CASIMIR FORCE ON NANOSTRUCTURED SURFACES: GEOMETRY AND FINITE CONDUCTIVITY EFFECTS

By

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To my parents, who sacrificed to provide me with a better life
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CASIMIR FORCE ON NANOSTRUCTURED SURFACES: GEOMETRY AND FINITE CONDUCTIVITY EFFECTS

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In this dissertation, we study the geometry and the finite conductivity effects on the Casimir force by measuring the interaction between a gold sphere and a heavily doped silicon plate with nano-scale, rectangular corrugations.

The Casimir force is a quantum effect that strongly depends on not only the material properties but also the shape of the boundary of the interacting objects. The majority of past experiments focus on simple geometries such as plate-sphere, two parallel plates or two cylinders, where the interactions are not expected to deviate significantly from the pairwise additive approximation (PAA) and proximity force approximation (PFA). To demonstrate the strong shape dependence of the Casimir force, we use artificial strongly deformed surfaces, which consist of nano-scale, periodic rectangular trenches. We fabricated three sets of samples. One of them is an shallow trench array with a depth of 100 nm and a periodicity of 400 nm. The other two are high aspect ratio trenches with a depth of 1 µm and a periodicity of 400 nm and 1 µm respectively.

A microelectromechanical torsional oscillator was used in our experiments to precisely measure the force. To improve the detection sensitivity, we use a dynamic approach, where the the Casimir force gradient is measured by the shifts in the resonant frequency of the oscillator.

At distance between 150 nm and 500 nm, the measured force gradient shows significant deviations from the value expected from the PAA and the PFA, demonstrating
that the Casimir force cannot be obtained from pairwise addition of van der Waals forces between particles. The observed deviation has a good agreement with the theoretical calculations based on scattering theory that includes the finite conductivity of the material, demonstrating the strong shape dependence of the Casimir force. Compared to the calculated values for perfectly conducting surfaces, the deviation is $\sim 50\%$ smaller, revealing the interplay between the material and the geometry effects.
CHAPTER 1
INTRODUCTION

The Casimir force, predicted by H. G. Casimir in 1948, arises from the change of the zero point energy of the electromagnetic field in the presence of boundaries. Between two perfect conducting parallel flat plates, the force is attractive and is given by

\[ F_c = \frac{\pi^2 \hbar c A}{240 z^4}, \]  

(1–1)

where \( A \) is the area of the plates, \( c \) is the speed of light, \( \hbar \) is the reduced Planck’s constant and \( z \) is the separation between the plates. It is quantum vacuum fluctuations which causes interaction between neutral bodies in vacuum. The Casimir force is one of few examples where changes in the zero point energy is experimentally observable and is thus of great interest.

The Casimir force is relevant to a diverse range of subjects from the condensed matter physics [1], elementary particle physics [2] to gravitation and cosmology [3]. In particular, one of the most important applications is in nanotechnology [4]. The Casimir force becomes the dominant force between two neutral conductors at a submicron scale since it increases rapidly with decrease of the distance. For example, the Casimir effect produces a pressure of approximately 1.3 mPa between two parallel, perfectly conducting plates at a separation of 1 \( \mu \)m and increases to \( 10^5 \) Pa (1 atmosphere) at 10 nm separation. The Casimir force has been primarily considered as a possible cause of stiction between moveable parts in micro- and nano-electromechanical systems (MEMS and NEMS). On the the hand, the Casimir force can also be put in good use, such as the actuation of a MEMS or NEMS device. Thus, controlling the Casimir force can have significant effects on the design, fabrication and function of MEMS and NEMS devices.

Equation 1–1 is only valid when the two flat surfaces are made of perfect conductors. In reality, the Casimir force depends on not only the geometry of the interacting
bodies but also dielectric properties of the boundary materials. In this dissertation, we investigate the geometry and the material effects of the Casimir force by measuring the force between a gold sphere and a silicon plate with a nano-scale periodic rectangular trench array. A microelectromechanical torsional oscillator was used to detect the force. We use a dynamic approach where the Casimir force gradient is measured from the shift in the resonant frequency of the oscillator to improve the detection sensitivity. Our results are in good agreement with the theoretical calculations which include both the shape and the finite conductivity of the material. We were able to experimentally demonstrate not only the geometry dependence of the Casimir force but also the profound interplay between the geometry effects and the material properties.

The structure of this dissertation is as follows. The sample design is described in Chapter 2. In Chapter 3, the methods of the sample fabrication and the special preparation for silicon surface are presented. In Chapter 4, the microelectromechanical device used in our experiments: a microelectromechanical torsional oscillator, is introduced. This is followed by the description of the experimental setup and the detection scheme. In Chapter 5, the sample characterization methods are provided. In Chapter 6, the detailed force measurements are discussed, including the calibration for the measurement system. In Chapters 7 and 8, the measurement results are compared to the theory. Chapter 9 summarizes the main body of the dissertation.

The rest of this chapter gives a brief introduction to the Casimir force and explains why the experimental investigation of the geometry and material dependence of the Casimir force is an interesting topic.

1.1 Introduction to the Casimir Force

The Casimir force is purely a quantum effect. In the classical picture, the electromagnetic field can be equal to zero in the absence of charges and currents when temperature is lowered to absolute zero. Thus there is no electromagnetic field between neutral bodies in the vacuum, yielding a zero interacting force of electromagnetic origin. However,
the quantum theory has totally changed the notion of the vacuum. According to the energy-time uncertainty relation
\[ \Delta E \cdot \Delta t \geq \frac{\hbar}{2}, \] (1–2)
particles, or strictly speaking virtual particles, with energy fluctuation \( \Delta E \) above the vacuum level may be created for a short amount of time in vacuum. Thus, the quantum vacuum is not truly empty, but instead contains electromagnetic fields and virtual particles that pop into and out of existences. The creation and the annihilation of virtual particles result in the vacuum fluctuation. In other words, electromagnetic fields have fluctuations and are not zero in vacuum.

The zero point energy is a crucial concept to understand the Casimir force. In the quantum picture, the energy of any electromagnetic mode with frequency \( \omega \) is given by:
\[ E_n = \hbar \omega (n + \frac{1}{2}), \] (1–3)
where \( n \) is an integer value and represents the number of photons in the mode. At the ground state \( (n = 0) \), each electromagnetic mode contributes half the energy of a photon even though there is no photons in the mode. Thus, the total zero point energy is given by
\[ E_0 = \frac{1}{2} \sum_j \hbar \omega_j, \] (1–4)
where \( j \) labels the allowed modes. In free space, modes with all frequencies exist. In the presence of conducting surfaces, the boundary conditions alter the frequency spectrum and therefore alter the zero point energy density. The change in the vacuum energy
\[ \Delta E_0 = E_{0,free space} - E_{0,surfaces} \] (1–5)
leads to the force on the surfaces.

**An example.** The simplest case of the Casimir force is that of two parallel perfectly conducting mirrors at zero temperature with a separation \( d \). We consider a box with two
Figure 1-1. A box with two sides ($x$ and $y$) of length $L$, and the third ($z$) of length $d$, in the case $d \ll L$.

sides ($x$ and $y$) of length $L$, and the third ($z$) of length $d$, in the case $d \ll L$ (Figure 1-1). The presence of the cavity allows only discrete modes, with a density of modes

$$k_x = \frac{\pi}{L} n_x, \quad k_y = \frac{\pi}{L} n_y \quad \text{and} \quad k_z = \frac{\pi}{d} n_z.$$  

Thus, we can write the zero point energy as

$$E_0 = 2 \sum_j \frac{1}{2} \hbar \omega_j = \hbar c \sum_{n_x, n_y, n_z} \sqrt{\left(\frac{\pi n_x}{L}\right)^2 + \left(\frac{\pi n_y}{L}\right)^2 + \left(\frac{\pi n_z}{d}\right)^2},$$  \hspace{1cm} (1–6)

where the factor 2 results from the two possible polarizations per $k$. Because modes in $x$ and $y$ directions are continuous, the sum over $n_x$ and $n_y$ transform into integrals. One obtains the zero point energy between two mirrors:

$$E_{0, \text{surfaces}} = \frac{L^2 \hbar c}{\pi^2} \sum_{n_x=1}^\infty \int_0^\infty dk_x \int_0^\infty dk_y \left( k_x^2 + k_y^2 + \frac{\pi^2}{d^2} n_z^2 \right)^{1/2}. \hspace{1cm} (1–7)$$

In free space, modes in all directions are continuous so we need to integrate over all three directions:

$$E_{0, \text{freespace}} = \frac{L^2 d \hbar c}{\pi^3} \int_0^\infty dk_x \int_0^\infty dk_y \int_0^\infty dk_z \left( k_x^2 + k_y^2 + k_z^2 \right)^{1/2}. \hspace{1cm} (1–8)$$

Both Eq. 1–7 and Eq. 1–8 diverge due to contributions from large momenta. In order to calculate the change of the zero point energy in presence of boundaries (Eq. 1–5), some regularizations such as a damping function of the frequency or modern zeta
function need to be introduced [5]. Using the Euler-Maclaurin summation formula [6, 7]

\[
\sum_{n=1}^{\infty} F(n) - \int_0^{\infty} dk F(k) = -\frac{1}{2} F(0) - \frac{1}{12} F'(0) + \frac{1}{720} F'''(0),
\]

and then removing the regularization, we obtain the finite value

\[
\Delta E = -\left(\frac{\pi^2 \hbar c}{720 d^3}\right) L^2.
\]

Therefore, the Casimir force can be written as

\[
F_c = -\Delta E'(d) = -\left(\frac{\pi^2 \hbar c}{240 d^4}\right) L^2.
\]

### 1.2 Geometry and Material Dependence of the Casimir Force

Historically, the prediction of the Casimir force is related to the investigation of the van der Waals (vdW) force. In fact, the theory of the Casimir force and the vdW force can be obtained in a unified way in the microscopic approach. It is well known that the vdW force is the interaction between neutral atoms due to quantum vacuum fluctuations. In the picture of quantum vacuum fluctuations, virtual photons which carry the energy from place to place, pop in and out of existence from the vacuum state. The lifetimes of the virtual photons are determined by the Heisenberg uncertainty relation (Eq. 1–2). When the distance \( R \) between two atoms is much smaller than the characteristic absorption wavelength, a photon emitted by one atom can propagate between two atoms during its lifetime. The correlated oscillations of the instantaneously induced dipole moments of those atoms give rise to the nonretarded vdW force, which is inversely proportional to \( R^7 \). If we increase the distance between two atoms to be on the order of or larger than the characteristic absorption wavelength, the relativistic retardation plays an important role because of the finite velocity of light. At such a separation, the virtual photon emitted by one atom cannot reach the other atom during its lifetime. However, the quantized electromagnetic field at the points where the two atoms are situated are correlated in the vacuum state. Thus, the induced atomic dipole
moments are correlated through the field, resulting in the retarded vdW force. In the microscopic view, the retarded vdW force is also referred to as the Casimir force.

Even though both the forces originate from quantum vacuum fluctuations, the Casimir force is rather different from the vdW force. The Casimir force between extended bodies cannot be simply obtained from the pairwise summation of the retarded vdW force of their constituents. While the vdW force between molecules is always attractive, the Casimir force can even change its sign depending on the shape of the boundaries. For example, the Casimir energy for an ideal metal shell has an opposite sign to parallel plates which indicates the possibility of generating repulsive Casimir forces [8]. Another example of the geometry dependence was found on a perfect conducting rectangular cavity with dimension $a_1 \times a_2 \times a_3$, where the calculated Casimir energy can be either positive or negative depending on the ratio of the size of the sides [9, 10]. Motivated by these results, there have been an extensive theoretical studies of the shape dependence of the Casimir force for perfect conductors [2, 11, 12].

Another important characteristic of the Casimir force is its material dependence. In Eq. 1–1, H.G. Casimir considered an ideal configuration with two perfectly conducting parallel plates. In this ideal situation, the surfaces reflect all frequencies perfectly. However, for real surfaces, they reflect some frequencies well while others are reflected poorly depending on the material properties. In addition, metal surfaces become transparent for electromagnetic modes with frequency $\omega > \omega_p$, where $\omega_p$ is the plasma frequency. Thus, the modes with frequency $\omega > \omega_p$ are not subject to any boundary conditions. In other words, the imperfect reflection of the real surfaces yields a decrease of the difference in total zero point energy between inside and outside the boundaries. Consequently, the strength of the Casimir force is smaller than the prediction for perfect metals. The frequency dependence of the material is essential for calculating the actual Casimir force.
Lifshitz et al. extended the Casimir formula to real materials in planar geometry based on the fluctuation-dissipation theorem [13, 14]. The Lifshitz theory relies on the knowledge of the dielectric permittivity of boundary materials along the imaginary frequency axis. For two parallel plates with dielectric function $\epsilon_1$ and $\epsilon_2$ immersed in a media with dielectric function $\epsilon_3$, the Casimir energy per unit area based on the Lifshitz theory can be given by

$$E_c = \frac{\hbar}{4\pi^2c^2} \int_1^{\infty} pdp \int_0^{\infty} \xi^2 \epsilon_3 [\ln(1 - \Delta_{31}^{(1)} \Delta_{32}^{(1)} e^{-x}) + \ln(1 - \Delta_{31}^{(2)} \Delta_{32}^{(2)} e^{-x})]d\xi$$

where

$$\Delta_{3k}^{(1)} = \frac{s_k \epsilon_3 - s_3 \epsilon_k}{s_k \epsilon_3 + s_3 \epsilon_k}, \quad \Delta_{3k}^{(2)} = \frac{s_k - s_3}{s_k + s_3}, \quad x = \frac{2d\sqrt{\epsilon_3}\xi p}{c}, \quad s_k = \sqrt{p^2 - 1 + \frac{\epsilon_k}{\epsilon_3}}.$$

With the knowledge of the frequency-dependent dielectric susceptibility of the material, Lifshitz theory can be applied to any material body between planar surfaces. It has been generally accepted and been supported by recent precise measurements.

1.3 Casimir Force Measurements

Due to the limitation of measurement techniques, the measurement of the Casimir force has lagged behind the theory for decades since the first prediction by H. B. G. Casimir. The first attempt to observe the Casimir force by Sparnaay in 1958 [15] was not conclusive due to 100% uncertainty in the measurements. After many years of pure theoretical research, the phenomenon has blossomed in the experimental field from the late 90’s. In addition, a new generation of modern techniques allowed the Casimir force to be measured at short separation distance with an accuracy of a few percent. The effects of the Casimir force on nanomachinery and the potential applications reignited the interest of experimental scientists.

The situation originally discussed by Casimir involved two parallel conducting surfaces. However, the two-plate configuration is seldom used in experiments. The only
recent experiment that measured the Casimir force in this configuration was carried out by G. Bressi et al \cite{bressi2019} which agreed to within 15\% to the theoretical prediction. That is because the Casimir force measurement in such a configuration requires accurate alignment to ensure the two surfaces are parallel, which is difficult in practice. For experimental convenience, the simplification of the alignment is achieved by replacing one or both of the plates by a curved surface such as a lens, cylinder or sphere. The majority of precise modern Casimir force measurements are performed in a sphere-plate geometry.

Two main modern techniques have been widely used in the Casimir force measurements. One of them is atomic force microscopy (AFM). As shown in Fig. 1-2, a metalized sphere is mounted on the tip of the AFM cantilever \cite{afm2020}. The force between the sphere and the plate is detected based on measuring the deflection of the cantilever. The deflection is measured by monitoring the intensity of the laser beam using photodiodes after it is reflected off the top of the cantilever. Another technique involves using a microelectromechanical torsional oscillator to detect the interaction capacitively (Fig. 1-3). This technique was first introduced by H. B. Chan et al.\cite{chan2010}. Then, F. Chen et al. adapted a laser interferometer to the setup to control the separation, achieving the most precise measurements so far \cite{chen2015}.
Using the modern techniques, the material dependence of the Casimir force has been thoroughly investigated in a sphere-plate geometry. Starting from 1998, the first series of precise measurements of the Casimir force were performed between a metalized sphere and plate. Different metal surfaces have been tested, such as Al-Al \[17\], Au-Au \[18\] and for dissimilar metals Au-Cu \[19\]. Those experiments demonstrated the role of the finite conductivity corrections to the Casimir force. For example, the Casimir force between a gold sphere and a gold plate at a separation of 150 nm can be more than 30\% smaller than the force for perfect metal. In addition, the effect of the surface roughness on the Casimir force was observed experimentally. In Ref. \[17\], the roughness can contribute up to 20\% to the Casimir force at close separations. The next series of measurements was aimed at the investigation of the Casimir force between a metalized sphere and a silicon plate. In the first stage of the research, F. Chen et al. demonstrated that the magnitude of the Casimir force can be modified by using silicon samples with different carrier densities \[20, 21\]. Then, they measured the Casimir force between a gold coated sphere and a silicon membrane. The modification of the Casimir force was achieved by changing the carrier density of the silicon membrane using a light pulse \[22\]. Since silicon is the basic material in nanofabrication, controlling the Casimir force using silicon surfaces can lead to many applications in nanotechnology. Another

Figure 1-3. A schematic diagram (not to scale) of the experimental setup using an microelectromechanical torsional oscillator.
important series of measurements was performed in fluid. In an early attempt, Munday et al. measured the Casimir force between a gold coated sphere and plate in ethanol, yielding an attractive force which is approximately 80% smaller than the force for ideal metals in vacuum [23]. By choosing the media carefully, Munday et al. succeeded to obtain the repulsive Casimir force between a gold sphere and a silica plate immersed in bromobenzene [24], which is the first experimental observation of the repulsive Casimir force.

1.4 Approximation for Non-planar Geometries

For a long time, there has been lack of exact calculation results of the Casimir force including material dependence for geometries other than two parallel plates. Proximity force approximation (PFA) and pairwise additive approximation (PAA) are two methods that are widely used to estimate the Casimir force between bodies that deviates only slightly from planar geometries. Both methods neglect the nonadditivity of the Casimir force. For relatively smooth objects, such as sphere-plate and cylinder-plate at separations much smaller than the radius, these approximations are highly accurate. For nontrivial geometries, where the nonadditivity of the Casimir force plays an important role, clear deviations can be observed from PFA and PAA. The experimental verification of the nonadditivity is a central theme of this thesis.

