CONSUMPTION AND INVESTMENT DECISION: AN ANALYSIS OF AGGREGATE AND TIME-ADDITIVE MODELS

By

LIANG FU

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To my parents, Chengde Fu and Bangliang Ma

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Liang Fu

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In the presence of an investment opportunity, a borrower decides whether to invest. We find that - depending on the borrower's personal wealth and the presence of limited liability - the investment is not always undertaken, which leads to investment distortion. Furthermore, when the borrower has intertemporal consumption preference rather than aggregate consumption preference, the potential investment distortion problem is weakly exacerbated. The change of the borrower's risk preference from constant absolute risk aversion to decreasing absolute risk aversion lowers the occurrence of the potential investment distortion as his personal wealth increases. Furthermore, when the borrower is privately informed about his personal wealth, the Revelation Principle applies and the naive contract which is derived from public information case is optimal.

CHAPTER 1
INTRODUCTION

This dissertation examines consumption and investment behavior associated with an investment project. Two parties are involved: a risk-averse borrower and a risk-neutral lender. The dissertation starts with a borrower who displays *constant absolute risk aversion* (CARA hereafter) with certain initial wealth and either aggregate consumption preference or time-additive consumption preference as a benchmark. It focuses on an investment decision in the presence of limited liability and the resulting potential investment distortion as well as its interaction with the borrower's consumption preference.

By design, the investment project is always taken when the borrower's effort is publicly observable and the associated first-best contract always exists. However, when the borrower's effort choice becomes private, he does not always undertake the project, which leads to investment distortion. In this dissertation investment distortion is the borrower's passing up the profitable investment opportunity. Based on this condition and depending on the borrower's personal wealth, investment distortion occurs if any of the following situations arise:

A. The lack of a loan agreement between the borrower and the lender when the borrower is wealth-constrained.

B. The wealth-constrained borrower prefers the status quo to the best available loan agreement.

C. The wealth-abundant borrower prefers the status quo to (a) borrowing from the lender (if feasible) and (b) self-financing.

In Situation A and Situation B, the *wealth-constrained borrower* refers to the borrower who does not have sufficient personal wealth to finance the investment. Situation A implies that there is no loan arrangement that motivates the borrower and lets the lender recoup her investment at the same time. The culprit for the non-existence of such an agreement is the joint force from two factors in the design. The borrower is the party who "owns" the outcomes of the investment project, which implies that after the lender is paid off the borrower claims the remainder of the profits. Therefore, the borrower's expected consumption is bounded above by a fixed amount, which is the sum of his personal wealth and the expected *net present value* (NPV hereafter) of the investment. This factor alone does not necessarily cause the non-existence of the loan arrangement. However, in the presence of the borrower's limited liability constraints, which require the borrower's consumption to be non-negative, the borrower's fixed expected consumption plays a vital role in determining the existence of the loan agreement. Intuitively, when limited liability constraints are imposed and the borrower's expected consumption has an upper bound, there is a feasibility issue concerning how the consumption schedules can be designed to motivate the borrower to work diligently. Stated differently, unlike the conventional standard agency problem where any compensation differential can usually be applied to fend off the incentive problem when the agent's action supply is private, the contract in this dissertation restricts the applicability of the consumption differential and, as a result, potential investment distortion arises.

Situation B suggests that for the borrower to decide between borrowing from the lender (given that such an arrangement is in place) and staying with the status quo, he considers his net payoff from the investment. In particular, the borrower receives the entire expected NPV from the investment but he incurs a personal cost of effort; therefore, if his net payoff is not sufficiently large, he might end up giving up the investment and decide to settle with the status quo. The economic implication behind this form of investment distortion is that the loan agreement, given its existence, is too risky for the borrower to engage in financing with the lender.

In Situation C, the *wealth-abundant borrower* refers to the borrower who has sufficient personal endowment to finance the investment project on his own without resorting to the lender. If the loan agreement fails, the borrower is left with the choice between no-lender investment and no-investment. Since the borrower is risk averse, the investment might be too risky for him to take on his own as risk-sharing is inadmissible. There is no conclusive answer to which option - borrowing or self-financing - the borrower prefers if the investment project is undertaken.

This dissertation analyzes Situations A through C for the borrower with either aggregate consumption preference or time-additive consumption preference. The discussion for the aggregate borrower provides a benchmark for the interplay between the borrower's investment and consumption behavior by deriving conditions under which investment distortion arises. The benchmark is important in the following two respects: (a) the role of the borrower's limited liability constraints on his consumption is made explicit in determining the feasibility of the loan arrangement between the borrower and the lender when the former has private information about his effort supply and (b) the role of the borrower's personal wealth is nontrivial in potential investment distortion.

With the benchmark for the borrower with aggregate consumption preference, the analysis reveals that the borrower's consumption preference has an effect on his investment decision, especially on the occurrence of the potential investment distortion. The investment distortion under aggregate consumption preference implies the distortion under time-additive preference, but not vice versa. In particular, if the borrower's personal wealth is strictly less than the investment requirement, the borrower's preferences have no influence on his investment behavior. Intuitively, the borrower displays CARA, which would imply that his personal wealth has no effect on his attitude towards risk at each instant if the consumption problem were to extend to multiperiods. Therefore, conditions under which investment distortion arises are exactly the same for the borrower with aggregate and time-additive consumption preferences. However, if the borrower has sufficiently large personal wealth to engage in self-financing, the concern for intertemporal consumption exacerbates the potential investment distortion problem. The aggravation is pronounced for the borrower's no-lender investment. The reason is that with no-lender investment, the magnitude of the borrower's personal wealth is important in shaping the borrower's intertemporal consumption in the absence of personal banking. In particular, questions such as "Would it be optimal for the borrower to save up for the future?" or "Would the borrower be able to do so if it is optimal?" are crucial in determining investment distortion. On the contrary, the magnitude of the borrower's personal wealth is not so compelling when a loan arrangement between the borrower and the lender is feasible because the external financing helps the borrower implement his (optimal) reserve for the future even though his personal wealth might prohibit such behavior. These findings with respect to the time-additive borrower not only reinforce the non-trivial role of the borrower's personal wealth but also introduce the importance of the borrower's consumption preference in affecting his investment decision. The interplay between the borrower's personal endowment and intertemporal consumption is highlighted in potential investment distortion.

With the CARA-borrower as a benchmark, the subsequent analysis introduces a borrower with *decreasing absolute risk aversion* (DARA hereafter) with either aggregate or time-additive consumption preference to investigate the change of the borrower's risk preference on his consumption and investment behavior. In general, for the DARA-borrower, the unique distinction between the CARA- and DARA-borrower is that, *ceteris paribus*, the latter is less likely to encounter investment distortion problems associated with the second-best loan contract. The attribute of the mitigated potential investment distortion is that the DARA-borrower changes his attitude towards risk as his personal wealth changes. In particular, as the DARA-borrower becomes wealthier, he becomes less risk averse, which in turn leads him to undertake the investment project that would otherwise be rejected by the CARA-borrower.

Although the borrower's hierarchical optimal investment decision is inconclusive between borrowing and self-financing, the loan agreement with the risk-neutral lender provides risk sharing, which is important to the risk-averse borrower. Therefore, were the borrower's personal wealth to be private information, how the loan contract would be affected is subsequently discussed. The borrower is then required to self-report his personal wealth and his consumption behavior in the presence of private wealth information is emphasized. The analysis reveals that the Revelation Principle applies and a truthful direct mechanism can be designed to motivate the CARA-borrower to report his personal wealth truthfully. The naive contract which applies the consumption schedule if the CARA-borrower's personal wealth were public is a feasible and optimal loan agreement with private information. Intuitively, since the lender is assumed to operate in a competitive financial market; then independent of the knowledge of the CARA-borrower's personal wealth, she eventually breaks even. Therefore, the CARA-borrower's expected consumption is similar to the Chapter 3 case and is upper-bounded by the sum of his (reported) wealth and the expected NPV from the investment project. Furthermore, the moral hazard aspect requires the CARA-borrower to be motivated to supply high effort. The CARA-borrower then faces the same incentive compatibility and expected consumption constraints as the ones for the public information case. Moreover, in equilibrium, the CARA-borrower's truth-telling incentive compatibility constraints hold at equality; therefore, the CARA-borrower solves a reduced program which is equivalent to the one in public information scenario. It then follows that the naive contracts are optimal.

Investment activities are vital to a prosperous and well-functioning economy; however, we often see sound investment opportunities being passed up. This dissertation designs a simple tractable two-party model to investigate the potential attribution to potential investment distortion. The findings are specifically valuable to the entrepreneur-type of borrower, who may be wealth-constrained or restricted by risk preference but who seeks ways to engage in the project. The borrower with aggregate consumption preference can be considered to represent a traditional entrepreneur, whose key focus is the aggregate return of an investment rather than the intertemporal gain. On the other hand, the borrower with time-additive consumption preference is more likely to represent small business owners, whose concerns include not only the investment he makes, but also the period-by-period consumption level which is necessary to sustain himself and/or his family. By taking into consideration the borrower's different risk and consumption preferences, this dissertation provides an in-depth analysis which provides insights into (1) how potential investment distortion may be avoided and (2) how the change in the borrower affects the corresponding consumption and investment problems. The findings also have policy implications. By identifying the idiosyncracy of each type of borrower, the policy maker is in a better position to address the borrower's financing requirement and promote the engagement of the investment project.

CHAPTER 2
LITERATURE REVIEW

The stylized model in this dissertation partly resembles the conventional agency problem (e.g., Grossman and Hart (1983)) in that the lender-borrower relationship and the corresponding optimal consumptions are counterparts of the labor market principal-agent wage contract. Since the borrower has the potential to finance the investment from the lender, the loan contract between the borrower and the lender features moral hazard in the debt market. For example, Gale and Hellwig (1985) focus on a contract between investors and entrepreneurs, both of whom operate in a competitive financial market. Entrepreneurs are just borrowers who wish to undertake risky ventures but lack the necessary resources so they turn to the investors for external finance. Investors are banks or other financial institutions. They find that the optimal, incentive-compatible debt contract in a model of borrowing and lending with asymmetric information is the standard debt contract. In this dissertation, engaging in borrowing is endogenous, especially for the wealth-constrained borrower; the lender has rational expectations of the borrower's incentives to shirk (given his private effort supply) and misreport (given his private information about wealth); the interaction between the borrower and the lender unavoidably raises potential investment distortion, inefficient risk sharing, and the borrower's concurrent concern for intertemporal consumption when he has time-additive consumption preference.

The benchmark consumption and investment behaviors for the CARA-borrower in this dissertation are closely related to Tirole (2006)'s basic model, where he models a risk-neutral lender and a risk-neutral borrower and investigates credit rationing in the presence of a well-functioning financial market. Tirole finds that the driving force of potential credit rationing when the borrower has private information about his action choice is the level of his wealth at hand. This dissertation extends Tirole (2006) by introducing the risk-sharing aspect of the loan agreement. The risk-averse borrower, together with the limited liability constraint,[1] implies that investment distortion, which is a form of credit rationing, is not a result of pure wealth effect. It is the joint product of the feasibility of the incentive contract and the borrower's personal wealth level. The driving force of such infeasibility is further investigated. This dissertation also differs from Tirole (2006) by considering the role the borrower's consumption preference plays in potential investment distortion. The findings of this dissertation are more compelling in two ways. First, it is perhaps more realistic to assume a risk-averse borrower and non-negative consumption when the problem at hand is investment and consumption behavior. Second, since consumption behavior is not independent of the investment decision, how the former affects the latter is itself of particular interest.

This dissertation focuses on the under-investment problem that arises when a profitable project is not undertaken. The finance literature explains under-investment from a capital structure perspective, which implies that under-investment is a result of information asymmetry between the borrower and the lender (e.g., Myers and Majluf (1984)) and the project is not undertaken due to costly external financing. This dissertation is distinguished from the finance literature by focusing on the moral hazard aspect rather than the adverse selection aspect of the information problem. In particular, the borrower's moral-hazard-associated incentive problem, together with the

[1] Tirole (2006) also assumes the non-negativity of the borrower's income. The limited liability contract in the literature takes different forms. Sappington (1983) shows that the principal deliberately induces less-than-efficient output in a state that is not the mostly productive when compelled to respect the agent's ex post limited liability. Lewis and Sappington (2000) derive the optimal contract when the agent has private information about his wealth. Che and Gale (2000) and Lewis and Sappington (2001) analyze models where the agent is privately informed both of his ability and of his wealth. A more recent paper by Lu (2009) finds that firms experience overinvestment rather than underinvestment when bankruptcy risk (calculated as Altman's Z-score, Altman (1968)) is higher.

upper-bounded profit from the investment and the borrower's limited liability, leads to potential under-investment.

The accounting literature focuses on the use and the "*quality*" of accounting information in mitigating investment inefficiencies directly. For example, Bens and Monahan (2004) and Bushman et al. (2006) study how a firm's disclosure policy and quality mitigate the under-investment problem. Biddle et al. (2008) and Lu (2009) explore the effects of accrual quality and the disclosure of non-financial-statement information on over-investment, respectively. Another stream of accounting literature investigates the connection between accounting information and investment behavior indirectly from the connection between accounting and the cost of capital in general. Francis et al. (2000) and Aboody et al. (2005) emphasize the role of accrual (or more generally, earnings) quality on the cost of capital. Bertomeu et al. (2008) further endogenously connect disclosure policy to its capital structure. However, the role of the borrower associated with the investment project is largely overlooked in the accounting literature. As an economic agent, the borrower is important in the investment decision and the corresponding financing arrangement. In particular, his unobservable effort choice and his personal wealth endowment both significantly affect the existence of the loan agreement, which in turn, influences the occurrence of investment inefficiency.

Investment decision and consumption behavior are hardly two isolated events. Breeden (1979) is the pioneer of contemporary research on consumption and financial market investment in what has become known as the "consumption *capital asset pricing model* (CAPM)". The intertemporal CAPM is developed by Merton (1973). Merton (1973) concludes that when securities have stochastic returns, an individual's portfolio holdings are found in terms of his indirect utility function for wealth and equilibrium expected asset returns are correspondingly found in terms of aggregate wealth and the returns on assets that are perfectly correlated with changes in the various state variables.

Breeden (1979) extends Merton (1973) and introduces the consumption-beta[2] and argues that in a rational expectations equilibrium asset prices in the economy are functions of the consumption preferences of individuals and time, which are non-stochastic. The individual's problem is to choose an optimal rate of consumption and an optimal portfolio of risky assets to maximize the expected value at each instant.

The risky investment project is quite similar to the risky portfolios in Breeden's work. However, while Breeden focuses more on the investment of securities in the financial market, this dissertation emphasizes the potential aspect of investment distortion when the investment decision is more economic and involves real technology and labor. Moreover, this dissertation not only discusses the connection between consumption behavior and investment but also explores the impact of particular consumption preferences on the potential investment distortion problem. *Consumption spending*, as discussed by Danthine and Donaldson (2005), defines a "standard of living," and most individuals, if not all, prefer a stable standard of living from time to time.[3] Therefore, intertemporal consumption preference is not an unrealistic focus. Although the borrower prefers smooth consumption, when the investment decision is considered at the same time, consumption smoothness is not without a catch. In particular, intertemporal consumption preference makes the borrower more susceptible to investment inefficiency. Intuitively, to avoid investment distortion the aggregate borrower faces a few hurdles. First, whether the personal wealth is sufficiently large to finance the project without borrowing. Second is the feasibility of the loan agreement. However, *ceteris paribus*, for the borrower with

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[2] Consumption-beta is named to contrast Sharpe (1964) and Lintner (1965) CAPM beta; the former defines *asset betas* to be measured relative to changes in the aggregate consumption rate, rather than relative to the market, which is the definition of the latter.

[3] Indeed, consumption has long been found to be surprisingly smooth over time. Hall (1988) finds that a rise in the interest rate is not accompanied by an increase in consumption. Similarly, Deaton (1987) also finds that consumption is smooth relative to income and prices.

time-additive consumption preference one more hurdle is added for those with aggregate consumption preference, i.e., the desire to smooth his consumptions over time. This additional pressure makes the borrower more constrained in solving his investment problem, which in turn, makes him more inclined to investment distortion. However, if it comes to the borrower's attention that concern for consumption smoothing makes the investment decision more vulnerable, his personal wealth endowment becomes important since it affects the borrower's viable investment options, which, in turn, affects whether consumption preference matters in the presence of investment decision.

In reality, the lender may not have access to the precise information about how much personal wealth the borrower possesses mainly for two reasons: first, the borrower can simply conceal such information and it may become almost impossible or prohibitively expensive for the lender to consult a third party to reveal such information; second, even though the lender can observe the borrower's private personal wealth information with no additional cost, it can be problematic for the lender to determine his exact net worth. For example, the borrower's personal wealth can take the forms of cash, real assets, or a mix of both. While the value of cash is easily determined, there may not be a market value for the assets or the borrower may have better information of the marketability of the assets, both of which make it difficult for the lender to have the same information about the borrower's personal wealth as the borrower.

In the economics literature dealing with investment behavior and privately informed personal wealth is not uncommon. Lewis and Sappington (2001) investigate an optimal contract between a principal who is the owner of a project and an agent who has the skills required to operate the project. The agent is privately informed about his ability to operate the project, his wealth, and his effort supplies. Lewis and Sappington (2001) find that the power of the incentive scheme does not always increase as his wealth or ability alone increases. In other words, ability and wealth act as perfect complements in determining the power of the incentive scheme and an agent requires the higher level of both to secure a more powerful compensation structure. The intentional sacrifice of surplus is designed to mitigate the agent's incentive to understate his private information of personal wealth and ability. The analysis in Chapter 5 is quite different from Lewis and Sappington (2001). The owner of the investment project is the borrower, not the lender. Given the borrower's ownership of the project and the fact that he may be wealth-constrained, a moral hazard problem in the borrowing sphere is of primary concern; therefore, the borrower's ability is not considered as a choice variable. This dissertation is more closely related to Lewis and Sappington (2000), where they find that the principal who is also the owner of the project can induce the agent, who is also the project operator, to truthfully reveal his privately informed constrained personal wealth by promising a higher probability of operation and/or a greater share of realized profit the larger the bond that a potential operator posts when his effort is essentially non-contractible. In both studies, the asymmetric information about the personal wealth leads to an adverse selection problem and the non-contractible personal effort supply represents a moral hazard problem. Furthermore, the Revelation Principle (Myerson (1981)) applies and a truthful direct mechanism induces truth-telling. The ultimate goal for Lewis and Sappington (2000)'s agent and the borrower in Chapter 5 is to maximize the expected profit from the operation/project; however, Lewis and Sappington (2000) find that the agent truthfully reveals his wealth because he is promised a higher probability of operation and/or a greater share of realized profit the larger the bond the agent posts. In Chapter 5, the borrower chooses truthful revelation of his wealth because he does not gain based on the limited reporting strategy and the form of his utility function.

CHAPTER 3
CARA-BORROWER

3.1 Introduction

The basic model involves a risk-neutral lender and a risk-averse borrower. The timeline follows a two-date structure: at time $t = 0$, the borrower has an innovative project that requires a fixed and known investment I. The borrower has initial personal wealth of $W \geq 0$. The borrower decides how much to invest and consumes the rest of his wealth.

To implement the project, the borrower may approach the lender to borrow $D_0 \geq 0$. Upon assessing the project proposed by the borrower, the lender either rejects the proposal or agrees to finance, where the corresponding financing agreement specifies the ewpayments $\{D_H, D_L\}$ to the lender. To be consistent with Tirole (2006), the lender operates in a competitive financial market.[1] After financing is secured, the borrower exerts an effort which leads to a binary cash flow $X \in \{X_H, X_L\}$, the value of which is realized in either a good state or a bad state and $X_H > X_L$ implies that the cash flow from the good state is strictly higher than that from the bad state.[2] The borrower can either behave by exerting the desired effort $a_H > 0$, which yields a probability $p_H \in (.5, 1)$ of X_H,[3] or misbehave by taking a shirking action $a_L = 0$ with a normalized zero personal

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[1] The competitive financial market is assumed to have perfect competition. In other words, there are many lenders that operate in the financial market and they all provide homogenous lending to the borrower. The possibility of monopoly or oligopoly is not considered.

[2] States can be considered as uncertain future economic environments, where a good future economic environment brings higher cash flow from investment than what a bad future economic environment tends to offer. The repayment to the lender is assumed to be binary to be consistent with the binary cash flow from the investment project. In particular, when the cash flow from good state is realized, the lender is paid D_H and if the cash flow from bad state is realized, the lender is paid D_L.

[3] The borrower is risk-averse, by supplying high personal effort level, he is expected to have more than a half probability to achieve the cash flow from a good state.

cost, resulting in a probability $p_L < p_H$ of X_H. At $t = 1$, cash flow from the investment X is realized and publicly observed. The lender receives her repayments, and the remainder goes to the borrower.[4]

The investment project is assumed to be more valuable when the borrower exerts a high-level effort. Formally, let

$$\Pi_H = p_H X_H + (1 - p_H) X_L - I, \text{ and}$$
$$\Pi_L = p_L X_H + (1 - p_L) X_L - I$$

denote the expected net present values (NPV hereafter) for the investment project when the borrower works diligently or when the borrower shirks, respectively. The following assumptions are held throughout this dissertation:

Assumption 1 $\Pi_H - a_H > \Pi_L \geq 0$

Assumption 1 suggests that high effort is always preferred in equilibrium and the investment project is attractive to the borrower. Since the lender operates in a competitive financial market, all the expected NPV from the investment accrues to the borrower; Assumption 1 restricts the borrower's equilibrium "net monetary gain" from the project to be strictly greater than zero.

