TEACHING STUDENTS WITH AUTISM TO SOLVE ADDITIVE WORD PROBLEMS USING SCHEMA-BASED STRATEGY INSTRUCTION

By

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To all of the children with autism who have inspired me to do this work

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LIST OF ABBREVIATIONS

ASD	Autism Spectrum Disorder
AYP	Adequate Yearly Progress
IEP	Individualized Education Program
NCLB	No Child Left Behind Act
SBI	Schema-based Strategy Instruction

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Students with autism often struggle with mathematical word problem solving due to executive dysfunction and communication impairment. The purpose of this study is to provide evidence of the efficacy of using schema-based strategy instruction (SBI) to improve the addition and subtraction word problem solving performance of elementary school students with autism. A first-grade student with autism and a sixth-grade student with autism were taught to use schematic diagrams to solve three types of addition and subtraction word problems based on the semantic structure of the problems. A multiple probes across behaviors single-case design was used with solving each of the three problem types treated as a separate behavior. This design was replicated across participants. In addition, participants' behaviors while solving mathematics word problems were analyzed prior to and following SBI. Finally, participants and their parents completed satisfaction scales regarding SBI. Results indicated that problem solving performance improved following SBI, improvements were maintained over time, and participants and their parents were satisfied with the SBI. Observations of changes in problem solving performance suggest that SBI may result in increased use of problem solving strategies and self-monitoring.

CHAPTER 1
INTRODUCTION

Autism Spectrum Disorders (ASDs) are becoming increasingly prevalent within the school-aged population. In 2002, one in 152 children were diagnosed with an ASD (Croen, Grether, Hoogstrate, & Selvin, 2003). By 2006, the number of children diagnosed with an ASD had increased to one in 110 (Centers for Disease Control and Prevention, 2009). Due to the increasing prevalence of ASDs among school-aged children, it is important to consider the impact of characteristic and secondary features of autism on students' learning. According to the *Diagnostic and Statistical Manual of Mental Disorders, Fourth Edition, Text Revision* (DSM-IV-TR; American Psychiatric Association, 2000), autism is characterized by communication and social difficulties, as well as repetitive and ritualistic interests and behaviors.

In addition, executive dysfunction is considered to be a secondary characteristic of autism such that Attention Deficit Hyperactivity Disorder (ADHD) and autism cannot be comorbidly diagnosed. These two disorders are characterized by differences in both the severity and profile of executive dysfunction, as well as disparate diagnostic criteria and treatment regimens demonstrating the uniqueness of the disorders (Pennington & Ozonoff, 2006). Intellectual disabilities are also typical of individuals with autism. According to Ghaziuddin (2000), approximately 75% of individuals with autism also have intellectual disabilities as characterized by a full-scale intelligence quotient of less than 70. However, compared to individuals with intellectual disabilities who do not have autism, those with autism are more likely to have visual strengths as noted by a performance or nonverbal intelligence quotient in the average range.

Language and Executive Functioning Profiles of Children with Autism

According to Nation and Norbury (2005), language and reading comprehension difficulties associated with autism are related and include difficulties integrating information, accessing prior knowledge, resolving anaphoric references, and monitoring comprehension. Several studies have assessed the language and comprehension profiles of children with autism. For instance, Frith and Snowling (1983) conducted multiple experiments to assess the reading ability and comprehension skills of children with autism as compared to ability-matched readers without autism. One of their findings indicated that children with autism had difficulty making use of semantic cues when comprehending sentences. Other researchers found that compared to typically developing children, children with high functioning autism had comprehension and recall deficits, such as difficulty making inferences (Norbury & Bishop, 2002). The authors hypothesized that pragmatic language deficits associated with autism interfered with students' comprehension and recall difficulties. Furthermore, O'Connor and Klein (2004) found that students with autism have difficulty resolving anaphoric references when reading and were able to improve their reading comprehension when given cues to resolve these references.

Executive dysfunction may include difficulties with planning, organization, switching cognitive set, and working memory (Zentall, 2007). Switching cognitive set involves the ability to quickly change focus from one activity to another and working memory involves the ability to hold information in immediate awareness while manipulating that information (Geary, Hoard, Nugent, & Byrd-Craven, 2008). For instance, being able to remember the numerical information contained in a mathematics word problem while also determining which computational operations to perform to solve the problem requires working memory. Researchers have found that children with autism are differentially impaired in executive functioning in the areas of vigilance, working memory, planning, and set shifting when compared to typically developing children and children with ADHD (Corbett, Constantine, Hendren, Rocke, & Ozonoff, 2009). When compared to ability matched and language matched children, children with autism were found to be differentially impaired in the areas of working memory, planning, set shifting, and inhibitory control (Ozonoff, Pennington, & Rogers, 1991; Hughes, Russel, Robbins; 1994).

Additional research has explored the working memory abilities of children with autism. According to Gabig (2008), children with autism have greater difficulties with verbal working memory than do age-matched controls. These difficulties become more pronounced as vocabulary and language processing demands increase. Compared to ability-matched controls, children with autism are less likely to use verbal rehearsal strategies leading to poor verbal working memory (Joseph, Steele, Meyer, & Tager-Flusberg, 2005) and are less likely to use organized search strategies leading to poor spatial working memory (Steele, Minshew, Luna, & Sweeney, 2007).

Impact of Language and Executive Functioning on Mathematics Performance

According to Donlan (2007), one of the areas in which language impairment impacts students' mathematics performance is word problem solving. Solving word problems requires that students use semantic mapping to determine the relationships between known and unknown quantities to determine the correct operation to use to solve for the unknown (Christou & Phillippou, 1999). The failure of students with autism to use semantic cues (Frith & Snowling, 1983) has important implications for teaching word problem solving because it will likely make semantic mapping more difficult for these students.

In addition, language comprehension deficits impact the ability of students to solve word problems. In a longitudinal study, Jordan, Hanich, and Kaplan (2003) compared children with reading disabilities, reading and math disabilities, and math disabilities. They found that language comprehension deficits led to poorer performance on word problems for students with reading disabilities compared to those with math disabilities only. In addition, Cowan, Donlan, Newton, and Lloyd (2005) found that children with language impairments had greater difficulty solving word problems than did typical children due to phonological working memory and language deficits.

Executive dysfunction, particularly in the areas of sustained attention and working memory, also impacts children’s mathematics progress and word problem solving performance (Zentall, 2007). For instance, Zentall and Ferkis (1993) found that children with executive dysfunction had difficulty filtering and manipulating relevant information to facilitate word problem solving due to working memory and reading difficulties. Working memory deficits may also lead to the use of immature problem solving strategies and procedures, and difficulty inhibiting irrelevant information leading to errors in problem solving (Geary, Hoard, Nugent, & Byrd-Craven, 2008). In fact, working memory deficits may underlie many mathematics difficulties (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Geary and his colleagues compared first grade students with mathematics difficulties to average and low-achieving first graders. They found that the students with math difficulties scored one standard deviation lower than did the other students on measures of working memory and that these deficits in working memory partially or fully mediated deficits in mathematics cognition.

Need for Research-Based Mathematics Instruction for Students with Autism

Problem solving is a primary goal of mathematics instruction and a critical process for successful functioning in an increasingly technological and mathematically oriented society (National Council of Teachers of Mathematics [NCTM], 2002). Moreover, federal legislation has mandated that schools become increasingly accountable for the academic progress of all students. The No Child Left Behind Act (NCLB, 2002) requires that schools demonstrate Adequate Yearly Progress (AYP) toward proficiency of all students on assessments of reading and mathematics. Additionally, the Individuals with Disabilities Education Improvement Act (IDEA; Assistance to States for the Education of Children with Disabilities, 2004) requires that students with disabilities have access to and make progress in the general education curriculum. NCLB also requires that states assess and document the progress of at least 95% of students with disabilities according to statewide standards. Although some students with disabilities are assessed using alternate standards, the majority of these students are evaluated against the same high academic standards as their non-disabled peers.

Within the area of mathematics, the focus of the general education curriculum and of AYP assessments is increasingly on conceptual understanding and problem solving rather than on computational proficiency (Bottge, 2001). According to Bottge, helping students become proficient problem solvers has proven challenging. Students with disabilities advance at a slower rate than their non-disabled peers resulting in ever increasing discrepancies and limited proficiency on tests of mathematics competence. For instance, on the National Assessment of Educational Progress, only 40% of fourth-grade students who took the assessment in Florida were proficient on complex mathematical problem solving tasks (Institute of Educational Statistics, 2007). Therefore, fostering successful problem solving performance in mathematics for students with disabilities, including those with autism, is critical.

Despite the need to identify scientifically based practices for teaching mathematical problem solving to students with autism, few studies have attempted to improve the word problem solving performance of these students. However, a body of research has evaluated the efficacy of interventions aimed at improving the math problem solving performance of students with math difficulties and learning difficulties. In their meta-analysis, Gersten et al. (2009) reviewed 42 studies aimed at improving students' mathematics performance. The studies focused on problem solving used various interventions including instruction in general problem solving procedures or heuristics, multiple strategy instruction, peer-assisted instruction, direct instruction, and instruction using visual representations. Results of the meta-analysis indicated that instruction using visual representations resulted in large effect sizes, particularly when visual representations were paired with heuristics and direct instruction.

Schema-based strategy instruction (SBI) is an intervention characterized by visual representations and direct instruction for teaching students to solve mathematical word problems. The results from several studies have demonstrated the efficacy of using SBI to improve the additive word problem solving performance of students with learning disabilities and low performing students in resource and inclusive settings (e.g., Griffin & Jitendra, 2009; Jitendra et al., 2007b; Jitendra et al., 1998). SBI may hold particular promise for students with autism due to the difficulties these children experience with communication, executive functioning, and working memory. Semantic mapping during word problem solving may be facilitated through the use of visual supports like those included in SBI (Mesibov & Howley, 2003; Tissot & Evans, 2003). In addition, schema induction and rule automization have been found to reduce cognitive load (Cooper & Sweller, 1987), and schema induction following SBI has been found to reduce working memory demands and cognitive load (Capizzi, 2007). Finally, the problem solving heuristic component of SBI, which includes verbal rehearsal and a mnemonic, provides support for attention and organization (Deshler, Alley, Warner, & Schumaker, 1981). Despite the potential benefits of using SBI to teach students with autism to solve additive word problems, only one such study has been identified. This preliminary, but promising, study conducted by Rockwell, Griffin, and Jones (2011) examined SBI with a fourth-grade student with autism. Therefore, further research on using SBI with students with autism is warranted.

Purpose and Research Questions

This study was designed to evaluate the use of SBI to teach additive word problem solving to students with autism. Research questions included (a) Will students with autism demonstrate improvement in their ability to solve one-step additive word problems with the unknown in the final position following SBI? (b) Will changes in students' correct problem solving generalize to problems with the unknown in the initial position? (c) Will changes in students' correct problem solving generalize to problems with the unknown in the medial position? (d) Will changes in students' correct problem solving generalize to problems with irrelevant information? (e) Will students' correct problem solving be maintained eight weeks following the intervention? (f) How do the problem solving behaviors of students with autism change following SBI? (g) How do students with autism perceive SBI? and (h) How do parents of students with autism perceive SBI?

Definitions of Terms

- SCHEMA-BASED STRATEGY INSTRUCTION (SBI). An intervention that involves using direct instruction to teach students to discriminate types of mathematical word problems found in the theoretical literature based on their semantic structure and to use schematic diagrams to facilitate problem solving (Willis & Fuson, 1988).
- DIRECT INSTRUCTION. Teacher-directed, explicit approach to instruction involving teacher modeling, guided practice, independent practice, and continuous teacher feedback (Rosenshine & Stevents, 1986).
- SCHEMATIC DIAGRAMS. Visual representations of the semantic structure of group,
- change, and compare word problems by visually depicting the relationships among the known quantities and the unknown quantity for which students must solve.
- ADDITIVE WORD PROBLEMS. Those word problems that require addition or subtraction to achieve a problem solution.
- GENERAL PROBLEM SOLVING HEURISTICS. A sequence of steps used to encourage students to monitor their problem solving process. For instance, a four-step heuristic represented by the mnemonic FOPS has been used to ensure that students *f*ound the information in the problem, *o*rganized the information using a diagram, *p*lanned to solve the problem, and *s*olved the problem (e.g., Jitendra et al., 2007b).

Problem Type Definitions

Group problems consist of two smaller groups, termed parts, which are combined to form a larger group, or all. Consider the story situation: Jen bought 12 apples and 15 oranges. Jen bought 27 pieces of fruit. In this problem the two parts are 12 apples and 15 oranges while the all is 27 pieces of fruit.

<table>
<tr><td>Part</td><td>Part</td></tr>
<tr><td colspan="2">All</td></tr>
</table>

Figure 1-1. Group diagram adapted from Willis and Fuson (1988).

Change problems consist of a beginning amount, change amount indicated by an action, and an ending amount. The change amount in the problem can indicate addition or subtraction. The label “get more” will be applied to additive changes, while the label “get less” will be applied to subtractive changes (Willis & Fuson, 1988). Consider the story situation: Jen had 12 oranges. She ate 3 of them. Now she has 9 oranges. This is a “get less” situation with the beginning amount of 12 oranges. The change amount is -3 oranges and the ending amount is 9 oranges. In this “get less” situation, the change diagram can be adapted to reflect the larger beginning amount and smaller ending amount by increasing the width of the box representing the beginning amount. Now, consider the story situation: Jen had 12 oranges. She picked 14 more. Now she has 29 oranges. This is a “get more” situation with a beginning amount of 12 oranges, change amount of +14 oranges, and ending amount of 29 oranges. In this “get more” situation, the change diagram can be adapted to reflect the smaller beginning amount and larger ending amount by increasing the width of the box representing the ending amount.

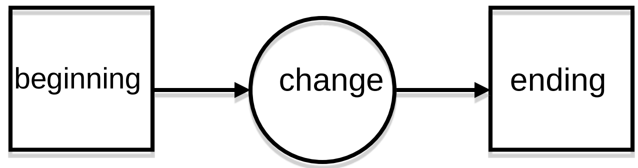


Figure 1-2. Change diagram adapted from Willis and Fuson (1988)

Compare problems consist of a larger amount, a smaller amount, and a difference, which results from comparing the smaller and larger amounts. Compare problems can be written in two forms. Consider the story situation: Jen has 22 apples. She has 15 oranges. Jen has 7 more apples than oranges. This story situation can also be written: Jen has 22 apples. She has 15 oranges. Jen has 7 fewer oranges than apples. In both cases, 22 apples is the larger amount, 15 oranges is the smaller amount, and 7 oranges is the difference.

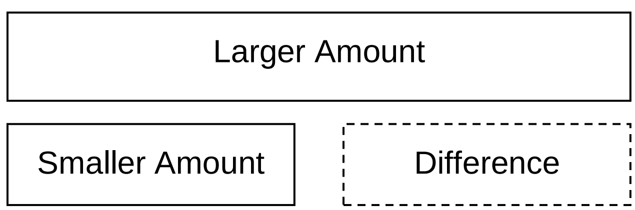


Figure 1-3. Compare diagram adapted from Willis and Fuson (1988).

Unknown Quantities and Algebraic Reasoning.

Algebraic reasoning involves the ability to “represent and analyze mathematical situations and structures using algebraic symbols” (NCTM, 2002, p. 37). Additive word problems include two known quantities and one unknown quantity for which students must solve. When this unknown quantity is located in the final position of the schematic diagram (i.e., the all in group problems, ending in change problems, and difference in compare problems), the schematic diagram can be translated into a number sentence that can be solved without algebraic reasoning. When the unknown quantity is in the initial or medial position in the schematic diagram, the number sentence derived from the schematic diagram can be solved only through the application of algebraic reasoning.

Delimitations and Limitations of the Study

This study was intended to investigate the use of SBI to teach students with autism who have no comorbid intellectual disabilities to solve one-step additive word problems. This study did not address the use of other problem solving interventions. It also did not address the use of SBI with students with comorbid intellectual disabilities or with disabilities other than autism. Furthermore, this study did not address multiplicative word problems, multi-step word problems, or other types of mathematical problem solving.

This study has several limitations. First, the inclusion only of students with autism but without comorbid intellectual disabilities, the use of a single-subject design, and the inclusion of only one-step additive word problems limit the generalizability of the findings. Generalizability is further limited because participants were recruited from the Center for Autism and Related Disabilities at the University of Florida, which serves children with autism in North Central Florida. Furthermore, because SBI was conducted by the researcher in participants' homes rather than by classroom teachers in a school setting, the feasibility of training teachers to conduct SBI with fidelity could not be determined. Social validity was also reduced because typical intervention agents did not conduct the SBI in a typical instructional setting.

CHAPTER 2
REVIEW OF THE LITERATURE

Schema-Based Instruction and Additive Word Problem Solving

Additive word problem solving requires that students be able to apply their knowledge of whole number operations while simultaneously manipulating information presented in written form (Verschaffel, Greer, & De Corte, 2007). Students who struggle with this type of problem solving may do so for a variety of reasons. Geary and Hoard (2005) present a conceptual framework for viewing mathematics difficulties that takes into account the supporting competencies and underlying systems that support successful mathematics performance. According to the authors, children may experience mathematics difficulties due to weaknesses in conceptual understanding or in procedural knowledge. Such weaknesses may lead to difficulties with actual problem solving. In their model, the central executive provides the attention control and working memory needed to carry out procedures for successful problem solving. In addition, the language system and visuospatial system allow students to represent and manipulate information in order to develop conceptual knowledge and carry out procedures. Because students with autism have deficits in the language system and central executive, these students will likely have difficulties building concepts for whole number operations and carrying out procedures necessary for word problem solving. To address the word problem solving difficulties of students with autism, it may be helpful to consider ways to mediate deficits of the language system and central executive, and to directly teach necessary conceptual and procedural knowledge.

SBI is a promising intervention because it addresses deficits of the language system and central executive while directly teaching conceptual and procedural knowledge. The schematic diagrams included in SBI provide support for the language system by allowing students to visually represent the semantic information presented in word problems. In addition, the problem solving heuristics included as part of SBI provide procedural supports for students. Finally, by teaching the critical features of different problem types, SBI helps students develop conceptual knowledge needed to efficiently solve word problem types.

SBI is a schema knowledge-mediated method of teaching mathematical word problem solving. All schema knowledge-mediated instruction has its basis in research on analogical problem solving from the field of cognitive psychology. Gick and Holyoak (1983) define analogical problem solving as applying the solution strategy of a known problem to a novel problem with similar underlying structure. Accomplishing this transfer of solution strategy requires that an individual be able to delete irrelevant dissimilarities between problems while preserving relevant commonalities. After exposure to multiple similar problems, individuals are able to develop a schema, or mental representation, of a generalized problem solution strategy for that type of problem. This generalized problem schema then acts as a mediator that facilitates transfer to analogous problems leaving additional cognitive capacity available to allow the individual to handle novel features of problems (Cooper & Sweller, 1987; Gick & Holyoak).

To determine whether instruction in abstract problem schemas would facilitate transfer to novel problems, Gick and Holyoak (1983) conducted a study with college students. Findings suggest that when participants were provided with just one analog problem from which to abstract the generalized problem schema, additional instruction was not helpful. However, when participants were provided with two problems from which to abstract the generalized problem schema, they benefited from provision of verbal and diagrammatic representations of the generalized problem schema. Provision of verbal and diagrammatic representations resulted in higher quality schema and more successful solution of analogous problems.

In a similar study, Chen and Daehler (1989) sought to determine whether explicit instruction in problem schema would facilitate transfer to novel problems for six-year-old children. They found that explicit instruction in problem schema did lead to improved transfer to analogous problems; however, children continued to apply the learned schema even when problems were not analogous. Chen and Daehler concluded that children might need additional instruction to prevent misapplication of learned schema to dissimilar problems. To extend these findings, Chen (1999) conducted a study with elementary school students to assess whether exposure to multiple problems with disparate procedural features would help children build higher quality and broader schemas. Children were asked to solve Luchin's water jar problems, wherein a container must be filled using several smaller containers without any water being spilled or left in a smaller container. He compared the schema formation of children who were given several problems that could be solved using the same combination of smaller containers to those given problems that required different combinations of smaller containers. He found that exposure to multiple problems with varying procedural features such that each problem required a different combination of smaller containers resulted in higher quality schemas and facilitated transfer to a wider variety of problems.

Taken together, these three studies of analogical problem solving indicate that both children and adults can benefit from multiple exposures to problem solving tasks and explicit instruction in generalized problem solution schema when solving problems. Due to the promise of research on schema induction in analogical problem solving, additional studies were conducted to assess applicability to mathematical word problem solving. First, studies with children and studies involving computer simulations of word problem solving were conducted to determine the schematic structures underlying one-step additive word problems (Verschaffel et al., 2007). These studies revealed 14 underlying schemas for one-step additive word problems that were condensed into three primary categories of word problems: combine or group, change, and compare.

Additional research has demonstrated that children develop understanding of these three primary additive schemas gradually as they mature. For instance, Carpenter and Moser (1984) found that children’s schema development progressed from solving change problems, to combine problems, and finally to compare problems. In contrast, Christou and Phillippou (1999) found that children’s understanding of the underlying schemas progressed from combine, to change, to compare. In both of these studies, children were first able to solve problems from a given schema in which unknowns were located in the final position (missing sum, missing difference) and were later able to solve problems with the unknown in the initial position (missing addend). Both sets of researchers also hypothesized that instruction in problem schemas might help students move along the trajectory of schema development more quickly.

Purpose of the Literature Review

Due to the contribution of schema theory to the field of mathematics problem solving instruction, this review will focus on studies of schema-based instruction to enhance mathematical problem solving. The intent of the review is to synthesize the research base on using schema-based instruction to improve additive word problem solving skills. The goal is to determine instructional features that lead to successful schema induction and problem solving transfer on additive word problems, and to determine which instructional features can be adapted for use with students with autism. The question guiding this inquiry is: Which features of interventions that enhance schema induction and additive word problem solving can be successfully applied to a schema-based mathematical problem solving intervention with students with autism? In addressing this question, the unique mathematical difficulties of children with autism as well as instructional strategies that have generally been found to be effective with these students and others with disabilities will be considered. Relevant research studies will be reviewed and major conceptual, methodological, and design/measurement issues will be discussed. In conclusion, implications for future research will be addressed.

Literature Review Search Procedures

Due to the contribution of schema theory to the field of mathematics problem solving instruction, this review will focus on studies of SBI to enhance mathematical problem solving. In identifying relevant research for this review, several criteria were used. First, only studies that utilized SBI to teach the three types of additive word problems were formally reviewed. Also, in an effort to review more recent research, only studies published since 1983, when Gick and Holyoak published their seminal study on analogical problem solving, were included. Finally, only studies published or submitted for publication in refereed journals were reviewed to ensure that rigorous research was included.

First, literature was obtained through searches of the following databases: *Academic Search Premiere, Professional Development Collection, PsychInfo,* and *Psychological and Behavioral Sciences Collection.* Keyword searches were conducted using combinations of the terms *schema, problem solving,* and *mathematics*. These searches yielded 18 empirical studies. Seven of these studies, which used SBI to teach additive word problem solving, were included in this review. Three studies were excluded because they used SBI to teach multiplicative word problem solving to middle school students. An additional seven studies used schema-broadening instruction, which uses verbal representations to teach problem types derived from third grade basal curricula, rather than SBI. These studies will be discussed separately. Because databases were found to be incomplete, an ancestral search of the reference lists of the published literature was performed. This search yielded one additional empirical study that met the criteria for inclusion in this review. Three additional studies that were known to the author were also included. The final sample contained 12 empirical studies, of which one employed a case study design; four a single subject research design; and seven a group experimental, quasi-experimental, or pre-experimental design. A complete summary of SBI and schema-broadening studies included in this review can be found in Appendix A.

Synthesis of SBI Research

As discussed previously, SBI consists of using direct instruction to teach students to discriminate types of mathematical word problems based on their semantic structure, using schematic diagrams to facilitate problem solving. Although all 12 studies included direct instruction, schematic diagrams, and problem types obtained from the theoretical literature in their definitions of SBI, they differed in the specific procedures they used to implement SBI and the populations with whom they implemented the intervention. In two studies (Fuson & Willis, 1989; Willis & Fuson, 1988), the focus was on using highly representative diagrams and whole class instruction to teach average and high-achieving students. In three of the studies (Jitendra & Hoff, 1996; Jitendra et al., 1998; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007a), a self-monitoring component was added to the intervention, which was primarily used to teach students with high incidence disabilities. Another two studies (Jitendra et al., 2007b; Griffin & Jitendra, 2009) extended SBI to teaching students in inclusive settings to solve multi-step problems. SBI in two other studies (Fuchs et al., 2008; Xin, Wiles, & Lin, 2008) involving students with high incidence disabilities was combined with other types of instruction to achieve greater gains in mathematics achievement. In the final three studies (Jitendra, George, Sood, & Price, 2010; Neef, Nelles, Iwata, & Page, 2003; Rockwell, Griffin, & Jones, 2011), SBI was used with individuals with significant disabilities.

SBI with Representative Diagrams

In two descriptive studies Willis and Fuson (1988) and Fuson and Willis (1989) sought to demonstrate that SBI could be used to teach single-step group, change-get-more, change-get-less, and compare story problems to high- and average-achieving second graders using whole-group instruction. Each problem type and its diagram was introduced and taught separately. For group and compare problems, the researchers developed diagrams that were highly representative of the relationships among the magnitudes of the quantities in the word problems. However, the diagram used for change problems represented only the semantic structure of the word problem, rather than the relationships among the magnitudes of the quantities.

The researchers taught teachers to conduct the SBI with 43 students in the 1988 study and then 76 students in the 1989 study. The results of both studies indicated that SBI when implemented with fidelity could assist young children in solving complex addition and subtraction story problems. Results also indicated that children had more success solving problems in which the final quantity in the diagram was unknown. The researchers hypothesized that this differential difficulty might have been due to the increased algebraic reasoning required to solve problems in which the unknown was not the final quantity. Unfortunately, results of the Fuson and Willis (1989) study indicated that teachers implemented SBI with differential fidelity leading to outcomes for students in different classrooms that could not be explained by the effects of SBI alone.

