TWO-STAGE CHANCE AND EXPECTED VALUE CONSTRAINED STOCHASTIC UNIT COMMITMENT: FORMULATIONS, ALGORITHMS AND CASE STUDIES

By

QIANFAN WANG

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2013
To my parents
ACKNOWLEDGMENTS

I would like to express my earnest appreciation to my advisor Dr. Yongpei Guan for his generous support, advice, and encouragement during my doctoral studies. I thank Dr. Guan for offering me the invaluable opportunity to join his research group to pursue my research interests. Without his inspiration, guidance and supervision throughout my research, this dissertation work would never be finished. Many thanks to Dr. Jianhui Wang for his insightful discussion to develop the research idea in this work and for his helpful suggestions to improve the quality of our three research papers. I am indebted to the rest of my committee members, Dr. Joseph C. Hartman, Dr. William W. Hager and Dr. Guanghui Lan, who made valuable comments and suggestions on this dissertation.

I am grateful to the staff at Department of Industrial and Systems Engineering, Ms. Cynthia Blunt, Ms. Leslie Suzanne Redding, for their administrative supports.

I thank my friends at University of Florida for their friendship, my mentors and managers of the internships at Argonne National Laboratory, SAS Institute and Alstom Grid for sharing their industry experiences with me.

Finally, I appreciate my girlfriend Xiaofei Yue and my family for their encouragement, support and love.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>4</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>8</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>10</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>11</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>1  INTRODUCTION</td>
<td>12</td>
</tr>
<tr>
<td>Challenges of Wind Power Integration</td>
<td>13</td>
</tr>
<tr>
<td>Effect of Wind Power on ISO</td>
<td>13</td>
</tr>
<tr>
<td>Effect of Wind Power on GENCO</td>
<td>15</td>
</tr>
<tr>
<td>Stochastic Programming</td>
<td>16</td>
</tr>
<tr>
<td>Chance-Constrained Stochastic Program</td>
<td>17</td>
</tr>
<tr>
<td>Expected Value Constrained Stochastic Program</td>
<td>17</td>
</tr>
<tr>
<td>Dissertation Outline</td>
<td>18</td>
</tr>
<tr>
<td>2  STOCHASTIC UC WITH UNCERTAIN WIND POWER OUTPUT</td>
<td>21</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>21</td>
</tr>
<tr>
<td>Background and Literature Review</td>
<td>22</td>
</tr>
<tr>
<td>Mathematical Formulation</td>
<td>24</td>
</tr>
<tr>
<td>Problem Formulation</td>
<td>24</td>
</tr>
<tr>
<td>Chance Constraint Description</td>
<td>27</td>
</tr>
<tr>
<td>Policy 1</td>
<td>27</td>
</tr>
<tr>
<td>Policy 2</td>
<td>27</td>
</tr>
<tr>
<td>Policy 3</td>
<td>27</td>
</tr>
<tr>
<td>Sample Average Approximation</td>
<td>28</td>
</tr>
<tr>
<td>Scenario Generation</td>
<td>28</td>
</tr>
<tr>
<td>Solution Validation</td>
<td>30</td>
</tr>
<tr>
<td>Upper bound</td>
<td>30</td>
</tr>
<tr>
<td>Lower bound</td>
<td>31</td>
</tr>
<tr>
<td>Summary of the Combined SAA Algorithm</td>
<td>31</td>
</tr>
<tr>
<td>Methods To Solve The SAA Problem</td>
<td>32</td>
</tr>
<tr>
<td>Sorting Approach</td>
<td>32</td>
</tr>
<tr>
<td>Strong Formulation</td>
<td>32</td>
</tr>
<tr>
<td>Computational Result</td>
<td>34</td>
</tr>
<tr>
<td>Six-Bus System</td>
<td>35</td>
</tr>
<tr>
<td>A 5-scenario example</td>
<td>36</td>
</tr>
<tr>
<td>Experiments at different risk levels</td>
<td>37</td>
</tr>
<tr>
<td>Experiments at different scenario sizes</td>
<td>38</td>
</tr>
</tbody>
</table>
3 STOCHASTIC PRICE-BASED UC

Nomenclature ................................................................. 42
Background and Literature Review ........................................ 44
Mathematical Formulation .................................................... 45
  Market Framework ......................................................... 45
  Problem Formulation ....................................................... 46
Sample Average Approximation ............................................. 49
  SAA Problem ................................................................. 50
  Convergence Analysis and Solution Validation ......................... 51
  SAA Algorithm Framework ............................................... 52
  Heuristics for Solving Each SAA Problem .............................. 53
    Inner upper bound ..................................................... 54
    Inner lower bound .................................................... 54
Computational Results ....................................................... 54
  Scenario Generation for Uncertain Wind Power and Price .......... 55
  Three-Generator System .................................................. 56
    Optimal solution with ten scenarios ................................ 57
    Sensitivity analysis for different risk levels and scenario sizes 58
  Computational Results for a Complicated System ..................... 58
  Multi-Bus System .......................................................... 59
Concluding Remarks .......................................................... 61

4 STOCHASTIC EXPECTED VALUE CONSTRAINED UC

Nomenclature ................................................................. 63
Motivation ..................................................................... 65
Mathematical Formulation .................................................. 66
Solution Methodology ......................................................... 68
  Scenario Generation ....................................................... 68
    MILP reformulation of chance constraint .......................... 69
    Reformulation of SAA problem ....................................... 70
  Solution Validation ....................................................... 70
    Upper bound ............................................................ 71
    Lower bound ........................................................... 72
  Summary of the Combined SAA Algorithm .............................. 74
Computational Results ....................................................... 75
  Six-Bus System ............................................................. 77
  Revised 118-Bus Systems .................................................. 78
    SAA algorithm analysis for 118SW ................................ 78
    Distributed wind power system 118DW ............................... 79
Concluding Remarks .......................................................... 80
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Bus data</td>
<td>35</td>
</tr>
<tr>
<td>2-2</td>
<td>Generator data</td>
<td>36</td>
</tr>
<tr>
<td>2-3</td>
<td>Fuel data</td>
<td>36</td>
</tr>
<tr>
<td>2-4</td>
<td>Transmission line data</td>
<td>36</td>
</tr>
<tr>
<td>2-5</td>
<td>Optimal unit commitment</td>
<td>37</td>
</tr>
<tr>
<td>2-6</td>
<td>Computational results for the six-bus system with different risk levels - using policy 1</td>
<td>38</td>
</tr>
<tr>
<td>2-7</td>
<td>Computational results for the six-bus system with different risk levels - using policy 2</td>
<td>38</td>
</tr>
<tr>
<td>2-8</td>
<td>Computational results for the six-bus system with different risk levels - using policy 3</td>
<td>38</td>
</tr>
<tr>
<td>2-9</td>
<td>Computational results for the 118-bus system</td>
<td>40</td>
</tr>
<tr>
<td>2-10</td>
<td>Computational time for the 118-bus system: MILP and strong formulation</td>
<td>41</td>
</tr>
<tr>
<td>3-1</td>
<td>Generator data</td>
<td>56</td>
</tr>
<tr>
<td>3-2</td>
<td>Fuel data</td>
<td>57</td>
</tr>
<tr>
<td>3-3</td>
<td>Pumped-storage</td>
<td>57</td>
</tr>
<tr>
<td>3-4</td>
<td>Optimal unit commitment</td>
<td>57</td>
</tr>
<tr>
<td>3-5</td>
<td>Computational results for a complicated system for each SAA problem - heuristic method (risk level: 10%)</td>
<td>60</td>
</tr>
<tr>
<td>3-6</td>
<td>Results of solution validation for a complicated system (risk level: 10%)</td>
<td>60</td>
</tr>
<tr>
<td>3-7</td>
<td>Bus settings</td>
<td>61</td>
</tr>
<tr>
<td>3-8</td>
<td>Computational results for distributed system</td>
<td>61</td>
</tr>
<tr>
<td>4-1</td>
<td>Computational results for the six-bus system with different utilization rates</td>
<td>77</td>
</tr>
<tr>
<td>4-2</td>
<td>Computational results for the six-bus system with different risk levels</td>
<td>77</td>
</tr>
<tr>
<td>4-3</td>
<td>Computational results for the 118-bus system with different combinations of iterations and sample sizes</td>
<td>79</td>
</tr>
<tr>
<td>4-4</td>
<td>Computational results for 118DW - without expected value constraint</td>
<td>80</td>
</tr>
</tbody>
</table>
4-5  Computational results for 118DW - with expected value constraint  . . . . . .  80
5-1  Deterministic vs. stochastic . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  85
5-2  Computational results for different risks . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  86
5-3  Computational results for different elasticities . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  87
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Proposed combined SAA algorithm</td>
<td>33</td>
</tr>
<tr>
<td>2-2</td>
<td>Six-bus system</td>
<td>35</td>
</tr>
<tr>
<td>2-3</td>
<td>Wind utilization in each operating hour for the 5 scenario case</td>
<td>37</td>
</tr>
<tr>
<td>2-4</td>
<td>Plotting the solutions of SAA with different scenario sizes</td>
<td>39</td>
</tr>
<tr>
<td>3-1</td>
<td>Proposed SAA algorithm</td>
<td>53</td>
</tr>
<tr>
<td>3-2</td>
<td>Obj.($) of the SAA problem with different risk levels</td>
<td>58</td>
</tr>
<tr>
<td>3-3</td>
<td>Obj.($) of the SAA problem with different scenario sizes</td>
<td>59</td>
</tr>
<tr>
<td>4-1</td>
<td>Proposed SAA algorithm</td>
<td>76</td>
</tr>
<tr>
<td>5-1</td>
<td>Step-wise approximation of price-elastic demand curve</td>
<td>85</td>
</tr>
</tbody>
</table>
Stochastic programming is a common approach to solving decision-making problems under uncertainty in power systems optimization. In this dissertation, we first present a unit commitment model with uncertain wind power output from the perspective of independent system operators (ISOs). The problem is formulated as a chance-constrained two-stage (CCTS) stochastic program. Our model ensures that, with high probability, a large portion of the wind power output at each operating hour will be utilized. Second, CCTS is proposed to study the optimal bidding strategy for independent power producers in the deregulated electricity market. Third, an expected value constrained stochastic unit commitment formulation is considered to handle integrating wind power. Finally, we study the stochastic unit commitment problem with uncertain demand response to enhance the reliability unit commitment process for ISOs. Accordingly, Sample Average Approximation (SAA) algorithms are developed for these stochastic unit commitment models. The computational results indicate the proposed algorithms can solve large-scale power grid optimization problems.
Mathematical optimization has seen a broad range of applications in the electric power grid operation and planning. Applications included optimal power flow, unit commitment, security-constrained economic dispatch, power system expansion, transmission planning, contingency analysis, etc. [72], [76]. Recently, rapid developments in renewable energy and smart grid encourage new research of advanced formulations and algorithms from the optimization society. In particular, as the most crucial decisions for power system operators, unit commitment (UC) is of great concern to the practice in power industry.

The UC problem aims to minimize the power generation cost subject to a set of physical constraints in power system operations. The system operator runs the UC optimization model to obtain the optimal scheduling (commitment and power generation) solution for each generator to satisfy electricity demand. Mixed integer programming (MIP) is applied to model and solve the UC problem in most U.S. electricity markets such as PJM, Midwest ISO (MISO) and ISO New England (ISO-NE) [25] for their market clearings. The reader is referred to [70] for comprehensive introductions of MIP. The deterministic UC problem is formulated as follows:

\[
\begin{align*}
\min_{x,y} & \quad b^T x + c^T y \\
\text{s.t.} & \quad Fx \leq f, \quad (1-2) \\
& \quad Gy \leq g, \quad (1-3) \\
& \quad Ax + By \leq h, \quad (1-4)
\end{align*}
\]

In the above formulation, \( x \) represents the unit commitment decision and \( y \) represents the economic dispatch decision. Constraints (1–2) describe the unit physical constraints.
(e.g., start-up/shut-down, min up/down-time constraints). Constraints (1–3) represent the dispatch constraints (e.g., reserve requirements, transmission limits, demand balance constraints). Finally, constraints (1–4) describe the coupling constraints for decisions $x$ and $y$ (e.g. generation upper/lower limits, ramping-up/down limits).

**Challenges of Wind Power Integration**

A number of initiatives have been launched to increase the utilization of wind power in different countries and regions (e.g., [23] and [1]). With the explosive growth of installed wind power capacity, power system operation paradigms have faced profound challenges. As the electricity market monitor and coordinator, the Independent System Operator (ISO) usually establishes effective wind power integration through market regulation. A market participant, e.g., Independent Power Producer (IPP) or Generation Company (Genco), can actively contribute to wind power integration using their generation portfolio combinations. In this dissertation, we focus on the uncertain wind power integration from the perspectives of ISO and IPP.

**Effect of Wind Power on ISO**

High penetration of wind power has greatly challenged the way the power system has been operated. On one hand, wind power is sustainable and has zero carbon emissions. On the other hand, wind power is intermittent and very difficult to predict. The fluctuation in wind power output requires sufficient ramping capability available in the system to address the inherent variability and uncertainty. The traditional power system operation methods, which were designed to address limited uncertainty in the system such as load variation, have failed to consider the variation from the unprecedented scale of wind power utilization. Hence, large-scale use of wind power production calls for advanced power system operation methods to maintain the security of system operations by better scheduling generation sources.

Research has been done to improve power system operation methods such as unit commitment to accommodate large amounts of wind power. A short-term generation
scheduling model for Wind Power Integration in the Liberalised Electricity Markets (WILMAR) was proposed in [11], [59], and [58]. WILMAR is a stochastic rolling unit commitment model with wind power scenarios. The model has been successfully used in several wind integration studies. Ummels et al. [60] analyzed the impacts of wind power on thermal generation unit commitment and dispatch in the Dutch system, which has a significant share of combined heat and power (CHP) units. Bouffard and Galiana [17] used a stochastic unit commitment model to calculate the reserve requirements by simulating the wind power realization in the scenarios in comparison with the traditional pre-defined reserve requirements. Ruiz et al. [50] proposed a stochastic formulation to manage uncertainty in the unit commitment problem and extended the model to consider uncertainty and variability in wind power by using the same stochastic framework [49]. Wang et al. [63] presented a security-constrained unit commitment (SCUC) algorithm that takes into account the intermittency and variability of wind power generation. Benders’ decomposition was used to decrease the computational requirements brought by a large number of wind power scenarios. A stochastic unit commitment model was proposed in [61]. Various wind power forecasts and different levels of reserve requirements were simulated. It was found that wind power forecast errors have significant impact on unit commitment and dispatch. More recently, the interval optimization approach and the scenario-based method are compared to solve stochastic SCUC with uncertain wind power output in [73]. In [45], both load and wind power uncertainties are considered, and the adaptive particle swarm optimization algorithm is applied to solve the stochastic unit commitment problem. In [54], a quantile-based scenario tree is discussed and compared with other scenario tree formulations through the stochastic unit commitment framework with high wind penetration. In [22], a computational framework incorporating the Weather Research and Forecast (WRF) model is presented. Wind power uncertainty quantification is
addressed by WRF and then taken into account in the stochastic optimization as scenarios.

Most of the models presented so far aim to minimize the overall operating cost, which allows the curtailment of wind power. The wind power curtailment occurs when transmission congestion exists or there is oversupply of wind power due to the technical constraints of the other conventional units such as capacity or minimum on/off time constraints. In this case, the unused wind power becomes a waste. The wind power curtailment will dampen the incentive of wind power investment in the long run and may cause more emissions from the alternative energy sources. For these reasons, it is desirable that the system operators are able to utilize as much wind power as possible. In practice, the system operators in some regions like Germany are required to use renewable energy such as wind power as a priority over the other conventional generation sources [26]. Hence, because of the wind power uncertainty and variability, the system operators have reliability concerns in dispatching their systems with large amounts of wind power while wanting to utilize wind to the largest possible extent at the same time. Therefore, the system operators need to determine a proper unit commitment strategy which can balance the need to spill wind power due to reliability and other reasons while still taking the most advantage of wind power.

**Effect of Wind Power on GENCO**

Instead of incorporating wind power into the unit commitment at the electricity market clearing, an alternative way for an ISO is to distribute the wind power integration to market participants. One common approach is to introduce green certificates to ensure utilization of renewable energy as effectively as possible [32]. This approach is basically imposing the national target for renewable energy utilization on either the demand side including consumers or distribution companies (e.g., Denmark and Germany) [41], or the generation side (e.g., Italy) [41]. If the regulation is applied to the demand side, consumers or distributors will be required to prove that they consume
at least the specified amount of renewable energy by submitting certificates to the authorities at a given time. If the regulation is applied to the generation side, every supplier, except renewable energy producers or importers, is required to ensure that a certain percentage of the energy produced by them, is renewable energy.

This regulation has exerted a large impact on electricity market economics and operations, in particular market participants such as independent power producers (IPPs) that own thermal units as well as renewable generation resources like wind power. Under this regulation, each producer has to utilize as much renewable energy as possible for possible extra profit obtained from the green certificate market. On the other side, an IPP owning traditional thermal units and wind power turbines has to face two-fold uncertainties - price and wind power output uncertainties when submitting bids to the market. If there is a mismatch between the amount submitted in day-ahead and the real-time outputs [28], a penalty will be imposed (e.g., [40] and [46]). Due to the intermittent nature of wind power, significant penalties can be generated. To avoid such significant penalties, an efficient approach to handle the uncertainties is based on the mixed utilization of wind power and pumped-storage units [28].

**Stochastic Programming**

Stochastic programming is a well-known method to tackle the uncertainty in the decision-making process. In the stochastic programming, the uncertain parameter is usually associated with a probability distribution. As an example, consider the classical two-stage stochastic program [14]. The first stage (here-and-now) decisions are made before the uncertainty occurs; the second stage (wait-and-see) decisions vary according to the scenario realization. The general formulation of the two-stage stochastic program with fixed recourse can be described as follows:

\[
\min \ c^T x + E[Q(x, \xi)] \quad s.t. \ Ax = b, \ x \geq 0,
\]

where
\[ Q(x, \xi) = \min \{qy(\xi)|Wy(\xi) = h - Tx, y(\xi) \geq 0\}. \] 

(1–6)

Here, \( x \) denotes the first stage decision variable, \( y(\xi) \) denotes the second-stage decision variable, and \( \xi \) is a random vector. This formulation minimizes the objective function which contains the expected value of recourse cost on second-stage variables.

In the following sections, we introduce the other two branches of stochastic programming: chance-constrained (or probability constrained) stochastic program and expected value constrained stochastic program.

**Chance-Constrained Stochastic Program**

Pioneering work in chance-constrained optimization was done by Charnes, Cooper and Symonds [20]. Chance constraints have been applied to a large class of optimization problems including facility location [12], call center staffing service [30], financial portfolio optimization [27], and optimal power flow [75]. The general chance-constrained stochastic program can be described as follows:

\[ \min_{x \in \mathbb{X}} f(x) \quad s.t. \quad Pr\{G(x, \xi) \leq 0\} \geq 1 - \epsilon. \]

(1–7)

where \( \mathbb{X} \subset \mathbb{R}^n \) denotes the deterministic feasible region, \( f(x) \) represents the objective value to be minimized, \( \xi \) is a random vector whose probability distribution is supported on set \( \Xi \subset \mathbb{R}^d \), \( G : \mathbb{R}^n \times \mathbb{R}^d \to \mathbb{R}^m \) is a constraint mapping, \( 0 \) is an \( m \) dimensional vector of zeros, and \( \epsilon \in (0, 1) \) is given and usually called the risk level of the chance-constrained optimization. This formulation will minimize the objective function over a deterministic feasible set while \( G(x, \xi) \leq 0 \) should be satisfied with a probability of at least \( 1 - \epsilon \).