1.4.1 Proximity Force Approximation

The proximity force approximation is an effective approximation method to calculate the Casimir force between bodies with smooth geometries. It has been widely accepted for comparing of the measurement results with the theory for the sphere-plate geometry. Generally speaking, the approach of PFA is to assume that the curved surface of the test bodies is made of infinitesimal planar elements. If we assume two test bodies \( V_1 \) and \( V_2 \), at an arbitrary point \((x, y)\), the curved surface elements on \( V_1 \) and \( V_2 \) around \( z_1(x, y) \) and \( z_2(x, y) \) are replaced by parallel planar elements \( dx dy \) as shown in Fig. 1-4. In doing so, we can replace the unknown pressure between the curved surface elements
Figure 1-4. An example that illustrates the PFA. The curved surface elements on the test bodies are replaced by parallel planar elements $dx dy$. $z$ is the closest distance between two bodies. $w$ is the separation between small elements.

with a known pressure $P(x, y)$ between two parallel planar elements. Consequently the interaction between two bodies can be represented as a summation of the planar interactions between parallel surface elements [5, 25]

$$F_{PFA}(z) = \int_s ds P(w),$$  \hspace{1cm} (1–13)

where $P$ is the known pressure between two parallel plates at a separation of

$$w = z_1(x, y) - z_2(x, y) (z_1 > z_2).$$  \hspace{1cm} (1–14)

The variable $z$ is the closest distance between the two bodies. In other words, $z$ is the smallest $w$.

In the configuration of a sphere (with a radius of $R$) and a large plate, the Casimir force for separation $z << R$ is given by

$$F_{PFA}(z) = 2\pi RE(z),$$  \hspace{1cm} (1–15)

where $E$ is the Casimir energy per unit area between two parallel plates.
1.4.2 Pairwise Additive Approximation

Pairwise additive approximation (PAA) is another commonly used approximation. The basic concept of PAA is that the interaction between two bodies can be obtained from the pairwise summation of a two body potential between atoms or molecules ($U(r)$). Under PAA, the Casimir energy between two bodies at a separation $z$ can be given by the summation of the retarded vdW potentials over all atoms of the interacting bodies

$$U_{PAA}(z) = -\int_{V_1} d^3r_1 \int_{V_2} d^3r_2 U(r), \quad (1-16)$$

where $r = |r_1 - r_2|$. However, the additive result overestimated the Casimir energy since it does not take into account the non-additive effects, which results from the fact that the interaction between two particles are affected by the presence of the third one. In order to approximately account for the non-additivity, a normalization procedure is normally used [2]. Thus, $U(r)$ can be chosen to use a “renormalized” retarded van der Waals potential [5, 26]

$$U(r) = -\frac{\pi \hbar c}{24} r^{-7}, \quad (1-17)$$

such that in the configuration of two perfectly conducting parallel plates the exact Casimir energy is recovered.

Applying the normalized PAA to the configuration of a sphere and a large plate at separations $z << R$, we obtain

$$U_{PAA}(z) = -\frac{\pi^3 \hbar c R}{720z^2}. \quad (1-18)$$

Thus, the force can be given by

$$F_{PAA}(z) = -\frac{\pi^3 \hbar c R}{360z^2}. \quad (1-19)$$

Recent theoretical approaches provide exact analytical [27–29] and numerical [30] results for the Casimir force in sphere-plate or two cylinders geometries, which demonstrate that the deviations of the exact force from PFA are less than $z/R$. Thus, the
PFA provides good accuracy when the separation $z$ is much smaller than the radius of the sphere $R$.

### 1.5 Experimental Demonstration of the Geometry and Material Dependence

All the precise experiments of the Casimir force mentioned above used a simple geometry, such as a sphere and a plate, two parallel plates and two cylinders, to investigate the material dependence. For these smooth geometries the Casimir force is not expected to show significant deviations from PFA and PAA. In other words, the geometry dependence cannot be revealed by these experiments.

The first attempt to demonstrate the strong geometry dependence of the Casimir force was performed by Roy and Monideen, where the force was measured between a large sphere and a plate with small sinusoidal corrugations at the separation between 0.1 and 0.9 $\mu$m. As shown in Fig. 1-5, the amplitude of the corrugation $a \approx 60$ nm and the period $\lambda \approx 1.1$ $\mu$m. The observed force shows clear deviations from PFA/PAA. However, the theoretical calculation in this geometry without the assumption of pairwise additivity indicates that the expected deviations are not significant enough to account for the observed deviation. Instead, the lateral movement of the two surfaces may be able to account for the deviations. The theoretical predictions suggest that the shape dependence of the Casimir force can be revealed from corrugations with smaller periods.

Due to the difficulty in the sample fabrication, no other experiments have been reported on the demonstration of the geometry dependence of the Casimir force normal to the surface. Although relevant theoretical research has studied the geometry effect comprehensively, there is no conclusive experimental verification of the geometry dependence of the Casimir force. Thus, investigating the strong dependence of the Casimir force for real material experimentally becomes an attractive topic. It will not only demonstrate the validity of the theory by comparing the experimental results with the
Figure 1-5. Geometry used for calculating the Casimir energy of a spherical surface and a corrugated plate at separation $z$.

Theoretical predictions, but can also be applied to the field of nanotechnology, such as nanomechanical devices [4, 31], noncontact friction [32] and carbon nanotubes [33].
CHAPTER 2
SAMPLE DESIGN

To demonstrate the strong geometry dependence of the Casimir force, a crucial step in the experimental setup is to choose a proper geometry configuration. In the well known configuration, two parallel plates or a sphere and a plate, the Casimir force is attractive which has been observed experimentally. Theoretical calculations indicate that the Casimir energy for geometries such as a spherical shell could have opposite sign compared to parallel plates possibly leading to repulsive Casimir forces. Although it is substantially interesting to demonstrate the repulsive Casimir force for such geometries, in practice, relevant measurements are experimentally difficult for such closed structures. On the other hand, the Casimir force measurements between two parallel plates or a sphere and a plate have already been well developed. One of the most promising approach is to replace the flat plate with an artificially deformed surface in the parallel plate or sphere-plate geometry. Thus, the strong shape dependence of the Casimir force can be demonstrated by revealing a strong deviation from the usual pairwise additive approximation (PAA) or proximity force approximation (PFA).

Following the theoretical predictions from Büscher and Emig [11, 34], we choose one of the surfaces to be a plate with nano-scale periodic rectangular trenches. The other surface is chosen to be a spherical surface for experimental convenience, as discussed in Sec. 1.3. In this chapter, I will first introduce the expected Casimir force under PFA and PAA for a geometry of rectangular trench array. Then, I will discuss the theoretical prediction and describe the samples that we choose to measure.

2.1 Approximation

We consider the interaction between a periodic rectangular trench array with a 50% duty cycle and a flat parallel plate, as shown in Fig. 2-1.

As discussed in Sec. 1.4.1, under PFA, the interaction between two bodies can be represented as a summation of the planar interactions between parallel surface
elements. Thus, for the trench array with a 50% duty cycle, half of the flat plate interacts with the top of the trenches at distance $z$ and the other half interacts with the bottom of the trenches at distance $z + t$. The total force under PFA can be given by

$$F_{PFA}(z) = \frac{1}{2} F_{\text{flat}}(z) + \frac{1}{2} F_{\text{flat}}(z + t).$$  \hspace{1cm} (2–1)$$

Under PAA, the force $F_{PAA}$ on a 50% duty cycle rectangular corrugation can be considered as the contribution of two parts: the force $F_{\text{array}}(z)$ on the array and the force $F_{\text{bottom}}$ on the trench bottom (Fig. 2-2a). $F_{PAA}$ can be obtained in the following procedure. We consider the forces between a flat plate and two separate trench arrays, as shown in Fig. 2-2b. All parameters and material properties for the two trench arrays are identical except that array II is laterally shifted from array I by half the period. Thus, the force $F_{\text{arrayI}}$ on array I and $F_{\text{arrayII}}$ on array II are the same. If array I, array II and a bottom plate are superimposed on each other, a solid flat surface is recovered. In the PAA picture,

$$F_{\text{flat}}(z) = F_{\text{arrayI}}(z) + F_{\text{arrayII}}(z) + F_{\text{bottom}},$$  \hspace{1cm} (2–2)$$

Figure 2-1. Geometry consisting of a flat plate and a plate with rectangular trench array.
Figure 2-2. (a) The trench structure can be considered as the superimposition of an array and a bottom plate. (b) If array I, array II and a bottom plate are superimposed on each other, a solid flat surface is recovered.

where $F_{\text{arrayI}}(z) = F_{\text{arrayII}}(z) = F_{\text{array}}(z)$. Therefore, under PAA, the force on the corrugated surface can be given by

$$F_{\text{PAA}}(z) = \frac{1}{2} F_{\text{flat}}(z) + \frac{1}{2} F_{\text{flat}}(z + \ell).$$  \hfill (2–3)

Generally, PFA and PAA do not produce the same results in the calculation of the Casimir force. However, for the plate-trench situation in our experiments, the PAA (Eq. 2–1) and the PFA (Eq. 2–3) predict the same force regardless of the periodicity $\lambda$ and material of the trench arrays.

2.2 Nonperturbative Approach to the Casimir Force in Plate-Trench Structure

After Roy and Mohidden’s first attempt to demonstrate the geometry dependence of the Casimir force [35], Büscher and Emig performed a series of theoretical calculation to probe the strong geometry dependence of the Casimir force [11, 12, 34]. Our samples were designed based on their calculation of the Casimir interaction between a flat plate
and a plate with a rectangular corrugation for perfect metal. In this section, I will explain a limiting case of this theoretical prediction.

Büscher and Emig calculate the Casimir force using a nonperturbative approach based on path integral quantization of the electromagnetic field. With the numerical computation of this approach, the Casimir force can be calculated precisely without any approximation.

Let us first consider perfectly conducting trench array with small \( \lambda \). As we known, the quantum fluctuations of the electromagnetic modes with characteristic wavelength comparable to the separation \( z \) give the main contribution to the Casimir force. With decreasing \( \lambda \), the field that penetrates into the trenches will be affected. In the extreme situation \( \lambda \to 0, \lambda \ll z \), the field can no longer get into the narrow trenches. The force between the flat surface and the trench array is equal to the force between two parallel plates at a separation \( z \). The force per unit area can be given by

\[
F_0/A = F_{\text{flat}}/A = -\frac{\pi^2}{480} \frac{1}{z^4}. \tag{2-4}
\]

Next, we look at the opposite limitation where \( \lambda \) is very large. In case of \( \lambda \gg z \), the diffraction of the dominant modes from the trenches can be neglected. Thus in the limit \( \lambda \to \infty \), the force can be given by PFA

\[
F_\infty/A = F_{\text{PFA}}/A = -\frac{\pi^2}{480} \frac{1}{z^4} + \frac{1}{(z+t)^4}. \tag{2-5}
\]

In Fig. 2-3, the ratio of the force calculated using nonperturbative approach to the force expected from PFA as a function of \( z/t \) is plotted. Note that since the PFA and the PAA predict the same force for the plate-trench geometries, I only use the notion of PFA for the rest of the dissertation. The force \( F_0 \) provides an upper bound of the Casimir force in plate-trench structure while \( F_\infty \) provides a lower bound (shown by the solid line and the dash line in Fig. 2-3 respectively). At a fixed \( z/t \), the force converges to the
Figure 2-3. The Casimir force calculated by B"uscher and Emig between a geometry as shown in Fig. 2-1 for perfect metal. The force converges to the upper bound for small $\lambda/t$. For large $\lambda/t$ the lower bound is approached.

lower bound for large $\lambda/t$, whereas the upper bound is approached for small $\lambda/t$ which strongly deviates from the PFA.

2.3 Sample Design

The plates with periodic rectangular trenches are designed based on the following criteria: (1) achieving strong deviation from PFA that can be detected by the experimental setups and (2) feasibility to be fabricated based on available microfabrication techniques.

2.3.1 Material Chosen

At the beginning of my doctoral program, the only available theoretical calculation on trench structures was for perfect metals. The first choice of the sample material was a metal such as gold. However, there is no easy method to fabricate well defined
nano-scale high aspect ratio trench structures in metal. On the other hand, microfabrication
techniques have been well developed on silicon structures. Electron Beam Lithography
or deep ultra-violet stepper lithography allows us to generate patterns with resolutions
better than 200 nm. Reactive ion etching, plasma dry etching, provides anisotropic
silicon etch. After deciding that creating well-defined corrugated structures is experimentally
feasible, we started creating our structures using heavily doped silicon substrate.

In addition, silicon is the basic fabrication material for nanotechnology. It can lead
many applications to control the Casimir force using silicon surface through the interplay
of the geometry and finite conductivity effects.

### 2.3.2 High Aspect Ratio Rectangular Corrugations

The first set of samples that we designed for the Casimir force measurements
are deep, rectangular trenches with a depth of 1 µm. Two periodicity $\lambda = 1 \mu m$ and
$\lambda = 400$ nm are chosen, which lead to $\lambda/t = 1$ and $\lambda/t = 0.4$. According to theoretical
prediction, strong deviation from PFA can be observed for these high aspect ratio
trenches.

In the PFA view, the total interaction between the trench structure and the parallel
plate can be given by Eq. 2–4, which is a sum of two contributions: (1) the interaction
between half of the flat surface and the top surface of the trench array separated by
distance $z$ and (2) the interaction between half of the flat surface and the bottom of the
trench array at distance $z + t$. For deep trenches, the second part is negligible since the
Casimir force at this separation ($z + t > 1 \mu m$) is too small to be detected. Therefore, the
force on deep trenches under PFA is half of the force between two parallel flat surfaces
at separation $z F_{PFA} = \frac{1}{2} F_{flat}(z)$. Consequently, the distance dependence of the force
under the PFA is the same as a flat surface.
2.3.3 Shallow Rectangular Corrugations

After demonstrating the strong deviation from PFA using the high aspect ratio trenches described in Sec. 8.1, another set of samples were designed to be shallow trenches with a depth of 100 nm and a period of 400 nm.

The dimensions lead to $\lambda/t = 4$, which will give rise to a weaker deviation from PFA than the previous deep trenches. However, since the depth of the trenches is comparable to the separation between the surfaces, both the top and bottom surfaces of the corrugations contribute to the force under PFA. $F_{PFA}(z) = \frac{1}{2}F_{flat}(z) + \frac{1}{2}F_{flat}(z + t)$ yields a distance dependence that is distinct from a flat surface. Thus, this structure provides additional evidence for the geometry dependence of the Casimir force.
CHAPTER 3
SAMPLE FABRICATION

In this chapter the samples used in the experiments and the fabrication procedures are described. One of the measurement surface is chosen to be a spherical gold surface. The other surface is a silicon plate with rectangular trench arrays or a flat surface. Three kinds of silicon samples with corrugated surface were fabricated. Two of them (sample A with a periodicity of 400 nm and sample B with a periodicity of 1 µm) are deep trenches with a depth of around 1 µm and the other (sample C with a periodicity of 400 nm) is a shallow trench array with a depth of around 100 nm. Samples A and B are from the same wafer while sample C is from another wafer. In addition to the silicon samples with rectangular corrugations, two other samples with a flat surface are also fabricated. One is from the same wafer as sample A/B and the other is from the same wafer as sample C.

3.1 Gold-Coated Spheres

In our experiments, the spherical gold surface is made from a glass sphere coated with a 5 nm layer of titanium followed by a 400 nm layer of gold using sputtering deposition. The goal of the fabrication is two-fold: (1) maintaining a smooth gold surface to ensure that the separation between two measured surfaces can be reduced to below 100 nm and (2) providing a uniform gold coverage of the sphere to ensure the good electrical connection. A detailed fabrication procedure can be found in Appendix A.

3.1.1 Base Sphere

The base sphere was selected carefully from a variety of micro-spheres by measuring the morphology via atomic force microscopy (AFM). The diameter of the test spheres ranges from 100 µm to 300 µm and the materials include ceramic, glass, polystyrene and PMMA. Most spheres are very rough with a variation in surface height exceeding 300 nm (Fig. 3-1). Some types of glass beads are smooth locally but with scattered particles (Fig. 3-2). Such surface morphology would prevent us from
measuring the Casimir force in submicron regions. The smoothest spheres are the glass spheres with a diameter of 103 µm from Microspheres-Nanospheres. As shown in Fig. 3-3, the peak-to-peak variation in surface height is around 30 nm and the rms value is 2.5 nm.

3.1.2 Sputter Deposition

We used a Kurt J. Lesker CMS-18 Multi-Target Sputter Deposition system to coat the glass sphere in our experiments. The Sputter deposition process involves the following steps: (1) An inert gas, usually argon, is ionized generating a plasma. (2) The ions are directed at a target material and sputter atoms from the target. (3) The sputtered atoms are transported to the substrate through a region of reduced pressure. (4) The sputtered atoms condense on the substrate, forming a thin film. The system has both DC and radio frequency (RF) sputter power supplies. Usually, electrically
conducting metal targets that cause no ion charging issues are sputtered with DC power. RF sputtering is typically used for target material with poor electrical conductivity to prevent excessive charge build-up on the target surface. RF sputtering provides a slower deposition rate compared to DC sputtering.

For our purpose, sputter deposition has two main advantages comparing to other deposition methods. The first advantage is that the process provides a good coverage of the sphere. Because of the diffusive transport (characteristic of sputtering), the atoms approach the substrate’s surface from partially randomized directions, producing a reasonably uniform film thickness across a textured substrate’s surface. In other words, both the top and the side of the sphere are reasonably uniformly coated. The other advantage is a smooth surface will be generated on the film due to the small grain size from the sputter deposition process.

Figure 3-2. Surface morphology of a 200 µm diameter glass sphere measured by AFM. The maximum height in surface variation is 280 nm and the rms value is 35.4 nm.
To generate as smooth a deposited film as possible, we use a radio frequency (RF) magnetron sputter deposition and tune the parameters such as the power and chamber pressure. Although sputter deposition can typically produce relatively smooth films over a short distance, the process of sputtering also generates scattered clumped particles. Here RF sputtering is used for the metal target since the diminution of active atoms can lead to less clumping \[36\], which helps to produce a smooth film surface over a long distance range. The best condition for gold sputter deposition were found to be 3 mTorr using a 100 W RF power supply.

In addition, the sputtering system allows us to perform an \textit{in situ} pre-treatment of the spheres immediately before the film deposition. The pre-treatment, which improve process consistency, includes a 20-second oxygen plasma clean and a 5-second argon plasma clean. The oxygen plasma helps to clean off the organic contamination on the
The maximum height in surface variation is 33 nm and the rms value is 2.4 nm.

surface and the argon plasma physically removes small particles by bombarding the surface.

Under the conditions discussed above, gold coated spheres (Fig. 3-4) with peak-to-peak variations of $\sim 30$ nm and rms $\sim 3$ nm were fabricated for the Casimir force measurements.

