A UNIFIED STATE-SPACE AND SCENARIO TREE FRAMEWORK FOR MULTI-STAGE STOCHASTIC OPTIMIZATION: AN APPLICATION TO EMISSION-CONSTRAINED HYDRO-THERMAL SCHEDULING

By

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A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2010

Für meine Eltern

ACKNOWLEDGMENTS

Panagote M. Pardalos was not only my PhD advisor but also a mentor for life. He was able to find a perfect balance between close supervision and academic freedom, allowing me to explore different research directions. It was his foresight to introduce me to the energy application field. Furthermore, he was able to open seemingly locked doors for me – and that more than just once.

Mario V.F. Pereira invited me to his company PSR where I spend a very productive and enjoyable time over a period of more than three months. The insight I have gained during these visits on the power systems optimization practice in general and on SDDP in particular, are of unmeasurable wealth to me. His scientific discussions were of great value and I thank him for co-advising this dissertation.

Niko A. Iliadis allowed me to gain first hand research experience during my time as an intern at his company EnerCoRD. This was the start of my exposure to real world energy system optimization problems, leading to this dissertation topic.

Bruno Flach was the safe port for my research. He answered all my technical questions on stochastic programming, hydro-thermal scheduling and SDDP with great skill, knowledge, and patience; shaping this dissertation in a unique way.

Josef Kallrath is a steady, candid and sincere supporter of my professional career ever since I was his intern at the BASF cooperation in 2004. I want to thank him not only for his numerous scientific collaborations, but also for his trust and faith in me.

Lothar Kountz was my math teacher in high school during my 9th, 10th, and 11th grade. It was his dedication to math, his skills in teaching, his abilities to motivate me and his faith in me which lead me down this wonderful path. I cannot express how much this means to me.

Timothy J. Anderson, Jean Philippe Richard and J. Cole Smith shaped this research through various comments and suggestions as their role of my dissertation committee members.

My parents are the anchor in my life. I thank them for their infinite and unconditional support.

During my four years of graduate studies at the University of Florida, I was fortunate to research with very bright, knowledgable and dedicated scientists. Next to the scholars mentioned above, I especially want to thank Saed Alazamir, Ashwin Arulselvan, Nikita Boyko, Lily Elefteriadou, Joshua Grasso, Gregory Holmes, Chang-Chia Liu, Artyom Nahapetyan, Ingrida Radziukyniene, Oleg Prokopyev, Basim Uthman, Petros Xanthopoulos, Vitaliy Yatsenko and Qipeng Phil Zheng for their collaboration.

Last, I want to thank Luiz Augusto Barroso, Rafael Kelman, Luiz Carlos Costa, and Fernanda Thomé for their fruitful discussions on this research as well as Ashwin Arulselvan, Bruno Flach, Sandra Garcia, Donatella Granata, and Josef Kallrath for their careful proof read of my dissertation.

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ADP	Approximate Dynamic Programming
cf.	confer
CO_2	Carbon dioxide
CVaR	Conditional Value at Risk
DP	Dynamic Programming
e.g.	exempli gratia
et al.	et alii
ETS	Emission Trading Scheme
EVS	Expected Value Solution
GHG	Green House Gas
GWh	Gigawatt hour
i.e.	id est
LP	Linear Programming
MILP	Mixed Integer Linear Programming
MW	Megawatt
MWh	Megawatt hour
PAR(k)	Periodic Autoregressive Model of lag-k
RHS	Right Hand Side
s.t.	subject to
SDDP	Stochastic Dual Dynamic Programming
SDDPT	Stochastic Dual Dynamic Programming with scenario Tree

SDP	Stochastic Dynamic Programming
VaR	Value at Risk
vs.	versus
VSS	Value of the Stochastic Solution

Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

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August 2010

Chair: Panagote M. Pardalos
Major: Industrial and Systems Engineering

In the hydro-thermal scheduling problem, one is interested in determining the optimal operating policy for the use of hydro and thermal resources in order to minimize total expected costs of fulfilling the demand for electricity over a given time horizon. Originally proposed in 1991 by Pereira and Pinto, Stochastic Dual Dynamic Programming (SDDP) remains to date the most efficient algorithm which is able to cope with inflow uncertainty and a detailed representation of a system's characteristics.

In this dissertation, we propose several extensions of the SDDP methodology: We embed the SDDP algorithm into a scenario tree framework, incorporate CO_2 emission allowance constraints, and supplement the profit maximization models to account for CO_2 emission allowance markets.

These extensions allows us to additionally deal with uncertainties related to the evolution of demand and fuel prices. From a practical standpoint, this is an innovation as fuel price and electricity demand uncertainty could not be taken into account efficiently in hydro-thermal power systems so far, and from a technical standpoint, this is a new approach unifying the state-space and scenario tree framework.

The importance of such an approach was made evident by the global economic crisis of 2008 when several countries experienced huge variations in demand and faced

sudden and sharp increases in fuel costs due to oil price swings, with implications not only on total incurred costs but also regarding security of supply.

Despite the uncertainty surrounding the design of a mechanism which is ultimately accepted by nations worldwide, the necessity to implement measures to curb emissions of greenhouse gases on a global scale is consensual. The electricity sector plays a fundamental role in this puzzle and countries may soon have to revise their operating policy directives in order to make them compatible with additional constraints imposed by such regulations.

Managing an annual emission allowance is somewhat similar to managing water reservoirs since one must determine the optimal trade-off between consuming parts of the limited amount of a resource in the present moment or saving it for future use. The decision to deplete the CO_2 stock on hand may only be assessed in terms of its expected future costs, which depend on the evolution of hydrological conditions. Thus, a reservoir model for the CO_2 emission quota has been proposed, respecting the stage decomposition framework of stochastic dynamic programming methods. This reservoir model allows for CO_2 allowances to expire at given times. This is practically of high importance as this model reflects the currently implemented policy of the EU Emission Trading Scheme.

The deregulation of the electricity markets made it necessary to incorporate uncertain electricity prices into the optimization models. Those models are typically solved using a hybrid method of the stochastic dynamic programming and stochastic dual dynamic programming. We extend those methods by incorporating stochastic CO_2 emission allowance prices and stochastic fuel prices. The input data are derived by a fundamental model which allows us to capture the joint correlation of electricity market prices, CO_2 emission allowance prices, fuel prices and hydro inflows.

CHAPTER 1
INTRODUCTION

The Three Gorges Dame in Eastern China is currently the world's largest electricity generator with an installed capacity of 18,200 MW in 2009 and an estimated construction cost of $26.3 billion [62]. Many countries around the world rely on hydro-electric power as one of their electricity sources. Hydro-electric power can be a large electricity source with basically zero marginal electricity production cost. Furthermore, hydro-electric power is a clean energy source emitting negligible amounts of greenhouse gases. However, the investment cost are typically immense.

In conventional power systems, the goal is to minimize expected electricity generation cost, while maintaining an adequate security of supply [124]. In this light, if we are given a purely hydro-electric power system, then the security of supply is the only concern. However, if there are thermal generation units next to the hydro-electric power stations, then there is trade-off between short-term costs (*e.g.* within a month) and long-term economic and security considerations (*e.g.* years). That is, there is a trade-off between using the water for electricity generation in the current period (leading to lower generation cost in this period and maybe a power shortage in the future if a draught occurs) or saving the water for future periods (leading to higher immediate cost and maybe spillage in the future if a very wet season occurs). As the water inflows are not known with a fair amount of certainty over a longer time period (*e.g.* years), those inflows have to be considered uncertain.

Hydro-thermal power system operators face different uncertainties when decisions have to be made. The water inflows are the most prominent among those uncertainties - especially when planning in a mid-term to long-term horizon. Until today, there are no models that can predict the hydro inflows with a good enough precision for such a time period the optimization models desire; *e.g.*, one year. However, those inflows are not totally random, as there are some seasonal patterns and as the inflows tend to depend on previous inflows. This enables the use of stochastic optimization methods, where the common assumption is that the “real” distribution of the data can be approximated fairly well.

The basic idea of robust optimization à la Ben-Tal and Nemirovski [14, 15] is to ensure that an optimal solution to a (convex) optimization problem remains feasible, even though there might be some noise in the data; *i.e.*, the data in the optimization model might not be correct as it is subject to uncertainty. Hence, the feasibility is the main concern. Most popular are the following two methods treating uncertainty in the way “robust” optimization is understood. The first method assumes that the data vary in a certain, known interval. This leads to very conservative solutions. The second method does also not assume that the distribution of the uncertainty is known, but instead, the mean and the standard deviation are available. Then, a robust counterpart to the original model is formed, which ignores some rare events with the help of safety parameters. Again, the resulting solutions are robust against certain changes in the uncertain parameter(s). In both cases, the robust counterparts remain convex problems [18]. Floudas and co-workers [91, 103] consider the robustness of scheduling problems under uncertainty. Specifically, two cases are considered which fit into the classification above: bounded uncertainty and uncertainty with known probability distribution.
Their robust optimization framework is applied to mixed-integer linear programming techniques for short-term scheduling problems to derive solutions which are, in a sense, immune against data uncertainty. Similarly, Verderame and Floudas [171] propose a robust optimization approach towards demand uncertainty for the operational planning of batch plants. The authors argue that the time horizon of several months make it mandatory to consider demand uncertainties. The integration of both the operational planning and scheduling for batch plants under demand and processing time uncertainty has been proposed by Verderame and Floudas [172] as well.

In contrast, stochastic optimization assumes that the uncertainty can be modeled (or more realistically, approximated) via some known distribution. For the purpose of this dissertation, we are restricted to discrete distributions and we concern ourselves mainly with sampling-based methods, while mentioning also the scenario-based approaches towards uncertainty modeling and solving.

Comparing robust and stochastic optimization, one recognizes that the advantages of robust optimization are that the distributions of the data do not have to be known and that the resulting problems are typically more computationally tractable than stochastic programming problems [151]. Especially in applications where a robust solution is essential (such as system designs), stochastic programming methods might fail [6]. However, when the distributions of the data can be estimated fairly well, then the stochastic programming approach might be favorable; especially when decisions are made on a routinely basis, a robust optimization approach might be too conservative. One might also want to combine the advantages of both the stochastic and the robust approach when it comes to design questions [142].

Despite the advantages of robust optimization, stochastic optimization is the standard in hydro-thermal scheduling. The stochastic programming methodology has been used for more than 30 years ago and is accepted as the tool of choice in the hydro-thermal scheduling community. The primary reason is that the water inflows seem to be indeed "stochastic;" that is, the inflows follow a distribution which can be estimated fairly well with the data available.

Using an analytic approach, investment decisions (*e.g.*, in new power plants) have to take into account the whole life time of the assets at hand. This leads to long-term optimization problems, having a typical horizon of 15-20 years. Reservoir management of hydro-thermal energy plants is a typical mid-term horizon problem with a length of 1 to 3 years. In contrast, short-term optimization problems cover typically up to one week (sometimes also days). Examples are unit commitment [152], economic dispatch

[173], and optimal power flow problems [132]. As mentioned before, hydro inflows are not know in advance over the horizon of mid-term or long-term optimization problems. Therefore, those problems are typically treated with stochastic approached whereas the short-term optimization problems are typically deterministic.

1.1 Electricity Market Deregulation

The energy industry went through a revolution in the last two decades: it was deregulated around the world. Electricity used to be sold by public utilities, seeking to meet the demand of their customers at minimal cost. Therefore, the utility used its own power plants and contracts with other power supplier. The price of the electricity was set (fixed) by the utility or government. In the early 1990s, this started to change significantly. The first (deregulated) electricity markets appeared in Chile in 1990. Electricity was traded in this market as a commodity: it could be sold and bought at a price determined by the market; *i.e.*, the prices of electricity varied according to demand and supply of electricity.

In the late 1990s and the beginning of this century, the deregulation of the electric power sector took place around the world. `NordPool` was established in 1996 and is the electricity market for the scandinavian countries Norway, Denmark, Sweden and Finland. The European Energy Exchange (EEX), established in 2002 (the result of a merge of the Power Exchange and the European Energy Exchange) is located in Leipzig and is Germany's energy exchange. The USA has several wholesale electricity markets – for example, ERCOT Market, New York Market, Midwest Market or California ISO – where PJM, located in Valley Forge, Pennsylvania, is the world's largest competitive wholesale electricity market, deregulated since 2002/2004.

The deregulation of energy markets made it more difficult for the power producers to operate their assets in an optimal way. The main reason is that a(nother) source of uncertainty has been introduced: electricity prices. To serve the electricity demand of its customers, a company can now choose to use its own physical assets (generating decision) or buy in the market (buying decision). In a liberalized market, the company's objective is no longer the minimization of its operating costs but the maximization of its profits – under a determined risk profile. Hence, along with the liberalization of the electricity market, another complication has been added: risk control.

The risk exposure for an electric power supplier caused by volatility of the electricity markets has to be hedged using financial tools; *i.e.*, by using a systematic approach to transfer risk from one portfolio to another through the electricity spot and derivatives markets. The main operational risks faced by power utilities are (i) electricity wholesale prices, (ii) fuel prices, (iii) volumetric risks, and (iv) credit risks; *cf.* Iliadis [85]. For electric utilities, the generation and hedging decisions should be made jointly [58, 113] as they are correlated and, hence, the separation theorem by Holthausen [80] does not apply. Various different risk measures for power utilities have been introduced. Value at Risk (VaR) being the established standard in the banking industry and other major application areas [92], is also an important risk measure in the power industry. Ever since Rockafellar and Uryasev [145] introduced their variant called Conditional Value at Risk (CVaR), it gained increasing popularity in the risk management community and recently also in the energy industry [180]. Its convexity and coherence [5] make CVaR an attractive risk measure for energy applications, especially in the context of hydro-thermal scheduling [82, 85].

Energy systems around the world are in different states of the deregulation process. There are still countries which are centrally dispatched, *e.g.*, Central and South American countries, while others have facets of both the regulated and the deregulated markets. In Germany, for instance, some public utilities are confronted with the situation that they have to serve the electricity demand of their customers while using their own assets as well as different types of energy contracts. However, they are not allowed to sell electricity in the spot-market. Such a situation is described by Rebennack et al. [141], where a short-term portfolio optimization problem with a quarter hour resolution for a period of one day is discussed. A deterministic mixed integer programming model is proposed and the implemented program is available in the GAMS [23] model library under the name "poutil" (Portfolio Optimization for electric UTILities) [61]. This model was then extended by Rebennack et al. [139] to the trading in the balancing market where the important feature of the model remains the dispatch of the power plant in discrete steps.

The deregulation of energy markets has a great impact on the energy system and is not without failures [97]. Due to market power abuse by dominant players, poor market design, and thin trading of forward and futures contracts, deregulated electricity markets may fail to supply electricity reliably and cheaply. Examples of such failures were reported in UK, Norway, Alberta (Canada) and California (USA) [176]. Hence, these markets have to be designed properly, monitored and analyzed with great care.

In sums, in a deregulated electricity market, the classical least-cost minimization problem for hydro-thermal power suppliers is replaced by revenue maximization models. In other words, the least cost operation of the system, once the concern of the central dispatcher, is replaced by the challenge of optimally bidding in the electricity market. However, under the hypothesis of absence of market power, the system operation where agents are free to submit price and quantity bids is shown to be equivalent to that which results from a centralized least-cost scheduling. Hence, as a price taker, the optimal bidding strategy is given by the marginal system cost, which can be derived through a cost minimization model [36, 69]. Furthermore, through a "fundamental approach," the electricity spot prices can be estimated through the centrally dispatched least-cost scheduling problem; again, assuming the absence of market power [166]. We will discuss that more detailed in Chapter 6 of this dissertation.

1.2 CO_2 Emissions

Another challenge is already on its way facing the electric power producer, but even more, the whole humanity: climate change due to human activities.

It seems indisputable that there is a climate change currently taking place at the Earth. However, it is very controversially discussed among scientists, how large the human activities really contribute to this development. Especially under these circumstances, the Kyoto Protocol, signed on December 11th in 1997 in Kyoto, Japan, can be seen as a milestone in fighting global warming, as it commits industrialized countries to reduce greenhouse gas emissions [1]. The Kyoto Protocol aims to reduce emissions of six greenhouse gases: CO_2, methane, nitrous oxide, hydrofluorocarbons perfluorocarbons, and sulphur hexafluoride [2]. We focus in this dissertation on CO_2 emissions; though the concepts discussed apply in general.

The main question, however, is only answered partly by the Kyoto Protocol: how to achieve these reduction targets for the global green house gas emissions economically and ecologically worthwhile?

There are basically two alternative to achieve an emission reduction on a country level: emission taxes or “Cap-and-Trade” systems. A tax system does exactly do what its name suggests: enforcing a tax on emissions. Such a tax can be put into place with a small amount of bureaucracy, it is easily controllable and the price is fixed. However, this tax seems politically infeasible and has the disadvantage of not limiting the emissions directly. In contrast, a Cap-and-Trade mechanism puts a cap on the emissions of a whole system whereas the emission price is determined by a market mechanism [45]. The main motivation behind a carbon tax or a Cap-and-Trade mechanism comes from the fact that climate change is recognized as a global problem and can only be solved globally. Hence, it is not important at which exact location the greenhouse gas emissions reduction is achieved, but more importantly, that the reduction is done in a cost efficient way.

Willi Sutton, an infamous US bank robber, once replied to the question why he robbed banks: “Because that’s where the money is.” Exactly this is the question for CO_2 emissions, too. Where do human-made CO_2 emissions come from? The U.S.

Energy Information Administration (EIA) publishes annually a report on the emissions of greenhouse gases in the United States [48]. Figure 1-1 shows the CO_2 emissions in percentage for five different sectors for the whole US in 2007. The figure reveals that the electricity production takes the largest share of the CO_2 emissions with 38% followed by the transportation sector with 36%. Hence, aiming for CO_2 emission reductions in the power industry is a natural choice – not only because it is the largest CO_2 emitter but also because there are clean technologies readily available, *e.g.*, through sustainable energy sources such as wind, solar, biomass or nuclear.

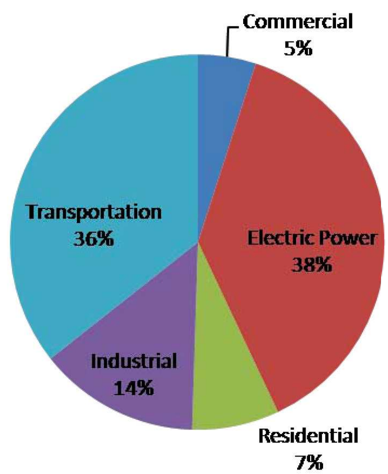


Figure 1-1. CO_2 emissions shares for different sectors in the USA in 2007; data source [48]
Electric Power Transportation Industrial Residential Commercial

The energy systems in the US vary significantly between the different states which explains Figure 1-2. The state of Vermont (VL) has the highest rate of nuclear-generated power in the US while having no coal-fired power plant. This makes their energy system "clean." On the other spectrum is the State of West Virginia which generates most of its electricity with coal-fired power plants.

The US emitted in 2007 approximately 6,017 million metric tons carbon dioxide [48]. The emitted metric tons CO_2 vary widely among the different states of the US, as shown in Figure 1-3. In the light of Figure 1-2, this can partly be explained by the

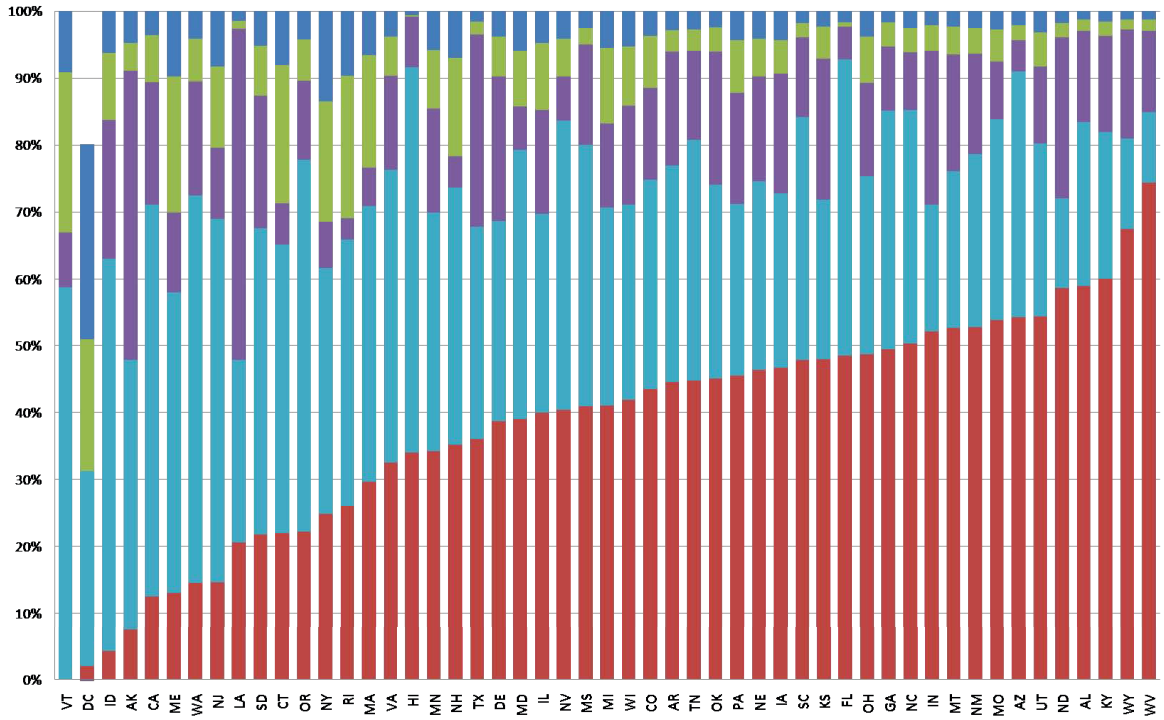


Figure 1-2. CO_2 emissions shares for different sectors for the USA states in 2007; data source [48]
■ Electric Power ■ Transportation ■ Industrial ■ Residential
■ Commercial

different energy mix but also by the different energy demands and economies among the US states.

The largest multinational emission trading scheme in the world is the European Union Emission Treading Scheme (EU ETS) for CO_2 [51]. The governments of the EU member states agreed on national emission caps and allocate the allowances to their industrial operators via so-called "national allocation plans." Plant operators have to monitor and annually report their CO_2 emissions and they have to return the used emission allowances of CO_2 in each year; although the CO_2 emissions are given for several years in advance in order to avoid annual anomalies. Those installations which have allowances left over can sell them in the market or save them for future use. Those that exceed their total emissions have to pay a fine of 100 € per ton of CO_2 emissions and their names are published.

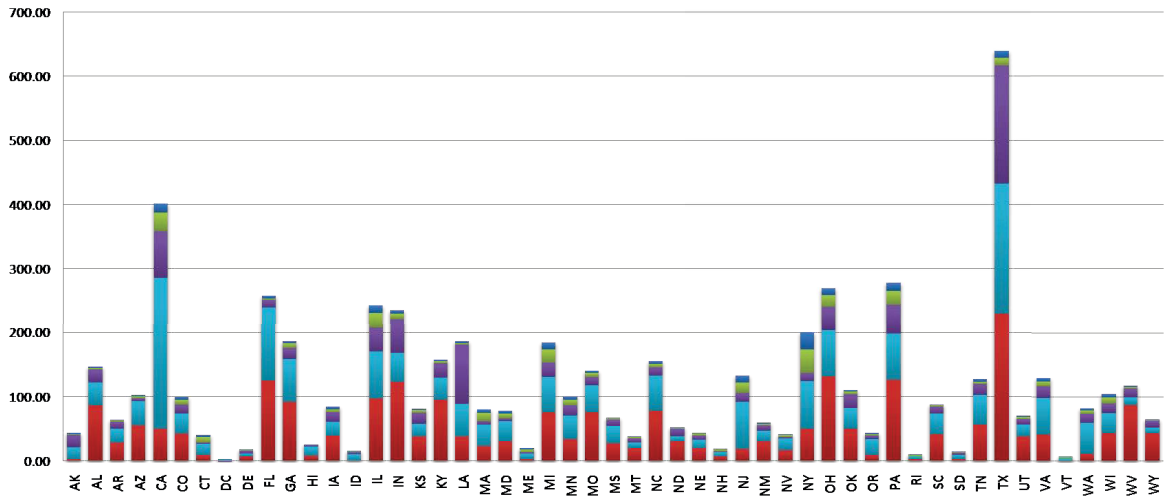


Figure 1-3. CO_2 emissions per US state in 2007 in million metric tons; data source [48]
■ Electric Power ■ Transportation ■ Industrial ■ Residential
■ Commercial

Setting the CO_2 emission allowances in a Cap-and-Trade system at an appropriate level is a non-trivial task as the experience with the European ETS shows [157]. It turns out that the trading system is very sensitive to changes in the amount of CO_2 emission allowances issued. If not done properly, there are either too many allowances available, leading to a market price of zero for the CO_2 emission allowances or too few and the market prices hits the level of the fine. Hence, appropriate tools and mechanisms are required for setting those levels meaningful.

Electricity companies within an emission trading scheme for CO_2 allowances face new challenges, managing their portfolio of assets in an optimal way. The need for optimization tools was also empirically shown by Ehrhart et al. [46], where a simulation of CO_2 prices was performed.

1.3 Types of Uncertainties

Next to the uncertain hydro inflows, power producer face electricity price uncertainty in the liberalized market. That is, the hourly (or quarterly) buy prices and the sell prices for electricity in the spot market are unknown in advance. In the light of the discussion above, power producer may face uncertain CO_2 emission allowance prices, as well, posting important challenges on the daily operation decisions and the strategic planning.

During the global economic crisis of 2008, several countries experienced huge variations in demand and faced sudden and sharp increases in fuel costs due to oil price swings, with implications not only on total incurred costs but also regarding security of supply. Hence, fuel price and electricity demand uncertainty might be of great importance, dependent on the energy system and the optimization horizon.

We group the uncertainties into the following four categories:

1. within a stage and independent of previous stages;
2. dependent on previous stages; seasonal dependencies;
3. dependent only on previous stage; vary every state;
4. political/macro-economical.

Examples for each uncertainty are

1. outage of plants, load fluctuations;
2. hydro inflow;
3. electricity spot prices, CO_2 spot prices;
4. fuel prices, electricity demand.

These sources of uncertainty can be treated as follows:

1. within each stage, separate of each other – typically treated via Monte Carlo sampling – this is not the focus of this dissertation and we refer to Costa [31];
2. via "multivariate periodic autoregressive models," due to their linearity, this is the standard approach used within the stochastic dual dynamic programming scheme, discussed in Chapter 3;
3. via Markov-Chains, transition matrices provide the conditional probability, used in Chapter 6;
4. via scenario trees, topic of Chapter 4.

Fuel prices and the evolution of electricity demand are heavily influenced by political and macro-economic decisions. For instance, imagine that the OPEC (Organization of the Petroleum Exporting Countries) announces an increase in oil extraction or that the global economy slows down. In both cases, the oil price is expected to decrease. For such events, a Markov Chain approach toward price modeling does not seem to be appropriate. In contrast, the idea of different political and macro-economic scenarios seems appealing and natural.

1.4 Dissertation Structure

In Chapter 2, we briefly review the concepts of multi-stage stochastic (linear) programming and apply it to hydro-thermal scheduling. The intention is to embed the hydro-thermal scheduling problems of the following chapters in a well defined framework.

Chapter 3 reviews the solution techniques commonly used and existing in the literature. Especially the concepts of stochastic dynamic programming (SDP) and stochastic dual dynamic programming (SDDP) are discussed in details. The latter method is adopted to solve problems described in the later chapters.

The incorporation of fuel cost uncertainty and electricity demand uncertainty in the context of hydro-thermal scheduling and their incorporation in the dynamic programming framework is discussed in Chapter 4. The proposed method combines a tree approach for handling certain types of uncertainty with the classical autoregressive models for the water inflows. Computational tests are performed on the real power systems of Panama and Costa Rica.

CO_2 emission caps on a hydro-thermal energy system is the subject of Chapter 5. Those quotas are modeled using reservoirs, allowing the incorporation of emission system caps in the dynamic programming framework, regardless of the length of the stages in the model, the time horizon of the CO_2 emission allowances issued and their expiration date. A case study on the real power system of Guatemala shows the economic effects of different quota levels on the electricity prices and the change in operation policy resulting from emission allowance restrictions.

A profit maximization model under the presence of an electricity market and a CO_2 emission quota market is discussed in Chapter 6. The CO_2 emission quota prices are forecasted using a fundamental approach, where the whole electricity power system is dispatched centrally in an optimal way. This is done by using a cost minimization model, incorporating the tree framework for fuel price and electricity price uncertainty (developed in Chapter 4) as well as the CO_2 emission constraints (developed in Chapter 5) into the standard least-cost hydro-thermal scheduling model. This model allows capturing the joint correlations of the different stochasticities in the energy system: inflow, electricity price, fuel cost, electricity demand and CO_2 emission allowance price.

This dissertation is concluded in Chapter 7.

1.5 Dissertation Contributions

The main contributions of this dissertation are several extensions to the classical hydro-thermal scheduling problem. Those extensions focus on modeling aspects allowing the usage of dynamic programming algorithms tailored to stochastic hydro-thermal scheduling problems. Each of the chapters in this dissertation build on each other in a natural way, leading finally to a profit maximization model dealing with the whole set of uncertainties at the same time while capturing the major correlations among those. All the extensions and proposed models have been implemented in the modeling language Mosel [71, 94]. In particular, the dissertation has the following major contributions:

1. Incorporation of the scenario tree approach towards the modeling of uncertainty into the dynamic programming framework. This is novel and can be seen as a unification of the discrete, "tree-community" and the continuous, "Markov Chain-community;" *cf.* Chapter 4. This extends the research of Pereira and Pinto [125] to a new algorithm we call "SDDPT" and is an improvement of the results of Pereira et al. [121].

2. Modeling of CO_2 emission quotas on a centrally dispatched hydro-thermal power system. The proposed optimization model allows the calculation of marginal CO_2 emission prices for a power system via the dual multipliers of the reservoir constraints. This provides insights on the effects of CO_2 caps on the operational part; *cf.* Chapter 5. The proposed reservoir model is capable of accommodating a detailed representation of emissions and related constraints. It is thus an alternative formulation to the work of Belsnes et al. [13].

3. Joint modeling of several uncertainties along with their correlation in a deregulated electricity market. A sub-model is used to generate those price forecasts and a stochastic optimization model is proposed; *cf.* Chapter 6. This extends the work of Belsnes et al. [13] and Mo et al. [112].

CHAPTER 2
MULTI-STAGE STOCHASTIC LINEAR PROGRAMMING APPLIED TO HYDRO-THERMAL SCHEDULING

Let us go back to the roots of stochastic programming with a quotation of Dantzig in his famous Management Science article in 1955: “linear programming methods [have to] be extended to include the case of uncertain demands for the problem of optimal allocation of a carrier fleet to airline routes to meet an anticipated demand distribution” [35]. This sentence captures the very idea of stochastic programming that decisions have to be made under uncertain/unknown data. Throughout the following 55 years, a rich theory and applications of stochastic programming have been established.

We begin in Section 2.1 with a generic multi-stage stochastic programming formulation and discuss the underlying assumptions of the problem and their formulations. This section is based on the work of Birge and Louveaux [21] as well as Kall and Wallace [93]. Afterwards, the hydro-thermal scheduling problem is introduced and framed in the previously presented context of multi-stage stochastic programming in Section 2.2. We discuss extensions and the validity of the model presented. These discussions are the basis for the proceeding chapters.

2.1 Multi-stage Stochastic Programming Formulation

We are facing the following general problem. At stage one, we have to make a decision[1] $\mathbf{x}$ satisfying the constraints $A\mathbf{x} = b$ while experiencing the cost $c\mathbf{x}$. After this decision $\mathbf{x}$ has been made, a random event ω_2 of the set of possible outcomes[2] Ω occurs. Then, in the second stage, we have to adjust our decision from stage one using variables $\mathbf{x}_2$. Iteratively, when the decisions for stage $t-1$ are made and the random

[1] For notational convenience, we consistently do not distinguish between row and column vectors throughout this dissertation.

[2] To be more precise, one would have to argue with random variables on a probability space. However, as we are dealing only with a discrete and finite set of outcomes, this notation should suffice.

outcome, corresponding to this stage $t-1$, has been observed, then we have to adjust our decision from stage $t-1$ using variables $\mathbf{x}_t$.

The overall goal is to minimize the cost of the first stage, $c\mathbf{x}$, plus the expected cost of the optimal decision of all subsequent stages

$$\min \mathbb{E}_{\omega_2 \in \Omega} \left[q_2(\omega_2)\mathbf{x}_2(\omega_2) + \ldots + \min \mathbb{E}_{\omega_t \in \Omega | \omega_{t-1}, \ldots, \omega_2} [q_t(\omega_t)\mathbf{x}_t(\omega_t)] + \ldots + \right.$$
$$\left. + \min \mathbb{E}_{\omega_T \in \Omega | \omega_{T-1}, \ldots, \omega_2} [q_T(\omega_T)\mathbf{x}_T(\omega_T)] \ldots \right], \qquad (2\text{–}1)$$

resulting from our decision of the first stage, our recourse decisions and the (conditional) random outcomes $\omega_2, \ldots, \omega_T$, assuming T stages in the problem.

In the first stage, decision variables $\mathbf{x}$ have to satisfy the constraint set $A\mathbf{x} = b$, and in all other stages, the constraints

$$T_t(\omega_t)\mathbf{x}_{t-1}(\omega_{t-1}) + W_t\mathbf{x}_t(\omega_t) = h_t(\omega_t), \qquad t = 2, \ldots, T \qquad (2\text{–}2)$$

have to be met by the stochastic decision variables $\mathbf{x}_t(\omega)$.

Then, the multi-stage stochastic program can be written as

$$\min \ c\mathbf{x} + \min \mathbb{E}_{\omega_2 \in \Omega} [q_2(\omega_2)\mathbf{x}_2(\omega_2) + \ldots +$$
$$+ \ \min \mathbb{E}_{\omega_t \in \Omega | \omega_{t-1}, \ldots, \omega_2} [q_t(\omega_t)\mathbf{x}_t(\omega_t)] + \ldots +$$
$$+ \ \min \mathbb{E}_{\omega_T \in \Omega | \omega_{T-1}, \ldots, \omega_2} [q_T(\omega_T)\mathbf{x}_T(\omega_T)] \ldots] \qquad (2\text{–}3)$$

$$\text{s.t. } A\mathbf{x} = b \qquad (2\text{–}4)$$

$$T_2(\omega_2)\mathbf{x} + W_2\mathbf{x}_2(\omega_2) = h_2(\omega_2) \qquad (2\text{–}5)$$

$$\vdots$$

$$T_T(\omega_T)\mathbf{x}_{T-1}(\omega_{T-1}) + W_T\mathbf{x}_T(\omega_T) = h_T(\omega_T) \qquad (2\text{–}6)$$

$$\mathbf{x} \geq 0, \quad \mathbf{x}_t(\omega_t) \geq 0, \qquad t \in \mathbb{T}_1, \qquad (2\text{–}7)$$

where $\mathbb{T}_1$ denotes the set of stages without the first stage; *i.e.*, $\mathbb{T}_1 := \mathbb{T} \setminus \{1\}$.

We can interpret the multistage stochastic program as follows: in stage 1, we want an optimal decision $\mathbf{x}^*$ in the sense that for all future outcomes in the following $T-1$ periods, the decision is on average the best. Hence, for each stage, we "simulate" the decision process. Suppose that we are in stage t. Then, we already know the realization of the stochastic outcome $\omega_t, \omega_{t-1}, \omega_{t-2}, \dots, \omega_2$ corresponding to all previous periods $t-1, t-2, \dots, 2$. Recognize that the random event ω_t has been observed at stage t when the decision $\mathbf{x}_t$ has to be made. This is known as the "Wait-and-See" formulation; a detailed discussion is presented in Section 2.2 on the example of hydro-thermal scheduling.

Constraints (2–5) - (2–6) of the multistage program have a special structure known as nonanticipativity constraints. The idea is that for decisions at stage t, the random events $\omega_{t+1}, \dots, \omega_T$ are not known, meaning that the future stages have not been observed. In other words, the decisions in the first stage, $\mathbf{x}$, are independent of the random events $\omega_2, \dots, \omega_T$ and have to be made a priori before they are observed.

2.1.1 Assumptions

For our purposes, we make the following assumptions:

the multi-stage stochastic programs considered have

1. fixed recourse; *i.e.*, the constraint matrix W is fixed and not random, which is shown in formulation (2–3) - (2–7) as W does not depend on the random outcomes ω;

2. deterministic "stochastic" constraint matrix $T_t(\omega_t)$; *i.e.*, just like W, matrix $T_t(\omega_t)$ is actually fixed and reduces to T_t;

3. continuous decision variables in linear expressions; *i.e.*, only non-negative continuous variables are included in the model, represented by constraints (2–7).

4. discrete, finite and known distribution of the random outcomes; as this is a very strong assumption, it is sufficient to assume that we can approximate the (discrete or continuous) distribution of the data by a discrete distribution "fairly" well;

5. to meet constraints (2–5) - (2–7) in a probabilistic sense; *i.e.*, they have to be met "almost surely;" as we assume a finitely, discrete distribution, this assumption is obsolete;

6. that the distribution of the random outcome of stage t depends on the outcome of all previous stages. Later on, we will assume the Markov property; *i.e*, the distribution of stage t depends only on the outcome of the previous stage $t-1$.

For the hydro-thermal scheduling problems considered in this dissertation, the linearity of all decision variables is assumed. The resulting multi-stage stochastic linear programs are much easer to solve compared to stochastic integer programs with integer variables occurring not only in the first stage decisions. One explanation of this challenge is the fact that the expected tth–stage value functions (see below) are discontinuous and non-convex; *cf.* to Blair and Jeroslow [22].

The research on solution methods for multi-stage stochastic integer programming problems have been driven by the research groups of Schultz and Sen. For reviews, we refer to Schultz et al. [154], Schultz [153] and Sen [156].

The validity of those assumptions for our application to hydro-thermal scheduling is discussed later on in Section 2.2.2.

2.1.2 Deterministic Equivalent Programming

The multi-stage stochastic programs of the form (2–3) - (2–7) become quickly very large, and hence, they are difficult to solve. One idea to tackle this problem and to exploit its block diagonal structure is to write the multi-stage stochastic program as a "one-stage program" in the following way

$$\min\ c\mathbf{x} + \mathcal{Q}_2(\mathbf{x}) \tag{2–8}$$

$$\text{s.t. } A\mathbf{x} = b \tag{2–9}$$

$$\mathbf{x} \geq 0 \tag{2–10}$$

where $\mathcal{Q}_2(\mathbf{x})$ is the expected cost function (for the future stages) depended on the first stage decision $\mathbf{x}$.

The definition of $\mathcal{Q}_2(\mathbf{x})$ is obtained recursively as follows. Define the Tth–stage value function

$$
\begin{aligned}
&Q_T(\mathbf{x}_{T-1}, \omega_T) := \\
&\qquad \min_{x_T} \{ q_T(\omega_T)\mathbf{x}_T \mid W_T\mathbf{x}_T = h_T(\omega_T) - T_{T-1}(\omega_T)\mathbf{x}_{T-1},\, \mathbf{x}_T \geq 0 \}\,.
\end{aligned} \tag{2–11}
$$

Let us formally take the expectation of (2–11) for $t = T-1, \dots, 1$, leading to the expected $t+1$th–stage value function

$$
\mathcal{Q}_{t+1}(\mathbf{x}_t) := \mathbb{E}_{\omega_{t+1}\in\Omega|\omega_t,\dots,\omega_2}\left[Q_{t+1}(\mathbf{x}_t, \omega_{t+1})\right]. \tag{2–12}
$$

Now, define the tth–stage value function for $t = 2, 3, \dots, T-1$

$$
\begin{aligned}
&Q_t(\mathbf{x}_{t-1}, \omega_t) := \\
&\qquad \min_{x_t} \{ q_t(\omega_t)\mathbf{x}_t + \mathcal{Q}_{t+1}(\mathbf{x}_t) \mid W_t\mathbf{x}_t = h_t(\omega_t) - T_{t-1}(\omega_t)\mathbf{x}_{t-1},\, \mathbf{x}_t \geq 0 \}\,.
\end{aligned} \tag{2–13}
$$

$\mathcal{Q}_{t+1}$ is then the expected future cost function as we will call it in the next chapters.

If $\mathcal{Q}_2(\mathbf{x})$ is an analytically known function of variable $\mathbf{x}$, then the program (2–8) - (2–10) is deterministic. Hence, if the expected second-stage value function $\mathcal{Q}_2(\mathbf{x})$ is analytically given, then the multi-stage stochastic program (2–3) - (2–7) reduces to a deterministic (non)linear program, because $\mathcal{Q}_2(\mathbf{x})$ might be a nonlinear function in $\mathbf{x}$. Formulation (2–8) - (2–10) is therefore also called deterministic equivalent program to (2–3) - (2–7).

If we add to the assumptions listed in Section 2.1.1 also that the cost coefficients $q_t(\omega_t)$ are fixed, then $\mathcal{Q}_2(\mathbf{x})$ is a convex function in variable $\mathbf{x}$. The reason is that the stochastic part only affects the right hand side of constraints (2–5) – (2–6). This yields to piecewise linear expected future cost functions.

2.2 Application: Hydro-Thermal Scheduling Problem

In the hydro-thermal scheduling problem, one is interested in determining the optimal operating policy for the use of hydro and thermal resources in order to minimize total expected costs of fulfilling the demand for electricity over a given time horizon. The problem of interest is a medium-term optimization problem spanning a time horizon of one to four years, where monthly decisions have to be made. As previously mentioned, these hydro-thermal systems are characterized by unknown water inflows for the future stages. Next, we frame this problem as a multi-stage stochastic programming problem.

Given are I hydro plants $i \in \mathbb{I} = \{1, \dots, I\}$ and J thermal plants $j \in \mathbb{J} = \{1, \dots, J\}$. The known electricity demand d_t for each period[3] $t \in \mathbb{T} = \{1, \dots, T\}$ can be either satisfied through electricity generated by turbined water $\mathbf{u}_{ti}$ of any hydro plant i or through thermal power generation $\mathbf{g}_{tj}$. For simplicity, we assume that the generated electricity from hydro plant i is given through the linear relation $\rho_i \mathbf{u}_{ti}$, where ρ_i is the constant production coefficient for hydro plant i. A shortage in electricity supply of δ_t is allowed, in principle, but leads to a high penalty cost via the coefficient Υ – this is practically important but also motivated to ensure feasibility of the problem.

