

DESIGNING MORE ACCEPTABLE AND EQUITABLE CONGESTION PRICING  
SCHEMES FOR MULTIMODAL TRANSPORTATION NETWORKS

By

DI WU

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL  
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT  
OF THE REQUIREMENTS FOR THE DEGREE OF  
DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2011

© 2011 Di Wu

To my family

## ACKNOWLEDGMENTS

First of all, I would like to express my gratitude to my advisor, Dr. Yafeng Yin, for his constant support and encouragement. He has been a wonderful advisor, teacher, colleague and friend since the first day I stepped into the campus of University of Florida. It is such an honor working with him.

I would like to thank Dr. Siriphong Lawphongpanich for serving as my committee member, but also for providing numerous valuable comments and suggestions on my research. I am also very grateful to Dr. Lily Elefteriadou, Dr. Scott Washburn and Dr. Sivaramakrishnan Srinivasan for serving as my committee members. Their support and guidance are also motivations for my Ph.D. study. I would also like to thank all the fellow students and my colleagues in Transportation Research Center for the discussions and their friendship. They really made my life in UF full of joy and happiness.

Last but certainly not least, I would thank my family for their selfless support. I would have never made any achievement without them. I would specially thank my wife, Xue, for her selfless love and support.

# TABLE OF CONTENTS

	<u>page</u>
ACKNOWLEDGMENTS.....	4
LIST OF TABLES.....	8
LIST OF FIGURES.....	10
ABSTRACT .....	11
CHAPTER	
1 INTRODUCTION .....	13
1.1 Background.....	13
1.2 Objectives .....	17
1.3 Organization of This Dissertation.....	19
2 LITERATURE REVIEW .....	20
2.1 Traffic Assignment Problem.....	20
2.1.1 User Equilibrium and System Optimal Conditions .....	20
2.1.2 Alternative Formulations of User Equilibrium Traffic Assignment Problem.....	23
2.1.3 Traffic Assignment with Elastic Demand .....	24
2.1.4 Stochastic Traffic Assignment Problem .....	25
2.1.5 Traffic Assignment Problem on Multimodal Networks .....	28
2.2 Congestion Pricing.....	29
2.2.1 Marginal Cost Pricing .....	29
2.2.1.1 Canonical model of marginal cost pricing.....	29
2.2.1.2 Extensions to general transportation networks .....	31
2.2.1.3 First-best toll set.....	32
2.2.2 Second-Best Road Pricing .....	33
2.2.3 Network Toll Design Problem .....	34
2.2.4 Congestion Pricing in Multimodal Networks .....	36
2.3 Empirical Results for Public Acceptance of Congestion Pricing.....	36
2.3.1 Effectiveness of Congestion Pricing .....	37
2.3.2 Equity Issues .....	37
2.3.3 Revenue Distribution .....	38
2.4 Equity Effects of Congestion Pricing.....	39
2.4.1 Social Welfare and Equity .....	39
2.4.1.1 Definitions of equity.....	40
2.4.1.2 Measures of equity.....	41
2.4.2 Equity Effects of Congestion Pricing.....	41
2.5 Design of More Equitable and Acceptable Congestion Pricing Schemes .....	44
2.6 Contributions of This Dissertation .....	46

3	PARETO-IMPROVING CONGESTION PRICING ON MULTIMODAL TRANSPORTATION NETWORK .....	51
3.1	Motivation .....	51
3.1.1	Unattractiveness of First-Best Pricing.....	51
3.1.2	Pareto-Improvement and Pareto-Improving Congestion Pricing .....	52
3.2	Problem Description.....	54
3.2.1	Underlying Network Structure.....	54
3.2.2	Strategy-Based Transit Route Choice .....	56
3.2.3	Feasible Region .....	58
3.3	Multimodal Traffic Assignment Models .....	60
3.3.1	Multimodal User Equilibrium Model .....	60
3.3.2	Existence of Multimodal User Equilibrium .....	65
3.3.3	Multimodal System Optimal Model .....	66
3.4	Multimodal Pareto-Improving Toll Problem .....	67
3.5	Solution Algorithm.....	70
3.6	Numerical Example.....	73
3.7	Summary .....	74
4	MULTIMODAL PARETO-IMPROVING CONGESTION PRICING WITH TRIP-CHAINING .....	83
4.1	Tour-Based Network Equilibrium Analysis Models .....	84
4.1.1	Activity-Based Traffic Assignment .....	84
4.1.2	Representing Trip Chaining .....	84
4.1.3	Feasible Region .....	85
4.1.4	Tour-Based Multimodal User Equilibrium Model .....	86
4.1.5	System Optimum Model .....	88
4.2	Tour-Based Model for Pareto-Improving Strategies.....	88
4.2.1	Model Formulation.....	88
4.2.2	Solution Algorithm .....	91
4.2.3	Optimum Policy Model.....	93
4.3	Numerical Example.....	94
4.3.1	Example Network .....	94
4.3.2	Optimal and Pareto-Improving Policies .....	94
4.3.3	Benefits of Capacity Expansion.....	95
4.3.4	Comparison of Tour-Based and Trip-Based Models.....	96
4.4	Summary .....	96
5	DESIGN OF MORE EQUITABLE CONGESTION PRICING SCHEME BY CONSIDERING INCOME EFFECTS.....	104
5.1	Problem Description.....	105
5.2	Mathematical Model and Solution Algorithm.....	106
5.2.1	Feasible Region .....	107
5.2.2	Multimodal User Equilibrium .....	108
5.2.3	Welfare and Equity Measures .....	113

5.2.3.1 Individual welfare measure .....	113
5.2.3.2 Social benefit measure.....	114
5.2.3.3 Equity measure .....	115
5.2.4 Model Formulation.....	116
5.2.5 Solution Algorithm .....	116
5.3 Numerical Examples .....	117
5.3.1 Nine-Node Network .....	117
5.3.2 Seattle Network .....	118
5.3.2.1 Network characteristic.....	118
5.3.2.2 User behavior.....	118
5.3.2.3 Demand estimation .....	119
5.3.2.4 Existing traffic condition and social welfare .....	120
5.3.2.5 Optimal pricing policies .....	120
5.4 Summary .....	123
<b>6 EQUITABLE AND TRADABLE CREDIT SCHEMES .....</b>	<b>143</b>
6.1 Introduction to Tradable Credit Schemes.....	143
6.1.1 Alternative Demand Management Schemes .....	143
6.1.2 Tradable Credit Schemes and Its Public Acceptability .....	145
6.1.3 The Tradable Credit Distribution and Charging Scheme .....	146
6.2 Mathematical Model.....	147
6.2.1 Representation of User Utility under Tradable Credit Scheme .....	147
6.2.2 Single-Modal User Equilibrium under the Tradable Credit Scheme .....	148
6.2.3 User Equilibrium with Heterogeneous Users and Nonlinear User Utility Function .....	150
6.2.4 Multimodal Stochastic User Equilibrium under Tradable Credit Scheme.....	152
6.2.4.1 Problem Description.....	152
6.2.4.2 Mathematical Model.....	153
6.3 Tradable Credit Scheme Design Problem.....	155
6.4 Solution Algorithm.....	158
6.5 Numerical Example.....	158
6.6 Summary .....	161
<b>7 CONCLUSIONS AND FUTURE RESEARCH.....</b>	<b>167</b>
7.1 Conclusions .....	167
7.2 Future Research .....	169
<b>LIST OF REFERENCES .....</b>	<b>171</b>
<b>BIOGRAPHICAL SKETCH.....</b>	<b>185</b>

## LIST OF TABLES

<u>Table</u>	<u>page</u>
2-1 Existing research on Pareto-improving pricing schemes .....	49
3-1 System optimal, user equilibrium and Pareto-improving conditions.....	76
3-2 Link characteristics of Sioux Falls network .....	78
3-3 Summary of results in Sioux Falls network .....	79
4-1 Link parameters of five-node network.....	98
4-2 Comparisons of user equilibrium, optimal and Pareto-improving policies.....	99
4-3 Pareto-improving and optimal policies without capacity expansion .....	101
4-4 Trip-based Pareto-improving pricing scheme .....	102
4-5 Comparison of system performances .....	102
5-1 Link parameters.....	125
5-2 Origin-destination demands.....	125
5-3 Utility function parameters .....	126
5-4 Results of Nine-Node network .....	127
5-5 Link parameters of Seattle network .....	128
5-6 Parameters for the Translog utility function .....	129
5-7 Demand matrix .....	130
5-8 User equilibrium condition without toll .....	131
5-9 Results for Seattle network.....	132
5-10 Mode-specific demand for Seattle network.....	133
5-11 Toll rates for Seattle network .....	134
5-12 Results for policies with negative revenues for Seattle network .....	134
5-13 Mode-specific demand for policies with negative revenues for Seattle network.....	135

5-14	Toll rates for policies with negative revenues for Seattle network .....	135
6-1	Results of optimal tradable credit schemes .....	163
6-2	Optimal credit distribution and charging schemes .....	164

## LIST OF FIGURES

<u>Figure</u>	<u>page</u>
2-1 Economics foundation of marginal cost pricing .....	50
3-1 A five-link network .....	80
3-2 Auto and transit network.....	80
3-3 Common lines in a transit network.....	80
3-4 Nine-node network .....	81
3-5 Sioux Falls network .....	82
4-1 Five-node network.....	103
5-1 Nine-node network .....	136
5-2 Relation of social benefit and Gini coefficient of the results.....	136
5-3 Seattle sketch network .....	137
5-4 Seattle surrounding area .....	138
5-5 Compositions of productions for the centroid nodes.....	139
5-6 Compositions of attractions for the centroid nodes.....	139
5-7 Travelers' mode choice percentage by origin .....	140
5-8 Travelers' mode choice percentage by income group .....	140
5-9 Relation of social benefit and Gini coefficient of the results.....	141
5-10 Changes in equivalent income.....	141
5-11 Relation of social benefit and Gini coefficient of the results.....	142
5-12 Changes in equivalent income.....	142
6-1 Pareto frontier of tradable credit schemes .....	165
6-2 Changes in equivalent income.....	165
6-3 Comparison of tradable credit schemes and congestion pricing schemes .....	166

Abstract of Dissertation Presented to the Graduate School  
of the University of Florida in Partial Fulfillment of the  
Requirements for the Degree of Doctor of Philosophy

DESIGNING MORE ACCEPTABLE AND EQUITABLE CONGESTION PRICING  
SCHEMES FOR MULTIMODAL TRANSPORTATION NETWORKS

By

Di Wu

August 2011

Chair: Yafeng Yin  
Major: Civil Engineering

Congestion pricing is to impose tolls on transportation facilities to influence people's travel choices in order to alleviate traffic congestion. Although recent successful implementations around the world have gained more support for the policy from transportation authorities and government officials, congestion pricing is still facing strong objection among the general public. This dissertation explores technical approaches to improve the public acceptability of congestion pricing and develops more acceptable and equitable pricing schemes on general multimodal transportation networks. The models proposed in this dissertation provide good tools for government agencies to develop congestion mitigation policies that proactively address the concerns of the general public and thus are more likely to gain their supports.

Two pricing approaches are investigated in this dissertation. The first approach designs Pareto-improving pricing schemes that make nobody worse off compared to the no-toll condition. Two pricing models are proposed based on trip-based and tour-based demand models, respectively. The resulting schemes are expected to be more attractive to travelers because the travel time savings will be more than enough to justify the toll rates charged from them. In contrast, conventional congestion pricing schemes,

e.g., marginal-cost pricing, may result in much higher toll rates for less travel time savings.

In the second approach, we aim to optimize pricing schemes against an equity measure. This approach considers the effects of income on travelers' choices of trip generation, mode and route in the presence of toll and explicitly captures the impacts of pricing schemes across different income and geographic groups. A pricing model is proposed to seek for a balance between efficiency and equity.

Lastly, this dissertation investigates the equity implications of tradable credit schemes. Tradable credit schemes are alternatives to congestion pricing, which charge credits instead of money. The credits are distributed to the travelers by the government and can be freely traded among travelers. A model is proposed in this dissertation for designing a credit distribution and charging scheme that achieves a balance between equity and efficiency. The scheme is demonstrated to have more potential than congestion pricing scheme in achieving superior performance on both equity and efficiency measures.

## CHAPTER 1 INTRODUCTION

### 1.1 Background

Traffic congestion has become one of the most serious social problems in modern societies. In its 2009 Annual Urban Mobility Report, Texas Transportation Institute (TTI) reports 36 hours of average annual delay per peak traveler as of year 2007 and \$87.2 billion of delay and fuel cost in total due to traffic congestion in the U.S. (Schrank and Lomax, 2009). Yet congestion has grown in cities around the world of every size in the past ten years in extent, duration and intensity. In August 2011, China National Highway 110 experienced a complete gridlock for more than 11 days and stretched more than 100 kilometers, which is considered the world's worst traffic jam ever (Hickman, 2010).

In general, congestion can be tackled from both the supply and demand sides. Supply-side tactics expand the means that travelers can use for travel, such as building more roads to increase roadway capacities and providing more transit services. Demand-side approaches manage the total number of vehicles traveling during peak periods, via travel demand management (TDM) or mixed land-use development, etc. Supply-side tactics, although very straightforward, are subject to many spatial and financial constraints. Furthermore, providing more road space has been proven to be self-defeating in congested areas because the increased capacity will soon be absorbed by the induced travel demand (Goodwin and Noland, 2003; Hansen and Huang, 1997). On the demand side, TDM has gained increasing attentions as a more effective and cost-efficient solution to the congestion problem.

Congestion pricing can be viewed as a major component in the TDM system. The concept of congestion pricing can be traced back to the 1920s, when Pigou (1920) and

Knight (1924) proposed the idea of charging tolls to internalize the externalities travelers imposed to the others in order to better distribute the traffic flow. Although being long advocated, congestion pricing was not adopted until recent advance in technologies that made electronic tolling feasible. Existing implementations around the world, e.g., Singapore, London and Oslo, have proved that congestion pricing is able to significantly reduce traffic congestion within the tolling area. For example, for the first six months after the introduction of congestion charges in London, the traffic delays inside the charging zone reduced by about 30% (Transport for London, 2003). In Singapore, the initial drop in traffic in the tolling area was 45% after the implementation of the Area Licensing Scheme (ALS) in 1975. The traffic volume dropped by 17% in 1998 after Electronic Road Pricing (ERP) replaced ALS in Singapore (Goh, 2002).

In the United States, although toll roads commonly existed during the nineteenth century, road pricing only started to gain attentions as an instrument for travel demand management recently. A prevalent form of road pricing in the U.S. is high occupancy/toll (HOT) lanes or express toll lanes with the first implementation in 1995 on State Route 91 in Orange County, California. In May 2006, the U.S. Department of Transportation launched a national congestion relief initiative to further promote congestion pricing implementations (U.S. Department of Transportation, 2006). Now, HOT lane or express lane facilities are already operational in more than seven states and many more are being planned. Evaluations of the existing HOT lane projects indicate that they can significantly reduce traffic delay and improve travel time reliability, despite that they are different in form from other congestion pricing implementations around the world (Burriss and Sullivan, 2006; Sullivan and Burriss, 2006).

Despite the proven ability of congestion pricing in reducing congestion delays and the increasing supports from governments and transportation agencies, opposition from the general public remains strong. In 2007, an online petition against road pricing in UK attracted over 1.8 million signatures within three months (Roberts, 2007). Hong Kong drew back after a pilot test on an electronic congestion pricing system between 1983 and 1985 (Hau, 1990). Residents in Cambridge (England) and Edinburgh also voted against congestion pricing schemes proposed for their cities. For Stockholm, where congestion pricing was tested and implemented permanently, only 51% of voters voted “yes” for the permanent implementation while 46% voted “no” (Stockholms stad, 2006).

While there are many reasons for the public opposition, much of this opposition centers on the perceived inequalities of congestion pricing (Taylor et al., 2010). Several empirical studies, e.g., Jaensirisak et al. (2005), clearly indicate that one of the main concerns with congestion pricing is the possible loss of welfare associated with, e.g., having to switch to less desirable routes, departure times or transportation modes because the more desirable ones are no longer affordable. For example, along with other schemes, marginal-cost pricing (Walters, 1961), a pricing scheme ubiquitous in the transportation literature, generally charges a toll on every link and significantly increases travel costs of many road users. In other words, marginal-cost pricing makes many users worse off when compared to the situation without pricing (Hau, 2005). New York State Assemblyman Richard Brodsky released a report in July 2007 arguing that congestion pricing would be regressive, disproportionately burdening working and middle class residents (Berger, 2008).

Various efforts have been made in designing more acceptable congestion pricing schemes in the literature. Dial (2000) considered an alternative pricing scheme that charges minimum tolls while minimizing system travel times, which would make road users “better off” compared to marginal cost pricing. Small (1992) proposed a package of revenue uses that can create net benefits for a wide spectrum of people and interest groups through travel allowances and tax reductions, and uses the remaining to improve transportation throughout the area. Yang and Zhang (2002) explicitly considered social and spatial equity by imposing an equity constraint in their toll design problem. Lawphongpanich and Yin (2010) proposed a Pareto-improving pricing scheme that will make nobody worse-off compared to the no-toll condition.

However, most studies in the literature a) rely on oversimplified assumptions and small size networks, and ignores the synergy among different travel modes, especially transit services; b) lack of systematical approaches to provide comprehensive policies that include multiple instruments, such as tolling, revenue refunding, subsidies and development of highway system and transit services; c) ignore the effects of income on travelers’ choice of trip generation, travel mode and route, and not able to capture the impacts of congestion pricing on different income and geographical groups. This dissertation attempts to overcome the shortcomings of the existing literature and address the acceptability problem of congestion pricing. More specifically, this dissertation develops more acceptable and equitable policies in general multimodal transportation networks that can either 1) make nobody worse off, i.e. Pareto-improvement, compared to the no-toll scenario, or 2) achieve a balance between equity and efficiency by directly optimizing equity measures.

## 1.2 Objectives

The objective of this dissertation is to analyze the impact of congestion pricing on the transportation systems and develop pricing schemes that are more acceptable, equitable and efficient in general multimodal transportation networks. In particular, we will investigate three types of schemes. They are Pareto-improving pricing schemes, toll optimization against equity measures and tradable credit schemes.

**Pareto-improving pricing.** Pareto-improving pricing scheme maximizes the social benefit while guaranteeing that nobody will be made worse off in term of the generalized travel cost, i.e. the cost of travel time and toll charges, under the toll scheme when compared to the no-toll condition. Lawphongpanich and Yin (2007; 2010) and Song et al. (2009) investigated the existence of such a pricing scheme in general networks and provided models to design such schemes in various network settings. However, their numerical results show that although the Pareto-improving scheme usually exists, it may not lead to significant improvement of system performance. One reason for this is the lack of revenue redistribution in their designs. With a similar objective, Guo and Yang (2010), Liu et al. (2009) and Nie and Liu (2010) investigated the existence of Pareto-improving pricing schemes under different revenue redistribution schemes and users' value of time distributions. Their research offers valuable insights to the development of such schemes that incorporate revenue usage, but their analysis either rely on oversimplified network representations or do not consider the interactions among travel modes. In this dissertation, we will investigate the possibility of achieving Pareto-improvement under more general multimodal networks where regular toll lanes, HOT lanes and transit services exist. Furthermore,

we will consider multiple instruments, such as tolling, revenue refunding, subsidy, transit and highway development using the toll revenue.

**Toll optimization against equity measures.** Although equity has long been viewed as an important issue in congestion pricing, there are only a limited number of studies that explicitly consider equity measures in their pricing models, e.g., Yang and Zhang, 2002; Connors et al., 2005; Sumalee, 2003, etc. None of the existing research is able to capture the distribution effects of congestion pricing on groups with different incomes. Therefore, they are not able to address vertical equity in a sensible way. In this dissertation, we focus on the pricing scheme that explicitly takes into consideration the effects of income on the choices of trip generation, mode and route and captures the distributional impacts of congestion pricing across different income and geographic groups. We will investigate how to incorporate equity measures into the design of pricing models and try to develop a pricing strategy that can maximize social benefit while being progressive.

**Equitable and tradable credit scheme.** Tradable credit scheme is an alternative approach to manage traffic congestion, originating from the idea of emission credit systems. A tradable credit scheme charges travelers credits instead of money to travel on the road network. The credits are distributed to the travelers by the government and can be traded among travelers at a market determined price. Under tradable credit schemes, the traffic volume can be managed by the government by carefully determining the total number of credits to be distributed. Drivers do not necessarily need to pay anything under such schemes to use the roadways as they receive credits from the government. Travelers only need to pay out-of-pocket money when their travel

needs require more credits than they are initially distributed. Thus such schemes are likely to gain more support from the general public.

### **1.3 Organization of This Dissertation**

This dissertation is organized as follows. Chapter 2 reviews the literature on the fundamentals of congestion pricing and transportation network modeling as well as their acceptability issues. Chapter 3 presents a model for the Pareto-improving congestion pricing scheme on multimodal transportation networks. Chapter 4 extends the Pareto-improving pricing model by considering the tour-based mode choice behavior. Chapter 5 considers toll optimizations against equity measures. The model presented in this chapter aims to design more equitable congestion pricing schemes by capturing their distributional effects across different income and geographic groups. An alternative approach to congestion pricing, more specifically the tradable credit scheme, is investigated in Chapter 6. A summary of the major contributions of this dissertation is presented in Chapter 7.

## CHAPTER 2 LITERATURE REVIEW

There is an extensive literature on congestion pricing and its public acceptability from both empirical and theoretical studies. In this chapter, we will start from reviewing existing models of traffic assignment problems under different settings, which is fundamental to the design and analysis of road pricing. We will then briefly summarize the economics principles of congestion pricing and review the congestion pricing design problems. We then turn our focus to analyze the factors that may affect the public acceptability of congestion pricing and review the existing models that aim to design more acceptable pricing schemes.

### **2.1 Traffic Assignment Problem**

Traffic assignment is the process that allocates a given set of trips to a specific transportation network or system (Patriksson, 1994). Traffic assignment problem is the problem that uses given origin-destination (OD) demand to estimate traffic conditions, such as traffic flow, travel time or cost on each road link, in the transportation system. Traffic assignment problem is an essential part of transportation system analysis, as it serves as the basis for various analysis needs. In this section, we briefly review the history of the traffic assignment problem development and existing traffic assignment models.

#### **2.1.1 User Equilibrium and System Optimal Conditions**

Wardrop (1952) introduced two principles in travel behavior in transportation systems:

##### **Wardrop's first principle:**

*The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route.*

**Wardrop's second principle:**

*The average journey time is a minimum.*

Wardrop's first principle describes the travelers' selfish non-corporative route choice behavior, where all users try to minimize their own travel times in the network. This principle will result in a state that the travel time on all utilized paths are equal and less than that on any unused paths for any specific OD pair. Under such a condition, no user can further reduce their travel time by unilaterally switching their travel paths. This situation is believed to be most close to reality and is usually used to predict the real traffic conditions in practice. Wardrop's second principle states a corporative route choice behavior of travelers in the network such that all users decide their travel routes in order to achieve minimum total system travel time. This condition is the best system performance that can be ever achieved under a certain demand level, but is unlikely to happen without the coordination or intervention from a central government or regulator. The traffic conditions under these two principles are later termed as user equilibrium (UE) and system optimum (SO).

Beckmann et al. (1956) in their seminal work formulated the traffic assignment problems as convex nonlinear optimization problems with linear constraints. Let  $N$  and  $L$  denote the sets of nodes and links in a general transportation network. Let  $W$  be the set of OD pairs. For each travel link  $l$  in the network,  $x_l^w$  denotes the OD specific flows for OD pair  $w$ ,  $v_l$  is the aggregate link flow. The travel time on link  $l$  depends on the aggregate link flow and can be represented by a link travel time function  $t_l(v_l)$ .  $A$  is the link-node incidence matrix and  $E$  is an "input-output" vector with  $|N|$  elements which has exactly two non-zero elements: one has a value 1 corresponding with the origin node of

the OD pair  $w$  and the other has a value of -1 for the destination node. Assuming the OD demand  $d^w$  is given and fixed for every OD pair  $w$ . We have the following UE flow assignment problem:

P-UE:

$$\min_{v,x} \sum_{l \in L} \int_0^{v_l} t_l(\omega) d\omega$$

$$s. t. Ax^w = E^w d^w, \quad \forall w \in W \quad (2-1)$$

$$v_l = \sum_{w \in W} x_l^w, \quad \forall l \in L \quad (2-2)$$

$$x_l^w \geq 0, \quad \forall l \in L, w \in W \quad (2-3)$$

where  $x^w$  is the vector of all link flows  $x_l^w$  for each OD pair  $w$ .

Beckmann et al. (1956) proved the equivalence between the optimality condition of the above mathematical programming problem and Wardrop's first principle. Moreover, when the link travel time function  $t_l(v_l)$  is continuous and non-decreasing with respect to the aggregate link flow  $v_l$ , the optimal solution to the above UE assignment will be the unique UE flow distribution for the network (Patriksson, 1994).

Similarly, the SO flow assignment problem can be formulated as:

P-SO:

$$\min_{v,x} \sum_{l \in L} t_l(v_l) v_l$$

$$s. t. (x, v) \in X$$

where  $X$  is the feasible region of link flow  $x$  defined by Constraints (2-1), (2-2) and (2-3).

### 2.1.2 Alternative Formulations of User Equilibrium Traffic Assignment Problem

In this section, we will present two alternative mathematical formulations, i.e. nonlinear complementarity problem and variational inequality problem (Patriksson, 1994) of UE traffic assignment problem other than P-UE. These models are commonly used in the literature to solve UE assignment problems. We will also demonstrate the equivalence of the three models as well as their relationship to the Wardrop's first principle.

UE-VI:

$$\sum_{w \in W} \sum_{l \in L} t_l(v_l^*) \cdot (x_l^w - x_l^{w*}) \geq 0, \quad \forall x \in X \quad (2-4)$$

$$(x^*, v^*) \in X$$

The problem UE-VI is a variational inequality (VI) problem (Facchinei and Pang, 2003). The VI formulation of traffic assignment problem was first proposed by Smith (1979) and independently developed by Dafermos (1980) and Florian and Spiess (1982). It can be easily shown that the Karush-Kuhn-Tucker (KKT) conditions of UE-VI are equivalent to the KKT condition of P-UE. Thus, the solution  $(x^*, v^*)$  that satisfy condition (2-4) is a valid UE flow distribution.

The following problem is proposed by Aashtiani and Magnanti (1981) as an alternative formulation to the VI for general traffic assignment problem.

UE-NCP:

$$x_l^w (t_l(v_l) + \rho_i^w - \rho_j^w) = 0, \quad \forall l \in L, l \in L_i^+, l \in L_j^-, w \in W \quad (2-5)$$

$$t_l(v_l) \geq \rho_j^w - \rho_i^w, \quad \forall l \in L, l \in L_i^+, l \in L_j^-, w \in W \quad (2-6)$$

$$(x, v) \in X$$

where  $\rho_i^w$  is called the node potential (Ahuja et al., 1993) for node  $i$  and OD pair  $w$ . The difference of two node potentials for different nodes indicates the shortest travel time between the two nodes for the specific OD pair.  $L_i^+$  and  $L_i^-$  are sets of links start from node  $i$  and end with node  $i$ , respectively.

The above system is a nonlinear complementarity problem (NCP). Constraint (2-6) guarantees that the travel time for any link in the network is greater or equal to the shortest travel time between the starting point and end point of the link of any directed route as defined by the difference of the node potentials. Further, Constraint (2-5) states that there will be positive flow on a certain link only when the travel time of that link is equal to the shortest travel time between the two nodes. These two conditions are actually the definition of the Wardrop's first principle. Therefore, a feasible solution to the system UE-NCP is a UE distribution (Patriksson, 1994).

Moreover, UE-NCP is exactly the KKT conditions of problem P-UE, which again shows that the optimal solution to P-UE satisfies Wardrop's first principle.

### 2.1.3 Traffic Assignment with Elastic Demand

In transportation system analysis, the traffic assignment problem is sometimes formulated as a problem with elastic (or variable) demands, in order to capture the impact of travel time on trip generation. Assume travel demand  $d^w$  follows a demand function  $D^w(\mu^w)$  with respect to the smallest travel time  $\mu^w$  between OD pair  $w$ , define the inverse demand function as  $D_w^{-1}$ . The UE problem with elastic demand is formulated as follows:

UE-ED:

$$\begin{aligned} \max_{v,x,d} \quad & \sum_{w \in W} \int_0^{d^w} D_w^{-1}(\omega) d\omega - \sum_{l \in L} \int_0^{v_l} t_l(\omega) d\omega \\ \text{s. t.} \quad & (x, v, d) \in X \end{aligned}$$

Wardrop's second principle is limited in fixed demand situation where the total demand for the network is assumed to be fixed. In order to apply a similar concept to elastic demand case, the SO condition is extended as the traffic flow distribution that maximizes total social benefit. The mathematical program for SO condition can be then formulated as:

SO-ED:

$$\begin{aligned} \max_{v,x,d} \quad & \sum_{w \in W} \int_0^{d^w} D_w^{-1}(\omega) d\omega - \sum_{l \in L} t_l(v_l) v_l \\ \text{s. t.} \quad & (x, v, d) \in X \end{aligned}$$

The elastic demand UE model UE-ED is similar in structure with the fixed demand case. Actually, it is possible to transform the elastic demand problem to a fixed demand problem by introducing artificial excess-demand links (Gartner, 1980). The excess-demand links connect the origin and destination of each OD pair. An overestimate of demand is defined as the fixed demand of each OD pair. The excess demand is carried on the excess-demand link whose travel time function is obtained from the inverse demand function for each OD pair.

#### 2.1.4 Stochastic Traffic Assignment Problem

The aforementioned UE problems assume that all travelers have accurate information about travel time on each link and all travelers being uniform and rational in their decision making. In other words, every traveler's behavior in such cases can be

accurately predicted. Such models are therefore known as deterministic user equilibrium (DUE) models. However, such assumptions may not hold in reality, e.g., travelers' perceptions of travel times may subject to variations. In 1977, Daganzo and Sheffi (1977) extended Wardrop's first principle as:

*At stochastic user equilibrium (SUE), no user believes he can improve his travel time by unilaterally changing routes.*

SUE describes the traffic condition as a result of that travelers do not necessarily have accurate travel time information. Assume the perceived travel time for a traveler in the network is:

$$C_k^w = \tilde{C}_k^w + \xi_k^w \quad (2-7)$$

where  $C_k^w$  is the perceived travel time for path  $k$  connecting OD pair  $w$ ,  $\tilde{C}_k^w$  is the actual path travel time.  $\xi_k^w$  is a random variable that represents the perception error of the travelers.

Let  $P_k^w$  be the probability that route  $k$  is perceived as the shortest path for OD pair  $w$  and  $f_k^w$  be the traffic flow on path  $k$  for OD pair  $w$ . The user equilibrium condition can be characterized by the following equation:

$$d^w P_k^w = f_k^w, \quad \forall w \in W, k \in K^w$$

The corresponding SUE model can be formulated as the following unconstraint mathematical program (Sheffi and Powell, 1982):

P-SUE:

$$\min_v - \sum_{w \in W} d^w E[\min_{k \in K^w} \{C_k^w(v)\}] + \sum_{l \in L} t_l(v_l) v_l - \sum_{l \in L} \int_0^{v_l} t_l(\omega) d\omega$$

where  $E[\min_{k \in K^w} \{C_k^w(v)\}]$  is the expectation of the smallest perceived travel time for all paths connecting OD pair  $w$ .

DUE condition is a special case of SUE condition by letting the variance of the random variable  $\xi$  to be zero.

The SUE problem is one application of the discrete choice model (Ben-Akiva, 1985). The most commonly used SUE model is the multinomial logit model, which is based on the assumption that the random residuals  $\xi$  in Equation (2-7) are independently and identically distributed (i.i.d.) as Gumbel random variables with zero mean and scale parameter  $\theta$ . Based on this assumption, the probability that a route is chosen can be written as:

$$P_k^w = \frac{\exp\left(\frac{\tilde{u}_k^w}{\theta}\right)}{\sum_{k' \in K} \exp\left(\frac{\tilde{u}_{k'}^w}{\theta}\right)}$$

The use of logit model in traffic assignment problem was first introduced by Dial (1971) for the case of constant travel times. In 1980, Fisk (1980) presented the following logit based SUE assignment model:

SUE-L:

$$\begin{aligned} \min_{v,f} \quad & \sum_{l \in L} \int_0^{v_l} t_l(\omega) d\omega + \frac{1}{\theta} \sum_{w \in W} \sum_{k \in K^w} f_k^w \ln f_k^w \\ \text{s. t.} \quad & \sum_{k \in K} f_k^w = d^w, \quad \forall w \in W \\ & \sum_{w \in W} \sum_{k \in K^w} \delta_{lk} f_k^w = v_l, \quad \forall l \in L \\ & f_l^w \geq 0, \quad \forall l \in L, w \in W \end{aligned}$$

where  $\delta_{lk}$  is the link-path incidence.  $\delta_{lk} = 1$  when link  $l$  is on path  $k$ ,  $\delta_{lk} = 0$  otherwise.

When  $\theta = \infty$ , problem SUE-L reduces to P-UE, which suggests that the DUE assignment problem is a special case of the logit based SUE problem.

### **2.1.5 Traffic Assignment Problem on Multimodal Networks**

There is a long history of developing multimodal transportation network models. For a review, see e.g., Uchida et al. (2007), De Cea et al. (2005) and Ceder (2007). Early developments of multimodal traffic assignment problem mainly focused on the representation of the static strategic route choice of transit passengers (Chriqui and Robillard, 1975; Nguyen and Pallottino, 1988; Spiess and Florian, 1989; De Cea and Fernandez, 1989). This problem is usually addressed as the common line problem. Such problem arises from the fact that transit users usually have more complicated route choice behaviors with the opportunities of transfer and waiting. Instead of choosing a single shortest path, transit passengers may pre-select a set of “attractive routes” and board on the first vehicle arrived that serves one of those “attractive routes”. In order to tackle this problem, Spiess and Florian (1989) and Nguyen and Pallottino (1988) introduced the notion of strategies or hyperpaths where the user’ route choice is described as a set of possible routes.

Later developments in transit models mainly focus on capturing congestion effects of the transit service. De Cea and Fernandez (1993) and Wu et al. (1994) considered congested networks where waiting time and in-vehicle costs (crowding effect) are functions of passenger flows. Lam et al. (1999) considered explicit capacity constraints of transit lines and estimated the additional waiting times by examining the Lagrangian multipliers associated with the capacity constraints. These models can be called “semi-congested” models because they only consider the increase of waiting time (Cepeda et al., 2006). Congestion at stops does not only increase the waiting times but also affects the flow splits among the attractive lines. The models that capture the latter effect are called “full-congested” (Cepeda et al., 2006). Examples of the full-congested models

include Bouzaiene-Ayari et al. (2001), Cominetti and Correa (2001) and Cepeda et al. (2006). In the latter two, an effective frequency function is introduced, which decreases with the passenger flows and vanishes when flows exceed the capacity of the bus line.

In the mid-1990, researchers started to consider schedule-based transit assignment problem where the timetable of transit services are given (Hickman and Bernstein, 1997). Hamdouch and Lawphongpanich (2008; 2010) recently developed a model for schedule-based transit assignment problem that considers the capacity of transit service.

## **2.2 Congestion Pricing**

The principle of congestion pricing was first introduced by Pigou (1920) and Knight (1924) in the early 1920s. Their work was later elaborated and extended by many researchers, e.g., Walters (1961), Mohring and Harwitz (1962), Bechmann et al. (1956) and Vickrey (1969). The original idea of congestion pricing is based on the fact that road use has negative externality. Thus by charging the marginal cost to internalize the externalities, social benefit is able to be maximized (Button, 1993).

### **2.2.1 Marginal Cost Pricing**

#### **2.2.1.1 Canonical model of marginal cost pricing**

The concept of congestion pricing mainly arose from the field of economics. Assuming a single road network with homogenous travelers and elastic demand, Figure 2-1 explains the effect of marginal cost pricing. The MC curve represents the marginal social cost with respect to traffic flow. The marginal social cost is the additional cost of adding one more traveler to the road, which includes the cost for the newly-added traveler herself and the extra cost experienced by other travelers already on the road. The AC curve is the average private cost that only considers the cost of the new traveler

herself. As the addition of a traveler will always cause extra delay to the others, the MC curve is always higher than AC curve. The D curve is the inverse demand function. The social benefit can be calculated as the area between the inverse demand (D) curve and marginal cost (MC) curve (Hau, 2005). As road users do not perceive the marginal social cost but rather the average private cost when they make their travel decisions, an equilibrium point in this condition exists when the AC curve intersects with the D curve (point a) with traffic flow  $h$  and average travel cost  $f$ , where total social benefit is the area of  $bde$  minus area of  $abc$ . However, the social optimal point is at point b, the intersection of MC curve and D curve, with a total social benefit as the area of  $bde$ , where flow is  $l$  and average travel cost is  $j$ .

The idea of marginal cost pricing is to charge the road users with a toll rate set as the difference of the marginal cost and average cost. In the case shown in Figure 2-1, the toll rate will be set to be the value of  $g-j$ , the difference of marginal cost and average cost at point  $b$ . Under such a tolling scheme, the average cost curve will be moved up (AC') and intersect with D curve at point  $b$ . As a result, a new equilibrium will be established at point  $b$ , which maximizes the social benefit (Walters, 1961).

