STUDY ON THE EFFECTS OF ANISOTROPIC DISORDER ON SUPERFLUID HELIUM THREE IN HIGH POROSITY AEROGEL USING LONGITUDINAL ULTRASOUND

By

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To my parents and Hyerin
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Longitudinal sound attenuation measurements in superfluid $^3$He in 98% aerogel were conducted at pressures between 14 and 33 bar and in magnetic fields up to 4.44 kG. The temperature dependence of the ultrasound attenuation in the $A$-like phase was determined for the entire superfluid region by exploiting the field induced meta-stable $A$-like phase at the highest field. In lower fields, the $A – B$ transition in aerogel was identified by a smooth jump in attenuation on both cooling and warming. Based on the transitions observed on warming, a phase diagram as a function of pressure ($P$), temperature ($T$) and magnetic field ($B$) is constructed. The transitions obtained by isothermal field sweeps are consistent with those by temperature sweeps at constant magnetic fields. The $A – B$ phase boundary in aerogel recedes to the corner of zero temperature and melting pressure in response to an increasing magnetic field, which is drastically different from the bulk.

The presence of elastic impurity scattering by aerogel limits the growth of the mean free path at low temperature. In this case, the dominance of temperature independent elastic scattering keeps the system from entering into collisionless limit on cooling. Therefore, it is expected that the sound attenuation obeys the $\omega^2$-dependence. However, our result reveals that non-trivial frequency dependencies, departing from the $\omega^2$-dependence appear as temperature lowers into the superfluid regime. This tendency
is more evident at higher pressure and lower temperature. We attribute this property to the gapless behavior of superfluid $^3$He in aerogel.
CHAPTER 1
INTRODUCTION

Liquid helium is a fascinating condensed matter system that has attracted a vast amount of interest owing to its unique low temperature properties. It is the only material in nature that exists as liquid even at absolute zero temperature due to the large zero point energy and the weak attractive interatomic interaction. The boiling points are 4.21 K and 3.19 K for $^4$He and $^3$He, respectively. This unique property allows physicists to investigate quantum phenomena in liquid state.

The most intriguing part is that both liquids undergo phase transitions to superfluid states. However, different quantum statistics applicable to $^3$He and $^4$He make striking differences in the physical behavior in superfluid as well as in normal fluid. In contrast to $^4$He, which is a composite boson with spin 0, $^3$He obeys Fermi statistics with spin $1/2$. As a result, $^3$He becomes a superfluid through BCS pairing. Because of its charge neutrality and the absence of the lattice structure, the attractive pair interaction is mediated through the spin polarization, giving rise to the spin triplet pairing ($S = 1$) rather than the spin singlet pairing ($S = 0$) as in conventional superconductors. It has been experimentally known that three distinct stable superfluid phases exist in bulk $^3$He, referred to as the $A$-, the $B$- and the $A_1$-phases. Figure 1-1 shows the $T - P$ phase diagram of liquid $^3$He in the absence of magnetic field. Since the discovery of the superfluidity in $^3$He [1], a tremendous amount of theoretical and experimental works have been performed and revealed rich quantum phenomena associated with spontaneous symmetry breaking. Moreover, its exceptional purity has offered an opportunity to test theoretical ideas such as the generalized BCS theory and Fermi liquid theory.

In addition to the exceptional purity, since the structures of order parameters of superfluid $^3$He were well known, it was expected that this system would provide a much better understanding for the impurity effects on unconventional Cooper pairing systems.
Unlike the case of conventional superconductors, all types of impurities are detrimental to Cooper pairs with a non-zero angular momentum. Therefore, superfluid $^3$He with $p$-wave pairing is expected to be strongly influenced by any types of impurity. However, a systematic investigation on effects of impurity or disorder had not been achieved until high porosity silica aerogel was employed in superfluid $^3$He for the first time by Porto and Parpia [2]. Since then, aerogel has played a major role as an impurity in superfluid $^3$He. Most physical quantities such as transition temperature, order parameter, and transport parameters are rescaled or influenced dramatically in this system due to the pair-breaking by scattering off the aerogel. In addition, the correlation of aerogel structure and their unusual length scales make the system more interesting and also unique.

Ultrasound ($\sim$ MHz) has been one of the most useful tools to probe the properties of liquid $^3$He. There exist several sound modes both in pure and impure (aerogel)
liquids, and their different responses to different phases of liquid have been utilized to characterize them. Most of all, the coupling of the zero sound to the order parameter collective modes (OPCM) lends us an exceptional tool in studying the superfluid phases of $^3$He.

In this work, the effects of high porosity aerogel on superfluid as well as normal $^3$He were investigated using various ultrasound techniques. In chapter 2, some of basic properties of normal and superfluid $^3$He are discussed. In chapter 3, the impurity effects of aerogel on superfluid $^3$He are discussed. We will survey the current status of the field by reviewing some of the important experimental results and theoretical ideas. Chapter 4 describes our longitudinal ultrasound experiment conducted in superfluid $^3$He in 98% porosity aerogel. The sound attenuation measurements in bulk liquid $^3$He using broadband transducers are discussed in chapter 5. Finally, in the conclusion, a summary and a few suggestions are given in connection with the direction for the future works.
CHAPTER 2
BASIC PROPERTIES OF LIQUID HELIUM THREE

2.1 Normal Liquid $^3$He

Since it has a spin 1/2 nucleus and no net angular momentum from the electrons, $^3$He atom is a fermion. Below about 100 mK, liquid $^3$He is in the degenerate regime where the thermally excited particles are limited to a very narrow region near the Fermi energy, $\epsilon_F$. However, unlike a non-interacting ideal Fermi gas, $^3$He atoms in liquid feel rather strong interactions and are strongly correlated. In 1957, Landau formulated a phenomenological theory for an interacting fermion system, called a Fermi liquid, in a series of seminal papers [3–5]. Liquid $^3$He is a perfect example of this Fermi liquid theory and has served as the paradigm for many decades.

2.1.1 Fermi Liquid Theory

Landau made two assumptions in his theory. The first was that at low temperature, $T \ll T_F \sim 1$ K, $^3$He atoms would behave as quasiparticles of effective mass, $m^*$, which are low-lying and long-lived excitations of strongly interacting fermionic particles. The quasiparticles have one-to-one correspondence to the excitations of the noninteracting Fermi gas, therefore, they carry the same spin 1/2 and the same Fermi momenta, $\vec{p}_F = \hbar \vec{k}_F$. Secondly, he introduced a set of effective interaction between the quasiparticles. When a quasiparticle with energy $\xi_{\vec{k}\sigma}$ relative to the chemical potential, $\mu$, is added to the system, it induces a change in the distribution function, $\delta n_{\vec{k}\sigma}$. Here, $\sigma$ represents the spin quantum number. Then, the quasiparticle energy $\epsilon_{\vec{k}\sigma}$ is no longer identical to $\xi_{\vec{k}\sigma}$ due to interaction. The difference, $\delta \epsilon_{\vec{k}\sigma} = \epsilon_{\vec{k}\sigma} - \xi_{\vec{k}\sigma}$, can be then written as

$$\delta \epsilon_{\vec{k}\sigma} = \sum_{\vec{k}'\sigma'} f_{\vec{k}\sigma\vec{k}'\sigma'} \delta n_{\vec{k}'\sigma'},$$

where the interaction function can be parametrized for isotropic system by

$$f_{\vec{k}\sigma\vec{k}'\sigma'} = \frac{1}{D(\epsilon_F)} \sum_l P_l(\cos \theta) [F_l^s + F_l^o \sigma \sigma'],$$

(2–1)
where $D(\epsilon_F) = V m^* p_F/(\pi^2 \hbar^3)$ is the density of states, $\hat{k} = \vec{k}/|\vec{k}|$, $\sigma = \pm 1$ and $P(\cos \theta)$ is the Legendre polynomial. The parameters $F_i^s$ and $F_i^a$ represent the spin-symmetric and spin-antisymmetric dimensionless Landau parameters, respectively. Most of the physical quantities in this low temperature region are renormalized by these interaction parameters and the effective mass from those of a Fermi gas. For example, the specific heat ($C$), sound velocity ($c$) and susceptibility ($\chi$) of the normal Fermi liquid (denoted by subscript $L$) and Fermi gas ($g$) are related through normalization:

\[
C_L = \frac{m^*}{m} C_g = \frac{m^* k_F^2 k_B T}{3 \hbar^2}, \tag{2–3}
\]

\[
c_L^2 = c_g^2 (1 + F_0^s) \left(1 + \frac{F_1^s}{3}\right) = \frac{p_F^2}{3 m^2} (1 + F_0^s) \left(1 + \frac{F_1^s}{3}\right), \tag{2–4}
\]

\[
\chi_L = \chi_g \frac{m^*}{m} (1 + F_0^a)^{-1} = \frac{\hbar^2 \gamma^2}{4} D(\epsilon_F) (1 + F_0^a)^{-1}, \tag{2–5}
\]

where $\gamma$ is the gyromagnetic ratio. Since, from the Galilean invariance

\[
\frac{m^*}{m} = 1 + \frac{1}{3} F_1^s, \tag{2–6}
\]

$F_0^a$, $F_0^s$ and $F_1^s$ can be determined from the heat capacity, sound velocity and susceptibility measurements. For example, $F_0^a \approx -0.74$, $F_0^s \approx 94$ and $F_1^s \approx 15.7$ at melting pressure [6], indicating the existence of strong interactions.

2.1.2 Collective Modes

One of the successful achievements of Landau’s theory was to predict the existence of zero sound modes. Normally, the ordinary sound (hydrodynamic or first sound) propagates by compression and rarefaction of particles in the low frequency range satisfying $\omega \tau \ll 1$ where $\omega$ is the angular frequency of the sound wave and $\tau$ is the quasiparticle relaxation time of the system. The effect of Fermi liquid interaction is also reflected in the first sound velocity and attenuation [7]:

\[
\frac{m^*}{m} = 1 + \frac{1}{3} F_1^s, \tag{2–6}
\]
\[ c_1^2 = \frac{1}{3} v_F^2 (1 + F_0^s) \left( 1 + \frac{F_1^s}{3} \right), \]  
(2–7)

\[ \alpha_1 = \frac{2 v_F^2}{15c^3} \left( 1 + \frac{F_1^s}{3} \right) \left( 1 + \frac{F_2^s}{5} \right) \omega^2 \tau_{\eta}, \]  
(2–8)

where \( \tau_{\eta} \) is the relaxation time for viscosity. Comparing this attenuation with a classical expression,

\[ \alpha_1 = \frac{2 \omega^2 \eta}{3 \rho c^3}, \]  
(2–9)

one can find the expression for the shear viscosity of a Fermi liquid as

\[ \eta = \frac{1}{5} \rho v_F^2 \left( 1 + \frac{F_1^s}{3} \right) \left( 1 + \frac{F_2^s}{5} \right) \tau_{\eta}. \]  
(2–10)

Therefore, when \( \omega \tau \gg 1 \), the first sound mode experiences increasing damping and eventually ceases to propagate. Landau argued that a new mode of sound could emerge in this collisionless limit in a Fermi liquid in which the essential restoring force would arise from the molecular field. Wölfle [8] calculated the zero sound velocity and attenuation using a kinetic equation and a collision integral,

\[ c_0 = c_1 \left[ 1 + \frac{2 m^*}{5} \left( 1 + \frac{1}{5} F_2^s \right) \left( \frac{v_F}{c_1} \right)^2 + O \left( \frac{v_F}{c_1} \right)^4 \right], \]  
(2–11)

\[ \alpha_0 = \frac{8}{45} \frac{m^* v_F^2}{m c_0^3} \left( 1 + \frac{F_2^s}{5} \right)^2 (1 - \lambda_2)(\tau_{\eta}^0)^{-1}, \]  
(2–12)

where \( \tau_{\eta}^0 \) is the quasiparticle life time on the Fermi surface and related to \( \tau_{\eta} \) by
\[
\tau_\eta = \left[ \frac{\pi^2}{12} + \frac{3}{4} \lambda_2 (1 - \lambda_2)^{-1} \right] \tau_N^0, \quad (2\text{–}13)
\]

where the parameter \(\lambda_2\) is an angular average of the scattering cross section corresponding to \(l = 1\) in Legendre polynomial, \(P_l(\cos \theta)\). \(\lambda_2 \approx 0.72\), giving \(\tau_\eta = 2.75 \tau_N^0\) [8]. In fact, his theory can be applied for an arbitrary value of \(\omega \tau\). In the \(\omega \tau \ll 1\) limit, the theory recovers exactly the same results as Eq. 2–7 and Eq. 2–8 when ignoring \(F_2^s\).

Since \(\tau_N^0 \propto \frac{1}{T^2}\) in a Fermi liquid, the attenuation of zero and first sound show very different temperature dependences. From Eqs. 2–8 and 2–12, \(\alpha_1 \sim 1/T^2\) and \(\alpha_0 \sim T^2\). Therefore, the unmistakable crossover between two sound regimes should appear as temperature changes, which was conclusively confirmed experimentally by Abel et al. [9].

Later, Rudnick [10] developed a theory on zero sound based on the fact that zero sound is a viscoelastic effect, as was recognized by Lea [11]. He was able to express the sound velocity and attenuation in one equation for each instead of having two expressions for the two limits, respectively. The viscoelastic model provided excellent fits to the measurements of Abel et al. [9] at low pressures and Ketterson et al. [12] at high pressures for a wide temperature range including the crossover region with

\[
c_l = c_1 + (c_0 - c_1) \frac{\omega^2 \tau_\eta^2}{1 + \omega^2 \tau_\eta^2}, \quad (2\text{–}14)
\]

\[
\alpha_l = \frac{c_0 - c_1}{c_1^2} \frac{\omega^2 \tau_\eta}{1 + \omega^2 \tau_\eta^2}. \quad (2\text{–}15)
\]

Note that these are classical results without the consideration of a Fermi liquid.

