TRANSCEIVER DESIGN AND SIGNAL PROCESSING FOR UWB SIGNALS

By

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I dedicate this to my parents, my sister and my wife
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TRANSCEIVER DESIGN AND SIGNAL PROCESSING FOR UWB SIGNALS

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Due to their huge bandwidth, ultra-wideband (UWB) systems can guarantee both
the large channel capacity and the high timing resolution. For this reason, we are
interested in the transceiver design and timing synchronization for UWB systems.
With the results presented in this dissertation, we can build UWB systems for various
applications.

We will first present the multi-carrier differential (MCD) technique for the UWB
impulse radio (IR). MCD-UWB can bypass the explicit channel estimation without
the need for analog delay elements and simultaneously enable variable data rates.
In MCD-UWB and also in other UWB systems, we have observed that heterogeneity
emerges between UWB transmitters and receivers at different levels of complexity. Our
analysis reveals that this kind of asymmetric UWB link can be model as a multiple-input
and multiple-output (MIMO) system. Then, we can apply existing multiantenna
techniques to our design for better performance or lower complexity.

As synchronization is indispensable for wireless communication systems, we also
considered the timing issue for UWB-based systems. In particular, we investigated
and evaluated timing with dirty template (TDT) algorithms for digital IR receivers
with low-resolution analog-to-digital converters (ADCs) and IRs with the orthogonal
bi-pulse modulation. Analysis shows that for digital IRs, the ADC resolution has very
small impact on the synchronization performance. For the bi-pulse UWB, the pulse
orthogonality enables a significant enhancement of the synchronization speed of TDT when no training sequence is transmitted.

We will further investigate the non-coherent combining and coherent combining time-of-arrival (ToA) estimation methods for the multi-band orthogonal frequency division multiplexing (MB-OFDM) UWB system. For the non-coherent combining based energy detection technique, the diversity gain of the timing performance increases as the number of signal bands increases. For the coherent combining, we proposed a new criterion that the ToA is estimated by suppressing the energy leakage in the channel estimate taps prior to the first path. Simulations have confirmed that high ToA estimation accuracy can be achieved by this criterion.
CHAPTER 1
INTRODUCTION

According to the FCC’s definition, ultra-wideband (UWB) refers to a wireless technology that employs a bandwidth larger than 500 MHz or 20% of the center frequency. The UWB spectrum can be accessed either by the generation of a series of extremely short duration pulses or by the aggregation of a number of narrowband subcarriers having the bandwidth over 500 MHz. Shannon’s channel capacity theorem proves that the huge bandwidth of UWB can guarantee a large channel capacity without invoking a high transmit power. Therefore, the spectrum occupied by existing technologies can be used by UWB without causing harmful interference. This motivates us to utilize UWB technology for information transmission since the spectrum resource is becoming scarce.

In this dissertation, we investigate UWB transceiver design techniques and apply signal processing methods to facilitate the design. We incorporate the differential multi-carrier modulation technique to realize a high-efficiency low-complexity digital impulse radio (IR) UWB. Then we investigate the asymmetry and heterogeneity that widely exist in both our digital IR UWB and other UWB-based systems. We derive an effective modeling method of these asymmetric links which makes it very convenient to optimize the system performance. As timing synchronization is indispensable for wireless communications, we also investigate the timing issue of UWB signals to facilitate our transceiver design. We analyze and evaluate the timing with dirty template (TDT) algorithms in two application scenarios. First, we prove the feasibility of TDT algorithms for digital IR receivers with low-resolution (2- or 3-bit) analog-to-digital converters (ADC). Then, we extend TDT which was original proposed for single-pulse UWB system to IRs with orthogonal bi-pulse modulation. Analysis and simulations show that due to the orthogonality of pulses, the timing speed of TDT algorithms can be enhanced. We will also exploit the unique feature of UWB signals – the high precision...
timing capability. Especially, we will investigate the time-of-arrival (ToA) estimation technique for UWB signals. With the results of our research, we are ready to build UWB systems for various applications.

1.1 Digital Multi-Carrier Differential (MCD) Modulation Impulse Radio (IR) Transceivers

For UWB IRs, RAKE reception is often referred to as the optimal coherent detector, provided that the channel information is available at the receiver. However, it is very challenging to estimate the extremely rich multipath of the UWB channel. To bypass the explicit channel estimation, semi-coherent approaches including the transmitted-reference (TR) (see e.g., [10, 24, 34, 75]) and differential (see e.g., [31–33]) systems as well as noncoherent techniques [115] have been proposed. These approaches result in simple transceiver structures. However, the required ultra-wideband analog delay element may be difficult to implement at the IC level.

To amend this problem, frequency-shifted reference (FSR) UWB was recently proposed [26, 118], where the reference and information-conveying signals are transmitted simultaneously on two orthogonal frequency tones to avoid the analog delay line. However, FSR-UWB still induces severe energy loss by allocating equal energies on the information-conveying tone and the reference tone. In addition, FSR-UWB was developed under the assumption that the inter-frame interference (IFI) and inter-symbol interference (ISI) are negligible [25, 26]. This can limit the data rate of the UWB system. Moreover, FSR-UWB modulates the transmitted UWB pulses with analog carriers, which induce bandwidth expansion and can give rise to frequency offset by having mismatched oscillators at the transmitter and receiver.

To address these limitations, we introduce digital multi-carrier differential (MCD) signaling schemes for UWB systems [103]. Different from exiting multi-carrier UWB techniques that use coherent detectors [21, 94, 111], our MCD-UWB bypasses the channel estimation via frequency-domain differential (de-)modulation. Inspired by the
FSR-UWB schemes, MCD-UWB avoids the analog delay line. However, our approach outperforms the FSR-UWB by considerably reducing energy loss, and the bandwidth expansion induced by the analog-carrier modulation. By allowing for (possibly severe) IFI and ISI, the data rates of MCD-UWB can be further boosted. The MCD-UWB system can be implemented with fast Fourier transform (FFT) and discrete cosine transform (DCT) circuits. It is worth stressing that all these digital processing deals with the discrete time signal sampled at the frame-rate, as opposed to the pulse-rate or Nyquist-rate which can be easily several Gigahertz. Finally, we will also prove that our MCD-UWB can effectively collect the multipath diversity even in the presence of IFI.

1.2 Transceiver Design for Asymmetric Ultra-Wideband (UWB) Links

In our MCD-UWB system and also in other wireless systems, we have observed the asymmetry between the transmitter and the receiver (see, e.g., [4, 46, 103, 121]). For example, for the complex low-rate MCD-UWB (see [103]), the modulation process includes the differential modulation and the multi-carrier modulation with the FFT operation at the transmitter. However, at the receiver, the multi-carrier demodulation and differential demodulation can be carried out only by a single mixing operation, which is much simpler than the modulation process of the transmitter. As a result, the transmitter and receiver can be realized at quite different complexity levels.

This kind of asymmetric and heterogeneity is becoming more common since UWB has been proposed as the physical layer realization for various networks which enable both the high data rate and low data rate transmission (see e.g., [2, 42, 61, 68, 120]). As these systems can provide various services, heterogeneity emerges among network nodes either inside a network or among different ones. To realize the seamless network operation, transceiver design needs to take into account the heterogeneity among these nodes.

For UWB links, heterogeneity and asymmetry can be induced by the different numbers of signal bands for multiband operation, different pulse rates or different pulse
shapers, between the transmitter and the receiver. Among these factors, the first two, i.e., the number of signal bands and the pulse rate will usually determine the complexity of the device. Generally, nodes of high complexity can provide high data rate services such as the multimedia data transfer which needs more power and computational resources. Nodes of low complexity are typically small and rely on limited battery power, such as wireless sensors.

In this dissertation, we investigate the transceiver design for the asymmetric UWB link with a single transmitter and a single receiver [104, 106]. Analysis reveals that the asymmetric link can be represented by a multiple-input and multiple-output (MIMO) system model which has originally been proposed and investigated for the multiple Tx- and Rx-antenna systems. Once the asymmetric UWB link is modeled as an MIMO system, we can apply existing multiantenna communication techniques to our UWB transceiver design for better performance or lower complexity. This is very attractive since many multiantenna techniques are available in the literature which are optimal in terms of system throughput, error rate or complexity, etc (see, e.g., [66, 73, 80]). Then, we will show how these techniques can be integrated into our asymmetric link model. Our analyses, together with the simulations, confirm the feasibility and effectiveness of the modeling and transceiver design for the asymmetric UWB link.

### 1.3 Timing Synchronization of UWB Signals

Timing synchronization is an indispensable technique for both our UWB IRs and general wireless communication systems. The system's error rate performance can be largely dependent on the performance of the timing synchronizer (see e.g., [29, 43, 101]). For this reason, we need to consider the synchronization of signals for UWB-based systems. In particular, we analyze and evaluate TDT [110, 114] algorithms in two application scenarios: digital IR receivers with low-resolution ADCs and IRs with the orthogonal bi-pulse modulation.
Compared to the analog UWB system, the digital UWB has the advantage of more flexible operations which can enable the convenient channel equalization, multiple access, higher order modulation and system optimization (see e.g., [51, 57, 112]), etc. This has also been partially shown by our research on the UWB transceiver design. However, due to the huge bandwidth of UWB signals, the all-digital realization of UWB receivers may only be possible by using very low-resolution (2-bit or 3-bit) ADCs based on the current hardware technology (see e.g., [35, 58, 87]). In order to facilitate rapid synchronization in digital IR receivers with low-resolution ADCs and digital receivers proposed in our research, we adopt TDT algorithms which perform synchronization by correlating two consecutive symbol-long segments of the received signal. For digital IR receivers with high-resolution ADCs, the extension of TDT from analog receivers [110, 114] is straightforward. For digital IR receivers with low-resolution ADCs, the application of TDT needs to be verified since the dominant quantization noise is closely dependent on the analog waveform. In this dissertation, we prove that the digital TDT algorithms remain operational without knowledge of the spreading codes or the multipath propagation channel, even in the presence of both the additive noise and the quantization error. Our simulations show that the resolution of the ADCs has very little effect on the synchronization performance [105].

Besides digital UWB IR receivers, we also exploit the merit of TDT algorithms for the orthogonal bi-pulse modulation UWB system which uses an even pulse and an odd pulse to convey information symbols in an alternating manner [65]. Although proposed for the single-pulse UWB, TDT is proved to be operational for the bi-pulse UWB system. In addition, due to the employment of orthogonal pulses, the bi-pulse based TDT can avoid the random symbol effect of the original non-data-aided (NDA) TDT [114], which was originally accomplished by transmitting training sequence in the data-aided (DA) mode. It is interesting to notice that for the bi-pulse orthogonal UWB IR, the idea of TDT can readily enable a demodulation scheme when information bits are
differentially modulated on adjacent symbols. Similar to TR-UWB and other techniques, our approach remains operational when channel estimation is bypassed. In addition, by using orthogonal pulses, our noncoherent algorithm completely avoids ISI even in the presence of timing errors. As a result, our algorithms only entail simple differential demodulator, while retaining the maximum likelihood (ML) optimality [63–65].

1.4 Time-of-Arrival (ToA) Estimation for Multi-Band Orthogonal Frequency Division Multiplexing (MB-OFDM) UWB

Cramer-Rao bound (CRB) analysis shows that UWB can guarantee a high timing resolution and accordingly the more accurate ranging and location capability, due to its huge bandwidth. This feature has been partially exploited when we apply TDT algorithms to the synchronization of UWB signals [63–65, 105]. In order to further exploit its high resolution capability, we will investigate the ToA estimation techniques for the multi-band orthogonal frequency division multiplexing (MB-OFDM) UWB system. We will first estimate the channel ToA with the noncoherent combining or the so-termed energy detection method. Based on the analysis of the pairwise mistiming probability in simple equally-spaced models, we have obtained meaningful results which show that the timing performance can be improved with multi-band (MB) signals by exploiting the diversity across subbands [108]. Since the analysis is conducted in the general Nakagami-\(m\) channel, the obtained results apply to a wide range of fading environments, such as the Rayleigh distribution (\(m = 1\)) and the one-sided Gaussian distribution (\(m = 0.5\)).

The energy detection-based ToA estimator has assumed that the channel is equally-spaced with a finite number of taps. In the real situation, this assumption may not hold due to the strong energy leakage induced by the missampling of the leading channel paths. This implies that large ToA estimation error could emerge if the estimator erroneously picks up taps that contain the strong energy leakage. For this reason, energy leakage needs to be efficiency suppressed before ToA estimation. Motivated by this, we have designed a new ToA estimation rule that ToA will be estimated by choosing
the optimal channel estimate such that the energy leakage is minimized among the estimated taps prior to the first multipath component. Analysis and simulations have shown that compared to traditional methods, our proposed method will provide a higher precision ToA estimate due to its better convergence [107].

1.5 Dissertation Organization

The organization of this dissertation is as follows. The MCD signaling scheme for UWB systems is introduced in Chapter 2. Transceiver design and system modeling are investigated for asymmetric UWB links in Chapter 3. In Chapter 4, the TDT algorithm is analyzed and evaluated in both digital IR receivers with low-resolution ADCs and IRs with the orthogonal bi-pulse modulation. In Chapter 5, ToA estimation techniques are investigated for the MB-OFDM UWB system. Summarizing remarks and future work are given in Chapter 6.
CHAPTER 2
DIFFERENTIAL UWB COMMUNICATIONS WITH DIGITAL MULTI-CARRIER MODULATION

2.1 Motivation

To collect the ample multipath diversity, RAKE reception is often referred to as the optimal coherent detector, provided that the channel information is available at the receiver. However, the extremely rich multipath of the UWB channel poses challenges in channel estimation and, accordingly, in the realization of the RAKE receiver. To bypass the explicit channel estimation, semi-coherent approaches including the TR-UWB (see e.g., [10, 24, 34, 75]) and differential (see e.g., [31–33]) systems as well as noncoherent techniques [115] have been proposed. These approaches result in simple transceiver structures. However, the required UWB analog delay element may be difficult to implement at the integrated-circuit (IC) level.

To amend this problem, FSR-UWB was recently proposed [26], where the reference and information-conveying signals are transmitted simultaneously on two orthogonal frequency tones. Unlike TR-UWB and existing semi/non-coherent approaches, FSR-UWB does not need the analog delay line. However, similar to TR-UWB which uses half of the total energy on the reference pulses, FSR-UWB induces the energy loss by allocating equal energies on the information-conveying tone and the reference tone. In [118], a multi-differential (MD) FSR-UWB scheme was proposed to improve the data rate of the original FSR-UWB by using a single reference tone together with multiple data tones. However, this approach requires that all carriers locate within the channel coherence bandwidth. This constraint restricts the number of usable carriers and thereby limits the overall data rate. Although FSR-UWB has been shown to be robust to the interframe interference in simulations, it was developed under the assumption that IFI and ISI are negligible [25], [26]. This can limit the data rate of the UWB system. Moreover, (MD) FSR-UWB modulates the transmitted UWB pulses with analog carriers,
which induce bandwidth expansion and can give rise to frequency offset by having mismatched oscillators at the transmitter and receiver.

To address these limitations, we introduce in this dissertation digital MCD signaling schemes for UWB systems. Different from exiting multi-carrier UWB techniques that use coherent detectors [21, 94, 111], our MCD-UWB bypasses the channel estimation via frequency-domain differential (de-)modulation. Inspired by the FSR-UWB schemes, our MCD-UWB avoids the analog delay line. However, our approach outperforms the FSR-UWB by considerably reducing energy loss, and the bandwidth expansion induced by the analog-carrier modulation. Equally attractive is that our MCD-UWB allows for high and variable data rates without increasing the spacing among the reference and data tones. This is to be contrasted with the MD-FSR-UWB where the average spacing between data and reference tones increases with the data rate. By allowing for (possibly severe) IFI and ISI, the data rates of MCD-UWB can be further boosted. Our MCD-UWB systems can be implemented with FFT and DCT circuits. It is worth stressing that all these digital processing deals with the discrete time signal sampled at the frame-rate, as opposed to the pulse-rate or Nyquist-rate which can be easily several Gigahertz (GHz). Finally, we will also prove that our MCD-UWB can effectively collect the multipath diversity even in the presence of IFI.

2.2 Digital Multi-Carrier Transmission Model

In this section, we will introduce the transmitted signal model using multiple digital carriers. We will start from the real carriers and then generalize to the complex case.

2.2.1 Real MCD Modulation

Here, we adopt the \((N_f/2 + 1)\) column vectors \(g_n := [g_n(0), \ldots, g_n(N_f - 1)]^T\), \(n = 0, 1, \ldots, N_f/2\), as our digital carriers

\[
g_n(k) = \begin{cases} \sqrt{\frac{2}{N_f+2}} \cos(2\pi f_n k), & n = 0, \text{ or } n = \frac{N_f}{2} \\ \sqrt{\frac{4}{N_f+2}} \cos(2\pi f_n k), & n \in [1, \frac{N_f}{2} - 1] \end{cases},
\]

(2–1)
where \( N_f \) is the number of frames transmitted during each block period and \( f_n := n/N_f. \)

These real digital carriers are reminiscent of those introduced in [111] to facilitate multiple access in UWB systems. However, instead of the \( N_f \) carriers in [20], we have only \((N_f/2 + 1)\) in (1); that is, the \((N_f/2 - 1)\) sin-based carriers in [20] are dropped here. Intuitively, the sin- and cos-based digital carriers share the same frequency tones with differing polarity. In [111], coherent detection is used to separate these carriers. However, due to the phase ambiguity inherent to differential detectors, the sin and cos carriers become indistinguishable. Hence, we only adopt the \((N_f/2 + 1)\) cosine waveforms. Stacking the \((N_f/2 + 1)\) vectors into a matrix, we have \( \mathbf{G} := [g_0, \ldots, g_{N_f/2}] \).

Evidently, it follows that \( \mathbf{G}^T \mathbf{G} = 2N_f/(N_f + 2)\) by construction.

Consider a block-by-block transmission where, during the \( k \)th block, the real digital carriers are modulated by \((N_f/2 + 1)\) real symbols \( \mathbf{d}_k^R := [d_k^R(0), \ldots, d_k^R(N_f/2)]^T \), with \( d_k^R \) satisfying \( \mathbb{E}\{d_k^R(n)d_k^R(m)\} = \delta_{m,n} \) and the superscript ‘R’ indicating the ‘real’ carrier case. The resultant \( N_f \) signals collected by \( \mathbf{a}_k^R := [a_k^R(0), \ldots, a_k^R(N_f - 1)]^T \) is obtained as \( \mathbf{a}_k^R = \mathbf{Gd}_k^R \). Adopting the widely-accepted notation in the UWB literature, we let each \( a_k^R(n), n \in [0, N_f - 1] \), be transmitted over one frame of duration \( T_f. \) Accordingly, each block consists of \( N_f \) such frames, and has a duration \( T_s = N_f T_f. \) Using the ultra-short UWB pulse shaper \( p(t) \), we obtain the following transmitted signal model:

\[
\mathbf{x}_R(t) = \sqrt{\mathcal{E}_p} \sum_{k=0}^{N_f - 1} \sum_{n=0}^{N_f - 1} a_k^R(n)p(t - kT_s - nT_f),
\]

(2-2)

where \( \mathcal{E}_p \) is the energy per pulse. In the ensuing sections, we will discuss how \( d_k^R(n)\)'s are generated using differential encoding to facilitate variable data rates and how they are differentially demodulated.

### 2.2.2 Complex MCD Modulation

As we discussed before, the maximum number of real carriers is \((N_f/2 + 1)\). Later, we will see that this also dictates the maximum number of distinct symbols transmitted per block. To increase the number of digital carriers, one can resort to the set of \( N_f \)
complex carriers $\{f_n\}_{n=0}^{N_f-1}$ which are simply the columns of the $N_f \times N_f$ FFT matrix $F^H$. During the $k$th block, these digital carriers are modulated by $N_f$ complex symbols $d_k^C := [d_k^C(0), \ldots, d_k^C(N_f-1)]^T$ to generate $N_f$ signals collected by $a_k^C := a_k^r + j a_k^i = F^H d_k^C = [a_k^C(0), \ldots, a_k^C(N_f-1)]^T$, where the superscript ‘C’ indicates the ‘complex’ carrier case. Notice that $a_k^C$ is generally complex. This implies that the carrier-less signal model (2–2) is not directly applicable. But even without an analog carrier, the real and imaginary parts of the vector $a_k^C$ can be transmitted over two consecutive blocks each of duration $T_s$:

$$x^C(t) = \sqrt{E_p} \sum_{k=0}^{\infty} \sum_{n=0}^{N_f-1} a_k^r(n) p(t - 2kT_s - nT_f)$$

$$+ \sqrt{E_p} \sum_{k=0}^{\infty} \sum_{n=0}^{N_f-1} a_k^i(n) p(t - (2k+1)T_s - nT_f).$$

Notice that each of the two summands in (2–3) is essentially the same as (2–2).

Unlike the real MCD that only requires frame-level synchronization, the complex case also entails symbol-level synchronization to locate the starting point of each real-imaginary pair. Using these transmitted signal models, we will next introduce the channel propagation effects and the received signal models.

### 2.3 Channel Effects and Received Signal Model

In the preceding section, we have seen that $a_k^R$, $a_k^r$ and $a_k^i$ share the same transmitted signal model. Therefore, when deriving the received signal in this section we will first consider the generic transmitted block $a_k := [a_k(0), \ldots, a_k(N_f-1)]^T$, and then specify the model for the real and complex cases towards the end of this section. The transmitted signal $x(t) = \sqrt{E_p} \sum_{k=0}^{\infty} \sum_{n=0}^{N_f-1} a_k(n) p(t - kT_s - nT_f)$ [c.f. (2–2) and (2–3)] propagates through the multipath channel with impulse response $\sum_{l=0}^{L-1} \beta(l) \delta(t - \tau(l))$, where $\{\beta(l)\}_{l=0}^{L-1}$ and $\{\tau(l)\}_{l=0}^{L-1}$ are amplitudes and delays of
the $L$ multipath elements, respectively. Then, the received waveform is given by

$$r(t) = \sqrt{E_p} \sum_{k=0}^{N_f-1} \sum_{n=0}^{L-1} a_k(n) h(t - kT_s - nT_f) + \eta(t),$$

where $h(t) = \sum_{l=0}^{L-1} \beta(l)p(t - \tau(l))$ is the composite pulse waveform after the multipath propagation and $\eta(t)$ is the additive white Gaussian noise. We assume $\tau(0) = 0$, which means perfect timing synchronization is achieved at the receiver. It is worth noting that we are not imposing any constraint on the frame duration $T_f$. In other words, to facilitate high data rates, the frame duration is allowed to be less than the channel delay spread ($T_f < \tau(L-1) + T_p$). Notice that in such cases, IFI and ISI emerge.

At the receiver, a bank of $L_c$ correlators are used to collect the multipath energy. Each correlator uses a single delayed pulse $p(t)$ as the template and the correlator output is sampled at the frame-rate. This can be interpreted as the parallel counterpart of the single correlator with a $T_s$-long template in FSR-UWB. Let $\{\tau_c(l)\}_{l=1}^{L_c}$ denote the delays associated with the $L_c$ correlators. As the correlation is carried out on a per frame basis, the correlator delays are upper bounded by the frame duration $T_f$. In addition, they should not exceed the channel delay spread to ensure effective energy collection. Hence, we have $\tau_c(L_c) \leq \max\{\tau(L-1), T_f - T_p\}$.

During the $n$th frame of the $k$th block, the template for the $l$th correlator is the pulse $p(t - kT_s - nT_f - \tau_c(l))$, and the correlator output is

$$y_k(l; n) := \int_{kT_s+nT_f}^{kT_s+nT_f+T_f} r(t) p(t - kT_s - nT_f - \tau_c(l)) dt.$$ 

Denoting the correlation between the template and the received composite impulse waveform $h(t)$ as $\beta_c(l; m) := \int_{mT_f}^{mT_f+T_f} h(t) p(t - mT_f - \tau_c(l)) dt = \sum_{n=0}^{L-1} \beta(n) R_p(\tau(n) - mT_f - \tau_c(l))$, where $R_p(\tau) := \int_0^{T_f} p(t) p(t-\tau) dt$ is the auto-correlation function of $p(t)$, we can re-express the frame-rate samples of the $l$th correlator as

$$y_k(l; n) = \sqrt{E_p} \sum_{m=0}^{n} \beta_c(l, m)a_k(n - m) + \sqrt{E_p} \sum_{m=n+1}^{M_p} \beta_c(l, m)a_{k-1}(N_f - m + n) + \eta_k(l; n),$$

(2–5)
where \( \eta_k(l; n) \) is the noise sample at the correlator output. From (2–5), it follows that 
\[
\{ \beta_c(l, m) \}_{m=0}^{M_l}
\]
can be regarded as the discrete-time equivalent impulse response of the channel. The order of this channel can be determined as 
\[
M_l := \max\{ m : \tau_c(l) + mT_f < \tau(L_c) + T_p \}.
\]
Notice that, as long as \( M_l > 0 \), for any \( l \in [1, \ldots, L_c] \), IFI and ISI are both present. Hereafter, we will let \( M_h := \max_i\{M_l\} \) denote the maximum order of the discrete-time equivalent channel.

Stacking the outputs corresponding to the \( k \)th block from the \( l \)th correlator to form the vector 
\[
y_k(l) := [y_k(l; 0), \ldots, y_k(l; N_f - 1)]^T,
\]
we obtain the input-output (I/O) relationship in a matrix-vector form:
\[
y_k(l) = \sqrt{\mathcal{E}_p} \mathbf{H}_l \mathbf{a}_k + \sqrt{\mathcal{E}_p} \mathbf{H}_l^{(1)} \mathbf{a}_{k-1} + \eta_k(l), \tag{2–6}
\]
where \( \eta_k(l) \) is the noise vector, \( \mathbf{H}_l^{(0)} \) is an \( N_f \times N_f \) lower triangular Toeplitz matrix with the first column being 
\[
[\beta_c(l; 0), \ldots, \beta_c(l; M_l), 0, \ldots, 0]^T,
\]
and \( \mathbf{H}_l^{(1)} \) is an \( N_f \times N_f \) upper triangular Toeplitz matrix with the first row being 
\[
[0, \ldots, 0, \beta_c(l; M_l), \ldots, \beta_c(l; 1)].
\]
Due to the multipath channel effect, \( y_k(l) \) depends on both \( \mathbf{a}_k \) and \( \mathbf{a}_{k-1} \). The IFI and ISI we mentioned before now take the form of inter-block interference (IBI).

To facilitate block-by-block detection, one could use either the cyclic prefix (CP) or padding zeros (ZP) to remove IBI [95]. With the CP option, IBI can be eliminated by inserting a CP of length \( M_h \) at the transmitter and discarding it at the receiver. Correspondingly, the system I/O relationship is given by
\[
\tilde{y}_k(l) = \sqrt{\mathcal{E}_p} \tilde{\mathbf{H}}_l \mathbf{a}_k + \tilde{\eta}_k(l), \tag{2–7}
\]
where the channel matrix \( \tilde{\mathbf{H}}_l \) becomes a column-wise circulant matrix with the first column being 
\[
[\beta_c(l; 0), \ldots, \beta_c(l; M_l), 0, \ldots, 0]^T.
\]

Now, we are ready to specify the received signals for the real and complex carriers. For the real carriers, the received signal on the \( l \)th correlator after CP removal is simply 
\[
y_k^R(l) = \tilde{y}_k(l).
\]
combined ones: \( y_k^C(l) = \tilde{y}_{2k}(l) + j\tilde{y}_{2k+1}(l) \). From (2–7), it follows that their respective I/O relationships are:

\[
\begin{align*}
y_k^R(l) &= \sqrt{\varepsilon_p} \tilde{H}_l a_k^R + \eta_k^R(l), \\
y_k^C(l) &= \sqrt{\varepsilon_p} \tilde{H}_l a_k^C + \eta_k^C(l),
\end{align*}
\]

(2–8)

where \( \eta_k^R(l) \) and \( \eta_k^C(l) \) are the real and complex frame-rate noise samples, respectively.

As IBI is eliminated, we will drop the block index \( k \) hereafter.

The \( N_f \times N_f \) circulant matrix \( \tilde{H}_l \) can be diagonalized by pre- and post-multiplication with the \( N_f \)-point FFT and inverse fast Fourier transform (IFFT) matrices; that is,

\[
F \tilde{H}_l F^H = D_H := \text{diag}\{H_l(0), \ldots, H_l(N_f-1)\},
\]

where \( H_l(n) = \sum_{m=0}^{M_l} \beta_c(l, m) \exp(-j2\pi nm/N_f) \) is the frequency response of the equivalent channel. The measurement results of UWB channels in [23] and [39] show that the root-mean-square (rms) delay spread \( \sigma_{\text{rms}} \) is at the level of 5ns. Accordingly, the coherence bandwidth is about \( 1/(5\sigma_{\text{rms}}) \approx 40\text{MHz} \) [74].

Considering a UWB system with \( N_f = 32 \) and \( T_f = 24\text{ns} \), with which our simulations are carried out, the carrier spacing \( 1/(N_fT_f) \approx 1.3\text{MHz} \) is much smaller than the channel coherence bandwidth. Therefore, it holds that \( H_l(n) \approx H_l(n+1), n \in [0, \ldots, N_f-2] \). In other words, the channel for adjacent carriers are approximately identical.

Denoting the complex multi-carrier demodulation results of the frame-rate sampled output for the \( l \)th correlator by \( v^C(l) := F y^C(l) = [v^C(l; 0), \ldots, v^C(l; N_f-1)]^T \) and applying FFT on both sides of (2–7), we obtain the system I/O relationship as

\[
v^C(l) = F y^C(l) = \sqrt{\varepsilon_p} D_H(l) d^C + \zeta^C(l),
\]

(2–9)

where \( \zeta^C(l) \) is the noise vector.

For the case of real carriers, it can be easily proved that \( G^T \tilde{H}_l G \) is an \( (N_f/2 + 1) \times (N_f/2 + 1) \) diagonal matrix with the diagonal entries given by

\[
\Re\{F_{1:N_f/2+1}^H \beta_c(l)\} = 2N_f/(N_f + 2) \Re\{D_H(l)\},
\]

where \( \beta_c(l) = [\beta_c(l; 0), \ldots, \beta_c(l; M_l), 0, \ldots, 0]^T \).
Defining the real multi-carrier demodulation results of the frame-rate sampled output for the \(l\)th correlator by \(v^R(l) := G^T y^R(l)\), we obtain the equivalent system I/O relationship as

\[
v^R(l) = G^T y^R(l) = \frac{2N_f}{N_f + 2} \sqrt{\mathcal{E}_p^R} \Re\{D_{\mu}(l)\} \mathbf{d}^R + \zeta^R(l), \tag{2–10}
\]

where \(\zeta^R(l)\) is the noise vector.

Using the I/O models (2–9) and (2–10), we will next introduce the construction of \(\mathbf{d}^C\) and \(\mathbf{d}^R\) (the differential encoding) at the transmitter and the restoration of the information (the differential decoding) at the receiver.

### 2.4 Differential Demodulation with Variable Data Rates

In the preceding sections, we have established the I/O relationships for real and complex multi-carrier UWB systems. With typical UWB system parameters, the equivalent channel coefficients, \(H_l(n)\) for complex carriers and \(\Re\{H_l(n)\}\) for real carriers, vary slowly across \(n\) in the frequency domain. This allows for differential (de-)modulation across neighboring carriers.

This idea is reminiscent of the FSR-UWB [26]. However, our design will turn out to facilitate variable data rates, without any bandwidth expansion and with considerably improved energy efficiency. More importantly, under certain circumstances, no reference tone is needed. This translates to zero energy loss.

Since FFT-based complex multi-carrier communication is better understood, we will present the differential demodulation for the complex carriers followed by that for the real carriers at different data rate levels.
2.4.1 High-Rate MCD-UWB

2.4.1.1 Complex carriers

In the high-rate implementation, the entries of vector $d^C$ are differentially encoded as follows:

$$d^C(n) = \begin{cases} 
1, & n = 0 \\
& \\
& \\
d^C(n-1)s(n-1), & n \in [1, \ldots, N_f - 1] 
\end{cases}, \quad (2-11)$$

where $\{s(n)\}_{n=0}^{N_f-2}$ are the $M$-ary phase-shift keying (PSK) information symbols. As a result, only the first carrier is used as an unmodulated reference, while the rest $(N_f - 1)$ carriers each conveys a distinct information symbol. Notice that, though the usage of an unmodulated reference carrier is reminiscent of the FSR-UWB, our digital carriers facilitate considerably higher rate with no bandwidth expansion. Furthermore, in typical UWB systems with large $N_f$, the cost of the reference carrier can be neglected and the bandwidth and energy efficiencies approach 100% as $N_f$ increases.

