ADVANCED TECHNIQUES FOR SYNTHETIC APERTURE RADAR IMAGE
RECONSTRUCTION

By

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A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
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To God and family

ACKNOWLEDGMENTS

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Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
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This dissertation discusses advanced aspects of Synthetic Aperture Radar (SAR).

SAR is a crucial capability for radar systems in both civilian as well as government

sectors. This dissertation focuses on two actively researched aspects of SAR systems.

First, a framework for reconstructing SAR images from a spectral analysis perspective

is introduced. This approach is more robust and more accurate due to lower sidelobes

level and can be used in situations that full data collection is not permissible. Second, a

framework for identifying movers directly from SAR data is also discussed. Furthermore,

this framework takes in account SAR systems with multiple antennas and is shown

to offer superior performance in such cases. The dissertation also provides a basic

overview of the signal processing involve in constructing SAR images. This background

knowledge lays the foundation to which further advanced concepts are built upon.

We first focus on a new approach to reconstructing SAR images. This new

approach is from the spectral estimation community. In particular we utilize the Bayesian

framework as it results in a sparser SAR image that is more amenable for identification

and classification purposes. We utilize an algorithm named SLIM, which can be thought

of as a sparse signal recovery algorithm with excellent sidelobes suppression and high

resolution properties. For a given sparsity promoting prior, SLIM cyclically minimizes

a regularized least square cost function. We show how SLIM can be used for SAR

image reconstruction as well as used for SAR image enhancement. We evaluate

the performance of SLIM using realistically simulated complex-valued backscattered data from a backhoe vehicle. The numerical results show that SLIM satisfactorily suppress the sidelobes and yield higher resolution than the conventional matched filter or delay-and-sum (DAS) approach. SLIM outperform the widely used compressive sampling matching pursuit (CoSaMP) algorithm, which requires the delicate choice of user parameter. Beside bringing SLIM to the field of SAR image reconstruction, we show how SLIM can be made more computationally efficient by utilizing the Fast Fourier Transform (FFT) and the conjugate gradient (CG) method.

In the event that full data collection is not possible, we propose several methods to reconstruct SAR images based on missing data samples. This is coined as interrupted SAR. For this scenario we consider nonparametric adaptive spectral analysis of complex-valued data sequences with missing samples occurring in arbitrary patterns. We once again utilize SLIM and IAA. However these algorithms were not adapted for the case of missing data samples. We consider how these algorithms can be adapted to the missing data sample case. Furthermore, we consider fast implementations of these algorithms using the Conjugate Gradient (CG) technique and the Gohberg-Semencul-type (GS) formula. Our proposed implementations fully exploit the structure of the steering matrices and maximize the usage of the Fast Fourier Transform (FFT), resulting in much lower computational complexities as well as much reduced memory requirements. The effectiveness of the adaptive spectral estimation algorithms is demonstrated via several numerical examples including both 1-D spectral estimation and 2-D interrupted synthetic aperture radar (SAR) imaging examples.

We then shift gears to discuss how SAR data can be used to detect moving targets. Mover detections, whether in post-processing or the main focus of a surveillance system is increasingly more common in SAR systems. The detection of small movers is a challenging task, detection of small movers in high clutter environment is even more difficult. While mover detection algorithms exist, they are catered toward a 2-channels

system. We explore how multi-channels can be used to detect smaller moving targets. We consider moving targets detection and velocities estimation for multi-channels synthetic aperture radar (SAR) based ground moving target indication (GMTI). Via forming velocity versus cross-range images, we show that small moving targets can be detected even in the presence of strong stationary ground clutter. Furthermore, the velocities of the moving targets can be estimated, and the misplaced moving targets can be placed back to their original locations based on the estimated velocities. An iterative adaptive approach (IAA), which is robust and user parameter free, is used to form velocity versus cross-range images for each range bin of interest. Moreover, we discuss calibration techniques to estimate the relative antenna distances and antenna gains in practical systems. Furthermore, we present a simple algorithm for stationary clutter cancellation. We conclude by demonstrating the effectiveness of our approaches by using the Air Force Research Laboratory (AFRL) publicly-released Gotcha airborne SAR based GMTI data set. Finally, with the emergence of Multiple-Input-Multiple-Output (MIMO) radar, we investigate a framework for the detection of ground moving targets using a MIMO radar system. We look at how phase histories collected from a MIMO Synthetic Aperture Radar (SAR) system can be used for target detection in a Ground Moving Target Indication (GMTI) mode while SAR images are being formed. We propose the usage of the recently developed Iterative Adaptive Approach to detect movers and estimate their velocities in the post-Doppler domain. We show how the waveform diversity afforded by a MIMO radar system enables its superiority over its Single-Input-Multiple-Output (SIMO) counterpart.

CHAPTER 1
INTRODUCTION TO SYNTHETIC APERTURE RADAR

The history of RADAR (Radio Detection and Ranging) is rich and is considered to be one of the high points of human innovations. A melting pot of multitudes of disciplines resulted in a system that enables us the peace and prosperity that we enjoy today. From weather applications that enables us to be one step ahead of mother nature, to area surveillance that keeps our troops and borders safe. While the history of radar is rooted in the principles of electromagnetics that can be traced back to one of the greatest mind of the 19th century, James Clerk Maxwell, the processing of its data which we now call the field of radar signal processing, is a very recent field that grew due to the oncoming of the digital age. Now, fast and cheap computing hardware allows us to be truly majestic with the processing of its returned signals.

Within the field of radar signal processing, one of the most elegant development is that of the concept of Synthetic Aperture Radar (SAR). The invention of SAR can be considered one of the most ingenious result of signal processing, as it allows a system to overcome a physical limitations by careful manipulations of the signals at the output of the radar. By using SAR, a radar which up to that point is mainly used in the 1-D domain, is transformed into to a system that is capable of forming both 2-D and 3-D images. This enables unprecedented achievements in terrain mapping applications and wide area surveillance for time of peace and time of war.

This dissertation revolves around the development of SAR and proposes advanced techniques on how to better reconstruct SAR data and how we can use novel signal processing techniques to overcome some of the shortcoming of SAR. It is a culmination of several years of research that resulted in a set of academic publications [1–10]. The dissertation begins with an introductory chapter to Synthetic Aperture Radar from a signal processing perspective. It highlights the founding principles behind SAR and identifies different aspects that can be exploited for improvements. Then we move

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on to talk about moving targets indications using SAR data directly. We extend the applications of moving target indications to multi-channel systems.

In Chapter 1, we begins by backtracking the development of SAR and explore in details the standard techniques that are employed today in modern SAR systems. We discuss the important concept of range and cross-range resolutions which are two essentials criteria in development of a SAR system. We then explain how these techniques are currently employed and discussed its shortcoming. We also make the connection between these techniques to more generalized spectral estimation techniques and how advanced algorithms in spectral estimation may be used to improve the modern SAR systems.

In Chapters 2 and 3, we proposes several adaptive signal processing techniques to the reconstruction of SAR images. In particular, we focus on the reconstruction of SAR images in a Bayesian framework, we show that this method can produce sparse result with great sidelobe suppression capability. In the chapter following, we also discuss how this approach can be used in the application of Interrupted SAR. This is a special case where mode-switching in modern SAR systems results in missing data, or data are thrown away on purpose to reduce the amount of data transfer.

In Chapters 4 and 5, we give a thorough treatment on the usage of SAR data to detect moving targets. Currently a special collection mode is required for movers detection. We propose to use SAR data to simultaneously identify moving targets, as well as to estimate their parameters–such as speed and direction. We introduce several adaptive techniques that can be used to identify the movers and estimate its parameters. We extend these methods to the case of using an antenna array to form SAR images and how having multiple elements increases the ability to correctly estimate the speed of the targets. We then conclude this dissertation with a theoretical treatment of using Multiple-Input-Multiple-Output (MIMO) Radar to form SAR images as well as to detect movers. The waveform diversity afforded by MIMO radar leads to new break

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through in the suppressing of ground clutter which leads to better accuracy in identifying our parameters of interests.

1.1   The Signal Processing Aspect of SAR

The development of SAR was motivated by the desire to have constant visuals of the terrain of interests during war time. Before we have satellites orbiting the earth with high resolution optical imaging and thermal imaging equipment, the only way to obtain visuals of the ground is with an optical imaging equipment flying on board an aircraft. This poses two problem, first, high resolution optical cameras were not yet available, and second, optical cameras only work under ideal weather conditions and when light was available. This means covert operations done at night relies on data that may be quite obsolete. This important barrier gave rise to the development of SAR. For with SAR imaging, vision of the ground is possible in all weather conditions, night or day, and can be obtain with rather long standoff ranges–safe from any enemy threat. When discussing a photograph, a natural question is to inquire about its resolution. What does a pixel in a photo represents in its spatial domain ? This is a question of resolution, see figure 1-1. In terms of radar terminology, dy is the resolution in range, and dx is the resolution in cross-range. We now talk about the two component individually and how the resolution is determined.

1.2   Resolution in Range

Range resolution relates back to the most fundamental usage of a radar. The goal of a radar is to first detect a presence of a target, and secondly to determine its distance. To achieve this, the radar emits a pulse of energy in the direction of interest and measure the time delay of the returning echo. Since we know the speed of light and we can measure the time delay, we can effectively determine the range of the object associated with the echo. If there was only one object, and only one pulse was sent, there is no problem. However, if there are more than one object, and more than one
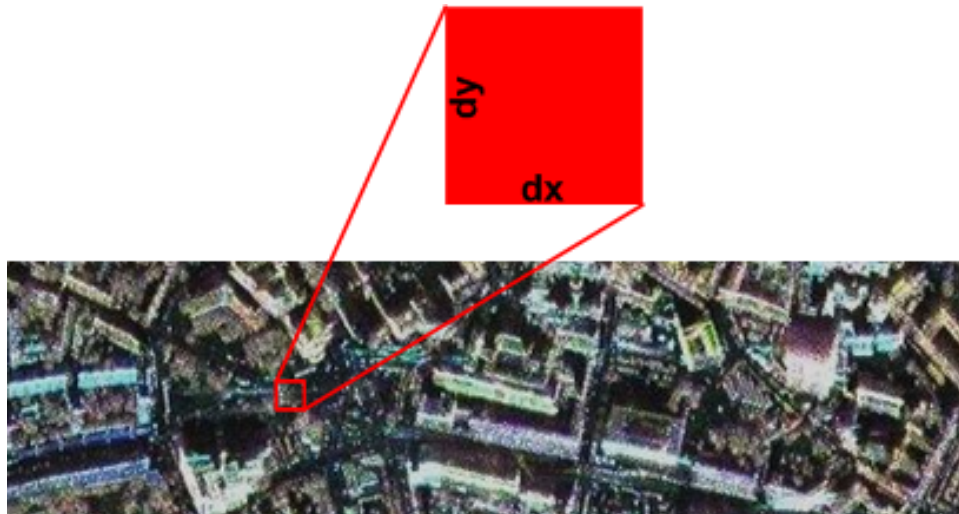
Figure 1-1. Resolution of a pixel
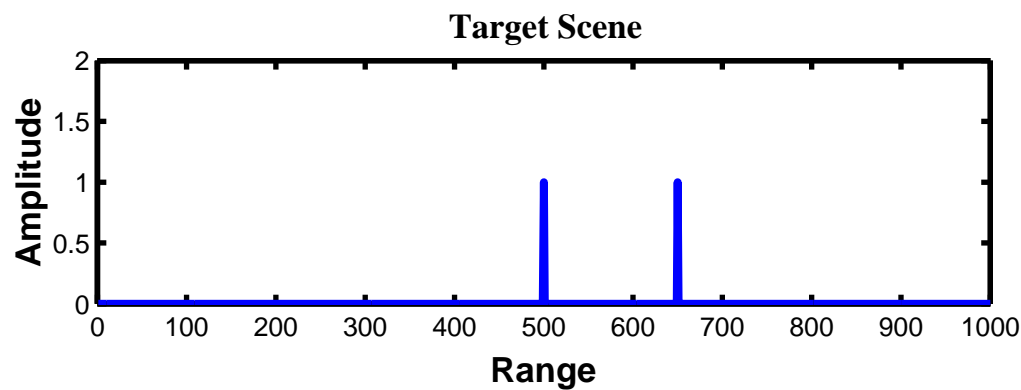
Figure 1-2. Scene with Impulse Targets

pulse of energy is sent, there exist some ambiguity between discerning the return of one pulse from the other.

Let $s(t)$ be a simple sinusoidal pulse that is transmitted from the transmitter of a radar.

$$s(t) = \begin{cases} Ae^{2i\pi f_0 t} & \text{if } 0 \le t \le T; \\ 0 & \text{otherwise}. \end{cases}$$

It has an amplitude of A operating at a carrier frequency f_0 and has a duration of T. We fire this pulse from our transmitter to a scene of interest that has impulse targets that occupies only one range bins. See Figure 1-2 The returned signal from the following

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scene would be an attenuated, time delayed version of the signal we sent. See Figure 1.2.1. If our return signals contains no noise, it is easy to see that the time delay of the returned signal contains the distance information, as we know that the wave travels at the speed of light. However, in the presence of noise, the signal is no longer clean and in practice we have to perform some signal processing on the returned signal.

1.2.1   Matched Filtering

The standard method to detect the returned signals is called the *matched filter*. Since we know what was sent out we can *matched* it with the return signal. From a filtering perspective, we can represent the matched filter as,

$$h(t) = s^*(-t) \tag{1--1}$$

where $s^*(t)$ is the time reversed and conjugated version of the transmitted signal. Given that the Fourier Transform of the transmitted signal and the filter is $S(f)$ and $H(f)$, respectively, the matched filter output is,

$$Y(f) = H(f)S(f) \tag{1--2}$$

in the time domain, the output is,

$$y(t) = h(t) \otimes s(t) \tag{1--3}$$

where the $\otimes$ is the convolution operator, and then can be written out as,

$$y(t) = \int_{-\infty}^{\infty} h(t - \tau)s(\tau)\,d\tau \tag{1--4}$$

substituting the definition of $h(t)$ we have the definition of the autocorrelation of a signal.

$$y(t) = \int_{-\infty}^{\infty} s^*(t + \tau)s(\tau)\,d\tau \tag{1--5}$$

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Transmitted Signal
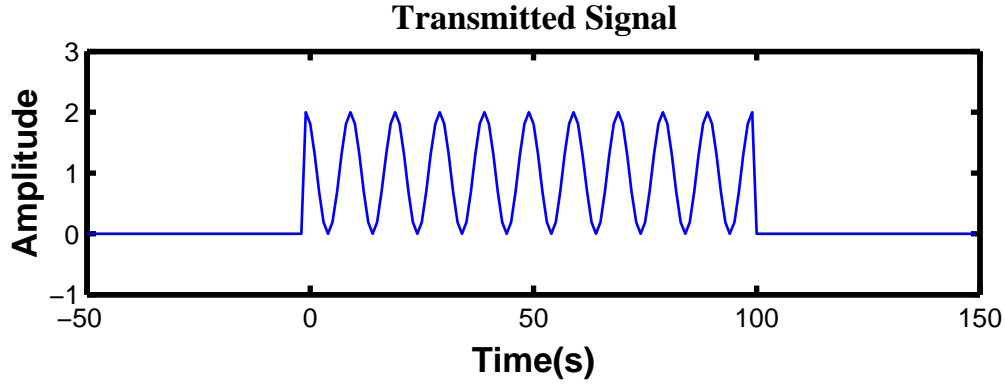
Figure 1-3. Transmitted Signal

Returns from Target Scene
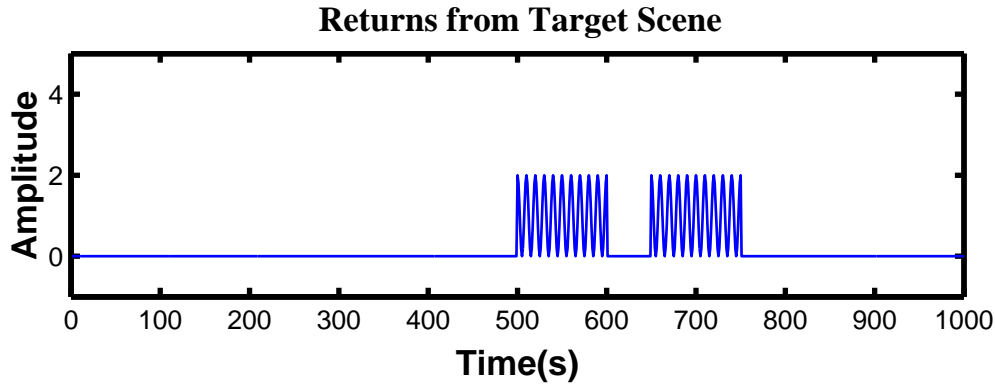
Figure 1-4. Returned Signal

Let the matched filter result be defined as,

$$y[n] = \sum_{k=-\infty}^{k=\infty} r[n-k]s[k] \tag{1–6}$$

where $r[n]$ is the returned signal, $s[k]$ is the transmitted signal and k is the lag. The matched filter result is given in figure 1-5, if we apply a threshold detector, we can pin point where the target lies. From this we can determine the exact delay time of each target. Now suppose if we put two targets closer together in range, such as that given in Figure 1-6. First we note that the raw returned signals (Figure 1-7) shows an overlap between the returns of the two targets. After filtering, (Figure 1-8) we only see one peak. A threshold detector would then give false detections as the location of the two peaks are ambiguous.

Target Return due to Matched Filtering
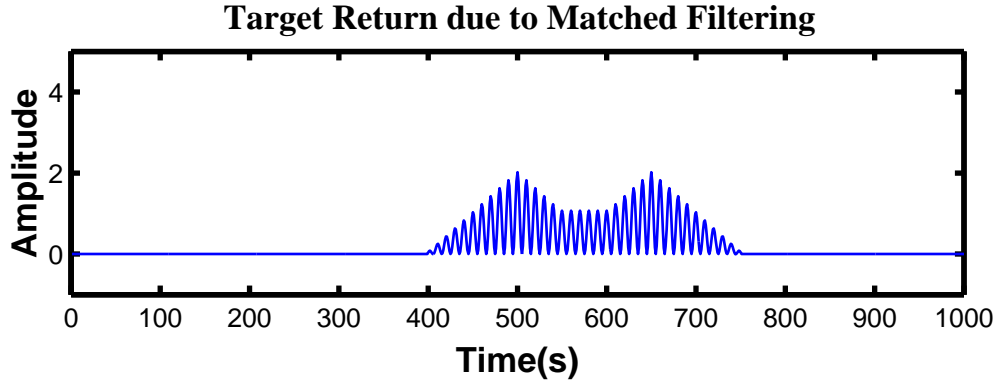
Figure 1-5. Matched Filter of Return Signals

From this example, we see that for two targets to be separable, the returns of the two targets must be at least separated by T, the duration of a pulse. Since the wave travel at the speed of light, c, and the duration of the wave is T, the distance traversed by the wave is cT. Not forgetting that T is a round-trip time delay, we conclude that the range resolution of a sinusoidal pulse with finite duration T is,

$$\frac{1}{2}cT \qquad (1-7)$$

So to get finer resolution, we need to decrease the pulse duration, T. An ideally, we would like to have an infinitely short pulse. However, the duration of the pulse T contains the transmitted energy. Reminded that an energy of a signal is defined as:

$$E = \sum_{n=0}^{n=T} |s[n]|^2 = A^2 T \qquad (1-8)$$

so if we make T small, we are reducing the amount of energy output of the radar. This is important as the amount of energy returned is an attenuated amount of the energy impinged on the target. Therefore if the amount of energy is too low, the return signal could be below the noise level, and no amount of signal processing can recover the signal. At this juncture, without any other tools available to us, it seems that to get the desired range resolution, a careful trade off between pulse duration and output energy must be made.

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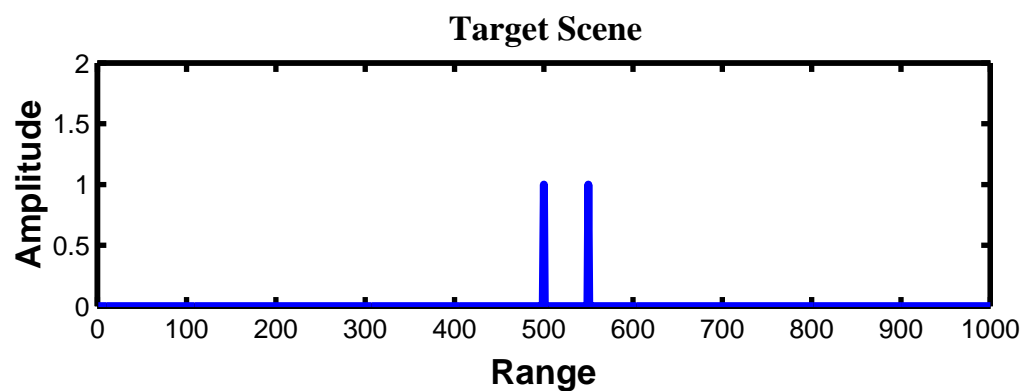
Figure 1-6. Two Close Targets
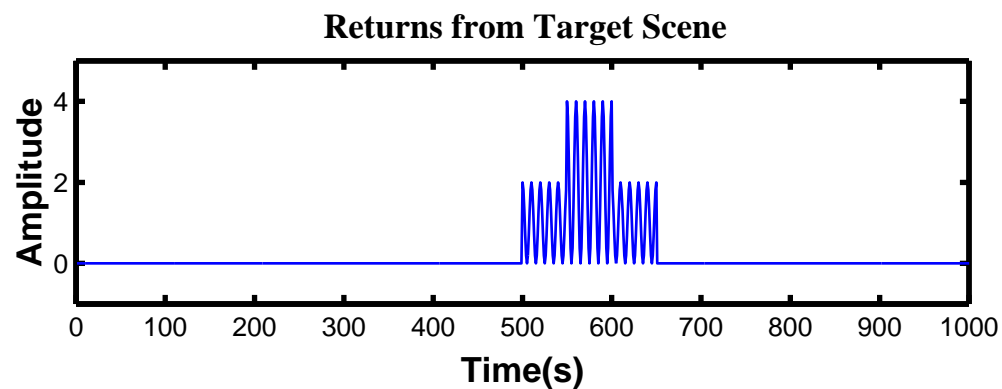
Figure 1-7. Returned Signals of Two Close Targets
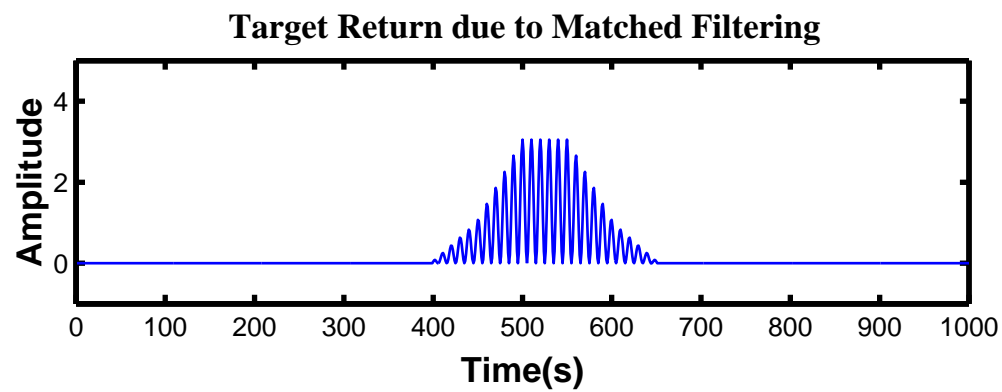
Figure 1-8. Matched Filter Return of Two Close Targets

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1.2.2 Pulse Compression

During this time, most of the effort within the practicing radar community evolve around the development of high-peak power radar tubes. In the discussion above it is noted that one approach is to make A much bigger. Increasing A requires a power source with high peak power. Radar tubes with high power output are costly, and weight becomes an issues for airborne platform. With such a high peak power, safety was also a concern for its operator. Around the 1950s and 1960s, a more elegant approach was found and it is called *pulse compression* . The development of pulse compression was not widely acknowledge at the time, because it was first file in patent form by R. H Dicke in 1953, S. Darlington in 1954, and later in published form by Charles Cook, in 1960. All concepts were arrived independently. The first radar to use the concept of pulse compression was built by MIT Lincoln Laboratory also in the 1960s by the Radar Techniques Group named the AN/FPS-17.

While the intuition behind pulse compression is a bit complicated due to it being thought of from a analog filtering perspective, it is quite simple in term of signal processing. In the above discussion, a truncated rectangular pulse filtered return gave us a triangle function. We note that the triangle function spans $2T$, and for two triangular function to be separated, it needs to be at separated by T. From a signal processing perspective, a clear objective is how to design a pulse such that the convolution of that pulse and its returns yield an impulse like function. It turns out that the first radar waveform that has this desirable property is the *chirp* .

1.2.3 Linear Frequency Modulated Signal

The chirp is the most famous example of a linear Frequency Modulated signal, see figure 1-9. It is the most widely used signal in pulsed radar system. A time limited chirp with a duration of T_p and a chirp rate of α can be expressed as follow,

$$s(t) = \begin{cases} Ae^{j(2\pi f_c t + \pi \alpha t^2)} & \text{if } -T_p/2 \le t \le T_p/2; \\ 0 & \text{otherwise}. \end{cases}$$

Transmitted Chirp Signal
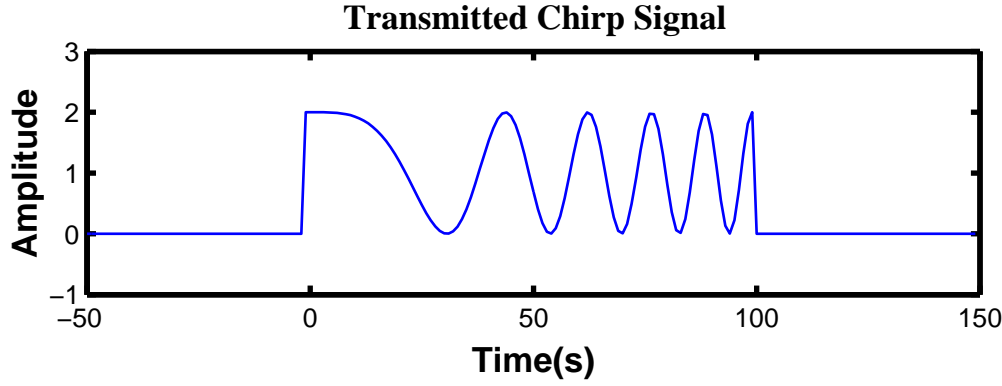
Figure 1-9. Example of a Chirp Signal

 Now supposed we send this as the transmitted signal instead of a pure sinusoidal pulse, then after matched filtering,

$$y(t) = \int_0^{T_p} s^*(t + \tau)s(\tau)d\tau \tag{1-9}$$

$$= \int_0^{T_p} e^{i(2\pi f_c \tau + \pi\alpha\tau^2)} e^{-i(2\pi f_c(t+\tau) + \pi\alpha(t+\tau)^2)} d\tau$$

$$= e^{-i(2\pi f_c t + \pi\alpha t T_p + \pi\alpha t^2)} T_p \frac{sin(\pi\alpha t T_p)}{\pi\alpha t T_p}$$

we see that the magnitude of this function is a sinc function, refer to figure 1-10. There are several things to note here. First, we note that for a sinc function, its 3 dB beamwidth corresponds to the first zero crossing in time domain. This happens when the argument is equal to 1, i.e $\alpha t T_p = 1$. This 3 dB width is also refer to as the effective compressed pulse width, T_e. Recall that for a uncompressed rectangular pulse, if two targets are separated in time by T_p then it is distinguishable. However for the chirp, they are separable if the two targets are separated in time by T_e. For any modulated signal, we can define its effective compressed pulse duration. In the case of the chirp it is,

$$T_e = \frac{1}{\alpha T_p} \tag{1-10}$$

Going back to our discussion on the separability of the targets, if we apply Equation 1-7 and instead of the pulse duration time T, we use the effective pulse duration T_e, then for

a chirp the target can be separable if it is separated in distance by,

$$D = \frac{cT_e}{2} \tag{1–11}$$
$$= \frac{c}{2\alpha T_p}$$

This means if we increase the transmission time of a chirp, we can increase the system's resolution, contrary to a rectangular pulse. A better metric for resolution is to speak in terms of a signal's bandwidth. For a chirp, its bandwidth is defined as,

$$B_c = \alpha T_p \tag{1–12}$$

and the effective pulse duration is,

$$T_e = \frac{1}{B} \tag{1–13}$$

and our resolution is,

$$\frac{c}{2B} \tag{1–14}$$

So by transmitting a chirp longer in time, we decrease its effective duration, and thus forced a higher bandwidth requirement. From the bandwidth perspective, we can see that in the earlier case of a rectangular burst, $T_e = T_p$, its bandwidth is equal to the inverse of its duration and its expression for bandwidth is then given in Equation 1–14. To illustrate what we have discussed, to resolve the two closed targets, instead of transmitting a sinusoidal burst, we employ a chirp signal with a chirp rate of 10 Hz/s of the same duration. What we see from figure 1-11 is that the returns from a chirp at the radar receiver is no longer recognizable, so a threshold detector would be of no use in this case and further post-processing is needed. After matched filtering however (figure 1-12), we were able to resolve the two closed targets with the same time duration! Essentially we traded system performance in exchange for increased system complexity.
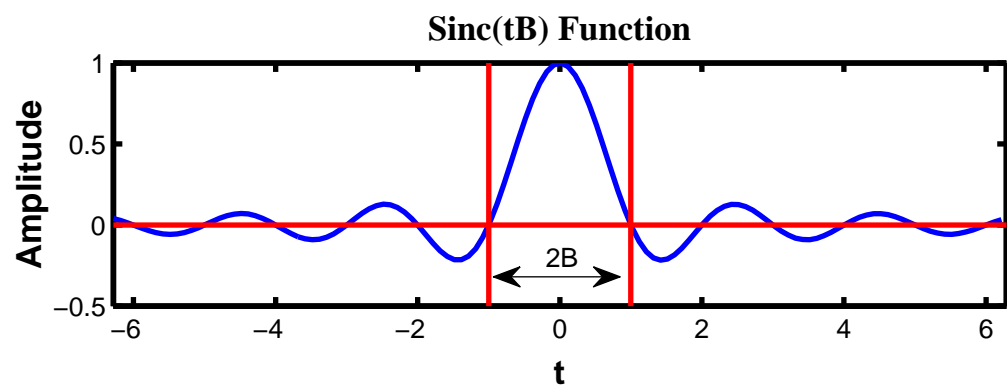
Figure 1-10. Sinc Function
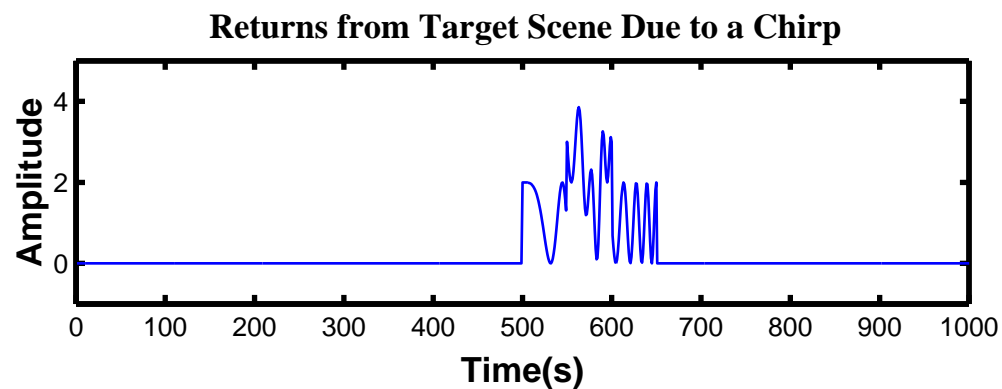
Figure 1-11. Return From Scene Due to a Chirp
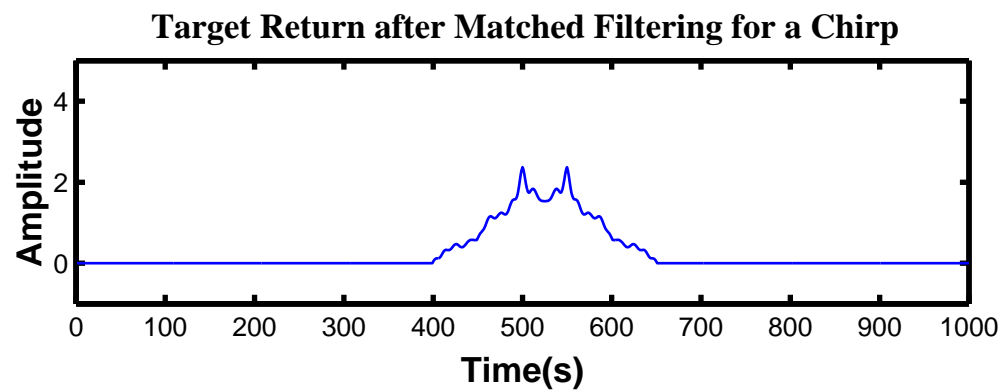
Figure 1-12. Returns from Scene After Matched Filtering for a Chirp
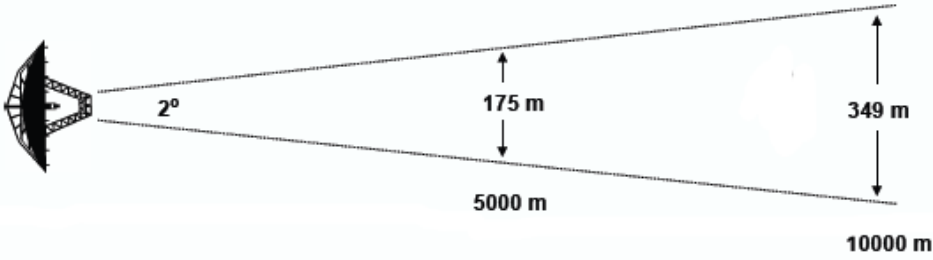
Figure 1-13. Cross Range Width of a Real Aperture Antenna

1.3 Resolution in Cross Range

All this time we were only focusing on how to improve the resolution along one dimension of our data, the range. Now we need to talk about the other dimension, the cross-range. The whole development of SAR is centered around increasing resolution in the cross range. The antenna that we use to transmit our signals has a beamwidth associated with it, which is a function of the antenna's effective aperture. When we send one pulse, and process its returns, we can only resolve the targets in the range direction. For targets that are next to one another in cross-range, or the azimuthal direction, if they are within the main beam of the antenna there is no way to separate them. In Figure 1-13, for an antenna with a $2°$ beamwidth, at 5km meters in range, the illuminated patch is 175m wide, and at 10000 meters it is 349m wide. So any object, at 10km in range, cannot be distinguished if it is separated in cross-range by 349 meters or less. To increase resolution in the cross range we need to use an antenna with a smaller beamwidth. The beamwidth is a function of an antenna's effective aperture and the relationship is given as follow,

$$\beta = \frac{\lambda}{D} \tag{1--15}$$

where β is the beamwidth in radians, and D is the physical size of the antenna aperture, and λ is the wavelength of our carrier frequency. For a given range, R, the width of the illuminated swath is,

$$W = R\beta \tag{1--16}$$
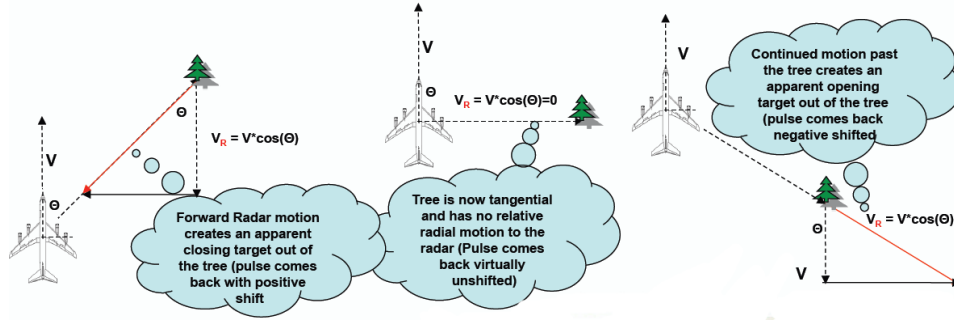
Figure 1-14. Concept of Doppler

and such,

$$W = \frac{R\lambda}{D} \tag{1--17}$$

so if we want to resolve a car that is 3m wide, at X-band (10Ghz) and 10km away, we would need the airplane to carry an antenna that is 100 meter long. This is certainly impractical and this beamwidth restriction is a hindrance to high resolution imagery.

