MODELING AND CONTROL OF A QUAD-ROTOR HELICOPTER

By

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To all of my thankful professors, family, friends and the Republic of Korea (ROK) Army

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Abstract of Thesis Presented to the Graduate School
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MODELING AND CONTROL OF A QUAD-ROTOR HELIIOPTER

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Chair : Rick Lind
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The quad-rotor helicopter is a UAV (Un-manned Air Vehicle) and has four rotors. To get efficient stabilization status, data measurement and analysis phase, and simulation and control phase are needed. During data measurement and analysis phase, people are struggling with inaccurate data due to the external disturbance or the battery issue. The author tried to get the moment of inertia of the quad-rotor helicopter as accurate as possible based on the experimental data. During simulation and control phase, the author applied PD (Proportional, Derivative) controller. First of all, trying to satisfy stabilization for the ideal case, which means all motors have same thrusts and speeds. Then the real motor values were applied to see the success of the controller's performance.

CHAPTER 1
INTRODUCTION

1.1 Motivation

Recently, there are many researches about helicopters, UAV and MAV (Micro Air Vehicle). Quad-rotor helicopters which are inexpensive to prepare, are of less mechanical complexity, better safety and higher payload. Especially this research is related to the real helicopters, so it is great topic to the author as a current army aviation officer. Therefore the author decided to study and research for VTOL (Vertical Take-Off and Landing) aircraft field with a quad-rotor helicopter.

Quad-rotor helicopter can be utilized such as surveillance, transporting or researching geography, and so on. And it is a good research project topic, because there is not a unique way to define control the quad-rotor helicopter, then anybody can try to approach different methods to find control a quad-rotor helicopter. Fundamentally the helicopter has six degrees of freedom and in case of the quad-rotor model, it has four motors.

In order to complete this project, all courses such as control theory, robot geometry and dynamics, those are required to graduate master's degree program. During preparing and researching this topic, the author could review and understand the relationship among all courses and utilized them to complete this research.

First of all, the author measured and searched all specifications from the real quad-rotor helicopter model. Then the author built a system model which is based on its dynamics and state space form. In order to satisfy stabilization and proper control, the author applied PD (Proportional, Derivative) controller with an ideal case and the measured values.

The main purpose of this thesis is showing stabilization and control through MATLAB and SIMULINK. The author will match with a real model in the future.

There are many conference papers, thesis about quad-rotor helicopters. Those projects try to set up their model and simulation for effective controlling based on different quad-rotor models.

- A nonlinear dynamic MIMO (Multi Input Multi Output) model of a 6-DOF (Degrees Of Freedom) quad-rotor helicopter is derived based on Newton-Euler formalism. [1]
- Derivation about quad-rotor helicopter dynamics and control [2]
- Present the mechanical design, dynamical modeling, sensing, and controlling of an indoor quad-rotor helicopter [3]
- Different types of controller (Lyapunov, PID, LQ, back-stepping and sliding-mode concepts are used) are implemented for validating through various flight experiments [4]
- Present the results of two nonlinear control techniques (back-stepping and sliding-mode) applied to a quad-rotor helicopter. [5]
- Describes the control approach (Integral Back-stepping) and the scheme for full control of quad-rotor helicopters (attitude, altitude and position). [6]
- The dynamic system modeling and the control algorithm evaluation for a quad-rotor helicopter [7]
- A problem of attitude stabilization and robust regulation of an indoor quad-rotor helicopter (The paper shows the design of continuous-time controller based on Dynamic Contraction Method) [8]
- A dynamic model of such a vehicle using bond graphs [9]
- A quad-rotor helicopter using custom-built chassis and avionics with off-the-shelf motors and batteries, to be a highly reliable experimental platform (A linear SISO controller was designed to regulate flyer attitude) [10]

1.2 Problem Statement

- What are basic movements of the quad-rotor helicopter?
- What is the equation of motion for the quad-rotor helicopter?
- How to control the quad-rotor helicopter properly and accurately when the different inputs / thrust (roll, pitch and yaw) are given?
- How to develop the controller when the ideal cases (pure movements) are implemented and when the measured values are applied?

1.3 Contribution

- Setting up the computer model which is based on the real quad-rotor helicopter model
- Applying PD (Proportional, Derivative) and PID (Proportional, Integral and Derivative) controllers for satisfying stabilization and controlling the model properly and accurately
- Verifying the PD and PID controllers by applying the ideal cases and the measured values

CHAPTER 2
QUAD-ROTOR HELICOPTER SPECIFICATIONS

2.1 Model Picture

Figure 2-1 shows the real quad-rotor helicopter which was used in this thesis. This quad-rotor helicopter model belongs to Geo-matics lab. There are four motors, and each motor has a propeller.

2.2 Specifications

Table 2-1 and Table 2-2 show the specifications of the model and the motor. And the motor model was Turnigy SK 4250-650. Compared to other general quad-rotor helicopter, this model is big and it is not easy to control without controller.

2.3 Basic Instruments

Table 2-3 shows the instruments that were needed during this research. During using digital tachometer and thrust meter, the motor from the quad-rotor helicopter must be linked to the solid body. Then the meter equipment could measure the maximum and the minimum value and it showed on the screen.

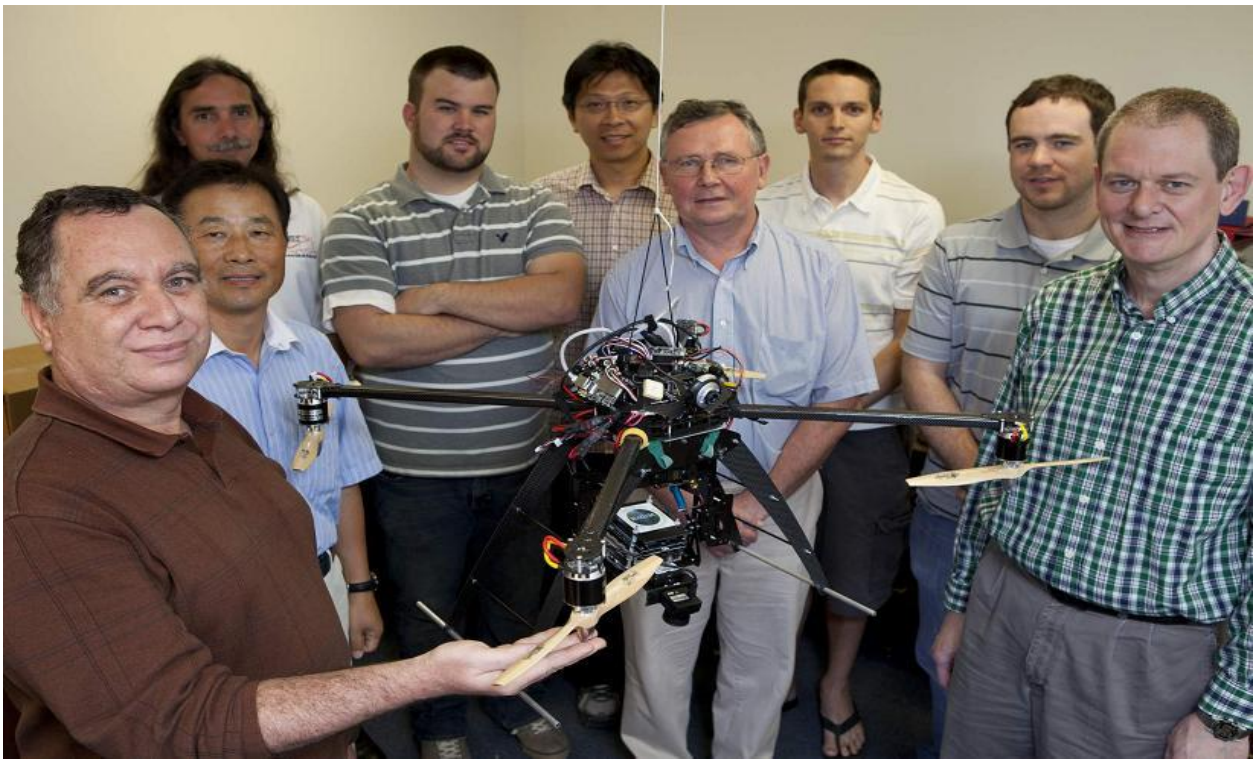


Figure 2-1. Quad-rotor helicopter model picture

Table 2-1. Weights

	Motor	Battery	Frame (Whole body)	Hub
Unit (Kg)	0.288	0.356	0.701	0.293

Table 2-2. Lengths

	To the center of the motor	To the end of the arm
From the beginning of the arm (mm)	415	440
From the center of the frame (mm)	480	505

Table 2-3. Basic instruments

	Name (Model)	Full scale (Range)	Accuracy
	Electronic balance (Kern 440-45N)	1 Kg	0.2 g
	measuring tape (Assist 3M)	3 m	0.001 m
	Digital tachometer (DT-2234C+)	2.5 ~ 99,999 RPM	0.05% + 1 digit
	Digital multi-meter (CEN-TECH 98025)	200mV/2000mV 20/200/1000V	(@200mV) 0.5% ± 1D (@2000mV-200V) 1% ± 2D (@1000V) 1% ± 2D
	Thrust meter (Tahmazo)	5 ~ 9,995 g	5 g

CHAPTER 3
EQUATIONS OF MODEL

3.1 Basic Movements

The quad-rotor helicopter can be defined as a VTOL UAV whose take-off or lift is powered by four motors and each motor has its own rotors. According to Figure 3-1, rotor number 1 and 3 (or front and rear rotor) rotates CCW (Counter Clock Wise), and rotor number 2 and 4 (or right and left rotor) rotates CW (Clock Wise). Because of this arrangement, aerodynamic torque is canceled by each other.

Initially the helicopter is on the ground before taking-off. There are five status of movement condition for the quad-rotor helicopter after taking-off. Those are hovering, vertical (or thrust), rolling, pitching and yawing movements. Each movement condition is characterized by different motor speeds.

With different colors, shapes and lengths, each movement condition can be described simply and clearly. The following indicators will be used generally for any of the different cases.

- Black lines (and circles) – Frame and four rotors
- Green lines – Body fixed frame
- Blue lines (straight and its length) – Velocity
- Blue lines (curve) – Rotation direction
- Red line (straight or curve) – Whole body moving direction

3.1.1 Hovering

Figure 3-1 shows the quad-rotor helicopter under hovering condition. In this case, the four rotors have the same speed, so generate the same thrust. The total thrust from all four motors equals the weight, so the helicopter hovers at an altitude. The helicopter maintains its equilibrium and balance, so it does not move in any direction unless acted upon by other forces.

Clearly, in hovering movement condition, there is no red arrow because the quad-rotor helicopter does not move anywhere (up, down, roll, pitch or yaw) but just maintains its current position.

3.1.2 Vertical Movement

When increasing or decreasing all rotor speeds by the same amount, the quad-rotor helicopter will move upward or downward along the z-axis. Figure 3-2 shows this vertical movement.

3.1.3 Roll Movement

In the case of roll movement, front and rear rotors have the same speed, but left and right rotors have different amounts of speed. If the left rotor increases (decreases) its speed, the right rotor decreases (increases) its speed simultaneously. Therefore the quad-rotor helicopter rotates (or rolls) left and right.

Figure 3-3 shows a simple diagram of roll movement as distinguished by the length of each straight arrow. Clearly front and rear arrows have the same length but the left and right arrows have different lengths.

3.1.4 Pitch Movement

Pitch movement is characterized by left and right rotors having same speed but front and rear rotors having different speed. If the front rotor increases (decreases) its speed, the rear rotor decreases (increases) its speed simultaneously. Therefore the quad-rotor helicopter rotates (or pitches) front and rear. The quad-rotor helicopter cannot distinguish four directions, so it must be marked which one is front, rear, right and left. Then roll movement and pitch movement can be characterized by its initial setting.

Figure 3-4 shows a simple diagram of pitch movement. Clearly right and left arrows have same length but front and rear arrows have different lengths. If the observer or the experimenter confuses the direction of each motor, pitching and rolling movements must be flipped over. So, the direction of each motor has to be decided firmly and clearly.

3.1.5 Yaw Movement

In case of yaw movement, the front-rear rotors have the same speed and the left-right rotors have same speed but each pair of rotors have different amounts of speed. If the front-rear rotor pair decreases (increases) its speed, then the left-right rotor pair increases (decreases) its speed simultaneously. Therefore the quad-rotor helicopter rotates CCW (CW) along z-axis.

3.2 Three Different Types of Moments

According to the general equations of motion for the quad-rotor helicopter [4], quad-rotor helicopter movements are caused by the series of moments and forces. Table 3-1 through Table 3-3 shows the three different types of moments of a quad-rotor helicopter.

3.3 Equations of Motion

Combined moments consists equations of motion for six degrees of freedom (6-DOF) dynamics of a quad-rotor helicopter. [4]

$$\ddot{x} = (\cos f \sin\theta \cos\psi + \sin f \sin\psi)\frac{1}{m}U_1$$

$$\ddot{y} = (\cos f \sin\theta \sin\psi - \sin f \cos\psi)\frac{1}{m}U_1$$

$$\ddot{z} = -g + (\cos f \cos\theta)\frac{1}{m}U_1$$

$$\ddot{\phi} = \dot{\theta}\dot{\psi}(\frac{I_y - I_z}{I_x}) - \frac{J_r}{I_x}\dot{\theta}\Omega + \frac{l}{I_x}U_2$$
$$\ddot{\theta} = \dot{f}\dot{\psi}(\frac{I_z - I_x}{I_y}) + \frac{J_r}{I_y}\dot{f}\Omega + \frac{l}{I_y}U_3 \qquad (3\text{-}1)$$
$$\ddot{\psi} = \dot{f}\dot{\theta}(\frac{I_x - I_y}{I_z}) + \frac{l}{I_z}U_4$$

3.4 Moments of Inertia

In order to calculate the moments of inertia of the quad-rotor helicopter for this thesis, two types of basic formula are needed. Those are the moment of inertia for the solid cylinder shape (for motor), and the moment of inertia for the solid cuboid shape (for central hub). The quad-rotor helicopter for this thesis is approximated with four solid cylinder-shape motors and solid cuboid-shape central hub.

I_h of the solid cuboid (central hub) can be calculated with I_z of the solid cylinder (motor) because they share same z-axis. Likewise I_w can be compared to I_x, because they share same x-axis. Lastly, I_d can be calculated with I_y, because they share same y-axis.

Thanks to above equations, the moments of inertia for the quad-rotor helicopter for this thesis are,

$$I_{xx} = 0.1328$$
$$I_{yy} = 0.1324 \qquad (3\text{-}2)$$
$$I_{zz} = 0.2638$$

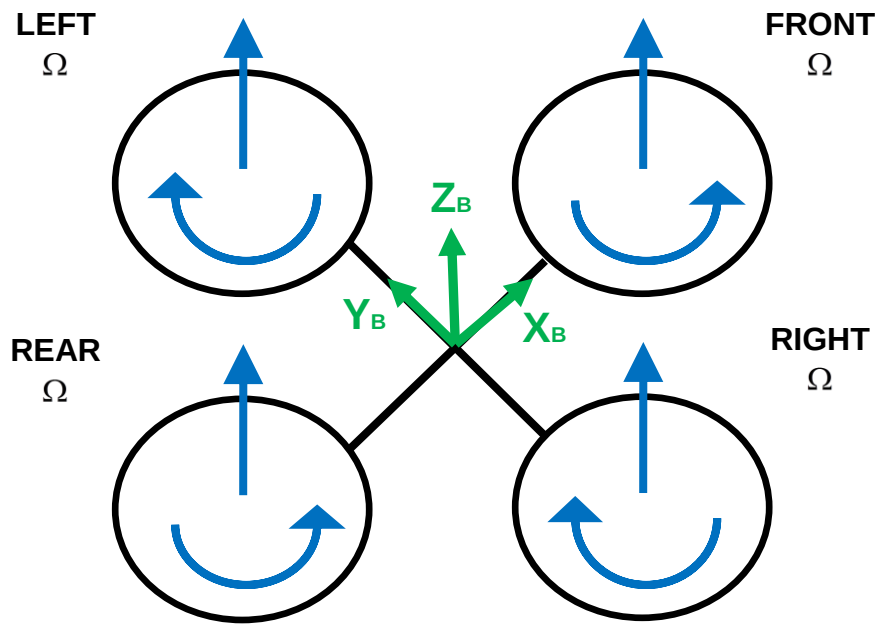


Figure 3-1. Quad-rotor Helicopter Schematic (Hovering)