Based on (2-11), and using the approximation that $H(n) \approx H(n + 1), \forall n \in [0, \ldots, N_f - 2]$, differential decoding can be performed on the FFT of the frame-rate samples $v^C(l)$ to recover the transmitted symbols without attempting to estimate the channel. Specifically, in the absence of noise, we have

$$v^C(l; n+1)(v^C(l; n))^* = E_p H_l(n+1) H_l^*(n) d(n+1) d^*(n) \approx E_p |H_l(n)|^2 s(n). \quad (2-12)$$

To establish a more convenient representation, we define the following $N_f \times N_f$ circularly shifting matrix

$$J = \begin{bmatrix} 
0 & 1 & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots & \vdots \\
\vdots & \ddots & 1 & 0 \\
0 & \cdots & 0 & 1 \\
1 & 0 & \cdots & 0 
\end{bmatrix}, \quad (2-13)$$
Figure 2-1. Receiver diagram for high-rate multi-carrier differential (MCD) ultra-wideband (UWB), where we use \( p_l(t) \) to denote \( p(t - \tau_c(l)) \) and omit all superscripts for notational brevity.

and formulate the \( N_f \times N_f \) matrix

\[
U^C = \sum_{l=1}^{L_c} v^C(l)(v^C(l))^H J, \tag{2–14}
\]

where, to effectively collect energy, we sum up the differential decoding results from all correlators. Clearly, the decision statistics for \( [s(0), \ldots, s(N_f - 2)]^T \) are the last \((N_f - 1)\) diagonal entries of \( U^C \); that is

\[
u_H^C(n) = [U^C]_{n,n} \approx \mathcal{E}_p \sum_{l=1}^{L_c} |H_l(n - 1)|^2 s(n - 1) \xi_H^C(n), \quad n \in [1, \ldots, N_f - 1], \tag{2–15}
\]

where \( \xi_H^C(n) \) is the noise term and the ‘\( \approx \)’ comes from the approximation \( H_l(n) \approx H_l(n + 1), n \in [0, \ldots, N_f - 2] \) in UWB channels.

The receiver structure is illustrated in Fig. 2-1. After passing through a bank of \( L_c \) correlators, the received signal \( r(t) \) is sampled at the frame-rate to generate the discrete-time signal \( \{y^C(l)\}_{l=1}^{L_c} \). With the multi-carrier demodulation by multiplying \( F \) and the differential decoding for each correlator branch \( l \), the resultant signals are summed up across all correlators to obtain the decision statistics for the transmitted information symbols.
2.4.1.2 Real carriers

For the case of real carriers, the entries of $d^R$ are differentially constructed as follows

$$d^R(n) = \begin{cases} 
1, & n = 0 \\
1 - d^R(n-1)b(n-1), & n \in [1, \ldots, N_f/2]
\end{cases}, \quad (2-16)$$

where the information bits $\{b(n)\}_{n=0}^{N_f/2}$ are binary phase shift-keying (BPSK) modulated.

Similar to the complex carriers, only the first carrier is used as the unmodulated reference, and each of the rest $N_f/2$ carriers conveys a distinct information bit.

Similar to (2–12), we have $v^R(l; n+1)v^R(l; n) \approx 4E_p N_f^2/(N_f + 2)^2 \Re\{H_l(n)\}^2 b(n)$. Then, differential decoding can be performed on $v^R(l)$ for each correlator branch $l$.

Let $J^R$ denote the $(N_f/2 + 1) \times (N_f/2 + 1)$ circularly shifting matrix, which has the same structure as $J$ in (2–13) but with a smaller dimension. We can then formulate the following $(N_f/2 + 1) \times (N_f/2 + 1)$ matrix

$$U^R = \sum_{l=1}^{L_c} v^R(l)(v^R(l))^T J^R. \quad (2-17)$$

As in (2–14), we sum up the differential demodulation results from all correlators to effectively collect the channel energy. Then, the decision statistics for $[b(0), \ldots, b(N_f/2-1)]^T$ are the last $N_f/2$ diagonal entries of $U^R$

$$u_{R_i}^R = [U^R]_{n,n} \approx \frac{4E_p N_f^2}{(N_f + 2)^2} \sum_{l=1}^{L_c} \Re\{H_l(n-1)\}^2 \cdot b(n-1)\xi_{R_i}^R(n), \quad n \in [1, \ldots, N_f/2], \quad (2-18)$$

where, $\xi_{R_i}^R(n)$ is the noise term and the ‘$\approx$’ comes from the approximation $\Re\{H_l(n)\} \approx \Re\{H_l(n+1)\}$.

For the case of real carriers, the receiver structure is similar to that for the complex carriers (see Fig. 2-1), except that the multi-carrier demodulation is performed by multiplying $G$. Again, the decoding results are summed up across all correlators to obtain the decision statistics for the transmitted information symbols.
Remark 1: The MD-FSR-UWB [118] scheme can also enable a higher data rate than the original FSR-UWB [26] by using a single reference tone together with multiple data tones. However, this approach requires that all carriers remain within the channel coherence bandwidth. This constraint restricts the number of usable carriers and therefore limits the data rate. Our high-rate MCD-UWB techniques relax the constraint by only requiring $H_l(n) \approx H_l(n + 1), n \in [0, \ldots, N_f - 2]$, and therefore can use all carriers to enable the maximum data rate. Furthermore, the development of MD-FSR-UWB in [118] assumes that IFI is absent by choosing a $T_f$ that is sufficiently long. This also limits the data rate. In MCD-UWB we develop here, severe IFI is allowed to enable high-rate transmission. In addition, higher data rate in MD-FSR-UWB comes at the price of having more analog carriers. This implies a larger bandwidth expansion. To be specific, when $N$ data tones are used with MD-FSR-UWB scheme, the bandwidth expansion is $2N / T_s$. On the contrary, our MCD-UWB relies on digital carriers that do not induce any bandwidth expansion.

2.4.2 Low-Rate MCD-UWB

Letting $s(n) = s, \forall n \in [0, N_f - 2]$, in (2–11), and $b(n) = b, \forall n \in [0, N_f/2 - 1]$, in (2–16), we obtain the low-rate version of our multi-carrier differential scheme. The resultant data rate $1 / T_s$ symbols/sec turns out to be the same as the FSR-UWB in [26]. However, we will show, both analytically and by simulations, that with more carriers conveying the same information, our low-rate schemes can capture the multipath diversity, and can considerably reduce the energy loss encountered by FSR-UWB. More importantly, under certain conditions, no reference carriers are needed and the energy loss can be completely avoided.

2.4.2.1 Complex carriers

When all carriers carry the same information symbol $s$, (2–11) becomes

$$d^C(n) = \begin{cases} 1, & n = 0 \\ d^C(n - 1)s, & n \in [1, \ldots, N_f - 1] \end{cases}.$$
where $s$ is the $M$-ary phase-shift keying (PSK) information symbol. Since (2–19) is a special case of (2–11), the same demodulator (2–14) can be applied here. However, taking into account that the diagonal entries of $U^C$ are the decision statistics for the same symbol $s$, we have

$$u^C_L = \sum_{n=1}^{N_f-1} [U^C]_{n,n} = \sum_{l=1}^{L_c} \sum_{n=1}^{N_f-1} \sum_{l=1}^{L_c} \text{H}^2 |H_l(n-1)|^2 s + \xi^C_L,$$

(2–20)

where $\xi^C_L$ is the noise term. Along the lines of [71, Ch. 5], the variance of $\xi^C_L$ can be shown to be

$$\sigma^2_{\xi} = 2N_0 \varepsilon_p \sum_{l=1}^{L_c} \sum_{n=1}^{N_f-1} |H_l(n-1)|^2,$$

where $N_0$ is the variance of the time-domain sampled complex noise at the correlator outputs.

Notice that, since one reference carrier is used, the energy of reference carrier of MCD-UWB can be neglected as $N_f$ increases. This is to be contrasted with the FSR-UWB, where half of the energy is used by the reference carrier. In fact, the energy loss entailed by the reference carrier can be completely avoided. To see this, notice that when the digital carriers are employed we have also $H(0) \approx H(N_f - 1)$, as a by-product of the discrete-time FFT.

Hence, if the signals riding on the first and last carriers are also differentially related with respect to the information symbol $s$, that is, $d^C(0) = d^C(N_f - 1)s$, then the noise-free product $v^C(l; 0)(v^C(l; N_f - 1))^*$ turns out to be $\varepsilon_p H_l(0) H_l^*(N_f - 1)(d^C(0)(d^C(N_f - 1))^* \approx \varepsilon_p |H_l(N_f - 1)|^2 s$. Recalling that $d^C(0) = 1$ [c.f. (2–19)], we deduce that the symbol $s$ has to satisfy $s^{N_f} = 1$. In other words, as long as the PSK modulation size $M$ is an integer factor of $N_f$\(^1\), our differential decoding can be carried out between all adjacent pairs of digital carriers without any reference tone. In this case, the decision statistic can be

\(^1\) With typically large and even $N_f$, this condition is not very restrictive for practical modulation sizes.
Figure 2-2. Receiver diagram for low-rate MCD-UWB, where we use \( p_1(t) \) to denote \( p(t - \tau_c(l)) \) and omit all superscripts for notational brevity.

alternatively expressed as:

\[
\begin{align*}
\hat{u}^C_L &= \sum_{n=0}^{N_f-1} |U^C|_{n,n} = \sum_{l=1}^{L_c} (\mathbf{v}^C(l))^\mathbb{H} \mathbf{J} \mathbf{v}^C(l) \approx E_p \sum_{l=1}^{L_c} \sum_{n=0}^{N_f-1} |H_l(n)|^2 s + \xi^C_L. \quad (2–21)
\end{align*}
\]

Notice that the summand limits in (2–21) are different from those in (2–20).

Interestingly, the decision statistic in (2–21) implies a simpler receiver implementation.

As a circulant matrix, \( \mathbf{J} \) can be diagonalized by pre- and post-multiplication with the FFT and IFFT matrices; i.e., \( \mathbf{F}^\mathbb{H} \mathbf{J} \mathbf{F} = \mathbf{D}_J \), whose \( n \)th diagonal entry is \( [\mathbf{D}_J]_{n,n} = \exp(j2\pi n/N_f) \).

In other words, the diagonal of \( \mathbf{D}_J \) is exactly the second column of \( \sqrt{N_f} \mathbf{F}^\mathbb{H} \). Substituting \( \mathbf{J} = \mathbf{F} \mathbf{D}_J \mathbf{F}^\mathbb{H} \) into (2–21), and recalling that \( \mathbf{v}^C(l) = \mathbf{F} \mathbf{y}^C(l) \), one can re-express \( \hat{u}^C_L \) as

\[
\begin{align*}
\hat{u}^C_L &= \sum_{l=1}^{L_c} (\mathbf{y}^C(l))^\mathbb{H} \mathbf{D}_J \mathbf{y}^C(l) = \sum_{l=1}^{L_c} \sum_{n=0}^{N_f-1} |\mathbf{y}^C(l; n)|^2 e^{j\frac{2\pi n}{N_f}} + \xi^C_L. \quad (2–22)
\end{align*}
\]

Eq. (2–22) reveals that the FFT operation at the receiver can be avoided.

The simplified receiver structure is illustrated in Fig. 2-2. It is essentially the digital version of the low-rate FSR-UWB receiver in [26]. It is worth stressing that this digital receiver operates on the frame-rate samples generated when the received signal \( r(t) \) passes through the correlator bank. The differential decoding is then performed by correlating \( \{|y^C(l; n)|^2\}_{n=0}^{N_f-1} \) with the complex carrier \( \{\exp(j2\pi n/N_f)\}_{n=0}^{N_f-1} \) on each correlator branch \( l \), and then summing them up across the \( L_c \) correlators.
2.4.2.2 Real carriers

When all the real carriers carry the same information bit \( b \), (2–16) becomes

\[
d^R(n) = \begin{cases} 
1, & n = 0 \\
d^R(n-1)b, & n \in [1, \ldots, N_f/2] 
\end{cases}, 
\]

where the information bit \( b \) is a BPSK symbol. As (2–23) is a special case of (2–16), the same demodulator (2–17) can be applied here. However, taking into account that the diagonal entries of \( U^R \) are the decision statistics for the same symbol \( b \), we have

\[
u^R_L = \frac{4E_p N_f^2}{(N_f + 2)^2} \sum_{l=1}^{L_c} \sum_{n=1}^{N_f/2} 4 \mathcal{R}\{H_l(n-1)\}^2 \cdot b + \xi^R_L, 
\]

where \( \xi^R_L \) is the noise term. The variance of \( \xi^R_L \) is \( \sigma^2_\xi = 4N_0E_p N_f^2/(N_f + 2)^2 \sum_{l=1}^{L_c} \sum_{n=1}^{N_f/2} |H_l(n-1)|^2 \), with \( N_0/2 \) being the variance of the time-domain sampled noise at the correlator output.

As in the complex-carrier case, one would expect a simpler decoder similar to (2–22) to hold also for the real-carrier case. This, however, is not straight forward, and the derivations we used for the complex case does not carry over. Intuitively, the low-complexity decoder in (2–22) relies on the fact that \( H_l(0) \approx H_l(N_f - 1) \). The counterpart condition for the real-carrier case would be \( H_l(0) \approx H_l(N_f/2) \), which is evidently not true. Nevertheless, we can prove in the following that the following low-complexity decoder can be employed for the real-carrier low-rate setup:

\[
u^R_L = \sum_{l=1}^{L_c} \sum_{n=0}^{N_f-1} |y^R(l; n)|^2 \cos\left(\frac{2\pi n}{N_f}\right). 
\]

**Proof.** To avoid terms containing \( H_l(0)H_l(N_f/2) \) to appear in the decision statistic, we will resort to the \( N_f \times N_f \) shifting matrix \( J \). It appears that we can use \( \sum_{l=1}^{L_c} (y^R)^T(l) \tilde{G}J\tilde{G}^T y^R(l) \) as the decision statistic like the complex-carrier case [c.f. (2–20)], where the \( N_f \times N_f \)
matrix $\bar{G} = [g_0, \ldots, g_{N_f/2}, 0, \ldots, 0]$ is constructed by concatenating $G$ with $(N_f/2 - 1)$ zero vectors of size $N_f$. However, unlike the complex case where $F^H J F$ is a diagonal matrix [c.f. (2–22)], $\bar{G}J\bar{G}^T$ does not have a form which can lead to the simple receiver structure as the complex-carrier case.

For these reasons, we use FFT and IFFT operations with the shifting matrix $J$ to enable a simple receiver for the real-carrier low-rate case. Denoting the FFT of $y^R(l)$ as $\bar{v}^R(l) := Fy^R(l)$, we have

$$\bar{u}^R_l = \sum_{l=1}^{L_c} (\bar{v}^R(l))^H J\bar{v}^R(l) = \sum_{l=1}^{L_c} (y^R(l))^H F^H JFy^R(l)$$

$$\approx \sum_{l=1}^{L_c} \mathcal{E}_p(d^R)^T G^TF^H J|D_{H(l)}|^2 FGd^R + \xi^R_L,$$

where the approximation comes from $H(n) \approx H(n+1), n \in [0, \ldots, N_f - 1]$, and $\xi^R_L$ is the noise term.

Let $J_{H(l)}$ denote the real part of $G^TF^H J|D_{H(l)}|^2 FG$ which is an $(N_f/2+1) \times (N_f/2+1)$ tridiagonal matrix with the main diagonal being zero, and vectors $N_f/(N_f + 2)[\sqrt{2}|H_l(N_f - 1)|^2, |H_l(N_f - 2)|^2, \ldots, |H_l(N_f + 1/2)|^2, \sqrt{2}|H_l(N_f/2)|^2]$ and $N_f/(N_f + 2)[\sqrt{2}|H_l(0)|^2, |H_l(1)|^2, \ldots, |H_l(N_f/2 - 2)|^2, \sqrt{2}|H_l(N_f/2 - 1)|^2]$ sitting on the subdiagonals. Then, the decision statistic of $b$ can be obtained as the real part of $\bar{u}^R_l$

$$u^R_l = \Re(\bar{u}^R_l) \approx \mathcal{E}_p \sum_{i=1}^{L_c} d^T J_{H(l)}d + \xi^R_L$$

$$= \frac{\sqrt{2}N_f}{N_f + 2} \mathcal{E}_p \sum_{i=1}^{L_c} [H_i(0)^2 + |H_i(N_f/2)|^2]$$

$$+ |H_i(N_f - 1)|^2 + |H_i(N_f - 1/2)|^2] b + \xi^R_L$$

$$+ \frac{N_f}{N_f + 2} \mathcal{E}_p \sum_{i=1}^{L_c} \sum_{n=1}^{N_f/2-1} [N_f^2 + |H_i(n)|^2 + |H_i(n + N_f/2)|^2] b,$$

where $\xi^R_L$ is the real part of $\xi^R_L$. The noise-free part of $u^R_L$ contains the information bit $b$. 
As with the case of complex carriers, the receiver structure for real carriers can be further simplified. By replacing $F^H J F$ with $D_J$ in (2–26), $u^R_L$ can be re-expressed by Eq. (2–25).

The receiver structure is illustrated in Fig. 2-2. Similar to the complex case, the differential decoding can be performed by correlating $\{|y^R(I; n)|^2\}_{n=0}^{N_r-1}$ with the real carrier $\{\cos(2\pi n/N_f)\}_{n=0}^{N_r-1}$ on each correlator branch $I$, and then summing up all the $L_c$ correlation results.

**Remark 2:** At low data rate, our MCD-UWB enjoys a simple receiver reminiscent of the FSR-UWB in [26]. However, FSR-UWB entails an inherent energy loss, simply because half of the energy per symbol is allocated to the reference tone. On the other hand, only a single reference tone (or even none) is employed in MCD-UWB, which consumes $1/N_f$ (or $2/(N_f + 2)$ in the real carrier case) of the energy per symbol. In UWB systems with large $N_f$ values, this implies considerable efficiency improvement.

### 2.4.3 Variable-Rate MCD-UWB

So far, we have seen that our MCD-UWB allows for both high-rate transmissions with $(N_f - 1)$ symbols per block, and low-rate transmissions with a low-complexity receiver. In fact, the MCD-UWB framework also enables transmissions with variable data rates to facilitate a desirable rate-performance tradeoff.

#### 2.4.3.1 Complex carriers

At the transmitter, with $N_s < N_f$ symbols being encoded in one block, the entries of the carrier modulating vector $d^C$ can be differentially constructed group by group as follows:

$$d^C(n) = \begin{cases} 1, & n = 0 \\ d^C(n-1)s(1), & n \in [1, \ldots, p^C(1)] \\ \vdots \\ d^C(n-1)s(j), & n \in [p^C(j-1)+1, \ldots, p^C(j)] \\ \vdots \\ d^C(n-1)s(N_s), & n \in [p^C(N_s-1)+1, \ldots, p^C(N_s)] \end{cases} \quad (2–28)$$
where $p^C(j) = \sum_{i=1}^{j} n_s(i), j \in [1, \ldots, N_s]$, $p^C(N_s) = N_f - 1$, and $\{s(j)\}_{j=1}^{N_s}$ are the $N_s$ information symbols each encoded on a group of $n_s(j)$ carriers. As with the high-rate case, the first carrier is used as the unmodulated reference [c.f. (2–9) and (2–28)]. However, we will next show that it is possible to avoid the reference carrier, as in the low-rate case.

The decoding procedure for variable-rate MCD-UWB is similar to that of the high-rate case, except that the decision statistic of each symbol now relies on a group of carriers instead of a single carrier. To effectively collect energy, for each symbol $s(j)$, we sum up the decoding results from all $L_c$ correlators and its corresponding $n_s(j)$ carriers. Using (2–14), the decision statistic for the $j$th information symbol is

$$u^C_V(j) = \sum_{n=p^C(j)-n_s(j)+1}^{p^C(j)} [U^C]_{n,n}, j \in [1, \ldots, N_s].$$

With $H(n) \approx H(n+1), n \in [0, \ldots, N_f - 1]$, the noise-free part of $u^C_V(j)$ can be explicitly expressed as

$$u^C_V(j) \approx \sum_{n=p^C(j)-n_s(j)+1}^{p^C(j)} \sum_{l=1}^{L_c} E_p |H_l(n)|^2 s(j), j \in [1, N_s].$$

Recall that, as a manifestation of digital multi-carrier modulation, we have also $H(0) \approx H(N_f - 1)$. Similar to the low-rate case, this feature can be exploited to eliminate the need for any reference carrier by letting $d^C(N_f - 1)s(N_s) = d^C(0)$. Together with $d^C(0) = 1$ in (2–28), this condition implies $\prod_{j=1}^{N_s} s(j)^{n_s(j)} = 1$. Considering the independence among $\{s(j)\}_{j=1}^{N_s}$, we have equivalently $s(j)^{n_s(j)} = 1, j \in [1, \ldots, N_s]$. Therefore, as long as the modulation size of $s(j)$ is an integer factor of $n_s(j)$, our differential demodulation can be carried out between all adjacent pairs of digital carriers without any reference tone, and the noise free part of the decision static for $s(N_s)$ can be modified into

$$\sum_{n=p^C(N_s-1)+1}^{N_f-1} |H_l(n)|^2 s(N_s) + |H_l(0)|^2 s(N_s).$$
2.4.3.2 Real carriers

In the variable-rate case, the entries of the carrier modulating vector $d^R$ can be constructed group by group as follows

$$d^R(n) = \begin{cases} 
1, & n = 0 \\
 d^R(n-1)b(1), & n \in [1, \ldots, p^R(1)] \\
 \vdots \\
 d^R(n-1)b(j), & n \in [p^R(j-1)+1, \ldots, p^R(j)] \\
 \vdots \\
 d^R(n-1)b(N_b), & n \in [p^R(N_b-1)+1, \ldots, p^R(N_b)] 
\end{cases} \tag{2–31}$$

where $p^R(j) = \sum_{i=1}^j n_b(i), j \in [1, \ldots, N_b], p^R(N_b) = N_f/2$ and $\{b(j)\}_{j=1}^{N_b}$ are $N_b$ information bits each of which is encoded on a group of $n_b(j)$ carriers.

Similar to the reception in variable-rate case of complex carriers, to effectively collect energy, for each bit, we sum up the decoding results from all correlators and all corresponding carriers. Using the definition of $U^R = \sum_{l=1}^{L_c} v^R(l)(v^R(l))^T J^R$ of (2–17), the decision static for the $j$th information bit can be expressed as

$$u^R_V(j) = \sum_{n=p^R(j)-n_b(j)+1}^{p^R(j)} [U^R]_{n,n}, j = [1, \ldots, N_b]. \tag{2–32}$$

For $\Re\{H_l(n)\} = \Re\{H_l(n+1)\}, n \in [0, \ldots, N_f/2]$, the noise-free parts of the decision statistics are

$$u^R_V(j) \approx \frac{4\varepsilon_p N_f^2}{(N_f + 2)^2} \sum_{n=p^R(j)-n_b(j)+1}^{p^R(j)} \sum_{l=1}^{L_c} \Re\{H_l(n)\}^2 \cdot b(j), j = [1, \ldots, N_b]. \tag{2–33}$$

However, unlike the complex-carrier case where $H(0) \approx H(N_f - 1)$, for real-carrier MCD-UWB, it does not hold that $H(0) \approx H(N_f/2)$. Therefore, the first and last carriers cannot be differentially related with respect to the information symbol $b(N_b)$, and a reference tone should always be used.
Though both the complex and the real MCD-UWB can enable variable-rate communications, it is worth noting that the maximum number of carriers in the complex MCD-UWB \( N_f \) nearly doubles that of the real MCD-UWB \( N_f/2 + 1 \). Therefore, for variable-rate MCD-UWB, when the data rate is fixed, the complex-carrier case can use more carriers per symbol. We will see later that the number of carriers per symbol is closely related to the diversity gain and the demodulation performance. Another difference between the real and complex MCD-UWB is that, the complex case requires the channel to be invariant in \( 2T_s \), but the real case only needs the channel to be invariant in \( T_s \).

2.4.4 MCD-UWB vs. Frequency-Shifted Reference (FSR-) UWB

So far, we have established the differential coding and decoding of our MCD-UWB for both high-rate and low-rate implementations. As the FSR-UWB, our approach also avoids analog delay lines. However, the energy loss of the FSR-UWB is considerably reduced in our MCD-UWB especially for typically large \( N_f \) values. For the high-rate case, both MD-FSR-UWB and MCD-UWB use a single carrier as the reference tone, and multiple carriers for information symbols. Unlike the MD-FSR-UWB, however, the number of information-conveying carriers of MCD-UWB is not limited by the coherence bandwidth. Furthermore, thanks to the digital operation, our MCD-UWB does not induce any bandwidth expansion. In addition, MCD-UWB allows for (possibly severe) IFI that is inevitable in high-rate transmissions.

2.5 Performance Analysis

So far, we have introduced the multi-carrier differential UWB schemes which enable the variable data rates. In addition, at low data rate, the demodulation can be further simplified. In this section, we analyze the error performance of MCD-UWB. Specifically, we will study the effects of the variable data rates on the bit-error rate (BER) performance for the cases of both complex and real carriers.
2.5.1 Complex MCD-UWB

In the cases of high, low and variable rates, the major difference lies in the different numbers of carriers each symbol occupies. Without loss of generality, let us now focus on the complex multi-carrier case, and consider the pairwise error probability (PEP) of erroneously decoding \( s \) as \( \bar{s} \neq s \) with an ML detector, assuming that \( s \) is differentially encoded on the first \( n_s \) carriers. The PEP can be upper bounded at high signal-to-noise ratio (SNR) using the Chernoff bound \(^2\)

\[
P(s \rightarrow \bar{s}) \leq \exp \left( \frac{-d^2(u^C, \bar{u}^C)}{4\sigma^2} \right),
\]

where \( \sigma^2 \) is the variance of the complex noise, and \( u^C(\bar{u}^C) \) is the noise-free part of the demodulation result corresponding to \( s(\bar{s}) \), and \( d(u^C, \bar{u}^C) \) is the Euclidean distance between \( u^C \) and \( \bar{u}^C \).

Using \( u^C \approx \mathcal{E}_p \sum_{l=1}^{L_c} \sum_{n=0}^{n_s-1} |H_l(n)|^2 s(n) \) [c.f. (2–30)], we have

\[
d^2(u^C, \bar{u}^C) = \left( \mathcal{E}_p \sum_{l=1}^{L_c} \sum_{n=0}^{n_s-1} |H_l(n)|^2 \right) |\epsilon|^2,
\]

where \( \epsilon := s - \bar{s} \). At high SNR, (2–34) can be re-expressed as

\[
P(s \rightarrow \bar{s}) \leq \exp \left( \frac{-\mathcal{E}_p |\epsilon|^2}{8N_0} \sum_{l=1}^{L_c} \sum_{n=0}^{n_s-1} |H_l(n)|^2 \right)
\]

\[
= \exp \left( -\frac{N_f \mathcal{E}_p |\epsilon|^2}{8N_0} \sum_{l=1}^{L_c} \sum_{n=0}^{n_s-1} \beta_c(l) \left( \Theta_l \beta_c(l) \right) \right),
\]

where \( \beta_c(l) = [\beta_c(l, 0), \ldots, \beta_c(l, M_l)]^T \) and \( \Theta_l^C := F_{l,0:n_s-1,0:M_l}^H F_{l,0:n_s-1,0:M_l} \), with \( F_{l,0:n_s-1,0:M_l} \) denoting the matrix consisting of the first \( n_s \) rows and the first \( (M_l + 1) \) columns of \( F \).

With \( \beta_c(l, m) \) being zero-mean, i.i.d. Gaussian with variance \( \beta = \mathbb{E}\{\beta^2_c(l, m)\} \), the

\(^2\) Though the noise-by-noise terms in our decision statistics are not strictly Gaussian distributed, they are approximately so under the Central Limit Theorem. In addition, these noise-by-noise terms are negligible under the high SNR scenario of interest here.
average PEP can be upper bounded by (see e.g. [96], [111])
\[
\overline{P}(s \rightarrow \bar{s}) \leq \prod_{l=1}^{L_c} \prod_{m=0}^{M_l} \left[ 1 + \frac{N_f B |\epsilon|^2}{4N_0} \lambda^C_l(m) \right]^{-\frac{1}{2}},
\]  
(2–37)

where \( \{\lambda^C_l(m)\}_{m=0}^{M_l} \) are the eigenvalues of \( \Theta^C_l \) in the nonincreasing order. Letting \( r^C_l \) denote the rank of \( \Theta^C_l \), we have \( \lambda^C_l(m) \neq 0 \) if and only if \( m \in [0, \ldots, r^C_l - 1] \). The maximum rank of \( \Theta^C_l \) is \( r^C_l = \min\{n_s, M_l + 1\} \). Therefore, at high SNR \((\mathcal{E}_p/N_0 \gg 1)\), eq. (2–37) becomes
\[
\overline{P}(s \rightarrow \bar{s}) \leq \left( \frac{N_f B |\epsilon|^2}{4N_0} \right)^{-\frac{1}{2}} \prod_{l=1}^{L_c} \prod_{m=0}^{r^C_l - 1} \lambda^C_l(m)^{-\frac{1}{2}}.
\]  
(2–38)

The average PEP can usually be expressed with the following general form
\[
\overline{P}(\epsilon \neq 0) \leq \left( \frac{\mathcal{E}_p}{N_0} \frac{G_{c,\epsilon}}{G_d} \right)^{-G_d},
\]  
(2–39)

where
\[
G_d = \frac{1}{2} \sum_{l=1}^{L_c} \min\{M_l + 1, n_s\},
\]  
(2–40)
\[
G_{c,\epsilon} = \frac{N_f B |\epsilon|^2}{4} \left( \prod_{l=1}^{L_c} \lambda^C_l \right)^{-\frac{1}{2G_d}}.
\]

Evidently \( G_d \) determines the PEP slope as a function of log-\( \mathcal{E}_p/N_0 \) and is the so-called diversity gain, and \( G_{c,\epsilon} \) denotes the coding gain which determines the horizontal shift. Notice that \( G_d \) is independent of the specific error \( \epsilon \), whereas \( G_{c,\epsilon} \) is \( \epsilon \) dependent.

To account for all possible pair-wise errors, we define the coding gain as \( G_c := \min_{\epsilon \neq 0} \{G_{c,\epsilon}\} \).

We are now ready to specify the diversity and coding gains at each data rate level. For the high-rate case, where the carrier number per symbol is \( n_s = 1 \), the rank of \( \Theta^C_l \) is one, and therefore the diversity gain is \( G_d = L_c/2 \). For the variable-rate case, the diversity gain analysis is the same as the preceding analysis under the general condition, i.e., \( G_d = 0.5 \sum_{l=1}^{L_c} \min\{M_l + 1, n_s(j)\}, \forall j \in [1, \ldots, N_s] \), for the \( j \)th transmitted
symbol. For the low-rate case where only one information symbol is transmitted, we obtain that \( G_d = 0.5 \sum_{i=1}^{L_c} (M_i + 1) \). Comparing the diversity gains at different data rate levels, we deduce that the more carriers are used to carry one symbol, the higher diversity gain can be achieved. This, however, is at the price of reduced data rate. Therefore, there is a tradeoff between the data rate and the error performance.

**Remark 3:** Eq. (2–40) shows that the diversity order \( G_d \) depends on both the number of carriers per symbol \( n_s \) and the channel orders \( \{ M_i \}_{i=1}^{L_c} \), which capture the severity of IFI. In the general case where IFI is present (\( M_i \neq 0 \)), the diversity gain is \( G_d = 0.5 \sum_{i=1}^{L_c} (M_i + 1) \), as long as \( n_s \geq M_i + 1, \forall l \). On the other hand, one may opt to avoid IFI in the first place, by choosing \( T_f \) to be large enough. In this case, we have \( M_i = 0, \forall l \). It follows that the diversity order is \( G_d = L_c/2 \) regardless of \( n_s \). Therefore, with fixed number of correlators \( L_c \), the IFI actually works to our advantage by enabling a greater \( G_d \).

**Remark 4:** To reduce the peak-to-average power ratio (PAPR) of the transmitted waveform, we can also use \( N_c < N_f \) carriers. For example, for the low-rate case, the diversity gain is \( G_d = 0.5 \sum_{i=1}^{L_c} \min\{ M_i + 1, N_c \} \). By using less carriers, the PAPR effects can be suppressed, and the diversity gain will decrease consequently. Therefore, there is also a tradeoff between the PAPR suppression and the symbol error rate performance. Notice that, as long as \( N_c \) is greater than the maximum order of the equivalent channel, the maximum diversity gain \( 0.5 \sum_{i=1}^{L_c} (M_i + 1) \) can always be achieved.

### 2.5.2 Real MCD-UWB

Similar to the complex-carrier MCD-UWB, for the case of real carriers, we first consider the general case when one bit \( b \) is conveyed by \( n_b \) carriers. The average PEP can be upper bounded by

\[
\bar{P}(b \rightarrow \bar{b}) \leq \prod_{l=1}^{L_c} \prod_{m=0}^{M_l} \left[ 1 + \frac{2N_f^3 \epsilon_p B \epsilon_b^2}{N_0(N_f + 2)^2} \lambda^R(m) \right]^{-\frac{1}{2}},
\]  

(2–41)
where $\epsilon_b = (b - \bar{b})$ and $\{\lambda_i^R(m)\}_{m=0}^{M_l}$ are the eigenvalues of $\Theta_i^R := \Re\{F_{0:n_b-1:0:M_l}\}^T \times \Re\{F_{0:n_b-1:0:M_l}\}$ in the nonincreasing order. The maximum rank of $\Theta_i^R$ is $r_i^R = \min\{M_l + 1, n_b\}$. Therefore, the maximum achievable diversity order is $0.5 \sum_{l=1}^{L_c} \min\{M_l + 1, n_b\}$. Specifically, for the high-rate case, where every bit is riding on a single carrier, the diversity gain is $G_d = L_c/2$. For the variable-rate case, we have $G_d = 0.5 \sum_{l=1}^{L_c} \min\{M_l + 1, n_b(j)\}$ for the $j$th bit, $\forall j \in [1, \ldots, N_b]$. Similarly, for low-rate real-carrier MCD-UWB, where all $N_f/2$ carriers are modulated by a single bit, we have $G_d = 0.5 \sum_{l=1}^{L_c} \min\{M_l + 1, N_f/2\}$. Similar to the complex MCD-UWB, using variable number of carriers to carry one bit also allows for a flexible tradeoff between the data rate and the diversity order.