In a real experimental situation, the presence of confining wall causes an additional attenuation and a correction to the velocity because the fluid starts to slip at the walls when the mean free path is comparable to the sample size. Nagai and Wölfle [13] calculated the sound velocity and attenuation using a set of hydrodynamic equations,
including the effect of cylindrical wall in the limit where the viscous penetration depth
\( \delta = \left( \frac{2\eta}{\rho \omega} \right)^{1/2} \) is smaller than the dimension of the resonator and the wavelength of the sound. They obtained the general expression for the sound velocity and attenuation,

\[
c = c_1 \left[ 1 + \frac{2\omega}{3\rho c_1^2} Im \left[ \frac{\eta}{1 - i\omega \tau_\eta} \right] + \frac{1}{\rho R \omega} Im \left[ Z(\omega) \right] \right],
\]

(2–16)

\[
\alpha = \frac{2\omega^2}{3\rho c_1^2} Re \left[ \frac{\eta}{1 - i\omega \tau_\eta} \right] + \frac{1}{\rho R c_1} Re \left[ Z(\omega) \right],
\]

(2–17)

where \( Z(\omega) \) is the complex surface impedance, and \( R \) and \( L \) are the radius and the length of the cylindrical acoustic resonator.

One of the most fascinating aspects of zero sound in a Fermi liquid is the possibility of propagating transverse zero sound (TZS) mode [3–5]. In general, the transverse wave in liquids decays within a length comparable to the wavelength. Therefore, the prediction by Landau on transverse zero sound mode in a Fermi liquid is extremely interesting. However, the relevant Fermi liquid interaction for TZS in liquid \(^3\)He is marginally strong to support this mode. This results in the speed of TZS very close to \( v_F \) causing strong Landau damping. Although there is no unequivocal experimental evidence of TZS in the normal state of \(^3\)He, the existence of propagating TZS in the B-phase of \(^3\)He was beautifully demonstrated by Lee et al. [14].

### 2.2 Superfluid \(^3\)He

Above the superfluid transition, the normal liquid possesses all the symmetries that condensed matter can have. In the superfluid \(^3\)He, most of the symmetries are broken spontaneously and the system exhibits dramatic changes in its physical properties. All three superfluid phases in liquid \(^3\)He are known to have \( p \)-wave spin triplet pairing. Unlike conventional \( s \)-wave superfluid, therefore, the order parameter of superfluid \(^3\)He is complex. In this section, we will briefly describe the order parameter structure of the superfluid phase of \(^3\)He and important properties of the superfluid phases pertinent to this work.
2.2.1 Superfluid Phases of $^3$He

The order parameter of a $p$-wave triplet superfluid can be written in the basis of symmetric Pauli matrices, $(i \sigma_x \sigma_y, i \sigma_y \sigma_y, i \sigma_z \sigma_y)$, in terms of the vector $\vec{d}(\hat{k})$ in spin space, which is defined through the gap parameter $\Delta_{\vec{k}\alpha\beta}$ as

$$\Delta_{\vec{k}\alpha\beta} = \sum_{\mu} d_{\mu}(\vec{k}) (\vec{\sigma}_{\mu} i \vec{\sigma}_{2})_{\alpha\beta} = (-d_x + id_y) | \uparrow\uparrow > + (d_x + id_y) | \downarrow\downarrow > + d_z | \uparrow\down + \down\uparrow >,$$

where $\hat{k}$ is a unit vector in momentum space. The vector $\vec{d}(\hat{k})$ can be expanded in $\hat{k}_j$ with a tensor quantity $d_{\mu j}$, a $3 \times 3$ matrix of complex components, where $\mu$ and $j$ correspond to the values of -1, 0, +1 of the quantum numbers $S_z$ and $L_z$, respectively,

$$d_{\mu}(\vec{k}) = \sum_{j} d_{\mu j} \hat{k}_j.$$

In the absence of a magnetic field, the free energy minimum is obtained by the following spherically symmetric order parameter,

$$d_{\mu j} = \Delta e^{i \phi} \delta_{\mu j} \rightarrow \Delta_{\vec{k}} = \Delta e^{i \phi} \left( \frac{-\hat{k}_x + i \hat{k}_y}{\sqrt{2}} | \uparrow\uparrow > + \frac{(\hat{k}_x + i \hat{k}_y)}{\sqrt{2}} | \downarrow\down > + \hat{k}_z | \uparrow\down + \down\uparrow > \right),$$

where $\Delta$ is an isotropic energy gap. This state was first discussed by Balian and Werthamer [15] and is referred to as the ‘$BW$ state’, which corresponds to the $B$-phase of superfluid $^3$He. This state consists of the superposition of all spin triplet states and has an isotropic gap. Since the free energy is invariant under the rotation of the spin space relative to the orbital space, the general form of the $BW$ state can be written by

$$d_{\mu j} = \Delta e^{i \phi} R_{\mu j}(\hat{n}, \theta),$$

where $R_{\mu j}(\hat{n}, \theta)$ describes a relative rotation of spin and orbital spaces around $\hat{n}$ by an angle $\theta$. Therefore, the $BW$ state is invariant under simultaneous rotation in spin and orbital space and only the relative spin-orbit symmetry is broken.
In the weak coupling theory, the $BW$ state is always stable over any other possible states with $p$-wave spin triplet pairing. However, the inclusion of strong coupling effects such as spin fluctuations weakens the stability of the $BW$ state relative to another state, called the axial state that was first proposed by Anderson and Morel [16]. Later, it was realized by Anderson and Brinkman [17] that the experimentally observed $A$-phase at high pressures is consistent with the theoretical axial phase which is now called the $ABM$ state. Unlike the $BW$ state, the $ABM$ state is highly anisotropic and has Cooper pairs with only $S_x = \pm 1$. This is why this state is referred as an equal-spin pairing state.

The order parameter discussed by Anderson and Morel [16] is

$$d_{\mu j} = \Delta_0 \hat{d}_\mu (\hat{m}_j + i\hat{n}_j),$$

where $\hat{l} = \hat{m} \times \hat{n}$ indicates the orbital angular momentum of the Cooper pair and $\hat{m}$ and $\hat{n}$ are mutually orthogonal unit vectors in orbital space.

In the presence of a magnetic field, the phase diagram changes drastically. Since the magnetic susceptibility of the $BW$ state is lower than that of the $ABM$ state (due to the $S_z = 0$ component), the magnetic energy is lower in the $ABM$ state than in the $BW$ state. Therefore, it is reasonable to think that the $ABM$ state would gain its ground against the $BW$ state as the strength of magnetic field increases and would eventually become more stable. However, the effect of a magnetic field on the phase diagram is more subtle and profound [18]. Even an infinitesimally weak magnetic field opens up a sliver of the $A$-phase region below the superfluid transition at all pressures. The $A$-phase region continuously grows as the strength of magnetic field increases and eventually pushes the $B$-phase out of the phase diagram around $B \approx 0.6$ T. The phase diagram shown in Fig. 2-1 clearly demonstrates this behavior. The profound effect of magnetic field will be discussed in detail in the following chapter.

The third phase of superfluid $^3$He, the $A_1$-phase, appears only in the presence of a magnetic field by splitting the superfluid transition into two second order transitions.
Figure 2-1. Phase diagram of liquid $^3$He in the magnetic field.

The Zeeman energy separates the transition temperature for the spin up and down components because of the minute particle-hole asymmetry. As a result, in the $A_1$-phase only the spin up component forms Cooper pairs into fully polarized superfluid state. The width of the $A_1$-phase is rather small and almost proportional to the strength of magnetic field, $\sim 60 \mu K/T$ at the melting pressure.

The presence of a magnetic field causes a distortion of the otherwise isotropic $BW$ state gap, the so called gap distortion. Tewordt and Schopohl [19] studied this effect and showed that the gap component perpendicular to magnetic field, $\Delta_\perp$, increases with increasing magnetic field, while the parallel component, $\Delta_\parallel$, decreases and suddenly falls to zero at a certain critical value of magnetic field. A prospective distortion process is illustrated in Fig. 2-2.
2.2.2 Orientational Effects

The order parameters of the BW and the ABM state (Eqs. 2–21 and 2–22) reveal their anisotropic nature in the form of \( \hat{n}, \hat{d}, \) and \( \hat{l} \). The appearance of these vectors indicates that the system will choose a preferred direction. However, the degeneracy of a specific direction still remains. Several external or internal perturbations which couple to the order parameter can lift this degeneracy: magnetic field, electric field, wall, superflow, and dipole-dipole interactions. When only one perturbation is considered, the order parameter aligns uniformly throughout the system showing a uniform texture. In reality, multiple sources of perturbation compete each other. In this case, the system will find the lowest energy configuration incorporating spatial variations of the preferred direction, producing a non-uniform texture.

For example, the dipole energy density in the \( A \)- and the \( B \)-phases are found as [20]

\[
\Delta f_D^A = -\frac{3}{5} g_D(T)(\hat{d} \cdot \hat{l})^2, \tag{2–23}
\]

\[
\Delta f_D^B = \frac{8}{5} g_D(T) \left( \cos \theta + \frac{1}{4} \right)^2, \tag{2–24}
\]

where \( g_D(T) \) is the dipole coupling constant defined by

\[
g_D(T) \equiv \lambda_D D(\epsilon_F) \Delta^2(T), \quad \lambda_D = \frac{3\pi}{2} \frac{1}{l(l+1)} \frac{\gamma^2 h^2/a^3}{E_F} \frac{1}{(D(\epsilon_F)|V_i|)^2}. \tag{2–25}
\]
Here, $V_l$ is the pair-interaction constant for a given $l$ (= 1 for a $p$-wave) and $a$ the average atomic spacing. Therefore, the minimum energy configuration is given by $\hat{d} \parallel \hat{l}$ for the $A$-phase and the relative spin-orbit rotation angle $\theta_L = \cos^{-1}(-\frac{1}{4}) \approx 104^\circ$ for the $B$-phase where $\theta_L$ is called "Leggett angle". In magnetic fields, the Leggett angle is modified by the gap distortion,

$$\theta_0(H) = \cos^{-1}\left( -\frac{1}{4} \frac{\Delta}{\Delta_\perp} \right). \quad (2–26)$$

The effect of a magnetic field is obtained by considering the magnetic energy density as [20]

$$\Delta f_H^A \propto (\hat{d} \cdot \vec{H})^2, \quad (2–27)$$

$$\Delta f_H^B \propto -(\hat{n} \cdot \vec{H})^2. \quad (2–28)$$

Therefore, the preferred orientations would be $\hat{d} \perp \vec{H}$ and $\hat{n} \parallel \vec{H}$.

The wall also has a significant effect on the texture. The order parameter, component, $\Delta_\perp$, is strongly suppressed within a layer of thickness of the coherence length, which renders both vectors, $\vec{l}$ in the $A$-phase and $\hat{n}$ in the $B$-phase, aligned to the surface normal ($\hat{s}$). In general, although the order parameter is restored to its bulk value within a few coherence length, the order parameter pertains the direction determined by the surface for a much larger length scale the healing length [21].

It is useful to define the healing length to have an idea of how a continuous configuration of the order parameter field, called a texture, forms in various situations. The healing length are defined for several orienting forces such as dipole, magnetic field, wall, and etc. by equating their corresponding energy gains to the bending energy cost. The healing lengths for three orienting forces are summarized in Table 2-1. Note that the surface healing length in the $B$-phase is quite long so that in a typical experimental
cell, the boundary condition at surface is not so demanding in contrast to the $A$-phase in which $\hat{l} \parallel \hat{s}$ should be strictly satisfied.

Table 2-1. Healing lengths for various orienting forces

<table>
<thead>
<tr>
<th>Orienting forces</th>
<th>Healing length</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole</td>
<td>$\xi_D \approx 8\mu m$, $\xi_B \approx 7\mu m$</td>
<td>in the G-L regime at melting $P$</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>$\xi_A \propto \frac{1}{H} \xi_0$, $\xi_B \propto \frac{1}{H} \xi_0$</td>
<td>$\xi_A \approx \xi_D$ at $\approx 28$ G, $\xi_B \gg \xi_A$ and $\xi_D$</td>
</tr>
<tr>
<td>Surface</td>
<td>$\xi_S \approx 0.2(1 - T/T_c)^{1/2} \text{ cm}$</td>
<td>$\xi_S \approx 1\text{ mm}$ at $T = 0.7T_c$</td>
</tr>
</tbody>
</table>
CHAPTER 3
SUPERFLUID HELIUM THREE IN 98% AEROGEL

Observation of superfluid transition in liquid $^3$He impregnated in high porosity silica aerogel has opened a way to introducing static disorder/impurities in this system and triggered immediate theoretical and experimental activities. The unique structure of aerogel formed by an entangled network of nanometer sized $SiO_2$ strands presents more than conventional randomly distributed isotropic scattering centers. The network of the impurity scattering can be altered by modifying the composition of the surface layers from magnetic to purely potential scattering. Furthermore, the correlated strand-like structure inevitably introduces local random anisotropy. Therefore, the effect of disorder is not simply limited to the suppression of superfluid by pair-breaking. There are numerous interesting phenomena observed and expected in this system.

In this chapter, we will survey the current status of the field by reviewing some of the important experimental results and theoretical ideas.

3.1 Aerogel and Scattering Models

Aerogel, known for more than seventy years, is a highly porous material comprised of a fractal network of the $SiO_2$ strands of $\delta \approx 3 - 5$ nm diameter. Aerogel can be synthesized with porosity up to 99.9%. Owing to their unique topological nano-porous structure and extremely low density, aerogels have found applications in various areas of industry as super heat insulators, light transmitting fibers, non-reflecting material, and transparent walls as well as in academies as Cerenkov counter and study of quenched disorder in quantum fluids.

Most of the experiments including this work in liquid $^3$He used 98% porosity aerogels. For a typical aerogel sample with 98% porosity, the average distance between the strands, which is essentially the correlation length of the aerogel ($\xi_a$), is in the range of 30 - 40 nm. The geometric mean path is $\ell_a \approx 120 - 150$ nm. Another important length is the coherence length of superfluid, $\xi_o$, defined by $\xi_o = \frac{\hbar v_F}{2\pi k_B T_c}$, where $T_c$ is the
superfluid transition in bulk. $\xi_o$ spans between 16 nm (melting pressure) and 77 nm (zero pressure) [22]. It is expected that the changes in the Fermi liquid parameters and the dipole-dipole interaction constant ($g_D$) due to the scattering off the aerogel are negligibly small since the Fermi wavelength, $\lambda_F = \frac{2\pi}{k_F} \approx 0.7$ nm, is much smaller than the length scales of aerogel [23].