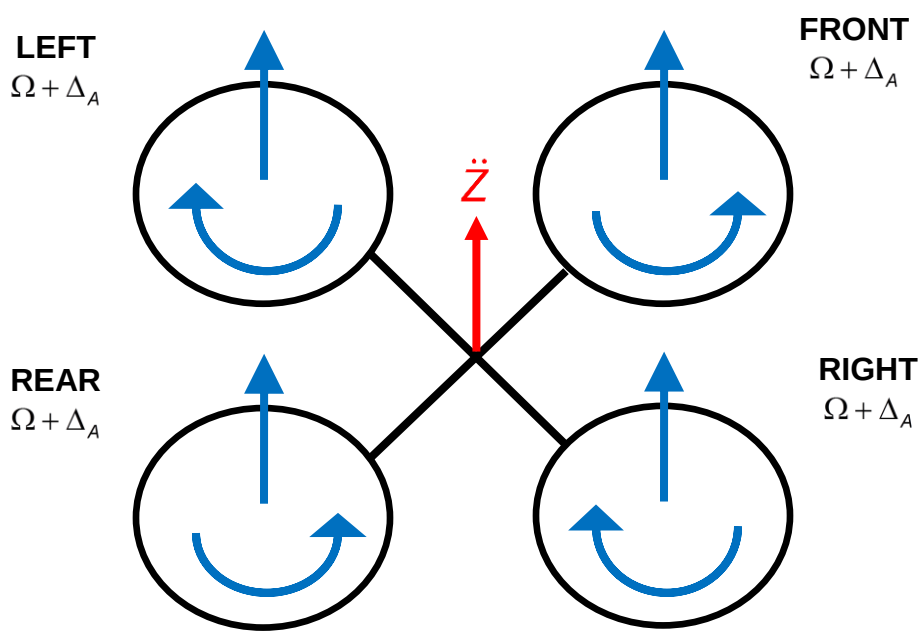


Figure 3-2. Vertical movement

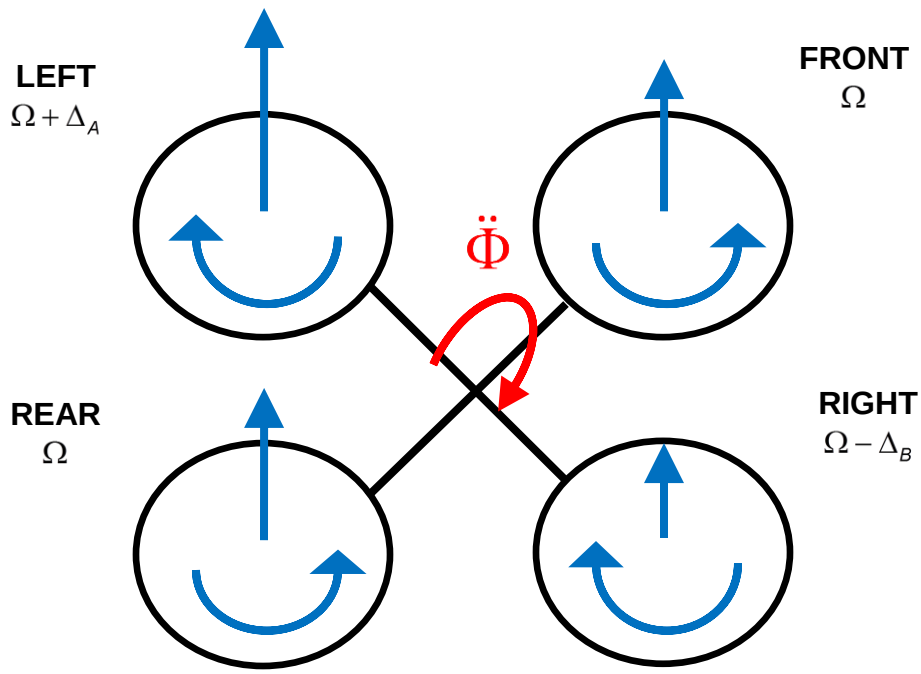


Figure 3-3. Roll movement

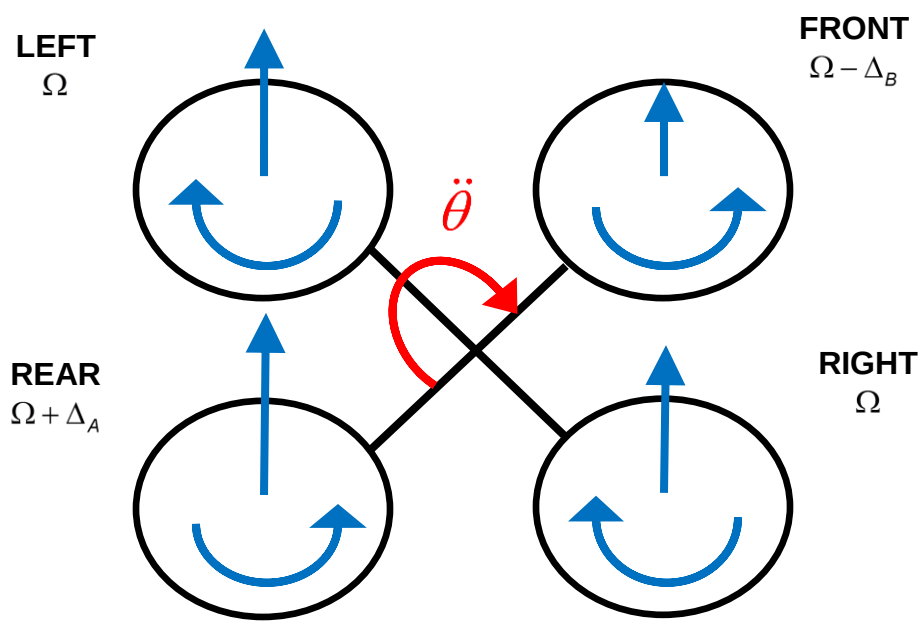


Figure 3-4. Pitch movement

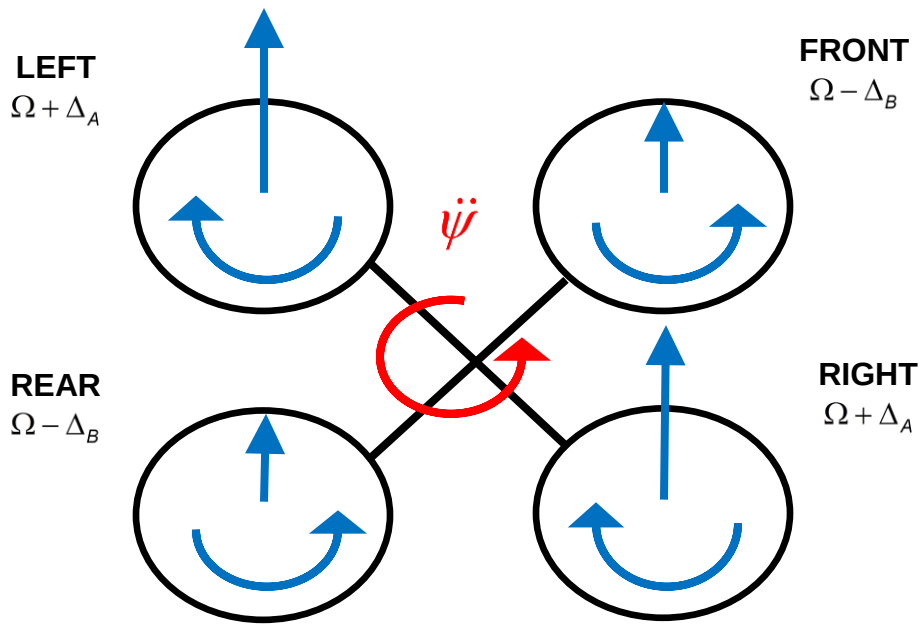


Figure 3-5. Yaw movement

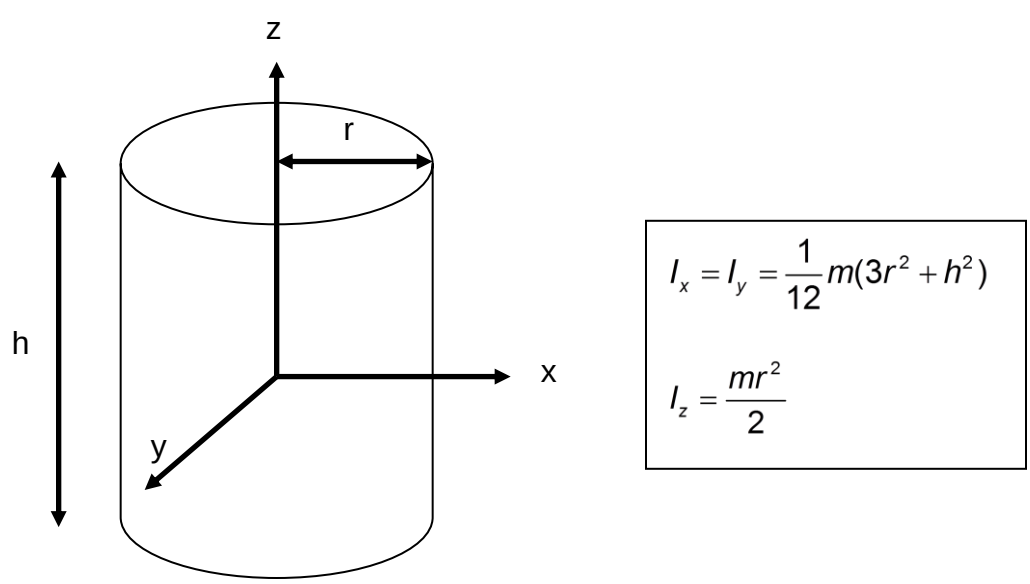
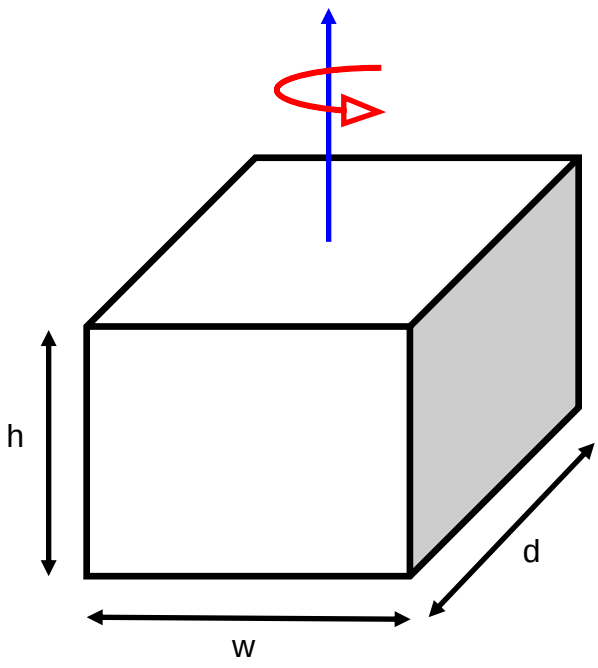


Figure 3-6. Moment of inertia for solid cylinder



$$I_h = \frac{1}{12} m(w^2 + d^2)$$

$$I_w = \frac{1}{12} m(h^2 + d^2)$$

$$I_d = \frac{1}{12} m(h^2 + w^2)$$

Figure 3-7. Moment of inertia for the solid cuboid

Table 3-1. Rolling moments

Effect	Equations
Body gyro effect	$\dot{\theta}\dot{\psi}(I_{yy} - I_{zz})$
Rolling moment due to forward flight	$(-1)^{i+1}\sum_{i=1}^{4} R_{mxi}$
Propeller gyro effect	$J_r\dot{\theta}\Omega_r$
Hub moment due to sideward flight	$h(\sum_{i=1}^{4} H_{yi})$
Roll actuators action	$l(-T_2 + T_4)$

Table 3-2. Pitching moments

Effect	Equations
Body gyro effect	$\dot{\theta}\dot{\psi}(I_{zz} - I_{xx})$
Pitching moment due to forward flight	$h(\sum_{i=1}^{4} H_{xi})$
Propeller gyro effect	$J_r\dot{\phi}\Omega_r$
Hub moment due to sideward flight	$(-1)^{i+1}\sum_{i=1}^{4} R_{myi}$
Pitch actuators action	$l(T_1 - T_3)$

Table 3-3. Yawing moments

Effect	Equations
Body gyro effect	$\dot{\theta}\dot{\phi}(I_{xx} - I_{yy})$
Yawing moment due to forward flight	$l(H_{x2} - H_{x4})$
Propeller gyro effect	$J_r\dot{\Omega}_r$
Hub moment due to sideward flight	$l(-H_{y1} + H_{y3})$
Yaw actuators action	$(-1)^{i}\sum_{i=1}^{4} Q_i$

CHAPTER 4
EXPERIMENTAL MODELING

4.1 Thrust Test

In order to do the thrust test, the author attached the thrust meter to a solid frame and linked each motor from the quad-rotor helicopter. Then the half throttle was given to the motor and the thrust meter measured the thrust at the status. The unit is Ns^2 and Table 4-1 shows the results of the thrust test.

4.2 Rotor Speed Test

In case of the motor speed test, digital tachometer was located in front of the motor. It measured the highest rpm while the rotors rotate. Each motor's rotors have almost the same speeds, if the same amount of throttle is given.

Table 4-1. Thrusts (Experimental Results)

Motor No.	1st result	2nd result	3rd result	4th result	mean	Standard Deviation
1	1.275	1.100	1.165	1.145	1.1713	0.0743
2	1.325	1.060	1.125	1.160	1.1675	0.1129
3	1.310	1.265	1.235	1.265	1.2688	0.0309
4	1.435	1.295	1.375	1.280	1.3463	0.0724

Table 4-2. Rotor speed (Experimental Results)

	1st result	2nd result	3rd result	4th result	mean	Standard Deviation
Unit (rpm)	2807	2727	2676	2579	2697	95.5070

CHAPTER 5
SYSTEM CONTROL

5.1 State Space Model

In order to get a system control, the quad-rotor helicopter dynamics model (3-1) can be expressed by state-space form as, $\dot{x} = f(X,U)$

$$
\begin{aligned}
x_1 &= x & x_7 &= \phi \\
x_2 &= \dot{x}_1 = \dot{x} & x_8 &= \dot{x}_7 = \dot{\phi} \\
x_3 &= y & x_9 &= \theta \\
x_4 &= \dot{x}_3 = \dot{y} & x_{10} &= \dot{x}_9 = \dot{\theta} \\
x_5 &= z & x_{11} &= \psi \\
x_6 &= \dot{x}_5 = \dot{z} & x_{12} &= \dot{x}_{11} = \dot{\psi}
\end{aligned}
\tag{5-1}
$$

From (3-1) and (5-1), we obtain :

$$
f(X,U) = \begin{pmatrix}
x_2 \\
(\cos x_7 \sin x_9 \cos x_{11} + \sin x_7 \sin x_{11}) \dfrac{U_1}{m} \\
x_4 \\
(\cos x_7 \sin x_9 \sin x_{11} - \sin x_7 \cos x_{11}) \dfrac{U_1}{m} \\
x_6 \\
-g + (\cos x_7 \cos x_9) \dfrac{U_1}{m} \\
x_8 \\
x_{12} x_{10} (\dfrac{I_y - I_z}{I_x}) - \dfrac{J_R}{I_x} x_{10} \Omega + \dfrac{l}{I_x} U_2 \\
x_{10} \\
x_{12} x_8 (\dfrac{I_z - I_x}{I_y}) + \dfrac{J_R}{I_y} x_8 \Omega + \dfrac{l}{I_y} U_3 \\
x_{12} \\
x_{10} x_8 (\dfrac{I_x - I_y}{I_z}) + \dfrac{l}{I_z} U_4
\end{pmatrix}
\tag{5-2}
$$

Where the system's inputs as,

$$
\begin{aligned}
U_1 &= b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\
U_2 &= b(\Omega_4^2 - \Omega_2^2) \\
U_3 &= d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \\
U_4 &= \Omega_2 + \Omega_4 - \Omega_1 - \Omega_3
\end{aligned}
\tag{5-3}
$$

5.2 Linearization of the System

State space model (5-2) is not a linear system, so in order to apply PID control technique, the system must be linearized.

Equilibrium points are,

$$
X = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } U = \begin{bmatrix} mg \\ 0 \\ 0 \\ 0 \end{bmatrix}
\tag{5-4}
$$

Therefore, linearized state space model of the system is,

$$
\begin{aligned}
\dot{X} &= AX + BU \\
y &= CX + DU
\end{aligned}
\tag{5-5}
$$

Where,

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17.1419 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -17.1419 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5.4358\times10^{-4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5.4522\times10^{-4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5-6}$$

$$B=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 7.3419 & 7.3419 & 7.3419 \\ 0 & 0 & 0 & 0 \\ 0 & 0.6384 & 0.6384 & 0.6384 \\ 0 & 0 & 0 & 0 \\ 0 & 0.3525 & 0.3525 & 0.3525 \\ 0 & 0 & 0 & 0 \\ 0 & 0.1336 & 0.1336 & 0.1336 \end{bmatrix} \tag{5-7}$$

$$C=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{5-8}$$

$$D=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{5-9}$$

5.3 PID (Proportional, Integral and Derivative) Control Technique

There are many kinds of controlling methods to satisfy stabilization or control properly for the quad-rotor helicopter. According to other papers and thesis, Lyapunov / optimal control theories and PID (Proportional, Integral and Derivative) / LQR (Linear-quadratic regulator) / back-stepping sliding-mode techniques were used for controlling the quad-rotor helicopter. [3-6]

In this thesis, the author only focused on PID control technique. Because it is the most general control method, and its performance is satisfied with the expectation for this thesis. Additionally PD controller was introduced for comparing two different types of controllers.

Figure 5-1 shows a block diagram of a PID controller. Basically, P (Proportional term) controls the system based on the current error proportionally. I (Integral term) contributes reducing the error in steady-state. D (Derivative term) prevents the rapid change of the controller's output. It contributes decreasing the overshoot and improving the stability of the system.

For this thesis, PD and PID controllers were applied.

5.4 Simulation Results

5.4.1 PD (Proportional and Derivative) Controller for the Ideal Case

The input of the PD controller is the error which is combined the expected angle and the output angle. Kr, Kp and Ky are the proportional control gains for rolling, pitching and yawing errors. Kdr, Kdp and Kdy are the derivative control gains for rolling, pitching and yawing errors after implementing derivative controller.

The output of rolling for the rolling input is same as the output of pitching for the pitching input and the output of pitching for the rolling input is same as the output of

rolling for the pitching input. Yawing movement does not occur in response to the rolling and pitching commands. Also, rolling and pitching movements are not created in response to a yawing command.

5.4.2 PD Controller without Mixing for the Real Case

The purpose of this chapter is that the author tried to show the bad result of the PD controller without mixing for the real case. It is the reason why the mixing process is needed to control the system for the real case properly. In this case, the real actuator was implemented between PD controller and the plant model. U1 through u4 mean the real calculated input thrusts.

According to the results, output of the roll and pitch angle is oscillated, so we can define this result as a failure of controlling.

5.4.3 PD Controller with Mixing for the Real Case

In this chapter, the real measured and calculated thrust values are applied for the system. Thanks to math process, the whole system are not affected by the real thrust values. Therefore, this model can be controlled by the expected movements such as the pure rolling, pitching and yawing because the mixing process was added.

U1 is the summation of all four motor's thrust (speed). U2 is the combination of difference between 4th motor and 2nd motor. U3 is the combination of difference between 3rd motor and 1st motor. Lastly, U4 is the combination of difference between the summation of 2nd motor and 4th motor, and the summation of 1st motor and 3rd motor.

5.4.4 PID Controller for the Ideal Case

Figure 5-32 shows Simulink block diagram for PID controller. Each control gains affect all output values. Therefore tuning process is required.

Likewise previous results, the output of rolling for the rolling input is same as the output of pitching for the pitching input and the output of pitching for the rolling input is same as the output of rolling for the pitching input. Yawing movements does not occur when the rolling and pitching commends, and rolling and pitching movements does not create when the yawing commend.

5.4.5 PID Controller without Mixing for the Real Case

The author intentionally put this chapter due to explain the reason why the tuning or mixing process is needed for the real case. Like the previous results of PD controller without mixing for the real case, all output angles were oscillated, so the PID controller cannot perform well without mixing.

5.4.6 PID Controller with Mixing for the Real Case

Similar to the result of Simulink block diagram for the real case of PD controller, each control gains affect all output values, so tuning process is required. Thanks to same process, the whole system can be controlled by the expected input value, not because of the real thrust values.

5.4.7 Robustness to Actuator Variation

PID controller with mixing for the real case performed almost same even if the different real measured thrusts were given. Figure 5-62 through Figure 5-64 show the results of the robustness to actuator variation.

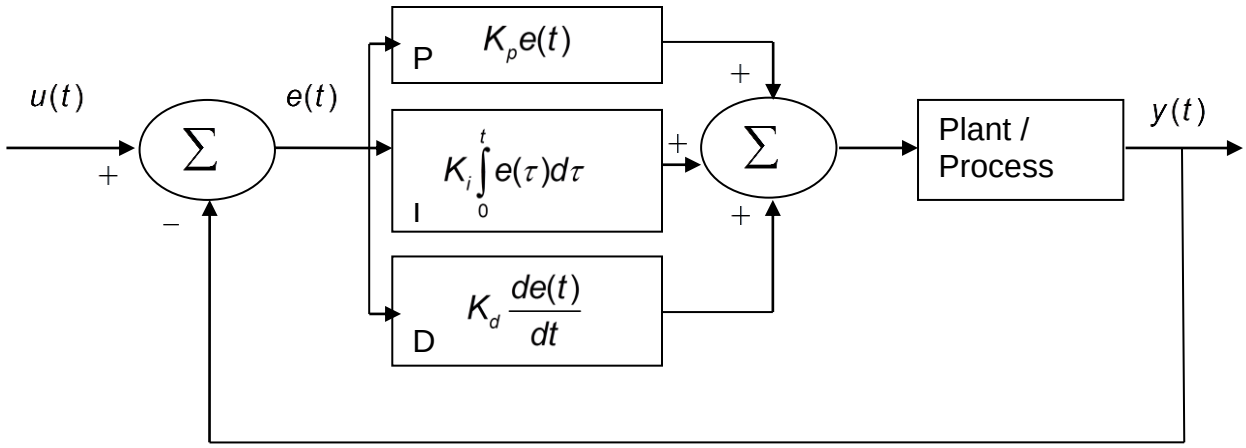


Figure 5-1. PID controller (Block diagram)

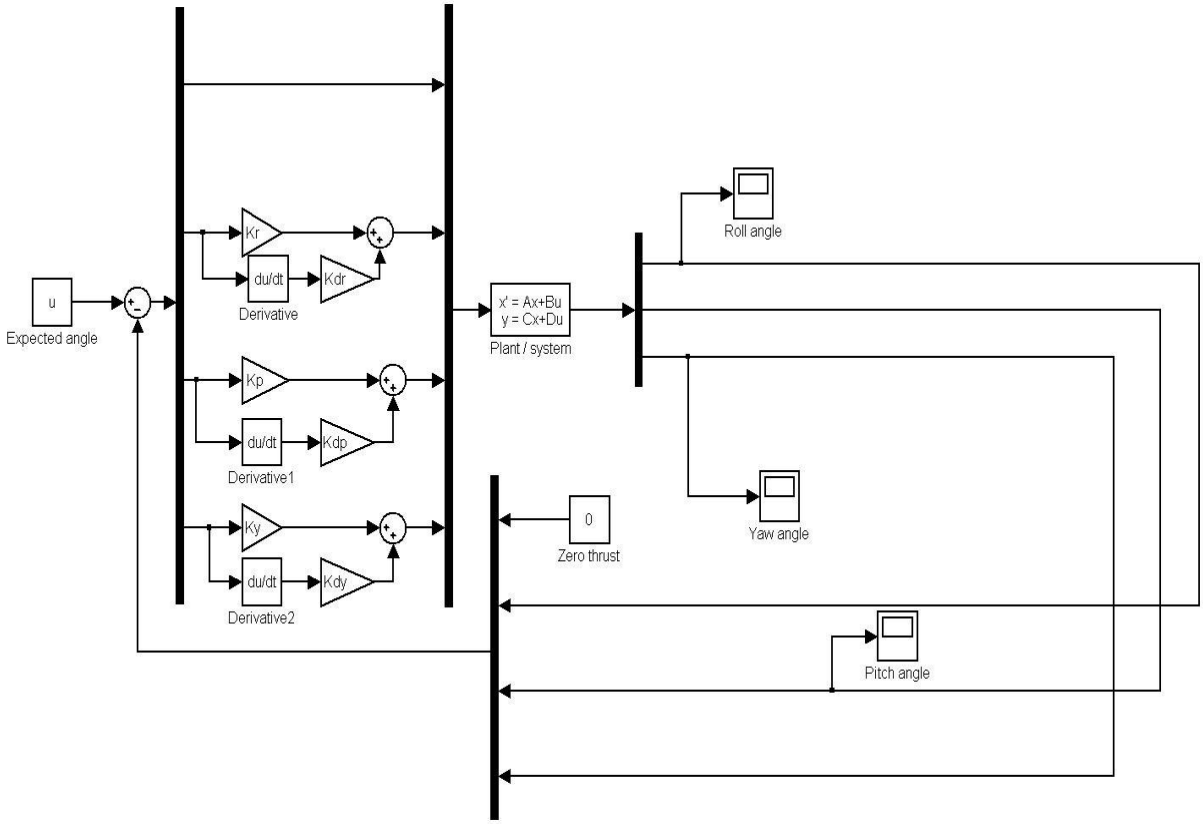


Figure 5-2. Simulink block diagram (PD controller : ideal case)

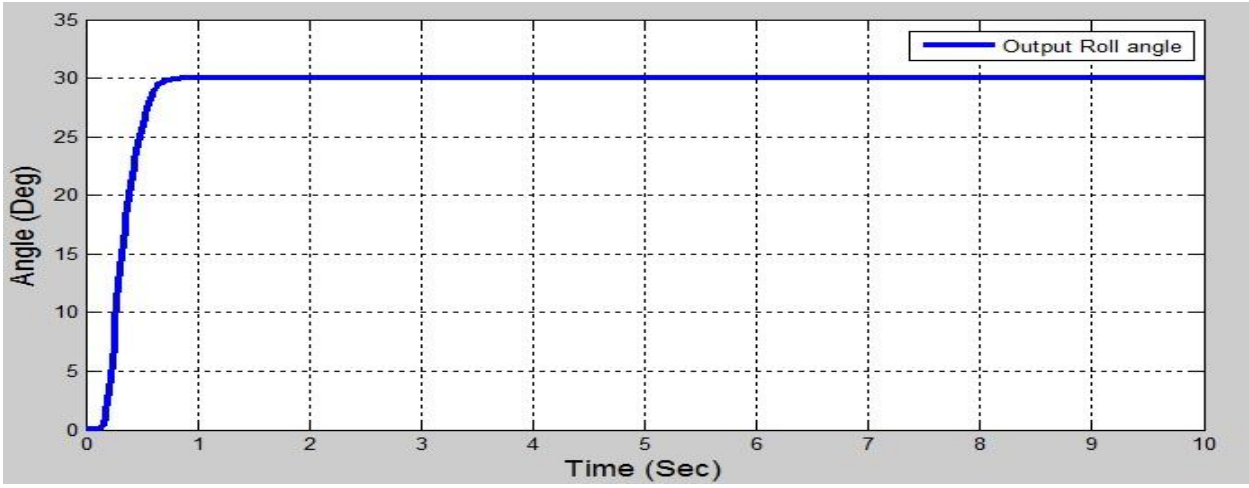


Figure 5-3. Output roll angle (input : roll 30 degree, PD controller)

