R_1}{R_2} \right) \right) + \frac{P \left( 4R_2^2 \ln \left( \frac{R_1}{R_2} \right) - R_1^4 + R_2^4 \right)}{64 D_{11}^{(2)}} + \frac{P R_1^4 - b_1^{(1)} R_1^2}{2}.$$  \hspace{2cm} (B.36)

By substituting the constant obtained in Eq. (B.36), in Eq. (B.33), we have the expression for vertical deflection in the central composite region as

$$w_0^{(1)}(r) = \left\{ b_1^{(1)} \left( \frac{r^2 - R_1^2}{2} \right) - \frac{P \left( r^4 - R_1^4 \right)}{64 D_{11}^{(1)}} + b_1^{(2)} \left( \frac{R_1^2 - R_2^2}{2} - R_2^2 \ln \left( \frac{R_1}{R_2} \right) \right) \right\} +$$

$$\frac{P \left( 4R_2^2 \ln \left( \frac{R_1}{R_2} \right) - R_1^4 + R_2^4 \right)}{64 D_{11}^{(2)}}.$$  \hspace{2cm} (B.37)

**Analytical Expression for Short-Circuit Acoustic Compliance $C_{AS}$**

The expression for short-circuit acoustic compliance is obtained by finding the ratio of the volume displaced by the unimorph due to applied pressure to the magnitude of the pressure load (i.e. $V = 0$);
Substituting Eqs. (B.37) and (B.30) in Eq. (B.38), we obtain

\[
C_{AS} = \frac{\left( \int_0^{R_1} w_0^{(1)}(r)2\pi rdr + \int_{R_1}^{R_2} w_0^{(2)}(r)2\pi rdr \right)}{P} \bigg|_{v=0} .
\]  

(B.38)

The above expression on simplification yields

\[
C_{AS} = \frac{2\pi}{P} \left[ \int_0^{R_1} \left( b_1^{(1)} \left( \frac{r^2-R_1^2}{2} - \frac{P(r^4-R_1^4)}{64D_1^{(1)}} \right) + b_1^{(2)} \left( \frac{R_1^2-R_2^2-R_2^2\ln\left( \frac{R_1}{R_2} \right)}{2} \right) \right) rdr + \right.
\]

\[
\left. \int_{R_1}^{R_2} \left( b_1^{(2)} \left( \frac{r^2-R_2^2}{2} - R_2^2\ln\left( \frac{r}{R_2} \right) \right) + \frac{P\left(4R_2^4\ln\left( \frac{R_2}{R_1} \right) - r^4 + R_2^4\right)}{64D_1^{(2)}} \right) rdr \bigg] \bigg|_{v=0} .
\]  

(B.39)

which is simplified further to yield the following expression:

\[
C_{AS} = \frac{2\pi}{P} \left[ \int_0^{R_1} \left( b_1^{(1)} \left( \frac{-R_1^4}{8} \right) + b_1^{(2)} \left( \frac{(R_1^2-R_2^2)^2}{8} \right) \right) rdr + \right. \]

\[
\left. \int_{R_1}^{R_2} \left( -P\left( r^4-R_1^4 \right) \frac{P\left(4R_2^4\ln\left( \frac{R_2}{R_1} \right) - r^4 + R_2^4\right)}{64D_1^{(1)}} + \frac{64D_1^{(2)}}{64D_1^{(2)}} \right) rdr \bigg] \bigg|_{v=0}
\]  

(B.40)
Computing the definite integrals found in the above expression, we obtain

\[
C_{AS} = \frac{2\pi}{P} \left[ \int_{0}^{R_1} \left( \frac{-R_1^4}{8} + b_1^{(2)} \frac{\left( R_2^2 - R_1^2 \right)^2}{8} + P \frac{r^5 - rR_1^4}{64D_{11}^{(1)}} + \frac{rP \left( 4R_2^4 \ln \left( \frac{R_1}{R_2} \right) - R_1^4 + R_2^4 \right)}{64D_{11}^{(2)}} \right) \, dr \right]_{V=0}.
\]

The above expression on simplification yields the analytical expression for the short-circuit acoustic compliance of the piezoelectric unimorph given by

\[
C_{AS} = \frac{2\pi}{P} \left[ \left( b_1^{(1)} \left( \frac{-R_1^4}{8} \right) + b_1^{(2)} \frac{\left( R_2^2 - R_1^2 \right)^2}{8} + \frac{P^* R_1^4}{12D_{11}^{(1)}} + \frac{P' \left( 12R_2^4 \ln \left( \frac{R_1}{R_2} \right) - 3R_1^4 + 3R_2^4 \right)}{24D_{11}^{(2)}} \right) \right]_{V=0}.
\]

where

\[
C_{AS} = \frac{2\pi}{P} \left[ \left( b_1^{(1)} \left( \frac{-R_1^4}{8} \right) + b_1^{(2)} \frac{\left( R_2^2 - R_1^2 \right)^2}{8} + \frac{P^* R_1^4}{12D_{11}^{(1)}} + \frac{P' \left( -R_1^4 + 3R_2^4 - 2R_1^4 / R_2^2 \right)}{12D_{11}^{(2)}} \right) \right]_{V=0}.
\]
\[ b_{1}^{(2)} = \frac{p'}{g_{1}g_{3} - g_{2}^{2}} \left[ g_{2} \left( \frac{B_{1}^{eq}}{D_{11}^{(1)}} - \frac{1}{R_{1}^{4}} - \frac{1}{R_{1}^{3}} + \frac{1}{R_{1}^{2}} - \frac{1}{R_{1}} - \frac{1}{t_{1}} - \frac{\alpha A_{1}^{eq}}{D_{11}^{(1)}} \right) \right] \]  

(B.44)

and

\[ b_{1}^{(1)} = \frac{P' \left( 1 - \frac{R_{2}^{4}}{R_{1}^{4}} \right)}{g_{1}g_{3} - g_{2}^{2}} \left[ g_{2} \left( \frac{B_{1}^{eq}}{D_{11}^{(2)}} - \frac{1}{R_{1}^{4}} + \frac{1}{R_{1}^{3}} + \frac{1}{R_{1}^{2}} - \frac{1}{R_{1}} - \frac{1}{t_{2}} - \frac{\alpha A_{1}^{eq}}{D_{11}^{(2)}} \right) \right] \]  

(B.45)

### Analytical Expression for Effective Acoustic Piezoelectric Coefficient \( d_{A} \)

The expression for effective acoustic piezoelectric coefficient is obtained by finding the ratio of the volume displaced by the unimorph due to an applied voltage with zero pressure loading:

\[
\frac{V}{P=0} \int_{0}^{R_{1}} \left( \int_{0}^{R_{2}} \left( \begin{array}{c}
w_{0}^{(1)}(r)2\pi r dr + w_{0}^{(2)}(r)2\pi r dr \end{array} \right) \right) r dr \]  