**Expected Value Constrained Stochastic Program**

Expected value constrained stochastic program is a relatively new method in the stochastic programming. A related concept is the recently developed conditional value-at-risk (CVaR) decision criterion approach [48]. It has been shown recently in [69]...
that the CVaR constrained problem is a special case of the expected value constrained problem. The general expected value constrained stochastic program propose an additional constraint into (1–5) as follows:

$$E[G(x, y(\xi), \xi)] \leq \ell$$

(1–8)

Expected value constrained stochastic program has been applied to supply chain network design and portfolio optimization [68].

**Dissertation Outline**

In Chapter 2, we present a unit commitment problem with uncertain wind power output. The problem is formulated as a chance-constrained two-stage (CCTS) stochastic program. Our model ensures that, with high probability, a large portion of the wind power output at each operating hour will be utilized. The proposed model includes both the two-stage stochastic program and the chance-constrained stochastic program features. These types of problems are challenging and have never been studied together before, even though the algorithms for the two-stage stochastic program and the chance-constrained stochastic program have been recently developed separately. In this chapter, a combined sample average approximation (SAA) algorithm is developed to solve the model effectively. The convergence property and the solution validation process of our proposed combined SAA algorithm is discussed and presented in this chapter. Finally, computational results indicate that increasing the utilization of wind power output might increase the total power generation cost, and our experiments also verify that the proposed algorithm can solve large-scale power grid optimization problems.

In Chapter 3, we propose an optimal bidding strategy for IPPs in the deregulated electricity market. The IPPs are assumed to be price takers, whose objectives are to maximize their profits considering price and wind power output uncertainties, while ensuring high wind power utilization. The problem is formulated as a two-stage
stochastic price-based unit commitment problem with chance constraints to ensure wind power utilization. In our model, the first stage decision includes unit commitment and quantity of electricity submitted to the day-ahead market. The second stage decision includes generation dispatch, actual usage of wind power, and amount of energy imbalance between the day-ahead and real-time markets. The chance constraint is applied to ensure a certain percentage of wind power utilization so as to comply with renewable energy utilization regulations. Finally, the SAA approach is applied to solve the problem, and the computational results are reported for the proposed SAA algorithm showing the sensitivity of the total profit as the requirement of wind power utilization changes.

Chapter 4 proposes an expected value and chance constrained stochastic optimization approach for the unit commitment problem with uncertain wind power output. In our model, the utilization of wind power can be adjusted by changing the utilization rate in the proposed expected value constraint. Meanwhile, the chance constraint is used to restrict the probability of load imbalance. We use the SAA method to transform the expected objective function, the expected value constraint and the chance constraint into sample average reformulations. Furthermore, we discuss a combined SAA framework that considers both the expected value and the chance constraints to construct statistical upper and lower bounds for the optimization problem. Finally, we test the performance of the proposed algorithm with different utilization rates and different risk levels for a six-bus system. We also study a revised IEEE 118-bus system to show the scalability of our model and algorithm.

Chapter 5 studies the stochastic unit commitment problem with uncertain demand response to enhance the reliability unit commitment process for ISOs. Although demand response (DR) encourages customers to voluntarily schedule electricity consumption based on price signals, the response from the consumer side could be uncertain due to a variety of reasons. In this chapter, we use a stochastic representation of DR by
scenario, and each scenario corresponds to a price-elastic demand curve. Contingency constraints are considered and in addition, a chance constraint is applied to ensure the loss of load probability (LOLP) lower than a pre-defined risk level. Then, the SAA method is applied to solve the problem.

Finally, Chapter 6 concludes the dissertation and provides general suggestions for future research.
CHAPTER 2
STOCHASTIC UC WITH UNCERTAIN WIND POWER OUTPUT

Nomenclature

A. Sets and Indices

$BG$ Set of buses with thermal generation units.

$BW$ Set of buses with wind farms.

$B$ Set of all buses.

$\mathcal{E}$ Set of transmission lines linking bus pairs.

$\Lambda_b$ Set of generators at bus $b$.

$T$ Time horizon (e.g., 24 hours).

B. Parameters

$k_{ij}^b$ Line flow distribution factor for transmission line linking bus $i$ and bus $j$ due to the net injection at bus $b$.

$U_{ij}$ Transmission flow limit on transmission line which links bus $i$ and bus $j$.

$D_{bt}$ Demand at bus $b$ in time period $t$.

$\mu_i^b$ Start-up cost for generator $i$ at bus $b$.

$\theta_i^b$ Shut-down cost for generator $i$ at bus $b$.

$\alpha_i^b$ Cost of generating minimum power output for generator $i$ if it is turned on at bus $b$.

$F_c(q_{it}^b)$ Fuel cost for generator $i$ at bus $b$ in time period $t$ when its generation is $q_{it}^b$.

$\gamma_t$ Penalty cost per unit of energy shortage in time period $t$.

$G_i^b$ Minimum-up time for generator $i$ at bus $b$.

$H_i^b$ Minimum-down time for generator $i$ at bus $b$.

© [2012] IEEE. REPRINTED, WITH PERMISSION, FROM [64]
\( R_t \) Amount of spinning reserve needed for the whole power system in time period \( t \).

\( UR^b_i \) Ramp-up rate limit of generator \( i \) at bus \( b \).

\( DR^b_i \) Ramp-down rate limit of generator \( i \) at bus \( b \).

\( LB^b_i \) Lower bound of electricity generated by generator \( i \) at bus \( b \).

\( UB^b_i \) Upper bound of electricity generated by generator \( i \) at bus \( b \).

\( w_{xt}(\xi) \) A random parameter indicating the wind power output or “available capacity” at bus \( b \) in time period \( t \).

C. Decision Variables

\( Q^G_t \) Total amount of electricity generated by thermal units in time period \( t \).

\( Q^{WW}_t \) Total amount of wind power committed to be utilized (delivered) in time period \( t \).

\( \hat{q}^b_t \) Amount of wind power committed to be utilized (delivered) at bus \( b \) in time period \( t \).

\( q^b_{it} \) Amount of electricity generated by generator \( i \) at bus \( b \) in time period \( t \).

\( \alpha^b_{it} \) Binary variable to indicate if generator \( i \) at bus \( b \) is on in time period \( t \).

\( u^b_{it} \) Binary variable to indicate if generator \( i \) at bus \( b \) is started up in time period \( t \).

\( v^b_{it} \) Binary variable to indicate if generator \( i \) at bus \( b \) is shut down in time period \( t \).

\( S^b_t(\xi) \) Amount of energy shortage at bus \( b \) in time period \( t \) (second-stage decision variable).

Background and Literature Review

In this chapter, we present a novel unit commitment model that can take into account wind power forecasting errors while maintaining the system reliability in case
of sudden fluctuations in wind power output. In our model, the system operators can request a portion of the wind power output to be utilized at a certain probability. In this way, the risk of a large amount of wind being curtailed will be adjustable by the operators. We use the chance-constrained optimization technique to formulate the problem to ensure that, with high probability, a large portion of the wind power output at each operating hour will be utilized. Since the wind power uncertainty is captured by a number of wind power scenarios in our approach, a large part of the wind power output, defined by the system operators, will be utilized in a large portion of scenarios. The system operators can refine the unit commitment solution by defining appropriate attitudes toward risk and cost [39]. Some system operators may prefer a lower risk of curtailing the wind power while the others may be prone to spill wind power when system constraints take effect. The risk preference reflects the various treatments of wind power in reality [15].

Chance-constrained optimization has been previously studied to solve the stochastic unit commitment problem with uncertain load in [42] and transmission planning problem in [74]. In [42], with the consideration of the hourly load uncertainty and its correlation structure, the unit commitment problem is initially formulated as a chance constrained optimization problem in which the load is required to be met with a specified high probability over the entire time horizon. In the solution approach, the probability constraint is replaced by a set of separate probability constraints each of which could be inverted to obtain a set of equivalent deterministic linear inequalities. Finally, the deterministic form of the stochastic constraint is used in solving the problem iteratively. In [74], the chance constraint is applied to transmission planning, and it is in the form that the not-overload-probability for the transmission line is required to be more than a specified probability. Two-step optimization process with a genetic algorithm is applied to solve the problem.
In this chapter, the chance constraint is applied to describe policies to ensure the utilization of wind power output. In our approach, different policies lead to different types of chance constraints. Some of these types of constraints could not be inverted to obtain equivalent deterministic linear inequalities. Thus, the algorithm developed in [42] could not be directly applied here to solve our problem. In addition, the algorithm proposed in [74] was not designed to solve two stage stochastic programs, and it also could not be directly applied here to solve our problem. Therefore, we propose to study a sample average approximation (SAA) algorithm to solve the problem. Our approach can provide a solution that converges to the optimal one as the number of samples increases.

Mathematical Formulation

Problem Formulation

Our model contains both chance-constrained and two-stage stochastic program features. The general two-stage and chance-constrained stochastic programs are described in Section 1.

In this chapter, we develop a chance-constrained two-stage stochastic unit commitment formulation, combining (1–7), (1–5), and (1–6), to address uncertain wind power output. We call it a CCTS program, which contains both chance-constrained and two-stage stochastic program features. In our model, the only uncertainty considered is the wind power availability. The first stage of the stochastic program consists of the traditional unit commitment problem with transmission constraints and the decision on the total amount of wind power committed to be utilized (delivered), formed in light of a probabilistic wind power forecast. The second stage represents the penalty cost due to energy shortage once the actual wind power output is known. The chance constraint ensures the utilization of wind power output. The detailed formulation is described as follows (denoted as the true problem).
\[
\min \sum_{b \in B} \sum_{t=1}^{T} \sum_{i \in \Lambda_b} (\mu_i b u_{it}^b + \theta_i b v_{it}^b + \alpha_i b o_{it}^b + F_c(q_{it}^b)) + \sum_{t=1}^{T} \sum_{b \in BW} S_t^b(\xi) \\
\text{s.t.} \quad \text{LB}_i^b o_{it}^b \leq q_{it}^b \leq UB_i^b o_{it}^b, \; \forall i \in \Lambda_b, \forall b, \forall t \tag{2-2}
\]
\[
-o_{i(t-1)}^b + o_{it}^b - o_{ik}^b \leq 0, \quad 1 \leq k - (t - 1) \leq G_i^b, \; \forall i \in \Lambda_b, \forall b, \forall t \tag{2-3}
\]
\[
o_{i(t-1)}^b - o_{it}^b + o_{ik}^b \leq 1, \quad 1 \leq k - (t - 1) \leq H_i^b, \; \forall i \in \Lambda_b, \forall b, \forall t \tag{2-4}
\]
\[
-o_{i(t-1)}^b + o_{it}^b - u_{it}^b \leq 0, \; \forall i \in \Lambda_b, \forall b, \forall t \tag{2-5}
\]
\[
o_{i(t-1)}^b - o_{it}^b - v_{it}^b \leq 0, \; \forall i \in \Lambda_b, \forall b, \forall t \tag{2-6}
\]
\[
q_{it}^b - q_{i(t-1)}^b \leq (2 - o_{i(t-1)}^b - o_{it}^b)LB_i^b + (1 + o_{i(t-1)}^b - o_{it}^b)UR_i^b, \quad \forall i \in \Lambda_b, \forall b, \forall t \tag{2-7}
\]
\[
q_{i(t-1)}^b - q_{it}^b \leq (2 - o_{i(t-1)}^b - o_{it}^b)LB_i^b + (1 - o_{i(t-1)}^b + o_{it}^b)DR_i^b, \quad \forall i \in \Lambda_b, \forall b, \forall t \tag{2-8}
\]
\[
Q_t^G + Q_t^W = \sum_{b \in B} D_{bt}, \; \forall t \tag{2-9}
\]
\[
\sum_{b \in BW} \hat{q}_{it}^b = Q_t^W, \; \forall t \tag{2-10}
\]
\[
\sum_{b \in BW} \sum_{i \in \Lambda_b} q_{it}^b = Q_t^G, \; \forall t \tag{2-11}
\]
\[
\sum_{b \in BG} \sum_{i \in \Lambda_b} UB_i^b o_{it}^b \geq R_t + \sum_{b \in B} D_{bt}, \; \forall t \tag{2-12}
\]
\[
-U_{ij} \leq \sum_{b \in B} k_b^j (\hat{q}_{it}^b + \sum_{n \in \Lambda_b} q_{im}^b - D_{bt}) \leq U_{ij}, \; \forall (i,j) \in E, \forall t \tag{2-13}
\]
\[
Pr(G(x, \xi) \leq 0) \geq 1 - \epsilon \tag{2-14}
\]
\[
S_t^b(\xi) = \max\{0, \hat{q}_{it}^b - w_{bt}(\xi)\}, \forall b, \forall t, \xi \in \Xi \subset \mathbb{R}^{\mathbb{B} \times T} \tag{2-15}
\]
\[
q_{it}^b, \hat{q}_{it}^b \geq 0; o_{it}^b, u_{it}^b, v_{it}^b \in \{0, 1\}, \forall t, \forall i, \forall b. \tag{2-16}
\]
In the above formulation, the decision variables $S_i^b(\xi)$ are second-stage variables, corresponding to $y$ in (1–7)–(1–6), and others are first stage variables, corresponding to $x$ in (1–7)–(1–6). We define $F_c(q^b_k) = c_i^b(q^b_k)^2 + b_i^b q^b_k$. Note here because of constraints (2–2), we have $\alpha_i^b o_i^b + F_c(q^b_k) = (c_i^b(q^b_k)^2 + b_i^b q^b_k + \alpha_i^b) o_i^b$. The objective function (2–1) is composed of power generation costs in the first stage and penalty cost due to energy shortage in the second stage. In our model, for each time period in the second stage, if the wind power output is larger than the amount of wind power committed to be utilized, the excess wind power can be curtailed without penalty, because the utilization of wind power usage is guaranteed by the chance constraint (2–14). Excess wind power will not be sold to a third party with the consideration of potential transmission congestion and other constraints. If the wind power output is less than the amount of wind power committed to be utilized, penalty cost will be triggered due to energy shortage. The hourly UC constraints listed above include the unit generation capacity constraints (2–2), unit minimum-up time constraints (2–3) (e.g., when $o_i^{k+1} = 0$ and $o_i^t = 1$, it means that the unit is turned on in time period $t$. Then in the following $G_i^b$ time periods, it should be on, because we have $-0 + 1 - o_i^k \leq 0$ at this moment, and $o_i^k = 1$ can be guaranteed for at least $G_i^b$ time periods), unit minimum-down time constraints (2–4) (the explanation is similar to the one for (6)), unit start-up constraints (2–5), unit shut-down constraints (2–6), unit ramping up constraints (2–7), unit ramping down constraints (2–8), system power balance constraints (2–9, 2–10, 2–11), system spinning reserve requirements (2–12), and transmission capacity constraints (2–13) (Note that the calculation of $k_{ij}^b$ is the same as the one described in [67]). Constraints (2–2)-(2–13) are first stage constraints; constraint (2–14) indicates that $G(x, \xi) \leq 0$ (for notation brevity, we use $x$ to represent all the first stage decision variables) should be satisfied with a probability of at least $1 - \epsilon$; constraint (2–14) is described in detail in the following part B; constraints (2–15) are the second-stage constraints which indicate the amount of
energy shortage, in case the wind power output is less than the amount of wind power committed to be utilized.

**Chance Constraint Description**

We apply three policies to guarantee the utilization of wind power and develop the corresponding three types of chance constraints with constraint mappings $G^1$, $G^2$ and $G^3$, respectively.

**Policy 1**

Constraint mapping $G^1$ defines that for the entire time planning horizon (24h), there is at least $1 - \epsilon$ chance the usage of the total wind power generation is larger than or equal to $\beta$, $0 < \beta < 100\%$.

$$Pr(\beta \sum_{t=1}^{T} \sum_{b \in BW} w_{bt}(\xi) - \sum_{t=1}^{T} Q_{t}^{W} \leq 0) \geq 1 - \epsilon.$$  

(2–17)

**Policy 2**

Constraint mapping $G^2$ defines that for each particular operating hour on the time planning horizon, there is at least $1 - \epsilon$ chance the usage of the wind power is larger than or equal to $\beta$, $0 < \beta < 100\%$.

$$Pr(\beta \sum_{b \in BW} w_{bt}(\xi) - Q_{t}^{W} \leq 0) \geq 1 - \epsilon, \forall t.$$  

(2–18)

**Policy 3**

Constraint mapping $G^3$ considers the joint probability which is at least $1 - \epsilon$ chance the usage of wind power is larger than or equal to $\beta$, $0 < \beta < 100\%$ for every operating hour.

$$Pr(\beta \sum_{b \in BW} w_{b}(\xi) - Q_{t}^{W} \leq 0) \geq 1 - \epsilon,$$  

(2–19)

where $w_{b}(\xi) = [w_{b1}(\xi), w_{b2}(\xi), \ldots, w_{bT}(\xi)]^T$, $Q_{t}^{W} = [Q_{1}^{W}, Q_{2}^{W}, \ldots, Q_{T}^{W}]^T$, and $0$ is a $T$ dimensional vector of zeros.
From above, it can be observed that policy 3 is most restrictive, while policy 1 is the least restrictive one. In policy 3, we require that at least $\beta$ of wind power is utilized during each of the 24 operating hours to make an outcome of the random wind generation amount a qualified one (i.e., satisfying the chance constraint). Thus, policy 3 is more restrictive than policy 2, which is more restrictive than policy 1.

**Sample Average Approximation**

Sample average approximation (SAA) is an effective method to solve chance-constrained and two-stage stochastic problems. The basic idea of SAA is to approximate the true distribution of random variables with an empirical distribution by Monte Carlo sampling technology. A number of theoretical research and computational studies of SAA have been developed for chance-constrained stochastic problems (e.g., [43] and [37]) and two-stage stochastic problems (e.g., [34] and [5]). However, there is no existing SAA method to solve the model that contains both chance-constrained and two-stage stochastic program features. In this section, we develop a combined SAA algorithm to solve the CCTS program. The combined SAA framework contains three parts: scenario generation, convergence analysis, and solution validation. For each SAA problem, we solve the corresponding mixed-integer-linear program (MILP) efficiently by developing a strong formulation. The details are shown in the following subsections.

**Scenario Generation**

In SAA, the true distribution of wind power generation is replaced by an empirical distribution using computer simulation. We use Monte Carlo simulation to generate scenarios. Assume the wind power is subject to a multivariate normal distribution $\mathcal{N}(\mu, \Sigma)$ (one of many possible distributions of wind power) for every time period $t$, where vector $\mu$ is chosen as the forecasted wind power and matrix $\Sigma$ describes the volatility. The Monte Carlo simulation generates a large number of scenarios, each with the same probability $1/N$. In each scenario, there are 24 hourly, random wind power outputs based on the forecasted generation. To decrease the variance of simple Monte
Carlo simulation, the Latin hypercube sampling (LHS) is employed to make the statistical distribution fit the real distribution better [63].

