ELECTRICITY BLACKOUT AND POWER SECURITY: SURVEY AND ANALYSIS

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A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL Fulfillment
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
UNIVERSITY OF FLORIDA
2012
This work is dedicated to my parents who have supported me in all my endeavors.
ACKNOWLEDGMENTS

First, I would like to thank Dr. Panos M. Pardalos, the chairman of my graduate committee, for providing me this opportunity to work under his advising on this energy topic. I appreciate his guidance and help throughout my Ph.D. degree studies. I am grateful to Dr. William W. Hager, Dr. Guanghui Lan, and Dr. Vladimir Boginski, members on my graduate committee, for their valuable time and attention.

I would also like to thank all my friends who have been providing me with academic help and moral support throughout my studies in Department of Industrial and Systems Engineering at University of Florida. I extend my special thanks to Dr. Amar Sapra, whom it is that inspired my enthusiasm of pursuing a Ph.D. degree here.

Above all, I would like to thank my family for their continuous faith, love and support on me. It is their bless, care and encouragement that give me the motivation to achieve my goals.
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Electricity power systems are critical to any country’s economy and security. Blackout, the most severe form of power loss to a relatively wide area, posting tremendous societal consequences, could be resulted from different causes from failure fault at a power transmission line to an intentional attack. Therefore, it is of importance to study the behavior of power grid (a complex network) during power blackout with its failure pattern. Based on these findings, we can find the suitable plan for normal daily management/expansion, emergency response and restoration after blackout. However, there are many schools of scholars using different ways to model the Blackout phenomenon, with different conclusions. Since those researches are done over several decades, it is important to organize them in a systematic way so that it could help the future research. Also, several minmax optimization models are proposed with different scenarios to represent the impact of blackout.
Since the industrial age, human-being has created many networks, which are tightly related to our daily life. These networks could be categorized as physical or engineered networks, information networks, biological networks, cognitive and semantic networks and social networks, which for example include but not limit to electrical network, computer network, biological network, artificial neural network, social network, business networking, radio network, telecommunications network, television network.

As the concept of network reaches every aspects of the human society, the new discipline of network theory has been developed to study their interconnection phenomena, and to explore common principles, algorithms and tools that govern network behavior. Several parameters such as density, size, average degree, average path length, diameter of a network, clustering coefficient and connectedness, which are rooted from graph theory, have been utilized to analyze the properties and characteristics of the network. Based on different parameters, generic network could be described by different network models, such as Small World model, Scale-Free network model, Preferential Attachment model and SIR model etc. The Small World model, Scale-Free network model, and Preferential Attachment model are randomly-generated graphs, while the first two observe the degree-distribution fitted with a power law distribution. The Small-World network is the one which is studied mostly and widely since it has the small-world properties which are found in many real-world phenomena, including road maps, food chains, electric power grids, metabolite processing networks, networks of brain neurons, voter networks, telephone call graphs, and social influence networks. With this property, networks are likely to have cliques, quasi-cliques, and/or clubs, meaning high connectivity between any two nodes within sub-networks due to a high clustering coefficient, and at the meanwhile most pairs of nodes will be connected by at least one short path via some “hub” nodes with a high degree.
Among those example networks, electricity network or power grid has been on the table drawing lots of eyeballs, and one reason is that it is the oldest and most traditional of the various mega structures. The electricity network has been evolving from several close-circuit systems within some particular geographic areas, where all energy was produced near the end-users which demand that energy. During the past decades the power infrastructure has evolved into what many experts consider the largest and most complex system of the technological age. Geographically, the North American power grid forms a network of over 6100 generation plants, over 365,000 mile of high voltage lines, over 6 million mile of low voltage lines and over 45,000 transformers that are continuously regulated by sophisticated control equipments. Consisting of millions of miles of lines operated by over 500 companies, the Continental U.S. power grid (Fig. 1-1) is one of largest electricity infrastructure in the world, and it is a complex network system of power plants and transmission lines which are independently owned and operated. However, despite of the advance in technology and design of the electrical grid, its power delivery infrastructures suffer aging issue across the whole world. Aged equipments and facilities have higher failure rates and subject to higher maintenance
and further repair/restoration costs; aged areas require additional substation sites
more than available to meet the demand of ever growing customers; problems caused
by aged equipments & facilities, obsolete system layouts, and modern deregulated
electricity distribution patterns could not be effectively addressed by traditional concepts,
planning, engineering, operation of power network systems.

The other reason why electricity network is so important is that it recently attracted
attention from the House Foreign Affairs Committee about its security issue. Delivery
of electricity power is critical to any country’s economy and security. As a result of the
recent deregulation of power generation and transmission, about one-half of all domestic
generation is now sold over ever-increasing distances on the wholesale market before it
is delivered to customers, and consequently the power grid is witnessing power flows in
unprecedented magnitudes and directions. Therefore surges in power lines can cause
massive network failures and permanent damage to multimillion dollar equipment in
power generation plants.

The electrical network has its own characteristics to make itself special than other
networks. First, it has to observe the following physical laws: (1)Kirchhoff’s current law:
The sum of all currents entering a node is equal to the sum of all currents leaving the
node; (2) Kirchhoff’s voltage law: The directed sum of the electrical potential differences
around a loop must be zero; (3) Ohm’s law: The voltage across a resistor is equal to
the product of the resistance and the current flowing through it; (4) Norton’s theorem:
Any network of voltage or current sources and resistors is electrically equivalent to
an ideal current source in parallel with a single resistor; and (5) Thévenin’s theorem:
Any network of voltage or current sources and resistors is electrically equivalent to
a single voltage source in series with a single resistor. Second, electric energy is
instantaneously consumed and it is currently too expensive to store. Third, as the
modern trends in for the 21st century, the electric utility industry seeks to take advantage
of novel approaches to meet growing energy demand with everything interconnected.
Within this kind of wide area synchronous grid, alternating current (AC) with frequencies synchronized can be transmitted throughout the wide area, connecting a large number of electricity generators and consumers. However, in such a synchronous grid all the generators run not only at the same speed but also at the same phase, and generation and consumption must be balanced across the entire grid. Hence, a single failure in a local area could cause power flow to re-route over transmission lines of insufficient capacity, which may result in further failures in other parts of the grid, in other word, the possibility of cascading failure and widespread power outage.

A electricity blackout is the situation where there is a total loss of power to a relatively wide area, and it is the most severe form of power outage. There are many causes of blackout in an electricity network, including faults at power plant stations, damage to power lines, a short circuit, or the overloading of electricity transmission systems. Blackouts are especially difficult to recover quickly, and may last from a few hours to a few weeks depending on the nature of the blackout and the configuration of the electrical network. Restoring power after a wide-area blackout needs to be done with the help of power from other grid. In the extreme case where there is total absence of grid power, a so-called black start needs to be performed to bootstrap the power grid into operation, which depends greatly on local circumstances and operational policies. Since localized “power islands” are progressively coupled together, in order to maintain supply frequencies within tolerable limits during this process, demand must be reconnected at the same pace that generation is restored, requiring close coordination between power stations, transmission and distribution organizations.

In the United States, the system’s vulnerability to physical disruptions from natural disasters and other causes has long been studied. However, this vulnerability has increased in recent years because infrastructure has not expanded as quickly as demand has, thereby reducing the system’s tolerance against deteriorated, failed, or unavailable system components. During the most severe form of power outage,
Blackout which is a total loss of power to a relatively wide area, tremendous societal consequences and substantial economical loss would be incurred. The possible causes for blackout may be faults at power stations, damage to power lines, a short circuit, or the overloading, the entire procedure could be so complicated due to the cascading phenomenon which is resulted from the self-organizing dynamical forces driving the system. Moreover, the threat of human attacks on the system has become more serious, too. Therefore, the methodology and algorithms proposed here could have rich applications used in daily operation management, emergency strategy and expansion planning for power grid.
CHAPTER 2
BLACKOUT ANALYSIS SURVEY

2.1 Background

The U.S. Electric Power System has been serving this nation for more than a century ever since Thomas Edison designed and built the world’s first central power station in New York City in 1882. Today, however, the aged but scattered infrastructure combined with an increasing demand in domestic electricity consumption, which are extremely vulnerable to even small scale of unintentional outage failure (say nothing of intentional contingency), has forced us to critically examine the condition and health of the nation’s electrical systems before it could be outdated and unprepared to deliver reliable electricity to the consumers. Therefore, there is a lot of academic research which aims at predicting the occurrence of blackout and mitigating the impact of blackout by designing and operating power systems in the way that the power grid system could tolerate with emergency events such as the loss of power transmission route, changing in the demand and generation patterns, etc.

2.2 Probabilistic and Reliability Model

2.2.1 Cascading Property

Instead of considering the power grid as an integrated complex system in a macroscopic way, some researchers have begun to use probabilistic and reliability model to describe the system in the microscopic way. Most of the papers on probabilistic model related to power grid blackout are contributed by Ian Dobson and his co-authors, and there are mainly three models they developed and/or largely used to explore the aspects of blackouts. The CASCADE and Branching process models are used to represent the dynamic features of the cascading failure in a tractable way, while the OPA model describes the power transmission system in an abstract way.

The cascading failure is a built-in property of a power grid system in which the failure of a component can trigger the failure of successive components by load-shifting.
Cascading is the key factor for large-scale power blackout, and it deserves the close attention paid from many researchers.

- **Distribution of Interval between Blackouts**
  Different papers have made controversial conclusions about the distribution of the time between blackouts. Some ([29], [58], [132]) conclude with exponential tail or similar, while others think about negative binomial [115] or Poisson distribution [71]. The probability distribution of the time between blackouts is at least determined by the probability of the trigger, and the time between blackouts is a mixture of gamma distributions [58].

- **Distribution of Blackout Size**
  The most question people would ask when hearing of power blackout is about the size and its distribution. One of the most possible answer would be the exponential; analysis of North American blackout statistics from NERC (North American Electric Reliability Council) Disturbance Analysis Working Group (DAWG) Database (http://www.nerc.com) show that it has an approximate power law region with an exponent between -1 and -2 ([9], [29]). The power law implies that blackouts can occur in all sizes, and most importantly at all places. Different power systems in different countries ([21], [71], [131]) show roughly similar forms of blackout size distribution in power law dependence. Dobson and his coworkers have some papers ([42], [133]) estimating the average propagation of failures and the size of the initial disturbance and to predict the distribution of blackout size.

- **Self-organization**
  Self-organization is the process where a structure or pattern appears in a system without a central authority or external element imposing it through planning. Many scholars have argued that, through computer simulation and historical data, power grids are self-organized critical systems, in which the evolving process is quite complicated. Based on a simple model of DC load flow and LP dispatch and NERC data on North American blackouts, ([29], [28]), the dynamics of blackouts have some features of self-organized critical systems [115].

- **Criticality**
  As the load increases, the average blackout size increases very slowly, until, at a loading called the critical loading, there is a sharp change and average blackout size starts increasing much more quickly ([27], [78],[97]).

### 2.2.2 CASCADE Model

The CASCADE model consists of finitely large number of components with a failure threshold, an initial system loading, a disturbance over certain component(s), and the addition loading on component caused by failure of other component(s).
In the CASCADE model, there are $n$ identical components with random initial loading, and for each component the minimum and maximum initial loading is $L^{\text{min}}, L^{\text{max}}$ respectively. Component $j$ is assumed to have an initial loading of $L \in [L^{\text{min}}, L^{\text{max}}]$ with a uniform distribution, and $L_j, j = 1, 2, \ldots, n$ are identically distributed. They normalized the initial load by

$$\ell_j = \frac{L_j - L^{\text{min}}}{L^{\text{max}} - L^{\text{min}}}$$

therefore $\ell_j$ is a random variable with a standard uniform distribution, $\ell_j = 1$ being the failure load.

To begin cascade process, they also assume that there is an initial disturbance $D$ on each component ($D$ could be 0 for certain components), and failures caused by $D$ on some components would add an extra load $P$ on other components resulting in further failures as a cascading process. $P$ and $D$ can be normalized as

$$p = \frac{P}{L^{\text{max}} - L^{\text{min}}}$$

$$d = \frac{D + L^{\text{max}} - L^{\text{fail}}}{L^{\text{max}} - L^{\text{fail}}}$$

The distribution for the total number $S$ of failed components is

$$P[S = r] = \binom{n}{r} d(d + rp)^{r-1}(1 - d - rp)^{n-r}$$

When considering the the interaction mechanism among individual components, they proposed an algorithm for normalized CASCADE with $k$ interactions (Fig. 2-1):

1. Set all $n$ components with initial load $\ell_1, \ell_2, \ldots, \ell_n$ iid from standard uniform distribution.

2. Sample the $n$ components $k$ times independently and uniformly with replacement, add the initial disturbance $d$ to the load of the sampled components, and set the stage index $i$ to zero.

3. Test each components which are not marked as failed before sampling in the previous step. If component $j$ is not marked as failed and its load $\ell_j > 1$, then mark it as failed. Let $M_i$ be the number of total failed components in stage $i$. 
4. Including those $M_i$ components, sample the $n$ components $k$ times independently and uniformly with replacement, and add $p$ to the load of the sampled components.