As with the complex MCD-UWB, Remarks 3 and 4 also hold in the real carrier case. In particular, consider MCD-UWB with two real carriers ($N_c = 2$), which can be regarded as the digital version of the FSR-UWB scheme. The noise-free part of its decision statistic is [c.f. (2–27)]

$$u_L^R \approx \frac{N_f \epsilon_p}{2 \sqrt{2}} \sum_{l=1}^{L_c} \left( |H_l(0)|^2 + |H_l(N_f - 1)|^2 \right) b.$$

(2–42)

Accordingly, the maximum diversity gain can be obtained as $0.5 \sum_{l=1}^{L_c} \min\{M_l + 1, 2\}$. Clearly, when more than two frames are involved in the IFI, i.e., $M_l > 1$, the MCD-UWB with $N_c > 2$ carriers carrying each bit can provide a greater diversity gain.

**Remark 5:** As mentioned before, the maximum number of carriers in the complex MCD-UWB ($N_f$) is greater than that of the real MCD-UWB ($N_f/2 + 1$). Therefore, for a fixed data rate, the complex case can use more carriers per symbol. According to the preceding diversity analysis, this implies a better error performance with a larger diversity gain $G_d$. On the other hand, the complex MCD-UWB enables a higher data rate than the real MCD-UWB when $G_d$ is fixed.

### 2.6 Simulations

In our simulations, the UWB pulse $p(t)$ is the second derivative of the Gaussian pulse with $T_p \approx 1$ns. Each $T_s$ consists of $N_f = 32$ frames. Simulations are performed...
Figure 2-3. Complex MCD-UWB with various data rates, and in the presence and absence of IFI.

using the Saleh-Valenzuela channel model with parameters $(1/\Lambda, 1/\lambda, \Gamma, \gamma) = (2, 0.5, 30, 5)\text{ns}$ [79]. The maximum delay spread of the multipath channel is about 90ns. The real- and complex-carrier cases adopt BPSK and quadrature phase-shift keying (QPSK) modulations, respectively. For both cases, we use $L_c = 6$ correlators with $\tau_c(l) = (l - 1)\text{ns}$, $\forall l \in [1, \ldots, L_c]$.

In the first test scenario, we compare the BER performance with and without IFI. As mentioned in the preceding section, one can either allow for IFI and remove the IBI via CP, or avoid IFI by choosing a large $T_f$. For the former case, we let $T_f = 24\text{ns}$ and employ CP of length 4 (frames). For the latter case, we let $T_f = 96\text{ns}$.

Figs. 2-3 and 2-4 show the comparisons between the CP-assisted and IFI-free cases for both complex and real MCD-UWB, respectively. In both figures, the BER curves are nearly parallel at high $E_p/N_0$ for the IFI-free cases. This indicates that the diversity gain does not change when the number of carriers per symbol increases. However, with the use of CP, the slope of the BER curve drops as the number of carriers per symbol increases. These confirm that the CP-assisted schemes outperform the
IFI-free ones in terms of the diversity gain, as predicted by the performance analysis in the preceding section. It is also worth noting that the CP-based scheme provides $3.66$ times the data rate of the IFI-free case. In the following, we will set $T_f = 24$ ns.

To validate the effects of the number of carriers per symbol on the error performance, we plot in Figs. 2-5 and 2-6 the BER curves for the low-rate complex and real MCD-UWB with various $N_c$ values. In all cases, only one symbol is transmitted per block of duration $T_s$. First, we observe that the complex- and real-carrier schemes have similar performances as predicted by the analysis in the preceding section. In both figures, the curves corresponding to larger $N_c$ values exhibit better BER performance. This is in part due to their greater diversity gains which give rise to steeper slopes of the BER curves, as indicated in our performance analysis. Another reason for the performance improvement with increasing $N_c$ is the improved energy efficiency. In particular, notice that the case with $N_c = 2$ can be regarded as the digital version of FSR-UWB, in the sense that half of the energy is used by the reference tone. This further illustrates that MCD-UWB can be more energy efficient than the FSR-UWB by enabling a larger $N_c$. 
Figure 2-5. Low-rate complex MCD-UWB.

Figure 2-6. Low-rate real MCD-UWB.
As mentioned before, the maximum number of the complex carriers is nearly twice that of the real carriers. Therefore, with a fixed data rate, complex MCD-UWB can allocate more carriers to each symbol. Indeed, Fig. 2-7 shows that, with a fixed data rate of 2 bits/block, the complex case outperforms the real case by $3\text{ dB}$ at $\text{BER} = 10^{-4}$. With both cases using 8 carriers per symbol, the complex and real MCD-UWB schemes achieve the same diversity order and nearly identical performance, as shown in Fig. 2-7. However, the complex MCD-UWB provides a data rate of 4 bits per block, which doubles that of the real MCD-UWB.

In Fig. 2-8, we also compare the performance of the MD-FSR-UWB with our variable-rate MCD-UWB. For MD-FSR-UWB, the one-sided noise bandwidth is 4GHz. The rms delay spread of the simulation channel is $\sigma_{\text{rms}} \approx 9\text{ ns}$, which gives rise to a coherence bandwidth of about 22MHz. With the carrier spacing of $1/(N_r T_r) \approx 1.3\text{ MHz}$, the MD-FSR-UWB can transmit a maximum of 9 bits per block. With 17 real carriers, our MCD-UWB can transmit up to 16 bits per block. In addition, notice that the 9 bits per block of MD-FSR-UWB is achieved at the price of bandwidth expansion, where
as MCD-UWB does not entail any bandwidth expansion. Fig. 2-8 shows that the BER curves for the MD-FSR-UWB are flat at low SNR, have steep slopes at moderate SNR, and flatten again at high SNR. The low-SNR flatness may be induced by the dominant double-noise term. The steep slope at moderate SNR reveals the considerable diversity order achieved by the MD-FSR-UWB. At high SNR values where the noise can be neglected, the BER performance of MD-FSR-UWB is limited by the discrepancy between the channel responses at the reference tone and the date-conveying tones. This limitation results in the flatness at high SNR. Since the higher data rate in MD-FSR-UWB is achieved at the price of increased frequency separation among the reference and data-conveying tones, the error floor level increases with the increasing data rate, as shown in Fig. 2-8. The MCD-UWB BER curves exhibit a more consistent performance, simply because the spacing between the reference and data tones remains invariant at any data rate.

Figure 2-8. MCD-UWB versus multi-differential (MD) frequency-shifted reference (FSR) UWB at various data rates.
Comparing the BER curves in Fig. 2-8, we also notice that, at relatively low data rate (4 bits per block), MCD-UWB outperforms MD-FSR-UWB at low and high SNR, while the latter outperforms the former at moderate SNR. At a higher data rate of 8 bits per block, the MCD-UWB outperforms the MD-FSR-UWB at all SNR values. At their respective maximum rates of 16 and 9 bits/block, our MCD-UWB still outperforms the MD-FSR-UWB at almost all SNRs while providing twice the data rate. It is worth noting that only 6 correlators are used for the MCD-UWB, which collects energy over only $6/24 = 25\%$ of each frame duration $T_f$. The error performance of MCD-UWB is expected to be further enhanced by increasing the number of correlators.

2.7 Conclusions

In this Chapter, we have introduced the digital multi-carrier differential signaling schemes for UWB communications. Our frequency-domain differential approach is inspired by the FSR-UWB, and can avoid the challenging UWB channel estimation without imposing the analog delay elements. However, with the employment of multiple digital carriers, our MCD-UWB outperforms FSR-UWB by avoiding the bandwidth expansion as well as the energy loss. Compared to the high-rate version of FSR-UWB (a.k.a. MD-FSR-UWB), our approach allows for higher data rates without being constrained by the channel coherence bandwidth, and without degrading the demodulation performance, by maintaining the minimum spacing between the data tone and its corresponding ‘reference’ tone. Our MCD-UWB enables diversity combining in a differential manner and ensures effective collection of the multipath diversity, even in the presence of severe IFI and ISI. The proposed multi-carrier modulation can be realized with standard FFT and DCT circuits, both operating at the frame-rate.
CHAPTER 3
MODELING AND TRANSCEIVER DESIGN FOR ASYMMETRIC UWB LINKS WITH HETEROGENEOUS NODES

3.1 Motivation

In our MCD-UWB system and also in other wireless systems, we have observed the asymmetry between the transmitter and the receiver (see, e.g., [4, 46, 103, 121]). For example, for the complex low-rate MCD-UWB (see [103]), the modulation process includes the differential modulation and the multi-carrier modulation with the FFT operation at the transmitter. However, at the receiver, the multi-carrier demodulation and differential demodulation can be carried out only by a single mixing operation, which is much simpler than the modulation process of the transmitter. As a result, the transmitter and receiver can be realized at quite different complexity levels.

This kind of asymmetric and heterogeneity is becoming more common since UWB has been proposed as the physical layer realization for various networks including both the high data rate\(^1\) and low data rate wireless personal area networks (WPANs) (see, e.g., [2, 20, 68]), the wireless body area networks (WBANs) (see, e.g., [42]) and the wireless sensor networks (WSNs) (see, e.g., [61, 120]). As these systems can provide various services, heterogeneity emerges among network nodes either inside a network or among different ones. To realize the seamless network operation, transceiver design needs to take into account the heterogeneity among these nodes.

Heterogeneity has been investigated for the higher layer issues including scheduling, polling and routing, etc., for the UWB-based or general wireless networks (see, e.g., [4, 46, 121]). In this dissertation, we focus on the physical layer heterogeneity of the asymmetric UWB links. For UWB links, heterogeneity and asymmetry can be induced by the different numbers of signal bands for multiband operation, different

\(^1\) IEEE 802.15.3a Task Group was officially disbanded in 2006, but the products are still being made in the industry.
pulse rates or different pulse shapers, between the transmitter and the receiver. Here, we are more interested in the first two factors, i.e., the number of signal bands and the pulse rate, because these factors usually determine the complexity of the device. Generally, nodes of high complexity can provide high data rate services such as the multimedia data transfer which needs more power and computational resources. Nodes of low complexity are typically small and rely on limited battery power, such as wireless sensors. Besides these, some researchers have also considered the complexity-performance tradeoff of UWB transceivers by choosing different ADC resolutions (see e.g., [9, 36, 88]). However, since the ADC resolution does not affect our system modeling and the transceiver design, here we will not discuss this issue.

In this dissertation, we investigate the transceiver design for the asymmetric UWB link with a single transmitter and a single receiver. We first introduce the general transceiver model that allows for the information exchange on multiple bands at variable pulse repetition rates. By varying the band number or the pulse rate, the complexity level and accordingly the data rate of the link can be changed. Analyzing this mathematical model, we find that the asymmetric link can be represented by an MIMO system model which has originally been proposed and investigated for the multiple Tx- and Rx-antenna systems. The similarity between the asymmetric links and the multiantenna systems reveals that the structure asymmetry of these UWB transceivers is essentially equivalent to the spatial asymmetry of multiantenna systems. As far as we know, we are among the first to observe and exploit the similarity between these two types of asymmetric systems (see, e.g., [104, 109]).

Once the asymmetric UWB link is modeled as an MIMO system, we can apply existing multiantenna communication techniques to our UWB transceiver design for better performance or lower complexity. This is very attractive since many multiantenna techniques are available in the literature which are optimal in terms of system throughput, error rate or complexity, etc (see, e.g., [66, 73, 80]). Then, we will show how these
techniques can be integrated into our asymmetric link model. It should be noted that
the MIMO modeling actually allows us to exploit the multipath diversity of the UWB
channel in very flexible manners. In addition, by changing the parameters of our design,
the MIMO modeling also enables the convenient rate-diversity tradeoff. Our analyses,
together with the simulations, confirm the feasibility and effectiveness of the modeling
and transceiver design for the asymmetric UWB link.

### 3.2 Transceiver Structures

Fig. 3-1 shows the UWB transmitter and receiver structures used in this Chapter.
Note that our transceiver does not necessarily represent any specific one utilized
by existing standards, but combines functions of an array of transceivers into one
framework. Both the transmitter and receiver can incorporate multiband operation.
At the transmitter, $B_t$ parallel data streams $\{v(n; b)\}_{b=0}^{B_t-1}$ are modulated on the unit-energy pulse shaper $p_t(t)$ with the duration of $T_t$, the bandwidth of which is approximately $1/T_t$. With a pulse rate of $1/(N_t T_t)$ and $N_t$ being an integer, the $B_t$ data modulated pulse trains are fanned out simultaneously on $B_t$ bands $\{f_c + f_t(b)\}_{b=0}^{B_t-1}$, with $f_c$ being the center frequency of the first signal band and $f_t(b) = b/T_t$, $b \in [0, B_t - 1]$. The transmitted signal is then given by:

$$v(t) = \sum_{b=0}^{B_t-1} \sum_{n=0}^{\infty} v(n; b)p_t(t - nN_t T_t) \exp(j2\pi(f_c + f_t(b))t) . \quad (3-1)$$

At the receiver, the received waveform $r(t)$ is carrier demodulated into $B_r$ signal bands by multiplying it with $e^{-j2\pi(f_c + f_r(b))t}$, $b \in [0, B_r - 1]$, and then sampled by a correlator with the template $p_r(t)$ at intervals of $N_r T_r$, with the width of $p_r(t)$ being $T_r$ and $N_r$ being an integer. We can then obtain $B_r$ discrete-time data streams $x(n; b)$, $b \in [0, B_r - 1]$

$$x(n; b) = \int_{nN_r T_r}^{(nN_r+1)T_r} r(t) \exp(-j2\pi(f_c + f_r(b))t)p_r(t)dt , \quad b \in [0, B_r - 1] , \quad (3-2)$$

where $f_r(b) = b/T_r$, $b \in [0, B_r - 1]$. The receiver could be different from the transmitter in terms of the pulse width, pulse rate or band number, which will result in the asymmetry between the transmitter and the receiver.

The multiband operation at the receiver was actually inspired by the idea of the “channelized UWB receiver” (see, e.g., [17, 57, 93]). Since the bandwidth of UWB signals is very large, it is not desirable to sample the received signal at the Nyquist rate with a single ADC. In order to solve this problem, the channelized receiver analyzes the received waveform into several narrower subbands and samples each subband with a much lower rate ADC. Then, the multiple digital signal streams are synthesized in the digital domain to reconstruct the digitalized version of the original waveform. Similarly, the limited digital-to-analog (DAC) rate could be a hurdle to the digital UWB transmitter when the signal bandwidth is huge. To address this problem, we propose to process the
subband digital streams with low-rate DAC’s and then synthesize the resultant subband waveforms in analog domain. In this sense, the proposed multiband transmitter can be regarded as the counterpart of the channelized receiver. By this means, the all-digital UWB receiver proposed in [51] can hopefully be realized at a relatively low complexity level.

At our receiver, mixers and correlator-based samplers act as the bandpass filter bank and the Nyquist rate samplers in the channelized UWB receiver. Unlike the channelized UWB receiver, the oversampling or Nyquist sampling is not required by our receiver. As a result, the complexity can be reduced by our receiver with acceptable performance loss (see, e.g., [30]). When $B_t > 1$, our transmitter can be regarded as the channelized UWB transmitter. With the pulse rate of $1/(N_t T_t)$, the same information transmission rate can be achieved as the single-band UWB transmitter with a higher pulse rate $B/(N_t T_t)$. Notice that the drawback of multiband operation of both our transceiver and the channelized UWB receiver (see, e.g., [17, 57]) is that the frequency skew needs to be tracked and calibrated for all frequency generators.

### 3.3 Modeling of the Asymmetric UWB Link

In the preceding section, we have introduced the structure of the UWB transceiver that forms the asymmetric UWB link. Heterogeneity and asymmetry can emerge when the transmitter and the receiver have different system parameters, especially when one node uses multiple bands while the other one uses a single band. Generally, the multiple frequency generators of the multiband node requires higher hardware cost than the single-band node. Therefore, the single-band node is more suitable for the low-complexity and small-sized device, such as the wireless sensor. In the following part of this section, we will investigate the I/O relationship of the asymmetric link and establish a feasible model for information transmission.
3.3.1 Multiband Transmitter and Single-Band Receiver

First, let us consider a scenario where the transmitter operates with \( B_t > 1 \) signal bands and pulse rate \( 1/(N_t T_t) \), and the receiver operates with a single band and pulse rate \( 1/(N_r T_r) \). As described in section 3.2, the bandwidth occupied by the transmitted signal is approximately \( B_t/T_t \), and the bandwidth captured by the receiver is approximately \( 1/T_r \). In order to access the same bandwidth at both the transmitter and the receiver, we require their pulse width to satisfy \( T_t = B_t T_r \).

Given the channel impulse response (CIR) \( h(t) \), the received waveform can be expressed as \( r(t) = v(t) * h(t) + n(t) \), with \( n(t) \) being the additive noise. Carrier demodulating \( r(t) \) by multiplying it with \( \exp(-j2\pi f_c t) \), we have

\[
\begin{align*}
  x(t) &= \sum_{b=0}^{B_t-1} \sum_{n=0}^{\infty} v(n; b) \hat{p}(t - nN_t T_t; b) + \eta(t),
\end{align*}
\]

where \( \hat{p}(t; b) = \exp(j2\pi f_b(b)t) \int p_r(t - \tau) \exp(-j2\pi(f_c + f_b(b))\tau)h(\tau)d\tau \) can be regarded as the equivalent pulse shaper on the \( b \)th band and \( \eta(t) \) is the filtered noise.

To establish Eq. (3–3), we used the fact that \( \exp(j2\pi f_b(b)N_t T_t) = 1 \). The continuous-time signal \( x(t) \) is sampled by the correlator \( p_r(t) \) at intervals of \( N_r T_r \) to generate the discrete-time sequence

\[
\begin{align*}
  x(k) &= \sum_{b=0}^{B_t-1} \sum_{n=0}^{\infty} v(n; b) R_{t,r}(kN_r T_r - nN_t T_t; b) + \eta(k),
\end{align*}
\]

where \( R_{t,r}(t) = \int \hat{p}_r(t + \tau)p_r(\tau)d\tau \).

In Eq. (3–4), the signal sequence \( x(k) \) is the superposition of \( B_t \) data streams riding on the \( B_t \) signal bands. However, even for each band, the equivalent discrete time channel may be time-varying because the transmitter pulse interval could be different from that of the receiver \( (N_r T_r \neq N_t T_t) \), and Eq. (3–4) can not be expressed as the convolution of signal sequences. Generally, \( N_t T_t \) and \( N_r T_r \) may not divide each other and

\[
\begin{align*}
  \frac{N_t T_t}{N_r T_r} = \frac{B_t N_t}{N_r} = \tilde{N}_t \frac{N_t}{N_r}.
\end{align*}
\]
where \( \tilde{N}_t = B_t N_t / \gcd\{B_t N_t, N_r\} \) and \( \tilde{N}_r = N_r / \gcd\{B_t N_t, N_r\} \).

With Eq. \((3-5)\), we can factorize the indices of the transmitted and received samples by \( k = m \tilde{N}_t + q, q \in [0, \tilde{N}_t - 1], n = g \tilde{N}_r + d, d \in [0, \tilde{N}_r - 1] \). Then, we have [c.f. \((3-4)\)]

\[
\tilde{x}(m; q) = \sum_{b=0}^{B_t-1} \sum_{d=0}^{\tilde{N}_t-1} \sum_{g=0}^{\infty} \tilde{v}(g; d, b) R_{t,r} \left( m \tilde{N}_t N_r - g \tilde{N}_r N_t T_r + q N_r T_r - d N_t T_t; b \right) + \tilde{\eta}(m; q),
\]

\((3-6)\)

where \( \tilde{x}(m; q) = x(m \tilde{N}_t + q) \), \( \tilde{\eta}(m; q) = \eta(m \tilde{N}_t + q) \) and \( \tilde{v}(g; d, b) = v(g \tilde{N}_r + d; b) \).

Defining \( \tilde{h}(l; q, d, b) = R_{t,r}(l \tilde{N}_r N_t T_t + q N_r T_r - d N_t T_t; b) \) as the amplitude of the discrete-time equivalent channel taps, we have the following I/O relationship

\[
\tilde{x}(m; q) = \sum_{b=0}^{B_t-1} \sum_{d=0}^{\tilde{N}_t-1} \sum_{g=0}^{\infty} \tilde{v}(g; d, b) \tilde{h}(m - g; q, d, b) + \tilde{\eta}(m; q) \quad q \in [0, \tilde{N}_t - 1].
\]

\((3-7)\)

In deriving Eq. \((3-7)\), we used the relationship \( \tilde{N}_t N_r T_r = \tilde{N}_r N_t T_t \). If the \((d, b)\) pair is mapped to a single index \( p \) by \( p = d + b \tilde{N}_r \), we can rewrite Eq. \((3-7)\) as

\[
\tilde{x}(m; q) = \sum_{p=0}^{B_t \tilde{N}_r - 1} \sum_{g=0}^{\infty} \tilde{v}(g; p) \tilde{h}(m - g; q, p) + \tilde{\eta}(m; q) \quad q \in [0, \tilde{N}_t - 1].
\]

\((3-8)\)

where \( \tilde{v}(g; p) = \tilde{v}(g; d, b) \) and \( \tilde{h}(m - g; q, p) = \tilde{h}(m - g; q, d, b) \).

Eq. \((3-8)\) indicates that when \( B_t \tilde{N}_r \) data streams \( \tilde{v}(n; p), p \in [0, B_t \tilde{N}_r - 1] \) are transmitted, \( \tilde{N}_t \) data streams \( \tilde{x}(n; q), q \in [0, \tilde{N}_t - 1] \) will be received at the receiver. This relationship constitutes an MIMO system with \( B_t \tilde{N}_r \) input ports and \( \tilde{N}_t \) output ports, which has also been utilized for multiantenna systems. The minimum interval for Eq. \((3-8)\) to hold is \( \tilde{N}_r N_t T_r \), during which only one symbol is transmitted through each subchannel of the equivalent MIMO system. For multiantenna systems, the MIMO model captures the spatial characteristic by deploying multiple Tx- and Rx-antennas.

While, for asymmetric UWB links, the MIMO model reflects the transceiver asymmetry due to the different band numbers and different sampling rates. In addition, when the
transmitter and the receiver both use a single band, the asymmetric link becomes a multirate system which has been well investigated in the multirate signal processing area (see, e.g., [12]).

There are benefits when we use Eq. (3–8) to model the asymmetric UWB link. First, although the real channel is time-invariant, the overall discrete time channel $R_{t,r}(\cdot)$ is time-varying, due to the asymmetry between the transmitter and the receiver. However, the equivalent channel for each subchannel remains time-invariant, which makes the subsequent signal processing more convenient. More importantly, after the conversion to the MIMO model, existing system design methods for multiantenna systems can be applied to optimize the asymmetric UWB link in terms of throughput, error performance or complexity (see, e.g., [66, 73, 80]).

Based on the preceding discussions, we have the following result

**Proposition 1.** For the asymmetric UWB link consisting of single-antenna transceivers illustrated in Fig. 3-1, when the transmitter operates with $B_t$ bands and pulse rate of $1/(N_tT_t)$, and the receiver operates with a single band and pulse rate of $1/(N_rT_r)$, the system can be modeled and optimally designed as an MIMO system corresponding to the multiantenna system with $B_t\tilde{N}_t$ Tx-antennas and $\tilde{N}_t$ Rx-antennas, with $\tilde{N}_t = B_tN_t/\gcd\{B_tN_t, N_r\}$ and $\tilde{N}_r = N_r/\gcd\{B_tN_t, N_r\}$.

### 3.3.2 Single-Band Transmitter and Multiband Receiver

Next, let us investigate the modeling of the asymmetric UWB link where the transmitter operates with a single band and the receiver captures the received signal over $B_r$ bands. As stated in the preceding section, the receiver can be regarded as another type of the channelized UWB receiver proposed in [17, 57]. Suppose that the pulse rate of the transmitter is $1/(N_tT_t)$ and the sampling rate at the receiver is $1/(N_rT_r)$, with $T_t$ and $T_r$ being the width of the pulse shapers at the transmitter and receiver, respectively. We choose $T_r = B_rT_t$ such that the transmitter and receiver can access the same bandwidth.
Following the multiband transmitter to single-band receiver case, we can derive the equivalent MIMO system model for the single-band transmitter and multiband receiver link.

**Proposition 2.** For the asymmetric UWB link consisting of single-antenna transceivers illustrated in Fig. 3-1, when the transmitter operates with a single band and pulse rate of $1/(N_t T_t)$, and the receiver operates with $B_r$ bands and pulse rate of $1/(N_r T_r)$, the system can be modeled and optimally designed as an MIMO model corresponding to the multiantenna system with $\tilde{N}_r$ Tx-antennas and $B_r \tilde{N}_t$ Rx-antennas, with $\tilde{N}_t = N_t / \gcd\{N_t, B_r N_r\}$ and $\tilde{N}_r = B_r N_r / \gcd\{N_t, B_r N_r\}$.

### 3.3.3 Special Cases

It has been shown that the asymmetric UWB link can be converted to an equivalent MIMO system model. By selecting some special values of the band number, the Tx pulse rate and the Rx pulse rate, we can realize the conversion with simpler models including both the multiple-input and single-output (MISO) and the single-input and multiple-output (SIMO) models. In particular, an MISO system model with $M$ input ports can be realized by setting $B_t = 1$, $B_r = 1$ and $N_t / N_r = 1/M$, and an SIMO system with $M$ output ports can be realized by setting $B_t = 1$, $B_r = 1$ and $N_t / N_r = M$, with $M$ being an integer. If both the transmitter and receiver operate with a single band and the Tx pulse rate is equal to the Rx sample rate, i.e. $T_r = T_t$ and $N_r = N_t$, the UWB link becomes symmetric and can be modeled by the normal single-input and single-output (SISO) model.

### 3.4 Block Transmission

In Eq. (3–8), the minimum time interval for the MIMO model to hold is $T_s := \tilde{N}_r N_t T_t = \tilde{N}_t N_r T_r$, during which only one symbol is transmitted over each of the equivalent subchannels. Hence, in a block-by-block transmission and processing of information symbols, the block size should be integer multiples of $T_s$. 

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3.4.1 Multiband Transmitter and Single-Band Receiver

Let us consider the block transmission where during each transmission period, $N_s$ information symbols are transmitted over each subchannel of the equivalent MIMO system. During the $n$th transmission period $[nN_s T_s, (n+1)N_s T_s)$, the transmitted symbols are $\tilde{v}(n; p) = [\tilde{v}(nN_s; p), \tilde{v}(nN_s + 1; p), \ldots, \tilde{v}((n+1)N_s - 1; p)]^T$, $p \in [0, B_t \tilde{N}_r - 1]$, and the received signals are $\tilde{x}(n; q) = [\tilde{x}(nN_s; q), \tilde{x}(nN_s + 1; q), \ldots, \tilde{x}((n+1)N_s - 1; q)]^T$, $q \in [0, \tilde{N}_t - 1]$. The block transmission can then be expressed in matrix form as [c.f. (3–8)]

$$\tilde{x}(n; q) = \sum_{p=0}^{B_t \tilde{N}_t - 1} \left[ H(q, p) \tilde{v}(n; p) + \tilde{H}(q, p) \tilde{v}(n-1; p) \right] + \tilde{\eta}(n; q), \quad q \in [0, \tilde{N}_t - 1], \quad (3–9)$$

where $H(q, p)$ and $\tilde{H}(q, p)$ are $N_s \times N_s$ lower and upper triangular Toeplitz matrices, and $\tilde{\eta}(n; q)$ is the associated noise vector. In Eq. (3–9), terms inside the square bracket constitute the block-by-block I/O relationship for a single Tx- and Rx-antenna system, with $\tilde{v}(n - 1; p)$ reflecting the IBI induced by the channel multipath effect (see, e.g., [95]).

With $(q, p)$ varying in the ranges of $q \in [0, \tilde{N}_t - 1]$ and $p \in [0, B_t \tilde{N}_r - 1]$, Eq. (3–9) represents the MIMO block-by-block I/O model of the asymmetric UWB link, and $H(q, p)$ as well as $\tilde{H}(q, p)$ corresponds to the equivalent subchannel between the $p$th input port and $q$th output port of the MIMO model.

With the MIMO representation of Eq. (3–9), we can easily adopt existing multiantenna techniques for the optimal design of the asymmetric UWB link. However, in Eq. (3–9), neither the transmitted nor the received signal is arranged in the order in which they are transmitted or received. In order to complete the Tx and Rx processing, we need to know the relationship between Eq. (3–9) and the “real” system I/O relationship given as follows

$$x(n) = \mathcal{H}v(n) + \tilde{\mathcal{H}}v(n-1) + \eta(n), \quad (3–10)$$
where $\mathbf{x}(n)$ and $\mathbf{v}(n)$ are arranged in the order in which they are transmitted or received; $\mathbf{H}$ and $\tilde{\mathbf{H}}$ are two $N_t \times N_r$ channel matrices. We term Eq. (3–10) as the SISO representation of the asymmetric UWB link consisting of a single Tx-antenna and a single Rx-antenna. Because the discrete time channel in Eq. (3–4) is time-varying, $\mathbf{H}$ and $\tilde{\mathbf{H}}$ are not Toeplitz matrices, which is the essential difference between the MIMO representation (3–9) and the SISO representation (3–10).

In order to establish the relationship between Eqs. (3–9) and (3–10), we first rewrite the MIMO representation Eq. (3–9) with a single channel matrix of larger dimension. Stacking the transmitted and the received signals into vectors $\tilde{\mathbf{v}}(n) = \begin{bmatrix} \tilde{\mathbf{v}}^T(n; 0), \tilde{\mathbf{v}}^T(n; 1), \ldots, \tilde{\mathbf{v}}^T(n; B_t \tilde{N}_r - 1) \end{bmatrix}^T$ and $\tilde{\mathbf{x}}(n) = \begin{bmatrix} \tilde{\mathbf{x}}^T(n; 0), \tilde{\mathbf{x}}^T(n; 1), \ldots, \tilde{\mathbf{x}}^T(n; \tilde{N}_t - 1) \end{bmatrix}^T$, we can rewrite Eq. (3–9) as

$$\tilde{\mathbf{x}}(n) = \mathbf{H}\tilde{\mathbf{v}}(n) + \tilde{\mathbf{H}}\tilde{\mathbf{v}}(n-1) + \tilde{\mathbf{\eta}}(n),$$

where $\tilde{\mathbf{\eta}}(n)$ is the noise vector associated with $\tilde{\mathbf{x}}(n)$. The channel matrix $\mathbf{H}$ collects the channel information of all $\tilde{N}_t \times B_t \tilde{N}_r$ subchannels of the MIMO model as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}(0, 0) & \mathbf{H}(0, 1) & \cdots & \mathbf{H}(0, B_t \tilde{N}_r - 1) \\ \mathbf{H}(1, 0) & \mathbf{H}(1, 1) & \cdots & \mathbf{H}(1, B_t \tilde{N}_r - 1) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}(\tilde{N}_t - 1, 0) & \mathbf{H}(\tilde{N}_t - 1, 1) & \cdots & \mathbf{H}(\tilde{N}_t - 1, B_t \tilde{N}_r - 1) \end{bmatrix}$$

and $\tilde{\mathbf{H}}$ has the same size and structure as $\mathbf{H}$, but with $\mathbf{H}(q, p)$ replaced by $\tilde{\mathbf{H}}(q, p)$, $p \in [0, B_t \tilde{N}_r - 1]$, $q \in [0, \tilde{N}_t - 1]$.

In the Appendix, we will derive the relationship between the MIMO and SISO representations: $\mathbf{v}(n) = \mathbf{P}_r \tilde{\mathbf{v}}(n)$, $\mathbf{x}(n) = \mathbf{P}_r \tilde{\mathbf{x}}(n)$, $\mathbf{H} = \mathbf{P}_r^T \mathbf{H} \mathbf{P}_t$ and $\tilde{\mathbf{H}} = \mathbf{P}_r^T \tilde{\mathbf{H}} \mathbf{P}_t$, with $\mathbf{P}_t$ and $\mathbf{P}_r$ being the Tx and Rx permutation matrices, respectively. Examples of $\mathbf{H}$ and $\mathbf{H}$ are shown in Fig. 3-2 and Fig. 3-3. We notice that after the permutation operation, the equivalent channel matrix $\mathbf{H}$ has a block-wise square and Toeplitz structure.
Combining Eqs. (3–10) and (3–11), we have the following result:

**Proposition 3.** For the asymmetric UWB link captured by an equivalent $B_t \tilde{N}_r \times \tilde{N}_t$ MIMO model, the channel matrices $\mathcal{H}$ and $\tilde{\mathcal{H}}$ can be converted into $(N_s \tilde{N}_t) \times (N_s B_t \tilde{N}_r)$ block-wise Toeplitz matrices as follows:

$$
\mathbf{H} = \mathbf{P}_t^T \mathcal{H} \mathbf{P}_t, \quad \tilde{\mathbf{H}} = \tilde{\mathbf{P}}_r^T \tilde{\mathcal{H}} \tilde{\mathbf{P}}_r, \quad (3–12)
$$

where $\mathbf{P}_t$ and $\tilde{\mathbf{P}}_r$ are permutation matrices. The resulting channel matrices $\mathcal{H}$ and $\tilde{\mathcal{H}}$ both consist of $B_t \tilde{N}_r \tilde{N}_t$ blocks, with their corresponding $(q, p)$th blocks being $N_s \times N_s$ lower and upper triangular Toeplitz matrices generated by the subchannel between the $p$th input port and the $q$th output port of the MIMO model.

