ACOUSTIC MEASUREMENT TECHNIQUES
FOR SUSPENDED SEDIMENTS AND BEDFORMS

By

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A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2004
To my family.
ACKNOWLEDGMENTS

I wish to express my appreciation to my chairman, Dr. Andrew B. Kennedy, for the freedom granted to me while pursuing my research interests, and for the guidance and support. I also wish to thank the members of my graduate committee, Dr. Robert G. Dean, Dr. Robert J. Thieke, Dr. Donald N. Slinn, for their intelligence and comprehensive supervision. My special thanks go to Dr. Renwei Mei, the external member of the committee, for his interest and involvement. I thank Dr. Peter D. Thorne for the guidance and great discussions on various aspects of acoustic measurements. I also thank Dr. Christopher R. Sherwood for providing the acoustic backscatter system and technical help.

I thank my father, Alexander Mouraenko, for his patience and love.

My special thanks go to the staff at the Coastal Engineering Lab, especially to Sidney Schofield, Jim Joiner, Victor Adams, and Chuck Broward, for their help, support, and technical guidance, without which this research would not have been possible.

I thank Jamie MacMahan and Justin Davis for their help and endless support, with special thanks to Brian Barr for his exceptional knowledge, great vision of life and numerous discussions.

This work was supported by the ONR Coastal Geosciences program and the National Science Foundation through the National Ocean Partnership Program (NOPP).
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<tr>
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<tr>
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<td>sediment sorting, standard deviation of sediment sizes</td>
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<td>[volt]</td>
<td>signal from mirror-reflected surface</td>
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<td>[volt]</td>
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</tr>
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<td>[volt]</td>
<td>amplitude envelope of recorded voltage</td>
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<td>$V_{rms}$</td>
<td>[volt]</td>
<td>recorded root-mean-squared voltage</td>
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<td>[volt]</td>
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<td>reference main lobe diameter at $r = 1$m</td>
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<tr>
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<tr>
<td>$\alpha$</td>
<td>[Np/m]</td>
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<td>attenuation coefficient due to suspended sediments</td>
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<td>water attenuation coefficient</td>
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<tr>
<td>$\gamma$</td>
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<td>modified scattering correction function</td>
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<tr>
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<td>scattering correction function</td>
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\( \Delta r \) \hspace{1em} [m] \hspace{1em} \text{thickness of insonified layer}

\( \varepsilon \) \hspace{1em} \text{relative error}

\( \varepsilon_m^2 \) \hspace{1em} \text{variance of estimated concentrations}

\( \varepsilon_s \) \hspace{1em} [m^2/s] \hspace{1em} \text{sediment diffusivity}

\( \zeta \) \hspace{1em} \text{sediment attenuation constant}

\( \theta \) \hspace{1em} [rad] \hspace{1em} \text{latitude angle from central axis of transducer}

\( \Theta \) \hspace{1em} [rad] \hspace{1em} \text{half intensity beam width}

\( \Theta_{\text{main}} \) \hspace{1em} [rad] \hspace{1em} \text{width of main lobe}

\( \lambda \) \hspace{1em} [1/m] \hspace{1em} \text{sound wave length}

\( \mu_{\phi} \) \hspace{1em} [\text{phi}] \hspace{1em} \text{mean sediment size}

\( \xi \) \hspace{1em} \text{correction factor, calculated from data}

\( \rho \) \hspace{1em} [kg/m^3] \hspace{1em} \text{water density}

\( \rho_A \) \hspace{1em} \text{acoustic density}

\( \rho_s \) \hspace{1em} [kg/m^3] \hspace{1em} \text{sediment density}

\( \sigma_t \) \hspace{1em} [m^2] \hspace{1em} \text{total scattering cross-section}

\( \sigma_{\phi} \) \hspace{1em} [\text{phi}] \hspace{1em} \text{sediment sorting, standard deviation of sediment sizes}

\( \tau \) \hspace{1em} [s] \hspace{1em} \text{sound pulse duration}

\( \varphi \) \hspace{1em} \text{phi scale}

\( \chi \) \hspace{1em} \text{normalized scattering cross-section}

\( \psi \) \hspace{1em} \text{near-field correction function}

\( \Psi_D \) \hspace{1em} \text{integrated beam pattern}
The acoustic backscatter system provides a nonintrusive method for measuring profiles of suspended sediment size and concentrations. Therefore, an inversion model is required to convert the recorded series of intensities into parameters of sediment suspension. The measurement precision highly depends on the choice of the inversion model. In this work the high-pass backscattering model is modified to allow accurate inversion for both sediment sizes and concentrations. The modified correction function with a single parameter is introduced for the form function and normalized cross-section. The parameters can be found from the calibration of the system in the chamber with generated homogeneous suspension. To account for the difference in shape and mineralogy, a set of two parameters can be determined for a particular sediment. Their use greatly improves the accuracy of mean sediment size measurements obtained with the three-frequency acoustic backscatter system.
The inversion model was applied to data obtained in the chamber with settling mixture of two sediment samples. The results were compared to the predictions by the 1D advection-diffusion model. Good agreement was shown for concentrations and sediment size measurements, with the error for sediment size approximation lying within the standard deviation of size distribution. The accuracy of the concentration measurements was different among the transducer frequencies, and was found to depend on the sensitivity of the transducer to the particular sediment size. The sediment sizes were estimated by minimizing the variance between concentrations, determined from different frequencies, and found to depend on the accuracy of estimated concentrations. For correction of recorded voltage data with applied time-varying gain, the system functions were found to be better approximated by second order polynomials.

A multiple reflection model was developed based on the multiple layer approximation. The numerical simulation with the initial uniform concentration profile of 3g/l shows that the effect in intensities from the multiple reflections can be up to 14%. Also, a bottom echo removal algorithm was developed in order to eliminate the effect of high intensity scattering on the concentration measurements close to the bottom.
CHAPTER 1
INTRODUCTION

The application of acoustics for the study of sediment transport and development of prediction models for morphological changes of coastlines and bathymetry has been widely recognized and developed during the last few decades [Thorne and Hanes, 2002]. By providing remote, no intrusive methods of measurements of flow velocities, bottom topography, and profiles of suspended sediment concentration, acoustic instruments have proven a reliable way of obtaining high resolution data during field and laboratory experiments [Hanes, 1991; Hanes et al., 2001; Hanes and Vincent, 1987; Hanes et al., 1988; Jansen, 1979; Libicki et al., 1989; Traykovski et al., 1999; Vincent and Green, 1990; Young et al., 1982].

The ultrasound at megahertz frequency range is used for the measurements of sediment suspension parameters within a meter range above the bottom [e.g., Smerdon et al., 1998]. The system consists of a sound transmitter and receiver. The properties of water-sediment mixture along an insonified path can then be estimated from the changes in sound attenuation and from the intensity of scattered sound [Morse and Ingard, 1968]. Two configurations of such systems are usually applied: bistatic, when the transmitter and receiver are spatially separated, and monostatic, when the same transducer serves as a transmitter and a receiver. The later configuration measures the backscattered intensity from the scatters in suspension sound.

The acoustic backscatter system allows measurement of both suspended sediment concentration profiles and distribution of sediment sizes [e.g., Crawford and Hay, 1993;
Compared to other techniques, acoustics are a nonintrusive way of measuring suspended sediments with a high temporal (in the order of a second) and spatial (in order of a centimeter) resolutions [Wren et al., 2000], with comparable accuracy [Hanes et al., 1988; Thorne and Hardcastle, 1997; Thorne et al., 1995].

Since the acoustic backscatter system is an indirect method of measurement, an inversion algorithm is required for converting measured backscattered sound intensities into parameters of suspension. The acoustic backscatter equation [e.g., Medwin and Clay, 1997, p.353] provides the basis for the development of such an algorithm.

The difficulties arise from the dependence of a backscattered signal from the properties of scatterers, their concentration, directivity of the sound beam, and also particular electronic implementation. Semi-empirical approximations of these parameters [e.g., Tamura and Hanes, 1986] can be used to convert the data into concentration profiles. Although a better description of the parameters can be found from theoretical approximation of sound scattering properties of natural sediments.

The acoustic properties of scatterers depend on many parameters, such as size, shape, mineralogy, distribution of those parameters in a sample, speed and direction of motion, their concentration, relative position to the sound transmitter and receiver, and others. Only a limited set of properties can be considered for the inversion. The simplified model for acoustic backscattering allows a user to obtain the most important properties for the study of sediment transport, while others can be approximated by general expressions. Only suspended sediment concentration, mean sediment size and sorting are considered to provide the necessary description of the water-sediment suspension, while the lognormal distribution of sediment sizes in a sample is assumed. The scattering pa-
rameters then are approximated by two functions—the form function and normalized cross-section. The choice of these two functions determines the accuracy and reliability of inversion model. The following works define the necessary framework for development of the inversion model: [Downing et al., 1995; Faran, 1951; Hay, 1991; Hay and Mercer, 1985; Hay and Schaafsma, 1989; Neubauer et al., 1974; Sheng and Hay, 1988; Thorne and Campbell, 1992; Thorne et al., 1993a].

A multiple frequency setup needs to be used in order to estimate sediment sizes. Schaafsma [1989] used a bistatic configuration of variable frequency transducers, which covered the frequency range between 1–100MHz, to measure concentration and sediment sizes of a mixture of two sediment samples. He showed that concentrations could be obtained with 20% error, while the sediment sizes can be resolved within a factor of 2. For field measurements monostatic configurations are usually used. A system consisting of a set of three or more transducers with frequencies 1–5MHz is able to estimate the mean sediment sizes along the insonified column [Crawford and Hay, 1993; Hay and Sheng, 1992; Schat, 1997; Thosteson and Hanes, 1998]. The accuracy of the estimation highly depends on correct approximation of scattering properties of the sediments, as well as on the accuracy of estimations of suspended sediment concentrations from each of the frequencies. It has been shown that the accuracy and stability of estimations of sediment sizes can be achieved by averaging among a series of consecutive profiles, which leads to significant loss in temporal resolution [e.g., Thorne and Hardcastle, 1997].

Several problems regarding measurement accuracy were addressed by the current dissertation. Their resolution improves both the accuracy and the temporal resolution of the measurements, and provides a basis for future improvements.
Usually, only measurements at low suspended sediment concentrations (<10g/l) are performed. Therefore, the multiple scattering of the sound waves can be ignored [e.g., Sheng and Hay, 1988]. Although it is true for natural sand, in the presence of stronger scatterers, like air bubbles, or at higher concentrations, as in the region near the sand bottom, multiple scattering can become important. Here a multiple reflection model is developed and the effects of multiple scattering on resulted errors are shown.

The acoustic properties of different natural sediments vary from sample to sample. By using the approximated parameters, such as form function and cross-section, obtained for a particular type of scatters like glass spheres or quartz sand, the inversion algorithm can lead to erroneous results if the actual sediment differs from the approximation in mineralogy and shape [Thorne and Buckingham, 2004]. Therefore, acoustical properties of the actual sediments need to be well defined before the inversion process can be done. The detailed study of each sediment sample, obtained from the experimental site, for mineralogical content, distribution of shapes, and overall acoustical performance is an even more difficult problem, which even in a case of positive solution cannot guarantee correct results, since the sediment content can change during the period of measurements.

The current work describes a new scattering model, which modifies a correction function originally presented by Thorne and Buckingham [2004], with a technique to determine necessary parameters. The description of the acoustic backscatter equation, an implementation of the multiple layer solution, and the newly developed model are discussed in Chapter 2.

The calibration of an acoustical backscatter system is performed in order to obtain system transfer function, which describes the dependence of sound intensity and recorded voltage levels. Different methods of calibration can be applied, such as by using thin-wire
targets [Sheng and Hay, 1993], calibration with a particle-laden turbulent jet [Hay, 1991], or in a calibration chamber with a homogeneous suspension [Tamura and Hanes, 1986; Thorne et al., 1993a]. The time varying gain (TVG) is usually applied for the amplification of low level sound signals, arriving from the farther distances from the transducer. With the proper correction for the TVG, the system function should be uniform with the range and therefore, can be determined from the calibration. In fact the system functions found from the calibrations are not always uniform with the range and may depend of particular sediment sample used for the calibration [Thorne and Buckingham, 2004]. The variations in system constant can reach 25%, which result in a corresponding error in estimation of suspended sediment concentrations. By providing proper corrections to the system functions developed here, these errors can be lowered.

The solution of the acoustic backscatter equation for suspended sediment concentration can be obtained explicitly with some limitations [Lee and Hanes, 1995]. Further development of the method [Thosteson and Hanes, 1998] allowed us to estimate the profiles of sediment sizes. The implicit method [Thorne et al., 1993a] provides a more flexible way of solution, although it can become unstable; when the concentrations become large and the sediment attenuation becomes significant, even small 5–10% errors can result in unbounded profiles of concentrations from the iterative solution [Thorne et al., 1995]. If the concentration profile is approximately known, for example, from the pump sampler measurements [e.g., Thorne et al., 1993a], the correction for sediment attenuation can be performed more accurately.

In Chapter 3 calibration of the acoustic backscatter system together with a technique of evaluation of the system parameters and the parameters for the correction functions is described. Also, the results of a series of laboratory tests are shown and compared
to the 1D advection-diffusion model predictions for sediment size and concentration profiles.

The bed echo contains important information, which can be used to improve accuracy of the acoustic backscatter system [Thorne et al., 1995]. Also, the intensity of the bed echo at low concentrations is much greater than the intensity of the sound backscattered from suspended particles. Because the acoustic backscatter system requires averaging and filtering of the input signal, signals from sand bottom interchange with the signals from suspended sediments. The algorithm for separation of these signals can provide one with the information on bed location and allow to determine concentrations of suspended sediments in the vicinity of the bottom. The algorithm is discussed in Chapter 4.
CHAPTER 2
MULTIPLE REFLECTION SOLUTION FOR ACOUSTIC BACKSCATTER

Acoustic Backscatter Equation

The acoustic backscatter equation is derived from the time integral of squared pressure backscattered from the objects in an insonified volume with following assumptions [Medwin and Clay, 1997]:

1. The objects in the insonified volume are alike, which implies that they are of a similar shape, size and mineral structure, and can be substituted with a population of objects with average properties.

2. The distribution of the objects within a gated volume is random and uniform. The gated volume is a volume of fluid, insonified within the short time interval—time gate.

3. The time gate is assumed to be large compared to the sound pulse duration. In practice, however, the time gate is often chosen to be equal to half of the pulse duration [e.g., Smerdon et al., 1998].

4. The attenuation of sound within the gated volume is ignored.

The far-field sound pressure from a circular piston source in isospeed medium is then equal to [Medwin and Clay, 1997]

\[ P(t) = D_t(\theta) P_0 \left( t - \frac{r_0}{c} \right) \frac{r_0}{r} e^{-\alpha r} \]  \hspace{1cm} (2-1)

where \( D_t(\theta) \) is the transmitter directivity function given by 2-65, \( P_0(t) \) is the sound pressure at the range \( r_0 \) along the central axis, \( \alpha \) is the total sound attenuation coefficient given by 2-42, \( c \) is the speed of sound in water, and \( r \) is the range from the transducer. After the transducer emits a sound pulse, it travels through the volume of water with suspended sediments. A portion of the sound is scattered by any particle in the direction of
the receiver. The pressure at the receiver is equal to

\[ P_r(t) = D_r(\theta)P_i(t)\frac{L_s}{r}e^{-\alpha r} \quad (2-2) \]

where \( D_r(\theta) \) is the directivity of the receiver, \( L_s \) is the complex acoustic scattering length, and the factor \( 1/r \) accounts for the spherical spreading of the sound wave. The complex acoustic scattering length describes the scattering properties of the particle. It is a function of angles between incident and reflected waves relative to the particle, and the sound wave number \( k \). For spheres it can be rewritten as [e.g., \textit{Sheng and Hay}, 1988]

\[ L_s = \frac{af_\infty(\theta,x)}{2} \quad (2-3) \]

where \( a \) is the particle radius, and \( f_\infty(\theta,x) \) is the nondimensional form function with \( x = ka \).

In general, the form function is a complex valued function. For scatterers of simple shapes, like cylinders and spheres, it can be found analytically [\textit{Morse and Ingard}, 1968]. But for particles of irregular shapes, such as sand grains, approximations are used to describe the amplitude of backscattered form function [\textit{Crawford and Hay}, 1993; \textit{Neubauer et al.}, 1974; \textit{Sheng and Hay}, 1988; \textit{Thorne and Buckingham}, 2004; \textit{Thorne et al.}, 1993b]. It relates to the form function as

\[ f_\infty = |f_\infty(0,x)| \quad (2-4) \]

The total pressure at the receiver, \( P_\Sigma \), is a sum of all backscattered pressures from individual particles. The squared pressure amplitude can be calculated as
\[ |P_\xi(t)|^2 = \sum_{i=1}^{N} P_{r,i}(t) P_{r,i}^*(t) + \sum_{i=1}^{N} \sum_{j\neq i}^{N} P_{r,i}(t) P_{r,j}^*(t) \quad (2-5) \]

where \( N \) is the total number of particles, and asterisk denotes complex conjugate value.

The second term is the sum of the cross-products of the pressures. In the case of a homogeneous suspension it can be shown that this term approaches zero either with increasing the number of particles or by ensemble averaging of the total backscattered pressure realizations [Libicki et al., 1989; Smerdon et al., 1998; Thorne et al., 1993a]. Previously mentioned assumptions 1 and 2 define the homogeneity of the water-sediment suspension. In practice, both methods of averaging are used to obtain the root-mean-squared (RMS) pressure profiles [e.g., Smerdon et al., 1998].

The RMS backscattered pressure for a given range \( r \) is calculated as is the integral over the gated volume of the total backscattered pressure squared, which for the gated time equal to sound pulse duration, \( \tau \), can be written as [Hay, 1991]

\[ P_{rms}^2 = P_0^2 \int_0^{\tau} \left( a^2 f_w^2 \right) \rho_n \Delta r \Psi D_r^2 \frac{1}{r^4} \sinh B \frac{e^{-4ar}}{B} \quad (2-6) \]

where \( P_0^2 = \int_{z-\frac{\tau}{2}}^{z+\frac{\tau}{2}} |P_0(t-z)|^2 dt \) is the reference pressure, \( \rho_n = N/V \) is the density of the particles in gated volume, \( \Delta r = \tau c/2 \) is the extent of the sample volume in \( r \), and \( \langle \ldots \rangle \) implies expected value over the sediment size distribution.

The term \( \sinh B/B \) with \( B = \alpha \tau c \) is the correction factor if the attenuation over the gated volume is large [Hay, 1991; Thosteson and Hanes, 1998].

The integrated beam pattern is calculated as

\[ \Psi_D = \int_0^{2\pi} \int_0^{\pi/2} D_\phi^2(\phi, \theta) D_\theta^2(\phi, \theta) \sin \theta d\theta d\phi \quad (2-7) \]
which for the monostatic system, when transmitter and receiver are located in the same
point and their directivities are the same, can be approximated as [Thorne and Hardcastle,
1997]

\[ \Psi_{d} \approx 2\pi \left( \frac{0.96}{ka_i} \right)^2, \quad ka_i \geq 10 \]  

(2-8)

where \( a_i \) is the transducer radius.

The density of the particles can be calculated by approximation of the particle
shape by spheres with radius \( a \). Therefore, it can be written as a function of particle mass
concentration, \( M \), as

\[ n_b = \frac{3M}{4\pi \langle a^3 \rangle \rho_s} \]  

(2-9)

where \( \rho_s \) is the density of sediments.

The root-mean-square voltage, recorded by the acoustic backscatter system can be
written in the form [Thorne and Hanes, 2002; Thorne and Hardcastle, 1997]

\[ V_{rms} = \Re T v \frac{1}{\Psi} P_{rms} = K_s K_t \frac{M^{1/2}}{\Psi r} e^{-2\omega r} \]  

(2-10)

\[ K_s = \left( \frac{1}{\rho_s} \frac{\langle a^2 f_a^2 \rangle^{1/2}}{\langle a^3 \rangle} \right) \]  

(2-11a)

\[ K_t = K_v \left( \frac{3\pi c}{16} \right)^{1/2} \frac{0.96}{ka_i} \]  

(2-11b)

\[ K_v = \Re T v P_o r_0 \]  

(2-11c)

where \( \Re \) is the transducer receive sensitivity, \( T_v \) is the voltage transfer function of the
system, \( \Psi \) is the near-field correction factor given by equation 2-67. \( K_t \) is the system
parameter, which is a constant if all other parameters of the system are fixed, or a function, if the time varying gain (TVG) is applied. It can be determined by the calibration with known concentrations and sediment properties [Hay, 1991; Thorne and Hanes, 2002; Thorne et al., 1993a].