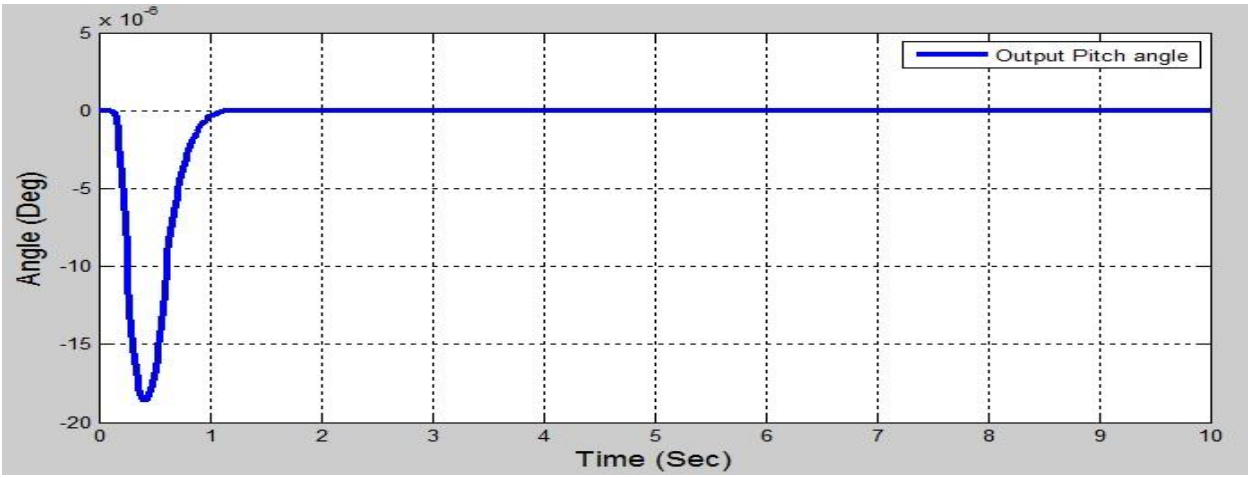


Figure 5-4. Output pitch angle (input : roll 30 degree, PD controller)

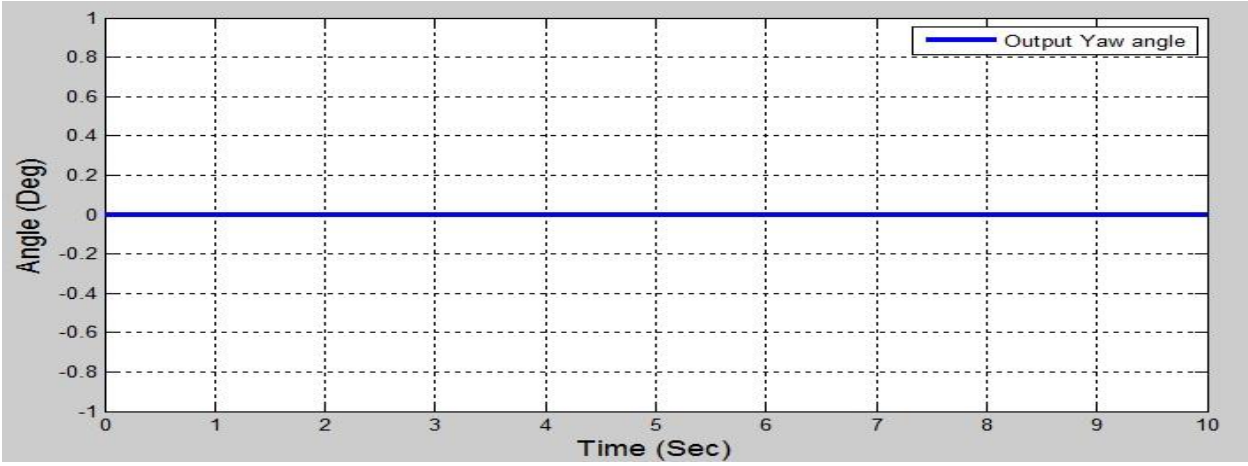


Figure 5-5. Output yaw angle (input : roll 30 degree, PD controller)

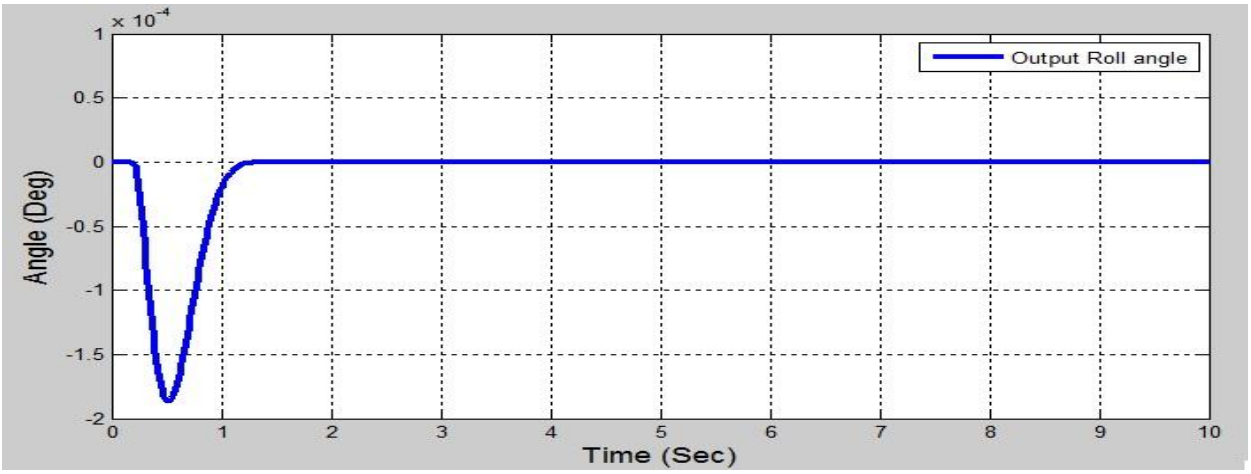


Figure 5-6. Output roll angle (input : pitch 30 degree, PD controller)

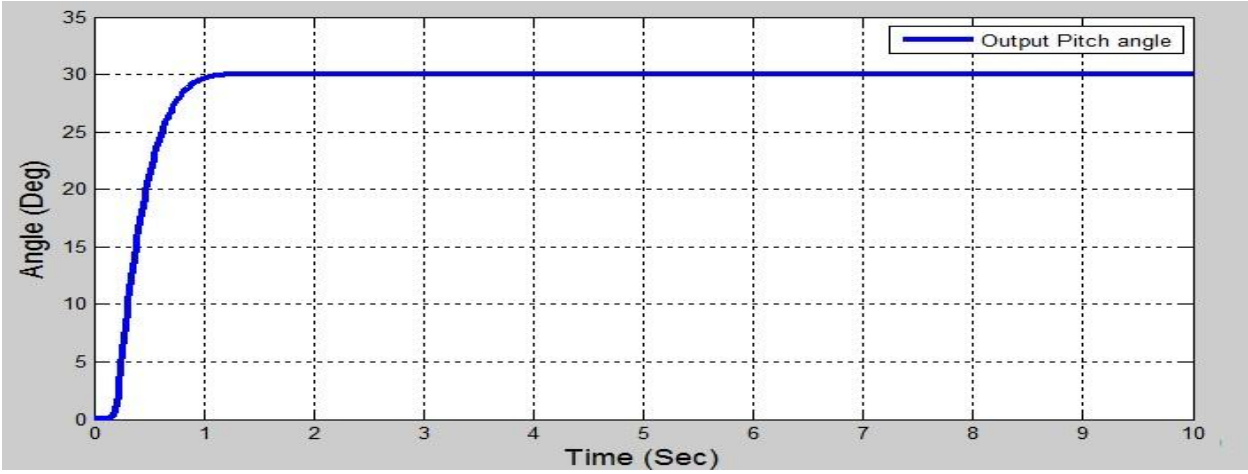


Figure 5-7. Output pitch angle (input : pitch 30 degree, PD controller)

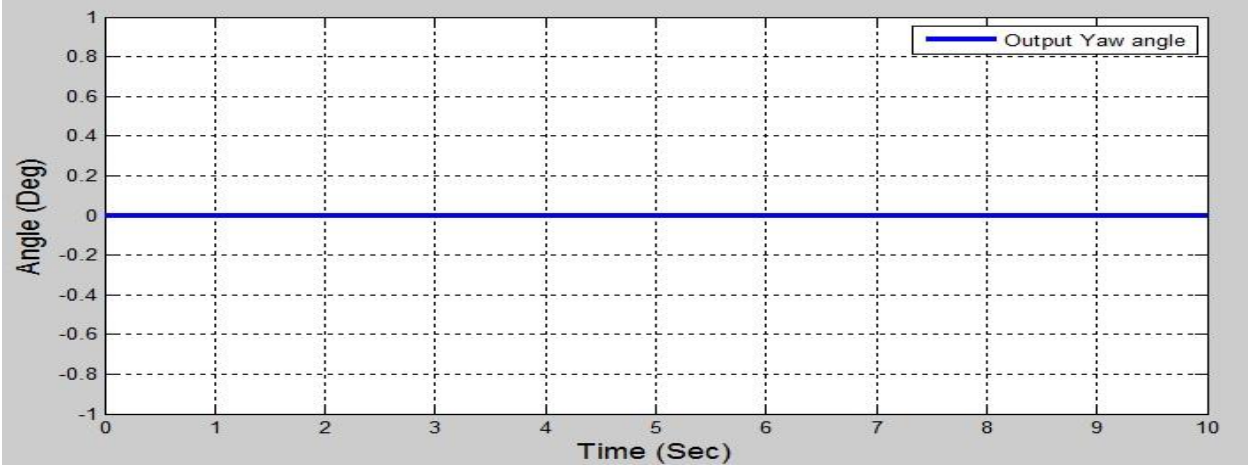


Figure 5-8. Output yaw angle (input : pitch 30 degree, PD controller)

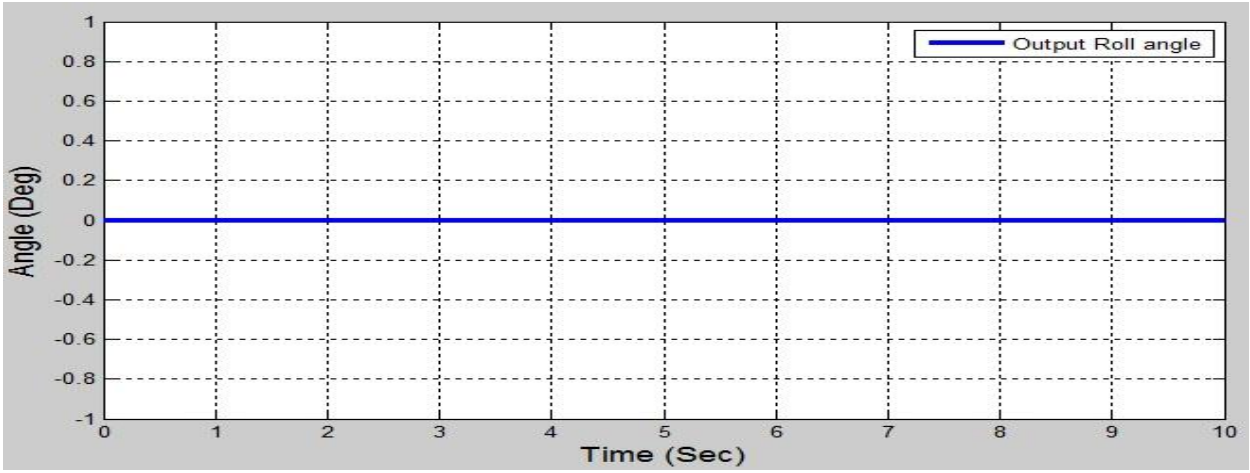


Figure 5-9. Output roll angle (input : yaw 30 degree, PD controller)

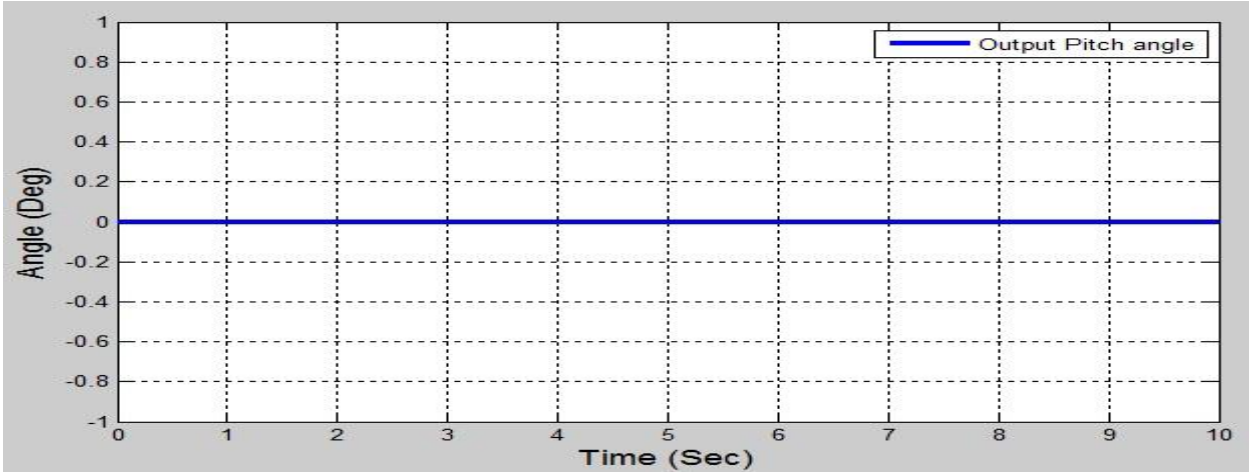


Figure 5-10. Output pitch angle (input : yaw 30 degree, PD controller)

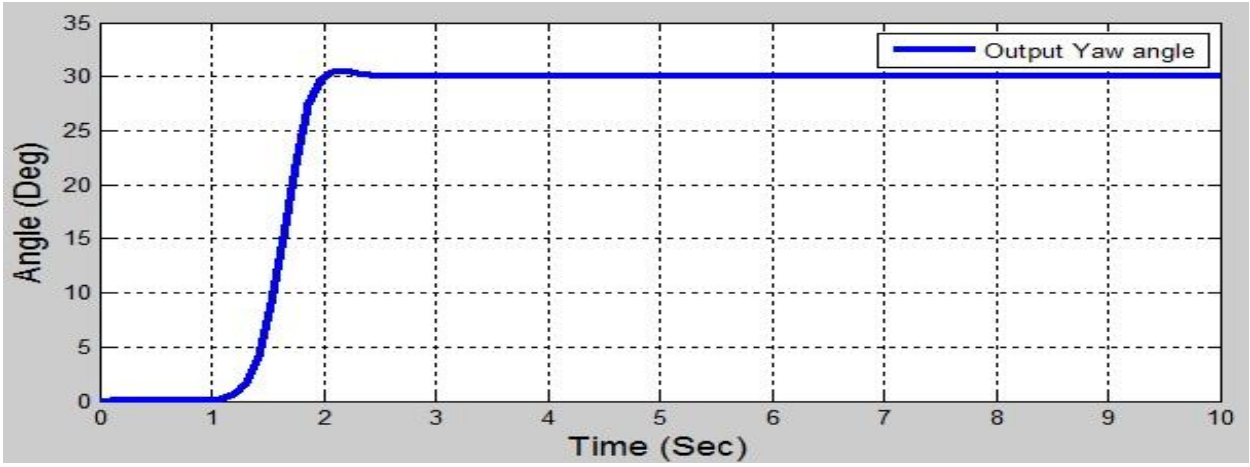


Figure 5-11. Output yaw angle (input : yaw 30 degree, PD controller)

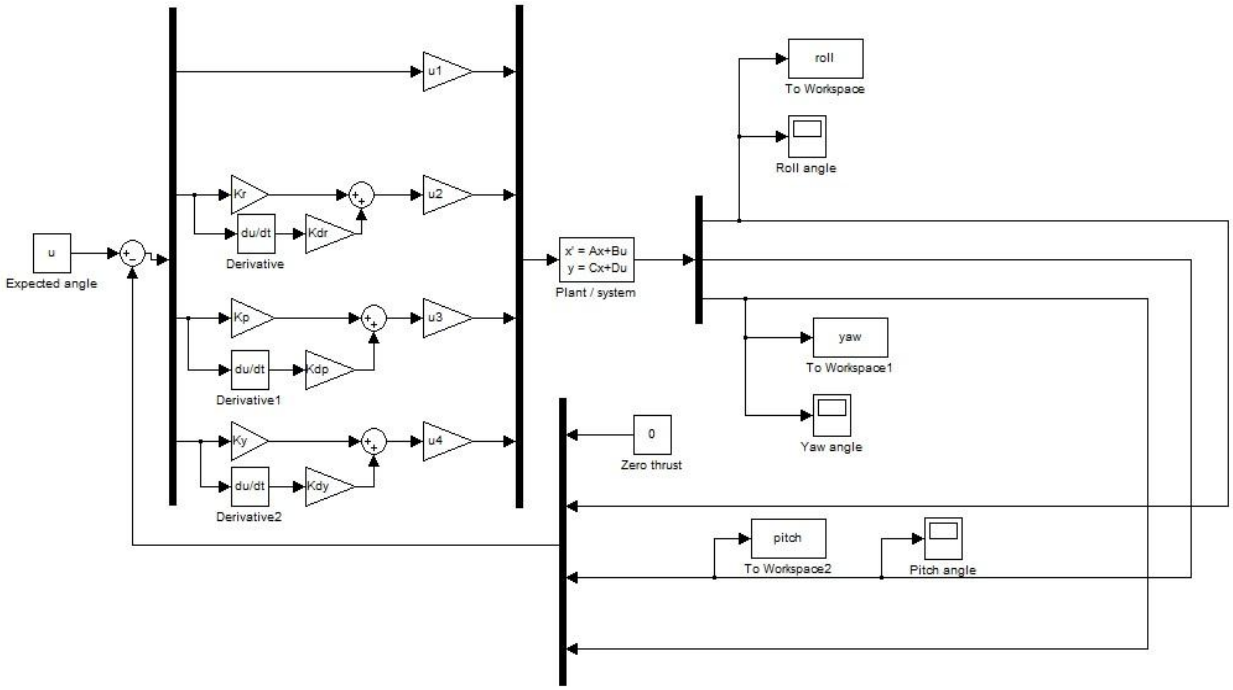


Figure 5-12. Simulink block diagram (PD controller : real case, without mixing)

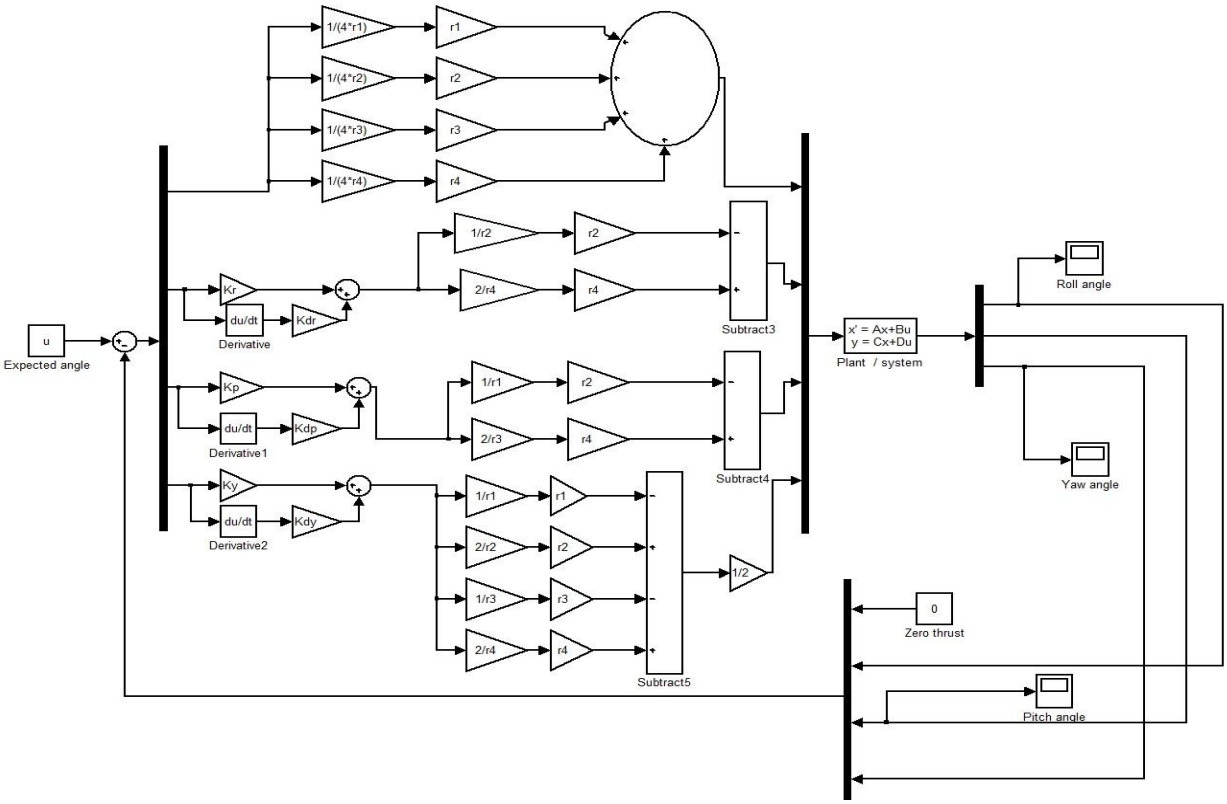


Figure 5-13. Simulink block diagram (PD controller : real case, with mixing)

