ADAPTIVE FORCE AND MOTION CONTROL OF ROBOT MANIPULATORS IN CONSTRAINED MOTION WITH DISTURBANCES

By

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A THESIS PRESENTED TO THE GRADUATE SCHOOL OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

UNIVERSITY OF FLORIDA

2013
To my wife, Hsiao-Chia Hu
ACKNOWLEDGMENTS

I would like to thank my committee chair, Dr. Carl D. Crane III, for his patient
guidance and for giving me the great chance to study at the Center for Intelligent
Machines and Robotics (CIMAR). I also would like to thank my committee members, Dr.
Warren Dixon and Dr. Gloria J. Wiens, for their valuable advice to this study. In addition,
I would like to thank my companions at CIMAR. From them I learned lot knowledge
about robotics. Thanks also go out to my family and wife for their support and love.
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The goal of this study is to design a controller to reject the disturbances caused by sensor noise and unmodeled effects during hybrid adaptive force and motion control of robot manipulators in constrained motion. A continuous robust integral of the sign of the error (RISE) feedback term is incorporated with adaptive force and motion control to yield asymptotic tracking results in the presence of disturbances and unknown system parameters and dry contact surface friction coefficient. The main reason to use the RISE method is that it can enhance disturbance rejection capabilities.

It is assumed that the system parameters of the robot manipulator and the dry friction coefficient of contact surface are unknown. These unknown parameters can be updated by the adaptive update law. The suggested controller can guarantee semi-global asymptotic motion and force tracking results which are supported through Lyapunov-based stability analysis under the condition that the position and velocity of end-effector and the normal contact force between end-effector and contact surface are measurable.

The contact surface of the environment is modeled by the set of m rigid and mutually independent hypersurfaces. The dynamic model of constrained robot
manipulators is developed through an Euler-Lagrange formulation. Two degree of freedom (DOF) robot manipulator simulation results are given to illustrate the efficacy of the suggested controller.
CHAPTER 1  
INTRODUCTION

Today, more and more robot manipulators are applied in many different areas. Only position control of manipulators is not enough to satisfy various tasks. Thus, in order to broaden the tasks of manipulators, it will be necessary to control force exerted between the manipulator end-effectors and the contact environment. The major tasks requiring force control are deburring, grinding, contour following, and assembly tasks. In order to perform these tasks successfully, it is important to simultaneously control the motion of the end-effectors and the force of the contact surfaces. A way to achieve simultaneously motion and force control is hybrid position/force control which controls the position of end-effectors in certain directions and controls the force in other directions which is orthogonal to the direction of position [1-3]. During the contact with a rigid surface, the contact object may impose some constraints on robot manipulators. There are some directions that the end-effectors cannot move to. This situation is denoted as constrained motion [4-6]. It must be noted, however, that the orthogonality conditions used in some of these hybrid control algorithms have mixed units. This causes the algorithm to not be invariant to changes in the choice of units of length [21].

There are many other challenging problems to perform complex tasks well, such as unstructured environments, unmodeled effects, and external and force sensor disturbances. Many researches have focused on how to compensate for these effects. Different methods have been proposed to deal with these conditions, such as sliding mode control [7] and robust control [8]. To address unstructured environments and unknown parameters of robot manipulators, an adaptive motion and force control method [9-10] is adopted in this study.
An adaptive law is updated by position and force tracking errors to yield semi-global asymptotic tracking results. To address external and force sensor disturbances, a recently developed continuous Robust Integral of the Sign of the Error (RISE) technique is used [11-12]. The reason to choose RISE technique is that this method can deal with the system with sufficiently smooth bounded disturbances and guarantee asymptotic stability results [13-14].

In this study, the dynamic equation of a general rigid link robot manipulator having n degrees of freedom is modeled using the Euler-Lagrange formulation. The contact surface of the environment is modeled by the set of m rigid and mutually independent hypersurfaces [4, 5 17]. Adaptive motion and force control with RISE feedback terms is used to account for disturbances in the sensor and unmodeled effects, and the presence of uncertain parameters in the robot manipulator and environment. Due to the friction force appearing in the contact surface, the dynamic model proposed by [4] cannot be used. A new transformed dynamic model proposed by [6] is adopted. This model is particularly suitable for the presence of friction force in the contact surface. The uncertain parameters of the robot manipulator and the dry friction coefficient of the contact surface are updated by an adaptive law. The proposed controller can yield semi-global asymptotic tracking results. According to the papers [15-16], the experimental results propose that the PI type force feedback control yields the best force tracking result. The controller proposed in this study has the same structure suggested in the papers. Simulation results are given to illustrate the suggested controller.
This study is organized as follows. The dynamic model of the robot manipulator is developed using the Euler-Lagrange formulation in Chapter 2. The filtered tracking errors and the suggested adaptive motion and force controller with the RISE feedback term is given in Chapter 3. Simulation results with different gains and external disturbances are presented in Chapter 4 and conclusions and future works are given in Chapter 5.
CHAPTER 2
CONSTRAINED ROBOT MANIPULATOR DYNAMICS

2.1 Euler-Lagrange Dynamic Model

The nonlinear dynamic system to be controlled is a general rigid link manipulator with n degrees of freedom. This system can be described using the Euler-Lagrange formulation:

\[ M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + \tau_d(t) = \tau(t) \quad (2-1) \]

where \( M(q) \in \mathbb{R}^{n \times n} \) is the generalized inertial matrix, \( V_m(q, \dot{q})\dot{q} \in \mathbb{R}^n \) is the generalized Centrifugal and Coriolis force, \( G(q) \in \mathbb{R}^n \) is the generalized gravitational force, \( \tau_d(t) \in \mathbb{R}^n \) is the generalized nonlinear disturbances (e.g., unmodeled effects or force sensor noise), \( \tau(t) \in \mathbb{R}^n \) is the applied torque input, and \( q(t), \dot{q}(t), \ddot{q}(t) \in \mathbb{R}^n \) are the generalized joint position, velocity, and acceleration, respectively.

When the robot manipulator makes contact with its environment, the contact forces occur between the end-effector and contact surface. The dynamic Equation 2-1 can be modified to [4]

\[ M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + J^T(q)F + \tau_d(t) = \tau(t) \quad (2-2) \]

where \( J(q) = \frac{\partial x(q)}{\partial q} \in \mathbb{R}^{n \times n} \) is the Jacobian matrix, \( x \in \mathbb{R}^n \) is the position and orientation of the end-effector in Cartesian space, and \( F \in \mathbb{R}^n \) is the forces/moments exerted by the end-effector on the contact surface.

The following development is based on the assumption that \( q(t), \dot{q}(t), \) and the normal contact force are measurable, that \( J(q) \) is nonsingular in a finite work space \( \Omega_q \), that the end-effector is in contact with the rigid surface at first and the contact forces will keep the end-effector on the contact surface, that the parameters of \( M(q), V_m(q, \dot{q}), G(q) \) are unknown and that \( \tau_d \) is unknown and that the robot manipulator is nonredundant. In
addition, the following assumptions will be made to facilitate the subsequent Lyapunov-based stability analysis.

Assumption 2-1: The inertial matrix $M(q)$ is symmetric, positive definite, and satisfies the following inequality $\forall \xi(t) \in R^n$:

$$k_1 \|\xi\|^2 \leq \xi^T M(q) \xi \leq k_2 \|\xi\|^2$$

(2-3)

where $k_1, k_2 \in R$ are known positive constant and $\|\cdot\|$ denotes the standard Euclidean norm.

Assumption 2-2: The matrix $N(q,\dot{q}) = \dot{M}(q) - 2V_m(q,\dot{q})$ is a skew-symmetric matrix satisfied the following relationship

$$\xi^T \left( \dot{M}(q) - 2V_m(q,\dot{q}) \right) \xi = 0 \ \forall \xi \in R^n$$

(2-4)

Assumption 2-3: The desired trajectory is designed as $q_d(t), \dot{q}_d(t), \ddot{q}_d(t), \dddot{q}_d(t), \dddot{q}_d(t), \dot{\lambda}_d(t), \ddot{\lambda}_d(t), \dddot{\lambda}_d(t)$, which exist, and are bounded.

Assumption 2-4: The nonlinear disturbance is satisfied such that $\tau_d(t), \dot{\tau}_d(t), \ddot{\tau}_d(t)$ are bounded by known constants.

Assumption 2-5: $M(q), V_m(q,\dot{q}), G(q)$ can be linearly parameterized in terms of suitably selected parameters for robot manipulator.

$$M(q) + V_m(q,\dot{q}) + G(q) = Y(q,\dot{q})\beta, \ \ Y \in R^{n \times p}, \ \ \beta \in R^p$$

where $Y$ is regression matrix and $\beta$ represents the parameters of the robot manipulator.

Assumption 2-6: If $q(t), \dot{q}(t) \in L_\infty$, then $V_m(q,\dot{q})$ and $G(q)$ are bounded. In addition, if $q(t), \dot{q}(t) \in L_\infty$, then the first and second partial derivatives of the elements of $M(q), V_m(q,\dot{q})$ and $G(q)$ with respect to $q(t)$ exist and are bounded, and the first and
second partial derivatives of the elements of $V_m(q, \dot{q})$ with respect to $\dot{q}(t)$ exist and are bounded.