**Inverse Problem for Sediment Concentration**

The parameters of the suspension such as sediment concentration and sediment size distribution parameters can be evaluated by solving the inverse to equation 2-10 problem [Thorne and Hanes, 2002]. For the concentration, the solution can be obtained from the equation

\[
M(r) = \left( \frac{V_{\text{rms}}}{K_s K_i} \psi r \right)^2 e^{4\alpha r} \tag{2-12}
\]

It cannot be solved explicitly, since the reflection parameter \( K_s \) and total attenuation coefficient \( \alpha \) are functions of sediment size distribution and concentration. The solution by iterations is usually applied. As can be seen from the equation, the solution for the concentration can be unstable, since it has positive exponential dependence on attenuation.

**Multiple Layer Approximation**

In the derivation of the equation 2-6 for RMS pressure, the multiple scattering between the particles was ignored. Multiple scattering for the suspension of sand can be neglected for suspended sediment concentrations of less than 10 g/l [Sheng and Hay, 1988]. At higher concentrations, which can be found at the ranges close to the bottom [e.g., Dohmen-Janssen and Hanes, 2002], or in the presence of stronger scatterers in the fluid, such as a wire or bubbles, multiple reflections can affect the measurements.

After the transducer emits a sound pulse, it travels within a narrow directed beam. Part of the sound wave is scattered by the suspended particles, and it may be rescattered
by other particles or reflected from the transducer surface multiple times before the back-scattered sound is recorded by the receiver. The receiver detects only the sound arriving from the direction within the narrow beam, which is defined by the directivity of the transducer (2-65). Therefore, the sound scattered backwards in the direction of the incident sound will most likely remain within the beam and be detected by the receiver (see Figure 2-1). Table 2-2 shows the approximate beam widths for transducers of different frequencies.

Figure 2-1. Example of the scattered sound within the narrow beam.

The RMS pressure (2-6) is calculated as the integral of the backscattered pressure over the time gate interval, which is again assumed to be equal to half of the pulse duration. During each time interval, the sound wave travels the distance $\tau c / 2$. Therefore, the whole range from the transducer surface can be represented as a set of layers with thicknesses $\tau c / 2$ (see Figure 2-2). The center of the $n$th layer is located at distance
$$r_n = n \frac{\tau c}{2}$$  \hspace{1cm} (2-13)

where $n = 0$ corresponds to the transducer surface.

Figure 2-2. The multilayer approximation for a possible sound pulse traveling path. The time interval corresponds to $\tau c/2$.

The sound waves can travel along many possible paths within the beam and scatter from the layers of suspended particles. In the multiple layer approximation all the paths begin at the layer 0 and pass between the layers with possible changes in direction. The mean squared pressure at the transducer at time $t_n = r_n/c$ is equal to the sum of mean squared pressures of the sound followed all possible paths

$$p^{2}_{rms} = \sum_{i=1}^{C_n} p^{2}_{rms,i}$$  \hspace{1cm} (2-14)

where the $C_n$ is the total number of the paths given by equation 2-26. Obviously, the length of every path in the summation should be equal to $r_n$. 
Let the sound pressure corresponding to an individual path at level $i$ be $P_i$. With spherical spreading and attenuation of the sound, the backscattered pressure from level $j$ becomes

$$P_j = P_i \frac{R_j}{r_{ij}} T_{ij} \quad (2-15)$$

where $r_{ij}$ is the distance between layers, $T_{ij}$ is the transmission coefficient, and $R_j$ is the backscattering coefficient. The backscattering coefficient defines a part of sound which is scattered from the layer of suspended particles in the direction of the incident sound wave. It is proportional to the number of scatterers within the layer and their scattering length (2-3). Using the same assumptions as for the acoustic backscatter equation, the coefficient can be written as

$$R_j^2 = \frac{\left\langle a^2 f_{\infty}^2 \right\rangle_j}{4} \frac{3M_j}{4\pi \left\langle a^3 \right\rangle_j \rho_s} \frac{\tau c}{2} \Psi_j r_j^2 \quad (2-16)$$

where the term $\left\langle a^2 f_{\infty}^2 \right\rangle_j/4$ approximates the scattering length of the scatterers in the layer $j$, and the term $\Psi_j r_j^2 \tau c/2$ defines the insonified volume at layer $j$. The transmitting coefficient $T_{ij}$ is equal to

$$T_{ij} = e^{-\alpha_{ij} r_{ij}} \quad (2-17)$$

where the attenuation coefficient is calculated using the trapezoidal rule as

$$\alpha_{ij} = \frac{\alpha_i}{2} + \alpha_{i+1} + \cdots + \frac{\alpha_{j-1}}{2} + \frac{\alpha_j}{2} \quad (2-18)$$

Therefore, the RMS pressure corresponding to a path of length $r$ is equal to

$$P_{rms}^2 = P_{i,li}^2 \prod_{r_{ij}} \frac{R_j^2}{r_{ij}^2} e^{-2\alpha_{ij} r_{ij}} \quad (2-19)$$
where $P_{i,j}$ is initially transmitted pressure (2-1) with the near-field correction factor applied (2-67). The multiplication is made over all sub paths, whose total length is 

$$\sum_{i,j} r_{ij} = r.$$ 

Numerical Realization of Multiple Reflections

The main difficulty in numerical realization is to account for all possible paths that sound waves can travel along. In the newly developed algorithm the binary tree is used to solve this problem. The generation of the binary tree is possible, because from any layer a sound wave is assumed to be traveling only along the directional beam in two directions—up or down. For the given number of layers, the binary tree will look like it is shown in Figure 2-3. The nodes of the tree correspond to the centers of the layers, and their numeration was done in a way that simplifies the generation of the binary tree. The root of the tree is located at the layer corresponding to the transducer surface. There is a set of parameters associated with every node. They include the indices of the nodes for possible up and down directions, as well as calculated reflection (2-16) and attenuation (2-42) coefficients. The same reflection and attenuation coefficients are applied to all nodes located at the same layer.

After the binary tree is generated, a fast algorithm of tree-walk is used [e.g., Knuth, 1997]. The walk over the binary tree starts with the node at the transducer surface, where the reflection coefficient is assumed to be 1 and the attenuation coefficient is equal the value for plain water. This implies complete reflection of the sound wave from the transducer surface and zero suspended sediment concentration. After the first step “down” is made (from the node “1” to “2” in Figure 2-3), neither reflection nor attenuation coefficients are known at layer 1 (node “2”); therefore they are initially assigned with the val-
ues 1 and 0 correspondingly. The intensity of the sound at layer 1 becomes

\[ I'_2 = I_1 \left( \frac{1}{\psi_1 \Delta r} \right)^2 e^{-\alpha_0 \Delta r} \]  

(2-20)

where \( I_1 \) is the initial intensity of sound emitted by the transducer with the index corresponding to the node number, and \( \psi_1 \) is the near-field correction factor calculated at \( r = \Delta r \).

Figure 2-3. Generation of a binary tree for numerical realization of multiple reflections.

A step “up” is made next (from the node “2” to “8” in Figure 2-3), and the calculated intensity of sound arriving back to layer 0 becomes

\[ I'_8 = I'_2 \frac{1}{\Delta r^2} e^{-\alpha_0 \Delta r} \]  

(2-21)

where “primes” are used for the intensities calculated with initial values \( R'_1 = 1 \) and \( \alpha'_1 = 0 \). The calculated intensity \( I_8 \) should be equal to the measured intensity correspond-
ing to bin one and recorded at time $\tau$, $I_{m,1}$. This provides us with an expression from
which the concentration at layer 1 can be found, and hence, corresponding reflection and
attenuation coefficients

$$\frac{I_{m,1}}{I'} = R^2 e^{-2\alpha_1\Delta r} \quad (2-22)$$

After the reflection and attenuation coefficients for layer 1 are determined, the in-
tensities along all the paths within the layers 0 and 1 can now be calculated and accumu-
lated according to the equation 2-14. The next step “down” is made (from the node “2” to
“3”) and the procedure is repeated until the last layer is reached. The expression to find
the concentration in general form becomes

$$\frac{I_{m,j} - I_{\Sigma,j}}{I'_{n_i}} = R^2 e^{-2\alpha_j\Delta r} \quad (2-23)$$

where $I_{\Sigma,j}$ is the sum of intensities calculated for the paths which arrive at the transducer
surface at the time corresponding to bin $i$. The node number is found as

$$n_i = \left( n - \frac{i-1}{2} \right) i + 1 \quad (2-24)$$

with $n$ number of layers.

**Determination of Sediment Size and Concentration**

Three parameters need to be known at every layer to calculate reflection and at-
tenuation coefficients: suspended sediment concentration $M$, and parameters of sediment
size distribution, $M_a$ and $S_a$.

If suspended sediment concentration is the only unknown parameter and mean
sediment size, $M_a$, and standard deviation, $S_a$, are known somehow (for example, from
additional measurements or assumed values are used), then equation 2-23 can be solved
via iterations [Thorne et al., 1993a]. To determine the sediment size parameters and the suspended sediment concentration more than one measurement with different sound frequencies are needed. The techniques were developed to obtain concentration, $M$, and mean sediment size, $M_a$, by using 3 transducers, while standard deviation, $S_a$, is assumed to be known [Hay and Sheng, 1992], [Crawford and Hay, 1993], [Thosteson and Hanes, 1998]. To obtain all the parameters—$M$, $M_a$, and $S_a$—a system with six transducers can be used [Schat, 1997].

More often the 3-transducer setup is used. It usually requires sorting, $S_a$, of the sediments to be known. The mean sediment size can be estimated from the minimization of the variance of sediment concentration approximated by each of the transducers

$$\varepsilon^2_M (M_a) = \sum_{k=1}^{3} (M_k - M)^2 |_{M_a}$$

where $M = \sum_{k=1}^{3} M_k / 3$.

Number of Paths

The number of possible paths grows very quickly with the number of layers $n$. For a specified $n$ the total number of paths is equal to $(n-1)$th Catalan number [Weisstein]. The $n$th Catalan number is equal to

$$C_n = \frac{(2n)!}{(n+1)!n!} \tag{2-26}$$

For illustration the first 24 numbers are given in Table 2-1. The asymptotic form of $n$th Catalan number is
$$C_n \sim \frac{4^n}{\sqrt{\pi n^{3/2}}} \quad (2-27)$$

which for the number of layers $n = 120$ gives $C_{120} \sim 7.58 \cdot 10^{68}$.

Table 2-1. First 24 Catalan numbers

<table>
<thead>
<tr>
<th>$n$</th>
<th>$C_n$</th>
<th>$n$</th>
<th>$C_n$</th>
<th>$n$</th>
<th>$C_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>9</td>
<td>1,430</td>
<td>17</td>
<td>35,357,670</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>10</td>
<td>4,862</td>
<td>18</td>
<td>129,644,790</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>11</td>
<td>16,796</td>
<td>19</td>
<td>477,638,700</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>12</td>
<td>58,786</td>
<td>20</td>
<td>1,767,263,190</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>13</td>
<td>208,012</td>
<td>21</td>
<td>6,564,120,420</td>
</tr>
<tr>
<td>6</td>
<td>42</td>
<td>14</td>
<td>742,900</td>
<td>22</td>
<td>24,466,267,020</td>
</tr>
<tr>
<td>7</td>
<td>132</td>
<td>15</td>
<td>2,674,440</td>
<td>23</td>
<td>91,482,563,640</td>
</tr>
<tr>
<td>8</td>
<td>429</td>
<td>16</td>
<td>9,694,845</td>
<td>24</td>
<td>343,059,613,650</td>
</tr>
</tbody>
</table>

Because of the large number of the paths, the execution time becomes very long even for a small number of layers. Therefore, not all the paths can be included in calculation. The number of reflections which a sound wave is exposed can be chosen as a criteria for the path to be included in the calculation. The number of reflections for each path varies between 1 and $n(n-1)/2$, where $n$ is the number of layers. The reflection coefficient for the layer of scatterers is determined by equation 2-16. To approximate the order of magnitude of the coefficient, the following values can be used: $r = 1$ m, $\tau = 10$ $\mu$s, $c = 1500$ m/s, $\Psi_\ell = 0.015$, $\rho_s = 2650$ kg/m$^3$, $M = 3$ g/l, $a = 0.2$ mm, for which $R \approx 0.01$. Two reflections result in attenuation of incident pressure on the order of $10^{-4}$, three—$10^{-6}$. Although, the number of the paths can be very large, the sound waves of small amplitude will be absorbed by the suspension. Therefore, the number of reflections can be limited to 3 if the number of layers is large ($n > 15$).

Effect of Multiple Reflections

Four tests were performed to analyze the relative effect of multiple reflections on measurement of sediment concentration. The ratios of intensities calculated with one and
three reflections using the model described above are plotted in Figures 2-4 and 2-5. For uniform concentration profiles (see Figure 2-4 a–d), the effect of multiple reflections is almost uniform with distance. The difference reaches its maximum of 12–14% at 3g/l, but at 1g/l it is only around 5%. For the concentration profiles, which linearly change from 0g/l to maximum values of 0.1–3g/l with distance (see Figure 2-4 e–h), the difference is even smaller—on the order of 1–3%. This is probably the most common situation for field measurements, when the concentration near the transducer is much smaller than it is near the bottom.

However, it is possible for a cloud of sediments to be injected by a turbulent eddy into the water column with the concentration at higher elevations to become greater than the concentration near the bottom. Figure 2-5 a–d shows the results when uniform concentration profiles were modified by a jet-type profile with maximum concentration of 2g/l, located at around 30cm distance from the transducer. The difference becomes more visible, especially at the ranges just after the jet location. Compared to uniform concentration profiles, the intensity increases around 1.2 times.

In the next test, a wire-type response was modeled with results shown in Figure 2-5 e–h. For the profiles at 0.1 and 0.5g/l there are “ghost” peaks present, which correspond to the second reflection of sound between the transducer surface and “wire”. This situation can be modeled in the lab with a strong reflector, like steel wire (see Chapter 4). Although the second reflection is present in Figure 2-5 b–d, there are no such “ghost” peaks in the tests with the sediment mixture jet (see Chapter 4). In the field, air bubbles can be present in the water column, and since the bubbles are strong reflectors the second reflection from them can affect the measurements at longer ranges.
Figure 2-4. Difference between intensities modeled with one and three reflections for selected profiles of concentrations: a) constant profiles, e) linear profiles, b–d) and f–h) percent difference.
Figure 2-5. Difference between intensities modeled with one and three reflections for selected profiles of concentrations: a) profiles of constant concentration with “bump”, e) profiles of constant concentrations with “spike”, b–d) and f–h) same as in Figure 2-4.
Special Parameters for Acoustic Backscatter Equation

Form Function and Normalized Cross-Section for Elastic Spheres

The theoretical far-field form function for elastic spheres can be expressed as

\[ f_{\omega,s}(x) = \frac{2}{\lambda x} \sum_{n=0}^{\infty} (-1)^n (2n+1) b_n \] (2-28)

where

\[ b_n = \begin{vmatrix} \beta_1 & \alpha_{12} & \alpha_{13} \\ \beta_2 & \alpha_{22} & \alpha_{23} \\ 0 & \alpha_{32} & \alpha_{33} \end{vmatrix} \] (2-29a)

\[ \beta_1 = x_i^2 \left( \frac{\rho}{\rho_s} \right) j_n(x_i), \quad \beta_2 = xj'_n(x_i), \]

\[ \alpha_{11} = x_i^2 \left( \frac{\rho}{\rho_s} \right) h_n^{(i)}(x_i), \quad \alpha_{21} = -x h_n^{(i)'}(x_i), \]

\[ \alpha_{12} = \left[ 2n(n+1) - x_i^2 \right] j_n(x_i) - 4x_i j'_n(x_i), \]

\[ \alpha_{22} = x_i j'_n(x_i), \quad \alpha_{32} = 2\left[ j_n(x_i) - x_i j'_n(x_i) \right], \] (2-29b)

\[ \alpha_{13} = 2n(n+1) \left[ x_i j'_n(x_i) - j_n(x_i) \right], \]

\[ \alpha_{23} = n(n+1) j_n(x_i), \]

\[ \alpha_{33} = 2x_i j'_n(x_i) + \left[ x_i^2 - 2n(n+1) + 2 \right] j_n(x_i), \]

where \( x_i = x(c/c_i), \quad x_i = x(c/c_i), \quad c \) is the sound speed in the water, \( c_i \) and \( c_s \) are the speeds of shear and compressional waves in the spherical particle, \( \rho \) and \( \rho_s \) are the fluid and particle densities, \( j_n \) is the spherical Bessel function, and \( h_n^{(i)} \) is the spherical Hankel function of first kind. The derivatives of the functions can be found by the recurrence relation \([Abramowitz and Stegun, 1965]\)
\[ g'_n = g_{n+1} (x) + \frac{n}{\chi} g_n (x) \]  

where \( g (x) \) is either a spherical Bessel or Hankel function.

The normalized scattering cross section \( \chi \) is defined as

\[ \chi = \frac{\sigma_t}{2 \pi a^2} \]  

where \( \sigma_t = |L_s|^2 \) is the total scattering cross section of a single particle of radius \( a \). For the spherical particles it can be found to be

\[ \chi_s (x) = \frac{2}{x^2} \sum_{n=0}^{\infty} (2n+1)|b_n|^2 \]  

Equations 2-28 and 2-32 involve infinite series, but only a finite number of terms can be calculated. The following approximate expression can be used to calculate the number of terms sufficient for the resulting accuracy of \( O(10^{-6}) \) for \( x < 50 \)

\[ N_{10^n} = 1.2246x + 6.4232 \]  

High-Pass Model for Quartz Sand

The modified high-pass model for backscattered intensity [Crawford and Hay, 1993; Johnson, 1977; Sheng and Hay, 1988; Thorne et al., 1993b] provides simpler expressions for the form function and normalized scattering cross section for quartz sand

\[ f_{\omega,h} (x) = C_0 \frac{K_j x^4}{1 + K_j x^2} \]  

\[ C_0 = \left( 1 - \nu_1 \exp \left\{ -\left( \frac{y_{\omega,h}}{\eta} \right)^2 \right\} \right) \left( 1 + \nu_2 \exp \left\{ -\left( \frac{y_{\omega,h}}{\eta} \right)^2 \right\} \right) \]  

\[ \chi_h (x) = \frac{\frac{4}{3} K_\alpha x^4}{1 + x^2 + \frac{4}{3} K_\alpha x^4} \]
where \( \nu_1 = 0.25 \), \( x_1 = 1.4 \), \( \eta_1 = 0.5 \) and \( \nu_2 = 0.37 \), \( x_2 = 2.8 \), \( \eta_2 = 2.2 \), \( K_f = 1.1 \),
\[ K_d = 0.18 \text{, and } x = ka. \]

The functions given by equations 2-34 and 2-35 are shown in Figure 2-6 together with corresponding curves for quartz spheres (2-28 and 2-32). The values of the velocities of shear and compressional waves in quartz spheres were taken to be \( c_1 = 3760\text{m/s} \) and \( c_i = 5980\text{m/s} \).

![Figure 2-6. Form function and normalized cross section for quartz sand (blue) and spheres (red): a) form function, b) normalized cross-section.](image)

**Modified Model for Elastic Spheres**

Based on the spherical model, modified expressions were developed by Thorne and Buckingham [2004] to better describe the scattering properties of natural sediments. The following empirical function was introduced

\[
\gamma_0(x, \beta) = \frac{\beta x^3 + 0.5x + 3.5}{x^3 + 3.5} \tag{2-36}
\]

and applied to the low-pass filtered spherical form function and normalized cross section in form.
The $\beta$ parameters describe the deviation of the modified model from the spherical model due to irregularities in shapes and sizes, which occur in natural sediments. While the parameters $\beta_f$ and $\beta_\chi$ are generally independent, their values are close and usually change between 1.3 and 2.2. The value of $\beta = 1.9$ can be taken for both of the parameters to provide a reasonable fit with the data [Thorne and Buckingham, 2004]. It can be seen, that the function $\gamma_0(x, \beta)$ is greater than 1 for all values of $x$. It implies that the values of modified form function and normalized cross section are greater than the original ones calculated by spherical model.

$$f_{\omega,\gamma}(x) = \gamma_0(x, \beta_f) f_{\omega,\gamma}$$  \hspace{1cm} (2-37)

$$\chi' \gamma(x) = \gamma_0(x, \beta_\chi) \chi$$ \hspace{1cm} (2-38)

![Figure 2-7. Modified $\gamma$ function for selected values of parameter $\beta$.](image)

**Modified High-Pass Model for Natural Sediments**

Similarly, the high-pass model can be modified to better model the properties of particular sediments. Therefore, the original function $\gamma_0(x, \beta)$ (2-36) was currently
modified as

$$
\gamma(x, \beta) = \begin{cases} 
\gamma_0(x, \beta), & \beta > 1 \\
\frac{x^3 + (\beta - 0.5)x + 3.5}{x^3 + 3.5}, & \beta_0 < \beta \leq 1 
\end{cases}
$$

(2-39)

The function \( \gamma(x, \beta) \) is equal to 1 for \( \beta = 0.5 \). The lower limit \( \beta_0 \) implies

\[ \gamma(x, \beta) > 0 \] and is equal to \( 0.5 - 5.25\sqrt{1.75} = -3.85 \). The graphs of the function \( \gamma(x, \beta) \) for selected \( \beta \) parameters equal to –1, 0.5, 1, and 1.9 are shown in Figure 2-7.