5. Increase the stage index by 1 and go to step 2.

![Flowchart](chart.png)

Figure 2-1. Cascading failure power system model

### 2.2.3 Branching Process Model

By introducing some general processing models such as Galton-Watson branching process with generalized Poisson distribution, these scholars could approximate the CASCADE model with simplified mathematical model so that each step of the cascading failure propagation could be calculated. The basic idea behind the Galton-Watson branching process is that the failure on each component in each stage will independently result in future failures in the next stage associated with a probability
distribution, and it could be represented in mathematics as following:

\[ M_{i+1} = M^{(1)}_{i+1} + M^{(2)}_{i+1} + \cdots + M^{(M_i)}_{i+1} \]

where \( M_{i+1}^k \) is identically independent by assumption and defined as the failures caused by \( k \)th component failure in stage \( i \), so the total number of failures is

\[ M = \sum_{k=0}^{\infty} M_k \]

In addition to above model, with the assumption that each component failure would cause further failures according to a Poisson distribution of mean \( \lambda \), the branching process could be modeled as a transient discrete time Markov process with its behavior governed by the parameter \( \lambda \). Therefore, the total number of failures becomes

\[ P[M = r] = (r \lambda)^{r-1} \frac{e^{-r \lambda}}{r!}, \quad 0 \leq \lambda \leq 1 \]

They also have a high level version of probabilistic model of the cascading process which utilizes a continuous state branching process [133]. With all these models, they propose some statistical estimators to measure the extent to which the load shedding is propagated.

However, this approximation can only work in a system with many components and many component interactions so that series of failures propagating in parallel can be assumed not to interact, and it can not reflect the mechanism and complexities of loading dependent cascading failure which does exist in real power network system.

### 2.2.4 Simulation and Complex System Model

In order to dynamically predict the behavior of the gigantic power grid under various circumstances, with the help of advanced information technology, people can use simulation model to not only better understand the entire power grid itself, but to obtain some experimental result in a timely manner as well. Most of these research involves complex system analysis with characteristics of cascading, therefore they are related to
power blackout analysis since that power grid is usually treated as a complex network, cascading is a universal phenomenon on power grid.

The complex system or networks has already drew a lot of attraction from researchers in multiple files, even before the power security issue has been exposed to public attention. Newman [98], Albert and Barabási [6] review the recent advances in the field of complex networks, including modeling the network and predicting its dynamic behavior. They discussed several main network models with network topology and network’s robustness against failures and attacks. Albert, Jeong and Barabási [4] especially focused on the tolerance against errors or attacks among different types of complex networks. Watts and Strogatz [129] studies the dynamics of ‘small-world’ networks, where they explicitly claimed that the power grid of the western United States in such category.

In the field of cascading failure simulation, various methods and models are proposed to capture the cascading phenomenon, and most of them focus on certain aspects of cascading which could be represented by successive static models, while most of which are related to such as static line overloading at the level of DC or AC load flow. There are several papers ([11], [19], [32], [33], [64], [88],[109], [120]) concentrating on hidden failures with protection control and operator reaction.

Due to the difficulties and complexities of modeling and the heavy computation burden, dynamic analysis such as self-organization during blackout has not been well studies. However, it is critical to study, in more detail, the transient status of the evolving power grid that is continually upgrading to couple with the changing load and generation demand ([83], [122]). Simulation such as the OPA could help to better understand transient reliability ([28], [130]), and identify some high-risk cascades as well ([33], [36], [108], [96]).

Besides those researches, there is one which caught a lot of eyeballs, even the ones from Larry M. Wortzel, a military strategist and China specialist. On March 10,
2009, he presented the U.S. House on Foreign Affairs committee that it should be concerned on how to attack a small U.S. power grid sub-network in a way that would cause a cascading failure of the entire U.S. grid [134], and his statement is based from a paper published [126] on the journal of Safety Science. The purpose of the research, explained by the author Mr. Jianwei Wang and his colleagues from Dalian University of Technology China, is to try to find ways to enhance the stability of power grids by exploring potential vulnerabilities.

In all the other studies cited below, the load on a node (or an edge) was generally estimated by its degree or betweenness and the redistribution load were usually forwarded following the shortest path. Wang proposed a new measure to assign the initial load of a node and to redistribute load among nodes after attacking, in order to reduce the computing complexity using some other measure such as the betweenness. The author assume the the initial load $L_j$ of a node $j$ in the power grid is a function of its degree $k_j$ and the degrees of its neighbor nodes $k_m(m \in \Gamma_j)$, where $\Gamma_j$ is the set of all neighboring nodes of the node $j$. The initial load of node $j$ is defined as:

$$L_j = [k_j(\sum_{m \in \Gamma_j} k_m)]^\alpha,$$

and the redistribution (Fig 2-2) between two adjacent nodes is defined as:

$$\Delta_{ij} = L_i \frac{[k_j(\sum_{m \in \Gamma_j} k_m)]^\alpha}{\sum_{m \in \Gamma_i} [k_n(\sum_{f \in \Gamma_n} k_f)]^\alpha}.$$

They also set the the capacity $C_j$ of a node $j$ proportional to its initial load, i.e., $C_j = TL_j$, and the cascading process begins when $L_j + \Delta_{ij} > C_j$. They evaluate the effect of attacking by the normalized avalanche size (broken nodes) $CF_{\text{attack}} = \frac{\sum_{i \in A} CF_i}{N_a(N-1)}$, where $CF_i$ is the avalanche size induced by removing node $i$. By adjusting the parameter $\alpha$ and $T$ which can influence the initial load and node tolerance respectively, they numerically studied the electrical power grid of the western United States with 4941 nodes and 6594 edges to investigate the network robustness under attacks.
Wang and Chen [127] proposed a cascading model with a local weighted flow redistribution rule and studied on weighted scale-free and small-world networks. In their model, they assume the weight (flow) of an edge $ij$ as $w_{ij} = (k_i k_j)^{\theta}$, where $\theta$ is the parameter for the strength of the edge weight, and $k_i$ is the degrees of nodes $i$. The redistribution model, which is the key to their simulation research, is defined as

$$
\Delta F_{im} = F_{ij} \frac{w_{im}}{\sum_{a \in \Gamma_i} w_{ia} + \sum_{b \in \Gamma_j} w_{jb}},
$$

where $\Gamma_i$ is the set of adjacent nodes of $i$. In the case study, the authors only consider one edge $ij$ attacking, and obtain the evaluation result by normalized avalanche size $S_N = \sum s_{ij} / N_{edge}$, where $s_{ij}$ is the avalanche size induced by cutting out edge $ij$, and $N_{edge}$ is the total number in the network. They furthermore explore the statistical characteristics of the avalanche size of a network, thus obtaining a power-law avalanche size distribution.

The concept of load entropy, which can be an average measure of a networks heterogeneity in the load distribution was introduced by Bao, Cao, and etc. They
consider that a certain degree of instantaneous node overload is permissible in reality complex networks and node brokage is mainly caused by the accumulative effect of overload. They also disagree the simple and unreasonable strategy of immediate removal of an instantaneously overloaded node. By their strategy of the overloaded node removal, it is assumed that the density of the removal probability \( P(L) \) of a node obeys uniform distribution in the discrete interval \([0, T]\). At each time period, the removal probability is calculated as:

\[
p_i(t) = \begin{cases} 
    p_i(t - 1) + P(L_i(t))/T & \text{if } L_i(t) > C_i; \\
    0 & \text{if } L_i(t) \leq C_i 
\end{cases}
\]

where \( L_i(t) \) is the load of node \( i \) at time \( t \). At each iteration, the load of all nodes is compared with a random \( \beta \in (0, 1) \) to determined whether the corresponding node should be removed. When the loads of all nodes are not larger than their corresponding capacity, a cascading failure stops. They defines the load entropy to evaluate the robustness of the network under an initial removal on the node with the largest load, but they do not mention about their load redistribution model after certain node’s removal.

In addition, a number of aspects of cascading failures have been discussed in some literatures, including the cascade control and defense strategy ([139], [119], [94], [124], [15]), the model for describing cascade phenomena ([128], [135]), the analytical calculation of capacity parameter ([125], [140]), and so on.

With the help of modern advanced computers, the simulation of power grid as complex networks has been developed so well that it could help us better understand the dynamics of power network in a way more accurate and less costly in time, therefore it could be applied for blackout analysis and prediction. However, there are some issues associated with that application. First, although there are a lot of proposed measurements for the nodes’ degree or edges’ weight, there is no scientific proof on the validity of these measurements. Second, the values for the tunable
parameters in the model are not well-recognized, and all those numerical studies are basically experimental. Third, in all the redistribution models, they only consider the conservation of power flow (the modified form of Kirchhoff’s nodal rule), but power grid modeled without applying Kirchhoff’s mesh rule is far away from reality. Forth, all those model only consider the power transmission. When dealing with power blackout with cascading, at least generators and consumer should be considered into the integrated planning.

Due to those shortcoming above, even numerous recent papers have applied complex network and topology methods with cascading property to study the structure and dynamic function of power grids, results are not so identical even contrast to each other. Here is a paper from Hines, Cotilla-Sanchez and Blumsack showing this argument [69]. The authors compare the analysis results from a variety of power networks subjected to random failures and directed attacks with the vulnerability measures of characteristic path length, connectivity loss and blackout sizes. They chose several contingency methods including random failure, degree attack, maximum/minimum-traffic attack and betweenness attack. Then conclude that topological measures can provide some general vulnerability indication, but it can also be misleading since individual simulations show only a mild correlation. Most importantly, they suggest that results from physics-based models are more realistic and generally more useful for infrastructure risk assessment.

2.3 Optimization Model

Mathematical optimization methods have been used to solve many power system problems such as planning, operation and control problems for many years. In order to apply optimization methods for the power problems in reality, some assumptions must be made to derive the mathematical model. However, even under this circumstances, optimization over large-scale power system is still a computation-intensive task within the scope of contemporary information technology. There are so many uncertain factors
such as uncontrollable system separation, angle instability and generation tripping in these large, complex, and wide-spread power system that would make the above model more complicated, without considering the new issues introduced by the deregulation of power utilities.

2.3.1 Pre-Optimal Power Flow Model

Before the development of Optimal Power Flow (OPF) model from its inception in 1961 and several solution methods in existence in 1978, there were some optimization models related to economic power flow dispatch. Megahed et al. \cite{86} propose the conversion of the nonlinearly constrained dispatch problem to a series of constrained linear programming problems. System voltages, active and reactive generation, and the phase angles are considered as prototype part of the OPF problem. These quantities are used in the loss formula. According to the authors, the method is fast and has good convergence characteristics.

2.3.2 OPF Model

Optimal Power Flow (OPF) model serves as the center and critical part in the mathematical optimization applied blackout problem over power system. The history of research on OPF model could date back to the early 1960’s \cite{26}, and it was derived from the solution of the economic dispatch by the equal incremental cost method.

Economic dispatch is defined as the process of allocating generation levels to the generating units in the mix, so that the system load may be supplied entirely and most economically. Research on optimal dispatch could go as far back as the early 1920’s, when engineers were concerned with the problem of economic allocation of generation or the proper division of the load among the generating units available. Generation dispatch has been widely studied and reported by several authors in books on power system analysis (\cite{61}, \cite{47}, \cite{39}, \cite{16}).

Although both of economic dispatch and OPF model are optimization problems with the same minimum cost objective, economic dispatch only considers real power
generation and transmission with only power balance equation as the constraint. On
the other side, the OPF is a static nonlinear optimization problem which can take nearly
all electrical variables, flow balance, power flow physics, generator and demand node
bounds and physical laws in consideration, to compute the optimal settings in a power
network, given settings of loads and system parameters. A typical OPF model is:

$$\min_{g,s,f,\theta} \sum_{i \in I} (h_i g_i + r_i s_i)$$

$$\sum_{(i,j) \in \delta^+(i)} f_{ij} - \sum_{(j,i) \in \delta^-(i)} f_{ji} = \begin{cases} P_i & i \in \mathcal{C} \\ -D_i & i \in \mathcal{D} \\ 0 & \text{otherwise} \end{cases}$$

$$\sin(\theta_i - \theta_j) - x_{ij} f_{ij} = 0 \quad \forall (i,j)$$

$$|f_{ij}| \leq u_{ij} \quad \forall (i,j)$$

$$P_i \leq P_{i} \leq P_i \quad \forall i \in \mathcal{C}$$

$$0 \leq D_j \leq D_j \quad \forall j \in \mathcal{D}$$

where a grid is represented by a directed network $\mathcal{G}$, and:

- Each node corresponds to a “generator” (i.e., a supply node), or to a “load” (i.e.,
a demand node), or to a node that neither generates nor consumes power(i.e., a
transmission or distribution node). We denote by $\mathcal{C}$ the set of generator nodes.

- If node $i$ corresponds to a generator, then there are values $0 \leq P_{i} \leq P_i$.
If
the generator is operated, then its output must be in the range $[P_i, P_i]$; if the
generator is not operated, then its output is zero. In general, we expect $P_{i} \geq 0$.

- If node $i$ corresponds to a demand, then there is a value $D_j$ (the “nominal
demand value at node $i$). We will denote the set of demands by $\mathcal{D}$.

- The arcs of $\mathcal{G}$ represent power lines. For each arc $(i,j)$, we are given a parameter
$x_{ij} > 0$ (the resistance) and a parameter $u_{ij}$ (the capacity).

