ELECTRON-ELECTRON INTERACTIONS IN TWO-DIMENSIONAL SPIN-ORBIT COUPLED SYSTEMS

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The work presented in this dissertation attempts to bring together two very active research venues in condensed matter physics. The first one is the field of strongly correlated electron systems and quantum phase transitions in itinerant electron systems. The second one is the field of semiconductor spintronics and quantum computation, in particular, a subfield of the latter that studies coherence in semiconductor spin qubits. To explore interesting overlap regions between these two venues, we extend the Landau's Fermi liquid (FL) theory to include a spin-orbit (SO) coupling. It is shown that although "charge-part" quantities, such as the charge susceptibility and effective mass, are determined solely by the quasi-particles, "spin-part" quantities, such as the spin susceptibility, have contributions from the damped states in between the two spin-split Fermi surfaces. Properties of such FLs are discussed in detail. We also study the linear response of an $SU(2)$ symmetric FL to an SO perturbation and predict new types of collective modes in FLs with SO coupling: chiral spin-waves. We find the equations of motion for these modes and propose an experimental setup suitable to observe them.

CHAPTER 1
INTRODUCTION: SPIN-ORBIT INTERACTION IN CONDENSED MATTER SYSTEMS

Many materials of current interest are characterized by non-trivial correlations between the spin and orbital degrees of freedom. To name just a few, these are two-dimensional (2D) electron and hole gases in semiconductor heterostructures with broken inversion symmetry [1], surface/edge states of three-dimensional (3D)/2D topological insulators [2–4], conducting (and superconducting) states at oxide interfaces [5], etc. Spin-orbit (SO) coupling inherent to all these systems locks electron spins and momenta into patterns characterized by chirality, and we will refer to materials of this type as to ”electron-chiral materials”.[1] Electron-chiral materials are endowed with unique properties that are interesting from both the fundamental and applied points-of-view. On the fundamental side, electron-chiral materials provide an ideal laboratory to study such important consequences of magneto-electric coupling as the spin-Hall effect [7–9] and spin textures (helices) [10–12]. On the applied side, these materials are studied as potential platforms for spintronic devices that allow electric manipulation of magnetic properties. In addition, the topological subclass of electron-chiral materials is being actively investigated as a platform for non-Abelian quantum computation [13].

The possibility of full control over electron spin entirely via electric means is the main reason why SO coupled systems are in general among the most interesting systems under research. Therefore, it is instructive to study different types of SO interactions. In what follows, a brief introduction to SO interactions is given, with a focus on low dimensions.

[1] Pseudospins of Dirac fermions in graphene are also correlated to their momenta, and thus graphene can be viewed as a pseudochiral system [6].

SO coupling, having its origin in relativistic quantum mechanics, was first observed in atomic physics. The effect arises when electrons of the atom, in their rest frame, experience a magnetic field while orbiting the charged nucleus. This effect is captured mainly in the Pauli SO term in the expansion of the Dirac equation, and also generates SO coupling in condensed-matter (CM) systems. One would classify the SOC in CM systems as either symmetry-independent or symmetry-dependent. While the former is largely the same as in atomic physics and exist in all types of crystals, the latter is only present in systems with broken inversion symmetry. Depending on the type of broken inversion symmetry, whether in the bulk or the surface, the effect goes under generic names of Dresselhaus or Rashba, respectively. Unlike the Dresselhaus SO interaction, the structure of the Rashba term does not depend on the symmetry of the host crystal and hence, it is simpler to construct the Rashba Hamiltonian on general grounds. In what follows, first a derivation of the Rashba SO Hamiltonian is given, and other types of SO interaction are discussed briefly later.

Instead of a detailed derivation of the SO Hamiltonian, starting with the original Pauli SO term, one can explore the class of Hamiltonians that allow for coupling between the spin and momentum given certain symmetries. Consider a single-particle Hamiltonian that depends only on the momentum and the spin of the particle and preserves time reversal, $SO(2)$ rotational symmetry and reflection in vertical planes.[2] Given these symmetries, and the fact that higher than linear powers of the spin operator are absent, the most general SO Hamiltonian is of the form [14]

$$i\gamma(p_-^{2n+1}\sigma_+ - p_+^{2n+1}\sigma_-), \tag{1–1}$$

[2] A direct product of $SO(2)$ and reflection groups is isomorphic to the group $C_{\infty v}$.

where $a_\pm = a_x \pm i a_y$, γ is a SO coupling constant, and σ represents projections of $j_z = \pm(n + \frac{1}{2})$. To prove the rotational invariance, we examine the transformation of such a term under an $SO(2)$ element R_θ (rotation of 2D real vectors by an angle θ),

$$R_\theta \vec{p} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} p_x \\ p_y \end{pmatrix}. \tag{1–2}$$

Therefore,

$$R_\theta p_\pm = e^{\pm i\theta} p_\pm, \tag{1–3}$$

and hence,

$$R_\theta p_\pm^{2n+1} = e^{\pm i(2n+1)\theta} p_\pm^{2n+1}.$$

(1–4)

The spins must be transformed accordingly

$$R_\theta \sigma_\pm = e^{\mp i(2n+1)\theta} \sigma_\pm. \tag{1–5}$$

This is identical to transforming the expectation value of the spins.[3] Therefore there are two rotationally invariant terms available: $p_-^{2n+1}\sigma_+$, and $p_+^{2n+1}\sigma_-$. The relative prefactor between the two terms is determined by enforcing the in-plane reflection symmetry; for instance, reflection in the $y - z$ plane results in $k_\pm \to -k_\mp$ and $\sigma_\pm \to \sigma_\mp$, and hence one arrives at the linear combination in Eq.

[3] This is to be distinguished from rotation of the spins in the spin space $R_\theta^{\text{spin}}|\sigma\rangle = e^{i(n+\frac{1}{2})\theta}|\sigma\rangle$.

In Eq. 1–1, the case of $n = 0$ describes the Rashba SO interaction [15] for electrons with spin $1/2$, which can be written as

$$H_R = \alpha\,(\boldsymbol{\sigma} \times p) \cdot \mathbf{e}_z, \tag{1–6}$$

where $\mathbf{e}_z$ is the normal vector out of the plane of the system, and α is the linear Rashba SO coupling constant.

To find the strength of the SO coupling, one has to start by adding the Pauli SO term to the full non-relativistic Hamiltonian of a single electron inside a solid, including external confining potentials and use approximate methods to capture the relevant SO terms. For instance in the case of a cubic crystal, a minimal effective theory contains 8-bands in the $\mathbf{k} \cdot \mathbf{p}$ theory (the so-called 8×8 Kane model) and Envelope Function Approximation suitable for slowly varying (compared to the lattice spacing) external potentials. The leading order result for the Rashba term for electrons in a cubic crystal, produced by this scheme, is:

$$\hbar\alpha \frac{\boldsymbol{\nabla} V}{|\boldsymbol{\nabla} V|} \cdot (\mathbf{p} \times \boldsymbol{\sigma}) = \frac{P^2}{3}\left[\frac{1}{E_g^2} - \frac{1}{(E_g + \Delta_0)^2}\right] \boldsymbol{\nabla} V \cdot (\mathbf{p} \times \boldsymbol{\sigma}), \tag{1–7}$$

where $\boldsymbol{\nabla} V = e\mathrm{E}_z \hat{z}$ is the expectation value of the confining electric field E_z in the *valence* band (this is a subtle but fundamental point; see Section 6.3.2 of Ref. [16]) times the charge of the electron e, E_g and Δ_0 are the band gap and SO splitting of the valence band, respectively, and P is a matrix element of the momentum in the confining direction. It is worth mentioning that the Rashba spin splitting for holes is totally different from that for electrons. For instance, in the same model, a similar expression for the Rashba coupling for holes includes a different combination of band structure parameters than the one inside the brackets in Eq. 1–7; for a thorough discussion see [16].

The natural unit for $\hbar\alpha$ is meV·Å; when measured in these units $\hbar\alpha$ is of order unity in most semiconductors. For instance, $\hbar\alpha = 3.04$ meV·Å in GaAs, while $\hbar\alpha = 245.7$ meV·Å in InSb [16]. Hence, for number density $n = 10^{12}$ cm^{-2}, the SO energy scale

$\alpha p_F = 0.07$ meV in GaAs and $\alpha p_F = 6.14$ meV in InSb, where $p_F = \sqrt{2\pi n}$ is the Fermi momentum.

The most common way to experimentally measure SO coupling is by the study of Shubnkov de Haas oscillations at small magnetic fields and low temperatures, in which beating in these oscillations signals the difference in spin sub-band populations.

Another type of structural SO interaction is the Dresselhaus SO, which originates from a lack of inversion symmetry in the bulk of material, and causes spin-splitting of the bulk conduction band. The 2D form of the Dresselhaus interaction depends on the direction of the confinement plane. For instance, in the case of a confinement in the (001) direction, the so-called linear Dresselhaus SO is given by [17]

$$H_D = \beta \left(\sigma_x p_x - \sigma_y p_y \right), \tag{1–8}$$

where β is the Dresselhaus coupling constant. As is clear from the expression, as long as the rest of the Hamiltonian is isotropic, any system with only one of the two (Rashba or Dresselhaus) couplings is equivalent to a system having only the other, since they are related by a unitary transformation. However, a system with having both couplings cannot be transformed into a system having only one.

There are also systems in which the orbital angular momentum enters the SO interaction as well. A well-known example of such system is holes in GaAs, which are formed by the states with orbital momentum $l = 1$. In the bulk, the bands of heavy holes with $j_z = \pm 3/2$ and of light holes with $j_z = \pm 1/2$ are degenerate at $k = 0$. In a quantum well, degeneracy is lifted and the light-hole band is split off the heavy-hole one. If the Fermi energy lies within the heavy-hole band, we have a system with pseudospin $j_z = \pm 3/2$. A particular form of the SO coupling in such a system is the case of $n = 2$ in Eq. 1–1,

$$H_{R3} = i\alpha_3 (p_-^3 \sigma_+ - p_+^3 \sigma_-), \tag{1–9}$$

where α_3 is a SO coupling constant that can be found in a similar fashion to the linear coupling α [16]. This is the so-called cubic Rashba interaction. Inclusion of an in-plane magnetic field in such a systems is not trivial because of the particular rotation properties of the effective spin $3/2$. Magnetic field is a pseudovector, i.e., rotates like a vector but reflects with an additional sign compared to a vector. Hence, to linear order in $B_\pm = B_x \pm B_y$, one can construct the following Hamiltonian consistent with the symmetry group $C_{\infty v}$:

$$H_{B3} = \alpha_3^{'}(p_-^2 B_- \sigma_+ + p_+^2 B_+ \sigma_-), \tag{1–10}$$

where $\alpha_3^{'}$ is a coupling constant that again can be determined from the $\mathbf{k} \cdot \mathbf{p}$ theory [14]. In addition to heavy holes in the (001) quantum well in AlGaAs/GaAs, this Hamiltonian is relevant for the (001) surface state in $SrTiO_3$ [18] and, possibly, for the 2D superconducting state at the $LaAlO_3/SrTiO_3$ interface (LAO/STO) [19].[4]

While the single-particle properties of electron-chiral systems are well understood by now, we are only starting to appreciate the importance of the interplay between SO/chirality and the electron-electron (ee) interaction, which is necessarily present in all these systems. The ee interaction is the driving force of phase transitions (be they finite- or zero-temperature) in electron systems. For example, a familiar ferromagnetic (Stoner) instability in the Hubbard model occurs because the increase of the kinetic energy due to spontaneous spin polarization is offset by the decrease of the interaction energy due to (partial or complete) elimination of minority-spin electrons. In our current picture of ferromagnetic and other magnetic instabilities in electron systems, the SO interaction plays an important but secondary role: for example, it helps to pin the direction of the magnetization in a ferromagnet but does not alter the properties of

[4] Although the analysis of quantum magnetoresistance in LAO/STO was carried out assuming a linear Rashba coupling [20], a demonstrated cubic Rashba coupling in the parent material [18] suggest that a cubic coupling is also relevant for LAO/STO.

the ferromagnetic phase itself. This picture works if the SO energy is the smallest energy scale in the problem. However, the current tendency − driven in part by the need to develop semiconductor-based spintronic devices−is to enhance the strength of the SO interaction relative to other important scales, namely, the Fermi energy, and Coulomb repulsion [21].[5] If these three energy scales are comparable to each other, we venture into an almost unexplored regime where ee interaction can drive the system into a magnetically-ordered state, yet strong SO coupling makes this state qualitatively different from familiar ferro- and anti-ferromagnets. For example, a strong enough Coulomb repulsion can force all electrons to occupy only one Rashba subband [14, 29] or to form a spin-nematic with a spin-split Fermi surface but zero net magnetization [30–32].

Given the presence of very strong SO coupling (compared to the Fermi level) in materials such as in BiTel [6] , p-GaAs with low carrier density, InSb, etc., and strong ee correlations in some of these systems, e.g., heavy-hole p-GaAs [7] , a theory that is non-perturbative either in SO or in ee interactions, can prove to be very useful.

In Chapter 2 of this dissertation, we construct a general theory of chiral Fermi liquids (FLs). It is hard to overestimate the importance of such a theory. The well-established theory of non-chiral (or SU(2)-invariant) FLs [34–36] allows one to classify possible

[5] In semiconductor heterostructures, this can be achieved by reducing the carrier number density to the point when only the lowest of the spin-split Rashba subbands is occupied. This possibility has been demonstrated in the process of searching for Majorana fermions in p-GaAs [22], InSb [23], and InAs [24] quantum wires. In other systems, such as a graphene/1 ml Au/Ni heterostructure [25] and surface states of bismuth tellurohalides (studied at UF [26]) [27, 28], the SO coupling is increased due to the presence of heavy (Au and Bi) atoms.

[6] Fermions at the surface BiTel differ from electrons in, e.g., n-GaAs heterostructures, by the absence of the quadratic term in the spectrum. Consequently, only one Rashba subband in this system is occupied.

[7] Parameter r_s reaches 10-40 in p-GaAs heterostructures [33] .

Pomeranchuk instabilities that break rotational but not translational symmetry of the Fermi surface and derive the stability conditions in terms of the Landau parameters. It also provides full classification and non-perturbative treatment of collective excitations in the charge and spin sectors (which are decoupled in the non-chiral case). With the exception of one-dimensional (1D) systems and surface states of 3D topological insulators at the Dirac point, all other examples of electron-chiral electron materials are believed to behave as FLs with well-defined quasi-particles near the Fermi surface (or multiple Fermi surfaces in case of spin-split states). This conclusion is supported by a number of studies in which various FL quantities−effective mass, quasi-particle lifetime, spin susceptibility, spin-Hall conductivity, spin-Coulomb drag correction to the charge conductivity, plasmon spectrum, etc.−were calculated within perturbation theory with respect to the ee interaction [14, 29, 37–50]. In addition, several previous attempts [51–53] to construct FL theories with SO coupling are discussed further in the next Chapter.

CHAPTER 2
THEORY OF CHIRAL FERMI LIQUIDS

2.1 Introduction

Being the reason for many spin-dependent phenomena particularly in low dimensional systems, the effects of Spin-Orbit (SO) interaction remains a subject of intense research. An interesting area is the role of ee interaction in systems with SO coupling. A classic problem that seems to have been neglected previously, is the construction of a Fermi liquid (FL) theory for systems with SO coupling. Landau's FL theory describes the system as an ensemble of almost free excitations, i.e., quasiparticles (QPs), near the Fermi surface (FS) but with a set of renormalized parameters, such as the effective mass and Lande g factor. "High-energy" physics, i.e., physics of states away from the Fermi energy which is formidable to account for in perturbation theory, is encapsulated in a number of phenomenological parameters. Given the success of FL theory in describing the properties in many fermionic systems (He^3, simple metals and their alloys, degenerate semiconductors, cold fermionic atoms), it is desirable to extend the theory to include the SO interaction. We shall only consider a linear Rashba coupling in two dimensional (2D) FLs, as the simplest SO coupling possible. Some quasiparticle parameters, such as the lifetime, Z-factor, and effective mass, have been previously calculated including the long-range Coulomb interaction within the random phase approximation (RPA) [48]. Such a theory can shed light on classification of possible Poemranchuck instabilities (corresponding to spatially uniform phases) in the presence of SO interactions. Perturbative studies make up a significant body of existing literature on the *ee* interactions in SO coupled systems such as possible first order phase transitions in such systems [29, 54], studies of the fermionic self-energy [55], spatially non-uniform continuous phase transitions [56], charge susceptibility and SO renormalization of Friedel oscillations in the screened potential [38], and spin susceptibilities [44, 45], to name a few.

Explicit inclusion of SO in the framework of FL theory has previously been presented, to our knowledge, only in Refs. [52] and [51]. The former preserves the spatial inversion symmetry and considers generic tensor interaction in three dimensional FLs, and we argue that this work misses the subtleties arising due to the broken SU(2) symmetry induced by non-spin-conserving forces such as SO. Reference [51] includes the effects of the Rashba or Dresselhaus SO only as a perturbing force to the $SU(2)$-symmetric (SU2S) FL to study spin resonance and spin Hall conductivity.

This work aims to address the question of whether there is a FL theory for an electron system with a SO interaction. We argue that the answer depends on whether the property under study is spin-independent, e.g., charge susceptibilities *or* spin-dependent, e.g., spin susceptibilities. For spin-independent quantities with local properties in SU(2) spin space, the original Landau's FL theory can be extended to include the effects of SO with rather straightforward modifications similar to any two-band FL theory. For spin-dependent phenomena on the other hand, any extension of a conventional FL theory would fail as these properties of the liquid are controlled by the damped many-body states in between the two FSs. A similar conclusion was reached for a partially spin-polarized FL [57]. However, to account for first order effects in SO, one can neglect the finite distance between the two FSs and keep SO merely in the energy splitting induced by SO.

This Chapter is organized as follows. In the next section, we construct the Landau interaction function (LF) as the main tool for phenomenological study of the FL. Equipped with the LF, we proceed to calculate the effective masses of the FL, in Sec. 2.3. We discuss an Overhauser-type splitting of the effective masses similar to effective mass splitting in a partially spin-polarized metal. The compressibility of the liquid and the Pomeranchuk stability condition are derived in Sec. 2.4. Zero sound collective modes are shown to be determined by a set of coupled equations for the scattering amplitude. In Sec. 2.5 we use these equations to relate the phenomenological and

microscopic theories. An explicit form of the LF is evaluated microscopically in a model with short range interactions in Sec. 2.5.2, confirming the form proposed in Sec. 2.2. To illustrate the subtlety in the spin sector, the out-of-plane spin susceptibility is evaluated as an example in Sec. 2.4.3 and the importance of the contribution of damped states in between the two FSs is shown explicitly. It is argued that this feature is present for all spin-dependent properties of any FL with broken SU(2) symmetry, and hence they should be considered as FLs without a conventional FL theory. In Sec. 2.6, we consider a new type of collective modes in chiral FLs, namely, chiral spin waves and propose an experiment in which they can be observed. These collective modes are found in the spin sector of the theory, using SO as a perturbation to the SU2S FL.

2.2 Landau Function

We consider a 2D system of electrons in the presence of the Rashba SO interaction, described by the Hamiltonian

$$\hat{H} = \hat{H}_f + \hat{H}_{\text{int}} = \frac{p^2}{2m}1 + \alpha\left(\hat{\boldsymbol{\sigma}} \times \mathbf{p}\right) \cdot \mathbf{e}_z + \hat{H}_{\text{int}}, \tag{2–1}$$

where m is the effective electron mass, $\hat{\boldsymbol{\sigma}}$ are the Pauli matrices, $\mathbf{e}_z$ is the unit vector along the normal to the 2DEG plane, and $\hat{H}_{\text{int}}$ entails a non-relativistic, density-density *ee* interaction. (Here and in the rest of the Chapter, α is chosen to be positive, and $\hbar$ and k_B are set to unity. Also, the index f will denote the properties of a free system.) The space group of Hamiltonian 3–1 is $C_{\infty v}$, and the Rashba SO term is the only combination of the spin and momentum that is invariant under the symmetry operations of this group– see Chapter 1. Eigenvectors and eigenenergies of $\hat{H}_f$ are given by

$$|s, \mathbf{p}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -ise^{i\theta_{\mathbf{p}}} \end{pmatrix} \tag{2–2}$$

and

$$\epsilon_s^{\mathbf{p},f} = p^2/2m + s\alpha p, \tag{2–3}$$

where $\theta_{\rm p}$ is the polar angle of the momentum $\mathbf{p}$, and $s = \pm 1$ denotes the chirality, i.e. the winding direction of spins around the FS.

In the rest of this Chapter, we assume that the chemical potential, μ, intersects both Rashba subbands. The (bare) Fermi momenta and Fermi velocities of the individual subbands are given by

$$p^f_\pm = \sqrt{(m\alpha)^2 + 2m\mu} \mp m\alpha = m\,(v_0 \mp \alpha) \tag{2–4}$$

$$v^f_{F\pm} = v_0 \equiv \sqrt{\alpha^2 + 2\mu/m}. \tag{2–5}$$

In a SU2S theory, the only relevant objects in spin space are spins of the two QPs and the invariant tensor 1 (2×2 Kronecker δ), out of which the tensorial structure of the LF can be built respecting the SU(2) symmetry,

$$\hat{f}(\mathbf{p}, \mathbf{p'}) = f^s(\mathbf{p}, \mathbf{p'}) 11' + f^a(\mathbf{p}, \mathbf{p'}) \hat{\boldsymbol{\sigma}} \cdot \hat{\boldsymbol{\sigma}}', \tag{2–6}$$

written in arbitrary spin basis. Scalar combinations $f^s(f^a)$ are the spin-symmetric (anti-symmetric) parts of the LF, $\mathbf{p}$ and $\mathbf{p}'$ represent momenta of the two quasiparticles. Clearly, spin and charge parts are completely separated. However, this picture is incorrect upon inclusion of SO. The central object of the FL theory is the Landau function describing the interaction between quasiparticles with momenta $\mathbf{p}$ and $\mathbf{p}'$, and spins $\hat{\boldsymbol{\sigma}}$ and $\hat{\boldsymbol{\sigma}}'$. Since SO coupling reduces $SU(2)$ spin symmetry down to $U(1)$, the Landau function will contain more invariants compared to the SU2S case [Eq. 2–6. The task of finding all invariants is simplified by noting that the Rashba coupling is equivalent to the effect of a non-Abelian magnetic field $\mathbf{B}_R(\mathbf{p}) = (2\alpha/g\mu_B)(p_y, -p_x, 0)$, where g is the electron's g-factor and μ_B is the Bohr magneton. The most general form of the Landau function must include scalar products formed out of the six objects– 1, $\hat{\boldsymbol{\sigma}}$ and $\mathbf{B}_R(\mathbf{p})$ for each of the two quasiparticles–that are invariant under $C_{\infty v}$, time reversal, and permutations of quasiparticles. The Landau function of a FL in the real magnetic field $\mathbf{B}$ contains extra "Zeeman" terms, $\hat{\boldsymbol{\sigma}}' \cdot \mathbf{B}$ and $\hat{\boldsymbol{\sigma}} \cdot \mathbf{B}$, as well as their products. In addition

to these couplings, the Rashba field can be coupled to spins in more ways because, in contrast to the real-field case, the cross product $\mathbf{B}_R(\mathbf{p}) \times \mathbf{B}_R(\mathbf{p}') \propto \hat{z}|\mathbf{p} \times \mathbf{p}'|$ is non-zero and acts on spins as another effective magnetic field along the z axis.