1.3.1 Synthetic Aperture

In the early 1950s, Carl Wiley came up with a solution to this problem. He filed a patent titled "Doppler Beam Sharpening", it is with this work that provided the basis for what we now call Synthetic Aperture Radar. In it he posits that by taking advantage of the phase information of the return signals, we can resolve targets within the beam of an illuminated patch. In the same manner that a moving object has a Doppler shifts associated with its velocity, stationary objects within the same range, has a Doppler shifts that is associated with the aircraft's velocity. Figure 1-14, taken from [11] illustrates the well known Doppler effect.

In order to see what affects the azimuthal resolution, considers the case in Figure 1-15, also taken from [11], where θ is large, and thus θ' is small. By the small angle

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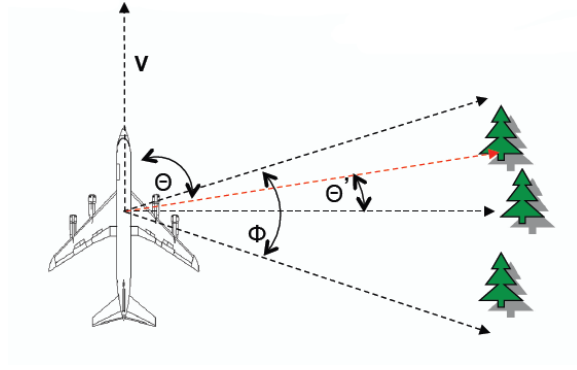
Figure 1-15. Doppler Resolution

approximation we have

$$f_d = \frac{2v\cos\theta}{\lambda} \tag{1--18}$$

$$= \frac{2v\sin\theta'}{\lambda}$$

$$= \frac{2v\theta'}{\lambda}$$

$$\theta' = \frac{f_d\lambda}{2v} \tag{1--19}$$

$$\rho'_\theta = \frac{\rho_{f_d}\lambda}{2v} \tag{1--20}$$

so the resolution of θ' depends on the resolution of the Doppler frequency, f_d, and since we sample the azimuthal Doppler at the pulse repetition frequency, PRF, ρ_{f_d} is related to PRF by,

$$\rho_{f_d} = \frac{PRF}{N_{pulses}} \tag{1--21}$$

one pulse repetition interval, PRI, is inversely related to the PRF,

$$PRI = \frac{1}{PRF} \tag{1--22}$$

and since we transmitted N pulses, the total time duration that we assume coherency, is

$$CPI = N \times PRI \tag{1--23}$$

which is the coherent processing interval. Plugging these definitions into

$$\rho'_\theta = \frac{\lambda}{2v \times CPI} \tag{1-24}$$

and since we know v, the aircraft velocity, and the CPI, the total distance that the aircraft travel during this time is,

$$L_{distance} = v \times CPI \tag{1-25}$$

plugging it into the previous equation for angular resolution, we have

$$\rho_{\theta'} = \frac{\lambda}{2L_{distance}} \tag{1-26}$$

in terms of the spatial resolution this is,

$$\rho_{azimuth} = \frac{R\lambda}{2L_{distance}} \tag{1-27}$$

in terms of the beamwidth of the real antenna aperture is,

$$\rho_{\theta'} = \frac{\lambda}{2\phi} \tag{1-28}$$

so our ability to resolve an object in azimuth, depends only on the distance travel! This distance travel $L_{distance}$ is in fact, the synthetic aperture. To further understand why this is so, we look at it in terms of the Doppler bandwidth that is analogous to how we define range resolution previously. Looking at Figure 1-16, suppose that there are two targets at the top and bottom edges of the illuminated beam, these two targets possess two different Doppler shifts.

$$f_{top} = \frac{2v\cos\left(\theta - \frac{\phi}{2}\right)}{\lambda} \tag{1-29}$$

$$f_{bottom} = \frac{2v\cos\left(\theta + \frac{\phi}{2}\right)}{\lambda} \tag{1-30}$$
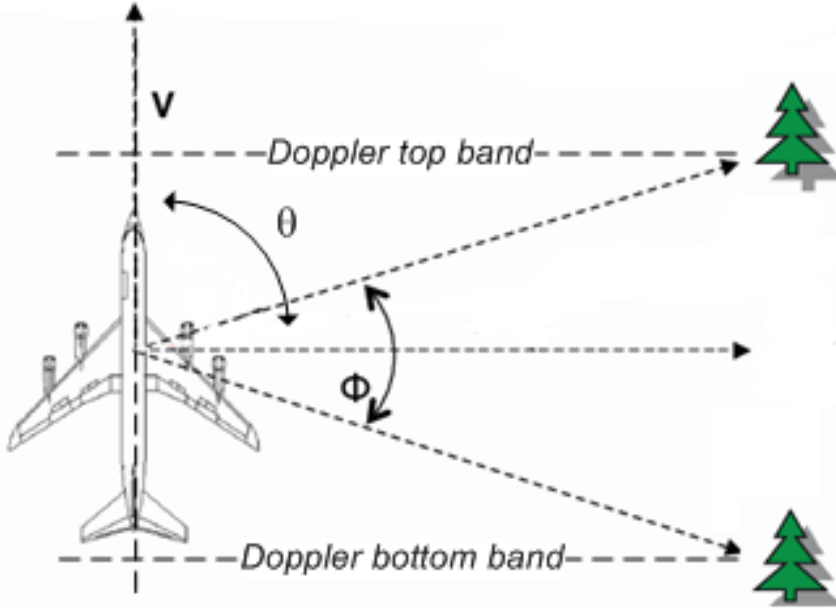
Figure 1-16. Cross Range Width of a Real Aperture Antenna

These two bounds form what is known as the azimuthal Doppler bandwidth,

$$BW_{azimuth} = f_{top} - f_{bottom} \tag{1–31}$$

$$= \frac{2v\cos(\theta - \frac{\phi}{2})}{\lambda} - \frac{2v\cos(\theta + \frac{\phi}{2})}{\lambda}$$

$$= \frac{4v\sin(\frac{\phi}{2})\sin(\theta)}{\lambda}$$

$$= \frac{2vL\sin(\theta)}{\lambda R} \tag{1–32}$$

this expression of the Doppler bandwidth matches what we concluded earlier regarding the improved resolution, that the longer the aperture length the higher the resolution–due to the increase in bandwidth. Another important aspect that is necessary in order to process the azimuth returns is to understand the nature of the return signal. In range compression, we used the matched filter to extract the target since the transmitted signal is pulsed compressed. It turns out that in azimuth we have to do the same thing. To see this, we note that for a given target, as it moves through the illuminated beam, its associated Doppler shifts changes at a constant rate. This rate is given as,

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$$\gamma = \frac{BW_{azimuth}}{T_a} \tag{1-33}$$

$$= \frac{4v^2 \sin(\frac{\phi}{2}) \sin(\theta)}{\lambda L} \tag{1-34}$$

where $T_a = \frac{L}{v}$ The returns in azimuth is actually a chirp, with a chirp rate, γ. To extract the target from the return signal, we have to also perform a matched filter in azimuth, where we matched the return to a chirp in *slow time*, which is the waveform that exist pulse to pulse, instead of the intra-pulse in the range case.

In short, to increase the resolution in azimuth, we need to fly a longer distance. There are however, certain limits to this distance. First, the object of interest must remain in the illuminated beam of the radiating antenna during the CPI. This means that the broader the antenna beamwidth the more time the object spends within its beam, which is opposite of the real antenna aperture case. Second, because we effectively are sampling the Doppler frequencies, we must sample at Nyquist, meaning our PRF must be twice as high as the azimuthal Doppler bandwidth.

1.3.2 Processing of the Return Signals

Up to this point, we still haven't discuss how we can form SAR images from the return signals. There is a very elegant relationship between the returned signals, and the physical returns of the scene of interest. In our earliest discussion where we speak of transmitting a constant pulse, the returns were obvious and can simply be detected using a threshold detector. Or in the case of SAR images, the returns from a rectangular pulse is the actual scene, but only a slice in range. However as we have shown, when we use a compressed pulse, the returns are no longer direct, but some form of pulse compression must be used. We will now show how the compressed pulsed are related to the physical scene. Again, let the transmitted signal be a chirp,

$$s(t) = e^{j(\omega_o t + \alpha t^2)} \tag{1-35}$$

and is equal to 0 everywhere outside of the interval $-\tau_c/2 \leq t \leq \tau_c/2$. This signal has a carrier frequency of ω_o and has a bandwidth of,

$$B_c = \frac{\alpha \tau_c}{\pi} \tag{1–36}$$

by our earlier discussion, the real time return at the radar receiver is a convolution of the reflectivity of the scene and the transmitted signal,

$$r_c(t) = Re\left\{ \int_{-\tau_1}^{\tau_1} g_1(\tau)s(t-\tau)d\tau \right\} \tag{1–37}$$

before we can make any sense of this returns we need to perform some form of pulse compression. We discussed the matched filter, which is also the autocorrelator, here we will perform something call a *deramp*. For the chase of the chirp, we can mix the return signal with its *in-phase* (I) and *quadrature* (Q) component. This is done in practice as it can be implemented cheaply in hardware. We mix the real, or the in-phase component with

$$c_I(t) = \cos(\omega_o(t) + \alpha t^2) \tag{1–38}$$

and the imaginary, or the quadrature with

$$c_Q(t) = -\sin(\omega_o(t) + \alpha t^2) \tag{1–39}$$

after mixing and performing a low pass filter and getting rid of some residual term, the output is,

$$\frac{1}{2} \int_{-u1}^{u1} g(u) e^{j\left[-\frac{2u}{c}(\omega_o + 2\alpha t)\right]} du \tag{1–40}$$

and if we define the following,

$$U = \frac{2}{c}\omega \tag{1–41}$$
$$= \frac{2}{c}(\omega_o + 2\alpha t)$$

we can see that pulse compressed signal,

$$\frac{1}{2} \int_{-u1}^{u1} g(u) e^{-juU} du \qquad (1\text{–}42)$$

which is nothing more than a Fourier transform of the reflectivity of the scene evaluated of a range of frequency specified by the time support of transmitted signal. This signal will have the support of,

$$\frac{2}{c}(\omega_o - \alpha\tau_c) \le U \le \frac{2}{c}(\omega_o + \alpha\tau_c) \qquad (1\text{–}43)$$

make use of the fact that $2\pi B_c = 2\alpha\tau_c$, the support can be written in term of the bandwidth of the transmitted signal,

$$\frac{2}{c}(\omega_o - \pi B_c) \le U \le \frac{2}{c}(\omega_o + \pi B_c) \qquad (1\text{–}44)$$

essentially when we send a compressed pulse, such as a chirp, we are sampling the Fourier transform of the reflectivity function for a set of frequency that spans ΔU centered at an offset frequency, ω_o which is the carrier frequency. To get the actual reflectivity of the scene, we just need to do an inverse Fourier transform! Since the azimuth returns is also a chirp, after azimuth compression, we just take the Fourier transform with respect to the azimuth. Or a 2D Fourier transform to the entire 2D return data set of fast time and slow time samples.

In this chapter we have thoroughly explain the concept of SAR, its signal model, and how to transmit as well as to how to handle the return signal. We have talked about how one can achieve resolution in range, and also in cross-range. We learn that higher resolution in range can be achieved if we send a modulated pulse with a high bandwidth. We learn that resolution in cross-range is fully dependent on the length of the flight path, which is limited to the real aperture beamwidth. We also make the important connection between the return signals and the reflectivity of the scene. We see that by transmitting a compress pulsed, we are essentially sampling the spatial frequencies of the scene, and to get our reflectivity, we just need to perform an inverse Fourier transform.

CHAPTER 2
SAR RECONSTRUCTION USING A BAYESIAN APPROACH

We introduce a new approach using the Bayesian framework for the reconstruction of sparse Synthetic Aperture Radar (SAR) images. The algorithm, named SLIM, can be thought of as a sparse signal recovery algorithm with excellent sidelobe suppression and high resolution properties. For a given sparsity promoting prior, SLIM cyclically minimizes a regularized least square cost function. We show how SLIM can be used for SAR image reconstruction as well as used for SAR image enhancement. We evaluate the performance of SLIM using realistically simulated complex-valued backscattered data from a backhoe vehicle. The numerical results show that SLIM satisfactorily suppress the sidelobes and yield higher resolution than the conventional matched filter or delay-and-sum (DAS) approach. SLIM outperform the widely used compressive sampling matching pursuit (CoSaMP) algorithm, which requires the delicate choice of user parameter. Compared with the recently developed iterative adaptive approach (IAA), which iteratively solves weighted least squares problem. Due to computational complexity involved with SAR imaging, we show how SLIM can be made more computationally efficient by utilizing the Fourier transform (FFT) and conjugate gradient (CG) method. Furthermore, since the two algorithms were derived under the Bayesian model, the *a posteriori* distribution given by the algorithms provides us with a confident measure regarding the statistical properties of the SAR image pixels.

The usage of a synthetic aperture in radar systems to overcome the azimuthal resolution limitation has played a ubiquitous role in both government and commercial industries. These class of radars, conveniently named, Synthetic Aperture Radar (SAR), has wide ranging applications. While most applications are found in the security and defense industries which includes, reconnaisance, surveillance, target identifications, foliage penetrations, and moving target indications, the commercial industries also make uses of SAR systems for navigations, mappings, and environmental monitoring.

At present, satellites imaging (optical) are often used for real time response to environmental disasters, SAR and its ability to image in all weather conditions offers a complimentary view of the situations. Using various SAR modes and SAR image reconstruction techniques certain details and observations can be made that otherwise would be missed when using optical imaging.

While a significant part of SAR image quality depends on the hardware parameters of the radar systems, the SAR image construction algorithms plays a pivotal role in making the most of the collected data. The SAR community has accepted various methods for SAR image reconstruction, all of which are data-independent approaches, such as the Fourier Transforms, and backprojection [12]. There are variants improvement to these basic algorithms via pre and post processes, but the area has been well developed. The attractiveness of these algorithms are that they are simple to implement and computationally efficient but they suffer from high sidelobe levels and thus yield low resolutions. To mitigate this problem we trade its low complexity for higher resolutions. In this chapter we will focus on data-adaptive approaches for SAR image reconstruction, with emphasis on algorithms that promotes sparsity in the resulting image. As sparser images are more beneficial for feature extractions and targets recognitions applications.

Various data-adaptive approaches has been proposed for SAR imaging [13–18]. However, some of these methods, such as Capon [18, 19] and amplitude and phase estimation(APES) [15, 16] require multiple snapshots to form the sample covariance matrix. This requirement is hard to satisfy in SAR systems, due to the moving nature of the platform. Another category of algorithm, the sparse signal recovery algorithms [20–22], including the popular regularized l_1-norm based methods and the greedy-type approaches, such as compressive sampling matching pursuit (CoSaMP) (see, e.g., [23] and the references therein), can be used to recover sparse radar images with high resolution and they do not require multiple snapshots; nevertheless, their performance is sensitive to user parameters which are hard to choose in practice [24]. In addition, these

approaches lack robustness to noise levels, which we will show later in the numerical examples. One possible data-adaptive algorithm, the iterative adaptive approach (IAA) [25–28] has been introduced for the SAR imaging application. IAA is robust and user parameter free and works with few or even a single snapshot. Compared with the data-independent approaches, IAA is shown to produce images with significantly reduced sidelobes and much enhanced resolution [29]. However, IAA is computationally intensive, which to some extent limits its applicability.

We consider a Bayesian-based sparse signal recovery algorithms, named SLIM (sparse learning via iterative minimization). SLIM is a cyclic algorithm that iteratively fits the signal to the observation while simultaneously satisfying the sparsity-promoting constraint. In another perspecive, SLIM is essentially a l_b-norm ($0 < b \leq 1$) constrained optimization. It has been shown that in some applications, l_b-norm (with $b < 1$) constrained optimization outperforms its l_1-norm counterparts such as basis pursuit (BP) [20] and least absolute shrinkage and selection operator (LASSO) [30] in that the former requires less measurements for reconstruction than the latter [31]. Moreover, when implemented with fast Fourier transform (FFT) and conjugate gradient (CG), can yield much faster convergence than many existing SAR imaging algorithms. Compare to other l_b-norm approaches, SLIM has only one user paramter, and it is easy to choose. One additional benefit, as a Bayesian-type approach, SLIM facilitate the performance prediction by providing the *a posteriori* distributions of the reconstructed image pixels. Furthermore, SLIM have been shown to perform well for MIMO radar imaging [32] so a natural extension of SLIM is applying it to the case of Synthetic Aperture Radar imaging.

This chapter is structured as follows. In Section II, we introduce the preliminaries of SAR system and derive the data model for SAR imaging problem. In Section III, we briefly touch on other data-adaptive algorithms then proceeds to introduce the SLIM algorithm to the SAR imaging problem and discuss its merits and limitations. In Section IV, the performance of SLIM is compared with those of two existing SAR imaging
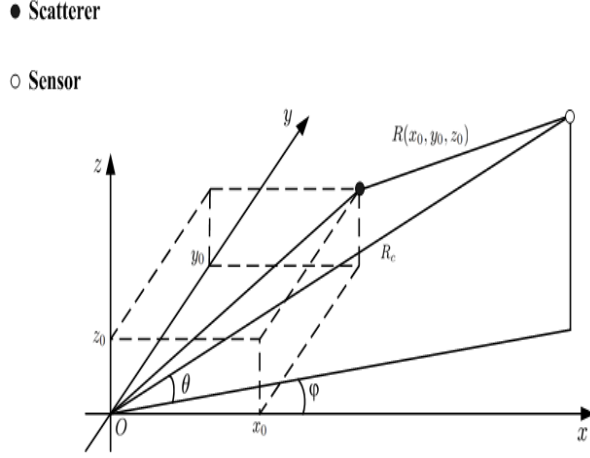
Figure 2-1. SAR imaging schematics.

algorithms, namely CoSaMP and IAA, using ideal scatterers as well as the "Backhoe Data Dome" provided by DARPA IXO and AFRL/SNAS (realistically simulated using computational electromagnetics software). Finally, we conclude the chapter in Section V.

2.1   Preliminaries and Data Model

In this section, we introduce the principles of SAR imaging, with special emphasis placed on the geometry of the physical scene and data acquisition mechanism. Based on the physical interpretation of the data acquisition process, we will apply some reasonable approximations to derive an appropriate data model, with which various methods can be applied to form the SAR image.

The data acquisition geometry for spot-light mode SAR is illustrated in Fig. 2-1. A 3-D Cartesian coordinate system is established, centered at the scene patch illuminated by the radar beam. The non-shaded circle represents a sensor at elevation angle θ and azimuth angle ϕ. A scatterer in the scene patch located at (x_0, y_0, z_0) is denoted by the filled circle.

In SAR radar system, the antenna transmits an electromagnetic pulse toward the scene of interest (SOI), and the reflected signal bears the information of the potential scatterers in the SOI. The time delay $\tau(x_0, y_0, z_0)$ of the returned signal associated with a scatterer located at (x_0, y_0, z_0) is directly determined by the distance $R(x_0, y_0, z_0)$, which can be calculated as follows:

$$
\begin{aligned}
R(x_0, y_0, z_0) &= \sqrt{(R_c \cos\theta\cos\phi - x_0)^2 + (R_c\cos\theta\sin\phi - y_0)^2 + \cdots} \\
&\quad \overline{\cdots (R_c\sin\phi - z_0)^2} \\
&= R_c\sqrt{1 + (\tfrac{x_0}{R_c})^2 + (\tfrac{y_0}{R_c})^2 + (\tfrac{z_0}{R_c})^2 - 2\tfrac{x_0}{R_c}\cos\theta\cos\phi - \cdots} \\
&\quad \overline{\cdots 2\tfrac{y_0}{R_c}\cos\theta\sin\phi - 2\tfrac{z_0}{R_c}\sin\theta} \,.
\end{aligned}
\tag{2--1}
$$

Since the sensor is usually air-borne and the distance R_c is much larger than the dimension of the scene patch, the expression above can be simplified using the following approximations:

$$
\begin{aligned}
R(x_0, y_0, z_0) &\approx R_c\sqrt{1 - 2\tfrac{x_0}{R_c}\cos\theta\cos\phi - 2\tfrac{y_0}{R_c}\cos\theta\sin\phi - 2\tfrac{z_0}{R_c}\sin\theta} \\
&\approx R_c(1 - \tfrac{x_0}{R_c}\cos\theta\cos\phi - \tfrac{y_0}{R_c}\cos\theta\sin\phi - \tfrac{z_0}{R_c}\sin\theta) \\
&= R_c - (x_0\cos\theta\cos\phi + y_0\cos\theta\sin\phi + z_0\sin\theta)\,.
\end{aligned}
\tag{2--2}
$$

The expression in (2--2) gives the distance between the scatterer and the sensor relative to R_c.

Let $p(t) = a(t)e^{j2\pi f_c t}$ denote the pulse (e.g., a chirp waveform) transmitted toward the SOI, where $a(t)$ is the complex envelope and f_c is the carrier frequency. The reflected signal $r(t, \theta, \phi)$ can be interpreted as the sum of delayed versions of $p(t)$ with the amplitudes and phases modified proportionally to the reflectivity $g(x, y, z)$ of the scatterers. Mathematically, this relation can be expressed as:

$$
r(t, \theta, \phi) = \sum_{x,y,z} g(x, y, z) p(t - \tau(x, y, z)),
\tag{2--3}
$$

where $\tau(x, y, z) = \frac{2R(x,y,z)}{c}$ and c is the speed of light. With this model, the matched filter output $y(t, \theta, \phi)$ can be expressed as

$$y(t, \theta, \phi) = \sum_{x,y,z} g(x, y, z) s(t - \tau(x, y, z)) + e(t), \qquad (2\text{--}4)$$

where $s(t)$ is the auto-correlation function of the transmitted signal $p(t)$ and $e(t)$ is the additive noise. By taking the Fourier transform of (2–4) and using the relations in (2–2) above, the phase history data are related to the scatterer reflectivity function by:

$$\begin{aligned}
Y(f, \theta, \phi) &= \sum_{x,y,z} g(x, y, z) S(f) e^{-j2\pi f \frac{2R(x,y,z)}{c}} + E(f) & (2\text{--}5a) \\
&= C(f) \sum_{x,y,z} g(x, y, z) e^{j\frac{4\pi}{c}(xf \cos\theta \cos\phi + yf \cos\theta \sin\phi + zf \sin\theta)} + \\
& \quad E(f) & (2\text{--}5b)
\end{aligned}$$

where $E(f)$ is the Fourier transform of $e(t)$ and $C(f) = S(f) e^{-j2\pi f \frac{2R_c}{c}}$ is common to all scatterers in the scene patch so that we will drop it from the expression in the following discussion for the sake of notational simplicity. Substituting the transformation pairs $u(f, \theta, \phi) = f \cos\theta \cos\phi$, $v(f, \theta, \phi) = f \cos\theta \sin\phi$ and $w(f, \theta, \phi) = f \sin\theta$ into (2–5b), the output signal can be further written as

$$Y(f, \theta, \phi) = \sum_{x,y,z} g(x, y, z) e^{j\frac{4\pi}{c}(xu(f,\theta,\phi) + yv(f,\theta,\phi) + zw(f,\theta,\phi))} + E(f). \qquad (2\text{--}6)$$

With (2–6), the phase history data can thus be interpreted as the 3-D Fourier transforms of the reflectivity function of the scene patch. In other words, the collected phase history data form a polar raster in the transformed spatial frequency space.

Consequently, we have a set of samples nonuniformly placed in the spatial frequency domain. Since there is no fast algorithm for computing Fourier transforms on a non-Cartesian space, we next seek to reformat the data samples. Basically, the approaches for this purpose fall into the following two categories: approximation and interpolation. The approximation-based approach is appropriate when the frequency

span Δf, the elevation aperture $\Delta\theta$ and the azimuth aperture $\Delta\phi$ are relatively small. In such a case, we can view the polar grid points as approximately uniform in a Cartesian cube, which facilitates the application of fast Fourier transforms defined on a Cartesian coordinate system.

However, for wide-aperture SAR phase history data, the approximation above is inaccurate. In addition, this method creates technical problems for the 3-D SAR image fusion application, as is discussed in Section IV. In this situation, we can resort to the interpolation-based approach to transform the data from the polar coordinate into the Cartesian coordinate. One particular interpolation-type method involves using non-uniform fast Fourier transforms (NUFFT) [33]. In this case, the non-uniform data samples in the 3-D spatial frequency space are first backprojected to the intermediate image space. Since the image samples are uniform in the 3-D Cartesian coordinate system, they can be conveniently projected again to the spatial frequency space using 3-D FFT. After this two-step procedure, the data samples in the spatial frequency space are uniform in the dimensions of u, v, and w.

Based on the scene geometry and the data reformation above, we can derive the data model as follows. Let M_1, M_2 and M_3 denote the numbers of image grid points in the x, y and z directions, respectively. Let N_1, N_2 and N_3 denote the number of frequency grid points, the number of azimuth angle grid points and the number of elevation angle grid points, respectively. The total number of unknowns is $M = M_1 \times M_2 \times M_3$ and the total number of observed data points is $N = N_1 \times N_2 \times N_3$. For a scatterer located at (x_j, y_k, z_l), the N-element steering vector can be written as:

$$\mathbf{a}_{j,k,l} = [b_{j,k,l}(1,1,1), \cdots, b_{j,k,l}(N_1, N_2, 1), \cdots, b_{j,k,l}(1,1,N_3), \cdots, b_{j,k,l}(N_1, N_2, N_3)]^T,$$
$$1 \leq j \leq M_1, 1 \leq k \leq M_2, 1 \leq l \leq M_3,$$

$$(2\text{--}7)$$

where $b_{j,k,l}(p, q, r) = e^{j\frac{4\pi}{c}(u_p x_j + v_q y_k + w_r z_l)}$ for $1 \leq p \leq N_1$, $1 \leq q \leq N_2$ and $1 \leq r \leq N_3$. By using these definitions, the steering matrix can be written as:

$$\mathbf{A} = [\mathbf{a}_{1,1,1}, \cdots, \mathbf{a}_{M_1,M_2,1}, \cdots, \mathbf{a}_{1,1,M_3}, \cdots, \mathbf{a}_{M_1,M_2,M_3}]. \tag{2–8}$$

We define $\mathbf{s} = [g(x_1, y_1, z_1), \cdots, g(x_{M_1}, y_{M_2}, z_1), \cdots, g(x_1, y_1, z_{M_3}), \cdots, g(x_{M_1}, y_{M_2}, z_{M_3})]^T$ to be an $M \times 1$ vector containing the reflectivity coefficients, and we stack the observed data into the vector $\mathbf{y}$. The data model can be compactly expressed in matrix form as:

$$\mathbf{y} = \mathbf{As} + \mathbf{e}, \tag{2–9}$$

where $\mathbf{e}$ accounts for the additive noise.

To form SAR images, we essentially need to estimate $\mathbf{s}$ from $\mathbf{y}$ and $\mathbf{A}$. The most straightforward method is to perform the inverse Fourier transform, which gives the conventional delay-and-sum (DAS) or matched filtering approach [34]. In spite of the computational efficiency, the quality of the DAS image is usually unsatisfactory due to the low resolution and high sidelobes in the image. Compared with the data-independent DAS method, the data-adaptive approaches, which utilize the information contained in the data acquired, usually yield better imaging results. In the following sections, we discuss several data-adaptive algorithms that can be used to enhance the imaging quality.

2.2 Data-Adaptive Algorithms

In this section, we first review two data-adaptive algorithms, namely CoSaMP and IAA, as they have the closest ties to SLIM and comment on their performance and applicability. Then we introduce SLIM and discuss how it can be adapted to the SAR imaging problem.

2.2.1 Compressed Sampling Matching Pursuit (CoSaMP)

CoSaMP [23] is a sparse signal recovery algorithm, which iteratively identifies the support of the sparse signal vector and estimates the corresponding entry values.

CoSaMP has two user parameters, which determine the sparsity of the estimation

result and the dimension of the potential support. Usually, the signal sparsity is set to

be half of the support dimension, hence we will only give values of the signal sparsity

in the numerical examples. CoSaMP is shown to generate SAR images with higher

resolution than the conventional DAS algorithm [35]. However, the performance of

CoSaMP is highly dependent on the user parameters, whose choice, unfortunately lacks

a clear guidance. Therefore, the performance of CoSaMP can be poor with an improper

selection of the user parameters, which is illustrated later on in the numerical examples.

Furthermore, the performance of CoSaMP degrades as the noise level increases, as

illustrated by the numerical examples.

2.2.2 Iterative Adaptive Approach (IAA)

IAA [25] is a nonparametric adaptive algorithm originally proposed for the array

signal processing application. IAA has been shown to provide much improved

performance over many other data-adaptive algorithms as well as DAS in applications

such as MIMO radar, active sonar, and medical imaging. In a recentwork [29],

IAA is introduced to the SAR imaging problem and it is shown there that IAA can

significantly reduce the sidelobe levels and produce much cleaner images. Though

the performance of IAA is satisfactory, its high computational complexity to some

extent limits its applicability to large dimensional problems. So far, reducing the

computational complexities of IAA is still an open problem. We will discuss further

about the performance of IAA with the numerical examples later on.

2.2.3 Sparse Learning via Iterative Minimization (SLIM)

In this section, we introduce our main algorithm that is based on a Bayesian model.

We named it SLIM (Sparse Learning via Iterative Minimization) since it fits itself to

a given sparse promoting prior iteratively. The SLIM algorithm can be cast into the

Bayesian framework by considering the following hierarchical Bayesian model:

$$\mathbf{y}|\mathbf{s}, \eta \sim \mathcal{CN}(\mathbf{As}, \eta\mathbf{I})$$

$$f(\mathbf{s}) \propto \prod_{m=1}^{M} e^{-\frac{2}{b}(|s_m|^b - 1)} \tag{2-10}$$

$$f(\eta) \propto 1,$$

where η is the noise power and $f(\mathbf{s})$ is a sparsity promoting prior for $0 < b \le 1$. When $b = 1$, the prior becomes the Laplacian prior $f(\mathbf{s}) \propto e^{-2||\mathbf{s}||_1}$ which has a finite peak at $\mathbf{0}$. When $b \to 0$, the prior becomes $f(\mathbf{s}) \propto \prod_{m=1}^{M} \frac{1}{|s_m|^2}$, which has an infinite peak at $\mathbf{0}$. Therefore, $\mathbf{s}$ and η can be estimated using the following MAP approach:

$$\max_{\mathbf{s},\eta} f(\mathbf{y}|\mathbf{s}, \eta) f(\mathbf{s}) \propto \frac{1}{\eta^N} e^{-\frac{1}{\eta}||\mathbf{y}-\mathbf{As}||^2} \prod_{m=1}^{M} e^{-\frac{2}{b}(|s_m|^b - 1)} . \tag{2-11}$$

This MAP problem is equivalent to the following regularized minimization problem:

$$\min_{\mathbf{s},\eta} g(\mathbf{s}, \eta) \triangleq N \log \eta + \frac{1}{\eta}||\mathbf{y} - \mathbf{As}||^2 + \sum_{m=1}^{M} \frac{2}{b}[|s_m|^b - 1], \tag{2-12}$$

where $0 < b \le 1$. Note that the cost function above can be interpreted as the sum of the negative log-likelihood $-\log f(\mathbf{y}|\mathbf{s}, \eta)$ and a regularization term $\sum_{m=1}^{M} \frac{2}{b}[|s_m|^b - 1]$ that controls the sparsity of $\mathbf{s}$. When $b = 1$, the regularization term becomes $2||\mathbf{s}||_1 - 2M$, which is the widely used l_1-norm constraint. When $b \to 0$, the regularization term becomes $\sum_{m=1}^{M} 2 \log |s_m|$, which promotes sparsity since, when $s_m \to 0$, $2 \log |s_m| \to -\infty$.