The modified high-pass model becomes

$$
f_{w,h}^f(x) = \gamma(x, \beta_f) f_{w,h} 
$$

(2-40)

$$
\chi_h^f(x) = \gamma(x, \beta_h) \chi_h
$$

(2-41)

**Sound Attenuation Coefficient**

The sound attenuation coefficient defines the exponential rate of decay of sound intensity with distance. It depends on temperature, salinity, pressure, presence of suspended particles and microbubbles, and varies with sound frequency [Richards, 1998]. The total attenuation coefficient can be written as a sum of the attenuation coefficient in clear water, \( \alpha_w \), and the attenuation due to scattering by suspended sediments, \( \alpha_s \),

$$
\alpha = \alpha_w + \alpha_s
$$

(2-42)

The attenuation of sound in the megahertz range in seawater at small (<30m) depths is primarily a function of water temperature. The formula by Fisher and Simmons [1977] gives the approximation to within 4%

$$
\alpha_w = f^2 \left( 55.9 - 2.37T + 4.77 \cdot 10^{-2}T^2 - 3.44 \cdot 10^{-4}T^3 \right) \cdot 10^{-3}
$$

(2-43)

where \( f \) is the sound frequency in megahertz, \( T \) is the water temperature in degrees centigrade.
The sediment attenuation coefficient at a range $r$ is given by

$$\alpha_s = n_b \frac{\sigma_s}{2}$$  \hspace{1cm} (2-44)

where $n_b$ is the number of particles per unit volume, $\sigma_s$ is the total scattering cross-section of the particle. Using the equations 2-9 and 2-31 and averaging over the sediment sizes, the sediment attenuation coefficient becomes

$$\alpha_s(r) = \frac{3\langle a^2 \chi \rangle}{4\rho_s \langle a^3 \rangle} M(r) = \zeta(r) M(r)$$  \hspace{1cm} (2-45)

where $\zeta$ is the sediment attenuation constant evaluated at range $r$

$$\zeta = \frac{3\langle a^2 \chi \rangle}{4\rho_s \langle a^3 \rangle}$$  \hspace{1cm} (2-46)

The attenuation of the sound traveling within interval $[r_1, r_2]$ due to scattering and absorption by suspended sediments can be approximated by

$$\alpha_s = \frac{1}{r_2 - r_1} \int_{r_1}^{r_2} \zeta(r) M(r) \, dr$$  \hspace{1cm} (2-47)

If the sediment attenuation constant is not changing with range, then the attenuation is equal to

$$\alpha_s = \zeta \bar{M}$$  \hspace{1cm} (2-48)

where $\bar{M}$ is the mean sediment concentration between ranges $[r_1, r_2]$.

**Lognormal and Normal Distributions of Sediment Sizes**

Sediment sizes vary from very fine (clay) to coarse (sand, gravel, shell) [e.g., Sleath, 1984]. Lognormal distribution can be used to approximate the actual distribution
of sediment sizes. Its probability density function is given by

\[
p_L(a|M_a, S_a) = \frac{1}{a\sqrt{2\pi S_a}} \exp \left\{ -\frac{(\ln a - M_a)^2}{2S_a^2} \right\}
\]  

(2-49)

To obtain the probability density function in terms of sediment radius \( a \) in meters the following substitutions were made

\[
M_a = -(1 + \mu_\phi) \ln 2 - 3 \ln 10
\]
\[
S_a = \sigma_\phi \ln 2
\]

(2-50)

where \( \mu_\phi \) and \( \sigma_\phi \) are the mean and standard deviation in phi units. The phi scale is defined as the logarithm of sediment size in millimeters. The sediment size, given in meters, can be written as

\[
\phi = -\log_2 \left( 2a \cdot 10^3 \right)
\]

(2-51)

If the probability density function of \( a \) is given by equation 2-49, then the quantity \( \ln a \) is normally distributed. In terms of \( \phi \) the probability density function is given by

\[
p_N(\phi|\mu_\phi, \sigma_\phi) = \frac{1}{\sqrt{2\pi \sigma_\phi}} \exp \left\{ -\frac{(\phi - \mu_\phi)^2}{2\sigma_\phi^2} \right\}
\]

(2-52)

The parameters \( \mu_\phi \) and \( \sigma_\phi \) can be approximated from the sieve analysis as [Sleath, 1984]

\[
\mu_\phi = -\log_2 \left( d_{84.1},d_{15.9} \right)^{\frac{1}{2}} \quad \text{and} \quad \sigma_\phi = \log_2 \left( \frac{d_{84.1}}{d_{15.9}} \right)^{\frac{1}{2}}
\]

(2-53)

where \( d_n \) is the diameter in millimeters for which \( n \% \) of the sample by weight is finer.

The raw moments of the lognormal distribution are

\[
\mu_1 = \langle a \rangle = \int_0^\infty a p_L(a|M_a, S_a) da = \exp \left\{ M_a + \frac{S_a^2}{2} \right\}
\]

(2-54)
\[ \mu_2 = \langle a^2 \rangle = \int_0^\infty a^2 p_\ell (a | M_a, S_a) \, da = \exp \left\{ 2 \left( M_a + S_a^2 \right) \right\} \]  
\[ (2-55) \]
\[ \mu_3 = \langle a^3 \rangle = \int_0^\infty a^3 p_\ell (a | M_a, S_a) \, da = \exp \left\{ 3M_a + \frac{9S_a^2}{2} \right\} \]  
\[ (2-56) \]

### Averaged Form Factor and Normalized Cross-Section Terms

The averaged terms in the equations 2-11 and 2-46 can be calculated with the assumption that sediment sizes obey some known distribution. Assuming the lognormal distribution (2-49), the following simplification can be made.

The term \( \langle a^2 | f_\infty (ka) |^2 \rangle \), where \( \langle \ldots \rangle \) denotes averaging over the sediment sizes, is equal to

\[ \langle a^2 | f_\infty (ka) |^2 \rangle = \int_0^\infty a^2 | f_\infty (ka) |^2 p(a) \, da \]  
\[ (2-57) \]

where the sediment size is distributed with the density function \( p(a) \). If \( p(a) \) is the lognormal distribution, the expression above simplifies to

\[ \langle a^2 | f_\infty (ka) |^2 \rangle = \int_0^\infty a^2 | f_\infty (ka) |^2 p_\ell (a | M_a, S_a) \, da \]  
\[ (2-58a) \]

\[ = \left[ \ln a = y, \frac{da}{a} = dy \right] \]  
\[ (2-58b) \]

\[ = \int_{-\infty}^{+\infty} e^{2y} | f_\infty (ke^y) |^2 \frac{1}{\sqrt{2\pi S_a}} \exp \left\{ -\frac{(y-M_a)^2}{2S_a^2} \right\} dy \]  
\[ (2-58c) \]

\[ = \int_{-\infty}^{+\infty} | f_\infty (ke^y) |^2 \exp \left\{ 2 \left( M_a + S_a^2 \right) \right\} \frac{1}{\sqrt{2\pi S_a}} \exp \left\{ -\frac{\left[ y - \left( M_a + 2S_a^2 \right) \right]^2}{2S_a^2} \right\} dy \]  
\[ (2-58d) \]

\[ = \mu_2 \int_{-\infty}^{+\infty} | f_\infty (ke^y) |^2 p_N (y | \mu, \sigma) \, dy \]  
\[ (2-58e) \]
\[
\langle a^2 | f_{\infty}(ka) \rangle^2 \rangle = \mu_2 \left( \left[ f_{\infty}(ke^y) \right]^2 \right)_{p_N(y, \hat{\sigma}, \hat{\mu})}
\]

(2-58f)

Here \( \langle \cdots \rangle_{p(x)} \) is the expected value of the argument with the probability density function \( p(x) \). \( p_N(y|\hat{\mu}, \hat{\sigma}) \) is the normal distribution probability density function

\[
p_N(y|\hat{\mu}, \hat{\sigma}) = \frac{1}{\sqrt{2\pi \hat{\sigma}}} \exp \left\{ -\frac{(y - \hat{\mu})^2}{2\hat{\sigma}^2} \right\}
\]

(2-59)

with \( \hat{\mu} = M_a + 2S_a^2 \) and \( \hat{\sigma} = S_a \).

With backward substitution the term \( \langle a^2 | f_{\infty}(ka) \rangle^2 \rangle \) becomes

\[
\langle a^2 | f_{\infty}(ka) \rangle^2 = \left[ \ln a = y, \frac{da}{a} = dy \right]
\]

(2-60a)

\[
= \mu_2 \left[ \int_0^\infty \left| f_{\infty}(ka) \right|^2 \frac{1}{a\sqrt{2\pi \hat{\sigma}}} \exp \left\{ -\frac{(\ln a - \hat{\mu})^2}{2\hat{\sigma}^2} \right\} \, da \right]
\]

(2-60b)

\[
= \mu_2 \int_0^\infty \left| f_{\infty}(ka) \right|^2 p_L(a|\hat{\mu}, \hat{\sigma}) \, da
\]

(2-60c)

\[
= \mu_2 \left\langle \left| f_{\infty}(ka) \right|^2 \right\rangle_{p_L(a|\hat{\mu}, \hat{\sigma})}
\]

(2-60d)

\[
= \left\langle a^2 \left| f_{\infty}(ka) \right|^2 \right\rangle_{p_L(a|\hat{\mu}, \hat{\sigma})}
\]

(2-60e)

The modified term

\[
\frac{\left\langle a^2 | f_{\infty}(ka) \right|^2 \rangle}{\left\langle a^3 \right\rangle} = \frac{\mu_2 \left\langle \left| f_{\infty}(ka) \right|^2 \right\rangle_{p_L(a|\hat{\mu}, \hat{\sigma})}}{\mu_3}
\]

(2-61a)

\[
= \frac{\exp\left\{ 2M_a + 2S_a^2 \right\} \left\langle \left| f_{\infty}(ka) \right|^2 \right\rangle_{p_L(a|\hat{\mu}, \hat{\sigma})}}{\exp\left\{ 3M_a + 9/2S_a^2 \right\} \left\langle \left| f_{\infty}(ka) \right|^2 \right\rangle_{p_L(a|\hat{\mu}, \hat{\sigma})}}
\]

(2-61b)
The expressions for the averaged normalized total scattering cross-section can be obtained following similar derivations

\[ \langle a^2 \chi(ka) \rangle = \frac{\langle a^2 \rangle \langle \chi(ka) \rangle}{\mu_1(\alpha^2, \varphi)} \]  \hspace{1cm} (2-62) \]

\[ \frac{\langle a^2 \chi(ka) \rangle}{\langle a^3 \rangle} = \frac{\mu_2 \langle \chi(ka) \rangle}{\mu_3(\alpha^2, \varphi)} \]  \hspace{1cm} (2-63a) \]

\[ = e^{-2S^2} \frac{\langle \chi(ka) \rangle}{\langle a \rangle} \]  \hspace{1cm} (2-63b) \]

Circular Transducer Directivity and Half Intensity Beam Width

The directivity of the transducer can be defined as the ratio of sound pressure at point \((r, \theta)\), where \(\theta\) is latitude angle measured from central axis of the transducer, to the axial pressure at the same range \(r\)

\[ D(\theta) = \frac{P(r, \theta)}{P(r, 0)} \]  \hspace{1cm} (2-64) \]

Let \(a_r\) be the radius of transducer and \(k = 2\pi/\lambda\) be the sound wave number, where \(\lambda\) is the sound wavelength. At great distances from the transducer \((rk \gg 1)\), the far-field directivity of a circular piston source is [Medwin and Clay, 1997, p.139; Morse and Ingard, 1968, p.381]

\[ D(\theta) = \frac{2J_1(ka_r \sin \theta)}{ka_r \sin \theta} \]  \hspace{1cm} (2-65) \]
where \( J_1(x) \) is the first order Bessel function of first kind. The curve of the squared directivity function, \( D^2(\theta) \), is shown in Figure 2-8.

![Figure 2-8. Circular piston directivity function, \( D^2(\theta) \).](image)

The time averaged acoustic intensity, \( I \), relates to the RMS pressure as [Medwin and Clay, 1997]

\[
I = \frac{p_{\text{rms}}^2}{\rho_A c} \tag{2-66}
\]

where \( \rho_A \) is the acoustic density that relates to the bulk modulus of elasticity, \( E \), as \( \rho_A = E/c^2 \).

The half intensity beam width, \( \Theta \), can be defined as \( \Theta = 2\theta \) where \( \theta \) is the angle from the center axis to where the acoustic intensity is equal to the half of the axial value. For a circular piston source, the directivity function of which is given by equation 2-65, \( D_1^2(\theta) = 0.5 \) at \( ka, \sin \theta = 1.6163 \). Although most of the sound energy is transmitted within the cone defined by the half intensity beam width, as shown in Figure 2-1, the low frequency transducers can have wider main lobe and significant side lobes.
Similar to the half intensity beam width, the width of the main lobe, $\Theta_{\text{main}}$, can be determined from $D_i^2(\theta) = 0$, which is satisfied at $ka_i \sin \theta = 3.8317$.

The beam width at a given range, $r$, from the transducer, for example at $r=1$ m, can be found as $W(r) = 2r \sin (\Theta/2)$ and $W_{\text{main}}(r) = 2r \sin (\Theta_{\text{main}}/2)$. The values of the beam widths $\Theta$ and $W$ for frequencies 1–5MHz are shown in Table 2-2. It can be seen, that for higher sound frequencies the beam widths are narrower.

Table 2-2. Beam width for a circular piston transducer ($a_i = 5$ mm and $c = 1500$ m/s).

<table>
<thead>
<tr>
<th>Frequency</th>
<th>1 MHz</th>
<th>2 MHz</th>
<th>3 MHz</th>
<th>4 MHz</th>
<th>5 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ka_i$</td>
<td>21.0</td>
<td>41.9</td>
<td>62.8</td>
<td>83.8</td>
<td>104.7</td>
</tr>
<tr>
<td>$\Theta$, deg</td>
<td>8.9</td>
<td>4.4</td>
<td>2.9</td>
<td>2.2</td>
<td>1.8</td>
</tr>
<tr>
<td>$\Theta_{\text{main}}$, deg</td>
<td>21.1</td>
<td>10.5</td>
<td>7.0</td>
<td>5.2</td>
<td>4.2</td>
</tr>
<tr>
<td>$W$, m</td>
<td>0.15</td>
<td>0.08</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>$W_{\text{main}}$, m</td>
<td>0.37</td>
<td>0.18</td>
<td>0.12</td>
<td>0.09</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Near-Field Correction Factor

Close to the transducer, the sound waves from different parts of the transducer surface interact with each other, both destructively and constructively. The structure of sound pressure in the near-field is different from the form given by 2-1. Regardless of the complexity, it still can be calculated accurately [Lockwood and Willette, 1973]. For the application to remote measurements of suspended sediment concentration and sediment size, the detailed information about the sound field close to the sensor is usually [Downing et al., 1995; Thorne et al., 1993a] substituted by the near-field correction factor, $\psi(r)$, which is given by [Downing et al., 1995]

$$
\psi(z) = \frac{1+1.32z+(2.5z)^{3.2}}{1.32z+(2.5z)^{3.2}}
$$

(2-67)

where $z = r/r_{nf}$ is the normalized distance, and $r_{nf}$ is the near-field range as given by
The value of $r_{nf}$ depends on the transducer radius and the sound wavelength. For a fixed transducer size, the near-field range increases linearly with the transducer frequency. For illustration, the values of the near-field range for frequencies 1–5 MHz are shown in Table 2-3.

Table 2-3. Near-field range $r_n$ for a circular piston source ($a_t = 5$ mm and $c = 1500$ m/s).

<table>
<thead>
<tr>
<th>Frequency</th>
<th>1 MHz</th>
<th>2 MHz</th>
<th>3 MHz</th>
<th>4 MHz</th>
<th>5 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{nf}$, m</td>
<td>0.05</td>
<td>0.10</td>
<td>0.16</td>
<td>0.21</td>
<td>0.26</td>
</tr>
</tbody>
</table>

![Graph](image)

Figure 2-9. Near-field correction factor, $1 / \psi(r)$, for $a_t = 5$ mm and $c = 1500$ m/s.

The near-field correction factor is applied to the far-field pressure in form of $1 / \psi(r)$. The corresponding curves for 1–5 MHz frequency transducers are shown in Figure 2-9. For higher frequencies the effect of the proximity to the transducer on transmitted sound is larger, and the near-field region is farther.
CHAPTER 3
CALIBRATION OF THE ACOUSTIC BACKSCATTER SYSTEM

Calibration of the acoustic backscatter system is necessary to obtain system function given by equation 2-11 [Thorne and Hanes, 2002]. For this purpose, the measurements of profiles of backscattered sound intensity can be taken in an environment with known characteristics. Different scatterers, such as thin wire targets, glass spheres or sand, are used in the calibration in various experimental setups [e.g., Hay, 1991; Jansen, 1979; Sheng and Hay, 1993]. Calibration in a chamber with a homogeneous suspension of sand or glass spheres is the most common technique [e.g., Ludwig and Hanes, 1990; Schaafsma, 1989; Tamura and Hanes, 1986; Thorne et al., 1993a; Thosteson and Hanes, 1998]. It has several advantages over other methods. The calibration with similar scatterers to those found in the field requires no changes in backscattering parameters such as form function or normalized cross-section. The experiments with the thin wire require modification of the acoustic backscatter equation due to cylindrical spreading of backscattered sound compared to spherical spreading from the sand grains and spheres. Also, the definition of sample volume for the wire is different from the one done for the suspension of particles. Although a modified form of equation 2-10 needs to be applied [Sheng and Hay, 1993], the method with the thin wire targets provides the easiest calibration which requires only a simple setup (similar to shown in Figure 4-2 in Chapter 4).

In experiments with a turbulent jet [Hay, 1991], a mixture of suspended particles and water is used. The suspension within the cross-section of the jet is not homogeneous and therefore modifications to the acoustic backscatter equation (equation 2-10) are
needed. The turbulence provides proper mixing and makes the orientation of the particles random relative to the transducer. Freely falling particles [Jansen, 1979] can have predefined orientation relative to the positions of transmitter and receiver, although the effect of this is not significant for particles with approximately spherical shapes [Sheng and Hay, 1988].

For calibration and other tests, two acoustic backscatter systems were used. Both systems were developed by Centre for Environment, Fisheries and Aquaculture Science (CEFAS), Lowestoft, UK. The first system, hereafter referred as “ABS1”, has three 10mm transducers with operational frequencies of 1.08, 2.07 and 4.70MHz. The data collection was carried out by a specially developed acquisition system [Thosteson, 1997]. The acquisition system operates at maximum rate of 88 profiles per second, and records a maximum of 4 root-mean-square (RMS) profiles a second by averaging over 22 consecutive raw voltage profiles. The second system, hereafter referred as “ABS2”, also has three 10mm transducers, with frequencies 1.0, 2.5 and 5.0MHz. Aquatec Electronics Ltd, UK, developed the data logger, which allows obtaining raw ABS profiles at a maximum of 80 profiles per second for each frequency.