Given a set $\mathcal{C}$ of f operating generators, a power flow is a solution to the system of
constraints given above. In this system, for each arc $(i,j)$, we use a variable $f_{ij}$ represent
the (power) flow on $(i,j)$ (negative if power is effectively flowing from $j$ to $i$). In addition,
for each node \(i\) we will have a variable \(\theta_i\) (the “phase angle” at \(i\)). Finally, if \(i\) is a generator node, then we will have a variable \(P_i\), while if \(i\) represents a demand node, we will have a variable \(D_i\). Given a node \(i\), we represent with \(\delta^+(i)\) (\(\delta^-(i)\)) the set of arcs oriented out of (respectively, into) \(i\).

The above constraints is from Ohm’s equation in direct current (DC) network. In the case of an AC network, they can only approximates a complex system of nonlinear equations. The issue of whether to use the more accurate nonlinear formulation, or the approximate DC formulation, is quite not easy. On the one hand, the linearized formulation certainly is an approximation only. On the other hand, a formulation that models AC power flows can prove intractable or may reflect difficulties inherent with the underlying real-life problem.

First, AC power flow models typically include equations of the form:

\[
\sin(\theta_i - \theta_j) - x_{ij}f_{ij} = 0 \quad \forall (i,j)
\]

Here, the \(f\) quantities describe active power flows and the \(\theta\) describe phase angles. In normal operation of a transmission system, one would expect that \(\theta_i \approx \theta_j\) for any arc \((i,j)\) and thus it can be linearized. Hence the linearization is only valid if we additionally impose that \(|\theta_i - \theta_j|\) be very small. However, in the literature one sometimes sees this “very small” constraint relaxed when the network is not in a normal operative mode. The nonlinear formulation gives rise to extremely complex models, but studies that require multiple power flow computations tend to rely on the linearized formulation to get some useful and straight-forward information.

Second, no matter we use an AC or DC power flow model, the resulting problems have a far more complex structure than traditional single- or multi-commodity flow models, which would lead to counter-intuitive behavior similar to Braess’s Paradox.

Originally, classical optimization methods were capable to effectively solve DC OPF, and even AC OPF with certain linear approximation methods. But more recently,
due to the wide application of Flexible A.C. Transmission system (FACTS) devices and deregulation of power grid, it is difficult to deal effectively with many power system problems through strict old-fashion mathematical formulation. Following paragraphs briefly discusses about the important mathematical optimization techniques used in power systems problems:

2.3.2.1 Linear programming (LP) and quadratic programming (QP)

When the objective function, constraints are linear, and decision variables are nonnegative, problems can be formulated as the LP ([117], [8]). T.S.Chung et al. [37] proposed a recursive linear programming based approach for minimizing line losses and finding the optimal capacitor allocation in a distribution system. E.Lobato et al. [81] use LP based OPF to minimize the transmission losses and Generator reactive margins of the Spanish power system. LP can also be used in various power systems applications, including reactive power planning [101], active and reactive power dispatch ([30], [31]). Problems formulated as LP can usually be solved by simplex and Interior Point (IP) methods, while the scale of problems could be up to thousands of variables and constraints even using inexpensive computers.

Both the simplex and IP methods can be extended to a quadratic objective function while the constraints maintain linear, which are called QP. J.A.Momoh [89] showed the extension of basic Kuhn-Tucker conditions to employ a generalized Quadratic-Based model for solving OPF, where the conditions for feasibility, convergence and optimality are discussed. The same author [91] also published another paper on applying Interior Point methods to solve quadratic power system optimization problem. N. Grudinin [62] proposed a reactive power optimization model based on Successive Quadratic Programming (SQP) methods, which turned to have the best performance while being compared with other five optimization methods. Nanda [95] developed a new algorithm for OPF using Fletcher’s QP method, and G.P.Granelli et al. [59] proposed Security-constrained economic dispatch using dual sequential quadratic programming,
which was compared with SQP to demonstrate a better result in computation time and accuracy.

2.3.2.2 Nonlinear programming (NLP)

When the objective function and/or constraints are non linear, problems can be formulated as the NLP. NLP can be applied to various areas of power systems, such as optimal power ([90], [118]). J.A. Momoh et al. [142] proposed a nonlinear convex network flow programming (NLCNFP) model and algorithm for solving the security-constrained multi-area economic dispatch problem. D. Pudjianto et al. [107] used NLP based reactive OPF for distributing reactive power among competing generators in a deregulated market.

To solve most of the NLP, the most regular way is to start from an initial point and to improve along a certain “descent” direction in which objective function decreases in case of minimization problem, and there are a lot of researches about how to obtain a better initial point and/or “descent” direction associated with its step length ([74], [121]). IP methods originally developed for LP can be applicable here. Sergio Granville [60] presented application of an Interior Point Method to the optimal reactive power dispatch problem. Wei Yan et al. [137] presented the solution of the Optimal Reactive Power Flow (ORPF) problem by the Predictor Corrector Primal Dual Interior Point Method (PCPDIPM).

2.3.2.3 Integer programming (IP) and mixed-integer programming (MIP)

For some power system related problems (e.g. generator/transmission-line ON status = 1, and generator/transmission-line OFF status = 0), when all or some of the decision variables are can take only integer values, such problem is called integer programming, or mixed integer programming respectively. They can be applied to many areas of power systems, such as optimal reactive power planning, power systems planning, unit commitment and, generation scheduling ([12], [2], [55], [41], [46]).
The most used approach to solve integer problems using mathematical programming techniques is “branch and bound”, and “cutting plane methods” ([74], [79]). When the size and complexity of models are large enough, while the structure of the problem is not in specific form (such as transportation network), decomposition technique usually is applied ([7], [40]).

2.3.2.4 Dynamic programming (DP)

DP based on the principle of optimality states that a sub-policy of an optimal policy must in itself be an optimal sub-policy. DP can be applied to various areas of power systems, such as reactive power control, transmission planning, and unit commitment ([70], [104]), but it is very inefficient due to the curse of dimensionality.

2.3.3 Unit Commitment

Since power generators cannot instantly turn on to produce power, unit commitment (UC) requirement must be followed in advance so that adequate power generation is always available to meet the demand, especially in the event that generators or transmission lines go out or load demand increases. Unit commitment handles the unit generation schedule in a power system for minimizing operating cost and satisfying prevailing constraints such as load demand over a set of time periods. Unit commitment in power operation planning concerns the scheduling of start-up/shut-down decisions and operation levels for power generation units such that the fuel costs over some time horizon are minimal.

UC is not the same as dispatching. Dispatching focuses on assigning a given set of power plants to another certain set of electric demand, while UC determines the start-up and shutdown schedules of thermal units to meet forecasted demand over certain time periods. The difference between both issues is time. The usual common objectives of unit commitment schedule include minimization of total production cost, minimization of emissions and maximization of reliability and security, and the most important non-linear constraints are the unit’s minimum up-time and down-time restriction.
Although the planning horizon for unit commitment in principle should be continuous in time, the unit commitment models typically are in discrete time due to the availability of data, the execution time for scheduling decisions and computation limitation on the complex MIP in continuous time. Here we demonstrate a very basic UC formulation:

\[
\begin{align*}
\min_{p_t^i, s_t^j, w_t^j} & \quad \sum_{t=1}^{T} \sum_{i=1}^{I} C_i(p_t^i, u_t^i) + \sum_{t=1}^{T} \sum_{i=1}^{I} S_t^i(u_t^i) \\
\text{subject to} & \quad u_{t-1}^i - u_t^i \leq 1 - u_t^i, \quad i = 1, 2, \ldots, I; \quad t = 2, 3, \ldots, T - 1 \\
& \quad l = t + 1, t + 2, \ldots, \min\{t + \tau - 1, T\} \\
& \quad l_t^j = l_{t-1}^j - (s_t^j - \eta_j w_t^j), \quad j = 1, 2, \ldots, J \\
& \quad l_0^j = l_0^j, l_T^j = l_{T+1}^j, \quad t = 1, 2, \ldots, T \\
& \quad \sum_{i=1}^{I} (p_t^i p_t^{i^\text{max}} - p_t^i) \geq R_t, \quad t = 1, 2, \ldots, T \\
& \quad \sum_{i=1}^{I} p_t^i + \sum_{j=1}^{J} (s_t^j - w_t^j) \geq D_t, \quad t = 1, 2, \ldots, T \\
& \quad p_t^{\text{min}} u_t^i \leq p_t^i \leq p_t^{\text{max}} u_t^i, \quad i = 1, 2, \ldots, I; \quad t = 1, 2, \ldots, T \\
& \quad 0 \leq s_t^j \leq s_t^{\text{max}}, \quad j = 1, 2, \ldots, J; \quad t = 1, 2, \ldots, T \\
& \quad 0 \leq w_t^j \leq w_t^{\text{max}}, \quad j = 1, 2, \ldots, J; \quad t = 1, 2, \ldots, T \\
& \quad 0 \leq l_t^j \leq l_t^{\text{max}}, \quad j = 1, 2, \ldots, J; \quad t = 1, 2, \ldots, T 
\end{align*}
\]

Here, \( T \) denote the number of subintervals of the optimization horizon and suppose there are \( I \) thermal as well as \( J \) pumped-storage hydro units. The variable \( u_t^i \in \{0, 1\}, \quad i = 1, 2, \ldots, I; \quad t = 1, 2, \ldots, T \) indicates whether the thermal unit \( i \) is in operation at time \( t \). Variables \( p_t^j, s_t^j, w_t^j, \quad j = 1, 2, \ldots, J; \quad t = 1, 2, \ldots, T \) are the output levels for the thermal units, the hydro units in generation and in pumping modes, respectively. The variables \( l_t^j \) denote the fill (in energy) of the upper dam of the hydro unit \( j \) at the end of interval \( t \), \( j = 1, 2, \ldots, J; \quad t = 1, 2, \ldots, T \). The objective is the sum of the fuel cost and start-up cost with parameter \( C_i, S_t^i \) respectively, and the constraints
include the power output bounds of units and the fill of the upper dam, load coverage, reserve management of the thermal units, balances for pumped-storage plants, and minimum down times for thermal stresses in the coal fired blocks.

The most straightforward way to solve the UC economic optimization is brute force, which enumerates all possible combinations, eliminates the possibilities that do not meet the obligations set, and finally chooses the best of all the remaining possibilities. Even though this algorithm is computationally intensive, most of current methodologies are a variation on the brute force, in which some procedures are added to reduce the number of possibilities enumerated.

The unit commitment problem belongs to the class of complex combinational optimization problems. Several mathematical programming techniques have been proposed to solve this time-dependent problem.

2.3.3.1 Dynamic programming (DP)

DP searches the solution space that consists of the units’ status for an optimal solution [116]. The search can be carried out in a forward or backward direction. The time periods of the study horizon are known as the stages of the problem. The combinations of units within a time period are known as the states ([72], [63]). Lowery [70] starts from a previously-determined optimal UC planning and gradually adds power plants to obtain optimal solutions for higher demands. Hobbs et al. [82] initialize their approach with options calculated for preceding periods. Cohen and Yoshimura [38] proposed a branch-and-bound model which starts from a previously obtained optimum. The UC problem may also be decomposed into smaller subproblems that are easily managed and solved with DP, where the master problem is optimized, linking the sub-problems by Lagrange multipliers. Van den Bosch and Honderd [24] decomposed the main problem into several sub-problems that are easier to solve. The decomposition proposed by Snyder et al. [116] consists of grouping power plants from the same type.
The advantage of DP is its ability to maintain solution feasibility. DP builds and evaluates the complete “decision tree” to optimize the problem at hand. But it suffers from the “curse of dimensionality” because the problem size (number of states) increases rapidly with the number of generating units to be committed, which results in an unacceptably long solution time. To reduce the dimension, search space and execution time, several approaches have been developed, including DP-SC (dynamic programming-sequential combination) [103], DP-TC (dynamic programming-truncated combination) [102], DP-STC (which is a combination of the DP-SC and DP-TC approaches) [76] and DP-VW (variable window truncated dynamic programming) [102]. The variation of window size according to load demand increment indicates a substantial saving in computation time without sacrificing the quality of the solution, and the solution of all of these DP methods is sub-optimal.

2.3.3.2 Dynamic and linear programming

The UC problem can be solved by using regular dynamic programming (DP) or DP with successive approximation of the solution space. Linear programming (LP) solves the economic dispatch within UC for optimal allocation of fuel/generation. Dantzig-Wolfe decomposition, when used, partitions the linear program into smaller, easily manageable LP subproblems ([85], [51]). The primary disadvantage of LP solutions is the numerous variables needed to represent the piece-wise linear input-output curves.

2.3.4 Contingency and Interdiction

Contingency and Interdiction analysis, which assesses the ability of the power grid to sustain various combinations of power grid component failures based on state estimates, is a critical part of the energy management system. Here, the contingency means a set of unexpected events happening within a short duration. The unexpected events can be failures of buses (generators, substations, etc) or transmission and distribution lines. Optimization is used to maximize the blackout size due to contingencies caused by by attacked with limited resources ([13], [45], [111]).
In the past, due to the heavy computation involved, the contingency analysis can be only analyzed for only a select set of $N - 1$ contingency, or $N - 1$ reliability case, which is the failure of one component (a bus or a line) has been an active research area. Milano et al. uses $N - 1$ contingency criterion as an initial optimal operating condition to estimate the system-wide available transfer capability [87]. Hedman et al. analyze the $N - 1$ reliable DC optimal dispatch with transmission switching by modifying economic dispatch optimization problems to incorporate the flexibility of transmission assets’ states [67]. While ensuring $N - 1$ reliability, the same authors also present a co-optimization formulation of transmission switching problem and the generation unit commitment[66].