The following cyclic approach is then employed to minimize the cost function at the ith iteration:

1. Fix the value of $\eta^{(i-1)}$, and compute the optimal value of $\mathbf{s}^{(i)}$ as

$$\mathbf{s}^{(i)} = \mathbf{P}^{(i)}\mathbf{A}^H(\mathbf{A}\mathbf{P}^{(i)}\mathbf{A}^H + \eta^{(i-1)}\mathbf{I})^{-1}\mathbf{y}, \tag{2-13}$$

where $\mathbf{P}^{(i)} = \mathrm{diag}\{|s_1^{(i-1)}|^{2-b}, \cdots, |s_M^{(i-1)}|^{2-b}\}$.

2. For a fixed value of $\mathbf{s}^{(i)}$, the optimal value of $\eta^{(i)}$ can be calculated as

$$\eta^{(i)} = \frac{1}{N}||\mathbf{y} - \mathbf{As}^{(i)}||^2 . \tag{2-14}$$

SLIM is usually initialized by DAS and terminated when the norm of the difference between two consecutive estimates of **s** falls below a pre-specified threshold.

The *a posteriori* distribution of **s** can be approximated by the following conditional probability distribution:

$$
\begin{aligned}
f(\mathbf{s}|\mathbf{y}, \overset{\circ}{\mathbf{p}}, \overset{\circ}{\eta}) \;\;&\propto f(\mathbf{s}, \mathbf{y}, \overset{\circ}{\mathbf{p}}|\overset{\circ}{\eta}) = f(\mathbf{y}|\mathbf{s}, \overset{\circ}{\eta})f(\mathbf{s}|\overset{\circ}{\mathbf{p}})f(\overset{\circ}{\mathbf{p}}) \\
&\propto \exp\{-\tfrac{1}{2\overset{\circ}{\eta}}||\mathbf{y} - \mathbf{As}||^2\}\exp\{-\tfrac{1}{2}\mathbf{s}^{\mathrm{H}}(\overset{\circ}{\mathbf{P}})^{-1}\mathbf{s}\} \\
&= \exp\{-\tfrac{1}{2\overset{\circ}{\eta}}\mathbf{y}^{\mathrm{H}}\mathbf{y} + \tfrac{1}{2\overset{\circ}{\eta}}\mathbf{s}^{\mathrm{H}}\mathbf{A}^{\mathrm{H}}\mathbf{y} + \tfrac{1}{2\overset{\circ}{\eta}}\mathbf{y}^{\mathrm{H}}\mathbf{As} - \\
&\quad \tfrac{1}{2}\mathbf{s}^{\mathrm{H}}(\tfrac{1}{\overset{\circ}{\eta}}\mathbf{A}^{\mathrm{H}}\mathbf{A} + (\overset{\circ}{\mathbf{P}})^{-1})\mathbf{s})\},
\end{aligned}
\tag{2-15}
$$

where $\overset{\circ}{\mathbf{p}}$ and $\overset{\circ}{\eta}$ are the estimated values at convergence. From (2–15), we observe that the conditional distribution of **s** given **y**, $\overset{\circ}{\mathbf{p}}$ and $\overset{\circ}{\eta}$ is a Gaussian distribution. Let $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ denote the mean and covariance matrix for this conditional Gaussian random vector, i.e., $\mathbf{s}|\mathbf{y}, \overset{\circ}{\mathbf{p}}, \overset{\circ}{\eta} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Expanding the expression above and equating the corresponding terms, we can find:

$$
\boldsymbol{\Sigma} = \overset{\circ}{\mathbf{P}} - \overset{\circ}{\mathbf{P}}\mathbf{A}^{\mathrm{H}}(\overset{\circ}{\eta}\mathbf{I} + \mathbf{A}\overset{\circ}{\mathbf{P}}\mathbf{A}^{\mathrm{H}})^{-1}\mathbf{A}\overset{\circ}{\mathbf{P}}, \quad \text{and} \quad \boldsymbol{\mu} = \overset{\circ}{\mathbf{P}}\mathbf{A}^{\mathrm{H}}(\mathbf{A}\overset{\circ}{\mathbf{P}}\mathbf{A}^{\mathrm{H}} + \overset{\circ}{\eta}\mathbf{I})^{-1}\mathbf{y}.
\tag{2-16}
$$

These are the *a posteriori* mean and covariance matrix of the signal vector **s** and the estimate of **s** is taken to be the *a posteriori* mean $\boldsymbol{\mu}$. With this *a posteriori* distribution, the statistical properties of the estimates can be utilized for performance prediction purposes for applications such as automatic target recognition.

2.3 Numerical Examples

In this section, we show the SAR imaging capability of SLIM and compare its performances with those of CoSaMP and IAA as well as the data-independent DAS algorithm which we will implement via FFT. We first consider a simulated example of ideal point scatterers, with which imaging properties of the various adaptive algorithms introduced above are studied. We then examine the imaging capability of the various algorithms using the realistically simulated "Backhoe Data Dome, Version 1.0" data set.

2.3.1 An Example of Ideal Point Scatterers

In this example, the SOI consists of 12 ideal point-like scatterers with uniform amplitudes 1, denoted by black circles in Figure 2-2(a). Based on the data model in (2–9), we define

$$\mathrm{SNR} \triangleq \frac{||\mathbf{As}||^2}{||\mathbf{e}||^2} = \frac{||\mathbf{As}||^2}{N\sigma^2},\qquad(2\text{–}17)$$

where σ^2 is the average noise power. Throughout this example, the SNR is set to be 10 dB. IAA is set to run for 10 iterations, after which no significant change in image quality is observed. SLIM and CoSaMP are set to run for 20 iterations.

Figure 2-2(b) shows the SAR image obtained using DAS. Though computationally efficient, DAS yields an image with high sidelobe levels and poor resolution. Note that the four closely spaced scatterers in the center of the SOI smear with each other and appear as one single scatterer. Figures 2-2(d) and (f) show the SAR images generated by CoSaMP, which has one user parameter that controls the SOI sparsity. Since this user parameter directly determines the number of non-zero pixels present in the image and can significantly affect the image quality, the lack of proper guidance for the choice of this parameter restricts the applicability of this algorithm. As is seen from Figures 2-2(d) and (g), different values of this user parameter can lead to very different results, which is undesirable in practice. Figures 2-2(c), (e) and (g) show the SAR images produced by IAA and SLIM, respectively. As evident from the figures, IAA and SLIM all possess excellent sidelobe suppression capability, producing clean images where the presence of the scatterers is clearly indicated and the closely spaced scatterers are distinguished.

Next, we consider the robustness of the various algorithms against additive noise. Again by looking at Figure 2-2, with SNR at 10dB, CoSaMP is more sensitive to the noise level than SLIM and IAA. When the user parameter does not match the scatterer sparsity, the performance of CoSaMP can be rather poor, as is seen from Figure 2-2(d). In contrast,SLIM and IAA are all much more robust to the noise level.

Regarding computational complexity, DAS is the most efficient algorithm since it can purely be implemented via FFTs. SLIM can be made computationally efficient via leveraging FFTs and the Conjugate Gradient method, details of which are too long and sidetracks the topic of this paper. Our implementations of SLIM incorporates these speed-up methods, and the computation time is as follow. For a typical run (Computer Specifications: Xeon 2.33 GHz CPU, 16.0 GB RAM, Matlab R2009a 64 bit), it takes SLIM less than 4 seconds to generate an image shown in Figure 2-2. CoSaMP is slightly faster than SLIM, and the computational time is around 3 seconds. IAA is computationally the most intensive among all the algorithms considered herein, and the typical running time is about 2 hours.

2.3.2 Imaging of the Backhoe

We now use the "Backhoe Data Dome, Version 1.0" to evaluate the SAR imaging capability of the SLIM algorithm. From this point on we drop the comparison with DAS and IAA. First we drop DAS because the high sidelobes resulted in significant smearing which makes the object unrecognizable. Secondly, we drop IAA because of its computation times. Both of these reasons are well founded by the ideal point scatterers example above. Interested reader who wish to see the perfromance of IAA can refer to [29]. We will will solely focus on SLIM and its nearest benchmark, CoSaMP. The data set consists of realistically simulated, fully polarized, complex backscattered data from a backhoe vehicle in free space. The data set was generated by X-Patch, a leading CEM (Computational Electromagnetic) code. The 3-D CAD model of the backhoe and an illustration of the backhoe data dome are shown in Figure 2-3. We will now look at the 3-D (range-azimuth-elevation) imaging of the backhoe object.

2.3.3 3-D SAR Imaging

In this section, we consider the 3-D SAR imaging problem. The phase history data used here consist of the portion with a bandwidth $B = 0.5$ GHz centered at $f_0 = 12$ GHz. The elevation aperture of $25°$ centered at $\theta_0 = 30°$ is equally divided into 5
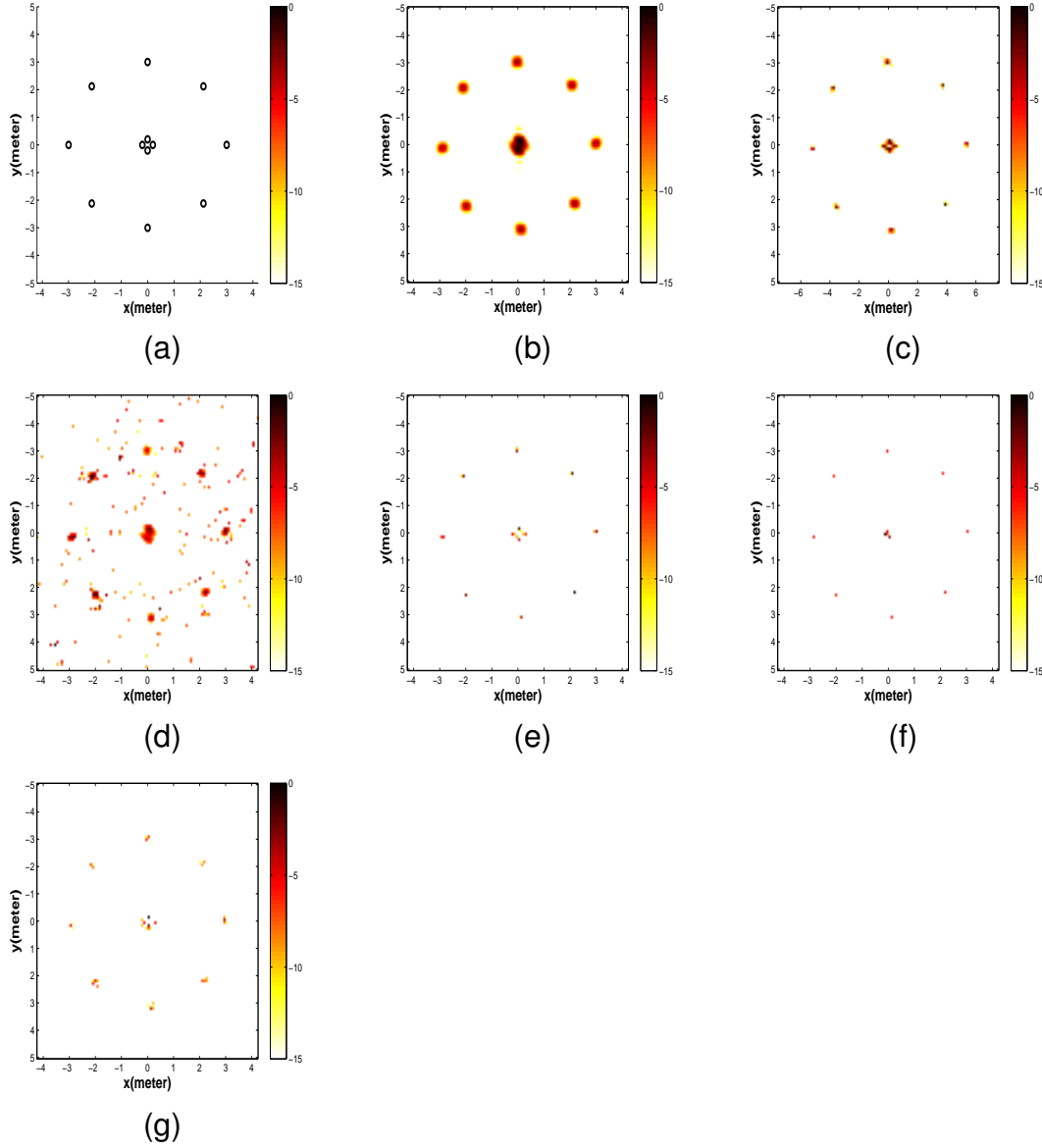
Figure 2-2. Imaging of Scatterers A) The true scatterer distribution B) DAS. C) IAA. D) CoSaMP (300). E) SLIM ($b = 1$). F) CoSaMP (12). G) SLIM ($b \to 0$).

non-overlapping subapertures, and the azimuth aperture of $45°$ centered at $\phi_0 = 90°$ is equally divided into 9 non-overlapping subapertures.

We will focus on comparing SLIM with CoSaMP. For CoSaMP, four different values of the user parameter are considered. The images shown here are projections onto a 2-D plane from a certain azimuth angle. As can be seen from Figure 2-4, SLIM with $b = 1$ can produce clean images representing the structure of the backhoe vehicle.

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(a)   (b)
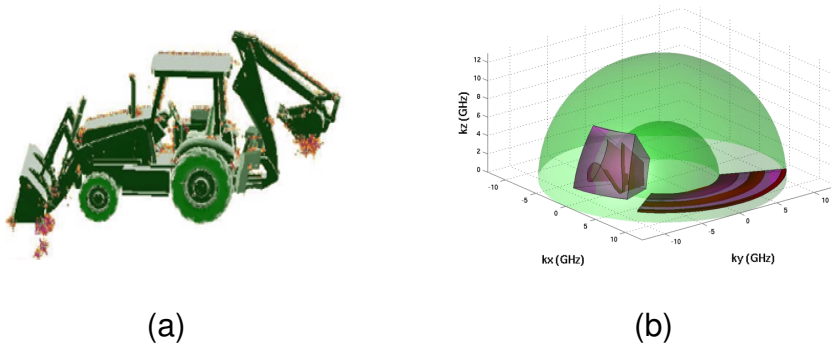
Figure 2-3. Visualizing the Backhoe Data Set A) 3-D CAD model of the backhoe. B) The backhoe data dome.

CoSaMP, on the other hand, tends to distort the image with some target features (such as the line around the -1 elevation) that appear and then disappear as the user parameter increases.

In Figure 2-5 we show the fused 3-D SAR images of the backhoe vehicle generated using the SLIM algorithm. We apply NUFFT to interpolate the phase history data samples onto a uniform 3-D Cartesian grid, which is used for all subapertures. The fused image is obtained by combining the subimages with the maximum-magnitude operator as follows:

$$\mathrm{Image}(x, y, z) = \max_{1 \le q \le 9, 1 \le r \le 5} |\mathrm{Subimage}(x, y, z; \phi_q, \theta_r)| . \tag{2–18}$$

As can be seen from the figure, the images generated are clean and focused, giving minimum artifacts and nicely showing the imaged vehicle. Moreover, SLIM with either $b \to 0$ or $b = 1$ works well in this 3-D fused imaging application.

With the aforementioned numerical examples, the advantages of SLIM over DAS, IAA and CoSaMP become apparent. First SLIM give results with high fidelity, nicely indicating the underlying structure of the targets. With the added benefits of a user parameter that is easy to choose. Furthermore, SLIM is robust to additive noise levels. In addition, SLIM is computationally efficient and preferable for problems with large data
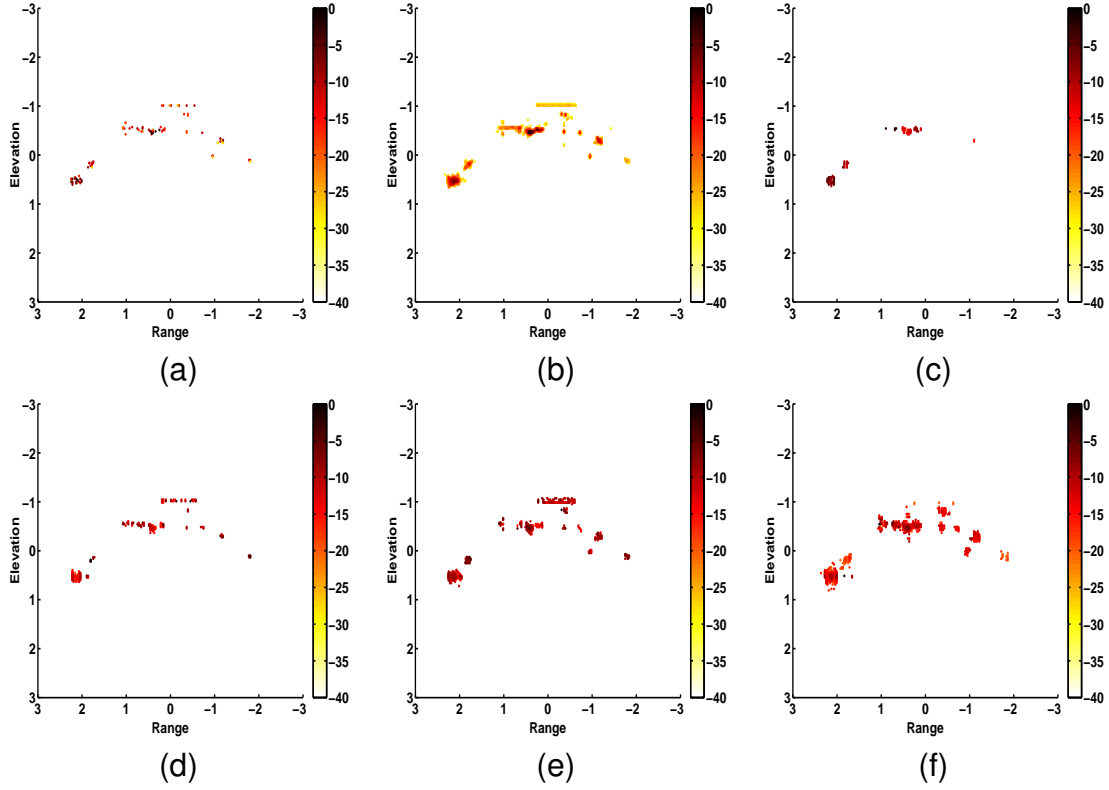
Figure 2-4. Backhoe 3-D SAR image reconstruction. A) SLIM ($b \to 0$) B) SLIM ($b = 1$) C) CoSaMP (128) D) CoSaMP (256) E) CoSaMP (512) F) CoSaMP (1024)
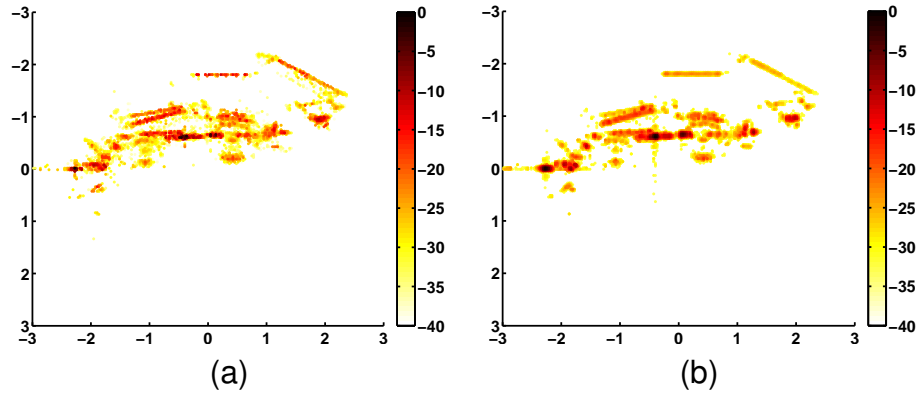
Figure 2-5. Fused 3-D Image Using SLIM. A) SLIM ($q \to 0$) B) SLIM ($q = 1$)

sets. The main competing algorithm, CoSaMP, is a good algorithm but since its user parameter directly ties with the number of non-zeros in the signal space, it requires some clairvoyant on the side of the user.

We have discussed the data acquisition process and data model of SAR imaging problem. We have introduced a sparse signal recovery algorithm named SLIM to the SAR imaging application for the purpose of sidelobe suppression and resolution enhancement. SLIM simply cyclically maximizing the *a posteriori* probability of the underlying parameters given the observations. SLIM can also be viewed as the iterative minimization of the regularized negative log-likelihood function. SLIM is easy to use (does not require the delicate tuning of user parameters), provides accurate and sparse signal estimates in a computationally efficient manner, and is robust to additive noise levels. We have tested the performances of SLIM using various numerical examples. It provides better resolution than the traditional DAS/FFT approach, it is computationally more feasible than IAA, and its user parameter, unlike CoSaMP, does not greatly influence its performance.

CHAPTER 3
SAR RECONSTRUCTION OF INTERRUPTED DATA

We consider nonparametric adaptive spectral analysis of complex-valued data sequences with missing samples occurring in arbitrary patterns. We first present two high-resolution missing-data spectral estimation algorithms: the Iterative Adaptive Approach (IAA) and the Sparse Learning via Iterative Minimization (SLIM) method. Both algorithms can significantly improve the spectral estimation performance, including enhanced resolution and reduced sidelobe levels. Moreover, we consider fast implementations of these algorithms using the Conjugate Gradient (CG) technique and the Gohberg-Semencul-type (GS) formula. Our proposed implementations fully exploit the structure of the steering matrices and maximize the usage of the Fast Fourier Transform (FFT), resulting in much lower computational complexities as well as much reduced memory requirements. The effectiveness of the adaptive spectral estimation algorithms is demonstrated via several numerical examples including both 1-D spectral estimation and 2-D interrupted synthetic aperture radar (SAR) imaging examples.

Spectral estimation is important in many fields including astronomy, communications, medical imaging, radar, and underwater acoustics. Most existing spectral estimation algorithms are devised for uniformly sampled complete-data sequences. However, in many practical applications, the measured data may be incomplete due, for example, to sensor failures, outliers, the data compression needs, etc. Missing data problems are also encountered in modern radar systems. These systems have multiple duties including searching, tracking and the automatic classification of targets. Switching in and out of these modes leads to an incomplete phase history data for synthetic aperture radar (SAR) imaging [36–38]. This loss of the phase history data is also prevalent during SAR missions encountering high radio frequency (RF) interferences [39, 40].

Many missing-data spectral estimation techniques have been developed previously. A conceptually and computationally simple method is the matched filter (MF), implemented

via using the fast Fourier transform (FFT), with the missing samples set to zero. However, as a data-independent approach, the FFT approach, as well as its windowed FFT (WFFT) variations, suffers from poor resolution and high sidelobe level problems. This high sidelobe level problem is especially severe for the missing data case. The CLEAN algorithm [41] is used to estimate the spectrum by deconvolving the missing-data FFT spectrum into the true signal spectrum and the Fourier transform of the missing pattern windowing function via an iterative approach. Although the CLEAN algorithm works for both missing and irregularly sampled data sequences, it cannot resolve closely spaced spectral lines, and hence it may not be a suitable tool for high-resolution spectral estimation. The multi-taper methods [42, 43] compute spectral estimates by assuming certain quadratic functions of the available data samples. The coefficients in the corresponding quadratic functions are optimized according to certain criteria, but it appears that this type of approaches cannot overcome the resolution limit of FFT. To achieve high resolution, several parametric algorithms, e.g., those based on autoregressive (AR) or autoregressive moving-average (ARMA) models, were used to handle the missing-data problem [44–47]. Although these parametric methods can provide improved spectral estimates, they are sensitive to model errors.

Several nonparametric adaptive filtering based techniques have been developed for the missing-data spectral estimation problem [38]. For example, in [48] and [38], two nonparametric missing-data amplitude and phase estimation (MAPES) algorithms are derived by using a "maximum likelihood (ML) fitting" criterion. The well-known expectation maximization (EM) technique [49] is used to solve the estimation problem iteratively. MAPES works for the general spectral analysis problem with missing samples occurring in arbitrary patterns. However, MAPES is conceptually complicated and it is computationally intensive. Furthermore, it is well-known that EM may converge rather slowly, and, it may even fail to converge globally. As we will show via numerical

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examples, the spectral estimation performance of MAPES degrades severely in high data missing ratio cases.

Recently, sparse signal recovery (SSR) techniques (see, e.g., [23] and the references therein) have attracted much attention in the signal processing community. Different from the conventional data-adaptive approaches, such as Capon [50] and APES [51], SSR can work for very few or even one snapshot, and hence can be used for spectral analysis of irregularly sampled data. However, most of the existing SSR algorithms, including the well-known CoSaMP (Compressive Sampling Matching Pursuit) [23], require the bases of the signal components to obey a "restricted isometry property" [52], which hardly holds in high-resolution spectral estimation problems. As we will show via numerical examples, CoSaMP fails to work properly in both 1-D spectral estimation and 2-D interrupted SAR imaging problems.

In this chapter, we will first present two recently-developed high-resolution spectral estimation algorithms, i.e., the iterative adaptive approach (IAA) [25] and the sparse learning via iterative minimization (SLIM) method [32, 53]. Both algorithms can work for both complete and incomplete data samples, and are able to achieve excellent spectral estimation performance under various conditions. However, both IAA and SLIM are computationally prohibitive, especially for high-dimensional spectral estimation problems such as interrupted SAR imaging. We will consider the fast implementation of IAA for the spectral estimation with missing samples. Moreover, we will propose two fast implementations of SLIM within the missing-data spectral estimation framework. We will apply the proposed algorithms to both 1-D spectral estimation and 2-D interrupted SAR imaging problems. Via several numerical examples, we show that IAA and SLIM outperform the conventional FFT and MAPES approaches, as well as CoSaMP, significantly. Compared to the original IAA and SLIM algorithms, the fast implementation approaches can dramatically reduce the computational complexities and memory requirements.

The remainder of this chapter is organized as follows. Section 3.1 introduces the IAA and SLIM algorithms. In Section 3.2, we propose three fast missing-data spectral estimation methods. Several 1-D spectral estimation and 2-D interrupted SAR imaging examples are presented in Section 3.3. Finally, Section 3.3.2 contains our conclusions.

3.1 Nonparametric Spectral Analysis

In this section, we first present two recently-developed adaptive spectral estimation algorithms, i.e., IAA and SLIM, within the missing data framework. Both algorithms can work for 1-D and higher dimensional spectral estimation problems with complete or incomplete data samples.

3.1.1 Data Model

We consider a spectral estimation problem with M given complex-valued data samples, and stack the M data samples into an $M \times 1$ column vector $\mathbf{x}$. Let β_l ($l = 0, 1, \cdots, L - 1$) be the complex amplitude at the lth frequency grid point of the spectrum, and $\mathbf{a}_l$ ($l = 0, 1, \cdots, L - 1$) be the normalized contribution of the lth frequency grid point to the M available data samples. Then, the given data samples can be modeled as follows:

$$\mathbf{x} = \mathbf{A}\boldsymbol{\beta} + \mathbf{n}, \tag{3–1}$$

where $\mathbf{n}$ denotes the noise vector,

$$\mathbf{A} = [\mathbf{a}_0, \mathbf{a}_2, \cdots, \mathbf{a}_{L-1}], \tag{3–2}$$

and

$$\boldsymbol{\beta} = [\beta_0, \beta_1, \cdots, \beta_{L-1}]^T, \tag{3–3}$$

with $(\cdot)^T$ denoting the transpose. In (3–1), $\mathbf{A}$ is referred to as the steering matrix, and the column vector $\boldsymbol{\beta}$ represents the spectrum to be estimated. Note that the data model in (3–1) can be used for both 1-D and higher dimensional spectral estimation problems. In particular, in the SAR imaging application, $\boldsymbol{\beta}$ contains the complex-valued reflection coefficients of scatterers within an imaging area (or volume) of interest. Note also that

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in high-resolution spectral estimation problems, the grid point number L is usually much larger than the number of data samples. Hence, the conventional approaches based on (3–1), such as the maximum likelihood (ML) and least-square (LS) approaches, fail to work completely.

3.1.2 Iterative Adaptive Approach (IAA)

IAA is an iterative adaptive algorithm based on a nonparametric weighted least squares (WLS) approach. Let $p_l = |\hat{\beta}_l|^2$ ($l = 0, 1, \cdots, L-1$) with $\hat{\beta}_k$ being an intermediate estimate of β_l. The noise and interference covariance matrix $\mathbf{\Omega}_l$ for the lth frequency grid point can be estimated as:

$$\mathbf{\Omega}_l = \sum_{k \neq l} p_k \mathbf{a}_k \mathbf{a}_k^H = \mathbf{R} - p_l \mathbf{a}_l \mathbf{a}_l^H, \tag{3–4}$$

where $(\cdot)^H$ denotes the conjugate transpose,

$$\mathbf{R} = \sum_{k=0}^{L-1} p_k \mathbf{a}_k \mathbf{a}_k^H = \mathbf{A}\text{diag}(\mathbf{p})\mathbf{A}^H, \tag{3–5}$$

and

$$\mathbf{p} = \begin{bmatrix} p_0 & p_1 & \cdots & p_{L-1} \end{bmatrix}^T \tag{3–6}$$

with $\text{diag}(\cdot)$ being a diagonal matrix formed by the elements of a given vector.

Given the noise and interference covariance matrix $\mathbf{\Omega}_l$, the estimate of β_l can be refined using the WLS criterion as follows:

$$\hat{\beta}_l = \arg\min \|\mathbf{x} - \mathbf{a}_l \beta_l\|^2_{\mathbf{\Omega}_l^{-1}}, \tag{3–7}$$

where $\|\mathbf{x}\|^2_{\mathbf{\Omega}_l^{-1}} = \mathbf{x}^H \mathbf{\Omega}_l^{-1} \mathbf{x}$. Solving the optimization problem of (3–7) yields

$$\hat{\beta}_l = \frac{\mathbf{a}_l^H \mathbf{\Omega}_l^{-1} \mathbf{x}}{\mathbf{a}_l^H \mathbf{\Omega}_l^{-1} \mathbf{a}_l} = \frac{\mathbf{a}_l^H \mathbf{R}^{-1} \mathbf{x}}{\mathbf{a}_l^H \mathbf{R}^{-1} \mathbf{a}_l}, \tag{3–8}$$

where we have used (3–4) and the matrix inversion lemma [54].

The final spectrum estimate is obtained via iterating (3–5) and (3–8). The initialization of $\{\beta_l\}_{l=0}^{L-1}$ can be done by a standard MF:

$$\hat{\beta}_{l,\text{MF}} = \frac{\mathbf{a}_l^H \mathbf{x}}{\|\mathbf{a}_l\|^2}, \quad \text{for} \quad l = 1, 2, \cdots, L, \tag{3–9}$$

with $\|\cdot\|$ denoting the Euclidean norm of a vector.

We remark that in the above derivation we have implicitly taken into account the effect of noise. A regularized version of IAA, referred to as IAA-R [55], can be used to take into account the noise effect explicitly. The fast implementation of IAA proposed in Section 3.2 can be extended to IAA-R straightforwardly. So we will focus on the former herein. We formally outline the steps in the IAA algorithm below.

The IAA Algorithm:

- **Initialization:** Estimate $\{\beta_k\}$ using MF.

- **Iteration:** Repeat the following steps:

- **Step 1** - Compute $\mathbf{p} = [|\beta_0|^2, |\beta_1|^2, \cdots, |\beta_{L-1}|^2]^T$.

- **Step 2** - Update the signal covariance matrix $\mathbf{R} = \mathbf{A}\text{diag}(\mathbf{p})\mathbf{A}^H$.

- **Step 3** - Update the spectrum estimate $\hat{\beta}_l = \frac{\mathbf{a}_l^H \mathbf{R}^{-1} \mathbf{x}}{\mathbf{a}_l^H \mathbf{R}^{-1} \mathbf{a}_l}$ for $l = 0, 1, \cdots, L-1$.

3.1.3 Sparse Learning via Iterative Minimization (SLIM)

The SLIM algorithm [32, 53] considers the following optimization problem:

$$(\hat{\boldsymbol{\beta}}, \hat{\eta}) = \min_{\boldsymbol{\beta}, \eta} g(\boldsymbol{\beta}, \eta), \tag{3–10}$$

where

$$g(\boldsymbol{\beta}, \eta) \triangleq M \log \eta + \frac{1}{\eta}\|\mathbf{x} - \mathbf{A}\boldsymbol{\beta}\|^2 + \sum_{l=0}^{L-1} \frac{2}{q}\left[(|\beta_l|^2)^{q/2} - 1\right], \tag{3–11}$$

with $\|\cdot\|$ denoting the Euclidean norm of a vector. In (3–11), q is a user parameter that ranges between 0 and 1. In our numerical examples in Section 3.3, we use $q = 1$ for both 1-D spectral estimation and 2-D interrupted SAR imaging problems. The SLIM

algorithm can be viewed as a maximum *a posteriori* (MAP) approach with η being the noise variance and the prior distribution of $\{\beta_l\}$ being $f(\beta_l) \propto e^{-\frac{2}{q}|\beta_l|^q-1}$.

The optimization problem can be solved iteratively by using the cyclic minimization (CM) and majorization-minimization (MM) techniques [56]. Let $\hat{\boldsymbol{\beta}}^{(t)}$ and $\hat{\eta}^{(t)}$ be intermediate estimates of $\boldsymbol{\beta}$ and η, respectively, for the tth iteration. For $0 \le q \le 1$, $\frac{2}{q}\left[(|\beta_l|^2)^{q/2}-1\right]$ is a concave function with respect to $|\beta_l|^2$. Hence, the function $\frac{2}{q}\left[(|\beta_l|^2)^{q/2}-1\right]$ is majorized by its tangent linear function at $|\hat{\beta}_l^{(t)}|^2$, i.e.,

$$\frac{2}{q}\left[(|\beta_l|^2)^{q/2}-1\right] \le \frac{2}{q}\left[|\hat{\beta}_l^{(t)}|^q-1\right] + \frac{1}{p_l^{(t)}}\left[|\beta_l|^2-|\hat{\beta}_l^{(t)}|^2\right], \tag{3–12}$$

where

$$p_l^{(t)} = |\hat{\beta}_l^{(t)}|^{2-q}, \tag{3–13}$$

and the equation holds when $|\beta_l| = |\hat{\beta}_l^{(t)}|$. Therefore, we have:

$$\begin{aligned}
g(\boldsymbol{\beta}, \eta) &\le M\log\eta + \frac{1}{\eta}\|\mathbf{x}-\mathbf{A}\boldsymbol{\beta}\|^2 + \sum_{l=0}^{L-1} p_l^{(t)}\left[|\beta_l|^2-|\hat{\beta}_l^{(t)}|^2\right] + \sum_{l=0}^{L-1}\frac{2}{q}\left[|\hat{\beta}_l^{(t)}|^q-1\right] \\
&\triangleq h(\boldsymbol{\beta}, \eta|\hat{\boldsymbol{\beta}}^{(t)}),
\end{aligned} \tag{3–14}$$

where, again, the equation holds when $|\beta_l| = |\hat{\beta}_l^{(t)}|$ for $l = 0, 1, \cdots, L-1$.

