OPTIMAL CONTROL AND DESIGN USING GENETIC ALGORITHMS ACCELERATED BY NEURAL NETWORKS

By

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To Vadivazhaghia Nambi Arasappan Pillai

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The main feature of this paper is the incorporation of Artificial Neural Networks (ANN) to accelerate the processing time of Genetic Algorithms (GA). In this paper, we use ANN for data mining and approximation of the initial range required GA such that its search efficiency increases. Deterministic/Gradient-based methods have proven to be difficult to produce optimal solutions to problems whose objective functions are discontinuous, non-differentiable, non-linear and stochastic. On the other hand, GA can produce an accurate search for an optimal solution for such problems. The hybridization of a GA with ANN though complex and expensive than the deterministic methods, is found to be more efficient in regard to actual operation time and near-optimal solutions for a given optimization problem. The method developed in this paper is a two-stage approach. The initial population required by GA is obtained using an ANN. Using this initial population, an optimal solution is obtained by GA. The approach is demonstrated in two types of problem - an optimal control problem and a fin design problem. It was found to be a successful method for generating optimal solutions in a constrained environment with minimal input from the user.

CHAPTER 1
INTRODUCTION

Over the years several techniques for optimization of aircraft and spacecraft trajectories have been developed. Optimization theories are not restricted to Aerospace applications alone but are found in use for Industrial Engineering, Business Management, Mechanical design and other streams as well [1]. The methods have been largely classified as gradient-based approach and heuristic approach. Initially theories for optimization were proposed using mathematical models and later automated tools were designed, developed and modified to produce solutions for all optimization problems. The automated optimization tools for numerical methods in recent times have been known to solve unconstrained, constrained, non-linear, linear, quadratic, non-linear least squares, sparse and structured objective and multi-objective problems. Extensive research has been carried out in designing and developing tools for optimization problems with applications in aerospace engineering. In general, optimization problems do not have any analytical solutions and are solved using either numerical methods or heuristic methods. Optimization of a trajectory using global collocation [2] where finite and infinite horizon problems have been solved using Legendre-Gauss-Radau (LGR) points. Space trajectories have been optimized using Particle Swarm Optimization which is a stochastic method inspired by behavior of birds and ants while searching for food [3]. Similarly a multi-objective problem using deterministic Collaborative Robust Optimization has been developed and is assisted by an approximation for uncertain intervals [4]. These researchers used methods involving gradient-based and deterministic approaches since the objective function was continuous and differentiable over a specified interval which constrained the solutions to be local. Problem arises when the objective function in real-time applications are discontinuous and non-differentiable. In literature, a handful of heuristic techniques have been published [5],[6], [7],[8]which overcome the disadvantage of gradient-based methods,

discontinuous and non-differentiable objectives. A near-optimal solution [9] was achieved for an Earth-Mars Trajectory using a stochastic approach (Genetic Algorithm). The literature published was to identify whether a GA Optimizer was suitable for a realistic model. In fact, a similar approach using collocation and non-linear programming was solved [10]. The main reason behind the implementation was that GA Optimizer was simple and required no initial guesses or prior information. Similarly, GA was developed as a preprocessing algorithm to formulate an initial guess of the solution for a direct collocation with non-linear programming method (semi-DCNLP). Several articles based on an augmented GA and ANN have been proposed, of which all problems pertain to training Neural Network for a particular set of design data (fluid and thermal science or structures). After the training has been accomplished, the approximated data is used as the objective to be optimized by GA Optimizer.

To address the problem dealing with objective functions which are not smooth and have discontinuities or large derivatives, an approach is developed in this paper such that the solution is optimal and consistent, obtained using Genetic Algorithms (GA) accelerated by Artificial Neural Networks (ANN). Genetic Algorithms depend upon natural selection and natural genetics. Initially a set of solutions are created by random generation and this set is called population. The solution to a specific problem is chosen from this current population depending on their best fitness. New populations are created for every generation by choosing the best individuals as parents from current population and using reproduction, crossover and mutation operators to produce offspring (individuals) for next generation. Over evolving successive generations, the population converges to an optimal solution. Moreover an initial guess of solution or any other input provided by the user is not required for Genetic Algorithms, as is the case for numerical methods. GA uses an initial range for population and finds an optimal solution from the population depending upon their best fitness value. If an initial range is really huge when compared to the range in which the optimal solution

lies, GA tends to lead a very slow optimization process which might prolong for many long hours and never converge to a solution. The other possibility of the range being too small will result in premature convergence of an optimal solution. So it is highly essential that a proper initial range is chosen such that evaluations carried out by GA are inexpensive. Literature works show that determining optimal trajectories and optimal design parameters of space vehicles in minimal computational time with less or no input from the user is highly important for improving the efficiency of spacecrafts and Unmanned Aerial Vehicles (UAVs). In this paper, we solve for optimal solutions using multi-stage approach heuristically. The solutions for optimal control and design problems are obtained using a hybrid solver - Genetic Algorithms and Artificial Neural Networks. ANN generates the initial range for GA from which an initial population is created randomly. Using this initial range, GA solves for optimal solutions of the control and design problem. The solutions obtained using this approach are compared with optimal solutions obtained using numerical methods,hybridization of GA and a single ANN and GA in the absence of ANN.

CHAPTER 2
GENETIC ALGORITHMS

2.1 Overview

Genetic Algorithms are heuristic search algorithms based on natural genetics theory. It was developed by John Holland and his students at University of Michigan in 1975 [11]. It is an artificial system that follows the mechanisms of natural genetics and natural selection. The main advantage of this method has been its robustness and flexibility in complex spaces. It is known for its simplicity which involves copying, crossing over and mutating the strings. They do not work with the parameters that are to be optimized but with a coded form of the parameter set. Initially the parameters are coded to binary strings (individuals). The coding of parameters is not restricted to a single string but also by applying transition rules (Selection 2.2.1,Reproduction 2.2.2 and Mutation (2.2.3) to generate new strings for each trial. These individuals together form the initial population of size 'n', where n is the number of trials used to generate new strings. This is followed by successive generation of populations. Searching for an optimal solution in a population of points (parallel computing) reduces the probability of finding a false solution instead of moving gingerly from a single point to another using calculus-based transition rule which ultimately leads to location of false solutions in multimodal search spaces. Taking into consideration the direct coding of parameters, search from a population of strings, and no need for auxiliary information together account for the robustness of the GA optimizer.

2.2 Transition Rules

Genetic Algorithms uses three main operators to create successive generations from current population are:

2.2.1 Selection

These are rules which select the best fit individuals for the given objective. It follows the artificial version of natural selection - Darwinian theory where individuals with more

fit have better potential to survive and most popularly known by the phrase 'survival of the fittest'. These best fit individuals become the parents of current generation and will reproduce again to form the next generation.

There are different types of function for the process - Stochastic uniform, Remainder, Uniform, Roulette and Tournament.

In Stochastic uniform type of function, a line is laid and each parent corresponds to a section of line which is proportional to its scaled value.

In Remainder function, parents are assigned deterministically from an individual's scaled value and roulette selection is used to for the remaining fractional part. After the parents are assigned, they are chosen stochastically. The probability that a parent is being chosen in this method is proportional to its fractional part of its scaled value.