The thermal power generation involves the known variable production cost c_{tj} in dollars per MWh produced and the thermal plants' generation is subject to lower bounds $\underline{g}_{tj}$ and upper bounds $\bar{g}_{tj}$, allowing also to model must-run thermal plants. The hydro plant has no variable operation cost but the hydro power generation is subject to minimal turbined water $\underline{u}_{ti}$ and maximal turbined water $\overline{u}_{ti}$. The minimum turbining requirements come typically from a water demand downstream the reservoir during each period.

[3] In our notation, there is no difference between a period and a stage. However, a stage $t \in \mathbb{T}$ is a point in time, when certain decisions have to be made. In contrast, a period spans the time between two stages. If we speak of a period $t \in \mathbb{T}$, then we refer to the time span between the stages $t-1$ and t.

Then, for a given turbined outflow $\mathbf{u}_t$ (as a vector in i), the "thermal complement function" $c_t(\mathbf{u}_t)$ for stage t is given through the following linear programming problem

$$c_t(\mathbf{u}_t) := \min \sum_{j\in\mathbb{J}} c_{tj}\mathbf{g}_{tj} + \Upsilon\delta_t \tag{2–14}$$

$$\text{s.t. } \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}_{tj} + \delta_1 = d_t - \sum\nolimits_{i\in\mathbb{I}} \rho_i \mathbf{u}_{ti} \tag{2–15}$$

$$\underline{g}_{tj} \le \mathbf{g}_{tj} \le \bar{g}_{tj}, \qquad j \in \mathbb{J} \tag{2–16}$$

$$\mathbf{g}_{tj} \ge 0, \qquad \delta_t \ge 0, \qquad j \in \mathbb{J}. \tag{2–17}$$

For stage t, the thermal complement function $c_t(\mathbf{u}_t)$ provides the minimal cost of electricity production to meet the electricity demand, given the turbined outflow of the hydro plants. Observe that the objective function (2–14) only depends indirectly on $\mathbf{u}_t$. Constraints (2–15) ensure that the remaining electricity demand is satisfied through the thermal power plants production. We assume that the thermal plants have only the capacity constraints (2–16) restricting their operations. As the variable operation costs as well as the electricity demand are assumed to be known, the thermal complement function is a deterministic optimization problem; though dependent on the stochastic variables $\mathbf{u}_t$ for $t \ge 2$. To ensure feasibility for any given hydro generation levels $\mathbf{u}_t$, one can replace the equality (2–15) by the "$\ge$" inequality.

We can naturally assume that the operation cost c_{tj} per unit energy are positive. This implies that $c_t(\mathbf{u}_t)$ is a non-negative, convex function in $\mathbf{u}_t$. More precisely, $c_t(\cdot)$ is a piecewise linear function.

Without loss of generality, we can further assume that each hydro plant i has a hydro reservoir which is subject to a lower bound $\underline{v}_{ti}$ and an upper bound $\bar{v}_{ti}$. Given the set of immediate upstream hydro plants $\mathbb{U}_i$ for plant i, then, for each stage, there is a water balance equation, stating that the water level $\mathbf{v}_{t+1i}$ at the end of stage t for reservoir i has to equal to the water level $\mathbf{v}_{ti}$ at the beginning of stage t minus the turbined water $\mathbf{u}_{ti}$ and the spilled water $\mathbf{s}_{ti}$ plus the water inflow a_{ti} and the water released from the immediate upstream plants. Furthermore, there is a minimum spillage $\underline{s}_{ti}$ and a maximum spillage $\bar{s}_{ti}$ per time stage t and hydro plant i. In this framework, run-of-the-river hydro plants can be modeled by setting $\underline{v}_{ti} = \bar{v}_{ti}$, while a reservoir without electric generators can be modeled by setting $\rho_i \equiv 0$.

The hydro-thermal scheduling problem asks to find a release policy for the hydro reservoirs such that the thermal complement generation meets the electricity demand and the overall, expected operation cost are minimized. This can be stated in the following multi-stage stochastic programming problem

$$
\begin{aligned}
z := \min c_1(\mathbf{u}_1) + \min \mathbb{E}_{\omega_2 \in \Omega_2} \left[c_t(\mathbf{u}_t(\omega_t)) + \ldots + \min \mathbb{E}_{\omega_t \in \Omega_t} \left[c_t(\mathbf{u}_t(\omega_t)) \right] + \ldots + \right. \\
\left. + \min \mathbb{E}_{\omega_T \in \Omega_T} \left[c_T(\mathbf{u}_T(\omega_T)) \right] \ldots \right] \qquad \text{(2–18)}
\end{aligned}
$$

$$
\text{s.t. } \mathbf{v}_{2i} = v_{1i} - \mathbf{u}_{1i} - \mathbf{s}_{1i} + \sum\nolimits_{h \in \mathbb{U}_i} (\mathbf{u}_{1h} + \mathbf{s}_{1h}) + a_{1i}, \qquad i \in \mathbb{I} \qquad \text{(2–19)}
$$

$$
\begin{aligned}
\mathbf{v}_{t+1i}(\omega_t) &= \mathbf{v}_{ti}(\omega_{t-1}) - \mathbf{u}_{ti}(\omega_t) - \mathbf{s}_t(\omega_t) + \\
&+ \sum\nolimits_{h \in \mathbb{U}_i} \big(\mathbf{u}_{th}(\omega_t) + \mathbf{s}_{th}(\omega_t)\big) + a_{ti}(\omega_t), \qquad t \in \mathbb{T}_1,\ i \in \mathbb{I} \qquad \text{(2–20)}
\end{aligned}
$$

$$
\begin{aligned}
&\underline{u}_{1i} \le \mathbf{u}_{1i} \le \bar{u}_{1i}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti}(\omega_t) \le \bar{u}_{ti}, \\
&\underline{v}_{2i} \le \mathbf{v}_{2i} \le \bar{v}_{2i}, \qquad \underline{v}_{t+1i} \le \mathbf{v}_{t+1}(\omega_t) \le \bar{v}_{t+1i}, \\
&\underline{s}_{1i} \le \mathbf{s}_{1i} \le \bar{s}_{1i}, \qquad \underline{s}_{ti} \le \mathbf{s}_{ti}(\omega_t) \le \bar{s}_{ti}, \qquad t \in \mathbb{T}_1,\ i \in \mathbb{I},\ j \in \mathbb{J}, \qquad \text{(2–21)}
\end{aligned}
$$

with the notation $\mathbf{v}_{2i}(\omega_1) \equiv \mathbf{v}_{2i}$ and $\Omega_t := \Omega | \omega_{t-1}, \ldots, \omega_2$.

At each stage $t \geq 2$, a random event $\omega_t \in \Omega_t$ is taking place, determining the hydro inflow a_{ti} during stage t. This stochastic event is coupled with the variables via the constraints (2–25) and (2–26), modeling the water balance constraints. For the scheduled hydro-electric generation $\mathbf{u}_1$ and $\mathbf{u}_t(\omega_t)$, respectively, the thermal complement functions $c_1(\mathbf{u}_1)$ and $c_t\big(\mathbf{u}_t(\omega_t)\big)$ determine the cost of meeting the electricity demand during the planning horizon.

Recognize that the stochastic program (2–18) - (2–21) does not (directly) include any thermal generation decisions. This makes this formulation very generic. In principal, any “complement generation function” could replace functions $c_t(\cdot)$ in (2–18).

Inserting the thermal complement function (2–14) - (2–17) into (2–18) - (2–21), leads to the following multi-stage stochastic programming problem

$$
\begin{aligned}
z := \min \sum_{j\in\mathbb{J}} c_{1j}\mathbf{g}_{1j} + \Upsilon\delta_1 + \min \mathbb{E}_{\omega_2\in\Omega_2}\Bigg[\sum_{j\in\mathbb{J}} c_{2j}\mathbf{g}_{2j}(\omega_2) + \Upsilon\delta_2(\omega_2) + \ldots + \\
+ \min \mathbb{E}_{\omega_t\in\Omega_t}\left[\sum_{j\in\mathbb{J}} c_{tj}\mathbf{g}_{tj}(\omega_t) + \Upsilon\delta_t(\omega_t)\right] + \ldots + \\
+ \min \mathbb{E}_{\omega_T\in\Omega_T}\left[\sum_{j\in\mathbb{J}} c_{Tj}\mathbf{g}_{Tj}(\omega_T) + \Upsilon\delta_T(\omega_T)\right]\ldots\Bigg] \qquad (2\text{–}22)
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t. } & \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}_{1j} + \sum\nolimits_{i\in\mathbb{I}} \rho\mathbf{u}_{1i} + \delta_1 = d_1 && (2\text{–}23)\\
& \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}_{tj}(\omega_t) + \sum\nolimits_{i\in\mathbb{I}} \rho\mathbf{u}_{ti}(\omega_t) + \delta_t(\omega_t) = d_t, \qquad t\in\mathbb{T}_1 && (2\text{–}24)\\
& \mathbf{v}_{2i} = v_{1i} - \mathbf{u}_{1i} - \mathbf{s}_{1i} + \sum\nolimits_{h\in\mathbb{U}_i}(\mathbf{u}_{1h} + \mathbf{s}_{1h}) + a_{1i}, \qquad i\in\mathbb{I} && (2\text{–}25)\\
& \mathbf{v}_{t+1i}(\omega_t) = \mathbf{v}_{ti}(\omega_{t-1}) - \mathbf{u}_{ti}(\omega_t) - \mathbf{s}_t(\omega_t) + \\
& \qquad + \sum\nolimits_{h\in\mathbb{U}_i}\big(\mathbf{u}_{th}(\omega_t) + \mathbf{s}_{th}(\omega_t)\big) + a_{ti}(\omega_t), \qquad t\in\mathbb{T}_1,\ i\in\mathbb{I} && (2\text{–}26)\\
& \underline{g}_{1j} \le \mathbf{g}_{1j} \le \bar{g}_{tj}, \qquad \underline{g}_{tj} \le \mathbf{g}_{tj}(\omega_t) \le \bar{g}_{tj},\\
& \underline{u}_{1i} \le \mathbf{u}_{1i} \le \bar{u}_{1i}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti}(\omega_t) \le \bar{u}_{ti},\\
& \underline{v}_{2i} \le \mathbf{v}_{2i} \le \bar{v}_{2i}, \qquad \underline{v}_{t+1i} \le \mathbf{v}_{t+1}(\omega_t) \le \bar{v}_{t+1i},\\
& \underline{s}_{1i} \le \mathbf{s}_{1i} \le \bar{s}_{1i}, \qquad \underline{s}_{ti} \le \mathbf{s}_{ti}(\omega_t) \le \bar{s}_{ti},\\
& \delta_1 \ge 0, \qquad \delta_t(\omega_t) \ge 0, \qquad t\in\mathbb{T}_1,\ i\in\mathbb{I},\ j\in\mathbb{J}. && (2\text{–}27)
\end{aligned}
$$

Now, the full stochastic nature of the problem is revealed, as the thermal generation variables are involved. Hence, the thermal power generation, hydro power generation, storage, spillage and rationing decisions are stochastic variables, dependent on the random outcome. The bounds on the thermal power generation, the turbined water, the water reservoir levels, the spilled water and the electricity rationing are given by constraints (2–27). The objective function (2–22) is the sum of the minimum expected operation costs per stage.

Looking at the first stage, we realize that the water inflow is assumed to be known, making the first stage decisions deterministic. The same holds true at any stage t. This way of modeling is motivated by the idea that the water inflows throughout this period (remember, we are looking at time horizons of one week to one month) can be observed and the decisions on the hydro operation can be adopted [162]. This is known in the literature as "Wait-and-See" model. We "wait" for the stochastic event (*e.g.* inflow) to be "seen" and make the decision then. In contrast, the "Here-and-Now" models need a decision before the outcome of the random event is known, [54]. We will see in Chapter 3 how a Here-and-Now approach changes the model formulation.

In contrast to deterministic mathematical programming, problem (2–22) - (2–27) does not provide a "single" solution of the problem. For stages 2 to T, a whole set of solutions is presented, dependent on the random events occurring. Hence, this solution set is naturally called a policy. In practice, one is most interested in the first stage solutions. Looking at a weekly or monthly resolution of problem (2–22) - (2–27) with a time horizon of a few years, the first stage solution carries the information of the future stages; *e.g.*, balancing the risks of rationing, draughts or spillage. This first stage solution is then used as targets (in one or the other sense) for the short-term optimization problems. Hence, one typically runs optimization problem (2–22) - (2–27) every week or month to exploit the new information available on the stochastic components.

We now discuss how (2–22) - (2–27) fits into the framework of multi-stage stochastic programming as given by formulation (2–3) - (2–7). The decision variables $\mathbf{x}$ are given as vectors in $\mathbf{g}_1$, $\mathbf{u}_1$, $\mathbf{v}_2$, $\mathbf{s}_1$, and δ_1; similarly for variables $\mathbf{x}_t(\omega_t)$. With this notation, first stage constraints (2–4) are given through the first stage electricity demand constraints (2–23), the first stage water balance constraints (2–25) and the first stage limits on the decision variables (2–27), defining constraint matrix A and the right hand side vector b. For a given $t \in \mathbb{T}_1$, the tth stage stochastic constraints (2–2) are given by the corresponding electricity demand constraints (2–24), water balance constraints (2–26) and variables bounds (2–27). The recourse matrix W_t corresponds to the coefficients given by the variables $\mathbf{g}_t$, $\mathbf{u}_t$, $\mathbf{v}_{t+1}$, $\mathbf{s}_t$, and δ_t, while matrix T_t is given by the coefficients of $\mathbf{v}_t$. Hence, both matrixes are fix; *i.e.*, they do not change with the random water inflow. In contrast, the right hand side value $h_t(\omega_t)$ is indeed stochastic, representing the uncertain hydro inflows $a_t(\omega_t)$. This is the only stochastic component among the input data. With the notations above, the objective function fits naturally into the generic stochastic framework.

Hence, the program has fixed recourse and a fix stochastic matrix T_t. The reason is that the "technology" does not change in the system; *e.g.*, the thermal and hydro generation efficiency remains the same throughout the optimization horizon. The linearity assumption is discussed in Section 2.2.2 while the assumptions on the distributions of the uncertain inflows are part of Chapter 3.

A survey of general stochastic programming models for energy problems is given by Wallace and Fleten [173]. Labadie [99] surveys various different approaches how to attack uncertain hydro inflows in multi-reservoir systems.

2.2.1 Additional Constraints

In order to make the stochastic model formulation (2–22) - (2–27) more practical, additional constraints have been proposed [130, 131]:

Electricity Demand.

- Load blocks: the monthly demand is separated into different load blocks.

Hydro.

- Reservoir security constraints: lower bound on the water reservoir level becomes a soft constraint;
- limits on total outflow: minimum and maximum outflow limits on the hydro reservoir outflow;
- peak modulation constraints in run-of-the-river plants: most run-of-the-river plants have small reservoirs to allow the storage within a day to regulate the daily demand pattern;
- run-of-the-river plants generation: to make the electricity generation for run-of-the-river plants more realistic, “look-up” tables provide a relation between the total inflow and the turbinable inflow;
- irrigation for hydro reservoirs;
- initial fill-up of reservoirs: this can be achieved by a so-called minimum reservoir storage curve;
- tailwater elevation: the production coefficient for the hydro plants vary according to different hydro reservoir levels;
- risk aversion: constraint on the hydro reservoir levels for the whole system.

Thermal.

- Piecewise linear cost: thermal plant’s generation efficiency depends on the generation level which is approximated via piecewise linear cost functions;
- must-run thermal plants: force to generate at a certain level;
- fuel consumption limits: availability of fuels is limited at each stage;
- fuel consumption rate limits: upper limit on the fuel consumption rate per stage;
- minimum generation constraint for a set of thermal plants: a lower generation limit on those plants is given;
- multiple fuels: plants may be able to operate on different fuels;
- unit commitment: start-up cost and minimum generation limits once started.

Generation Reserve.

- Spinning reserve: lowers the generation capacity of hydro and thermal plants;
- generation reserve: as percentage of system load or as compensation for generation outages.

Power Transmission Network.

- Interconnection model: each network system has its own energy demand and the interconnection between the models is limited;
- linearized power flow model: the transmission network is divided into buses and circuits where the physical properties of power flow are modeled through (linearized) First and Second Kirchhoff laws, circuit limit flows and DC links;
- transmission losses: modeled as piecewise linear and as additional loads at the circuit terminal buses.

Natural Gas Network.

- Production limits: gas production is limited above and below;
- pipeline flow limits: the gas flow between nodes of the gas network are limited;
- supply and demand balance: supply and demand has to equal at each node of the gas network.

All the constraints above have the following common properties: they are linear(ized) and span only one stage. Hence, the block diagonal structure of the constraint matrix (2–23) - (2–26) remains preserved; *i.e.*, the resulting stochastic programming formulation respects the structure of (2–3) - (2–7). This is very important, as solution techniques like Dynamic Programming (DP) exploit this structure. We will come back to this in Chapter 5 where we discuss the modeling of CO_2 emission constraints, spanning multiple stages.

So far, we restricted ourselves to linear relationships between, and for, the variables. Sometimes, the structure may allow to include a discrete structure (most likely in a two-stage problem) which reduces to a linear program. Examples for such structures are network type constraints. More generally, such a reduction to linear programming is always possible, if the constraint set of the problem in extensive form is totally unimodular [3, 26, 143].

2.2.2 Is the "Hydro-thermal Scheduling World" Linear?

The correct answer is NO[4] . There is no doubt about it. We next justify why, or to which extend, a linear approach is meaningful in our context.

Let us have a look at the purpose of the proposed stochastic models; *i.e.*, the desired deliverables. As a reminder, for short-term hydro-thermal scheduling, one needs to have handy a meaningful value associated with reservoir water levels in order to balance the risk of rationing (dry seasons), risk of spillage (wet season) and cost of thermal operation. Such water values are indirectly delivered by the proposed stochastic models via operation policies[5] . With a weekly or monthly resolution, the operation horizon in each stage is pretty large. Hence, one can argue that the non-linear drivers present can be averaged out over a period of one week or month. For instance, the ramp-up constraints for thermal power plants might be negligible on a weekly scale, while they cannot be ignored in an hourly model. Still, the transmission networks (electricity as well as gas) are a vital part in a hydro-thermal energy system and the non-linearities involved are significant. The past experiences show that the electricity transmission network is a more and more frequent source for power outages rather than the generation itself. Similarly, the gas network involves highly non-linear, non-convex structures [182].

The daily experience tells us that the world of decisions is not linear. However, even computer scientists have to admit that the world is not black and white. The same holds true for hydro-thermal scheduling. The problem of interest might not be linear,

[4] Among others, the most "urgent" non-linearities are given by the non-linear thermal unit commitment and generation cost, the non-linear forces driving the transmission grid (Second Kirchhoff law) and natural gas network (Weymouth panhandle equations), and the non-linear hydro-electric generation.

[5] The SDDP algorithm calculates – next to the operation policy and an estimate of the expected operation costs – future cost function cuts for the second stage; *cf.* Chapter 3. These cuts provide the expected water value for all future stages considered.

however, piecewise linear turns out to be a fairly good approximation for the most common non-linearities. Wolf and Smeers [175], for instance, introduced a piecewise linear approximation of the nonlinear Weymouth panhandle equations to model and solve a gas network optimization problem. The computational results on the Belgium Gas network demonstrated the effectiveness of this model. The common linearization of the active and reactive power flow equations are discussed by Rubio-Barros et al. [150]. A comparison of different power flow models is presented by Overbye et al. [120]. Even though these relations are non-convex and non-smooth, these linearizations work very well in practice, when the power system is in steady state as then the three main assumptions on this linearization are satisfied: losses are neglectable, phase angle differences at adjacent buses are small, and the reactance is much greater than the resistance.

Despite the argument above, one can pragmatically claim that the reason is mainly driven by the operations research science: Large scale, non-linear, non-convex, non-smooth multi-stage stochastic programming problems are for the future generation, optimistically.

CHAPTER 3
SOLUTION TECHNIQUES: A SURVEY

A methodological survey of the literature on hydro-thermal scheduling algorithms until the mid 1980s was given by Yeh [181]. Since this time, significant algorithmic improvements have been made. We survey the highlights of the latest developments in this area to solve the hydro-thermal scheduling problems discussed in Chapter 2.

The survey by Labadie [99] in 2004 covers a huge variety of different models for multi-reservoir optimization problems and reviews their solution methods. Discrete, non-linear and multi-objective problems are discussed next to linear models; astonishingly, SDDP has not been mentioned. Hence, this chapter is a good complement to the main focus is given on DP methods such as SDDP.

A fundamental difference is the way inflow uncertainty is treated. We classify the solution methods into three different groups:

1. deterministic models,
2. scenario-based methods,
3. sampling-based methods.

Deterministic models treat the hydro inflows (and other possible “uncertainties”) as known. As this is not the main focus of this dissertation, we only briefly discuss different deterministic approaches in Section 3.1 which have been applied to the hydro-thermal scheduling problems.

Scenario-based methods generate up-front a set of realizations of the random space. The realizations are then used to generate the extensive form of the stochastic program, yielding for our application to a (very) large-scale (linear) programming programs. Those mathematical programs are then typically solved exactly; *i.e*, to global (= local) optimality. Hence, the solution quality depends on the approximation of the realizations to the original, stochastic program. For two-stage stochastic programs, the number of realizations can be fairly large leading to good solutions. However, for multi-stage problems, scenario-based methods are typically limited in the number of realizations per stage. In Section 3.2, we review scenario-based methods which have been successfully applied to the hydro-therm scheduling problems. General solution methods include the L-shaped method [160], the diagonal quadratic approximation method [115] and the augmented Lagrangian decomposition method [149].

In contrast, sampling-based methods generate samples of the random space on-the-fly and solve the resulting problems approximately. Often, probabilistic convergence results are established for the proposed solution methods while statistical methods stop the algorithms after a finite number of steps. There is a rich class of such methods available in the literature. The most relevant methods applied to hydro-thermal scheduling are reviewed in Section 3.3. Other methods include the stochastic linearization method [53], the auxiliary function method [33], the stochastic decomposition method [78], the sample path optimization [144], or the separable approximation method [128].

Some authors classify the solution methods for stochastic hydro-thermal scheduling problems into Linear Programming (LP) and DP methods. The classification into scenario-based and sampling-based methods is slightly broader, as LP methods are typically scenario-based methods and DP methods are sampling-bases methods.

3.1 Deterministic Models

Solving deterministic problems has the great advantage that, for a given amount of computational time, more system details can be incorporated compared to a stochastic model. Especially non-linear relations (*e.g.* the one discussed in Chapter 2) can be taken into account. However, those models are typically restricted to Mixed Integer Linear Programming (MILP) problems or convex quadratic programming problems, as large-scale general non-convex models are currently not computational tractable. Nevertheless, unit commitment, start-up costs as well as minimum up and down time constraints fit into the framework of MILP. Hence, this approach allows the incorporation of short-term aspects into the mid-term hydro-thermal scheduling problems. The models solved are often on the border of short-term and mid-term optimization problems.

Christoforodis et al. [30] used interior-point methods to solve a scheduling problem for the hydro-dominated energy system of the Swiss Railways. Medina et al. [111] presented a clipping-off interior point algorithm for solving a deterministic, large-scale, MILP mid-term hydro-thermal scheduling problem. Results on the Spanish power system reveal the superiority of the algorithm over standard interior point methods.

A semidefinite programming approach towards a convex quadratic hydro-thermal scheduling problem was developed by Fuentes and Quintana [59]. Their approach runs in polynomial time due to a convex quadratic modeling of the cost function and constraints, thus, avoiding integer variables.

Pereira et al. [123] successfully combined analytical models and Monte Carlo simulation to overcome computational challenges posed on the analytical model. The idea is to solve easier problems analytically and leave the detailed model to the simulation scheme. Test on a multi-reservoir hydro-electric power system illustrate the method. The idea of Monte Carlo simulation, as used by Baslis et al. [8], is to generate a fairly large number of scenarios which are then used as input data in a deterministic model. The authors use deterministic MILP problems and apply it to the Greek power system. Computational tests were performed with 100 Monte Carlo scenarios using hourly time steps, providing a distribution of the optimal solution.

3.2 Scenario-Based Methods

Scenario-based methods approximate the random space by a set of possible outcomes, called “scenarios.” These scenarios are then used to transform the stochastic program into the deterministic equivalent which can then be solved using deterministic optimization techniques [174]. The computed scenarios have typically the shape of a “tree” – branching at each stage – or a “fan” – set of individual scenarios.

A great advantage of scenario-based modeling of uncertainties is that various, uncertainties (correlated and uncorrelated) can be incorporated into the model; *e.g.*, hydro inflows, electricity spot prices, contract prices, electricity demand, and fuel prices. Whereas sample-based methods (discussed in the following section) typically rely on simpler (in most cases linear) models, the scenario generation process can be quite complex for scenario-based methods.

Rockafellar and Wets [146] pioneered the work on scenario analysis and aggregation with their work in 1991. Given a set of scenarios, rather than solving the deterministic equivalent in the extensive form, they proposed a progressive hedging algorithm, which iteratively generates policies by modified scenario subproblems. Specific conditions are derived for which the obtained policy (solution to the stochastic optimization problem) converges to the optimal policy corresponding to the deterministic equivalent.

3.2.1 Scenario Generation and Reduction

Next to running time improvements, one of the main goals in scenario generation is to obtain scenario trees satisfying certain statistical properties.

The generation of scenarios for multi-stage problems are described by Dupačová et al. [43]. In their work, different possibilities for scenario-tree generation are stressed, *i.e.*, using cluster analysis in case of external scenario path generator, or importance sampling to build a whole tree from scratch. Their method is widely applicable as convexity with respect to the random parameters is not required.

Høyland and Wallace [84] present a scenario generation method for multi-stage problems. The limited number of discrete outcomes satisfy statistical properties which have to be given as input. Using non-linear programming, the appealing idea is to "minimize some measure of distance between the specifications and the statistical properties of the discrete approximation."

Høyland et al. [83] present a heuristic method for the generation of scenarios for multi-stage stochastic optimization problems. As the critical properties of the generated scenario trees, the first four marginal moments and the correlations are taken into account, though higher moments are possible as well. The algorithm is iterative, combining simulation, transformations and Cholesky decomposition.

The research group of Römisch published a series of methods on scenario tree approaches for multi-stage stochastic optimization. The stability results of Römisch [147] as well as Heitsch et al. [77] for stochastic programming problems allow the analysis of the generated scenario trees by measuring the closeness of the obtained solution to the (unknown) solution of the original stochastic program. Based on this stability theory, Dupačová et al. [44] as well as Heitsch and Römisch [74] developed a framework for scenario generation and reduction, approximating the underlying discrete probability distribution which replaced the (continuous) stochastic processes. These scenario tree approximation schemes were further developed by Heitsch and Römisch [76]. The developed scenario reduction algorithms have been implemented in GAMS [23] having the solver name SCENRED [60].

3.2.2 Applications to Reservoir Management

The previously described scenario generation method by Høyland and Wallace has been applied to the Nordic electricity market by Fleten et al. [58]. The authors used a scenario tree approach towards an portfolio management problem in the deregulated hydro-power electricity market. Numerical results are presented for a five-stage scenario tree model with 256 scenarios spanning a two year time horizon. Similarly, Shrestha et al. [159] used a scenario tree with 243 scenarios over six periods to model and solve a hydro-power profit maximization problem.

Heitsch and Römisch [75] discuss a stochastic power management problem via multivariate scenario trees. The scenarios carry the information of uncertainties from electrical load, stream flows to hydro units, market prices of fuel and electricity. In order to reduce the deterministic equivalent of the multi-stage stochastic program, a scenario reduction technique based on recursive deletion and bundling of scenarios was used.

Similarly, Gröwe-Kuska et al. [70] discuss scenario generation and scenario reduction on a hydro power profit maximization problem. Again, a multivariate scenario tree is used, capturing uncertainties in electrical load, spinning reserve, inflows, fuel/electricity prices. The software SCENRED was used for the scenario tree reduction.

Eichhorn et al. [49, 50] walk the reader nicely through the process of scenario reduction, scenario tree generation (approximation) and multi-stage stochastic programming modeling via a risk management problem for electricity portfolios. Different risk measures (linear and mixed integer) are discussed, allowing the maximization of expected revenues and the minimization of risk simultaneously. The resulting mathematical programs are large scale linear or mixed integer programs.

In order to capture the inflow uncertainty, a large scenario tree may be required, leading to very large scale deterministic equivalent programs; *cf.* [57, 159]. Especially if the number of hydro reservoirs is large, the correlation among the hydro inflows requires a fairly large number of scenarios. Thus, sampling-based methods received great attention in the literature as well as in practice to solve mid-term and long-term hydro-therm scheduling problems.

3.3 Sampling-Based Methods

Sampling-based optimization methods for hydro-thermal scheduling problems are essentially variations of Dynamic Programming methods applied to the stochastic case. DP approaches use the decomposition of the multi-stage program into recursive, one-stage programs as given in equations (2–12) – (2–13), see Bellman [11]. This so-called Bellman recursion is a powerful decomposition of the problem and is the basis for many (mainly sampling-based) solution algorithms of stochastic programs. A survey of DP techniques for water reservoir management was presented by Lamond and Boukhtouta [100] in 1996.

One of the key components of each DP algorithm for water reservoir management problems is the approximation of the expected tth-stage value function[1] , *cf.* to Chapter 2. The literature until the 1980's focused mainly on stochastic dynamic programming methods, interpolating the tth-stage value function while discretizing the state space. However, this leads to an exponential increase in the size of the problem to solve, known as the curse of dimensionality; these concepts will be carefully addressed in Section 3.3.1.

In order to overcome this problem, a remarkable series of research followed based on nested Benders' decomposition method [16]. Among them are the papers by Pereira and Pinto [124], Jacobs et al. [89] and Morton [114], while Velásquez et al. [170] presented a modification of the dual dynamic programming algorithm of Pereira and Pinto for the multi-stage case. The curse of dimensionality was then finally broken by the stochastic dual dynamic programming method of Pereira [122] and Pereira and Pinto [125]. This is the subject of Section 3.3.2. The research after 1991 on hydro-thermal scheduling problems was very much driven by this SDDP method.

Recently, the so-called Approximate Dynamic Programming (ADP) algorithms have been applied to a variety of multi-stage stochastic optimization problems with a dynamic structure. Similar to stochastic dual dynamic programming method, the curse of dimensionality can be broken by ADP approaces; *cf.* Powell [129]. ADP has been successfully applied to various real world optimization problems including problems arising from energy applications; *cf.* to Enders et al. [52] and Löhndorf and Minner [107].

3.3.1 Stochastic Dynamic Programming: Expected Future Cost Interpolation

Stochastic dynamic programming applies the Bellman recursion to the stochastic case by taking the expectation. This is the key idea of the one-stage stochastic

[1] In the context of hydro-thermal scheduling, the expected $t+1$th-stage value function is called "future cost function." Observe that we drop the word "expected"

programming approach discussed in Chapter 2 in the form of equations (2–8) - (2–13). Let us now have a closer look at the methodology of SDP. The following is mainly based on [122, 130, 131].

3.3.1.1 Methodology

To formulate the multi-stage stochastic programming problem (2–22) - (2–27) in the one-stage recursion framework, consider a specific stage t. Then, the task is to find optimal hydro-thermal scheduling decisions for this stage at an expected minimal cost; *i.e.*, minimal expected operation cost for stage t plus minimal expected future costs. Per assumption, at this stage t, the "past" has occurred which means that the previous inflows as well as the hydro reservoir levels v_t are known; with v_t being a vector in the hydro reservoirs $i \in \mathbb{I}$.

Then, the multi-stage stochastic program (2–22) - (2–27) can be decomposed into so-called one-stage dispatch problems as follows

$$z_t(v_t) := \min \mathbb{E}_{\omega\in\Omega_t}\left[\sum_{j\in\mathbb{J}} c_{tj}\mathbf{g}_{tj}(\omega) + \Upsilon\delta_t(\omega) + z_{t+1}\big(\mathbf{v}_{t+1}(\omega)\big)\right] \tag{3–1}$$

$$\text{s.t. } \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}_{tj}(\omega) + \sum\nolimits_{i\in\mathbb{I}} \rho_i\mathbf{u}_{ti}(\omega) + \delta_t(\omega) = d_t \tag{3–2}$$

$$\mathbf{v}_{t+1i}(\omega) = v_{ti} - \mathbf{u}_{ti}(\omega) - \mathbf{s}_{ti}(\omega) + \sum\nolimits_{h\in\mathbb{U}_i}\big(\mathbf{u}_{th}(\omega) + \mathbf{s}_{th}(\omega)\big) + a_{ti}(\omega), \quad i \in \mathbb{I} \tag{3–3}$$

$$\begin{aligned} &\underline{g}_{tj} \le \mathbf{g}_{tj}(\omega) \le \bar{g}_{tj}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti}(\omega) \le \bar{u}_{ti}, \\ &\underline{v}_{t+1i} \le \mathbf{v}_{t+1i}(\omega) \le \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \le \mathbf{s}_{ti}(\omega) \le \bar{s}_{ti}, \\ &\delta_t(\omega) \ge 0, \qquad i \in \mathbb{I},\, j \in \mathbb{J}. \end{aligned} \tag{3–4}$$

Equation (3–2) ensures that the electricity demand corresponding to period t is satisfied, with the possibility of rationing. The water balance equations are modeled through (3–3) where v_{ti} are the "initial" water reservoir levels for stage t. The bounds and domain for each variable are given in (3–4). The objective function (3–1) is the sum of expected immediate costs, given by the thermal generation costs and the rationing penalty, and the future cost. The future cost $z_{t+1}(\cdot)$ is a function dependent on the water reservoir levels $\mathbf{v}_{t+1}(\omega)$ at the end of the stage t, which is a (stochastic) decision variable. This function also depends indirectly (via set Ω_t) on the realizations of the random parameters occurred in the previous stages.

This decomposition of the problem leads to a natural solution method following the idea of the Bellman recursion. Starting at the final stage T, the future cost function $z_{T+1}(\cdot) \equiv 0$ and the cost function $z_T(v_T)$ can be calculated for some water reservoir level v_T (and some past inflows). One can then move backwards in time and solve each one-stage dispatch problem with the previously calculated future cost functions. This way, a solution to the original stochastic programming problem (2–22) - (2–27) can be obtained in the sense that $z \equiv z_1(v_1)$ with the initial water reservoir level v_1.

However, the tricky part is how to correctly "guess" the right hydro reservoir levels v_t for each one-stage dispatch problem (3–1) - (3–4). As those one-stage dispatch problems cannot be solved computationally for the whole continuum of reservoir levels v_t, a discretization into N values v_t^n, $n \in \mathbb{N} = \{1, \ldots, N\}$, may be chosen. Problem (3–1) - (3–4) is then solved for those N values v_t^n. Typically, SDP algorithms interpolate the solution values corresponding to the N reservoir levels to obtain a "future" cost function for the previous stage. The same concept applies to the past inflows.

As mentioned in Chapter 2, we denote by Ω_t the set of possible outcomes, conditioned on the past outcomes. For the sake of this dissertation, we restrict ourselves to random processes dependent exclusively on the outcome of the previous stage. Such models are called models of lag-1. Hence, Ω_t is now the set of all possible outcomes $\omega_t \in \Omega$, conditioned only on the previous outcome ω_{t-1}; *i.e.*, $\Omega_t := \Omega|\omega_{t-1}$. In this case, the one-stage dispatch problem (3–1) - (3–4) depends on another state variable ω_{t-1} as follows

$$z_t(v_t, \omega_{t-1}) := \min \mathbb{E}_{\omega \in \Omega|\omega_{t-1}} \left[\sum_{j \in \mathbb{J}} c_{tj} \mathbf{g}_{tj}(\omega) + \Upsilon \delta_t(\omega) + z_{t+1}\big(\mathbf{v}_{t+1}(\omega), \omega\big) \right] \tag{3–5}$$

$$\text{s.t.} \ \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{tj}(\omega) + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{ti}(\omega) + \delta_t(\omega) = d_t \tag{3–6}$$

$$\mathbf{v}_{t+1i}(\omega) = v_{ti} - \mathbf{u}_{ti}(\omega) - \mathbf{s}_{ti}(\omega) + \sum\nolimits_{h \in \mathbb{U}_i} \big(\mathbf{u}_{th}(\omega) + \mathbf{s}_{th}(\omega)\big) + a_{ti}(\omega), \quad i \in \mathbb{I} \tag{3–7}$$

$$\begin{aligned} &\underline{g}_{tj} \le \mathbf{g}_{tj}(\omega) \le \bar{g}_{tj}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti}(\omega) \le \bar{u}_{ti}, \\ &\underline{v}_{t+1i} \le \mathbf{v}_{t+1i}(\omega) \le \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \le \mathbf{s}_{ti}(\omega) \le \bar{s}_{ti}, \\ &\delta_t(\omega) \ge 0, \qquad i \in \mathbb{I},\ j \in \mathbb{J}. \end{aligned} \tag{3–8}$$

So far, we have not discussed how to computationally deal with the random inflows $a_{ti}(\omega)$. Naturally, the conditioned distribution of the random variables are not known and expected to be continuous. However, as discussed in Chapter 2, we assume that we can approximate the distribution with discrete and finite samples of a known distribution. In the context of SDP (later on also for SDDP), these inflows can be modeled as a linear autoregressive model via a continuous Markov process (in contrast to a Markov chain), taking into consideration the correlation to the inflows of the previous stage(s). As we assume a lag-1 model, the inflows in stage t depend only on the previous inflows in stage $t-1$. A simple version of this inflow model is then given through

$$a_t = \varsigma_t \left(\phi_1 \cdot \frac{a_{t-1} - \mu_{t-1}}{\varsigma_{t-1}} + \phi_2 \cdot \zeta \right) + \mu_t \tag{3–9}$$

with inflow mean μ_t, standard deviation ς_t, model parameters ϕ_1 and ϕ_2, and independent random variables ζ sampled from an appropriate distribution; typically a standard normal distribution. We discuss this inflow model in greater detail in Section 3.3.1.1.

The conditional distribution of the water inflows $\Omega|\omega_{t-1}$ is then "approximated" via a discrete sample of L inflow scenarios a_t^l, $l \in \mathbb{L} = \{1, \dots, L\}$, each having equal probability $p^l = 1/L$. In this context, these inflow scenarios are also called "backward openings."

The stochastic programming problem (3–5) - (3–8) reduces then to a one-stage deterministic optimization problem

$$z_t(v_t, a_{t-1}) := \min \sum_{l \in \mathbb{L}} p^l \left[\sum_{j \in \mathbb{J}} c_{tj} \mathbf{g}_{tj}^l + \Upsilon \delta_t^l + z_{t+1}(\mathbf{v}_{t+1}^l, a_t^l) \right] \tag{3–10}$$

$$\text{s.t. } \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{tj}^l + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{ti}^l + \delta_t^l = d_t \tag{3–11}$$

$$\mathbf{v}_{t+1i}^l = v_{ti} - \mathbf{u}_{ti}^l - \mathbf{s}_{ti}^l + \sum\nolimits_{h \in \mathbb{U}_i} (\mathbf{u}_{th}^l + \mathbf{s}_{th}^l) + a_{ti}^l, \qquad i \in \mathbb{I} \tag{3–12}$$

$$\underline{g}_{tj} \le \mathbf{g}_{tj}^l \le \bar{g}_{tj}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti}^l \le \bar{u}_{ti},$$

$$\underline{v}_{t+1i} \le \mathbf{v}_{t+1i}^l \le \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \le \mathbf{s}_{ti}^l \le \bar{s}_{ti},$$

$$\delta_t^l \ge 0, \qquad i \in \mathbb{I},\ j \in \mathbb{J},\ l \in \mathbb{L}, \tag{3–13}$$

where we assume

$$z_t(v_t, a_{t-1}) \approx z_t(v_t, \omega_{t-1}); \tag{3–14}$$

i.e., the deterministic model is a good approximation of the "real" stochastic model. Observe that problem (3–10) - (3–13) decomposes into L independent problems, one for each water inflow sample a_{ti}^l.

In order to "simulate" the stochastic inflow throughout the planning horizon, a sample of M so-called "forward inflows" a_t^m, $m \in \mathbb{M} = \{1, \dots, \mathbb{M}\}$, is derived from the linear autoregressive model (3–9). The forward inflows are typically equally probable; *i.e.*, $p^m = 1/M$. Those forward inflow samples are then the "past" inflows. Hence, we have for each storage discretization $n \in \mathbb{N}$, forward inflow $m \in \mathbb{M}$ and backward opening

$l \in L$

$$z_t^{lmn}(v_t^n, a_{t-1}^m) := \min \sum_{j \in \mathbb{J}} c_{tj} \mathbf{g}_{tj} + \Upsilon \delta_t + z_{t+1}\left(\mathbf{v}_{t+1}, a_t^{lm}\right) \tag{3–15}$$

$$\text{s.t. } \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{tj} + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{ti} + \delta_t = d_t \tag{3–16}$$

$$\mathbf{v}_{t+1i} = v_{ti}^n - \mathbf{u}_{ti} - \mathbf{s}_{ti} + \sum\nolimits_{h \in \mathbb{U}_i} \left(\mathbf{u}_{th} + \mathbf{s}_{th}\right) + a_{ti}^{lm}, \qquad i \in \mathbb{I} \tag{3–17}$$

$$\underline{g}_{tj} \le \mathbf{g}_{tj} \le \bar{g}_{tj}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti} \le \bar{u}_{ti},$$

$$\underline{v}_{t+1i} \le \mathbf{v}_{t+1i} \le \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \le \mathbf{s}_{ti} \le \bar{s}_{ti},$$

$$\delta_t \ge 0, \qquad i \in \mathbb{I},\ j \in \mathbb{J}, \tag{3–18}$$

and

$$z_t(v_t^n, a_{t-1}^m) := \sum_{l \in \mathbb{L}} p^l z_t^{lmn}(v_t^n, a_{t-1}^m). \tag{3–19}$$

The question how function $z_t(\cdot)$ can be evaluated for any water reservoir level v_t and past inflow a_{t-1} is answered by SDP as mentioned above: for all $n \in \mathbb{N}$ and $m \in \mathbb{M}$, the function values are interpolated, constructing an approximated future cost function.