Given $\eta = \hat{\eta}^{(t)}$, minimizing the majorization function of $g(\boldsymbol{\beta}, \eta)$, i.e., $h(\boldsymbol{\beta}, \eta|\hat{\boldsymbol{\beta}}^{(t)})$, with respect to $\boldsymbol{\beta}$ yields:

$$\begin{aligned}
\hat{\boldsymbol{\beta}}^{(t+1)} &= \left[\mathbf{A}^H\mathbf{A} + \eta^{(t)}\,\text{diag}^{-1}\left(\mathbf{p}^{(t)}\right)\right]^{-1}\mathbf{A}^H\mathbf{x} \\
&= \text{diag}\left(\mathbf{p}^{(t)}\right)\mathbf{A}^H\boldsymbol{\Sigma}^{-1}\mathbf{x},
\end{aligned} \tag{3–15}$$

where

$$\boldsymbol{\Sigma} = \mathbf{A}\,\text{diag}\left(\mathbf{p}^{(t)}\right)\mathbf{A}^H + \hat{\eta}^{(t)}\mathbf{I}, \tag{3–16}$$

and $\mathbf{p}^{(t)}$ is defined similarly to $\mathbf{p}$ in (3–6), i.e.,

$$\mathbf{p}^{(t)} = \left|\hat{\boldsymbol{\beta}}^{(t)}\right|^{2-q} \triangleq \left[\left|\hat{\beta}_0^{(t)}\right|^{2-q}\quad \left|\hat{\beta}_1^{(t)}\right|^{2-q}\quad \cdots\quad \left|\hat{\beta}_{L-1}^{(t)}\right|^{2-q}\right]^T. \tag{3–17}$$

Furthermore, given $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}^{(t+1)}$, minimizing $h(\boldsymbol{\beta}, \eta | \hat{\boldsymbol{\beta}}^{(t)})$ with respect to η yields:

$$\hat{\eta}^{(t+1)} = \frac{1}{M} \left\| \mathbf{x} - \mathbf{A}\hat{\boldsymbol{\beta}}^{(t+1)} \right\|^2. \tag{3–18}$$

The SLIM algorithm estimates $\boldsymbol{\beta}$ via iterating (3–15), (3–17), and (3–18).

By using the CM and MM properties, we readily have:

$$
\begin{aligned}
g\left(\hat{\boldsymbol{\beta}}^{(t+1)}, \hat{\eta}^{(t+1)}\right) &\leq h\left(\hat{\boldsymbol{\beta}}^{(t+1)}, \hat{\eta}^{(t+1)} | \hat{\boldsymbol{\beta}}^{(t)}\right) \leq h\left(\hat{\boldsymbol{\beta}}^{(t+1)}, \hat{\eta}^{(t)} | \hat{\boldsymbol{\beta}}^{(t)}\right) \\
&\leq h\left(\hat{\boldsymbol{\beta}}^{(t)}, \hat{\eta}^{(t)} | \hat{\boldsymbol{\beta}}^{(t)}\right) = g\left(\hat{\boldsymbol{\beta}}^{(t)}, \hat{\eta}^{(t)}\right),
\end{aligned}
\tag{3–19}
$$

i.e., the cost function decreases monotonically.

Again, we initialize $\{\beta_l\}_{l=0}^{L-1}$ using MF in (3–9). The variable η is initialized as:

$$\hat{\eta} = \frac{1}{\gamma L} \left\| \mathbf{x} - \mathbf{A}\hat{\boldsymbol{\beta}}_{MF} \right\|^2, \tag{3–20}$$

where $\hat{\boldsymbol{\beta}}_{MF}$ is a column vector formed by $\{\hat{\beta}_{l,MF}\}_{l=0}^{L-1}$ similarly to (3–35), and γ is chosen to be 10 in our numerical examples in Section 3.3. The SLIM algorithm is summarized below.

The SLIM Algorithm:

- **Initialization:** Estimate $\boldsymbol{\beta}$ using MF in (3–9), and set an initial value of $\hat{\eta}$ using (3–20).

- **Iteration:** Repeat the following steps:

- **Step 1** - Compute $\mathbf{p} = |\hat{\boldsymbol{\beta}}|^{2-q}$.

- **Step 2** - Compute $\boldsymbol{\Sigma} = \mathbf{A}\text{diag}(\mathbf{p})\mathbf{A}^H + \hat{\eta}\mathbf{I}$.

- **Step 3** - Update $\hat{\boldsymbol{\beta}} = \text{diag}(\mathbf{p}) \mathbf{A}^H \boldsymbol{\Sigma}^{-1}\mathbf{x}$.

- **Step 4** - Update $\hat{\eta} = \frac{1}{M}\|\mathbf{x} - \mathbf{A}\hat{\boldsymbol{\beta}}\|^2$.

3.1.4 Computational Complexities and Memory Requirements

For the IAA algorithm, it is computationally intensive to compute the covariance matrix $\mathbf{R}$ in Step 2 and $\mathbf{a}_l^H\mathbf{R}^{-1}\mathbf{a}_l$ (for $l = 0, 1, \cdots, L-1$) in Step 3, with the computational complexities being $O(M^2L)$ per iteration. Similarly, SLIM requires $O(M^2L)$ flops per

iteration to compute $\boldsymbol{\Sigma}$ in Step 2. Furthermore, both IAA and SLIM need to pre-compute and store the steering matrix **A** of size $M \times L$. Both the computational complexity and memory requirement of the algorithms, if implemented directly, would be prohibitive, especially for high-dimensional spectral estimation problems such as SAR imaging, where M and L could be up to 10^5 and 10^7, respectively. We present below several fast implementations of IAA and SLIM to dramatically reduce their computational complexities and memory requirements.

3.2   Missing Data Spectral Analysis and Fast Implementations

Hereafter, we focus our discussion on a 2-D spectral estimation problem, since the interrupted SAR imaging application is of particular interest to us. The 1-D spectral estimation problem is a special case of its 2-D counterpart. Moreover, our results can be readily extended to 3- or higher-dimensional spectral estimation problems.

3.2.1   Data Model

We consider the following 2-D spectral estimation problem:

$$x(m_1, m_2) = \sum_{l_1=0}^{L_1-1} \sum_{l_2=0}^{L_2-1} \beta_{l_1, l_2} z_1^{m_1 l_1} z_2^{m_2 l_2} + n(m_1, m_2), \qquad (3\text{–}21)$$

where

$$z_1 = e^{j \frac{2\pi}{L_1}} \quad \text{and} \quad z_2 = e^{j \frac{2\pi}{L_2}}. \qquad (3\text{–}22)$$

In (3–21), $\{x(m_1, m_2)\}$ ($m_1 = 0, 1, \cdots, M_1 - 1$, and $m_2 = 0, 1, \cdots, M_2 - 1$) are the complex-valued data samples, $\{\beta_{l_1, l_2}\}$ ($l_1 = 0, 1, \cdots, L_1 - 1$, and $l_2 = 0, 1, \cdots, L_2 - 1$) are the complex amplitudes of signal spectrum at frequency grid $\{(l_1, l_2)\}$, and $\{n(m_1, m_2)\}$ denote the noise.

For notional convenience, we organize the complete data samples $\{x(m_1, m_2)\}$ and the complex-valued amplitudes $\{\beta_{l_1, l_2}\}$ into an $M_1 \times M_2$ matrix $\mathbf{X_C}$ and an $L_1 \times L_2$ matrix

B, respectively, as follows:

$$\mathbf{X_C} = \begin{bmatrix} x(0,0) & x(0,1) & \cdots & x(0, M_2-1) \\ x(1,0) & x(1,1) & \cdots & x(1, M_2-1) \\ \vdots & \vdots & \ddots & \vdots \\ x(M_1-1,0) & x(M_1-1,1) & \cdots & x(M_1-1, M_2-1) \end{bmatrix}, \tag{3–23}$$

and

$$\mathbf{B} = \begin{bmatrix} \beta_{0,0} & \beta_{0,1} & \cdots & \beta_{0,L_2-1} \\ \beta_{1,0} & \beta_{1,1} & \cdots & \beta_{1,L_2-1} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{L_1-1,0} & \beta_{L_1-1,1} & \cdots & \beta_{L_1-1,L_2-1} \end{bmatrix}. \tag{3–24}$$

Then, the complete data model in (3–21) can be rewritten as:

$$\mathbf{X_C} = \mathbf{F}_1 \mathbf{B} \mathbf{F}_2^T + \mathbf{N_C}, \tag{3–25}$$

where $\mathbf{N_C}$ is defined similarly to $\mathbf{X_C}$ in (3–23) for the noise, and $\mathbf{F}_1$ and $\mathbf{F}_2$ are $M_1 \times L_1$ and $M_2 \times L_2$ inverse Fourier transform matrices, respectively, i.e.,

$$\mathbf{F}_k = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & z_k & z_k^2 & \cdots & z_k^{L_k-1} \\ 1 & z_k^2 & z_k^4 & \cdots & z_k^{2(L_k-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & z_k^{M_k-1} & z_k^{2(M_k-1)} & \cdots & z_k^{(M_k-1)(L_k-1)} \end{bmatrix}, \quad \text{for} \quad k = 1, 2. \tag{3–26}$$

By applying the matrix vectorization operator to both sides of (3–25), the complete data model can be rewritten as:

$$\mathbf{x_C} = \mathbf{F} \mathrm{vec}(\mathbf{B}) + \mathrm{vec}(\mathbf{N_C}), \quad \text{with} \quad \mathbf{F} = \mathbf{F}_2 \otimes \mathbf{F}_1 \quad \text{and} \quad \mathbf{x_C} = \mathrm{vec}(\mathbf{X_C}), \tag{3–27}$$

where $\mathbf{F}$ is the 2-D inverse Fourier transform matrix, $\mathrm{vec}(\cdot)$ denotes the vectorization operator (i.e., stacking the columns of a matrix on top of each other), and $\otimes$ denotes

the Kronecker product of matrices. In (3–27), we have used the fact that $\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\text{vec}(\mathbf{B})$ (see, e.g., [54]).

Now we assume that only M out of $M_1 M_2$ samples are available. Let $\mathbf{x}$ be an $M \times 1$ column vector formed by the given data samples, and $\mathbf{J_g}$ be the corresponding $M \times (M_1 M_2)$ selection matrix, i.e.,

$$\mathbf{x} = \mathbf{J_g}\, \mathbf{x_c}. \tag{3–28}$$

The selection matrix $\mathbf{J_g}$ has elements 0 and 1. Each row of the selection matrix contains one and only one element equal to 1, whose position indicates the selected row/cloumn index. As an example, let

$$\mathbf{J_g} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \tag{3–29}$$

It can be easily verified that $\mathbf{J_g C}$ is a 2-row matrix formed by the second and fifth rows of a 6-row matrix $\mathbf{C}$, while $\mathbf{D J_g^T}$ is a 2-column matrix formed by the second and fifth columns of a 6-column matrix $\mathbf{D}$. Note also that $\mathbf{J_g^T C}$ expands a 2-row matrix $\mathbf{C}$ to a 6-row matrix whose second and fifth rows are equal to the first and second rows of $\mathbf{C}$, respectively, with the remaining rows being zero. Also, $\mathbf{D J_g}$ expands a 2-column matrix $\mathbf{D}$ to a 6-column matrix similarly.

From (3–27) and (3–28), we get the data model for the 2-D missing-data spectral estimation problem:

$$\mathbf{x} = \left(\mathbf{J_g}\, \mathbf{F} \right) \text{vec}(\mathbf{B}) + \mathbf{n}. \tag{3–30}$$

Comparing (3–30) with (3–1), we can see that the aforementioned IAA and SLIM algorithms in Section 3.1 can be applied to (3–30) directly with the steering matrix $\mathbf{A}$ being $\mathbf{J_g}\, \mathbf{F}$. If implemented directly, the computational complexity and memory requirement of IAA and SLIM are $O(M^2 L_1 L_2)$ and $O(M L_1 L_2)$, respectively, which are prohibitive for practical applications. We present below several computationally efficient implementations of IAA and SLIM that exploit the structure of $\mathbf{J_g}\mathbf{F}$.

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3.2.2 Fast Iterative Adaptive Approach (Fast-IAA)

We present below an efficient implementation of IAA via fully exploiting the structure of the steering matrix $\mathbf{J_g F}$. We refer to this fast implementation of IAA as Fast-IAA.

3.2.2.1 Efficient Computation of the IAA Covariance Matrix

We first consider how to compute the IAA covariance matrix $\mathbf{R}$ (step 2 of the IAA algorithm outline) efficiently.

Let $\hat{\mathbf{B}}$ be an intermediate estimate of $\mathbf{B}$, and $\mathbf{P} = |\hat{\mathbf{B}}|^2$ be the corresponding power estimate, where $|\cdot|^2$ denotes the element-wise modulus-square operation. Then, the IAA covariance matrix $\mathbf{R}$ in (3–5) can be written as:

$$
\begin{aligned}
\mathbf{R} &= \left(\mathbf{J_g F}\right) \operatorname{diag}(\operatorname{vec}(\mathbf{P})) \left(\mathbf{J_g F}\right)^H \\
&= \mathbf{J_g R_C J_g^T},
\end{aligned}
\tag{3–31}
$$

where

$$
\mathbf{R_C} = \mathbf{F} \operatorname{diag}(\operatorname{vec}(\mathbf{P})) \mathbf{F}^H
\tag{3–32}
$$

represents the IAA covariance matrix of the complete data samples corresponding to $\mathbf{P}$.

By using the 2-D inverse Fourier transform property of $\mathbf{F}$, it can be readily verified that $\mathbf{R_C}$ is a Toeplitz-Block-Toeplitz matrix [57], which can be written as follows:

$$
\mathbf{R_C} = \begin{bmatrix}
\mathbf{R}_0 & \mathbf{R}_1^H & \mathbf{R}_2^H & \cdots & \mathbf{R}_{(M_2-1)}^H \\
\mathbf{R}_1 & \mathbf{R}_0 & \mathbf{R}_1^H & \cdots & \mathbf{R}_{(M_2-2)}^H \\
\mathbf{R}_2 & \mathbf{R}_1 & \mathbf{R}_0 & \cdots & \mathbf{R}_{(M_2-3)}^H \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\mathbf{R}_{M_2-1} & \mathbf{R}_{M_2-2} & \mathbf{R}_{M_2-3} & \cdots & \mathbf{R}_0
\end{bmatrix},
\tag{3–33}
$$

with

$$
\mathbf{R}_{m_2} = \begin{bmatrix} r_{0,m_2} & r_{-1,m_2} & r_{-2,m_2} & \cdots & r_{-(M_1-1),m_2} \\ r_{1,m_2} & r_{0,m_2} & r_{-1,m_2} & \cdots & r_{-(M_1-2),m_2} \\ r_{2,m_2} & r_{1,m_2} & r_{0,m_2} & \cdots & r_{-(M_1-3),m_2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{M_1-1,m_2} & r_{M_1-2,m_2} & r_{M_1-3,m_2} & \cdots & r_{0,m_2} \end{bmatrix}. \tag{3--34}
$$

In (3–34), r_{m_1,m_2} (for $m_1 = -M_1 + 1, \ -M_1 + 2, \ \cdots, \ M_1 - 1$; and $m_2 = 0, \ 1, \ \cdots, \ M_2 - 1$) is the 2-D inverse Fourier transform of $\mathbf{P}$, which can be computed efficiently by using the 2-D inverse FFT (IFFT) technique. Furthermore, from (3–31), by the selection matrix property, $\mathbf{R}$ is a submatrix of $\mathbf{R_C}$ formed by the rows and columns specified by the selection matrix $\mathbf{J_g}$. By applying 2-D IFFT to $\mathbf{P}$ to obtain $\mathbf{R_C}$ first and then get $\mathbf{R}$, we can reduce the computational complexity from $O(M^2 L_1 L_2)$ to $O(L_1 L_2 \log(L_1 L_2))$.

3.2.2.2 Efficient Computation of $\hat{\mathbf{B}}$

Now we consider an efficient method to compute $\hat{\mathbf{B}}$ given $\mathbf{R}$. Applying the WLS method (i.e., (3–8)) to the 2-D spectral estimation data model in (3–30), after some straightforward manipulations, we readily have:

$$
[\hat{\mathbf{B}}]_{l_1,l_2} = \frac{c_{l_1,l_2}}{d_{l_1,l_2}}, \quad \text{for} \quad l_1 = 0, \ 1, \ \cdots, \ L_1 - 1 \quad \text{and} \quad l_2 = 0, \ 1, \ \cdots, \ L_2 - 1, \tag{3--35}
$$

where

$$
\begin{aligned}
c_{l_1,l_2} &= \left[\mathbf{J_g}\,\mathbf{F}\right]^{H}_{.,l_1+M_1 l_2} \mathbf{R}^{-1}\mathbf{x} \\
&= [\mathbf{F}]^{H}_{.,l_1+M_1 l_2} \left(\mathbf{J_g}^{T}\mathbf{R}^{-1}\mathbf{x}\right),
\end{aligned} \tag{3--36}
$$

and

$$
\begin{aligned}
d_{l_1,l_2} &= \left[\mathbf{J_g}\,\mathbf{F}\right]^{H}_{.,l_1+M_1 l_2} \mathbf{R}^{-1} \left[\mathbf{J_g}\,\mathbf{F}\right]_{.,l_1+M_1 l_2} \\
&= [\mathbf{F}]^{H}_{.,l_1+M_1 l_2} \left(\mathbf{J_g}^{T}\mathbf{R}^{-1}\mathbf{J_g}\right) [\mathbf{F}]_{.,l_1+M_1 l_2},
\end{aligned} \tag{3--37}
$$

with $[\cdot]_{l_1,l_2}$ and $[\cdot]_{,l}$ denoting the $(l_1,\ l_2)$th element and the lth column of a given matrix, respectively.

Given $\mathbf{R}^{-1}$, c_{l_1,l_2} in (3–36) can be computed efficiently as follows: (1) compute $\mathbf{R}^{-1}\mathbf{x}$; (2) expand the so-obtained $M \times 1$ vector to an $M_1 M_2 \times 1$ vector by using the selection matrix property; (3) organize the vector into an $M_1 \times M_2$ matrix, and (4) apply 2-D FFT to the so-obtained $M_1 \times M_2$ matrix.

We now consider how to compute d_{l_1,l_2} in (3–37) efficiently. Let $\mathbf{Q} = \mathbf{J}_g^T \mathbf{R}^{-1} \mathbf{J}_g$, which, obviously, can be obtained from $\mathbf{R}^{-1}$ by inserting some zero rows and columns specified by the selection matrix $\mathbf{J}_g^T$. For convenience, we rewrite $\mathbf{Q}$ as an $M_2 \times M_2$ block matrix formed by $M_1 \times M_1$ sub-matrices, i.e.,

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{0,0} & \mathbf{Q}_{0,1} & \cdots & \mathbf{Q}_{0,M_2-1} \\ \mathbf{Q}_{1,0} & \mathbf{Q}_{1,1} & \cdots & \mathbf{Q}_{1,M_2-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Q}_{M_1-1,0} & \mathbf{Q}_{M_1-1,1} & \cdots & \mathbf{Q}_{M_1-1,M_2-1} \end{bmatrix}. \tag{3–38}$$

Then, d_{l_1,l_2} can be expressed as a polynomial function of $z_1^{l_1}$ and $z_2^{l_2}$ as follows:

$$d_{l_1,l_2} = \sum_{m_1=-(M_1-1)}^{M_1-1} \sum_{m_2=-(M_2-1)}^{M_2-1} e_{m_1,m_2} z_1^{-l_1 m_1} z_2^{-l_2 m_2}, \tag{3–39}$$

where

$$e_{m_1,m_2} = \begin{cases} \sum_{k_1=m_1}^{M_1-1} \sum_{k_2=m_2}^{M_2-1} [\mathbf{Q}_{k_2,k_2-m_2}]_{k_1,k_1-m_1}, & \text{for } m_1 \geq 0 \text{ and } m_2 \geq 0, \\ \sum_{k_1=0}^{M_1-1+m_1} \sum_{k_2=m_2}^{M_2-1} [\mathbf{Q}_{k_2,k_2-m_2}]_{k_1,k_1-m_1}, & \text{for } m_1 < 0 \text{ and } m_2 \geq 0, \\ \sum_{k_1=m_1}^{M_1-1} \sum_{k_2=0}^{M_2-1+m_2} [\mathbf{Q}_{k_2,k_2-m_2}]_{k_1,k_1-m_1}, & \text{for } m_1 \geq 0 \text{ and } m_2 < 0, \\ \sum_{k_1=0}^{M_1-1+m_1} \sum_{k_2=0}^{M_2-1+m_2} [\mathbf{Q}_{k_2,k_2-m_2}]_{k_1,k_1-m_1}, & \text{for } m_1 < 0 \text{ and } m_2 < 0. \end{cases} \tag{3–40}$$

From (3–39), d_{l_1,l_2} (for $l_1 = 0,\ 1,\ \cdots,\ L_1 - 1$ and $l_2 = 0,\ 1,\ \cdots,\ L_2 - 1$) can be computed efficiently via applying 2-D FFT to the polynomial coefficients $\{e_{m_1,m_2}\}$ in (3–40), which reduces the computational complexity from $O(M^2 L_1 L_2)$ to $O(L_1 L_2 \log(L_1 L_2))$.

The Fast-IAA algorithm is summarized beow.

The Fast-IAA Algorithm:

- **Initialization:** Apply 2-D FFT to $\mathbf{x}$ to get an initial estimate of $\mathbf{B}$.

- **Iteration:** Repeat the following steps:

- **Step 1** - Compute $\mathbf{P} = |\hat{\mathbf{B}}|^2$.

- **Step 2** - Apply 2-D IFFT to $\mathbf{P}$, and construct the matrix $\mathbf{R}$ using (3–33) and 3–34).

- **Step 3** - Compute the matrix inverse of $\mathbf{R}$.

- **Step 4** - Compute $\mathbf{R}^{-1}\mathbf{x}$, and then apply 2-D FFT to $\mathbf{J}^T\mathbf{R}^{-1}\mathbf{x}$ to get $\{c_{l_1,l_2}\}$.

- **Step 5** - Compute $\{e_{l_1,l_2}\}$ from $\mathbf{R}^{-1}$ using (3–40).

- **Step 6** - Apply 2-D FFT to $\{e_{l_1,l_2}\}$ to compute $\{d_{l_1,l_2}\}$.

- **Step 7** - Update $\hat{\mathbf{B}}$ by using (3–35) and the results of Steps 4 and 6.

Note that the computationally most intensive steps of the Fast-IAA algorithm are the matrix inverse of $\mathbf{R}$ in Step 3 and 2-D FFT/IFFT, with computational complexities $O(M^3)$ and $O(L_1 L_2 \log(L_1 L_2))$, respectively. Hence, the overall computational complexity of the IAA algorithm is reduced to $O(M^3) + O(L_1 L_2 \log(L_1 L_2))$, which is much smaller than $O(M^2 L_1 L_2)$ of the original IAA algorithm. We remark that in the complete data case, $\mathbf{R}$ in (3–31) is a Toeplitz-Block-Toeplitz matrix [57], whose inverse can be computed efficiently by using the Gohberg-Semencul (GS) type formula [58, 59]. We refer to this GS-type formula based Fast-IAA algorithm as Fast-IAA-GS [60]. The computational complexity of Fast-IAA-GS is $O(L_1 L_2 \log(L_1 L_2))$. Furthermore, we note that Fast-IAA, as well as Fast-IAA-GS, does not use the steering matrix explicitly. Hence, it does not need to pre-compute and store the steering matrix, which significantly reduces the memory requirement.

3.2.3 Fast SLIM Using Conjugate Gradient (CG-SLIM)

Next, we consider a fast implementation of the SLIM algorithm by using the conjugate gradient (CG) technique [61]. We refer to this algorithm as CG-SLIM.

CG is an efficient method to solve a linear system iteratively. The number of iterations needed by CG is usually much smaller than the problem size, which can significantly reduce the computational complexity. In this section, we employ the CG technique to compute the most intensive step of SLIM, i.e., $\boldsymbol{\Sigma}^{-1}\mathbf{x}$. Let

$$\mathbf{y} = \boldsymbol{\Sigma}^{-1}\mathbf{x}. \tag{3–41}$$

Then, the unknown vector $\mathbf{y}$ in (3–41) can be computed efficiently by using the CG approach which is outline below.

The Conjugate Gradient Algorithm:

- **Initialization:** Set an initial vector $\mathbf{y}$, and $\mathbf{r}_0 = \mathbf{r}_1 = \mathbf{g} = \mathbf{x} - \mathbf{R}\mathbf{y}$.

- **Iteration:** Repeat the following steps until convergence:

- **Step 1** - Compute $\mathbf{h} = \boldsymbol{\Sigma}\mathbf{g}$.

- **Step 2** - Compute $\alpha = \frac{\|\mathbf{r}_1\|^2}{\mathbf{g}^H \mathbf{h}}$.

- **Step 3** - Update $\mathbf{y} = \mathbf{y} + \alpha\mathbf{g}$.

- **Step 4** - Update $\mathbf{r}_0 = \mathbf{r}_1$ and $\mathbf{r}_1 = \mathbf{r}_1 - \alpha\mathbf{h}$.

- **Step 5** - Update $\mathbf{g} = \mathbf{r}_1 + \frac{\|\mathbf{r}_1\|^2}{\|\mathbf{r}_0\|^2}\mathbf{g}$.

For the CG algorithm above, Step 1 is computationally the most intensive, which involves a matrix-vector product. However, in the 2-D spectral estimation problem in (3–30), the computational complexity of this step can be reduced by using the structure of the steering vector. Let $\hat{\mathbf{B}}$ be an intermediate estimate of $\mathbf{B}$, and $\mathbf{P} = |\hat{\mathbf{B}}|^{2-q}$. Replacing $\mathbf{A}$ and $\mathbf{p}$ in (3–16) by $\mathbf{J_g}\,\mathbf{F}$ and $\text{vec}(\mathbf{P})$, respectively, we have:

$$\begin{aligned} \boldsymbol{\Sigma} &= \left(\mathbf{J_g}\,\mathbf{F}\right)\text{diag}\left(\text{vec}(\mathbf{P})\right)\left(\mathbf{J_g}\,\mathbf{F}\right)^H + \hat{\eta}\mathbf{I} \\ &= \mathbf{J_g}\,[\mathbf{F}\,\text{diag}\left(\text{vec}(\mathbf{P})\right)\,\mathbf{F}]^H\,\mathbf{J_g}^T + \hat{\eta}\mathbf{I}. \end{aligned} \tag{3–42}$$

Then, Step 1 of the CG algorithm can be written as:

$$
\begin{aligned}
\mathbf{h} &= \left(\mathbf{J_g}\,\mathbf{F} \right) \operatorname{diag}\left(\operatorname{vec}(\mathbf{P}) \right) \left(\mathbf{J_g}\,\mathbf{F} \right)^{H} \mathbf{g} + \hat{\eta}\,\mathbf{g} \\
&= \mathbf{J_g}\mathbf{F} \left[\operatorname{vec}(\mathbf{P}) \odot \left(\mathbf{F}^{H}\,\mathbf{J_g^T}\mathbf{g} \right) \right] + \hat{\eta}\mathbf{g} \\
&= \mathcal{F}_{2d}^{-1}\left[\operatorname{vec}(\mathbf{P}) \odot \mathcal{F}_{2d}(\mathbf{g}) \right] + \hat{\eta}^{(t)}\mathbf{g},
\end{aligned}
\tag{3–43}
$$

where $\odot$ denotes the Hadamard product of matrices, and the functions $\mathcal{F}_{2d}^{-1}(\cdot)$ and $\mathcal{F}_{2d}(\cdot)$ are defined, respectively, as follows:

$$
\mathcal{F}_{2d}^{-1}(\mathbf{b}) = \mathbf{J_g}\mathbf{F}\mathbf{b},
\tag{3–44}
$$

and

$$
\mathcal{F}_{2d}(\mathbf{g}) = \mathbf{F}^{H}\mathbf{J_g^T}\mathbf{g}.
\tag{3–45}
$$

By the properties of the 2-D inverse Fourier transform matrix $\mathbf{F}$ and the selection matrix $\mathbf{J_g}$, (3–44) and (3–45) can be computed efficiently by using the 2-D FFT/IFFT technique. For the $\mathcal{F}_{2d}^{-1}(\cdot)$ function in (3–44), we can re-organize the $L_1 L_2 \times 1$ vector $\mathbf{b}$ into an $L_1 \times L_2$ matrix, apply 2-D IFFT to the so-obtained matrix, and then pick up the elements of the 2-D IFFT output specified by the selection matrix to form a column vector. For the $\mathcal{F}_{2d}(\cdot)$ function in (3–45), we can first expand the $M \times 1$ column vector to an $M_1 M_2 \times 1$ vector via inserting zeros at the positions specified by the selection matrix $\mathbf{J_g^T}$, re-organize the vector into an $M_1 \times M_2$ matrix, and then apply 2-D FFT to the so-obtained matrix. By using these 2-D FFT and IFFT techniques, the computational complexity of Step 1 of the Conjugate Gradient algorithm is reduced to $O(L_1 L_2 \log(L_1 L_2))$.

By combining the SLIM algorithm and the CG algorithm, the CG-SLIM algorithm is summarized below.

The CG-SLIM Algorithm:

- **Initialization:** Apply 2-D FFT to $\mathbf{x}$ to get an initial estimate of $\mathbf{B}$, initialize $\hat{\eta}$, and $\mathbf{y} = 0$.

- **Iteration:** Repeat the following steps:

- **Step 1** - Compute $\mathbf{P} = |\hat{\mathbf{B}}|^{(2-q)}$.

- **Step 2** - Initialize CG using $\mathbf{r}_1 = \mathbf{x} - \mathcal{F}_{2d}^{-1}\left(\text{vec}(\mathbf{P}). * \mathcal{F}_{2d}(\mathbf{y})\right)$ and $\mathbf{g} = \mathbf{r}_1$.

- **Step 3** - Repeat the following steps until $\frac{\|\mathbf{r}_1\|^2}{\|\mathbf{x}\|^2} \leq \epsilon_{\text{CG}}$:

  - **Step 3.a** - Compute $\mathbf{h} = \mathcal{F}_{2d}^{-1}\left(\text{vec}(\mathbf{P}). * \mathcal{F}_{2d}(\mathbf{g})\right)$.
  - **Step 3.b** - Compute $\alpha = \frac{\|\mathbf{r}_1\|^2}{\mathbf{g}^H \mathbf{h}}$.
  - **Step 3.c** - Update $\mathbf{y} = \mathbf{y} + \alpha \mathbf{g}$.
  - **Step 3.d** - Update $\mathbf{r}_0 = \mathbf{r}_1$ and $\mathbf{r}_1 = \mathbf{r}_1 - \alpha \mathbf{h}$.
  - **Step 3.e** - Update $\mathbf{g} = \mathbf{r}_1 + \frac{\|\mathbf{r}_1\|^2}{\|\mathbf{r}_0\|^2}\mathbf{g}$.

- **Step 4** - Update $\text{vec}(\hat{\mathbf{B}}) = \text{vec}(\mathbf{P}). * (\mathcal{F}_{2d}(\mathbf{y}))$.

- **Step 5** - Update $\hat{\eta} = \frac{1}{M}\left\|\mathbf{x} - \mathcal{F}_{2d}^{-1}\left(\text{vec}(\hat{\mathbf{B}})\right)\right\|^2$.

We terminate the CG iterations when the ratio of the Euclidean norm of the residue vector $\mathbf{r}_1$ to the Euclidean norm of the data vector $\mathbf{x}$ becomes less than a predefined threshold ϵ_{CG}. In the numerical examples in Section 3.3, we choose $\epsilon_{\text{CG}} = 10^{-6}$. We note that computationally the most intensive step in the CG-SLIM algorithm is now 2-D FFT/IFFT, with a computational complexity $O(L_1 L_2 \log(L_1 L_2))$. Hence, the overall computational complexity of CG-SLIM is $O(K L_1 L_2 \log(L_1 L_2))$ per SLIM iteration, with K being the average CG iteration number, which is usually much smaller than M. For instance, in the interrupted SAR example in Section 3.3, the data sample number M is up to 1600, but the CG algorithm usually converges in less than $K = 30$ iterations.

3.2.4 Fast SLIM Using the Gohberg-Semencul-Type Formula (GS-SLIM)

For a complete-data spectral estimation problem, i.e., when $\mathbf{J_g} = \mathbf{I}$, similarly to $\mathbf{R}_c$ in (3–33), the matrix $\mathbf{\Sigma}$ in (3–42) will be a Toeplitz-Block-Toeplitz matrix [57, 60]. In this case, the vector $\mathbf{y} = \mathbf{\Sigma}^{-1}\mathbf{x}$ in (3–41) can be computed efficiently by using the GS-type formula [58, 59], whose complexity is around $\min(M_1^3 M_2^2, M_1^2 M_2^3)$, which is much lower than $O(M_1^2 M_2^2 L_1 L_2)$ of the original SLIM algorithm. Inspired by this, we introduce below a new fast implementation of SLIM, referred to as GS-SLIM, for the missing data

spectral estimation problem. This algorithm includes two steps: (1) estimate missing samples from an intermediate spectral estimate, and (2) estimate the signal spectrum from the given data samples, as well as the estimated missing samples. Note that the second step of the GS-SLIM algorithm is essentially a complete-data spectral estimation problem, where the aforementioned GS technique can be utilized.