### 3.2 Fabrication of Silicon Sample

The silicon plates with trench arrays were fabricated on a highly p-doped silicon wafer. The process consists of two stages. The first stage involves creating a resist pattern which will serve as the etch mask for trenches etching (performed at Bell Labs). The second stage is to create trenches by transferring the pattern into silicon using dry etch (performed at University of Florida Nanofabrication Facility (UFNF)). The general
sample prepare procedure and the etching procedures are listed in Appendix B, C and D.

3.2.1 Silicon Sample with Silicon Oxide Etch Mask

The silicon wafers with silicon oxide etch mask were provided by our collaborators at Bell Labs. The samples were prepared by first depositing on a layer of silicon oxide (0.2 µm) on the highly p-doped blank silicon wafer by chemical vapor deposition. Then, a photoresist pattern was made by lithography using a deep ultraviolet stepper. Following this, the pattern was transferred from the photoresist to the silicon oxide using reactive ion etching. After stripping the remaining photoresist, the whole wafer is coated with a layer of photoresist for protection.

3.2.2 Deep Reactive Ion Etch

Sample A and B were fabricated using deep reactive ion etch (DRIE). DRIE is a highly anisotropic plasma etch process [37]. The basic Bosch process in DRIE can be separated into two cycling steps (Fig. 3-5a) [38]: \( SF_6 \) plasma etching and \( C_4 F_8 \) plasma passivation. During the passivation process, the silicon surface was coated with a thin teflon-like polymer film which resulted from \( C_4 F_8 \). This layer protects the entire substrate from chemical attack and prevents further etching. In the etching process, the bottom of the features was exposed to the fluorine radicals while the sidewalls were still protected because the directional ions bombard the substrate which remove the bottom passivation film at a much higher rate than the side walls. The fluorine radicals attack and etch the silicon isotropically. By repeating the etch/passivate steps, the bottom of the feature is etched by small isotropic etches while the side walls are protected, which results an anisotropic etch. The two-phase process provides high mask selectivity and high anisotropic etch which can be hundreds micron deep. But it also causes the sidewalls to have an undulating shape. A typical image of undulate sidewall of a silicon feature created by DRIE Bosch process is shown in Fig. 3-6a.
Figure 3-5. (a) The scheme of Bosch process. Two cycling steps: $SF_6$ isotropic etch and $C_4F_8$ passivation. The two-phase process results unwanted sidewall scalloping and undercut. (b) The scheme of a simultaneous etch/passivation process. The teflon-like film is formed to protect the side wall simultaneously during the etch.

Figure 3-6. (a) A typical image of undulate sidewall of a silicon structure created by DRIE Bosch process. (b) Our first attempt to etch a silicon trench using Bosch process. The silicon structure is 2.2 $\mu$m in depth and the amplitude of the scalloping is 200 nm.
The samples that we aimed to fabricate are trench arrays with a periodicity of 400 nm or 1 µm and a depth of 1 µm on silicon wafer. We determine that the Bosch process is not suitable for creating the trench arrays because the amplitude of the undulated sidewalls is comparable to the width of the trenches. Figure 3-6b shows an preliminary attempt using the cycled process to create a 2.2 µm trench.

To minimize the unwanted sidewall scalloping and undercut, we use a simultaneous etch/passivation recipe instead of the cycled etch/passivation process (Fig. 3-5b). The plasma is generated from a mixture of SF$_6$ and C$_4$F$_8$ gas. The passivation gas form a thin teflon-like film on the silicon structure. Simultaneously, the directional ions bombard the substrate to prevent the passivation building up on the bottom of the trench structures, which allows the fluorine radicals to etch the silicon. The process results in straight sidewalls, with little undercut and no scalloping. By adjusting the bias power and the ratio of etching gas and passivation gas, we can control the slope of the side walls and tune it to be nearly vertical.

Figure 3-7 shows the cross section view of sample A with a period of 1 µm and sample B with a period of 400 nm. The side walls of the trenches are smooth and nearly vertical. However, we notice that a certain degree of rounding that shows up in the bottom sections makes the structure imperfect rectangular shape. The detailed characterization will be described in Sec. 5.2.

3.2.3 Reactive Ion Etch

For shallow trenches (sample C), the rounding from DRIE process becomes non-negligible compared to the trench depth. To fabricate trench arrays with flat bottom surface, we developed a recipe using a reactive ion etcher with an inductively coupled plasma module (ICP-RIE).

A schematic diagram of ICP-RIE is shown in Fig. 3-8. The system consists of two radio frequency (RF) sources. One is coupled inductively to create a plasma of ionized atoms and radicals of reactive gas. The other is applied to the lower electrode to
produce a substrate bias which can extract and accelerate the ions from plasma to etch the sample.

We developed the recipe using \( \text{Ar} \) and \( \text{SF}_6 \) as the etching gas without any passivation gas. The etching process can be considered as two parts: physical milling from the bombardment of directional ions and chemical etch from the reactive radicals resulted from the \( \text{SF}_6 \) plasma. Because the electric field accelerates reactive radicals towards the surface, the etching caused by these radicals is much stronger than those traveling in other directions. Argon plasma is used to provide only ions to give purely physical milling without any chemical reaction. \( \text{SF}_6 \) plasma provides mainly directional etching with some attack of the side walls depending on the bias power. By tuning the ratio of \( \text{SF}_6/\text{Ar} \) and adjusting the bias RF power a nearly straight side wall can be achieved.

One of the difficulties in developing the recipe is to suppress the etching rate to a low level. Normally the etching rate for silicon using \( \text{SF}_6 \) is a couple microns per minutes. In our case, however, a rate of a couple hundreds nanometer per minutes is desired. The etching rate needs to be reduced by at least a factor of 10. To achieve this, the gas
flow, chamber pressure and the RF power are all minimized to the extent that a plasma is just able to be generated.

Figure 3-9 shows the cross section view of sample C with a period of 400 nm. The bottom structure of the features is much flatter using this etching process than the result from DRIE. However, such a process consumes the masks at a much higher rate due to directional ions bombardments. The selectivity of silicon oxide to silicon is about 1:1, which make it impossible to etch trenches that are deeper than 200 nm.

3.3 Preparation of Silicon Surface

Since the silicon surface is very reactive in air, a thin layer of native oxide is present on the surface. To prepare the silicon surface for the Casimir force measurements, hydrofluoric acid (HF) is used to etch the samples. HF can remove the native oxide on the silicon surface. In addition, HF leads to hydrogen termination of the surface, which temporarily prevent oxide formation at ambient pressure. Following this step, the silicon
sample is baked at 120°C for more than 15 minutes. This eliminates residual water that might be trapped in the trenches. After this process the measurement set up is immediately assembled and kept in a vacuum chamber at a pressure of $10^{-6}$ Torr. In such an environment, the silicon surface can be stable for more than one week.
CHAPTER 4
THE EXPERIMENTAL SETUP

Micro-electromechanical systems (MEMS) are devices with moving parts linked to electrical components for detections and actuations. The advances in MEMS technology have produced ultra sensitive transducers, making it possible to explore novel interactions with high sensitivity between surfaces. All force measurements discussed in this dissertation were performed using MEMS. In this chapter, the general fabrication of MEMS is discussed. The specific device used in our measurements, a microelectromechanical torsional oscillator, is introduced. Then, a detail description is given with respect to device preparation, including dicing, releasing, metal deposition and packaging. Finally, the experimental setup and detection scheme are presented.

4.1 Fabrication of MEMS

The MEMS devices used in our experiments were fabricated at the commercial foundry MEMSCAP. The fabrication process, known as PolyMUMPS, is a three-layer polysilicon surface micromachining process [39].

The process starts with a heavily n-doped silicon wafer on which a thin layer (600 nm) of silicon nitride which act as an electrical isolation layer, is deposited (Fig. 4-1a). The first layer of polysilicon (POLY 0) with a thickness of 500 nm is deposited using low pressure chemical vapor deposition (LPCVD) (Fig. 4-1b). It is then patterned by the following steps. First, a layer of photoresist is spun on the wafer (Fig. 4-1c). The photoresist is then exposed to ultraviolet light which is shield through the appropriate photomask (Fig. 4-1d). The sections of photoresist that are not covered by the mask have their chemical properties alter, which makes their removal either easier or more difficult than the unexposed regions. The developer is then used to wash away the photoresist at the undesired regions (Fig. 4-1e). Next, a reactive ion etch (RIE) is used to transfer the pattern from photoresist into polysilicon. Finally, the remaining photoresist is stripped from the wafer (Fig. 4-1f). This method including depositing, patterning
Figure 4-1. These steps are repeated for all seven layers in the PolyMUMPS process. (a) A thin layer of silicon nitride is deposited on the highly n-doped silicon wafer as an electrical isolation layer. (b) A layer of polysilicon is deposited using LPCVD. (c) A layer of photoresist is spun on the wafer. (d) The photoresist is exposed to ultraviolet light using a photomask. (e) The undesired photoresist is washed away and RIE is used to etch the polysilicon. (f) The remaining photoresist is strip away.

The wafer with photoresist, etching and stripping remaining photoresist is repeated for all the layers in the process. Following POLY 0, a 2.0 µm phosphosilicate glass (PSG) is deposited by LPCVD and annealed @ 1050 °C for 1 hour. This layer serves as the sacrificial layer and the dopant source. The anneal dopes the polysilicon with phosphorus from the PSG layer and also significantly reduces the net stress in the polysilicon layer.
A cross section view of all seven layers of the PolyMUMPS process is shown in Fig. 4-2a. It includes: (1) polysilicon which is used as structure material, (2) deposited oxide (PSG) used as sacrificial layer, (3) silicon nitride which serves as electrical isolation between the polysilicon and the substrate and (4) metal which provides for probing, bonding, electrical routing and highly reflective mirror surfaces. A MEMS device with moveable components can be obtained by removing the sacrificial layers (Fig. 4-2b).

### 4.2 Microelectromechanical Torsional Oscillator

The MEMS device used in our experiments was a microelectromechanical torsional oscillator fabricated using PolyMUMPS process, as shown in Fig. 4-3. The oscillator consists of a 3.5 $\mu$m thick, $500 \times 500$ $\mu$m$^2$ heavily doped polysilicon plate, which is suspended by two torsional rods. The rods are anchored to the silicon nitride covered Si platform and connected to a bonding pad to provide electrical connection to the plate. The size of the spring used in the experiments discussed in this dissertation is $20 \mu m \times 3 \mu m \times 2 \mu m$. Underneath the top plate, there are two separate electrodes with an
Figure 4-3. (a) Scanning electron microscope image of a microelectromechanical oscillator. The top plate (big square) is suspended by two torsional rods which are anchored to the silicon nitride surface. The holes on the top plate are etching holes to make the wet etch more efficient. Three bonding pads (small square) provide electrical connections to the top plate and two separate electrodes underneath the top plate. (b) A magnified view of the oscillator. Underneath the top plate, there are two separate electrodes.

area of approximately half of that of the top plate. A gap of 2 µm between the electrodes and the top plate is created by the silicon oxide sacrificial layer.

The two electrodes and the oscillator top plate can be considered as two variable capacitors. Here, I demonstrate the basic behavior of the device by measurements of the capacitance. When the top plate is tilted by a small angle \( \theta \) (Inset of Fig. 4-4), the capacitance of the left side capacitor \( C_1 \) and the right side \( C_2 \) can be approximately given by

\[
C_1 = \frac{\epsilon_0 A}{d - b_1 \theta} \quad (4-1)
\]

\[
C_2 = \frac{\epsilon_0 A}{d + b_1 \theta} \quad (4-2)
\]

where \( A \) is the area of the electrode, \( \theta \) is the angle of rotation, \( d \) is the fixed gap distance between the movable top plate and the electrodes, \( \epsilon_0 \) is the electrical permittivity of free space and \( b_1 \) is the the effective moment arm of the top plate.
When we apply a DC voltage to one of the bottom electrodes, the movable top plate tilts toward this electrode due to the attractive electrostatic torque. This external torque is balanced by the restoring torque of the torsional spring of the oscillator:

\[ T_{osc} = k\theta \]  

(4–3)

In Fig. 4-4, the capacitance between the top plate and one of the bottom electrode is plotted as a function of the DC voltage applied to the same electrode. With the increase of the DC voltage, the measured capacitance also increase, which corresponds to the change of tilted angle \( \theta \) (the red dots in Fig. 4-4). Above a certain voltage, the capacitance suddenly jumps and then stays at a fixed value for higher DC voltages. This inherent instability situation is known as the pull-in effect [40], which happens when the external torque exceeds the restoring torque of the oscillator. As shown by the black dots in Fig. 4-4, the capacitance remains the same even if the voltage is decreased back
down, because the top plate is stuck at the snapped down position. The permanent stiction is likely resulted from the adhesion forces due to the residual electrostatic charges.

After an oscillator is snapped down, there are two ways to free the top plate. One is to manually free the device using a glass capillary with a very fine tip controlled by a micromanipulator. This needs to be performed under a microscope, as such the device need to be disconnected from the experimental setup. The other method is to shake the top plate by applying a large AC voltage to the top plate for a short amount of time (less than 2 s). This method is not always successful and might damage the torsional spring due to the large current.

4.3 Device Preparation

The procedure to prepare a MEMS device for experiments involves the following steps: dicing, releasing, metal deposition and packaging.

**Dicing.** The die received from MEMSCAP is a 1 cm$^2$ square and composed of several chips. A dicing saw is used to separate the die into smaller chips. The chip designed for these experiments is a 2.5 mm$^2$ square. Each chip consists of three oscillator devices.

**Releasing.** At this stage, the chip is protected by a layer of photoresist and the movable parts of the devices are supported by the sacrificial PSG layer. The device needs to be released by etching away the PSG layer. The process is described as below:

Strip the photoresist on diced chip. First the chip is rinsed by DI water to remove small particles on the surface. Then, the chip is rinsed with acetone and soaked in acetone for 5 minutes. The chip should be transferred into fresh acetone at least twice during the soaking to prevent the accumulation of particles. Following this, the chip is transferred into IPA and blown dry using $N_2$ gas. In the end, the chip is etched using $O_2$ plasma to remove possible organic particles.
Etch the sacrificial PSG layer. The chip is soaked in 49% HF for 5 minutes to remove the sacrificial PSG layer followed by a 10 minutes DI water rinsing to stop the etching process.

Dry the released chip. If the device is dried in air, the surface tension at the solid-liquid interface pulls against the layers the liquid is attached to, which might result in the two layers (the movable top plate and the electrodes) sticking together. To avoid this, a critical point dryer is used. After the etching is stopped by DI water, methanol is first used to wash away all water around the sample. Then, the sample is submerged in a methanol bath located inside the chamber of the critical point dryer. While the chamber is pressurized and kept at 10°C, methanol is gradually replaced by liquid carbon dioxide. After more than six replacing cycles, the majority of liquid inside the chamber is liquid carbon dioxide. In the next step, the temperature and the pressure are slowly increased. Instead of crossing the phase boundary, the transition from liquid to gas passes through the supercritical region, where the distinction between gas and liquid ceases to apply. This process can dramatically decreases the surface tension effect and effectively prevents the two surfaces from sticking together.

**Metal deposition.** The gold coated sphere as one of the measurement surface, is attached on the oscillator top plate. Our early attempts by gluing the sphere directly on the polysilicon top plate encountered a problem of poor electrical connections. This is because the silicon surface is very reactive in air such that a layer of native oxide can be formed on the surface which isolates the sphere from the heavily doped silicon plate. To solve this problem, we first modified the design of the device by adding one small gold square on one side of the oscillator top plate at a distance of 210 µm from the rotation axis. Then, a thin layer of gold (a thickness of 50 nm) is deposited on the top plate through a shadow mask after the device is released. We performed the deposition ourselves instead of adding a whole metal layer on MEMS design. The reason is that the metal layer provided by MEMSCAP has a thickness of 2 µm. The stress generated
between a thick gold layer and the silicon surface will bend the moveable plate. The small gold area ensures the connection without affecting the flatness of the silicon plate.

**Packaging.** After the gold deposition, the chip is glued to a 16-pin ceramic package using conductive silver epoxy. The device is then wire bonded to the package. To ensure the device function properly a capacitance measurement is performed on the device.

### 4.4 Experimental Setup

The schematic of the experimental setup is shown in Fig. 4-5. The silicon sample represents one of the surfaces in the Casimir force measurements, which can be a silicon plate with a corrugated structure or a flat surface. The silicon sample is glued on an aluminum holder which is mounted on a coarse z direction positioner. The coarse z positioner also allows us to adjust the orientation of the silicon sample. For the silicon samples with corrugated structures, the sample is positioned in a way that the trench array is perpendicular to the rotation axis of the moveable oscillator plate. Since the spring constant for the translation along the torsional axis is orders of magnitude larger than the orthogonal direction in the plane of the substrate, such arrangements can eliminate the motion of the moveable plate in response to lateral Casimir forces [41, 42].
Two gold coated spheres, each with a radius $R$ of $51.5\mu m$, are stacked and attached onto one side of the top plate using conductive epoxy at a distance of $b = 210\mu m$ from the rotation axis. The difficulty of this step is to apply a small amount of epoxy without breaking the torsional springs that support the top plate. To achieve this purpose, we take advantage of the manipulators on a wire bonding machine. A short length of gold wire is pulled out from the bonding tip and serves as the epoxy applying tool. The bonding machine can precisely position the wire while the size of the wire allows us to apply only a small drop of epoxy. The flexibility of the wire ensures the pushing force will not break the spring when the wire touches the movable top plate. The sphere is then manually placed on the glue using a fine tip tweezer. We use the tweezer to pick up the sphere at a $45^\circ$ angle and then tilt the tweezer by $45^\circ$ to place the sphere on the glue. There is no gold film deposited near the contact region between the bottom of the spheres and the double size tape. By rotating the sphere, the conductive epoxy can contact with the deposited gold film which provides electrical connection.

The ceramic package containing the oscillator device is mounted on a stage with a combination of $xy$ direction manual translation stage and $z$ direction closed-loop piezoelectric translation stage. The maximum extension of the closed-loop piezo is $10\mu m$.

During the assembling, the silicon sample is first moved to the sphere within a distance of $10\mu m$ using the coarse $z$ positioner under the microscope. Meanwhile the spheres are positioned under the center of the silicon sample by the $xy$ manual translation stage. In the next step, the whole assembled stage (Fig. 4-6a) is moved to the platform and the device is connected to the electrical circuits. A preliminary electrostatic force measurement is performed to check the performance of the device and to estimate the separation between the two surfaces. Since the separation between the two surfaces will change during pumping down of the chamber, a safe separation is empirically determined to be between $3\mu m$ to $7\mu m$. After the electrostatic force
verification, the coarse z positioner is fixed by the modest tightening of the top screw and the stage is covered with a metal shield. The separation typically decreases by \( \sim 1 \, \mu m \) after tightening the screw. In the end, the glass bell jar is placed over the apparatus and the chamber is evacuated using a vacuum pump to a pressure of \( 10^{-6} \) torr (Fig. 4-6b). After pumping down, the separation typically increases by \( \sim 1 \, \mu m \).