However, as electricity demand continues to grow and renewable energy increases its penetration in the power grid, analysis of the $N - 1$ reliability is not sufficient for many real applications with multiple failures to discover the vulnerabilities of power grids. Although the combinatorial number of contingency states imposes a substantial computational burden for analysis, the $N - k$ contingency analysis for failures of multiple components (totally $k$ buses and lines) can reflect a larger variation of vulnerabilities of a power system and attract a lot of research focus. Salmeron, Wood, and Baldick applied a linearized power flow model and used a bi-level optimization framework along with mixed-integer programming to analyze the security of the electric grid and to obtain the worst contingency selection, where the interdiction model is to “identify critical sets of a power grid’s components, e.g., generators, transmission lines, and transformers, by identifying maximally disruptive, coordinated attacks on a grid, which a terrorist group might undertake”[111]. Pinar et al. modeled the power grid vulnerability analysis as a mixed integer nonlinear programming (MINLP) problem, and used a special structure in the formulation to avoid nonlinearity and approximate the original problem as a pure combinatorial problem [106]. Bienstock used the two approaches of the integer programming and a new, continuous nonlinear programming formulation for comparison on vulnerability evaluation over large-scale power grid failures ([22], [23], [123]).
In the paper from Fan et al. [50], the critical node method, originally used for research in general graphs [14], is in the first time, to be applied to study on power grids, which is one of the main contribution of this paper. More importantly, this paper is among of the first few papers to evaluate different $N - k$ contingency selection by graph algorithms and interdiction methods through the economic objective (generating and load shedding cost) of those contingency states. In this paper, several graph algorithms and interdiction methods for contingency selection are surveyed and compared with our new method represented in formulation. Their new model can select the contingency state including both buses and lines, which is a big plus than the others. Also the evaluation measurement on economical emphasis is quite new, and lead to some interesting conclusions.

2.4 Blackout with Distributed Generation

Traditionally, electricity industries generate most of their power in large and centralized facilities, from such as fossil fuel, nuclear, large solar or hydropower sources. This kind of business strategies has excellent economies of scale and some other considerations, such as of health & safety, logistics, environment, geography and geology, but usually electricity transmission over long distances would negatively affect the environment. The other approach is via distributed generation, which can reduce the amount of energy lost in transmission and the size of transmission infrastructure due to closeness between supply and demand. A very good application example of distributed generation is microgrid, which is a localized grouping of electricity generation, energy storage, and loads. Under normal operating condition, it is connected to a traditional centralized grid, however it could be disconnected from there functioning autonomously. From the point of view of the electricity network operator, a microgrid could be treated as one entity, but instead of simply receiving electricity supply, it could sustain itself for a long time without external electricity source or even sometimes output electricity for others.
The blackout is always preceded by a sequence of cascading failures which breaks down transmission lines and generators, and thus leads to large variation in power flow, its routing and bus voltage due the mechanism of load balance in a massively interconnected network system. With the emerging of distributed generation, the phenomena of islanding must be taken in consideration during blackout analysis, especially for real-time decision making in the early stage of possible blackout. Islanding is the situation where a distributed generator continues to power a neighborhood while electrical grid power from the electric utility is not available.

By proper management of intentional islanding, the operator disconnects those “islands (localized grouping of electricity generation, energy storage, and loads)” from the grid, and forces the distributed generator to supply power for the local demand. Strategy like this could largely reduce the burden of substations and generators which are already crumbling due to emergent power rerouting to meet the remote power demand during power outage, and relieve the overloading transmission lines during the critical restoration phase [65]. Distributed generation could increase the reliability and security of power supply [10] by providing electricity to the medium-voltage and low-voltage networks where it is most needed, in case of higher voltage network failures. A lot of conceptual but pioneer papers have been published on how to apply islanding operation to mitigate the spread of power outage and prevent possible blackout ([48], [43], [80], [3], [49]), however due to the complexity of interacting mechanism within the “island” and “inter-islands”, there are quite a few preliminary researches on real-time algorism to quickly detect the possible optimal islanding strategy when power system vulnerability is approaching to an extreme emergency state ([138], [141], [105], [92]).
CHAPTER 3
KEY CONCEPT USED IN THIS DISSERTATION

In this chapter, some basic but key concepts are introduced here.

3.1 Power Flows

A brief introduction about DC power flow model is presented here. For the purposes of our problem, a grid is represented by a directed network $G$, where:

- Each node corresponds to a “generator” (i.e., a supply node), or to a “load” (i.e., a demand node), or to a node that neither generates nor consumes power (i.e., a transmission or distribution node). We denote by $P$ the set of generator nodes.

- If node $i$ corresponds to a generator, then there are values $0 \leq P_{i_{\min}} \leq P_{i_{\max}}$. If the generator is operated, then its output must be in the range $[P_{i_{\min}}, P_{i_{\max}}]$; if the generator is not operated, then its output is zero. In general, we expect $P_{i_{\min}} \geq 0$.

- If node $i$ corresponds to a demand, then there is a value $D_{i_{\text{nom}}}$ (the “nominal” demand value at node $i$). We will denote the set of demands by $D$.

- The arcs of $G$ represent power lines. For each arc $(i, j)$, we are given a parameter $x_{ij} > 0$ (the resistance) and a parameter $u_{ij}$ (the capacity).

Given a set $C$ of operating generators, a power flow is a solution to the system of constraints given next. In this system, for each arc $(i, j)$, we use a variable $f_{ij}$ represent the (power) flow on $(i, j)$ (negative if power is effectively flowing from $j$ to $i$). In addition, for each node $i$ we will have a variable $\theta_i$ (the “phase angle” at $i$). Finally, if $i$ is a generator node, then we will have a variable $P_i$, while if $i$ represents a demand node, we will have a variable $D_i$. Given a node $i$, we represent with $\delta^+(i)$ ($\delta^-(i)$) the set of arcs oriented out of (respectively, into) $i$.

In the system given below, constraints are typical for network flow models respectively: flow balance, power flow physics, generator and demand node bounds.
\[
\sum_{(i,j) \in \delta^+(i)} f_{ij} - \sum_{(j,i) \in \delta^-(i)} f_{ji} = \begin{cases} 
P_i & i \in C \\
-D_i & i \in D \\
0 & \text{otherwise}
\end{cases}
\]

\[
\theta_i - \theta_j - x_{ij} f_{ij} = 0 \quad \forall (i,j)
\]

\[
|f_{ij}| \leq u_{ij} \quad \forall (i,j)
\]

\[
P_i^{\min} \leq P_i \leq P_i^{\min} \quad \forall i \in C
\]

\[
0 \leq D_j \leq D_j^{\text{nom}} \quad \forall j \in D
\]

### 3.2 DC and AC power flows

The above constraints is from Ohm’s equation in direct current (DC) network. In the case of an AC network, they can only approximates a complex system of nonlinear equations. The issue of whether to use the more accurate nonlinear formulation, or the approximate DC formulation, is quite not easy. On the one hand, the linearized formulation certainly is an approximation only. On the other hand, a formulation that models AC power flows can prove intractable or may reflect difficulties inherent with the underlying real-life problem.

First, AC power flow models typically include equations of the form:

\[
\sin(\theta_i - \theta_j) - x_{ij} f_{ij} = 0 \quad \forall (i,j)
\]

Here, the \(f\) quantities describe active power flows and the \(\theta\) describe phase angles. In normal operation of a transmission system, one would expect that \(\theta_i \approx \theta_j\) for any arc \((i,j)\) and thus it can be linearized. Hence the linearization is only valid if we additionally impose that \(|\theta_i - \theta_j|\) be very small. However, in the literature one sometimes sees this “very small” constraint relaxed when the network is not in a normal operative mode. The nonlinear formulation gives rise to extremely complex models, but studies that require
multiple power flow computations tend to rely on the linearized formulation to get some useful and straight-forward information.

Second, no matter we use an AC or DC power flow model, the resulting problems have a far more complex structure than traditional single- or multi-commodity flow models, which would lead to a counter-intuitive behavior similar to Braess’s Paradox.

### 3.3 Preliminary Model

The approach to identifying critical system components first develops a network vulnerability model to represent the optimal-attack problem that a terrorist group might face (which is the simplest). The general model is a $\max - \min$ problem:

$$\max_{\delta \in \Delta} \min_{p} c^T p$$

subject to $g(p, \delta) \leq b$, $p \geq 0$.

An interdiction plan is represented by the binary vector $\delta$, whose $k$th entry is 1 if component $k$ of the system is attacked and is 0 otherwise. For a given plan, the inner problem is an optimal power-flow model that minimizes generation costs plus the penalty associated with unmet demand, together denoted by $c^T p$. Here, $p$ represents power flows, generation outputs, phase angles and unsatisfied demand; $c$ represents linearized generation costs, and the costs of unsatisfied demand. The outer maximization chooses the most disruptive, resource-constrained interdiction plan $\delta \in \Delta$, where $\Delta$ is a discrete set representing attacks that a terrorist group might be able to execute. In this model, $g$ corresponds to a set of functions that are nonlinear in $(p, \delta)$. The inner problem involves a simplified optimal power-flow model, with constraint $g(p, \delta)$ functions that are linear in for a fixed $\delta = \delta$. 

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### 3.3.1 Optimal Power Flow Model

We approximate active power flows with a DC model, which neglects reactive power effects and nonlinear losses. This approximation is normally acceptable in the context of long-term.

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<td>$d_i$</td>
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<tr>
<td>$P^\text{Line}_l$</td>
</tr>
<tr>
<td>$P^\text{Gen}_g$</td>
</tr>
<tr>
<td>$x_l, r_l$</td>
</tr>
<tr>
<td>$h_g$</td>
</tr>
<tr>
<td>$f_i$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P^\text{Line}_l$</td>
</tr>
<tr>
<td>$P^\text{Gen}_g$</td>
</tr>
<tr>
<td>$\theta_i$</td>
</tr>
<tr>
<td>$S_i$</td>
</tr>
</tbody>
</table>
Formulation of Optimal Power-Flow model:

\[
\begin{align*}
\min_{P_{\text{Line}}, P_{\text{Gen}}, \theta, S} & \quad \sum_{g} h_{g} P_{g}^{\text{Gen}} + \sum_{i} f_{i} S_{i} \\
\text{subject to} & \quad P_{i}^{\text{Line}} = B_{i}(\theta(i) - \theta(j)) \forall (i, j) \\
& \quad \sum_{g} P_{g}^{\text{Gen}} - \sum_{l(i, j)} P_{l}^{\text{Line}} + \sum_{l(j, i)} P_{l}^{\text{Line}} = \sum_{i} (d_{i} - S_{i}) \forall i \\
& \quad - \bar{P}_{i}^{\text{Line}} \leq P_{i}^{\text{Line}} \leq \bar{P}_{i}^{\text{Line}} \forall l \\
& \quad 0 \leq P_{g}^{\text{Gen}} \leq \bar{P}_{g}^{\text{Gen}} \forall g \\
& \quad 0 \leq S_{i} \leq d_{i} \forall i
\end{align*}
\]

The objective is to minimize generating plus shedding costs measured. First constraint approximates active power flows on the lines. Second constraint maintains power balance at the nodes. Third and fourth constraint set maximum line power flows and generating-unit outputs. Minimum power outputs are set to zero for all generating units for simplicity here, but extensions to nonzero minima are straight-forward. Fifth constraint states that load shedding cannot exceed demand. This will be a a subproblem of the attacking model described next.

### 3.3.2 Contingency Model

In the contingency model, a group of terrorists, will make a coordinated set of resource-constrained attacks on the power grid. We make the following assumptions on the effect of each attack. However, we would extend the meaning of “attacking” to a broader scope such that a component failure, impact of daily maintenance or planning expansion would be included in this form to universal application purpose.

- **Line attack**: lines under an attack are opened.
- **Generator attack**: The generator is disconnected from the grid.
- **Node attack**: All lines, generation, and load connected to the node are disconnected.
- **Substation attack**: All consumer nodes connected to the substation are disconnected. (especially important for blackout caused by component failures)
Terrorist resource constraints can accommodate information from intelligence sources, here we model this feature through a simple knapsack constraint.