### 2.2 Model of Contact Surface

Suppose that the contact surface of the environment can be described by the following set of $m$ rigid and mutually independent hypersurfaces \[4, 5, 17\]

$$\Phi(x) = 0 \quad \Phi(x) = [\phi_1(x), ..., \phi_m(x)]^T \quad m \leq n$$

where $\Phi(x)$ is assumed to be twice differentiable with respect to $x$. At the point of contact on the surface, the contact force $F$ including the normal contact force and friction force can be described by the following equation

$$F = F_n + F_t = D^T(x)\lambda + A_t f_t(\mu, v_{\text{end}}, \lambda) = [D^T(x) + L^T(\mu, x, \dot{x})]\lambda$$

$$D(x) = \frac{\partial \Phi(x)}{\partial x} \quad L(\mu, x, \dot{x}) = A_t f_t(\mu, v_{\text{end}}, \lambda) \quad D, L \in \mathbb{R}^{m \times n}$$

where $\lambda \in \mathbb{R}^m$ is a vector of normal contact force components, $F_n$ is normal contact force in Cartesian space, $F_t$ is friction force in Cartesian space and $D(x)$ is the direction of $F_n$, $A_t$ is the direction of $F_t$. The magnitude of $F_t$ is depended on $\lambda$ and the friction coefficient $\mu$, and the direction of $F_t$ is the opposite direction of the end-effector velocity $v_{\text{end}}$. $L$ is differentiable with respect to $x$ except at the point when $v_{\text{end}}$ changes direction.

When the robot manipulator is in contact with environment, the end-effector of the robot manipulator is constrained to be on that contact surface and only $(n - m)$ degree of freedom can be control independently. Thus, motion control can be described by the following set of $(n - m)$ mutually independent curvilinear coordinates

$$\Psi(x) = [\psi_1(x), ..., \psi_{n-m}(x)]^T$$
where \( \psi_i(x) \) is assumed to be twice differentiable. Because the position control cannot be done in the \( m \) rigid and mutually independent hypersurfaces \( \Phi(x), \psi_i(x) \) is independent of \( \phi_i(x) \). \( \Psi(x) \) and \( \Phi(x) \) can uniquely determine the configuration of the robot manipulator.

### 2.3 Dynamic Model of Constrained Robot Manipulator

According to the above definition, the configuration of the robot manipulator can be defined by \([4]\)

\[
\mathbf{r} = [r_f^T, r_p^T]^T
\]

where

\[
r_f = [\phi_1(x), ..., \phi_m(x)]^T \quad r_p = [\psi_1(x), ..., \psi_{n-m}(x)]^T
\]

Differentiating Equation 2-8 with respect to time, gives

\[
\dot{\mathbf{r}} = J_x \dot{x} = J_q \dot{\mathbf{q}}
\]

where

\[
J_x = \frac{\partial \mathbf{r}(x)}{\partial x}, \quad J_x = [D(x)^T \quad J_{xp}]^T, \quad J_{xp} = \frac{\partial \mathbf{r}(x)}{\partial x} \in R^{(n-m)\times n}
\]

\[
J_q = \frac{\partial \mathbf{r}(x(q))}{\partial q}, \quad J_q = J_x(x(q))J(q), \quad J_q, J_x \in R^{n\times n}
\]

Substituting Equation 2-8 and 2-9 into the joint space dynamic model Equation 2-2 and multiplying both sides by \( J_q^{-T} \), gives the operational space dynamic model \([1]\) with the constraints Equation 2-5 and the contact force Equation 2-6 as

\[
M(\mathbf{r}) \ddot{\mathbf{r}} + V(\mathbf{r}, \dot{\mathbf{r}}) \dot{\mathbf{r}} + G(\mathbf{r}) + B'(\mu, r, \dot{r}) \lambda + T_d(t) = \tau_r
\]

\[
\mathbf{r} = \begin{bmatrix} 0 \\ r_p \end{bmatrix}, \quad B' = \begin{bmatrix} l_m \\ 0 \end{bmatrix} + B(\mu, r, \dot{r})
\]

Equation 2-11 can be expressed as

\[
M_{12}(\mathbf{r}) \dddot{r}_p + V_{12}(\mathbf{r}, \dot{r}) \dddot{r}_p + G_1(\mathbf{r}) + (l_m + B_1) \lambda + T_{d1}(t) = \tau_{r1}
\]
\[
M_{22}(r) \ddot{r}_p + V_{22}(r, \dot{r}) \dot{r}_p + G_2(r) + B_2 \lambda + T_{d2}(t) = T_{r2}
\]  

(2-12)

where

\[
M(r) = J_q^T(q)M(q)J_q^{-1}(q) = \begin{bmatrix} M_{11}(r) & M_{12}(r) \\ M_{21}(r) & M_{22}(r) \end{bmatrix}
\]

\[
V(r, \dot{r}) = J_q^{-T}(q)V(q, \dot{q})J_q^{-1}(q) - J_q^{-T}(q)M(q)J_q^{-1}(q)\dot{q}(q)J_q^{-1}(q)
\]

\[
= \begin{bmatrix} V_{11}(r, \dot{r}) & V_{12}(r, \dot{r}) \\ V_{21}(r, \dot{r}) & V_{22}(r, \dot{r}) \end{bmatrix}
\]

\[
G(r) = J_q^{-T}G(q) = \begin{bmatrix} G_1(r) \\ G_2(r) \end{bmatrix}
\]

\[
B(\mu, r, \dot{r}) = J_x^{-T}L^T = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}
\]

\[
T_d(t) = J_q^{-T}\tau_d(t) = \begin{bmatrix} T_{d1} \\ T_{d2} \end{bmatrix}
\]

\[
T_r = J_q^{-T}(q)\tau = \begin{bmatrix} T_{r1} \\ T_{r2} \end{bmatrix}
\]

In Equation 2-11, the constraint set by the environment can be simply presented by \(r_f = 0\). The position of the robot manipulator is described by \(r_p\). Note that in operational space dynamic model, normal contact force has a simple structure, i.e., \(J_q^{-T}F_n = [l_m \ 0]^T\lambda\). Also, the contact surface friction force only appears in the second equation of Equation 2-12. Due to this condition, motion and force control cannot be dealt with separately. In other words, they are coupled. The strategy developed by [6] is adopted to deal with this condition.

According to [6], \(M_{21}(r)K_f\lambda\) is added and subtracted to the left hand side of the second equation of Equation 2-12, where \(K_f = \text{diag}\{k_{f1}, ..., k_{fm}\}\), and \(G_f\lambda\) is added to the both sides of the first equation of Equation 2-12, where \(G_f = \text{diag}\{g_{f1}, ..., g_{fm}\}\). In this way, Equation 2-12 can be reformed as the following
\[ H(r_p)v + V_p(r_p, \dot{r}_p)\dot{r} + G(r_p) + B_p(\mu, r_p, \dot{r}_p)\lambda + T_d(t) = T_r + \tilde{G}_r\lambda \]  

(2-13)

where

\[ v = \begin{bmatrix} K_f \lambda \\ \dot{r}_p \end{bmatrix}, \quad H(r_p) = \begin{bmatrix} (I_m + G_f)K_f^{-1} & M_{12}(r) \\ M_{21} & M_{22}(r) \end{bmatrix} \]

\[ V_p(r_p, \dot{r}_p) = \begin{bmatrix} 0 & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \]

\[ B_p(\mu, r_p, \dot{r}_p) = B(\mu, r, \dot{r}) + B'(r_p) \]

\[ B'(r_p) = -\begin{bmatrix} 0 \\ M_{21}(r)K_f \end{bmatrix}, \quad \tilde{G}_f = \begin{bmatrix} G_f \\ 0 \end{bmatrix} \]

According to the assumption 2-1, 2-2, and 2-5, the following properties can be obtained for Equation 2-13 in the Appendix.

**Properties 2-1:** The matrix \( M(r) \) is symmetric, positive definite, and satisfies the following inequality \( \forall \xi(t) \in R^n : \)

\[ k_3 \|\xi\|^2 \leq \xi^T M(r) \xi \leq k_4 \|\xi\|^2 \]  

(2-14)

where \( k_3, k_4 \in R \) are known positive constant and \( \|\cdot\| \) denotes the standard Euclidean norm.

**Properties 2-2:** The matrix \( N(r, \dot{r}) = \dot{M}(r) - 2V(r, \dot{r}) \) is a skew-symmetric matrix which satisfies the following relationship

\[ \xi^T \left( \dot{M}(r) - 2V(r, \dot{r}) \right) \xi = 0 \quad \forall \xi \in R^n. \]  

(2-15)

**Properties 2-3:** \( H(r_p), V_p(r_p, \dot{r}_p), G(r_p) \), and \( B'(\mu, r_p, \dot{r}_p) \) can be linear parameterized in terms of a suitably selected set of parameters for robot manipulators.
Properties 2-4: The matrix $H(r_p)$ is a symmetric positive definite matrix on the assumption that the maximum eigenvalue of $K_f$ is small enough.

