# DESIGN AND COMPARISON OF INTELLIGENT AUTO-ADJUSTING MECHANISMS

By

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A THESIS PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
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MASTER OF SCIENCE

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To my parents and friends

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The task of self adjustment of a platform mounted with a laser pointer to point to a desired location on the ground is to be satisfied. Two different types of mechanisms are presented to perform this task and a comparative study between them is presented.

A simple tripod with three degrees of freedom with variable leg lengths is first studied in order to perform the task. It is found that a tripod is very easy to model mathematically. An infinitesimal twist in order to orient the laser pointer towards the desired location is calculated. It was found that the workspace of allowable orientations of the laser pointer line was very limited. A SolidWorks model of the same mechanism was checked to confirm the mathematical model. It was found that the mechanism covers only a small area on ground even after adjusting leg lengths to their maximum extent.

A tensegrity mechanism was considered next. The mechanism is studied with regards to geometric and force balance constraints. The solution presents the change in lengths of the ties and struts to orient the laser pointer to point to the desired location on the ground. After checking equilibrium conditions, the tensegrity mechanism seems to work without a problem satisfying the desired goal.

# CHAPTER 1
# INTRODUCTION

An (Improvised Explosive Device (IED) can be almost anything with any type of material and initiator. It is a “Homemade” device that is designed to cause death or injury by using explosives alone or in combination with toxic chemicals, biological toxins, or radiological material. In 2007 during Afganistan War, 230 IED incidences were reported killing over 12 members of coalition forces. This number increased to 49 coalition members with 830 IED’s used. During Iraq war, 2317 IED incidences were reported. The death rate for IEDs was 90 troops in 915 IEDs in 2007.

As these explosives can be constructed in wide varieties, it is not possible to deactivate them even after they have been detected. Therefore, space charge is used to blow up these explosives. A shape charge creates plasma from molten metal and transforms it into a forceful jet stream.

The goal here is to develop a mechanism that can aim a projectile towards a user specified point on the target IED. The mechanisms used and compared in this research were designed and tested for this goal and the results are presented. The two mechanisms chosen for the comparison are a tripod and a three-three tensegrity mechanism.

## Literature Survey

A tripod is a word generally used for a three legged object. It comes from a Greek work ‘’tripous” which means three legs. A tripod is used more often for its stability while moving up and down as well as sideways. A very common use of a simple tripod is a stand for machine guns. It absorbs the recoil from the gun and helps soldiers to stabilize the gun while firing. This tripod is more of a foldable structure where as it can be used as a moving foldable mechanism with some basic modifications. As in case of the tripod stands used for photography, they can be folded to fit into a small rectangular box. They are mostly very rigid and heavy. This is the reason they are used where stability is more important.

Another mechanism chose for comparison is a tensegrity mechanism. They are a more complex mechanism and need a lot of study to understand their working. This chapter will present more about tensegrity mechanisms and their comparison with a simple tripod.

Tensegrity is a blend of words tension and integrity (Edmonson, 1987 and Fuller, 1975). Tensegrity structures refer to balanced light weight components in tension and compression. They are different from a truss structure in a way that all tensile elements of the mechanism are replaced by strings (R.E.Skelton, University of California). The components in tension are know as 'Ties' and those in compression are known as '. Struts' in case of tensegrity mechanisms do not touch each other and are connected by ties subjected to tension. This type of design is known as Class-I tensegrity mechanisms. Kenneth Snelson (2004) defined tensegrity as "a closed structural system composed of a set of three or more elongated compression struts within a network of tension ties".

Special attention should be given to mechanical stability and static construction while designing the tensegrity mechanism. It is assumed that all the components of these mechanisms are either in pure tension or in pure compression (Pallegrino and Calladine, 1985). If the mechanism is subjected to buckling or yielding, it may fail. If subjected to unbalanced tensile and compressive loads, the mechanism may not be in equilibrium.

One of the advantages of a tensegrity structure is that it can be made with elastic ties. Hence, the structure can be folded into a small set of struts and when released, it will regain its original shape.

The shape of a tensegrity mechanism is very unique. It is formed by rotating a parallel prism about its axis by a specific angle. The mechanisms formed by clockwise rotations are known as left-handed tensegrity mechanisms and those rotated counter-clockwise are right-handed ones. The angle of rotation is very specific and has been determined by Kenner to be

$$\alpha = \pi\left(\frac{1}{2} - \frac{1}{n}\right) \tag{1.1}$$

where n is the number of struts and α is the minimum angle of rotation. Therefore, for a three strut mechanism, the value of α is (π/6).The maximum value of the angle of rotation is (π/2), regardless of n. Examples of tensegrity mechanisms are as shown in Figure 1-1.

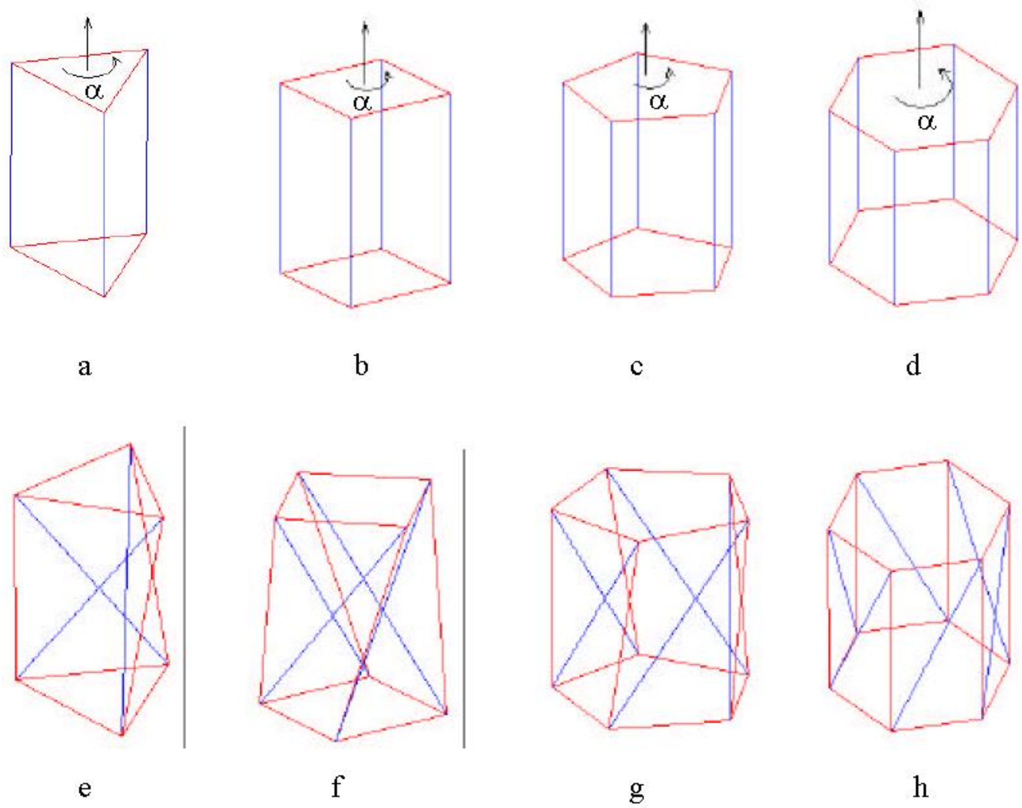


Figure 1-1. Tensegrity Mechanisms

In this figure, the struts are represented by blue lines and the ties are represented by red ties.

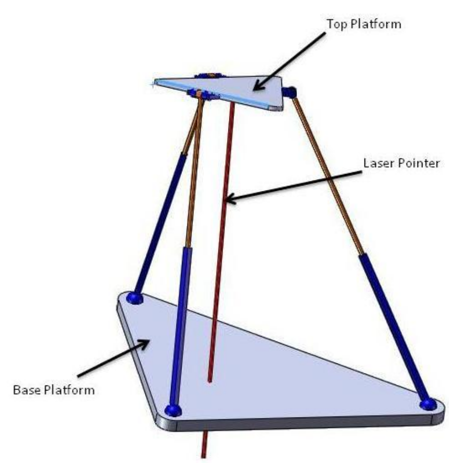


Figure 1-2. Simple tripod with laser pointer

A tripod is a simple mechanism comparatively. It has a very small top surface on which a laser pointer is mounted as shown in the figure. Three base points form one more triangular surface on the ground called base platform. For both the tripod and the tensegrity mechanism, it is assumed that the distance between two vertices of the base will remain constant. A detailed mathematical analysis needs to be done to study this mechanism to perform the required task of aiming a projectile towards the target improvised explosive device.

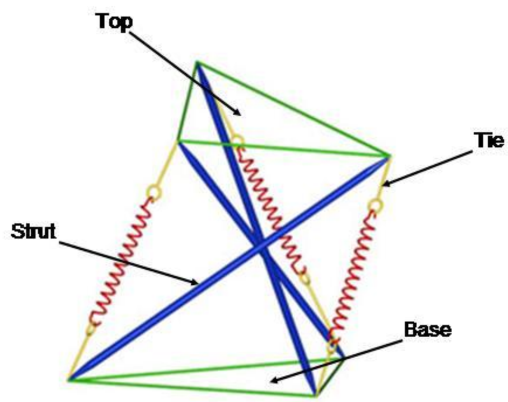


Figure 1-3. Three Strut Tensegrity Mechanism

A common tensegrity mechanism consists of a top surface and a base surface of the same shape connected by struts and ties. Many studies have been done to determine the equilibrium position of a tensegrity mechanism. During these studies, two parameters are determined related to these mechanisms. One of the parameters is the effect of external forces and moments. The other is the position of top plane with respect to base. Kenner, Yin and Tobie studied the relationship between top and base platform. Length of struts and ties is an important parameter to calculate equilibrium position. Many people have derived the equations to find the length of struts and ties for the equilibrium position of a tensegrity mechanism. Most recently, University of Florida, Vikas determined all the equilibrium positions of a parallel three-strut tensegrity mechanism.

All these studies have been done when the top and base surfaces are parallel to each other as shown in figure 1-1. This is not possible all the time. There are many applications that require different orientations of the top platform with respect to the base, as shown in Figure 1-3. Also, whenever any operation is performed or an external force/moment is applied, the top platform never stays parallel to the base.

The relationship between orientations of the top platform with respect to the base is studied in this research and one of the solutions is presented. This solution is then compared with that of a simple tripod and the results are analyzed.

To obtain the change in orientation in a specific pre-determined manner, many mechanical adjustments were needed for a tensegrity mechanism. One of the options is to make struts from pneumatic cylinders to adjust their length and change their orientation. The other option is to have a prismatic joint inside each strut connected to a tie. This mechanism allows all adjustments in the length of each tie and the strut resulting in reduced tensile force in the ties and change in orientation of the top platform. This research combines the study of tensegrity mechanisms and parallel platforms to form a structure which has adjustable ties and struts.

Ties are constructed from compliant and non-compliant parts. The non-compliant parts are subjected to a change in length. A reverse displacement analysis of such a mechanism has already been studied by Tran, 2002, University of Florida. His work presents the set of equations to calculate the change in length of non-compliant part. This can only be applied if the final orientation of the top platform with respect to the base is already known and also the orientation satisfies specific linearity constraints. This research presents the way to find out that orientation and hence, use a part of Tran's working concepts to find the change in length of a non-compliant element of each tie and change in length of the strut.

**Project Objective:**

Both the tripod and the tensegrity mechanism have their own advantages and disadvantages. In both the mechanisms, positioning of the top platform to a required orientation is studied. There are many applications found where positioning the top platform with respect to the base is considered to be a very useful tool. One of the applications taken into consideration for this research is the aiming of a shape charge projectile at a user specified point on an improvised explosive device (IED). A robot will be used to place the mechanism over the target IED.  A laser pointer will be mounted on the top platform.  Its directional axis will be parallel to the line of action of the shape charge projectile which is also attached to the top platform.  The laser pointer shows the initial aiming point.  The objective is to move this point to the desired aiming point.

There are many pros and cons of both the mechanisms. Following is the list of pros for a simple tripod:

1. It is a very simple mechanism. Mathematically modeling a simple tripod is very easy.

2. It is very stable mechanism due to its heavy and strong legs. Hence, stability is not a concern.

3. It can be folded to fit in a box and is easy to carry.

Following is the list of pros for a tensegrity mechanism:

1. Elastic elements can be used for the ties.  The potential energy that is contained in these elements can be used to change the shape of the mechanism from the stowed to the deployed shape.  This potential energy can also be used to change the orientation of the top platform to obtain the desired aiming point.

2. The mechanism can be folded to fit all the struts into a small rectangular box without deforming its original shape and becomes easy to carry due to less volume.

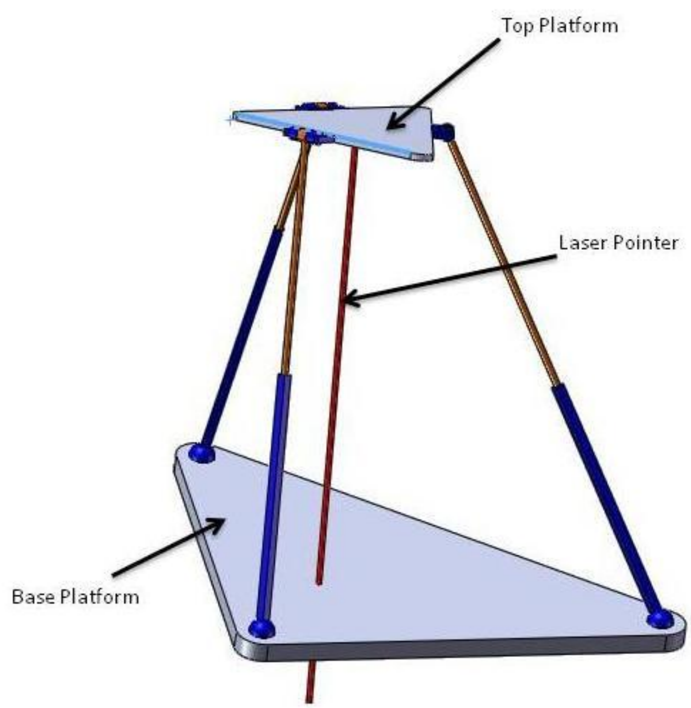


Figure 1-4. Simple tripod mechanism

The tripod taken into consideration for this research is as shown in Figure 1-4.

