THE ANALOGY BETWEEN
FLUID FLOW AND ELECTRIC CIRCUITRY

F. RODRIGUEZ
Cornell University
Ithaca, New York 14853

The behavior of fluids in pipe networks resembles that of currents in electric circuits. Of course, in the area of Process Control, extensive use has been made of the parallels between control systems and electric circuits. Also, the analogy between thermal conductivity and electric conductivity is often invoked to introduce or reinforce concepts of conductive heat transfer. However, none of the texts usually used by chemical engineers appears to have used electric circuitry as a tool for teaching fluid flow.

Most college students encounter fluid flow for the first time as sophomores or juniors, long after they have been introduced to Ohm's and Kirchhoff's Laws. In fact, many have had a multiple exposure to the concepts of electrical circuitry in high school and in freshman college physics.

The analogy is most useful in dealing with pipe networks with laminar flow, but it has some advantage even in turbulent flow. A factor favoring the use of the analogy is the growing adoption of SI units which make the parallel between mechanical and electric systems more obvious. One common misunderstanding which the analogy helps to clear up comes from the usual form of the mechanical energy balance for a flowing fluid. The friction term in energy per unit of flowing mass often becomes identified by students as a resistance whereas it is, in fact, in the nature of a potential. Perhaps because engineers often express the friction term as "head" in feet or meters (where force and mass units have been cancelled out), the image of a barrier or resistance seems to occur naturally.

In Table 1, the identification of kg in a mass flow system is made with coulombs in the electric circuit. When the familiar volts, ohms, and amperes are expressed as joules, coulombs, and seconds, the analogy becomes more apparent. The usual mechanical energy balance for a fluid flow process is:

\[
\frac{\Delta V^2}{2g_e} + \frac{\Delta p}{\rho} + \frac{g\Delta z}{g_e} + h_t + \frac{W_t}{\eta_t} - W_p\eta_p = 0 (1)
\]

where the last two terms represent contributions of turbines and pumps. The term \(h_t\) represents frictional dissipation of energy. In the absence of pumps or turbines, and with negligible changes in kinetic energy:

\[
-h_t = \frac{\Delta p}{\rho} + \frac{g\Delta z}{g_e} (2)
\]

The identification of \(h_t\) as a potential rather than as a resistance should be obvious from equation 2, but, as previously noted, the common units may confuse some students.

BRANCHED FLOW

In the elementary case (Table 2) of parallel resistances, almost every student has been told...
### TABLE I

**The Fluid Flow—Electric Current Analogy**

<table>
<thead>
<tr>
<th>Physical system:</th>
<th>ELECTRIC CURRENT</th>
<th>EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLUID FLOW</td>
<td>Resistor: ← E →</td>
<td>←L_e,D→</td>
</tr>
<tr>
<td>Flowing unit:</td>
<td>mass, kilogram</td>
<td>electricity, C</td>
</tr>
<tr>
<td>Flow rate:</td>
<td>m, kg/s</td>
<td>I, C/s (A)</td>
</tr>
<tr>
<td>Potential:</td>
<td>h, J/kg</td>
<td>E, J/C (V = W/A)</td>
</tr>
<tr>
<td>Resistivity:</td>
<td>(\frac{32\mu}{(D^2\rho g_c)}), J-m·s/kg·m²</td>
<td>(\rho', \frac{J-m·s}{C²}) (ohm·m)</td>
</tr>
<tr>
<td>Resistance:</td>
<td>(R = \frac{4L}{\pi D^2}), J-s/kg·m²</td>
<td>(R = \rho' L/A = \rho' \left[\frac{4L}{\pi D^2}\right]), J-s/C² (ohm)</td>
</tr>
<tr>
<td>Ohm's Law:</td>
<td>(h_t = R_t(m))</td>
<td>(E = R(I))</td>
</tr>
<tr>
<td>Power:</td>
<td>(P = h_t(m) = R_t(m)^2)</td>
<td>(P = E(I) = R(I)^2)</td>
</tr>
</tbody>
</table>

---

that an equivalent resistance, \(R_e\), is given by the reciprocal of the sum of the reciprocals of the individual resistances \(R_i\) and \(R_e\). It is a simple consequence of Kirchhoff's laws. The potential \(E\) across each resistance is the same, but the total current, \(I\), is given by the sum of the individual currents \(I_1\) and \(I_2\). Thus,

\[
E = R_1I_1 = R_2I_2 \quad \text{and} \quad I = I_1 + I_2 \tag{3}
\]

Rearrangement gives

\[
\frac{I}{R_1} = \frac{I_1}{R_e} \quad \text{and} \quad \frac{I}{R_2} = \frac{I_2}{R_e} \tag{4}
\]

Combining equations 4 and 3 to eliminate currents gives

\[
(R_e)^{-1} = (R_1)^{-1} + (R_2)^{-1} \tag{5a}
\]

The general case for \(n\) resistances in parallel is

\[
(R_e)^{-1} = \sum_{n} (R_i)^{-1} \tag{5b}
\]

The power (energy/time) to cause the flow is given by the product of total flow and the friction term (equation 1-4, Table 1).