Exploring all possible independent invariants, we arrive at the following Landau function:

$$\begin{aligned}
\hat{f} &= f^{\mathrm{s}} 1 1' + f^{\mathrm{a}\parallel}(\hat{\sigma}_x \hat{\sigma}'_x + \hat{\sigma}_y \hat{\sigma}'_y) + f^{\mathrm{a}\perp} \hat{\sigma}_z \hat{\sigma}'_z \\
&+ \frac{1}{2} g^{\mathrm{ph}} \left[1 \hat{\boldsymbol{\sigma}}' \times (\mathbf{p}' - \mathbf{p}) \cdot \mathbf{e}_z - 1' \hat{\boldsymbol{\sigma}} \times (\mathbf{p}' - \mathbf{p}) \cdot \mathbf{e}_z \right] \\
&+ \frac{1}{2} g^{\mathrm{pp}} \left[1 \hat{\boldsymbol{\sigma}}' \times (\mathbf{p}' + \mathbf{p}) \cdot \mathbf{e}_z + 1' \hat{\boldsymbol{\sigma}} \times (\mathbf{p}' + \mathbf{p}) \cdot \mathbf{e}_z \right] \\
&+ h^{(1)} (\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}} \cdot \mathbf{e}_z)(\hat{\boldsymbol{\sigma}}' \times \hat{\mathbf{p}} \cdot \mathbf{e}_z) + h^{(2)} (\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}}' \cdot \mathbf{e}_z)(\hat{\boldsymbol{\sigma}}' \times \hat{\mathbf{p}} \cdot \mathbf{e}_z) \\
&+ \frac{1}{2} h \left[(\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}} \cdot \mathbf{e}_z)(\hat{\boldsymbol{\sigma}}' \times \hat{\mathbf{p}} \cdot \mathbf{e}_z) + (\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}}' \cdot \mathbf{e}_z)(\hat{\boldsymbol{\sigma}}' \times \hat{\mathbf{p}}' \cdot \mathbf{e}_z) \right],
\end{aligned} \tag{2–7}$$

where the scalar functions f^{s} through h share the same argument $(\mathbf{p}, \mathbf{p}')$, which we omit for brevity. In Sec. 2.5, we show that all the terms in Eq. 2–7 are produced by the perturbation theory for the interaction vertex. The superscripts ph and pp in the "g" terms indicate that, in the perturbation theory, the corresponding terms come from the interaction in the particle-hole and particle-particle (Cooper) channels, respectively.

As is to be expected, SO coupling breaks spin-rotational invariance of the exchange, $\hat{\boldsymbol{\sigma}} \cdot \hat{\boldsymbol{\sigma}}'$, term of an SU2S FL. Anisotropy in the exchange part of $\hat{f}$ comes from the combination $[\hat{\boldsymbol{\sigma}} \cdot (\mathbf{B}_R \times \mathbf{B}'_R)] \left[\hat{\boldsymbol{\sigma}}' \cdot (\mathbf{B}_R \times \mathbf{B}'_R)\right] \propto \hat{\sigma}_z \hat{\sigma}'_z$, which affects only the $\hat{\sigma}_z \hat{\sigma}'_z$ part of the exchange interaction. [Here, $\mathbf{B}_R \equiv \mathbf{B}_R(\mathbf{p})$ and $\mathbf{B}'_R \equiv \mathbf{B}_R(\mathbf{p}')$.] Consequently, the anisotropic exchange interaction [first line of Eq. 2–7 contains different couplings ($f^{\mathrm{a}\parallel}$ and $f^{\mathrm{a}\perp}$) for interaction between in-plane and out-of-plane spins. Recent perturbative calculations [14, 44, 45] show that anisotropy is of the Ising type, i.e, that $f^{\mathrm{a}\perp} > f^{\mathrm{a}\parallel}$. In addition to breaking the rotational symmetry of the exchange interaction, the SO

coupling generates the effective Zeeman terms [with coupling constants g^{ph}, g^{pp}, $h^{(1)}$, and $h^{(2)}$], which depend explicitly on $\mathbf{p}$ and $\mathbf{p}'$. It is worth pointing out that the "h" terms can be written in two equivalent forms. For example, the $h^{(1)}$ term can be written down either as $(\hat{\boldsymbol{\sigma}} \times \mathbf{p} \cdot \mathbf{e}_z)(\hat{\boldsymbol{\sigma}}' \times \mathbf{p}' \cdot \mathbf{e}_z)$ or as $(\hat{\boldsymbol{\sigma}} \cdot \mathbf{p})(\hat{\boldsymbol{\sigma}}' \cdot \mathbf{p}')$, which results in equivalent LFs upon re-defining the scalar function $f^{\mathrm{a}\|}$.

Spins in Eq. 2–7 are not yet represented in any particular basis. However, in order to project electrons' momenta on the FSs, we need to specify the basis. Since the *ee* interaction commutes with spins, the spin structure of quasiparticles' states is still governed by the Rashba term, as long as the interaction is below the critical value for a Pomeranchuk instability. This argument is nothing more than the usual assumption that symmetries of the system do not change if the interaction is switched on adiabatically. Microscopic calculations in Sec. 2.5 indeed show that the spin structure of quasiparticles is the same as of free electrons. Therefore, we take the chiral basis of Eq. 2–2 as the eigenbasis for quasiparticles of a Rashba FL.

2.3 Effective Mass

Landau's derivation of the FL effective mass is restricted to Galilean invariant systems, and thus cannot be applied to our case because the SO term breaks Galilean invariance. A generalization of the Landau's derivation for a relativistic FL [58] is also not applicable here because our Hamiltonian Eq. 3–1 is not fully Lorentz invariant either. Therefore, we need to devise an argument which involves neither Galilean nor Lorentz boosts. To this end, we notice that, since $\hat{H}_{\mathrm{int}}$ depends only on the positions of electrons but not on their velocities, the position operator commutes with $\hat{H}_{\mathrm{int}}$. Therefore, the velocity operator is the same as in the absence of the *ee* interaction:

$$\hat{\mathbf{v}}_j = -i[\hat{\mathbf{x}}_j, \hat{H}] = -i[\hat{\mathbf{x}}_j, \hat{H}_f] = \frac{\mathbf{p}_j}{m}1 + \alpha(\mathbf{e}_z \times \hat{\boldsymbol{\sigma}}). \tag{2–8}$$

Summing Eq. 2–8 over all particles gives

$$\mathrm{Tr}\sum_j \mathbf{v}_j = \frac{1}{m}\mathrm{Tr}\sum_j [\mathbf{p}_j 1 + m\alpha(\mathbf{e}_z \times \hat{\boldsymbol{\sigma}})]. \tag{2–9}$$

For the non-relativistic case ($v_j \ll c$, $\forall j$) considered here, the left-hand side of Eq. 2–9 is the total flux of particles. (Relativistic corrections to this result are of order v^2/c^2, which is the same order as the relativistic correction to the spectrum, assumed to be much smaller than the SO correction.) By definition, the total flux of particles is equal to that of quasiparticles. The same is true for the total momentum and total spin. Hence the sums over particles can be converted into sums over quasiparticles. Using a 2×2 occupation number of quasiparticles $\hat{n}^{\mathrm{p}}$, we have

$$\mathrm{Tr}\int [\mathbf{p}1 + m\alpha(\mathbf{e}_z \times \hat{\boldsymbol{\sigma}})]\, \hat{n}^{\mathrm{p}}(dp) = m\mathrm{Tr}\int \partial_{\mathrm{p}}\hat{\varepsilon}^{\mathrm{p}}\hat{n}^{\mathrm{p}}(dp), \tag{2–10}$$

where $(dp) \equiv d^2p/(2\pi)^2$, $\hat{\varepsilon}^{\mathrm{p}}$ is the energy functional for quasiparticles, and $\partial_{\mathrm{p}}\hat{\varepsilon}^{\mathrm{p}}$ is their velocity. Since both the Hamiltonian and $\hat{n}^{\mathrm{p}}$ are diagonal in the chiral basis, Eq. 2–10 in the chiral basis reads

$$\sum_s \int [\mathbf{p} + m\alpha(\mathbf{e}_z \times \hat{\boldsymbol{\sigma}}_{ss})] n_s^{\mathrm{p}}(dp) = m\sum_s \int \partial_{\mathrm{p}}\hat{\varepsilon}_s^{\mathrm{p}} n_s^{\mathrm{p}}(dp), \tag{2–11}$$

where $n_s^{\mathrm{p}} \equiv n_{ss}^{\mathrm{p}}$. Now we apply arbitrary independent variations on the diagonal elements of the density matrix, keeping off-diagonal elements to be zero: $n_s^{\mathrm{p}} = n_s^{0\mathrm{p}} + \delta n_s^{\mathrm{p}}$, where $n_s^{0\mathrm{p}} = \Theta(p_s - p)$ at $T = 0$ and p_s is the Fermi momentum of fermions with chirality s. (Here, superscript 0 refers to the unperturbed FL with no variations in the occupation number.) A variation of the diagonal element of the quasiparticle energy $\varepsilon_s^{\mathrm{p}} \equiv \varepsilon_{ss}^{\mathrm{p}}$ is related to δn_s^{p} via

$$\delta\varepsilon_s^{\mathrm{p}} = \sum_{s'} \int f_{s,s'}(\mathbf{p}, \mathbf{p}')\delta n_{s'}^{\mathrm{p}'}(dp'), \tag{2–12}$$

where $f_{s,s'}(\mathbf{p}, \mathbf{p}') \equiv f_{s,s';s,s'}(\mathbf{p}, \mathbf{p}')$ and $f_{s_1,s_2;s_3,s_4}(\mathbf{p}, \mathbf{p}')$ is the Landau function from Eq. 2–7 in the chiral basis (all the unprimed Pauli matrices come with with $s_1 s_3$ and all the primed

ones with $s_2 s_4$). Varying both sides of Eq. 2–11, we obtain

$$\sum_s \int [\mathbf{p} + m\alpha(\mathbf{e}_z \times \hat{\boldsymbol{\sigma}}_{ss})]\, \delta n_s^{\mathrm{p}}(dp)$$
$$= m \sum_s \int \partial_{\mathrm{p}} \varepsilon_s^{0\mathrm{p}} \delta n_s^{\mathrm{p}}(dp) + m \sum_s \int \partial_{\mathrm{p}} \delta \varepsilon_s^{\mathrm{p}} n_s^{0\mathrm{p}}(dp). \tag{2–13}$$

The LF is given by

$$\begin{aligned} f_{s_1,s_2;s_3,s_4} = & \quad f^{\mathrm{s}} 1_{s_1 s_3} 1'_{s_2 s_4} + f^{\mathrm{a}\parallel}(\hat{\sigma}_x \hat{\sigma}'_x + \hat{\sigma}_y \hat{\sigma}'_y) + f^{\mathrm{a}\perp} \hat{\sigma}_z \hat{\sigma}'_z \\ + & \quad \tfrac{1}{2} g^{ph} \left[1 \hat{\boldsymbol{\sigma}}' \times (\mathbf{p} - \mathbf{p}') \cdot \mathbf{e}_z - 1' \hat{\boldsymbol{\sigma}} \times (\mathbf{p}' - \mathbf{p}) \cdot \mathbf{e}_z \right] \\ + & \quad \tfrac{1}{2} g^{pp} \left[1 \hat{\boldsymbol{\sigma}}' \times (\mathbf{p} + \mathbf{p}') \cdot \mathbf{e}_z + 1' \hat{\boldsymbol{\sigma}} \times (\mathbf{p} + \mathbf{p}') \cdot \mathbf{e}_z \right] \\ + & \quad h^1 (\hat{\boldsymbol{\sigma}} \times \mathbf{e}_{\mathrm{p}} \cdot \mathbf{e}_z)(\hat{\boldsymbol{\sigma}}' \times \mathbf{e}_{\mathrm{p}'} \cdot \mathbf{e}_z) + h^2 (\hat{\boldsymbol{\sigma}} \times \mathbf{e}_{\mathrm{p}'} \cdot \mathbf{e}_z)(\hat{\boldsymbol{\sigma}}' \times \mathbf{e}_{\mathrm{p}} \cdot \mathbf{e}_z) \\ + & \quad \tfrac{1}{2} h \left[(\hat{\boldsymbol{\sigma}} \times \mathbf{e}_{\mathrm{p}} \cdot \mathbf{e}_z)(\hat{\boldsymbol{\sigma}}' \times \mathbf{e}_{\mathrm{p}} \cdot \mathbf{e}_z) + (\hat{\boldsymbol{\sigma}} \times \mathbf{e}_{\mathrm{p}'} \cdot \mathbf{e}_z)(\hat{\boldsymbol{\sigma}}' \times \mathbf{e}_{\mathrm{p}'} \cdot \mathbf{e}_z) \right], \end{aligned} \tag{2–14}$$

where $\mathbf{e}_{\mathrm{p}} = \mathbf{p}/p$. Integration by parts in the first term of the right-hand side of Eq. 2–13 gives, upon relabeling $s \longleftrightarrow s'$ and $p \longleftrightarrow p'$ and using the symmetry $f_{s,s'}(\mathbf{p}, \mathbf{p}') = f_{s',s}(\mathbf{p}', \mathbf{p})$,

$$\sum_s \int [\mathbf{p} + m\alpha(\mathbf{e}_z \times \hat{\boldsymbol{\sigma}}_{ss})]\, \delta n_s^{\mathrm{p}}(dp) \tag{2–15}$$
$$= m \sum_s (\partial_{\mathrm{p}} \varepsilon_s^{0\mathrm{p}}) \delta n_s^{\mathrm{p}} - m \sum_{s,s'} \int f_{s,s'}(\mathbf{p}, \mathbf{p}')(\partial_{\mathrm{p}'} n_{s'}^{0\mathrm{p}'}) \delta n_s^{\mathrm{p}}(dp)(dp').$$

Since variations are arbitrary, the integrands themselves must be equal. Using $\partial_{\mathrm{p}'} n_{s'}^{0\mathrm{p}'} = -\delta(p_{s'} - p')\mathbf{e}_{\mathrm{p}'}$ and $\partial_{\mathrm{p}} \varepsilon_s^{0\mathrm{p}} = \mathbf{p}_s / m_s^*$ as a definition of the quasiparticle effective mass (with $\mathbf{p}_s \equiv p_s \mathbf{p}/p \equiv p_s \mathbf{e}_{\mathrm{p}}$), projecting Eq. 2–15 onto $\mathbf{e}_{\mathrm{p}}$ and setting $\mathbf{p} = \mathbf{p}_s$, we obtain

$$p_s \left[1 + \frac{m\alpha}{p_s} (\mathbf{e}_z \times \hat{\boldsymbol{\sigma}}_{ss}) \cdot \mathbf{e}_{\mathrm{p}} \right]$$
$$= p_s \frac{m}{m_s^*} + m \sum_{s'} \int f_{s,s'}(\mathbf{p}_s, \mathbf{p}') \delta(p_{s'} - p') \mathbf{e}_{\mathrm{p}} \cdot \mathbf{e}_{\mathrm{p}'}(dp'), \tag{2–16}$$

To simplify the final form of m_s^*, we define

$$F_{s,s'}(\mathbf{p},\mathbf{p}') = \nu_s f_{s,s'}(\mathbf{p},\mathbf{p}') \tag{2–17}$$

with $\nu_s = m_s^*/2\pi$ being the density of states of the subband s. All the scalar functions in the $\hat{f}$ matrix can be re-defined in the same fashion, e.g.

$$F^{\mathrm{s}}(\mathbf{p},\mathbf{p}') = \nu_s f^{\mathrm{s}}(\mathbf{p},\mathbf{p}') \tag{2–18}$$

and similarly for the $F^{\mathrm{a}\parallel}$, $F^{\mathrm{a}\perp}$, G^{ph}, G^{pp}, $H^{(1)}$, $H^{(2)}$, and H functions. With these definitions, Eq. 2–16 can be written as

$$\frac{m_s^*}{m}\left[1+\frac{m\alpha}{p_s}(\mathbf{e}_z\times\hat{\boldsymbol{\sigma}}_{ss})\cdot\mathbf{e}_{\mathrm{p}}\right] = 1+\sum_{s'}\frac{p_{s'}}{p_s}F^{,1}_{ss'}, \tag{2–19}$$

where

$$F^{,\ell}_{s,s'} \equiv \int F_{s,s'}(\mathbf{p}_s,\mathbf{p}_{s'})\cos(\ell\theta_{\mathrm{pp}'})\frac{d\theta_{\mathrm{pp}'}}{2\pi} \tag{2–20}$$

and $\theta_{\mathrm{p,p'}}$ is the angle between $\mathbf{p}$ and $\mathbf{p}'$. Using explicit forms of the Pauli matrices in the chiral basis 2–2

$$\hat{\sigma}_x = \begin{pmatrix} \sin\theta_{\mathrm{p}} & i\cos\theta_{\mathrm{p}} \\ -i\cos\theta_{\mathrm{p}} & -\sin\theta_{\mathrm{p}} \end{pmatrix}, \hat{\sigma}_y = \begin{pmatrix} -\cos\theta_{\mathrm{p}} & i\sin\theta_{\mathrm{p}} \\ -i\sin\theta_{\mathrm{p}} & \cos\theta_{\mathrm{p}} \end{pmatrix}, \hat{\sigma}_z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{2–21}$$

we find $(\mathbf{e}_z\times\hat{\boldsymbol{\sigma}}_{ss'})\cdot\mathbf{e}_{\mathrm{p}} = s\delta_{ss'}$. Projecting Eq. 2–7 onto the chiral basis, we obtain

$$\begin{aligned} F_{s,s'}(\mathbf{p}_s,\mathbf{p}_{s'}) &= F^{\mathrm{s}}(\mathbf{p}_s,\mathbf{p}_{s'}) + F^{\mathrm{a}\parallel}(\mathbf{p}_s,\mathbf{p}_{s'})ss'\cos\theta_{\mathrm{pp}'} \\ &+ G^{\mathrm{ph}}(\mathbf{p}_s,\mathbf{p}_{s'})\left[sp_s+s_{s'}-(sp_{s'}+s_s)\cos\theta_{\mathrm{pp}'}\right]/2 \\ &+ G^{\mathrm{pp}}(\mathbf{p}_s,\mathbf{p}_{s'})\left[sp_s+s_{s'}+(sp_{s'}+s_s)\cos\theta_{\mathrm{pp}'}\right]/2 \\ &+ H^{(1)}(\mathbf{p}_s,\mathbf{p}_{s'})ss'\cos(2\theta_{\mathrm{pp}'}) + H^{(2)}(\mathbf{p}_s,\mathbf{p}_{s'})ss'\cos^2\theta_{\mathrm{pp}'} \\ &+ H(\mathbf{p}_s,\mathbf{p}_{s'})ss'\cos\theta_{\mathrm{pp}'}. \end{aligned} \tag{2–22}$$

We see that, in contrast to the case of a SU2S FL, when the effective mass contains only the $\ell = 1$ harmonic of F^{s}, the $\ell = 1$ harmonic of $F_{ss'}$ in Eq. 2–19 contains also

higher harmonics of partial components of $F_{s,s'}$; namely, it contains the $\ell = 2$ harmonics of $F^{a\|}$, G^{ph}, G^{pp}, and H, and the $\ell = 3$ harmonics of $H^{(1)}$ and $H^{(2)}$. The only component of $F_{s,s'}$ which does not contribute to m_s^* is $F^{a\perp}$. This happens because $\hat{\sigma}_z$ is off-diagonal in the chiral basis, and hence $F^{a\perp}$ drops out from the diagonal part of the LF. The final result for the effective masses can be cast into the following form:

$$\frac{m_s^*}{m_s} = 1 + \sum_{s'} \frac{p_{s'}}{p_s} F^{,1}_{ss'}, \tag{2–23}$$

where

$$m_s \equiv \frac{m}{1 + s\frac{m\alpha}{p_s}}. \tag{2–24}$$

Notice that the Luttinger theorem [59] guarantees only that the total area of the FS, $\pi(p_+^2 + p_-^2)$, is not renormalized by the interaction but says nothing about the partial areas. Therefore, p_+ and p_- in Eqs. 2–23 and 2–24 must be considered as renormalized Fermi momenta. The mass m_s in Eq. 2–24 has the same form as the effective mass of free Rashba fermions $m_s^f = p_s^f/\partial_p \epsilon_s^f = m/(1 + sm\alpha/p_s^f)$, except for the Fermi momenta of a free system p_s^f are now replaced by the renormalized Fermi momenta p_s. From Eqs. 2–23 and 2–24, we see that a Rashba FL liquid is different from an SU2S one in that the effective mass (and, as will see later, other FL quantities) is not determined only by the Landau parameters: one also needs to know the *renormalized* Fermi momenta. In this respect, a Rashba FL is similar to other two-component FLs [60, 61].

Due to hidden symmetry of the Rashba Hamiltonian–conservation of the square of the velocity [62]– the group velocities of free Rashba fermions with opposite chiralities are the same, cf. Eq. 2–5. (However, the masses are different because of the difference in the Fermi momenta). It is reasonable to expect that the *ee* interaction breaks hidden symmetry and leads to splitting of the subbands' velocities. However, no such splitting occurs to first order in the SOI [48], even if the *ee* interaction is treated to an arbitrary order [55]. In what follows, we present a microscopic calculation of the effective masses

and show that the velocities are indeed split even by a weak *ee* interaction but only at larger values of the SOI.

We adopt a simple model of a screened Coulomb potential in the high-density limit ($r_s \ll 1$). The leading-order result for mass renormalization $m^*/m - 1 \propto r_s \ln r_s$ is obtained already for a static screened Coulomb potential $U(q) = 2\pi e^2/(q + p_{\mathrm{TF}})$, where $p_{\mathrm{TF}} = \sqrt{2} r_s p_F$ is the Thomas-Fermi screening momentum. The same form of $U(q)$ is valid also in the presence of the SOI because the total density of states remains the same as long as both Rashba subbands are occupied. The self-energy of the subband s is given by

$$\begin{aligned} \Sigma_s(\mathbf{p}, \omega) = & - \frac{1}{2} \sum_{s'} \int (dq) \int \frac{d\Omega}{2\pi} U(q) \\ & \times \; [1 + ss' \cos(\theta_{\mathrm{p+q}} - \theta_{\mathrm{p}})] g_{s'}(\mathbf{p} + \mathbf{q}, \omega + \Omega), \end{aligned} \tag{2–25}$$

where

$$g_s(\mathbf{p}, \omega) = \frac{1}{i\omega - \epsilon_s^{\mathrm{p},f} + \mu} \tag{2–26}$$

is the free Green's function in the chiral basis with $\epsilon_s^{\mathrm{p},f}$ given by Eq. 2–3. At small r_s, typical momentum transfers are small: $q \sim p_{\mathrm{TF}} \ll p_F$; therefore, the difference between $\theta_{\mathrm{p+q}}$ and θ_{p} can be neglected and intersubband scattering ($s = -s'$) drops out. Simplifying Eq. 2–26 in the way specified above and subtracting the value of the self-energy at the FS, we obtain

$$\begin{aligned} \Delta\Sigma_s &= \Sigma_s(\mathbf{p}, \omega) - \Sigma_s(\mathbf{p}_s, 0) \\ &= -\sum_{q,\Omega} U(q) \left[g_s(\mathbf{p} + \mathbf{q}, \omega + \Omega) - g_s(\mathbf{p}_s + \mathbf{q}, \Omega) \right]. \end{aligned} \tag{2–27}$$

Integration over Ω gives (for $\epsilon_s^{\mathrm{p}} > 0$)

$$\begin{aligned} \Delta\Sigma_s &= \int_{-\frac{\epsilon_s^{\mathrm{p}}}{v_F q} < \cos\theta_{\mathrm{p}} < 0} \frac{d^2 q}{(2\pi)^2} U(q) \\ &= \int_{\frac{\epsilon_{p,s}}{v_F}}^{p_s} \frac{dq}{2\pi^2} q U(q) \left[\cos^{-1}\left(-\frac{\epsilon_s^{\mathrm{p}}}{v_F q} \right) - \frac{\pi}{2} \right]. \end{aligned} \tag{2–28}$$

Expanding Eq. 2–28 to linear order in ϵ_s^{p} and solving the resulting integral over q to logarithmic accuracy, we find

$$\Delta\Sigma_s = \frac{\epsilon_s^{\mathrm{p}}}{\pi v_F} e^2 \ln \frac{p_s}{p_{\mathrm{TF}}}. \tag{2–29}$$

where the subband Fermi momentum, $p_\pm^f$, was chosen as the upper cutoff. Accordingly, the effective mass is renormalized as

$$\frac{m_\pm^*}{m_\pm} = 1 - \frac{e^2}{\pi v_F} \ln \frac{p_\pm^f}{p_{\mathrm{TF}}}. \tag{2–30}$$

This result is the same as for a 2D electron gas without SOI, except that the Fermi momentum, entering the logarithmic factor, is specific for a given band. This is already enough to produce splitting of the Fermi velocities

$$v_+ - v_- = \frac{p_+^f}{m_+^*} - \frac{p_-^f}{m_-^*} = -v_0 \frac{e^2}{\pi v_F} \ln \frac{p_-}{p_+}. \tag{2–31}$$

The mechanism for this splitting is similar to Overhauser-type splitting of effective masses in a partially spin-polarized metal [63–65].