As for Uniform function, parents are chosen according to expectations and number of parents. This method is not an effective search strategy but can be used for debugging and testing.

In Roulette, parents are chosen using a simulated roulette wheel, where the area of a section of the wheel (of an individual) is proportional to the individual's expectation.

The parents in Tournament function are chosen by choosing a random tournament size and best individuals out of that tournament set are chosen as parents.

2.2.2 Reproduction

This process specifies how the children are being created for the next generation. They are of two types

2.2.2.1 Elitism

It defines the number of individuals that will survive the next generation and is user specified and can be denoted as a real number.

2.2.2.2 Crossover

A set of rules which swap the characters or traits between two strings or individuals which result in partial string exchanges. So crossing randomly selected best-fit

individuals result in a new population constructed by varying their traits. There are different options for crossover function such as Scattered, Single point, Two Point, Intermediate, Heuristic and Arithmetic.In Scattered option, a random binary string is created, the ones in the binary string are replaced by the elements of $parent_1$, corresponding to ones' position. In a similar manner the zeros of the string are replaced by the elements of $parent_2$. For example, if

$$v_1 = \begin{bmatrix} q & w & e & r & t & y \end{bmatrix} \tag{2–1}$$

and

$$v_2 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix} \tag{2–2}$$

are parents, then the created bit string being

$$b = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \tag{2–3}$$

then the child would be

$$c = \begin{bmatrix} q & 2 & e & r & 5 & 6 \end{bmatrix} \tag{2–4}$$

As for Single point crossover, a random number from 1 to n is chosen, where n is the number of variables is chosen. Then the vector elements lesser than or equal to the random number is chosen from $parent_1$, similarly vector elements greater than or equal to random number is chosen from $parent_2$. These elements are concatenated to form a child vector. For example, if

$$v_1 = \begin{bmatrix} q & w & e & r & t & y \end{bmatrix} \tag{2–5}$$

and

$$v_2 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix} \tag{2–6}$$

If the random number is 4, then the child is

$$c = \begin{bmatrix} q & w & e & r & 5 & 6 \end{bmatrix} \tag{2–7}$$

Two-point crossover is similar to Single-point crossover but two random numbers are selected from 1 to number of variables. Using the same example, if v_1 and v_2 are parents, then

$$v_1 = \begin{bmatrix} q & w & e & r & t & y \end{bmatrix} \tag{2–8}$$

and

$$v_2 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix} \tag{2–9}$$

If the random number is 2 and 4, then the child is

$$c = \begin{bmatrix} q & w & 3 & 4 & t & y \end{bmatrix} \tag{2–10}$$

In Intermediate crossover, the child is created by using the weighted average of the parents. The weight is user-defined and is a single parameter which can be a scalar or a vector. The following formula is used to create the child.

$$c_1 = v_1 + p\,(v_2 - v_1) \tag{2–11}$$

where c_1 is the child, p is the weight, v_2 is $parent_2$, and v_1 is $parent_1$. If the weight lies between the range $[0,1]$, the children generated lie within the hypercube created by parents who are placed in opposite vertices. If otherwise, the children lie outside the hypercube generated. If the weight is a scalar, then the children lie in the same line connecting the vertices.

As for Heuristic method, the children lie in a line containing the two parents, a very small distance from the better fitted parent and away from the worst fitted parent. The distance from the better fitted parent is user defined and this function follows the equation

$$c_1 = v_2 + h\,(v_1 - v_2) \tag{2–12}$$

where c_1 is the child, h is the distance from $parent_1$, v_2 is $parent_2$ (worst-fitted), and v_1 is $parent_1$ (better-fitted). In the Arithmetic option, the children are reproduced which are the

weighted mean of the actual parents. These children are always feasible even with linear constraints and bounds.

2.2.3 Mutation

Mutation is a set of rules which randomly alters the value in a string. It is needed because of the fact that crossover and selection occasionally may lose potentially useful information retained in a string. The different types of mutation are Gaussian and Uniform.

In the Gaussian type, a Gaussian distribution is created with a mean of zero. A random number is chosen from this distribution and added to the entries of the parent. The distribution is created using the initial range specified by the user. So if the initial range is a vector of two rows and columns equal to the number of variables, then a standard deviation is generated using the formula

$$SD = p_{sd}\left(v_{i,2} - v_{i,1}\right) \tag{2–13}$$

where p_{sd} is a parameter defined by the user which can be an integer and determines the initial value of standard deviation in the first generation, v_2 is $parent_2$, v_1 is $parent_1$ and i is the co-ordinate corresponding to the parent vectors.

Another parameter g_{sd} determines or controls the spread of the standard deviation. The standard deviation for k^{th} is given by

$$SD_{i,k} = SD_{i,k-1}\left(1 - g_{sd}\frac{k}{generations}\right) \tag{2–14}$$

where i is the co-ordinate corresponding to the parent vectors.

In the Uniform Mutation process, initially a fraction of vector entries of the parents are chosen for mutation and each entry has a probability rate of being mutated. After the entries are chosen, they will be replaced by a random number selected (uniform selection) from the range of that entry.

There are other Genetic operators as well; however the above three are the main operators which have proved to be effective and efficient in solving many optimization problems. The three main parameters which decide upon the efficiency of a GA Optimizer are the population, cross-over fraction and mutation. The nominal rates for cross-over and mutation are 0.8 and 0.035 respectively. It is a known fact that GA performance requires the choice of a high crossover probability and a low mutation property and a moderate population size.

2.2.4 Population

The population size defines how many individuals are generated for each generation. The determination of population occurs from a predefined initial population range. The $initialrange$ can be user defined and it specifies the range of the vectors in the $initialpopulation$ that is to be generated. It is of utmost importance because of the fact the search space is a function of the initial range. If the given search space is large enough, the solutions of GA are known to be smooth and unimodal or if the search space is not large enough, it would conduct an exhaustive search and find a solution, but it would be a local optimum rather than global optimum and if the search space is exorbitantly large the GA would continue solving and eventually would take infinite time to arrive at an optimal solution. The types of population for the entries can be generated as bit strings or vectors. If the population is a vector, multiple subpopulations are generated which equals the length of the vector defined. In such case, Migration specifies how many individuals are passed on to the next generation. The worst individuals of a specific population are replaced by the best individuals of another population. There are three types of possible ways in which migration can be carried out - by Direction, Interval and Fraction.

Direction - as the name indicates, the direction in which the migration should take place is specified. It can either be forward or both. If migration is to take from n^{th} generation, then in forward migration, $n+1^{th}$ generation is replaced by n^{th} generation.

If the migration takes place in both directions then the $n+1^{th}$ generation and $n-1^{th}$ generation are replaced by n^{th} generation.

Interval - The migration takes place after a specific interval. If the interval is chosen to be 10 then migration takes place every 10 generations of the optimization process.

Fraction - The number of individuals which can migrate between populations is defined by the fraction function.