Let  $t(v)$  denote the travel cost function of the toll road, where  $v$  is the traffic flow. The marginal cost is defined as the change of total travel cost with respect to flow and can be calculated as

$$MC(v) = \frac{\partial t(v) \cdot v}{\partial v} = t(v) + \frac{v \partial t(v)}{\partial v}$$

The marginal cost toll is the difference between marginal cost  $MC(v)$  and average cost  $t(v)$ . Therefore, the marginal cost toll  $\tau(v)$  is calculated as

$$\tau(v) = v \frac{\partial t(v)}{\partial v} \quad (2-8)$$

### 2.2.1.2 Extensions to general transportation networks

Now, we come back to the mathematical formulation of the traffic assignment problems and examine the effect of marginal cost pricing scheme. We first present the tolled UE problem. The tolled UE condition is the traffic condition such that every traveler minimizes his/her own generalized travel cost  $\bar{t}$ . For each travel link, the generalized travel cost is the sum of travel time and toll charge, which can be represented as:

$$\bar{t}_l(v_l) = t_l(v_l) + \tau_l$$

With the above definition, the tolled UE problem can be formulated as:

TUE-MP:

$$\begin{aligned} \min_{v,x} \quad & \sum_{l \in L} \int_0^{v_l} t_l(\omega) d\omega + \sum_{l \in L} \tau_l v_l \\ \text{s. t.} \quad & (x, v) \in X \end{aligned}$$

Problem TUE-MP is a simple modification of the original UE assignment problem P-UE by replacing the link travel time function  $t_l(v_l)$  with the generalized link travel cost  $\bar{t}_l(v_l)$ . As P-UE generates the UE flow distribution associated with the travel time function  $t_l(v_l)$ , the optimal solution of problem TUE-MP will be UE flow distribution subject to the generalized travel cost. Therefore, TUE-MP is the tolled UE assignment problem.

By setting the toll rate to the marginal cost toll as in Equation (2-8), it can be easily shown that the tolled UE assignment problem TUE-MP generates the same optimal solution as the SO problem. This confirms mathematically the fact that marginal cost pricing will drive the system from UE to SO condition.

### 2.2.1.3 First-best toll set

Under the marginal cost pricing, the flow under the tolled UE condition is identical to the SO condition. This pricing scheme is called first-best pricing scheme in the literature, as it drives the traffic condition to the best possible one (SO). However, as pointed out by Hearn and Ramana (1998), marginal cost pricing, defined by Equation (2-8), is not the only pricing scheme that can achieve SO condition. Actually, marginal cost pricing is only one element of the so-called first-best toll set, which contains all the first-best pricing schemes.

To derive the first-best toll set, one may consider the problem as to find a toll vector  $\tau$  such that the SO flow satisfies the UE conditions. More specifically,

$$x_l^w (t_l(v_l^{SO}) + \tau_l + \rho_i^w - \rho_j^w) = 0, \quad \forall l \in L, l \in L_i^+, l \in L_j^-, w \in W$$

$$t_l(v_l^{SO}) + \tau_l \geq \rho_j^w - \rho_i^w, \quad \forall l \in L, l \in L_i^+, l \in L_j^-, w \in W$$

$$v_l^{SO} = \sum_{w \in W} x_l^w, \quad \forall l \in L$$

Conditions (2-1), (2-3)

where  $v_l^{SO}$  is the traffic flow in SO condition.

This formulation can be further simplified by summing over all the OD pairs and links:

$$\sum_{l \in L} (t_l(v_l^{SO}) + \tau_l) v_l^{SO} = \sum_{w \in W} d^w (E^w)' \rho^w$$

$$\sum_{l \in L} \delta_{lk} (t_l(v_l^{SO}) + \tau_l) \geq (E^w)' \rho^w, \quad \forall w \in W, k \in K^w$$

where  $K^w$  is the set of all paths between OD pair  $w$ .

### 2.2.2 Second-Best Road Pricing

While the first-best pricing schemes can achieve the maximum system efficiency, there usually are certain restrictions for congestion pricing schemes for political reasons or due to the difficulties of real-world implementation that prevent the implementation of first-best pricing. For example, first-best pricing requires charging on every link of the network, which may be practically difficult and inefficient. For the case of heterogeneous travelers, first-best pricing scheme may require to charge different toll rates for different types of travelers, who may not be practically distinguishable (Arnott and Kraus, 1998). The existence of these restrictions prevents the tolling scheme to achieve the optimal (or first-best) traffic condition. The pricing schemes that maximize social benefit under such restrictions are usually called second-best pricing schemes, as they cannot achieve the best condition (Florian and Hearn, 2003).

Practically, all pricing schemes are second best. Examples of second-best pricing schemes include area-based pricing, e.g., Maruyama and Sumalee (2007), and cordon pricing, e.g., Zhang and Yang (2004). HOT lanes are also second-best implementations of congestion pricing as only part of all the road users are charged. Parking charges can also be considered second-best pricing as parking space can be considered as toll links (Verhoef, 2002).

### 2.2.3 Network Toll Design Problem

The network toll design problem has received numerous interests in the transportation science community, especially in the past two decades. The toll design problem is usually formulated as a bi-level mathematical program (Clegg et al., 2001; Yang and Bell, 1997). A general form of the bi-level formulation is presented below:

TDP-BL:

$$\begin{aligned} \max_{\tau} \quad & z(x, v, \tau) \\ \text{s. t.} \quad & h(\tau) \geq 0 \\ & (x, v) \text{ solves TUE-MP} \end{aligned} \tag{2-9}$$

The objective  $z$  is some system performance measure, such as social benefit (Yang and Lam, 1996; Ferrari, 1999; Verhoef, 2002), toll revenue (Dial, 1999) or the total number of toll roads (Hearn and Ramana, 1998). Constraints (2-9) are some restrictions to the pricing scheme, e.g., only certain links in the network can be tolled (Yang and Lam, 1996), or tolls can be charged for only a certain type of road travelers, such as solo drivers on HOT lanes. The resulting pricing scheme will be first-best if no such constraint exists in the model (Hearn and Ramana, 1998). The lower level problem is the tolled user equilibrium problem TUE-MP or other more general UE assignment problems, e.g., SUE.

We can use the alternative formulation, i.e. NCP, of the tolled UE problem to replace the mathematical programming model TUE-MP and the formulation for the toll design problem is changed to (Lawphongpanich and Yin, 2010):

TDP-MPCC:

$$\max_{x,v,\tau} z(x, v, \tau)$$

s. t. Constraint (2-9)

$$x_l^w(t_l(v_l) + \tau_l + \rho_i^w - \rho_j^w) = 0, \quad \forall l \in L, l \in L_i^+, l \in L_j^-, w \in W$$

$$t_l(v_l) + \tau_l \geq \rho_j^w - \rho_i^w, \quad \forall l \in L, l \in L_i^+, l \in L_j^-, w \in W$$

$$(x, v) \in X$$

Problem TDP-MPCC is a mathematical program with equilibrium constraints (MPEC) (Luo et al., 1996). Although TDP-MPCC appears to be a single level problem, the existence of the equilibrium constraints still makes the problem hard to solve (Friesz et al., 1990). In particular, TDP-MPCC violates the Magasarian-Fromovitz constraint qualification (MFCQ). This renders standard algorithms for nonlinear programs ineffective.

In order to solve the problem, researchers have developed various algorithms. Most of the algorithms are heuristics, including sensitivity-analysis-based approach (Yang and Lam, 1996; Ying and Yang, 2005), pattern-search-based algorithm (Abdulaal and LeBlanc, 1979), linearization method (LeBlanc and Boyce, 1986), relaxation method (Ban et al., 2006), simulated annealing method (Friesz et al., 1992), genetic-algorithms-based approach (Yin, 2000; Shepherd and Sumalee, 2004), cutting plane method (Hearn and Yildirim, 2001), heuristic method based on “location index” (Verhoef, 2002), cutting constraint method (Lawphongpanich and Hearn, 2004) and manifold suboptimization (Lawphongpanich and Yin, 2010).

#### **2.2.4 Congestion Pricing in Multimodal Networks**

Multimodal network equilibrium modeling has been well studied in the literature, e.g., Florian (1977); Abdulaal and Leblanc (1979); Florian and Nguyen (1978); Garcia and Marin (2005). For a recent overview, see, e.g., De Cea et al. (2005). Although the same principle still applies, the pricing models previously developed for vehicular traffic networks do not lend themselves for multimodal transportation networks. The asymmetric nature of travel costs in multimodal transportation networks imposes a great challenge for both model formulation and solution algorithm. Among existing studies, Gentile et al. (2005) presented a multiclass equilibrium model for multimodal networks and compare cordon pricing and rationing policies in an effort to improve social welfare. Hamdouch et al. (2007) extended the toll pricing framework for vehicular traffic network to multimodal transportation networks and demonstrate its usefulness in estimating the impact on the transit ridership due to toll pricing and transit fare adjustment.

#### **2.3 Empirical Results for Public Acceptance of Congestion Pricing**

Although congestion pricing has been demonstrated both theoretically and empirically to be able to successfully reduce traffic congestion and are gaining more and more support from politicians and transportation officials, the general public is still quite skeptical about road pricing (Jaensirisak et al., 2005; Schade and Baum, 2007). Research suggests that acceptability is essential to guarantee the success of congestion pricing and has identified acceptability as the main obstacle for any implementation of road pricing (Sikow-Magny, 2003). In this section, we will briefly review the factors that may affect public acceptability of congestion pricing suggested by empirical studies (for more extensive review, see e.g., Jaensirisak et al., 2005).

### **2.3.1 Effectiveness of Congestion Pricing**

The effectiveness of pricing schemes is viewed by travelers as how congestion is reduced or how they can benefit from the implementation. Steg (2003) identified the perceived effectiveness of the pricing measure as one of the important factors that affect public acceptance. In addition, Rienstra et al. (1999) found that the acceptance of policy measures increases if people are more convinced about the effectiveness of these measures based on a survey conducted in Netherlands on transportation policy. It is also found that the higher the toll charges are, as expected, the less acceptable the pricing scheme is (Ubbels and Verhoef, 2006). So it is preferred that the congestion pricing scheme can improve system efficiency by charging the minimum toll rates.

However, a traveler usually considers the benefit of himself from the pricing scheme more than the society as a whole (Jaensirisak et al., 2003). This is because the individual usually does not see himself/herself as responsible for causing the problem of congestion (Schade and Schlag, 2003). It can be described as the so-called “the tragedy of the commons” (Hardin, 1968). This suggests that in order to make congestion pricing more acceptable, it is very important to increase every individual’s utility besides the total social benefit.

### **2.3.2 Equity Issues**

Equity issues are considered one of the most relevant objections to congestion pricing. Many equity problems have been raised in practice concerning the impact of pricing to low-income travelers and the use of toll revenue. Jakobsson et al. (2000) and Fujii et al. (2004) analyzed responses from car owners in European and Asian countries and found equity (or fairness) plays an important role in the acceptance of road pricing. Other research also found that the perceived or real unfairness due to income and

regional differences may also result in a low level of acceptance (Emmerink et al., 1995; Langmyhr, 1997). Weinstein and Sciara (2006) analyzed equity concerns associated with HOT lane projects and concluded that equity issues may arise at any stage of the project development, thus continuous monitoring of equity implications of the projects is required both before and after the opening of the HOT lanes.

### **2.3.3 Revenue Distribution**

How to use the toll revenue also plays a significant role in the public acceptability of road pricing. Verhoef (1996) surveyed morning commuters for their opinion on road pricing. An overwhelming majority (83%) stated that his/her opinion depends on the allocation of toll revenues. Jaensirisak et al. (2005) summarized previous studies on congestion pricing acceptability and found the mean acceptability was 35 percent when there was no explicit mention of revenue usage and 55 percent when some specific plan to use the revenue is presented.

Among all the possible usage of the toll revenue, enhancing the performance of alternative travel modes is one of the most favorable options by travelers. Jones (1991) found acceptability increased from 30 percent where there was no explicit mention of the use to which the revenue would be put to 57 percent when it was stated that the revenue would be used for improving public transport and facilities for pedestrians and cyclists. In another empirical study on the public acceptability of different pricing strategies in four European cities: Athens, Como, Dresden and Oslo (Schade and Schlag, 2003), it was found that the use of toll revenue for improvements of public transportation is favored by the vast majority of respondents.

## 2.4 Equity Effects of Congestion Pricing

The previous section reviews empirical results on the public acceptance of congestion pricing. The empirical evidences show that the acceptability of a congestion pricing proposal centers on its perceived equity. In this section, we elaborate on the equity issues within the transportation system, particularly on congestion pricing.

### 2.4.1 Social Welfare and Equity

Social welfare is an economic term that can be traced back over one and a half century (Button, 1979). Welfare refers to the overall well-being of people, either as individuals or collectively. In a recent review, Levinson (2010) gave the following description of social welfare<sup>1</sup>:

*Social welfare comprises both efficiency, a measure of the degree to which system outputs achieve a theoretical maximum (minimum) using the same level of inputs, and equity, a measure of the distribution of outputs (or inputs) across some population.*

For transportation system, efficiency is a relatively well-defined concept. An efficient system means less delay, lower travel cost or higher utility level. However, equity is more subjective and may be viewed differently from different perspectives. A particular decision may seem equitable when evaluated one way but inequitable when evaluated another way. Moreover, equity is more of a descriptive and normative term. It is generally considered based on comparing of outcomes of different policies, i.e. policy A is more equitable than policy B rather than saying policy A is equitable and policy B is not.

---

<sup>1</sup> Social welfare is distinguished from social benefit. In this dissertation, the term “social benefit” only concerns efficiency of the society.

### 2.4.1.1 Definitions of equity

Various types of equity have been defined in the literature (for review, see, e.g., Levinson, 2010, Banister, 1994). The definitions that are more relevant to transportation and road pricing are listed below:

- Horizontal equity (fairness): The distribution of impacts between similar (in e.g., income, gender, ability, need) individuals and groups. According to this definition, individuals and groups within the same class should receive equal shares of resources and opportunities, bear equal costs and in other ways be treated similarly. A policy is horizontally equitable if similar individuals are provided with equal opportunities or are made equally well off under the policy.
- Vertical equity (justice): The distribution of impacts between different individuals and groups. This definition says that individuals and groups with different backgrounds should be treated the same way (given same opportunities, social resources, etc.). In other words, primary social goods (liberty, opportunity and wealth) should be distributed equally or to favor less advantaged people (Rawls, 1971). Consequently, it is generally considered vertically equitable if a policy gives more benefit to economically and socially disadvantaged (i.e. low-income) individuals and groups.

Other equity definitions also exist such as spatial equity, describes how benefits are distributed over space, e.g., different OD pairs, (Viegas, 2001); temporal equity (longitudinal or intergenerational equity), describes how benefits are distributed to the present or the future (Viegas, 2001); market equity, describes how benefits are distributed proportional to the price paid (Levinson, 2010); social equity, describes how benefits are allocated proportional to need (Jones, 2003).

The different types of equity often overlap and conflict with each other. For example, spatial equity requires individuals at different location be treated similarly, which may be considered as one kind of vertical equity as individuals can be grouped by locations. Horizontal equity requires that everybody bear the costs of their services, but vertical equity often requires subsidies for disadvantaged people.

It may depend on what type of equity is used when deciding if a policy is equitable or not. Actually, demanding equality in one space implies inequality in some other space (Sen, 1992). For example, consider the transportation financing approach by using tax revenue. As roads are financed using local tax revenue, non-local users basically get the service for free. Therefore, financing the road using toll revenue is considered more equitable as everybody using the service pays. However, such toll charges will create a larger burden for low-income users thus hurt the low-income users more than the others and are less equitable when considering vertical equity.

#### **2.4.1.2 Measures of equity**

Ramjerdi (2006) summarized a number of potential measures of inequality: range, variance, coefficient of variation, relative mean deviation, logarithmic variance, variance of logarithms, Gini coefficient, Theil's entropy, Atkinson index, Kolm's measure. Other equity measures may also consider factors other than cost, such as accessibility (Feng et al., 2010) and environmental impact. However, there is no consensus among researchers that which dimension of equity should be considered or which equity measure should be used when analyzing equity effects. In fact, most studies in the literature use their own measures depending on their objectives and definitions of equity. It is difficult, if not impossible, for one single study to consider all of the dimensions and all measures for each dimension (Levinson, 2010).

#### **2.4.2 Equity Effects of Congestion Pricing**

First of all, congestion tolls can be a substantial part of household expenditure, therefore significantly affecting the welfare of travelers. The U.K. Government Statistical Survey (1992) reported that the average household in U.K. spent 15.4 percent of their weekly expendable income to transportation for an average of £39.70. A toll charge of

£3-5 thus can substantially affect the household budget. Moreover, the toll charge will have more significant impact for families with lower income. For a £4 daily toll price, the transportation expenditure would account for more than 30 percent of household expenditure for low income groups, while the toll charges may only be less than 5 percent of household budget for high income families (Banister, 1994). This suggests that even for a relatively low toll rate, the impact of congestion pricing on the travelers can be significant and may substantially affect the equity of the society.

There are two major equity concerns for congestion pricing:

**Distribution of benefit among road users and the rest of the community**

**(horizontal equity):** If the toll revenue is spent outside the transportation system, it can be considered as the road users are cross-subsidizing the other part of the society. And the net benefit of the pricing scheme may be compromised. Thus it is generally suggested that the toll revenue to be spent within the transportation system, such as on improving transit service and increasing highway capacity.

**Distribution of welfare between users with different incomes (vertical equity):**

This is generally referred to as the progressivity and regressivity of congestion pricing. A transportation policy is progressive if it favors the disadvantaged travelers (e.g., travelers with lower income), while it is regressive if it imposes more burden to the disadvantaged travelers.

Numerous efforts have been made to analyze the equity effect of congestion pricing. Niskanen (1987) analyzed the consumer welfare of road users with different valuations of time and concluded that uncompensated congestion charge may unambiguously worsen the road users' aggregate position. Small (1983) and Hau (2005)

also found that the “average commuters” under congestion pricing would be somewhat worse off without revenue redistribution policies. However, the results by Santos and Rojey (2004) show that effect of congestion pricing are location specific and depend on where people live, where people work and what mode of transport they use, and the average impact can be beneficial even without redistribution of toll revenue.

Moreover, uncompensated pricing schemes are usually regressive. Layard (1977) analyzed the equity effect of congestion pricing and suggested that the toll charges deter travel of low-income travelers and encourage high-income travelers to travel more, thus it becomes more likely that the pricing scheme would be regressive. The majority of existing studies (e.g., Glazer, 1981; Small, 1983; Jones, 1992; Hau, 2005; Arnott et al., 1994; Evans, 1992 and many more) suggested that uncompensated user charge can only benefit users with high value of travel time, and low-income car owners will suffer the greatest detrimental impact, particularly if adequate public transport is not available. Giuliano (1992) further pointed out by citing the work of Gomez-Ibanez (1992) that the loser groups to congestion pricing are potentially large and represent a wide range of income levels.

At the same time, Foster (1974) viewed congestion pricing as progressive by arguing that richer people generally paid more as they produced more trips, which is later challenged by Richardson (1974). Schweitzer and Taylor (2008) compared congestion pricing and sales taxes for financing transportation facilities and concluded that congestion pricing was more equitable than sales taxes as low-income residents group, on average, pay more out-of-pocket with sales taxes.

Although there is still no definite answer for the equity impact of congestion pricing (due to the different ways equity is framed and evaluated), researchers do share the same belief that equity can be improved by redistributing the toll revenue. Small (1983) suggested that every income class can benefit from congestion pricing under certain revenue redistribution methods. He (Small, 1992) later showed a lump sum refunding scheme (an equal travel allowance for all commuters) can make congestion pricing progressive. Arnott et al. (1994) also concluded that although uncompensated pricing is regressive, the effect of pricing with revenue refunding can be progressive.

### **2.5 Design of More Equitable and Acceptable Congestion Pricing Schemes**

Most of the pricing models in the literature focus on maximizing social benefit or the revenue of the toll road operators. However, such schemes are generally lack of public supports due to equity concerns. In order to account for this shortcoming, various studies have implicitly or explicitly incorporated equity measures in their design models. Dial (1999; 2000) considers an alternative pricing scheme that minimizes revenue as well as the system travel times. Daganzo (1995) proposed a hybrid scheme that involves both rationing the right to travel on the highway for free and charging the users being rationed. He concluded, by analyzing a bottleneck model, that such hybrid scheme can benefit everyone even if the collected revenues are not returned to population. However, a later study by Nakamura and Kockelman (2002) suggested that it is hard to find such a hybrid scheme in an application to the San Francisco Bay Bridge corridor.

Adler and Cetin (2001) considers a revenue redistribution scheme that directly transfers toll revenue from users on more desirable routes to users on less desirable routes. By analyzing a simple parallel link network they claimed that this scheme can

reduce travel cost for all travelers while maintaining the total system efficiency. Eliasson (2001), based on a similar setting, suggested that any toll reform that reduces aggregate travel time and redistributes the toll revenues equally to all users makes everyone better off.

Bernstein (1993) proposed a revenue-neutral pricing scheme that charges for peak hour while subsidizing users traveling during the off-peak. He stated that this approach will correct the distributional impacts of congestion pricing.

More recently, Lawphongpanich and Yin (2007) investigated Pareto-improving tolls on general road networks that will make nobody worse off compared to the no-toll condition. Their computational experiments (Lawphongpanich and Yin, 2010; Song et al., 2009) suggest that such schemes are relatively prevalent, although they may not lead to a significant level of improvement. Guo and Yang (2010), Liu et al. (2009) and Nie and Liu (2010) further investigated the existence of Pareto-improving schemes with revenue redistribution in networks with multiple user classes or with travelers with continuously distributed value of time (VOT). Their results suggested that a Pareto-improving pricing scheme usually exists. However, this scheme may require class-specific refunding, i.e. refunding differently to users with different value of time or from different OD pairs. The existence of anonymous refunding schemes usually requires more restrictive conditions, e.g., the difference of travel time changes among OD pairs should not be too large (Guo and Yang, 2010) or the VOT distributions of the users are concave (Nie and Liu, 2010).

In addition to the aforementioned efforts, various network toll design models have been proposed to explicitly consider social and spatial equity (impact of pricing between

different OD pairs), e.g., Yang and Zhang (2002), using the Theil measure of equity, e.g., Connors et al. (2005) or Gini coefficient, e.g., Sumalee (2003). Szeto and Lo (2006) developed a general time-dependent design model based on a gap function that measures the degree of intergeneration equity to design the equity-based optimal tolls. Ho and Sumalee (2010) proposed a continuum model for designing pricing strategies that are equitable, acceptable and socially beneficial.

## **2.6 Contributions of This Dissertation**

In the previous sections, we reviewed several factors that have been identified in the literature to have significant impact on the public acceptability of congestion pricing. Although efficiency is viewed as a very important factor, a successful congestion pricing scheme cannot only consider the overall efficiency of the transportation system. The literature suggests that a traveler usually considers the benefit of himself from the pricing scheme more than the society as a whole. Thus it is important to increase every individual's utility as long as the total social benefit for a congestion pricing design.

Based on this idea, researchers have proposed Pareto-improving pricing schemes. Such schemes maximizes the social benefit while guarantees that nobody will be worse off in term of generalized travel cost (travel time plus toll charge) under the pricing scheme when compared to the no-toll condition. Table 2-1 shows the existing literature on the design of Pareto-improving pricing schemes.

Previous studies on Pareto-improving pricing schemes mainly focus on analyze the existence of such a tolling scheme under various network conditions. Their research either relies on the analysis of either a rather small network or a single travel mode, and none of them have considered using the toll revenue to expand the capacity of the existing network in their design models. Their results can be only used as guidelines for

congestion pricing development but are not able to develop a comprehensive congestion pricing strategies that include multiple instruments, such as tolling, revenue refunding, subsidies and development of transit and highway networks. In this dissertation, we try to fill this gap by considering multiple travel modes, subsidizing users and possible network capacity expansions using the toll revenue in the design of Pareto-improving pricing strategies for general transportation networks.

Another factor that substantially affects public acceptability on congestion pricing is the equity effect. Although equity problem has been raised for a long time, only a limited number of studies have explicitly considered equity measures in their pricing design models. Moreover, all of these models assume constant value of travel time for each individual traveler and ignore the impact of incomes. As a result, their equity analysis can only measure the spatial equity and cannot capture the distribution effects of congestion pricing on groups with different incomes. According to many empirical studies, e.g., Jara-Diaz and Videla (1989) and Franklin (2006), the ignorance of income effect may lead to an underestimate of the regressivity of the pricing schemes. Therefore, these models cannot address vertical equity in a sensible way. Furthermore, most of these analyses are based on small networks and usually do not consider instruments other than pricing, such as revenue refunding and transit development. The transit service, as an alternative to auto networks, is also critical in evaluating the equity effect of any pricing schemes. So, in this dissertation, we develop a more equitable pricing scheme that directly takes into account the effects of income on the choices of trip generation, mode and route, and explicitly captures the distribution of impacts of pricing schemes across different income and geographic groups. This scheme

considers the nonlinear income effects of user utilities, thus can accurately address the vertical equity among travelers with different incomes.

Table 2-1. Existing research on Pareto-improving pricing schemes

Study	Travel mode	Heterogeneous users	Network size	Revenue redistribution	Network expansion
Lawphonpanich and Yin (2007,2010)	Auto only	No	Large	No	No
Song et al. (2009)	Auto only	Yes	Large	No	No
Guo and Yang (2010)	Auto only	Yes	N/A	Yes	No
Liu et al. (2009)	Auto and transit	Yes	Small	Yes	No
Nie and Liu (2010)	Auto and transit	Yes	Small	Yes	No

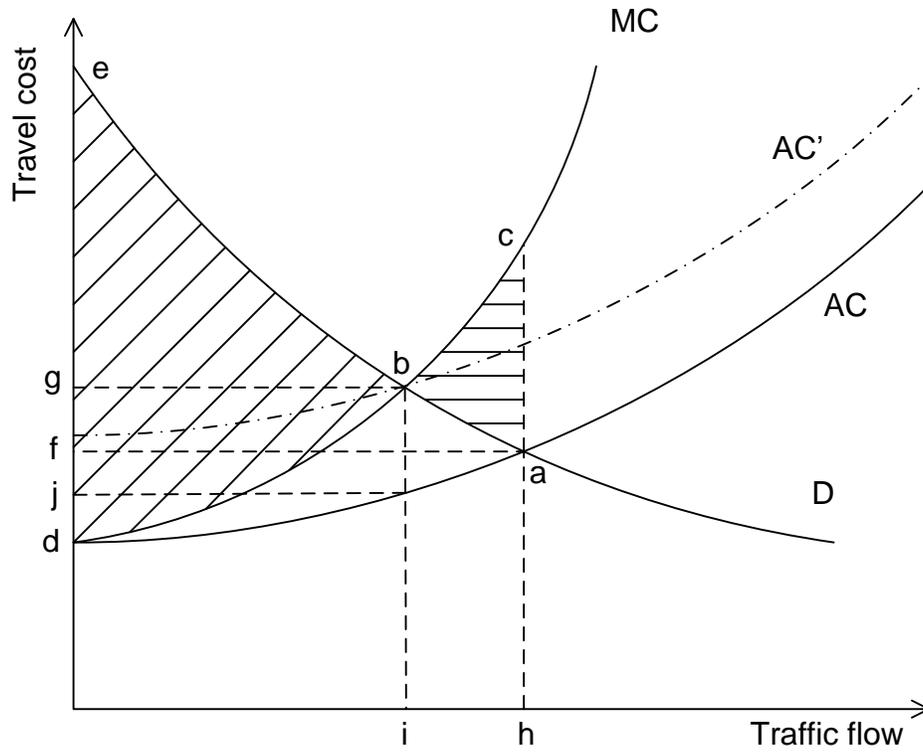


Figure 2-1. Economics foundation of marginal cost pricing

CHAPTER 3  
PARETO-IMPROVING CONGESTION PRICING ON MULTIMODAL  
TRANSPORTATION NETWORK

**3.1 Motivation**

**3.1.1 Unattractiveness of First-Best Pricing**

First-best pricing can increase social welfare to the maximum level, which is beneficial to the society. However, the standard arguments in favor of first-best pricing schemes tend to ignore the distributional effects, by concentrating on the benefits to the society as a whole (Banister, 1994). This means such schemes do not guarantee that every traveler in the network is made better off and consequently makes the pricing scheme less equitable. Actually, it is possible that the only stakeholder that gains from the pricing scheme is the government who collects toll revenues. The road users may be made worse off because they have to pay more than the travel time they save or have to switch to less desirable routes or modes to avoid the toll charges. Hau (2005) pointed out that marginal cost pricing (or more general first-best pricing) is “most likely doomed to be political failures” because of the increased travel cost for travelers.

The following example demonstrates how marginal cost pricing can increase road users' travel cost (Lawphongpanich and Yin, 2007). Consider the network in Figure 3-1 where there is only one OD pair (node 1 to node 4) with a total travel demand of 3.6. The link travel time functions are listed besides each travel link. By solving the UE assignment problem for this network, the equilibrium travel time under UE condition without toll intervention is 71.06 with only two paths used (1-3-4, 1-3-2-4). The total travel time is 255.82. The system is not efficient under UE condition as the capacity of link 1-2 is not utilized. The system performance can be improved using congestion pricing. After introducing marginal cost pricing, the total travel time reduces to 227.11

with all three paths (1-3-4, 1-3-2-4, 1-2-4) utilized. However, considering the toll charges, the travel cost for each traveler increases to 101.70. In this case, the toll charges make the travelers pay an extra 139.02. Therefore, the marginal cost pricing actually makes all travelers worse off. As a result, the travelers are very unlikely to accept the implementation of such a pricing scheme even it can significantly reduce traffic congestion.

Yang and Zhang (2002) in their paper also provided another simple example but considering multiple user classes and OD pairs that showed marginal cost pricing would increase travelers' travel cost. They demonstrated that congestion pricing may also provide spatial inequality. In other words, users from certain OD pair might suffer from the pricing while others benefit from the reduced congestion.

### **3.1.2 Pareto-Improvement and Pareto-Improving Congestion Pricing**

Pareto optimality (or Pareto efficiency) is an economic concept named after Italian economist Vilfredo Pareto. Pareto optimality refers to the condition that no one can be made better off without making anybody else worse off. Correspondingly, Pareto improvement means the result of a policy change that makes at least one individual better off without making any other individual worse off. Pareto optimality is one of the key concepts in social welfare analysis in social sciences (Feldman and Serrano, 2006).

A Pareto-improving congestion pricing scheme is a congestion pricing scheme that improves the performance of the transportation system without making any individual, including the government, worse off. The Pareto-improving congestion pricing scheme will not increase any traveler's travel cost, thus is able to overcome the drawbacks of marginal cost pricing, which make such schemes more attractive to the general public.

In the literature, many researchers have investigated the possibility of achieving Pareto improvement of the transportation system using congestion pricing approaches. One of the early efforts on this topic is by Daganzo (1995) and Daganzo and Garcia (2000), who proposed a Pareto-improving hybrid scheme on a bottleneck network using both the instruments of congestion pricing and rationing. More recently, Guo and Yang (2010), Liu et al. (2009) and Nie and Liu (2010) analyzed Pareto-improving pricing schemes with revenue refunding. Their approaches use the toll revenue to subsidize the travelers on less favorable routes and OD pairs in order to achieve Pareto-improvement. Lawphongpanich and Yin (2010) and Song et al. (2009), from a different perspective, focused their research on Pareto-improving schemes in general networks that require neither rationing nor revenue redistribution. These schemes are derived based on the fact that the original Wardropian user equilibrium flow distribution may not be strongly Pareto optimal (Hagstrom and Abrams, 2002). Their computational experiments suggest that such Pareto-improving tolls are relatively prevalent, although they may not lead to a significant level of improvement.

Although these studies provide valuable insights in the development of Pareto-improving pricing schemes, they either focus on small networks or only consider single travel mode. To achieve higher levels of improvement through the synergy among different modes, the goal of this study is to determine Pareto-improving tolls for general multimodal transportation networks that includes, e.g., transit services, high-occupancy/toll (HOT) and general-purpose or regular lanes. To achieve a Pareto improvement, our approach adjusts transit fares and charges tolls on highway links to better distribute travel demands among the available modes of transportation. Revenue

cross-subsidizing between transit and vehicle users is another key component in our schemes for achieving Pareto improvement.

### 3.2 Problem Description

We consider in this study three modes of transportation: single occupancy vehicle (SOV), high occupancy vehicle (HOV) and transit. The travel models herein assume the following:

- The total demand for each origin-destination (OD) pair is fixed. However, travelers have the flexibility to choose one among all three transportation modes.
- Travelers are homogenous.
- Travelers' mode choice can be represented using a multinomial logit model.
- Each transit line has a fixed service frequency and capacity.

#### 3.2.1 Underlying Network Structure

The overall multimodal network consists of two (sub) networks, auto and transit network. The links in the auto network represent freeways or highways. When a freeway segment contains both regular and HOV/HOT lanes, they are represented as two parallel links. Therefore, the auto network contains two sets of links, regular and HOV/HOT, and they are denoted as  $L^S$  and  $L^H$ , respectively. Henceforth, the superscripts  $S$  and  $H$  denote the regular (or SOV mode) and HOV/HOT links (or HOV mode), respectively. We assume that every link is tollable. All vehicles on the regular links pay the same amount of toll while only SOVs need to pay a toll on HOT links. Note that restricting SOV flow to be zero on HOT links makes them HOV-only links.

In addition to tolling on the auto network, transportation authority can also adjust the fares on various transit lines of the transit network. Transit network is derived from the auto network and allows for passengers to board, alight, wait and transfer between

various transit lines. Figure 3-2 displays an example of a multimodal network with three transit lines 1–2–3 (line a), 1–2 (line b), and 2–3 (line c). For each transit line, “station” nodes are added for all the bus stops (e.g., 1a, 2a and 1b) as well as embarking (e.g., link 1–1a) and alighting links (e.g., link 2a–2). Buses run on the transit links, and the passengers have to use the embarking and alighting links to access and egress. Let  $L^T$  denote transit and embarking/alighting links (henceforth we use  $T$  or superscript  $T$  to represent variables associated with transit). In our model, we assume transit vehicles share the right of way with other vehicles. Therefore, a transit link has the same in-vehicle travel times as the corresponding regular or HOV/HOT link that the transit service uses. For example, transit link 1a–2a and 1b–2b has the same travel time as link 1–2.

When integrated together, the transit and auto network form a multimodal network  $G(N, L)$ , where  $N$  is the set of nodes and stops and  $L$  is set of auto links and transit lines. Each link is associated with an in-vehicle travel time. For each link in  $L^T$ , there is an associated waiting time that travelers have to spend in waiting for the transit service. For transit and alighting links, the waiting time is zero. The embarking link is assigned a waiting time for the transit line it connects. Assuming that transit vehicles’ inter-arrival times to transit stations follow exponential distribution and passenger arrival rate is uniform, the average waiting time for transit line  $l$  is  $1/f_l$ , where  $f_l$  is the frequency of line  $l$  (e.g., Lam et al., 1999). For links in the auto sub-network, the waiting time is set to be infinity, suggesting that there is no transit service available.

### 3.2.2 Strategy-Based Transit Route Choice

Unlike travelling on the auto network, transit users may have more complicated route choices. Instead of choosing a single shortest path, travelers may pre-select a set of “attractive lines” and board on the first arrival vehicle that serves one of those attractive lines. Such a behavior is addressed as the common lines problem in the literature of transit assignment, e.g., Chriqui and Robillard, (1975).

Alternatively, recent studies have introduced the notion of strategies or hyperpaths to describe more realistic transit users’ route choice behaviors, e.g., Spiess and Florian, (1989); Nguyen and Pallottino, (1988). In this notion, passengers choose an optimal strategy by specifying at each node a set of immediate successor nodes that allow them to reach to their destinations with a minimum expected total travel time. Consider a transit network in Figure 3-3, where each link is a transit line. Then a strategy for travelers with OD pair (node 1 to node 4) specifies the successors of node 1 as nodes 2 and 4, and the successors of node 2 as nodes 3 and 4. With such a strategy, the travelers will board the first arrival vehicle at node 1. If the first bus arrived is for line 1–2, the travelers will have to transfer at node 2 to either Line 2–3 or 2–4, depending whichever comes first. Another strategy may specify node 4 as the only successor of node 1. Consequently the travelers will only take line 1–4 at node 1. Generally, different strategies lead to different expected waiting and in-vehicle travel times. An optimal strategy is the one that minimizes the expected total travel time.

Assuming that each transit user chooses the optimal strategy, the user equilibrium transit assignment problem can be equivalently formulated using the following link based formulation (Spiess and Florian, 1989):

$$\begin{aligned}
& \min_{x, \omega} \sum_{l \in L} t_l x_l + \sum_{i \in N} \sum_{w \in W} \omega_i^w \\
& \text{s. t.} \quad \sum_{l \in L_i^+} x_l^w - \sum_{l \in L_i^-} x_l^w = q_i^w, \quad \forall i \in N, w \in W \\
& \quad x_l^w \leq f_l \omega_i^w, \quad \forall i \in N, l \in L_i^+, w \in W \\
& \quad x_l = \sum_{w \in W} x_l^w, \quad \forall l \in L \\
& \quad x_l^w \geq 0, \quad \forall l \in L, w \in W
\end{aligned}$$

where  $W$  is the set of OD pairs;  $\omega_i^w$  represents the total expected waiting time experienced by passengers of OD pair  $w$  at node  $i$ ;  $x_l^w$  is the passenger flow of OD pair  $w$  on link  $l$  and  $q_i^w$  is the supply or demand of node  $i$  for OD pair  $w$ .  $L_i^+$  is the set of outgoing links from node  $i$  and  $x_l$  is the total passenger flow on link  $l$  and  $t_l$  is the travel time or cost of link  $l$ .