Considering all of these aerogel properties, Thuneberg et al. [22] and Thuneberg [23] discussed various scattering models based on the quasiclassical theory. Homogeneous scattering model (HSM) is the simplest among those and assumes that the medium is isotropic, i.e., the mean free path ($\ell$) is independent of the quasiparticle momentum direction, and the scattering center distribution is uniform and random. This model conveniently gives the same formalism as in bulk for the Ginzburg-Landau theory and Leggett’s theory on NMR [24] with renormalized parameters. Although HSM predicts suppression of $T_c$ and superfluid density ($\rho_s$) in the right direction, it fell short in explaining the experimental results for the entire pressure range. The superfluid transition in aerogel ($T_{ca}$) according to this model is given by

$$\ln \frac{T_c}{T_{ca}} = 2 \sum_{n=1}^{\infty} \left( \frac{1}{2n - 1} - \frac{1}{2n - 1 + x} \right),$$

where $x = \xi_o/\ell$ is the Abrikosov-Gorkov depairing parameter [22].

When $\xi_a$ is comparable to $\xi_o$, as is at high pressures, it is important to consider inhomogeniety and anisotropy of aerogel. Hänninen and Thuneberg [25] studied inhomogeneous but isotropic scattering model (IISM) extensively. This model gives better agreement with experiments but predicts a significant temperature dependence of suppression factor for the order parameter.

Sauls and Sharma [28] proposed a phenomenological IIS model by redefining the depairing parameter $x = \hat{x}/(1 + \xi_a^2/\hat{x})$, where $\xi_a = \xi_a/\ell$, $\hat{x} = \xi_o/\ell$. This model produced an excellent fit of the superfluid transition in 98% aerogel to the experimentally determined
transition temperatures by the Cornell group [27] and the Northwestern group [26] (see Fig. 3-1).

The anisotropic HSM emphasizes the importance of anisotropic nature of aerogel strands. The main effect of anisotropy can be incorporated into the Ginzburg-Landau free energy through the quadratic order parameter term which shifts the transition temperature [23]. In the limit of $\xi_a \leq \xi_o$, this local anisotropy is averaged out recovering HISM with the modified Ginzberg-Landau coefficients, $\beta_i$ [22]. It is believed that anisotropic scattering stabilizes the $A$-phase. Recently, the effects of anisotropic scattering have attracted attention following the observation of the existence of the $A$-phase at low pressure in aerogel [29]. This subject will be discussed in detail later.
3.2 Superfluidity and Superfluid Phases

The first observation of a superfluid transition of liquid $^3$He in 98% aerogel was made by Porto and Parpia [2] using a tortional oscillator. They measured the resonance frequency of the oscillator and found that it showed an abrupt increase at a temperature slightly lower than the bulk superfluid transition, indicating decoupling of mass inside the oscillator. The superfluid fraction obtained from the shift in resonance frequency revealed striking features. Unlike bulk, the superfluid fraction was found to reach a value much less than unity in the zero temperature limit, which monotonically decreases with pressure (see Fig. 3-2). In the same year, Sprague et al. [30] also reported superfluid transitions in 98% aerogel using a pulsed NMR technique. Based on the frequency shift and magnetization, they concluded that the observed superfluid phase was an equal spin pairing state. A year later, Sprague et al. [31] observed a transition from an ESP to a non-ESP state. Alles et al. [32] showed that this non-ESP state could be identified as the $B$-phase of the bulk, based on the analyses of their NMR spectra.

Along with the systematic suppression of the superfluid transition temperature ($T_{ca}$) in 98% aerogel, the gap suppression is also expected. According to HSM, the ratio of the gap suppression is the same as the ratio of suppression of the superfluid temperature. However, several experiments estimated much severer gap suppression than predicted by HSM. For example, Barker et al. [33] estimated about 50% of gap suppression from the NMR frequency shift at 32 bar, and Halperin et al. [34] showed similar suppression factors utilizing the data of NMR and specific heat by Sprague et al. [30, 31] and Choi et al. [35], respectively (see Fig. 3-3), demonstrating that HSM is not a suitable model for superfluid $^3$He in aerogel.

Two distinct superfluid phases have been observed in $^3$He in aerogel in the absence of a magnetic field, called the $A$-like (ESP) and the $B$-like (non ESP) phases. In contrast to the spin structures, the identification of the orbital structures for both the $A$-like and the $B$-like phases are still inconclusive. Presently, it is believed that the $B$-like phase has
the same order parameter as that of the $B$-phase in bulk. In addition to the work of Alles et al. [32], Dmitriev et al. provided the most convincing evidence for this identification by observing the sharp NMR frequency shift at Leggett angle ($\beta_L \approx 104^\circ$) [36] as well as the homogeneous spin precession domain (HPD) in the $B$-like phase [37]. By contrast, the identification of the orbital structure of the $A$-like phase is far from conclusive.

According to Imry and Ma [38], an arbitrarily weak field destroys long-range order since it is energetically favorable to have the system break into domains at large distances. Based on this argument, Volovik [39] pointed out that the $A$-like phase of superfluid $^3$He in aerogel is a globally isotropic state without a long range order, called glass or LIM (Larkin-Imry-Ma) state. This LIM state has no superfluidity, in other words superfluid density ($\rho_s$) is zero. However, the superfluidity can be restored by an
Suppression of the transition temperature and the order parameter. The amplitude of order parameter is determined from the NMR frequency shifts \[30, 31\] and the specific heat jump measurements \[35\]. [Figure reproduced with permission from W. P. Halperin et al., J. Phys. Soc. Jap. 77, 111002 (2008). Copyright (2008) by the Physical Society of Japan.]

Application of a small magnetic field (\(~30\) G) in the dipole locked case in which \(L_0 > \xi_D\) or large superflow \[40\], where \(L_0\) is the LIM length and \(\xi_D\) (\(~10\) \(\mu\)m) is the dipole length. Recently, it was noted that the regular anisotropy introduced by controlled deformation of aerogel can also restore the superfluidity of the \(A\)-like phase. For example, Kunimatsu et al. \[41\] observed a large negative NMR frequency shift in the \(A\)-like phase and Sato et al. \[42\] also found the stabilized coherent precession of magnetization in the \(A\)-like phase. Both experiments were performed in compressed aerogels and their results are consistent with the configuration of \(\vec{l}\) parallel to magnetic field, indicating that the long range order of \(\vec{l}\) is restored in aerogel.
From a different point of view, Fomin [43] suggested another phase as a candidate for the A-like phase, called the robust phase, which has the form of an ESP state,

\[ A_{\mu j} = \frac{\Delta}{3} [\hat{d}_\mu (m_j + i n_j) + \hat{e}_\mu (l_j + i p_j)], \tag{3–2} \]

where \( \hat{d}_\mu \) and \( \hat{e}_\mu \) are the mutually orthogonal unit vectors, and this order parameter should satisfy the condition,

\[ \bar{A}_{\mu l} \bar{A}^*_{\mu j} + \bar{A}_{\mu j} \bar{A}^*_{\mu l} = \delta_{jl} \cdot \text{const.}, \tag{3–3} \]

where \( \bar{A}_{\mu j} \) is the mean order parameter over the large length scale compared to the distance between aerogel strands. Under this condition, the interaction with a random field (by aerogel) vanishes and long-range order is preserved. \( \bar{A}_{\mu j} \) for the A-like phase needs to be quasi-isotropic with equal spin pairing (ESP). There are some theoretical [44] and experimental [45] results that might support the idea of the robust phase, but it appears that this phase is not considered as the thermodynamically favored state [46–48].

### 3.3 Phase Diagram

The A-like phase in aerogel shows a metastable property as the A-phase in bulk. A large supercooling effect which is an indication of metastability has been observed [26, 30, 49–52] even at low pressures below the bulk polycritical point (PCP). Gervais et al. [26] have performed systematic measurements using a transverse ultrasound technique at several pressures and magnetic fields. Based on their tracking experiment at 33.4 bar, they concluded that the A-like phase region should lie in a very narrow temperature window (\( \leq 20 \mu K \)) just below the superfluid transition, \( T_{ca} \). Furthermore, from the field dependent suppression of the \( A - B \) transition, they determined a strong coupling parameter, \( g_a(\beta) \) (see Eq. 4–2) for five pressures, and concluded that the poly critical point (PCP) did not exist in aerogel since no divergence in \( g_a(\beta) \) as a function of pressure was observed. Later, Vicente et al. [29] performed tracking experiments
Figure 3-4. The relative size of the steps for the supercooled aerogel $A - B$ transition as a function of the turn-around temperature. The dashed vertical lines indicate the aerogel superfluid transition temperature. The $A$- and the $B$-like phases coexist in the shaded regions (see Ref. [29]). [Figure reproduced with permission from C. L. Vicente et al., Phys. Rev. B 72, 094519 (2005). Copyright (2005) by the American Physical Society.]

at 28.4 and 33.5 bar and identified the warming $A - B$ transitions in zero magnetic field (see Fig. 3-1). A couple of interesting features have been addressed by those authors: the coexistence of the $A$-like and the $B$-like phases in the narrow temperature region below $T_{ca}$ (see Fig. 3-4) and the positive slope of the $A - B$ transition line. The observation of a positive slope was also made by Baumgardner and Osheroff [50], and Kado et al. [52] in a low magnetic field, 28.4 mT. According to the Clasius-Clapeyron equation,

\[
\left( \frac{dP}{dT} \right)_{AB} = \frac{s_B - s_A}{v_B - v_A},
\]

(3–4)
since \( s_B < s_A \) due to more excitations of quasiparticles along the nodes in the \( A \)-like phase, the sign change in the slope should stem from the relative specific volume, where \( s \) is the molar entropy and \( v \) is the molar volume.

Vicente et al. made an interesting argument that the bulk \( P - T \) phase diagram is shifted up in pressure in the presence of aerogel due to the overall reduction of strong coupling effects. They also conjectured that the local anisotropic scattering due to the aerogel structure might be responsible for the existence of a finite region of the \( A \)-like phase even below the PCP at \( B = 0 \). Basically, combining those two effects makes the slope positive as illustrated in Fig. 3-5. Then, they proposed the idea of uniaxially deforming aerogel to induce global anisotropy in the system, which would allow a
systematic study of this idea. There is an interesting theoretical consideration about
the existence of PCP below the melting pressure. Aoyama and Ikeda [46] have shown
theoretically that local anisotropy tends to lower the PCP instead of opening the $\mathcal{A}$-like
phase all the way down to the pressure such as magnetic field or global anisotropy do.
Experimental confirmation about this issue has not been achieved clearly yet.

3.4 Gapless Superfluidity

Abrikosov and Gor'kov [53] considered magnetic impurity scattering effects in
$s$-wave superconductors in the Born limit (phase shift $\delta_0 \ll 1$). They showed that the
bound states due to pair-breaking by impurity scattering are formed inside the gap
at the expense of smoothing out the square-root singularities at the gap edge. The
number of bound states increases with disorder, which eventually leads to gapless
superconductivity. In this weak scattering limit, the non-magnetic impurity scattering
in an isotropic $p$-wave superconductor (or superfluid) has the similar effects to that
of the magnetic impurity scattering in an $s$-wave superconductor. In the unitary limit,
Buchholtz and Zwicknagl [54] calculated the density of states for the isotropic $p$-
wave superconductor. They found that even a small density of impurities generate
an island of impurity bound states centered at the Fermi energy in the absence of a
magnetic field. In close to but not exactly the unitary limit, the impurity bound states are
formed at a position centered at a finite energy. These theoretical results are exactly
applicable to the superfluid $^3\text{He}$ $B$-like phase in aerogel, and qualitatively the same
features of density of states (DOS) were obtained by Sharma and Sauls [55] and
Higashitani et al. [56]. Sharma and Sauls showed that in the unitary limit a band of
excitations formed, centered at the Fermi level, with energies, $\epsilon \leq \gamma \approx 0.67 \Delta \xi_o/\ell$,
and eventually bridged the superfluid gap completely leading to a gapless superfluid.
Similarly, the calculation by Higashitani et al. is shown in Fig. 3-6. Because of the
impurity states, the physical quantities such as thermal conductivity, specific heat,
and sound attenuation are expected to follow power laws rather than the exponential
Figure 3-6. Upper panel: The temperature dependence of shear viscosity coefficient normalized at $T = T_c$ for the various impurity scattering parameter $\tau T_c$. Lower panel: The density of states at $T = 0$ for $\tau T_c$. Note that the viscosity is nonzero at $T = 0$ when density of state is nonzero. [Figure reproduced with permission from S. Higashitani et al., Phys. Rev. B 71, 134508 (2005). Copyright (2005) by the American Physical Society.]

temperature dependence in the deep superfluid region. This deviation from the BCS prediction becomes more significant at lower pressures where the pair-breaking effect is more severe. However, since the profile of the impurity states are less sensitive to the specific ordering symmetry, the difference between the different phases becomes more evident at higher pressures. The thermal conductivity measurements in the superfluid
$^3$He in aerogel have been performed by Fisher et al. [57, 58]. They observed that the normalized thermal conductivity reached a finite value in the zero temperature limit. This behavior was attributed to the gapless superfluidity, specifically, non-zero DOS at the Fermi energy. Another evidence for gapless behavior was claimed by Choi et al. [35] who measured heat capacity ($C_a$) and showed a non-zero intercept of $C_a/T$ as $T \to 0$. A recent ultrasound attenuation measurement from our group by Choi et al. [59] provided another evidence for gapless superfluidity of $^3$He in aerogel. They obtained finite sound attenuation in the zero temperature limit and showed that it increased as pressure decreased, which is consistent with other results.

Despite these experimental evidence of gapless superfluidity in $^3$He/aerogel, no methods equivalent to the tunneling spectroscopy in superconductors are available for this system.

### 3.5 Longitudinal Sound

Unlike other porous material, aerogel is highly compliant. Because of this property, the movement of fluid trapped in aerogel causes a motion of aerogel matrix as well through viscous drag. Therefore, the sound mode of liquid $^3$He is effectively coupled to the mode of aerogel matrix which has a relatively low sound velocity. This property results in two sound modes: the so called fast and slow sound modes. In fast sound mode, the fluid moves together in phase with aerogel, which is in between first and fourth sound modes. On the other hand, the fluid oscillates out of phase with aerogel in slow sound mode, which is reminiscent of second sound mode. In a rigid porous material, only fourth sound exists. This phenomena can be described by a phenomenological model proposed by McKenna et al. [60]. They modified the conventional two fluid hydrodynamic equations allowing the motion of the aerogel matrix, and obtained the following secular equation for the sound velocity:

$$
(c_x^2 - c_1^2)(c_x^2 - c_2^2) + \frac{\rho_n}{\rho_a} (c_x^2 - c_4^2)(c_x^2 - c_3^2) = 0,
$$

(3–5)
where \( c_1, c_2 \) and \( c_4 \) are the velocities of first, second and fourth sound in bulk, and \( \rho_a \) and \( \rho_n \) are the densities of aerogel and the normal fluid, respectively. They found good agreement of this model with the observed temperature dependence of the sound velocity in superfluid \(^4\)He [60].