Properties 2-5: The matrix $N_p(r,\dot{r}) = \dot{H}(r_p) - 2V_p(r, \dot{r})$ is a skew-symmetric matrix which satisfies the following relationship

$$
\xi^T(\dot{H}(r_p) - 2V_p(r, \dot{r}))\xi = 0 \quad \forall \xi \in \mathbb{R}^n.
$$

The controller proposed in this study is based on Equation 2-13 which designs the control torque $T_r(t)$ to let the position and force follow the desired trajectory. The controller is also incorporated with a RISE feedback structure to reject the disturbance term $T_d(t)$ in Equation 2-13.

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1 See Appendix for the proof of matrix $H(r_p)$ is a symmetric positive definite matrix

2 See Appendix for the proof of $N_p(r, \dot{r})$ is skew-symmetric matrix.
3.1 Open-Loop Tracking Error System

The goal of this study is to design an adaptive motion and force controller with a RISE feedback term to track the desired time-varying motion and force trajectory with disturbance, based on the system described by Equation 2-14. Let the position and force tracking error be defined as

\[ e_{p1}(t) = r_d(t) - r(t), \quad e_{p1}(t) \in R^n \]
\[ e_{f1}(t) = \lambda_d(t) - \lambda(t), \quad e_{f1}(t) \in R^n \] (3-1)

where \( r_d(t) \) is the desired robot manipulator trajectory and \( \lambda_d(t) \) is the desired constrained force trajectory. To facilitate the following Lyapunov-based stability analysis, the filtered tracking errors are defined as

\[ e_{p2}(t) = \dot{e}_{p1}(t) + \alpha_{p1} e_{p1}(t), \quad e_{p2}(t) \in R^n \] (3-2)
\[ e_{p3}(t) = \dot{e}_{p2}(t) + \alpha_{p2} e_{p2}(t), e_{p3}(t) \in R^n \] (3-3)
\[ e_{f2} = e_{f1} + K_f e_{f1}, \quad e_{f2} \in R^n \] (3-4)
\[ s = e_{p3} + e_{f2}, s \in R^n \] (3-5)

where \( \alpha_{p1}, \alpha_{p2} \in R \) denote positive constants. The proposed controller is based on the assumption that position, velocity, and normal contact force are measurable and acceleration is not measurable, so the filtered tracking error \( e_{p3}(t) \) and \( e_{f2} \) are not measurable.

Multiplying both sides of Equation 3-5 by \( H(r_p) \) and utilizing Equation 2-13, 3-1, 3-2, 3-3 and 3-4 gives the open-loop tracking error system as

\[ H(r_p)s = H(r_p)v_d + V_p(\hat{r}_p, \dot{r}_p)\dot{r} + G(r_p) + B_p(\mu, r_p, \dot{r}_p)\lambda + T_d(t) \]
By adding and subtracting $H(r_d) v_d + V_p(r_d, \dot{r}_d) \dot{r}_d + G(r_d) + B_p(\mu, r_d, \dot{r}_d) \lambda_d$ to Equation 3-6, gives

$$H(r_p)s = Y_d \beta + S_1 + T_d(t) - T_r(t) - \bar{G}_f \lambda$$

(3-7)

where $Y_d \beta \in \mathbb{R}^n$ is defined as

$$Y_d \beta = H(r_d) v_d + V_p(r_d, \dot{r}_d) \dot{r}_d + G(r_d) + B_p(\mu, r_d, \dot{r}_d) \lambda_d.$$  

(3-8)

In Equation 3-8, $\beta \in \mathbb{R}^p$ represents the unknown parameters needed to be updated on-line and $Y_d \in \mathbb{R}^{nxp}$ is the desired regression matrix containing the desired robot manipulator position, velocity, acceleration, and force, respectively. In Equation 3-7, the auxiliary function $S_1 \in \mathbb{R}^n$ is defined as

$$S_1 = H(r_p) v_d - H(r_d) v_d + V_p(r_p, \dot{r}_p) \dot{r} - V_p(r_d, \dot{r}_d) \dot{r}_d + G(r_p) - G(r_d) + B_p(\mu, r_p, \dot{r}_p) \lambda - B_p(\mu, r_d, \dot{r}_d) \lambda_d + H(\alpha p_1 \dot{e}_p + \alpha p_2 e_{p2} + e_{f1}).$$

(3-9)

Based on the Equation 3-7, the control torque is designed to be

$$T_r(t) = Y_d \hat{\beta} + u + \bar{G}_f \lambda$$

(3-10)

where $\hat{\beta} \in \mathbb{R}^p$ is the estimated system parameter vector and $u \in \mathbb{R}^n$ is the RISE feedback term. $u$ is defined as

$$u(t) = (k_1 + 1) e_{p2}(t) - (k_1 + 1) e_{p2}(0) + (k_2 + 1) e_{f1}(t) - (k_2 + 1) e_{f1}(0) + \int_0^t (k_1 + 1) \alpha p_2 e_{p2}(\delta) + \gamma_1 sgn(e_{p2}(\delta)) + (k_2 + 1) K_f e_{f1}(\delta) + \gamma_2 sgn(e_{f1}(\delta)]) d\delta$$

(3-11)

where $k_1, \gamma_1, k_2, \gamma_2 \in \mathbb{R}$ are positive constant control gains.

The estimated system parameter vector $\hat{\beta}$ is generated by the following update law

$$\dot{\hat{\beta}} = \Gamma Y_d^T s$$

(3-12)
where $\Gamma \in \mathbb{R}^{p \times p}$ is a known, constant, diagonal, positive-definite gain matrix. Because Equation 3-12 contains the unknown signal, acceleration and derivative of force error, $\hat{\beta}$ can be integrated by parts as follows to get rid of the immeasurable signal

$$
\hat{\beta}(t) = \hat{\beta}(0) + \Gamma \hat{Y}_d^r s_{int}(\sigma)|_0^t - \Gamma \int_0^t \hat{Y}_d^r s_{int}(\sigma) \, d\sigma,
$$

(3-13)

$$
s_{int} = e_{p2} + \alpha p_2 e_{p1} + e_{f1} + \int_0^t (K_f e_{f1}(\delta) + \alpha p_2 \alpha p_1 e_{p1}(\delta)) \, d\delta.
$$

(3-14)

According to the above equations, the terms in the control torque $T_c(t)$ are all measurable and thus the control torque is implementable.

### 3.2 Closed-Loop Tracking Error System

The closed-loop tracking error system can be obtained by putting Equation 3-10 into Equation 3-7 as

$$
H(r_p) s = Y_d \tilde{\beta} + S_1 + T_d(t) - u(t)
$$

(3-15)

where $\tilde{\beta}(t) \in \mathbb{R}^p$ denotes the estimated system parameter error vector defined as

$$
\tilde{\beta} = \beta - \hat{\beta}.
$$

(3-16)

According to the subsequent Lyapunov-stability analysis, the time derivative of Equation 3-15 is obtained as

$$
H(r_p) \dot{s} = -\frac{1}{2} H(r_p) s + Y_d \tilde{\beta} + \tilde{N}(t) + N_d(t) - \dot{u}(t) - e_{p2} - e_{f1}
$$

(3-17)

where the auxiliary term $\tilde{N}(t) \in \mathbb{R}^n$ is defined as

$$
\tilde{N}(t) = \dot{s}_1 - Y_d \Gamma \hat{Y}_d^r s - \frac{1}{2} \dot{H}(r_p) s + e_{p2} + e_{f1}
$$

(3-18)

and $N_d(t) \in \mathbb{R}^n$ is defined as

$$
N_d(t) = \dot{T}_d(t).
$$

(3-19)

The time derivative of $u(t)$ is obtained as

$$
\dot{u}(t) = (k_1 + 1) e_{p3} + \gamma_1 sgn(e_{p2}) + (k_2 + 1) e_{f2} + \gamma_2 sgn(e_{f1}).
$$

(3-20)
According to [18], the Mean Value Theorem can be applied to Equation 3-18 to obtain the upper bound\(^1\) of \(\bar{N}(t)\) as

\[
\|\bar{N}(t)\| \leq \rho(\|z\|)\|z\| \tag{3-21}
\]

where \(z(t) \in \mathbb{R}^{5n}\) is defined as

\[
z(t) = [e_{p1}^T \ e_{p2}^T \ e_{p3}^T \ e_{f1}^T \ e_{f2}^T]^T. \tag{3-22}
\]

The upper bound of disturbance term \(N_d(t)\) and its time derivative \(\dot{N}_d(t)\) can be obtained based on assumption 2-4 as

\[
\|N_d(t)\| \leq \zeta_{N_d}, \ |\dot{N}_d(t)| \leq \zeta_{N_d2} \tag{3-23}
\]

where \(\zeta_{N_d}, \zeta_{N_d2} \in \mathbb{R}\) are known positive constants.