Assume the borrower is an expected utility maximizer with constant absolute risk aversion (CARA hereafter). His utility function takes the form of negative exponential, which is increasing and concave in his gross consumption.

[4] Note that in the timeline depicted above, the borrower only has access to the external capital market at $t = 0$ if the lender agrees to finance; no other personal banking access is available thereafter. The lack of personal banking does not prohibit the borrower from storing his initial wealth at $t = 0$ for later consumption. As will be discussed later, saving up for the future is trivial for the borrower with aggregate consumption preference but *important* for the borrower with time-additive consumption preference.

3.2 Consumption Preference Case 1: Aggregate

The borrower is assumed to either have an aggregate consumption preference or a time-additive consumption preference towards consumption. The aggregate consumption preference leads to a one-period consumption problem, while the time-additive consumption preference results a two-period consumption problem.[5]

3.2.1 The Model

Let C and U denote the borrower's aggregate monetary holdings across periods and his utility function, respectively. Then C represents the borrower's gross consumption and $C - a$ his net consumption. The risk-averse borrower's utility is thus expressed as

$$U = U\left(C - a\right) = -\exp\left[-\rho(C - a)\right]$$

where $\rho > 0$ is the Arrow-Pratt risk-aversion parameter and a is the borrower's personal cost of effort. The borrower's objective is to maximize his expected utility from his consumption.

3.2.2 Investment Behaviors: Borrow, Self-Invest, No-Invest

When facing an investment opportunity, the borrower has the option to invest where investment can take the form of borrowing from the lender or self-financing if his personal wealth holdings are sufficient and no investment leads to investment distortion, which is discussed in Section 3.2.2.3.

Before formally discussing the borrower's investment options, the borrower's consumption behavior when no investment opportunity is present is briefly discussed for completeness.

When there is no investment opportunity, the borrower simply consumes his personal wealth, and his optimal consumption is characterized as follows:

[5] Christensen and Feltham (2005) discuss consumption preferences for multi-period contracts in detail in Chapter 25.

Fact 1 In the absence of the investment project, the borrower's optimal aggregate consumption is $C = W$.

The intuition for this consumption schedule is straightforward. Given the lack of personal banking access the borrower's optimal choice is to consume all the wealth at hand. Moreover, since the borrower has aggregate consumption preference the total consumption rather than period-by-period consumption is the focus. The borrower can arbitrarily allocate his personal wealth W between two consumption instants of time; therefore, there is no unique solution for his period-by-period consumption.

3.2.2.1 Benchmark: first-best contract

When the investment project is available, the *first-best contract* refers to the financing arrangement between the borrower and the lender when the borrower's effort supply is publicly observable and contractible. The lender's break-even condition, which is considered as her individual rationality constraint (IR), is

$$p_H D_H + (1 - p_H) D_L \geq D_0 \tag{3–1}$$

The risk-neutral lender behaves competitively in the financial market in the sense that the financial agreement, if it exists, makes zero profit. Therefore, in equilibrium, the lender's (IR) constraint is binding.[6] The borrower's consumption is binary, i.e., $C \in \{C_H, C_L\}$, which is bounded above by the sum of his personal wealth (W) and the stochastic cash

[6] The assumption that the lender operates in a competitive financial market implies that multiple prospective lenders are competing for investment. If the most attractive financial agreement made a positive profit, the borrower could turn to an alternative lender and offer to switch for a zero-profit agreement. Were the lender to decide to invest, her investment, together with the borrower's personal wealth, would not be less than what is required for the investment project, i.e.,

$$I \leq W + D_0$$

It turns out that the borrower's investment D_0 is indeterminate in the analysis because she only cares about breaking even.

flow (X_H or X_L), less his share of investment in the project ($I - D_0$) and the lender's repayments (D_H or D_L):

$$C_H \leq W - (I - D_0) - D_H + X_H$$

$$C_L \leq W - (I - D_0) - D_L + X_L$$

With these borrower's consumption constraints, together with the lender's binding (IR) constraint and the defined Π_H, it follows that

$$p_H C_H + (1 - p_H) C_L \leq W + \Pi_H \tag{3–2}$$

Finally, C_H and C_L are the borrower's monetary holdings, which are used for consumption purposes; they are assumed to be non-negative. These non-negativity constraints are essentially similar to the limited liability constraints imposed in the agency problems (e.g., Sappington (1983) and Innes (1990)); therefore, they are referred to as *limited liability constraints* hereafter.[7]

Based on Assumption 1, the first-best contract [Program FB] is designed to maximize the borrower's expected utility when high effort a_H is supplied, subject to (3–2) and the limited liability constraints.

[7] Given that it is evident that the cash flow of the project from the good state X_H is strictly positive, there is no explicit requirement for the cash flow from the bad state X_L to be positive. This implies that X_L might be negative. Assumption 1 and inequality (3–2) all suggest that the magnitude of the expected NPV Π_H is what is important to the analysis; in other words, it is the magnitude of X_L *relative to* that of X_H that matters, not the magnitude of X_L itself that plays a role in the loan agreement. However, in the no-lender investment analysis later on, the sign of X_L is important in determining the feasibility of the no-lender investment in certain circumstances.

[Program FB]

$$\max_{C_H \geq 0, C_L \geq 0} -p_H \exp\left[-\rho\left(C_H - a_H\right)\right] - \left(1 - p_H\right) \exp\left[-\rho\left(C_L - a_H\right)\right]$$

$$\text{s.t. } p_H C_H + \left(1 - p_H\right) C_L \leq W + \Pi_H$$

Although modeled differently, the designed contract in this chapter is similar to the conventional standard agency problem, e.g., Grossman and Hart (1983) and Holmstrom (1979). However, it also has distinct characteristics. Inequality (3–2) suggests that were the investment to be taken the borrower's expected consumption would be no greater than the sum of his personal wealth endowment and the expected NPV from the investment. For a given project, the borrower's personal wealth W and the expected NPV Π_H are both fixed, which implies that the borrower's consumption in expectation has an *upper bound*. This is quite different from the conventional agency problem where the borrower's individual rationality constraint de facto imposes a *lower bound* on the agent's compensation.[8] As will later become more clearer in the second-best contract, the fact that the borrower's equilibrium expected consumption is bounded above by a fixed amount plays an important role in the potential investment distortion problem.

Similar to what is established in the conventional agency problem, the first-best contract [Program FB] would provide full insurance for the borrower were the borrower to supply high effort. As a consequence, the risk-averse borrower does not bear any risk associated with the cash flow from the project and his consumption.

Fact 2 The first-best contract is characterized as

$$C_H = C_L = W + \Pi_H \tag{3–3}$$

[8] Although the individual rationality constraint in the standard agency problem focuses on the agent's expected utility from his monetary receipts, it does not change the implication that a lower bound, rather than an upper bound of what the agent receives from the contract, is imposed.

where $\Pi_H = p_H X_H + (1 - p_H) X_L - I$.

Assume the investment project is viable, the consumption schedule is *riskless*. This is because Jensen's inequality implies that risky consumption is not optimal in the first-best contract. Moreover, the borrower's utility function is strictly increasing and concave; therefore, we have

$$U(C - a_H) = U(W + \Pi_H - a_H)$$

which implies (3–3).

Note that the borrower's option to self-finance the investment project if he is wealth-abundant is not discussed in this section. It turns out that when the borrower's effort supply is publicly observable and contractible, self-financing is inferior to borrowing. Intuitively, the loan agreement provides a full-insured riskless consumption for the borrower, which would be infeasible were the borrower to choose no-lender investment as the project results in risky cash flow. Therefore, the risk-averse borrower always prefers borrowing to self-financing when his effort supply is publicly observable.

3.2.2.2 Second-best environment

In this section attention is turned to the case in which the borrower's effort is not observable. First, it is not guaranteed that the loan agreement between the lender and the borrower exists. Second, since the borrower has personal wealth, it is also not clear whether the investment project will be undertaken if no loan agreement is reached. Moreover, whether the financing arrangement is always preferred is inconclusive. The definition of *second-best environment* is first introduced.

Definition 1. *The second-best environment refers to the environment where the borrower's effort choice is not publicly observable.*

The contract between the borrower and the lender in the second-best environment is referred to as the *second-best contract*. The timeline for the potential loan agreement in the second-best environment is slightly changed. In particular, the borrower privately chooses his effort supply after the financing has been secured. In the design it is the risk-averse borrower who "owns" the outcomes of the project and he seeks to obtain capital from and share his risk with the risk-neutral principal. Therefore, the second-best contract, were it to exist, would be written on the realized risky cash flow from investment because it is observable and contractible.

Unlike the first-best contract, the second-best environment features potential investment distortion and consumption distortion, which are defined as follows:

Definition 2. *An Investment distortion arises if the first-best investment is not undertaken.*

Definition 3. *A Consumption distortion arises if the borrower does not experience a riskless and/or smooth intertemporal consumption.*

Consumption distortion involves two aspects of the borrower's consumption behavior. The presence of risky consumption for the aggregate consumption preference and the presence of risky intertemporal consumption and the lack of smooth intertemporal consumption for time-additive consumption preference. Depending on the amount of the borrower's personal wealth, there are two possible situations for investment distortion:

- For the borrower with $W < I$, the borrower requires a mutually agreeable second-best contract with the lender for the project to be invested. The lack of such a contract leads to investment distortion.
- For the borrower with $W \geq I$, the borrower can either seek financing from the lender or finance the project without borrowing from the lender. The failure of both resorts leads to investment distortion.

The lack of a mutually agreeable second-best contract in the first situation implies that no contract can be found that motivates the borrower to work diligently and lets the lender recoup her investment at the same time. Two possibilities can contribute to this type of investment distortion: 1) the loan agreement is infeasible because there does not exist a consumption schedule that solves the borrower's maximization problem with respect to the loan contract and 2) given the existence of such a consumption schedule that maximizes the borrower's expected utility in the second-best loan agreement, the borrower prefers the status quo instead. In other words, either the borrower cannot borrow or he decides not to borrow. When the borrower has $W \geq I$, investment distortion suggests that the borrower prefers the status quo to (a) self-financing (or no-lender investment[9]) and (b) borrowing from the lender if such a contract is feasible. Note that the definition of investment distortion implies that investment is all or nothing and it is an extreme form of under-investment. When comparing each investment strategy, if the borrower is indifferent to the choice between the second-best contract (or the no-lender investment) and the status quo, he is assumed to choose the former.[10]

With respect to the borrower with aggregate consumption preference, his choice of the second-best contract is no longer riskless. On the contrary, the consumption associated with the second-best contract is risky in the sense that consumption varies with the realized cash flow from the investment. Similar to the standard agency problem, when the borrower has private information about his effort choice, the loan arrangement lets him bear some risk to induce him to exert high effort, and the consumption structure features consumption differential, which refers to the difference between consumption associated with the realization of the cash flow from good and bad states. Even for the borrower's self-financing scheme, consumption differential is still unavoidable because the cash flow from the investment is stochastic and risky; therefore, in the absence of any other personal banking, the borrower's consumption is risky. The smoothing aspect of the consumption series applies to the borrower with time-additive consumption preference, which is discussed in Section 3.3.

[9] Self-financing and no-lender investment are used interchangeably.

[10] Since the expected NPV from the investment project is assumed to be non-negative; a rational investor always invests when it is feasible. Therefore, the borrower's investment activity (regardless of whether it is borrowing or self-financing) is considered as *optimal* investment strategy and his decision to not invest is considered as investment distortion.

In the first-best environment, given Assumption 1, a full insurance contract ensures that the borrower's consumption is always positive; therefore, the borrower's limited liability constraints are trivial because they are never binding. This may not be the case for the second-best contract. In particular, as the borrower's action becomes unobservable and the corresponding moral-hazard-related incentive problem (referred to as the *control problem* as well) becomes unavoidable, the existence of the limited liability constraints implies that it becomes infeasible to penalize the borrower sufficiently (i.e., let his consumption go negative when the incentive problem gets severe). For this reason there is a feasibility issue of the second-best contract, which potentially leads to investment distortion.

The only difference between the second-best and the first-best loan agreement is the moral hazard concern brought about by the borrower's unobservable action, which requires the corresponding financing arrangement to grant the borrower sufficient stake in the project to induce him to work diligently. With $a_L = 0$, in equilibrium, the borrower behaves if the following incentive compatibility constraint (IC hereafter) holds:

$$-p_L \exp(-\rho C_H) - (1-p_L) \exp(-\rho C_L) \leq -p_H \exp[-\rho(C_H - a_H)] - (1-p_H) \exp[-\rho(C_L - a_H)] \tag{3–4}$$

The left- and right-hand sides of (3–4) are the borrower's expected utility from his consumption when he chooses to shirk and to deliver high effort, respectively; therefore, (3–4) ensures that the borrower is motivated to work diligently. Rearranging the terms in (3–4) yields

$$-\exp(-\rho C_L)[(1-p_L) - (1-p_H)\exp(\rho a_H)] \leq -\exp(-\rho C_H)[p_H \exp(\rho a_H) - p_L] \tag{3–5}$$

Note that the right-hand side of (3–5) is negative because $p_H > p_L$ and $a_H > 0$ ensure the square-bracket term is strictly positive. On the left-hand side of (3–5), since

$-\exp(-\rho C_L) < 0$, we must have the term in the square bracket positive, i.e.,

$$(1-p_L) - (1-p_H)\exp(\rho a_H) > 0$$

which implies that

$$a_H < \frac{1}{\rho}\ln\frac{(1-p_L)}{(1-p_H)} \tag{3–6}$$

The following assumption is held throughout this chapter.

Assumption 2 Condition $a_H < \frac{1}{\rho}\ln\frac{(1-p_L)}{(1-p_H)}$ always holds.

Given Assumption 2, inequality (3–5) is simplified as

$$C_L + \frac{1}{\rho}\ln\Omega \le C_H \tag{3–7}$$

where $\Omega = \frac{p_H\exp(\rho a_H)-p_L}{(1-p_L)-(1-p_H)\exp(\rho a_H)}$. In other words, Assumption 2 implies that the borrower's personal cost of equilibrium effort is not prohibitively large and the feasibility of the borrower's (3–7) constraint is ensured.[11]

The second-best contract [Program SB] for the borrower is the first-best contract [Program FB] with the (3–7) constraint added, which is

[Program SB]

$$\max_{C_H\ge 0, C_L\ge 0} -p_H\exp\left[-\rho\left(C_H-a_H\right)\right] - (1-p_H)\exp\left[-\rho\left(C_L-a_H\right)\right]$$
$$\text{s.t. } p_H C_H + (1-p_H)C_L \le W + \Pi_H$$
$$C_L + \frac{1}{\rho}\ln\Omega \le C_H$$

Solving the above program provides the following conclusions:

[11] Furthermore, given $p_H > p_L$, $a_H > 0$, and (3–6), Ω is well defined for the natural logarithm function in (3–7).

Proposition 1. *A solution exists for [Program SB] if*

$$W + \Pi_H - \frac{p_H}{\rho} \ln \Omega \geq 0 \tag{3–8}$$

and the solution takes the form of

$$C_H = W + \Pi_H + \frac{(1 - p_H)}{\rho} \ln \Omega$$
$$C_L = W + \Pi_H - \frac{p_H}{\rho} \ln \Omega$$

where $\Omega = \frac{p_H \exp(\rho a_H) - p_L}{(1-p_L) - (1-p_H)\exp(\rho a_H)}$.

Condition (3–8) in Proposition 1 ensures the constraint set can be satisfied (i.e., the constraint set is feasible), which, in turn, determines the existence of the second-best contract [Program SB]. Closer investigation shows that condition (3–8) is also a necessary condition for a solution to exist for [Program SB].[12] It is evident that given the existence of the second-best solution the borrower's limited liability constraints are both satisfied. Stated differently, the limited liability constraints are trivial if the constraints for [Program SB] are feasible because negative consumption never arise. Proposition 1 implies that a consumption differential arises at the solution to the problem, i.e., $C_H > C_L$, which is consistent with the conventional agency problem that the borrower bears some risk to be motivated to deliver high effort.

Under condition (3–8), the borrower's optimal consumption for the second-best contract shows the following characteristics:

$$C_H > C_L \geq 0$$

[12] This is formally proved in the Appendix. However, the feasibility condition only ensures the solution exists for [Program SB].

where $C_L = 0$ if condition (3–8) holds at equality. The borrower's limited liability constraint for C_H is obviously never binding. Thus, the following corollary is developed to address the limited liability constraints for [Program SB]:

Corollary 1. *When a solution to [Program SB] exists, the borrower's limited liability constraint does not bind at this solution.*

Corollary 1 concludes that the second-best contract either has an interior solution or no solution; a corner solution never arises.[13] Assuming C_H and C_L are the optimal solutions from the conventional agency problem, other things being equal, constraint (3–2) is replaced by the usual individual rationality constraint that the borrower's expected utility is no less than his reservation utility. The limited liability constraints in the conventional agency problem work in the following way: were C_L to be negative, the limited liability constraint for C_L would be binding and the corresponding C_H would go up by the amount that keeps the consumption differential constant at what it would be if C_L were allowed to be negative. However, this practice is not applicable for [Program SB] and the reason is in constraint (3–2), which imposes an upper bound on the borrower's

[13] For a constraint in nonlinear programming to be not binding, it implies that the Lagrangian multiplier assigned to that constraint is zero. Based on Kuhn-Tucker analysis, let L_C be the first-order derivative of C with respect to the Lagrangian function L, where $C \in \{C_H, C_L\}$. Then in the presence of the limited liability constraints for C, the complementary slackness requires that

$$CL_C \leq 0$$

Then an "interior solution" refers to

$$C \geq 0 \text{ and } L_C = 0$$

and a "corner solution" refers to

$$C = 0 \text{ and } L_C < 0$$

This also implies that the limited liability constraints for C are not binding when the interior solution arises, whereas they are binding for the corner solution.

expected consumption. Rearranging terms in (3–2) provides

$$C_H \leq \frac{W + \Pi_H - (1 - p_H)\, C_L}{p_H} \tag{3–9}$$

Inequality (3–9) implies that there is an upper bound for C_H. Therefore, for any $C_L < 0$ it is infeasible to adjust C_H upwards to induce $C_L = 0$. This states that the limited liability constraint for C_L is never binding and $C_L = 0$ when (3–8) is satisfied at equality is still part of the interior solutions. Intuitively, Corollary 1 states that no loan arrangement exists that provides a solution of $C_L < 0$ that can motivate the borrower and let the lender recoup her investment at the same time.[14]

In the following numerical example, we vary W while fixing the other parameter values at $\rho = .01$, $p_H = .7$, $p_L = .1$, $X_H = 350$, $X_L = 150$, $I = 150$, and $a_H = 90$. The second column in Table 3-1 shows whether the existence condition (3–8) for the second-best contract holds. The third column shows the second-best contract (labeled as "SB") described in Proposition 1.[15] The last column shows the borrower's consumption from [Program SB] but without the limited liability constraints, i.e., unrestricted second-best contract (labeled as "SBU"). Assumptions 1 and 2 are all satisfied with these parameter values:

$$\begin{aligned}
\Pi_H &= p_H X_H + (1 - p_H)\, X_L - I = 140 \\
\Pi_L &= p_L X_H + (1 - p_L)\, X_L - I = 20 \\
a_H &= 90 < \frac{1}{\rho} \ln \frac{(1 - p_L)}{(1 - p_H)} = 109.9
\end{aligned}$$

[14] The upper-bounded C_H in (3–9) alone does not lead to the non-existence of the loan contract, it is a joint force with the limited liability constraint on C_L that potentially causes the failure of the loan agreement. Were C_L to be allowed to go negative, a feasible C_H would always be found to satisfy (3–9).

[15] The first-best contract characterized in Fact 2 is not shown because of its straightforwardness.

Table 3-1. Second-Best Contracts for the Borrower with Aggregate Consumption

		SB		SBU	
W	(3–8)	C_H	C_L	C_H	C_L
20	No	N/A	N/A	229	-1
50	Yes	259	29	259	29
200	Yes	409	179	409	179

Consistent with Proposition 1, the second-best contract is not always feasible. It is evident from Table 3-1 that when the borrower's personal wealth W is 20, there is no feasible solution for the second-best contract. This is reflected by the failure to satisfy condition (3–8) in Proposition 1.[16] Taking $W = 20$ as an example, we compare two second-best contracts characterized in Table 3-1.

At $W = 20$, the contract SB is obviously not feasible. The alternative second-best contract SBU, which features the lack of the borrower's limited liability constraints, has solutions of $C_H = 229$ and $C_L = -1$. However, this consumption schedule is not feasible when the limited liability constraints are imposed. Were the borrower's limited liability constraint for C_L to be binding, i.e., $C_L = 0$, it would follow from the borrower's IC constraint that the corresponding C_H is adjusted upwards by 1 to be 230 to keep the borrower incentive compatible. Unfortunately, substituting $C_H = 230$ and $C_L = 0$ into the (3–2) constraint yields

$$p_H C_H + (1 - p_H) C_L = 161 \geq W + \Pi_H = 160$$

which is a violation of the constraint.