### 4.5 Detection Scheme

In this section the preliminary scheme of the force detection and relevant enhancements to increase the measurement sensitivity are described. The basic principle of the force detection is to measure the capacitance change which is proportional to the force using an oscillator on the capacitor bridge.

As shown in Fig. 4-5(b), the force \( F \) acting between the sphere and silicon sample produces an external torque \( \tau = Fb \), which is balanced by the restoring torque of the torsional spring of the oscillator

\[
\tau = Fb = k\theta.
\]
As we discussed above, the oscillator’s top plate and the bottom electrodes can be considered as two variable capacitors with the capacitance given by Eq. 4–1 and 4–2. For all the force measurements reported in this dissertation \( \theta \leq 2 \times 10^{-5} \text{ rad}, \) i.e., \( \theta \ll 1, \ b_1 \theta \ll d. \) Under this circumstance, Eq. 4–1 and 4–2 can be written as

\[
C_1 = \frac{\epsilon_0 A}{d - b_1 \theta} = C_0 \left(1 + \frac{b_1 \theta}{d}\right) \quad (4–5)
\]

\[
C_2 = \frac{\epsilon_0 A}{d + b_1 \theta} = C_0 \left(1 - \frac{b_1 \theta}{d}\right), \quad (4–6)
\]

where \( C_0 = \frac{\epsilon_0 A}{d}. \) Therefore \( \theta \propto \Delta C = C_1 - C_2. \) Consequently, the force between the two surfaces at a separation \( z \) can be given by

\[
F(z) = A \Delta C. \quad (4–7)
\]

The electrical circuits to measure the capacitance change are schematically shown in Fig. 4-7 [43]. We apply a 102 kHz AC voltage to each bottom electrodes as the carrier signal. The AC voltages, \( V_{AC1} \) and \( V_{AC2} \), have an equal magnitude \( V_{ac} \) and frequency \( \omega \), but are 180° out of phase. The electrical representation of the oscillator is shown in Fig. 4-8. When the top plate is flat, the two capacitors are identical with a capacitance \( C_0. \) Thus the bridge circuit is in balance and the output current is zero. If there is an external force applied to the top plate that tilt the plate by a small angle \( \theta \), the induced capacitance change is linearly proportional to the angle \( \theta \) under the condition \( \theta \ll 1. \) Consequently, the output current \( i \) is also linearly proportional to \( \theta \).

The top plate is connected to an Amptek 250 charge sensitive preamplifier, which detects the charge variation and generates an amplified voltage. The amplified voltage is then sent to a SRS830 lock-in amplifier, which measures the voltage at the frequency of the carrier wave. The lock-in amplifier essentially performs a homodyne phase detection between the reference signal and a local oscillator to extract the rms voltage via phase-locking. Therefore, noise at any other frequency can be rejected by the lock-in amplifier such that the generated voltage is effectively isolated from the noise sources.
Figure 4-7. The diagram of the electronic circuit for basic detection scheme. A high frequency carrier AC voltage is applied to each of the bottom electrodes. $V_{AC1}$ and $V_{AC2}$ have the same magnitude and frequency, but $180^\circ$ out of phase. The detected signal is fed into a charge sensitive preamplifier.

In our setup we use a HP3314 function generator to create the AC voltages. An analog splitter manufactured by MiniCircuits is used to produce AC signals of equal magnitude and frequency, but $180^\circ$ out of phase. For a certain ac voltage input, the magnitude of the split voltage is a fixed value. Therefore, we built a tuneable divider to adjust the magnitude of the voltage (Fig. 4-9). Since adding a divider will shift the phase of the voltage, another fixed divider is added to the other arm of the splitter to keep the two signal at $180^\circ$ out of phase.

**Dynamic measurements.** To improve the sensitivity of the force detection, we set up a dynamic measurement scheme which takes advantage of the high quality factor of the microelectromechanical torsional oscillator. The diagram for the electronic circuit is...
Figure 4-8. Electrical representation of the oscillator. Two ac excitations ($V_{AC1}$ and $V_{AC2}$), with an equal magnitude and frequency but 180° out of phase, are applied to each bottom electrode. When no force is applied, the symmetrical gaps give way to two identical capacitances $C_0$, the output current is zero. When a force is applied, the top plate of the oscillator tilts by $\theta$. For a small $\theta$, the changes of the capacitances are linear with $\theta$. A current $i$, which is linear to $\theta$, will be detected.

Figure 4-9. The circuit diagram of a fixed-ratio amplitude divider (left) and a variable amplitude divider (right). Each output of the splitter is connected with an amplitude divider which adjusts the amplitude to the desired value to reach the bridge balance as well as maintains the phase difference between the two arms.
shown in Fig. 4-10. In addition to the approach we discussed above, a driving signal is applied to one of the bottom electrodes to drive the oscillator at its resonant frequency. As the device oscillates in response to this excitation the output signal from the capacitor is the amplified voltage with the carrier frequency modulated by the driving signal. To extract the driving signal from the capacitor output, we use two lock-in amplifiers: The first one is locked to the carrier frequency such that it filters out noise signals and its output is proportional to the mechanical response of the oscillator at the driving frequency. The local oscillator of the second lock-in amplifier is phase-locked to the mechanical response signal and is then fed back to keep the driving signal fixed at the resonant frequency. This entire circuit works as a phase-locked loop to track the shift of the resonant frequency of the oscillator [44].

In this approach, the top plate of the oscillator is excited at its resonant frequency. At small oscillation where non-linear effect can be neglected, the motion of the oscillator is given by

\[ f_r = f_0(1 - \frac{b^2}{8\pi^2 I f_0^2} \frac{\partial F}{\partial z}), \]  

(4–8)

where \( f_r \) is the resonant frequency of the oscillator in the presence of the external force \( F \) and \( f_0 \approx \sqrt{k/I} \) is the intrinsic resonant frequency of the oscillator (\( \sim 1783 \text{Hz} \)). \( b \) is the distance between the sphere and the axis of the oscillator and \( I \) is the moment of inertia of the top plate together with two spheres. The distance \( z \) is given by \( z = z_0 - z_{\text{piezo}} - b\theta \), where \( z_0 \) is the initial gap between two surfaces, \( z_{\text{piezo}} \) is the piezo extension and \( b\theta \) is the correction due to the tilting of the oscillator top plate. The force gradient is measured through the resonance frequency change. The Eq. 4–8 can be rewritten as

\[ \Delta f = C \frac{\partial F}{\partial z}, \]  

(4–9)

where \( C = -\frac{b^2}{8\pi^2 I f_0} \). The proportional constant \( C \) and the initial gap \( z_0 \) are two parameters that need to be calibrated for the measurement system.
Figure 4-10. The diagram of the electronic circuit for a dynamic measurement scheme. In addition to the high frequency carrier AC voltage applied to each of the bottom electrodes, a driving signal is applied to one of the electrodes to modulate the top plate at its resonant frequency. The detected signal is amplified by a charge sensitive preamplifier and then fed into the first lock-in. The first lock-in outputs a voltage that is proportional to the tilt angle of the oscillator, which is filtered, amplified and locked to the resonant frequency of the oscillator by the second lock-in and fed back to drive the oscillator at its resonant frequency. The phase lock loop maintains a specific phase difference of $\pi/2$ between the drive and the vibrations of the device, ensuring that the MEMS oscillator is driven at resonance.
CHAPTER 5
SAMPLE CHARACTERIZATION

In this chapter, the detailed characterization of the samples is described. First, the topographies of both the gold spheres and the silicon plates, obtained using an atomic force microscopy (AFM), are presented. Then the methods to measure the actual dimensions of the trench arrays are introduced. While the period of the trench arrays is a known parameter, determined by the mask design, the other parameters, such as the fraction of solid volume, the depth of the trench and the side wall angle, must be obtained from measurements using SEM or AFM. Different approaches were used to characterize for high aspect ratio trenches and shallow trenches to achieve the best results.

5.1 Topographies of Gold Spheres and Silicon Plates

We measured the topographies of both the gold spheres and the silicon plates using an AFM. Those AFM images are used to determine the surface roughness of the samples, which has to be taken into account when we compare the measurements with theoretical predictions. In Fig. 5-1, we show a typical AFM image for the surface of a gold coated sphere with a 1 µm × 1 µm scanning area.

Figure 5-1. 1 µm × 1 µm AFM image of a gold coated sphere.
Figure 5-2. Histograms of the surface height in a scan size of 10 µm × 10 µm for (a) flat Si surface and (b) gold coated sphere surface. The peak to peak variation in the surface height of the Si surface is less than 4 nm and that of the Au surface is 55 nm. The rms value for the Si surface is 0.3 nm and that for the Au surface is 3 nm.

To analyze the surface roughness of the samples, we take AFM scans with a 10 µm × 10 µm scanning area and then obtain a histogram of the surface height for each set of image data. In figure 5-2, the histogram of the surface height of one AFM image for (a) a gold coated sphere and (b) a flat silicon plate is shown. The statistical analysis indicates that the peak to peak variation in the surface height of the Si surface is less than 4 nm and that of the Au surface is 55 nm. The rms value for the Si surface is 0.3 nm and that for the Au surface is 3 nm.

In practice, we cannot measure the topographies of the actual sphere or silicon plate used in the force measurements. To ensure that the obtained AFM images represent the surface roughness for the sphere used in the measurements, we measured multiple spheres from the same batch of sputtering deposition. We note
that all the investigated spheres yield consistent statistical results. Similar examinations were also performed for silicon plates. Again the results were statistically consistent. Therefore, the average of multiple AFM image data were used to represent the surface roughness of the actual samples.

5.2 Dimension of Samples with High-aspect Ratio Trench Arrays

We aimed to fabricate trench arrays with a duty cycle at 50%. However, in practice, the trenches were fabricated at a duty cycle close to but not exactly at 50%. By analyzing SEM images, the fraction of solid volume $p$ can be determined for samples with high-aspect ratio trench arrays using the following steps. (1) Top views (Fig. 5-3b) are taken by SEM at ten different locations for each sample. Each image contains more than 30 periods. (2) Each image is loaded into Matlab which returns a M-by-N array (M and N are determined by the dimension of the image). Each point in the array represents the grayscale data for a pixel in the top view image. In Fig. 5-4, a cross section of the structure consisting of a 1-D array of the gray scale data is plotted. The high values in grayscale represent the top of the trench array while the low values represent the bottom. Although a couple transition points exist in the data, the majority of the grayscale data are clearly separated such that the top and the bottom are well defined. (3) We choose a fixed number of pixels which contains integer multiples of one period along a single row of pixels. The grayscale data for those pixels are used...
Figure 5-4. The cross section of SEM data for the periodic trench array plotted as 1-D array in grayscale.

to create a histogram. As shown in Fig. 5-5, bright pixels (large number in x axis) represent the top solid part of the trench while dark pixels represent the bottom part. The middle value (125 in Fig. 5-5) is set as the dividing line of the top surface and the bottom surface. $p$ is defined as the sum of the counts for pixels with grayscale data that is larger than 125 divided by the total pixel counts. The fraction of solid volume $p$ is calculated from each histogram and averaged for all ten images.

The trench depth and the angle of the side walls to the top surface are measured from the SEM images of the cross section of the trenches. For these high aspect ratio trenches, an AFM tip cannot reach the bottom of the trenches. Therefore, AFM cannot be used to measure the depth of these deep trenches. A summary of the dimensions for samples with deep trenches is listed in Table 5-1.

Table 5-1. The physical dimensions of the high aspect ratio trench arrays. $\lambda$ is the period of the trench array, $p$ is the fraction of solid volume, $t$ is the depth of the trenches and $\theta$ is the angle of the side walls to the top surfaces.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\lambda$ ($\mu m$)</th>
<th>$p$</th>
<th>$t$ ($\mu m$)</th>
<th>$\theta$ ($^\circ C$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0.510</td>
<td>0.98</td>
<td>90.3</td>
</tr>
<tr>
<td>B</td>
<td>0.4</td>
<td>0.478</td>
<td>1.01</td>
<td>91.0</td>
</tr>
</tbody>
</table>
Figure 5-5. Histogram of the grayscale data for a SEM top view image of deep trenches. The middle value (125) is set as the dividing line of the top surface and the bottom surface.

In all theoretical analysis, the trenches are assumed to have a perfectly rectangular shape. But in practice, the bottom sections show a certain degree of rounding. For such high aspect ratio trenches, the overall electrostatic force is insensitive to small variations in the depth of the trenches. In fact, the calculated electrostatic force changes only by 0.01% when the depth of the trenches varies by 10%. This also shows that the determination of the depth is not as critical as the fraction of solid part \( p \) in the theoretical comparison.

### 5.3 Dimension of Samples with Shallow Trench Arrays

The method used for deep trenches to obtain the fraction of solid volume \( p \) cannot be applied to shallow trenches as the characteristic of the SEM images for two kinds of trenches are totally different. In figure 5-6, we compare the SEM image of deep trenches and shallow trenches in a zoomed in top view. For deep trenches, the contrast of the top surfaces and the bottom surfaces in the image is sharp (Fig. 5-6b) since the bottom...
Figure 5-6. The SEM image of (a) shallow trenches and (b) deep trenches in a zoomed in top view. For shallow trenches, the contrast of the top and the bottom are comparable. In addition, the bright lines on the edges of the shallow trenches make the edge hardly defined.

surfaces is far away. In that case, the two surfaces are well defined in the grayscale data returned by Matlab. However, for shallow trenches the bottom surfaces are too close to the top surfaces resulting in less distinct tones (Fig. 5-6a). In addition, there are bright lines on the edges of trenches in the image of shallow trenches, which may have resulted from edge effects or the side walls of the trenches. These effects make it impossible to obtain precise dimensions of the shallow trenches from the SEM top view image.

Figure 5-7. (a) A SEM image and (b) a schematic of the cross section view of the shallow trenches.

We use the SEM image of the cross section view to obtain the actual dimension of the shallow trenches. Ten SEM images of the cross section were taken at different
Figure 5-8. An AFM image of shallow trenches with a scanning area of $1 \mu m \times 1 \mu m$.

positions. Instead of using a ruler to measure it manually, we load the image using a Matlab program. As described above, the grayscale data for each pixel is presented by a M-by-N array. As labelled in Fig. 5-7 (b), we count the pixel number for the top $L_1$, the bottom $L_2$ and period $\lambda$. Since the period of the trenches is fixed by lithography to be $\lambda = 400$ nm, we can obtain the length $L_1$ and $L_2$ by scaling in the pixel numbers to the period. The disadvantage of this method is the limited sampling. Since we can only cleave the plate with trench structures once, the cross section view is limited to this line instead of randomly covering the whole sample. An AFM image of the shallow

Figure 5-9. A cross section of the AFM image shown in Fig. 5-8.
trenches is shown in Fig. 5-8 and one cross section of this image is shown in figure 5-9. The flat bottom region, as shown in Fig. 5-9, demonstrates that the AFM tip can reach the bottom of the trenches and provide correct depth measurements. The depth is calculated from the average of twenty AFM scans which are randomly taken on the sample. Ten of these scans are with a scanning area of 1 $\mu$m $\times$ 1 $\mu$m, the others are 2 $\mu$m $\times$ 2 $\mu$m. A summary of the dimensions for samples with shallow trenches are listed in Table 5-2.

Table 5-2. The actual dimensions of the shallow trench arrays. $\lambda$ is the period of trench array, $L_1$ is the length of the top surface, $L_2$ is the length of the bottom surface, $t$ is the depth of the trenches and $\theta$ is the angle of the side walls to the top surfaces.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\lambda$ ($\mu$m)</th>
<th>$L_1$ (nm)</th>
<th>$L_2$ (nm)</th>
<th>$t$ (nm)</th>
<th>$\theta$ ($^\circ$ C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.4</td>
<td>185.3</td>
<td>199.1</td>
<td>98</td>
<td>94.6</td>
</tr>
</tbody>
</table>
CHAPTER 6
FORCE MEASUREMENTS

In our experiments, both the electrostatic force and the Casimir force were measured. In fact, the electrostatic force plays an essential role in the Casimir force measurements. Through the measurements of the electrostatic force, we can determine specific calibration parameters as well as the residual voltage. In this chapter we start by introducing the electrostatic force for our geometries. Then, the residual voltage in our system, which results from the work function difference between the two surfaces, is discussed. This residual voltage will be used in the Casimir force measurements to eliminate the effect of electrostatic force. This is followed by the description of the calibration of our measurement system. Two parameters need to be determined. One is the proportionality constant between the force gradient and the measured resonant frequency shift. The other is the initial separation \( z_0 \) between the two surfaces. Finally, the measurements of the Casimir force are presented.

6.1 Electrostatic Force

6.1.1 Electrostatic Force between a Sphere and a Flat Surface

The electrostatic force between the grounded gold sphere and the flat plate at voltage \( V \) is given by [45]:

\[
F_{e, flat} = 2\pi \epsilon_0 (V - V_0)^2 \sum_{n=1}^{\infty} \frac{[\coth(\alpha) - n \coth(n\alpha)]}{\sinh(n\alpha)},
\]

(6–1)

where \( \epsilon_0 \) is the permittivity of vacuum, \( V_0 \) is the residual voltage, \( \alpha = \cosh^{-1}(1 + z/R) \) and \( z \) is the separation between the sphere and the plate. In practice an approximate expansion is sufficient to calculate the numerical value. Using the perturbative expansion, we can rearrange the equation as follows

\[
F_{e, flat}(z) = -2\pi \epsilon_0 (V - V_0)^2 \sum_{i=-1}^{6} c_i \left(\frac{z}{R}\right)^i
\]

(6–2)
where \( C_{-1} = 0.5, C_0 = -1.18260, C_1 = 22.2375, C_2 = -571.366, C_3 = 9592.45, C_4 = -90200.5, C_5 = 383084 \) and \( C_6 = -300357 \). Compared to the complete sphere-plate expression, the relative error introduced by using Eq. 6–2 is \( 4.7 \times 10^{-5} \) and \( 1.5 \times 10^{-5} \) at separation of 1.5 \( \mu \)m and 5 \( \mu \)m respectively (assume \( R = 200 \mu \text{m} \)) [46].

### 6.1.2 Electrostatic Force between a Sphere and a Plate with Periodical Trenches

There is no analytic expression for the electrostatic force between a sphere and a plate with corrugated structures. Thus we calculate the electrostatic force in such a configuration by numerical methods.