Golov et al. [61, 62] applied this model to the case of superfluid \(^3\)He in aerogel. From Eq. 3–5, they obtained simplified relations for the fast and the slow sound velocities.

\[
c_f^2 = c_1^2 \left( 1 + \frac{\rho_a \rho_s(T)}{\rho \rho_n(T)} \right) / \left( 1 + \frac{\rho_a}{\rho_n(T)} \right), \quad (c_2^2 \ll c_f^2 \text{ and } c_a^2 \ll c_f^2),
\]

(3–6)

\[
c_s^2 = c_a^2 \frac{\rho_a \rho_s(T)}{\rho}, \quad (c_2^2 \ll c_s^2 \ll c_a^2 \ll c_1^2).
\]

(3–7)

The sound velocity, \( c_f \), is about 80% of the bulk \(^3\)He first sound velocity and increases slightly in the superfluid state. The inequalities in the equations are justified by \( c_f \approx 350 \text{ m/s, } c_s \approx 13 \text{ m/s, } c_2 < 0.1 \text{ m/s at 29 bar and 1 mK, } c_a \approx 50 \text{ m/s for 98% aerogel} [61]. Note that the slow sound is much faster than the second sound suggesting that the restoring force mainly comes from the aerogel. From the slow sound velocity measurements, they determined superfluid fraction, \( \rho_s/\rho \), which is comparable to those reported by Porto and Parpia [2].

Unlike sound velocity, the damping mechanism in liquid \(^3\)He in aerogel is diverse and complex. To date, only one theoretical model was proposed to describe sound attenuation in this system. Higashitani et al. [56] and Miura et al. [63] introduced a phenomenological expression for the collision drag force density \( \vec{F} \) as an additional damping source due to the relative motion of normal fluid and aerogel,

\[
\vec{F} = \frac{1}{\tau_f} \rho_n(\vec{v}_n - \vec{v}_a),
\]

(3–8)

where \( \tau_f \) is the frictional relaxation time that needs to be determined through a microscopic calculation and \( \vec{v}_{n(a)} \) is the velocity of the normal fluid (aerogel strand).
Its temperature dependence is shown in Fig. 3-7 [56]. After introducing this drag force and viscosity (η) into the hydrodynamic equations of McKenna et al. [60], Miura et al. [63] obtained the extended dispersion relation,

$$\left(z^2_x - c_1^2\right)\left(z^2_x - c_2^2\right) + \left(i\frac{4\eta\omega}{3\rho_n} + \frac{i}{\omega\tau_f} + \frac{z^2}{\omega\tau_f \rho_a \chi_a}\right)\left(z^2_x - c_3^2\right) = 0,$$

(3–9)

where $z = \omega/q; \chi_a = \omega/(\omega^2 - \omega_q^2) = z^2/(z^2 - c_a^2); \omega_q = c_4q$. This equation is more general than Eq. 3–5 since it deals with the case of $v_n \neq v_a$. From this dispersion relation, the attenuation for fast sound was derived,

$$\alpha_f = \frac{\omega^2/2c_f}{1 + \rho_a \rho_s/\rho_n \rho} \left(\frac{\rho_n \tau_f/\rho \rho_n}{1 + \rho_a/\rho_n} + \frac{4\eta/3\rho c_1^2}{1 + \rho_a \rho_s/\rho_n \rho}\right).$$

(3–10)

This equation will be used extensively in our later discussions.
Figure 3-7. The reduced temperature ($T/T_c$) dependence of the frictional relaxation time $\tau_f$ normalized at $T = T_c$ for the $B$-like phase. $F_s^2$ is taken to be 10.01 corresponding to the pressure of 16 bar. Note that $\tau_f$ vanishes at zero temperature but when the sound propagates to the node direction in the $A$-phase, it depends on the parameter $\tau \Delta_A$, where $\tau$ is the mean free time in the normal state and $\Delta_A$ is the maximum value of the order parameter in the $ABM$ state. i) In the Born limit, $\tau_f = 0$ for $\tau \Delta_A \geq \pi/4$, and $\tau_f \neq 0$ for $\tau \Delta_A \leq \pi/4$. ii) In the unitary limit, $\tau_f \neq 0$ for any value of $\tau \Delta_A$. In the limit of $\tau \Delta_A \ll 1$, it takes the normal state value for both Born and unitary limits. This note is from the private communication with Seiji Higashitani. [Figure reproduced with permission from S. Higashitani et al., Phys. Rev. B 71, 134508 (2005). Copyright (2005) by the American Physical Society.]
CHAPTER 4
LONGITUDINAL SOUND ATTENUATION IN SUPERFLUID HELIUM THREE IN 98% AEROGEL AND PHASE DIAGRAM

4.1 Experiments

The presence of the compliant aerogel complicates the sound propagation because the sound modes of the liquid $^3$He and the aerogel matrix are effectively coupled [60]. As a result, two longitudinal sound modes emerge in this composite medium: one with the speed of sound close to, but slightly lower than, that of the liquid (fast mode) and the other with a significantly lower speed of sound (slow mode) [61]. In this experiment, the longitudinal fast sound attenuation in superfluid $^3$He in 98% aerogel was measured at frequencies between 3.69 and 11.3 MHz. The employment of the multiple frequency excitations turned out to be extremely valuable in this work. The experiment was performed at the High B/T Facility of the National High Magnetic Field Laboratory located in University of Florida.

Figures 4-1 and 4-2 show a schematic diagram and pictures of the cell. The bottom part of the cell is made out of pure silver and contains about 14 m$^2$ of silver powder heat exchanger. The top part of the cell made out of coin silver was glued to the bottom part using Stycast 2850FT, Emerson and Cuming. The top part of the sample cell forms a diaphragm so the pressure of the cell can be measured capacitively. The variation in the cell pressure during the measurement was around $\pm$ 0.1 bar. Two best-matched LiNbO$_3$ transducers (9.6 mm diameter) with fundamental resonances of 1.1 MHz were selected from six transducers tested using a broadband commercial spectrum analyzer and a home-made CW spectrometer. Figure 4-3 shows the lock-in output of the CW spectrometer for the frequency sweeps around the 11$^{th}$ harmonics of the transducers. We chose the transducers 1 (T1) and 2 (T2) as a transmitter and a receiver. The transducers were supported by a MACOR spacer forming a 3.02 mm size acoustic cavity. Aerogel with 98% porosity was grown in and around this cavity to ensure optimal acoustic coupling between the aerogel and the transducers. The aerogel grown
Figure 4-1. Schematic diagram of the experimental cell.

Figure 4-2. Pictures of the acoustic cavity and the assembled cell.
outside of the cavity was carefully removed, and copper wires were attached to the outer surfaces (electrodes) of the transducers using silver epoxy. In order to reduce the ringing of the transducers, a thin layer of silver epoxy was applied to the electrode.

A small piece of a cigarette paper with numerous needle holes was placed between each transducer and the cell wall to interrupt back reflections from the wall through the bulk liquid. The sample cell housing the cavity was placed on the top gold plated Cu-flange which is thermally connected to the Cu-demag stage through the three Cu-rods welded to it. A homemade superconducting solenoid magnet located in the inner vacuum space enclosed the cell. The magnet was thermally anchored to the mixing chamber (see Fig. 4-4). The magnetic field, $\vec{B}$, was chosen to be perpendicular to the sound wave vector $\vec{q}$, $\vec{B} \perp \vec{q}$, expecting $\vec{i} \parallel \vec{q}$ in the $A$-like phase.

Two different spectrometers were used in this work. A MATEC broadband pulsed spectrometer was used for the measurement at 29 and 19.5 bar, and a commercial
spectrometer, LIBRA/NMRKIT II (Tecmag Inc., Houston, TX) was used for the rest of
the pressures, 33, 25, and 14 bar. Both spectrometers transmitted $3\,\mu s$ pulses and
detected the transmitted signals. A MATEC 310 broadband gated amplifier mixes gating
pulses with a continuous sinusoidal wave ($1\sim2\,V_{rms}$ required to properly trigger the
internal synchronization circuits) to produce an RF pulse of a desired frequency. This
RF pulse was fed to the transmitter transducer through a variable (0 - 34 dB) attenuator.
The signal obtained by the receiver transducer was amplified by a MITEQ AU-1534
preamplifier and delivered to the first stage input of MATEC 625 broadband receiver.
The overall scheme is shown in Fig. 4-5.
With LIBRA/NMRKIT II spectrometer, each measurement was obtained by averaging eight transmitter signals produced in a phase alternating pulse sequence. The level of excitation used in this experiment was set in the range where neither self-heating nor nonlinearity was observed. A typical setting of this spectrometer and the origin scripts for handling data can be found elsewhere [64]. In one temperature sweep, the measurements at four pre-determined frequencies were performed in a cyclic manner. The temperature was monitored by a melting curve thermometer (MCT) for $T \geq 1$ mK and a Pt-NMR thermometer for $T \leq 1$ mK.

In spite of the effort to spoil the quality factor of the transducers, sustained ringings were observed and we were unable to resolve echoes following the initial received
signal. Consequently, by integrating a portion of the received signal, only the relative attenuation could be determined. Figure 4-6 shows a typical receiver signal and an integration scheme. The region of integration was carefully chosen not to include any echoes. Our method produced consistent relative attenuation for various choices of the integration range within the safe window described above. The relative attenuation in reference to the value at the aerogel superfluid transition temperature \( T_{ca} \) was determined by

\[
\Delta \alpha = \alpha(T) - \alpha(T_{ca}) = -\frac{1}{d} \ln \frac{A(T)}{A(T_{ca})},
\]

where \( d \) is the sound path length and \( A(T) \) is the integrated area of the transmitter signal at temperature \( T \).
4.2 Ultrasound Attenuation and a P-B-T Phase Diagram

4.2.1 Overview

The influence of high porosity aerogel as quenched disorder has been studied in various systems such as liquid $^4$He [65], $^3$He-$^4$He mixture [66, 67], $^3$He [2, 30], and liquid crystals [68, 69]. The effect of aerogel on superfluid $^3$He is exceptionally interesting because it is a $p$-wave triplet anisotropic superfluid possessing continuous symmetry. Since the discovery of superfluidity of $^3$He in high porosity aerogel [2, 30], more than a decade of theoretical and experimental efforts have been invested to understand this system and have revealed many interesting phenomena. The fragile nature of $p$-wave pairing against impurity scattering was immediately recognized by the significant depression of superfluid transition [2, 27, 30], and the theoretical descriptions based on various isotropic impurity scattering models have provided a successful account for the observed behavior [22, 25, 28]. A wide variety of experimental evidence reflecting the role of aerogel as an effective pair-breaking agent are now well documented [34].

For the past few years, attention has been shifted to understanding phenomena related to an energy scale smaller than the condensation energy. For example, the relative stability among possible superfluid phases, specifically the transition between two superfluid phases observed in this system, the $A$-like and the $B$-like phases, has been investigated. In the absence of a magnetic field, the supercooled $A$-like phase appears at all pressures studied, even below the bulk polycritical point (PCP) [26, 51, 64], while only a very narrow region where the two phases coexist was identified on warming [29]. In the presence of low magnetic fields, the $B$-like to $A$-like transition was observed, on warming, to follow a quadratic field dependence [26, 49, 50], which is reminiscent of the bulk $A-B$ transition, $1 - T_{AB}/T_c = g(\beta)(B/B_c)^2$, where $T_{AB}$ and $T_c$ are the $A-B$ transition and the superfluid transition temperatures, respectively and see Eq. 4–3 for $B_c$. However, the systematic field and pressure dependence study by Gervais et al. [26] found a monotonic increase in $g(\beta)$ with pressure without showing
any anomalies. This observation raised a question on the position or the existence of the PCP in aerogel. It is important to emphasize that the $A$ and the $B$ phases of bulk $^3$He are highly competing phases separated by first order transition with a minute free energy difference and have identical intrinsic superfluid transition temperatures. These properties are at the heart of many intriguing phenomena showing subtle modifications of the $A-B$ transition in the presence of weak external perturbations such as a magnetic field. Therefore, it is reasonable to expect that the presence of impurities or disorder will have a similar influence on the $A-B$ transition.

In 1996, Volovik [39] discussed the significance of the quenched random anisotropic disorder presented by the strand-like aerogel structure and its interaction with the anisotropic order parameter. This coupling is thought to be particularly important in the $A$-phase, where the order parameter is doubly anisotropic in the sense that the rotational symmetries in spin and orbital space are broken separately. Vicente et al. [29] argued that the aerogel strands generated orbital fields emulating the role of a magnetic field, thereby giving rise to similar profound effects on the $A$-like to $B$-like transition. They further suggested the use of uniaxially deformed aerogel to amplify and to systematically investigate the effect of the anisotropic disorder [29]. A series of calculations by Aoyama and Ikeda [46, 70] are consonant with these ideas and predict a widened $A$-like phase region in a uniaxially deformed aerogel, the appearance of a novel superfluid phase in uniaxially stretched aerogel, and a change of the PCP location in the phase diagram.

Unlike the $B$-like phase, the clear identification of the $A$-like phase in aerogel has not been made. However, some of the recent NMR measurements using uniaxially deformed aerogels [41, 71] provide compelling evidence that the $A$-like phase possesses the $ABM$ pairing symmetry, albeit with unusual textural configurations. The free energy calculation by Ikeda and Aoyama [72] also found the disordered $ABM$ phase as the most stable among the various plausible pairing states, such as the
Imry-Ma [73], the planar, and the robust [43] phases. Furthermore, the third superfluid phase observed in 98% aerogel in the presence of high magnetic fields [74] fortifies this identification. Therefore, we will continue our discussion with the assumption that the $A$-like phase observed at least in 98% aerogel has the same pairing symmetry as the bulk $A$-phase.