Results of these two studies indicate that SBI could be a valuable method for teaching young children to solve word problems. In addition, they indicate that students could benefit from whole-group instruction using SBI, provided that the instruction is implemented with fidelity. Ensuring that teachers implement SBI with fidelity may require ongoing training and support rather than just the initial training provided by Fuson and Willis (1989).

Despite their promising results, the studies have several limitations. First, the primary use of descriptive statistics in these studies provided some evidence of the practical significance of the intervention, but did not indicate statistical significance. Using significance testing would have allowed the authors to minimize that possibility that their results were due to chance and allowed them to attribute the experimental effects to the SBI. The decision to use descriptive statistics was likely based on the small sample sizes included in these studies and on the number of outcomes in which the researchers were interested; however, it is still a significant limitation. Another limitation of the studies is the lack of a control or comparison group thus limiting the certainty with which observed effects can be attributed to SBI.

The applicability of these studies to teaching additive word problem solving to students with autism is questionable because these studies did not include students with disabilities. However, there are elements of these studies that hold promise for working with students with autism. For instance, the inclusion of schematic diagrams that clearly represent the relationships among the quantities in the word problems provide visual support that may help facilitate semantic mapping for students with autism (Tissot & Evans, 2003). Furthermore, instructing students in one problem type at a time may be beneficial because it allows students to practice to mastery before introducing new information (Truelove, Holoway-Johnson, Mangione, & Smith, 2007). Finally, separating change-get-more and change-get-less problems may help students determine which operation to use when solving word problems.

SBI with Self-Monitoring

In three studies, Jitendra and colleagues (1996, 1998, 2007a) sought to determine whether SBI could be used to improve the ability of students with high incidence disabilities to solve single-step group, change, and compare word problems. SBI in these studies consisted of using direct instruction to teach students to use schematic diagrams and to use a problem solving mnemonic to self-monitor their performance. Students received instruction in all problem types at once; however, they first received problem schemata instruction using story situations in which all quantities were given and then received problem solving instruction in which one quantity was unknown.

Jitendra and Hoff (1996) conducted SBI one-to-one with three students with learning disabilities. Results of an adapted multiple probes across students single-case design indicated no change in level from baseline to just after problem schemata instruction; however, all three participants demonstrated improvement following problem solving instruction and maintained improvement over time. These results suggest that exposure to schema types alone was not sufficient to improve problem solving performance and that explicit instruction in using schematic diagrams to solve problems was necessary to effect improvement in problem solving performance. Interviews with students indicated that they enjoyed using schematic diagrams, found them useful, and would recommend SBI to friends.

In order to replicate these findings with larger numbers of students, Jitendra et al. (1998) conducted an experimental study with 34 students with mathematics disabilities or at-risk for math failure in which they compared SBI and traditional basal instruction. Participants were randomly assigned to receive either SBI or basal instruction in small groups from trained graduate students. Results indicated that although both groups improved from pretest to generalization. The SBI group improved more than did the basal instruction group with their performance approaching that of a normative sample of third graders. In addition, students in the SBI group consistently rated their instruction higher than did the basal instruction group.

In order to determine the feasibility of having teachers conduct SBI in classroom settings, Jitendra et al. (2007a) conducted two design experiments, or flexible studies conducted prior to formal experimental studies that are intended to provide a deeper understanding of the realities of independent variable implementation in classroom settings (Gersten, Baker, & Lloyd, 2000). The first included 38 students attending two low-ability third-grade classrooms, their two teachers, and the special education teacher. The second design experiment involved 56 students in two heterogeneously grouped third-grade classrooms and their two teachers. Results indicated that teachers in both experiments implemented SBI with high fidelity and that students in both design studies improved their problem solving performance following SBI; however, effect sizes were larger in the first experiment than in the second. In addition gains in performance were more evident for low-achieving students in the second study than for average or high-achieving students. Finally, student responses to a strategy satisfaction questionnaire administered to participants in the first experiment indicated that students with learning disabilities rated SBI higher than low-achieving students.

Taken together, the results of these three studies indicate that SBI with a self-monitoring component holds particular promise for teaching additive word problem solving to students with high incidence disabilities and students with poor math performance. They also suggest that it may be beneficial to include separate components of instruction to address schema induction and problem solving. When conducted in small groups, SBI may be more effective than traditional basal instruction (Jitendra et al., 1998). The design experiments (Jitendra et al., 2007a) showed that, when provided with training and ongoing support, teachers could effectively implement SBI in their classrooms with high levels of fidelity. These design experiments also allowed the researchers to adapt the SBI in response to teacher concern. This ultimately led to a more externally valid intervention in experiment two that included problems with information presented in tables, graphs, and pictographs; and instruction in mathematical vocabulary and writing descriptions of problem-solution strategies. Finally, participants’ positive responses during interviews and on questionnaires suggest that SBI is a socially valid intervention.

Despite the promising results of these studies, they have several limitations. First, results in the Jitendra and Hoff (1996) study should be interpreted with caution because the use of only one probe following problem schemata instruction and of just two probes at maintenance precludes the establishment of stable trend and level at these times. In the Jitendra et al. (1998) study, intervention effects may have been confounded by small differences in IQ between groups and by missing IQ scores for some participants. Also, the results of the design experiments (Jitendra et al., 2007a) should be interpreted with caution due to classroom differences prior to intervention and the lack of control or comparison groups, which make it difficult to determine whether the effects found were due to SBI or to other factors such as history or maturation. Furthermore, the aggregation of data across different mild disability categories in the Jitendra et al. studies does not allow for the statistical analysis of differential effectiveness based upon disability category. With regards to external validity, all three studies have limited generalizability due to small sample size. Finally, the inclusion of only one-step additive word problems may not be representative of the variety of problem types found in the general education curriculum.

Despite these methodological flaws, the studies provide insight into how SBI may be adapted for use with students with autism. First, the use of one-to-one or small group instruction may be particularly beneficial for students with disabilities (Truelove et al., 2007). The inclusion of a self-monitoring component also has applicability to students with autism because research suggests that self-monitoring may help students with autism increase desirable behaviors and decrease competing behaviors (Shabani, Wilder, & Flood, 2001). Finally, the separation of problem schemata instruction and problem solving instruction may be helpful because it allows students to achieve mastery at semantically mapping the critical features of word problems to schematic diagrams before requiring that they engage in problem solving. This may benefit students with autism, in particular, by ensuring the solid formation of problem schema thereby reducing working memory demands associated with word problem solving (Capizzi, 2007).

SBI with Multi-Step Problems

Researchers have also conducted quasi-experimental studies to demonstrate the effectiveness of SBI in inclusive, general education classrooms (Griffin & Jitendra, 2009; Jitendra et al., 2007b). In these studies, SBI was extended to include two-step, as well as one-step, additive word problems. Following problem schemata instruction and problem solving instruction using one-step word problems, students were taught to simultaneously use two schematic diagrams to solve two-step word problems. The effectiveness of SBI was compared to general strategy instruction (GSI), which involved teaching general problem solving heuristics found in the mathematics basal text. Classroom teachers conducted all instruction in a whole-group format. In the Jitendra et al. study instruction was conducted for 25 minutes each day, while in the Griffin and Jitendra study instruction was conducted once per week for 100 minutes.

The two studies included 154 students in inclusive, general education classrooms. Participants were assigned to either SBI or GSI using random assignment with matching based on initial math problem solving performance. Results of the Jitendra et al. (2007b) study indicated statistically significant effects and medium effect sizes in favor of SBI over GSI on all measures, including a statewide achievement test. Results of the Griffin and Jitendra (2009) study indicated that both the SBI and GSI groups demonstrated similarly improved problem solving performance over time; however, follow up tests indicated that the SBI group acquired word problem solving skills more quickly than did the GSI group. It is possible that main effects favoring SBI over GSI in this study because instruction was conducted only once per week.

Taken together, the results of these two studies indicate that classroom teachers can successfully conduct SBI with whole groups of students with high fidelity when provided with adequate training, minimal ongoing support, and scripted materials. The results also suggest that SBI benefits students with and without disabilities in inclusive classrooms. Furthermore, the findings indicating differential effects favoring SBI on a nationally standardized and statewide test of achievement (Jitendra et al., 2007b) are promising because these tests assess a variety of math skills not limited to problem solving and represent an authentic measure of generalization. Moreover, because SBI resulted in more rapid acquisition of problem solving skills than did general strategy instruction in the Griffin and Jitendra (2009) study, it is possible that SBI may have potential as a short-term intervention to help struggling students catch up to more capable peers. Finally, because teachers were asked to study rather than read scripts, the use of scripted lessons did not limit the flexibility or personal style with which teachers conducted lessons.

Although the results of these studies suggest that SBI can be used effectively to improve the math problem solving performance of students in inclusive settings, they have several limitations. Despite the inclusion of students with disabilities in these studies, the small numbers of students with learning disabilities (n=9) and minimal analysis of the differential progress of students with learning disabilities make the results related to differential effectiveness of SBI for students with learning disabilities less clear. Additional studies should assess further the effectiveness of SBI for students with disabilities in inclusive settings. Moreover, because the authors did not match based on reading ability or control for students' reading abilities as a covariate, this confounding variable could have impacted the internal validity of the findings. Furthermore, despite the inclusion of two-step word problems, this study did not address the range of problem solving tasks found in the general education. Finally, the use of whole class instruction and scripted lessons may have limited the extent to which teachers could differentiate instruction to meet student needs.

These studies also have important implications for instructing students with autism. SBI as operationalized in these studies addressed a wider range of problem types while continuing to include components that may benefit students with autism (e.g., visual supports, direct instruction, self-monitoring). In addition, these studies clearly demonstrate that teachers can effectively implement SBI in inclusive classroom settings with minimal support. SBI, therefore, has the potential to increase access to the general education curriculum and increase time spent in the general education classroom for students with disabilities including those with autism.

SBI with Additional Instruction

Xin et al. (2008) conducted a study using a multiple probes across participants single case design to determine whether combining direct instruction in using schematic diagrams with explicit instruction in story problem grammar would improve the pre-algebraic concept formation and additive word problem solving performance of three fourth- and fifth-grade students with math difficulties. Story problem grammar instruction was aimed at helping students understand the part-part-whole nature of additive word problems. The authors developed cue cards for the different types of additive word problems to help students identify sentences or parts of sentences containing relevant information and to help students map this information to part-part-whole diagrams. The authors hypothesized that by helping students understand the underlying part-part-whole structure of additive word problems; algebraic reasoning and operation choice would be facilitated.

Results indicated that the intervention was effective in improving students' performance on experimenter-developed word problem solving probes and equation solving probes, and on probes derived from the KeyMath-R/NU (Connolly, 2007). Although the results of this study are promising, the administration of the KeyMath-R/NU and equation solving probes at pre- and post-test only may not have been adequate to control for threats to internal validity in the within-subjects single-case design. Because SBI with and without story problem grammar instruction was not compared in this study, it is unclear that story problem grammar instruction provides an added benefit. Future research may seek to evaluate the differential impact of SBI with and without story problem grammar instruction.

In their 2008 experimental study, Fuchs et al. reconceptualized SBI as a means of providing preventative tutoring to students with mathematics and reading difficulties within the secondary tier of a response-to-intervention (RtI) model. RtI is defined by Vaughn and Fuchs (2003) as a three-tier model for identifying and remediating learning problems. The primary tier includes implementation of research-based general instructional programs and screening of all students by the general education teacher to identify those at-risk of learning problems. These at-risk students then receive short-term small-group tutoring accompanied by progress monitoring within the secondary tier. Students who fail to respond to this intensive tutoring are referred for special education evaluation as part of the tertiary tier, while those who respond are returned to the primary tier.

Students with mathematics and reading difficulties were randomly assigned to either continue in their general education curriculum using a math basal text or receive SBI. In this study, SBI included instruction in the three additive problem schema as well as additional schema-broadening instruction, which involved instruction in four superficial transfer features (e.g. includes irrelevant information, uses 2-digit operands, has missing information in the first or second position, presents information in charts, graphs, or pictures). Results indicated statistically significant differences and large effect sizes favoring students who received SBI over those who stayed in the general curriculum on problem solving measures.

The results of this study suggest that SBI may have potential as a secondary intervention for struggling students within and RtI model. However, the use of one-to-one tutoring rather than small group tutoring is a limitation because within RtI secondary preventative interventions are generally conducted in small groups of three to six students (Vaughn & Fuchs, 2003). Also, the lack of a maintenance measure makes it uncertain whether the tutoring intervention led to continued success in the general curriculum, which the authors suggest is the goal of secondary preventative tutoring within the RtI framework. Furthermore, because SBI with and without schema-broadening instruction was not compared, it is not clear that the additional instruction in superficial transfer features led to improved outcomes. Future research may seek to evaluate the differential impact of SBI with and without schema-broadening instruction.

The additional features added to SBI in these studies have important implications for working with students with autism. For instance, research has shown that students with autism benefit from story grammar instruction and cues to support reading comprehension (O'Connor & Klein, 2004). It is therefore possible that instruction in story problem grammar and the use of cue cards to help students identify relevant information within word problems may be helpful to students with autism. It is also possible that this instruction may facilitate algebraic reasoning and operation choice for students with autism. However, the use of a part-part-whole diagram to represent all additive word problems may provide less semantic support for students with autism than the provision of schematic diagrams particular to each problem type. Students will also benefit from explicit instruction for generalization (Cowan & Allen, 2007) and may therefore benefit from instruction on transfer features.

SBI for Individuals with Developmental Disabilities

Despite the evident promise of using SBI to improve additive word problem solving performance, only three such studies have been identified that included participants with significant disabilities. In the first, Neef, Nelles, Iwata, & Page (2003) used a behavioral approach to teach two males ages 19 and 21 with below average intellectual abilities to solve change word problems. Although the authors did not identify their study as using SBI, their intervention included the critical components SBI, including direct instruction, use of diagrams to represent problems, and use of a problem type from the theoretical literature. Their study was unique in that it addressed precurrent skills for problem solving. Precurrent skills are the underlying skills that an individual must successfully and simultaneously execute in order to complete a complex task successfully, in this case identifying the initial value, change value, operation, and ending value in change problems. The participants received instruction on each precurrent skill separately before being instructed in solving the problem. A multiple baseline across behaviors single-case design was used wherein each precurrent skill represented a separate behavior. Probes were conducted at baseline and following training on each precurrent skill.

Results of the Neef et al. (2003) study indicated that both participants improved in their ability to identify components of change story problems from baseline to post-training on each component. In addition, following training on all precurrent skills, both participants improved in their ability to solve change problems. These results suggest that teaching students with developmental disabilities to identify the component parts of word problems could effectively improve their ability to solve novel problems. However, because only two participants were included in this study, causality can be inferred, but generalization is limited due to a lack of replications. Additional research will be needed to verify the results of these findings, determine whether problem solving gains are maintained over time, and extend these results to school-aged children with developmental disabilities.

In another study, Jitendra et al. (2010) conducted a case study to illustrate the use of SBI with students with emotional behavioral disorders (EBD). Their case study included a fourth-grade student and a fifth-grade student. Both of these students were diagnosed with EBD, were attending a special school for students with behavioral problems, were struggling in mathematics, and were receiving mathematics instruction from the same special education teacher. SBI in this study included a problem solving heuristic, addressed one- and two-step problems, and was delivered daily by the special education teacher over a 20-week period. Students were administered experimenter developed word problem solving pre- and post-tests, and word problem solving fluency probes. Both students improved their performance on word problem solving fluency probes as instruction progressed. In addition, both students showed gains from pre- to post-test. The authors hypothesized that SBI was effective with these students with EBD because it focuses on conceptual understanding and links procedures to that understanding, involves systematic instruction, and uses continuous progress monitoring to inform instruction.

Although the results of this case study provide some insight into the use of SBI with students with EBD, they should be interpreted with caution. Because of the lack of experimental control in a case study, it is not clear that improvements noted during and following SBI were due to the intervention and not to a confounding variable such as maturation. In addition, the inclusion of only two participants limits the generalizability of the findings to other students with EBD. Additional research using larger numbers of students with EBD and using more rigorous designs will need to be conducted to provide convincing evidence of the efficacy of using SBI with this population.

The third study was conducted by Rockwell et al. (2011) to evaluate the efficacy of using SBI for improving the additive word problem solving performance of a fourth-grade student with autistic disorder and low average intellectual abilities who was fully included in the general education classroom, but was performing below grade level in mathematics. SBI in this study used highly representative diagrams paired with a four-step problem solving mnemonic for self-monitoring. In addition, problem types were taught separately and instruction in each problem type included problem schemata instruction and then problem solving instruction. Results of a multiple probes across problem types design indicated that the participant improved word problem solving performance across all three additive problem types when unknowns were in the final location. These gains were maintained over time and were generalized to problems requiring algebraic reasoning to solve for an unknown quantity not located in the final position; however, the participant demonstrated greater difficulty using algebraic reasoning to solve change problems. The authors hypothesized that this may have been due to the less representative nature of using a single diagram to represent change-get-more and change-get-less problems.

As in the Jitendra et al. (2010) study, there are several limitations to this study. The inclusion of only one participant limits the generalizability of the findings. In addition, the use of a multiple probes across behaviors design may not have been optimal because solving the three types of additive word problems were related behaviors. As a result, the participant did increase her performance on change problems slightly following instruction in solving group problems and using self-monitoring. Finally, because this study included only single-step addition and subtraction story problems, it may not adequately represent the range of problem solving tasks found in the general education curriculum.

These three studies have important implications for using SBI to teach additive word problem solving to students with autism. Results of the Neef et al. (2003) study suggest that individuals with developmental disabilities, and perhaps students with autism, may benefit from a behavioral approach to problem solving instruction that involves task analysis and discrete training in precurrent skills. The Jitendra et al. (2010) case study indicates that students with behavior problems may benefit from the systematic instruction and continuous progress monitoring included in SBI. This is particularly promising given that the behavioral problems exhibited by many students with autism can make it difficult for them to benefit from classroom instruction (Mesibov & Howley, 2003). The Rockwell et al. (2011) study provides preliminary evidence that SBI may be an effective intervention for students with autism. In addition, it suggests that using highly representative schematic diagrams and a self-monitoring mnemonic may be beneficial for students with autism. Furthermore, this study provides some evidence that teaching each problem type separately and instructing for schema induction and problem solving separately may be effective with students with autism. Finally, it is possible that teaching change-get-more and change-get-less problems separately using more highly representative diagrams would help facilitate algebraic reasoning.

Synthesis of Schema-Broadening Research

According to Fuchs, Fuchs, Hamlett, and Appleton (2002), schema-broadening instruction involves teaching the problem-solution rules for problem types commonly found in the third-grade mathematics curriculum using verbal representations of the problem-solution schema. Four problem types were obtained through discussions with teachers and a review of the basal mathematics textbook. These include shopping list, buying bags, half, and pictograph problems. These problem types include single- and multi-step problems and both additive and multiplicative operations. Schema-broadening instruction also involves explicit instruction in superficial problem features (e.g. different format, different keyword, additional question, larger problem solving context) and transfer.

Fuchs and colleagues (2002; 2003a; 2003b; 2004a, 2004b, 2004c, 2006) have conducted a line of research using quasi-experimental designs to evaluate the efficacy of using schema-broadening instruction to improve the mathematics word problem solving performance of third-grade students. These studies included a total of 1,973 participants ranging from typically-achieving students to students with mild disabilities. The studies compared schema-broadening instruction delivered by researchers or teachers using scripted lessons to instruction delivered by teachers using the basal textbook, and one study in which computer assisted instruction was used (Fuchs et al., 2002). They also explored the relative contribution of both schema instruction and transfer instruction (Fuchs et al., 2003a; Fuchs, Fuchs, Finelli, Courey, & Hamlett 2004c). In addition, schema-broadening instruction has been combined with self-regulated learning strategies (Fuchs, Fuchs, & Prentice, 2004b; Fuchs et al., 2003b), problem sorting activities (Fuchs et al., 2004a), and real-life problem solving strategy instruction (Fuchs et al., 2006).

Problem solving performance in these studies was assessed using researcher-developed outcome measures. These outcome measures assessed immediate transfer using problems like those used for instruction, near transfer using problems with varying superficial features, and far transfer using real-life problem solving contexts that combine multiple problem schema and varying superficial features. In all studies, statistically significant effects favoring schema-broadening instruction over comparison groups were found. Effect sizes were often large, sometimes surpassing three standard deviations. In addition, the inclusion of specific instruction on transfer features resulted in greater gains than did instruction in problem schema alone. Furthermore, self-regulated learning strategies, problem sorting activities, and real-life problem solving strategy instruction all were found to enhance the efficacy of schema-broadening instruction.

Overall, the findings of the seven studies on using schema-broadening instruction indicate that this intervention holds promise for enhancing students' mathematical problem solving skills. The use of problem types found in the general curriculum improves the external validity of the intervention. Also, the use of measures with increasing demands for transfer mirrors the types of tasks that students will be exposed to in the general education curriculum. Moreover, the use of primarily intact, heterogeneously grouped classrooms demonstrates the feasibility of conducting the schema-broadening intervention in general education classrooms. The studies also indicate that schema-broadening instruction can be implemented with high treatment fidelity by both researchers and teachers, particularly when teachers are provided with scripted lessons. Finally, schema-broadening instruction has been used effectively in a variety of settings. It has been used with students with disabilities in pull out settings, with at-risk students in pull out settings, and with students of varying ability in inclusive settings.

Although these studies demonstrate that schema-broadening instruction has great potential, there are limitations that merit discussion. For instance, although using problems drawn from the general education curriculum enhances the external validity of the intervention, the way these problems were grouped for instruction is not consistent with schema theory which specifies specific problem types within additive and multiplicative formats (Christou & Philippou, 1999). The theoretical basis of the intervention would be strengthened if existing schema theory were used to categorize problem types from the general education curriculum. In addition, because only researcher-developed assessments were used, it is unclear whether the effects of the intervention would generalize to classroom assessments or standardized assessments used to document AYP. The value of schema-broadening instruction would be more evident if standardized measures were used in addition to researcher-developed measures.

Limitations aside, the research on schema-broadening instruction has implications for working with students with autism. The inclusion of explicit transfer instruction may be particularly beneficial for students with autism who typically have difficulty generalizing skills and require explicit instruction to do so (Cowan & Allen, 2007). However, the lack of maintenance assessments in all of these studies is concerning, particularly when considering that students with autism tend to have difficulty with maintenance as well as generalization (Kendall, 1989). In addition, the use of verbal instruction and verbal representations of problem schema may be less effective for students with autism who tend to benefit from the use of visual supports (Mesibov & Howley, 2003; Tissot & Evans, 2003).

Implications for Future Research

Although there are several studies providing evidence to support the use of SBI to improve the additive word problem solving performance of students with high incidence disabilities and students without disabilities, additional research is needed to establish SBI as an evidence-based practice for teaching mathematical word problem solving to children with developmental disabilities. Future studies should seek to replicate the results of the Rockwell et al. (2011) study using larger numbers of participants and more rigorous designs. In addition, information from the Rockwell et al. study can be used to develop and evaluate more effective SBI procedures for use with students with autism. Finally, methods that have been used in SBI studies that may hold promise for students with autism should be evaluated with this population.

This study seeks to address some of the identified limitations in the research base by using a multiple probes across behaviors design replicated across participants to ensure internal validity, assessing generalization to problems with irrelevant information, and assessing maintenance of treatment effects over an extended period of time to improve social validity. The proposed study will also improve upon the procedures used in the Rockwell et al. (2011) study by teaching students to adapt the *change* diagram to reflect the larger beginning amount in a “get less” situation and the larger ending amount in a “get more” situation as in the Willis and Fuson (1988) study. In addition, explicit instruction for generalization will involve teaching students to use the part-part-whole structure of the word problems as an aid to algebraic reasoning when determining which operation to use as in the Xin et al. (2008) study. Generalization instruction will also address superficial transfer features like those included in the Fuchs et al. (2008) study (i.e. includes irrelevant information, has the unknown in the initial or medial position).

CHAPTER 3
METHOD

Participants

Participants were recruited through the Center for Autism and Related Disabilities (CARD). Initially, families of six students with autism agreed to allow their children to participate, however, only two students met the inclusion criteria. These two study participants will be referred to by the pseudonyms Daniel and Justin.

Inclusion and Exclusion Criteria

In order to be included in this study, students had to be in grades one through six, have a diagnosis of autistic disorder as indicated in a report from a qualified professional, and have nonverbal cognitive abilities within two standard deviations of the mean as indicated in a report from a qualified professional. They also had to participate in grade-level Annual Yearly Progress (AYP) assessments as indicated by their Individual Education Programs (IEPs), but be performing below grade level on mathematics as measured by previous AYP assessments or other standardized tests of mathematics achievement. Students' IEPs also had to include goals or objectives addressing mathematics problem solving. Furthermore, students had to be able to decode at a second grade reading level and be able to perform addition and subtraction computations involving two digit numbers without regrouping with at least 90% accuracy as assessed using curriculum based measures of second-grade oral reading accuracy and addition and subtraction computation accuracy obtained from the Basic Skill Builders Series (Beck, Conrad, & Anderson, 1997). Students were excluded from participation in this study if they had a concomitant diagnosis of mental retardation, were unable to communicate verbally, or were unable to attend to one-to-one group instructional sessions lasting 30 minutes.

Participant Information for Daniel

Daniel is a seven-year-old male who has a diagnosis of autistic disorder from a developmental behavioral pediatrician. A school psychologist employed by the local school district conducted his most recent psychoeducational evaluation in March 2009. At that time, Daniel's behaviors were so severe that a valid assessment of his intellectual and academic abilities could not be completed. Daniel's IEP includes a goal to address his behavioral issues, including attending to classroom instruction and activities, remaining in his seat, refraining from talking in class without permission, and transitioning from one activity to another with the rest of the class.