Capturing congestion effects is very critical in transit assignment. In the above, the travel times or costs of transit links are assumed to be constant. In order to capture the discomfort “cost” of passengers in crowded vehicles, the times or costs can be assumed to be continuous and increasing functions of passenger flows. Spiess and Florian (1989) introduced such crowding functions and formulated another equilibrium transit model. However, the model may underestimate the waiting times because passengers may have to wait for another bus if the first arrival one reaches its capacity. Wu et al. (1994) extended the model to consider both waiting time and in-vehicle cost as functions of the passenger flows. Lam et al. (1999) considered explicit capacity constraints of transit lines and estimate the additional waiting times by examining the Lagrangian multipliers associated with the capacity constraints. Both models can be

called “semi-congested” because they only consider the increase of waiting times (Cepeda et al., 2006). Congestion at stops does not only increase the waiting times but also affects the flow splits among the attractive lines. The models that capture the latter effect are called “full-congested” (Cepeda et al., 2006). Examples of the full-congested models include Bouzaiene-Ayari et al. (2001), Cominetti and Correa (2001) and Cepeda et al. (2006). In the latter two, an effective frequency function is introduced, which decreases with the passenger flows and vanishes when flows exceed the capacity of the bus line.

In this study, we focus on deriving optimal pricing strategies for multimodal transportation networks. The problem of interest is generally larger and more complicated than transit assignment. Our consideration of transit network modeling is more in line with Spiess and Florian (1989) and Lam et al. (1999). More specifically, the strategy-based route choice behavior is considered and congestion effects are captured by incorporating explicit capacity constraints for transit lines. Although the resulting transit model is more a semi-congested version and applies in a more restrictive setting, it offers computational advantages over the full-congested ones (Cepeda et al., 2006).

### 3.2.3 Feasible Region

We now define the feasible region for the multimodal network flow models. Let  $x_l^{w,m}$  be the passenger flow on link  $l$  for OD pair  $w$  by mode  $m$ ;  $A$  be the node-link incidence matrix and  $M$  be the set of transportation modes, i.e.,  $M = \{S, H, T\}$ . We further denote the travel demand between OD pair  $w$  by mode  $m$  as  $d^{w,m}$ . To specify the origin and destination of OD pair  $w$  in the flow balance constraints, let  $E^{w,m}$  be its “input-output” vector, i.e., a vector that has exactly two non-zero components: one has

a value of 1 in the component corresponding the origin node and the other is -1 in the component for the destination. Let  $D^w$  be the total demand for OD pair  $w$  and  $c_l^T$  be the transit service capacity on link  $l$ . The feasible region can be represented as the following linear system:

$$Ax^{w,m} = E^{w,m}d^{w,m}, \quad \forall w \in W, m \in M \quad (3-1)$$

$$\sum_{w \in W} x_l^{w,T} \leq c_l^T, \quad \forall l \in L^T \quad (3-2)$$

$$x_l^{w,T} \leq f_l \omega_i^w, \quad \forall i \in N, l \in L_i^+, w \in W \quad (3-3)$$

$$\sum_{m \in M} d^{w,m} = D^w, \quad \forall w \in W \quad (3-4)$$

$$x_l^{w,m} \geq 0, \quad \forall l \in L, w \in W, m \in M \quad (3-5)$$

$$x_l^{w,T} = 0, \quad \forall l \in L^S \cup L^H, w \in W \quad (3-6)$$

$$x_l^{w,m} = 0, \quad \forall l \in L^T, w \in W, m \in \{S, H\} \quad (3-7)$$

Equation (3-1) is the flow balance constraint and Constraint (3-2) ensures that the transit passenger flow on each link is less than the service capacity. Equation (3-3) originates from the strategy-based transit assignment problem (Spiess and Florian, 1989) and regulates the relationship between transit link flow and expected waiting time  $\omega_i^w$ . Equation (3-4) implies that the sum of OD demand of each mode is the total demand, which is given as a constant. The Constraint (3-5) requires that the link flow by mode should be nonnegative, and the last two constraints ensure that transit and auto users use the corresponding facilities. There is redundancy in these three constraints because Constraint (3-5) is only necessary for the variables not specified by Equations (3-6) and (3-7). Note that, if there are HOV-only lanes in the network, another additional constraint,  $x_l^{w,S} = 0$  should be included for those links. We further note that with the

assumption that travelers' mode choice can be formulated as a multinomial logit model,  $d^{w,m}$  is always strictly positive. To reflect this, additional constraint,  $d^{w,m} \geq \varepsilon$  can be added, where  $\varepsilon$  is a small positive constant. We denote the set of all  $(x, d, \omega)$  feasible to the above system as  $\Phi$ , which is a polyhedron.

### 3.3 Multimodal Traffic Assignment Models

#### 3.3.1 Multimodal User Equilibrium Model

This section presents user equilibrium and system optimum traffic assignment models for the multimodal networks. We assume that link travel time increases monotonically with the vehicular flow on the link, e.g., in a form of BPR function as follows:

$$t_l = fft_l \cdot \left( 1 + 0.15 \left( \frac{v_l}{cap_l} \right)^4 \right)$$

where  $fft_l$  and  $cap_l$  are the free flow (in-vehicle) travel time and vehicular capacity of link  $l$ , respectively;  $v_l$  is the total vehicular flow on link  $l$  which is the sum of SOV, HOV and transit vehicular flows that using link  $l$ . As previously mentioned, the frequency of transit service is fixed and thus the vehicular flow can be estimated as

$$v_l = \sum_{w \in W} x_l^{w,S} + \frac{\sum_{w \in W} x_l^{w,H}}{o^H} + \sum_{l' \in L_l^T} f_{l'}, \quad \forall l \in L$$

where  $o^H$  is the average occupancy of HOVs and  $L_l^T$  is the set of transit lines sharing the right-of-way with auto link  $l$ . Therefore, link travel time is an increasing function of its person flow, denoted as  $t_l(x)$ . Note that the function is asymmetric, e.g., Patriksson (1994).

We assume that users behave in a manner consistent with the Wardropian user equilibrium principle within the same transportation mode, and thus the user equilibrium conditions for each mode can be expressed as

$$C_p^{w,m} - t^{w,m} \begin{cases} = 0, & \text{if } b_p^{w,m} > 0, \\ \geq 0, & \text{if } b_p^{w,m} = 0, \end{cases} \quad \forall p \in P^{w,m}, w \in W, m \in M$$

where  $P^{w,m}$  is the set of paths (or hyperpaths) between OD pair  $w$  for mode  $m$ ;  $C_p^{w,m}$  is the (expected) travel time on path  $p$  for OD pair  $w$  and mode  $m$  and  $b_p^{w,m}$  is the corresponding path flow;  $t^{w,m}$  is the smallest or equilibrium travel time for OD pair  $w$  by mode  $m$ .

It is assumed that travelers' mode choice can be described by a multinomial logit model as follows:

$$\frac{d^{w,m}}{D^w} = \frac{\exp(-\theta \cdot t^{w,m} - \beta^m)}{\sum_{m \in M} \exp(-\theta \cdot t^{w,m} - \beta^m)}, \quad \forall w \in W$$

where  $\theta$  and  $\beta^m$  are parameters that need to be calibrated.  $\beta^m$  is a mode-specific constant, which can be set as 0 for one of the three modes. It follows from the above equation that:

$$\begin{aligned} \theta \cdot t^{w,S} + \beta^S + \ln d^{w,S} &= \theta \cdot t^{w,H} + \beta^H + \ln d^{w,H} \\ &= \theta \cdot t^{w,T} + \beta^T + \ln d^{w,T} \end{aligned} \quad (3-8)$$

**Theorem 3-1.** The solution  $(x^*, d^*, \omega^*)$  of the following variational inequality (VI) problem satisfies the user equilibrium conditions in the multimodal network.

MUE-VI:

$$\begin{aligned}
& \sum_{w \in W} \sum_{m \in M} \sum_{l \in L} t_l(x^*) (x_l^{w,m} - x_l^{w,m*}) \\
& \quad + \frac{1}{\theta} \sum_{w \in W} \sum_{m \in M} (\ln d^{w,m*} + \beta^m) (d^{w,m} - d^{w,m*}) \\
& \quad + \sum_{w \in W} \sum_{i \in N} (\omega_i^w - \omega_i^{w*}) \geq 0, \quad \forall (x, d, \omega) \in \Phi
\end{aligned}$$

$$(x^*, d^*, \omega^*) \in \Phi$$

**Proof.** The KKT conditions for MUE-VI are as follows:

$$t_l(x) + \rho_i^{w,m} - \rho_j^{w,m} - \eta_l^{w,m} = 0, \quad (3-9)$$

$$\forall l \in \{L^S \cup L^H\}, w \in W, m \in \{S, H\}$$

$$t_l(x) + \rho_i^{w,T} - \rho_j^{w,T} + \mu_l^w + \gamma_l - \eta_l^{w,T} = 0, \quad \forall l \in L^T, w \in W \quad (3-10)$$

$$\frac{1}{\theta} (\ln d^{w,m} + \beta^m) + \lambda^w - E^{w,m} \rho^{w,m} = 0, \quad \forall w \in W, m \in M \quad (3-11)$$

$$\sum_{l \in L_i^+} f_l \mu_l^w = 1, \quad \forall i \in N, w \in W \quad (3-12)$$

$$\gamma_l \left( \sum_{w \in W} x_l^{w,T} - c_l^T \right) = 0, \quad \forall l \in L^T \quad (3-13)$$

$$\mu_l^w (x_l^{w,T} - f_l \omega_i^w) = 0, \quad \forall i \in N, l \in L_i^+, w \in W \quad (3-14)$$

$$\eta_l^{w,m} x_l^{w,m} = 0, \quad \forall l \in L, w \in W, m \in M \quad (3-15)$$

$$\gamma_l \geq 0, \quad \forall l \in L^T$$

$$\mu_l^w \geq 0, \quad \forall l \in L^T, w \in W$$

$$\eta_l^{w,m} \geq 0, \quad \forall l \in \{L^S \cup L^H\}, w \in W, m \in \{S, H\}$$

$$\eta_l^{w,T} \geq 0, \quad \forall l \in L^T, w \in W$$

$$(x, d, \omega) \in \Phi$$

where  $\rho_i^{w,m}$  is the multiplier (or node potential) associated with the flow balance constraint (3-1);  $\gamma_l$ ,  $\mu_l^w$  and  $\lambda^w$  are the multipliers associated with Equations (3-2) to (3-4), respectively. Furthermore,  $\gamma_l$  can be interpreted as the extra waiting time on link  $l$  when the demand for the transit line exceeds its service capacity (Lam et al., 1999). Similarly,  $\eta_l^{w,m}$  is the multiplier associated with Equations (3-5) to (3-7).

Assume that the above conditions hold for a solution  $(x, d, \omega) \in \Phi$ . For any utilized path  $p$  with respect to the flow distribution for SOV or HOV, the flow on each link along the path must be positive, i.e.,  $x_l^{w,m} > 0$  if  $\delta_{lp} = 1$  (where  $\delta_{lp}$  equals one, indicating that link  $l$  is part of path  $p$  and zero otherwise). According to Equation (3-15),  $\eta_l^{w,m} = 0$ . Thus, summing Equation (3-9) along the path yields:

$$\sum_{l \in L} \delta_{lp} t_1(x) - (\rho_{d(w)}^{w,m} - \rho_{o(w)}^{w,m}) = 0 \quad (3-16)$$

Therefore, every utilized path for SOV or HOV has the same travel time and it equals  $\rho_{d(w)}^{w,m} - \rho_{o(w)}^{w,m}$ , the minimum travel time between a given OD pair  $w$ . Using a similar argument, it follows from Equation (3-13) to (3-15) and (3-10) that every utilized hyperpath  $l_p$  between OD pair  $w$  satisfies:

$$\sum_{l \in L} \delta_{lp} (t_l + \gamma_l + \mu_l^w) - (\rho_{d(w)}^{w,T} - \rho_{o(w)}^{w,T}) = 0 \quad (3-17)$$

From Equation (3-14), it follows:

$$\mu_l^w x_l^{w,T} = \mu_l^w f_l \omega_l^w$$

Summing the above equation over all links and using Equation (3-12) yields:

$$\sum_l \mu_l^w x_l^{w,T} = \sum_i \omega_i^w$$

The right side of the above is the total waiting time for OD pair  $w$ . Therefore,  $\mu_l^w$  can be interpreted as a “decomposed” waiting time at link  $l$  such that  $\sum_l \delta_{lp}(t_l + \gamma_l + \mu_l^w)$  is the expected total travel time of the utilized hyperpath. Equation (3-17) implies that each utilized hyperpath for the same OD pair has the same minimum expected travel time  $\rho_{d(w)}^{w,T} - \rho_{o(w)}^{w,T}$ .

According to Equation (3-16) and (3-17), it is straightforward to show that Equation (3-11) is equivalent to Equation (3-8), i.e., the demand distribution follows the multimodal logit model. Therefore, the solution  $(x, d, \omega)$  satisfies the multimodal user equilibrium conditions. ■

As the travel time function is asymmetric, MUE-VI cannot be readily reformulated as a mathematical program. Although the solution algorithms proposed for solving VI in the literature, e.g., Lawphongpanich and Hearn (1984), can be applied to solve MUE-VI, although they may not be particularly efficient (Aghassi et al., 2006). In this paper, we apply a new technique developed by Aghassi et al. (2006) that reformulates an asymmetrical VI as a nonlinear optimization problem.

Applying Theorem 2 in Aghassi et al. (2006), MUE-VI can be reformulated as the following optimization problem:

MUE-NLP:

$$\begin{aligned} \min_{x,d,\omega,\tilde{c},\tilde{d},\tilde{D},\tilde{\omega}} \quad & \sum_{w \in W} \sum_{m \in M} \sum_{l \in L} t_l(x) \cdot x_l^{w,m} + \frac{1}{\theta} \sum_{w \in W} \sum_{m \in M} (\ln d^{w,m} + \beta^m) \cdot d^{w,m} \\ & + \sum_{w \in W} \sum_{i \in N} \omega_i^w - \sum_{l \in L} c_l \cdot \tilde{c}_l - \sum_{w \in W} D^w \cdot \tilde{D}^w \end{aligned}$$

$$\begin{aligned}
s. t. \quad & \sum_{i \in N} A'_l \cdot \tilde{d}_i^{w,m} \leq t_l(x), \quad \forall w \in W, m \in \{S, H\} \\
& \sum_{i \in N} A'_l \cdot \tilde{d}_i^{w,m} + \tilde{c}_l + \tilde{\omega}_l^w \leq t_l(x), \quad \forall w \in W \\
& - \sum_{l \in L^+} f_l \cdot \tilde{\omega}_l^w \leq 1, \quad \forall i \in N, w \in W \\
& -E^{w,m} \cdot \tilde{d}^{w,m} + \tilde{D}^w \leq \frac{1}{\theta} (\beta^m + \ln d^{w,m}), \quad \forall w \in W, m \in M \\
& \tilde{c}_l \leq 0, \quad \forall l \in L^T \\
& \tilde{\omega}_l^w \leq 0, \quad \forall l \in L^T, w \in W \\
& (x, d, \omega) \in \Phi
\end{aligned}$$

where  $\tilde{c}_l$ ,  $\tilde{D}^w$ ,  $\tilde{d}^{w,m}$ ,  $\tilde{\omega}_l^w$  are auxiliary variables and  $A_l$  is the  $l$ th column of the incidence matrix  $A$ .

The above program is a regular nonlinear optimization problem and can be solved using commercial nonlinear optimization solvers. If the optimal objective value is zero, one part of the optimal solution vector to MUE-NLP,  $(x, d, \omega)$ , will solve MUE-VI.

### 3.3.2 Existence of Multimodal User Equilibrium

**Theorem 3-2.** The variational inequality problem for multimodal user equilibrium (MUE-VI) has at least one solution.

**Proof.** We first introduce a restricted VI formulation (MUE-VIR) to the original MUE-VI problem by imposing an upper bound for variable  $\omega_l^w$  as follows:

$$\omega_l^w \leq \bar{\omega}$$

where  $\bar{\omega}$  is large enough to ensure that the feasible region is nonempty.

With the upper bound constraint, the feasible region for MUE-VIR problem is a compact and convex region. Given that all the functions are continuous, the MUE-VIR problem has at least one solution, see, e.g., Harker and Pang (1990).

It can be easily verified that any solution  $(x^*, d^*, \omega^*)$  to the MUE-VIR problem also solves the MUE-VI problem because for any  $\omega_i^w > \bar{\omega}$ ,  $\omega_i^w - \omega_i^{w*} > \bar{\omega} - \omega_i^{w*} \geq 0$  and the additional upperbound constraint in MUE-VIR does not affect variables  $x$  and  $d$ .

Therefore, the solution to MUE-VI exists. ■

### 3.3.3 Multimodal System Optimal Model

We now define the system optimum problem that maximizes the total social benefit. Given a user equilibrium solution, the total expected utility can be easily computed as

$$\sum_{w \in W} \frac{1}{\theta} \cdot \ln \left( \sum_{m \in M} \exp(-\theta \cdot t^{w,m} - \beta^m) \right) \cdot D^w \quad (3-18)$$

The above quantity is a valid measure for the total social benefit, as shown in, e.g., Small and Rosen (1981) and Ying and Yang (2005).

Correspondingly, the multimodal system optimum problem can be formulated as

MSO:

$$\begin{aligned} \min_{x, d, \omega} \quad & \sum_{w \in W} \sum_{m \in M} \sum_{l \in L} t_l(x) x_l^{w,m} + \frac{1}{\theta} \sum_{w \in W} \sum_{m \in M} (\ln d^{w,m} + \beta^m) d^{w,m} \\ & + \sum_{w \in W} \sum_{i \in N} \omega_i^w \end{aligned}$$

$$s. t. \quad (x, d, \omega) \in \Phi$$

The objective function is to maximize the total social benefit and can be derived similarly as in Yang (1999), Maher et al. (2005) and Ying and Yang (2005). MSO is a nonlinear convex program that is easy to solve.

### 3.4 Multimodal Pareto-Improving Toll Problem

This section discusses the use of congestion pricing to improve the social welfare in the multimodal network. It is assumed that a transportation authority determines tolls for auto links and adjusts transit fares for transit lines. The toll is nonnegative while the transit fare adjustment is unrestricted. A negative adjustment means that transit users are subsidized. As previously mentioned, both SOVs and HOVs need to pay tolls on regular links while only the former pay tolls on HOT links. The feasible toll set  $\Psi$  can thus be defined as

$$\tau_l^H = \tau_l^S, \quad \forall l \in L^S$$

$$\tau_l^S = 0, \quad \forall l \in L^T$$

$$\tau_l^H = 0, \quad \forall l \in \{L^H \cup L^T\}$$

$$\tau_l^T = 0, \quad \forall l \in \{L^S \cup L^H\}$$

$$\tau_l^H, \tau_l^S \geq 0, \quad \forall l \in L$$

where  $\tau_l^H$  and  $\tau_l^S$  are tolls on link  $l$  for HOVs and SOVs respectively. Note that if a HOT link is not tolled, it becomes a regular link. HOT links in this paper essentially provide a mean for discriminatory tolling. Similarly,  $\tau_l^T$  is transit fare adjustment. Given a toll vector  $\tau$  (in the unit of time), the tolled multi-modal user equilibrium problem can be formulated as the following nonlinear complementarity problem:

TMUE:

$$\begin{aligned} (t_l(x) + \tau_l^m - (\rho_j^{w,m} - \rho_i^{w,m})) x_l^{w,m} &= 0, \\ \forall l \in \{L^S \cup L^H\}, w \in W, m \in \{S, H\}, l \in L_i^+, l \in L_j^- \end{aligned} \quad (3-19)$$

$$\begin{aligned} (t_l(x) + \tau_l^T + \mu_l^w + \gamma_l - (\rho_j^{w,T} - \rho_i^{w,T})) x_l^{w,T} &= 0, \\ \forall l \in L^T, w \in W, l \in L_i^+, l \in L_j^- \end{aligned} \quad (3-20)$$

$$\frac{1}{\theta} (\ln d^{w,m} + \beta^m) + \bar{\lambda}^w - E^{w,m} \rho^{w,m} = 0, \quad \forall w \in W, m \in M \quad (3-21)$$

$$\sum_{l \in L_i^+} f_l \mu_l^w = 1, \quad \forall i \in N, w \in W \quad (3-22)$$

$$\gamma_l \left( \sum_{w \in W} x_l^{w,T} - c_l^T \right) = 0, \quad \forall l \in L^T \quad (3-23)$$

$$\mu_l^w (x_l^{w,T} - f_l \omega_i^w) = 0, \quad \forall i \in N, l \in L_i^+, w \in W \quad (3-24)$$

$$\begin{aligned} t_l(x) + \tau_l^m - (\rho_j^{w,m} - \rho_i^{w,m}) &\geq 0, \\ \forall l \in L, w \in W, m \in \{S, H\}, l \in L_i^+, l \in L_j^- \end{aligned} \quad (3-25)$$

$$\begin{aligned} t_l(x) + \tau_l^T + \mu_l^w + \gamma_l - (\rho_j^{w,T} - \rho_i^{w,T}) &\geq 0, \\ \forall l \in L, w \in W, l \in L_i^+, l \in L_j^- \end{aligned} \quad (3-26)$$

$$\gamma_l \geq 0, \quad \forall l \in L^T \quad (3-27)$$

$$\mu_l^w \geq 0, \quad \forall l \in L^T, w \in W \quad (3-28)$$

$$\tau \in \Psi$$

$$(x, d, \omega) \in \Phi$$

TMUE is derived from adding a toll vector to the KKT condition of MUE-VI. In determining a Pareto-improving toll vector, each individual user cannot be made worse off when compared to the situation under MUE. To ensure this, we require the tolled

equilibrium travel cost between each OD pair for each mode to be less than or equal to the equilibrium travel cost without toll. If the condition holds, the users who remain on the same mode will not be made worse off and those who switch to other modes will be better off. We further require the total revenue to be nonnegative (or greater than a certain value to cover the toll operation cost) to ensure that the expense of transportation authority does not increase. Mathematically, the above two requirements can be written as

$$-E^{w,m} \rho^{w,m} \leq t_{UE}^{w,m}, \quad \forall w \in W, m \in M \quad (3-29)$$

$$\sum_{w \in W} \sum_{m \in M} \sum_{l \in L} \tau_l^m x_l^{w,m} \geq 0, \quad \forall w \in W, m \in M, l \in L \quad (3-30)$$

where  $t_{UE}^{w,m}$  is the equilibrium travel time without toll. Note that we do not consider the economic behaviors of transit operator in this study. However, because we assume fixed transit frequencies, transit operating costs would remain approximately the same after the tolling. On the other hand, the transit operator is more likely to have more revenue due to a higher transit ridership.

The multimodal Pareto-improving toll problem can be formulated as follows:

MPIT:

$$\begin{aligned} \max_{x,d,\omega,\tau,\rho} \quad & \sum_{w \in W} \frac{1}{\theta} \cdot \ln \left( \sum_{m \in M} \exp(-\theta(\rho_j^{w,m} - \rho_i^{w,m}) - \beta^m) \right) \cdot D^w \\ & + \sum_{w \in W} \sum_{m \in M} \sum_{l \in L} \tau_l^m x_l^{w,m} \end{aligned}$$

s. t. (3-19) to (3-30)

$$\tau \in \Psi$$

$$(x, d, \omega) \in \Phi$$

The objective of the above problem is to maximize the total social benefit. There are two parts in the objective function. The first term is the user benefit while the second term is the total toll revenue representing the benefit for the government (or the producer benefit). As formulated, the MPIT problem is always feasible. In particular, the solution to MUE-VI and a zero toll vector is feasible to MPIT. On the other hand, Pareto-improving tolls may not exist. If the optimal objective value of MPIT is greater than the social benefit associated with the original user equilibrium, the optimal toll vector will be Pareto-improving.

### 3.5 Solution Algorithm

As formulated, MPIT is a mathematical problem with complementarity constraints (MPCC), a class of problems difficult to solve, particularly when the problem is large. This paper applies a manifold suboptimization algorithm developed by Lawphongpanich and Yin (2010) to solve MPIT. The basic idea of the algorithm is to solve a sequence of restricted nonlinear optimization problems to obtain a strongly stationary solution to the original MPCC. The restriction is based on two set of index sets defined as  $\Omega_x^{w,m,1} = \{l: x_l^{w,m} = 0\}$ ,  $\Omega_\gamma^1 = \{l: \gamma_l = 0\}$ ,  $\Omega_\mu^{w,1} = \{l: \mu_l^w = 0\}$  and  $\Omega_{\bar{x}}^{w,m,1} = \{l: x_l^{w,m} > 0\}$ ,  $\Omega_{\bar{\gamma}}^1 = \{l: \gamma_l > 0\}$ ,  $\Omega_{\bar{\mu}}^{w,1} = \{l: \mu_l^w > 0\}$ .

The restricted MPIT (or R-MPIT) can thus be formulated as follows:

$$\begin{aligned} \max_{x,d,\omega,\tau,\rho} \quad & \sum_w \frac{1}{\theta} \cdot \ln \left( \sum_m \exp(-\theta(\rho_j^{w,m} - \rho_i^{w,m}) - \beta^m) \right) \cdot D^w + \sum_m \sum_l \tau_l^m \sum_w x_l^{w,m} \\ \text{s. t.} \quad & (3-1), (3-4), (3-21), (3-22), (3-25), (3-26), (3-29), (3-30) \end{aligned}$$

$$t_l(x) + \tau_l^m - (\rho_j^{w,m} - \rho_i^{w,m}) = \bar{x}_l^{w,m},$$

$$\forall l \in \{L^S \cup L^H\}, w \in W, m \in \{S, H\}, l \in L_i^+, l \in L_j^-$$

$$t_l(x) + \tau_l^T + \mu_l^w + \gamma_l - (\rho_j^{w,T} - \rho_i^{w,T}) = \bar{x}_l^{w,T},$$

$$\forall l \in L^T, w \in W, l \in L_i^+, l \in L_j^-$$

$$\sum_{w \in W} x_l^{w,T} - c_l^T = \bar{\gamma}_l, \quad \forall l \in L^T$$

$$x_l^{w,T} - f_l \omega_i^w = \bar{\mu}_l^w, \quad \forall l \in L^T, w \in W \quad (3-31)$$

$$x_l^{w,m} = 0, \quad \forall l \in \Omega_x^{w,m,k}, w \in W, m \in W \quad (3-32)$$

$$\gamma_l = 0, \quad \forall l \in \Omega_\gamma^k \quad (3-33)$$

$$\mu_l^w = 0, \quad \forall l \in \Omega_\mu^{w,k}, w \in W$$

$$\bar{x}_l^{w,m} = 0, \quad \forall l \in \Omega_{\bar{x}}^{w,m,k}, w \in W, m \in W$$

$$x_l^{w,m} \geq 0, \quad \forall l \notin \Omega_x^{w,m,k}, w \in W, m \in W$$

$$\bar{x}_l^{w,m} \geq 0, \quad \forall l \notin \Omega_{\bar{x}}^{w,m,k}, w \in W, m \in W$$

$$\bar{\gamma}_l = 0, \quad \forall l \in \Omega_{\bar{\gamma}}^k$$

$$\gamma_l \geq 0, \quad \forall l \notin \Omega_\gamma^k$$

$$\bar{\gamma}_l \geq 0, \quad \forall l \notin \Omega_{\bar{\gamma}}^k$$

$$\bar{\mu}_l^w = 0, \quad \forall l \in \Omega_{\bar{\mu}}^{w,k}, w \in W$$

$$\mu_l^w \geq 0, \quad \forall l \notin \Omega_\mu^{w,k}, w \in W$$

$$\bar{\mu}_l^w \geq 0, \quad \forall l \notin \Omega_{\bar{\mu}}^{w,k}, w \in W$$

$$\tau \in \Psi$$

where  $\bar{x}_l^{w,m}$ ,  $\bar{\gamma}_l$  and  $\bar{\mu}_l^w$  are auxiliary variables.

The manifold suboptimization algorithm solves a sequence of R-MPIT and use the associated KKT multipliers to update the index sets  $\Omega$ . The procedure can be described as follows:

Step 1: Solve the problem MUE-NLP to a zero optimal objective value to obtain a user equilibrium solution.

Step 2.1: Let  $x_l^{w,m}$ ,  $\gamma_l$  and  $\mu_l^w$  be the link demand and node waiting time from Step 1.

Set  $k = 1$ ,  $\Omega_x^{w,m,1} = \{l: x_l^{w,m} = 0\}$ ,  $\Omega_\gamma^1 = \{l: \gamma_l = 0\}$ ,  $\Omega_\mu^{w,1} = \{l: \mu_l^w = 0\}$  and  $\Omega_{\bar{x}}^{w,m,1} = \{l: x_l^{w,m} > 0\}$ ,  $\Omega_\gamma^1 = \{l: \gamma_l > 0\}$ ,  $\Omega_{\bar{\mu}}^{w,1} = \{l: \mu_l^w > 0\}$ .

Step 2.2: Let  $(x^{w,m,k}, \gamma^k, \mu^{w,k}, \bar{x}^{w,m,k}, \bar{\gamma}^k, \bar{\mu}^{w,k}, \rho^{w,m,k}, \tau^{m,k})$  solve the relaxed problem R-MPIT.

Step 2.3: Let  $\Gamma_x^{w,m,k} = \{l \in \Omega_x^{w,m,k}: \hat{x}_l^{w,m,k} < 0 \text{ and } \bar{x}_l^{w,m,k} = 0\}$ ,  $\Gamma_\gamma^k = \{l \in \Omega_\gamma^k: \hat{\gamma}_l^k <$

$0 \text{ and } \bar{\gamma}_l^k = 0\}$  and  $\Gamma_\mu^{w,k} = \{l \in \Omega_\mu^{w,k}: \hat{\mu}_l^{w,k} < 0 \text{ and } \bar{\mu}_l^{w,k} = 0\}$ , where  $\hat{x}_l^{w,m,k}$ ,  $\hat{\gamma}_l^k$  and  $\hat{\mu}_l^{w,k}$  are the multipliers associated with (3-31)-(3-33) respectively. If  $\Gamma_x^{w,m,k}$ ,  $\Gamma_\gamma^k$

and  $\Gamma_\mu^{w,k}$  are all empty for all  $w$  and  $m$ , stop and

$(x^{w,m,k}, \gamma^k, \mu^{w,k}, \bar{x}^{w,m,k}, \bar{\gamma}^k, \bar{\mu}^{w,k}, \rho^{w,m,k}, \tau^{m,k})$  is strongly stationary

(Lawphongpanich and Yin, 2010). Otherwise, do the following and go to Step 2.2:

(a) Set  $\Omega_x^{w,m,k+1} = \Omega_x^{w,m,k} - \Gamma_x^{w,m,k}$ ,  $\Omega_\gamma^{k+1} = \Omega_\gamma^k - \Gamma_\gamma^k$ ,  $\Omega_\mu^{w,k+1} = \Omega_\mu^{w,k} - \Gamma_\mu^{w,k}$ .

(b) Set  $\Omega_{\bar{x}}^{w,m,k} = \{l: \bar{x}_l^{w,m,k} = 0\}$ ,  $\Omega_{\bar{\gamma}}^k = \{l: \bar{\gamma}_l^k = 0\}$ ,  $\Omega_{\bar{\mu}}^{w,k} = \{l: \bar{\mu}_l^{w,k} = 0\}$ ,

(c) Set  $k = k + 1$ .

Lawphongpanich and Yin (2010) proved that the above algorithm will stop after a finite number of iterations when the KKT multipliers associated with R-MPIT are unique.

### 3.6 Numerical Example

We solved MUE, MSO and MPIT using GAMS (Brooke et al., 2005) and CONOPT (Drud, 1995) on a nine-node network shown in Figure 3-4:

The two-tuple near each link is (link free flow travel time, link capacity). And there are 10 transit lines: 1–5, 2–6, 5–6, 5–7, 6–5, 6–8, 7–3, 7–8, 8–4, 8–7. The service frequency is 1 for each transit line. The BPR function is used to calculate the in-vehicle travel time for each link in the example.

Table 3-1 reports the Pareto-improving tolls and compares the resulting traffic condition with UE and SO. The travel time or cost in the table includes transit waiting time, and travel costs under SO includes the marginal-cost tolls, which are listed under the column of SO. The Pareto-improving tolls in the table increase the social benefit up to 73.5% of the maximum possible level and reduce the total system travel time by 21.4%. At the same time, no individual user is made worse off and a majority is better off. The transportation authority also gains some positive revenue. Pareto-improving tolls encourage users to switch to transportation modes with higher occupancy. In contrast, although marginal-cost pricing achieve a maximum improvement of social benefits and more revenue, some users, SOVs in particular, will be made worse off. Also note that marginal-cost pricing requires discriminatory tolling for all links.

The models were also implemented on the Sioux Falls network as shown in Figure 3-5. The link number is shown in the figure and each link is assumed to have a transit service line operating with a frequency of 1. The free flow travel time and link capacity are listed in Table 3-2. The OD demands are the ones reported in LeBlanc et al. (1975) multiplied by 0.2.

In Table 3-3 the Pareto-improving tolls in this example with mild congestion increase social benefit by 24.4% of the maximum possible level. The total vehicle travel time is reduced by 4.4%, suggesting that traffic condition on the highway network is improved. However, the total passenger travel time remains approximately the same. This is because SOV and HOV users switch to transit due to the tolling and transit subsidy and will experience additional transit waiting time. The reduction of in-vehicle travel time is only large enough in this example to compensate the increase of transit waiting time, leading to approximately the same total passenger travel time. We expect that a more significant Pareto improvement can be achieved if (a) highway congestion is more severe or (b) the transit operator reacts adaptively to increase the transit frequency to serve higher level of ridership or (c) individual users can be made worst off to a limited level (by 3.21% in this particular example). On the other hand, marginal-cost pricing can improve the system performance significantly. However, travel costs for some users may increase.

### **3.7 Summary**

In order to develop congestion pricing schemes more appealing to the public, this study determines a Pareto-improving charging scheme on a multimodal transportation network. Such a scheme improves the social benefit without making any individual user and transportation authority worse off when compared to a situation before pricing.

Pareto improvement is achieved by appropriately charging tolls on highway links and adjusting the fares of transit lines. To better distribute travel demands among the available transportation modes, the revenue from tolling highway links is used to subsidize the fare adjustments on transit lines. The Pareto-improving scheme can be

obtained by solving a mathematical program with complementarity constraints using a manifold suboptimization technique.

Table 3-1. System optimal, user equilibrium and Pareto-improving conditions

		UE			Pareto-improving			SO			
Social benefits		-2603.27			-2068.95			-1875.90			
System travel time		2851.63			2241.59			1994.13			
Revenue		0.00			70.22			513.97			
		OD	SOV	HOV	Transit	SOV	HOV	Transit	SOV	HOV	Transit
Demand	1-3	20.80	7.65	1.55	10.97	12.60	6.42	7.79	13.56	8.65	
	1-4	21.51	7.91	0.58	16.38	7.30	6.32	14.67	11.56	3.77	
	2-3	21.49	7.91	0.61	13.29	15.26	1.44	10.95	17.85	1.20	
	2-4	27.74	10.20	2.06	16.85	19.35	3.80	16.58	21.59	1.83	
Demand change (%)	1-3	-	-	-	-47.25	64.67	315.80	-62.56	77.25	459.63	
	1-4	-	-	-	-23.83	-7.74	987.35	-31.80	46.17	548.26	
	2-3	-	-	-	-38.14	93.08	138.68	-49.03	125.73	98.78	
	2-4	-	-	-	-39.24	89.65	84.32	-40.21	111.54	-11.15	
Travel cost	1-3	22.40	22.40	25.40	21.22	15.53	13.90	24.59	16.82	14.07	
	1-4	22.22	22.22	30.27	19.90	18.94	14.66	23.22	19.41	20.02	
	2-3	20.34	20.34	28.19	18.65	12.96	19.75	21.91	14.47	22.95	
	2-4	22.07	22.07	25.07	22.07	16.37	19.52	23.38	17.06	24.40	
Travel cost change (%)	1-3	-	-	-	-5.27	-30.68	-45.29	9.80	-24.91	-44.60	
	1-4	-	-	-	-10.45	-14.76	-51.58	4.51	-12.64	-33.88	
	2-3	-	-	-	-8.29	-36.27	-29.93	7.71	-28.87	-18.58	
	2-4	-	-	-	0.00	-25.79	-22.14	5.94	-22.70	-2.67	

Table 3-1. CONTINUED

	Link	UE			Pareto-improving			SO		
		SOV	HOV	Transit	SOV	HOV	Transit	SOV	HOV	Transit
Toll	1-5	-	-	-	0.72	0.72	-4.71	1.71	0.68	0.00
	1-6	-	-	-	0.00	0.00	-1.74	2.39	0.96	0.00
	2-5	-	-	-	0.00	0.00	-0.77	1.15	0.46	0.00
	2-6	-	-	-	0.00	0.00	0.00	0.12	0.05	0.00
	5-6	-	-	-	0.00	0.00	-3.25	0.00	0.00	0.00
	5-7	-	-	-	7.92	7.92	6.54	9.84	3.94	0.00
	5-9*	-	-	-	0.00	0.00	0.00	0.50	0.20	0.00
	6-5	-	-	-	0.00	0.00	-1.30	0.00	0.00	0.00
	6-8	-	-	-	0.39	0.39	2.37	1.28	0.51	0.00
	6-9*	-	-	-	0.00	0.00	-2.29	0.00	0.00	0.00
	7-3	-	-	-	0.00	0.00	-3.52	1.41	0.56	0.00
	7-4	-	-	-	0.79	0.79	-0.76	0.84	0.34	0.00
	7-8	-	-	-	0.00	0.00	-0.64	0.00	0.00	0.00
	8-3	-	-	-	0.69	0.69	-11.40	0.00	0.00	0.00
	8-4	-	-	-	0.00	0.00	-7.20	0.50	0.20	0.00
	8-7	-	-	-	0.13	0.13	-6.36	0.00	0.00	0.00
	9-7*	-	-	-	0.00	0.00	-1.44	0.25	0.10	0.00
	9-8*	-	-	-	0.00	0.00	-2.05	0.00	0.00	0.00
	1-5*	-	-	-	0.00	0.00	-5.42	2.07	0.83	0.00
	2-6*	-	-	-	0.00	0.00	-0.06	0.12	0.05	0.00
	5-7*	-	-	-	5.69	0.00	4.31	10.56	4.22	0.00
	6-8*	-	-	-	0.78	0.00	3.18	1.31	0.52	0.00
	7-3*	-	-	-	1.60	0.00	-3.52	1.74	0.70	0.00
	8-4*	-	-	-	0.18	0.00	-7.71	0.55	0.22	0.00

\* HOT link.