Following are some of the characteristics of this mechanism:

1. Three legs of the tripod are connected to the top platform by revolute joints. The axes of these joints are parallel to three edges of the top platform.

2. Three legs are made up of two concentric cylinders. These two cylinders are connected by a prismatic joint with each other. This changes length of each leg and helps orienting the laser pointer mounted on top platform.

3. The mechanism is designed in a special way. The revolute joints have a brake on them. Whenever the leg length is changing, the respective revolute joint is allowed to rotate and the other two act as fixed joints.

4. As all three legs of the mechanism are far away from each other, they have enough space between them to point the laser pointer to the required orientation.

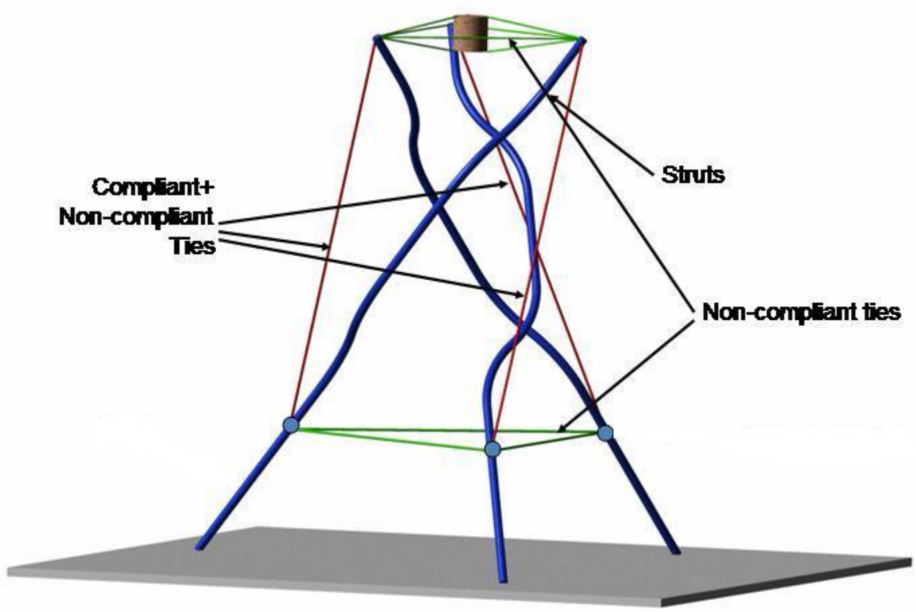


Figure 1-5. Three Strut Tensegrity Mechanism

The tensegrity mechanism taken into consideration for this research is as shown in Figure 1-5. Following are some of the characteristics of this mechanism:

1. It's a simple three strut tensegrity mechanism. The ties connecting the top platform with the base are made up of two elements; compliant springs and non-compliant strings.

2. The struts are designed to be curved hollow cylindrical rods. This gives more space for laser pointer to point to the required location.

3. Prismatic joints are used to connect the strut with side ties. This helps to change the length of non-compliant part of the tie which changes orientation of top platform.

4. Ties forming the top platform are completely non-compliant and have high tensile strength.

5. The base platform is formed by the surface of soil. It is assumed that the mechanism stands on top of the soil. The connection between the struts and ground may be modeled by spherical joints. Ties connecting the bottom of three struts are also non-compliant and are made up of same material as those in pt. 4.

This thesis presents the mathematical model of this new type of special tensegrity mechanism and the modified simple tripod to perform the required task. Further chapters will present the mathematical equations to calculate the required orientation of the top platform and the change in length of legs in case of simple tripod and the change in tie length and strut length in the tensegrity mechanism.

CHAPTER 2
PROBLEM FORMULATION

The simple tripod and tensegrity mechanisms discussed in the last chapter have many parameters that can be adjusted to change the orientation of the top platform. To understand the workings of both mechanisms, it is necessary to understand the nomenclature for both of them.

## Nomenclature

### Simple Tripod:

The tripod is very easy to analyze in a way that it does not have a long list of different parameters and variables. There are only three values; i.e. the leg lengths, which can be changed to change orientation of top platform.

Each leg of a tripod is numbered as 1, 2 and 3. The following nomenclature is used:

1. Length of each leg is named as L1, L2 and L3.
2. Vertices of base are named as $P_1$, $P_2$ and $P_3$ and that of top platform are named as $P_4$, $P_5$ and $P_6$.

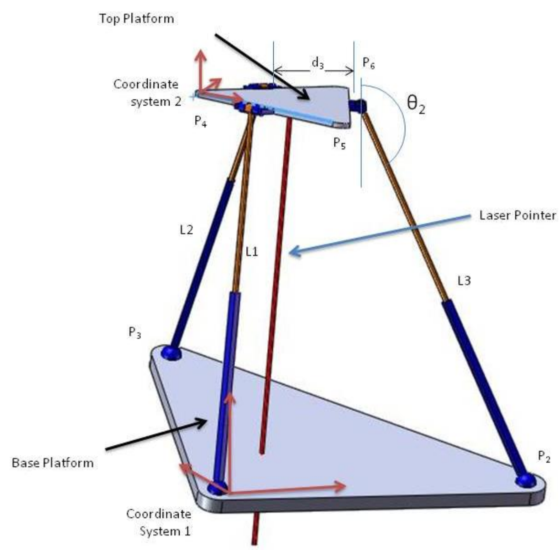


Figure 2-1. Nomenclature for simple tripod

There are two coordinate systems; the base coordinate system and the top coordinate system. The base coordinate system is named as 1 and the top coordinate system is named as 2. The origin of the base coordinate system is located at vertex $P_1$ and that of the top coordinate system is located at vertex $P_4$ of top platform. The transformation matrix can be calculated using simple geometry.

**Tensegrity Mechanism:**

The tensegrity mechanism is a very complex mechanism considering its stability issues. There are many parameters and variables that can be changed to change the orientation of the top platform. The nomenclature used for this mechanism is discussed here.

The three struts are named A, B and C and the respective ties are numbered as 1, 2 and 3. The compliant and non-compliant parts of each tie are given the similar nomenclature. Tie 1 connects the end point of strut A with top vertex of strut B. The two ends of the tie are vertex $P_1$ and $P_4$ as shown in figure 2-1. All the vertices, ties and struts are also shown in figure 2-2.

| | |
|---|---|
| $L_A$, $L_B$, $L_C$ | Length of each strut |
| $K_1$, $K_2$, $K_3$ | Spring constants |
| l01, l02, l03 | Free length of springs |
| L1, L2, L3 | Actual length of each spring |
| l1, l2, l3 | Length of non-compliant part of each tie |
| $P_1$, $P_2$, $P_3$ | Vertices of the base platform |
| $P_4$, $P_5$, $P_6$ | Vertices of the top platform |

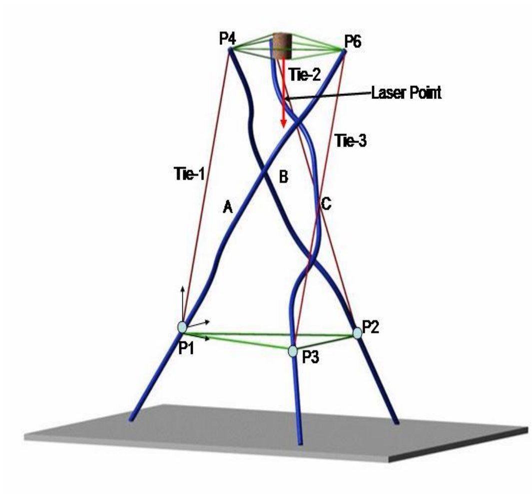


Figure 2-2. Nomenclature for tensegrity mechanism

Tie 2 connects vertex $P_2$ and $P_5$ and tie 3 connects $P_3$ and $P_6$. The following nomenclature is used:

1. The free lengths of three springs and spring constants are represented by l01, l02, l03 and $K_1$, $K_2$, $K_3$, respectively.

2. Length of three variable length compressive struts is represented by $L_A$, $L_B$ and $L_C$, respectively.

3. Length of each spring in tension i.e. actual length of each spring is written as L1, L2 and L3, respectively.

4. Length of non-compliant part of each tie is given as l1, l2 and l3, respectively.

5. Total length of side ties is given by $l_{1total}$, $l_{2total}$ and $l_{3total}$.

There are two coordinate systems located on top and base platforms respectively. The origin of base coordinate system is located at one of the vertices of base, $P_1$. The X-axis of the base coordinate system is assumed to be along the line joining points $P_1$ and $P_3$. The origin of the top coordinate system is assumed at the laser pointer with its Z-axis pointed in the same and opposite direction as that of laser pointer. The X-axis and Y-axis of top coordinate system are assumed to be parallel to that of base coordinate system. The top coordinate system is represented by ‘T’ as a subscript and that of base is represented as ‘B’.

**General Assumptions**

For the purpose of this research, it is assumed that the robot is used to detect landmines and keeping the tripod at correct location. It also provides the basic data required for the calculations.

The tripod is kept at correct location by this robot. It is also assumed that the tripod stands still on uneven soil surface and the joint between soil and leg behave as spherical joints.

Assuming all the above data is known and the coordinate systems defined, the problem statement can be specified.

## Problem Statement

### Given Data

Following is the data provided by Robot's vision system as well as some fixed initial parameters of the mechanism.

### Simple Tripod

For a simple tripod, very less number of parameters is already known. They can be listed as follows:

- Coordinates of points P1, P2 and P3 in coordinate system 1; P11, P21 and P31
- Coordinates of points P4, P5 and P6 in coordinate system 2; P42, P42 and P62
- Initial lengths of three legs; L1, L1 and L3
- Coordinates of desired point of intersection of laser pointer with XY-plane of coordinate system 1; PdesiredB
- Angle made by each leg with a vertical line; Ө1, Ө2 and Ө3

### Tensegrity Mechanism

As discussed earlier about the complexity of a tensegrity mechanism, there are many parameters in the mechanism which can be used to orient the top platform with respect to base. The given data are:

1. Initial position of origin of top coordinate system represented in base coordinate system; $P0T_B$ (i)*
2. Coordinates of points $P_1$, $P_2$ and $P_3$ in base coordinate system; PB1, PB2, PB3 (i)**
3. Coordinates of points $P_4$, $P_5$ and $P_6$ in top coordinate system; PT4, PT5, PT6 (i)**
4. Initial lengths of struts; $L_A$, $L_B$, $L_C$ (i)*

5. Initial lengths of compliant and non-compliant parts of ties; L1, L2, L3 and l1, l2, l3 respectively (i)*

6. Coordinates of desired point of intersection of laser pointer with XY-Plane of base coordinate system, $P_{desiredB}$ (r)**

**To Find:**

To achieve the required goal, following data needs to be calculated:

1. Desired orientation of top platform with respect to base; $T_{BT}$

2. Length of three legs ( for simple tripod)

3. Length of three variable struts; LA, LB, LC (for tensegrity mechanism)

4. Length of three non-compliant parts of the string; l1, l2, l3 (for tensegrity mechanism)

The change in length of a tie, strut, or leg will be very difficult to actuate in an exact and continuous manner. Hence, for this research, it was decided to have the change in length at specific intervals. It was decided to use a clicking type of mechanism that will click and change the length by specific number of intervals. Therefore, after the exact change in length has been determined, the number of clicks is decided. This data is sent to an inexpensive microprocessor mounted on the mechanism which will give a signal to change the orientation of the top platform. This is also discussed in further chapters.

[(i): known due to same initial construction of a mechanism

(r): data provided by robot’s visual system

* : constant value parameter

** : variable]

CHAPTER 3
ANALYSIS OF THE SIMPLE TRIPOD

A simple tripod is easier to study and adjust than a tensegrity mechanism. Once the required position of the top platform is known, the only task is to calculate the change in leg length. In this research, the concept of a twist is used to calculate the desired orientation of the top platform. A small infinitesimal twist can be applied to top platform that will move the laser pointer towards the desired point by adjusting the leg lengths. The calculations to get this twist are presented in this chapter.

## Defining Coordinate Systems

Three different coordinate systems are considered for this mechanism. The mechanism is as shown in Figure 3-1:

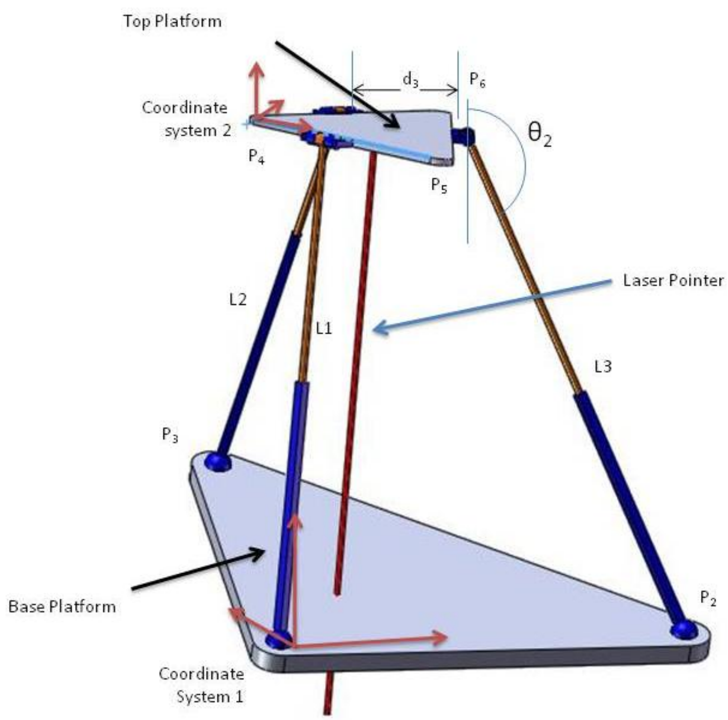


Figure 3-1. Coordinate systems for the simple tripod

Following are the three coordinate systems:

**Third Coordinate System:**

This coordinate system moves with the top platform. Its origin is a point on the top platform. The laser line is assumed to be passing through the origin of third coordinate system. Therefore, it can be said that this system represents the orientation of the laser line. The Z-axis of this coordinate system is along the laser line pointing towards the ground. If checked at the initial position of the mechanism, then the Z-axis points vertically downwards. The XY-plane of this coordinate system is the top platform itself. It is assumed that the two axes, X and Y are parallel with those of the second coordinate system.