The result in Eq. 2–31 is correct only to logarithmic accuracy, when $p_+ \ll p_-$, which implies strong SOI. In Appendix A, we compute the self-energy beyond the logarithmic accuracy and show that Eq. 2–31 is reproduced as a leading term. Also, we present there a simple way to generate an expansion of the self-energy in α, which confirms previous results for velocity splitting [48, 66].

2.4 Thermodynamic Properties

In principle, Eq. 2–7 enables one to compute all thermodynamic properties of a chiral FL. In this section, we illustrate how this program can be carried out in the charge sector by calculating the isothermal compressibility (Sec. 2.4.1) and deriving Pomeranchuk stability conditions (Sec. 2.4.2).

2.4.1 Compressibility

The $T = 0$ compressibility of a FL is given by

$$\kappa = \frac{1}{N^2}\left(\frac{\partial N}{\partial \mu}\right)_{\mathcal{N}}, \tag{2–32}$$

where $N = \sum_s \int (dp) n_s^{\mathrm{p}} = \sum_s N_s$ is the number density and $\mathcal{N}$ is the total particle number.

The compressibility of a chiral FL is obtained by a simple generalization of the original Landau's argument [34]. If the exact dispersions of Rashba subbands are $\epsilon_+(p) = \varepsilon_+^{\mathrm{p}} + \mu$ and $\epsilon_-(p) = \varepsilon_-^{\mathrm{p}} + \mu$, the chemical potential is defined as $\mu = \epsilon_+(p_+) = \epsilon_-(p_-)$. The variation of the chemical potential consists of two parts: the first one is due to a change in the Fermi momenta in each of the two subbands and the second one is due to a variation of the quasiparticles' dispersions. Since the variations of the chemical potential are the same for both subbands, we have

$$\delta\mu = \frac{\partial \epsilon_+(p)}{\partial p}\delta p_+ + \delta\varepsilon_+^{\mathrm{p}} = \frac{\partial \epsilon_-(p)}{\partial p}\delta p_- + \delta\varepsilon_-^{\mathrm{p}}, \tag{2–33}$$

where $\delta\varepsilon_\pm^{\mathrm{p}}$ are given by Eq. 2–12, $\frac{\partial \epsilon_s(p)}{\partial p} = \frac{\partial \varepsilon_s^{\mathrm{p}}}{\partial p_s} = v_s$, and $\delta p_s = (2\pi/p_s)\delta N_s$. Assuming that the variations of $n_\pm^{\mathrm{p}}$ in the p-space are localized near the corresponding Fermi momenta, we integrate $n_\pm^{\mathrm{p}}$ over the magnitude of the momenta to obtain $\varepsilon_s^{\mathrm{p}} = \sum_{s'} F_{ss'}^{,0}\delta N_{s'}/\nu_s$. Combining Eq. 2–33 with the constraint $\delta N = \delta N_+ + \delta N_-$, we solve the resulting system for $\delta N_\pm$ in terms of $\delta\mu$ and δN to obtain

$$\kappa = \frac{1}{N^2}\frac{\nu + \nu_+\left(F_{--}^{,0} - F_{-+}^{,0}\right) + \nu_-\left(F_{++}^{,0} - F_{+-}^{,0}\right)}{\left(1 + F_{++}^{,0}\right)\left(1 + F_{--}^{,0}\right) - {F_{+-}^{,0}}^2} \tag{2–34}$$

with $\nu = \sum_s \nu_s$. The result for the SU2S case is recovered in the limit when the intra- and intersubband components of $F_{ss'}^{,0}$ become the same. Indeed subsitituting $F_{++}^{,0} = F_{--}^{,0} = F_{+-}^{,0} = F_{-+}^{,0} = F^{\mathrm{s},0}/2$ into Eq. 2–34, we obtain $\kappa = \nu/(1 + F^{\mathrm{s},0})N^2$.

Notice that Eq. 2–34 is different from the compressibility of a two-component FL [67] because of the additional assumption used in Ref. [67], namely, that not only the total

number of particles but also the numbers of particles of a given type remains constant under compression. While this assumption is justified in the context of a degenerate electron-proton plasma considered in Ref. [67], it is not applicable to our case when particles of opposite chiralities can be exchanged between the Rashba subbands, and thus the number of particles with given chirality is not conserved.

2.4.2 Stability conditions

To obtain stability conditions in the charge sector, we must require that the free energy Ω be a minimum with respect to arbitrary deformations of the FSs which do not affecting their spin structure [68]:

$$\bar{p}_s(\theta) - p_s = \sum_{n=-\infty}^{\infty} \Delta_{s,n} e^{in\theta}. \tag{2–35}$$

Corresponding variations in the occupation numbers can be written as

$$\delta\hat{n}_{s,\mathrm{p}} = \Theta(\bar{p}_s(\theta) - p) - \Theta(p_s - p), \tag{2–36}$$

A change in the free energy

$$\delta\Omega = \mathrm{Tr}\int \hat{\epsilon}_p \delta\hat{n}_{\mathrm{p}}(dp) + \frac{1}{2}\mathrm{Tr}\mathrm{Tr}'\int \hat{f}(\mathbf{p},\mathbf{p}')\delta\hat{n}_{\mathrm{p}}\delta\hat{n}_{\mathrm{p}'}(dp)(dp') \tag{2–37}$$

must be positive definite with respect to such variations. Using $\hat{\varepsilon}_{ss'} = \delta_{ss'}\frac{p_s}{m_s^*}(p - p_s)$, the equation for $d\Omega$ is simplified to

$$\delta\Omega = \sum_{s,n} \frac{p_s^2}{4\pi m_s^*}\Delta_{s,n}^2 + \sum_{s,s',n} \frac{p_s p_{s'}}{4\pi m_s^*}\hat{F}_{ss'}^{,n}\Delta_{s,n}\Delta_{s',-n}. \tag{2–38}$$

We thus arrive at the following stability conditions in the charge sector:

$$1 + F_{ss}^{,n} > 0, \tag{2–39a}$$

$$(1 + F_{++}^{,n})(1 + F_{--}^{,n}) > (F_{s,-s}^{,n})^2. \tag{2–39b}$$

Condition 2–39a is the same as for the single-component case, while condition 2–39b indicates that the two-component FL is stable only if the inter-band interaction is sufficiently weak [67].

2.4.3 Spin susceptibility of a chiral Fermi liquid

As it was mentioned in Sec. 2.1, the subtleties of a non SU2S FL are all in the spin sector. To illustrate this point more clearly, we proceed with evaluating the spin susceptibility. In order to do this within the framework of FL theory, one needs to properly find the change in the occupation number of quasiparticles due to an external magnetic field.

Consider an external magnetic field in the $\mathbf{e}_z$ direction and of magnitude much smaller than the effective SO field

$$g\mu_B H \ll \alpha p_F, \tag{2–40}$$

where g is effective g-factor. (We neglect here the diamagnetic response of electrons.) In the absence of an external field, quasiparticles occupy chiral states $|\mathbf{p}, s\rangle$ (with $s = \pm 1$) filled up to the Fermi momenta p_s. The presence of a magnetic field affects the spin structure of quasiparticle states. Suppose that the states in the presence of the field are $|\mathbf{p}, h\rangle$ (with $h = \pm 1$) filled up to the Fermi momenta $\tilde{p}_h$. (To distinguish between the quantities in the absence and in the presence of the field, we denote the latter with a tilde over a corresponding symbol.) The energy functional of a quasiparticle in the absence of the field is diagonal in the $|\mathbf{p}, s\rangle$ basis with eigenvalues $\varepsilon_\pm$:

$$\varepsilon^{\mathrm{p}}_{ss'} = \langle \mathbf{p}, s|\hat{\varepsilon}^{\mathrm{p}}|\mathbf{p}, s'\rangle = \delta_{ss'}\varepsilon^{\mathrm{p}}_s \tag{2–41}$$

The occupation number in the absence of the field is also diagonal

$$n^{\mathrm{p}}_{ss'} = \langle \mathbf{p}, s|\hat{n}^{\mathrm{p}}|\mathbf{p}, s'\rangle = \delta_{ss'}n^{\mathrm{p}}_s, \tag{2–42}$$

where $n^{\mathrm{p}}_s = \Theta(p_s - p)$. In the presence of the field, the $|\mathbf{p}, s\rangle$ basis is not an eigenstate of the Hamiltonian. Therefore, the Zeeman part of the energy functional is not diagonal in this basis:[1]

$$\tilde{\varepsilon}^{\mathrm{p}}_{ss'} = \varepsilon^{\mathrm{p}}_{ss'} + \delta\varepsilon^{\mathrm{p}}_{ss'}, \tag{2–43}$$

where

$$\delta\varepsilon^{\mathrm{p}}_{ss'} = \frac{1}{2} g^*(\mathbf{p}) \mu_B H \sigma^z_{ss'}, \tag{2–44}$$

$g^*(\mathbf{p})$ is the renormalized g-factor which depends on the electron momentum, and σ^z in the chiral basis is given by the last formula in Eq. 2–21. Both the energy functional of quasiparticles in the presence of the field and their occupation number are diagonal in the $|\mathbf{p}, h\rangle$ basis

$$\begin{aligned} \tilde{\varepsilon}^{\mathrm{p}}_{hh'} &= \langle \mathbf{p}, h | \hat{\varepsilon}^{\mathrm{p}} | \mathbf{p}, h' \rangle = \delta_{hh'} \tilde{\varepsilon}^{\mathrm{p}}_h \\ \tilde{n}^{\mathrm{p}}_{hh'} &= \langle \mathbf{p}, h | \hat{n}^{\mathrm{p}} | \mathbf{p}, h' \rangle = \delta_{hh'} \tilde{n}^{\mathrm{p}}_h \end{aligned} \tag{2–45}$$

There exists a unitary matrix $\hat{U}$ that diagonalizes $\tilde{\varepsilon}^{\mathrm{p}}_{ss'}$

$$\tilde{\varepsilon}^{\mathrm{p}}_{hh'} = U^{\dagger}_{hs} \tilde{\varepsilon}^{\mathrm{p}}_{ss'} U_{s'}. \tag{2–46}$$

or, equivalently,

$$\tilde{\varepsilon}^{\mathrm{p}}_{ss'} = U^{\dagger}_{sh} \tilde{\varepsilon}^{\mathrm{p}}_{hh'} U_{h'}. \tag{2–47}$$

To first order in H,

$$\hat{U} = 1 + H\hat{M} + \mathcal{O}(H^2), \tag{2–48}$$

[1] In the SU2S case, the choice of the spin quantization axis is arbitrary, and the Zeeman energy of a quasiparticle can always be written in the diagonal form as $\sigma_z H$. In the chiral case, the choice of the spin quantization axis is unique, and the Zeeman energy cannot always be reduced to diagonal form.

and $\hat{M}$ is an anti-Hermitian matrix parameterized as

$$\hat{M} = \begin{pmatrix} ia & \beta \\ -\beta^* & ib \end{pmatrix} \tag{2–49}$$

with real a and b. The matrix $\hat{M}$ is determined from the condition that the linear-in-H part of $\tilde{\varepsilon}^{\mathrm{p}}_{ss'}$ is given by Eq. 2–44. For a diagonal $\tilde{\varepsilon}^{\mathrm{p}}_{hh'}$, the diagonal elements of $U^{\dagger}_{sh}\tilde{\varepsilon}^{\mathrm{p}}_{hh'}U_{h'}$ are equal to zero, while Eq. 2–44 contains only off-diagonal elements. This fixes β in Eq. 2–49 to be real and equal to

$$\beta = \frac{g^*(\mathbf{p})\mu_B}{2}\frac{1}{\varepsilon^{\mathrm{p}}_{+} - \varepsilon^{\mathrm{p}}_{-}}. \tag{2–50}$$

The diagonal components of $\hat{M}$ remain undefined to first order in H but, using that $\hat{U} = e^{H\hat{M}}$, they are found to be zero to all higher orders as well. Consequently, $\hat{M}$ can be written as

$$\hat{M} = \beta \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{2–51}$$

Since the same matrix $\hat{U}$ diagonalizes also the occupation number matrix, we have

$$\tilde{n}^{\mathrm{p}}_{ss'} = U^{\dagger}_{sh}\tilde{n}^{\mathrm{p}}_{hh'}U_{h'}. \tag{2–52}$$

To find a linear-in-H correction to $\tilde{n}^{\mathrm{p}}_{ss'}$, it suffices to approximate $\tilde{n}^{\mathrm{p}}_{hh'}$ as $\mathrm{diag}\,(n^{\mathrm{p}}_{+}, n^{\mathrm{p}}_{-})$ with field-independent $n^{\mathrm{p}}_{\pm}$. Then

$$\begin{aligned} \delta\tilde{n}^{\mathrm{p}}_{ss'} &= \beta\left[\hat{M}^{\dagger}\mathrm{diag}\,(n^{\mathrm{p}}_{+}, n^{\mathrm{p}}_{-}) + \mathrm{diag}\,(n^{\mathrm{p}}_{+}, n^{\mathrm{p}}_{-})\,\hat{M}\right] \\ &= \beta H(n^{\mathrm{p}}_{+} - n^{\mathrm{p}}_{-})\hat{\sigma}^{z}_{ss'}. \end{aligned} \tag{2–53}$$

It is at this point where the main difference between the SU2S and chiral FLs occurs: for the former, the change in the occupation number is localized near the FS; for the latter, it is proportional to a *difference* of the occupation numbers in the absence of the field, and is thus finite for *all* momenta in between the FSs of chiral subbands.

With this remark in mind, we still proceed with a derivation of the equation for the renormalized g-factor by decomposing the energy variation into the Zeeman part and the part due to a variation in the occupation numbers:

$$\begin{aligned}\delta\tilde{\varepsilon}^{\mathrm{p}}_{ss'} &= \frac{1}{2}g^*(\mathbf{p})\mu_B H\sigma^z_{ss'} \\ &= \frac{1}{2}g\mu_B H\sigma^z_{ss'} + \sum_{tt'}\int(dp')f_{st,s'}(\mathbf{p},\mathbf{p}')\delta\tilde{n}^{\mathrm{p}'}_t .\end{aligned} \tag{2–54}$$

Since $\delta\tilde{n}^{\mathrm{p}'}_t$ is proportional to σ^z_t, the sum over t and t' selects the only component of the LF in Eq. 2–7 that contains σ^z, i.e., $f^{\mathrm{a}\perp}$. The renormalized g-factor remains isotropic in the momentum space until a Pomeranchuk instability is reached: $g^*(\mathbf{p}) = g^*(p)$. With this simplification, Eq. 2–54 reduces to

$$\begin{aligned}\frac{g^*(p)}{g} &= 1 + 4\int(dp')f^{\mathrm{a}\perp}(\mathbf{p},\mathbf{p}')(n^{\mathrm{p}'}_+ - n^{\mathrm{p}'}_-)\frac{\beta}{\mu_B g} \\ &= 1 - 2\int_{p_+}^{p_-}\frac{dp'p'}{2\pi}\frac{f^{\mathrm{a}\perp,0}(p,p')}{\varepsilon^{\mathrm{p}'}_+ - \varepsilon^{\mathrm{p}'}_-}\frac{g^*(p')}{g},\end{aligned} \tag{2–55}$$

where $f^{\mathrm{a}\perp,0}(p,p')$ is the $\ell = 0$ angular harmonic of $f^{\mathrm{a}\perp}$ (we remind the reader that $\varepsilon^{\mathrm{p}}_{\pm}$ depend only on the magnitude of $\mathbf{p}$). In contrast to the SU2S case, the equation for $g^*(p)$ remains an integral one, even if the external momentum is projected onto one of the FSs.

The out-of-plane spin susceptibility is then found as

$$\begin{aligned}\chi_{zz} &= \frac{g\mu_B}{2H}\sum_{ss'}\int(dp)\sigma^z_{ss'}\delta\tilde{n}_{s's} = \frac{g\mu_B^2}{2}\int_{p_+}^{p_-}\frac{pdp}{2\pi}\frac{g^*(p)}{\varepsilon^{\mathrm{p}}_+ - \varepsilon^{\mathrm{p}}_-} \\ &= \frac{g\mu_B^2}{2}\left[\int_{p_+}^{p_-}\frac{dpp}{2\pi}\frac{1}{\varepsilon^{\mathrm{p}}_+ - \varepsilon^{\mathrm{p}}_-} - 2\int_{p_+}^{p_-}\frac{dpp}{2\pi}\int_{p_+}^{p_-}\frac{dp'p'}{2\pi}\frac{f^{\mathrm{a}\perp,0}(p,p')}{(\varepsilon^{\mathrm{p}}_+ - \varepsilon^{\mathrm{p}}_-)\left(\varepsilon^{\mathrm{p}'}_+ - \varepsilon^{\mathrm{p}'}_-\right)}\right].\end{aligned} \tag{2–56}$$

In the last line, we used Eq. 2–55 for $g^*(p)$. Equations 2–55 and 2–56 explicitly demonstrate that the out-of-plane spin susceptibility is determined entirely by the states

in between the two spin-split FSs. This is already true for a non-interacting system when the second term in Eq. 2–56 vanishes while the first one gives a correct result for the out-of-plane susceptibility of the Rashba Fermi gas with both subbands occupied, i.e., $\chi^f_{zz} = g\mu_B^2 m/4\pi$ (cf. Ref. [44]). (In Appendix B, we show explictly that the contribution from states near the FSs of a Rashba Fermi gas to χ^f_{zz} vanishes.)

To apply Eq. 2–56 for the FL case, one needs to know (renormalized) dispersions, $\varepsilon^{\mathrm{p}}_{\pm}$, and the $f^{\mathrm{a},\perp}$ component of the LF in the entire interval of momenta in between the FSs. If the SO coupling is not weak, one thus needs to know the properties of chiral QPs far away from their respective FSs, which is outside the scope of the FL theory. This problem is not merely technical but fundamental because QPs decay away from their FSs, and one thus cannot formulate the FL theory as a theory of well-defined QPs. This does not mean that a chiral electron system is a non-FL; on the contrary, all microscopic calculations as well as experimental evidence point at the FL-nature of chiral electron systems. However, they are FLs with the spin sector that cannot be described within the FL theory. This conclusion is not restricted to the case of a FL with Rashba SOI but is also true for any non-SU2S FL, including a partially spin-polarized FL, e.g., He^3 in a magnetic field [57]. Nevertheless, we show in Sec. 2.5.4 that Eq. 2–56 reproduces correctly both limiting cases of a weak *ee* interaction and a weak SO coupling.

As a concluding remark for this section, the in-plane spin susceptibility ($\chi_{xx} = \chi_{yy}$) can be shown to contain both on- and off-the FS contributions. However, since there is always a finite contribution from the damped states in between the two FSs, the problem remains the same as for χ_{zz}. In addition, the in-plane spin susceptibility contains higher angular harmonics of all Landau parameters except $f^{\mathrm{a}\perp}$.

2.5 Zero-Sound Collective Modes and Perturbation Theory for the Landau Function

The purpose of this section is to establish the connection between the Landau function and interaction vertices of the microscopic theory, and also to confirm the form of the Landau function in Eq. 2–7 using perturbation theory.

2.5.1 Relation between the Landau function and microscopic vertices

As in the theory of a SU2S FL, the relation between the Landau function and microscopic interaction vertices is established by deriving the equations of motion for zero-sound modes. With no loss in the generality of the concluding statements of the section, we ignore the effect of impurities and complications arising from the electric charge of the electrons.

To study the collective modes within the framework of a phenomenological FL theory, we start with the (collisionless) quantum Boltzmann equation for the non-equilibrium part of the occupation number matrix:

$$\partial_t \delta\hat{n}^{\mathrm{p}} + i[\hat{n}, \hat{\varepsilon}^{\mathrm{p}}_{-} + \mathbf{v}\cdot\boldsymbol{\nabla}_{\mathrm{r}}\delta\hat{n}^{\mathrm{p}} - \frac{1}{2}\left[\boldsymbol{\nabla}_{\mathrm{r}}\delta\hat{\varepsilon}^{\mathrm{p}}, \partial_{\mathrm{p}}\hat{n}^{0\mathrm{p}}\right]_{+} = 0, \tag{2–57}$$

where $\left[\hat{A}, \hat{B}\right]_{\pm}$ denotes the (anti)commutator of $\hat{A}$ and $\hat{B}$. (For brevity, the dependences of $\delta\hat{n}^{\mathrm{p}}$ and $\hat{\varepsilon}^{\mathrm{p}}$ on $\mathbf{r}$ and t are not displayed.) We will be interested in zero-sound modes in the charge sector which correspond to variations in the diagonal elements of $\delta\hat{n}$. In the chiral basis,

$$\delta n^{\mathrm{p}}_{s}(\mathbf{r}, t) \equiv \delta n^{\mathrm{p}}_{ss}(\mathbf{r}, t) = \delta(\varepsilon^{\mathrm{p}}_{s} - \mu)a_s(\theta_{\mathrm{p}})e^{i(\mathrm{q}\cdot\mathrm{r}-\Omega t)}, \tag{2–58}$$

where $a_s(\theta_{\mathrm{p}})$ describes the angular dependence of δn^{p}. Hence, Eq. 2–127 reduces to

$$\partial_t \delta n^{\mathrm{p}}_{s} + \mathbf{v}_s\cdot\boldsymbol{\nabla}_{\mathrm{r}}\delta n^{\mathrm{p}}_{s} + \delta(\varepsilon^{\mathrm{p}}_{s} - \mu)\mathbf{v}_s\cdot\boldsymbol{\nabla}_{\mathrm{r}}\delta\varepsilon^{\mathrm{p}}_{s} = 0, \tag{2–59}$$

where

$$\boldsymbol{\nabla}_{\mathrm{r}}\delta\varepsilon^{\mathrm{p}}_{s} = \sum_{s'}\int f_{s,s'}(\mathbf{p}, \mathbf{p}')\boldsymbol{\nabla}_{\mathrm{r}}\delta n^{\mathrm{p}'}_{s'}(dp'). \tag{2–60}$$

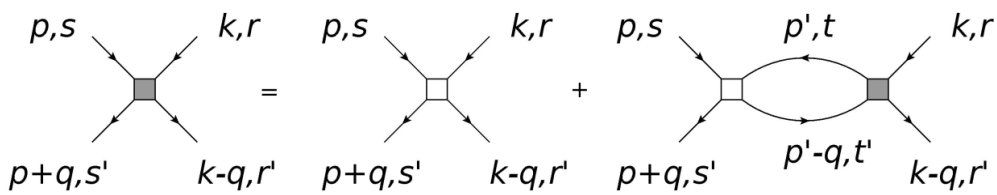

Figure 2-1. The Dyson equation for the scattering vertex. The first term on the right side represents the regular vertex, Γ^{Ω}.

Using Eq. 2–58, we obtain

$$A_s = \sum_{s'} \int F_{ss'} \Psi_{s'} A_{s'} \frac{d\theta'}{2\pi}, \tag{2–61}$$

where

$$A_s = a_s \Psi_s^{-1} \tag{2–62}$$

and

$$\Psi_s = \frac{\mathbf{v}_s \cdot \mathbf{q}}{\Omega - \mathbf{v}_s \cdot \mathbf{q}}. \tag{2–63}$$

Next, we derive Eq. 2–61 from the microscopic theory. Using the Dyson equation for the interaction vertex– see Fig. 2-1–we arrive at

$$\begin{aligned} \Gamma_{s,r;s',r'}(P,K;Q) &= \Gamma^{\Omega}_{s,r;s',r'}(P,K) \\ &+ \int_{P'} \Gamma^{\Omega}_{s,t;s',t'}(P,P') \Phi_{t,t'}(P';Q) \Gamma_{t',r;t,r'}(P',K;Q), \end{aligned} \tag{2–64}$$

where $s \ldots t' = \pm 1$ label the Rashba subbands, the $2+1$ momenta are defined as $P = (\omega, \mathbf{p})$, $P' = (\omega', \mathbf{p})$, etc., and $\int_P$ is a shorthand notation for $(2\pi)^{-3} \int d\omega \int d^2p \ldots$. Furthermore, Γ is the exact vertex which contains the poles corresponding to the collective modes, Γ^{Ω} is a regular vertex, obtained from Γ in the limit of $q/\Omega \to 0$ and $\Omega \to 0$, [34] and $\Phi_{ss'}$ is the particle-hole correlator. The off-diagonal components of $\Phi_{ss'}$ are gapped because of spin-orbit splitting of the Rashba subbands, while diagonal ones contain singular parts given by

$$\Phi_{ss}(P,Q) = (2\Psi i Z_s^2 / v_s) \delta(\omega) \delta(p - p_s) \Psi_s, \tag{2–65}$$

where Z_s is the Z-factor of subband s. The Landau function is related to the vertex $\Gamma^{\Omega}_{s,r;s,r}$. Since Eq. 2–64 must hold for any K, vertex can be written as the product of two independent contributions:

$$\Gamma_{s,r;s,r}(P,K;Q) = \eta_{ss}(P;Q)\eta_{rr}(K;Q). \tag{2–66}$$

Near the poles of Γ, we have

$$\eta_{ss} = \sum_t Z_t^2 \nu_t \int \Gamma^{\Omega}_{s,t;s,t}(\theta,\theta')\Psi_t \eta_t \frac{d\theta'}{2\pi}, \tag{2–67}$$

Comparing the kernels in Eqs. 2–67 and 2–61, we identify

$$\hat{f}_{ss'} = \frac{\nu_{s'}}{\nu_s} Z_{s'}^2 \Gamma^{\Omega}_{ss'}. \tag{2–68}$$

We see that, except for the normalization factor, the relation between the vertex and the Landau function is the same as in the SU2S theory.