**Proof.** The signal vectors $\mathbf{v}(n)$ and $\mathbf{x}(n)$ have the same elements as $\tilde{\mathbf{v}}(n)$ and $\tilde{\mathbf{x}}(n)$ but arranged in a different order. Therefore, $\tilde{\mathbf{v}}(n)$ and $\tilde{\mathbf{x}}(n)$ can be obtained from $\mathbf{v}(n)$ and...
Using the \( N_s \tilde{N}_r \times B_t \) and \( N_b \tilde{N}_t \times N_s \tilde{N}_r \) permutation matrices
\[
P_t = I_{B_t} \otimes [e_1, e_{N_t+1}, \ldots, e_{(N_s-1)N_t+1}, e_2, e_{N_t+2}, \ldots, e_{(N_s-1)N_t+2}, \ldots, e_{N_t}, e_{2N_t}, \ldots, e_{N_sN_t}]
\]
\[
P_r = [e_1, e_{N_t+1}, \ldots, e_{(N_s-1)N_t+1}, e_2, e_{N_t+2}, \ldots, e_{(N_s-1)N_t+2}, \ldots, e_{N_t}, e_{2N_t}, \ldots, e_{N_sN_t}]
\]
by
\[
x(n) = P_r \tilde{x}(n), \quad v(n) = P_t \tilde{v}(n) .
\] (3–13)

These relationships imply that \( \text{c.f. (3–10)} \) \( P_r \tilde{x}(n) = \mathcal{H}P_t \tilde{v}(n) + \hat{\mathcal{H}}P_t v(n-1) + \eta(n) \). As the permutation matrix is orthogonal, we can rewrite this equation as
\[
\tilde{x}(n) = P_t^T \mathcal{H}P_t \tilde{v}(n) + P_t^T \hat{\mathcal{H}}P_t v(n-1) + P_t^T \eta(n) .
\] (3–14)

Comparing Eq. (3–14) with the MIMO representation (3–11), we obtain the relationships of channel matrices between the MIMO and SISO representations which yield Eq. (3–12).

Proposition 3 indicates that, by simply employing two permutation operators at the transmitter and receiver, the time-varying discrete-time channel turns into an MIMO frequency-selective channel with \( B_t \tilde{N}_r \) input ports and \( \tilde{N}_t \) output ports. It is worth mentioning that the permutation \( P_t \) and \( P_r^T \) can be easily implemented with interleavers \( \pi \{ N_s, B_t \tilde{N}_r \} \) and \( \pi \{ \tilde{N}_t, N_s \} \), respectively.

### 3.4.2 Single-Band Transmitter and Multiband Receiver

For this case, we consider a block transmission during which the number of symbols transmitted through each subchannel is \( N_s \). Following the previous subsection, we can also establish the MIMO and SISO representations as Eqs. (3–10) and (3–11) for the asymmetric UWB link composed of the single-band transmitter and the multiband receiver. Similarly, we have the following result:

**Proposition 4.** For the asymmetric UWB link captured by an equivalent MIMO system with \( \tilde{N}_t \) input ports and \( B_t \tilde{N}_t \) output ports, the channel matrices \( \bar{\mathcal{H}} \) and \( \bar{\mathcal{H}} \) can be
converted into \((N_s B_r, \tilde{N}_t) \times (N_s \tilde{N}_r)\) block-wise Toeplitz matrices as follows:

\[
\bar{H} = \bar{P}_t^T \bar{\mathcal{H}} \bar{P}_t, \quad \bar{\bar{H}} = \bar{P}_r^T \bar{\mathcal{H}} \bar{P}_t. \tag{3–15}
\]

where \(\bar{P}_t\) and \(\bar{P}_r\) are the Tx and Rx permutation matrices. The resulting channel matrices \(\bar{H}\) and \(\bar{\bar{H}}\) in the MIMO representation both consist of \(\tilde{N}_t B_r \tilde{N}_r\) blocks, with their corresponding \((q, p)\)th blocks being \(N_s \times N_s\) lower and upper triangular Toeplitz matrices generated by the subchannel between the \(p\)th input port and the \(q\)th output port of the MIMO model, \(p \in [0, \tilde{N}_r - 1], q \in [0, B_r \tilde{N}_t - 1]\). It can be readily proved that \(P_t = \bar{P}_r, \quad P_r = \bar{P}_t\).

### 3.5 Channel Equalization with Multiple-Input and Multiple-Output (MIMO) Signal Processing

In preceding sections, we have seen that the asymmetric UWB link can be represented by an MIMO model. As a result, the time-varying multipath channel can be converted into a group of time-invariant subchannels of the equivalent MIMO system. Therefore, the asymmetric UWB transceiver design problem amounts to the transmission of information through the multipath MIMO channel. Next, we will give an example of integrating existing MIMO techniques for multipath channels into our transceiver design. We will show that the rich multipath diversity in UWB can be harvested in much more flexible ways other than the widely adopted Rake receiver.

Several MIMO communication techniques have been proposed in the multipath channel, such as the generalized delay diversity code (GDD)[27], Lindskog-Paulraj scheme [44] and MIMO-OFDM (see, e.g., [47, 85]), etc. Generally, channel equalizers can be realized in two manners: the time domain equalizer (TDE) such as the maximum-likelihood sequence estimator (MLSE), and the frequency domain equalizer (FDE) including OFDM and the single-carrier frequency domain equalizer (SC-FDE). Same as the SISO system [16], by using the FFT, FDE of MIMO signals [47, 85] can be realized at a complexity much lower than that of TDE [27, 44]. For this reason, in
recent years, almost all multipath channel MIMO techniques are based on the OFDM framework.

With OFDM, the multipath channel can be converted into a group of flat fading subchannels each corresponding to an OFDM subcarrier. Then, MIMO technique is independently applied to each subcarrier. There are generally two types of MIMO techniques: space-time coding (STC) which can improve the error performance by exploiting the diversity gain, and the spatial multiplexing that aims at throughput enhancement. In this dissertation, we will use STC-based MIMO-OFDM as an example of our transceiver design. Its validity can be further verified by comparing the diversity gain with different system parameters via simulations. Note that this diversity is actually due to the multipath effect which is different from the multiantenna diversity exploited by the traditional multiantenna system. Since people are quite familiar with the MIMO-OFDM-based system, we will just briefly introduce how to integrate it into our transceiver design.

3.5.1 Multiband Transmitter and Single-Band Receiver

In the $n$th transmission duration, at the $p$th input port of the MIMO model, a block of $N_c$ information symbols $s(n; p) = [s_0(n; p), s_1(n; p), \ldots, s_{N_c-1}(n; p)]^T$ are multicarrier modulated on $N_c$ orthogonal digital subcarriers to form $\tilde{u}(n; p) = F^H s(n; p)$, where $F$ is the FFT matrix. By doing that, we actually partition the frequency selective channel into $N_c$ frequency flat fading subchannels. As a result, we have $N_c$ parallel independent MIMO systems each based on one subcarrier.

For each subcarrier $k \in [0, N_c - 1]$, the symbols $s_k(n; p)s$, $p \in [0, B_t \tilde{N}_r - 1]$ are independently generated by the STC encoder. The $N_g = (N_s - N_c)$ point guard interval (GI) in the form of ZP or CP is added to each block to mitigate ISI. If ZP is adopted, the symbol vector is generated by $\tilde{v}(n; p) = T_{N_s, N_g} \tilde{u}(n; p)$, with $T_{N_s, N_g} = [I_{N_c}, 0_{N_c \times N_g}]^T$ being the ZP-inserting matrix. Then, the resulting $\tilde{N}_r \times B_t$ data streams $\tilde{v}(n; p)s$ are interleaved and transmitted from the antenna.
At the receiver, remove the GI of the received signal vector at the \( q \)th output port of the MIMO model by multiplying \( \tilde{x}(n; q) \) with the ZP-wrapping matrix \( R_{N_c,N_g} = [I_{N_c}, T_{N_c,N_g}] \). The resulting signal vector is then multicarrier demodulated with FFT operation to generate sequence \( y(n; q) = [y_0(n; q), y_1(n; q), \ldots, y_{N_c}(n; q)]^T = F_{N_c} R_{N_c,N_g} \tilde{x}(n; q) \). With some coarse synchronization, it can be easily shown that [c.f. (3–9)]

\[
y(n; q) = \sum_{p=0}^{B_t N_r - 1} \sqrt{N_c} D(q, p) s(n; p) + \zeta(n; q), \quad q \in [0, \tilde{N}_t - 1],
\]

(3–16)

where \( D(q, p) \) is the \( N_c \times N_c \) diagonal matrix with the diagonal entries being the FFT coefficients of the discrete-time channel between the \( p \)th input port and the \( q \)th output port, and \( \zeta(n; q) \) is the noise vector. For each subcarrier \( k \in [0, N_c - 1] \), the received signals \( y_k(n; q)s, q \in [0, \tilde{N}_t - 1] \) are independently STC decoded to obtain the decision statistics.

Stack \( s(n; p)s \) and \( y(n; q)s \) into vectors \( s(n) = [s^T(n; 0), s^T(n; 1), \ldots, s^T(n; B_t \tilde{N}_r - 1)]^T \) and \( y(n) = [y^T(n; 0), y^T(n; 1), \ldots, y^T(n; \tilde{N}_t - 1)]^T \). Using Proposition 3, we have the Tx and Rx processing for the MIMO-OFDM-based transceiver

\[
v(n) = P_t (I_{N_r,B_r} \otimes T_{N_r,N_g}) (I_{N_r,B_r} \otimes F^H_{N_c}) s(n)
\]

\[
y(n) = (I_{\tilde{N}_r} \otimes F_{N_c}) (I_{\tilde{N}_r} \otimes R_{N_c,N_g}) P^T_r x(n)
\]

(3–17)

3.5.2 Single-Band Transmitter and Multiband Receiver

Similar to the preceding subsection, the MIMO-OFDM transmission can also be realized for the single-band transmitter and multiband receiver link. The required Tx and Rx processing is given by

\[
v(n) = \tilde{P}_t (I_{\tilde{N}_r} \otimes T_{N_r,N_g}) (I_{\tilde{N}_r} \otimes F^H_{N_c}) s(n)
\]

\[
y(n) = (I_{\tilde{N}_r,B_r} \otimes F_{N_c}) (I_{\tilde{N}_r,B_r} \otimes R_{M_e,L_e}) \tilde{P}^T_r x(n)
\]

(3–18)
### 3.6 Simulations

In this section, the transceiver design of asymmetric UWB links will be evaluated via simulations. We use the second-order derivative Gaussian pulse with varying pulse width as the pulse shaper. Simulations are carried out in the IEEE 802.15.3a line-of-sight (LoS) office channel model (CM1) in [19] with a maximum delay spread of about 60ns. In order to avoid the ISI, we use a total of 60ns padding zeros. Length of each signal block is 300ns including ZP. STC techniques are adopted to exploit the diversity which is equal to the product of the input and output port numbers, i.e., the subchannel number of the MIMO system (see, e.g., [89]). It should be noted that the diversity gain exploited here is essentially the channel multipath diversity instead of the multiantenna diversity of the conventional multiantenna system. By altering the system parameters including \((B_t, N_t, T_t)\) and \((B_r, N_r, T_r)\), we can obtain an MIMO system with various structures. All parameter combinations in our simulations together with their corresponding link data rates are shown in table 3-1.

First, we compare the BER performance of different scenarios when the data rate is fixed (see Fig. 3-4). Alamouti code [3] is adopted for the \(2 \times 1\) MIMO model and the maximum ratio combining (MRC) is used for the \(1 \times 2\) one. As a performance benchmark, we also include the BER curve of the SISO model. Given that the duration of each binary phase-shift keying (BPSK) symbol block is 300ns, all three links can

---

**Table 3-1. Multiple-input and multiple-output (MIMO) model of ultra-wideband (UWB) link with varying system parameters**

<table>
<thead>
<tr>
<th>MIMO model</th>
<th>(B_t)</th>
<th>(B_r)</th>
<th>(T_t) (ns)</th>
<th>(T_r) (ns)</th>
<th>(N_t)</th>
<th>(N_r)</th>
<th>Data rate (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 \times 1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>1 \times 2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>400</td>
</tr>
<tr>
<td>2 \times 1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>1 \times 2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>400</td>
</tr>
<tr>
<td>2 \times 1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>3 \times 1</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>266.7</td>
</tr>
<tr>
<td>3 \times 2</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>133.3</td>
</tr>
<tr>
<td>3 \times 1</td>
<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>266.7</td>
</tr>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>133.3</td>
</tr>
</tbody>
</table>
achieve a transmission rate of 400Mbps. Fig. 3-4 shows that with a larger subchannel number, the equivalent $1 \times 2$ SIMO and $2 \times 1$ MISO models outperform the SISO one in terms of diversity gain, which is shown by their larger slopes of the BER curves. This subchannel number versus diversity gain relationship has been observed in multiantenna-based MIMO systems (see, e.g., [67]). The SIMO link outperforms the MISO link in terms of coding gain, which is defined as the horizontal shift of the BER curves with the same coding gain. This implies that increasing the complexity of the receiver can improve the BER performance, which has been seen in multiantenna systems too (see, e.g., [67]).

In Fig. 3-5, the same system models are realized as those in Fig. 3-4 but with a single band at both the transmitter and the receiver. Same as Fig. 3-4, Alamouti code and MRC are used for the $2 \times 1$ and $1 \times 2$ systems, respectively. Similar results can be obtained in all three cases. It is interesting to notice that when MIMO model is established by frequency domain asymmetry (Fig. 3-4), i.e., with different numbers

Figure 3-4. Bit-error rate (BER) performance for asymmetric UWB links for fixed data rate.
Figure 3-5. BER performance for asymmetric UWB links for fixed data rate.

of bands at the transmitter and the receiver, the diversity gain is slightly smaller than that when the MIMO relationship is realized by the unequal pulse rates of single-band transceivers (Fig. 3-5). This is because for the latter case, subchannels are independent since multipaths of the real channel are independent. However, for the former case, subchannels are correlated because the frequency response of the real channel is correlated in frequency domain. As the subchannel dependency induces diversity loss (see, e.g., [90]), systems in Fig. 3-5 will outperform those of Fig. 3-4 in terms of diversity gain.

In Figs. 3-6 and 3-7, we compare the performance of the more general asymmetric UWB links. Orthogonal space-time block codes (OSTBC) are used to exploit the diversity provided by the random channel. OSTBC guarantees that the detection of different symbols can be decoupled, and at the same time the full diversity order can be achieved (see, e.g., [89]) which is equal to subchannel number. From the figures, we can see that the diversity order of the asymmetric link increases as the subchannel
Figure 3-6. BER performance for asymmetric UWB links with orthogonal space-time block code (OSTBC).

Figure 3-7. BER performance for asymmetric UWB links with OSTBC.
number increases. Notice that the data rate decreases as the diversity gain increases. This implies that our transceiver design enables a convenient rate-diversity tradeoff for the single Tx-antenna and single Rx-antenna link. In addition, similar to Figs. 3-4 and 3-5, due to the subchannel dependency, the coding gain of the link between a single-band node and a multiband node is lower than that of the link only consisting of single-band nodes.

3.7 Conclusions

In this Chapter, we established the general UWB transceiver model for asymmetric UWB links. It turns out that the asymmetric UWB link can be modeled as an MIMO system, which allows us to exploit the multipath diversity of the single Tx-antenna and single Rx-antenna system in a very flexible manner. Based on these results, we use OFDM-UWB as an example to show how to integrate MIMO signal processing into our transceiver design. Analysis and simulation results confirm that our transceiver design can enable flexible collection of the rich multipath diversity, as well as facilitate convenient rate-diversity tradeoff of the system.
4.1 Motivation

In UWB IR systems, information is conveyed by low-power impulse-like waveforms. Therefore, timing synchronization for UWB systems is more challenging than for conventional narrowband systems (see e.g., [29, 43]). In [110, 114], TDT algorithms were proposed for synchronization in analog UWB systems. By correlating two consecutive symbol-long signal segments, TDT effectively collects the multipath energy and ensure rapid synchronization without assuming any knowledge on the spreading codes or the propagation channel. The TDT algorithms encompass both DA and NDA modes. The analysis and simulations in [110, 114] reveal that the DA TDT outperforms the NDA TDT; and the performance of the DA TDT can be enhanced at the price of decreased bandwidth efficiency.

We will extend the basic idea of TDT in two different implementation scenarios. First, we investigate the possibility of applying TDT to digital UWB receivers with low-resolution (2-bit or 3-bit) analog-to-digital converters (ADCs) to facilitate the low-complexity digital design (see e.g., [35, 58, 87]). The advantage of digital TDT is that compared to the analog TDT, the delay operation can be readily realized at digital UWB receivers without invoking the ultra-wideband analog delay line (see e.g., [87]). Secondly, we exploit the merit of TDT algorithms for the orthogonal bi-pulse modulation UWB system which uses an even pulse and an odd pulse to convey information symbols in an alternating manner [65]. Due to the employment of orthogonal pulses, the bi-pulse based TDT can avoid the random symbol effect of the original NDA TDT, which was originally accomplished by transmitting training sequence in the DA mode. Therefore, the bi-pulse TDT can improve the synchronization speed and simultaneously preserve the energy efficiency by only slightly increasing the transceiver complexity.
4.2 Timing with Dirty Templates (TDT) Algorithm for Low-Resolution Digital UWB Receivers

It is well known that for wireless communication systems, the correct reception of information depends on the correct synchronization of signals at the receiver and the system’s error rate performance is largely determined by the performance of the timing synchronizer (see e.g., [29, 43, 101]). For this reason, we will apply TDT to digital UWB receivers with low-resolution ADCs. We will prove that the digital TDT algorithms remain operational without knowledge of the spreading codes or the multipath propagation channel, even in the presence of both the additive noise and the quantization error. In addition, we evaluate and compare the performance of the timing algorithms at DA and NDA modes for digital UWB receivers at ADCs of different resolution levels.

4.2.1 Digital IR UWB

In IR UWB systems, every information symbol is transmitted over a $T_s$ period that consists of $N_f$ frames. During each frame of duration $T_f$, a data-modulated short pulse $p(t)$ with duration $T_p \ll T_f$ is transmitted. Though TDT was shown to be able of handling pulse position modulation (PPM) and multi-user applications [110, 114], we will focus on a peer-to-peer scenario with pulse amplitude modulation (PAM). Specifically, the transmitted waveform is

$$v(t) = \sqrt{E} \sum_{k=0}^{\infty} s(k)p_T(t - kT_s), \quad (4-1)$$

where $E$ is the energy per pulse, $s(k)$ is the $k$th information symbol with $E\{s^2(k)\} = 1$ and $p_T(t)$ denotes the transmitted symbol-long waveform

$$p_T(t) = \sum_{n=0}^{N_f-1} c_{ds}(n) \cdot p(t - nT_f - c_{th}(n)T_c). \quad (4-2)$$

Notice that $p_T(t)$ can be regarded as the symbol-level pulse shaper which accounts for the time-hopping (TH) and/or direct-sequence (DS) spreading.
Let \( g(t - \tau_0) \) denote the multipath channel with propagation delay \( \tau_0 \). Then, the received waveform can be expressed as

\[
r(t) = \sqrt{E} \sum_{k=0}^{\infty} s(k)p_R(t - kT_s - \tau_0) + n(t),
\]

where \( p_R(t) := p_T(t) * g(t) \) is the aggregate symbol-long received waveform with * denoting convolution, and \( n(t) \) represents the zero-mean additive white Gaussian noise (AWGN) with power spectral density \( N_0/2 \). As in [110, 114], we assume that the support of \( p_R(t) \) is bounded by \( T_s \), which means that there is no ISI. However, it is worth emphasizing that IFI is allowed and can be significant, especially when TH codes are employed.

The received signal \( r(t) \) is matched-filtered by \( p(-t) \) and sampled every \( T_t \) seconds with a low-resolution ADC [87]. Without loss of generality, we assume that \( T_s = N_T T_t \). Then, the waveform received in one symbol duration becomes \( N_T \) digital samples given by [c.f. (4–3)]

\[
u(n) = \sqrt{E} \sum_{k=0}^{\infty} s(k)\tilde{p}_R(n - kN_T - N_{\tau_0}) + \eta(n) + e(n)
\]

where \( N_{\tau_0} = \lfloor \frac{\tau_0}{T_t} \rfloor \) is the propagation delay, \( \{\tilde{p}_R(n)\}_{n=0}^{N_T-1} \) are the ADC outputs corresponding to the symbol-long waveform \( p_R(t) \), and \( e(n) \) is the quantization error due to the finite resolution of the ADC.

4.2.2 Digital TDT Algorithms

In [114], TDT algorithms were developed to synchronize UWB signals using the continuous-time continuous-value received waveform. In the following, we will introduce digital TDT algorithms to estimate \( N_{\tau_0} \). Following the idea of dirty templates [114], these algorithms will rely on pairs of successive groups each consisting of \( N_T \) samples \( u(n) \), taken at candidate time shifts \( N_\tau \in [0, \ldots, N_T - 1] \). The groups in each pair then serve
as templates for each other to generate the symbol-rate samples

$$x(k; N_r) = \sum_{n=0}^{N_T-1} u(n + 2kN_T + N_r) u(n + (2k - 1)N_T + N_r)$$

(4–5)

$$\forall k \in [1, +\infty), \ N_r \in [0, \ldots, N_T - 1].$$

Let $\chi(k; N_r)$ and $\rho(n)$ denote the noise-free parts of $x(k; N_r)$ and $u(n)$, respectively. In the absence of noise, applying the Cauchy-Schwartz inequality to (4–5), we obtain

$$\chi^2(k; N_r) \leq \sum_{n=0}^{N_T-1} \rho^2(n + 2kN_T + N_r) \sum_{n=0}^{N_T-1} \rho^2(n + (2k - 1)N_T + N_r).$$

(4–6)

Following the proof in [114], we know that equality holds in (4–6) for any sequence of information symbols if and only if $N_{\tau_0} = N_r$; i.e., $\chi^2(k; N_r)$ reaches its maximum when $N_{\tau_0} = N_r$.

With an analog UWB receiver, this observation is proved to remain valid even in the presence of noise [114]. However, the only noise present there is the additive Gaussian noise independent of the received waveform. For digital UWB receivers with low-resolution ADC, the noise component in the digital samples $u(n)$ is much dependent on the noise-free part of the waveform. Next, we will analyze the statistical characteristics of the noise at the ADC output and prove that the observation of (4–6) holds true even in the presence of both the additive Gaussian noise $\eta(n)$ and quantization error $e(n)$.

4.2.2.1 Noise analysis

Let $\tilde{N}_{\tau_0} := \lfloor N_{\tau_0} - N_r \rfloor N_r$ denote the residual synchronization error, where $[a]_b$ represents the modulo operation on $a$ with base $b$. The $k$th symbol-rate sample defined in (4–5) can be more explicitly expressed as

$$x(k; N_r) = s(2k - 1) \left( s(2k - 2)\mathcal{E}_A(\tilde{N}_{\tau_0}) + s(2k)\mathcal{E}_B(\tilde{N}_{\tau_0}) \right) + \zeta(k; N_r).$$

(4–7)
where \( \mathcal{E}_A(\tilde{N}_r) := \mathcal{E} \cdot \sum_{n=\tilde{N}_r-N_T}^{N_T-1} \tilde{p}_R^2(n) \), \( \mathcal{E}_B(\tilde{N}_r) := \mathcal{E} \cdot \sum_{n=0}^{N_T-\tilde{N}_r-1} \tilde{p}_R^2(n) \), and the noise term \( \zeta(k; N_r) \) consists of two parts
\[
\zeta_1(k; N_r) := \sum_{n=0}^{\tilde{N}_r-1} \sqrt{\mathcal{E}} \left( s(2k-2)\nu(n + 2kN_T + N_r) + s(2k-1)\nu(n + (2k-1)N_T + N_r) \right) \tilde{p}_R([n + N_T - \tilde{N}_r]_{N_T})
\]
\[
+ \sum_{n=0}^{\tilde{N}_r-1} \nu(n + (2k-1)N_T + N_r) \nu(n + 2kN_T + N_r),
\]
(4–8)
\[
\zeta_2(k; N_r) := \sum_{n=\tilde{N}_r}^{N_T-1} \sqrt{\mathcal{E}} \left[ s(2k-1)\nu(n + 2kN_T + N_r) + s(k)\nu(n + (2k-1)N_T + N_r) \right] \tilde{p}_R([n + N_T - \tilde{N}_r]_{N_T})
\]
\[
+ \sum_{n=\tilde{N}_r}^{N_T-1} \nu(n + (2k-1)N_T + N_r) \nu(n + 2kN_T + N_r),
\]
(4–9)
where \( \nu(n) := \eta(n) + e(n) \) captures the aggregate effects of the additive noise and the quantization error.

Unlike the high resolution ADC, the uniform quantization noise assumption does not hold for the low-resolution 2- or 3-bit ADCs [84]. In fact, for the low-resolution ADCs used at digital UWB receivers [87], the assumption that quantization error is uniform and independent of the input signal is not valid. For low-resolution ADCs, we found that:

**Proposition 5.** The noise terms \( \zeta_1(k; N_r) \) and \( \zeta_2(k; N_r) \) are independent random variables with means
\[
m_1(k; N_r) := \mathbb{E}\{\zeta_1(k; N_r)\} = s(2k-2)s(2k-1)m_1(N_r)
\]
\[
m_2(k; N_r) := \mathbb{E}\{\zeta_2(k; N_r)\} = s(2k-1)s(2k)m_2(N_r),
\]
where \( m_1(N_r) \) and \( m_2(N_r) \) are independent of \( k \) and \( m_k := m_1(N_r) + m_2(N_r) \) is a constant independent of \( N_r \). In addition, their variances \( \sigma_1^2(N_r) := \text{var}\{\zeta_1(k; N_r)\} \) and \( \sigma_2^2(N_r) := \text{var}\{\zeta_2(k; N_r)\} \) are independent of \( k \), and \( \sigma_k^2 := \sigma_1^2(N_r) + \sigma_2^2(N_r) \) is a constant.
Proof. To prove Proposition 5, we will start with the mean of the noise term $\zeta_1(k; N_r)$.

The quantization error $e(n)$ and the zero-mean Gaussian noise $\eta(n)$ are independent. Therefore, given the transmitted symbols $s(2k)$, $s(2k-1)$ and $s(2k-2)$, the mean of $\zeta_1(k; N_r)$ is given by:

$$
E\{\zeta_1(k; N_r)\} = \sum_{n=0}^{\tilde{N}_r-1} \sqrt{\varepsilon} \{ s(2k-2)E\{e(n+2kN_T+N_r)\} + s(2k-1) \\
\times E\{e(n+(2k-1)N_T+N_r)\}\} \tilde{p}_R([n+N_T-\tilde{N}_r]N_r) \\
+ \sum_{n=0}^{\tilde{N}_r-1} E\{e(n+(2k-1)N_T+N_r)\} E\{e(n+2kN_T+N_r)\}.
$$

(4–10)

To find out the mean of the quantization error $e(n+kN_T+N_r)$ for any $k$ and $n \in [0, \ldots, \tilde{N}_r-1]$, we notice that its corresponding input signal

$$
\gamma(n; k; N_r) := \sqrt{\varepsilon} s(k-1) \tilde{p}_R([n+N_T-\tilde{N}_r]N_r) + \eta(n+kN_T+N_r)
$$

is Gaussian distributed with mean $\sqrt{\varepsilon} s(k-1) \tilde{p}_R([n+N_T-\tilde{N}_r]N_r)$ and variance $N_0/2$. For a $b$-bit roundoff ADC of uniform resolution, the resolution is $\Delta = 2\sqrt{\varepsilon}n/2^b$. According to the quantization error analysis in [84], the mean of $e(n+kN_T+N_r)$ is given by

$$
E\{e(n+kN_T+N_r)\} = \sum_{i=1}^{\infty} \frac{\Delta}{\pi i} \frac{(-1)^i}{\pi i} \exp \left( -\frac{\pi^2 i^2 N_0}{\Delta^2} \right) \\
\times \sin \left( \frac{2\pi i \sqrt{\varepsilon} s(k-1) \tilde{p}_R([n+N_T-\tilde{N}_r]N_r)}{\Delta} \right).
$$

(4–11)

Notice that for the PAM modulation, the transmitted symbol $s(k)$ takes $\pm 1$ values. Hence, $E\{e(n+kN_T+N_r)\}$, $n \in [0, \ldots, \tilde{N}_r-1]$ can be re-expressed as

$$
E\{e(n+kN_T+N_r)\} = s(k-1)m_e(n, N_r)
$$

(4–12)

where

$$
m_e(n, N_r) := \sum_{i=1}^{\infty} \frac{\Delta}{\pi i} \frac{(-1)^i}{\pi i} \exp \left( -\frac{\pi^2 i^2 N_0}{\Delta^2} \right) \sin \left( \frac{2\pi i \sqrt{\varepsilon} \tilde{p}_R([n+N_T-\tilde{N}_r]N_r)}{\Delta} \right).
$$

(4–13)
As a result, the mean of $\zeta_1(k; N_r)$ can be expressed as

$$E\{\zeta_1(k; N_r)\} = s(2k - 2)s(2k - 1)m_1(N_r)$$

(4–14)

$$m_1(N_r) := \sum_{n=0}^{\tilde{N}_r-1} \left(2\sqrt{\mathcal{E}}m_e(n, N_r)\tilde{p}_R([n + N_T - \tilde{N}_r]_{N_T}) + m_e^2(n, N_r)\right),$$

where $m_1(N_r)$ is independent of $k$ (i.e., independent of the transmitted symbols).

Likewise, for $n \in [\tilde{N}_r, \ldots, N_T - 1]$, we have

$$E\{\zeta_2(k; N_r)\} = s(2k - 1)s(2k)m_2(N_r)$$

(4–15)

$$m_2(N_r) := \sum_{n=\tilde{N}_r}^{N_T-1} \left(2\sqrt{\mathcal{E}}m_e(n, N_r)\tilde{p}_R([n + N_T - \tilde{N}_r]_{N_T}) + m_e^2(n, N_r)\right).$$

Therefore, the summation of $m_1(N_r)$ and $m_2(N_r)$,

$$m_\zeta := m_1(N_r) + m_2(N_r) = \sum_{n=\tilde{N}_r}^{N_T-1} \left[2\sqrt{\mathcal{E}}m_e(n, N_r)\tilde{p}_R([n + N_T - \tilde{N}_r]_{N_T}) + m_e^2(n, N_r)\right]$$

is independent of both $k$ and $N_r$.

Next, let us consider the variance of the noise at the ADC output. Given the transmitted symbols $s(2k - 2), s(2k - 1)$ and $s(2k)$, the variance of $\zeta_1(k; N_r)$ can be expressed as

$$\sigma_1^2(N_r) := \text{var}\{\zeta_1(k; N_r)\} = \sum_{n=0}^{\tilde{N}_r-1} \text{var}\{\zeta_1(n; k; N_r)\}$$

(4–16)

$$= \sum_{n=0}^{\tilde{N}_r-1} \left(E\{\zeta_1^2(n; k; N_r)\} - E^2\{\zeta_1(n; k; N_r)\}\right),$$

where $\zeta_1(n; k; N_r) := \sqrt{\mathcal{E}}[s(2k - 2)\nu(n + 2kN_T + N_r) + s(2k - 1)\nu(n + (2k - 1)N_T + N_r)]\tilde{p}_R([n + N_T - \tilde{N}_r]_{N_T}) + \nu(n + (2k - 1)N_T + N_r)\nu(n + 2kN_T + N_r).$ In the analysis of the mean of noise, we have known that $E^2\{\zeta_1(n; k; N_r)\}$ is independent of the transmitted symbols. We will next show that $E\{\zeta_1^2(n; k; N_r)\}$ is also independent of the transmitted symbols.
symbols. The mean of $\zeta_1^2(n; k; N_r)$ can be expressed as

$$
E\{\zeta_1^2(n; k; N_r)\} = \mathcal{E} \left[ E\{\nu^2(n + (2k - 1)N_T + N_r)\} + 2m_e^2(n; N_r) \right. \\
+ E\{\nu^2(n + 2kN_T + N_r)\} \tilde{\rho}_R^2([n + N_T - \tilde{N}_{\tau_0}]_{N_T}) \\
+ E\{\nu^2(n + (2k - 1)N_T + N_r)\nu^2(n + 2kN_T + N_r)\} \\
+ 2\sqrt{\mathcal{E}} E\{\nu^2(n + (2k - 1)N_T + N_r)m_e(n; N_r) \} \\
+ \nu^2(n + 2kN_T + N_r)m_e(n; N_r),
$$

(4–17)

where

$$
E\{\nu^2(n + kN_T + N_r)\} = \frac{N_0}{2} + 2E\{\gamma(n; k; N_r)e(n + kN_T + N_r)\} \\
- 2\sqrt{\mathcal{E}}s^2(k - 1)\tilde{\rho}_R([n + N_T - \tilde{N}_{\tau_0}]_{N_T})m_e(n; N_r) \\
+ E\{\nu^2(n + kN_T + N_r)\},
$$

(4–18)

Following [84], we have

$$
E\{e^2(n + kN_T + N_r)\} = \frac{\Delta^2}{12} + \sum_{l=1}^{\infty} \frac{\Delta^2}{\pi^2 l^2} (-1)^l \exp \left( -\frac{\pi^2 l^2 N_0}{\Delta^2} \right) \\
\times \cos \left( \frac{2\pi l\sqrt{\mathcal{E}}s(k - 1)\tilde{\rho}_R([n + N_T - \tilde{N}_{\tau_0}]_{N_T})}{\Delta} \right),
$$

(4–19)

which is independent of $s(k - 1)$ when $s(k - 1)$ takes $\pm 1$ values. As a result, we have

$$
E\{\gamma(n; k; N_r)e(n + kN_T + N_r)\} \\
= \frac{\Delta}{\pi} \sum_{l=1}^{\infty} (-1)^l \left[ \frac{\sqrt{\mathcal{E}}s(k - 1)\tilde{\rho}_R([n + N_T - \tilde{N}_{\tau_0}]_{N_T})}{l} \right. \\
\times \sin \left( \frac{2\pi l\sqrt{\mathcal{E}}s(k - 1)\tilde{\rho}_R([n + N_T - \tilde{N}_{\tau_0}]_{N_T})}{\Delta} \right) \\
+ \frac{\pi N_0}{\Delta} \cos \left( \frac{2\pi l\sqrt{\mathcal{E}}s(k - 1)\tilde{\rho}_R([n + N_T - \tilde{N}_{\tau_0}]_{N_T})}{\Delta} \right) \left. \right] \\
\times \exp \left( -\frac{\pi^2 l^2 N_0}{\Delta^2} \right)
$$

(4–20)

which is also independent of $s(k - 1)$ when $s(k - 1)$ takes $\pm 1$ values. Substituting (4–17)-(4–20) to (4–16), we obtain that $\sigma_1^2(N_r)$ is independent of the transmitted
symbols, and so is true for $\sigma_1^2(N_r)$. Because $\tilde{p}_R([n + N_T - \tilde{N}_{\tau_0}]N_T)$ is a periodic function with the period $N_T$, $\sigma_1^2 = \sigma_1^2(N_r) + \sigma_2^2(N_r)$ is independent of $N_r$ and the transmitted symbols. This concludes the proof.