Recall that we used the backward openings in order to sample from the distribution of the random variable responsible for the water inflows. This enables us to replace the expected operational cost in formulation (3–5) - (3–8) by the average operational cost. Hence, this is not necessary for the first stage and we obtain for the one-stage dispatch problem of the first stage

$$z_1^m(v_1, a_0^m) := \min \sum_{j \in \mathbb{J}} c_{1j} \mathbf{g}_{1j} + \Upsilon \delta_1 + z_2 \left(\mathbf{v}_2, a_1^m\right) \tag{3–20}$$

$$\text{s.t.} \ \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{1j} + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{1i} + \delta_1 = d_1 \tag{3–21}$$

$$\mathbf{v}_{2i} = v_{1i}^n - \mathbf{u}_{1i} - \mathbf{s}_1 + \sum\nolimits_{h \in \mathbb{U}_i} \left(\mathbf{u}_{1h} + \mathbf{s}_{1h}\right) + a_{1i}^m, \qquad i \in \mathbb{I} \tag{3–22}$$

$$\underline{g}_{1j} \leq \mathbf{g}_{1j} \leq \bar{g}_{1j}, \qquad \underline{u}_{1i} \leq \mathbf{u}_{1i} \leq \bar{u}_{1i},$$

$$\underline{v}_{2i} \leq \mathbf{v}_{2i} \leq \bar{v}_{2i}, \qquad \underline{s}_{1i} \leq \mathbf{s}_{1i} \leq \bar{s}_{1i},$$

$$\delta_1 \geq 0, \qquad i \in \mathbb{I}, \ j \in \mathbb{J}. \tag{3–23}$$

Solving these M one-stage dispatch problems for the first stage, with initial hydro reservoir levels v_1 and previous inflows a_0^m, provides an estimate of the expected operation cost z of the original problem (2–22) - (2–27) as the average value over the M inflow scenarios; *i.e.*,

$$z_1 := \sum_{m \in \mathbb{M}} p^m z_1^m(v_1, a_0^m) \approx z. \tag{3–24}$$

Recall that the multi-stage stochastic programming problem (2–22) - (2–27) for hydro-thermal scheduling introduced in Chapter 2 assumes a deterministic first stage decision. Hence, the water inflows a_1 of stage one are deterministic and not stochastic. However, in order to calculate z_1 (as an estimate of z), M one-stage dispatch problems (3–20) - (3–23) are solved. Again, the concept of Wait-and-See comes here into play where we assume to know the inflows of the first stage when the decision is made. One can further observe that the first stage does not depend on l, no backward openings are present. These backward openings reflect the stochastic water inflow in the future stages.

The SDP is summarized in Algorithm 3-1. The running time is dominated by the number of linear programming problems solved; which is $M \cdot N \cdot L \cdot (T-1) + M$. That does not look too bad. However, the problem is that the state space (given by v_t and

Algorithm 3-1. SDP: Stochastic Dynamic Programming (Generic)

// Initialize
Generate inflow data; *cf.* Section 3.3.1.1
Initialize the future cost function for stage $T+1$ to 0

// Backwards iteration to compute
// 1.) hydro-thermal scheduling policy
// 2.) (estimated) average operational cost z_1
for each stage $t = T, T-1, \ldots, 2$ do
  for each forward inflow scenario $m \in \mathbb{M}$ do
    for each storage and inflow discretization $n \in \mathbb{N}$ do
      for each backwards opening inflow scenario $l \in \mathbb{L}$ do
        solve the one-stage dispatch problem (3–15) - (3–18)
      end for
    end for
  end for
  construct the future cost function $z_t(\cdot)$ for stage $t-1$ using the discrete values $z_t(v_t^n, a_{t-1}^m)$ of (3–19)
end for
for each forward inflow scenario $m \in \mathbb{M}$ do
  solve the one-stage dispatch problem (3–20) - (3–23)
end for
calculate average operational cost z_1 via (3–24)

a_{t-1}) has to be discretized in order to solve the recursion. This is encoded in the set $\mathbb{N}$. Now, if one chooses to discretize the storage vector into $\mathcal{N}_1$ values, the inflow into $\mathcal{N}_2$ values and this for each of the I components of the state space vector, there are $(\mathcal{N}_1 \cdot \mathcal{N}_2)^I$ states in each stage; *i.e.*, $N = (\mathcal{N}_1 \cdot \mathcal{N}_2)^I$. Hence, the state space increases exponentially with the number of hydro reservoirs in the system[2] . This is knows as the so-called curse of dimensionality [12, 101]. Therefore, most applications of SDP to hydro-thermal systems are restricted to single reservoir systems or systems with a few reservoirs.

[2] This analysis of the exponential increase of the state space is only rough in the sense that in higher dimensions, the number of grid points in each dimension reduces, see [135]. However, asymptotically, the growth remains exponential

SDP as presented in Algorithm 3-1 is a static algorithm in the sense that the discretization of the state space has to be chosen in advance. As the quality of the obtained solution depends crucially on the choice of this discretization, there is certainly a trade-off between computational time and solution quality. Furthermore, there is no mechanism available measuring the quality of the obtained solution.

In sum, there are three main challenges for SDP algorithms: curse of dimensionality, static discretization of state space and lack of solution quality measure.

Inflow Model. In the SDP (and SDDP) algorithm, a linear autoregressive model of lag-1 might be used to generate the forward inflows and backward openings. Such a model is described in equation (3–9). Let us have a closer look how that works.

Beforehand, a series of inflow scenarios are generated for each stage t. This can be done by applying (3–9) an appropriate number of times while sampling from the distribution of ζ. However, we actually want the inflow “decomposed” into the reservoirs $i \in \mathbb{I}$. This can be done by applying (3–9) for each of the I reservoirs, taking into account that the random variables ζ are correlated. For each forward scenario m and stage t, we sample I independent random variables

$$\bar{\zeta}_{ti}^{m} \sim \mathcal{N}(0, 1) \tag{3–25}$$

in order to generate correlated random variables

$$\zeta_{ti}^{m} := \sum_{\iota=1}^{I} C_{i\iota} \bar{\zeta}_{t\iota}^{m}, \tag{3–26}$$

with the correlation matrix C. These correlated random variables can then be used within (3–9) to generate correlated inflows for each of the reservoirs. The same can be done to obtain the L backward openings for each forward inflow scenario. Algorithm 3-2 summarizes these ideas.

The input data μ_t, ς_t, ϕ_1 and ϕ_2 are estimated using historical data. Typically, such data are available over a relatively large time period, let us say 30 to 50 years. However, Algorithm 3-2. Inflow Scenario Generation

// generate forward inflows a_{ti}^m for $t = 1, \dots, T$
for each stage $t = 1, \dots, T$ do
  for each inflow scenario $m = 1, \dots, M$ do
    for each reservoir level $i = 1, \dots, I$ do
      sample independent random variable $\bar{\zeta}_{ti}{}^m$ via (3–25)
    end for
    for each reservoir level $i = 1, \dots, I$ do
      generate correlated random variables ζ_{ti}^m via (3–26)
    end for
    for each reservoir level $i = 1, \dots, I$ do
      calculate a_{ti}^m conditioned on previous inflow a_{t-1i}^m and with correlated random variable ζ_{ti}^m via (3–9)
    end for
  end for
end for

// generate backward openings a_{ti}^{lm} for $t = 2, \dots, T$
for each stage $t = 2, \dots, T$ do
  for each inflow scenario $m = 1, \dots, M$ do
    for each inflow scenario $l = 1, \dots, L$ do
      for each reservoir level $i = 1, \dots, I$ do
        sample independent random variable $\bar{\zeta}_{ti}{}^{lm}$ via (3–25)
      end for
      for each reservoir level $i = 1, \dots, I$ do
        generate correlated random variables ζ_{ti}^{lm} via (3–26)
      end for
      for each reservoir level $i = 1, \dots, I$ do
        calculate a_{ti}^{lm} conditioned on previous inflow a_{t-1i}^m and with correlated random variable ζ_{ti}^{lm} via (3–9)
      end for
    end for
  end for
end for

the inflow mean μ_t and the standard deviation ς_t are not typically not derived for every stage $t \in \mathbb{T}$. Instead, only one value per season or month is calculated from historic data.

"Wait-and-See" vs. "Here-and-Now". Let us go back to the one-stage dispatch problem (3–10) - (3–13). The underlying assumption was that the hydro inflow is stochastic, but that the uncertain inflow reveals itself during the stage the decision has to be made. Therefore, we assume that in stage t at the time when we make the decision of the water releases and the thermal production, the actual hydro inflow is known. This is the reason why all the decision variables have the scenario index $l = 1, \ldots, L$. In other words, an optimal decision for stage t can be made. Recognize that there are still L inflow scenarios as we do not know the inflow in advance (before stage t). This is known in the literature as Wait-and-See model. We "wait" for the stochastic event to occur and make the decision afterwards.

This way of modeling the hydro-thermal scheduling problems is motivated by the idea that the water inflows throughout this stage (remember, we are looking at time horizons of one week to one month for each stage) can be observed and the decisions on the operation of the hydro-thermal system can be adjusted.

In contrast, assuming that the water inflows for stage t are indeed stochastic during stage t, then the one-stage dispatch problems (3–10) - (3–13) change as follows

$$z_t(v_t, a_{t-1}) := \min \sum_{j\in\mathbb{J}} c_{tj}\mathbf{g}_{tj} + \Upsilon\delta_t + \sum_{l\in\mathbb{L}} p^l z_{t+1}\left(\mathbf{v}^l_{t+1}, a^l_t\right) \tag{3–27}$$

$$\text{s.t.} \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}_{tj} + \sum\nolimits_{i\in\mathbb{I}} \rho_i\mathbf{u}_{ti} + \delta_t = d_t \tag{3–28}$$

$$\mathbf{v}^l_{t+1i} = v_{ti} - \mathbf{u}_{ti} - \mathbf{s}^l_{ti} + \sum\nolimits_{h\in\mathbb{U}_i}\left(\mathbf{u}_{th} + \mathbf{s}^l_{th}\right) + a^l_{ti}, \qquad i \in \mathbb{I} \tag{3–29}$$

$$\underline{g}_{tj} \le \mathbf{g}_{tj} \le \bar{g}_{tj}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti} \le \bar{u}_{ti},$$

$$\underline{v}_{t+1i} \le \mathbf{v}^l_{t+1i} \le \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \le \mathbf{s}^l_{ti} \le \bar{s}_{ti},$$

$$\delta_t \ge 0, \qquad i \in \mathbb{I},\ j \in \mathbb{J},\ l \in \mathbb{L}. \tag{3–30}$$

The decision on the electricity generation have to be made before the water inflow is observed; *i.e.*, water release for electricity generation $\mathbf{u}_{ti}$, the thermal plant electricity generation $\mathbf{g}_{tj}$ and the demand rationing δ_t are deterministic decision variables not depending on the inflow scenario index l. Thus, the immediate cost are observed with certainty and are outside the expected value sum.

To ensure the feasibility of the water balance equations (3–29), the decision variables on the spillage $\mathbf{s}^l_{ti}$ and the water reservoir levels $\mathbf{v}^l_{t+1i}$ at the end of stage t are stochastic (they have scenario index l). Recognize that problem (3–27) - (3–30) does not decompose into L independent linear programming problems in contrast to the Wait-and-See formulation.

Models of the type (3–27) - (3–30), requiring a decision before the outcome of the random event is known, are called Here-and-Now models; *cf.* Faber and Stedinger [54] as well as Stedinger et al. [162] for further discussion on this topic. For the remainder of this dissertation, we restrict ourselves to the Wait-and-See models.

3.3.1.2 Extensions, variations and related methods

Early research on SDP methods was performed by Hall and Buras [72], Buras [24], and Turgeon [167, 168]. Surveys are given by Yakowitz [177], Yeh [181], and Stedinger [161]. A more recent review of SDP methods, their applications and comparisons to other stochastic approaches was given by Sahinidis [151].

One of the strengths of SDP methods is that they can handle non-convex, non-smooth one-stage programs; *i.e.*, the one-stage dispatch problems (3–10) - (3–13) can basically be of any type. Those one-stage problems could even be solved approximately, for instance, with heuristic methods.

A thorough discussion of computational aspects of SDP methods was given by Hanson [73]. Special focus is given on computational ways to overcome the curse of dimensionality; *i.e*, with grid computing, parallelization and better data structures.

A modified two-stage dynamic programming approach is presented by Ferrero et al. [55] for a long-term hydro-thermal scheduling problem. The proposed algorithm overcomes several drawbacks of DP methods; *e.g.*, the discretization of the state and control variables is not required.

A variant of DP are the differential dynamic programming methods originally introduced by Mayne [110] for deterministic optimal control problems. The basic idea is a quadratic approximation, using successive approximations. Extensions to the stochastic case are given by Jacobson and Mayne [90]. We refer to Yakowitz [178] for more details on differential dynamic programming methods.

Chaer and Monzón [28] discuss the stability region for SDP algorithms. More specifically, they derive a relationship between the time integration step, the space discretization step and the maximal incoming and outcoming flows. The Uruguayan hydro-thermal system is chosen for the computational tests.

A computational comparison of nested Benders decomposition and DP methods for hydro-electric optimization problems was performed by Archibals et al. [4]. The tested problems are of mid-size (having up to 17 reservoirs) with a relative short time horizon of five periods. The results of DP algorithms where within 3.2% of the optimally computed solutions using the simplex method.

A survey of stochastic (and deterministic) dynamic programming algorithms is given by Yakowitz [177] in 1982. He discussed different optimization problems related to water resources among which are also reservoir management problems – those are very close to our hydro-thermal scheduling problems. We close this section with a remarkable citation from the 1982 article: “The situation with respect to stochastic dynamic programming is that there are, as yet, no widely applicable computational devises other than discrete dynamic programming (DDP). Because of their curse of dimensionality, . . . DDP is not adequate for solving many water resource problems of interest. The largest numerical stochastic dynamic programming solutions . . . are for problems having at most two or three state variables."

3.3.2 Stochastic Dual Dynamic Programming: Expected Future Cost Extrapolation

The stochastic dual dynamic programming algorithm developed by Pereira [122] and Pereira and Pinto [125] was finally able to overcome the biggest drawback of the previously used methods (like SDP): the curse of dimensionality. SDDP uses dual information to underestimate the future cost function via extrapolation techniques, not only to reduce the size of the state space, but also to make the discretization process dynamic and to generate bounds on the optimal solution at the same time, thus eliminating all major drawbacks of the SDP methods.

SDDP is now an established method and it is still state-of-the-art in solving hydro-thermal scheduling problems. It is extensively used in operations studies and dispatch centers in more than 30 countries across the world spanning all five continents, including South, Central and North America, Austria, Spain, Norway, Turkey, New Zealand and China. In 2008, a 10 year success story of SDDP for the Brazilian system was presented by Maceira et al. [109].

There are several excellent descriptions of the SDDP algorithm available in literature next to the original publications of Pereira [122] and Pereira and Pinto [125]. Among them are the articles by de Oliveira et al. [37], Tilmant and Kelman [165], Gjelsvik et al. [65], as well as Bezerra et al. [19]. The documents of the company PSR for their hydro-thermal optimization tools, named after the algorithm SDDP, are especially detailed on the modeling aspects [130, 131].

3.3.2.1 Methodology

As described in greater detail in this section, the SDDP algorithm can be seen as an enhancement of the SDP algorithm. The main difference between SDP and SDDP is that SDDP is essentially composed of two phases: backward pass and forward simulation. In the case of minimization problems, the backward pass relies on the convexity of the objective function with respect to the Right Hand Side (RHS). This ensures that the successive generation of the so-called Benders cuts constructs a piecewise linear approximation of the future cost function, thus providing a lower bound in the backward pass which can be compared against the upper bound obtained in the forward simulation phase.

Future Cost Function Cuts. Let us now derive the convexity of the future cost function. Hence, we have to show that the function $z_t(\cdot,\cdot)$ defined in (3–10) - (3–13) is jointly convex in v_t and a_{t-1}. Therefore, first ignore the functions $z_{t+1}(\cdot,\cdot)$ in (3–10). Then, $z_t(\cdot,\cdot)$ is a convex function jointly in v_t and a_{t-1}, as they both appear in the RHS of a linear program. For the last stage, per construction, we have $z_{T+1} \equiv 0$. With the latter argument, $z_T(\cdot,\cdot)$ is convex, jointly in v_t and a_{t-1}. As the sum of convex functions is convex ($p_t^l \geq 0$), the function $z_T(\cdot,\cdot)$ is convex. Iterating this argument proves that $z_t(\cdot,\cdot)$ is a convex function in the reservoir level v_t and the past inflows a_{t-1}.

The convexity of the future cost function is important as this allows SDDP to underestimate the future cost function via extrapolation. This works as follows. Evaluating function $z_t(\cdot,\cdot)$ at the specific points v_t^n and a_{t-1}^m for all $n \in \mathbb{N}$ and $m \in \mathbb{M}$, leads to the function values $z_t(v_t^n, a_{t-1}^m)$. If we can also obtain the slopes γ_{tmn}^{ν} and γ_{tmn}^{a} of the function $z_t(v_t^n, a_{t-1}^m)$ at this point (recall, these are both vectors in the reservoirs i), then we can extrapolate the whole function $z_t(\cdot,\cdot)$, due to its convexity. In other words, we can underestimate the function $z_{ts}(\cdot,\cdot)$ via the (linear) slopes of the planes at the points v_t^n and a_{t-1}^m. Hence, we obtain the following linear program, defining a lower bound on the "true" function $z_t(\cdot,\cdot)$

$$\underline{z}_t(v_t, a_{t-1}) := \min \alpha \tag{3–31}$$

$$\text{s.t. } \alpha \geq \gamma_{tmn}^{\nu} v_t + \gamma_{tmn}^{a} a_{t-1} + \gamma_{tmn}^{c}, \qquad n \in \mathbb{N},\ m \in \mathbb{M}, \tag{3–32}$$

where γ^c_{tmn} is the constant term corresponding to the plane for stage $t \in \mathbb{T}_1$, forward inflow $m \in \mathbb{M}$, and water level discretization $n \in \mathbb{N}$. Decision variable α is a continuous variable which might be negative!

The slopes are obtained by using dual information as well as the decomposition of problem (3–10) - (3–13) into L independent problems (3–15) - (3–18). The slopes γ^ν_{tmn} and γ^a_{tmn} are then given by

$$\gamma^\nu_{tmn} := \left.\frac{\partial z_t(v_t,\cdot)}{\partial v_t}\right|_{v_t=v^n_t} \overset{(3\text{–}19)}{=\!=} \left.\frac{\partial\left(\sum_{l=1}^L p^l z^l_t(v_t,\cdot)\right)}{\partial v_t}\right|_{v_t=v^n_t} = \sum_{l=1}^L p^l \left.\frac{\partial z^l_{ts}(v_t,\cdot)}{\partial v_t}\right|_{v_t=v^n_t}, \tag{3–33}$$

$$\begin{aligned}\gamma^a_{tmn} &:= \left.\frac{\partial z_t(\cdot,a_{t-1})}{\partial a_{t-1}}\right|_{a_{t-1}=a^m_{t-1}} \overset{(3\text{–}19)}{=\!=} \left.\frac{\partial\left(\sum_{l=1}^L p^l z^l_t(\cdot,a_{t-1})\right)}{\partial a_{t-1}}\right|_{a_{t-1}=a^m_{t-1}} \\ &= \sum_{l=1}^L p^l \left.\frac{\partial z^l_t(\cdot,a_{t-1})}{\partial a_{t-1}}\right|_{a_{t-1}=a^m_{t-1}} = \sum_{l=1}^L p^l \left.\frac{\partial z^l_t(\cdot,a_{t-1})}{\partial a_t}\cdot\frac{\partial a_t}{\partial a_{t-1}}\right|_{a_{t-1}=a^m_{t-1}} \\ &\overset{(3\text{–}9)}{=\!=} \sum_{l=1}^L p^l \left.\frac{\partial z^l_t(\cdot,a_{t-1})}{\partial a_t}\cdot\varphi_t\right|_{a_{t-1}=a^m_{t-1}},\end{aligned} \tag{3–34}$$

for $t \in \mathbb{T}_1,\ m \in \mathbb{M}$, and $n \in \mathbb{N}$, where we define $\varphi_t := \varsigma_t\phi_1/\varsigma_{t-1}$. Now, let η^{lmn}_{ts} be the dual multipliers of constraints (3–17) as a row vector of the water reservoirs $i \in \mathbb{I}$, for a given storage value v^n_t and previous water inflows a^m_{t-1}. Then we derive

$$\gamma^\nu_{tmn} = \sum_{l=1}^L p^l\eta^{lmn}_t \quad \text{and} \quad \gamma^a_{tmn} = \sum_{l=1}^L p^l\varphi_t\eta^{lmn}_t, \tag{3–35}$$

for all $t \in \mathbb{T}_1,\ m \in \mathbb{M}$, and $n \in \mathbb{N}$. For notational convenience, we define

$$\gamma^\nu_{T+1mn} \equiv \gamma^a_{T+1mn} \equiv \gamma^c_{T+1mn} \equiv 0, \qquad m \in \mathbb{M},\ n \in \mathbb{N} \tag{3–36}$$

consistent with the zero future cost for the last stage T.

As for the evaluation points v^n_t and a^m_{t-1} the linear plane and the function $z_t(\cdot,\cdot)$ touch each other, one obtains

$$z_t(v^n_t, a^m_{t-1}) = \gamma^\nu_{tmn}v^n_t + \gamma^a_{tmn}a^m_{t-1} + \gamma^c_{tmn}, \tag{3–37}$$

leading to the definition of the constant term as

$$\gamma^{c}_{tmn} = \gamma^{\nu}_{tmn} v^{n}_{t} + \gamma^{a}_{tmn} a^{m}_{t-1} - z_t(v^{n}_{t}, a^{m}_{t-1}). \tag{3–38}$$

Recognize, however, that the backward openings are used as an approximation of the stochastic water inflow. Hence, $z_t(\cdot,\cdot)$ in (3–37) is also an approximation of the "true" expected operational cost for and after stage t. Even at the evaluation point of v^n_t and a^m_{t-1}, function $z_t(v^n_t, a^m_{t-1})$ may not reflect the true cost, in general.

The piecewise linear approximation of the future cost function via (3–31) - (3–32) allows us, based on formulation (3–15) - (3–18), to derive the following linear programming formulation for the one-stage dispatch problem

$$z^{lmn}_t(v^n_t, a^m_{t-1}) := \min \sum_{j\in\mathbb{J}} c_{tj}\mathbf{g}_{tj} + \Upsilon\delta_t + \alpha_{t+1} \tag{3–39}$$

$$\text{s.t. } \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}_{tj} + \sum\nolimits_{i\in\mathbb{I}} \rho_i \mathbf{u}_{ti} + \delta_t = d_t \tag{3–40}$$

$$\mathbf{v}_{t+1i} = v^n_{ti} - \mathbf{u}_{ti} - \mathbf{s}_t + \sum\nolimits_{h\in\mathbb{U}_i} \left(\mathbf{u}_{th} + \mathbf{s}_{th}\right) + a^l_{ti}, \qquad i \in \mathbb{I} \tag{3–41}$$

$$\alpha_{t+1} \geq \gamma^{\nu}_{t+1\tilde{m}\tilde{n}}\mathbf{v}_{t+1} + \gamma^{a}_{t+1\tilde{m}\tilde{n}} a^l_t + \gamma^{c}_{t+1\tilde{m}\tilde{n}}, \qquad \tilde{n}\in\mathbb{N},\ \tilde{m}\in\mathbb{M} \tag{3–42}$$

$$\underline{g}_{tj} \leq \mathbf{g}_{tj} \leq \bar{g}_{tj}, \qquad \underline{u}_{ti} \leq \mathbf{u}_{ti} \leq \bar{u}_{ti},$$

$$\underline{v}_{t+1i} \leq \mathbf{v}_{t+1i} \leq \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \leq \mathbf{s}_{ti} \leq \bar{s}_{ti},$$

$$\delta_t \geq 0, \qquad i\in\mathbb{I},\ j\in\mathbb{J}. \tag{3–43}$$

Similar to the SDP algorithm, the last stage T is different as the future costs are zero. With definition (3–36), setting the slopes of the piecewise linear approximation of the future cost function for stage T as zero, the minimization in (3–39) forces decision variable α_{T+1} to value zero in any optimal solution.

Putting Everything Together: The SDDP Algorithm. Using the idea of (stochastic) dynamic programming to recursively solve the problem, we are able to calculate an (approximate) lower bound on the objective function value z by using the future function cuts derived. This estimate of z is given through the first stage solutions of problems (3–20) - (3–23) via relation (3–24) by using the piecewise linear approximation of the future function cuts, leading to

$$z_1^m(v_1) := \min \sum_{j \in \mathbb{J}} c_{1j} \mathbf{g}_{1j}^m + \Upsilon \delta_1^m + \alpha_2 \tag{3–44}$$

$$\text{s.t. } \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{1j}^m + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{1i}^m + \delta_1^m = d_1 \tag{3–45}$$

$$\mathbf{v}_{2i}^m = v_{1i} - \mathbf{u}_{1i}^m - \mathbf{s}_1^m + \sum\nolimits_{h \in \mathbb{U}_i} \left(\mathbf{u}_{1h}^m + \mathbf{s}_{1h}^m \right) + a_{1i}^m, \qquad i \in \mathbb{I} \tag{3–46}$$

$$\alpha_{2\theta} \geq \gamma_{2\tilde{m}\tilde{n}}^{\nu} \mathbf{v}_2^m + \gamma_{2\tilde{m}\tilde{n}}^{a} a_1^m + \gamma_{2\tilde{m}\tilde{n}}^{c}, \qquad \tilde{n} \in \mathbb{N},\ \tilde{m} \in \mathbb{M} \tag{3–47}$$

$$\underline{g}_{1j} \leq \mathbf{g}_{1j}^m \leq \bar{g}_{1j}, \qquad \underline{u}_{1i} \leq \mathbf{u}_{1i}^m \leq \bar{u}_{1i},$$

$$\underline{v}_{1i} \leq \mathbf{v}_{2i}^m \leq \bar{v}_{2i}, \qquad \underline{s}_{1i} \leq \mathbf{s}_{1i}^m \leq \bar{s}_{1i},$$

$$\delta_1^m \geq 0, \qquad i \in \mathbb{I},\ j \in \mathbb{J}. \tag{3–48}$$

Hence, in this regard, SDDP differs from SDP in the sense that the obtained operational costs define (approximate) lower bound on the true overall operational costs through

$$\underline{z} \equiv z_1(v_1) \approx \sum_{m \in \mathbb{M}} \frac{1}{M} z_1^m(v_1). \tag{3–49}$$

Once the backward pass is complete and those future function cuts have been computed, a Monte Carlo simulation is performed, using the same M forward inflow scenarios as used in the backwards phase. Starting at time $t = 1$, we know the initial water reservoir levels. For all other stages, we have to use the previously obtained reservoir levels to simulate the system for a given inflow series. Hence, for each time $t = 1, \ldots, T$ and each inflow scenario $m = 1, \ldots, M$, the following one-stage dispatch problem has to be solved

$$z_t^m(v_t^m) := \min \sum_{j \in \mathbb{J}} c_{tj} \mathbf{g}_{tj}^m + \Upsilon \delta_t^m + \alpha_{t+1} \tag{3–50}$$

$$\text{s.t.} \ \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{tj}^m + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{ti}^m + \delta_t^m = d_t \tag{3–51}$$

$$\mathbf{v}_{t+1i}^m = v_{ti}^m - \mathbf{u}_{ti}^m - \mathbf{s}_t^m + \sum\nolimits_{h \in \mathbb{U}_i} \left(\mathbf{u}_{th}^m + \mathbf{s}_{th}^m\right) + a_{ti}^m, \qquad i \in \mathbb{I} \tag{3–52}$$

$$\alpha_{t+1} \geq \gamma_{t+1\tilde{m}\tilde{n}}^{\nu} \mathbf{v}_{t+1}^m + \gamma_{t+1\tilde{m}\tilde{n}}^{a} a_t^m + \gamma_{t+1\tilde{m}\tilde{n}}^{c}, \qquad \tilde{n} \in \mathbb{N},\ \tilde{m} \in \mathbb{M} \tag{3–53}$$

$$\underline{g}_{tj} \leq \mathbf{g}_{tj}^m \leq \bar{g}_{tj}, \qquad \underline{u}_{ti} \leq \mathbf{u}_{ti}^m \leq \bar{u}_{ti},$$

$$\underline{v}_{t+1i} \leq \mathbf{v}_{t+1i}^m \leq \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \leq \mathbf{s}_{ti}^m \leq \bar{s}_{ti},$$

$$\delta_t^m \geq 0, \qquad i \in \mathbb{I},\ j \in \mathbb{J}. \tag{3–54}$$

Recognize that (3–50) does not have a backward opening scenario index l, as those where used to construct the future cost functions in the backward pass. Furthermore, (3–50) is no longer dependent on the previous water inflow(s). In the backward pass, those previous inflows are important, as they carry information on the stochastic water inflow in the current stage. In the forward simulation phase, this is encoded indirectly in the M forward inflow scenarios.

Tracking the encountered operational cost for a given inflow scenario m while running through the stages provides the actual cost of operation for this inflow scenario. and calculates as

$$z^m := \sum_{t \in \mathbb{T}} \left(\sum_{j \in \mathbb{J}} c_{tj}^s g_{tj}^{m*} + \Upsilon \delta_t^{m*} \right), \qquad m \in \mathbb{M} \tag{3–55}$$

where g_{tj}^{m*} and δ_t^{m*} are optimal solutions of the corresponding variables in problem (3–50) - (3–54). Hence, the average cost over all M inflow scenarios is then given by

$$\hat{z} = \frac{1}{M} \sum_{m \in \mathbb{M}} z^m, \tag{3–56}$$

which is an estimator of the sample mean of the real operation costs when the actual stochastic inflows occur. This enables us to calculate the standard deviation through

$$\hat{\sigma} := \sqrt{\tfrac{1}{M-1} \sum\nolimits_{m \in \mathbb{M}} \left(z^m - \hat{z}\right)^2}. \tag{3–57}$$

We can stop the backward-forward iterations, once the encountered operation cost $\hat{z}$ lies inside the confidence interval

$$\left[\underline{z} - \eta \frac{\hat{\sigma}}{\sqrt{M}}, \underline{z} + \eta \frac{\hat{\sigma}}{\sqrt{M}}\right] \tag{3–58}$$

with a desired parameter η.

The SDDP algorithm is summarized in Algorithm 3-3. One observes that the computational complexity of the algorithm is dominated (similar to SDP) by the number of LP problems to be solved. In each main iteration, there are $M\left(1 + N \cdot L \cdot |\mathbb{T}_1|\right)$ LP problems for each backward pass and $M \cdot T$ LP problems for each forward simulation.

In contrast to the SDP algorithm, where N grows exponentially in the reservoir size (see Section 3.3.1), at the beginning, one typically assigns in SDDP one storage value discretization for each forward inflow scenario; *i.e.*, $N = 1$. During the Monte Carlo simulation phase, new storage values are generated which dynamically update the discretization set $\mathbb{N}$ to be used in the next backward pass, if the algorithm does not converge before. This leads to a linear grows of the size of the storage discretization in the number of main iterations. Practical experience shows that the SDDP algorithm converges in a few iterations (less than 20) even for large-scale power systems with more than 30 hydro reservoirs. Thus, the SDDP algorithm is able to break the curse of dimensionality.

3.3.2.2 Convergence analysis

Linowsky and Philpott [106] present the first convergence analysis of sampling-based multistage stochastic linear Benders decomposition methods where the uncertain parameters occur only in the constraint righthand sides (needed for convexity reasons).

Algorithm 3-3. SDDP: Stochastic Dual Dynamic Programming (Generic)

```
// Initialize
Generate inflow data; cf. Section 3.3.1.1
// Main iterations
repeat
  // Run the backwards pass to compute
  // 1.) (approx.) lower bound on the operation cost $\underline{z}$
  // 2.) (approx.) future cost function cuts
  for each stage $t = T, T-1, \ldots, 2$ do
    for each forward inflow scenario $m \in \mathbb{M}$ do
      for each storage discretization $n \in \mathbb{N}$ do
        for each backwards opening inflow scenario $l \in \mathbb{L}$ do
          solve the one-stage dispatch problem (3–39) - (3–43)
        end for
        calculate the coefficients $\gamma^{v}_{tmn}$, $\gamma^{a}_{tmn}$ and the constant term $\gamma^{c}_{tmn}$ for the future
        cost function cut in stage $t$ for stage $t-1$ via (3–35) and (3–38)
      end for
    end for
  end for
  for each forward inflow scenario $m \in \mathbb{M}$ do
    solve the one-stage dispatch problem (3–44) - (3–48)
  end for
  calculate lower bound $\underline{z}$ via (4–30)
  // Run the forward Monte-Carlo simulation to obtain
  // 1.) estimated operation cost $\hat{z}$
  // 2.) standard deviation $\hat{\sigma}$
  // 3.) new storage discretization levels
  for each forward inflow scenario $m \in \mathbb{M}$ do
    for each stage $t \in \mathbb{T}$ do
      solve the one-stage dispatch problem (4–31) - (4–35)
      update the storage discretization levels
    end for
    calculate the encountered operation cost for inflow scenario $m$ via (3–55)
  end for
  estimate the expected operation cost $\hat{z}$ via (3–56)
  calculate $\hat{\sigma}$ via (3–57)
until $\hat{z}$ lies in confidence interval (3–58)
```

The SDDP algorithm falls into this category. The authors proof that this class of algorithms converge with probability one (no guarantees on the convergence speed) if the following two assumptions are met:

- future cost function cuts have to be computed for every stage,
- forward inflows are also used when generating the future cost function cut.

In 2008, Philpott and Guan [127] proved the convergence of sampling-based multi-stage stochastic linear programs under some milder assumptions which are satisfied for SDDP algorithms.

Linderoth et al. [104] computationally test the convergence behavior of sample average approximations methods for two-stage stochastic linear programs with recourse. The reported results are very encouraging as good solutions were computed for all tested problems with reasonable computational effort. A very thorough discussion on the convergence behavior of the SDDP algorithm is given by Shapiro [158]. Again, the analysis is restricted to the case of two-stages. However, the author reports a theoretically very slow convergence; *cf.* also Lemaréchal et al. [102]. Even though no analytical results for the multi-stage case are presented, the authors conjecture that for the multi-stage case, it is computationally impossible to compute the exact solutions, in general. For practical problems, however, Shapiro admits: "with a reasonable computational effort the SDDP algorithm could produce a practically acceptable and implementable policy". As a practical recommendation, a stronger stopping criteria as the confidence interval (3–58) for the SDDP algorithm is suggested; *i.e.*, the application of a t-test on the expected value corresponding to different policies.

3.3.2.3 Extensions, variations and related methods

As mentioned before, the SDDP algorithm is an established method for the solution of hydro-thermal scheduling problems, for mid-term as well as long-term optimization problems. The initial SDDP algorithm lead to a new stream of research, covering several extensions, enhancements and variations of the SDDP algorithm. Therefore, we refer to the SDDP algorithm of Pereira and Pinto in the following as the "classical" SDDP algorithm.

Diniz and dos Santos [39] as well as dos Santos and Diniz [42] propose an extension of the classical SDDP algorithm by incorporating the information of several future stages into one stage. This is done by inclusion of additional variables and constraints of the stages ahead. The process is controlled by the so-called time step aggregation factor. The aim of this technique is to reduce the number of future function cuts to be generated, yielding a speed-up of computational time. However, due to the increase of the problem sizes, there is a trade-off between the time step aggregation factor and the convergence increase. It is worth mentioning that this approach has the same accuracy (with respect to the time discretization) as the the classical SDDP algorithm. Numerical results for the Brazilian system are presented, demonstrating a significant speed-up compared to the classical SDDP algorithm.

In order to enhance the inflow models, Infanger and Morton [88] propose cut sharing techniques among different scenario subproblems at the same stage. This allows sampling based methods like the classical SDDP to have serial dependencies of the random outcomes.

Maceira and Damázio [108] present a paradox case for the Brazilian hydro-thermal system where counter-intuitive results where obtained by using a Periodic Autoregressive Model of lag-k; PAR(k). The key observations are the negative coefficients in the stream flow model and the proposed solution is to reduce the order of the model in those cases. Similar observations have been made by Noakes et al. [116].

For the case of the power system of Brazil, Granville et al. [68] suggested an electricity transmission constrained hydro-thermal stochastic optimization model. The Fist and Second Kirchoff's laws driving the power flow have been linearized.

The natural gas supply, demand and transportation network has been incorporated into the framework of SDDP by Bezerra et al. [20]. The gas pressure difference between any two nodes drives the gas transportation in the corresponding pipeline, leading to non-linear expressions. In order to respect the properties of the models SDDP can solve, a linearization of the flow balance equations have been proposed. A case study on the Brazilian hydro dominated power system revealed the influence of the gas supply constraints on the electricity generation costs.

Tilmant and Goor [164] consider a static and a dynamic hydropower-irrigation management problem. In the static problem, a fixed amount of water is assigned to the irrigation systems while in the dynamic approach, the water irrigation is part of the optimization process. The resulting problems where solved using SDDP. Computational study showed the benefits of the dynamic model over the static approach on the example of the Euphrates river in Turkey and Syria with an expected profit increase of 6%.

Donohue [40] developed the so-called Abridged Nested Decomposition (AND) method originally designed to solve the dynamic vehicle allocation problem. This method is similar to the SDDP algorithm though requiring a smaller sample size for convergence as described by Donohue and Birge [41]. Birge applied this method to the real power system of Colombia; *cf.* [126].

Chen and Powell [29] introduced a CUtting-Plane and Partial-Sampling (CUPPS) algorithm. The basic idea is to use forward sampling passes to generate valid cuts which support the future cost function (the authors call it “expected recurse function” which is the established term in the stochastic programming community) from below (for minimization problems). In contrast to SDDP, there is no backward pass and the cuts are generated in the forward simulation phase. The proposed method is convergent with probability one.

The concept of Generalized Dual Dynamic Programming (GDDP) was introduced by Velásquez [169]. Similar to SDDP, the GDDP approach uses Benders cuts. Control theory aspects allow the algorithm to distinguish between state variables and control variables, reducing the size of the subproblems. The author considers this an extension of SDDP and the constructive dual dynamic programming approach discussed next.

Constructive Dual Dynamic Programming. A similar but different technique to SDDP was developed by Read et al. [134] and Read [133] in 1984 which is called Constructive Dual Dynamic Programming (CDDP). The basic idea is to solve the dual of the dynamic programming formulation directly. This allows to construct the marginal value surface exactly, defining an operating policy over the whole state-space. This is the major difference to the classical SDDP algorithm where a locally accurate approximation is computed instead. A very detailed description of the CDDP algorithm discussing also efficient implementation techniques is given by Read and Hindsberger [135].

CDDP algorithms are limited to lower/medium dimensional problems as the curse of dimensionality is not broken. Thus, if the number of hydro reservoirs in the system is large (*e.g.*, as for the Brazilian power system), then CDDP methods are not computational feasible and SDDP methods are preferable. However, CDDP algorithms poses several desirable features. First, by constructing the marginal value surface exactly, simulation studies can be performed. Second, linearity is not required, allowing nonlinear risk measure or game-theoretic components.

Several extensions to the original CDDP have been proposed in the literature. To deal with inflow correlations, Yang and Read [179] suggested the use of an additional dimension. Again, with an increase in the dimension, Kerr et al. [96] embedded risk measures into the framework of CDDP. Gaming components were successfully added by Scott and Read [155] which was then followed by a series of publications, namely by Batstone and Scott [10], Stewart et al. [163], as well as Read et al. [136].

A CDDP algorithm has been applied to the New Zealand hydro-power system by Culy et al. [34] and later by Craddock et al. [32]. CDDP has also been applied to the Nordic power systems, [135].

Hybrid SDP/SDDP. As mentioned in Chapter 1, the deregulation of the electricity market adds another stochastic component to the hydro-thermal scheduling problem: Electricity spot prices. This stochastic component is typically modeled as a Markov Process, making the spot price(s) additional state variable(s) next to the reservoir levels and past inflows.

In the profit maximization model, the future cost function becomes a future benefit function which is a concave function, jointly in the reservoir levels and past inflows. Recall that the reason was the concavity of linear maximization problems in right hand side variations. However, with respect to the spot prices, the future benefit function becomes convex. The reason is that the spot prices appear in the objective function. Thus, the future benefit function is saddle-shaped, which makes it impossible to apply the SDDP algorithm directly. This comes from the concavity requirement on the future benefit function in order to approximate it by a piecewise linear function.

In order to overcome this difficulty, Gjelsvik and Wallace [66] introduced a hybrid SDP/SDDP method. The appealing idea is to combine the individual strength of the SDP and the SDDP method – the lack of concavity assumption and the break of the curse of dimensionality, respectively – in order to overcome their individual weaknesses. In this scheme, the spot price forecasts are treated via Markov Chains in a discrete manner (in the SDP framework) while the reservoir levels and water inflows are modeled by continuous approximations (in the SDDP framework).

A description of the method is given by Gjelsvik et al. [64] and by Gjelsvik et al. [63] as well as in the technical report by Pereira et al. [121]. A detailed description of the hybrid SDP/SDDP method with application to the Nordic countries is given by Gjelsvik et al. [65].

A penalty function approach towards risk management for hydro-thermal profit maximization was introduced by Kristiansen [98]. The resulting models where solved using a hybrid SDP/SDDP approach. Computational results for one of Norway's power producer where performed for different levels of risk appetite. Iliadis et al. [87] also used a hybrid SDP/SDDP approach to solve a hydro-thermal profit maximization problem subject to risk constraints. Using the same methodology, Iliadis et al. [86] benchmarked different risk measures for hydro-electric agents in a deregulated market with focus on the CVaR risk measure.

CHAPTER 4
SDDPT: FUEL COST AND ELECTRICITY DEMAND UNCERTAINTY IN THE FRAMEWORK OF SDDP

As proposed by Pereira and Pinto, the stochastic dual dynamic programming algorithm deals with stochastic multi-stage linear programs in which the uncertain parameters lie on the RHS of the constraint matrix. While originally applied in the context of the least-cost hydro-thermal scheduling problem, it has since been extended to a diverse set of applications; *cf.* Iliadis [85], Flach et al. [56], Chabar et al. [27], as well as Costa [31] and Chapter 3.

Uncertainty in inflows was at the heart of the original application and some of the aforementioned extensions to the algorithm were devised in order to cope with uncertainties of a different nature which could not be incorporated into the original framework in a straightforward manner. This chapter intends to contribute to this body of work by further extending the SDDP algorithm in order to take into account additional sources of stochasticity.

The central issues regarding uncertainty and its effects on the decisions to be taken may vary depending on the time horizon and characteristics of the system under consideration. Predominantly hydro systems are more concerned with inflow uncertainty, since that directly affects the system's capacity of sustained energy production. Thermal systems, on the other hand, are usually focused on guaranteeing reliability at times of peak demand, thus making unit outages an important issue.

