# FOR WHOM THE NET TOLLS: A TWO-SIDED MARKET ANALYSIS AND PUBLIC POLICY IMPLICATIONS FOR THE NET NEUTRALITY DEBATE

By

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To my parents, Xiaoyan Guo and Changjie Guo,
for a lifetime of love, trust, and support

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Net neutrality is a widely debated policy issue that could fundamentally alter the dynamics of providing and accessing online content through the Internet. As opposed to the status quo of Internet data transmission where requested packets are delivered from the local broadband service provider (BSP) to online users with no discrimination, several BSPs have proposed charging online content providers for priority content delivery. Opponents of the proposal (backers of the "net neutrality" principle) want Congress to implement legislation to prevent the proposed priority delivery and pricing mechanism. My dissertation provides a comprehensive analysis of the net neutrality debate. I propose a technological-economic framework of net neutrality and analyze two different forms of network discrimination: content provider discrimination and user discrimination. Using game-theoretical models, I study five critical economic consequences of the potential net neutrality regulation: consumer surplus, social welfare, capacity expansion, market coverage, and vertical integration.

Major findings include: If content provider discrimination replaced net neutrality, the monopolist BSP gains while content providers are worse off. Consumer surplus and social welfare increase or remain unchanged. In most cases, the BSP has higher incentive to expand its infrastructure capacity (bandwidth) under network neutrality. While content provider

discrimination increases the BSP's market coverage (more consumers are served), it may hinder innovations at the edge. When a BSP is vertically integrated with a content provider, social welfare may increase or decrease depending on how effectively the vertically integrated firm generates revenue from its content compared to the competing independent content provider.

For user discrimination, the impact of net neutrality depends on the characteristics of the Internet data consumption market. With net neutrality, the BSP would prefer to charge a two-part tariff for Internet access. Without net neutrality, a BSP may choose to charge a uniform price and degrade heavy users or charge a higher price to heavy users for preferential delivery of their data packets. Interestingly, without net neutrality, degrading the experience of heavy users – a practice that the FCC recently banned – increases social welfare. These findings have important policy implications and shed light for regulators to determine the fate of net neutrality.

# CHAPTER 1
# INTRODUCTION

## What is Net Neutrality?

Net neutrality is a widely debated policy issue that could fundamentally alter the dynamics of providing and accessing online content through the Internet. Many of us have opinions regarding the subject of net neutrality, and if the public discourse on the issue is any indicator, our opinions have definite ideological overtones: do we want – as the proponents of net neutrality would frame the issue – the Internet to be a neutral carrier of information packets, without any regard to what the packets contain, where it originated from or where might it end – and therefore want legislation to enforce that principle? Or – if the question is posed by supporters on the other side of the debate – do we want the free market to decide what services should be charged and how, and who should be charged for these services?

The debate started – and received widespread media attention – when some broadband service providers (BSPs) like Verizon, Comcast and AT&T (among others) proposed charging popular websites for preferential access to their residential and commercial customers (Helm 2006; Waldmeir 2006). The proposal encountered stiff resistance from those who were to be charged, and thus erstwhile competitors like Google, Yahoo! and Microsoft (and seemingly disparate organizations like the Christian Coalition of America and the American Civil Liberties Union) were soon lobbying before the United States Congress to pass legislation that would prevent the broadband service providers from carrying out their proposed plan (WSJ 2006), and thereby maintain what was termed the "neutrality" of the Internet[1] (the term "net neutrality" itself is attributed to the Columbia Law School professor Tim Wu). This would involve the

[1] A "neutral" Internet is one that treats every packet with equal priority, regardless of its content, origin, destination or platform.

designing of "rules that prevent network operators and ISPs[2] from using their power over the transmission technology to negatively affect competition in complementary markets for applications, content and portals" (van Schewick 2007).

The supporters of net neutrality believe that a "maximally useful public information network aspires to treat all content, sites, and platforms equally" (Wu 2003), and while a formal definition of the operationalization of the principle does not exist, Hahn and Wallsten (2006) point out that it "usually means that broadband service providers charge consumers only once for Internet access, do not favor one content provider over another, and do not charge content providers for sending information over broadband lines to end users."