2.2.5 Fitness Scaling

The fitness function values which are being returned after each generation can be scaled. Raw fitness scores will be returned by the fitness function and they can be scaled down to values which will be suitable for the selection function and this is called fitness scaling. The different types of fitness scaling are rank, proportional, and shift linear. In the type rank, the raw score are scaled according their rank (its position depending on sorted scores). This type of scaling removes effect of spread of the scores. In proportional scaling the scaled value of an individual is proportional to its raw fitness score. As for shift linear, the raw scores are scaled such that the expected fittest individual is equal to a constant (user defined) multiplied by average fitness score.

The above said parameters are the different options with which a Genetic Algorithm Optimizer can be designed. Parameters can also be user defined and custom made according to a specific problem. The discussed options are the most commonly defined and essential options. These clearly define the rules using which an optimization process can be carried out without any discrepancies. They improve the performance of the function towards an optimal point i.e. an approach which leads to improvement towards an optimal point. Calculus-based methods have always focused on convergence and not on the interim performance. On the contrary, Genetic Algorithms strive to improve the fitness function using the above discussed options to attain an optimal solution.

CHAPTER 3
ARTIFICIAL NEURAL NETWORKS

3.1 Overview

Artificial Neural Networks are mathematical models of human nervous system. They are composed of artificial neurons or nodes. The effect of synaptic terminals is carried out by weight functions which modulate the effect of inputs and the nonlinearity of the nodes is represented by a transfer function. The inputs defined here in the picture are equivalent to inputs received by the dendrites of a human neuron and the output transmitted is equivalent to the output transmitted by the axon of a human neuron. The output of the artificial neuron is given by the Universal Approximation Property

$$y_i = W \cdot \sigma(b, x_i) + \varepsilon \tag{3–1}$$

where W is the weight vector, x is the input vector, bias weight b, ε is the approximation error and σ is the activation function. The transmission of signals is calculated as a weighted sum of the inputs and is transformed by the transfer function. By altering the values of the weights a neural network can be trained to perform a particular function. They can be essentially trained to perform different functions such as fitting a function, recognizing a pattern, clustering datasets, approximation of data and controlling systems. The transfer functions or the activation functions are mainly classified by three types. They are Step function, Linear function and Sigmoid function.

The Step function returns zero if the output is equal to zero or lesser than zero and returns one if it is greater than zero. It represented by the equation

$$y = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \tag{3–2}$$

where y is the output function and x is the input function.

The Linear Transfer Function usually spans between $-\infty$ to $+\infty$ and denoted by straight line. It is a function of a single variable input. It output is same as the input or a

function of addition and multiplication of the input. The equation which represents the function is given by

$$y = mx + c \tag{3–3}$$

where m and c are real constants, y the output and x the input.

As for the Sigmoid function, is a progressive function which spreads from small values and within given period of time accelerates to reach a maximum value. The equation which represents the function is given by

$$y(x) = \frac{1}{1 + e^{-x}} \tag{3–4}$$

where, y the output and x the input.

A basic neural network has three layers namely input layer, hidden layer and output layer. The input layer can be a scalar input or input of vectors. Here the hidden layer is function of the weights w and bias b. An ANN is to be designed such that the inputs produce a desired output and this wholly depends upon the weights of the network. Various methods exist to strengthen the connection; one of them is to set the weights explicitly and another is that by training the neural networks and using a learning rule.

In this particular paper we are using Radial Basis Neural Networks to carry out approximation of initial range from the generation data obtained from GA. The approximation of the generation data results in gives an estimate set of values for the next generation, from which an initial range for the next generation can be deduced. The Radial Basis Neural Network has three layers namely input, hidden radial basis layer and output linear layer. The Radial Basis Function is a Gaussian distribution which is mainly known for its approximation technique in statistics. The Neural Network used in this literature is for the same approximation of clustered data. The entire working of the neural net is explained in this chapter.

The net input to the transfer function is that the vector distance between cluster center c and input vector x multiplied by bias b. This type of setting weights is called

clustering technique. The concept is that patterns present in the input vector form clusters. The center of the clusters is known and the Euclidean distance between the input vector and the cluster center is measured. As the input moves away from the center, the activation value reduces. The transfer function is given as

$$\sigma(b,x) = e^{-n} \tag{3–5}$$

where $n = \|c - x\| \cdot b$ is the coefficient of the transfer function of the hidden layer, c is the center of the cluster, x is the input vector, b is the spread of the transfer function.

The radial basis function has a maximum of 1 when the input is 0, i.e. it produces 1 when the input x is equal to its weight w. The bias vector b allows the sensitivity of the radial basis neuron to be adjusted. The factor $n = \|c - x\| \cdot b$ is the weight of the layer which is a width parameter that controls the spread of the curve. The hidden layer transforms the inputs to a nonlinear function from the input space to the hidden space, whereas the output layer applies a linear space from hidden to output space. The hidden layer uses the Radial Basis Function while the output uses Linear Function.

The use of this approximation to approximate the values obtained after the initial run of the Genetic Algorithms Solver for determining the range of initial population will be discussed in detail in the coming chapter.

CHAPTER 4
PROBLEM FORMULATION

4.1 Overview

There are are two kind of problems solved in this paper to test the working of the proposed algorithm, developed using MATLAB. The objective of one problem is to find the optimal trajectory of a moving vehicle using the control angle and the other problem deals with finding an optimal trajectory of a model rocket using the fin design. The problem formulation for the design of optimal trajectory is as follows.

- In Section (4.2), the equations of motion are formulated.
- In Section (4.3), the boundary conditions are developed.
- In Section (4.4), the control constraints are formulated.
- Section (4.5), describes the objective function that is to be minimized.

4.2 Equations of Motion

4.2.1 Problem 1

The equations of motion for the vehicle moving at a velocity a over flat Earth are given as follows

$$\dot{x} = u \tag{4–1}$$

$$\dot{y} = v \tag{4–2}$$

$$\dot{u} = a\cos(\beta) \tag{4–3}$$

$$\dot{v} = a\sin(\beta) \tag{4–4}$$

where $x(t)$ and $y(t)$ are the horizontal and vertical components of position, u and v are the horizontal and vertical components of velocity a. β is the control angle.

4.2.2 Problem 2

The equations of motion for the model rocket with propellant mass m and total mass, M, with thrust T acting on it, with a drag of D, and moving with an acceleration a is given by

$$\ddot{y} = \frac{T(t) - m(t,a)\ g - D(t)}{m(t,a)} \tag{4–5}$$

where the Drag equation is given by

$$D = \frac{1}{2}\ \rho\ v_2\ S_i C_{d_i} \tag{4–6}$$

the Thrust is given by

$$T = \dot{m}\ v_e \tag{4–7}$$

the mass flow rate of the propellant is given by

$$\dot{m} = \frac{m\ (a-g) - D\ {v_e}^2}{v_e} \tag{4–8}$$

where $y(t)$ is the vertical component of position, v is the velocity, a is the acceleration of the rocket. v_e is the exit velocity of the propellant with respect to the rocket, S_i is the surface area of the rocket parts and C_D is the co-efficient of drag, D and g is the acceleration due to gravity.