The resolutions of the two systems are different. For the ABS1, the voltage is measured in a 12-bit range, while the ABS2 has 16-bit resolution. Spatial resolution is defined by the pulse duration. It is 10µs for the ABS1 and 13µs for the ABS2, which results in a bin size of approximately 7.5 and 10mm, respectively. Both systems use time varying gain (TVG). The form of the TVG was set to account for the spherical spreading, and is nominally given by a linear function of the range \( r \), although the exact form of the TVG may deviate from linear.
Calibration Chamber

The recirculating chamber at the University of Florida Coastal Laboratory was used to perform the calibration of the acoustic backscatter systems. This chamber was previously used in a number of projects [Lee, 1994; Ludwig and Hanes, 1990; Tamura and Hanes, 1986; Thosteson, 1997]. The chamber is schematically shown in Figure 3-1. It consists of a vertical 2m acrylic tube with inside diameter of 0.19m. A rotary pump moves a water-sediment mixture through a 4cm PVC pipe upward, where it discharges into the tank through the four nozzles. The nozzles are at the same level, about 10cm below the free surface. The jets are directed towards the center of the tank from 90-degree sectors (see Figure 3-3), where they collide and generate a turbulent flow, mixing the suspension. The mixture flows down the chamber with a mean cross-section velocity of approximately 4cm/s. The bottom of the tank has a funnel, which leads the flow through a pipe back into the pump. The funnel prevents the sediments from accumulating at the bottom. The suction ports were closed with metal plugs, which protruded into the interior of the chamber for about 5mm.

Before taking measurements, the chamber was filled with tap water and left for about 20 hours to allow air bubbles to leave the system. For additional 2–4 hours the system was left with the pump turned on.

To establish the predefined level of suspended sediment concentration, portions of sand were added into the water. Every portion was initially dried, weighed and placed for several hours in container with a small amount of water and detergent. The detergent was used to detach air bubbles from the surface of the sand grains. Excess water with detergent was removed from the container before the sand was added into the chamber.
Figure 3-1. Calibration chamber—side view.
ABS Tests in the Chamber

The chamber has a different environment for measurements than field experiments. Because of the laboratory setup, various artificial conditions can affect the measurements, which may lead to erroneous conclusions about estimated parameters. A series of tests were performed in the University of Florida Coastal Laboratory to investigate the effects of the chamber on calibration measurements.

For every test, profiles of root-mean-square voltage were obtained. Each profile was calculated as the average of 120 individual RMS profiles recorded at 4Hz for 30 seconds. To obtain each individual RMS profile, the ABS was set to average 22 raw profiles. Tests were made with fewer averaged profiles to check if the setting has an effect on the measurements. The higher sampling rate theoretically can lead to the interaction between several consecutive pulses, when previous sound pulses were not fully attenuated. The tests did not reveal this possibility. Additionally, to the average profiles, the 10th and 90th percentile profiles were calculated to illustrate the scatter among the measured RMS profiles.

Tests in Air and Still Water

Two tests were performed to examine the lower bound for voltage profiles. For this purpose the measurements were taken with the transducers left in the air and placed in the chamber with still clear water. The resulting curves are shown in Figure 3-2. The curves, which correspond to the measurements in the air, take on the lowest values except for the 1.08MHz transducer, which has a peak near approximately 4cm range. None of the profiles start with zero values—on the contrary they start from the value around 50–100 in relative units and decay within a few bins. Within this range there is no difference between profiles measured in the air and in the water. Most likely this is an effect of resid-
ual vibration of transducer surface after the sound pulse was emitted. The measurements in the water show that the scatter of the RMS profiles (in figures plotted with dashed lines) can be significant especially for high frequency transducers. The peaks around 9cm for 1.08 and 2.07MHz are caused by the reflection of emitted sound from the chamber walls, which will be illustrated further.

Figure 3-2. RMS voltage profiles (solid lines) with scatter intervals (dashed line) from the measurements in air and still clear water: a) 1.08MHz, b) 2.07MHz, and c) 4.70MHz.

Homogeneity of Sediment Suspension and Effect of Chamber Walls

The jet system and high flow rate serve to generate a homogeneous suspension. To investigate the homogeneity of the mixture in vertical and horizontal directions, samples were taken from the four openings in the side of the chamber wall (see Figure 3-1). The results of this investigation was previously published by Ludwig and Hanes [1990]. From their tests, it was concluded that the suspension in the chamber is homogeneous within the measurement error.

Additional measurements were currently conducted to test effects of the proximity of a transducer to the jet system and sidewalls. The ABS1 system with 3 frequencies
(1.08, 2.07, and 4.70MHz) was used for the tests. One transducer at a time was placed at different vertical levels relative to the nozzles (see Figure 3-1) and at different distances from the sidewall (see Figure 3-3).

![Figure 3-3. Calibration chamber—top view. The jet system is shown with transducer positions for the sidewall proximity tests.](image)

For the wall proximity tests, measurements were taken at the center of the chamber corresponding to a distance of 95mm, and also at 30 and 50mm distances from the sidewall. In the vertical, the transducer was placed at 3 levels: 0, 15 and 30cm. The 0cm level corresponds to the upper edge of the nozzles, therefore the jets of sand-water mixture were colliding just under the transducer surface.

The profiles of RMS voltage were recorded for several different levels of suspended sand concentration (see Tables 3-1, 3-2, and 3-3). Sand collected from Jacksonville Beach, FL with sieve fraction of size 0.212–0.250mm was used for the test.

<table>
<thead>
<tr>
<th>Distance from wall (row) and level (column)</th>
<th>0cm</th>
<th>15cm</th>
<th>30cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>30mm</td>
<td>0, 0.2, 0.5g/l</td>
<td>0, 0.2, 0.5g/l</td>
<td>0g/l</td>
</tr>
<tr>
<td>50mm</td>
<td>0, 0.2, 0.5g/l</td>
<td>0, 0.2, 0.5g/l</td>
<td>0g/l</td>
</tr>
<tr>
<td>95mm (center)</td>
<td>0, 0.2, 0.5, 0.8g/l</td>
<td>0, 0.2, 0.5, 0.8g/l</td>
<td>0, 0.2, 0.5, 0.8g/l</td>
</tr>
</tbody>
</table>
Table 3-2. Locations and concentration levels in tests for 2.07MHz transducer.

<table>
<thead>
<tr>
<th>Distance from wall (row) and level (column)</th>
<th>0cm</th>
<th>15cm</th>
<th>30cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>30mm</td>
<td>0.2, 0.5g/l</td>
<td>0.2, 0.5g/l</td>
<td>–</td>
</tr>
<tr>
<td>50mm</td>
<td>0.2, 0.5g/l</td>
<td>0.2, 0.5g/l</td>
<td>–</td>
</tr>
<tr>
<td>95mm (center)</td>
<td>0.2, 0.5, 0.8g/l</td>
<td>0.2, 0.5, 0.8g/l</td>
<td>0.2, 0.5, 0.8g/l</td>
</tr>
</tbody>
</table>

Table 3-3. Locations and concentration levels in tests for 4.70MHz transducer.

<table>
<thead>
<tr>
<th>Distance from wall (row) and level (column)</th>
<th>0cm</th>
<th>15cm</th>
<th>30cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>95mm (center)</td>
<td>0.5, 0.8g/l</td>
<td>0.5, 0.8g/l</td>
<td>0.5, 0.8g/l</td>
</tr>
</tbody>
</table>

Figure 3-4. Profiles of RMS voltage (solid lines) with scattering ranges (dashed lines) for 1.08MHz transducer for sidewall proximity tests.
Figures 3-4 and 3-5 show the results of measurements with the 1.08 and 2.07MHz transducers for different concentrations of sand. For the 1.08MHz transducer the profiles have several peaks, which correspond to the positions of the suction port plugs. As shown earlier (Table 2-2), the 1.08MHz transducer has the widest directional beam. Therefore, a part of the sound can reflect from the plugs and be detected by the receiver. The profiles for 2.07MHz transducer do not have similar peaks.

![Figure 3-5. Profiles of RMS voltage (solid lines) with scattering ranges (dashed lines) for 2.07MHz transducer for sidewall proximity tests.](image)

The measurements along the chamber centerline at different vertical levels are shown in Figures 3-6, 3-7, and 3-8. The curves for the 1.08MHz have peaks at approximately 9 and 19cm. Comparing to the profiles at other distances from the wall (see Figure 3-4), these peaks appear only when the transducer is placed in the center of the chamber. The ranges at which the peaks appear correspond to multiple chamber radiuses. They can
result from multiple reflection of the sound wave from side lobes between the chamber wall and transducer surface. This also demonstrates that the side lobes of the 1.08MHz transducer are quite significant. This can affect the measurements in the field as well, because other transducers may not detect the scatterers located within the side lobes of the 1.08MHz transducer.

Figure 3-6. Profiles of RMS voltage (solid lines) with scattering ranges (dashed lines) measured along centerline with 1.08MHz transducer

Figure 3-7. Profiles of RMS voltage (solid lines) with scattering ranges (dashed lines) measured along centerline with 2.07MHz transducer.
The effect of proximity to the nozzles is noticeable only for the 2.07 and 4.70MHz transducers. Figures 3-7 and 3-8 show that the profiles at level 0cm are lower than at levels 15 and 30cm. The drop in the intensity can be due to either Doppler shift of the frequency of the backscattered sound from faster moving sand grains, which cannot be resolved by the receiver with fixed frequency, or due to inhomogeneity of suspended sediment concentration in the upper part of the chamber. Although separate measurements of the concentration in the jet part of the chamber were not performed, the later seems to have more effect, since the maximum speed from the jets is not very high (approximately 3m/s) and resulted Doppler shift would be about 0.2% of the transducer frequency. However, the measurements show that if the transducer is placed lower than the jets, the difference between profiles from different layers is very small. Hence, the vertical homogeneity of the concentration in the chamber is confirmed and by the current tests.

Figure 3-8. Profiles of RMS voltage (solid lines) with scattering ranges (dashed lines) measured along center line with 4.70MHz transducer

Figures 3-9 and 3-10 show the difference between the measurements at different levels along the chamber, but now the transducers are placed at 30 and 50mm from the chamber wall. Similar to the tests when the transducers were located at the center of the chamber, the tests for 2.07MHz transducer show a slight decrease in voltage at 0cm level.
However, the profiles for 1.08MHz do not have peaks at ranges equal to multiples of the chamber radius.

Figure 3-9. Profiles of RMS voltage (solid lines) with scattering ranges (dashed lines) measured near the chamber wall with 1.08MHz transducer.

Figure 3-10. Profiles of RMS voltage (solid lines) with scattering ranges (dashed lines) measured near the chamber wall with 2.07MHz transducer.
Results

The tests show that the jet system generates homogeneous suspension in the region approximately 15cm below the nozzles. The chamber wall effects are not significant for 2.07 and 4.70MHz transducers, but for the measurements with the 1.08MHz transducer, it should not be placed in the center of the chamber to avoid multiple reflections from the side lobes of the beam. To obtain the smooth profile, the peaks corresponding to the positions of the suction port plugs can be filtered out by interpolating between the nearest data points. A few bins at the beginning of the profile do not contain useful information on the suspended sediment concentration, since it is contaminated by an electronic noise or residual vibration of the transducer membrane.

Estimation of System Function

The system functions of the acoustic backscatter, $K_v$, can be found from the calibration by using the expression

$$K_v = V_{rms} \frac{\psi r}{K_v M^{1/2}} \frac{ka_r}{0.96} \left( \frac{16}{3\tau c} \right)^{1/2} e^{2\alpha r}$$  \hspace{1cm} (3-1)

where $K_v$ is calculated by equation 2-11, $M$ is the level of suspended sand concentration in the chamber, $V_{rms}$ is the measured root-mean-square profile of voltage, $\psi$ is the near-field correction factor (2-67), $k$ and $c$ are the sound wavenumber and the sound speed in water, $a_r$ is the transducer radius, $\tau$ is the pulse duration, and $\alpha$ is the total attenuation coefficient (2-42)

Since the transducer frequency is preset and changes in the speed of sound in water are relatively small, the system parameter $K_v$, given by equation 2-11a, is usually obtained from the calibration instead of $K_v$ [Thorne and Hanes, 2002]. However, the effec-
tive transducer radius, which determines the directivity of the sound beam and extent of the near-field region, is usually 5–20% less than the geometrical radius. By using the parameter $K_r$, the best-fit value for $a_i$ may be found from the calibration.

The system function $K_v$ is constant if the time varying gain (TVG) is not applied, otherwise, $K_v$ is a function of range $r$. The system function describes the properties of the system and, therefore, does not vary with sediment size or concentration. However, the functions estimated from the measurements taken for different concentrations of sediments of different size distributions will most likely be not the same due to variations in sediment shapes, mineralogy, etc., which are not accounted for in the acoustic equation 2-10. The modified form function and normalized cross-section function with parameters $\beta_f$ and $\beta_x$, given by equations 2-40. and 2-41, may be used to account for these differences, as well as to better approximate the system function.

Let $K_v^i$ be the $i$th estimation of the inherited system function, $K_v$, where each $K_v^i$ is calculated by equation 3-1 at the concentration level $M_{meas}^i$ from the measured voltage profile $V_{rms}^i$ for a given sediment size distribution. The average function can be found as the mean of estimates as

$$\bar{K}_v = \frac{1}{N} \sum_{i=1}^{N} K_v^i$$

(3-2)

where $N$ is the number of estimations, to provide an estimation of the system function for a particular sediment size distribution.

**Approximation by 2nd Order Polynomial**

The system function calculated by equation 3-2 is not necessarily a smooth function. Therefore, $\bar{K}_v$ was fitted with a 2nd order polynomial over the range of bins.
\( n_{nf} \leq j \leq n \) (\( n_f \) is the near-field bin index and \( n \) is the total number of bins) to produce an estimation of system function \( K' \) as

\[
K' = p_2r^2 + p_1r + p_0
\]  

(3-3)

where \( p_i, i = 0, 1, \text{ or } 2 \), are the coefficients of the best-fit polynomial.

**Calibration of the ABS1**

The ABS1 was calibrated with quartz sand, collected at SISTEX'99 experiment [Vincent et al., 2001], with mean size \( \mu_{\phi} = 2.17 \) and standard deviation \( \sigma_{\phi} = 0.27 \) estimated from the sieve analysis. From the fall velocity measurements, the mean sediment size was approximated to be 1.96 in phi units. Both measurements and corresponding normal distribution curves are shown in Figure 3-11. The transducer radius was taken to be 90% of the geometrical radius. The modified form function and normalized cross section were calculated by equations 2-34 and 2-35 with initial value of 0.5 for parameters \( \beta_\chi \) and \( \beta_f \).

![Sieve and Fall Velocity Data](image)

Figure 3-11. Estimation of sediment size distribution used for the calibration of ABS1 by: a) sieve analysis, b) fall velocity.
The resulted system functions are shown in Figures 3-12–3-14. Individual estimations, $K_v^i$, are shown together with the curves for $K_v'$ (marked as “fit”) on plots “b” in Figures 3-12–3-14. The errors on plot “b” were calculated as the average of relative root-mean-square deviations from the 2nd order fit and are given by the expression

$$
\varepsilon_{K_v} = \frac{1}{n - n_{nf} + 1} \sum_{j=n_{nf}}^{n} \left( \frac{1}{N} \sum_{i=1}^{N} \frac{K_v^i(r_j) - K_v'(r_j)}{K_v'(r_j)} \right)^2
$$

(3-4)

The near-field bin index $n_{nf}$ for current calculations was taken to be equal to 15, but can be adjusted for any particular system. The value was chosen from the sidewall proximity tests which showed that the measurements at bins <15 are not accurate.

Calculated by 2-10, voltage profiles were compared to the measured profiles. Their ratios are plotted on plots “a.” Using 2-12 the inverted profiles of the suspended sediment concentration are shown on plots “c.” Relative errors for each of the concentration levels between inverted and measured concentrations are also shown with the mean error, which was calculated as the average of individual relative errors:

$$
\varepsilon_M = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{n - n_{nf} + 1} \sum_{j=n_{nf}}^{n} \left| \frac{M_i^\text{calc}(r_j) - M_i^\text{meas}(r_j)}{M_i^\text{meas}(r_j)} \right|
$$

(3-5)

where the subscript “meas” denotes measured or expected concentration in the chamber and subscript “calc” denotes estimated concentration profiles calculated with 2-12.
Figure 3-12. Estimation of the system function for 1.08MHz transducer: a) ratio of measured and calculated voltage profiles, b) estimated system functions, and c) inverted sediment concentration.

Figure 3-13. Estimation of the system function for 2.07MHz transducer: a–c) same as in Figure 3-12.
Figure 3-14. Estimation of the system function for 4.70MHz transducer: a–c) same as in Figure 3-12.

The estimated system functions show that the expected linear dependence due to the TVG is present only for the 2.07MHz transducer, although the 2nd order coefficient is still significant. The curves of voltage ratios on plot “a” for the 4.70MHz transducer (see Figure 3-14) are not uniform with range compared to the curves for the other two transducers. This may indicate erroneously estimated attenuation at each level of concentration. The errors corresponding to the different concentration levels are not uniform, especially for 1.08 and 4.70MHz. Since the estimated system functions were applied, the errors grow for smaller and larger concentrations.

Calibration of the ABS2

The calibration of the ABS2 was performed similarly to the ABS1. The sand from Jacksonville Beach, FL was used for the calibration. The estimations of sediment size
distribution parameters by sieve analysis and based on fall velocity are shown in Figure 3-15.

Figure 3-15. Estimation of sediment size distribution for sand used in calibration of ABS2 by: a) sieve analysis, b) fall velocity.

Figure 3-16. Estimation of the system function for 1MHz of ABS2: a–c) same as in Figure 3-12.
The resulting system functions are shown in Figure 3-16. Unlike the 1.08MHz transducer of the ABS1, the curves for the system functions are very close to each other and the correction of $\beta_z$ parameter was not needed.

The 1MHz transducers of the both systems are very similar, and one would expect to obtain similar results in calibration of both systems. However, the current tests show that the results are very different. The calibration of the 1.08MHz transducer of the ABS1 does not result in a unique system function. The application of the average of estimated system functions can lead to large errors in inverted concentrations. For these cases, a semi-empirical model was developed to provide the solution for suspended sediment concentration. This model is described in Appendix B.

Sensitivity Analysis For Estimation of System Function

A particular estimate of system function $K_v$ depends on several parameters, which can be initially approximated with some error or uncertainty. In order to investigate the sensitivity of the system function $K_v$ to changes in the parameters, such as form function, normalized cross-section, and sediment distribution, the following analysis was carried out. Using equation 3-1, the system function $K_v$ can be rewritten as a function of parameter $\beta_z$, transducer radius $a_r$, and sediment size distribution parameters $\mu_\phi$ and $\sigma_\phi$ as

$$K_v = F\left(\beta_z, a_r, \mu_\phi, \sigma_\phi\right)$$

The form function and normalized cross section were estimated by equations 2-40 and 2-41. The dependence of $K_v$ on listed parameters is not explicit, but through the form function, normalized cross-section, beam directivity, and near-field correction factor. Generally for calibration, these parameters need to be known and fixed. For current
analysis, a subset of the listed parameters set to be free variables (i.e. by letting them vary within some intervals). This produces the new system functions, $K_{v,\text{fit}}$, estimated by minimizing the error calculated by equation 3-4.

The results of the calculations, based on the measurements for the ABS1, are gathered in tables in Figures 3-17–3-19. For each test, some of four parameters $\beta_x$, $a_t$, $\mu_\phi$, and $\sigma_\phi$ were fixed (marked with “dots”). Others were variable, for them the best-fit values are presented. Bounds for free parameters are shown in the last column of the table. For fixed parameters, their initial values were used. The errors of inverted concentrations together with the mean error (black line) were calculated and plotted for each test. The values of the mean errors are shown in the table in “Mean error, %” row. Values in row “Fit errors, %” were calculated by 3-4.

The bounds of free parameters were chosen to cover a realistic, but still wide, range of values. In many cases, the minima were reached at boundaries. If unconstrained minimization would be performed, the minimum could be reached at unrealistic values, such as 0mm for transducer radius.

It can be seen from the figures that not all the parameters have the same effect on calibration. The parameter $\beta_x$ and mean sediment size $\mu_\phi$ have the most significant effects, while transducer radius $a_t$ and sediment size sorting $\sigma_\phi$ have almost no effect.

Since both the mean sediment size and parameter $\beta_x$ are present in the expression for normalized cross-section, minimization on one of them can be compensated by an equivalent change in another. Because of this relationship, use of incorrect values for $\beta_x$ may lead to erroneous approximation of sediment size from the measurements.
Figure 3-17. Values of fitted parameters $\beta_x$, $a_t$, $\mu_\phi$, and $\sigma_\phi$ with corresponding errors in inverted concentrations for the 1.08MHz transducer of the ABS1.