Lobbying by both sides of the issue has been intense, and the United States Congress is currently considering proposals to introduce network neutrality legislation (Dunbar 2006; McCullagh and Broache 2006; Windhausen 2006). The U.S. House of Representatives and the Senate have held several hearings on the subject (Representatives 2005; Senate 2006). The Federal Trade Commission has also chimed in, and has recently published a report that has advised a wait-and-watch approach on the matter (FTC 2007).

The exposition so far might lead a reader to think that the net neutrality issue is relevant only within the United States, but as the Internet is a global entity, such a conclusion would be misleading. While the net neutrality debate initially started in the United States, the intensity of the discussions has led other countries across the world to consider its repercussions. Following the lead of US broadband service providers, BSPs like Deutsche Telekom in Germany and Telecom Italia in Italy are lobbying the European Commission to allow charging content providers for preferential treatment of their packets. The Commission, in turn, has expressed "its

[2] The terms Internet service providers (ISPs) and broadband service providers (BSPs) are oftentimes used interchangeably in reality and thus in the remainder of this dissertation.

readiness to closely monitor attempts to call into question the neutral character of the Internet" (EC 2006). In several other countries like Canada or Japan, the concerned regulatory agencies have begun to study the issue and its implications, since any policy directives emerging from the United States can serve as a precedent for other countries.

## The Two Sides of the Debate

Both sides in this debate, it would seem, have good arguments to back up their claim. Backers of the net neutrality principle would like to maintain the status quo, whereby the BSPs do not look into the packets transmitted through their network and therefore no discrimination exists. Independent commentators have asked the Federal Communications Commission to impose rules on the BSPs that would prevent them to discriminate against the third-party content providers (Coalition 2002; Wu and Lessig 2003). This, they claim, would preserve the egalitarian philosophy on which the Internet was founded (Lessig 2001). Other backers of the principle are many online start-ups, who claim that it would be almost impossible for them to pay these proposed fees when their revenue streams are almost non-existent, since they have to give away most of their content for free in order to build a loyal customer base (Sydell 2006). There are venture capitalists who have argued that abandoning net neutrality would result in would-be entrepreneurs becoming more hesitant to start a business, a state of affairs that might even hurt the competitiveness of the American online firms in the long run (Sydell 2006; Wu 2006a).

Vint Cerf, the renowned computer scientist who is commonly referred to as one of the "founding fathers of the Internet", contends that such a payment structure would result in the Internet resembling more and more like the controlled mass media today, with a few broadband service providers controlling what the customers effectively have access to (Waldmeir 2006). Tim Berners-Lee, the founder of the World Wide Web, also favors keeping net neutrality in place, since "[the Internet] is the basis of a fair competitive market economy" (Berners-Lee

2006). Finally, some people have voiced their fears of the Internet service providers starting to offer competing services (like Internet telephony) to their consumers at rates that undercut other rival providers, which can effectively smother competition (Senate 2006). This again might result in stagnation in what has so far remained one of the most open of marketplaces.

Opponents of network neutrality legislation have denied its need in the first place (National Cable & Telecommunications Association 2003; Owen and Rosston 2003; Yoo 2006), since they feel that broadband service providers do not have an incentive to discriminate against content providers (Speta 2000a; Speta 2000b). Broadband service providers themselves have argued that it is they who have put their resources to maintain and upgrade the physical services that they provide to the consumers, and that the popular web sites have so far got a "free ride" on their resources (Waldmeir 2006), and that the "Internet service providers should be allowed to strike deals to give certain Web sites or services priority in reaching computer users" (Krim 2005). With online content increasing exponentially over the years, and consumers increasingly becoming used to broadband access, the priority delivery charges will be necessary to meet the rising costs of increasing the capacity and serving the expanded consumer base. Not having these sources of revenue might act as a disincentive to upgrade their infrastructure and affect the service providers' plans of increasing their existing capacities. That, in turn, would affect many emerging online services like real-time broadband video that by design require preferential treatment of their packets.