(B.46)

Substituting Eqs. (B.37) and (B.30) in Eq.(B.46), we obtain

\[
d_{A} = \frac{2\pi}{V} \left( \int_{0}^{R_{1}} \left( \begin{array}{c}
b_{1}^{(1)}(r)2\pi r dr + b_{1}^{(2)}(r)2\pi r dr \end{array} \right) r dr \right) \]  

(B.47)

Grouping terms containing \( r \), we obtain

\[
d_{A} = \frac{2\pi}{V} \left( \begin{array}{c}
\int_{0}^{R_{1}} \left( \begin{array}{c}
b_{1}^{(1)}(r)2\pi r dr + b_{1}^{(2)}(r)2\pi r dr \end{array} \right) r dr \end{array} \right) \]  

(B.48)
Computing the definite integrals found in the above expression, we obtain

\[
d_A = \frac{2\pi}{V} \left[ b_1^{(1)} \left( \frac{-R_i^4}{8} \right) + b_2^{(2)} \left( \frac{R_i^2}{2} \right) \left( \frac{R_i^2 - R_s^2}{2} - R_s^2 \ln \left( \frac{R_i}{R_s} \right) \right) \right] + \left[ b_1^{(2)} \left( \frac{R_i^4 - R_s^4}{8} \right) + \frac{R_i^2 R_s^2}{2} \ln \left( \frac{R_i}{R_s} \right) \right] \bigg|_{P=0},
\]

(B.49)

The above expression on simplification yields the analytical expression for the effective acoustic piezoelectric coefficient of the piezoelectric unimorph given by

\[
d_A = \frac{2\pi}{V} \left[ b_1^{(1)} \left( \frac{-R_i^4}{8} \right) + b_2^{(2)} \left( \frac{R_i^2 \left( R_i^2 - R_s^2 \right)^2}{8} \right) \right] \bigg|_{P=0},
\]

(B.50)

where,

\[
b_1^{(1)} = \frac{g_2 N_i^p + g_4 M_i^p}{g_3 g_1 - g_2^2},
\]

(B.51)

\[
b_1^{(2)} = \frac{g_2 N_i^p + g_4 M_i^p \left( 1 - R_i^2 \right)}{g_3 g_1 - g_2^2},
\]

(B.52)

\[
N_i^p = \frac{E_p}{1-V_p} d_{31} V,
\]

(B.53)

and

\[
M_i^p = \frac{E_p}{1-V_p} d_{31} V \left( h_p + h_s \right).
\]

(B.54)

Analytical expression for acoustic mass was too complicated and hence was obtained numerically.
APPENDIX C
MATLAB CODES

Three MATLAB codes were programmed to implement the electroacoustic modeling discussed in the thesis. First code is used to derive the response of a particular system. The second code is used to extract curves shown in Chapter 5 of the thesis. The third code uses the direct analytical expressions obtained in Chapter 3. The MATLAB codes are split up into different modules, which are referred to as sub-routines from the three main programs. The three main programs direct the tasks to different sub-routines and output the necessary parameters. The sub-routines and the main programs used are presented in this appendix.

Subroutines used by Program 1

**shim.m**

```matlab
Es = 89.63e9;    %Young's modulus of shim(Brass)
vs = 0.324;      %Poisson's ratio of shim
Ts = 2.0574e-004; %thickness of the shim layer(annular)
R2 = 0.923*0.0254/2; %radius of the shim layer
mu = 1;         %const of proportionality of shear stress vs. shear strain
densy = 8700;   %Density of the shim
rhos = densy*Ts; %areal density of shim
```

**piezo.m**

```matlab
Es = 89.63e9;    %Young's modulus of shim(Brass)
vs = 0.324;      %Poisson's ratio of shim
Ts = 2.0574e-004; %thickness of the shim layer(annular)
R2 = 0.923*0.0254/2; %radius of the shim layer
mu = 1;         %const of proportionality of shear stress vs. shear strain
densy = 8700;   %Density of the shim
rhos = densy*Ts; %areal density of shim
```
Ep = 63.00E9; %Young’s modulus of the piezoelectric material(PZT-5H)(APC855)
vp = 0.33; %Poisson’s ratio of the piezoelectric material
tp = 2.3368e-004; %thickness of the piezoelectric material
R21 = 0.724; %piezoelectric material radii expressed as a fraction of shim radius
R1 = R21*R21; %Piezoelectric radius
d31 = -2.7e-10; %electromechanical transduction const of the piezoelectric material
densityp = 7700; %Density of the piezoelectric material
rhop = densityp*tp; %Areal Density of the piezoelectric material
dielectricconstant = 3250; %relative permitivity of the piezoelectric material
epsilon = dielectricconstant*epsilon0; %absolute permitivity of the piezoelectric material
P = 1; %Pressure in Pa applied on the top surface of the structure
V = 0; %Voltage in V applied
Ef = V/tp; %electric field in V applied in direction 3(Z)

% reading in electrical and mechanical loads

P = input('
Enter Pressure exerted on the composite section : '); %Pressure in Pa applied on the top surface of the structure
Ef = input('
Enter strength of Electric field applied across the piezoelectric : '); %electric field in V applied in direction 3(Z)

customisedpiezo.m % Material Properties of the piezoelectric material
% Ep = input('Enter Young’s modulus E of piezoelectric material layer (Eshim=89.63E9) in Pascals : '):
% vp = input('Enter Poisson’s ratio of piezoelectric material(Poisson’s ratio of shim=0.324) : '):
% tp = input('Enter Piezoelectric material Thickness(shim thickness=0.00023) in meters : '):
% R21 = input('Enter Radius of piezoelectric layer as a fraction of radius of shim(0.0127m): '):
% d31 = input('Enter d31 (electromagnetic transduction coefficient) for the piezoelectric : '):
% Densityp = input('Enter Density of piezoelectric(shim=16000kg/m3) : '):
% epsilon0 = input('Enter Relative Permitivity(Dielectric Constant of Piezoelectric : '):
% epsilon = epsilon*8.85e-12; %absolute permitivity of piezoelectric
% rhop = rhop*tp; %Areal Density
% R1 = R2*R21; %Radius of piezoelectric

% initialise.m % Initializing variables
% Initialising variables
Cp = 0;
Fp = 0;
num1=1400;
r=linspace(0.1,num1+1);
sr=size(r);
U0=zeros(sr);
theta=zeros(sr);
%
%%%%%%%%%%%%%%%%%%%%%%%% %%%%%%%%%%%%%%%%%%%%%%%%%
adshim.m
%%%%%%%%%%%%%%%%%%%%%%%% %%%%%%%%%%%%%%%%%%%%%%%%%