Table 3-2. Link characteristics of Sioux Falls network

Link No.	Free-flow travel time	Capacity	Link No.	Free-flow travel time	Capacity	Link No.	Free-flow travel time	Capacity
1	3.6	6.02	31	3.6	9.82	61	2.4	6.05
2	2.4	9.01	32	3.0	20.00	62	3.6	10.12
3	3.6	12.02	33	3.6	9.82	63	3.0	10.15
4	3.0	15.92	34	2.4	9.75	64	3.6	10.12
5	2.4	36.81	35	2.4	36.81	65	1.2	10.46
6	2.4	24.22	36	3.6	9.82	66	1.8	9.77
7	2.4	36.81	37	1.8	51.80	67	2.4	20.63
8	2.4	25.82	38	1.8	51.80	68	3.0	10.15
9	1.2	18.25	39	2.4	10.18	69	1.2	10.46
10	3.6	9.04	40	2.4	9.75	70	2.4	10.00
11	1.2	36.85	41	3.0	10.26	71	2.4	9.85
12	2.4	13.86	42	2.4	9.85	72	2.4	10.00
13	3.0	10.52	43	3.6	17.02	73	1.2	10.16
14	3.0	9.92	44	3.0	10.26	74	2.4	11.38
15	2.4	9.90	45	2.4	9.64	75	1.8	9.77
16	1.2	21.62	46	2.4	20.63	76	1.2	10.16
17	1.8	15.68	47	3.0	10.09	77	2.4	10.00
18	1.2	36.81	48	3.0	10.27	78	2.4	10.00
19	1.2	9.80	49	1.2	10.46	79	2.4	10.00
20	1.8	15.68	50	1.8	29.36	80	1.2	10.00
21	2.0	10.10	51	4.2	9.99	81	1.2	10.00
22	3.0	10.09	52	1.2	10.46	82	1.2	10.00
23	3.0	20.00	53	1.2	9.65	83	1.8	10.00
24	2.0	10.10	54	1.2	36.81	84	1.8	10.00
25	1.8	17.83	55	1.8	29.36	85	3.6	10.00
26	1.8	17.83	56	2.4	8.11	86	2.4	10.00
27	3.0	20.00	57	2.4	4.42	87	3.6	10.00
28	3.6	17.02	58	1.2	9.65	88	1.8	10.00
29	3.0	10.27	59	2.4	10.01	89	1.2	10.00
30	4.2	9.99	60	2.4	8.11	90	1.8	10.00

Table 3-3. Summary of results in Sioux Falls network

	SO			UE			Pareto-improving		
Social benefit	-3890.3			-4147.4			-4084.6		
Total vehicle travel time	2907.8			3859.2			3691.0		
Total passenger travel time	4173.5			4406.3			4397.0		
Revenue	463.8			0.0			30.4		
	SOV	HOV	Transit	SOV	HOV	Transit	SOV	HOV	Transit
Max travel cost increase (%)	3.21	0.66	0.00	-	-	-	0.00	0.00	0.00
Max travel cost decrease (%)	27.51	27.51	49.08	-	-	-	0.68	0.68	39.78

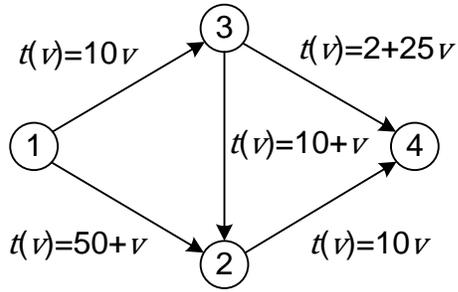


Figure 3-1. A five-link network

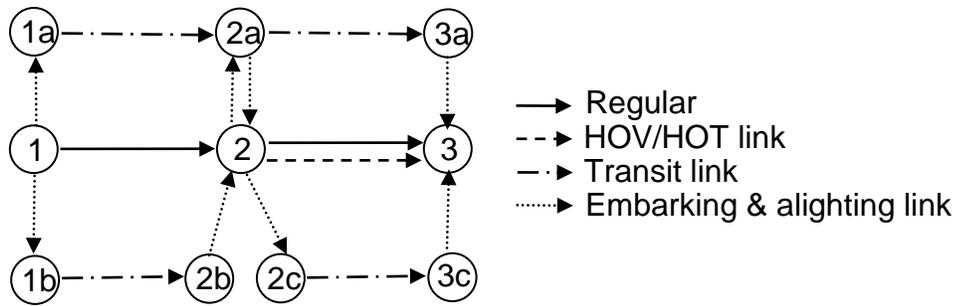


Figure 3-2. Auto and transit network

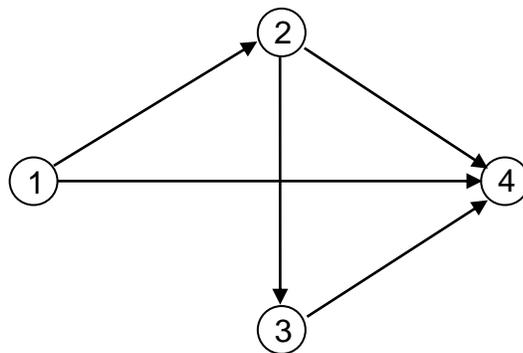


Figure 3-3. Common lines in a transit network

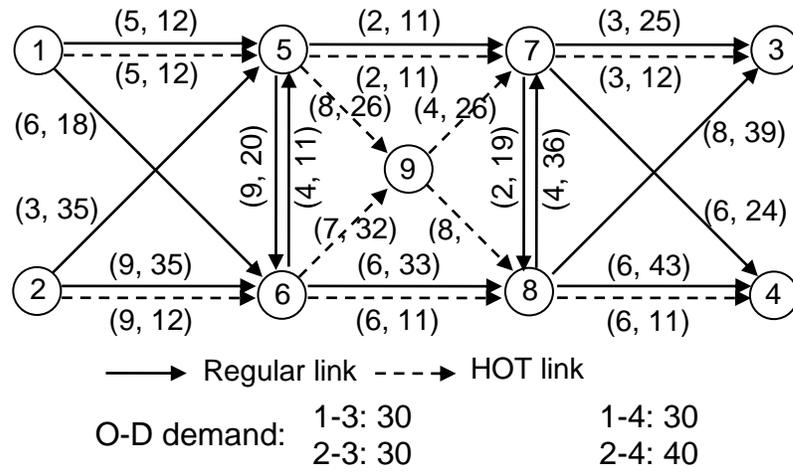


Figure 3-4. Nine-node network

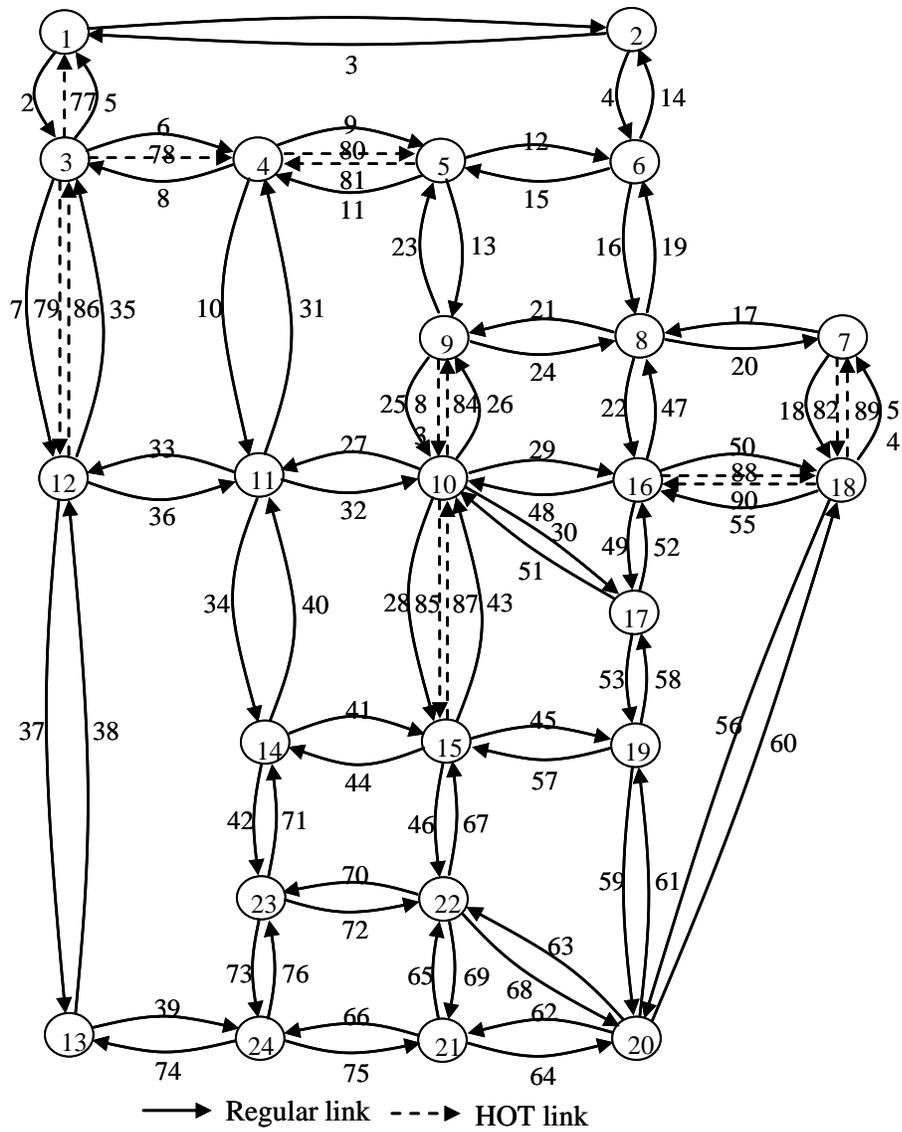


Figure 3-5. Sioux Falls network

## CHAPTER 4 MULTIMODAL PARETO-IMPROVING CONGESTION PRICING WITH TRIP- CHAINING

Trip chaining is a phenomenon that we know exists but rarely consider in pricing models (Adler and Ben-Akiva, 1979). For multimodal transportation networks, modeling trip chaining is particularly relevant in determination of pricing policies, because: 1) a trip chain or tour refers to a sequence of individual trips between a pair of anchor activities (Primerano et al., 2008). For mode choice, travelers typically decide which mode to take for the entire tour, taking into account all the trip segments. Constrained by the schedules, routes and uncertainty associated with transit service, travelers may be more likely to use private automobiles to pursue multiple activities in a single journey (Frank et al., 2008; Ye et al., 2007). A pricing model without considering trip chaining may overestimate the mode shift to transit and thus the benefit of pricing; 2) For certain forms of pricing, such as area-based pricing, a tour-based modeling framework is essential (Maruyama and Sumalee, 2007). Area-based pricing charges travelers for an entry permit (e.g., per day), i.e., a tour is charged a toll (or daily usage fee) only once regardless of the number of times the restricted zone is accessed during the entire tour. If individual trips are used as the basic unit of analysis, the impact of the pricing may be miscalculated.

Based on these considerations, this chapter extends Chapter 3 by developing a model to represent trip chaining in multimodal transportation networks. We further consider using the toll revenue to subsidize transit passengers and expand highway capacity. Such arrangements could help to achieve even higher level of improvement in social benefit without making anyone worse off.

## 4.1 Tour-Based Network Equilibrium Analysis Models

### 4.1.1 Activity-Based Traffic Assignment

Activity-based travel demand models are developed based on the recognition of the interaction of activities and travelers' travel behaviors. Activity-based travel demand analysis views travel as derived demand from the travelers' need or desire to fulfill certain activities (Bhat and Koppelman, 2003). Different from trip-based models, activity-based traffic analysis models are based on the analysis of a complete tour including several trips for different activities by constructing a trip chain.

Although activity-based travel demand models are gaining more attention in the literature, such models are rarely considered in traffic assignment models. Lam and Yin (2001) proposed a dynamic user equilibrium model that integrates route choice and sequential activity choice. Maruyama and Harata (2006) and Maruyama and Sumalee (2007) proposed a path-based model with an explicit representation of trip-chains to analyze the effect of cordon-based and area-based congestion pricing schemes.

### 4.1.2 Representing Trip Chaining

It is assumed that travelers can be classified into groups and each group has a specific tour. Travelers in the same group begin at the origin of their trip chains, travel to each node in the chain consecutively, and terminate their trip at the last node in the chain, i.e., their destination. The demand for each group is known and fixed. But the travelers can freely choose any available travel modes to fulfill their travel needs.

Let  $q$  denote a trip chain or tour,  $w$  denote an OD pair and  $\Omega^q$  represent the set of OD pairs of all the trips or legs in the trip chain. For example, if tour  $q$  is home  $\rightarrow$  work  $\rightarrow$  shopping  $\rightarrow$  home, then  $\Omega^q = \{(home, work), (work, shopping), (shopping, home)\}$ .

We further introduce an index  $\sigma^{w,q}$  to facilitate converting the tour demand to OD demand. If OD pair  $w$  belongs to  $\Omega^q$ ,  $\sigma^{w,q} = 1$ ; otherwise,  $\sigma^{w,q} = 0$ .

In a network with trip chaining, it is assumed that user equilibrium will be achieved if for each tour the travel costs of all utilized paths (or hyperpaths) are equal and not greater than those of unutilized ones. Moreover, we assume that travelers will choose a single travel mode for their whole tours and the travel mode choice can be formulated as a multinomial Logit model:

$$\frac{y^{q,m}}{Y^q} = \frac{\exp(-\theta \cdot h^{q,m} - \beta^m)}{\sum_{m \in M} \exp(-\theta \cdot h^{q,m} - \beta^m)} \quad \forall w \in W \quad (4-1)$$

where  $y^{q,m}$  is the demand of tour  $q$  using mode  $m$ ;  $Y^q$  is the total travel demand for tour  $q$ ;  $h^{q,m}$  is the equilibrium travel cost of tour  $q$  by mode  $m$ ;  $\theta$  and  $\beta^m$  are parameters to be calibrated.  $\beta^m$  is a mode-specific constant and can be set as 0 for one of these three modes. Similar to the Chapter 3, another formulation equivalent to (4-1) can be written as:

$$\begin{aligned} \theta \cdot h^{q,S} + \beta^S + \ln y^{q,S} &= \theta \cdot h^{q,H} + \beta^H + \ln y^{q,H} \\ &= \theta \cdot h^{q,T} + \beta^T + \ln y^{q,T} \end{aligned} \quad (4-2)$$

### 4.1.3 Feasible Region

The feasible region of our tour-based model is defined as:

$$\begin{aligned} Ax^{w,m} &= E^{w,m}d^{w,m}, \quad \forall w \in W, m \in M \\ \sum_{w \in W} x_l^{w,T} &\leq c_l^T, \quad \forall l \in L^T \\ x_l^{w,T} &\leq f_l \omega_i^w, \quad \forall i \in N, l \in L_i^+, w \in W \\ \sum_{m \in M} y^{q,m} &= Y^q, \quad \forall q \in Q \end{aligned} \quad (4-3)$$

$$d^{w,m} = \sum_{q \in Q} \sigma^{w,q} \cdot y^{q,m}, \quad \forall w \in W, m \in M \quad (4-4)$$

$$x_l^{w,m} \geq 0, \quad \forall l \in L, w \in W, m \in M$$

$$x_l^{w,T} = 0, \quad \forall l \in L^S \cup L^H, w \in W$$

$$x_l^{w,m} = 0, \quad \forall l \in L^T, w \in W, m \in \{S, H\}$$

The above feasible region is similar to the one defined in Chapter 3 with Constraints (4-3) and (4-4) replacing the total demand Constraint (3-4). Constraint (4-3) requires that the sum of tour demand of each mode is the total demand. Constraint (4-4) converts tour demand to OD demand. All the other variables are defined in Chapter 3. The set of all  $(x, y, d, \omega)$  feasible to the above system is denoted as  $\Theta$ .

#### 4.1.4 Tour-Based Multimodal User Equilibrium Model

A tour-based multimodal user equilibrium features the following conditions: 1) the flow distribution is feasible; 2) in the auto modes (SOV and HOV), the travel costs of all utilized paths for each tour are equal and not greater than those of unutilized ones; 3) transit passengers choose cheapest hyperpaths for each tour and 4) the mode split for each tour is the function of the equilibrium tour costs of those three modes.

The user equilibrium flow distribution can be obtained by solving the following variational inequality (VI) problem.

**Theorem 4-1.** The solution  $(x^*, y^*, d^*, \omega^*)$  of the following VI problem satisfies the tour-based user equilibrium conditions in the multimodal network.

$$\begin{aligned}
& \sum_{w \in W} \sum_{m \in M} \sum_{l \in L} t_l(x^*) (x_l^{w,m} - x_l^{w,m*}) \\
& + \frac{1}{\theta} \sum_{q \in Q} \sum_{m \in M} (\ln y^{q,m*} + \beta^m) (y^{q,m} - y^{q,m*}) \\
& + \sum_{w \in W} \sum_{i \in N} (\omega_i^w - \omega_i^{w*}) \geq 0, \quad \forall (x, y, d, \omega) \in \Theta \\
& (x^*, y^*, d^*, \omega^*) \in \Theta
\end{aligned}$$

The above theorem can be proved by analyzing the KKT conditions of the VI problem and comparing them to the tour-based user equilibrium condition. The proof is similar to the proof of Theorem 3-1, thus is omitted in this section.

To solve the VI problem, we adopt the technique proposed by Aghassi et al. (2006). The problem is converted to the following regular nonlinear mathematical program using a gap function.

$$\begin{aligned}
\min_{x, d, \omega, \tilde{c}, \tilde{d}, \tilde{\omega}, \tilde{y}, \tilde{Y}} & \sum_{w \in W} \sum_{m \in M} \sum_{l \in L} t_l(x) \cdot x_l^{w,m} + \frac{1}{\theta} \sum_{q \in Q} \sum_{m \in M} (\ln y^{q,m} + \beta^m) \cdot y^{q,m} \\
& + \sum_{w \in W} \sum_{i \in N} \omega_i^w - \sum_{l \in L} c_l^T \cdot \tilde{c}_l - \sum_{q \in Q} Y^q \cdot \tilde{Y}^q \\
\text{s. t.} & \sum_{i \in N} A'_i \cdot \tilde{d}_i^{w,m} \leq t_l(x), \quad \forall w \in W, m \in \{S, H\} \\
& \sum_{i \in N} A'_i \cdot \tilde{d}_i^{w,m} + \tilde{c}_l + \tilde{\omega}_l^w \leq t_l(x), \quad \forall w \in W \\
& - \sum_{l \in L^+} f_l \cdot \tilde{\omega}_l^w \leq 1, \quad \forall i \in N, w \in W \\
& -E^{w,m} \cdot \tilde{d}^{w,m} + \tilde{y}^{w,m} \leq 0, \quad \forall w \in W, m \in M \\
& -E^{w,m} \cdot \tilde{d}^{w,m} + \tilde{D}^w \leq \frac{1}{\theta} (\beta^m + \ln d^{w,m}), \quad \forall w \in W, m \in M
\end{aligned}$$

$$\begin{aligned}\tilde{c}_l &\leq 0, \quad \forall l \in L^T \\ \tilde{\omega}_l^w &\leq 0, \quad \forall l \in L^T, w \in W \\ (x, y, d, \omega) &\in \Theta\end{aligned}$$

Where  $\tilde{c}_l$ ,  $\tilde{d}^{w,m}$ ,  $\tilde{\omega}_l^w$ ,  $\tilde{y}^{w,m}$  and  $\tilde{Y}^q$  are auxiliary variables and  $A_l$  is the  $l$ th column of the incidence matrix  $A$ .

The reformulated problem is a regular nonlinear optimization program.

Commercial nonlinear optimization solvers can be used to solve the above program. If the optimal objective value reaches zero, one portion of the optimal solution, i.e.,  $(x, y, d, \omega)$  will solve the VI problem.

#### 4.1.5 System Optimum Model

The system optimum model of the tour-based assignment can be written as follows:

$$\begin{aligned}\text{TSO} \\ \min_{x,y,d,\omega} & \sum_{w \in W} \sum_{m \in M} \sum_{l \in L} t_l(x) x_l^{w,m} + \frac{1}{\theta} \sum_{q \in Q} \sum_{m \in M} (\ln y^{q,m} + \beta^m) y^{q,m} \\ & + \sum_{w \in W} \sum_{i \in N} \omega_i^w \\ \text{s. t. } & (x, y, d, \omega) \in \Theta\end{aligned}$$

### 4.2 Tour-Based Model for Pareto-Improving Strategies

#### 4.2.1 Model Formulation

In this study, an improving strategy refers to a policy for tolling roads and highways, adjusting fares on various transit lines and expanding highway link capacities using toll revenues. For mathematical formulation, the toll rates should be nonnegative while the adjustments to transit fares are unrestricted. A negative adjustment implies that transit

passengers are subsidized. The toll revenue can also be used to expand existing highway links to further reduce congestion. It is further assumed that the transportation authority is able to leverage the stream of toll revenue to finance the proposed capacity expansion through the financing market. Denote  $\kappa_l$  as the amortized cost for providing additional unit of capacity to link  $l$ , and the cost includes the construction and financial costs.

The Pareto-improving strategies can be obtained by solving the following mathematical program:

TPIT:

$$\begin{aligned} \max_{x,d,\omega,\tau,\rho,c} \quad & \sum_{q \in Q} \frac{1}{\theta} \cdot \ln \left( \sum_{m \in M} \exp(-\theta \cdot h^{q,m} - \beta^m) \right) \cdot Y^q \\ & + \sum_{w \in W} \sum_{m \in M} \sum_{l \in L} \tau_l^w x_l^{w,m} - \sum_{m \in M} \sum_{l \in L^m} \kappa_l (c_l^m - \bar{c}_l^m) \end{aligned}$$

$$s. t. \quad \left( t_l(x, c) + \tau_l^m - (\rho_j^{w,m} - \rho_i^{w,m}) \right) x_l^{w,m} = 0, \quad (4-5)$$

$$\forall l \in L^m, m \in \{S, H\}, w \in W, l = (i, j)$$

$$\left( t_l(x, c) + \tau_l^T + \mu_l^w + \gamma_l - (\rho_j^{w,T} - \rho_i^{w,T}) \right) x_l^{w,T} = 0, \quad (4-6)$$

$$\forall l \in L^T, w \in W, l = (i, j)$$

$$\frac{1}{\theta} (\ln y^{q,m} + \beta^m) + \lambda^q - h^{q,m} = 0, \quad \forall q \in Q, m \in M \quad (4-7)$$

$$h^{q,m} = \sum_{w \in W} \sigma^{w,q} (E^{w,m} \rho^{w,m}), \quad \forall q \in Q, m \in M \quad (4-8)$$

$$\sum_{\forall l \in L_i^+} f_l \mu_i^w = 1, \quad \forall i \in I, w \in W \quad (4-9)$$

$$\gamma_l \left( \sum_{w \in W} x_l^{w,T} - c_l^T \right) = 0, \quad \forall l \in L^T \quad (4-10)$$

$$\mu_l^w (x_l^{w,T} - f_l \omega_l^w) = 0, \quad \forall l \in L^T, w \in W \quad (4-11)$$

$$t_l(x) + \tau_l^m - (\rho_j^{w,m} - \rho_i^{w,m}) \geq 0, \quad (4-12)$$

$$\forall l \in L^m, m \in \{S, H\}, w \in W, l = (i, j)$$

$$t_l(x) + \tau_l^T + \mu_l^w + \gamma_l - (\rho_j^{w,T} - \rho_i^{w,T}) \geq 0, \quad (4-13)$$

$$\forall l \in L^T, w \in W, l = (i, j)$$

$$\gamma_l \geq 0, \quad \forall l \in L^T \quad (4-14)$$

$$\mu_l^w \geq 0, \quad \forall l \in L^T, w \in W \quad (4-15)$$

$$h^{q,m} \leq h_{UE}^{q,m}, \quad \forall q \in Q, m \in M \quad (4-16)$$

$$\sum_w \sum_m \sum_l \tau_l^m x_l^{w,m} - \sum_m \sum_l k_l^m (c_l^m - \bar{c}_l^m) \geq 0 \quad (4-17)$$

$$c_l^m \geq \bar{c}_l^m, \quad \forall l \in L, m \in M \quad (4-18)$$

$$\tau_l^S, \tau_l^H \geq 0, \quad \forall l \in L \quad (4-19)$$

$$\tau_l^S = \tau_l^H, \quad \forall l \in L^S \quad (4-20)$$

$$(x, y, d, \omega) \in \Theta$$

where  $\tau_l^m$  is the toll rate or fare adjustment on link  $l$  for mode  $m$ ;  $h_{UE}^{q,m}$  is the equilibrium tour cost without any tolling intervention;  $c_l^m$  is the capacity on link  $l$  for travel mode  $m$ , and  $\bar{c}_l^m$  indicates the existing capacity.

In the above, the objective function consists of three components. The first component is a logsum measurement of user benefits (Ying and Yang, 2005). The second and third components represent the producer benefits, i.e., the profit received by the government. More specifically, the second component is the total toll revenue

and the last one represents the total cost for capacity expansion. Constraint (4-5) to (4-15) are the tour-based multimodal tolled user equilibrium conditions, which can be easily derived from the KKT condition of the VI problem in Theorem 4-1 by adding tolls to travel costs. Constraints (4-16) and (4-17) are Pareto-improving constraints. The former ensures that the tour cost after policy implementation is no greater than the cost without the tolling intervention. If travelers remain at the same mode, they will not be made worse off. If they voluntarily switch to another mode, they will be better off. Constraint (4-17) ensures that the toll revenue is enough to cover the cost for capacity expansion such that the government will not be worse off. Of the remaining constraints, Constraint (4-18) ensures that capacity expansion should be nonnegative; Constraints (4-19) and (4-20) require the charging scheme to be feasible. For example, Constraint (4-20) guarantees a uniform toll for both SOVs and HOVs on a regular link.

#### **4.2.2 Solution Algorithm**

The problem stated above is a mathematical programming problem with complementarity constraints (MPCC), a class of problems difficult to solve. We use a relaxation approach to solve the MPCC (Scholtes, 2001; Ban et al., 2006). The basic idea of this algorithm is to relax the complementarity constraints, which make the problem hard to solve, to regular nonlinear constraints. The approach solves a sequence of relaxed problems where one has a tighter relaxation and thus generates a better solution than its predecessor. The approach is sketched below:

Step 1: Initialization

Choose an initial nonnegative value for the auxiliary variable  $r_0$ . Set the iteration limit  $M$ , updating factor  $0 < u < 1$ , and set  $k = 1$ .

Step 2: Solving the following relaxed problem:

R-TPIT:

$$\begin{aligned} \max_{x,d,\omega,\tau,\rho,c} \quad & \sum_{q \in Q} \frac{1}{\theta} \cdot \ln \left( \sum_{m \in M} \exp(-\theta \cdot h^{q,m} - \beta^m) \right) \cdot Y^q \\ & + \sum_{w \in W} \sum_{m \in M} \sum_{l \in L} \tau_l^w x_l^{w,m} - \sum_{m \in M} \sum_{l \in L^m} \kappa_l (c_l^m - \bar{c}_l^m) \end{aligned}$$

s. t. (4-7) to (4-9), (4-12) to (4-20)

$$\left( t_l(x, c) + \tau_l^m - (\rho_j^{w,m} - \rho_i^{w,m}) \right) x_l^{w,m} \leq r_k,$$

$$\forall l \in L^m, m \in \{S, H\}, w \in W, l = (i, j)$$

$$\left( t_l(x, c) + \tau_l^T + \mu_l^w + \gamma_l - (\rho_j^{w,T} - \rho_i^{w,T}) \right) x_l^{w,T} \leq r_k,$$

$$\forall l \in L^T, w \in W, l = (i, j)$$

$$\gamma_l \left( \sum_{w \in W} x_l^{w,T} - c_l^T \right) \leq r_k, \quad \forall l \in L^T$$

$$\mu_l^w (x_l^{w,T} - f_l \omega_i^w) \leq r_k, \quad \forall l \in L^T, w \in W$$

$$(x, y, d, \omega) \in \Theta$$

Step 3: Parameter updating

If  $k < M$ , set  $r_{k+1} = u \cdot r_k$ ,  $k = k + 1$ , go to step 2. Otherwise, go to step 4.

Step 4: Solving the original R-PIT

Solve the exact formulation of R-TPIT as a regular nonlinear program using the solution from step 2 as initial solution. If successful, an exact solution is obtained; otherwise, an approximate solution is obtained from the last iteration of step 2.

The relaxed problem R-TPIT is a regular nonlinear program, easier to solve as compared to the original MPCC. The optimal solution will approximate better the

solution to the original MPCC as the relaxation parameter  $r_k$  decreases. The relaxed problem becomes the original MPCC problem when the relaxation parameter  $r_k$  reaches zero. Ralph and Wright (2004) proved that under certain conditions the relaxation scheme can solve optimally the original MPCC.

#### 4.2.3 Optimum Policy Model

For the comparison purpose, the section presents an optimal policy model without Pareto-improving constraints. We note that the system optimal pricing scheme can be derived from the optimality condition of the system optimum model TSO. However, the toll scheme is differentiated and will require charging different toll rates to different vehicles, even at regular links. For fair comparison, the solution to the following optimal policy model is used:

$$\begin{aligned} \max_{x,d,\omega,\tau,\rho,c} \quad & \sum_{q \in Q} \frac{1}{\theta} \cdot \ln \left( \sum_{m \in M} \exp(-\theta \cdot h^{q,m} - \beta^m) \right) \cdot Y^q \\ & + \sum_{w \in W} \sum_{m \in M} \sum_{l \in L} \tau_l^w x_l^{w,m} - \sum_{m \in M} \sum_{l \in L^m} \kappa_l (c_l^m - \bar{c}_l^m) \end{aligned}$$

$$s. t. \quad (4-5) \text{ to } (4-15), (4-18) \text{ to } (4-20)$$

$$(x, y, d, \omega) \in \Theta$$

The optimal policy problem maximizes the total social benefit subject to tolled user equilibrium conditions. The problem has a similar structure as TPIT, and can be solved using the same relaxation approach. In fact, the absence of Pareto-improving constraints makes it much easier to obtain a feasible solution to this problem, thereby expediting the solution process.

## 4.3 Numerical Example

### 4.3.1 Example Network

The models are solved for the toy network shown in Figure 4-1. The network consists of five nodes and sixteen links. Among all the links, fourteen are regular links with uniform toll rates and the others are links for HOVs only. The BPR function is used as the link travel time function and the free-flow travel time and capacity of each link are listed in Table 4-1. The average occupancy of the HOV mode is assumed to be 2.5. There are five tours in the network, i.e.,  $\{(1, 3), (3, 1)\}$ ,  $\{(1, 3), (3, 5), (5, 1)\}$ ,  $\{(1, 3), (3, 4), (4, 1)\}$ ,  $\{(2, 4), (4, 2)\}$  and  $\{(2, 4), (4, 5), (5, 2)\}$ . The demands are 10, 10, 9, 20 and 14 correspondingly.

There is one transit service on each link in the network. The service frequency for each line is 0.5 with a capacity of 15. The mode choice parameters are  $\beta^S = 0$ ;  $\beta^H = 1$ ;  $\beta^T = 4$  and  $\theta = 0.2$ . The amortized cost for providing unit capacity expansion is 5 and 10 for regular and HOV links respectively.

### 4.3.2 Optimal and Pareto-Improving Policies

Table 4-2 compares three scenarios, i.e., user equilibrium without any policy implementation, optimal policy, and Pareto-improving policy. In addition to the values reported in the table, the optimal and Pareto-improving policies use a portion of the toll revenue to expand Link 5 by 6.72 and 7.35 respectively.

The total social benefit, user travel time, vehicle travel time and net toll revenue of those three scenarios are reported in Table 4-2. Under the optimal policy, SOV and HOV users of tour 4 and 5 will suffer from an increase in travel cost while Pareto-improving policy makes every one better off. At the same time, the Pareto-improving policy significantly increases the social benefit by 10.9% (as the social benefit is a

relative value, the percentage is only meaningful for the particular reference point in the example), and reduces the total user and vehicle travel times by 13.1% and 20.1% respectively.

We further note that in the above example the toll revenue is not used by full amount to fund capacity expansion. This is because the benefit incurred by additional capacity may decrease as congestion decreases. At a certain level, the cost of the additional capacity will exceed its benefit and thus adding more capacity will result in a loss in social benefit.

#### **4.3.3 Benefits of Capacity Expansion**

Table 4-3 presents the system performances under the Pareto-improving and optimal pricing (policy without capacity expansion). Compared to the results presented in Table 4-2, allowing capacity expansion reduces the total user travel time from 1564.13 to 1518.28, a 2.9% reduction. However, total vehicle travel time increases by 4.8% from 1160.72 to 1215.96. This is because when additional capacity is provided, more travelers switch to SOV mode, thereby increasing the total vehicle time. Note that with capacity expansion, the social benefit increases by 2.3% as compared to the pure pricing policy.

Another observation worth noting is that without capacity expansion, the optimal pricing policy tends to charge more tolls on highway links in order to redistribute traffic flow and force travelers to switch to travel modes with higher occupancy. Consequently, the trip cost for auto modes will become much higher than before, implying that the auto travelers are made worse off substantially when compared to the original condition without toll intervention. In contrast, the Pareto-improving pricing scheme can still maintain the same or even less travel cost, which makes it much more appealing to the

public. Indeed, Pareto-improving pricing is generally more appealing in the situation where providing extra capacity is impossible or expensive.

#### **4.3.4 Comparison of Tour-Based and Trip-Based Models**

We now compare the tour-based model with the trip-based counterpart where users are assumed to make their mode choice based on travel cost of each individual O-D trip. On the same 5-node network, the trip-based Pareto-improving pricing model (without capacity expansion) leads to the toll matrix shown in Table 4-4 with the social benefit of -1450.40. With the same toll structure and travel demand, the flow distribution from the tour-based will achieve a social benefit of -1600.46, much smaller than the trip-based estimate. The comparison of the results of both models is presented in Table 4-5.

The values given in Table 4-5 are results of these two different models based on the same toll matrix in Table 4-4. The toll matrix is an optimal Pareto-improving pricing scheme with the trip-based model and is estimated to increase the total social benefits by 11.9%. However, with the tour-based model, the performance of the pricing scheme is greatly reduced. The total social benefit is -1600.46, little improvement from the user equilibrium value before the policy implementation (-1646.24 as shown in Table 4-2). Moreover, it will result in a deficit of 109.53, making the government worse off.

#### **4.4 Summary**

The model presented in this chapter extended Chapter 3 by incorporating a activity-based travel demand model. The model represents the travelers' mode choice by considering the fact that travelers make their decision based on their entire tours instead of single trips. In contrast to the trip-based counterpart, the tour-based model captures more realistic travel behaviors, thus is expected to be able to more accurately estimate the socioeconomic impacts that transportation policies may incur. Results from

the numerical example demonstrate that Pareto-improvement is achievable by adjusting transit fares, charging tolls on highway links and financing additional capacity to the highway network through the toll revenue.