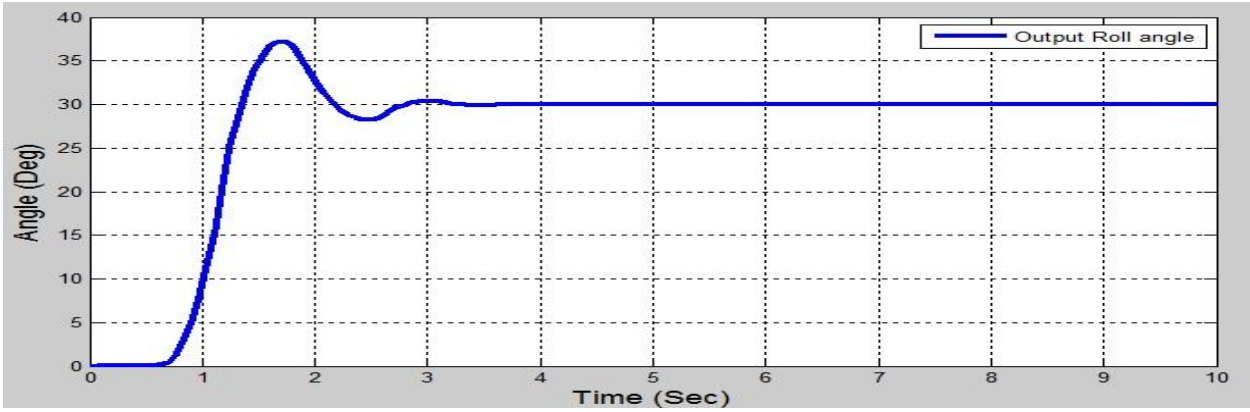


Figure 5-14. Output roll angle (input : roll 30 degree, PD controller, without mixing)

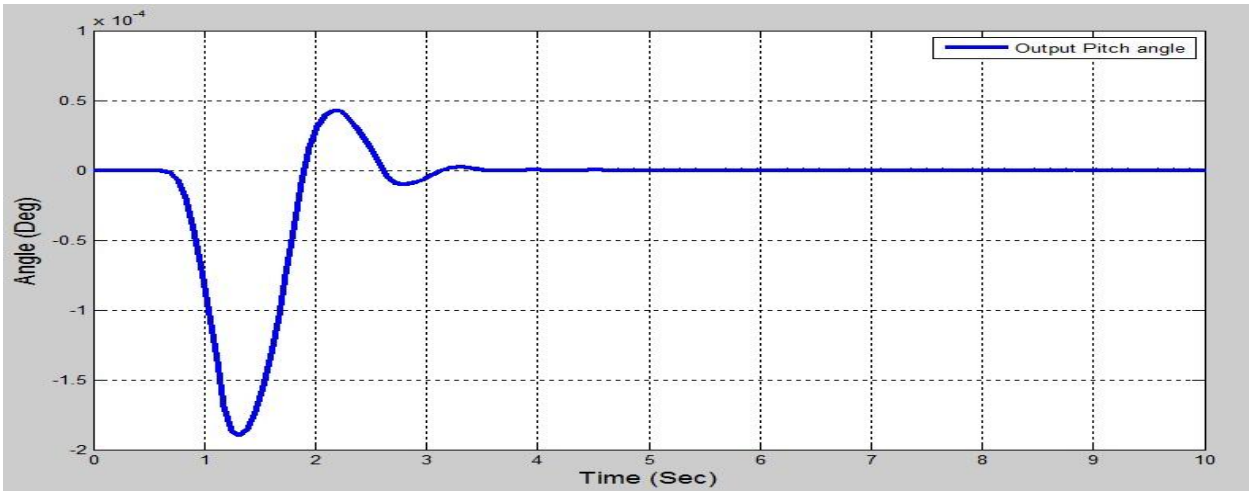


Figure 5-15. Output pitch angle (input : roll 30 degree, PD controller, without mixing)

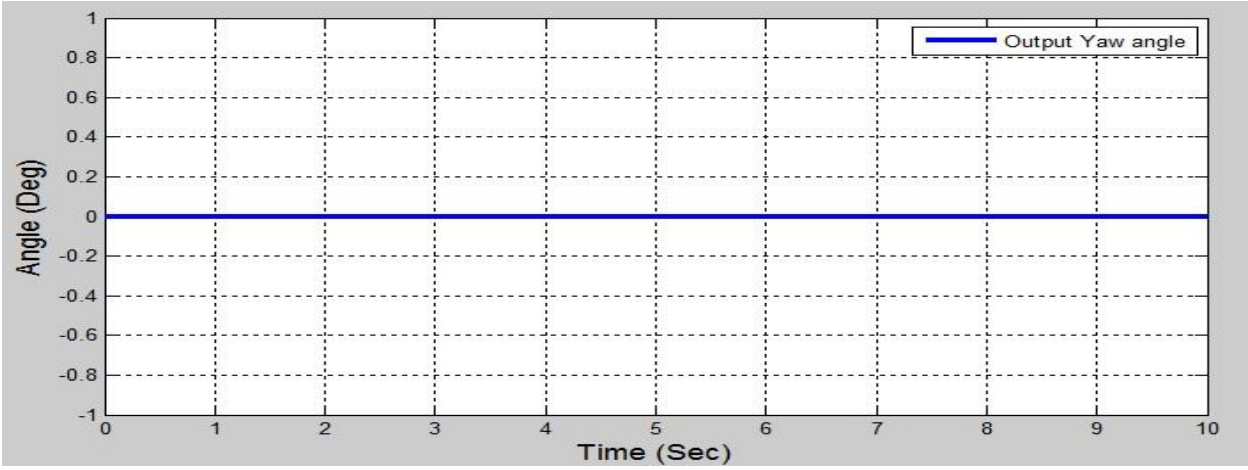


Figure 5-16. Output yaw angle (input : roll 30 degree, PD controller, without mixing)

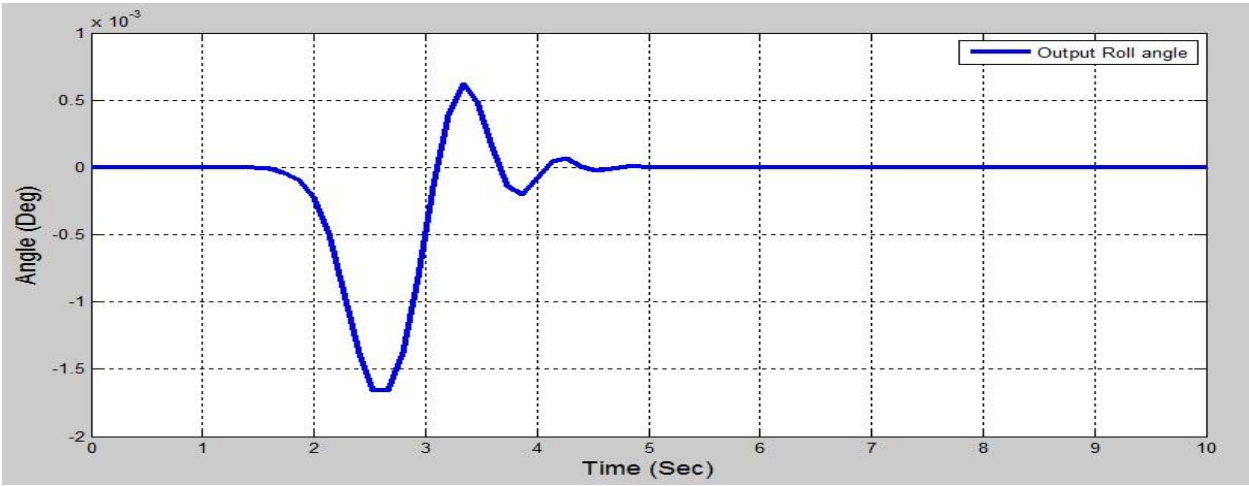


Figure 5-17. Output roll angle (input : pitch 30 degree, PD controller, without mixing)

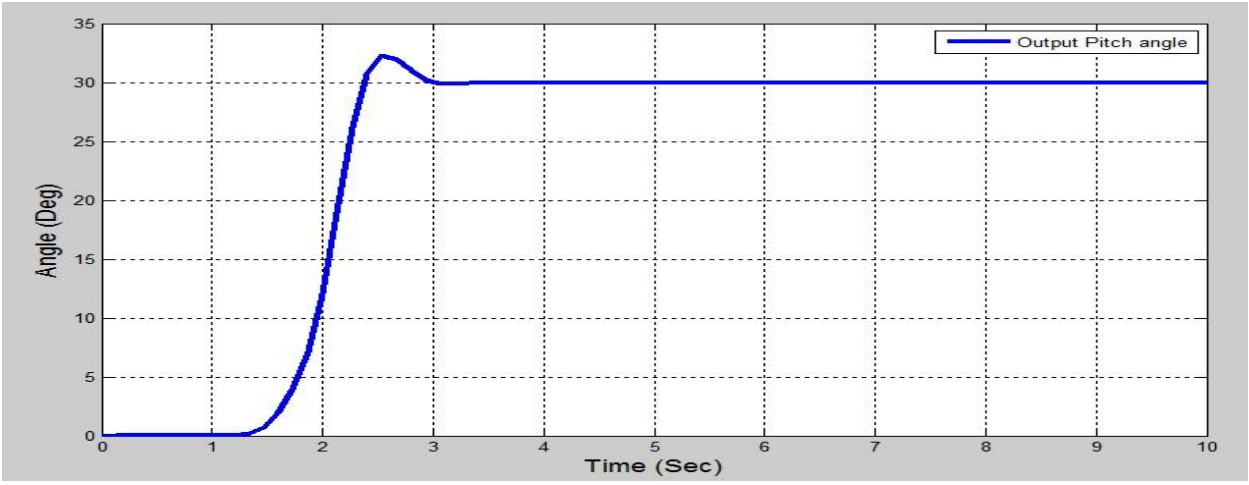


Figure 5-18. Output pitch angle (input : pitch 30 degree, PD controller, without mixing)

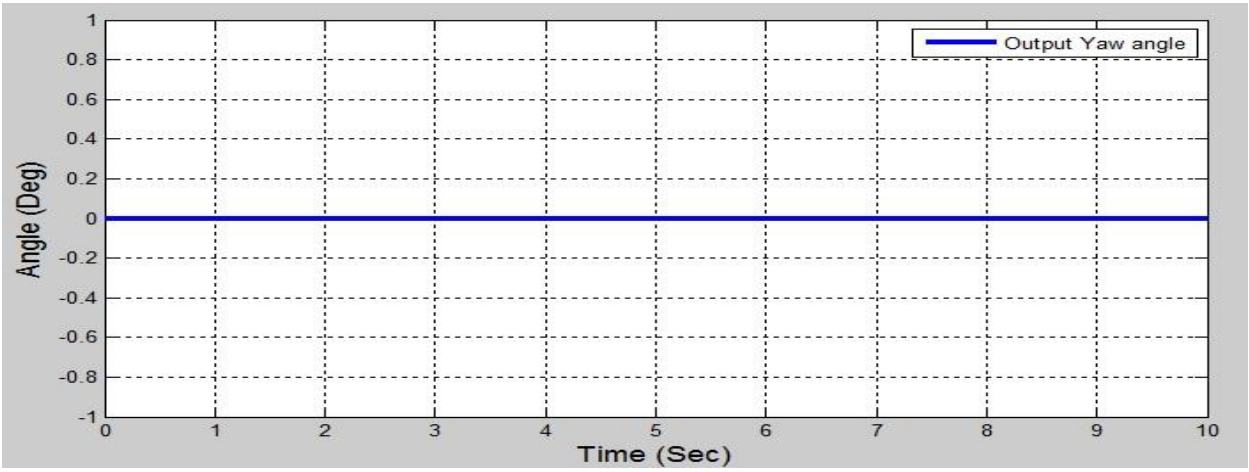


Figure 5-19. Output yaw angle (input : pitch 30 degree, PD controller, without mixing)

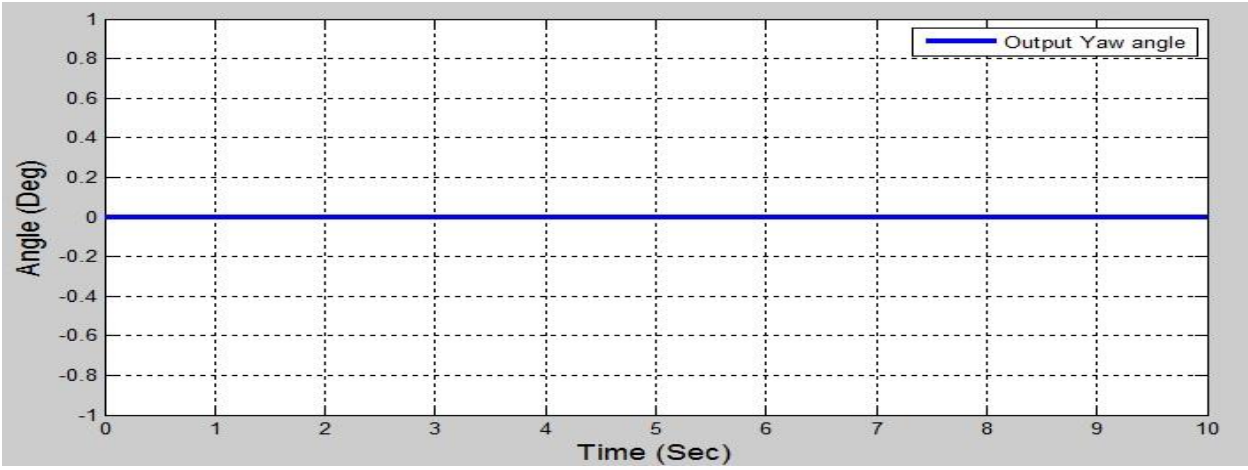


Figure 5-20. Output roll angle (input : yaw 30 degree, PD controller, without mixing)

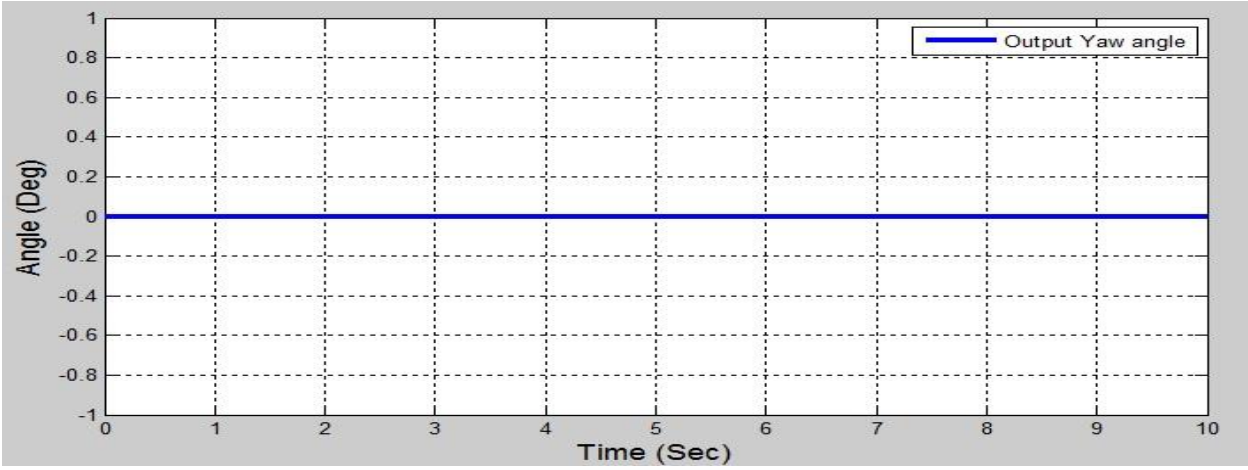


Figure 5-21. Output pitch angle (input : yaw 30 degree, PD controller, without mixing)

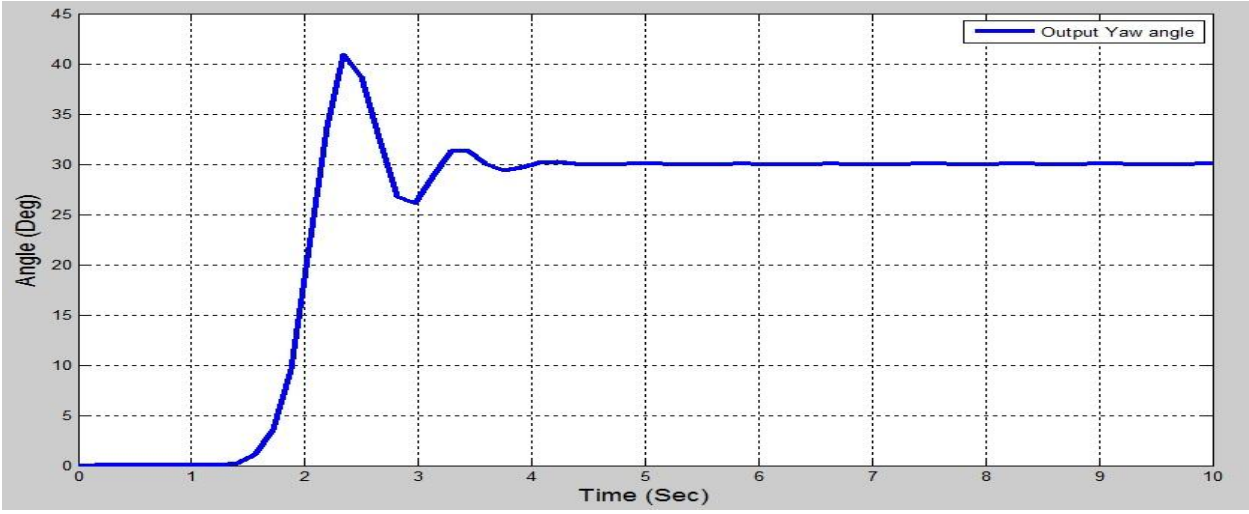


Figure 5-22. Output yaw angle (input : yaw 30 degree, PD controller, without mixing)

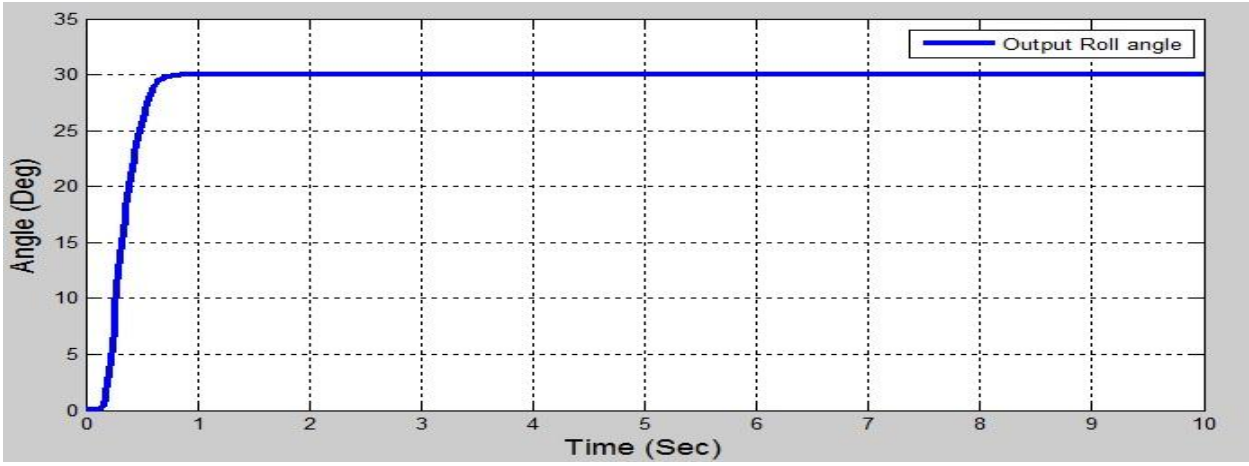


Figure 5-23. Output roll angle (input : roll 30 degree, PD controller, real case)

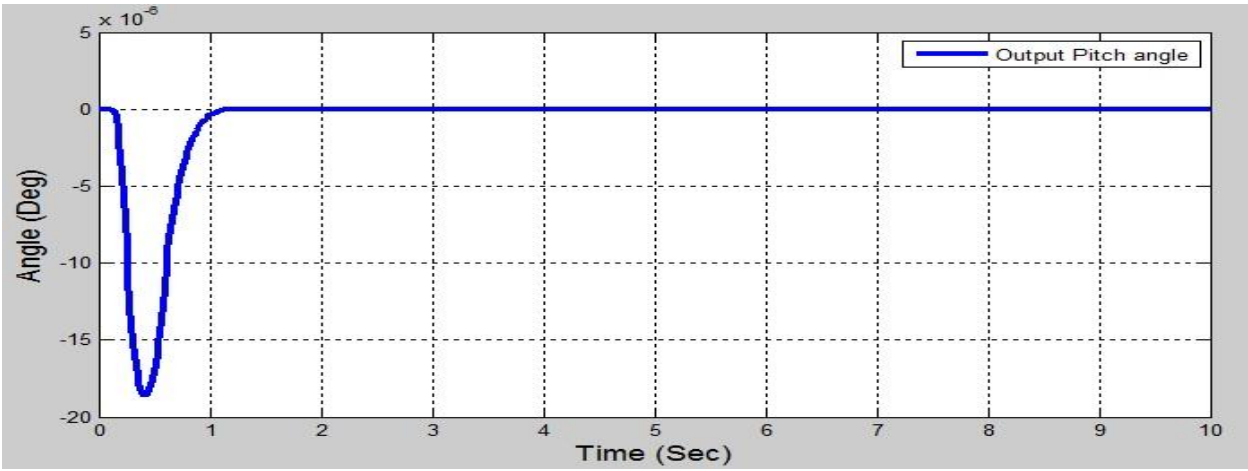


Figure 5-24. Output pitch angle (input : roll 30 degree, PD controller, real case)