Figure 3-18. Same as on Figure 3-17 for the 2.07MHz transducer of the ABS1.
Figure 3-19. Same as on Figure 3-17 for the 4.70MHz transducer of the ABS1.

The biggest decrease in mean error (3-5)—from 18.9 to 14.0%—is shown in Figure 3-19 for the 4.70MHz transducer by varying parameter $\beta_\chi$. After the corrections to sediment attenuation, not only the mean error was reduced, but also the errors for most of concentration levels. The sediment attenuation for transducer 1.08MHz was insignificant; neither $\mu_\phi$ nor $\beta_\chi$ have any influence on errors. For the 2.07MHz transducer parameter $\beta_\chi$ was estimated to be $-0.77$, which results in decreasing of the originally estimated (by equation 2-35) normalized cross-section and therefore sediment attenuation. The corresponding value of $\beta_\chi$ for the 4.7MHz is 1.20, which implies that the sediment attenuation is larger than estimated by equation 2-35.

The system functions for the 4.70MHz transducer with corrected $\beta_\chi$ are shown in Figure 3-20. The errors in inverted concentrations at low concentration levels (0.1 and 0.2g/l) were decreased more than 2 times, and also became more uniform with range. The
spreading among $K_{v,fit}$ curves was significantly reduced resulting in a mean relative error of 2.4%.

![Figure 3-20. Estimation of system function $K_v$ for 4.70MHz transducer with corrected parameter $\beta_z$: a–c) same as in Figure 3-12.](image)

**Dependence on Sediment Sorting**

The dependence of $K_v$ on sorting is rather complicated, because of filtering the data (either theoretical or experimental) used in obtaining the form function and normalized cross-section. From equations 2-61 and 2-63, $K_v$ can be simplified to

$$K_v = C_i(r) \left( \frac{|a_j^2f_c(x_o)|^2}{\langle a^2 \rangle} \right)^{-i/2} \cdot \exp \left\{ -C_2(r) \left( \frac{x^2f_c(x_o)}{\langle a \rangle} \right) \right\} =$$

$$= C_i(r) e^{S_a^2} \left( \frac{|a_j^2f_c(x_o)|^2}{\langle a^2 \rangle} \right)^{-i/2} \cdot \exp \left\{ -C_2(r) e^{-2S_a^2} \left( \frac{x^2}{\langle a \rangle} \right) \right\} =$$

$$= C_i(r) \exp \left\{ S_a^2 - C_4(r) e^{-2S_a^2} \right\}$$

(3-7)

where $S_a = \sigma_e \ln 2$, and functions $C_i(r)$ do not depend on $\sigma_e$, but on other parameters.
By approximating exponents to 2nd order, the following expression can be obtained

\[ K_r \approx C_1(r) \left\{ 1 - C_4(r) + \sigma^2 \ln 4 \left( 1 + 2C_4(r) \right) \right\} \quad (3-8) \]

where \( K_r \) now depends on \( \sigma \) to second order. Therefore, small changes in \( \sigma \) will have small effect on minimized relative error. This is supported by Figures 3-17–3-19, where there is no consistency in calculated values for \( \sigma \); the errors sometime reach their minima at either limits of the constraint interval for \( \sigma \). Based on current analysis, it can be concluded that if the sediments are well sorted, then it becomes nearly impossible to successfully approximate sorting from measurements using this minimization technique.

**Dependence on Transducer Radius**

The dependence of \( K_r \) on transducer radius \( a_t \) can be explicitly found and derivative becomes

\[
\frac{\partial K_r}{\partial a_t} = C(r) \left\{ \frac{\partial \psi}{\partial a_t} a_t + \psi \right\} = C(r) \left\{ -2z \frac{\psi'}{\partial a_t} \right\} \approx C(r) \left\{ 1.01 - 0.01(z - \pi) \right\}
\]

(3-9)

where \( C(r) \) is the coefficient not depending on \( a_t \). For far-field \( (z > \pi) \) the relation between \( K_r \) and \( a_t \) is almost linear with a very small second order term, and therefore does not significantly affect the relative error (3-4). However, if the near-field region extends significantly, as in the case of 4.70MHz transducer, changes in transducer radius can be noticeable.

**Determination of Parameter \( \beta_f \)**

For a homogeneous suspension of sediments, coefficient \( K_r \) (equation 2-11a) is constant with range. Therefore, any uncertainties in determination of \( K_r \) affect only the
magnitude of the estimated system function $K_v$. Since the system function for a given transducer is unique, any deviation of the estimated system functions can be compensated by using the modified form functions (2-37 or 2-40) with the appropriate parameter $\beta_j$.

Because the coefficient $K_v$ appears as a multiplier in equation 3-1, the parameter $\beta_j$ cannot be found from measurements with a homogeneous suspension of one sediment sample. Thus measurements with different sediment samples are required.

From equations 2-11 and 2-40 it can be found that there exists a constant $\xi$

$$\xi^2 \left\langle a^2 f_{w,h}^2 \right\rangle = \left\langle a^2 f_{w,h}^2 \right\rangle = \left\langle a^2 \gamma^2 (ka, \beta_j) f_{w,h}^2 \right\rangle$$

which for small standard deviation $\sigma$ gives

$$\xi \approx \gamma(ka, \beta_j)$$

(3-11)

Similar to the estimation of the system functions, $K_v'$, which were obtained from the measurements with particular sediments (see equations 3-2 and 3-3), let the system function $K_v$ be written in the form of a 2nd order polynomial over the range $n_{nf} \leq j \leq n$

$$K_v = \bar{p}_2 r^2 + \bar{p}_1 r + \bar{p}_0$$

(3-12)

with each coefficient $p_k$, $k = 0, 1, 2$, found as

$$\bar{p}_k = \frac{1}{m} \sum_{j=1}^{m} p_{k,j}$$

(3-13)

where $p_{k,j}$ are the corresponding coefficients of the estimation for one sediment size system function $K_v'$, and $m$ is the number of estimations.
Therefore, $\xi$ can also be written as

$$\xi = \frac{K'_{\xi}}{K_{\xi}} = \frac{p_2 r^2 + p_1 r + p_0}{\bar{p}_2 r^2 + p_1 r + \bar{p}_0} \tag{3-14}$$

which needs to be satisfied for all values of $r$. This may not be possible for all times, therefore the estimation of $\xi$ using mean quantities can be used and the following expression obtained

$$\frac{1}{r_0} \int_0^{r_0} \left( r^2 \left( p_2 - \xi \bar{p}_2 \right) + \left( p_1 - \xi \bar{p}_1 \right) r + \left( p_0 - \xi \bar{p}_0 \right) \right) dr = 0 \tag{3-15}$$

with the solution for $\xi$ as

$$\xi = \frac{\frac{1}{3} p_2 r_0^2 + \frac{1}{2} p_1 r_0 + p_0}{\frac{1}{3} \bar{p}_2 r_0^2 + \frac{1}{2} \bar{p}_2 r_0 + \bar{p}_0} \tag{3-16}$$

where $r_0$ is some reference range. After obtaining the solution for $\xi$, parameter $\beta_f$ can be found by inverting equation 3-11.

**Estimation of System Function for ABS1**

To obtain the system function for the ABS1, the tests were carried out with four different sediment samples. The sediments were predominantly quartz sand. The parameters of the size distributions were estimated from sieve analysis and are shown in Table 3-4. The sample from Case 3 is the same sample used for the calibration of the ABS1 and described in the previous section. Its size distribution is shown in Figure 3-11. The sediments for Case 4 were the mix of two sieve fractions—0.125–0.149 and 0.297–0.250mm. The estimations of distribution parameters are shown in Figure 3-21.

The system functions were calculated for all four series of tests. The results of the calculations are shown in Appendix A (see Figures A-13–A-36).
ters $\beta_x$ and $\beta_f$, the mean errors (calculated by equations 3-4 and 3-5), and parameter $\zeta$ are gathered into Table 3-5. The parameters $\beta_f$ were approximated in both cases—for initial values of $\beta_x=0.5$ and for best-fit values, estimated from the minimization of error 3-4. It was previously noted that for a wide sediment size distribution, the value of sorting $\sigma_\phi$ can have a noticeable effect on estimated quantities. For example, Figure A-2 shows that if $\sigma_\phi=0.44$, then the corresponding parameter $\beta_x$ is equal to $-1.98$. But if the sorting would be estimated as $\sigma_\phi=0.06$, which is the best-fit value when $\sigma_\phi$ is taken as a free parameter, the parameter $\beta_x$ is equal to $-0.19$. In Table 3-5 the values of parameter $\beta_x$ correspond to the estimated from measurements values of $\sigma_\phi$ (from the column for Test#8 in Figures A-1–A-12 in Appendix A).

Table 3-4. Sediment sizes.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_\phi$</td>
<td>2.67</td>
<td>2.74</td>
<td>2.17</td>
</tr>
<tr>
<td>$\sigma_\phi$</td>
<td>0.44</td>
<td>0.25</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Figure 3-21. Estimation of sediment size distribution for Case 4 used for calibration of ABS1 by: a) sieve analysis, b) fall velocity.
The results in Table 3-5 show that for the fitted values of $\beta_x$, all mean errors decrease up to 50%. Case 1 for the 4.70MHz transducer is the only one, where the mean error in inverted concentrations increases slightly. Although the changes in parameters $\beta_x$ and $\beta_f$ are also small.

Table 3-5. Estimation of parameters $\beta_x$ and $\beta_f$ for different sediment sizes.

<table>
<thead>
<tr>
<th>Frequency, MHz</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial</td>
<td>Fitted</td>
<td>Initial</td>
<td>Fitted</td>
</tr>
<tr>
<td>1.08MHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_x$</td>
<td>0.5</td>
<td>-2.5</td>
<td>0.5</td>
<td>-2.5</td>
</tr>
<tr>
<td>$\varepsilon_{K_v}$, %</td>
<td>6.9</td>
<td>5.3</td>
<td>3.6</td>
<td>3.3</td>
</tr>
<tr>
<td>$\varepsilon_M$, %</td>
<td>12.3</td>
<td>9.5</td>
<td>6.8</td>
<td>6.0</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.04</td>
<td>1.04</td>
<td>1.51</td>
<td>1.53</td>
</tr>
<tr>
<td>$\beta_f$</td>
<td>0.90</td>
<td>0.91</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>2.07MHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_x$</td>
<td>0.5</td>
<td>-1.98</td>
<td>0.5</td>
<td>0.37</td>
</tr>
<tr>
<td>$\varepsilon_{K_v}$, %</td>
<td>7.3</td>
<td>3.6</td>
<td>2.6</td>
<td>2.6</td>
</tr>
<tr>
<td>$\varepsilon_M$, %</td>
<td>11.4</td>
<td>6.8</td>
<td>5.9</td>
<td>5.9</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.08</td>
<td>1.06</td>
<td>1.15</td>
<td>1.21</td>
</tr>
<tr>
<td>$\beta_f$</td>
<td>0.92</td>
<td>0.85</td>
<td>1.85</td>
<td>2.5</td>
</tr>
<tr>
<td>4.70MHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_x$</td>
<td>0.5</td>
<td>0.33</td>
<td>0.5</td>
<td>1.47</td>
</tr>
<tr>
<td>$\varepsilon_{K_v}$, %</td>
<td>3.2</td>
<td>3.1</td>
<td>5.8</td>
<td>2.1</td>
</tr>
<tr>
<td>$\varepsilon_M$, %</td>
<td>16.8</td>
<td>17</td>
<td>21.9</td>
<td>11.8</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.16</td>
<td>1.13</td>
<td>1.01</td>
<td>1.09</td>
</tr>
<tr>
<td>$\beta_f$</td>
<td>1.10</td>
<td>1.04</td>
<td>0.55</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Table 3-6. Values of $x = ka$ for test cases.

<table>
<thead>
<tr>
<th>Frequency, MHz</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.08</td>
<td>0.36</td>
<td>0.34</td>
<td>0.51</td>
<td>0.51</td>
</tr>
<tr>
<td>2.07</td>
<td>0.69</td>
<td>0.66</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>4.70</td>
<td>1.57</td>
<td>1.49</td>
<td>2.22</td>
<td>2.23</td>
</tr>
</tbody>
</table>

It was previously noticed, that even large changes in parameter $\beta_x$ for the 1.08MHz transducer have small effects on estimated errors—the fitted values for $\beta_x$
drop to −2.5 (the lower bound), the value chosen to be the lower limit in the calculations. For the 1.08MHz frequency, the values of \( x = ka \) (see Table 3-6) are less than 0.51 for the Cases 1–4. This gives the values for the normalized cross-section of 0.01 or less (see equation 2-35 and Figure 2-6), which results in very small total attenuation (2-42) at low concentrations. Since, the parameter \( \beta_\chi \) is found from an expression like

\[
\gamma(ka, \beta_\chi) = \frac{\chi_b(ka)}{\chi'}
\]

(3-17)

where normalized cross-sections—\( \chi' \) is estimated from data and \( \chi_b \) is given by equation 2-35—are small numbers, approximated from their ratio values of \( \gamma \) can have large errors. Therefore, the initial value for \( \beta_\chi = 0.5 \) can be used.

Figure 3-22. Estimated system functions and their means for Cases 1–4 with \( \beta_\chi = 0.5 \) (dashed) and fitted (solid): a) 1.08, b) 2.07, and c) 4.70MHz transducers of the ABS1.
The estimated system functions for all four cases are shown in Figure 3-22 together with their means calculated by equation 3-12. The dashed lines correspond to the functions estimated by assuming the initial value $\beta_\chi = 0.5$. The solid lines correspond to the system functions estimated with fitted values for $\beta_\chi$ (see Table 3-5). The parameter $\xi$ was estimated by equation 3-16 with $r_0 = 0.6$ m. Corresponding values of $\beta_f$ were found by inverting equation 3-11. Both parameters are shown in Table 3-5.

![Graphs of system functions](image)

Figure 3-23. Ratios between individual system functions for Case 1–4 and mean system function with $\beta_\chi = 0.5$ (dashed) and fitted (solid): a) 1.08, b) 2.07, and c) 4.70MHz transducers of the ABS1.

Although the system function should be unique for a given system, the estimated curves have some scatter, which results from the different sediments used in each case. For a homogeneous suspension, the curves differ approximately by a constant (equation 3-14). Therefore, the ratios of individual estimations to the mean were calculated with the results shown in Figure 3-23. For the 1.08 and 2.07MHz transducers, the calculated ratios
are close to constant for $r > 0.1\text{m}$. In most cases for these two transducers, the ratios are more uniform for the system functions estimated with fitted values of $\beta_x$, compared to ones estimated by using $\beta_x = 0.5$. This means that the corrections to the normalized cross-section (equation 2-41) lead to a better approximation of sediment attenuation. Also it should be noted that for the 4.70MHz transducer, the ratios are less uniform with range than for other two frequencies.

Analysis of Relationship between $\beta_x$ and $\beta_f$

As can be seen from the results in Table 3-5, by choosing appropriate values of $\beta_x$ and $\beta_f$, corrections to the form function and normalized cross-section can be made (equations 2-40 and 2-41), which improve the accuracy of suspended sediment concentrations. Since the parameters $\beta_x$ and $\beta_f$ were found individually for each frequency and each sediment sample, the following questions may arise:

- Can a single value $\beta$ be found, which could be used for correction of both the form function and the normalized cross-section for every individual or for all used frequencies?
- Can a single value of $\beta_f$ be found which would represent a sediment sample and be unique for an applicable range of sound frequencies?
- Can a single value of $\beta_x$ be found which would represent a sediment sample and be unique for an applicable range of sound frequencies?

In Figure 3-24, the estimated parameters $\beta_f$ are plotted versus corresponding fitted values of $\beta_x$ for frequencies 2.07 and 4.70MHz. It can be seen that in most cases $\beta_f > \beta_x$, as it obtained by Thorne and Buckingham [2004]. Because of large scatter and an insufficient number of available data points, the functional relationship between $\beta_f$
and $\beta_x$ can only be approximated with a large degree of uncertainty. Nevertheless, it can be concluded that $\beta_f$ and $\beta_x$ are most likely different even for a single frequency.

Figure 3-24. Fitted parameters $\beta_x$ and corresponding parameters $\beta_f$ for Cases 1–4.

Values of $\beta_f$ and $\beta_x$ for Cases 1–4 are plotted in Figure 3-25. It can be seen, that even with the large scatter, there is an overall trend in the parameters.

Figure 3-25. Values of parameters $\beta_f$ and $\beta_x$ for Cases 1–4 for different frequencies.
Figure 3-26 shows calculated values for $\xi$ together with $\gamma(ka, \beta_x)$. Dashed lines correspond values of $x = ka$ (see Table 3-6). Note that values of $\gamma(ka, \beta_x)$ for the 1.08MHz transducer were not plotted since all fitted $\beta_x = -2.5$. Solid lines correspond to the function $\gamma(ka, \beta)$ (equation 2-39) with $\beta$ estimated as

$$\beta = \frac{1}{2}(\beta_f, 2.07 + \beta_f, 4.70) \tag{3-18}$$

where $\beta_f, 2.07$ and $\beta_f, 4.70$ are the values of $\beta_f$ for the 2.07 and 4.70MHz transducers. The lines fit data points for $\xi$ relatively well, especially in Cases 1 and 4. Although in Case 3 the point corresponding to the 1.08MHz frequency is much lower than points for the other two frequencies and also lower than the approximated curve. Overall, equation 3-18 gives a good approximation for parameter $\beta_f$ for a given sediment sample.

Figure 3-26. Values of $\xi$ and $\gamma(ka, \beta_x)$ as functions of $ka$ with approximated functions $\gamma(ka, \beta)$ for Cases 1–4 (solid lines).
A similar approximation for $\beta_z$ will not be possible. The values of $\gamma(ka, \beta_z)$ are close to each other only for Case 4, while other three cases look random.

“Fun-Pump” Tests

The homogeneous sediment suspension, generated in the chamber (Figure 3-1) for calibration, will most likely not be found during field or lab experiments, when the measurements are taken above a sandy bottom, and the suspension is generated by waves and currents. The following tests were conducted in order to simulate the “real” environment, which have variable concentration and size distribution profiles of sediment suspension. A simple technique is used to generate such nonuniform profiles in the chamber. With the sediments added to the chamber, the pump system generates a homogeneous suspension throughout the water column. If at any time the pump is turned off, the sediments will settle with some terminal velocity. Since there are no other sources of sediment near the top of the chamber, a nonzero gradient in sediment concentration will be generated. Also, if there are several sediment sizes present, then the sediments with different sizes will settle at different velocities. This will also generate a nonuniform profile of sediment size distributions.

The measurements were taken with the ABS1 system. Three transducers were placed at the top of the chamber at about 10cm below the free water surface pointed downwards (see Figure 3-1). The ABS was operating at 88 profiles per second. Every 22 profiles were averaged to obtain a single root-mean-square (RMS) profile of voltage. The resulting RMS voltage profiles were recorded by the data logger at 4Hz. The duration of every run was three minutes, including 30 seconds of data with homogeneous suspension with the pump turned on. Then the pump was turned off, and the measurements were re-
corded for another two minutes, while the sediments were settling down. After that, the pump was turned back on and data were collected for another 30 seconds.

**Sample Sediment Size Distribution**

A mixture of two sediment samples was used for the tests. These are the same sediments as were used for the calibration (see Case 4). The first sample is the sieve fraction of size 0.125–0.149mm (2.47–2.75 phi units) of the sand from Jacksonville, FL. It will be referred as Sand 1. The other sample is the sieve fraction of size 0.297–0.250mm (1.25–1.75 phi units) will be referred as Sand 2.

Assuming a uniform distribution of sediment sizes within each fraction, one can approximate the mean and standard deviation of a mixture of two sediment samples. Let the mixture contains two fractions $\alpha_1$ and $\alpha_2$ of Sand 1 and Sand 2. Therefore,

$$\alpha_1 + \alpha_2 = 1$$ \hfill (3-19)

The size distributions are given by

$$p_i(\varphi) = \begin{cases} \frac{1}{b_i - a_i}, & \varphi \in [a_i, b_i] \\ 0, & \varphi \notin [a_i, b_i] \end{cases}$$ \hfill (3-20)

where $i$ corresponds to Sand 1 or Sand 2, and $[a_i, b_i]$ are the size ranges. The mean and variance of sediment sizes in mixture becomes

$$\mu_\varphi = \alpha_1 \mu_1 + \alpha_2 \mu_2, \quad \sigma_\varphi^2 = \alpha_1 \left( \sigma_1^2 + \mu_1^2 \right) + \alpha_2 \left( \sigma_2^2 + \mu_2^2 \right) - \mu_\varphi^2$$ \hfill (3-21)

where $\mu_i$ and $\sigma_i$ are the mean and standard deviation of Sand 1 and Sand 2, determined as

$$\mu_i = \frac{a_i + b_i}{2}, \quad \sigma_i = \frac{(a_i - b_i)^2}{12}$$ \hfill (3-22)
Different concentration levels were acquired by adding proper amounts of Sand 1 or Sand 2 into the chamber. The fractions of each sample are shown in Table 3-7 together with the resulting size distributions. Based on the fall velocity analysis, a 1:1 mixture of two samples had the mean of 1.99 and standard deviation of 0.60 (see Figure 3-21). Note that the values of parameters of the distribution for the 1:1 mixture in Figure 3-21 from the sieve analysis were different from Table 3-7. It is because for the calibration in Case 4 average values of the parameters were used.