With this notion, we conducted longitudinal ultrasound attenuation measurements in the superfluid phases of $^3$He in 98% porosity silica aerogel. Our measurements were performed in the presence of magnetic fields, 0 to 4.44 kG, and at various sample pressures ranging from 14 to 33 bar. At the highest field, the existence of the meta-stable $A$-like phase persisted to the lowest temperatures, thereby allowing the sound attenuation in the $A$-like phase to be measured over the entire range of the temperatures studied. In lower magnetic fields, we were able to identify the transitions between the two phases on cooling and warming, and herein, a $P$-$B$-$T$ phase diagram of this system is presented.

4.2.2 Results and Discussion

4.2.2.1 Longitudinal Sound Attenuation and the A-B Transition in Aerogel

Figures 4-7, 4-9, 4-11, and 4-13 show the relative ultrasound attenuations obtained at 29, 19.5, 33, and 25 bar in the presence of magnetic fields ranging from zero to 4.44 kG, respectively. All the data shown were taken on warming after cooling though the supercooled $A$-like to $B$-like transition at a fixed external magnetic field, except for $B = 4.44$ kG, where no supercooled transition was observed down to $\approx 200$ $\mu$K.

Therefore, the warming trace at the highest field should be in the $A$-like phase for the entire temperature range, probably in the meta-stable $A$-like phase in the low temperature region. The superfluid transition is marked by a slight decrease in attenuation around 1.95 mK for 29 bar (Fig. 4-7), 1.61 mK for 19.5 bar (Fig. 4-9), 2.1 mK for 33 bar (Fig. 4-11), and 1.85 mK for 25 bar (Fig. 4-13). The zero field attenuation, which essentially represents the $B$-like phase attenuation except for a
Figure 4-7. Temperature dependence of relative longitudinal sound attenuations using a 6.22 MHz excitation at 29 bar in the presence of various magnetic fields. All the data were taken on warming after cooling through the $A$-like to $B$-like transition except for $B = 4.44$ kG, where no supercooled transition was observed. The arrows point the positions where the $B$-like to $A$-like phase transitions occur. Inset: Magnified view of zero field attenuation near the superfluid transition indicated by the vertical line.

very narrow region ($\approx 100 \, \mu$K) right below $T_{ca}$, can be directly compared with the absolute attenuation measurements by Choi et al. [75] performed under almost identical experimental conditions. The features observed in the current experiment, namely the broad shoulder structure appearing in the range $1.0 < T < 1.5$ mK and the absence of attenuation peak associated with the pair-breaking and the order parameter collective modes (OPCM), are consistent with those reported earlier [75] and also with the calculations by a Hiroshima group [56].

Establishing the attenuation in the $A$-like ($B = 4.44$ kG) and the $B$-like ($B = 0$) phases for the entire temperature range in the superfluid, one can envision a transition
Figure 4-8. The $A-B$ transition features in sound attenuation using a 6.22 MHz excitation at 29 bar. The red (black) trace represents the attenuation in the $A$-like ($B$-like) phase. The switching behavior between the two traces is demonstrated for each field as marked by an arrow.

between the two phases at any intermediate field where a switching from one trace to another occurs. It is expected that the attenuation in the $A$-like phase is higher than in the $B$-like phase under the assumption that it is the $ABM$ state, since the sound presumably propagates along the node direction in our experimental configuration. However, unlike in the bulk, the difference in attenuation between the $A$-like and the $B$-like phases is much smaller and subtle because of the absence of the order parameter collective modes, which are the fingerprints of specific pairing symmetry, and the presence of the impurity states residing in the gap. One can see the subtle difference in the attenuation between two phases in Figs. 4-7, 4-9, 4-11, and 4-13. At all temperatures, the attenuation in the $A$-like phase is slightly larger than in the $B$-like phase, while the largest difference is observed in the zero temperature limit. For this
Figure 4-9. Temperature dependence of relative longitudinal sound attenuations using a 6.22 MHz excitation at 19.5 bar in the presence of various magnetic fields. All the data were taken on warming after cooling through the $A$-like to $B$-like transition except for $B = 4.44$ kG, where no supercooled transition was observed. The arrows point to the positions where the $B$-like to $A$-like phase transitions occur. Inset: Magnified view of zero field attenuation near the superfluid transition indicated by the vertical line.

reason, the acoustic signature of the $A - B$ transition in aerogel is not as clear as in the bulk. Despite this small difference in attenuation, the $B$-like to $A$-like transition features are noticeable in most of the cases (indicated by the arrows in Figs. 4-7 ~ 4-14).

However, in the temperature region where two phases show almost identical attenuation, as in $0.7 < T < 1.0$ mK or very close to $T_{ca}$, the transition feature is rather vague. When this situation arose, the transition temperature $T_{ABa}$ was determined from the attenuation measurements conducted at other frequencies for 33 and 25 bar. For instance, at $B = 3.33$ and 3.85 kG, the transitions are much more evident in the attenuation at 8.73 MHz, as shown in the Figures 4-12 and 4-14. This phenomenon is due to the
non-trivial frequency dependencies of the attenuation observed in aerogel and will be discussed in next section of this chapter.

The lowest finite magnetic field used in this experiment was 1.11 kG, and two attenuation measurements performed in this field at 33 bar are shown in Fig. 4-15. These data were collected with two different warming rates of 1.4 µK/min (inverted triangles) and 1.7 µK/min (regular triangles). Both measurements produced the same transition temperature despite the difference in the warming rate by about 20%.

In Fig. 4-16, the width of the $A$-like phase, $\Delta T = T_{ca} - T_{ABA}$, as a function of $B^2$, along with the results obtained in the low field region by Gervais et al., is plotted. Within the Ginzburg-Landau (G-L) limit, we can perform analysis that is similar to work used to describe the bulk liquid [76]. Specifically, the suppression of the $B$-like phase in finite
Figure 4-11. Temperature dependence of relative longitudinal sound attenuations using a 6.22 MHz excitation at 33 bar in the presence of various magnetic fields. All the data were taken on warming after cooling through the $A$-like to $B$-like transition except for $B = 4.44$ kG, where no supercooled transition was observed. The arrows point the positions where the $B$-like to $A$-like phase transitions occur. Inset: Magnified view of zero field attenuation near the superfluid transition indicated by the vertical line.

Magnetic fields can be written as

$$1 - T_{ABa}(T)/T_{ca} = g(\beta)(B/B_c)^2 + O(B/B_c)^4.$$  \hfill (4–2)

Here, $B_c$ represents a characteristic field scale directly related to the transition temperature, namely

$$B_c = \sqrt{\frac{8\pi^2}{7\zeta(3)} \frac{k_B T_{ca}}{\gamma\hbar} (1 + F_0^a)},$$  \hfill (4–3)

where $k_B$, $\gamma$, $\zeta(x)$, and $F_0^a$ are the Boltzmann constant, the gyromagnetic ratio for a $^3$He nuclei, the Riemann zeta function, and a Fermi liquid parameter, respectively. In addition, the strong coupling parameter $g(\beta)$ is a function of the pressure-dependent
Figure 4-12. The $A - B$ transition features in sound attenuation using a 6.22 MHz excitation at 33 bar. The red (black) trace represents the attenuation in the $A$-like ($B$-like) phase. The top (bottom) panels show the traces taken using 6.22 MHz (8.73 MHz) excitations. The switching behavior between the two traces is demonstrated for each field as marked by an arrow.

$\beta$-parameters, the coefficients of the quartic terms in the G-L free energy expansion \[77\], and can be written as

$$g(\beta) = \frac{\beta_{345}}{2(2\beta_{345} - 3\beta_{13})} \left( 1 + \sqrt{\frac{(3\beta_{13} + \beta_{345})(2\beta_{13} - \beta_{345})}{\beta_{245}\beta_{345}}} \right), \quad (4-4)$$

where $\beta_{ijk} = \beta_i + \beta_j + \beta_k$. In the weak coupling limit, $g(\beta) \to 1$, and the strong coupling effects cause it to increase.

In order to illuminate the overall field dependence, the data presented in Figure 4-16 are recasted as $\Delta T/B^2$ in Figure 4-17. As noted by Tang et al. \[76\], one of the advantages of this plot is that the intersection of the curve with the $B = 0$ axis gives the strong coupling parameter, $g(\beta)$, and the slope of the curve is related to the coefficient
Figure 4-13. Temperature dependence of relative longitudinal sound attenuations using a 6.22 MHz excitation at 25 bar in the presence of various magnetic fields. All the data were taken on warming after cooling through the $A$-like to $B$-like transition except for $B = 4.44$ kG, where no supercooled transition was observed. The arrows point the positions where the $B$-like to $A$-like phase transitions occur. Inset: Magnified view of zero field attenuation near the superfluid transition indicated by the vertical line.

of the higher order correction, as can be seen in Eq. 4–2. Our $g(\beta)$ values extracted by extrapolating to zero field are shown in Figure 4-18. The parameters used are summarized in Table 4-1.

Table 4-1. Parameters for determining $g(\beta)$.

<table>
<thead>
<tr>
<th>P (bar)</th>
<th>$T_{ca}$ (mK)</th>
<th>$F_0^a$</th>
<th>$B_c$ (kG)</th>
<th>$g(\beta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>2.10</td>
<td>-0.74786</td>
<td>10.42</td>
<td>13.13</td>
</tr>
<tr>
<td>29</td>
<td>1.95</td>
<td>-0.75289</td>
<td>9.50</td>
<td>9.49</td>
</tr>
<tr>
<td>25</td>
<td>1.85</td>
<td>-0.75642</td>
<td>8.86</td>
<td>6.41</td>
</tr>
<tr>
<td>19.5</td>
<td>1.61</td>
<td>-0.75752</td>
<td>7.68</td>
<td>3.95</td>
</tr>
</tbody>
</table>

In the same figure, $g(\beta)$ of the bulk by Tang et al. (open circles) and of 98% aerogel by Gervais et al. (solid circles) are included for comparison. Additionally, we
reproduced the theoretical calculation [26] based on the homogeneous scattering model (HSM) [22] with the rescaled strong coupling corrections by the factor of $T_{ca}/T_c$ for two different mean free path values of $\ell = 150$ (dot-dashed line) and 200 nm (dashed line). Although our $g(\beta)$ value at 19.5 bar is in good agreement with that of Gervais et al., the discrepancy between the two sets of the data becomes larger at higher pressures. However, $g(\beta)$ in aerogel from both measurements is substantially smaller than that of the bulk value at the corresponding pressure. For the bulk, $g(\beta)$ grows quickly and approaches the PCP as predicted by the G-L theory. However, no such behavior is seen in aerogel. Although the error bars in our data are rather large, our results lie between the two theoretical curves. It is also interesting to observe that the sign of the quartic...
Figure 4-15. Temperature dependence of attenuation at 33 bar using 6.22 MHz excitation. The attenuation in the $B$-like ($B = 0$) and the $A$-like ($B = 4.44$ kG) phases are already shown in Figures 4-7 $\sim$ 4-13. For $B = 1.11$ kG, the attenuation was measured with two warming rates of 1.4 $\mu$K (inverted triangles) and 1.7 $\mu$K (triangles). Inset: Magnified view of the region of the $A - B$ transition in aerogel.

correction is negative at higher pressures and seems to change its sign at $P \approx 19.5$ bar (see Figure 4-17), which needs to be compared with the bulk case where the sign crossover occurs at $P \approx 6.7$ bar [76]. Based on these observations, one could argue that the presence of aerogel reduces the strong coupling effects and, in effect, the phase diagram of this system is shifted up in pressure.

4.2.2.2 The $A - B$ Transition in Aerogel by Isothermal Field Sweeps

The $A - B$ transition can be also induced through an isothermal field sweep (IFS). Although it is a time-consuming process, an IFS offers an independent way of determining this phase transition and is especially valuable in the region where the slope of the transition curve in the T-P phase diagram becomes small. During an IFS in either the up or down direction, heating was observed due to the eddy currents in the
Figure 4-16. Magnetic field dependence of the width of the $A$-like phase, $\Delta T = T_{ca} - T_{ABA}$. For comparison, our results are plotted along with those from Gervais et al. (solid circles) [26]. The data points from Gervais et al. were taken at the slightly different pressures of 33.4, 28, 25, and 20 bar, respectively.

silver cell body. To alleviate this problem, we slowly demagnetized the main magnet of the nuclear demagnetization stage during a field sweep (typically $\approx 0.14$ G/min). This passive procedure limited the temperature variation during an IFS to $\approx 50 \mu$K.

In Fig. 4-19, the magnitudes of the integrated acoustic signals taken at four different frequencies during an isothermal field sweep at 25 bar and 0.3 mK are displayed. The temperature variation during this process is also shown in the same figure. The sample was cooled from the normal fluid in the presence of a magnetic field of 4.44 kG to $\approx 0.3$ mK. After establishing equilibrium, the magnetic field was slowly reduced at the rate of 4 G/min [78] to go through the $A$-like to $B$-like transition. Therefore, the $B$-like phase was supposed to be induced through a primary nucleation, and this case is the only instance of a primary nucleation transition observed by IFS in our work. For the
Figure 4-17. Magnetic field dependence of the width of the $A$-like phase scaled by $B^2$. The quadratic coefficient, $g(\beta)$ is determined by the intersection of each curve with the $B = 0$ axis, Eq. 4–2.

entire sweep process, the temperature remained within $\approx 30 \, \mu K$ around 0.27 mK. The smooth change in magnitudes at all frequencies can be observed from $\approx 4.3$ to 4.0 kG, indicating the transition from the $A$-like to $B$-like phase. The difference in the magnitude of the acoustic signal between two phases matches well with the attenuation difference determined from the temperature sweep measurements.

For $B \lesssim 4.0$ kG (in the B-like phase), the attenuation exhibits a weak field dependence, most notably at 11.3 MHz. This behavior can not be simply attributed to the temperature variations during the field sweep because the attenuation shows a very weak temperature dependence around 0.3 mK (see Figs. 4-7 and 4-14). One can speculate that this variation in attenuation might be related to the progressive distortion of the gap induced by magnetic field, as the isotropic $BW$ state evolves through the distorted $BW$ state to the planar state and eventually to the $ABM$ phase with the node
Figure 4-18. Pressure dependence of $g(\beta)$. The present data (solid squares) are given with the data by Gervais et al. (solid circles) [26] for aerogel and by Tang et al. (open circles) [76] for the bulk liquid. The dashed and dot-dashed lines are from homogeneous scattering model (HSM) with the transport mean free path, $\ell = 200$ and $150$ nm, respectively (see Ref. [26] for details).

along the sound propagation direction [19]. The increase (decrease) in the magnitude (attenuation) in the low field region could be due to the enhancement (reduction) in the component of the gap perpendicular (parallel) to the magnetic field. In the $A$-like phase at the highest field, the sound propagates in the node direction, resulting in a higher attenuation.