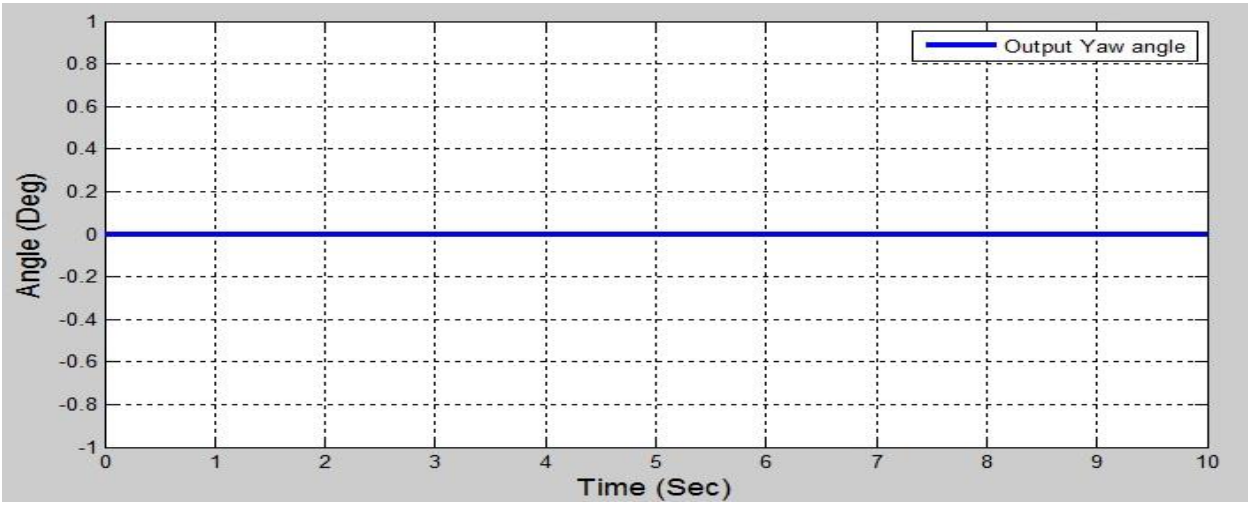


Figure 5-25. Output yaw angle (input : roll 30 degree, PD controller, real case)

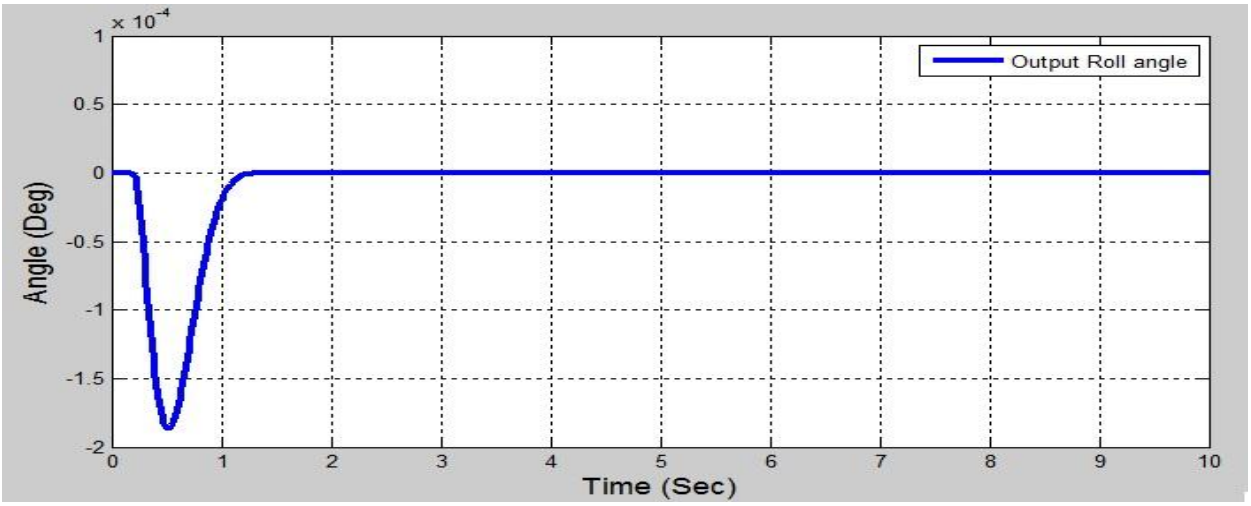


Figure 5-26. Output roll angle (input : pitch 30 degree, PD controller, real case)

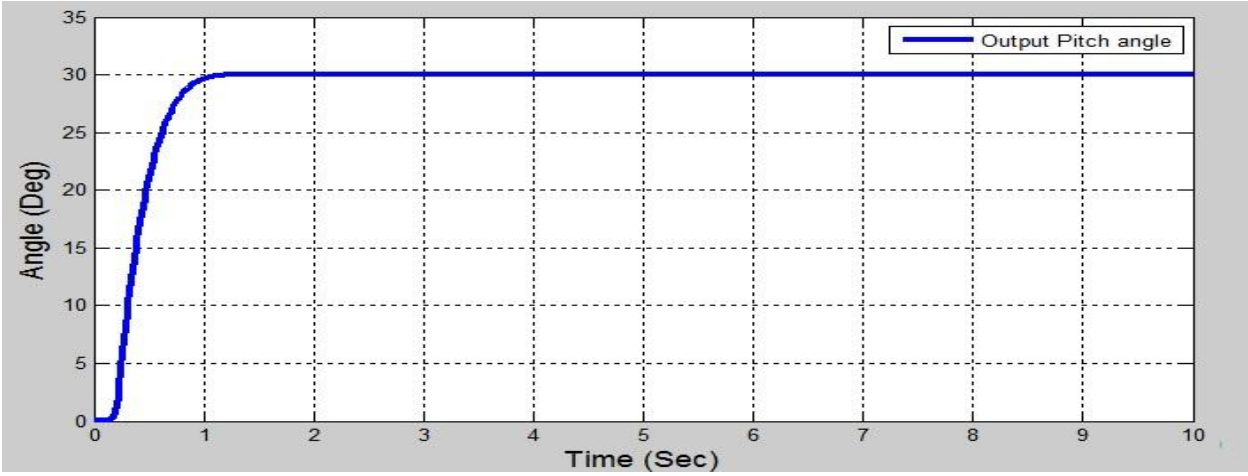


Figure 5-27. Output pitch angle (input : pitch 30 degree, PD controller, real case)

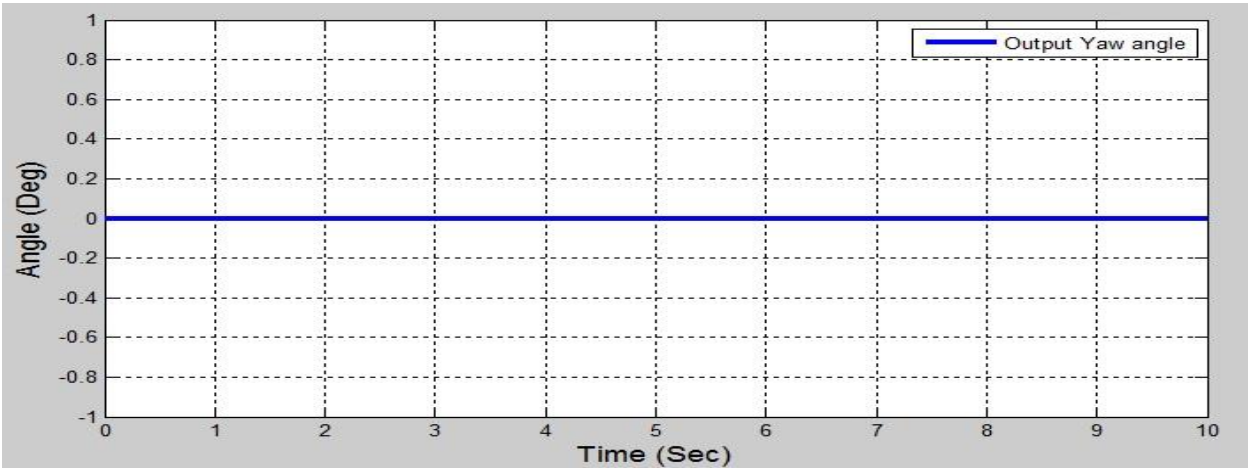


Figure 5-28. Output yaw angle (input : pitch 30 degree, PD controller, real case)

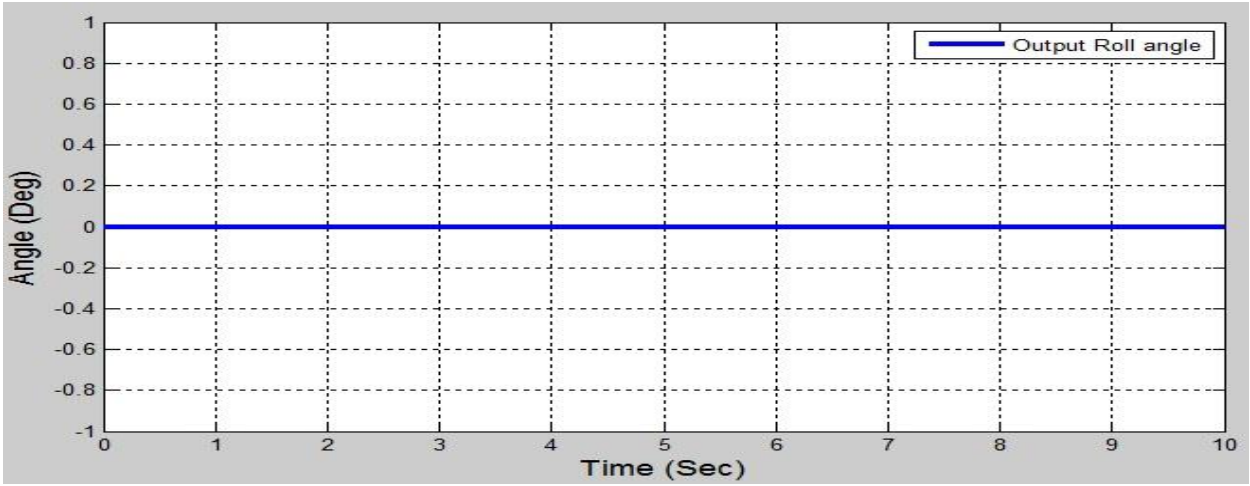


Figure 5-29. Output roll angle (input : yaw 30 degree, PD controller, real case)

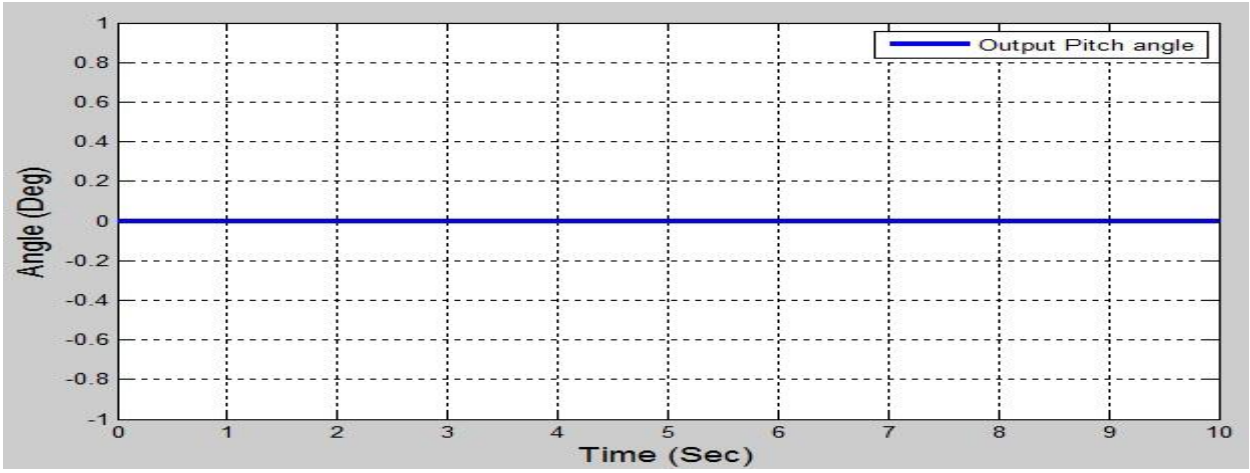


Figure 5-30. Output pitch angle (input : yaw 30 degree, PD controller, real case)

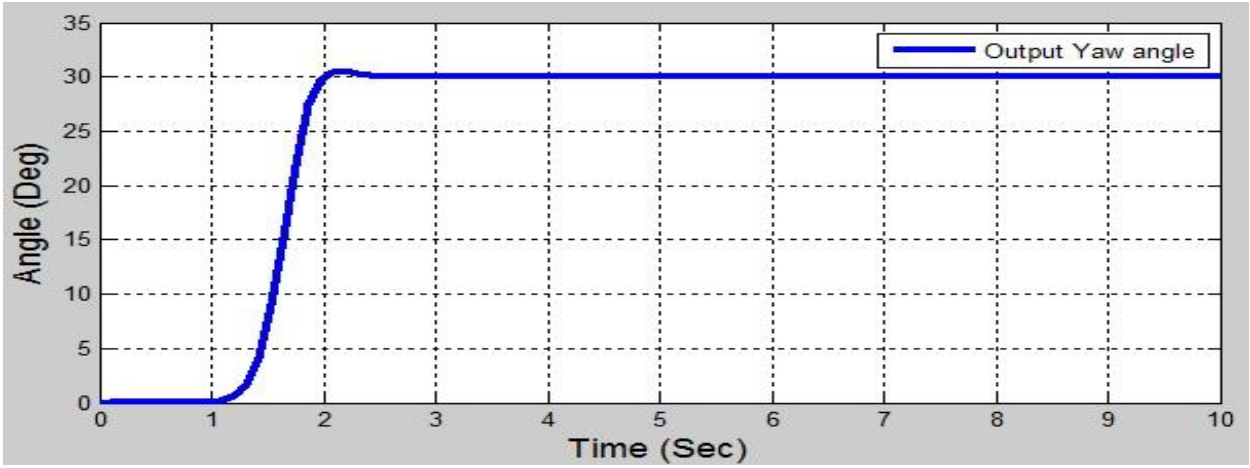


Figure 5-31. Output yaw angle (input : yaw 30 degree, PD controller, real case)

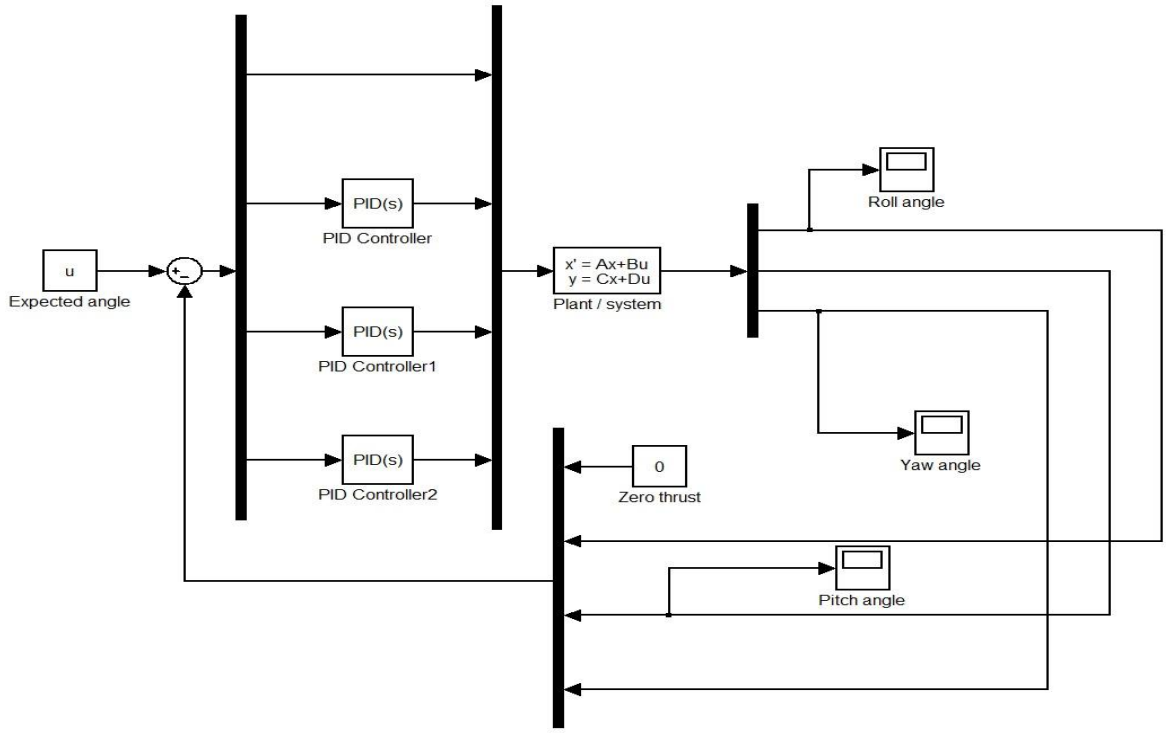


Figure 5-32. Simulink block diagram (PID controller : ideal case)

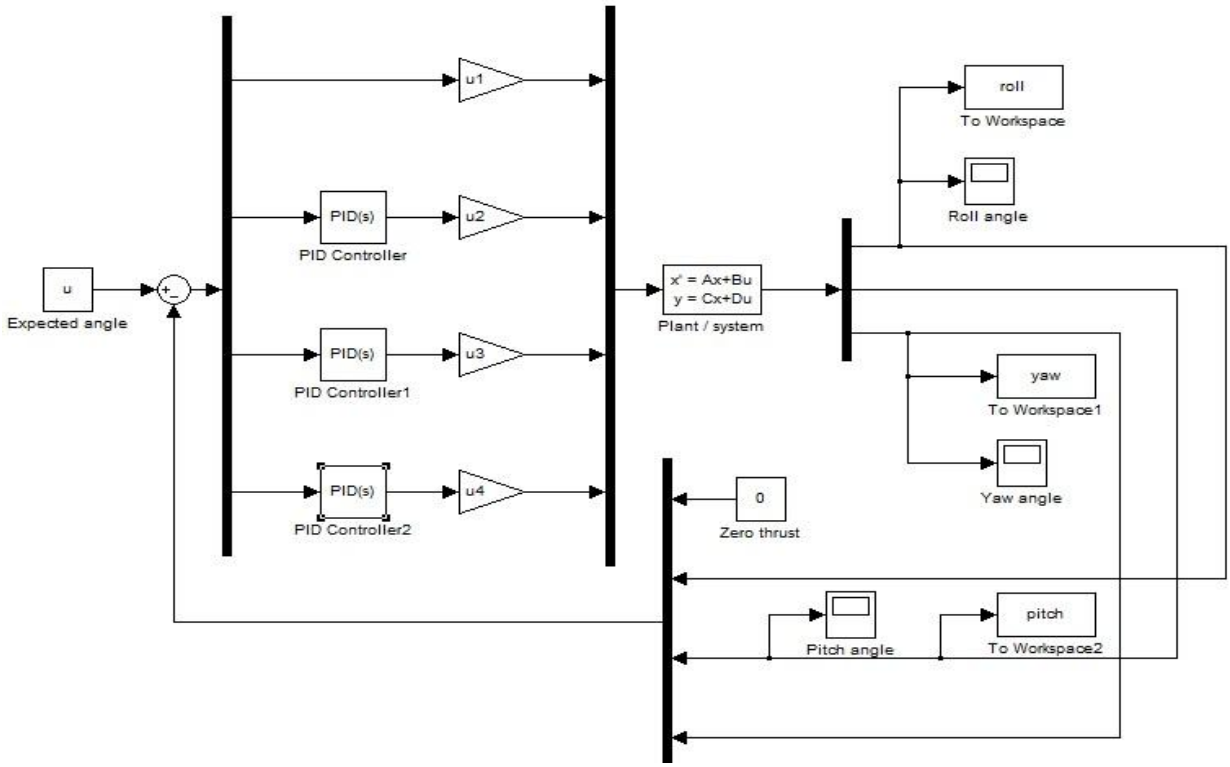


Figure 5-33. Simulink block diagram (PID controller : real case, without mixing)