We follow the spirit by Zimmermann [183] and Chapter 1 to classify the uncertainties for our real world optimization problem with respect to its context. In general, uncertainties related to the hydro-thermal scheduling problem may be broadly classified into four groups. For each one of these groups, the way to mathematically represent these uncertainties has an immediate impact on the methodologies to efficiently solve the resulting problems. The first group may include, for example, the availability of each generating unit. In this case, the best approach is usually to perform a probabilistic evaluation based on Monte Carlo sampling; *cf.* Costa [31]. The second group includes sources of uncertainty to which a time series model may be accurately fitted and expected to provide reasonable forecasts - uncertainty in inflows is an example that lies on this category. The third group relates to random variables whose evolution in time is better represented by Markov Chains. That is sometimes the case of electricity spot prices [27, 112]. Finally, the fourth group deals with sources of uncertainty that are more closely related to structural, political or macro-economical conditions and can only be characterized in the form of a scenario tree, particularly when one is interested in long-term projections rather than short-term forecasts. Growth in electricity demand or the evolution of fuel prices are the most prominent examples in this group, and are exactly the motivation for our work in this chapter.

Given the recent global economic crisis and huge swings in oil prices, it became evident that relying on point estimates for key variables such as demand in future time stages and fuel costs for thermal plants may result in biased and risky decisions.

Pereira et al. [121] proposed the modeling of the electricity demand uncertainty as a linear auto-regressive process. This is theoretically possible and amenable to the application of the SDDP algorithm since, as will be shown in Section 4.1, demand appears at the RHS of the constraint matrix and, hence, the future cost function is a convex function in demand. In practice, however, a linear auto-regressive model seems not to be a good predictor for demand since these are mean-reverting processes and are not able to capture the possibility that future demand may follow structural regimes which are completely different from that of the present. As the fuel price appear in the objective function, an auto-regressive process model leads to a future cost function having a saddle shape, destroying the necessary convexity of the problem which allows it to be solved with SDDP. Hence, a Markov Chain approach seems to be the natural way and was proposed by Pereira et al. as well, leading to fuel price clusters with transition probabilities. We will discuss the differences to our proposed approach in

Section 4.1.4. Again, such a model is difficult to calibrate and it seems not to capture the fuel price development completely. More complex models as, for instance, proposed by Batlle and Barquín [9] seem to be more appropriate to capture the fuel price uncertainty, which then in turn can be transformed into a scenario tree.

In the literature, there is a wide range of publications suggesting scenario tree approaches for the stochastic load process and the stochastic fuel prices; *cf.* Nowak and Römisch [117]. There are different efficient methodologies for the generation of scenarios and the reduction of the size of the tree is computationally very important; *cf.* Chapter 3. A lagrangian relaxation technique for a short-term hydro-thermal scheduling problem under demand uncertainty was developed by Dentcheva and Römisch [38].

The main contribution of this chapter is the extension of DP based algorithms, like SDDP, by embedding it into a scenario tree framework, thus capturing additional sources of uncertainty which cannot currently be dealt with in an efficient and meaningful manner. Among the DP methods, SDDP is the method of choice if it comes to hydro-thermal power systems with a significant share of hydro power and a large number of hydro reservoirs; *cf.* Chapter 3. Thus, we discuss the joint handling of scenario-based and sampling-based stochastic modeling on the SDDP method. In other words, we embed the scenario tree framework in the classical SDDP framework and call this extension "SDDPT" (SDDP with scenario Tree). A paper based on this chapter has been submitted for publication to a scientific journal [138].

The remainder of this chapter is organized as follows. An extension of the classical multi-stage stochastic hydro-thermal optimization problem to uncertainties modeled via scenario trees is presented in Section 4.1 along with the necessary algorithmic modifications in order to solve this model with SDDP approaches. Case studies for the real power systems of Panama and Costa Rica are performed in Section 4.2. We conclude this chapter with Section 4.3.

4.1 Uncertainties via Scenario Trees

In this section, we extend the classical hydro-thermal scheduling problem (2–22) - (2–27), where the electricity demand and fuel prices were considered to be deterministic, see Chapter 2. This problem essentially consists of determining the optimal operating policy for the use of hydro and thermal resources so as to minimize total expected costs in order to fulfill the known demand. We assume that we are given a hydro-thermal power system which has to centrally dispatch the generation. The resulting problem has still its justification also in the context of deregulated electricity markets, as it is the core of many optimization problems such as the profit maximization problem by Mo et al. [112] or the optimal expansion problem by Gorenstin et al. [67]. Furthermore, many hydro-thermal systems are still centrally dispatched; *e.g.*, Central and South American countries. Cost minimization models for a centrally dispatched power system might also be used in a fundamental model in order to forecast electricity prices. This is discussed in Chapter 6.

Let us discuss now how to include uncertainties into the hydro-thermal scheduling problem (2–22) - (2–27), which are best captured via scenario trees. Candidates for such uncertainties are fuel price uncertainty and electricity demand uncertainty; *i.e.*, the data c_{tj} or d_t are now stochastic: $c_{tj}(\xi)$ or $d_t(\xi)$, respectively, with $\xi \in \Xi_t$. Following the notation for the hydro inflow uncertainty of Chapter 2, we denote by Ξ_t the set of possible fuel price outcomes conditioned on all fuel price outcomes previous to stage t. We assume that the hydro inflow and the uncertainty treated here via a tree are independent; *i.e.*, random outcome $\omega \in \Omega$ and $\xi \in \Xi_t$ are statistically independent. This is justified in the context of our hydro-thermal application as the hydro inflows should have no influence on the fuel prices and the electricity demand. Without loss of generality, in this section, we discuss this concept of scenario-based modeling of uncertainty on the example of fuel prices. The idea generalizes naturally for other

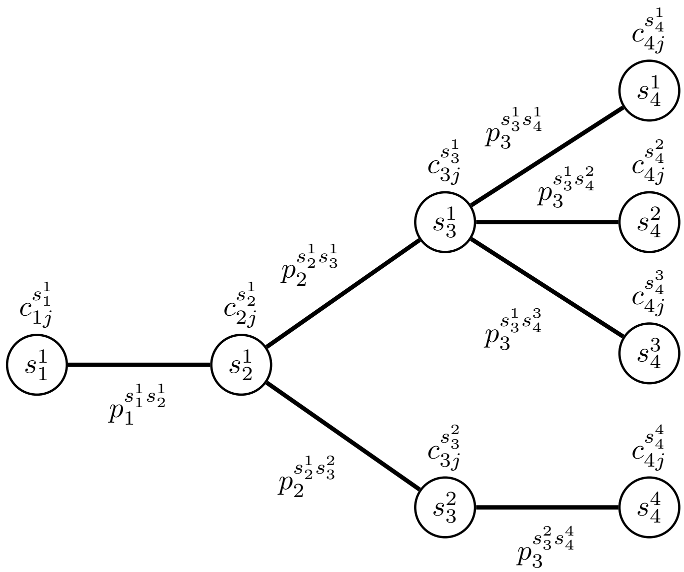


Figure 4-1. Scenario tree with 4 stages

$$\mathbb{S}_1 = \{s_1^1\},\, \mathbb{S}_2 = \{s_2^1\},\, \mathbb{S}_3 = \{s_3^1, s_3^2\},\, \mathbb{S}_4 = \{s_4^1, s_4^2, s_4^3, s_4^4\},$$
$$\boldsymbol{\Theta}_2^{s_1^1} = \{s_1^1\},\, \boldsymbol{\Theta}_3^{s_2^1} = \{s_3^1, s_3^2\},\, \boldsymbol{\Theta}_4^{s_3^1} = \{s_4^1, s_4^2, s_4^3\},\, \boldsymbol{\Theta}_4^{s_3^2} = \{s_4^4\},$$
$$p_1^{s_1^1 s_2^1} = p_3^{s_3^2 s_4^4} = 1,\, p_2^{s_2^1 s_3^1} + p_2^{s_2^1 s_3^2} = 1,\, p_3^{s_3^1 s_4^1} + p_3^{s_3^1 s_4^2} + p_3^{s_3^1 s_4^3} = 1$$

variations in the RHS, the objective function and even for changes in the technology or recourse matrix; *cf.* Section 4.1.3.

In Chapter 2, we define c_{tj} as the operational cost for electricity generation of one MWh for thermal plant $j \in \mathbb{J}$ in stage t. These operational cost are typically calculated as the product of fuel price and fuel consumption[1] . While the fuel consumption is a technological parameter for each thermal plant remaining basically constant throughout the life cycle of the plant, the fuel price is subject to uncertainty. Thus, one typically considers fuel cost uncertainty in order to capture the thermal plants' generation cost uncertainty. However, to keep the notation simpler, we assume the thermal generation cost c_{tj} to be stochastic.

[1] Next to the cost from fuel consumption, variable operation and management cost per MWh are usually added; but they are typically small compared to the fuel cost

Therefore, let $\mathbb{S}_t$ be the set of different scenarios for the stochastic fuel price $c_{tj}(\xi)$ and $c_{tj}^{s_t}$ be its realization with $s_t \in \mathbb{S}_t = \{1, \dots, S_t\}$. For notational convenience, we skip the index t for the scenarios s_t. Let $\boldsymbol{\Theta}_{t+1}^s$ be the set of Θ_{t+1}^s successor scenarios at stage $t+1$ belonging to scenario $s \in \mathbb{S}_t$ – each having (conditional) probability $p_{t+1}^{s\theta} (\geq 0)$ for $\theta \in \boldsymbol{\Theta}_{t+1}^s \subseteq \mathbb{S}_{t+1}$ with $\sum_{\theta \in \Theta_{t+1}^s} p_{t+1}^{s\theta} = 1$. Recognize that each of the θ corresponds to one of the scenarios $\mathbb{S}_{t+1}$ of stage $t+1$. Furthermore, let p_t^s be the probability that scenario $s \in \mathbb{S}_t$ occurs at time t. An example of a tree with $T = 4$ stages is shown in Figure 4-1.

Then, at stage t for a scenario $s \in \mathbb{S}_{t-1}$, we observe the fuel price c_{tj}^s corresponding to scenario s and the one-stage dispatch problem (3–1) - (3–4) is given by

$$z_{ts}(v_t) := \min \mathbb{E}_{\omega \in \Omega_t} \left[\sum_{j \in \mathbb{J}} c_{tj}^s \mathbf{g}_{tj}^s(\omega) + \Upsilon \delta_t^s(\omega) + \sum_{\theta \in \Theta_{t+1}^s} p_{t+1}^{s\theta} z_{t+1\theta}\big(\mathbf{v}_{t+1}^s(\omega)\big) \right] \tag{4–1}$$

$$\text{s.t.} \ \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{tj}^s(\omega) + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{ti}^s(\omega) + \delta_t^s(\omega) = d_t \tag{4–2}$$

$$\mathbf{v}_{t+1i}^s(\omega) = v_{ti} - \mathbf{u}_{ti}^s(\omega) - \mathbf{s}_t^s(\omega) + \sum\nolimits_{h \in \mathbb{U}_i} \big(\mathbf{u}_{th}^s(\omega) + \mathbf{s}_{th}^s(\omega)\big) + a_{ti}(\omega), \quad i \in \mathbb{I} \tag{4–3}$$

$$\begin{aligned} &\underline{g}_{tj} \leq \mathbf{g}_{tj}^s(\omega) \leq \bar{g}_{tj}, \qquad \underline{u}_{ti} \leq \mathbf{u}_{ti}^s(\omega) \leq \bar{u}_{ti}, \\ &\underline{v}_{t+1i} \leq \mathbf{v}_{t+1i}^s(\omega) \leq \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \leq \mathbf{s}_{ti}^s(\omega) \leq \bar{s}_{ti}, \\ &\delta_t^s(\omega) \geq 0, \qquad i \in \mathbb{I},\ j \in \mathbb{J}. \end{aligned} \tag{4–4}$$

Here, ω represents the inflow uncertainty while the fuel price uncertainty is captured via the Θ_t^s scenarios s. Hence, the fuel price does not depend on the random event ω; *i.e.*, c_{tj}^s has a deterministic value for scenario s.

Recognize that the thermal power generation $\mathbf{g}_{tj}^s(\omega)$ is stochastic. This has nothing to do with the fuel price uncertainty but the stochasticity is caused by the stochastic water inflows, making the water usage/release a stochastic variable affecting the complementary thermal power generation. Furthermore, notice that formulation (4–1) -

(4–4) is of type Wait-and-See for the fuel price uncertainty; *i.e.*, we assume to know the random fuel price during stage t when we have to make the decision; this is similar to the hydro inflow modeling as discussed in Chapter 2 and Chapter 3.

For a given inflow scenarios $l \in \mathbb{L}$ and initial water reservoir level v_t, the expected cost at stage t and for a scenario $s = s_t \in \mathbb{S}_t$ can then be calculated from

$$z^l_{ts}(v^n_t, a^m_{t-1}) := \min \sum_{j\in\mathbb{J}} c^s_{tj}\mathbf{g}^{ls}_{tj} + \Upsilon\delta^{ls}_t + \sum_{\theta\in\Theta^s_{t+1}} p^{s\theta}_{t+1} z_{t+1\theta}\big(\nu^{ls}_{t+1}, a^l_t\big) \tag{4–5}$$

$$\text{s.t.} \ \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}^{ls}_{tj} + \sum\nolimits_{i\in\mathbb{I}} \rho_i \mathbf{u}^{ls}_{ti} + \delta^{ls}_t = d_t \tag{4–6}$$

$$\mathbf{v}^{ls}_{t+1i} = v^n_{ti} - \mathbf{u}^{ls}_{ti} - \mathbf{s}^{ls}_t + \sum\nolimits_{h\in\mathbb{U}_i}\left(\mathbf{u}^{ls}_{th} + \mathbf{s}^{ls}_{th}\right) + a^l_{ti}, \qquad i \in \mathbb{I} \tag{4–7}$$

$$\underline{g}_{tj} \le \mathbf{g}^{ls}_{tj} \le \bar{g}_{tj}, \qquad \underline{u}_{ti} \le \mathbf{u}^{ls}_{ti} \le \bar{u}_{ti},$$

$$\underline{v}_{t+1i} \le \mathbf{v}^{ls}_{t+1i} \le \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \le \mathbf{s}^{ls}_{ti} \le \bar{s}_{ti},$$

$$\delta^{ls}_t \ge 0, \qquad i \in \mathbb{I},\ j \in \mathbb{J}, \tag{4–8}$$

and

$$z_{ts}(v_t, a_{t-1}) := \sum_{l\in\mathbb{L}} p^l z^l_{ts}(v_t, a_{t-1}), \tag{4–9}$$

$$z_{ts}(v_t) \approx \sum_{m\in\mathbb{M}} \frac{1}{M} z_{ts}(v_t, a^m_{t-1}) \tag{4–10}$$

The value of interest z, for known initial water reservoir level v_1, is then approximated by $z_{1s_1}(v_1)$. Recognize that in the first stage, there is exactly one scenario $s_1 \in \mathbb{S}_1$, making $z_{1s_1}(v_1)$ a well defined expression. Let us know have a closer look at how to approximate the future cost functions $z_{t+1\theta}(\cdot,\cdot)$ in (4–5). To derive hose future function cuts, we follow the central theme of Chapter 3.

4.1.1 Future Cost Function Cuts for SDDPT

For each one-stage dispatch problem (4–5) - (4–8), for a given scenario $s \in S_t$, we now have Θ^s_{t+1} future cost functions: one for each future scenario θ proceeding from scenario s. In order to adopt the ideas of SDDP, we first have to prove that the function $z_{ts}(\cdot,\cdot)$ is jointly convex in v_t and a_{t-1}.

Let us first ignore the functions $z_{t+1\theta}(\cdot,\cdot)$ in (4–5). Then, $z^l_{ts}(\cdot,\cdot)$ is a convex function jointly in v_t and a_{t-1}, as they both appear in the RHS of an LP problem (the same argument holds true for the electricity demand d_t). For the last stage, $z_{T+1} \equiv 0$. With the latter argument, $z^l_{Ts}(\cdot,\cdot)$ is convex in v_t and a_{t-1}. As the sum of convex functions is convex ($p^l_t \geq 0$), the function $z_{Ts}(\cdot,\cdot)$ is convex. Iterating this argument and exploiting that $p^{s\theta}_{t+1} \geq 0$ as well, we discover that $z_{ts}(\cdot,\cdot)$ is a convex function in the reservoir level v_t and the past inflows a_{t-1}. This leads to the following

Proposition 4.1. *The function $z_{ts}(v_t, a_{t-1})$ defined in (4–9) is a convex function, jointly in the reservoir levels v_t and the previous inflows a_{t-1}.*

Evaluating this function $z_{ts}(\cdot,\cdot)$ at the specific points v^n_t and a^m_{t-1} for all $n \in \mathbb{N}$ and $m \in M$, leads to the function values $z_{ts}(v^n_t, a^m_{t-1})$. If we have also the slopes γ^ν_{tmns} and γ^a_{tmns} of the function $z_{ts}(v^n_t, a^m_{t-1})$ at this point, then we can extrapolate the whole function $z_{ts}(\cdot,\cdot)$, due to its convexity. In other words, we can underestimate the function $z_{ts}(\cdot,\cdot)$ via the (linear) slopes of the planes at the points v^n_t and a^m_{t-1}. Hence, we obtain the following linear program, defining a lower bound on the true function $z_{ts}(\cdot,\cdot)$

$$\underline{z}_{ts}(v_t, a_{t-1}) := \min \alpha \tag{4–11}$$

$$\text{s.t. } \alpha \geq \gamma^\nu_{tmns} v_t + \gamma^a_{tmns} a_{t-1} + \gamma^c_{tmns}, \qquad n \in \mathbb{N},\ m \in \mathbb{M}, \tag{4–12}$$

where γ^c_{tmns} is the constant term corresponding to the plane for stage $t \in \mathbb{T}$, forward inflow $m \in \mathbb{M}$, water level discretization $n \in \mathbb{N}$, and fuel price scenario $s \in \mathbb{S}_t$.

According to their definition, the slopes γ^{ν}_{tmns} and γ^{a}_{tmns} are given by

$$\gamma^{\nu}_{tmns} := \frac{\partial z_{ts}(v_t, \cdot)}{\partial v_t}\bigg|_{v_t = v_t^n} \overset{(4\text{–}9)}{=\!=} \frac{\partial \left(\sum_{l=1}^{L} p^l z^l_{ts}(v_t, \cdot)\right)}{\partial v_t}\Bigg|_{v_t = v_t^n} = \sum_{l=1}^{L} p^l \frac{\partial z^l_{ts}(v_t, \cdot)}{\partial v_t}\bigg|_{v_t = v_t^n}, \quad (4\text{–}13)$$

$$\begin{aligned}\gamma^{a}_{tmns} &:= \frac{\partial z_{ts}(\cdot, a_{t-1})}{\partial a_{t-1}}\bigg|_{a_{t-1} = a^m_{t-1}} \overset{(4\text{–}9)}{=\!=} \frac{\partial \left(\sum_{l=1}^{L} p^l z^l_{ts}(\cdot, a_{t-1})\right)}{\partial a_{t-1}}\Bigg|_{a_{t-1} = a^m_{t-1}} \\ &= \sum_{l=1}^{L} p^l \frac{\partial z^l_{ts}(\cdot, a_{t-1})}{\partial a_{t-1}}\bigg|_{a_{t-1} = a^m_{t-1}} = \sum_{l=1}^{L} p^l \frac{\partial z^l_{ts}(\cdot, a_{t-1})}{\partial a_t} \cdot \frac{\partial a_t}{\partial a_{t-1}}\bigg|_{a_{t-1} = a^m_{t-1}} \\ &\overset{(3\text{–}9)}{=\!=} \sum_{l=1}^{L} p^l \frac{\partial z^l_{ts}(\cdot, a_{t-1})}{\partial a_t} \cdot \varphi_t\bigg|_{a_{t-1} = a^m_{t-1}}, \end{aligned} \quad (4\text{–}14)$$

for $t \in \mathbb{T}$, $m \in \mathbb{M}$, $n \in \mathbb{N}$, and $s \in \mathbb{S}_t$, where we define $\varphi_t := \varsigma_t \phi_1 / \varsigma_{t-1}$. Now, let η^{lmn}_{ts} be the dual multipliers of constraints (4–7) as a row vector of the water reservoirs $i \in \mathbb{I}$, for a given storage value v^n_t and previous water inflows a^m_{t-1}. Then we obtain

$$\gamma^{\nu}_{tmns} = \sum_{l=1}^{L} p^l \eta^{lmn}_{ts} \quad \text{and} \quad \gamma^{a}_{tmns} = \sum_{l=1}^{L} p^l \varphi_t \eta^{lmn}_{ts}. \quad (4\text{–}15)$$

As for the evaluation points v^n_t and a^m_{t-1} the linear plane and the function $z_{ts}(\cdot, \cdot)$ touch each other, one obtains

$$z_{ts}(v^n_t, a^m_{t-1}) = \gamma^{\nu}_{tmns} v^n_t + \gamma^{a}_{tmns} a^m_{t-1} + \gamma^{c}_{tmns}, \quad (4\text{–}16)$$

leading to the definition of the constant term as

$$\gamma^{c}_{tmns} = \gamma^{\nu}_{tmns} v^n_t + \gamma^{a}_{tmns} a^m_{t-1} - z_{ts}(v^n_t, a^m_{t-1}). \quad (4\text{–}17)$$

For notational convenience, we set the future cost functions cuts for the last stage T equal to zero; *i.e,*

$$\gamma^{\nu}_{tmns} \equiv \gamma^{a}_{tmns} \equiv \gamma^{c}_{tmns} \equiv 0. \quad (4\text{–}18)$$

Combining the LP problem (4–11) - (4–12) with the optimization problem (4–5) - (4–8) leads to the linear programming problem

$$z^l_{ts}(v^n_t, a^m_{t-1}) := \min \sum_{j\in\mathbb{J}} c^s_{tj}\mathbf{g}^{ls}_{tj} + \Upsilon\delta^{ls}_t + \sum_{\theta\in\Theta^s_{t+1}} p^{s\theta}_{t+1}\alpha_{t+1\theta} \tag{4–19}$$

$$\text{s.t.} \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}^{ls}_{tj} + \sum\nolimits_{i\in\mathbb{I}} \rho_i\mathbf{u}^{ls}_{ti} + \delta^{ls}_t = d_t \tag{4–20}$$

$$\mathbf{v}^{ls}_{t+1i} = v^n_{ti} - \mathbf{u}^{ls}_{ti} - \mathbf{s}^{ls}_t + \sum\nolimits_{h\in\mathbb{U}_i}\left(\mathbf{u}^{ls}_{th} + \mathbf{s}^{ls}_{th}\right) + a^l_{ti}, \qquad i\in\mathbb{I} \tag{4–21}$$

$$\alpha_{t+1\theta} \geq \gamma^{\nu}_{t+1\tilde{m}\tilde{n}\theta}\mathbf{v}^{ls}_{t+1} + \gamma^{a}_{t+1\tilde{m}\tilde{n}\theta}a^l_t + \gamma^{c}_{t+1\tilde{m}\tilde{n}\theta},$$

$$\tilde{n}\in\mathbb{N},\ \tilde{m}\in\mathbb{M},\ \theta\in\mathbf{\Theta}^s_{t+1} \tag{4–22}$$

$$\underline{g}_{tj} \leq \mathbf{g}^{ls}_{tj} \leq \bar{g}_{tj}, \qquad \underline{u}_{ti} \leq \mathbf{u}^{ls}_{ti} \leq \bar{u}_{ti},$$

$$\underline{v}_{t+1i} \leq \mathbf{v}^{ls}_{t+1i} \leq \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \leq \mathbf{s}^{ls}_{ti} \leq \bar{s}_{ti},$$

$$\delta^{ls}_t \geq 0, \qquad i\in\mathbb{I},\ j\in\mathbb{J}. \tag{4–23}$$

Recall that for the last stage T, the future costs are zero. Therefore, we obtain

$$\sum_{\theta\in\Theta^s_{T+1}} p^{s\theta}_{T+1}\alpha_{T+1\theta} \equiv 0, \tag{4–24}$$

with some appropriate definition of the sets Θ^s_{T+1} and probabilities $p^{s\theta}_{T+1}$. Let us now have a closer look at the complete algorithm in the next section.

4.1.2 SDDPT: SDDP with Scenario Tree

Using the basic ideas of dynamic programming to attack the problem backwards in time, we are able to calculate an (approximate) lower bound on the objective function value z by using the future function cuts derived in Section 4.1.1. This works as follows. In the first stage, no objective function cuts have to be calculated; remember that the obtained future function cuts in stage t show up in the formulation of stage $t-1$. Hence, no backward openings are needed for the first stage (no index “l” anymore). Furthermore, the initial water reservoir levels represent the state of the current hydro-system and are assumed to be known. Assuming a Wait-and-See model for the stochasticity modeled via a tree leads to $\mathbb{S}_1 = \{s_1\}$, and LP problem (4–19) - (4–23) reduces to

$$z^m_{1s_1}(v_1) := \min \sum_{j\in\mathbb{J}} c_{1j}\mathbf{g}^m_{1j} + \Upsilon\delta^m_1 + \sum_{\theta\in\Theta^{s_1}_2} p^{s_1\theta}_2 \alpha_{2\theta} \tag{4–25}$$

$$\text{s.t. } \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}^m_{1j} + \sum\nolimits_{i\in\mathbb{I}} \rho_i \mathbf{u}^m_{1i} + \delta^m_1 = d_1 \tag{4–26}$$

$$\mathbf{v}^m_{2i} = v_{1i} - \mathbf{u}^m_{1i} - \mathbf{s}^m_1 + \sum\nolimits_{h\in\mathbb{U}_i} \left(\mathbf{u}^m_{1h} + \mathbf{s}^m_{1h}\right) + a^m_{1i}, \qquad i \in \mathbb{I} \tag{4–27}$$

$$\alpha_{2\theta} \geq \gamma^{\nu}_{2\tilde{m}\tilde{n}\theta}\mathbf{v}^m_2 + \gamma^a_{2\tilde{m}\tilde{n}\theta} a^m_1 + \gamma^c_{2\tilde{m}\tilde{n}\theta}, \qquad \tilde{n}\in\mathbb{N},\ \tilde{m}\in\mathbb{M},\ \theta\in\mathbf{\Theta}^{s_1}_2 \tag{4–28}$$

$$\underline{g}_{1j} \leq \mathbf{g}^m_{1j} \leq \bar{g}_{1j}, \qquad \underline{u}_{1i} \leq \mathbf{u}^m_{1i} \leq \bar{u}_{1i},$$

$$\underline{v}_{1i} \leq \mathbf{v}^m_{2i} \leq \bar{v}_{2i}, \qquad \underline{s}_{1i} \leq \mathbf{s}^m_{1i} \leq \bar{s}_{1i},$$

$$\delta^m_1 \geq 0, \qquad i\in\mathbb{I},\ j\in\mathbb{J}. \tag{4–29}$$

Consistent with relation (4–10), we obtain a lower bound on the overall operation cost through

$$\underline{z} \equiv z_{1s_1}(v_1) \approx \sum_{m\in\mathbb{M}} \frac{1}{M} z^m_{1s_1}(v_1). \tag{4–30}$$

Once the backward pass is complete and those future function cuts have been computed, a Monte Carlo simulation can be performed, using the same M forward inflow scenarios as used in the backwards pass. Starting at time $t = 1$, we know the initial water reservoir levels. For all other stages, we have to use the previously obtained reservoir levels to simulate the system for a given inflow series. Hence, for each time $t = 1, \ldots, T$, each inflow scenario $m = 1, \ldots, M$ and each tree scenario $s = 1, \ldots, |\mathbb{S}_t|$, we have to solve the following one-stage dispatch problem

$$z^m_{ts}(v^s_{tm}) := \min \sum_{j\in\mathbb{J}} c^s_{tj}\mathbf{g}^{ms}_{tj} + \Upsilon\delta^{ms}_t + \sum_{\theta\in\Theta^s_{t+1}} p^{s\theta}_{t+1}\alpha_{t+1\theta} \tag{4–31}$$

$$\text{s.t. } \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}^{ms}_{tj} + \sum\nolimits_{i\in\mathbb{I}} \rho_i \mathbf{u}^{ms}_{ti} + \delta^{ms}_t = d_t \tag{4–32}$$

$$\mathbf{v}^{ms}_{t+1i} = v^s_{tmi} - \mathbf{u}^{ms}_{ti} - \mathbf{s}^{ms}_t + \sum\nolimits_{h\in\mathbb{U}_i}\left(\mathbf{u}^{ms}_{th} + \mathbf{s}^{ms}_{th}\right) + a^m_{ti}, \qquad i\in\mathbb{I} \tag{4–33}$$

$$\alpha_{t+1\theta} \geq \gamma^{v}_{t+1\tilde{m}\tilde{n}\theta}\mathbf{v}^{ms}_{t+1} + \gamma^{a}_{t+1\tilde{m}\tilde{n}\theta}a^m_t + \gamma^{c}_{t+1\tilde{m}\tilde{n}\theta}, \quad \tilde{n}\in\mathbb{N},\ \tilde{m}\in\mathbb{M},\ \theta\in\mathbf{\Theta}^s_{t+1} \tag{4–34}$$

$$\underline{g}_{tj} \leq \mathbf{g}^{ms}_{tj} \leq \bar{g}_{tj}, \qquad \underline{u}_{ti} \leq \mathbf{u}^{ms}_{ti} \leq \bar{u}_{ti},$$

$$\underline{v}_{t+1i} \leq \mathbf{v}^{ms}_{t+1i} \leq \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \leq \mathbf{s}^{ms}_{ti} \leq \bar{s}_{ti},$$

$$\delta^{ms}_t \geq 0, \qquad i\in\mathbb{I},\ j\in\mathbb{J}. \tag{4–35}$$

Tracking the encountered operational cost for a given inflow scenario m while running through time provides the actual cost of operation for this scenario and calculates as

$$z^m := \sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}_t} p^s_t \left(\sum_{j\in\mathbb{J}} c^s_{tj} g^{ms*}_{tj} + \Upsilon\delta^{ms*}_t\right), \qquad m\in\mathbb{M} \tag{4–36}$$

where g^{ms*}_{tj} and δ^{ms*}_t are optimal solutions of the corresponding variables in problem (4–31) - (4–35). Hence, the average cost over all M inflow scenarios is then given by

$$\hat{z} = \frac{1}{M}\sum_{m\in\mathbb{M}} z^m, \tag{4–37}$$

which is an estimator of the sample mean of the real operation costs when the actual stochastic inflows occur. The standard deviation is calculated via relation (3–57) and the SDDPT algorithm can stop the backward-forward iterations, once the encountered operation cost $\hat{z}$ lies inside the confidence interval (3–58).

A pseudo-code of the SDDPT method is given in Algorithm 4-1. One observes that the computational complexity of the SDDPT algorithm is dominated by the number of LP problems to be solved. In each main iteration, there are $M\left(1 + N\cdot L\sum_{t\in\mathbb{T}_1} S_t\right)$ LP problems for each backward pass and $M\sum_{t\in\mathbb{T}} S_t$ LP problems for each forward

Algorithm 4-1. SDDPT: Stochastic Dual Dynamic Programming with Scenario Tree

// Initialize
Generate inflow data; *cf.* Chapter 3
// Main iterations
repeat
// Run the backwards pass to compute
// 1.) (approx.) lower bound on the operation cost $\underline{z}$
// 2.) (approx.) future cost function cuts
for each stage $t = T, T-1, \ldots, 2$ do
for each forward inflow scenario $m \in \mathbb{M}$ do
for each storage discretization $n \in \mathbb{N}$ do
for each tree scenario $s \in \mathbb{S}_t$ do
for each backwards opening inflow scenario $l \in \mathbb{L}$ do
solve the one-stage dispatch problem (4–19) - (4–23)
end for
end for
calculate the coefficients γ^{v}_{tmns}, γ^{a}_{tmns} and the constant term γ^{c}_{tmns} for the future cost function cut in stage t for stage $t-1$ via (4–15) and (4–17)
end for
end for
end for
for each forward inflow scenario $m \in \mathbb{M}$ do
solve the one-stage dispatch problem (4–25) - (4–29)
end for
calculate lower bound $\underline{z}$ via (4–30)
// Run the forward Monte-Carlo simulation to obtain
// 1.) estimated operation cost $\hat{z}$
// 2.) standard deviation $\hat{\sigma}$
// 3.) new storage discretization levels
for each forward inflow scenario $m \in \mathbb{M}$ do
for each stage $t \in \mathbb{T}$ do
for each tree scenario $s \in \mathbb{S}_t$ do
solve the one-stage dispatch problem (4–31) - (4–35)
update the storage discretization levels
end for
end for
calculate the encountered operation cost for inflow scenario m via (4–36)
end for
estimate the expected operation cost $\hat{z}$ via (4–37)
calculate $\hat{\sigma}$ via (3–57)
until $\hat{z}$ lies in confidence interval (3–58)

simulation. This implies that the total number of LP problems to be solved using SDDPT depends linearly on the size of the tree. This does not sound too bad, however, one has to remember that the size of the tree, and also S_t, can be exponential in the number of stages.

Theoretically, the LP problems defined through (4–19) - (4–23) are independent for each inflow scenarios $m \in \mathbb{M}$, $l \in \mathbb{L}$ and scenarios $s \in \mathbb{S}_t$. (Due to the structure of the decomposition, the LP problems are time dependent.) This suggests a natural way to parallelize the SDDPT Algorithm 4-1. However, one has to recognize that a perfect speed-up cannot be expected, as one typically exploits the similarity of the LP problems to be solved; *i.e.*, for given stage t, the LP problems vary only in the RHS values (observe that $\gamma^{a}_{t+1\tilde{m}\tilde{n}\theta} a^{l}_{t}$ is a constant for stage t as well). Hence, warm-starts using dual simplex algorithms are typically used to exploit that structure.

4.1.3 Multivariate Scenario Trees

In brief, the embedding of the scenario tree into SDDP works by defining a tree structure “on top” of the forward water inflows. Hence, for each forward inflow $m \in \mathbb{M}$ (as well as storage discretization and backward opening) at stage t, there are S_t one-stage dispatch problems to solve – one for each scenario at stage t. For scenario $s \in \mathbb{S}_t$, the future cost are then given by the conditional probability of any other scenario being the successor of s. Hence, the future cost function of the classical SDDP becomes a sum of future cost functions.

In contrast to the SDP methods, SDDP requires the future cost functions to be convex in the state variables in order to underestimate these function via extrapolation techniques; *cf.* Chapter 3. Recall that we used the argument of LP problems being convex in variations in the RHS to prove the convexity. Hence, adding a state variable corresponding to some data other than the RHS may destroy the convexity property; *e.g.*, a state variable affecting the objective function leads to a concave future cost function in that variable.

Per construction, the tree approach of SDDPT towards uncertainty modeling does not require any state variable (which has the nice affect that no discretization of this state space is required). Going through the analysis of Section 4.1.1 again, it is apparent that the uncertainty captured via the tree does not affect the convexity of the future const functions in the state variables v_t and a_{t-1}. This implies that the uncertainty captured by the tree can affect any coefficient of the one-stage dispatch problem (4–5) - (4–8) (*e.g.*, thermal generation cost, power coefficient for hydro plants, electricity demand, lower and upper bounds on thermal generation decisions, water release, water spillage reservoir level and spillage, as well as coefficients of all the constraints discussed in Chapter 2). The only change in LP problems (4–5) - (4–8) is that the appropriate coefficients have a scenario index s. Furthermore, those uncertainties can be handled jointly. Thus, any multivariate scenario tree could be used to model various uncertainties simultaneously.

4.1.4 Scenario Tree vs. Markov Chain

Pereira et al. [121] proposed a Markov Chain to cope with fuel price uncertainty. This Markov Chain has a state space size of K and is time-homogeneous. Hence, the transition probability distribution does not depend on the stages and can be represented by a right stochastic matrix. This leads to K price clusters for each stage t with stage independent transition probability. The incorporation with the classical SDDP works as follows. Each forward inflow $m \in \mathbb{M}$ is assigned exactly one such price cluster (hence, $M \geq K$) and the future cost function in (3–15) is substituted by the expected value of the future cost functions for each price cluster. This leads to the very nice property that the number of LP problems to be solved remains the same as in the classical SDDP. However, the main drawback of this method is that it is practically very tricky to define the initial values for the cost clusters and to derive meaningful transition probabilities. Furthermore, it is questionable if the fuel prices really evolve according to a time-homogeneous Markov Chain.

In contrast, the scenario tree approach provides a natural way to forecast fuel prices and/or electricity demand. Government agencies such as the US Energy Information Administration (EIA) or the International Energy Agency (IEA) publish regularly fuel price and electricity demand forecasts on a scenario basis. The World Energy Outlook by the IEA, for instance, based its forecasts in 2007 on three different scenarios: A reference scenario, an alternative policy scenario and a high growth scenario. Those data are readily available and can be transformed into a scenario tree straight forward. This practical advantage comes with the cost that the number of LP problems to be solved increases with the size of the tree; *cf.* Section 4.1.1. Hence, one wants to make sure to use a tree as small as practically feasible. However, as described in Section 4.1.2, the implementation of a parallelization scheme would be simple and possibly capable of achieving computational times comparable to those in which uncertainty in demand or fuel prices is not considered. Furthermore, all the techniques for scenario generation and scenario reduction readily available in the literature (*cf.* Chapter 3) can be used to generate thin scenario trees, keeping the running time of the SDDPT algorithm computationally feasible.

4.2 Case Study for Central America

In this section, we present and discuss the results of the application of the methodology proposed in this chapter by studying the cases of Panama and Costa Rica power systems for electricity demand uncertainty. The consideration of demand and fuel price uncertainty are specially important not only to Panama and Costa Rica, but also to a number of countries in Central America due to three main factors:

- Significant share of hydro resources and existence of reservoirs. The fact that a considerable part of the system's installed capacity comes from hydro plants and the existence of reservoirs which are capable of seasonal regulation stresses the necessity of taking into account the uncertainties related to demand and fuel prices. By having a more detailed representation of the evolution of these parameters; *i.e.*, instead of relying on single point estimates for demand and fuel prices throughout the horizon – one is capable of having a more accurate calculation of the opportunity cost of water, which ultimately determines the system's operating policy.

- High dependence on international markets. Resources such as oil, coal and natural gas are not usually abundant in these countries and, consequently, they experience a severe dependence on international markets, being exposed to both availability and price issues. By factoring into the problem the possibility that there might be a stronger need for these fuels in the future or that they might be a lot more expensive then, the obtained solution may be hedged against extreme events that would otherwise compromise security of supply or lead to unbearable costs.
- Supply adequacy issues. There are countries in which the whole system is designed and dimensioned according to a pre-defined reliability criterion which may be, for example, a maximum percentage risk of running into a situation where part of the load has to be shed. In such cases, the need for the installation of new generation capacity is assessed by means of successive simulations of the system for a given set of inflow scenarios (and usually fixed demand and fuel prices): More capacity is added as long as the results indicate a risk of deficit greater than 5%. In such cases, a simulation of the exact same supply configuration associated with an increase in fuel prices would lead to deficit risks greater than those previously calculated.

We study the effects of electricity demand uncertainty on the first stage decisions. Assuming a Wait-and-See model, we know the electricity demand for the first stage with certainty. However, all future electricity demands per stage are not known and have to be forecasted. For mid-term optimization models, the first stage decisions are the information of interest. In hydro-thermal scheduling, mid-term optimization problems provide information for the water reservoir management; *i.e.*, water reservoir levels are priced via the future function cuts, see Wallace and Fleten [173]. We use those cuts in our studies to obtain solutions for the first stage. This allows us to study the demand effects on an annual basis for different first stage decisions.

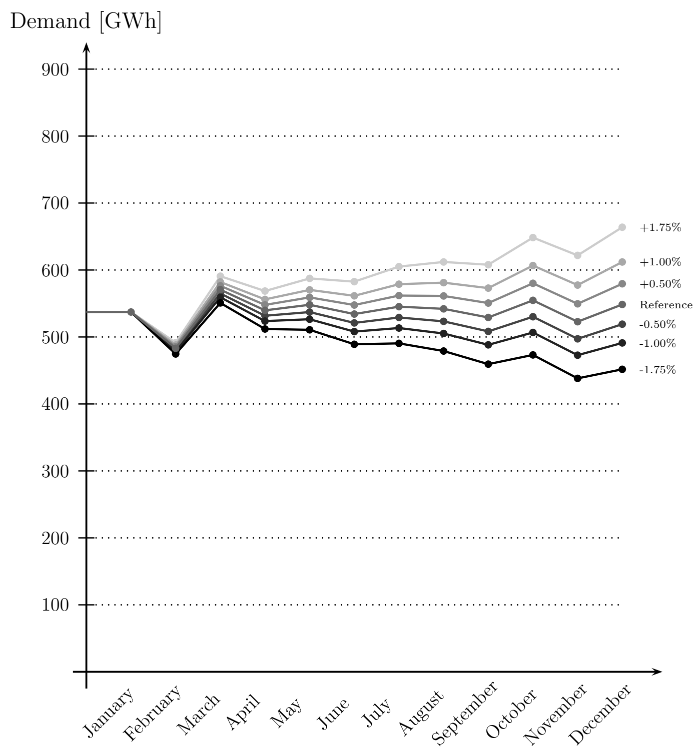
In our computational results, we consider 7 different electricity demand scenarios. The electricity demand for January is the same for each scenario while the demand for all other months are given by the cumulative percentage change relative to the reference scenario at $\pm 1.75\%$, $\pm 1.00\%$ and $\pm 0.50\%$ for the Panama system and $\pm 1.00\%$, $\pm 0.50\%$ and $\pm 0.25\%$ for the Costa Rica system, respectively; *i.e.*, in stage t, $1 < t \leq T$, a

$x\%$ change for the demand d_t of the reference scenario leads to the new demand of $(1+x)^{t-1}d_t$. The different electricity demand scenarios for the case of Panama are shown in Figure 4-2 (a) and for Costa Rica in Figure 4-2 (b), respectively.

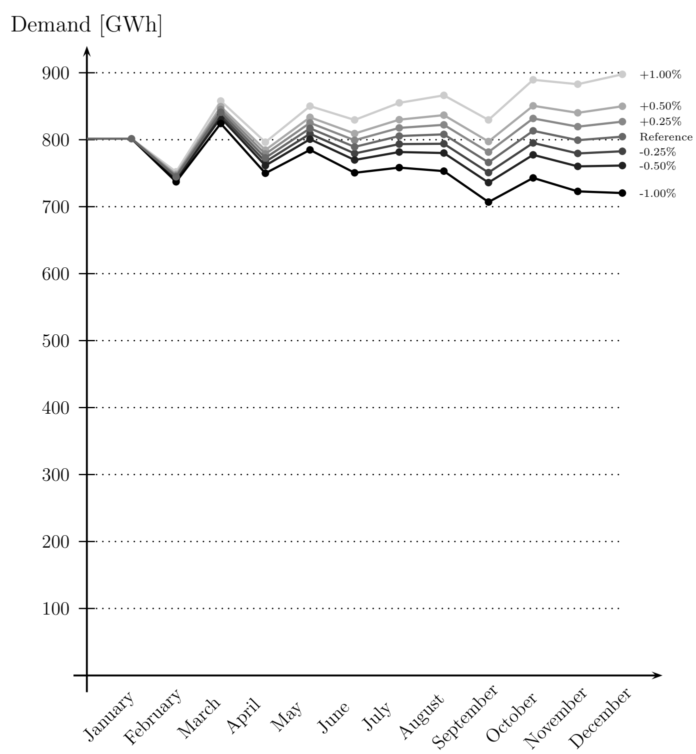
The presented SDDPT algorithm has been implemented in Mosel (Version 3.0.0), an algebraic modeling language developed by Y. Comlombani and S. Heipcke for Dash Optimization (now FICO). The resulting LP problems from the decomposition are solved using Xpress Optimizer (Version 20.00.05). The time horizon of choice is one year with monthly stages where the first month is January and the last month considered is December. To achieve accurate computational results and to reduce noise, we use $M = 100$ forward inflow scenarios and $L = 50$ backward openings for the SDDP algorithm. The inflows are forecasted via a linear autoregressive model of lag-1; the corresponding parameters of equation (3–9) are estimates using real data.