[16] As is evident from (3–8), it is not just the variation of W that may lead to the non-existence of the second-best contract. For expositional simplicity the example (in Table 3-1) focuses on the minimal variation of one parameter while fixing the other parameters.

It is evident that contract SBU is similar to the one from the standard agency model in that solutions always exist as long as the borrower's IC constraint is feasible. This implies that the joint force of the limited liability constraints and the borrower's upper-bounded expected consumption (i.e., constraint (3–2)) play an important role in determining the feasibility of contract SB.

When the second-best loan SB is feasible the optimal consumption features consumption differential $C_H > C_L$ to ensure that the borrower is motivated. Furthermore, the consumption spread always stays at 230 (i.e., $C_H - C_L = 230$), reflecting the borrower's CARA risk preference. The consumption behavior between contracts SB and SBU are identical since the borrower's consumption are positive and the limited liability constraints are then trivial.

3.2.2.3 Investment distortion

In the second-best environment, investment distortion occurs when the borrower prefers the status quo to investment. When the borrower's personal wealth is not sufficiently large to engage in self-financing, investment distortion arises when either there is no solution for the second-best contract or the status quo is preferable to the borrower. Based on Proposition 1, the failure to satisfy condition (3–8) contributes to the infeasibility of the loan agreement. When the second-best contract is feasible, (3–2) suggests that the expected NPV from the investment project accrues to the borrower; therefore, the borrower's net expected gain from the investment is $\Pi_H - a_H$. It turns out that not any level of $\Pi_H - a_H$ ensures the investment behavior. In particular, there is a boundary condition for the borrower's net gain under which he is better off without the investment. The conclusions are summarized in the following proposition:

Proposition 2. *For $W < I$, investment distortion occurs if and only if any of the following conditions hold:*

a. $W + \Pi_H - \frac{p_H}{\rho} \ln \Omega < 0.$

b. $\Pi_H - a_H < \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi.$

where $\Omega = \frac{p_H \exp(\rho a_H) - p_L}{(1-p_L)-(1-p_H)\exp(\rho a_H)}$ and $\Phi = \frac{p_H - p_L}{p_H \exp(\rho a_H) - p_L}$.

Condition a of Proposition 2 addresses the infeasibility of the second-best contract. The lack of a feasible second-best contract is essentially a form of credit rationing for the wealth-constrained borrower. Tirole (2006) points out that insufficient personal wealth may contribute to credit rationing when the borrower's effort supply is privately observed. The argument is that since the borrower enjoys the entire benefit of shirking but shares the investment project with the lender, the contract between the borrower and the lender then requires the borrower to contribute more endowment to ensure he is motivated. Condition a is consistent with Tirole's credit-rationing argument. Basically, for the borrower with personal wealth

$$W < \frac{p_H}{\rho} \ln \Omega - \Pi_H$$

no financial contract can be found to address the incentive problem and let the lender recoup her investment.

Condition b of Proposition 2 implies that the second-best contract exists, but is too risky for the borrower; therefore, investment distortion arises. Similar to the conventional agency problem, other things being equal, the increase in a_H implies that the increase in the severity of the control problem in the second-best contract. As the incentive problem becomes worse, the second-best contract, if it exists, provides a riskier consumption schedule for the borrower, which is undesirable for the risk-averse borrower. Note that the left-hand side of Condition b is decreasing while the right-hand side is increasing in a_H; therefore, Condition b is more likely to hold as the control problem gets more severe, which further implies that it is the *riskiness* of the second-best contract that leads to investment distortion.

The riskiness of the loan agreement itself is also reflected in the borrower's expected utility from the contract. Proposition 1 implies that when condition (3–8) is satisfied at equality, $C_L = 0$. Given the borrower's concave utility function, the borrower's expected utility is strictly lower for $C_L = 0$ than for $C_L > 0$; in other words, the "most risky" second-best contract from the borrower's perspective is the one that solves at $C_L = 0$. Regrouping (3–8) in Proposition 1, we have

$$W \geq \frac{p_H}{\rho} \ln \Omega - \Pi_H \tag{3–10}$$

For the wealth-fixed borrower, as the right-hand side of (3–10) increases, condition (3–8) is more likely to hold at equality. Therefore, as the second-best contract becomes more risky, the borrower's expected utility from the second-best contract is decreasing; he is then more likely to prefer the status quo to the loan arrangement. Note that the increase in the right-hand side of (3–10) is more likely to trigger Condition b to hold, which makes the borrower more vulnerable to investment distortion.

When the borrower has sufficient personal wealth, i.e., $W \geq I$, the borrower's decision between the second-best contract, given its existence, and the status quo is not conclusive to determine investment distortion since the borrower has the option to self-finance the investment project. When the second-best contract is infeasible, the choice is only between no-lender investment and the status quo. Let CE_{NL} denote the borrower's certainty equivalent consumption from no-lender investment, it then follows from his consumption behavior[17] that

$$CE_{NL} = W - I + \left\{ -\frac{1}{\rho} \ln \left[(1 - p_H) \exp(-\rho X_L) + p_H \exp(-\rho X_H) \right] \right\}$$

Note that the term in the curly bracket can be considered as the certainty equivalent cash flow from the investment project. Let $CE(X) = -\frac{1}{\rho} \ln \left[(1 - p_H) \exp(-\rho X_L) + p_H \exp(-\rho X_H) \right]$,

[17] The borrower's consumptions under no-lender investment are

$$\begin{aligned} \widehat{C}_H &= W - I + X_H \\ \widehat{C}_L &= W - I + X_L \end{aligned}$$

and the above equation is then simplified as

$$CE_{NL} = W - I + CE(X)$$

Given that the borrower's certainty equivalent consumption from the status quo is just his personal wealth W, his decision to determine whether self-finance is a sound choice boils down to comparing the borrower's net certainty equivalent consumption from no-lender investment with W. In other words, we compare the stochastic cash flow from the investment $CE(X)$ with the incurred investment cost, which is $I + a_H$. If the benefit is less than the cost, i.e.,

$$CE(X) < I + a_H \tag{3–11}$$

the project is too risky for the borrower to engage in self-financing; therefore, investment distortion arises.

When the second-best contract is feasible, investment distortion requires the failure of both no-lender investment and loan agreement with the lender. We conclude the potential investment distortion with a wealth-abundant borrower as follows:

Proposition 3. *For $W \geq I$, investment distortion occurs if and only if*

a. $CE(X) < I + a_H$ when no solution to [Program SB] exists.

b. $CE(X) < I + a_H$ and $\Pi_H - a_H < \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi$ when a solution to [Program SB] exists

where $CE(X) = -\frac{1}{\rho} \ln [(1 - p_H) \exp(-\rho X_L) + p_H \exp(-\rho X_H)]$, $W - I + X_L \geq 0$, $\Omega = \frac{p_H \exp(\rho a_H) - p_L}{(1-p_L)-(1-p_H)\exp(\rho a_H)}$ and $\Phi = \frac{p_H - p_L}{p_H \exp(\rho a_H) - p_L}$.

The second condition in Part b of Proposition 3 directly follows from Condition b of Proposition 2; it is the condition under which the status quo is preferable to the loan agreement.

When the second-best contract is feasible, the next logical question is whether the borrower always prefers borrowing to self-financing. The following corollary is developed:

Corollary 2. *When a solution to [Program SB] exists, the investment preference for the borrower' with* $W \geq I$ *is*

$$\textit{No Invest} \succ \textit{Second-Best Loan} \succ \textit{No-Lender Invest} \tag{3–12}$$

if the following condition holds

$$\Pi_H < \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi < \Pi_H + I - CE(X) \tag{3–13}$$

where $CE(X) = -\frac{1}{\rho} \ln [(1 - p_H) \exp(-\rho X_L) + p_H \exp(-\rho X_H)]$, $\Omega = \frac{p_H \exp(\rho a_H) - p_L}{(1-p_L)-(1-p_H)\exp(\rho a_H)}$ *and* $\Phi = \frac{p_H - p_L}{p_H \exp(\rho a_H) - p_L}$.

Note that Condition b of Proposition 3 is only necessary but not sufficient for (3–12). Furthermore, Corollary 2 implies that borrowing is not always preferable to self-financing when the borrower is wealth-abundant.

Let EU_{SB} (EU_{NL})and RU denote the borrower's expected utility from the second-best contract (no-lender investment) and his reservation utility from no-investment.[18] The following example is designed to illustrate investment distortion (or the lack of investment distortion) discussed in Proposition 2 and Proposition 3. The parameter values in Table 3-2 follow those in Table 3-1 unless otherwise specified. With the parameter values, the certainty equivalent cash flow from the investment project is calculated as $CE(X) = 243$. Furthermore, let "ID" stand for whether there is investment distortion and C_H and C_L be the optimal second-best consumption derived in Proposition 1.

The upper exhibit represents the wealth-constrained borrower. Case A and Case B both suggest investment distortion; however, the Case A scenario reflects Condition a of Proposition 2 that the second-best contract is infeasible ($W + \Pi_H - \frac{p_H}{\rho} \ln \Omega = -54 < 0$), where Case B reflects Condition b of Proposition 2 that the second-best contract exists

[18] "EU" and "RU" denote "expected utility" and "reservation utility" and the subscripts "SB" and "NL" stand for "second-best" and "no-lender", respectively.

Table 3-2. Investment Distortion for the Borrower with Aggregate Consumption

$W = 20$						
$Case$	a_H	C_H	C_L	EU_{SB}	RU	ID
A	100	N/A	N/A	N/A	$-.82$	Yes
B	88	226	6	$-.86$	$-.82$	Yes
C	30	176	123	$-.28$	$-.82$	No

$W = 180$							
$Case$	a_H	C_H	C_L	EU_{SB}	RU	EU_{NL}	ID
D	109	N/A	N/A	N/A	$-.16$	$-.19$	Yes
E	107	450	16	$-.76$	$-.16$	$-.19$	Yes
F	60	341	221	$-.09$	$-.16$	$-.12$	No

but is too risky for the borrower to engage in. There is no investment distortion for Case C because none of the conditions in Proposition 2 holds and the borrower achieves a higher expected utility with borrowing from the lender than consuming his personal holdings (i.e., $EU_{SB} = -.28 > RU = -.82$).

The lower figure focuses on the wealth-abundant borrower with $W > I$. Case D and Case E represent investment distortion but based on different grounds. Case D reflects Part a of Proposition 3, where the second-best contract is not feasible; therefore, investment distortion arises because no-lender investment is not preferable to the status quo ($CE(X) < I + a_H = 259$). Stated differently, the investment project is too risky for the borrower and he simply is not interested when sharing risk with the lender is inadmissible. For Case E, there is a solution for the second-best contract. However, investment distortion arises because Part b of Proposition 3 holds. In particular, no-lender investment is not attractive to the borrower because the certainty equivalent cash flow $CE(X)$ is less than the costs he would incur to finance the project. Furthermore, the comparison between no-investment and the loan agreement shows that the borrower prefers the former, which suggests that the second-best contract is too risky for the borrower to consider. However, the investment preference ordering in Corollary 2 does not apply to Case E. In Corollary 2, the first inequality of condition (3–13) ensures

$$\text{No Invest} \succ \text{Second-Best Loan} \tag{3–14}$$

and the second inequality indicates that

$$\text{Second-Best Loan} \succ \text{No-Lender Invest} \tag{3–15}$$

For Case E, (3–14) holds but (3–15) is violated. Therefore, as reflected by Table 3-2, the borrower's expected utility from no-lender investment is higher than that from the second-best contract for Case E. Intuitively, in the presence of the relatively large personal cost of high effort supply ($a_H = 107$), although the loan agreement provides risk-sharing it also enlarges the consumption differential to motivate the borrower, which makes the loan agreement very risky and in consequence reduces the borrower's expected utility dramatically given his concave utility function. Were the borrower to choose self-financing, his consumption from no-lender investment would be 380 and 180 when high and low cash flows are realized, respectively. This consumption schedule is not feasible for the second-best loan agreement because the borrower IC constraint would be violated. When the borrower weighs the benefit brought about by risk-sharing with the lender and the cost of respecting the imposed (IC) constraint, the borrower might be worse off signing a contract with the lender than just investing on his own.

On the contrary, Case F reflects the situation where the second-best contract is the borrower's optimal investment decision. In particular, condition (3–15) is satisfied with Case F. Here, the moderate personal effort cost of the borrower ($a_H = 60$) makes the benefit from risk-sharing outweigh the cost from applying the consumption differential to keep the borrower motivated.

3.3 Consumption Preference Case 2: Time-Additive

Section 3.2 analyzes an investment/consumption problem for the borrower with aggregate consumption preference and provides benchmark conditions under which investment distortion arises. However, the timeline indicates that although the investment decision does not involve staged investments, the design is a two-date problem. This raises the question of whether the borrower has intertemporal consumption concerns. From this section on this chapter brings in the borrower's intertemporal consumption preference and investigates the effect of consumption smoothing on the potential investment distortion. The timeline is specified as follows and a superscript "A" is added to each of the previous notation to represent "time additive". Before $t = 0$, the contract between the borrower and the lender, were it to exist, would be signed; then the borrower consumes C_0^A at $t = 0$. Afterwards he exerts effort for the investment if the project is taken, and, finally, he receives C_1^A for consumption at $t = 1$. Assume no discount and based on Koopman's stationarity axiom,[19] the borrower has an aggregate utility across periods

$$U^A = U^A\left(C_0^A\right) + U^A(C_1^A, a) = -\exp(-\rho C_0^A) - \exp\left[-\rho(C_1^A - a)\right]$$

When the project is undertaken, the borrower's monetary holdings C_1^A at $t = 1$ are also his gross consumption and $C_1^A - a$ is his net consumption. For the borrower's $t = 0$ consumption, since no effort is supplied, C_0^A is the borrower's net consumption.

The analysis for the borrower with time-additive consumption preference shares quite a few similarities with that for the borrower with aggregate consumption preference in the previous section. In particular, regardless of the borrower's consumption preference, he faces similar investment options and his investment behavior features no investment distortion or potential investment distortion.

3.3.1 No Investment Distortion

When no investment opportunity is present, similar to the borrower with aggregate consumption preference, the following conclusion is drawn:

Fact 3 In the absence of the investment project, the borrower's optimal aggregate consumption is $C = W$ and his period-by-period consumptions are equal.

[19] Koopmans (1960) stationarity axiom implies that intertemporal preferences between two periods remain the same. Therefore, the borrower with time-additive consumption preference has an aggregate utility function that can be expressed as the sum of his period-by-period utility.

That is, the borrower's optimal consumption allocation is $C_0^A = C_1^A = \frac{W}{2}$ in this case.

The first-best contract for the borrower with intertemporal consumption concern is modeled similarly to [Program FB]; however, the borrower's consumption constraints are different. At $t = 0$, the borrower has a personal wealth level of W, this does not necessarily indicates he consumes the entire W at $t = 0$, he can choose to consume $W_0 \leq W$ and save $W_1 = W - W_0 \geq 0$ for the future period $t = 1$.[20] Therefore at $t = 0$, the upper bound for the borrower's consumption is his initial personal wealth net of his investment in the project and his stored wealth, i.e.,

$$C_0^A \leq W - (I - D_0) - W_1$$

At $t = 1$, the borrower's consumption is bounded above by the sum of his stored wealth and the project return net of the repayments for the lender.

$$\begin{aligned} C_H^A &\leq W_1 + X_H - D_H \\ C_L^A &\leq W_1 + X_L - D_L \end{aligned}$$

Again, given the lender's binding IR, it can be derived that the borrower's expected consumption across time is bounded above by the sum of his personal wealth holding and the expected NPV from the investment project, i.e.,

$$C_0^A + p_H C_H^A + (1 - p_H)\, C_L^A \leq W + \Pi_H \tag{3–16}$$

[20] As discussed later, a negative W_1 is not optimal for the borrower with time-additive consumption preference. For the borrower with time-additive consumption preference, the lender's investment D_0 is also indeterminate. However, it is possible for the lender to lend more than what is required. Intuitively, for the wealth-constrained borrower, the lender's investment not only ensures the investment, but also facilitates smoothing the consumption series.

Therefore, the first-best problem for the borrower with time-additive consumption preference is

[Program FB^A]

$$\max_{C_0^A\geq 0, C_H^A\geq 0, C_L^A\geq 0} -\exp(-\rho C_0^A) - p_H \exp\left[-\rho\left(C_H^A - a_H\right)\right] - (1-p_H)\exp\left[-\rho\left(C_L^A - a_H\right)\right]$$

$$\text{s.t. } C_0^A + p_H C_H^A + (1-p_H)\, C_L^A \leq W + \Pi_H$$

Fact 4 The first-best contract for the time-additive borrower is characterized as

$$C_0^A = \frac{W + \Pi_H - a_H}{2}$$
$$C_H^A = C_L^A = \frac{W + \Pi_H + a_H}{2}$$

where $\Pi_H = p_H X_H + (1-p_H)\, X_L - I$.

The borrower with time-additive consumption preference has a consumption schedule that resembles that of the borrower with aggregate consumption preference, and in particular the borrower's intertemporal *net* consumption are smooth, which suggests

$$C_0^A = C_H^A - a_H = C_L^A - a_H$$

3.3.2 Second-Best Contract and Potential Investment Distortion

Similar to the borrower with aggregate consumption preference, potential investment distortion only arises in the second-best environment. The borrower's incentive compatibility (i.e.,IC^A) constraint, given Assumption 2, is

$$C_L^A + \frac{1}{\rho}\ln\Omega \leq C_H^A \tag{3–17}$$

where $\Omega = \frac{p_H\exp(\rho a_H) - p_L}{(1-p_L)-(1-p_H)\exp(\rho a_H)}$. The second-best contract for the borrower with time-additive consumption preference is the following program.

[Program SB^A]

$$\max_{C_0^A\geq 0, C_H^A\geq 0, C_L^A\geq 0} -\exp(-\rho C_0^A) - p_H \exp\left[-\rho\left(C_H^A - a_H\right)\right] - (1-p_H)\exp\left[-\rho\left(C_L^A - a_H\right)\right]$$

$$\text{s.t. } C_0^A + p_H C_H^a + (1-p_H)\, C_L^A \leq W + \Pi_H$$
$$C_L^A + \frac{1}{\rho}\ln\Omega \leq C_H^A$$

The optimal loan arrangement for the borrower under aggregate consumption shows that no corner solutions exist. For the borrower under time-additive consumption, this is not the case. Therefore, the optimal consumption are characterized as either "interior solution" or "corner solution," where the former refers to neither of the borrower's limited liability constraints is binding and the latter refers to any one of the borrower's limited liability constraints being binding.

Proposition 4. *A solution exists for [Program SB^A] if*

$$W + \Pi_H - \frac{p_H}{\rho}\ln\Omega \geq 0 \tag{3–18}$$

and the consumption takes the following forms

a. If $W + \Pi_H - \frac{p_H}{\rho}\ln\Omega \geq -a_H - \frac{1}{\rho}\ln\Phi$, the second-best optimal interior consumption is

$$\begin{aligned} C_0^A &= \frac{W + \Pi_H - \frac{p_H}{\rho}\ln\Omega - a_H - \frac{1}{\rho}\ln\Phi}{2} \\ C_H^A &= \frac{W + \Pi_H - \frac{p_H}{\rho}\ln\Omega + a_H + \frac{1}{\rho}\ln\Phi}{2} + \frac{1}{\rho}\ln\Omega \\ C_L^A &= \frac{W + \Pi_H - \frac{p_H}{\rho}\ln\Omega + a_H + \frac{1}{\rho}\ln\Phi}{2} \end{aligned}$$

b. Otherwise, the optimal corner consumption is

$$C_0^A = W + \Pi_H - \frac{p_H}{\rho} \ln \Omega$$
$$C_H^A = \frac{1}{\rho} \ln \Omega$$
$$C_L^A = 0$$

where $\Omega = \frac{p_H \exp(\rho a_H) - p_L}{(1-p_L)-(1-p_H)\exp(\rho a_H)}$ and $\Phi = \frac{p_H - p_L}{p_H \exp(\rho a_H) - p_L}$.

Closer investigation reveals that in the corner consumption situation only the constraint for C_L^A is binding. The driving force of this possibility is the borrower's time-additive consumption preference. Recall that for the borrower with aggregate consumption preference, there is an upper bound on C_H, which is derived from (3–2) constraint. Similarly, when the borrower has a time-additive consumption preference, inequality (3–16) also suggests that C_H^A is bounded from above

$$C_H^A \leq \frac{W + \Pi_H - C_0^A - (1-p_H)\, C_L^A}{p_H} \tag{3–19}$$

Moreover, the second-best optimal consumption in Proposition 4 implies that

$$C_H^A > C_0^A \geq C_L^A \tag{3–20}$$

Therefore, from (3–19) and (3–20) it is possible for C_H^A to be adjusted upwards when the limited liability constraint for C_L^A is binding because of the presence of C_0^A. Intuitively, if the control problem gets severe, the consumption differential needs to increase to ensure the borrower delivers high effort. This does not impose any additional restriction as long as the optimal consumption is still interior; however, when the limited liability constraint of C_L^A is binding, the question of whether there exists a feasible consumption schedule that satisfies the borrower's IC^A constraint becomes important. As (3–19) shows, C_0^A provides some range which makes C_H^A still be feasible when the limited liability constraint of C_L^A is binding. Therefore, the borrower's intertemporal consumption behavior helps to take some pressure off his $t = 1$ consumption when the consumption differential in $t = 1$ is widening.