After the scenarios are generated (e.g., \( N \) scenarios), the expected value function \( E[\sum_{b \in BW} S^b_t(\xi)] \) is estimated by the sample average function \( N^{-1} \sum_{j=1}^N \sum_{b \in BW} S^b_t(\xi^i) \) (see, e.g., [34]). On the other hand, in general, the chance constraint can be estimated by an indicator function \( N^{-1} \sum_{j=1}^N 1_{(0, \infty)}(G(x, \xi^i)) \leq \epsilon \) (see, e.g., [43]), which requires that a certain percentage of the samples satisfy the chance constraint. The value of the indicator function \( 1_{(0, \infty)}(G(x, \xi^i)) \) is equal to one when \( G(x, \xi^i) \leq \epsilon \) or zero when \( G(x, \xi^i) \leq 0 \). The corresponding formulation is shown as follows (denoted as the SAA problem):

\[
\begin{align*}
\min & \quad \sum_{b \in BW} \sum_{t=1}^T \sum_{i \in A_b} (\mu^b_i u^b_i + \theta^b_i v^b_i + \alpha^b_i o^b_i + \gamma^b_i q^b_i) + N^{-1} \sum_{t=1}^T \sum_{j=1}^N \sum_{b \in BW} S^b_t(\xi^i) \\
\text{s.t.} & \quad (2 - 2) - (2 - 13) \\
& \quad N^{-1} \sum_{j=1}^N 1_{(0, \infty)}(G(x, \xi^i)) \leq \epsilon \\
& \quad q^b_i \leq w_{bt}(\xi^i) + S^b_t(\xi^i) \\
& \quad (\forall t; \forall b \in BW; j = 1, 2, \ldots, N) \\
& \quad q^b_i, q^b_t, S^b_t(\xi^i) \geq 0; u^b_i, v^b_i, o^b_i \in \{0, 1\}, \forall t, \forall i, \forall b.
\end{align*}
\]

First, as the sample size \( N \) goes to infinity, we can prove that the objective of the above formulation converges to that of the true problem as shown in the following proposition.

**Proposition 2.1.** Let \( \hat{\theta}_N \) represent the objective value of the SAA problem, and \( \theta^* \) represent the objective value of the true program. We have \( \hat{\theta}_N \rightarrow \theta^* \) and \( D(\hat{x}_N, x^*) \rightarrow 0 \) w.p.1 as \( N \rightarrow \infty \), where \( D(\hat{x}_N, x^*) \) represents the distance between the optimal solution \( \hat{x}_N \) for the SAA problem and the optimal solution \( x^* \) for the true problem.

**Proof.** The details of the convergence proof are given in Appendix A. \( \square \)

Next, we discuss the solution validation process in the following subsection.
Solution Validation

Solution validation for the two-stage and chance-constrained problems have been well studied in [43] and [5], respectively. In this section, we develop a combined algorithm that embeds the solution validation of the chance-constrained problem into that of the two-stage problem.

Assume that \( \bar{x} \) is an optimal solution for the SAA problem, and \( \bar{v} \) is the corresponding objective value. For a given candidate solution for the SAA problem, solution validation provides a scheme to validate its quality by obtaining upper and lower bounds for the corresponding optimal objective value. We construct the upper and lower bounds as follows:

**Upper bound**

Since CCTS contains a chance constraint, we start with the verification of feasibility of the given solution \( \bar{x} \). To do this, we first estimate the true probability function of the chance constraint

\[
q(\bar{x}) = Pr\{G(\bar{x}, \xi) > 0\}. \quad (2–20)
\]

Following the method described in [6] and [43], we construct a \((1 - \tau)\)-confidence upper bound on \(q(\bar{x})\):

\[
U(\bar{x}) = \hat{q}_{N'}(\bar{x}) + z_\tau \sqrt{\hat{q}_{N'}(\bar{x})(1 - \hat{q}_{N'}(\bar{x}))/N'}, \quad (2–21)
\]

where \( N' \) is the sample size for the validation of the chance constraint, and \( \hat{q}_{N'}(\bar{x}) \) is the estimated value of \( q(\bar{x}) \) for the given sample size \( N' \).

If this upper bound of \( q(\bar{x}) \) is less than the risk level \( \epsilon \), then \( \bar{x} \) is feasible with confidence level \((1 - \tau)\). Then, we can evaluate the corresponding upper bound of the optimal value for the second-stage part in CCTS, the same as the validation process for the normal two-stage stochastic problem as described in [5]:

\[
U(\bar{v}) = c^T \bar{x} + \frac{1}{N'} \sum_{n=1}^{N'} Q(x, \xi^n). \quad (2–22)
\]
It is easy to see that $U(\bar{v})$ is the upper bound for CCTS.

**Lower bound**

To get the lower bound for the objective value $\bar{v}$, we take $\hat{S}$ iterations. For each iteration $1 \leq s \leq \hat{S}$, we run the $N$ scenario SAA problem $M$ times. For these $M$ runs, we follow the same scheme as the one described in [6] and [43] to pick the $Lth$ smallest optimal value, denoted as $\bar{v}_{ls}$, as the approximated lower bound for the chance-constrained part with confidence level $(1 - \tau)$, where $L$ is calculated as described in [43]. Finally, taking the average of $\{\bar{v}_{ls}, 1 \leq s \leq \hat{S}\}$ provides the lower bound for CCTS.

**Summary of the Combined SAA Algorithm**

In the algorithm, we put the calculation of the upper bound for CCTS in the loop of the calculation of the lower bound for the chance-constrained part in order to speed up the algorithm. The proposed combined SAA algorithm is summarized in the following steps (also see flowchart in Fig. 2-1).

1. For $s = 1, 2, \ldots, \hat{S}$, repeat the following steps:
   
   (a) For $m = 1, 2, \ldots, M$, repeat the following steps:
   
   i. Solve the associated SAA with $N$ scenarios. Denote the solution as $\bar{x}_m$ and the optimal value as $\bar{v}_m$;
   
   ii. Generate scenarios $\xi^1, \xi^2, \ldots, \xi^{N'}$. Estimate $q(\bar{x}_m)$ by $\hat{q}_{N'}(\bar{x}_m)$ and use (2–21) to get $U(\bar{x}_m)$;
   
   iii. If $U(\bar{x}_m) \leq \epsilon$, go to (d); else, skip (d) and go to next iteration;
   
   iv. Estimate the corresponding upper bound for CCTS using (2–22), based on the $N'$ scenarios generated in (b);
   
   (b) Pick the smallest upper bound in Step (1) as the approximated upper bound $\hat{g}^\xi$;
   
   (c) Sort the $M$ optimal values obtained in Step (1) in nondecreasing order, e.g., $\bar{v}_1 \leq \bar{v}_2 \leq \ldots \bar{v}_M$. Pick the $Lth$ optimal value $\bar{v}_L$ and denote it as $\bar{v}_{ls}$.

2. Taking the average of $\bar{v}_{l_1}, \bar{v}_{l_2}, \ldots, \bar{v}_{l_{\hat{S}}}$, we get the lower bound $\bar{v} = \frac{1}{\hat{S}} \sum_{s=1}^{\hat{S}} \bar{v}_{ls}$.
3. Taking the minimum of $\hat{g}^1, \hat{g}^2, \ldots, \hat{g}^S$, we get the upper bound $\hat{g} = \min_{1 \leq s \leq S} \hat{g}^s$.

4. Estimate the optimality gap given by $(\hat{g} - \bar{\nu})/\bar{\nu} \times 100\%$.

**Methods To Solve The SAA Problem**

The SAA problem is an MILP. We can use the standard branch and bound algorithm which is implemented in most commercial solvers. The main problem is how to solve the SAA problem effectively with the chance constraints under different policies. In this section, we introduce a sorting approach for policies 1 and 2. For policy 3, we derive a strong formulation as studied in [37] to speed up the algorithm.

**Sorting Approach**

After taking samples, we can simplify (2–17) by sorting the right-hand-side values of the constraints for each sample, (i.e., wind power in each day) and picking the $\lceil (1 - \epsilon) \times N \rceil$th right-hand-side value to construct a deterministic constraint. Similarly, after taking samples, we can simplify (2–18) by sorting the right-hand-side values of the constraints for each sample (i.e., wind power in each hour) and picking the $\lceil (1 - \epsilon) \times N \rceil$th right-hand-side value to construct a deterministic constraint.

**Strong Formulation**

It can be observed that the sorting method does not work for (2–19) because the sorting algorithm cannot handle the joint probability case described in (2–19). Instead, reformulating as an MILP allows solution of the problem incorporating (2–19). For a given sample size $N$, constraint (2–19) can be reformulated as follows:

$$\beta \sum_{b \in BW} w_b(\xi^j) - Q_t^W \leq M \times z_j$$  \hspace{1cm} (2–23)

$$\sum_{j=1}^{N} z_j \leq N \times \epsilon$$  \hspace{1cm} (2–25)

$$z_j \in \{0, 1\}$$  \hspace{1cm} (2–26)

$$j = 1, 2, \ldots, N).$$  \hspace{1cm} (2–27)
Figure 2-1. Proposed combined SAA algorithm
The star-inequalities can speed up the computation of the above MILP model (see, e.g., [43], [37], and [29]). Moreover, it has been proved that the MILP model can be transformed into a strong formulation after adding the star-inequalities [37]. To do this, we introduce a new set of binary variables \( \{ r_{ij} : j = 1, 2, \ldots, q; t = 1, 2, \ldots, T \} \) and define \( h_{tj} = \beta \sum_{b \in B_b} w_{bt}(\xi^t) \). Without loss of generality, we assume that \( h_{t1} \geq h_{t2} \geq \cdots \geq h_{tN} \).

The strong formulation is described as follows:

\[
\begin{align*}
    r_{tj} - r_{t(j+1)} & \geq 0 \quad (j = 1, 2, \ldots, q; t = 1, 2, \ldots, T) \\
    z_{[j]} - r_{tj} & \geq 0 \quad (j = 1, 2, \ldots, q; t = 1, 2, \ldots, T) \\
    Q_t^W + \sum_{j=1}^q (h_{tj} - h_{t(j+1)})r_{tj} & \geq h_{t1} \quad (t = 1, 2, \ldots, T) \\
    \sum_{j=1}^N z_j & \leq N \times \epsilon \\
    r_{tj}, z_j & \in \{0, 1\},
\end{align*}
\]

where \([j]\) represents the scenario index corresponding to the \(j\)th largest \( h \) value in time period \( t \), and inequalities (2–31) are the star inequalities.

**Computational Result**

In this section, a six-bus system and a revised 118-bus system are studied to illustrate the proposed algorithms. In the six-bus system, we run the computational experiments at different risk levels to compare the results of different policies. The solution validation is neglected for simplicity, and computational experiments at different sample sizes are tested to verify the convergence property of the combined SAA algorithm. In the revised 118-bus system, we run the computational experiments to test the entire combined SAA algorithm described in Sections III and IV. The algorithm
Table 2-1. Bus data

<table>
<thead>
<tr>
<th>Bus ID</th>
<th>Type</th>
<th>Unit</th>
<th>Wind Farm</th>
<th>Hourly Load (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B₁</td>
<td>Thermal</td>
<td>G₁</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B₂</td>
<td>Thermal</td>
<td>G₂</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B₃</td>
<td>Thermal</td>
<td>-</td>
<td>W₁</td>
<td>300</td>
</tr>
<tr>
<td>B₄</td>
<td>Wind</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B₅</td>
<td>Thermal</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>B₆</td>
<td>Thermal</td>
<td>G₃</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

is coded in C++ using CPLEX 12.1. All experiments are implemented on a computer workstation with Intel Quad Core 2.40GHz and 8GB memory.

**Six-Bus System**

The six-bus system includes three generators, one wind farm, three loads, and six transmission lines. The layout of the system is depicted in Fig. 2-2, and the characteristics of the buses, thermal units, and transmission lines are described in Tables 2-1-2-4.

![Six-bus system](image)

Figure 2-2. Six-bus system

We assume the wind farm is located at bus $B₄$, $β = 85\%$, and $γ_t = 600$, $∀t$. The wind power is assumed multivariate normal distributed, with the hourly mean forecasted outputs ranging between $10 - 100$ and a standard deviation of $45\%$ of the expected values. To run the model in CPLEX effectively, we use the interpolation method [57] to
approximate the fuel cost function. A piecewise linear function replaces the fuel cost function in (2–1).

A 5-scenario example

We first give an example with 5 scenarios to illustrate Policy 3. Based on the chance constraint description in Policy 3 with $\epsilon = 20\%$, there is only one scenario in which there are time periods whose wind utilisations are less than 85%. From the results shown in Figure 2-3, we can observe that the chance constraint is satisfied and only scenario 3 can have utilization below 85%. The unit commitment results in the optimal solution are listed in Table 2-5.
Figure 2-3. Wind utilization in each operating hour for the 5 scenario case

Table 2-5. Optimal unit commitment

<table>
<thead>
<tr>
<th>Hours (1-24)</th>
<th>G₁</th>
<th>G₂</th>
<th>G₃</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>1 1 1 1 0 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0</td>
</tr>
</tbody>
</table>

Experiments at different risk levels

Next, we use 200 scenarios to run experiments on different risk levels. The results are reported in Tables 2-6, 2-7, and 2-8 for comparison.

It can be observed that the total cost (column “obj.” in Table 2-8) is reduced as the risk level increases from 15% to 100%. This is reasonable because the fuel cost for thermal plants might be higher if the policy on wind power generation is more restrictive and less wind power will be curtailed. An extreme case is $\epsilon = 100\%$ in which the chance constraint can be neglected. In such a case, the optimal cost is smaller than that at any other risk level. Meanwhile, the wind utilization is at its lowest value as well (below 50%). Here the wind utilization is measured as the average wind usage under all scenarios, which is equal to the ratio between $\sum_{t=1}^{T} \sum_{j=1}^{200} \sum_{b \in BW} \min \{ \tilde{q}_t^b, w_{bt}(\xi) \}$ and $\sum_{t=1}^{T} \sum_{j=1}^{200} \sum_{b \in BW} w_{bt}(\xi)$. From Tables 2-6-2-8, we can also observe that Policy 3 is
the most restrictive one. For the same given risk level, the wind power utilization is the highest among all three polices.

Table 2-6. Computational results for the six-bus system with different risk levels - using policy 1

<table>
<thead>
<tr>
<th>Risk Level $\epsilon$</th>
<th>Obj. ($)</th>
<th>Utilization</th>
<th>CPU Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>53038.3</td>
<td>84.01%</td>
<td>0.29</td>
</tr>
<tr>
<td>20%</td>
<td>52541.3</td>
<td>83.04%</td>
<td>0.31</td>
</tr>
<tr>
<td>40%</td>
<td>51473.3</td>
<td>81.92%</td>
<td>0.25</td>
</tr>
<tr>
<td>70%</td>
<td>48235.5</td>
<td>75.01%</td>
<td>0.29</td>
</tr>
<tr>
<td>100%</td>
<td>46213.8</td>
<td>48.9%</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table 2-7. Computational results for the six-bus system with different risk levels - using policy 2

<table>
<thead>
<tr>
<th>Risk Level $\epsilon$</th>
<th>Obj. ($)</th>
<th>Utilization</th>
<th>CPU Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>63315.5</td>
<td>90.00%</td>
<td>0.02</td>
</tr>
<tr>
<td>20%</td>
<td>60462.5</td>
<td>88.75%</td>
<td>0.07</td>
</tr>
<tr>
<td>40%</td>
<td>53782.1</td>
<td>83.57%</td>
<td>0.08</td>
</tr>
<tr>
<td>70%</td>
<td>48699.1</td>
<td>75.09%</td>
<td>0.10</td>
</tr>
<tr>
<td>100%</td>
<td>46213.8</td>
<td>48.9%</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table 2-8. Computational results for the six-bus system with different risk levels - using policy 3

<table>
<thead>
<tr>
<th>Risk Level $\epsilon$</th>
<th>Obj. ($)</th>
<th>Utilization</th>
<th>CPU Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>79372.5</td>
<td>93.86%</td>
<td>1.47</td>
</tr>
<tr>
<td>20%</td>
<td>76850.5</td>
<td>93.5%</td>
<td>2.07</td>
</tr>
<tr>
<td>40%</td>
<td>69268.3</td>
<td>92.2%</td>
<td>3.85</td>
</tr>
<tr>
<td>70%</td>
<td>62789.5</td>
<td>90.14%</td>
<td>53.28</td>
</tr>
<tr>
<td>100%</td>
<td>46213.8</td>
<td>48.9%</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Experiments at different scenario sizes

We have shown the convergence property of the combined SAA algorithm in Section III. Here, we set different sample sizes for the SAA algorithm to verify that the optimal solution indeed converges as the scenario size increases (e.g., see Fig. 2-4). Policy 3 is applied and the risk level is set to $\epsilon = 10\%$ for comparison.
A modified IEEE 118-bus system, based on the one given online at motor.ece.iit.edu/data, is used to test the SAA algorithm. We select all the generators that use coal as the fuel. In total, there are 33 thermal generators. We take all 186 transmission lines and 91 loads. Since there are only 33 generators, we reduce the value of the load at each bus to ensure the existence of a solution. Additionally, we consider a wind farm at Bus 3, the risk level $\epsilon = 10\%$, and $\beta = 85\%$. The detailed revised 118-bus system data is given online at cso.ise.ufl.edu/data_r118.xls.

We then apply policy 3, and the computational results are reported in Table 2-9. The first column represents the combination of iteration numbers ($\hat{S}, M$) (i.e., iteration numbers for obtaining the lower bound and for the chance constraint part), and validation’s scenario size ($N'$). The second column represents the scenario size of the SAA problem. The third column represents the lower bound obtained by the SAA algorithm. The fourth column represents the upper bound obtained by the SAA algorithm.

Figure 2-4. Plotting the solutions of SAA with different scenario sizes

**Modified 118-Bus System**

![Graph showing the relationship between Optimal Objective Value and Scenario Size.](image-url)
algorithm. The fifth column represents the gap which is calculated by \( \frac{UB - LB}{LB} \times 100\% \). Finally, the sixth column represents the CPU time of the algorithm.

Table 2-9. Computational results for the 118-bus system

<table>
<thead>
<tr>
<th>((S \times M, N'))</th>
<th>(N)</th>
<th>LB</th>
<th>UB</th>
<th>Gap</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((5 \times 5, 1000))</td>
<td>10</td>
<td>470290</td>
<td>477184</td>
<td>1.46%</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>472839</td>
<td>477210</td>
<td>0.9%</td>
<td>6.75</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>473578</td>
<td>477857</td>
<td>0.9%</td>
<td>10.2</td>
</tr>
<tr>
<td>((5 \times 20, 2000))</td>
<td>10</td>
<td>471994</td>
<td>475233</td>
<td>0.6%</td>
<td>15.8</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>474271</td>
<td>476325</td>
<td>0.4%</td>
<td>25.4</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>474560</td>
<td>477562</td>
<td>0.6%</td>
<td>40.3</td>
</tr>
<tr>
<td>((20 \times 20, 3000))</td>
<td>10</td>
<td>472356</td>
<td>473995</td>
<td>0.34%</td>
<td>64.5</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>474692</td>
<td>476230</td>
<td>0.32%</td>
<td>103.8</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>474701</td>
<td>476125</td>
<td>0.30%</td>
<td>145.8</td>
</tr>
</tbody>
</table>

From Table 2-9, we can observe that as the iteration number and the sample size increase, the optimality gap decreases. For the last case in which the sample size is 100, the iteration numbers are \( \hat{S} = M = 20 \), and the validation scenario size \( N' = 3000 \), the optimality gap is around 0.30\%. That is, the proposed algorithm converges fast and can solve the problem effectively.

Finally, we compare the performance between the default MILP and the strong formulation approaches, and report the results in Table 2-10. It can be observed from the table that the strong formulation approach takes much less time than the default MILP approach when the risk level is not trivial (e.g., \( 0 < \epsilon < 100\% \)). The results show the scalability of the strong formulation approach to solve large-scale problems. It can also be observed from the table that the optimal objective value decreases as the risk level increases.