<table>
<thead>
<tr>
<th>Additional Sets and Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I^* \in I$, $G^* \in G$, $L^* \in L$, $S^* \in S$</td>
</tr>
<tr>
<td>$M_{Node}^<em>, M_{Gen}^</em>, M_{Line}^<em>, M_{Sub}^</em>$</td>
</tr>
<tr>
<td>$M$</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Decision Variables for Attacking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I^* \in I$, $G^* \in G$, $L^* \in L$, $S^* \in S$</td>
</tr>
<tr>
<td>$\delta_{Node}^i$, $\delta_{Gen}^g$, $\delta_{Line}^l$, $\delta_{Sub}^s$</td>
</tr>
</tbody>
</table>

Formulation of Contingency model:

\[
\min_{\delta_{Node}^i, \delta_{Gen}^g, \delta_{Line}^l, \delta_{Sub}^s} \tilde{F}(\delta_{Node}^i, \delta_{Gen}^g, \delta_{Line}^l, \delta_{Sub}^s)
\]

subject to

\[
\sum_{i \in I^*} M_{Node}^i \delta_{Node}^i + \sum_{g \in G^*} M_{Gen}^g \delta_{Gen}^g + \sum_{l \in L^*} M_{Line}^l \delta_{Line}^l + \sum_{s \in S^*} M_{Sub}^s \delta_{Sub}^s \leq M
\]

\[
\delta_k = 0, \text{ if } k \in (I^* \cup G^* \cup L^* \cup S^*)
\]

and where

\[
\tilde{F}(\delta_{Node}^i, \delta_{Gen}^g, \delta_{Line}^l, \delta_{Sub}^s) = \min_{P_{Line}, P_{Gen}, S_i} \sum_{g} h_g P_{Gen}^g + \sum_{i} f_i S_i
\]
subject to \( P_{i}^{\text{Line}} = B_i(\theta(i) - \theta(j))(1 - \delta_{i}^{\text{Line}})(1 - \delta_{j}^{\text{Node}})(1 - \delta_{i}^{\text{Node}}) \prod_{s|l \in L^s_{\text{Sub}}} (1 - \delta_{s}^{\text{Sub}}) \forall (i, j) \)

\[
\sum_g P_{g}^{\text{Gen}} - \sum_{l(i,j)} P_{i}^{\text{Line}} + \sum_{l(j,i)} P_{l}^{\text{Line}} = \sum_i (d_i - S_i) \forall i
\]

\[
- \bar{P}_{i}^{\text{Line}}(1 - \delta_{i}^{\text{Line}})(1 - \delta_{j}^{\text{Node}})(1 - \delta_{j}^{\text{Node}}) \prod_{s|l \in L^s_{\text{Sub}}} (1 - \delta_{s}^{\text{Sub}}) \leq \bar{P}_{i}^{\text{Line}}(1 - \delta_{i}^{\text{Line}})(1 - \delta_{j}^{\text{Node}})(1 - \delta_{j}^{\text{Node}}) \prod_{s|l \in L^s_{\text{Sub}}} (1 - \delta_{s}^{\text{Sub}}) \forall l
\]

\[
0 \leq P_{g}^{\text{Gen}} \leq \bar{P}_{g}^{\text{Gen}}(1 - \delta_{g}^{\text{Gen}}) \forall g
\]

\[
0 \leq S_i \leq d_i \forall i
\]

This model maximizes generation costs plus load-shedding costs, which is evaluated through the inner minimization problem that consists of the optimal power-flow model with attacked components removed. At the outer level, the first inner constraint reflects the terrorists’ options to attack different combinations of components in the grid without exceeding their resources. The second inner constraint defines terrorist actions as binary variables and ensure non-attacking of certain grid components.

The constraints of the outer problem are similar as the pure optimal power-flow one, with taking into consideration the components that have been attacked through the binary status variables. The computational challenge is from the max-min structure of the problem. The optimal objective value of the linearized version of the inner minimization, as a function of continuous \( \delta \), is convex. Hence the entire program is the maximization of a convex function, which is usually a difficult task, while it is just the simplest scenario considered. However, we can still do some linerization to make the final calculation easier.
For the constraints of the outer problem with \(1 - \delta_i^{\text{Line}}\), \(1 - \delta_i^{\text{Node}}\), \(1 - \delta_g^{\text{Gen}}\) and \(1 - \delta_s^{\text{Sub}}\), since the decision variable are binary, they could be linearized. Here is a simplified proof.

Given \(X\), \(Y\) as binary variables, \(W\) as a continuous variable, \(c\) as a parameter, we have that these two constraints (1) and (2) are equivalent:

\[
W \leq cXY; \quad (3-1)
\]

\[
\begin{align*}
\text{let } Z &= XY, \text{ then } W \leq cZ; \\
Z &\leq X; \\
Z &\leq Y; \\
Z &\geq X + Y - 1; \\
Z &\geq 0;
\end{align*} \quad (3-2)
\]

Therefore, we can claim that these two constraints (3) and (4) are equivalent. Here, \(X\), \(Y\), \(Z\) are binary variables, \(W\) as a continuous variable, \(c\) as a parameter.

\[
W \leq cXYZ; \quad (3-3)
\]

\[
\begin{align*}
W &\leq cV; \\
V &\leq X \text{ and } V \leq Y \text{ and } V \leq Z; \\
V &\geq X + Y + Z - 2; \\
V &\geq X - 1 \text{ and } V \geq Y - 1 \text{ and } V \geq Z - 1; \\
V &\geq 0;
\end{align*} \quad (3-4)
\]
Here is a proof: Obviously, using the fact from (1) and (2), we can have:

\[
\begin{align*}
\text{let } U = XY, \text{ then } W &\leq cUZ; \\
U &\leq X; \\
U &\leq Y; \\
U &\geq X + Y - 1; \\
U &\geq 0;
\end{align*}
\]

(3–5)

Continuing the similar procedure, we can have:

\[
\begin{align*}
\text{let } V = UZ, \text{ then } W &\leq cV; \\
V &\leq U \Rightarrow V \leq X \text{ and } V \leq Y; \\
V &\leq Z; \\
V &\geq U + Z - 1 \Rightarrow V \geq Z - 1 \text{ and } V \geq X + Y + Z - 2; \\
V &\geq 0;
\end{align*}
\]

(3–6)

By induction, those constraints for the outer problem with \((1 - \delta^\text{Line}_i), (1 - \delta^\text{Node}_i), (1 - \delta^\text{Gen}_g)\) and \((1 - \delta^\text{Sub}_s)\) are linearizable.
CHAPTER 4
ECONOMIC ANALYSIS OF THE $N-K$ POWER GRID CONTINGENCY SELECTION
AND EVALUATION

Contingency analysis is important for providing information about the vulnerability of power grids. Many methods have been purposed to use topological structures of power grids for analyzing contingency states. Considering failures of buses and lines, we present and compare several graph methods for selecting contingencies in this paper. A new method, called critical node detection, is introduced for selecting contingencies consisting of failures on buses. Besides these methods, we include an interdiction model which provides the worst case contingency selection. Our measurement for contingency evaluation is to maximize the social benefit, or to minimize the generating and load shedding cost. Comparing with other measurements for contingency selection, our model is based on economic analysis and is reasonable for evaluating the selected contingency state. Additionally, a contingency consisting of both buses and lines is also studied. This chapter is based on a published paper [50].

4.1 Nomenclature

Indices and Index Sets

- $I$: set of buses, including substations, generators and load consumers
  - Let $1^g_i$ for $i \in I$ be the indicator such that $1^g_i = 1$ if $i$ is a generator, and $1^g_i = 0$ otherwise; and let $1^d_i$ be the indicator such that $1^d_i = 1$ if $i$ is a load consumer, and $1^d_i = 0$ otherwise

- $(i, j) \in L$: transmission lines, $i, j \in I$ and $i < j$

- $N = (I, L)$: an undirected network representing a power grid where $I$ is the set of buses/nodes and $L$ is the set of transmission lines/edges.

Parameters

- $D_i$: load at bus $i$
- $\bar{F}_{ij}$: transmission capacity for line $(i, j) \in L$
- $\bar{G}_i$: generation capacity at bus $i \in I$
• $b_{ij}$: susceptance of line $(i,j)$; $b_{ij}$ is computed from the resistance and the reactance of line $(i,j) \in L$

• $h_i$: generation cost at bus $i \in I$

• $r_i$: penalty cost for load-shedding at $i \in I$

• $k$: the number of components (buses or lines) to be failed

Decision variables

• $g_i$: power generation at node $i \in I$

• $f_{ij}$: power flow on line $(i,j) \in L$

• $s_i$: load shed at node $i \in I$

• $\theta_i$: phase angle at node $i \in I$

• $\delta_i$: $\delta_i = 1$ if the bus $i$ is selected as one of $k$ components for failures; $\delta_i = 0$ otherwise

• $\sigma_{ij}$: $\sigma_{ij} = 1$ if the line $(i,j)$ is selected as one of $k$ components for failures; $\sigma_{ij} = 0$ otherwise

4.2 Background

Contingency analysis is a key function in the Energy Management System (EMS), which assesses the ability of the power grid to sustain various combinations of power grid component failures based on state estimates. The contingency means a set of unexpected events happening within a short duration. The unexpected events can be failures of buses (generators, substations, etc) or transmission and distribution lines. In the past, the $N-1$ contingency, or $N-1$ reliability, analyzing the failure of one component (a bus or a line) has been an active research area [66, 100]. However, analysis of the $N-1$ reliability is not sufficient for many real applications with multiple failures to discover the vulnerabilities of power grids.

The $N-k$ contingency analysis for failures of multiple components (totally $k$ buses and lines) can reflect a larger variation of vulnerabilities of a power system. However, combinatorial number of contingency states imposes a substantial computational
burden for analysis. Generally, contingency analysis consists of two steps: contingency selection and evaluation [93]. Contingency selection is to select a set of potential contingencies, and to reduce the set of contingencies efficiently; Contingency evaluation examines severe contingencies in details. Sometimes, vulnerability analysis, which aims to find a set of small group of lines whose loss can cause a severe blackout, is also used.

Methods for contingency selection include automatic methods, online security analysis, ranking by approximate power flow solutions, concentric selection, sparse vector methods, genetic methods, hybrid methods and etc. These methods are briefly reviewed in [35, 93]. Recently, probability analysis [33], power flow traffic [5, 126], limitations on generation and line capacity [27], and several other methods by studying the topology structures of power grids [68] are used for vulnerability analysis. In [68], Hines et al. reviewed and compared three vulnerability measures for contingency evaluation: characteristic path length, connectivity, blackout size. Additionally, they also compared the contingency selection methods, such as random failures, degree attack, maximum-traffic attack, minimum-traffic attack and betweenness attack, under three measures. The analysis considered the failures of buses only. In [68], they concluded that under the measure of characteristic path length, those buses selected by the node betweenness method are most crucial for the power grid; under the measure of connectivity loss, those buses selected by degree based method are most crucial; and under the measure of blackout size, those buses selected by maximum-traffic method are most crucial.

In this paper, besides methods compared in [68], we introduce the critical node detection method [14] to find buses who plays important roles in a power grid. For failure on buses, we consider these graph algorithms: random, degree based, maximum-traffic, minimum-traffic, node betweenness, and critical node detection methods; For failures on lines, the graph algorithm considers the edge betweenness.
To evaluate selected contingencies, economic analysis of cascading failures should be included. After selecting contingencies by these graph algorithms, we evaluate the economic consequence of these contingency selection by using the direct current optimal power flow (OPF). The OPF minimizes the generating and load shedding cost or maximizes the social benefit in fact. In the past, the economic analysis for reliability or contingency analysis have been studied in some problems, for example, $N - 1$ reliability in [100], and unit commitment [136]. In this paper, based the OPF model whose objective is assumed to minimize the generating and load shedding cost, we use this objective as economic analysis to evaluate selected contingency states. By this analysis, we assume that under failures on some buses or lines, the power system is still operated in an optimized way. Especially, here we consider the load shedding cost, which can measure the unsatisfactory demand in case of failures in a system.

Additionally, the worst contingency selection can be obtained and evaluated by interdiction methods. We adapt the interdiction model introduced by [111, 112] to select $k$ components for failures and also evaluate them by economic analysis in one model. The interdiction model is to “identify critical sets of a power grid’s components, e.g., generators, transmission lines, and transformers, by identifying maximally disruptive, coordinated attacks on a grid, which a terrorist group might undertake” [111]. By means of “maximally disruptive”, the model is the worst case economic analysis of the contingency.

The critical node detection method is formulated by a mixed integer linear program (MILP), and also the interdiction method by a mixed integer nonlinear program. Some heuristic method for critical node detection, node and edge betweenness are reviewed, while linearization techniques are proposed for nonlinear programs. All discussed graph algorithms and interdiction methods are compared on two IEEE test systems.
4.3 Optimal Power Flow Model for Economic Analysis

As discussed in previous section, after contingency selection by graph algorithms, which decide the values of \( \delta = (\delta_1, \cdots, \delta_{|I|}) \), \( \sigma = (\sigma_{ij} : i = 1, \cdots, |I|, j = i + 1, \cdots, |I|) \), the OPF model is used for evaluating the selected contingencies. For a given IEEE test system and given value of \( k \) (usually ranges from 2 to 10), each graph algorithms presents a contingency \( \delta, \sigma \). Thus in the following model (6–1a)-(4–6), \( \delta, \sigma \) are the known failed components in a power grid. To maximize the social benefit, we minimize the generating and load shedding cost \( \sum_{i \in I} (h_i g_i + r_i s_i) \) in the OPF model in the power grids with failed buses or lines. The economic analysis model for power grids with failed components can be formulated as follows:

\[
\begin{align*}
\min \quad & z(\delta, \sigma) = \min_{g,s,f,o} \sum_{i \in I} (h_i g_i + r_i s_i) \\
\text{s.t.} \quad & f_{ij} = b_{ij}(\theta_i - \theta_j)(1 - \delta_i)(1 - \delta_j)(1 - \sigma_{ij}), \forall (i, j) \in L \\
& -\bar{F}_{ij} \leq f_{ij} \leq \bar{F}_{ij}, \forall (i, j) \in L \\
& \sum_j f_{ji} + g_i = \sum_j f_{ij} + (D_i - s_i), \forall i \in I \\
& 0 \leq g_i \leq \bar{G}_i(1 - \delta_i), \forall i \in I \\
& 0 \leq s_i \leq D_i, \forall i \in I
\end{align*}
\]

For constraint (4–2) on line \((i, j)\), it is an approximate active power flow on transmission line. If one ending bus of this line is failed (i.e., \( \delta_i = 1 \) or \( \delta_j = 1 \)), or the line is failed (i.e., \( \sigma_{ij} = 1 \)), the power flow on this line is 0. The constraint (4–3) ensures the maximum power flow on each line. The constraint (4–4) ensures the the power balance at each bus, where the served demand at bus \( i \) is \((D_i - s_i)\). The constraint (4–5) limits the maximum generating output \( \bar{G}_i(1 - \delta_i) \). If the generator at bus \( i \) is failed (i.e., \( \delta_i = 1 \)), no power is provided by this generator. The constraint (4–6) states the limitation of load shedding by the maximum load.
This formulation is a linear program with given values of $\delta$, $\sigma$, and can be solved efficiently by simplex method in CPLEX.