**Second Coordinate System**

This coordinate system is also mounted on the top platform. This represents the orientation of the top platform. The origin is assumed at one of the vertices of the top platform; $P_4$. The X-axis of this coordinate system is assumed along the line joining the two vertices $P_4$ and $P_5$. The Z-axis of this system is assumed to be perpendicular to the plane of the top platform and parallel to that of first coordinate system initially. Orientation of the Y-axis can be easily calculated as it is perpendicular to both the X and Z axes. The coordinates of all three vertices of the top platform are assumed to be known in this coordinate system.

As shown in the Figure3-1, the revolute joints are along the sides of the top platform and not at the vertices. The mechanism is constructed in such a way that these revolute joints will lie at the will be same as that of each side of the top platform. Let $d_1$ be the distance of the first revolute joint from point $P_4$ (origin of second coordinate system). Therefore, $d_1$ can be calculated using the following equation:

$$d_1 = \text{distance } (\mathbf{P}_4 \text{ to } \mathbf{P}_5)/2 \tag{3.1}$$

Similarly, let $d_2$ and $d_3$ be distances of mid-points of joints 2 and 3 from vertices $\mathbf{P}_5$ and $\mathbf{P}_4$, respectively. These distances can be calculates as:

$$d_2 = \text{distance } (\mathbf{P}_5 \text{ to } \mathbf{P}_6)/2 \tag{3.2}$$

$$d_3 = \text{distance } (\mathbf{P}_4 \text{ to } \mathbf{P}_6)/2 \tag{3.3}$$

**First Coordinate System**

This coordinate system represents the firm ground on which the mechanism stands. The origin of this coordinate system is assumed at one of the vertices of the base, $\mathbf{P}_1$. The X-axis of the system is assumed along the line joining vertices $\mathbf{P}_1$ and $\mathbf{P}_2$. The Z-axis points vertically upwards. The Y-axis can be calculated in a similar way as that in case of second coordinate system.

The main step is to get the accurate transformation matrix of the third coordinate system with respect to the first coordinate system. This initial transformation matrix will be the same for all the mechanisms.

**Constant Mechanism Parameters:**

Listed below are the parameters of the mechanism which will never change as they depend on the initial construction of the mechanism.

1. Transformation matrix between coordinate system 3 and 2
2. Angle between leg 1 and a line parallel to Z-axis and passing through center of revolute joint 1
3. Angle between leg 2 and a line parallel to Z-axis and passing through center of revolute joint 2
4. Angle between leg 3 and a line parallel to Z-axis and passing through center of revolute joint 3
5. Distances between vertices of the top platform and the center of the respective revolute joints

6. Coordinates of points $P_1$, $P_2$ and $P_3$ in the first coordinate system and coordinates of points $P_5$ and $P_6$ in the second coordinate system

These parameters are referred as ‘params’ in the program. These parameters are known throughout the program and except for the three angles associated with the revolute joints, all others are constant throughout the program.

**Coordinates of Points $P_1$, $P_2$ and $P_3$ in Second Coordinate System**

It is a little complicated to understand and calculate the coordinates of all three points at the same time as the transformation matrix that related the first and third coordinate systems is unknown. Therefore, a single point is considered each time with separate cases.

**Coordinates of Point $P_1$ in Second Coordinate System**

For calculating the coordinates of this point, one more coordinate system is assumed along leg L1. This coordinate system as its Z and Y axes parallel to second coordinate system but the X-axis along the length of the leg. In this case, the angle $\Theta_1$ will be the angle between X-axis of this new coordinate system and Z-axis of second coordinate system. The origin of this new coordinate system is assumed at the center of revolute joint. Therefore, the transformation matrix can be calculated as:

$$TL1to2 := \begin{bmatrix} 0 & 0 & 1 & d_1 \\ -\sin(\theta_1) & -\cos(\theta_1) & 0 & 0 \\ \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.4}$$

Coordinates of point $P_1$ in this new coordinate system will be given as:

$$P_1 := \begin{bmatrix} L1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.5}$$

where L1 is the length of leg 1. From above two equations, point $P_1$ is known in new coordinate system and transformation matrix between new coordinate system and second coordinate system is known. Therefore, following equation can be used to calculate coordinates of point $P_1$ in second coordinate system.

$$P1_2 := TL1to2 \cdot \begin{bmatrix} L_1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{3.6}$$

These are coordinates of point $P_1$ in second coordinate system.

**Coordinates of Point $P_2$ in Second Coordinate System:**

The calculations for this part are similar to those for $P_1$ with little variation. The temporary coordinate system is assumed along leg 2 and its origin is assumed at center of revolute joint 2. Similar to previous part, the transformation matrix can be calculated. The same equation is used but, in this case, that equation gives a transformation matrix that will move the temporary coordinate system to vertex $P_5$ instead of the origin of second coordinate system. This can be given as:

$$TL2to5corner := \begin{bmatrix} 0 & 0 & 1 & d_2 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & 0 \\ \cos(\theta_2) & -\sin(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.7}$$

This transformation matrix will bring the origin of temporary coordinate system at vertex $P_5$. Therefore, the origin of temporary coordinate system is in XY-plane of second co-ordinate system. Now, it is easy to break the steps of calculating transformation matrix as rotation and translation.

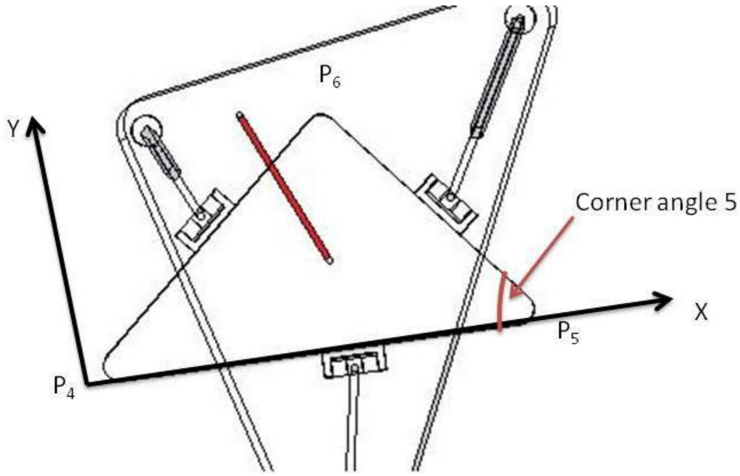


Figure 3-2. Top view (XY-plane of top coordinate system)

From the figure, the angle between side of the top platform ($P_5$ to $P_6$) and X-axis of second coordinate system can be calculated using the equation:

$$cornerangle5 := \arctan\left(P6_2[2], P6_2[1] - P5_2[1]\right) \tag{3.7}$$

This equation will give the angle of rotation for the temporary coordinate system about Z-axis and the translation part is jus the X-coordinate of point $P_5$ (obtained from the figure). Therefore, the transformation matrix is:

$$T5cornerto2 := \begin{bmatrix} \cos(cornerangle5) & -\sin(cornerangle5) & 0 & \mathrm{P5}_2[1] \\ \sin(cornerangle5) & \cos(cornerangle5) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.8}$$

Similar to point $P_1$, coordinates of point $P_2$ in temporary coordinate system will be:

$$P_2 := \begin{bmatrix} \mathrm{L2} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.9}$$

Therefore, the coordinates of point P2 can be given by following equation:

$$P2_2 := T5cornerto2\ .\ TL2to5corner\ .\ \begin{bmatrix} L_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{3.10}$$

**Coordinates of Point $P_3$ in Second Coordinate System:**

This part follows the exact steps as that in calculations for $P_2$. As there is no difference except for the calculation of angle of rotation, the equations and figure are as given below:

$$TL3tocorner6 := \begin{bmatrix} 0 & 0 & 1 & d_3 \\ -\sin(\theta_3) & -\cos(\theta_3) & 0 & 0 \\ \cos(\theta_3) & -\sin(\theta_3) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.11}$$

$$cornerangle6 := arctan\left(-P6_2[2],\ -P6_2[1]\right) \tag{3.12}$$

$$T6cornerto2 := \begin{bmatrix} \cos(cornerangle6) & -\sin(cornerangle6) & 0 & P6_2[1] \\ \sin(cornerangle6) & \cos(cornerangle6) & 0 & P6_2[2] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.13}$$

$$P3_2 := T6cornerto2\ .\ TL3tocorner6\ .\ \begin{bmatrix} L_3 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{3.14}$$

**Calculation of Transformation Matrix:**

Any transformation matrix can be represented in a specific manner. It can be represented as four separate parts making first three elements of four columns. The first column is X-axis of a coordinate system represented in another coordinate system. In this case, the X-axis of first coordinate system is represented in second coordinate system. This becomes first three elements of first column of a transformation matrix. The elements of second and third column are Y and Z axes of first coordinate system represented in second coordinate system. The last and fourth column is coordinates of origin of first coordinate system in second.

For using this method, the things required can be calculated as the coordinates of all three vertices of base are already known in second coordinate system.

1. X-axis of first in terms of second coordinate system:

   Unit vector along line joining $P_2$ and $P_1$

$$x1vec_2 := \frac{(P2_2[1..3] - P1_2[1..3])}{dist(P2_2, P1_2)} \tag{3.15}$$

2. Z-axis of first in terms of second coordinate system:

   Z-axis is a unit vector along the direction obtained by taking cross product of any vector in XY-plane with X-axis. Let the vector in XY-plane be ‘*tempvec*’.

‘*tempvec*’ can be calculated as a vector along points $P_3$ and $P_1$:

$$tempvec_2 := \frac{(P3_2[1..3]-P1_2[1..3])}{dist(P3_2, P1_2)} \tag{3.16}$$

Equation of Z-axis is:

$$z1vec_2 := \frac{(x1vec_2 \text{ X } tempvec_2)}{mag(x1vec_2 \, X \, tempvec_2)} \tag{3.17}$$

1. Y-axis of first in terms of second coordinate system:

   Y axis can be obtained simply by taking a cross product of X-axis with Z- axis.

$$y1vec_2 := z1vec_2 \, X \, x1vec_2 \tag{3.18}$$

3. Origin of first in terms of second coordinate system:

4. Origin of first coordinate system is point $P_1$ which is already known in second coordinate system.

As all the parameters of transformation matrix are known, the matrix can be written as:

$$ {}^{A}_{B}\boldsymbol{T} = \begin{bmatrix} & {}^{A}_{B}\boldsymbol{R} & & {}^{A}\boldsymbol{P}_{Bo} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.19} $$

where ${}^{A}_{B}\boldsymbol{R}$ can be given as:

$$ {}^{A}_{B}\boldsymbol{R} = \begin{bmatrix} {}^{A}\boldsymbol{x}_B & {}^{A}\boldsymbol{y}_B & {}^{A}\boldsymbol{z}_B \end{bmatrix} \tag{3.20} $$

**Coordinates of Laser Line in First Coordinate System**

The laser line is along Z-axis of third coordinate system. Therefore, coordinates of laser line in third coordinate system are given as:

$$ \boldsymbol{\$}_{\text{laser}} = \{0, 0, 1; 0, 0, 0\} \qquad 3.21) $$

As the line passes through origin, the moment of line about origin will be zero.

To calculate coordinates of this line in first coordinate system, transformation matrix of third coordinate system with respect to first one must be known.

This can be calculated as:

$$ {}^{3}T_1 = {}^{3}T_2 . {}^{2}T_1 \tag{3.21} $$

Once the transformation matrix is known, the equations to calculate coordinates of a line in another coordinate system are given by Dr. Crane and Dr Duffy in “Screw Theory and Its Applications”. The equations are:

$$ \{{}^{A}S_1; {}^{A}S_{0L1}\} = \{{}^{A}_{B}\boldsymbol{R}\,{}^{B}S_1; {}^{A}\boldsymbol{P}_{Bo} \times {}^{A}_{B}\boldsymbol{R}\,{}^{B}S_1 + {}^{A}_{B}\boldsymbol{R}\,{}^{B}S_{0L1}\} \tag{3.22} $$

This equation will give the coordinates of laser line in first coordinate system.

**Point of Intersection**

Point of intersection is a point where laser line intersects ground i.e. XY-plane of first coordinate system. XY plane of first coordinate system can be given by:

$$[D_0; \mathbf{S}_{xy}] = [0; 0, 0, 1] \quad (3.23)$$

The point of intersection lies in this plane. Therefore, it should satisfy following equation:

$$\mathbf{S}_{xy} \times \mathbf{P} + D_0 = 0 \quad (3.24)$$

Also, the point lies in the laser line. Let Plücker Coordinates of line be {$\mathbf{S}_{laser}$; $\mathbf{SoL}_{laser}$}. In this case, the point of intersection should also satisfy following equation:

$$\mathbf{P} \times \mathbf{S}_{laser} = \mathbf{SoL}_{laser} \quad (3.25)$$

Taking cross product with $\mathbf{S}_{xy}$ on both sides of the equation above:

$$\mathbf{S}_{xy} \times (\mathbf{P} \times \mathbf{S}_{laser}) = \mathbf{S}_{xy} \times \mathbf{SoL}_{laser} \quad (3.26)$$

Simplifying above equation and substituting the values from equation obtained from plane,

$$(\mathbf{S}_{xy} . \mathbf{S}_{laser}) \mathbf{P} + D_0 \mathbf{S}_{laser} = \mathbf{S}_{xy} \times \mathbf{SoL}_{laser} \quad (3.27)$$

The only unknown in the equation above is **P** which is point of intersection. Therefore, coordinates of point of intersection can be obtained using above equation.