As indicated by (3–20), the limited liability constraint for the borrower's $t = 1$ consumption C_H^A is never binding. Furthermore, Proposition 4 also suggests that the limited liability constraint for C_0^A is not binding as well. The reason is similar to the reason, discussed earlier, why the limited liability constraint of C_L never binds for the borrower with aggregate consumption preference. Again, the upper bound in (3–19) prohibits the existence of such C_H^A that can provide a mutually agreeable second-best contract to respect the required consumption differential when both C_L^A and C_0^A are forced to be corner solutions.

From Proposition 4, it is evident the consumption differential is unavoidable in the second-best contract; furthermore, consumption across periods is not smoothing, which is consistent with the consumption distortion definition. Define *utility smoothing* when the following equality holds:

$$-\exp(-\rho C_0^A) = -p_H \exp\left[-\rho\left(C_H^A - a_H\right)\right] - (1 - p_H)\exp\left[-\rho\left(C_L^A - a_H\right)\right]$$

We then can derive the following corollary:

Corollary 3. *The second-best interior consumption for the borrower with time-additive consumption preference features utility smoothing.*

The economic implication behind utility smoothing is the following. Define *efficiency of a loan contract* as the level of the expected utility the borrower can achieve; in other words, the greater the expected utility the borrower achieves, the more efficient the corresponding loan contract is. Therefore, the first-best contract is the most efficient contract because the period-by-period consumption is riskless and smooth, which leads to a highest expected utility for the borrower. Since it is necessary for a second-best contract to provide risky consumption to motivate the borrower to work diligently, for a second-best contract to replicate the efficiency of the first-best contract, the best it can do is achieve a smooth local certainty equivalent consumption across time, where *local certainty equivalent consumption* at time $t = 0, 1$ refers to the certain consumption levels that make the borrower indifferent to the consumption schedule provided by the second-best contract and the guaranteed consumption. Since the second-best contract with interior consumption is efficient, this implies that the borrower has equal local certainty equivalent consumption; therefore, it has to be the case that period-by-period utilities are smooth.

Define CE_t as the local certainty equivalent consumption for the borrower with time-additive consumption preference at time $t = 0, 1$, respectively, Corollary 3 then states that

$$-\exp(-\rho CE_0) = -\exp(-\rho C_0^A)$$
$$-\exp\left(-\rho CE_1 - a_H\right) = -(1 - p_H)\exp\left[-\rho\left(C_L^A - a_H\right)\right] - p_H \exp\left[-\rho\left(C_H^A - a_H\right)\right]$$

which further implies

$$CE_0 = CE_1 - a_H \tag{3–21}$$

Equation (3–21) indicates that the borrower's net $t = 1$ local certainty equivalent consumption equals his $t = 0$ local certainty equivalent consumption. Therefore, even though there is consumption distortion with respect to the second-best consumption schedule, the borrower's desire to have smooth consumption is preserved in the sense that smoothing utility is not distorted.

Recall that in formulating the time-additive borrower's consumption problem, the borrower is assumed to put $W_1 \geq 0$ aside as a reserve for his t_1 consumption. We argue that once the second-best contract [Program SB^A] is in place, it is optimal for the borrower to always store additional consumption W_1. Consider the following two potential consumption schedules:

A.

$$C_0^A = W - (I - D_0)$$
$$C_H^A = X_H - D_H$$
$$C_L^A = X_L - D_L$$

and

B.

$$C_0^A = W - (I - D_0) - W_1$$
$$C_H^A = W_1 + X_H - D_H$$
$$C_L^A = W_1 + X_L - D_L$$

where $W_1 \geq 0$.

It is evident that Consumption Behavior A stores no additional consumption for $t = 1$ while Schedule B delays some consumption till t_1. For the borrower with time-additive consumption preference, the consumption differential at $t = 1$ is determined by his attitude towards risk, which is invariant with his personal wealth holdings; therefore, whether W_1 is positive or not has no effect on his consumption differential for $t = 1$.

For Consumption Schedule A, the borrower's period-by-period expected utility is

$$-\exp\left[-\rho\left(W - (I - D_0)\right)\right] \tag{3–22}$$

$$-p_H \exp\left[-\rho\left(X_H - D_H\right)\right] - (1 - p_H)\exp\left[-\rho\left(X_L - D_L\right)\right] \tag{3–23}$$

and for Consumption Schedule B, his period-by-period expected utility is

$$-\exp\left[-\rho\left(W - (I - D_0) - W_1\right)\right] \tag{3–24}$$

$$-p_H \exp\left[-\rho\left(W_1 + X_H - D_H\right)\right] - (1 - p_H)\exp\left[-\rho\left(W_1 + X_L - D_L\right)\right] \tag{3–25}$$

It is evident that

$$(3–22) \geq (3–24)$$

$$(3\text{–}23) \leq (3\text{–}25)$$

However, given the concavity of the borrower's utility function, the following relationship holds

$$(3\text{–}22)-(3\text{–}24) \leq (3\text{–}25)-(3\text{–}23)$$

which is equivalent to

$$(3\text{–}22)+(3\text{–}23) \leq (3\text{–}24)+(3\text{–}25)$$

where the equality holds at $W_1 = 0$. This indicates that the borrower's expected aggregate utility is weakly higher when he has Consumption Schedule B rather than A, which implies that it is optimal for the borrower to store additional consumption at $t = 0$.

Before proceeding to investment distortion, the following example is provided in comparison with Table 3-1. The parameter values follow those in Table 3-1 unless otherwise specified. Other than the column for the borrower's personal wealth levels, Table 3-3 consists of three blocks. The first and the second blocks are the second-best contract characterized in Proposition 4 with interior and corner consumption indicated, which are labeled as "SBI" and "SBC," respectively. For minimal variation, only a_H is changed to trigger the corner consumption.[21] The last block is the second-best contract if the limited liability constraints are absent given the same borrower's personal cost of effort a_H as for the corner consumption, which is labeled as "SBU^A." The first-best contract for the time-additive borrower follows Fact 4 and is smoothing in the borrower's net period-by-period consumption; therefore, it is not tabulated.

[21] For the second-best contract with interior consumption, the borrower's personal cost of high effort is $a_H = 90$. The borrower's personal costs of effort supply for corner solution to arise for each wealth level are $a_H = 90$ for 20, $a_H = 96$, and 108 for $W = 50$, and 200, respectively.

Table 3-3. Second-Best Contracts for the Borrower with Time-Additive Consumption

	SBI			SBC			SBU^A		
W	C_0^A	C_H^A	C_L^A	C_0^A	C_H^A	C_L^A	C_0^A	C_H^A	C_L^A
20	N/A	N/A	N/A	N/A	N/A	N/A	4.1	225	-5.3
50	19.1	240	9.7	1.2	269.7	0	5.5	265.4	-4.3
200	94.1	315	84.7	5.9	477.2	0	8.2	475	-2.2

When the borrower's personal wealth holdings is 20, the second-best contract is not feasible.[22] This is because for $W = 20$, condition (3–18) is violated.

When the second-best contract exists, it is evident the consumption differential is present and period-by-period consumption is not smoothing, both of which indicate consumption distortion. The second block in Table 3-3 provides corner solution for the second-best contract. Intuitively, as the borrower's personal cost of equilibrium effort grows, his disutility from working diligently increases. The incentive contract needs to offer a wider consumption differential to ensure the borrower is motivated, which makes it more likely to set off the limited liability constraint for C_L^A. The magnitude of C_0^A provides some cushion for such consumption differential adjustment. In the presence of limited liability, once the limited liability constraint for C_L^A is binding, the second-best contract reallocates the distribution of consumption across periods to ensure the feasibility of such a loan agreement.

The reallocation is not without its boundary since the limited liability constraint exists for C_0^A as well. The last block of Table 3-3 for $W = 20$ reflects such a case. Were C_L^A to be restricted to be non-negative, we would see

$$C_H^A = 225 + |-5.3| = 230.3$$

[22] The cutoff personal wealth to ensure the feasibility of the second-best loan contract is approximately 22.

to satisfy the borrower's IC^A constraint and

$$C_0^A = 4.1 - 5.3 = -1.2$$

to ensure the constraint (3–16) holds; however, C_0^A's limited liability constraint is violated, which implies there is no solution for [Program SB^A].

After characterizing the second-best contract, we next tackle the potential investment distortion problem in the second-best environment. Similar to the borrower with aggregate consumption preference, for the wealth-constrained borrower (i.e., $W < I$) infeasibility of the second-best loan agreement, which is reflected by the failure to satisfy condition (3–18), leads directly to investment distortion. In addition, investment distortion arises when the borrower prefers the status quo to the loan agreement when the latter is feasible but too risky.

Since the second-best contract described in Proposition 4 can either provide interior or corner consumption, the investment decisions between borrowing and staying put are discussed separately. If the second-best contract results in interior consumption, based on Corollary 3, the condition under which investment distortion occurs is the same as that in Condition b of Proposition 2 for the borrower that has aggregate consumption preference. Intuitively, the borrower's intertemporal consumption problems are identical in terms of their period-by-period expected utility when interior consumption is realized for the loan agreement, which suggests that the investment decision is not qualitatively different from that for the borrower with aggregate consumption preference; there is just one more repeated identical period. On the other hand, if the second-best contract has a corner solution, we argue that the condition under which the investment distortion arises in the presence of interior second-best solutions is sufficient for investment distortion associated with the corner consumption. The borrower maximizes his expected utility from intertemporal consumption only when the utility across periods is smooth given the concavity of the utility function. Since smooth utility is only achievable when interior consumption is realized, the borrower's expected utility associated with interior consumption is higher than that associated with corner consumption. Therefore, if the borrower's reservation utility from no investment outperforms his expected utility from the second-best loan with respect to interior consumption, it certainly is preferred to a second-best loan contract that results from corner consumption. We summarize with the following corollary:

Corollary 4. *For $W < I$, investment distortion occurs for the borrower with time-additive consumption preference if and only if any of the following conditions hold*

a. $W + \Pi_H - \frac{p_H}{\rho} \ln \Omega < 0$.

b. $\Pi_H - a_H < \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi$.

where $\Omega = \frac{p_H \exp(\rho a_H) - p_L}{(1-p_L)-(1-p_H)\exp(\rho a_H)}$ and $\Phi = \frac{p_H - p_L}{p_H \exp(\rho a_H) - p_L}$.

3.3.3 $W \geq I$ Investment and Potential Investment Distortion

When the borrower has sufficiently large personal wealth, i.e., $W \geq I$, the failure of both the second-best contract and no-lender investment leads to investment distortion. The comparison between no-investment and the loan agreement follows the preceding discussion. The no-lender investment for the borrower with time-additive consumption preference is more intriguing because the borrower's consumption strategy at $t = 0$ is important in shaping his investment decision.

In Section 3.2.2.3, the condition under which the status quo is preferable to no-lender investment for the borrower with aggregate consumption preference is derived as

$$CE(X) < I + a_H \tag{3–26}$$

where $CE(X) = -\frac{1}{\rho} \ln\left[(1-p_H)\exp(-\rho X_L) + p_H \exp(-\rho X_H)\right]$ is the certainty equivalent cash flow from the investment project. It turns out (3–26) is still a sufficient but not a necessary condition for the status quo to win over no-lender investment when the borrower has time-additive consumption preference.[23] In other words, even if

$$CE(X) \geq I + a_H \tag{3–27}$$

investment distortion can still arise. This implies that projects that are not considered risky for the borrower with aggregate consumption preference may be considered risky for the borrower with time-additive consumption preference. In particular, the borrower who does not achieve smoothing utilities across time is more vulnerable to the riskiness of the investment project. Since for the risk-averse borrower the investment resulting in non-smoothing intertemporal utilities is riskier than the one which provides otherwise, it is important for the borrower to smooth his utilities across time. However, without access to personal banking, it is not always feasible for the borrower to do so.

With no-lender investment, the borrower's intertemporal consumption schedule is

$$C_0^A \leq W - I$$
$$C_H^A \leq W - C_0^A - I + X_H$$
$$C_L^A \leq W - C_0^A - I + X_L$$

Since it is always optimal for the borrower to store some of his personal wealth at $t = 0$ for future consumption as a practice to smooth intertemporal utilities, it follows that the borrower prefers C_0^A in to be not binding.[24] However, whether C_0^A is binding or not depends on the borrower's personal wealth level relative to the investment requirement.

[23] Condition (3–26) is a sufficient and necessary condition for Proposition 3 in Section 3.2.2.3 to determine the borrower's preference of status quo to no-lender investment.

[24] In equilibrium, the borrower's $t = 1$ consumption C_H^A or C_L^A are binding. Furthermore, it is possible that the borrower's intertemporal utilities are smoothed when

$$C_0 = W - I$$

However, this is an extreme case.

For a specific investment project, a wealthier borrower is more likely to save up for the future and achieves utility smoothing; this suggests that the borrower's personal wealth endowment is important in shaping his consumption behavior at $t = 0$, which in turn affects the potential investment distortion problem.

It turns out that C_0^A is not binding when the following relationship is satisfied

$$CE(X) \leq W - I + a_H \tag{3–28}$$

Otherwise, the borrower's $t = 0$ consumption is equal to $W - I$. Intuitively, other things being equal, (3–28) is more likely to be realized if W is relatively large. For $W >> I$, C_0^A is not binding and by reserving some of his personal wealth for $t = 1$ consumption, the borrower is more likely to achieve smoothing period-by-period utilities, which is strictly preferred. Once the borrower has smoothing utilities, as discussed earlier, any investment decision with time-additive consumption preference is not qualitatively different from that with aggregate consumption preference; therefore, (3–26) is all that matters to determine whether the borrower prefers the status quo to no-lender investment. However, it is inconclusive to determine the relationship between inequalities (3–28) and (3–26); therefore, condition (3–26) is no longer necessary to ensure potential investment distortion. The investment distortion for the wealth-abundant borrower with time-additive consumption preference is summarized in the following proposition:

Proposition 5. *For the time-additive borrower with $W \geq I$,*

A. *If [Program SB^A] is infeasible, investment distortion occurs if*

$$CE(X) < \min\{I + a_H, W - I + a_H\}$$

B. *If [Program SB^A] is feasible, investment distortion occurs if conditions A. and C. hold:*

C. $\Pi_H - a_H < \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi$.

where $CE(X) = -\frac{1}{\rho} \ln [(1 - p_H) \exp(-\rho X_L) + p_H \exp(-\rho X_H)]$, $\Omega = \frac{p_H \exp(\rho a_H) - p_L}{(1-p_L)-(1-p_H)\exp(\rho a_H)}$, *and* $\Phi = \frac{p_H - p_L}{p_H \exp(\rho a_H) - p_L}$.

Similar to the borrower with aggregate consumption, the borrower with time-additive consumption preference has an inconclusive investment preference ordering between borrowing and no-lender investment, which implies that the second-best contract with the lender is not always preferred.

The next numerical example shows investment distortion for the borrower with time-additive consumption preference. The parameter values mainly follow those in Table 3-2 except for the borrower's personal wealth holdings, which are specified in the Table. The superscript "A" is added to the notation to distinguish the borrower with time-additive consumption preference from aggregate consumption preference. The upper exhibit is for the wealth-constrained borrower and the lower exhibit is for the wealth-abundant borrower; his consumption from the second-best contract and no-lender investment are not tabulated.

Case A investment distortion occurs because the second-best contract is infeasible, which means that condition (3–18) fails to hold. Note that condition (3–18) is equivalent to the one that determines the non-existence of the second-best contract for the borrower with aggregate consumption preference. Case B represents investment distortion due to the riskiness of the second-best contract, which suggests that the borrower prefers the status quo to the investment. Close calculation reveals that the borrower's "net gain" (i.e., $\Pi_H - a_H$) is 52, which is not sufficiently large for the borrower to undertake the investment project (the required boundary is 56). This boundary condition is also equivalent to the one that determines passing up on the second-best contract based on its riskiness for the borrower with aggregate consumption preference. Therefore, the borrower's investment decision is invariant with his consumption preference if he is wealth-constrained. Finally, Case C shows no investment distortion as the borrower's expected utility from the second-best contract is higher than his reservation utility.

For the borrower with $W \geq I$, Case D and Case E in Table 3-4 both reflect investment distortion. The second-best contract is infeasible for Case D for $X_L = 130$;

Table 3-4. Investment Distortion for the Borrower with Time-Additive Consumption

$Case$	W	a_H	EU^A_{SB}	RU^A	ID^A
A	50	100	N/A	-1.56	Yes
B	50	88	-1.59	-1.56	Yes
C	50	50	-1.05	-1.56	No

$Case$	W	a_H	X_L	EU^A_{SB}	RU^A	EU^A_{NL}	ID^A
D	250	109	130	N/A	$-.57$	$-.67$	Yes
E	250	90	150	-61	$-.57$	$-.58$	Yes
F	500	90	300	$-.14$	$-.16$	$-.10$	No

therefore the borrower only has two options: no-lender investment and no-investment. For no-lender investment, the project is too risky for the borrower because the certainty equivalent cash flow from investment is less than the total cost incurred for self-investment (i.e., $CE(X) < I + a_H$); therefore, the borrower prefers the status quo, which leads to investment distortion. For Case E, the second-best contract is feasible, however, it is too risky for the borrower to resort to the lender; therefore, no-investment is preferable to the loan arrangement. With respect to no-lender investment, even though condition (3–26) (i.e., $CE(X) < I + a_H$) is not satisfied, which implies that the borrower with aggregate consumption preference would invest, the borrower with time-additive consumption preference still considers the project too risky and prefers no-investment (Condition b in Proposition 5 is satisfied). The borrower's consumption schedule for no-lender investment is $C^A_0 = 100$, $C^A_H = 350$, and $C^A_L = 150$, this clearly shows non-smoothing utilities across time, which suggests that the borrower is more susceptible to investment distortion.

Case F reflects the lack of investment distortion and both second-best contract and no-lender investment are preferable to the status quo. The comparison between the loan agreement and self-financing reveals that the latter wins. When the borrower is relatively wealthy ($W = 500$) and the investment project is relatively less risky (with $X_H = 350$ and $X_L = 300$), he becomes less concerned about risk-sharing. The borrower then prefers no-lender investment to the loan agreement because the latter is subject to the borrower's IC^A constraint.[25] Therefore, similar to the borrower with aggregate consumption preference, the second-best contract is *not always optimal* in the absence of investment distortion.

3.4 Investment Distortion and Consumption Smoothing

Comparing the investment distortion problems for the borrower with aggregate and time-additive consumption preferences, respectively, enables me to draw the following conclusion naturally. *Ceteris paribus*, define *investment distortion aggravation* as the scenario where one type of borrower is more susceptible to investment distortion, the following conclusion is developed from this chapter's analysis:

Proposition 6. *Investment distortion is weakly aggravated when the borrower has time-additive consumption preference.*

For the borrower's investment decision between the second-best loan contract and no investment, investment distortion arises (a) when the second-best contract is infeasible or (b) the second-best contract is too risky for the borrower. The conditions under which investment distortion arises for the borrower with aggregate consumption preference are the same as those under which investment distortion occurs for the borrower with time-additive consumption preference; therefore, consumption preference has no effect on the occurrence of the investment distortion if the borrower is wealth-constrained.

When the borrower is wealth-abundant, other than seeking a loan agreement and staying put, he can also choose to self-finance the investment project. Given the borrower's concave and increasing utility function, the borrower with time-additive consumption preference maximizes his expected utility when his period-by-period utilities are smoothing. However, the concern for consumption smoothing makes the borrower

[25] The no-lender investment consumption schedule features $C_0^A = 296$, $C_H^A = 404$, and $C_L^A = 354$ and the second-best loan contract consumption schedule is solved at $C_0^A = 267$, $C_H^A = 487$, and $C_L^A = 257$. Substituting the consumption schedule from no-lender investment into the IC^A constraint in the loan contract directly shows a violation.

with time-additive consumption preference more susceptible to investment distortion when investment decision is associated with no-lender investment. In particular, whether the borrower is able to smooth utilities across time in the absence of personal banking is important. The borrower's inability to do so may lead him to forego projects that are not considered too risky in the eyes of his aggregate consumption preference counterpart. Therefore, regardless of the borrower's personal wealth holdings, potential investment distortion is weakly aggravated when the borrower has time-additive consumption preference.

CHAPTER 4
DARA-BORROWER

To distinguish the Chapter 3 borrower from the borrower discussed in this chapter, the borrower in Chapter 3 is labeled as the *CARA-borrower*. Chapter 3 analyzes the CARA-borrower's consumption and investment behaviors while varying his consumption preferences. The general conclusion is that potential investment distortion is weakly aggravated when the CARA-borrower has time-additive consumption preference. Moreover, if the second-best loan agreement is the optimal investment decision for both consumption preferences, conditions under which investment distortion occurs are identical regardless of the CARA-borrower's consumption preference. This special case is mainly caused by the nature of the CARA-borrower's risk preference. In particular, time-additive consumption and investment problem may be simply considered as two identical one-period consumption and investment problems for the CARA-borrower. However, were the borrower's risk preference to be different, such repeated consumption and investment decision feature may not be present. In this chapter the borrower is assumed to have decreasing absolute risk aversion (DARA hereafter), whose utility function takes the form of square-root on the net consumption. Unlike the negative exponential utility function which is the unique utility form in the CARA class, the square-root utility function is only one of the utility forms in the DARA class. For expositional simplicity and with a little abuse of terminology, the borrower with square root function in this chapter is sometimes referred to as the *DARA-borrower* to avoid confusion with the CARA-borrower; otherwise, "borrower" in this chapter refers to the DARA-borrower.