2.5.2 Landau function from perturbation theory

Next, we show how the various components of the phenomenological LF in Eq. 2–7 are reproduced by perturbation theory for the interaction vertex. In this section, we consider a second order perturbation theory in a finite-range interaction U_q. In Sec. 2.5.3, we derive some particular results for the case of a point-like interaction, $U_q = U$.

According to Eq. 2–68, the LF is proportional to a certain limit of the (antisymmetrized) interaction vertex Γ^{Ω}. In the spin basis,

$$\Gamma^{\Omega}_{\alpha,\beta;\gamma,\delta}(\mathbf{p},\mathbf{p}') = \lim_{\Omega\to 0} \lim_{\frac{q}{\Omega}\to 0} \Gamma_{\alpha,\beta;\gamma,\delta}(P,P';Q)\,|_{\omega=\omega'=0}\ . \tag{2–69}$$

To first order in the interaction, we have

$$\Gamma^{\Omega}_{\alpha,\beta;\gamma,\delta} \;=\; \left(U_0 - \frac{U_{|\mathrm{p}-\mathrm{p}'|}}{2}\right)\delta_{\alpha\gamma}\delta_{\beta\delta} \;-\; \frac{U_{|\mathrm{p}-\mathrm{p}'|}}{2}\hat{\boldsymbol{\sigma}}_{\alpha\gamma}\cdot\hat{\boldsymbol{\sigma}}_{\beta\delta}, \tag{2–70}$$

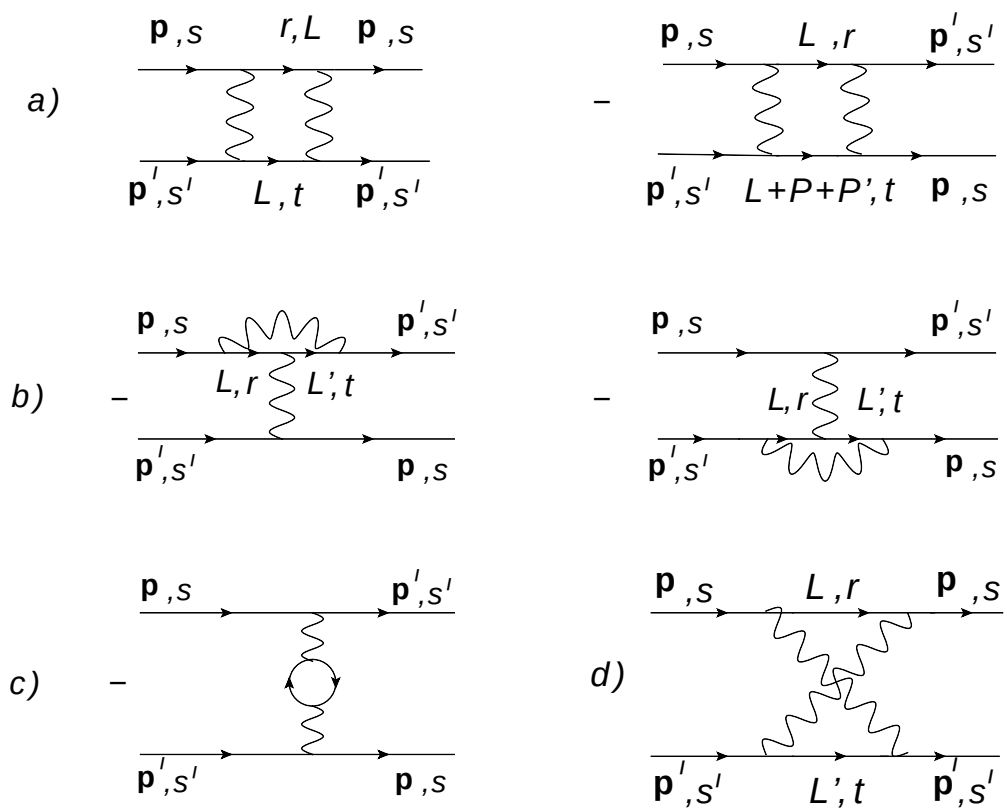

Figure 2-2. All second-order diagrams for the Γ^{Ω} vertex related to the Landau function. The rest of the second-order diagrams for Γ vanish in the $q/\Omega \to 0$ limit. The internal $2+1$-momentum L' is given by $L' = L + P' - P$.

which is the same as in the SU2S case. To see the non-SU2S terms, one needs to go to at least second order.

We start with a particle-particle channel diagram in Fig. 2-2a, the direct ($a1$) and exchange ($a2$) parts of which are given by

$$\Gamma^{\Omega,a1}_{\alpha,\beta;\gamma,\delta}(\mathbf{p},\mathbf{p}') = -\int_L U^2_{|\mathbf{p}-\mathbf{l}|}\, G^f_{\alpha\gamma}(L)\, G^f_{\beta\delta}(L') \tag{2–71a}$$

$$\Gamma^{\Omega,a2}_{\alpha,\beta;\gamma,\delta}(\mathbf{p},\mathbf{p}') = -\int_L U_{|\mathbf{p}-\mathbf{l}|}\, U_{|\mathbf{p}'-\mathbf{l}|}\, G^f_{\alpha\delta}(L)\, G^f_{\beta\gamma}(L'), \tag{2–71b}$$

correspondingly. Here, integration goes over $L = (\mathbf{l}, \omega)$, $L' = -L + K$ with $K = (0, \mathbf{k}) = (0, \mathbf{p} + \mathbf{p}')$, and the free Green's function in the spin basis reads

$$\hat{G}^f(P) = \sum_s \frac{1}{2}\left(1 + s\hat{\boldsymbol{\sigma}} \times \mathbf{e}_\mathrm{p} \cdot \mathbf{e}_z\right) g_s(P) \tag{2–72}$$

where $g_s(P)$ is the same as in Eq. 2–26. First, we focus on the direct term, $a1$. The cross product of the unity matrices obviously only renormalizes the 1×1 term in the the LF of SU2S FL. Consider now terms involving one unity matrix and one Pauli matrix:

$$\begin{aligned}\left(\Gamma^{\Omega,a1}_{\alpha,\beta;\gamma,\delta}\right)_g &= -\frac{1}{4}[1_{\alpha\gamma}\hat{\boldsymbol{\sigma}}_{\beta\delta} \times \sum_{s,s'} s'\mathbf{A}'_{ss'}(\mathbf{k}) \cdot \mathbf{e}_z \\ &+ 1_{\beta\delta}\hat{\boldsymbol{\sigma}}_{\alpha\gamma} \times \sum_{s,s'} s\mathbf{A}_{ss'}(\mathbf{k}) \cdot \mathbf{e}_z],\end{aligned} \tag{2–73}$$

where

$$\mathbf{A}_{ss'}(\mathbf{k}) = \int_L \mathbf{l} U^2_{|\mathrm{p}-\mathrm{l}|} g^f_s(L) g^f_{s'}(L'), \tag{2–74a}$$

$$\mathbf{A}'_{ss'}(\mathbf{k}) = \int_L \mathbf{l}' U^2_{|\mathrm{p}-\mathrm{l}|} g^f_s(L) g^f_{s'}(L'). \tag{2–74b}$$

Since the Green's function depends only on the magnitude of the electron momentum, the vectors in Eq. 2–74a and 2–74b are related to each other by

$$\mathbf{A}_{ss'}(\mathbf{k}) = \mathbf{A}'_s(\mathbf{k}). \tag{2–75}$$

For the same reason, the directions of both vectors must coincide with that of $\mathbf{k}$. Using these two properties, we obtain

$$\sum_{s,s'} s\mathbf{A}_{ss'} = \sum_{s,s'} s'\mathbf{A}'_{ss'} = \mathbf{k}\mathcal{A}_1(k), \tag{2–76}$$

where $\mathcal{B}_1(k)$ is some scalar function of k, the precise form of which depends on the choice of U_q. Using the last result, we finally arrive at

$$\begin{aligned}
\left(\Gamma^{\Omega,a1}_{\alpha,\beta;\gamma,\delta}\right)_g &= -\frac{1}{4}\mathcal{B}_1(|\mathbf{p}+\mathbf{p}'|) \\
&\times [1_{\alpha\gamma}\hat{\boldsymbol{\sigma}}_{\beta\delta} + 1_{\beta\delta}\hat{\boldsymbol{\sigma}}_{\alpha\gamma}] \times (\mathbf{p}+\mathbf{p}')\cdot\mathbf{e}_z \\
&\equiv [1_{\alpha\gamma}\hat{\boldsymbol{\sigma}}_{\beta\delta} + 1_{\beta\delta}\hat{\boldsymbol{\sigma}}_{\alpha\gamma}] \times \mathbf{C}_1,
\end{aligned} \tag{2–77}$$

which coincides with the tensor form of the g^{pp} term in Eq. 2–7.

Likewise, exchange diagram $a2$ gives

$$\begin{aligned}
\left(\Gamma^{\Omega,a2}_{\alpha,\beta;\gamma,\delta}\right)_g &= -\frac{1}{4}\mathcal{A}_2(|\mathbf{p}+\mathbf{p}'|) \\
&\times [1_{\alpha\delta}\hat{\boldsymbol{\sigma}}_{\beta\gamma} + 1_{\beta\gamma}\hat{\boldsymbol{\sigma}}_{\alpha\delta}] \times (\mathbf{p}+\mathbf{p}')\cdot\mathbf{e}_z \\
&\equiv [1_{\alpha\delta}\hat{\boldsymbol{\sigma}}_{\beta\gamma} + 1_{\beta\gamma}\hat{\boldsymbol{\sigma}}_{\alpha\delta}] \times \mathbf{C}_2,
\end{aligned} \tag{2–78}$$

where $\mathcal{A}_2$ is defined by the same relations as 2–74a and 2–88, except for a replacement

$$U^2_{|\mathbf{p}-\mathbf{l}|} \to U_{|\mathbf{p}-\mathbf{l}|}U_{|\mathbf{p}'-\mathbf{l}|}. \tag{2–79}$$

Although the tensor form of the exchange vertex seems to be different from that of the direct one, it is, in fact, the same. To see this, it is convenient to transform the vertex into the chiral basis using

$$\begin{aligned}
\Gamma^{\Omega}_{ss'}(\mathbf{p},\mathbf{p}') &= \sum_{\alpha\beta\gamma\delta}\Gamma^{\Omega}_{\alpha,\beta;\gamma,\delta}(\mathbf{p},\mathbf{p}')\langle\alpha|s,\mathbf{p}\rangle\langle\beta|s',\mathbf{p}'\rangle \\
&\quad\times\langle s,\mathbf{p}|\gamma\rangle\langle s',\mathbf{p}'|\delta\rangle,
\end{aligned} \tag{2–80}$$

where $|s, \mathbf{k}\rangle$ is the Rashba spinor defined by Eq. 2–2. Applying 2–80 to Eqs. 2–77 and 2–78, we obtain for the direct and exchange contributions, correspondingly

$$\Gamma^{\Omega,a1}_{ss'} = (\langle s, \mathbf{p}|\hat{\boldsymbol{\sigma}}|s, \mathbf{p}\rangle + \langle s', \mathbf{p}'|\hat{\boldsymbol{\sigma}}|s', \mathbf{p}'\rangle) \times \mathbf{C}_1 \tag{2–81a}$$

$$\begin{aligned}\Gamma^{\Omega,a2}_{ss'} &= \frac{1}{2}\left[1 + ss^{i\left(\theta_{\mathrm{p}}-\theta_{\mathrm{p}'}\right)}\right]\langle s, \mathbf{p}|\hat{\boldsymbol{\sigma}}|s', \mathbf{p}'\rangle \times \mathbf{C}_2 \\ &\quad + \frac{1}{2}\left[1 + ss^{i\left(\theta'_{\mathrm{p}}-\theta_{\mathrm{p}}\right)}\right]\langle s', \mathbf{p}'|\hat{\boldsymbol{\sigma}}|s, \mathbf{p}\rangle \times \mathbf{C}_2.\end{aligned} \tag{2–81b}$$

Calculating the matrix elements of the Pauli matrices, it can be readily shown that the vectors to the left from $\mathbf{C}_1$ in 2–81a and from $\mathbf{C}_2$ in 2–81b are the same.

Next, we consider the term involving two Pauli matrices in Eq. 2–71a,

$$\left(\Gamma^{\Omega,a1}_{\alpha,\beta;\gamma,\delta}\right)_h = -\frac{1}{4}(\mathbf{e}_z \times \hat{\boldsymbol{\sigma}}_{\alpha\gamma})_i(\mathbf{e}_z \times \hat{\boldsymbol{\sigma}}_{\beta\delta})_j S^{(1)}_{ij}(\mathbf{p}, \mathbf{p}'), \tag{2–82}$$

where $S^{(1)}_{ij}$ is a second rank tensor

$$S^{(1)}_{ij}(\mathbf{p}, \mathbf{p}') = \sum_{s,s'} ss' \int_L \hat{\mathbf{l}}_i \hat{\mathbf{l}}^2_j \, |\mathrm{p}-\mathrm{l}| g^f_s(L) g^f_{s'}(L'). \tag{2–83}$$

This tensor depends only on $\mathbf{k} = \mathbf{p} + \mathbf{p}'$, and can be formed only from of the components of $\mathbf{k}$ as

$$S^{(1)}_{ij}(\mathbf{k}) = k_i k_j \mathcal{S}^{(1)}(k). \tag{2–84}$$

Hence

$$\begin{aligned}\left(\Gamma^{\Omega,a1}_{\alpha,\beta;\gamma,\delta}\right)_h &= -\frac{1}{4}\mathcal{S}^{(1)}(|\mathbf{p} + \mathbf{p}'|) \\ &\times [\hat{\boldsymbol{\sigma}}_{\alpha\gamma} \times (\mathbf{p} + \mathbf{p}') \cdot \mathbf{e}_z)] \, [\hat{\boldsymbol{\sigma}}_{\beta\delta} \times (\mathbf{p} + \mathbf{p}') \cdot \mathbf{e}_z] \,,\end{aligned} \tag{2–85}$$

which reproduces the tensor structure of the h terms in Eq. 2–7 (with $h^{(1)} = h^{(2)} = h/2$ to this order of the perturbation theory). As before, the exchange diagram produces same type of terms. Since the tensor structures of the direct and exchange particle-particle

diagrams are the same, the non-SU2S part of the total particle-particle vertex $\Gamma^{\Omega,a} = \Gamma^{\Omega,a1} - \Gamma^{\Omega,a2}$ vanishes for a point-like interaction, $U_q = U$. Therefore, the $g^{\rm pp}$ term in the LF is absent in this case, whereas the h terms come from only the particle-hole channel, considered below.

Next, we show that the crossed diagram in the particle-hole channel (Fig. 2-2 d) produces both the $g^{\rm ph}$ and h terms in Eq. 2–7. The term involving one unity matrix and one Pauli matrix resulting from the direct diagram is processed in the same way as the analogous term from the particle-particle diagram. The only difference is that vectors $\mathbf{A}_{ss'}$ and $\mathbf{A}'_{ss'}$ are now replaced by

$$\mathbf{B}_{ss'}(\mathbf{k}) = \int_L \mathbf{l} U^2_{|\mathbf{p}-\mathbf{l}|} g^f_s(L) g^f_{s'}(L''), \tag{2–86a}$$

$$\mathbf{B}'_{ss'}(\mathbf{k}) = \int_L \mathbf{l}' U^2_{|\mathbf{p}-\mathbf{l}|} g^f_s(L) g^f_{s'}(L''), \tag{2–86b}$$

where $L'' = L + P' - P$, and $\mathbf{B}$ and $\mathbf{B}'$ are related by

$$\mathbf{B}_{ss'}(\mathbf{k}) = -\mathbf{B}'_s(\mathbf{k}). \tag{2–87}$$

Defining a scalar function $\mathcal{B}(k)$ via

$$\sum_{s,s'} s\mathbf{B}_{ss'} = -\sum_{s,s'} s'\mathbf{B}'_{ss'} = \mathbf{k}\mathcal{B}(k), \tag{2–88}$$

we obtain

$$\begin{aligned}\left(\Gamma^{\Omega,d}_{\alpha,\beta;\gamma,\delta}\right)_g &= -\frac{1}{4}\mathcal{B}(|\mathbf{p}-\mathbf{p}'|) \\ &\times \left[1_{\alpha\gamma}\hat{\boldsymbol{\sigma}}_{\beta\delta} \times (\mathbf{p}'-\mathbf{p})\cdot\mathbf{e}_z - 1_{\beta\delta}\hat{\boldsymbol{\sigma}}_{\alpha\gamma} \times (\mathbf{p}'-\mathbf{p})\cdot\mathbf{e}_z\right],\end{aligned} \tag{2–89}$$

which coincides with the tensor form of the $g^{\rm ph}$ term in Eq. 2–7. The term involving two Pauli matrices is cast into a form similar to Eq. 2–85:

$$\left(\Gamma^{\Omega,d}_{\alpha,\beta;\gamma,\delta}\right)_h = -\frac{1}{4}\mathcal{T}(|\mathbf{p}-\mathbf{p}'|)$$
$$\times\left[\hat{\boldsymbol{\sigma}}_{\alpha\gamma}\times(\mathbf{p}-\mathbf{p}')\cdot\mathbf{e}_z)\right]\left[\hat{\boldsymbol{\sigma}}_{\beta\delta}\times(\mathbf{p}-\mathbf{p}')\cdot\mathbf{e}_z\right], \qquad (2\text{–}90)$$

where $\mathcal{T}(k)$ is defined similarly to Eqs. 2–83 and 2–84 except L' is replaced by $L'' = L + P' - P$. The last expression reproduces again the tensorial structure of the $h^{(1,2)}$ and h terms, except for now $h^{(1)} = h^{(2)} = -h/2$. The exchange counterpart to diagram $b1$ does not depend on the external momenta. Thefore, the direct and exhange part of diagram b do not cancel each other even for the case of $U_q = U$.

The rest of the second-order diagrams renormalizes only the the SU2S part of the LF. We postpone analysis of these diagrams until the next section.

2.5.3 Example: mass renormalization

In this section, we apply the formalism developed in previous sections to a microscopic calculation of mass renormalization for Rashba fermions. We consider a weak, short-range *ee* interaction ($U_q = U$), and assume also that the SOI is weak as well, i.e., $\alpha \ll v_F$. Our focus will be on the leading, $\mathcal{O}(U^2\alpha^2)$ correction to the effective masses of Rashba fermions. Although such correction seems to be perfectly analytic, in fact, it is not. It will be shown that the correction to the effective mass of the subband s comes as $sU^2\alpha^2$. Since the eigenenergies contain s and α only as a combination of $s\alpha$, the second-order term in the regular expansion in $s\alpha$ is the same for $s = \pm 1$. In this sense, the $sU^2\alpha^2$ correction is non-analytic, and will be shown to come from the Kohn anomalies of the various vertices.

To simplify notations, we will suppress the superscript Ω in the vertices because all the vertices considered in this section are of the Γ^{Ω} type. Also, we will suppress the superscript f labeling the quantities pertinent to the free system: in a straightforward perturbation theory, considered here, the interaction shows only as the U^2 prefactor.

A non-analytic part of the LF to comes from particle-hole diagrams for Γ^{Ω} in Fig. 2-2, i.e., from diagrams b-d. To maximize the effect of the Kohn anomaly, one needs to select those initial and final states that correspond to the minimal number of small matrix elements for intra-band backscattering. Detailed calculations of the diagrams are presented in Appendix D; here, we illustrate how an $\mathcal{O}(U^2\alpha^2)$ correction occurs using diagram c as an example. Explicitly, this diagram reads

$$\Gamma^{c}_{ss'}(\mathbf{p}_s, \mathbf{p}'_s) = -\frac{U^2}{2}[1 + ss' \cos(\theta_{\mathrm{p}'} - \theta_{\mathrm{p}})]\Pi(|\mathbf{p}_s - \mathbf{p}'_{s'}|), \tag{2–91}$$

where $\Pi(q)$ is the polarization bubble of free Rashba fermions. Without the SOI, the Kohn anomaly of Π is located at $q = 2p_F$, where p_F is the Fermi momentum at $\alpha = 0$. In 2D, the polarization bubble is independent of q for $q \leq 2p_F$ and exhibits a characteristic square-root anomaly for $q > 2p_F$. SO coupling splits the spectrum into two Rashba subbands with Fermi momenta $p_\pm$ given by Eq. 2–4. In Ref. [42], it was shown that the static polarization bubble $\Pi(q)$ remains constant for $q \leq 2p_+$ for an arbitrary value of α (but as long as both subbands are occupied). In the region $2p_+ \leq q \leq 2p_-$, the polarization bubble exhibits a non-analytic dependence on q. At small α, all three Fermi momenta are close to each other: $p_+ \approx p_- \approx p_F$. In this case, the singular parts of Π can be written as (see Appendix C)

$$\Pi(q) = -\nu + \Pi_{++}(q) + \Pi_{+-}(q) \tag{2–92}$$

$$\Pi_{++}(q) = -\frac{\nu}{6}\Theta(q - 2p_+)\left(\frac{q - 2p_+}{p_F}\right)^{3/2} \tag{2–93}$$

$$\Pi_{+-}(q) = \frac{\nu}{2}\Theta(q - 2p_F)\left(\frac{q - 2p_F}{p_F}\right)^{1/2}. \tag{2–94}$$

The Kohn anomalies of Π affects the amplitudes of backscattering processes with $\mathbf{p}' \approx -\mathbf{p}$, hence $\theta_{\mathrm{p}'} - \theta_{\mathrm{p}} = \pi - \theta$ with $|\theta| \ll 1$. Therefore,

$$\Gamma^{c}_{ss'} = -\frac{U^2}{2}\left(1 + ss' - ss'\frac{1}{2}\theta^2\right)\Pi(|\mathbf{p}_s - \mathbf{p}'_{s'}|), \tag{2–95}$$

The Kohn anomaly of Π_{++} affect the processes with $2p_+ \leq q \leq 2p_-$. Since Π_{++} already contains a small factor of $(q - 2p_+)$, reflecting the smallness of the matrix element of intraband backscattering, the singular contribution from Π_{++} is maximal for interband processes ($s = -s'$), when the angular factor in 2–95 is almost equal to 1. The only vertex of this type is

$$\Gamma^c_{+-} = -U^2\Pi_{++}(q) = \frac{\nu U^2}{6}\Theta(q - 2p_+)\left(\frac{q - 2p_+}{p_F}\right)^{3/2}. \tag{2–96}$$

The Kohn anomaly of Π_{+-} gives an effect of the same order as in 2–96 but for intraband processes:

$$\Gamma^c_{--} = -U^2\frac{\theta^2}{2}\Pi_{+-}(q). \tag{2–97}$$

For a backscattering process,

$$q = |\mathbf{p}_s - \mathbf{p}'_{s'}| \approx p_s + p_{s'} - p_F\theta^2/4, \tag{2–98}$$

where the dependence of the prefactor of the θ^2 term on α can be and was neglected. For Γ^c_{--}, we have $q = 2p_- - p_F\theta^2/4$. Expressing θ^2 in terms of q, we arrive at

$$\Gamma^c_{--} = -\nu U^2\Theta(q - 2p_F)\frac{(q - 2p_-)\,(q - 2p_F)^{1/2}}{p_F^{3/2}}. \tag{2–99}$$

Now it is obvious that Γ^c_{+-} and Γ^c_{--} are of the same order.