For the 2-D spectral estimation problem in (3–30), the SLIM formulation in (3–10) and (3–11) can be written as follows:

$$(\hat{\mathbf{B}}, \hat{\eta}) = \min_{\mathbf{B}, \eta} g(\mathbf{B}, \eta), \tag{3–46}$$

where

$$g(\mathbf{B}, \eta) \triangleq M \log \eta + \frac{1}{\eta} \|\mathbf{x} - \mathbf{J_g} \mathbf{Fvec}(\mathbf{B})\|^2 + \sum_{l_1=0}^{L_1-1} \sum_{l_2=0}^{L_2-1} \frac{2}{q} \left[(|\beta_{l_1,l_2}|^2)^{q/2} - 1 \right]. \tag{3–47}$$

Below, we employ the maximization-minimization (MM) technique to solve the optimization problem iteratively.

Let $\mathbf{x_m}$ and $\mathbf{J_m}$ be the missing samples and the associated selection matrix, respectively. Let $\hat{\mathbf{B}}^{(t)}$ and $\hat{\eta}^{(t)}$ be the intermediate estimates of $\mathbf{B}$ and η at the tth iteration, respectively. Naturally, the missing data can be estimated as follows:

$$\hat{\mathbf{x}}_{\mathbf{m}}^{(t)} = \mathbf{J_m} \mathbf{Fvec}(\hat{\mathbf{B}}^{(t)}). \tag{3–48}$$

We consider below an efficient method to update the $\mathbf{B}$ and η estimates such that the cost function in (3–47) decreases monotonically.

From (3–47), we readily have:

$$g(\mathbf{B}, \eta) \ = \ M \log \eta + \frac{1}{\eta}\|\mathbf{x} - \mathbf{J_g}\mathbf{F}\mathrm{vec}(\mathbf{B})\|^2 + \sum_{l_1=0}^{L_1-1}\sum_{l_2=0}^{L_2-1} \frac{2}{q}\left[(|\beta_{l_1,l_2}|^2)^{q/2} - 1\right] \quad (3\text{–}49)$$

$$\leq \ M \log \eta + \frac{1}{\eta}\|\mathbf{x} - \mathbf{J_g}\mathbf{F}\,\mathrm{vec}(\mathbf{B})\|^2 + \frac{1}{\eta}\|\hat{\mathbf{x}}_m^{(t)} - \mathbf{J_m}\mathbf{F}\mathrm{vec}(\mathbf{B})\|^2$$

$$+ \sum_{l_1=0}^{L_1-1}\sum_{l_2=0}^{L_2-1} \frac{2}{q}\left[(|\beta_{l_1,l_2}|^2)^{q/2} - 1\right] \quad (3\text{–}50)$$

$$= \ M \log \eta + \frac{1}{\eta}\|\hat{\mathbf{x}}_C^{(t)} - \mathbf{F}\mathrm{vec}(\mathbf{B})\|^2 + \sum_{l_1=0}^{L_1-1}\sum_{l_2=0}^{L_2-1} \frac{2}{q}\left[(|\beta_{l_1,l_2}|^2)^{q/2} - 1\right] \quad (3\text{–}51)$$

$$\leq \ M \log \eta + \frac{1}{\eta}\|\hat{\mathbf{x}}_C^{(t)} - \mathbf{F}\mathrm{vec}(\mathbf{B})\|^2 +$$

$$\sum_{l_1=0}^{L_1-1}\sum_{l_2=0}^{L_2-1} p_{l_1,l_2}^{(t)}\left[|\beta_{l_1,l_2}|^2 - |\hat{\beta}_{l_1,l_2}^t|^2\right] + \sum_{l_1=0}^{L_1-1}\sum_{l_2=0}^{L_2-1} \frac{2}{q}\left[|\hat{\beta}_{l_1,l_2}^{(t)}|^q - 1\right] \quad (3\text{–}52)$$

$$\triangleq \ h(\mathbf{B}, \eta | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_C^{(t)}), \quad (3\text{–}53)$$

where $\hat{\mathbf{x}}_C^{(t)}$ is the complete data composed by the estimated missing sample vector $\hat{\mathbf{x}}_m^{(t)}$, as well as the given sample vector $\mathbf{x}$. The inequality in (3–52) is obtained similarly to (3–14), and

$$\mathbf{p}_{l_1,l_2}^{(t)} = |\hat{\beta}_{l_1,l_2}^{(t)}|^{2-q}. \quad (3\text{–}54)$$

Obviously, the equality in $g(\mathbf{B}, \eta) \leq h(\mathbf{B}, \eta | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_C^{(t)})$ holds when $\mathbf{B} = \hat{\mathbf{B}}^{(t)}$. In other words, the function $g(\mathbf{B}, \eta)$ is majorized by $h(\mathbf{B}, \eta | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_C^{(t)})$ at the point $\mathbf{B} = \hat{\mathbf{B}}^{(t)}$. Therefore, we can obtain the estimates of $\mathbf{B}$ and η via minimizing $h(\mathbf{B}, \eta | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_C^{(t)})$ with respect to $\mathbf{B}$ and η.

Similarly to (3–15), given $\eta = \hat{\eta}^{(t)}$, minimizing $h(\mathbf{B}, \eta | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_C^{(t)})$ with respect to $\mathbf{B}$, yields:

$$\mathrm{vec}(\hat{\mathbf{B}}^{(t+1)}) = \mathrm{vec}(\hat{\mathbf{P}}^{(t)}) \odot \left(\mathbf{F}^H \mathbf{\Sigma}_C^{-1} \hat{\mathbf{x}}_C^{(t)}\right), \quad (3\text{–}55)$$

where

$$\mathbf{\Sigma}_C = \mathbf{F}\mathrm{diag}\left(\mathrm{vec}(\hat{\mathbf{P}}^{(t)})\right) \mathbf{F}^H + \hat{\eta}^{(t)}\mathbf{I}. \quad (3\text{–}56)$$

As discussed above, $\boldsymbol{\Sigma}_{\mathsf{C}}$ is a Toeplitze-Block-Toeplitz matrix [57, 60] having the form:

$$\boldsymbol{\Sigma}_{\mathsf{C}} = \begin{bmatrix} \boldsymbol{\Sigma}_0 & \boldsymbol{\Sigma}_1^H & \boldsymbol{\Sigma}_2^H & \cdots & \boldsymbol{\Sigma}_{(M_2-1)}^H \\ \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_0 & \boldsymbol{\Sigma}_1^H & \cdots & \boldsymbol{\Sigma}_{(M_2-2)}^H \\ \boldsymbol{\Sigma}_2 & \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_0 & \cdots & \boldsymbol{\Sigma}(M_2-3)^H \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Sigma}_{M_2-1} & \boldsymbol{\Sigma}_{M_2-2} & \boldsymbol{\Sigma}_{M_2-3} & \cdots & \boldsymbol{\Sigma}_0 \end{bmatrix}, \tag{3–57}$$

and

$$\boldsymbol{\Sigma}_{m_2} = \begin{bmatrix} \boldsymbol{\sigma}_{0,m_2} & \boldsymbol{\sigma}_{-1,m_2} & \boldsymbol{\sigma}_{-2,m_2} & \cdots & \boldsymbol{\sigma}_{-(M_1-1),m_2} \\ \boldsymbol{\sigma}_{1,m_2} & \boldsymbol{\sigma}_{0,m_2} & \boldsymbol{\sigma}_{-1,m_2} & \cdots & \boldsymbol{\sigma}_{-(M_1-2),m_2} \\ \boldsymbol{\sigma}_{2,m_2} & \boldsymbol{\sigma}_{1,m_2} & \boldsymbol{\sigma}_{0,m_2} & \cdots & \boldsymbol{\sigma}_{-(M_1-3),m_2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\sigma}_{M_1-1,m_2} & \boldsymbol{\sigma}_{M_1-2,m_2} & \boldsymbol{\sigma}_{M_1-3,m_2} & \cdots & \boldsymbol{\sigma}_{0,m_2} \end{bmatrix} + \hat{\eta}\delta(m_2)\mathbf{I}, \tag{3–58}$$

where $\delta(m_2)$ is a Kronecker delta function, and $\{\boldsymbol{\sigma}_{m_1,m_2}\}$ is the 2-D Fourier transform of $\mathbf{P}$ and can be computed efficiently by using 2-D FFT. Furthermore, via exploiting the Toeplitze-Block-Toeplitz structure of $\boldsymbol{\Sigma}_{\mathsf{C}}$, $\boldsymbol{\Sigma}^{-1}\hat{\mathbf{x}}_{\mathsf{C}}$ in (3–55) can be computed efficiently by using the generalized GS algorithm [57, 60].

On the other hand, given $\mathbf{B} = \hat{\mathbf{B}}^{(t+1)}$, minimizing $h(\mathbf{B}, \eta | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_{\mathsf{C}}^{(t)})$ with respect to η yields:

$$\tilde{\eta} = \frac{1}{M}\left\| \hat{\mathbf{x}}_{\mathsf{C}}^{(t)} - \mathbf{F}\mathrm{vec}\left(\hat{\mathbf{B}}^{(t+1)}\right) \right\|^2. \tag{3–59}$$

Equation (3–59) is a natural estimate of η given $\mathbf{B} = \hat{\mathbf{B}}^{(t+1)}$. However, we find empirically that the GS-SLIM algorithm suffers from a slow convergence problem using (3–59). Instead, we propose to update $\hat{\eta}$ as follows:

$$\hat{\eta}^{(t+1)} = \rho\tilde{\eta} + (1-\rho)\hat{\eta}^{(t)} = \frac{\rho}{M}\left\| \hat{\mathbf{x}}_{\mathsf{C}}^{(t)} - \mathbf{F}\mathrm{vec}\left(\hat{\mathbf{B}}^{(t+1)}\right) \right\|^2 + (1-\rho)\hat{\eta}^{(t)}, \tag{3–60}$$

where ρ is choose to be the given-to-complete data ratio, i.e., $\rho = \frac{M}{M_1 M_2}$, in the numerical examples in Section 3.3.

The 2-D spectrum $\mathbf{B}$ is estimated via iterating (3–48), (3–55), (3–57) and (3–60). We prove below that the cost function $g(\mathbf{B}, \eta)$ decreases monotonically. First, we readily have the first-order derivative of $h(\hat{\mathbf{B}}^{(t+1)}, \eta | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_{\mathbf{C}}^{(t)})$ as follows:

$$
\begin{aligned}
\frac{\partial h(\hat{\mathbf{B}}^{(t+1)}, \eta | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_{\mathbf{C}}^{(t)})}{\partial \eta} &= \frac{M}{\eta} - \frac{\left\| \hat{\mathbf{x}}_{\mathbf{C}}^{(t)} - \mathbf{Fvec}\left(\hat{\mathbf{B}}^{(t+1)}\right) \right\|^2}{\eta^2} \\
&= \frac{M}{\eta^2}(\eta - \tilde{\eta}),
\end{aligned}
\tag{3–61}
$$

where $\tilde{\eta}$ is defined in (3–59). From (3–61), we can see that $\frac{\partial h(\hat{\mathbf{B}}^{(t+1)}, \eta | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_{\mathbf{m}}^{(t)}, \hat{\eta}^{(t)})}{\partial \eta} < 0$ for $0 < \eta < \tilde{\eta}$ and $\frac{\partial h(\hat{\mathbf{B}}^{(t+1)}, \eta | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_{\mathbf{m}}^{(t)}, \hat{\eta}^{(t)})}{\partial \eta} > 0$ for $\eta > \tilde{\eta}$. In other words, the function $h(\hat{\mathbf{B}}^{(t+1)}, \eta | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_{\mathbf{m}}^{(t)}, \hat{\eta}^{(t)})$ monotonically decreases with respect to η for $0 < \eta < \tilde{\eta}$, and monotonically increases for $\eta > \tilde{\eta}$. On the other hand, obviously we have $\hat{\eta}^{(t)} \leq \hat{\eta}^{(t+1)} \leq \tilde{\eta}$ when $\hat{\eta}^{(t)} \leq \tilde{\eta}$, and $\tilde{\eta} \leq \hat{\eta}^{(t+1)} \leq \hat{\eta}^{(t)}$ when $\eta^{(t)} > \tilde{\eta}$. That immediately implies:

$$
h(\hat{\mathbf{B}}^{(t+1)}, \hat{\eta}^{(t+1)} | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_{\mathbf{C}}^{(t)}) \leq h(\hat{\mathbf{B}}^{(t+1)}, \hat{\eta}^{(t)} | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_{\mathbf{C}}^{(t)}).
\tag{3–62}
$$

Using (3–62) and the fact that $h(\mathbf{B}, \eta) | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_{\mathbf{C}}^{(t)})$ majorizes $g(\mathbf{B}, \eta)$ (see (3–53)), we immediately have:

$$
\begin{aligned}
g(\mathbf{B}^{(t+1)}, \eta^{t+1}) &\leq h(\hat{\mathbf{B}}^{(t+1)}, \hat{\eta}^{(t+1)} | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_{\mathbf{C}}^{(t)}) \\
&\leq h(\hat{\mathbf{B}}^{(t+1)}, \hat{\eta}^{(t)} | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_{\mathbf{C}}^{(t)}) \\
&\leq h(\hat{\mathbf{B}}^{(t)}, \hat{\eta}^{(t)} | \hat{\mathbf{B}}^{(t)}, \hat{\mathbf{x}}_{\mathbf{C}}^{(t)}) = g(\mathbf{B}^{(t)}, \hat{\eta}^{(t)}),
\end{aligned}
\tag{3–63}
$$

i.e., the cost function $g(\mathbf{B}, \eta)$ is monotonically decreasing. The GS-SLIM algorithm is summarized below.

The GS-SLIM Algorithm:

- **Initialization:** Set an initial value for η, let $\hat{\mathbf{x}}_{\mathbf{m}} = 0$, and apply 2-D FFT to $\mathbf{x}$ to get an initial estimate of $\mathbf{B}$.
  Iteration: Repeat the following steps:

- **Step 1** - Compute $\mathbf{P} = |\hat{\mathbf{B}}|^{(2-q)}$.

- **Step 2** - Apply 2-D FFT to **P** to compute $\mathbf{\Sigma_C}$ in (3–57) and (3–58).

- **Step 3** - Compute $\mathbf{y} = \mathbf{\Sigma_C^{-1}}\hat{\mathbf{x}}_\mathbf{C}$ using the generalized GS algorithm.

- **Step 4** - Update $\text{vec}(\hat{\mathbf{B}}) = \text{vec}(\mathbf{P}) \odot \mathcal{F}_{2d}(\mathbf{y})$.

- **Step 5** - Compute $\tilde{\mathbf{x}}_\mathbf{C} = \mathcal{F}_{2d}^{-1}(\text{vec}(\mathbf{B}))$.

- **Step 6** - Update $\hat{\eta} = \rho\tilde{\eta} + (1-\rho)\hat{\eta}$ with $\tilde{\eta} = \frac{1}{M}\|\hat{\mathbf{x}}_\mathbf{C} - \tilde{\mathbf{x}}_\mathbf{C}\|^2$.

- **Step 7** - Update missing data estimate $\hat{\mathbf{x}}_\mathbf{m} = \mathbf{J_m}\tilde{\mathbf{x}}_\mathbf{C}$ and the associated complete data $\hat{\mathbf{x}}_\mathbf{C}$.

3.3 Numerical Examples

We present below several 1-D spectral estimation and 2-D interrupted SAR imaging examples to demonstrate the performance of the missing data spectral estimation algorithms and their fast implementations.

3.3.1 1-D Spectral Estimation

To demonstrate the performance improvements of the new missing data spectral analysis approaches over some existing ones, we first consider a simple example in Figure 3-1. The complete data has 128 samples, and contains 8 complex sinusoids. The frequencies and amplitudes of the sinusoids are indicated by circles in Figure 3-1. The data is corrupted by a zero-mean circularly symmetric complex white Gaussian noise with variance $\sigma_0^2 = 0.001$. In Figures 3-1(a)-3-1(e), we provide the spectra estimates obtained by windowed FFT (WFFT), APES/MAPES, CoSaMP, IAA and SLIM, respectively. A Taylor window with a -30 dB sidelobe level is used for WFFT. The iteration numbers for all iterative approaches, including MAPES, CoSaMP, IAA, and SLIM, are fixed to 20. (In our examples, we did not find significant performance improvement after 20 iterations for all algorithms.) We choose the user parameter of CoSaMP to be 32. For SLIM, we choose $q = 1$. The subplots at the left, middle and right columns in Figure 3-1 show the results obtained from the complete data, 50% and 30% data, respectively. The available data samples are selected randomly. For Figure 3-1(b), we use APES for the complete data case, and MAPES for the two missing data cases.

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From Figure 3-1(a), it is clear that WFFT suffers from the low resolution and high sidelobe level problems. The sidelobe level increases significantly in the missing data case. Due to the high sidelobe levels, several targets fail to be detected. In Figure 3-1(b), APES provides a good spectral estimation performance for the complete data case. Its missing data variant, i.e., MAPES, behaves well when 50% of the data samples are available. However, its performance degrades significantly when the available data percentage decreases to 30%. Moreover, MAPES is computationally very intensive. The computation times needed by MAPES, when implemented on an ordinary personal computer (Intel Xeon CPU at 2.67 GHz; 12 GB RAM), for the two missing-data examples in Figure 3-1(b) are 969 and 847 seconds, respectively, while the computation times needed by all the other algorithms are less than 1 second. We can also see in Figure 3-1(c) that CoSaMP produces many false peaks in the missing data cases. From Figures 3-1(d) and 3-1(e), we note that both IAA and SLIM provide good spectral estimation performance under various conditions. All of the 8 sinusoids can be identified clearly even with only 30% data samples available. We remark that the amplitude estimates of SLIM are significantly biased downward. However, with the correctly identified sinusoidal number and frequencies, the amplitude estimates can be refined readily using, for example, the standard least-squares (LS) method or the RELAX algorithm [62].

We next consider a more challenging problem of analyzing a long data sequence where 800 out of 1000 data samples are available for spectral estimation. The data sequence contains 100 complex-valued sinusoids, and is corrupted by a zero-mean circularly symmetric complex white Gaussian noise with variance $\sigma_0^2 = 0.001$. Figures 3-2(a)-3-2(f) show the spectral estimates (zoomed-in to show details within the $[0.21, 0.24]$ Hz band) obtained by WFFT, IAA, SLIM, Fast-IAA, CG-SLIM, and GS-SLIM, respectively. Again, the data-adaptive algorithms outperform the conventional data-independent WFFT approach significantly. The performance of the fast implementations
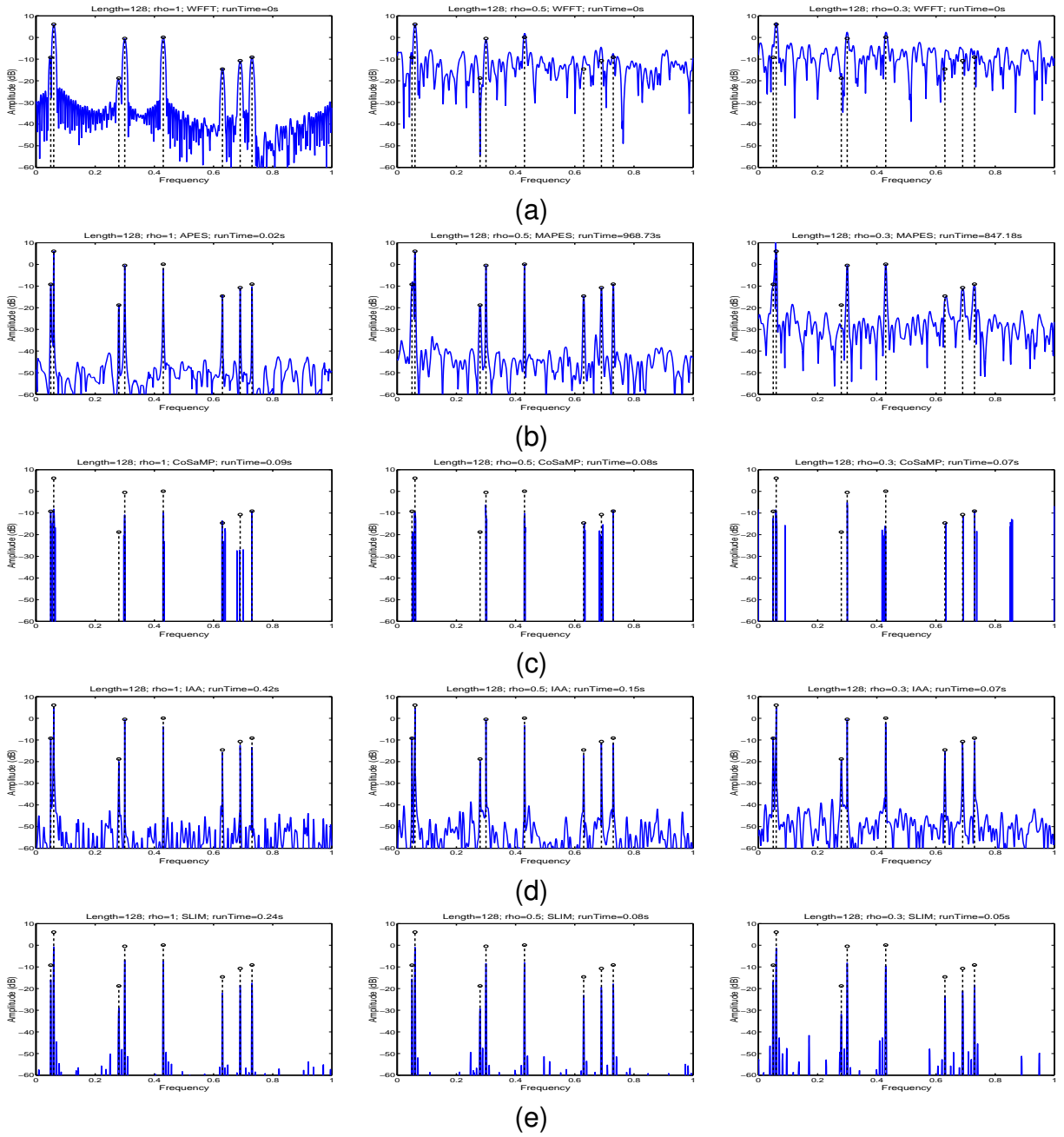
Figure 3-1. Nonparametric spectral estimation without and with missing samples, where the complete data length is 128. The subplots in the left, middle and right columns are obtained with the complete data, 50% data, and 30% data, respectively. A) WFFT B) MAPES C) CoSaMP D) IAA E) SLIM

Figure 3-2. Nonparametric spectral estimates for data sequences with missing samples when 800 out of 1000 data samples are available for spectral analysis. A) WFFT B) IAA C) Fast-IAA D) SLIM E) CG-SLIM F) GS-SLIM

of IAA and SLIM are very close to those of their original algorithms. From Figure 3-2, we see again that the amplitude estimates of SLIM and its variants are biased downward, while IAA and its variant provide rather accurate amplitude estimates.

The missing data samples can be recovered by simply applying FFT (see Equation (3–48)) to the spectral estimates of SLIM or its fast implementations. The missing-data IAA (MIAA) algorithm in [63] can also be used to recover the missing samples from the IAA or Fast-IAA spectra estimates, which, however, is more complicated both conceptually and computationally than its SLIM counterpart. Figures 3-3(a) and 3-3(b)
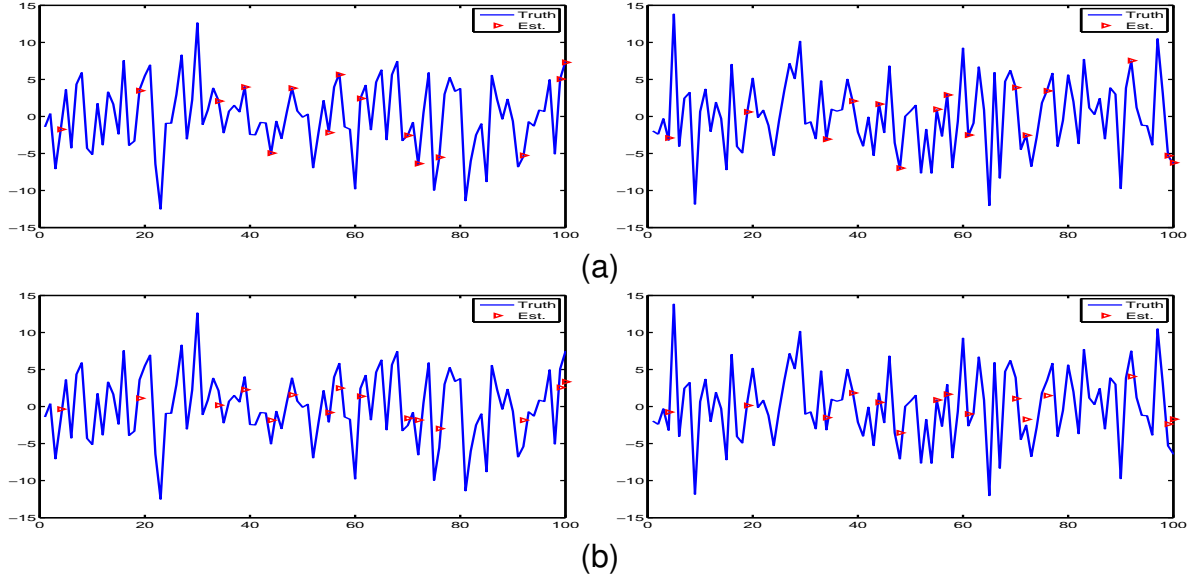
Figure 3-3. Missing samples estimation by SLIM, the subplots in the left and right columns represent the real and imaginary parts of the data samples, respectively. A) CG-SLIM B) GS-SLIM

show the missing samples estimated by CG-SLIM and GS-SLIM, respectively. As we can see, the missing samples can be estimated quite accurately. To mitigate the biased amplitude estimation problem of CG-SLIM and GS-SLIM, we can apply the efficient complete-data IAA algorithm discussed in Section 3.2.2, i.e., Fast-IAA-GS, to the estimated missing samples obtained using the SLIM variants, as well as the given samples. We refer to these two new spectral estimation procedures (i.e., CG-SLIM followed by Fast-IAA-GS, and GS-SLIM followed by Fast-IAA-GS) as CG-SLIM-IAA and GS-SLIM-IAA, respectively. Note that due to the usage of the GS-type formula, Fast-IAA-GS is computationally quite efficient. Furthermore, the Fast-IAA-GS step can be initialized by using the CG-SLIM or GS-SLIM spectral estimate to reduce the needed number of iterations. Figures 3-4(a) and 3-4(b) show the spectral estimates obtained by CG-SLIM-IAA and GS-SLIM-IAA, respectively. The Fast-IAA-GS iteration number is set to be 5 for both algorithms. From Figures 3-4(a) and 3-4(b), we can see that CG-SLIM-IAA and GS-SLIM-IAA provide more accurate amplitude estimates than CG-SLIM and GS-SLIM. In particular, comparing Figures 3-2(c) and 3-4(a), we see

that CG-SLIM-IAA provides a similar spectral estimate as Fast-IAA. However, as we will show below, CG-SLIM-IAA is computationally much more efficient than Fast-IAA.
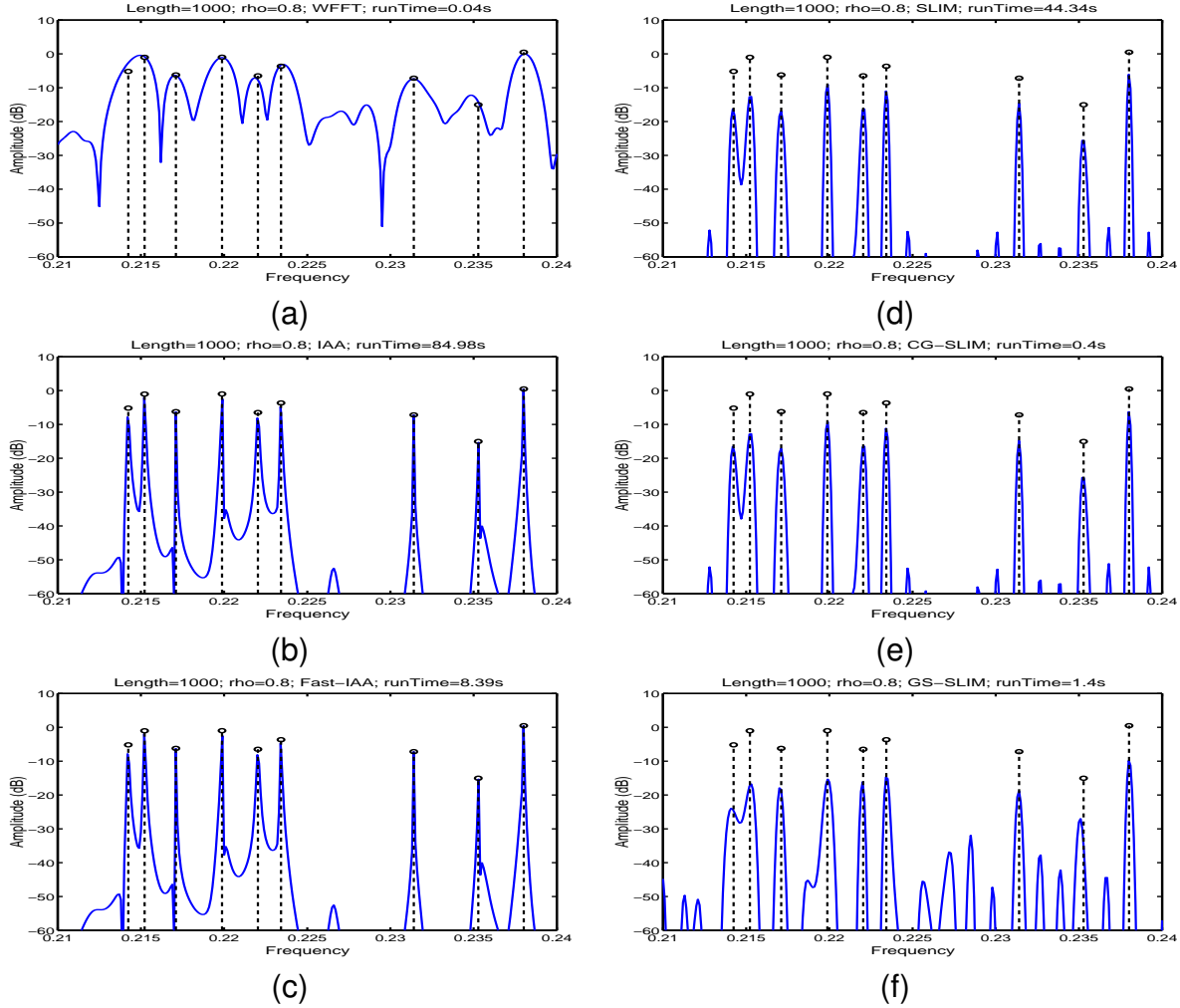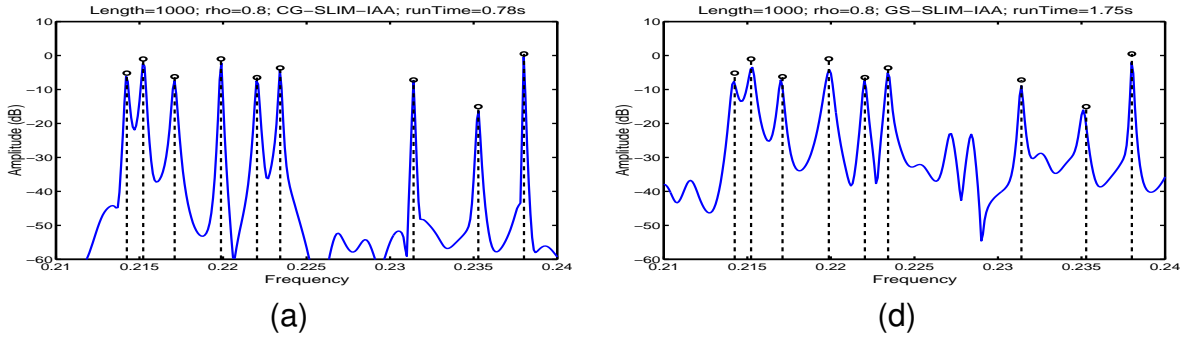

Figure 3-4. Nonparametric spectral estimates for data sequences with missing samples when 800 out of 1000 data samples are available for spectral analysis. A) CG-SLIM-IAA B) GS-SLIM-IAA

The computation times needed by the 1-D spectral estimation methods to obtain Figures 3-2 and 3-4 are shown in Table 3-1. Clearly, the proposed fast implementations of IAA and SLIM are computationally much more efficient than their original counterparts. In particular, the proposed CG-SLIM and GS-SLIM algorithms reduce the computation times needed by SLIM from 44.34 seconds to 0.4 and 1.4 seconds, respectively. The computational efficiencies of CG-SLIM-IAA and GS-SLIM-IAA are close to those of CG-SLIM and GS-SLIM, respectively. In particular, CG-SLIM-IAA needs only 0.78 seconds, which amount to only 9.3% of the computation time needed by Fast-IAA (8.39 seconds, while providing a similar spectral estimation performance to Fast-IAA. Although for this 1-D spectral estimation example, CG-SLIM is computationally more efficient than GS-SLIM, as we will show in Section 3.3.2, GS-SLIM can be computationally more efficient than CG-SLIM for 2-D interrupted SAR imaging.

Table 3-1. Computation times needed by IAA, SLIM and their fast implementations.

Alg.	WFFT	IAA	SLIM	Fast-IAA	CG-SLIM	GS-SLIM	CG-SLIM-IAA	GS-SLIM-IAA
Time (s)	0.04	84.98	44.34	8.39	0.4	1.4	0.78	1.75

3.3.2 2-D Interrupted SAR Imaging

In the following examples, we illustrate the applications of the missing-data spectral analysis algorithms to interrupted SAR imaging using incomplete phase history data.

Figure 3-5. Slicy object and benchmark SAR image A) Photograph of the object (taken at $45°$ azimuth angle). B) benchmark SAR image formed with a 288×288 (not 40×40) data matrix.

We use the two-dimensional phase-history data of an object called Slicy generated at $0°$ azimuth angle using XPATCH [64], a high frequency electromagnetic scattering prediction code for complex 3-D objects. A photo of the Slicy object taken at $45°$ azimuth angle and a SAR image benchmark obtained from a complete 288×288 data matrix are shown in Figures 3-5(a) and 3-5(b), respectively. Below, we use only a 40×40 center block of the phase-history data for SAR imaging.