4.4 Graph Algorithms for Contingency Selection

In this section, graph algorithms are used for contingency selection of buses or lines, respectively.

4.4.1 Selection of $K$ Buses by Critical Node Method and Other Methods

Using the contingency selection methods in [68], we add a new method, called critical node detection, for contingency selection of buses. For contingency selection on buses, other methods, such as power generation limits in [27] and ranking index [93], are also used for analysis.

In this section, we assume all failures, or attacks happen on buses. In the $N - k$ contingency analysis model, the number of failed buses is $k$, i.e., $\sum_{i \in I} \delta_i = k$.

The random failure [68], selects $k$ buses for failure with equal probability. Thus, $k$ buses from $I$ are randomly selected and fixed $\delta_i = 1$. All other $\delta_i$s are set to be 0. The degree based method is to select $k$ buses starting with the highest degree bus, till the $k$th highest degree, and set $\delta_i = 1$ for these corresponding $k$ buses.

The maximum-traffic and minimum-traffic [5, 126] methods are based on the power flow of each bus:

$$ T_i = |g_i - (D_i - s_i)| + \sum_{j \in I} |b_{ij}(\theta_i - \theta_j)|, $$

where the term $|g_i - (D_i - s_i)|$ is the absolute value of net power injection into bus $i$ by generators and load consumers, and the summation part is the sum of power flows in and out of bus $i$. The maximum-traffic method selects $k$ buses with highest $T_i$s, while the minimum-traffic method selects $k$ buses with lowest $T_i$s. In this paper, we use the OPF model (see section below) at normal state to find the traffic $T_i$ on each bus.
The node betweenness method [5, 35] is to find \( k \) buses with highest node betweenness. The node betweenness [54] for bus \( i \) is defined as

\[
C_B(i) = \sum_{s \neq i \neq t \in I} \frac{n_{st}(i)}{n_{st}},
\]

where \( n_{st} \) is the number of shortest paths from \( s \) to \( t \), and \( n_{st}(i) \) is the number of shortest paths from \( s \) to \( t \) that pass through a bus \( i \). To calculate the betweenness of all buses in a power grid needs calculating the shortest paths between all pairs of buses in a power grid. By modified FloydWarshall algorithm, and Johnson’s algorithm for sparse graphs, the computational complexity can be \( O(|I|^3) \) and \( O(|I|^2 \log |I| + |I| |L|) \), respectively. By using Brandes’ algorithm [25], it can be more efficient with complexity \( O(|I| |E|) \).

The critical node detection problem (CNP), introduced in [14], is to detect a set of vertices in a graph whose deletion results in the graph having the minimum pairwise connectivity between the remaining vertices. Assume \( k \) buses are failed with \( \delta_i = 1 \), and \( \delta_i = 0 \) for all other buses. In CNP, the set \( \{ i : \delta_i = 1 \} \) of buses is to be selected. Let \( x_{ij} = 1 \) denote that bus \( i \) and \( j \) are in the same component after deletion of \( k \) buses with incident lines, and \( x_{ij} = 0 \) otherwise. The constraints of CNP include that the number of buses in the deleted subset is \( k \), i.e., \( \sum_{i : \delta_i = 1} = k \), and all three lines in \( L \) have the relation that if two lines are in the resulted graph, another line is also in the resulted graph, i.e., \( \max\{x_{ij} + x_{jt} - x_{it}, x_{ij} - x_{jt} + x_{it}, -x_{ij} + x_{jt} + x_{it}\} \leq 1 \). The relation between \( x_{ij} \) and \( \delta_i \) can be expressed as \( x_{ij} + \delta_i + \delta_j \geq 1 \) under the objective of minimization. The objective of CNP is to minimize the pairwise connectivity between the remaining vertices. Therefore, the CNP for selecting \( k \) buses in a power grid \( N = (I, L) \) can
formulated as follows:

$$\min \sum_{i,j:v_i,v_j \in V} x_{ij} \quad (4-7)$$

s.t.  
$$x_{ij} + \delta_i + \delta_j \geq 1, \forall (i,j) \in L \quad (4-8)$$
$$x_{ij} + x_{jt} - x_{it} \leq 1, \forall i, j, t \in I \quad (4-9)$$
$$x_{ij} - x_{jt} + x_{it} \leq 1, \forall i, j, t \in I \quad (4-10)$$
$$-x_{ij} + x_{jt} + x_{it} \leq 1, \forall i, j, t \in I \quad (4-11)$$
$$\sum_{i \in I} \delta_i = k \quad (4-12)$$
$$\delta_i, x_{ij} \in \{0, 1\}, \forall i, j \in I \quad (4-13)$$

This problem is NP-hard and the formulation (4–7)-(4–13) for CNP is a mixed integer linear program, which can be solved by CPLEX. In [14], heuristic methods based maximum independent set problem is proposed with computational complexity $O(|I|^2|L|)$. In this paper, for small IEEE test system, MILP by CPLEX is used and for large systems, heuristic methods are used. In (4–13), if $i$ is required to satisfy $\delta_i = 0$ for $i \in \{i \in I : 1^g = 0\}$, all $k$ failures are happening on generators.

4.4.2 Selection of $K$ Lines by Edge Betweenness

For failures on lines, we use edge betweenness [35] for contingency selection. The edge betweenness is adapted from node betweenness, and it can be expressed as

$$C_{(i,j)B} = \sum_{s,t \in I} \frac{n_{st}(i,j)}{n_{st}},$$

where $n_{st}$ is the number of shortest paths from bus $s$ to bus $t$, and $n_{st}(i,j)$ is the number of shortest paths from bus $s$ to $t$ that pass through line $(i,j)$. The algorithms for finding the node betweenness can also be used to find the edge between for each line of a power grid. After obtaining $k$ lines with highest edge betweenness, we set $\sigma_{ij} = 1$ for these lines and set $\sigma_{ij} = 0$ for all other lines.
Recently, studies of vulnerability for lines in power grid can be found in [23, 44, 106], where optimization methods are used to find small groups of lines whose failure can cause severe blackout. Additionally, contingency selection of lines is also studied by fitness function of line capacity [77], line capacity method [27] and etc.

4.5 A $N - k$ Contingency Selection by Worst Case Interdiction Analysis

In previous section, by graph algorithms, we study the contingency selection for both buses and lines. However, except the maximum and minimum traffic methods for selecting buses, all methods are from the prospective of graphs, or topology structures of power grids. In the following, the interdiction model is used for the worst case analysis.

The interdiction model for power grid was proposed in [111, 112], where a contingency of maximally disruption to a power grid can be derived. The interdiction model has been used to model intelligent adversaries, e.g., terrorists. If $\{i : \delta_i = 1\}$ and $\{(i, j) : \sigma_{ij} = 1\}$ denote the sets of destroyed buses and lines, respectively, the constraint $\sum_{i \in I} \delta_i + \sum_{(i, j) \in L} \sigma_{ij} = k$ can be interpreted as a resource constraint on terrorists. The decision variables to maximally destroy a power grid include $\delta, \sigma$. After the disruptive, the system operator solves OPF by readjusting power generations, demands and power flows to minimize the consequence. The interdiction model is formulated as follows:

\[
\max_{\delta, \sigma} \min_{g, s, f, \theta} \sum_{i \in I} (h_i g_i + r_i s_i) \quad \text{(4–14)}
\]

s.t. Constraints (4–2)-(4–6) \quad \text{(4–15)}

\[
\sum_{i \in I} \delta_i + \sum_{(i, j) \in L} \sigma_{ij} = k \quad \text{(4–16)}
\]

\[
\delta_i, \sigma_{ij} \in \{0, 1\}, \forall i \in I, (i, j) \in L \quad \text{(4–17)}
\]

If the constraint $\sigma_{ij} = 0$ for all $(i, j) \in L$ is added to this model, all interdictions can only happen on buses. Similarly, if the constraint $\delta_i = 0$ is added, all interdictions can
only happen on lines. Additionally, if we add \(\sum_{i \in I} \delta_i = k_1\) and \(\sum_{(i,j) \in L} \sigma_{ij} = k_2\), where \(k_1 + k_2 = k\), the number of interdicted buses (or lines) are limited.

In this interdiction analysis model, for fixed \(\delta, \sigma\), the \(\min\) part in (5–1) with constraints (5–2) construct the OPF model. Assume the associated dual variables for constraint (4–2) is \(\alpha_{ij}\), and \(\beta_{ij}, \gamma_{ij}\) for (4–3), \(\eta_i\) for (4–4), \(\zeta_i\) for (4–5), \(\mu_i\) for (4–6). Taking the duality of this part and combining unknown variables \(\delta, \sigma\), we can obtain the following equivalent formulation for the problem (5–1)-(5–7):

\[
\max_{\delta, \sigma, \alpha, \beta, \gamma, \eta, \zeta, \mu} \sum_{(i,j) \in L} \bar{F}_{ij}(-\beta_{ij} + \gamma_{ij}) + \sum_{i \in I} D_i(\eta_i + \mu_i) + \sum_{i \in I} \bar{G}_i(1 - \delta_i)\zeta_i
\]

\[
\text{s.t.} \quad \eta_i + \zeta_i \leq h_i \quad \leftarrow g_i \quad (4–19)
\]

\[
\eta_i + \mu_i \leq r_i \quad \leftarrow s_i \quad (4–20)
\]

\[
\alpha_{ij} + \beta_{ij} + \gamma_{ij} - \eta_i + \eta_j = 0 \quad \leftarrow f_{ij} \quad (4–21)
\]

\[
-\sum_{j \; j > i} b_{ij}(1 - \delta_i)(1 - \delta_j)(1 - \sigma_{ij})\alpha_{ij} + \sum_{j \; j < i} b_{ji}(1 - \delta_j)(1 - \delta_i)(1 - \sigma_{ji})\alpha_{ji} = 0 \quad \leftarrow \theta_i \quad (4–22)
\]

\[
\beta_{ij} \geq 0, \gamma_{ij} \leq 0, \zeta_i \leq 0, \mu_i \leq 0 \quad (4–23)
\]

Constraints (5–3), (5–7)

where parameters on the right hand side denote corresponding variables of the original problem. This formulation is a mixed integer nonlinear program, and these terms, \(\alpha_{ij}(1 - \delta_i)(1 - \delta_j)(1 - \sigma_{ij})(1 - \sigma_{ij})\zeta_i\) where \(i = 1, \ldots, I, j = i + 1, \ldots, I\), are nonlinear. To linearize them, we introduce some variables: \(u_{ij} = (1 - \delta_i)(1 - \delta_j)(1 - \sigma_{ij})\), \(\nu_{ij} = \alpha_{ij} u_{ij}\) and \(\omega_i = (1 - \delta_i)\zeta_i\). The term \(u_{ij} = (1 - \delta_i)(1 - \delta_j)(1 - \sigma_{ij})\) can be linearized by

\[
u_{ij} \leq 1 - \delta_i, \quad u_{ij} \leq 1 - \delta_j, \quad u_{ij} \leq 1 - \sigma_{ij}, \quad u_{ij} \geq 1 - \delta_i - \delta_j - \sigma_{ij}, \quad u_{ij} \geq 0, \quad (4–24)
\]
the term \( v_{ij} = \alpha_{ij} u_{ij} \) by

\[
\begin{align*}
    v_{ij} &\geq \alpha_{ij} - U_b \cdot (1 - u_{ij}), \\
    v_{ij} &\geq L_b \cdot u_{ij}, \\
    v_{ij} &\leq \alpha_{ij} - L_b \cdot (1 - u_{ij}), \\
    v_{ij} &\leq U_b \cdot u_{ij},
\end{align*}
\]

(4–26)

and the term \( w_{i} = (1 - \delta_i) \zeta_i \) by

\[
\begin{align*}
    w_{i} &\geq \zeta_i - U_b \cdot \delta_i, \\
    w_{i} &\geq L_b \cdot (1 - \delta_i), \\
    w_{i} &\leq \zeta_i - L_b \cdot \delta_i, \\
    w_{i} &\leq U_b \cdot (1 - \delta_i),
\end{align*}
\]

(4–27)

where \( L_b, U_b \) are lower and upper bounds for \( \alpha_{ij}, \zeta_i \) and can be chosen as small enough or large enough constants, respectively.