The remaining diagrams are evaluated in Appendix D with the following results

$$\Gamma^b_{--} = 0 \tag{2–100}$$

$$\Gamma^b_{+-} = -\Gamma^c_{+-} = U^2\Theta\left(q-2p_F\right)\left(\frac{q-2p_F}{p_F}\right)^{3/2} \tag{2–101}$$

$$\Gamma^d_{--} = \frac{U^2\nu}{12}\Theta(q-2p_F)\left(\frac{q-2p_F}{p_F}\right)^{3/2} \tag{2–102}$$

$$\Gamma^d_{+-} = \frac{U^2\nu}{24}\Theta(q-2p_+)\left(\frac{q-2p_+}{p_F}\right)^{3/2} \tag{2–103}$$

$$\Gamma^{b,c,d}_{++} = 0. \tag{2–104}$$

For q, one substitutes $q = |\mathbf{p}_- - \mathbf{p}'_-|$ into Eqs. 2–99, 2–100 and 2–102, and $q = |\mathbf{p}_+ - \mathbf{p}'_-|$ into Eqs. 2–96, 2–101 and 2–103.

Now we can calculate the angular harmonics of the vertices. Combining the vertices in the $--$ channel and transforming back from q to θ, we obtain for the nth harmonic of the total vertex in this channel

$$\begin{aligned}\Gamma^{,n}_{--} = \Gamma^{c,n}_{--} + \Gamma^{d,n}_{--} &= -(-1)^n\frac{U^2\nu}{4}\int_0^{2\sqrt{\frac{2m\alpha}{p_F}}}\frac{d\theta}{\pi}\theta^2\left(\frac{2m\alpha}{p_F}-\frac{\theta^2}{4}\right)^{1/2} \\ &+ (-1)^n\frac{U^2\nu}{12}\int_0^{2\sqrt{\frac{2m\alpha}{p_F}}}\frac{d\theta}{\pi}\left(\frac{2m\alpha}{p_F}-\frac{\theta^2}{4}\right)^{3/2} \\ &= -\frac{3}{8}(-1)^n U^2\nu\left(\frac{m\alpha}{p_F}\right)^2.\end{aligned} \tag{2–105}$$

Equation 2–105 is valid for $1 \leq n \ll \sqrt{p_F/8m\alpha}$. Likewise, we find for the $+-$ channel

$$\Gamma^{,n}_{+-} = \Gamma^{d,n}_{+-} = \frac{1}{16}(-1)^n U^2\nu\left(\frac{m\alpha}{p_F}\right)^2. \tag{2–106}$$

The harmonics of the LF are related to those of vertices via Eq. 2–68 with the proportionality coefficient $\nu_{s'}Z^2_{s'}/\nu_s$, which is taken to be equal to one to lowest order in U and α.

As mentioned in Sec. 2.3, the formula for the effective mass, Eq. 2–23, contains not only the components of the LF but also the ratio of the subband Fermi momenta which, in general, are renormalized by the interaction. However, combining previous results from Refs. [45, 55, 66], one can show that this effect occurs only to higher orders

in α and U. Indeed, an analytic part of the ground state energy of the electron system with Rashba SOI can be written as $E_{\text{an}} = C\left(\delta n/n - 2\alpha/v_F^f\right)^2$, where $\delta N = N_- - N_+$ is the difference in the number of electrons occupying the Rashba subbands and v_F^f is the *bare* Fermi velocity [55]. This result is valid to *all* orders in the *ee* interaction of arbitrary type, and to lowest order in α. Therefore, the minimum of E_{an} corresponds to the same value of δN as for a free electron gas, which means that the Fermi momenta are not renormalized. Renormalization of δN occurs because of non-analytic terms in the ground state energy [45, 66]. For a short-range interaction, the first non-analytic correction occurs to fourth order in U: $E_{\text{na}} \propto U^4|\alpha|^3 \ln^2|\alpha|$. (A cubic dependence on $|\alpha|$ is because a non-analytic part of the energy in 2D scales as the cube of the parameter controlling non-analyticity; [69] an additional factor of $\ln^2|\alpha|$ comes from renormalization of the interaction in the Cooper channel.) Then the minimum of $E_{\text{an}} + E_{\text{na}}$ corresponds to a change in the Fermi momenta $p_- - p_+ \propto \delta N \propto U^4\alpha^2 \ln^2|\alpha|$, which is beyond the order of the perturbation theory considered here. To lowest order in α, we can also take p_+ to be equal to p_-. Substituting Eqs. 2–105 and 2–106 into Eq. 2–23, we obtain for the renormalized masses ($\Delta m_\pm^*$ denotes the difference between the renormalized and bare values):

$$\frac{\Delta m_+^*}{m_+} = F_{++}^{,1} + F_{+-}^{,1} = -\frac{1}{16}\left(\frac{\nu U m\alpha}{p_F}\right)^2 \tag{2–107}$$

$$\frac{\Delta m_-^*}{m_-} = F_{--}^{,1} + F_{-+}^{,1} = \frac{5}{16}\left(\frac{\nu U m\alpha}{p_F}\right)^2, \tag{2–108}$$

We see that mass renormalization is different for different Rashba subbands. Since the Fermi momenta are not renormalized to this order, this implies that the group velocities $v_\pm^* = p_\pm/m_\pm^*$ are also different.

As we pointed out in Sec. 2.3, the equality of the subbands' group velocities is preserved to an arbitrary order in the *ee* interaction, provided that the SOI is treated to first order [55]. Within the RPA for a Coulomb potential, velocity splitting occurs because of a non-analytic $r_s|\alpha|^3 \ln|\alpha|$ term in the electron self-energy [66] whereas in our model

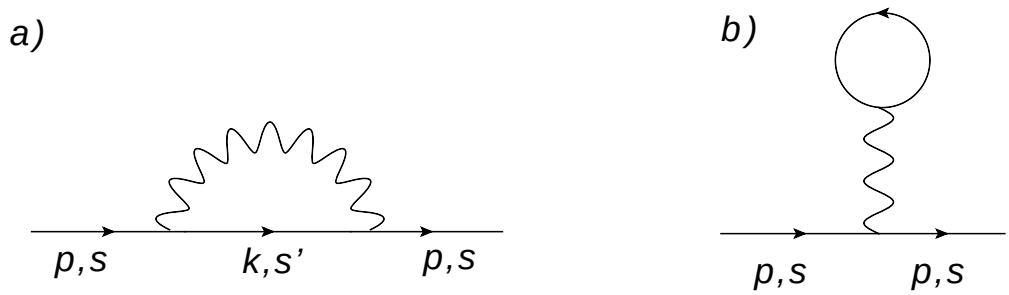

Figure 2-3. Self energy diagrams to first order in the interaction.

velocity splitting occurs already at order $U^2\alpha^2$. The difference between our result and that of Ref. [66] is that our renormalization comes from the Kohn anomaly which was not considered in Ref. [66]. Note that the model assumption of $U_q = U$ used in this section is not essential: for momentum-dependent interaction U_q, one simply needs to replace U by U_{2p_F} in the final results. In the high-density limit ($r_s \ll 1$), the $2p_F$ component of a screened Coulomb potential is of order r_s^2; therefore, our result for the Coulomb case would translate into $v_+ - v_- \propto r_s^2\alpha^2$. Terms of order r_s^2 were not considered in Ref. [66]. In reality, both the $r_s|\alpha|^3 \ln|\alpha|$ and $r_s^2\alpha^2$ terms are present, and the competition between the two is controlled by the ratio of two small parameters: $|\alpha| \ln|\alpha|/r_s$.

2.5.4 Perturbation theory for χ_{zz}

In this section, we show that Eq. 2–56 reproduces the known results for χ_{zz} in the limiting cases of a weak *ee* interaction or a weak SO coupling.

2.5.4.1 Weak electron-electron interaction

First, we examine the limit of a weak and short-range *ee* interaction (to first order in the interaction amplitude U without making any assumption about the strength of the SO coupling.) It follows from Eq. 2–70 that $f^{a\perp} = -U/2$ to first order in the interaction. We also need the self-energy of the quasi-particles to first order in the interaction given by (see Fig. 2-3)

$$\Sigma_s(P) = \Sigma_s^a(P) + \Sigma_s^b(P), \tag{2–109}$$

$$\Sigma_s^a(P) \;=\; -\frac{U}{2}\sum_{s'}\int_K [1+ss'\cos(\theta_{\rm k}-\theta_{\rm p})g_s(K) = -\frac{U}{2}N, \tag{2–110}$$

and similarly $\Sigma_s^b(P) = UN$, where N is the total number density of electrons. Therefore, the self-energies of both Rashba branches are constant and equal to each other. Since the shift in the energy of quasiparticles with opposite chiralities is the same, we have $\varepsilon_+^{\rm p} - \varepsilon_-^{\rm p} = 2\alpha p$ and $p_- - p_+ = 2m\alpha$, which are the same relations as for a free system. Substituting these results into Eq. 2–56, we find that to first order in U

$$\chi_{zz} = \chi_0\left(1+\frac{mU}{2\pi}\right). \tag{2–111}$$

As is to be expected, this result coincides with that given by the first ladder diagram of the perturbation theory [44]. Notice that χ_{zz} in Eq. 2–111 does not depend on α. This feature survives to all orders in U within the ladder approximation. Beyond the ladder approximation, the leading dependence on α occurs to order U^2 as a non-analytic correction: $\delta\chi_{zz} = (2/3)\chi_0(mU/4\pi)^2|\alpha|p_F/\epsilon_F$ (Refs. [44, 45]).

2.5.4.2 Weak spin-orbit coupling

Next, we consider the opposite limit of an arbitrary *ee* interactions but infinitesimally small SO coupling. In this limit, one must recover the result of an SU2S FL. To obtain χ_{zz} in this limit, one needs to evaluate integrals of the type

$$\int_{p_+}^{p_-}\frac{dp'}{2\pi}\frac{f^{{\rm a}\perp,0}(p,p')g^*(p')}{\varepsilon_+^{\rm p'}-\varepsilon_-^{\rm p'}},$$

to zeroth order in α. Infinitesimal SO coupling implies that the region of integration is infinitesimally small compared to the Fermi momentum, and it suffices to consider only the linear part of the dispersion near the FS $\varepsilon_s^{\rm p} = v_s(p-p_s)$, where $v_s = \partial_p\varepsilon_s^{\rm p}|_{p=p_s}$. Note that in the presence of the interaction, in general, $v_+ \neq v_-$. However, to obtain χ_{zz} to zeroth order in α, one can keep the SO coupling only in the Fermi momenta of the two subbands and set α to zero everywhere else. As a result, v_s can be set to equal to v_F,

the Fermi velocity in the absence of the SO coupling. Therefore,

$$\int_{p_+}^{p_-} \frac{dp'}{2\pi} \frac{f^{\mathrm{a},0}(p,p')g^*(p')}{\varepsilon_+^{\mathrm{p}'} - \varepsilon_-^{\mathrm{p}'}} = \frac{p_F}{2\pi v_F} f^{\mathrm{a}\perp,0}(p_F,p_F)g^*(p_F), \tag{2–112}$$

where $f^{\mathrm{a},0}(p_F,p_F) = f^{\mathrm{a}\perp,0}(p_F,p_F)\,|_{\alpha=0}$ is the spin-asymmetric component of the LF in the absence of the SO coupling. Substituting this result into Eq. 2–55 and solving for $g^*(p_F)$, we reproduce the result for an SU2S FL

$$g^*(p_F) = \frac{g}{1+\nu_F f^{\mathrm{a}\perp}(p_F,p_F)}, \tag{2–113}$$

where $\nu_F = p_F/\pi v_F$ is the (renormalized) density of states at the Fermi level. Similarly,

$$\chi_{zz} \approx \frac{g^2\mu_B^2}{2} \int_{p_+}^{p_-} \frac{dp p_F}{2\pi} \frac{g^*(p_F)}{\varepsilon_+^{\mathrm{p}} - \varepsilon_-^{\mathrm{p}}} = \frac{g^2\mu_B^2}{4} \frac{\nu_F}{1+\nu_F f^{\mathrm{a},0}}. \tag{2–114}$$

Notice that the SO coupling is eliminated as an *anomaly*, i.e., as the cancellation between a small denominator ($\varepsilon_+^{\mathrm{p}} - \varepsilon_-^{\mathrm{p}}$) and a narrow integration range ($p_- - p_+$).

2.6 Chiral Spin Waves

In this section, we study collective modes in the spin sector of a chiral FL, and obtain the spectrum and equations of motion for chiral spin-wave collective modes. As discussed in Sec. 2.4.3, the spin sector of a chiral FL with arbitrary strong SOI does not admit a description in terms of well-defined QPs. However, if the SOI is weak, it can be treated as a perturbation to the SU2S-FL; this approach was employed in Ref. [51] for studying the $q = 0$ chiral spin resonances. In what follows, we extend this approach to the finite-q case and study the dispersion relations in the entire q space.

2.6.1 Dispersion of chiral spin-waves

To find the spectrum of the collective modes, we consider the Dyson equation for the interaction vertex in Eq. 2–64 with the kernel

$$\Phi_{ss'}(P,Q) = (2\pi i Z^2/v_F)\delta(\omega)\delta(p-p_F)\Psi_{ss'}, \tag{2–115}$$

where Z is the QP renormalization factor in the absence of SO,

$$\Psi_{-+} = \frac{\Delta + \mathbf{v}\cdot\mathbf{q}}{\omega - \Delta - \mathbf{v}\cdot\mathbf{q}},$$
$$\Psi_{+-} = \frac{-\Delta + \mathbf{v}\cdot\mathbf{q}}{\omega + \Delta - \mathbf{v}\cdot\mathbf{q}},$$
$$\Psi_{++} = \Psi_{--} = \frac{\mathbf{v}\cdot\mathbf{q}}{\omega - \mathbf{v}\cdot\mathbf{q}}, \tag{2–116}$$

and $\Delta = 2\alpha p_F$. To investigate the collective modes in the spin sector, we need to keep only the spin part of Γ^{Ω}. In the perturbation theory with respect to the SOI, we neglect its effect on the LF which is taken the same as for an SU2S FL, Eq. 2–6. We further adopt the s-wave approximation, in which $F^a = \text{const} \equiv F^{a,0}$. This approximation allows one to obtain a closed-form solution of Eq. 2–127 without affecting the results qualitatively. One of the effects not captured by this approximation is renormalization of α, which is controlled by $F^{a,1}$ [38, 51] and is thus absent in the s-wave approximation. To account for this effect, one can simply replace α by α^* in the final results. In the s-wave approximation, Γ^{Ω} as expressed as

$$Z^2\nu_F\Gamma^{\Omega}_{s,t;s'}(P,P') = F^{a,0}\langle s'\mathbf{p}|\boldsymbol{\sigma}|s\mathbf{p}\rangle\cdot\langle t'\mathbf{p}'|\boldsymbol{\sigma}|t\mathbf{p}'\rangle, \tag{2–117}$$

and $\langle s'\mathbf{p}'|\boldsymbol{\sigma}|s\mathbf{p}\rangle$ are the Pauli matrices in the chiral basis, given by Eq. 2–21. Note that even in the s-wave approximation, $\Gamma^{\Omega}_{s,t;s'}$ depends on the directions of the electron momenta via the Pauli matrices. Since Eq. 2–64 holds for any K, the exact vertex can be factorized as

$$\Gamma_{s,r;s',r'}(P,K;Q) = \eta_{ss'}(P;Q)\eta_{rr'}(K;Q) \tag{2–118}$$

Near the poles of Γ, we have

$$\eta_{ss'} = \frac{F^{a,0}}{2}\sum_{t,t'}\int\frac{d\theta'}{2\pi}\langle s'\mathbf{p}_F|\boldsymbol{\sigma}|s\mathbf{p}_F\rangle\cdot\langle t'\mathbf{p}'_F|\boldsymbol{\sigma}|t\mathbf{p}'_F\rangle\Psi_{tt'}\eta_t. \tag{2–119}$$

Changing the variables as $\mu_{ss'} = \Psi_s \eta_{ss'}$, we obtain a set of equations for normal modes

$$\begin{aligned}
\Psi_{-+}^{-1}\mu_{+-} &= \frac{F^{a,0}}{2}\left[\int \frac{d\theta_{\mathrm{p}'}}{2\pi}(\mu_{+-}+\mu_{-+}) + \int \frac{d\theta_{\mathrm{p}'}}{2\pi}\cos\vartheta(\mu_{+-}-\mu_{-+})\right. \\
&+ \left. i\int \frac{d\theta_{\mathrm{p}'}}{2\pi}\sin\vartheta(\mu_{++}-\mu_{--})\right] \\
\Psi_{+-}^{-1}\mu_{-+} &= \frac{F^{a,0}}{2}\left[\int \frac{d\theta_{\mathrm{p}'}}{2\pi}(\mu_{+-}+\mu_{-+}) - \int \frac{d\theta_{\mathrm{p}'}}{2\pi}\cos\vartheta(\mu_{+-}-\mu_{-+})\right. \\
&- \left. i\int \frac{d\theta_{\mathrm{p}'}}{2\pi}\sin\vartheta(\mu_{++}-\mu_{--})\right] \\
\Psi_{++}^{-1}\mu_{++} &= \frac{F^{a,0}}{2}\left[i\int \frac{d\theta_{\mathrm{p}'}}{2\pi}\sin\vartheta(\mu_{+-}-\mu_{-+}) + \int \frac{d\theta_{\mathrm{p}'}}{2\pi}\cos\vartheta(\mu_{++}-\mu_{--})\right] \\
\Psi_{--}^{-1}\mu_{--} &= -\frac{F^{a,0}}{2}\left[i\int \frac{d\theta_{\mathrm{p}'}}{2\pi}\sin\vartheta(\mu_{+-}-\mu_{-+}) + \int \frac{d\theta_{\mathrm{p}'}}{2\pi}\cos\vartheta(\mu_{++}-\mu_{--})\right],
\end{aligned} \tag{2–120}$$

where

$$\vartheta \equiv \theta_{\mathrm{p}'} - \theta_{\mathrm{p}}. \tag{2–121}$$

The last two equation of set 2–120 imply that $\mu_{++} = -\mu_{--}$. Expanding the normal msodes into angular harmonics as

$$\mu_{ss'} = \sum_{n=0}^{\infty} \mu_{ss'}^{'n}\cos(n\theta) + \sum_{n=1}^{\infty} \bar{\mu}_{ss'}^{'n}\sin(n\theta), \tag{2–122}$$

we see that $\bar{\mu}_{++}^{'n}$ couples only to $\mu_{s,-s}^{'m}$ while $\mu_{++}^{'n}$ couples only to $\bar{\mu}_{s,-s}^{'m}$. Introducing new coordinates

$$\begin{aligned}
X &= \frac{\mu_{+-}^{'1} - \mu_{-+}^{'1}}{2} + i\bar{\mu}_{++}^{'1} \\
Y &= \frac{\bar{\mu}_{+-}^{'1} - \bar{\mu}_{-+}^{'1}}{2} - i\mu_{++}^{'1} \\
Z &= \mu_{+-}^{'0} + \mu_{-+}^{'0},
\end{aligned} \tag{2–123}$$

we obtain the following equations for the reduced set of variables

$$\begin{aligned}
2Z &= F^{a,0}\int\frac{d\vartheta}{2\pi}\left[\Psi_{+-}+\Psi_{-+}\right]Z \\
&+ F^{a,0}\int\frac{d\vartheta}{2\pi}\cos\vartheta\left[\Psi_{+-}-\Psi_{-+}\right]X \\
4Y &= F^{a,0}\int\frac{d\vartheta}{\pi}\left[\sin^2\vartheta(\Psi_{+-}+\Psi_{-+})+2\cos^2\vartheta\Psi_{++}\right]Y \\
4X &= F^{a,0}\int\frac{d\vartheta}{\pi}\left[\cos^2\vartheta(\Psi_{+-}+\Psi_{-+})+2\sin^2\vartheta\Psi_{++}\right]X \\
&+ F^{a,0}\int\frac{d\vartheta}{\pi}\cos\vartheta\left[\Psi_{+-}-\Psi_{-+}\right]Z. \qquad (2\text{–}124)
\end{aligned}$$

The integrals in Eq. 2–124 are solved exactly to obtain an algebraic equation containing the spectrum for three collective chiral modes for arbitrary values of q. The spectra of these modes for $F^{a,0} = -0.5$, are shown in Panel (a) and (b) of Fig. 2-4. It is shown in the next section that the modes labelled as x, y, and z, correspond to waves of magnetizations M_x, M_y, and M_z, respectively. The x-mode and y-mode share the same resonance frequency and the spectra for both run into the particle-hole continuum at different values of $q < \Delta/v_F$, while the z-mode merges with the continuum at $v_F q = \Delta$. Inclusion of higher-harmonics of the Landau function beyond the s-wave, results in extra modes with higher resonance frequencies. However, the higher the frequency of the mode, the heavier its damping by particle-hole excitations.

It is instructive to analyze the behavior of the collective modes at $q \to 0$. Expanding the solutions of Eqs. 2–124 to order q^2, we find for the modes' dispersions

$$\Omega_i^2(q) = \Delta^2(1 + F^{a,0}\delta_i) + D_i q^2, \qquad (2\text{–}125)$$

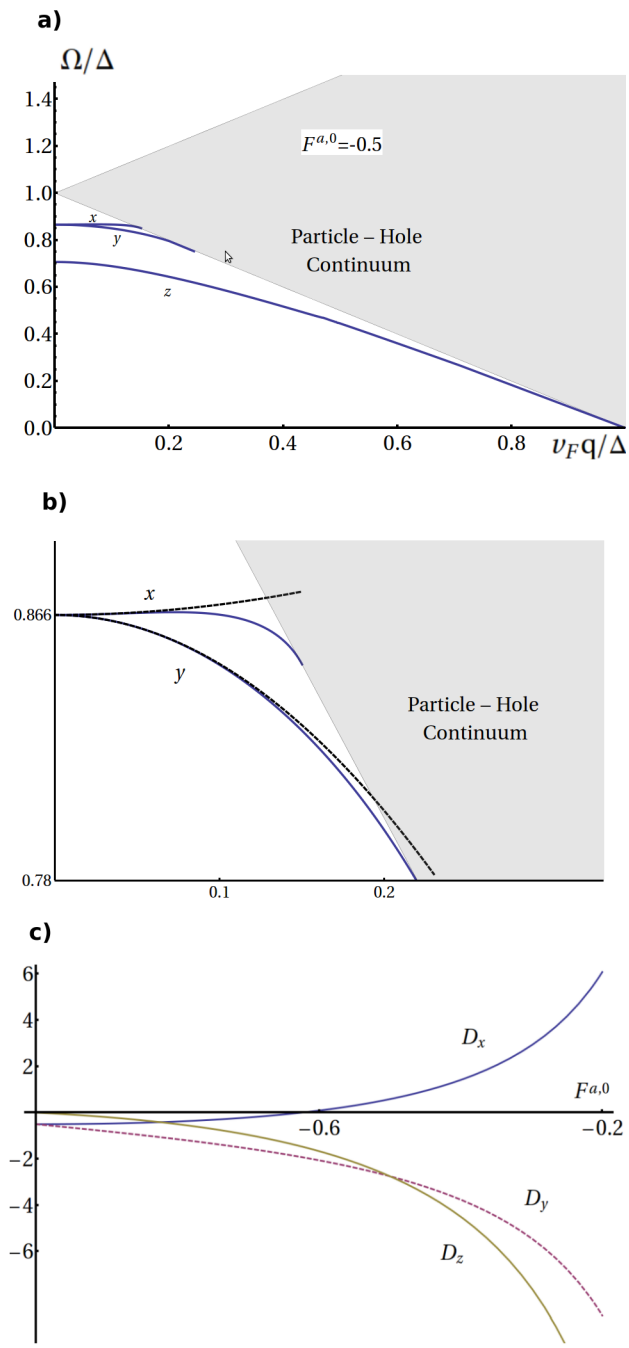

Figure 2-4. a) Spectrum of chiral spin waves for $F^{a,0} = -0.5$. b) zoom of the small-q region for the x-mode and y-mode. The dashed curves represent the parabolic approximation. c) Stiffnesses of x, y, z-modes as a function of $F^{a,0}$.

where $\delta_{x,y} = 1/2$, $\delta_z = 1$, and the mode stiffnesses depend on $F_{a,0}$ as

$$D_x = -\left[\frac{2}{F^{a,0}} + \frac{17}{4} + \frac{13}{8}F^{a,0} - \frac{(F^{a,0})^2}{16(1+F^{a,0}/2)}\right] v_F^2 \tag{2–126a}$$

$$D_y = \left[\frac{2}{F^{a,0}} + \frac{5}{4} + \frac{F^{a,0}}{8} + \frac{3(F^{a,0})^2}{16(1+F^{a,0}/2)}\right] v_F^2 \tag{2–126b}$$

$$D_z = \left[\frac{4}{F^{a,0}} + \frac{13}{2} + \frac{5}{2}F^{a,0}\right] v_F^2. \tag{2–126c}$$

At $q = 0$, these equations reduce to chiral spin resonances in the s-wave approximation [51]. Although chiral waves are somewhat analogous to spin-waves in a partially polarized FL [70–72], they exhibit a number of distinct features. First, chiral spin-waves have at least three branches: this is a direct consequence of broken $SU(2)$ symmetry: since in the in-plane and out-of-plane spin susceptibilities are now not the same, their singularities, corresponding to collective modes, also occur in different regions of the Ω, q plane. Second, the q dependences of these x-modes is of the opposite *sign* compare to that of the other two modes, in a wide interval of the *ee* coupling constant, $F^{a,0}$: for $-0.625 < F^{a,0} < 0$, D_z and D_y are negative while D_x is positive; see Fig. 2-4c. The mode stiffness plays the role of the (inverse) effective mass; therefore, chiral spin-waves behave as massive particles with opposite signs of masses. In the next section, we will develop the analogy between massive particles and chiral spin-waves further.