### 3.3 Stability Analysis

**Theorem 3-1:** The proposed controller given in Equation 3-10, 3-11, and 3-13 ensures that the signals of the constrained robot manipulator described by Equation 2-13 are bounded under closed-loop operation and the position and force tracking error is regulated as

\[
\|e_{p1}(t)\| \to 0, \ |e_{f1}(t)| \to 0 \text{ as } t \to \infty \tag{3-24}
\]

on the condition that gain \(k_1\) and \(k_2\) are chosen large enough based on the initial conditions of the constrained robot manipulator system, \(\alpha_{p1}\) and \(\alpha_{p2}\) are chosen according to the subsequent proof as

\[
\alpha_{p1} > \frac{1}{2}, \ \alpha_{p2} > 1 \tag{3-25}
\]

and \(\gamma_1\) and \(\gamma_2\) are chosen according to the Lemma 2 in the Appendix as

\[
\gamma_1 > \zeta_{N_d} + \frac{\zeta_{N_d2}}{\alpha_{p2}}, \gamma_2 > \zeta_{N_d} + \frac{\zeta_{N_d2}}{k_f}. \tag{3-26}
\]

\(^1\) See Lemma 1 of the Appendix for the proof of the upper bound of \(\bar{N}(t)\).
Proof: A positive definite function is selected as

\[ V = e_{p1}^T e_{p1} + \frac{1}{2} e_{p2}^T e_{p2} + \frac{1}{2} s^T H s + \frac{1}{2} e_{f1}^T e_{f1} + P_1 + P_2 + \frac{1}{2} \beta^T \Gamma^{-1} \beta \]  \hspace{1cm} (3-27)

where the auxiliary function \( P_1(t), P_2(t) \in R \) are defined as

\[ P_1(t) = \beta_1 \sum_{i=1}^n |e_{p2i}(0)| - e_{p2}(0)^T N_d(0) - \int_0^t L_1(\tau) d\tau \]  \hspace{1cm} (3-28)

\[ P_2(t) = \beta_2 \sum_{i=1}^m |e_{f1i}(0)| - e_{f1}(0)^T N_d(0) - \int_0^t L_2(\tau) d\tau \]  \hspace{1cm} (3-29)

and the subscript \( i \) denotes the \( i \)th element of the vector. In Equation 3-28 and 3-29, the auxiliary function \( L_1(t), L_2(t) \in R \) are defined as

\[ L_1(t) = e_{p3}^T (N_d(t) - \gamma_1 sgn(e_{p2})) \]  \hspace{1cm} (3-30)

\[ L_2(t) = e_{f2}^T (N_d(t) - \gamma_2 sgn(e_{f1})). \]  \hspace{1cm} (3-31)

The derivative of \( P_1(t), P_2(t) \) with respect to time can be obtained as

\[ \dot{P}_1(t) = -L_1(t) = -e_{p3}^T (N_d(t) - \gamma_1 sgn(e_{p2})) \]  \hspace{1cm} (3-32)

\[ \dot{P}_2(t) = -L_2(t) = -e_{f2}^T (N_d(t) - \gamma_2 sgn(e_{f1})). \]  \hspace{1cm} (3-33)

If the sufficient condition in Equation 3-26 is satisfied, the following inequality can hold\(^2\)

\[ \int_0^t L_1(\tau) d\tau \leq \beta_1 \sum_{i=1}^n |e_{p2i}(0)| - e_{p2}(0)^T N_d(0) \]  \hspace{1cm} (3-34)

\[ \int_0^t L_2(\tau) d\tau \leq \beta_2 \sum_{i=1}^m |e_{f1i}(0)| - e_{f1}(0)^T N_d(0). \]  \hspace{1cm} (3-35)

Thus, \( P_1(t), P_2(t) \geq 0 \) are obtained.

Taking the time derivative of \( V \), the following equation can be obtained

\[ \dot{V} = 2 e_{p1}^T \dot{e}_{p1} + \frac{1}{2} e_{p2}^T \dot{e}_{p2} + s^T H s + \frac{1}{2} e_{f1}^T \dot{e}_{f1} - L_1 - L_2 - \beta^T \Gamma^{-1} \dot{\beta} \]  \hspace{1cm} (3-36)

\[ = 2 e_{p1}^T (e_{p2} - \alpha_1 e_{p1}) + e_{p2}^T (e_{p3} - \alpha_2 e_{p2}) + s^T (-\frac{1}{2} H s + \gamma_d \dot{\beta}^{\dagger} + \dot{N}(t)) \]

\[ + N_d(t) - (k_1 + 1) e_{p3} - \gamma_1 sgn(e_{p2}) - (k_2 + 1) e_{f2} \]

\[^2\) See Lemma 2 of the Appendix for the proof of the inequality in Equation 3-26
\[-\gamma_2 sgn(e_{f1}) - e_{p2}e_{f1} + e_{f1}^T(e_{f2} - \bar{K}_f e_{f1}) + \frac{1}{2} s^T \hat{H} s \]

\[-e_{p3}^T (N_d(t) - \gamma_1 sgn(e_{p2})) - e_{f2}^T (N_d(t) - \gamma_2 sgn(e_{f1})) - \beta^T \hat{Y}_d^T s \]  

\[= s^T \hat{N}(t) - 2\alpha_{p1}e_{p1}^T e_{p1} - \alpha_{p2}e_{p2}^T e_{p2} - \bar{K}_f e_{f1}^T e_{f1} + 2e_{p1}^T e_{p2} \]

\[-(k_1 + 1)e_{p3}^T e_{p3} - (k_2 + 1)e_{f2}^T e_{f2} \]  

Using Equation 3-22 and the following inequality

\[e_{p1}^T e_{p2} \leq \frac{1}{2} \|e_{p1}\|^2 + \frac{1}{2} \|e_{p2}\|^2 \]  

(3-39)

\[\dot{V} \leq -(2\alpha_{p1} - 1)\|e_{p1}\|^2 - (\alpha_{p2} - 1)\|e_{p2}\|^2 - \bar{\eta}_1 \|s\|^2 \]

\[+\|s\|\rho\|z\|\|z\| \leq -\eta_2 \|z\|^2 - \]

\[(\eta_1 \|s\|^2 - \rho(\|z\|) \|s\| \|z\|) \leq -\left(\eta_2 \|z\|^2 - \frac{\rho^2(\|z\|)^2}{4\eta_1} \right) \|z\|^2 \]  

(3-40)

where

\[\eta_1 = \min\{k_1 + 1, k_2 + 1\}, \quad \eta_2 = \min\{2\alpha_{p1} - 1, \alpha_{p2} - 1, 1\}\]

and \(\rho(\|z\|)\) is a positive globally invertible nondecreasing function.

According to Equation 3-27 and 3-40, \(V > 0\) and \(\dot{V} \leq 0\). Thus, \(e_{p1}(t), e_{p2}(t), e_{p3}(t), e_{f1}(t), e_{f2}(t), \bar{\beta} \in L_\infty\). Since \(e_{p1}(t), e_{p2}(t), e_{p3}(t), e_{f1}(t)\) and \(e_{f2}(t) \in L_\infty\), then according to Equation 3-1, 3-2, 3-3 and 3-4, \(\hat{e}_{p1}, \hat{e}_{p2}, \hat{e}_{f1} \in L_\infty\). Since \(\bar{\beta} \in L_\infty\) and \(\beta\) is unknown constant vector, then according to Equation 3-16, \(\hat{\beta} \in L_\infty\). Since \(e_{p1}(t), e_{p2}(t), e_{p3}(t), e_{f1}(t), e_{f2}(t) \in L_\infty\) and assumption 2-3 that \(q_d(t), \dot{q}_d(t), \ddot{q}_d(t), \lambda_d(t), \dot{\lambda}_d(t), \ddot{\lambda}_d(t)\) exist and are bounded, then \(r(t), \dot{r}(t), \ddot{r}(t), \lambda(t), \dot{\lambda}(t) \in L_\infty\). Since \(q_d(t), \dot{q}_d(t), \ddot{q}_d(t), \lambda_d(t), \dot{\lambda}_d(t), \ddot{\lambda}_d(t)\) exist and are bounded, then
$Y_d(q_d, \dot{q}_d, \ddot{q}_d, \lambda_d), \dot{Y}_d(q_d, \dot{q}_d, \ddot{q}_d, \lambda_d, \dot{\lambda}_d), \ddot{Y}_d(q_d, \dot{q}_d, \ddot{q}_d, \dddot{q}_d, \lambda_d, \dot{\lambda}_d, \dddot{\lambda}_d) \in \mathcal{L}_c$. Since $e_{p1}(t), e_{p2}(t), e_{p3}(t), e_{f1}(t), e_{f2}(t) \in \mathcal{L}_c$, then input torque $T_r(t) \in \mathcal{L}_c$ and is implementable. Since $e_{p1}(t), \dot{e}_{p1}(t), e_{f1}(t), \dot{e}_{f1}(t) \in \mathcal{L}_c$, then $e_{p1}(t)$ and $e_{f1}(t)$ are uniformly continuous. Moreover, since

$$V(t) - V(0) = - \int_0^t \left( \eta_2 \| z \|^2 - \frac{\rho^2 \| z \|^2}{4\eta_1} \right) \| z \|^2 \leq 0 \rightarrow e_{f1}(t), e_{p1}(t) \in \mathcal{L}_2 \quad (3-41)$$

applying Barbalat’s lemma, if $\eta_2 \| z \|^2 - \frac{\rho^2 \| z \|^2}{4\eta_1} \geq 0$, then $e_{p1}(t)$ and $e_{f1}(t)$ semi-globally asymptotically regulates to zero, $e_{p1}(t), e_{f1}(t) \rightarrow 0$ as $t \rightarrow \infty$. □
4.1 Simulation Environment

The simulation environment depicted in Figure 4-1 was used to test the proposed controller.