Table 3-7. Concentration levels, mean and standard deviation of the mixture for “fun-pump” tests.

<table>
<thead>
<tr>
<th>Concentration, g/l</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\mu_\phi$</th>
<th>$\sigma_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>1.00</td>
<td>0.00</td>
<td>2.61</td>
<td>0.08</td>
</tr>
<tr>
<td>0.8</td>
<td>0.50</td>
<td>0.50</td>
<td>2.06</td>
<td>0.57</td>
</tr>
<tr>
<td>1.2</td>
<td>0.67</td>
<td>0.33</td>
<td>2.24</td>
<td>0.53</td>
</tr>
<tr>
<td>1.6</td>
<td>0.50</td>
<td>0.50</td>
<td>2.06</td>
<td>0.57</td>
</tr>
<tr>
<td>2.0</td>
<td>0.60</td>
<td>0.40</td>
<td>2.17</td>
<td>0.55</td>
</tr>
<tr>
<td>2.4</td>
<td>0.50</td>
<td>0.50</td>
<td>2.06</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Figure 3-27. Sediment size distribution for Sand 1, Sand 2, and 1:1 mixture.

The fall velocity analysis provided estimations of size distribution functions (PDF) for both sand samples and the 1:1 mixture. The results are shown in Figure 3-27. Note that the size distribution for Sand 1 is wider than that approximated from the sieve analysis.
Prediction of Concentration and Sediment Size Distribution

The profiles of sediment concentration and size distribution parameters at any given time after the pump was turned off can be predicted with a 1D advection-diffusion model (see Appendix C for a more detailed description of the model). The sediment diffusivity parameter (equation C-1) can be taken to be zero, because of the absence of turbulence production when the pump is off. When the pump has just been turned off, the turbulence is not zero. Therefore, a small constant value can be used for the sediment diffusivity to account for the residual turbulence. The sediment diffusivity was taken to be 0.0005m²/s. Other values within 50-200% interval were also tested; this did not result in significant difference in predicted profiles.

Figure 3-28. Predicted with 1D advection-diffusion model profiles of concentration and sediment size distributions for maximum concentration of 0.8g/l at times 0–70s: size distributions at a) 30, b) 50, and c) 75cm below the free surface, d) sediment concentration profiles, e) profiles of mean sediment size in phi units, and f) profiles of standard deviations in phi units.
The suspended sediment concentration profiles and profiles of size distribution parameters predicted with the 1D advection-diffusion model for the initial concentration in the chamber of 0.8g/l are shown in Figure 3-28. All the profiles are shown from 0 to 70 seconds at 10-second intervals. The initial size distribution was approximated from the fall velocity analysis (see Figure 3-27). The parameters of size distribution were estimated by fitting the normal distribution curve (equation 2-52) to the calculated by the model distribution curves.

The model results show that most of the concentration profiles have a step, which originates from the difference in terminal settling velocity of two sediment samples; the coarse sediment, Sand 2, settles faster than the finer sediment, Sand 1. A similar irregularity can be seen in the profiles for the approximated mean sediment sizes and standard deviations. The mean sediment size at every range increases (in phi units) with time, while the sorting decreases. The difference in the sorting along the profile can be quite significant. It can be seen especially at times 10 and 20 seconds after the pump was turned off, when the STD changes from approximately 0.2 to 0.5 along a single profile. This can affect the accuracy of the inversion of the measurements taken by the ABS into concentration and sediment sizes, as already was previously discussed.

Parameters of Inversion

For the inversion of RMS voltage profiles, the model defined by equations 2-10 and 2-11, was used. The modified high-pass model for backscattered intensity, defined by equation 2-40 for the form function, and by equation 2-41 for the normalized cross-section were applied with the parameters $\beta_f$ and $\beta_x$ determined from the calibration (see parameters for Case 4 in Table 3-5). The system function $K_v$ was approximated by equation 3-12 based on all available calibration profiles (see section “Determination of
Parameter $\beta_f$). The resulting coefficients of the second order polynomial are given in Table 3-8.

<table>
<thead>
<tr>
<th>Frequency, MHz</th>
<th>$p_2$</th>
<th>$p_1$</th>
<th>$p_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.08</td>
<td>24.65</td>
<td>-1.162</td>
<td>11.62</td>
</tr>
<tr>
<td>2.07</td>
<td>-4.118</td>
<td>55.37</td>
<td>18.56</td>
</tr>
<tr>
<td>4.70</td>
<td>-199.9</td>
<td>359.6</td>
<td>27.18</td>
</tr>
</tbody>
</table>

The form function and normalized cross-section are shown in Figure 3-29. Both the initial unmodified high-pass model curves for $f_{\omega,h}$ and $\chi_{\omega,h}$ (equations 2-34 and 2-35) and the modified versions $f'_{\omega,h}$ and $\chi'_{\omega,h}$ (equations 2-40 and 2-41) are plotted. With the provided parameters $\beta_f$ and $\beta_\chi$, the modified functions have lower values than the initial ones. It should be pointed out that according to the plots the 1.08MHz frequency is the least sensitive to the sediment size represented by Sand 1. For this sand, the sediment attenuation, $\alpha_s$ (equation 2-47), is also close to zero. Therefore in the test with 0.4g/l initial concentration when only Sand 1 was used (see Table 3-7), the inverted concentration for the 1.08MHz transducer was much lower (with an average approximately equal to 0.07g/l) than for other two transducers (see Figure 3-30). The mean concentration profiles were also estimated to be lower.

The profiles of mean sediment sizes were found by minimization of the error between estimated concentration profiles for each frequency at a given range (equation 2-25). The standard deviation of sediment sizes needs to be known in advance. Theoretically, the sediment sorting can be specified at each time for every profile. But since field measurements usually lack of such information, and the sorting is estimated from the
sieve or fall velocity analyses. Therefore, a single value equal to 0.54 $\phi$ was used in all calculations.

![Graph](image)

Figure 3-29. High-pass model for backscattered intensity and its modified version used for “fun-pump” test: a) form functions, b) normalized cross-sections.

The empirical model described in Appendix B was also used to invert the ABS measurements into concentration profiles. The required parameters of the model were estimated by using the same calibration data as for system functions and parameters $\beta_f$ and $\beta_x$.

Results of Inversion for Concentration

The results of inverting the concentrations from the “fun-pump” test with a maximum concentration of 0.4g/l are shown in Figure 3-30. The profiles of suspended sediment concentration estimated from the 1.08MHz frequency transducer are shown together with the mean profiles. The mean profiles were calculated as an average of the estimations from the three frequencies. It was already noted in the previous section, that the resulting concentrations from 1.08MHz data are almost four times lower than expected. There may be several causes of this discrepancy. Some of them are:
• Erroneous initial values for parameters of sediment size distribution, used for calibration, which can result in biased values for parameter $\beta_f$ and system functions;
• Lower sensitivity of the 1.08MHz transducer to the used sediment sizes.

The mean sediment size specified for the calibration was taken as an average of the sediment sizes for all used concentration levels. It was equal to $2.16\varphi$ (see Table 3-4). But since a mixture of two sediment samples was used for the calibration, none of sample mean sizes was equal to it. As can be seen from Figure 3-29, the 1.08MHz transducer is less sensitive to the finer sample than to the coarse sediments. Since two sample sizes were not equally resolved, the use of their mean value can result in overestimation of the system function and in obtaining a biased value for $\beta_f$. By exclusion of the 1.08MHz frequency, one can improve the accuracy in estimation of the mean concentration profiles. But the decision should be made based on ability of the transducer’s particular frequency to resolve sediment sizes present in the sample.

Figure 3-30. Inversion by unmodified high-pass model concentration profiles for “fun-pump” test with maximum concentration of 0.4g/l: mean profiles (thick lines) and 1.08MHz profiles (thin lines).
Figures 3-31 and 3-32 show inverted concentration profiles for the “fun-pump” test with a maximum concentration of 0.8g/l. The profiles in Figure 3-31 were calculated with the initial unmodified high-pass model, which corresponds to the modified model with values of parameters $\beta_f$ and $\beta_x$ equal to 0.5 for all frequencies. The profiles in Figure 3-32 were calculated with the modified model with $\beta_f$ and $\beta_x$ estimated from calibration (see Table 3-5).

At the initial time, when the sediment suspension in the chamber was homogeneous (profiles labeled “pump off”) both models give similar results, since the system functions were calibrated by using these profiles, and the correction for $\beta$’s is not yet necessary.

The high jumps in the concentration at the ranges near the transducer show that the inversion is not accurate there for either model. After the pump was turned off and the sediments started to settle, the difference in the inverted profiles between two models become more evident. The unmodified model overestimates the concentration over the range
$r > 0.5 \text{m}$ at time 10 seconds after the pump was turned off. The profiles also have “steps” which correspond to erroneously determined sediment sizes (see discussion in the following section).

Figure 3-32. Inversion by modified model concentration profiles for “fun-pump” test with maximum concentration of 0.8g/l: mean profiles (thick lines) and 2.07MHz profiles (thin lines).

The concentration profiles, both inverted using the modified model and calculated by the 1D advection-diffusion model, are shown in Figure 3-33. The mean profiles calculated with empirical model are shown as green lines. The mean concentration profiles were calculated by averaging of the profiles inverted from all three frequencies. The relative errors were calculated as

$$
\varepsilon = \frac{1}{N} \sum_{j} \left| \frac{M_{\text{calc}}(r_j) - M_{\text{model}}(r_j)}{M_{\text{model}}(r_j)} \right|
$$

(3-23)

where $M_{\text{model}}$ is the concentration estimated by the 1D advection-diffusion model, $M_{\text{calc}}$ is the inverted concentration, with the summation carried over the indices at which $M_{\text{model}}(r_j)>0.05\text{g/l}$, and $N$ is the number of included indices.
Figure 3-33. Comparison of concentration profiles inverted by modified high-pass model with profiles calculated by the 1D advection-diffusion and empirical models (labeled as “ABn”) for the “fun-pump” test with maximum concentration of 0.8g/l at 10 second intervals.

The sequence of plots in Figure 3-33 shows the dynamics of the two sediment samples settling. The profiles corresponding to individual frequencies (blue lines) show increasing stratification of sediment suspension over time due to the different settling velocity. Again, when the content of coarse sediments gets low, the concentration estimated from the 1.08MHz frequency is much lower than obtained by the other frequencies. The 4.70MHz becomes the most accurate predictor of the concentration when only fine sedi-
ments are present. The concentrations estimated from the 2.07MHz measurements are closest to the estimated mean concentration.

The empirical model provides the best estimations for suspended sediment concentration. Although it does not use the information on sediment size, it still can be applied for the inversion.

**Results of Inversion for Sediment Size**

The sediment size distribution parameters were also estimated. The results for the test with the maximum concentration of 0.8g/l are shown in Figures 3-34 and 3-35 for unmodified and modified models; they correspond to the results presented in Figures 3-31 and 3-32. It can be seen from Figure 3-34, that the unmodified model provides estimations of mean sediment size which are closest to the expected values at very low concentrations (at time>30 seconds), but fails at higher concentrations, for example, profiles at 10 and 20 seconds. The model predicts sediment sizes which were not present in the chamber (see Figure 3-27), aliasing the estimations towards finer sediment sizes. The only approximation which is reasonable is at time \( t=0 \) seconds; and that profile was used for the calibration of the system, therefore the inversion is much more accurate.

The estimations with the modified model are shown in Figures 3-35 and 3-36. Several definite improvements can be noticed. The model now captures correctly the dynamics of the changes in mean sediment sizes along the chamber with time. It was predicted by 1D advection-diffusion model (see Figure 3-28e) that the mean sediment sizes decrease with time at any horizontal level after the pump was turned off. Figure 3-35 shows that the inverted profiles behave similarly to those predicted by the numerical (1D) model. Also, there is a limit for estimated sediment size at approximately 2.4 \( \phi \), which is close to the mean size of Sand 1 sample, used in the test.
Figure 3-34. Inverted profiles of mean sediment sizes for “fun-pump” test with maximum concentration of 0.8g/l by unmodified high-pass model.

Figure 3-35. Inverted by modified high-pass model profiles of mean sediment sizes for “fun-pump” test with maximum concentration of 0.8g/l.

In Figure 3-36 the comparison of estimated sediment sizes with the 1D advection-diffusion model results are shown for each 10-second interval. Although the mean absolute errors are within the initially approximated standard deviation for the sediment samples (see Table 3-7), there is a steady shift of 0.4 $\phi$. This shift is most likely to be due to the underestimation of parameter $\beta_f$ for the 1.08MHz frequency. Figure 3-26 shows that
the value of $\beta_j$ estimated for 1.08MHz results in lower values of $\gamma(x, \beta)$ than for the other two frequencies. Also the 1.08MHz transducer has the largest uncertainty in determination of the system function (see Figure 3-12), which would result in a biased $\beta_j$ estimation.

![Graphs showing comparisons](image)

Figure 3-36. Comparison of inverted profiles of mean sediment sizes by modified high-pass model with profiles calculated by the 1D advection-diffusion model for the “fun-pump” test with maximum concentration of 0.8g/l at 10-second intervals.

Similar results were obtained for other tests with different concentration levels by using the same values of $\beta_j$ and $\beta_x$ (see Appendix D). Overall, the predictions of sedi-
ment sizes and profiles of suspended sediment concentration are quite accurate. This is achieved by the applying the following improved techniques:

- More accurate estimation of system functions by using 2nd order polynomials, which help to account for nonlinear effects resulting from using the time varying gain factor (TVG).
- Modification of the high-pass model for backscattered intensity by using the correction function \( \gamma(x, \beta) \) with parameters \( \beta_x \) and \( \beta_z \) estimated for a particular sediment sample.

The developed empirical model also shows very good results in obtaining of profiles of sediment concentrations. Among the other applied methods (modified and unmodified high-pass models), it gives the best accuracy in inversion.
CHAPTER 4
MEASUREMENTS OF TARGET POSITION AND BOTTOM ECHO REMOVAL ALGORITHM

A Simplified Bottom Echo Model

The acoustic backscatter system allows measurement of suspended sediment concentration based on the intensity of backscattered sound from the scatterers in the water column. At low concentrations the sound pulse emitted by the transmitter reaches the bottom and reflects back where it is recorded by the receiver. The recorded root-mean-square (RMS) voltage profiles contain information from the scatterers in the water and the bottom. The later can be used, for example, as a reference for correction of the sediment attenuation [Thorne et al., 1995]. Because the acoustic properties of the sandy bottom are different from the properties of the suspended sediments [e.g., Medwin and Clay, 1997, p.610], the acoustic backscatter equation with the parameters described in Chapter 2 cannot be applied to the measurements contaminated by the bottom echo; in vicinity of the bottom the signal becomes saturated since the intensity of the sound reflected from the bottom is higher than the intensity of the sound backscattered from the suspended sediment particles.

Here an algorithm is developed which allows to separate these signals and provides a technique for accurate measurement of bottom location.

After the signal is registered by the ABS, it passes through a lowpass filter, which removes high frequency noise. The resulting signal is a convolution of the registered signal with the response function of the filter
\[ V_{\text{rms}}(t) = \int_{0}^{t} V_{\text{env}}(t - t') h(t') \, dt' \]  

where \( V_{\text{rms}} \) is the recorded RMS voltage, \( V_{\text{env}} \) is the amplitude envelope of the amplified signal registered by the receiver, and \( h \) is the filter impulse response function. The registered signal is assumed to be a linear combination of the bottom echo, \( V_b \), and the signal backscattered from suspended sediments, \( V_s \),

\[ V_{\text{env}}(t) = V_b(t) + V_s(t) \]  

which with the substitution \( t = \frac{r}{c} \) results in

\[ V_{\text{rms}}(r) = \overline{V_b}(r) + \overline{V_s}(r) \]  

where “overbar” means filtered signals. To check the validity of the linear assumption a series of test were performed. The results of the tests are discussed in the following section. The second term is given by equation 2-10. The RMS voltage from the bottom echo can be approximated as [Medwin and Clay, 1997, p.594]

\[ V_b(t) = R_b V_o(t) e^{-\frac{R_a}{2}} \]  

where \( R_b \) is the bottom reflection coefficient, \( g_a \) is the acoustical roughness parameter, and \( V_o \) is the recorded voltage from the perfect mirror-reflected surface. The acoustical roughness is a function of an incident angle and the physical roughness of the surface. By adopting a simplified expression for the bottom echo voltage

\[ \overline{V_b}(r) = v_b e^{-\left(\frac{r - r_b}{\eta_b}\right)^2} \]  

where \( v_b \) is the magnitude of the bottom echo return, \( r_b \) is the bottom location relative to
the transducer, and $b$ is the spreading parameter, equation 4-3 can be used to extract the bottom echo return from the measured signal.

A set of the parameters $v_b$, $r_b$, and $b$ required to the bottom echo model can be approximated from a series of measured RMS profiles. The magnitude of the bottom echo, $v_b$, depends on the total attenuation of the signal, which is mainly a function of sediment concentration in the water (2-42). When the measurements are obtained in the field, the suspended sediment concentration changes with time as waves pass over the measurement site [Hanes, 1991]. Therefore over a period of time a set of profiles can be found which have the same mean concentration level. For these profiles a single value for $v_b$ can be determined. From a voltage profile, the relative attenuation can be approximated by calculating the integral over the range from 0 to the approximate bottom location. Figure 4-1 shows an example of the measured RMS profile with the integral equal to the shaded area.

![Figure 4-1. A sample of voltage profile with definition of "A"-point.](image)

The bottom location is usually assumed to correspond to the location of the maximum intensity, however, the position of the maximum can be shifted and its magnitude
can be increased if other scatterers are located close to the bottom (from equation 4-3). Another approximation can be used—after the maximum is reached, the signal decays and the location where the profile reaches some level of intensity can be found. This location is labeled as “A” in Figure 4-1. An arbitrary level can be chosen to locate the position of the point “A”. The level 500 is shown in the figure and has been used for the 2.07MHz transducer (400 for the 1.08MHz and 100 for the 4.70Mhz).

A set of the profiles with approximately equal attenuation is used to determine the profile for the bottom echo. Since the location of the point “A” changes with time, the relative shift, $\Delta r_b$, needs to be calculated for each profile, so the “A” points for all profiles coincide. The lower envelope found as the lowest signal levels can now be used as an estimation of the bottom return. Assuming a constant total attenuation, higher levels of signal should relate to the presence of suspended sediments, since all profiles should have a similar bottom return. After the bottom echo profile is determined, the parameters in equation 4-5 can be found as the best-fit values. Finally, a correction for $r_b$ needs to be done with the previously calculated shift, $\Delta r_b$. The resulting RMS profiles for suspended sediments can be found from the initially measured RMS profiles minus the bottom return, as

$$\bar{V}_s (r) = V_{rms} (r) - \bar{V}_b (r) \quad (4-6)$$

Tests with a Steel Wire

A series of the tests was conducted at the University of Florida Coastal Laboratory with the acoustic backscatter system. The purpose of the tests was to test the ability of the ABS to distinguish presence of a target such as a steel wire above the sand surface and to measure its elevation. The tests were carried out in a tank filled with tap water (see Fig-
ure 4-2). A pan with quartz sand with the mean size of approximately 0.5mm was placed on the bottom of tank and the surface was made flat. The target was a steel wire with the diameter of 0.33mm, which was strained on a frame attached to the ruler. The ruler could move up or down with 1mm precision.

Figure 4-2. Setup for the tests with steel wire.

A series of 31 tests were performed at different elevations of the wire above the sand surface (see Table 4-1). The measurements were taken with the ABS1 system (see Chapter 3). The RMS voltage profiles were collected at 2Hz for 15 seconds at every position of the wire. The resulting 30 profiles were averaged to obtain mean profiles, which are shown in Figures 4-3 and 4-4. The location of the sand surface corresponds to 452mm distance and on plots is shown with a solid line. The wire position is shown as a dashed line. The data from three transducers—1.08, 2.07, and 4.70MHz—are presented.