4.2.1 Panama

For our computational studies, we use the real power system of Panama. Panama's electricity power system consisted of 12 thermal plants, 4 plants with hydro-reservoirs as well as 1 run-of-the-river plant. The thermal plants' data are given in Table 4-1. We can see that the generation cost per MWh ranged from \$71.3 to \$313.4 for the first month. The "ACP1" to "ACP4" fuels are a special mix of different fuels for those particular thermal plants only. For the consecutive months, we assume the same fuel prices, while an annual discount rate of 10% applies. For the first month considered, the fixed thermal generation is 82.4 GWh with a generation cost of million \$10.3 and the thermal capacity is 426.4 GWh. Over a one year horizon, the fixed generation cost accumulates to million \$115.5. A schematic diagram of the hydro system of Panama is shown in Figure 4-3. The installed hydro-electric capacity for January was 529.4 GWh. The electricity demand for January is assumed to be known at 537.3 GWh. Further, we assume an electricity demand presenting seasonal effects with a difference of 2.1% between January and December. This pattern can be seen in Figure 4-2.



A



B

Figure 4-2. Electricity demand scenarios for the power systems of Panama and Costa Rica. A) Panama. B) Costa Rica

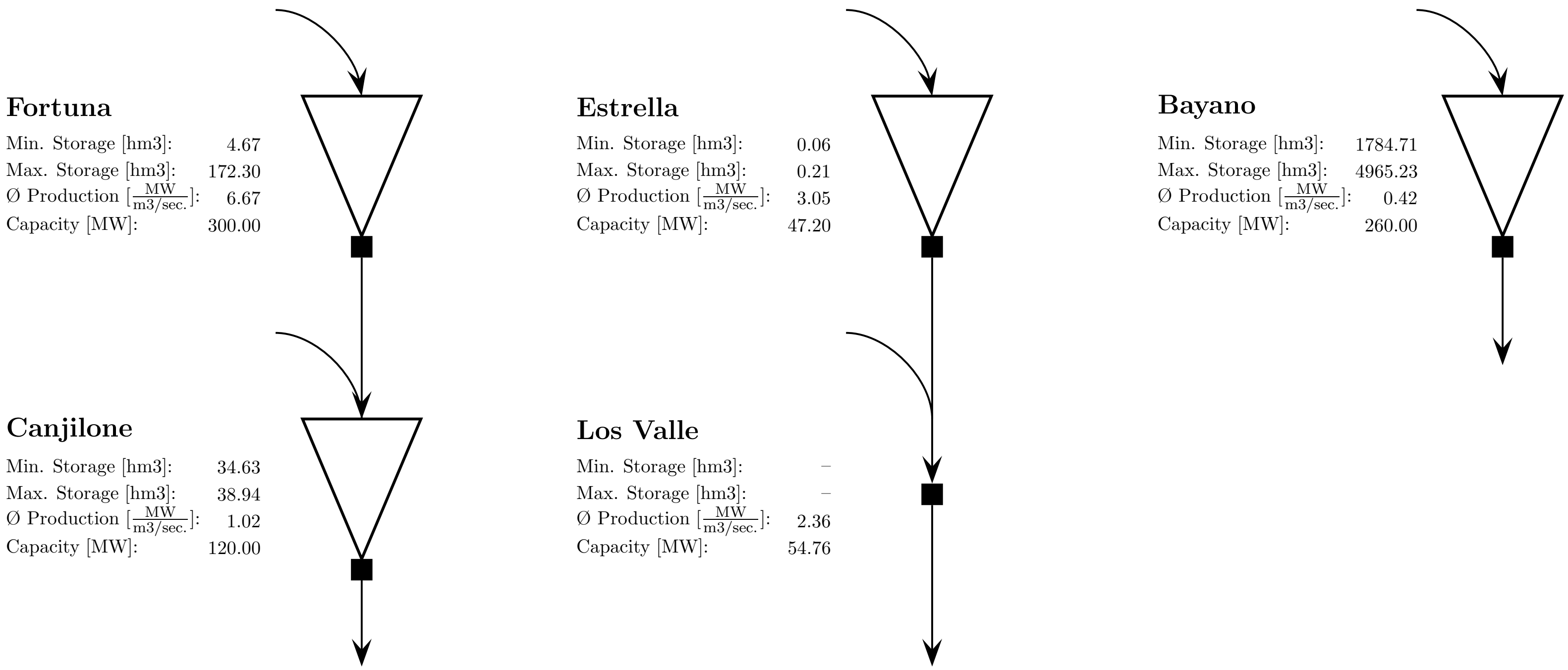


Figure 4-3. Hydro-electric system of Panama

Consider now Table 4-2. Computational results are shown for 7 different electricity demand scenarios, an average of those and a stochastic scenario corresponding to Figure 4-2. The second row gives the thermal generation for the first stage for the best computed solution, while the row labeled “Hydro” indicates the hydro-electric generation in GWh for January. In the row “Demand,” the yearly electricity demand is given, while in row labeled “Cost” the yearly generation cost are provided (excluding the fixed generation cost). “EVS” stands for Expected Value Solution and the corresponding row indicates the corresponding yearly generation cost. The rows “Δ” provide the percentage change with respect to the reference scenario.

The results in Table 4-2 reveal that the first stage decisions are very sensitive to changes in electricity demand; recall that the demand of the first stage is the same in each scenario. This is explained by the very idea of hydro-thermal electricity systems, where one wants to hedge against dry seasons where the installed thermal capacity might not be sufficient to meet the electricity demand (or very expensive thermal generation units might be needed). Relatively full hydro-reservoirs can prevent electricity shortages during those seasons. However, this comes with the “risk” that some water might have to be spilled if a wet season occurs. This explains the trends in the higher (lower) thermal electricity generation for demand increases (decreases).

However, the thermal electricity generation does not increase (with a demand increase) with the same rate as it decreases (with a demand decrease); the same holds true for the annual cost. The first reason is given by the hedging against dry seasons; *i.e.*, the increase in future demand leads to an proportional higher increase of the hydro electricity than a decrease for the case of demand decreases. The second reason is that a decrease in demand may allow to use some hydro-electric power in the first stage to avoid the production using the most expensive thermal plants. The third reason explaining the relative similarity in electricity production of the case of +1.00% and

+1.75% is the fact that all the “cheap” thermal plants have already been used in the first stage generation.

Let us know have a look at the solutions for the “average” case and the “stochastic” case. With the “average” case, we mean the average decision taken from all individual decisions for the 7 scenarios. The resulting annual cost of the average case represents the case of perfect information of the electricity demand; *i.e.*, doing the decision over and over again with known electricity demand over the whole year would lead to this average cost. This cost are given by million \$156.721. The difference between the cost when having perfect information and the cost of the stochastic approach yields to \$653,429 which is 0.42% with respect to the case of perfect information, representing what is called the expected value of perfect information.

As shown in Table 4-2, the thermal and hydro electric generation decisions for the first stage differ significantly for the “average” and “stochastic” case. The reason is once again that the extreme cases of electricity demand increases may lead to electricity shortages in future stages which are penalized heavily. Hence, the higher reservoir levels in the stochastic case compared to the “average” case is a hedging against future electricity shortages.

Using the expected electricity demand as the single scenario of choice leads in our case to the same first stage decisions as the reference scenario. Now, using this first stage solution in any of the 7 demand scenarios leads to the so-called EVS. Recognize that we use this term with respect to the “two-stage uncertainty” in the electricity demand embedded in a multi-stage stochastic optimization context. Hence, this is can be seen as an adoption of this recognized terminology [21]. The increase in annual operation costs by ignoring the random variations in the electricity demand compared to the stochastic approach is called the value of the stochastic solution (VSS). For our data, we have that the VSS is million \$1.279, corresponding to a 0.81% cost decrease compared to the EVS.

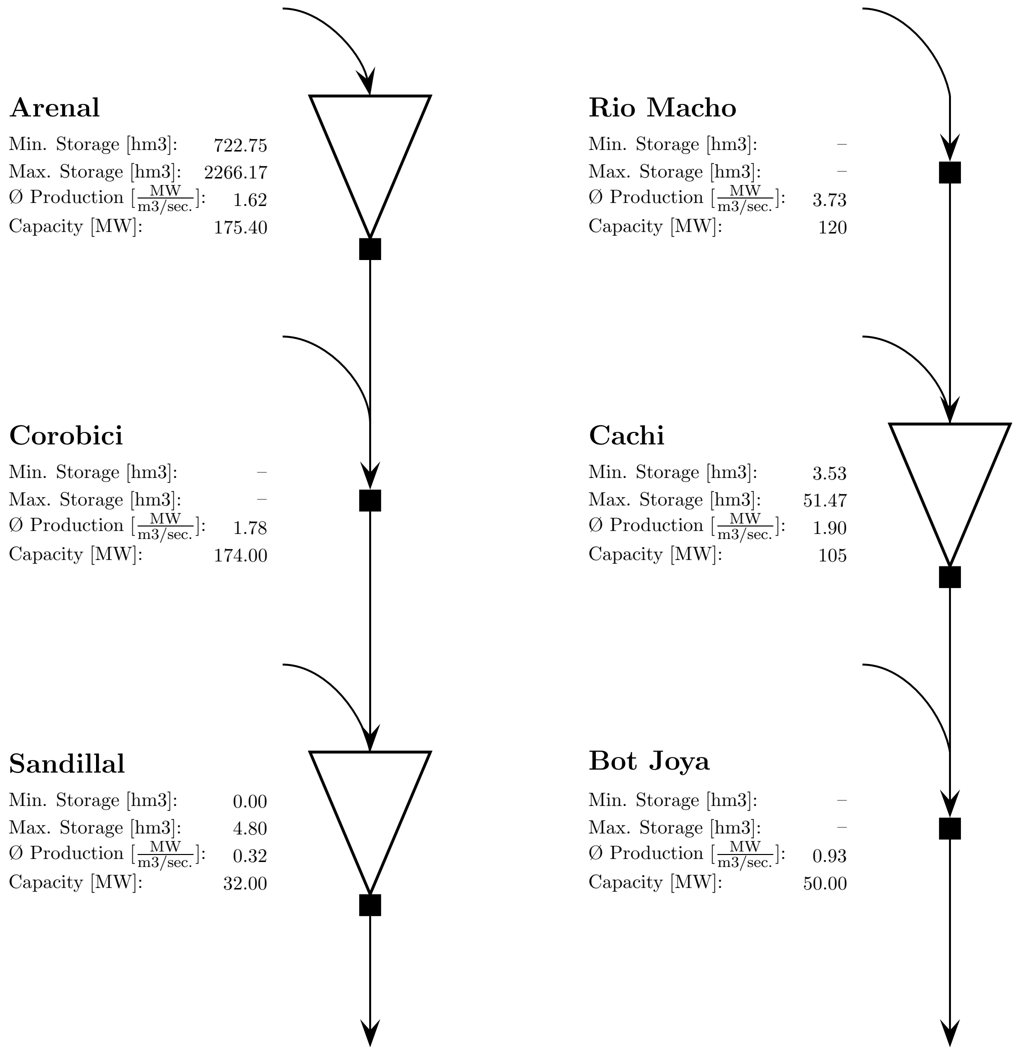


Figure 4-4. Hydro-electric reservoir system of Costa Rica; excluding additional 24 run-of-the river plants

4.2.2 Costa Rica

For our second case study, we use the power system of Costa Rica. The properties of the 13 thermal plants considered are listed in Table 4-3. For January, the thermal capacity amounted to 490.4 GWh while the fixed generation cost per annum is million \$13.6. The hydro system of Costa Rica consists of 3 reservoirs with electric generators and 26 run-of-the-river plants. The hydro-system for the 3 hydro-reservoirs is shown in Figure 4-4. The installed hydro-electric capacity was 2,438 MW.

The computational results in Table 4-4 show a very similar pattern to the results obtained for the Panama system. Following the conventions for the Panama case study, the expected value of perfect information is $915,870 which is 0.92% of the cost incurred under perfect information. The VSS amounts to million $1.142, or a 1.12% cost saving compared to the EVS.

4.3 Discussion

The SDDP algorithm has been traditionally used for the solution of the least-cost hydro-thermal scheduling problem since it allows for a relatively detailed representation of the system's characteristics – in particular, it becomes possible to represent hydro plants individually – while considering uncertainty in inflow scenarios and remaining computationally tractable. The method relies on the approximation of the future cost function by a set of linear inequalities, which can be iteratively constructed until a convergence criterion is achieved.

In this chapter, we proposed an extension of this methodology which permits us to incorporate additional sources of uncertainty whose evolution in time is more accurately represented in the form of scenario trees, such as demand and fuel prices. In fact, this can be seen as a unified state-space and scenario tree framework for hydro-thermal scheduling.

Recall that we classified the solution methods for stochastic hydro-thermal scheduling problems in Chapter 3 into two broad classes: scenario-based and sampling-based. Those two approaches towards uncertainty modeling seem to be contrary to each other and divide the research community. However, the hybrid method proposed in this Chapter employs the best features of both methods which allows efficiently solving large-scale real world problems. More specifically, the advantages of scenario-based modeling (*e.g.*, coping with complex forecasting tools) and sampling-based modeling (*e.g.*, coping with large number of inflow samples with a detailed reservoirs system while avoiding the curse of dimensionality) are merged to a single hybrid approach. This way, the convexity of the future cost functions is preserved and the SDDPT method maintains the algorithmic properties of SDDP.

The proposed approach amounts to parallelizable runs of the SDDP algorithm, each corresponding to the data associated with a branch of the scenario tree of a given stage. Whenever two or more branches of the scenario tree are joined at a node, the corresponding future cost functions are also merged, following the probability of occurrence associated with each branch. Therefore, at these nodes a single future cost function is obtained and the algorithm continues in the same manner until it reaches the scenario tree root node.

The impact of taking into account demand and fuel price uncertainty may reach beyond the operational scheduling problem and extend into supply adequacy and load supplying capability issues. The importance of taking into account electricity demand uncertainties was explored by using the real power systems of Panama and Costa Rica as case studies.

The incorporation of fuel price uncertainty might be especially useful when maximizing profits in a deregulated energy market. Typically, risk constraints are taken into account in such models, “penalizing” a certain risk exposure. There are the hybrid SDP/SDDP algorithms for hydro-thermal profit maximization models which can adopt our methodology in a straight forward way. We see a big potential of our approach for fuel price uncertainty in such an environment.

Table 4-1. Thermal plants considered for the Panama power system

Min. Generation [MW]	15	15	15	40	20	12	12	0	10	0	0	0
Capacity [MW]	40	40	40	158	42.8	44	96	30	53.5	7	12	18
Fuel Type	1	1	1	2	3	3	1	4	1	5	6	7
Cost [$/MWh]	109.6	109.6	109.6	122.0	230.3	152.4	74.5	96.2	71.3	160.7	313.4	171.2

Fuel Type 1: Bunker, 2: Diesel M., 3: Diesel L., 4: ACP1, 5: ACP2, 6: ACP3, 7: ACP4

Table 4-2. Computational results for the power system of Panama with different electricity demand scenarios. The generated electricity is given in GWh and the cost are given in $1000

Scenario	-1.75%	-1.00%	-0.50%	Reference	+0.50%	+1.00%	+1.75%	Average	Stochastic
Thermal	257.8	257.5	264.1	276.1	268.2	340.4	340.2	286.3	302.0
Δ	-6.63%	-6.74%	-4.34%	–	-2.85%	23.31%	23.23%	3.71%	9.38%
Hydro	279.5	279.8	273.2	261.2	197.1	196.9	269.1	251.0	235.3
Δ	7.01%	7.12%	4.58%	–	3.02%	-24.64%	-24.55%	-3.92%	-9.92%
Demand	5,866.5	6,110.6	6,280.2	6,455.6	6,636.9	6,824.3	7,117.5	6,470.2	–
Δ	-9.12%	-5.34%	-2.72%	–	2.81%	5.71%	10.25%	0.23%	–
Cost	109,702	128,330	141,329	151,664	162,295	189,495	214,229	156,721	157,374
Δ	-27.67%	-15.39%	-6.81%	–	7.01%	24.94%	41.25%	–	–
EVS	109,794	128,371	141,333	151,664	163,198	193,536	222,675	158,653	–
Δ	0.08%	0.03%	0.00%	0.00%	0.56%	2.13%	3.94%	–	–

Table 4-3. Thermal plants considered for the Costa Rica power system

Min. Generation [MW]	0	0	0	0	97.5	0	0	19.6	0	1	0	0	0
Capacity [MW]	14	34	36	26	130	130.5	78	26.1	42	10	66	24	25
Fuel Type	1	7	7	1	8	7	7	8	8	7	9	1	8
Cost [$/MWh]	103.3	223.3	223.3	84.5	2.7	198.1	186.7	2.7	2.7	155.1	13.68	68.7	2.7

Fuel Type 1: Bunker, 7: Diesel, 8: Geo ice, 9: Geo CR

Table 4-4. Computational results for the power system of Costa Rica with different electricity demand scenarios. The generated electricity is given in GWh and the cost are given in $1000

Scenario	-1.00%	-0.50%	-0.25%	Reference	+0.25%	+0.25%	+1.00%	Average	Stochastic
Thermal	187.0	195.8	207.0	215.8	229.7	236.9	318.1	227.2	232.9
Δ	-13.37%	-9.30%	-4.12%	–	9.75%	47.40%	23.23%	5.26%	7.89%
Hydro	614.5	605.7	594.5	585.7	571.8	564.6	483.4	574.3	568.6
Δ	4.93%	3.43%	1.52%	–	-2.37%	-3.59%	-17.47%	-1.94%	-2.91%
Demand	9,052.9	9,303.7	9,432.3	9,563.1	9,696.1	9,831.3	10,108.5	9,569.7	–
Δ	-5.34%	-2.71%	-1.37%	–	1.39%	2.80%	5.70%	0.07%	–
Cost	70,574	78,815	84,199	91,002	103,768	113,684	158,176	**100,032**	**100,947**
Δ	-22.45%	-13.39%	-7.48%	–	14.03%	24.92%	73.82%	–	–
EVS	71,575	79,689	84,478	91,002	105,685	116,616	165,576	**102,089**	–
Δ	1.42%	1.11%	0.33%	0.00%	1.85%	2.58%	4.68%	–	–

CHAPTER 5
CO_2 EMISSION CONSTRAINED SDDP

Despite the uncertainty surrounding the design of a mechanism which is ultimately accepted by nations worldwide, the necessity to implement measures to curb emissions of greenhouse gases on a global scale is consensual. The electricity sector plays a fundamental role in this puzzle and countries may soon have to revise their operating policy directives in order to make them compatible with additional constraints imposed by such regulations. In this chapter, we present a modeling approach for Green House Gas (GHG) emission quotas which allows these constraints to be incorporated into the stochastic dual dynamic programming algorithm, vastly used to solve the hydro-thermal scheduling problem. The proposed method is flexible and capable of accommodating a detailed representation of emissions and related constraints.

When deciding on the imposition of emission quotas, one of the main concerns for each country is the impact of such limitations on the competitiveness of its industrial activities and the potential side effects on its economy. The existence of a quota and penalties associated with its violation may have huge effects in terms of decreasing economic activity and additional costs related to energy efficiency projects, higher energy costs and eventual acquisition of additional quotas in international markets.

Since the emission quotas are to be established for each country as a whole, it will be up to each government to decide how they are going to be divided among each sector – and this is exactly where the problem we study in this chapter comes into play. Recall from Chapter 1 that the electricity sector contributes a significant share of the overall CO_2 emissions and thus, is a natural choice for CO_2 emission regulations.

Having an annual limit on the total CO_2 emission allowances on a power system directly affects the way system operators define the operating schedule of each plant since a new element must now be factored into the equation on top of the usual sources of uncertainty such as demand and inflows. While it is desirable that the generation of dirty plants is replaced by that of cleaner alternatives, this comes at a cost which must be borne by society. It thus becomes imperative that policy makers are able to estimate the increase in costs when defining the share of quotas to be allocated to the electricity sector and the fines associated with their violations.

Managing an annual emission allowance is somewhat similar to managing water reservoirs since one must determine the optimal trade-off between consuming parts of the limited amount of a resource in the present moment or saving it for future use. The decision to deplete the CO_2 stock on hand may only be assessed in terms of its expected future costs which depend on the evolution of hydrological conditions. For example, consuming emission quotas in the present – thus preventing their use in future time stages – may prove useful if a high inflow scenario occurs and hydro plants are able to meet a higher share of demand.

Belsnes et al. [13] model CO_2 reservoirs in SDDP via a “hydro” reservoir, where thermal plants are interpreted as hydro-electric stations using CO_2 allowances (instead of water) and transmission line cost are used to model the thermal generation cost. In this paper, we introduce an alternative reservoir model, allowing complicating constraints (*e.g.*, fuel availability) on the thermal plants and allowing CO_2 emissions to expire as determined by the EU ETS regulations. Other than the article by Belsnes et al., the literature on CO_2 emission constrained hydro-thermal cost minimization problems is very thin. There are a few articles for the case of liberalized electricity markets and CO_2 emission trading. However, this is subject of Chapter 6 where we review those methods.

The contribution of this chapter is on the modeling aspect of the emission-constrained hydro-thermal scheduling problem. We propose a representation of GHG emission quotas as reservoirs, thus allowing it to be readily embedded into the stochastic dual dynamic programming algorithm with the addition of state variables into the future cost functions. The proposed approach is flexible and capable of handling the representation of emission constraints both at a system-wide level and in a more detailed view that encompasses different quotas for each technology or set of plants. While there are numerous economic assessments and policy analysis stemming from the results of the problem being studied, that is not the focus of this chapter. A paper based on this chapter has been submitted for publication to a scientific journal [137].

Although we adopt the role of a system operator that determines the dispatch of both hydro and thermal plants in a centralized fashion, the utilized model may also provide insights of the potential consequences of imposing emission quotas in liberalized markets. Under the hypothesis of absence of market power, the system operation where agents are free to submit price and quantity bids is shown to be equivalent to that which results from a centralized least-cost scheduling; *cf.* Chapter 6.

The remainder of this chapter is organized as follows. In Section 5.1, we formulate the problem of interest as a stochastic hydro-thermal scheduling problem. In order to solve this model, a CO_2 reservoir is presented in Section 5.2 along with the derivation of the future function cuts, necessary to incorporate this methodology into the framework of DP. A case study for the real power system of Guatemala is presented in Section 5.3. We conclude with a discussion in Section 5.4.

5.1 CO_2 Emission Constrained Stochastic Hydro-Thermal Scheduling

In this section, we describe the hydro-thermal scheduling problem – discussed in Chapter 2 – which is subject to CO_2 emission caps.

Given is a hydro-thermal system with I hydro power plants $i \in \mathbb{I} = \{1, \dots, I\}$ and J thermal plants $j \in \mathbb{J} = \{j, \dots, J\}$. Decisions can be made at discrete stages $t \in \mathbb{T} = \{1, \dots, T\}$ (*e.g.*, monthly) on the electricity generation $\mathbf{g}_{tj}$ of the thermal plants $j \in \mathbb{J}$ and the electricity generation $\rho_i u_{ti}$ of the hydro power plants $i \in \mathbb{I}$.

The objective is the minimization of the expected operational cost z of the system in the long-term horizon, particularly taking into account CO_2 emission quotas. The operation cost consists of the variable cost for the electricity production of the thermal plants and the fees to be paid if the emission quotas are exceeded. The variable production cost of hydro plants are assumed to be zero.

CO_2 is emitted at a constant factor B_j, whenever the thermal plant $j \in \mathbb{J}$ is used to generate electricity. In order to avoid short term anomalies, CO_2 emission allowances are not given at every stage (*e.g.*, monthly), but for a longer time period (*e.g.*, yearly); *i.e.*, at stages $y \in \mathbb{Y}_g \subseteq \mathbb{T}$. If the given quota $E_y^{\mathrm{CO_2}}$ is not met, then a penalty fee of $C^{\mathrm{CO_2}}$ for the emissions exceeding the quota has to be paid. We distinguish two models in this chapter where the emissions allowances expire at certain stages $y \in \mathbb{Y}_e$ and where the emission allowances have no expiration date once they are issued. For notational convenience, we assume that $\mathbb{Y}_e \subseteq \mathbb{Y}_g$; *i.e.*, the emission allowances expire only then when new allowances are issued.

Without loss of generality, we assume that each hydro plant has a water reservoir and that each reservoir has a hydro plant. We will not distinguish between reservoirs and hydro plants except when explicitly specified and assign both the index $i \in \mathbb{I}$; *cf.* Chapter 2. The inflows a_{ti} per hydro reservoir $i \in \mathbb{I}$ and stage $t \in \mathbb{T}$ are assumed to be stochastic while all other technical specifications of the system are known; in particular, the generation cost c_{tj} per thermal plant $j \in \mathbb{J}$ and stage $t \in \mathbb{T}$ as well as the electricity demand d_t per stage $t \in \mathbb{T}$ are given as an average value over the time discretization length; *i.e.*, monthly.

We next formalize this problem as a mathematical program.

Stochastic Programming Model.

Let Ω_t be the set of all random outcomes conditioned on the previous stages and ω_t be a realization; *cf.* Chapter 2. Each random outcome reveals a certain hydro inflow for the corresponding stage. Then, the described mid-term hydro-thermal scheduling problem can be modeled as the following multi-stage stochastic linear programming problem

$$
\begin{aligned}
z := \min \sum_{j\in\mathbb{J}} c_{1j}\mathbf{g}_{1j} + \Upsilon\delta_1 + C^{\mathrm{CO_2}}\mathbf{f}_1 + \\
+ \min \mathbb{E}_{\omega_2\in\Omega_2}\Bigg[\sum_{j\in\mathbb{J}} c_{2j}\mathbf{g}_{2j}(\omega_2) + \Upsilon\delta_2(\omega_2) + C^{\mathrm{CO_2}}\mathbf{f}_2(\omega_2) + \ldots + \\
+ \min \mathbb{E}_{\omega_t\in\Omega_t}\Bigg[\sum_{j\in\mathbb{J}} c_{tj}\mathbf{g}_{tj}(\omega_t) + \Upsilon\delta_t(\omega_t) + C^{\mathrm{CO_2}}\mathbf{f}_t(\omega_t) + \ldots + \\
+ \min \mathbb{E}_{\omega_T\in\Omega_T}\Bigg[\sum_{j\in\mathbb{J}} c_{Tj}\mathbf{g}_{Tj}(\omega_T) + \Upsilon\delta_T(\omega_T) + C^{\mathrm{CO_2}}\mathbf{f}_T(\omega_T)\Bigg]\ldots\Bigg]
\end{aligned}
\tag{5–1}
$$

$$
\text{s.t. } \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}_{1j} + \sum\nolimits_{i\in\mathbb{I}} \rho_i\mathbf{u}_{1i} + \delta_1 = d_1 \tag{5–2}
$$

$$
\sum\nolimits_{j\in\mathbb{J}} \mathbf{g}_{tj}(\omega_t) + \sum\nolimits_{i\in\mathbb{I}} \rho_i\mathbf{u}_{ti}(\omega_t) + \delta_t(\omega_t) = d_t, \qquad t\in\mathbb{T}_1 \tag{5–3}
$$

$$
\mathbf{v}_{2i} = v_{1i} - \mathbf{u}_{1i} - \mathbf{s}_{1i} + \sum\nolimits_{h\in\mathbb{U}_i}\big(\mathbf{u}_{1h} + \mathbf{s}_{1h}\big) + a_{1i}, \qquad i\in\mathbb{I} \tag{5–4}
$$

$$
\begin{aligned}
\mathbf{v}_{t+1i}(\omega_t) = \mathbf{v}_{ti}(\omega_{t-1}) - \mathbf{u}_{ti}(\omega_t) - \mathbf{s}_t(\omega_t) + \\
+ \sum\nolimits_{h\in\mathbb{U}_i}\big(\mathbf{u}_{th}(\omega_t) + \mathbf{s}_{th}(\omega_t)\big) + a_{ti}(\omega_t), \qquad t\in\mathbb{T}_1,\ i\in\mathbb{I}
\end{aligned}
\tag{5–5}
$$

$$
\sum\nolimits_{t|y}\sum\nolimits_{j\in\mathbb{J}} B_j\mathbf{g}_{tj}(\omega_t) - \mathbf{f}_y(\omega_t) \le E_y^{\mathrm{CO_2}}, \qquad y\in\mathbb{Y}_g \tag{5–6}
$$

$$
\begin{aligned}
&\underline{g}_{1j} \le \mathbf{g}_{1j} \le \bar{g}_{tj}, \qquad \underline{g}_{tj} \le \mathbf{g}_{tj}(\omega_t) \le \bar{g}_{tj},\\
&\underline{u}_{1i} \le \mathbf{u}_{1i} \le \bar{u}_{1i}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti}(\omega_t) \le \bar{u}_{ti},\\
&\underline{v}_{2i} \le \mathbf{v}_{2i} \le \bar{v}_{2i}, \qquad \underline{v}_{t+1i} \le \mathbf{v}_{t+1}(\omega_t) \le \bar{v}_{t+1i},\\
&\underline{s}_{1i} \le \mathbf{s}_{1i} \le \bar{s}_{1i}, \qquad \underline{s}_{ti} \le \mathbf{s}_{ti}(\omega_t) \le \bar{s}_{ti},\\
&\mathbf{f}_1 \ge 0, \qquad \mathbf{f}_t(\omega_t) \ge 0,\\
&\delta_1 \ge 0, \qquad \delta_t(\omega_t) \ge 0, \qquad t\in\mathbb{T}_1,\ i\in\mathbb{I},\ j\in\mathbb{J},
\end{aligned}
\tag{5–7}
$$

with the notation $t|y$, indicating if stage t corresponds to the emission allowance period starting at y and ending at the period before $y+1$; ***i.e.***, $x := y+1 \in \mathbb{Y}_g$ and $t\in\mathbb{T}$ with $t+1 = x$.

Modeled in a Wait-and-See fashion (*cf.* Chapter 2), the hydro inflows a_{1i} are known with certainty in the first stage when the generation decisions are made. The decisions in all other stages are stochastic.

Let us now have a look at the future stages $t > 1$. There, the known electricity demand d_t has to be met by thermal and/or hydro electricity production as stated in constrains (5–3) where a rationing of $\delta_t(\omega_t)$ is possible but penalized in the objective. The water balance equations (5–5) ensure that the reservoir level $\mathbf{v}_{t+1i}(\omega_t)$ for reservoir i at the end of stage t equals the reservoir level $\mathbf{v}_{ti}(\omega_{t-1})$ at the beginning of the stage plus the stochastic water inflow $a_{ti}(\omega_t)$ minus the turbined water $u_{ti}(\omega_t)$ minus the spilled water s_{ti} plus the inflows from the plants immediately upstream of plant i, either from upstream hydro power generation or spillage; with the notation $\mathbf{v}_{2i}(\omega_1) \equiv \mathbf{v}_{2i}$.

Constraints (5–6) model the emission allowances per horizon, where the emitted tons CO_2 due to thermal generation have to be less then or equal to the CO_2 quota $E_y^{CO_2}$ plus the additional CO_2 allowances “bought” via fines $\mathbf{f}_y$. In this formulation, in order to reduce the notational burden, we assume that the emission allowances expire right before new once are issued. However, the generalization is straightforward and the reservoir model proposed in Section 5.2 is more precise in this regard. Recognize that constraints (5–6) may span multiple stages. This is particularly problematic for the decomposition methods available in the literature. We discuss this in greater detail below where we provide an alternative model of the CO_2 emission constraints (5–6).

The objective function (5–1) is then given as the sum of the first stage generation cost plus the expected cost of thermal power generation in the future stages, including possible fees for rationing and CO_2 emission quota violations.

Additional linear operational constraints can be added to the model (5–1) - (5–7) in order to make it more practical; *e.g.*, linearized electricity and gas network constraints, multiple load blocks, and sub-systems. For a comprehensive list of constraints proposed in the literature, refer to Chapter 2.

5.2 CO_2 Emission Quota Modeling via Reservoirs

As mentioned before, constraint (5–6) may span multiple stages and, hence, destroys the block diagonal structure of the original hydro-thermal scheduling problem (2–22) - (2–27). Dynamic programming methods (and other decomposition techniques) exploit this structure and cannot deal with this constraint in its present form. Hence, we suggest a formulation of the CO_2 emission quotas respecting the stage decomposition framework of dynamic programming methods like SDP and SDDP.

The quota on the CO_2 emission, modeled via constraint (5–6), can be interpreted as a reservoir as follows: At any given time $y \in \mathbb{Y}_g$ (*e.g.*, at the beginning of the year) it "rains" CO_2 emission rights, filling the emissions reservoir, see Figure 5-1. At each time stage $t \in \mathbb{T}$, there is a balance equation for the CO_2 emissions as follows

$$\mathbf{e}_{t+1} = \mathbf{e}_t - \sum\nolimits_{j \in \mathbb{J}} B_j \mathbf{g}_{tj} + \mathbf{f}_t, \qquad t \in \mathbb{T} \setminus \mathbb{Y}_g \tag{5–8}$$

$$\mathbf{e}_{t+1} = \widetilde{\mathbf{e}}_t - \sum\nolimits_{j \in \mathbb{J}} B_j \mathbf{g}_{tj} + \mathbf{f}_t + E_t^{CO_2}, \qquad t \in \mathbb{Y}_g \tag{5–9}$$

$$\mathbf{e}_{t+1} \geq 0, \qquad \mathbf{f}_t \geq 0, \qquad t \in \mathbb{T}, \tag{5–10}$$

with the initial emission allowances e_1 at the beginning of the planning period; *e.g.*, $e_1 \equiv 0$.

In equations (5–8), the emissions allowances do not expire at the end of the stage t, $\mathbf{e}_{t+1}$, are the CO_2 emissions allowances at the beginning of stage t, $\mathbf{e}_t$, minus the emission allowances used via thermal electricity generation, $\sum_{j \in \mathbb{J}} B_j \mathbf{g}_{tj}$, plus the emission rights fined, $\mathbf{f}_t$ plus the emission allowances issued, $E_t^{CO_2}$. Notice that we need to have a (non-negative) variable $\mathbf{f}_t$ for the emissions exceeding the quota for each stage $t \in \mathbb{T}$ (in contrast to $y \in \mathbb{Y}_g$) in order to ensure that the emission reservoir level $\mathbf{e}_{t+1}$ is nonnegative at the end of each stage. In particular, variables $\mathbf{f}_t$ ensure feasibility of the CO_2 reservoir constraints in the one-stage dispatch problem for the SDDP algorithm, derived in Section 5.2.1.

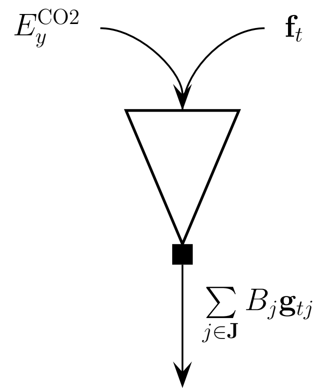


Figure 5-1. CO_2 emission reservoirs

With equation (5–9), we are able to model the two cases (i) the emission allowances expire at stages $y \in \mathbb{Y}_e$ or (ii) there is no expiration date. This is realized via definition

$$\widetilde{\mathbf{e}}_t := \begin{cases} 0, & \text{if the emissions expire; } \textit{i.e.}, \ t \in \mathbb{Y}_e \\ \mathbf{e}_{t-1}, & \text{if the emissions do not expire} \end{cases} \tag{5–11}$$

for all $t \in \mathbb{T}$ and $e_0 = e_1$ being the initial CO_2 emission allowances. This model allows that emission allowances expire whenever new allowances are issued or not, as it is the case in the EU ETS; *i.e.*, $\mathbb{Y}_e \subseteq \mathbb{Y}_g$.

5.2.1 One-Stage Dispatch Programming

The modeling of the CO_2 emission quotas as reservoirs through constraints (5–8) - (5–10) allows a stage decomposition of the hydro-thermal scheduling problem; *cf.* Chapter 3. Being at stage t, the coupling between the previous and future stages is then given through the hydro reservoir levels v_t as the "initial" reservoir level for stage t, the CO_2 emissions level e_t, and the previous inflows a_{t-1} – the so-called state variables.

Following the logic of the LP problems (3–1) - (3–4) and (3–5) - (3–8), decomposing the problem into stages and applying backward opening inflow scenarios $l \in \mathbb{L}$ each with (conditional) probability p^l, the following deterministic one-stage dispatch problem is obtained

$$z_t(v_t, e_t, a_{t-1}) := \min \sum_{l \in \mathbb{L}} p^l \left[\sum_{j \in \mathbb{J}} c_{tj} \mathbf{g}_{tj}^l + \Upsilon \delta_t^l + C^{\mathrm{CO_2}} \mathbf{f}_t^l + z_{t+1}\big(\mathbf{v}_{t+1}^l, \mathbf{e}_{t+1}^l, a_t^l\big) \right] \tag{5–12}$$

$$\text{s.t.} \ \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{tj}^l + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{ti}^l + \delta_t^l = d_t \tag{5–13}$$

$$\mathbf{v}_{t+1i}^l = v_{ti} - \mathbf{u}_{ti}^l - \mathbf{s}_{ti}^l + \sum\nolimits_{h \in \mathbb{U}_i} \left(\mathbf{u}_{th}^l + \mathbf{s}_{th}^l\right) + a_{ti}^l, \qquad i \in \mathbb{I} \tag{5–14}$$

$$\mathbf{e}_{t+1}^l = e_t - \sum\nolimits_{j \in \mathbb{J}} B_j \mathbf{g}_{tj}^l + \mathbf{f}_t^l, \qquad \text{if } t \in \mathbb{T} \setminus \mathbb{Y}_g \tag{5–15}$$

$$\mathbf{e}_{t+1}^l = \widetilde{e}_t - \sum\nolimits_{j \in \mathbb{J}} B_j \mathbf{g}_{tj}^l + \mathbf{f}_t^l + E_t^{\mathrm{CO_2}}, \qquad \text{if } t \in \mathbb{Y}_g \tag{5–16}$$

$$\underline{g}_{tj} \le \mathbf{g}_{tj}^l \le \bar{g}_{tj}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti}^l \le \bar{u}_{ti},$$

$$\underline{v}_{t+1i} \le \mathbf{v}_{t+1i}^l \le \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \le \mathbf{s}_{ti}^l \le \bar{s}_{ti},$$

$$\mathbf{e}_{t+1}^l \ge 0, \qquad \mathbf{f}_t^l \ge 0, \qquad \delta_t^l \ge 0, \qquad j \in \mathbb{J},\ i \in \mathbb{I}. \tag{5–17}$$

Recognize that this is a generalization of the one-stage dispatch problems (3–10) - (3–13) as we add the CO_2 emission reservoir constraints (5–8) - (5–10) and the state variables e_t. Like problem (3–10) - (3–13), the above LP problem decomposes into L independent problems, one for each water inflow sample a_t^l.

Along the lines of LP problems (3–15) - (3–18), M forward inflows are used to simulate the stochastic hydro inflows while the reservoir storage values v_t and the CO_2

reservoir level e_t are discretized, one obtains

$$z_t^{lmn}(v_t^n, e_t^n, a_{t-1}^m) := \min \sum_{j\in\mathbb{J}} c_{tj}\mathbf{g}_{tj} + \Upsilon\delta_t + C^{\mathrm{CO_2}}\mathbf{f}_t + z_{t+1}\big(\mathbf{v}_{t+1}, \mathbf{e}_{t+1}, a_t^{lm}\big) \tag{5–18}$$

$$\text{s.t. } \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}_{tj} + \sum\nolimits_{i\in\mathbb{I}} \rho_i \mathbf{u}_{ti} + \delta_t = d_t \tag{5–19}$$

$$\mathbf{v}_{t+1i} = v_{ti}^n - \mathbf{u}_{ti} - \mathbf{s}_{ti} + \sum\nolimits_{h\in\mathbb{U}_i} \big(\mathbf{u}_{th} + \mathbf{s}_{th}\big) + a_{ti}^{lm}, \qquad i \in \mathbb{I} \tag{5–20}$$

$$\mathbf{e}_{t+1} = e_t^n - \sum\nolimits_{j\in\mathbb{J}} B_j \mathbf{g}_{tj} + \mathbf{f}_t, \qquad \text{if } t \in \mathbb{T}\setminus\mathbb{Y}_g \tag{5–21}$$

$$\mathbf{e}_{t+1} = \widetilde{e}_t^n - \sum\nolimits_{j\in\mathbb{J}} B_j \mathbf{g}_{tj} + \mathbf{f}_t + E_t^{\mathrm{CO_2}}, \qquad \text{if } t \in \mathbb{Y}_g \tag{5–22}$$

$$\underline{g}_{tj} \le \mathbf{g}_{tj} \le \bar{g}_{tj}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti} \le \bar{u}_{ti},$$

$$\underline{v}_{t+1i} \le \mathbf{v}_{t+1i} \le \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \le \mathbf{s}_{ti} \le \bar{s}_{ti},$$

$$\mathbf{e}_{t+1} \ge 0, \qquad \mathbf{f}_t \ge 0, \qquad \delta_t \ge 0, \qquad j \in \mathbb{J},\ i \in \mathbb{I}. \tag{5–23}$$

Taking the expected value with respect to the L backward inflow scenarios leads to

$$z_t(v_t^n, e_t^n, a_{t-1}^m) := \sum_{l\in\mathbb{L}} p^l z_t^{lmn}(v_t^n, e_t^n, a_{t-1}^m). \tag{5–24}$$

Assuming a Wait-and-See model, the first stage problems are a special case, as the backward openings are not required; *cf.* Chapter 3 and Chapter 4. Formulation (3–20) -

([3–23](#)) changes in this context then to

$$z_1^m(v_1, e_1, a_0^m) := \min \sum_{j\in\mathbb{J}} c_{1j}\mathbf{g}_{1j} + \Upsilon\delta_1 + C^{CO_2}\mathbf{f}_1 + z_2\big(\mathbf{v}_2, a_1^m\big) \tag{5–25}$$

$$\text{s.t.} \ \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}_{1j} + \sum\nolimits_{i\in\mathbb{I}} \rho_i\mathbf{u}_{1i} + \delta_1 = d_1 \tag{5–26}$$

$$\mathbf{v}_{2i} = v_{1i}^n - \mathbf{u}_{1i} - \mathbf{s}_1 + \sum\nolimits_{h\in\mathbb{U}_i} \big(\mathbf{u}_{1h} + \mathbf{s}_{1h}\big) + a_{1i}^m, \qquad i\in\mathbb{I} \tag{5–27}$$

$$\mathbf{e}_2 = e_1^n - \sum\nolimits_{j\in\mathbb{J}} B_j\mathbf{g}_{1j} + \mathbf{f}_1, \qquad \text{if } t = 1 \in \mathbb{T}\setminus\mathbb{Y}_g \tag{5–28}$$

$$\mathbf{e}_2 = \tilde{e}_1^n - \sum\nolimits_{j\in\mathbb{J}} B_j\mathbf{g}_{1j} + \mathbf{f}_1 + E_1^{CO_2}, \qquad \text{if } t = 1 \in \mathbb{Y}_g \tag{5–29}$$

$$\underline{g}_{1j} \le \mathbf{g}_{1j} \le \bar{g}_{1j}, \qquad \underline{u}_{1i} \le \mathbf{u}_{1i} \le \bar{u}_{1i},$$

$$\underline{v}_{2i} \le \mathbf{v}_{2i} \le \bar{v}_{2i}, \qquad \underline{s}_{1i} \le \mathbf{s}_{1i} \le \bar{s}_{1i},$$

$$\mathbf{e}_2 \ge 0, \qquad \mathbf{f}_1 \ge 0, \qquad \delta_1 \ge 0, \qquad j\in\mathbb{J},\ i\in\mathbb{I}. \tag{5–30}$$

and the expected operation cost z can be estimated by

$$z_1 := \sum_{m\in\mathbb{M}} p^m z_1^m(v_1, e_1, a_0^m) \approx z. \tag{5–31}$$

It remains to derive the future cost function cuts; in particular, with respect to the additional state variables e_t.