% Computing A,D for the outer annular plate
% AA = Es*ts/(1-(vs^2));
% DD = Es*ts^3/(12*(1-(vs^2)));
%
abdpiezo1.m
%%%%%%%%%%%%%%%%%%%%%%%% %%%%%%%%%%%%%%%%%%%%%%%%%

% Computing A,B,D for the central composite section
% Constitutive Relations for isotropic circular plates
Qs = [1 vs; vs 1].*(Es/(1-(vs^2)));
Qp = [1 vp; vp 1].*(Ep/(1-(vp^2)));
Qsi= [1 vsi; vsi 1].*(Esi/(1-(vsi^2)));
%
% Computing A,B,D for the central composite section
% taking reference plane as center of the shim layer
z1 = -ts/2; % distance of bottom of shim layer from reference
zi = ts/2; % distance of interface from reference
zi2 = ts/2 + tp; % distance of top of the piezoelectric layer from reference
z2 = ts/2 + tp + tsi/2; % distance of top of silver from reference

A = Qs.*(zi-z1) + Qp.*(zi2-zi) + Qsi.*(zi2-zi);
B = Qs.*((zi^2-z1^2)/2) + Qp.*((zi2^2-zi^2)/2) + Qsi.*((zi2^2-zi^2)/2);
D = Qs.*((zi^3-z1^3)/3) + Qp.*((zi2^3-zi^3)/3) + Qsi.*((zi2^3-zi^3)/3);
%
% Computing Delta(determinant of matrix mapping defined variables y1,y2 to U0 theta
delta = A(1,1)*D(1,1)-((B(1,1))^2);

% Computing fictitious forces due to piezoelectric

Cp = Ef* (Ep/(1-\nu_p)) * d31 * (z_{i2}^2 - z_i^2)/2;
Fp = Ef* (Ep/(1-\nu_p)) * d31 * (z_{i2} - z_i);

% solver.m

% Actual Module, which sets up the system of equations and superposes the solutions

% annular; (Sub-routine call – annular.m) – sets up system of equations for the annular section
% composite; (Sub-routine call – composite.m) – sets up system of equations for the composite section

% Computing deflections and Forces at the interface by superposition

if R1 ~= 0
    if R1 ~= R2
        Keff = (KMat1 - KMat2);
        Peff = -(PMat1 - PMat2);
        XMat = inv(Keff) * (Peff);
    end;
end;

if R1 ~= 0
    Const1 = inv amat1 *(XMat-f1);
end;

% Finding constants in the system of equations for Deflection in the annular region

if R1 ~= R2
    Const2 = inv amat2 *(XMat-f2);
    FMat = bMat2 * Const2 + g2;
end;

% Initializing deflections to zero

r = linspace(0,1,num1+1);
sr = size(r);
U0 = zeros(sr);
theta = zeros (sr);

% Computing deflections at discrete “num1” number of points

jj = floor(R21*num1)+1; % finding the argument at the interface
for i=1:num1+1
    rad1 = r(i)*R2;
    if (i<jj+1)
\[ U_0(i) = \text{Const1}(1) \cdot \text{rad1} - B(1,1) \cdot P \cdot (\text{rad1}^3)/(16 \cdot \text{delta}); \]
\[ \theta(i) = \text{Const1}(2) \cdot \text{rad1} - A(1,1) \cdot P \cdot (\text{rad1}^3)/(16 \cdot \text{delta}); \]
\[ \text{else} \]
\[ U_0(i) = \text{Const2}(1) \cdot (\text{rad1} - (R2^2)/\text{rad1}); \]
\[ \text{funcr} = (\text{rad1}^3) - (R2^4)/\text{rad1}; \]
\[ \theta(i) = \text{Const2}(2) \cdot (\text{rad1} - (R2^2)/\text{rad1}) - P \cdot \text{funcr}/(16 \cdot \text{DD}); \]
\[ \text{end}; \]
\[ \text{end}; \]

% Determining vertical deflection by integration of theta at "num1" number of points

\[ W_0=\text{zeros}(	ext{sr}); \]

\[ \text{for } i=\text{jj}+1: \text{num1}+1 \]
\[ W_0(i) = -(R2^2) \cdot \text{log}(r(i)) + ( (r(i)^2-1) \cdot 0.5 \cdot (R2^2) ) \cdot \text{Const2}(2) + (P^*(R2^4) \cdot \text{log}(r(i))/(16 \cdot \text{DD})) - \]
\[ P^*((r(i)^4)-1) \cdot (R2^4)/(64 \cdot \text{DD}); \]
\[ \text{end}; \]

% Vertical deflection at the interface

\[ \text{Wint}=-(R2^2) \cdot \text{log}(R21) + ((R21^2-1) \cdot 0.5 \cdot (R2^2)) \cdot \text{Const2}(2) + (P^*(R2^4) \cdot \text{log}(R21)/(16 \cdot \text{DD})) - \]
\[ P^*((R21^4)-1) \cdot (R2^4)/(64 \cdot \text{DD}); \]
\[ \text{WConst2}=\text{Wint}-\text{((Const1(2)*(R1^2)/2) - (A(1,1)*P*(R1^4))/(64 \cdot \text{delta}))}; \]
\[ \text{W0(jj)=Wint}; \]

\[ \text{for } i=1: \text{jj}-1 \]
\[ W_0(i) = (\text{Const1}(2) \cdot ((r(i)R2)^2)/2) - (A(1,1) \cdot P \cdot ((r(i)R2)^4))/(64 \cdot \text{delta}) + \text{WConst2}; \]
\[ \text{end}; \]