Table 4-1. Link parameters of five-node network

Link	Free-flow travel time	Capacity
1	7	18
2	4	18
3	10	35
4	4	15
5	10	16
6	7	11
7	4	26
8	13	21
9	6	13
10	7	32
11	2	25
12	5	24
13	4	19
14	6	19
15	7	10
16	10	10

Table 4-2. Comparisons of user equilibrium, optimal and Pareto-improving policies

	User equilibrium				Optimal policy			Pareto-improving policy		
Social benefits	-1646.25				-1451.83			-1467.03		
Total user travel time	1748.12				1502.36			1518.28		
Total vehicle travel time	1522.46				1188.10			1215.96		
Net toll revenue	0.00				161.39			98.56		
	Tour	SOV	HOV	Transit	SOV	HOV	Transit	SOV	HOV	Transit
Demand	1	7.26	2.67	0.07	4.85	4.93	0.22	5.23	4.61	0.17
	2	7.29	2.68	0.03	4.91	4.99	0.10	5.28	4.65	0.08
	3	6.56	2.41	0.03	4.41	4.48	0.11	4.75	4.19	0.06
	4	14.51	5.34	0.15	13.72	6.10	0.18	14.06	5.21	0.73
	5	10.18	3.75	0.07	9.63	4.28	0.09	9.96	3.69	0.35
Tour cost	1	28.85	28.85	31.85	25.94	20.85	21.29	24.76	20.39	22.02
	2	36.92	36.92	43.92	34.02	28.94	33.31	34.25	29.89	35.31
	3	42.70	42.70	49.70	39.92	34.84	38.50	38.07	33.70	39.89
	4	23.20	23.20	26.20	24.18	23.24	25.80	23.09	23.05	17.87
	5	23.20	23.20	28.20	24.18	23.24	27.35	23.09	23.05	19.81

Table 4-2. CONTINUED

	User equilibrium			Optimal policy			Pareto-improving policy			
	Link	SOV	HOV	Transit	SOV	HOV	Transit	SOV	HOV	Transit
Toll	1	-	-	-	4.78	4.78	0.00	4.22	4.22	-7.26
	2	-	-	-	5.97	5.97	0.00	3.43	3.43	0.00
	3	-	-	-	0.94	0.94	0.00	0.00	0.00	0.00
	4	-	-	-	0.94	0.94	0.00	0.00	0.00	0.00
	5	-	-	-	2.71	2.71	0.00	2.26	2.26	7.00
	6	-	-	-	0.38	0.38	0.00	0.04	0.04	0.00
	7	-	-	-	0.03	0.03	0.00	1.44	1.44	-3.99
	8	-	-	-	0.00	0.00	0.00	0.00	0.00	0.88
	9	-	-	-	0.40	0.40	0.00	0.04	0.04	1.18
	10	-	-	-	0.00	0.00	0.00	0.00	0.00	0.00
	11	-	-	-	0.00	0.00	0.00	0.00	0.00	0.00
	12	-	-	-	0.88	0.88	0.00	0.12	0.12	0.88
	13	-	-	-	0.03	0.03	0.00	0.00	0.00	5.21
	14	-	-	-	0.00	0.00	0.00	0.00	0.00	0.00
	15	-	-	-	-	0.00	0.00	-	0.00	0.00
	16	-	-	-	-	0.00	0.00	-	0.00	-10.01

Table 4-3. Pareto-improving and optimal policies without capacity expansion

		Pareto-improving policy				Optimal policy		
Social benefits		-1502.26				-1482.53		
Total user travel time		1564.13				1539.77		
Total vehicle travel time		1160.72				1135.99		
Net toll revenue		46.10				322.85		
		Tour	SOV	HOV	Transit	SOV	HOV	Transit
Demand	1	3.14	5.86	1.00	3.38	5.94	0.68	
	2	3.32	6.19	0.48	3.51	6.17	0.32	
	3	2.97	5.53	0.50	3.15	5.53	0.31	
	4	14.27	5.25	0.48	13.72	6.10	0.18	
	5	10.07	3.70	0.23	9.63	4.28	0.09	
Tour cost	1	28.47	20.36	14.21	34.41	26.60	22.43	
	2	36.50	28.38	26.16	43.46	34.65	34.44	
	3	41.75	33.64	30.64	48.00	40.19	39.55	
	4	23.20	23.20	20.12	24.18	23.24	25.80	
	5	23.20	23.20	22.06	24.18	23.24	27.35	

Table 4-4. Trip-based Pareto-improving pricing scheme

Link	SOV	HOT	Transit
1	0.00	0.00	-7.67
2	0.00	0.00	0.00
3	0.00	0.00	-6.20
4	0.00	0.00	1.84
5	3.71	3.71	-11.89
6	0.00	0.00	2.00
7	0.00	0.00	0.00
8	0.00	0.00	0.00
9	0.05	0.05	-0.53
10	0.00	0.00	0.00
11	0.00	0.00	0.00
12	0.00	0.00	0.00
13	0.01	0.01	-0.35
14	0.00	0.00	-4.75
15	-	0.00	-7.03
16	-	0.00	-6.16

Table 4-5. Comparison of system performances

	Trip-based	Tour-based
Social benefits	-1450.40	-1600.46
Total travel time	1648.57	1593.00
Total vehicle travel time	1315.52	1144.22
Toll revenue	0.00	-109.53

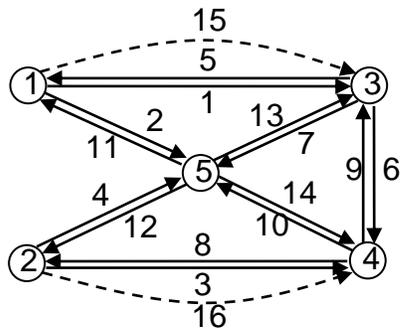


Figure 4-1. Five-node network

## CHAPTER 5 DESIGN OF MORE EQUITABLE CONGESTION PRICING SCHEME BY CONSIDERING INCOME EFFECTS

The previous two chapters discussed the implementation of Pareto-improving pricing strategies in transportation networks. The idea of Pareto-improving tolls originates from the fact that travelers in the networks are selfish and only care their own travel cost instead of the society as a whole. Pareto-improving pricing schemes guarantee that the travel cost will not increase for any travelers, thus can be more attractive to the general public. In this chapter, we will investigate another pricing scheme that will not only improve the overall efficiency of the transportation systems but also be able to improve the equity within the society. This scheme can successfully address the equity issue of congestion pricing, thus will have higher acceptability than conventional pricing schemes.

The design of more equitable congestion pricing scheme has been well studied in the literature. For instance, Yang and Zhang (2002) developed an optimal pricing model for multiclass networks with social and spatial equity constraints. Lawphongpanich and Yin (2010), Guo and Yang (2010) and Wu et al. (2011) adopted Pareto-improving approaches to ensure that none is worse off in the presence of tolls. This study differs from previous research in the following aspects: (a) it directly takes into account the effects of income on choices of trip generation, mode and route in the presence of tolls; and (b) it explicitly captures the impacts of pricing schemes across different income and geographic groups. In the literature, Franklin (2006) considered the income effects on travel choice behaviors and estimated the welfare impacts of tolling on the SR520 Bridge in Seattle, Washington. Bureau and Glachant (2008) simulated and compared

the distributional effects of nine toll scenarios in Paris. However, both studies focused on evaluation of pricing schemes instead of design.

### **5.1 Problem Description**

We consider a general multimodal transportation network that includes three types of facilities, i.e., transit services, high-occupancy/toll (HOT) and regular toll lanes. In this setting, a pricing scheme refers to a strategy for tolling roads as well as adjusting fares on various transit lines. The toll on a regular toll lane is anonymous or uniform while single-occupancy vehicles (SOV) and high-occupancy vehicles (HOV) may be charged different toll rates on HOT lanes. Tolls on highway segments are nonnegative while adjustments to transit fares are unrestricted. Herein, a negative fare adjustment means that transit users are subsidized by the toll revenue. Therefore, a component of the pricing scheme in this paper can be viewed as revenue recycling.

We further consider a discrete set of user groups with different incomes and preferences among four travel modes, i.e., no travel, transit, drive alone and car pool. It is assumed that users' travel decisions can be represented by a nested logit model whose utility functions are nonlinear in income. This specification allows us to more accurately capture the income effects on the choice behavior in the presence of tolls. According to Jara-Diaz and Videla (1989), Franklin (2006) and Bureau and Glachant (2008), the traditional specification with constant marginal utility of income may lead to an underestimate of the regressivity of a pricing scheme.

The equivalent variation is adopted in this paper as the benefit measure. It is the dollar amount that has to be taken away from a traveler in the no-tolling scenario to leave him as well off as he would be in the tolling scenario (Nicholson, 1998). Intuitively, if the traveler benefits from the pricing scheme, the equivalent variation will be negative.

The equivalent income, defined as the original income minus the equivalent variation, can thus serve as a welfare measure. It is a measure of how wealthy the traveler feels under the pricing scheme. As the user utility is monotonically increasing with income, an equivalent income higher than the original income of the traveler implies that the pricing scheme increases the traveler's utility level. Calculating equivalent income is difficult in the random utility framework if the utility is a nonlinear function of income. In this chapter, we adopt the formula derived by Dagsvik and Karlstrom (2005) to compute the expected equivalent income.

Although there is no consensus definition of equity in the context of congestion pricing, this chapter deems a pricing scheme to be more equitable if it leads to a more uniform distribution of wealth across different groups of population categorized by income and geographic locations. Such a notion of equity combines some aspects of both vertical and spatial equity discussed in the literature. The Gini coefficient (Gini, 1912) is adopted to measure the equity effect of a pricing scheme. The coefficient measures the inequality of income distribution across a population, a value of 0 expressing complete equality and a value of 1 implying complete inequality.

With the above consideration, we intend to design a pricing scheme to maximize the social benefit and minimize the income inequality simultaneously. Since these two objectives are often conflicting, we seek for a balance between them.

## **5.2 Mathematical Model and Solution Algorithm**

The notations used in this chapter are slightly different from the previous chapters, and will be redefined at the time of using.

### 5.2.1 Feasible Region

Let  $(N, L)$  be a directed transportation network where  $N$  is the set of nodes and  $L$  the set of directed links. As aforementioned, there are three types of facilities in the network, i.e., regular (toll) lanes, HOT lanes and transit lines. There are three travel modes, i.e., SOV, HOV and transit, as well as a no-travel option for each traveler to choose. Hereinafter, we use the superscript  $S$ ,  $H$  and  $T$  to denote regular lanes (or SOV mode), HOT lanes (or HOV mode) and transit lines (or transit mode) and the superscript  $R$  to denote the no-travel option. Each link  $l \in L$  has an associated travel time  $t_l$  that depends on link flow  $v_l$  and link capacity  $c_l$ . The transit services are assumed to share the roadway with the other auto modes, and thus have the same travel times. However, transit users may experience additional waiting and transferring times at bus stops. Let  $G$  be the set of all traveler groups and  $W$  denote the set of origin-destination (OD) pairs. The demand of traveler group  $g$  for OD pair  $w$  is denoted as  $D^{w,g}$ . The demand  $D^{w,g}$  is assumed to be fixed.

With the above notations, the set of all feasible flow distributions  $\Phi$  for the network can be described as follows:

$$\sum_{k \in K^{w,m}} f_k^{w,m,g} = d^{w,m,g}, \quad \forall w \in W, m \in M, g \in G \quad (5-1)$$

$$\sum_{m \in M} d^{w,m,g} = D^{w,g}, \quad \forall w \in W, g \in G \quad (5-2)$$

$$f_k^{w,m,g} \geq 0, \quad \forall w \in W, m \in M, g \in G, k \in K^{w,m} \quad (5-3)$$

where  $f_k^{w,m,g}$  is the flow of user group  $g$  on path  $k$  of mode  $m$  between OD pair  $w$ ,  $d^{w,m,g}$  is the demand of user group  $g$  between OD pair  $w$  using mode  $m$ , and  $K^{w,m}$  is the set of all paths connecting OD pair  $w$  for mode  $m$ . Constraints (5-1) and (5-2)

ensure that demands are satisfied and Constraint (5-3) requires path flow variables to be nonnegative.

The feasible toll set  $\Psi$  can be represented as:

$$\sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^{w,g}} f_k^{w,m,g} \sum_{l \in L} \Delta_{k,l} \tau_l^m \geq 0 \quad (5-4)$$

$$\tau_l^m \geq 0, \quad \forall l \in L, m \in \{S, H\} \quad (5-5)$$

$$\tau_l^S = \tau_l^H, \quad \forall l \in L_{RT} \quad (5-6)$$

$$\tau_l^R = 0, \quad \forall l \in L \quad (5-7)$$

where  $\tau_l^m$  is the toll rate or fare adjustment for mode  $m$  on link  $l$  and  $\Delta_{k,l}$  is the path-link incident matrix, i.e.,  $\Delta_{k,l} = 1$  if path  $k$  uses link  $l$  and  $\Delta_{k,l} = 0$  otherwise. Constraint (5-4) requires that the total toll revenue is nonnegative. Constraints (5-5) and (5-7) ensure that the tolls for auto modes are nonnegative and the toll for the no-travel option must be zero.

### 5.2.2 Multimodal User Equilibrium

It is assumed that travelers' choices of modes and routes follow a nested logit model where travelers first decide their travel modes and then choose the routes among those available for the selected modes. The utility function for a group  $g$  traveler between OD pair  $w$  by mode  $m$  on route  $k$  is defined as:

$$u_k^{w,m,g} = v_k^{w,m,g} + \varepsilon_k^{w,m,g}$$

where  $u_k^{w,m,g}$  is the utility for the traveler;  $v_k^{w,m,g}$  is the deterministic observable portion of the utility and  $\varepsilon_k^{w,m,g}$  is a random error representing the unobservable factors. As aforementioned, the deterministic utility function is assumed to be nonlinear, and its specification can be, e.g., the generalized Leontief model (Diewert, 1971):

$$v_k^{w,m,g} = \beta_0^{m,g} + \beta_1 T_k^{w,m} + \beta_2 \sqrt{T_k^{w,m}} + \beta_3 (y^g - C_k^{w,m}) \\ + \beta_4 \sqrt{y^g - C_k^{w,m}} + \beta_5 \sqrt{T_k^{w,m}} \sqrt{y^g - C_k^{w,m}}$$

Alternatively, the Translog model (Christensen et al., 1971):

$$v_k^{w,m,g} = \beta_0^{m,g} + \beta_1 \ln T_k^{w,m} + \beta_2 \ln^2 T_k^{w,m} + \beta_3 \ln(y^g - C_k^{w,m}) \\ + \beta_4 \ln^2(y^g - C_k^{w,m}) + \beta_5 \ln T_k^{w,m} \ln(y^g - C_k^{w,m})$$

where  $y^g$  is the income of the traveler;  $\beta_0^{m,g}$ ,  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$ ,  $\beta_4$  and  $\beta_5$  are parameters to be calibrated;  $T_k^{w,m}$  and  $C_k^{w,m}$  are the travel time and toll of path  $k$  between OD pair  $w$  by mode  $m$ . They can be calculated as follows:

$$C_k^{w,m} = \sum_{l \in L} \Delta_{k,l} \tau_l^m, \quad \forall k \in K^{w,m}, w \in W, m \in M$$

$$T_k^{w,m} = \sum_{l \in L} \Delta_{k,l} t_l, \quad \forall k \in K^{w,m}, w \in W, m \in \{S, H\}$$

$$T_k^{w,T} = \sum_{l \in L} \Delta_{k,l} t_l + \tilde{t}_k^w, \quad \forall k \in K^{w,T}, w \in W$$

$$T_k^{w,R} = 0, \quad \forall k \in K^{w,R}, w \in W$$

where  $\tilde{t}_k^w$  is the expected waiting and transferring time for transit users on path  $k$  between OD pair  $w$ . The transit frequency is assumed to be fixed and known, and thus the expected waiting and transferring time can be estimated beforehand for each transit path. For the no-travel mode (mode  $R$ ), the travel time and toll are always zero.

As per the nested logit model, the probability for the group  $g$  traveler between OD pair  $w$  to choose mode  $m$  and path  $k$  is given by the following:

$$P_k^{w,m,g} = \frac{\exp\left(\frac{v_k^{w,m,g}}{\theta^{w,m,g}}\right)}{\sum_{j \in K^{w,m}} \exp\left(\frac{v_j^{w,m,g}}{\theta^{w,m,g}}\right)} \cdot \frac{\exp(\bar{v}^{w,m,g})}{\sum_{m' \in M} \exp(\bar{v}^{w,m',g})}$$

where  $\bar{v}^{w,m,g}$  is the expected utility for choosing mode  $m$  and calculated as follows:

$$\bar{v}^{w,m,g} = \ln \left( \sum_{j \in K^{w,m}} \exp\left(\frac{v_j^{w,m,g}}{\theta^{w,m,g}}\right) \right)^{\theta^{w,m,g}} \quad (5-8)$$

where  $\theta^{w,m,g}$  is a measure of the degree of independence in the error terms of the utility function for different paths of mode  $m$  for OD pair  $w$  and user group  $g$  (Train, 2003).

The above behavioral considerations lead to a multimodal user equilibrium where the perceived utility for each traveler is maximized. The corresponding flow distribution can be obtained by solving the following variational inequality (VI) problem.

**Theorem 5-1.** The solution  $(f^*, d^*)$  of the following VI problem is in user equilibrium.

$$\begin{aligned} & \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^{w,m}} (-v_k^{w,m,g} + \theta^{w,m,g} \ln f_k^{w,m,g}) \cdot (f_k^{w,m,g} - f_k^{w,m,g*}) \\ & + \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} (1 - \theta^{w,m,g}) \ln d^{w,m,g} \cdot (d^{w,m,g} - d^{w,m,g*}) \geq 0, \end{aligned} \quad (5-9)$$

$$\forall (f, d) \in \Phi$$

$$(f^*, d^*) \in \Phi$$

**Proof.** The optimality conditions of the above IV problem include the following:

$$\begin{aligned} -v_k^{w,m,g} + \theta^{w,m,g} \ln f_k^{w,m,g} - \rho^{w,m,g} &= 0, \\ \forall k \in K^{w,m}, w \in W, m \in M, g \in G \end{aligned} \quad (5-10)$$

$$(1 - \theta^{w,m,g}) \ln d^{w,m,g} + \rho^{w,m,g} - \zeta^{w,g} = 0, \quad (5-11)$$

$$\forall w \in W, m \in M, g \in G$$

where  $\rho^{w,m,g}$  and  $\zeta^{w,g}$  are multipliers associated with constraints (5-1) and (5-2).

From (5-10),

$$f_k^{w,m,g} = \exp\left(\frac{\rho^{w,m,g}}{\theta^{w,m,g}}\right) \cdot \exp\left(\frac{v_k^{w,m,g}}{\theta^{w,m,g}}\right), \quad (5-12)$$

$$\forall k \in K^{w,m}, w \in W, m \in M, g \in G$$

Summing (5-12) for all paths for the same  $w$ ,  $m$  and  $g$ , and considering constraint (5-1), we have:

$$\rho^{w,m,g} = \theta^{w,m,g} \ln \left( \frac{d^{w,m,g}}{\sum_{k \in K^{w,m}} \exp\left(\frac{v_k^{w,m,g}}{\theta^{w,m,g}}\right)} \right), \quad (5-13)$$

$$\forall w \in W, m \in M, g \in G$$

Comparing (5-8) and (5-13) yields:

$$\rho^{w,m,g} = \theta^{w,m,g} \ln d^{w,m,g} - \bar{v}^{w,m,g}, \quad \forall w \in W, m \in M, g \in G \quad (5-14)$$

Substitute (5-14) into (5-11), and use Constraint (5-2), we obtain:

$$\zeta^{w,g} = \ln D^{w,g} - \ln \left( \sum_{m \in M} \exp \bar{v}^{w,m,g} \right), \quad \forall w \in W, g \in G$$

Therefore:

$$d^{w,m,g} = \frac{\exp \bar{v}^{w,m,g}}{\sum_{m' \in M} \exp \bar{v}^{w,m',g}} \cdot D^{w,g}, \quad \forall w \in W, m \in M, g \in G$$

$$f_k^{w,m,g} = \frac{\exp\left(\frac{v_k^{w,m,g}}{\theta^{w,m,g}}\right)}{\sum_{j \in K^{w,m}} \exp\left(\frac{v_j^{w,m,g}}{\theta^{w,m,g}}\right)} \cdot d^{w,m,g},$$

$$\forall k \in K^{w,m}, w \in W, m \in M, g \in G$$

The above two equations demonstrate that the solution to the VI, i.e., the demand-flow distribution follows the nested-Logit-based user equilibrium condition. ■

The above VI can be solved by a variety of existing algorithms in the literature. In this dissertation, we reformulate it as a regular nonlinear programming using a gap function introduced by Aghassi et al. (2006). If the reformulated program can be solved with the optimal objective value of zero, part of the solution, i.e.  $(f, d)$ , will solve the VI problem. The reformulated program is as follows:

$$\begin{aligned} \min_{f,d,\rho,\zeta} \quad & \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^{w,m}} (-v_k^{w,m,g} + \theta^{w,m,g} \ln f_k^{w,m,g}) \cdot f_k^{w,m,g} \\ & + \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} (1 - \theta^{w,m,g}) \ln d^{w,m,g} \cdot d^{w,m,g} \\ & - \sum_{g \in G} \sum_{w \in W} D^{w,g} \cdot \zeta^{w,g} \end{aligned}$$

$$s. t. \quad \rho^{w,m,g} \leq -v_k^{w,m,g} + \theta^{w,m,g} \ln f_k^{w,m,g},$$

$$\forall w \in W, m \in M, g \in G, k \in K^{w,m}$$

$$-\rho^{w,m,g} + \zeta^{w,g} \leq (1 - \theta^{w,m,g}) \ln d^{w,m,g},$$

$$\forall w \in W, m \in M, g \in G$$

## 5.2.3 Welfare and Equity Measures

### 5.2.3.1 Individual welfare measure

To represent the welfare change of each individual traveler under the pricing scheme, we use the equivalent variation, which is the dollar amount that the traveller would be indifferent about accepting in lieu of the toll charge. More specifically, it is the change in his or her wealth that would be equivalent to the toll charge in terms of its welfare impact. Let  $u^{w,g}(y^g, \tau)$  be the utility for a traveler in group  $g$  between OD pair  $w$  with income  $y^g$  in the presence of toll rate  $\tau$ . The equivalent variation, i.e.,  $ev^{w,g}(\tau)$ , is defined as:

$$ev^{w,g}(\tau) = \arg\{z: u^{w,g}(y^g - z, 0) = u^{w,g}(y^g, \tau)\}$$

Based on the equivalent variation, we can calculate the equivalent income level that allows the individual to experience the same level of utility before tolling as the original income does after tolling. The equivalent income  $e^{w,g}(\tau)$  for the individual is defined as:

$$e^{w,g}(\tau) = \arg\{z: u^{w,g}(z, 0) = u^{w,g}(y^g, \tau)\}$$

It follows that  $ev^{w,g}(\tau) = y^g - e^{w,g}(\tau)$ .

The equivalent income measures the impact of congestion pricing on each individual traveler in monetary term. For a traveler, if the equivalent expenditure is higher than his or her original income, the pricing scheme is considered to be beneficial to the traveller.

Due to the existence of the random error term in the utility function, the exact equivalent income cannot be calculated for each individual traveler. Instead, we adopt the method proposed by Dagsvik and Karlstrom (2005) to compute the expected

equivalent income. Although the formula derived by Dagsvik and Karlstrom calculates the expected compensating expenditure, it can be easily modified to calculate the expected equivalent income.

Let  $v_k^{w,m,g}(y, \tau)$  be the deterministic portion of the utility for users in group  $g$  traveling between OD pair  $w$  on mode  $m$  and path  $k$  with income level  $y$  in the presence of toll level  $\tau$ . The expected equivalent income for a traveler in the group can be calculated as:

$$\begin{aligned}
 & E(e^{w,g}(\tau)) \\
 &= \sum_{m^0 \in M} \sum_{k^0 \in K^{w,m^0}} \int_0^{p_{k^0}^{w,m^0,g}(\tau)} \frac{\left( \sum_{k \in K^{w,m^0}} \exp\left(\frac{h_k^{w,m^0,g}(z, \tau)}{\theta^{m^0}}\right) \right)^{\theta^{m^0}-1} \cdot \exp\left(\frac{v_{k^0}^{w,m^0,g}(y^g, 0)}{\theta^{m^0}}\right)}{\sum_{m \in M} \left( \sum_{k \in K^{w,m}} \exp\left(\frac{h_k^{w,m,g}(z, \tau)}{\theta^m}\right) \right)^{\theta^m}} d
 \end{aligned} \tag{5-15}$$

where  $p_{k^0}^{w,m^0,g}(\tau)$  and  $h_k^{w,m^0,g}(z, \tau)$  are defined as:

$$v_{k^0}^{w,m^0,g}(y^g, \tau) = v_{k^0}^{w,m^0,g}\left(p_{k^0}^{w,m^0,g}(\tau), 0\right)$$

$$h_k^{w,m^0,g}(z, \tau) = \max\left(v_k^{w,m^0,g}(y^g, \tau), v_k^{w,m^0,g}(z, 0)\right)$$

Equation (5-15) provides a way to calculate the expected equivalent income for each individual traveler without knowing his or her exact mode choice.

### 5.2.3.2 Social benefit measure

The sum of the expected equivalent income of all travellers can serve as a benefit measure as it represents, at an aggregate level, how wealthy travelers feel under a pricing scheme. In designing a pricing scheme, we attempt to maximize the social benefit, which consists of two components: the first is the total expected equivalent

income of all travelers in the network while the second is the total toll revenue collected by the government. The social benefit associated with a pricing scheme  $\tau$  can thus be calculated as:

$$SB(\tau) = \sum_{g \in G} \sum_{w \in W} E(e^{w,g}(\tau)) \cdot D^{w,g} + \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^{w,m}} C_k^{w,m} f_k^{w,m}$$

A measure of net benefit can also be defined, which differs from the above by the total original income, a constant. Both measures thus lead to the same solution.

### 5.2.3.3 Equity measure

We use the Gini coefficient to represent the equity impact of a pricing scheme. The coefficient measures the inequality of a wealth distribution. A Gini coefficient value of 0 means total equality and a value of 1 expresses maximal inequality. Given the expected equivalent income of travelers in the presence of a pricing scheme, it is straightforward to compute the Gini coefficient as follows:

$$GN(\tau) = \frac{1}{2 \cdot (\sum_{g \in G} \sum_{w \in W} D^{w,g})^2 \cdot \overline{E(e(\tau))}} \cdot \sum_{g_1, g_2 \in G} \sum_{w_1, w_2 \in W} (D^{w_1, g_1} \cdot D^{w_2, g_2} \cdot |E(e^{w_1, g_1}(\tau)) - E(e^{w_2, g_2}(\tau))|)$$

where  $\overline{E(e(\tau))}$  is the average expected equivalent expenditure for all travelers.

The Gini coefficient is calculated based on equivalent income. A more equitable pricing scheme will lead to a smaller value of the Gini coefficient. Moreover, if the value is smaller than the Gini coefficient without tolling, the pricing scheme is considered progressive. Otherwise, it is regressive.

It is worth noting that our modeling framework does not impose any restriction on which equity measure can be used. Other equity measure such as Theil's entropy and Atkinson index can be used instead.

#### 5.2.4 Model Formulation

We now formulate a mathematical program to determine a pricing scheme that improves both the equity and social surplus. As these two objectives are often conflicting, we seek for a balance between them. The formulation maximizes a weighted sum of the social benefit and the Gini coefficient as follows:

$$\max_{\tau, d, f} \quad \alpha \cdot \frac{SB(\tau)}{SB(0)} - (1 - \alpha) \cdot \frac{GN(\tau)}{GN(0)}$$

$$s. t. \quad (5-1) \text{ to } (5-7) \text{ and } (5-9)$$

where both terms of the objective function are normalized by the corresponding measures without tolling and  $\alpha$  is a positive weighting parameter.

As formulated, the problem is a mathematical programming program with equilibrium constraints (MPEC), a class of problem difficult to solve. Compounding to the difficulty is that Equation (5-15) involves numerical integration.

#### 5.2.5 Solution Algorithm

We adopt the compass search algorithm (Kolda et al., 2003) to solve the above toll optimization model. The algorithm solves a series of multimodal user equilibrium problems, i.e., the VI formulation (5-9) with different toll vectors. The algorithm varies the toll rates one at a time by a step size, and evaluates the resulting objective function in order to find a better feasible solution. If no better solution is found, the step size is then halved and the evaluation procedure will be repeated. The process continues until the step size is sufficiently small. We implement the algorithm in GAMS (Brooke et al.,

2005) in conjunction with Matlab (Wong, 2009) and use CONOPT (Drud, 1995) as the solver for the multimodal user equilibrium problem.

### 5.3 Numerical Examples

We solve the toll optimization model for two networks, the nine-node network and the Seattle regional network.

#### 5.3.1 Nine-Node Network

Figure 5-1 illustrates the network, which consists of 9 nodes and 4 OD pairs. The link travel time is the BPR function and the free-flow travel time and capacity of each link are displayed in Table 5-1. There are four groups of travelers in the network and their demands for each OD pair are listed in Table 5-2.

The following Generalized Leonief utility function is used in this example:

$$v_k^{w,m,g} = \beta_0^{m,g} - \beta_1^g T_k^{w,m} + \beta_2^g \sqrt{y^g - C_k^{w,m}}$$

The values of these parameters are reported in Table 5-3.

Under the no toll condition, the total income, Gini coefficient and total travel time of the network are 12765, 0.1328 and 3041.5 respectively. It is assumed that only ten links in the network can be tolled. They are link 1, 2, 3, 4, 6, 9, 11, 12, 14 and 15. Four pricing schemes are obtained using the proposed algorithm with different weighting factors. Policy I focuses on maximizing the social benefit with the parameter  $\alpha$  equal to one. In contrast, Policy IV solely minimizes the Gini coefficient. Policy II and III are intermediate with the weighing parameter equal to 0.91 and 0.80, respectively.

Table 5-4 compares these four schemes. The net benefit and Gini coefficient for each pricing policies as well as those in the no-toll condition are plotted in Figure 5-2.

In Figure 5-2, Policy I, the pricing scheme that maximizes the social benefit, is regressive and does harm the poor. However, all the other policies are progressive. In fact, they improve the system in all three aspects, i.e., social benefit, system travel time and equity.

We further note that the system travel time does not change with the weighting factor  $\alpha$  in a monotone fashion. As shown in Table 5-4, Policy I, the most efficient pricing scheme, yields a total travel time of 2196.1 while Policy II, a more equitable scheme, understandably leads to a higher system travel time of 2242.8. However, Policy III, which is more equitable than Policy II, produces a smaller total travel time of 2196.3.

### **5.3.2 Seattle Network**

#### **5.3.2.1 Network characteristic**

The network shown in Figure 5-3 is a sketch of the regional freeway network of the Seattle area in Washington. The network consists of 16 nodes, 44 links and 30 OD pairs. It is assumed the freeway links (solid lines in the figure) have travel time functions in the form of the BPR function. The free-flow travel time and capacity of all the freeway links are listed in Table 5-5 as calibrated by Boyles (2009). The link travel times for all the other links are assumed to be fixed at 5 minutes. It is further assumed that the transit services share the same roadways with auto users and thus have the same in-vehicle travel time. However, an extra 10 minutes of waiting and transfer time is added to all transit paths.

#### **5.3.2.2 User behavior**

The user utility function is assumed to be a Translog function as follow:

$$v_k^{w,m,g} = \beta_0^R \log \gamma y^g + \beta_1 \ln(y^g - C_k^{w,m}) + \beta_2 \ln T_k^{w,m} + \beta_3 \ln^2 T_k^{w,m}$$

where  $y^g$  is the daily income for user group  $g$ ,  $\gamma$  converts daily income to annual income.

Franklin (2006) has calibrated the above parameters for auto and transit mode using Puget Sound Regional Council's 1999 Household Activity Survey (NuStats Research and Consulting, 1999). Based on these parameters, we set the parameters for the Translog utility function as shown in Table 5-6.

The travel time for the no-travel option is undefined. To calculate its utility, we use the following formulation:

$$v_k^{w,R,g} = \beta_0^R \log \alpha y^g + \beta_1 \ln(y^g - C_k^{w,R}) + \beta_2 \ln \tilde{T}^w + \beta_3 \ln^2 \tilde{T}^w$$

where  $\tilde{T}^w$  is the shortest free-flow travel time for OD pair  $w$ .

The parameter  $\beta_0^N$  is selected that approximately 15% of the total demand would choose the no-travel option in the multimodal user equilibrium condition with no toll charges. There is not enough information to calibrate the parameters for the HOV mode, so only one auto mode is considered in the example.

### 5.3.2.3 Demand estimation

The OD demand matrix is generated in order to match the PM peak traffic flow count on the freeway links in the Seattle area as reported by WSDOT (2004). To further specify the demand for each user groups, additional information regarding household income and wages in the surrounding area is obtained from County Business Patterns (U.S. Census Bureau, 2004) and 2000 Decennial Census (U.S. Census Bureau, 2002). The surrounding region of Seattle is divided to six areas by zip codes to their nearby demand nodes as shown in Figure 5-4. The zip-code-based data are then used to calculate the income and wage distribution for the six areas. The user group specified

demand is estimated based on the assumption that the income of the travelers should have similar distribution as the region they depart from or arrive to.

The travellers of the network are aggregated into four groups based on their annual household incomes, i.e., \$20,000, \$40,000, \$70,000 and \$120,000, respectively. The estimated demand matrix is listed in Table 5-7. Figure 5-5 and Figure 5-6 present the production and attraction of each traffic analysis zone, including Everett, Seattle, Bellevue, Tacoma, Lynnwood and Renton.

#### **5.3.2.4 Existing traffic condition and social welfare**

Table 5-8 reports the multimodal user equilibrium condition for the existing no-toll condition. Figure 5-7 and Figure 5-8 show the traveler's mode choice decisions under the no-toll condition by origin and by income group. For different origins, the percentages of travelers' mode choice decision among different modes are similar, with about 80% of the total population choosing to drive and 4% for transit service. However, there is an obvious trend in Figure 5-8 that as the income increasing, a higher percentage of travelers will choose auto modes.

#### **5.3.2.5 Optimal pricing policies**

We assume that 12 links can be charged. They are link 3, 4, 5, 6, 8, 9, 12, 13, 15, 16, 17 and 20, which are marked as red in Figure 5-4. The maximum link toll is \$20 and the maximal subsidy for transit users is also \$20 for each link.

By varying the weighing factor  $\alpha$ , we solve the toll optimization model to obtain a Pareto frontier for the network, as shown in Figure 5-9. The corresponding results and tolling schemes are summarized in Table 5-9, Table 5-10 and Table 5-11. It can be observed that policy 1 is the most efficient scheme, providing a net benefit of \$97.1 million annually. However, it increases the Gini coefficient by 4.4% at the same time. In

fact, only government benefits from the scheme, collecting \$180.5 million in toll revenue annually. All the travelers experience a decrease in the equivalent income. The poorest group, i.e., Group 1, suffers the most loss of 7.5% or \$1500 annually. In contrast, the richest group, i.e. group 4, experiences a negligible loss of 0.06% or \$72 annually. It is thus safe to say that policy 1 achieves the maximal system efficiency at the price of low-income travelers, thereby compromising the equity. The basic mechanism of policy 1 is to charge a high toll to discourage travel and reduce traffic congestion. The reduction in travel time is more valuable to the high-income groups, which is the reason why the richest travelers are almost not harmed by the policy. However, the high toll prices off a significant portion of low-income travelers. As shown in Table 5-10, the trips made by the lowest-income group decrease by 30.0% where auto trips reduce by 44.6% and the transit trips are almost tripled. In the meanwhile, the travel demand of group 4 remains the same due to the negligible change in their utility level.

When the equity is more favored, the optimal pricing policies tend to charge less and subsidize more. As the poor being the largest transit rider group, this mechanism acts as a mean to offer more benefit to low-income travelers. Consequently, the toll revenue for the government becomes lower. Under policy 2 and 3, although the rich still suffers less than the poor, the difference in loss is smaller as compared to policy 1. Note that the richest people in group 1 actually benefit from the pricing policies, as shown in Figure 5-10.

With the equity measure given enough weight in the objective function, policy 4 exhibits a different and interesting pattern. The lowest-income travelers become the ones that enjoy the most benefit. Moreover, as shown in Figure 5-10, the low and high-

income travelers obtain more benefit than those mid-income travelers, from different sources though. The former benefits more from the transit subsidy while the latter values more the reduced travel time. All user groups are better off compared to the no-toll condition, suggesting that policy 4 is Pareto-improving.

Unfortunately, our computation experiments do not find a pricing policy that reduces the Gini coefficient. All the policies discussed previously are essentially regressive and lead to a more uneven distribution of social wealth. By carefully examining the results, we find that the constraint that restricts the total toll revenue to be nonnegative is the one that preventing the pricing scheme from achieve higher equity levels. Thus we try to further investigate the performance of the pricing schemes by eliminating the nonnegative revenue constraint.

Figure 5-11 shows the results of optimal policies obtained by varying the weighing factor  $\alpha$  and allowing the toll revenue to be negative. This is done by relaxing Constraint (5-4). When efficiency is favored, i.e., the value of  $\alpha$  is large, ignoring Constraint (5-4) will not change the solution. However, when more weight is put towards equity, the resulting optimal pricing schemes (Policy N1 and Policy N2) yields negative total toll revenues as shown in Table 5-12, which suggest that the government subsidizes transit users from additional external sources other than the toll revenue to further improve equity and efficiency. The negative toll revenue may help achieve a better overall performance by providing more subsidy to transit riders who are more likely to be low-income travelers and encouraging more people to switch to transit services. These new pricing policies are reported in Table 5-12, Table 5-13 and Table 5-14. It is easy to observe that policy N1 and N2 are both progressive with lower Gini coefficients.

Moreover, Policy N1 improves the network in all three aspects, i.e., the Gini coefficients, social welfare and total system travel time. Policy N2 charges no toll at all but offers the maximal amount of subsidies allowed to the transit links. This certainly cannot be done without significant external support.

Figure 5-12 shows the changes in average equivalent income for different income groups under different pricing policies. Although the results display similar trend as with the nonnegative toll revenue constraint in effect where high-income travelers get the most benefit when the policy weights efficiency higher and both high-income and low-income travelers gain more than the mid-income groups when the equity weights more, the low-income travelers becomes the single user groups that can obtain significantly higher benefit under policies N1 and N2. This is because policies N1 and N2 offers substantial subsidies for transit service, and the benefit from the subsidy significantly outweighs the benefit from congestion reduction which valued the most by the high-income traveler.