Some BSPs contend that the new payment mechanisms might herald the beginning of new business models that demand preferential treatment of their packets, and that the "vertical integration of new features and services by broadband network operators is an essential part of

the innovation strategy companies will need to use to compete and offer customers the services they demand" (Thierer 2004).

The issue of the incentive to expand broadband capacity is definitely of great interest to observers who have noted the gradual decline of the position of the United States in the ranks of countries that lead in broadband deployment (Yang et al. 2004) – investment in telecommunications as a percentage of Gross Domestic Product (0.169% as compared to the leader South Korea's 1.327%) (ITU 2006), per-capita broadband subscription (falling from fourth among all nations in 2001 to twelfth in late 2006 to fifteenth in April 2007) (Ricadela 2007), and average broadband speeds (the average download speed in the United States is around 4.8 Mbps, compared to the leader Japan where it is 61 Mbps) (Cauley 2007). If the BSPs can utilize the extra profit from charging the content providers to upgrade the broadband infrastructure, it would indeed be a welcome development. It is to be noted here that in all the countries that currently lead in the various measures of broadband deployment, none of the BSPs have implemented any mechanism for priority delivery of online content.

The debates have been so fierce that even the proper usage and context of the term "net neutrality" has been subject to confusion (Wu 2006b), and an extensive discussion of the issues might be found in (Wu 2003). In brief, network neutrality aims to address concerns raised by some specific behavior of the broadband service providers: (1) blocking of some content providers; (2) preferential treatment of one provider over another and (3) transparency failures, whereby a broadband provider fails to notify its customers and content providers what service they offer in terms of estimated bandwidth, latency, etc. (Wu 2006b). The current proposals by the broadband service providers has raised concerns around the second issue – i.e., the possibility that one content or application provider pays the broadband service provider for preferential

treatment of its packets, as the latter acts effectively as a gatekeeper between the content providers and the customers it serves.

The entire debate has raised a number of unanswered questions that are of interest to researchers and practitioners alike, not to mention the regulatory agencies. The stakes involved in the issue was brought into focus during a House Committee hearing in April 2006 (Wu 2006a), where it was pointed out that "[the Internet] has become part of America's basic infrastructure. It has become as essential to people and to the economy as the roads, the electric grid, or the telephone…Given this infrastructure, Americans are accustomed to basic rights to use the network as they see fit."

The debate over net neutrality has so far been argued mainly from two angles – the legal perspective, which considers access to the Internet as a basic right that should be legislated; and the regulatory perspective, which considers any initiative to legislate pricing by a private entity as a hindrance to market mechanisms that ultimately hinder social welfare. In sharp contrast to much of the lofty rhetoric presented by the two sides in the debate, my dissertation presents the issues from a technological-economic perspective.

## Structure of the Dissertation

The rest of the dissertation is arranged as follows. In Chapter 2, I propose a technological-economic framework to provide a holistic view of the net neutrality debate. Based on the proposed framework, I further review the existing literature and position this dissertation within the literature. Chapter 3 proposes a two-sided market model of digital content provision and consumption through the Internet and analyzes one form of network discrimination – content provider discrimination. Winners and losers are identified by comparing the payoff for the BSP and content providers, the consumer surplus, and the social welfare under net neutrality versus payment mechanism model without net neutrality. Within the same framework I also examine

the broadband service provider's incentive to expand their capacity. In Chapter 4, I further study the impact of content provider discrimination upon market coverage on the consumer's side and application innovation on the content provider's side. In Chapter 5, I consider a special case of content provider discrimination – when the broadband service provider vertically integrates with content providers. Chapter 6 presents a different form of network discrimination – user discrimination and examines six potential mechanisms of user discrimination. Finally, I conclude in Chapter 7.