Daniel attends a regular education first grade class where he is assisted by a paraprofessional and participates in regular assessments with accommodations. He also receives language therapy and occupational therapy each for 30 minutes each week. According to Daniel's classroom teacher, classroom assessments, and his IEP, he is performing at or above grade level in most areas of the curriculum; however, he struggles with mathematics word problem solving and written expression. Daniel's IEP includes a goal related to answering open-ended questions. Objectives for this goal address oral language, solving mathematics word problems, answering reading comprehension questions, and responding to writing prompts. At the time data collection was started, Daniel's class had just completed a chapter on addition and subtraction that included addition and subtraction word problem solving. Word problem solving instruction consisted primarily of associating keywords with operations. On the chapter test that included multiple choice computation questions and open-ended word problem solving tasks, Daniel scored a 90%; however, item analysis revealed that he responded incorrectly to all of the word problems. On screening assessments obtained from the Basic Skill Builders Series (Beck et al., 1997) and conducted as part of this study, Daniel read a second-grade passage with 98% accuracy and performed two-digit addition and subtraction without regrouping with 100% accuracy.

Daniel is a cheerful and friendly boy. He loves to watch Disney Pixar movies and enjoys reenacting scenes from them using toys. He also enjoys drawing and coloring. He draws pictures of the characters from his favorite movies, recreates visual displays that can be found in his classroom, and writes in various fonts when completing schoolwork and homework. He has an acute visual memory and is able to duplicate pictures he has seen only a few times with great detail. Daniel is also able to benefit from verbal instructions. He often repeats things that his teacher has taught him in class while completing his homework. He is able to follow one- and two-step verbal instructions. Although Daniel uses speech as his primary means of communication, his speech is often formulaic and echolalic. He is able to make his wants and needs known, but struggles to answer even simple questions unless he is given choices.

Participant Information for Justin

Justin is a 12-year-old male who has a diagnosis of autistic disorder from a developmental behavioral pediatrician. A team of evaluators at the University of Florida conducted his most recent psychoeducational evaluation in July 2011. Results of that evaluation indicated that Justin’s nonverbal cognitive abilities are within the low average range while his verbal cognitive abilities are within the impaired range. On executive functioning measures, Justin performed in the average range on tasks assessing sustained attention, inhibition of competing behaviors, and set shifting. However, he performed in the impaired range on tasks assessing verbal working memory. His ability to perform mathematics calculations correctly and fluently falls within the average range. However, his ability to solve applied problems falls within the impaired range.

Justin attends regular education sixth-grade classes for language arts, mathematics, science, and social studies. He also attends a special education reading class, has a study hall period, and participates in regular assessments with accommodations. He is assisted throughout the school day by a paraprofessional whom he shares with another student with autism. He also receives language therapy for 60 minutes each week and receives consultative services from an occupational therapist. On previous assessments of AYP, Justin performed below grade level in both reading and mathematics. Justin's IEP includes goals related to reading comprehension and mathematics problem solving. On screening assessments obtained from the Basic Skill Builders Series (Beck et al., 1997) and conducted as part of this study, Justin read a second grade passage with 97% accuracy and performed two-digit addition and subtraction without regrouping with 100% accuracy.

Justin is a kind, gentle boy who is eager to please. He smiles and claps his hands when he is told that he has done something well. He enjoys playing Wii Sports with his older brother, swimming in the family's pool, and cuddling with his cat. He also enjoys having predictable routines in his life and especially enjoys school because of its structure. Justin is able to benefit from verbal instructions and is able to follow multi-step verbal directions. His primary means of communication is speech and he is able to make his wants and needs clearly known. He is also able to answer open-ended questions if given ample time to formulate his response. Although Justin communicates verbally, he is often frustrated by his inability to quickly formulate responses to questions. When he becomes frustrated he sometimes cries and says that he “can’t do it” or “can’t talk right” and requires a break to regain his composure. Despite requiring these breaks, Justin is reluctant to take breaks when they are offered. He seems to want to complete his tasks quickly and without breaks and please the adults with whom he is working.

Independent Variable

Setting

All instruction took place during 30-min. sessions conducted Monday through Friday from 3:00-3:30 PM at the dining room tables in the participants’ homes. The author conducted all instruction with individual participants. For Daniel instruction was conducted after he had completed his school day and finished his assigned homework. For Justin, instruction was conducted after he had finished participating in various activities during his summer vacation including a part-time summer-school session, language therapy, occupational therapy, and a church-based summer camp.

Treatment

Lesson checklists and scripts were developed based on those used in previous SBI studies (Griffin & Jitendra, 2009; Rockwell et al., 2011) to ensure that all instructional components were addressed during each session. Lessons included an average of 8 (range 7-10) additive story situations or word problems. Lessons that involved problem sorting included an additional 6 word problems for sorting.

Behavioral consequences were an important component of the treatment. Although the specific behavioral consequences employed were individualized for each participant, some behavioral consequences were employed universally. Verbal praise such as “I like how you’re working” or “you’re working so nicely” was provided intermittently for on task behavior and for problem completion. In addition, participants were allowed up to two, 3-minute breaks during each instructional session. Breaks were provided non-contingently, but only at the participants’ request. When participants paused in their work or in answering a question for longer than a 5-second wait-time, the researcher gestured toward visual supports of the RUNS steps or schematic diagrams to prompt the participant while saying “what do you need to do next?” or “what goes here?”. When a participant completed a schematic diagram incorrectly or performed a computation incorrectly during guided or independent practice, the researcher prompted him to check his work.

For Daniel, whose behaviors were more severe, food items were paired with verbal praise to reinforce on-task behavior on a variable schedule. Additional behavioral accommodations included the use of an air disk to allow Daniel to move while staying seated, and proximity and touch to encourage on-task behavior. Daniel sometimes asked to sit in the researcher’s lap while completing work. He also requested that the researcher place a hand firmly on his back or shoulder while he was working.

Although Justin did not need additional reinforcement for on-task behavior, he often needed additional reassurance about his performance. When Justin became agitated as evidenced by whining, crying, or statements like “it’s too hard” or “I can’t do it,” the researcher would offer additional verbal reinforcement such as “you’re doing great” or “you can do it” or “you’re doing very well.” These phrases were often paired with a pat on the shoulder or back.

Treatment was divided into three phases with each phase addressing one of the three problem types identified by Carpenter and Moser (1984). Word problem types were addressed in this study based on the sequence proposed by Christou and Philippou (1999): first group, then change, and finally compare problem types. Prior to the start of instruction, baseline problem solving and generalization probes were conducted. Instruction was initiated after stability of data had been established on baseline probes.

The SBI intervention procedures used in this study were based on those used in previous SBI studies (e.g., Jitendra et al., 2007a, Jitendra et al., 2007b, Rockwell et al., 2011). Prior to beginning the SBI, the participant learned a four-step procedure for problem solving adapted from that used by Jitendra et al. using the mnemonic RUNS. The RUNS steps consist of: (1) Read the problem, (2) Use a diagram, (3) Number sentence, and (4) State the answer. The researcher taught the participant to write the RUNS mnemonic prior to solving a problem, and to check off the steps as he completed them. The researcher modeled writing the RUNS mnemonic and checking off the problem solving steps throughout the teacher modeling portions of SBI. The researcher also instructed the participant to write the RUNS mnemonic and check off the problem solving steps during the guided practice portions of SBI, and reminded him to engage in these behaviors as needed during independent practice. The researcher included the RUNS mnemonic to encourage the participants to self-monitor their problem solving behavior; which can benefit students with autism by increasing desirable behaviors and decreasing competing behaviors (Shabani et al., 2001).

As in previous SBI studies, this study used direct instruction (Rosenshine & Stevens, 1986) consisting of modeling with think-alouds, guided practice, and independent practice throughout the SBI. Modeling with think-alouds involved the researcher in solving problems for the participant while telling him the thought processes she used to solve the problems (Examples of researcher think-alouds are found in the sample lesson script in Appendix B). Guided practice involved the participant in solving problems while the researcher verbally guided him through the steps and thought processes required to arrive at a correct answer. The researcher gradually reduced the amount of verbal guidance she provided as the participant demonstrated his ability to solve the problems more independently. Independent practice involved the participant in problem solving with the researcher providing feedback regarding any missed steps after he had solved each problem.

The researcher introduced and taught each problem type separately starting with group problems, moving on to change problems, and finally teaching compare problems. Instruction in solving each problem type first involved instruction using story situations in which all components of the problem consisted of known quantities in order to facilitate schema induction. The researcher used the direct instruction sequence to teach each participant to identify the critical components of each type of word problem. For group problems this involved teaching participants to identify the two distinct parts and the all obtained from putting those parts together. For change problems, the researcher taught participants to identify the beginning, the change, and the action word that indicates whether this change is “get more” or “get less,” and the ending obtained. For *compare* problems, the researcher taught each participant to identify the larger amount, the smaller amount, the comparative word or phrase used to compare them, and the difference obtained. The researcher then taught participants to put the critical components into the diagram for the problem type being taught. When teaching each participant to use the change diagram, the researcher also taught him to adapt the sizes of the ending and beginning boxes based on whether the action word indicated “get more” or “get less.” This was intended to help participants visually represent the structure and the relationship between addition and subtraction in change problems. According to Van de Walle, Karp, and Bay-Williams (2010), understanding the relationship between addition and subtraction is critically important conceptual knowledge for solving additive word problems. If the action word indicated “get more” the researcher taught the participant to make the ending box larger, but if the action word indicated “get less” she taught the participant to make the beginning box larger. This allowed the change diagram to clearly represent which quantity in the problem was the largest making the relationship among the quantities more visually evident. Next, the researcher taught the participants to translate their diagrams into number sentences, write the answer, and label that answer with a noun. For example, if a group story situation said, “John picked 12 tomatoes and 15 cucumbers. John picked 27 vegetables,” the participants were taught to write the answer 27 and label it with the noun, vegetables. The researcher then exposed each participant to story situations representing all three problem types and helped him to practice determining whether or not each problem fit the schema being currently taught, one of the previously taught schemas, or a schema that had not been taught yet. These sorting activities were based on those used in the Fuchs et al. (2004a) study to help reinforce discrimination of the schema types. Once each participant demonstrated schema induction on formative assessments, the researcher applied the direct instructional sequence to teaching that participant to solve for an unknown all, ending, or difference using problem situations. Once the participant had demonstrated his ability to solve problems of the type being taught on formative assessments, the researcher initiated a probe phase. When stability of data on probes had been established, the researcher began the next phase of instruction by teaching the next problem type following instruction in solving group and change problems or by initiating generalization probes to assess for spontaneous generalization following instruction in solving compare problems.

The researcher initiated generalization instruction for Daniel, the participant who did not demonstrate spontaneous generalization on generalization probes. Generalization lessons were not necessary for Justin because he performed at ceiling on generalization probes, indicating spontaneous generalization. Generalization instruction for Daniel included one lesson each on generalizing to solving problems with unknowns in the initial position, solving problems with unknowns in the medial position, and solving problems with irrelevant information. These were brief, 15-minute lessons wherein the researcher modeled the use of schematic diagrams with generalization problems. The researcher presented six generalization problems and modeled how to solve these problems using think-alouds. Guided practice and independent practice were not included in the generalization mini-lessons. The researcher administered a generalization probe immediately following each lesson. Had Daniel not demonstrated mastery on this probe, the researcher would have initiated a full generalization lesson with guided and independent practice. This was not necessary given Daniel’s performance on generalization probes following each mini-lesson The researcher conducted a final set of problem solving and generalization probes with each participant 8 weeks later to assess maintenance of treatment effects over time.

Treatment Integrity

Analysis of the fidelity of independent variable implementation is a desirable component for single-subject research in special education (Horner et al., 2005). All treatment sessions were video taped. Two doctoral students familiar with SBI directly observed (i.e., Doctoral Student T) or viewed tapes (i.e., Doctoral Student J) of 32% of lessons within and across phases and participants. These doctoral students used detailed lesson checklists to evaluate the percentage of instructional components addressed during the sessions. A sample checklist can be found in Appendix C. According to Peterson, Homer, and Wonderlich (1982), ideal assessment of treatment integrity occurs continuously and accounts for problems with observational data such as reactivity and observer drift. Assessing treatment integrity through direct observation ensured that fidelity of implementation was continuously assessed and that lack of fidelity was addressed promptly. The additional assessment of treatment integrity using videotapes ensured that high levels of treatment integrity were not obtained due to reactivity.

Dependent Variable

Researcher-developed materials were used to assess problem solving progress throughout this study. Practice sheets were used as a formative assessment of student progress during instruction. Problem solving probes were used to evaluate treatment effects using a multiple probes across behaviors design with the solving of each problem type (i.e., change, group, and compare) treated as separate behaviors. This design was replicated across two participants.

Formative Assessment

During training on each problem type, the researcher assessed participant progress using a formative assessment that was administered at the end of each treatment session at the table where instruction was conducted. These practice sheets consisted of two problems of the type taught. During story situation instruction, practice sheets included story situations in which all three quantities were known. During problem solving instruction, practice sheets included problem situations in which the final quantity was unknown. Each of the problems was worth a possible three points: one point for using the correct schematic diagram, one point for writing the correct number sentence to solve the problem, and one point for correctly computing the answer. These scoring procedures were derived from those used by Jitendra and Hoff (1996) and Willis and Fuson (1988). The researcher used student scores on practice sheets administered at the end of each training session for making decisions about the pace of instruction. Only after the direct instruction sequence had been completed and a participant had earned all six possible points on at least one practice sheet did the researcher progress from story situation to problem solving instruction or from problem solving instruction to a probe phase. Appendix D contains sample practice sheets for each problem type with hypothetical student responses and scores.

Probes

To evaluate the effectiveness of SBI, problem solving and generalization probes were administered to participants at baseline, following instruction on each problem type, following generalization instruction, and eight weeks later to assess maintenance of treatment effects. Baseline probe sessions and probe sessions following instruction on all three problem types were video taped so that participants' problem solving behaviors before and after SBI could be analyzed and compared. During each probe phase, the researcher administered as many problem solving probes as were necessary to obtain stability of data as well as one generalization probe addressing each type of generalization task to assess for spontaneous generalization. During probe phases, no training sessions took place. Instead, the participants completed one problem solving probe and one generalization probe during the time scheduled for training sessions.

Participants were given the following instructions prior to beginning each probe: "Here are some math word problems I want you to solve by yourself. Remember to use what you have learned to solve these problems. If you have trouble reading a problem, let me know and I will help you. Do your best." Participants were given up to 20 min. to complete each probe. Each of the problems was worth a possible two points. One point was awarded for writing the correct number sentence to solve the problem. Assuming this number sentence was correct, an additional point could be earned for correctly computing the answer. These scoring procedures are derived from those used by Jitendra and Hoff (1996) and Willis and Fuson (1988). See Appendix E for sample problem solving and generalization probes with hypothetical student responses and scores.

Problem solving probes. Problem solving probes consisted of nine-item experimenter-developed tests. These probes were adapted from probes used in a previous SBI study (Rockwell et al., 2011). Each problem solving probe presented three story problems from each problem type for a total of nine problems. Problem solving probes included story problems with the unknown quantity located in the final position in the schematic diagram: the all in group problems, the ending in change problems, and the difference in compare problems. Generalization probes also consisted of three story problems from each problem type for a total of nine problems. These probes include items used in generalization probes in the Rockwell et al. study. Because three generalization tasks were targeted in this study, three types of generalization probes were developed.

Generalization probes. Generalization probes assessing generalization to problems containing irrelevant information included problems with unknowns in the final position but provided text-based information on a fourth, irrelevant quantity. Generalization probes assessing generalization to problems with the unknown in the initial position involved story problems with the unknown quantity located in the initial position in the schematic diagram: the first part in group problems, the beginning quantity in change problems, and the larger quantity in compare problems. Generalization probes assessing generalization to problems with the unknown in the medial position included story problems with the unknown quantity located in the medial position in the schematic diagram: the second part in group problems, the change quanitity in change problems, and the smaller quantity in compare problems.

Reliability and Validity Evidence

According to Crocker and Algina (2006), it is desirable to use measurement instruments that have evidence of producing reliable scores and are valid assessments of the intended construct.

Inter-rater agreement. The researcher scored all of the probes and practice sheets. Two other doctoral students who were familiar with SBI independently scored 30% of the probes (i.e., Doctoral Student M) and 30% of the practice sheets (i.e., Doctoral Student J) within and across phases and participants. Doctoral Student M was blind to the phases from which probes were sampled. Agreement was calculated on a point-by-point basis such that each probe could have a maximum of 18 agreements and each practice sheet could have a maximum of six agreements. The total number of agreements was divided by the total number of agreements and disagreements and then multiplied by 100 to obtain the percent agreement.

Equivalent forms reliability. The problem solving forms were pilot tested with a fourth-grade student with autism (Rockwell et al., 2011). Because differences between the probes would have led to instability of the data, the stability of the data from the Rockwell et al. study provide some evidence that the probes yielded equivalent scores.

Validity evidence based on test content. Dr. Cynthia Griffin, an expert in SBI, reviewed the problem solving probes to ensure that the problems contained the critical features of the schema intended.

Validity evidence based on response processes. Results of the Rockwell et al. (2011) study indicated that a fourth-grade student with autism was able to respond to problem solving probes and accompanying instructions as intended. This student, who read at a second-grade level, was able to read the problems included on the probes, identify the corresponding problem types following SBI, and respond to the problems in the space provided.

Data Collection and Analysis

A single case, multiple probes across behaviors design was used in this study (Kennedy, 2005). The researcher chose a multiple probes design because the use of probe sessions rather than continual assessment of the dependent variable minimizes test fatigue. In addition, the multiple probes design allows for the demonstration of experimental control following an instructional intervention that cannot be easily removed for a return to baseline. The length of baselines and implementation of the intervention was staggered across the three problem types for a total of three demonstrations of experimental control per participant and six total demonstrations of experimental control. Data from probes and practice sheets were graphed following each training or probe session to facilitate decision-making. Instructional phases were initiated only after stability, trend, and level were established on all previous baselines.

In addition, participants were videotaped while completing probes at baseline and following instruction on all problem types. The researcher viewed these tapes and problem solving behaviors (e.g., reading the problem aloud, underlining parts of the problem, using diagrams or drawings to solve the problem) were coded. A second doctoral student viewed 46% of these tapes within and across probe phases and coded behaviors as a validity check. Problem solving behaviors observed during probe sessions conducted at baseline were compared to those conducted following SBI to determine if and how participants' problem solving behaviors changed following SBI.

Finally, participants and their parents completed Likert-type satisfaction questionnaires during the week maintenance probe sessions were conducted to assess the social validity of the SBI (Kennedy, 2005). Samples of these questionnaires can be found in Appendix F.

CHAPTER 4
RESULTS

Treatment Integrity and Inter-Rater Reliability

Doctoral Student T directly observed 7% of lessons to assess treatment integrity. Treatment integrity for these lessons averaged 98.5% (range 97% to 100%). Doctoral Student J viewed videotapes of 25% of lessons. Treatment integrity for these lessons averaged 98% (range 94% to 98%). A total of 32% of lessons were assessed for treatment integrity. Overall treatment integrity averaged 98% (range 94% to 100%).

Doctoral Student M scored 30% of probes within and across participants and phases. Inter-scorer reliability as assessed on a point-by-point basis averaged 99% (range 88% to 100%). Doctoral Student J scored 30% of practice sheets within and across participants and phases. Inter-rater reliability as assessed on a point-by-point basis was 100% for all probes scored. All discrepancies in scoring were resolved through discussion.

Break Use

During the first instructional session, the researcher offered Daniel two break cards. He used both of these breaks. During the breaks, the researcher set a timer for three minutes and engaged Daniel in stretching activities. Daniel engaged in these activities willingly, but then took several minutes after the break ended to transition back to the table to work. Daniel's mother suggested that Daniel not be given breaks. She explained that he prefers to complete all of his work quickly and then have a long period of free time. She also explained that Daniel has great difficulty transitioning back to work after free time making short breaks more problematic than helpful. After that session, the researcher did not offer Daniel breaks. He did ask to use the restroom during two additional sessions and was given a break to do so. He was able to transition back to work after using the restroom without difficulty.

Justin was given two break cards at the beginning of each session, but rarely used these breaks. He requested breaks during 6 instructional sessions. During these breaks, the researcher set a timer for 3-minutes and engaged Justin in stretching activities. Justin willingly participated in these activities and smoothly transitioned back to work when the breaks were finished. He used both break cards during five of the six instructional sessions in which he requested breaks. The sessions during which Justin asked for breaks all occurred following a day at summer school.

Daniel's Problem Solving Performance

During the first set of baseline probes, Daniel was unable to solve any of the problems on problem solving or generalization probes. For each problem he wrote two number sentences that were members of the same single-digit fact family. Because Daniel's handwriting is large, two number sentences filled the space allotted for student work beneath each word problem. Daniel appeared aware that completing the task required him to use mathematics. He also seemed to make an effort to finish the task in an acceptable manner.

Group Problems

Daniel responded quickly to instruction in solving *group* problems. He immediately performed at ceiling on practice sheets. On problem solving probes following instruction in solving group problems, he was able to correctly solve all group problems. His average performance on group problems improved from 0/6 at baseline to 6/6 following instruction on solving this problem type.

Change Problems

Following instruction on solving group problems, Daniel's performance on change problems improved from an average of 0/6 to 1.33/6. This improvement was the result of overgeneralization of the group schema to change problems. Daniel used the group diagram to solve change problems. This allowed him to inadvertently obtain correct number sentences and answers for change problems wherein the action indicated "get more" and a larger ending quantity. For instance, Daniel correctly solved the problem "There are 23 pigs on the farm. If 15 pigs are born, how many pigs are on the farm now?" by using a group diagram rather than a change diagram. Although he used the incorrect diagram, he obtained a correct answer because the problem required addition as would a group problem with an unknown all.

Daniel responded quickly to instruction in solving change problems. He immediately performed at ceiling on practice sheets. On problem solving probes following change instruction he was able to correctly solve all but one of the change problems. His average performance on change problems improved from 0.67/6 at baseline to 5.67/6 following instruction on solving change problems.

Compare Problems

Although Daniel showed the same overgeneralization of the group problem schema to compare problems as he did to change problems, this did not allow him to obtain correct number sentences or answers to compare problems. As a result his ability to solve compare problems did not improve following instruction in solving group problems. However, Daniel demonstrated some improvement in his ability to solve compare problems following instruction in solving change problems (from an average of 0/6 prior to change instruction to 1.33/6). Daniel was not observed using group or change diagrams when solving these problems. Instead, he repeated the words “more than” aloud. A discussion with Daniel’s mother revealed that she had been working with Daniel on studying for a unit test in mathematics and had been reviewing key words with him. The classroom teacher had taught Daniel that the key words “more than” meant to subtract and his mother had been reinforcing this with him. It is therefore possible that the improvement in compare problem solving performance was due to classroom instruction in key words and review of that instruction at home. This keyword instruction likely did not result in Daniel correctly solving all compare problems correctly because many of the compare problems in probes did not use the keywords “more than” and instead used comparing phrases like “fewer than,” “less than,” or “older than.”

Daniel did not attain mastery during instruction as quickly with compare problems as he did with group and change problems. It appeared he had difficulty distinguishing between group and compare problem types, perhaps due to similarities between the group and compare diagrams. He repeatedly said “upside down group” when shown the compare diagram. The group and compare diagrams both consist of one large rectangle and two smaller rectangles. In the group diagram the larger rectangle is below the smaller rectangles; however, in the compare diagram the larger rectangle is above the smaller rectangles. If one were to flip the group diagram upside down it would look very similar to the compare diagram with the exception that all of the rectangles in the group diagram are solid whereas the rectangle representing the difference in compare problems is dashed. It is therefore not surprising that Daniel had some difficulty distinguishing between these diagrams and thought that the compare diagram was a group diagram that had been turned upside down. Daniel required an additional lesson with problem sorting activities and independent practice to demonstrate mastery on compare practice sheets, perhaps due to this apparent confusion,. Daniel's average performance on compare problems improved from 0.07/6 at baseline to 5.33/6 following instruction.

Generalization to Problems with Irrelevant Information

Spontaneous generalization to problems with irrelevant information was not observed. Following instruction on solving group problems, Daniel overgeneralized the group problem schema to all problems with irrelevant information. However, rather than excluding the irrelevant information, he treated these problems as story situations in which all of the components of the problem were known problems. He also tended to use the quantities in the order in which they were given in the problem. Following instruction on solving change problems, he began to apply the schema corresponding to the problem correctly; however, rather than excluding the irrelevant information he continued to treat these problems as story situations in which all of the components of the problem were known quantities. He also continued to place the quantities into the schematic diagram in the order in which they were given in the word problem without considering whether those quantities represented that particular component of the problem. This led Daniel to create number sentences that represented inequalities rather than equalities. For example, when given the problem, "Tim looks out the window. He counts 42 boys, 4 teachers, and 36 girls on the playground. How many children are on the playground?" Daniel used the number sentence 42+4=36. Sometimes Daniel did not seem to notice the disparity, but at other times he would try to change one of the quantities in the original number sentence or change the order in which quantities were placed into the diagrams to correct an inequality. For instance, when given the problem, “A scout troop leader is taking his troop of 28 scouts camping, but 5 scouts get sick. The campground can hold 23 tents. How many scouts are going camping?” Daniel first tried to change 28 to 18 so that the number sentence 18+5=23 would hold true. One exception to the lack of spontaneous generalization was seen in the generalization probe given during session 17, which read, “A school is taking 98 students to the museum. If 16 students get sick and it costs $5 per student to enter the museum, how many students are going to the museum?” The irrelevant quantity in this change problem was a monetary amount rather than a numeral. Daniel appeared to realize that this monetary amount did not apply and was able to correctly solve the problem.

Due to his lack of spontaneous generalization, Daniel was provided with one 15-minute lesson on solving problems with irrelevant information. During this lesson, the researcher modeled identifying the important information and underlining it and then crossing out any extra information. Following this lesson, Daniel’s performance on problems with irrelevant information improved to ceiling level for all problems types. This resulted in an average improvement from 0/6 to 6/6 for group problems, 0.5/6 to 6/6 for change problems and 0/6 to 6/6 for compare problems.