Using Proposition 5, and by the central limit theorem, $x(k; N_r)$ in (4–7) can be approximated as a Gaussian random variable with variance $\text{var}\{x(k; N_r)\} = \sigma_1^2$ and mean

$$\mathbb{E}\{x(k; N_r)\} = s(2k - 1)(s(2k - 2)\tilde{E}_A(\tilde{N}_{\tau_0}) + s(2k)\tilde{E}_B(\tilde{N}_{\tau_0})),$$

where

$$\tilde{E}_A(\tilde{N}_{\tau_0}) := E_A(\tilde{N}_{\tau_0}) + m_1(N_r)$$

$$\tilde{E}_B(\tilde{N}_{\tau_0}) := E_B(\tilde{N}_{\tau_0}) + m_2(N_r).$$

With proper setting of the scale of the low-resolution ADC, both $\tilde{E}_A(\tilde{N}_{\tau_0})$ and $\tilde{E}_B(\tilde{N}_{\tau_0})$ are positive $\forall \tilde{N}_{\tau_0} \in [0, \ldots, N_T - 1]$. Accordingly, the mean square of the samples in (4–7) can be obtained as

$$\mathbb{E}\{x^2(k; N_r)\} = \mathbb{E}^2\{x(k; N_r)\} + \text{var}\{x(k; N_r)\} = \frac{1}{2} \mathcal{E}_D(\tilde{N}_{\tau_0}) + \frac{\mathcal{E}_R}{2} + \sigma_1^2,$$

where $\mathcal{E}_R := \tilde{E}_A(\tilde{N}_{\tau_0}) + \tilde{E}_B(\tilde{N}_{\tau_0})$ is a constant and $\mathcal{E}_D(\tilde{N}_{\tau_0}) := \tilde{E}_A(\tilde{N}_{\tau_0}) - \tilde{E}_B(\tilde{N}_{\tau_0})$. When $\tilde{N}_{\tau_0} = 0$, $\mathcal{E}_D(\tilde{N}_{\tau_0})$ reaches its maximum value $\mathcal{E}_R$. This proves that, even in the presence of both the additive noise and quantization error, the mean-square of the symbol-rate samples $x(k; N_r)$ approaches the maximum when perfect synchronization is achieved.

Based on this principle, the digital version of the TDT algorithms can be established, enabling both the NDA and the DA modes.

### 4.2.2.2 Timing synchronizer

A general expression of the timing offset (propagation delay) estimator is given by:

$$\hat{N}_{\tau_0} = \arg \max_{N_r} \mathcal{Y}(K; N_r),$$

(4–22)
where \( y(K; N_T) \) is the objective function. Depending on the operating mode of the estimator, the objective function \( y(K; N_T) \) can be formed differently, as we will specify next.

In the NDA mode, the objective function is formulated as:

\[
y_{nda}(K; N_T) = \frac{1}{K} \sum_{k=1}^{K} \left( \sum_{n=2kN_T}^{(2k+1)N_T-1} u(n + N_T)u(n + N_T - N_T) \right)^2,
\]

which enables timing acquisition even when TH and/or DS codes are present and the UWB multipath channel remains unknown. Notice that (4–23) is essentially the sample mean form of the ensemble expression in (4–21). Hence, the maximum of \( y_{nda}(K; N_T) \) will give the estimate of \( N_{T_0} \).

One major advantage of \( y_{nda}(K; N_T) \) is that it can be applied to any information symbol sequence transmitted. However, the timing acquisition process can be expedited when the training sequence is employed [114]:

\[
s(k) = (-1)^{\lfloor k/2 \rfloor}.
\]

Using this sequence, the objective function can still be formulated using (4–23). To distinguish from the NDA mode, however, we will henceforth denote it with \( y_{dA1}(K; N_T) \).

In fact, with the training sequence in (4–24), the averaging and the squaring operations can be exchanges to obtain better estimates of the mean-square of \( x(k; N_T) \) in (4–21). Such variants give rise to the following formulations of the objective function

\[
y_{dA2}(K; N_T) = \left( \frac{1}{K} \sum_{k=1}^{K} \sum_{n=2kN_T}^{(2k+1)N_T-1} u(n + N_T)u(n + N_T - N_T) \right)^2,
\]

and

\[
y_{dA3}(K; N_T) = \left( \sum_{n=0}^{N_T-1} \bar{u}(n + N_T)\bar{u}(n + N_T - N_T) \right)^2,
\]

where

\[
\bar{u}(n) := \frac{1}{K} \sum_{k=1}^{K} (-1)^k u(n + 2kN_T).
\]
It should be noted that, different from the ones in [114] for analog UWB receivers, the digital TDT algorithms here do not entail any analog delay lines. In the next subsection, we will compare the detection performance of these digital TDT algorithms.

### 4.2.3 Comparison of Digital TDT Algorithms

We consider a coarse timing setup where, instead of estimating the true $N_\tau_0$, the receiver partitions the symbol duration $N_T$ into $N_i$ intervals each of duration $N_i := N_T / N_i$. The timing acquisition then amounts to finding $n^*$ that maximizes the objective function among possible candidates $n \in [0, N_i - 1]$; that is

$$
\hat{n}^* = \arg \max_{n \in [0, N_i - 1]} y(K; n N_i).
$$

(4–27)

The probability of detection is then given by

$$
P_d = \Pr\{\hat{n}^* = n^*\} = \Pr\{y(K; n^* N_i) = \max_n y(K; n N_i)\}.
$$

(4–28)

Denoting the pdf of $y(K; n N_i)$ as $f_{K,n}(y)$, the probability of detection can be explicitly written as

$$
P_d = \int_{-\infty}^{+\infty} f_{K,n}(y_0) \prod_{n \neq n^*} \int_{-\infty}^{y_0} f_{K,n}(y_1) dy_1 dy_0,
$$

which involves $N_i$-fold integration and is cumbersome to evaluate. Hence, we employ a lower bound of $P_d$

$$
P_d := \prod_{n \neq n^*} \int_{-\infty}^{+\infty} f_{K,n}(y_0) \int_{-\infty}^{y_0} f_{K,n}(y_1) dy_1 dy_0.
$$

(4–29)

This bound has been proved to be universally tighter than the union bound [37, 114]. Arguing through the central limit theorem, we can regard $y(K; n N_i)$s as Gaussian distributed [114]. As a result, we have

$$
P_d = \prod_{n \neq n^*} Q \left( \frac{m_y(K; n^* N_i) - m_y(K; n N_i)}{\sqrt{\sigma_y^2(K; n^* N_i) + \sigma_y^2(K; n N_i)}} \right),
$$

(4–30)

where $m_y(K; n N_i)$ and $\sigma_y^2(K; n N_i)$ are the mean and variance of the objective function $y(K; n N_i)$. 

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The means and variances of the different objective functions can be readily obtained using Proposition 5 as follows:

\[ m_{y_{nda}}(K; N_r) = \frac{1}{2} \mathcal{E}_D^2(\tilde{N}_r) + \frac{\mathcal{E}_R^2}{2} + \sigma^2_\zeta \]
\[ m_{y_{da1}}(K; N_r) = \mathcal{E}_D^2(\tilde{N}_r) + \sigma^2_\zeta \]
\[ m_{y_{da2}}(K; N_r) = \mathcal{E}_D^2(\tilde{N}_r) + \frac{1}{K} \sigma^2_\zeta \]
\[ m_{y_{da3}}(K; N_r) = \mathcal{E}_D^2(\tilde{N}_r) + \sigma^2_\zeta \]

and

\[ \sigma^2_{y_{nda}}(K; N_r) = \frac{2\sigma^2}{K} \left[ \mathcal{E}_D^2(\tilde{N}_r) + \mathcal{E}_R^2(\tilde{N}_r) + \sigma^2_\zeta \right] + \frac{4}{K} \mathcal{E}_A^2(\tilde{N}_r) \mathcal{E}_B^2(\tilde{N}_r) \]
\[ \sigma^2_{y_{da1}}(K; N_r) = \frac{2\sigma^2}{K} \left[ 2\mathcal{E}_D^2(\tilde{N}_r) + \sigma^2_\zeta \right] \]
\[ \sigma^2_{y_{da2}}(K; N_r) = \frac{2\sigma^2}{K} \left[ 2\mathcal{E}_D^2(\tilde{N}_r) + \frac{\sigma^2_\zeta}{K} \right] \]
\[ \sigma^2_{y_{da3}}(K; N_r) = 2\sigma^2_\zeta \left[ 2\mathcal{E}_D^2(\tilde{N}_r) + \sigma^2_\zeta \right] \]

where \( \sigma^2_\zeta := \frac{\sigma^2_1(N_r)}{K} + \frac{\sigma^2_2(N_r)}{K^2} \) and, in deriving the mean and variance of \( y_{da3}(K; N_r) \), we assumed the independence between the noise terms \( \zeta_1(k; N_r) \) and \( \zeta_2(k; N_r) \).

From (4–31) and (4–32), it follows that

\[ \sigma^2_{y_{nda}}(K; nT_i) > \sigma^2_{y_{da1}}(K; nT_i) > \sigma^2_{y_{da2}}(K; nT_i) > \sigma^2_{y_{da3}}(K; nT_i) \]

where we used \( \sigma^2_\zeta = \frac{\sigma^2_1(N_r)}{K} + \frac{\sigma^2_2(N_r)}{K^2} < \frac{\sigma^2_1(N_r)}{K} + \frac{\sigma^2_2(N_r)}{K} \) in establishing the last inequality. This is verified by Fig. 4-5.

From these mean and variance values, together with (4–30), it readily follows that

\[ P_{d,nda} < P_{d,da1} < P_{d,da2} < P_{d,da3} \]

The analysis shows that the DA-TDT outperforms the NDA-TDT. In addition, even with the same training pattern, the DA estimators can have different performance with slightly different objective functions. However, unlike the analog TDT synchronizers
where the performance improvement comes at the price of increased complexity, all DA-TDT algorithms here have nearly the same complexity. Therefore, the digital TDT with objective function \( y_{nda3}(K; N_r) \) is preferable when training is possible.

Besides the type of the timing estimator, the ADC resolution will also affect the acquisition performance. To gain some intuition, we notice that, as the resolution of the ADC increases, the quantization error becomes approximately uniformly distributed on \([-\frac{\Delta}{2}, \frac{\Delta}{2}]\). The total noise variance (4–18) then approaches \( \frac{N_0}{2} + \frac{\Delta^2}{12} \), where \( \frac{N_0}{2} \) comes from the additive noise and \( \frac{\Delta^2}{12} \) from the quantization error. Therefore, the smaller the quantization step \( \Delta \), the smaller the noise variance is and the better the timing acquisition performance.

**4.2.4 Simulations**

In this section, we will evaluate the performance of the four digital TDT algorithms with simulations. We will refer to algorithms using objective functions \( y_{nda}, y_{da1}, y_{da2} \) and \( y_{da3} \) as Algorithm 1-4, respectively. The simulations are performed in a Saleh-Valenzuela
Figure 4-2. Probability of detection versus ADC resolution, *Algorithm 4*, $K = 8$.

Figure 4-3. Probability of detection vs. lower bounds, $K = 8$. 
Figure 4-4. Probability of detection versus K with $\mathcal{E}/N_0 = -2$ dB, 2-bit ADC.

Figure 4-5. Variances of $y_{nda}(K; N_T)$, $y_{da1}(K; N_T)$, $y_{da2}(K; N_T)$ and $y_{da3}(K; N_T)$; $K=8$, $N_T=32$; $\mathcal{E}/N_0 = 2$ dB.
channel [79] with parameters \( (1/\Lambda, 1/\lambda, \Gamma, \gamma) = (2, 0.5, 30, 5) \text{ns} \). The sample frequency of the low resolution ADC is 1GHz. Each symbol duration contains \( N_f = 32 \) frames with \( T_f = 50 \text{ns} \). We use a random TH code uniformly distributed over \( [0, N_c - 1] \) with \( N_c = 50 \) and \( T_c = 1 \text{ns} \). In all simulations, only frame-level coarse timing is performed.

First, we test the probability of detection for all synchronizers. With a 2-bit ADC, the digital signal only has three levels: 0 and \( \pm V_m \), where \( V_m \) is the maximum scale of the ADC. In this Chapter, we set \( V_m \) as the amplitude of the transmitted pulse \( p(t) \). Fig. 4-1 compares the performance of the synchronizers with the 2-bit digital signal and the sampled signal with an ideal ADC of infinite resolution. The results show that under all SNR values, the detection performance improves from Algorithm 1 to Algorithm 4. We also observe that, even with the same training pattern, Algorithm 4 outperforms the other two DA-TDT algorithms. It also shows that even with a low-complexity 2-bit ADC, the acquisition performance degrades by only 1dB. Increasing the ADC’s resolution can improve the probability of detection (see Fig. 4-2). However, difference of the acquisition performance with ADCs of 2-bit, 3-bit and infinite resolution is very small. The applicability of TDT algorithms is justified with the use of a low-cost and low-resolution ADC.

In Fig. 4-3, we plot the simulated probability of detection together with the lower bounds. The bounds can predict the relative performance of Algorithm 1-4 very well. The theoretical analysis has proved that our estimators are mean-square-sense (MSS) consistent; i.e., the variance of detection variables decreases as \( K \) increases (see (4–32)). This is corroborated by Fig. 4-4. We observe that with \( K = 32 \), even at a very low \( E/N_0 \) value, Algorithm 4 can enable a detection probability close to 100%. More importantly, with a low-resolution digital UWB receiver, this is achieved without increasing the complexity as with the analog TDT in [114].
4.3 Timing and Differential (De)Modulation for Orthogonal Bi-Pulse UWB

In the next, we will continue investigating the timing synchronization of UWB signals. In particular, we exploit the merit of TDT algorithms for the orthogonal bi-pulse modulation UWB system which uses an even pulse and an odd pulse to convey information symbols in an alternating manner [65]. Although proposed for the single-pulse UWB, TDT is proved to be operational for the bi-pulse system. As the original TDT [114], our timing algorithm also relies on correlating adjacent waveform segments. In particular, a synchronization will be asserted when the correlation function reaches its maximum. Due to the employment of orthogonal pulses, the bi-pulse based TDT can avoid the random symbol effect of the original NDA TDT, which was originally accomplished by transmitting training sequence in the DA mode. Therefore, the bi-pulse TDT can improve the synchronization speed and simultaneously preserve the energy efficiency by only slightly increasing the transceiver complexity.

It is interesting to notice that for the bi-pulse orthogonal UWB IR, the idea of TDT can readily enable a noncoherent demodulation scheme when information bits are differentially modulated on adjacent symbols. Similar to the TR UWB and the differential UWB [33, 75], our approach remains operational when channel estimation is bypassed. It is known that performance of these two systems degrades when the timing error exists. This is because mistiming induces ISI, while these methods both ignore the ISI. To amend this problem, the noncoherent UWB was proposed to explicitly deal with the ISI using Viterbi algorithm, which enables a maximum-likelihood (ML) demodulation [116]. Compared to these existing techniques, by using orthogonal pulses, our noncoherent algorithm completely avoids the ISI even in the presence of timing errors. As a result, our algorithms only entail simple differential demodulator of differentially modulated UWB signals, while retaining the ML optimality.

Both the timing synchronization and demodulation can be realized in a single framework which is characterized by extracting and correlating symbol-level waveforms.
Due to the use of the orthogonal pulse pair, both algorithms can be realized in a more convenient manner than the original IR. In the following, advantages of orthogonal bi-pulse modulation will be gradually revealed as we introduce the synchronization and demodulation algorithms.

### 4.3.1 Bi-Pulse UWB IR

For the IR UWB system, each information symbol is transmitted over a $T_s$ period that consists of $N_f$ frames. During each frame of duration $T_f$, a data-modulated pulse $p(t)$ with duration $T_p \ll T_f$ is transmitted from the antenna. The transmitted signal is [c.f. (4–1)]

$$v(t) = \sqrt{\mathcal{E}} \sum_{n=0}^{\infty} \tilde{s}(n) \cdot p_T(t - nT_s) \quad (4–35)$$

where $\mathcal{E}$ is the energy per pulse, $\tilde{s}(n) := s(n)\tilde{s}(n - 1)$ are differentially encoded symbols with $s(n)$ denoting the binary PAM information symbols and $p_T(t)$ denotes the symbol-level transmitted waveform which is defined by Eq. (4–2).

The transmitted signal propagates through the multipath channel with impulse response

$$g(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l) \quad (4–36)$$

where $\{\alpha_l\}_{l=0}^{L-1}$ and $\{\tau_l\}_{l=0}^{L-1}$ are amplitudes and delays of the $L$ multipath elements, respectively. Among $\{\tau_l\}_{l=0}^{L-1}$, $\tau_0$ represents the propagation delay of the channel.

Then, the received waveform after synchronization is given by

$$r(t) = \sqrt{\mathcal{E}} \sum_{n=0}^{\infty} \tilde{s}(n) \cdot p_R(t - nT_s - \tau_0 + \hat{\tau}_0) + \eta(t) \quad (4–37)$$

where $\eta(t)$ is the additive noise, $\hat{\tau}_0$ is the candidate time shift introduced by the synchronizer and $p_R(t)$ denotes the aggregate symbol-level received waveform:

$$p_R(t) = \sum_{n=0}^{N_f-1} c_{ds}(n) \cdot p(t - nT_f - c_{th}(n)T_c) \ast g(t + \tau_0) \quad (4–38)$$
Figure 4-6. Transmitted waveforms using orthogonal modulation scheme and the time-hopping (TH) code $[0, 1, 0]$. where $\ast$ denotes the convolution operation\(^1\). Let us define $\Delta \tau =: \hat{\tau}_0 - \tau_0$ as the timing error. Without loss of generality, we assume that $\Delta \tau$ is in the range of $[0, T_s)$. As we will show in the rest of this Chapter, this assumption will not affect the timing synchronization and the differential demodulation.

Pulse shape modulation (PSM) has been proposed for UWB IR as a supplement to the traditional PAM and PPM. By using multiple orthogonal pulses for information transmission, PSM can improve the spectral efficiency when jointly implemented with PAM and PPM (see e.g., [15, 59, 91]). In this dissertation, instead of using multiple pulses for data-rate enhancement, we adopt a pair of orthogonal pulses, an odd waveform and an even waveform, for UWB IR system to facilitate faster timing synchronization and lower-complexity demodulation than the original IR.

\(^1\) Due to the distortion effects of transmit and receive antennas, the receive pulse shaper could be different from $p(t)$ [55]. However, this will not affect the design and operation of our bi-pulse timing synchronizer and demodulator in this dissertation.
In our system, every encoded information symbol is still transmitted over a $T_s$ duration. However, each symbol duration is divided into two equal halves. The two halves will use a pair of pulse shapers: an even waveform $p_0(t)$ and an odd waveform $p_1(t)$. The even and odd pulses can be chosen as Gaussian pulses $[100]$, Hermite pulses $[52]$, or their derivatives. The optimal design of these pulses has been widely investigated in the literature (see e.g., $[48, 62]$).

For the symbol with an even index, the pulse $p_0(t)$ is data-modulated and transmitted in the first half of the symbol duration and pulse $p_1(t)$ in the second half. For the symbol with an odd index, $p_1(t)$ is used for the former half and $p_0(t)$ for the latter half of the symbol duration (see Fig. 4-6). For this approach, we use the same time-hopping and spreading codes for the two halves, i.e., $c_{th}(i) = c_{th}(N_f/2 + i)$ and $c_{ds}(i) = c_{ds}(N_f/2 + i)$, $i \in [0, N_f/2 - 1]$.

Denote the symbol-level transmitted waveform with an even index by $p_{T0}(t)$ and by $p_{T1}(t)$ otherwise. The received signal is given by:

$$r(t) = \sqrt{E} \sum_{n=0}^{\infty} \tilde{s}(n) \cdot p_{Ri_n}(t - nT_s + \Delta \tau) + \eta(t)$$

(4–39)

where $p_{Ri_n}(t)$ is the noise-free received symbol-level waveform with $i_n = [n]_2$. Here we use $[A]_B$ for the modulo operation with base $B$. To facilitate the noncoherent UWB modulation, we select $T_f$ and the TH code of the $(N_f/2 - 1)$ frame to satisfy $(c_{th}(N_f/2 - 1)T_c + T_p + \tau_{L-1}) < T_f$ so that there is no interference between the even and odd waveforms ($p_0(t)$ and $p_1(t)$) even after multipath propagation. However, it is worth noting that no constraint is imposed on the inter-frame interference and inter-pulse interference within each $T_s/2$ duration exclusively containing $p_0(t)$ and $p_1(t)$. Then, we can express
$\Delta \tau$.

\[ R_i(t) \]

\[ 0 \quad \frac{T_s}{2} \]

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4-7.png}
\caption{The $T_s/2$-long signal segments.}
\end{figure}

$p_{R0}(t)$ and $p_{R1}(t)$ as follows:

\begin{align}
\begin{aligned}
p_{R0}(t) &= \begin{cases} 
R_0(t) & \text{if } t \in [0, \frac{T_s}{2}) \\
R_1(t - \frac{T_s}{2}) & \text{if } t \in [\frac{T_s}{2}, T_s)
\end{cases} \\
p_{R1}(t) &= \begin{cases} 
R_1(t) & \text{if } t \in [0, \frac{T_s}{2}) \\
R_0(t - \frac{T_s}{2}) & \text{if } t \in [\frac{T_s}{2}, T_s)
\end{cases}
\end{aligned}
\end{align}

(4-40)

where

\[ R_0(t) = \sum_{n=0}^{N_i/2-1} c_{ds}(n) \cdot p_0(t - nT_f - c_{th}(n)T_c) \ast g(t + \tau_0) \]

and

\[ R_1(t) = \sum_{n=0}^{N_i/2-1} c_{ds}(n) \cdot p_1(t - nT_f - c_{th}(n)T_c) \ast g(t + \tau_0) \]

denote the parts of symbol-level waveforms involving $p_0(t)$ and $p_1(t)$, respectively.

For illustrative purpose, $R_i(t)$ is plotted in Fig. 4-7 as a triangle with the maximum non-zero support of $\frac{T_s}{2}$ and the frame-level repetition is ignored. Notice that due to the timing error, every $R_i(t)$ can be partitioned into two segments $q^a_i(t, \Delta \tau)$ and $q^b_i(t, \Delta \tau)$.
(see Fig. 4-7):

\[
q^a_i(t, \Delta \tau) = \begin{cases} 
0, & t \in [0, T_s/2 - [\Delta \tau]T_s/2) \\
R_i(t - T_s/2 + [\Delta \tau]T_s/2), & t \in [T_s/2 - [\Delta \tau]T_s/2, T_s/2)
\end{cases}
\] (4–41)

and

\[
q^b_i(t, \Delta \tau) = \begin{cases} 
R_i(t + [\Delta \tau]T_s/2), & t \in [0, T_s/2 - [\Delta \tau]T_s/2) \\
0, & t \in [T_s/2 - [\Delta \tau]T_s/2, T_s/2)
\end{cases}
\] (4–42)

The waveforms \(q^a_i(t, \Delta \tau)\) and \(q^b_i(t, \Delta \tau)\) constitute the complete waveform \(R_i(t)\) as:

\[
R_i(t) = q^a_i(t + T_s/2 - [\Delta \tau]T_s/2, \Delta \tau) + q^b_i(t - [\Delta \tau]T_s/2, \Delta \tau).
\] (4–43)

Timing synchronization and channel estimation are two major challenges for the realization of UWB IR. In the next two sections, we will discuss how to solve these problems for our proposed orthogonal bi-pulse UWB IR system. In particular, TDT in [114] will be adopted for timing and the noncoherent scheme in [116] will be used to bypass the explicit channel estimation. Both timing synchronization and demodulation rely on correlating adjacent waveform segments. As we will show next, with orthogonal pulses, the correlation result in our bi-pulse IR only contains information of a single symbol regardless of mistiming \(\Delta \tau\). As a result, our timing synchronizer can avoid the interference between adjacent random symbols and achieve a faster synchronization than [114]. For the same reason, our system can avoid the Viterbi algorithm required by the original noncoherent UWB [116] and result in a simpler differential demodulator.

### 4.3.2 Timing Bi-Pulse UWB Signals

In the following, we will derive an NDA timing synchronizer based on the orthogonal bi-pulse UWB system. Following the idea of TDT [114], our timing algorithm also relies on correlating the neighboring signal segments. Instead of taking \(T_s\)-long segments, however, TDT for bi-pulse IR relies on half-symbol \((T_s/2)\)-long segments of the received waveform.
Our demodulation starts with extracting the $T_s/2$-long segments from the received waveform. These segments are given by:

$$\begin{align*}
    r_n(t) &= r(t + nT_s), \quad t \in [0, T_s/2), \\
    r_n(t + T_s/2) &= r(t + nT_s + T_s/2)
\end{align*}$$  \hfill (4–44)

Then, adjacent segments are correlated to calculate the correlation function $x(n, \Delta \tau)$:

$$x(n, \Delta \tau) = \int_0^{T_s/2} r_n(t) r_n(t - \frac{T_s}{2}) \, dt$$  \hfill (4–45)

The next question is how to recover the timing error $\Delta \tau$ from this correlation function.

Let us first introduce two Lemmas which will be used to derive the timing algorithm from Eq. (4–45).

**Lemma 1.** Let $p_0(t)$ and $p_1(t)$ constitute an even and odd pulse pair. After propagating through any real channel, the received waveforms corresponding to $p_0(t)$ and $p_1(t)$ are still orthogonal.

**Proof.** Without loss of generality, we assume that the pulse $p_0(t)$ is evenly symmetric and $p_1(t)$ is oddly symmetric, both with respect to the origin [52]. Therefore, $p_0(t)$ and $p_1(t)$ are two orthogonal pulses.

Let $c(t)$ denote the channel impulse response. Then the received pulses corresponding to $p_0(t)$ and $p_1(t)$ can be expressed as

$$q_0(t) = c(t) * p_0(t) \quad \text{and} \quad q_1(t) = c(t) * p_1(t).$$

Next let us prove that $q_0(t)$ and $q_1(t)$ are two orthogonal pulses, which is equivalent to showing that

$$\int_{-\frac{T_p}{2}}^{\frac{T_p}{2} + T_h} q_0(t)q_1(t) \, dt = 0$$  \hfill (4–46)

where $T_h$ is the excess delay spread of the channel.

Using Parseval’s theorem, we can express the left hand side of (4–46) as

$$\int_{-\frac{T_p}{2}}^{\frac{T_p}{2} + T_h} q_0(t)q_1(t) \, dt = \int_{-\frac{B_T}{2}}^{\frac{B_T}{2}} Q_0(f)Q_1^*(f) \, df$$  \hfill (4–47)
where \( Q_0(f) = \mathcal{F}\{q_0(t)\} \) and \( Q_1(f) = \mathcal{F}\{q_1(t)\} \) represent the Fourier transform for \( q_0(t) \) and \( q_1(t) \), \( z^* \) is the complex conjugate, and \( B \) is the bandwidth of the UWB pulse.

Due to the basic properties of Fourier transform with convolution operation, we have
\[
Q_0(f) = C(f)P_0(f) \quad \text{and} \quad Q_1(f) = C(f)P_1(f),
\]
with \( C(f) = \mathcal{F}\{c(t)\} \), \( P_0(f) = \mathcal{F}\{p_0(t)\} \) and \( P_1(f) = \mathcal{F}\{p_1(t)\} \). Then (4–47) can be re-expressed as
\[
\int_{-\frac{B}{2}}^{\frac{B}{2}} Q_0(f)Q_1^*(f)df = \int_{-\frac{B}{2}}^{\frac{B}{2}} P_0(f)P_1^*(f)|C(f)|^2 df.
\]

(4–48)

We know that if a function \( f(t) \) is a real and even function, then its Fourier transform \( F(f) \) is a real and even function; if \( f(t) \) is a real and odd function, then \( F(f) \) is a purely imaginary and odd function. Based on these, \( P_0(f) \) turns out to be a real and even function, \( P_1(f) \) is a purely imaginary and odd function, and \( |C(f)|^2 \) is a real and even function. Therefore, \( P_0(f)P_1^*(f)|C(f)|^2 \) is a purely imaginary and odd function. As a result, we have
\[
\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} q_0(t)q_1(t)dt = \int_{-\frac{B}{2}}^{\frac{B}{2}} P_0(f)P_1^*(f)|C(f)|^2 df = 0
\]
which proves that \( q_0(t) \) and \( q_1(t) \) are orthogonal.

Lemma 1 implies that
\[
\int_{0}^{\frac{T_s}{2}} q_0^a(t, \Delta \tau)q_1^a(t, \Delta \tau)dt = 0, \quad \alpha \in \{a, b\}.
\]

In addition, since the non-zero supports of \( q_0^a(t, \Delta \tau) \) and \( q_1^a(t, \Delta \tau) \) do not overlap, the following always holds true:

**Lemma 2.** When the non-zero support of \( R_i(t) \) is upper bounded by \( T_s/2 \), we have

\[
\int_{0}^{\frac{T_s}{2}} q_i^a(t, \Delta \tau)q_i^a(t, \Delta \tau)dt = 0, \quad i \in \{0, 1\}.
\]

(4–50)

This is evident from Eqs. (4–41) and (4–42).

With these results, we are ready to recover the timing error \( \Delta \tau \) from the correlation function \( x(n, \Delta \tau) \) in Eq. (4–45). Using Lemmas 1 and 2, \( x(n, \Delta \tau) \) can be expressed in a
very simple form which contains the energy of either $q_i^a(t)$ or $q_i^b(t)$, $i \in \{0, 1\}$, depending on whether $\Delta \tau \in [0, T_s/2)$ or $\Delta \tau \in [T_s/2, T_s)$.

### 4.3.2.1 Case I: $\Delta \tau \in [0, T_s/2)$

In this case, the noise-free signal segments involved in timing synchronization are [cf. (4–39) and (4–44)]

\[
\begin{align*}
\bar{r}_n(t - T_s/2) &= \tilde{s}(n-1)q_i^b(t, \Delta \tau) + \tilde{s}(n)q_i^a(t, \Delta \tau) \quad , \quad i \neq j \in \{0, 1\} \\
\bar{r}_n(t) &= \tilde{s}(n)q_i^b(t, \Delta \tau) + \tilde{s}(n)q_j^a(t, \Delta \tau)
\end{align*}
\]

where $\bar{r}_n(t - T_s/2)$ and $\bar{r}_n(t)$ are the noise-free parts of $r_n(t - T_s/2)$ and $r_n(t)$, respectively.
As a result, the noise free part of the correlation function $\chi(n, \Delta \tau)$ can be expressed as [cf. (4–45)]:

$$\bar{\chi}(n, \Delta \tau) = \int_0^{T_s} \left( \tilde{s}(n-1)q_{b}(t, \Delta \tau) + \tilde{s}(n)q_{a}(t, \Delta \tau) \right) \left( \tilde{s}(n)q_{b}(t, \Delta \tau) + \tilde{s}(n)q_{a}(t, \Delta \tau) \right) dt. \tag{4–52}$$

Using Lemmas 1 and 2, and the differential modulation relationship, $\bar{\chi}(n, \Delta \tau)$ becomes:

$$\bar{\chi}(n, \Delta \tau) = \int_0^{T_s} s_n(q_{b}(t, \Delta \tau))^2 dt. \tag{4–53}$$

Accordingly, the absolute value of $\bar{\chi}(n, \Delta \tau)$ can be equivalently expressed as:

$$|\bar{\chi}(n, \Delta \tau)| = \int_0^{T_s} (q_{b}(t, \Delta \tau))^2 dt := E_{b}(\Delta \tau) \tag{4–54}$$

where $E_{b}(\Delta \tau)$ is defined as the energy of the waveform segment $q_{b}(t, \Delta \tau)$. In the random channel environment, it is reasonable to assume that $q_{b}(t, \Delta \tau)$ and $q_{a}(t, \Delta \tau)$ approximately have the same energy $E_{b}(\Delta \tau)$. It should be noted that $E_{b}(\Delta \tau)$ decreases as $\Delta \tau$ increases (see Fig. 4-7). Therefore, we have the following result:

**Proposition 6.** For the orthogonal bi-pulse UWB IR system, when the timing error $\Delta \tau$ is in the range of $[0, T_s/2)$, the absolute value of the noise-free correlation function $|\bar{\chi}(n, \Delta \tau)|$ equals the energy $E_{b}(\Delta \tau)$ of the waveform segment $q_{b}(t, \Delta \tau)$, which is a decreasing function in $\Delta \tau$.