## Concept and Calculations for Infinitesimal Twist

The top platform will move with some velocity while trying to achieve the desired orientation with respect to base. Let us assume that there is a twist which will represent this motion of top platform. If the top platform is applied this particular twist, it will change its orientation and point the laser pointer at desired point.

This twist can be formed by two parts; the angular velocity vector ω and the linear velocity of any one point on top platform, v. for ease of calculations, lets assume that this process is carried out in several number of small steps. In this case, the twist is very small and will only move the point of intersection closer to desired point of intersection in one step.

This infinitesimal twist has six unknowns; vector of angular velocity and a vector for linear velocity. As there are six unknown parameters, six constraint equations are required to determine their values. Following are the six equations chosen:

**Constraint 1:** Lets assume that the angular velocity is ω = 1 rad/sec. Therefore, the angular velocity vector will be a unit vector.

**Constraint 2:** The twist is supposed to move point of intersection towards desired point of intersection. Therefore, the linear velocity vector has to lie in the plane formed by laser line and desired point of intersection. In that case, the twist should satisfy following equation:

$$[\mathbf{v} + \boldsymbol{\omega} \times \mathbf{P}_{intersect}] . [(\mathbf{P}_{desired} - \mathbf{P}_{intersect}) \times \mathbf{S}_{laser}] = 0 \qquad (3.28)$$

**Constraint 3, 4 and 5:** For next constraint equation, a concept of reciprocal screws is used. A reciprocal product of the wrench and twist gives the virtual power of wrench about twist. This wrench is applied to a body about a screw. If the direction of the screw is same as that of a revolute joint, then this wrench will have no effect on the body and the reciprocal product of twist and such wrench will be zero. Similarly, if the screw about which the wrench is applied passes through the center of a spherical joint (i.e. point of intersection of axes of all three revolute joints), then it will have no effect on the motion of the body. This concept can be used to get three more equations for a simple tripod.

Consider three wrenches applied to the mechanism. All three of them pass through the three vertices of base respectively. This will nullify the effects of wrench on motion of spherical joint. Also, these three wrenches are parallel to the axis of three respective revolute joints joining each leg with top platform. Therefore, the reciprocal product of these three wrenches with the twist applied to top platform will be zero.

This can be represented mathematically as follows:

$[\boldsymbol{\omega}; \mathbf{v}]^T \circ \mathbf{\$}_{1b} = 0$

$[\boldsymbol{\omega}; \mathbf{v}]^T \circ \mathbf{\$}_{2b} = 0$

$$[\boldsymbol{\omega}; \mathbf{v}]^T \circ \mathbf{\$}_{3b} = 0 \qquad (3.29)$$

Where $\mathbf{\$}_{1b}$, $\mathbf{\$}_{2b}$ and $\mathbf{\$}_{3b}$ are the screws along which a wrench is applied. If the reciprocal product is zero, then the magnitude of force does not make any difference in calculations. Therefore, a unit force is assumed in this case.

These equations can be simplified to obtain three separate equations to calculate three parameters out of six unknowns.

**Constraint 6:** There are only five constraint equations and the sixth equation comes to free choice. For ease, the Z-coordinate of ω is considered as '0' as that will not really affect the effect of twist.

Using above six equations, the twist can be calculated. Once the twist is known, the change in leg length can also be calculated. Many trials were carried out to calculate a finite twist but it is observed that the twist comes to be infinity. This means that the velocity required to move the laser pointer to point to desired point of intersection is infinity. This result was double checked with a solid works model. It was found that the mechanism can only move laser pointer in a very small circular motion no matter how much the leg length changes. Due to this result the mechanism was not pursued after calculation of velocity till the calculations of leg lengths.

It was proved mathematically as well as using a solid works simulation model that a simple tripod is not suitable for these kinds of jobs though its mathematical calculations are a lot easier than that of a tensegrity mechanism.

## Numerical Example

For a numerical example, the values of basic constant mechanism parameters are assumed. The length of each leg, the coordinates of point $P_5$ and that of point $P_6$ will be constant. They are:

$$P5_2 := \begin{bmatrix} 30. \\ 0 \\ 0 \end{bmatrix} : \# \ cm \quad (Point\ 5\ in\ coordinate\ system\ 2)$$

$$P6_2 := \begin{bmatrix} 15. \\ 9. \\ 0 \end{bmatrix} : \# \ cm \quad (Point\ 6\ in\ coordinate\ system\ 2)$$

$$L_1 := 70. : \ \# \ cm$$

$$L_2 := 70. :$$

$$L_3 := 70. :$$

Desired point of intersection is assumed to be:

$$Pintersectdesired_1 := \langle 63., 20., 0 \rangle :$$

Transformation matrix between second and third co-ordinate system is obtained as:

$$T3to2 := \begin{bmatrix} 1 & 0 & 0 & 15.00000000 \\ 0 & -1 & 0 & 4.500000000 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From the equations 3.6, 3.10 and 3.14, the coordinates of points $P_1$, $P_2$ and $P_3$ are calculated in second coordinate system. These coordinates are:

$$P1_2 := \begin{bmatrix} 15.00000000 \\ -60.62177828 \\ -34.99999998 \\ 1. \end{bmatrix}$$

$$P2_2 := \begin{bmatrix} 45.64980478 \\ 43.08300794 \\ -53.62311103 \\ 1. \end{bmatrix}$$

$$P3_2 := \begin{bmatrix} -13.15718496 \\ 38.92864157 \\ -57.34064309 \\ 1. \end{bmatrix}$$

The transformation matrix of first coordinate system with respect to second coordinate system can be calculated once the coordinates of vertices of base are known. This transformation matrix was obtained as:

$$T2to1 := \begin{bmatrix} 0.298019712413817082 & 0.920526655746304101 & -0.252616167456478857 & 39.1376443573145281 \\ -0.954325878040565123 & 0.293181339559288490 & -0.0575049722619127638 & 28.9496382956575218 \\ 0.0211274865603692970 & 0.258215760869250188 & 0.965856226517634142 & 45.6088066047878300 \\ 0. & 0. & 0. & 1. \end{bmatrix}$$

Using this transformation matrix, all the points can be expressed in either of the coordinate systems. Desired point of intersection is known in first coordinate system. Therefore, the initial point of intersection is also calculated in first coordinate system. The initial point of intersection is obtained as:

$$Pintersect_1 := \begin{bmatrix} 63.7833136904575824 \\ 18.6122951143576998 \\ -0. \end{bmatrix}$$

Using the six constraint equations, a function is written that takes constant mechanism parameters, the transformation matrix and the coordinates of desired point of intersection as input and calculates the twist as output.

The twist can be represented as:

[ω; v] = [L, M, N; P, Q, R]

From the last constraint, N = 0. The other five values are calculated using the function and are obtained as:

$$[L = 0.9599642756, M = 0.2801224546, P = 2.316642391\,10^{16}, Q = 1.731843997\,10^{16}, R = -1.334015266\,10^{17}]$$

It is very clear from the above values that the linear velocity required to move the top platform infinitesimally towards desired point of intersection is almost equal to infinity. This proves that a simple tripod is not a solution of the problems involving orientation of top platform.

# CHAPTER 4
# ANALYSIS OF THE TENSEGRITY MECHANISM

This chapter focuses more on finding the proper orientation of the top platform with respect to the bottom. The tensegrity mechanism is a very complicated mechanism. For equilibrium of this mechanism without application of any external force, it has to satisfy certain conditions. One of the conditions is force balance. As the base vertices are considered to be nailed to ground, the top platform has to be tested for balanced forces. There is no possibility of applying an external force in this case. Therefore, the orientation of the top platform has to be chosen carefully.

## Initial Transformation Matrix

To express any transformation matrix, it is first assumed that the two coordinate systems coincide with each other. One of the systems is then translated and/or rotated to its new position. This way, calculating the transformation matrix becomes easy. It takes into account each translation and rotation of the coordinate system and any transformation matrix can be represented using one translation followed by three rotations about three axes.

As discussed earlier, the axes of top and base coordinate systems are parallel to each other. Assuming the two coordinate systems were coinciding with each other, the new position of the top coordinate system can be explained as simple translation matrix. The transformation matrix is calculated by assuming the top coordinate system has been translated to a point defined in the base coordinate system. As the initial origin of the top coordinate system is known in the base coordinate system, the initial transformation matrix is given as:

$$T_{BT} := \begin{bmatrix} 1 & 0 & 0 & \mathrm{P0T}_B[1] \\ 0 & 1 & 0 & \mathrm{P0T}_B[2] \\ 0 & 0 & 1 & \mathrm{P0T}_B[3] \\ 0 & 0 & 0 & 1 \end{bmatrix}; \tag{4.1}$$

where **P0T**$_B$ is the origin of the top coordinate system in the base coordinate system.

**P0T**$_B$ is one of the parameters which depend on the manufacturing and initial positioning of mechanism. This is assumed to be same for all the mechanisms. Hence, it can be said that **P0T**$_B$ is constant for all the cases as it completely depends on the construction of mechanism. The position of top platform with respect to base is going to be the same initially for every mechanism.

### Point Transformations

Using the initial transformation matrix calculated in previous part, all the points from top coordinate system can be expressed in terms for base coordinate system. Also, the equation of line along laser pointer can also be expressed in the base coordinate system.

The coordinates of every point are represented in homogeneous coordinates. In a homogeneous coordinate system, a three-dimensional point given by X, Y and Z are represented by four scalar values, that is, x, y, z and w. The three-dimensional and homogeneous coordinates are related by

$$X := \frac{x}{w};\ Y := \frac{y}{w} : Z := \frac{z}{w} : \tag{4.2}$$

Thus, when w=1, the first three components of the homogeneous coordinates of a point are the same as the three-dimensional coordinates of the point. By using homogeneous coordinates, the equation of all the points is given by following generalized form:

$$[X\ Y\ Z\ w]^T \quad (4.3)$$

Equation of Point ‘P’ is written as:

$$\begin{bmatrix} P \\ 1 \end{bmatrix} \quad (4.4)$$

**P** in the above equation is a 3×1 matrix representing three-dimensional coordinates of a point.

The equation of every point is represented in this form in this report.

Knowing the transformation matrix, homogeneous coordinates of all the points can be calculated in base coordinate system. Vertices of the base already can be stated as constants as they are nailed to the ground and do not change. Their special properties are as follows:

1. Point $\mathbf{P}_1$ is the origin of base coordinate system. Therefore, it’s X, Y and Z coordinates will be ‘0’.

2. Point $\mathbf{P}_2$ lies on X-axis of base coordinate system. Therefore, it’s Y and Z coordinates will be ‘0’.

3. Point $\mathbf{P}_3$ does not lie on any of the axes of base coordinate system but it lies in XY-plane of base coordinate system. Therefore, it’s Z-coordinate will still be ‘0’.

Initial position of points $\mathbf{P}_4$, $\mathbf{P}_5$ ad $\mathbf{P}_6$ i.e. vertices of top platform, in terms of the base coordinate system can be calculated from the initial transformation matrix calculated in previous part and is given by following equations:

$$PB4 := T_{BT} \,.\, PT4;$$

$$PB5 := T_{BT} \,.\, PT5 :$$

$$PB6 := T_{BT} \,.\, PT6 : \quad (4.5)$$

where PB4, PB5 and PB6 are the coordinates of points $\mathbf{P}_4$, $\mathbf{P}_5$ ad $\mathbf{P}_6$ in base coordinate system, respectively.

These values are constant for every mechanism as the initial transformation matrix depends on the construction of the mechanism. The construction is assumed to be same for all the mechanisms. With these equations, coordinates of all vertices of base as well as top coordinate system are known in base coordinate system.

The coordinates of desired point of intersection of the laser line with the plane defined by the base points are already known in the base coordinate system and will be provided by the operator. These will vary according to the position of mechanism with respect to the position of ordnance.

## The Concept of Desired Transformation Matrix

There are two steps to calculate the desired transformation matrix. In first step, the transformation matrix is calculated with considering only one constraint i.e. considering that the laser is supposed to point to the desired point. In second step, the equilibrium of the mechanism is considered and the matrix is modified.