%

\textbf{annular.m}

\% Module, which sets up the system of equations for the outer annular plate

\% Compute stiffness due to annular plate KMat2 and

\% Setting up the system of equations KMat2*XM=PMat2

\textbf{if} R1==R2 \textbf{then}
\text{R12}=1/R21; \%
\text{factor}=(1+(R12^2))+\text{vs*(1-(R12^2))}; \%
\text{bbmat2}=[\text{factor*AA} 0;0 \text{-factor*DD}]; \%
\text{factor2}=-(R2^2-R1^2)/R1; \%
\text{amat2}=[\text{factor2} 0;0 \text{factor2}]; \%
\text{func} = (R1^3) - (R2^4)/R1; \%
\text{f2}=[0;-\text{P*func}/(16*\text{DD})]; \%
\text{difffunc} = 3*(R1^2) + (R2^4)/(R1^2); \%
\text{g2}=[0;\text{P}/16]*\text{difffunc*vs*func}/R1); \%
\text{KMat2}=\text{bbmat2*inv(amat2)}; \%
\text{PMat2}=g2-\text{bbmat2*inv(amat2)*f2}; \%
\text{end};
else
    XMat=[0;0];
    PMat1=[0;0];
    KMat1=[0 0;0 0];
end;
%
%  Module, which sets up the system of equations for the central composite plate
%
%  Compute stiffness due to composite section KMat2 and
% Setting up the system of equations KMat1*XMat=PMat2
if R1~=0
    amat1=[R1 0;0 R1];   %Deflection XMat= amat*Consts + f
    temp1=P*(R1^3)/(16*delta);
    f1=temp1*[-B(1,1); -A(1,1)];
    bmat1=[A(1,1)+A(1,2) -(B(1,1)+B(1,2));B(1,1)+B(1,2) -(D(1,1)+D(1,2))];
    temp2=P*R1*R1/(16*delta);
    temp3=-B(1,1) * A(1,2) + A(1,1) * B(1,2);
    temp5=-B(1,1) * B(1,2) + 3*delta + A(1,1) * D(1,2);
    fict = [Fp;Cp];
    g1=[temp2*temp3;temp2*temp5];
    KMat1=bmat1*inv(amat1);  %stiffness matrix (FMat=KMat*Xmat+PMat)
    KMat1=KMat1*[1 0;0 1];
    PMat1=g1-bmat1*inv(amat1)*f1;
    PMat1=PMat1+fict;
    PMat1=[1 0;0 1]*PMat1;
else
    XMat=[0;0];
    PMat1=[0;0];
    KMat1=[0 0;0 0];
end;
%
%  Lumped Element Modeling – Parameter Extraction
%
%  Computing Strain energy and hence compliance
% Numerical integration using Trapezoidal rule
temp1=r(1)*W0(1)*0.5+r(num1+1)*W0(num1+1)*0.5;
for i=2:num1
    temp1=temp1+W0(i)*r(i);
end;
temp1=((temp1^2)^0.5)*R2*(R2/num1);
se=pi*P*(temp1);  % strain energy
Area=pi*(R2^2);  % total Area
Weff=se/(0.5*P*Area);  % Deflection - volume velocity convention
Cme=Weff/(P*Area);  % Compliance - volume velocity convention
wr=W0(1)/Weff;  % ratio of the Displacement in the two conventions
Cms=Cme*(wr^2); % Compliance - max deflection convention

Computing Mass
% Numerical integration using Trapezoidal rule

temp3=(r(1)*(W0(1)^2)+r(jj)*(W0(jj)^2));
temp4=(r(jj)*(W0(jj)^2)+r(num1+1)*(W0(num1+1)^2));
for i=2:jj-1
    temp3=temp3+2*(W0(i)^2)*r(i);
end;
for i=jj+1:num1
    temp4=temp4+2*(W0(i)^2)*r(i);
end;
temp3=temp3*R2*(R2/num1)*0.5*(rhos+rhop);
temp4=temp4*R2*(R2/num1)*0.5*rhos;
Mms=2*pi*(temp3+temp4)/(W0(1)^2);  % Mass - max deflection Convention
Mme=Mms*(wr^2);

oparam1.m

Lumped Element Modeling – Other Miscellaneous Parameter Extraction

Area1=pi*R1*R1;    % Area of the inner composite section
Area=pi*R2*R2;     % Total Area
actmass=rhos*Area+(rhop)*Area1;    % Actual Mass of the plate
massratio=Mme/actmass;     % Effective Mass/Actual Mass
frequency=(1/(2*pi))*(Cme*Mme)^(-0.5);   % Resonant Frequency
sensitivity=Cme*frequency;

plotdef.m

Sub-routine to plot deflection curves
plotdef1.m

% Sub-routine to plot deflection curves
% figure(1);
plot(r,U0,'r');
Title(' Radial Deflection Vs Radius red(V=1,P=0) blue(P=1,V=0) ');
ylabel('Deflection u_0 ------->')
xlabel('r / R2 ------->');
hold on;
figure(2);
plot(r,theta,'r');
Title(' Angular Deflection Vs Radius red(V=1,P=0) blue(P=1,V=0) ');
ylabel('Angular Deflection \theta ------->')
xlabel('r / R2 ------->');
hold on;
figure(3);
plot(r,W0,'r');
Title(' Vertical Deflection Vs Radius red(V=1,P=0) blue(P=1,V=0) ');
ylabel('Deflection w_0 ------->')
xlabel('r / R2 ------->');
hold on;

integG1.m
Two-port Electromechanical model – parameter extraction

Computing Strain energy and hence compliance

Numerical integration using Trapezoidal rule

temp1=r(1)*W0(1)*0.5+r(num1+1)*W0(num1+1)*0.5;
for i=2:num1
    temp1=temp1+W0(i)*r(i);
end;
temp1=((temp1^2)^0.5)*R2*(R2/num1);
se=pi*(temp1);
Area=pi*(R2^2); %total Area
Weff=se/(0.5*Area); %Defection - volume velocity convention
G=Weff/(V); %Transduction Impedance -volume velocity
de=W0(1); %transduction capacitance
leverage=G/d31; %how much of d31 acts in the other direction
wre=W0(1)/Weff; %ratio of the Displacement in the two conventions
Cme=Cme*(wr^2); %Compliance - max deflection convention
Area1=pi*(R1^2); %Area of the piezo
Cef=epsilon*Area1/tp; %Electrical Free Capacitance

Program 1: Program used to derive Response of a particular Piezoelectric Transducer

MAIN PROGRAM 1

Programmed By
Suryanarayana A.N. Prasad
Dept. of AeMES
University of Florida

clear all;
shim; %Reading shim properties(Subroutine call - shim.m)
choice = input('nFor customised input enter 1, For standard set of input enter 2
: '); if (choice==1)
customizedpiezo; %Reading piezo properties(Subroutine call - customizedpiezo.m)
else
    piezo; %Reading piezo props from console(Subroutine call - piezo.m)
end;
initialise; %initialising sampling etc.(Subroutine call - initialise.m)
adshim; %} subroutine call - adshim.m}
% Response of the piezoelectric unimorph to unit pressure loading

tsi=0;
abdpiezo1;  %A,B,D Matrix extraction (subroutine call - abdpiezo1.m)
solver;  % (subroutine call - solver.m)

% Lumped Element Modeling – parameter extraction

integ1;  % (subroutine call - integ1.m)
oparam1;  % (subroutine call - oparam.m)- Miscellaneous parameters

% output

-W0(1)  %Centre Deflection(Maximum)
plotdef;  % (subroutine call - plotdef.m)
M1=FMat(2)  %Interface Bending Moment
F1=FMat(1)  %Interface Radial Force
Cme  %Equivalent Mechanical Compliance
Mme  %Equivalent Mechanical Mass
Massratio  %Massratio
frequency  %Short-circuit resonant Frequency
sensitivity  %Gain - Bandwidth-product
dV1=Weff*Area  %Volume displacement