4.3 Boundary Conditions

4.3.1 Problem 1

The boundary conditions for the vehicle are given as follows

$$x(t_0) = 0 \tag{4–9}$$

$$y(t_0) = 0 \tag{4–10}$$

$$u(t_0) = 0 \tag{4–11}$$

$$v(t_0) = 0 \tag{4–12}$$

where t_0 is the initial time. Next, the terminal conditions of each conditions of each aircraft are given as

$$x\left(t_f\right) = x_f \tag{4–13}$$

$$y\left(t_f\right) = y_f = 0 \tag{4–14}$$

$$u\left(t_f\right) = u_f \tag{4–15}$$

$$v\left(t_f\right) = v_f = 0 \tag{4–16}$$

where $t_f = 10$ is the final time of the moving vehicle.

4.3.2 Problem 2

The boundary conditions for the rocket are given as follows

$$y\left(t_0\right) = 0 \tag{4–17}$$

$$v\left(t_0\right) = 0 \tag{4–18}$$

$$a\left(t_0\right) = 0 \tag{4–19}$$

where t_0 is the initial time. Next, the terminal conditions of each conditions of each aircraft are given as

$$y\left(t_f\right) = y_f = 0 \tag{4–20}$$

$$v\left(t_f\right) = v_f \tag{4–21}$$

$$a\left(t_f\right) = a_f = 0 \tag{4–22}$$

where $t_f = 13\ seconds$ is the final time of the moving vehicle.

4.4 Control and Design Constraints

4.4.1 Problem 1

The polynomial used for parameterizing the control angle β is given by

$$\beta \approx \sum_{i=0}^{N} c_i \, \phi_i(t) \tag{4–23}$$

where $\phi_(t) = 1 + t + t^2 + ...$ and c_i is some constant.

4.4.2 Problem 2

The design constraints for designing the fin of the model rocket is given by the center of gravity and center of pressure.

The center of pressure is the point on the rocket where all of the aerodynamic forces act. This was determined by simplifying the traditional calculations. The center of pressure times the total projected area, A, is equal to the sum of the center of pressure of each component, d, times the projected area of each component,a

$$C_p \, A = a_{nose} d_{nose} + a_{fuselage} d_{fuselage} + a_{fins} d_{fins} \tag{4–24}$$

The center of gravity was computed using the basic equations for determining the center of mass of an object with multiple components.

$$C_G = \sum \frac{m_i \, d_i}{M} \tag{4–25}$$

where the total mass of the object, M, is equal to the sum of the product of each individual components mass, m, with its corresponding distance, d, from a reference line.

4.5 Cost Functional

4.5.1 Problem 1

The cost functional for the moving vehicle is to maximize its final horizontal position, which is given by

$$J = -x_{t_f} \tag{4–26}$$

4.5.2 Problem 2

The cost functional for the rocket is a combination of the static margin,SM and height of apogee, given by

$$J = \sqrt{\left(\frac{1500 - y}{1500}\right)^2 + \left(\frac{2 - SM}{SM}\right)^2} \tag{4–27}$$

Using these cost functionals the trajectories of both the problems are determined subject to the dynamic constraints(Section (4.2)), boundary constraints(Section (4.3)), control and design constraints (Section (4.4)). The optimal problems are solved using MATLAB and Simulink version of Genetic Algorithm and Neural Networks using default feasibility and optimality tolerances.

CHAPTER 5
AUGMENTATION OF GENETIC ALGORITHMS AND ARTIFICIAL NEURAL NETWORKS

5.1 Overview

Over the years, many works in literature have amalgamated Neural Networks with Genetic Algorithms. Most of these literary works have been used in the fields of Aerospace Engineering [14],[20],[12], Computational Fluid Dynamics [13],[15], Solid Mechanics [19] or other Design problems [17], [18]. The general outlines of these problems have been that an optimal design has been achieved by approximating the results obtained from the Genetic Algorithm using Neural Networks. These works which have also been optimized by Neural Networks alone or the data obtained using Genetic Algorithm have been used for training the Neural Networks and then creating an off-line method to obtain optimal solutions. As far as this research is concerned, the optimization is carried out by Genetic Algorithms and the approximation is done by Neural Networks, but a novel way is being implemented. The problem addressed here is a) processing time and b) consistency of producing near-optimal solutions of Genetic Algorithms. Though there are several papers which have previously addressed the same problem [16] but the solutions provided are different. In [16] the objective function values from Genetic Algorithms are stored in a database and then are approximated to function or set of values and then strings are selected from this particular approximation. The main feature to be noted is that objective values obtained from approximation are inserted in the population set of every generation and the values are again obtained for every modified generation. Through this solution is feasible and makes perfect sense, it seems to highly expensive and still accounts to take up much of the processing time. Secondly, the idea of spoiling the entire population is hindering the main idea of natural selection and reproduction of genetics. The main design of this paper is that the data obtained from the GA Optimizer is approximated by a Neural Network to determine

the initial range of the population of a GA, thereby not hindering the natural theory of genetics and building an inexpensive algorithm to carry out the process.

5.2 Augmentation

Firstly a Genetic Algorithm Optimizer is built according to the user's definition. To implement a search for optimal solution all parameters of the problem are initially coded into a chromosome or string where each parameter is a part of the string. The

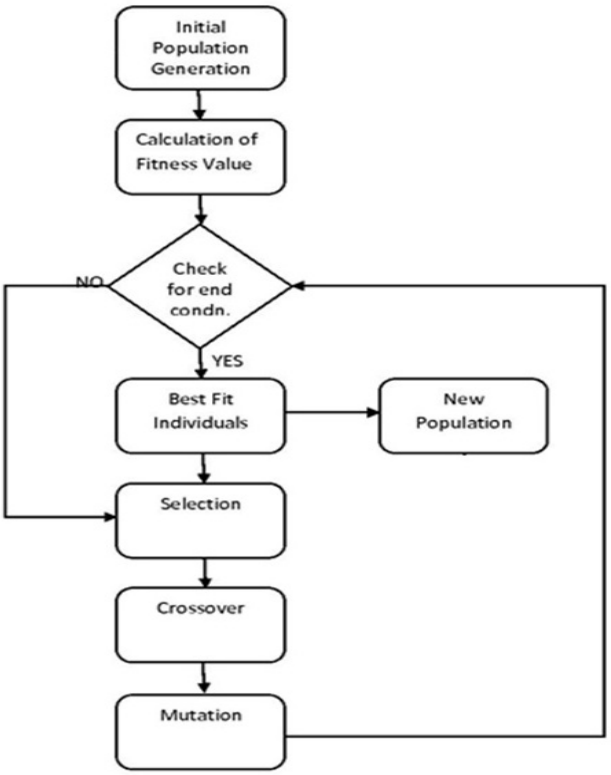


Figure 5-1. Genetic Algorithm Flowchart

algorithm for generating a GA Optimizer is as follows:

- Step 1: As discussed before a random initial population of strings is generated.
- Step 2: The fitness of each string/chromosome is calculated.
- Step 3: The strings are checked for end-conditions. If they have been met, the strings are selected as 'best-fit' individuals for final population; else they are selected to generate a new population.

- Step 4: A population is generated using selection, crossover, and mutation operators.
- Step 5: Two strings are selected as parents based on their fitness value to generate new set of offspring.
- Step 6: A crossover probability is used to carry out crossover function between the two parents to generate a new offspring.
- Step 7: A mutation probability is used to mutate the new offspring generated.
- Step 8: Close-the loop; go to Step 2.