The CARA-borrower consumption and investment analyses in Chapter 3 show the importance of the borrower's personal wealth. In particular, the wealth level determines the borrower's potential investment strategy and whether the loan contract is feasible. However, once different investment strategies (i.e., borrow, no-lender investment, and no-investment) are in place, the potential investment distortion problem is not differentiated for different CARA-borrower that may possess different level of personal wealth. This implicitly assumes that every CARA-borrower is homogenous. To relax this assumption and further investigate what role, other than that already examined for the CARA-borrower, the borrower's personal wealth plays in determining potential investment distortion, this chapter introduces the DARA-borrower. The difference in the two risk preference, as will be later discussed in this chapter, clearly has influence on the borrower's investment behavior. For example, with the knowledge of the borrower's risk preference, the lender can better design the loan contract to reduce the incidence of potential investment distortion. This is especially important for the wealth-constrained borrower when investing is his optimal investment choice given the non-negative expected NPV from the investment project.

The DARA-borrower faces similar investment decisions as the CARA-borrower, he has the choice of borrowing from the lender, engaging in no-lender investment, and consuming his personal wealth and forgoing the investment project. The investment project, if engaged, again provides a binary cash flow $X \in \{X_H, X_L\}$. The lender is risk neutral and breaks even from her involvement in the project. The borrower is expected to work diligently because Assumption 1 in Chapter 3 applies for this chapter as well, which indicates that high effort is always preferred in equilibrium. Let $\widehat{C}$ and $\widehat{U}$ denote the borrower's aggregate money holdings across periods and his utility function, respectively. The risk-averse borrower's utility is expressed as

$$\widehat{U} = \widehat{U}\left(\widehat{C} - a\right) = \sqrt{\widehat{C} - a}$$

where a is the borrower's personal cost of effort. The borrower's objective is to maximize his expected utility from his consumption. Unlike the CARA-borrower in Chapter 3, the DARA-borrower's consumption not only needs to be non-negative (i.e., $\widehat{C} \geq 0$), but it is also required to be not less than his personal cost of effort supply (i.e., $\widehat{C} \geq a_H$) for his utility function to be feasible,

This chapter proceeds as follows. In Section 4.1 the first-best contract and no-lender investment for the DARA-borrower with aggregate and time-additive consumption preferences are discussed as benchmarks. Section 4.2 characterizes the second-best loan agreements when the borrower's effort supply is unobservable depending on his consumption preference and identifies the second-best loan contract associated with potential investment distortions. Section 4.2 also provides comparisons between the DARA- and the CARA-borrower in their investment behavior and the incidence of investment distortion. This chapter concludes with Section 4.3, which provides a discussion of the effect of the borrower's consumption preference on his optimal investment policy.[1]

4.1 First-Best Contract and No-Lender Investment

In the first-best contract scenario, the borrower's effort supply is observed publicly. Since the lender operates in a competitive financial market, she is able to "buy" the entire investment project and reward the borrower with the expected NPV of the investment if the borrower works diligently. To distinguish the DARA-Borrower's consumption from those of the CARA-Borrower in Chapter 3, let $\left\{\widehat{C}_H, \widehat{C}_L\right\}$ and $\left\{\widehat{C}_0^A, \widehat{C}_H^A, \widehat{C}_L^A\right\}$ denote the DARA-Borrower's consumption when he has aggregate and time-additive consumption preferences, respectively. The borrower solves the following optimization programs for the first-best loan agreements depending on his consumption preferences:

[1] Other than the form of the utility function, the basic model setup follows that in Chapter 3; moreover, definitions in Chapter 3 apply to this chapter as well.

[Program FB^D]

$$\max_{\widehat{C}_H \geq a_H, \widehat{C}_L \geq a_H} p_H \sqrt{\widehat{C}_H - a_H} + (1 - p_H) \sqrt{\widehat{C}_L - a_H}$$
$$\text{s.t. } p_H \widehat{C}_H + (1 - p_H) \widehat{C}_L \leq W + \Pi_H$$

if he has aggregate consumption preference or

[Program FB^{DA}]

$$\max_{\widehat{C}_0^A \geq 0, \widehat{C}_H^A \geq a_H, \widehat{C}_L^A \geq a_H} \sqrt{\widehat{C}_0^A} + p_H \sqrt{\widehat{C}_H^A - a_H} + (1 - p_H) \sqrt{\widehat{C}_L^A - a_H}$$
$$\text{s.t. } \widehat{C}_0^A + p_H \widehat{C}_H^A + (1 - p_H) \widehat{C}_L^A \leq W + \Pi_H$$

if he has time-additive consumption preference.

The first-best contracts provide the following optimal consumption for the borrower.

Fact 5 The first-best contracts for the borrower are characterized as

1. For aggregate consumption preference

$$\widehat{C}_H = \widehat{C}_L = W + \Pi_H$$

2. For time-additive consumption preference

$$\begin{aligned} \widehat{C}_0^A &= \frac{W + \Pi_H - a_H}{2} \\ \widehat{C}_H^A &= \widehat{C}_L^A = \frac{W + \Pi_H + a_H}{2} \end{aligned}$$

In the absence of the lender, when the borrower has sufficient personal wealth he has the option of engaging in self-financing. The borrower who has aggregate consumption preference maximizes his expected utility from self-financing, which leads to the following program:

[Program NL^D]

$$\max_{\widehat{C}_H \geq a_H, \widehat{C}_L \geq a_H} p_H \sqrt{\widehat{C}_H - a_H} + (1 - p_H)\sqrt{\widehat{C}_L - a_H}$$

$$\text{s.t. } \widehat{C}_H \leq W - I + X_H$$

$$\widehat{C}_L \leq W - I + X_L$$

Similar to the analysis in Chapter 3, in equilibrium, both constraints are binding.

$$\widehat{C}_H = W - I + X_H \tag{4–1}$$

$$\widehat{C}_L = W - I + X_L \tag{4–2}$$

However, from the limited liability constraint, it is required that

$$\widehat{C}_L \geq a_H$$

Therefore, a solution exists if

$$W + X_L \geq I + a_H$$

whose satisfaction ensures the feasibility of both the consumption and the limited liability constraints. Note that for the borrower to engage in no-lender investment, his personal wealth is required to be $W \geq I$; therefore, the following assumption is imposed to facilitate the analysis:

Assumption 3 $X_L \geq a_H$

Assume the borrower's personal wealth is just sufficient to cover the investment requirement (i.e., $W = I$), for the borrower's no-lender investment to be his optimal investment decision, based on consumption schedule (4–1) and (4–2), it is required that

$$p_H \sqrt{X_H - a_H} + (1 - p_H)\sqrt{X_L - a_H} > \sqrt{I}$$

As the borrower's personal wealth increases to greater than I, both his expected utility from no-lender investment and no-investment increase. However, it turns out that were he to prefer no-lender investment at $W = I$, he would make the same investment decision for any $W > I$.

Let $CE(X)$ denote the borrower's certainty equivalent consumption when $W = I$, it follows that

$$\sqrt{CE(X) - a_H} = p_H\sqrt{X_H - a_H} + (1 - p_H)\sqrt{X_L - a_H}$$

the following conclusion with respect to investment distortion is derived:

Proposition 7. *The sufficient condition for the wealth-abundant borrower with aggregate consumption preference to prefer no-lender investment to no investment is*

$$CE(X) - a_H \geq I \tag{4–3}$$

where $\sqrt{CE(X) - a_H} = p_H\sqrt{X_H - a_H} + (1 - p_H)\sqrt{X_L - a_H}$ *and* $W \geq I + a_H - X_L$.

Intuitively, the borrower chooses to invest if his consumption from the cash flows of the project net his incurred personal cost of high effort supply is sufficient to cover the investment requirement. However, condition (4–3) in Proposition 7 is not necessary for the borrower to prefer no-lender investment to no investment. The reason is that (4–3) is derived under the assumption that $W = I$; therefore, the failure to satisfy (4–3) does not imply the occurrence of investment distortion.

Comparing the DARA-borrower with the CARA-borrower in their no-lender investment strategies, it turns out that the condition which leads both types of borrower to prefer self-financing is quite similar. Let $CE(X)$ be the borrower's certainty equivalent consumption for $W = I$, for both types of borrower, the following corollary can be developed:

Corollary 5. *The wealth-abundant CARA- and DARA-borrower with aggregate consumption preference engages in no-lender investment in the absence of the lender if*

$$CE(X) - a_H \geq I$$

where $-\exp\left[-\rho\left(CE(X)-a_H\right)\right]=-p_H\exp\left[-\rho\left(X_H-a_H\right)\right]-(1-p_H)\exp\left[-\rho\left(X_L-a_H\right)\right]$ *for the CARA-borrower and* $\sqrt{CE(X)-a_H}=p_H\sqrt{X_H-a_H}+(1-p_H)\sqrt{X_L-a_H}$ *for the DARA-borrower, and* $X_L\geq a_H$*.*

Intuitively, when the CARA- or DARA-borrower's personal wealth is $W=I$, it is the cash flows from the investment project that both types of borrower are consuming. Since intertemporal consumption is not a concern for the borrowers with aggregate consumption preference; therefore, independent of their risk preference, they choose self-financing when their consumption from investing is sufficient to exceed his wealth at hand and the incurred personal cost of effort. However, the condition in Corollary 5 is also a necessary condition for the CARA-borrower but not necessary for the DARA-borrower.

Next, the potential investment distortion caused by the borrower's unwillingness to engage in no-lender investment is investigated when the borrower's consumption preference becomes time-additive.

When the borrower has time-additive consumption preference, he solves the following problem to justify his no-lender investment decision:

[Program NL^{DA}]

$$\max_{\widehat{C}_0^A\geq 0,\widehat{C}_H^A\geq a_H,\widehat{C}_L^A\geq a_H}\sqrt{\widehat{C}_0^A}+p_H\sqrt{\widehat{C}_H^A-a_H}+(1-p_H)\sqrt{\widehat{C}_L^A-a_H}$$

$$\text{s.t. } \widehat{C}_0^A\leq W-I$$

$$\widehat{C}_H^A\leq W-I-\widehat{C}_0^A+X_H$$

$$\widehat{C}_L^A\leq W-I-\widehat{C}_0^A+X_L$$

Similar to the CARA-borrower with time-additive consumption preference, the DARA-borrower's $t=1$ consumption $\left\{\widehat{C}_H^A,\widehat{C}_L^A\right\}$ is always binding, but not his $t=0$ consumption $\widehat{C}_0^A$. The following conclusion is developed to address the potential investment distortion associated with no-lender investment.

Proposition 8. *The wealth-abundant borrower with time-additive consumption preference engages in no-lender investment in the absence of the loan if*

$$CE(X) - a_H \geq 2I \tag{4–4}$$

where $\sqrt{CE(X) - a_H} = p_H\sqrt{X_H - a_H} + (1 - p_H)\sqrt{X_L - a_H}$ *and* $X_L \geq a_H$.

Comparing (4–4) with (4–3) implies that the borrower requires higher $CE(X)$ to engage in no-lender investment (twice to be exact). This is consistent with the fact that the borrower with time-additive consumption preference has intertemporal consumption concern, which leads to a more constrained problem.

4.2 Second-Best Loan Agreement

The previous section develops sufficient conditions for the borrower to invest in the absence of capital market access. This section focuses on the second-best loan contract where the lender is available; however, the borrower's effort supply is private information.

The DARA-borrower in this chapter is assumed to have domain-additive personal cost structure. This cost structure makes it quite difficult to derive closed-form solutions of the optimal consumption without imposing further assumptions; however, it is consistent with the CARA-borrower in Chapter 3 in the cost sphere. Furthermore, the lack of closed-form expression for the second-best loan contract consumption hinders the analysis of investment distortion associated with the second-best loan agreement. Therefore, in this section numerical examples are applied to facilitate revealing the insights of the second-best loan contract and potential investment distortion.

4.2.1 Aggregate Consumption Preference

Following the analysis in Chapter 3, the borrower maximizes his expected utility from his net consumption in the second-best loan agreement, with the associated expected consumption, incentive compatibility, and limited liability constraints.

[Program SB^D]

$$\max_{\widehat{C}_H \geq a_H, \widehat{C}_L \geq a_H} p_H \sqrt{\widehat{C}_H - a_H} + (1 - p_H) \sqrt{\widehat{C}_L - a_H}$$

$$\text{s.t. } p_H \widehat{C}_H + (1 - p_H) \widehat{C}_L \leq W + \Pi_H \tag{4–5}$$

$$p_L \sqrt{\widehat{C}_H} + (1 - p_L) \sqrt{\widehat{C}_L} \leq p_H \sqrt{\widehat{C}_H - a_H} + (1 - p_H) \sqrt{\widehat{C}_L - a_H} \tag{4–6}$$

The borrower's (4–6) is binding in equilibrium, otherwise, the first-best consumption apply which leads to directly violation of (4–6) constraint given $p_H > p_L$. Following Chapter 3, the lender breaks even and all expected cash flows from the investment project accrue to the borrower; therefore, the borrower consumes at most the sum of his personal wealth at hand and the expected cash flows from the investment. This implies that the (4–5) constraint is also binding in equilibrium. Then, the optimal consumption is derived from the following set of equations:

$$p_H \widehat{C}_H + (1 - p_H) \widehat{C}_L = W + \Pi_H$$
$$p_L \sqrt{\widehat{C}_H} + (1 - p_L) \sqrt{\widehat{C}_L} = p_H \sqrt{\widehat{C}_H - a_H} + (1 - p_H) \sqrt{\widehat{C}_L - a_H}$$

The closed-form expressions for the optimal consumption are difficult to derive; however, presuming the existence of the solution to the above [Program SB^D], the following sensitivity tests can be performed.

Corollary 6. *The borrower's expected utility with respect to the second-best loan is decreasing in his personal cost of high effort supply and increasing in his personal wealth.*

Assume the existence of the second-best loan contract, the borrower bears risk associated with his unobservable personal cost. And as his personal cost of equilibrium high effort increases, the risk he is bearing increases at the same time, which results a lower expected utility for the borrower. On the other hand, since the borrower is entitled to consume his personal wealth, the wealthier he is, the more consumption he has, which increases his expected utility as a result.

Given the lender's presence and the existence of a solution to the second-best loan contract, the next logical issue is investment distortion. The lack of the closed-form expressions of the borrower's optimal consumption from the second-best loan contract clearly imposes difficulties in deriving conditions under which investment distortion occurs. However, it is possible to compare the nature of potential investment distortions between the two types of borrowers, i.e., the CARA- and the DARA-borrower.

Intuitively, personal wealth is of importance in the incidence of investment distortion associated with the second-best loan contract. The CARA-borrower's risk premium for the loan agreement is invariant with his personal wealth holdings. However, the DARA-borrower's risk premium decreases when he gets wealthier. Therefore, *ceteris paribus*, the potential investment distortion is less severe for the DARA-borrower as his personal wealth grows.

Let $\widehat{RP}$ and RP denote the risk premium of the DARA- and the CARA-borrower's in the second-best loan contract, respectively. Furthermore, let $\widehat{EU}_{SB}$ (EU_{SB}), $\widehat{RU}$ (RU), $\widehat{ID}$ (ID) denote the DARA (CARA)-borrower's expected utility from the second-best loan contract, his *reservation utility*, which is defined as his utility from consuming his personal wealth, and whether investment distortion is present. The parameter values follow those in Chapter 3 except for $X_H = 310$.[2]

Table 4-1 compares the incidence of investment distortion of the DARA-borrower with those of the CARA-borrower. In general, Table 4-1 indicates that investment distortion can be mitigated by changing the borrower's risk preference. In particular, when the borrower's personal wealth is $W = 50$, Case 1 implies that no solution exists for the DARA-borrower; therefore, investment distortion is present. Furthermore, even though the loan agreement is feasible for the CARA-borrower, investment distortion still occurs because borrowing is too risky for the CARA-borrower to engage in. Therefore, Case

[2] Assumption 1 in Chapter 3 is satisfied: $\Pi_H - a_H > \Pi_L \geq 0$ for each cases in Table 4-1.

Table 4-1. Investment Distortion for Aggregate Consumption

$Case$	W	a_H	$\widehat{EU}_{SB}$	$\widehat{RP}$	$\widehat{RU}$	$\widehat{ID}$	EU_{SB}	RP	RU	ID
1	50	85	N/A	N/A	7.1	Yes	$-.8$	49.3	$-.6$	Yes
2	135	95	11.5	19.3	11.6	Yes	$-.5$	78.8	$-.3$	Yes
3	200	95	14.4	8.7	14.1	No	$-.3$	78.8	$-.1$	Yes

1 represents investment distortion regardless of the borrower's risk preference although the underlying causes are different. For Case 2, the borrower's personal wealth level is $W = 135$, the loan arrangement is feasible regardless of the borrower's attitude towards risk, but investment distortion is present for both types of borrowers. Case 3 increases the borrower's personal wealth holdings to $W = 200$, and shows that the DARA-borrower experiences no investment distortion while the CARA-borrower still encounters investment distortion. Note that for the CARA-borrower the risk premium remains invariant with the increase in the personal wealth; this is consistent with the CARA-borrower's risk preference. For the DARA-borrower, the increase in the borrower's personal wealth lowers his risk premium, which leads him to be more likely to prefer investing by borrowing from the lender. Therefore, the presence of the DARA-borrower weakly mitigates the potential investment distortion.

4.2.2 Time-Additive Consumption Preference

In this section the borrower's concern for consumption smoothing is introduced in the second-best loan agreement and the associated potential investment distortion is investigated.

The DARA-borrower with time-additive consumption preference solves the following optimization problem:

[Program SB^{DA}]

$$\max_{\widehat{C}_0^A \geq 0, \widehat{C}_H^A \geq a_H, \widehat{C}_L^A \geq a_H} \sqrt{\widehat{C}_0^A} + p_H \sqrt{\widehat{C}_H^A - a_H} + (1 - p_H) \sqrt{\widehat{C}_L^A - a_H}$$

$$\text{s.t. } \widehat{C}_0^A + p_H \widehat{C}_H^A + (1 - p_H) \widehat{C}_L^A \leq W + \Pi_H$$

$$p_L \sqrt{\widehat{C}_H^A} + (1 - p_L) \sqrt{\widehat{C}_L^A} \leq p_H \sqrt{\widehat{C}_H^A - a_H} + (1 - p_H) \sqrt{\widehat{C}_L^A - a_H}$$

Recall that in Chapter 3 it is sometimes the violation of the limited liability for the CARA-borrower consumption at $t = 1$ when low cash is realized (C_L^A) that leads to investment distortion. For the DARA-borrower with time-additive consumption preference, it is the limited liability on his consumption at $t = 0$ $(\widehat{C}_0^A)$ that is more problematic. For the CARA-borrower, his expected utility is maximized when he has smoothing utility; therefore, given his risk preference, he intends to smooth utilities if not consumption across time. It follows that it is rarely the case that the limited liability of the CARA-borrower's $t = 0$ consumption (C_0^A) is violated because it works against the utility smoothing purpose. However, for the DARA-borrower, utility smoothing is no longer desirable for him if he has time-additive consumption preference. The reason is that the DARA-borrower's attitude towards risk changes with his personal wealth level, since his $t = 1$ consumption is risky, the rational DARA-borrower, in equilibrium, moves his consumption forward in time to reduce the risk premium he faces at $t = 1$. To better investigate the DARA-borrower's tendency to move consumption forward in time to reduce the risk premium in period $t = 1$, let $\widehat{CE}_t$ denote the DARA-borrower's local certainty equivalent consumption and $\widehat{RP}_t$ denote the local risk premium at time $t = 0, 1$, respectively. Given the definitions of certainty equivalent consumption and risk premium, the following relationships are present

$$\begin{aligned}
\sqrt{\widehat{CE}_1 - a_H} &= p_H \sqrt{\widehat{C}_H^A - a_H} + (1 - p_H) \sqrt{\widehat{C}_L^A - a_H} \\
\widehat{RP}_1 &= E\left(\widehat{C}_1\right) - \widehat{CE}_1
\end{aligned}$$

The following numerical example is now provided with that of the CARA-borrower as a comparison. The borrower's personal cost of supply high effort is $a_H = 60$ and other parameters in Table 4-2 follow those in Table 4-1. Let $\triangle\widehat{CE}$ ($\triangle CE$) and $\triangle\widehat{NCE}$ denote the DARA (CARA)-borrower's difference in local certainty equivalent consumption and the DARA-borrower's difference in net local certainty equivalent consumption, respectively.

The italic column in Table 4-2 indicates that the DARA-borrower's $t = 1$ local risk premium systematically decreases with the increase of his personal wealth holdings. However, for the CARA-borrower, his local risk premium is invariant with his personal wealth holdings. Specifically, for CARA-borrower, any increase of his personal wealth is divided equally into two parts, the first half goes to his $t = 0$ consumption C_0 and the second half goes to his expected $t = 1$ consumption, which is $E(C_1)$.[3] ,[4] Furthermore, the CARA-borrower's local certainty consumption differences are equal and independent of his personal wealth holdings. This is reflected by the fact that

$$\triangle CE = 60 = a_H$$

This is also consistent with the CARA-borrower's intention to smooth utility across time.