5.2.2 Future Cost Function Cuts for CO_2 Reservoirs in SDDP

The SDDP algorithm uses the property of the future cost function $z_t(\cdot,\cdot,\cdot)$ to be convex, jointly in its state variables v_t, e_t and a_{t-1}. The main reasons are that the inflow model used is linear in the previous inflows and all three state variables appear as RHS in (5–12) - (5–17). Following the principles of Chapter 4, evaluating this function z_t at a specific point v_t^n, e_t^n and a_{t-1}^n leads to a function value $z_t(v_t^n, e_t^n, a_{t-1}^m) \in \mathbb{R}$. If we know also the slopes γ_{tmn}^{v}, γ_{tmn}^{e} and γ_{tmn}^{a} of z_t at this point v_t^n, e_t^n and a_{t-1}^n, then we can extrapolate the function z_t – just like in the classical SDDP. In other words, we can underestimate the function z_t via the (linear) slopes of the points v_t^n, e_t^n and a_{t-1}^m. Hence, we obtain the following linear program, defining a lower bound

$$\underline{z}_t(v_t, e_t, a_{t-1}) = \min \alpha \tag{5–32}$$

$$\text{s.t. } \alpha \geq \gamma^{\nu}_{tmn} v_t + \gamma^{e}_{tmn} e_t + \gamma^{a}_{tmn} a_{t-1} + \gamma^{c}_{tmn}, \qquad n \in \mathbb{N},\ m \in \mathbb{M} \tag{5–33}$$

on the "true" function $z_t(\cdot,\cdot,\cdot)$, where $n \in \mathbb{N}$ denotes the n-th linear segment of the convex underestimation and γ^{c}_{tmn} is the corresponding constant term.

The slopes γ^{ν}_{tmn} and γ^{a}_{tmn} are basically determined through relations (3–33) and (3–34). Let us denote by $\tilde{\eta}^{lmn}_t$ the dual multipliers of constraints (5–20), then we obtain

$$\gamma^{\nu}_{tmn} = \sum_{l=1}^{L} p^l \tilde{\eta}^{lmn}_t \quad \text{and} \quad \gamma^{a}_{tmn} = \sum_{l=1}^{L} p^l \varphi_t \tilde{\eta}^{lmn}_t, \tag{5–34}$$

for all $t \in \mathbb{T}_1,\ m \in \mathbb{M}$, and $n \in \mathbb{N}$.

Similarly, γ^{e}_{tmn} can be derived for $t \in \mathbb{T}_1,\ m \in \mathbb{M}$, and $n \in \mathbb{N}$ through

$$\gamma^{e}_{tmn} = \left.\frac{\partial z_t(\cdot, e_t, \cdot)}{\partial e_t}\right|_{e_t = e^n_t} = \left.\frac{\partial \left(\sum_{l\in\mathbb{L}} p^l z^l_t(\cdot, e_t, \cdot)\right)}{\partial e_t}\right|_{e_t = e^n_t} = \sum_{l\in\mathbb{L}} p^l \left.\frac{\partial z^l_t(\cdot, e_t, \cdot)}{\partial e_t}\right|_{e_t = e^n_t} \tag{5–35}$$

with probability p^l of inflow scenario l conditioned on the previous inflows. Now, let π^{lmn}_t be the dual multipliers of the constraint (5–21) for $t \in \mathbb{T} \setminus \mathbb{Y}_g$ and the dual multipliers of constraint (5–22) for $t \in \mathbb{Y}_g$, respectively, for given emission reservoir level e^n_t. Then, we obtain

$$\gamma^{e}_{tmn} = \sum_{l\in\mathbb{L}} p^l \pi^{lmn}_t, \qquad t \in \mathbb{T} \setminus \mathbb{Y}_g,\ m \in \mathbb{M}\ n \in \mathbb{N} \tag{5–36}$$

and for $y \in \mathbb{Y}_g$ and $n \in \mathbb{N}$

$$\gamma^{e}_{ymn} = \begin{cases} 0, & \text{if the emissions expire; } \textit{i.e.}, y \in \mathbb{Y}_e \\ \sum_{l\in\mathbb{L}} p^l \pi^{lmn}_y, & \text{if the emissions do not expire} \end{cases}. \tag{5–37}$$

The constant term γ^c_{tmn} is then calculated from

$$\gamma^c_{tmn} = \gamma^\nu_{mtn} v^n_t + \gamma^e_{tmn} e^n_t + \gamma^a_{tmn} a^m_{t-1} - z_t(v^n_t, e^n_t, a^m_{t-1}) \qquad t \in \mathbb{T}_1,\ m \in \mathbb{M},\ n \in \mathbb{N}. \tag{5–38}$$

Recall that the future cost function for the last stage is zero. This can be achieved by defining

$$\gamma^\nu_{tmn} \equiv \gamma^e_{tmn} \equiv \gamma^a_{tmn} \equiv \gamma^c_{tmn} \equiv 0. \tag{5–39}$$

This piecewise linear approximation replaces the convex future cost functions in (5–18), yielding to the LP problems

$$z^{lmn}_t(v^n_t, e^n_t, a^m_{t-1}) := \min \sum_{j \in \mathbb{J}} c_{tj} \mathbf{g}_{tj} + \Upsilon \delta_t + C^{\mathrm{CO_2}} \mathbf{f}_t + \alpha_{t+1} \tag{5–40}$$

$$\text{s.t. } \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{tj} + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{ti} + \delta_t = d_t \tag{5–41}$$

$$\mathbf{v}_{t+1i} = v^n_{ti} - \mathbf{u}_{ti} - \mathbf{s}_t + \sum\nolimits_{h \in \mathbb{U}_i} \left(\mathbf{u}_{th} + \mathbf{s}_{th}\right) + a^l_{ti}, \qquad i \in \mathbb{I} \tag{5–42}$$

$$\mathbf{e}_{t+1} = e^n_t - \sum\nolimits_{j \in \mathbb{J}} B_j \mathbf{g}_{tj} + \mathbf{f}_t, \qquad \text{if } t \in \mathbb{T} \setminus \mathbb{Y}_g \tag{5–43}$$

$$\mathbf{e}_{t+1} = \tilde{e}^n_t - \sum\nolimits_{j \in \mathbb{J}} B_j \mathbf{g}_{tj} + \mathbf{f}_t + E^{\mathrm{CO_2}}_t, \qquad \text{if } t \in \mathbb{Y}_g \tag{5–44}$$

$$\alpha_{t+1} \geq \gamma^\nu_{t+1\tilde{m}\tilde{n}} \mathbf{v}_{t+1} + \gamma^e_{t+1\tilde{m}\tilde{n}} \mathbf{e}_{t+1} + \gamma^a_{t+1\tilde{m}\tilde{n}} a^l_t + \gamma^c_{t+1\tilde{m}\tilde{n}}, \tag{5–45}$$

$$\tilde{n} \in \mathbb{N},\ \tilde{m} \in \mathbb{M} \tag{5–46}$$

$$\begin{aligned} &\underline{g}_{tj} \leq \mathbf{g}_{tj} \leq \bar{g}_{tj}, \qquad \underline{u}_{ti} \leq \mathbf{u}_{ti} \leq \bar{u}_{ti}, \\ &\underline{v}_{t+1i} \leq \mathbf{v}_{t+1i} \leq \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \leq \mathbf{s}_{ti} \leq \bar{s}_{ti}, \\ &\mathbf{e}_{t+1} \geq 0, \qquad \mathbf{f}_t \geq 0, \qquad \delta_t \geq 0, \qquad j \in \mathbb{J},\ i \in \mathbb{I}, \end{aligned} \tag{5–47}$$

for $t \in \mathbb{T}_1$, $l \in \mathbb{L}$, $m \in \mathbb{M}$ and $n \in \mathbb{N}$. We are now able to derive the SDDP algorithm with CO_2 reservoir.

5.2.3 SDDP with CO_2 Reservoir

As for the classical SDDP algorithm, the one-stage dispatch problems for the first stage do not carry the backward inflow index $l \in \mathbb{L}$, reducing to

$$z_1^m(v_1, e_1) := \min \sum_{j \in \mathbb{J}} c_{1j} \mathbf{g}_{1j} + \Upsilon \delta_t + C^{\mathrm{CO_2}} \mathbf{f}_1 + \alpha_2 \tag{5–48}$$

$$\text{s.t. } \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{1j} + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{1i} + \delta_1 = d_1 \tag{5–49}$$

$$\mathbf{v}_{2i} = v_{1i}^n - \mathbf{u}_{1i} - \mathbf{s}_1 + \sum\nolimits_{h \in \mathbb{U}_i} \left(\mathbf{u}_{1h} + \mathbf{s}_{1h}\right) + a_{1i}^m, \qquad i \in \mathbb{I} \tag{5–50}$$

$$\mathbf{e}_2 = e_1^n - \sum\nolimits_{j \in \mathbb{J}} B_j \mathbf{g}_{1j} + \mathbf{f}_1, \qquad \text{if } t = 1 \in \mathbb{T} \setminus \mathbb{Y}_g \tag{5–51}$$

$$\mathbf{e}_2 = \widetilde{e}_1^n - \sum\nolimits_{j \in \mathbb{J}} B_j \mathbf{g}_{1j} + \mathbf{f}_1 + E_1^{\mathrm{CO_2}}, \qquad \text{if } t = 1 \in \mathbb{Y}_g \tag{5–52}$$

$$\alpha_2 \geq \gamma_{2\tilde{m}\tilde{n}}^{\nu} \mathbf{v}_2 + \gamma_{2\tilde{m}\tilde{n}}^{e} \mathbf{e}_2 + \gamma_{2\tilde{m}\tilde{n}}^{a} a_1^m + \gamma_{2\tilde{m}\tilde{n}}^{c}, \qquad \tilde{n} \in \mathbb{N},\ \tilde{m} \in \mathbb{M} \tag{5–53}$$

$$\underline{g}_{1j} \leq \mathbf{g}_{1j} \leq \bar{g}_{1j}, \qquad \underline{u}_{1i} \leq \mathbf{u}_{1i} \leq \bar{u}_{1i},$$

$$\underline{v}_{2i} \leq \mathbf{v}_{2i} \leq \bar{v}_{2i}, \qquad \underline{s}_{1i} \leq \mathbf{s}_{1i} \leq \bar{s}_{1i},$$

$$\mathbf{e}_2 \geq 0, \qquad \mathbf{f}_1 \geq 0, \qquad \delta_1 \geq 0, \qquad j \in \mathbb{J},\ i \in \mathbb{I}, \tag{5–54}$$

The solution of these M one-stage dispatch problems is an (approximate) lower bound on z by taking the expected value

$$\underline{z} \equiv z_1(v_1, e_1) \approx \sum_{m \in \mathbb{M}} \frac{1}{M} z_1^m(v_1, e_1). \tag{5–55}$$

The forward Monte Carlo simulation uses then the derived future cost function cuts to solve the following LP problems

$$z_t^m(v_t^m, e_t^m) := \min \sum_{j \in \mathbb{J}} c_{tj} \mathbf{g}_{tj}^m + \Upsilon \delta_t^m + C^{\mathrm{CO_2}} \mathbf{f}_t^m + \alpha_{t+1} \tag{5–56}$$

$$\text{s.t.} \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{tj}^m + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{ti}^m + \delta_t^m = d_t \tag{5–57}$$

$$\mathbf{v}_{t+1i}^m = v_{ti}^m - \mathbf{u}_{ti}^m - \mathbf{s}_t^m + \sum\nolimits_{h \in \mathbb{U}_i} \left(\mathbf{u}_{th}^m + \mathbf{s}_{th}^m \right) + a_{ti}^m, \qquad i \in \mathbb{I} \tag{5–58}$$

$$\mathbf{e}_{t+1}^m = e_t^m - \sum\nolimits_{j \in \mathbb{J}} B_j \mathbf{g}_{tj}^m + \mathbf{f}_t^m, \qquad \text{if } t \in \mathbb{T} \setminus \mathbb{Y}_g \tag{5–59}$$

$$\mathbf{e}_{t+1}^m = \widetilde{e}_t^m - \sum\nolimits_{j \in \mathbb{J}} B_j \mathbf{g}_{tj}^m + \mathbf{f}_t^m + E_t^{\mathrm{CO_2}}, \qquad \text{if } t \in \mathbb{Y}_g \tag{5–60}$$

$$\alpha_{t+1} \geq \gamma_{t+1\tilde{m}\tilde{n}}^{\nu} \mathbf{v}_{t+1}^m + \gamma_{t+1\tilde{m}\tilde{n}}^{e} \mathbf{e}_{t+1}^m + \gamma_{t+1\tilde{m}\tilde{n}}^{a} a_t^m + \gamma_{t+1\tilde{m}\tilde{n}}^{c}, \tag{5–61}$$

$$\tilde{n} \in \mathbb{N}, \ \tilde{m} \in \mathbb{M} \tag{5–62}$$

$$\underline{g}_{tj} \leq \mathbf{g}_{tj}^m \leq \bar{g}_{tj}, \qquad \underline{u}_{ti} \leq \mathbf{u}_{ti}^m \leq \bar{u}_{ti},$$

$$\underline{v}_{t+1i} \leq \mathbf{v}_{t+1i}^m \leq \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \leq \mathbf{s}_{ti}^m \leq \bar{s}_{ti},$$

$$\mathbf{e}_{t+1}^m \geq 0, \qquad \mathbf{f}_t^m \geq 0, \qquad \delta_t^m \geq 0, \qquad i \in \mathbb{I}, \ j \in \mathbb{J}, \tag{5–63}$$

with

$$v_1^m := v_1 \quad \text{and} \quad e_1^m := e_1, \qquad m \in \mathbb{M} \tag{5–64}$$

$$v_{t+1}^m := v_t^{m*} \quad \text{and} \quad e_{t+1}^m := e_t^{m*}, \qquad t \in \mathbb{T}_1, \ m \in \mathbb{M}, \tag{5–65}$$

in order to obtain an estimation of the minimal operation cost

$$z^m := \sum_{t \in \mathbb{T}} \left(\sum_{j \in \mathbb{J}} c_{tj} g_{tj}^{m*} + \Upsilon \delta_t^{m*} + C^{\mathrm{CO_2}} \mathbf{f}_t^{m*} \right), \tag{5–66}$$

per forward inflow scenario $m \in \mathbb{M}$. The average cost is then obtained by taking the expectation over the forward inflow scenarios as in equation (3–56). Just like in the classical SDDP algorithm, the standard deviation of those M costs are calculated via relation (3–57) and the algorithm can stop, as soon as the estimated operation cost $\hat{z}$ lie inside the confidence interval (3–58).

Algorithm 5-1. Stochastic Dual Dynamic Programming with CO_2 Reservoir

// Initialize
Generate inflow data; *cf.* Chapter 3

// Main iterations
repeat
  // Run the backwards pass to compute
  // 1.) (approx.) lower bound on the operation cost $\underline{z}$
  // 2.) (approx.) future cost function cuts
  for each stage $t = T, T-1, \dots, 2$ do
    for each forward inflow scenario $m \in \mathbb{M}$ do
      for each storage discretization $n \in \mathbb{N}$ do
        for each backwards opening inflow scenario $l \in \mathbb{L}$ do
          solve the one-stage dispatch problem (5–40) - (5–47)
        end for
        calculate the coefficients γ^{ν}_{tmn}, γ^{e}_{tmn}, γ^{a}_{tmn} and the constant term γ^{c}_{tmn} for the future cost function cut in stage t for stage $t-1$ via (5–34), (5–36) or (5–37), and (5–38)
      end for
    end for
  end for
  for each forward inflow scenario $m \in \mathbb{M}$ do
    solve the one-stage dispatch problem (5–48) - (5–54)
  end for
  calculate lower bound $\underline{z}$ via (5–55)

  // Run the forward Monte-Carlo simulation to obtain
  // 1.) estimated operation cost $\hat{z}$
  // 2.) standard deviation $\hat{\sigma}$
  // 3.) new storage (water and CO_2) discretization levels
  for each forward inflow scenario $m \in \mathbb{M}$ do
    for each stage $t \in \mathbb{T}$ do
      solve the one-stage dispatch problem (5–56) - (5–63)
      update the storage discretization levels via (5–64) - (5–65)
    end for
    calculate the encountered operation cost for inflow scenario m via (5–66)
  end for
  estimate the expected operation cost $\hat{z}$ via (3–56)
  calculate $\hat{\sigma}$ via (3–57)
until $\hat{z}$ lies in confidence interval (3–58)

A summary of the method described is presented in pseudo-code form through Algorithm 5-1. Analytically, the running time – in the number of LP problems to be solved in each main iteration – of Algorithm 5-1 remains the same as the generic SDDP Algorithm 3-3: $M(1 + N \cdot L \cdot |\mathbb{T}_1|) + M \cdot T$. However, the number of main iterations may increase, as the state variable of the CO_2 reservoir has to be discretized, too. This may also lead to an increase in the size of the discretization set $\mathbb{N}$.

5.2.4 SDDPT with CO_2 Reservoir

In Chapter 4, we embed the scenario tree modeling of uncertainty into the framework of SDDP and obtained the SDDPT algorithm. This algorithm can also be combined with the CO_2 reservoir constraints (5–8) - (5–10).

The state-space of the one-stage dispatch problems solved by the SDDPT algorithm is then increased by the CO_2 reservoir levels e_t along with the corresponding CO_2 emission constraints. Consistent with the increase in the state-space, the future function cuts – *e.g.*, in constraints (4–22) – are extended by the additional variable, combining the CO_2 reservoir constraints through the various stages. The future cost function cuts corresponding to the CO_2 reservoir depend then also on each scenario $s \in \mathbb{S}_t$. In other words, γ^e_{t+1mn} has an additional scenario index s: γ^e_{t+1mns}.

5.2.5 Multiple GHG Reservoirs

The concept of the CO_2 emission reservoirs can be applied to any other type of emissions, simply by changing the emission factors B_j. Furthermore, multiple reservoirs – one for each desired GHG emission – can be added to the model. This way, the "CO_2" reservoir level variables e_t generalize to a vector in the number of reservoirs considered.

As mentioned in the introduction of this chapter, the least-cost minimization model considered has also its justification in a deregulated environment, under the price-taker assumption. In this case, the cost minimization model can predict electricity prices dependent on the inflow scenarios via a so-called fundamental model. Those prices can then be used in profit maximization models; *cf.* Chapter 6. Typically, those profit

Table 5-1. CO_2 emission factors for different types of thermal plants. All other plants are assumed to have zero emissions

type	CO_2 factor	unit	source
coal	2.86	[ton CO_2 / ton]	[81]
diesel	22.38	[pounds CO_2 / gallon]	[47]
bunker	78.8	[kg CO_2 / MMBtu]	[47]

maximization models include only a (small) subset of power plants of the whole power system; *e.g.*, the plants being owned by one of the several power producers in a country. In order to cap the GHG emissions on a country level, GHG emission quotas might be assigned to each producer separately– as it is done in the EU ETS. Hence, in order to obtain an accurate fundamental model, the cost minimization model should be divided into different regions (*e.g.*, one for each power supplier) subject to certain quota. This can be modeled by assigning each region its own reservoir, where allowances are used up only by the corresponding power plants.

5.3 Case Study for Guatemala

For our computational studies, we use the real power system of Guatemala. The electricity power system consisted of 41 thermal plants as summarized in Table 5-2. We assume a constant fuel price throughout the planning horizon, leading to a constant production price for each plant. Guatemala had two hydro-reservoirs plants, schematically shown in Figure 5-2, as well as ten run-of-the-river plants. The run-of-the-river plants' total installed capacity was 354.9 MW.

The CO_2 emissions factors per type of plant used to calculate the CO_2 emissions per plant of the Guatemala power system are provided in Table 5-1.

The SDDP algorithm with the CO_2 emission reservoir constraints has been implemented using the modeling language Mosel (Version 3.0.0). The resulting linear programming problems from the decomposition are solved using Xpress Optimizer (Version 20.00.05). The time horizon of choice is one year with monthly stages where the first month is January and the last month considered is December. We apply

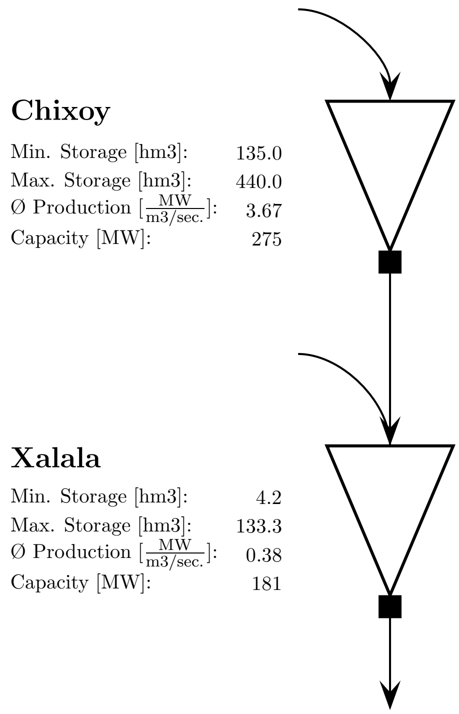


Figure 5-2. Hydro-electric reservoir system of Guatemala

an annual discount rate of 10%. The inflow uncertainty is captured within SDDP via scenarios which are generated by linear autoregressive models with correlated innovations – estimation of the parameters was carried out by fitting a lag-1 model to real inflow data of the past 38 years. We use $M = 100$ inflow scenarios for our simulations during the SDDP simulation phase while $L = 25$ inflow scenarios are used for the backward pass. Hence, the presented results are "averages" over those 100 forward inflow scenarios.

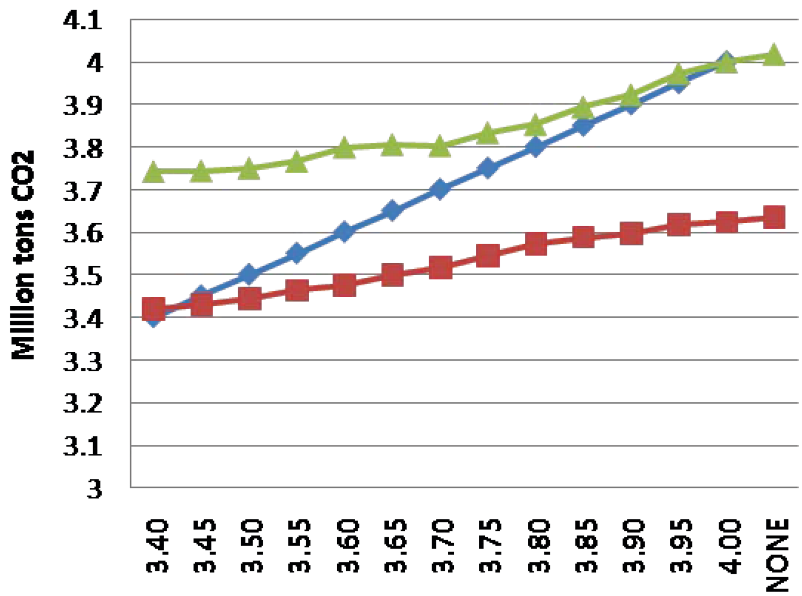
The electricity demand for the planning horizon of one year is assumed to be known and follows seasonal pattern; see Table 5-3. Both the emission fine for exceeding a given annual quota and the penalty for rationing were "arbitrarily" set, the former being fixed to $100 and the latter set to ten times the largest generation cost.

Operating the Guatemala power system without CO_2 emission quota leads to (average) annual operation cost of million $348 and (average) emissions of 3.635 million tons CO_2. We then proceed by imposing different quota levels on the system as shown in Figure 5-3. One observes in Figure 5-3A that the average CO_2 emissions decrease at a slower rate than the quota levels are decreased. Figure 5-3A also shows the CO_2 emissions of the worst case scenario occurred; *i.e.*, the case with the lowest inflows observed. Interestingly, the quota level of \$100 does not push the worst case emissions below the quota level. Hence, the probability of exceeding the quota limit is above zero for all tested quota levels less than 3.95; the values are shown in Table 5-4 in row four. Given an acceptable risk of exceeding the yearly allowance, one could easily identify the "optimal" penalty level by running the model several times.

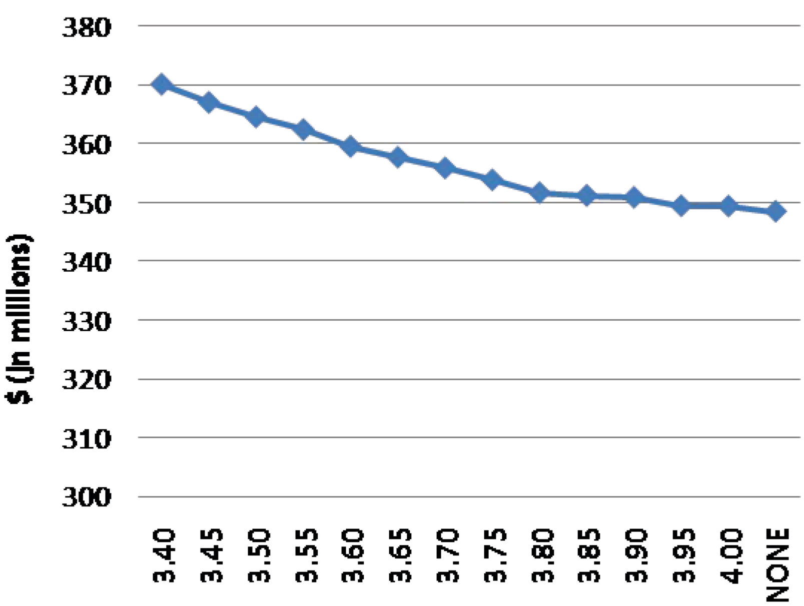
Table 5-4 shows also the average annual operation cost for different quota levels in row two, including the eventually incurred fines. Row three provides the average amount of CO_2 emissions exceeding the allocated quota. In row five and six, the risk of not meeting the electricity demand are shown, given as the maximum rationing percentage with respect to the total electricity demand among the 100 scenarios and as the number of scenarios with demand rationing, respectively; for the particular optimal operational policy calculated. Reviewing the numbers, the rationing risk is negligible.

The average annual operation costs over all 100 scenarios are shown in Figure 5-3B. The important information in this figure is given by the slope of the operational cost curve: The slope can be interpreted as the incremental/marginal cost for CO_2 reduction. When excluding CO_2 emission fines, one gains a good approximation of incremental CO_2 emission reduction cost which are for the first 17,000 tons CO_2 roughly \$52 per ton and for the last 10,000 tons over \$150 per ton. As these cost are operational cost, they are short-term CO_2 reduction cost. Again, these results could be used by policy makers as guidelines and compared to society's willingness to reduce emissions in order to reach a plausible compromise.

Figure 5-4 shows the generation mix over the span of the whole year for the different quota levels imposed. While the production of the CO_2 emission free sources



A



B

Figure 5-3. Annual CO_2 emissions and operational cost for different quota levels. A) Average and maximum CO_2 emissions. ■ CO_2 emission quota ■ average CO_2 emissions ■ maximum CO_2 emissions B) Average operational cost

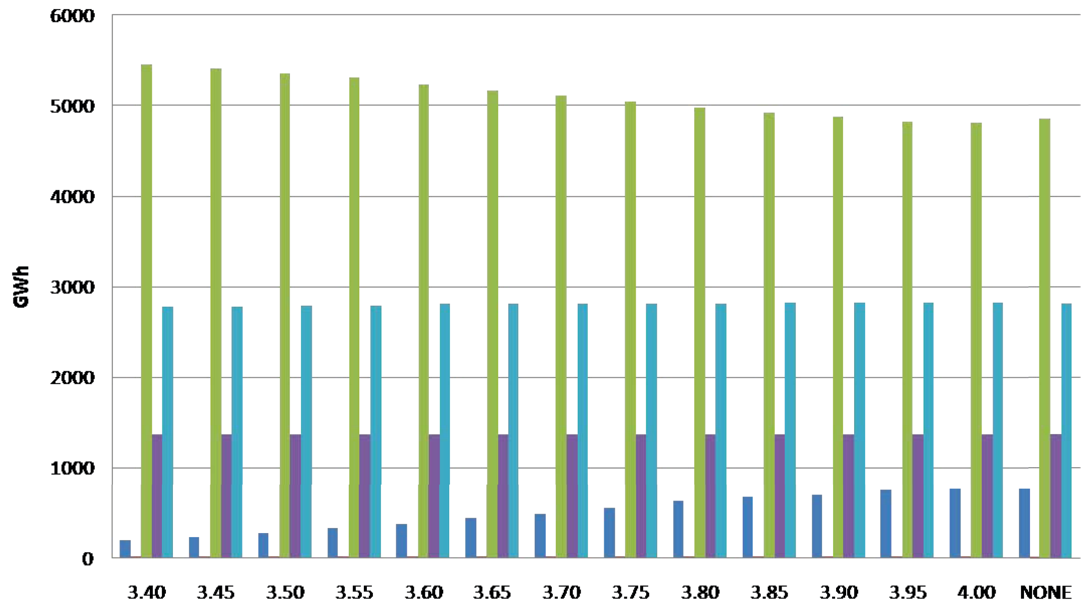
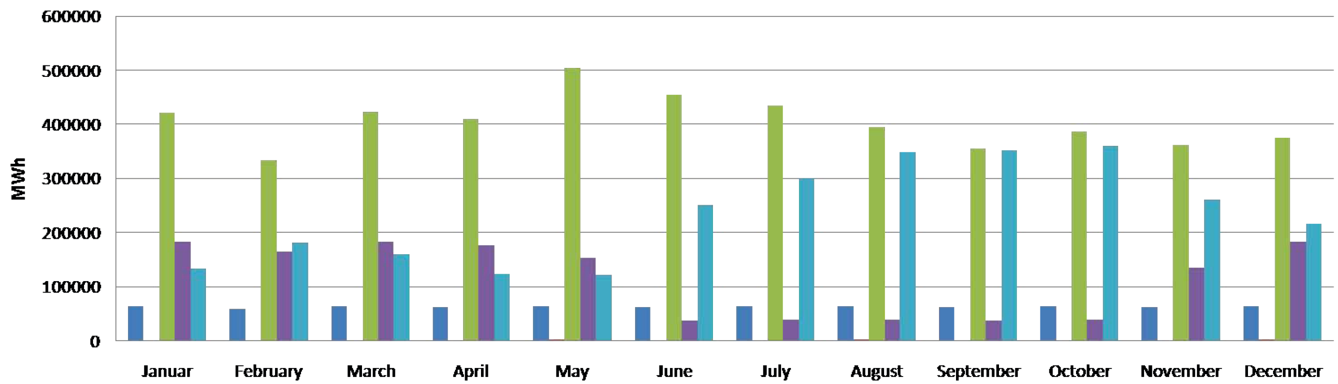


Figure 5-4. Yearly generation mix for Guatemala power system with different quota levels.
■ coal ■ diesel ■ bunker ■ geo and co-generation ■ hydro

("geo," "co-generation" and "hydro") remain basically unchanged, the dirty coal is replaced steadily by the more expensive but cleaner bunker.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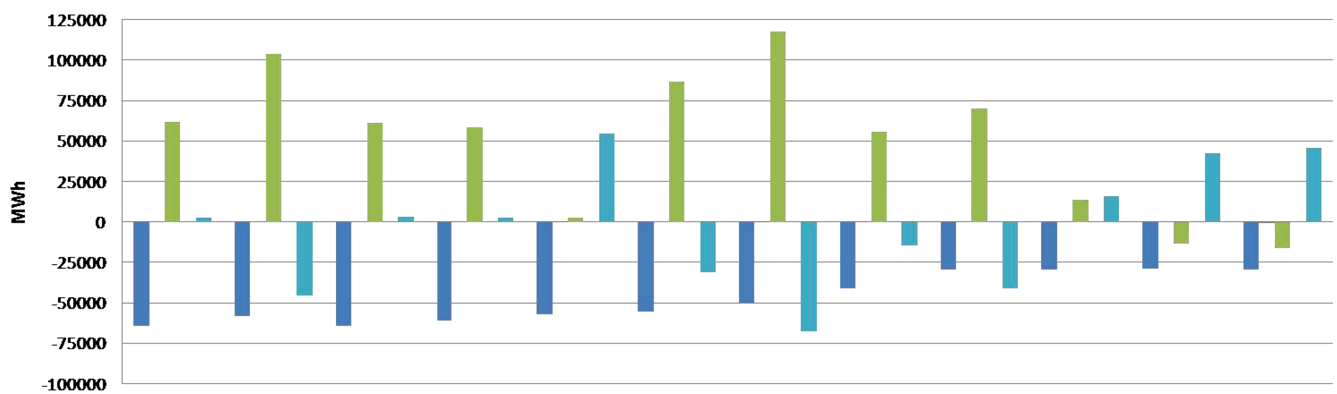
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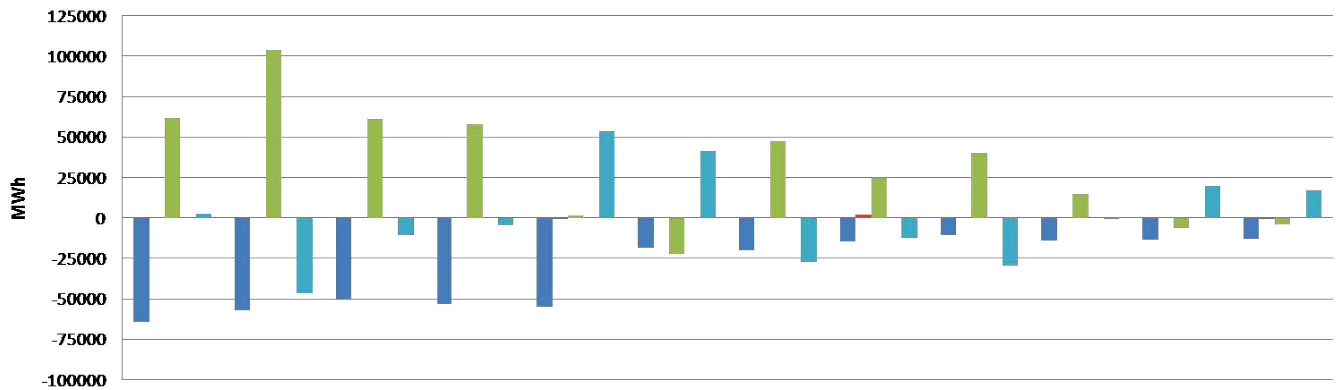

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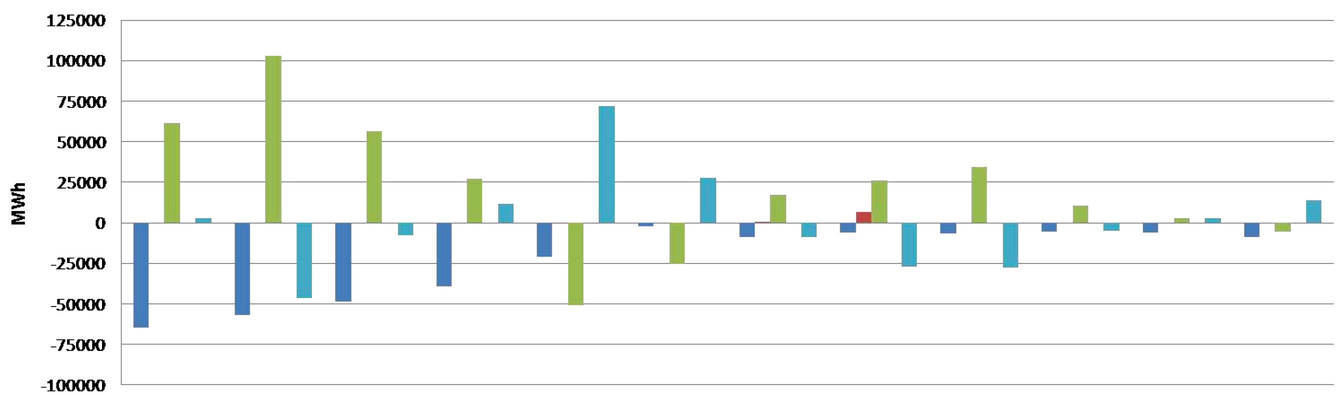

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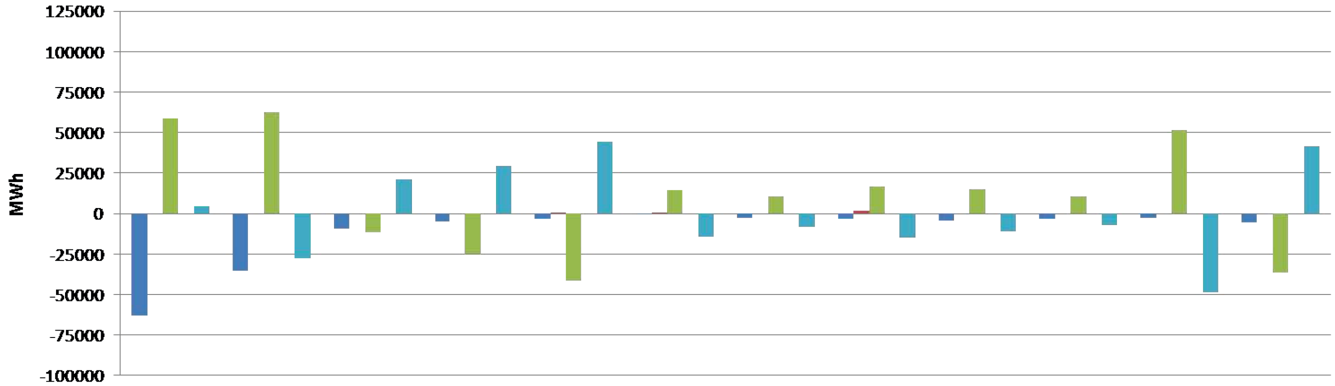
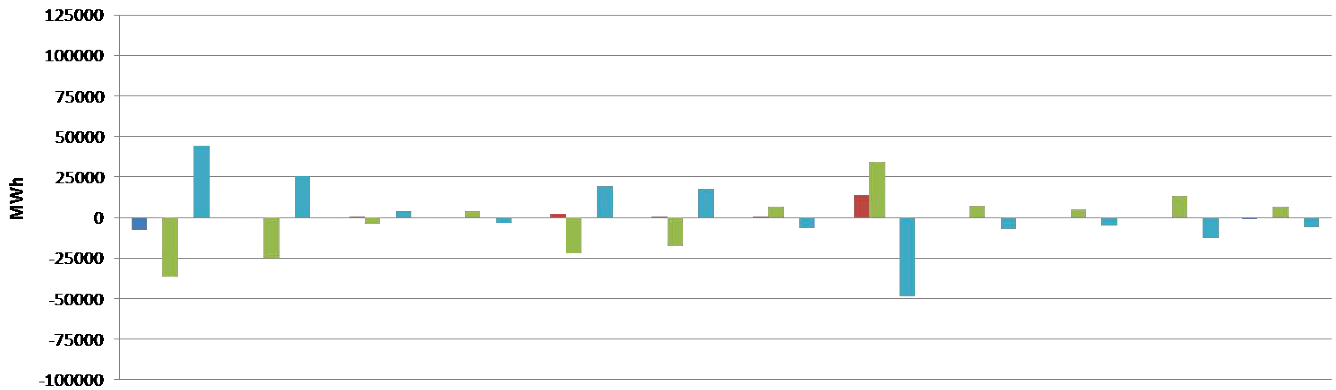
MWh

D



E



F

Figure 5-5. Monthly dispatching decisions for the quota free case, which is then used as the base case showing the monthly difference in electricity production. A) Quota: None. B) Quota: 3.4 Million tons. C) Quota: 3.6 Million tons. D) Quota: 3.7 Million tons. E) Quota: 3.8 Million tons. F) Quota: 3.95 Million tons.

■ coal ■ diesel ■ bunker ■ geo and co-generation ■ hydro

Figures 5-5A to 5-5F show the generation mix for each month over the planning horizon of one year for different quota levels. Most obvious are the changes in the electricity production with coal fired plants in the first stages when implying high quotas. It can be clearly seen that the coal generation is replaced by bunker in most of the cases. However, for the cases of relative moderate emission quota levels of 3.9, 3.95 or 4.0 million tons of CO_2, the hydro resources are operated more aggressively, leading to increases in the rationing risk; *cf.* Table 5-4. One observes an increase in the hydro generation for the first stages (and hence a decrease in the water reservoir levels) leading to this higher risk. Controversy, for lower quota levels, less water is used in the first stages but instead the capacity of the bunker plants is used to produce electricity which in later stages could replace dirtier plants, such as coal or diesel. The more the inflow uncertainty unfolds, the more coal can be used when expected that enough CO_2 emissions are available.