After the best-fit individuals are obtained they are decoded to their original values.

The factors which were taken into consideration for developing a GA Optimizer were Selection, Reproduction, Mutation and Population.

Initially the GA is allowed to compute from a initial range which can either be exorbitantly large or infinitesimally small. The initial population, $generation_1$ is formed

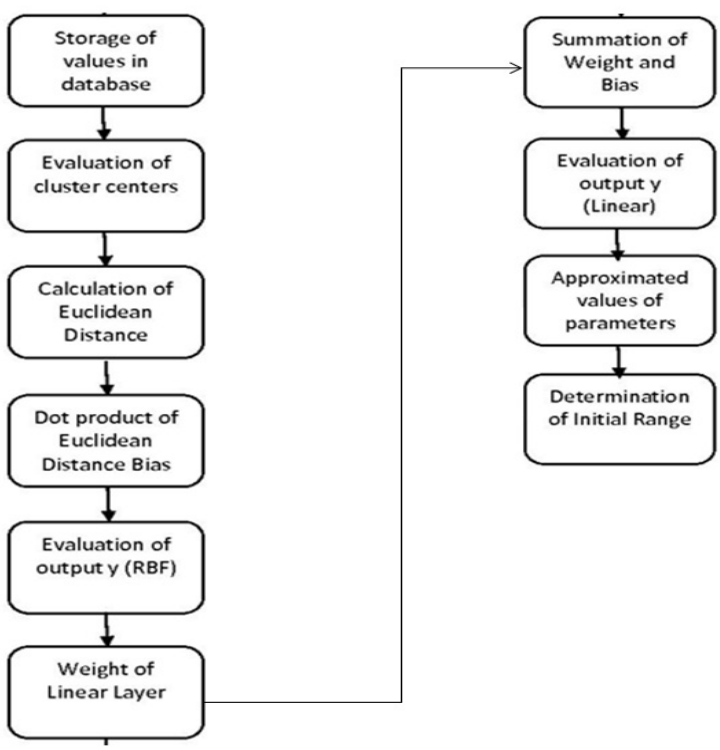


Figure 5-2. Neural Network Flowchart

from the initial range specified in the solver and can be considered to be range of $generation_0$. Similarly, Genetic Algorithms is allowed to compute $generation_2$ from $generation_1$ following the transition rules. Using,

$$x = generation_0 \ as \ the \ input \ vector \ and \tag{5–1}$$

$$t = \ generation_1 \ as \ the \ targetvector \tag{5–2}$$

a Neural Network is built with a hidden layer using the transfer function

$$\sigma(V,x) = e^{-n} \tag{5–3}$$

where $n = \|c - x\| \cdot V$ is the coefficient of the transfer function of the hidden layer, c is the center of the cluster, x is the input vector, V is the weight of the transfer function.

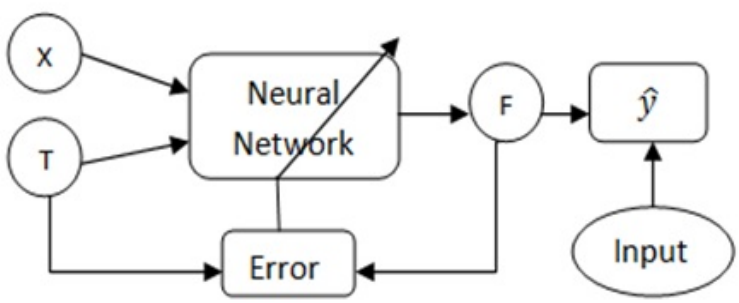


Figure 5-3. Neural Network for every 3 generation - Actual Model

Now the size of the transfer function is determined. Let nr and nc be the respective values for number of rows and number of columns. Using this information, the weights of the network are determined using

$$t = \begin{bmatrix} W & \varepsilon \end{bmatrix} \times \begin{bmatrix} \sigma(V,x) \\ ones(1,nc) \end{bmatrix} \tag{5–4}$$

Using the calculated weights, the formed Neural Network, and $generation_2$ as a new input - i to the Neural Network, an output y is formed

$$y = W \cdot \sigma(V, x) + \varepsilon \tag{5–5}$$

where ε is the approximation error.

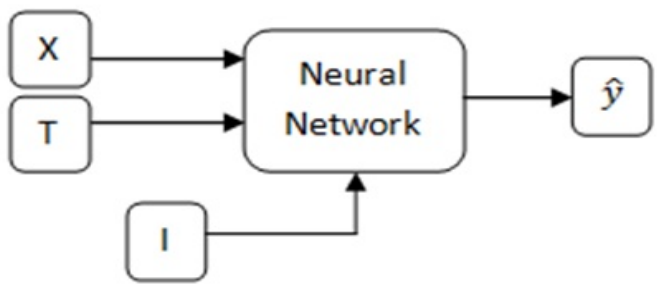
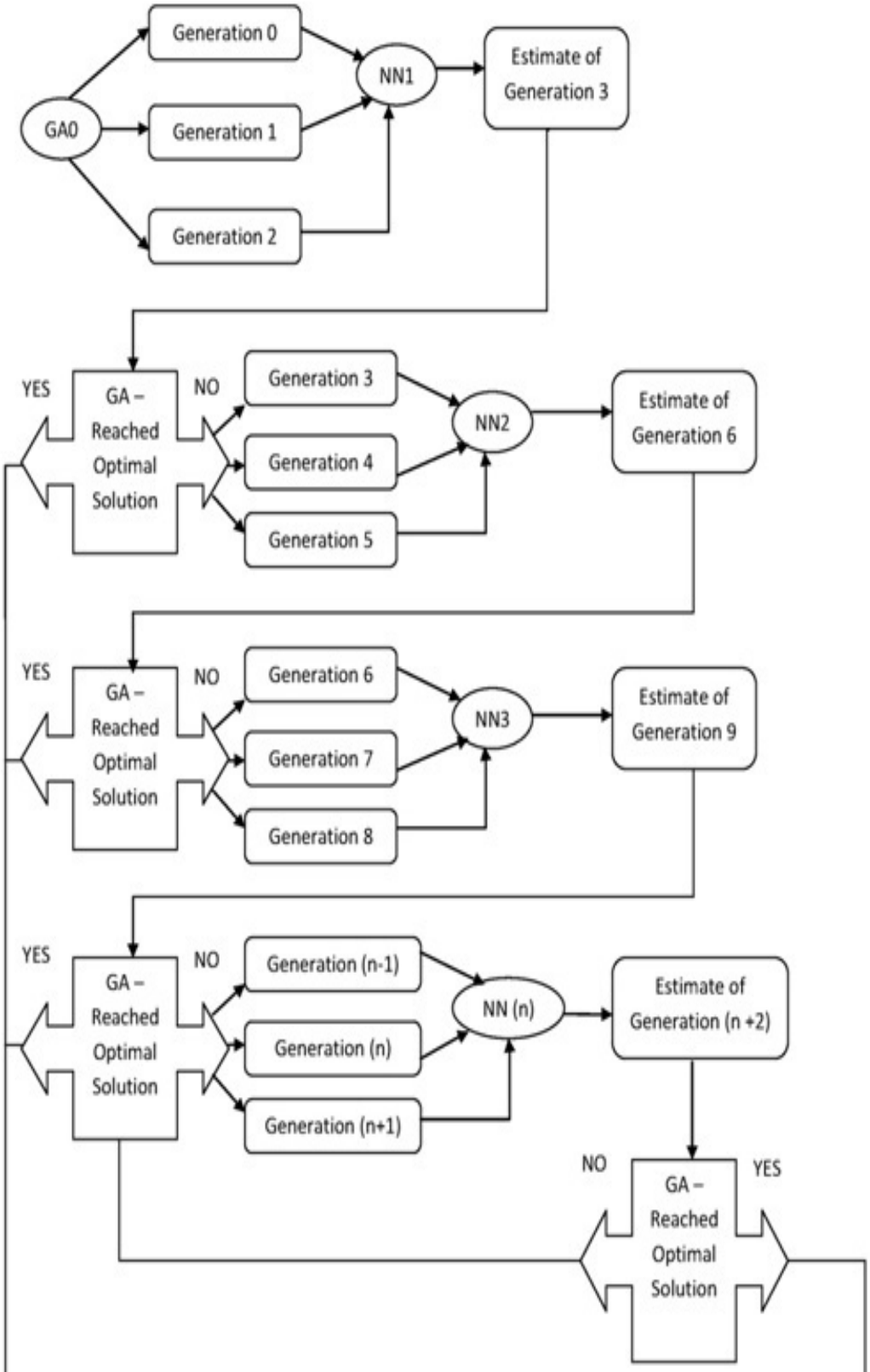