A two degree of freedom robot manipulator is chosen for the simulation. The dynamic system matrices, $M(q), V_m(q, \dot{q})$, forward kinematics and Jacobian matrix are given by

$$M(q) = \begin{bmatrix} \beta_2 + 2\beta_1 C_{q2} & \beta_2 + \beta_1 C_{q2} \\ \beta_2 + \beta_1 C_{q2} & \beta_2 \end{bmatrix}$$

$$V_m(q, \dot{q}) = \begin{bmatrix} -\beta_1 \dot{q}_2 S_{q2} & -\beta_1 (\dot{q}_1 + \dot{q}_2) S_{q2} \\ \beta_1 \dot{q}_1 S_{q2} & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 C_{q1} + l_2 C_{q12} - d \\ l_1 S_{q1} + l_2 S_{q12} \end{bmatrix}$$

$$J(q) = \frac{\partial x(q)}{\partial q} = \begin{bmatrix} -l_1 S_{q1} - l_2 S_{q12} & -l_2 S_{q12} \\ l_1 C_{q1} + l_2 C_{q12} & l_2 C_{q12} \end{bmatrix}$$
where \( p \) is system parameter calculated by

\[
A_1(x) = \frac{\partial \phi}{\partial x} = \begin{bmatrix} \cos(\theta) \\ -\sin(\theta) \end{bmatrix}, \quad D(x) = \frac{\partial \phi}{\partial x} = \begin{bmatrix} \sin(\theta) \end{bmatrix}, \quad L(q, u, \tau) = A_1 \psi(q) + g(q) + B_1 u
\]

The actual values of system parameters are

\[
\begin{bmatrix} c_1 \\ c_2 \\ \alpha \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}
\]

The task space of the robot manipulator is described as

where \( \dot{r} \) is orthogonal to the curvilinear coordinate of \( r \).

The surface is described as

As seen in Figure 4.1, the contact surface between the robot manipulator and the environment is a semi-circle surface \( S \) with a dry friction coefficient \( \mu = 0.2 \). The surface

\[
T_d(t) = \begin{bmatrix} 10 \sin(\theta) \\ 0 \end{bmatrix}
\]

where \( c_1 = \cos(q_1), c_2 = \cos(q_2), \alpha = \sin(q_1 + q_2), \beta = [\beta_1, \beta_2, \beta_3]^T \) is system parameter calculated by

\[
\begin{cases}
\beta_1 = \beta_2 = \beta_3 = \sin(q_1 + q_2), \\
\beta_1 = [\beta_1, \beta_2, \beta_3]^T
\end{cases}
\]

The actual value is unknown in simulation and its initial estimate value is \( \beta = [0.18, 0.18, 1.8]^T \).
\[ J_x = \begin{bmatrix} \sin(\theta) \\ \cos(\theta) \end{bmatrix}, \quad J_q = J_x(x(q))J(q) \]

\[ F = F_n + F_t, \quad F_n = D(x)f_n = \begin{bmatrix} \sin(\theta) \\ \cos(\theta) \end{bmatrix}f_n, \]

\[ F_t = L(\mu, x, \dot{x})f_n = \begin{bmatrix} \cos(\theta) \\ -\sin(\theta) \end{bmatrix}\mu \text{sgn}(r_i)f_n \]

where \( f_n \in \mathbb{R} \), normal contact force component, can be measured by the force sensor.

Based on the above equations, Operational space dynamics can be derived and the dynamics Equation 2-13 controlled by the proposed controller can be obtained. The following simulation is based on the assumption that the position and velocity of the end-effector and the normal contact force are measurable, that \( J(q) \) is nonsingular in a finite work space \( \Omega_q \), that the end-effector is in contact with the rigid surface with dry friction force at first, and the end-effector will not leave the contact surface.

The control torque is given by Equation 3-10, 3-11, and 3-13. The parameter estimated vector \( \hat{\beta} \) is defined as

\[ \hat{\beta} = [\hat{\beta}_1 \quad \hat{\beta}_2 \quad \hat{\beta}_3 \quad \hat{\mu}]^T. \]  \hspace{1cm} (4-1)

The parameter estimated vector Equation 4-1 is generated by the updated law Equation 3-13. The regression matrix \( Y_a \) can be obtained by Equation 3-8. Parameter values of the controller are \( \alpha_1 = 100, \alpha_2 = 1.1, k_1 = 1500, g_1 = 40, g_2 = 40, \tilde{\alpha}_f = [0.01 \quad 0]^T, K_f = 0.1, K_2 = 10 \) and \( \Gamma = \text{diag}[5 \quad 5 \quad 5 \quad 5] \). The desired position and force trajectory are given by

\[ r_d = -\frac{R\pi}{6}(1 + \cos(0.5\pi t)) \]

\[ \lambda_d = -15 + 5\cos(\pi t) \]

Sampling time is 0.001 sec
4.2 Simulation Results without Disturbances

The desired position trajectory is shown in Figure 4-2 and desired force trajectory is shown in Figure 4-3. The time response of position trajectory $r_p$ and its position tracking errors $e_{p1}$ in the presence of unknown system parameters but without disturbance are shown in Figure 4-4 and Figure 4-5, respectively. Figure 4-6 is the enlarged diagram of Figure 4-5. The time response of force trajectory $\lambda$ and its force tracking errors $e_{f1}$ in the presence of unknown system parameters but without disturbance are shown in Figure 4-7 and Figure 4-8, respectively. Figure 4-9 is the enlarged diagram of Figure 4-8. The time response of friction force is shown in Figure 4-10. The estimated system parameters $\beta_1, \beta_2, \beta_3$ and dry contact surface friction $\mu$ are shown in Figure 4-11 and Figure 4-12, respectively. Figure 4-13 shows the control torque of the robot manipulator. As shown in the Figure 4-5 and Figure 4-6, the position tracking error falls between -0.001m and 0.001m. The proposed controller has a good position tracking ability. According to Figure 4-8 and Figure 4-9, the proposed controller also has good force tracking ability. The force tracking error falls between -1N and 1N. In Figure 4-11 and Figure 4-12, the estimated parameters do not converge to their actual value. This phenomenon may be addressed by using composite adaptive update law which may be studied in the future work.
Figure 4-2. Desired position trajectory.

Figure 4-3. Desired force trajectory.
Figure 4-4. Time response of position trajectory $r_p$ in the presence of unknown system parameters but without disturbance.

Figure 4-5. The time response of position tracking errors $e_{p1}$ in the presence of unknown system parameters but without disturbance.
Figure 4-6. Enlarged diagram of Figure 4-5.

Figure 4-7. The time response of force trajectory $\lambda$ in the presence of unknown system parameters but without disturbance.
Figure 4-8. The time response of tracking errors $e_{f1}$ in the presence of unknown system parameters but without disturbance.

Figure 4-9. Enlarged diagram of Figure 4-8.
Figure 4-10. The time response of friction force in the presence of unknown system parameters but without disturbance.

Figure 4-11. Estimated system parameters $\beta_1, \beta_2, \beta_3$ in the presence of unknown system parameters but without disturbance.
Figure 4-12. Estimated dry contact surface friction $\mu$ in the presence of unknown system parameters but without disturbance.

Figure 4-13. Control torque of the robot manipulator in the presence of unknown system parameters but without disturbance.

4.3 Simulation Results with Disturbances

The desired position and force trajectory is the same as shown in Figure 4-2 and Figure 4-3, respectively. The disturbance is shown in Figure 4-14 and Figure 4-15. As per assumption 2-4, the nonlinear disturbance is bounded by known constants. The time response of the position trajectory $r_p$ and its position tracking errors $e_{p1}$ in the
presence of unknown system parameters and disturbance are shown in Figure 4-16 and Figure 4-17, respectively. Figure 4-18 is the enlarged diagram of Figure 4-17. The time response of force trajectory $\lambda$ and its force tracking errors $e_{f_1}$ in the presence of unknown system parameters and disturbance are shown in Figure 4-19 and Figure 4-20, respectively. Figure 4-21 is the enlarged diagram of Figure 4-20. The time response of friction force is shown in Figure 4-22. The estimated system parameters $\beta_1, \beta_2, \beta_3$ and dry contact surface friction $\mu$ are shown in Figure 4-23 and Figure 4-24, respectively. Figure 4-25 shows the control torque of the robot manipulator. As shown in the Figure 4-17 and Figure 4-18, the position tracking error falls between -0.001m and 0.001m like Figure 4-8 and Figure 4-9. The proposed controller has good position disturbance rejection ability. According to Figure 4-20 and Figure 4-21, the proposed controller also has good force disturbance rejection ability. The force tracking error falls between -1N and 1N. Like the estimated system parameter vector in Figure 4-11 and Figure 4-12, the estimated parameters of system with disturbances do not converge to their actual value.