Several additional IFS studies were conducted at various combinations of pressure and temperature, where the sample was cooled from the normal state at a fixed field to a temperature in the $B$-like phase via the superfluid and the supercooled $A$-like to $B$-like transitions. Then, the magnetic field was ramped up through the $B$-like to $A$-like transition and decreased again back through the transition, if necessary. Figure 4-20 shows the IFS results at 14 bar and $T \approx 0.27$ mK. The phase transition occurs
Figure 4-19. Results of the isothermal field sweep (IFS) at 0.3 mK and $P = 25$ bar. The magnitudes of the integrated acoustic signals, $A(T)$, measured using 4 different excitation frequencies are displayed as a function of magnetic field. The temperature variation during the IFS is also shown in the bottom panel.

over a rather broad range of field ($\Delta B \approx 0.5$ kG), but no appreciable hysteresis was observed. The results of two other IFS studies at 29 bar ($T \approx 0.86$ and 1.38 mK) are shown in Figs. 4-21 and 4-22. For $T \approx 0.86$ mK (Fig. 4-21), the transition can only be identified in the 3.69 MHz measurements ($\Delta B \approx 0.2$ kG). Brussaard et al. [49] observed hysteretic behavior in the field driven $A - B$ transition in their measurements at $T \approx 0.335$ mK and $P = 7.4$ bar using an oscillating aerogel sample attached to a vibrating wire. The magnetic field sweep was performed in the presence of a field gradient in which a single $A - B$ phase boundary was moving through the sample during the process. They proposed the pinning of the $A - B$ phase boundary by the aerogel strands as a mechanism for the observed hysteresis. Furthermore, based
Figure 4-20. Results of the isothermal field sweep at 14 bar.

Figure 4-21. Results of the isothermal field sweep at 29 bar and $T \approx 0.86$ mK.
Figure 4-22. Results of the isothermal field sweep (ramp up only) at 29 bar and $T \approx 1.38$ mK.

on this scenario, they made an argument that the $A - B$ transitions determined by a conventional temperature sweep method, specifically those by Gervais et al., might not provide reliable thermodynamic transition points due to supecooling and superwarming caused by the pinning, suggesting the finite width of the transition is an evidence of the existence of a range of pinning potential strengths [79]. We would like to point out that the experiments by Gervais et al. and by us were performed without a designed field gradient. In this case, it is also plausible that the random disorder presented by aerogel, more specifically anisotropic disorder, could cause the broadening of the transition [29, 80]. The effect of rounding by disorder is also apparent in the superfluid transition, which is a second order transition and does not involve an interfacial boundary. Imry and Wortis [80] have made a heuristic argument about the influence of random impurities on a first order transition. They predicted various degrees of rounding in the transition due to fluctuations (inhomogeneities) of the random microscopic impurities through the
simple generalization of the Harris criterion [81] valid for second order transition. It is worth noting that the Lancaster group also reported a similar degree of hysteresis in field (≈ mT) in the bulk A – B transition induced by a similar method [33]. The field sweep performed at 29 bar around 0.86 mK in Fig. 4-21 seems to show a glimpse of hysteresis in the 3.69 MHz data. However, we acknowledge that hysteresis at the level of mT can not be resolved from our measurements, and the width of the transition is certainly larger than any hysteresis that might exist.

4.2.2.3 Phase Diagram

The A – B transitions in aerogel identified by the temperature sweep at constant field (TSCF) and the IFS are plotted in the P-T phase diagram in Figure 4-23. For both methods, the mid-point of the transition in T or B was chosen as the transition point and the actual width of the transition is represented by the error bar. The width in B is translated into the temperature width using the measured field dependence of the A – B transition in aerogel (see Figs. 4-7 ~ 4-14). The transition points determined by the two different methods exhibit self-consistency within the resolution of our measurements. For example, the IFS transition point at 14 bar was observed at 3.33 kG and lies on the extension of the TSCF measurements at 3.33 kG, and the 3.7 kG IFS point is right on the line for 3.85 kG from the TSCF. We could not have obtained the IFS point at 4.21 kG at 25 bar by the conventional TSCG at this field.

The emerging phase diagram, Fig. 4-23, from our measurements unambiguously reveals that the A – B phase boundary in 98% aerogel recedes toward the melting pressure and zero temperature corner in response to the increasing field. This tendency is robust even when allowing for the possibility of superwarming, which might shift the transition temperature down. This phase diagram is in drastic contrast to that of the bulk [82]. Firstly, the slope of the constant field phase boundary is positive in aerogel but negative in bulk for most of the corresponding pressure range. Secondly, the phase boundary in the bulk recedes toward P ≈ 19 bar, which is in close proximity to the bulk
Figure 4-23. Phase diagram of superfluid $^3$He in 98% aerogel. The solid triangles represent the aerogel superfluid transition. The $A - B$ transitions in aerogel obtained by the TSCF are in solid circles and by the IFS in solid stars. The solid lines going through the data points are guides for eyes but conforms to the constant field phase boundaries for 1.11, 2.22, 2.75, 3.33, and 3.85 kG, respectively from right to left. For comparison, the constant field $A - B$ phase boundaries for the bulk liquid are shown by the dotted lines [82] for 1, 3, 5, 5.5, and 5.8 kG, respectively. The numbers right next to the star symbols indicate the mid-field strength of the transition.

PCP, rather than toward the meting pressure. It is noteworthy that the slope of the bulk $A - B$ phase transition line actually changes its sign around the PCP, with a positive slope for $P < P_c$. The observed behavior of the strong coupling parameter, $g(\beta)$, and these differences can be accounted for qualitatively and naturally by recognizing the reduction in strong coupling effects due to impurity scattering [22, 46, 48, 83]. Briefly and simply stated, these effects combine to effectively shift the phases and features of the bulk phase diagram up in the pressure to yield the phase diagram for $^3$He in 98% aerogel.
In G-L theory, the free energy (relative to the normal state) of the \(A(B)\) phase is
\[ f_{A(B)} = -\frac{\alpha^2}{2\beta_{A(B)}}, \]
where \(\alpha = D(\epsilon_F)(T/T_c - 1)\) is the coefficient of the quadratic term in the G-L free energy expansion, \(D(\epsilon_F)\) is the density of states at the Fermi surface, and \(\beta_A = \beta_{245}, \beta_B = \beta_{12} + \beta_{345}/3\). In zero field, the two phases share the same superfluid transition temperature and the PCP is determined by the condition \(\beta_A(P_c) = \beta_B(P_c)\). The presence of a magnetic field introduces an additional term in the G-L expansion given by
\[ f_z = g_z B_\mu A_{\mu i} A_{\nu i}^* B_\nu. \tag{4–5} \]

Here, \(A_{\mu i}\) represents the order parameter of a superfluid state with spin \((\mu)\) and orbital \((i)\) indices [20]. The magnetic field couples through the spin channel of the order parameter. With two distinct symmetries in the \(A\) and \(B\) phase order parameters, this quadratic contribution lifts the degeneracy in the superfluid transition temperature, thereby pushing the \(A\)-phase \(T_c\) slightly above that of the \(B\) phase. As a result, a narrow region of the \(A\)-phase must be wedged between the normal and the \(B\) phase for \(P < P_c\), even for an infinitesimally small magnetic field. The degree of this effect is inversely related to the free energy difference between two phases, \(g(\beta) \propto (\beta_A - \beta_B)^{-1}\), giving rise to the diverging behavior in \(g(\beta)\) as \(P \to P_c\).

In the presence of aerogel, the impurity scattering warrants various corrections to both the \(\alpha\) and \(\beta\) parameters. The first order corrections obviously come from the suppression of \(T_c\) by pair-breaking and incur the reduction of the strong coupling effects in the \(\beta\)-parameters simply scaled by \(T_{ca}/T_c\). The most extensive calculation of the \(\beta\)-parameters including various vertex corrections was done by Aoyama and Ikeda [83]. Their theoretical phase diagram based on those corrections indeed resembles the bulk phase diagram that is, in effect, shifted to lower temperature and, simultaneously, to higher pressure, resulting in the relocation of the PCP to a higher pressure.

Aoyama and Ikeda have also incorporated the anisotropic nature of the aerogel through the angular dependence of the scattering amplitude [46]. In a uniaxially
deformed aerogel, the calculation shows the unambiguous effect of global anisotropy as uniform orbital field, represented by an additional quadratic free energy term [22],

\[ f_a = g_a \hat{a}_i A_{\mu i} A_{\mu j}^* a_j. \]  

(4–6)

where \( \hat{a} \) is a unit vector pointing in the direction of the aerogel strand. The similarity between Eqs. 4–5 and 4–6 is apparent. The effect of the orbital field produced by the aerogel strands was estimated to be comparable to the effect produced by a magnetic field \( \sim 1 \text{ kG} \) in the case of complete alignment [29]. It has been experimentally demonstrated that uniaxial compression indeed induces optical birefringence proportional to the strain and, consequently, global anisotropy into the system [84, 85].

In a globally isotropic aerogel, however, the local anisotropy comes into play only when \( \xi_o \lesssim \xi_a \), where \( \xi_a \) represents the correlation length of the aerogel and \( \xi_o \) is the pair correlation length [29]. In the other limit, the local anisotropy is simply averaged out to produce no effect. As discussed by Vicente et al., this net local anisotropy should emulate the effect of magnetic field even in the absence of magnetic field in a globally isotropic aerogel. Furthermore an inhomogeneity in the local anisotropy would cause a broadening of the \( A - B \) transition in aerogel in which the mixture of the \( A \) and \( B \) phases coexists [80]. Considering \( \xi_a \approx 40 \text{ - } 50 \text{ nm} \) in 98% aerogel, this local anisotropy effect in a globally isotropic aerogel should be more pronounced at higher pressures but is expected to tail off as the pressure decreases to the point where \( \xi_o \sim \xi_a \), which occurs around 10 bar. The impressive agreement in \( T_{ca} \) between the experiments and the theory of Sauls and Sharma [28] was achieved by incorporating the aerogel correlation length into the depairing parameter of the homogeneous isotropic scattering model [22].

Although the aerogel sample used in this work is supposed to be isotropic, we cannot rule out the possibility of having a weak global anisotropy built into this sample from the sample preparation or the shrinkage occurring during condensation of \(^3\text{He} \).
In either case, the observed behavior in this work as well as others can be explained coherently [84, 86].

4.2.2.4 Attenuation Properties of $A$-like Phase

In Fig. 4-24, we plot the relative attenuation on cooling and warming in zero magnetic field along with the warming trace at 4.44 kG. The metastable $A$-like phase region extends to 1.6 mK in zero field. It is noteworthy that the attenuation in the zero field metastable $A$-like phase is practically identical to that of the $A$-like phase at the highest field. Considering our experimental configuration, we expect to have $\vec{H} \perp \hat{l} \parallel \vec{q}$ throughout the sample at 4.44 kG and a uniform $\hat{l}$-texture in a dipole-locked state since the surface orientation effect is in harmony with the field coupling. Consequently, the sound attenuation would exhibit anisotropic behavior [87]. In contrast, the Imry-Ma
state proposed by Volovik \cite{39, 88} should show isotropic attenuation in zero field since the $\hat{l}$-texture in this state is disordered. Therefore, if the zero field $A$-like phase was the Imry-Ma state, it is reasonable to expect different attenuation in the high field $A$-like phase unless it is completely dipole unlocked state. On the other hand, it has been demonstrated that the sound attenuation in aerogel is strongly modified by the presence of impurity states, especially at low pressures where the pair-breaking effect is stronger \cite{59}. The anisotropy in the attenuation in high fields might be weakened by the presence of impurity states. It will be interesting to directly measure the anisotropy in the attenuation in zero and finite magnetic fields as well as in uniaxially compressed aerogel where interesting textural configurations have been observed \cite{41, 71, 89}.

4.3 Frequency Dependent Ultrasound Attenuation

4.3.1 Overview

The effective scattering mean free path in this system, $\ell$, should incorporate both the elastic scattering from the aerogel as well as the inelastic quasiparticle scattering, following the Matthiessen’s rule, $1/\ell = 1/\ell_a + 1/\ell_i$, where $\ell_a$ is the elastic scattering mean free path set by the geometrical mean free path in aerogel and $\ell_i$ is the inelastic quasiparticle-quasiparticle scattering mean free path. At low temperatures, the temperature independent elastic scattering dominates the scattering process, preventing the divergent growth of the mean free path in a Fermi liquid. As a consequence, the classic first to zero sound crossover in the normal fluid of $^3$He was found to be effectively inhibited in aerogel, keeping the system from entering into the collisionless limit ($\omega \tau > 1$) on cooling, where $\omega$ is the angular sound frequency and $\tau = \ell/v_f$ ($v_f$: Fermi velocity) is the relaxation time \cite{90}. Sound propagation in the superfluid phases is also expected to remain in the hydrodynamic limit \cite{91, 92} but is further complicated by the presence of impurity states induced by pair-breaking scattering \cite{59, 92}. The details of impurity states depend on the type of pairing mechanism and scattering strength. Therefore, the full understanding of sound propagation in this system is not trivial and should
Figure 4-25. Absolute attenuations for pressures from 8 to 34 bar as a function of temperature at 9.5 MHz (on warming except 8 bar).

be followed by extensive theoretical and experimental studies. A recent ultrasound attenuation measurement performed in the $B$-like phase at 9.5 MHz exposed many interesting features such as the absence of the order parameter collective modes and the finite zero temperature attenuation evincing the existence of impurity states and gapless superfluidity [59] (see Fig. 4-25). These observations are consistent with the theoretical predictions based on a hydrodynamic two-fluid model as mentioned earlier [92]. Figure 4-26 shows the attenuation ratio $\alpha_0/\alpha_c$ obtained from the results shown in Fig. 4-25, where $\alpha_0$ is the attenuation in the zero temperature limit and $\alpha_c$ attenuation at the superfluid transition. This attenuation ratio provides the lower bound of the density of states at the Fermi level according to the calculation by Higashitani et al. [56], and it shows that the density of states starts increasing significantly below $\sim$ 15 bar.
In this section, we present ultrasound attenuation of the $B$-like phase of superfluid $^3$He in 98% porosity using 4 different frequencies between 3.6 to 11.3 MHz. Our results reveal non-trivial frequency dependences in attenuation, gradually departing from the $\omega^2$-dependence expected in this system.