Figure 5-4. Neural Network for every 3 generations - Condensed form

The mean-square error,(MSE) between the linear layer output,y and the target vector, t is calculated such that weight W and the approximation ε would be adjusted such that the MSE converges to zero. A Neural Network is built and is trained for every 3 generations until the optimization by Genetic Algorithms leads a near-optimal solution using the initial range provided by the developed neural networks. The flowchart which depicts the working of the augmented solver is shown in Figure(5-5) using the condensed form of Neural Networks Figure(5-4).


GA0
Generation 0
Generation 1
Generation 2
NN1
Estimate of Generation 3
YES
GA – Reached Optimal Solution
NO
Generation 3
Generation 4
Generation 5
NN2
Estimate of Generation 6
YES
GA – Reached Optimal Solution
NO
Generation 6
Generation 7
Generation 8
NN3
Estimate of Generation 9
YES
GA – Reached Optimal Solution
NO
Generation (n-1)
Generation (n)
Generation (n+1)
NN (n)
Estimate of Generation (n +2)
NO
YES
GA – Reached Optimal Solution

CHAPTER 6
RESULTS

The problems were solved using the MATLAB version of Genetic Algorithms combined with an encoded Neural Network using MATLAB. The following table provides the options and input parameters used

Table 6-1. GA Parameters for Problem 1 and Problem 2

Options/Parameters	Values
Initial Population Range	TBD by Neural Network
Population Size	20
Elite Count	2
Crossover Fraction	0.75
Generations	100
Time Limit	$+\infty$
Fitness Limit	$-\infty$
Stall Generations	50
Stall Time Limit	$+\infty$
Tolerance Limit	10^{-6}
Creation Function	Uniform
Fitness Scaling	Rank
Selection Function	Stochastic Uniform
Crossover Function	Scattered
Mutation Function	Gaussian
Plot Function	Best - Fit
Upper Bound - Problem 1	100
Lower Bound - Problem 1	-100
Upper Bound - Problem 2	10
Lower Bound - Problem 2	0

The following figures are the plots for problem which depict states, control, comparison of methods and best-fit,using

- a) only GA,
- b) GA and a single NN for all generations created and
- c) GA with NN for every 3 generations respectively.

Figures (6-1)- (6-2) compare the 3 different methods for the optimal solution which almost matches results obtained using a NLP solver. Figures (6-3) - (6-8) depict the state and control plot for the three different methods. From the control plots it is clear

Table 6-2. GA Parameters for Problem 1 and Problem 2

Options/Parameters	Values
Minimum Rocket Length	50.62 in
Rocket Outer Diameter	2.3 in
Mass propellant	1.2 lbs
Burn time	1.7 s
Density of air	1.225 kg/m^3
Density of balsa wood (fin material)	160.0 kg/m^3
Number of fins	3.0
Fin span	4.6 in
Fin thickness	0.25 in
Cd,nose	0.02
Cd,fuselage	0.05
Cd,interference	0.02
Cd,fin	0.005
Ctip	0.0 in
Croot	4.6 in
Upper Bound - Problem 2	10.0
Lower Bound - Problem 2	0.0

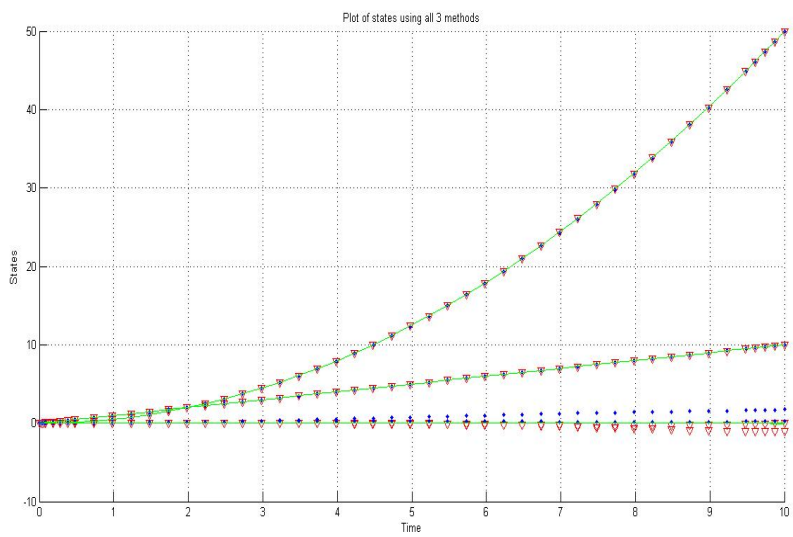


Figure 6-1. States of all 3 methods

that the method of using NN for every 3 generations of GA yields a consistent and stable result.

The encircled areas of Figure 6-3 and Figure 6-4 clearly depict that the curves are not consistent for consecutive 5 runs of the solver using same parametric conditions.