![Graph](image)

Figure 4-14. Disturbance used for the simulation.
Figure 4-15. Disturbance used for the simulation.

Figure 4-16. The time response of position trajectory $r_p$ in the presence of unknown system parameters and disturbance.
Figure 4-17. The time response of position tracking errors $e_p$ in the presence of unknown system parameters and disturbance.

Figure 4-18. Enlarged diagram of Figure 4-17.
Figure 4-19. The time response of force trajectory $\lambda$ in the presence of unknown system parameters and disturbance.

Figure 4-20. The time response of force tracking errors $e_{f1}$ in the presence of unknown system parameters and disturbance.
Figure 4-21. Enlarged diagram of Figure 4-20.

Figure 4-22. The time response of friction force in the presence of unknown system parameters and disturbance.
Figure 4-23. Estimated system parameters $\beta_1, \beta_2, \beta_3$ in the presence of unknown system parameters and disturbance.

Figure 4-24. Estimated dry contact surface friction $\mu$ in the presence of unknown system parameters and disturbance.
4.4 Discussion

Figures 4-26 and 4-27 show the position tracking error $e_{p1}$ without disturbance and with disturbance in the same figure. From these figures, the position tracking error with disturbance is almost the same as the position tracking error without disturbance. It shows that, due to the RISE feedback term, the disturbance rejection ability is good.

Figures 4-28 and 4-29 show the force tracking error $e_{f1}$ without disturbance and with disturbance in the same figure. From these figures, the force tracking error with disturbance is a little bigger than the force tracking error without disturbance. Despite this situation, the proposed controller also has a good force disturbance rejection ability. Noted that in these figures, there are abrupt changes at $t = 0.5, 2, 4, 6, 8s$. Because the direction of the friction force depends on the direction of end-effector velocity, the abrupt direction change of the end-effector velocity can be seen in Figures 4-4 and 4-16.

To make comparison, the simulation is run with the same parameters as before except $K_f$ to show how the different gain will affect the result in Figure 4-30 and 4-31.

Figure 4-25. Control torque of the robot manipulator in the presence of unknown system parameters and disturbance.
The position tracking errors look alike, but the force tracking errors have much
difference. It agrees with the Properties 2-4 that \( K_f \) has to be small enough.

The simulation is also run with the same parameters as before except \( \alpha_{p2} \) in
Figures 4-32, 4-33, and 4-34. The transient position tracking error improves a lot, but
the force tracking error doesn’t ameliorate in this condition and the joint torque become
bigger.

In the last simulation, the system is run to test the validity of Theorem 3-1.
Figures 4-35 and 4-36 are given to show that the position and force tracking error don’t
converge to zero under the condition that \( \alpha_{p1} < \frac{1}{2}, \alpha_{p2} < 1, \gamma_1 < \zeta_{Nd} + \frac{\zeta_{Nd2}}{\alpha_{p2}}, \gamma_2 < \zeta_{Nd} + \frac{\zeta_{Nd2}}{K_f} \). These figures show that if \( \alpha_{p1}, \alpha_{p2}, \gamma_1, \gamma_2 \) do not satisfy Theorem 3-1 conditions, the
system would not be stable.

![Graph](image)

Figure 4-26. Position tracking error with disturbance and without disturbance.
Figure 4-27. Enlarged diagram of Figure 4-26.

Figure 4-28. Force tracking error with disturbance and without disturbance.
Figure 4-29. Enlarged diagram of Figure 4-28.

Figure 4-30. The time response of position tracking errors $e_{p1}$ with different $K_f$ gains.
Figure 4.31. The time response of force tracking errors $e_{f1}$ with different $K_f$ gains.

Figure 4.32. The time response of position tracking errors $e_{p1}$ with different $\alpha_{p2}$ gains.
Figure 4-33. The time response of force tracking errors $e_f^1$ with different $\alpha_p^2$ gains.

Figure 4-34. The time response of control torque with different $\alpha_p^2$ gains.
Figure 4-35. The time response of position tracking errors $e_{p1}$ with $\alpha_p < \frac{1}{2}, \alpha_p < 1, \gamma_1 < \zeta_{Nd} + \frac{\zeta_{Nd2}}{\alpha_p}$.\[ \gamma_2 < \zeta_{Nd} + \frac{\zeta_{Nd2}}{k_f}.\]

Figure 4-36. The time response of force tracking errors $e_{r1}$ with $\alpha_p < \frac{1}{2}, \alpha_p < 1, \gamma_1 < \zeta_{Nd} + \frac{\zeta_{Nd2}}{\alpha_p}$.\[ \gamma_2 < \zeta_{Nd} + \frac{\zeta_{Nd2}}{k_f}.\]
CHAPTER 5
CONCLUSIONS AND SUGGESTED FUTURE WORKS

5.1 Conclusions

This work focused on designing an adaptive controller for constrained robot manipulator with unknown parameters both in the robot manipulator and the contact surface with disturbance caused by force sensor noise and unmodeled effects.

First, the dynamic equation of a general rigid link robot manipulator having $n$ degrees of freedom modeled by Euler-Lagrange formulation is used in this study. Because of the friction force occurring between contact surface and end-effector, the position and force control cannot be controlled separately. The Euler-Lagrange dynamic model is modified to suit this condition.

Second, the contact surface of the environment is modeled by the set of $m$ rigid and mutually independent hypersurfaces. When the robot manipulator is in contact with environment, the end-effector of robot manipulator is constrained to be on that contact surface. Thus, only $(n-m)$ position coordinates and $m$ force coordinates need to be controlled.

Third, the proposed controller is an adaptive hybrid position/force controller with RISE feedback structure. By using the Lyapunov-stability analysis, the suggested controller can guarantee semi-global asymptotic position and force tracking result even if in the presence of the disturbance. The only assumption about the disturbance is that it has to be bounded by known constants.

In conclusion, the suggested force feedback term in the controller is in accordance with PI type. Intensive simulation results were given to show the validity of the proposed controller.
5.2 Suggested Future Works

The contact surface model in this study is assumed to be a rigid contact surface, but in most application areas, the contact surface may not be rigid. The stiffness of the contact surface needs to be considered in the contact model [19].

In addition, the robot manipulator is assumed to be non-redundant in this study. The future work can focus on how to extend the controller proposed in this study to the redundant robot manipulator which has some advantages. For example, a problem occurs, when the Jacobian matrix becomes linearly dependent. In this case, a non-redundant robot manipulator is at a singular configuration and loses the degree of freedom in some direction [1]. Redundant robot manipulator can deal with this situation.

Another suggested future work is to improve the estimated system parameters and dry contact surface friction coefficient. In this study, the estimated vector $\hat{\beta}$ doesn’t converge to their actual values. According to [20], the composite adaptive control skill may be incorporated with the controller proposed in this study. The main idea of composite adaptive control skill is that it includes the prediction error of the system parameters estimation in the adaptive updated law. The prediction error is defined as the difference between the estimated system parameters and actual system parameters. Thus, including prediction error in the Lyapunov-stability analysis can prove that estimated system parameters converge to their actual values as time goes to infinity.
Lemma 1. The Mean Value Theorem can be applied to prove the upper bound of $\tilde{N}(t)$ in Equation 3-21

\[ \|\tilde{N}(t)\| \leq \rho(\|z\|)\|z\| \]

where $z(t) \in \mathbb{R}^{5n}$ is defined as

\[ z(t) = [e^T_{p1} \ e^T_{p2} \ e^T_{p3} \ e^T_{f1} \ e^T_{f2}]^T \]