**Concluding Remarks**

In this chapter, a chance constrained two-stage stochastic program considering the uncertain wind power output was studied. In our approach, the chance constraint guarantees the minimum usage of the wind power by setting a risk level, which limits
Table 2-10. Computational time for the 118-bus system: MILP and strong formulation

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>$N$</th>
<th>Obj.(§)</th>
<th>MILP (sec)</th>
<th>Strong (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>100</td>
<td>477463</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>481947</td>
<td>0.2</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>482059</td>
<td>0.23</td>
<td>0.21</td>
</tr>
<tr>
<td>20%</td>
<td>100</td>
<td>472759</td>
<td>8.08</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>472833</td>
<td>54.16</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>473072</td>
<td>154.51</td>
<td>0.57</td>
</tr>
<tr>
<td>100%</td>
<td>100</td>
<td>468712</td>
<td>0.19</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>468924</td>
<td>0.21</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>468883</td>
<td>0.25</td>
<td>0.29</td>
</tr>
</tbody>
</table>

the chance that a large amount of wind power might be curtailed. We studied three different types of policies and compared the wind utilizations by these policies. The results verified that Policy 3 is the most restrictive one. Then, we studied a combined SAA algorithm that can derive an optimal solution when the sample size increases. The final computational results verify the effectiveness of the proposed SAA algorithm and the related solution validation process, and show that the proposed model can help increase the usage of wind power.
A. Sets and Indices

\( N \) Number of scenarios.

\( T \) Time horizon (24 hours).

\( B \) Number of buses.

\( \Lambda_b, \Gamma_b \) Sets of thermal generators and hydro units in bus \( b \), respectively.

B. Parameters

\( SU^b_i \) Start-up cost of thermal generator \( i \) in bus \( b \).

\( SD^b_i \) Shut-down cost of thermal generator \( i \) in bus \( b \).

\( F_c(p^b_t) \) Fuel cost of thermal generator \( i \) in time \( t \) at bus \( b \) when its generation is \( p^b_t \).

\( MU^b_i \) Minimum-up time for thermal generator \( i \) in bus \( b \).

\( MD^b_i \) Minimum-down time for thermal generator \( i \) in bus \( b \).

\( UR^b_i \) Ramp-up rate limit for thermal generator \( i \) in bus \( b \).

\( DR^b_i \) Ramp-down rate limit for thermal generator \( i \) in bus \( b \).

\( L^b_i \) Lower limit of electricity generated by thermal generator \( i \) in bus \( b \).

\( U^b_i \) Upper limit of electricity generated by thermal generator \( i \) in bus \( b \).

\( S_{ib}^{begin} \) Water reserve level of pumped-storage unit \( i \) in bus \( b \) in the first time period.

\( S_{ib}^{end} \) Water reserve level of pumped-storage unit \( i \) in bus \( b \) in the last time period.

\( L^H_{ib} \) Lower limit of power pumped in/out by pumped-storage unit \( i \) in bus \( b \) in one time period.

\( U^H_{ib} \) Upper limit of power pumped in/out by pumped-storage unit \( i \) in bus \( b \) in one
Penalty cost per MW for energy imbalance in time $t$ in bus $b$.

Efficiency of generating power by the pumped-storage unit.

Efficiency of absorbing power by the pumped-storage unit.

C. Decision Variables

Electricity generation amount by thermal generator $i$ in time $t$ in bus $b$.

Binary decision variable: “1” if thermal generator $i$ is on in time $t$ in bus $b$; “0” otherwise.

Binary decision variable: “1” if thermal generator $i$ is started up in time $t$ in bus $b$; “0” otherwise.

Binary decision variable: “1” if thermal generator $i$ is shut down in time $t$ in bus $b$; “0” otherwise.

Binary decision variable to indicate whether pumped-storage unit $i$ generates (e.g., “1”) or consumes (e.g., “0”) power in time $t$ in bus $b$.

Water reserve level of pumped-storage unit $i$ in time $t$ in bus $b$.

Electricity bid into the day-ahead market in time $t$ in bus $b$.

Total amount of thermal unit generation in time $t$ in bus $b$.

Wind power sold in the real-time market in time $t$ in bus $b$.

Power generated from pumped-storage unit $i$ in time $t$ in bus $b$.

Power consumed by pumped-storage unit $i$ in time $t$ in bus $b$.

Power imbalance in time $t$ in bus $b$.

Note: Some of these decision variables are second stage variables when they are followed by $(\xi)$, where $\xi$ represents a random vector following a certain probabilistic distribution.

D. Random Numbers
$R_{tb}^{DA}(\xi)$  Day-ahead market price of electricity in time $t$ in bus $b$.

$R_{tb}^{RT}(\xi)$  Real-time market price of electricity in time $t$ in bus $b$.

$W_{tb}(\xi)$  Wind power output in time $t$ in bus $b$.

**Background and Literature Review**

In this chapter, we propose to study the optimal bidding strategy for an IPP whose generation portfolio may consist of thermal, hydro and wind power units. The objective of an IPP’s self-scheduling problem is to maximize its profit while ensuring high utilization of wind power output to comply with related regulations. In our approach, the IPP is assumed to be a price taker in the market, which means the IPP does not have control over the market prices by bidding strategically. One of the reasons for the IPP to be a price taker is its relatively small share of generation in the total generation capacity in the market. Since the IPP is a price-taker, the market prices are purely input to the IPP’s own profit maximization problem. Thus, the objective of the IPP is to optimize its own generation portfolio based on the forecasted day-ahead and real-time price information. With this price-based decision making, the IPP tries to come up with its best generation schedule that can be bid into the market subject to physical constraints of the generators such as min on/off, capacity limits, etc. Along with the generation quantity obtained from the optimization, the IPP can bid a low price to ensure the acceptance of its bids in the market. This problem is typically defined as the Price-based Unit Commitment (PBUC) problem as shown in the literature (see, e.g., [9, 35, 36, 52]). In this chapter, the IPP is considered to operate and schedule a few number of thermal generators, several wind farms and pumped-storage units. We propose a novel bidding strategy that can maximize the expected profit with the consideration of wind power forecasting errors and ensure high utilization of wind power output for IPPs.

The chance constraint requires a certain probability at which a given portion of wind power must be utilized. For example, we can define a probability of 95% at which the utilization of wind power is no smaller than a certain number (e.g., 85%). Applications
of chance-constrained optimization in power system have been studied recently [42], [74], [64] and [65]. However, none of the previous studies has investigated wind power bidding strategies with chance constraints, which is the main focus of this chapter.

In this chapter, we study a sample average approximation (SAA) method to solve the chance constrained power producer bidding problem. As compared to the SAA algorithms recently developed for the two-stage stochastic program described in [64], [34] and [5], and for the chance-constrained single-stage stochastic program described in [43] and [37], we develop a different SAA algorithm due to the binary decision variables for the hydro units in the second stage. The validation process and convergence proof of the proposed approach are also investigated. The case studies show the proposed algorithm in this chapter provides tight lower and upper bounds, which illustrate the effectiveness of our approach.

The contributions of this chapter are summarized as follows:

1. Much of the previous research studied the stochastic unit commitment problem from an ISO’s perspective. In this chapter, our contributions focus on the price-based unit commitment (PBUC) from a price-taking IPP’s perspective. A two-stage stochastic programming model is studied to solve the problem.

2. Compared with other works in PBUC [35, 36], this is the first work to apply the chance-constrained stochastic programming to address bidding strategies on a thermal-wind-hydro generation portfolio.

3. The SAA framework is adjusted specifically for our chance-constrained stochastic programming model. We also propose a heuristic-based SAA algorithm for this specific problem structure.

**Mathematical Formulation**

**Market Framework**

In a deregulated electricity market, the IPPs submit bids each day, and the market operator provides the market clearing prices of electricity. The IPPs with both thermal and wind power units in the generation portfolio face at least two main sources of uncertainties: market prices and variable wind power output. IPPs have to consider
the uncertainty of wind power in their bidding as their wind power forecasts will not be perfectly accurate and errors always exist. In addition, some of the electricity markets (such as PJM [4]) are enforcing penalties on the mismatch between IPPs’ day-ahead bids and their actual generation in the real-time market. One can easily observe that uncertain wind power output contributes significantly to the mismatch. In the meantime, IPPs can not choose to abandon too much wind power to avoid its fluctuation as they may be subject to certain regulations that require renewable energy like wind power accounting for a certain share of their total generation output [41]. Hence, IPPs need to optimize their bidding strategies that can balance the objective of profit maximization and the risks associated with wind power realizations, while making sure the wind power is utilized to the greatest degree. Based on the above discussion, the proposed method in this chapter models such a bidding process for IPPs by formulating the problem as a two-stage stochastic program. To maximize the total expected profit, the IPPs decide the quantity to be submitted to the market in the first stage (day-ahead market), considering the possible realizations of uncertain market prices and wind power output in the second stage (real-time market). Chance constraints are used to model the least percentage of wind power utilization. Our modeling framework also captures the two-settlement (day-ahead and real-time) market procedure as in most of the U.S. markets by modeling the two markets in the objective function (e.g., PJM [4]).

Problem Formulation

The PBUC problem is formulated as a two-stage chance constrained stochastic programming problem. The first stage of the model is a unit commitment problem with decisions on commitment and quantity of the electricity offered to the day-ahead market. The second stage of the model is an economic dispatch problem with decisions on thermal and hydro unit dispatch, the actual usage of wind power, and energy imbalance. We consider the chance constraint at the second stage, in which the actual wind power
used could be different from the wind power output. We describe the final formulation of the problem (denoted as the true problem) as follows.

\[
\begin{align*}
\max & - \sum_{t=1}^{T} \sum_{b=1}^{B} \sum_{i \in \Lambda_b} (SU^b_{it}o^b_{it} + SD^b_{it}v^b_{it}) \\
& + E[Q(y, o, v, q^B, \xi)] \\
\text{s.t.} & \quad -y^b_{i(t-1)} + y^b_{it} - y^b_{ik} \leq 0, \\
& \quad 1 \leq k - (t - 1) \leq MU^b_{it}, \forall i \in \Lambda_b, \forall b, \forall t \tag{3-2} \\
& \quad y^b_{i(t-1)} - y^b_{it} + y^b_{ik} \leq 1, \\
& \quad 1 \leq k - (t - 1) \leq MD^b_{it}, \forall i \in \Lambda_b, \forall b, \forall t \tag{3-3} \\
& -y^b_{i(t-1)} + y^b_{it} - o^b_{it} \leq 0, \\
& \quad \forall i \in \Lambda_b, \forall b, \forall t \tag{3-4} \\
& \quad y^b_{i(t-1)} - y^b_{it} - v^b_{it} \leq 0, \\
& \quad \forall i \in \Lambda_b, \forall b, \forall t \tag{3-5} \\
& \quad y^b_{it}, o^b_{it}, v^b_{it} \in \{0, 1\}, \forall i \in \Lambda_b, \forall b, \forall t, \tag{3-6}
\end{align*}
\]

where \(Q(y, o, v, q^B, \xi)\) is equal to

\[
\begin{align*}
\max & \sum_{t=1}^{T} \sum_{b=1}^{B} (R^{DA}_{tb}(\xi)q^B_{tb} + R^{RT}_{tb}(\xi)q^{imb}_{tb}(\xi)) \\
& - \sum_{t=1}^{T} \sum_{b=1}^{B} \sum_{i \in \Lambda_b} F_c(p^b_{it}(\xi)) - \sum_{t=1}^{T} \sum_{b=1}^{B} \gamma^b_{tb} |q^{imb}_{tb}(\xi)| \\
\text{s.t.} & \quad L^h_b y^b_{it} \leq p^b_{it}(\xi) \leq U^b_{it} y^b_{it}, \forall i \in \Lambda_b, \forall b, \forall t \tag{3-7} \\
& \quad p^b_{it}(\xi) - p^b_{i(t-1)}(\xi) \leq (2 - y^b_{it(t-1)} - y^b_{it})L^b_{it} + (1 + y^b_{i(t-1)} - y^b_{it})UR^b_{it}, \forall i \in \Lambda_b, \forall b, \forall t \tag{3-8} \\
& \quad p^b_{i(t-1)}(\xi) - p^b_{it}(\xi) \leq (2 - y^b_{i(t-1)} - y^b_{it})L^b_{it} + (1 - y^b_{i(t-1)} + y^b_{it})DR^b_{it}, \forall i \in \Lambda_b, \forall b, \forall t \tag{3-9}
\end{align*}
\]
\[
\sum_{i \in I_b} p^b_i(\xi) = q^G_{tb}(\xi), \forall b, \forall t \quad (3-11)
\]
\[
q^W_{tb}(\xi) + q^G_{tb}(\xi) + \sum_{i \in I_o} (q^{H+}_{tb}(\xi) - q^{H-}_{tb}(\xi)) = q^B_{tb} + q^{imb}_{tb}(\xi), \forall b, \forall t \quad (3-12)
\]
\[
s^b_i(\xi) = s^{begin}_{tb}(\xi) + \eta R q^{imb}_{tb}(\xi) - \frac{q^{H+}_{tb}(\xi)}{\eta}, \quad \forall i \in I_b, \forall b, \forall t \quad (3-13)
\]
\[
h^b_i(\xi)L^H_{ib} \leq q^{H+}_{tb}(\xi) \leq h^b_i(\xi)U^H_{ib}, \forall i \in I_b, \forall b, \forall t \quad (3-14)
\]
\[
(1 - h^b_i(\xi))L^H_{ib} \leq q^{H-}_{tb}(\xi) \leq (1 - h^b_i(\xi))U^H_{ib}, \forall i \in I_b, \forall b, \forall t \quad (3-15)
\]
\[
s^b_{tb}(\xi) = s^{end}_{tb}, \quad s^b_{tb}(\xi) = s^{begin}_{tb}, \forall i \in I_b, \forall b \quad (3-16)
\]
\[
Pr(\beta \sum_{b=1}^B W_{tb}(\xi) \leq \sum_{b=1}^B q^W_{tb}(\xi), \forall t) \geq 1 - \epsilon \quad (3-17)
\]
\[
p^b_i(\xi), q^W_{tb}(\xi), q^G_{tb}(\xi), q^{H+}_{tb}(\xi), q^{H-}_{tb}(\xi), s^b_{tb}(\xi) \geq 0, \quad (3-18)
\]

The objective function (3-1) is to maximize the expected total profit. It is equal to the expected revenue \( E[\sum_{i=1}^T \sum_{b=1}^B (R^D_{tb}(\xi)q^B_{tb} + R^R_{tb}(\xi)q^{imb}_{tb}(\xi))] \) which follows the two-settlement market procedure in most U.S. electricity markets, minus the expected power generation cost \( \sum_{i=1}^T \sum_{b=1}^B \sum_{i \in I_b} (SU^p_i + SD^p_i + \gamma^b_i q^{imb}_{tb}(\xi)) \), and the expected penalty cost \( E[\sum_{i=1}^T \sum_{b=1}^B \gamma^b_i q^{imb}_{tb}(\xi)] \). It should be noted here that \( q^{imb}_{tb}(\xi) \) captures the amount of imbalance between the day-ahead bid amount and the real-time generation output. As this imbalance is mainly caused by the variable wind power and inaccurate wind power forecasting, the additional penalty \( \gamma^b_t \) which is imposed by market operators can reduce the uncertainty of the market related to uncertain wind generation. The unit commitment constraints at the first stage listed above include constraints (3-2) to (3-5), representing the unit minimum-up time requirement when the unit is turned on (e.g., constraints (3-2)), the unit minimum-down time requirement when the unit is turned off (e.g., constraints (3-3)), the unit start-up condition (e.g., constraints
(3–4)), and the unit shut-down condition (e.g., constraints (3–5)). The hourly economic dispatch constraints at the second stage include unit generation upper and lower limit constraints (3–8), unit ramping up constraints (3–9), unit ramping down constraints (3–10), total thermal generation output (3–11) (it sums up all the generation by thermal units), power balance constraints (3–12) (the total system generation should be equal to the amount of energy offered in the day-ahead market plus the imbalance), hydro water inventory balance constraints (3–13), hydro unit pump in/out limit constraints ((3–14) and (3–15)), and first/last period water reservation amount constraints (3–16). The chance constraint (3–17) is associated with a risk level $\epsilon$ (e.g., $\epsilon = 10\%$), which means the total utilization of wind power has to be larger than or equal to $\beta$ (e.g., $\beta = 85\%$) for at least $100(1 - \epsilon)$ percent of chance. As can be seen, adding this constraint can help IPPs comply with regulations which require a certain percentage of wind power utilization at a high probability. In addition, $v_{it}^b$, $o_{it}^b$, and $v_{it}^b$ are first-stage decision variables, and others are second-stage decision variables.

In the objective function of our model, there exists an absolute value which indicates the imbalance penalty. It can be reformulated by using linear programming as shown in [18]. For instance, the following minimization problem

$$\min \gamma_t^b |q_{tb}^{imb}|, \text{ subject to } Ax = b$$

(3–19)

can be reformulated as follows:

$$\min \{\gamma_t^b d_{tb} - d_{tb} \leq q_{tb}^{imb} \leq d_{tb} \text{ and } Ax = b\}$$

(3–20)

after introducing an auxiliary variable $d_{tb}$. Thus, we can replace the absolute value part in (3–19) with a linear function (3–20).

Sample Average Approximation

In this section, we apply a sample average approximation (SAA) method to solve the stochastic program shown above. SAA is composed of three steps: 1) scenario
generation to approximate the true distribution, 2) convergence analysis to show
the convergence property of the algorithm, and 3) solution validation to verify that the
solution converges to the optimal one. The readers are referred to [5] for more details
regarding the traditional SAA method description. For notation brevity, the proposed
mathematical model can be abstracted as follows:

$$\min_{x^f \in \mathcal{X}} f(x^f) + E[Q(x^f, \xi)]$$

where

$$Q(x^f, \xi) = \min c x^s(\xi)$$

$$s.t. \quad Ax^s(\xi) = g - Dx^f, x^s(\xi) \geq 0$$

$$Pr\{H(x^f, x^s(\xi), \xi) \leq 0\} \geq 1 - \epsilon.$$ 

In the above formulation, $x^f$ and $x^s$ represent the first and second stage decision
variables, and $f(x^f)$ and $Q(x^f, \xi)$ represent the first and second stage objective
functions. In addition, $\mathcal{X}$ represents the feasible region of $x^f$, $c, g, A, D$ are vectors/matrices
of parameters, and $H$ is the constraint mapping.