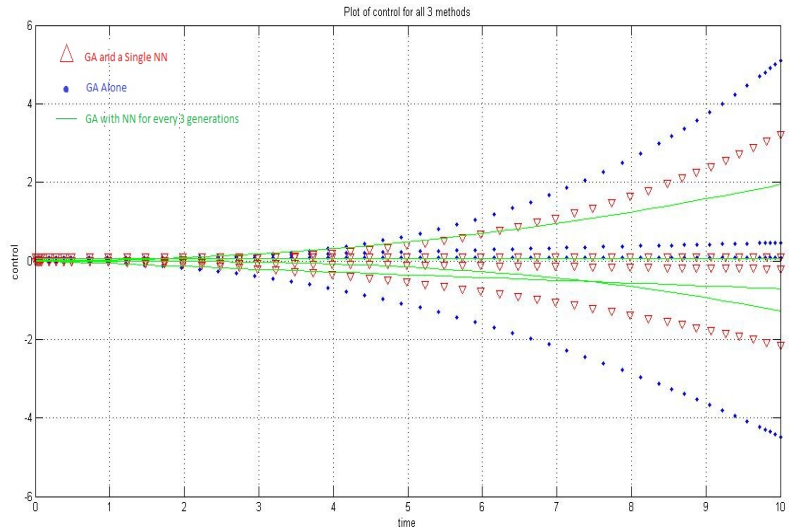


Figure 6-2. Control of all 3 methods

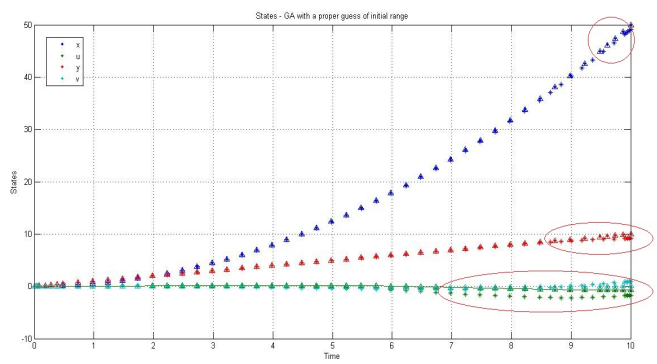


Figure 6-3. States - using GA alone

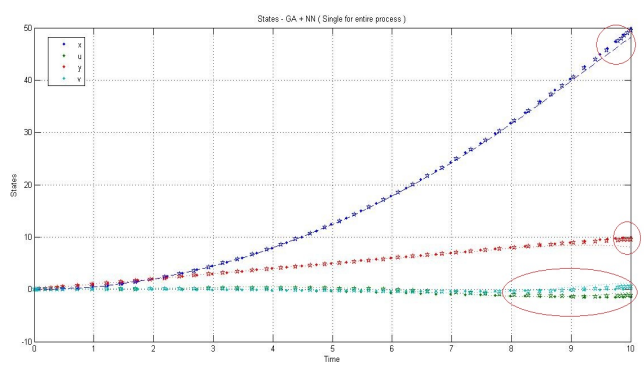


Figure 6-4. States - using GA and a single NN

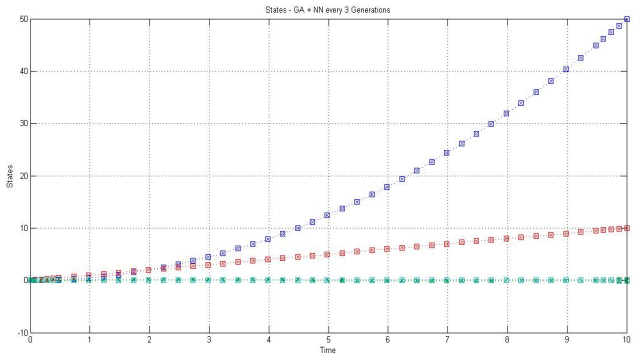


Figure 6-5. States - using GA and a NN for every 3 generations

In Figure 6-5 we can observe that the state curve is consistent and stable for 5 consecutive runs of the augmented solver.Similar results where observed in the control plots and are demonstrated in Figures 6-6 - 6-8.

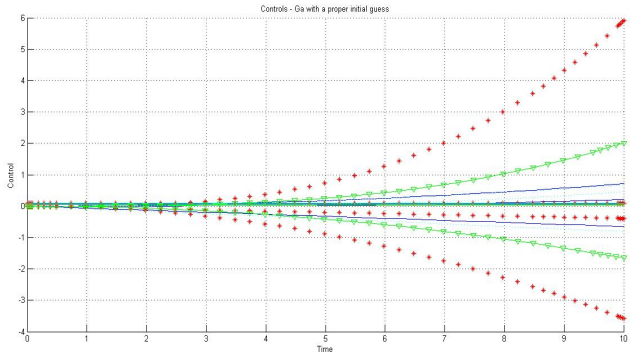


Figure 6-6. Control - using GA alone

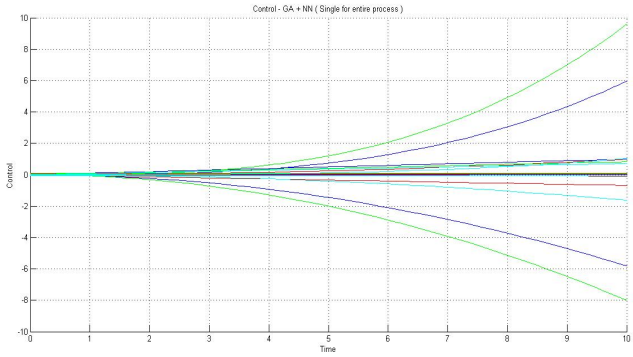


Figure 6-7. Control - using GA and a single NN

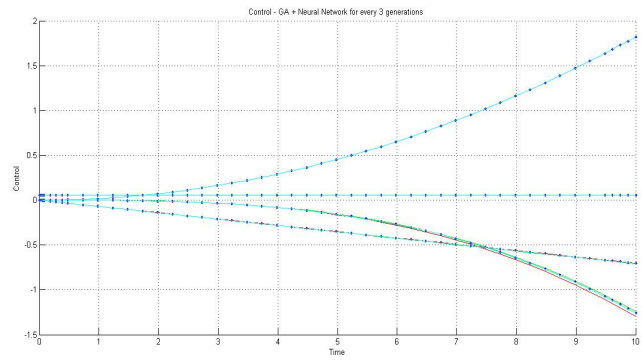


Figure 6-8. Control - using GA and a NN for every 3 generations

Similar results were observed for Problem 2 and the final results of the states, and optimal fin parameters are depicted in Figures 6-9 Table 6 respectively.