Proof: $\tilde{N}(t)$ can be written in the following form

\[
\tilde{N}(t) = N(r, \dot{r}, \ddot{r}, \lambda, \dot{\lambda}, \ddot{\lambda}, e_{p1}, e_{p2}, e_{p3}, e_{f1}, e_{f2}) - N(r_d, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, 0,0,0,0,0) \\
+ N(r, \dot{r}, \ddot{r}, \lambda, \dot{\lambda}, \ddot{\lambda}, 0,0,0,0,0) - N(r, \dot{r}_d, \ddot{r}_d, \lambda, \dot{\lambda}_d, \ddot{\lambda}_d, 0,0,0,0,0) \\
+ N(r, \dot{r}, \ddot{r}, \lambda, \dot{\lambda}_d, \ddot{\lambda}_d, 0,0,0,0,0) - N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, 0,0,0,0,0) \\
+ N(r, \dot{r}_d, \ddot{r}_d, \lambda, \dot{\lambda}_d, \ddot{\lambda}_d, 0,0,0,0,0) - N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, 0,0,0,0,0) \\
+ N(r, \dot{r}, \ddot{r}_d, \lambda, \dot{\lambda}_d, \ddot{\lambda}_d, 0,0,0,0,0) - N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, 0,0,0,0,0) \\
+ N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, 0,0,0,0,0) - N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, 0,0,0,0,0) \\
+ N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, e_1, 0,0,0,0,0) - N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, 0,0,0,0,0) \\
+ N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, e_{p1}, e_{p2}, 0,0,0,0) - N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, e_{p1}, 0,0,0,0) \\
+ N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, e_{p1}, e_{p2}, e_{p3}, 0,0) - N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, e_{p1}, e_{p2}, 0,0) \\
+ N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, e_{p1}, e_{p2}, e_{p3}, e_{f1}, 0) - N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, e_{p1}, e_{p2}, e_{p3}, 0) \\
+ N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, e_{p1}, e_{p2}, e_{p3}, e_{f1}, e_{f2}) - N(r, \dot{r}_d, \ddot{r}_d, \lambda_d, \dot{\lambda}_d, \ddot{\lambda}_d, e_{p1}, e_{p2}, e_{p3}, e_{f1}, 0) \\
\]
The Mean Value Theorem can be applied to $\tilde{N}(t)$ and rewritten as

$$\tilde{N}(t) = \frac{\partial N(\sigma_1, r, \dot{r}, \ddot{r}, \lambda, \dot{\lambda}, \ddot{\lambda}, 0, 0, 0, 0, 0)}{\partial \sigma_1} |_{\sigma_1 = v_1} (r - r_d)$$

$$+ \frac{\partial N(r, \sigma_2, r, \ddot{r}, \lambda, \dot{\lambda}, \ddot{\lambda}, 0, 0, 0, 0, 0)}{\partial \sigma_2} |_{\sigma_2 = v_2} (\dot{r} - \dot{r}_d) + \frac{\partial N(r, \ddot{r}, \sigma_3, \lambda, \dot{\lambda}, \ddot{\lambda}, 0, 0, 0, 0, 0)}{\partial \sigma_3} |_{\sigma_3 = v_3} (\ddot{r} - \ddot{r}_d)$$

$$+ \frac{\partial N(r, \ddot{r}, \sigma_4, \lambda, \ddot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_4} |_{\sigma_4 = v_4} (\dddot{r}_d - \dddot{r}_d) + \frac{\partial N(r, \dddot{r}, \sigma_5, \lambda, \dddot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_5} |_{\sigma_5 = v_5} (\lambda - \lambda_d)$$

$$+ \frac{\partial N(r, \dddot{r}, \sigma_6, \lambda, \dddot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_6} |_{\sigma_6 = v_6} (\dot{\lambda}_d - \dot{\lambda}_d) + \frac{\partial N(r, \dddot{r}, \sigma_7, \lambda, \dddot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_7} |_{\sigma_7 = v_7} (\dddot{\lambda}_d - \dddot{\lambda}_d)$$

$$+ \frac{\partial N(r, \dddot{r}, \sigma_8, \lambda, \dddot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_8} |_{\sigma_8 = v_8} (e_{p1} - 0) + \frac{\partial N(r, \dddot{r}, \sigma_9, \lambda, \dddot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_9} |_{\sigma_9 = v_9} (e_{p2} - 0)$$

$$+ \frac{\partial N(r, \dddot{r}, \sigma_{10}, \lambda, \dddot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_{10}} |_{\sigma_{10} = v_{10}} (e_{p3} - 0) + \frac{\partial N(r, \dddot{r}, \sigma_{11}, \lambda, \dddot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_{11}} |_{\sigma_{11} = v_{11}} (e_{f1} - 0) + \frac{\partial N(r, \dddot{r}, \sigma_{12}, \lambda, \dddot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_{12}} |_{\sigma_{12} = v_{12}} (e_{f2} - 0)$$

where

$$v_1 \in (r_d, r), v_2 \in (\dot{r}_d, \dot{r}), v_3 \in (\ddot{r}_d, \ddot{r}_d), v_4 \in (\dddot{r}_d, \dddot{r}_d), v_5 \in (\lambda, \lambda), v_6 \in (\dot{\lambda}_d, \dot{\lambda}_d)$$

$$v_7 \in (\dddot{\lambda}_d, \dddot{\lambda}_d), v_8 \in (0, e_{p1}), v_9 \in (0, e_{p2}), v_{10} \in (0, e_{p3}), v_{11} \in (0, e_{f1}), v_{12} \in (0, e_{f2})$$

Thus, $\tilde{N}(t)$ can be upper bounded as

$$\|\tilde{N}(t)\| \leq \left\| \frac{\partial N(\sigma_1, r, \dddot{r}, \dot{r}, \dddot{\lambda}, \dot{\lambda}, \dddot{\lambda}, 0, 0, 0, 0, 0)}{\partial \sigma_1} |_{\sigma_1 = v_1} \right\| \|e_1\|$$

$$+ \left\| \frac{\partial N(r, \dddot{r}, \dot{r}, \dddot{\lambda}, \dot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_2} |_{\sigma_2 = v_2} \right\| \|e_{p2} - \alpha_{p1} e_{p1}\| + \left\| \frac{\partial N(r, \dddot{r}, \dot{r}, \dddot{\lambda}, \dot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_3} |_{\sigma_3 = v_3} \right\| \|e_{p3}\|$$

$$+ \left\| \frac{\partial N(r, \dddot{r}, \dot{r}, \dddot{\lambda}, \dot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_4} |_{\sigma_4 = v_4} \right\| \|e_{p1}\| + \left\| \frac{\partial N(r, \dddot{r}, \dot{r}, \dddot{\lambda}, \dot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_5} |_{\sigma_5 = v_5} \right\| \|e_{p2}\|$$

$$+ \left\| \frac{\partial N(r, \dddot{r}, \dot{r}, \dddot{\lambda}, \dot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_6} |_{\sigma_6 = v_6} \right\| \|e_{p3}\| + \left\| \frac{\partial N(r, \dddot{r}, \dot{r}, \dddot{\lambda}, \dot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_7} |_{\sigma_7 = v_7} \right\| \|e_{f1}\|$$

$$+ \left\| \frac{\partial N(r, \dddot{r}, \dot{r}, \dddot{\lambda}, \dot{\lambda}, 0, 0, 0, 0, 0, 0)}{\partial \sigma_8} |_{\sigma_8 = v_8} \right\| \|e_{f2}\|$$

Noted that the partial derivatives can be upper bounded as
\[
\left\| \frac{\partial N(\sigma_1, r, \sigma_2, r, \sigma_3, \lambda_1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)}{\partial \sigma_1} \right\|_{\sigma_1 = v_1} \leq \rho_1(e_{p1})
\]
\[
\left\| \frac{\partial N(r, \sigma_2, r, \sigma_3, \lambda_1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)}{\partial \sigma_2} \right\|_{\sigma_2 = v_2} \leq \rho_2(e_{p1}, e_{p2})
\]
\[
\left\| \frac{\partial N(r, r, \sigma_3, \lambda_1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)}{\partial \sigma_3} \right\|_{\sigma_3 = v_3} \leq \rho_3(e_{p1}, e_{p2}, e_{f1})
\]
\[
\left\| \frac{\partial N(r, r, r, \lambda_1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)}{\partial \sigma_4} \right\|_{\sigma_4 = v_4} \leq \rho_4(e_{p1}, e_{p2}, e_{f1})
\]
\[
\left\| \frac{\partial N(r, r, r, r, \lambda_1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)}{\partial \sigma_5} \right\|_{\sigma_5 = v_5} \leq \rho_5(e_{p1}, e_{p2}, e_{f1})
\]
\[
\left\| \frac{\partial N(r, r, r, r, r, \lambda_1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)}{\partial \sigma_6} \right\|_{\sigma_6 = v_6} \leq \rho_6(e_{p1}, e_{p2}, e_{p3}, e_{f1})
\]
\[
\left\| \frac{\partial N(r, r, r, r, r, r, \lambda_1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)}{\partial \sigma_7} \right\|_{\sigma_7 = v_7} \leq \rho_7(e_{p1}, e_{p2}, e_{p3}, e_{f1})
\]
\[
\left\| \frac{\partial N(r, r, r, r, r, r, r, \lambda_1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)}{\partial \sigma_8} \right\|_{\sigma_8 = v_8} \leq \rho_8(e_{p1}, e_{p2}, e_{p3}, e_{f1}, e_{f2})
\]