### Desired Transformation Matrix without Considering Conditions for Equilibrium of Mechanism

For understanding the concept behind this research, let us assume a few changes. Let us assume that the robot does not use mechanism for blowing the land mines. In that case, the robot will use a robotic arm. The motion of the robotic arm will be easy to imagine. Imagine a laser pointer attached the fixed robotic arm by a spherical joint as shown in following figure:

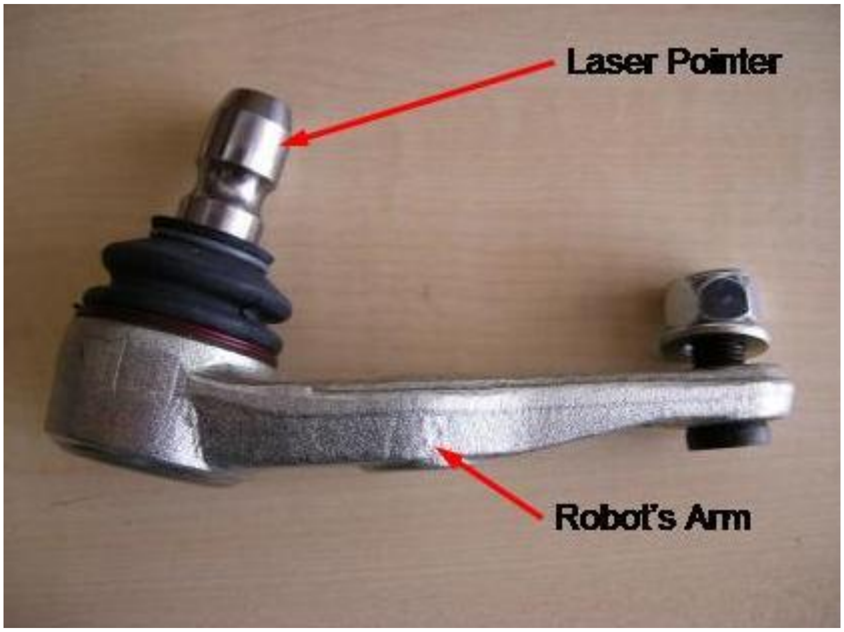


Figure 4-1. Ball joint between laser pointer and robotic arm

The laser pointer has three degrees of freedom when attached to a fixed robotic arm by ball and socket joint; rotation about X-axis, rotation about Y-axis and rotation about Z-axis. In this case, the laser pointer rotates about the mid point of spherical joint. This point of rotation is fixed.

Now let us assume that instead of joining only the laser pointer, there is a small triangular plate which is joined to the fixed robotic arm with the laser pointer. This triangular plate will have the same motion as that of laser pointer. The fixed point of rotation of both, the triangular plate and the laser pointer, is the midpoint of the triangular plate. This is where the laser pointer is attached to it. This triangular plate is the top platform of the mechanism. For the purpose of this research, it is assumed to be the top platform as the motion explained above. The fixed point of rotation is the origin of the top platform. The line along the laser pointer passes through the origin too. Therefore, one of the points on this line will always be known.

The origin of top coordinate system is assumed to be fixed. Hence, the only change in transformation matrix of top with respect to base coordinate system will be rotation. There will not be any translation of the origin.

Also, the laser pointer is fixed to the top platform. The angle between the line along the laser pointer and top platform will always be constant. This angle is assumed to be 90°. In technical terms, the above statement can be rephrased as: “The angle between the XY-plane of the top coordinate system and the laser pointer will always be 90°.” Therefore, once the desired position of the laser pointer is known, the position of the XY-plane of the top coordinate system can be calculated.

The position of the XY-plane can be used to calculate the angle of rotation about the X and Y axes of the original top coordinate system to reach the desired top coordinate system position. The concept of compound transformations can be used to calculate the transformation matrix of the base coordinate system with respect to the desired top coordinate system.

**Conditions for Equilibrium of Mechanism**

For any tensegrity mechanism to be in equilibrium, the top platform of the mechanism should be in static equilibrium. As discussed earlier, all struts in a tensegrity mechanism have compressive forces and all the ties have tensile forces. All of these forces act on the top platform and base platform. The forces acting on the base platform do not matter as all the vertices of the base platform are fixed. But, for the top platform to be in equilibrium, the forces acting on top platform must be balanced.

In the case of this research, making arrangements for application of any kind of external force is impossible. Therefore, the top platform has to be in static equilibrium without application of an external force. All the forces acting on the platform (compressive + tensile) must be cancelled by each other.

The total wrench acting on the top platform can be written as:

$$\hat{\mathbf{w}} = f_1\ B\hat{\mathbf{s}}_1 + f_2\ \hat{\mathbf{s}}_2 + f_3\ \hat{\mathbf{s}}_3 + f_4\ \hat{\mathbf{s}}_4 + f_5\ \hat{\mathbf{s}}_5 + f_6\ \hat{\mathbf{s}}_6 \tag{4.6}$$

where $f_i$ (i = 1…6) are the force magnitudes in each leg of the leg connectors (struts or side ties) and $\hat{\mathbf{s}}_i$ (i = 1...6) are the Plücker coordinates of the lines of action of each of the six legs. When there is no eternal wrench applied to the top platform, $\hat{w} = 0$, and the above equation reduces to

$$\mathbf{f}_1\ \hat{\mathbf{s}}_1 + \mathbf{f}_2\ \hat{\mathbf{s}}_2 + \mathbf{f}_3\ \hat{\mathbf{s}}_3 + \mathbf{f}_4\ \hat{\mathbf{s}}_4 + \mathbf{f}_5\ \hat{\mathbf{s}}_5 + \mathbf{f}_6\ \hat{\mathbf{s}}_6 = \mathbf{0} \tag{4.7}$$

Ignoring the trivial solution of $f_i$=0, it is apparent that when there is no external wrench applied to the top platform that equilibrium can only occur if the Plücker coordinates of the six legs are linearly dependant. This implies that an equilibrium solution with no external wrench will exist only for certain positions and orientations of the top platform.

**Calculation of Desired Transformation Matrix without Force Balance**

There are five basic steps followed to calculate the desired transformation matrix of the top coordinate system with respect to base coordinate system.

**Desired Plücker Coordinates of Line along Laser Pointer:**

With the origin of the second coordinate system fixed, only one more point must be known to define the desired position and orientation of the laser pointer. The goal is to move the laser pointer to the desired point on the ground. Therefore, an equation of desired laser pointer line can be calculated as the line joining the origin of the top coordinate system and the desired point of intersection. Also, as the coordinates of all the points are expressed with respect to the base coordinate system, the coordinates of this desired line are calculated in the base coordinate system. Therefore, the desired Plücker coordinates of laser pointer line are calculated as:

$$\$_{laser} := \{S_{\mathrm{D}}; S0l\} \tag{4.8}$$

$$S_{\mathrm{D}} := P0T_B - P_{desiredB} \tag{4.9}$$

$$S0l := P0T_{\boldsymbol{B}}\, X\, S_{\mathrm{D}} \tag{4.10}$$

where **$**$_{laser}$ is the equation of line in which **S**$_{D}$ is the desired direction of the line along laser pointer and **S0l** is moment of the desired line about origin of base coordinate system. This value will vary for every case depending on the location of desired point of intersection.

**Desired Orientation of XY-plane of Top Coordinate System**

Once the desired coordinates of the laser line is known, the equation of the desired orientation of the XY-plane of the top coordinate system i.e. top platform, can be easily calculated. The only two conditions are, the plane has to pass through the origin and it should be perpendicular to the desired line along the laser pointer.

For calculating the equation of the plane, the direction perpendicular to the plane must be known. This direction is the same as the direction of line **$**$_{laser}$. Therefore, the equation of the desired orientation of the XY-plane can be calculated in the base coordinate system as:

$$\mathrm{XY} := [\mathrm{D0}_B; S_{\mathrm{D}}] \tag{4.11}$$

$$S_{\mathrm{D}} := P0T_B - P_{desiredB} \tag{4.12}$$

$$D0_B := P0T_B \,.\, S_{\mathrm{D}} \tag{4.13}$$

where $\mathbf{S}_D$ is same as that in the equation of line. This will vary depending on the desired point of intersection.

**Transformation Matrix without Applying Equilibrium Conditions**

Now that the desired position of the XY-plane of the top coordinate system is known, the transformation matrix of the desired top coordinate system and the initial top coordinate system can be calculated.

If the lines of intersection of any plane-M with the YZ-plane and the XZ-plane are known, then it becomes easy to align its XY-plane with plane-M. The angle between line of intersection of the YZ-plane and the Y-axis is the angle of rotation of the coordinate system about the X-axis. The angle between the line of intersection of the XZ-plane and the X-axis is the angle of rotation of the coordinate system about the Y-axis. Once the angle of rotation about the X and Y axes are known, the rotation matrices can be calculated. Using the compound transformation, the transformation matrix of desired orientation of the top coordinate system with respect to the base can be calculated.

As the equation of the desired XY-plane is known, its intersection with the initial YZ-plane and the XZ-plane of the top-coordinate system can be calculated.

Let $\mathbf{Y}_d$ be the direction of the line of intersection of the YZ-plane and the desired XY-plane and let x be the required angle of rotation about the X-axis. $\mathbf{Y}_d$ can be easily calculated by using the theory given in “Screw Theory and its Application to Spatial Robot Manipulators” by Dr. Crane and Dr. Duffy, section 1.4. According to this section, if the equations of two planes intersecting each other are known, then the equation of the line of intersection of those two planes can be calculated. The Plücker coordinates of the line of intersection can be given by:

$$\mathbf{r} \times (\mathbf{S}1 \times \mathbf{S}2) = (-D02)\mathbf{S}1 - (-D01)\mathbf{S}2 \qquad (4.14)$$

Comparing the above equation with a standard equation of a line, the direction of the line of intersection is given by (**S**1 × **S**2). Applying this theory to calculate $\mathbf{Y}_d$, the following equation can be written:

$$Y_d := CrossProduct\left(\langle 1, 0, 0\rangle, \langle S_D[1], S_D[2], S_D[3]\rangle\right) : \tag{4.15}$$

where <1, 0, 0> is the direction perpendicular to the YZ-plane i.e. the direction along the initial X-axis.

The angle between $\mathbf{Y}_d$ and the x-axis can be found using a simple vector scalar product. A scalar product of two vectors is given by:

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}|.|\mathbf{b}|.\cos(\Theta) \tag{4.16}$$

where Θ is angle between vectors **a** and **b**. Using the same equation, the angle between the direction vectors of the line of intersection and the x-axis can be calculated.

$$x = \text{acos}(\mathbf{X}.\ \mathbf{Y}_d) \tag{4.17}$$

The transformation matrix with this angle of rotation about the X-axis is given by:

$$T_{TX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(x) & -\sin(\mathrm{x}) & 0 \\ 0 & \sin(x) & \cos(x) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \tag{4.18}$$

Let’s call this stage of transformation ‘X’. With the ‘X’ stage, the top coordinate system is first moved to position ‘X’ and then moved to the desired orientation. The transformation matrix above gives the transformation between initial top coordinate system and ‘X’ stage.

Let’s calculate further the transformation matrix in this new stage. For doing that, the direction vector $\mathbf{S}_D$ should be known in the X-stage. It can be calculated using the following equations:

$$R := \begin{bmatrix} T_{TX}[1,1] & T_{TX}[1,2] & T_{TX}[1,3] \\ T_{TX}[2,1] & T_{TX}[2,2] & T_{TX}[2,3] \\ T_{TX}[3,1] & T_{TX}[3,1] & T_{TX}[3,3] \end{bmatrix} \tag{4.19}$$

$$S_{DX} := R.\langle S_D[1], S_D[2], S_D[3] \rangle \tag{4.20}$$

where $\mathbf{S}_{DX}$ is direction vector $\mathbf{S}_D$ in 'X' stage.

After this rotation, the X-axis of the top-coordinate system is already in the desired XY-plane. Now, consider the line of intersection of the XZ-plane and Y-axis in a similar way to calculate the angle of rotation about the Y-axis.

Let **X**d be the direction of the line of intersection of the XZ-plane and the desired XY-plane and let y be the required angle of rotation about the Y-axis.

Following these equations, the equation for $\mathbf{X}_d$ can be given as :

$$X_d := CrossProduct\left(\langle 0, 1, 0\rangle, \langle S_{DX}[1], S_{DX}[2], S_{DX}[3] \rangle\right) \tag{4.21}$$

where <0, 1, 0> is the direction along the new Y-axis. With $\mathbf{X}_d$ known, the angle of rotation about the Y-axis can be given by:

$$y = \mathrm{acos}(\mathbf{X}_d \cdot \mathbf{Y}) \tag{4.22}$$

The transformation matrix for this rotation is given by:

$$T_{XD} := \begin{bmatrix} \cos(y) & 0 & \sin(y) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(y) & 0 & \cos(y) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \tag{4.23}$$

Using compound transformations, the final transformation matrix of the initial top coordinate system with the desired top coordinate system can be given by the following equation:

$$T_{TD} := T_{TX}.T_{XD} \tag{4.24}$$

where, $T_{TX}$ is transformation matrix of the initial top coordinate system with stage ‘X’ and ‘$T_{XD}$’ is the transformation matrix of stage ‘X’ with the desired top coordinate system.

Using the same principle of compound transformations, the transformation matrix of the desired top coordinate system with that of base can be calculated. The equation used is same as that used for the compound transformations used for the top coordinate system. It is given as:

$$T_{BD} := T_{BT} \cdot T_{TD} \tag{4.25}$$

**Calculation of Final Desired Transformation Matrix**

With the above transformation matrix, the laser pointer is pointing at the desired location on the ground, but it is not necessary the case that the mechanism is stable. For a mechanism to be stable, the determinant of the following matrix should be zero.

$$J := \begin{bmatrix} S14[1] & S15[1] & \mathrm{S25}[1] & \mathrm{S26}[1] & \mathrm{S34}[1] & \mathrm{S36}[1] \\ S14[2] & S15[2] & \mathrm{S25}[2] & \mathrm{S26}[2] & \mathrm{S34}[2] & \mathrm{S36}[2] \\ S14[3] & S15[3] & \mathrm{S25}[3] & \mathrm{S26}[3] & \mathrm{S34}[3] & \mathrm{S36}[3] \\ SO14[1] & SO15[1] & \mathrm{SO25}[1] & \mathrm{SO26}[1] & \mathrm{SO34}[1] & \mathrm{SO36}[1] \\ SO14[2] & SO15[2] & \mathrm{SO25}[2] & \mathrm{SO26}[2] & \mathrm{SO34}[2] & \mathrm{SO36}[2] \\ SO14[3] & SO15[3] & \mathrm{SO25}[3] & \mathrm{SO26}[3] & \mathrm{SO34}[3] & \mathrm{SO36}[3] \end{bmatrix}; \tag{4.26}$$

where, **S**ij (i=1...6, j=1...6) are Plücker coordinates of the lines along the three struts and the three side ties. If the determinant of the above matrix is ‘0’ then one can say that all these six lines are linearly dependant. As discussed earlier, if the lines are linearly dependant, the top platform is in static equilibrium.