Figures 3-6 - 3-8 show the SAR images obtained from the 40×40 complete data, and two randomly interrupted data. The available data patterns for the two interruption cases are shown in Figures 3-7(a) and 3-8(a), where the available data ratios are 68% and 30%, respectively. We provide the SAR images obtained by IAA, SLIM, and their fast variants, i.e., Fast-IAA/Fast-IAA-GS, CG-SLIM and GS-SLIM, as well as WFFT and CoSaMP. Note that we use Fast-IAA-GS for the complete data case in Figure 3-6, and Fast-IAA for the two missing data cases in Figures 3-7 and 3-8. Again, the iteration numbers for all iterative approaches, including CoSaMP, IAA, and SLIM, are all fixed to

10. We set the CoSaMP user parameter to be 3000, the SLIM parameter $q = 1$, and $\epsilon_{CG} = 10^{-6}$ for CG-SLIM. The noise variance estimate η of SLIM is initialized using (3–20).

Figures 3-6(b), 3-7(b) and 3-8(b) show the SAR images formed by WFFT, where a 2-D Taylor window with a -30 dB sidelobe level is applied. Again, WFFT has low resolution and high sidelobe problems, especially for the interrupted scenarios. The object features are smeared under the interrupted data conditions. We can also see that the CoSaMP approach fails to work properly under all conditions. IAA, SLIM, and their fast variants, i.e., Fast-IAA, CG-SLIM and GS-SLIM, significantly improve the interrupted SAR imaging quality, in terms of both higher imaging resolution and lower sidelobe levels, as evidenced by sharper SAR images with less ghost artifacts. In this example, SLIM and its variants provide the best imaging performance, as compared to the benchmark in Figure 3-5 for all interruption conditions. Note that only 0.56% of the data samples was used to form the SAR images in Figure 3-8, compared to the entire 288×288 data samples in Figure 3-5.

Table 3-2 provides the computation times needed by the aforementioned algorithms under various interruption conditions. Again, the proposed fast implementation approaches have significantly enhanced computational efficiencies. In particular, for the interruption example in Figure 3-7, GS-SLIM needs only 2.4 seconds, which amount to only 0.4% of the computation time needed by SLIM (584.1 seconds).

Table 3-2. Computation times needed by IAA, SLIM and their fast implementations for interrupted SAR imaging under various interruption conditions.

Alg.	WFFT	CoSaMP	IAA	SLIM	Fast-IAA-GS	CG-SLIM	GS-SLIM
Fig. 3-6 (s)	0.01	173.0	11933	9260	2.63	7.08	1.87
Fig. 3-7 (s)	0.32	192.5	1119	584	12.90	6.50	2.40
Fig. 3-8 (s)	0.03	163.7	240	131	3.80	4.10	1.80

We have considered using data-adaptive methods, namely IAA and SLIM, for spectral estimation of data sequences with missing samples. Both methods can provide significantly improved spectral estimates, in terms of spectral resolution and lower
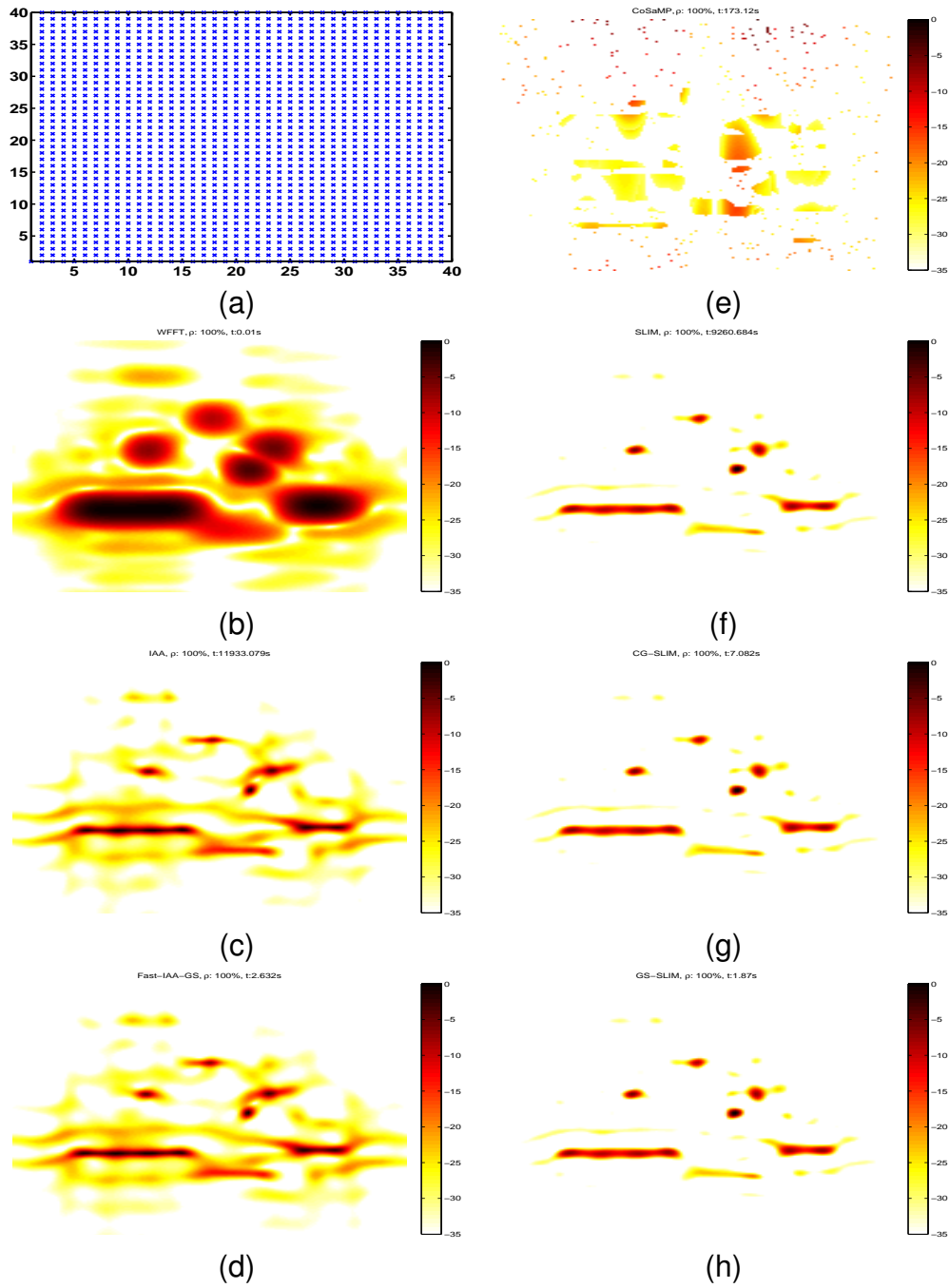
Figure 3-6. Modulus of the SAR images of the Slicy object obtained from a 40×40 complete data matrix. A) Available data pattern. B) WFFT C) IAA D) Fast-IAA-GS E) CoSaMP F) SLIM G) CG-SLIM H) GS-SLIM
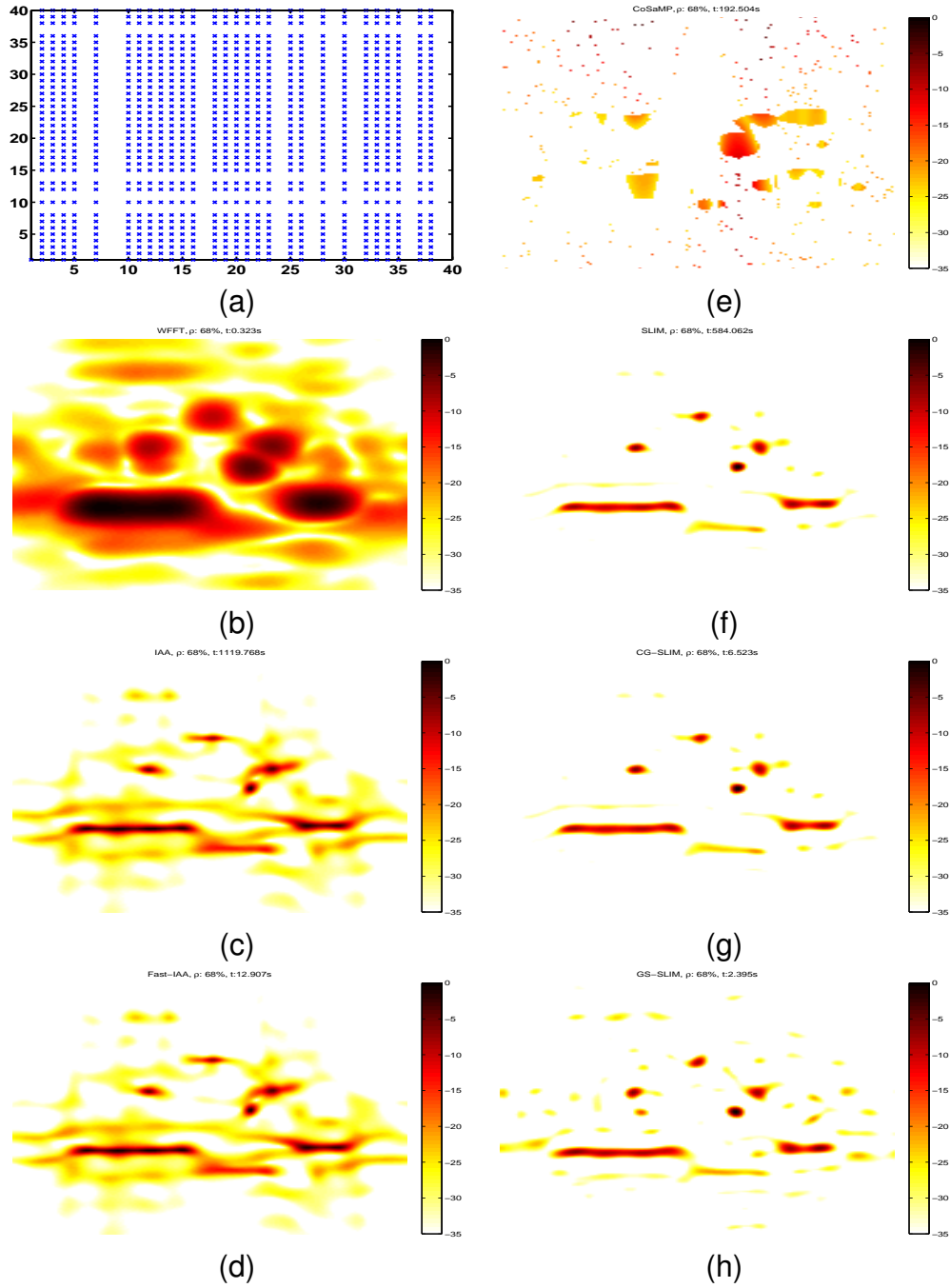
Figure 3-7. Modulus of the interrupted SAR images of the Slicy object under a random interruption condition (68% data available compared to Figure 3-6). A) Available data pattern. B) WFFT C) IAA D) Fast-IAA E) CoSaMP F) SLIM G) CG-SLIM H) GS-SLIM
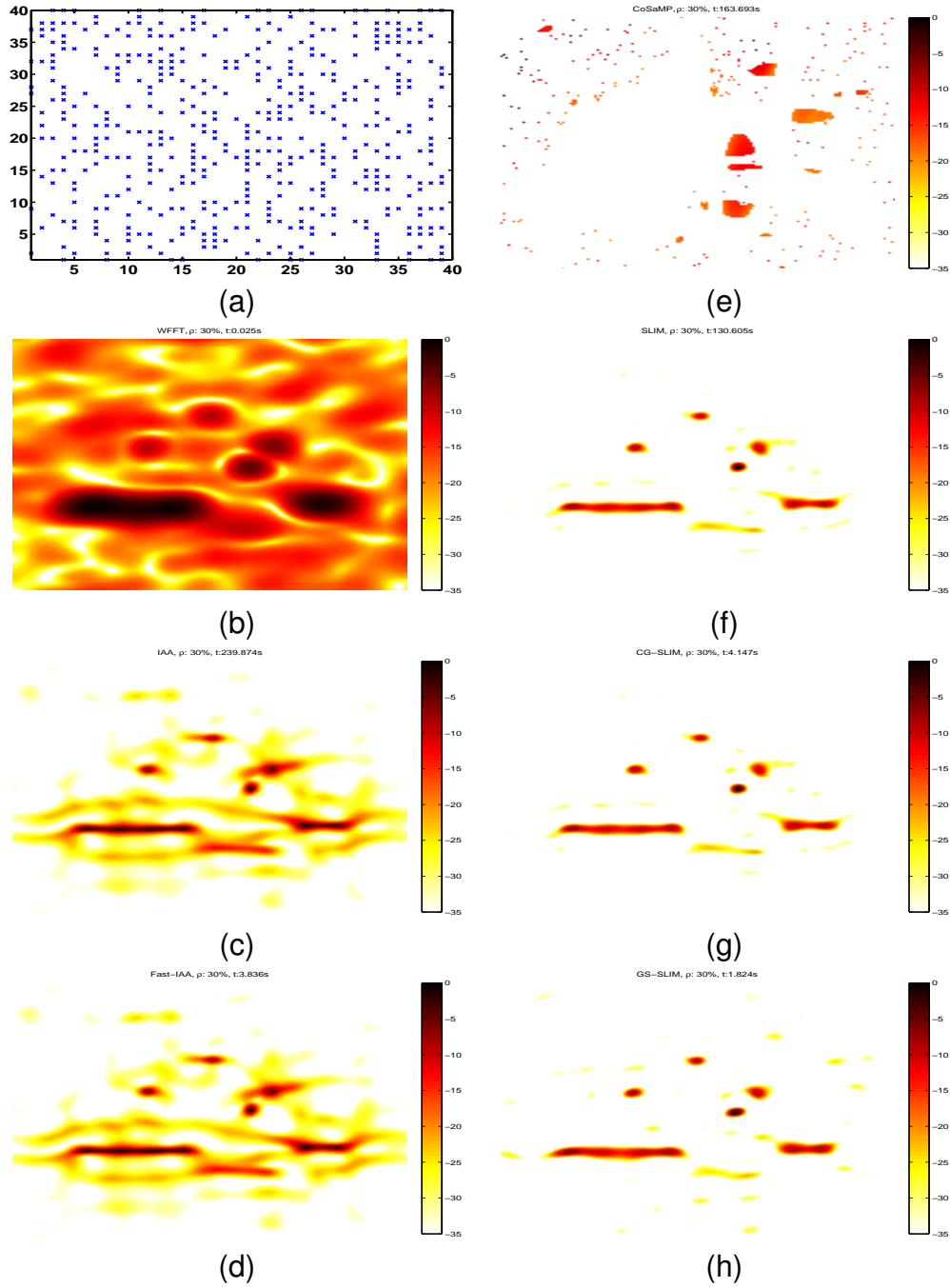
Figure 3-8. Modulus of the interrupted SAR images of the Slicy object under a random interruption condition (30% data available compared to Figure 3-6). A) Available data pattern. B) WFFT C) IAA D) Fast-IAA E) CoSaMP F) SLIM G) CG-SLIM H) GS-SLIM

sidelobe levels, as compared to the conventional data-independent approaches. We have introduced several fast implementations of these algorithms, namely Fast-IAA, CG-SLIM and GS-SLIM, by exploiting the structures of the steering matrices and maximizing the utility of FFTs. Via several numerical examples of both 1-D spectral analysis and 2-D interrupted SAR imaging, we have demonstrate the effectiveness of IAA and SLIM for spectral estimation and the computational efficiencies of their fast implementations.

CHAPTER 4
SAR GROUND MOVING TARGET INDICATION

Synthetic aperture radar (SAR) imaging has been widely used in many civilian and military applications [12, 65]. Since SAR was originally developed for imaging the stationary scene, a moving target is usually misplaced due to the motion-induced Doppler-shift in the SAR image [66–69]. The goal of SAR-based ground moving target indication (GMTI) is twofold: detection of moving targets in the SAR images, and parameter estimation, including velocity and location determination of the moving targets. This problem is rather challenging when the moving targets are slow and small and are buried in strong stationary ground clutter.

GMTI techniques have been developed for most of the recent history of radar systems. GMTI can be performed incoherently with a single channel SAR [70, 71], or coherently with a two- or multi-channel SAR [72–75]. Techniques such as displaced phase center antenna (DPCA) and along-track interferometry (ATI) have been widely used in the surveillance community [74, 76–78]. The traditional data-independent DPCA and ATI investigations are mostly limited to two-channel SAR. Multi-channel DPCA and ATI have been presented recently [79–81]. However, they may fail to work properly for challenging GMTI problems, such as for small moving target detection in the presence of strong stationary clutter. With the availability of increased computational power, adaptive SAR based GMTI can make up for the shortcomings of conventional SAR based GMTI methods. Via adaptively forming angle-Doppler images, moving targets can be detected effectively from ground clutter via space-time adaptive processing (STAP) [82, 83]. STAP can significantly improve the low-velocity and small target detection capability in the presence of strong clutter and jamming interferences. However, it is well-known that conventional STAP has a high computational requirement [82, 83]. Moreover, it needs a large amount of target-free homogeneous secondary data, which is hardly available in practice [82, 83]. An l_1-norm based method, which belongs to the category

of compressive sensing (CS) algorithms, is used for STAP in [84]. However, the CS algorithms usually require user-parameters, which are not easy to tune, and have rather high computational cost.

In this chapter, new moving target indication techniques for multi-channel airborne SAR are presented. Via adaptively forming the velocity versus cross-range images for each range bin of interest, small moving targets can be detected effectively even in the presence of strong stationary ground clutter. An iterative adaptive approach (IAA) [25, 55] is used to form velocity versus cross-range images for each range bin of interest. This weighted least squares based IAA algorithm is robust and user parameter free. IAA can provide better performance than DPCA and ATI, while having much lower computational requirement than conventional STAP and obviating the need of homogeneous target-free secondary data. The performances of the proposed algorithms are demonstrated using the Air Force Research Laboratory (AFRL) Gotcha airborne SAR based GMTI data set [85].

The remainder of this chapter is organized as follows. In Section 4.1, we introduce the SAR imaging geometry and the data model of this SAR based GMTI problem. We then discuss the need for array calibrations in Section 4.2. We present our approach to both calibrate the antenna gains and determine the array inter-element spacings. Stationary clutter cancelation, target detection and velocity estimation algorithms are presented in Section 4.3. In Section 4.4, we discuss the AFRL Gotcha GMTI data set and present several examples to demonstrate the performance of the proposed algorithms. Finally, concluding remarks are given.

4.1 Geometry and Data Model

We consider the 3-D geometry of an airborne SAR system as shown in Figure 4-1. The airplane moves with a velocity v_p along the y direction. There are M receive channels placed along the flight path, and the distance between the mth antenna and the first antenna is denoted as L_m. Hence $L_1 = 0$. Assume that a moving target in the kth
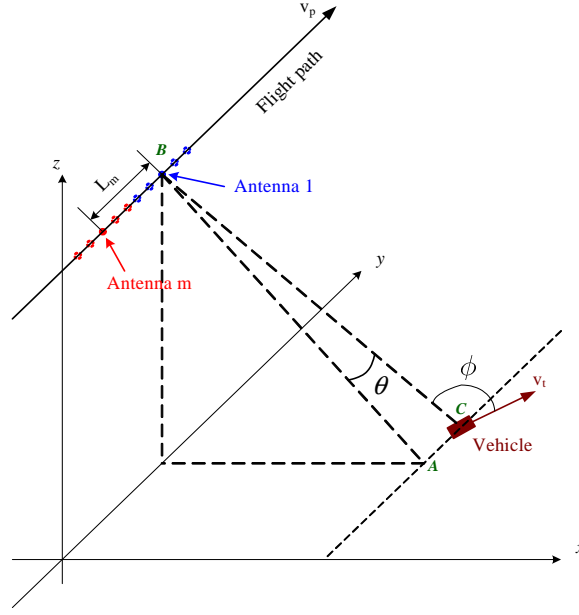
Figure 4-1. Geometry of an airbore multi-channel SAR system.

range bin with radial velocity v_t is at the angle θ relative to the normal of the synthetic aperture, as shown in Figure 4-1. The angle between the radial velocity of the target and the slant range line BC in Figure 4-1 is denoted as ϕ. Then, after range compression, the received data due to the moving target for the mth antenna and the nth probing pulse can be modeled as:

$$y_m(k, n) = g_m b e^{j2\pi\left[\frac{2v_p}{\lambda f_{PRF}}\sin(\theta) + \frac{2v_t}{\lambda f_{PRF}}\cos(\phi)\right]n} e^{j2\pi\frac{L_m}{\lambda}\sin(\theta)}, \tag{4-1}$$

$$m = 1, 2, \ldots, M, \quad k = 1, 2, \ldots, K, \quad n = 1, 2, \ldots, N, \tag{4-2}$$

where g_m is the antenna gain of the mth antenna relative to the first antenna (and hence $g_1 = 1$), b is the complex-valued response from the moving target, λ is the wavelength of the radar, and f_{PRF} is the pulse repetition frequency (PRF). In (4–2), $e^{j2\pi\frac{2v_p}{\lambda f_{PRF}}n\sin(\theta)}$ denotes the phase delay due to the motion of the platform, $e^{j2\pi\frac{2v_t}{\lambda f_{PRF}}n\cos(\phi)}$ denotes the phase delay due to the motion of the moving target, and $e^{j2\pi\frac{L_m}{\lambda}\sin(\theta)}$ denotes the phase delay due to the antenna distance L_m. By letting

$$f_d = \frac{2v_p}{\lambda f_{PRF}}, \tag{4-3}$$

$$f_v = \frac{2v_t}{\lambda f_{\text{PRF}}} \cos(\phi), \tag{4–4}$$

and

$$l_m = \frac{L_m}{\lambda}, \tag{4–5}$$

we have

$$y_m(k, n) = g_m b e^{j2\pi[f_d \sin(\theta) + f_v]n} e^{j2\pi l_m \sin(\theta)}. \tag{4–6}$$

Let

$$f_t = f_d \sin(\theta) + f_v \tag{4–7}$$

denote the total Doppler caused by both platform and target motions, and let

$$\eta_m = \frac{l_m}{f_d}. \tag{4–8}$$

Then we can rewrite $y_m(k, n)$ as

$$y_m(k, n) = g_m b e^{j2\pi f_t(n + \eta_m)} e^{-j2\pi f_v \eta_m}. \tag{4–9}$$

Note that we have $f_v = 0$ for the stationary ground clutter, η_m can be uniformly or non-uniformly spaced depending on the array geometry, and $\eta_1 = L_1/f_d = 0$.

For each m, applying the discrete-time Fourier transform along n to (4–9), we get the SAR image for the mth antenna:

$$z_m(k, s) = g_m b e^{-j2\pi f_v \eta_m} e^{j2\pi f_s \eta_m} \gamma(f_s - f_t), \quad s = 1, 2, \ldots, N, \tag{4–10}$$

where $f_s = (s - 1)/N$ is the normalized Doppler frequency for the sth Doppler bin (or cross-range bin), and

$$\gamma(f_s - f_t) = \frac{\sin[\pi N(f_s - f_t)]}{\sin[\pi(f_s - f_t)]} e^{-j(N-1)\pi(f_s - f_t)}. \tag{4–11}$$

When the distances and relative gains among the antennas are known, the gains and the second phase shift term in Equation (4–10) can be compensated out as follows:

$$\tilde{z}_m(k,s) = \frac{e^{-j2\pi f_s \eta_m}}{g_m} z_m(k,s) = b e^{-j2\pi f_v \eta_m} \gamma(f_s - f_t). \tag{4–12}$$

Because $f_v = 0$ for the stationary ground clutter, the data model in the presence of the stationary ground clutter and noise can be written as:

$$\tilde{z}_m(k,s) = b e^{-j2\pi f_v \eta_m} \gamma(f_s - f_t) + \sum_{s'} c_{s'} \gamma(f_s - f_{s'}^C) + e_m(f_s), \tag{4–13}$$

where $f_{s'}^C$ is the Doppler shift of the stationary or ground clutter, $c_{s'}$ is the complex-valued response of the clutter in the s'^{th} Doppler bin, and $e_m(f)$ is the noise term. Stacking $\{\tilde{z}_m(k,s)\}_{m=1}^M$ into a vector $\tilde{\mathbf{z}}(k,s)$, we have:

$$\tilde{\mathbf{z}}(k,s) = \mathbf{a}(f_v) b \gamma(f_s - f_t) + \mathbf{a}(0) \sum_{s'} c_{s'} \gamma(f_s - f_{s'}^C) + \mathbf{e}(f_s), \tag{4–14}$$

where

$$\tilde{\mathbf{z}}(k,s) = [\ \tilde{z}_1(k,s) \quad \tilde{z}_2(k,s) \quad \cdots \quad \tilde{z}_M(k,s) \ ]^T, \tag{4–15}$$

and

$$\mathbf{a}(f_v) = [\ 1 \quad e^{-j2\pi f_v \eta_2} \quad \cdots \quad e^{-j2\pi f_v \eta_M} \ ]^T, \tag{4–16}$$

is the steering vector.

Our problem of interest is to detect the moving target and estimate its velocity from $\tilde{\mathbf{z}}(k,s)$.

4.2 Array Calibration

For applications that rely on accurate phase histories, array calibration is crucial [86, 87]. For our data model, this means having correct values for the antenna inter-element spacing, L_m, and the individual antenna gain, g_m. Even with prior knowledge of the array geometry and the antenna specifications, array calibration is still necessary. Depending

on the physical array configurations and platform geometry, in-situ antenna performance is never the same as its free-space counterparts.

4.2.1   Distances Among Antennas

The antenna inter-element spacing may be unknown in practice. We consider estimating the spacings using the Rank-1 method [86] in this subsection. Consider the estimation of the distance between Antennas 1 and 2. For a given Doppler bin s, we have (in the absence of noise):

$$\mathbf{z}_{1,2}(k,s) = \begin{bmatrix} z_1(k,s) \\ z_2(k,s) \end{bmatrix} = \begin{bmatrix} 1 \\ g_2 e^{j\pi \frac{s-1}{N} \frac{L_2}{D}} \end{bmatrix} z_1(k,s),$$

$$k = 1, \cdots, K, \qquad (4\text{--}17)$$

where $D = v_p/f_{\text{PRF}}$ is the distance the radar traveled within a pulse repetition interval (PRI). In the derivation of (4–17), we dropped the term $e^{j2\pi f_v \eta_m}$ in (4–10) by assuming $f_v = 0$, since the target power is negligible compared to the stationary ground clutter power. For each Doppler bin s, the ideal (noiseless) covariance matrix for $\mathbf{z}_{1,2}(k,s)$ is

$$\begin{aligned} \mathbf{R}_{1,2}(s) &= E\{\mathbf{z}_{1,2}(k,s)\mathbf{z}_{1,2}^H(k,s)\} \\ &= p(s)\mathbf{a}_{1,2}(s)\mathbf{a}_{1,2}^H(s), \quad s = 1, \cdots, N, \qquad (4\text{--}18) \end{aligned}$$

where $p(s) = \sum_{k=1}^{K} |z_1(k,s)|^2$ is the power of $z_1(k,s)$ ($k = 1, \cdots, K$), and

$$\mathbf{a}_{1,2}(s) = \begin{bmatrix} 1 & g_2 e^{j\pi \frac{s-1}{N} \frac{L_2}{D}} \end{bmatrix}^T \qquad (4\text{--}19)$$

is the steering vector. Note that, $\mathbf{a}_{1,2}(s)\mathbf{a}_{1,2}^H(s)$ is a 2×2 Rank-1 matrix. Note also that the steering vector contains the variable L_2. The steering vector can be estimated from the eigen-decomposition of the covariance matrix $\mathbf{R}_{1,2}(s)$, which is usually replaced by the sample covariance matrix [88] in practice. Then the antenna separation can be determined from the estimated steering vector. More details can be found in references

[86]. This process can be repeated for each antenna element to determine all of the inter-element distances of the array.

4.2.2 Antenna Gains

To estimate the gains of the individual antenna element (relative to the first antenna), we follow a similar regime. With the estimated antenna distances, the phase delay due to this distance can be compensated out. We assume a constant gain across all Doppler bins, but a varying gain as a function of range bin. We found that this assumption helps improve the GMTI performance for our applications. For a fixed range bin k ($k = 1, \cdots, K$), we consider the following data model (in the absence of noise):

$$
\begin{aligned}
\bar{\mathbf{z}}(k, s) &= \begin{bmatrix} z_1(k, s) \\ z_2(k, s) \cdot e^{-j\pi \frac{s-1}{N} \frac{L_2}{D}} \\ \vdots \\ z_M(k, s) \cdot e^{-j\pi \frac{s-1}{N} \frac{L_M}{D}} \end{bmatrix} \\
&= \begin{bmatrix} 1 \\ g_2(k) \\ \vdots \\ g_M(k) \end{bmatrix} z_1(k, s), \quad s = 1, \cdots, N,
\end{aligned}
\tag{4--20}
$$

where M is the number of array elements. The ideal (noiseless) covariance matrix is

$$
\begin{aligned}
\bar{\mathbf{R}}(k) &= E\{\bar{\mathbf{z}}(k, s)\bar{\mathbf{z}}^H(k, s)\} \\
&= \bar{p}(k)\bar{\mathbf{a}}(k)\bar{\mathbf{a}}^H(k), \quad k = 1, \cdots, K,
\end{aligned}
\tag{4--21}
$$

where $\bar{p}(k) = \sum_{s=1}^{N} |z_1(k, s)|^2$ is the power of $z_1(k, s)$ ($s = 1, \cdots, N$), and

$$
\bar{\mathbf{a}}(k) = [1 \ g_2(k) \ \cdots \ g_M(k)]^T
\tag{4--22}
$$

is the steering vector at the kth range bin. Note that, $\bar{\mathbf{a}}(k)\bar{\mathbf{a}}^H(k)$ is also a Rank-1 matrix. Again, the sample covariance matrix $\hat{\bar{\mathbf{R}}}(k)$ is used to replace the covariance matrix $\bar{\mathbf{R}}(k)$ in practice, where

$$\hat{\bar{\mathbf{R}}}(k) = \frac{1}{N}\sum_{s=1}^{N}\bar{\mathbf{z}}(k,s)\bar{\mathbf{z}}^H(k,s), \quad k = 1, \cdots, K. \tag{4-23}$$

Let

$$\hat{\bar{\mathbf{R}}}(k) = \sum_{i=1}^{M}\bar{\lambda}_i(k)\bar{\mathbf{e}}_i(k)\bar{\mathbf{e}}_i^H(k) \tag{4-24}$$

denote the eigen-decomposition of $\hat{\bar{\mathbf{R}}}(k)$, and let

$$\bar{\mathbf{e}}_1(k) = [\bar{e}_{11}(k), \ \bar{e}_{12}(k), \ \cdots, \ \bar{e}_{1M}(k)]^T \tag{4-25}$$

be the eigenvector corresponding to the largest eigenvalue. Then the estimation of the steering vector $\bar{\mathbf{a}}(k)$ can be written as

$$\hat{\bar{\mathbf{a}}}(k) = \left[1 \ \frac{\bar{e}_{12}(k)}{\bar{e}_{11}(k)} \ \cdots \ \frac{\bar{e}_{1M}(k)}{\bar{e}_{11}(k)}\right]^T, \tag{4-26}$$

and the mth antenna gain at the range bin k is estimated as:

$$\hat{g}_m(k) = \frac{\bar{e}_{1m}(k)}{\bar{e}_{11}(k)}, \quad m = 2, \cdots, M. \tag{4-27}$$

4.3   Ground Moving Target Indication (GMTI)

After antenna calibration, our goal is to detect the moving target and to estimate f_v (the Doppler shift induced by the motion of the moving target) and f_t (the total Doppler shift which includes both the location information in the cross-range dimension and the velocity) of the moving target. Once we have f_v and f_t, the true location f_d of the moving target in the Doppler dimension can be calculated by using Equation (4–7). We estimate the value of b at all possible f_v values for each pixel in the SAR image domain. The peak values and the corresponding peak locations of these images can be used to indicate

the presence of the moving target and the associated target parameter estimates, $\hat{f}_v$ and $\hat{f}_t$, of f_v and f_t.

4.3.1   Ground Clutter Cancelation Using RELAX

The detection of weak moving targets in urban environments can be challenging due to the strong response of the stationary ground clutter such as buildings, trees, and other reflective structures. This problem is exacerbated by the Doppler induced displacement of the moving targets onto existing strong stationary ground return, such as return from a building. The removal of stationary ground clutter would then improve the probability of correctly separating the moving targets from the background.

We consider the following stationary clutter rejection approach to address this challenging problem by using the RELAX algorithm [62, 89]. By taking the stationary ground clutter and a moving target into consideration, the data model in Equation (4–14) can be rewritten as:

$$\tilde{\mathbf{z}} = \mathbf{a}(f_v)\tilde{b} + \mathbf{a}(0)\tilde{c} + \tilde{\mathbf{e}}, \tag{4–28}$$

where $\tilde{\mathbf{z}} = \tilde{\mathbf{z}}(k, s)$, $\tilde{b} = b\gamma(f_s - f_t)$ and $\tilde{c} = \sum_{s'} c_{s'}\gamma(f_s - f_{s'}^c)$ are, respectively, the complex-valued responses of the moving targets and stationary clutter at the given normalized Doppler frequency f, and $\tilde{\mathbf{e}} = \mathbf{e}(f_s)$ is the noise term. For notational convenience, we have omitted the dependence of $\tilde{\mathbf{y}}$, $\tilde{b}$, $\tilde{c}$, and $\tilde{\mathbf{e}}$ on k and s.

By applying the nonlinear least-squares (NLS) criterion, $\tilde{c}$ as well as the $\tilde{b}$ and f_v can be estimated by minimizing the following cost function:

$$F[\tilde{c}, \tilde{b}, f_v] = \left\| \tilde{\mathbf{z}} - \mathbf{a}(f_v)\tilde{b} - \mathbf{a}(0)\tilde{c} \right\|^2, \tag{4–29}$$

where $\| \cdot \|$ denotes the Frobenius norm. The optimization problem can be solved by using the RELAX algorithm [62, 89] as follows.