Generalization to Problems with Unknowns in the Initial Position

Following instruction in solving group problems, Daniel tended to overgeneralize the group problem schema with an unknown all to every problem. As a result, Daniel was unable to solve any group problems with the unknown in the initial position because he continued to add the given quantities together. For instance, when given the word problem, “Mr. Smith bought pants and a shirt. The shirt cost $15. If he spent $47, how much did the pants cost?” Daniel completed a group diagram using $15 and $47 as the parts. He then used the number sentence 15+47=52. However, this overgeneralization of the group schema with an unknown all to every problem allowed Daniel to inadvertently obtain the correct number sentence and answer for change "get-less" problems and for compare problems with the unknown in the initial position. For instance, when given the following change "get less" problem with an unknown beginning, "Many children ride the school bus home. At the first stop 7 children get off. Now there are 21 students on the bus. How many students ride the bus home?", Daniel used a group diagram with 7 and 21 as the two parts resulting in the correct number sentence and answer 7+21=28. When given the following compare problem with an unknown larger amount, "There are some pigs in the barn and 12 pigs outside in the mud. If there are 32 more pigs in the barn than in the mud, how many pigs are in the barn?", Daniel again used a group diagram with 12 and 32 as the two parts resulting in the correct number sentence and answer 12+32=44. This overgeneralization of the group schema with an unknown all allowed Daniel to obtain the correct answer to these problems by chance, not because he had an understanding of how to solve change or compare problems with unknowns in the initial position. As a result, Daniel's average performance on change and compare problems with the unknown in the initial position improved from 0/6 at baseline to 3.33/6 and 4.67/6 respectively following instruction on solving group problems. After one 15-minute lesson, during which the researcher modeled how to identify and solve problems with the unknown in the initial position, Daniel was able to correctly solve all problems on subsequent probes assessing generalization to problems with unknowns in the initial position. When comparing the probes assessing generalization to problems with unknowns in the initial position administered prior to generalization instruction to those administered after generalization instruction, Daniel's average performance improved from 0/6 to 6/6 for group problems, from 2.5/6 to 6/6 for change problems, and from 3.5/6 to 6/6 for compare problems.

Generalization to Problems with Unknowns in the Medial Position.

Spontaneous generalization to problems with the unknown in the medial position was not seen. Rather, Daniel tended to overgeneralize the group problem schema to these problems as though he was solving for an unknown all. This resulted in incorrect number sentences and answers for all problems. However, after one 15-minute lesson, during which the researcher modeled how to solve for an unknown in the medial position, Daniel was able to correctly solve all problems on subsequent probes assessing generalization to problems with unknowns in the initial position. His performance on group, change, and compare problems with the unknown in the medial position improved from 0/6 prior to generalization instruction to 6/6 following generalization instructions.

Maintenance

Maintenance probes were conducted 8 weeks after SBI was completed. Daniel's performance on group, change, and compare problems with unknowns located in the final position remained at ceiling level on these probes indicating that treatment effects from the SBI were maintained over time. He also demonstrated maintencance of treatment effects on generalization probes involving problems with uknowns in the initial or medial position as his performance remained at ceiling level save one computational error on these problems. Daniel's performance on problems involving irrelevant information did not remain at ceiling level as he crossed out relevant information and included irrelevant information on one change problem. However, his performance on these problems still remained well above what they were prior to generalization instruction.

Figure 4 is a graph of Daniel's performance on problem solving probes, generalization probes, and practice sheets. Data are graphed for each session so that trend and level can be visualized.

Daniel's Problem Solving Behaviors

Prior to receiving SBI, Daniel engaged in similar behaviors on all problem solving and generalization probes administered. On the first problem solving probe, Daniel responded to problems without reading them or waiting for the researcher to read them aloud. This behavior continued despite the researcher prompting Daniel to read the problems or wait for her to read them. As a result, the researcher held Daniel's pencil while reading the problem aloud to him beginning with the 6th problem on the first problem solving probe and continuing for all problems on subsequent problem solving and generalization probes conducted prior to SBI. After she had finished reading the problem, the researcher gave Daniel back the pencil so that he could show his work. The researcher made this accomodation in an effort to obtain a more valid assessment of Daniel's problem solving performance. Despite this accommodation, Daniel did not use the information provided in the problem prior to SBI. Instead, immediately after getting the pencil he quickly wrote horizontal number sentences, usually two number sentences from the same fact family for each problem. The majority of the time these fact families consisted of single digit numbers that were not included in the word problem just read. Daniel verbally recited the number sentences while writing them.

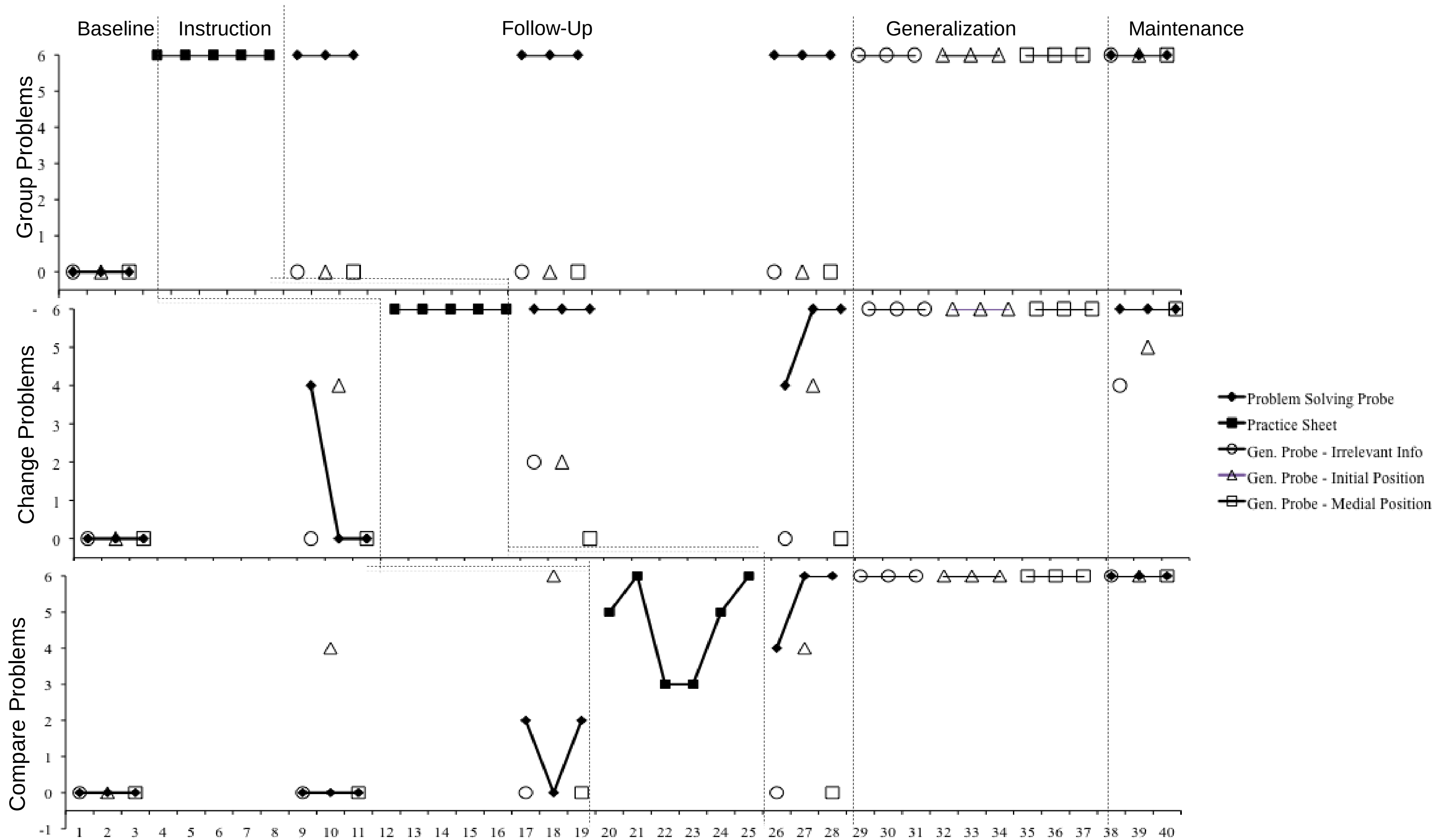


Figure 4-1. Points earned by Daniel on probes and practice sheets.

For instance, as he was writing 3+3=6 and 6-3=3 he said, “Three plus three equals six. Six minus three equals three.” Daniel frequently made computational mistakes in number sentences and sometimes noticed these mistakes and self-corrected verbally and by erasing and re-writing all or part of the number sentence.While Daniel worked on probes, the researcher provided intermittent verbal and edible reinforcers for on-task behavior. She would say “good job doing your word problems” or “I like how you’re sitting and working” while handing Daniel a piece of popcorn, corn chip, or potato chip. Daniel would smile and eagerly eat the edibles when reinforcers were delivered.

Following SBI, Daniel waited for the researcher to finish reading each word problem on problem solving and generalization probes before trying to solve it. As a result, the researcher no longer implemented the accommodation of holding Daniel’s pencil while reading the problem aloud. After hearing the problem, Daniel did not immediately start writing. Instead he quietly looked up from the probe to the RUNS and schematic diagram cards placed on the table above the probe for several seconds before beginning to talk and write. During the majority of problems Daniel used self-talk that included SBI vocabulary (i.e., group problem, change get more, comparing words, put together all). When solving *compare* problems, Daniel was observed to circle the comparing words in the problem. On problem solving probes, Daniel was not observed making any additional marks on the word problems or using schematic diagrams to solve the problems. Instead he wrote correct, vertical number sentences to solve the problems. On generlization probes including irrelevant information, Daniel crossed out the irrelevant information in each problem and circled the comparing words in *compare* problems. Again, he did not use schematic diagrams to solve the problems, but instead wrote the correct vertical number sentence for each problem. On generalization probes with the unknowns in the initial and medial positions, Daniel continued to circle the comparing words in *compare* problems. He also used schematic diagrams to solve these problems, writing vertical number sentences only after completing the diagram. Although Daniel continued to make errors when solving problems post-SBI, he was able to self-correct these mistakes consistently. After completing a number sentence, Daniel often did not write his answer on the answer line and did not include a label with his answer. The researcher again provided intermittent verbal and edible reinforcers for on-task behavior and Daniel responded to these reinforcers by smiling and eagerly eating the edibles.

Daniel's problem solving behaviors changed dramatically following SBI. Following the intervention, he began to use the information contained in the word problems in his solutions as evidenced by his willingness to listen to the problem before starting his work, his use of number sentences containing quantities from the word problems, and his circling of comparing words and crossing out of irrelevant information. Daniel's knowledge and use of the schema for group, change, and compare problems also improved following SBI as evidenced by his use of SBI vocabulary, schematic diagrams, and correct number sentences. Finally, Daniel's ability to self-monitor his problem solving increased following SBI as evidenced by his more consistent self-correction of errors.

Justin's Problem Solving Performance

During the first set of baseline probes, Justin was able to successfully solve several of the *group* and *change* problems. He earned an average of 3/6 points on both of these problem types. However, he only correctly solved one compare problem, earning an average score of 0.5/6 on these problems. Justin appeared to be aware that the problems he was given were additive word problems. However, his strategy for determining which operation to use to solve the problem seemed to be based on his perception of the complexity of the problem's syntax. After reading a problem he would say, "This one's easy. I'll add" or, "This one's hard. I'll subtract." He then would apply his chosen operation to the two quantities included in the problem.

Justin responded quickly to instruction on solving each type of word problem. He immediately performed at ceiling on practice sheets and continued to perform at this level throughout instruction on each problem type. Following instruction on solving group problems, Justin's performance on *change* and *compare* problems declined. He would refuse to solve these problems on probes by saying, "No, not a group problem" and writing "no" on the answer line for those problems. Following instruction in solving *change* problems, Justin continued to refuse to attempt solving *compare* problems on probes by saying, "No, not a *group* or *change* problem" and writing "no" on the answer line for compare problems. Following instruction on solving each type of word problem, Justin's performance on probes for that problem type reached and remained at ceiling level for the remainder of instruction. Justin's improved his average performance prior to instruction from 3/6, 1.7/6 and 0.6/6 for group, change, and compare problems respectively to 6/6 for each problem type following instruction on solving that type of word problem.

Generalization to Problems with Irrelevant Information

Prior to receiving SBI, Justin was able to successfully solve only one problem containing irrelevant information. For the majority of problems, Justin appeared to choose an operation – either addition or subtraction – at random. After reading a problem he would say, “This one’s easy. I’ll add” or, “This one’s hard. I’ll subtract.” He then proceeded to apply his chosen operation to all three quantities included in the problem. The *change* problem that he correctly solved contained two quantities that referred to a count of items and one quantity that referred to a cost (i.e., A teacher is taking 28 students to the zoo. If 6 students get sick and each zoo ticket costs $5, how many students did the teacher take to the zoo?). He appeared to apply the same strategy to this problem as to others, but omitted the monetary amount, chose the correct operation presumably by chance, and was therefore able to obtain the correct answer.

Following instruction on solving group problems, Justin began to apply the group schema to solving group problems with irrelevant information and refused to solve change or compare problems by saying “no, not a group problem” and writing “no” on the answer line for these problems. For two of the group problems he added all three quantities together, but for one of the group problems, in which the relevant quantities were both 2-digit numbers and the irrelevant quantity was a 1-digit number, he omitted the irrelevant quantity and obtained the correct answer (i.e., Jim looks out the window. He counts 42 boys, 4 teachers, and 36 girls on the playground. How many children are on the playground?).

Following change instruction, Justin demonstrated understanding that there was irrelevant information contained in the problems. He would say, “But this is wrong. It doesn’t fit. I don’t like it.” He actually became agitated and started to cry, repeatedly saying, “It’s wrong. I don’t like it.” When Justin began to cry the researcher intervened by saying, “It’s okay. If some of the problems have a mistake in them, you can still use your diagrams. If you want to, you can cross out the mistake." After the researcher's reassurance, Justin was able to calm down and complete the generalization probe, crossing out the irrelevant information in each problem. He applied the group problem schema correctly to group problems containing irrelevant information and his average performance on these problems improved from 1/6 to 6/6. Justin applied the change problem schema to change and compare problems containing irrelevant information. His average performance improved from 1/6 to 6/6 for change problems and from 0/6 to 2/6 for compare problems. Justin's improved performance on change problems likely represents spontaneous generalization of the change schema to problems containing irrelevant information. His slight improvement in solving compare problems is likely due to change rather than due to spontaneous generalization. Justin correctly solved the problem, "There are 27 students on the monkey bars, 12 students on the swings, and 15 students in the sand box. How many more students are on the monkey bars than in the sand box?" by using a change diagram wherein the change was "get less" with 27 as the beginning amount and -15 as the change amount. This led Justin to obtain the correct number sentence and answer (27-15=12) despite using the wrong schematic diagram.

Following instruction in solving compare problems, Justin continued to cross out the irrelevant information in the problems. He would say, "No, this is a mistake" as he crossed out the irrelevant information. Justin also began to correctly apply the group, change, and compare schemas correctly to problems containing irrelevant information. This resulted in his average performance on group and change problems containing irrelevant information remaining at ceiling level, while his performance on compare problems containing irrelevant information improved form 0.67/6 to 6/6.

Generalization to Problems with Unknowns in the Initial Position

Prior to receiving SBI, Justin was able to successfully solve several problems with the unknown in the initial position of the schematic diagram. For the majority of problems, Justin appeared to choose an operation – either addition or subtraction – at random. After reading a problem he would say, “This one’s easy. I’ll add” or, “This one’s hard. I’ll subtract.” He then proceeded to apply his chosen operation to the two quantities included in the problem. As a result, Justin was able to correctly solve two group problems, one change problem, and two compare problems; presumably by chance.

Following instruction on solving group problems, Justin began to apply the group schema to solving group problems with the unknown in the initial position. When applying the group schema to these problems, he did not seem to realize that the initial part was the unknown and instead completed the diagram as though both parts were known and the all was the unknown resulting in incorrect answers on all group problems leading to a decline from 4/6 at baseline to 0/6 following group instruction. Justin also refused to solve the majority of change or compare problems by saying, “no, not a group problem” and writing “no” on the answer line. However, Justin applied the group problem schema to the change problem, “Kim had some money in her piggy bank. She spends $20 on a new doll. Now she has $38. How much money did Kim have in her piggybank?” Because Justin used $20 and $38 as the two parts, this resulted in him obtaining the correct number sentence and answer (20+38=58) to this problem presumably by chance. As a result his performance on change problems was maintained at 2/6 while his performance on compare problems declined from 4/6 to 0/6 following instruction in solving group problems.

Following change instruction, Justin demonstrated understanding that the unknown was in a different location in the diagram. When he read the first problem (A teacher is planning to take some students to the museum. 9 students cancelled. Now 50 students are going to the museum. How many students was the teacher planning to take?) Justin pointed to the sentence “9 students cancelled,” looked at the researcher, and said, “This is the change, not the beginning. OK?” When the researcher did not respond, Justin began to become agitated and start to cry so the researcher said, “You are doing a great job. Put things in the diagram where you think they belong and then finish using your RUNS steps.” Justin was able to calm down after the researcher reassured him. He then correctly completed the problems with the unknown in the initial position of the change diagram with a larger beginning amount. He then proceeded to correctly solve the remaining group and change problems using the correct diagrams and placing the unknowns in the initial position. As a result, his average performance on group and change problems improved from 2/6 to 6/6 following change instruction. On this generalization probe, when solving compare problems with an unknown in the initial position, Justin used the change diagram. However, he completed the change diagram for these problems as though he was solving for an unknown ending. As a result, Justin’s performance on compare problems improved from 2/6 to 4/6. However, this change in performance was likely due to chance as Justin’s misuse of the change schema sometimes resulted in the correct number sentence and answer for a compare problem. For instance, when given the problem, “There are some cows in the barn and

74 cows in the pasture. If there are 22 more cows in the barn than in the pasture, how many cows are in the barn?" Justin's use of the change diagram with 74 as the beginning amount and +22 as the change resulted in the correct number sentence and answer 74+22=96.

Following instruction in solving compare problems, Justin correctly applied the group, change, and compare schemas. He also continued to demonstrate an understanding of the location of the unknown by correctly completing schematic diagrams and by orally labeling each quantity as representing a particular part of the problem (i.e., "this is the all" or "this is the difference."). This resulted in his average performance on group and change problems remaining at ceiling level and his performance on compare problems improved form 2.67/6 to 6/6 following instruction on solving compare problems.

Generalization to Problems with Unknowns in the Medial Position

Prior to receiving SBI, Justin was able to successfully solve all problems with the unknown in the medial position of the schematic diagram. Justin read the first three problems aloud and after each one said, "This is hard. I'll subtract." He proceeded to use subtraction for all of the remaining problems without reading them aloud. As a result, Justin was able to obtain correct number sentences and answers for all of the problems. However, his ability to obtain the correct answer did not appear to be due to understanding, but rather to his assumption that the problems were difficult thus requiring subtraction rather than addition. Following instruction on solving group problems, Justin refused to solve any problems with the unknown in the medial position. He would say, "no, not a group problem," write "no" on the answer line, and then proceed to the next problem. As a result, Justin's performance on all problem types declined from 6/6 to 0/6 following instruction on solving group problems.

Following change instruction, Justin demonstrated understanding that the unknown was in a different location in the diagram. When he read the first problem (A teacher is taking 25 students to the museum. Some students cancelled. Now 21 students are going to the museum. How many students canceled?) Justin pointed to the sentence "Now 21 students are going to the museum," looked at the researcher, and said, "This is not the change. It's the ending." He then correctly completed this and subsequent group and change problems with the unknowns in the medial position of the diagram using the correct schematic diagram for each problem. His average performance on group and change problems improved from 3/6 to 6/6 following instruction on solving change problems. Following change instruction, Justin began to apply the change schema to compare problems with an unknown in the medial position. However, he completed the change diagram for these problems as though he was solving for an unknown ending. Using this strategy, Justin was able to correctly solve one compare problem (Jan's book has 89 pages. If Paul's book has 32 fewer pages than Jan's, how many pages does Paul's book have?). Using 89 as the beginning amount and -32 as the change, Justin obtained the correct number sentence and answer (89-32=57) despite using the incorrect diagram. As a result he earned 2/6 points on compare problems with the unknown in the medial position following instruction on solving change problems.

Following instruction on solving compare problems, Justin correctly applied the group, change, and compare schemas to problems with unknowns in the medial position. He also continued to demonstrate an understanding of the location of the unknown by correctly completing schematic diagrams and by orally labeling each quantity as representing a particular part of the problem (i.e., “this is the ending” or “this is the difference.”). This resulted in his average performance on group and change problems remaining at ceiling level. Justin’s average performance on compare problems with unknowns in the medial position improved form 2.67/6 to 6/6 following instruction on solving compare problems.

Maintenance

Maintenance probes were conducted eight weeks after SBI was completed. Justin’s performance on group, change, and compare problems with unknowns located in the final position remained at ceiling level on these probes indicating that treatment effects from the SBI were maintained over time. He also demonstrated maintencance of treatment effects on generalization probes involving problems with irrelevent information and problems with uknowns in the initial or medial position as his performance remained at ceiling level on these probes as well.

Figure 4-2 is a graph of Justin’s performance on probes and practice sheets. Data are graphed for each session so that trend and level can be visualized.

Justin’s Problem Solving Behaviors

Prior to receiving SBI, Justin tended to work quickly and quietly on all probes. When the researcher offered to read the problems to him, Justin said, “That’s OK. I’ll do it.” He then focused on the paper and moved his lips as though he was reading quietly to himself. Sometimes, after reading a word problem he would say, “This is an easy one. I’ll add” or “This one’s hard. I’ll subtract.” Justin tended to work quickly and quietly. He would write a vertical addition or subtraction number sentence for each word

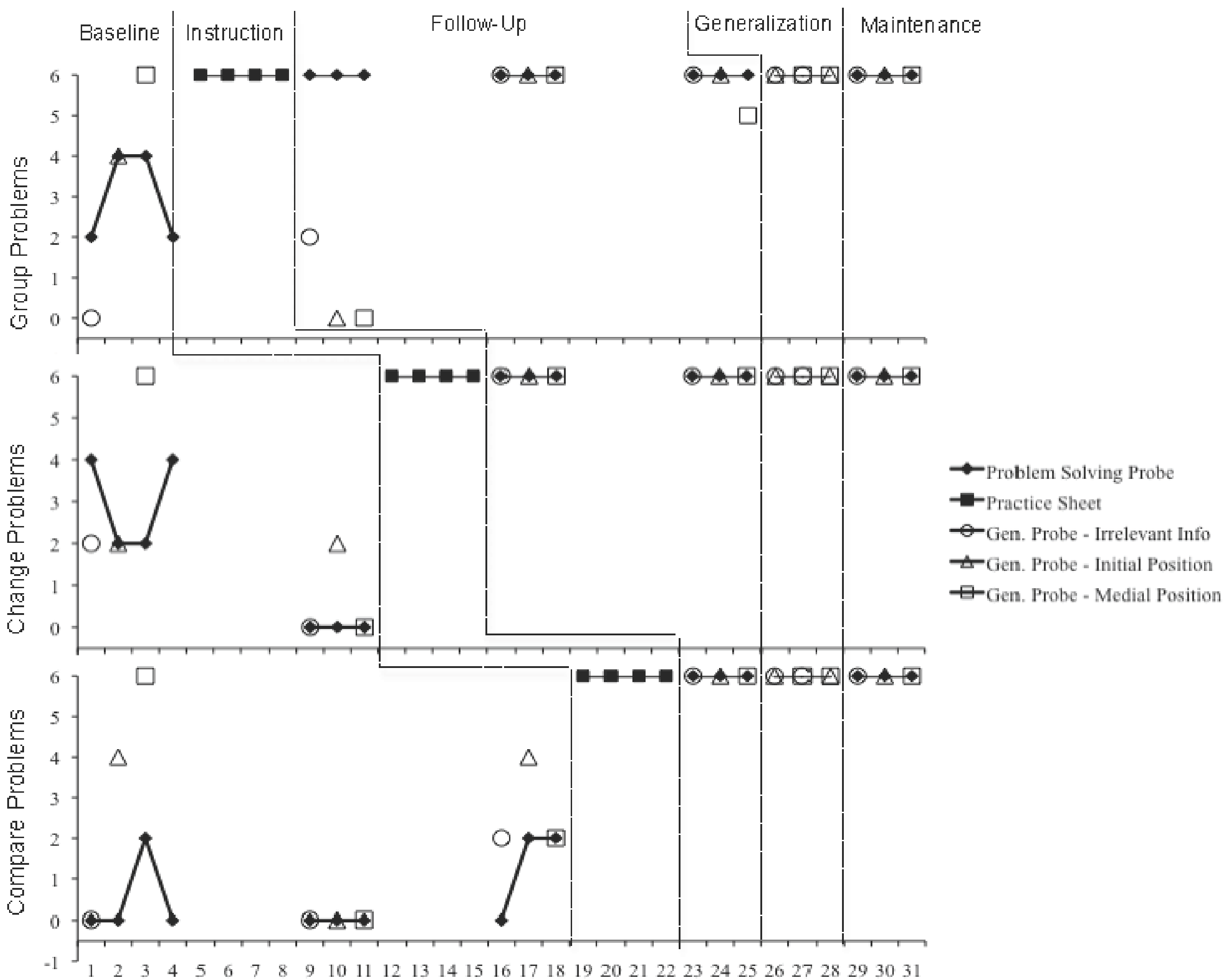


Figure 4-2. Points earned by Justin on probes and practice sheets.

Following SBI, Justin would write “RUNS” before starting each problem and would cross off each step as it was completed. Justin also accepted the researcher’s offer to read the problems aloud to him. He appeared to follow along with his eyes as she read the problems. After the researcher read each problem, Justin pointed to a phrase in the problem and verbalized which critical component that phrase represented and to which problem type the word problem belonged. For instance, he would point to the words *how many more birds than* in a problem and announce, “Comparing words, it’s a compare problem” or he would point to the words “5 children got sick” in another problem and say, “I think that’s the change get less.” When completing generalization probes with problems containing irrelevant information, Justin would cross out the irrelevant information in the word problem. He would then draw the corresponding schematic diagram, complete the diagram correctly, and write the number sentence corresponding to the diagram. He would then write his answer and a label on the answer line. When Justin made errors in his diagram or number sentence he would say “oh no” while erasing and correcting the mistakes. Again, the researcher provided intermittent verbal praise for on-task behavior to which Justin responded by smiling and repeating the praise.