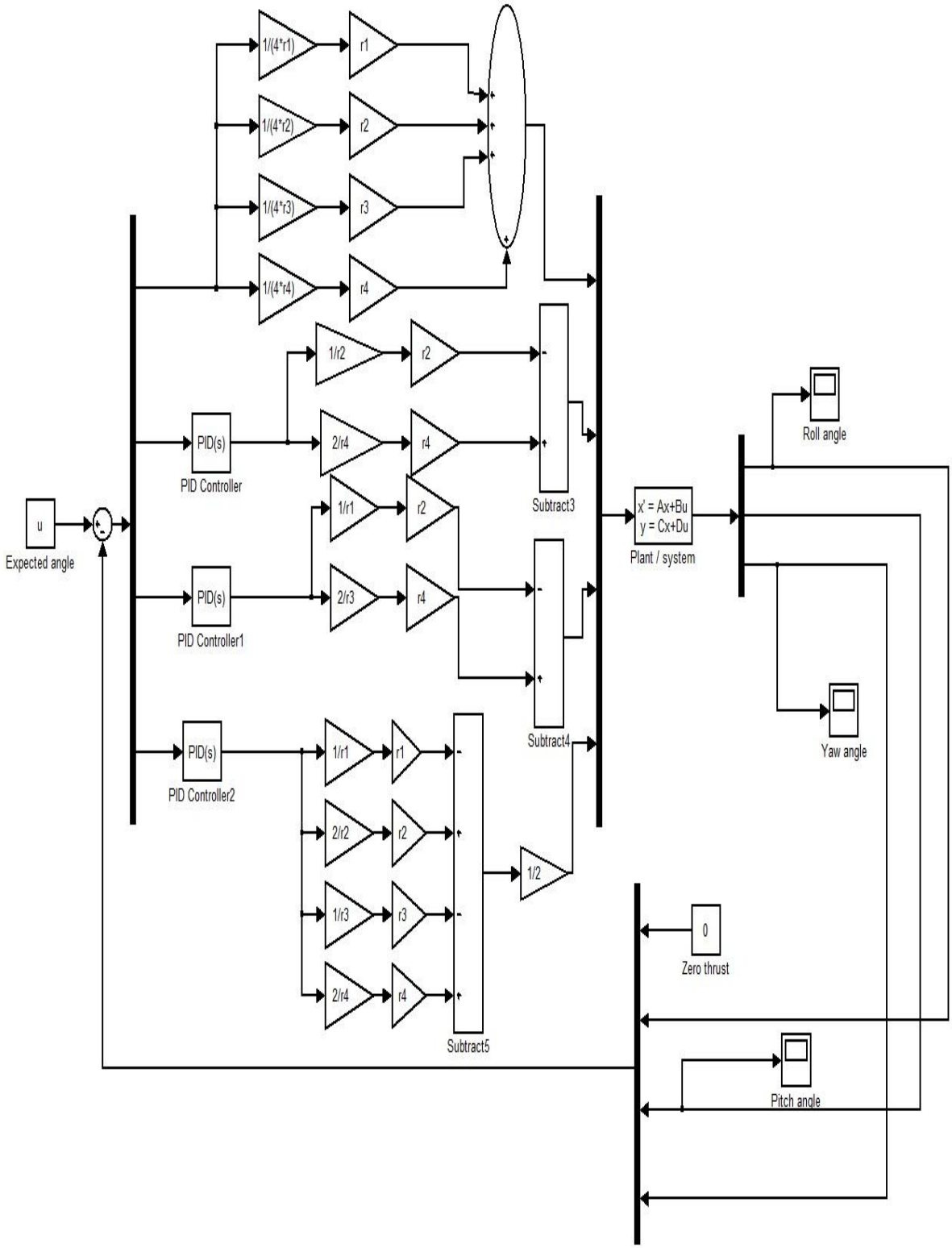


Figure 5-34. Simulink block diagram (PID controller : real case, with mixing)

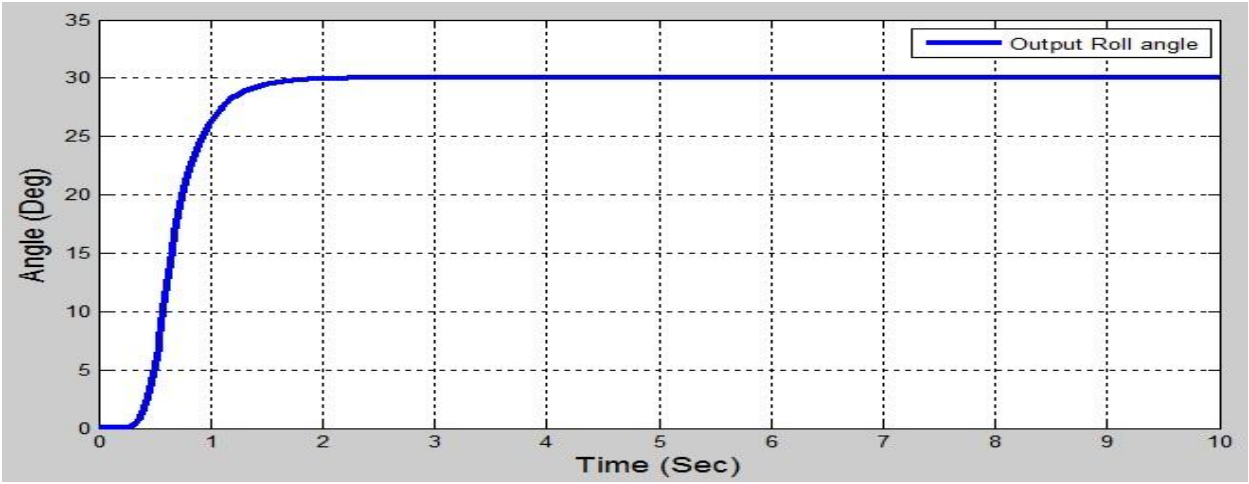


Figure 5-35. Output roll angle (input : roll 30 degree, PID controller)

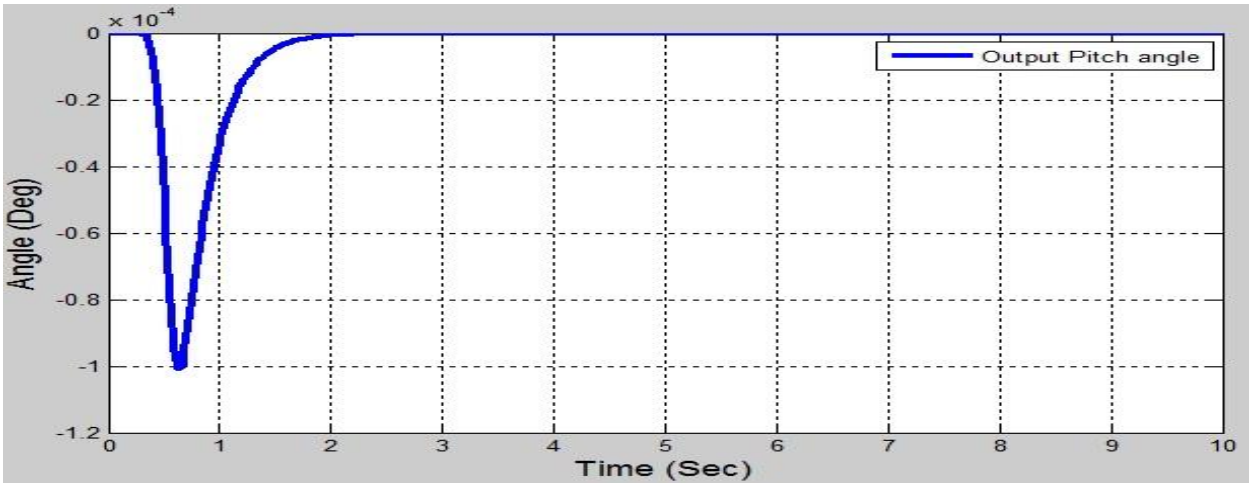


Figure 5-36. Output pitch angle (input : roll 30 degree, PID controller)

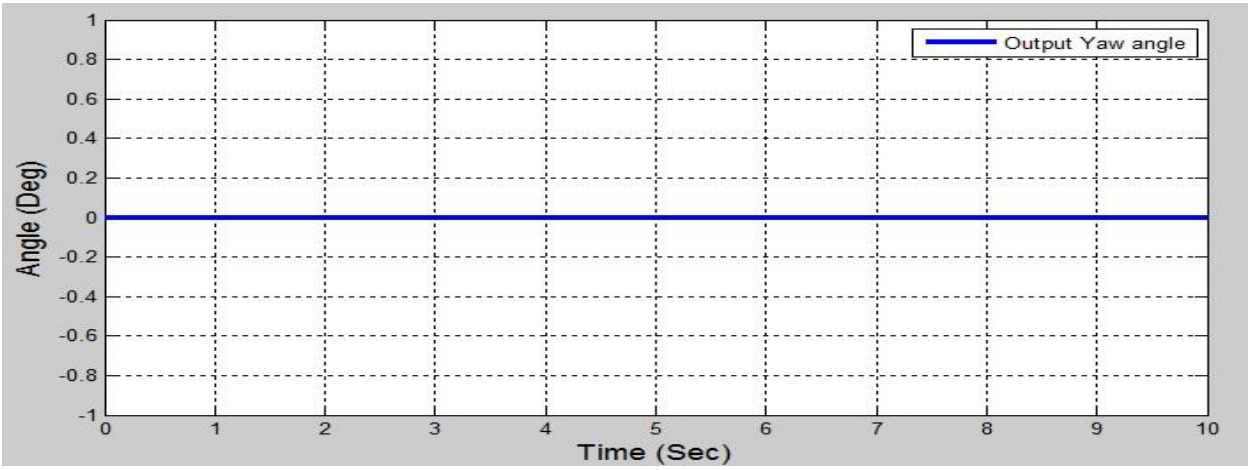


Figure 5-37. Output yaw angle (input : roll 30 degree, PID controller)

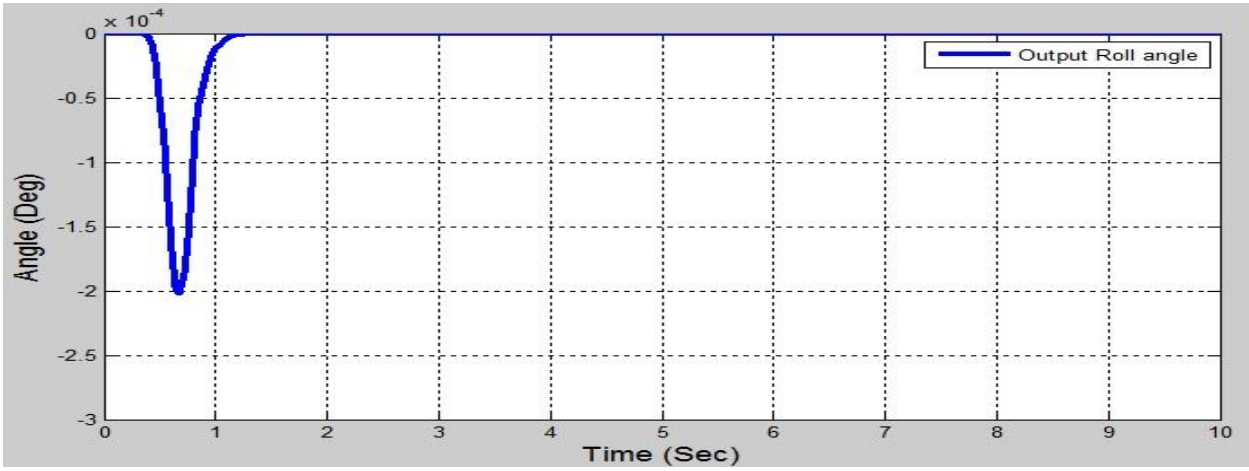


Figure 5-38. Output roll angle (input : pitch 30 degree, PID controller)

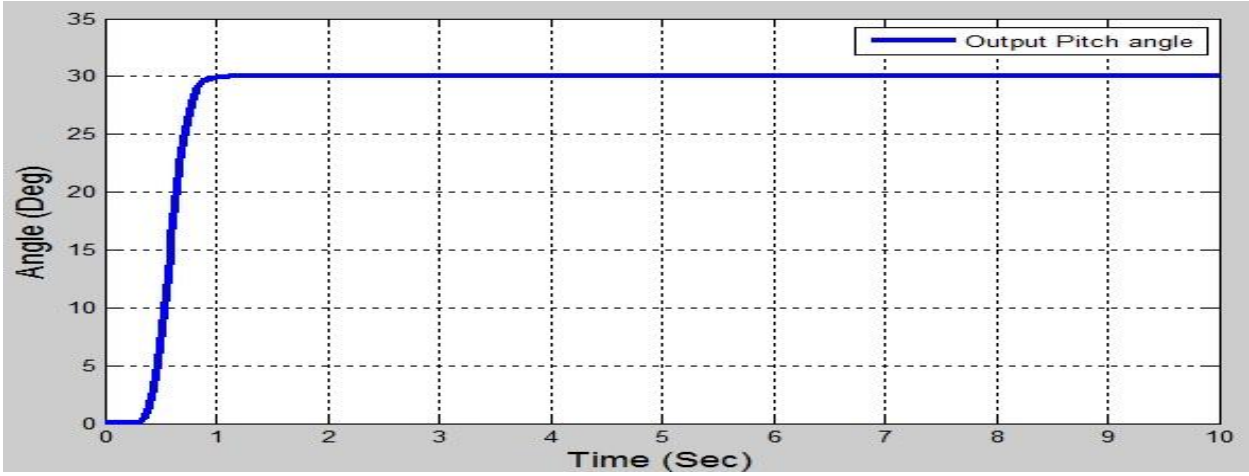


Figure 5-39. Output pitch angle (input : pitch 30 degree, PID controller)

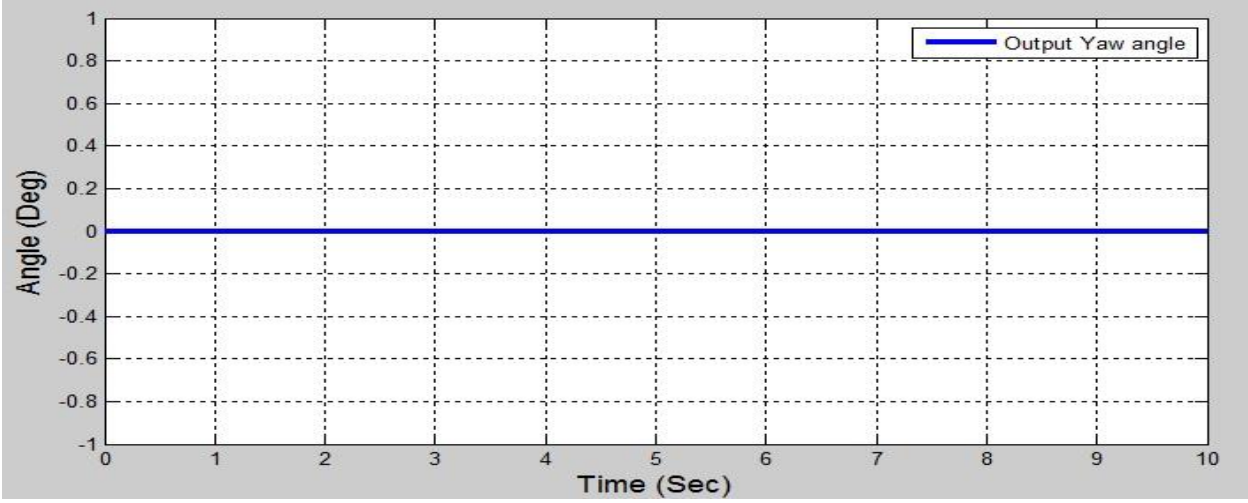


Figure 5-40. Output yaw angle (input : pitch 30 degree, PID controller)

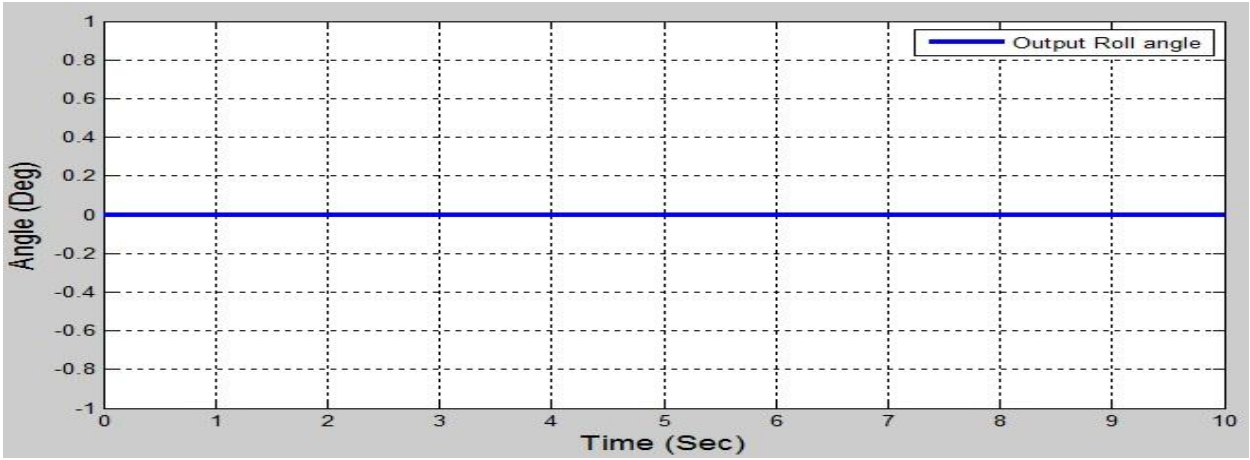


Figure 5-41. Output roll angle (input : yaw 30 degree, PID controller)

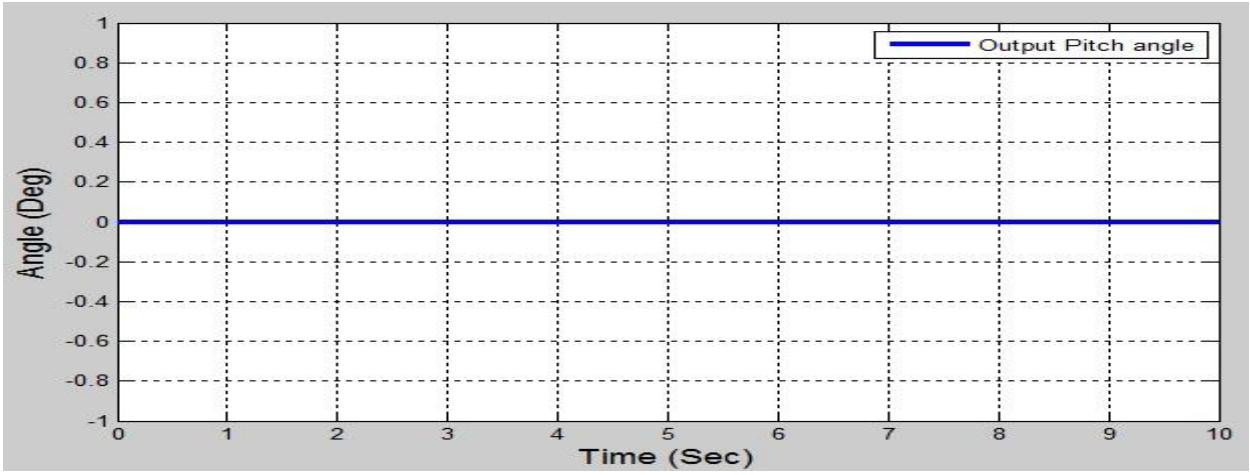


Figure 5-42. Output pitch angle (input : yaw 30 degree, PID controller)

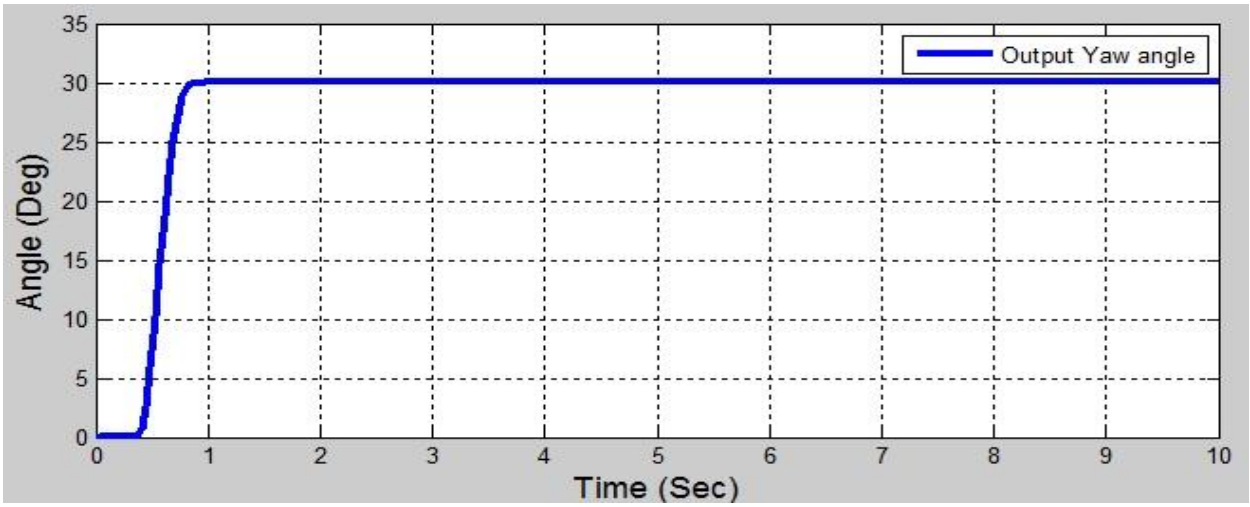


Figure 5-43. Output yaw angle (input : yaw 30 degree, PID controller)

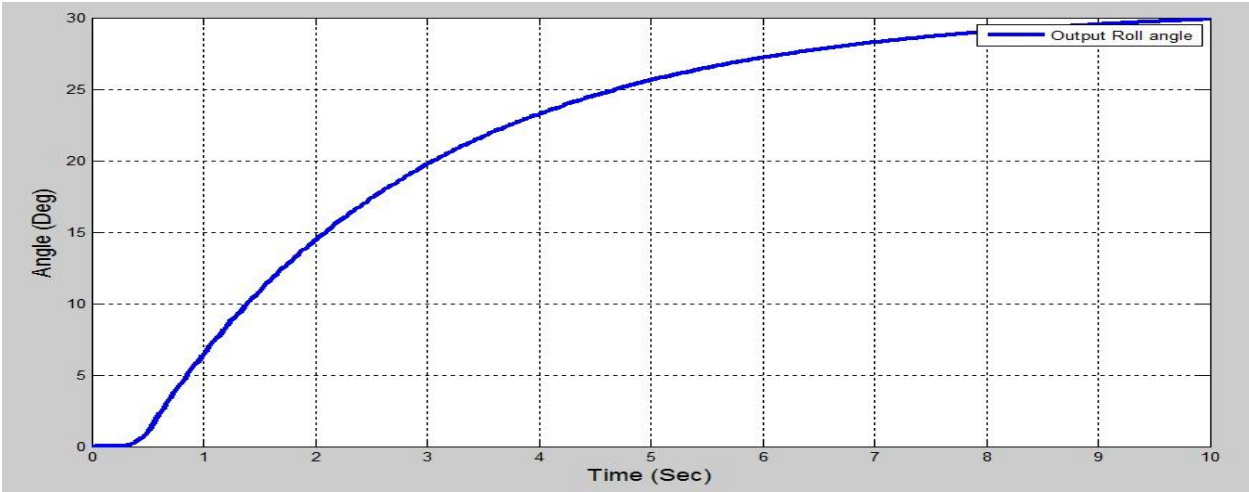


Figure 5-44. Output roll angle (input : roll 30 degree, PID controller, without mixing)