2.6.2 Equations of motion for chiral spin-waves

To find the equations of motion, we consider the Boltzmann equation in Eq. 2–127 in the presence of electron-impurity scattering

$$\frac{\partial \delta\hat{n}}{\partial t} + i[\hat{n}, \hat{\varepsilon} + \mathbf{v}\cdot\partial_{\mathrm{r}}\delta\hat{n} - \frac{1}{2}\{\partial_{\mathrm{r}}\delta\hat{\varepsilon}, \partial_{\mathrm{p}} n^0\} = \left(\frac{\partial \delta\hat{n}}{\partial t}\right)_{\mathrm{coll}}. \tag{2–127}$$

For brevity, the dependences of $\delta\hat{n}$ on $\mathbf{p}$, $\mathbf{r}$, and t are suppressed. In the equilibrium $\partial_{\mathrm{r}}\hat{\varepsilon}^0 = \partial_{\mathrm{r}}\hat{n}^0 = 0$. Considering the SOI as a perturbation to the SU2S FL, we follow the

notations of Ref. [51] and represent $\delta\hat{n}$ as a sum of two terms:

$$\delta\hat{n} = \delta\hat{n}_{\mathrm{SO}} + \delta\hat{n}_{\mathrm{ext}} \tag{2–128}$$

where $\delta\hat{n}_{\mathrm{SO}} = \partial_\varepsilon n^0 \delta\hat{\varepsilon}_{\mathrm{SO}}$ is a perturbation due to the SOI,

$$\delta\hat{\varepsilon}_{\mathrm{SO}} = \frac{\Delta(x)}{2}\hat{\boldsymbol{\sigma}} \times \mathbf{e}_{\mathrm{p}} \cdot \mathbf{e}_z, \tag{2–129}$$

where we allow for Δ to have a slow (compared to Fermi wavelength) position dependence in the plane of the 2D system, n^0 is the equilibrium Fermi function, and $\delta\hat{n}_{\mathrm{ext}}$ is a perturbation due to external forces. The external part of the perturbation can be parameterized as $\delta\hat{n}_{\mathrm{ext}} = \partial_\varepsilon n^0 \hat{u}_{\mathrm{p}}$ where

$$\hat{u}_{\mathrm{p}} = u_i{}^{\mathrm{p}}(\mathbf{r}, t)\hat{\tau}_i^{\mathrm{p}}, \tag{2–130}$$

and $\hat{\boldsymbol{\tau}}^{\mathrm{p}}$ is a vector of rotated Pauli matrices $\hat{\tau}_1^{\mathrm{p}} = -\hat{\sigma}_z$, $\hat{\tau}_2^{\mathrm{p}} = \hat{\boldsymbol{\sigma}} \cdot \mathbf{e}_{\mathrm{p}}$, and $\hat{\tau}_3^{\mathrm{p}} = \hat{\boldsymbol{\sigma}} \times \mathbf{e}_{\mathrm{p}} \cdot \mathbf{e}_z$. Matrices $\hat{\boldsymbol{\tau}}^{\mathrm{p}}$ obey the following algebra

$$[\hat{\tau}_3^{\mathrm{p}}, \hat{\tau}_3^{\mathrm{p}'}] = 2i\hat{\sigma}_z \sin\vartheta \tag{2–131a}$$

$$[\hat{\tau}_3^{\mathrm{p}}, \hat{\tau}_2^{\mathrm{p}'}] = 2i\hat{\sigma}_z \cos\vartheta \tag{2–131b}$$

$$[\hat{\tau}_3^{\mathrm{p}}, \hat{\tau}_1^{\mathrm{p}'}] = -2i\hat{\tau}_2^{\mathrm{p}}, \tag{2–131c}$$

where ϑ is defined by Eq. 2–121.

A change in the QP energy is given by

$$\delta\hat{\varepsilon} = \delta\hat{\varepsilon}_{\mathrm{SO}} - \frac{\nu_F}{2}\mathrm{Tr}' \int \frac{d\theta'}{2\pi}\hat{f}(\mathbf{p}, \mathbf{p}')\hat{u}' \tag{2–132}$$

where $\hat{f}(\mathbf{p}, \mathbf{p}')$ is the Landau function, $\Delta = 2\alpha^* p_F$, $\nu_F = m^*/\pi$ is the density of states, and prime refers to the quantum numbers of an electron with momentum$\mathbf{p}'$. Notice that the $\mathrm{Tr}' \int \hat{f}(\mathbf{p}, \mathbf{p}')\delta\hat{n}_{\mathrm{SO}} d\mathbf{p}'$ contribution to $\delta\hat{\varepsilon}$ is included via renormalization of the SO coupling α^* $\alpha \to \alpha^* = \alpha/(1 + F^{a,1}/2)$, where $F^{a,1}$ is the first harmonic of the spin part of the Landau function [38, 51]. The components of magnetization are expressed via $\hat{u}_{\mathrm{p}}$,

projected onto the FS, as

$$M_i = \frac{g\mu_B}{4}\nu_F \int \frac{d\theta}{2\pi} \mathrm{Tr}\left(\hat{\sigma}_i \hat{u}_{\mathrm{p_F}}\right). \tag{2–133}$$

Expanding u_i^{p} over angular harmonics as

$$u_i^{\mathrm{p}} = \sum_n u_i^{\prime n} \cos n\theta_{\mathrm{p}} + \bar{u}_i^{\prime n} \sin n\theta_{\mathrm{p}}, \tag{2–134}$$

we obtain for the dimensionless components of magnetization where

$$\begin{aligned} \mathcal{M}_x &= \frac{1}{2}(u_2^{\prime 1} + \bar{u}_3^{\prime 1}) \\ \mathcal{M}_y &= \frac{1}{2}(\bar{u}_2^{\prime 1} - u_3^{\prime 1}) \\ \mathcal{M}_z &= u_1^{\prime 0} \end{aligned} \tag{2–135}$$

with $M_{x,y} = (g\mu_B\nu_F/2)\mathcal{M}_{x,y}$ and $M_z = -(g\mu_B\nu_F/2)\mathcal{M}_z$.

To leading order in SO, the collision integral due to scattering at short-range impurities can be written as

$$\left(\frac{\partial \hat{n}}{\partial t}\right)_{\mathrm{coll}} = -\frac{\hat{n} - \bar{\hat{n}}}{\tau}, \tag{2–136}$$

where $\bar{\hat{n}}$ denotes the angular average, τ is the impurity mean-free time.

After all these steps, in the s-wave approximation, we obtain a system of coupled Boltzmann equations

$$(\hat{\Omega} - \mathbf{v}\cdot\hat{\mathbf{q}})iu_1^{\mathrm{p}} - \Delta u_2^{\mathrm{p}} = \Delta F^{a,0}(\cos\theta_{\mathrm{p}}\mathcal{M}_x + \sin\theta_{\mathrm{p}}\mathcal{M}_y) + \left(F^{a,0}\mathbf{v}\cdot\hat{\mathbf{q}} + \frac{i}{\tau}\right) i\mathcal{M}_z \tag{2–137}$$

$$(\hat{\Omega} - \mathbf{v}\cdot\hat{\mathbf{q}})u_2^{\mathrm{p}} - \Delta i u_1^{\mathrm{p}} = \Delta F^{a,0} i\mathcal{M}_z + \left(F^{a,0}\mathbf{v}\cdot\hat{\mathbf{q}} + \frac{i}{\tau}\right)(\cos\theta_{\mathrm{p}}\mathcal{M}_x + \sin\theta_{\mathrm{p}}\mathcal{M}_y) \tag{2–138}$$

$$(\hat{\Omega} - \mathbf{v}\cdot\hat{\mathbf{q}})u_3^{\mathrm{p}} = \left(F^{a,0}\mathbf{v}\cdot\hat{\mathbf{q}} + \frac{i}{\tau}\right)(\sin\theta_{\mathrm{p}}\mathcal{M}_x - \cos\theta_{\mathrm{p}}\mathcal{M}_y) \tag{2–139}$$

where $\hat{\Omega} \equiv i\partial_t + i/\tau$, and $\hat{\mathbf{q}} \equiv -i\nabla_{\mathrm{r}}$. The right-hand sides of Eqs. 2–137-2–139 contain only the lowest angular basis functions, which allows one to derive closed equations for

the harmonics of u_i^{p} or, equivalently, for the components of magnetization:

$$\begin{aligned} 2iM_z &= F^{a,0}\int\frac{d\theta_{\mathrm{p}}}{2\pi}\left[\hat{\mathcal{P}}_{+-}\left(iM_z+\cos\theta_{\mathrm{p}}M_x+\sin\theta_{\mathrm{p}}M_y\right)\right. \\ &+ \left.\hat{\mathcal{P}}_{-+}\left(iM_z-\cos\theta_{\mathrm{p}}M_x-\sin\theta_{\mathrm{p}}M_y\right)\right], \end{aligned} \tag{2–140a}$$

$$\begin{aligned} 4M_x &= F^{a,0}\int\frac{d\theta_{\mathrm{p}}}{\pi}\cos\theta_{\mathrm{p}}\left(\hat{\mathcal{P}}_{+-}-\hat{\mathcal{P}}_{-+}\right)iM_z \\ &+ F^{a,0}\int\frac{d\theta_{\mathrm{p}}}{\pi}\left[\cos^2\theta_{\mathrm{p}}\left(\hat{\mathcal{P}}_{+-}+\hat{\mathcal{P}}_{-+}\right)+2\sin^2\theta_{\mathrm{p}}\hat{\mathcal{P}}_{++}\right]M_x \\ &+ F^{a,0}\int\frac{d\theta_{\mathrm{p}}}{\pi}\left[\cos\theta_{\mathrm{p}}\sin\theta_{\mathrm{p}}\left(\hat{\mathcal{P}}_{+-}+\hat{\mathcal{P}}_{-+}-2\hat{\mathcal{P}}_{++}\right)\right]M_y, \end{aligned} \tag{2–140b}$$

$$\begin{aligned} 4M_y &= F^{a,0}\int\frac{d\theta_{\mathrm{p}}}{\pi}\sin\theta_{\mathrm{p}}\left(\hat{\mathcal{P}}_{+-}-\hat{\mathcal{P}}_{-+}\right)iM_z \\ &+ F^{a,0}\int\frac{d\theta_{\mathrm{p}}}{\pi}\left[\sin\theta_{\mathrm{p}}\cos\theta_{\mathrm{p}}\left(\hat{\mathcal{P}}_{+-}+\hat{\mathcal{P}}_{-+}-2\hat{\mathcal{P}}_{++}\right)\right]M_x \\ &+ F^{a,0}\int\frac{d\theta_{\mathrm{p}}}{\pi}\left[\sin^2\theta_{\mathrm{p}}\left(\hat{\mathcal{P}}_{+-}+\hat{\mathcal{P}}_{-+}\right)+2\cos^2\theta_{\mathrm{p}}\hat{\mathcal{P}}_{++}\right]M_y \end{aligned} \tag{2–140c}$$

where

$$\hat{\mathcal{P}}_{+-}=(\hat{\Omega}-\Delta-\mathbf{v}\cdot\hat{\mathbf{q}})^{-1}(\Delta+\mathbf{v}\cdot\hat{\mathbf{q}}+\frac{i}{F^{a,0}\tau}). \tag{2–141}$$

The operators $\hat{\mathcal{P}}_{-+}$ and $\hat{\mathcal{P}}_{++}$ are obtained from $\hat{\mathcal{P}}_{+-}$ by substituting $\Delta\to-\Delta$ and $\Delta=0$, respectively. Note that $\Delta(x)$ and $\hat{\mathbf{q}}$ do not commute; therefore, the order of operators must be kept intact. Keeping M_i to the right of $\hat{\mathcal{P}}_{ss'}$ emphasizes operations in space and time. Moreover, we choose $\hat{\mathbf{q}}$ to be along the x-axis. Several integrals in Eqs. 2–140a, 2–140b, and 2–140c vanish due to this choice, and as a result M_y is decoupled from the M_x-M_z system.

At $q=0$, Eqs. 2–140a, 2–140b, and 2–140c describe chiral resonances at frequencies $\Omega_{x,y}$ and Ω_z, which are the starting points of the dispersion relations for the collective modes. For small q, the mode frequencies are still close to Ω_x and Ω_z. To obtain dispersion relation in this region, we formally expand Eqs. 2–140a, 2–140b,

and 2–140c to order $\hat{q}_x^2$. For eigenvalues, it means that we are working in the regime $\mathbf{v}\cdot\mathbf{q} \ll \max\{\Omega-\Delta, 1/\tau\}$, for instance

$$\begin{aligned}\hat{\mathcal{P}}_{+-} &= \frac{\Delta}{\Omega-\Delta} + \frac{\Omega}{(\Omega-\Delta)^2}\mathbf{v}\cdot\hat{\mathbf{q}} + \frac{\Omega}{(\Omega-\Delta)^3}(\mathbf{v}\cdot\hat{\mathbf{q}})^2 \\ &+ \frac{\Omega}{(\Omega-\Delta)^4}\mathbf{v}\cdot(\hat{\mathbf{q}}\Delta)\mathbf{v}\cdot\hat{\mathbf{q}},\end{aligned} \tag{2–142}$$

where $\hat{\mathbf{q}}$ in $(\hat{\mathbf{q}}\Delta)$ acts only on $\Delta(x)$.

Using $\Delta^{-1}(x)v_F\hat{q}_x \to \hat{q}_x$ and $\hat{\Omega}/\Delta(x) \to \hat{\Omega}$, we find that

$$\begin{aligned}2iM_z &= \frac{F^{a,0}}{2}\left[\frac{2}{\Omega^2-1} + \frac{1}{(\Omega-1)^2} + \frac{1}{(\Omega+1)^2}\right]\hat{q}_x M_x + \frac{i}{2\tau}\left[\frac{1}{(\Omega-1)^2} - \frac{1}{(\Omega+1)^2}\right]\hat{q}_x M_x \\ &+ 2\left[\frac{F^{a,0}+\frac{i\Omega}{\tau}}{\Omega^2-1}\right] iM_z + \frac{F^{a,0}\Omega+\frac{i}{\tau}}{2}\left[\frac{1}{(\Omega-1)^3} + \frac{1}{(\Omega+1)^3}\right]\hat{q}_x^2 iM_z \\ &+ \frac{F^{a,0}\Omega}{2}\left[\frac{1}{(\Omega-1)^4} - \frac{1}{(\Omega+1)^4}\right](\hat{q}_x\Delta)\hat{q}_x iM_z\end{aligned} \tag{2–143}$$

$$\begin{aligned}4M_x &= F^{a,0}\left[\frac{2}{\Omega^2-1} + \frac{1}{(\Omega-1)^2} + \frac{1}{(\Omega+1)^2}\right]\hat{q}_x iM_z + \frac{i}{\tau}\left[\frac{1}{(\Omega-1)^2} - \frac{1}{(\Omega+1)^2}\right]\hat{q}_x iM_z \\ &+ 2\left[\frac{i}{\Omega\tau} + \frac{F^{a,0}+\frac{i\Omega}{\tau}}{\Omega^2-1}\right] M_x + \frac{3}{4}(F^{a,0}\Omega + \frac{i}{\tau})\left[\frac{1}{(\Omega-1)^3} + \frac{1}{(\Omega+1)^3}\right]\hat{q}_x^2 M_x \\ &+ \left[\frac{F^{a,0}+\frac{i}{\Omega\tau}}{2\Omega^2}\right]\hat{q}_x^2 M_x + \frac{3F^{a,0}\Omega}{4}\left[\frac{1}{(\Omega-1)^4} - \frac{1}{(\Omega+1)^4}\right](\hat{q}_x\Delta)\hat{q}_x M_x\end{aligned} \tag{2–144}$$

$$\begin{aligned}4M_y &= 2\left[\frac{i}{\Omega\tau} + \frac{F^{a,0}+\frac{i\Omega}{\tau}}{\Omega^2-1}\right] M_y + \frac{F^{a,0}\Omega+\frac{i}{\tau}}{4}\left[\frac{1}{(\Omega-1)^3} + \frac{1}{(\Omega+1)^3} + \frac{6}{\Omega^3}\right]\hat{q}_x^2 M_y \\ &+ \frac{F^{a,0}\Omega}{4}\left[\frac{1}{(\Omega-1)^4} - \frac{1}{(\Omega+1)^4}\right](\hat{q}_x\Delta)\hat{q}_x M_y.\end{aligned} \tag{2–145}$$

To obtain macroscopic equations of motion in the ballistic limit, one takes the limit of $\tau \to \infty$ and solve the system of 2–143, 2–144, and 2–145 to leading order in $\hat{q}_x$. The

solutions describe the three modes discussed in the last section. In the z-mode,

$$-\partial_t{}^2 M_z = \Delta^2(1+F^{a,0})M_z - D_z\partial_x{}^2 M_z - C_z\frac{\Delta'}{\Delta}\partial_x M_z, \tag{2–146a}$$

$$M_x = \frac{2+2F^{a,0}}{\Delta F^{a,0}} v_F \partial_x M_z, \tag{2–146b}$$

where $\Delta' = d\Delta/dx$, the mode stiffness D_i can be read from Eqs. 2–126a-2–126c, and

$$C_z = \left(\frac{2}{(F^{a,0})^2} + \frac{1}{F^{a,0}}\right)\left(\frac{1}{F^{a,0}} + 1\right) v_F^2. \tag{2–147}$$

The z-mode is a wave of the out-of-plane magnetization propagating in-plane. However, at finite q, a small $M_x(\propto \frac{v_F q}{\Delta} M_z)$ magnetization component is developed along the direction of propagation. The x-mode is described by the following equations of motion

$$-\partial_t{}^2 M_x = \Delta^2\left(1+\frac{F^{a,0}}{2}\right) M_x - D_x\partial_x^2 M_x - C_x\frac{\Delta'}{\Delta}\partial_x M_x, \tag{2–148a}$$

$$M_z = \frac{4+2F^{a,0}}{\Delta F^{a,0}} v_F \partial_x M_x, \tag{2–148b}$$

where

$$C_x = 12\left(\frac{2}{(F^{a,0})^2} + \frac{1}{2F^{a,0}}\right)\left(\frac{1}{F^{a,0}} + \frac{1}{2}\right) v_F^2. \tag{2–149}$$

This chiral mode describes a wave of the M_x magnetization propagating longitudinally in the x direction. However, at finite q, a small $M_z(\propto \frac{v_F q}{\Delta} M_x)$ magnetization component is produced. In the y-mode,

$$-\partial_t{}^2 M_y = \Delta^2\left(1+\frac{F^{a,0}}{2}\right) M_y - D_y\partial_x^2 M_y - C_y\frac{\Delta'}{\Delta}\partial_x M_y, \tag{2–150}$$

where $C_y = C_x/3$. This chiral mode is a wave of in-plane magnetization M_y propagating transversely in the x direction.

The main damping effect is contained in the $q = 0$ limit of Eqs. 2–144 and 2–143, by which one arrives at Eqs. 2–151a and 2–151b.

In the presence of damping, the modes are broadened almost independent of the wavelength. Therefore the effect of damping is mainly contained in the $q = 0$ form of the equations of motion

$$-\partial_t \left(\partial_t + \frac{1}{\tau}\right)^2 M_{x,y} = \Delta^2 \left[\left(1 + \frac{F^{a,0}}{2}\right)\partial_t + \frac{1 + F^{a,0}}{2\tau}\right] M_{x,y}. \tag{2–151a}$$

$$-\partial_t \left(\partial_t + \frac{1}{\tau}\right) M_z = \Delta^2 (1 + F^{a,0}) M_z. \tag{2–151b}$$

These equations describe Dyakonov-Perel spin relaxation [73] renormalized by the *ee* interaction. The modes are well resolved in the ballistic limit, $\Delta\tau \gg 1$.

Another source of damping for chiral spin-waves is due to electron-electron scattering, characterized by

$$\frac{1}{\tau_{\text{ee}}} \sim A \left(\frac{v_F q}{\Delta}\right)^2 [\Delta^2 + \pi^2 T^2], \tag{2–152}$$

where A is the pre-factor in the imaginary part of the electron self-energy, which in a FL scales as $\text{Im}\Sigma(\omega, T) = A(\omega^2 + \pi^2 T^2)$. In a GaAs heterostructure with number density $n \approx 10^{11}\text{cm}^{-2}$, using experimental results in Ref. Eisenstein1995, we estimate that $A = 0.1\text{meV}^{-1}$. In such system, $\Delta = 0.1\text{meV}$.[74] Therefore, given that in the long wavelength regime $v_F q \ll \Delta$, the damping due to e-e collisions are negligible compared to the damping due to disorder. The two damping sources are comparable only at high temperatures $T \sim \Delta/\pi v_F q\sqrt{A\tau}$.

2.6.3 Standing chiral waves

For standing-wave solutions, $M_i \sim \exp(i\Omega_i t)$, Eqs. 2–146a, 2–148a, and 2–150 are transformed into the "Schroedinger equations" for massive particles

$$\left[-\frac{1}{2\mathfrak{m}_i}\partial_x^2 + V_i(x)\right] M_i = E_i M_i, \tag{2–153}$$

where $i = \{x, y, z\}$, the "effective masses" are related to the stiffnesses in Eqs. 2–126a, 2–126b, and 2–126c via $\mathfrak{m}_i = 1/2D_i$, $E_i = \Omega_i^2$, and $V_i(x) = \Delta^2(x)(1 + F^{a,0}\delta_i)$ are the "potential energies", which we now allow to vary slowly (compared to the electron wavelength) in the 2DEG plane. By means of the lateral profile of Δ, the potential V_i confines the spin-chiral modes. These confined modes, i.e. standing chiral waves, are described by the bound states of Eq. 2–153. The extra term due to the gradient of Δ is ignored in the potential V_i. These terms are considered in Sec. 2.6.4, where we show that they cause no qualitative change in the energy levels for the bound states while adding a slowly varying envelope to the M_i wave function, i.e. $M_i \to M_i e^{bV_i(x)/2}$, where b is a constant proportional to the relative strength of the gradient term -see Sec. 2.6.4. However, these changes do not affect the qualitative arguments presented in this section.