Prior to SBI, Justin used information contained in the problem to determine a solution. However, his solution strategy did not allow him to obtain correct answers consistently. He appeared to read the problem, make comments indicating that he chose which operation to use based on the complexity of the problem’s syntax, and write number sentences using quantities contained in the word problems. Following SBI, Justin continued to use the information contained in the problems to determine a solution; however, his use of the strategies he learned in SBI allowed him to consistently obtain correct answers on word problems. He used the RUNS mnemonic to self-monitor his problem solving, used critical problem features to determine to which schema a problem belonged, and used schematic diagrams to help formulate his number sentences. Justin used self-correction both before and after SBI to correct computational errors; however, following SBI he also self-corrected errors in schema use and operation choice.

Satisfaction Data

Both participants and their mothers completed satisfaction scales following SBI. Copies of these scales can be found in Appendix F.

On the participant satisfaction scale, Daniel indicated feeling positive about solving math word problems and about using diagrams to solve word problems. He also indicated that he thought other children might also feel positive about using diagrams to solve word problems. On the other hand, Daniel indicated feeling neutral about using the RUNS steps and indicated that other children might feel negative about using the RUNS steps. This is consistent with his reference to, but failure to write down or completely follow the RUNS steps following SBI.

Justin, on the other hand, indicated feeling positively about solving math word problems, using the RUNS steps, and using diagrams. He indicated that he thought other children would feel neutral about using the RUNS steps and using diagrams to solve word problems. This is consistent with his use of the RUNS steps and of diagrams when solving problems following SBI.

On the parent satisfaction scale, Daniel’s mother responded that she agreed strongly that SBI addressed important skills, was a valuable use of instructional time, was enjoyable for Daniel, and resulted in improvements in Daniel’s problem solving performance. She also agreed strongly that she would recommend SBI to others and would like Daniel’s teachers to learn to use SBI. She agreed that Daniel used diagrams to solve word problems following SBI and was neutral regarding his use of the RUNS steps following SBI. On the open-ended section of the questionnaire, Daniel’s mother provided evidence of spontaneous generalization of SBI to the school setting. She indicated that on the mathematics unit test that was completed after SBI, Daniel correctly answered all four of the open-ended addition and subtraction word problems included. She said that his paraprofessional reported that he used vocabulary from SBI while completing these items and used a schematic diagram for one of the problems.

Justin’s mother responded that she agreed strongly that SBI addressed important skills, was a valuable use of instructional time, and was enjoyable for Justin. She also agreed that she would recommend SBI to others and would like Justin’s teachers to learn to use SBI. She responded neutrally regarding Justin’s use of the RUNS mnemonic and diagrams when solving word problems, and regarding improvements in his abilities to solve problems. On the open-ended section of the questionnaire, she indicated that because it was summer break she had not had an opportunity to observe Justin’s problem solving performance or strategies outside of SBI sessions and therefore was not certain whether Justin would continue to show improvement or use SBI strategies. Justin’s mother is taking college courses to become an elementary school teacher and indicated that she was grateful for being able to observe most of the SBI sessions as she hopes to be able to use SBI with her own students in the future.

Summary of Results

Although Justin and Daniel are children with Autistic Disorder, their profiles are very different. Daniel is a first-grade student with obvious visual strengths. He also has a number of behaviors that can make it challenging to work with him. For instance, Daniel needed frequent prompting and reinforcement to stay seated during instruction, had difficulty answering questions due to echolalic speech, and often seemed more interested in drawing pictures than in completing mathematics word problems. In addition, because Daniel was a first grader he had minimal experience with solving word problems. At the time of this study, he had recently completed his first classroom unit on word problem solving which included only keyword instruction. Justin, on the other hand, has visual strengths that are not as strong as Daniel's, yet he uses language more flexibly than does Daniel. During the study, Justin exhibited fewer behaviors that interfered with instruction and was very eager to please adults. As a sixth grader, Justin had been exposed to word problem solving instruction numerous times and likely to a variety of word problem solving strategies.

Despite having been exposed to some keyword instruction prior to SBI, Daniel did not seem to have any understanding of word problems other than to know that they were a mathematics task. He did not use any information from the word problems in his solutions and instead wrote number sentences for unrelated fact families. His performance for each problem type was at basal level on the first set of probes and remained at near-basal level until after instruction on each problem type. Justin, on the other hand, appeared to have a strategy for solving additive word problems. He seemed to use sentence complexity as an indicator of which operation to use and was able to correctly solve approximately half of the word problems on initial probes. On subsequent baseline probes, his performance declined as a result of his refusal to solve problems not belonging to schemas already addressed in SBI.

Following SBI, Justin's and Daniel's performance on problem solving probes reached ceiling level and was maintained at that level up to eight weeks following instruction. Although both participants achieved mastery on the word problems targeted in this study, Justin did so in fewer lessons than did Daniel. In addition, Daniel required additional lessons to help him generalize SBI to problems with irrelevant information and with the unknown in the initial or medial positions. Justin, on the other hand, demonstrated spontaneous generalization to these types of problems after receiving SBI on group and change problems. Justin also was observed to follow the SBI procedures more fully when completing probes after SBI. He used the RUNS mnemonic and schematic diagrams consistently to solve each problem. Daniel, on the other hand, did not write out the RUNS mnemonic, though he was observed to visually refer to it while solving problems. In addition, Daniel only used schematic diagrams when solving problems with unknowns in the initial and medial positions. Both participants used self-talk that included vocabulary taught as part of the SBI, and demonstrated increased use of self-correction and self-monitoring following SBI.

CHAPTER 5
DISCUSSION

SBI for Students with Autism

Although researchers have studied a variety of methods for improving the problem solving performance of students with mathematics difficulties or mild disabilities (Gersten et al., 2009), a literature search revealed few studies that included students with autism. The purpose of this study was to explore the use of SBI with students with autism and contribute to the small body of literature that has addressed ways to teach mathematical word problem solving to students with autism. Because this study is primarily a population expansion, it is important to consider how the characteristic features of autism may have impacted the SBI intervention and the participants' responses to it. Although the interaction between SBI and the characteristic features of autism can not be supported by the probe data, observations of the participants' behaviors and support from the literature can provide some insight into how the characteristic features of autism and the SBI intervention may interact.

Participant Profiles and Impact on Problem solving Performance

Daniel and Justin both have diagnoses of Autistic Disorder from developmental behavioral pediatricians. They both display difficulties with language and communication that are typical of autistic disorder (American Psychiatric Association, 2000). Although both participants use oral language as their primary means of communication, their language is impaired compared to other children of the same age. Daniel speaks primarily in echolalic phrases that he uses to convey his wants and needs. He only answers questions when given choices or when he can answer by pointing to an object or picture, or by drawing his own picture. Although Justin uses language more flexibly than does Daniel, his language is still severely impaired compared to other children his age. He can follow simple two-step instructions, but has difficulty with complex multi-step instructions. He can sometimes answer open-ended questions without being provided with choices or visual cues. However, he often becomes frustrated by his inability to formulate his thoughts into language. He also has difficulties with complex language comprehension skills such as making inferences and predictions.

In addition, both Daniel and Justin exhibit difficulties with executive functioning that often accompany ASD (American Psychiatric Association, 2000). For instance, Daniel has difficulties attending to classroom instruction (sustained attention), refraining from getting out of his seat or calling out without permission (inhibition), and transitioning from one activity to the next (shifting set). According to Justin's most recent psychoeducational evaluation, he too has difficulties with executive functioning particularly within the area of verbal working memory.

Language impairment and executive dysfunction can both contribute to difficulties with mathematics problem solving. Children with language impairments can be expected to have difficulties with mathematical word problem solving because solving problems requires that students use semantic mapping to determine the relationships between known and unknown quantities to determine the correct operation to use to solve for the unknown (Christou & Phillippou, 1999). The failure of students with autism to use semantic cues (Frith & Snowling, 1983) has important implications for teaching word problem solving because it will likely make semantic mapping more difficult for these students. Executive dysfunction, including deficits in working memory and sustained attention, can compound these difficulties by making it difficult for children to plan a solution strategy and filter and manipulate relevant information in word problems (Zentall, 2007; Zentall & Ferkis, 1993). Working memory deficits may also lead to the use of immature problem solving strategies and procedures and difficulties inhibiting irrelevant information leading to errors in problem solving (Geary et al., 2008).

Geary and Hoard (2005) suggest that difficulties in mathematics problem solving can be due to weaknesses in the underlying systems and supporting competencies that underlie mathematics problem solving. For students with autism, deficits in the language system and central executive may lead to difficulties building concepts for whole number operations and carrying out procedures necessary for word problem solving.

Observations of Daniel's and Justin's behaviors while solving additive word problems prior to starting SBI suggest that their language impairment and executive dysfunction may have been impacting their ability to solve problems. Daniel attempted to solve word problems without reading the problem or allowing the researcher to read the problem to him. It is possible that this impulsivity was the result of executive dysfunction associated with Daniel's autism. Even when the researcher offered him reminders to read the problem before starting to work, Daniel appeared to be unable to inhibit his impulsive behavior and attend to the word problem. When the researcher implemented an intervention to prevent this impulsivity, Daniel still did not make use of the information contained in the word problem. This may have been due to difficulties attending to the researcher or to difficulties making use of the semantic information contained in the word problems, but it is likely that language impairment and executive dysfunction both contributed to Daniel's poor baseline performance on probes.

When solving problems prior to SBI, Justin seemed to rely on his perception of the complexity of the syntax of the word problem to choose which operation to use. On those problems that he perceived contained more complex syntax, he subtracted. On problems that he perceived contained less complex syntax, he added. This strategy suggests that Justin may have had difficulty making use of the semantic information contained in word problems. It is possible that these difficulties were due to language impairments or verbal working memory deficits, but it is likely that both may have contributed to his difficulty arriving at an effective solution strategy.

Shortcomings of Traditional Problem solving Instruction

Daniel and Justin had both received mathematical word problem solving instruction in their public school classrooms prior to starting SBI. The county in which Daniel and Justin reside used the state adopted mathematics series *Harcourt Math Florida* (Harcourt School Publishers, 2002). Starting in kindergarten, this math series encourages students to draw pictures to represent mathematical word problems. The workbooks accompanying the text include pages with pictures of different objects and students are instructed to either draw more objects or cross some of the objects out as the teacher reads aloud change problems involving those objects. For instance, the worksheet might include a box with a picture of 5 fish in it. The teacher will then read the problem “John has 5 fish in his fish tank. He gets 2 more fish. How many fish does he have now?” The students are expected to draw 2 more fish, count the total number of fish, and write 7 on the answer line. The same worksheet might include a box with a picture of 8 cookies. The teacher will then read the problem “Amy has 8 cookies. She eats 4 cookies. How many cookies does Amy have left?” The students are expected to cross out 4 of the cookies, count the remaining cookies, and write 4 on the answer line.

These activities assume that students possess the prerequisite language and verbal working memory skills to keep in mind what the teacher read, understand whether the syntax of the problem indicates “get more” or “get less” and then alter the pictorial representation without additional instruction. By first grade, students are given written problems with accompanying pictures they can then alter. By second grade they are given space in which to draw their own pictorial representations to accompany word problems.

In addition to teaching students to create pictorial representations for word problems, the mathematics text also introduces keyword instruction starting in first grade, wherein students are taught that the keywords *more* and *together* indicate addition, and the keywords *less*, *fewer*, and *more than* indicate subtraction. Starting in second grade, students are taught a general problem solving heuristic that includes the following steps: (1) read the problem carefully, (2) look for clues, (3) choose a strategy, and (4) check your work. Keyword instruction, use of pictorial representations, and the general problem solving heuristic are reintroduced and reinforced in the texts for grades three to five. Additional keywords and types of pictorial representations are introduced for multiplicative word problem solving starting in grade three.

Presumably, both Daniel and Justin had been exposed to the mathematical word problem solving instruction included in the mathematics basal text prior to SBI. As a first grader, Daniel was likely introduced to pictorial representations in kindergarten. According to his mother, his class had just completed a chapter that reinforced the use of pictorial representations and introduced keywords for additive word problems when this study commenced. As a sixth grader, Justin had likely been taught to use pictorial representations to assist in solving mathematical word problems for the past six years. He had also likely been taught to identify keywords and to choose an operation based on those keywords when solving additive word problems for several years. Justin had also likely been taught to use the general problem solving heuristic.

Despite this prior additive word problem solving instruction, both Justin and Daniel exhibited difficulty solving word problems on probes prior to SBI. Daniel seemed completely unaware that the word problem contained information relevant to obtaining a correct answer as evidenced by his use of two unrelated number sentences from the same fact family to solve problems. He did however seem to be aware that the word problems were a mathematics task. He did not use either the pictorial representations or the keyword instruction that he was exposed to in school. Justin appeared to have more understanding of how to solve additive word problems than did Daniel. This is to be expected given the greater number of exposures he would have had to problem solving instructions in over six years of public schooling as compared to Daniel who had been in school for less than two years. Justin seemed to be aware that in order to solve the problem he needed to first read it. He also seemed to know that the syntax of the problem held clues as to whether to add or subtract. However, he seemed to rely on his perception of the complexity of the problem’s syntax rather than using the keyword strategy included in the mathematics basal text or using semantic mapping. Justin did not attempt to draw pictorial representations to assist with problem solving.

Initially it may seem surprising that neither Daniel nor Justin used the strategies they had presumably learned in school to solve word problems. However, it is possible that the strategies included in the mathematics basal text are not optimal for students with disabilities or mathematics difficulties. For instance, the basal text relies heavily on keyword instruction. According to Van de Walle et al. (2010), keyword instruction can be misleading because some problems do not contain keywords, and even those problems that do contain keywords may not always require the operation that keyword has been linked to during instruction. Keyword instruction is also ineffective because it fails to take into account the underlying meaning and semantic structure of the problem and fails to teach the relationships between addition and subtraction. The basal mathematics textbook also encourages students to use pictorial representations to solve word problems. Although visual representations can be helpful in teaching problem solving (Gersten et al., 2009), this strategy is only effective when students use visual representations that accurately reflect the underlying structure and relationships among quantities in the word problem (Hegarty & Kozhevnikov, 1999). In addition, although problem solving heuristics can be helpful to students (Gersten et al.), the heuristic included in the basal mathematics textbook does not provide a way for students to systematically approach the problem (Van de Walle et al.). Instead, the heuristic directs them to choose from among the less than optimal strategies to which they have been exposed. Students with autism, in particular, may flounder when using a problem solving heuristic that does not provide a systematic approach to problem solving. When asked to complete multi-step activities, students with autism benefit from structure in the form of explicit and systematic instructions that provide them with information regarding what steps to take, the order in which to do those steps, and what behaviors constitute completion of each step (Mesibov & Howley, 2003). Because the general problem solving heuristic included in the basal textbook does not provide the specific behaviors expected during each problem solving step, it is unlikely to be helpful to students with autism.

Benefits of SBI

SBI provides supports that may address several of the weaknesses identified in the basal textbook approach to teaching word problem solving. SBI includes a problem solving heuristic that represents a systematic approach to solving word problems. SBI also includes visual representations, an effective approach to improving mathematical problem solving performance (Gersten et al., 2009). However, unlike the basal text that expects students to create their own pictorial representation for each individual problem, SBI teaches students three specific diagrams that visually represent the underlying semantic structure of the word problems (Jitendra et al., 2007b). SBI combines visual representations, direct instruction, and a problem solving heuristic that supports a systematic approach to solving word problems. As such, SBI holds great promise as a means of supporting the problem solving performance of students for whom traditional classroom instruction has not been effective (Gersten et al., 2009; Jitendra et al., 2007b; & Van de Walle et al., 2010).

SBI may be particularly beneficial to students with autism because it seems to provide scaffolding to support the language system. Visual representations make use of strengths in the visuospatial system of students with autism while helping them to compensate for difficulties with language (Mesibov & Howley, 2003; Tissot & Evans, 2003). The diagrams used in SBI provide a visual representation of the semantic structure of the three types of additive word problems. Various studies of SBI have used different diagrams to represent the three additive word problem schemas. The diagrams used in this and the Rockwell et al. (2011) study were adapted from those used previously in an effort to most clearly represent the relationships among the quantities in each problem type. The diagrams used in this study clearly differentiate the larger quantity from the two smaller quantities in each problem type. For *change* problems, in which the larger quantity can be either the *beginning* or the *ending*, students were taught to adapt the diagram to clearly represent the larger quantity. The researcher chose to use diagrams that explicitly differentiated the larger quantity so that instruction could focus on the conceptual knowledge that addition results in a sum that is larger than the addends while subtraction results in a difference that is smaller than the minuend. Thus, the diagrams used in this study not only provided a visual representation of the semantic structure of additive word problems, but also provided a visual support that could be used to teach the underlying relationships between addition and subtraction and thus facilitate students' algebraic reasoning about addition and subtraction problems (Van de Walle et al., 2010). However, Daniel seemed to have some difficulty distinguishing the group and compare diagrams. It is possible that use of a different shape for the compare diagram would help students to more easily differentiate the two diagrams in the future.

SBI may also help to reduce working memory demands and thereby may provide support for the central executive of students with autism (Cooper & Sweller, 1987). First, SBI includes a problem solving heuristic represented in this study by the mnemonic RUNS. The RUNS mnemonic provides support for planning and organization (Deshler et al., 1981). Participants were taught to check off the RUNS steps as they were completed. This allowed participants to approach each problem systematically without skipping or omitting steps and without having to rely on verbal working memory to keep track of their problem solving plan. The RUNS heuristic, is paired with a specific problem solving strategy (i.e. using schematic diagrams); which allows students to know precisely what behaviors are expected during each step of the problem solving process. This may provide them with the structure necessary to plan, initiate, and complete the activity, in this case word problem solving, independently and with increased confidence (Mesibov & Howley, 2003). SBI also focuses on teaching the three schemas underlying the majority of single-step additive word problems. In traditional instruction, students often focus on the superficial features of word problems and approach each problem as requiring a unique solution (Fuchs et al., 2003a). In SBI, on the other hand, students are taught to look for the critical features that indicate to which schema a problem belongs and then choose from several known and rehearsed solution strategies. Because each problem does not require a unique solution and because the solution strategies are known, working memory loads are reduced (Capizzi, 2007).

Although SBI provides many supports that may be beneficial to students with autism, additional behavioral supports are needed to lead to an effective intervention package. The participants in this study both seemed to benefit from intermittent reinforcement for on-task behavior. Reinforcers were chosen individually for each participant. It is likely that students with autism in general will benefit from reinforcement for on-task behavior as a support for executive functioning. Both participants also seemed to benefit from prompting and corrective feedback that included gestures and references to visual supports. Prompting and corrective feedback are a necessary component of guided practice, but the additional use of gestures and visual supports may help scaffold the language of students with autism. Justin benefited from being allowed to request noncontingent breaks; however, Daniel had great difficulty transitioning back to instruction following breaks. Therefore, the provision of noncontingent breaks may be beneficial for some, but not all, students with autism. Finally, the researcher provided additional behavioral supports to meet the participants' individual needs. For instance, Justin was provided with verbal reassurance when he became agitated. Daniel was provided with various means of sensory stimulation such a physical touch and an air disk on which to sit. It may be important for intervention agents to assess students' behavioral needs to determine which individual supports will allow them to most benefit from instruction.

The results of this study indicate that SBI was effective in improving the word problem solving performance of two participants with autism. The participants in this study both made measureable gains in problem solving performance following SBI. Daniel's baseline performance was as low as basal level, but quickly reached ceiling level on problem solving probes following instruction in solving each type of word problem. Although he did not demonstrate spontaneous generalization, he was able to reach ceiling level on generalization probes with minimal instruction. Justin also made significant gains in his problem solving performance following SBI. While his baseline performance was above basal level, an analysis of his problem solving behaviors while completing probes suggests that he was getting the correct answers to problems due more to chance than to the use of an effective problem solving strategy. Following SBI, Justin's performance on problem solving probes improved to ceiling level. He also demonstrated spontaneous generalization requiring no specific instruction to reach ceiling level on generalization probes. Both Daniel and Justin maintained improvements in their problem solving performance eight weeks following the completion of SBI.

The improvements in problem solving performance Justin and Daniel demonstrated following SBI suggest that SBI may be an effective intervention for improving the problem solving performance of students with autism. Gains in problem solving performance were evident following only four weeks of instructional time. This suggests that students with autism who receive SBI can acquire word problem solving skills quickly. It is also important to note Daniel's ability to generalize the use of SBI to target generalization problems following minimal generalization instruction, Daniel's spontaneous use of SBI in the school setting and while completing homework, Justin's spontaneous generalization to target generalization problems, and both participants' maintenance of improvements in problem solving performance eight weeks after completing SBI. This evidence of maintenance and generalization is particularly promising given the challenges of programming for skill generalization and maintenance for individuals with developmental disabilities (e.g. Koegel & Rincover, 1977; Stokes & Baer, 1977).

Daniel and Justin also engaged in behaviors that lend support to the theory that SBI effectively makes use strengths in the visuospatial systems of students with autism while providing support for weaknesses in their language systems and central executives. Following SBI, Daniel was able to inhibit his impulsivity and attend to the researcher while she read problems aloud to him. He also referred to the RUNS steps and the schematic diagrams when solving problems. These behaviors suggest that he used these visual supports to scaffold his organization and planning while solving problems. Daniel also engaged in self-talk about the critical components of word problems, and circled some critical components and crossed out irrelevant information. This suggests that Daniel made use of schema knowledge when solving word problems and lends support to the assumption that SBI provided adequate support for Daniel's language system to allow him to make use of the semantic information contained in the word problems.

One of the most interesting behaviors Daniel displayed while solving probes following SBI was selective use of schematic diagrams. He did not use schematic diagrams at all when the unknown quantity was located in the final position of the schematic diagram. However, he used schematic diagrams to solve all problems wherein the unknown quantities were located in the initial or medial position of the schematic diagram. According to Christou and Phillipou (1999) it is more difficult for children to solve additive word problems with the unknowns located in the initial or medial position than in the final position. The researchers hypothesized that these problems were more difficult to solve because they required more complex algebraic reasoning. Daniel's use of the schematic diagrams to solve problems with unknowns in the initial or medial position, but not with problems with unknowns in the final position, suggests that the schematic diagrams may have provided him with necessary scaffolding to support his algebraic reasoning and to help make clear the relationship between addition and subtraction.

When solving problems following SBI, Justin was observed using the RUNS steps as instructed. He wrote the mnemonic RUNS before starting to solve each problem and crossed each letter off as he completed that step. Justin also indicated on satisfaction scales that he enjoyed using the RUNS steps. According to Mesibov and Howley (2003), an explicit heuristic like RUNS can provide structure that helps students with autism plan and complete work independently and with confidence. Justin's consistent use of the RUNS mnemonic and his positive feelings about it suggest that this heuristic may not only have supported Justin's central executive, but may also have allowed him to solve word problems with greater independence and confidence.

Following SBI, Justin also used schematic diagrams consistently to solve all problems and reported having positive feelings about using schematic diagrams. While solving problems and completing diagrams he engaged in self-talk about the critical problem features and sometimes circled critical problem features or crossed out irrelevant information. Justin's consistent use of diagrams suggests that schematic diagrams may have functioned as an effective visual support for his language system and may have helped him to manipulate and use the semantic information contained in word problems.

Not only did Justin use diagrams consistently, but he also generalized the use of schematic diagrams to problems with unknowns in the initial and medial position without any additional generalization instruction. This lends support to the theory that the diagrams may have sufficiently supported Justin's understanding of the relationship between addition and subtraction to allow him to spontaneously engage in algebraic reasoning without teacher modeling of the process. According to Geary and Hoard (2005), the language system and central executive are both critical in developing the underlying conceptual knowledge for problem solving. Justin's problem solving behaviors prior to SBI indicated that he did not have an adequate understanding of the relationship between addition and subtraction needed for additive word problem solving. Despite six years of public school instruction, Justin was still unable to make use of the semantic information contained in word problems or of the conceptual relationship between addition and subtraction when solving word problems. Justin's spontaneous generalization of schematic diagrams to solving problems with unknowns in the initial and medial position suggest that SBI may have improved his conceptual understanding of the relationship between addition and subtraction and allowed him to make use of the semantic information contained in word problems.

Limitations

Although this study suggests that SBI holds promise for teaching additive word problem solving to students with autism, there are several limitations to this study. The purpose of this study was to evaluate the use of SBI to teach additive word problem solving to students with autism. Only one previous study was identified (Rockwell et al., 2011) that addressed the use of SBI with children with autism. Both this study and the Rockwell et al. study included students who had diagnoses of Autistic Disorder but did not have comorbid intellectual disabilities. These participants, while having language impairments characteristic of Autistic Disorder, still used language as their primary means of communication. Finally, all the participants were included in regular education classrooms and participated in AYP assessments. The results of this study and of the Rockwell et al., study may not generalize to children with different profiles of intellectual, language, and academic functioning.

In addition, because this study included only single-step addition and subtraction story problems, it only addressed a subset of skills needed for successful mathematical word problem solving. Finally, SBI was conducted one-to-one in the participants' homes by a researcher familiar with the intervention. This study did not evaluate whether teachers in typical classroom settings could successfully implement SBI with students with autism. This further limits the generality of the findings.

A further limitation of this study is the inclusion of only two participants. Originally, the researcher planned to include four participants. However, due to unforeseen difficulties with recruiting, only two participants were included. Because a multiple probes across behaviors design was employed wherein solving each type of word problem was treated as a separate behavior, there were three demonstrations of experimental control for each participant for a total of six demonstrations of experimental control. As a result of using a multiple probes across behaviors design, the number of participants did not impact the internal validity of the results (Kennedy, 2005). However, the inclusion of only two participants limits the external validity of the findings. Ideally, a population expansion would include at least six and preferably more participants. According to Horner et al. (2005) replication across multiple participants is necessary to ensure the external validity of the results of single subject research.