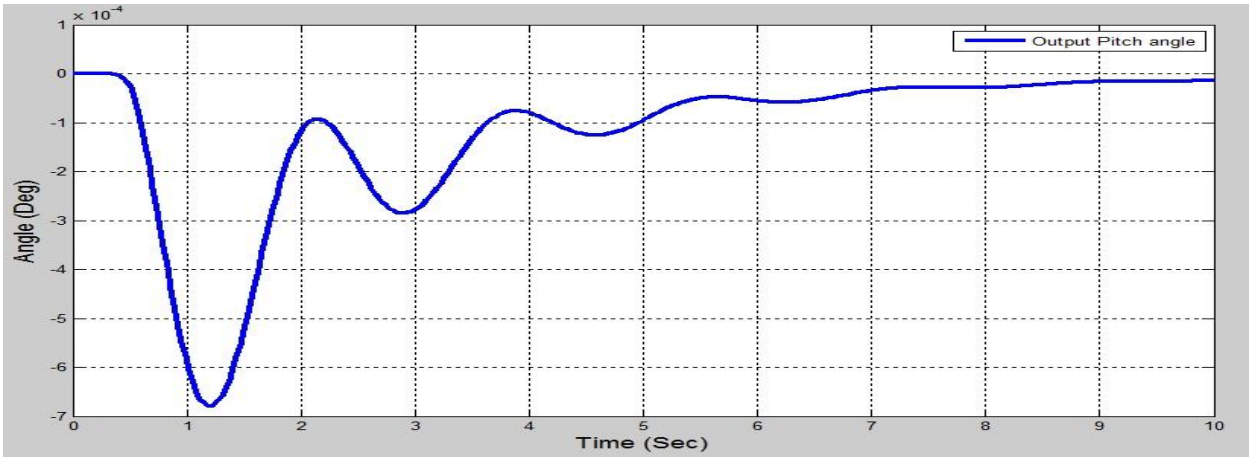


Figure 5-45. Output pitch angle (input : roll 30 degree, PID controller, without mixing)

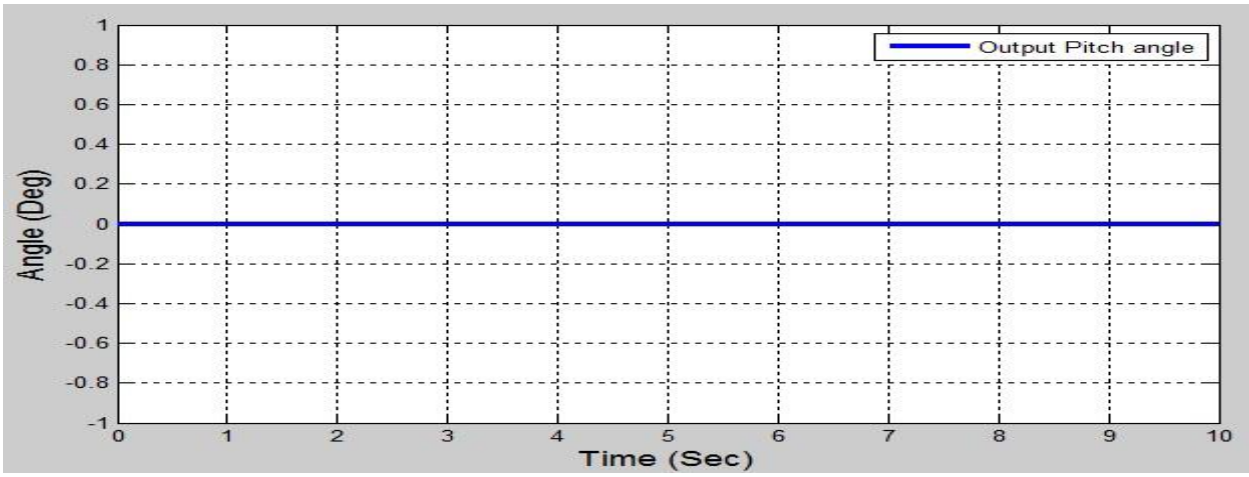


Figure 5-46. Output yaw angle (input : roll 30 degree, PID controller, without mixing)

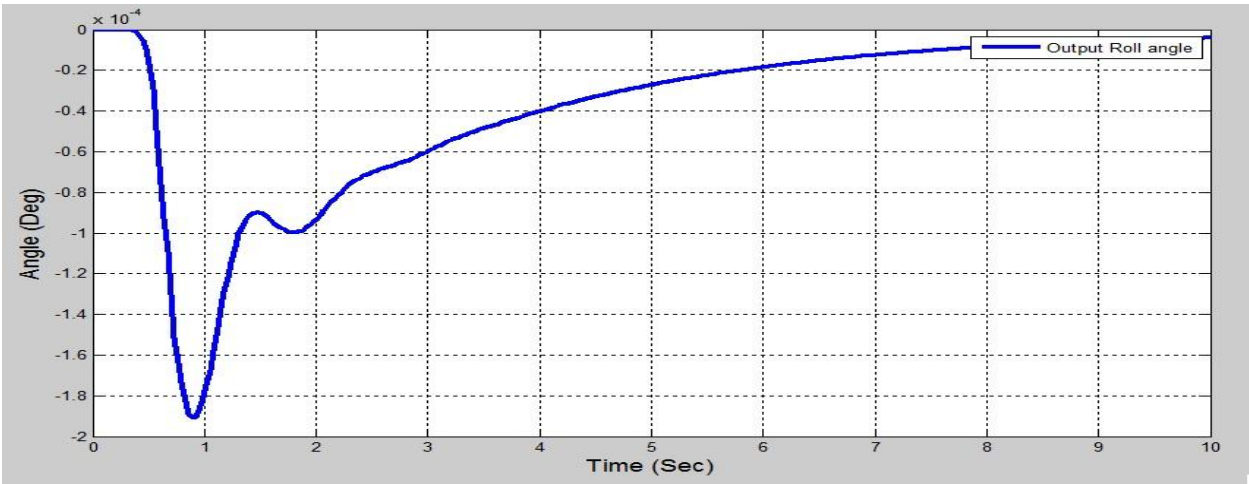


Figure 5-47. Output roll angle (input : pitch 30 degree, PID controller, without mixing)

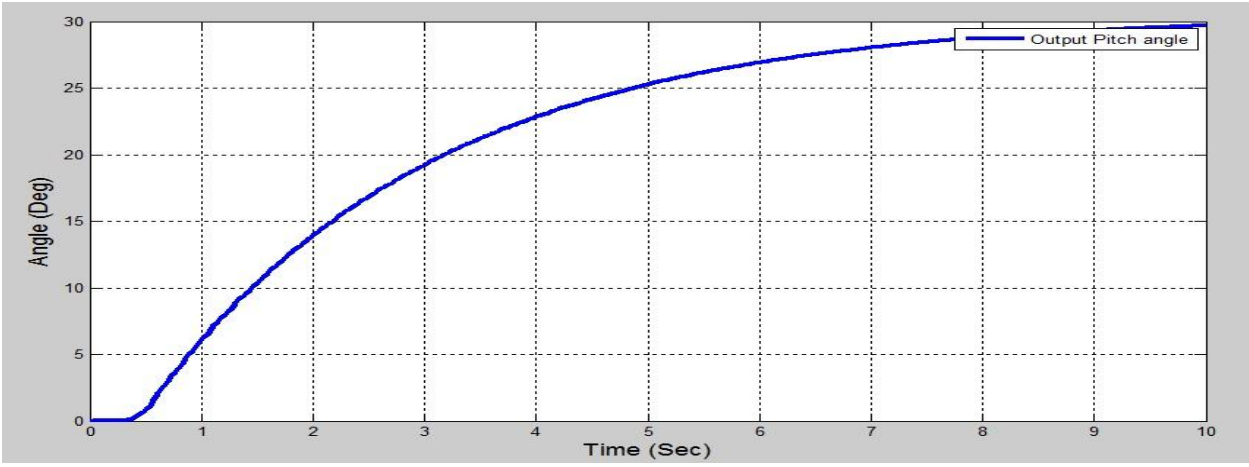


Figure 5-48. Output pitch angle (input : pitch 30 degree, PID controller, without mixing)

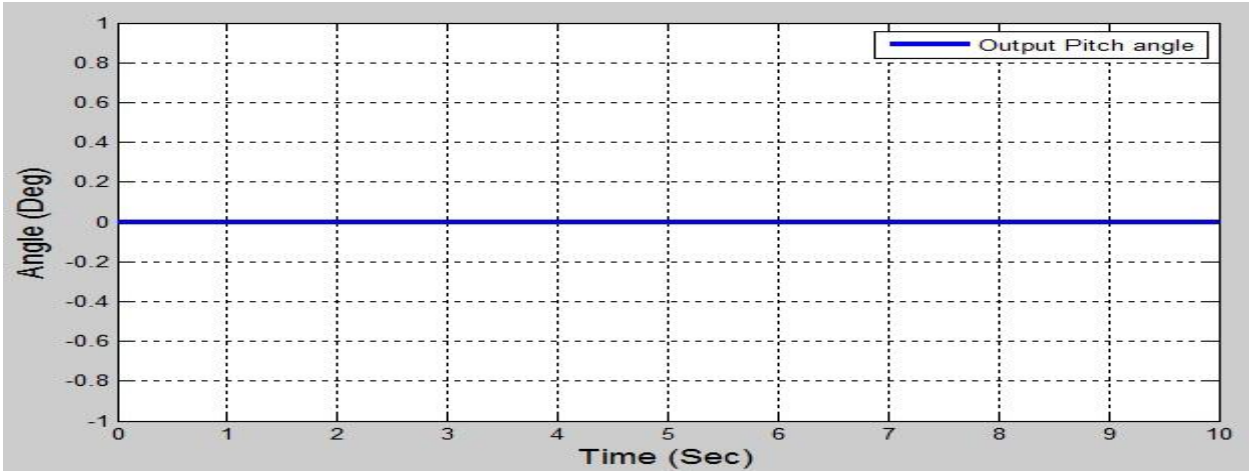


Figure 5-49. Output yaw angle (input : pitch 30 degree, PID controller, without mixing)

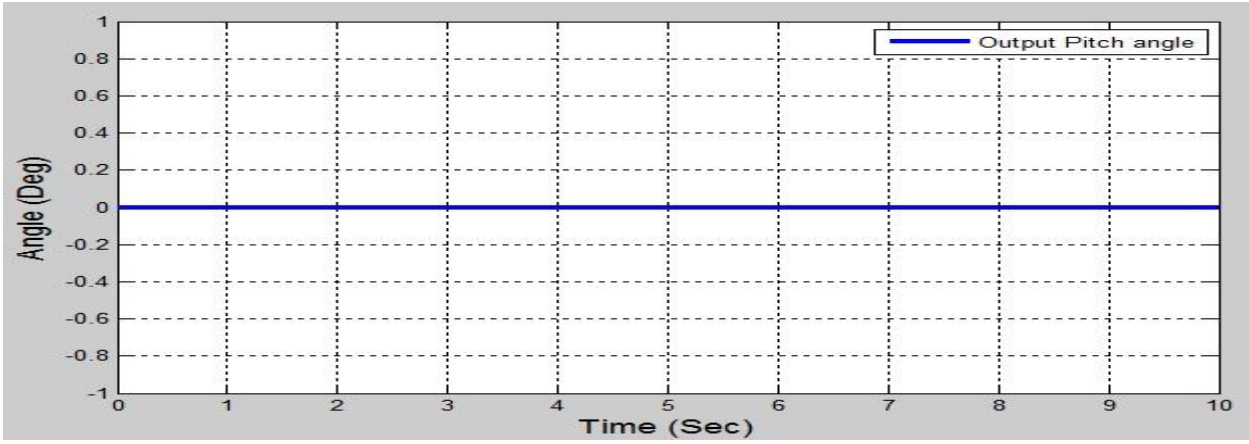


Figure 5-50. Output roll angle (input : yaw 30 degree, PID controller, without mixing)

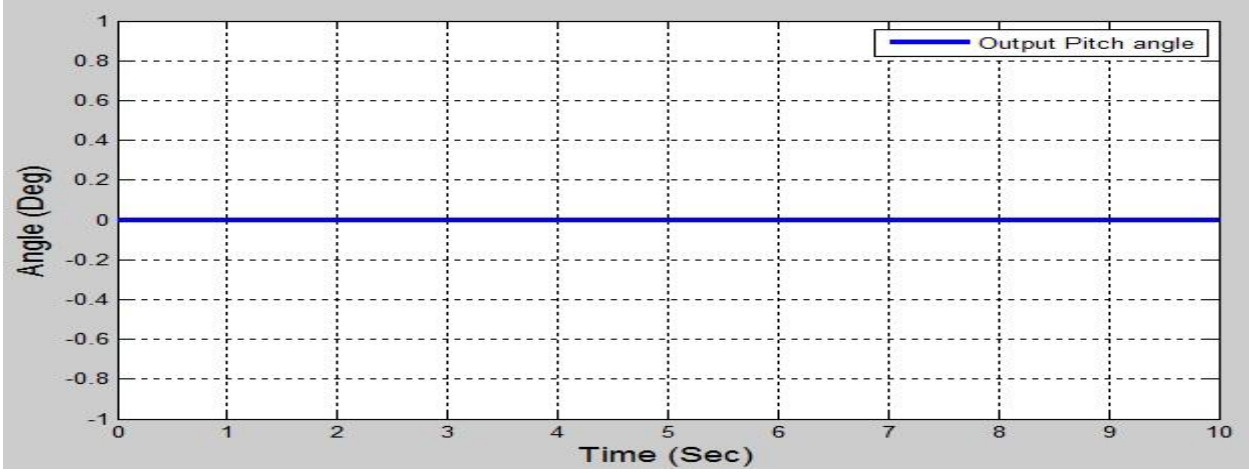


Figure 5-51. Output pitch angle (input : yaw 30 degree, PID controller, without mixing)

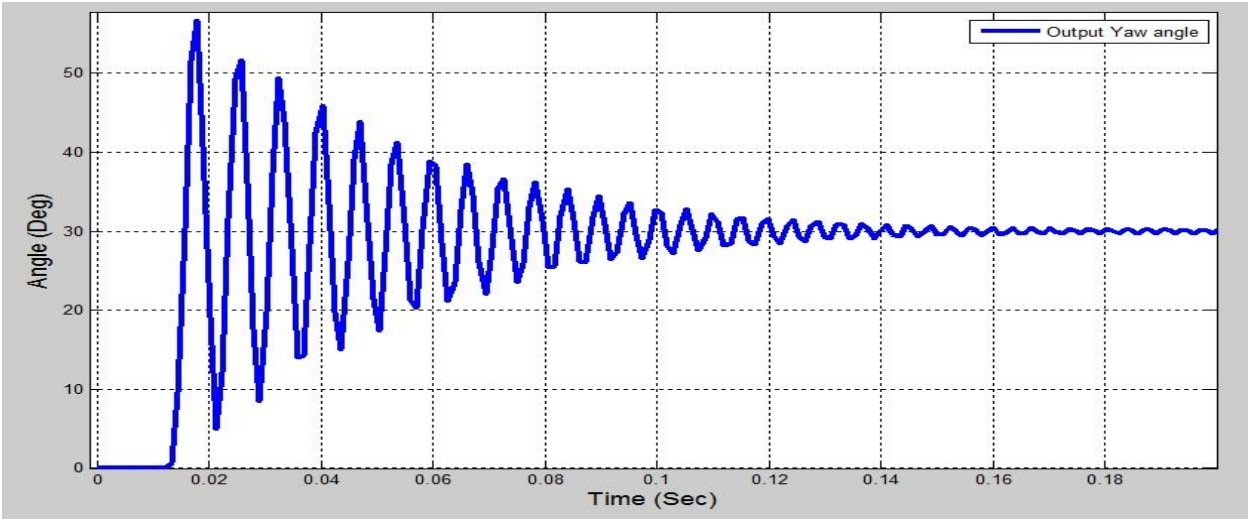


Figure 5-52. Output yaw angle (input : yaw 30 degree, PID controller, without mixing)

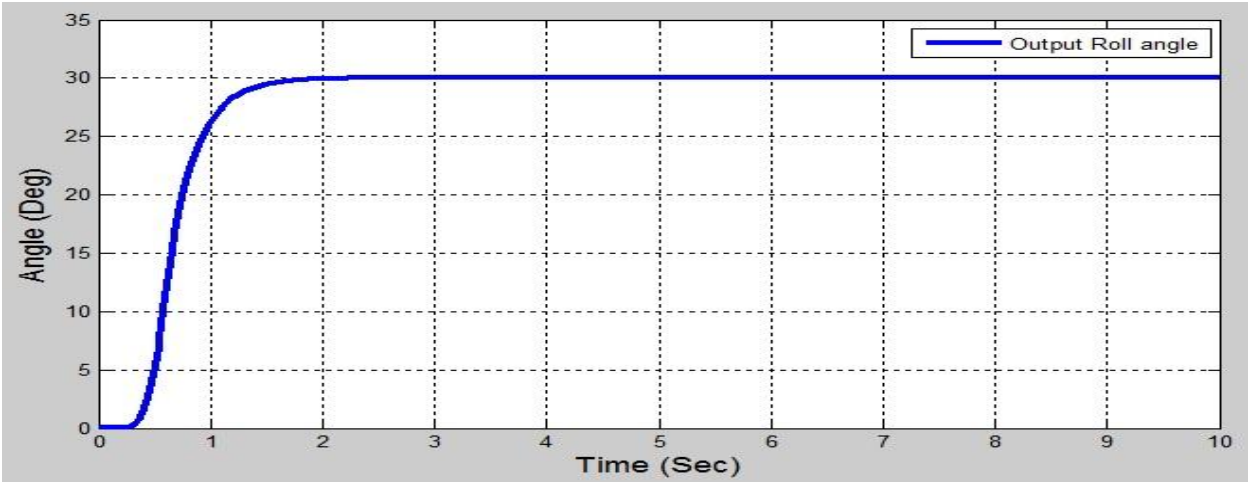


Figure 5-53. Output roll angle (input : roll 30 degree, PID controller, real case)

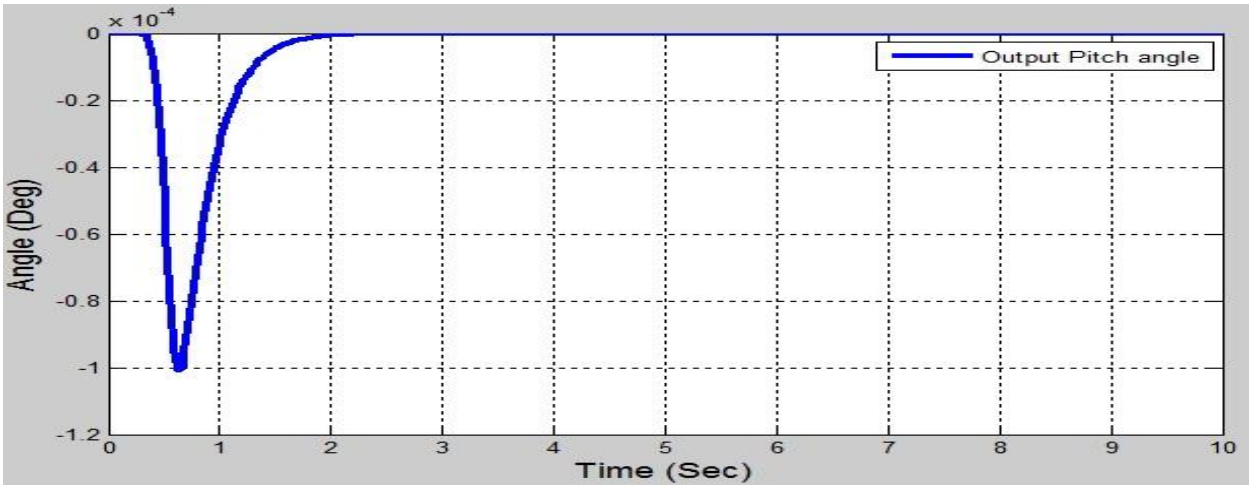


Figure 5-54. Output pitch angle (input : roll 30 degree, PID controller, real case)

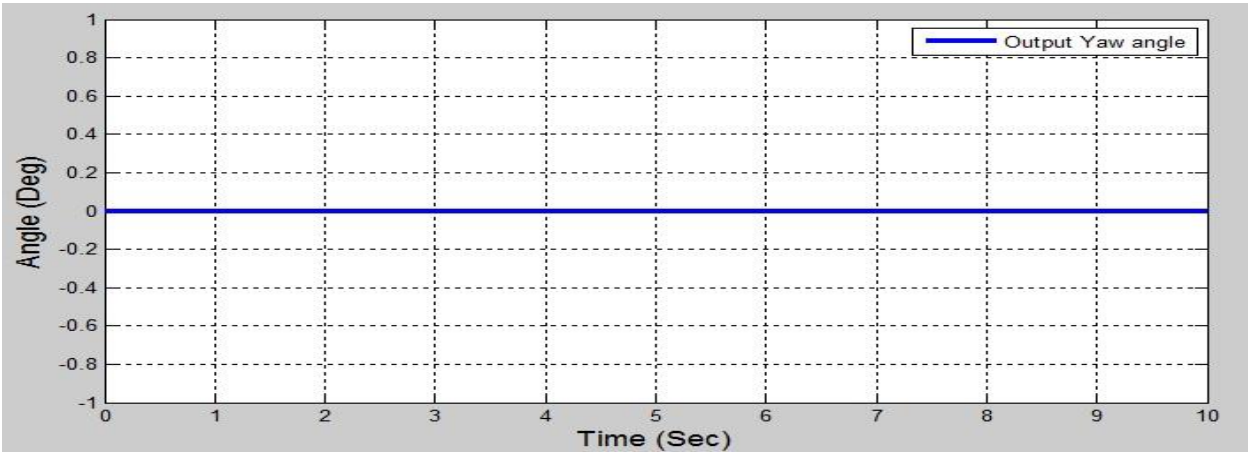


Figure 5-55. Output yaw angle (input : roll 30 degree, PID controller, real case)