# CHAPTER 2
# A TECHNOLOGICAL-ECONOMIC FRAMEWORK

## The Technological Perspective

Despite the extensive discussion on this topic, the proper usage and context of the term "net neutrality" itself has been subject to confusion (Wu 2006b). In this chapter, I frame the net neutrality debate from the technological and economic perspectives. From the technological perspective, the five-layer Internet architecture defines the underlying data transmission mechanism of the Internet and exemplifies the "end-to-end" design principle of the Internet. Starting from the bottom up, the physical layer defines the transmissions through the physical media. On top of this, the link layer is responsible for router-to-router transmissions. Following next is the network layer which establishes the logical connection between the source and the destination. Communications between the individual processes on the source and destination are defined by the transport layer. And finally, the application layer completes the data transmission between various applications running on the end systems. The status quo of data transmission on the Internet is a simple blind mechanism which treats all data packets equally resulting in no discrimination. However, the technology to discriminate packets and streamline Internet traffic has been available at a minimal fixed cost. It is feasible to adopt a more sophisticated mechanism which has the capability to recognize and distinguish the transmitted data packets based on its source and destination, content, and/or application. As a result, some data packets may be potentially favored over others resulting in four basic forms of discrimination – application discrimination, content discrimination, content provider discrimination, and user discrimination in the context of online content provision and consumption through the Internet. Based on the required information for discrimination, there is a natural mapping from these discrimination mechanisms to the five-layer Internet architecture.

As shown in Figure 2-1, no discrimination (net neutrality) corresponds to the two lower layers since the physical and link layers move data from a router to its adjacent router without the information about the source and the destination of the route. Potential discrimination (no net neutrality) corresponds to the three upper layers. Specifically, network layer contains the sender and the receiver information and thus corresponds to content provider discrimination and user discrimination. The transport and application layers recognize processes and applications on the end systems and therefore correspond to content discrimination, application discrimination, or both. Among the four possible forms of discrimination, the current proposal by the BSPs – "charge popular websites for preferential access to their customers" – is content provider discrimination which is also the focus of my dissertation and I will elaborate more in Chapters 3, 4, and 5. In Chapter 6, I analyze another form of network discrimination – user discrimination.

## The Economic Perspective

From the economic perspective, the entire net neutrality debate has raised a number of unanswered questions that are of interest to researchers and practitioners alike, not to mention the regulatory agencies. The regulatory agencies would like to know who are the winners and losers if the principle of net neutrality is abolished. Specifically, if the social welfare increases as a result of abandoning net neutrality – and more specifically, the end consumers are better off, the idea for the proposed payment mechanisms would gain traction among the policymakers; conversely, if abandoning the principle of net neutrality results in helping to extract more rent for only a few private agencies, the idea would find a much less sympathetic audience.

Another issue of interest is to check the veracity of a key claim of the BSPs – that under net neutrality, the incentive to expand the capacity of the existing infrastructure for the next generation of broadband services is much less as compared to when they are allowed to charge the online content providers for preferential treatment. For policymakers, this is indeed a key

issue. Higher capacity broadband services will enable many services that are deemed important for the society as a whole. Some examples of such services include disaster recovery, remote medical supervision and the like. For content providers, the next generation broadband services will enable instant delivery of high-definition movies, consumer interactivity, a richer online shopping experience and so forth, and in the process open many new channels of revenue generation.

Furthermore, new start-up firms may not be able to compete with incumbent firms and may be unlikely to win the prioritization auction, leading to less innovation. It is also interesting to consider the effect of the broadband service provider as a potential competitor to the content (or other service) providers. Such a situation already exists today (albeit with limited success so far) with broadband service providers like Comcast building their own modest Internet portals, or with providers like AT&T or Comcast offering VoIP digital phone services (Krim 2005). The BSPs can impose preferential treatment for their own content and applications over those of other providers. Policymakers would then like to ensure that the monopolist broadband service provider does not discriminate against or downgrade other content providers and enjoy unfair competitive advantage.

Answers to these critical economic issues will help shed light for policy makers to determine the fate of net neutrality. With the background in the underlying technology and the associated economic issues, IS researchers can bring forth a holistic perspective that has often been lacking in the net neutrality debate. This dissertation represents one such attempt.

## Literature Review

Formal analytical studies on net neutrality are very limited, regardless of the extensive discussions and debates on the topic. Based on the proposed technological-economic framework of the net neutrality debate, I review the existing literature in net neutrality. I summarize the

literature review in Table 2-1 based on types of discrimination and economic consequences examined in these studies.