For a certain structure with infinite length on one dimension, the structure can be represented a 2-D model. Since in our case the length of the trenches can be considered infinite compared to the width, we follow the approximation to numerically calculate the structure in a periodical 2-D model.

To calculate the electrostatic force, we start with evaluating the electric potential distribution between a flat surface and a plate with periodical trench array in which the boundary conditions on the flat surface and the trenches are already known. Then the potential energy per unit area \( E_{\text{plate-trench}} \) can be derived from the calculated potential distribution. Finally, the electrostatic force between a sphere and a plate with trenches can be given by the proximity force theorem (PFA).

Two different solvers have been used for numerical calculation. During the early part of my doctoral program, when we were aiming to measure the Casimir force using high aspect ratio trench samples, we used a free program that solves poisson equation by means of FEM in Matlab language. Later, we purchased a commercial program called COMSOL Multiphysics. Two solvers provide consistent results on the electrostatic force calculation for our structures.

#### 6.1.2.1 Poisson solver using matlab program

The poisson solver that we used is a package of Matlab code which applies the finite element method to solve a form of Poisson’s equation over an arbitrary triangulated...
Figure 6-1. A geometry consisting of a flat surface and a periodical trench surface in two periods. The area between them are nonuniformly divided into triangular mesh.

region. The Matlab code was running on the computer cluster of the University of Florida High Performance Computing Center. Here I will describe the numerical approach in detail.

**Define the geometry and boundary condition.** As shown in Fig. 6-1, the first step is to define a geometry consisting of a flat surface and a periodical trench surface in two periods. The boundary conditions can be described as follows: the flat surface is grounded, the trench surface is at potential $V$. Periodical boundary condition is applied to $AA'$, $CC'$ which is achieved by iteration. The initial boundary condition is set so that the potential of the left side $AA'$ and the right side $CC'$ can be considered linear, which means the magnitude of the electric field on these two sides are given by $V/z$. As the boundary conditions are all determined, the electric field on an arbitrary point within the structure can be evaluated by the Poisson equation. Then the boundary conditions on
line AA' and CC' can be replaced by obtained electric field on the line BB' due to the periodicity. Following this way, an iteration is used to obtain the numerical result of the periodical boundary conditions.

**Triangulate Mesh.** To discretize the domain of interest into elements, we divide the domain into a triangulated mesh. Although it is desirable to have as many triangles as possible, in practice we are limited by the memory capacity of the computer. To achieve the best result with a finite number of triangles, we use a nonuniform mesh distribution: The distribution is sparse on the bottom of the trenches while the triangles are concentrated around the areas where the force gradient is sensitive to the geometry (Fig. 6-1).

**Obtain the electric potential distribution.** The electric potential distribution can be obtained by numerically solving the Poisson equation

\[ \nabla^2 U(x, y) = 0, \]  

(6–3)

with the boundary conditions discussed above. The Poisson solver, written by John Burk, solves the Poisson equation in a triangulated region in the plane using FEM. The code uses continuous piecewise linear basis functions on triangles. Figure 6-2 and 6-3 demonstrate the potential distribution for the trench array with a periodicity of 400 nm and 1 \( \mu \)m, respectively.

**Electric potential energy.** In this paragraph we will evaluate the electric potential energy between the two surfaces starting from the 2-D distribution of the electrical potential \( \Phi(x, y) \). The negative gradient of the potential function yields the electric field \( E \):

\[
E = -\nabla \Phi(x, y) = -\frac{\partial \Phi}{\partial x} \hat{e}_x - \frac{\partial \Phi}{\partial y} \hat{e}_y.
\]

(6–4)
Figure 6-2. The potential distribution for the trench array with a periodicity of 400 nm and a depth of 1 \( \mu \text{m} \). The applied voltage on the trench surface is 1 V and the flat plate is grounded.

Figure 6-3. The potential distribution for the trench array with a periodicity of 1 \( \mu \text{m} \) and a depth of 1 \( \mu \text{m} \). The applied voltage on the trench surface is 1 V and the flat plate is grounded.
Since for each point \((x, y)\) in the area there is a corresponding potential value \(\Phi\) given by a matrix in Matlab, the partial derivative \(\frac{\partial \Phi}{\partial x}\) and \(\frac{\partial \Phi}{\partial y}\) can be represented by the matrix elements:

\[
\frac{\partial \Phi}{\partial x}(y) = \frac{\Phi(x_m, y) - \Phi(x_n, y)}{x_m - x_n},
\]
\[
\frac{\partial \Phi}{\partial y}(x) = \frac{\Phi(x, y_m) - \Phi(x, y_n)}{y_m - y_n}.
\]

(6–5)

Therefore the electric field \(E\) as a 2-D distribution function \(E(x, y)\) can be numerically obtained. To calculate the total electric potential energy \(W\) in this area, we use the formula

\[
W = \frac{1}{2}\epsilon_0 \int \int E(x, y)^2 \, dx \, dy,
\]

(6–6)

where the integral range is the entire area between the two surfaces. For discrete values of \(E(x, y)\) the integral is reduced to the summation:

\[
W = \frac{1}{2}\epsilon_0 \sum_{k=1}^{N} (E_x^2 + E_y^2).
\]

(6–7)

**Calculate electric static force using PFA.** The calculations discussed above are for the geometries between a flat surface and a trench array structure. In the actual force measurements, the flat surface is modified into a spherical surface. Thus, as described in Sec. 1.4.1 we use the PFA [47, 48] to relate the sphere-plane and the plane-plane geometries according to

\[
F_{\text{sphere-trench}} = 2\pi RE_{\text{plate-trench}},
\]

(6–8)

where \(E_{\text{plate-trench}}\) is the electric potential energy per unit area between a flat surface and a trench array structure. The \(E_{\text{plate-trench}}\) is obtained from \(E_{\text{plate-trench}} = W/(2\lambda)\), where \(W\) the total electric potential energy between a flat surface and a 2 period trench structure and \(\lambda\) is the period of the trench array. The calculated electrostatic forces are presented in Fig. 6-4.
Figure 6-4. The calculated electrostatic force between a sphere and a plate with a flat surface (blue line), sample A (red line), sample B (black line). The diameter of the sphere is 103 $\mu$m. The applied voltage on the plate is 1 V and the sphere is grounded.

**Convergence Test.** In the procedure of generating triangulate mesh, we divide the area between the two surfaces into $N$ triangles. To ensure that $N$ is sufficient large, we check the convergence of the numerical calculation. At a fixed separation ($z = 150$ nm), we calculate the electrostatic force for a trench array with a periodicity of 400 nm and a depth of 1 $\mu$m using different triangle numbers (Fig. 6-5). When $N$ increases from 5000 to 10,000, the calculated force changes by $\sim$1%. When $N$ increases from 10,000 to 20,000, the calculated force only changes by less than 0.1%. This convergence test is repeated at multiple separations and for different structures providing consistance results. Base on this test, we chose to use $N > 10,000$ for our numerical calculation.
Figure 6-5. The numerical calculated electrostatic force between a sphere and a plate with trench structures plotted as a function of divided triangle numbers. The periodicity of the trench array is 400 nm and the depth is 1 µm. The separation between the sphere and the trench surface is 150 nm. When the triangle number \( N \) increases from 10,000 to 20,000, the calculated force only changes by less than 0.1%.

### 6.1.2.2 COMSOL multiphysics

COMSOL Multiphysics is a simulation software for modeling and solving physics problems based on partial differential equations (PDEs). It is a package of numerical solvers that solve the PDEs using the finite element analysis together with adaptive meshing and error control.

The basic process of COMSOL includes the steps of building model geometries, creating a mesh for the finite elements, specifying the physics, solving the problem and postprocessing the solutions. Although the process and basic concepts are similar to the numerical approach discussed in Sec. 6.1.2.1, COMSOL is a more powerful software. It provides a flexible user interference, quick set-up of model and efficient solving.
Our lab purchased COMSOL Multiphysics in the later part of my doctoral program. The models for deep trenches (sample A and B) were setup and the electrostatic force was calculated. Consistent results are obtained compared to the calculation discussed above which demonstrate the validity of both methods. Then, COMSOL was applied to sample C (shallow trenches with a slightly trapezoidal shape) to obtain the numerical value of the electrostatic force.

In Fig. 6-6, a triangular mesh generated by COMSOL is plotted. The non-uniformity of the triangle distribution is different with that for deep trenches. Since the depth of the trenches are smaller than the separations of the measured force, the density of the triangles inside the trench is comparable to that in the gap between the flat surface and the trench. The sharp corners of the trench requires a finer triangle division. The calculated potential distribution is shown in Fig. 6-7. In the end, the electrostatic force can be calculated using the same post-process described above.

6.2 Residual Voltage

The residual voltage $V_0$ is the contact potential between two grounded surfaces resulting from the work function difference between the two surfaces. Since it can
Figure 6-7. The potential distribution for the trench array with a periodicity of 400 nm and a depth of 100 nm. The applied voltage on the trench surface is 1 V and the flat plate is grounded.

generate an attractive electrostatic force between two grounded surfaces, it is important to determine the residual voltage between our samples so that we can isolate the electrostatic force from the Casimir force or eliminate the electrostatic force effects in the Casimir force measurements.

The work function is the minimum energy needed to move an electron from the Fermi level into vacuum. If we choose the energy of a free electron (with no kinetic energy) in the vacuum as the zero energy, the work function can be written as

\[ W_{\text{work}} = -E_F, \]  

where \( E_F \) is the Fermi energy. For real surfaces, there are potential patches on the surfaces that can be caused by multiple factors including strains in the surface or contamination in the surface. These patches create a surface dipole layer, which generate a potential difference across the surface \( W_S \). The actual work function is given
by

\[ W_{\text{work}} = -E_F + W_S. \]  

(6–10)

In other words, the local changes in surface crystalline structure give rise to varying work functions, and hence cause the surface potential variation (patch effect).

There have been experiments studying the patch potential [49]. Ideally, it is desirable to measure the relevant patch potential \textit{in situ} so that we could characterize the electrostatic force generated by patch potential. There are techniques such as Kelvin electrometers and probes that might be able to offer sufficient resolution and sensitivity for this application. In our experiments, we use a simple method described below.

The total force between a grounded sphere and a flat plate with voltage \( V \) is

\[ F_{\text{total}} = F_0 + F_{e,\text{flat}}, \]  

(6–11)

where \( F_0 \) is the voltage independent offset corresponding to the Casimir part and \( F_{e,\text{flat}} \) is the electrostatic force defined by Eq. 6–2. Using the parabolic dependence of the total force on the applied voltage \( V \), we determine the residual voltage \( V_0 \) by finding the voltage between the two surfaces that minimizes the force. In our dynamic measurements, the force gradient is measured through the resonant frequency shift of the micro-mechanical oscillator. At separation \( z \), the resonant frequency of the oscillator is measured as a function of the applied voltage on the silicon plate. Then, we use a fitting procedure to obtain the voltage at which the frequency achieves the maximum in the parabola. At this point of system calibration, we only know the piezo extension and the exact value of \( z \) is uncertain. A preliminary test is usually performed to estimate the value of \( z \). In the actual measurements, the parabola is repeated at multiple \( z \) ranging from 2 \( \mu \)m to 100 nm. Four fitted parabolas are plotted in Fig. 6-8 with the purple line corresponding to \( \sim 100 \text{ nm} \) and the red line corresponding to \( \sim 400 \text{ nm} \). A plot of \( V_0 \) as a function of separation \( z \) is plotted in Fig. 6-9. \( V_0 \) is found to change by less than 2 mV for \( z \) ranging from 100 nm to 1 \( \mu \)m. Due to the decrease in the signal-to-noise ratio, the
Figure 6-8. The frequency signal plotted as a function of the applied voltage at a fixed separation distance $z$ for silicon flat plate. The circles are the measured frequency and the lines are the fitting data. The purple, black, blue and red curves correspond to the data set with a separation of approximate 100 nm, 150 nm, 200 nm and 400 nm, respectively.

random error increases with the increasing of the separation. We calculate the residual voltage by averaging the measured $V_0$ at the separation between 100 nm and 1 $\mu$m. In this set of data, the residual voltage $V_0$ is determined to be -0.499 V.

The same approach is used to obtain the residual voltage for the samples with trench structures. In Fig. 6-10, we plot the residual voltage for sample A, B and C.

6.3 Calibration of $C$ and $z_0$

As discussed in Sec. 4.5, the gradient of the force $F'(z)$ between the surfaces depends linearly on the shifts in the resonant frequency of the oscillator

$$\Delta f = CF'(z),$$

where $z$ is given by $z = z_0 - z_{\text{piezo}} - b\theta$. 

80
Figure 6-9. The residual sphere-plate voltage $V_0$ plotted as a function of the separation distance $z$. $V_0$ is found to change by less than 2 mV for $z$ ranging from 100 nm to 1 $\mu$m.

Figure 6-10. The frequency signal plotted as a function of the applied voltage at a fixed separation distance $z$ for (a) sample A, (b) sample B and (c) sample C. The circles are the measured frequency and the lines are the fitting data.
$b\theta$ is the distance change due to the rotation of the top plate, which need to be taken into account in the calculation of separation $z$. In order to calculate $b\theta$, the following steps are taken. We slowly extend the piezo until the gold sphere and the silicon plate come into contact. As we further extend the piezo, the distance change $\delta b\theta$, which equals to the piezo extension change, is a known value. We extend the piezo for a couple steps at a step size of 2.5 nm and record the corresponding change in capacitance signal of the oscillator. Thus, the change in the capacitance signal is calibrated with respect to the distance change which is corresponding to the rotation of the top plate.

Calibration of $z_0$ and $C$ is performed using electrostatic force measurements by applying a dc voltage $V$ to the silicon sample while the gold sphere is electrically grounded. In one calibration procedure, six sets of electrostatic force measurements were taken. The voltages were chosen to be $V_0 + 245$ mV, $V_0 + 283$ mV and $V_0 + 300$ mV. The force for each voltage is measured twice to check the consistency. Only voltages larger than $V_0$ are used to avoid depleting the surface of the p-doped silicon with charge carriers.

The total measured force gradient consists of two parts: the Casimir force and the electrostatic force. After subtracting the contribution of the Casimir force gradient to the frequency shift, the measured frequency is fitted by the calculated electrostatic force gradient using a two parameter least square fit.

In Fig. 6-11, the solid circles represent the measured electrostatic force gradient, with the proportionality constant $C$ and $z_0$ determined from fitting to Eq. 6–2 (solid line). The calibrated $C$ and $z_0$ for six electrostatic measurements are listed in Table 6-1. In this set of calibration, $C$ is determined to be $555 \pm 3$ m N$^{-1}$ s$^{-1}$ and uncertainties in the distance $z$ is found to be $\sim 0.4$ nm from fitting to the electrostatic force for six different measurements. For samples with trench structures, the numerical calculated
Figure 6-11. Measured gradient of the electrostatic force at \( V = V_0 + 0.3V \) on the flat silicon surface (solid circles) and sample B (hollow squares). The solid line is a fit using Eq. 6–2. The dashed line is a fit using the force gradient obtained from numerical calculation (Poisson solver using Matlab). Inset: A two period trench structure with a periodicity of 400 nm and a depth of 1 \( \mu \text{m} \). The space between the corrugated structure and a flat surface is divided into triangular mesh to solve the Poisson equation in 2D (\( z = 150 \text{ nm} \)).

electrostatic force values are used to fit the experimental data, as shown by the dash lines in Fig. 6-11 and Fig. 6-12.

Table 6-1. \( C \) and \( z_0 \) obtained from the fitting of the gradient of the electrostatic force. Six sets of force gradient are measured with three different voltages.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C ) (( m\text{ N}^{-1}\text{ s}^{-1} ))</td>
<td>555</td>
<td>550</td>
<td>559</td>
<td>554</td>
<td>557</td>
<td>552</td>
</tr>
<tr>
<td>( z_0 ) (( \mu \text{m} ))</td>
<td>0.1727</td>
<td>0.1719</td>
<td>0.1726</td>
<td>0.1721</td>
<td>0.1723</td>
<td>0.1716</td>
</tr>
</tbody>
</table>

6.4 The Casimir Force Measurements

The Casimir force gradient between the gold sphere and the silicon sample (flat or corrugated) is measured by grounding the sphere and applying the residual voltage \( V_0 \) to the silicon sample. By doing so, the effect of the electrostatic force can be effectively eliminated in the Casimir force measurements.
As we know, the Casimir force changes rapidly when the two surfaces are getting close. Thus, it is very important to precisely determine the distance between two surfaces when their separation is small. Given that the unknown calibration parameters are mainly determined by the electrostatic force measurements at close distances, both measurements for small separations need to be finished within a relatively short amount of time in order to minimize any potential system drift. This is achieved by the following approach. The electrostatic force is measured when the sphere is brought close to the silicon sample. When the electrostatic force measurement is finished, the sphere is kept at the smallest measurement separation (∼100 nm). Then the voltage applied on the silicon sample is immediately switched to the residual voltage and the Casimir force is measured from small to large separation.
There is one more parameter that needs to be chosen carefully, namely the amplitude of the driving signal. As described in Chapter 4, we use a dynamic approach to measure the Casimir force gradient. A driving signal is applied to one of the bottom electrodes to modulate the top plate at its resonant frequency

\[ z(t) = A_z \cos(\omega_r t), \]  

(6–13)

where \( \omega_r \) is the resonant frequency of the oscillator. For appropriately small \( A_z \), the motion of the oscillator can be given by Eq. 4–8. However, if the oscillation amplitude is large, the non-linear effect of the oscillator due to the Casimir force produces additional amplitude-dependent frequency shifts, as demonstrated in Ref. [31]. Thus \( A_z \) needs to be sufficiently small such that the non-linearities can be neglected. On the other hand, \( A_z \) should be as large as possible to provide a reliable signal. \( A_z \) is chosen based on preliminary measurements. For electrostatic force measurements, \( A_z \) is \( \sim 1.5 \) nm at the separations from 150 nm to 500 nm. For the Casimir force measurements, \( A_z \) is \( \sim 1.5 \) nm at the separations below 170 nm, \( \sim 6 \) nm at the separations from 170 nm to 270 nm and \( \sim 15 \) nm at the separations above 270 nm.

The data of the measurements will be presented in the following chapters.
The Casimir force, in the original configuration discussed by Casimir, is the interaction between ideal metal plates at zero temperature. In case of ideal metal, all frequencies can be reflected perfectly. For real surfaces, however, there are many nonidealities, such as finite conductivity, surface roughness and finite temperature. In this chapter, I will discuss the corrections to the Casimir force due to the nonideal surfaces. The measurement results for sphere-plate structure will be compared with theoretical predictions including real surface corrections.