**SAA Problem**

In our approach, the SAA problem is generated similarly to the one described
in [64] and [5]. For instance, a Monte Carlo simulation method is utilized for scenario
generation purposes. After the scenarios are generated (e.g., $N$ scenarios), the
objective function $E[Q(x^f, \xi)]$ can be linearized and replaced by the sample average
function $N^{-1} \sum_{j=1}^{N} Q(x^f, \xi^j)$ [34]. Meanwhile, an indicator function $\mathbf{1}_{(0, \infty)}(H(x^f, x^s(\xi^j), \xi^j))$
is introduced to estimate the chance constraint as described in [43]. We have

$$\mathbf{1}_{(0, \infty)}(H(x^f, x^s(\xi^j), \xi^j)) = \begin{cases} 
1, & \text{if } H(x^f, x^s(\xi^j), \xi^j) \in (0, \infty); \\
0, & \text{if } H(x^f, x^s(\xi^j), \xi^j) \notin (0, \infty). 
\end{cases}$$

By introducing binary decision variables $z$ to indicate if a constraint is satisfied, when
a sample size $N$ is given, we can linearize the chance constraint as the following
constraints (3–22)–(3–24), and the SAA problem can be described as follows:

\[
\begin{align*}
\max & - \sum_{t=1}^{T} \sum_{b=1}^{B} \sum_{i \in \Lambda_{b}} (SU_{i}^{b} o_{it}^{b} + SD_{i}^{b} v_{it}^{b}) + N^{-1} \sum_{j=1}^{N} \\
& \left[ \sum_{t=1}^{T} \sum_{b=1}^{B} \left( R_{t_{mb}}^{DA}(\xi_{t})(q_{tb}^{B} + R_{t_{mb}}^{RT}(\xi_{t})(q_{tb}^{im}(\xi_{t})) \right) \\
& - \sum_{t=1}^{T} \sum_{b=1}^{B} \sum_{i \in \Lambda_{b}} F_{c}(\rho_{t_{i}^{b}}(\xi_{t})) - \sum_{t=1}^{T} \sum_{b=1}^{B} \gamma_{t_{i}^{b}}(q_{tb}^{im}(\xi_{t})) \right]\end{align*}
\]

(3–21)

s.t. (3–22) – (3–24), and the SAA problem can be described as follows:

Convergence Analysis and Solution Validation

We use statistical methods to analyze the solution of the SAA problem and provide convergence analysis and solution validation. As the sample size \( N \) goes to infinity, we claim that the objective of the SAA problem converges to that of the true problem. To prove the convergence property, we need to first prove the convergence of the chance constrained part. This result can be achieved using a similar approach as described in [43]. Secondly, after converting the chance constraint into the MILP formulation, we should notice that the first-stage problem in the whole SAA problem of the true problem is a pure integer program, and the second-stage problem is a mixed integer linear program. According to [5], the solution of such an SAA problem will converge to that of the true problem.

In [5] and [43], the procedures for solution validation of SAA problems have been developed for the two-stage problem and for the chance-constrained problem, respectively. Let \( \bar{x} \) and \( \bar{y} \) be an optimal solution and the corresponding optimal objective value for the SAA problem, respectively. To validate the quality of \( \bar{x} \), the validation
process obtains upper and lower bounds for \( \tilde{\nu} \) of the true problem. Usually, the solution validation needs to consider the feasibility when dealing with chance constraints (e.g., chance constraints contain both first and second stage decision variables), because it is not guaranteed that the solution of the SAA problem always satisfies the chance constraints with a large scenario size. However, in this chapter, the chance constraints are only considered in the second stage. The second-stage decision is made after the scenarios are realized. Thus, the chance constraints can always be satisfied by tuning the second stage decision variables. We apply directly the validation process in [5] to construct statistical bounds for the objective value of our SAA problem.

**SAA Algorithm Framework**

To describe the SAA algorithm, we first introduce additional notation. For instance, we let \( N \) be the scenario size of the SAA problem, \( K \) be the iteration number, \( N' \) be the scenario size of the validation process to obtain a lower bound, \( \tilde{g} \) be the lower bound of the true problem, \( \bar{x}_k \) and \( \tilde{v}_k \) be the optimal solution and optimal objective value in iteration \( k \), and \( \tilde{\nu} \) be the upper bound of the true problem. The SAA algorithm can be summarized as follows with a flowchart in Fig. 3-1:

1. Set \( k = 1, 2, \ldots, K \) and repeat the following steps for each \( k \):
   
   (a) For a given sample size \( N \), generate a corresponding SAA problem and solve the SAA problem to obtain \( \bar{x}_k \) and \( \tilde{v}_k \);
   
   (b) For a given sample size \( N' \) for the validation process, generate independent scenarios \( \xi^1, \xi^2, \ldots, \xi^{N'} \), and estimate the lower bound of the problem using the following formula:

\[
\hat{g}^k = f(\bar{x}_k) + \frac{1}{N'} \sum_{n=1}^{N'} Q(\bar{x}_k^f, \xi^n),
\]  

(3–25)

where \( Q(\bar{x}_k^f, \xi^n) \) is the second-stage problem defined in (3–7)-(3–18) with \( x^f \) fixed as \( \bar{x}_k^f \).

2. Take the average of \( \tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_K \). The upper bound can be obtained as \( \tilde{\nu} = \frac{1}{K} \sum_{k=1}^{K} \tilde{v}_k \) following Theorem 1 in [38].
3. Take the maximum of $\hat{g}^1, \hat{g}^2, \ldots, \hat{g}^K$. The lower bound can be obtained as $\hat{g} = \max_{1 \leq k \leq K} \hat{g}^k$.

4. Estimate the optimality gap: $(\bar{v} - \hat{g})/\hat{g} \times 100\%$.

---

**Figure 3-1. Proposed SAA algorithm**

**Heuristics for Solving Each SAA Problem**

As shown in Section 3, each SAA problem contains many integer variables which make the problem hard to solve by commercial solvers like CPLEX [2] under default settings. In addition, there are many binary decision variables in the second stage due to hydro operations. When the system becomes larger, the number of integer variables will increase significantly. To reduce the computational complexity of the problem, we apply heuristic methods to obtain good feasible solutions, and tight inner
upper and lower bounds of the optimal objective value for each SAA problem. Based on optimization theory, in our heuristic approach, we solve a relaxation problem by relaxing the second stage binary decision variables to be fractional, which provides an upper bound for our maximization problem. In addition, any feasible solution leads to a corresponding lower bound.

**Inner upper bound**

The basic idea is to use the relaxation to get an inner upper bound for a maximization problem. Since the integer variables lead to the difficulty of solving each SAA problem, we relax the integrality for constraints (3–14) and (3–15) while maintaining the integrality of the \( z \) variables in (3–24) to get an inner upper bound for the given SAA problem. Note here that this approach is more effective for small sample sizes.

**Inner lower bound**

We create a feasible solution to get an inner lower bound for the given SAA problem in this part. Note that the first-stage solution from obtaining the inner upper bound in 3 should also satisfy the first-stage constraints in the SAA problem. Therefore, we can fix the first-stage solution obtained from the above part in 3, and solve the second-stage sub-problem to obtain a feasible solution and a corresponding inner lower bound for the SAA problem.

Moreover, when we use the inner upper bound and corresponding solution derived in 3 to replace \( \bar{v}_k \) and \( \bar{x}_k \) in Step 1.1 in 3 in the calculation of obtaining the upper bound in the validation process, we still obtain the upper bound for the original true problem. Similarly, using the inner lower bound and corresponding solution obtained in 3 to replace \( \bar{v}_k \) and \( \bar{x}_k \) in Step 1.1 in 3 in the calculation of obtaining the lower bound in the validation process provides the lower bound for the original true problem.

**Computational Results**

In this section, we first study a three-generator system in a single bus to illustrate the proposed algorithm. Second, we consider a more complicated large system in a
single bus to evaluate the performance of the heuristic approach. Finally, we evaluate the performance of our algorithm on a generalized multi-bus system (e.g., thermal, wind and hydro units are located in different buses), by comparing it with the case in which each bus is considered separately. It should be noted that our SAA solution framework can be applied to larger systems as well although the sizes of the test systems used in this chapter are moderate. The reason for us to use a moderate size instance is that an IPP with an excessively large generation portfolio may most likely be able to influence the market prices, which violates our assumption of a price-taker. For the three-generator system, we run the computational experiments on the SAA problems at different risk levels and different sample sizes for comparison. The SAA algorithm described in Section 3 is also tested for this system. For the complicated system, we consider the heuristic method described in Section 3 to solve the SAA problem and run the computational experiments to test the heuristic-based SAA algorithm. The codes are written in C++ and the problem is solved with CPLEX 12.1. All the experiments are implemented on a computer workstation with 4 Intel Cores and 8GB RAM.

**Scenario Generation for Uncertain Wind Power and Price**

In the SAA framework, we need to generate scenarios by Monte Carlo simulation. In our approach, MISO wind power and price historical data are utilized for our case studies. The wind power data is available in the National Renewable Energy Laboratory (NREL) 2006 eastern wind data set. The Locational Marginal Price (LMP) historical data is provided by MISO. The state-of-the-art time series models for wind power generation are in two categories: wind speed-based approaches and wind power-based approaches. The wind speed-based approaches (see, e.g., [19, 44]) apply the time series model to generate wind speed scenarios and convert them into wind power output. The wind power-based approaches consider wind power time series directly (see, e.g., [21]). To capture the wind power uncertainty, we now first apply the time series model to analyze the historical wind data available from NREL [3]. The method
in [21] is applied to construct the ARIMA-based model. A Monte Carlo simulation is then performed on random noise which is subject to a normal distribution in the ARIMA model to generate scenarios. As LMPs are very difficult to forecast themselves due to a variety of factors such as strategic bidding or transmission congestion, we assume the uncertainties of day-ahead and real-time LMPs to follow a Gaussian distribution and follow the method described in [16] to generate price scenarios. That is, we use the historical data to get the mean and variance for the Gaussian distribution for the day-ahead and real-time LMPs, respectively. Then, the iid samples are generated from the Gaussian distribution with the estimated mean and variance, plus the white noise following the standard normal distribution. Note that the proposed model in this chapter can be applied to other scenario generation approaches without loss of generality by changing the scenario generation approach accordingly. For example, as described in [63], the wind power can be assumed to follow a multivariate Gaussian distribution in Monte Carlo simulation.

Three-Generator System

In this subsection, we study a simple case in which an IPP owns and operates three thermal generators, one wind farm, and one pumped-storage unit. For this small instance, each SAA problem can be solved by CPLEX with default settings directly. Therefore, the heuristic method in Section 3 is not considered. We report the computational results at different risk levels and scenario sizes. The characteristics of thermal and pumped-storage units are described in Tables 3-1-3-3.

Table 3-1. Generator data

<table>
<thead>
<tr>
<th>Unit</th>
<th>Lower (MW)</th>
<th>Upper (MW)</th>
<th>Min-down (h)</th>
<th>Min-up (h)</th>
<th>Ramp (MW/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>50</td>
<td>100</td>
<td>2</td>
<td>4</td>
<td>40</td>
</tr>
<tr>
<td>$G_2$</td>
<td>100</td>
<td>150</td>
<td>3</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>$G_3$</td>
<td>20</td>
<td>50</td>
<td>3</td>
<td>2</td>
<td>15</td>
</tr>
</tbody>
</table>
In order to run the model in CPLEX effectively, we linearize the fuel cost function by using the interpolation method [57]. Accordingly, the fuel cost function in (3–7) is replaced by a piecewise linear function.

**Optimal solution with ten scenarios**

We report the optimal solution of the SAA algorithm with ten scenarios and risk level $\epsilon = 10\%$ in this subsection. Table 3-4 reports the unit commitment status for each generator. It can be observed that $G_1$ is committed mostly, The reason is that $G_1$ has more flexible lower/upper bounds and ramp limits than $G_2$ and $G_3$. The flexibility of these characteristics allows $G_1$ to accommodate the wind power better.

Table 3-4. Optimal unit commitment

<table>
<thead>
<tr>
<th>Hours (1-24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
</tr>
<tr>
<td>$G_2$</td>
</tr>
<tr>
<td>$G_3$</td>
</tr>
</tbody>
</table>

To show the effectiveness of the pumped-storage unit, we compare the average imbalance $a_{imb}$ value with and without the pumped-storage unit. The average imbalance decreases from 6.8 to 2.5 when we have the pumped-storage unit in the system.
Sensitivity analysis for different risk levels and scenario sizes

The numerical results on different risk levels and different scenario sizes are reported in the following Figs. 3-2 and 3-3. From Fig. 3-2, it can be observed that the total profit increases as the risk level increases. This is reasonable because the total profit will be lower if the utilization requirement of wind power output is more restrictive. To verify the convergence property of the SAA algorithm shown in Section 3, the SAA algorithm is tested numerically by setting different sample sizes. The results shown in Fig. 3-3 (with the risk level to be 10%) indicate that the objective function oscillates at the beginning when the sample size is small and then converges slowly to the optimal objective value.

![Figure 3-2. Obj.($) of the SAA problem with different risk levels](image)

Computational Results for a Complicated System

In this subsection, we report the case study result of a more complicated system. We assume the IPP owns five generators, five wind farms, and two hydro units. \( G_1 \) and \( G_2 \) used in the three-generator system are duplicated in this case study setting.
Figure 3-3. Obj.($) of the SAA problem with different scenario sizes

Under this setting, each SAA problem cannot be solved to optimality within the two-hour time limit when the scenario size reaches 50. The reason is that the wind power scenarios vary extensively such that the computational complexity is dominated by the chance constraint. However, our heuristic approach can still provide tight inner lower and upper bounds. As shown in Table 3-5, the lower bound matches the upper bound when the sample size is no larger than 50. When the sample size increases, we can stop our heuristic algorithm if the time limit is reached. Accordingly, we can still report the corresponding lower and upper bounds for each SAA. The drawback for this approach is that it potentially increases the optimality gap for each SAA problem. However, as we increase the iteration number $K'$ and the validation process sample size $N'$, the final estimated optimality gap for the SAA framework can still be reduced to a small number, as shown in Table 3-6.

Multi-Bus System

In the above larger system in Section 3, we assume all the IPP’s power generation resources are in a single bus, or aggregated. It is common in practice that the IPP’s power generation resources are distributed at different buses as described in the
Table 3-5. Computational results for a complicated system for each SAA problem - heuristic method (risk level: 10%)

<table>
<thead>
<tr>
<th>N</th>
<th>Inner LB</th>
<th>Inner UB</th>
<th>CPU Time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>128690</td>
<td>128690</td>
<td>3.8</td>
</tr>
<tr>
<td>30</td>
<td>123820</td>
<td>123820</td>
<td>605.6</td>
</tr>
<tr>
<td>50</td>
<td>124688</td>
<td>124688</td>
<td>2476.9</td>
</tr>
</tbody>
</table>

Table 3-6. Results of solution validation for a complicated system (risk level: 10%)

<table>
<thead>
<tr>
<th>(K, N')</th>
<th>LB</th>
<th>UB</th>
<th>Gap</th>
<th>CPU Time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10, 200)</td>
<td>122410</td>
<td>129672</td>
<td>5.9%</td>
<td>1334.5</td>
</tr>
<tr>
<td>(30, 500)</td>
<td>121040</td>
<td>128765</td>
<td>6.3%</td>
<td>3243.8</td>
</tr>
<tr>
<td>(50, 800)</td>
<td>122818</td>
<td>126637</td>
<td>3.1%</td>
<td>5453.2</td>
</tr>
</tbody>
</table>

model in Section 4. In such a case, the electricity prices at different buses might be different. One can separately solve the problem for each bus, which can save the computational time. In this subsection, we use our model to consider different buses simultaneously since the objective of the IPP should be to maximize the profit of its entire generation portfolio located at different buses. While each bus contains its own price information and power bidding balance constraints, one chance constraint is applied on the total wind utilization for the whole multi-bus generation portfolio. The chance constraint is the coupling constraint for all buses. In our experiment, we assume the IPP’s power generation resources are distributed in five different buses. $G_1$ used in the three-generator system is duplicated as the fourth generator in this case study. The detailed settings are summarized in Table 3-7.

The computational results are reported in Table 3-8. It can be observed that the proposed method which considers the coupling chance constraint provides a larger total profit. This matches the theoretical result. That is, any solution of the separated chance constrained problem must be a feasible solution of the coupling chance constrained problem, which leads to the fact that the coupling chance constrained problem provides
Table 3-7. Bus settings

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Wind</th>
<th>Thermal</th>
<th>Hydro</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

a better solution with a higher total profit. We therefore conclude that sharing resources inside the multi-bus system can tackle the uncertainties better and offer a higher profit in general.

Table 3-8. Computational results for distributed system

<table>
<thead>
<tr>
<th>Risk Level</th>
<th>Obj. with Coupling Chance Constraint</th>
<th>Obj. with Separating Chance Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>79873</td>
<td>62968</td>
</tr>
<tr>
<td>10%</td>
<td>92266</td>
<td>76452</td>
</tr>
<tr>
<td>40%</td>
<td>113419</td>
<td>104145</td>
</tr>
<tr>
<td>100%</td>
<td>153631</td>
<td>153631</td>
</tr>
</tbody>
</table>

**Concluding Remarks**

In this chapter, a stochastic programming model is proposed to address the price-based unit commitment problem with wind power utilization constraints. Our model incorporates day-ahead price, real-time price, and wind power output uncertainties. In the first stage, an IPP makes decisions on unit commitment and the amount of energy offered for the day-ahead market. The economic dispatch of generators is made in the second stage. A chance constraint is considered to ensure the utilization of the volatile wind power to a large extent. In other words, there is a great chance the usage of the wind power satisfies a pre-defined percentage. The chance constraint allows the power producer to adjust the utilization of wind power based on different regulations.
Our model maximizes the profit and accommodates the required usage of wind power output. An SAA algorithm is developed to solve the problem, and the objective value of the SAA problem converges to the optimal one as the scenario size increases. For more complicated systems, we propose a heuristic approach to accelerate the SAA algorithm. Our implementation provides the overall upper and lower bounds for the true problem. The reasonable estimated optimality gap and moderate computational time verify that our approach is effective in solving this problem.
# A. Indices and Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B)</td>
<td>Index set of all buses.</td>
</tr>
<tr>
<td>(\mathcal{E})</td>
<td>Index set of transmission lines linking two buses.</td>
</tr>
<tr>
<td>(BG)</td>
<td>Set of buses with thermal generation units.</td>
</tr>
<tr>
<td>(BW)</td>
<td>Set of buses with wind farms.</td>
</tr>
<tr>
<td>(\Lambda_b)</td>
<td>Set of thermal generators at bus (b).</td>
</tr>
<tr>
<td>(T)</td>
<td>Time horizon (e.g., 24 hours).</td>
</tr>
<tr>
<td>(SU_i^b)</td>
<td>Start-up cost of thermal generator (i) at bus (b).</td>
</tr>
<tr>
<td>(SD_i^b)</td>
<td>Shut-down cost of thermal generator (i) at bus (b).</td>
</tr>
<tr>
<td>(F_c(q^b_t(\xi)))</td>
<td>Fuel cost of thermal generator (i) at bus (b) in time (t) when its generation amount is (q^b_t(\xi)).</td>
</tr>
<tr>
<td>(MU_i^b)</td>
<td>Minimum up-time for thermal generator (i) at bus (b).</td>
</tr>
<tr>
<td>(MD_i^b)</td>
<td>Minimum down-time for thermal generator (i) at bus (b).</td>
</tr>
<tr>
<td>(UR_i^b)</td>
<td>Ramp-up rate limit for thermal generator (i) at bus (b).</td>
</tr>
<tr>
<td>(DR_i^b)</td>
<td>Ramp-down rate limit for thermal generator (i) at bus (b).</td>
</tr>
<tr>
<td>(L_i^b)</td>
<td>Lower bound of electricity generated by thermal generator (i) at bus (b).</td>
</tr>
<tr>
<td>(U_i^b)</td>
<td>Upper bound of electricity generated by thermal generator (i) at bus (b).</td>
</tr>
<tr>
<td>(C_{ij})</td>
<td>Transmission capacity for the transmission line linking bus (i) and bus (j).</td>
</tr>
<tr>
<td>(K_{ij}^b)</td>
<td>Line flow distribution factor for the transmission line linking bus (i) and bus (j), due to the net injection at bus (b).</td>
</tr>
<tr>
<td>(\gamma^{+}_t)</td>
<td>Penalty cost per MW for load curtailment in time (t).</td>
</tr>
<tr>
<td>(\gamma^{-}_t)</td>
<td>Penalty cost per MW for generation curtailment in time (t).</td>
</tr>
</tbody>
</table>
\( \beta \) The utilization rate of the wind power output.