However, for the DARA-borrower, smoothing utility is not present. This can be seen from the fact that

$$\triangle\widehat{CE} = \widehat{CE}_1 - \widehat{CE}_0 \neq a_H$$

Furthermore, column $\triangle\widehat{NCE}$, where

$$\triangle\widehat{NCE} = \left(\widehat{CE}_1 - a_H\right) - \widehat{CE}_0$$

[3] Note that equally allocating wealth across periods is only feasible for the CARA-borrower when interior solution is present.

[4] The borrower's consumption at $t = 0$ is not shown in the table.

Table 4-2. Local Certainty Equivalent Consumptions and Local Risk Premium

	DARA					CARA			
W	$E(\widehat{C}_1)$	$\widehat{CE}_1$	$\widehat{RP}_1$	$\triangle\widehat{CE}$	$\triangle\widehat{NCE}$	$E(C_1)$	CE_1	RP_1	$\triangle CE$
50	152.6	137.3	*15.3*	127.9	67.9	119.4	102.6	16.8	60
51	152.7	137.5	*15.2*	127.2	67.2	119.9	106.1	16.8	60

is decreasing with the growth of his personal wealth holdings. This implies that the DARA-borrower's required local certainty equivalent consumption at $t = 1$, where the risk premium is necessary to motivate him to supply high effort, is decreasing in magnitude with respect to his local certainty equivalent consumption at $t = 0$, where no risk premium is present. This means that the DARA-borrower requires less risk premium when his personal wealth holdings grow, which corresponds to the local risk premium $\widehat{RP}_1$ column.

In sum, when the borrower has time-additive consumption preference, the DARA-borrower moves his consumption forward in time to reduce the local risk premium associated with unobservable effort supply at $t = 1$ as his personal wealth grows, while the CARA-borrower has incentives to equally allocate the increase of his personal wealth across each consumption periods to smooth intertemporal consumption and retains the unchanged local risk premium.

Although the DARA-borrower's optimal consumption for the second-best loan agreement is difficult to derive because of the domain-additive cost preference, the following numerical is designed to show the difference in potential investment distortion for the DARA- and the CARA-borrower, respectively.

Let $a_H = 80$ and keep the other parameters the same as in the previous numerical examples. Table 4-3 compares the incidents of investment distortion for the DARA- and CARA-borrower when both have time-additive consumption preference. Denote $\widehat{EU}^A_{SB}$ (EU^A_{SB}), $\widehat{RP}^A$ (RP^A), $\widehat{RU}^A$ (RU^A), and $\widehat{ID}^A$ (ID) the DARA- (CARA-) borrower's

Table 4-3. Investment Distortion for Time-Additive Consumption

$Case$	W	$\widehat{EU}^A_{SB}$	$\widehat{RP}^A$	$\widehat{RU}^A$	$\widehat{ID}^A$	EU^A_{SB}	RP^A	RU^A	ID
1	100	13.1	21	14.1	Yes	-1.3	39.8	-1.2	Yes
2	150	17.9	15.6	17.3	No	$-.98$	39.8	$-.94$	Yes

expected utility from the second-best loan contract, his total risk premium associated with the contract,[5] his reservation utility, and whether investment distortion is present.

In Table 4-3, Case 1 implies investment distortion for both risk preferences while Case 2 displays a difference. First, the CARA-borrower still experiences investment distortion even though his personal wealth has grown, this is consistent with the conclusion in Chapter 3 that the CARA-borrower's risk preference does not vary with his personal wealth holdings; therefore, given that he has investment distortion to begin with, he would encounter the same investment distortion regardless of the change in the magnitude of his personal wealth. However, as the personal wealth increases, the DARA-borrower no longer experiences investment distortion; this is because his attitude towards risk changes with the increase of his personal endowment. As the DARA-borrower becomes wealthier, he is less risk-averse, which can be inferred from the decreasing risk premium; this follows that he is more likely to undertake the loan arrangement that would otherwise be too risky were he to have less personal wealth.

4.3 Consumption Preference and Investment Distortion

In Chapter 3, the investment distortion problem for the CARA-borrower shows that investment distortion under aggregate consumption preference implies investment distortion under time-additive consumption preference. In this chapter, although the closed-form expressions of the DARA-borrower's optimal consumption with respect

[5] Since the borrower only exerts effort at $t = 1$, the total risk premium is in fact the local risk premium at $t = 1$.

to the second-best loan contract are difficult to derive, the implication of investment distortion for the CARA-borrower retains. Intuitively, for the DARA-borrower with aggregate consumption preference, were the second-best loan agreement to be feasible, the importance of the contract would be to satisfy the borrower's incentive compatibility constraint. For the borrower with time-additive consumption preference, both types of borrowers save for the future, but for different reasons. The CARA-borrower reserves for the $t = 1$ consumption in an attempt to have smoothing utilities. However, it is not optimal for him to save as much as he can at $t = 0$ because doing so works against the smoothing purpose. The DARA-borrower prefers shifting his consumption forward in time to reduce the risk premium at $t = 1$, not smoothing utilities. Therefore, for the DARA-borrower, time-additive consumption preference not only requires him to ensure the feasibility of the second-best loan contract, but also leads him to 1) reduce the local risk premium $\widehat{RP}_1$ at $t = 1$ and (2) not violate the limited liability constraint on $\widehat{C}_0^A$ while shifting consumption forward in time. In other words, the DARA-borrower with time-additive consumption preference bears extra burdens in the second-best loan agreement; then *ceteris paribus*, the potential investment distortion is exacerbated when the DARA-borrower switches from aggregate consumption preference to time-additive consumption preference.

Let the DARA-borrower's personal wealth be fixed at $W = 140$. Table 4-4 compares his investment decisions between borrowing and no-investment when he has either aggregate or time-additive consumption preference.

Given the same personal wealth level, Table 4-4 clearly shows that investment distortion for the borrower with aggregate consumption preference implies investment distortion for the same borrower but with time-additive consumption preference. When the borrower's personal cost of effort is $a_H = 80$, no investment distortion is present for either consumption preference. However, as a_H grows to 98, investment distortion occurs for both consumption preferences. Moreover, the potential investment distortion

Table 4-4. Second-Best Contracts Investment Distortion for the DARA-Borrower

a_H	$\widehat{EU}_{SB}$	$\widehat{RU}$	$\widehat{ID}$	$\widehat{EU}_{SB}^{A}$	$\widehat{RU}^{A}$	$\widehat{ID}^{A}$
80	12.8	11.8	No	17.2	16.7	No
90	12.2	11.8	No	15.6	16.7	Yes
98	11.5	11.8	Yes	13	16.7	Yes

problem is exacerbated when the borrower has time-additive consumption preference. This is illustrated by $a_H = 90$, and investment distortion is only present for the borrower with time-additive consumption preference. Closer investigation reveals that the DARA-borrower's utility differential when he has time-additive consumption preference is increasing as his personal cost of effort grows, i.e., for $a_H = 80$ (90), the utility differential $EU\left(\widehat{C}_1\right) - EU\left(\widehat{C}_0^A\right) = 3.9$ (6.1). This is because as the borrower's personal cost of effort increases, the local risk premium $\widehat{RP}_1$ increases as well. The borrower has more incentive to delay consumption forward in time in an attempt to reduce $\widehat{RP}_1$, which widens his period-by-period utility even more.

If the borrower is wealth-abundant and self-finances the investment project, similar logic follows. In other words, since the borrower with time-additive consumption preference faces more hurdles with respect to the investment behavior, the potential investment distortion problem is worsened as the consumption preference emphasizes intertemporal consumption.

CHAPTER 5
PRIVATE PERSONAL WEALTH

Both Chapter 3 and Chapter 4 discuss the borrower's consumption and investment behavior under the assumption that the borrower's personal wealth is publicly observable but effort supply is unobservable. This chapter relaxes this assumption and focuses on the scenario where the borrower's personal wealth is private information while his effort supply is also private. Then the problem becomes an entanglement of moral hazard and adverse selection.

The analysis is focused on the CARA-borrower with aggregate consumption preference. The timeline is revised as follows: at time $t = 0$, a menu of contracts which specifies the CARA-borrower's consumption and the lender's investment and payoff from the project is offered. The CARA-borrower subsequently observes his personal wealth $W \in \{W_H, W_L\}$ while the lender remains uninformed. However, the lender requires the CARA-borrower report his personal wealth $\widetilde{W} \in \left\{\widetilde{W}_H, \widetilde{W}_L\right\}$ and post an up-front bond in the amount of his report. Let

$$\underline{W} = \frac{p_H}{\rho} \ln \Omega - \Pi_H$$

denote the minimum required personal wealth for the CARA-borrower to hold to ensure the existence of the loan contract. Other than the lender, the CARA-borrower does not have any other personal banking access, which implies that the CARA-borrower cannot borrow to post a bond that is higher than his observed personal wealth. Therefore, the CARA-borrower's only misreporting strategy is to understate his personal wealth. The CARA-borrower then chooses his effort supply of $a \in \{a_H, a_L\}$ and investment is made. Finally, at time $t = 1$, the investment outcome is realized and the CARA-borrower's share of consumption and the lender's share of payoff from investment are honored. The CARA-borrower's private information does not substantially change the relationship between him and the lender, in particular, the following three aspects apply: (1) the communication between two parties is not blocked and it is costless, (2) the lender does not change the rules of the loan contract after the report of the CARA-borrower's private information, i.e., the lender has perfect commitment; and (3) the CARA-borrower's consumption has no restriction on its form. These aspects indicate the Revelation Principle applies.[1]

Since neither the CARA-borrower nor the lender have information about the personal wealth at time the menu of contracts is offered, the lender's investment and payoff and the CARA-borrower's share of consumption all depend on the reported personal wealth $\widetilde{W}$. Although the CARA-borrower's personal wealth is private information, it is assumed that his effort supply a does not vary with his wealth holdings. Based on Assumption 1 in Chapter 3, the CARA-borrower is motivated to work diligently, and his objective is to maximize his expected utility from the loan contract while supplying high personal effort. However, his expected utility depends on the odds of his type. Let α be the probability that the CARA-borrower is wealthy and $(1-\alpha)$ be the odds that he is poor, where $\alpha \in (0,1)$. Since the Revelation Principle applies, the loan agreement focuses on a truthful direct mechanism.

We use the following notation to denote the choice variables in the CARA-borrower's optimizing program.

$C\left(\widetilde{W}_H\right) \in \left\{C_H\left(\widetilde{W}_H\right), C_L\left(\widetilde{W}_H\right)\right\}$	Consumption when reports $\widetilde{W}_H$
$C\left(\widetilde{W}_L\right) \in \left\{C_H\left(\widetilde{W}_L\right), C_L\left(\widetilde{W}_L\right)\right\}$	Consumption when reports $\widetilde{W}_L$
$D_0\left(\widetilde{W}\right)$	Lender's investment
$D\left(\widetilde{W}\right) \in \left\{D_H\left(\widetilde{W}\right), D_L\left(\widetilde{W}\right)\right\}$	Lender's payoff

where $\widetilde{W} \in \left\{\widetilde{W}_H, \widetilde{W}_L\right\}$.

[1] Harris and Townsend (1985) and Laffont and Martimort (2001) have detailed discusses of Revelation principle.

Let $EU\left(W^{RP}|a_H\right)$ denote the CARA-borrower's expected utility from the loan contract under a truth-telling mechanism. It follows that

$$EU\left(W^{RP}|a_H\right) = \alpha EU\left[C\left(\widetilde{W}_H\right)\right] + (1-\alpha)\,EU\left[C\left(\widetilde{W}_L\right)\right]$$

The lender's individual rationality constraint ensures that she breaks even for each wealth type of the CARA-borrower, which is described as

$$D_0\left(\widetilde{W}\right) \le ED\left(\widetilde{W}\right) \tag{5–1}$$

where $\widetilde{W} \in \left\{\widetilde{W}_H, \widetilde{W}_L\right\}$.

Once the CARA-borrower knows his wealth type, he makes reporting and effort supply choices. Let $EU\left(W, \widetilde{W}, a\right)$ denote the CARA-borrower's expected utility when he observes his personal wealth to be $W \in \{W_H, W_L\}$, but chooses to report $\widetilde{W} \in \left\{\widetilde{W}_H, \widetilde{W}_L\right\}$ and supply an effort of $a \in \{a_H, a_L\}$. For each of his wealth types, he has the following feasible strategies and the associated expected utilities:

Type	Report	Effort	$EU\left(W, \widetilde{W}, a\right)$
W_H	Truthful	a_H	$EU\left(W_H, \widetilde{W}_H, a_H\right)$
W_H	Misreport	a_H	$EU\left(W_H, \widetilde{W}_L, a_H\right)$
W_H	Truthful	a_L	$EU\left(W_H, \widetilde{W}_H, a_L\right)$
W_H	Misreport	a_L	$EU\left(W_H, \widetilde{W}_L, a_L\right)$
W_L	Truthful	a_H	$EU\left(W_L, \widetilde{W}_L, a_H\right)$
W_L	Truthful	a_L	$EU\left(W_L, \widetilde{W}_L, a_L\right)$

Note that since the CARA-borrower cannot overstate his personal wealth, misreporting is not a feasible reporting strategy when he observes his wealth type to be W_L. In order to implement a truthful direct mechanism, the CARA-borrower is subject to the following set of incentive compatibility constraints:

$$EU\left(W_H, \widetilde{W}_H, a_H\right) \geq EU\left(W_H, \widetilde{W}_L, a_H\right) \tag{5–2}$$

$$EU\left(W_H, \widetilde{W}_H, a_H\right) \geq EU\left(W_H, \widetilde{W}_H, a_L\right) \tag{5–3}$$

$$EU\left(W_H, \widetilde{W}_H, a_H\right) \geq EU\left(W_H, \widetilde{W}_L, a_L\right) \tag{5–4}$$

$$EU\left(W_L, \widetilde{W}_L, a_H\right) \geq EU\left(W_L, \widetilde{W}_L, a_L\right) \tag{5–5}$$

In Chapter 3, the CARA-borrower's consumption is upper-bounded by the sum of his remained wealth after putting his share of investment in the project and the cash flow from the investment net the lender's payoff. These consumption constraints also apply in this chapter:

$$C_H\left(\widetilde{W}_H\right) \leq \widetilde{W}_H - \left[I - D_0\left(\widetilde{W}_H\right)\right] + X_H - D_H(\widetilde{W}_H) \tag{5–6}$$

$$C_L\left(\widetilde{W}_H\right) \leq \widetilde{W}_H - \left[I - D_0\left(\widetilde{W}_H\right)\right] + X_L - D_L(\widetilde{W}_H) \tag{5–7}$$

$$C_H\left(\widetilde{W}_L\right) \leq \widetilde{W}_L - \left[I - D_0\left(\widetilde{W}_L\right)\right] + X_H - D_H(\widetilde{W}_L) \tag{5–8}$$

$$C_L\left(\widetilde{W}_L\right) \leq \widetilde{W}_L - \left[I - D_0\left(\widetilde{W}_L\right)\right] + X_L - D_L(\widetilde{W}_L) \tag{5–9}$$

The CARA-borrower with private information then solves the following problem with respect to the loan contract:

[Program $\widetilde{SB}$]

$$\max_{C\left(\widetilde{W}\right)\geq 0, D_0\left(\widetilde{W}\right), D\left(\widetilde{W}\right)} EU\left(W^{RP}|a_H\right)$$

$$\text{s.t.}(5\text{–}1)$$

$$(5\text{–}2)\text{-}(5\text{–}5)$$

$$(5\text{–}6)\text{-}(5\text{–}9)$$

Before the formal model is introduced, two benchmark cases are briefly discussed. The first benchmark is when the CARA-borrower's personal wealth is publicly observable; the analysis is then the same as the second-best loan contract discussed in Chapter 3, where the only private information is the CARA-borrower's effort supply. The second benchmark is when the CARA-borrower's personal wealth remains private but his effort supply is public. Under this scenario the risk-neutral lender can buy the investment project from the CARA-borrower and provide him a riskless consumption schedule; therefore, the first-best contract in Chapter 3 applies and the CARA-borrower's private personal wealth becomes irrelevant.

Define the "*naive contract*" as the optimal loan contract the CARA-borrower would achieve if his personal wealth were public; i.e., the optimal second-best loan agreement in Chapter 3 is the naive contract. The following conclusion is developed for [Program $\widetilde{SB}$]:

Proposition 9. *The naive contract is optimal for the CARA-borrower with aggregate consumption preference when he has private information about his personal wealth.*

Intuitively, whenever the CARA-borrower decides to understate his personal wealth, he consumes his concealed wealth privately. However, the feature of the naive contract is that any difference in the CARA-borrower's public personal wealth is reflected by a linear mapping onto his consumption; i.e., for any $W_H > W_L$, the difference in the CARA-borrower's consumption with the naive contract features $W_H - W_L$. Applying the naive contract to [Program $\widetilde{SB}$], we see the CARA-borrower does not benefit by the privately consumed wealth compared to what he would achieve were he to report truthfully. Recall that in Chapter 3, once the CARA-borrower has sufficient personal wealth to ensure the existence of the loan contract, his consumption from the loan contract is a linear function of his personal wealth. In other words, the CARA-borrower's personal wealth is "quasi-irrelevant" in the sense that it is not important other than determining the feasibility of the loan contract (i.e., condition 3–8 in Chapter 3). When the CARA-borrower has private information about his personal wealth, his reporting strategy is restricted to common knowledge wealth possibilities; therefore, given the quasi-irrelevance of the personal wealth, the optimal consumption if W were public would also be optimal were W to be private for the CARA-borrower.

Let the CARA-borrower's personal cost of effort be $a_H = 30$ and his binary personal wealth take the value of either $W_L = 140$ or $W_H = 300$. The rest of the parameter values follow those in Chapter 3.[2] The probability of the CARA-borrower being wealthy, i.e., he has a personal wealth of 300 is assumed to be $\alpha = .7$. The optimal contract for [Program $\widetilde{SB}$] shows a solution:

$$\begin{aligned} C_H\left(\widetilde{W}_H\right) &= 456 \text{ and } C_L\left(\widetilde{W}_H\right) = 403 \\ C_H\left(\widetilde{W}_L\right) &= 296 \text{ and } C_L\left(\widetilde{W}_L\right) = 243 \end{aligned}$$

with an expected utility of $-.0374$. This consumption schedule coincides with the naive contract in Chapter 3 when the CARA-borrower's public personal wealth is 300 and 140, respectively. Furthermore, note that the consumption differences between reporting truthfully (i.e., 300) and misreporting (i.e., 140) is the difference between the reported wealth (i.e., $300 - 140 = 160$). Therefore, by ignoring the CARA-borrower's private personal wealth source and offering him the consumption schedule that he would achieve if his wealth were public, the CARA-borrower has no incentive to misreport his wealth.

Chapter 3 demonstrates investment distortion arises when 1) no solution exists for the loan contract and 2) the loan agreement is too risky and the CARA-borrower prefers the status quo. Since the CARA-borrower's consumption behavior follows that in Chapter 3, the corresponding potential investment distortion problem is invariant to whether private information is present.

[2] That is, $p_H = .7$, $p_L =, 1$, $X_H = 350, X_L = 150$, $I = 150$.

If the CARA-borrower has time-additive consumption preference, Chapter 3's optimal contract features either interior or corner consumption. Let the naive contract indicate the interior consumption if the CARA-borrower's personal wealth is public. It turns out that the implication of Proposition 9 does not change, i.e., the naive contract is again optimal in the presence of privation information. Intuitively, since the CARA-borrower's effort supply is at $t = 1$, his incentive compatibility constraints associated with truthful reporting are not different from its aggregate preference counterpart because his $t = 0$ consumption is irrelevant. However, for the incentive compatibility constraint associated with misreporting, the CARA-borrower's $t = 0$ matters because it is not clear how he allocates his concealed wealth (i.e., $W_H - W_L$) across periods. Chapter 3 concludes that intertemporal utility smoothing is achievable with interior consumption and the CARA-borrower prefers smoothing to non-smoothing utility series; therefore, there is no loss of generality to assume that the CARA-borrower would equally allocate his concealed personal wealth across periods to ensure his expected utility from misreporting is maximized if his personal wealth were private. When equal allocation is applied, the characteristic of the CARA-borrower's incentive compatibility constraint for misreporting is equivalent to its aggregate preference counterpart. Then for the CARA-borrower with time-additive consumption, the naive contract would be optimal for the loan agreement if his wealth were private.

In Chapter 4, we find that it is difficult to derive closed-form consumption for the DARA-borrower because of the domain-additive cost structure, the same difficulty applies here. Unlike the CARA-borrower whose risk preference stays the same regardless of his personal wealth, the DARA-borrower changes his attitude towards risk based on his wealth. In particular, the more wealthier the DARA-borrower is, the less risk-averse he is, which implies that the risk premium associated with the loan contract is decreasing as his personal wealth increases. Therefore, the less personal wealth the DARA-borrower reports, the more risky loan agreement he faces. This implies that it is not in the

DARA-borrower's best interest to understate his personal wealth. Therefore, although the private information version of the DARA borrower remains an open issue at this point, the naive contract retains attractive truth incentives.

The borrower's misreporting strategy is one-directional in this chapter. However, over-reporting may be valuable to the borrower. In particular, when the borrower observes his personal wealth to be less than $\underline{W}$, the lender walks away from the loan contract. Therefore, from the borrower's perspective, if he has access to personal banking, he can borrow and over-report his personal wealth to exceeds $\underline{W}$ to secure the loan agreement and pay back the funds he borrows when the cash flows from the investment are realized.