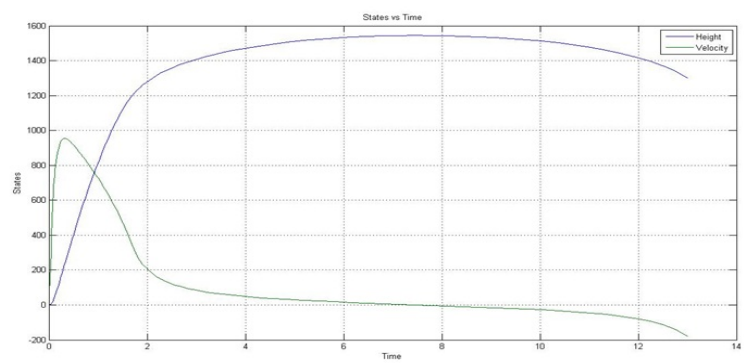


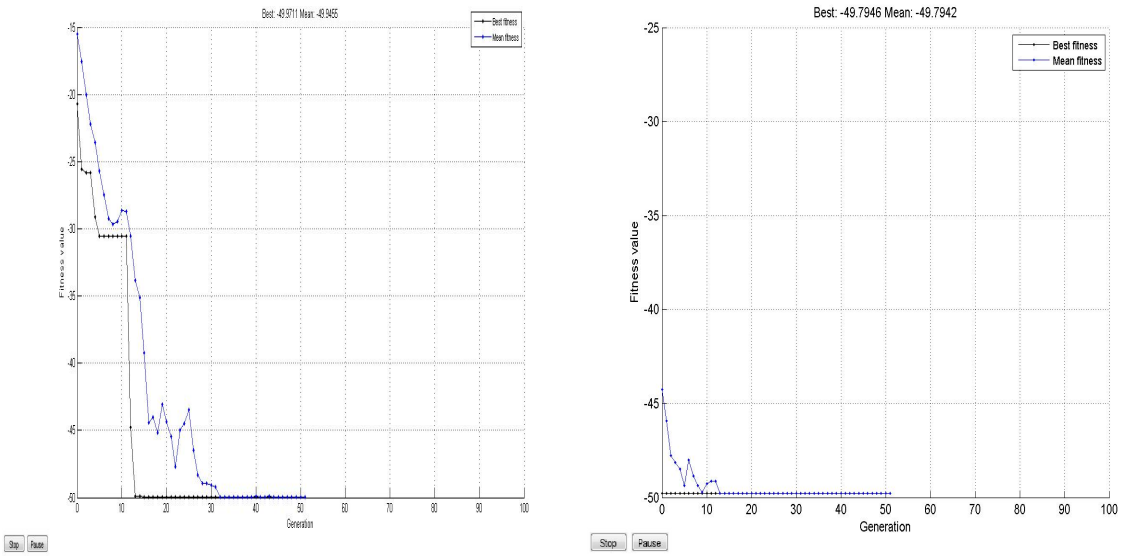
Figure 6-9. Control - using GA and a NN for every 3 generations

Table 6-3. Optimal Design Parameters obtained using GA and NN

Parameters	Values
Fin Root Chord	2.25 in
Fin Tip Chord	1.5 in
Fin Span	2.25 in
Rocket Length	74.375 in

The following plots show the best-fit individuals for 3 methods used in this research (Problem 1) and the best-fit for Problem 2 using GA and NN in every 3 generations.

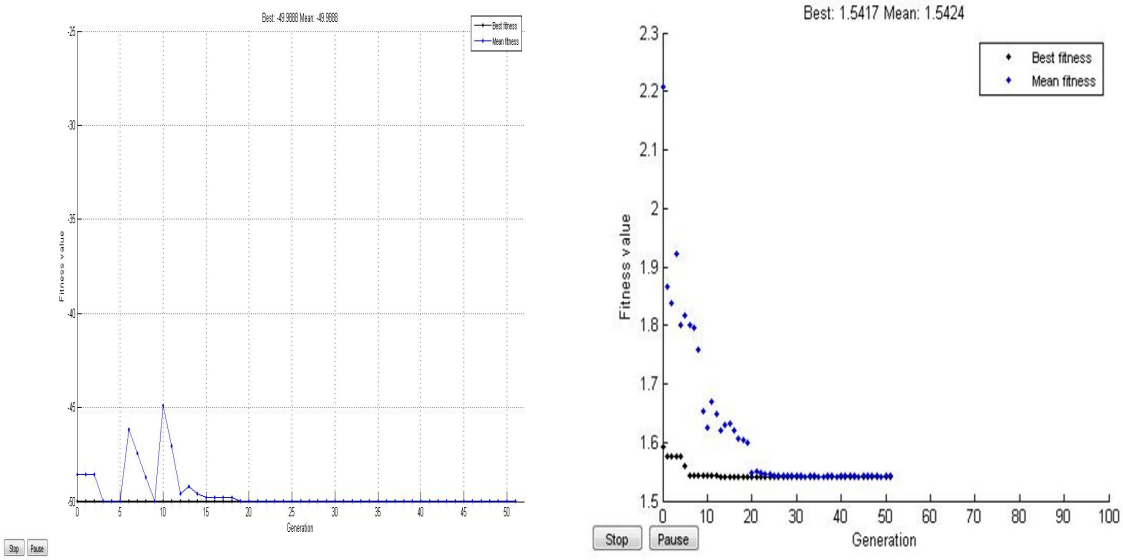
From the plots and discussion it is evident that GA can be accelerated by NN. It is also noted that it produces a global result. NN helps GA in avoiding poor convergence



A Best-fit using GA alone

B Best-fit using GA and a single NN

Figure 6-10. BEST-FIT Individuals



A Best-fit using GA and NN every 3 generations - Problem 1

B Best-fit using GA and NN every 3 generations - Problem 2

Figure 6-11. BEST-FIT Individuals

or no convergence for irregular initial population. NN approximated the initial range to almost perfect range such that search space is neither too small nor too large. The augmented solver was observed to run for about 126.36 seconds for Problem 1 and

134.61 seconds for Problem 2, less than using a Genetic Algorithms which takes about several hours before it converges to a solution. From these results it is evident that the augmentation of Genetic Algorithms and Neural Networks yields a consistent near-optimal solution using very little computational time.

CHAPTER 7
CONCLUSION

From the plots and discussion it is clear that an implementation of approximation by Neural Network to accelerate Genetic Algorithms is feasible. Apart from that it produces a global optimal result and is inexpensive as the other methods defined in literature. The values obtained for Problem 2 were used to design the fin for the rocket and the rocket had a successful flight and reached a maximum height of 1507 feet. The Genetic Algorithm accelerated by Neural Networks deduced the maximum height to be 1543 feet. As for the Problem 1, it had a local optimal solution deduced using Direct Shooting Method and when Genetic Algorithms with NN was used a global optimal solution was obtained. It is clear from the results that Genetic Algorithms is capable of a global solution and to avoid its poor convergence or no convergence for irregular initial population ranges the augmentation of Neural Networks proved to be successful. The Neural Networks has approximated the initial range to almost perfect range such that the search space is neither too large nor too small; it is tailor-made for both the problems discussed in this paper. In Chapter 2, Genetic Algorithms were discussed in detail along with the options used in solving the optimization problems. Chapter 3 gave an overview of Artificial Neural Networks and the specific NN used for approximation purposes. In Chapter 4, the optimal control and optimal design problems were formulated and described. Chapter 5 discussed the method of hybridization and the results of the problems of Chapter 4. In one problem we developed a solution for optimal control and in the other we developed a solution for optimal design but the objective for both the problems were such that maximum distance had to be covered.

In order to improve the feasibility of the optimizer, better user-defined applets can be designed. So to improve the accessibility, the augmentation can be used as a baseline to develop a Java Applet for control engineers who wish to optimize trajectories or design objects or design a business project. Since the Genetic Algorithm and Neural

Network are simple in structure and easier to understand, an applet design is not a difficult task. Modeling and designing such an applet would be the next goal in this line of research.

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BIOGRAPHICAL SKETCH

Saraswathi Nambi is a Graduate Student in Aerospace Engineering at University of Florida (UF). Her research interests encompass various fileds like trajectory optimization, neural networks, genetic algorithms and robot design.

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