The effective masses of the z-mode and the y-mode are negative for any $F^{a,0}$ within the interval from -1 to 0. Therefore, the z- and y-modes, as shown in the bottom part of Fig 2-5 b, can be confined by a *potential barrier* in Δ. The effective mass of the x-mode is positive for $F_c < F_0^a < 0$ and negative for $-1 < F^{a,0} < F_c$, where $F_c \approx -0.625$- see Fig 2-4c. In the region where the x-mode has a positive effective mass it can be confined by a potential well in Δ, as shown in the upper part of Fig 2-5b. In the case of negative $\mathfrak{m}_x$, the confining profile is the same as for the y- and z-modes. As shown in Fig 2-5b, it is proposed in Ref. [77] to modulate Δ by applying a gate voltage to a part of the 2DEG. The width of the gate should be much larger not to affect the electron wavelength. For $F_c < F^{a,0} < 0$, only a certain polarity of the gate voltage can confine the x-mode and only the reverse polarity can confine the other two modes. Discrete energy levels of the confined modes can be detected through a microwave absorption experiment where the distance between the absorption peaks would signal the difference in the effective masses of these confined modes. In the case of $-1 < F^{a,0} < F_c$, either all three modes are confined or all deconfined for a given polarity of the gate voltage and

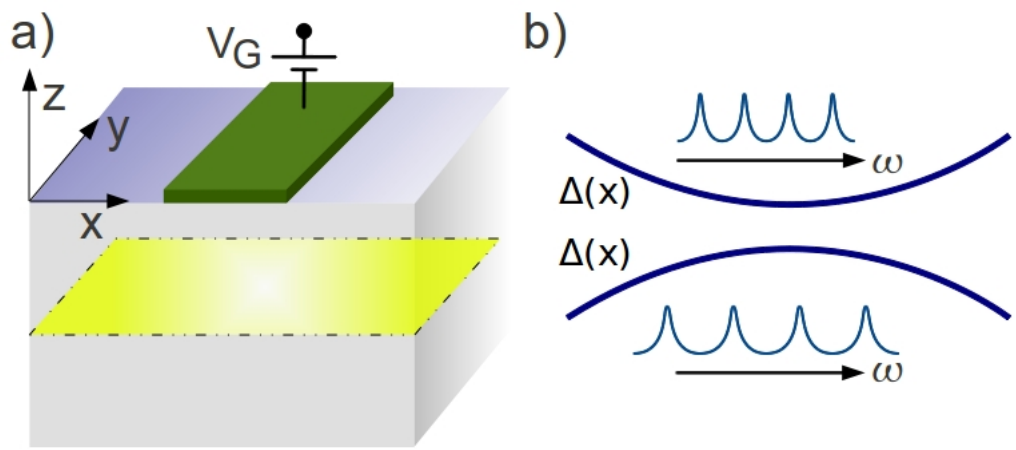

Figure 2-5. a) Sketch of the suggested experimental setup. Top gate modulates the SO splitting. b) Top: A minimum in Δ confines the x-mode with a positive effective mass. Bottom: A maximum in Δ confines the z-mode. The corresponding microwave absorption spectra are shown schematically in arbitrary units.

hence, reversing the polarity, would either strongly suppress or enhance the absorption spectrum. However, in all cases the two chiral spin resonances should be observed as strong peaks as they exist even in the absence of the interaction, and the spin-chiral waves.

2.6.4 Parabolic confinement of chiral spin-waves

In Sec. 2.6.3, the equations of motion for standing wave solutions of chiral-spin collective modes are transformed into a Schrdinger-like equation -see Eq. 2–153. However, the potential terms involving the gradient of the $\Delta(x)$ are ignored in that section. Here we restore the gradient terms and assess their effect. Using $M_i(x,t) = M_i(x)\exp(i\Omega_i t)$, Eqs. 2–146a, 2–148a, and 2–150 can be written as

$$\Omega_i^2 M_i = \left[-\frac{1}{2\mathfrak{m}_i}\partial_x^2 + \Delta^2(x)(1+F^{a,0}\delta_i) - C_i\frac{\Delta'}{\Delta}\partial_x\right] M_i, \tag{2–154}$$

which can in turn be written as

$$\varepsilon_i M_i = \left[-\frac{1}{2}\partial_x^2 + \frac{1}{2}U(x) + bU'(x)\partial_x\right] M_i, \tag{2–155}$$

where $\varepsilon_i = \mathfrak{m}_i[\Omega_i^2 - \Delta_0^2(1 + F^{a,0}\delta_i)]$, $U = 2\mathfrak{m}_i[\Delta^2 - \Delta_0^2](1 + F^{a,0}\delta_i)$, $U'(x) = dU(x)/dx$, and $b = C_i/2\Delta^2(1 + F^{a,0}\delta_i)$ is taken to be a constant. One can eliminate the gradient term by letting $M_i(x) \to M_i(x)e^{bU(x)/2}$ and arrive at

$$\varepsilon_i M_i = \left[-\frac{1}{2}\partial_x^2 + \frac{1}{2}\left(U(x) + \frac{b^2}{4}U'^2(x) - \frac{b}{2}U''(x)\right)\right] M_i. \tag{2–156}$$

This means that the gradient terms force a Gaussian envelope to the standing chiral spin-waves and also slightly modify their energy. For the case of a parabolic confinement, i.e. $U(x) = hx^2$,

$$\left(\varepsilon_i + \frac{bh}{2}\right) M_i = -\frac{1}{2}\partial_x^2 M_i + \frac{1}{2}\left(1 + b^2h^2\right) x^2 M_i, \tag{2–157}$$

which describes a harmonic oscillator with a renormalized frequency and shifted energy levels:

$$\varepsilon_{i,n} = \sqrt{1 + b^2h^2}\left(n + \frac{1}{2}\right) - \frac{bh}{2}. \tag{2–158}$$

2.6.5 Conditions for observation

To distinguish between the chiral resonances and the modes described here, one needs to observe several bound states, which requires the profile of $\Delta(x)$ to satisfy certain requirements. First, a realistic potential has a finite depth, W whose value determines the number of possible bound states. Suppose $V_i = \Delta_0^2(1 + F^{a,0}\delta_i) - W/\cosh^2(\frac{x}{x_0})$, where $\Delta_0 = \Delta(x{=}0)$. At $x \ll x_0$, this potential is identical to a parabolic potential

$$V_i = \Delta_0^2(1 + F^{a,0}\delta_i) + \frac{1}{2}\mathfrak{m}_i\omega_{\mathrm{H}}^2 x^2, \tag{2–159}$$

where $\omega_{\mathrm{H}} \equiv 2\sqrt{W/\mathfrak{m}_i x_0^2}$. To have more than one bound state, $W \geqslant \omega_{\mathrm{H}}/2$.[75] Therefore, the condition to have more than one bound state amounts to $\mathfrak{m}_i x_0^2 W \geqslant 1$.

Furthermore, for the bound state to exist, it should not be damped by particle-hole excitations. To find specific conditions, let us consider a parabolic confinement as in Eq. 2–159 The bound states for this case are found in Sec. 2.6.4. The characteristic size of

these bound states are $a \sim 1/\sqrt{\mathfrak{m}_i \tilde{\omega}_{\mathrm{H}}}$, where $\tilde{\omega}_{\mathrm{H}} = \omega_{\mathrm{H}}\sqrt{1+(\mathfrak{m}_i \omega_{\mathrm{H}} b)^2}$ the renormalized frequency due to the gradient terms. For $F^{a,0} = -0.5$, $q^*_{x,y} \approx 0.2/\lambda_{\mathrm{SO}}$ (cf. Fig. 2-4), where $\lambda_{\mathrm{SO}} \equiv 1/2m|\alpha|$, and $q^*_z = 1/\lambda_{\mathrm{SO}}$ independent of $F^{a,0}$.[76] In a GaAs heterostructure with $\alpha = 5$ meV $\cdot$ Å, [74] $\lambda_{\mathrm{SO}} \approx 1$ μm. Similarly, the condition to observe at least two bound states is that a should exceed $\sqrt{3}q^*_i$. This condition is obtained using the classical size of the wave function for $n = 1$ level of harmonic oscillator. Using the above values for λ_{SO} and $F^{a,0}$, $a \gtrsim 2$ μm ($a \gtrsim 9$ μm) for observation of the z-mode's (x- and y-modes') bound states. This condition also implies an upper limit on $\Delta\Omega = \sqrt{\omega_{\mathrm{H}}}$, i.e. the spacing between the absorption peaks:

$$\frac{\Delta\Omega}{\Omega_{i0}} \leqslant \frac{q^*_i \lambda_{\mathrm{SO}}}{\sqrt{3}} \sqrt{\frac{D_i}{(1+F^{a,0}\delta_i)v_F^2}}, \tag{2–160}$$

where Ω_{i0} are the resonance frequencies at $q = 0$.

Another necessary condition for observation of the bound states is that their broadening width, which is of order $1/\tau$ in the ballistic regime should not exceed their level-spacing. Therefore the condition to have well-resolved bound states amounts to $\Delta\Omega \gg 1/\tau$. For $|F^a_0| \sim 1$, this condition is the same as the condition for the ballistic limit, i.e., $\Delta\tau \gg 1$. Ignoring the dependence of α on the number density n, in a GaAs heterostructure we find that $\Delta\tau = 3.5 \times 10^{-6}\sqrt{n[10^{11}\mathrm{cm}^{-2}]}\mu[\mathrm{cm}^2/\mathrm{Vs}]$, where μ is the carrier mobility. To satisfy the condition for the ballistic regime μ must exceed 10^6 cm$^2/Vs$.

2.6.6 Excitation of chiral spin-waves

The obvious way to excite the chiral spin waves of M_i is by means of a magnetic field H_i, oscillating near the corresponding resonance frequency. One needs to simply add the term $-g\mu_B H_i \sigma_i$ to the right hand side of Eq. 2–132 and proceed similarly keeping H_i only to linear order, which results in

$$\hat{L}_i M_i = \frac{g^2}{4}\mu_B^2 \nu_F \Delta^2 \delta_i H_i, \tag{2–161}$$

where $\hat{L}_i = -\partial_t{}^2 - \Delta^2(1+\delta_i F^{a,0}) + D_i\partial_x^2$. However, the SO interaction allows for a coupling of spins to an in-plane electric field, **E**, which are experimentally easier to implement and manipulate. In order to include the external electric field into the equation of motion, the canonical momentum $\mathbf{p} \to \mathbf{p} - e\mathbf{A}/c$, where $A = ic\mathbf{E}/\omega$, and $\omega = -i\partial_t$. To include the effect of the external field in the equation of motion for chiral spin-waves, it suffices to only add the tensor part of the field-dependent Hamiltonian

$$\hat{\varepsilon}_{\mathrm{E}} = \alpha\left(\hat{\boldsymbol{\sigma}} \times \mathbf{e}_z\right) \cdot \left(\frac{e\mathbf{A}}{c}\right), \tag{2–162}$$

to the right hand side of Eq. 2–132. For the M_x mode only the linear term in **E** can be kept which appears in only the following commutation

$$\left[\delta\hat{\varepsilon}_{\mathrm{SO}}, \alpha\left(\hat{\boldsymbol{\sigma}} \times \mathbf{e}_z\right) \cdot \left(\frac{e\mathbf{A}}{c}\right)\right] = \frac{\alpha e \Delta}{\omega}\left(\hat{\mathbf{p}} \times \mathbf{E}\right) \cdot \mathbf{e}_z \sigma_z, \tag{2–163}$$

as a part of $[\hat{n}, \hat{\varepsilon}]$ in the Boltzmann equation. Consequently one arrives at the following equation,

$$\hat{L}_x M_x = -\frac{g\mu_B}{4}\nu_F \Delta^2 \frac{\alpha e E_y}{\Omega_{x0}}, \tag{2–164a}$$

$$\hat{L}_x M_y = +\frac{g\mu_B}{4}\nu_F \Delta^2 \frac{\alpha e E_x}{\Omega_{x0}}, \tag{2–164b}$$

where Ω_{0i} are the resonance frequencies at $q = 0$. For the M_z mode, however, the linear order term in **E** is absent and the higher orders can be kept through the following commutation

$$\left[\hat{u}, \alpha\left(\hat{\boldsymbol{\sigma}} \times \mathbf{e}_z\right) \cdot \left(\frac{e\mathbf{A}}{c}\right)\right], \tag{2–165}$$

as $\hat{u}$ is already proportional to the external field. Consequently, one arrive at

$$\hat{L}_z M_z = 4\left(1 - \frac{F_0^a \Delta^2}{\Omega_{z0}^2}\right)\left(\frac{\alpha e}{\Omega_{z0}}|\mathbf{E} \times \hat{z}|\right)^2 M_z. \tag{2–166}$$

Equation (2–166) describes a parametric resonance in M_z excited by the electric field with frequency Ω_{z0}. The initial amplitude of M_z can be provided by a pulse in

B_z. Therefore, while the x-mode couples linearly to the electric field, the z-mode is generated to second order in E.

It is worth noting that all of the results presented above remain the same if the Rashba SO interaction is replaced by the Dresselhaus one. If the Rashba and Dresselhaus interactions are present simultaneously, spin-chiral modes become non-sinusoidal. Hence, it is better to perform the experiment on a symmetric quantum well which has only the Dresselhaus but no Rashba interaction.

CHAPTER 3
CONCLUSIONS

A 2D FL system of electrons in the presence of Rashba SO interaction is studied. The presence of SO breaks the SU(2) symmetry of the system and on the basis of the remaining symmetries a suitable Landau function for this system is constructed in Eq. 2–7. In Sec. 2.5.2, this particular form of the LF was also obtained by a second order perturbation theory analysis. The key result of this dissertation is that while the charge sector of a chiral FL can be fully described by the Landau function projected on the two spin-split FSs, any quantity pertaining to the spin sector must involve the Landau function with momenta in between the two FSs. This feature is most explicitly demonstrated for the case of the static, out-of-plane spin susceptibility. Therefore, there is no conventional FL theory, i.e., a theory operating solely with free quasiparticles, for the spin sector of a chiral FL. This does not mean that we are dealing here with a non-Fermi liquid, because chiral quasiparticles are still well-defined near their respective FSs. However, the spin sector of a chiral FL does not allow for a FL-type description. In other words, we are dealing with a special class of FLs ("non-Landau FLs"), which are FLs without a full-fledged FL theory. Not only chiral but also any FL with broken $SU(2)$ symmetry, e.g., a 3D FL in the presence of finite magnetic field, belongs to this class. [57]

The properties of this system in the charge sector are similar to any two band system. One interesting effect of the interactions in a system with pure Rashba or Dresselhaus SO interaction, is to break the degeneracy of velocities of fermions with different chiralities. As the phenomenological formula for the effective masses Eq. 2–19 suggests, the effective masses and velocities of Rahsba Fermions are split due to the effect of interactions. We obtain the leading splitting for the case of Coloumb interactions in Eq. 2–29, as well as in Appendix A. In addition, in a model with short-range interactions we evaluate the leading non-analytic correction to the Landau

parameters and show that they are of the order $U^2\alpha^2$, where U is the amplitude of the short-range interaction.

In the spin sector, using linear response of a SU2S FL to an SO perturbation, a new type of collective mode is predicted: spin-chiral waves in a FL with Rashba SO coupling [77]. Already in the free case, precession of the electron spin around the Rashba SO field leads to absorption of electromagnetic radiation at the frequency given by the Rashba splitting of chiral subbands. This is a resonance in zero magnetic field [15, 78]. The ee interaction adds several qualitatively different elements to this story. First, it breaks spin-rotational invariance; hence the frequencies of normal modes with in- and out-of-plane components of the magnetization are different. This is an interaction-induced splitting of the spin-chiral resonance [51]. Second, the resonance frequencies acquire dispersion: these are spin-chiral waves that exist only in the presence of both ee and SO interactions [77]. The spectrum and equations of motion for chiral spin waves are obtained. In addition, we propose an experiment to observe these modes via lateral modulations of the SO coupling.

All the results of presented here also hold for a system with Dresselhaus SO interaction. However, if both Rashba and Dresselhaus are present, the SO(2) symmetry of the FSs is broken as well, manifesting itself in complications such as the presence of many more Landau parameters in phenomenological results, non-sinusoidal spin-waves, etc.

Our results can also be simply generalized to the case of hole systems. The Hamiltonian for free Rashba heavy-holes can be cast into the following from

$$\hat{H}_{\text{holes}} = \frac{p^2}{2m}1 + \frac{i\gamma}{2}\left(\hat{\boldsymbol{\sigma}}_+\mathbf{p}_-^3 - \hat{\boldsymbol{\sigma}}_-\mathbf{p}_+^3\right), \tag{3–1}$$

where $p_\pm = p_x \pm i p_y$, and similarly for $\hat{\boldsymbol{\sigma}}_\pm$. Equation 2–7 does not hold for a 2D FL system of heavy-holes. In fact the following form

$$
\begin{aligned}
\hat{f}_{\text{holes}} &= f^{\text{s}} 11' + f^{\text{a}\parallel}(\hat{\sigma}_x \hat{\sigma}'_x + \hat{\sigma}_y \hat{\sigma}'_y) + f^{\text{a}\perp} \hat{\sigma}_z \hat{\sigma}'_z \\
&+ \frac{1}{2} g^{\text{ph}} \Big[1 \left(\hat{\boldsymbol{\sigma}}'_+ (\mathbf{p}'_-{}^3 - \mathbf{p}_-{}^3) - \hat{\boldsymbol{\sigma}}'_- (\mathbf{p}'_+{}^3 - \mathbf{p}_+{}^3) \right) \\
&- 1' \left(\hat{\boldsymbol{\sigma}}_+ (\mathbf{p}'_-{}^3 - \mathbf{p}_-{}^3) - \hat{\boldsymbol{\sigma}}_- (\mathbf{p}'_+{}^3 - \mathbf{p}_+{}^3) \right) \Big] \\
&+ \frac{1}{2} g^{\text{pp}} \Big[1 \left(\hat{\boldsymbol{\sigma}}'_+ (\mathbf{p}'_-{}^3 + \mathbf{p}_-{}^3) - \hat{\boldsymbol{\sigma}}'_- (\mathbf{p}'_+{}^3 + \mathbf{p}_+{}^3) \right) \\
&+ 1' \left(\hat{\boldsymbol{\sigma}}_+ (\mathbf{p}'_-{}^3 + \mathbf{p}_-{}^3) - \hat{\boldsymbol{\sigma}}_- (\mathbf{p}'_+{}^3 + \mathbf{p}_+{}^3) \right) \Big] \\
&+ h^{(1)} \left(\hat{\boldsymbol{\sigma}}_+ \mathbf{p}^3_- - \hat{\boldsymbol{\sigma}}_- \mathbf{p}^3_+ \right) \left(\hat{\boldsymbol{\sigma}}'_+ \mathbf{p}'^3_- - \hat{\boldsymbol{\sigma}}'_- \mathbf{p}'^3_+ \right) \\
&+ h^{(2)} \left(\hat{\boldsymbol{\sigma}}'_+ \mathbf{p}^3_- - \hat{\boldsymbol{\sigma}}'_- \mathbf{p}^3_+ \right) \left(\hat{\boldsymbol{\sigma}}_+ \mathbf{p}'^3_- - \hat{\boldsymbol{\sigma}}_- \mathbf{p}'^3_+ \right) \\
&+ \frac{1}{2} h \Big[\left(\hat{\boldsymbol{\sigma}}_+ \mathbf{p}^3_- - \hat{\boldsymbol{\sigma}}_- \mathbf{p}^3_+ \right) \left(\hat{\boldsymbol{\sigma}}'_+ \mathbf{p}^3_- - \hat{\boldsymbol{\sigma}}'_- \mathbf{p}^3_+ \right) \\
&+ \left(\hat{\boldsymbol{\sigma}}_+ \mathbf{p}'^3_- - \hat{\boldsymbol{\sigma}}_- \mathbf{p}'^3_+ \right) \left(\hat{\boldsymbol{\sigma}}'_+ \mathbf{p}'^3_- - \hat{\boldsymbol{\sigma}}'_- \mathbf{p}'^3_+ \right) \Big] ,
\end{aligned}
\tag{3–2}
$$

can serve as the appropriate LF for a FL system of holes. The major difference from the case of electron liquids is the coupling to magnetic fields such as in linear spin susceptibilities, where the linear Zeeman term for holes is of the form $\left(\hat{\boldsymbol{\sigma}}_+ \mathbf{p}^2_- \mathbf{B}_- + \hat{\boldsymbol{\sigma}}_+ \mathbf{p}^2_+ \mathbf{B}_+ \right)$. However, the structure of the theory remains qualitatively the same.

APPENDIX A
MASS RENORMALIZATION VIA THE SELF-ENERGY

In Sec. 2.3, we analyzed mass renormalization of Rashba fermions to logarithmic accuracy in the parameter $p_{\rm TF}/p_{\pm}$ which controls the Random Phase Approximation for the Coulomb interaction, and showed that degeneracy of the subbands' Fermi velocities is lifted by the *ee* interaction, at least when SO coupling is so strong that $p_+ \ll p_-$ (but still both subbands are occupied). In this Appendix, we derive the result for mass renormalization still to leading order in the *ee* interaction but for an arbitrarily strong SO interaction. For the time being, we assume that the interaction is described by a non-retarded and spin-independent, but otherwise arbitrary potential U_q. To obtain the result beyond the logarithmic accuracy, one needs to keep the matrix elements in Eq. (2–26) and relax the assumption of small momentum transfers. The only small parameter will now be the quasiparticle energy, $\varepsilon_s^{{\rm p},f} = \epsilon_s^{{\rm p},f} - \mu$. From this point on, we suppress the superscript f since all dispersions considered below are for non-interacting electrons, i.e, $\varepsilon_s^{{\rm p},f} \to \varepsilon_s^{\rm p}$. The leading order renormalization is logarithmic, therefore the potential can be to taken as static since the dynamic part does not produce logarithmic integrals. Subtracting from Eq. (2–26) the self-energy, evaluated on the FS, we obtain for the remainder

$$\begin{aligned}\Delta\Sigma_s &= -\sum_{s'}\int\frac{d^2q}{(2\pi)^2}\int\frac{d\Omega}{2\pi}\frac{U_q}{2}[(1+ss'\cos\theta_{\rm p+q})g_{s'}(\mathbf{p}+\mathbf{q},\omega+\Omega)\\ &- (1+ss'\cos\theta_{{\rm p}_s+{\rm q}})g_{s'}(\mathbf{p}_s+\mathbf{q},\Omega)]. \qquad \text{(A–1)}\end{aligned}$$

Because of the in-plane rotational symmetry, it is convenient to measure all angles from the direction of $\mathbf{p}$ which, by definition, coincides with the direction of $\mathbf{p}_s$. Using an exact results

$$\epsilon_{s'}^{\rm p+q} = \frac{|\mathbf{p}+\mathbf{q}|^2}{2m} + s'\alpha|\mathbf{p}+\mathbf{q}| = \frac{p^2}{2m} + \frac{pq}{m}\cos\theta + \frac{q^2}{2m} + s'\alpha|\mathbf{p}+\mathbf{q}|, \qquad \text{(A–2)}$$

where θ is the angle between $\mathbf{p}$ and $\mathbf{q}$, and expanding $|\mathbf{p}+\mathbf{q}|$ to linear order in $\varepsilon_s^{\mathrm{p}}$ as

$$|\mathbf{p}+\mathbf{q}| = \left|\left(p_s + \frac{\varepsilon_s^{\mathrm{p}}}{v_0}\right)\mathbf{e}_{\mathrm{p}} + \mathbf{q}\right| = |\mathbf{p}_s+\mathbf{q}| + \frac{\varepsilon_s^{\mathrm{p}}}{v}\cos(\theta_{\mathrm{p}_s+\mathrm{q}}), \tag{A–3}$$

where v_0 is given by Eq. (2–5), we get

$$\varepsilon_{s'}^{\mathrm{p+q}} = \varepsilon_s^{\mathrm{p}}\left(1 + \frac{q}{mv_0}\cos\theta - \frac{s\alpha}{v_0} + s'\alpha\cos\theta_{\mathrm{p}_s+\mathrm{q}}\right) + \frac{p_s q}{m}\cos\theta + \frac{q^2}{2m} + s'\alpha|\mathbf{p}_s+\mathbf{q}| - s\alpha p_s, \tag{A–4}$$

and

$$\cos\theta_{\mathrm{p+q}} = \cos\theta_{\mathrm{p}_s+\mathrm{q}} + \frac{m\varepsilon_s^{\mathrm{p}}}{p_s|\mathbf{p}_s+\mathbf{q}|}\sin^2\theta_{\mathrm{p}_s+\mathrm{q}}. \tag{A–5}$$