% Response of the piezoelectric unimorph to unit voltage loading

% Initializing Voltage Loading

P=0;
V=1;
Ef=V/tp:

abdpiezo1;  % (subroutine call - abdpiezo1.m)
solver;  % (subroutine call - solver.m) – code to setup and solve the equations

% output

plotdef1;  % (subroutine call - plotdef1.m) – deflection plot

% Two-port parameter extraction

integG1;  % (subroutine call - integG1.m) – two-port network parameters

% output

Weff  %Effective Displacement
W0(1)  %Maximum Displacement
De = G  %Transduction Compliance
phi = Weff/Cme  %phi
phi1 = De/Cef  % phi prime
Cef  %Electrical free Compliance
K=De/((Cme*Cef)*0.5)  %lector-acoustic coupling factor
Ca=Cme*Area/Area  %Acoustic Compliance
Ma=Mme/Area/Area  %Acoustic Mass
dV2=Weff*Area  %Volume displacement
Subroutines used only by Program 2

shim.m

Material Properties of the Shim

Es = 89.63e9;    % Young's modulus of shim (Brass)
vs = 0.324;      % Poisson's ratio of shim
R2 = 2.54e-3/2;  % Radius of the shim layer
ts = R2*TSR2;    % Thickness of the shim layer (annular)
mu = 1;          % Const of proportionality of shear stress vs shear strain
densitys = 8700;  % Density of the shim
rhos = densitys*ts;  % Areal density of shim

piezo2.m

Material Properties of the Piezo

Ep = Es*EPES;    % Young's modulus of the piezo
vp = 0.3;        % Poisson's ratio of the piezo material
tp = ts*TPTS;    % Thickness of the piezo
R1 = R21*R2;     % Piezoelectric radius
d31 = -5.0e-11;  % Electromechanical transduction const of the piezo
densityp = 7600;  % Density of the piezo
rhop = densityp*tp;  % Areal Density of the piezo
epsilon0 = 8.85E-12;  % Permitivity of free space in F/m
dielectricconstant = 1000;  % Relative permitivity of the piezo
epsilon = dielectricconstant*epsilon0;  % Absolute permitivity of the piezo
P = 1;           % Pressure in Pa applied on the top surface of the structure
V = 0;           % Voltage in V applied
Ef = V/tp;       % Electric field in V applied in direction 3(Z)
silver;         % Load properties of silver (Sub-routine call – silver.m)

silver.m

Material Properties of the Silver Electrode

%
properties obtained from http://www.webelements.com/webelements/elements/text/Ag/phys.html

\[
\begin{align*}
E_s &= 83.9; \\
\nu_s &= 0.37; \\
\tau_{si} &= 0.0011 * 0.0254; \\
\text{density}_{si} &= 10490; \\
R_{si} &= 0.718 * 0.0254; \\
\rho_{si} &= \text{density}_{si} * \tau_{si} * (R_{si} / 0.02)^2;
\end{align*}
\]

\[
\text{Computing A,B,D for the central composite section}
\]

\[
Q_s = [1 \nu_s; \nu_s 1] * \left( \frac{E_s}{1-(\nu_s^2)} \right);
\]

\[
Q_p = [1 \nu_p; \nu_p 1] * \left( \frac{E_p}{1-(\nu_p^2)} \right);
\]

\[
\text{Computing A,B,D for the central composite section}
\]

\[
z_1 = -\frac{t_s}{2}; \quad \text{distance of bottom of shim layer from reference}
\]

\[
z_i = \frac{t_s}{2}; \quad \text{distance of interface from reference}
\]

\[
z_i2 = \frac{t_s}{2} + t_p; \quad \text{distance of top of the piezo layer from reference}
\]

\[
z_2 = \frac{t_s}{2} + t_p + \frac{\tau_{si}}{2}; \quad \text{distance of top of silver from reference}
\]

\[
A = Q_s * (z_i - z_1) + Q_p * (z_{i2} - z_i);
\]

\[
B = Q_s * \left( \frac{(z_i^2 - z_1^2)}{2} \right) + Q_p * \left( \frac{(z_{i2}^2 - z_i^2)}{2} \right);
\]

\[
D = Q_s * \left( \frac{(z_i^3 - z_1^3)}{3} \right) + Q_p * \left( \frac{(z_{i2}^3 - z_i^3)}{3} \right);
\]

\[
\text{Computing Delta (determinant of matrix mapping defined variables y1,y2 to U0 theta)}
\]

\[
\delta = A(1,1) * D(1,1) - (B(1,1))^2;
\]

\[
\text{Computing fictitious forces due to piezo}
\]

\[
C_p = \text{E}_f * (\text{E}_p / (1 - \nu_p)) * d_{31} * (z_{i2}^2 - z_i^2) / 2;
\]

\[
F_p = \text{E}_f * (\text{E}_p / (1 - \nu_p)) * d_{31} * (z_{i2} - z_i);
\]

\[
\text{Program 2: Program used to derive Response of a particular Piezoelectric Transducer}
\]

\[
\text{MAIN PROGRAM 2}
\]

Programmed By
clear all;
count=0;
EPES = 30/150;
TSR2 = 1/112.5;
rstep=0.005;
tstep=0.3;
tmax=1.3;
tmin=0.00001;
for TPTS=tmin:tstep:tmax
    count=count+1;
    count2=0;
    for R21=0:rstep:1
        if R21==0 R21=0.000001;
        else
            if R21==1 R21=0.999999;
        end;
        end;
        count2=count2+1;

    end;
end;

% Analysis with unit pressure loading

shim2; %Reading shim properties
piezo2; %Reading piezo props from console
initialise2; %initialising sampling etc. (Subroutine call - initialise.m)
adshim; % (subroutine call - adshim.m)
abdpiezo1; % (subroutine call – abdpiezo1.m)
solver; % (subroutine call – solver.m)
teg1; % (subroutine call – teg1.m)
oparam1; % (subroutine call – oparam1.m)
Wnd1=W0(1)*64*DD/R2/P/(R2^3); %Non-dimensional deflection for applied pressure
WN1(count,count2)=-Wnd1; %Non-dimensional deflection for applied pressure
Ca=Cme*Area/Area; % Acoustic Compliance
Ma=Mme/Area/Area; % Acoustic Mass
CND(count,count2)=Cme*Area*3*64*DD/R2^4; %Non-dimensional Compliance
MND(count,count2)=Mme/(1.8*rhos*Area); %Non-dimensional Mass
freqND(count,count2)=1/sqrt(CND(count,count2)*MND(count,count2));
%C1nd1=Const1*16*DD/R2^3
%C2nd1=Const2*16*DD/R2^3