The marginal CO_2 emission prices and electricity prices are shown for the different quota levels and the 12 stages in Figure 5-6 and Figure 5-7, respectively. The trend of decreasing prices is explained by the expiration of the CO_2 emissions at the end of the planning horizon of one year; *i.e.*, the CO_2 emission rights have no future value above the planning horizon. The stochastic water inflow drives this trend further. While at the beginning, one might have to be very conservative with respect to CO_2 emissions for most (or even all) of the inflow scenarios, while at the end of the horizon, the emissions quota might only affect a few of the scenarios. Possible end effects of the hydro-thermal generation plan where the water has less future value than at the beginning are very minor for this data set and do not alter the results.

Stage 6, 7 and 8 (June, July and August) are the crucial stages for this data set. Out of the 100 inflow scenarios, several droughts are occurring during these stages; leading to a shortage in water, increasing the thermal production up to their capacity limits and with that, increasing the CO_2 emissions and the electricity production cost. Once these

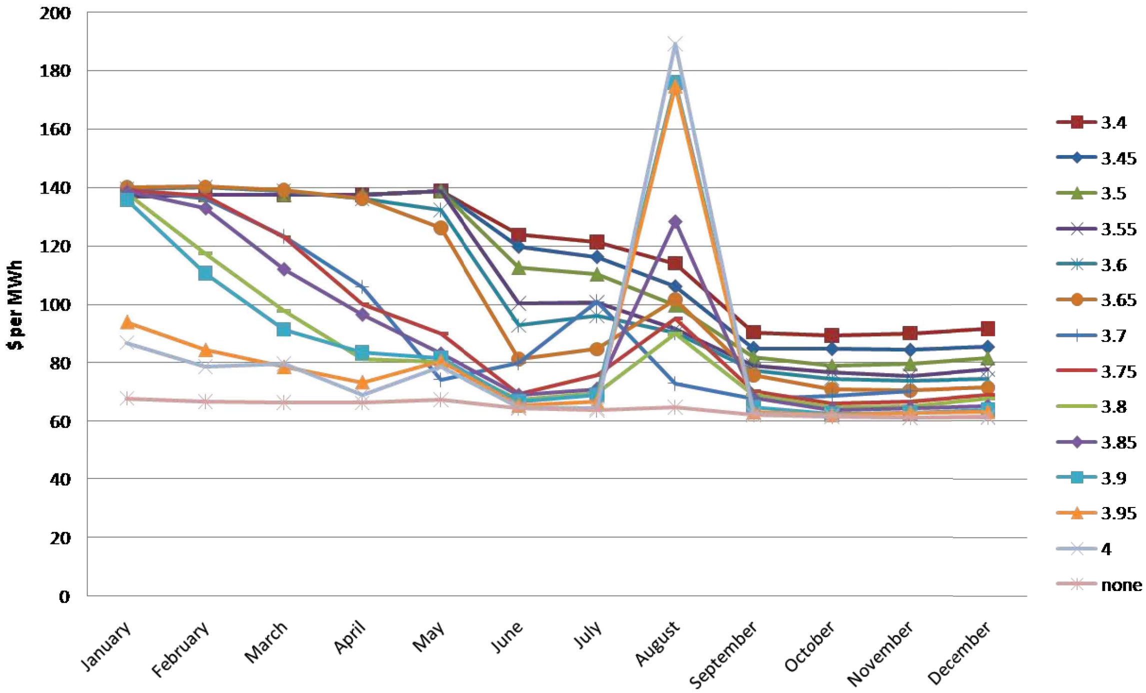


Figure 5-6. Average electricity marginal(=spot) prices for different quota levels

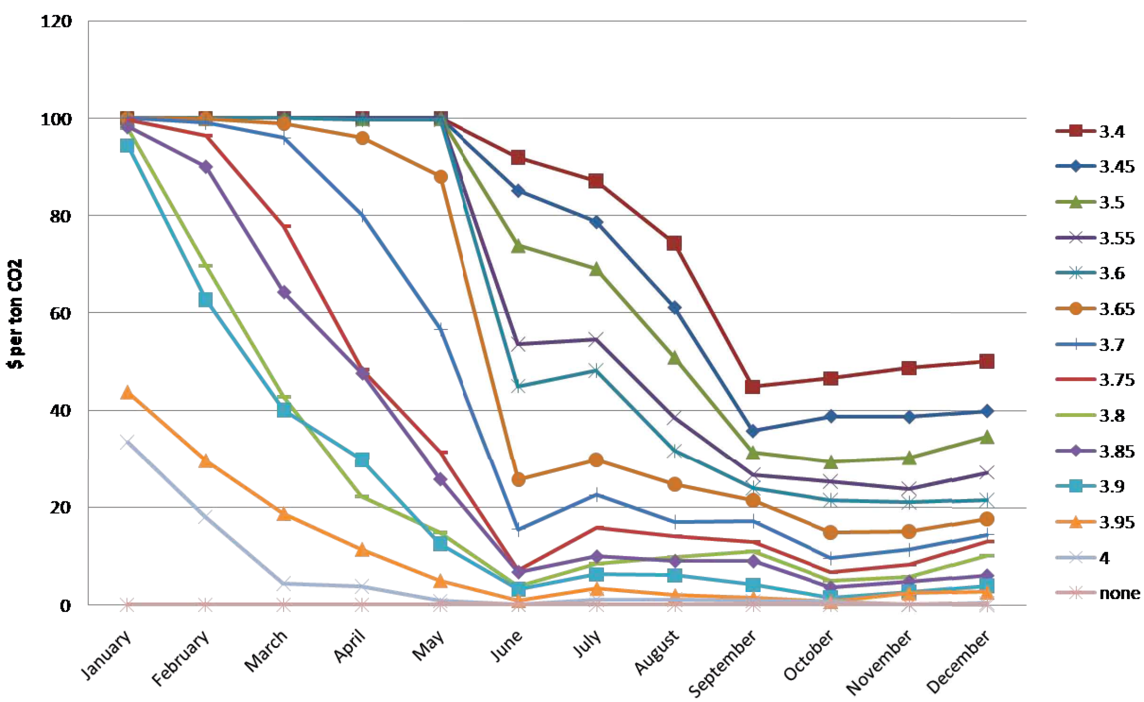


Figure 5-7. Average CO_2 emission allowance marginal(=spot) prices for different quota levels

stages are past, the prices, both CO_2 allowance and electricity, drop. The generally relative high CO_2 emission prices are explained by the energy system of Guatemala. To save one ton of CO_2 emissions generated by coal plants, one needs to replace roughly 2.2 MWh productions of coal fired plants by bunker fuels, which leads in the cheapest case to an increase of $44.6 for the electricity production.

The large electricity price spikes in stage 8 (August) for the moderate quota levels is explained by the few scenarios where a rationing occurs, letting the marginal electricity production cost sky rock.

5.4 Discussion

In this chapter, we discussed a CO_2 constrained hydro-thermal scheduling problem in the mid-term horizon. In order to allow the solution via dynamic programming methods such as stochastic dynamic programming or stochastic dual dynamic programming, a reservoir model for the CO_2 emissions has been proposed, respecting the stage decomposition framework of these methods. This reservoir model allows for CO_2 allowances to expire and a detailed modeling of the power systems characteristics.

Computational results on the Guatemala power system demonstrate the feasibility of the approach. The slope of the annual expected operational cost for different quota levels give the marginal operational CO_2 emission reduction price. This provides useful insights for policy makers to establish CO_2 emission quota levels acceptable for the society while considering their short-term affects. Furthermore, the influence of different CO_2 emission quotas on the electricity price are apparent.

The presented CO_2 reservoir model can be applied to optimal expansion planning problems for hydro-dominated power systems. This way, the marginal CO_2 emission price for jointly the investment planning and operation scheduling can be obtained.

Table 5-2. Thermal plants considered for the Guatemala power system

Number of Plants	1	3	1	18	3	1	1	3	10
Cumulative Capacity [MW]	24.0	120.4	41.4	729.8	91.5	132.4	13.0	58.0	227.0
Fuel Type	1	1	2	2	2	3	3	4	5
Cost [$/MWh]	129.9	132.0	61.6	67.1	68.7	41.2	45.9	2.7	1.0
CO_2 Emission [kg/MWh]	625.0	635.2	544.1	593.5	607.3	1001.0	1115.4	0	0
Fuel Type 1: Diesel, 2: Bunker, 3: Coal, 4: GEO, 5: Co-generation									

Table 5-3. Monthly electricity demand for the Guatemala power system

	January	February	March	April	May	June	July	August	September	October	November	December
GWh	801	738	828	771	844	805	840	847	806	850	820	838

Table 5-4. Computational results for different quota levels on the Guatemala power system

Quota [million tons]	3.40	3.45	3.50	3.55	3.60	3.65	3.70	3.75	3.80	3.85	3.90	3.95	4.00	NONE
(Average) Cost [million $]	370.1	367.0	364.5	362.4	359.5	357.6	355.8	353.8	351.6	351.1	350.8	349.3	349.3	348.4
CO_2 Emissions Above Quota [tons]	4914	3426	2425	1569	1109	634	427	274	162	57	23	24	0	-
CO_2 Emissions Above Quota [%]	46	36	28	25	19	18	14	10	10	4	1	1	0	-
Max. Demand Rationing [MWh %]	0	0	0	0	0	0.02	0.02	0.02	0.01	0.21	0.38	0.38	0.38	0
Demand Rationing [%]	0	0	0	0	0	1	1	1	1	3	6	6	7	0

CHAPTER 6
PROFIT MAXIMIZATION IN DEREGULATED ELECTRICITY MARKETS AND CO_2 EMISSION MARKETS

A power-producer in the liberalized (or deregulated) market faces new challenges. In the regulated market, the power producer's goal is to satisfy their customers' electricity demands while minimizing the expected cost of operating their assets; *e.g.*, optimizing the dispatch of the own power plants, while using predefined contracts with other power producers as well as trading with financial tools (*e.g.*, forwards) in the electricity markets. In the deregulated environment, the power producer has no longer to meet the electricity demand of their customers, but instead, trade the electricity in the market. However, this complicates matters practically and mathematically in two ways:

1. uncertainty of spot prices, and
2. risk appetite[1]

have to be included in the optimization models.

The presence of a CO_2 market, trading CO_2 emission allowances, makes the profit maximization models even more involved. CO_2 emission allowance prices are determined via market mechanisms, adding an additional source of uncertainty into the models: CO_2 emission allowance prices.

Thus, the optimization model of this chapter differs from the models discussed in the previous chapters in the following three main aspects:

- net profit maximization (instead of cost minimization),
- presence of electricity sport prices which are uncertain, and
- presence of CO_2 emission allowance markets with uncertain price (instead of system cap).

[1] Risk control is not included in this chapter. However, the methods readily available in the literature can be incorporated in out model in a straight forward manner.

Hence, the objective of optimization is the maximization of the expected net profit – revenues minus operational costs – of a power sub-system[2] in the mid-term horizon. The resulting multi-stage stochastic programming problem contains jointly the following uncertainties

- hydro inflows,
- fuel prices,
- electricity spot price, and
- CO_2 emission allowance price.

The hydro inflow uncertainty is modeled as a continuous Markov process in the form of a linear autoregressive model; *cf.* Chapter 3. Contrary, all other three uncertainties are modeled is a discrete way. The fuel price and electricity demand uncertainties are captured by a multivariate scenario tree, thus, employing the methodology of Chapter 4. A Markov Chain approach is used towards the electricity spot price and CO_2 emission allowance price uncertainty, defining price clusters with transition probabilities.

In order for the proposed model to be practically relevant, we have to make the following two assumptions:

1. The considered power sub-system as well as the whole power system have a significant share of installed hydro-electric capacity; *i.e.*, the systems are hydro-dominated.
2. All players corresponding to the sub-systems in the electricity market are price takers.

As we consider stochastic hydro inflows in the model, the first assumption is apparent. The price clusters for the electricity spot prices as well as the CO_2 emission allowance prices are modeled being static in the sense that they cannot be influenced by the

[2] We refer to an entire power system – *e.g.*, a whole country – as the power "system" whereas a "sub-system" is a part of this power system.

decisions of the power producer. This partly explains the second assumption. In addition, the second assumption has to do with the way we are "forecasting" the electricity prices and CO_2 emission allowance prices; *i.e.*, the derivation of the price clusters and their transition probabilities. Let us discuss that in more details now.

If market power is present and the power producer is a price-maker, then game-theoretic aspects come into play. In this case, the profits of the agent do no longer solely depend on the least-cost operation of the system and the corresponding bid in the market, but also on the bidding strategies of the other players. This adds a strategic component to the optimization model. Nash equilibrium models are commonly used in this context. Several different solution techniques have been proposed in the literature; *e.g.*, iterative methods by Otero-Novas et al. [118] and Otero-Novas et al. [119], complementarity-based models by Bushnell [25] and Hobbs and Helman [79], as well as SDP techniques by Kelman et al. [95] and Scott and Read [155].

In presence of market power, the least-cost solution (centrally dispatch model) is expected to differ from the Cournot-Nash equilibrium which more accurately model the reality. This was empirically shown for the Brazilian power system by Barroso et al. [7].

In contrast, in the case of perfect competition – *i.e.*, absence of market power – the centrally dispatched solution is the same as the solution of the market-based dispatch as empirically shown by Lino et al. [105]. Hence, as a price taker, the optimal bidding strategy is given by the marginal system cost, which can be derived through a cost minimization model as argued by Gross and Finlay [69].

This leads to the concept of fundamental modeling. Through a least-cost model of the whole system, marginal electricity prices as well as marginal CO_2 emission allowance prices can be derived. These prices are given as the dual multipliers from an optimal solution policy of the corresponding electricity demand constraints as well as CO_2 emission reservoir constraints. Given the absence of market power in the system, these prices can then be used as price forecasts in the sub-system's model.

But why bother? The difficulty is to obtain meaningful electricity price and CO_2 emission price forecasts. This problem is complex for several reasons. First, there are only very few historical data for electricity prices and, even less, for CO_2 emission allowance prices. Second, the prices depend on the state of the system; *i.e.*, if the reservoirs in a hydro-dominated power system are full, then both the electricity prices as well as the CO_2 emission allowance prices are expected to be lower than when the reservoirs are empty. Third, the electricity prices and CO_2 emission allowance prices are correlated to the hydro inflows, the fuel prices and the electricity demand. All these challenges are overcome using a fundamental model.

The closest work to our approach has been performed by Belsnes et al. [13]. The authors solved a hydro-thermal profit maximization problem in the mid-term to long-term horizon in the presence of CO_2 emission allowance markets. Similar to the approach proposed in this chapter, the prices are also forecasted using fundamental modeling. In this chapter, we consider additional uncertainties via the form of scenario trees and allow a detailed modeling of the CO_2 emission constraints; *cf.* Chapter 5.

In contrast to a fundamental model, Rong and Lahdelma [148] use a scenario tree approach to model jointly the uncertainties of heat demand, electricity spot prices and CO_2 emission allowance prices. Their multi-stage stochastic optimization model focuses on the CO_2 emissions trading of a combined heat and power producer. No hydro-plants are considered in their paper. Nevertheless, this paper is relevant to our work as the optimization of the operation and the emission trading are considered jointly in one model; *cf.* Section 6.1.

Benz and Trueck [17] consider different stochastic models to capture and predict the spot price dynamics of CO_2 emission allowances in the short-term. Short-term models have to be much more detailed than the models required for our purposes, as we assume "average" prices for each stage; *i.e.*, average monthly prices. Benz and Trueck suggest Markov switching and AR-GARCH models to capture the characteristics of the logreturns such as skewness, excess kurtosis and different phases of volatility behavior. For further literature of CO_2 emission allowance spot price forecast and market behavior, we refer to the literature in [17].

The main contribution of this chapter is the joint modeling of three different types of uncertainties in a single profit maximization problem while capturing their correlation. Those three types of uncertainties are captured by (linear) time series models (*e.g.*, hydro inflows), scenario trees (*e.g.*, fuel prices and electricity demand) and Markov Chains (*e.g.*, electricity spot prices and CO_2 emission allowance prices). The estimation of the market based data are derived via a fundamental model. Further, the resulting profit maximization model is solved using a hybrid SDP / SDDPT method. Parts of this chapter have been published in [140].

The remainder of this chapter is organized as follows. The profit maximization problem is formulated as a multi-stage stochastic programming problem in Section 6.1. A solution technique to solve the stochastic model is derived in Section 6.1.1. We conclude with a discussion in Section 6.2.

6.1 Multi-Stage Stochastic Programming Formulation

As discussed above, we are considering an electric power producer (*i.e.*, agent) in a deregulated electricity market with the presence of a CO_2 emission allowance market. Further, the agent's power portfolio is hydro-dominated and all players in the market are price takers. Then, the objective of the agent is the maximization of the revenues through optimally dispatch of the power plants and trading in the electricity spot market as well as in the CO_2 emission allowance market while meeting some given electricity demand quantity contracts. The computational challenge of this problem is given through the large variety of uncertainties which have to be considered: hydro inflows, fuel prices, electricity prices, and CO_2 emission allowance prices.

Let us denote by $\psi_t \in \Psi_t$ the set of all possible random outcomes at stage $t \in \mathbb{T}_1$, conditioned on the occurred outcomes of the previous stages. Each random outcome carries the information of the stochastic components hydro inflows, fuel prices, electricity prices, and CO_2 emission allowance prices. The profit maximization problem can be formulated as the following multi-stage stochastic (linear) program

$$
\begin{aligned}
z^{\mathrm{P}} := \max \Bigg[& P_1^{\mathrm{ES}} \mathbf{e}_1^{\mathrm{ES}} - \sum_{j \in \mathbb{J}} c_{1j} \mathbf{g}_{1j} + P_1^{\mathrm{CS}} \mathbf{e}_1^{\mathrm{CS}} - P_1^{\mathrm{CB}} \mathbf{e}_1^{\mathrm{CB}} + \\
& + \max \mathbb{E}_{\psi_2 \in \Psi_2} \Bigg[P_2^{\mathrm{ES}}(\psi_2) \mathbf{e}_2^{\mathrm{ES}}(\psi_2) - \sum_{j \in \mathbb{J}} c_{2j}(\psi_2) \mathbf{g}_{2j}(\psi_2) + \\
& \qquad + P_2^{\mathrm{CS}}(\psi_2) \mathbf{e}_2^{\mathrm{CS}}(\psi_2) - P_2^{\mathrm{CB}}(\psi_2) \mathbf{e}_2^{\mathrm{CB}}(\psi_2) + \ldots + \\
& + \max \mathbb{E}_{\psi_t \in \Psi_t} \Bigg[P_t^{\mathrm{ES}}(\psi_t) \mathbf{e}_t^{\mathrm{ES}}(\psi_t) - \sum_{j \in \mathbb{J}} c_{tj}(\psi_t) \mathbf{g}_{tj}(\psi_t) + \\
& \qquad + P_t^{\mathrm{CS}}(\psi_t) \mathbf{e}_t^{\mathrm{CS}}(\psi_t) - P_t^{\mathrm{CB}}(\psi_t) \mathbf{e}_t^{\mathrm{CB}}(\psi_t) + \ldots + \\
& + \max \mathbb{E}_{\psi_T \in \Psi_T} \Bigg[P_T^{\mathrm{ES}}(\psi_T) \mathbf{e}_T^{\mathrm{ES}}(\psi_T) - \sum_{j \in \mathbb{J}} c_{Tj}(\psi_T) \mathbf{g}_{Tj}(\psi_T) + \\
& \qquad + P_T^{\mathrm{CS}}(\psi_T) \mathbf{e}_T^{\mathrm{CS}}(\psi_T) - P_T^{\mathrm{CB}}(\psi_T) \mathbf{e}_T^{\mathrm{CB}}(\psi_T) \Bigg] \ldots \Bigg]
\end{aligned} \tag{6–1}
$$

$$\text{s.t.} \sum_{j\in\mathbb{J}} \mathbf{g}_{1j} + \sum_{i\in\mathbb{I}} \rho_i \mathbf{u}_{1i} = d_1^C + \mathbf{e}_1^{\mathrm{ES}} \tag{6–2}$$

$$\sum_{j\in\mathbb{J}} \mathbf{g}_{tj}(\psi_t) + \sum_{i\in\mathbb{I}} \rho_i \mathbf{u}_{ti}(\psi_t) = d_t^C + \mathbf{e}_t^{\mathrm{ES}}(\psi_t), \qquad t \in \mathbb{T}_1 \tag{6–3}$$

$$\mathbf{v}_{2i} = v_{1i} - \mathbf{u}_{1i} - \mathbf{s}_{1i} + \sum_{h\in\mathbb{U}_i} \left(\mathbf{u}_{1h} + \mathbf{s}_{1h}\right) + a_{1i}, \qquad i \in \mathbb{I} \tag{6–4}$$

$$\begin{aligned} \mathbf{v}_{t+1i}(\psi_t) &= \mathbf{v}_{ti}(\psi_{t-1}) - \mathbf{u}_{ti}(\psi_t) - \mathbf{s}_t(\psi_t) + \\ &\quad + \sum_{h\in\mathbb{U}_i} \left(\mathbf{u}_{th}(\psi_t) + \mathbf{s}_{th}(\psi_t)\right) + a_{ti}(\psi_t), \qquad t \in \mathbb{T}_1,\ i \in \mathbb{I} \end{aligned} \tag{6–5}$$

$$\sum_{t|y} \left(\sum_{j\in\mathbb{J}} B_j \mathbf{g}_{tj}\left(\psi_t\right) - \mathbf{e}_t^{\mathrm{CB}}(\psi_t) + \mathbf{e}_t^{\mathrm{CS}}(\psi_t)\right) \le E_y^{\mathrm{CO}_2}, \qquad y \in \mathbb{Y}_g \tag{6–6}$$

$$\begin{aligned} &\underline{g}_{1j} \le \mathbf{g}_{1j} \le \bar{g}_{tj}, \qquad \underline{g}_{tj} \le \mathbf{g}_{tj}(\psi_t) \le \bar{g}_{tj}, \\ &\underline{u}_{1i} \le \mathbf{u}_{1i} \le \bar{u}_{1i}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti}(\psi_t) \le \bar{u}_{ti}, \\ &\underline{v}_{2i} \le \mathbf{v}_{2i} \le \bar{v}_{2i}, \qquad \underline{v}_{t+1i} \le \mathbf{v}_{t+1}(\psi_t) \le \bar{v}_{t+1i}, \\ &\underline{s}_{1i} \le \mathbf{s}_{1i} \le \bar{s}_{1i}, \qquad \underline{s}_{ti} \le \mathbf{s}_{ti}(\psi_t) \le \bar{s}_{ti}, \\ &\mathbf{e}_1^{\mathrm{ES}} \ge 0, \qquad \mathbf{e}_t^{\mathrm{ES}}(\psi_t) \ge 0, \\ &\mathbf{e}_1^{\mathrm{CB}} \ge 0, \qquad \mathbf{e}_t^{\mathrm{CB}}(\psi_t) \ge 0, \\ &\mathbf{e}_1^{\mathrm{CS}} \ge 0, \qquad \mathbf{e}_t^{\mathrm{CS}}(\psi_t) \ge 0, \qquad t \in \mathbb{T}_1,\ i \in \mathbb{I},\ j \in \mathbb{J}. \end{aligned} \tag{6–7}$$

At each stage, one has to make the decision on the water releases $\mathbf{u}_t(\psi_t)$ for electricity generation as well as the water spillage $\mathbf{s}_t(\psi_t)$, which define the water reservoir levels $\mathbf{v}_{t+1}(\psi_t)$ at the end of each stage through equations (6–4) and (6–5). Furthermore, thermal plants can be used to generate electricity as well through variables $\mathbf{g}_{tj}(\psi_t)$ at operational costs of $c_{tj}(\psi_t)$, where each MWh produced releases B_j tons of CO_2 emissions. In order to meet the CO_2 emission allowance quota $E_y^{\mathrm{CO}_2}$, additional allowances can be bought $\mathbf{e}_t^{\mathrm{CB}}(\psi_t)$ in the CO_2 emission allowance market for the price of $P_t^{\mathrm{CB}}(\psi_t)$. If there are enough allowances available, then those allowances can also be sold in the CO_2 market via variables $\mathbf{e}_t^{\mathrm{CS}}(\psi_t)$ for a gain of $P_t^{\mathrm{CS}}(\psi_t)$. The generated electricity is used to meet the given quantity contracts' demand d_t^C. Access electricity can be sold in the electricity spot market for $P_T^{\mathrm{ES}}(\psi)$. The overall aim is then to maximize the expected profits, given as the difference between revenues and cost. The revenues from the quantity contracts are fix and, thus, are neglected in our model.

Consistent with the assumptions and notation in the previous chapters, we assume a Wait-and-See model along with deterministic first stage decisions. However, the sample-based solution algorithm discussed in this chapter simulates the inflow uncertainty throughout the planning horizon via M forward inflows. But again, a Wait-and-See formulation makes the first stage decisions "deterministic."

Recognize that model (6–1) - (6–6) does only allow the sales of electricity in the spot market but not to buy. In case that the purchase of electricity in the spot market is possible, the thermal generation decisions and the hydro-electricity generation decisions become decoupled! The reason is that in this case, the thermal plants are used whenever the electricity generation cost (including the CO_2 emission cost) are less than the electricity spot (sales) price. However, when including risk constraints, bounds on the electricity spot price trading, and / or derivative contracts, then the profit maximization problem has to be solved jointly for the thermal and the hydro generation.

Next to CO_2 emission allowance markets, the Kyoto protocol offers two additional mechanisms: Joint Implementation (JI) and Clean Development Mechanism (CDM). The idea is that part of the emission reduction can be achieved by conducting emission-reducing projects in other industrialized countries with Kyoto targets (JI) and in countries without targets (CDM). The possibility of such projects can be incorporated in the optimization model (6–1) - (6–7) via deterministic variables; *cf.* Rebennack et al. [140]. In addition, all the constraints discussed in Chapter 2 can be applied as well.

6.1.1 One-Stage Dispatch Programming

Let us now have a look at how the multi-stage stochastic program (6–1) - (6–7) can be solved. In order to exploit the block diagonal structure of problem, we use dynamic programming techniques to decompose the problem into one-stage dispatch problems.

As discussed in Chapter 3, SDDP methods rely on the convexity of the future cost function in its state variables. For maximization problems, this translates to the concavity requirements of the future benefit function in its state variables. Recall that the reason for the concavity of the future benefit function lies in the occurrence of the uncertainty as the RHS values of the LP problem (and the linear model for the water inflows). However, introducing a state variable for an objective function coefficient leads to a saddle shape future benefit function.

This problem has been solved by Gjelsvik and Wallace [66] by treating the uncertainty with respect to the electricity prices as Markov Chains. By introducing price clusters associated with a certain hydro inflow, the state space is discretized; *cf.* Chapter 3, thus, keeping the concavity of the future benefit function in the state variables. The electricity price uncertainty as well as the CO_2 emission allowance market price uncertainty are treated via Markov Chains.

In contrast, the Markov process and Markov Chain approach towards inflow and market price uncertainty, respectively, fuel price uncertainty is treated though a scenario tree. Consistent with Chapter 4, we assume that the fuel prices are independent from the hydro inflows. Though, our model captures the correlation among electricity prices, CO_2 emission allowance prices, hydro inflows and fuel prices.

The stochastic spot market electricity price and the stochastic CO_2 emission spot prices are modeled as discrete cost. These cost / prices are handled jointly and grouped into clusters λ_t^{sk} with $k \in \mathbb{K}_{st} = \{1, \dots, K_{st}\}$, one for each scenario $s \in \mathbb{S}_t$; we allow to have different price clusters for each stage t and fuel price scenario s. The scenario index allows us to capture the correlation of the fuel prices and the market prices for electricity as well as CO_2 emission allowances. Hence, for each cluster $k \in \mathbb{K}_{st}$ for scenario $s \in \mathbb{S}_t$, we have the tuple

$$\lambda_t^{sk} = [P_t^{sk,\mathrm{ES}}, P_t^{sk,\mathrm{CS}}],$$

with $P_t^{sk,\text{ES}}$ being the electricity spot price per MWh at stage t for price cluster k and scenario s, and $P_t^{sk,\text{CS}}$ being the CO_2 emission spot (sales) price per ton CO_2 emissions at stage t for price cluster k and scenario s, respectively. These prices are autocorrelated and hence, the prices at stage t depend on the prices of the previous stage. This is modeled via a two-dimensional discrete Markov Chain. The condition probability

$$\hat{p}_{t+1}^{sk\theta\kappa} = P\left(\lambda_{t+1}^{\theta\kappa} \,\middle|\, \lambda_t^{sk}\right)$$

is the transition probability of a price cluster λ_t^{sk} in stage t to a price cluster $\lambda_{t+1}^{\theta\kappa}$ in stage $t+1$.

We assume a fixed (positive) charge for trading in the emission market. This leads to a fixed absolute difference between the CO_2 emission allowance sales prices and buy price. Thus, by specifying the allowance sales price, the CO_2 emission allowance buy price $P_t^{sk,\text{CB}}$ in dollars per ton is given as well.

The one-stage dispatch problems for stage t depend then on the (initial) water reservoir levels v_t, the (initial) emission reservoir level e_t, the water inflows a_{t-1} observed at the previous stage $t-1$, as well as the price cluster k.

Combining the concepts of the one-stage dispatch problems (3–5) - (3–8), (4–5) - (4–8) and (5–12) - (5–12), we obtain a one-stage dispatch problem for each fuel price scenario $s \in \mathbb{S}_t$ and price cluster $k \in \mathbb{K}_{st}$

$$z^k_{ts}(v_t, e_t, a_{t-1}) := \max \sum_{l \in \mathbb{L}} p^l \Bigg[P^{sk,\mathrm{ES}}_t \mathbf{e}^{l,\mathrm{ES}}_t - \sum_{j \in \mathbb{J}} c^s_{tj} \mathbf{g}^l_{tj} + P^{sk,\mathrm{CS}}_t \mathbf{e}^{l,\mathrm{CS}}_t - P^{sk,\mathrm{CB}}_t \mathbf{e}^{l,\mathrm{CB}}_t$$
$$+ \sum_{\theta \in \Theta^s_{t+1}} p^{s\theta}_{t+1} \left(\sum_{\kappa \in \mathbb{K}_{\theta t+1}} \hat{p}^{sk\theta\kappa}_{t+1} z^{\kappa}_{t+1\theta}\big(\mathbf{v}^l_{t+1}, \mathbf{e}^l_{t+1}, a^l_t\big) \right) \Bigg] \tag{6–8}$$

$$\text{s.t.} \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}^l_{tj} + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}^l_{ti} = d^C_t + \mathbf{e}^{l,\mathrm{ES}}_t, \tag{6–9}$$

$$\mathbf{v}^l_{t+1i} = v_{ti} - \mathbf{u}^l_{ti} - \mathbf{s}^l_{ti} + \sum\nolimits_{h \in \mathbb{U}_i} \left(\mathbf{u}^{ls}_{th} + \mathbf{s}^l_{th}\right) + a^l_{ti}, \qquad i \in \mathbb{I} \tag{6–10}$$

$$\mathbf{e}^l_{t+1} = e_t - \sum\nolimits_{j \in \mathbb{J}} B_j \mathbf{g}^l_{tj} + \mathbf{e}^{l,\mathrm{CB}}_t - \mathbf{e}^{l,\mathrm{CS}}_t, \qquad \text{if } t \in \mathbb{T} \setminus \mathbb{Y}_g \tag{6–11}$$

$$\mathbf{e}^l_{t+1} = \widetilde{e}_t - \sum\nolimits_{j \in \mathbb{J}} B_j \mathbf{g}^l_{tj} + \mathbf{e}^{l,\mathrm{CB}}_t - \mathbf{e}^{l,\mathrm{CS}}_t + E^{\mathrm{CO_2}}_t, \qquad \text{if } t \in \mathbb{Y}_g \tag{6–12}$$

$$\underline{g}_{tj} \le \mathbf{g}^l_{tj} \le \bar{g}_{tj}, \qquad \underline{u}_{ti} \le \mathbf{u}^l_{ti} \le \bar{u}_{ti},$$
$$\underline{v}_{t+1i} \le \mathbf{v}^l_{t+1i} \le \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \le \mathbf{s}^l_{ti} \le \bar{s}_{ti},$$
$$\mathbf{e}^l_{t+1} \ge 0, \qquad \mathbf{e}^{l,\mathrm{ES}}_t \ge 0,$$
$$\mathbf{e}^{l,\mathrm{CB}}_t \ge 0, \qquad \mathbf{e}^{l,\mathrm{CS}}_t \ge 0, \qquad j \in \mathbb{J},\ i \in \mathbb{I}. \tag{6–13}$$

In each stage, the aim is to maximize the expected immediate profits plus the future expected profits. The future profits depend then on the proceeding scenarios θ as well as the prices clusters κ. Just like (5–12) - (5–17), all decision variables carry inflow index "l," decomposing the problem into L independent ones.

The future benefit functions $z^k_{t+1s}(\cdot,\cdot,\cdot)$ can be overestimated using the information of the dual multipliers.

6.1.2 Future Benefit Function Cuts for CO_2 Reservoirs and Scenario Tree

Recognize that the uncertain parameters of electiricty spot prices, CO_2 emission allowance prices and fuel prices appeared in the objective functions (6–1) and (6–8) and not in the RHS of the corresponding mathematical programs. In order to preserve the concavity of the future benefit function in the state variables, we treated those uncertainties as a discrete Markov Chain and scenario tree, respectively.

Thus, the future benefit function $z_{ts}^{k}(v_t, e_t, a_{t-1})$ is a concave function, jointly in its state variables v_t, e_t and a_{t-1}. If $z_{ts}^{k}(\cdot,\cdot,\cdot)$ can be evaluated for specific points v_t^n, e_t^n and a_{t-1}^n and the slopes γ_{tmnsk}^{ν}, γ_{tmnsk}^{e} and γ_{tmnsk}^{a} at these points are computable, then z_{ts}^{k} can be extrapolated. Together with the concavity property, this leads to an overestimation of the functions $z_{ts}^{k}(\cdot,\cdot,\cdot)$. An upper bound can then be obtained by solving the following LP problem

$$\bar{z}_{ts}^{k}(v_t, e_t, a_{t-1}) = \max \beta \tag{6–14}$$

$$\text{s.t. } \beta \leq \gamma_{tmnsk}^{\nu} v_t + \gamma_{tmnsk}^{e} e_t + \gamma_{tmnsk}^{a} a_{t-1} + \gamma_{tmnsk}^{c}, \qquad m \in \mathbb{M},\ n \in \mathbb{N}, \tag{6–15}$$

with γ_{tmnsk}^{c} being the constant term for the linear segment corresponding to the state discretization $n \in \mathbb{N}$, the forward inflows $m \in \mathbb{M}$, scenario $s \in \mathbb{S}_t$ and cluster $k \in \mathbb{K}_{st}$.

6.2 Discussion

In this chapter, we considered a profit maximization model for an electric utility which is subject to a CO_2 emission quota in a deregulated electricity market. Free CO_2 emission allowances are issued by the regulator on a regular basis and CO_2 emission allowances can be traded in the market. The considered electric utility has a hydro-dominated portfolio, whereas the market is free of price-makers.

The presented multi-stage stochastic programming formulation allows to consider jointly stochastic hydro inflows, fuel prices, electricity market prices and CO_2 emission allowance market prices while capturing all major correlations among them. To approximate the distribution of the stochastic electricity market prices and CO_2 emission allowance market prices, a fundamental model is proposed.

CHAPTER 7
CONCLUSIONS

This dissertation extends the classical SDDP algorithm by Pereira and Pinto [125] in several ways.

First, a scenario tree framework to capture uncertainties driven by political and macro-economical forces is embedded into the SDDP algorithm. Examples for this type of uncertainties are fuel prices and electricity demand. The feasibility and efficiency of the so-called SDDPT algorithm is demonstrated by two real case studies for the power systems of Panama and Costa Rica.

Second, a model for CO_2 emission caps on hydro-thermal power systems is proposed. The multi-stage constraints of the CO_2 emission allowances are re-formulated as a reservoir constraint, respecting the stage decomposition framework of SDDP. The dual multipliers corresponding to the CO_2 emission reservoir constraints provide marginal CO_2 emission allowance prices. Those prices are operational system prices for CO_2 emission reduction in the mid-term. Computational results are performed for the real power system of Guatemala.

Third, a profit maximization model for electric utilities in a deregulated electricity market has been proposed. The utility is subject to CO_2 emission quotas where allowances are traded in a market environment. The proposed model includes stochastic parameters in the hydro inflows, fuel prices, electricity market prices and CO_2 emission allowance prices.

The presented scenario tree model as well as the reservoir model for the CO_2 emission allowances can be applied to hydro-thermal expansion planning problems. In particular, long-term models subject to CO_2 emission allowance caps are of practical interest for regulators and society, as the marginal CO_2 prices represent the investment cost for CO_2 emission reductions.

APPENDIX: NOMENCLATURE

The nomenclature throughout this article is summarized in Tables A-1 - A-4. A "$*$" indicates an optimal solution value.

Table A-1. Indices and sets

Symbol	Size	Meaning
$i \in \mathbb{I}$	I	hydro plants/reservoirs in the system
$j \in \mathbb{J}$	J	thermal plants in the system
$k \in \mathbb{K}$	K	price clusters
$l \in \mathbb{L}$	L	inflow scenarios: backward openings
$m \in \mathbb{M}$	M	inflow scenarios: forward scenarios
$n \in \mathbb{N}$	N	reservoir level discretization
$\psi_t \in \Psi_t$	–	random outcome corresponding to uncertain hydro inflows, fuel prices, electricity prices, and CO_2 emission allowance prices for stage t conditioned to all previous outcomes
$s = s_t \in \mathbb{S}_t$	S_t	(fuel price) scenarios for stage t
$t \in \mathbb{T}$	T	time stages of the planning horizon
$\mathbb{T}_1$	T-1	$\mathbb{T}$ without the first stage
$h \in \mathbb{U}_i$	–	immediate upstream hydro plants for hydro plant i
$\theta \in \boldsymbol{\Theta}^s_{t+1}$	Θ^s_{t+1}	successor scenarios at stage $t+1$ belonging to scenario $s \in \mathbb{S}_t$
$\omega = \omega_t \in \Omega$	–	random outcome corresponding to hydro inflow uncertainty
$\omega \in \Omega_t$	–	random outcome for stage t conditioned to all previous random outcomes
$\xi \in \Xi_t$	–	random outcome corresponding to fuel price uncertainty for stage t conditioned to all previous random fuel price outcomes
$\mathbb{Y}_g$	Y_g	stages when the CO_2 allowances are "g"iven, $\mathbb{Y}_g \subseteq \mathbb{T}$
$\mathbb{Y}_e$	Y_e	stages when the CO_2 allowances "e"xpire, $\mathbb{Y}_e \subseteq \mathbb{Y}_g$
$\zeta \sim \mathcal{N}(0,1)$	–	normally distributed random variable for the hydro inflow model; may depend on hydro plant i, inflow l, m and/or stage t

Table A-2. Objective functions

Symbol	Unit	Meaning
$c_t(\mathbf{u}_t)$	\$	thermal complement function for stage t for hydro release $\mathbf{u}_t$; $\mathbf{u}_t$ may depend on random outcome ω_t
$Q_t(\mathbf{x}_{t-1}, \omega_t)$	–	tth–stage value function for stage $t \in \mathbb{T}_1$
$\mathcal{Q}_t(\mathbf{x}_{t-1})$	–	expected tth–stage value function for stage $t \in \mathbb{T}_1$
z	\$	minimal total expected operational cost; dependent on the initial water reservoir levels and the past inflows
z^{P}	\$	total expected profits; dependent on the initial water reservoir levels, initial CO_2 reservoir level, and the past inflows
$z_t(v_t)$	\$	minimal expected operation cost for stage t and after for given water reservoir levels v_t; may depend on scenario s and inflow m
$z_{t+1}(v_{t+1})$	\$	future cost function for stage t for reservoir levels v_{t+1}; may depend on θ
$z_t(v_t, a_{t-1})$	\$	minimal expected operation cost for stage t and after for given water reservoir levels v_t and past inflows a_{t-1}; the random water inflow has been approximated by a discrete sample; may depend on scenario s, inflow l, m and/or storage discretization n
$z_{t+1}(v_{t+1}, a_t)$	\$	future cost function for stage t for reservoir levels v_{t+1} and occurred inflow a_t; the random water inflow has been approximated by a discrete sample; may depend on θ
$z_t(v_t, \omega_{t-1})$	\$	minimal expected operation cost for stage t and after for given water reservoir levels v_t and past random event ω_{t-1}
$z_{t+1}(v_{t+1}(\omega), \omega)$	\$	future cost function for stage t for stochastic reservoir levels $v_{t+1}(\omega)$ and random event ω occurred during stage t
$z_t(v_t, e_t)$	\$	minimal expected operation cost for stage t and after for given water reservoir levels v_t and CO_2 reservoir level e_t; may depend on inflow m
$z_t(v_t, e_t, a_{t-1})$	\$	minimal expected operation cost for stage t and after for given water reservoir levels v_t, CO_2 reservoir level e_t and past inflows a_{t-1}; the random water inflow has been approximated by a discrete sample; may depend on inflow l, m and/or storage discretization n
$z_{t+1}(v_{t+1}, e_{t+1}, a_t)$	\$	future cost function for stage t for reservoir levels v_{t+1}, CO_2 reservoir level e_{t+1} and occurred inflow a_t; the random water inflow has been approximated by a discrete sample; may depend on θ

Table A-3. Decision variables and values obtained through optimization

Symbol	Unit	Meaning
α	\$	approximates future cost function; may depend on $t+1$ and θ
δ_t	MWh	load shedding at stage t; may depend on inflow l, m, scenario s or random outcome ω
$\mathbf{e}_t^{\mathrm{CB}}$	tons CO_2	CO_2 emission allowances bought in the market at stage t; may depend on random outcome ψ_t
$\mathbf{e}_t^{\mathrm{CS}}$	tons CO_2	CO_2 emission allowances sold in the market at stage t; may depend on random outcome ψ_t
$\mathbf{e}_t^{\mathrm{ES}}$	MWh	electricity sold in the spot market at stage t; may depend on random outcome ψ_t
$\mathbf{e}_{t+1}$	tons CO_2	CO_2 emissions reservoir level at the end of stage t, may depend on inflow l, m
$\mathbf{f}_t$	tons CO_2	CO_2 emissions above quota, may depend on inflow l, m or random outcome ω_t
$\mathbf{g}_t$	MWh	thermal plant electricity generation for stage t; may depend on thermal plant j, inflow l, m, scenario s or random outcome ω, ψ_t
$\mathbf{s}_t$	m^3	spilled water for stage t; may depend on hydro plant i, inflow l, m, scenario s or random outcome ω, ψ_t
$\mathbf{u}_t$	m^3	turbined water for stage t; may depend on hydro plant i, inflow l, m, scenario s or random outcome ω, ψ_t
$\mathbf{v}_{t+1}$	m^3	hydro reservoir level at the end of stage t; may depend on hydro plant i, inflow l, m, scenario s or random outcome ω, ψ_t
$\mathbf{x}$	–	first stage variables (of deterministic part)
$\mathbf{x}_t(\omega)$	–	variables of stage $t \in \mathbb{T}_1$
$\underline{z}$	\$	estimated lower bound on the overall objective function value
$\hat{z}$	\$	estimated mean objective function values
z^m	\$	operational cost over time horizon T corresponding to inflow scenario m
$\hat{\sigma}$	\$	standard deviation of operation cost of M inflow scenarios

Table A-4. Input data

Symbol	Unit	Meaning
A	–	constraint matrix of deterministic part
a_0	m^3	hydro inflows at the stage(s) prior to the start of the planning horizon as a vector of the hydro reservoirs i
a_{t-1}	m^3	past water inflows for stage t; state variable
a_t	m^3	water inflows during stage t; may depend on hydro reservoir i, inflow l, or random outcome ω, ψ_t
B_j	tons CO_2/MWh	(average) CO_2 emission factor for each thermal plant j
b	–	RHS of deterministic constraints
C	–	hydro reservoir inflow correlation matrix
C^{CO_2}	\$/ton CO_2	fine for exceeding the CO_2 emission quota
c	–	objective function coefficient of first stage
c_{tj}	\$/MWh	cost for power production at thermal plant j during stage t; may depend on scenario s or random outcome ξ, ψ_t
d_t	MWh	electricity demand during stage t; may depend on random outcome ξ
$E_y^{CO_2}$	tons CO_2	CO_2 emission quota per horizon
e_0, e_1	tons CO_2	initial CO_2 emission allowances
e_t	m^3	initial CO_2 reservoir level for stage t; may depend on the discretization n; state variable
η	–	factor for convergence of Algorithm 3.1
η_{ts}^{lmn}, $\tilde{\eta}_{ts}^{lmn}$	\$/m^3	dual multipliers of constraints (4–7)
ϕ_1	–	model parameter for the linear autoregressive inflow model
ϕ_2	m^3	model parameter for the linear autoregressive inflow model
φ_t	–	defined as $\varsigma_t \phi_1 / \varsigma_{t-1}$
$\underline{g}_{tj}$	MWh	minimum generation for thermal plant j during stage t
$\bar{g}_{tj}$	MWh	maximum generation for thermal plant j during stage t
γ_{tmn}^{a}	\$/m^3	slope for the future cost function approximations corresponding to the water inflow; may depend on scenario s
γ_{tmn}^{c}	\$	constant of the future cost function approximations; may depend on scenario s
γ_{tmn}^{e}	\$/ton CO_2	slope for the future cost function approximations corresponding to the CO_2 reservoir level

Table A-4. Continued

Symbol	Unit	Meaning
γ^{ν}_{tmn}	\$/m^3	slope for the future cost function approximations corresponding to the water reservoir level; may depend on scenario s
$h_t(\omega_t)$	–	stochastic RHS for stage $t \in \mathbb{T}_1$
λ^k_t	–	price clusters for stage t and cluster k
μ_t	m^3	inflow mean for stage t for the linear autoregressive inflow model
$\mathcal{N}_1$	–	storage vector discretization size; applies to each reservoir i
$\mathcal{N}_2$	–	inflow vector discretization size; applies to each hydro plant i
P^{CB}_t	\$/ton CO_2	CO_2 emission allowance market (buy) price; may depend on price cluster k, scenario s, or random outcome ψ_t
P^{CS}_t	\$/ton CO_2	CO_2 emission allowance market (sales) price; may depend on price cluster k, scenario s, or random outcome ψ_t
P^{ES}_t	\$/MWh	electricity market (sales) price; may depend on price cluster k, scenario s, or random outcome ψ_t
$p^{s\theta}_{t+1}$	–	conditional probability that scenario θ occurs at stage $t+1$ after scenario s has occurred at stage t
p^s_t	–	probability that scenario s occurs at time t
p^l	–	probability of backwards inflow scenario l
p^m	–	probability of forward inflow scenario m
$\hat{p}^{sk\theta\kappa}_{t+1}$	–	transition probability of price cluster λ^{sk}_t in stage t to price cluster $\lambda^{\theta\kappa}_{t+1}$ in stage $t+1$; $t \in \mathbb{T}_1$
$q_t(\omega_t)$	–	stochastic objective function coefficient of stage $t \in \mathbb{T}_1$
ρ	MWh/m^3	power coefficient for hydro plants; may depend on the hydro plant i
$\underline{s}_{ti}$	m^3	minimum spillage for hydro plant i during stage t
$\bar{s}_{ti}$	m^3	maximum spillage for hydro plant i during stage t
ς_t	m^3	standard deviation of inflow for stage t for the linear autoregressive inflow model
$T_t(\omega_t)$	–	constraint matrix of stochastic part of stage $t \in \mathbb{T}_1$; as we assume fixed technology, T_t is deterministic
$\underline{u}_{ti}$	m^3	minimum turbining for hydro plant i during stage t
$\bar{u}_{ti}$	m^3	maximum turbining for hydro plant i during stage t
Υ	\$/MWh	penalty for load shedding

Table A-4. Continued

Symbol	Unit	Meaning
v_0	m^3	initial hydro reservoir level, may depend on i
v_t	m^3	initial reservoir level for stage t; may depend on the discretization n; state variable
$\underline{v}_{t+1i}$	m^3	minimum reservoir level for hydro plant i at the end of stage t
$\bar{v}_{t+1i}$	m^3	maximum reservoir level for hydro plant i at the end of stage t
W_t	–	constraint matrix of stochastic part of stage $t \in \mathbb{T}_1$; as we assume fixed recourse, W_t is deterministic

REFERENCES

[1] 1998. *Kyoto Protocol to the United Nations Framework Convention on Climate Change*. United Nations.