By solving the problem (4–18)-(4–24) with linearizations (4–25), (4–26) and (4–27), which construct a mixed integer linear program, we can equivalently obtain solutions to the original problem (5–1)-(5–7). In this paper, we use CPLEX to solve the equivalent linear program formulation.

### 4.6 Numerical Experiments and Comparisons

The linear program formulations for OPF and interdiction models are implemented in C++ and using CPLEX 11.0 via ILOG Concert Technology 2.5. In this section, we test all proposed algorithms and methods for contingency selection and evaluation on two bus systems: IEEE-30-Bus system test case and RTS-96 test system [1].

In this section, we first consider \( \mathcal{N} - k \) contingency with \( k \) varying from 1 to 10. Bus and line failures are investigated separately for comparing costs. Finally, we consider failures of buses and lines in one contingency by interdiction model.

For experiments on IEEE-30-Bus System, the generating cost is not considered, i.e., setting \( h_i = 0 \) for \( i \in \mathcal{I} \). The relationship between load shedding cost and the number of failed buses under these graph algorithms and the interdiction method is shown in Fig. 4-2. From this figure, we can see, under the measure by minimizing load shedding, the interdiction method presents much worse power grid performance than other contingencies. The degree based, node betweenness, maximum-traffic and critical node detection (CNP) methods all select the crucial buses for this system, while the
minimum-traffic cannot present crucial selection. Except the random failures, all costs proposed by other methods are monotonously increasing as well as the number $k$ increases. Additionally, when $k \geq 6$ for the worst case interdiction method, the whole power system is completely failed (no generation and no satisfied demand).

In Fig. 4-3 for IEEE-30-Bus System, assume the contingency consisting only of lines, and assume load shedding is the only considered cost. Similarly, the interdiction method presents the worst case selection of lines as we expect. As the number $K$ of failures is increasing, the load shedding cost increases as well.

As discussed in previous section, the interdiction method can select the contingency consisting failures on both buses and lines. In Fig. 4-4, we show the cost of failures on both buses and lines. For given value of $k$, by adding $\sum_{i \in I} \delta_i = k_1$ and $\sum_{(i,j) \in L} \sigma_{ij} = k_2$ in
the interdiction model, where \( k_1 + k_2 = k \), we assume there are \( k_1 = \lceil k/2 \rceil \) failed buses and \( k_2 = \lfloor k/2 \rfloor \) failed lines. In this figure, we can find that the contingency consisting of only buses has the most influence for the system. When \( k \) is large enough (e.g., \( k \geq 6 \) for cases of “buses only” and “buses and lines”, \( k \geq 8 \) for the case of “lines only”), the whole system is completely failed. Analyzing the reason behind that buses are important nodes, one possible explanation is that our data sets assume that all lines have high capacity comparing those generations on buses.

To improve efficiency of the computation for MILP problems in large systems, for example, the interdiction method for RTS-96 System, the failed buses or lines with high appearances when \( K \) is small, can be fixed for decreasing the computational
time when $K$ is large. For experiments on RTS-96 System, both generating and load shedding costs are considered. In Fig. 4-5, the relationship between generating and load shedding cost and the number of failed buses under graph algorithms and the interdiction method is presented. From this figure, we can see, under the measure by minimizing generating and load shedding cost, the interdiction method presents the worst case selection of contingencies. The maximum-traffic method provides the second worst contingencies consistently, while the CNP method provides the third worst contingencies in most of values for $k$. Similarly, except the random failures, all costs proposed by other methods are monotonously increasing as well as the number $k$ increases.

Figure 4-5. Generating and load shedding cost vs. failed buses (RTS-96 System)
In Fig. 4-6 for RTS-96 System, assume the contingency consisting only of lines. Similarly, the interdiction method presents the worst case selection of lines as we expect. As the number $K$ of failures is increasing, the generating and load shedding cost increases as well.

![Generating and load shedding cost vs. failed lines (RTS-96 System)](image)

Figure 4-6. Generating and load shedding cost vs. failed lines (RTS-96 System)

4.7 Conclusive Remarks

We study the $N - k$ contingency selection and evaluation, by which the failures of multiple components can be selected and evaluated to reflect vulnerabilities of a power system. After reviewing some graph algorithms, we introduced the critical node detection method. These methods can find the contingencies consisting of failures only on buses, like generators, substations, etc. For contingency consisting of failures on transmission or distribution lines, the edge betweenness method is used. By maximizing the social benefit, equivalently minimizing the generating and load shedding cost, we use economic analysis for evaluating selected contingencies. For the worst case contingency analysis, the interdiction method with limited $k$ components’ failures is used for both contingency selection and evaluation. Comparing with graph algorithms for contingency selection, the interdiction model can select contingencies consisting both buses and lines. By numerical experiments on two IEEE power grid test cases, we find that interdiction always select the most crucial components, while the maximum-traffic
method and critical node detection method select the second most crucial buses. For all cases, the minimum-traffic method find the least crucial ones.

Future research directions include considering the security constraints in the OPF model, using alternating current power models, etc. In addition, using contingency analysis for finding vulnerabilities of power systems should provide suggestions for system planner and operator. To design a recovery plan for a power grid, or to design a warning system for systems reaching a kind level of failures is also important.
CHAPTER 5
EXTENSION WORK OF ECONOMIC ANALYSIS OF THE $N - K$ POWER GRID
CONTINGENCY WITH LINE SWITCHING

With the arising awareness of the security issues in power systems, contingency analysis for unexpected failures has been an important problem in recent years. In order to leverage grid controllability, the power grid operators can apply the technique of transmission line switching to mitigate the impact caused from the attacking. However, regarding the exponential number of contingencies and switching choices of lines, it brings some computational issue of nonconvex discrete optimization problem, which does not have well-developed non-exponential algorithm. In this section, results are shown to provide some insight about applying line switching with $N - k$ power grid contingency model.

5.1 Nomenclature

Indices and Index Sets

- $I$: set of buses, including substations, generators and load consumers
  - Let $1^g_i$ for $i \in I$ be the indicator such that $1^g_i = 1$ if $i$ is a generator, and $1^g_i = 0$ otherwise; and let $1^d_i$ be the indicator such that $1^d_i = 1$ if $i$ is a load consumer, and $1^d_i = 0$ otherwise
- $(i, j) \in L$: transmission lines, $i, j \in I$ and $i < j$
- $N = (I, L)$: an undirected network representing a power grid where $I$ is the set of buses/nodes and $L$ is the set of transmission lines/edges.

Parameters

- $D_i$: load at bus $i$
- $\bar{F}_{ij}$: transmission capacity for line $(i, j) \in L$
- $\bar{G}_i$: generation capacity at bus $i \in I$
- $b_{ij}$: susceptance of line $(i, j)$; $b_{ij}$ is computed from the resistance and the reactance of line $(i, j) \in L$
- $h_i$: generation cost at bus $i \in I$
• \( r_i \): penalty cost for load-shedding at \( i \in I \)
• \( k \): the number of components (buses or lines) to be failed

**Decision variables**

• \( g_i \): power generation at node \( i \in I \)
• \( f_{ij} \): power flow on line \((i, j) \in L\)
• \( s_i \): load shed at node \( i \in I \)
• \( \theta_i \): phase angle at node \( i \in I \)
• \( \delta_i \): \( \delta_i = 1 \) if the bus \( i \) is selected as one of \( k \) components for failures; \( \delta_i = 0 \) otherwise
• \( \sigma_{ij} \): \( \sigma_{ij} = 1 \) if the line \((i, j)\) is selected as one of \( k \) components for failures; \( \sigma_{ij} = 0 \) otherwise
• \( \zeta_{ij} \): \( \zeta_{ij} = 1 \) if the line \((i, j)\) is switched on by operator; \( \zeta_{ij} = 0 \) otherwise

**5.2 Background**

Traditionally, the system operator treats transmission assets (lines or transformers) as static assets within Optimal Power Flow (OPF) problems, which are the network flow problem for the electrical grid. Therefore, power grid is modeled as interdiction model with contingency, over which the system operator dispatches generators to minimize cost. In addition, system operators can change the topology of systems to improve grid controllability and increase transmission capacity. In the mean while, the already built-in line capacity redundance of the electric grid ensuring the mandatory reliability standard, can provide the possibility to improve the efficiency of the network against worst-case scenarios.

Glavitsch [56] introduces how to use the transmission switching in response to contingency conditions as a corrective mechanism. He models this problem with a mathematical formulation and compares some searching techniques solve the problem. Bakitzis et al.[20] uses a Mixed Integer Programming method (a continuous variable formulation for the switching decision as well as discrete control variables) to model the
the transmission switching problems. Bacher et al. [17] have a research on AC power network with transmission line switching in order to mitigate line overloading issue. However, they assume that the generation dispatch is already determined and fixed which do not take the advantage of optimization over the network topology. Rolim et al. [110] have a review paper for the transmission switching methods and the solution techniques. Mazi et al. [84] propose a method of a heuristic technique to solve the problem of alleviating transmission line overloading during contingency by the use of transmission switching.

The initial concept of the dispatchable Optimal Power Flow (OPF) network model was first proposed by O’Neill et al. [99]. Fisher et al. [52], further introduced the concept of incorporating the switching of transmission line into dispatch optimization formulations. Gorenstin et al. [57] study a similar problem with transmission switching, and they use a linear integer approximate OPF formulation to solve the problem based on branch and bound. Shao et al. ([113], [114]) work on the use of transmission switching to solve the problems of line overloading and voltage violations based. They propose a new solution using a sparse inverse technique and a fast decoupled power flow to find the best switching actions with less computation time.

Recently, transmission line switching has been analytically studied in order to reduce dispatch cost in power system scheduling, and up to 25% dispatch cost can be saved [52, 66, 67, 75]. In Bacher et al. [18], they demonstrates that compared with traditional technique, it could reduce electrical losses in the network by temporarily opening a transmission line. Fliscounakis et al. [53] proposed a mixed integer linear program to to minimize losses with optimal transmission line topology. Traditionally, model do search for the optimal topology but do not co-optimize the generation to maximize the market surplus. In contrast to these approaches, their optimal transmission switching model maximizes the market surplus by co-optimizing the transmission topology along with generation.
Special Protection Schemes (SPSs), are becoming a mainstream protocol in electric grid operations since they can be used to solve a variety of issues from maintaining voltage stability to a corrective action that is taken once a specific contingency occurs. Given the fact that the topology of the power grid will be changed if one or more transmission lines are attacked or disconnected, transmission line switchings can also be incorporated into system operators post-disruption decision for a better mitigation effect. Grid operators identify specific grid conditions where it can be advantageous to implement an automatic, predetermined corrective action in response to specific abnormal grid operations. For example, in PJM, Special Protection Schemes have included transmission line switching as one operation during contingencies [73].

5.3 Optimal Power Flow Model for Economic Analysis with Line Switching

\[
\begin{align*}
\max_{\delta, \sigma} \min_{g, s, f, \theta, \xi} & \sum_{i \in I} (h_i g_i + r_i s_i) \\
\text{s.t.} & \quad \text{Constraints (4–2)-(4–6)} \\
& \sum_{i \in I} \delta_i + \sum_{(i,j) \in L} \sigma_{ij} = k \\
& b_{ij} (\theta_i - \theta_j) - f_{ij} + (1 - \xi_{ij}) M_{ij} \geq 0 \\
& b_{ij} (\theta_i - \theta_j) - f_{ij} - (1 - \xi_{ij}) M_{ij} \leq 0 \\
& \sum_{(i,j) \in L} \xi_{ij} = m \quad \forall (i,j) \in L \\
& \delta_i, \sigma_{ij}, \xi_{ij} \in \{0, 1\}, \forall i \in I, (i,j) \in L
\end{align*}
\]

This is a bi-level problem, and the inner level could not be dualized as before because of the integer decision variable $\xi_{ij}$. Here in this model, we limit the total number of lines that could be switched to $m$ as a resource constraint.
5.4 Numerical Experiments and Analysis

The linear program formulations for OPF and interdiction models with line switching are implemented in C++ and using CPLEX 11.0 via ILOG Concert Technology 2.5. In this section, we test all proposed algorithms and methods for contingency selection and evaluation on the bus system: IEEE-30-Bus system test case, and we have some preliminary result here.

![Figure 5-1. IEEE-30-bus system with line switching](image)

Compared with figure 4-3 with $k = 2$ and $k = 6$, we can see that when there is no line switching allowed ($m = 0$), the objective value is the same as in last chapter. However, when there is line switching allowed, we can see some improvement over the objective value for the case of $k = 2$. The effect of line switching can NOT be observed for the case of $k = 6$, since the whole power grid is almost down and there is little room for the operator to manipulate.
CHAPTER 6
FUTURE RESEARCH

With the arising awareness of the security issues in power systems, contingency analysis for unexpected failures has been an important problem in recent years. However, regarding the exponential number of contingencies, the security checking for the feasible power flows under different situations is computationally expensive and almost intractable in short time periods. In this research, we plan to use some optimization approaches to classify the large contingency set into smaller ones based on the optimal power flow model. We want to show that checking smallest subsets of contingencies by the robust optimization method is enough for checking the whole set’s feasibility. By these approaches, the computational complexity for large power systems will be reduced significantly.