Using the following inequality
\[
\left\| e_{p2} - \alpha p_1 e_{p1} \right\| \leq \left\| e_{p2} \right\| + \alpha p_1 \left\| e_{p1} \right\|
\]
and \( z(t), \bar{N}(t) \) can be further upper bounded as
\[
\left\| \bar{N}(t) \right\| \leq \left( \rho_1(e_{p1}) + \alpha p_1 \rho_2(e_{p1}, e_{p2}) + \rho_3(e_{p1}, e_{p2}, e_{f1}) \right) \left\| e_{p1} \right\|
\]
\[
+ \left( \rho_2(e_{p1}, e_{p2}) + \rho_5(e_{p1}, e_{p2}, e_{f1}) \right) \left\| e_{p2} \right\| + \left( \rho_3(e_{p1}, e_{p2}, e_{f1}) + \rho_7(e_{p1}, e_{p2}, e_{p3}, e_{f1}) \right) \left\| e_{f1} \right\|
\]
\[
+ \rho_6(e_{p1}, e_{p2}, e_{p3}, e_{f1}) \left\| e_{p3} \right\| + \rho_8(e_{p1}, e_{p2}, e_{p3}, e_{f1}, e_{f2}) \left\| e_{f2} \right\|
\]
\[
\leq \left( \rho_1(e_{p1}) + \alpha p_1 \rho_2(e_{p1}, e_{p2}) + \rho_4(e_{p1}, e_{p2}, e_{f1}) + \rho_2(e_{p1}, e_{p2}) + \rho_5(e_{p1}, e_{p2}, e_{f1}) \right.
\]
\[
+ \rho_3(e_{p1}, e_{p2}, e_{f1}) + \rho_7(e_{p1}, e_{p2}, e_{p3}, e_{f1}) + \rho_6(e_{p1}, e_{p2}, e_{p3}, e_{f1}) + \rho_8(e_{p1}, e_{p2}, e_{p3}, e_{f1}, e_{f2}) \right) \left\| z(t) \right\|
\]

Thus,
\[ \|\tilde{N}(t)\| \leq \rho(\|z\|) \|z\| \]

where \(\rho(\|z\|)\) is some positive globally invertible nondecreasing function. \(\Box\)
**APPENDIX B**

**PROOF FOR EQUATION 3-26**

**Lemma 2** The function $L_1(t), L_2(t) \in R$ is defined as

$$L_1(t) = e_{p3}^T(N_d(t) - \gamma_1 sgn(e_{p2}))$$

$$L_2(t) = e_{f2}^T(N_d(t) - \gamma_2 sgn(e_{f1}))$$

If the following sufficient conditions is satisfied

$$\gamma_1 > \zeta_{Nd} + \frac{\zeta_{Nd2}}{\alpha_{p2}}, \quad \gamma_2 > \zeta_{Nd} + \frac{\zeta_{Nd2}}{\kappa_f}$$

Then

$$\int_0^t L_1(\tau)d\tau \leq \gamma_1 \sum_{i=1}^n |e_{p2i}(0)| - e_{p2}(0)^TN_d(0)$$

$$\int_0^t L_2(\tau)d\tau \leq \gamma_2 \sum_{i=1}^m |e_{f1i}(0)| - e_{f1}(0)^TN_d(0)$$

**Proof:** Integrating both sides of Equation 3-30

$$\int_0^t L_1(\tau)d\tau = \int_0^t e_{p3}^T(N_d(\tau) - \beta_1 sgn(e_{p2}(\tau)))d\tau$$

$$= \int_0^t e_{p2}^T(N_d(\tau))d\tau - \beta_1 \int_0^t e_{p2}^Tsgn(e_{p2}(\tau))d\tau + \int_0^t \alpha_{p2} e_{p2}^T(N_d(\tau) - \beta_1 sgn(e_{p2}))d\tau$$

Using integration by part, the first term of Equation A-1 can be expressed as

$$\int_0^t e_{p2}^T(N_d(\tau))d\tau = e_{p2}^T(t)N_d(t) - e_{p2}^T(0)N_d(0) - \int_0^t e_{p2}^TN_d(\tau)d\tau$$

Using the following property,

$$-\beta_1 \int_0^t e_{p2}^Tsgn(e_{p2})d\tau = \beta_1 \sum_{i=1}^n |e_{p2i}(0)| - \beta_1 \sum_{i=1}^n |e_{p2i}(t)|$$

Equation A-1 can be rewritten as

$$\int_0^t L_1(\tau)d\tau \leq \beta_1 \sum_{i=1}^n |e_{p2i}(0)| - e_{p2}(0)^TN_d(0) + \|e_{p2}(t)\|(\|N_d(t)\| - \beta_1)$$

$$+ \alpha_{p2} \int_0^t \|e_{p2}(\tau)\| \left(\|N_d(t)\| + \frac{1}{\alpha_{p2}} \|\dot{N}_d(t)\| - \beta_1 \right)d\tau$$

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Thus, according to Equation 3-23

\[ \text{if } \beta_1 > \zeta N_d + \frac{\xi N_d}{a_p} \text{, then } P_1(t) \text{ is positive definite function.} \]

In the similar way as above proof, integrating both sides of Equation 3-31

\[ \int_0^t L_2(\tau) d\tau = \int_0^t e_{f_2}^T(\tau) \left( N_d(\tau) - \beta_2 \text{sgn} \left( e_{f_1}(\tau) \right) \right) d\tau \]

\[ = \int_0^t \dot{e}_{f_1}^T(N_d(\tau)) d\tau - \beta_2 \int_0^t \dot{e}_{f_1}^T \text{sgn}(e_{f_1}(\tau)) d\tau + \int_0^t k_f \dot{e}_{f_1}^T \left( N_d(\tau) - \beta_1 \text{sgn}(e_{f_1}) \right) d\tau \]

Using integration by part, the first term of Equation A-2 can be expressed as

\[ \int_0^t \dot{e}_{f_1}^T(N_d(\tau)) d\tau = e_{f_1}^T(t) N_d(t) - e_{f_1}^T(0) N_d(0) - \int_0^t e_{f_1}^T \dot{N}_d(\tau) d\tau \]

Using the following property,

\[ -\beta_2 \int_0^t \dot{e}_{f_1}^T \text{sgn}(e_{f_1}) d\tau = \beta_2 \sum_{i=1}^m |e_{f_1i}(0)| - \beta_2 \sum_{i=1}^m |e_{f_1i}(t)| \]

Equation A-2 can be rewritten as

\[ \int_0^t L_2(\tau) d\tau \leq \beta_2 \sum_{i=1}^m |e_{f_1i}(0)| - e_{f_1}(0)^T N_d(0) + \| e_{f_1}(t) \| (\| N_d(t) \| - \beta_2) \]

\[ + k_f \int_0^t \| e_{f_1}(\tau) \| \left( \| N_d(t) \| + \frac{1}{k_f} \| \dot{N}_d(t) \| - \beta_2 \right) d\tau \]

Thus, according to Equation 3-23, if \( \beta_2 > \zeta N_d + \frac{\xi N_d}{k_f} \), then \( P_2(t) \) is positive definite function.  □
APPENDIX C
PROOF FOR PROPERTIES 2-4

Properties 2-4 The matrix $H(r_p)$ is a symmetric positive definite matrix on the assumption that the maximum eigenvalue of $K_f$ is small enough.

Proof: $H(r_p)$ can be rewritten as

$$H = \begin{bmatrix} (I_m + G_f)K_f^{-1} - M_{11}(t) & 0 \\ 0 & 0 \end{bmatrix} + M(r)$$

(A-3)

If the maximum eigenvalue of $K_f \leq \frac{1}{k_2}$, then the minimum eigenvalue of $(I + G_f)K_f^{-1} \geq k_2$. According to assumption 2-1, $(I_m + G_f)K_f^{-1} - M_{11}(t)$ is a symmetric positive semi-definite matrix. Thus, $H(r_p)$ is a symmetric positive definite matrix on the assumption that the maximum eigenvalue of $K_f$ is small enough. □
APPENDIX D
PROOF FOR PROPERTIES 2-5

Properties 2-5 The matrix \( N_p(r, \dot{r}) = \dot{H}(r_p) - 2V_p(r_p, \dot{r}_p) \) is a skew-symmetric matrix satisfied the following relationship

\[
\xi^T(\dot{H}(r_p) - 2V_p(r_p, \dot{r}_p))\xi = 0 \quad \forall \xi \in \mathbb{R}^n
\]

Proof: According to Equation A-3, \( N_p(r, \dot{r}) \) can be rewritten as the following

\[
N_p = \dot{M}(r) - 2V(r, \dot{r}) - \begin{bmatrix}
\dot{M}_{11}(r) & 0 \\
0 & 0
\end{bmatrix}
\]

From assumption 2-2 and the above equation lead to the properties 2-5. □
REFERENCES


BIOGRAPHICAL SKETCH

Yung-Sheng Chang was born in Taipei, the capital city of Taiwan. He received his bachelor's degree in the Department of Electrical and Control Engineering from the National Chiao Tung University in June 2009. He then began working in Ministry of Nation Defense in August 2009. After that, he continued his education for pursuing master’s degree in the Department of Mechanical Engineering from the University of Florida and joined the Center for Intelligent Machines and Robotics under the advisement of Dr. Carl Crane in August 2011.