\( \epsilon \) Risk level associated with the chance constraint on load imbalance.

\( \Delta \) The tolerance of load imbalance in the chance constraint.

**B. Decision Variables**

\( \alpha_i^b \) Binary decision variable: “1” if thermal generator \( i \) at bus \( b \) is on in time \( t \); “0” otherwise.

\( u_i^b \) Binary decision variable: “1” if thermal generator \( i \) at bus \( b \) is started up in time \( t \); “0” otherwise.

\( v_i^b \) Binary decision variable: “1” if thermal generator \( i \) at bus \( b \) is shut down in time \( t \); “0” otherwise.

\( q_i^b(\xi) \) Electricity generation amount by thermal generator \( i \) at bus \( b \) in time \( t \) corresponding to scenario \( \xi \).

\( \hat{q}_i^b(\xi) \) Amount of wind power utilized (delivered) at bus \( b \) in time \( t \) corresponding to scenario \( \xi \).

\( Q_t^G(\xi) \) Total generation amount by thermal units in time \( t \) corresponding to scenario \( \xi \).

\( Q_t^W(\xi) \) Total wind power committed to be utilized (delivered) in time \( t \) corresponding to scenario \( \xi \).

\( Q_{t}^{imb+}(\xi) \) Total load curtailment in time \( t \) corresponding to scenario \( \xi \).

\( Q_{t}^{imb-}(\xi) \) Total generation curtailment in time \( t \) corresponding to scenario \( \xi \).

**C. Random Numbers**

\( W_{bt}(\xi) \) A random parameter indicating the uncertain wind power output (decided by the weather) by wind farm at bus \( b \) in time \( t \) corresponding to scenario \( \xi \).

\( D_{bt}(\xi) \) A random parameter indicating the uncertain load at bus \( b \) in time \( t \) corresponding to scenario \( \xi \).
Motivation

In this chapter, besides the chance constraint enforcing a low energy imbalance probability, a general expected value constraint is introduced in the stochastic programming framework to ensure the overall expected amount of wind power usage. A related concept is the recently developed conditional value-at-risk (CVaR) decision criterion approach. CVaR is a risk control metric, which is known to possess better properties than value-at-risk (VaR) [48]. In [16], CVaR has been applied to optimal wind power trading strategies in LMP markets. In [7], a more general model considering the conventional unit on/off operations and ramping constraints is developed, in which CVaR is applied to study the trading of thermal energy and uncertain wind power output. Both the wind and thermal units are assumed to be connected to the same bus owned by an independent power producer. It has been shown recently in [69] that the CVaR constrained problem is a special case of the expected value constrained problem. In this chapter, we study the general expected value constrained problem. In our approach, an SAA method will be used to solve such a stochastic optimization problem involving both the chance and the expected value constraints. The developed SAA algorithm combines the statistical analysis of both the expected value constraint and the chance constraint.

The main contributions of this chapter are listed as follows:

1. We propose a stochastic optimization model that addresses both wind power output and load uncertainties.

2. We introduce the expected value constraint in the proposed stochastic optimization model to ensure wind power utilization. To the best of our knowledge, this is the first study on the expected value constrained stochastic unit commitment problem.

3. We develop a new combined SAA algorithm to solve the expected value and chance constrained stochastic unit commitment problem, which has never been studied before.

4. Our proposed approach will help enhance the unit commitment procedure by ISOs/RTOs to ensure the utilization of wind power output, while maintaining the system reliability.
Mathematical Formulation

In this section, we develop a two-stage stochastic unit commitment formulation considering both the expected value constraint and the chance constraint to address uncertain wind power output. The first stage is to determine the day-ahead unit commitment decisions that include turn-on/turn-off decisions of thermal power generating units by satisfying unit commitment physical constraints. The second stage contains the decisions on the real-time dispatch of thermal units and the actual amount of wind power usage. The penalty cost is introduced in the second stage to control the load imbalance. The detailed formulation is described as follows:

\[
\begin{align*}
\min & \quad \sum_{t=1}^{T} \sum_{b \in B} \sum_{i \in \Lambda_b} (SU_i^b u_i^b + SD_i^b v_i^b) + E[Q(o, u, v, \xi)] \\
\text{s.t.} \quad & -o_{i(t-1)}^b + o_{ik}^b - o_{ik}^b \leq 0, \\
& \forall k : 1 \leq k - (t - 1) \leq MU_i^b, \forall i \in \Lambda_b, \forall b \in B, \forall t, \\
& o_{i(t-1)}^b - o_{ik}^b + o_{ik}^b \leq 1, \\
& \forall k : 1 \leq k - (t - 1) \leq MD_i^b, \forall i \in \Lambda_b, \forall b \in B, \forall t, \\
& -o_{i(t-1)}^b + o_{ik}^b - u_i^b \leq 0, \forall i \in \Lambda_b, \forall b \in B, \forall t, \\
& o_{i(t-1)}^b - o_{ik}^b - v_i^b \leq 0, \forall i \in \Lambda_b, \forall b \in B, \forall t, \\
& o_{ik}^b, u_i^b, v_i^b \in \{0, 1\}, \forall i \in \Lambda_b, \forall b \in B, \forall t, \\
\end{align*}
\]

where

\[
Q(o, u, v, \xi) = \min \sum_{t=1}^{T} \sum_{b \in B} \sum_{i \in \Lambda_b} F_c(q_{ik}^b(\xi)) + \sum_{t=1}^{T} \gamma_t^+ Q_{imb}^+(\xi) + \sum_{t=1}^{T} \gamma_t^- Q_{imb}^-(\xi)
\]

\[
L_i^b o_i^b \leq q_{ik}^b(\xi) \leq U_i^b o_i^b, \forall i \in \Lambda_b, \forall b \in B, \forall t \\
q_{ik}^b(\xi) - q_{ik(t-1)}^b(\xi) \leq (2 - o_{ik(t-1)} - o_{ik}^b)L_i^b +
\]
\[(1 + \sigma_{\text{up}}^b(i \rightarrow i-1) - \sigma_{\text{down}}^b(i \rightarrow i-1))UR^b_i, \quad \forall i \in \Lambda_n, \forall b \in B, \forall t\]  
\[q_{i(t-1)}^b(\xi) - q_i^b(\xi) \leq (2 - \sigma_{\text{up}}^b(i \rightarrow i-1) - \sigma_{\text{down}}^b(i \rightarrow i-1))L_i^b +\]  
\[(1 - \sigma_{\text{up}}^b(i \rightarrow i-1) + \sigma_{\text{down}}^b(i \rightarrow i-1))DR^b_i, \quad \forall i \in \Lambda_n, \forall b \in B, \forall t\]  
\[Q_i^G(\xi) = \sum_{b \in B} \sum_{i \in \Lambda_n} q_i^b(\xi), \quad \forall t\]  
\[Q_i^W(\xi) = \sum_{b \in B} \hat{q}_i^b(\xi), \quad \forall t\]  
\[Q_i^{1\text{mb}+}(\xi) - Q_i^{1\text{mb}-}(\xi) = \sum_{b \in B} D_{bt}(\xi)\]  
\[(Q_i^W(\xi) + Q_i^G(\xi)), \quad \forall t\]  
\[-C_{ij} \leq \sum_{b \in B} K_{ij}^b(\hat{q}_i^b(\xi) + \sum_{r \in \Lambda_n} q_r^b(\xi) - D_{bt}(\xi))\]  
\[\leq C_{ij}, \quad \forall (i, j) \in \mathcal{E}, \forall t\]  
\[\hat{q}_i^b(\xi) \leq W_{bt}(\xi), \quad \forall b \in B, \forall t\]  
\[Pr(-\Delta \leq Q_i^W(\xi) + Q_i^G(\xi) - \sum_{b \in B} D_{bt}(\xi) \leq \Delta, \forall t)\]  
\[\geq 1 - \epsilon,\]  
\[E \left[ \sum_{t=1}^{T} Q_i^W(\xi) \right] \geq \beta E \left[ \sum_{t=1}^{T} \sum_{b \in BW} W_{bt}(\xi) \right],\]  
\[q_{i(t-1)}^b(\xi), \hat{q}_i^b(\xi), Q_i^G(\xi), Q_i^W(\xi), Q_i^{1\text{mb}+}(\xi) \geq 0.\]  

In the above formulation, we denote \(F_c(.)\) as the fuel cost function. The objective function (4–1) is composed of the unit commitment costs in the first stage, and the fuel cost as well as the penalty cost due to load imbalance in the second stage. Minimum up/down-time constraints (4–2) and (4–3) mean the status (on or off) of each unit should last for a minimum time once it is started up or shut down. Constraints (4–4) and (4–5) indicate the start-up and shut-down operations for each unit. Constraints (4–8) describe the upper and lower bounds of power output of each unit, and ramping constraints (4–9) and (4–10) limit the maximum increment or decrement of power generation of each unit between two adjacent periods. Constraints (4–11) and (4–12) represent the total thermal generation and the actual wind power utilized. Constraints (4–13) describe the possible
load imbalance which is penalized in the objective function. Constraints (4–14) indicate the transmission capacity constraints and constraints (4–15) describe that the wind power utilized should be no more than the maximum available wind power. The chance constraint (4–16) requires that the chance of load imbalance beyond the tolerance level should be below a pre-defined risk level. Finally, the expected value constraint (4–17) can guarantee the usage of wind power output is no less than a certain ratio of the maximum available wind power.

**Solution Methodology**

In this section, we develop a combined SAA algorithm to solve the expected value constrained and chance constrained two-stage stochastic program. Essentially, SAA is used to approximate the actual distribution with an empirical distribution corresponding to a random sample [43]. The basic framework of the combined SAA framework contains three parts: scenario generation, convergence analysis, and solution validation. For convergence properties, the proofs for the expected value constrained stochastic program and the chance constrained stochastic program are provided in [69] and [43] respectively, which can be directly applied here to show the convergence property of the combined SAA algorithm. Therefore we focus on the scenario generation and solution validation in the following analysis.

**Scenario Generation**

We use Monte Carlo simulation to generate scenarios for the wind power output and load. Assuming the wind power output follows a multivariate normal distribution \( N(\mu, \Sigma) \) [63], where \( \mu \) is the predicted value of the wind power output and matrix \( \Sigma \) denotes its volatility, we can run Monte Carlo simulation to generate \( N \) scenarios and each scenario has the same probability \( \frac{1}{N} \). Now we can replace the second-stage objective function by

\[
\frac{1}{N} \sum_{n=1}^{N} \left( \sum_{t=1}^{T} \sum_{b \in B} \sum_{i \in A_b} F_{t}^{n}(q_{it}^{n}(\xi_{i}^{n})) + \sum_{t=1}^{T} \gamma_{t}^{+} Q_{t}^{imb+}(\xi_{i}^{n}) \right)
\]
\[
+ \sum_{t=1}^{T} \gamma_t Q_t^{imb}(\xi^n).
\] (4–19)

For the expected value constraint, we use the Monte Carlo simulation to generate \(N\) scenarios to estimate constraint (4–17) by:

\[
\frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T} Q_t^W(\xi^n) \geq \frac{\beta}{N} \sum_{n=1}^{N} \sum_{t=1}^{T} \sum_{b \in B_W} W_{bt}(\xi^n).
\] (4–20)

Meanwhile, the chance constraint (4–16) can be estimated as follows [6]:

\[
\frac{1}{N} \sum_{n=1}^{N} 1_{[-\Delta, \Delta]}(Q_t^W(\xi) + Q_t^G(\xi) - \sum_{b \in B} D_{bt}(\xi), \forall t) \geq 1 - \epsilon,
\] (4–21)

where \(1_{[-\Delta, \Delta]}(\cdot)\) is an indicator function, i.e.,

\[
1_{[-\Delta, \Delta]}(x) = \begin{cases} 
1, & \text{if } x \in [-\Delta, \Delta]; \\
0, & \text{if } x \notin [-\Delta, \Delta].
\end{cases}
\]

Similarly, constraints (4–8)- (4–15) can be replaced respectively by scenario-based constraints, following the traditional SAA method for the two-stage stochastic program [5]. The ultimate SAA problem is provided in the following Subsection 4 after reformulating the indicator constraint (4–21).

**MILP reformulation of chance constraint**

After we generate the scenarios, the chance constraint is converted to an indicator function (4–21) in the SAA problem. We use an MILP model to reformulate this sampled chance constraint. For a given sample size \(N\), we introduce a binary decision variable \(z\) indicating whether the chance constraint is satisfied in the corresponding scenario. Then the chance constraint can be represented as follows:

\[
-\Delta - Mz_n \leq Q_t^W(\xi) + Q_t^G(\xi) - \sum_{b \in B} D_{bt}(\xi), \forall t, \forall n
\]

\[
\forall t, \forall n
\]

\[
\sum_{n=1}^{N} z_n \leq N\epsilon, z_n \in \{0, 1\}, \forall n.
\] (4–23)
Reformulation of SAA problem

The ultimate SAA problem formulation is as follows:

\[
\begin{align*}
\min & \sum_{t=1}^{T} \sum_{b \in B} \sum_{i \in A_b} (SU_i^b u_i^b + SD_i^b v_i^b) + \frac{1}{N} \sum_{n=1}^{N} \\
& \left( \sum_{t=1}^{T} \sum_{b \in B} \sum_{i \in A_b} F_c(q_t^b(\xi^a)) + \sum_{t=1}^{T} \gamma_t^+ Q_t^{imb+}(\xi^a) \\
& + \sum_{t=1}^{T} \gamma_t^- Q_t^{imb-}(\xi^a) \right) \\
\text{s.t.} & (4-2) - (4-6), \\
& (4-8) - (4-15), (4-18), \forall \xi \in \{\xi^1, \ldots, \xi^N\}, \\
& (4-20), (4-22), (4-23).
\end{align*}
\]

Solution Validation

The basic idea for the validation process is to apply statistical techniques to approximate the upper and lower bounds of the optimal objective value of the SAA problem. The optimality gap can be obtained from the validation process with a confidence level. For notation brevity to introduce our proposed algorithm, the mathematical model can be abstracted as follows:

\[
\begin{align*}
\min & c^T x + E[Q(x, \xi)] \\
\text{s.t.} & A x \leq b, \\
& E[G(x, y(\xi), \xi)] \leq \ell, \\
& Pr(H(x, y(\xi), \xi) \geq 0) \leq \epsilon, \\
& x \geq 0,
\end{align*}
\]

where \( Q(x, \xi) = \min d(\xi)^T y(\xi) \)

\[
\begin{align*}
\text{s.t.} & T(\xi) x + W y(\xi) = h(\xi), y(\xi) \geq 0.
\end{align*}
\]
Upper bound

First, we notice that the second stage problem might not have the complete recourse property \[5\]. That is, the second stage is not guaranteed to be feasible for a given unit commitment decision. If this is the case, we can enlarge the \(\Delta\) in the chance constraint to allow more imbalance. Such a setting can guarantee that for any first stage solution \(x\) (e.g. unit commitment decision) and any scenario \(\xi\), there always exists a feasible solution \(y(\xi)\) for the second stage, i.e., \(Q(x, \xi) < \infty\) for all \(x\) and \(\xi\). It can be also observed that \(Q(x, \xi) \geq 0\). Then we can assume that the expected value \(E[Q(x, \xi)]\) is well-defined and finite valued for a given distribution of \(\xi\).

We know that any feasible solution can provide an upper bound of the optimal objective value. Let \(\{\xi^1, \xi^2, \cdots, \xi^N\}\) be a sample of size \(N\), and \(\bar{x}\) be the optimal first stage solution of the SAA problem. We know it is possible that \(\bar{x}\) is infeasible to the original problem. Define \(p(x) := Pr\{H(x, y(\xi), \xi)\}\) and \(q(x) := E[G(x, y(\xi), \xi)]\), and use the following two constraints in the SAA problem:

\[
\begin{align*}
  p(x) & \leq \bar{\epsilon}, \\
  q(x) & \leq \bar{\ell},
\end{align*}
\]

where \(\bar{\epsilon} \leq \epsilon\) and \(\bar{\ell} \leq \ell\). Constraints (4–32) and (4–33) are equivalent to the chance constraint (4–28) and the expected value constraint (4–27) when \(\bar{\epsilon} = \epsilon\) and \(\bar{\ell} = \ell\). Note here if the sample size \(N\) is large enough, when \(\bar{\epsilon} \leq \epsilon\) and \(\bar{\ell} \leq \ell\), the feasible solution of the SAA problem is more likely to be feasible to the original problem. Now, for a given size \(N\), first based on the method described in [6], we construct the \((1 - \tau)\) confidence upper bound for the chance constraint (4–28):

\[
U_c(\bar{x}) = p_N(\bar{x}) + z_{\tau} \sqrt{\frac{p_N(\bar{x})(1 - p_N(\bar{x}))}{N}},
\]
where \( p_N(\bar{x}) = \frac{1}{N} \sum_{n=1}^{N} 1_{[0, \infty)}(H(\bar{x}, y(\xi^n), \xi^n)) \). If \( U_c(\bar{x}) \) is less than or equal to the risk level \( \epsilon \), then \( \bar{x} \) is feasible for the chance constraint with the confidence level \( (1 - \tau) \). If not, we will decrease the value of \( \bar{\epsilon} \) and solve the problem again to check whether the updated optimal solution leads to a new value \( U_c(\bar{x}) \), which is less than or equal to \( \epsilon \).

Secondly, we compute the \((1 - \tau)\) confidence upper bound for the expected value constraint:

\[
U_e(\bar{x}) = q_N(\bar{x}) + z_\tau \sqrt{\frac{\sum_{n=1}^{N} [G(\bar{x}, y(\xi^n), \xi^n) - q_N(\bar{x})]^2}{N(N - 1)}},
\]

(4–35)

where \( q_N(\bar{x}) = \frac{1}{N} \sum_{n=1}^{N} G(\bar{x}, y(\xi^n), \xi^n) \). If \( U_e(\bar{x}) \) is less than or equal to \( \ell \), we claim \( \bar{x} \) is feasible for the expected constraint with the confidence level \((1 - \tau)\). If not, we can decrease the value of \( \bar{\ell} \) to improve the chance of obtaining a feasible solution.

Since it is well-known that \( Pr(A \cap B) \geq Pr(A)Pr(B) \), we conclude that \( \bar{x} \) is feasible for the original problem with a confidence level \( \eta \), where \( \eta \geq (1 - \tau)^2 \). The corresponding upper bound of the true problem is given as follows:

\[
U(\bar{x}) = c^T \bar{x} + \frac{1}{N} \sum_{n=1}^{N} Q(\bar{x}, \xi^n).
\]

(4–36)

**Lower bound**

To obtain a lower bound, we first consider the following Lagrange relaxation formulation (denoted as \( P_0 \)) of the original problem:

\[
\min \{ c^T x + E[Q(x, \xi)] + \pi(E[G(x, y(\xi), \xi)] - \ell) \}
\]

s. t. Constraints (4–26) and (4–28).

(4–37)

Denote the optimal objective value of \( P_0 \) as \( v_0 \), and the optimal objective value of the original problem as \( v^* \). From Lagrangian duality, we have

\[
v_0 \leq v^* \]

(4–38)
for any $\pi \geq 0$. Especially, the equation holds when $\pi$ is the optimal Lagrange multiplier and there is no duality gap.