Furthermore, because this study looked only at problem solving as an outcome measure, it is not possible to draw data-based conclusions about whether SBI improved the conceptual knowledge or algebraic reasoning, reduced the working memory demands, or increased levels of inhibitory control or sustained attention of the participants. Although the results on outcome measures, observed behaviors, and literature may suggest outcomes other than changes in additive word problem solving performance, conclusions drawn about the impact of SBI on other outcomes are theoretical in nature and tentative at best.

Implications

Future research designed to replicate and expand this study is indicated to contribute to the evidence on using SBI to teach mathematical word problem solving to students with autism. To enhance external validity future studies may wish to replicate this study using larger numbers of participants and participants with more varying academic, intellectual, and language profiles. To allow conclusions to be drawn about other outcome variables, future studies of SBI with students with autism may wish to include additional dependent variables such as measures of attention, on-task behavior, working memory, conceptual knowledge, computation accuracy and fluency, or algebraic reasoning.

Once the results have been verified through replication with students with autism with varying profiles, teachers should be trained to conduct the SBI with small groups of students to ensure that teachers can successfully implement the intervention with fidelity in typical educational contexts. Additional studies may also be designed to teach children with autism to solve multiplicative word problems, multi-step word problems, and word problems that include information beyond the text in charts and graphs. Future studies may also wish to use instruments currently approved to assess AYP in order to ensure that improvements in mathematical word problem solving generalize to high-stakes assessments.

Furthermore, it is possible that schematic diagrams may have the potential to benefit students with autism in other areas of the curriculum. For instance, schematic diagrams may help students with autism to develop writing or reading comprehension skills. Additional research may wish to examine the impact of using schematic diagrams representing various text structures on the written expression and reading comprehension of students with autism. For instance, studies could assess whether the use of plot diagrams would improve the narrative-text reading comprehension or written expression of students with autism. Studies may also wish to evaluate the impact of using schematic diagrams to develop the conceptual knowledge of students with autism in other areas of the curriculum. For instance, schematic diagrams might be used to represent cause and effect relationships in science or history courses.

Although future research is needed to verify the results of this study and expand its external validity, the results of this and the Rockwell et al. (2011) study suggest that SBI may have important practical implications. First, SBI may be an effective means of teaching students with autism to solve mathematics word problems. It appears that SBI may provide support for the language systems and central executives of students with autism, thereby facilitating their ability to solve word problems. In addition, the schematic diagrams used in SBI provide visual supports that not only support problem solving, but may also support conceptual understanding of the underlying relationships between addition and subtraction. Because previous studies (Griffin & Jitendra, 2009; Jitendra et al., 2007a; Jitendra et al., 2007b; Jitendra et al., 2010) have demonstrated that teachers in classroom settings can effectively implement SBI with students with mild disabilities and math difficulties, it is likely that teachers will be able to implement SBI in classrooms with students with autism. Furthermore, visual supports and highly structured activities provide needed support for students with autism in classroom settings (Mesibov & Howley, 2003; Tissot & Evans, 2003). SBI provides visual supports through the use of schematic diagrams and provides structure through the use of the

RUNS heuristic. Therefore, SBI may be well suited to supporting the problem solving performance of students with autism in classroom settings.

Although the research base on using SBI with students with autism is still small, many studies have been conducted that indicate that SBI is an effective means of teaching word problem solving to students without disabilities, with mild disabilities, and those at-risk of mathematics failure. It may be valuable for textbook developers to consider using elements of the SBI approach when developing word problem solving instruction. For instance, mathematics textbooks typically use keyword instruction, which focuses on superficial features of word problems, despite the misleading nature of keywords. Focusing on the underlying structure of word problems may better facilitate schema induction, successful problem solving, and conceptual knowledge of all students. Furthermore, SBI may have particular promise for the tier 2 and 3 intervention materials included with textbooks as part of the RtI model. As a direct instruction approach that can be used with small groups or individual students, SBI may be ideal as an additional intervention for struggling students. Finally, it may be helpful for academic standards writers to consider SBI and schema-theory when writing mathematics standards. Understanding the underlying schematic structure of word problems can facilitate transfer of problem solving skills to novel problems and can encourage the development of conceptual knowledge. Standards developers may therefore wish to include schema induction as an objective.

APPENDIX A
STUDY SUMMARY TABLE

Table A-1: Summary Table

Author/Date	Purpose	Participants	Procedures
Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004c	Assess the added value of an expanded version of schema-broadening instruction including additional transfer features to enhance real-life problem solving strategies	351 students from 24 third grade classrooms in 7 urban schools. Students identified as low, average, or high performing. Classrooms were randomly assigned to general classroom instruction, schema-broadening instruction, or expanded schema-broadening instruction	Control consisted of teacher-delivered instruction from the textbook. Schema-broadening instruction involved instruction in solution rules for four problem types from the textbook and three superficial transfer features. Expanded schema-broadening instruction added three additional superficial transfer features. Research assistants conducted scripted lessons for treatment groups. Pre- and post-testing included four measures with varying degrees of transfer.
Results: Two-factor mixed-model ANOVAs conducted with teacher as the unit of analysis on pre-test and improvement scores. Results indicated that groups were comparable prior to treatment. For transfer-1 and transfer-2, the two treatment groups improved significantly more than did the control group. For transfer-3 and transfer-4, the expanded schema-broadening instruction group improved more than the schema-broadening instruction group, which improved more than the control group. Results were similar for students with disabilities. Effect sizes large for significant contrasts.			

Table A-1. Continued

Author/Date	Purpose	Participants	Procedures
Fuchs, Fuchs, Finelli, Courey, Hamlett, Sones, & Hope, 2006	Assess the contribution of instruction in strategies to promote real-life problem solving by comparing the effectiveness of schema-broadening instruction with and without real-life problem solving strategies	445 students in 30 third-grade classrooms from 7 schools in an urban district. Classrooms randomly assigned to control, schema-broadening instruction (SBI), or SBI plus real-life problem solving strategies (SBI-RL)	Teachers implemented all instruction, using the textbook in the control condition and scripted lessons in the treatment conditions. SBI involved teaching four problem types from the textbook and four superficial transfer features. The SBI-RL condition added instruction in five strategies for solving real-life problems. Students were pre- and post-tested using immediate, near, and far transfer measures.
Results: ANOVAs were conducted. Results indicated group comparability. On immediate transfer, near transfer, and questions two and four of far transfer, both SBI groups outgrew the control group. Effect sizes were large, often over 3 standard deviations. On question one of the far transfer measure all groups grew comparably. On question three of the far transfer measure the SBI-RL group outgrew the SBI and control groups. Again, effect sizes were large, hovering at approximately two standard deviations.			
Fuchs, Fuchs, Hamlett, & Appleton, 2002	Assess efficacy of schema broadening tutoring for improving mathematics problem solving performance of fourth grade students with mathematics disabilities. Compare the treatment effects of schema-broadening tutoring to computer-assisted tutoring.	40 fourth grade students with mathematical disabilities from three schools in a southeastern city assigned to schema-broadening tutoring, schema-broadening tutoring and computer-assisted tutoring, computer-assisted tutoring, and control groups	Teachers used the textbook with the control groups. In schema-broadening tutoring a research assistant used scripted lessons to teach schemas for four problem types obtained from the textbook and four transfer features to small groups of students. Computer-assisted tutoring involved guided practice using real-life problems and motivational scoring. Pre- and post-testing using story problems, transfer story problems, and real-life problems.
Results: Chi-square analysis and ANOVAs indicated group comparability before treatment. ANOVAs indicated statistically significant treatment effects on both the story problems and transfer story problems. On both measures, students in schema-broadening tutoring conditions outgrew students in the computer-assisted and control conditions. On the transfer story problem students in the computer-assisted condition also outgrew students in the control condition. Effect sizes comparing the schema-broadening tutoring and control group for these measures were large.			

Table A-1. Continued

Author/Date	Purpose	Participants	Procedures
Fuchs, Fuchs, & Prentice, 2004b	Assess the differential effects of schema-broadening instruction with self-regulated learning strategies (SRL) on complex mathematics problem solving tasks for students without disability risk (NDR), at risk of mathematics disability (MDR), at risk of mathematics and reading disabilities (MDR/RDR), and at risk of reading disability (RDR)	201 students who met criteria for inclusion in the NDR, MDR, RDR, or MDR/RDR category. These students were from 16 of the classrooms included in the previous study (Fuchs et al., 2003b) and were in the control group or the schema-broadening instruction plus SRL group.	For descriptions of conditions, see Fuchs et al., 2003b. Pre- and post-test measures included immediate and near transfer. A three-factor ANOVA assessed each performance dimension (conceptual underpinnings, computation, and labeling) for these measures. Exploratory analyses using subtests of the *TerraNova* were conducted to generate hypotheses about the relative contribution of reading difficulty and mathematics difficulty to the responsiveness of students with MDR/RDR.
Results: On the immediate transfer measure for conceptual underpinnings the NDR, RDR, and MDR students outgrew the MDR/RDR students; and for computation and labeling the NDR students outgrew the RDR, MDR, and MDR/RDR students. On the near transfer measure on conceptual underpinning the MDR/RDR students improved less than the NDR students; and on computation and labeling the MDR/RDR, MDR, and RDR students improved less than the NDR students. Across all measures and performance dimensions, the treatment group outgrew the control group. Computation deficits accounted for more of the variance in responsiveness of students with MDR/RDR compared to reading comprehension deficits (1.5% versus 21%).			

Table A-1. Continued

Author/Date	Purpose	Participants	Procedures
Fuchs, Fuchs, Prentice, Burch, Hamlett, Owen, Hosp, & Jancek, 2003a	Assess the contribution of explicitly teaching for transfer in a schema-broadening intervention that combined instruction in problem solution rules with instruction in transfer	375 students in 24 third grade classrooms at 6 schools in a southeastern urban school district. Students identified as at, above, or below grade level. Classes randomly assigned to control, solution rules, partial solution rules plus transfer, full solution rules plus transfer.	Teachers used the math textbook with the control group. In treatment conditions, teachers assisted research assistants who provided instruction using scripted lessons. Solution rules involved teaching the schemas for four problem types from the textbook. Transfer instruction involved teaching four superficial problem features. Pre- and post- testing was conducted using transfer, near transfer, and far transfer measures.
Results: ANOVAs indicated group comparability before treatment. ANOVAs indicated that for immediate and near transfer treatment groups outgrew the control group. For far transfer the two groups receiving transfer instruction significantly outgrew the control group. Effect sizes for significant post hoc contrasts were large (ranging from 0.78 to 2.25). Results were not mediated by initial achievement status. Students with disabilities did best in full solution plus transfer and worst in partial solution plus transfer.			
Fuchs, Fuchs, Prentice, Burch, Hamlett, Owen, & Schroeter, 2003b	Examine the contribution of self-regulated learning strategies (SRL) on problem solving improvement when combined with schema-broadening instruction	395 students in 24 3rd grade classrooms from 6 schools, designated as having high, average, or low math achievement. Each class randomly assigned to control, schema-broadening instruction, or schema-broadening instruction/SRL	Schema-broadening instruction taught solution rules for 4 problem types obtained from the textbook and 4 transfer features. SRL involved having students score their work, chart their progress, set goals, and report examples of transfer. Teachers used scripted lessons to conduct instruction. Pre- and post-tests included immediate, near, and far transfer. A self-regulation questionnaire was given at post-test.
Results: ANOVAs indicated that groups were comparable prior to treatment. For immediate transfer and near transfer with high-achieving students, the schema-broadening plus SRL group improved more than the schema-broadening group, which improved more than the control group. For near transfer with average and low-achieving students both treatment groups improved more than the control group. For far transfer, the schema-broadening plus SRL group outgrew the other two groups. On the student questionnaire, the schema-broadening plus SRL group indicated increased levels of self-regulation compared to the other groups. With respect to students with disabilities, ANOVAs indicated that for immediate transfer both treatment groups outperformed the control group, for near transfer the schema-broadening plus SRL group outperformed the control group, but there were no treatment effects for far transfer. Effect sizes for significant contrasts were moderate to large.			

Table A-1. Continued

Author/Date	Purpose	Participants	Procedures
Fuchs, Fuchs, Prentice, Hamlett, Finelli, & Courey, 2004a	Assess whether guided practice in sorting problems into schemas might add value to the schema-broadening intervention	366 students from 24 third grade classrooms at 6 schools in an urban southeastern school district. Students identified as high, average or low-achieving. Classes randomly assigned to contrast, schema-broadening, or schema-broadening plus sorting	Teachers conducted instruction for the contrast groups using the textbook. In the treatment conditions, teachers assisted research assistants who provided instruction using scripted lessons. Schema broadening instruction consisted of teaching four problem types from the textbook and four superficial problem features for transfer. Sorting instruction involved presenting sample problems and asking students to identify the problem type and transfer type. Pre- and post-testing with immediate, near, and far transfer measures evaluated for problem solving proficiency, and problem type schema and transfer type schema knowledge.
Results: ANOVAs indicated that groups were comparable prior to the intervention. Results of ANOVAs indicated main effects of condition on all measures. On problem solving and problem type schema knowledge, both treatment groups improved more than the contrast group. On transfer type schema knowledge, the schema-broadening plus sorting group improved more than the schema-broadening group, which improved more than the control group. Results of the regression indicated that schema development accounted for more variance in problem solving performance than initial problem solving level. For students with disabilities, there were main effects of condition on problem solving and transfer type schema with both treatment groups outperforming the control group. For problem type schema, there was no main effect of condition. Effect sizes were large, often over three standard deviations.			

Table A-1. Continued

Author/Date	Purpose	Participants	Procedures
Fuchs, Seethaler, Powell, Fuchs, Hamlett, & Fletcher, 2008	Assess the efficacy of using SBI as a secondary preventative tutoring protocol within tier two of a response to intervention model to address mathematical word problem solving difficulties of third grade students with math and reading difficulties	42 students with mathematics and reading difficulties from 29 classrooms in eight schools in an urban southeastern school district were randomly assigned to either continue in their general education curriculum (control) or receive schema-broadening tutoring	The tier two schema-broadening tutoring consisted of instruction in three problem schemas (e.g. total, difference, and change) and four superficial transfer features (e.g. includes irrelevant information, uses 2-digit operands, has missing information in the first or second position, presents information in charts, graphs, or pictures). University students conducted the one-to-one tutoring sessions using scripted lessons. Pre- and post-testing involved multiple measures of foundational skills and word problem solving.
Results: ANOVAs applied to pre-test scores indicated group comparibility prior to treatment. Results of ANOVAs on improvement scores indicated no main effect of treatment for addition and subtraction fact retrieval, double digit addition and subtraction, Simple Algebraic Equations, WRAT Reading, and KeyMath Problem Solving. Significant effects and large effect sizes favoring the schema-broadening tutoring were found for WRAT Arithmetic, Jordan's Story Problems, and Peabody Word Problems.			
Fuson & Willis, 1989	Determine whether regular classroom teachers could implement SBI sufficiently to allow students' problem solving performance to improve. To determine if student problem solving would improve following teacher implementation of the schematic diagram intervention.	76, second graders in 3 ability-grouped (1 average-achieving, 1 high/average-achieving, 1 high-achieving) classrooms at two schools in a small city near Chicago.	Intervention in schema identification, diagram choice/labeling, and problem solution conducted by teachers following an in-service. Intervention addressed put-together, change, and compare problems. Classroom observations to determine facility with which teachers implemented the intervention. Pre- and post- testing using a 24-item researcher developed test scored for diagram drawing and labeling, solution strategy choice, and correct answer.
Results: Observations indicated that teachers implemented the interventions with varying facility. Despite variance in the quality of instruction, descriptive statistics indicated that all students made gains in choosing the correct diagram, labeling the diagram correctly, choosing the correct solution strategy, and arriving at the correct answer. Student ability and quality of instruction may both influence effectiveness of instruction using schematic diagrams on problem solving.			

Table A-1. Continued

Author/Date	Purpose	Participants	Procedures
Griffin & Jitendra, 2009	Assess differential effects and maintenance effects of SBI instruction and general strategy instruction (GSI). Assess the influence of word problem solving instruction on computation skills	60 students in 3 inclusive 3rd grade classrooms in an elementary school in a college town in FL. 4 teachers (3 general, 1 special education)	Assignment to conditions based on initial SAT-9 scores. Two instructional groups within each condition. SBI consisted of instruction in using schematic diagrams and self-monitoring to solve one- and two-step additive story problems involving group, change, and compare schemas. GSI consisted of strategies taught in math basal text. Intervention conducted for 100 minutes one day per week.
Results: ANOVA and Chi-square tests indicated group equivalency. ANCOVA indicated that students in both groups made statistically significant gains and maintained those gains on word problem solving, word problem solving fluency, and computation fluency, but no effect based on group assignment was noted. However, further analysis indicated that the SBI group acquired problem solving skills more quickly than did the GSI group. Group differences decreased over time.			
Jitendra, Griffin, Deatline-Buchman & Scesniak, 2007a	Two design studies to evaluate the efficacy of SBI to solve one-step group, change, and compare addition and subtraction story problems in classroom settings before conducted formal experimental studies.	Study 1: 38 students in 2 low-ability 3rd grade classes 2 general & 1 special education teacher. Study 2: 56 students in 2 hetero-geneously grouped 3rd grade classes, 2 teachers	In-service training for teachers. Whole class instruction conducted by teachers with support from researchers. Pre- and post testing on word problem solving criterion referenced test, word problem solving fluency test, and basic mathematics calculation fluency test. Student satisfaction questionnaire
Results: Study 1: Repeated measures ANOVA indicated statistically significant main effects for time on word problem solving criterion-referenced test and word problem solving fluency test. Moderate to large effect sizes. Similar results for low-achieving and learning disabled students. Study 2: Repeated measures ANOVA indicated statistically significant main effects for time on word problem solving fluency test and basic mathematics computation fluency test. Small to moderate effect sizes. Gains following schema-based instruction were more apparent for the low-performing students. Treatment fidelity high for both studies (93%, 98%).			

Table A-1. Continued

Author/Date	Purpose	Participants	Procedures
Jitendra, George, Sood, & Price, (2010)	To describe how SBI was used to improve the additive word problem solving of two students with Emotional Behavioral Disorders	4th grade student with severe learning disability and emotional disturbance, 5th grade students with behavioral disorders	Following pre-testing using a curriculum based measure, students received SBI in solving one- and two-step additive word problems from their trained special education teacher for 45 minutes daily for 20 weeks.
Results: Both participants showed gains on experimenter-developed word problem solving fluency probes as instruction progressed, and showed gains from pre- to post-test on experimenter developed word problem solving tests.			
Jitendra, Griffin, Haria, Leh, Adams, & Kaduvettoor, 2007b	Assess the differential effects, maintenance effects, and generalization effects of SBI and general strategy instruction (GSI).	94 students in 5 inclusive 3rd grade classrooms from an elementary school in a northeastern school district. 6 teachers (5 general, 1 special education)	Assignment to conditions based on initial SAT-9 scores. 3 instructional groups within each condition. SBI consisted of instruction in using schematic diagrams to solve one- and two-step additive story problems involving group, change, and compare schemas. GSI consisted of strategies taught in math basal text. Intervention conducted for 100 minutes one day per week.
Results: ANOVA and Chi-square tests indicated group equivalency. ANCOVAs of word problem solving and SAT-9 post-test scores indicated statistically significant effects and medium effect sizes in favor of SBI over GSI. ANCOVA applied to PSSA scores found statistically significant effects in favor of SBI over GSI. Analysis of group effects based on disability status, English language learner status, and Title I status resulted in statistically significant covariates for the word problem solving maintenance test and one subtest of the SAT-9 in favor of SBI.			

Table A-1. Continued

Author/Date	Purpose	Participants	Procedures
Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley, 1998	Compare effectiveness of SBI and traditional basal instruction on the ability of students with and at risk of disabilities to solve story problems. Examine maintenance and generalization.	34 elementary students from four public school classrooms in a northeastern and southeastern school district (25 with mild disabilities, 9 at risk)	Random assignment to groups. Teachers conduct traditional basal instruction. Researchers conduct schema-based instruction to small groups using diagrams to teach group, change, and compare problem types. Pre-, post-, maintenance, and generalization testing using 15-item experimenter designed word problem solving tests. Student interviews to assess value of instruction.
Results: ANOVA indicated no between group differences at pre-test. ANOVAs indicated statistically significant main effects favoring the SBI group for post- and maintenance testing. Statistically significant interaction between group and test time on generalization where SBI group improved at a greater rate than did the traditional basal instruction group.			
Jitendra & Hoff, 1996	Determine if students with learning disabilities would improve in solving simple one step story problems following SBI.	Three students attending third or fourth grade at a northeastern private elementary school for students with learning disabilities	Adapted multiple probes across students design with probes administered at baseline, following problem schemata instruction using story situations, following schema intervention using story problems, and two to three weeks later to assess maintenance. Schema-based instruction used diagrams to teach group, change, and compare problem types.
Results: No change in level from baseline to just after problem schemata instruction, increased level following intervention using story problems and schematic diagrams, and maintenance level at or near the levels seen following the intervention phase.			

Table A-1. Continued

Author/Date	Purpose	Participants	Procedures
Neef, Nelles, Iwata, & Page, 2003	Evaluate effects of teaching students with developmental disabilities the precurrent skills of identifying the component parts of change problems on their ability to solve addition and subtraction story problems.	Two young adult males (ages 19 and 23) with below average intellectual ability (IQ: 46 and 72) as measured by the Wechsler Adult Intelligence Scale	Multiple baseline across behaviors design with each precurrent skill representing a separate behavior. Problem solving probes consisting of 5 change story problems conducted at baseline and following training on each precurrent skill. Instruction was conducted one-to-one by researchers using a teaching to mastery.
Results: Participants demonstrated improved ability to identify components of change story problems following training sessions. After all precurrent skills had been trained, participants improved in their ability to solve addition and subtraction story problems.			
Rockwell, Griffin, & Jones, 2011	Provide preliminary evidence of the efficacy of using SBI to teach additive word problem solving to an elementary student with autism.	A fourth grade student with autistic disorder and low-average nonverbal cognitive abilities.	Multiple probes across behaviors design. Problem solving probes consisting of six story problems were conducted at baseline and following training on each problem type. SBI was conducted one-to-one by researchers.
Results: The participant demonstrated improved ability to solve each type of additive story problem following SBI. After one lesson on generalizing to problems with unknowns in the initial or medial position, the participant was able to successfully solve such problems. Problem solving gains were maintained six weeks following the intervention.			

Table A-1. Continued

Author/Date	Purpose	Participants	Procedures
Willis & Fuson, 1988	Assess efficacy of SBI to teach average and high achieving second graders to additive story problems. Assess which problem types were within the students' zone of proximal development	43 second grade students in two ability grouped (one high achieving, one average achieving) public school classrooms in a city near Chicago.	Intervention in schema identification, diagram drawing/labeling, and problem solution conducted by researchers to teach put-together, change, and compare problem types. Pre- and post- testing using a 10-item researcher developed test scored for diagram drawing and labeling, solution strategy choice, and correct answer.
Results: Descriptive statistics indicate increases in percentages of students choosing correct diagram and correctly labeling it from pre- to post-test. Confusions with put-together and compare problems noted. Statistically significant improvements in choice of solution strategy and correct answer were found using t tests. ANOVA revealed that problems involving an unknown in the first position were significantly more difficult for students.			
Xin, Wiles, & Lin, (2008)	Determine if combining SBI with instruction in story problem grammar would improve pre-algebraic concept formation and additive word problem solving performance of students with math difficulties	2 fourth grade students with learning disabilities and math difficulties. A fifth grade student with math difficulties	Used a multiple probes across participants and problem types design with probes conducted at baseline, following part-part-whole lessons, following additive compare problem lessons, and to assess maintenance. Researchers conducted instruction in 20-35 min sessions 3 days week. Story problem grammar instruction used cue cards to help students identify and map relevant information to diagrams
Results: Results indicated that the intervention was effective in improving students' performance on experimenter-developed word problem solving probes and equation solving probes, and on probes derived from the KeyMath-R/NU			

APPENDIX B
SAMPLE LESSON SCRIPT

Intervention: Change Lesson 1 Script

Materials

Word Problem solving (RUNS) Poster
Group Diagram Poster
Compare Diagram Poster
2 Copies of Change problems 4.1-4.6 on note cards
2 Copies of Compare problems 4.1-4.2 on note cards
2 Copies of Group problems 4.1-4.2. on note cards
1 Dry erase board
1 Dry erase pen
1 Copy of Change Practice Sheet 1
2 Pencils

Teacher:	**So far, you have learned to use the RUNS steps to solve one type of word problem. Let's review our RUNS steps. (***Display Word Problem solving (RUNS) poster. Point to each step on the word problem solving (RUNS) poster and have student name the step.*) **Tell me each step when I point to it.**
Student:	R – Read the problem; U – Use a diagram; N – Number sentence; S – State the answer.
Teacher:	**We have also learned a diagram to help us solve one type of problem. What type of problem did we learn the diagram for?**
Student:	Group problems.
Teacher:	**Right, we learned the group diagram. Can we solve all problems with the group diagram?**
Student:	No, we can only solve group problems.
Teacher:	**That's right, the group diagram can only be used to solve group problems. Today, we are going learn about the diagram for change problems. A change problem has a beginning, a change, and an ending. The beginning, change, and ending all describe the same thing/object. Our change problems can change in two ways. They can get more, or they can get less. Whether the change is to get more or get less, the change is always an action.** (*Display the change diagram poster.*) **Where on this change diagram do you think the beginning should go?**
Student:	(*Point to first circle.*)
Teacher:	**That's right, the beginning amount will go in this first circle. Where do you think the change amount should go?**
Student:	(*Point to middle box.*)

Teacher: **Right, the change amount goes in the middle box. If the change means get more, what operation could I put in this circle to show "get more?" Would "get more" mean add or subtract?**

Student: Add

Teacher: **Right, if the change is get more that means add. So when the change is get more I would put a plus sign and the change amount in the circle. What operation would I use to show "get less?"**

Student: Subtract

Teacher: **Right, if the change is "get less" that means subtract. So when the change is "get less" I put a minus sign and the change amount in the circle. Where does the ending amount go in this change diagram?**

Student: (*Point to circle on right.*)

Teacher: **That's right, the ending amount goes in the last circle. (***Display Change problem 4.1*) **I will use my RUNS steps and the change diagram to help me solve change problems. First I will write my RUNS steps on my dry erase board. Now, we are ready for Step I: <u>Read the problem</u>.** *(Point to the first checkbox on the problem solving checklist)*. **Follow along as I read. (***Read the problem aloud.*) **Tammy likes to paint pictures. She has painted 8 pictures so far. If she paints 3 more pictures, she will have 11 pictures. Now I can check off the R. The next step is Use a Diagram. I have two diagrams.** (*Point to each diagram and name it.*) **I need to decide if this is a change problem, a group problem or neither. I see that this whole problem is talking about the same thing – Tammy's pictures. I also see that we have a beginning – 8 pictures. Then I see a change – She paints 3 more pictures. I know this is a change because it is an action "paints." I can see that this is a change "get more" because she paints more. So my change is plus 3 paintings. This problem also has an ending – 11 pictures. So, this is a change problem. I will draw my change diagram on the dry erase board and fill in the numbers. Now I am ready for Step 3: <u>Number sentence</u>. I can use my diagram to write my number sentence. 8 + 3 = 11. Finally, I will state the answer. 11 paintings. Notice that I labeled my answer so we know what I'm talking about**.