CHAPTER 6
CONCLUSION

This dissertation characterizes a borrower's investment and consumption behaviors with respect to an investment project and investigates whether the borrower's consumption preference affects the investment decision. It is concluded that the project may be inefficiently abandoned, which results in investment distortion, and the investment distortion problem is worsened when the borrower focuses more on the intertemporal consumption than on aggregate consumption. The borrower's initial wealth endowment is important in determining the form of potential investment distortion. For the wealth-constrained borrower, the joint force of upper-bounded expected profit from the investment and the limited liability constraints determines the feasibility of the loan arrangement between the borrower and the lender, which, in turn, is conclusive about the possibility of investment distortion. The borrower's consumption preference has no effect on the occurrence of investment distortion. However, the wealth-abundant borrower not only focuses on the force noted above but also weighs the certainty equivalent cash flow of the investment and the entire personal cost of implementing such investment, which is the riskiness of the investment itself, to determine whether the project is undertaken. The borrower's intertemporal consumption preference is no longer irrelevant in the investment decision. The conditions under which investment distortion arises for the borrower with aggregate consumption preference imply those for the borrower with time-additive consumption preference but not vice versa, which suggests that the investment distortion problem is exacerbated when the wealth-abundant borrower has intertemporal consumption structure.

However, as the borrower's risk preference changes, the associated potential investment distortion problem differs. The DARA-borrower experiences less severe investment distortion than the CARA-borrower because the increase in his personal wealth makes him less risk averse; therefore, other things being equal, the loan contract between the DARA-borrower and the lender is less likely to fail. The DARA-borrower's risk preference not only affects the occurrence of the potential investment distortion, it also changes his intertemporal consumption behavior when he has time-additive consumption preference.

When the CARA-borrower has private information about his personal wealth, the Revelation Principle applies which motivates the CARA-borrower to truthfully report his private information. A naive contract which ignores the CARA-borrower's private source of information and treats it as public information is a feasible and optimal loan agreement. The private information case of the DARA-borrower remains an open issue, though the naive contract retains attractive truth incentives.

In closing, a few issues are raised. It is assumed that the borrower has all the bargaining power while the role of the lender is quite passive. In real-world situations, the borrower, especially the wealth-constrained borrower, tends to be the party that does not have market power in the financial market. Therefore, whether and how the results of this dissertation change under alternative bargaining structures is still an open but interesting question. Second, the possibility of over-reporting when the borrower's personal wealth is privately observed warrants investigation. Over-reporting strategy in the presence of private information is particularly valuable to the wealth-constrained borrower, by borrowing to post the required up-front bond, the borrower is able to secure the loan agreement and implement the investment project.

APPENDIX: PROOFS

Proof of Proposition 1 and Corollary 1.

Given [Program SB]

$$\max_{C_H \geq 0, C_L \geq 0} -(1-p_H)\exp\left[-\rho\left(C_L - a_H\right)\right] - p_H \exp\left[-\rho\left(C_H - a_H\right)\right]$$

$$\text{s.t. } p_H C_H + (1-p_H) C_L \leq W + \Pi_H \tag{A-1}$$

$$C_L + \frac{1}{\rho}\ln\Omega \leq C_H \tag{A-2}$$

where $\Omega = \frac{p_H \exp(\rho a_H) - p_L}{(1-p_L)-(1-p_H)\exp(\rho a_H)}$.

Rearranging terms of the constraints provides

$$C_H \geq C_L + \frac{1}{\rho}\ln\Omega \tag{A-3}$$

$$C_H \leq \frac{W + \Pi_H - (1-p_H) C_L}{p_H} \tag{A-4}$$

If $W + \Pi_H - \frac{p_H}{\rho}\ln\Omega < 0$, we have

$$\frac{W + \Pi_H}{p_H} < \frac{1}{\rho}\ln\Omega \tag{A-5}$$

Given the limited liability constraint that $C_L \geq 0$, if (A-5) holds, the constraint set of (A-3) and (A-4) is never satisfied; which by contradiction, suggests that there exists no feasible solution if $W + \Pi_H - \frac{p_H}{\rho}\ln\Omega < 0$.

Therefore, [Program SB] has feasible solution if

$$W + \Pi_H - \frac{p_H}{\rho}\ln\Omega \geq 0 \tag{A-6}$$

Assume condition (A-6) holds, the following analysis shows the existence of the solution and its characterizations.

Note that the objective function in [Program SB] is concave and continuous, given condition (A-6), the constraint set is compact, then based on the Weierstrass Theorem, there exists a solution for [Program SB].

Both constraints in [Program SB] are binding. If (A-1) is not binding, the lender is getting rent from her repayment, which contradicts a competitive financial market assumption. If (A-2) is not binding, the first-best contract consumption schedule $C_H = C_L$ applies, which leads to a violation of (A-2) for any $a_H > 0$. Then the solution for [Program SB] is solved from the following set of equations

$$\begin{aligned} p_H C_H + (1 - p_H) C_L &= W + \Pi_H \\ C_L + \frac{1}{\rho} \ln \Omega &= C_H \end{aligned}$$

which is

$$C_H = W + \Pi_H + \frac{(1 - p_H)}{\rho} \ln \Omega \tag{A-7}$$

$$C_L = W + \Pi_H - \frac{p_H}{\rho} \ln \Omega \tag{A-8}$$

With solution (A-7) and (A-8), the first-order conditions with respect to C_H and C_L are both zero, which implies that (A-7) and (A-8) are interior solutions. QED.

Proof of Proposition 2.

Part a directly follows from the proof of Proposition 1 and Corollary 1.

Part b. Let CE_{SB} and CE_{RU} denote the borrower's certainty equivalent consumption from the second-best contract and from consuming his personal wealth at hand, respectively. Given Proposition 1's optimal second-best contract, CE_{SB} and CE_{RU} are characterized as

$$CE_{SB} = W + \Pi_H - \frac{p_H}{\rho} \ln \Omega - \frac{1}{\rho} \ln \Phi \tag{A-9}$$

$$CE_{RU} = W \tag{A-10}$$

where $\Phi = \frac{p_H - p_L}{p_H \exp(\rho a_H) - p_L}$.

Since the borrower incurs personal cost of effort when the investment project is undertaken, for the borrower to decide between investment and no-investment, the *net* certainty equivalent from investing is compared with his personal wealth.

If $\Pi_H - a_H < \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi$, (A-9) and (A-10) imply that

$$CE_{SB} - a_H < CE_{RU}$$

which is an indication of investment distortion. If $\Pi_H - a_H \geq \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi$, again from (A-9) and (A-10),

$$CE_{SB} \geq CE_{RU}$$

Therefore, $\Pi_H - a_H < \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi$ is a sufficient and necessary condition to induce investment distortion. QED.

Proof of Proposition 3.

Part a. If the borrower chooses no-lender investment, he forces his consumption to be

$$\begin{aligned} C_H &= W - I + X_H \\ C_L &= W - I + X_L \end{aligned}$$

and his expected utility from no-lender investment becomes

$$-p_H \exp\left[-\rho\left(W - I + X_H - a_H\right)\right] - \left(1 - p_H\right) \exp\left[-\rho\left(W - I + X_L - a_H\right)\right]$$

However, because of the limited liability constraint on the borrower's consumption, it is also required that

$$W - I + X_L \geq 0$$

otherwise, no-lender investment is not feasible

Let CE_{NL} denote the borrower's certainty equivalent consumption under no-lender investment. Together with the above consumption, it follows that

$$CE_{NL} = W - I + CE(X) \tag{A-11}$$

where $CE(X) = -\frac{1}{\rho}\ln\left[(1-p_H)\exp(-\rho X_L) + p_H\exp(-\rho X_H)\right]$.

Given

$$CE_{RU} = W$$

from the proof of Proposition 2, we compare no-lender investment with no-investment. Note that the personal cost of effort is considered for the former. Then, if $CE(X) < I + a_H$, based on (A-11),

$$CE_{NL} - a_H < CE_{RU}$$

Conversely, if $CE(X) \geq I + a_H$,

$$CE_{NL} - a_H \geq CE_{RU}$$

Therefore, condition $CE(X) < I + a_H$ is sufficient and necessary that results the borrower prefers no investment to no-lender investment.

Part b directly follows from the proof of part b of Proposition 2. QED.

Proof of Corollary 2.

Given (A-9), (A-10), and (A-11), and taking the borrower's personal cost of effort into consideration, if

$$\Pi_H - a_H < \frac{p_H}{\rho}\ln\Omega + \frac{1}{\rho}\ln\Phi < \Pi_H + I - CE(X)$$

then

$$CE_{RU} > CE_{SB} - a_H > CE_{NL} - a_H$$

QED.

Proof of Proposition 4 and Corollary 3.

Given [Program SB^A]

$$\max_{C_0^A \geq 0, C_H^A \geq 0, C_L^A \geq 0} - \exp(-\rho C_0^A) - p_H \exp\left[-\rho\left(C_H^A - a_H\right)\right] - (1-p_H)\exp\left[-\rho\left(C_L^A - a_H\right)\right]$$

$$\text{s.t. } C_0^A + p_H C_H^A + (1-p_H)\, C_L^A \leq W + \Pi_H \tag{A-12}$$

$$C_L^A + \frac{1}{\rho}\ln\Omega \leq C_H^A \tag{A-13}$$

where $\Omega = \frac{p_H \exp(\rho a_H) - p_L}{(1-p_L)-(1-p_H)\exp(\rho a_H)}$.

Rearranging terms of the constraints provides

$$C_H^A \geq C_L^A + \frac{1}{\rho}\ln\Omega \tag{A-14}$$

$$C_H^A \leq \frac{W + \Pi_H - C_0^A - (1-p_H)\, C_L^A}{p_H} \tag{A-15}$$

If $W + \Pi_H - \frac{p_H}{\rho}\ln\Omega < 0$, we have

$$\frac{W + \Pi_H}{p_H} < \frac{1}{\rho}\ln\Omega \tag{A-16}$$

Given the limited liability constraint that $C_0^A, C_L^A \geq 0$, if (A-16) holds, the constraint set (A-14) and (A-15) is never satisfied, which by contradiction, suggests that there exists no feasible solution if $W + \Pi_H - \frac{p_H}{\rho}\ln\Omega < 0$.

Therefore, [Program SB^A] has feasible solution if

$$W + \Pi_H - \frac{p_H}{\rho}\ln\Omega \geq 0 \tag{A-17}$$

Assume condition (A-17) holds, the following analysis shows the existence of the solution and its characterizations.

Note that the objective function in [Program SB^A] is concave and continuous, given condition (A-17), the constraint set is compact, then based on the Weierstrass Theorem, there exists a solution for [Program SB^A].

Note that $\Omega > 1$; it then follows from the borrower's IC^A constraint that

$$C_H^A \geq C_L^A + \frac{1}{\rho}\ln\Omega > 0$$

Therefore, the limited liability for C_H^A is not binding.

In equilibrium, both constraints in [Program SB^A] are binding. If (A-12) is not binding, the lender is getting rent from her repayment, which contradicts a competitive financial market assumption. If (A-13) is not binding, the first-best contract consumption schedule $C_0^A = C_H^A - a_H = C_L^A - a_L$ applies, which leads to a violation of (A-13) for any $a_H > 0$. Therefore, we have

$$C_L^A + \frac{1}{\rho}\ln\Omega = C_H^A \tag{A-18}$$

$$C_0^A + p_H C_H^A + (1 - p_H)\, C_L^A = W + \Pi_H \tag{A-19}$$

Substituting C_H^A in (A-18) into (A-19) and solving for C_0^A

$$C_0^A = W + \Pi_H - \frac{p_H}{\rho}\ln\Omega - C_L^A \tag{A-20}$$

Given (A-18) and (A-20), the borrower's period $t = 0$ expected utility is

$$-\exp\left[-\rho\left(W + \Pi_H - \frac{p_H}{\rho}\ln\Omega - C_L^A\right)\right] \tag{A-21}$$

and his expected utility for $t = 1$ is

$$-p_H \exp\left[-\rho\left(C_L^A + \frac{1}{\rho}\ln\Omega - a_H\right)\right] - (1 - p_H)\exp\left[-\rho\left(C_L^A - a_H\right)\right] \tag{A-22}$$

Substituting, the borrower's problem becomes

[Program SB$^{A'}$]

$$\max_{C_L^A \geq 0} -\exp\left[-\rho\left(W + \Pi_H - \frac{p_H}{\rho}\ln\Omega - C_L^A\right)\right]$$
$$-(1 - p_H)\exp\left[-\rho\left(C_L^A - a_H\right)\right] - p_H\exp\left[-\rho\left(C_L^A + \frac{1}{\rho}\ln\Omega - a_H\right)\right]$$

Let $L^{'}_{C^A_L}$ denote the first-order derivative of C^A_L with respect to the Lagrangian function of [Program SB$^{A'}$], it follows that

$$L^{'}_{C^A_L} \leq 0 \tag{A-23}$$

$$C^A_L L^{'}_{C^A_L} = 0 \tag{A-24}$$

where (A-24) is the complementary slackness condition. Solving (A-23) and (A-24) yields the optimal solution of C^A_L :

If $W + \Pi_H - \frac{p_H}{\rho} \ln \Omega + a_H + \frac{1}{\rho} \ln \Phi \geq 0$

$$C^A_L = \frac{W + \Pi_H - \frac{p_H}{\rho} \ln \Omega + a_H + \frac{1}{\rho} \ln \Phi}{2}, L^{'}_{C^A_L} = 0 \tag{A-25}$$

Otherwise

$$C^A_L = 0, L^{'}_{C^A_L} < 0 \tag{A-26}$$

where $\Phi = \frac{p_H - p_L}{p_H \exp(\rho a_H) - p_L}$.

Therefore, (A-25) and (A-26) are the interior and corner solutions for C^A_L, respectively. Substituting C^A_L in (A-25) into (A-21) and (A-22),

$$(A-21) = (A-22)$$

Moreover, substituting C^A_L in (A-25) into (A-19) and (A-20), optimal consumption for [Program SB^A] is

$$\begin{aligned} C^A_0 &= \frac{W + \Pi_H - \frac{p_H}{\rho} \ln \Omega - a_H - \frac{1}{\rho} \ln \Phi}{2} \\ C^A_H &= \frac{W + \Pi_H - \frac{p_H}{\rho} \ln \Omega + a_H + \frac{1}{\rho} \ln \Phi}{2} + \frac{1}{\rho} \ln \Omega \\ C^A_L &= \frac{W + \Pi_H - \frac{p_H}{\rho} \ln \Omega + a_H + \frac{1}{\rho} \ln \Phi}{2} \end{aligned}$$

Note that $a_H + \frac{1}{\rho} \ln \Phi$ is non-positive, therefore, $W + \Pi_H - \frac{p_H}{\rho} \ln \Omega + a_H + \frac{1}{\rho} \ln \Phi \geq 0$, the above consumption is an *interior* solution. Moreover, substituting C^A_L in (A-26) into

(A-19) and (A-20), the optimal *corner* consumption for [Program SB^A] is

$$C_0^A = W + \Pi_H - \frac{p_H}{\rho}\ln\Omega$$
$$C_H^A = \frac{1}{\rho}\ln\Omega$$
$$C_L^A = 0$$

Note that for the above consumption schedule, given (A-17), the borrower's limited liability constraints are all satisfied. Moreover, the first-order condition with respect to C_0^A for [Program SB^A] is 0; which implies that the optimal C_0^A is an interior solution. QED.

Proof of Corollary 4.

The conditions under which investment distortion arises for the borrower with time-additive consumption preference are restated:

a. $W + \Pi_H - \frac{p_H}{\rho}\ln\Omega < 0$

b. $\Pi_H - a_H < \frac{p_H}{\rho}\ln\Omega + \frac{1}{\rho}\ln\Phi$, where $\Omega = \frac{p_H\exp(\rho a_H) - p_L}{(1-p_L)-(1-p_H)\exp(\rho a_H)}$ and $\Phi = \frac{p_H - p_L}{p_H\exp(\rho a_H) - p_L}$.

The proof of Part a follows directly from the proof of Proposition 4 and Corollary 3.

Part b. Prove sufficiency. Given $\Pi_H - a_H < \frac{p_H}{\rho}\ln\Omega + \frac{1}{\rho}\ln\Phi$, we have

$$\frac{W + \Pi_H - \frac{p_H}{\rho}\ln\Omega - a_H - \frac{1}{\rho}\ln\Phi}{2} < \frac{W}{2} \tag{A-27}$$

Note that the left-hand side of (A-27) is the borrower's $t = 0$ interior consumption; then it follows that

$$-\exp(-\rho C_0^A) < -\exp(-\rho\frac{W}{2}) \tag{A-28}$$

When the second-best contract provides interior consumption, given Corollary 3,

$$-\exp(-\rho C_0^A) = -p_H\exp\left[-\rho\left(C_H^A - a_H\right)\right] - (1-p_H)\exp\left[-\rho\left(C_L^A - a_H\right)\right] \tag{A-29}$$

From (A-28) and (A-29), we have

$$-\exp(-\rho C_0^A) - p_H\exp\left[-\rho\left(C_H^A - a_H\right)\right] - (1-p_H)\exp\left[-\rho\left(C_L^A - a_H\right)\right] < -2\exp(-\rho\frac{W}{2})$$

Note that the left-hand side is the borrower's expected utility from the loan contract and the right-hand side is his reservation utility; therefore, given $\Pi_H - a_H < \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi$, investment distortion occurs for second-best interior consumption.

When the second-best loan contract provides corner consumption, given the concavity of the borrower's utility function, his expected utility is strictly lower than that under interior consumption. Therefore, *ceteris paribus*, if $\Pi_H - a_H < \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi$, investment distortion also arises for second-best corner consumption.

Prove necessary: We argue that given

$$\Pi_H - a_H \geq \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi \tag{A-30}$$

the second-best loan contract does not provide corner consumption. Based on Proposition 4, corner consumption arises when

$$W + \Pi_H + a_H < \frac{p_H}{\rho} \ln \Omega - \frac{1}{\rho} \ln \Phi \tag{A-31}$$

Note that because $\Omega > 1$ and $\Phi < 1$, then $\frac{p_H}{\rho} \ln \Omega > 0$ and $\frac{1}{\rho} \ln \Phi < 0$. It follows that

$$W + \Pi_H + a_H > \Pi_H - a_H \text{ and } \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi < \frac{p_H}{\rho} \ln \Omega - \frac{1}{\rho} \ln \Phi \tag{A-32}$$

However, given (A-32), inequalities (A-30) and (A-31) can not coexist. This implies that given $\Pi_H - a_H \geq \frac{p_H}{\rho} \ln \Omega + \frac{1}{\rho} \ln \Phi$, corner consumption for the second-best loan contract never arises.

For the second-best interior consumption, investment distortion is not present when

$$-\exp(-\rho C_0^A) \geq -\exp(-\rho \frac{W}{2})$$

then based on Corollary 3 and (A-29),

$$-\exp(-\rho C_0^A) - p_H \exp\left[-\rho\left(C_H^A - a_H\right)\right] - (1 - p_H) \exp\left[-\rho\left(C_L^A - a_H\right)\right] \geq -2\exp(-\rho \frac{W}{2})$$

which indicates a absence of investment distortion. QED.

Proof of Proposition 5.

The borrower's investment decision between the second-best contract and no-investment follows the proof of Corollary 4.

The following derives the conditions under which the borrower prefers no-investment to no-lender investment. The borrower's no-lender investment program when he has time-additive consumption preference is

[Program NL^A]

$$\max_{C_0^A \geq 0, C_H^A \geq 0, C_L^A \geq 0} - \exp(-\rho C_0^A) - p_H \exp\left[-\rho\left(C_H^A - a_H\right)\right] - (1 - p_H) \exp\left[-\rho\left(C_L^A - a_H\right)\right]$$

$$\text{s.t. } C_0^A \leq W - I$$

$$C_H^A \leq W - C_0^A - I + X_H$$

$$C_L^A \leq W - C_0^A - I + X_L$$

Let $\nu_{1-3} \geq 0$ be the Lagrangian multipliers for the constraints and $\nu_{4-6} \geq 0$ be the Lagrangian multipliers for the limited liability constraints. The first-order conditions with respect to the borrower's consumptions are

$$L_{C_0^A} = \rho \exp(-\rho C_0^A) - \nu_1 - \nu_2 - \nu_3 + \nu_4 = 0$$

$$L_{C_H^A} = \rho p_H \exp\left[-\rho\left(C_H^A - a_H\right)\right] - \nu_2 + \nu_5 = 0 \tag{A-33}$$

$$L_{C_L^A} = \rho(1 - p_H) \exp\left[-\rho\left(C_L^A - a_H\right)\right] - \nu_3 + \nu_6 = 0 \tag{A-34}$$

Rearranging terms for (A-33) and (A-34), given non-negative multipliers,

$$\nu_2, \nu_3 > 0$$

Then the borrower's consumption constraints at t_1 are both binding, i.e.,

$$C_H^A = W - C_0^A - I + X_H \tag{A-35}$$

$$C_L^A = W - C_0^A - I + X_L \tag{A-36}$$

First, assuming that $X_L \geq 0$; then the limited liability constraint on C_L^A holds.