7.1 Description of the Material

To account for the finite conductivity of materials, the dielectric permittivities along the imaginary axis \( \epsilon(i\xi) \) are used in the theoretical calculation of the Casimir force. However, there is often no simple form of the permittivity as a function of the frequency. Different approaches have been used to obtain \( \epsilon(i\xi) \).

One of the common approaches to estimate the optical properties of metals is the plasma model. According to this approach, the dielectric function can be given by [50, 51]

\[
\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2},
\]

(7–1)

where \( \omega_p \) is the plasma frequency. For imaginary frequency \( \omega = i\xi \)

\[
\epsilon(i\xi) = 1 + \frac{\omega_p^2}{\xi^2}.
\]

(7–2)

The plasma model is a description of the high frequency optical properties. It is unphysical for metal due to the divergence of \( \sim 1/\xi^2 \) for small frequencies. For metal, a \( 1/\omega \) behavior has already been clearly shown in the low frequency “tail” of the experimental data [52].
Another approach is based on the tabulated optical data. In this approach, \( \epsilon(i\xi) \) is found through the Kramers-Kronig relation [53, 54]

\[
\epsilon(i\xi) = \frac{2}{\pi} \int_0^{\infty} \frac{\omega \epsilon''(\omega)}{\omega^2 + \xi^2} d\omega + 1, \tag{7–3}
\]

where \( \epsilon'' \) is the complex permittivity given by

\[
\epsilon' + i\epsilon'' = n^2 - k^2 + 2ink. \tag{7–4}
\]

\( n \) is the real part of the complex index and \( k \) is the imaginary part. Both of them are tabulated as a function of frequency in several references [52]. Since the tabulated optical data are available only for particular regions, the complex permittivity needs to be extrapolated for lower frequencies. One common method is to use the Drude model, in which the dielectric function is given by

\[
\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}, \tag{7–5}
\]

where \( \gamma \) is the relaxation parameter. From Eq. 7–5 we can obtain

\[
\epsilon'' = \frac{\omega_p^2 \gamma}{\omega(\omega^2 + \gamma^2)}. \tag{7–6}
\]

The divergence \( \sim 1/\omega \) of the Drude model provides a proper low frequency description for metals. However, as pointed out by some theoretical analysis, if we use \( \epsilon(i\xi) \) obtained from the extrapolation in the Lifshitz theory, it can lead to a violation of the Nernst heat theorem for perfect crystal lattices [55] possibly resulting in the calculated Casimir force being in contradiction with the experiments. It is still a controversy on whether the plasma model or the Drude model is more appropriate for describing the optical properties of metals in Casimir force calculations.

For gold, the plasma frequency \( \omega_{p,g} \) is 9 eV and the relaxation rate \( \gamma \) is 35 meV [56]. In Fig. 7-1, the dielectric permittivities along the imaginary axis \( \epsilon_g(i\xi) \), which are obtained from the approaches discussed above, are plotted as a function of frequency.
Figure 7-1. The permittivity along the imaginary axis for gold calculated using the plasma model (red dash line) and the tabulated data together with Drude model (black solid line).

For intrinsic silicon with a resistivity $\rho_0 = 1000 \ \Omega \cdot \text{cm}$, the tabulated optical data are available for $\omega > 0.00496 \ \text{eV}$ [52]. Since the complex permittivity is available for sufficient low frequencies no extrapolation is need. Thus, Eq. 7–3 is used to obtain $\epsilon_{\text{si}}(i\xi)$, which is shown by the black solid line in Fig. 7-2.

However, in our experiments the silicon plates are heavily p-doped with a resistivity much lower than the one used in the tables. The tabulated optical data are adapted for our heavily doped silicon by adding the imaginary part of the Drude dielectric function to that obtained from tables

$$
\epsilon_{\text{si},d}(i\xi) = \epsilon_{\text{si},i}(i\xi) + \frac{\omega_{p,\text{si}}^2}{\xi(\xi + \gamma_{\text{si}})},
$$

(7–7)
where $\omega_{p,si}$ is the plasma frequency for silicon and $\gamma_{si}$ is the relaxation parameter. The value of $\omega_{p,si}$ and $\gamma_{si}$ can be given by

$$\omega_{p,si} = \frac{e\sqrt{n}}{\sqrt{\varepsilon_0 m^*}}$$

(7–8)

$$\gamma_{si} = \varepsilon_0 \rho \omega_{p,si}^2,$$

(7–9)

where $n$ is the carrier density, $\varepsilon_0$ is the permittivity of vacuum, $\rho$ is the resistivity and $m^* = 0.34m_e$ is the electron effective mass for p-doped silicon. Using a four-probe technique, the resistivity of our silicon sample is determined to be $\rho = 0.028 \ \Omega \cdot \text{cm}$, which leads to a carrier density $n = 2 \times 10^{18} \ \text{cm}^{-3}$. In Fig. 7-2 the dependence of $\epsilon_{si,d}(i\xi)$ on frequency is shown by the red dash line.

### 7.2 Finite Conductivity Correction

In our experiments, the Casimir force gradient $F'_c = \partial F_c / \partial z$ is measured between a gold sphere with a radius $R$ and a silicon plate. Since $R >> z$, we use
PFA (as described in Sec. 1.4.1) to relate the interaction for the sphere-plane and the plane-plane geometries according to \( F'_{c,s-p} = 2\pi RP_{c,p-p} \), where \( P_{c,p-p} \) is the force per unit area between two parallel infinite plates. To compare the measurement results with the theory, we first calculate the Casimir force including finite conductivity corrections using Lifshitz formula

\[
P_{c,p-p}(z) = \frac{-\hbar}{2\pi c^2} \int_0^\infty \xi^3 d\xi \int_1^\infty p^2 \left\{ \frac{(s_1+p)(s_2+p)}{(s_1-p)(s_2-p)} e^{2p\xi z/c} - 1 \right\}^{-1} + \left[ \frac{(s_1+\epsilon_1 p)(s_2+\epsilon_2 p)}{(s_1-\epsilon_1 p)(s_2-\epsilon_2 p)} e^{2p\xi z/c} - 1 \right]^{-1} dp, \tag{7–10}
\]

where \( \omega = i\xi \) is the complex frequency, \( \epsilon_j(i\xi) \) is the dielectric function of material and \( S_j = \sqrt{\epsilon_j - 1 + p^2} \).

With the determined \( \epsilon_j(i\xi) \), the Casimir force can be calculated by numerically integrating Eq. 7–10 \[54\]. The basic steps are described as below. In Eq. 7–10, we make a substitution

\[
x = 2p\xi z/c. \tag{7–11}
\]

First, at a fixed separation \( z \) the integration is done by integrating over \( x \) while fixing \( p \). The range and the step size for the \( x \) integration is determined by the range and step size of \( \epsilon_j(i\xi) \). This step yields an integral whose value depends on \( p \). Then, Eq. 7–10 is integrated over \( p \) at a adjusted step size \( dp \approx p/100 \) to obtain the Casimir force at separation \( z \). Finally, the integration is repeated as a function of \( z \).

In the theoretical calculation presented in this chapter, \( \epsilon_j(i\xi) \) is calculated from tabulated data as described in Sec. 7.1. This is commonly used in recent experiments to compare the measurement data with the theory. Note that it is still controversial among researchers on whether the plasma model or the tabulated data approach is more appropriate for describing the optical properties of gold, we compare the Casimir force for both approaches. The difference between the calculated forces is 2% at 150 nm separation and decreases to 0.3% at 300 nm separation. Such differences are too small to be resolved in our set up.
7.3 Roughness Correction

The roughness correction can play an important role in the Casimir force calculation. It has been reported that the roughness correction can contribute 20% of the measured force at the shortest separation in the measurements [19]. However, when special preparation procedures are performed to decrease the surface roughness, the correction can be reduced to less than 1% even at the shortest separation [20]. Thus, a careful analysis of the roughness correction is necessary for comparison of theory with experiments.

The investigation of the topography of the silicon plate and the gold coating on the sphere has been discussed in Sec. 5.1. To estimate the roughness correction, we first calculate the zero roughness level \( H_0 \) defined as

\[
\sum_i (H_0 - h_i) v_i = 0, \tag{7–12}
\]

where \( h_i \) is the roughness height and \( v_i \) is the fraction of the surface area with heights \( h_i \leq h < h_{i+1} \). Note that the separation between two surfaces is measured from the zero roughness level in our force measurements.

We estimate the roughness correction by averaging the different possible separation distances on the two surfaces resulting from the surface roughness

\[
P(z_i) = \sum_k \sum_l v_k v_l P_{c,p-p}(z_i + H_0^{(1)} + H_0^{(2)} - h_k - h_l), \tag{7–13}
\]

where \( P_{c,p-p} \) is the Casimir force including finite conductivity correction. As discussed in Section 5.1, gold coated surface dominates the roughness correction. We simplify the calculation to

\[
P(z_i) = \sum_l v_l P_{c,p-p}(z_i + H_0^{(Au)} - h_l). \tag{7–14}
\]

The calculation shows that the roughness correction is moderate in our measurements in the separations ranged from 150 nm to 500 nm. When the separation increases from 150 nm to 500 nm, the corrections to the Casimir force due to roughness decrease from
Figure 7-3. Dots: measured Casimir force gradient between a gold sphere and a flat silicon surface. Solid line: the theoretical Casimir force gradient including the finite conductivity and surface roughness corrections.

0.73% to 0.07%. Thus the maximum correction due to roughness is about 0.73% for our sample.

In Fig. 7-3, the theoretical Casimir force including the finite conductivity and surface roughness corrections is plotted as the solid line.

### 7.4 Thermal Correction

All the computation discussed above are done for zero temperature. However, the Casimir force measurements were performed at room temperature. In order to provide a comprehensive comparison between theory and experiments, it is necessary to estimate the thermal correction.

To calculate the thermal Casimir force between two parallel plates at a separation $z$ in thermal equilibrium, Eq. 7–10 is modified by replacing the integration in continuous $\xi$
with a summation over the discrete Matsubara frequencies $\xi_l$

$$\xi_l = \frac{2\pi k_B T}{\hbar} \sum_{l=0}^{\infty} l',$$

(7–15)

where $k_B$ is the Boltzmann constant and prime refers to the addition of multiple $1/2$ in the term $l = 0$.

Recently, there has been extensive discussion on thermal corrections to the Casimir force. Different approaches have been used to calculate the temperature effect on the Casimir force between real material with finite conductivity. These include using the Drude model along the imaginary axis in the Lifshitz formula [57, 58] and using the free-electron plasma dielectric function into the Lifshitz formula [50, 51], and (c) use the surface impedance boundary condition to describe the thermal Casimir force. However, the results produced by these approaches are not consistent with each other, due to the different ways that the zero Matsubara frequency of the electromagnetic field would contribute to the Casimir force. For example, using Drude model, unexpected large temperature corrections result at small separations.

Generally speaking, the influence of the thermal field fluctuations on the Casimir force is important for separations on the order of

$$\lambda_T = \frac{\hbar c}{k_B T}.$$  

(7–16)

At room temperature $T = 300 K$, then $\lambda_T \sim 7 \mu m$. Therefore the temperature correction should be negligible at small separations. Since the measurement range for our experiments are from 150 nm to 500 nm, the thermal corrections are less than 1% in this region, which is smaller than the measurement uncertainty in our setup.
CHAPTER 8
DEMONSTRATING THE GEOMETRY DEPENDENCE OF THE CASIMIR FORCE

In my doctoral research, the Casimir force gradient is measured on plates with periodic rectangular trenches in three different dimensions. Significant deviations from PFA/PAA are observed, demonstrating the strong geometry dependence of the Casimir force. The measured deviations are in good agreement with the theoretical calculations that take into account the finite conductivity of the materials. Our results demonstrate the interplay between geometry effects and the material effects. In this chapter, the experimental results are presented. Then a detailed comparison between the experimental data and the theoretical calculations is provided.

8.1 High Aspect Ratio Rectangular Corrugations

In our first series of experiments, the Casimir force was investigated using silicon plates with high aspect ratio rectangular corrugations with a depth \( \sim 1 \, \mu m \). Two sets of samples are measured, sample A with a periodicity of 1 \( \mu m \) and sample B with a periodicity of 400 nm. The experiment results in this section can be found in article Measurement of the Casimir Force between a Gold Sphere and a Silicon Surface with Nanoscale Trench Arrays, H. B. Chan et al., Phys. Rev. Lett. 101, 030401 (2008).

8.1.1 Prediction by PFA

To demonstrate the strong dependence of the Casimir force, we evaluate the deviation of the measured Casimir force from the prediction of PFA. As discussed in Chapter 2, under PFA the interaction in plate-trench geometry can be given by

\[
F_{PFA}(z) = \frac{1}{2} F_{\text{flat}}(z) + \frac{1}{2} F_{\text{flat}}(z + t). \tag{8–1}
\]

\( F_{\text{flat}} \) is the force between two parallel plates, where \( z \) is measured from the top part of the trench array and \( t \) is the depth of the trenches. For deep trenches, the contribution from the bottom part is negligible since the Casimir force at this separation \( (z + t > 1 \, \mu m) \)
is too small to be detected. Therefore, equation 8–1 is simplified into

\[ F_{PFA}(z) = \frac{1}{2} F_{\text{flat}}(z). \]  

(8–2)

Considering the fraction of solid volume of the trench array is \( p \) instead of \( 50\% \), the interaction under PFA can be given by

\[ F_{PFA}(z) = p F_{\text{flat}}(z). \]  

(8–3)

Another commonly used approximation is the pairwise additive approximation (PAA) which calculates the interaction by pairwise addition of the vdW force. As discussed in Sec. 2.1, the PFA and the PAA predict the same force for the plate-trench situation in our experiments. Thus, we only write the PFA for the remaining chapter.

In our experiments, the Casimir force gradient is measured in the sphere-trench geometry. This is connected to the Casimir force in plate-trench geometry in the following approach. According to the proximity force theorem [48]

\[ F_{\text{sphere-trench}}(z) = 2\pi R E_{\text{plate-trench}}(z), \]  

(8–4)

where \( E_{\text{plate-trench}}(z) \) is the Casimir energy per unit area for a plate-trench geometry. Differentiating Eq. 8–4 with respect to \( z \), we obtain

\[ F'_{\text{sphere-trench}}(z) = -2\pi R P_{\text{plate-trench}}(z), \]  

(8–5)

where \( P_{\text{plate-trench}}(z) \) is the Casimir force per unit area for a plate-trench geometry. Thus, we have

\[ F'_{PFA}(z) = p F'_{\text{flat}}(z). \]  

(8–6)

In Fig. 8-1, the measured Casimir force gradients are plotted for sample A and B together with the PFA prediction.
8.1.2 Deviations from PFA

To analyze the deviation from PFA, we consider the ratio of the Casimir force in trench geometry to the prediction under PFA

$$\rho = \frac{F'_{c,\text{trench}}}{\rho F'_{c,\text{flat}}}.$$ \hspace{1cm} (8–7)

The ratios $\rho_A = \frac{F'_{c,\text{trench}}}{\rho_A F'_{c,\text{flat}}}$ and $\rho_B = \frac{F'_{c,\text{trench}}}{\rho_B F'_{c,\text{flat}}}$ are plotted in Fig. 8-2.

Note that $\rho$ equals one if PFA is valid. For sample A with $\frac{\lambda}{t} = 0.94$, the measured force gradient deviates from PFA by $\sim 10\%$. The deviation increases to $\sim 20\%$ for sample B with $\frac{\lambda}{t} = 0.41$. Both samples show clear deviation from PFA prediction for the separation between 150 nm and 250 nm, demonstrating the strong geometry dependence of the Casimir force. The uncertainty increases considerably at larger separations because of the decrease of the force gradient. The observed deviations occur because the Casimir force is associated with confined electromagnetic modes with wavelength comparable to the separation between the interacting objects, which are affected by the presence of trenches. In the limitation $\lambda << z$, these modes can no longer penetrate into the trenches, rendering the Casimir force on the corrugated surface equal to a flat one, which lead to deviations from PFA by a factor of 2.

Figure 8-1. Measured Casimir force gradient between the same gold sphere and (a) sample A, $F'_{c,A}$ ($\lambda = 1 \mu m$) and (c) sample B, $F'_{c,B}$ ($\lambda = 400 \text{ nm}$). The lines represent the force gradients expected from PFA ($\rho F'_{c,\text{flat}}$).
Figure 8-2. Ratio of the Casimir force in trench geometry to the prediction under PFA 
\[ \rho = \frac{F_{c,\text{trench}}}{pF_{c,\text{flat}}} \] for sample A with \( \lambda/t = 0.94 \) (blue solid circles) and B 
with \( \lambda/t = 0.41 \) (red hollow squares). Theoretical predictions for perfect 
conducting surfaces are plotted as a solid line (\( \lambda/t = 1 \)) and a dash line 
(\( \lambda/t = 0.5 \)).

The experimental results are compared to the calculations by B"uscher and Emig. 
In their theoretical calculation, the Casimir force is calculated on the trench structures 
with \( p = 0.5 \) for perfect conductor. As shown in Fig. 8-2, the solid line and the dash line 
are the calculation on trench structure with \( \lambda/t = 1 \) and \( \lambda/t = 0.5 \) respectively. Note 
the geometry used in the theoretical calculation is slightly different from our sample. 
Nevertheless, the qualitative trends can be obtained from the comparison. First, the 
development from PFA in sample B is larger than in sample A, which is consistent with 
the theoretical prediction that the deviation from PFA is stronger when \( \lambda/t \) decreases. 
Second, even though up to 20% deviation is observed, the measured deviations for both 
samples are smaller than the theoretical prediction for a perfect conductor by \( \sim 50\% \). 
Such discrepancy of measurement from prediction based on perfect metal arises due 
to the interplay between geometry effects and finite conductivity, as discussed in the 
following sections.
8.1.3 Theoretical Calculation including material properties

The discrepancy between our measured deviation and that of predictions for perfect conductors stimulate the interests of theorists. Two groups calculated the Casimir force for our geometries based on scattering theory which take into account the finite conductivity of the materials.

8.1.3.1 Scattering theory approach to the Casimir force

The scattering theory approach to the Casimir force is a powerful method that can calculate the Casimir force for arbitrary geometries including the finite conductivity correction and the finite temperature correction [59, 60]. In this section, the basic concept of the scattering theory is briefly summarized.

Based on the scattering theory, the influence of a group of objects on the electromagnetic field is described by their scattering properties. In the framework of this approach, there are two key components: (1) the scattering amplitude of each object considering the object as isolated scatterer and (2) the translation matrices which depend only on the separations and orientations of the objects. At zero temperature, the interaction free energy between two bodies is given by

\[
E(z) = \frac{\hbar c}{2\pi} \int_0^\infty dk \text{Tr} \log[1 - R_1 \cdot X_{12}(z) \cdot R_2 \cdot X_{21}(z)],
\]

(8–8)

where \( R_i \) is the scattering operator, \( X_{12}(z) \) and \( X_{21}(z) \) are the translation operators.