Notice that Eq. (4–54) shows that $|\bar{\chi}(n, \Delta \tau)|$ is a constant independent of the symbol index $n$, even when random symbol sequence is transmitted. This consists of a fundamental difference between our method here and the original TDT in [114], where the correlator output varies with the particular symbol sequence transmitted. As a result, unlike the original TDT in [114] that entails significant averaging or special training patterns to remove the random symbol effect, our approach here completely avoids such a problem.
4.3.2.2 Case II: \( \Delta \tau \in [T_s/2, T_s) \)

In this case, the noise-free signal segments involved in timing are [cf. (4–39) and (4–44)]

\[
\bar{r}_n(t - T_s/2) = \bar{s}(n)q_i^n(t, \Delta \tau) + \bar{s}(n)q_j^n(t, \Delta \tau), \quad i \neq j \in \{0, 1\}. \tag{4–55}
\]

\[
\bar{r}_n(t) = \bar{s}(n)q_i^n(t, \Delta \tau) + \bar{s}(n + 1)q_j^n(t, \Delta \tau)
\]

The noise-free correlation function can be expressed as [cf. (4–45)]:

\[
\bar{x}(n, \Delta \tau) = \int_0^{T_s/2} (\bar{s}(n)q_i^n(t, \Delta \tau) + \bar{s}(n)q_j^n(t, \Delta \tau)) \times (\bar{s}(n)q_i^n(t, \Delta \tau) + \bar{s}(n + 1)q_j^n(t, \Delta \tau)) \, dt. \tag{4–56}
\]

Using Lemmas 1 and 2, \( \bar{x}(n, \Delta \tau) \) becomes:

\[
\bar{x}(n, \Delta \tau) = \int_0^{T_s/2} s_{n+1}(q_j^n(t, \Delta \tau))^2 \, dt. \tag{4–57}
\]

Accordingly, the absolute value of \( \bar{x}(n, \Delta \tau) \) can be equivalently expressed as:

\[
|\bar{x}(n, \Delta \tau)| = \int_0^{T_s/2} (q_j^n(t, \Delta \tau))^2 \, dt := E_a(\Delta \tau) \tag{4–58}
\]

where \( E_a(\Delta \tau) \) is the energy of waveform segment \( q_j^n(t, \Delta \tau) \), \( j \in \{0, 1\} \). Since \( E_a(\Delta \tau) \) increases as \( \Delta \tau \) increases (see Fig. 4-7), we have the following result:

**Proposition 7.** For the orthogonal bi-pulse UWB IR system, when the timing error \( \Delta \tau \) is in the range of \( [T_s/2, T_s) \), the absolute value of the noise-free correlation function \( |\bar{x}(n, \Delta \tau)| \) equals the energy \( E_a(\Delta \tau) \) of the waveform segment \( q_j^n(t, \Delta \tau) \), which is an increasing function in \( \Delta \tau \).

Notice that here \( |\bar{x}(n, \Delta \tau)| \) also remains a constant \( \forall n \), at any given \( \Delta \tau \). This again confirms that the random symbol effect is completely mitigated in our bi-pulse IR approach.
4.3.2.3 TDT for bi-pulse IR

Generally, it is unknown to the receiver whether $\Delta \tau$ is in $[0, T_s/2)$ or in $[T_s/2, T_s)$. However, we notice that for $\Delta \tau \in [0, T_s/2)$, $|\bar{x}(n, \Delta \tau)| = \mathcal{E}_b(\Delta \tau)$ increasingly approaches its maximum $\mathcal{E}_b(0) = \mathcal{E}_R$ when $\Delta \tau$ approaches 0 and decreasingly approaches its minimum $\mathcal{E}_b(T_s/2) = 0$ when $\Delta \tau$ approaches $T_s/2$ (see Fig. 4-7). In addition, for $\Delta \tau \in [T_s/2, T_s)$, $|\bar{x}(n, \Delta \tau)| = \mathcal{E}_a(\Delta \tau)$ decreasingly approaches its minimum $\mathcal{E}_a(T_s/2) = 0$ when $\Delta \tau$ approaches $T_s/2$ and increasingly approaches its maximum $\mathcal{E}_a(T_s) = \mathcal{E}_R$ when $\Delta \tau$ approaches $T_s$. Therefore, $|\bar{x}(n, \Delta \tau)|$ is continuous at $\Delta \tau = T_s/2$. Moreover, $|\bar{x}(n, \Delta \tau)|$ is a periodic function of $\Delta \tau$ with period of $T_s$; that is, $|\bar{x}(n, \Delta \tau)| = |\bar{x}(n, T_s + \Delta \tau)|$, $\forall \Delta \tau$. Therefore, $|\bar{x}(n, \Delta \tau)|$ reaches its minimum when $\Delta \tau = T_s/2$, and reaches its maximum when $\Delta \tau = 0$. This result leads to a timing synchronizer based on the sample mean of the symbol-rate sample $|x(k, \Delta \tau)|$.

In the following, we combine Propositions 6 and 7 to build a timing synchronizer for our proposed bi-pulse IR system:

**Proposition 8.** For the orthogonal bi-pulse IR system, a blind timing synchronizer can be built even when TH codes are present and the UWB multipath channel is unknown.

The timing algorithm can be summarized in the following steps:

**Step 1:** Extract the $T_s/2$-long segments $r(t + n T_s)$ and $r(t + n T_s - T_s/2)$ from the received waveform.

**Step 2:** Estimate the noise-free correlation function of adjacent segments with

$$y(M, \Delta \tau) = \frac{1}{M} \sum_{n=1}^{M} \left| \int_{0}^{T_s} r(t + n T_s) r(t + n T_s - T_s/2) dt \right| .$$

**Step 3:** The timing error can be estimated when $y(M, \Delta \tau)$ reaches its maximum

$$\widehat{\Delta \tau}_0 = \arg \max_{\Delta \tau \in [0, T_s)} y(M, \Delta \tau) .$$

Unlike the original NDA TDT, the estimation of the correlation function for our synchronizer is not affected by the ISI (see Eqs. (4–54) and (4–58)) even with random
information symbols. Therefore, the proposed synchronization approach is expected to achieve a better acquisition performance than that of the original TDT in the NDA mode.

4.3.3 Demodulating Bi-Pulse UWB Signals

Following the noncoherent UWB [63], our demodulation also builds on correlating the neighboring signal segments. The advantage is that due to the orthogonality between \( p_0(t) \) and \( p_1(t) \), our algorithm avoids ISI even in the presence of mistiming. The demodulation algorithm also starts from the extraction and correlation of \( T_s/2 \)-long waveform segments. The extraction is as described in the preceding section. However, the correlation is carried out in a different manner, as detailed in the following.

4.3.3.1 Extraction of decision statistics

Instead of \( x(n, \Delta \tau) \), we calculate two correlation functions \( x_1(n, \Delta \tau) \) and \( x_2(n, \Delta \tau) \) (see Fig. 4-9):

\[
x_1(n, \Delta \tau) = \int_0^{T_s/2} r_n(t) r_{n-1}(t + T_s/2) dt + \int_0^{T_s/2} r_n(t + T_s/2) r_{n-1}(t) dt \quad (4–59)
\]

\[
x_2(n, \Delta \tau) = \int_0^{T_s/2} r_n(t - T_s/2) r_{n-1}(t) dt + \int_0^{T_s/2} r_n(t) r_{n-1}(t - T_s/2) dt \quad (4–60)
\]

As shown in the following, we use both correlation functions to demodulate one symbol because either of them only contains part of the symbol energy in the presence of mistiming. Using \( x_1(n, \Delta \tau) \) and \( x_2(n, \Delta \tau) \) together, one can obtain the entire symbol energy from the received waveform.

For \( \Delta \tau \in [0, T_s/2) \), signal segments with index \( n \) involved in the demodulation are

\[
r_n(t) = \tilde{s}(n)q^b_i(t, \Delta \tau) + \tilde{s}(n)q^a_i(t, \Delta \tau) + \eta_1(t)
\]

\[
r_n(t - T_s/2) = \tilde{s}(n - 1)q^b_i(t, \Delta \tau) + \tilde{s}(n)q^a_i(t, \Delta \tau) + \eta_2(t) \quad (4–61)
\]

\[
r_n(t + T_s/2) = \tilde{s}(n)q^b_i(t, \Delta \tau) + \tilde{s}(n + 1)q^a_i(t, \Delta \tau) + \eta_3(t)
\]
and segments with index \((n - 1)\) are

\[
\begin{align*}
    r_{n-1}(t) &= \tilde{s}(n-1)q_a^p(t, \Delta \tau) + \tilde{s}(n-1)q_b^p(t, \Delta \tau) + \eta_4(t) \\
    r_{n-1}(t - Ts/2) &= \tilde{s}(n-2)q_a^p(t, \Delta \tau) + \tilde{s}(n-1)q_b^p(t, \Delta \tau) + \eta_5(t) \\
    r_{n-1}(t + Ts/2) &= \tilde{s}(n-1)q_b^p(t, \Delta \tau) + \tilde{s}(n)q_a^p(t, \Delta \tau) + \eta_6(t)
\end{align*}
\]

(4–62)

These notions seem to be rather redundant. However, because the orthogonal pulse pair used in our bi-pulse IR switches in order from symbol to symbol, \(r_n(t)\) is different from \(r_{n-1}(t)\) since they contain different combinations of \(q_i^a(t), i \in \{0, 1\}, \alpha \in \{a, b\}\), except for that \(r_n(t - Ts/2) = r_{n-1}(t + Ts/2)\).
Using Lemmas 1 and 2, correlation functions \( x_1(n, \Delta \tau) \) and \( x_2(n, \Delta \tau) \) can be simplified to:

\[
\begin{align*}
    x_1(n, \Delta \tau) &= 2s(n)\mathcal{E}_b(\Delta \tau) + \xi_1(n) \\
    x_2(n, \Delta \tau) &= 2s(n)\mathcal{E}_a(\Delta \tau) + \xi_2(n)
\end{align*}
\]  

(4–63)

where \( \mathcal{E}_a(\Delta \tau) \) and \( \mathcal{E}_b(\Delta \tau) \) are the energy of waveform segments \( q_i^a(t, \Delta \tau) \) and \( q_i^b(t, \Delta \tau) \) for \( i \in \{0, 1\} \) as defined by Eqs. (4–54) and (4–58), respectively, and \( \xi_1(n) \) as well as \( \xi_2(n) \) are noise terms.

From (4–63), we see that each correlation function only contains the information of one transmitted symbol. This is distinct from the original noncoherent UWB in [116] where the correlation result always contains two consecutive information symbols, when timing error is present. Therefore, due to the orthogonality of the UWB pulses, our noncoherent demodulator avoids the ISI even without Viterbi decoding required by [116]. In addition, each symbol \( s(n) \) is contained in both \( x_1(n, \Delta \tau) \) and \( x_2(n, \Delta \tau) \). The noise-free part of \( x_1(n, \Delta \tau) + x_2(n, \Delta \tau) \) contains all available energy of one symbol.

Similarly, for \( \Delta \tau \in [\frac{T_s}{2}, T_s) \), correlation functions \( x_1(n, \Delta \tau) \) and \( x_2(n, \Delta \tau) \) can be expressed as:

\[
\begin{align*}
    x_1(n - 1, \Delta \tau) &= 2s(n)\mathcal{E}_a(\Delta \tau) + \xi_3(n - 1) \\
    x_2(n, \Delta \tau) &= 2s(n)\mathcal{E}_b(\Delta \tau) + \xi_4(n)
\end{align*}
\]  

(4–64)

where \( \xi_3(n) \) and \( \xi_4(n) \) are noise terms. Notice that here we consider \( x_1(n - 1, \Delta \tau) \) instead of \( x_1(n, \Delta \tau) \) since the former contains the same symbol as \( x_2(n, \Delta \tau) \).

Clearly, to distinguish which of (4–63) and (4–64) is the case, one needs to separate the \( \Delta \tau \in [0, \frac{T_s}{2}) \) case from the \( \Delta \tau \in [\frac{T_s}{2}, T_s) \) case. For a random symbol sequence, these cases can be distinguished by using the following rule:

\( \Delta \tau \) is in the range of \([0, \frac{T_s}{2}) \) if

\[
E\{|x_1(n, \Delta \tau) + x_2(n, \Delta \tau)|\} \geq E\{|x_1(n - 1, \Delta \tau) + x_2(n, \Delta \tau)|\}
\]  

(4–65)
and $\Delta \tau \in [T_s/2, T_s)$ if otherwise.

This can be easily seen from (4–63) (or (4–64)) since $x_1(n, \Delta \tau)$ and $x_2(n, \Delta \tau)$ contain the same symbol if $\Delta \tau \in [0, T_s/2)$, while $x_1(n-1, \Delta \tau)$ and $x_2(n, \Delta \tau)$ contain the same symbol if $\Delta \tau \in [T_s/2, T_s)$. The expectation can be approximated by

$$E\{|x_1(n, \Delta \tau) + x_2(n, \Delta \tau)|\} \approx \frac{1}{M} \sum_{n=0}^{M-1} |x_1(n, \Delta \tau) + x_2(n, \Delta \tau)|$$

with $M$ being the length of symbol sequence.

### 4.3.3.2 Symbol detection

Before deriving the decoding algorithm, let us first investigate the statistical distribution of noise terms in Eqs. (4–63) and (4–64). For UWB pulses used in this dissertation, both the analysis along the lines of [113, Appendix I] and simulations confirm that $\xi_1(n)$ and $\xi_2(n)$ (also $\xi_3(n-1)$ and $\xi_4(n)$) are uncorrelated zero-mean Gaussian random variables with equal variances, for an arbitrary channel realization, symbol sequence and timing error $\Delta \tau \in [0, T_s)$. Based on these, we can combine $x_1(n, \Delta \tau)$ (or $x_1(n-1, \Delta \tau)$) and $x_2(n, \Delta \tau)$ with selective combining (SC), equal gain combining (EGC) or MRC.

For SC, the correlation output with more energy is selected. For $\Delta \tau \in [0, T_s/2)$, the information symbol can be estimated by

$$\hat{s}(n) = \begin{cases} 
\text{sign}(x_1(n, \Delta \tau)) & \text{if } \hat{E}_a(\Delta \tau) \geq \hat{E}_b(\Delta \tau) \\
\text{sign}(x_2(n, \Delta \tau)) & \text{if } \hat{E}_b(\Delta \tau) > \hat{E}_a(\Delta \tau)
\end{cases} \tag{4–66}$$

where $\hat{E}_a(\Delta \tau)$ and $\hat{E}_b(\Delta \tau)$ are estimates of the signal strength $E_a(\Delta \tau)$ and $E_b(\Delta \tau)$ contained in $x_1(n, \Delta \tau)$ and $x_2(n, \Delta \tau)$:

$$\hat{E}_b(\Delta \tau) = \frac{1}{M} \sum_{n=0}^{M-1} |x_1(n, \Delta \tau)|$$

$$\hat{E}_a(\Delta \tau) = \frac{1}{M} \sum_{n=0}^{M-1} |x_2(n, \Delta \tau)| \tag{4–67}$$
Similarly, for $\Delta \tau \in [T_s/2, T_s)$, $s(n)$ can be estimated by

$$\hat{s}(n) = \left\{ \begin{array}{ll}
sign(x_1(n-1, \Delta \tau)) & \text{if } \hat{E}_a(\Delta \tau) \geq \hat{E}_b(\Delta \tau) \\
\text{sign}(x_2(n, \Delta \tau)) & \text{if } \hat{E}_b(\Delta \tau) > \hat{E}_a(\Delta \tau) \\
\end{array} \right.. \quad (4-68)$$

For EGC, the two correlation functions are simply added together to obtain the decision statistic. For $\Delta \tau \in (0, T_s/2)$, the information symbol can be estimated by

$$\hat{s}(n) = \text{sign}(x_1(n, \Delta \tau) + x_2(n, \Delta \tau)) \quad (4-69)$$

and for $\Delta \tau \in [T_s/2, T_s)$

$$\hat{s}(n) = \text{sign}(x_1(n-1, \Delta \tau) + x_2(n, \Delta \tau)) \quad (4-70).$$

For MRC, the two correlation functions are weighted and added together to maximize the SNR of the decision statistic. The optimal weight coefficients of $x_1(n, \Delta \tau)$ and $x_2(n, \Delta \tau)$ are proportional to their signal amplitudes $E_a(\Delta \tau)$ and $E_b(\Delta \tau)$, respectively. In practice, estimates of $E_a(\Delta \tau)$ and $E_b(\Delta \tau)$ are used as the weight coefficients. Then, for $\Delta \tau \in (0, T_s/2)$, the information symbol can be estimated by

$$\hat{s}(n) = \text{sign}\left(\hat{E}_a(\Delta \tau)x_1(n, \Delta \tau) + \hat{E}_b(\Delta \tau)x_2(n, \Delta \tau)\right) \quad (4-71)$$

and for $\Delta \tau \in [T_s/2, T_s)$

$$\hat{s}(n) = \text{sign}\left(\hat{E}_a(\Delta \tau)x_1(n-1, \Delta \tau) + \hat{E}_b(\Delta \tau)x_2(n, \Delta \tau)\right). \quad (4-72)$$

Comparison of these algorithms has been widely carried out in the literature (see e.g., [71, 82, 83]). It is well-known that MRC achieves the best performance by maximizing the SNR of the decision statistic. However, whether EGC or SC is better depends on the relative strength of $x_1(n, \Delta \tau)$ and $x_2(n, \Delta \tau)$. EGC will outperform SC when strength of $x_1(n, \Delta \tau)$ and $x_2(n, \Delta \tau)$ is comparable, and SC will perform better if one correlation function is much stronger than the other one. A simple calculation will show
that with uncorrelated, zero-mean and equal-variance noises, when the symbol strength of one correlation function is \((1 + \sqrt{2})\) times or more that of the weaker one, SC will achieve a higher SNR and a better performance than EGC.

4.3.3.3 Special case: perfect synchronization

The demodulator with perfect synchronization is a special case of the demodulation algorithm in the presence of mistiming. For perfect timing synchronization \((\Delta \tau = 0)\), the
extracted segments with index $n$ are [cf. (4–61)] (see also Fig. 4-10)

$$r_n(t) = \tilde{s}(n) R_j(t) + \eta_1(t)$$

$$r_n(t - T_s/2) = \tilde{s}(n - 1) R_i(t) + \eta_2(t)$$

$$r_n(t + T_s/2) = \tilde{s}(n) R_i(t) + \eta_3(t)$$

(4–73)

and segments with index $(n - 1)$ are [cf. (4–62)]

$$r_{n-1}(t) = \tilde{s}(n - 1) R_i(t) + \eta_4(t)$$

$$r_{n-1}(t - T_s/2) = \tilde{s}(n - 2) R_i(t) + \eta_5(t)$$

$$r_{n-1}(t + T_s/2) = \tilde{s}(n - 1) R_j(t) + \eta_6(t)$$

(4–74)

where $r_{n-1}(t + T_s/2) = r_n(t - T_s/2)$, $i \neq j$, $i, j \in \{0, 1\}$ and $\eta_k(t)$ are noise terms.

Note that, unlike (4–61) and (4–62) each segment in Eqs. (4–73) and (4–74) contains a complete $R_i(t)$, $i \in \{0, 1\}$, thanks to the perfect synchronization.

Using Lemmas 1 and 2, and the differential modulation relationship, we can simplify $x_1(n, \Delta \tau = 0)$ and $x_2(n, \Delta \tau = 0)$ as:

$$x_1(n, \Delta \tau) = 2s(n) E_R + \xi_5(n)$$

$$x_2(n, \Delta \tau) = \xi_6(n)$$

(4–75)

where $E_R := \int_0^{T_s} (R_i(t))^2 dt$ for $i \in \{0, 1\}$, $\xi_5(n)$ and $\xi_6(n)$ are noise terms.

As shown in (4–75), correlation function $x_1(n, \Delta \tau = 0)$ captures the entire symbol energy. However, $x_2(n, \Delta \tau = 0)$ only consists of noise. So for this case, the optimal demodulation can be carried out in an SC manner by only using $x_1(n, \Delta \tau = 0)$ with a slicer,

$$\hat{s}(n) = \text{sign}(x_1(n, \Delta \tau = 0)).$$

(4–76)

Under perfect synchronization, the symbol detector in (4–76) is simply a normal differential demodulator. Therefore, they are expected to achieve the same BER
Figure 4-11. Acquisition probability comparison: proposed timing with dirty templates (TDT) versus original data-aided (DA) and non-data-aided (NDA) TDT [114].

performance. Of course, the correlation that generates the decision statistic \( x_1(n, \Delta \tau = 0) \) differs from [116] and [33] due to the special bi-pulse modulation.

### 4.3.4 Simulations

In this section, we will evaluate the performance of our proposed approaches with simulations. We select the orthogonal pulses \( p_0(t) \) and \( p_1(t) \) as two consecutive order Hermite pulses with duration \( T_p \approx 0.8 \)ns. Simulations are performed in a modified Saleh-Valenzuela channel [11, 79] with parameters \( \Lambda = 0.0233 \) (1/ns), \( \lambda = 2.5(1/\text{ns}) \), \( \Gamma = 7.1 \)ns and \( \gamma = 4.3 \)ns. The maximum channel delay spread is about 31ns. Each symbol contains \( N_f = 10 \) frames with duration \( T_f = 32 \)ns.

First, let us compare the acquisition probability of the proposed orthogonal bi-pulse TDT synchronizer with the original NDA and DA TDT algorithms in [114]. Thanks to the orthogonal pulses, the bi-pulse synchronizer remarkably outperforms the original NDA TDT algorithm and achieves a comparable performance to the DA TDT algorithm, especially when \( M \) is small (see Fig. 4-11).
In Fig. 4-12, we compare the mean square error (MSE) for all three TDT synchronizers. The MSE is normalized by the square of the symbol duration $T_s$. The performance of our modulation in NDA mode is comparable to that of the original DA TDT synchronization algorithm. Even without any training symbol sequence, our bi-pulse TDT synchronizer can greatly outperform the original NDA TDT especially when $M$ is small. This performance improvement is enabled at the price of slightly higher complexity by alternating the pulse shaper.

In Figs. 4-13 and 4-14, we evaluate the BER performance of our proposed bi-pulse differential demodulator. We also compare the bi-pulse UWB with the original noncoherent UWB [116]. In the case of mistiming, the timing error is uniformly distributed in $[0, T_s)$ and $[0, T_r)$ for Figs. 4-13 and 4-14, respectively.

From Fig. 4-13, we can see that when there is no mistiming, the bi-pulse noncoherent UWB achieves the same performance as the original noncoherent UWB. This is because with perfect timing, symbol detection of both is essentially differential UWB. By avoiding ISI, our bi-pulse noncoherent UWB leads to a simple demodulator with
Figure 4-13. Bi-pulse UWB versus noncoherent UWB [116]; timing error is in the range of $[0, T_s)$.

Figure 4-14. Comparison of selective combining (SC), equal gain combining (EGC) and maximum ratio combining (MRC) for bi-pulse UWB; timing error is in the range of $[0, T_f)$.
ML optimality even in the presence of unknown mistiming. However, under the same conditions, performance of original noncoherent UWB degrades drastically when Viterbi decoding is not used.

From both Figs. 4-13 and 4-14, we see that MRC outperforms EGC and SC in terms of BER. This has been well known because MRC can always maximize the SNR of the decision statistic. However, which of EGC and SC performs better depends on the range of timing error. For the small timing error range $[0, T_f)$, SC performs better, and for the large timing error range $[0, T_s)$, EGC outperforms SC. This is because that for the $[0, T_f)$ case, one of the two correlation functions almost contains the entire energy of the received symbol, but the other one is almost pure noise. By simply summing up these two terms, EGC can not achieve a SNR as high as SC where only the stronger correlation function is used. For the $\Delta T \in [0, T_s)$ case, the energy contained in both two correlation is comparable. Although the stronger one is used for detection, SC still loses a large portion of the symbol energy. Therefore, EGC outperforms SC when the timing error is in $[0, T_s)$. 

Figure 4-15. Joint simulation of timing and demodulation for $M = 2$. 
We evaluate the timing and demodulation in the same simulation (see Fig. 4-15), where $M$ is the length of symbol sequence. Specially, the timing error is first estimated and corrected for the received signal. Then differential demodulation is carried out by assuming that the signal is perfectly synchronized. Our bi-pulse IR with NDA TDT can perform almost the same as the original IR with DA TDT and significantly outperform the original IR with NDA TDT. This is in accord with the simulation result that for $M = 2$, the timing performance of our bi-pulse system is similar to the original TDT in DA mode and much better than the original TDT in NDA mode (see Figs. 4-13 and 4-14).

### 4.4 Conclusions

In this Chapter, we extended the TDT algorithm to two different implementation scenarios. First, we applied TDT algorithms to facilitate the design of low-resolution digital UWB receivers. Secondly, we applied TDT algorithms to the timing synchronization of orthogonal bi-pulse IR signals, which also enables an effective noncoherent demodulation process. The validity of TDT has been corroborated in both systems. For the digital IR receivers, simulations show that the low-resolution ADC only induces very small performance degradation. For the bi-pulse IR system, the timing synchronizer can achieve a better acquisition performance than the original TDT synchronizer in NDA mode. The simple demodulator achieves ML optimality without the Viterbi decoding required by the original noncoherent UWB.
High precision ToA estimation in the multipath wireless channel is crucial for timing-based wireless ranging and localization [22]. Because ToA is part of the channel information, ToA estimation is largely dependent on channel estimation. Intuitively, if one obtains all channel parameters including each path’s gain and delay, delay of the first multipath component automatically becomes the ToA estimate (see [7, 72]). Due to the huge bandwidth, UWB signals can resolve the channel multipath much better than narrowband signals. For this reason, ToA estimation can potentially be estimated in high accuracy by using UWB signals.

There are two types of UWB signals: IR which has been introduced in the preceding Chapters and MB-OFDM. For MB-OFDM, the UWB bandwidth is utilized by simultaneously transmitting symbols on narrow-band subcarriers. In recent years, more efforts have been put to the ToA estimation for IR, due to the strict requirement of timing accuracy for the implementation of the Rake receiver (see [98, 114]). However, ToA estimation for the MB-OFDM UWB has not been well studied since fine timing is not indispensable for OFDM. In particular, the system performance will not be affected as long as the guard interval can deal with the timing error. In order to exploit the timing capability of MB-OFDM, we will investigate the ToA estimation with MB-OFDM signals.

Because the MB-OFDM signal consists of several subbands each only containing part of the channel information, these subband signals need to be combined at the receiver to obtain full knowledge of the channel. Generally, two strategies can be taken to combine the MB channel information. Under the first strategy, the channel information from all subbands is jointly utilized for channel and ToA estimation, which is often termed as the coherent combining (see, e.g., [77, 102]). With the second strategy, the CIR is
first estimated for each subband and then channel estimates from all subbands are averaged for a better final ToA estimate [8], which is called the *noncoherent combining*.

The coherent combining can usually provide a better timing resolution than the noncoherent combining at the price of a higher computational complexity (see, e.g., [77, 102]). However, there are scenarios where one has to adopt the noncoherent combining. For example, the channel information can not be coherently combined when random phase rotations are present after carrier demodulation. Even worse is that the channel itself may be independent across subbands due to the frequency dependent fading as the channel fading and dispersion statistics vary with the frequency [49]. Therefore, we also need to investigate the noncoherent combining or the *energy detection* method for ToA estimation with MB-OFDM signals.

In this Chapter, we will first investigate the energy detection-based ToA estimation technique for a simplified situation where the wireless channel is assumed to has a finite number of multipaths and the number of multipaths is known at the receiver. Given these assumptions, we can prove that the timing performance of the ToA estimator has an increasing diversity order as the number of signal bands increases. As we dig deeper into the problem, we will find that the finite channel length assumption does not hold due to energy leakage phenomenon of the channel estimator which is caused by the limited signal bandwidth. In order to solve this problem, we propose to estimate the ToA by suppressing the energy leakage in the second half of this Chapter.

### 5.2 Energy Detection Based ToA Estimator for MB-OFDM

As the analysis of such a problem is very complicated for general channel environments, we will confine our discussion to the Nakagami-\(m\) fading channel.

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1 Note that the independent channel across subbands can also be induced by the dense multipath. However, this type of independency will not prevent one from coherently combining subband channel information.
This is because a series of channel campaigns have shown that the Nakagami-$m$ distribution can fit the channel measurement results very well [86]. In addition, the Nakagami-$m$ distribution is a general fading model, parameters of which can be adjusted to incorporate several fading environments, such as the Rayleigh distribution ($m = 1$) and the one-sided Gaussian distribution ($m = 0.5$). Therefore, the analysis and results are applicable to a wide range of channel fading environments.

Based on the analysis of the pairwise mistiming probability, we will prove that the timing performance can be improved with MB signals by exploiting the diversity across subbands. Although [8] touches upon a similar issue, it assumes a Rayleigh fading channel with independent multipath components and a uniform power delay profile (PDP). Instead of these limiting constraints, we consider the Nakagami-$m$ fading channel with arbitrary multipath correlation and arbitrary PDP.

Following the method in [82, Ch. 12], we calculate the pairwise mistiming probability based on the moment generating functions (MGF) of the probability density function (PDF) of the instantaneous SNR. With the union bound analysis, we can derive the diversity order of the mistiming probability curve.

### 5.2.1 ToA Estimation by Energy Detection

In this section, we will first introduce the MB-OFDM system that is investigated. Architectures of the MB-OFDM transmitter and receiver are illustrated in Fig. 5-1 (a) and (b) as specified in [1]. Different from the basic OFDM system, for MB-OFDM, the baseband signal is carrier modulated on one of the $B$ frequency bands at the transmitter according to the frequency hopping pattern, and the received waveform is carrier demodulated accordingly at the receiver.

Let $s_b(t)$ denote the baseband signal transmitted on the $b$th subband. The noise-free baseband received signal $r_b(t)$ can be expressed as

\[
r_b(t) = s_b(t) * g_b(t) * c_b(t), \quad b \in [1, B]
\]
Figure 5-1. Multi-band orthogonal frequency division multiplexing (MB-OFDM) transceiver block diagrams (a) Transmitter; (b) Receiver.

where * denotes the convolution operation, $g_b(t)$ is a linear time invariant filter including possible transmit and receive filters as well as the bandlimiting effect of transmitting a finite bandwidth signal, and $c_b(t)$ is the baseband channel for the $b$th subband

$$c_b(t) = \sum_{i=1}^{L_c} A_{b,i} \delta(t - \tau_{b,i}), \ b \in [1, B]$$

(5–2)

where $\{A_{b,i}\}_{i=1}^{L_c}$ and $\{\tau_{b,i}\}_{i=1}^{L_c}$ are amplitudes and delays of the $L_c$ channel paths. Note that due to the frequency dependent fading [49], $c_b(t)$ can be independent across subbands.

The baseband received signal $r_b(t)$ is sampled at the rate of $1/T_s$ which is essentially the bandwidth of the transmitted signal $s_b(t)$. The noise-free discrete time samples $r_{b,n} = r_b(t)|_{t=nT_s}$ can be expressed as

$$r_{b,n} = s_{b,n} * h_{b,n}, \ b \in [1, B]$$

(5–3)
where $s_{b,n}$ is the $n$th sample transmitted on the $b$th subband and $h_{b,n}$ is the equivalent discrete time channel

$$
h_{b,n} = [c_b(\tau) * g_b(\tau)] |_{\tau=nT_s}, \ b \in [1, B], \ n \in [0, L - 1]
$$

(5–4)

with $L$ being the number of channel taps. Representing the delay spread of the channel, $L$ is assumed to be known at the receiver.

In order to facilitate the ToA estimation, the discrete time channel (5–4) is first estimated for all $B$ subbands. The $n$th tap of the estimated channel for the $b$th subband can be expressed as follows:

$$
\tilde{h}_{b,n} = \begin{cases} 
    h_{b,n-L_1} + \eta_{b,n}, & n \in [L_1, L_1 + L - 1] \\
    \eta_{b,n}, & n \in [0, L_1 - 1] \\
    \text{and } n \in [L_1 + L, L_1 + L + L_2 - 1]
\end{cases}
$$

(5–5)

with $L_1 < L$ and $L_2 < L$. Due to the lack of timing information, the channel is estimated with a larger number of taps than $L$. Among the $(L_1 + L_2 + L)$ taps in (5–5), $L$ consecutive taps contain the noise-contaminated channel information and the rest $(L_1 + L_2)$ taps only contain noise. Following [8], we also assume that the noise $\eta_{b,n}$ is independent zero mean circularly symmetric complex Gaussian with unit variance.