As it can be seen from the figures, the position of the wire and the sand surface location coincide with corresponding maxima. A correction can be made to the estimation of the range vector, \( r \), which originally was calculated by equation 2-13 to ensure that
the measured locations correspond to the expected. A bin size is given as $\tau c/2$, where $\tau$ is the sound pulse duration and $c$ is the speed of sound in the water. For the given system, $\tau = 10^{-5}$s which with $c = 1480$ m/s results in the bin size of 7.4mm. Therefore one can expect that the wire will not be visible at an elevation of the wire lower than 7mm above the sand surface. This is supported by the data in Figures 4-3 and 4-4. Tests 11–23 show that the signal from the target is not yet separated from the signal from the sand bottom as it is in tests 23–31, but the recorded profile is not symmetrical and has a slightly higher intensity at the peak. This is support the assumptions discussed in the previous section.

The profiles in Figures 4-5 and 4-6 are calculated with equation 4-6, for which the bottom echo profile was taken from the measurements with no wire present (test 1). As it can be seen from the measurements with the 4.70MHz transducer, the ABS can detect the target at 4mm elevation above the sand surface, which is lower than 7.4mm bin size. Although the strength is much smaller than in cases of higher elevations, the used technique for separation of the signals reveal the difference between the cases with no target and with a target present close to the reflecting surface. Thee tests provide the basis for the bottom echo removal algorithm.

In the next tests with a setup as shown in Figure 4-2 a wire with the 2mm diameter was used as a target. It was positioned at 5cm intervals from the transducer surface. The resulting RMS profiles are shown in Figure 4-7. The wire is located at 10, 15, and 20cm distance from the transducer. Figure shows the presence of second “ghost-wire” reflection, especially for the 2.07 and 4.70MHz transducers. The location of the second reflection exactly correspond to the twice the distance between the wire and the transducer. This means that the sound should reflect three times—two from the wire and ones from the transducer in order to generate this type of record. As it was previously discussed in
Chapter 2, the multiple reflections are possible in presence of stronger scatterers. The current tests show the possibility of such event.

Table 4-1. Tests with 0.33mm steel wire.

<table>
<thead>
<tr>
<th>Position number</th>
<th>Time, min</th>
<th>Ruler, mm</th>
<th>Elevation above bottom, mm</th>
<th>Distance from the ABS, mm</th>
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</thead>
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<td>0.0</td>
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</tr>
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Figure 4-3. Root-mean-square voltage profiles for the tests with 0.33mm steel wire. Tests 1–16 with the wire location (dashed line) and the bottom location (solid line).
Figure 4-4. Same as in Figure 4-3 for tests 17–31.
Figure 4-5. Root-mean-square voltage profiles with subtracted bottom echo profile for the tests with 0.33mm steel wire. Tests 1–16 with the wire location (dashed line) and the bottom location (solid line).
Figure 4-6. Same as in Figure 4-5 for tests 17–31.
Figure 4-7. Root-mean-square voltage profiles (solid) for tests with 2mm steel wire with the wire locations (dashed) and “ghost wire” reflection locations (dash-dotted) from a) 1.08, b) 2.07, and c) 4.70MHz transducers.

Tests with a Water-Sediment Jet

Measurements in a larger tank with the water-sediment mixture jet were performed. A setup which is similar to the experiments of Hay [1991] is schematically shown in Figure 4-8. The mixture was generated in the calibration chamber (Figure 3-1) and was moved by the pump system through a flexible pipe connected to one of the chamber jets. The discharge of 2 liters per minute through a 3.2mm nozzle at the end of the pipe was large enough to prevent any accumulation of sediment in the pipe. The end of the pipe with the nozzle was attached to a ruler, which allowed positioning the jet with the 1mm precision. The sediment concentration in the chamber was 1g/l. Quartz sand with the
mean size of 2.17 in phi units (approximately 0.22mm) and standard deviation of 0.27 \( \varphi \) was used.

![Figure 4-8. Setup for the tests with the sediment jet.](image)

Figure 4-8. Setup for the tests with the sediment jet.

![Figure 4-9. Root-mean-square voltage profiles (solid) for tests for different water-sediment jet locations (dashed) from a) 1.08, b) 2.07, and c) 4.70MHz transducers.](image)

Figure 4-9. Root-mean-square voltage profiles (solid) for tests for different water-sediment jet locations (dashed) from a) 1.08, b) 2.07, and c) 4.70MHz transducers.

The measurements with the ABS1 (see Chapter 3) were taken at the rate of 4Hz during 20 seconds at each jet location. The recorded RMS voltage profiles were averaged...
to provide with a single profile for each jet position. For the tests shown in Figure 4-9 the jet was located at 10cm intervals with the closest distance to the transducers of 30cm.

As it can be seen from the measurements, the location of the jet exactly coincides with the peak of the profile. This is the same result as it was obtained in the tests with the wires. Therefore the assumption about the distance from the transducer to the bottom estimated from the ABS measurements as a location of the maximum of the peak is supported by these tests. Note that because of the low concentration the test with the jet showed no evidence of second reflection, which also was discussed in Chapter 2.

**Result of Bottom Echo Removal**

To illustrate the application of currently developed algorithm for the separation of signals from suspended scatterers and bottom echo data obtained in SandyDuck’97 field [Mouraenko and Hanes, 2003] experiment were used. A sample run with the data recorded on October 18th, 1997 (run “DAIH430”) at approximately 4m water depth over sandy bottom showed events of suspension of sand under the waves with the significant wave height of 1.9m. The measurements of suspended sediment concentration profiles were performed with the acoustic backscatter system. The profiles of RMS voltage were converted into the sediment concentration profiles with the model described in Chapter 2 for two cases—with the bottom echo present and removed.

The results are shown in Figure 4-10. As it can be seen, the estimated bottom elevation fluctuates slightly with the waves passing over the instrument location. It especially noticeable for the time 17–20 seconds when a high wave passed over and the suspended sediment concentration near the bottom increased. The concentrations for that moment estimated from the initial (with bottom echo present) profiles increase up to 200g/l and higher. Theoretically, this concentration can be found near the bottom—the concentration
for fully packed sand bottom can reach 1600g/l. But the attenuation of sound at that concentration level would be very high and the measured intensity should rapidly decrease after that concentration is reached. This is not true in the presented case. Therefore this estimation of the concentration is a result of the unbounded solution of inverse problem (equation 2-12) which is caused by very high intensity of the signal reflected from the bottom. The other plot shows the estimated concentration profiles with the bottom echo removed from the initial voltage profiles. The maximum concentration at the same time is now approximated as 50g/l with the signal decreasing afterwards. To obtain the total concentration, the bottom echo needs to be converted into concentration by a future inversion model.

![Figure 4-10. Time series of suspended sediment concentrations estimated with a) bottom echo and b) bottom echo removed; c) free surface elevation.](image)
CHAPTER 5
CONCLUSIONS

The accuracy and temporal resolution of measurements with the acoustic backscatter system depend strongly on the inversion model used for converting recorded data into profiles of suspended sediment concentrations and sediment sizes. The acoustic backscatter equation provides the basis for developing of the inversion model, but it is necessary to determine scattering parameters of sediment suspension before the inversion can be performed.

The form function and normalized cross-section obtained from the analytical expressions for glass spheres or from the measurements with quartz sand cannot describe natural sediments of all possible mineralogical contents and shapes. In current work, a modified scattering correction function is proposed. It provides a simple way to account for the differences in sediment properties.

The form of correction function depends on a single parameter $\beta$. Therefore, two different parameters can be defined—each for the form function and normalized cross-section. A series of laboratory tests with different sediment samples showed that the parameter $\beta$ varies in the range $-3 < \beta < 2.5$. The values of parameter $\beta$ for correction of the form function, $\beta_f$, and normalized cross-section, $\beta_\chi$, are most likely different. It was found that $\beta_f$ is greater than $\beta_\chi$. The parameters $\beta_f$ and $\beta_\chi$ obtained from different sound frequencies can be different, but single values for $\beta_f$ and $\beta_\chi$ can be found.
Application of the modified correction function leads to a significant increase in accuracy of estimating of sediment sizes. During the tests in the calibration chamber with a mixture of two sediment samples a series of measurements was collected with the acoustic backscatter system. The freely settling sediments were generating rapidly changing profiles with a gradient in sediment size and concentration. The results of inversion were compared to the 1D advection-diffusion model. The corrected inversion model predicts very well the overall dynamics of the profiles of sediment sizes with high temporal resolution (based on profiles averaged within every 3 seconds). The resulting errors lay within the sediment size standard deviation.

The correct approximation of parameter $\beta_x$ provides the necessary adjustment to the sediment attenuation, what makes the inversion with the implicit method more stable.

The calibration tests show that the system function is better approximated by a second order polynomial if the time varying gain (TVG) is applied. From the tests with different concentration levels the best-fit approximation can be found, and the corresponding parameters $\beta_x$ can be determined. Also, by testing over different sediment samples, the system function can be obtained. The system function should not be a function of a particular sediment sample. Although several approximations can exist, the general system function can be estimated by the mean of these approximations. From the ratios of each approximation and the general system function the parameters $\beta_f$ can be determined. Therefore, a pair of parameters—$\beta_f$ and $\beta_x$—can describe every sediment sample, while the system functions become unique for each of the transducers. Further research needs to be done to evaluate the universality of the correction function.
A sensitivity analysis was carried out for the system function, as a function of sediment size distribution parameters, parameter $\beta_\chi$, and the transducer radius. The effective acoustical radius of the transducer is usually 10-20% smaller than its geometrical radius, which was previously noticed by other investigators. Current analysis shows that the transducer radius has a very small (<1%) influence on inversion accuracy in far-field region.

The standard deviation of sediment size distribution has a second order effect on accuracy of inversion; therefore, for well-sorted sediments ($\sigma_\varphi < 0.5$) the ability of the instrument to successfully estimate the sorting from measurements becomes small. In case of wide or bimodal sediment size distribution the transducers unequally resolve presence of the sediments of all sizes. This can result in large variability of estimates for the sorting among the transducers and inability to resolve the sorting accurately. The predefined value of sorting, for example from sieve analysis, is recommended to use in the inversion.

Multiple scattering of the sound waves can be ignored for the concentrations of natural sediments of <10g/l, but at higher concentrations or in presence of air bubbles it can become important. Therefore, a model was developed to examine the effect of multiple scattering by using a multiple layer approximation. The transmission and reflection coefficients were defined by analogy with the acoustic backscatter equation. A binary tree walk algorithm was implemented to account for all possible paths along which sound wave can travel. The resulting sound intensities can be found as the sum of all intensities of the sound waves arriving at the same time. The number of different paths increases rapidly with the number of layers, but the number of reflections can be limited to 3 if the
number of layers is large (>15). The results show that, for the uniform concentration profiles, the difference can reach 12–14% at maximum concentration of 3g/l. With linearly changing concentration profiles, the difference does not exceed 1–3% at maximum concentration of 3g/l at range approximately 1m. “Bump” type profiles results in differences up to 20%, (at maximum concentration at “bump” of 5–7g/l) and produce the “ghost” picks, which can be seen from the wire tests. For further analysis, the model needs to be tested with data collected in presence of bubbles in order to investigate its validity for such applications.

To improve the accuracy of the acoustic backscatter system in measuring suspended sediment concentration in the vicinity of the bottom, the bottom echo removal algorithm was developed. When the suspended sediment concentrations along the water column are low, the bed echo signal saturates the signal corresponding to the sound back-scattered from suspended particles. The algorithm separates these two signals by assuming a bell-shaped form of bottom echo signal. This solution provides information on bottom location. It qualitatively well predicts the backscattered intensities from suspended sediments near the bottom, but requires future comparison with other types of measurements.
APPENDIX A
CALIBRATION TEST RESULTS

The calibration of the ABS1 with transducers of frequencies 1.08, 2.07, and 4.70MHz was performed with four sediment samples. The parameters of size distributions of each sample is gathered into Table 3-4. In Figures A-1–A-12 the best-fit parameters, calculated from minimization of error (equation 3-4) are shown. The results of estimated system functions are shown in Figures A-13–A-36.

Figure A-1. Case 1: Errors in inverted concentrations from varying of parameters $\beta_x$, $a_x$, $\mu_\varphi$, and $\sigma_\varphi$ for 1.08MHz transducer.
Figure A-2. Case 1: Errors in inverted concentrations from varying of parameters $\beta_x$, $\alpha_z$, $\mu_{\psi}$, and $\sigma_{\psi}$ for 2.07MHz transducer.

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Figure A-3. Case 1: Errors in inverted concentrations from varying of parameters $\beta_x$, $\alpha_z$, $\mu_{\psi}$, and $\sigma_{\psi}$ for 4.70MHz transducer.

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**Figure A-4.** Case 2: Errors in inverted concentrations from varying of parameters $\beta_\chi$, $a_\chi$, $\mu_\chi$, and $\sigma_\chi$ for 1.08MHz transducer.

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</table>

**Figure A-5.** Case 2: Errors in inverted concentrations from varying of parameters $\beta_\chi$, $a_\chi$, $\mu_\chi$, and $\sigma_\chi$ for 2.07MHz transducer.
Figure A-6. Case 2: Errors in inverted concentrations from varying of parameters $\beta$, $a$, $\mu$, and $\sigma$ for 4.70MHz transducer.

Figure A-7. Case 3: Errors in inverted concentrations from varying of parameters $\beta$, $a$, $\mu$, and $\sigma$ for 1.08MHz transducer.
Figure A-8. Case 3: Errors in inverted concentrations from varying of parameters $\beta_x$, $a_i$, $\mu_\varphi$, and $\sigma_\varphi$ for 2.07MHz transducer.

Figure A-9. Case 3: Errors in inverted concentrations from varying of parameters $\beta_x$, $a_i$, $\mu_\varphi$, and $\sigma_\varphi$ for 4.70MHz transducer.
Figure A-10. Case 4: Errors in inverted concentrations from varying of parameters $\beta_{\chi}$, $a_{\chi}$, $\mu_{\phi}$, and $\sigma_{\phi}$ for 1.08MHz transducer.

Figure A-11. Case 4: Errors in inverted concentrations from varying of parameters $\beta_{\chi}$, $a_{\chi}$, $\mu_{\phi}$, and $\sigma_{\phi}$ for 2.07MHz transducer.
Figure A-12. Case 4: Errors in inverted concentrations from varying of parameters $\beta_{\chi}$, $a_t$, $\mu_{\varphi}$, and $\sigma_{\varphi}$ for 4.70MHz transducer.

Figure A-13. Case 1: Estimation of the system function for the 1.08MHz transducer initial value of parameter $\beta_\chi$: a–c) same as in Figure 3-12.
Figure A-14. Case 1: Estimation of the system function for the 1.08MHz transducer with corrected parameter $\beta_\chi$: a–c) same as in Figure 3-12.

Figure A-15. Case 1: Estimation of the system function for the 2.07MHz transducer with initial value of parameter $\beta_\chi$: a–c) same as in Figure 3-12.
Figure A-16. Case 1: Estimation of the system function for the 2.07MHz transducer with corrected parameter $\beta^*_\chi$: a–c) same as in Figure 3-12.

Figure A-17. Case 1: Estimation of the system function for the 4.70MHz transducer with initial value of parameter $\beta^*_\chi$: a–c) same as in Figure 3-12.
Figure A-18. Case 1: Estimation of the system function for the 4.70MHz transducer with corrected parameter $\beta_x$: a–c) same as in Figure 3-12.

Figure A-19. Case 2: Estimation of the system function for the 1.08MHz transducer with initial value of parameter $\beta_x$: a–c) same as in Figure 3-12.
Figure A-20. Case 2: Estimation of the system function for the 1.08MHz transducer with corrected parameter $\beta_x$: a–c) same as in Figure 3-12.

Figure A-21. Case 2: Estimation of the system function for the 2.07MHz transducer with initial value of parameter $\beta_x$: a–c) same as in Figure 3-12.
Figure A-22. Case 2: Estimation of the system function for the 2.07MHz transducer with corrected parameter $\beta_\chi$: a–c) same as in Figure 3-12.

Figure A-23. Case 2: Estimation of the system function for the 4.70MHz transducer with initial value of parameter $\beta_\chi$: a–c) same as in Figure 3-12.
Figure A-24. Case 2: Estimation of the system function for the 4.70MHz transducer with corrected parameter $\beta_x$: a–c) same as in Figure 3-12.

Figure A-25. Case 3: Estimation of the system function for the 1.08MHz transducer with initial value of parameter $\beta_x$: a–c) same as in Figure 3-12.
Figure A-26. Case 3: Estimation of the system function for the 1.08MHz transducer with corrected parameter $\beta_X$: a–c) same as in Figure 3-12.

Figure A-27. Case 3: Estimation of the system function for the 2.07MHz transducer with initial value of parameter $\beta_X$: a–c) same as in Figure 3-12.
Figure A-28. Case 3: Estimation of the system function for the 2.07MHz transducer with corrected parameter $\beta_x$: a–c) same as in Figure 3-12.

Figure A-29. Case 3: Estimation of the system function for the 4.70MHz transducer with initial value of parameter $\beta_x$: a–c) same as in Figure 3-12.
Figure A-30. Case 3: Estimation of the system function for the 4.70MHz transducer with corrected parameter $\beta_x$: a–c) same as in Figure 3-12.

Figure A-31. Case 4: Estimation of the system function for the 1.08MHz transducer with initial value of parameter $\beta_x$: a–c) same as in Figure 3-12.
Figure A-32. Case 4: Estimation of the system function for the 1.08MHz transducer with corrected parameter $\beta_\chi$: a–c) same as in Figure 3-12.

Figure A-33. Case 4: Estimation of the system function for the 2.07MHz transducer with initial value of parameter $\beta_\chi$: a–c) same as in Figure 3-12.
Figure A-34. Case 4: Estimation of the system function for the 2.07MHz transducer with corrected parameter $\beta_x$: a–c) same as in Figure 3-12.

Figure A-35. Case 4: Estimation of the system function for the 4.70MHz transducer with initial value of parameter $\beta_x$: a–c) same as in Figure 3-12.
Figure A-36. Case 4: Estimation of the system function for the 4.70MHz transducer with corrected parameter $\beta_x$: a–c) same as in Figure 3-12.
The empirical models were previously used to convert measured by the acoustic backscatter system sound intensities into suspended sediment concentration profiles [e.g., Tamura and Hanes, 1986]. The currently developed model provides an accurate method to invert ABS measurements by using a set of coefficients. The coefficients for the empirical model can be obtained from the calibration in the chamber (Figure 3-1) with the sediments taken from the field site. Therefore this limits the application of the model to the used sediment. The solution of the acoustic backscatter equation described in Chapter 2 allows using the same system coefficients for the sediment of wide range sizes and properties, as long as the system parameters remain unchanged. The problem arises when the unique system coefficients cannot be found. This can lead to increasing errors in estimation of suspended sediment concentrations.

From the solution for the acoustic backscatter system given by equations 2-10 and 2-11 recorded voltage is determined as a function of suspended sediment concentration, sediment size, and system parameters. The total sound attenuation is given by 2-42. The attenuation due to the water is small compared to the sediment attenuation, thus can be neglected. The sediment attenuation at a given point is described by the sediment concentration and the sediment attenuation constant (2-47).

In order to obtain the system constants calibration of the system is performed. Measurements are obtained on homogeneous suspension at known concentration levels with the sediments taken from field site. With system and sediment parameters assumed...
to be constant during the measurements, voltage curves can be written as a function of
distance from the transducer. The approximation can be made as follows

\[ V_{rms} = A \frac{M^{1/2}}{\psi r} e^{Br} \]  

(B-1)

with two constants \( A \) and \( B \). The parameter \( A \) depends on sediment size and system
constant only, but not on concentration. The parameter \( B \) is defined by the attenuation
and relates to the concentration as

\[ B = -2\alpha_s = -2 \int_0^r \zeta M dr = -2\zeta M \]  

(B-2)

when \( \zeta \) and \( M \) are constants for homogeneous suspension.