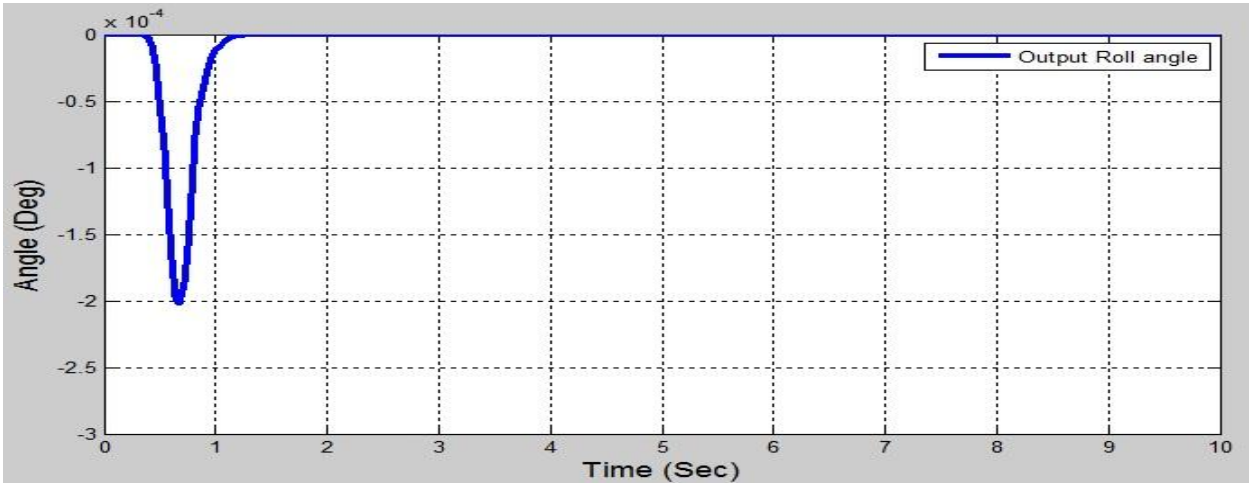


Figure 5-56. Output roll angle (input : pitch 30 degree, PID controller, real case)

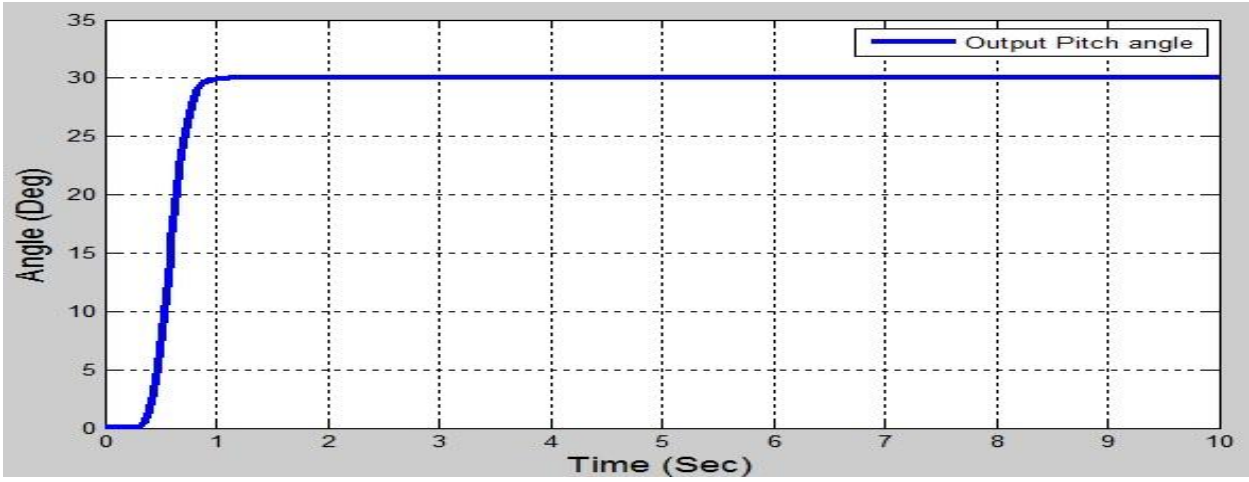


Figure 5-57. Output pitch angle (input : pitch 30 degree, PID controller, real case)

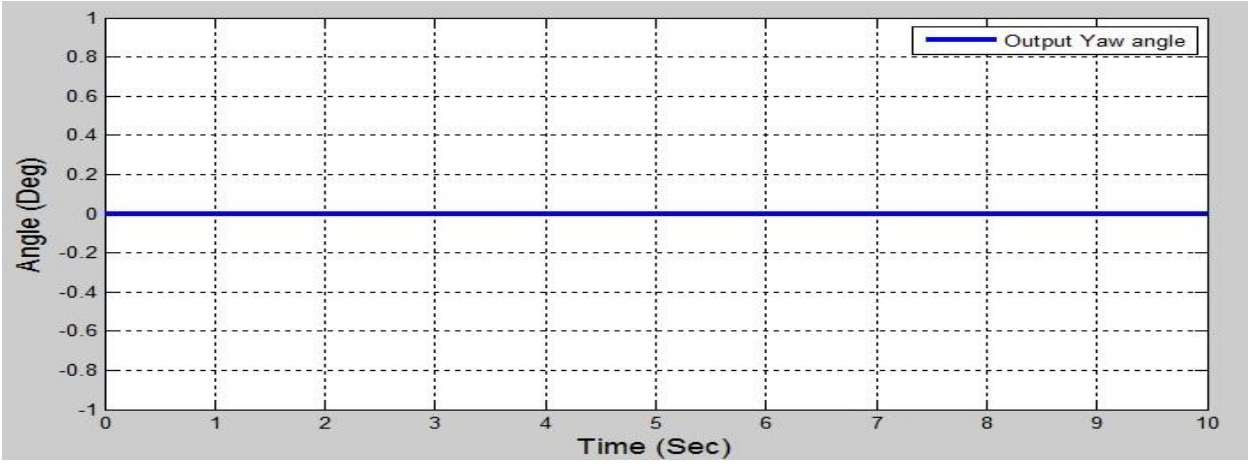


Figure 5-58. Output yaw angle (input : pitch 30 degree, PID controller, real case)

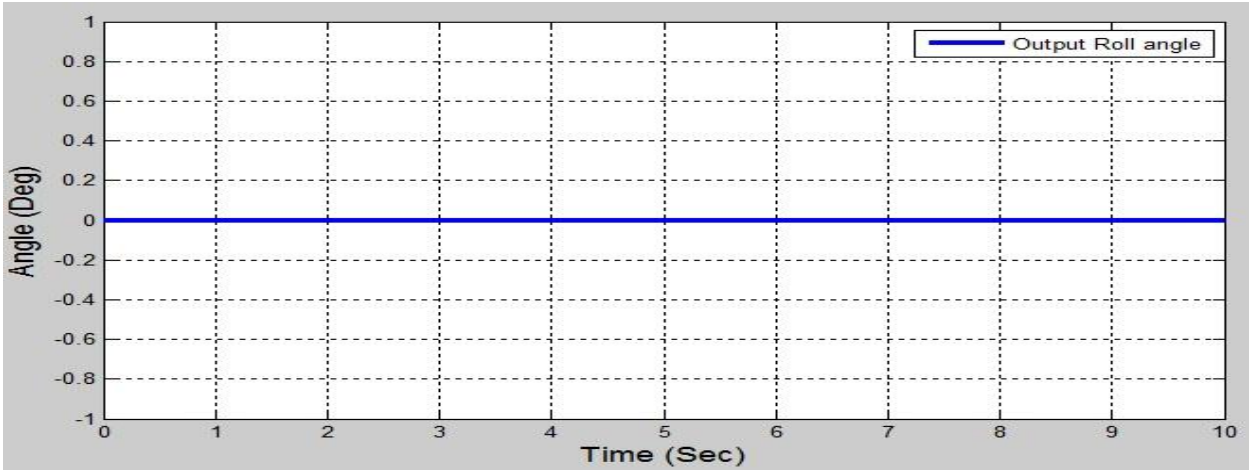


Figure 5-59. Output roll angle (input : yaw 30 degree, PID controller, real case)

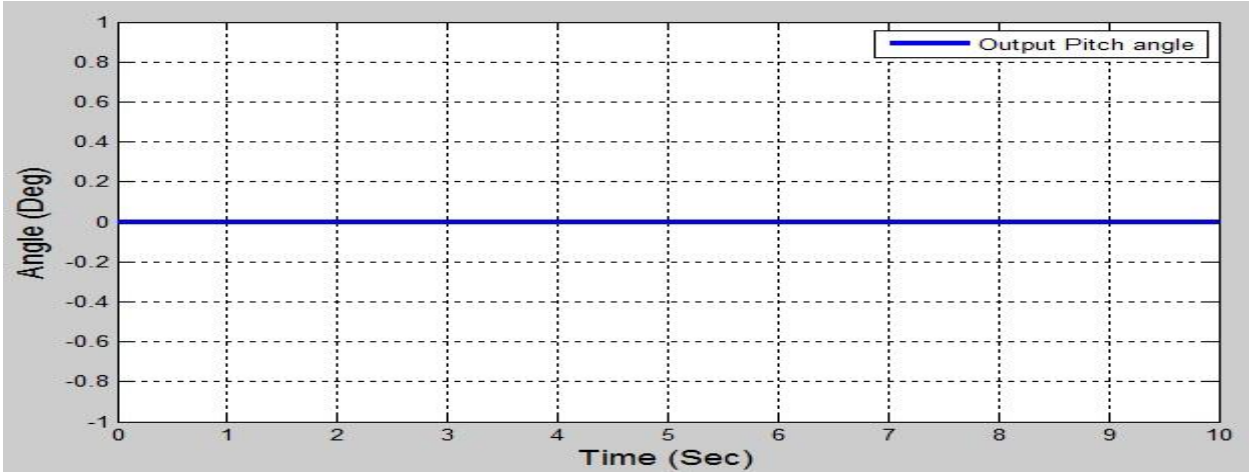


Figure 5-60. Output pitch angle (input : yaw 30 degree, PID controller, real case)

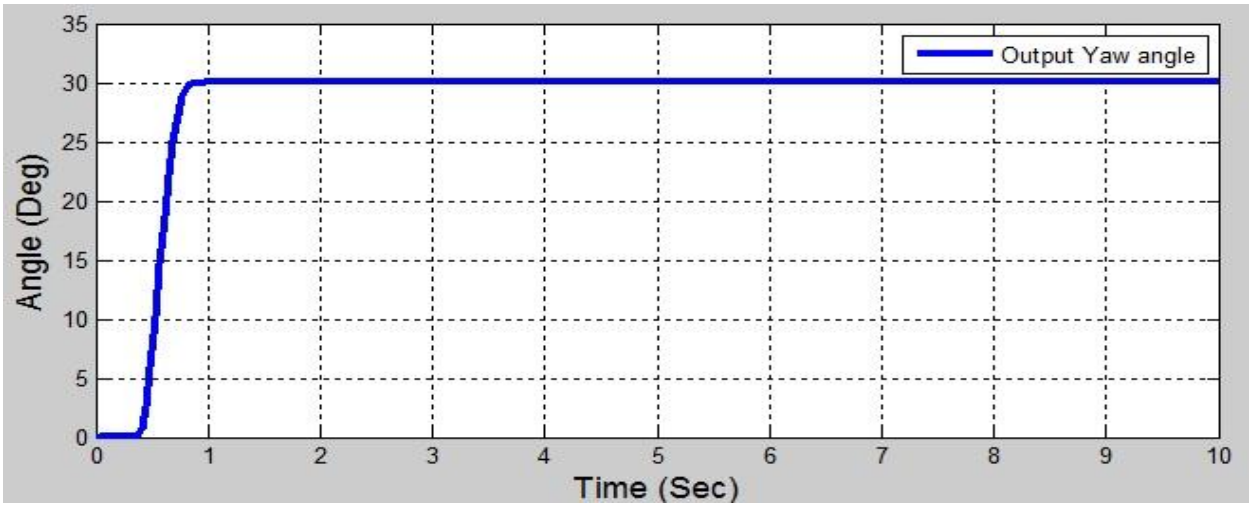


Figure 5-61. Output yaw angle (input : yaw 30 degree, PID controller, real case)

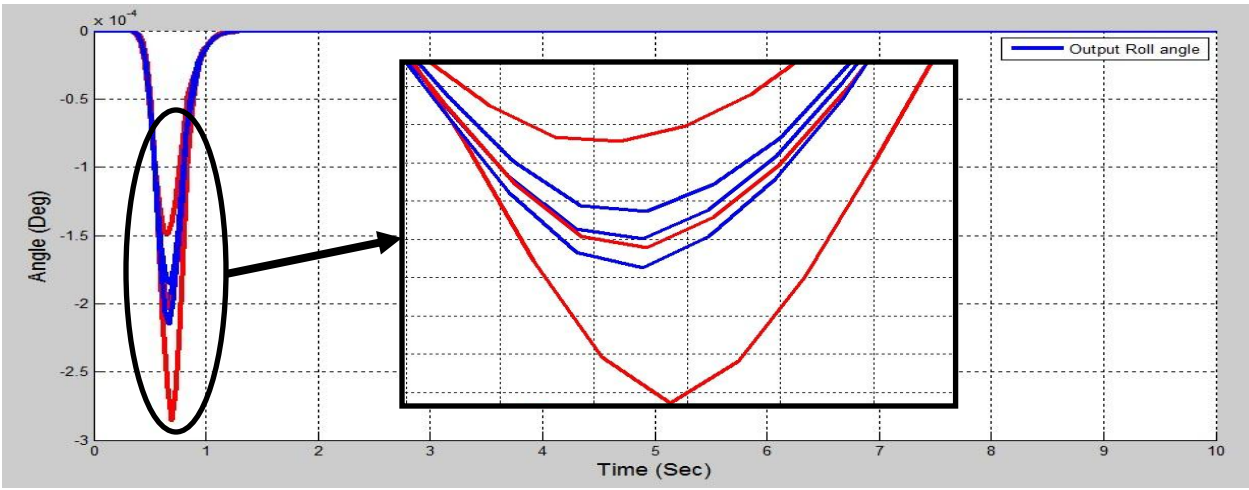


Figure 5-62. Robustness to actuator variation (input : pitch, output : roll)

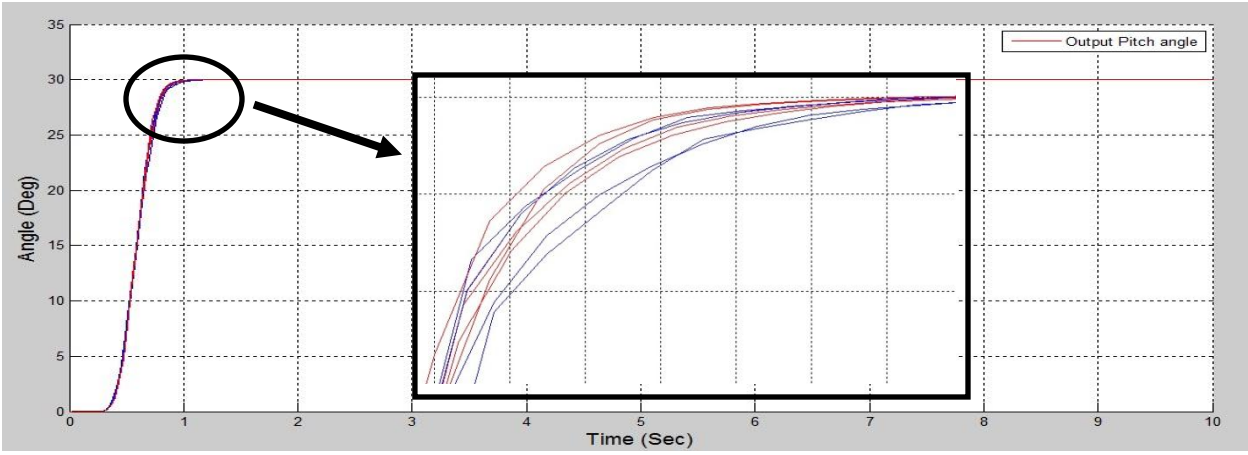


Figure 5-63. Robustness to actuator variation (input : pitch, output : pitch)

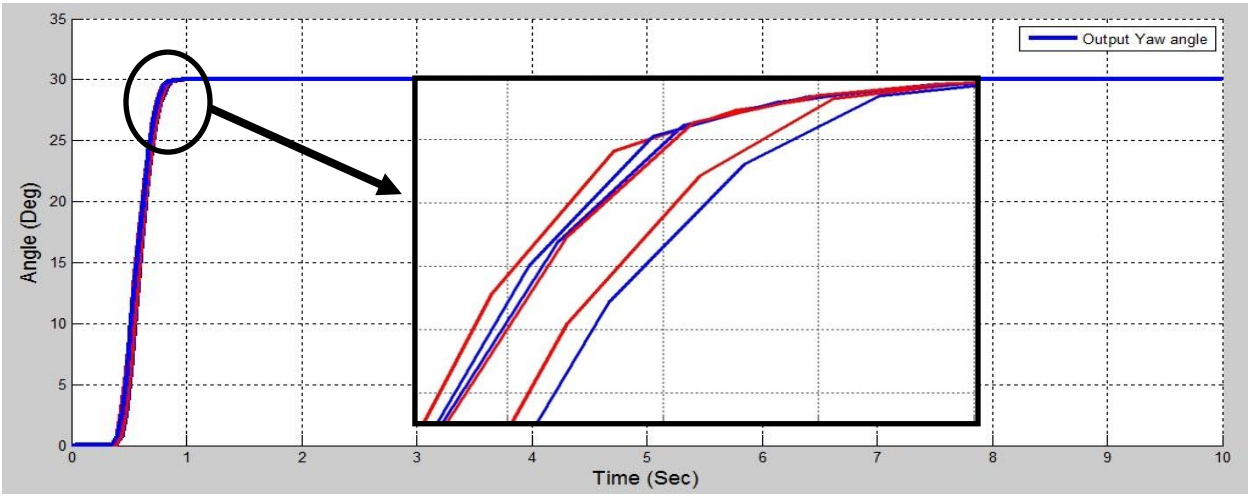


Figure 5-64. Robustness to actuator variation (input : yaw, output : yaw)

Table 5-1. Definition of symbols

Symbols	Definition
ζ	Position vector
R	Rotation matrix
$\hat{\omega}$	Skew symmetric matrix
ϕ	Roll angle
θ	Pitch angle
ψ	Yaw angle
Ω	Rotor speed
$I_{x,y,z}$	Body inertia
J_r	Rotor inertia
J_m	Motor inertia
J_p	Propeller inertia
τ_a	Torque on airframe body
b	Thrust factor
d	Drag factor
l	Lever

Table 5-2. Control gains (PD Controller case)

Input	Controller	P	D
Roll 30 degree	Roll	100	10
	Pitch	1000	100
	Yaw	1	1
Pitch 30 degree	Roll	100	10
	Pitch	1000	100
	Yaw	1	1
Yaw 30 degree	Roll	1	1
	Pitch	1	1
	Yaw	100	10

Table 5-3. Control gains (PID Controller case)

Input	Controller	P	I	D
Roll 30 degree	Roll	0.001	0.001	1
	Pitch	100	0.001	10
	Yaw	1	1	1
Pitch 30 degree	Roll	100	0.001	10
	Pitch	0.01	0.001	3
	Yaw	1	1	1
Yaw 30 degree	Roll	1	1	1
	Pitch	1	1	1
	Yaw	0.01	0.001	10

CHAPTER 6
CONCLUSION

The author got data fundamentally from the real experiment with basic measurement instruments that are announced in experimental setup. Even though the instruments have good accuracy range, there were errors among data. In the result of real experiment, each value was changed with a big difference. Therefore the standard deviation was high.

On the other hand, the author tried to generate one thousand random values (normal distributed random data) between the highest and the lowest value of the real experimental result. Through this process, the author could calculate less standard deviation than previous value.

Using this simple method, the author does not need to experiment over again and again. With several real data, the author calculates those means and standard deviations. Then with numbers of generating data based on real data's measured range, the author calculates those means comparing to real mean. When the author tries to calculate the moment of inertia and thrust factor, these modified values can be used.

In case of simulation and controlling phase, the author tried to get high performance for satisfying stabilization with PD and PID controller. Therefore, the quad-rotor helicopter model for this thesis can be controlled by the expected inputs. It implies that the helicopter can operate rolling, pitching and yawing movements properly when the specific commands are requested.

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BIOGRAPHICAL SKETCH

Sang Min Oh was born in Seoul, South Korea in 1981. He graduated from Soong-Sil University with his Bachelor of Science degree in 2004. During his junior and senior period in the University, Sang Min participated in Reserve Officer Training Corps (ROTC) program.

After he commissioned in 2004, he served in the army as a communication officer. Sang Min changed his branch to army aviation in 2005, and he became a pilot with about 300 flight hours of 500MD and UH-1H helicopters. Therefore, he will contribute for improving and developing his branch with his knowledge.

Thanks to the Korean army, he experienced training, commanding and S-4 (logistics) officer tasks. And he got advanced military education such as Military English Course (MEC) and Officers' Advanced Course (OAC). Recently, he is interested in the Peace Keeping Operations (PKO) and its related jobs in the United Nations (UN). He sincerely hopes that he can serve for the world peace.

Sang Min is proud of serving for his country and he always tries to do his best for improving the army. He wants to contribute for enhancing the relationship between the United States and Korea. He will do his best at any time, any location and any job.