% Analysis with unit voltage loading

piezo;
P=0;V=1;Ef=V/tp;
abdpiezo2; % (subroutine call – abdpiezo2.m)
solver; % (subroutine call – solver.m)
teg1; % (subroutine call – teg1.m)
Wnd2 = W0(1)*DD*2*(1+vs)/Cp/R2/R2; %Non-dimensional deflection for applied voltage
plot(radii,MND(count,:),k');
hold on;
drawnow;
figure(5);
plot(radii,phiND(count,:),k');
hold on;
drawnow;
figure(6)
%plot(radii,phiprimeND(count,:),k');
%hold on;
%drawnow;
%figure(11);
%plot(radii,phiND1(count,:),k');
%hold on;
%drawnow;
%figure(12)
plot(radii,phiprimeND1(count,:),k');
hold on;
drawnow;
%figure(13)
%plot(radii,phiprimeND2(count,:),k');
%hold on;
%drawnow;
%figure(14)
%plot(radii,phiprimeND3(count,:),k');
%hold on;
%drawnow;
%figure(15)
%plot(radii,phiprimeND4(count,:),k');
%hold on;
%drawnow;
figure(7);
plot(radii,phi(count,:),k');
hold on;
drawnow;
figure(8)
plot(radii,phiprime(count,:),k');
hold on;
drawnow;
figure(9)
plot(radii,K(count,:),k');
hold on;
drawnow;
figure(10);
%plot(radii,optim(count,:),k');
%hold on;
%drawnow;
%figure(16)
plot(radii,KND(count,:),k');
hold on;
drawnow;
figure(11)
plot(radii,chargeV(count,:),k');
hold on;
drawnow;
figure(12)
plot(radii2,inchargedensityV(count,:),'k'); hold on; drawnow; figure(13) plot(radii,freqND(count,:),'k'); hold on; drawnow; figure(14) plot(radii,chargedensityV(count,:), 'k'); hold on; drawnow; figure(15) plot(radii,chargeP(count,:),'k'); hold on; drawnow; figure(16) plot(radii,chargedensityP(count,:), 'k'); hold on; drawnow; figure(17) plot(radii2,inchargedensityP(count,:), 'k'); hold on; drawnow; end; figure(1); %title('plot of non-dimensional center deflection on application of unit pressure against Radius of piezo patch for Ep/Es=61/89.63 for different tp/ts'); xlabel('R_1/R_2'); ylabel('w(0)/(P*R_2^4/64D_2)'); figure(2); %title('plot of non-dimensional center deflection on application of unit voltage against Radius of piezo patch for Ep/Es=61/89.63 for different tp/ts'); xlabel('R_1/R_2'); ylabel('w(0)*2*D_2*(1+\nu_s)/C_pR_2^2'); figure(3); %title('plot of non-dimensional center deflection on application of unit pressure against Radius of piezo patch for Ep/Es=61/89.63 for different tp/ts'); xlabel('R_1/R_2'); ylabel('Non-Dimensional C_A'); figure(4); %title('plot of non-dimensional center deflection on application of unit voltage against Radius of piezo patch for Ep/Es=61/89.63 for different tp/ts'); xlabel('R_1/R_2'); ylabel('Non-Dimensional M_A'); figure(5); xlabel('R_1/R_2'); ylabel('Non-Dimensional \phi (leverage/C_N_D)'); figure(6); %xlabel('R_1/R_2'); %ylabel('Non-Dimensional \phi''(leverage)'); %figure(11); %xlabel('R_1/R_2'); %ylabel('Non-Dimensional \phi (W_N_D/C_N_D)'); %figure(12); xlabel('R_1/R_2'); ylabel('Non-Dimensional \phi'' (leverage/C_E_F_N_D)');
% figure(13);
% xlabel('R_1/R_2');
% ylabel('Non-Dimensional \phi'' (W_N_D/C_E_F_N_D)');
% figure(14);
% xlabel('R_1/R_2');
% ylabel('Non-Dimensional \phi'' (Leverage/C_E_F_N_D_2)');
% figure(15);
% xlabel('R_1/R_2');
% ylabel('Non-Dimensional \phi'' (W_N_D/C_E_F_N_D_2)');
figure(7);
xlabel('R_1/R_2');
ylabel('phi');
figure(8);
xlabel('R_1/R_2');
ylabel('phi''');
figure(9);
xlabel('R_1/R_2');
ylabel('K');
figure(10);
% title('plot of non-dimensional center deflection on application of unit pressure against Radius of piezo patch for Ep/Es=61/89.63 for different tp/ts');
% xlabel('R_1/R_2');
% ylabel('OPTIMAL PARAMETER');
% figure(16);
% title('plot of non-dimensional center deflection on application of unit pressure against Radius of piezo patch for Ep/Es=61/89.63 for different tp/ts');
xlabel('R_1/R_2');
ylabel('Universal K');
figure(11);
xlabel('R_1/R_2');
ylabel('Q(charge) applied V');
figure(12);
xlabel('R_1/R_2');
ylabel('incremental charge density V applied');
figure(13);
xlabel('R_1/R_2');
ylabel('Non-Dimensional Frequency');
figure(14);
xlabel('R_1/R_2');
ylabel('Q/A applied V');
figure(15);
xlabel('R_1/R_2');
ylabel('Q(charge) applied P');
figure(16);
xlabel('R_1/R_2');
ylabel('Q/A applied P');
figure(17);
xlabel('R_1/R_2');
ylabel('incremental charge density P applied');
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Program 3: Program implementing Direct Solution of a Particular Piezoelectric Transducer

directsolution.m
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Programmed By
%  Suryanarayana A.N. Prasad
%  Dept. of AeMES
%  University of Florida
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% M A I N  P R O G R A M   -   direct solution
%
shimg2;
piezog2;
initialise;
abdpiezo2;
Dstar=delta/A(1,1);
alpha=B(1,1)/A(1,1);
adshim;

% Intermediate constants defined in chapter -3

Aeq=A(1,1)+A(1,2);
Beq=B(1,1)+B(1,2);
Deq=D(1,1)+D(1,2);
f1=A(1,2)/Dstar;
f2=B(1,2)/Dstar;
f3=D(1,2)/Dstar;
Pstar=P*R1^2/16;
R21=R2/R1;
f4=(1+vs)+R21^2*(1-vs);
f5=(3+vs)+R21^4*(1-vs);
%f5=-f5;
t1=(f2-f1*alpha);
t2=(-alpha*f2+3+f3-f5);
g1=AA*f4-(1-R21^2)*Aeq;
g2=(1-R21^2)*Beq;
g3=DD*f4-(1-R21^2)*Deq;
temp=Pstar/(g3*g1-g2^2);