For a tensegrity mechanism to satisfy the equilibrium condition listed above, only two parameters can be changed by keeping the laser pointer pointing at the same location. One of the parameters is translation of the top coordinate system along its Z-axis and the other is its rotation about the Z-axis. It was checked mathematically and noticed that there is no change in the Jacobean matrix with the change in Z coordinate of the origin of the top coordinate system. The only other option left is the rotation about Z-axis of the top coordinate system.

A method was documented to calculate the transformation matrix by rotation of the coordinate system about an arbitrary axis other than the three standard axes of a coordinate system. Using this method, if the vector about which the coordinate system is rotated is known with the angle of rotation, the transformation matrix can be calculated. The rotation matrix given by this method is:

$$ {}^{A}_{B}\boldsymbol{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{4.27} $$

where the elements of the matrix are given as:

r11 = $(m_x)^2 (1 - \cos(t)) + \cos(t)$

r12 = $m_x.m_y.(1 - \cos(t)) - m_z . \sin(t)$

r13 = $m_x.m_z.(1 - \cos(t)) + m_y . \sin(t)$

r21 = $m_x.m_y. (1 - \cos(t)) + m_z . \sin(t)$

r22 = $(m_y)^2 (1 - \cos(t)) + \cos(t)$

r23 = $m_y.m_z. (1 - \cos(t)) - m_x . \sin(t)$

r31 = $m_x.m_z. (1 - \cos(t)) - m_y . \sin(t)$

r32 = $m_y.m_z. (1 - \cos(t)) + m_x . \sin(t)$

r33 = $(m_z)^2 (1 - \cos(t)) + \cos(t)$

In these equations, the following parameters are used:

$m_{ix}$: X coordinate of vector about which the coordinate system is rotated

$m_y$: Y coordinate of vector about which the coordinate system is rotated

$m_z$: Z coordinate of vector about which the coordinate system is rotated

In case of this research, that vector is a vector along the laser line. Therefore, $m_x$, $m_y$ and $m_z$ are known. There is only one unknown value in the equation; the angle of rotation 't'.

As discussed earlier, there is only one equilibrium condition that the transformation matrix has to satisfy and now there is only one unknown which is the angle of rotation. The total transformation matrix in terms angle 't' can be calculated using the following equation:

$$T = TR.T_{BD} \tag{4.28}$$

where TR is the transformation matrix in terms of (t). Using this matrix, all the elements of Jacobean matrix are recalculated and are found in terms of only one unknown variable, (t). The determinant of this matrix has to be equal to zero according to the equilibrium conditions. The solving of this equation to calculate the angle of rotation results in 12 different values. Out of these 12 values, some of them are imaginary and some of them are real. The number of imaginary and real values is not constant.

Once the value of angle of rotation is known, the final desired transformation matrix can be calculated by substituting that value.

## Calculation of Change in Struts and Ties Length

Once the transformation matrix is known, all the vertices of the top platform in its desired orientation can be calculated in the base coordinate system using the following equations:

$$PB4 := T_{BDesired} \cdot PT4;$$
$$PB5 := T_{BDesired} \cdot PT5;$$
$$PB6 := T_{BDesired} \cdot PT6; \tag{4.29}$$

The desired strut length can be very easily calculated as the distance between points (PB1 – PB6), (PB3 – PB5) and (PB2 – PB4) respectively. Similarly, the total length of ties 1, 2 and 3 are obtained as the distance between points (PB1 – PB4), (PB3 – PB6) and (PB2 – PB5). Further, unit vectors alone each of the lines defined by the ties and struts can readily be determined and will be written as $\mathbf{S}_{ij}$ where the subscripts i and j refer to the specific mechanism vertex points that define the vector from point i to point j, with this notation, it is apparent that

$$\mathbf{S}_{ij} = \mathbf{S}_{ji} \tag{4.30}$$

The analysis proceeds by performing a static force analysis. There are twelve unknown force magnitudes, i.e. the compressive or tensile forces of the three struts, the three compliant ties, three bottom ties and three top ties. Compressive forces in the three struts are written as Fa, Fb and Fc. The tensile forces in the three varying length ties are written as fa, fb and fc. The tensile forces in base ties between pair of points are written as T12, T23 and T13 and those in top ties are T45, T56 and T46.

**Force balance equation:** Each vertex provides three equations and the sum of these forces in X, Y and Z direction is zero. In vector form, writing force balance equation at points 1, 2, 4 and 6yields

$$\mathbf{fa} - \mathbf{Fa} + \mathbf{T12} + \mathbf{T13} = 0$$

$$\mathbf{fb} - \mathbf{Fb} + \mathbf{T12} + \mathbf{T23} = 0$$

$$\mathbf{fa} - \mathbf{Fc} + \mathbf{T45} + \mathbf{T46} = 0$$

$$\mathbf{fc} - \mathbf{Fb} + \mathbf{T56} + \mathbf{T46} = 0 \tag{4.31}$$

Equations for all the vertices will give (6X3 = 18) equations and there are only 12 unknown variables. Therefore, four vertices were arbitrarily chosen.

Let $\mathbf{S}_{ij}$ be unit vector along i and j, i.e. along the ties and struts, where j>I for i = 1,2,..5 and j = 2, 3..6. The system of equations given above can be written as:

$$fa\ \mathbf{S}_{14} - Fa\ \mathbf{S}_{15} + T12\ \mathbf{S}_{12} + T13\ \mathbf{S}_{13} = 0$$

$$fb\ \mathbf{S}_{25} - Fb\ \mathbf{S}_{26} + T12\ \mathbf{S}_{12} + T23\ \mathbf{S}_{23} = 0$$

$$fa\ \mathbf{S}_{14} - Fc\ \mathbf{S}_{34} + T45\ \mathbf{S}_{45} + T46\ \mathbf{S}_{46} = 0$$

$$fc\ \mathbf{S}_{36} - Fb\ \mathbf{S}_{26} + T56\ \mathbf{S}_{56} + T46\ \mathbf{S}_{46} = 0 \tag{4.32}$$

Since, the position of the structure is known, vectors, $\mathbf{S}_{ij}$ can be easily found. They are then broken down into x,y and z components: $\mathbf{S}_{ij}x$, $\mathbf{S}_{ij}y$ and $\mathbf{S}_{ij}z$. Therefore, 12 equations are obtained from the set of equations given above. These 12 equations can be written in the form as

$$\mathbf{J}\ \mathbf{v} = 0 \tag{4.33}$$

Where,

$$\mathbf{J} = \begin{bmatrix}
S_{14}x & 0 & 0 & -S_{15}x & 0 & 0 & S_{12}x & 0 & S_{13}x & 0 & 0 & 0 \\
S_{14}y & 0 & 0 & -S_{15}y & 0 & 0 & S_{12}y & 0 & S_{13}y & 0 & 0 & 0 \\
S_{14}z & 0 & 0 & -S_{15}z & 0 & 0 & S_{12}z & 0 & S_{13}z & 0 & 0 & 0 \\
0 & S_{25}x & 0 & 0 & -S_{26}x & 0 & -S_{12}x & S_{23}x & 0 & 0 & 0 & 0 \\
0 & S_{25}y & 0 & 0 & -S_{26}y & 0 & -S_{12}y & S_{23}y & 0 & 0 & 0 & 0 \\
0 & S_{25}z & 0 & 0 & -S_{26}z & 0 & -S_{12}z & S_{23}z & 0 & 0 & 0 & 0 \\
-S_{14}x & 0 & 0 & 0 & 0 & S_{34}x & 0 & 0 & 0 & S_{45}x & 0 & S_{46}x \\
-S_{14}y & 0 & 0 & 0 & 0 & S_{34}y & 0 & 0 & 0 & S_{45}y & 0 & S_{46}y \\
-S_{14}z & 0 & 0 & 0 & 0 & S_{34}z & 0 & 0 & 0 & S_{45}z & 0 & S_{46}z \\
0 & 0 & -S_{36}x & 0 & S_{26}x & 0 & 0 & 0 & 0 & 0 & -S_{56}x & -S_{46}x \\
0 & 0 & -S_{36}y & 0 & S_{26}y & 0 & 0 & 0 & 0 & 0 & -S_{56}y & -S_{46}y \\
0 & 0 & -S_{36}z & 0 & S_{26}z & 0 & 0 & 0 & 0 & 0 & -S_{56}z & -S_{46}z
\end{bmatrix}$$

and $$\mathbf{v} = [f_A, f_B, f_C, F_A, F_B, F_C, T_{12}, T_{23}, T_{13}, T_{45}, T_{56}, T_{46}]^T$$

Solving the 12 equations listed above, the values of 12 unknowns are obtained. By applying simple concept for force in a linear spring, the equation for the force in each spring can be written as:

fa = K1.x1

fb = K2.x2

fc = K3.x3 (4.34)

where x1, x2 and x3 are the change in length of each spring and can be calculated from the equation above. Finally the length of the three variable length tie segments l1, l2 and l3 can be determined from following equations:

la = $l_{1total}$ – len1 – x1

la = $l_{2total}$ – len2 – x2

la = $l_{3total}$ – len3 – x3 (4.35)

where, $l_{1total}$, $l_{2total}$ and $l_{3total}$ is total length of side ties (length of compliant part + length of non compliant part) and len1, len3 and len3 are spring free lengths.

**Numerical Example**

In this example, the values of constants and inputs are arbitrarily decided. They can be changed before starting the example, but will remain constant for all the similar mechanisms once selected. The three spring constants are considered to be same and free lengths of the springs are equal. This means the side ties will experience the same force at the mechanisms initial configuration. Assuming that the initial constant values are given:

$$PB1 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$PB2 := \begin{bmatrix} 0 \\ 25 \\ 0 \\ 1 \end{bmatrix}$$

$$PB3 := \begin{bmatrix} 20 \\ 16 \\ 0 \\ 1 \end{bmatrix};$$

$$PT4 := evalf\left(\begin{bmatrix} -3 \\ -3 \cdot \mathrm{sqrt}(3) \\ 0 \\ 1 \end{bmatrix}\right);$$

$$PT5 := \begin{bmatrix} 6 \\ 0 \\ 0 \\ 1 \end{bmatrix};$$

$$PT6 := \begin{bmatrix} -3. \\ 5.196152424 \\ 0. \\ 1. \end{bmatrix}$$

$$P0T_B := \begin{bmatrix} 5 \\ 10 \\ 30 \\ 1 \end{bmatrix}:$$

$$P_{desiredB} := \begin{bmatrix} 25 \\ 25 \\ 0 \\ 1 \end{bmatrix};$$

From all above values, the initial transformation matrix is calculated.

$$T_{BT} := \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 30 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using above transformation matrix, the vertices of top platform are calculated in base coordinate system.

$$PB4 := \begin{bmatrix} 2. \\ 4.803847576 \\ 30. \\ 1. \end{bmatrix}$$

$$PB5 := \begin{bmatrix} 11 \\ 10 \\ 30 \\ 1 \end{bmatrix}$$

$$PB6 := \begin{bmatrix} 2. \\ 15.19615242 \\ 30. \\ 1. \end{bmatrix}$$

Desired orientation of XY-plane of top coordinate system is obtained from the equations 4.11, 4.12 and 4.13 as:

$$D0_B := 650$$

$$S_D := \begin{bmatrix} -20 \\ -15 \\ 30 \\ 0 \end{bmatrix}$$

To calculate transformation matrix for changing orientation of top coordinate system to match desired position of XY-plane, the angle of rotation about X and Y axes is calculated using equations 4.15 and 4.22 . The values obtained are:

$x := 1.55334214$

$y := 1.55334214$

Using these values, the final transformation matrix (without considering equilibrium conditions) is obtained to be:

$$T_{BD} := \begin{bmatrix} 0.01745329258 & 0. & 0.9998476797 & 5. \\ 0.9996953826 & 0.01745329258 & -0.01745063409 & 10. \\ -0.01745063409 & 0.9998476797 & 0.0003046174219 & 30. \\ 0. & 0. & 0. & 1. \end{bmatrix}$$

To calculate the Jacobean matrix, a transformation matrix with purely rotational elements only is calculated in terms of angle of rotation ‘t’ and is used to obtain the final transformation matrix (considering equilibrium constraints) is calculated. Using this transformation matrix, all the vertices are recalculated in base coordinate system. Using these vertices, the six lines along three struts and three side ties are calculated to get a Jacobean matrix in terms of ‘t’. The determinant of this matrix gives following equation in terms of only one unknown,‘t’:

$$
\begin{aligned}
det := & 2.35070910^{20} \cos(t)^4 - 1.51969810^{21} \cos(t)^5 \sin(t) \\
& + 7.30086 10^{21} \cos(t)^4 \sin(t) + 6.0556 10^{21} \cos(t)^3 \sin(t)^2 \\
& + 6.95443136110^{24} \cos(t)^2 \sin(t)^3 \\
& - 4.39800791510^{22} \cos(t) \sin(t)^4 \\
& + 3.94901610^{22} \cos(t)^4 \sin(t)^2 \\
& - 2.31755424110^{24} \cos(t)^3 \sin(t)^3 + 2.58856910^{19} \cos(t)^6 \\
& - 1.30100410^{20} \cos(t)^5 + 2.19755934910^{22} \cos(t)^2 \sin(t)^4 \\
& - 1.\ 10^{9} \sin(t)^6 - 5.4039432810^{19} \sin(t)^5 \\
& + 5.4085580810^{19} \cos(t) \sin(t)^5 - 1.\ 10^{7} - 4.\ 10^{13} \cos(t) \\
& + 5.2527 10^{19} \cos(t)^2 - 1.249268710^{23} \sin(t)^2 \\
& - 1.83383710^{20} \cos(t)^3 + 2.31931857010^{24} \sin(t)^3 \\
& - 2.7483410^{21} \cos(t) \sin(t) + 9.75813 10^{21} \cos(t)^2 \sin(t) \\
& + 1.\ 10^{15} \sin(t) + 3.34889510^{23} \cos(t) \sin(t)^2 \\
& + 2.20044727410^{22} \sin(t)^4 - 1.279095210^{22} \cos(t)^3 \sin(t) \\
& - 2.55508410^{23} \cos(t)^2 \sin(t)^2 \\
& - 6.95619569210^{24} \cos(t) \sin(t)^3
\end{aligned}
$$

Solving this equation for ‘t’, following different values of ‘t’ are obtained.

$$
\begin{aligned}
& [[t = 0.1020686080], [t = 0.05090018412 + 0.03671270550 I], [t \\
& = 3.116468538 - 0.04342716775 I], [t = 0.03240545074 \\
& + 0.07108689189 I], [t = -0.02666020952 + 0.08913858502 I], [t \\
& = -0.08172615815 + 0.03807404331 I], [t = -3.128149943], [t = \\
& -0.08172615815 - 0.03807404331 I], [t = -0.02666020952 \\
& - 0.08913858502 I], [t = 0.03240545074 - 0.07108689189 I], [t \\
& = 3.116468538 + 0.04342716775 I], [t = 0.05090018412 \\
& - 0.03671270550 I]]
\end{aligned}
$$

It is observed that some of the values are imaginary and only some of the values are real. Neglecting the imaginary values and selecting one of the real value, the forces in each strut and side tie are calculated. Using these forces, the values of non-complaint parts of side ties are obtained as:

$lA := 13.9973952$

$lB := 19.4721712$

$lC := 10.5051177$4

These values are well within the limits.