According to the standards of the North American Electric Reliability Corporation (NERC), power systems are required to perform necessary adjustments under normal and contingency conditions to ensure system reliability. If only a single element is lost ($N - 1$ contingency), the system must be stable and all thermal and voltage limits must remain within applicable rating. The loss-of-load is not allowed for $N - 1$ contingency. In the case of multiple simultaneous failures ($N - k$ contingency), the system still has to meet the stable, thermal and voltage limits, but planned or controlled loss-of-load is allowed, to a limited degree [34]. In this model, we introduce the concept of $\epsilon$ to ensure the power grid survivability criterion, which requires at least $(1 - \epsilon)$ fraction of the initial total demand must be met when there are up to $k$ components failed within the grid. It means that for no-contingency state and contingency states with $k = 1$, no loss-of-load is allowed and $\epsilon = 0$; for contingency states with $k \geq 2$, a small fraction of total load demand can be shed and $0 < \epsilon \ll 1$. 

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6.1 Nomenclature

Index Sets and Indices

- \( I \): the set of buses (indexed by \( i, j \))
- \( G \): the set of generating units (indexed by \( g \))
  - \( G_i \): the set of generating units located at bus \( i \)
- \( E \): the set of directed transmission lines connecting buses (indexed by \( e \))
  - \( E_i \): the set of transmission lines oriented into bus \( i \)
  - \( E_i^\text{out} \): the set of transmission lines oriented out of bus \( i \)
  - \( e = (i_e, j_e) \): the tail bus \( i_e \) and head bus \( j_e \) of transmission line \( e \)
- \( \mathcal{C}(k) \): the set of all \( N-k \) contingencies with exactly \( k \) failed elements
  - \( c \): index for contingency and \( c \in \mathcal{C} \)

Parameters

- \( B_e, F_e \): susceptance and thermal capacity of transmission line \( e \in E \)
- \( D_i \): load demand at bus \( i \)
- \( P_g, P_g^{\text{max}} \): lower and upper bounds of generation level for unit \( g \in G \)
- \( N \): the number of total system elements consisting of generating units and transmission lines (i.e., \( N = |G| + |E| \))
- \( k \): the number of failed elements as appearing in \( N-k \) contingency, and usually positive integers (i.e., \( k = 1, 2, 3, \cdots \))
- \( \delta_g^c \in \{0, 1\} \): \( \delta_g^c = 1 \) means that the generating unit \( g \) is failed in contingency \( c \), and \( \delta_g^c = 0 \) otherwise
- \( \sigma_e^c \in \{0, 1\} \): \( \delta_e^{ct} = 1 \) means that the transmission line \( e \) is failed in contingency \( c \), and \( \delta_e^c = 0 \) otherwise

Decision Variables

- \( p_g \): the generation level of unit \( g \)
- \( f_e \): the power flow on transmission line \( e \)
• $\theta_i$: the phase angle on bus $i$

• $q_i$: load shedding (i.e., unsatisfied demand) at bus $i$

• $Q(c)$: the minimum amount of load shedding in contingency $c$

• $\delta_g, \sigma_e$: unknown variables corresponding to $\sigma^c_e \in \{0, 1\}$ for identifying a contingency

### 6.2 Optimal Power Flow with Contingency

During the contingency $c \in C(k)$, the following program (6–1) is used for controlled operations to find the optimal power flow. Controlled operations include load shedding, and $\epsilon$ is used for controlling the amount of load shedding.

\[
\text{OPF}(c, k, \epsilon) : \\
Q(c, k, \epsilon) = \min_{f, p, q, \theta} \sum_{i \in I} q_i \tag{6–1a}
\]

\[
\text{s.t. } \sum_{e \in E_i} f_e + \sum_{g \in G_i} p_g = \sum_{e \in E_i} f_e + (D_i - q_i), \forall i \in I \tag{6–1b}
\]

\[
B_e(\theta_{ie} - \theta_{je})(1 - \sigma^c_e) - f_e = 0, \forall e \in E \tag{6–1c}
\]

\[
-F_e(1 - \sigma^c_e) \leq f_e \leq F_e(1 - \sigma^c_e), \forall e \in E \tag{6–1d}
\]

\[
P_g(1 - \delta^c_g) \leq p_g \leq P_g(1 - \delta^c_g), \forall g \in G \tag{6–1e}
\]

\[
0 \leq q_i \leq D_i, \forall i \in I \tag{6–1f}
\]

\[
\sum_{i \in I} q_i \leq \epsilon \sum_{i \in I} D_i \tag{6–1g}
\]

The objective (6–1a) is to minimize the amount of load shedding in the process to find a feasible power flow among the system. The constraint (6–1b) for every bus $i$ is to ensure the flow balance of $i$, where the satisfied demand at $i$ is $D_i - q_i$. The constraint (6–1c) computes the flow amount on line $e$, depending the difference of phase angles of its two ending buses. If $e$ is in the contingency $c$ (i.e., $\sigma^c_e = 1$), the flow is zero, as well as limited by (6–1d). The flow on each line $e$ is limited by the line capacity shown in (6–1d). The constraint (6–1e) for generating unit $g$ is to ensure that the generation level is limited within the lower and upper bounds. When this generating unit $g$ is in the contingency.
c (i.e., \( \delta^c_g = 1 \)), the generation level is 0. The constraint (6–1f) ensures that the load shedding amount at each bus \( i \), while the last constraint (6–1g) presents a threshold for the total amount of load shedding.

Note when \( k = 0 \), i.e., \( c \in C(0) \), the formulation (6–1) is the optimal power flow for economic dispatch, where no load shedding is allowed and the objective is to minimize the generating cost.

6.3 Analysis

In the formulation (6–1), the vectors \( \delta^c, \sigma^c \) is known and defined by contingency \( c \). However, among the large number of contingencies within \( C(k) \), which one is the worst-case leading to large amount of load shedding is not known until you check all contingencies. Here, we use robust optimization method in the network interdiction model to find the worst-case contingency with respect to the load shedding. The network interdiction model for power grids was proposed in [112]. Following their model, we assume that exactly \( k \) failed components on generating units and/or transmission lines, and add the constraints for controlled load shedding by the threshold \( \epsilon \). For different values of \( k \) and \( \epsilon \), the model for worst-case contingency identification can be formulated
as follows,

\[
Q(k, \epsilon) = \max_{\delta, \sigma} \min_{f, p, q} \sum_{i \in I} q_i \quad (6-2a)
\]

s.t. \[
\sum_{e \in E} f_e + \sum_{g \in G} p_g = \sum_{e \in E_i} f_e + (D_i - q_i), \forall i \in I \\
B_e(\theta_i - \theta_{je}) - f_e - M\sigma_e \leq 0, \forall e \in E \\
B_e(\theta_i - \theta_{je}) - f_e + M\sigma_e \geq 0, \forall e \in E \\
- F_e(1 - \sigma_e) \leq f_e \leq F_e(1 - \sigma_e), \forall e \in E \\
P_g(1 - \delta_g) \leq p_g \leq P_g(1 - \delta_g), \forall g \in G \\
0 \leq q_i \leq D_i, \forall i \in I \\
\sum_{i \in I} q_i \leq \epsilon \sum_{i \in I} D_i \\
\sum_{g \in G} \delta_g + \sum_{e \in E} \sigma_e = k \\
\delta_g, \sigma_e \in \{0, 1\}, \forall g \in G, e \in E \\
\]

The objective \((6-2a)\) has two levels, where the outer level denotes the identification the worst-case contingencies, while the inner one denotes the optimal operations in the corresponding contingency. The constraint \((6-2i)\) ensures that there can exactly \(k\) failed components in the system. Constraints \((6-2c),(6-2d),(6-2e)\) ensure that the flow \(f_e\) is \(B_e(\theta_i - \theta_{je})\) and within the capacity limits if the line is not in the contingency, and 0 if in the contingency, where \(M\) is a large-enough constant.

Assume that optimized value for this program is \(c^* = (\delta^*, \sigma^*)\), which identifies a worst-case contingency with respect to the amount of load shedding.

6.4 Preliminary Result

6.4.1 “Critical” Sets

In order to have some basic idea about this “critical contingency subset”, the optimal contingency selection (transmission line only) is obtained by solving the previous model
using CPLEX on IEEE-300-Bus system test case. The results are shown in the following tables with different $k$ and $\epsilon$.

![IEEE-300-Bus system](image)

**Figure 6-1.** IEEE-300-bus system (From: Power Systems Test Case Archive)

From the tables below 6-1, 6-2, 6-3, 6-4, 6-5, 6-6, we can see that same transmission line(s) are in the solution for different $k$ within the same scenario of $\epsilon$. In the scenario of $\epsilon = 0.0005$, line (60,238) and (32,35) are shown in all the cases, (39,62) and (41,92) are in most of the cases; In the scenario of $\epsilon = 0.001$, line (60,238) is shown in all the cases, (32,35), (121,154) and (40,68) are in most of the cases; In the scenario of $\epsilon = 0.01$, line (60,238) is shown in all the cases, (35,36), (39,62) and (41,92) are in most of the cases; In the scenario of $\epsilon = 0.1$, line (60,238) and (41,92) are shown in all the cases, (32,35) is in most of the cases; In the scenario of $\epsilon = 0.15$, line (60,238) is shown in all the cases, (32,35), (40,68) and (41,92)
is in most of the cases; In the scenario of \( \epsilon = 0.2 \), line (60,238) is shown in all the cases, (35,36) and (40,68) are in most of the cases. Line (60,328) appears in all the solutions with different \( \epsilon \) and \( k \), so it is definitely the most critical component in this power grid.

6.4.2 Survivability Constraints and the Possible Cut

Similar work has focused on identifying a small set of arcs whose removal (to model complete failure) will result in a network unable to deliver a minimum amount of demand. A problem of this type can be solved using mixed-integer programming techniques [23]. An example of this approach is used in [23], where as an approximation to the \( N - k \) problem with AC power flows, a linear mixed-integer program to solve the following combinatorial problem: remove a minimum number of arcs, such that in the resulting network there is a partition of the nodes into two sets, \( N_1 \) and \( N_2 \), such that

\[
D(N_1) + G(N_2) + \text{cap}(N_1, N_2) \leq Q^{min}
\]

Here \( D(N_1) \) is the total demand in \( N_1 \), \( G(N_2) \) is the total generation capacity in \( N_2 \), \( \text{cap}(N_1, N_2) \) is the total capacity in the (non-removed) arcs between \( N_1 \) and \( N_2 \), and \( Q^{min} \) is a minimum amount of demand that needs to be satisfied. The quantity in the left-hand side in the above expression is an upper-bound on the total amount of demand that can be satisfied, the upper-bound can be strict because under power flow laws it may not be attained.

Typically, this kind of exact method could not handle large scale problems, because the exponential number of contingencies could make the problem intractable. However, combined with survivability constraint, it means that if by any selection of removal of \( k \) transmission lines from the power grid, the constraint above could be satisfied, that contingency is not feasible, and we can use this to generate possible cutting plane and/or to eliminate some possible branches.
By substitute the constraint of

\[ Q^{\min} = \epsilon \sum_{i \in I} D_i \]

in to

\[ D(N_1) + G(N_2) + \text{cap}(N_1, N_2) \leq Q^{\min} \]

we can get

\[ (\epsilon \sum_{i \in N_1} D_i - \sum_{i \in N_1} q_i) + \text{cap}(N_1, N_2) \leq (1 - \epsilon) \sum_{i \in N_2} D_i - \sum_{i \in N_2} G_i \]

\[ (\epsilon \sum_{i \in N_1} D_i - \sum_{i \in N_1} q_i) \text{ is difference between the actual load shedding and the averaged load shedding in set } N_1, \text{ and } (1 - \epsilon) \sum_{i \in N_2} D_i - \sum_{i \in N_2} G_i \text{ is the difference between averaged total demand and actual generation in set } N_2. \text{ This constraint means that the load shedding could not be too much unbalanced between sets } N_1 \text{ and } N_2. \]
Table 6-1. Contingency Selection with $\epsilon = 0.0005$

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Table 6-3. Contingency Selection with $\epsilon = 0.01$

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Table 6-5. Contingency Selection with $\epsilon = 0.15$

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Table 6-6. Contingency Selection with $\epsilon = 0.2$

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REFERENCES


PJM Interconnection. *Transmission operations*.


[141] Qianchuan Zhao, Kai Sun, Da-Zhong Zheng, Jin Ma, and Qiang Lu. A study of system splitting strategies for island operation of power system: a two-phase

BIOGRAPHICAL SKETCH

Hongsheng Xu was born and raised in Chengdu located in southwestern part of China, and he spent his first 18 years there. Hongsheng received his Bachelor of Science degree in civil engineering from Tsinghua University. He enrolled in the Ph.D. program of Department of Civil Engineering in 2006 to pursue his degree in transportation engineering. In 2008, with a Master of Science degree from civil engineering, he started his Ph.D. new program in the Department of Industrial & Systems Engineering. He spent the first one and a half years working on optimization related computation problems, and gained a lot of experience of programming. Then, Hongsheng joined the Center for Applied Optimization (CAO) in the spring of 2010, and focused on optimization over power security issues. He received his Ph.D. from the University of Florida in the summer of 2012. His research interests include power network optimization and its application, global optimization and applications, design and analysis of computer algorithms, parallel computing in mathematical programming, software design and development.