One can decompose $\Delta\Sigma_s$ into two parts as $\Delta\Sigma = \Delta\Sigma_s^1 + \Delta\Sigma_s^2$ with

$$\Delta\Sigma_s^1 = -\sum_{s'}\int\frac{d^2q}{(2\pi)^2}\int\frac{d\Omega}{2\pi}\frac{U_q}{2}(1+ss'\cos\theta_{\mathrm{p}_s+\mathrm{q}})\left[g_{s'}(\mathbf{p}+\mathbf{q},\omega+\Omega) - g_{s'}(\mathbf{p}_s+\mathbf{q},\Omega)\right], \tag{A–6}$$

$$\Delta\Sigma_s^2 = -\frac{m\varepsilon_s^{\mathrm{p}}}{p_s}\sum_{s'}ss'\int\frac{d^2q}{(2\pi)^2}\int\frac{d\Omega}{2\pi}\frac{U_q}{2}\frac{\sin^2\theta_{\mathrm{p}_s+\mathrm{q}}}{|\mathbf{p}_s+\mathbf{q}|}g_{s'}(\mathbf{p}_s+\mathbf{q},\omega+\Omega), \tag{A–7}$$

where the first part comes from only the immediate vicinity of the FS, while the second one comes from the entire band. Integrating over Ω in Eq. (A–6), we obtain the difference of two Fermi functions of the s' subband. Expanding the first Fermi function with the help of Eq. (A–4), we get

$$\begin{aligned}\Sigma_1^s = {} & \varepsilon_s^{\mathrm{p}}\sum_{s'}\int\frac{d^2q}{(2\pi)^2}\frac{U(q)}{2}(1+ss'\cos\theta_{\mathrm{p}_s+\mathrm{q}})\left(1+\frac{q}{mv_0}\cos\theta - \frac{s\alpha}{v} + s'\alpha\cos\theta_{\mathrm{p}_s+\mathrm{q}}\right)\\ & \times\delta\left(\frac{p_s q}{m}\cos\theta + \frac{q^2}{2m} + s'\alpha|\mathbf{p}_s+\mathbf{q}| - s\alpha p_s\right).\end{aligned} \tag{A–8}$$

The δ-function in (A–8) enforces the mass-shell condition $|\mathbf{p}_s + \mathbf{q}| = p_{s'}$, from which we find

$$\cos\theta = -\frac{q}{2p_s} + (s - s')\frac{m\alpha}{q}\left(1 + s\frac{m\alpha}{p_s}\right), \tag{A–9}$$

$$\cos\theta_{\mathrm{p_s+q}} = 1 - \frac{q^2}{2p_s p_{s'}} + |s - s'|\frac{(m\alpha)^2}{p_s p_{s'}}, \tag{A–10}$$

where we used that $\cos\theta_{\mathrm{p+q}} = (p + q\cos\theta)/|\mathbf{p} + \mathbf{q}|$ for arbitrary $\mathbf{p}$ and $\mathbf{q}$. Integrating over θ, we arrive at

$$\Delta\Sigma_s^1 = \frac{\varepsilon_s^{\mathrm{p}}}{2\pi^2 v_0}\left[\int_0^{2p_s} dqU_q\sqrt{1 - \frac{q^2}{4p_s^2}}\left(1 - \frac{q^2}{2p_s m v_0} - \frac{s\alpha}{v}\frac{q^2}{2p_s^2}\right)\right.$$
$$\left. + \int_{2m\alpha}^{2mv_0} dqU_q \frac{\left[\frac{q^2}{4p_F^2} - \left(\frac{m\alpha}{p_F}\right)^2\right]\left(1 - \frac{q^2}{2p_s m v_0} + s\frac{\alpha}{v_0}\frac{q^2}{2p_F^2} + \frac{2m\alpha^2}{v_0 p_s} - s\frac{2m^2\alpha^3}{v_0 p_F^2}\right)}{\left[1 - \frac{s\alpha}{v_0}\left(1 + \frac{p_s}{p_{-s}}\right)\right]\sqrt{1 - \left[\frac{q}{2p_s} - s\frac{2m\alpha}{q}\left(1 + \frac{sm\alpha}{p_s}\right)\right]^2}}\right], \tag{A–11}$$

where the first (second) term is a result of intra-band (inter-band) transitions. Equation (A–11) is exact in α. If U_q is replaced by the screened Coulomb potential $U(q) = 2\pi e^2/(q + p_{\mathrm{TF}})$, where $p_{\mathrm{TF}} = \sqrt{2}r_s p_F$ is the Thomas-Fermi screening momentum, the first term in Eq. (A–11) produces, in the small- r_s limit, the $r_s \ln r_s$ result of Eq. (2–29):

$$\Delta\Sigma_s^1 = \frac{\varepsilon_s^{\mathrm{p}}}{2\pi^2 v_0}\int_0^{2p_s} dqU_q^{\mathrm{TF}} = \frac{\epsilon_s^{\mathrm{p}}}{\pi v_F}e^2\left[\ln\frac{2p_s}{p_{\mathrm{TF}}} + \mathcal{O}\left(\frac{p_{\mathrm{TF}}}{p_s}\right)\right]. \tag{A–12}$$

Note that, in contrast to Eq. (2–29), where p_s occurs as an upper cutoff a logarithmically divergent integral, the upper limit of the integral in Eq. (A–12) is defined uniquely.

Equation (A–11) also allows one to study the dependence of the self-energy on α at small α, without assuming that this dependence is analytic. To first order in α, the integrand can be expanded as

$$\Delta\Sigma_s^1 = -\frac{\varepsilon_s^{\mathrm{p}}}{\pi^2 v_F}\frac{s\alpha}{v_F}\int_0^{2p_F} dqU_q\frac{q^2}{2p_F^2}\sqrt{1 - \frac{q^2}{4p_F^2}}. \tag{A–13}$$

where p_F and v_F are the Fermi momentum and Fermi velocity at $\alpha = 0$, correspondingly. On the other hand, integration over Ω in Eq. (A–7) gives

$$\begin{aligned}\Delta\Sigma_s^2 &= -\frac{m\varepsilon_s^{\mathrm{p}}}{p_s}\sum_{s'} ss' \sum_q \frac{U_q}{2}\frac{\sin^2\theta_{\mathrm{p}_s+\mathrm{q}}}{|\mathbf{p}_s+\mathbf{q}|}\Theta\left(-\varepsilon_{s'}^{\mathrm{p}_s+\mathrm{q}}\right)\\ &= -\frac{m\varepsilon_s^{\mathrm{p}}}{p_s}\sum_{s'} ss' \sum_q \frac{U_q}{2}\frac{\sin^2\theta_{\mathrm{p}_s+\mathrm{q}}}{|\mathbf{p}_s+\mathbf{q}|}\Theta\left(-\frac{p_s q}{m}\cos\theta - \frac{q^2}{2m} - s'\alpha|\mathbf{p}_s+\mathbf{q}| + s\alpha p_s\right).\end{aligned} \tag{A–14}$$

A term of order α can be obtained by expanding the Θ-function with respect to α

$$\begin{aligned}\Delta\Sigma_s^2 &= \frac{m^2\varepsilon_s^{\mathrm{p}}}{p_s^2}\alpha s\sum_{s'}\sum_q \frac{U_q}{2}\frac{q^2\sin^2\theta}{|\mathbf{p}_s+\mathbf{q}|^2}\delta\left(\frac{p_F q}{m}\cos\theta + \frac{q^2}{2m}\right)\\ &= \frac{\varepsilon_s^{\mathrm{p}}}{\pi^2 v_F}\frac{s\alpha}{v_F}\int_0^{2p_F} dq U_q \frac{q^2}{2p_F^2}\sqrt{1-\frac{q^2}{4p_F^2}}.\end{aligned} \tag{A–15}$$

Comparing Eqs. (A–13) and (A–15), we see that $\Delta\Sigma_s = \Delta\Sigma_s^1 + \Delta\Sigma_s^2 = 0$ to order α, which is an agreement with the previous results.[48, 55]

In the Coulomb case, Refs. [48, 66] also find an $r_s\alpha^2\ln\alpha$ correction to the mass, which is the same for two Rashba subbands. This correction is produced by the second term in Eq. (A–11). To see this, one can keep only the logarithmic integrals in (A–11). Differentiating (A–11) twice with respect to α, we obtain

$$\frac{\partial^2\Delta\Sigma_s}{\partial\alpha^2} = -\frac{e^2\varepsilon_s^{\mathrm{p}}}{\pi v_F^3}\int_{2m|\alpha|}^{2mv_F}\frac{dq}{q} = -\frac{e^2\varepsilon_s^{\mathrm{p}}}{\pi v_F^3}\ln\frac{v_F}{\alpha}. \tag{A–16}$$

Integrating Eq. (A–16) over α, we reproduce the result of Ref. [48, 66]. In addition, Ref. [66] obtains velocity splitting as $sr_s\alpha^3\ln\alpha$. Such a term is produced by both Σ_s^1 and Σ_s^2. Indeed, keeping again only logarithmic integrals and differentiating (A–11) three times with respect to α gives

$$\frac{\partial^3\Delta\Sigma_s^1}{\partial\alpha^3} = -s\frac{3e^2\varepsilon_s^{\mathrm{p}}}{\pi v_F^4}\int_{2m\alpha}^{2mv_F}\frac{dq}{q} = -s\frac{3e^2\varepsilon_s^{\mathrm{p}}}{\pi v_F^4}\ln\frac{v_F}{\alpha}. \tag{A–17}$$

There is also another contribution of the same order coming from $\Delta\Sigma_s^2$. It follows from Eq. (A–14) that

$$\Delta\Sigma_s^2 = -\frac{m\varepsilon_s^{\rm p}}{8\pi^2 p_s}\left[\int_0^{2p_s} qdqU(q)\int_{-1}^{-\frac{q}{2p_s}} d(\cos\theta)\frac{q^2\sin\theta}{(p_s^2+q^2+2qp_s\cos\theta)^{3/2}}\right.$$
$$\left.-\int_{4m|\alpha|}^{2p_{-s}} qdqU(q)\int_{-1}^{-\frac{q}{2p_s}+\frac{4ms\alpha}{q}} d(\cos\theta)\frac{q^2\sin\theta}{(p_s^2+q^2+2qp_s\cos\theta)^{3/2}}\right]. \qquad \text{(A–18)}$$

Keeping again only logarithmic integrals, we find

$$\Delta\Sigma_s^2 = \frac{m\varepsilon_s^{\rm p}}{16\pi^2 p_s}\int_{4m|\alpha|}^{2p_{-s}}\frac{q^3}{p_s^3}dqU(q)\sqrt{1-\left(\frac{q}{2p_s}-\frac{4ms\alpha}{q}\right)^2}\left(\frac{4ms\alpha}{q}-\frac{q}{2p_s}\right). \qquad \text{(A–19)}$$

Differentiating (A–19) three times with respect to α gives

$$\frac{\partial^3\Delta\Sigma_s^2}{\partial\alpha^3} = -s\frac{4e^2\varepsilon_s^{\rm p}}{\pi v_F^4}\int_{4m\alpha}^{2p_F}\frac{dq}{q} = -s\frac{4e^2\varepsilon_s^{\rm p}}{\pi v_F^4}\ln\frac{v_F}{\alpha}. \qquad \text{(A–20)}$$

Combining Eqs. (A–17) and (A–20), we obtain

$$\frac{\partial^3\Delta\Sigma_s}{\partial\alpha^3} = -s\frac{7e^2\varepsilon_s^{\rm p}}{\pi v_F^4}\ln\frac{v_F}{\alpha}, \qquad \text{(A–21)}$$

which, after integration over α, reproduces the main logarithmic term in the result of Ref. [66]. [We believe that a constant inside the logarithm, also obtained in Ref. [66], exceeds the overall accuracy of the calculation.] Note that the velocity splitting from both contributions in Eqs. (A–17) and (A–20) is a result of inter-band transitions only.

APPENDIX B
OUT-OF-PLANE SPIN SUSCEPTIBILITY OF NON-INTERACTING RASHBA ELECTRONS: A THERMODYNAMIC CALCULATION

In this Appendix, we show that the out-of-plane spin susceptibility, χ_{zz}, of a non-interacting electron gas with Rashba SO coupling is determined by the states in between from the two spin-split FSs. Since we are dealing only with free states here, the index f will be suppressed while quantities in the absence of the magnetic field will be denoted by the superscript 0.

In the presence of a weak magnetic field H in the z-direction, the electron spectrum changes to

$$\epsilon_s(p) = \epsilon_s^0(p) + s\frac{\Delta^2}{2\alpha p}, \tag{B–1}$$

where $\epsilon_s^0(p)$ coincides with Eq. (2–3) and $\Delta = g\mu_B H/2$. The ground-state energy is given by

$$E = \sum_s \int \frac{dpp}{2\pi}\epsilon_s(p)\Theta(p_s - p), \tag{B–2}$$

where the Fermi momenta of the subbands are found from

$$\epsilon_s(p_s) = \epsilon_F. \tag{B–3}$$

It is easy to check that, at fixed number density, ϵ_F is not affected by the magnetic field. One can thus replace ϵ_F by $\epsilon_s^0(p_s^0)$ in Eq. (B–3), which gives for the corrections to the Fermi momenta

$$p_s = p_s^0 + \delta p_s = p_s^0 - s\frac{\Delta^2}{2\alpha v_0 p_s^0}, \tag{B–4}$$

where is the Fermi velocity in each of the subbands, given by Eq. (2–5). It is convenient to subtract the field-independent part from E, and split the remainder into two parts as

$$E - E^0 = E_{\text{on}} + E_{\text{off}}, \tag{B–5}$$

where E_{on} is the contribution from the states near the FSs

$$E_{\text{on}} = \frac{1}{2\pi} \sum_s \int_{p_s^0}^{p_s^0+\delta p_s} dp p \epsilon_s^0(p) \tag{B–6}$$

and E_{off} is the contribution from the states away from the FSs

$$E_{\text{off}} = \frac{1}{2\pi} \sum_s \int_0^{p_s^0} dp p \left(s \frac{\Delta^2}{2\alpha p} \right). \tag{B–7}$$

The FS contribution vanishes to order Δ^2

$$E_{\text{on}} = \frac{1}{2\pi} \sum_s p_s \delta p_s \epsilon_s^0(p_s) = 0. \tag{B–8}$$

On the other hand, the off-FS contribution becomes

$$E_{\text{off}} = \frac{1}{2\pi} \int_{p_+^0}^{p_-^0} dp p \left(-\frac{\Delta^2}{2\alpha p} \right) = -\frac{m\Delta^2}{2\pi}, \tag{B–9}$$

which gives a correct result $\chi_{zz}^0 = g^2 \mu_B^2 m / 4\pi$ (Ref. [44]). Therefore, the out-of-plane spin susceptibility of non-interacting Rashba fermions comes entirely from the states in between the two FSs.

APPENDIX C
KOHN ANOMALY IN A FREE ELECTRON GAS WITH RASHBA SPIN-ORBIT COUPLING

In this Appendix, we derive Eqs. (2–92-2–94) for the Kohn anomalies in the polarization bubble of non-interacting Rashba fermions (Fig. C-1):

$$\Pi(Q) = \sum_{s,s'} \Pi_{ss'}(Q), \tag{C–1}$$

where

$$\begin{aligned}\Pi_{ss'}(Q) &= \frac{1}{2}\int_P [1 + ss'\cos(\theta_{\mathrm{p}} - \theta_{\mathrm{p+q}})] g_s(P) g_{s'}(P+Q) = \\ & \frac{1}{2}\int \frac{d^2p}{(2\pi)^2}[1 + ss'\cos(\theta_{\mathrm{p}} - \theta_{\mathrm{p+q}})]\Theta(-\epsilon_{s,\mathrm{p}}) \\ & \times \left[\frac{1}{i\Omega - \epsilon_{s',\mathrm{p+q}} + \epsilon_{s,k}} - \frac{1}{i\Omega + \epsilon_{s,\mathrm{p-q}} - \epsilon_{s',\mathrm{p}}}\right].\end{aligned} \tag{C–2}$$

(As in Appendices A and B, we suppress the superscript f denoting the properties of an interaction-free system.) For q near $2p_F$, we expand the difference of the quasiparticle energies as

$$\epsilon_s^{\mathrm{p+q}} - \epsilon_s^{\mathrm{p}} = -2\epsilon_s^{\mathrm{p}} + v_F q' + v_F p_F \theta^2, \tag{C–3}$$

where $q' = q - 2p_s$ and the angle $\theta = \angle(-\mathbf{p}, \mathbf{q})$ is taken to be small. To leading order in SO, the prefactors in the q' and θ^2 terms can be and were replaced by their values in the absence of SO. (For the second term in Eq. (C–2), the angle $\theta = \angle(\mathbf{p}, \mathbf{q})$ is taken to be

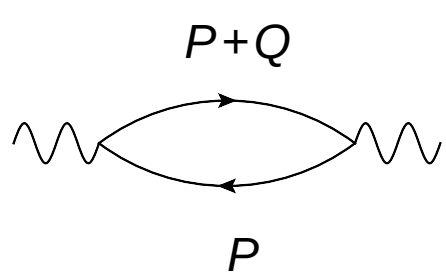

Figure C-1. Polarization bubble, given by Eq. C–1, with $(2+1)$ momentum transfer Q.

small.) Using Eq. (C–3), replacing $[1+\cos(\theta_{\mathrm{p}}-\theta_{\mathrm{p+q}})]$ by $2\theta^2$, and integrating over ϵ_{p} in Eq. (C–2), we obtain for $\Omega=0$ and $s=s'$

$$\Pi_{ss}(q)=\frac{\nu}{2\pi}\int_0^{\theta_c}\theta^2\ln\left(\theta^2+\frac{q'}{p_F}\right)d\theta, \tag{C–4}$$

where θ_c is a cutoff whose particular choice does not affect the singular part of the q-dependence. To extract the singular part, we differentiate $\Pi_{ss}(q)$ twice with respect to q, which makes the integral to converge in the limit of $\theta_c\to\infty$,and obtain

$$\Pi_{ss}(q)=\Pi_{ss}(q=2p_s)-\frac{\nu}{6}\Theta(q-2p_s)\left(\frac{q-2p_s}{p_F}\right)^{3/2}. \tag{C–5}$$

The $(q')^{3/2}$ anomaly in $\Pi_{ss}(q)$ is weaker than the square-root anomaly in the absence of SO. This is a consequence of the fact that backscattering within the same Rashba subband is forbidden, which is manifested by a small factor of θ^2 in the integrand of Eq. (C–4). A similar procedure is applied to $\Pi_{s,-s}$, which results from backscattering between different subbands and hence does not have a small factor of θ^2. With q' replaced now by $q-2p_F$, we obtain the usual square-root Kohn anomaly

$$\Pi_{s,-s}(q)=\Pi_{s,-s}(q=2p_F)+\frac{\nu}{2}\Theta(q-2p_F)\left(\frac{q-2p_F}{p_F}\right)^{1/2}. \tag{C–6}$$

Note that when the bubble is inserted into the interaction vertex, as in Fig. 2-2, the maximum momentum entering the bubble is $2p_-$ (when both the incoming electrons are the $-$ subband). Therefore, the singularity in Π_{--}, present only for $q>2p_-$, is outside the range of allowed momenta. With this in mind, the non-analytic part of Π, relevant for the calculation of the Landau function in Sec. 2.5.3, can be written as in Eqs. (2–92-2–94) of the main text.

APPENDIX D
NON-ANALYTIC CONTRIBUTIONS TO THE LANDAU FUNCTION

In this Appendix, we calculate $\mathcal{O}(s\alpha^2 U^2)$ terms in the Landau function produced by vertices b and d in Fig. 2-2. A contribution from diagram c is calculated in the main text of Sec. 2.5.3.

Diagram d reads

$$\Gamma^d_{ss'} = -\frac{U^2}{4}\sum_{r,t,L}[1+sr\cos(\theta_{\mathrm{p}}-\theta_{\mathrm{l}})][1+s't\cos(\theta_{\mathrm{p}'}-\theta_{\mathrm{l}'})]g_r(L)g_t(L'), \qquad \text{(D–1)}$$

where $L' = (\mathbf{l}+\mathbf{p}'-\mathbf{p},\omega_l)$. A non-analytic part of $\Gamma^d_{ss'}$ comes from backscattering processes with $\mathbf{p}\approx-\mathbf{p}'$. Therefore, internal momenta must be chosen such that $\mathbf{l}\approx\mathbf{p}$ and $\mathbf{l}'\approx\mathbf{p}'\approx-\mathbf{p}$, and both cosine terms in Eq. (D–1) are near $+1$. (The magnitudes of the internal momenta may differ from external ones, but taking this effect into account is not necessary to lowest order in α.) The $s=s'=+1$ channel does not contain a singularity, because the maximum momentum transfer in this channel is $2p_+$, while any convolution of two Green's functions does not depend on the difference of their momenta for $q\leq 2p_+$ (cf. Appendix C). We thus focus on the $s=s'=-1$ and $s=-s'$ channels. In the $s=s'=-1$ channel, the angular-dependent factors are maximal for $r=t=-1$; however, the singularity in the convolution of two g_- function is outside the allowed momentum transfer range. The next best choice is $r=-t$, when one of angular-dependent factors is small but the second one is large. The choice $t=r=1$ makes both angular-dependent factors to be small and is discarded. With all angles measured from $\mathbf{p}$ ($\theta_{\mathrm{p}}=0$), we denote $\theta'_{\mathrm{p}}=\pi-\theta$, $\theta_{\mathrm{l}}=\phi$, and $\theta_{\mathrm{l}'}=\pi-\phi'$ and take θ, ϕ, and ϕ' to small. After some elementary geometry, we find that $\phi'=\phi+\theta$, which implies that the angular-dependent factors in front of the g_+g_- and g_-g_+ combinations are the same. Therefore,

$$\Gamma^d_{--} = -\frac{U^2}{2}\int_L \phi^2 g_+(L)g_-(L'), \qquad \text{(D–2)}$$

and similarly

$$\Gamma^d_{+-} = -\frac{U^2}{4}\int_L \phi^2 g_+(L) g_+(L'). \tag{D–3}$$

The integrals in the equations above are the same as for the Π_{++} component of the polarization bubble [cf. Eq. (C–4], except for that the momentum transfer in Γ^d_{--} is measured from $2p_F$ rather than from $2p_+$. With this difference taken into account, we obtain the results for Γ^d_{--} and Γ^d_{+-} in Eqs. (2–102) and (2–103) of the main text.

The two "wine-glass" diagrams b) are equal to each other, and their sum is given by

$$\Gamma^b_{ss'} = \frac{U^2}{8}\sum_{r,t}\int_L [1 - ss' e^{i(\theta_{\mathrm{p}'}-\theta_{\mathrm{p}})}][1 + rte^{i(\theta_{\mathrm{l}}-\theta_{\mathrm{l}'})}][1 + sre^{i(\theta_{\mathrm{p}}-\theta_{\mathrm{l}})}][1 + s'te^{i(\theta_{\mathrm{l}'}-\theta_{\mathrm{p}'})}] g_r(L) g_t(L'). \tag{D–4}$$

Again, we have $\mathbf{l} \approx \mathbf{p}$ and $\mathbf{l}' \approx \mathbf{p}' \approx -\mathbf{p}$, so that $r = -t$ for the $s = s' = -1$ channel. Using the definitions of small angles above, we obtain

$$\Gamma^b_{--} = \frac{U^2}{2}\left(1 - e^{-i\theta}\right)\int_L \left(1 + e^{i(2\phi+\theta)}\right)\left(1 - e^{-2i\phi}\right)[g_+(L) g_-(L') + g_-(L) g_+(L')] \tag{D–5}$$

Expanding the internal angular-dependent factors to first order in ϕ and θ, we obtain an integral that vanishes by parity. The ϕ^2 term is finite but still comes with a small factor of θ from the external matrix element. Therefore, Γ^b_{--} does not contribute to order α^2.

For the $s = -s'$ channel, only the combination $r = t = +1$ contributes. Following the same steps as for Γ^b_{--}, we arrive at

$$\Gamma^b_{+-} = \frac{U^2}{8}\left(1 + e^{-i\theta}\right)\int_L \left(1 - e^{i(2\phi+\theta)}\right)\left(1 - e^{-2i\phi}\right) g_+(L) g_+(L'). \tag{D–6}$$

Expanding the angular factors to lowest order, we obtain

$$\Gamma^b_{+-} = U^2 \int_L \phi^2 g_+(L) g_+(L'), \tag{D–7}$$

which coincides with the integral for Π_{ss} in Eq. (C–4). Combining everything together, we obtain the vertices in Eqs. (2–100) and (2–101) of the main text.

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BIOGRAPHICAL SKETCH

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