4.3.2 Results and Discussion

A theoretical model for longitudinal sound propagation in liquid $^3$He in aerogel was developed by a Hiroshima group [93]. This model, based on the Landau-Boltzmann transport theory, considers the impurity scattering off aerogel as well as the relative motion between aerogel and $^3$He liquid. A $^3$He quasiparticle impinged on aerogel strand transfers momentum and causes a dragged motion of aerogel, the collisional drag effect. When this process generates relative motion between these two components, it gives rise to an additional damping mechanism. Their theory provided a satisfactory account for the longitudinal sound attenuation measurements conducted in the
normal fluid (16 bar) at 14.6 MHz [90]. The collisional drag effect is only pronounced in the hydrodynamic limit and the sound attenuation is expected to follow a quadratic frequency dependence (see Eq. 3–10). Although there is no direct experimental confirmation for the $\omega^2$-dependence of attenuation, this claim is supported by the observation of strong frequency dependence in attenuation [59, 90]. In addition, the elastic scattering mean free path has been experimentally determined in 98% aerogel by thermal conductivity (90 nm) [58], spin diffusion (130 nm) [94], and sound attenuation (120 nm) [59], which makes $\omega\tau < 1$ for $\omega < 20$ MHz for the whole pressure range and at all temperature.

With this notion, we deduced the absolute attenuation from the relative attenuation measured in this experiment using the normal fluid absolute attenuation at 9.5 MHz as a fixed point through

$$\alpha(T) = \frac{\alpha_r}{\omega_r^2} \omega^2 + \Delta \alpha$$  (4–7)

where $\alpha_r$ is the attenuation at 9.5 MHz at aerogel superfluid transition temperature, $T_{ca}$, from Ref. [59] and $\omega_r/2\pi = 9.5$ MHz.

Sound attenuation calculated following this recipe is displayed in Fig. 4-27 for 33 bar and in Fig. 4-28 for 25 and 14 bar. The traces shown were smoothed through the 10 points sliding average filter. The sound attenuation at 9.5 MHz for the corresponding pressure is also reproduced in the same panel. They all share similar qualitative features: the absence of collective modes and pair-breaking edge, the shoulder structure appearing around 0.6 $T/T_{ca}$, and non-exponential temperature dependence leading to the finite zero temperature attenuation. All of these features are in qualitative agreement with the theory of Higashitani et al. In their theory similar to the two-fluid hydrodynamic model described in Ref. [60], the collisional drag effect is included in the form of a mutual friction between the normal component and the aerogel. Therefore,
Figure 4-27. (a) Temperature dependence of relative attenuation at 3.69, 6.22, and 11.30 MHz taken at 33 bar. Inset: the temperature dependence of absolute attenuation at 9.5 MHz at the same sample pressure is reproduced from Ref. [59]. (b) Temperature dependence of absolute attenuation for all frequencies at 33 bar. See text for the details.
Figure 4-28. Temperature dependence of absolute attenuation for three frequencies along with the previous measurement at 9.5 MHz for 25 bar (a) and 14 bar (b) of sample pressures.
the damping of the fast mode [60] arises from the friction as well as the usual shear viscosity (\( \eta \)) (see Eq. 3–10).

Their calculation captures the important features observed in our experiments and the direct comparison with the experimental results at 9.5 MHz can be found in Ref. [59]. For example, the bump in attenuation around 0.6 \( T/T_{ca} \) is directly related to the frictional damping from the collisional drag effect. The frictional relaxation time, \( \tau_f \), initially increases right below \( T_{ca} \) (even above the normal fluid value for high pressure) due to rather rapid opening of superfluid gap. \( \tau_f \) eventually approaches zero as \( T \to 0 \), displaying a broad peak. This effect manifests in the size of the peak in \( \tau_f \) is more pronounced at a higher pressure where the pair-breaking effect is less significant, and accordingly, the bump structure in the attenuation gradually fades out as the sample pressure is lowered as shown in Figs. 4-27 and 4-28.

We plot the sound attenuation as a function of frequency at a constant temperature (Fig. 4-29). The sound attenuation is normalized by the one at \( T_{ca} \) (effectively by the square of the sound frequency, \( f^2 \)) and the data for the same set of the reduced temperatures are chosen for all pressures in this plot. Our assumption of the classic hydrodynamic behavior in the normal fluid enforces a flat line for \( T/T_{ca} = 1 \). One can clearly see the evolution of the frequency dependence deviating from the \( \omega^2 \)-dependence as temperature lowers. For 33 and 25 bar, the attenuation establishes a quite strong frequency dependence beyond the quadratic behavior in the zero temperature limit after going through a non-monotonic frequency dependence on cooling. For 14 bar, however, the attenuation shows a quite different behavior than the one observed at higher pressures. Down to the lowest temperature, the attenuation at 14 bar seems to possess a structure rather than following a monotonic frequency dependence. At 25 and 33 bar, the non-monotonic dependence seems to be associated to the broad bump structure in the attenuation. However, the similar frequency dependence persists
Figure 4-29. Sound attenuation as a function of frequency for select reduced temperatures at 33 (a), 25 (b), and 14 (c) bar. The sound attenuation is normalized by the one at $T_{ca}$, effectively by $f^2$. The lines going through the data points are guides for eyes.
Table 4-2. Important parameters estimated for three pressures used in this work.

<table>
<thead>
<tr>
<th>P (Bar)</th>
<th>$\omega T$</th>
<th>$\Delta_0(0)$</th>
<th>$\frac{\Delta_0(0)}{2\pi \hbar}$</th>
<th>$0.23T_{ca}$</th>
<th>$0.60T_{ca}$</th>
<th>$0.85T_{ca}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>0.30</td>
<td>96.17</td>
<td>62</td>
<td>0.18</td>
<td>0.20</td>
<td>0.26</td>
</tr>
<tr>
<td>25</td>
<td>0.27</td>
<td>91.13</td>
<td>57</td>
<td>0.20</td>
<td>0.22</td>
<td>0.29</td>
</tr>
<tr>
<td>14</td>
<td>0.23</td>
<td>77.37</td>
<td>31</td>
<td>0.36</td>
<td>0.39</td>
<td>0.53</td>
</tr>
</tbody>
</table>

down to the lowest temperature at 14 bar where the anomalous bump structure almost vanishes.

This non-monotonic behavior is not well understood yet but this persistent feature down to the lowest temperature seems to be related to the existence of the impurity bound states inside the gap as seen in Fig. 4-26. More specifically, we believe that the progressive deviation from the $\omega^2$-dependence is directly related to the gap structure.

Table 4-2 lists several important quantities pertinent to our discussion. The average gap in aerogel at zero temperature used in this table is from Halperin et al. [34]. The sound frequency used in this table is 11.3 MHz, which is the highest frequency employed in this work. The superfluid gaps at finite temperatures are obtained assuming the same temperature dependence as in the bulk and using the specific heat jump measurements by Choi et al. [35]. The superfluid gap is significantly suppressed at all pressures and the degree of the suppression is much severer at lower pressure, 14 bar. Although the sound is in the hydrodynamic limit, the sound frequency is yet comparable to the size of the superfluid gap at all temperatures. This uniques condition cannot be realized in bulk superfluid where a reasonably high frequency sound inevitably enters into the zero sound limit at low temperature. Therefore, the sound attenuation cannot be described purely from the hydrodynamic mechanism and should involve mechanisms of resonant quasiparticle and/or pair excitations. Therefore, the detailed spectrum of the impurity states should be considered.

Hirschfeld et al. [95] calculated the electromagnetic absorption in isotropic as well as anisotropic $p$-wave superconducting states considering various absorption
mechanisms involving impurity bound states. A sketch of various absorption mechanisms are illustrated in Fig. 4-30. The sound attenuation should possess the similar mechanisms as described in this diagram. A large electromagnetic absorption occurs when the resonant scattering takes place. For the scattering phase shift close to unitary limit ($\delta \neq \pi/2$), which is, we believe, the case of aerogel, there are three different resonant scattering processes which contribute to the total absorption: I. scattering from filled to empty bound states, II. scattering from filled states at gap edge to unoccupied bound states, III. scattering from filled bound states to the empty gap edge. They showed that the contribution from the process I would be relatively smaller than those from the processes II and III due to the large density of states at the gap edges. There is another possible process, pair-breaking mechanism for $\hbar \omega \approx 2\Omega_G$, where $2\Omega_G$ is the energy gap in the presence of impurities, but this does not contribute to determine the absorption
threshold [95]. In aerogel, the profile of the impurity states might be close to a single
dome structure centered at the Fermi energy rather than having two separate domes
as shown in Fig. 4-30. The relative contribution between the different processes could
be quite different at lower pressures where the severe pair-breaking causes significantly
smeared gap edges and considerable weight of impurity states. Furthermore, the
presence of thermally excited quasiparticles cannot be ignored considering the range of
sound frequency used in our work.

The fact that each strand of aerogel has a finite size ($\approx 3$ - $5$ nm) which is much
larger than $\lambda_F$ has a couple of important implications. It should put the quasiparticle
scattering at least close to the unitary limit and also in the intermediate regime between
the point-like impurity and surface scattering. It is interesting to ponder the latter aspect
further. Recently, Nagai et al. [96] considered the effects of surface Andreev bound
states on transverse acoustic impedance. In the case of diffusive scattering, weak
singularities are expected to appear when the excitation frequency is equal to the size of
the gap between the lower bound of the band of impurity states and the empty gap edge
($\Delta + \Delta^*$), and the upper bound of impurity states and the empty gap edge ($\Delta - \Delta^*$).
In high temperatures, the former contribution is supposed to be the dominant process.
This effect has been observed as a resonant peak in the transverse acoustic impedance
measurement by Aoki et al. [97]. In the low temperature limit where the gap is fully
developed, the contribution from the latter process is getting more significant at a given
excitation frequency. We believe that the ultrasound attenuation in superfluid $^3$He in
98% aerogel should reflect the similar processes described above resulting in non-trivial
frequency dependence in attenuation. We definitely need more detailed experimental
investigations to fully understand this phenomena such as the measurements in the
wider and finer range in frequency.

We observed that the property of aerogel, as reflected in sound attenuation, change
after going through a cool-down and warm-up cycle. Figure 4-31 shows the relative (a)
and absolute (b) attenuations at 29 bar, which were obtained during the first cool-down. The difference is clearly seen especially for the data at 11.3 MHz. The shoulder is much less prominent in this case compared to the results obtained in the subsequent cooling (see Fig. 4-27). No further significant changes in attenuation has been observed after multiple thermal cycles. However, the main features in the frequency dependence remain qualitatively the same (Figs. 4-29 and 4-32).
Figure 4-31. (a) Temperature dependence of relative attenuation at 3.69, 6.22, 8.73, 9.50 and 11.30 MHz taken at 29 bar. (b) Temperature dependence of absolute attenuation for all frequencies at 29 bar. The temperature dependence of absolute attenuation at 9.5 MHz at the same sample pressure is reproduced from Ref. [59].
Figure 4-32. Sound attenuation as a function of frequency for select reduced temperatures at 29 bar. The sound attenuation is normalized by the one at $T_{ca}$, effectively by $f^2$. The lines going through the data points are guides for eyes.
It is well known that zero sound propagates in a different manner from the ordinary sound in Fermi liquids where $\omega \tau \gg 1$. The restoring force responsible for zero sound arises from the molecular field which occurs when the external perturbation distorts the quasiparticle distribution from its equilibrium. In the case of liquid $^3$He, zero sound emerges below $\sim 10$ mK at 15 MHz and 29 bar out of the highly damped first sound at higher temperatures. This first-zero sound crossover occurs $\omega \tau \sim 1$. Since $\tau \propto 1/T^2$, the crossover point moves to a higher temperature for a higher frequency. The zero sound persists for the entire temperature range in superfluid phase due to strong Fermi liquid interaction. This zero sound couples to coherent oscillation of order parameters, called the order parameter collective modes (OPCM), in superfluid, resulting in strong attenuation peaks. In the $B$-phase, the eighteen OPCM's can be classified by the total angular momentum, $J$, and an additional degree of freedom, $\zeta$ from the real (+) and imaginary (-) parts of the order parameter [98]. Among them, the (imaginary) squashing and real squashing modes, which exist at $\sqrt{12/5} \Delta(T)$ and $\sqrt{8/5} \Delta(T)$, respectively, strongly couple to zero sound and cause significantly large zero sound attenuation.

For the $A$-phase, the gap is not isotropic, so that the pair-breaking occurs for any frequency due to the gap nodes. As a consequence, the classification of OPCM is different from the $B$-phase (see [20] for detail). There are three OPCMs that couple to zero sound, called clapping, normal flapping and super flapping modes. The excitation of these modes is strongly anisotropic. It depends on the relative orientation of $\hat{i}$ with respect to $\hat{q}$, the direction of the sound propagation. When they are parallel, the flapping and the clapping modes are absent, and when they are perpendicular, the flapping mode is absent.
In the present measurement in the absence of a magnetic field, the sound frequencies (6 ∼ 24 MHz) that we used are rather small so that the large attenuation by pair-breaking and OPCM’s appears just below $T_c$ and are not separable. As the temperature decreases, the attenuation drops quickly after the $A - B$ transition at high pressure and becomes frequency independent, approaching 0 as $T \to 0$. We used PVDF (polyvinylidene fluoride) transducers to make this frequency dependence measurement. Since a PVDF transducer has a broad frequency bandwidth and a low quality factor (Q), it has advantages over conventional resonant transducers in studying frequency dependent properties of materials. In this work, we constructed two acoustic cavities using PDVF transducers to extend our frequency dependent attenuation measurements in aerogel as well as in bulk liquid. However, we could not obtain any signals from the acoustic cavity with aerogel. Therefore, we present the results of attenuation measurement in bulk.