Equation 8–8 in general can represent extremely complicated situations. However, it can be simplified for certain systems. In particular, for the following geometry: a plate with a periodic, rectangular trench array and a parallel plate, a specific base can be chosen to simplify Eq. 8–8 based on the symmetry properties of periodic system. Thus, the Casimir force per unit area can be given by

\[
F(z) = -\hbar \int \int \int \text{Tr}((1 - M^{-1} \partial_z(M)d^2k_\perp d\xi),
\]

(8–9)
where $k_\perp$ gather the components of the wave vector in the plane of the objects, $\xi$ is the imaginary frequency. $M$ is defined as

$$M = R_1(\xi) e^{-\kappa z} R_2(\xi) e^{-\kappa z},$$  \hspace{1cm} (8-10)$$

where $\kappa = \sqrt{\xi^2/c^2 + k_\perp^2}$.

In order to calculate the Casimir force at non-zero temperature, the integral $\frac{\hbar c}{2\pi} \int_0^\infty dk$ is replaced by the summation over Matsubara (imaginary) frequencies $\frac{1}{\beta} \sum'$. Equation 8–8 is then modified into

$$E(z) = \frac{1}{\beta} \sum' \frac{\infty}{l} \text{Tr} \log[1 - R_1 \cdot X_{12}(z) \cdot R_2 \cdot X_{21}(z)].$$  \hspace{1cm} (8–11)$$

8.1.3.2 Theoretical calculation I

Davids et al. calculated the finite temperature Casimir force between a silicon plate with deep trenches and a parallel gold surface based on the scattering theory. In their approach, Eq. 8–11 is numerically calculated at $T = 300$ K using a modal expansion. The basic concept of the modal expansion is a plane wave expansion of the fields and a Fourier decomposition of the permittivity of the structure. In order to provide a precise comparison between the theoretical calculation and the experiment results, the optical properties of the samples used in our experiments have been included in the calculation code. The gold surface is modeled by the Drude model

$$\epsilon_g(i\xi) = 1 + \frac{\omega_{p,g}^2}{\xi(\xi + \gamma_g)},$$  \hspace{1cm} (8–12)$$

where $\omega_{p,g} = 1.27524 \times 10^{16}$ rad/s and $\gamma_g = 6.59631 \times 10^{13}$ rad/s. The intrinsic silicon permittivity is modeled by the Drude-Lorentz model

$$\epsilon_{si}(i\xi) = \epsilon_\infty + (\epsilon_0 - \epsilon_\infty) \frac{\omega_0^2}{\xi^2 + \omega_0^2},$$  \hspace{1cm} (8–13)$$
where $\epsilon_0 = 11.87$, $\epsilon_{\infty} = 1.035$ and $\omega_0 = 6.6 \times 10^{15}$ rad/s. To adapt it to the property of p-doped silicon, a Drude background is added to the intrinsic part

$$
\epsilon_{\text{si},d}(i\xi) = \epsilon_{\text{si}}(i\xi) + \frac{\omega_{p,\text{si}}^2}{\xi(\xi + \gamma_{\text{si}})}.
$$

(8–14)

where $\omega_{p,\text{si}} = 3.6151 \times 10^{14}$ rad/s and $\gamma_{\text{si}} = 7.868 \times 10^{13}$ rad/s. Note, these materials models and parameters are cited from [61], which is slightly different from the parameters that we used to calculate the Casimir force between a gold sphere and a silicon plate.

### 8.1.3.3 Theoretical calculation II

R. Guérout et al. calculated the zero temperature Casimir force in plate-trench geometries based on scattering theory for both the deep and shallow trenches. In this approach, the Casimir force per unit area between two reflecting objects can be given by Eq. 8–9 (Sec. 8.1.3). The description of the materials is done via the use of the dielectric functions evaluated at imaginary frequencies $\epsilon(i\xi)$ which enter into the description of the reflection operators. The dielectric functions are modeled using Eq. 7–3 and 7–4 with tabulated optical data as described in Sec. 7.1. For gold, optical data are extrapolated at low frequencies by the Drude model Eq. 7–6, with the plasma frequency $\omega_{p,g} = 9$ eV and the relaxation rate $\gamma_g = 35$ meV. The p-doped silicon is modeled by adding a Drude background to the dielectric function of the intrinsic silicon (Eq. 7–7), with $\omega_{p,\text{si}} = 1.36 \times 10^{14}$ rad/s and $\gamma_{\text{si}} = 4.75 \times 10^{13}$ rad/s.

### 8.1.4 Compare with Theory including Material Properties

In this step, a more precise comparison is performed between the experimental results and the exact calculations including finite conductivity corrections. In Fig. 8-3 and 8-4, the solid lines represent the ratio of the exact numerics to the theoretical PFA prediction calculated by Davids et al. and the dash lines represent the calculations by R. Guérout et al.. The exact numerics are calculated in geometries which are identical to our samples based on scattering theory including finite conductivity correction. The
in the theoretical prediction is calculated using the Lifshitz formula with the same optical data as the calculation in the trench structure. The hollow squares with error bars are the ratio of measured force gradient and the “experimental” PFA prediction, in which \( F_{c, flat}^{'} \) is obtained from a separate measurement in sphere-plane geometry. The flat surface and the trench array samples are from the same wafer and follow the same preparation procedure so that we can expect the same optical properties for them. In addition, both measurements in trench geometry and plane geometry used the same experimental setup (same micromechanical oscillator and gold sphere) to provide the same measurement environment. Given the uncertainty in the numerical calculation and the experimental measurements, the experimental data agrees well with the theoretical calculation. The measured force gradient at the closest measurement separation deviates from PFA by \( \sim 10\% \) for sample A and \( \sim 20\% \) for sample B.

8.2 Shallow Trenches

After observing the strong deviations from PFA in high aspect ratio trenches, we designed and measured another set of trench structure with a depth of 98 nm and a periodicity of 400 nm. For this geometry, both the top and bottom surfaces of the corrugations contribute to the PFA prediction. The experiment and the theoretical calculation results in this section can be found in *The Casimir force on a surface with shallow nanoscale corrugations: Geometry and finite conductivity effects*, Y. Bao et al., Phys. Rev. Lett. A detailed theoretical approach was presented in *Casimir Interaction of Dielectric Gratings*, Astrid Lambrecht and Valery N. Marachevsky, Phys. Rev. Lett. 101, 160403 (2008).

8.2.1 PFA Prediction

The depth of the shallow trenches (\( t = 97.8 \) nm) is smaller than the typical separation between the two interacting bodies, so that the contribution of the bottom part to the PFA prediction is not negligible. In addition, the trenches have a slightly trapezoidal shape instead of perfect rectangular. A schematic a schematic of the cross
Figure 8-3. The ratio $\rho_A$ of the exact Casimir force gradient in trench geometry to the PFA prediction for sample A. The hollow squares are the measured force gradient divided by the “experimental” PFA. The “experimental” PFA refers to the measured Casimir force in sphere-plane geometry multiplied by $p_A$. The solid and dash lines are the theoretical calculation in the exact geometry including finite conductivity corrections to the theoretical PFA calculated by P. S. Davids et al. for temperature of 300K and R. Guérout et al. for zero temperature respectively. The theoretical PFA is calculated using the Lifshitz formula.

The section of the shallow trenches is shown in Fig. 5-7, with $l_1$ is the top length in one period, $l_2$ is the bottom length, $\lambda$ is the periodicity and $t$ is the depth. To account for the contribution of both the side walls and the bottom parts, the interaction under PFA can be given by

$$F_{PFA} = \frac{1}{\lambda} \int_0^\lambda F_{\text{flat}}(z(x)) \, dx$$

$$= p_1 F_{\text{flat}}(z) + p_2 F_{\text{flat}}(z + t) + 2 \int_0^{p_3} F_{\text{flat}}(z_t x/p_3) \, dx,$$

(8–15)

where $p_1 = l_1/\lambda$, $p_2 = l_2/\lambda$ and $p_3 = (1 - p_1 - p_2)/2$. For “theoretical PFA”, $F_{\text{flat}}$ is calculated using the Lifshitz formula with the same optical parameters as the trench.
Figure 8-4. The ratio $\rho_B$ of the exact Casimir force gradient in trench geometry to the PFA prediction for sample B. The hollow squares are the measured force gradient divided by the “experimental” PFA. The “experimental” PFA refers to the measured Casimir force in sphere-plane geometry multiplied by $p_B$. The solid and dash lines are the theoretical calculation in the exact geometry including finite conductivity corrections to the theoretical PFA calculated by P. S. Davids et al. for temperature of 300K and R. Guéroud et al. for zero temperature respectively. The theoretical PFA is calculated use the Lifshitz formula.

geometry calculation. For “experimental PFA”, $F_{flat}$ is measured from a flat surface made of the same material as the sample with the shallow trench array.

8.2.2 Compare with Theory

To analyze the deviation from PFA, the ratio $\rho = F'_{c,trench}/F'_{c,PFA}$ is calculated. In Fig. 8-5, the hollow squares with error bars are the measured Casimir force in shallow trench geometry divided by the “experimental PFA”. The measured force gradient clearly deviates the PFA prediction by up to 15%. The solid line is the exact Casimir force calculated for shallow trench geometry divided by the “theoretical PFA”. The theoretical calculation, including material properties, yields a good agreement with measurements. If we replace the two surfaces with perfect conducting surface, as represented by the dash line in Fig. 8-5, the deviation of the calculated force exceeds the measured value.
Figure 8-5. The ratio $\rho_C$ of the exact Casimir force gradient in trench geometry to the PFA prediction for sample C, where the PFA prediction is calculated using Eq. 8–15. The hollow squares are the measured force gradient divided by the “experimental” PFA. The solid line is the theoretical calculation in the exact geometry to the theoretical PFA at zero temperature. The dashed line is a linear interpolation between the two theoretical values (solid circles) assuming perfect conductors.

by a factor of 2. This result demonstrate the non-trivial interplay between the material dependence and the geometry dependence of the Casimir force.

8.3 Determination of Errors

To estimate the theoretical precision and the experimental precision, the errors originated from different sources are analyzed in this section.

8.3.1 Determination of the Experimental Errors

Random errors. In the measurements, the random errors arise mainly from thermomechanical fluctuations of the oscillator. At the closest separation $z = 150$ nm, with the average of 10 measurements, the random error $\Delta_R = 3.5$ pN/µm and the relative random error $\delta_R = 2\%$. At the separation $z = 300$ nm, with the average of
50 measurements, the random error $\Delta R = 0.3 \text{ pN}/\mu \text{m}$ and the relative random error $\delta_R = 2\%$.

**Systematic errors.** There are three main sources of systematic error:

$$\delta R = \frac{\Delta R}{R}$$

(8–16)

$$\delta C = \frac{\Delta C}{C}$$

(8–17)

$$\delta z_0 = 3\frac{\Delta z_0}{z_0}.$$  

(8–18)

In our experiments, the value of $R$ was taken from the manufacture specification instead of actual measurements. There can be up to 10\% error resulting from the uncertainty of the determination of $R$. However, in our comparison of experiment and theory, the Casimir force has been normalized by the PFA prediction. For “experimental PFA”, the force gradient between a sphere and a plane is measured using the same sphere as the measurements of trench structures. Thus, the radius $R$ is ruled out from the division and no longer contributes to the error. From the electrostatic calibration, $\delta C \approx 0.6\%$. At the closest distance $z = 150 \text{ nm}$, $\delta z_0 \approx 1\%$.

### 8.3.2 Determination of the Theoretical Errors

**Numerical errors.** In the computation of the Casimir force, numerical errors can originate from multiple sources. Cited from Ref. [61], an example of numerical errors in the deep trench calculation is presented. There are three main error sources. One is the termination of the Matsubara frequency summation, which is mainly determined by the minimum separation between two test bodies. In Davids’ calculation, 36 Matsubara frequencies were used yielding a convergence of better than $10^{-4}$ at the separation of 100 nm. Another error source is the truncation of the discrete spatial frequency spectrum (diffraction orders) resulting in finite dimensional reflection matrices. Increasing the diffraction order $N$ will improve the modal approach, but also increase the computational
cost. Using $N = 5$, the convergence is at $\sim 1\%$ accuracy. The last error source is the numerical integration over the continuous wavevector in the first Brillouin zone, the relative error of which is less than $3\%$ for our experimental separation range. Thus, the estimated total error is less than $3\%$ for the separation range in our experiments.

**Error from the description of the material.** To include the finite conductivity corrections into the Casimir force calculation, the dielectric permittivities along the imaginary axis $\epsilon(i\xi)$ are used to describe the materials. So far, the actual optical properties have never been measured in each individual experiment for sufficient large range of frequencies. The $\epsilon(i\xi)$ is obtained from different models, such as plasma model [62], Drude Model [63] and generalized Drude or plasma model [64], resulting in an error in the theoretical computations. Based on our calculation between a gold sphere and a p-doped silicon plate (Sec. 7.2), the difference resulting from using plasma model or tabulated data for gold is $2\%$, $0.3\%$ and $0.2\%$ at 150, 300 and 500 nm respectively. Thus, we estimate the upper bound of the errors resulting from the model of the optical properties to be $2\%$.

**Error from proximity force theorem.** In the theoretical calculations of the trench geometry, the Casimir force gradient in sphere-trench geometry is calculated from the Casimir force plate-trench geometry using PFA $F'_{\text{sphere-trench}} = 2\pi RF'_{\text{plane-trench}}$. The approximation is valid at $R \gg z$, where $R$ is the radius of the sphere and $z$ is the separation between the test bodies. There have been a number of theoretical calculations to study the Casimir force between a sphere and a plane beyond PFA [62, 65]. To specify the accuracy of PFA, we consider the correction factor

$$
\rho_p-s = \frac{F_{p-s}}{F_{PFA}}.
$$

(8–19)

For small values of $(L/R)$, the expression of $\rho_{p-s}$ is assumed to be of the form [65–67]

$$
\rho_{p-s} = 1 - \beta \frac{z}{R} + O\left(\frac{z^2}{R^2}\right),
$$

(8–20)
where $\beta = \frac{5}{\pi^2} - \frac{1}{3}$. Thus, we consider the upper limit of error introduced by PFA is $z/R$. In our experiments, the radius $R = 51.5 \mu m$ and $z$ ranges from 150 nm to 500 nm so that the error is less than 1%.

**Error from the surface roughness.** In the comparison of experimental data and theory for trench structures, we did not take into account the contribution of the surface roughness correction. We estimate the error introduced by the surface roughness based on the roughness correction to the Casimir force between a sphere and a flat surface. From the calculation in Sec. 7.3, the error should be less than 1% in the measurement range.
CHAPTER 9
SUMMARY

In this dissertation the Casimir force is measured between a gold spherical surface and a silicon nanostructured surface, allowing for the investigation of the geometry and the finite conductivity effects of the Casimir force.

9.1 Summary of the Chapters

In Chapter 1, we introduced the Casimir force and developments in the Casimir force measurements. Two important properties of the Casimir force are its strong shape dependence and material dependence. While the material dependence has been widely studied in smooth geometries, few attempts have been made to demonstrate the shape dependence experimentally. The main focus of this dissertation was to experimentally investigate the strong geometry dependence of the Casimir force for real materials.

In Chapter 2, we proposed the experimental approach of replacing the flat plate with an artificially deformed surface for the well developed sphere-plate setup to reveal the strong geometry dependence. The deformed surface is introduced as a plate with an array of nano-scale periodic rectangular trenches. The dimensions of the trenches are chosen based on Büscher and Emig's theoretical predictions to exhibit significant deviation from the pairwise additive approximation and the proximity force approximation. Trenches with two depths were selected: high aspect ratio trenches with a depth of 1 μm and shallow trenches with a depth of 100 nm.

The detailed sample fabrication procedure was discussed in Chapter 3. The spherical gold surface is made from a 100 μm diameter glass sphere coated with a 5 nm layer of titanium followed by a 400 nm layer of gold using RF sputtering deposition, which provided the smoothest gold surface that we could achieve. The silicon samples with trench arrays were fabricated on a highly doped silicon wafer covered with a resist pattern and were provided by Bell Labs. Two different dry etch approaches were used. The deep reactive ion etch with a simultaneous etch/passivation recipe was used to
create high aspect ratio trenches. A reactive ion etcher with a inductively coupled plasma module was used to create shallow trenches. No passivation gas was used in the etching process of shallow trenches in order to minimize the rounding issue at the bottom of the trenches. Note that the silicon surface is very reactive in air, a quick hydrofluoric acid etching is necessary to remove the native oxide on the silicon surface and lead to hydrogen termination of the surface.

In Chapter 4 we first introduced MEMS, in particular, the microelectromechanical torsional oscillator used in our measurements which is fabricated using the PolyMUMPS process by MEMSCAP. The oscillator was shown to be able to provide sensitive force detection based on capacitive measurements. Then, the experimental setup was presented. This is followed by a description of the detection scheme. By utilizing the electrical circuit as described, the force gradient can be measured from the shift in the resonant frequency of the oscillator.

In order to calculate the electrostatic force and the Casimir force on the trench arrays, it is crucial to obtain accurate sample dimensions. A detailed sample characterization was presented in Chapter 5. AFM and SEM are two main technique that have been used. Different approaches were used to characterize the high aspect ratio trenches and shallow trenches to achieve the best results.

In Chapter 6 both the electrostatic force and the Casimir force measurements were discussed. The electrostatic force measurements were essential in the Casimir force measurements. This is because we use the electrostatic force to calibrate the measurement system. Three parameters can be obtained from the electrostatic force measurements: the residual voltage resulting from the work function difference between the interacting surfaces, the proportionality constant $C$ between the force gradient and the measured resonant frequency shift, and the initial separation $z_0$ between the two surfaces. While the electrostatic force between a sphere and a plate can be calculated from the analytic expression, a numerical approach was needed to calculate the force.
between the sphere and the trench arrays. Following this, the detail of the Casimir force measurements were presented.

In Chapter 7 the measured Casimir force gradient between a gold sphere and a silicon plate was compared with the theoretical calculation including the corrections due to real surfaces. In our calculation, the finite conductivity was taken into account using the Lifshitz formula. The roughness correction were estimated by averaging the Casimir force at different possible separation distances on the two surfaces resulting from the surface roughness, yielding a correction of 0.73% at 150 nm separation. In addition, since our measurements were performed at room temperature, the correction due to finite temperature was also discussed, which is less than 1% in our measurement range.