**EXAMPLE OF BRANCHED, LAMINAR FLOW**

**Statement:** A stream of 18 m³/hr is split into three pipes, A, B, and C, with diameters of 20, 30, and 40 mm respectively and lengths of 50 m each. What power is dissipated as friction?

**Data:** \(\mu = 0.10\) Pa·s, i.e. (1 poise), \(\rho = 1\) Mg/m³, i.e. (1 g/cm³)

**Calculations:**

- \(m = 5.0\) kg/sec
- Resistances calculated from equation 1-2 (Table 1):
  \(R_A = 12.7\) kJ·s/kg²
  \(R_B = 2.51\) kJ·s/kg²
  \(R_C = 0.794\) kJ·s/kg²
- Equivalent resistance, \(R_{eq}\), from equation 2-2 (Table 2):
  \(R_{eq} = 0.576\) kJ·s/kg²
- Potential, \(h_t\), from equation 1-3 (Table 1):
  \(h_t = 0.576 \times 5.0 = 2.88\) kJ/kg
- Power, \(P\), from equation 1-4 (Table 1):
  \(P = 2.88 \times 5.0 = 14.4\) kW
- Individual streams calculated from
  \(m_1 = h_t/(R_{eq})\):
  \(m_2 = 0.23\) kg/s,
  \(m_3 = 1.15\) kg/s,
  \(m_4 = 3.63\) kg/s
- Individual Reynolds numbers from equation 3-2 (Table 3):
  \(N_{eq1} = 146\), \(N_{eq2} = 488\), \(N_{eq3} = 1155\)

**S**pring 1979
TABLE 2
Branched Flows

<table>
<thead>
<tr>
<th>GENERAL CASE AT A JUNCTION:</th>
<th>FLUID FLOW</th>
<th>ELECTRIC CURRENT</th>
<th>EQ.</th>
</tr>
</thead>
<tbody>
<tr>
<td>m = ∑m_i</td>
<td>I = ∑I_i</td>
<td>(2-1)</td>
<td></td>
</tr>
<tr>
<td>Equivalent Resistance:</td>
<td>1/R_{equiv} = ∑ 1/R_i</td>
<td>(2-2)</td>
<td></td>
</tr>
</tbody>
</table>

TURBULENT FLOW

The resistivity in turbulent flow varies with the flow rate (Table 3). In laminar flow, the resistivity is a function only of μ, ρ, and D. In turbulent flow, the friction factor f decreases in non-linear fashion as the Reynolds number increases. For smooth pipes at N_{re} above about 5 x 10^4, the behavior is approximated by equation 3-2 (Table 3). A modified resistance M_{t} can be defined (Table 3) so as to be independent of flow rate. The analogy with equations 3, 4, and 5 can be extended to give:

\[ h_t = M_t (m)^{1.8} = M_{t1} (m_1)^{1.8} = M_{t2} (m_2)^{1.8} \]  
(6)

for two pipes with modified resistances M_{t1} and M_{t2} and individual flows of m_1 and m_2, respectively. The result for the general case where i = 1, 2, etc. is:

\[ (M_{re})^{-0.556} = ∑ (M_{i})^{-0.556} \]  
(7)

AN EXAMPLE OF BRANCHED, TURBULENT FLOW

Statement: Same conditions as in previous, laminar case except that μ = 1.0 mPa-s (that is, 1.0 centipoise)
Calculations:
Modified resistances calculated from equation 3-3 (Table 3):

| TABLE 3 |
| Turbulent Flow |

| Resistance: | \( R_t = h_t/m = \frac{32 f}{g_e \pi^2 D^3 \rho^2} \) | (3-1) |

If \( f = 0.046(N_{re})^{-0.2} \) and \( N_{re} = (4m)/(\pi D \mu) \)

Then \( R_t = \frac{0.1421(\mu)^{0.2} L}{g_e (D)^{4.8} \rho^{0.8} (m)^{0.6}} = M_t (m)^{0.8} \)  
(3-3)

\( M_{sa} = 255.0 \text{ J/(kg)} \text{ s/kg) }^{1.8} \)
\( M_{td} = 36.4 \text{ J/(kg)} \text{ s/kg) }^{1.8} \)
\( M_{tc} = 9.16 \text{ J/(kg)} \text{ s/kg) }^{1.8} \)

Equivalent modified resistance from eq. 7:
\( M_{sa} \)  
(same units)

Potential, \( h_t \), from eq. 3-1, 3-3 (Table 3):
\( h_t = 3.84 \times (5.0)^{1.8} = 69.6 \text{ J/kg} \)

Power, P, from equation 1-4 (Table 1):
\( P = 69.6 \times 5.0 = 348 \text{ W} \)

Individual streams calculated from:
\( m_i = (h_t/M_{t1})^{0.556} \):
\( m_1 = 0.49 \text{ kg/s} \)
\( m_2 = 1.43 \text{ kg/s} \)
\( m_3 = 3.09 \text{ kg/s} \)

Individual Reynolds numbers from equation 3-2 (Table 3):

CONCLUSIONS

The emphasis here has been on the use of electric circuits as analogs in teaching concepts of pipe flow in networks. Extensive computer programs have evolved for handling complex circuits. These can be adapted for fluid-handling systems, also.