After trying with all the real values of angle of rotation ‘t’, it has been observed that the minimum value of angle of rotation gives minimum change in the length of non-compliant part of the side tie. Therefore, out of all the real values, only the minimum value of angle of rotation is selected in the above example and answers are presented.

# CHAPTER 5
# FUTURE WORK

This research opens up to new challenges. One of them is to actually manufacture the designed mechanism. There are many things that need special consideration while doing so. To name a few material and manufacturing cost, a cheap microprocessor and wireless contacts are some of the important things under consideration. This is on design aspect side of it.

In addition to this, the change in length of struts and ties calculated is a real number. It will be very difficult to achieve such a great accuracy in real life. It will be great if the accuracy is mapped as a specific area around the desired point of intersection depending on the accuracy provided by moving mechanical parts of the mechanism. Cost consideration might be an issue where accuracy is expected. Also, if change in length is adjusted in the form of small clicks of specific distance, then number of clicks and its effect on the mechanism might be an interesting topic to study. The main concern in this case will be stability of the mechanism.

Also, only two mechanisms to achieve the desired goal are discussed in this research. There might be other mechanisms that might be more efficient than the ones discussed in this report. The world of mechanisms has no limits, so doesn’t the future work in this topic.

# APPENDIX
# MAPLE PROGRAM FOR TRIPOD:

> $restart$ ;

> $with(LinearAlgebra)$ :

> # $tripod\ aiming\ device$

## Constants:

> $R2D := evalf\left(\frac{180.0}{\mathrm{Pi}}\right)$ :

> $D2R := \frac{1}{R2D}$ :

## Procedures

> $dist := \mathbf{proc}(P1, P2)$
> $\mathrm{sqrt}\left((P1[1] - P2[1])^2 + (P1[2] - P2[2])^2 + (P1[3] - P2[3])^2\right)$
> $\mathbf{end\ proc}$ ;

> $mag := \mathbf{proc}(V1)\quad \mathrm{sqrt}\left(V1[1]^2 + V1[2]^2 + V1[3]^2\right)$ $\mathbf{end\ proc}$ ;

> $reciprocalProduct := \mathbf{proc}\ (screw1, screw2)$
> $screw1[1..3] . screw2[4..6] + screw1[4..6] . screw2[1..3]$ :
> $\mathbf{end\ proc}$ :

## Given

> $P5_2 := \begin{bmatrix} 30. \\ 0 \\ 0 \end{bmatrix}$ : # *cm (Point 5 in coordinate system 2)*

> $P6_2 := \begin{bmatrix} 15. \\ 9. \\ 0 \end{bmatrix}$ : # *cm (Point 6 in coordinate system 2)*

> $d_1 := \frac{P5_2[1]}{2}$ : # *hinge points are put at the mid points*

> $d_2 := \frac{dist(P5_2, P6_2)}{2}$ :

> $d_3 := \dfrac{dist\left(\begin{bmatrix}0\\0\\0\end{bmatrix}, P6_2\right)}{2}$ :

> $T3to2 := \begin{bmatrix} 1 & 0 & 0 & \dfrac{\mathrm{P5}_2[1]}{2} \\ 0 & -1 & 0 & \dfrac{\mathrm{P6}_2[2]}{2} \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

> $\theta_1 := 120 \cdot D2R$ : # *radians*

> $\theta_2 := 140 \cdot D2R$ :

> $\theta_3 := 145 \cdot D2R$ :

> $L_1 := 70.$ : # *cm*

> $L_2 := 70.$ :

> $L_3 := 70.$ :

> $Pintersectdesired_1 := \langle 63., 20., 0 \rangle$ :

> $params := [P5_2, P6_2, d_1, d_2, d_3, T3to2]$ ;
> # *constant mechanism parameters*

## Step 1 - get coordinates of P1, P2, and P3 in the 2nd coordinate system

> $TL1to2 := \begin{bmatrix} 0 & 0 & 1 & \mathrm{d}_1 \\ -\sin(\theta_1) & -\cos(\theta_1) & 0 & 0 \\ \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ :

> $P1_2 := TL1to2 . \begin{bmatrix} \mathrm{L}_1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

> $TL2to5corner := \begin{bmatrix} 0 & 0 & 1 & d_2 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & 0 \\ \cos(\theta_2) & -\sin(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}:$

> $cornerangle5 := \arctan\left(P6_2[2], P6_2[1] - P5_2[1]\right):$

> $T5cornerto2 := \begin{bmatrix} \cos(cornerangle5) & -\sin(cornerangle5) & 0 & P5_2[1] \\ \sin(cornerangle5) & \cos(cornerangle5) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}:$

> $P2_2 := T5cornerto2 \,.\, TL2to5corner \,.\, \begin{bmatrix} L_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

> $TL3tocorner6 := \begin{bmatrix} 0 & 0 & 1 & d_3 \\ -\sin(\theta_3) & -\cos(\theta_3) & 0 & 0 \\ \cos(\theta_3) & -\sin(\theta_3) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}:$

> $cornerangle6 := arctan\left(-P6_2[2], -P6_2[1]\right):$

> $T6cornerto2 := \begin{bmatrix} \cos(cornerangle6) & -\sin(cornerangle6) & 0 & P6_2[1] \\ \sin(cornerangle6) & \cos(cornerangle6) & 0 & P6_2[2] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}:$

> $P3_2 := T6cornerto2 \,.\, TL3tocorner6 \,.\, \begin{bmatrix} L_3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

## Step 2 - get T2to1 transformation matrix

> $x1vec_2 := \frac{(P2_2[1..3] - P1_2[1..3])}{dist(P2_2, P1_2)}$ :

> $tempvec_2 := \frac{(P3_2[1..3]-P1_2[1..3])}{dist(P3_2, P1_2)}$ :

> $z1vec_2 := \frac{(x1vec_2 \; \&x \; tempvec_2)}{mag(x1vec_2 \; \&x \; tempvec_2)}$ :

> $y1vec_2 := z1vec_2 \; \&x \; x1vec_2$ :

> $T1to2 := Matrix(4, 4)$ :

> $T1to2[1..3, 1] := x1vec_2$ : $T1to2[1..3, 2] := y1vec_2$ : $T1to2[1..3, 3] := z1vec_2$ : $T1to2[1..4, 4] := P1_2$ :

> $T2to1 := MatrixInverse(T1to2)$ ;

## Step 3 - determine coordinates of laser line in coordinate system 1

> $LaserLine_3 := \langle 0, 0, 1, 0, 0, 0 \rangle$ :
*# coordinates of laser line in the 3rd coord system*

> $T3to1 := T2to1 \,.\, T3to2$ :

> $LaserLine_1 := \langle 0, 0, 0, 0, 0, 0 \rangle$ : *# placeholder*

> $LaserLine_1[1..3] := T3to1[1..3, 1..3] \,.\, LaserLine_3[1..3]$ :

> $LaserLine_1[4..6] := T3to1[1..3, 1..3] \,.\, LaserLine_3[4..6] + T3to1[1..3, 4] \; \&x \; (T3to1[1..3, 1..3] \,.\, LaserLine_3[1..3])$ :

> $LaserLine_1$ *# coordinates of laser line in coord system 1*

## Step 4 - determine intersection point of laser line and base plane in coordinate system 1

> $Splane_1 := \langle 0, 0, 1 \rangle$ : $Dplane_1 := 0$ :

> $Sline_1 := LaserLine_1[1..3] : S0line_1 := LaserLine_1[4..6] :$

> $Pintersect_1 := \dfrac{\left(Splane_1 \,\&x\, S0line_1 - Dplane_1 \cdot Sline_1\right)}{DotProduct\left(Splane_1, Sline_1\right)}$

Now, given the desired intersection point in coordinate system 1, determine the infinitesimal twist that will move the intersection point towards the goal

```
GetCorrectiveTwist1 := proc(Pint, Pdesired, T2to1, P2in1, P3in1,
    params)

 description "returns corrective infinitesimal twist"
 local P5in2, P6in2, d1, d2, d3, T3to2, Sr1a, Sr1b, Sr2a, Sr2b, Sr3a,
    Sr3b, Pleg1topin1, Pleg2topin1, Pleg3topin1, P4in1, P5in1,
    P6in1, temp, v1, v2, v3, correctiveDirection, laserDirection,
    omegaTwist, mat1, vec1, v, screw, L, M, N, P, Q, R, srw, minv,
   ans, temp2;
P5in2 := params[1] : P6in2 := params[2] : d1 := params[3] : d2
    := params[4] : d3 := params[5] : T3to2 := params[6] :

P4in1 := T2to1[1..3, 4] :
temp := T2to1 . <P5in2, 1> : P5in1 := temp[1..3, 1] :
temp := T2to1 . <P6in2, 1> : P6in1 := temp[1..3, 1] :
Pleg1topin1 := P4in1 + d1·v1 :
Pleg2topin1 := P5in1 + d2·v2 :
Pleg3topin1 := P6in1 + d3·v3 :
v1 := (P5in1 - P4in1) / dist(P5in1, P4in1) : # unit vector from P4 to P5
v2 := (P6in1 - P5in1) / dist(P6in1, P5in1) :
v3 := (P4in1 - P6in1) / dist(P4in1, P6in1) :
Sr1b := <v1, 0, 0, 0> :
   # line through bottom of leg 1 parallel to distal revolute joint
   axis
Sr2b := <v2, P2in1 &x v2> :
Sr3a := < (Pleg3topin1 - P3in1) / dist(Pleg3topin1, P3in1), P3in1
   &x ((Pleg3topin1 - P3in1) / dist(Pleg3topin1, P3in1)) > :
Sr3b := <v3, P3in1 &x v3> :
#screw :=<L, M, N, P, Q, R> :
N := 0 :
```

```
correctiveDirection := Pdesired − Pint :
laserDirection := (T2to1 . T3to2)[1 ..3, 3] :
temp := correctiveDirection &x laserDirection :
mat1 := Matrix(1 ..3, 1 ..3) :
mat1[1, 1 ..3] := Sr1b[1 ..3, 1] :
mat1[2, 1 ..3] := Sr2b[1 ..3, 1] :
mat1[3, 1 ..3] := Sr3b[1 ..3, 1] :
vec1 := ⟨0, 0, 0⟩ :
vec1[1] := − (L · Sr1b[4, 1] + M · Sr1b[5, 1]) :
vec1[2] := − (L · Sr2b[4, 1] + M · Sr2b[5, 1]) :
vec1[3] := −(L · Sr3b[4, 1] + M · Sr3b[5, 1] ) :
v := MatrixInverse(mat1) . vec1 :
srw := ⟨L, M, 0⟩ :
temp2 := srw &x Pint :
N = 0 :
ans := solve({ L = sqrt(1 − M^2), temp2 . temp = 0, v[1] = P, v[2]
    = Q, v[3] = R}, [L, M, P, Q, R]) :

#screw:=⟨ans[1], ans[2], 0, ans[4], ans[5], ans[6]⟩ :

ans
end proc;
```

```
> P2_1 := T2to1 .P2_2 ;
  P3_1 := T2to1 . P3_2 ;

  myTwist := GetCorrectiveTwist1(Pintersect_1, Pintersectdesired_1,
      T2to1, P2_1[1 ..3], P3_1[1 ..3], params);

> myTwist[1];
```

## MAPLE PROGRAM FOR TENSEGRITY MECHANISM:

```
restart;

with(LinearAlgebra) :

# tripod approach four

> R2D := evalf(180.0/Pi) :

> D2R := 1/R2D :

> dist :=proc(P1, P2)
    sqrt((P1[1] - P2[1])^2 + (P1[2] - P2[2])^2 + (P1[3]-P2[3])^2)
  end proc ;

> mag := proc(V1)   sqrt(V1[1]^2 + V1[2]^2 + V1[3]^2) end proc ;

> reciprocalProduct := proc (screw1, screw2)
  screw1[1..3] . screw2[4..6] + screw1[4..6] . screw2[1..3] :
  end proc :
```

Given:

```
> L01 := 30 :

> L02 := 30 :

> L03 := 30 :

> x1 := 5 :

> x2 := 5 :

> x3 := 5 :
```