Now, we first generate $N$ scenarios, and estimate the chance constraint \((4–28)\) as

$$\frac{1}{N} \sum_{n=1}^{N} 1_{[0, \infty)}(H(x, y(\xi^n), \xi^n)) \leq \epsilon.$$  

The Lagrange relaxation problem can be approximated as follows (denoted as \(P_1\)):

$$\min \ c^T x + E[Q(x, \xi)] + \pi(E[G(x, y(\xi), \xi)] - \ell)$$

s.t. Constraint \((4–26)\),

$$\frac{1}{N} \sum_{n=1}^{N} 1_{[0, \infty)}(H(x, y(\xi^n), \xi^n)) \leq \epsilon. \tag{4–39}$$

Then, after the scenarios are generated (e.g., \(N\) scenarios), the expected objective value in the Lagrangian relaxation objective function is estimated by the sample average approximation function:

$$\frac{1}{N} \sum_{n=1}^{N} (Q(x, \xi^n) + \pi(G(x, y(\xi^n), \xi^n) - \ell)).$$

Now the Lagrange relaxation problem is approximated as the following SAA problem (denoted as \(P_2\)):

$$\min \left\{ f_N(x) = c^T x + \frac{1}{N} \sum_{n=1}^{N} (Q(x, \xi^n) + \pi(G(x, y(\xi^n), \xi^n) - \ell) \right\}$$

s.t. Constraint \((4–26)\),

$$\frac{1}{N} \sum_{n=1}^{N} 1_{[0, \infty)}(H(x, y(\xi^n), \xi^n)) \leq \epsilon. \tag{4–40}$$

We solve the SAA problem with \(N\) scenarios \(M\) times. For these \(M\) runs, we denote the optimal value of \(P_1\) problem to be $\hat{\nu}^{P_1}_1, \hat{\nu}^{P_1}_2, \ldots, \hat{\nu}^{P_1}_M$, and the optimal value of \(P_2\) problem to be $\hat{\nu}^{P_2}_1, \hat{\nu}^{P_2}_2, \ldots, \hat{\nu}^{P_2}_M$. We follow the same scheme as the one described in [6] and [43], and pick the \(L\)th smallest optimal value of \(P_1\). Without loss of generality, we
can assume the value is $\hat{v}_L^{p_1}$. From the conclusions in [6] and [43], we have:

$$Pr(\hat{v}_L^{p_1} \leq v_0) = 1 - \tau. \quad (4-41)$$

At the same time, for each run $i = 1, 2, \cdots, M$, we have $E[\hat{v}_L^{p_2}] \leq \hat{v}_L^{p_1}$ [51]. In particular, we have

$$E[\hat{v}_L^{p_2}] \leq \hat{v}_L^{p_1}. \quad (4-42)$$

Now we approximate $E[\hat{v}_L^{p_2}]$ by statistical techniques. We set $S$ as the number of total iterations. In each iteration $s$, $1 \leq s \leq S$, we solve the SAA with $N$ scenarios $M$ times to obtain $\hat{v}_L^{p_2}$, and re-denote it as $\bar{v}_L^{p_2}$. Let

$$\bar{v}_L = \frac{\bar{v}_L^{p_2} + \cdots + \bar{v}_L^{p_2}}{S}, \quad (4-43)$$

$$\bar{v} = \bar{v}_L - z_{\tau} \sqrt{\frac{\sum_{s=1}^{S} [\bar{v}_L^{p_2} - \bar{v}_L]^2}{S(S-1)}}. \quad (4-44)$$

We can obtain that:

$$Pr(\bar{v} \leq E[\hat{v}_L^{p_2}]) = 1 - \tau. \quad (4-45)$$

From inequalities (4–38), (4–41), (4–42) and (4–45), we can see that $Pr(\bar{v} \leq v^*) \geq (1 - \tau)^2$, i.e., $\bar{v}$ is the lower bound of $v^*$ with a confidence level $\eta$, where $\eta \geq (1 - \tau)^2$.

**Summary of the Combined SAA Algorithm**

In this subsection, we summarize the combined SAA algorithm as follows (the flow chart is shown in Fig. 2-1):

1. For $s = 1, 2, \cdots, S$, repeat the following steps:
   
   (a) For $m = 1, 2, \cdots, M$, repeat the following steps:
      
      i. Set $\delta_c > 0$, $\delta_\varepsilon > 0$, $\tau \in (0, 1)$, $\bar{\varepsilon} = \varepsilon$, and $\bar{\ell} = \ell$.  

ii. Solve the SAA problem with $N$ scenarios:

$$
\min \hat{f}_N(x) = c^T x + \frac{1}{N} \sum_{n=1}^{N} Q(x, \xi^n)
$$

s.t. $Ax \leq b$, $x \geq 0$,

$$
\frac{1}{N} \sum_{n=1}^{N} 1_{(0, \infty)}(H(x, y(\xi^n), \xi^n)) \leq \bar{c},
$$

$$
\frac{1}{N} \sum_{n=1}^{N} G(x, y(\xi^n), \xi^n) \leq \bar{\ell},
$$

where the second stage is the same as (4–30). Let $(\bar{x}^m, \bar{y}^m)$ be the optimal solution to the SAA problem, and $\bar{x}^m$ be the optimal Lagrange multiplier.

iii. Generate scenarios $\xi^1, \xi^2, \ldots, \xi^{N'}$ for a large number $N'$ and use (4–34) and (4–35) to obtain $U_c(\bar{x}^m)$ and $U_e(\bar{x}^m)$. Check whether $U_c(\bar{x}^m) \leq \epsilon$ and $U_e(\bar{x}^m) \leq \ell$ hold. If the former fails, reduce $\bar{c}$ by $\delta_c$. If the latter fails, decrease $\ell$ to $\ell - \delta_e$. Return to b).

iv. Estimate the corresponding upper bound using (4–36), based on the $N'$ scenarios generated in c).

(b) Pick the smallest upper bound in 1) as the approximated upper bound $\hat{g}^a$.

(c) By using the Lagrange multiplier obtained in b), sort the $M$ optimal values of $P_2$ in a nondecreasing order, e.g., $\hat{\nu}^{P_2}_1 \leq \hat{\nu}^{P_2}_2 \leq \cdots \leq \hat{\nu}^{P_2}_M$. Pick the $L$th smallest optimal value $\hat{\nu}^{P_2}_L$ and denote it as $\hat{\nu}_{Ls}$.

2. Taking the minimum of $\hat{g}^1, \hat{g}^2, \ldots, \hat{g}^S$, we get the upper bound $\hat{g} = \min_{1 \leq s \leq S} \hat{g}^s$.

3. Compute $\bar{\nu}_L$ based on (4–43) and the estimated lower bound $\bar{\nu}$ based on (4–44).

4. Estimate the optimality gap using $(\hat{g} - \bar{\nu}) / \bar{\nu} \times 100\%$ with the confidence level at least $(1 - \tau)^2$.

**Computational Results**

In this section, we perform case studies for a six-bus system and two revised IEEE 118-bus systems to show the effectiveness of the proposed approach. We first perform sensitivity analysis of the SAA problem on the six-bus system. Then the combined SAA algorithm described in Section 4 is applied to solve the two revised IEEE-118 bus systems. We use C++ with CPLEX 12.1 to implement the proposed formulations and
Initialization

Solve SAA problem

Get $U_c(\bar{x}^m)$, $U_e(\bar{x}^m)$

reduce $\tilde{\epsilon}$

no

$U_c(\bar{x}^m) \leq \epsilon$?

yes

reduce $\tilde{\ell}$

no

$U_e(\bar{x}^m) \leq \ell$?

yes

$m = m + 1$

Estimate UB using (36)

$m = M$?

no

yes

$m = 0$, pick $\hat{g}^s$

Sort and pick $\bar{v}_{L_s}^{P_2}$

$s = S$?

no

yes

$\hat{g} = \min_{1 \leq s \leq S} \hat{g}^s$

Estimate the lower bound $\bar{v}$

Estimate the optimality gap

$s = s + 1$

Figure 4-1. Proposed SAA algorithm
algorithms. All the experiments are conducted on a computer workstation with 4 Intel Cores and 8GB RAM.

**Six-Bus System**

The six-bus system contains three thermal generators and one wind farm. The settings of the wind farm, the thermal generators, the load forecasts, and the transmission lines are the same as the ones described in [64], except the wind farm is located at bus $B_2$. We provide a sensitivity analysis on the utilization rate in the expected value constraint and the risk level in the chance constraint.

We set the sample size $N$ to be 50. We also require zero probability of load imbalance. That is, the risk level of the chance constraint is zero. The computational results are reported in Table 4-1. It can be observed that the total cost (column “obj.” in Table 4-1) increases as the utilization rate increases from 60% to 90%. This is because the utilization policy becomes more restrictive as $\beta$ increases.

<table>
<thead>
<tr>
<th>Utilization $\beta$</th>
<th>Obj. ($)</th>
<th>CPU Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60%</td>
<td>26977.5</td>
<td>2.7</td>
</tr>
<tr>
<td>70%</td>
<td>27646.2</td>
<td>20.5</td>
</tr>
<tr>
<td>80%</td>
<td>32991.9</td>
<td>10.2</td>
</tr>
<tr>
<td>85%</td>
<td>39603.9</td>
<td>9.5</td>
</tr>
<tr>
<td>90%</td>
<td>46215.9</td>
<td>9.1</td>
</tr>
</tbody>
</table>

To investigate the significance of the settings on the chance constraint, we report the results in Table 4-2 with different risk levels. The sample size is set to be 50 and $\beta$ is set to be 70%. It can be observed that the total cost decreases as the risk level

<table>
<thead>
<tr>
<th>Risk Level $\epsilon$</th>
<th>Obj. ($)</th>
<th>CPU Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>27235.5</td>
<td>77.9</td>
</tr>
<tr>
<td>0.10</td>
<td>26649.0</td>
<td>161.4</td>
</tr>
<tr>
<td>0.15</td>
<td>26284.3</td>
<td>160.2</td>
</tr>
<tr>
<td>0.20</td>
<td>25928.5</td>
<td>821.6</td>
</tr>
</tbody>
</table>
increases from 0.05 to 0.2, because the load imbalance policy become more relaxed as the risk level increases.

Finally, we run the experiments with different scenario sizes and observe that the optimal objective value converges as the sample size increases to 70 (the risk level is set to be 0.1 and \( \beta \) is set to be 70%).

**Revised 118-Bus Systems**

Two revised IEEE 118-bus systems (named 118SW and 118DW) based on the one given online at [http://motor.ece.iit.edu/data](http://motor.ece.iit.edu/data) are studied in this section. Apart from the 33 thermal generators in the original IEEE 118-bus system, 118SW is created by adding a single centralized wind farm and 118DW is created by adding 10 wind farms at 10 different buses. All the 186 transmission lines in the original system are selected for both revised systems.

**SAA algorithm analysis for 118SW**

The computational results of our algorithm for 118SW are reported in Table 4-3. In the first column, we indicate that different combinations of validation settings (i.e., iteration numbers \( (S, M) \) and validation scenario number \( (N') \)) are considered in the experiments. The scenario size of the SAA problem is given in the second column. The lower bound and upper bound obtained by the SAA algorithm are reported in the third and fourth columns. The fifth column represents the gap which is calculated by \( \frac{UB - LB}{LB} \times 100\% \). Finally, the CPU time of the algorithm is reported in the sixth column.

From the table, we have the following observations: First, when the scenario size of the SAA problem is \( N = 10 \), we already obtain a small optimality gap at 0.44%. Such an observation suggests that the solution of the SAA problem with the scenario size \( N = 10 \) is already very close to the solution of the true problem. Secondly, the optimality gap decreases as the scenario size of the SAA problem and the validation problem increases. Finally, in the last row, when the scenario size of the SAA problem is \( N = 50 \)
Table 4-3. Computational results for the 118-bus system with different combinations of iterations and sample sizes

<table>
<thead>
<tr>
<th>$(S \times M, N')$</th>
<th>$N$</th>
<th>LB</th>
<th>UB</th>
<th>Gap</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(5 \times 5, 100)$</td>
<td>10</td>
<td>585578</td>
<td>588178</td>
<td>0.44%</td>
<td>127</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>585873</td>
<td>588178</td>
<td>0.39%</td>
<td>402</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>586651</td>
<td>588206</td>
<td>0.27%</td>
<td>1379</td>
</tr>
<tr>
<td>$(10 \times 5, 500)$</td>
<td>10</td>
<td>586133</td>
<td>587700</td>
<td>0.27%</td>
<td>316</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>586818</td>
<td>587700</td>
<td>0.15%</td>
<td>783</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>587144</td>
<td>587734</td>
<td>0.10%</td>
<td>2319</td>
</tr>
<tr>
<td>$(20 \times 10, 1000)$</td>
<td>10</td>
<td>586829</td>
<td>587758</td>
<td>0.15%</td>
<td>1364</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>587219</td>
<td>587758</td>
<td>0.09%</td>
<td>3325</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>587380</td>
<td>587793</td>
<td>0.07%</td>
<td>9005</td>
</tr>
</tbody>
</table>

with the iteration numbers and the validation scenario size being $(20 \times 10, 1000)$, the smallest optimality gap is achieved at 0.07%.

**Distributed wind power system 118DW**

We run the experiments on 118DW in this section and analyze the impact of the expected value constraint on distributed wind power resources. In total, there are 10 wind farms at 10 different buses (bus numbers are shown in Table 4-4). Under such a setting, the associated wind power uncertainties affect the power grid operations dramatically.

We first relax the expected value constraint and solve the stochastic optimization problem. The utilization rates of wind power at specific buses are summarized in Table 4-4. The lowest utilization and highest utilization are observed in $B_{36}$ and $B_{96}$, respectively. The reason of low utilization is that there is one thermal generator in $B_{36}$ and the capacity of the transmission line connected with this bus is relatively lower compared with others. The wind power has to be curtailed due to the limited transmission line capacity. Conversely, the total capacity of the transmission lines connected with $B_{96}$ is much larger (there are five transmission lines connected with $B_{96}$) and no generator is located there.
Table 4-4. Computational results for 118DW - without expected value constraint

<table>
<thead>
<tr>
<th>Bus NO.</th>
<th>Utilization</th>
<th>Bus NO.</th>
<th>Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_6$</td>
<td>90%</td>
<td>$B_{56}$</td>
<td>65%</td>
</tr>
<tr>
<td>$B_{16}$</td>
<td>78%</td>
<td>$B_{66}$</td>
<td>76%</td>
</tr>
<tr>
<td>$B_{26}$</td>
<td>73%</td>
<td>$B_{76}$</td>
<td>74%</td>
</tr>
<tr>
<td>$B_{36}$</td>
<td>40%</td>
<td>$B_{86}$</td>
<td>71%</td>
</tr>
<tr>
<td>$B_{46}$</td>
<td>90%</td>
<td>$B_{96}$</td>
<td>96%</td>
</tr>
</tbody>
</table>

Then we solve the problem with the expected value constraint enforced. The utilization rate is set to be 85%. The corresponding utilization rates at each studied bus are reported in Table 4-5. It can be observed that the utilization rates at all buses are increased compared with those in Table 4-4. On the other hand, we can also observe that the utilization increments at different buses are not the same as the largest increment is 22% ($B_{26}$) and the smallest increment is 2% ($B_{96}$), which shows the advantage of applying the proposed solution approach to minimize the total cost while ensuring the overall high wind power utilization rate. In other words, our model can help coordinate generation injections at different buses and provide the optimal generation strategy at each bus.

Table 4-5. Computational results for 118DW - with expected value constraint

<table>
<thead>
<tr>
<th>Bus NO.</th>
<th>Utilization</th>
<th>Bus NO.</th>
<th>Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_6$</td>
<td>97%</td>
<td>$B_{56}$</td>
<td>69%</td>
</tr>
<tr>
<td>$B_{16}$</td>
<td>84%</td>
<td>$B_{66}$</td>
<td>90%</td>
</tr>
<tr>
<td>$B_{26}$</td>
<td>95%</td>
<td>$B_{76}$</td>
<td>85%</td>
</tr>
<tr>
<td>$B_{36}$</td>
<td>55%</td>
<td>$B_{86}$</td>
<td>78%</td>
</tr>
<tr>
<td>$B_{46}$</td>
<td>93%</td>
<td>$B_{96}$</td>
<td>98%</td>
</tr>
</tbody>
</table>

**Concluding Remarks**

In this chapter, we proposed the expected value constrained stochastic program to study the uncertain wind power generation. By adjusting the utilization rate of wind resources, the system operator can change the utilization rates of wind resources...
through the expected value constraint. In addition, the chance constraint is applied to model the load imbalance with a small probability. Thus, our model incorporates both the expected value constraint and chance constraint. Accordingly, we proposed a combined SAA algorithm to solve the problem. We also performed the sensitivity analysis on different risk levels and wind utilization rates for the six-bus system. Finally, the computational results of the revised IEEE 118-bus systems exhibit the scalability of our proposed models and algorithms. It is worthwhile to note that by previous study [64], ISOs can address a high wind power utilization with a chance-constrained stochastic program. The expected value constraint, from another perspective, allows the ISOs enforce an overall high wind power usage. The ISOs can accordingly choose the different policies and corresponding constraints to enhance the wind power penetration level. In some extreme case, the ISOs might want to have both chance constraint and expected value constraint on the wind power utilization. The proposed combined SAA algorithm in this research can be applied to such a case without loss of generality.
CHAPTER 5
STOCHASTIC UC WITH UNCERTAIN DEMAND RESPONSE

The objective of an Independent System Operator (ISO) is to maximize the social welfare for electricity producers and customers. Customers participating in the Demand Response (DR) program can expect savings by reducing their electricity usage during peak periods [8]. In the literature, DR was mostly modeled as a fixed demand curve. However, due to a variety of reasons such as lack of attention, latency in communication, and change in consumption behavior, the actual response from the consumers to a price signal is uncertain in nature. Hence, the customer behavior is explicitly modeled by an uncertain demand elasticity in this chapter, which means customers have different responses to the electricity prices under different scenarios. We also consider generator outages and transmission line contingencies which can be addressed by DR programs to avoid or reduce forced load curtailment. Our proposed approach can be applied to enhance the reliability unit commitment process for ISOs.

We consider a two-stage stochastic programming formulation with unit commitment decisions at the first stage and real-time generation and load amount decisions at the second stage. The objective is to maximize the social welfare:

\[
\max - (c_{gs} + \sum_{t=1}^{T} \sum_{i} E[f_i(x_{i}(\xi))])
\]
\[
+ \sum_{t=1}^{T} \sum_{b} E[F_{t,b,\xi}(d_{b}^{l}(\xi))] - \sum_{t=1}^{T} E[\gamma_{t} w_{t}(\xi)],
\]

(5–1)

where \(c_{gs}\) denotes generator start-up and shut-down costs [64], \(f_i(x_{i}(\xi))\) represents the fuel cost for generator \(i\) at time \(t\) when the generation amount is \(x_{i}(\xi)\), \(F_{t,b,\xi}(\cdot)\) represents the consumer benefit at bus \(b\) at time \(t\) with \(d_{b}^{l}(\xi)\) representing the amount of elastic load at bus \(b\) at time \(t\) (note here each bus load includes both inelastic and elastic loads, and the consumer benefit for the inelastic load is zero, cf. [55]. Therefore,
the objective function only includes elastic loads), and \( w_t(\xi) \) and \( \gamma_t \) represent the total amount of load curtailment and unit penalty cost at time \( t \), respectively.