Now let's look at another problem. (*Display Change problem 4.2*). **First, I will write RUNS on board to help me remember my steps. The first step is read the problem. Follow along as I read. (***Read the problem aloud*). **Ryan had $10. He spent $6 on melons for a picnic. He has $4 left. I read the problem. The U stands for Use a diagram. I need to decide which diagram I will use. I can tell that this is a change problem because it talks about just one thing – Ryan's money. I can also see that it has a beginning – Ryan had $10; and a change – he spent $6. The word spent tells me that this is a change get less – -$6. The problem also has an ending – $4 left. So, now I will draw my change diagram and put my numbers in. The next step is number sentence. I just use my diagram to help me. 10 – 6 = 4. Now I can**

state my answer 4. Exactly so I will write $4. Now let's look at this problem. (_Display group problem 4.1_). **First I will write RUNS on my board. R stands for read the problem. Follow along with me as I read. Sandra has 4 pairs of blue jeans. She also has 5 pairs of khakis. Sandra has 9 pairs of pants. Now I need to use a diagram. I must decide which diagram to use. I do not think that this is a compare problem. It talks about more than one thing – jeans, khakis, and pants; and it does not have an action that shows a change. Let me see if this could be a group problem. Jeans and khakis could be the small parts, and since jeans and khakis are both types of pants, pants would be the all. So this is a group problem. It has small parts – jeans and khakis; and a big all – pants. I will draw a group diagram on my board and put the numbers in. Now I will write my number sentence. 4 + 5 = 9. Now I can state my answer. 9 jeans. I labeled my answer.**

Here is another problem. (_Display compare problem 4.1_). **First I will write RUNS on my board. R stands for read the problem. Follow along with me as I read. James has 28 marbles. Joe has 52 marbles. James has 24 fewer marbles than Joe has. Now I will use a diagram. I must decide which diagram to use. I do not think that this is a change problem. It talks about more than one thing – James's marbles and Joe's marbles; and it does not have an action that shows a change. Let me see if this could be a group problem. James's marbles and Joe's marbles could be the small parts. If I put those together I would get James's marbles and Joe's marbles as the all. This problem does not talk about the all. So this is not a group problem either. I do not have a diagram to solve this problem yet. I will move onto the next problem. Now I want you to help me solve some problems. Here is the first one. (**_Display change problem 4.3_). **What should I do first?**

Student: Write RUNS on your board.

Teacher: Exactly. Write RUNS. **Now what should I do?**

Student: Read the problem

Teacher: Good. Follow along with me as I read. John has 15 toy cars. He gets 7 more for his birthday. Now John has 22 toy cars. Now what should we do?

Student: Use a diagram.

Teacher: We need to decide which diagram to use. Do you think this is a change problem?

Student: Yes

Teacher: How do you know?

Student: It talks about one thing – John's toy cars. And there is an action that shows a change – gets more toy cars.

Teacher: You're right, this is a change problem because it talks about one thing and has an action that shows a change. Now I will draw the change diagram. What goes in this first circle at the beginning?

Student: 15 toy cars

Teacher: Yes, at the beginning, John has 15 toy cars. What goes in this middle box for the change?

Student: 7 toy cars

Teacher: 7 toy cars is the amount of change. Hmm, I still need to know if this change was "get more" or "get less." What do I need?

Student: John got more. So we need a plus. We need to add.

Teacher: Yes, it says John got more cars, so that is adding. What goes in this last circle at the ending?

Student: 22 toy cars.

Teacher: Right, John had 22 cars at the ending. I used a diagram, now what should I do?

Student: Number sentence.

Teacher: What number sentence should I write?

Student: 15 + 7 = 22

Teacher: Very good, you used the diagram to make that number sentence. Now what do we do?

Student: State the answer.

Teacher: What should I write?

Student: 22 toy cars.

Teacher: Great work! I love how you labeled that answer. Now let's look at the next problem. (*Display group problem 4.2*). **What should I do first?**

Student: Write RUNS on your board.

Teacher: Exactly*.* Write RUNS. **Now what should I do?**

Student: Read the problem.

Teacher: Good. Follow along with me as I read. Jan bought 12 carrots and 9 tomatoes at the store. She bought 21 vegetables at the store. Now what should we do?

Student: Use a diagram.

Teacher: We need to decide which diagram to use. Do you think this is a change problem?

Student: No

Teacher: How do you know?

Student: It talks about more than one thing – carrots, tomatoes and vegetables. And there is no action.

Teacher: You're right, this can't be a change problem because it talks about more than one thing and has no action that shows a change. Do you think this a group problem?

Student: Yes

Teacher: How do you know?

Student: It has two small parts – carrots and tomatoes. Those are both vegetables. It has the all – vegetables.

Teacher: You're right, this a group problem because it has two small parts and a big all. Now I will draw the group diagram. What goes in this part?

Student: 12 carrots

Teacher: What goes in this part?

Student: 9 tomatoes
Teacher: Yes, 12 carrots and 9 tomatoes are the small parts. What goes in the all?
Student: 21 vegetables.
Teacher: Right, carrots and tomatoes are both vegetables. So that is the all. What now?
Student: Number sentence.
Teacher: What number sentence should I write?
Student: 12 + 9 = 21
Teacher: Very good, you used the diagram to make that number sentence. Now what do we do?
Student: State the answer.
Teacher: What should I write?
Student: 21 vegetables.
Teacher: I like that you labeled that answer. Here's another problem. Display change problem 4.4. **What should I do first?**
Student: Write RUNS on your board.
Teacher: Exactly. Write RUNS. **Now what should I do?**
Student: Read the problem.
Teacher: Good. Follow along with me as I read. Donovan collects words. He has 55 words in his collection. If he gives 27 of his words away he will still have 28 words. Now what should we do?
Student: Use a diagram.
Teacher: We need to decide which diagram to use. Do you think this is a change problem?
Student: Yes
Teacher: How do you know?
Student: It talks about one thing – Donovan's words. And there is an action that shows a change – gives away words.
Teacher: You're right, this is a change problem because it talks about one thing and has an action that shows a change. Now I will draw the change diagram. What goes in this first circle for the beginning.
Student: 55 words
Teacher: Yes, at the beginning, Donovan has 55 words. What goes in this middle box for the change?
Student: 27 words
Teacher: 27 words is the amount of change. Hmm, I still need to know if this change was "get more" or "get less." What do I need.
Student: Donovan gave words away. So he got less. We need a minus sign to subtract.
Teacher: Yes, it says Donovan gave words away, so that is subtracting. What goes in this last circle at the ending?
Student: 28 words.
Teacher: Right, Donovan had 28 words at the ending. I used a diagram, now what should I do?
Student: Number sentence.

Teacher: **What number sentence should I write?**
Student: 55 – 27 = 28
Teacher: **Very good, you used the diagram to make that number sentence. Now what do we do?**
Student: State the answer.
Teacher: **What should I write?**
Student: 28 words.
Teacher: **Great. I like that you labeled that answer. Now let's look at this problem. (***Display compare problem 4.2*). **What should I do first?**
Student: Write RUNS on your board.
Teacher: **Exactly***.* Write RUNS. **Now what should I do?**
Student: Read the problem
Teacher: **Good. Follow along with me as I read. John has 15 toy cars. Mark has 7 toy cars. John has 8 more toy cars than Mark has. Now what should we do?**
Student: Use a diagram.
Teacher: **We need to decide which diagram to use. Do you think this is a change problem?**
Student: No
Teacher: **How do you know?**
Student: It talks about more than one thing – John's toy cars and Mark's toy cars. And there is no action that shows a change.
Teacher: **You're right, this is not a change problem because it talks about more than one thing and has no action that shows a change. Do you think this is a group problem?**
Student: No
Teacher: **How did you know that?**
Student: Well, it sort of has two small parts – Mark's toy cars and John's toy cars. If you put those together you get Mark's toy cars and John's toy cars. It doesn't talk about that, so there's no all.
Teacher: **Right there is no all so this is not a group problem either. We haven't learned a diagram for this type of problem yet, so let's move on to the next problem. (***Display change problem 4.5*). **What should I do first?**
Student: Write RUNS on your board.
Teacher: **Exactly***.* Write RUNS. **Now what should I do?**
Student: Read the problem
Teacher: **Good. Follow along with me as I read. Lydia had 26 flowers on the roof. She planted 58 more flowers. Now Lydia has 84 flowers on the roof. Now what should we do?**
Student: Use a diagram.
Teacher: **We need to decide which diagram to use. Do you think this is a change problem?**
Student: Yes
Teacher: **How do you know?**

Student: It talks about one thing – Lydia's flowers. And there is an action that shows a change – plants more.

Teacher: You're right, this is a change problem because it talks about one thing and has an action that shows a change. Now I will draw the change diagram. What goes in this first circle at the beginning?

Student: 26 flowers

Teacher: Yes, at the beginning, Lydia had 26 flowers. What goes in this middle box for the change?

Student: 58 flowers

Teacher: 58 flowers is the amount of change. Hmm, I still need to know if this change was "get more" or "get less." What do I need?

Student: Lydia planted more. So we need a plus. We need to add.

Teacher: Yes, it says Lydia planted more flowers, so that is adding. What goes in this last circle at the ending?

Student: 84 flowers.

Teacher: Right, Lydia had 84 flowers at the ending. I used a diagram, now what should I do?

Student: Number sentence.

Teacher: What number sentence should I write?

Student: 26 + 58 = 84

Teacher: Very good, you used the diagram to make that number sentence. Now what do we do?

Student: State the answer.

Teacher: What should I write?

Student: 84 flowers.

Teacher: Great job labeling that answer. Let's do one more problem together. Display change problem 4.6. **What should I do first?**

Student: Write RUNS on your board.

Teacher: Exactly*.* Write RUNS. **Now what should I do?**

Student: Read the problem

Teacher: Good. Follow along with me as I read. There are 9 cakes at the bakery. Customers buy 7 of the cakes. There are 2 cakes left at the bakery. Now what should we do?

Student: Use a diagram.

Teacher: We need to decide which diagram to use. Do you think this is a change problem?

Student: Yes

Teacher: How do you know?

Student: It talks about one thing – cakes. And there is an action that shows a change – customers buy cakes.

Teacher: You're right, this is a change problem because it talks about one thing and has an action that shows a change. Now I will draw the change diagram. What goes in this first circle at the beginning?

Student: 9 cakes

Teacher: Yes, at the beginning, there are 9 cakes at the bakery. What goes in this middle box for the change?

Student: 7 cakes

Teacher: 7 cakes is the amount of change. Hmm, I still need to know if this change was "get more" or "get less." What do I need?

Student: Customers bought cakes. So there are fewer cakes at the bakery. That's get less. We need a minus for subtract.

Teacher: Yes, it says bought 7 cakes. That means there are fewer cakes at the bakery. So this is a subtraction problem. What goes in this last circle at the ending?

Student: 2 cakes.

Teacher: Right, there are 2 cakes in the bakery at the ending. I used a diagram, now what should I do?

Student: Number sentence.

Teacher: What number sentence should I write?

Student: 9 – 7 = 2

Teacher: Very good, you used the diagram to make that number sentence. Now what do we do?

Student: State the answer.

Teacher: What should I write?

Student: 2 cakes.

Teacher: Great job labeling the answer. You're becoming a pro at helping me use the RUNS steps with the group and change diagrams. Now I would like for you to try to complete the R-U-N-S steps for two change problems. On this piece of paper are two problems. Do your best to complete the R-U-N-S steps using the change diagram. Tell me when you are finished. (*Give student Change Practice Sheet 1 and a pencil. Provide access to RUNS poster and Change Diagram poster. Allow up to 10 minutes to complete the worksheet. Score worksheet for correct number sentence and computation. Provide corrective feedback*.). **You did a great job using the change diagram and the RUNS steps to solve problems. Tomorrow we will work on some more problems together and you will do some more problems by yourself.**

APPENDIX C
SAMPLE LESSON CHECKLIST

Change Lesson 1 Checklist

____Review RUNS

____Review Group Problems/Diagram

    ____Student identifies that group problems talk about more than one thing

    ____Student identifies that group problems have small parts

    ____Student identifies that the small parts are put together to make one big all

____Introduce Change Problems/Diagram

    ____Change problems talk about one thing

    ____Change problems have a beginning, change, and ending

    ____The change is an action that can mean “get more” or “get less”

____Model with think alouds using RUNS and Change Diagram – 6.1

    _____Read problem aloud

    _____Identify critical features of change problems that match diagram

    _____Draw and fill out diagram

    _____Write number sentence

    _____Write answer with label

____Model with think alouds using RUNS and Change Diagram – 6.2

    _____Read problem aloud

    _____Identify critical features of change problems that match diagram

    _____Draw and fill out diagram

    _____Write number sentence

    _____Write answer with label

____Model using RUNS and using Group Diagram with Group Problem – 6.1

    _____Read problem aloud

    _____Identify critical features of group problems that match diagram

    _____Draw and fill out diagram

    _____Write number sentence

    _____Write answer with label

____Model using RUNS and not using a diagram with Compare Problem – 6.1

_____Read problem aloud

_____Identify lack of critical group or change problem features

____Have student help using RUNS and Change Diagram – 6.3

_____Have student identify the steps to be followed

_____Teacher prompts, praises, provides corrective feedback as needed

_____Read problem aloud

_____Identify lack of critical features of change problems that match diagram

_____Draw and fill out diagram

_____Write number sentence

_____Write answer with label

____Have student help using RUNS and Group Diagram with Group problem – 6.2

_____Have student identify the steps to be followed

_____ Teacher prompts, praises, provides corrective feedback as needed

_____Read problem aloud

_____Identify lack of critical features of group problems that match diagram

_____Draw and fill out diagram

_____Write number sentence

_____Write answer with label

____Have student help using RUNS and Change Diagram – 6.4

_____Have student identify the steps to be followed

_____Teacher prompts, praises, provides corrective feedback as needed

_____Read problem aloud

_____Identify lack of critical features of change problems that match diagram

_____Draw and fill out diagram

_____Write number sentence

_____Write answer with label

____Administer Change Practice Sheet 1

____Provide corrective feedback/praise

____Preview next lesson

____Provide intermittent reinforcement for on-task behavior

____Allow up to 2 movement breaks during lesson

APPENDIX D
SAMPLE PRACTICE SHEETS WITH HYPOTHETICAL RESPONSES AND SCORING

Group Story Situation Practice Sheet

1. There are 11 bath towels and 7 dish towels in the linen closet. There are 18 towels in the linen closet.

<table>
<tr><td>11</td><td>7</td></tr>
<tr><td colspan="2">18</td></tr>
</table>

11+7=18

Answer: 18 towels
Scoring: _1_Correct diagram, _1_Correct number sentence, _1_Correct computation

2. There are 16 puppies and 13 kittens at the pet store. There are 29 animals at the pet store.

<table>
<tr><td>16</td><td>13</td></tr>
<tr><td colspan="2">29</td></tr>
</table>

16+13=19

Answer: 19 animals
Scoring: _1_Correct diagram, _1_Correct number sentence, _1_Correct computation

Group Problem solving Practice Sheet

1. Jose has 46 legos and 53 wooden blocks. How many blocks does Jose have?

46	53
?	

46+53=98

Answer: 98 blocks

Scoring: 1 Correct diagram, 0 Correct number sentence, 0 Correct computation

2. There are 51 girls and 46 boys in the third grade. How many children are in the third grade?

51	46
?	

51+46=97

Answer: 97 children

Scoring: 1 Correct diagram, 1 Correct number sentence, 1 Correct computation

Change Story Situation Practice Sheet

1. There are 77 apples on a tree. Some people pick 32 apples. Now there are 45 apples on the tree.

$$\boxed{77} \rightarrow (-32) \rightarrow \boxed{45}$$

77 - 32=45

Answer: ________45 apples____

Scoring: _1_Correct diagram, _1_Correct number sentence, __1_Correct computation

2. Jack has $23 in his bank account. He gets $50 for his birthday. Now he has $73 in his bank account.

$$\boxed{23} \rightarrow (+50) \rightarrow \boxed{73}$$

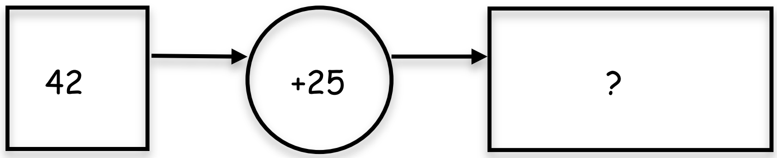
23+50=73

Answer: ______$73____

Scoring: _1_Correct diagram, _1_Correct number sentence, __1_Correct computation

Change Problem solving Practice Sheet

1. Julie has 42 silly bands. She buys 25 more silly bands. How many silly bands does Julie have now?

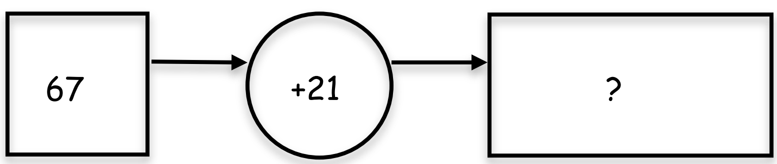


42+25=67

Answer: 67 silly bands

Scoring: _1_ Correct diagram, _1_ Correct number sentence, _1_ Correct computation

2. There were 67 roses in the garden. The gardener picked 21 roses to put in a vase. How many roses are in the garden now?



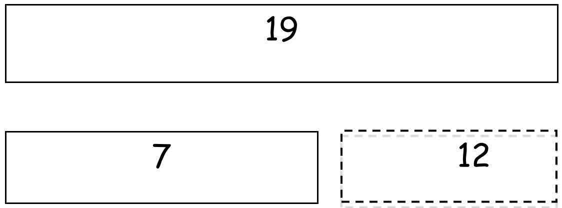
67+21=88

Answer: 88 roses

Scoring: _0_ Correct diagram, _0_ Correct number sentence, _0_ Correct computation

Compare Story Situation Practice Sheet

1. There are 19 bath towels and 7 dish towels in the linen closet. There are 12 more bath towels than dish towels in the linen closet.

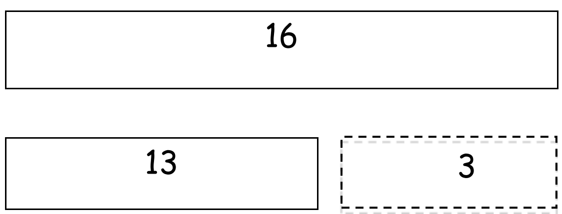


$$19 - 7 = 12$$

Answer: ____12 more dish towels________________________________

Scoring: _1__Correct diagram,_1__Correct number sentence, _1__Correct computation

2. There are 16 puppies and 13 kittens at the pet store. There are 3 fewer kittens than puppies at the pet store.



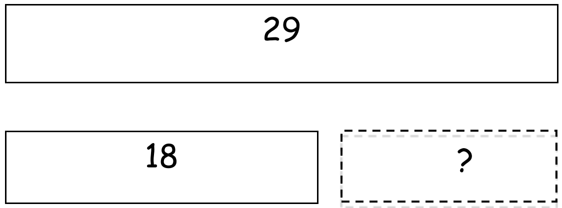
$$16 - 13 = 3$$

Answer: ____3 fewer kittens________________________________

Scoring: _1__Correct diagram,_1__Correct number sentence, __1_Correct computation

Change Problem solving Practice Sheet

1. Mrs. Smith baked 29 cookies. She also baked 18 cupcakes. How many more cookies than cupcakes did Mrs. Smith bake?

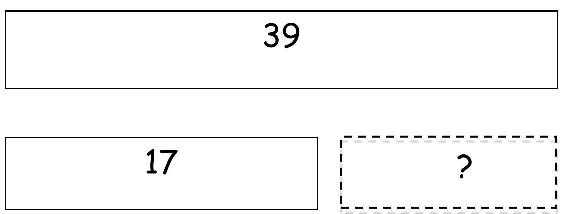


29 – 18=11

Answer: ______11 more cookies______________________________

Scoring: _1__Correct diagram,_1__Correct number sentence, _1__Correct computation

2. At the animal shelter there are 39 dogs and 17 cats. How many fewer cats are there than dogs at the animal shelter.



39 – 17=23

Answer: ______23 fewer cats______________________________

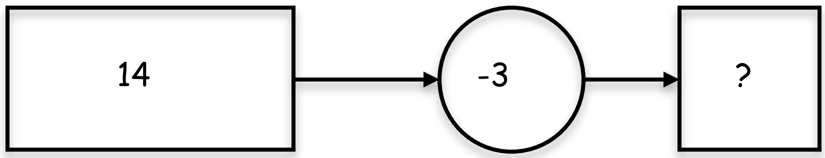
Scoring: _1__Correct diagram,_1__Correct number sentence, _0__Correct computation

APPENDIX E

SAMPLE PROBES WITH HYPOTHETICAL RESPONSES AND SCORING

Problem solving Probe

1. A scout troop leader is taking 14 scouts to the movies. Three scouts cancelled. How many scouts are going to the movie now?

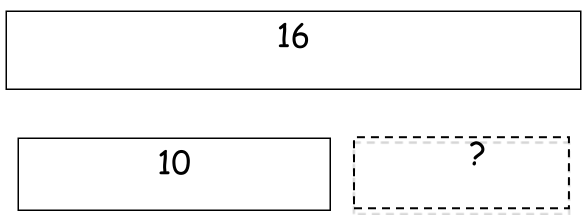


14 – 3 = 11

ANSWER: 11 scouts

Scoring: 1 Correct number sentence, 1 Correct computation

2. There are 10 crickets in the shed and 16 crickets outside. How many more crickets are outside than in the shed?

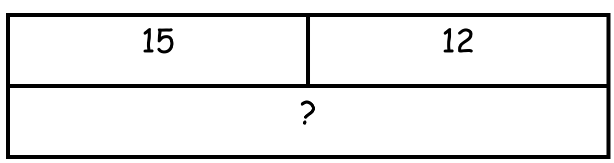


16 – 10 = 6

ANSWER: 6 crickets

Scoring: 1 Correct number sentence, 1 Correct computation

3. You spend $15 on games and $12 on balls. How much money did you spend?

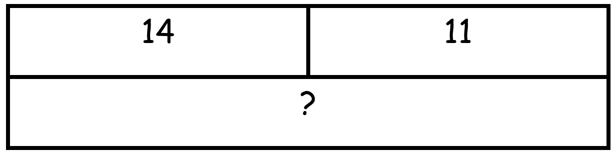


15 + 12 = 27

ANSWER: 27 dollars

Scoring: 1 Correct number sentence, 1 Correct computation

4. Paul picks 14 apples from one tree. He picks 11 apples from another. How many apples did he pick?

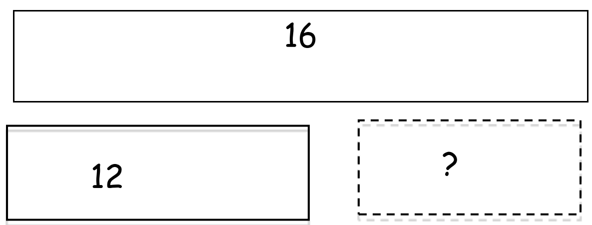


14 + 11 = 25

ANSWER: 25 apples

Scoring: 1 Correct number sentence, 1 Correct computation

5. There are 12 chicks in the shed and 16 chicks outside. How many fewer chicks are in the shed than outside?

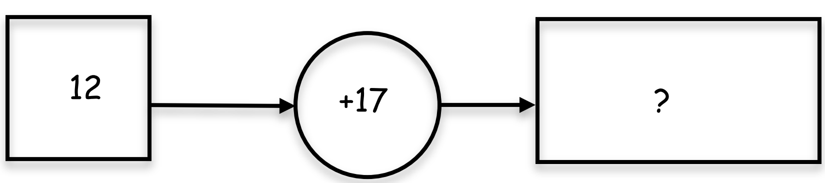


16 – 12 = 2

ANSWER: 2 chicks

Scoring: 1 Correct number sentence, 0 Correct computation

6. There are 12 chicks at the farm. If 17 more chicks are born, how many chicks are at the farm now?

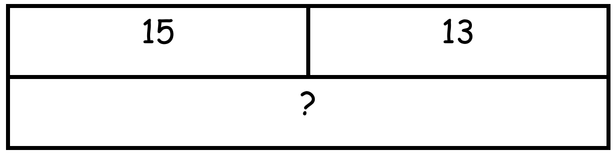


12 + 17 = 29

ANSWER: 29 chicks

Scoring: 1 Correct number sentence, 1 Correct computation

7. Mrs. Smith buys 15 carrots and 13 potatoes. How many vegetables did she buy?

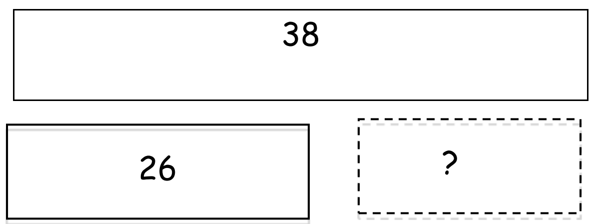


15 + 13 = 27

ANSWER: 27 vegetables

Scoring: 1 Correct number sentence, 0 Correct computation

8. Joe has 38 baseball cards. Tom has 26 baseball cards. How many more baseball cards does Joe have than Tom?

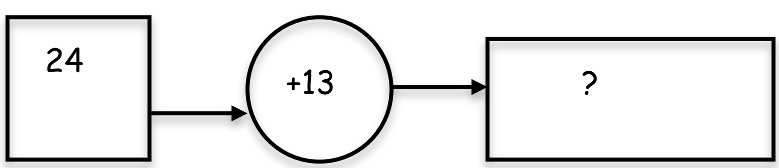


38 – 26 = 12

ANSWER: 12 baseball cards

Scoring: 1 Correct number sentence, 1 Correct computation

9. There are 24 children going on the field trip. If 13 more children decide to go, how many children will be going on the field trip then?