> $PB1 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

> $PB2 := \begin{bmatrix} 0 \\ 25 \\ 0 \\ 1 \end{bmatrix}$

> $PB3 := \begin{bmatrix} 20 \\ 16 \\ 0 \\ 1 \end{bmatrix};$

> $PT5 := \begin{bmatrix} 6 \\ 0 \\ 0 \\ 1 \end{bmatrix};$

> $PT4 := evalf\left(\begin{bmatrix} -3 \\ -3\cdot \mathrm{sqrt}(3) \\ 0 \\ 1 \end{bmatrix}\right);$

> $PT5 := \begin{bmatrix} 6 \\ 0 \\ 0 \\ 1 \end{bmatrix};$

> $PT6 := evalf\left(\begin{bmatrix} -3 \\ 3\cdot \mathrm{sqrt}(3) \\ 0 \\ 1 \end{bmatrix}\right);$

> $P0T_B := \begin{bmatrix} 5 \\ 10 \\ 30 \\ 1 \end{bmatrix}:$

> $P_{intersecB} := \begin{bmatrix} 5 \\ 10 \\ 0 \\ 1 \end{bmatrix};$

> $P_{desiredB} := \begin{bmatrix} 25 \\ 25 \\ 0 \\ 1 \end{bmatrix};$

Procedure to find desired transformation matrix

### Step 1: original transformation matrix

> *# Lets assume that the Top co-ordinate system is rotated by some angles about X,Y and Z axes and is moved to point P0T $_B$. so, transfermation matrix can be given as :*

> $T_{BT} := \begin{bmatrix} 1 & 0 & 0 & \mathrm{P0T}_B[1] \\ 0 & 1 & 0 & \mathrm{P0T}_B[2] \\ 0 & 0 & 1 & \mathrm{P0T}_B[3] \\ 0 & 0 & 0 & 1 \end{bmatrix};$

> *#Assuming these angles can be found out when tripod is kept on the floor by robot by image recognition*

### Step 2: find co-ordinates of points P4, P5, P6 in base co-ordinate system

> $PB4 := T_{BT} . PT4;$

> $PB5 := T_{BT} . PT5;$

> $PB6 := T_{BT} . PT6;$

### Step 3: find the equation of desired position of XY plane in base co-ordinate system

> *#direction of desired line i.e. line passing through desired point of intersection and origin of top co-ordinate system ( which is a constant point with respect to base co-ordinate system)*

> $S_{\mathrm{D}} := P0T_B - P_{desiredB};$

> *# This is same as the direction perpendicular to desired position of XY Plane.*

> $\# \; Also \; point \; P0T_B \; is \; on \; the \; desired \; XY \; plane$
> $. \; Hence \; we \; can \; find \; the \; equation \; of \; XY \; plane$

> $D0_B := P0T_B \,.\, S_D;$

> $\# \; Desired \; position \; of \; XY \; plane \; is \; \left[D0_B; S_D\right]$

## Step 4: Calculation of transformation matrix between top co-ordinate system and desired position of top plate

> *#direction of line of intersection of desired XY plane and YZ plane (we can align Y asis with this direct by rotating about X axis)*

> $Y_d := CrossProduct\left(\langle 1, 0, 0\rangle, \left\langle S_D[1], S_D[2], S_D[3]\right\rangle\right) :$

> *#Angle of rotation about X axis*

> $x := \arccos\left(D2R\left(DotProduct\left(Y_d, \langle 1, 0, 0\rangle\right)\right)\right);$

> *#Transformation matrix after rotation about X axis*

> $$T_{TX} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(x) & -\sin(x) & 0 \\ 0 & \sin(x) & \cos(x) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

> $$R := \begin{bmatrix} T_{TX}[1,1] & T_{TX}[1,2] & T_{TX}[1,3] \\ T_{TX}[2,1] & T_{TX}[2,2] & T_{TX}[2,3] \\ T_{TX}[3,1] & T_{TX}[3,1] & T_{TX}[3,3] \end{bmatrix} : P := \begin{bmatrix} T_{TX}[4,1] \\ T_{TX}[4,2] \\ T_{TX}[4,3] \end{bmatrix};$$

> $S_{DX} := R.\left\langle S_D[1], S_D[2], S_D[3]\right\rangle;$

> *#direction of line of intersection of desired XY plane and XZ plane (we can align X asis with this direct* **by** *rotating about y axis)*

> $X_d := CrossProduct\left(\langle 0, 1, 0\rangle, \left\langle S_{DX}[1], S_{DX}[2], S_{DX}[3]\right\rangle\right);$

> *#Angle of rotation about Y axis*

> $y := \arccos\left(D2R\left(DotProduct\left(X_d, \langle 0, 1, 0\rangle\right)\right)\right);$

> $T_{XD} := \begin{bmatrix} \cos(y) & 0 & \sin(y) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(y) & 0 & \cos(y) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$

> $T_{TD} := T_{TX}.T_{XD};$

**Step5: Calculation of required transformation matrix between Base and desired co-ordinate system.**

> $T_{BD} := T_{BT} \,.\, T_{TD};$

**Bringing determinant of transformation matrix to '0' with laser intersecting at desired point of intersection**

> $U := T_{BD};$

> $mx := S_{\mathrm{D}}[1] : my := S_{\mathrm{D}}[2] : mz := S_{\mathrm{D}}[3] : s := \sin(t) : c := \cos(t) :$
$v := (1 - \cos(t)) :$

> $Rot$
$:= [[\mathrm{mx}^2 \cdot v + c, \mathrm{mx} \cdot \mathrm{my} \cdot \mathrm{v} - \mathrm{mz} \cdot \mathrm{s}, \mathrm{mx} \cdot \mathrm{mz} \cdot \mathrm{v} + \mathrm{my} \cdot \mathrm{s}, 0$
$],$
$[\mathrm{mx} \cdot \mathrm{my} \cdot \mathrm{v} + \mathrm{mz} \cdot \mathrm{s}, \mathrm{my}^2 \cdot v + c, \mathrm{my} \cdot \mathrm{mz} \cdot \mathrm{v} - \mathrm{mx} \cdot \mathrm{s}, 0],$
$[\mathrm{mx} \cdot \mathrm{mz} \cdot \mathrm{v} - \mathrm{my} \cdot \mathrm{s}, \mathrm{my} \cdot \mathrm{mz} \cdot \mathrm{v} + \mathrm{mx} \cdot \mathrm{s}, \mathrm{mz}^2 \cdot v + c, 0],$
$[0, 0, 0, 1]] :$

> $U := Rot.U :$

> $PB4 := U.PT4 : PB5 := U.PT5 : PB6 := U.PT6 :$

> $S15 := PB1 - PB5; S26 := PB2 - PB6 : S34 := PB3 - PB4 : S14$
$:= PB1 - PB4 : S25 := PB2 - PB5 : S36 := PB3 - PB6 :$

> $SO15 := CrossProduct(\langle PB1[1], PB1[2], PB1[3]\rangle, \langle S15[1], S15[2], S15[3]\rangle):$
> $SO16 := CrossProduct(\langle PB1[1], PB1[2], PB1[3]\rangle, \langle S16[1], S16[2], S16[3]\rangle):$
> $SO25 := CrossProduct(\langle PB2[1], PB2[2], PB2[3]\rangle, \langle S25[1], S25[2], S25[3]\rangle):$
> $SO26 := CrossProduct(\langle PB2[1], PB2[2], PB2[3]\rangle, \langle S26[1], S26[2], S26[3]\rangle):$
> $SO34 := CrossProduct(\langle PB3[1], PB3[2], PB3[3]\rangle, \langle S34[1], S34[2], S34[3]\rangle):$
> $SO36 := CrossProduct(\langle PB3[1], PB3[2], PB3[3]\rangle, \langle S36[1], S36[2], S36[3]\rangle):$

> $$J := \begin{bmatrix} S14[1] & S15[1] & \mathrm{S25}[1] & \mathrm{S26}[1] & \mathrm{S34}[1] & \mathrm{S36}[1] \\ S14[2] & S15[2] & \mathrm{S25}[2] & \mathrm{S26}[2] & \mathrm{S34}[2] & \mathrm{S36}[2] \\ S14[3] & S15[3] & \mathrm{S25}[3] & \mathrm{S26}[3] & \mathrm{S34}[3] & \mathrm{S36}[3] \\ SO14[1] & SO15[1] & \mathrm{SO25}[1] & \mathrm{SO26}[1] & \mathrm{SO34}[1] & \mathrm{SO36}[1] \\ SO14[2] & SO15[2] & \mathrm{SO25}[2] & \mathrm{SO26}[2] & \mathrm{SO34}[2] & \mathrm{SO36}[2] \\ SO14[3] & SO15[3] & \mathrm{SO25}[3] & \mathrm{SO26}[3] & \mathrm{SO34}[3] & \mathrm{SO36}[3] \end{bmatrix};$$

> $det := Determinant(J);$

> $evalf(solve(\{det = 0\}, [t]));$

> $mx := S_{\mathrm{D}}[1] : my := S_{\mathrm{D}}[2] : mz := S_{\mathrm{D}}[3] : s := \sin(\mathrm{t}) : c := \cos(\mathrm{t}) :$
> $v := (1 - \cos(\mathrm{t}));$

> $$Rot := \begin{bmatrix} \mathrm{mx}^2 \cdot v + c & \mathrm{mx \cdot my \cdot v - mz \cdot s} & \mathrm{mx \cdot mz \cdot v + my \cdot s} & 0 \\ \mathrm{mx \cdot my \cdot v + mz \cdot s} & \mathrm{my}^2 \cdot v + c & \mathrm{my \cdot mz \cdot v - mx \cdot s} & 0 \\ \mathrm{mx \cdot mz \cdot v - my \cdot s} & \mathrm{my \cdot mz \cdot v + mx \cdot s} & \mathrm{mz}^2 \cdot v + c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

> $T_B := T_{BD}.Rot;$

> $PB4 := T_B.PT4; PB5 := T_B.PT5 : PB6 := T_B.PT6 :$

> $l_{Atotal} := PB1 - PB4 :$

> $l_{Btotal} := PB2 - PB5 : l_{Ctotal} := PB3 - PB6 :$

> $LA := PB1 - PB5 :$

> $LB := PB2 - PB6 :$

> $LC := PB3 - PB4:$

> $$S15 := \frac{LA}{mag(LA)}; S26 := \frac{LB}{mag(LB)}; S34 := \frac{LC}{mag(LC)}; S14 := \frac{l_{Atotal}}{mag(l_{Atotal})}; S25 := \frac{l_{Btotal}}{mag(l_{Btotal})}; S36 := \frac{l_{Ctotal}}{mag(l_{Ctotal})};$$
> $$S12 := \frac{(PB1 - PB2)}{mag(PB1 - PB2)}; S13 := \frac{(PB1 - PB3)}{mag(PB1 - PB3)}; S23 := \frac{(PB2 - PB3)}{mag(PB2 - PB3)}; S45 := \frac{(PB4 - PB5)}{mag(PB4 - PB5)}; S46 := \frac{(PB4 - PB6)}{mag(PB4 - PB6)}; S56 := \frac{(PB5 - PB6)}{mag(PB5 - PB6)};$$

> $assign(evalf(solve(\{S34[1] \cdot FC + S45[1] \cdot t45rans + S46[1] \cdot t46rans = S14[1], S34[2] \cdot FC + S45[2] \cdot t45rans + S46[2] \cdot t46rans = S14[2], S34[3] \cdot FC + S45[3] \cdot t45rans + S46[3] \cdot t46rans = S14[3]\}, [FC, t46rans, t45rans])));$

> $assign(solve(\{-S36[1] \cdot fC + S26[1] \cdot FB - S56[1] \cdot t56rans - S46[1] \cdot t46rans = 0, -S36[2] \cdot fC + S26[2] \cdot FB - S56[2] \cdot t56rans - S46[2] \cdot t46rans = 0, -S36[3] \cdot fC + S26[3] \cdot FB - S56[3] \cdot t56rans - S46[3] \cdot t46rans = 0\}, [fC, FB, t56rans]));$

> $assign(evalf(solve(\{S25[1] \cdot fB - S26[1] \cdot FB - S12[1] \cdot t12rans + S23[1] \cdot t23rans = 0, S25[2] \cdot fB - S26[2] \cdot FB - S12[2] \cdot t12rans + S23[2] \cdot t23rans = 0, S25[3] \cdot fB - S26[3] \cdot FB - S12[3] \cdot t12rans + S23[3] \cdot t23rans = 0\}, [fB, t12rans, t23rans])));$

> $assign(evalf(solve(\{-S15[2] \cdot FA + S12[2] \cdot t12rans + S13[2] \cdot t13rans = -S14[2], -S15[3] \cdot FA + S12[3] \cdot t12rans + S13[3] \cdot t13rans = -S14[3]\}, [FA, t13rans])));$

> $KE := 3 \cdot 20 \cdot (L01)^2;$

> $fA := \mathrm{sqrt}\left(\frac{(2 \cdot KE \cdot 20^3)}{(400 \cdot (1 + fB^2 + fC^2))}\right);$

> $fBactual := fB \cdot fA:$

> $fCactual := fC \cdot fA:$

> $lA := evalf\left(dist(PB1, PB5) - L01 - \frac{fA}{20}\right);$

> $\mathrm{lB} := dist(PB2, PB6) - L02 - \frac{fB}{20};$

> $lC := dist(PB3, PB4) - L03 - \frac{fC}{20};$

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# BIOGRAPHICAL SKETCH

Ms. Aasawari Deshpande was born in Mumbai, India in 1986. After completing her high-school studies in Mumbai, she pursued her undergraduate studies at University of Mumbai, India, where she received her Bachelor of Engineering in the field of mechanical engineering. She worked for Feed-Tech Eng, India and Larson and Toubro, India as an Automation Engineer intern. In August 2007, she came to the University of Florida to pursue her Master of Science degree in mechanical engineering.