Hermalin and Katz view the potential net neutrality regulation as product-line restrictions on the providers of last-mile Internet access services (Hermalin and Katz 2007). They found that a single-product restriction typically but not necessarily reduces the overall social welfare. Economides and Tag utilize a two-sided market model to examine the effect of two-sided pricing with content provider discrimination with emphasis on the cross-group network externality between applications and consumers (Economides and Tag 2007). They find that net neutrality regulation increases total industry surplus in the presence of a monopoly BSP or in the duopoly setting. Jamison and Hauge (2007) study the impact of a premium service for content providers without net neutrality and find the provision of premium service stimulates application innovations at the edge of the Internet. Musacchio et al. (2008) propose a two-sided market model where demand depends on prices and the ISPs' investment levels. They compare the social welfare effect of two-sided pricing and one-sided pricing and find that the result is determined by the ratio between advertising rates and end-user price sensitivity.

In my dissertation, I examine both content provider discrimination (Chapters 3-5) and user discrimination (Chapter 6). Specifically, I propose a game theoretical model of content provider discrimination (Chapter 3) to identify the winners and losers of implementing these different types of transmission discrimination and pricing mechanism and present their implications on the consumers and social welfare. Within the same framework, I also investigate whether broadband providers have lesser incentive to expand capacity (bandwidth) for the consumers if network neutrality legislation is in place. I further extend the model to study the special cases of content

provider discrimination: content provider discrimination with partial market coverage (Chapter 4) and vertical integration (Chapter 5).

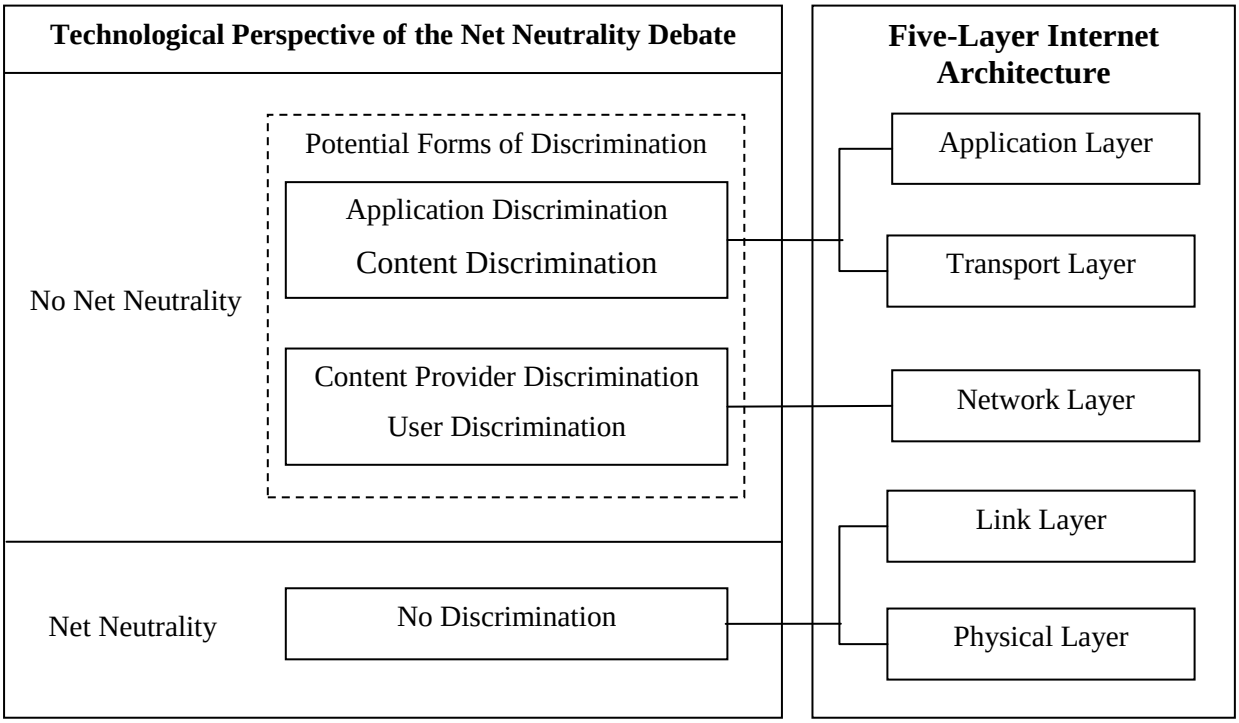



Figure 2-1. Mapping the net neutrality debate to the underlying Internet architecture (Technological perspective of the net neutrality debate)