In [8], the following assumptions are made: 1) channel taps are independent within each subband; 2) the channel is uncorrelated over all $B$ subbands; 3) channel taps are Gaussian random variables, and the channel has a uniform PDP. We will consider the correlation among channel taps both inside each subband and over all subbands. This is more realistic since subbands always share some common channel information. In addition, the analysis is performed when the channel tap $h_{b,n}$ has a Nakagami-$m$ distribution with arbitrary PDP.

After the channel is estimated, a simple energy detection based synchronizer can be adopted for ToA estimation. In particular, for a single band, this estimator detects the start of the channel by seeking the maximum total energy of a length $L$ segment in the
channel estimate sequence given by (5–5). With the availability of multiple subbands, this ToA estimator simply combines the energy from all subbands. Then, the index of the first channel tap can be estimated by

\[ \bar{k} = \arg \max_p \left( \sum_{b=1}^{B} \sum_{n=p}^{p+L-1} |\tilde{h}_{b,n}|^2 \right). \]  

(5–6)

According to the synchronization criterion (5–6), mistiming occurs when the following inequality holds for any \( l \in [1, L_2] \) or \( l \in [-L_1, -1] \):

\[ \sum_{b=1}^{B} \sum_{n=L_1+l}^{L_1+L+l-1} |\eta_{b,n}|^2 > \sum_{b=1}^{B} \sum_{n=L_1}^{L_1+L-1} |\tilde{h}_{b,n}|^2. \]  

(5–7)

After the common terms are canceled in (5–7), the inequality becomes

\[ \sum_{b=1}^{B} \sum_{n=L_1+l}^{L_1+L+l-1} |\eta_{b,n}|^2 > \sum_{b=1}^{B} \sum_{n=L_1}^{L_1+L-1} |\tilde{h}_{b,n} + \eta_{b,n}|^2, \text{ for } l \in [1, L_2] \]  

(5–8)

and

\[ \sum_{b=1}^{B} \sum_{n=L_1+l}^{L_1+L+l-1} |\eta_{b,n}|^2 > \sum_{b=1}^{B} \sum_{n=L_1+L+l}^{L_1+L-1} |\tilde{h}_{b,n} + \eta_{b,n}|^2, \text{ for } l \in [-L_1, -1]. \]  

(5–9)

5.2.2 Analysis of Mistiming Probability

In this section, we will analyze the mistiming probability of the energy detection based ToA estimator. We will first derive the pairwise probability of mistiming by an arbitrary number of \( l \) taps. Then we use the union bound to obtain an upper bound of the mistiming probability of the ToA estimator. The high SNR approximation of the upper bound will clearly reveal the relationship between the channel parameters and the timing performance. In the following, we will focus on the \( l > 0 \) case. The analysis can be directly applied to the \( l < 0 \) case.

Let \( v_l = \sum_{b=1}^{B} \sum_{n=L_1}^{L_1+l-1} |h_{b,n-L_1} + \eta_{b,n}|^2 \) and \( y_l = \sum_{b=1}^{B} \sum_{n=L_1+L}^{L_1+L+l-1} |\eta_{b,n}|^2, l \in [1, L_2] \).

The pairwise probability of mistiming by \( l \) taps is

\[ P_l = \Pr(y_l > v_l), \ l \in [1, L_2] \]  

(5–10)
which can be further expressed as

\[ P_l = \int_0^\infty f_V(v_l) \int_0^\infty f_Y(y_l) dy_l dv_l \]  

(5–11)

where \( f_V(v_l) \) and \( f_Y(y_l) \) are PDFs of \( v_l \) and \( y_l \), respectively. Since variance of the noise is unity, i.e., \( \mathbb{E}\{\eta_{b,n}^2\} = 1 \), \( \forall b \) and \( n \), the random variable \( y_l \) has a chi-square distribution with \( 2Bl \) degrees of freedom

\[ f_Y(y_l) = \frac{1}{(Bl-1)!} y_l^{Bl-1} \exp(-y_l). \]  

(5–12)

Using the result in [70], we have the pairwise probability of mistiming by \( l \) taps

\[ P_l = \sum_{n=0}^{Bl-1} \frac{1}{n!} \int_0^\infty f_V(v_l) v_l^n \exp(-v_l) dv_l. \]  

(5–13)

Before deriving \( P_l \), we first calculate the conditional pairwise probability of mistiming, given the channel fading coefficients. For certain \( \lambda_l = \sum_{b=1}^B \sum_{n=0}^{l-1} |h_{b,n}|^2 \), the random variable \( v_l \) obeys a noncentral chi-square distribution with PDF [82, Ch. 10]

\[ f_V(v_l; \lambda_l) = \left( \frac{v_l}{\lambda_l} \right)^{Bl-1} \exp(-v_l - \lambda_l) l_{Bl-1}(2\sqrt{v_l\lambda_l}) \]  

(5–14)

where \( l_a(x) \) is the modified Bessel function of the first kind.

Using (5–13) and property of the Laplace transform: \( \mathcal{L}\{v_l^n f_V(v_l; \lambda_l)\} = (-1)^n F_V^{(n)}(s; \lambda_l) \),

the conditional pairwise probability of mistiming \( P_l(\lambda_l) \) can be expressed as

\[ P_l(\lambda_l) = \sum_{n=0}^{Bl-1} \frac{1}{n!} \int_0^\infty f_V(v_l; \lambda_l) v_l^n \exp(-v_l) dv_l \]

(5–15)

\[ = \sum_{n=0}^{Bl-1} \frac{(-1)^n}{n!} F_V^{(n)}(1; \lambda_l) \]

where \( F_V(s; \lambda_l) \) is the Laplace transform of \( f_V(v_l; \lambda_l) \). From the MGF, the Laplace transform of \( f_V(v_l; \lambda_l) \) can be expressed as (see [82, Ch. 9])

\[ F_V(s; \lambda_l) = \mathcal{L}\{f_V(v_l; \lambda_l)\} = \mathbb{E}\{e^{-sv_l}; \lambda_l\} = \frac{\exp\left(-\frac{s\lambda_l}{1+s}\right)}{(1+s)^{Bl}}. \]  

(5–16)
The $n$th order derivative of $F_V(s; \lambda_i)$ can be expressed as

$$F_V^{(n)}(s; \lambda_i) = \sum_{\rho=0}^{n} \left( \frac{n}{\rho} \right) f_1^{(\rho)}(s; \lambda_i) f_2^{(n-\rho)}(s) \quad (5-17)$$

where $f_1(s; \lambda_i) = \exp\left(\frac{-s\lambda_i}{1+s}\right)$ and $f_2(s) = (1 + s)^{-Bl}$. According to the Faà di Bruno’s formula[38], the $p$th order derivative of $f_1(s; \lambda_i)$ can be expressed as

$$f_1^{(p)}(s; \lambda_i) = f_1(s; \lambda_i) \sum_{k=0}^{p} \lambda_i^k B_{p,k} \left(g'(s), g''(s), \ldots, g^{(p-k+1)}(s)\right) \quad (5-18)$$

with $B_{p,k}$ being the Bell polynomial and $g(s) = \frac{-s}{1+s}$. Applying (5–18) to (5–17) and combining terms with the same order of $\lambda_i$, we have

$$F_V^{(n)}(s; \lambda_i) = \exp\left(\frac{-s\lambda_i}{1+s}\right) \sum_{k=0}^{n} \lambda_i^k f_{n,k}(s) \quad (5-19)$$

with $f_{n,k}(s)$ being a function of $s$

$$f_{n,k}(s) = \sum_{\rho=k}^{n} \left( \frac{n}{\rho} \right) f_2^{(n-\rho)}(s) B_{p,k} \left(g'(s), g''(s), \ldots, g^{(p-k+1)}(s)\right) \quad (5-20)$$

The conditional pairwise probability of mistiming can then be simplified using (5–15) and (5–19)

$$P_i(\lambda_i) = \exp\left(\frac{-\lambda_i}{2}\right) \sum_{k=0}^{Bl-1} c_{B,I,k} \lambda_i^k \quad (5-21)$$

where $c_{B,I,k} = \sum_{n=k}^{Bl-1} (-1)^n f_{n,k}(1)$. The pairwise mistiming probability $P_i$ is the expectation of $P_i(\lambda_i)$

$$P_i = \int_{0}^{\infty} f_\lambda(\lambda_i) P_i(\lambda_i) d\lambda_i \quad (5-22)$$

where $f_\lambda(\lambda_i)$ is the PDF of the random variable $\lambda_i$. Same as Eq. (5–21), the integral of (5–22) can also be obtained with the aid of MGF.

The channel taps undergo Nakagami-\(m\) fading with PDF [82]

$$f(\gamma_{b,n}; m) = \frac{m^{m-1} \gamma_{b,n}^{m-1}}{(A_{b,n} \bar{\gamma})^m} \Gamma(m) \exp\left(-\frac{m \gamma_{b,n}}{A_{b,n} \bar{\gamma}}\right) \quad (5-23)$$
\[ F_\Lambda(s) = \prod_{b=1}^{B} \prod_{n=0}^{l-1} \left( 1 + \frac{sA_{b,n} \bar{\gamma}}{m} \right)^{-m} \]

\[
\begin{pmatrix}
\det \begin{pmatrix}
\frac{1}{\sqrt{\rho_{0,1}}} \left( 1 + \frac{m}{sA_{1,0} \bar{\gamma}} \right)^{-1} & \sqrt{\rho_{0,1}} \left( 1 + \frac{m}{sA_{1,1} \bar{\gamma}} \right)^{-1} & \cdots & \sqrt{\rho_{0,Bl-1}} \left( 1 + \frac{m}{sA_{1,B} \bar{\gamma}} \right)^{-1} \\
\vdots & \ddots & \ddots & \vdots \\
\sqrt{\rho_{0,Bl-1}} \left( 1 + \frac{m}{sA_{B,0} \bar{\gamma}} \right)^{-1} & \sqrt{\rho_{1,Bl-1}} \left( 1 + \frac{m}{sA_{1,0} \bar{\gamma}} \right)^{-1} & \cdots & \sqrt{\rho_{1,Bl-1}} \left( 1 + \frac{m}{sA_{B,B} \bar{\gamma}} \right)^{-1} \\
\end{pmatrix}
\end{pmatrix}^{-m}
\]

(5–24)

where \( \gamma_{b,n} = |h_{b,n}|^2 \), \( b \in [1, B] \), \( n \in [0, L - 1] \), \( A_{b,n} \) represents the PDP effect of the \( n \)th channel tap in the \( b \)th frequency band, \( \bar{\gamma} \) is the average SNR and \( \Gamma(x) \) is the Gamma function.

For correlated Nakagami-\( m \) distributed random variables \( \gamma_{b,n} \), a simple form PDF can not be found for \( \lambda_l = \sum_{b=1}^{B} \sum_{n=0}^{l-1} \gamma_{b,n} \) [6], but the Laplace transform of \( \lambda_l \) can be written as [40] where \( \rho_{i,j} \) is the power correlation between \( \gamma_{b_1,m_1} \) and \( \gamma_{b_2,m_2} \) with \( i = (b_1 - 1)l + n_1 \) and \( j = (b_2 - 1)l + n_2 \). Using property of the Laplace transform, we can obtain the pairwise mistiming probability \( P_l \) [cf. (5–21) and (5–22)]

\[ P_l = \sum_{k=0}^{Bl-1} (-1)^k c_{B,l,k} F_\Lambda^{(k)} \left( \frac{1}{2} \right) \]

(5–25)

where \( c_{B,l,k} \) is defined after Eq. (5–21).

At high SNR as \( \bar{\gamma} \to \infty \), the Laplace transform \( F_\Lambda(s) \) can be approximated by [40]

\[ F_\Lambda(s) \approx (\mathcal{D}_{B,l})^{-m} \prod_{b=1}^{B} \prod_{n=0}^{l-1} \left( \frac{sA_{b,n} \bar{\gamma}}{m} \right)^{-m} \]

(5–26)

where \( \mathcal{D}_{B,l} \) denotes the determinant term in (5–24) when \( \bar{\gamma} \to \infty \). Then \( P_l \) can be approximated at high SNR by

\[ P_l = \left( \frac{\bar{\gamma}}{2m} \right)^{-mBl} (\mathcal{D}_{B,l})^{-m} \prod_{b=1}^{B} \prod_{n=0}^{l-1} A_{b,n}^{-m} \sum_{k=0}^{Bl-1} c_{B,l,k} (-2)^k \frac{(mBl)!}{(mBl - k)!} . \]

(5–27)
From (5–27), the pairwise probability of mistiming by \( l \) taps can be approximated at high SNR by

\[
P_l = C_{B,m,l} (\bar{\gamma})^{-mB_l}, \quad l \in [1, L_2]
\]

(5–28)

where \( C_{B,m,l} = (2m)^{mB_l} (D_{B,l})^{-m} \prod_{b=1}^{B} \prod_{n=0}^{l-1} A_{b,n}^m \cdot \sum_{k=0}^{B_l-1} c_{B,l,k} (-2)^k \frac{(mB_l)!}{(mB_l-k)!} \) is a constant independent of \( \bar{\gamma} \). For \( l \in [-L_1, -1] \), the pairwise probability of mistiming can be derived in the same way which has the form of

\[
P_l = C_{B,m,l} (\bar{\gamma})^{mB_l}, \quad l \in [-L_1, -1].
\]

From the union bound, the probability of mistiming \( P_{mt} \) is upper bounded by the summation of the pairwise probability of mistiming. This means that

\[
P_{mt} < \sum_{l=1}^{L_2} C_{B,m,l} (\bar{\gamma})^{-mB_l} + \sum_{l=-L_1}^{-1} C_{B,m,l} (\bar{\gamma})^{mB_l}.
\]

(5–29)

When the SNR \( \bar{\gamma} \) is high, the right hand side of (5–29) approximates \( (C_{B,m,1} + C_{B,m,-1}) (\bar{\gamma})^{-mB} \). As a result, the slope of the probability of mistiming curves is \( G_d = mB \) in log-log scale. In the field of communications, this slope is often defined as the diversity of the system error performance. Eq. (5–29) implies that the diversity gain of the estimator is \( G_d = mB \) which increases with both the subband number and the Nakagami-\( m \) parameter. This shows that by use of the noncoherent combining, the energy detection based ToA algorithm can achieve a higher diversity gain, proportional to the number of subbands and the diversity achieved in each channel.

5.2.3 Simulations

Simulations are first carried out for the discrete time channel with \( L = 12 \) independent Nakagami-\( m \) distributed taps. The channel has an exponentially decaying PDP with the last tap being 20 dB weaker than the first tap. The average power of the first channel tap is normalized. Assume that due to the lack of timing information, \( L_1 = L_2 = 5 \) pure noise terms are involved in the estimated channel at the front and rear parts. The probability of mistiming is simulated for the energy detection based ToA estimator when the channel is independent for different subbands.
**Figure 5-2.** The probability of mistiming with energy detection for $B_m = 6$. The parameter $B$ is the number of subbands and $m$ is the Nakagami-$m$ fading parameter.

**Figure 5-3.** The probability of mistiming with energy detection for Saleh-Valenzuela channels. The parameter $B$ is the number of subbands.
Analysis has shown that by using $B$ subbands, the energy detection based ToA estimation can obtain a diversity gain of $mb$ in the Nakagami-$m$ channel. This is verified by simulations, as the probability of mistiming curves with the same $mb$ are roughly parallel to each other (see Fig. 5-2). In addition, given $mB$, the parallel curves have different horizontal shifts. In the literature of communications, this shift is defined as the coding gain of the probability of mistiming curves. The simulation result is reasonable since a higher coding gain can be obtained when more signal energy is collected from more subbands. Union bounds are also plotted in Fig. 5-2 by using Eq. (5–29), which confirms that these bounds are tight at high SNR.

We then simulate the ToA estimator in the more realistic Saleh-Valenzuela (S-V) channel model \[79\]. The channel has a cluster structure with the cluster arrival rate of $\Lambda = 1/200$ ns$^{-1}$ and the ray arrival rate of $\lambda = 1/20$ ns$^{-1}$. The clusters and rays inside each cluster decay exponentially with time constants $\Gamma = 60$ ns and $\gamma = 20$ ns, respectively. The channel rays are generated as random variables with independent Rayleigh distributed amplitudes and independent uniformly distributed phases. Note that in addition to the random and irregular PDP, closely spaced multipath arrivals will also give rise to correlations among the discrete time equivalent channel taps.

MB-OFDM signals with the bandwidth of 100 MHz are modulated to different subbands around the 1.5 GHz center frequency and passed through the channel. For each subband, the discrete time channel is generated by sampling the subband channel at the rate of $1/T_s = 100$ MHz. Since each subband only contains part of the entire channel information, the equivalent discrete time channels differ across subbands but are correlated. Numerical analysis shows that given these channel parameters, the correlation coefficient between subbands can be up to about 0.5. As shown by Fig. 5-3, given the Nakagami-$m$ parameter ($m = 1$), the diversity gain increases linearly with the number of subbands available ($B$). This again verifies the diversity gain analysis of the energy detection based ToA estimator.
5.3 ToA Estimation for MB-OFDM by Suppressing Energy Leakage

In the preceding section of this Chapter, we have investigated the energy detection-based ToA estimation for MB-OFDM. We have assumed that the channel has an equally-spaced CIR and the channel length is known at the receiver. Based on this simple situation, we proved that the diversity gain of the ToA estimation performance improves as the number of signal bands increases. However, later analysis will show that these assumptions actually do not hold due to the energy leakage phenomenon of the channel estimator. In particular, if channel multipaths are not perfectly sampled, the estimated channel will have an infinite number of taps. In order to solve this problem, we propose the ToA estimation technique by suppressing energy leakage.

Same as the energy detection-based ToA estimation (see, e.g., [7, 108]), we will first estimate the time domain channel by mapping the the frequency domain channel information to a sequence of equally-spaced channel taps with the inter-tap time interval being the inverse of the signal bandwidth. Compared to the more complicated ML based methods including the expectation maximization (EM) algorithm and the space alternating generalized EM (SAGE) algorithm (see, e.g., [18, 119]) as well as the subspace-based estimator (see, e.g., [50]), this method keeps its compatibility with the simple and widely adopted channel estimation method with the equally-spaced model and simultaneously can achieve a good ToA estimation accuracy.

It is known that channel estimation with the equally-spaced model will result in the energy leakage phenomenon. This means that energy of one channel path will disperse into all taps in the estimated channel when this path is mis-sampled by the estimated channel taps\(^2\) (see, e.g., [54, 92]). Energy leakage needs to be mitigated since it will induce large ToA estimation error if the estimator erroneously picks up taps that contain

\(^2\) We use taps for samples in the time domain estimated channel and paths for rays in the original physical channel
the strong energy leakage. Motivated by this, we have designed a new ToA estimation rule relying on the suppression of energy leakage. In particular, ToA will be estimated by searching the model parameters such that the energy leakage is minimized among the estimated taps prior to the first multipath component. If the tap spacing is set as the inverse of the signal bandwidth, the estimation can be performed by only optimizing the first tap delay of the equally-spaced model.

Although the performance analysis of this ToA estimator is mathematically intractable for an arbitrary multipath channel, meaningful results have been obtained based on the analysis of a simplified two-path channel. In the two-path channel, the first path carries the ToA information and the second path models the effect of the trailing paths in a multipath channel. Performance enhancement approaches proposed for the simplified channel have been proved effective for the ToA estimation for the general multipath channel. We have evaluated the ToA estimator with simulations and compared it with the SAGE-based estimator in all eight IEEE 802.15.4a channels. Simulation results show that the proposed ToA estimator performs very well in these channels.

5.3.1 Channel Estimation with Equally-Spaced Taps

MB-OFDM combines the basic OFDM technique and the frequency hopping technique. For an MB-OFDM system with $B$ frequency bands, a block of information symbols $s_b = [s_{b,1}, \ldots, s_{b,K}]^T$ are multi-carrier modulated to the $b$th frequency band, $b \in [1, B]$ on $K$ orthogonal digital subcarriers to form the signal block

$$x_b = F^H s_b$$

where $F$ is the FFT matrix. A GI in the form of ZP or CP is added to each block to mitigate the IBI (see [56, 95]). After the DAC, the signal is carrier modulated and transmitted from the antenna. The transmitted signal then propagates through the channel:

$$h(t) = \sum_{i=1}^{L} h_i \cdot \delta(t - \tau_i) ,$$
where $\{h_l\}_{l=1}^L$ and $\{\tau_l\}_{l=1}^L$ are amplitudes and delays of the $L$ channel paths. It should be noted that the delays are continuously valued numbers.

At the receiver, the arriving waveform is carrier demodulated and ADC to baseband discrete-time samples. After the symbol-level coarse timing, GI is removed and the baseband signal is multi-carrier demodulated with FFT operation to generate the frequency domain signals $\{r_{b,k}\}_{k=1}^K$. It can be readily shown that

\begin{equation}
  r_{b,k} = s_{b,k} H_{b,k} + \xi_{b,k} , \quad k = 1, \ldots, K, \quad b = 1, \ldots, B
\end{equation}

where $\xi_{b,k}$ is the AWGN, and $H_{b,k}$ is the FT coefficient of the channel:

\begin{equation}
  H_{b,k} = \sum_{l=1}^L h_l \cdot \exp(-j \omega_{b,k} \tau_l)
\end{equation}

where $\omega_{b,k}$ is the frequency of the $k$th subcarrier on the $b$th frequency band. Based on (5–32), the ML estimate of $H_{b,k}$ is formed as

\begin{equation}
  \hat{H}_{b,k} = \frac{r_{b,k}}{s_{b,k}} = H_{b,k} + \eta_{b,k} , \quad k = 1, \ldots, K, \quad b = 1, \ldots, B
\end{equation}

where $\eta_{b,k} = \xi_{b,k}/s_{b,k}$ is the noise term on the $(b,k)$th subcarrier (see [97]).

In order to facilitate the analysis in the next section, we can also express Eq. (5–34) in the vector form

\begin{equation}
  \hat{\mathbf{H}}_b = \mathbf{H}_b + \mathbf{\eta}_b
\end{equation}

where $\hat{\mathbf{H}}_b = [\hat{H}_{b,1}, \hat{H}_{b,2}, \ldots, \hat{H}_{b,K}]^T$, $\mathbf{H}_b = [H_{b,1}, H_{b,2}, \ldots, H_{b,K}]^T$ and $\mathbf{\eta}_b = [\eta_{b,1}, \eta_{b,2}, \ldots, \eta_{b,K}]^T$.

In the following, we will estimate the ToA of the channel $\tau_1$ from the estimate of the channel FT coefficients (5–35).

### 5.3.2 Energy Leakage in Channel Estimate

After the frequency domain channel information is obtained at the receiver, we first recover the time domain channel effect. Following the traditional method (see, e.g.,
an equally-spaced model is fitted to the frequency domain channel

\[
\tilde{h}(t) = \sum_{n=1}^{\bar{L}} \tilde{h}_n \cdot \delta(t - \bar{\tau}_n)
\]  

(5–36)

where \(\bar{\tau}_n = \bar{\tau}_1 + (n - 1) T_p, 1 \leq n \leq \bar{L}\) and \(\bar{L} = \lceil T_h / T_p \rceil\) with \(T_h\) being the maximum channel delay spread and \(T_p \ll T_h\) a preset tap interval which can be chosen as the inverse of the signal bandwidth. By doing this, the model (5–36) is solely determined by the delay of the first tap. Therefore, ToA of the channel can be estimated by optimizing the single free parameter \(\bar{\tau}_1\). This is in comparison with existing channel estimators where all channel path delays are individually searched in \([0, T_h]\) (see, e.g., [18, 41, 50]).

For every possible value of the first tap delay \(\bar{\tau}_1\), tap amplitudes \(\tilde{h}(\bar{\tau}_1)\) for the equally-spaced model are expected to satisfy [c.f. (5–33)]

\[
\sum_{n=1}^{\bar{L}} \tilde{h}_n(\bar{\tau}_1) \exp(-j\omega_{b,k} \bar{\tau}_n) = \hat{H}_{b,k}
\]  

(5–37)

where the dependance of \(\tilde{h}_n\)s on \(\bar{\tau}_1\) is explicitly shown. More compactly written, this relationship becomes

\[
G(\bar{\tau}_1)\tilde{h}(\bar{\tau}_1) = H + \eta
\]  

(5–38)

where \(H = [H_1^T, H_2^T, \ldots, H_B^T]^T\) and \(\eta = [\eta_1^T, \eta_2^T, \ldots, \eta_B^T]^T\) includes all the subband FT coefficients and noise terms, respectively, \(\tilde{h}(\bar{\tau}_1) = [\tilde{h}_1(\bar{\tau}_1), \tilde{h}_2(\bar{\tau}_1), \ldots, \tilde{h}_L(\bar{\tau}_1)]^T\) and \(G(\bar{\tau}_1) = [G_1^T(\bar{\tau}_1), G_2^T(\bar{\tau}_1), \ldots, G_B^T(\bar{\tau}_1)]^T\) with \(G_k(\bar{\tau}_1)\) being a \(K \times \bar{L}\) FT matrix with the \((k, n)\)th element being \(\exp(-j\omega_{b,k} \bar{\tau}_n)\). Based on Eq. (5–38), \(\tilde{h}(\bar{\tau}_1)\) can be obtained in an ML-optimum manner as

\[
\tilde{h}(\bar{\tau}_1) = G^H(\bar{\tau}_1)G(\bar{\tau}_1)^{-1} G^H(\bar{\tau}_1)H + \bar{\eta}
\]  

(5–39)

where \(\bar{\eta} = (G^H(\bar{\tau}_1)G(\bar{\tau}_1))^{-1} G^H(\bar{\tau}_1)\eta\) is the noise term (see [97]).
The frequency domain channel response $H_{b,k}$ is the superposition of contributions from all $L$ channel paths as (see (5–33))

$$H_{b,k} = \sum_{l=1}^{L} H_{b,k}(l)$$

(5–40)

with $H_{b,k}(l) = h_l \exp(-j\omega_{b,k}\tau_l)$ for $l \in [1, L]$. Therefore, the estimated time domain channel can also be expressed as

$$\bar{h}(\bar{\tau}_1) = \sum_{l=1}^{L} (G^H(\bar{\tau}_1)G(\bar{\tau}_1))^{-1} G^H(\bar{\tau}_1)H(l) + \bar{\eta}$$

(5–41)

where $H(l)$ is the frequency domain contribution from the $l$th channel path which can be constructed by $H(l) = [H^T_1(l), H^T_2(l), \ldots, H^T_B(l)]^T$ and $H_b(l) = [h_l \exp(-j\omega_{b,1}\tau_l), h_l \exp(-j\omega_{b,2}\tau_l), \ldots, h_l \exp(-j\omega_{b,K}\tau_l)]^T, b \in [1, B]$. In Eq. (5–39), multiple frequency bands are combined in a coherent manner. Same as the other ToA estimation techniques for MB-OFDM in the literature (see, e.g., [76]), the coherent combining in the proposed ToA estimator also requires that no random phase rotation exists in subband signals after carrier demodulation.

**Proposition 9.** For the $l$th channel path with amplitude and delay $(h_l, \tau_l), l \in [1, L]$, when Eq. (5–39) is used for channel estimation, only the $m$th tap of the channel estimate contains non-zero contribution from the $l$th path if this path is exactly sampled by the $m$th tap as $\tau_l = \bar{\tau}_m, \exists m \in [1, \bar{L}]$.

This holds because when $\tau_l = \bar{\tau}_m$, $H(l)$ will be the $m$th column of the matrix $G(\bar{\tau}_1)$ scaled by $h_l$. Then, $G^H(\bar{\tau}_1)H(l)$ becomes the $m$th column of $G^H(\bar{\tau}_1)G(\bar{\tau}_1)$ and only the $m$th element in $(G^H(\bar{\tau}_1)G(\bar{\tau}_1))^{-1} G^H(\bar{\tau}_1)H(l)$ is non-zero (see Eq. (5–41)). If the $l$th channel path is missampled, i.e., $\tau_l \neq \bar{\tau}_n, \forall n \in [1, \bar{L}]$, all channel estimate taps $\{\bar{h}_n\}_{n=1}^{\bar{L}}$ will generally be non-zero even if noise is absent. As a result, energy of this channel path will disperse into all taps in the time domain channel estimate. This is often known as the energy leakage phenomenon (see, e.g., [45, 54, 92]). In a multipath channel environment where channel path delays are continuously varying, the equally-spaced
Figure 5-4. Energy leakage due to the missampling.

The model cannot simultaneously sample all channel paths. Therefore, energy leakage will always exist as shown by Fig. 5-4 which illustrates the time domain channel estimate for a certain value of $\tau_1$. In impulse radio, we can observe the similar phenomenon as inter-pulse interference (IPI) emerges due to the limited bandwidth of the pulse (see [41, 99]).

The tap interval $T_p$ can be set as the inverse of the signal bandwidth, which is known as the time domain resolution of the system [81]. Intuitively, a smaller $T_p$ seems able to resolve the multipath channel better. However, when $T_p$ is smaller than the system resolution, the matrix $G^H(\tau_1)G(\tau_1)$ in (5–39) tends to be ill-conditioned and the problem becomes unsolvable.

From Eq. (5–41), the $n$th estimated channel tap is the summation of contributions from all paths of the physical channel:

$$\bar{h}_n(\tau_1) = \sum_{l=1}^{L} \bar{h}_n(\tau_1, l) + \bar{\eta}_n(\tau_1), \quad n = 1, \ldots, \bar{L} \quad (5–42)$$
where \( \tilde{h}_n(\bar{\tau}_1, l) \) contains the information of the \( l \)th channel path \( \{ h_i, \tau_i \} \) and \( \bar{\eta}_n(\bar{\tau}_1) \) is the related noise term with \( \bar{\eta} = [\bar{\eta}_1(\bar{\tau}_1), \bar{\eta}_2(\bar{\tau}_1), \ldots, \bar{\eta}_L(\bar{\tau}_1)]^T \). When \( T_p \) is chosen as the inverse of the signal bandwidth, the matrix \( G^H(\bar{\tau}_1)G(\bar{\tau}_1) \) becomes \( I_L / N \) with \( I_L \) being an \( L \times L \) identity matrix and \( N = BK \). The contribution from \( (h_i, \tau_i) \) can then be expressed by

\[
\tilde{h}_n(\bar{\tau}_1, l) = h_i \exp \left( j\pi \frac{(N - 1)(\bar{\tau}_n - \tau_i)}{NT_p} \right) \frac{\sin(\pi(\bar{\tau}_n - \tau_i)) / T_p}{N \sin(\pi(\bar{\tau}_n - \tau_i))/ (NT_p)}
\]

with the amplitude being

\[
|\tilde{h}_n(\bar{\tau}_1, l)| = \frac{|h_i| \sin(\pi(\bar{\tau}_n - \tau_i))/ T_p)}{N \sin(\pi(\bar{\tau}_n - \tau_i))/ (NT_p)} \tag{5–44}
\]

for \( n \in [1, \bar{L}] \) and \( l \in [1, L] \). Eq. (5–44) is actually the absolute value of the discrete sinc-function \( N \sin(\pi t / T_p) / \sin(\pi t / (NT_p)) \) scaled by \( h_i \) and sampled at \( t = (\bar{\tau}_n - \tau_i), n \in [1, \bar{L}] \) (see [60, 117]).

From the properties of the discrete sinc-function, if there is an \( m \)th tap in the estimated channel taps that satisfies \( \bar{\tau}_m = \tau_i, \ m \in [1, \bar{L}] \), i.e. the \( l \)th channel path is correctly sampled by the channel estimate, only the \( m \)th term in Eq. (5–43) contains the non-zero contribution from \( \{ h_i, \tau_i \} \) and the other terms are all zero. In particular, we have \( \tilde{h}_m(\bar{\tau}_1, l) = h_i \) when \( \bar{\tau}_m = \tau_i \). Otherwise, if \( \bar{\tau}_n \neq \tau_i, \ \forall n \in [1, \bar{L}], \) all taps have the non-zero contributions from the \( l \)th channel path and the energy leakage emerges. This is the special case of Proposition 9. Knowing that amplitude of the energy leakage is a sampled discrete sinc-function, we can also have the following properties of the energy leakage.

**Proposition 10.** The \( m \)th tap of the channel estimate will contain the strongest energy leakage from the \( l \)th channel path \( \{ \tau_i, h_i \} \) if \( m = \arg \min_{1 \leq n \leq L} |\tau_i - \bar{\tau}_n| \), i.e., \( \{ \bar{\tau}_m, \tilde{h}_m \} \) is the closest tap to \( \{ \tau_i, h_i \} \). Given this \( m \)th tap, the energy leakage on the other taps decreases as their tap indices \( n \ (n \neq m) \) deviate from \( m \).

**Proposition 11.** Given that the \( m \)th tap of estimated channel contains the strongest energy from \( \{ \tau_i, h_i \} \), the gain of this tap increases as \( |\tau_i - \bar{\tau}_m| \) decreases in \([0, T_p/2]\).
Figure 5-5. Energy leakage due to the missampling.

Amplitudes of the other taps that contain weaker contributions \( |\tilde{h}_n(\bar{\tau}_n, l)|, \ n \neq m \) approximately decrease when \( |\tau_l - \bar{\tau}_m| \) decreases in \([0, T_p/2)\).