#### **5.4 Summary**

We have presented a toll optimization model to design efficient and equitable pricing schemes for general multimodal transportation networks. The model considers the effects of income on travelers' choices of trip generation, mode and route and explicitly captures the impacts of pricing schemes on different income and geographic groups. The model is formulated as a MPEC and solved by a derivative-free algorithm.

Two numerical examples are reported in the paper to illustrate the model. The first example is on the nine-node network where optimal pricing schemes can be found to achieve higher efficiency and better equity. In the second example, we find it difficult to design a progressive pricing policy. However, by allowing a certain shortfall in the total

toll revenue, we obtain a pricing policy that reduce traffic congestion, increase social welfare and improve the equity.

Our examples demonstrate that the model provides a valuable mean to design pricing policies with a balance between efficiency and equity. The proposed model is very flexible in accommodating a variety of welfare and equity measures.

Table 5-1. Link parameters

Link	Free-flow travel time	Capacity
1	5	12
2	6	18
3	3	35
4	9	35
5	9	20
6	2	11
7	8	26
8	4	11
9	6	33
10	7	32
11	3	25
12	6	24
13	2	19
14	8	39
15	6	43
16	4	36
17	4	26
18	8	30

Table 5-2. Origin-destination demands

OD	1-3	1-4	2-3	2-4
Group 1	2	2	20	35
Group 2	1	3	15	25
Group 3	2	10	10	5
Group 4	10	20	5	10

Table 5-3. Utility function parameters

	$\gamma$	$\beta$	$\alpha$			
			SOV	HOV	Transit	No travel
Group 1	9.0	-0.2	0.0	-1.5	-2.1	-8.0
Group 2	10.0	-0.3	0.0	-1.5	-2.6	-9.0
Group 3	10.0	-0.4	0.0	-1.8	-3.5	-9.5
Group 4	10.0	-0.4	0.0	-2.5	-4.0	-10.0

Table 5-4. Results of Nine-Node network

		Policy I			Policy II			Policy III			Policy IV		
$\alpha$		1.00			0.91			0.80			0.00		
Net benefit		613.5			604.6			562.7			417.7		
Gini coefficient		0.1338			0.1316			0.1291			0.1272		
Total travel time		2196.1			2242.8			2196.3			2210.0		
Toll revenue		89.8			81.5			43.9			0.7		
Toll	Link	SOV	HOV	Transit	SOV	HOV	Transit	SOV	HOV	Transit	SOV	HOV	Transit
	1	0.00	0.00	0.00	1.00	1.00	0.75	2.25	2.50	1.25	3.00	6.00	2.00
	2	0.50	0.00	0.00	2.00	0.75	0.25	3.25	1.75	0.00	6.00	5.25	-2.00
	3	0.00	0.00	0.00	0.00	0.00	-1.25	0.00	0.00	-3.00	0.00	0.00	-2.00
	4	0.00	0.00	0.00	0.00	0.00	-1.50	0.00	0.00	-3.50	0.00	0.00	-4.00
	6	6.00	0.00	0.00	6.50	0.00	0.00	6.50	0.00	0.00	7.50	0.00	0.00
	9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	11	0.75	0.00	0.00	0.25	0.00	0.25	0.00	0.00	1.00	0.00	1.50	0.00
	12	1.00	0.00	0.00	0.25	0.00	0.00	0.00	0.00	0.25	0.00	0.00	0.00
	14	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00	0.00	-2.00
15	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00	0.00	-0.50	

Table 5-5. Link parameters of Seattle network

Link Number	Link	Free-flow travel time	Capacity
1	(1,2)	7	6864
2	(2,1)	7	6864
3	(2,3)	14	6864
4	(2,4)	16	5895
5	(3,2)	14	6864
6	(3,4)	10	3825
7	(3,5)	4	6864
8	(4,2)	16	5722
9	(4,3)	10	3825
10	(4,6)	3	5895
11	(5,3)	4	6864
12	(5,6)	10	6614
13	(5,7)	13	8762
14	(6,4)	3	5722
15	(6,5)	10	4944
16	(6,8)	9	5609
17	(7,5)	13	7577
18	(7,8)	2	5994
19	(7,9)	12	7449
20	(8,6)	9	5994
21	(8,7)	2	5609
22	(8,10)	10	4079
23	(9,7)	12	7449
24	(10,8)	10	3950

Table 5-6. Parameters for the Translog utility function

Parameter	Auto	Transit	No travel
$\beta_0$	-	-0.227	-0.227
$\beta_1$	9.014	9.014	9.014
$\beta_2$	1.512	1.512	1.512*
$\beta_3$	-0.560	-0.560	-0.560*

\* See Section 5.3.2.2 for more details

Table 5-7. Demand matrix

User group 1		Income: 20000				
	Everett	Seattle	Bellevue	Tacoma	Lynnwood	Renton
Everett	0	953	288	900	470	466
Seattle	625	0	1027	1764	479	1588
Bellevue	50	806	0	492	50	50
Tacoma	828	1889	739	0	669	751
Lynnwood	596	877	320	866	0	649
Renton	358	1263	50	1148	415	0

User group 2		Income: 40000				
	Everett	Seattle	Bellevue	Tacoma	Lynnwood	Renton
Everett	0	437	321	468	352	311
Seattle	729	0	1324	1597	625	1697
Bellevue	80	999	0	770	50	50
Tacoma	610	1313	713	0	492	538
Lynnwood	391	314	307	389	0	449
Renton	357	904	50	875	456	0

User group 3		Income: 70000				
	Everett	Seattle	Bellevue	Tacoma	Lynnwood	Renton
Everett	0	510	850	533	428	473
Seattle	1066	0	2249	2057	1096	2256
Bellevue	1086	2137	0	1899	651	691
Tacoma	462	1298	1152	0	479	611
Lynnwood	263	319	768	385	0	543
Renton	536	1215	581	1177	769	0

User group 4		Income: 120000				
	Everett	Seattle	Bellevue	Tacoma	Lynnwood	Renton
Everett	0	50	491	50	50	50
Seattle	50	0	2550	432	400	959
Bellevue	865	2559	0	1389	1070	509
Tacoma	50	50	646	0	50	50
Lynnwood	50	50	295	50	0	50
Renton	50	258	619	50	50	0

Table 5-8. User equilibrium condition without toll  
 Total Equivalent Income: \$19.2230 million  
 Gini Coefficient: 0.31288  
 Total Travel Time: 3104767 min/day

Link	Flow	Travel time	v/c Ratio	Toll
1	6733	7.97	0.98	-
2	7305	8.35	1.06	-
3	6063	15.28	0.88	-
4	4766	17.02	0.81	-
5	7114	16.42	1.04	-
6	5333	15.67	1.39	-
7	2846	4.02	0.41	-
8	5343	17.82	0.93	-
9	5106	14.76	1.33	-
10	2210	3.01	0.37	-
11	2603	4.01	0.38	-
12	7072	11.96	1.07	-
13	12031	19.93	1.37	-
14	2347	3.01	0.41	-
15	6066	13.40	1.23	-
16	7560	13.45	1.35	-
17	9563	17.95	1.26	-
18	4257	2.08	0.71	-
19	7913	14.29	1.06	-
20	6271	10.62	1.05	-
21	4717	2.15	0.84	-
22	5526	15.05	1.35	-
23	6057	12.79	0.81	-
24	4697	13.00	1.19	-

Table 5-9. Results for Seattle network

		Policy 1	Policy 2	Policy 3	Policy 4	No Toll
	$\alpha$	1.00	0.91	0.83	0.50	0.00
	Net benefit (million \$)	0.3884	0.3783	0.3298	0.2233	0
	Gini coefficient	0.32664	0.32358	0.31854	0.31341	0.31288
	Total travel time (hour)	45,511	46,331	47,863	49,009	51,746
	Toll revenue (\$)	721,878	593,198	321,787	40	0
Gain in income	Group 1	-7.54%	-5.33%	-1.35%	3.46%	0.00%
	Group 2	-4.36%	-3.11%	-1.19%	0.72%	0.00%
	Group 3	-1.83%	-1.12%	-0.05%	0.96%	0.00%
	Group 4	-0.06%	0.31%	0.86%	1.43%	0.00%

Table 5-10. Mode-specific demand for Seattle network

		Policy 1		Policy 2		Policy 3		Policy 4		No Toll	
		Auto	Transit	Auto	Transit	Auto	Transit	Auto	Transit	Auto	Transit
Group demand	Group 1	9225	3160	9988	4239	10603	6025	10206	8116	16642	939
	Group 2	11997	1501	12257	1708	12636	2038	12703	2520	14434	690
	Group 3	22194	1581	22336	1690	22615	1849	22764	2063	23517	984
	Group 4	11663	542	11667	562	11711	583	11757	599	11724	418
Travel demand		55078	6785	56249	8200	57565	10495	57430	13298	66317	3030

Table 5-11. Toll rates for Seattle network

Link	Policy 1		Policy 2		Policy 3		Policy 4	
	Auto	Transit	Auto	Transit	Auto	Transit	Auto	Transit
3	5.00	1.00	0.00	-1.00	0.00	-5.00	0.00	-15.00
4	5.00	1.00	1.00	0.00	0.00	-4.25	0.00	-15.00
5	10.25	0.00	8.25	-2.00	3.75	-10.00	2.25	-16.00
6	13.00	-1.00	13.00	-1.00	11.50	-4.00	12.50	-1.00
8	10.00	0.50	8.25	-2.50	3.25	-8.00	0.00	-15.50
9	13.00	-0.25	12.75	-2.25	10.75	-3.25	11.00	-0.25
12	10.00	0.00	10.00	0.00	7.75	-3.75	6.00	0.00
13	20.00	-2.00	20.00	-6.00	16.00	-12.00	12.00	-18.00
15	11.50	0.00	11.00	-1.75	9.00	-2.75	5.50	0.00
16	15.00	-2.00	15.00	-6.00	11.75	-10.00	7.00	-18.00
17	15.00	0.25	11.25	-5.50	6.75	-11.75	2.00	-20.00
20	8.00	-0.50	4.25	-4.50	0.75	-10.25	0.00	-16.50

Table 5-12. Results for policies with negative revenues for Seattle network

		Policy N1	Policy N2
$\alpha$		0.67	0
Net benefit (million \$)		0.1496	-0.0003
Gini coefficient		0.30882	0.30478
Total travel time (hour)		50,706	52,524
Toll revenue (\$)		-348,060	-711,219
Gain in income	Group 1	8.47%	13.27%
	Group 2	2.69%	4.38%
	Group 3	1.99%	2.81%
	Group 4	1.99%	2.39%

Table 5-13. Mode-specific demand for policies with negative revenues for Seattle network

		Policy N1		Policy N2	
		Auto	Transit	Auto	Transit
Group demand	Group 1	9885	9410	9677	10183
	Group 2	12788	2898	12909	3106
	Group 3	22908	2262	23066	2355
	Group 4	11797	636	11803	660
Travel demand		57368	15206	57456	16304

Table 5-14. Toll rates for policies with negative revenues for Seattle network

Link	Policy N1		Policy N2	
	Auto	Transit	Auto	Transit
3	0.00	-20.00	0.00	-20.00
4	0.00	-20.00	0.00	-20.00
5	0.00	-20.00	0.00	-20.00
6	7.00	-11.00	0.00	-20.00
8	0.00	-19.00	0.00	-20.00
9	5.00	-9.00	0.00	-20.00
12	2.25	-10.75	0.00	-20.00
13	6.00	-20.00	0.00	-20.00
15	3.25	-8.50	0.00	-20.00
16	5.25	-20.00	0.00	-20.00
17	0.00	-20.00	0.00	-20.00
20	0.00	-20.00	0.00	-20.00

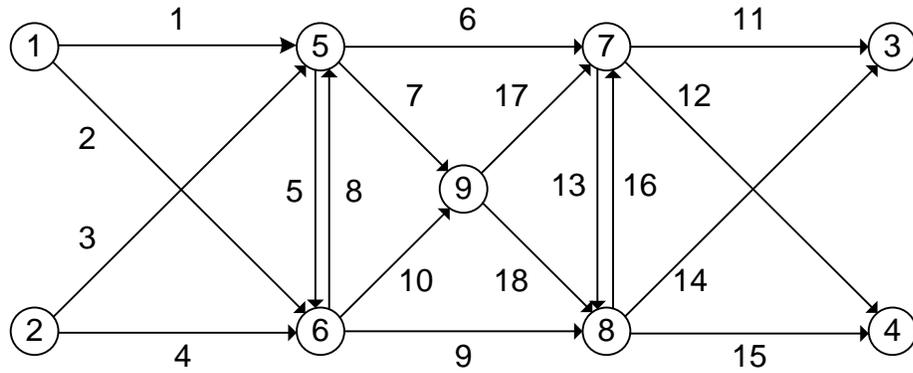


Figure 5-1. Nine-node network

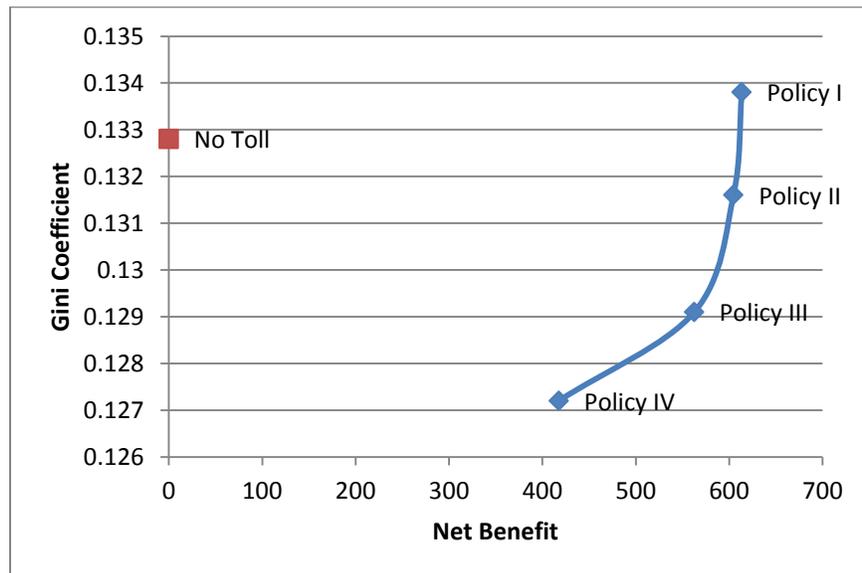


Figure 5-2. Relation of social benefit and Gini coefficient of the results

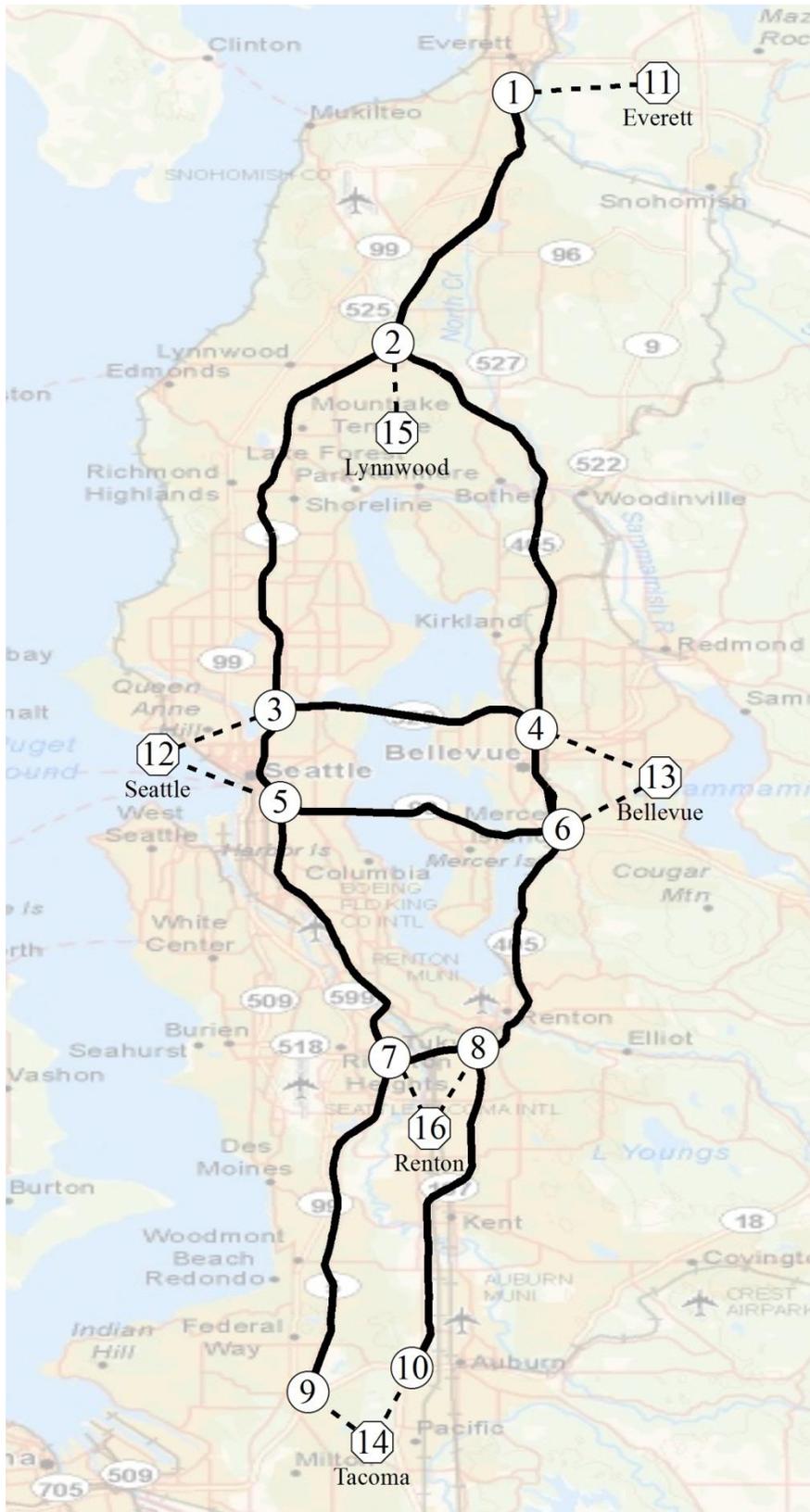


Figure 5-3. Seattle sketch network

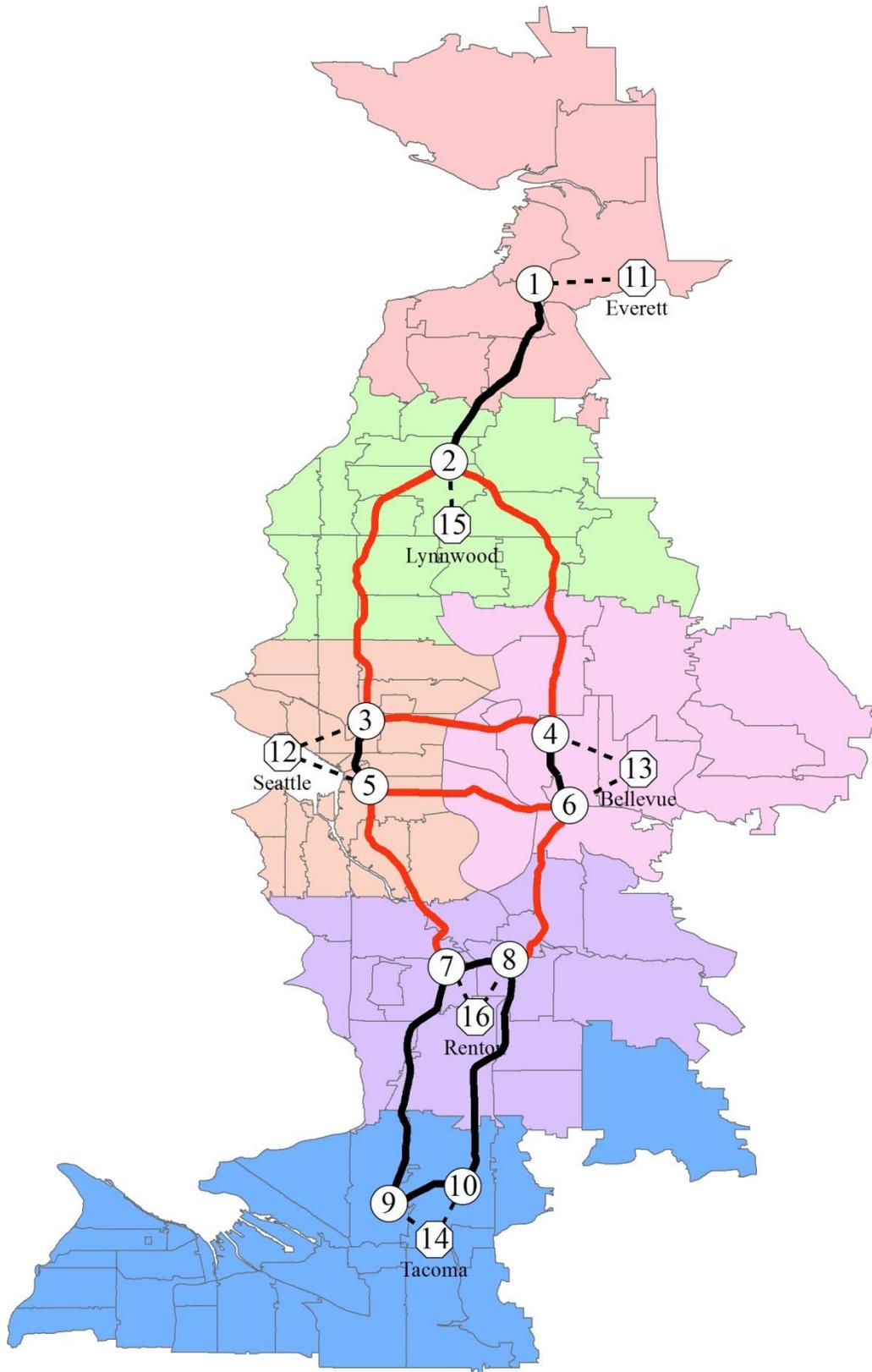


Figure 5-4. Seattle surrounding area

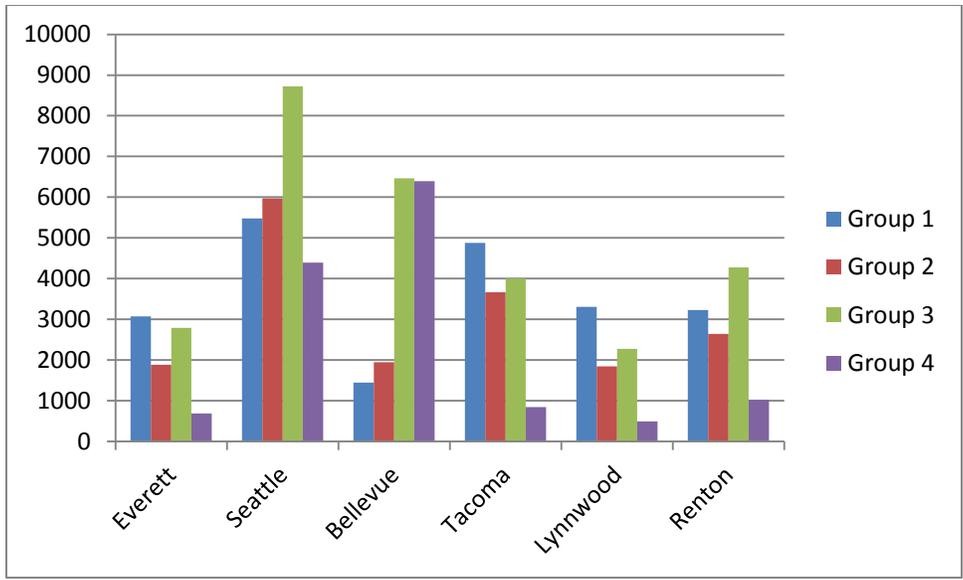


Figure 5-5. Compositions of productions for the centroid nodes

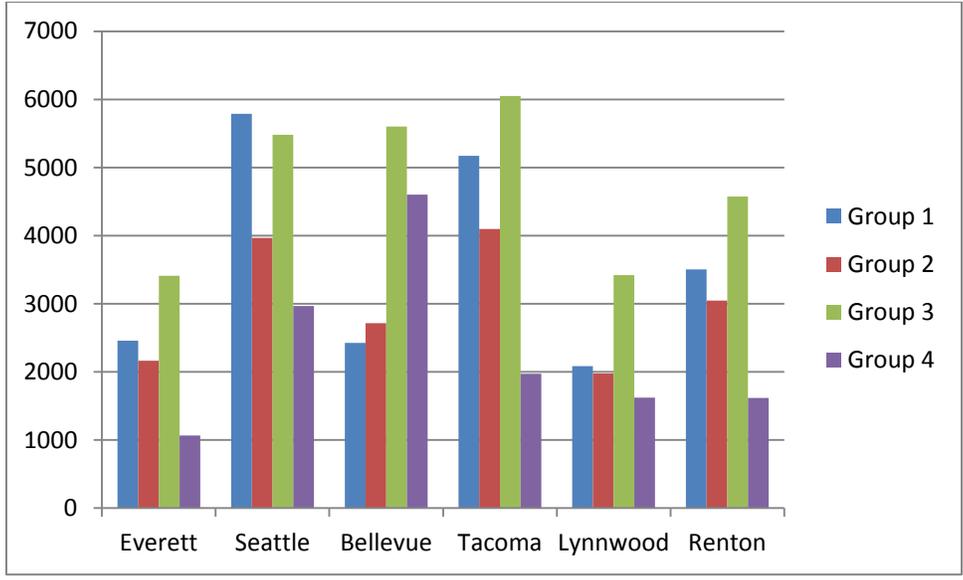


Figure 5-6. Compositions of attractions for the centroid nodes

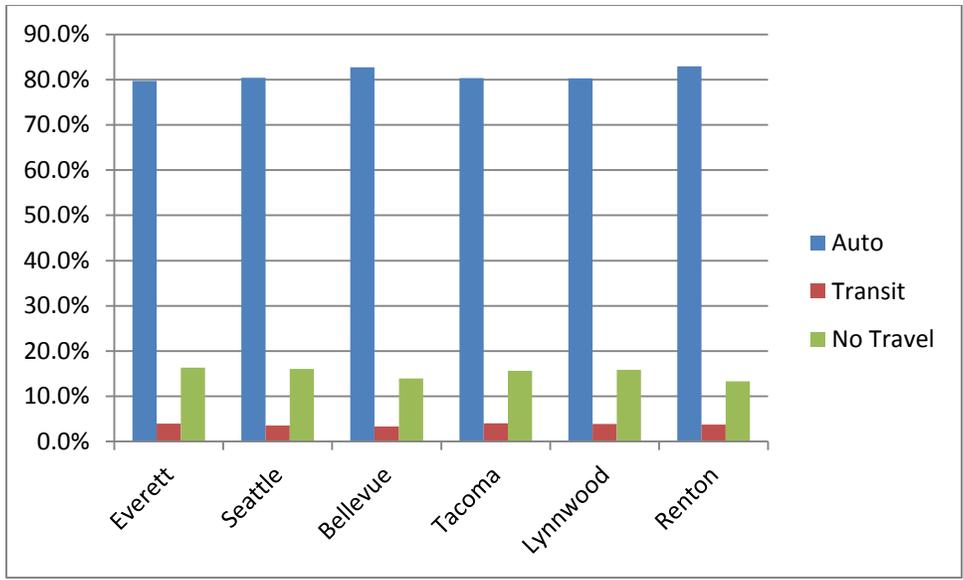


Figure 5-7. Travelers' mode choice percentage by origin

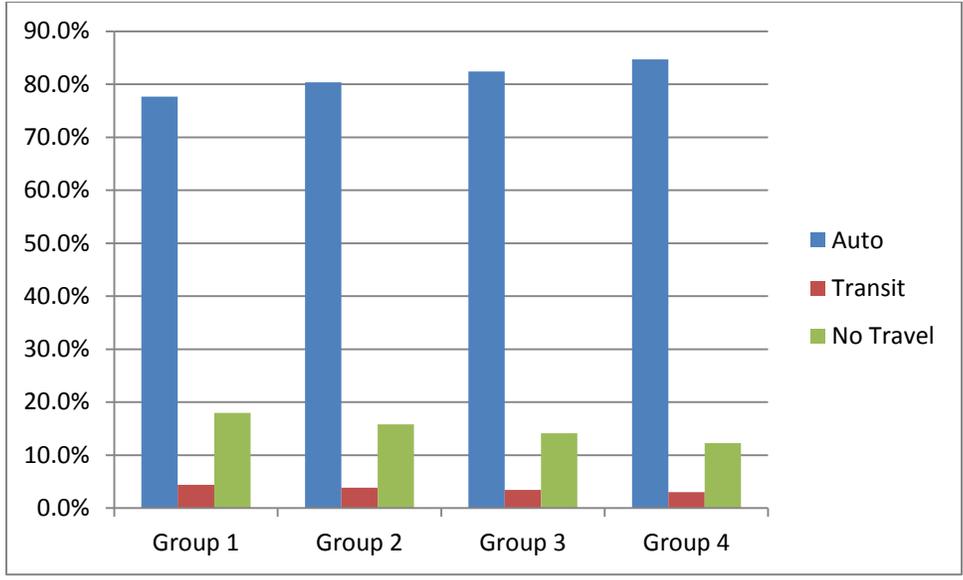


Figure 5-8. Travelers' mode choice percentage by income group

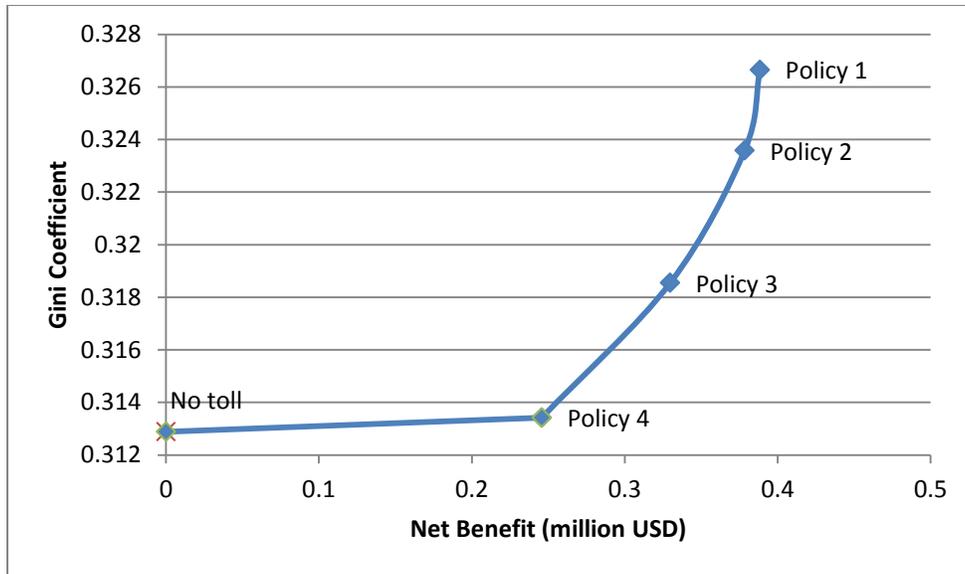


Figure 5-9. Relation of social benefit and Gini coefficient of the results



Figure 5-10. Changes in equivalent income

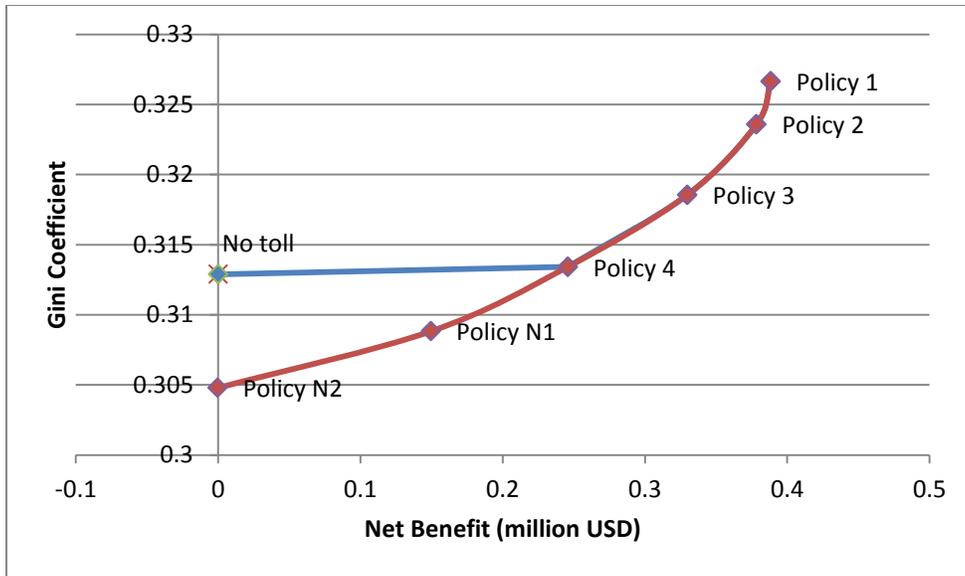


Figure 5-11. Relation of social benefit and Gini coefficient of the results

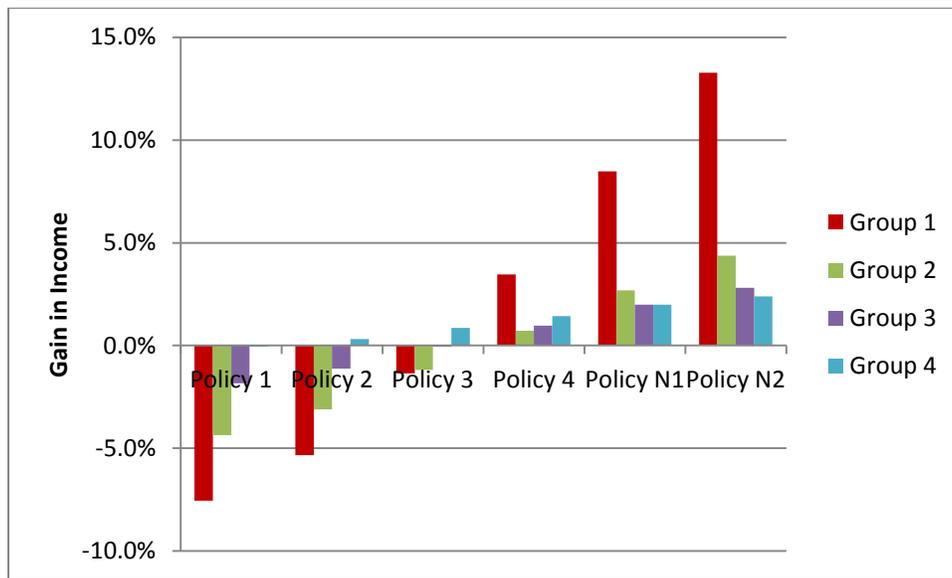


Figure 5-12. Changes in equivalent income

## CHAPTER 6 EQUITABLE AND TRADABLE CREDIT SCHEMES

### 6.1 Introduction to Tradable Credit Schemes

#### 6.1.1 Alternative Demand Management Schemes

Although pricing approach has been consistently advocated by economists and transportation researchers for nearly a century, the public objections toward congestion pricing still limit its implementations to a small number of cities worldwide. Given the ubiquitous political resistance and public objection to congestion charges, researchers and planners have turned to other demand management instruments to curtail the unrestricted use of private vehicles. A commonly used quantity control method is road space rationing. One implementation of such a scheme is the plate-number-based rationing strategy, which has been implemented in many cities in Latin American, e.g., Mexico City and Sao Paulo, and China, e.g., Beijing and Guangzhou. Such a method restricts cars from using the road space for certain days during the week by their license plate numbers. Recently, Han et al. (2010) and Wang et al. (2010) analyzed the user equilibrium condition and efficiency of such a plate-number-based rationing strategy. Their analytical results show that Pareto improvement is achievable in both short-term and long-term equilibrium conditions.

In practice, observable congestion reduction and air quality improvement have been reported under short-term rationing. However, in long term, pure plate-number-based rationing strategy tends to promote car ownership as commuters may seek a second car in order to circumvent the restriction. As a result, the effectiveness of such a scheme will be limited in long term. In particular, an increase in the total number of vehicles in circulation and a change toward old, cheap and polluting vehicles are

observed in Mexico City after the implementation of the rationing strategy (Davis, 2008; Mahendra, 2008).

A hybrid strategy is proposed by Daganzo (1995) by using both pricing and rationing. He proved the possibility to achieve Pareto optimum using this strategy without toll redistribution for a single bottleneck. However, analysis of such scheme on the San Francisco-Oakland bay bridge by Nakamura and Kockelman (2002) suggested that the possibility of achieving Pareto improvement for such a scheme is very small if not impossible.

Another quantity control method proposed in the literature is the tradable mobility permit (or credit) scheme. Under such a scheme, drivers are required to process a permit (or certain number of credits) in order to use (a certain area of) the road space. The permits (or credits) are tradable among road users. Earlier development of tradable permit schemes usually focuses on environmental objectives, partially due to the fact that the permit system gives the regulators direct control over the quantity of environmental impacts (e.g., emissions) (Cropper and Oates, 1992). Dales (1968) was among the earliest researchers on this topic, who proposed the creation of transferable rights to attain water quality targets in a cost-effective manner. Existing implementations of tradable permit schemes include the US emissions trading policy, which aims to reduce emissions by issuing emission reduction credits (Tietenberg, 1994). The Kyoto Protocol and the European Union Emission Trading Scheme also implemented similar schemes (Perrels, 2010).