The tests in the calibration chamber show that it is not always possible to reproduce
concentration profile by varying only two parameters, such as \( \beta_x \) and concentration. The
system parameter (2-11) is not always constant, especially when the time varying gain
(TVG) is used. The semi-empirical model for conversion of the ABS measurements into
concentrations only is introduced. The voltage curves from the calibration are approxi-
mated with the tree parameter model as

\[ V_{fit} = M^{1/2} A_M r^{n_M} e^{B_M r} \]  

(B-3)

where constants \( A_M \), \( B_M \), and \( n_M \) are chosen to minimize the error with measured data
for each of the concentration level \( M \). From the original backscatter equation for far-
field the parameter \( A_M \) is a constant, \( n_M \) is equal to \(-1\) if the TVG is not used, and \( B_M \)
depends on concentration linearly. In present model all three parameters are free with de-
pendence on concentration only.
Since the measurements for some discrete concentration levels can be obtained, the triple of parameters \( (A_M, B_M, n_M) \) are approximated for other concentrations by extrapolation from known values. The power law curves were used to extrapolate the values of parameters for concentrations up twice maximum concentration measured in form

\[
y = ax^b + c
\]

(B-4)

where \( y \) is one of the parameters \( A_M, B_M, \) or \( n_M \), and \( a, b, \) and \( c \) are the coefficients. Therefore, each of \( A_M, B_M, \) or \( n_M \) have corresponded set of parameters \( (a, b, c) \), which are fully specify the model.

Comparing to the initial solution, where only two parameters (mean sediment size and concentration) are variable and other parameters can be found from the theoretical or empirical expressions, the proposed model requires 9 parameters—coefficients \( (a, b, c) \) for each of \( (A_M, B_M, n_M) \)—obtained from the calibration. The sand used in the calibration of the model needs to be from the field site, since the information on sediment size is not directly included into model and cannot be extracted.

To invert the measured voltage into concentration iterative solution method is applied. Follow the algorithm described elsewhere (e.g. \([Thorne and Hanes, 2002]\)), the concentration near the transducer needs to be determined first. Since the measurements at the ranges just near the transducer surface usually contain electronic noise, the concentration can be taken to be zero. The concentrations at further ranges are calculated by iteration using formula

\[
M_{k+1} = \left( \frac{V_{rms,k+1}}{A_k e^{n_k}} \right)^2
\]

(B-5)
where $M_k$ is the suspended sediment concentration at range $r_k$, and $(A_k, B_k, n_k)$ are the parameters calculated using the corresponding power law curves (equation B-4). The concentration used in obtaining of $(A_k, B_k, n_k)$ is equal to the mean concentration at previously calculated ranges.

Figures B-1–B-3 show the calibration of the transducers with frequencies 1.08, 2.07, and 4.70MHz with calculated coefficients $(A_M, B_M, n_M)$. Measured voltages for different concentration levels (on plots “a”) are shown with approximating curves calculated by fitting equation B-3. The corresponding coefficients $(A_M, B_M, n_M)$ are gathered into the table on each figure. The values of the coefficients are plotted as circles on plots “b”–“d”. The extrapolated by the power law curves with coefficients $(a, b, c)$, which are also gathered into the table, show the approximations for $(A_M, B_M, n_M)$ at higher concentrations.

According to equation B-2 coefficients $B_M$ depend linearly on concentration. The power coefficient $b$ for the transducers with frequencies 2.07 and 4.70MHz are 0.82 and 1.17 correspondingly, which are closer to 1 than the power coefficient for 1.08MHz transducer, which is equal to 0.53. The parameter $B_M$ corresponds to the attenuation of the sound, and therefore has to be negative. Despite, it has positive values for all concentrations for 1.08MHz transducer, and up to 0.5g/l for 2.07MHz. This comes from the coupling of two parameters—$B_M$ and $n_M$—when the changes in one of the parameter can lead to the changes in another. Although, if one of the parameters is set to be constant, ability of the model to provide better fit to the voltage curves becomes limited, which therefore results in larger errors in inversion into the sediment concentration.
Fit curves: \( V_{\text{fit}} = \sqrt{M} A r^n e^{B r} \)

<table>
<thead>
<tr>
<th>M</th>
<th>A</th>
<th>B</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>11.13</td>
<td>2.3641</td>
<td>−1.5118</td>
</tr>
<tr>
<td>0.20</td>
<td>15.99</td>
<td>2.0960</td>
<td>−1.3626</td>
</tr>
<tr>
<td>0.30</td>
<td>18.62</td>
<td>2.1847</td>
<td>−1.3493</td>
</tr>
<tr>
<td>0.40</td>
<td>15.67</td>
<td>1.9793</td>
<td>−1.3780</td>
</tr>
<tr>
<td>0.50</td>
<td>21.86</td>
<td>1.7846</td>
<td>−1.2009</td>
</tr>
<tr>
<td>1.00</td>
<td>27.02</td>
<td>1.7256</td>
<td>−1.1824</td>
</tr>
<tr>
<td>1.50</td>
<td>28.94</td>
<td>1.6908</td>
<td>−1.1490</td>
</tr>
<tr>
<td>2.00</td>
<td>37.25</td>
<td>1.2816</td>
<td>−1.0444</td>
</tr>
<tr>
<td>3.00</td>
<td>39.65</td>
<td>1.1783</td>
<td>−1.0122</td>
</tr>
</tbody>
</table>

Coefficients \([a b c]\):
\[
A = \left[ 26.535906 \quad 0.380370 \quad 0.191433 \right];
B = \left[ −0.828189 \quad 0.529510 \quad 2.606169 \right];
n = \left[ 2.856120 \quad 0.051853 \quad −4.032534 \right];
\]

Figure B-1. Calibration coefficients for 1.08 MHz transducer: a) measured voltage (dots) and fitted by curves B-3 and calculated coefficients b) \( A \), c) \( B \), and d) \( n \).

Fit curves: \( y = a x^b + c \)

<table>
<thead>
<tr>
<th>M</th>
<th>A</th>
<th>B</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>172.50</td>
<td>0.5905</td>
<td>−0.8134</td>
</tr>
<tr>
<td>0.20</td>
<td>204.42</td>
<td>0.3838</td>
<td>−0.7435</td>
</tr>
<tr>
<td>0.30</td>
<td>230.66</td>
<td>0.2201</td>
<td>−0.6823</td>
</tr>
<tr>
<td>0.40</td>
<td>257.35</td>
<td>0.0624</td>
<td>−0.6308</td>
</tr>
<tr>
<td>0.50</td>
<td>238.02</td>
<td>0.1373</td>
<td>−0.6500</td>
</tr>
<tr>
<td>0.70</td>
<td>299.86</td>
<td>−0.2650</td>
<td>−0.5405</td>
</tr>
<tr>
<td>1.00</td>
<td>355.77</td>
<td>−0.6645</td>
<td>−0.4658</td>
</tr>
<tr>
<td>1.50</td>
<td>394.97</td>
<td>−1.0320</td>
<td>−0.3886</td>
</tr>
<tr>
<td>2.00</td>
<td>492.35</td>
<td>−1.6548</td>
<td>−0.2847</td>
</tr>
</tbody>
</table>

Coefficients \([a b c]\):
\[
A = \left[ 198.614575 \quad 0.380370 \quad 144.602059 \right];
B = \left[ −0.828189 \quad 0.529510 \quad 2.606169 \right];
n = \left[ 2.856120 \quad 0.051853 \quad −4.032534 \right];
\]

Figure B-2. Calibration coefficients for 2.07MHz transducer: a–d) same as in Figure B-1.
Figure B-3. Calibration coefficients for 4.70MHz transducer: a–d) same as in Figure B-1.

The method was applied to the measurements used for calibration of the ABS1 and the results are shown on Figure B-4. Though the initial concentration near the transducer should be approximated, it was taken to be 0.001g/l for all 3 transducers. The method shows very good accuracy for the 1.08 and 2.07MHz transducers compared to the results in Figures 3-12 and 3-13. For the 2.07 and 4.70MHz transducers the method works very well for low concentrations and underestimates the concentrations at higher levels. This would not happen if the initial concentration were determined properly (for example, known concentrations were used). For the 4.70MHz transducer reasonable results can be achieved at low (less than 0.5g/l) concentration levels.
Figure B-4. Concentration profiles converted with the empirical model from the measurements with a) 1.08, b) 2.07, and c) 4.70MHz transducer.

The disadvantage of the method is that the parameters such as sediment of the calibration system should be similar to the ones found at experiment site. To obtain better conversion the range of concentration levels for calibration should be wide.
APPENDIX C
1D ADVECTION-DIFFUSION MODEL

The distribution of suspended sediments in the chamber is modeled with the advection-diffusion equation [Nielsen, 1992, p.241]

\[
\frac{\partial M}{\partial t} + w_s \frac{\partial M}{\partial r} = \frac{\partial}{\partial r} \left( \varepsilon_s \frac{\partial M}{\partial r} \right)
\]  

\[(C-1)\]

where \( M \) is the suspended sediment concentration, \( \varepsilon_s \) is the sediment diffusivity, and \( w_s \) is the settling velocity. With the vertical coordinate, \( r \), directed downward the advection term is taken with the positive sign. The settling velocity is a function of the sediment size \( \varphi \), which varies with the range \( r \). In general the sediment diffusivity is a function of the range as well.

To simplify a solution of the equation, sediment sizes are assumed to obey some distribution, \( p(\varphi) \), or can be divided into fractions. The total concentration can be found as the superposition of mass concentrations for each fraction

\[
M(t,r) = \int_{-\infty}^{\infty} M^\varphi(t,r) d\varphi \quad \text{or} \quad M(t,r) = \sum_{\varphi} M^\varphi(t,r)
\]

\[(C-2)\]

where \( M^\varphi \) is the suspended sediment concentration of the fraction of sediments of size \( \varphi \). The sediment size distribution can be calculated as

\[
p(\varphi)_{t,r} = \frac{M^\varphi(t,r)}{M(t,r)}
\]

\[(C-3)\]
Now the equation can be rewritten for each sediment size fraction as

$$\frac{\partial M^\phi}{\partial t} + w^\phi_s \frac{\partial M^\phi}{\partial r} = \frac{\partial}{\partial r} \left( \varepsilon_s \frac{\partial M^\phi}{\partial r} \right)$$  \hspace{1cm} (C-4)

where $w^\phi_s$ is the settling velocity for the sediment size $\phi$. The settling velocity for each fraction is now can be assumed constant, and the sediment diffusivity is assumed to be a function of $r$ only. If the sediment diffusivity is zero than the equation is a pure advection.

The equation is solved for $r \in [0, D]$, where $D$ is the depth of a domain, with the initial condition for the suspended sediment concentration given by

$$M(t, r)\big|_{r=0} = M_0(r)$$  \hspace{1cm} (C-5)

and the boundary condition at the free surface

$$M^\phi(t, r)\big|_{r=0} = 0$$  \hspace{1cm} (C-6)

A form of the sediment diffusivity defines properties of the model. The following functions can be applied:

1. No diffusion $\varepsilon_s(r) = 0$.
2. Constant diffusivity $\varepsilon_s(r) = \varepsilon_s$.
3. Linear diffusivity $\varepsilon_s(r) = \kappa u_* (D - r)$.
4. Parabolic diffusivity $\varepsilon_s(r) = \kappa u_* (D - r) r/D$,

where $\kappa$ is von Karman constant which is equal to 0.4, and $u_*$ is the friction velocity.

The distribution of the sediment sizes at time $t = 0$ needs to be specified. The normal distribution (2-52) with mean $\mu_\phi$ and standard deviation $\sigma_\phi$ may be used as the initial.
The settling velocity for each sediment size was approximated by Gibbs formula [Gibbs et al., 1971]

\[
    w_s = \frac{-3\nu + \sqrt{9\nu^2 + gd^2(s-1)(0.003869 + 0.02480d)}}{0.011607 + 0.07440d}
\]  

(C-7)

where \( d \) is the sediment size in centimeters, \( w_s \) is the settling velocity in cm/s, \( \nu \) is the kinematic viscosity of fluid in cm\(^2\)/s, \( g \) is the acceleration of gravity in cm/s\(^2\), and \( s \) is the specific gravity of sediments.

**Numerical Solution**

A numerical solution of equation C-4 was obtained on a uniform grid with Crank-Nicolson 2nd order scheme [Fletcher, 1991, p.228] (indices “\( \phi \)” were omitted)

\[
    \frac{M_i^{n+1} - M_i^n}{\Delta t} + \left\{ w_s - \left( \frac{d\varepsilon_s}{dr} \right)_i \right\} \frac{1}{2} \left( \frac{M_{i+1}^{n+1} - M_{i-1}^{n+1}}{2\Delta r} + \frac{M_{i+1}^n - M_{i-1}^n}{2\Delta r} \right) \\
    - \left( \varepsilon_s \right)_i \frac{1}{2} \left( \frac{M_{i+1}^{n+1} - 2M_i^{n+1} + M_{i-1}^{n+1}}{\Delta r^2} + \frac{M_i^{n} - 2M_i^{n+1} + M_{i-1}^n}{\Delta r^2} \right) = 0
\]

(C-8)

The implicit approximations lead to the tridiagonal matrix form, which is solved with efficient Thomas algorithm,

\[
a_i M_{i-1}^{n+1} + b_i M_i^{n+1} + c_i M_{i+1}^{n+1} = f_i, \quad i = 2, N
\]

(C-9)

with the coefficients

\[
a_0 = 0, \quad b_0 = 1, \quad c_0 = 0, \quad f_0 = 0 \quad (C-10a)
\]

\[
a_i = -\frac{1}{4} \left\{ C_{\beta} - \frac{\Delta t}{\Delta r} \left( \frac{d\varepsilon_s}{dr} \right)_i \right\} - \frac{\Delta t}{2\Delta r^2} \left( \varepsilon_s \right)_i \quad (C-10b)
\]

\[
b_i = 1 + \frac{\Delta t}{\Delta r^2} \left( \varepsilon_s \right)_i \quad (C-10c)
\]

\[
c_i = \frac{1}{4} \left\{ C_{\beta} - \frac{\Delta t}{\Delta r} \left( \frac{d\varepsilon_s}{dr} \right)_i \right\} - \frac{\Delta t}{2\Delta r^2} \left( \varepsilon_s \right)_i \quad (C-10d)
\]
\[ f_i = M_i^n - \frac{1}{4} \left[ C_{fi} - \frac{\Delta t}{\Delta r} \left( \frac{d\varepsilon_i}{dr} \right)_i \right] (M_{i+1}^n - M_{i-1}^n) \]
\[ + \frac{\Delta t}{2\Delta r^2} (\varepsilon_i)_i (M_{i+1}^n - 2M_i^n + M_{i-1}^n) \]  
(C-10e)

\[ a_{N+1} = -\frac{C_{fl}}{2}, \quad b_{N+1} = 1 + \frac{C_{fi}}{2}, \quad c_{N+1} = 0, \quad f_{N+1} = M_{N+1}^n = \frac{C_{fi}}{2} (M_{N+1}^n - M_N^n) \]  
(C-10f)

where \( C_{fi} \) is Courant number equal to \( C_{fl} = w_s^p \Delta t / \Delta r \), \( N \) is the number of grid cells, and \( n \) and \( i \) are the running indexes. There are no constrains for the value of \( C_{fi} \) required for stability of the numerical scheme. But since the settling velocity depends on the sediment size, steps in time and vertical direction were chosen to satisfy \( C_{fl} = 1 \) for the coarsest fraction. For other sediment sizes Courant number are smaller.

**Vertical Sediment Size Distribution**

Four test cases were calculated to illustrate how sediment size distribution parameters change if different types of diffusion applied. The same initial parameters were used in all cases. A domain size, \( D \), was set to be equal 1.5m. Other parameters are gathered into Table C-1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Diffusion type</th>
<th>( M_i(r), g/l )</th>
<th>( \mu_\phi )</th>
<th>( \sigma_\phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>no diffusion</td>
<td>( \varepsilon_i = 0 \text{m}^2/\text{s} )</td>
<td>1.0</td>
<td>2.26</td>
</tr>
<tr>
<td>Case 2</td>
<td>constant</td>
<td>( \varepsilon_i = 0.001 \text{m}^2/\text{s} )</td>
<td>1.0</td>
<td>2.26</td>
</tr>
<tr>
<td>Case 3</td>
<td>linear</td>
<td>( u_s = 0.01 \text{m/s} )</td>
<td>1.0</td>
<td>2.26</td>
</tr>
<tr>
<td>Case 4</td>
<td>parabolic</td>
<td>( u_s = 0.01 \text{m/s} )</td>
<td>1.0</td>
<td>2.26</td>
</tr>
</tbody>
</table>

The results of the calculations are shown in Figures C-1–C-4. Although the sediment sizes were no longer normally distributed at any time \( t > 0 \), their distribution can be approximated by the normal distribution. The parameters of the distribution can be estimated by fitting the normal probability curve to one calculated by C-3. The approximated
parameters $\mu_\phi$ and $\sigma_\phi$ are shown on plots “e” and “f” in Figures C-1–C-4 as functions of a distance from the free surface. The plots show that the mean sediment size (in mm) decreases with time and distance, as well as the standard deviation. In the case of zero diffusivity, the rate of that change is higher than for other types. For the linear diffusivity case, the rate of diffusion is higher towards the free surface. This reduces settling speed of the sediments and lowers the rate of change in sediment sorting.

All the test cases show that the sorting is not uniform with vertical elevation, but if the diffusivity is higher (as in linear case) than more mixing occurs and the sorting become more uniform. For the linear diffusivity case, the sorting decreases to approximately 80% of the initial value, while for the zero diffusivity, it decreases down to approximately 27% of the initial value in 90 seconds.

The sorting is a key parameter for the correct inversion of sediment size and concentration from the ABS measurements. It is usually (see Chapter 2) assumed to be constant, though it is possible to use an average vertical profile of the standard deviations of sediment sizes to account for the changes in sorting.

The linear and parabolic diffusivities approximate the mixing in the wave boundary layer more accurately [e.g., Puleo et al., 2004]. Therefore, by applying these types of diffusivity a character of changes in the sediment sorting with time and distance in a wave boundary layer can be approximated.
Figure C-1. Case 1—zero diffusivity: sediment size distribution at distance a) 0.25, b) 0.5, and c) 1m; d) mass concentration; e) approximate mean, and f) standard deviation.

Figure C-2. Case 2—constant diffusivity: a–f) same as in Figure C-1.
Figure C-3. Case 3—linear diffusivity: a–f) same as in Figure C-1.

Figure C-4. Case 4—parabolic diffusivity: a–f) same as in Figure C-1.
APPENDIX D
RESULTS FOR “FUN-PUMP” TESTS

The following plots show the results of inversion of suspended sediment concentration and mean size profiles for “fun-pump” (see Chapter 3) tests for initial maximum concentration levels of 1.2, 1.6, 2.0, and 2.4 g/l.

Figure D-1. “Fun-pump” test results for concentration profiles with maximum concentration of 1.2 g/l: a–h) same as for 3-33.
Figure D-2. “Fun-pump” test results for mean sediment size profiles with maximum concentration of 1.2g/l: a–h) same as for 3-36.
Figure D-3. “Fun-pump” test results for concentration profiles with maximum concentration of 1.6 g/l: a–h) same as for 3-33.
Figure D-4. “Fun-pump” test results for mean sediment size profiles with maximum concentration of 1.6g/l: a–h) same as for 3-36.
Figure D-5. “Fun-pump” test results for concentration profiles with maximum concentration of 2.0g/l: a–h) same as for 3-33.
Figure D-6. “Fun-pump” test results for mean sediment size profiles with maximum concentration of 2.0g/l: a–h) same as for 3-36.
Figure D-7. “Fun-pump” test results for concentration profiles with maximum concentration of 2.4g/l: a–h) same as for 3-33.
Figure D-8. “Fun-pump” test results for mean sediment size profiles with maximum concentration of 2.4g/l: a–h) same as for 3-36.
LIST OF REFERENCES


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Weisstein, E.W., Catalan Number, from *MathWorld--A Wolfram Web Resource* 

BIOGRAPHICAL SKETCH

The author was born the second son of Nadezhda and Alexander Mouraenko in 1974 in the small town of Myaundzha in the Magadan region of northeast of Russia. From early on he spent a great deal of time with his father, a professional driver, from whom he inherited a passion for mechanics and learned to respect perfection at work. At the age of thirteen he and his family moved to Barnaul, Russia, where in 1991 he finished high school with a silver medal and entered the Mathematical Department of the Altai State University. In 1996 he graduated with honors and chose to continue his education in applied sciences by entering the graduate school at the Institute for Water and Environmental Problems. In 1999 he continued his study at the University of Florida, and in 2001 he received his master’s degree in coastal and oceanographic engineering. In his spare time, he enjoys long distance running and competing in various races including marathons.