Substituting the binding C_H^A and C_L^A into the objective function, the program becomes

$$\max_{C_0^A \geq 0} -\exp(-\rho C_0^A) + \exp\left(\rho C_0^A\right) \exp\left[-\rho\left(W - I - a_H\right)\right] EU(X)$$

$$\text{s.t. } C_0^A \leq W - I \tag{A-37}$$

where $EU(X) = -(1 - p_H)\exp\left(-\rho X_L\right) - p_H \exp\left(-\rho X_H\right)$. Taking the derivative and solving for C_0^A,

$$C_0^A = \frac{W - I - a_H + CE(X)}{2} \tag{A-38}$$

where $CE(X) = -\frac{1}{\rho}\ln\left[(1 - p_H)\exp(-\rho X_L) + p_H \exp(-\rho X_H)\right]$.

Given (A-37), (A-38) only applies if

$$CE(X) \leq W - I + a_H \tag{A-39}$$

Otherwise, for

$$CE(X) > W - I + a_H \tag{A-40}$$

it follows that

$$C_0^A = W - I$$

Case 1.When condition (A-39) holds, given (A-35), (A-36), and (A-38) the borrower's optimal consumption is derived as

$$C_0^A = \frac{W - I - a_H + CE(X)}{2}$$
$$C_H^A = \frac{W - I + a_H - CE(X)}{2} + X_H$$
$$C_L^A = \frac{W - I + a_H - CE(X)}{2} + X_L$$

However, given the limited liability on C_0^A, the following condition must hold: $CE(X) \geq I + a_H - W$.

Let EU_{NL}^{A} and RU^{A} denote the borrower's expected utilities from no-lender investment and no-investment, respectively. Given the above consumption schedule, we have

$$\begin{aligned} EU_{NL}^{A} &= -2\exp\left(-\rho C_0^A\right) = -2\exp(\left[-\rho\frac{W-I-a_H+CE(X)}{2}\right] \\ RU^{A} &= -2\exp\left(-\rho\frac{W}{2}\right) \end{aligned}$$

If $\frac{W-I-a_H+CE(X)}{2} < \frac{W}{2}$, i.e., $CE(X) < I + a_H$, investment distortion arises. However, this conclusion is derived under the assumption that 1) $X_L \geq 0$ and 2) $CE(X) \geq I + a_H - W$. Taken (A-39) into consideration, for $X_L \geq 0$, investment distortion arises if

$$I + a_H - W \leq CE(X) < \min\{I + a_H, W - I + a_H\}$$

which is equivalent to

$$CE(X) < \min\{I + a_H, W - I + a_H\}$$

given $W \geq I$.

Case 2.When condition (A-40) holds, the borrower's optimal consumption is derived as

$$C_0^A = W - I$$

$$C_H^A = X_H$$

$$C_L^A = X_L$$

The borrower's expected utility associated with no-lender investment is calculated as

$$EU_{NL}^{A} = -\exp\left[-\rho\left(W-I\right)\right] - p_H\exp\left[-\rho\left(X_H - a_H\right)\right] - \left(1-p_H\right)\exp\left[-\rho\left(X_L - a_H\right)\right]$$

Comparing with the borrower's expected utility from no-investment, $RU^{A} = -2\exp\left(-\rho\frac{W}{2}\right)$. Investment distortion occurs when

$$-\frac{2}{\rho}\ln\frac{\exp\left[-\rho\left(W-I\right)\right] + \exp\left[-\rho\left(CE(X) - a_H\right)\right]}{2} < W$$

Condition (A-40) has to hold, i.e., $CE(X) + I - a_H > W$. Summarizing case 2 implies that for $X_L \geq 0$, investment distortion arises if

$$-\frac{2}{\rho} \ln \frac{\exp\left[-\rho\left(W-I\right)\right] + \exp\left[-\rho\left(CE(X)-a_H\right)\right]}{2} < W < CE(X) + I - a_H$$

Note that in the context, we establish that it is strictly preferable for the borrower to delay his consumption forward in time in an attempt to smooth intertemporal consumption. Therefore, between the two cases that are discussed above, Case 1 scenario is strictly preferable to Case 2 scenario for the borrower in term of its realized expected utility. This follows that Case 1 investment distortion condition applies to Case 2.

Next, we consider the case where X_L might be negative. There are two different cases for $X_L < 0$.

First, if $C_0^A - (W - I) \leq X_L < 0$, which indicates that

$$C_L^A = W - C_0^A - I) + X_L \geq 0$$

Case 1 set of consumption schedule applies while Case 2 consumption schedule is not feasible because $C_L^A = X_L$ violates the limited liability constraint, i.e., when $CE(X) > W - I + a_H$, no solution exists for the borrower's no-lender investment behavior. When $CE(X) \leq W - I + a_H$, investment distortion analysis follows the analysis of Case 1 above.

Second, if $X_L < C_0^A - (W - I)$, it follows that $C_0^A < 0$, and no solution can be found for the borrower's no-lender investment problem. This leads directly to investment distortion. QED.

Proof of Proposition 6.

Directly follows from the proofs of Corollary 4 and Proposition 5. QED.

Proof of Proposition 7.

Let $EU_{NL}^D\left(W \geq I\right)$ denote the borrower's expected utility when he engages in self-financing. Given his consumption at equilibrium

$$
\begin{aligned}
\widehat{C}_H &= W - I + X_H - a_H \\
\widehat{C}_L &= W - I + X_L - a_H
\end{aligned}
$$

then

$$
EU_{NL}^{D}(W \geq I) = p_H\sqrt{W - I + X_H - a_H} + (1 - p_H)\sqrt{W - I + X_L - a_H}
$$

First, assume that $W = I$. Then the borrower's expected utility from no-lender investment corresponding to $W = I$ is

$$
p_H\sqrt{X_H - a_H} + (1 - p_H)\sqrt{X_L - a_H}
$$

Given $W = I$, the borrower's no-lender investment decision is justifiable if the following inequality holds:

$$
p_H\sqrt{X_H - a_H} + (1 - p_H)\sqrt{X_L - a_H} \geq \sqrt{W} = \sqrt{I} \tag{A-41}
$$

Note that at $W = I$, the borrower consumes the stochastic cash flows from his investment; therefore, let $CE(X)$ denote the borrower's certainty equivalent consumption at $W = I$, then it follows that

$$
\sqrt{CE(X) - a_H} = p_H\sqrt{X_H - a_H} + (1 - p_H)\sqrt{X_L - a_H}
$$

Therefore, at $W = I$, the borrower prefers no-lender investment if

$$
CE(X) - a_H \geq I \tag{A-42}
$$

For any $W \geq I$, given the borrower's decreasing absolute risk aversion preference, it is always the case that if (A-41) holds, the following relationship is true:

$$
EU_{NL}^{D}(W \geq I) \geq \sqrt{W}
$$

Therefore, condition (A-42) is the sufficient condition for the borrower to engage in no-lender investment over no investment in the absence of the lender for any $W \geq I$. QED.

Proof of Corollary 5: Directly follows the Proof of Proposition 3 in Chapter 3 and the Proof of Proposition 7 above. QED.

Proof of Proposition 8:

The borrower's consumption constraints at $t = 1$ are binding in equilibrium:

$$\widehat{C}_H^A = W - I - \widehat{C}_0^A + X_H$$
$$\widehat{C}_L^A = W - I - \widehat{C}_0^A + X_L$$

Assuming that $W = I$; it follows that the borrower's consumption schedule is

$$\widehat{C}_0^A = 0$$
$$\widehat{C}_H^A = X_H$$
$$\widehat{C}_L^A = X_L$$

The borrower's no-lender investment is justifiable in the absence of the lender if

$$p_H\sqrt{X_H - a_H} + (1 - p_H)\sqrt{X_L - a_H} \geq 2\sqrt{\frac{W}{2}} = 2\sqrt{\frac{I}{2}}$$

which can be simplified as

$$\sqrt{CE(X) - a_H} \geq 2\sqrt{\frac{I}{2}}$$

where $CE(X)$ follows the definition in the Proof of Proposition 7 and denotes the borrower's certainty equivalent consumption at $t = 1$. Therefore, at $W = I$, investment distortion arises if

$$CE(X) - a_H \geq 2I \tag{A-43}$$

Following a similar argument made in the Proof of Proposition 7, were the borrower to decide to engage in no-lender investment at $W = I$, he would choose self-financing when his personal wealth is $W > I$. Therefore, condition (A-43) is sufficient to avoid investment distortion in the absence of the lender. QED.

Proof of Corollary 6:

The borrower's second-best loan contract in equilibrium is the following program:

[Program SB^D]

$$\max_{\widehat{C}_H \geq a_H, \widehat{C}_L \geq a_H} p_H \sqrt{\widehat{C}_H - a_H} + (1 - p_H) \sqrt{\widehat{C}_L - a_H}$$

$$\text{s.t. } p_H \widehat{C}_H + (1 - p_H) \widehat{C}_L = W + \Pi_H$$

$$p_L \sqrt{\widehat{C}_H} + (1 - p_L) \sqrt{\widehat{C}_L} = p_H \sqrt{\widehat{C}_H - a_H} + (1 - p_H) \sqrt{\widehat{C}_L - a_H}$$

Substituting $\widehat{C}_L = \frac{W + \Pi_H - p_H \widehat{C}_H}{(1 - p_H)}$ into the objective function, which is denoted by obj.yields

$$obj = p_H \sqrt{\widehat{C}_H - a_H} + (1 - p_H) \sqrt{\frac{W + \Pi_H - p_H \widehat{C}_H}{(1 - p_H)} - a_H} \tag{A-44}$$

Taking the total derivative with respect to a_H yields

$$\begin{aligned} \frac{dobj}{da_H} &= -\frac{1}{2} \left[p_H \frac{1}{\sqrt{\widehat{C}_H - a_H}} + (1 - p_H) \frac{1}{\sqrt{\frac{W + \Pi_H - p_H \widehat{C}_H}{(1 - p_H)} - a_H}} \right] \\ &\quad + \frac{1}{2} p_H \left(\frac{1}{\sqrt{\widehat{C}_H - a_H}} - \frac{1}{\sqrt{\frac{W + \Pi_H - p_H \widehat{C}_H}{(1 - p_H)} - a_H}} \right) \frac{d\widehat{C}_H}{da_H} \end{aligned}$$

Note that the term in square bracket is positive and the term in the parentheses is negative given $\widehat{C}_H > \widehat{C}_L$; furthermore, $\frac{d\widehat{C}_H}{da_H}$ is strictly positive because as a_H increases, it becomes more costly to motivate the borrower given his risk-averse preference; therefore, the borrower is compensated more. These imply that

$$\frac{dobj}{da_H} < 0$$

Similar, taking the total derivative of (A-44) with respect to W implies that

$$\frac{dobj}{dW} > 0$$

QED.

Proof of Proposition 9.

Expanding the CARA-borrower's objective function, we have

$$\begin{aligned} EU\left(W^{RP}|a_H\right) &= \alpha\{-p_H \exp\left[-\rho\left(C_H\left(\widetilde{W}_H\right) - a_H\right)\right] \\ &\quad - (1-p_H)\exp\left[-\rho\left(C_L\left(\widetilde{W}_H\right) - a_H\right)\right]\} \\ &\quad + (1-\alpha)\{-p_H \exp\left[-\rho\left(C_H\left(\widetilde{W}_L\right) - a_H\right)\right] \\ &\quad - (1-p_H)\exp\left[-\rho\left(C_L\left(\widetilde{W}_L\right) - a_H\right)\right]\} \end{aligned}$$

Similarly, the lender's individual rationality constraints are expanded as:

$$D_0\left(\widetilde{W}_H\right) \leq p_H D_H\left(\widetilde{W}_H\right) + (1-p_H) D_L\left(\widetilde{W}_H\right) \tag{A-45}$$

$$D_0\left(\widetilde{W}_L\right) \leq p_H D_H\left(\widetilde{W}_L\right) + (1-p_H) D_L\left(\widetilde{W}_L\right) \tag{A-46}$$

And the CARA-borrower's expected utilities from each reporting and effort supply strategy are:

$$
\begin{aligned}
EU\left(W_H,\widetilde{W}_H,a_H\right) &= -p_H\exp\left[-\rho\left(C_H\left(\widetilde{W}_H\right)-a_H\right)\right] \\
&\quad -(1-p_H)\exp\left[-\rho\left(C_L\left(\widetilde{W}_H\right)-a_H\right)\right] \\
EU\left(W_H,\widetilde{W}_L,a_H\right) &= -p_H\exp\left[-\rho\left(C_H\left(\widetilde{W}_L\right)-a_H+W_H-W_L\right)\right] \\
&\quad -(1-p_H)\exp\left[-\rho\left(C_L\left(\widetilde{W}_L\right)-a_H+W_H-W_L\right)\right] \\
EU\left(W_H,\widetilde{W}_H,a_L\right) &= -p_L\exp\left[-\rho C_H\left(\widetilde{W}_H\right)\right] \\
&\quad -(1-p_L)\exp\left[-\rho C_L\left(\widetilde{W}_H\right)\right] \\
EU\left(W_H,\widetilde{W}_L,a_L\right) &= -p_L\exp\left[-\rho\left(C_H\left(\widetilde{W}_L\right)+W_H-W_L\right)\right] \\
&\quad -(1-p_L)\exp\left[-\rho\left(C_L\left(\widetilde{W}_L\right)+W_H-W_L\right)\right] \\
EU\left(W_L,\widetilde{W}_L,a_H\right) &= -p_H\exp\left[-\rho\left(C_H\left(\widetilde{W}_L\right)-a_H\right)\right] \\
&\quad -(1-p_H)\exp\left[-\rho\left(C_L\left(\widetilde{W}_L\right)-a_H\right)\right] \\
EU\left(W_L,\widetilde{W}_L,a_L\right) &= -p_L\exp\left[-\rho C_H\left(\widetilde{W}_L\right)\right] \\
&\quad -(1-p_L)\exp\left[-\rho C_L\left(\widetilde{W}_L\right)\right]
\end{aligned}
$$

Furthermore, the CARA-borrower's consumption constraints depending on his reported personal wealth are

$$C_H\left(\widetilde{W}_H\right) \le \widetilde{W}_H - \left[I - D_0\left(\widetilde{W}_H\right)\right] + X_H - D_H(\widetilde{W}_H) \tag{A-47}$$

$$C_L\left(\widetilde{W}_H\right) \le \widetilde{W}_H - \left[I - D_0\left(\widetilde{W}_H\right)\right] + X_L - D_L(\widetilde{W}_H) \tag{A-48}$$

$$C_H\left(\widetilde{W}_L\right) \le \widetilde{W}_L - \left[I - D_0\left(\widetilde{W}_L\right)\right] + X_H - D_H(\widetilde{W}_L) \tag{A-49}$$

$$C_L\left(\widetilde{W}_L\right) \le \widetilde{W}_L - \left[I - D_0\left(\widetilde{W}_L\right)\right] + X_L - D_L(\widetilde{W}_L) \tag{A-50}$$

Note that from inequalities (A-47) and (A-48), the following relationship holds:

$$p_H C_H\left(\widetilde{W}_H\right)+(1-p_H)\,C_L\left(\widetilde{W}_H\right) \le \widetilde{W}_H+\Pi_H+D_0\left(\widetilde{W}_H\right)-\left[p_H D_H(\widetilde{W}_H)+(1-p_H)\,D_L(\widetilde{W}_H)\right] \tag{A-51}$$

Given (A-45), (A-51) becomes

$$p_H C_H\left(\widetilde{W}_H\right) + (1-p_H)\, C_L\left(\widetilde{W}_H\right) \le \widetilde{W}_H + \Pi_H \tag{A-52}$$

Similarly, given (A-49) and (A-50), together with (A-46), we have

$$p_H C_H\left(\widetilde{W}_L\right) + (1-p_H)\, C_L\left(\widetilde{W}_L\right) \le \widetilde{W}_L + \Pi_H \tag{A-53}$$

Therefore, the CARA-borrower's consumption constraints (A-47) to (A-50) and the lender's individual rationality constraints (A-45) and (A-46) become the CARA-borrower's expected consumption constraints (A-52) and (A-53).

Ignoring the CARA-borrower's limited liability constraints, let $\lambda_1 - \lambda_6 \ge 0$ be the Lagrangian multipliers for the CARA-borrower's four incentive compatibility constraints and the expected consumption constraints (A-52) and (A-53). Taking the derivatives of the Lagrangian function with respect to the CARA-borrower's consumption when high wealth is reported $C\left(\widetilde{W}_H\right)$, we have

$$\begin{aligned}
\rho \exp\left[-\rho C_H\left(\widetilde{W}_H\right)\right]\left\{\exp\left(\rho a_H\right)(\alpha+\lambda_1+\lambda_2+\lambda_3) - \lambda_2 \frac{p_L}{p_H}\right\} &= \lambda_5 \\
\rho \exp\left[-\rho C_L\left(\widetilde{W}_H\right)\right]\left\{\exp\left(\rho a_H\right)(\alpha+\lambda_1+\lambda_2+\lambda_3) - \lambda_2 \frac{(1-p_L)}{(1-p_H)}\right\} &= \lambda_5
\end{aligned}$$

Suppose that $\lambda_5 = 0$, it follows that

$$\begin{aligned}
(\alpha+\lambda_1+\lambda_2+\lambda_3) &= \lambda_2 \frac{p_L}{p_H} \\
(\alpha+\lambda_1+\lambda_2+\lambda_3) &= \lambda_2 \frac{(1-p_L)}{(1-p_H)}
\end{aligned}$$

However, this is contrary to the assumption that $p_H > .5$; therefore, $\lambda_5 > 0$. Given that $\lambda_5 > 0$, if $\lambda_2 = 0$, we have

$$\begin{aligned}
\rho \exp\left[-\rho C_H\left(\widetilde{W}_H\right)\right]\exp\left(\rho a_H\right)(\alpha+\lambda_1+\lambda_3) &= \lambda_5 \\
\rho \exp\left[-\rho C_L\left(\widetilde{W}_H\right)\right]\exp\left(\rho a_H\right)(\alpha+\lambda_1+\lambda_3) &= \lambda_5
\end{aligned}$$

which implies that $C_H\left(\widetilde{W}_H\right) = C_L\left(\widetilde{W}_H\right)$. However, this directly violates the CARA-borrower's second incentive compatibility constraint

$$EU\left(W_H, \widetilde{W}_H, a_H\right) \geq EU\left(W_H, \widetilde{W}_H, a_L\right)$$

which can be simplified as

$$C_H\left(\widetilde{W}_H\right) \geq C_L\left(\widetilde{W}_H\right) + \frac{1}{\rho}\ln\Omega$$

where $\Omega = \frac{p_H \exp(\rho a_H) - p_L}{(1-p_L)-(1-p_H)\exp(\rho a_H)}$. Given that $\lambda_2, \lambda_5 > 0$, the following two constraints are binding:

$$\begin{aligned} C_H\left(\widetilde{W}_H\right) &= C_L\left(\widetilde{W}_H\right) + \frac{1}{\rho}\ln\Omega \\ p_H C_H\left(\widetilde{W}_H\right) + (1-p_H)\, C_L\left(\widetilde{W}_H\right) &= \widetilde{W}_H + \Pi_H \end{aligned}$$

The optimal solution is

$$\begin{aligned} C_H\left(\widetilde{W}_H\right) &= \widetilde{W}_H + \Pi_H - \frac{1-p_H}{\rho}\ln\Omega \\ C_L\left(\widetilde{W}_H\right) &= \widetilde{W}_H + \Pi_H - \frac{p_H}{\rho}\ln\Omega \end{aligned}$$

Similarly, taking the derivatives of the Lagrangian function with respect to the CARA-borrower's consumption when low wealth is reported $C\left(\widetilde{W}_L\right)$, we derive that $\lambda_4, \lambda_6 > 0$, which implies that the fourth incentive compatibility constraint and (A-53) are binding:

$$\begin{aligned} EU\left(W_L, \widetilde{W}_L, a_H\right) &= EU\left(W_L, \widetilde{W}_L, a_L\right) \\ p_H C_H\left(\widetilde{W}_L\right) + (1-p_H)\, C_L\left(\widetilde{W}_L\right) &= \widetilde{W}_L + \Pi_H \end{aligned}$$

Solving for the optimal consumption associated with $\widetilde{W}_L$ yields

$$
\begin{aligned}
C_H\left(\widetilde{W}_L\right) &= \widetilde{W}_L + \Pi_H - \frac{1-p_H}{\rho}\ln\Omega \\
C_L\left(\widetilde{W}_L\right) &= \widetilde{W}_L + \Pi_H - \frac{p_H}{\rho}\ln\Omega
\end{aligned}
$$

Given that

$$\widetilde{W}_H > \widetilde{W}_L \geq \underline{W} = \frac{p_H}{\rho}\ln\Omega - \Pi_H$$

the limited liability constraints on $C\left(\widetilde{W}\right)$ are satisfied. The remaining incentive compatibility constraints are also satisfied.

Note that the optimal solutions for [Program $\widetilde{SB}$] are exactly the same as the optimal consumption from the naive contract when the CARA-borrower's public observable personal wealth are W_H and W_L, respectively. Therefore, the naive contract is optimal for [Program $\widetilde{SB}$]. QED.

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BIOGRAPHICAL SKETCH

Liang Fu earned her Bachelor of Arts in International economics from Southwestern University of Finance and Economics in 1998. She earned her Master of Arts in economics from Kent State University in 2002. She joined the Ph.D. program of Fisher School of Accounting at the University of Florida and she received her degree of Doctor of Philosophy in Accounting in 2009.