One derivative of such permit schemes for congestion mitigation is the vehicle ownership quota system where the governmental authority imposes a limit for the

number of new vehicle registrations allowed each year. Such a scheme was implemented in Singapore (Chin and Smith, 1997) and Beijing (Yan and Wills, 2010).

### **6.1.2 Tradable Credit Schemes and Its Public Acceptability**

A tradable credit scheme for traffic congestion management involves: a) initial distribution of credits; b) credit trading among travelers; c) credit charging schemes. Under such a scheme, the road users will be charged certain number of credits to use road facilities. However, users can decide whether to use the credits for their travel or to sell them to other users.

Verhoef et al. (1997) analyzed the potential of such a scheme in the regulation of road transportation externalities and discussed many practical applications on both the demand side (user-oriented) and the supply side (the automobile and fuel industries). Kockelman and Kalmanje (2005) investigated responses from a survey conducted in Austin, Texas. Their analysis suggests that the tradable credit scheme can emerge as a viable, cost-effective and strongly supported strategy for congestion mitigation. Such a scheme promises substantial benefits for both network efficiency and equity, and addresses key issues that can undermine other methods. A personal tradable permit system that aims to reduce carbon emission is evaluated by Wadud (2010). The results showed that the policy can be progressive if the allocation strategies are appropriately selected (such as the per adult based uniform allocation).

Mathematically, tradable credit schemes have been formulated as variational inequalities by Nagurney and Dhanda (1996) and Nagurney (2000). Recently, Yang and Wang (2011) proposed a simple mathematical program to model the equilibrium condition under the tradable credit scheme where the price of the credit is determined by the market. The model is briefly introduced in Section 6.2.2.

### **6.1.3 The Tradable Credit Distribution and Charging Scheme**

We assume that the credits are universal for all links but link-specific in the amount of credit charge. The design of tradable credit distribution and charging scheme consists of three major parts: a) initial distribution of credits by the government; b) the determination of the number of credits to be charged for each link in the network; c) design of the trading mechanism that enables transfer of credits among road users.

It is assumed that the government will distribute the credits to eligible travelers free of charge. However, the credits can only be used for a specific time period (a month, for example), thus no one can gain by banking or stocking credits for future use. The government needs to determine the total number of credits to be distributed and how to distribute the credits among all the road users. The distribution can be uniform, but it may be more beneficial to implement more complicated schemes based on the objective of the government.

The credits that travelers received can be used for travel on any link in the network. However, the number of credits needed may vary from link to link. In order to travel on the network, the traveler must submit the required number of credits for all the links he or she visited. The credit charging scheme is designed by the government in order to manage traffic condition in the network.

If the available credits for a traveler are less than the number required, the traveler cannot travel on the specific link unless additional credits are obtained. These extra credits can be purchased from other travelers who have leftover credits or simply prefer to receive the money in exchange for travel rights. It is assumed that the government does not interfere with the credit trading market but solely acts as a manager to monitor the system. It is also assumed the transaction cost is minimal that it can be ignored in

the model. The price of the credit, on the other hand, is market determined and solely based on the supply and demand on the market.

## 6.2 Mathematical Model

### 6.2.1 Representation of User Utility under Tradable Credit Scheme

In the conventional pricing schemes, travelers need to directly pay a certain amount of money to use the roadways. Thus their income level is directly affected by the amount of toll charges they paid. For a utility function  $u(t, y)$  where  $t$  is the travel time and  $y$  is the income level, the utility under a pricing scheme can be easily calculated by adjusting the user income level with the toll rates, i.e.  $u(t, y - C)$ , where  $C$  is the total toll cost for the user.

However, under a tradable credit scheme, road users do not pay directly to access the roadway. But their effective income levels are still affected by the charging scheme through the realized value of the tradable credits, i.e. the market determined credit price. For example, if a traveler needs more credits than the ones he/she was distributed, he/she needs to buy those credits from the market and pay the corresponding price. The travelers can also sell their extra unused credits to increase their income. Thus, the effective income for the travelers are decided by their initial income, the number of credits initially distributed to them, the number of credits required for their travel and also the market determined price of the tradable credits. Mathematically,

$$y' = y + p \cdot (q - C)$$

where  $y'$  is the effective income level under the tradable credit scheme,  $p$  is the market determined credit price,  $q$  is the initial number of credits distributed,  $C$  is the total number of credits charged for the traveler.

The user utility level then can be calculated using the effective income level. The following equations are some commonly used utility functions:

Linear-In-Income:

$$u = \theta \cdot T + (y + p \cdot (q - C))$$

Generalized Leonief:

$$u = \beta_0 + \beta_1 T + \beta_2 \sqrt{T} + \beta_3 (y + p \cdot (q - C)) \\ + \beta_4 \sqrt{y + p \cdot (q - C)} + \beta_5 \sqrt{T} \sqrt{y + p \cdot (q - C)}$$

Translog:

$$u = \beta_0 + \beta_1 \ln T + \beta_2 \ln^2 T + \beta_3 \ln(y + p \cdot (q - C)) \\ + \beta_4 \ln^2(y + p \cdot (q - C)) + \beta_5 \ln T \ln(y + p \cdot (q - C))$$

For the linear-in-income utility function, as the initial income level  $y$  is a constant for every traveler, the removal of  $y$  from the utility function will not affect the travelers' decision making. Thus the utility function can be simplified as:

$$\dot{u} = \theta \cdot T + p \cdot (q - C)$$

### 6.2.2 Single-Modal User Equilibrium under the Tradable Credit Scheme

Now we introduce the single-modal user equilibrium formulation proposed by Yang and Wang (2011). This model considers one single travel mode and fixed OD demand. A linear utility function is used that the travelers will choose the path with the smallest general travel cost (travel time plus the cost of credits).

Given the initial distribution and the charging scheme, the feasible flow and credit price set  $\Phi$  can be represented as:

$$\sum_{k \in K^w} f_k^w = d^w, \quad \forall w \in W$$

$$x_l = \sum_w \sum_{k \in K^w} \Delta_{k,l} f_k^w, \quad \forall l \in L$$

$$\sum_l \kappa_l x_l \leq Q$$

$$f_k^w \geq 0, \quad \forall w \in W, k \in K^w$$

where  $W$  is the set of all OD pairs,  $L$  is the set of all links,  $K^w$  is the set of all paths connecting OD pair  $w$ .  $f_k^w$  is the flow on path  $k$  for OD pair  $w$ ,  $x_l$  is the aggregate flow on link  $l$ .  $d^w$  is the demand for OD pair  $w$ ,  $\Delta_{k,l}$  is the path-link incident matrix.  $Q$  is the total number of credits distributed and  $\kappa_l$  is the number of credits to be charged on link  $l$ .

Based on the feasible region  $\Phi$ , the user equilibrium condition can be represented as:

$$\sum_{l \in L} (t_l(x_l) + p\kappa_l) \Delta_{k,l} = \mu^w, \quad \text{if } f_k^w > 0, w \in W, k \in K^w \quad (6-1)$$

$$\sum_{l \in L} (t_l(x_l) + p\kappa_l) \Delta_{k,l} \geq \mu^w, \quad \text{if } f_k^w = 0, w \in W, k \in K^w \quad (6-2)$$

$$\sum_{l \in L} \kappa_l x_l = Q, \quad \text{if } p > 0 \quad (6-3)$$

$$\sum_{l \in L} \kappa_l x_l \leq Q, \quad \text{if } p = 0 \quad (6-4)$$

where  $t_l(x_l)$  is the travel time on link  $l$ ,  $\mu^w$  is the smallest generalized travel cost for OD pair  $w$ ,  $p$  is the market determined credit price.

Constraints (6-1) and (6-2) are simple extensions of the Wardrop's user equilibrium condition, which state that the paths with positive flows must be the ones

with the lowest generalized travel cost. Constraints (6-3) and (6-4) state the relationship between the credit price and the total number of credits being charged. Because the credit can be freely traded without transaction cost, the price will only be positive if all the credits distributed are charged back by the government.

The above equilibrium conditions can be formulated as a nonlinear mathematical programming problem as follow:

$$\begin{aligned} \min_{x,f} \quad & \sum_{l \in L} \int_0^{x_l} t_l(\omega) d\omega \\ \text{s. t.} \quad & \sum_{l \in L} \kappa_l v_l \leq Q \\ & (x, f) \in \Phi \end{aligned}$$

For the proof of the equivalence of the above minimization problem and the credit based user equilibrium conditions, please refer to Yang and Wang (2011).

### 6.2.3 User Equilibrium with Heterogeneous Users and Nonlinear User Utility Function

The above model has a simple structure and easy to solve. However, it is based on the strong assumption that the road users are homogeneous and the user utility function is linear with respect to travel time and credit cost. In this subsection, we extend the user equilibrium model to consider the nonlinear user utility function with heterogeneous users.

Assuming we have travelers in different income groups traveling on different OD pairs. Let  $d^{w,g}$  be the demand of users in group  $g$  of OD pair  $w$  and  $u_k^{w,g}$  be the user utility for the users in group  $g$  of OD pair  $w$  choosing path  $k$ , which is a function of the path travel time, number of credits to be charged and credit price. The users then will

choose their travel paths in order to maximize their utility. The user equilibrium condition under this situation can be written as:

$$u_k^{w,g} = \mu^{w,g}, \quad \text{if } f_k^{w,g} > 0, w \in W, g \in G, k \in K^w$$

$$u_k^{w,g} \leq \mu^{w,g}, \quad \text{if } f_k^{w,g} = 0, w \in W, g \in G, k \in K^w$$

$$\sum_{g \in G} \sum_{w \in W} \sum_{k \in K^w} \sum_{l \in L} \Delta_{l,k} \kappa_l f_k^{w,g} = Q, \quad \text{if } p > 0$$

$$\sum_{g \in G} \sum_{w \in W} \sum_{k \in K^w} \sum_{l \in L} \Delta_{l,k} \kappa_l f_k^{w,g} \leq Q, \quad \text{if } p = 0$$

The user equilibrium problem can be reformulated as the following VI problem:

$$\begin{aligned} & \sum_{g \in G} \sum_{w \in W} \sum_{k \in K^w} -u_k^{w,g} \cdot (f_k^{w,g} - f_k^{w,g*}) \\ & + \left( Q - \sum_{g \in G} \sum_{w \in W} \sum_{k \in K^w} \sum_{l \in L} \Delta_{l,k} \kappa_l f_k^{w,g*} \right) \cdot (p - p^*) \geq 0, \end{aligned}$$

$$\forall (f, p) \in \Phi'$$

$$(f^*, p^*) \in \Phi'$$

where the feasible region  $\Phi'$  is defined as.

$$\sum_{k \in K^w} f_k^{w,g} = d^{w,g}, \quad \forall w \in W, g \in G \tag{6-5}$$

$$f_k^{w,g} \geq 0, \quad \forall w \in W, g \in G, k \in K^w$$

$$p \geq 0$$

Note that credit price  $p$  is now a decision variable in the model.

The equivalency of the VI formulation and the user equilibrium conditions can be shown by analyzing the KKT conditions of the VI as shown below:

$$\rho^{w,g} - u_k^{w,g} \geq 0, \quad \forall k \in K, w \in W, g \in G \quad (6-6)$$

$$f_k^{w,g} \cdot (\rho^{w,g} - u_k^{w,g}) = 0, \quad \forall k \in K, w \in W, g \in G \quad (6-7)$$

$$Q - \sum_{g \in G} \sum_{w \in W} \sum_{k \in K^w} \sum_{l \in L} \Delta_{l,k} \kappa_l f_k^{w,g} \geq 0 \quad (6-8)$$

$$p \cdot \left( Q - \sum_{g \in G} \sum_{w \in W} \sum_{k \in K^w} \sum_{l \in L} \Delta_{l,k} \kappa_l f_k^{w,g} \right) = 0 \quad (6-9)$$

$$(f, p) \in \Phi$$

where  $\rho^{w,g}$  is the multipliers associated with Constraint (6-5).

Constraints (6-6) and (6-7) ensure the path flow follows the user equilibrium condition that every traveler in the network tries to maximize his/her utility level, where  $\rho^{w,g}$  is the maximal utility for travelers in group  $g$  of OD pair  $w$ . Constraints (6-8) and (6-9) regulate the credit price and guarantee the total number of credits charged is not more than the amount distributed.

## 6.2.4 Multimodal Stochastic User Equilibrium under Tradable Credit Scheme

### 6.2.4.1 Problem Description

With the aforementioned tradable credit scheme, we consider a general multimodal transportation network that includes three types of facilities, i.e., transit services, high-occupancy/toll (HOT) and regular toll lanes. Instead of charging tolls, HOT and regular toll lanes will only charge travelers credits. Further, HOT lanes can charge different number of credits from SOV and HOV users. A tradable credit scheme in this setting involves a credit distribution scheme that distribute the initial credits to the travelers based on certain criteria, and a link-based credit charging schemes that

specifies the number of credits to be charged on each link for different travel modes.

The number of credits being charged must be nonnegative.

We further consider multiple user groups with different incomes and preferences among four travel modes, i.e., no travel, transit, drive alone and car pool. It is assumed that users' travel decisions can be represented by a nested logit model whose utility functions are nonlinear in income. Under the nested logit model, travelers will first choose their travel modes based on the expected utility for those modes and then choose among all the available paths within the selected mode.

#### 6.2.4.2 Mathematical Model

In the aforementioned setting, the multimodal user equilibrium problem under the tradable credit scheme can be formulated as following VI problem:

$$\begin{aligned}
& \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^{w,m}} (-v_k^{w,m,g} + \theta^{w,m} \ln f_k^{w,m,g*}) \cdot (f_k^{w,m,g} - f_k^{w,m,g*}) \\
& + \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} (1 - \theta^{w,m}) \ln d^{w,m,g*} (d^{w,m,g} - d^{w,m,g*}) \\
& + \left( Q - \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^{w,m}} \sum_{l \in L} \Delta_{l,k} k_l^m f_k^{w,m,g*} \right) \cdot (p - p^*) \\
& \geq 0, \quad \forall (f, d, p) \in \Phi'' \\
& (f^*, d^*, p^*) \in \Phi''
\end{aligned} \tag{6-10}$$

where  $v_k^{w,m,g}$  is the observable portion of the user utility. The feasible region  $\Phi''$  is

defined as:

$$\sum_{k \in K^{w,m}} f_k^{w,m,g} = d^{w,m,g}, \quad \forall w \in W, m \in M, g \in G \tag{6-11}$$

$$\sum_{m \in M} d^{w,m,g} = D^{w,g}, \quad \forall w \in W, g \in G \quad (6-12)$$

$$p \geq 0 \quad (6-13)$$

The KKT conditions of the VI problem can be written as:

$$-v_k^{w,m,g} + \theta^{w,m,g} \ln f_k^{w,m,g} - \rho^{w,m,g} = 0,$$

$$\forall k \in K^{w,m}, w \in W, m \in M, g \in G$$

$$(1 - \theta^{w,m,g}) \ln d^{w,m,g} + \rho^{w,m,g} - \zeta^{w,g} = 0,$$

$$\forall w \in W, m \in M, g \in G$$

$$Q - \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^w} \sum_{l \in L} \Delta_{l,k} \kappa_l^m f_k^{w,m,g} \geq 0$$

$$p \cdot \left( Q - \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^w} \sum_{l \in L} \Delta_{l,k} \kappa_l^m f_k^{w,m,g} \right) = 0$$

$$(f, d, p) \in \Phi$$

where  $\rho^{w,m,g}$  and  $\zeta^{w,g}$  are multipliers associated with Constraints (6-11) and (6-12), respectively.

It is straightforward to prove that the above KKT conditions are the multimodal user equilibrium conditions under tradable credit scheme.

To solve the problem, we reformulate the VI problem as a nonlinear programming problem (Aghassi et al., 2006). The reformulated problem is as follow:

$$\begin{aligned}
& \min_{f,d,p,\rho,\zeta} \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^{w,m}} (-v_k^{w,m,g} + \theta^{w,m} \ln f_k^{w,m,g}) \cdot f_k^{w,m,g} \\
& \quad + \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} (1 - \theta^{w,m}) \ln d^{w,m,g} \cdot d^{w,m,g} \\
& \quad + \left( Q - \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^{w,m}} \sum_{l \in L} \Delta_{l,k} \kappa_l^m f_k^{w,m,g} \right) \cdot p \\
& \quad - \sum_{g \in G} \sum_{w \in W} D^{w,g} \cdot \zeta^{w,g} \\
& \text{s. t. } \rho^{w,m,g} \leq -v_k^{w,m,g} + \theta^{w,m,g} \ln f_k^{w,m,g}, \\
& \quad \forall w \in W, m \in M, g \in G, k \in K^{w,m} \\
& \quad -\rho^{w,m,g} + \zeta^{w,g} \leq (1 - \theta^{w,m,g}) \ln d^{w,m,g}, \\
& \quad \forall w \in W, m \in M, g \in G \\
& \quad Q \geq \sum_{g \in G} \sum_{w \in W} \sum_{m \in M} \sum_{k \in K^{w,m}} \sum_{l \in L} \Delta_{l,k} \kappa_l^m f_k^{w,m,g} \\
& \quad (f, d, p) \in \Phi''
\end{aligned}$$

If the above problem can be solved with a zero objective value, part of the solution, i.e.  $(f, d, p)$ , will solve the VI problem.

### 6.3 Tradable Credit Scheme Design Problem

The design of a tradable credit scheme involves specifying both the credit distribution scheme and the credit charging scheme. We assume only positive credit number can be charged for each link and travel mode. That is, travelers will not be given the chance to obtain more credits by traveling. So that the transportation agency can have strict control over the total number of credits available to the travelers. Further, the credit charges are mode specific that they can be different for different travel modes

even on the same link. The initial credit is distributed based on certain criteria such as travelers' income, travel origins and/or destinations. The objective of the credit scheme is to improve both the efficiency and equity of the transportation system.

The feasible region of the credit scheme can be described as:

$$\tau_l^m \geq 0, \quad \forall m \in \{S, H, T\}, l \in L \quad (6-14)$$

$$\tau_l^R = 0, \quad \forall l \in L \quad (6-15)$$

$$q^{w,g} \geq 0, \quad \forall w \in W, g \in G \quad (6-16)$$

$$\sum_{w \in W} \sum_{g \in G} q^{w,g} D^{w,g} = Q \quad (6-17)$$

where  $\tau_l^m$  is the number of credits to be charged on link  $l$  for travel mode  $m$ , and  $q^{w,g}$  is the number of credits distributed to the travelers in group  $g$  of OD pair  $w$ .

Constraint (6-14) ensures the only positive number of credit can be charged, while Constraint (6-15) guarantees travelers will not be charged anything if they decide not to travel. Constraint (6-16) says the amount of the initial distributed credits is nonnegative. Constraint (6-17) requires the total credits distributed to be a predetermined amount  $Q$ .

The tradable credit scheme design problem can be formulated as follows:

$$\min_{f,d,p,\tau,q} \alpha \cdot \frac{SB(\tau, q)}{SB(0,0)} - (1 - \alpha) \cdot \frac{GN(\tau, q)}{GN(0,0)}$$

$$s. t. \quad (6-10)-(6-17)$$

where  $SB(\tau, q)$  and  $GN(\tau, q)$  are social benefit and Gini coefficient under the credit scheme as specified by  $(\tau, q)$ .  $\alpha$  is a weighting factor.

We still use equivalent income to calculate the social benefit and Gini coefficient.

The equivalent income  $e^{w,g}(\tau, q)$  under tradable credit scheme  $(\tau, q)$  is defined as:

$$e^{w,g}(\tau, q) = \arg\{z: u^{w,g}(z, 0, 0) = u^{w,g}(y^g, \tau, q)\}$$

Where  $u^{w,g}(y, \tau, q)$  is the user utility of user group  $g$  of OD pair  $w$  given income level  $y$  and tradable credit scheme  $(\tau, q)$ .

Using the result from Dagsvik and Karlstrom (2005), we can calculate the expected equivalent income for each user group and OD pair in a similar ways as in Chapter 5.

$$E(e^{w,g}(\tau, q))$$

$$= \sum_{m^0 \in M} \sum_{k^0 \in K^{w,m^0}} \int_0^{p_{k^0}^{w,m^0,g}(\tau,q)} \frac{\left( \sum_{k \in K^{w,m^0}} \exp\left(\frac{h_k^{w,m^0,g}(z, \tau, q)}{\theta^{m^0}}\right) \right)^{\theta^{m^0}-1} \exp\left(\frac{v_{k^0}^{w,m^0,g}(y^g, 0, 0)}{\theta^{m^0}}\right)}{\sum_{m \in M} \left( \sum_{k \in K^{w,m}} \exp\left(\frac{h_k^{w,m,g}(z, \tau, q)}{\theta^m}\right) \right)^{\theta^m}} dz$$

Under the tradable credit scheme, the government will not receive or pay any money. Thus the social benefit only includes user benefit, which is the sum of the expected equivalent income of all the travelers.

$$SB(\tau, q) = \sum_{g \in G} \sum_{w \in W} E(e^{w,g}(\tau, q)) \cdot D^{w,g}$$

The Gini coefficient, as the equity measure, is also calculated based on the expected equivalent income.

$$GN(\tau, q) = \frac{1}{2 \cdot \left( \sum_{g \in G} \sum_{w \in W} D^{w,g} \right)^2 \cdot \overline{E(e(\tau, q))}} \cdot \sum_{g_1, g_2 \in G} \sum_{w_1, w_2 \in W} (D^{w_1, g_1} \cdot D^{w_2, g_2} \cdot |E(e^{w_1, g_1}(\tau, q)) - E(e^{w_2, g_2}(\tau, q))|)$$

where  $\overline{E(e(\tau, q))}$  is the average expected equivalent expenditure for all travelers under the credit scheme  $(\tau, q)$ .

## 6.4 Solution Algorithm

Similar to the pricing scheme design problem as described in Chapter 5, the tradable credit scheme problem is also a MPCC problem. A derivative-free algorithm such as the compass search algorithm discussed in Section 5.2.5 can be used to solve this problem. However, as the initial credit distribution also need to be determined in addition to the credit charging scheme, the number of the decision variables is significantly larger, which renders the compass search algorithm inefficient. In order to efficiently solve the problem, we adopt the SID-PSM (A Pattern Search Method Guided by Simplex Derivatives) algorithm (Custódio and Vicente, 2007; Custódio et al., 2010). SID-PSM algorithm is a derivative-free algorithm for constrained and unconstrained optimization problem. The algorithm use simplex derivatives to approximate the exact derivatives of the objective function and use them to guide the search directions.

The solution algorithm is developed as that the SID-PSM algorithm on the upper lever decides the credit distribution and charging scheme, while the lower level multimodal traffic assignment problem based on any given credit scheme is solved using the nonlinear reformulation. We implement the algorithm in GAMS (Brooke et al., 2005) in conjunction with Matlab (Wong, 2009) and use CONOPT (Drud, 1995) as the solver for the multimodal user equilibrium problem.

## 6.5 Numerical Example

The model and solution algorithm is implemented in the Seattle sketch network as described in Section 5.3.2. We use the same network parameters and demand data in this example. We assume the initial credits can only be distributed by the origin of each traveler. That is to say, all the travelers from the same origin will be given the same

number of initial credits. Such a credit distribution scheme is easy to implement. The government can simply distribute the credits by the travelers' home address.

The user utility function is assumed to be a form of the Translog utility function as follow:

$$v_k^{w,m,g} = \beta_0^R \log \alpha y^g + \beta_1 \ln \left( y^g - p \cdot (q^w - C_k^{w,m}) \right) + \beta_2 \ln T_k^{w,m} + \beta_3 \ln^2 T_k^{w,m}$$

where  $y^g$  is the daily income for user group  $g$ ,  $\alpha$  converts daily income to annual income,  $C_k^{w,m}$  is the total number of credits charged on path  $k$  for OD pair  $w$  of travel mode  $m$ ,  $q^w$  is the number of initial distributed credits and  $p$  is the market determined credit trading price.

The values of parameters  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  are the same as shown in Figure 5-6. The minimal OD travel time  $\tilde{T}^w$  is used in place of  $T_k^{w,m}$  for the no travel option when calculating the user utility function. We further assumed that the total number of credits distributed to the travelers are fixed and averages 10 credit per traveler.

The problem is solved by varying the value of the weighing factor  $\alpha$ . The results are shown in Table 6-1. Figure 6-1 plots the results of the obtained optimal credit schemes and the original UE result without the credit scheme. It can be seen that all the obtained credit schemes can improve the social benefit, including Scheme 6 that only considers the equity measure. Those schemes with higher weight in equity measures can also achieve a better level of equity.

Figure 6-2 shows the changes in equivalent income for different traveler groups under the obtained optimal policies. Under the most efficient credit scheme, travelers from all four groups benefit from the scheme. The ones that enjoy the most benefit

increase are the low-income and high-income travelers. For high-income travelers, they obtained most of the benefit from the decrease in the travel time due to the decrease in the demand for the auto travel mode (an 18% decrease from 66317 trips to 54426 trips). The low-income travelers, however, benefit the most by giving up traveling by auto mode or just choosing not to travel and selling unused credits to the others.

As the credit schemes give more weight to the equity measure, the only gainers of the schemes become the low-income travelers. Their gains in equivalent income increase dramatically to 13% of their total income under Scheme 6. At the same time, the high-income travelers are made worse off. In this scenario, a significant amount of low-income travelers decide not to take their trip and sell their credit to others as this offers them the most value. However, the travel demand from high-income travelers is only slightly affected, which means these travelers with higher income are buying credits from others.

Table 6-2 reports the credit distribution and charging schemes obtained by solving the credit scheme design problem. In contrast to the congestion pricing schemes as discussed in Chapter 5, with the total number of credits distributed fixed, a more equitable credit charging scheme generally charges more than the more efficient ones. For the most equitable scheme (Scheme 6), the credit charges on almost all links of auto mode are close to the upper bound (20 credits per link). Moreover, the transit travel mode will also be charged a lot of credits for certain links, which is the exact opposite of what is observed from the congestion pricing scheme designs.

The reason behind this phenomenon is that the mechanism to achieve higher equity level is different in pricing and credit schemes. In general, in order to improve

equity level, subsidies from high-income travelers to low-income travelers is required. Under congestion pricing, the only way to subsidize the travelers is to subsidize transit service. As transit is the mode that has the highest composition of low-income travelers, subsidizing the transit service can indeed improve equity. Under tradable credit scheme, the subsidization appears in a totally different form by the trading of credits. During any credit trading transaction, the credit buyer pays directly to the seller, which essentially acts as subsidizing the seller. Because high-income travelers value the travel rights (or credits) higher, they generally gain by buying credit at the market price. On the other hand, low-income travelers are also willing to sell their credits in exchange for money. Thus, the higher the credit price, the more money low-income travelers will receive during the transactions. When a credit charging scheme charges more credit for travel, it will increase the demand of credits. As a result, the credit price will be higher, which effectively offers the credit sellers (i.e. low-income travelers) more subsidies.

Figure 6-3 shows the comparison of the results of tradable credit schemes and congestion pricing schemes. It can be observed that the Pareto frontier of the tradable credit schemes is strictly dominating that of congestion pricing schemes. In other words, for whatever the combination of the weighing parameters, there is always a tradable credit scheme that can perform better than the best congestion pricing scheme available. Moreover, tradable credit schemes can easily provide progressive results while congestion pricing schemes cannot reach equity levels that are better than the original condition without external subsidy.

## **6.6 Summary**

In this chapter, we considered an alternative to congestion pricing, namely, the tradable credit scheme. Different from congestion pricing scheme, tradable credit

scheme charges credits instead of money from road travelers. The credits are distributed to the travelers in a way to maximize system efficiency and/or equity. The credits distributed can be freely traded among travelers at a market determined price. A mathematical model is developed in this chapter for the design of more efficient and equitable tradable credit scheme. The model is formulated as an MPCC problem and solved using a derivative-free algorithm SID-PSM. A numerical example is given based on the Seattle sketch network. The results from the solution algorithm show that tradable credit scheme can significantly improve the condition of the transportation system by both reducing congestion and improving system equity. Compared to congestion pricing scheme as discussed in Chapter 5, the tradable credit scheme can achieve higher system achievement in both efficiency and equity measures and does not require any kind of external subsidy.

Table 6-1. Results of optimal tradable credit schemes

		Scheme 1		Scheme 2		Scheme 3		Scheme 4		Scheme 5		Scheme 6	
$\alpha$		10		0.98		0.91		0.67		0.5		0	
Net benefit (million \$)		0.3977		0.3950		0.3843		0.3742		0.3103		0.1499	
Gini coefficient		0.3233		0.3187		0.3127		0.3107		0.3089		0.3072	
Total travel time		45,201		45,298		45,380		45,097		44,458		42,831	
Gain in income	Group 1	2.62%		5.63%		5.79%		7.48%		10.07%		13.08%	
	Group 2	1.17%		2.06%		2.31%		2.33%		2.30%		1.60%	
	Group 3	2.04%		1.90%		1.53%		1.29%		0.64%		-0.69%	
	Group 4	2.34%		1.32%		1.45%		1.14%		0.31%		-0.98%	
		Auto	Transit										
Group demand	Group 1	8937	2785	9091	2809	9716	2695	9099	2768	7721	2801	5697	1316
	Group 2	11822	1446	11886	1440	11958	1416	11645	1478	10655	1635	9169	1084
	Group 3	22035	1567	22050	1567	21975	1576	21720	1636	20818	1811	19697	1366
	Group 4	11632	542	11628	543	11581	553	11531	567	11317	615	11161	496
Travel demand		54426	6340	54655	6359	55230	6239	53995	6448	50511	6863	45723	4261

Table 6-2. Optimal credit distribution and charging schemes

		Scheme 1		Scheme 2		Scheme 3		Scheme 4		Scheme 5		Scheme 6	
Credit distributed	Everett	0.0		2.0		8.2		15.8		15.3		14.4	
	Seattle	0.0		0.0		13.1		9.2		6.8		3.4	
	Bellevue	13.1		0.0		0.0		0.0		0.0		0.0	
	Tacoma	0.0		0.8		7.0		11.6		15.6		21.8	
	Lynnwood	0.0		7.9		14.2		18.8		21.5		23.6	
	Renton	54.1		65.1		19.6		13.8		12.7		11.9	
		Auto	Transit										
Credit charged	Link 3	11.3	2.6	10.8	1.9	5.5	0.1	5.8	0.3	4.1	0.6	6.6	1.5
	Link 4	8.0	2.1	8.4	1.8	8.8	0.2	14.9	1.3	19.9	2.1	19.9	14.5
	Link 5	11.8	0.6	12.1	0.6	14.0	1.6	12.3	0.3	11.5	0.8	14.6	4.1
	Link 6	12.9	0.0	13.3	0.0	18.6	0.0	19.9	1.7	19.9	2.5	19.9	14.9
	Link 8	13.1	0.8	13.4	1.0	15.3	1.6	15.2	0.2	19.9	2.2	19.9	14.4
	Link 9	15.8	0.5	15.7	0.2	15.7	0.2	16.8	1.1	20.0	3.5	19.9	15.8
	Link 12	9.7	0.0	10.2	0.0	16.1	0.0	16.6	2.0	19.9	3.1	19.9	15.4
	Link 13	20.0	0.0	20.0	0.0	20.0	0.0	20.0	0.2	20.0	0.9	20.0	5.0
	Link 15	14.3	0.6	13.7	0.2	13.4	0.5	14.9	1.2	18.9	3.6	19.9	15.9
	Link 16	16.9	0.0	15.5	0.0	12.3	0.0	12.5	0.0	16.4	0.0	17.9	10.9
	Link 17	17.3	0.7	17.5	0.6	14.4	0.9	13.9	0.6	13.4	1.0	16.3	6.4
Link 20	6.3	0.0	5.7	0.0	4.4	0.0	4.5	0.0	7.9	0.0	20.0	11.3	

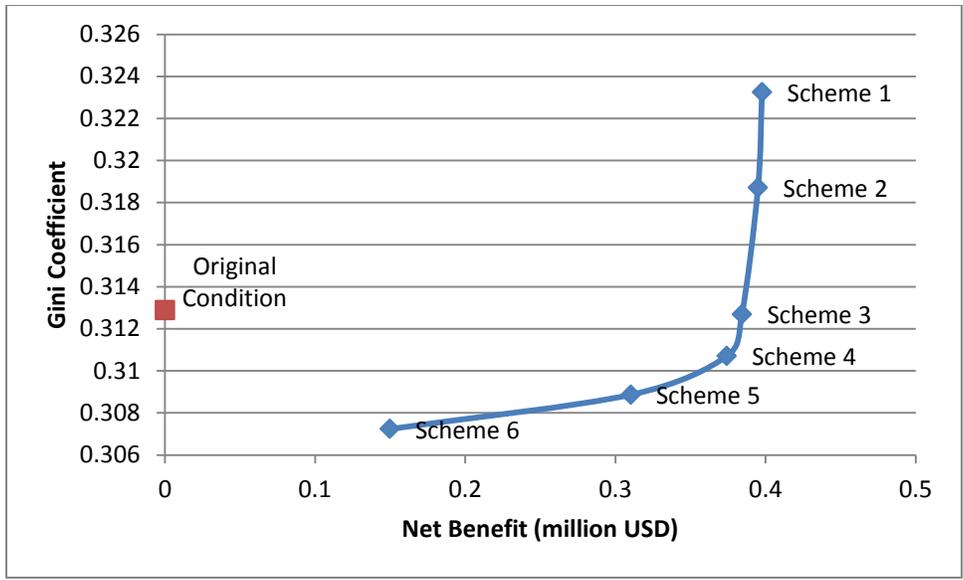


Figure 6-1. Pareto frontier of tradable credit schemes

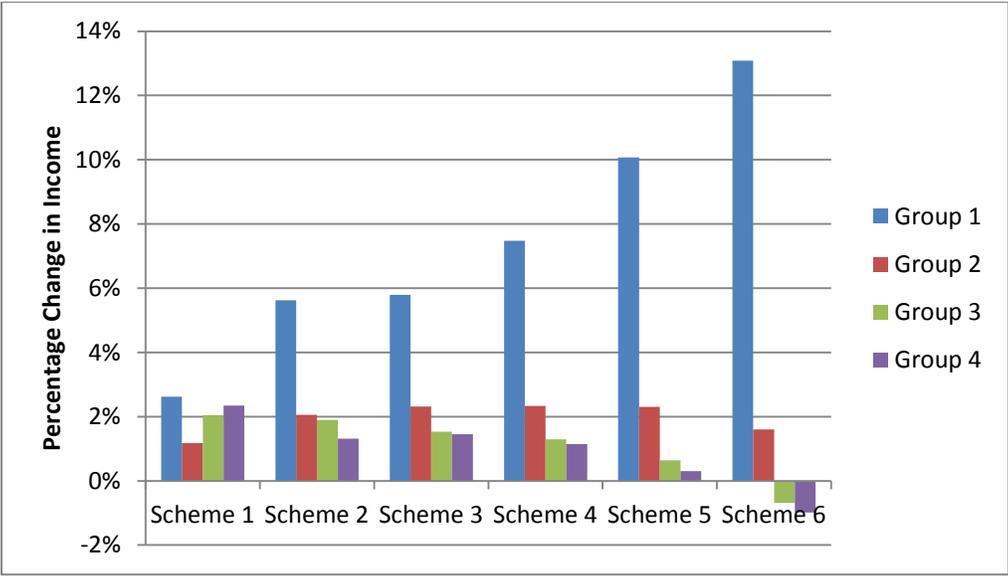


Figure 6-2. Changes in equivalent income

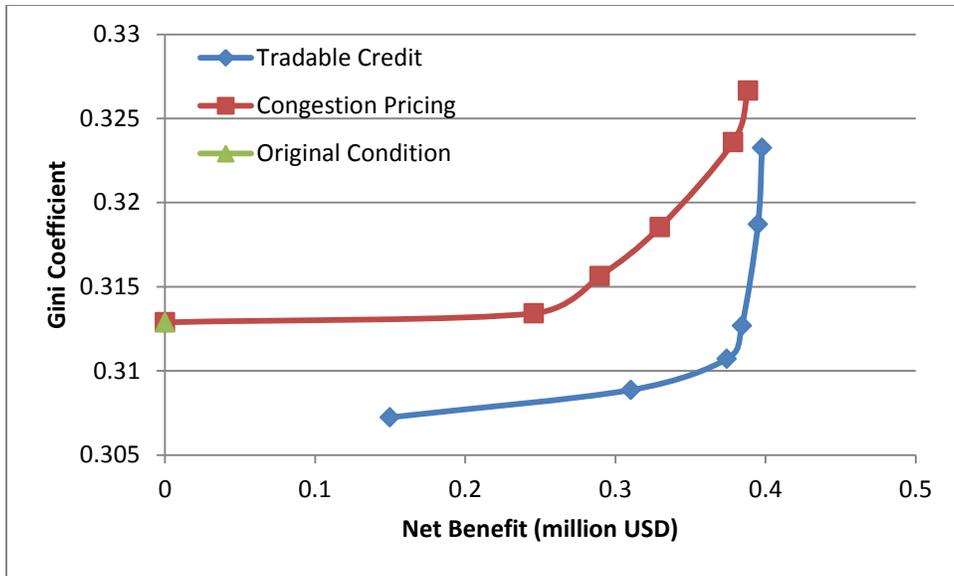


Figure 6-3. Comparison of tradable credit schemes and congestion pricing schemes

## CHAPTER 7 CONCLUSIONS AND FUTURE RESEARCH

### 7.1 Conclusions

This dissertation focuses on technical approaches to improve the public acceptability of congestion pricing schemes in multimodal transportation networks. The networks being considered in this dissertation include transit services, high occupancy vehicle/high occupancy toll (HOV/HOT) facilities (which can implement differentiating toll charges) and regular toll lanes. We try to improve the acceptability of congestion pricing in such networks by carefully designing where to charge and how much to charge and incorporate multiple innovative instruments such as differentiating tolls, subsidizing transit service and expanding highway capacity. We proposed design models for three types of schemes in this dissertation. They are Pareto-improving pricing, toll optimization with explicit equity considerations and tradable credit schemes.