5.2 Properties of PVDF Transducers

PVDF shows piezoelectric properties once it is polarized in a large external electric field of $\approx 30$ MV/m while it is stretched at a high temperature. Even though its electromechanical coupling factors are smaller than those of piezoelectric ceramics such as PZT (piezoelectric transducers) family, it has several distinctive advantages over PZT such as a broad bandwidth (resolution) due to the low quality factor ($\sim 10$) and flexibility to be formed into various shapes. The disadvantage of the low coupling factors may be compensated by using a large excitation owing to a very high breakdown voltage ($\sim 1000$ V). However, the linearity of the PVDF transducer should be tested when it is used at a high voltage. PVDF has a negative $d_{33}$ (piezoelectric strain constant) unlike other piezoelectric materials, which means that it compresses or expands in the opposite way in response to an electric field compared to the usual piezoelectric materials. PVDF also has a pyroelectric property. Since the pyroelectric constant is positive, heating a PVDF film causes a positive voltage. Therefore, the easiest way
to check the polarity of the film is to bring a multimeter and connect the leads to the electrodes of the PVDF film, and send a warm breath to the film. If it shows a positive voltage, the side connected to the positive multimeter lead is the positive poling side.

The resonance frequency and the bandwidth of a PVDF transducer depend on the thickness \(d\) and the backing material. It is well known that PVDF transducers are operated at thickness modes of \(d = (2n - 1)\lambda/4\) for heavy backing and \((2n - 1)\lambda/2\), for polymer backing \((n = 1, 2, 3, \ldots)\), where \(\lambda\) is the wavelength and \(v\) is the sound velocity (\(\sim 2200\) m/s) in PVDF (Fig. 5-1). These modes correspond to the resonance frequencies of \(f_r = (2n - 1)v/4d\) and \((2n - 1)v/2d\), respectively. When there is a quarter-wave backing plate with a larger acoustic impedance than PVDF, it exhibits new resonance peaks near the frequencies that meet the \(\lambda/2\) condition. In effect, the backing plate plays the role of a complete reflector [99, 100]. A PVDF transducer with a polymer backing has the widest bandwidth but the lowest sensitivity. It is important that the bonding layer should be significantly thinner \((< d/50)\) than the PVDF film thickness. Otherwise, it may cause undesirable back wall reflections.
Since they have large dielectric and mechanical internal (viscoelastic) losses, which are several tens of times larger than those of ceramic transducer materials, PVDF transducers have a significant amount of electric power dissipation and internal damping. Ohigashi introduced the loss factors, $\tan\delta_e$ and $\tan\delta_m$ to account for the dielectric and viscoelastic losses, respectively [101]. These factors are incorporated into the models such as Mason’s model and KLM (Krimholtz-Leedom-Matthaei) model for theoretical consideration [102]. To improve the efficiency of a PVDF transducer, it is sometimes synthesized with other material. For example, P(VDF-TrFE) is a copolymer of PVDF and trifluoroethylene. This material is superior to PVDF in many aspects and comparable to classical PZT transducers except for its mechanical strength [103].

5.3 **Acoustic Cell and Experiment**

We have made four transducers out of 28 $\mu$m PVDF films for two acoustic cavities used for bulk liquid $^3$He and liquid $^3$He in aerogel. Each transducer used in this study was constructed following the steps described below.

- A 0.38" diameter circular shape was cut out of a PVDF film (Measurement Specialties, Inc.) with a sharp scissor. It is important to make sure not to have a short between the two electrodes. The cut-out film was then glued to a cylindrical
backing piece with silver epoxy (EPO-TEK, H20E). The backing piece was made out of copper. The opposite side of the copper cylinder was machined out in a cone shape to prevent the direct back reflection \[104\] (see Fig. 5-2). The side of the copper cylinder was also threaded to diffuse the wall reflections.

- The electrical connection was made by attaching a wire to the apex of the cone cavity using silver epoxy and then the cone shape cavity was filled with Stycast 2850FT to minimize the dead volume for liquid $^3$He and also to secure the wire connection.

- A spacer ring was made out of copper (ID 8.1 ± 0.03 mm, OD 9.6 ± 0.05 mm). It has the thickness of 0.078" (2 ± 0.02 mm) which defines the acoustic cavity size.

- Aerogel was cut into a circular button shape using a high speed diamond cutter with the thickness slightly larger than the spacer ring. The aerogel was inserted into the ring and then was sandwiched by two transducers with Cu backing. The spacer ring and the transducers were glued together using silver epoxy. Therefore, the spacer ring could serve as the common ground for both transducers. The second acoustic cavity was constructed in a similar way but without aerogel and was used for bulk liquid study.

Figure 5-3 shows these two acoustic cavities, left one with aerogel and right one without aerogel. The top and bottom views of this assembly are shown in Fig. 5-4. The
cell was installed in Thule, the ultra low temperature cryostat in Lee’s group equipped with a Cu demagnetization stage (Fig. 5-5). Temperature was monitored by a melting curve thermometer (MCT). The Matec broadband spectrometer used in the previous work was also used for this study. Unfortunately, we could not detect any acoustic responses from the aerogel cavity suggesting that the coupling between the aerogel and the transducers established by simple compression was not good enough. Therefore, in this work, we present the data obtained only from the bulk liquid $^3$He without aerogel.

5.4 Results and Discussion

Figure 5-6 shows the results of the sensitivity study conducted at 12.1 mK using the bulk $^3$He cavity. The magnitude was calculated by integrating the area of the first received signal at a given excitation frequency while keeping the excitation level of the transmitter constant. Since it has a heavy backing, PVDF should operate in $(2n - 1)\lambda/4$ mode and therefore the first resonance $(n = 1)$ is expected to appear at $\sim 20$ MHz ($= v/4d$) with a very low Q as shown in Fig. 5-6. Similar responses have been observed by Frankel and Granroth [105, 106] using 9 $\mu$m thick PVDF films. Figure 5-7 shows the sensitivity of a PVDF transducer in liquid $^4$He at 1 bar and 30 mK obtained by Granroth et al. The position of their main peak (resonance) around 50 MHz is consistent with ours considering the difference in the film thickness.
Due to the varying sensitivity, the linear response regimes are also different for different frequencies. Figures 5-8 and 5-9 show the results of the linearity test conducted at $\sim 0.7$ mK at 6 MHz and 20 MHz. Each figure displays the receiver signals obtained at different levels of transmitter excitation. The $28 \, \text{V}_{\text{rms}}$ pulse was fed to the transmitter through various levels of attenuation. Each receiver signal was normalized in reference to a given receiver signal by properly compensating the attenuation differences. For 6 MHz, the receiver signal at 0 dB excitation was chosen as the reference and the normalized signals are displayed in the inset of Fig. 5-8. Based on these measurements, we conclude that the linear output regime lies between 1 V
and 5 V. In our study, we maintained the receiver signal levels within the linear response regime by adjusting the attenuation level of the excitation.

Figure 5-10 shows the sound attenuation in normal and superfluid $^3$He for 6, 11, 17 and 24 MHz at 28.1 bar. The relative attenuation was obtained by integrating the area of the first received signal and was calibrated using the fact that the attenuation is frequency independent in the zero sound regime except near $T_c$ and it becomes almost zero below the temperature, $0.5 T_c$ [107]. Based on our analyses, 0.2 cm$^{-1}$ baseline shift in attenuation affected the fitting parameters described below by $\sim 1\%$. The crossover between the first and zero sound regimes is clearly seen in attenuation and moves to the higher temperature with the frequency as expected. Below $T_c$, the attenuation peaks associated with the pair-breaking and the OPCM are manifest.
Figure 5-7. Integrated magnitude (in arbitrary units) versus frequency response measured by Granroth et al. in liquid $^4$He at 1 bar and 30 mK. This plot appeared in the poster presentations of Refs. [105] and [106] but were not included in the proceedings.

Table 5-1. Fitting parameters

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>P1 (cm$^{-1}$)</th>
<th>P2 ($\mu$s mK$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>$2.479 (1.581) \times 10^{-5}$</td>
<td>0.67 (0.84)</td>
</tr>
<tr>
<td>17</td>
<td>$2.544 (1.581) \times 10^{-5}$</td>
<td>0.77 (0.99)</td>
</tr>
<tr>
<td>11</td>
<td>$2.478 (1.581) \times 10^{-5}$</td>
<td>0.70 (0.85)</td>
</tr>
<tr>
<td>6</td>
<td>$3.121 (1.581) \times 10^{-5}$</td>
<td>0.78 (0.91)</td>
</tr>
</tbody>
</table>

$$\alpha_l = P_1 \frac{(P_2)^2 T^2}{T^4 + (P_2)^2 \omega^2},$$

(5–1)

The results of our fit (red curve) are shown in Figs. 5-11~5-14, and summarized in Table 5-1.

The behavior of longitudinal sound in the normal fluid is well described by the viscoelastic model of Rudnick [10] (see Eq. 2–15). Since $\tau_\eta \propto 1/T^2$, we used the equation shown below to fit our data with two fitting parameters of P1 and P2. P1
Figure 5-8. Linearity test for 6 MHz pulse. Inset: normalized output signals against the one for 0 dB attenuated input (black).

Figure 5-9. Linearity test for 20 MHz pulse. Inset: normalized output signals against the one for 24 dB attenuated input (red).
Figure 5-10. Sound attenuation in normal and superfluid $^3$He at four different sound frequency. Inset: Magnified view near $T_c$.

parameterizes $(c_0 - c_1)/c_1^2$ and $P^2 = \eta T^2$. Adopting the values of $c_1 = 394 \text{ m/s}$ and $(c_0 - c_1)/c_1 = 0.00623$ at 28 bar [6], $(c_0 - c_1)/c_1^2 = P1 \approx 1.581 \times 10^{-5}$ which is substantially smaller than our values. We have also tried a fit to our data with $P1$ fixed at $1.581 \times 10^{-5}$ (blue curves). This procedure produced unsatisfactory fits to our data (blue curves). There are several additional mechanisms for attenuation that we need to consider. The first is the contribution from the wall scattering of the quasiparticles. When the sound frequency is of the order of or larger than the quasiparticle relaxation rate $(1/\tau)$, or the mean free path ($\ell = v_F\tau$) is comparable to the viscous penetration depth $(\delta = (2\eta/\rho\omega)^{1/2})$, the fluid begins to slip at the wall. This slip effect causes an additional damping, and the corresponding attenuation $(\alpha_w)$ is represented in the hydrodynamic
Figure 5-11. Data fitting for 24 MHz. Red curve is a theoretical fitting with two free parameters, P1 and P2 and blue curve with P1 fixed.

Figure 5-12. Data fitting for 17 MHz. Red curve is a theoretical fitting with two free parameters, P1 and P2 and blue curve with P1 fixed.
Figure 5-13. Data fitting for 11 MHz. Red curve is a theoretical fitting with two free parameters, P1 and P2 and blue curve with P1 fixed.

Figure 5-14. Data fitting for 6 MHz. Red curve is a theoretical fitting with two free parameters, P1 and P2 and blue curve with P1 fixed.
The values of $\tau_\eta T^2$ obtained by the several groups are summarized in Table 5-2.

<table>
<thead>
<tr>
<th>$\tau_\eta T^2$ ($\mu s \cdot mK^2$)</th>
<th>Carless et al. [110]</th>
<th>Wheatly [111]</th>
<th>Rudnick [10]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.96 (at 30 bar)</td>
<td>0.73 (at 30 bar)</td>
<td>0.62 (at 29.3 bar)</td>
<td></td>
</tr>
</tbody>
</table>

This value tends to increase as the pressure decreases. Our P2 values are in good agreement with these results.
agreement with that of Wheatley and comparable to the value obtained by Rudnick. It is not a trivial task to obtain absolute attenuation. All possible loss mechanisms such as diffraction, coupling, wedging, and non-parallelism as well as slip effects in our case should be considered. Including these losses are especially crucial when the sample has low attenuation. Moreover, since PVDF transducers have large internal losses, a model calculation is needed to make accurate determination of absolute attenuation.
CHAPTER 6
CONCLUSION

Longitudinal ultrasound attenuation measurements were conducted in a 98% uncompressed aerogel in the presence of magnetic fields. Utilizing the meta-stable $A$-like phase that extended down to the lowest temperature in 4.44 kG, we were able to establish the temperature dependence of the attenuation in the $A$-like phase over the entire superfluid region. This arrangement allowed us to determine the $A \rightarrow B$ transitions in aerogel in various magnetic fields. Based on the transition points on warming, a $P \rightarrow T \rightarrow B$ phase diagram of this system is constructed. The key features of the phase diagram can be understood on the basis of two fundamental points: firstly, the strong coupling effect is significantly reduced in this system by impurity scattering, and secondly, the anisotropic disorder presented in the form of aerogel strands plays an important role that emulates the effect of a magnetic field.

In the absence of a magnetic field, the $A$-like phase has not been understood at the satisfactory level yet. Our measurements suggest that the practically identical behavior of longitudinal ultrasound attenuation to the one in the highest magnetic field (4.44 kG) may indicate a dipole unlocked LIM state for the $A$-like phase. If this is the case, it would be interesting to study about the mechanisms that two isotropic states show the different attenuation (see Fig. 4-24).

Our results of frequency dependent ultrasound attenuation offer the decisive evidence of the gapless superfluidity in superfluid $^3$He in aerogel. According to Fig. 4-29, the impurity bound states are much more conspicuous at lower pressure, turning the phase into the completely gapless regime, and the resonant impurity scatterings may play an important role for the observed breakdown of $\omega^2$ dependence of sound attenuation in superfluid state in aerogel. In order to understand the detail of attenuation mechanism, the measurement in a wider and finer frequency range is required.
Longitudinal ultrasound attenuation measurements were performed in bulk liquid $^3$He using PVDF broadband transducers. We could see nice attenuation crossover between first and zero sounds in normal liquid but the attenuation was much larger than expected. Utilizing PVDF has many advantages but it is not trivial to use it in the measurement of absolute attenuation because of many factors to consider (Chapter 5). One of the possible solution to this problem may be to use two identical acoustic cavities with different pathlengths.
REFERENCES


[78] The same sweep rate was used for all isothermal field seeps in this work (2008).


BIOGRAPHICAL SKETCH

Byoung Hee Moon was born in small rural district in Chunchengbuk-Do of South Korea. He went to Seoul city for college education and graduated from Yonsei University. He studied theoretical nuclear physics in master course in the same university. After serving the army for 26 months, he decided to go abroad and came to University of Florida in 2002 and joined the experimental low temperature group of Dr. Yoon Lee in 2003 and received his PhD in May 2010.