GLOSSARY:

- \( D \): Diameter, m
- \( E \): Electric potential, volt ( = J/C)
- \( g_s \): Acceleration due to gravity, 9.81 m/s^2
- \( g_c \): Proportionality constant, 1.06, dimensionless in SI system
- \( h_t \): Energy loss in pipe flow, J/kg
- \( I \): Electric current, ampere (C/s)
- \( m \): Mass flow rate, kg/s
- \( M_t \): Modified resistance term, equat. 3-3, Table 3, (J/kg)(s/kg)^{1.8}
- \( N_{re} \): Reynolds number
- \( \Delta p \): Pressure drop, Pa
- \( P \): Power, W
- \( R \): Electric resistance, ohm (= (J-s)/(C^2)
- \( R_{fp} \): Fluid flow resistance, (J-s)/kg^2
- \( V \): Fluid velocity (average), m/s
- \( W_s \): Energy supplied to system by pumps, J/kg
- \( W_t \): Energy taken from system by turbines, J/kg
- \( \Delta z \): Change in elevation, m.
- \( \eta_p \): Pump efficiency
- \( \eta_t \): Turbine efficiency
- \( \rho \): Density, kg/m^3
- \( \rho_c \): Electric resistivity, ohm-m
- \( \mu \): Viscosity, Pa-s

98

CHEMICAL ENGINEERING EDUCATION
BOOK REVIEW: Amplifiers
Continued from page 75.

The all important operational amplifier integrator is explained briefly along with several device parameters that influence the quality of the integration. An excellent discussion of the OA differentiator is given with an example of a practical circuit to control frequency response. Figure 3-25 presents the gain function during either integration or differentiation in a manner that readily characterizes the response of the circuit. The remainder of chapter III treats logarithmic circuits and the operational amplifier comparator.

Somewhat less familiar OA circuits are described in chapter IV. The use of zener diodes to produce limited or otherwise bound circuits is noted followed by a brief description of constant amplitude phase shifting. Operational amplifier active filters for passing or rejecting selected frequency bands are discussed, and commercially available units are listed. The rectification of low level ac signals and its use in phase-selective detection are also treated for OA-based circuits. The chapter concludes with a discussion of function generators and oscillators, sample-and-hold circuits for analog memories, and operational amplifier regulators that offer PID action. I found a surprising lack of figures to support the rather complex ideas introduced in this chapter, and I frequently had to sketch the circuit myself in order to follow the author’s reasoning.

Chapter V deals with the extensive and often subtly differentiated list of parameters that characterize modern operational amplifiers. Offset and drift along with circuit models of these effects, circuits to measure them, and circuits to compensate for them are presented in excellent detail. Difference amplifier properties are also noted. Noise that originates both within the operational amplifier and from external sources is discussed and classified as to frequency. Several standard techniques for minimizing the noise problem are suggested. A rather lengthy section on frequency response, dynamic properties and amplifier stability completes the chapter. Some of this material would be tedious for the non-electrical engineer, but certain properties such as unity gain bandwidth, slewing rate, and full power response need to be understood.

Chapter VI lists the specifications of several commercial operational amplifiers (current to 1975) and distinguishes between bipolar and FET-input units. Other specialized devices that are discussed include the chopper-stabilized, electrometric, programmable gain, and the so-called instrumentation amplifiers. Again, a more liberal use of figures would have made the reading go more rapidly.

Chapter VII is devoted to applications mainly in the field of electro-analytical chemistry. It provides an excellent review of the ways in which these interesting and powerful devices have been used to control, detect and measure physico-chemical properties in systems that also concern chemical engineers. The applications cited are well referenced to the literature and provide a broad base for further utilization of operational amplifiers in chemical instrumentation.

Chapter VIII discusses some of the commercially available modular OA systems that permit the user to construct his own OA-based instrumentation array. Unfortunately, the Malmstadt-Enke system is no longer available as noted, but to my knowledge, the other American-made systems can still be obtained in a form similar to that described. With the wide availability of both discrete component and integrated circuit operational amplifiers in today's electronics market, many educators and experimentalists are assembling their own modular systems with excellent results.

For the experimentally oriented chemical engineer who deals with chemical instrumentation and who feels the need to understand the applications of operational amplifiers in detail, this book provides excellent resource material, and I certainly recommend it. For the most part its content is well within the grasp of any engineer, scientist or educator with a modest background in basic electronics, and the nearly 200 references to the literature permit considerable study beyond the scope of the book. Since the book is oriented to applications in electrochemistry, the chemical engineer would occasionally have to reapply the same operational amplifier principles to systems more common to his own work.

The translation from the original Czech is generally well done. One occasionally finds a non-idiomatic expression or a term that has been translated too literally. There are very few actual errors and a correspondingly small number of typographical errors. More figures would have helped in the comprehension as well as certain repeated figures that would save the reader from constantly paging back through the text. My overall impression of the book is quite good, and I think the author has done an excellent job.