For a given estimated $\hat{\tilde{c}}$ (the mean value of $\tilde{\mathbf{z}}$ can be used as the initial value of $\hat{\tilde{c}}$ in practice), let

$$\tilde{\mathbf{z}}_b = \tilde{\mathbf{z}} - \mathbf{a}(0)\hat{\tilde{c}}. \tag{4–30}$$

Then, the cost function of (4–29) can be simplified as

$$F_b[\tilde{b}, f_v] = \left\| \tilde{z}_b^H - \mathbf{a}(f_v)\tilde{b} \right\|^2 . \tag{4–31}$$

Minimizing (4–31) with respect to $\tilde{b}$ and f_v yields:

$$\hat{\tilde{b}} = \left. \frac{\mathbf{a}^H(f_v)\tilde{z}_b}{M} \right|_{f_v=\hat{f}_v}, \tag{4–32}$$

where M is the antenna number, and

$$\begin{aligned}
\hat{f}_v &= \arg\min_{f_v} \left\| \left[\mathbf{I} - \frac{\mathbf{a}(f_v)\mathbf{a}^H(f_v)}{N} \right] \tilde{z}_b \right\|^2 \\
&= \arg\max_{f_v} |\mathbf{a}^H(f_v)\tilde{z}_b|^2 .
\end{aligned} \tag{4–33}$$

The cost function in (4–33) can be computed by applying DTFT (discrete-time Fourier transform) to $\tilde{z}_b$. On the other hand, given the estimated $\hat{f}_v$ and $\hat{\tilde{b}}$, $\tilde{c}$ can be re-estimated by minimizing the cost function

$$F_c[\tilde{c}] = \left\| \tilde{\mathbf{y}}_c - \mathbf{a}(0)\tilde{c} \right\|^2 , \tag{4–34}$$

where

$$\tilde{z}_c = \tilde{z} - \mathbf{a}(\hat{f}_v)\hat{\tilde{b}}. \tag{4–35}$$

Solving this optimization problem yields:

$$\hat{\tilde{c}} = \frac{\mathbf{a}^H(0)\tilde{z}_c}{M} . \tag{4–36}$$

Therefore, the original NLS problem can be solved via iterating (4–32), (4–33) and (4–36) until convergence. The steps of the RELAX algorithm are illustrated below.

The RELAX Algorithm:

- **Initialize** $\hat{\tilde{c}} = \frac{\mathbf{a}^H(0)\tilde{z}}{M}$

- **Iterate:**

    - Step 1:
        $$\tilde{z}_b = \tilde{z} - \mathbf{a}(0)\hat{\tilde{c}}$$

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$$\hat{f}_v = \arg\max_{f_v} |\mathbf{a}^H(f_v)\tilde{\mathbf{z}}_b|^2$$

$$\hat{\tilde{b}} = \left.\frac{\mathbf{a}^H(f_v)\tilde{\mathbf{z}}_b}{M}\right|_{f_v = \hat{f}_v}$$

– Step 2:

$$\tilde{\mathbf{z}}_c = \tilde{\mathbf{z}} - \mathbf{a}(\hat{f}_v)\hat{\tilde{b}}$$

$$\hat{\tilde{c}} = \frac{\mathbf{a}^H(0)\tilde{\mathbf{z}}_c}{M}$$

- **until** convergence or a certain number of iterations is reached

After $\hat{\tilde{c}}$ is estimated, the stationary clutter can be removed using Equation (4–30).

In this RELAX algorithm, we have fixed the steering vector of the stationary clutter as $\mathbf{a}(0)$. In practice, because of various errors and perturbations, the true steering vector of the ground clutter may not be exactly $\mathbf{a}(0)$. RELAX can still be used to remove a significant portion of the ground clutter as long as its steering vector is close to $\mathbf{a}(0)$. After using RELAX to remove a major portion of the stationary ground clutter, we use the robust non-parametric IAA algorithm for moving target detection and target velocity estimation.

4.3.2 Moving Target Detection Using IAA

Since we have used RELAX to estimate the intensity of the stationary clutter return, we can subtract it out from $\tilde{\mathbf{z}}$, as shown in Equation (4–30), to get

$$\tilde{\mathbf{z}}_b = \mathbf{a}(f_v)\tilde{b} + \check{\mathbf{e}}, \tag{4–37}$$

where $\check{\mathbf{e}}$ includes the residual of the ground clutter and noise.

The goal now is to detect moving targets. The recently proposed IAA algorithm [25, 55] is considered herein to make an estimate of $\tilde{\mathbf{b}} = [\tilde{b}_1, \cdots, \tilde{b}_N]^T$, where $\tilde{b}_i = \tilde{b}(f_{vi})$ ($i = 1, \cdots, N$) are the responses at different potential velocities, and f_{vi} ($i = 1, \cdots, N$) are scanning points of f_v. IAA has also been presented in Chapter 2, interested readers should follow the reference for a detailed development of IAA. Let

$$\mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_N] \tag{4–38}$$

denote the steering matrix, where $\mathbf{a}_i = \mathbf{a}(f_{vi})$, we have the following compact data model:

$$\tilde{\mathbf{z}}_b = \mathbf{A}\tilde{\mathbf{b}} + \breve{\mathbf{e}}. \tag{4–39}$$

IAA solves for $\tilde{\mathbf{b}}$ by using a weighted least squares approach, which involves the minimization of the following cost function,

$$\min_{\tilde{b}_i} \|\tilde{\mathbf{z}}_b - \tilde{b}_i \mathbf{a}_i\|_{\tilde{\mathbf{Q}}_i^{-1}}^2, \qquad i = 1, \cdots, N, \tag{4–40}$$

where $\|\mathbf{u}\|_{\tilde{\mathbf{Q}}^{-1}}^2 \triangleq \mathbf{u}^H \tilde{\mathbf{Q}}^{-1}\mathbf{u}$. In Equation (5–21), the interference and noise covariance matrix $\tilde{\mathbf{Q}}_i$ is given by:

$$\tilde{\mathbf{Q}}_i = \tilde{\mathbf{R}} - |\tilde{b}_i|^2 \mathbf{a}_i \mathbf{a}_i^H, \tag{4–41}$$

where

$$\tilde{\mathbf{R}} = \sum_{i=1}^{N} |\tilde{b}_i|^2 \mathbf{a}_i \mathbf{a}_i^H \tag{4–42}$$

is the IAA covariance matrix. By letting $\tilde{\mathbf{P}}$ denote an $N \times N$ diagonal matrix whose diagonal contains the signal power as $\tilde{p}_i = |\tilde{b}_i|^2$, we can rewrite Equation (5–23) in the following matrix form:

$$\tilde{\mathbf{R}} = \mathbf{A}\tilde{\mathbf{P}}\mathbf{A}^H. \tag{4–43}$$

The minimization of Equation (5–21) with respect to $\tilde{b}_i$ gives:

$$\tilde{b}_i = \frac{\mathbf{a}_i^H \tilde{\mathbf{Q}}_i^{-1} \tilde{\mathbf{z}}_b}{\mathbf{a}_i^H \tilde{\mathbf{Q}}_i^{-1} \mathbf{a}_i}. \tag{4–44}$$

Using the definition of $\tilde{\mathbf{Q}}_i$ and the matrix inversion lemma, Equation (5–25) can be written as [25]:

$$\tilde{b}_i = \frac{\mathbf{a}_i^H \tilde{\mathbf{R}}^{-1} \tilde{\mathbf{z}}_b}{\mathbf{a}_i^H \tilde{\mathbf{R}}^{-1} \mathbf{a}_i}, \tag{4–45}$$

which avoids the computation of $\tilde{\mathbf{Q}}_i^{-1}$ for each $i = 1, \cdots, N$, and significantly saves the computation time. Since the *a priori* knowledge of $\tilde{\mathbf{P}}$ is required in Equation (5–24) to calculate the covariance matrix $\tilde{\mathbf{R}}$, IAA needs to be implemented iteratively. Usually, IAA is initialized with the periodogram.

Once $\tilde{\mathbf{b}}$ is estimated by IAA, the moving target will be detected based on the peak value of $\hat{\tilde{\mathbf{b}}}$, the estimate of $\tilde{\mathbf{b}}$. The elements, whose f_v are near zero, inside $\hat{\tilde{\mathbf{b}}}$ will be removed from $\hat{\tilde{\mathbf{b}}}$ to further reduce the effect of the residue stationary ground clutter. Then the peak value of $\hat{\tilde{\mathbf{b}}}$ is compared with a threshold, and a moving target will be indicated if the peak value is greater than the threshold. Moreover, the $\hat{f}_v$ corresponding to the peak value of $\hat{\tilde{\mathbf{b}}}$ can be used to estimate the radial velocity of the target. The radial velocity of the moving target can be calculated from (4–4) as:

$$v_t = \frac{\lambda f_{\text{PRF}}}{2 \cos(\phi)} \hat{f}_v \quad (\text{meters/second}). \tag{4–46}$$

Note that ϕ can be determined from the flight information of the platform when collecting the data.

The flow chart of the entire processing chain for the proposed adaptive SAR based GMTI is given in Figure 4-2.
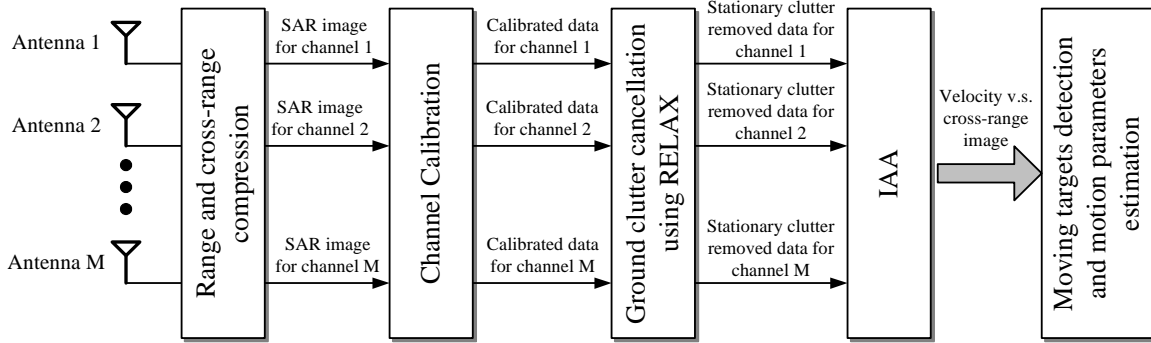
Figure 4-2. The flow chart of the entire processing chain for the proposed adaptive SAR based GMTI.

4.4 Analysis of the AFRL GOTCHA Data Set

4.4.1 Description of the AFRL Gotcha GMTI Data Set

To demonstrate the effectiveness of our approach, we apply it to the AFRL Gotcha open data set [85]. This data set is an airborne X-band SAR data collected in an urban environment. Three phase-history sets, all with HH polarization corresponding to three different channels, are contained in this data set. The carrier frequency used in this

system is 9.6 GHz, the probing signal is a linear frequency modulated (LFM) signal with a bandwidth of 640 MHz, and the PRF is 2171.6 Hz.
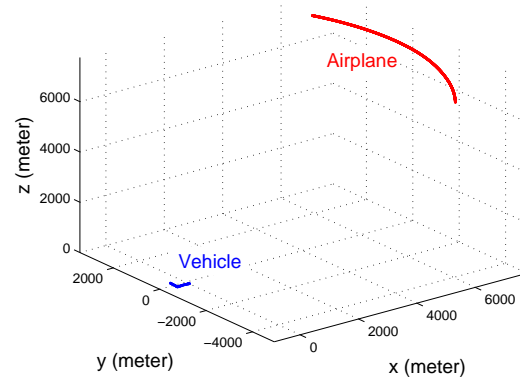
Figure 4-3. AFRL Gotcha data scene setup. A) 3D view. B) 2D view.

The data collection duration is about 71 seconds. There are multiple vehicles moving on the roads near buildings in the scene 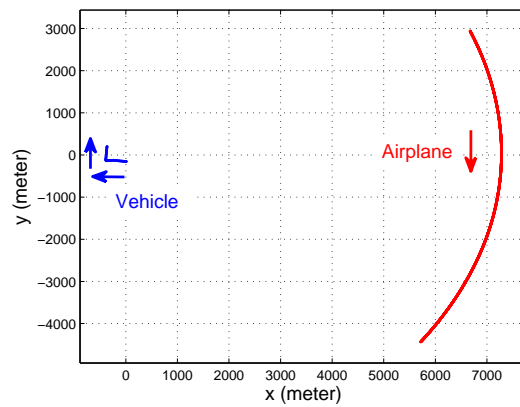under interrogation. The ground truth of one of the moving vehicles is provided in the data set. The locations of the radar platform and the vehicle of interest, which is referred to as the target vehicle, are given in the auxiliary data of the AFRL data set. The tracks of the radar platform and the target vehicle during the 71-second duration are plotted in Figure 4-3. The vehicle is moving away from the radar platform, which induces a negative Doppler-shift. There are a total of 5400 original range bins in the data set, but only 384 range bins, range-gated

to center around the target vehicle, are provided in the data set. The diagram of the range-gating is shown in Figure 2 of the reference [85].

4.4.2 SAR Imaging

We begin the data processing by forming the SAR images. Due to the truncation of the data set for distribution purposes, the 384 range bins given by this data set are centered around the moving target vehicle. These range bins are not aligned, as shown in Figure 3 of the reference [85]. Fortunately, the offset value for each probing pulse was given in the data set. So the first step after we load the data is to shift the range bins and align them. A simple 2D FFT of the aligned data can give a reasonable SAR image, as shown in Figure 4-4(a) at the 46th second for a CPI of 1 second for one of the 3 channels. However, the image is smeared because of the high sidelobe problem of FFT. To reduce the sidelobes and hence smearing, before the 2D FFT, a 2D Taylor window with parameter 4 and -80 dB sidelobe level is applied to the data, followed by zero padding. The resulting SAR image with Taylor windowing and zero padding is shown in Figure 4-4(b). However, the range migration induced by the target motion will cause coupling between the range and cross-range dimensions and this smears the image of the moving target. The Keystone transform method [90, 91] is used to reformat the SAR data, correct the range migration, and image the moving target. The resulting SAR image is shown in Figure 4-4(c), with the imaging quality visibly enhanced.

4.4.3 Array Calibration

The Gotcha SAR based GMTI system has three antennas, but unfortunately, the information about the antenna array is not provided in the data set. So we calibrated the array by estimating the distances among the antennas and their corresponding gains (relative to the first antenna) from the data set using the approach presented in Section 4.2. The estimated distances between antennas for the entire 71 seconds are shown in Figure 4-5. It is shown in this figure that the distance estimates are stable and almost
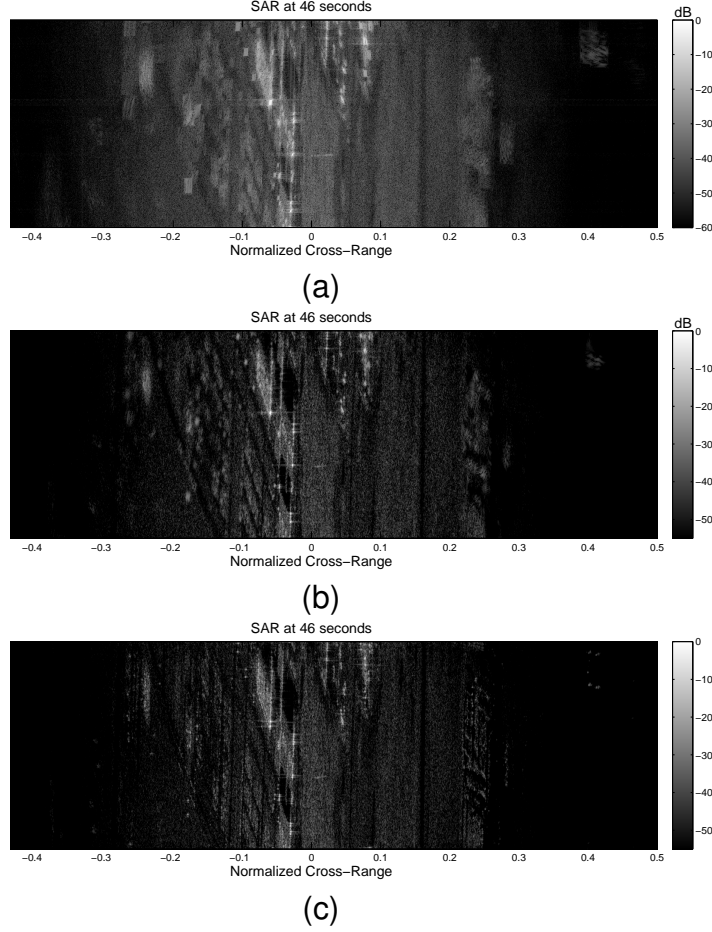
Figure 4-4. SAR images using data at the 46th second. A) 2D FFT only. B) 2D FFT with 2D Taylor window and zero padding. C) 2D FFT with 2D Taylor window, zero padding and Keystone reformatting.

constant over time. From Figure 4-5, the estimated distance between Antennas 1 and 2 is around 0.238 meters, and between Antennas 1 and 3 is 0.476 meters.

4.4.4 Velocity Ambiguity Analysis

Equations (4–46) shows that for a given f_v, the estimated target velocity is dependent on the wavelength and the PRF. Table 4-1 shows various f_v values and the corresponding velocities for the AFRL Gotcha data set, where the wavelength is $\lambda = 0.0312$ meters, and the PRF is $f_{\mathrm{PRF}} = 2171.6$ Hz.

However, because of the large inter-antenna spacing, velocity ambiguity problems exist. To better understand the ambiguity problem, we consider the spectral window

Figure 4-5. Estimated distance between Antennas.

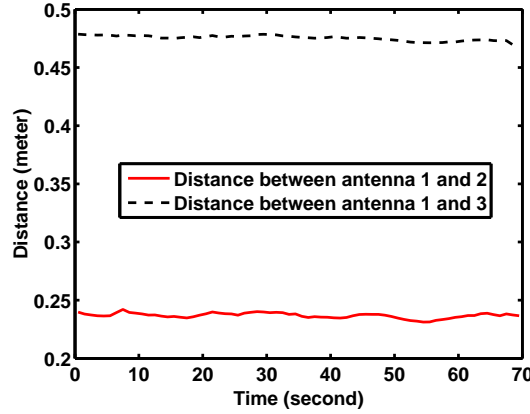| f_v	Velocity (meters/second)	Velocity (miles/hour)
0.1	3.3925	7.6332
0.2	6.7850	15.2664
0.3	10.1776	22.8995
0.4	13.5701	30.5327
0.5	16.9626	38.1659
0.6	20.3551	45.7991
0.7	23.7477	53.4323
0.8	27.1402	61.0654
0.9	30.5327	68.6986
1.0	33.9252	76.3318

Table 4-1. Target velocity versus f_v.

defined as [92]

$$W(f) = \left| \sum_{m=1}^{M} e^{j2\pi f \eta_m} \right|^2. \tag{4–47}$$

For the AFRL Gotcha GMTI data set at the 46th second, we have $\eta_1 = 0$, $\eta_2 = 4.952$, and $\eta_3 = 9.904$. The spectral window with $f \in [-0.5, 0.5]$ is plotted in Figure 4-6. It is clearly shown that the peak repetition occurs at a period of around 0.2. We define the maximal frequency without ambiguity as $f_{\max}$, which means that there are no ambiguity within $f \in [-f_{\max}, f_{\max}]$. From Figure 4-6, we find that $f_{\max} = 0.101$, which corresponds to the vehicle velocity of about 3.425 meters/second (based on Table 4-1), which is far below a normal vehicle speed on the road. For a fixed number of receive channels, the inter-antenna spacing is constrained by two opposing factors. On the one hand, large
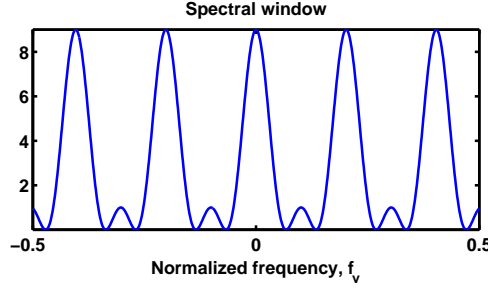
100

Spectral window

Figure 4-6. Spectral windows for the array geometry of the AFRL Gotcha GMTI system.

inter-antenna spacing increases vehicle velocity estimation accuracy and resolution as a result of longer aperture. On the other hand, to avoid velocity ambiguity problems, the inter-antenna spacing should be small.

The above analysis shows that because of the limitations of the AFRL Gotcha GMTI data set, the unambiguous velocity range is much smaller than the velocity range of a normal moving vehicle. We will suggest later on some methods to resolve the ambiguity problem based on scene context.

4.4.5   Adaptive GMTI

The center range bin of the AFRL Gotcha GMTI data set at the 46th second is used to evaluate our approach first, because the ground truth of the target vehicle is known. For comparison purposes, the results of DPCA and ATI are also presented. Because DPCA and ATI investigations so far are limited to two-channel SAR, only the data of Antennas 1 and 2 are used for all methods. At the 46th second, the vehicle of interest, a Durango, is located at the center range bin, and the ground truth of the vehicle is known. This vehicle is moving away from the radar platform, which induces a negative Doppler-shift. For all the SAR images in this section, we use circles to indicate the uncorrected locations of the detected moving targets. The multiple potential corrected locations (caused by the ambiguity problem discussed above) of the moving targets are marked by triangles "$\triangle$" and "∇", with the directions of the triangles indicating the leaving or closing of the targets.

We plot the DPCA output in the complex plane in Figure 4-7(a), and the SAR image with detected targets in Figure 4-7(b). We see that the traditional DPCA fails to identify the mover and falsely identify two non-movers. The output of ATI is given in Figure 4-8(a), and the corresponding SAR image with detected targets in Figure 4-8(b). The ATI technique is able to find the mover, but also gives two false alarms. The GMTI results of the proposed adaptive approach are given in Figure 4-9. Figure 4-9(a) shows the velocity versus normalized cross-range image of IAA. In this image, the locations and values of the peaks indicate the presence of the moving target and the associated target parameter estimates. The SAR image with the detected moving targets is shown in Figure 4-9(b). It can be seen that the mover was identified correctly without any false alarm by using the adaptive approach. However, the mover velocity cannot be uniquely determined due to the velocity ambiguity problem caused by the large inter-antenna spacing. As shown in Figure 4-9(a), there are more than one peak in this velocity versus normalized cross-range image. We next attempt to find the correct peak location of the target vehicle by shifting the target vehicle to its potential original locations based on the estimated possible velocities. The potential original locations are discarded if they are outside the range $[-0.25, 0.25]$, which is the range of the normalized cross-range of the SAR image. The results are shown in Figure 4-9(c), where two potential original locations of the vehicle are denoted by the triangle, and the true location of the vehicle is also given in this SAR image. The right triangle matches the true location and is close to a road. Based on the scene context of the SAR image, we can discard the left triangle because it is not close to a road [93].

In the second example, we consider the scenario at the 46th second again, but data from all three antennas at all range bins are processed by IAA. The SAR image with IAA detected moving targets is shown in Figure 4-10. Multiple targets, including the vehicle of interest, are detected in this scenario. We next attempt to find the correct peak locations of the moving targets by shifting the moving targets to their potential original
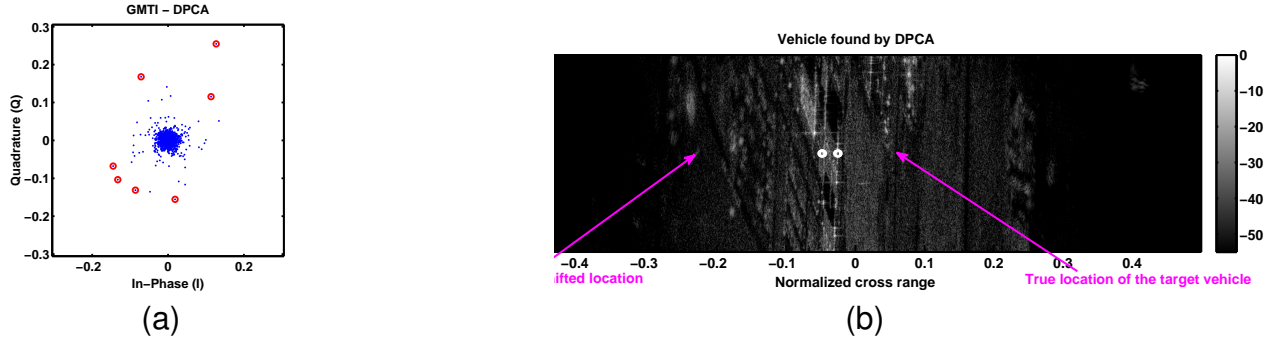
Figure 4-7. GMTI results of DPCA at the 46th second. "○" indicates the detected moving targets by DPCA, with ground truth overlay. A) DPCA detection. B) Detected moving targets in the SAR image.
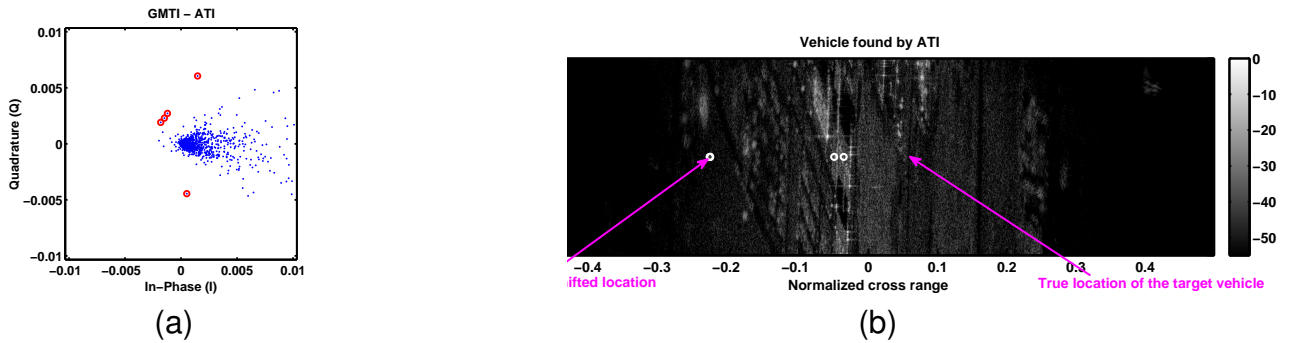

Figure 4-8. GMTI results of ATI at the 46th second. "○" indicates the detected moving targets by ATI, with ground truth overlay. A) ATI detection. B) Detected moving targets in the SAR image.

locations based on the estimated possible velocities. The target velocity and location cannot be determined uniquely due to the ambiguities caused by the large inter-antenna spacing [92, 93]. Again, the potential estimated locations are discarded if they are outside the range $[-0.25, 0.25]$, which is the range of the normalized cross-range of the SAR image. The potential estimated locations of the moving targets and the ground truth of the moving vehicle of interest, the Durango, are also indicated in Figure 4-10. The left box indicates the Durango's Doppler-shifted location in the image, and the right box indicates the Durango's actual location in the image. The figure shows that the moving vehicle of interest is detected correctly. From the auxiliary data of the Gotcha data set, we know that the Durango is moving away from the airplane at a rate of 13.95 meters/second, this translates to a radial velocity of 9.772 meters/second. The velocity
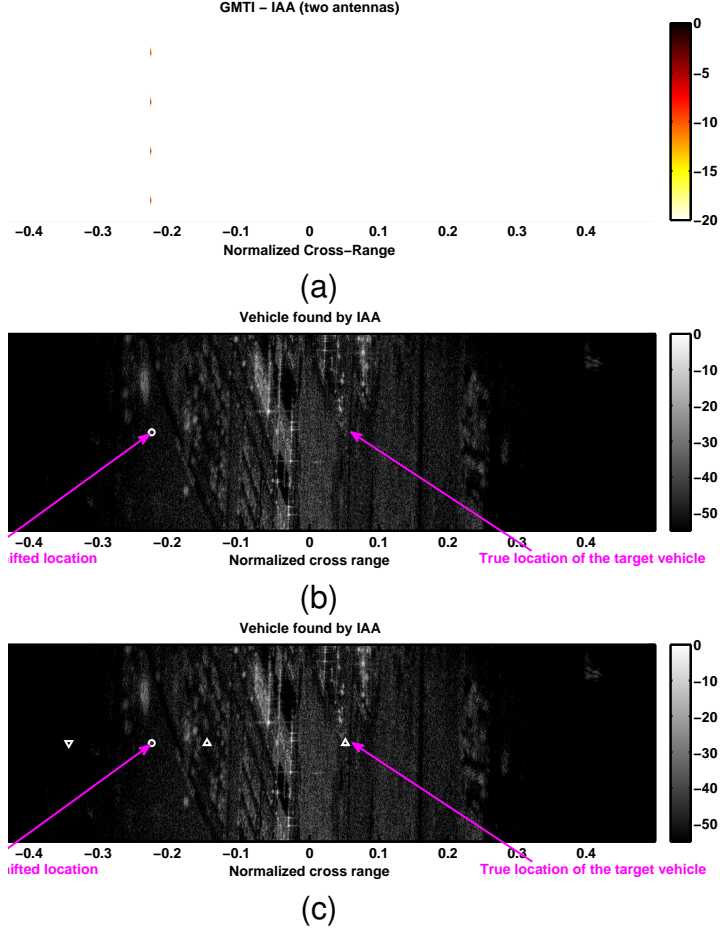
Figure 4-9. GMTI results of IAA at the 46ᵗʰ second. "○" indicates the detected moving targets by IAA, and "△" and "▽" indicate the estimated original locations of the moving target, with ground truth overlay. A) Velocity versus normalized cross-range image of IAA. B) Detected moving targets in the SAR image. C) Estimated potential locations of the moving target.

estimated by IAA is 9.824 meters/second, which matches the true velocity very well.

We comment in passing that, with data from three channels, we can also perform STAP for GMTI. However, the computation of STAP involves estimating the covariance matrix for the data vector of length MN ($3 \times 2172 = 6516$ in this example) and computing its inversion. In contrast, the proposed algorithm here only requires solving N times the problem with data dimension M, which is computationally much more efficient.

The IAA results at the 51ˢᵗ second and the 68ᵗʰ second are shown in Figures 4-11(a) and 4-11(b), respectively. These two cases are more challenging. The Doppler-shifted
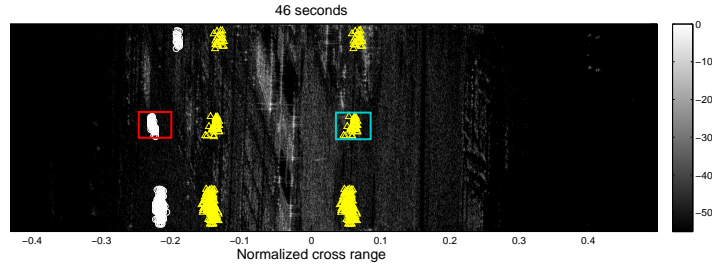
Figure 4-10. GMTI results using IAA with all three antennas at the 46th second. "○" indicates the detected moving target, and "△" and "∇" indicate the estimated original locations of the moving target. Right (blue) and left (red) boxes indicate the actual target location and the Doppler shifted location of the vehicle Durango, respectively.

(a)

(b)
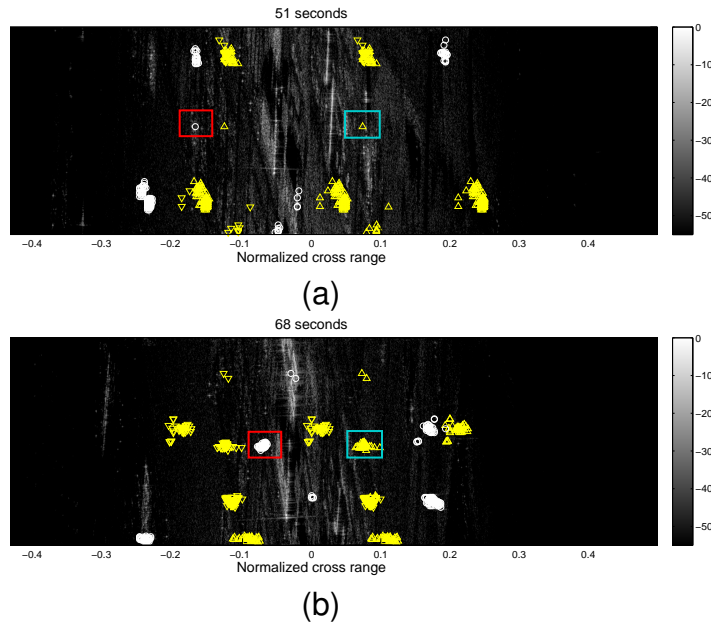
Figure 4-11. GMTI results using IAA with all three antennas. "○" indicates the detected moving target, and "△" and "∇" indicate the estimated original locations of the moving target. Right (blue) and left (red) boxes indicate the actual target location and the Doppler shifted location of the vehicle Durango, respectively. A) Result at the 51st second. B) Result at the 68th second.

image of the moving target of interest is mixed with a strong stationary clutter at the 51st second, and the road is downhill at the 68th second. The figures show that our IAA algorithm can detect the moving target and estimate its true location accurately even for these challenging scenarios.

We have presented a framework for adaptive SAR GMTI for an airborne platform. By combining a clutter canceling technique using RELAX, and an adaptive technique for Doppler spectral estimation using IAA, we are able to outperform existing data-independent techniques including DPCA and ATI. This framework works beyond the customized DPCA or ATI radar architecture, since it provides an accurately estimated target velocity, works with more than two channels, and is able to detect moving targets in the presence of strong stationary urban clutter. We have demonstrated the effectiveness of this framework using the publicly available AFRL Gotcha GMTI data set, and our GMTI results match the ground truth provided in the data set. We have also presented techniques for array calibration, which work well even when the array hardware specifications are completely unknown.

CHAPTER 5
MULTIPLE-INPUT MULTIPLE-OUTPUT SAR GMTI

Ground Moving Target Indication (GMTI) mode is an essential part of any real-time surveillance radar system, since it provides target movement information to the users, giving hints as to the overall intent of the ground targets. Detection of ground moving targets is a tricky endeavour, unlike the general MTI mode, which is used mainly to detect airborne objects. GMTI has to deal with the challenging effects of ground clutter. In urban environments, ground returns from buildings and other stationary structures can significantly drown out returns from small moving targets. Furthermore, since the surveillance platform is airborne and travels at a given velocity this causes the stationary ground structures to exhibit Doppler shifts that are functions of the platform velocity and their respective cross-range locations. These Doppler shifts complicate moving target detections as they may overlap with the Doppler shifts of the targets of interests.