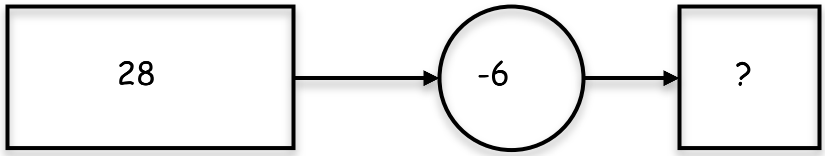
24 + 13 = 37

ANSWER: 37 children

Scoring: 1 Correct number sentence, 1 Correct computation

Generalization Probe – Irrelevant Information

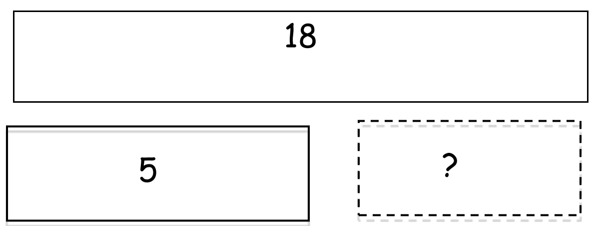
1. A teacher is taking 28 students to the museum. If 6 students get sick and it costs $5 per person to enter the museum, how many students went to the museum?



28 – 6=22

ANSWER: 22 students
Scoring: _1_ Correct number sentence, _1_ Correct computation

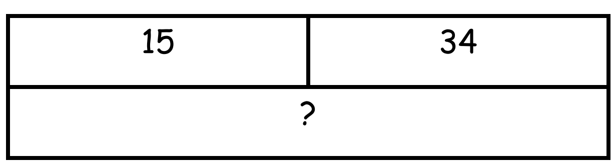
2. There are 18 students on the monkey bars, 6 students on the swings, and 5 students in the sand box. How many more students are on the monkey bars than in the sand box?



88 – 6 = 13

ANSWER: 13 more students
Scoring: _0_ Correct number sentence, _0_ Correct computation

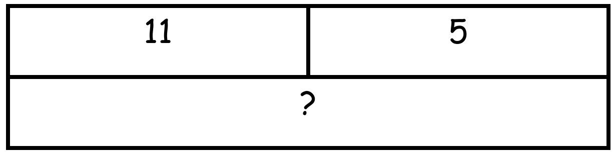
3. Mrs. Jones spent $15 on a shirt, $34 on a dress, and $7 on lunch. How much money did she spend on clothing?



15 + 34 = 48

ANSWER: $48
Scoring: _1_ Correct number sentence, _0_ Correct computation

4. Peg bought 11 apples, 5 pears, and 2 loaves of bread. How many pieces of fruit did Peg buy?

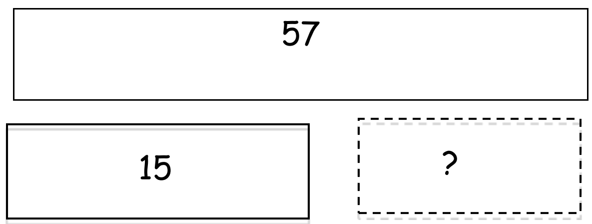


11+5-15

ANSWER: 15 pieces of fruit

Scoring: __1 Correct number sentence, _0_ Correct computation

5. There are 57 cows in the pasture, 15 cows in the barn and, and 6 horses in the stable. How many more cows are in the pasture than in the barn?

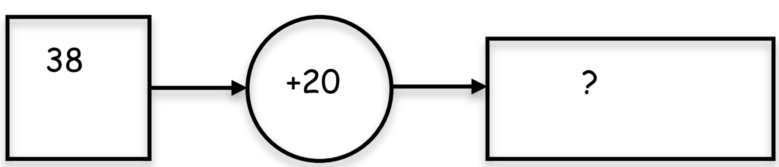


57 - 15 = 42

ANSWER: 42 more cows

Scoring: _1_ Correct number sentence, _1_ Correct computation

6. Kelly has $38 in her piggybank. She gets $20, some books, and a bicycle for her birthday. How much money does Kelly have now?

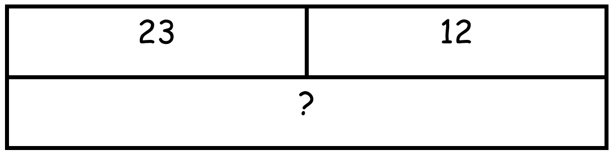


38+20=58

ANSWER: $58

Scoring: _1_ Correct number sentence, _1_ Correct computation

7. Jim looks out his class window. He sees 23 cars, 12 trucks, and 5 trees in the parking lot. How many vehicles does Jim see?

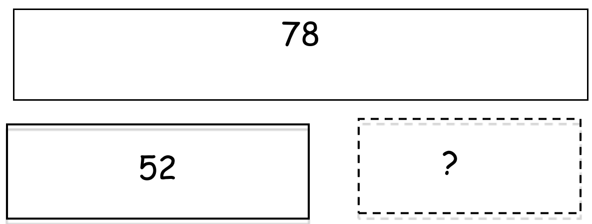


23+12=35

ANSWER: 35 vehicles

Scoring: 1 Correct number sentence, 1 Correct computation

8. Jan's book has 52 pages, Paul's book has 78 pages, and Amy's book has 92 pages. How many fewer pages does Jan's book have than Paul's?

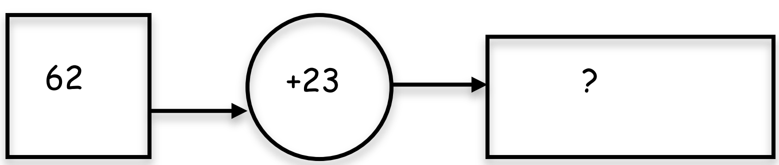


78 - 52=26

ANSWER: 26 fewer pages

Scoring: 0 Correct number sentence, 0 Correct computation

9. There are 62 cows and 37 pigs on a farm. If 23 more cows are born, how many cows will be on the farm?



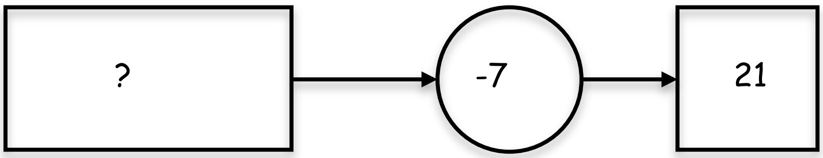
62 + 23 = 85

ANSWER: 85 cows

Scoring: 1 Correct number sentence, 1 Correct computation

Generalization Probe – Initial Position

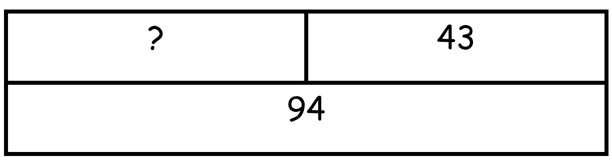
1. Many children ride the school bus home. At the first stop 7 children get off the bus. Now there are 21 students on the bus. How many children ride the bus home?



21+7=28

ANSWER: 28 children
Scoring: 1 Correct number sentence, 1 Correct computation

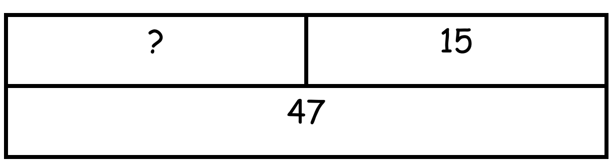
2. There are some boys and 43 girls in the first grade. If there are 94 children in first grade, how many boys are there?



94 – 43=61

ANSWER: 61 boys
Scoring: 1 Correct number sentence, 0 Correct computation

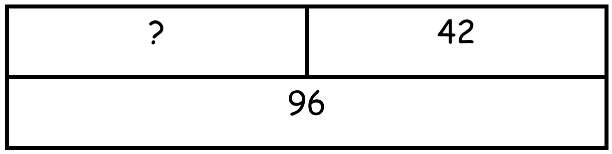
3. Mr. Smith bought pants and a shirt. The shirt cost $15. If he spent $47, how much did the pants cost?



47 – 15=32

ANSWER: $32
Scoring: 1 Correct number sentence, 1 Correct computation

4. A farmer has some pigs and 42 cows. The farmer has 96 animals. How many pigs does he have?

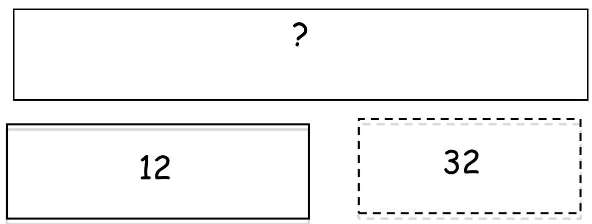


96 – 42=54

ANSWER: 54 pigs

Scoring: 1 Correct number sentence, 1 Correct computation

5. There are some pigs in the barn and some 12 pigs outside in the mud. If there are 32 more pigs in the barn than in the mud, how many pigs are in the barn?

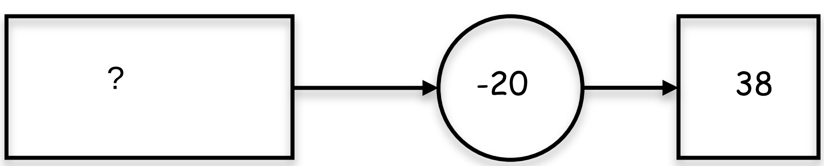


32 – 12 = 20

ANSWER: 20 pigs

Scoring: 0 Correct number sentence, 0 Correct computation

6. Kim has some money in her piggybank. She spends $20 on a new doll. Now she has $38. How much money did Kim have in her piggybank?



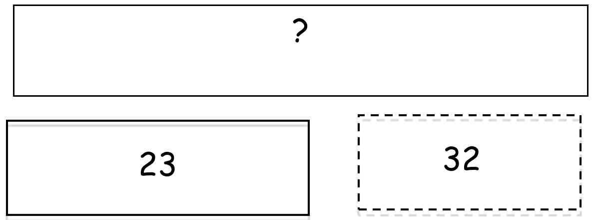
38 + 20=58

ANSWER: $58

Scoring: 1 Correct number sentence, 1 Correct computation

7. There are some goldfish and 23 guppies in a fish tank. If there are 32 fewer guppies than goldfish in the tank, how many goldfish are there?

1.



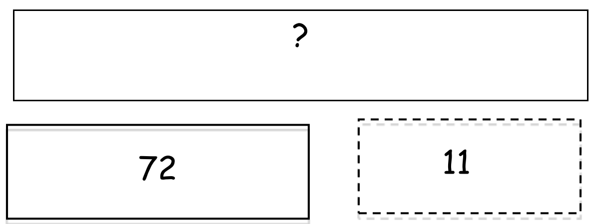
23+32=55

ANSWER: 55 goldfish

Scoring: _1_Correct number sentence, _1_Correct computation

8. Jim can type faster than Pete. Pete can type 72 words a minute. If Pete can type 11 words fewer than Jim, how many words can Jim type in a minute?

2.

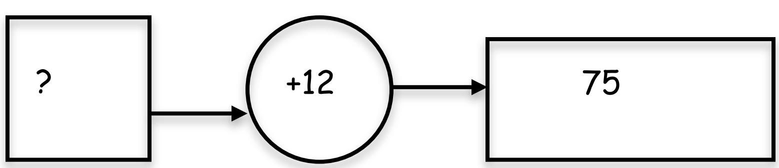


72+11=83

ANSWER: 83 pages

Scoring: _1_Correct number sentence, _1_Correct computation

9. Ellen has read a lot of books this year. If she reads 12 more books, she will have read 75 books. How many books has she already read?



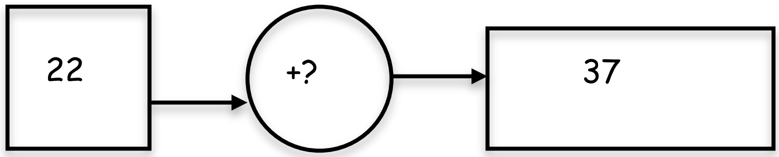
75 – 12=63

ANSWER: 63 books

Scoring: _1_Correct number sentence, _1_Correct computation

Generalization Probe – Medial Position

1. There were 22 children in little league. Some more children signed up. Now there are 37 children in little league. How many children signed up?

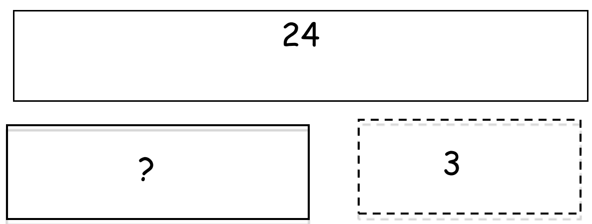


37+22=59

ANSWER: 59 children

Scoring: 0 Correct number sentence, 0 Correct computation

2. There are 24 students in Ms. Kent's class. There are 3 more students in Ms. Kent's class than Ms. Lee's class. How many students are in Ms. Lee's class?

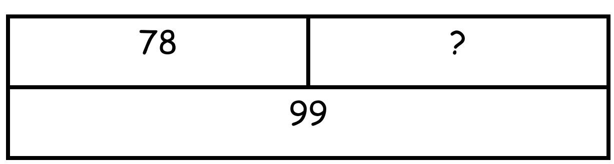


24 – 3=21

ANSWER: 21 children

Scoring: 1 Correct number sentence, 1 Correct computation

3. A mom bought her son a bike and a helmet for his birthday. The bike cost $78. If she spent $99, how much did the scooter cost?

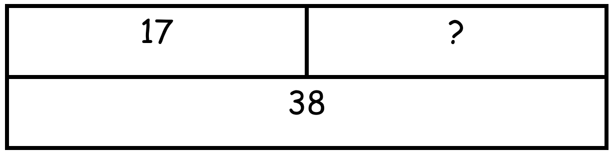


99 – 78=21

ANSWER: $21

Scoring: 1 Correct number sentence, 1 Correct computation

4. There are 38 children on a soccer team. If there are 17 boys on the team, how many girls are on the team?

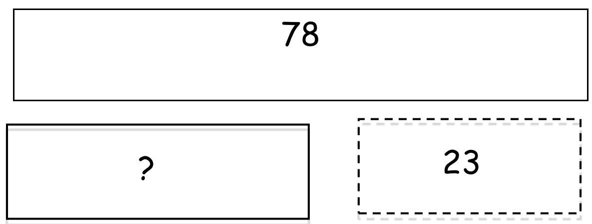


38 – 17=21

ANSWER: 21 girls

Scoring: _1_ Correct number sentence, _1_ Correct computation

5. 78 first graders ride the bus to school. If 23 more first graders ride the bus than get dropped off, how many first graders get dropped off?

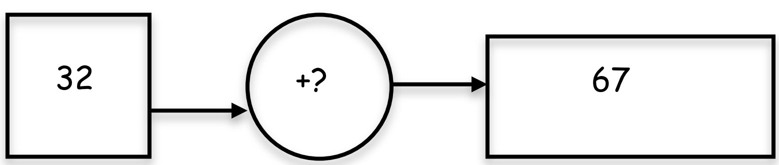


78 – 23=55

ANSWER: 55 first graders

Scoring: _1_ Correct number sentence, _1_ Correct computation

6. Kelly has 32 silly bands. She uses her allowance to buy some more silly bands. Now she has 67 silly bands. How many silly bands did Kelly buy?

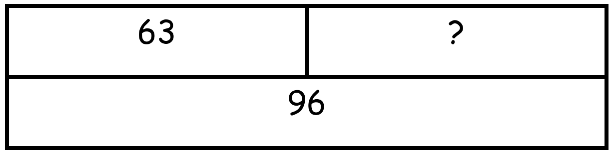


67 - 32=35

ANSWER: 35 silly bands

Scoring: _1_ Correct number sentence, _1_ Correct computation

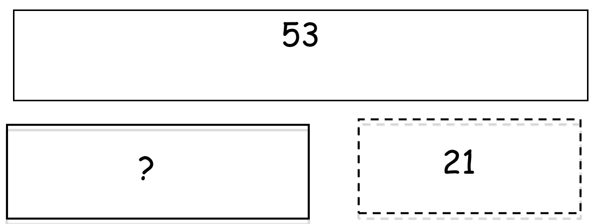
7. 63 children bought lunch today. If there are 96 children eating lunch in the school cafeteria, how many of them brought lunches from home?



96 – 63=36

ANSWER: 36 children
Scoring: _1_Correct number sentence, _1_Correct computation

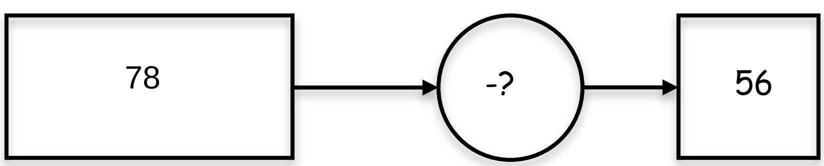
8. The apple tree has 53 pieces of fruit on it. The orange tree has 21 fewer pieces of fruit than the apple tree. How many pieces of fruit does the orange tree have?



53-21 = 32

ANSWER: 32 pieces of fruit
Scoring: _1_Correct number sentence, _1_Correct computation

9. 78 students tried out for the high school football team. After cuts, 56 students were on the team. How many students got cut?



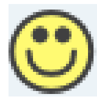
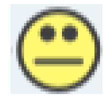
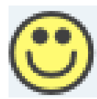
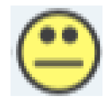
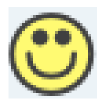
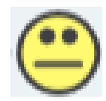
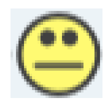
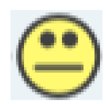
78 - 56=22

ANSWER: 22 students
Scoring: _1_Correct number sentence, _1_Correct computation

APPENDIX F
SATISFACTION SCALES

Student Satisfaction Scale

Circle the face that mathes each statement.

How I feel about solving math word problems			
How I feel about using the RUNS steps to solve math word problems			
How other kids might feel about using the RUNS steps to solve math word problems			
How I feel about using diagrams to solve math word problems			
How other kids might feel about using diagrams to solve math word problems?			

Parent Satisfaction Scale

Please place an X in the box indicating the extent to which you agree or disagree with each of the following statements about Schema-Based Strategy Instruction (SBI).

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
SBI addresses important skills for my child					
SBI was a valuable use of instructional time					
My child seemed to enjoy SBI					
My child uses the RUNS mnemonic when solving math word problems					
My child uses schematic diagrams when solving math word problems					
My child has shown improvement in solving math word problems					
I would like my child's teacher to learn to implement SBI with my child					
I would recommend SBI to others					

Please provide any additional comments regarding SBI:

APPENDIX G
INFORMED CONSENT DOCUMENTS

The following informed consent and assent documents were approved by the University of Florida's Institutional Review Board on July 28, 2010 and renewed for use through July 28, 2012. This study is Protocol #2010-U-0649.

Child Consent Script

Investigator: Hi! My name is Sarah Rockwell and this is (other doctoral student). We are students at the University of Florida. We are working on a special project to help us learn more about how to teach children to solve math word problems.

For the next nine weeks, we will be working with you every day to help you learn to solve math word problems. We are also going to ask you to solve some word problems on your own. Is it OK for us to teach you to solve word problems?

(Child's response)

Is it OK if we ask you to solve some word problems on your own?

(Child's response)

Do you have any questions about what we want you to do?

(Child's response)

Investigator: I need to tell you one more thing. After we get started, if you decide that you don't want to be in this study any more, let me know. It is okay if you decide not to be in the study.

Is it okay to start now?

(Child's response)

Parent Informed Consent Letter

September 16, 2010

Dear Parent/Guardian:

I am a doctoral candidate in the School of Special Education, School Psychology, and Early Childhood Studies at the University of Florida. I will be conducting a study to evaluate the effectiveness of an approach to teaching children with autism to solve addition and subtraction word problems. This approach is called Schema-Based Strategy Instruction and involves teaching children to use diagrams to solve math word problems.

The procedures used in this study are as follows. First, your child will participate in screenings to determine if he/she is able to perform addition and subtraction computations and read word problems. Then your child will participate in pre-testing conducted one-to-one using nine-item problem solving tests. Next, I and another doctoral student will conduct one-to-one lessons to teach your child to use diagrams to solve addition and subtraction word problems. This instruction will take place during 30-minute sessions conducted daily for approximately nine weeks. Instruction and assessments will take place after school at a time that is convenient for you. The Center for Autism and Related Disabilities (CARD) has agreed to provide space for this study, or you may choose a location that will be more convenient for you. Instruction will address three types of addition and subtraction word problems. Your child will be tested using the same nine-item tests after learning each to solve each type of problem. After instruction, you and your child will complete short questionnaires asking your perceptions of the instruction. Finally, your child will participate in follow-up testing eight weeks later conducted one-to-one using the same nine-item tests.

I will need your permission to obtain background information about your child from existing school records. Specifically, I would like to have access to current scores from standardized tests of achievement and tests of learning aptitude. The information will be used to identify your child’s skill level. No identifying information about your child will be reported. In addition, I will need your permission to videotape lessons and assessments conducted with your child. I and another doctoral student will view these videotapes in order to collect data on the quality of instruction provided to your child and on your child’s problem solving strategies and behaviors. When not in use, the videotapes will be stored in a secured and locked cabinet. I will also need your permission to conduct individual problem solving assessments with your child. The results of these assessments will be used to determine your child’s mathematics progress. No identifying information about your child will be reported. Finally, I will need your permission to conduct satisfaction questionnaires with you and your child. The results of these questionnaires will be used to ensure that instruction was enjoyable for and beneficial to your child and no information about you or your child will be reported.

If you agree to allow your child to participate in this study, both you and your child retain the right to withdraw consent for participation at any time without penalty. This will be explained to your child. No compensation will be given to your child for participation in this project. In addition, no risks or benefits to you or your child are anticipated as a result of participation in this study. If you should have any questions about your child's participation, please feel free to call me at (352) 284-6000 or contact my faculty supervisor, Dr. Cynthia Griffin at (352) 273-4265. We would be happy to talk to you about the project. Questions or concerns about research participant's rights may be directed to the UFIRB office, Box 112250, University of Florida, Gainesville, FL 32611-2250; phone: (352) 392-0433.

Sincerely,

Sarah B. Rockwell, M.Ed.
Doctoral Candidate
School of Special Education, School Psychology, and Early Childhood Studies
University of Florida

_ _

Please return this section as soon as possible.

I have received a copy of the letter from Sarah B. Rockwell describing the study of using schema-based strategy instruction to teach addition and subtraction word problem solving to students with autism. I have read the procedures described. I agree to allow my child to participate in the project.

Student's Name ___________________________

Parent's or Guardian's Signature _______________________________

2nd Parent/Witness Signature ___________________________

Date: _________________________

In addition, I give my permission for Sarah B. Rockwell or another doctoral student to obtain my child's school records to collect my child's most recent scores on tests of academic achievement and learning aptitude.
(Please check "yes" or "no") ___________ yes _____________ no

I also give my permission for Sarah B. Rockwell or another doctoral student to videotape mathematics lessons and assessments with my child.
(Please check "yes" or "no") ___________ yes _____________ no

I also give my permission for Sarah B. Rockwell or another doctoral student to assess my child's ability to read word problems and perform addition and subtraction computations.
(Please check "yes" or "no") ___________ yes _____________ no

I also give my permission for Sarah B. Rockwell or another doctoral student to assess my child's mathematics progress using nine-item problem solving tests.
(Please check "yes" or "no") ___________ yes _____________ no

I also give my permission for Sarah B. Rockwell or another doctoral student to assess my and my child's perceptions of instruction using satisfaction questionnaires.
(Please check "yes" or "no") ___________ yes _____________ no

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BIOGRAPHICAL SKETCH

Sarah B. Rockwell has 16 years of experience working with children with autism and other developmental disabilities. While in high school, she logged over 300 hours volunteering in the homes and classrooms of children with disabilities. While working on her undergraduate degree, she continued her volunteer work in early intervention and pre-kindergarten classes for children with disabilities. She also worked as substitute teacher in special education classes and as a Personal Care Assistant and Behavior Technician for Special Friends, Inc., a company providing services for children with the Medicaid waiver due to their disability status. After receiving her undergraduate degree in special education with a focus on early childhood at the University of Florida in 2004, Sarah began working as a teacher in a pre-kindergarten class for students with disabilities. She later taught students in grades two thru five with Autism and worked as an educational diagnostician and consultant at the Multidisciplinary Diagnostic and Training Program in the department of Pediatric Neurology at the University of Florida. Sarah was awarded her master's degree in special education with a focus on reading disabilities from the University of Florida in 2007. She immediately began working as a full-time doctoral student focusing her energy on mathematics instruction for students with autism. When Sarah became a mother in January 2010, she developed an interest in baby-wearing and breastfeeding education. She is a trained baby-wearing educator, and would like to combine her interests in special education, child development, and baby-wearing by conducting future research on the role of baby-wearing in sensory integration, muscle tone, and social and communication behavior of children with disabilities.