Ar1=Aeq/Dstar;
Ar2=Aeq/DD;
Br1=Beq/Dstar;
Br2=Beq/DD;
Dr1=Deq/Dstar;
Dr2=Deq/DD;
% Analytical expression for constants

temp2=(+Br1*alpha+Dr2*(1-R21^4)-Dr1+t2+Cp/Pstar);
temp3=(Br2*(1-R21^4)-Br1+t1+Fp/Pstar+alpha*Ar1);
a12=temp*(g2*temp2+g3*temp3);
a11=a12*(1-R21^2)+Pstar*alpha/Dstar;
% temp4=(-Br2*(1-R21^4)+Br1-t1-Fp/Pstar-alpha*Ar1);
% temp5=(-Br1*alpha-Dr2*(1-R21^4)+Dr1-t2-Cp/Pstar);
b12=-temp*(g1*temp2+g2*temp3);
b11=b12*(1-R21^2)+Pstar/Dstar-Pstar*(1-R21^4)/DD;

interface_i=round(num1/R21);
r=0:R2/(num1-1):R2;
for i=1:num1
    if i<=interface_i
        u0P(i)=a11*r(i)-alpha*P*(r(i)^3)/16/Dstar;
        thetaP(i)=b11*r(i)-P*(r(i)^3)/16/Dstar;
        w0P(i)=b11*((r(i)^2-R1^2)/2)-P*(r(i)^4-R1^4)/64/Dstar+b12*((R1^2-R2^2)/2-R2^2*log(r(i)/R2))+P*(4*R2^4*log(r(i)/R2)-r(i)^4+R2^4)/64/DD;
    else
        u0P(i)=a12*(r(i)-R2^2/r(i));
        thetaP(i)=b12*(r(i)-R2^2/r(i))-P*((r(i)^3)-(R2^4)/r(i))/16/DD;
        w0P(i)=b12*((r(i)^2-R2^2)/2-R2^2*log(r(i)/R2))+P*(4*R2^4*log(r(i)/R2)-r(i)^4+R2^4)/64/DD;
    end;
end;

% Evaluation of short-circuit acoustical compliance

temp4=Pstar/12/Dstar;
temp5=(R1^4);
temp6=(b12*)((R2^2)-(R1^2))^2/8-(b11*(R1^4))/8);
temp7=Pstar/12/DD;
temp8=-(R1^4)+(3*(R2^4))-2*(R2^6)/(R1^2);
Cas=2*pi*(temp6+temp4*temp5+temp7*temp8)/P;

% output module

figure(1);
plot(r/R2,u0P,'b');
xlabel('Normalized Radius r/R_2');
ylabel('radial displacement u_0(r)');
figure(2);
hold on;
plot(r/R2,thetaP,'b');
xlabel('Normalized Radius r/R_2');
ylabel('slope \theta (r)');
figure(3);
hold on;
plot(r/R2,w0P,'b');
xlabel('Normalized Radius r/R_2');
ylabel('Deflection w_0(r)');
P=0;V=1;Ei=E/V/tp;
abdpiezo2;

% Analytical expressions for constants
\[ b_{12v} = \frac{(g_2 F_p + g_1 C_p)}{(g_3 g_1 - g_2^2)}; \]
\[ b_{11v} = b_{12v} \times (1 - R_{21}^2); \]
\[ a_{12v} = \frac{(g_3 F_p + g_2 C_p)}{(g_3 g_1 - g_2^2)}; \]
\[ a_{11v} = a_{12v} \times (1 - R_{21}^2); \]

```matlab
for i=1:num1
    if i<=interface_i
        u0V(i)=a_{11v} r(i)-alpha*P*(r(i)^3/16/Dstar;
        thetaV(i)=b_{11v} r(i);
        w0V(i)=b_{11v} ((r(i)^2-R1^2)/2)+b_{12v} ((R1^2-R2^2)/2-R2^2 log(R1/R2));
    else
        u0V(i)=a_{12v} (r(i)-R2^2/r(i));
        thetaV(i)=b_{12v} (r(i)-R2^2/r(i));
        w0V(i)=b_{12v} ((r(i)^2-R2^2)/2-R2^2 log(r(i)/R2));
    end;
end;
```

```matlab
% Evaluation of effective acoustic piezoelectric coefficient

temp6v=(b_{12v} (R2^2-R1^2)^2/8-b_{11v} R1^4/8);
da=2*pi*(temp6v)/V;
```

```matlab
figure(1);
plot(r/R2,u0V,'r');
xlabel('Normalized Radius r/R_2');
ylabel('radial displacement u_0(r)');
figure(2);
hold on;
plot(r/R2,thetaV,'r');
figure(3);
hold on;
plot(r/R2,w0V,'r');
```

```matlab
%%
```

```matlab
% output module
```

```matlab
% Temporal analysis
```
APPENDIX D
FINITE ELEMENT MODEL (ABAQUS) INPUT FILE

Pressure Loading only (Normalized Piezoelectric Patch Radius = 0.2)

*Heading
** Job name: piezoplate_pre_R21_0_2_for_thesis Model name: Model-1
*Preprint, echo=YES, model=YES, history=YES, contact=YES
**
** PARTS
**
*Part, name=pzt
*End Part
*Part, name=shim
*End Part
**
** ASSEMBLY
**
*Assembly, name=Assembly
**
*Instance, name=shim-1, part=shim
*Node
1, 0., 0.
2, 0.025, 0.
3, 0.05, 0.
4, 0.075, 0.
5, 0.1, 0.
6, 0.125, 0.
7, 0.15, 0.
8, 0.175, 0.
9, 0.2, 0.
10, 0.225, 0.
11, 0.25, 0.
12, 0.275, 0.
13, 0.3, 0.
14, 0.325, 0.
15, 0.35, 0.
16, 0.375, 0.
17, 0.4, 0.
18, 0.425, 0.
19, 0.45, 0.
20, 0.475, 0.
21, 0.5, 0.
22, 0., 0.001
23, 0.025, 0.001
24, 0.05, 0.001
25, 0.075, 0.001
26, 0.1, 0.001
27, 0.125, 0.001
28, 0.15, 0.001
<p>| | | |</p>
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** Region: (shim: Picked)
* Elset, elset = _I1, internal, generate
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** Section: shim
* Solid Section, elset = _I1, material = shim
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* End Instance
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** Region: (pzt:Picked)
*Elset, elset=_I1, internal, generate
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** Section: pzt
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*End Instance
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*Elset, elset=_G31, internal, instance=pzt-1, generate
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*Nset, nset=_G32, internal, instance=pzt-1
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*Elset, elset=_G32, internal, instance=pzt-1, generate
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*Elset, elset=_G33, internal, instance=shim-1, generate
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*Surface, type=ELEMENT, name=__G29_S1, S1

*Elset, elset=__G30_S3, internal, instance=shim-1, generate
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** MATERIALS 