Proposition 10 means that energy leakage of one path has less impact on the taps that are far from it. Proposition 11 implies that when the channel path is better sampled by the estimated channel as \( |\tau_l - \bar{\tau}_m| \) decreases, more energy from the \( l \)th path will be captured by the \( m \)th tap. Therefore, we can observe that when the \( m \)th estimated channel tap becomes closer to the \( l \)th channel path, energy leakage of this path becomes more peaky with the peak located at \( \bar{\tau}_m \). Eventually, when the \( l \)th channel path is exactly sampled by the \( m \)th tap as \( \bar{\tau}_m = \tau_l \), the estimated channel perfectly recovers the \( l \)th channel tap.

Fig. 5-5 shows the channel estimate which uses a different value of \( \bar{\tau}_1 \) other than that for Fig. 5-4. Comparing Figs. 5-4 and 5-5, we can see that when the first tap delay \( \bar{\tau}_1 \) is chosen such that one tap in the estimated channel is very close to the first channel path, Fig. 5-5 has a much weaker energy leakage prior to the first path \((h_1, \tau_1)\) of the real channel. This interesting observation can be explained with the aforementioned
properties of the energy leakage. First, from Proposition 10, energy leakage for each channel path decreases as the estimated channel tap index deviates from the one that contains the strongest energy of this channel path. Since the estimated channel taps prior to the first path are always closer to the first channel path than the trailing paths, energy leakage prior to the first path tends to be dominated by the first path. Secondly, from Proposition 11, when one tap of the estimated channel is close to the first channel path, energy of the first channel path is mostly captured by this tap and energy leakage prior to this tap is weak. For these reasons, energy leakage prior to the first path can be mitigated when certain tap of the estimated channel is close to the first channel path.

5.3.3 Proposed ToA Estimator

We have analyzed the energy leakage phenomenon when the equally-spaced model is used to estimate the time domain channel for the MB-OFDM UWB. Since channel multipaths can not be simultaneously sampled at their true arrival times by the equally-spaced model, energy leakage will always exist. However, by comparing Figs. 5-4 and 5-5, we see that energy leakage can be effectively suppressed when a proper value of $\bar{\tau}_1$ is chosen. Furthermore, for the ToA estimation purpose, Fig. 5-5 is clearly more preferable than Fig. 5-4. This is because when the channel estimate in Fig. 5-4 is used by the threshold-based ToA estimator [13], one of the several strong energy leakage taps (marked with the ellipse) prior to the real channel ToA can be mistakenly picked out as the first channel path due to their considerable strength. This may cause a severe ToA estimation error. Therefore, we need to suppress the energy leakage by choosing a proper value of $\bar{\tau}_1$ before estimating the ToA.

\footnote{Actually, when the first two paths are close as shown in Figs. 5-4 and 5-5, they can not be resolved due to limited bandwidth and the strongest tap will capture most energy from both of these two paths.}
For multipath channels, when the energy leakage is mitigated prior to the first channel path, one tap of the estimated channel should be sufficiently close to the first channel path in order to capture most of the energy from the first path\(^4\). When this occurs, a sharp jump of the tap amplitude will emerge near the leading edge of the channel which can be used to estimate the ToA (see Fig. 5-5). This sharp jump of amplitude can be detected by searching the value of \(\tau_1\) to maximize the following energy ratio between two adjacent taps

\[
\gamma_n(\tau_1) = \frac{|\hat{h}_n(\tau_1)|^2}{|\hat{h}_{n-1}(\tau_1)|^2}, \quad n \in [L_1, L_2]
\]

where \([L_1, L_2]\) represents the ambiguity region of the first channel path after the coarse timing synchronization. The ToA estimate is then obtained as the delay of the tap where (5–45) is maximized:

\[
\hat{\tau}_1 = \hat{\tau}_1 + (\hat{n} - 1) T_p
\]

with \((\hat{\tau}_1, \hat{n}) = \arg\max_{0 \leq \tau_1 < T_h, L_1 \leq n \leq L_2} \gamma_n(\tau_1)\). The advantage of this criterion is that the ToA estimator can avoid the channel dependent threshold required by the threshold-based ToA estimators (see [13, 78]).

### 5.3.3.1 Analysis of ToA estimation criterion

Due to the nonlinearity of Eq. (5–42), the analysis of (5–45) is mathematically intractable for an arbitrary multipath channel. For this reason, we consider a simplified case where the channel contains two paths and the interval between these two paths is \((p + 0.5) T_p\) with \(p\) being an integer number\(^5\). In this simplified model, the first path carries the ToA information and the second path models the joint effect of the trailing

\(^4\) This tap will also capture energy from the other paths when these paths are close to the first path.

\(^5\) We do not consider the \(p = 0\) case where the two paths can not resolved by the limited bandwidth.
paths. For this case, the strongest energy leakage arises from the second path when the first path is sampled at its true arrival instant. If the two paths are $pT_p$ apart, they can be simultaneously sampled and there is no inter-path interference.

Suppose that the two channel paths have amplitudes $[h_1, h_2]$ and delays $[\tau_1, \tau_2]$. Then the energy ratio (5–45) can be expressed as

$$\gamma_n(\bar{\tau}_1) = \left| \frac{\bar{h}_n(\bar{\tau}_1, 1) + \bar{h}_n(\bar{\tau}_1, 2)}{\bar{h}_{n-1}(\bar{\tau}_1, 1) + \bar{h}_{n-1}(\bar{\tau}_1, 2)} \right|^2, n \in [L_1, L_2]$$

(5–47)

where $\bar{h}_n(\bar{\tau}_1, 1)$ and $\bar{h}_n(\bar{\tau}_1, 2)$ are contributions from the two channel paths which can be expressed by [c.f. (5–43)]

$$\bar{h}_n(\bar{\tau}_1, 1) = h_1 \exp \left( j\pi(N - 1)(\bar{\tau}_n - \tau_1) / NT_p \right) \cdot \frac{\sin(\pi(\bar{\tau}_n - \tau_1) / T_p)}{N \sin(\pi(\bar{\tau}_n - \tau_1) / (NT_p))}$$

(5–48)

and

$$\bar{h}_n(\bar{\tau}_1, 2) = h_2 \exp \left( j\pi(N - 1)(\bar{\tau}_n - \tau_2) / NT_p \right) \cdot \frac{\sin(\pi(\bar{\tau}_n - \tau_2) / T_p)}{N \sin(\pi(\bar{\tau}_n - \tau_2) / (NT_p))}$$

(5–49)

where the parameter $\bar{\tau}_1$ shows that given the tap interval, the estimated channel is determined by the first tap delay.

Using $\tau_2 = (p + 0.5)T_p + \tau_1$ and $\exp(j\pi(p + 0.5)T_p/(NT_p)) \approx 1$ when $N \gg p$, we have the following approximation

$$\bar{h}_n(\bar{\tau}_1, 2) = h_2 \exp \left( -j\pi(N - 1)(p + 0.5) / N \right) \cdot \frac{\sin(\pi(\bar{\tau}_n - \tau_1) / (NT_p)) \sin(\pi(\bar{\tau}_n - \tau_2) / T_p)}{\sin(\pi(\bar{\tau}_n - \tau_1) / T_p) \sin(\pi(\bar{\tau}_n - \tau_2) / (NT_p))} \bar{h}_n(\bar{\tau}_1, 1)$$

(5–50)

$$\approx h_2 (-1)^{\delta_j} \cdot \frac{\sin(\pi(\bar{\tau}_n - \tau_1) / (NT_p)) \sin(\pi(\bar{\tau}_n - \tau_2) / T_p)}{\sin(\pi(\bar{\tau}_n - \tau_1) / T_p) \sin(\pi(\bar{\tau}_n - \tau_2) / (NT_p))} \bar{h}_n(\bar{\tau}_1, 1).$$

Therefore, energy of the $n$th tap can be approximated by

$$\left| \bar{h}_n(\bar{\tau}_1) \right|^2 = \left| \bar{h}_n(\bar{\tau}_1, 1) \right|^2 + \left| \bar{h}_n(\bar{\tau}_1, 2) \right|^2$$

(5–51)
and the energy ratio can be expressed as

\[ \gamma_n(\bar{\tau}_1) = \frac{|\bar{h}_n(\bar{\tau}_1, 1)|^2 + |\bar{h}_n(\bar{\tau}_1, 2)|^2}{|\bar{h}_{n-1}(\bar{\tau}_1, 1)|^2 + |\bar{h}_{n-1}(\bar{\tau}_1, 2)|^2}, \quad n \in [L_1, L_2]. \quad (5-52) \]

Since Eq. (5–52) is still too complicated to analyze, we need to further simplify it. Suppose that $|\bar{h}_n(\bar{\tau}_1, 1)|$ and $|\bar{h}_n(\bar{\tau}_1, 2)|$ reach their maximum values at the $m$th and $m'$th taps ($m < m'$), respectively, such that $|\bar{h}_m(\bar{\tau}_1, 1)| = \max(|\bar{h}_n(\bar{\tau}_1, 1)|)$ and $|\bar{h}_{m'}(\bar{\tau}_2, 1)| = \max(|\bar{h}_{n'}(\bar{\tau}_1, 2)|), n, n' \in [1, N]$. First, we assume that the energy ratio will only achieve its maximum value at the $m$th or $m'$th tap of the estimated channel. This assumption will be confirmed by numerical analysis later. Based on this, the ToA estimate will be either $\bar{\tau}_m$ or $\bar{\tau}_{m'}$. From Proposition 10 of energy leakage analysis, we know that $|\tau_1 - \bar{\tau}_m| \in [0, T_p/2)$ and $|\tau_2 - \bar{\tau}_{m'}| \in [0, T_p/2)$. Certainly, we only want to keep $\bar{\tau}_m$ as the ToA estimate because $\bar{\tau}_{m'}$ will induce a large estimation error. For this reason, we will later use an approach to avoid the energy ratio $\gamma_n(\bar{\tau}_1)$ to be maximized at the $m'$th tap. For now, we just assume that the ToA estimate is $\bar{\tau}_m$ and focus on the analysis of $\gamma_n(\bar{\tau}_1)$ at the $m$th tap where $|\bar{h}_n(\bar{\tau}_1, 1)|$ is maximized.

5.3.3.2 Estimate ToA with $\bar{\tau}_m$

From Proposition 11 of energy leakage analysis, we know that $|\bar{h}_m(\bar{\tau}_1, 1)|$ is a decreasing function of $|\tau_1 - \bar{\tau}_m|$. Similarly, $|\bar{h}_m(\bar{\tau}_1, 2)|$ and $|\bar{h}_{m-1}(\bar{\tau}_1, 2)|$ which are the energy captured from the second path are increasing functions of $|\tau_2 - \bar{\tau}_{m'}|$. Due to the relationship $\tau_2 = (\rho + 0.5) T_p + \tau_1$, $|\tau_2 - \bar{\tau}_{m'}|$ decreases as $|\tau_1 - \bar{\tau}_m|$ increases. Based on these, both $|\bar{h}_m(\bar{\tau}_1, 2)|$ and $|\bar{h}_{m-1}(\bar{\tau}_1, 2)|$ are increasing functions of $|\bar{h}_m(\bar{\tau}_1, 1)|$. This implies that when the ToA estimation error $|\tau_1 - \bar{\tau}_m|$ decreases, the energy captured from the first path increases and simultaneously the interference from the second path also increases. As a result, the energy ratio (5–52) may not be maximized at the correct ToA estimate, i.e. $\bar{\tau}_m = \tau_1$; hence ToA estimation error emerges.

In the following, we will further analyze the relationship between the ToA estimation error $|\tau_1 - \bar{\tau}_m|$ and the energy ratio $\gamma_m(\bar{\tau}_1)$. Since $|\bar{h}_m(\bar{\tau}_1, 1)|$ is a decreasing function
of $|\tau_1 - \bar{\tau}_m|$, this turns out to be the analysis of the relationship between $|\bar{h}_m(\bar{\tau}_1, 1)|$ and $\gamma_m(\bar{\tau}_1)$. From (5–48), $|\bar{h}_m(\bar{\tau}_1, 1)|$ reaches its maximum value $h_1$ when $\bar{\tau}_m = \tau_1$ and the first path is exactly sampled. When $\bar{\tau}_m = (\tau_1 \pm 0.5 T_p)$, $|\bar{h}_m(\bar{\tau}_1, 1)|$ reaches its minimum value

$$|\bar{h}_{1,n}(\bar{\tau}_1)| = \left| \frac{h_1}{N} \cdot \frac{\sin(0.5\pi)}{\sin(0.5\pi/N)} \right| \approx \frac{2h_1}{\pi}$$

(5–53)

where the approximation holds when $N \gg 1$.

In order to address the monotonic relationship between $|\bar{h}_m(\bar{\tau}_1, 1)|$ and $|\bar{h}_m(\bar{\tau}_1, 2)|$ as well as $|\bar{h}_{m-1}(\bar{\tau}_1, 2)|$, we will use the following approximation

$$|\bar{h}_n(\bar{\tau}_1, 2)|^2 = \left( |\bar{h}_m(\bar{\tau}_1, 1)| - \frac{2h_1}{\pi} \right)^2 c_e(n), \ n = m, m - 1, \ |\bar{h}_m(\bar{\tau}_1, 1)| \in [2h_1/\pi, h_1]$$

(5–54)

where $2h_1/\pi$ is the minimum value of $|\bar{h}_m(\bar{\tau}_1, 1)|$ and $\{c_e(n)\}_{n=m-1}^m$ are positive coefficients. Values of $\{c_e(n)\}_{n=m-1}^m$ reflect the interference strength from the second path, which are determined by the amplitude of the second path and the interval between the first and second paths. In particular, the stronger the second path or the
smaller the inter-path interval, the larger \( \{ c_e(n) \}_{n=m-1}^m \). Numerical analysis in Fig. 5-6 shows the normalized \( \{ c_e(n) \}_{n=m-1}^m \) given \( h_1 = h_2 = 1 \) for different inter-path intervals. For different values of \( |\tau_1 - \bar{\tau}_m| \), the change of \( c_e(n) \) is very small for \( n = m - 1, m \). Therefore, \( c_e(n) \) can be approximated as a constant to \( |\tau_1 - \bar{\tau}_m| \) and accordingly a constant to \( |\bar{h}_m(\bar{\tau}_1, 1)| \). In Fig. 5-6, \( c_e(n) \) varies slowly with respect to \( n \) especially when the inter-path interval is large. Therefore, we can assume that \( c_e(m - 1) \approx c_e(m) \).

From Proposition 11 of the energy leakage analysis, \( |\bar{h}_{m-1}(\bar{\tau}_1, 1)| \) decreases as \( |\bar{h}_m(\bar{\tau}_1, 1)| \) increases. In order to model this monotonicity, we use the simple approximation \( |\bar{h}_{m-1}(\bar{\tau}_1, 1)|^2 \approx h_1^2 - |\bar{h}_m(\bar{\tau}_1, 1)|^2 \). After this, the energy ratio at the \( m \)th tap of the estimated channel (5–52) is simplified as

\[
\gamma_m(\bar{\tau}_1) \approx \frac{|\bar{h}_m(\bar{\tau}_1, 1)|^2 + (|\bar{h}_m(\bar{\tau}_1, 1)| - 2h_1/\pi)^2 c_e(m)}{h_1^2 - |\bar{h}_m(\bar{\tau}_1, 1)|^2 + (|\bar{h}_m(\bar{\tau}_1, 1)| - 2h_1/\pi)^2 c_e(m)}, \quad |\bar{h}_m(\bar{\tau}_1, 1)| \in [2h_1/\pi, h_1].
\] (5–55)

Eq. (5–55) only contains a single independent variable \( |\bar{h}_m(\bar{\tau}_1, 1)| \) with the coefficient \( c_e(m) \) modeling the interference from the second path. For each \( c_e(m) \), the ToA estimate can be calculated by searching \( |\bar{h}_m(\bar{\tau}_1, 1)| \) to maximize the energy ratio (5–55). Fig. 5-7 shows that the ToA estimation error decreases as \( c_e(m) \) decreases. This is a quite reasonable results since \( c_e(m) \) reflects the interference from the second path. In addition, when \( c_e(m) \) is smaller than a certain value (10 dB in Fig. 5-7), the ToA estimation criterion by maximizing (5–55) can always guarantee an accurate estimation result. As \( c_e(m) \) becomes larger, the ToA estimation error will rapidly increase because the value of (5–55) is dominated by the second path.

Fig. 5-8 shows the numerical analysis result for the two path simplified channel where the ToA is estimated by maximizing the energy ratio of (5–52) at the tap that contains the strongest contribution from the first path. As predicted by the theoretical analysis in Fig. 5-7, the ToA estimation error decreases either as the inter-path interval increases or the second path strength decreases, both of which result in the decrease of \( c_e(m) \) (see (5–54)). Also, when the inter-path interval is large, the ToA estimation is very
Figure 5-7. Time-of-arrival (ToA) estimation error by maximizing Eq. (5–55).

Figure 5-8. ToA estimation by maximizing the energy ratio at the strongest component tap of the first path; two path channel.
Figure 5-9. Avoid picking out the second path; \( M = 4 \).

accurate since inference from the second path has a small impact on the locating of the first path. As the second path strength increases, the ToA estimation error will rapidly increase since the inter-path inference is dominant. These results again match with the theoretical analysis based on Eq. (5–55) very well.

5.3.3.3 Avoid the fake ToA estimate \( \bar{\tau}_{m'} \)

In the preceding analysis, we assume that the ToA can be estimated by searching \( \bar{\tau}_1 \) to maximize the energy ratio \( \gamma_n(\bar{\tau}_1) \) at the \( m \)th tap of the estimated channel. This \( m \)th tap contains the strongest energy from the first channel path. However, as shown by Fig.5-8, the energy leakage \( \gamma_n(\bar{\tau}_1) \) can also reach the maximum at the \( m' \)th tap where the energy from the second path is maximized if the second path is stronger. When this happens, \( \bar{\tau}_{m'} \) will be erroneously chosen as the ToA estimate.

In order to solve this problem, we need to avoid the energy ratio \( \gamma_n(\bar{\tau}_1) \) being maximized at the \( m' \)th tap. We will use the following modified energy ratio instead of the one defined by (5–52)

\[
\gamma_n(\bar{\tau}_1, M) = \frac{|\bar{h}_n(\bar{\tau}_1)|^2}{\frac{1}{M} \sum_{i=n-1}^{n-M} |\bar{h}_i(\bar{\tau}_1)|^2}, \quad n \in [L_1, L_2].
\] (5–56)
Figure 5-10. ToA estimation by maximizing the energy ratio among the reconstructed taps for the two path channel; $\tau_2 = \tau_1 + 3.5 T_p$.

Different from (5–52), the denominator in (5–56) also includes $(M - 1)$ taps prior to the current one. For the $m$'th path that the numerator of $\gamma_m(\bar{\tau}_1, M)$ captures the strongest energy contribution of the second path, if $M$ is sufficiently large, the denominator will include not only the weaker energy contributions of both the channel paths but also the strongest energy contribution of the first path (see Fig. 5-9). However, for the $m$th tap that the numerator of $\gamma_m(\bar{\tau}_1, M)$ contains the strongest energy contribution of the first path, the denominator only contains the weaker energy leakage contributions from both paths. As a result, it will be less likely for Eq. (5–56) to reach its maximum value at the $m$'th tap, in comparison with (5–52). Simulations show that this approach will also work for the IEEE 802.15.4a UWB channels.

This method has been proved to be efficient by numerical analysis for the two-path channel where the ToA is estimated by searching for the maximum value of $\gamma_m(\bar{\tau}_1, M)$ among all taps of the estimated channel (see Fig. 5-10). As a performance benchmark, the ToA estimation based on maximizing the energy ratio at the $m$th tap which contains
the strongest energy contribution of the first path is also included. Fig. 5-10 shows that the ToA estimate can always satisfy either \(|\hat{\tau}_1 - \tau_1| < 0.5 T_p\) or \(|\hat{\tau}_1 - \tau_2| < 0.5 T_p\). This validates the previous assumption that the energy ratio will only achieve its maximum value at the taps that capture the strongest energy from the two paths.

In Fig. 5-10, an optimal value of \(M\) exists that enables the best ToA estimation performance. When \(M\) is smaller than this value, the ToA estimation accuracy improve as \(M\) increases because it becomes less likely to locate at the trailing paths when more leading paths are included in the denominator of \(\gamma_n(\bar{\tau}_1, M)\). As \(M\) continuously increases, the ToA estimation performance will again degrade. This is because as more weaker energy leakage taps prior to the first path are included, the denominator of \(\gamma_n(\bar{\tau}_1, M)\) and therefore the estimator becomes less sensitive to ToA estimation error.

In the later simulations, similar phenomena can be observed in IEEE 802.15.4a UWB channels.

5.3.3.4 ToA estimation for multipath channels

We have obtained some interesting results by analyzing properties of energy leakage and the energy ratio for a two-path channel. First, we find that energy leakage prior to the first channel path can be mitigated by choosing a proper value of the first tap delay \(\bar{\tau}_1\) of the equally-spaced model. Based on this, we propose the algorithm which estimates the ToA by searching the estimated channel tap where the energy ratio is maximized. Secondly, performance of the ToA estimator is largely dependent on the interference from the trailing paths. In particular, the stronger the interference, the larger the ToA estimation error. Thirdly, in order to improve the ToA estimation performance, we use the modified energy leakage Eq. (5–56) to avoid the estimator erroneously locating at the strong trailing paths.

Based on these discussions, we have the following ToA estimation algorithm for the general multipath channels.
Proposed ToA Estimation Algorithm: For each $\bar{\tau}_1 \in [0, T_p)$, we evaluate the energy ratio $\gamma_n(\bar{\tau}_1, M) = (|\bar{h}_n(\bar{\tau}_1)|^2) / (\frac{1}{M} \sum_{i=n-M}^{n-1} |\bar{h}_i(\bar{\tau}_1)|^2)$ at the $n$th tap, $n \in [L_1, L_2]$. 

$[L_1, L_2]$ represents the ambiguity region of the first channel path after the coarse timing synchronization. Then find the $(\bar{\tau}_1, n)$ pair that maximizes the energy ratio $\gamma_n(\bar{\tau}_1, M)$:

$$
(\hat{\bar{\tau}}_1, \hat{n}) = \arg\max_{0 \leq \bar{\tau}_1 < T_p, L_1 \leq n \leq L_2.} \gamma_n(\bar{\tau}_1, M).
$$

(5–57)

The ToA estimate can then be obtained as:

$$
\hat{\tau}_1 = \hat{\bar{\tau}}_1 + (\hat{n} - 1) T_p.
$$

(5–58)

The range of $\bar{\tau}_1$ in (5–57) has been reduced to $[0, T_p)$ as compared to the $[0, T_h]$ in Eq. (5–46) where $T_h$ is the maximum channel delay spread. This is because for the equally-spaced model, the energy ratio satisfies $\gamma_n(\bar{\tau}_1 + kT_p, M) = \gamma_{n+k}(\bar{\tau}_1, M)$ for $\bar{\tau}_1 \in [0, T_p)$ with $k$ being an integer (see (5–43)). Therefore, it is sufficient to limit $\bar{\tau}_1$ in $[0, T_p)$ when $\gamma_n(\bar{\tau}_1, M)$ is also maximized with respect to the tap index.

Similar to the impulse radio\cite{41, 99}, the interference from trailing paths is essentially due to the limited signal bandwidth of the system. The energy leakage can be mitigated either by using more bandwidth (see \cite{102, 108}) or a windowing function to the channel FT coefficient sequence in (5–34) before it is converted to the time domain. Windowing has the advantage of suppressing the side lobes at the price of inducing a wider main lobe \cite{60}. For our problem, this implies that the energy leakage becomes weaker but there will be multiple strong taps even if the channel path is exactly sampled at its arrival instant. Therefore, as will be illustrated in simulation results, windowing may or may not improve the performance of ToA estimation depending on whether the strength of the weaker energy leakage taps or the main lobe width is dominant.

5.3.4 Simulations

In this section, we simulate the performance of the proposed ToA estimator based on the MB-OFDM UWB system specified in the ECMA-368 standard \cite{1}. In
Figure 5-11. ToA estimation performance versus $M$.

this standard, the entire UWB spectrum is divided into 14 equally-sized subbands. For
each subband with a bandwidth of 528 MHz, the multicarrier modulation/demodulation
is performed with a 128 point IFFT/FFT. A total of 122 subcarriers are used as data,
guard and pilot subcarriers. Each simulation is carried out in 2000 randomly generated
channel realizations. Assume that the remaining timing ambiguity is $\pm 3.3$ ns after
the symbol-level coarse synchronization (see, e.g., [53]) which corresponds to the
spatial distance of $\pm 1$ meter. We simulate the proposed ToA estimator in all eight IEEE
802.15.4a channel models and compare it with the SAGE algorithm. Performance of the
subspace-based algorithm is usually quite sensitive to channel order mismatch [69] and
turns out to be not as good as the other two algorithms. Therefore, we will not include
the subspace-based algorithm in simulations.

We first evaluate the mean absolute error performance for the proposed ToA
estimator in the IEEE 802.15.4a channel model CM1 with different values of $M$ in Eq.
(5-56). Simulation results in Fig. 5-11 show that the ToA estimator performs better for
a larger $M$. However, when $M \geq 5$, the performance only improves slightly. When $M$
Figure 5-12. ToA estimation performance for the residential channels.

continuously increases, the performance may even degrade, because the estimator becomes less sensitive to the ToA estimation error. This has also been observed in the simplified two-path channel. For the other seven channel models, the ToA estimator also works very well with $M = 5$.

We then evaluate the proposed ToA estimator in the eight IEEE 802.15.4a channel models (see Figs. 5-12-5-15) with $M = 5$. These channel models include four different types of environments with both the LoS and non-line-of-sight (NLoS) propagations: residential LoS and NLoS (CM1&CM2), office LoS and NLoS (CM3&CM4), outdoor LoS and NLoS (CM5&CM6) as well as industrial LoS and NLoS (CM7&CM8) channel models. Channel estimate results obtained by (5–39) with a random $\bar{\tau}_1$ are fed to the SAGE-based ToA estimator as the initial state of the iterative algorithm.

Simulation results in Figs. 5-12-5-15 show that in general the proposed ToA estimator performs very well, given that the receiver sampling interval is 1.89 ns = 1/528 MHz. Performance in the LoS channel is better than the corresponding NLoS channel.
Figure 5-13. ToA estimation performance for the office channels.

Figure 5-14. ToA estimation performance for the outdoor channels.
This is because in the NLoS channel, the first channel path may not be a strong one and the synchronization is more likely to be affected by noise and the trailing paths (see, e.g., [14, 28]). However, there is an exception for the outdoor environment where performance for the NLoS channel (CM6) is better than the LoS channel (CM5) at high SNR. Comparing channel realizations generated by these two channel models, we find that channel paths in CM6 are weaker but sparser than CM5. As a result, the CM6 channel can be better resolved than CM5 and the localization of its first path will be less interfered by its trailing paths. At low SNR, as the performance is dominated by noise, the LoS channel CM5 will still be better than the NLoS channel CM6.

Performance of the proposed ToA estimator is quite different for various channel environments. We find that it performs best in the outdoor LoS channel (CM7). This is because CM7 contains an extremely strong first path. As a result, the ToA estimation performance is nearly not affected by the noise in the entire simulated SNR range. Compared to the other channel models, the proposed ToA estimator can obtain a
relatively good performance in both the residential LoS and NLoS channels. This is because channel paths of these two channels are sparser than the others. Performance in the industrial NLoS channel (CM8) is the worst one, since the CM8 channel is much denser than the other seven channels and its first path is weak.

For all channel models, we applied the Hamming windowing function to the frequency domain FT coefficient sequence and then convert it back to the time domain for ToA estimation. By doing that, energy leakage on each tap is reduced and due to the wider main lobe, more than one taps contain the strongest energy leakage when the channel path is exactly sampled. Simulations show that the ToA performance degrades at low SNR. This is because due to the wider main lobe, energy captured by the tap which is used for ToA estimation is weaker. Therefore, the ToA estimator has worse resistibility against the noise. At high SNR, the ToA estimator with windowing tends to outperform the original one, since the interference from trailing paths is effectively suppressed. We have also tried other windowing functions such as the Blackman window which can further reduce the strength of energy leakage with an even wider main lobe. Simulations show that the performance severely degrades because multipaths can not be resolved.

The proposed ToA estimator is also compared with the SAGE-based ToA estimator in the IEEE 802.15.4a channel models. Simulations show that the proposed estimator outperforms SAGE in terms of the mean absolute ToA estimation error. SAGE is a generalized form of the EM algorithm and it is a feasible method to estimate multiple parameters due to its faster convergence than the EM algorithm. Similar to the EM algorithm, SAGE may not converge to the globally optimum solution, especially when the number of unknown parameters is large. As a result, waveform of the estimated channel may not be very close to the real channel, in terms of each path’s delay and amplitude. The proposed ToA estimator directly addresses the delay estimation of the first channel path, without unnecessarily caring about the estimation errors for the trailing paths.
Therefore, our estimator can locate the first channel path better and enable a better ToA estimation performance.

5.4 Conclusions

In this Chapter, we investigated both the non-coherent combining and coherent combining ToA estimation techniques for MB-OFDM signals. For the non-coherent energy detection-based ToA estimator, we analyzed the probability of mistiming performance in Nakagami-$m$ channels. Theoretical analysis shows that a higher diversity gain can be achieved by using more subbands in channels with a lower amount of fading (larger $m$). Simulations in both the Nakagami-$m$ discrete time channel and the S-V channel model have corroborated our theoretical analysis. In order to address the energy leakage problem, we proposed a novel ToA estimator which locate the first channel path by minimizing the energy leakage prior to the first path. By directly addressing the estimation of the first channel path, the proposed ToA estimator can outperform the traditional SAGE algorithm in terms of accuracy. The proposed ToA estimator operates in baseband which avoids the complicated manipulation of the analog or oversampled waveform at the receiver.
CHAPTER 6
SUMMARY AND FUTURE WORK

6.1 Summary

In this dissertation, we have investigated UWB transceiver design techniques and applied signal processing methods to facilitate the design. We introduced MCD-UWB technique which can bypass the explicit channel estimation without invoking the analog delay line. Compared with FSR-UWB, MCD-UWB has shown advantages of higher power efficiency and more flexible operations. Then we utilized the MIMO system to model asymmetric UWB links. This modeling method turns out to be very effective in optimizing the system performance. To facilitate our transceiver design, we also investigated the timing synchronization of UWB signals with TDT algorithms. In particular, we proved the feasibility and effectiveness of TDT algorithms in digital IR receivers with low-resolution ADCs and IRs with orthogonal bi-pulse modulation. Simulations have also been carried out to corroborate our analysis. In order to further exploit the high timing resolution of UWB signals, we investigated ToA estimation techniques for MB-OFDM UWB signals. We proposed two ToA estimators, namely, the energy detection-based estimator and the ToA estimator by suppressing the energy leakage, which adopt the non-coherent and coherent combining of channel information, respectively. Analysis and simulations have been provided to confirm the validity of the ToA estimators.

6.2 Future Work

In Chapter 5, we have assumed that the random phase rotation has been calibrated when channel information over subbands is coherently combined. In our future work, we will address the estimation problem of the random phase rotation. Since the phase rotation will induce randomness among subbands, the channel estimation which is performed prior to ToA estimation, will not fully achieve the timing resolution provided by the coherent combining of multiple subbands. As a result, stronger energy leakage
will emerge for a larger phase rotation. Following this idea, we can solve the problem by searching for the phase rotation and the first tap delay $\bar{\tau}_1$ which is a nuisance parameter such that the energy leakage prior to the first channel path is minimized. This is actually the same method that has been used to estimate the ToA for the MB-OFDM UWB system in Chapter 5. Figs. 6-1-6-4 show the normalized (by $\pi$) phase rotation estimation results for the eight IEEE 802.15.4a channels. Since the criterion is to minimize the energy leakage prior to the first path, the phase estimation algorithm only uses the information of leading paths. In the future, we should also consider the phase estimation with all channel information.

Besides the ToA estimation, we are recently interested in tracking the moving target by measuring the Doppler frequency induced by its radial velocities with respect to the anchor nodes. This problem exists in both the UWB system and general mobile communication systems. However, due to the huge signal bandwidth, the UWB receiver is able to extract the first channel path and use it to estimate the Doppler frequency and consequently the radial velocity. Based on the geometric relationship between the moving target and anchor nodes, we can find out the x- and y-axis coordinates of the target. This approach relies on the angle-of-arrival (AoA) information. Therefore, no common time base is required either between the moving target and each anchor node or among all anchor nodes. In addition, since the AoA information is acquired with Doppler frequency, our approach does not need the antenna array that is required by the AoA-based method [5].
Figure 6-1. Phase rotation estimation for the residential channels.

Figure 6-2. Phase rotation estimation for the office channels.
Figure 6-3. Phase rotation estimation for the outdoor channels.

Figure 6-4. Phase rotation estimation for the industrial channels.
REFERENCES


BIOGRAPHICAL SKETCH

Huilin Xu was born in Qiqihar, Heilongjiang Province, China. He received Bachelor of Science and Master of Science degrees from the University of Science and Technology of China, Hefei, China, in 2002 and 2005, respectively, both in electrical engineering. From Fall 2005 to Summer 2010, he was a PhD student in the Department of Electrical and Computer Engineering, University of Florida, Gainesville, Florida. He received his PhD degree in 2010. His research interests are in the areas of wireless communications and signal processing.