A Pareto-improving pricing scheme improves the transportation system efficiency with the guarantee that no traveler will be made worse off compared to the condition without any pricing intervention. Such a scheme overcomes the drawback of marginal cost pricing and is expected to receive more approval from the general public. In this dissertation, two pricing scheme design models are proposed based on trip-based and tour-based travel demand models, respectively. Our models also allows the use of toll revenues 1) to subsidize transit service in order to encourage travelers to switch to higher occupancy travel modes, 2) to expand highway capacities to reduce traffic congestion. These models are formulated as MPEC problems and solved via manifold suboptimization method and relaxation algorithm. Numerical examples indicate Pareto-improvement can be achieved using the proposed model and solution algorithm in the

example networks. The Pareto-improving pricing schemes can lead to significant reduction in traffic congestion while guarantee that nobody is made worse-off. The tour-based model offers a more realistic representation of the travelers mode choice behavior, thus will provide more accurate results. However, the trip-based model may be used as an approximation when there is insufficient data to support the tour-based model.

As equity is one of the most important factors that affect public acceptance on congestion pricing, the next part of this dissertation develops a model that explicitly takes into consideration the distributional impacts of congestion pricing across different income and geographic groups. More importantly, this model considers the usually overlooked effect of income on travelers' choice of trip generation, mode and route. The proposed model provides transportation agencies a powerful tool in congestion pricing design with both efficiency and equity objectives. The model provides a framework for congestion pricing scheme designs that can incorporate various demand models and equity measures. The model is implemented in a test network as well as a realistic freeway network in the Seattle area. The results show that although a progressive pricing scheme may not be available, our model can always provide pricing schemes that achieve a better balance between efficiency and equity, which may be favored by the general public.

Lastly, we investigated an innovative tradable credit scheme as an alternative to traditional congestion pricing schemes. In contrast to congestion pricing, under tradable credit scheme, travelers do not necessarily need to pay out-of-pocket money in order to use the roadway being regulated. Moreover, tradable credit scheme provides another

way to subsidize low-income travelers. We tested such a scheme in the Seattle area freeway network, the results are very promising that tradable credit schemes can achieve better performance than congestion pricing in both efficiency and equity simultaneously. With the advance of electrical tolling technologies, tradable credit scheme could be implemented without much technical difficulty. This scheme provides a very attractive alternative for congestion pricing.

## **7.2 Future Research**

The literature in transit modeling has grown extensively since the introduction of the strategy-based model as adopted in Chapters 3 and 4. There are more advanced models that can be incorporated in the design models proposed in this dissertation in order to more accurately represent the transit network. Moreover, the models presented in this dissertation only consider simple travel mode choices that only consist of one mode for a trip or tour. It is worth to extend the models to capture more complicated mode choice behaviors such as park-and-ride (a tour that includes both transit and auto modes) and carpooling with travelers from different OD pairs (a tour consists of trips with both SOV and HOV modes).

For the tour-based Pareto-improving design models, another important future research direction is to consider the time dimension. Travelers do not make all the trips at once. Instead, they take their trips by sequence and their departure times could be subject to the traffic condition and service availability of public transit at different times of day. Other factors, such as trip purpose, parking price, propensity for flexibility could also affect travelers' mode choice decisions. Further research could incorporate these considerations into the pricing design models to provide more accurate and more meaningful analysis of the impacts of pricing schemes.

The models for the design of more equitable pricing and tradable credit schemes as proposed in Chapters 5 and 6 are generally hard to solve, especially when the number of toll links is large. The algorithms proposed in this dissertation, although being able to solve the problems to good solutions, are usually time-consuming and the result quality highly depends on the initial solutions given to the algorithms. Future work may explore how to further improve the efficiency and reliability of the algorithms.

The pricing or credit charging schemes considered in this dissertation is link based, where tolls or credits are charged when travelers traveling on certain links of the network. Many real world implementations, however, may appear in different forms such as area-based and distance-based pricing schemes. Thus, it is meaningful to consider extend the proposed model to accommodate different types of pricing or tradable credit schemes.

Lastly, the long-term effect of congestion pricing or tradable credit schemes could also be investigated. The models presented in this dissertation mainly focus on short-term situations where travelers' origins and destinations are assumed to be fixed. In the long-term, these congestion mitigation strategies could have substantial impacts to travelers' choices of origins and destinations such as residential and work locations. People may prefer to live or work at places where they can avoid the toll charges and their travel pattern may also be significant altered. A pricing or tradable credit model that can take into consideration the long-term effect could become a very helpful tool for the government agencies in developing their traffic management strategies.

## LIST OF REFERENCES

- Aashtiani, H.Z. and Magnanti, T.L., 1981. Equilibria on a congested transportation network. *SIAM Journal on Algebraic and Discrete Methods*, 2(3), pp.213-226.
- Abdulaal, M. and LeBlanc, L.J., 1979. Methods for combining modal split and equilibrium assignment models. *Transportation Science*, 13(4), pp.292-314.
- Adler, T. and Ben-Akiva, M., 1979. A theoretical and empirical model of trip chaining behavior. *Transportation Research Part B*, 13(3), pp.243-257.
- Adler, J.L. and Cetin, M., 2001. A direct redistribution model of congestion pricing. *Transportation Research Part B*, 35(5), pp.447-460.
- Aghassi, M., Bertsimas, D. and Perakis, G., 2006. Solving asymmetric variational inequalities via convex optimization. *Operations Research Letters*, 34(5), pp.481-490.
- Ahuja, R.K., Magnanti, T.L. and Orlin, J.B., 1993. *Network Flows: Theory, Algorithms, and Applications*. Englewood Cliffs, NJ: Prentice Hall.
- Arnott, R., de Palma, A. and Lindsey, R., 1994. The welfare effects of congestion tolls with heterogeneous commuters. *Journal of Transport Economics and Policy*, 28(2), pp.139-161.
- Arnott, R. and Kraus, M., 1998. When are anonymous congestion charges consistent with marginal cost pricing? *Journal of Public Economics*, 67(1), pp.45-64.
- Banister, D., 1994. Equity and acceptability question in internalising the social costs of transport. In *Internalising the Social Costs of Transport*. Paris: OECD. pp.153-175.
- Ban, J.X., Liu, H.X., Ferris, M.C. and Ran, B., 2006. A general MPCC model and its solution algorithm for continuous network design problem. *Mathematical and Computer Modelling*, 43(5-6), pp.493-505.
- Bechmann, M., McGuire, C.B. and Winsten, C.B., 1956. *Studies in the Economics of Transportation*. New Haven, CT: Yale University Press.
- Ben-Akiva, M.E., 1985. *Discrete Choice Analysis: Theory and Application to Travel Demand*. Cambridge, MA: MIT Press.
- Berger, J., 2008. Congestion Pricing: Just Another regressive Tax? *New York Times*, 20 April.
- Bernstein, D., 1993. Congestion pricing with tolls and subsidies. In *Proceedings of the Pacific Rim Transportation Technology Conference*, pp.145-151.

- Bhat, C. and Koppelman, F., 2003. Activity-based modeling of travel demand. In R.W. Hall, ed. *Handbook of Transportation Science*. Norwell, Massachusetts: Kluwer Academic Publishers. pp.39-65.
- Bouzaiene-Ayari, B., Gendreau, M. and Nguyen, S., 2001. Modeling bus stops in transit networks: A survey and new formulations. *Transportation Science*, 35(3), pp.304-321.
- Boyles, S.D., 2009. *Operational, Supply-Side Uncertainty in Transportation Networks: Causes, Effects, and Mitigation Strategies*. Ph.D. Dissertation. Austin: University of Texas.
- Brooke, A., Kendrick, D., Meeraus, A. and Raman, R., 2005. *GAMS: A User's Guide*. Washington, DC: GAMS Development Corporation.
- Bureau, B. and Glachant, M., 2008. Distributional effects of road pricing: Assessment of nine scenarios for Paris. *Transportation Research Part A*, 42(7), pp.994-1007.
- Burris, M. and Sullivan, E., 2006. Benefit-cost analysis of variable pricing projects: quickride HOT lanes. *ASCE Journal of Transportation Engineering*, 132(3), pp.183-190.
- Button, K.J., 1979. An historical survey of the early debate on consumer's surplus theory. *Journal of Economic Studies*, 16(2), pp.155-181.
- Button, K., 1993. Overview of internalising the social costs of transport. In *Internalising the Social Costs of Transport*. Paris: OECD. pp.7-30.
- Ceder, A., 2007. *Public Transit Planning and Operation: Theory, Modeling and Practice*. Boston, MA: Elsevier.
- Cepeda, M., Cominetti, R. and Florian, M., 2006. A frequency-based assignment model for congested transit networks with strict capacity constraints: characterization and computation of equilibria. *Transportation Research Part B*, 40(6), pp.437-459.
- Chin, A.T. and Smith, P., 1997. Automobile ownership and government policy: The economics of Singapore's vehicle quota scheme. *Transportation Research Part A*, 31(2), pp.129-140.
- Chriqui, C. and Robillard, P., 1975. Common bus lines. *Transportation Science*, 9(2), pp.115-121.
- Christensen, L.R., Jorgenson, D.W. and Lau, L.J., 1971. Conjugate duality and the transcendental logarithmic production function. *Econometrica*, 39, pp.255-256.
- Clegg, J., Smith, M., Xiang, Y. and Yarrow, R., 2001. Bilevel programming applied to optimising urban transportation. *Transportation Research Part B*, 35(1), pp.41-70.

- Cominetti, R. and Correa, J., 2001. Common-lines and passenger assignment in congested transit networks. *Transportation Science*, 35(3), pp.250-267.
- Connors, R., Sumalee, A. and Watling, D., 2005. Equitable network design. *Journal of the Eastern Asia Society for Transportation Studies*, 6, pp.1382-1397.
- Cropper, M.L. and Oates, W.E., 1992. Environmental economics: A survey. *Journal of Economic Literature*, 30(2), pp.675-740.
- Custódio, A.L., Rocha, H. and Vicente, L.N., 2010. Incorporating minimum Frobenius norm models in direct search. *Computational Optimization and Applications*, 46(2), pp.265-278.
- Custódio, A.L. and Vicente, L.N., 2007. Using sampling and simplex derivatives in pattern search methods. *SIAM Journal on Optimization*, 18(2), pp.537-555.
- Dafermos, S., 1980. Traffic equilibrium and variational inequalities. *Transportation Science*, 14(1), pp.42-54.
- Daganzo, C.F., 1995. A Pareto optimum congestion reduction scheme. *Transportation Research Part B*, 29(2), pp.139-154.
- Daganzo, C.F. and Garcia, R.C., 2000. A Pareto improving strategy for the time-dependent morning commute problem. *Transportation Science*, 34(3), pp.303-311.
- Daganzo, C.F. and Sheffi, Y., 1977. On stochastic models of traffic assignment. *Transportation Science*, 11(3), pp.253-274.
- Dagsvik, J.K. and Karlstrom, A., 2005. Compensating variation and Hicksian choice probabilities in random utility models that are nonlinear in income. *Review of Economic Studies*, 72(1), pp.57-76.
- Dales, J.H., 1968. Land, water, and ownership. *Canadian Journal of Economics*, 1(4), pp.791-804.
- Davis, L.W., 2008. The effect of driving restrictions on air quality in Mexico City. *Journal of Political Economy*, 116(1), pp.38-81.
- De Cea, J. and Fernandez, E.J., 1989. Transit assignment to minimal routes: An efficient new algorithm. *Traffic Engineering & Control*, 30(10), pp.491-494.
- De Cea, J. and Fernandez, E.J., 1993. Transit assignment for congested public transport system: an equilibrium model. *Transportation Science*, 27(2), pp.133-147.
- De Cea, J., Fernandez, E.J., Dekock, V. and Soto, A., 2005. Solving network equilibrium problems on multimodal urban transportation networks with multiple user classes. *Transport Reviews*, 25(3), pp.293-317.

- Dial, R.B., 1971. A probabilistic multipath traffic assignment model which obviates path enumeration. *Transportation Research*, 5(2), pp.83-111.
- Dial, R.B., 1999. Minimal-revenue congestion pricing part I: A fast algorithm for the single-origin case. *Transportation Research Part B*, 33(3), pp.189-202.
- Dial, R.B., 2000. Minimal-revenue congestion pricing Part II: An efficient algorithm for the general case. *Transportation Research Part B*, 34(8), pp.645-665.
- Diewert, W.E., 1971. An application of the Shephard duality theorem: A generalized Leontief production function. *The Journal of Political Economy*, 79(3), pp.481-507.
- Drud, A.A., 1995. *A System for Large Scale Nonlinear Optimization. Tutorial for CONOPT Subroutine Library*. Bagsvaerd, Denmark: ARKI Consulting and Development A/S.
- Eliasson, J., 2001. Road pricing with limited information and heterogeneous users: A successful case. *The Annals of Regional Science*, 35(4), pp.595-604.
- Emmerink, R.H.M., Nijkamp, P. and Rietveld, P., 1995. Is congestion pricing a first-best strategy in transport policy? A critical review of arguments. *Environment and Planning B*, 22(5), pp.581-602.
- Evans, A.W., 1992. Road congestion pricing: When is it a good policy? *Journal of Transport Economics and Policy*, 26(3), pp.213-243.
- Facchinei, F. and Pang, J.-S., 2003. *Finite-Dimensional Variational Inequalities and Complementarity Problems*. New York, NY: Springer.
- Feldman, A.M. and Serrano, R., 2006. *Welfare Economics and Social Choice Theory*. 2nd ed. New York, NY: Springer.
- Feng, T., Zhang, J., Fujiwara, A. and Timmermans, H.J.P., 2010. Incorporating accessibility-based equity into stochastic road network design problem: Sensitive analyses and policy implications. *Journal of the Eastern Asia Society for Transportation Studies*, 8, pp.997-1010.
- Ferrari, P., 1999. A model of urban transport management. *Transportation Research Part B*, 33(1), pp.43-61.
- Fisk, C., 1980. Some developments in equilibrium traffic assignment. *Transportation Research Part B*, 14(3), pp.243-255.
- Florian, M., 1977. A Traffic Equilibrium Model of Travel by Car and Public Transit Modes. *Transportation Science*, 11(2), pp.166-179.

- Florian, M. and Hearn, D., 2003. Network equilibrium and pricing. In Hall, R.W. *Handbook of Transportation Science*. Norwell, MA: Kluwer Academic Publisher. pp.373-412.
- Florian, M. and Nguyen, S., 1978. A combined trip distribution, modal split and trip assignment model. *Transportation Research*, 12(4), pp.241-246.
- Florian, M. and Spiess, H., 1982. The convergence of diagonalization algorithms for asymmetric network equilibrium problems. *Transportation Research Part B*, 16(6), pp.477-483.
- Foster, C., 1974. The regressiveness of road pricing. *International Journal of Transport Economics*, 1, pp.133-141.
- Frank, L. et al., 2008. Urban form, travel time, and cost relationships with tour complexity and mode choice. *Transportation*, 35(1), pp.37-54.
- Franklin, J.P., 2006. The equity effects of roadway tolls: An application of Hicksian welfare measures with income effects. In *Paper Presented in the 11th International Conference on Travel Behavioral Research*, Kyoto.
- Friesz, T.L. et al., 1992. A simulated annealing approach to the network design problem with variational inequality constraints. *Transportation Science*, 26(1), pp.18-26.
- Friesz, T.L., Tobin, R.L., Cho, H.-J. and Mehta, N.J., 1990. Sensitivity analysis based heuristic algorithms for mathematical programs with variational inequality constraints. *Mathematical Programming*, 48(1-3), pp.265-284.
- Fujii, S., Garling, T., Jakobsson, C. and Jou, R.-C., 2004. A cross-country study of fairness and infringement on freedom as determinants of car owners' acceptance of road pricing. *Transportation*, 31(3), pp.285-295.
- Garcia, R. and Marin, A., 2005. Network equilibrium with combined modes: Models and solution algorithms. *Transportation Research Part B*, 39(3), pp.223-254.
- Gartner, N.H., 1980. Optimal traffic assignment with elastic demands: a review. Part II: algorithmic approaches. *Transportation Science*, 14(2), pp.192-208.
- Gentile, G., Papola, N. and Perisa, L., 2005. Advanced pricing and rationing policies for large scale multimodal networks. *Transportation Research Part A*, 39(9), pp.612-631.
- Gini, C., 1912. *Variabilita e Mutabilita*. Bologna, Italy.
- Giuliano, G., 1992. An assessment of the political acceptability of congestion pricing. *Transportation*, 19(4), pp.335-358.

- Glazer, A., 1981. Congestion tolls and consumer welfare. *Public Finance*, 36(1), pp.77-83.
- Goh, M., 2002. Congestion management and electronic road pricing in Singapore. *Journal of Transport Geography*, 10(1), pp.29-38.
- Gomez-Ibanez, J.A., 1992. The political economy of highway tolls and congestion pricing. In *Exploring the Role of Pricing as a Congestion Management Tool: Summary of Proceedings of the Seminar on the Application of Pricing Principles to Congestion Management*. Washington,DC: U.S. Federal Highway Administration. pp.5-9.
- Goodwin, P. and Noland, R.B., 2003. Building new roads really does create extra traffic: a response to Prakash et al. *Applied Economics*, 35(13), pp.1451-1457.
- Guo, X. and Yang, H., 2010. Pareto-improving congestion pricing and revenue refunding with multiple user classes. *Transportation Research Part B*, 44(8-9), pp.972-982.
- Hagstrom, J.N. and Abrams, R.A., 2002. Characterizing Braess's paradox for traffic networks. In *Proceedings of IEEE 2001 Conference on Intelligent Transportation Systems*, Oakland,CA pp.837-842.
- Hamdouch, Y., Florian, M., Hearn, D.W. and Lawphongpanich, S., 2007. Congestion pricing for multi-modal transportation systems. *Transportation Research Part B*, 41(3), pp.275-291.
- Hamdouch, Y. and Lawphongpanich, S., 2008. Schedule-based transit assignment model with travel strategies and capacity constraints. *Transportation Research Part B*, 42(7-8), pp.663-684.
- Hamdouch, Y. and Lawphongpanich, S., 2010. Congestion pricing for schedule-based transit networks. *Transportation Science*, 44(3), pp.350-366.
- Hansen, M. and Huang, Y., 1997. Road supply and traffic in California urban areas. *Transportation Research Part A*, 31(3), pp.205-218.
- Han, D., Yang, H. and Wang, X., 2010. Efficiency of the plate-number-based traffic rationing in general networks. *Transportation Research Part E*, 46(6), pp.1095-1110.
- Hardin, G., 1968. The Tragedy of the Commons. *Science*, 162(3859), pp.1243-1248.
- Harker, P.T. and Pang, J.-S., 1990. Finite-dimensional variational inequality and nonlinear complementarity problems: A survey of theory, algorithms and applications. *Mathematical Programming*, 48(1-3), pp.161-220.

- Hau, T.D., 1990. Electronic road pricing: Developments in Hong Kong 1983-1989. *Journal of Transport Economics and Policy*, 24(2), pp.203-214.
- Hau, T.D., 2005. Economic fundamentals of road pricing: a diagrammatic analysis, Part I - fundamentals. *Transportmetrica*, 1(2), pp.81-117.
- Hearn, D.W. and Ramana, M.V., 1998. Solving congestion toll pricing models. In Marcotte, P. and Nguyen, S. *Equilibrium and Advanced Transportation Modeling*. Boston, MA: Kluwer Academic Publishers. pp.109-124.
- Hearn, D.W. and Yildirim, M.B., 2001. A toll pricing framework for traffic assignment problems with elastic demand. In M. Gendreau and P. Marcotte, eds. *Current Trends in Transportation and Network Analysis: Current Trends*. Netherlands: Kluwer Academic Publishers. pp.135-146.
- Hickman, L., 2010. Welcome to the world's worst traffic jam. *The Guardian*, 23 August.
- Hickman, M.D. and Bernstein, D.H., 1997. Transit service and path choice models in stochastic and time-dependent networks. *Transportation Science*, 31(2), pp.129-146.
- Ho, H.W. and Sumalee, A., 2010. An equitable, acceptable and socially beneficial congestion pricing design: A continuum modeling approach. *Journal of the Eastern Asia Society for Transportation Studies*, 8, pp.200-215.
- Jaensirisak, S., May, A.D. and Wardman, M., 2003. Acceptability of road user charging: The influence of selfish and social perspectives. In J. Schade and B. Schlag, eds. *Acceptability of Transport Pricing Strategies*. Oxford,UK: Elsevier. pp.203-218.
- Jaensirisak, S., Wardman, M. and May, A.D., 2005. Explaining variations in public acceptability of road pricing schemes. pp.127-54.
- Jakobsson, C., Fujii, S. and Garling, T., 2000. Determinants of private car users' acceptance of road pricing. *Transport Policy*, 7(2), pp.153-158.
- Jara-Diaz, S.R. and Videla, J., 1989. Detection of income effect in mode choice: Theory and application. *Transportation Research Part B*, 23(6), pp.393-400.
- Jones, P., 1991. Gaining public support for road pricing through a package approach. *Traffic Engineering & Control*, 32(4), pp.194-196.
- Jones, P., 1992. *Review of available evidence on public reactions to road pricing*. Report to London Transportation Unit. Department of Transport.
- Jones, P., 2003. Acceptability of road user charging: Meeting the challenge. In J. Schade and B. Schlag, eds. *Acceptability of Transport Pricing Strategies*. Oxford,UK: Elsevier. pp.27-62.

- Knight, F.H., 1924. Some fallacies in the interpretation of social cost. *The Quarterly Journal of Economics*, 38, pp.582-606.
- Kockelman, K.M. and Kalmanje, S., 2005. Credit-based congestion pricing: a policy proposal and the public's response. *Transportation Research Part A*, 39(7-9), pp.671-690.
- Kolda, T., Lewis, R. and Torczon, V., 2003. Optimization by direct search: New perspectives on some classical and modern methods. *SIAM Review*, 45(3), pp.385-482.
- Lam, W.H., Gao, Z.Y., Chan, K.S. and Yang, H., 1999. A stochastic user equilibrium assignment model for a congested transit networks. *Transportation Research Part B*, 33(5), pp.351-368.
- Lam, W.H.K. and Yin, Y., 2001. An activity-based time-dependent traffic assignment model. *Transportation Research Part B*, 35(6), pp.549-574.
- Langmyhr, T., 1997. Managing equity : The case of road pricing. *Transport Policy*, 4(1), pp.25-39.
- Lawphongpanich, S. and Hearn, D.W., 1984. Simplicial decomposition of the asymmetric traffic assignment problem. *Transportation Research Part B*, 18(2), pp.123-133.
- Lawphongpanich, S. and Hearn, D.W., 2004. An MPEC approach to second-best toll pricing. *Mathematical Programming*, 101(1), pp.33-55.
- Lawphongpanich, S. and Yin, Y., 2007. *Pareto-improving congestion pricing for general road networks*. Technical Report. Gainesville,FL: Department of Industrial and Systems Engineering, University of Florida.
- Lawphongpanich, S. and Yin, Y., 2010. Solving the Pareto-Improving Toll Problem via Manifold Suboptimization. *Transportation Research Part C*, 18(2), pp.234-246.
- Layard, R., 1977. The distributional effects of congestion taxes. *Economica*, 44(175), pp.297-304.
- LeBlanc, L.J. and Boyce, D.E., 1986. A bilevel programming algorithm for exact solution of the network design problem with user-optimal flows. *Transportation Research Part B*, 20(3), pp.259-265.
- LeBlanc, L.J., Morlok, E.K. and Pierskalla, W.P., 1975. An efficient approach to solving the road network equilibrium traffic assignment problem. *Transportation Research*, 9(5), pp.309-318.
- Levinson, D., 2010. Equity effects of road pricing: A review. *Transport Reviews*, 30(1), pp.33-57.

- Liu, Y., Guo, X. and Yang, H., 2009. Pareto-improving and revenue-neutral congestion pricing schemes in two-mode traffic networks. *Netnomics*, 10(1), pp.123-140.
- Luo, Z.-Q., Pang, J.-S. and Ralph, D., 1996. *Mathematical Programs with Equilibrium Constraints*. Cambridge, UK: Cambridge University Press.
- Mahendra, A., 2008. Vehicle restrictions in four Latin American cities: Is congestion pricing possible? *Transport Review*, 28(1), pp.105-133.
- Maher, M., Stewart, K. and Rosa, A., 2005. Stochastic social optimum traffic assignment. *Transportation Research Part B*, 39(8), pp.753-767.
- Maruyama, T. and Harata, N., 2006. Difference between area-based and cordon-based congestion pricing: Investigation by trip-chain-based network equilibrium model with nonadditive path costs. *Transportation Research Record*, 1964, pp.1-8.
- Maruyama, T. and Sumalee, A., 2007. Efficiency and equity comparison of cordon- and area-based road pricing schemes using a trip-chain equilibrium model. *Transportation Research Part A*, 41(7), pp.655-671.
- Mohring, H. and Harwitz, M., 1962. *Highway Benefits: An Analytical Framework*. Northwestern University Press.
- Nagurney, A., 2000. Marketable pollution permits in oligopolistic markets with transaction costs. *Operations Research*, 48(3), pp.424-435.
- Nagurney, A. and Dhanda, K., 1996. A variational inequality approach for marketable pollution permits. *Computational Economics*, 9(4), pp.363-384.
- Nakamura, K. and Kockelman, K.M., 2002. Congestion pricing and roadsapce rationing: an application to the San Francisco Bay Bridge corridor. *Transportation Research Part A*, 36(5), pp.403-417.
- Nguyen, S. and Pallottino, S., 1988. Equilibrium traffic assignemnt for large scale transit networks. *European Journal of Operational Research*, 37(2), pp.176-186.
- Nicholson, W., 1998. *Microeconomic Theory: Basic Principles and Extensions*. New York, NY: The Dryden Press.
- Nie, Y.M. and Liu, Y., 2010. Existence of self-financing and Pareto-improving congestion pricing: Impact of value of time distribution. *Transportation Research Part A*, 44(1), pp.39-51.
- Niskanen, E., 1987. Congestion tolls and consumer welfare. *Transportation Research Part B*, 21(2), pp.171-174.
- NuStats Research and Consulting, 1999. *1999 Puget Sound Household Travel Survey: Final Report*. Puget Sound Regional Council.

- Patriksson, M., 1994. *The Traffic Assignment Problem: Models and Methods*. Utecht, Netherlands: V.S.P. Intl Science.
- Perrels, A., 2010. User response and equity considerations regarding emission cap-and-trade schemes for travel. *Energy Efficiency*, 3(2), pp.149-165.
- Pigou, A., 1920. *The Economics of Welfare*. London: Macmillan.
- Primerano, F., Taylor, M., Pitaksringkarn, L. and Tisato, P., 2008. Defining and understanding trip chaining behavior. *Transportation*, 35(1), pp.55-72.
- Ralph, D. and Wright, S.J., 2004. Some properties of regularization and penalization schemes for MPECs. *Optimization Methods and Software*, 19(5), pp.527-556.
- Ramjerdi, F., 2006. Equity measures and their performance in transportation. *Transportation Research Record: Journal of the Transportation Research Board*, 1983, pp.67-74.
- Rawls, J., 1971. *A Theory of Justice*. Cambridge,MA: Belknap Press of Harvard University Press.
- Richardson, H.W., 1974. A note on the distributional effects of road pricing. *Journal of Transport Economics and Policy*, 8(1), pp.82-85.
- Rienstra, S.A., Rietveld, P. and Verhoef, E.T., 1999. The social support for policy measures in passenger transport: A statistical analysis for the Netherlands. *Transportation Research Part D*, 4(3), pp.181-200.
- Roberts, P., 2007. *Petition to: Scrap the planned vehicle tracking and road pricing policy*. [Online] Available at: <http://petitions.number10.gov.uk/traveltax/> [Accessed 21 December 2010].
- Santos, G. and Rojey, L., 2004. Distributional impacts of road pricing: The truth behind the myth. *Transportation*, 31(1), pp.21-42.
- Schade, J. and Baum, M., 2007. Reactance or acceptance? Reactions towards the introduction of road pricing. *Transportation Research Part A*, 41(1), pp.41-48.
- Schade, J. and Schlag, B., 2003. Acceptability of urban transport pricing strategies. *Transportation Research Part F*, 6(1), pp.45-61.
- Scholtes, S., 2001. Convergence properties of a regularization scheme for mathematical programs with complementarity constraints. *SIAM Journal on Optimization*, 11(4), pp.918-936.
- Schrank, D. and Lomax, T., 2009. *2009 Urban Mobility Report*. College Station, Texas: Texas Transportation Institute.

- Schweitzer, L. and Taylor, B.D., 2008. Just pricing: The distributional effects of congestion pricing and sales taxes. *Transportation*, 35(6), pp.797-812.
- Sen, A., 1992. *Inequality Reexamined*. Cambridge,MA: Harvard University Press.
- Sheffi, Y. and Powell, W.B., 1982. An algorithm for the equilibrium assignment problem with random link times. *Networks*, 12(2), pp.191-207.
- Shepherd, S. and Sumalee, A., 2004. A genetic algorithm based approach to optimal toll level and location problems. *Network and Spatial Economics*, 4(2), pp.161-179.
- Sikow-Magny, C., 2003. Efficient pricing in transport: Overview of European Commission's transport research programme. In J. Schade and B. Schlag, eds. *Acceptability of Transport Pricing Strategies*. Oxford,UK: Elsevier. pp.13-26.
- Small, K.A., 1983. The incidence of congestion tolls on urban highways. *Journal of Urban Economics*, 13(1), pp.90-111.
- Small, K.A., 1992. Using the revenues from congestion pricing. *Transportation*, 19(4), pp.359-381.
- Small, K.A. and Rosen, H.S., 1981. Applied welfare economics with discrete choice models. *Econometrica*, 49(1), pp.105-130.
- Smith, M.J., 1979. The existence, uniqueness and stability of traffic equilibria. *Transportation Research Part B*, 13(4), pp.295-304.
- Song, Z., Yin, Y. and Lawphongpanich, S., 2009. Nonnegative Pareto-improving tolls with multiclass network equilibria. *Transportation Research Record*, 2091, pp.70-78.
- Spiess, H. and Florian, M., 1989. Optimal strategies: A new assignment model for transit networks. *Transportation Research Part B*, 23(2), pp.83-102.
- Steg, L., 2003. Factors influencing the acceptability and effectiveness of transport pricing. In J. Schade and B. Schlag, eds. *Acceptability of Transport Pricing Strategies*. Oxford,UK: Elsevier. pp.187-202.
- Stockholms stad, 2006. *Resultat från folkomröstningen - hela staden*. [Online] Available at: <http://www.stockholm.se/Extern/Templates/PageWide.aspx?id=109698> [Accessed 18 July 2007].
- Sullivan, E. and Burris, M., 2006. Benefit-cost analysis of variable pricing projects: SR-91 express lanes. *ASCE Journal of Transportation Engineering*, 132(3), pp.191-198.

- Sumalee, A., 2003. Optimal toll ring design with spatial equity impact constraint: An evolutionary approach. *Journal of Eastern Asia Society for Transportation Studies*, 5, pp.1813-1828.
- Szeto, W.Y. and Lo, H.K., 2006. Transportation network improvement and tolling strategies: The issue of intergeneration equity. *Transportation Research Part A*, 40(3), pp.227-243.
- Taylor, B.D., Iseki, H. and Kalasuskas, R., 2010. Addressing equity in political debates over road pricing: lessons from recent projects. In *Paper Presented at the 89th Transportation Research Board Annual Meeting*, Washington, DC.
- Tietenberg, T.H., 1994. *Environmental Economics and Policy*. New York, NY: HarperCollins.
- Train, K., 2003. *Discrete Choice Methods with Simulation*. Cambridge, NY: Cambridge University Press.
- Transport for London, 2003. *Congestion Charging Six Months on*.
- U.K Government Statistical Survey, 1992. *Family Spending: A Report on the 1991 Family Expenditure Survey*. London: HMSO.
- U.S. Census Bureau, 2002. *Census 2000 Summary File 3: Technical Documentation*. U.S. Census Bureau.
- U.S. Census Bureau, 2004. *County Business Patterns*. U.S. Census Bureau.
- U.S. Department of Transportation, 2006. *National strategy to reduce congestion on America's transportation network*.
- Ubbels, B. and Verhoef, E., 2006. Acceptability of road pricing and revenue use in the Netherlands. *European Transport*, 32, pp.69-94.
- Uchida, K., Sumalee, A., Watling, D. and Connors, R., 2007. A study on network design problems for multi-modal networks by probit-based stochastic user equilibrium. *Networks and Spatial Economics*, 7(3), pp.213-240.
- Verhoef, E., 1996. *Economic Efficiency and Social Feasibility in the Regulation of Road Transport Externalities*. Amsterdam, Netherlands: Thesis Publishers.
- Verhoef, E.T., 2002. Second-best congestion pricing in general networks. Heuristic algorithms for finding second-best optimal toll levels and toll points. *Transportation Research Part B*, 36(8), pp.707-729.
- Verhoef, E., Nijkamp, P. and Rietveid, P., 1997. Tradable permits: Their potential in the regulation of road transport externalities. *Environment and Planning*, 24(4), pp.527-548.

- Vickrey, W.S., 1969. Congestion theory and transport investment. *American Economic Review*, 59(2), pp.251-260.
- Viegas, J.M., 2001. Making urban road pricing acceptable and effective: searching for quality and equity in urban mobility. *Transport Policy*, 8(4), pp.289-294.
- Wadud, Z., 2010. Personal tradable carbon permits for road transport: Why, why not and who wins? *Transportation Research Part A*. DOI:10.1016/j.tra.2010.03.002.
- Walters, A.A., 1961. The theory and measurement of private and social cost of highway congestion. *Econometrica*, 29(4), pp.676-699.
- Wang, X., Yang, H. and Han, D., 2010. Traffic rationing and short-term and long-term equilibrium. In *Papar presented at Transportation Research Board Annual Meeting*, Washington, DC.
- Wardrop, J.G., 1952. Some theoretical aspects on road traffic research. In *Proceedings of the Institution of Civil Engineers, Part II, Vol. 1*, pp.325-378.
- Washington State Department of Transportation, 2004. *Annual Peak Hour Report: Year 2004*.
- Weinstein, A. and Sciara, G.-C., 2006. Unraveling equity in HOT lane planning: A view from practice. *Journal of Planning Education and Research*, 26(2), pp.174-184.
- Wong, L., 2009. Linking Matlab and GAMS: A supplement. *REPA Working Paper 2009-03*. Department of Economics, University of Victoria.
- Wu, J., Florian, M. and Marcotte, P., 1994. Transit equilibrium assignment: A model and solution algorithms. *Transportation Science*, 28(3), pp.193-203.
- Wu, D., Yin, Y. and Lawphongpanich, S., 2011. Pareto-improving congestion pricing on multimodal transportation networks. *European Journal of Operational Research*, 210(3), pp.660-669.
- Yang, H., 1999. System optimum, stochastic user equilibrium, and optimal link tolls. *Transportation Science*, 33(4), pp.354-360.
- Yang, H. and Bell, M.G.H., 1997. Traffic restraint, road pricing and network equilibrium. *Transportation Research Part B*, 31(4), pp.303-314.
- Yang, H. and Lam, W.H.K., 1996. Optimal road tolls under conditions of queueing and congestion. *Transportation Research Part A*, 30(5), pp.319-332.
- Yang, H. and Wang, X., 2011. Managing network mobility with tradable credits. *Transportation Research Part B*, 45(3), pp.580-594.

- Yang, H. and Zhang, X., 2002. Multiclass network toll design problem with social and spatial equity constraints. *Journal of Transportation Engineering*, 128(5), pp.420-428.
- Yan, F. and Wills, K., 2010. Beijing city to limit new car quotas in 2011. *Reuters*, 23 December.
- Ye, X., Pendyala, R.M. and Gottardi, G., 2007. An exploration of the relationship between mode choice and complexity of trip chaining patterns. *Transportation Research Part B*, 41(1), pp.96-113.
- Yin, Y., 2000. Genetic-algorithms-based approach for bilevel programming models. *Journal of Transportation Engineering*, 126(2), pp.115-120.
- Ying, J. and Yang, H., 2005. Sensitivity analysis of stochastic user equilibrium flows in a bi-modal network with application to optimal pricing. *Transportation Research Part B*, 39(9), pp.769-795.
- Zhang, X. and Yang, H., 2004. The optimal cordon-based network congestion pricing problem. *Transportation Research Part B*, 38(6), pp.517-537.

## BIOGRAPHICAL SKETCH

Di Wu received his Bachelor of Science degree in automation from Tsinghua University in 2005 in China. He then joined the master's degree program in space center of Tsinghua University and received his Master of Science degree in aerospace engineering in summer 2007. After that, Di started to pursue his Ph.D. degree in Department of Civil and Coastal Engineering at University of Florida under the supervision of Dr. Yafeng Yin. Since then, Di has been awarded two Master of Science degrees in civil engineering and industrial engineering at University of Florida, respectively. Di Wu's primary research interest is in transportation system analysis and modeling.