** Material, name=pzt 
*Density 7500. 
*Dielectric 8.85e-09, 
*Elastic 3e+10, 0.3 
*Piezoelectric, type=E 0., 0., 0., 0., 0., 0., -5e-11, 0.  
-5e-11, 0., 0., 0., 0., 0., 0., 0.  
0., 0. 
** Material, name=shim 
*Density 2500. 
*Elastic 9e+10, 0.3 
** BOUNDARY CONDITIONS 

** Name: axisymmetric Type: Symmetry/Antisymmetry/Encastre 
*Boundary _G34, ZSYM 
** Name: fixed Type: Symmetry/Antisymmetry/Encastre 
*Boundary _G33, ENCASTRE 
** STEP: Voltage_loading_only 
** *Step, name=Voltage_loading_only, perturbation 
*Static 
** BOUNDARY CONDITIONS 

** Name: bottom_pzt Type: Electric potential 
*Boundary _G32, 9, 9 
** Name: top_pzt Type: Electric potential 
*Boundary _G31, 9, 9 
** LOADS 

** Name: negative_pressure_load   Type: Pressure
*Dsload
_G39, P, 1.
**
** OUTPUT REQUESTS
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*Restart, write, frequency=1
**
** FIELD OUTPUT: F-Output-1
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*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-1
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*Output, history, variable=PRESELECT
*El Print, freq=999999
*Node Print, freq=999999
*End Step

Voltage Loading only (Normalized Piezoelectric Patch Radius = 0.55)

*Heading
** Job name: piezoplate_pot_R21_0_55_for_thesis Model name: Model-1
*Preprint, echo=YES, model=YES, history=YES, contact=YES
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** PARTS
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*Part, name=pzt
*End Part
*Part, name=shim
*End Part
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** ASSEMBLY
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*Assembly, name=Assembly
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*Instance, name=shim-1, part=shim
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  3, 3, 4, 16, 15, 44, 45, 46, 42
  4, 4, 5, 17, 16, 47, 48, 49, 45
  5, 5, 6, 18, 17, 50, 51, 52, 48
  6, 6, 7, 19, 18, 53, 54, 55, 51
  7, 7, 8, 20, 19, 56, 57, 58, 54
  8, 8, 9, 21, 20, 59, 60, 61, 57
  9, 9, 10, 22, 21, 62, 63, 64, 60
 10, 10, 11, 23, 22, 65, 66, 67, 63
 11, 11, 12, 24, 23, 68, 69, 70, 66
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 17, 18, 19, 31, 30, 55, 82, 83, 80
 18, 19, 20, 32, 31, 58, 84, 85, 82
 19, 20, 21, 33, 32, 61, 86, 87, 84
 20, 21, 22, 34, 33, 64, 88, 89, 86
 21, 22, 23, 35, 34, 67, 90, 91, 88
 22, 23, 24, 36, 35, 70, 92, 93, 90

** Region: (pzt:Picked)
*Elset, elset=_I1, internal, generate
  1, 22, 1
** Section: pzt
*Solid Section, elset=_I1, material=pzt
1.,
*End Instance
*Nset, nset=_G31, internal, instance=pzt-1
25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 72, 75, 77, 79
81, 83, 85, 87, 89, 91, 93
*Elset, elset=_G31, internal, instance=pzt-1, generate
12, 22, 1
*Nset, nset=_G32, internal, instance=pzt-1
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 37, 41, 44, 47
50, 53, 56, 59, 62, 65, 68
*Elset, elset=_G32, internal, instance=pzt-1, generate
1, 11, 1
*Nset, nset=_G33, internal, instance=shim-1
21, 42, 63, 84, 105, 126, 186, 227, 268, 309, 350
*Elset, elset=_G33, internal, instance=shim-1, generate
20, 100, 20
*Nset, nset=_G34, internal, instance=shim-1
1, 22, 43, 64, 85, 106, 130, 190, 231, 272, 313
*Nset, nset=_G34, internal, instance=pzt-1
1, 13, 25, 40, 73
*Elset, elset=_G34, internal, instance=shim-1, generate
1, 81, 20
*Elset, elset=_G34, internal, instance=pzt-1
1, 12
*Elset, elset=_G29_S1, internal, instance=pzt-1, generate
1, 11, 1
*Surface, type=ELEMENT, name=_G29, internal
_G29_S1, S1
*Elset, elset=_G30_S3, internal, instance=shim-1, generate
81, 100, 1
*Surface, type=ELEMENT, name=_G30, internal
_G30_S3, S3
*Elset, elset=_G39_S1, internal, instance=shim-1, generate
1, 20, 1
*Surface, type=ELEMENT, name=_G39, internal
_G39_S1, S1
** Constraint: Constraint-1
*Tie, name=Constraint-1, adjust=yes
_G30, _G29
*End Assembly
**
** MATERIALS
**
*Material, name=pzt
*Density
7500.,
*Dielectric
8.85e-09,
*Elastic
3e+10, 0.3
*Piezoelectric, type=E
0., 0., 0., 0., 0., 0., -5e-11, 0.
-5e-11, 0., 0., 0., 0., 0., 0., 0.
0., 0.
*Material, name=shim
*Density
2500.,  

*Elastic
   9e+10, 0.3
**
** BOUNDARY CONDITIONS
**
** Name: axisymmetric Type: Symmetry/Antisymmetry/Encastre
*Boundary
   _G34, ZSYMM
** Name: fixed Type: Symmetry/Antisymmetry/Encastre
*Boundary
   _G33, ENCASTRE
** -------------------------------
**
** STEP: Voltage_loading_only
**
*Step, name=Voltage_loading_only, perturbation
*Static
**
** BOUNDARY CONDITIONS
**
** Name: bottom_pzt Type: Electric potential
*Boundary
   _G32, 9, 9
** Name: top_pzt Type: Electric potential
*Boundary
   _G31, 9, 9, 1.
**
** LOADS
**
** Name: pressure_load Type: Pressure
*Dsload
   _G39, P, 1e-12
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=1
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*El Print, freq=999999
*Node Print, freq=999999
*End Step
LIST OF REFERENCES


BIOGRAPHICAL SKETCH

Suryanarayana A.N. Prasad was born on the 12th of March 1979, in Bombay, Maharashtra, India. He completed his schooling from Shrine Vailankanni Senior Secondary School, Madras, India, in 1996. He obtained his baccalaureate degree (B. Tech. Naval Architecture) from Indian Institute of Technology - Madras, Tamil Nadu, India, in 2000. He joined the Department of Aerospace Engineering, Mechanics and Engineering Science, University of Florida, U.S.A in the term fall 2000 with a graduate research scholarship and is currently pursuing his master's degree with a major in aerospace engineering and a minor in electrical and computer engineering.