[2] 2003. *Kyoto Protocol, MEM/03/154*. Brussels.

[3] A.-Ghouila-Houri. 1962. Caracterization des matrices totalement unimodulaires. *C. R. Academy of Sciences of Paris* **254** 1192–1194.

[4] Archibals, T.W., C.S. Buchanan, K.I.M. McKinnon, L.C. Thomas. 1999. Nested Bendes decomposition and dynamic programming for reservoir optimization. *Journal of the Operational Research Society* **34**(2) 215–229.

[5] Artzner, P., F. Delbaen, J-M Eber, D. Heath, H. Ku. 2007. Coherent Multiperiod Risk Adjusted Values and Bellman's Principle. *Annals of Operations Research* **152**(1) 5–22.

[6] Bai, D., T. Carpenter, J. Mulvey. 1997. Making a Case for Robust Optimization Models. *Management Science* **43**(7) 895–907.

[7] Barroso, L.A.N., M.H.C. Fampa, R. Kelman, M.V.F. Pereira, P. Lino. 2002. Market Power Issues in Bid-Based Hydrothermal Dispatch. *Annals of Operations Research* **117** 247–270.

[8] Baslis, C.G., S. E. Papadakis, A.G. Bakirtzis. 2009. Simulation of Optimal Medium-Term Hydro-Thermal System Operation by Grid Computing. *IEEE Transactions on Power Systems* **24**(3) 1208–1217.

[9] Batlle, C., J. Barquín. 2004. Fuel prices scenario generation based on a multivariate GARCH model for risk analysis in a wholesale electricity market. *International Journal of Electrical Power & Energy Systems* **26**(4) 273–280.

[10] Batstone, S., T. Scott. 1998. Long term contracting in a deregulated electricity industry: simulation results from a hydro management model. *ORSNZ Proceedings*. 147–156.

[11] Bellman, R.E. 1957. *Dynamic Programming*. Princeton University Press, Princeton, N.J.

[12] Bellman, R.E. 1961. *Adaptive Control Processes: A Guided Tour*. Princeton University Press, Princeton, NJ.

[13] Belsnes, M.M., A. Haugstad, B. Mo, P. Markussen. 2003. Quota Modeling in Hydrothermal Systems. *IEEE PowerTech*.

[14] Ben-Tal, A., A. Nemirovski. 1998. Robust convex optimization. *Mathematics of Operations Research* **23**(4) 769–805.

[15] Ben-Tal, A., A. Nemirovski. 2002. Robust optimization methodology and applications. *Mathematical Programming, Series B* **92** 453–480.

[16] Benders, J.F. 1962. Partitioning procedures for solving mixed variables programming problems. *Numererische Mathematik* **4** 238–252.

[17] Benz, E., S. Trueck. 2009. Modeling the Price Dynamics of CO2 Emission Allowances. *Energy Economics* **31**(1) 4–15.

[18] Bertsimas, D., D.B. Brown, C. Caramanis. 2007. Theory and applications of Robust Optimization. Tech. rep., Sloan School of Management and Operations Research Center, Massachusetts Institute of Technology.

[19] Bezerra, B., L.A. Barroso, R. Kelman, B. Flach, M.L. Latorre, N. Campodonico, M.V.F. Pereira. 2010. Integrated ElectricityGas Operations Planning in Long-term Hydroscheduling Based on Stochastic Models. S. Rebennack, P.M. Pardalos, M.V.F. Pereira, N.A. Iliadis, eds., *Handbook of Power Systems*. Energy Systems, Springer-Verlag Berlin Heidelberg.

[20] Bezerra, B., R. Kelman, L.A. Barroso, B. Flach, M.L. Latorre, N. Campodonico, M.V.F. Pereira. 2006. Integrated electricity-gas operations planning in hydrothermal systems. *Specialists in Electric Operational and Expansion Planning (SEPOPE)*. Brazil.

[21] Birge, J.R., F. Louveaux. 2000. *Introduction to Stochastic Programming*. Operations Research and Financial Engineering, Springer.

[22] Blair, C.E., R.G. Jeroslow. 1982. The value function of an integer program. *Mathematical Programming* **23** 237–273.

[23] Brooke, A., D. Kendrick, A. Meeraus. 1992. *GAMS - A User's Guide*. Boyd & Fraser Publishing Company, Danvers, Massachusetts.

[24] Buras, N. 1966. Dynamic programming and water resources development. *Advanced Hydroscience* **3** 372–412.

[25] Bushnell, J. 2003. A mixed-complementarity model of hydrothermal electricity competition in the Western US. *Operations Research* **51** 80–93.

[26] Camion, P. 1965. Characterization of Totally Unimodular Matrices. *Proceedings of the American Mathematical Society* **16**(5) 1068–1073.

[27] Chabar, R.M., M.V.F. Pereira, S. Granville, L.A. Barroso, N.A. Iliadis. 2006. Optimization of Fuel Contracts Management and Maintenance Scheduling for Thermal Plants under Price Uncertainty. *IEEE Power Systems Conference and Exposition*. 923–930.

[28] Chaer, R., P. Monzón. 2008. Stability conditions for a stochastic dynamic optimizer for optimal dispatch policies in power systems with hydroelectrical generation.

IEEE/PES Transmission and Distribution Conference and Exposition: Latin America, 2008. 1–5.

[29] Chen, Z.-L., W.B. Powell. 1999. A Convergent Cutting-Plane and Partial-Sampling Algorithm for Multistage Stochastic Linear Programs with Recourse. *Journal of Optimiztion Theory and Applications* **103** 497–524.

[30] Christoforodis, M., M. Aganagic, B. Awobamise, S. Tong, A. Rahimi. 1996. Long-Term/Mid-Term Resource Optimization of a Hydro Dominant Power System Using Interior Point Method. *IEEE Transactions on Power Systems* **11**(1) 287–294.

[31] Costa, L.C. 2008. Considering Reliability Constraints in the Optimal Power Systems Expansion Planning Problem. Master's thesis, COPPE/UFRJ.

[32] Craddock, M., A.D. Shaw, B. Graydon. 1999. Risk-averse reservoir management in a de-regulated electricity market. *ORSNZ Proceedings*. 157–166.

[33] Culioli, J.-C., G. Cohen. 1990. Decompostion/coordination algorithms in stochastic optimization. *SIAM Journal of Control and Optimization* **28** 1372–1403.

[34] Culy, J., V. Willis, M. Civil. 1990. Electricity modeling in ECNZ re-visited. *ORSNZ Proceedings*. 9–14.

[35] Dantzig, G.B. 1955. Linear programming under uncertainty. *Management Science* **1**(3 & 4) 197–206.

[36] David, A.K. 1993. Competitive bidding in electricity supply. *IEE Proceedings: Generation, Transmission and Distribution*, vol. 140. 421–426.

[37] de Oliveira, G.C., S. Granville, M. Pereira. 2002. Optimization in electrical power systems. P.M. Pardalos, M.G.C. Resende, eds., *Handbook of applied optimization*. Oxford University Press, London, 770–807.

[38] Dentcheva, D., W. Römisch. 1997. Optimal Power Generation under Uncertainty via Stochastic Programming. K. Marti, P. Kall, eds., *Stochastic Programming Methods and Technical Applications*. Lecture Notes in Economics and Mathematical Systems, Springer-Verlag, 22–56.

[39] Diniz, A.L., T.N. dos Santos. 2008. Multi-Period Stage Definition for the Multi Stage Benders Decomposition Approach Applied to Hydrothermal Scheduling. *EngOpt 2008 - International Conference on Engineering Optimization*. Rio de Janeiro, Brazil. 01 - 05 June 2008.

[40] Donohue, C.J. 1996. Stochastic network programming and the dynamic vehicle allocation problem. Ph.D. thesis, University of Michigan.

[41] Donohue, C.J., J.R. Birge. 2006. The Abridged Nested Decomposition Method for Multistage Stochastic Linear Programs with Relatively Complete Recourse. *Algorithmic Operations Research* **1**(1) 20–30.

[42] dos Santos, T.N., A.L. Diniz. 2009. A New Multiperiod Stage Definition for the Multistage Benders Decomposition Approach Applied to Hydrothermal Scheduling. *IEEE Transactions on Power Systems* **24**(3) 1383–1392.

[43] Dupačová, J., G. Consigli, S.W. Wallace. 2000. Scenarios for multistage stochastic programs. *Annals of Operations Research* **100** 25–53.

[44] Dupačová, J., N. Gröwe-Kuska, W. Römisch. 2003. Scenario reduction in stochastic programming: An approach using probability metrics. *Mathematical Programming* **95** 493–511.

[45] Easterbrook, G., C.F. Bales, R. Duke. 27 February 2009. Carbon Tax V. Cap and Trade. McKinsey & Company: The Debate Zone.

[46] Ehrhart, K-M., C. Hoppe, J. Schleich, S. Seifert. 2003. Strategic Aspects of CO2-Emissions Trading: Theoretical Concepts and Empirical Findings. *Energy and Environment* **14**(5) 579–597.

[47] EIA. 2009. Voluntary reporting of greenhouse gases program (fuel and energy source codes and emission coefficients). Energy Information Admisitration: Ofiicial Energy Statistics from the U.S. Government.

[48] EIA, U.S. Energy Information Administration. 3 December 2009. Emissions of Greenhouse Gases in the United States 2008. Report #: DOE/EIA-0573(2008).

[49] Eichhorn, A., H. Heitsch, W. Römisch. 2009. Scenario Tree Approximation and Risk Aversion Strategies for Stochastic Optimization of Electricity Production and Trading. J. Kallrath, P.M. Pardalos, S. Rebennack, M. Scheidt, eds., *Optimization in the Energy Industry*, *Energy Systems*, vol. 1. Springer.

[50] Eichhorn, A., H. Heitsch, W. Römisch. 2010. Stochastic Optimization of Electricity Portfolios: Scenario Tree Modeling and Risk Management. S. Rebennack, P.M. Pardalos, M.V.F. Pereira, N.A. Iliadis, eds., *Handbook of Power Systems*. Energy Systems, Springer-Verlag Berlin Heidelberg.

[51] Ellerman, A.D., B.K. Buchner. 2007. The European Union Emissions Trading Scheme: Origins, Allocation, and Early Results. *Review of Environmental Economics and Policy* **1**(1) 66–87.

[52] Enders, J., W.B. Powell, D. Egan. 2010. A dynamic model for the failure replacement of aging high-voltage transformers. *Energy Systems* **1**(1) 31–59.

[53] Ermoliev, Y. 1983. Stochastic quasigradient methods and their application to system optimization. *Stochastics* **9** 1–36.

[54] Faber, B.A., J.R. Stedinger. 2001. Reservoir optimization using sampling SDP with ensemble streamflow prediction (ESP) forecasts. *Journal of Hydrology* **249**(1–4) 113–133.

[55] Ferrero, R.W., J.F. Rivera, S.M. Shahidehpour. 1998. A dynamic programming two-stage algorithm for long-term hydrothermal scheduling of multireservoir systems. *IEEE Transactions on Power Systems* **13**(4) 1534–1540.

[56] Flach, B.C., L.A. Barroso, M.V.F. Pereira. 2010. Long-term optimal allocation of hydro generation for a price-maker company in a competitive market: latest developments and a stochastic dual dynamic programming approach. *IET Generation, Transmission and Distribution* **4**(2) 1–16.

[57] Fleten, S.-E., S.W. Wallace. 2009. Delta-Hedging a Hydropower Plant Using Stochastic Programming. J. Kallrath, P.M. Pardalos, S. Rebennack, M. Scheidt, eds., *Optimization in the Energy Industry*, *Energy Systems*, vol. 1. Springer Berlin Heidelberg, 507–524.

[58] Fleten, S.-E., S.W. Wallace, W.T. Ziemba. 2002. Hedging electricity portfolios via stochastic programming. C. Greengard, A. Ruszczynski, eds., *Decision Making Under Uncertainty: Energy and Power*, *IMA Volumes on Mathematics and Its Applications*, vol. 128. Springer-Verlag, New York, 71–93.

[59] Fuentes, R., V. Quintana. 2002. Semidefinite Programming: A Practical Application to Hydro-Thermal Coordination. *14th PSCC, Sevilla, 24-28 June 2002*.

[60] General Algebraic Modeling System (GAMS). 2002. SCENRED.

[61] General Algebraic Modeling System (GAMS). 2008. The GAMS Model Library Index. Model name 'poutil' (Portfolio Optimization for electric UTILities).

[62] Gerin, R. 2008. Rolling on a River. Beijing Review, No. 50.

[63] Gjelsvik, A., M.M. Belsnes, A. Haugstad. 1999. An algorithm for stochastic medium-term hydrothermal scheduling under spot price uncertainty. *13th power systems computation conference: Proceedings, vol. 2*.

[64] Gjelsvik, A., M.M. Belsnes, M. Håland. 1997. A case of hydro scheduling with a stochastic price model. E. Broch, D.K. Lysne, N. Flatabø, E. Helland-Hansen, eds., *Procedings of the 3rd international conference on hydropower*. Trondheim/Norway/30 June2 July 1997. A.A. Balkema, Rotterdam, 211–218.

[65] Gjelsvik, A., B. Mo, A. Haugstad. 2010. Long- and Medium-term Operations Planning and Stochastic Modelling in Hydro-dominated Power Systems Based on Stochastic Dual Dynamic Programming. S. Rebennack, P.M. Pardalos, M.V.F. Pereira, N.A. Iliadis, eds., *Handbook of Power Systems*. Energy Systems, Springer-Verlag Berlin Heidelberg.

[66] Gjelsvik, A., S.W. Wallace. 1996. Methods for stochastic medium-term scheduling in hydro-dominated power systems. Tech. rep., Norwegian Electric Power Research Institute, Trondheim. EFI TR A4438.

[67] Gorenstin, B., J.P. Costa, M.V.F. Pereira, N.M. Campodónico. 1993. Power System Expansion Planning Under Uncertainty. *IEEE Transactions on Power Systems* **8**(1) 129–136.

[68] Granville, S., G.C. Oliveira, L.M. Thome, N. Campodonico, M.L. Latorre, M.V.F. Pereira, L.A. Barroso. 2003. Stochastic optimization of transmission constrained and large scale hydrothermal systems in a competitive framework. *IEEE Power Engineering Society General Meeting*, vol. 2. Toronto.

[69] Gross, G., D. Finlay. 2000. Generation supply bidding in perfectly competitive electricity markets. *Computational & Mathematical Organizations Theory* **6** 83–98.

[70] Gröwe-Kuska, N., H. Heitsch, W. Römisch. 2003. Scenario Reduction and Scenario Tree Construction for Power Management Problems. *IEEE Power Tech Conference*. Bologna, Italy.

[71] Guéret, C., C. Prins, M. Sevaux, S. Heipcke. 2002. *Applications of Optimization with XpressMP*. Dash Optimization Ltd.

[72] Hall, W.A., N. Buras. 1961. The Dynamic Programming Approach to Water-Resources Development. *Journal of Geophysical Research* **66**(2) 517–520.

[73] Hanson, F.B. 1996. *Stochastic Digital Control System Techniques*, chap. Techniques in Computational Stochastic Dynamic Programming. Academic Press, New York, NY, 103–162.

[74] Heitsch, H., W. Römisch. 2003. Scenario reduction algorithms in stochastic programming. *Computational Optimization and Applications* **24**(2–3) 187–206.

[75] Heitsch, H., W. Römisch. 2005. Generation of multivariate scenario trees to model stochasticity in power management. *IEEE St. Petersburg PowerTech Proceedings*.

[76] Heitsch, H., W. Römisch. 2009. Scenario tree modeling for multistage stochastic programs. *Mathematical Programming* **118**(2) 371–406.

[77] Heitsch, H., W. Römisch, C. Strugarek. 2006. Stability of multistage stochastic programs. *SIAM Journal on Optimization* **17** 511–525.

[78] Higle, J.L., S. Sen. 1991. Stochastic decompostion: An algorithm for two stage linear programs with recourse. *Mathematics of Operations Research* **16**(3) 650–669.

[79] Hobbs, B.F., U. Helman. 2004. Complementarity-based equilibrium modeling for electric power markets. D. Bunn, ed., *Modeling Prices in Competitive Electricity Markets*. John Wiley.

[80] Holthausen, D.M. 1979. *Hedging and the competitive firm under price uncertainty*. American Economic review.

[81] Hong, B.D., E.R. Slatick. 1994. Carbon Dioxide Emission Factors for Coal . *Energy Information Administration, Quarterly Coal Report* **Q1** 1–8. DOE/EIA-0121(94/Q1).

[82] Hongling, L., J. Chuanwen, Z. Yan. 2008. A review on risk-constrained hydropower scheduling in deregulated power market. *Renewable and Sustainable Energy Reviews* **12** 1465–1475.

[83] Høyland, K., M. Kaut, S.W. Wallace. 2003. A heursitics for generating scenario trees for multistage decision problems. *Computational Optimization and Applications* **23**(2-3) 169–185.

[84] Høyland, K., S. W. Wallace. 2001. Generating Scenario Trees for Multistage Decision Problems. *Management Science* **47**(2) 295–307.

[85] Iliadis, N. 2008. Financial risk modelling in electricity portfolio optimisation. Ph.D. thesis, Doctoral School of EPFL.

[86] Iliadis, N.A., M.V.F. Pereira, S. Granville, M. Finger, P.A. Haldi, L.A. Barroso. 2006. Benchmarking of hydroelectric stochastic risk management models using financial indicators. *General Meeting of the Power-Engineering-Society, Montreal, Canada, 18-22 June*, vol. 1-9. 4449–4456.

[87] Iliadis, N.A., A. Tilmant, R. Chabar, S. Granville, M. Pereira. 2005. Optimal Operation of Hydro-dominated Multireservoir Systems in Deregulated Electricity Markets. Y. Chen, ed., *Proceedings of the International Conference for Reservoir Operation and River Management*. Guanghzou, China.

[88] Infanger, G., D.P. Morton. 1996. Cut sharing for multistage stochastic linear programs with interstage dependency. *Mathematical Programming* **75**(2) 241–256.

[89] Jacobs, J., G. Freeman, J. Grygier, D.P. Morton, G. Schultz, K. Staschus, J.R. Stedinger. 1995. SOCRATES - A system for scheduling hydroelectric generation under uncertainty. *Annals of Operations Research* **59**(1) 99–133.

[90] Jacobson, D.H., D.Q. Mayne. 1970. *Differential Dynamic Programming*. American Elsevier, New York.

[91] Janak, S.L., X. Lin, C.A. Floudas. 2007. A new robust optimization approach for scheduling under uncertainty II. Uncertainty with known probability distribution. *Computers & Chemical Engineering* **31** 171–195.

[92] Jorion, P. 2000. *Value at Risk*. Mc Graw-Hill.

[93] Kall, P., S.W. Wallace. 1994. *Stochastic Programming*. John Wiley & Sons, Chichester.

[94] Kallrath, J., ed. 2004. *Modeling Languages in Mathematical Optimization*. Springer.

[95] Kelman, R., L.A.N. Barroso, M.V.F. Pereira. 2001. Market power assessment and mitigation in hydrothermal systems. *IEEE Transactions on Power Systems* **16**(3) 354–359.

[96] Kerr, A.L., E.G. Read, R.J. Kaye. 1998. Reservoir management with risk aversion. *ORSNZ Proceedings*. 167–176.

[97] Kraus, M. 2005. *Applied Research in Environmental Economics*, *ZEW Economic Studies*, vol. 31, chap. Liberalised Energy Markets Do We Need Re-Regulation? Physica-Verlag HD, 197–218.

[98] Kristiansen, T. 2004. Financial risk management in the hydropower industry using stochastic optimization. *Advanced Modeling and Optimization* **6** 17–24.

[99] Labadie, J. 2004. Optimal operation of multireservoir systems: State-of-the-art review. *Journal of Water Resources Planning and Management* **130**(2) 93–111.

[100] Lamond, B.F., A. Boukhtouta. 1996. Optimizing long-term hydro-power production using markov decision processes. *International Transactions in Operational Research* **3**(3-4) 223–241.

[101] Larson, R. 1967. A survey of dynamic programming computational procedures. *IEEE Transactions on Automatic Control* **12**(6) 767–774.

[102] Lemaréchal, C., A. Nemirovskii, Y Nesterov. 1995. New variants of bundle methods. *Mathematical Programming, Series B* **69**(1-3) 111–147.

[103] Lin, X., S.L. Janak, C.A. Floudas. 2004. A new robust optimization approach for scheduling under uncertainty: I. Bounded uncertainty. *Computers & Chemical Engineering* **28** 1069–1085.

[104] Linderoth, J., A. Shapiro, S. Wright. 2006. The empirical behavior of sampling methods for stochastic programming. *Annals of Operations Research* **142**(1) 215–241.

[105] Lino, P., L.A.N. Barroso, M.V.F. Pereira, R. Kelman, M.H.C. Fampa. 2003. Bid-Based Dispatch of Hydrothermal Systems in Competitive Markets. *Annals of Operations Research* **120** 81–97.

[106] Linowsky, K., A.B. Philpott. 2005. On the Convergence of Sampling-Based Decomposition Algorithms for Multistage Stochastic Programs. *Journal of Optimization Theory and Applications* **125**(2) 349–366.

[107] Löhndorf, N., S. Minner. 2010. Optimal day-ahead trading and storage of renewable energies-an approximate dynamic programming approach. *Energy Systems* **1**(1) 61–77.

[108] Maceira, M.E.P., J.M. Damázio. 2006. Use of the PAR(p) model in the Stochastic Dual Dynamic Programming Optimization Scheme used in the Operation Planning of the Brazilian Hydropower System. *Probability in the Engineering and Informational Sciences* **20** 143–156.

[109] Maceira, M.E.P., V.S. Duarte, D.D.J. Penna, L.A.M. Moraes, A.C.G. Melo. 2008. Ten Years of Application of Stochastic Dual Dynamic Programming in Official and Agent Studies in Brazil - Description of the NEWAVE Program. *16th PSCC, Glasgow, Scotland, July 14-18, 2008*.

[110] Mayne, D.Q. 1966. A Second-Order Gradient Method for Determining Optimal Control of NonLinear Discrete Time Systems. *International Journal of Control* **3** 85–95.

[111] Medina, J., V.H. Quintana, A.J. Conejo. 1999. A clipping-off interior-point technique for medium-term hydro-thermal coordination. *IEEE Transactions of Power Systems* **14**(1) 266–273.

[112] Mo, B., A. Gjelsvik, A. Grundt. 2001. Integrated Risk Management of Hydro Power Scheduling and Contract Management. *IEEE Transactions on Power Systems* **16**(2) 216–221.

[113] Mo, B., A. Gjelsvik, A. Grundt, K. Karesen. 2001. Optimisation of hydropower operation in a liberalised market with focus on price modelling. *PowerTech Proceedings, Porto*.

[114] Morton, D.P. 1996. An enhanced decomposition algorithm for multistage stochastic hydroelectric scheduling. *Annals of Operations Research* **64** 211–235.

[115] Mulvey, J.M., A. Ruszczynski. 1991. A diagonal quadratic approximation method for large scale linear programs. *Operations Research Letters* **12** 205–215.

[116] Noakes, D.J., A.I. McLeod, K.W. Hipel. 1985. Forecasting monthly riverflow time series. *International Journal of Forecasting* **1**(2) 179–190.

[117] Nowak, M. P., W. Römisch. 2000. Stochastic Lagrangian Relaxation Applied to Power Scheduling in a Hydro-Thermal System under Uncertainty. *Annals of Operations Research* **100**(1–4) 251–272.

[118] Otero-Novas, I., C. Meseguer, J.J. Alba. 1999. An iterative procedure for modeling strategic behavior in competitive generation markets. *Proceedings of the 13th Power System Conference*. Trondheim, Norway.

[119] Otero-Novas, I., C. Meseguer, C. Batlle, J.J. Alba. 2000. A simulation model for a competitive generation market. *IEEE Transactions on Power Systems* **15** 250–256.

[120] Overbye, T.J., X. Cheng, Y. Sun. 2004. A Comparison of the AC and DC Power Flow Model for LMP Calculations. *Proccedings of the 37th Hawaii International Conference on System Sciences*.

[121] Pereira, M., N. Campodnico, R. Kelman. 1999. Application of stochastic dual dp and extensions to hydrothermal scheduling. Tech. Rep. 2.0, PSRI. PSRI Technical Report 012/99.

[122] Pereira, M.V.F. 1989. Optimal stochastic operations scheduling of large hydroelectric systems. *International Journal of Electrical Power & Energy Systems* **11** 161–169.

[123] Pereira, M.V.F., M.E.P. Maceira, G.C. Oliviera, L.M.V.G. Pinto. 1992. Combining analytical models and Monte-Carlo techniques in probabilistic power system analysis. *IEEE Transactions on Power Systems* **7**(1) 265–272.

[124] Pereira, M.V.F., L.M.V.G. Pinto. 1985. Stochastic Optimization of a Multireservoir Hydroelectric System: A Decomposition Approach. *Water Resources Research* **21**(6) 779–792.

[125] Pereira, M.V.F., L.M.V.G. Pinto. 1991. Multi-stage stochastic optimization applied to energy planning. *Mathematical Programming* **52** 359–375.

[126] Philpott, A.B. 2004. On models for estimating the effect on prices of CO2 charges. Tech. rep., The University of Auckland.

[127] Philpott, A.B., Z. Guan. 2008. On the convergence of stochastic dual dynamic programming and related methods. *Operations Research Letters* **36** 450–455.

[128] Powell, W., A. Ruszczyński, H. Topaloglu. 2004. Learning algorithms for separable approximations of discrete stochastic optimization problems. *Mathematics of Operations Research* **29**(4) 814–836.

[129] Powell, W.B. 2007. *Approximate Dynamic Programming: Solving the Curses of Dimensionality*. Wiley-Interscience.

[130] PSRI. 2006. *SDDP Model - Methodology Manual*. Version 9.0.

[131] PSRI. 2008. *SDDP Methodology Manual*. Version 9.2.

[132] Radziukyniene, I., S. Rebennack, P.M. Pardalos. 2010. *Optimal Power Flow: A Bibliographic Survey*. Submitted (journal).

[133] Read, E.G. 1989. A dual approach to stochastic dynamic programming for reservoir release scheduling. A.O. Esogbue, ed., *Dynamic programming for optimal water resources system management*. Prentice Hall, NY, 361–372.

[134] Read, E.G., J.G. Culy, T.S. Halliburton, N.L. Winter. 1987. A simulation model for long-term planning of the new zealand power system. G.K. Rand, ed., *Operational research*. North Holland, New York, 493–507.

[135] Read, E.G., M. Hindsberger. 2010. Constructive Dual DP for Reservoir Optimization. S. Rebennack, P.M. Pardalos, M.V.F. Pereira, N.A. Iliadis, eds., *Handbook of Power Systems*. Energy Systems, Springer-Verlag Berlin Heidelberg.

[136] Read, E.G., P. Stewart, R. James, D. Chattopadhyay. 2006. Offer construction for generators with intertemporal constraints via markovian dynamic programming and decision analysis. *EPOC Winter Workshop, Auckland*.

[137] Rebennack, S., B. Flach, M.V.F. Pereira, P.M. Pardalos. 2010. *Hydro-Thermal Scheduling under CO2 Emission Constraints*. Submitted (journal).

[138] Rebennack, S., B. Flach, M.V.F. Pereira, P.M. Pardalos. 2010. *Scenario Tree Approach for Stochastic Dual Dynamic Programming*. Submitted (journal).

[139] Rebennack, S., N.A. Iliadis, J. Kallrath, P.M. Pardalos. 26-30 July 2009. Short term portfolio optimization for discrete power plant dispatching. *Power & Energy Society General Meeting, 2009. PES '09. IEEE*. Calgary, AB, 1–6.

[140] Rebennack, S., N.A. Iliadis, M.V.F. Pereira, P.M. Pardalos. 2009. Electricity and CO2 Emissions System Price Modeling. *IEEE PowerTech 2009, Bucharest*. Romania, 1–6.

[141] Rebennack, S., J. Kallrath, P.M. Pardalos. 2010. Energy Portfolio Optimization for Electric Utilities: Case Study for Germany. E. Bjørndal, M. Bjørndal, P.M. Pardalos, R. Mikael, eds., *Energy, Natural Resources and Environmental Economics*, chap. 14. Energy Systems, Springer. DOI 10.1007/978-3-642-12067-1_14.

[142] Rebennack, S., J. Kallrath, P.M. Pardalos. 2010. *Optimal Storage Design for a Multi-Product Plant: A Non-Convex MINLP Formulation*. Computers & Chemical Engineering (2010), doi:10.1016/j.compcheng.2010.04.002.

[143] Rebennack, S., O. Prokopyev, P.M. Pardalos. 2009. *On the Two-Stage Stochastic Minimum s-t Cut Problem*. Submitted (journal).

[144] Robinson, S.M. 1996. Analysis of sample path optimization. *Mathematics of Operations Research* **21**(3) 513–528.

[145] Rockafellar, R.T., S. Uryasev. 2000. Optimization of Conditional Value-at-Risk. *Journal of risk* **2**(3) 21–42.

[146] Rockafellar, R.T., R. J.-B. Wets. 1991. Scenarios and policy aggregation in optimization under uncertainty. *Mathematics of Operations Research* **16**(1) 119–147.

[147] Römisch, W. 2003. Stability of stochastic programming problems. A. Ruszczyński, A. Shapiro, eds., *Stochastic Programming*, *Handbooks in operations research and management*, vol. 10, chap. 8. Elsevier, Amsterdam, 483–554.

[148] Rong, A., R. Lahdelma. 2007. CO2 emissions trading planning in combined heat and power production via multi-period stochastic optimization. *European Journal of Operational Research* **176**(3) 1874–1895.

[149] Rosa, C., A. Ruszczynski. 1996. On augmented lagrangian decomposition methods for multistage stochastic programs. *Annals of Operations Research* **64** 289–309.

[150] Rubio-Barros, R., D. Ojeda-Esteybar, A. Vargas. 2011. Energy Carrier Networks: Interactions and Integrated Operational Planning. P.M. Pardalos, S. Rebennack, M.V.F. Pereira, N.A. Iliadis, A. Sorokin, eds., *Handbook of Networks in Power Systems: Optimization, Modeling, Simulation and Economic Aspects*. Energy Systems, Springer.

[151] Sahinidis, N.V. 2004. Optimization under uncertainty: state-of-the-art and opportunities. *Computers & Chemical Engineering* **28**(6–7) 971–983. FOCAPO 2003 Special issue.

[152] Salam, S. 2010. Solution to Short-term Unit Commitment Problem. S. Rebennack, P.M. Pardalos, M.V.F. Pereira, N.A. Iliadis, eds., *Handbook of Power Systems*. Energy Systems, Springer-Verlag Berlin Heidelberg.

[153] Schultz, R. 2003. Stochastic programming with integer variables. *Mathematical Programming* **97** 285–309.

[154] Schultz, R., L. Stougie, M.H. Van Der Vlerk. 1996. Two-stage stochastic integer programming: a survey. *Statistica Neerlandica* **50** 404–416.

[155] Scott, T.J., E.G. Read. 1996. Modelling hydro reservoir operation in a deregulated electricity sector. *International Transactions in Operations Research* **3** 209–221.

[156] Sen, S. 2005. Algorithms for stochastic mixed-integer programming models. *Handbooks in OR & MS*, vol. 12, chap. 9. Elsevier, Dordrecht.

[157] Sépibus, J.D. 2007. Scarcity and Allocation of Allowances in the EU Emissions Trading Scheme - A Legal Analysis. NCCR Trade Regulation Working Paper No. 2007/32.

[158] Shapiro, A. 2009. Analysis of Stochastic Dual Dynamic Programming Method. Tech. rep., Georgia Institute of Technology.

[159] Shrestha, G.B., B.K. Pokharel, T.T. Lie, S.-E. Fleten. 2005. Medium term power planning with bilateral contracts. *IEEE Transactions on Power Systems* **20**(5) 627–633.

[160] Slyke, R. Van, R. Wets. 1969. L-shaped linear programs with applications to optimal control and stochastic programming. *SIAM Journal of Applied Mathematics* **17** 638–663.

[161] Stedinger, J.R. 1998. Stochastic multi-reservoir hydroelectric scheduling. *28. Internationales Wasserbau-Symposium, Wasserwirtschaftliche Systeme-Konzepte, Konflikte Kompromisse*. Institut fuer Wasserbau und Wasserwirtschaft, Rheinisch-Westfaelischen Technischen Hochschule Aachen, Aachen, Germany, 89–117.

[162] Stedinger, J.R., B.F. Sule, D.P. Loucks. 1984. Stochastic dynamic programming models for reservoir optimization. *Water Resources Research* **20**(11) 1499–1505.

[163] Stewart, P.A., R.J.W. James, E.G. Read. 2004. Intertemporal considerations for supply offer development in deregulated electricity markets. *6th European IAEE conference: Zurich*.

[164] Tilmant, A., Q. Goor. 2010. Dynamic Management of Hydropower-Irrigation Systems. S. Rebennack, P.M. Pardalos, M.V.F. Pereira, N.A. Iliadis, eds., *Handbook of Power Systems*. Energy Systems, Springer-Verlag Berlin Heidelberg.

[165] Tilmant, A., R. Kelman. 2007. A stochastic approach to analyze trade-offs and risks associated with large-scale water resources systems. *Water Resources Research* **43** 11. W06425, doi:10.1029/2006WR005094.

[166] Tipping, J., E.G. Read. 2010. Hybrid Bottom-Up/Top-Down Modeling of Prices in Deregulated Wholesale Power Markets. S. Rebennack, P.M. Pardalos, M.V.F. Pereira, N.A. Iliadis, eds., *Handbook of Power Systems*. Energy Systems, Springer-Verlag Berlin Heidelberg.

[167] Turgeon, A. 1981. A decomposition method for the long-term scheduling of reservoirs in series. *Water Resources Research* **17**(6) 1565–1570.

[168] Turgeon, A. 1981. Optimal short-term hydro scheduling from the principle of pregressive optimality. *Water Resources Research* **17**(3) 481–486.

[169] Velásquez, J. 2002. GDDP: generalized dual dynamic programming theory. *Annals of Operations Research* **117** 21–31.

[170] Velásquez, J., P.J. Restrepo, R. Campo. 1999. Dual Dynamic Programing: A Note on Implementation. *Water Resources Research* **35**(7) 2269–2271.

[171] Verderame, P.M., C.A. Floudas. 2009. Operational Planning of Large-Scale Industrial Batch Plants under Demand Due Date and Amount Uncertainty. I.

Robust Optimization Framework. *Industrial & Engineering Chemistry Research* **48** 7214–7231.

[172] Verderame, P.M., C.A. Floudas. 2010. Integration of Operational Planning and Medium-Term Scheduling for Large-Scale Industrial Batch Plants under Demand and Processing Time Uncertainty. *Industrial & Engineering Chemistry Research* **49** 4948–4965.

[173] Wallace, S.W., S.-E. Fleten. 2003. Stochastic programming models in energy. A. Ruszczynski, A. Shapiro, eds., *Stochastic Programming*, *Handbooks in Operations Research and Management Science*, vol. 10. North-Holland, 637–677.

[174] Wets, R.J.-B. 1974. Stochastic programs with fixed recourse: The equivalent deterministic program. *SIAM Review* **16**(3) 309–339.

[175] Wolf, D. De, Y. Smeers. 2000. The Gas Transmission Problem Solved by an Extension of the Simplex Algorithm. *Management Science* **46**(11) 1454–1465.

[176] Woo, C.-K., D. Lloyd, A. Tishler. 2003. Electricity market reform failures: UK, Norway, Alberta and California. *Energy Policy* **31**(11) 1103–1115. Themes in current Asian energy.

[177] Yakowitz, S. 1982. Dynamic programming applications in water resources. *Water Resources Research* **18**(4) 673–696.

[178] Yakowitz, S. 1989. Algorithms and Computational Techniques in Differential Dynamic Programming. C.T. Leondes, ed., *Control and Dynamical Systems: Advances in Theory and Applications*, vol. 31. Academic Press, New York, 75–91.

[179] Yang, M., E. Read. 1999. A constructive dual dynamic programming for a reservoir model with correlation. *Water Resources Research* **35**(7) 2247–2257.

[180] Yatsenko, V., N. Boyko, S. Rebennack, P.M. Pardalos. 2010. Space Weather Influence on Power Systems: Prediction, Risk Analysis, and Modeling. *Energy Systems* **2** 197–207.

[181] Yeh, W.W-G. 1985. Reservoir management and operations models: A state of the art review. *Water Resources Research* **21**(12) 1797–1818.

[182] Zheng, Q.P., S. Rebennack, N.A. Iliadis, P.M. Pardalos. 2010. Optimization Models in The Natural Gas Industry. S. Rebennack, P.M. Pardalos, M.V.F. Pereira, N.A. Iliadis, eds., *Handbook of Power Systems*. Energy Systems, Springer.

[183] Zimmermann, H.-J. 2000. An application-oriented view of modeling uncertainty. *European Journal of Operational Research* **122**(2) 190–198.

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