Our model includes generation upper/lower bound constraints, min-up/-down time constraints, start-up/shut-down constraints, ramp-up/-down constraints, spinning reserve constraints, and transmission capacity constraints (cf. [64]). Both the generation upper/lower bound and transmission capacity constraints consider contingencies. For instance, the generation bound constraints with uncertain generator contingency consideration are modeled as follows:

\[
L_i y_{it} (1 - C_i(\xi)) \leq x_{it}(\xi) \leq U_i y_{it} (1 - C_i(\xi)) \quad \forall i, \forall t, \quad (5-2)
\]

\[
Pr(C_i(\xi) = 1) = \tau_i \quad \forall i, \quad (5-3)
\]

where \( L_i \) and \( U_i \) are lower and upper bounds of generator \( i \), \( y_{it} \) is a binary variable to indicate if generator \( i \) is on during time period \( t \), \( C_i(\xi) \) is a random binary parameter indicating the contingency of generator \( i \), and \( \tau_i \) is the given probability value that the contingency happens for generator \( i \). Constraints (5–2) enforce the generation output to be zero during the contingency. Finally, a chance constraint is introduced to formulate the loss of load probability (LOLP) as follows:

\[
Pr \left( \sum_b (d^b_t(\xi) + \hat{d}^b_t(\xi)) \leq \sum_i x_{it}(\xi), \forall t \right) \geq 1 - \epsilon, \quad (5-4)
\]

where \( \epsilon \) is defined as risk level and \( \hat{d}^b_t(\xi) \) represents the amount of inelastic load at bus \( b \) at time \( t \).

Solution Methodology and Case Study

Solution Methodology

An SAA method is utilized to solve the problem. In our approach, a Monte Carlo method is first applied to generate scenarios (e.g., \( N \) scenarios). Then, the expected value function is replaced with the sample average function, and accordingly the chance constraint is replaced with an MILP reformulation as in [43]. The price-elastic
demand curve for each ISO could be different with the common part that the demand is a non-increasing function of price (cf. [55] and [33]). This curve can be obtained by simulation and historical data analysis. Without loss of generality, in this chapter, the price-elastic demand curve is described as

\[ d_t^b(\xi) = A_t^b \rho_t^b \alpha_t^b(\xi) \]  

(cf. [56]) with the purpose to illustrate our proposed solution approach. For a given elasticity \( \alpha_t^b(\xi) \), \( A_t^b \) can be decided by the given reference point \( (D_{t,b}^{\text{ref}}, P_{t,b}^{\text{ref}}) \). Then, a step-wise function is applied to approximate this demand-price function as described in [56]:

\[ F_{t,b}(d_t^b(\xi)) = \sum_{k=1}^{K} p_t^k r_{t,b}^k(\xi) \]  

(5–5)

\[ d_t^b(\xi) = \sum_{k=1}^{K} r_{t,b}^k(\xi), \quad 0 \leq r_{t,b}^k(\xi) \leq l_{t,b}^k, \]  

(5–6)

where \( K \) is the number of steps (see Fig. 5-1), \( p_t^k \) and \( l_{t,b}^k \) are given for each \( k \), and \( r_{t,b}^k(\xi) \) is an auxiliary decision variable. Based on (5–5) and (5–6), and the max objective, we have \( r_{t,b}^k(\xi) = l_{t,b}^k \) in the solution if \( d_t^b(\xi) \geq \sum_{u=1}^{K} l_{t,b}^u \). Thus, the discrepancy between the approximation and the integral of the curve (consumer benefit) is equal to the difference between the shaded area above the curve and the one below the curve, and this discrepancy converges to zero as \( K \to +\infty \). In addition, \( \alpha_t^b(\xi) \) is assumed to follow a normal distribution (our methodology can also be applied to other distributions) and a Monte Carlo method is applied to generate \( \alpha_t^b(\xi) \) for obtaining \( F_{t,b,\xi} \) under different scenarios.

For the probability constraints, during the scenario generation process, we randomly set generator \( i \) under contingency status for \( \tau_i \times N \) scenarios, and transmission line \( (m, n) \) under contingency status for \( \kappa_{mn} \times N \) scenarios, where \( \kappa_{mn} \) is the given probability value that the contingency happens for transmission line \( (m, n) \) and \( N \) represents the total number of scenarios.

**Case Study**

We study the revised IEEE 118-bus system (online at ece.iit.edu/data) with 33 generators to illustrate the results.
Figure 5-1. Step-wise approximation of price-elastic demand curve

**Deterministic DR vs. stochastic DR**

We first set the risk level to be zero and compare the results based on the deterministic DR and stochastic DR representations, to show how stochastic DR works better. We assume possible contingencies occur on two transmission lines and two generators in this subsection. For the deterministic DR, the price-elastic demand curve is certain in which the mean value of the elasticity is taken to generate the curve. Several indicators are given in Table 5-1 for comparison purposes. It can

Table 5-1. Deterministic vs. stochastic

<table>
<thead>
<tr>
<th></th>
<th>Deterministic</th>
<th>Stochastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Start-ups</td>
<td>82</td>
<td>144</td>
</tr>
<tr>
<td>Expected Reserve Amount (MW)</td>
<td>1191</td>
<td>2100</td>
</tr>
<tr>
<td>Expected Load Loss (MW)</td>
<td>833</td>
<td>120</td>
</tr>
<tr>
<td>Solution Time (sec.)</td>
<td>35.2</td>
<td>50.6</td>
</tr>
</tbody>
</table>
be observed that the stochastic formulation approach puts more generators online to provide additional capacity for unexpected consumption behaviors. This approach provides more reserves (for each scenario, it is measured as the difference between the total generation capacity of online generators and the load) which lead to less load curtailment.

**Risk levels and demand response effect**

The optimal objective values corresponding to different risk levels are reported in Table 5-2. The risk level is represented by the probability defined in the chance constraint (5–4), which indicates the possibility of the load being curtailed. It can be observed that the social welfare increases when the risk level increases, because allowing load curtailment provides more flexibility for generation scheduling.

Table 5-2. Computational results for different risks

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>Obj. ($)</th>
<th>Time (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1252530</td>
<td>24.5</td>
</tr>
<tr>
<td>10%</td>
<td>1323200</td>
<td>57.8</td>
</tr>
<tr>
<td>30%</td>
<td>1464530</td>
<td>131.9</td>
</tr>
</tbody>
</table>

To show the effectiveness of DR, we assume $\alpha_i^b$ the same for each $b$ and $t$ and compare the optimal social welfare using a group of elasticities with different mean and standard deviation values (e.g., $\mu_\alpha$ and $\sigma_\alpha$). It can be observed in Table 5-3 that the total social welfare has a tendency to increase as the demand elasticity increases. But it is not guaranteed that there is always a positive correlation between elasticity and welfare. It depends on each specific price-elastic demand curve. Also, our conclusion is based on the “reference point” modeling approach we used. It may not be generalized to other modeling methods. However, the general modeling framework we described in this chapter can accommodate other demand side modeling approaches. Our proposed solution approach can solve theses models efficiently and numerically.
Table 5-3. Computational results for different elasticities

<table>
<thead>
<tr>
<th>(µ, σ)</th>
<th>Obj. ($)</th>
<th>Time (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-0.8, 0.2)</td>
<td>1252530</td>
<td>24.90</td>
</tr>
<tr>
<td>(-2, 1)</td>
<td>1494270</td>
<td>30.28</td>
</tr>
<tr>
<td>(-3, 2)</td>
<td>1738670</td>
<td>14.57</td>
</tr>
</tbody>
</table>

Concluding Remarks

In this chapter, we provided a general modeling framework that considers the uncertain demand-side response in which price-elastic demand curves vary by scenario. This framework can accommodate different demand side modeling approaches. In addition, the proposed chance constraint controls the LOLP and the sample average approximation method can solve the IEEE 118-bus system efficiently. Final case studies indicate that the stochastic representation of uncertain demand response can lead to more available generation capacity, as compared to its deterministic counterpart.
CHAPTER 6
CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

In this dissertation, several stochastic programming formulations are proposed to capture the uncertainty in the unit commitment problem. In particular, a chance-constrained two-stage stochastic program is developed to study the stochastic unit commitment with uncertain wind power output for ISO. This idea is extended to study the price-based unit commitment for electricity market participant IPP. Then, an expected value constrained stochastic program is considered to guarantee high utilization of wind power in ISO’s unit commitment problem from a different perspective. Finally, the chance-constrained two-stage stochastic program is applied to study the uncertain demand response. We develop corresponding SAA frameworks for the above hybrid models. The case studies indicate the significance of the new stochastic unit commitment models and effectiveness of the new SAA algorithms. Next, we briefly discuss some future research directions.

Unit Commitment with Uncertain Contingency

Unexpected contingencies of power grid elements, such as transmission lines and generators, can result in dramatic electricity shortages or even large-scale blackouts (see, e.g., [10, 24, 47, 71]). Given the topology of a power grid structure, unit commitment is among the most crucial decisions for a system operator to handle post-contingency. The well-known $N$-1 and $N$-2 security criteria are implemented in industry practice with stochastic unit commitment models (see, e.g., [50, 62]). These criteria have also been generalized to consider multiple contingency cases (e.g., $N$-$k$ criterion [13]). With the $N$-$k$ rule, a power grid with the $N$ components will continue to meet demand whenever any $k$ or fewer components suffer a contingency. However, very limited research [31, 53] has been done to incorporate the general $N$-$k$ rule in the unit commitment using the stochastic programming or robust optimization due to the computational intractability.
Robust optimization is another state-of-the-art approach to solve decision-making under uncertainty. Compared with stochastic programming, robust optimization does not require an explicit probability distribution of the uncertain parameter. The uncertainty representation is a deterministic set in the robust optimization. Robust optimization provides the optimal solution under the worst-case scenario, thus its solution immunizes all the possible scenario realizations (i.e., always be feasible). It is an important and interesting topic to formulate and solve the contingency-constrained unit commitment with stochastic programming/robust optimization and compare their performances.

**Other Recourses to Hedge Wind Power Uncertainty**

Since demand response can be considered as an additional reserve capacity, it will be interesting to use the demand response to balance the uncertain wind power generation. Recently, some promising energy storage techniques such as plug-in hybrid electric vehicle (PHEV) in the smart grid can accommodate the wind power in a significant manner. As a future subject, we are interested in modeling and solving more advanced stochastic unit commitment with diversified resources to hedge the wind power uncertainty.
APPENDIX: PROOF OF PROPOSITION 2.1

Convergence proofs for the chance-constrained and the two-stage stochastic programs have been studied in [43] and [5], respectively. This appendix, however, provides the first proof for the case that contains both chance-constrained and two-stage stochastic program features.

In our CCTS program, the samples of the random variable (wind generation) are used in the approximations for both the second-stage value and the chance constraint part. Once the samples are generated, the stochastic problem will become a deterministic problem. We show in this part that the objective of the deterministic problem is convergent to that of the stochastic problem as the scenario size goes to infinity.

Recall that our CCTS program (e.g., the true problem) can be expressed as follows:

\[
\begin{align*}
\min & \sum_{b \in BG} \sum_{t=1}^{T} \sum_{i \in \Lambda_b} (\mu_i^b u_i^b + \theta_i^b \nu_i^b + \alpha_i^b o_i^b + F_c(q_i^b)) \\
& + \sum_{t=1}^{T} \gamma_t E[ \sum_{b \in BW} S_t^b(\xi)] \\
\text{s.t.} \quad & (2 - -2) - (2 - -13) \\
& Pr\{G(x, \xi) \leq 0 \} \geq 1 - \epsilon \\
& S_t^b(\xi) = \max\{0, q_t^b - w_{bt}(\xi)\}, \forall t, \forall \xi, \forall b \in BW \\
& q_t^b, \hat{q}_t^b \geq 0; o_t^b, u_t^b, \nu_t^b \in \{0, 1\}, \forall t, \forall i, \forall b.
\end{align*}
\]

Before starting the proof of the convergence, we introduce the following two approximated models:

(i) Replace the chance-constrained part by the sample approximation.

\[
\begin{align*}
\min & \sum_{b \in BG} \sum_{t=1}^{T} \sum_{i \in \Lambda_b} (\mu_i^b u_i^b + \theta_i^b \nu_i^b + \alpha_i^b o_i^b + F_c(q_i^b)) \\
\end{align*}
\]
\[ + \sum_{t=1}^{T} \gamma_t E[ \sum_{b \in BW} S_t^b(\zeta)] \]  
\text{(A–2)}

s.t.

\[ (2 - 2) - (2 - 13) \]

\[ N^{-1} \sum_{j=1}^{N} 1_{(0, \infty)}(G(x, \xi^j)) \leq \epsilon \]

\[ S_t^b(\zeta) = \max\{0, \hat{q}_t^b - w_{bt}(\zeta)\}, \forall t, \forall \xi, \forall b \in BW \]

\[ q_{it}^b, \hat{q}_t^b \geq 0; \sigma_{it}^b, u_{it}^b, v_{it}^b \in \{0, 1\}, \forall t, \forall i, \forall b. \]

(ii) Replace both the chance-constrained and the second-stage parts by the sample approximation.

\[
\min \sum_{b \in BG} \sum_{t=1}^{T} \sum_{i \in \Lambda_b} (\mu_t^b u_{it}^b + \theta_t^b v_{it}^b + \alpha_t^b o_{it}^b + F_c(q_{it}^b))
\]

\[ + N^{-1} \sum_{t=1}^{T} \sum_{j=1}^{N} \gamma_t \sum_{b \in BW} S_t^b(\xi^j) \]  
\text{(A–3)}

s.t.

\[ (2 - 2) - (2 - 13) \]

\[ N^{-1} \sum_{j=1}^{N} 1_{(0, \infty)}(G(x, \xi^j)) \leq \epsilon \]

\[ \hat{q}_t^b \leq w_{bt}(\xi^j) + S_t^b(\xi^j) \]

\( (\forall t; \forall b \in BW; j = 1, 2, \ldots, N) \)

\[ q_{it}^b, \hat{q}_t^b, S_t^b(\xi^j) \geq 0; \sigma_{it}^b, u_{it}^b, v_{it}^b \in \{0, 1\}, \forall t, \forall i, \forall b. \]

Before we give the detailed proof of the proposition, we show a corollary based on an assumption and Proposition 2 in [43].

**Assumption 1.** There is an optimal solution \( \bar{x} \) of the true problem (1–7) such that for any \( \nu > 0 \) there is \( x \in X \), where \( X \) is the feasible region for the problem, with \( \|x - \bar{x}\| \leq \nu \) and \( q(x) \leq \epsilon \).

**Proposition A.1.** Suppose that the significance levels of the true and SAA problems are the same (i.e., \( \epsilon = \epsilon_N \)), the set \( X \) is compact, the function \( f(x) \) and decision variables are
continuous, $G(x, \zeta)$ is a Caratheodory function, and the above assumption holds, then
$
\hat{\theta}_N \rightarrow \theta^* \text{ and } D(\hat{\theta}_N, x^*) \rightarrow 0 \text{ w.p.} 1 \text{ as } N \rightarrow \infty.
$

In Proposition 2, the convergence property holds for the continuous case. Now we show the convergence property holds for the mixed integer case in the following corollary.

**Corollary 1.** For the mix-integer case, suppose the objective function is $f(x, y)$, where $x \in X$ is the set of binary variables, and $y \in Y$ is the set of continuous variables. If $X$ is finite, and the other assumptions in the above proposition hold, then, we still have
$
\hat{\theta}_N \rightarrow \theta^* \text{ and } D(\hat{\theta}_N, x^*) \rightarrow 0 \text{ w.p.} 1 \text{ as } N \rightarrow \infty.
$

**Proof.** Let $|X| = \Gamma$. We denote the elements of set $X$ in order: $x^1, x^2, \ldots, x^\Gamma$. For each fixed $x^i$, we can apply Proposition 2 for continuous variable $y$ and get a corresponding convergent solution by solving the SAA problem, i.e.,

$$\min \{f_N(x^i, y)\} \rightarrow \min \{f(x^i, y)\} \equiv g(x^i), \quad (A-4)$$

where $f_N(., .)$ represents the objective function for the SAA problem when the sample size is $N$.

Without loss of generality, we assume

$$\sigma = \min_{1 \leq i, j \leq \Gamma} \| g(x^i) - g(x^j) \| > 0. \quad (A-5)$$

Now let $x^*$ be the integer part in the optimal solution of the true problem, and $\hat{x}_N$ be the integer part in the optimal solution of the SAA problem when the sample size is $N$. Based on (A-4), there exists a large constant number $N_0$ such that

$$\| f_N(\hat{x}_N, \hat{y}_N) - g(\hat{x}_N) \| < \frac{\sigma}{2}, \quad (A-6)$$

and

$$\| f_N(x^*, \hat{y}_N) - g(x^*) \| < \frac{\sigma}{2}, \quad (A-7)$$

92
when $N > N_0$. Meanwhile, we should have

$$f_N(x^*, \hat{y}_N) \geq f_N(\hat{x}_N, \hat{y}_N), \quad (A-8)$$

since $(\hat{x}_N, \hat{y}_N)$ is the optimal solution for the SAA problem when the sample size is $N$.

On the other hand, it is obvious that

$$g(\hat{x}_N) \geq g(x^*), \quad (A-9)$$

based on the definition of $x^*$. If $\hat{x}_N \neq x^*$, then based on (A–8) and (A–9),

$$0 \leq g(\hat{x}_N) - g(x^*) \leq g(\hat{x}_N) - g(x^*) + f_N(x^*, \hat{y}_N) - f_N(\hat{x}_N, \hat{y}_N).$$

Thus

$$\| g(\hat{x}_N) - g(x^*) \| \leq \| g(\hat{x}_N) - g(x^*) + f_N(x^*, \hat{y}_N) - f_N(\hat{x}_N, \hat{y}_N) \| \leq \| g(\hat{x}_N) - f_N(\hat{x}_N, \hat{y}_N) \| + \| g(x^*) - f_N(x^*, \hat{y}_N) \| < \sigma,$$

where the third inequality follows from (A–6) and (A–7). This contradicts with (A–5) and the original conclusion holds.

Now we prove our proposition in two steps.

First, we prove that the solution of (A–3) converges to that of (A–2). Notice that (A–2) is a pure two-stage stochastic program, where the first stage decision variables are continuous or discrete and the expectation function in the objective function is continuous. Based on the conclusion in [5], the SAA of this problem converges to the true value of the two-stage stochastic program.
Second, we prove that the solution of \((A^-2)\) converges to that of \((A^-1)\). It is easy to see that our model satisfies all conditions in the above corollary. Then, accordingly, the solution of \((A^-2)\) converges to that of \((A^-1)\).

Therefore, the solution of our SAA problem \((A^-3)\) converges to that of the true problem \((A^-1)\). The conclusion holds.
REFERENCES


[66] ———. “Price-Based Unit Commitment With Wind Power Utilization Constraints.”


(2007).


[73] Wu, L., Shahidehpour, M., and Li, Z. “Comparison of Scenario-Based and Interval


BIOGRAPHICAL SKETCH

Qianfan Wang received his B.S. in computer science and B.S. in economics from the Peking University, Beijing, China in 2009. He received his Ph.D. in industrial and systems engineering from the University of Florida, Gainesville, FL in spring, 2013. He was a visiting student at the Argonne National Laboratory in fall, 2010 and spring, 2011. He also interned with SAS Institute, Cary, NC and Alstom Grid, Redmond, WA in summer, 2011 and summer, 2012.