In this chapter we propose a new GMTI framework for the MIMO radar system architecture by leveraging the advantages of a MIMO system [94–97] and the advances in MIMO waveform design [98] for enhanced ground clutter suppression. We demonstrate the MIMO radar potential via several numerical examples, comparing its performance to its SIMO counterpart. We also discuss existing GMTI methods, and motivate as to why they necessitate an alternative.

5.1   Existing GMTI Methods

Current methods in detection of ground moving targets include Displaced Phase Center Antenna (DPCA) and Along-Track Interferometry (ATI). These systems require two dedicated antennas placed in a pre-defined configuration. In DPCA, the intensity is compared between the first and second channels, while in ATI the phase is being compared between them. Since the same scene is sampled by the two channels at two different times, stationary targets will yield the same returns across both channels, while moving targets will have different returns. DPCA and ATI are computationally efficient,

but they are not data adaptive and cannot effectively suppress strong clutter. More importantly, for radar systems with more than 2 channels, they do not make the most of the collected data.

For systems with more than 2 channels, one solution is to use the Space-Time Adaptive Processing (STAP) [99, 100] technique. STAP is suitable for adaptive ground clutter suppression, but due to its high computational complexity and its requirement of a clutter-and-noise covariance matrix,its usage in a realistic GMTI scenario.

We propose a new GMTI method that utilizes more than 2 channels effectively, and since it operates on SAR phase history data, it can be used simulatneously for both SAR imaging and GMTI. Our method also takes advantage of the MIMO radar waveform diversity for enhanced GMTI performance. In order to illustrate the effectiveness of the proposed GMTI method, we first develop a return signal model of our MIMO radar system and then we explain further the steps necessary to detect the movers from the set of return signals.
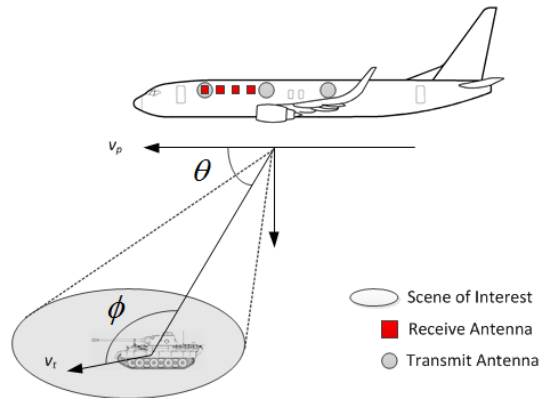
5.2   MIMO GMTI System Model

Figure 5-1. Illustration of ground moving target indication (GMTI) using a multiple-input and multiple-output (MIMO) radar system.

To facilitate our discussions, we develop an airborne MIMO radar system model. In this section we describe the various components of this model, and the physics involved

in generating and receiving the transmitted signals. There are various components to this model and we will break them down as follows.

5.2.1 Scene of Interest

As shown in Figure 5-1, we consider an airborne radar system attached to a flying platform. Let v_p denote the moving velocity of the radar platform. We assume that the imaging area is far away from the radar platform, and hence the return signals are plane waves. Let θ and τ be the target angle and the round-trip time delay between the radar and the target, respectively. The moving speed of the target is v_t, and the angle between the target moving direction and signal arrival direction is ϕ.

5.2.2 Antenna Array and Transmission Waveforms

We consider an airborne radar system equipped with two uniform linear arrays for transmission and reception. The configuration of the antennas are special, by having a sparse transmit array, and a dense receive array, in the MIMO case, a larger dense virtual array can be achieved with good waveforms. The transmitting array has N antenna elements with the inter-element spacing being d_t, and the receiving array has M antenna elements with the inter-element spacing being d_r.

Let $s_n(t)$, with $n = 0,\ 1,\ \cdots,\ N-1$ and $t = 0,\ 1,\ \cdots,\ T-1$, be the transmitted pulse waveform from the nth transmitting antenna. For simplicity, we assume that the transmitted waveforms $\{s_n(t)\}$ have perfect auto- and cross-correlation properties within a certain time lag T_0, i.e.,

$$\sum_{t=0}^{T-1} s_{n_1}(t)s_{n_2}^*(t-\tau) = \begin{cases} 1, & \text{when } \tau = 0 \text{ and } n_1 = n_2, \\ 0, & |\tau| \leq T_0. \end{cases} \tag{5–1}$$

The signals are being transmitted with a pulse repetition interval (PRI) of T_{PRI} and modulated to a carrier frequency corresponding to the wavelength λ. To achieve the requirement of having a good waveform set with good auto- and cross-correlation properties, we used the PeCAN algorithm [101] to design the initial sequence then

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use shifted copies of the sequence to form a set of N sequences with the prescribed zero-correlation zone(ZCZ).

5.2.3   Received Signal Model

Let $x_{m,p}(t)$ ($m = 0, 1, \cdots, M-1$, and $p = 0, 1, \cdots, P-1$) be the received data at the mth receive antenna output due to the pth probing pulse with P being the number of pulses within a coherent processing interval (CPI). Ignoring the intra-pulse Doppler effect as well as any quantization and array callibration errors, the received data samples for a given point contains a Doppler shift associated with the platform movement, $\frac{2v_p \sin(\theta)}{\lambda}$, and a Doppler shift due to the target movement, $\frac{2v_t \cos(\phi)}{\lambda}$. By transmitting from the nth transmit antenna and then receiving at the mth receive antenna, we also have phase shifts of $2\pi \frac{d_r}{\lambda} m \sin(\theta)$ and $2\pi \frac{d_t}{\lambda} n \sin(\theta)$ relative to the reference point. This results in the following signal model,

$$x_{m,p}(t) = \beta_{\tau,\theta} \sum_{n=0}^{N-1} e^{j2\pi\left(\frac{2v_p \sin(\theta)}{\lambda} + \frac{2v_t \cos(\phi)}{\lambda}\right) p\, T_{PRI}} e^{j2\pi \frac{d_r}{\lambda} m \sin(\theta)} e^{j2\pi \frac{d_t}{\lambda} n \sin(\theta)} s_n(t - \tau), \qquad (5\text{--}2)$$

where $\beta_{\tau,\theta}$ denotes the complex-valued reflection coefficient of the target. For notational simplicity, let

$$b = \frac{2v_p}{\lambda} T_{PRI}, \qquad (5\text{--}3)$$

and

$$v = \frac{2v_t \cos(\phi)}{\lambda} T_{PRI}. \qquad (5\text{--}4)$$

Then, the data model in (5–2) can be rewritten as:

$$x_{m,p}(t) = \beta_{\tau,\theta} \sum_{n=0}^{N-1} e^{j2\pi f_d (p + \eta_r m + \eta_t n)} e^{-j2\pi v(\eta_r m + \eta_t n)} s_n(t - \tau), \qquad (5\text{--}5)$$

where

$$f_d = b \sin(\theta) + v \qquad (5\text{--}6)$$

is the overall Doppler frequency caused by both platform and target movements and

$$\eta_r = \frac{d_r}{\lambda b} \quad \text{and} \quad \eta_t = \frac{d_t}{\lambda b} \tag{5–7}$$

are the normalized inter-element spacings of the receiving and the transmitting arrays, respectively. Note that for stationary ground clutter, we have $v = 0$ and $f_d = b\sin(\theta)$, then the second term in (5–5), i.e., $e^{-j2\pi v(\eta_r m + \eta_t n)}$, is equal to 1.

Taking the stationary ground clutter, noise and interference from other targets into account, the observed data samples can be modeled as:

$$x_{m,p}(t) = \beta_{\tau,\theta} \sum_{n=0}^{N-1} e^{j2\pi f_d(p + \eta_r m + \eta_t n)} e^{-j2\pi v(\eta_r m + \eta_t n)} s_n(t - \tau) + z_{m,p}(t), \tag{5–8}$$

where $z_{m,p}(t)$ contains the noise, ground clutter, and interferences from other moving targets. The problem of interest is to detect moving targets from the observed data samples $\{x_{m,p}(t)\}$. This problem is challenging considering the fact that the stationary ground clutter is much stronger than the moving targets.

5.3 GMTI Algorithm

In this section, we detail our GMTI algorithm for a MIMO radar system architecture. It involves three tasks: range compression, Doppler processing and phase compensation, and moving targets detection. Our moving target detector is based on the Iterative Adaptive Approach (IAA), a high resolution spectral estimation technique, that we will apply to a special range-Doppler-velocity domain. We will use the Delay-and-Sum approach as our benchmark. Each task is detailed as follows.

5.3.1 Range Compression

The return signals must first be range compressed via, for example, the matched filtering. Let,

$$y_{m,n}(\tau, p) \triangleq \sum_{t=\tau}^{T+\tau-1} x_{m,p}(t) s_n^*(t - \tau) \tag{5–9}$$

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be the range compressed output at the m receiving antenna for the nth transmitting waveform at the pth pulse, where $(\cdot)^*$ denotes the complex conjugate.

From (5–5), (5–9) and (5–1), the range-compressed output can be modeled as,

$$y_{m,n}(\tau, p) = \beta_{\tau,\theta}e^{j2\pi f_d(p+\eta_r m+\eta_t n)}e^{-j2\pi v(\eta_r m+\eta_t n)} + \tilde{z}_{m,n}(\tau, p). \tag{5–10}$$

5.3.2 Doppler Processing and Phase Compensation

To work in the post-Doppler domain, we apply the Fourier transform to both sides of (5–10):

$$\tilde{Y}_{m,n}(\tau, f) = \beta_{\tau,\theta}e^{-j2\pi v(\eta_r m+\eta_t n)}e^{j2\pi f(\eta_r m+\eta_t n)}\delta(f - f_d) + \tilde{Z}_{m,\tau}(\tau, f), \tag{5–11}$$

where $\tilde{Y}_{m,n}(\tau, f)$ and $\tilde{Z}_{m,\tau}(\tau, f)$ are the Fourier transforms of $\{y_{m,n}(\tau, p)\}$ and $\{\tilde{z}_{m,n}(\tau, p)\}$ with respect to p, respectively, and $\delta(\cdot)$ is the Kronecker delta, i.e., $\delta(0) = 1$ and $\delta(f) = 0$ for $f \neq 0$.

In order to look at the returns of the same scene for all channels, we must undo the phase shifts that was caused by the inter-element spacing. We apply a phase compensation to $\tilde{Y}_{m,n}(\tau, f)$, i.e.,

$$Y_{m,n}(\tau, f) = e^{-j2\pi f(\eta_r m+\eta_t n)}\tilde{Y}_{m,n}(\tau, f). \tag{5–12}$$

Then, (5–11) becomes:

$$Y_{m,n}(\tau, f) = \beta_{\tau,\theta}e^{-j2\pi v(\eta_r m+\eta_t n)}\delta(f - f_d) + Z_{m,n}(\tau, f), \tag{5–13}$$

where

$$Z_{m,n}(\tau, f) = e^{-j2\pi f(\eta_r m+\eta_t n)}\tilde{Z}_{m,n}(\tau, f). \tag{5–14}$$

Let

$$\mathbf{Y}(\tau, f) = \begin{bmatrix} Y_{0,0}(\tau, f) & Y_{1,0}(\tau, f) & \cdots & Y_{M-1,0}(\tau, f) & Y_{0,1}(\tau, f) & \cdots & Y_{M-1,N-1}(\tau, f) \end{bmatrix}^T,$$
$$\tag{5–15}$$

and

$$\mathbf{a}(v) = \begin{bmatrix} 1 & e^{-j2\pi v \eta_r} & \cdots & e^{-j2\pi v \eta_r (M-1)} & e^{-j2\pi v \eta_t} & e^{-j2\pi v(\eta_t+\eta_t)} & \cdots & e^{-j2\pi v(\eta_t(N-1)+\eta_t(M-1))} \end{bmatrix}^T,$$

(5–16)

where $(\cdot)^T$ denotes the transpose.

From (5–13), we have

$$\mathbf{Y}(\tau, f) = \mathbf{a}(v)\beta_{\tau,\theta}\delta(f - f_d) + \mathbf{Z}(\tau, f),$$

(5–17)

where $\mathbf{Z}(\tau, f)$ is defined similarly to $\mathbf{X}(\tau, f)$ in (5–15).

Note that the steering vector of the stationary ground clutter is

$$\mathbf{a}(0) = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T.$$

(5–18)

This implies that after phase compensation, all the ground clutter lies in the subspace spanned by $\mathbf{a}(0)$, and hence can be suppressed effectively by using adaptive techniques. Meanwhile, $\sum_{m,n} Y_{m,n}(\tau, f)$ provides a SAR (synthetic aperture radar) image of the stationary scene.

5.3.3 Moving Target Detection in the Range-Doppler-Velocity Domain

To detect the movers, we look at the range-Doppler-velocity domain. For a given range-Doppler cell, we look at the "virtual" returns for every possible velocities. To estimate the virtual returns at these velocities, we propose to use a recently developed adaptive algorithm to make an estimate of these returns. The recently proposed IAA algorithm [25] is considered herein to make an estimate of the vector $\beta_{(\tau,\theta)} = [\beta(\tau, \theta)_1, \cdots, \beta(\tau, \theta)_K]^T$, where $\beta(\tau, \theta)_i = \beta_{\tau,\theta}(v_i)$ $(i = 1, \cdots, K)$ are the responses at different potential velocities of the mover, and v_i $(i = 1, \cdots, K)$ are the different velocities. K is the number scanning points of the target's velocity. We propose to use IAA due to its high resolution and low sidelobe properties and thus its ability to resolve slowly moving targets from stationary ground clutter. Similar to the steps in Chapter 4, we now show how IAA can be applied our MIMO received model.

Let

$$\mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_K] \tag{5–19}$$

denote the steering matrix, where $\mathbf{a}_i = \mathbf{a}(v_i)$, we have the following compact model, where for notational expediency, we have drop the dependency on τ and θ:

$$\mathbf{y} = \mathbf{A}\boldsymbol{\beta} + \mathbf{e}. \tag{5–20}$$

IAA solves for $\boldsymbol{\beta}$ by using a weighted least squares approach, which involves the minimization of the following cost function,

$$\min_{\beta_i} \|\mathbf{y} - \beta_i \mathbf{a}_i\|_{\tilde{\mathbf{Q}}_i^{-1}}^2, \qquad i = 1, \cdots, K, \tag{5–21}$$

where $\|\mathbf{u}\|_{\tilde{\mathbf{Q}}^{-1}}^2 \triangleq \mathbf{u}^H \tilde{\mathbf{Q}}^{-1} \mathbf{u}$ and $(\cdot)^H$ denotes the cojugate transpose. In Equation (5–21), the interference and noise covariance matrix $\mathbf{Q}_i$ is given by:

$$\mathbf{Q}_i = \mathbf{R} - |\beta_i|^2 \mathbf{a}_i \mathbf{a}_i^H, \tag{5–22}$$

where

$$\mathbf{R} = \sum_{i=1}^{N} |\beta_i|^2 \mathbf{a}_i \mathbf{a}_i^H \tag{5–23}$$

is the IAA covariance matrix. By letting $\mathbf{P}$ denote an $MN \times MN$ diagonal matrix whose diagonal contains the signal power as $p_i = |\beta_i|^2$, we can re-write Equation (5–23) in the following matrix form:

$$\mathbf{R} = \mathbf{A}\mathbf{P}\mathbf{A}^H. \tag{5–24}$$

The minimization of Equation (5–21) with respect to β_i gives:

$$\beta_i = \frac{\mathbf{a}_i^H \mathbf{Q}_i^{-1} \mathbf{y}}{\mathbf{a}_i^H \mathbf{Q}_i^{-1} \mathbf{a}_i}. \tag{5–25}$$

Using the definition of $\mathbf{Q}_i$ and the matrix inversion lemma, Equation (5–25) can be written as [25]:

$$\beta_i = \frac{\mathbf{a}_i^H \mathbf{R}^{-1} \mathbf{y}}{\mathbf{a}_i^H \mathbf{R}^{-1} \mathbf{a}_i}, \tag{5–26}$$

which avoids the computation of $\mathbf{Q}_i^{-1}$ for each $i = 1, \cdots, K$. Since the *a priori* knowledge of $\mathbf{P}$ is required in Equation (5–24) to calculate the covariance matrix $\mathbf{R}$, IAA has to be implemented iteratively. Usually, IAA is initialized with the periodogram and it is iterated for a set number of iterations (usually 10 iterations).

Once β is estimated by IAA, we obtain a 3D cube denoted by $P(\tau, f, v)$, whose voxel value represents the reflection coefficient of the virtual target. To detect the movers from this cube of possible moving targets we compute the following metric:

$$\rho(\tau, f, v) = \frac{P(\tau, f, v)}{\sigma_v^2}, \qquad (5\text{–}27)$$

where σ_v^2 represents the average power of the clutter that has velocity v. In this way we make the assumption that the clutter power is homogenous over various range-Doppler cells for a given velocity. Hence σ_v^2 can be estimated using the mean value of $P(\tau, \theta, v)$ which is just Equation (5–27) with J (suspected number of movers, say $J = 10$) largest values removed. Then we can look at the 3D cube $P(\tau, \theta, v)$ where its peaks indicates the existence of the moving targets. The associated range $\hat{\tau}$, Doppler frequency $\hat{f}_d$ and velocity $\hat{v}$ can then be related back to the true target angle, $\hat{\theta}$ by the following equation,

$$\hat{\theta} = \sin^{-1}\left(\frac{\hat{f}_d - \hat{v}}{b}\right). \qquad (5\text{–}28)$$

5.4 Numerical Examples

We present below several numerical examples to demonstrate the advantages of the proposed MIMO GMTI approach over its SIMO counterpart. The realistic ground clutter and the associated moving targets are shown in Figure 5-2. This ground clutter was collected via the GOTCHA program sponsored by the Air Force Research Lab. We modified the clutter returns by embedding five moving targets with velocities $v_t \cos(\phi)$ being $-8.5, -5.1, 5.1, 6.8$ and 8.1 meters per second. The corresponding normalized

velocities v are -0.25, -0.15, 0.15, 0.20, and $.24$ respectively. As we can see, the challenge is to locate the two targets that are very close to each other.
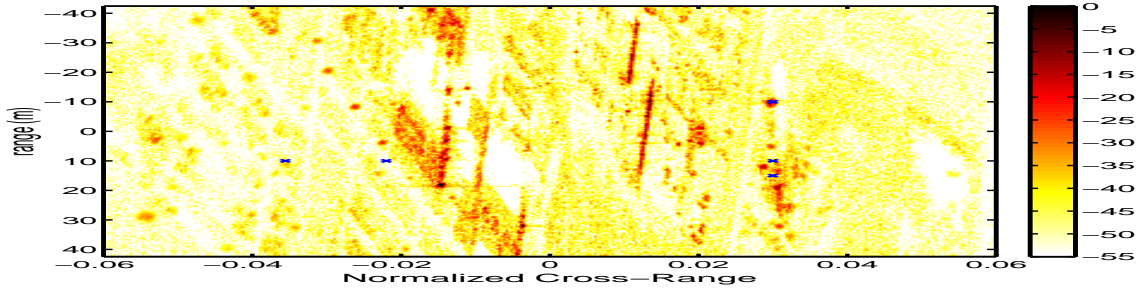

Figure 5-2. Ground clutter, and simulated target locations and velocities.

We simulate a radar system with a carrier frequency of 9.6 GHz, with the bandwidth being 640 MHz, which corresponds to a 0.23 meter range resolution. The Pulse Repetition Frequency (PRF) is around 2.17 KHz. We assume that the velocity of the radar platform is 100 meters per second, with the corresponding normalized velocity b being 2.95. The unimodular sequence design algorithm in [101], PeCAN, is utilized to design the transmitted pulse waveforms $s_n(t)$. For the SIMO case, we design a 4096-chip unimodular sequence, then cyclically shift it with various delays for each transmit antennas in the MIMO case.

5.4.1   Single-Input and Multiple-Output (SIMO) System

First, we consider the performance of a SIMO radar system. We compare its performance using two detection algorithms. First we discuss the performance when the Delay-and-Sum (DAS) method is used for mover detection, the DAS is also known as the *matched filter,* and is widely popular in practice. We first consider the conventional SIMO radar system. Figures 5-3(a)-(c) show the Doppler-Velocity images of the mover using DAS for a given range bin. The circles indicate the true cross-range locations and normalized velocities of the moving targets. As we can see, the so-obtained Doppler-Velocity images suffer from high sidelobe problem which may cause false detections and poor velocity estimation. This is exascerbated further via the poor resolution of DAS, and as we can see DAS was only able to detect one target

successfully. With respect to the two closely spaced targets, DAS failed to resolve the two but did indicate the presents of a mover. For clarity the detection result is transpose onto the SAR image in Figure 5-3(d). As we can see the combination of a low resolution method like DAS combined with the small aperture problem of SIMO yields many false results.

Figures 5-4(a)-(c) shows the same setup as above, but instead of using DAS we used IAA to detect the movers. As mentioned earlier, IAA is a high-resolution approach and should be able to perform better than DAS, but due to the small aperture limitations of SIMO, we see that while it correctly picks up some target, there are also some false detection as well.

Figure 5-3. Detection via DAS. Velocity-Doppler image for the SIMO case at various ranges. A) 10 meters. B) 15 meters. C) -10 meters. (D) Range-Doppler image via DAS for SIMO.

5.4.2 Multiple-Input and Multiple-Output (MIMO) System

For the MIMO case, we use a 4-element Uniform Linear Array (ULA) with an inter-element spacing of 0.1 meters for receiving, and a 3-element ULA with inter-element spacing of 0.5 meters for transmitting. The corresponding η_r and η_t are 1.09 and 5.43, respectively. We provide below the GMTI results by using the proposed MIMO
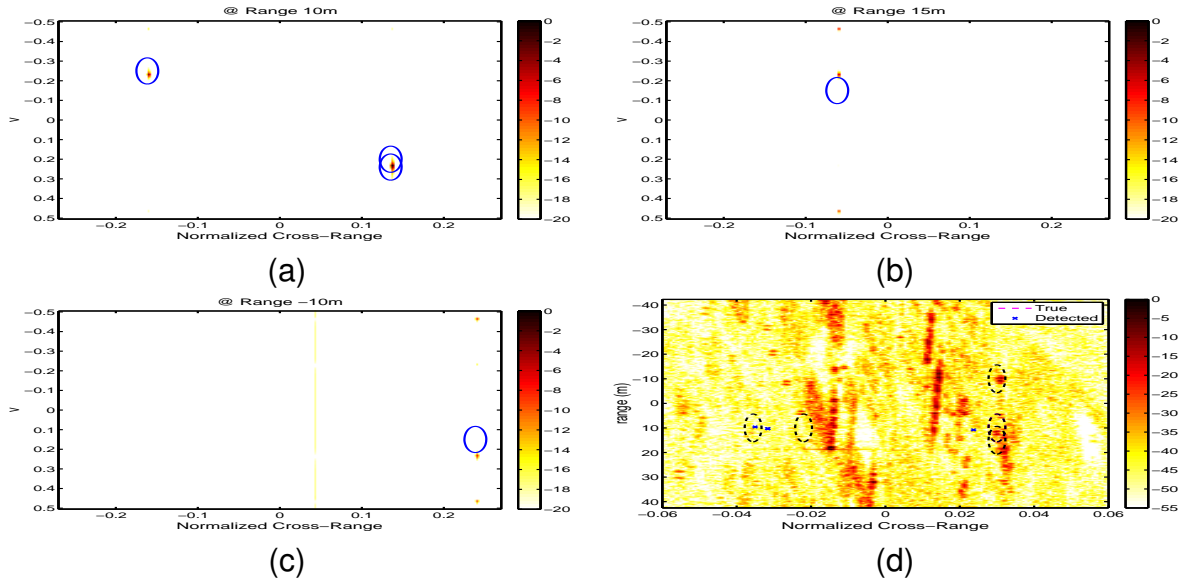
Figure 5-4. Detection via IAA. Velocity-Doppler image for the SIMO case at various ranges. A) 10 meters. B) 15 meters. C) -10 meters. D) Range-Doppler image via IAA for SIMO.

radar system. We first look at using DAS in this MIMO setup, Figures 5-5(a)-(c) show the Doppler-velocity images at a given range. As we can see, the so-obtained Doppler-Velocity images have much higher resolution than their SIMO counterparts. However, due to the high sidelobe nature of DAS, many false alarms exust, and DAS is not able to resolve the two closely spaced targets.

Figure 5-6 shows the performance of IAA with a MIMO setup. Due to the high resolution and low sidelobe nature of IAA in addition to the increased aperture of the MIMO configuration, IAA is successful in detecting all 5 targets with no false alarm (in the given dynamic range), as well as correctly estimating their velocities.

We have considered using MIMO radar for moving target detection in the presence of strong stationary ground clutter. A MIMO SAR based GMTI system has been introduced. With this framework, target detection can be made in parallel with SAR imaging. We have shown detection performance using DAS as well as a high resolution detection algorithm, IAA. With the right waveforms, the MIMO configuration and its larger virtual aperture allow for more advanced receiver side algorithms such as IAA to not
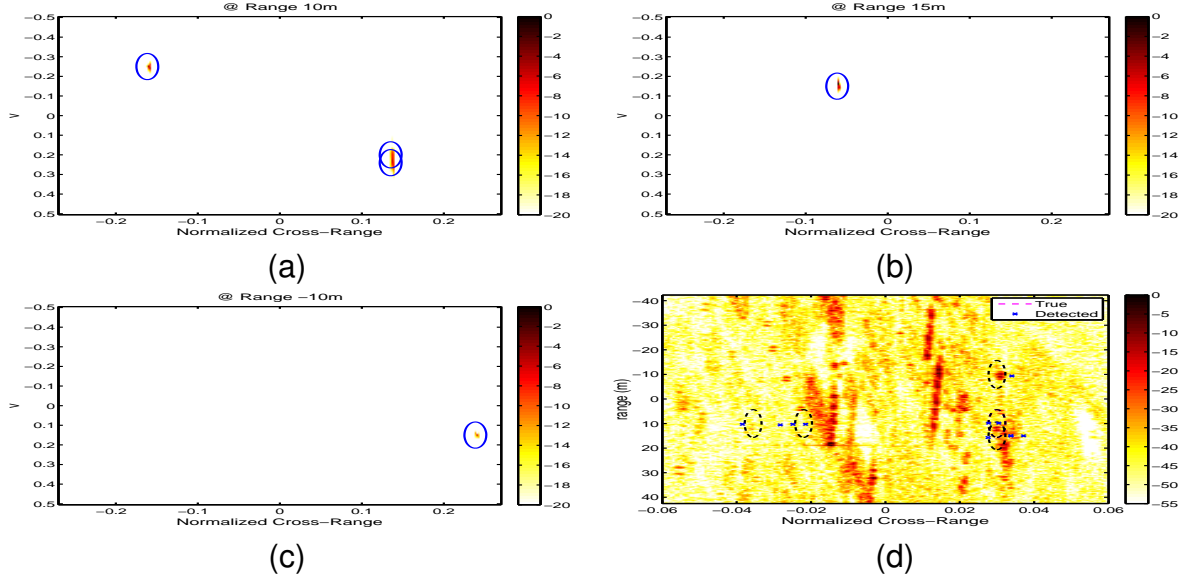
118

Figure 5-5. Detection via DAS. Velocity-Doppler image for the MIMO case at various ranges. A) 10 meters. B) 15 meters. C) -10 meters. (D) Range-Doppler image via DAS for MIMO.

only detect the movers but also correctly estimate their velocities, something that was not possible using a SIMO configuration.
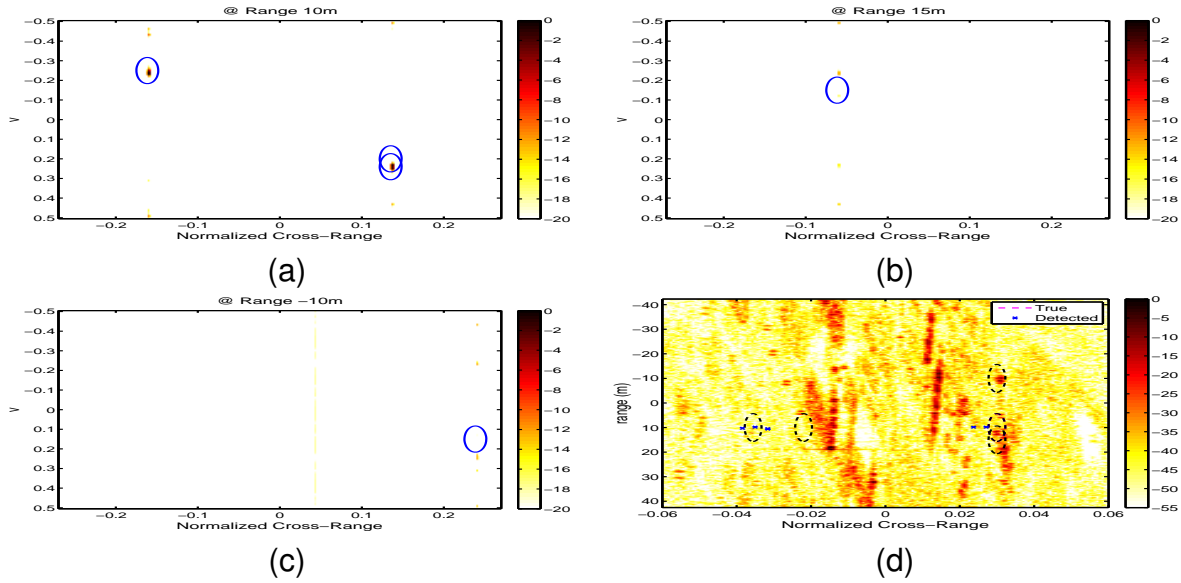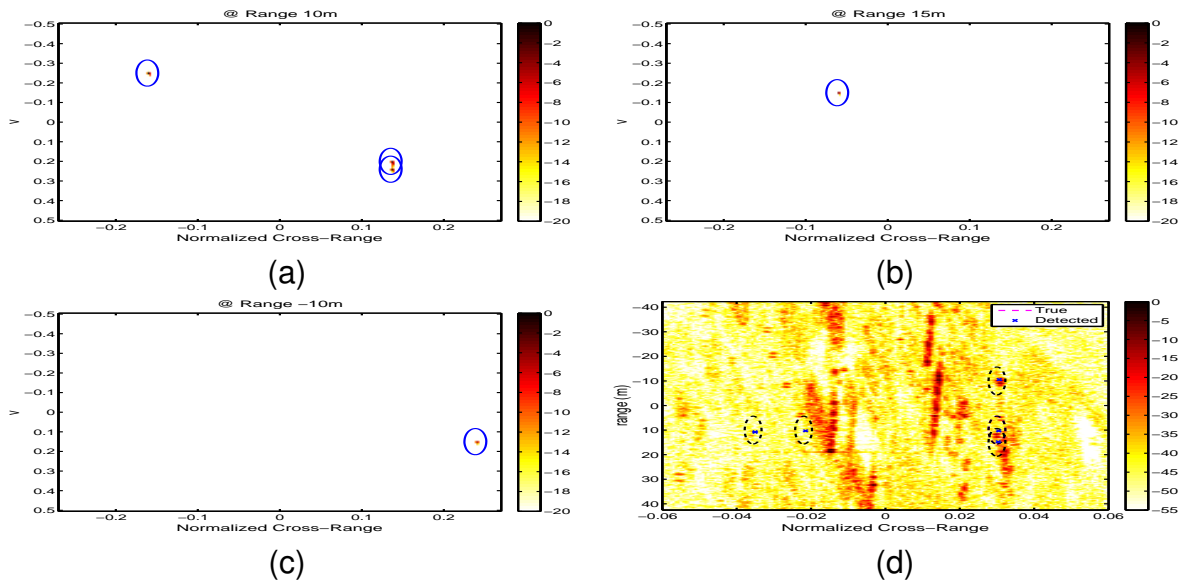
Figure 5-6. Detection via IAA. Velocity-Doppler image for the SIMO case at various ranges. A) 10 meters. B) 15 meters. C) -10 meters. D) Range-Doppler image via IAA for MIMO.

CHAPTER 6
CONCLUDING REMARKS AND FUTURE WORKS

This dissertation has presented a new approach to forming SAR images, an alternative to the traditionally used Fourier Transform. The new method based on a Bayesian framework, is more robust and more accurate and uses spectral estimation as its founding principles. We showed how it can be used on SAR data that are incomplete and we showed how its computational complexity can be made more competitive via exploiting the structure of the sensing matrices, as well as various numerical techniques–such as the Fast Fourier Transforms and the Conjugate Gradient Method. The significance of this contribution lies in the amount of data that is needed to form acceptable SAR images that can be used not only for reconnaissance but are more amenable to automatic target recognitions algorithms. This lower data requirements means less time is spent sensing, which reduces risks to sensing systems. The lower time requirement also results in an increase in time allotted for analysis and response.

The second body of contribution that this dissertation presents is a framework on how to detect movers directly from SAR data. Movers detection as discussed in Chapter 4 are specific to certain SAR systems. The dissertation presents a framework on how movers can be detected solely based on the measured SAR data. This remove the need for a dedication GMTI mode as well as the ability to operate on reconnaisance data directly. The disseration also opens up a venue of using MIMO systems for mover detections. For targets that are small and is close to the endo-clutter, a MIMO systems increases the minimum detectable velocity of a system. It also allows for increased parameter identifiability. With a MIMO SAR system, we can not only detect the state of the targets but also its velocity.

SAR research is in some sense a small niche. This is largely in part because of the applications of SAR, which are more geared toward the defense community. However another reason is the lack of measured SAR data. SAR data in its complex

phase histories format is almost non-existent. Communities that works with SAR data for environmental research works only with complex SAR data. Complex SAR data are post-processed data and many of its useful data characteristics which are exploited using spectral estimation has been lost. Compouding this with the difficulty of gathering SAR data makes this almost a closed field. To spearhead research in this area, more phase-history SAR data should be made available to researchers. At the time of this writing, we are in the process of building a prototype MIMO Radar. This initial 2x2 system can be used to gather both 1-D and 2-D data. With this data we can more reliably verify the workings of the proposed algorithms as well as make more contribution to the field of MIMO SAR.

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BIOGRAPHICAL SKETCH

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