In Chapter 8 we discussed the comparison of the measurements and the theory for sphere-grating geometry. The measured force gradient on trench arrays deviates from the PFA and PAA by ∼10%, ∼20% and ∼10% for sample A, B and C respectively. The observed deviation agrees with the theoretical calculations based on scattering theory including the finite conductivity correction, but smaller than the prediction for perfect metals. The errors originated from different sources both in measurements and in theoretical calculations were also analyzed in this chapter.

9.2 Future Experiments

The results of this dissertation laid the foundation for future experiments to investigate samples with different structures and materials to provide a complete understanding of the geometry and the material dependence. In particular, metamaterials, which are artificial microstructured materials with designed properties, have been theoretically proposed as a good candidate to provide significant modulation of the amplitude and even the sign of the Casimir force. It is of great interest to generate repulsive Casimir forces since such force can be useful in MEMS, such as preventing stiction between moveable components. It has been shown that a perfect lens (made of left-hand material with \( \varepsilon = \mu = -1 \) over a broad range of frequencies) sandwiched
between two mirrors can lead to a repulsive Casimir force between the mirrors [68]. In addition, Rosa et al. discussed the pursuit of repulsive Casimir force via the configuration of a metallic half space and a metallic metamaterial [69]. There have been a number of theoretic researches on the Casimir force involving metamaterials [70, 71]. However, Casimir force experiments on these structures are still challenging in practice. Lifshitz formula for the Casimir force needs well defined permittivities and permeabilities. For the theoretical work on metamaterials, $\varepsilon$ and $\mu$ are obtained using effective-medium approximation. This requires the field wavelengths, which contributes significantly to the force, to be much larger than the structure dimension of the metamaterial. Currently, the Casimir force can be accurately measured at the separation less than 500 nm, which means that the metamaterial unit cell needs to be much smaller so that the theories predicting repulsive forces are valid. Fabricating such structures at lengths scale of $< 100$ nm for the Casimir force measurements is a challenging goal.

9.3 Conclusion

In conclusion, we measured the Casimir force gradient between a gold spherical surface and a heavily doped silicon plate with an array of nano-scale, rectangular trenches using a MEMS-based technique. Significant deviations of up to 20% from the PFA and the PAA were observed. Recently theoretical advances allow the Casimir force to be accurately calculated on nanostructured surfaces for real materials. The good agreement of the experimental results and the theoretical calculations demonstrate the strong geometry dependence of the Casimir force. In addition, our results demonstrate the controlling of the Casimir force through the interplay between the geometry and the finite conductivity effect, which can lead to a variety of applications in the nanotechnology.
APPENDIX A
FABRICATION PROCEDURE FOR GOLD COATED SPHERES

1. Place a piece of removable double side tape on a glass side and pour a small amount of spheres on the tape.

2. Load the sample into the sputtering chamber (Kurt J. Lesker CMS-18 Multi-Target Sputter Deposition system) and perform the cleaning process. The process includes a 5 sec argon plasma clean followed by a 20 sec oxygen plasma clean. The recipe is shown in Table A-1.

Table A-1. The recipe parameters of $O_2$ plasma and $Ar$ plasma for sputtering system. It is used to clean the glass spheres before the deposition.

<table>
<thead>
<tr>
<th>process</th>
<th>Gas</th>
<th>Power</th>
<th>pressure</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar clean</td>
<td>Ar</td>
<td>100 W</td>
<td>10 mT</td>
<td>5 s</td>
</tr>
<tr>
<td>$O_2$ clean</td>
<td>$O_2$</td>
<td>200 W</td>
<td>10 mT</td>
<td>20 s</td>
</tr>
</tbody>
</table>

3. Deposition. In the same chamber, without removing the sample from vacuum, deposit a 5 nm layer of titanium followed by a 400 nm layer of gold on the spheres. The recipe is listed in Table A-2. The sample holder continuously rotates during the deposition to give a uniform coverage on the side of the spheres.

Table A-2. The recipe parameters of gold deposition and titanium deposition.

<table>
<thead>
<tr>
<th>process</th>
<th>Target</th>
<th>Power</th>
<th>pressure</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti deposition</td>
<td>Ti at gun2</td>
<td>200 W DC</td>
<td>5 mT</td>
<td>50 s</td>
</tr>
<tr>
<td>Au deposition</td>
<td>Au at gun1</td>
<td>100 W RF</td>
<td>3 mT</td>
<td>4000 s</td>
</tr>
</tbody>
</table>

4. Unload sample.
APPENDIX B
GENERAL PREPARE PROCEDURE FOR SILICON SAMPLE

The following procedures are performed on the silicon wafers with silicon oxide etch mask provided by our collaborators at Bell Labs.

1. Pre-etch clean
   (a) Clean the sample in Acetone to strip away the photoresist. Soak the sample in acetone for 5 minutes, then transfer it into IPA and blow dry with \(N_2\).
   (b) Piranha clean to remove organic compounds from substrates. The sample is soaked in a mixture solution of 3:1 concentrated sulfuric acid to 30% hydrogen peroxide at 110 \(^{\circ}\)C for 15 minutes. Then the sample is rinsed using DI water for 15 minutes and finally blown dry using \(N_2\).
   (c) After a quick etch using 1:1000 HF to DI for 30 s, the sample is rinsed using DI water for 15 minutes and blown dry using \(N_2\). It is then baked at 120 \(^{\circ}\)C for 15 minutes. The quick etch is to remove the thin oxide layer that builds on silicon substrate without damaging the oxide mask. The baking is to eliminate the residual water on the substrate.

2. Etching
   (a) DRIE (Deep Reactive Ion Etch system from STS) is used for high aspect ratio trenches etch.
   (b) RIE (Unaxis ICP-RIE etcher) is used for shallow trench etch.

3. Post-etch
   (a) SEM (Scanning Electron Microscope) and AFM (Atomic Force Microscope) are used to obtain necessary data files for sample profile analysis.
   (b) The sample is coated with photoresist 1813. The sample is spun at 500 rpm for 5 s to dispense photoresist and 3500 rpm for 45 s for coating. The sample is then baked on a hotplate at 115 \(^{\circ}\)C for 1 minutes. The purpose of the photoresist is to protect the sample surface and prepare it for dicing. This step is usually taken after SEM and AFM images are taken to avoid additional sample cleaning steps.
   (c) Sample is diced into 0.7 mm by 0.7 mm pieces for the force measurements.
   (d) Strip the photoresist on diced samples. Samples are first rinsed using DI water. After being sprayed by Acetone, the samples are soaked in acetone for 5 minutes. After all the photoresist have been removed, samples are transferred into IPA and dried by \(N_2\).
(e) An $O_2$ dry etch using Unaxis ICP etcher is performed to remove possible organic compounds. The etching recipe is shown in Table B-1.

Table B-1. The $O_2$ etching recipe parameter for Unaxis ICP etcher. It is used to remove organic residual compounds on the sample.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>RF1</th>
<th>RF2</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_2$ Pressure</td>
<td>60 sccm</td>
<td>10 mT</td>
<td>100 W</td>
<td>300 W</td>
</tr>
</tbody>
</table>

(f) Right before the force measurements, the diced sample is soaked in 1:100 HF to DI for 2 minutes to get rid of all possible native oxide on the Si sample. After etching, the sample is rinsed using DI water for 15 minutes and blown dry using $N_2$.

(g) Glue the sample onto an Aluminum holder using silver epoxy and bake it at 120$^\circ$C for 30 minutes. The purpose of baking is to cure the silver epoxy and eliminate the possible water residual in trench array.

(h) Load sample, align and pump down the chamber. Since silicon surface is very reactive in air, the time between HF etch and pumping down is usually limited to 3 hours.
APPENDIX C
ETCHING PROCEDURE FOR SAMPLES WITH DEEP TRENCH ARRAYS

The samples with deep trench arrays are etched by DRIE (deep reactive ion etch system from STS) using a continuous etch recipe. The advantages of using DRIE are that the passivate gas protects the side wall which provides a deep and straight sidewall and that etching in the system consumes very little oxide mask which will ensure the pattern profile. The disadvantage is that the bottom of the trenches are rounded due to passivation.

1. Glue the sample onto a 4 inches Si wafer (carrier wafer). The DRIE etcher in UFNF only accept 4 inches wafer while the sample we used is about 1 cm square. Cover the carrier wafer with Kapton tape and only leave a opening for the sample. Glue the sample using a heat conductive paste to ensure the helium flow underneath the carrier wafer will cool the sample during the etch. Make sure no glue paste will be exposed to etching gas during the etch.

2. Load a blank wafer into the DRIE etcher. Perform a oxygen etch to clean the chamber, as shown in Table C-1. Then, run etch recipe on the blank wafer to verify that the etcher functions properly.

Table C-1. The $O_2$ etching recipe parameters for STS DRIE. It is used to clean the chamber before real sample etch.

<table>
<thead>
<tr>
<th>$O_2$ Pressure</th>
<th>RF1</th>
<th>RF2</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 sccm</td>
<td>40 mT</td>
<td>15 W</td>
<td>700 W</td>
</tr>
</tbody>
</table>

3. Unload the blank wafer and load the wafer with sample. Run the etch recipe as shown in Table C-2

Table C-2. The recipe parameters for DRIE. It it used to create deep trenches on silicon wafer.

<table>
<thead>
<tr>
<th>Sample</th>
<th>period</th>
<th>$C_4F_8$</th>
<th>$SF_6$</th>
<th>RF1</th>
<th>RF2</th>
<th>Pressure</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>sample A</td>
<td>500 nm</td>
<td>90 sccm</td>
<td>45 sccm</td>
<td>30 W</td>
<td>700 W</td>
<td>10 mT</td>
<td>140 s</td>
</tr>
<tr>
<td>sample B</td>
<td>200 nm</td>
<td>90 sccm</td>
<td>45 sccm</td>
<td>30 W</td>
<td>700 W</td>
<td>10 mT</td>
<td>160 s</td>
</tr>
</tbody>
</table>

4. Remove the sample from the carrier wafer. Spray the sample with Acetone and soak the sample in Acetone for 5 minutes to remove the backside glue. After Acetone, transfer sample to IPA and then blown dry using $N_2$ gas.

5. $O_2$ clean to remove possible polymer layer produced from the passivation during DRIE etch using Unaxis ICP (Table B-1).
6. Remove oxide mask using 1:10 HF to DI for 4 minutes.
ETCHING PROCEDURE FOR SAMPLES WITH SHALLOW TRENCH ARRAYS

The samples with shallow trench arrays are etched by RIE etcher (RIE etch system from Unaxis). The rounding bottom problem from DRIE becomes non-negligible for shallow trenches. The recipe used for RIE mainly is physical milling from ions. Such a recipe provides a flat bottom, but consume the etching mask much faster.

1. Load a clean wafer and run a chamber clean recipe (Table D-1). $O_2$ gas is used to etch all possible organic compounds while $Ar$ gas is used to provide some physical milling on residual particles.

<table>
<thead>
<tr>
<th></th>
<th>$O_2$</th>
<th>Ar</th>
<th>Pressure</th>
<th>RF1</th>
<th>RF2</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>sccm</td>
<td>60</td>
<td>5</td>
<td>10 mT</td>
<td>100</td>
<td>300</td>
<td>15 min</td>
</tr>
</tbody>
</table>

2. Run etching recipe on blank wafer for testing.

3. Unload blank wafer and reload it with sample. The sample is just set on the blank wafer without any glue.

4. Run etching recipe on the sample (Table D-2).

<table>
<thead>
<tr>
<th></th>
<th>Sample</th>
<th>period</th>
<th>$SF_6$</th>
<th>Ar</th>
<th>Pressure</th>
<th>RF1</th>
<th>RF2</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>200 nm</td>
<td>3 sccm</td>
<td>6 sccm</td>
<td>1 mT</td>
<td>200</td>
<td>600</td>
<td>22s</td>
</tr>
</tbody>
</table>

5. Remove oxide mask using 1:10 HF to DI for 4 minutes.
APPENDIX E
MATLAB CODE USED IN COMSOL

The following code calculate the electrostatic energy per unit area on shallow trench structure (sample C) in COMSOL using matlab language.

```matlab
% COMSOL Multiphysics Model M-file
% Generated by COMSOL 3.5a (COMSOL 3.5.0.603, $Date: 2008/12/03 17:02:19 $)
clear all
n=0;
for i=2E-6:50e-9:2e-6
    fprintf('n=
end

% COMSOL version
clear vrsn
vrsn.name = 'COMSOL 3.5';
vrsn.ext = 'a';
vrsn.major = 0;
vrsn.build = 603;
vrsn.rcs = '$Name: $';
vrsn.date = '$Date: 2008/12/03 17:02:19 $';
fem.version = vrsn;

% Geometry
a=1;
g1=rect2(400e-9,a,'base','corner','pos',{
    '0',
    '0',
    '0'}),
    'rot',
    '0');
g2=rect2(199.26e-9,98e-9,...
    'base','corner','pos',{
    '100.42e-9',
    '-98e-9'},
    'rot',
    '0');
g3=geomcoerce('solid',{g1,g2});
gg=geomedit(g3);
gg(4)=beziercurve2([1.0042E-7,9.266E-8],[-1.0042E-7,9.266E-8],[1,1]);
g4=geomedit(g3,gg);
gg=geomedit(g4);
gg(8)=beziercurve2([2.9958E-7,3.0734E-7],[-9.8E-8,0],[1,1]);
g5=geomedit(g4,gg);

%geomplot(g5)
% Geometry objects
clear s
s.objs={g5};
s.name={'CO1'};
s.tags={'g5'};

fem.draw=struct('s',s);
fem.geom=geomcsg(fem);

% (Default values are not included)
% Initialize mesh
fem.mesh=meshinit(fem,'hmax',0.04e-7)

% Refine mesh
%fem.mesh=meshrefine(fem,...
%   'mcase',0,...
%   'rmethod','regular');
```
Refine mesh
fem.mesh = meshrefine(fem, ...
    'mcase', 0, ...
    'rmethod', 'regular');

Refine mesh
fem.mesh = meshrefine(fem, ...
    'mcase', 0, ...
    'rmethod', 'regular');

figure, meshplot(fem), axis equal

Application mode 1
set boundary condition
clear appl
appl.mode.class = 'Electrostatics';
appl.assignsuffix = '_es';
clear bnd
bnd.V0 = [0, 0, 0, 1];
bnd.name = {'plate', '', 'gap', 'trench'};
bnd.type = {'V0', 'cont', 'nD0', 'V'};
bnd.ind = [3, 4, 1, 4, 2, 4, 2, 4, 2, 4, 3];
appl.bnd = bnd;
fem.appl{1} = appl;
fem.frame = {'ref'};
fem.border = 1;
clear units;
units.basesystem = 'SI';
fem.units = units;

ODE Settings
clear ode
clear units;
units.basesystem = 'SI';
ode.units = units;
fem.ode = ode;

Multiphysics
fem = multiphysics(fem);

Extend mesh
fem.xmesh = meshextend(fem);

Solve problem
fem.sol = femstatic(fem, ...
    'solcomp', {'V'}, ...
    'outcomp', {'V'}, ...
    'blocksize', 'auto');

Save current fem structure for restart purposes
fem0 = fem;

Plot solution
% postplot(fem, ...
% 'tridata', {'V', 'cont', 'internal', 'unit', 'V'}, ...
% 'trimap', 'Rainbow', ...
% 'title', 'Surface: Electric potential [V]', ...
% 'axis', [-1.2371541501976275E-7, 5.237154150197628E-7, -1.5E-7, 2.0E-7]);

% Integrate
l1=postint(fem,'We_es', ...
    'unit', 'N', ...
    'recover', 'off', ...
    'dl', [1, 2])
    n=n+1;
    Energy(n)=l1;
    g(n)=a;
    makef_output='energy_d100w200.txt'

    output_unit = fopen ( makef_output , 'at' );

    fprintf ( output_unit , '%e %e
' , l1 , a );
    fclose ( output_unit );

end
dat=[Energy; g];
The following matlab codes calculate the Casimir Force using Lifshitz Formula.

```matlab
function make_f_two(astart, astep, astop)

%astart = 60e-9;
%astep = 10e-9;
%astop = 60e-9;

hbar = 1.055e-34;
c = 3e8;

for aa = astart:astep:astop
    f = hbar/(2*pi^2*c^3) * int_p_two(aa);
    fp = fopen('sif1.dat', 'a');
    fprintf(fp, '%e %t %e\n', aa, f);
    fclose(fp);
end

function [output] = int_p_two(a)

    uplimit = 2000;
    p = 1;
    total = 0;

    while (p<uplimit)
        dp = p/100;
        total = total + int_x_two(a, p) * p^2 * dp;
        p = p + dp
    end
    output = total;

function [output] = int_x_two(a, p)

    load('au_ei.dat');
    xx1 = au_ei(:,1);
    yy1 = au_ei(:,2);
    clear au_ei;

    load('si_ei.dat');
    xx2 = si_ei(:,1);
    yy2 = si_ei(:,2);
    clear si_ei;
```
```matlab
% Given values
c = 3e8;

% Calculations
step = mean(diff(xx1));
[m,n] = size(xx1);
total = 0;

for nn = 1:m
    y1 = yy1(nn);
    x1 = xx1(nn);
    y2 = yy2(nn);
    x2 = xx2(nn);

    s1 = sqrt(y1 - 1 + p^2);
    s2 = sqrt(y2 - 1 + p^2);

    term1 = ((s1 + p)*(s2 + p))/((s1 - p)*(s2 - p)) * exp((2*p*a/c)*x1-1);
    term2 = ((s1 + y1 * p)*(s2 + y2 * p))/((s1 - y1 * p)*(s2 - y2 * p)) * exp((2*p*a/c)*integrand = x1^3* (1/(term1) + 1/(term2));
    total = total + integrand * step;
end

output = total;
```

REFERENCES


[38] F. Lu, Introduction to Deep reactive ion etching (Applied Quantum Technologies, Duke University, 2008).


BIOGRAPHICAL SKETCH

Yiliang Bao was born in Jilin in the northeast of China, but moved to Shanghai, when she was a kid. She spent most of her childhood and adolescence in Shanghai and graduated from Fudan University with Bachelor's degree in science in 2003. In her senior year, she decided to apply for graduate school to pursue her PhD degree as well as to expand her career of scientific research. She was accepted by the Department of Physics at the University of Florida with teaching assistantship in 2003. In the summer of 2004, she joined HoBun Chans research group and began working with him on the Casimir force measurements using microelectromechanical devices. This work demonstrated the strong shape and material dependence of the Casimir force. After seven years of research, she graduated from the University of Florida with a Doctore of Philosophy in physics.