Table 2-1. Literature review

| The Net Neutrality Debate | | Economic Consequences | | | | |
|---|---|---|---|---|---|---|
| | | **Consumer Surplus** | **Social Welfare** | **Application Innovations** | **Capacity Expansion** | **Vertical Integration** |
| **Different Forms of Discrimination** | **Application Discrimination** | | | | | |
| | **Content Discrimination** | | | | | |
| | **Content Provider Discrimination** | Economides and Tag 2007; This dissertation | Economides and Tag 2007; Hermalin and Katz 2007; Musacchio et al. 2008; This dissertation | Jamison and Hauge 2007 | Jamison and Hauge 2007; Musacchio et al. 2008; This dissertation | This dissertation |
| | **User Discrimination** | This dissertation | This dissertation | | This dissertation | |

# CHAPTER 3
# CONTENT PROVIDER DISCRIMINATION

The status quo of prohibiting broadband service providers from charging websites for preferential access to their customers – the bedrock principle of net neutrality – is under fierce debate. In this chapter, I study the focal point of the net neutrality debate – content provider discrimination. Specifically I develop a game-theoretic model to address two critical issues of net neutrality: (1) Who are winners and losers of abandoning net neutrality; and (2) Will broadband service providers have greater incentive to expand their capacity without net neutrality?

## The Model

To analyze the problem at hand, I consider a stylized model with three types of players: (1) a monopolist broadband service provider that not only serves consumers in a specific geographic market by providing them with Internet access but also serves content providers by delivering their content to the consumers in this market; (2) two competing content providers that provide their service for "free" to the end users as they generate revenues from advertisers and associated "click-throughs" of the consumers; and (3) consumers who consume content from their preferred content provider through the Internet access provided by the local BSP (as shown in Figure 3-1). I develop the model both under net neutrality (hereafter shortened to NN), and when net neutrality is abolished in favor of the regime where the BSP can charge the content providers (i.e., no net neutrality, or NNN for short).

The advertisement-supported revenue model I consider here is overwhelmingly prevalent among online content providers, which is very different from traditional online retailers like Amazon.com. The specifics of the revenue model change from one provider to another, but in general terms, it involves no upfront fees from the customer, but rather a customer's value is encapsulated in the entire lifecycle of one's relationship with the firm. The idea was first

proposed by the noted journalist and commentator on digital technologies, Esther Dyson (1994), and is now considered mainstream by industry observers like Chris Anderson (2008) and Nicholas Carr (2008), economics researchers like Paul Krugman (2008) and Hal Varian (McKinsey McKinseyQuarterly 2009), and IS researchers like Eric Clemons (Knowledge@Wharton 2008). In this framework, the firm gets its revenues not directly from a particular customer, but rather in a stochastic sense when these customers indirectly generate revenues through a variety of means such as banner advertisements, affiliate revenues, rental of subscription lists, sale of aggregate information, to name a few.

Without loss of generality, I normalize the total number of end consumers (i.e., the total number of consumers served by the monopolist BSP) to 1. This unit mass of customers is uniformly distributed on line segment $[0,1]$ in terms of their ideal content. There are two competing online content providers, Y and G, where content provider Y is located at zero, while content provider G is located at the opposite end of the interval (see Figure 3-2). Let $x$ be the marginal consumer that is indifferent to the content between Y and G. Then, the market shares for Y and G are $x$ and $1-x$ respectively. This amounts to the online content market of end consumers being fully covered by content providers Y and G. This assumption of full market coverage will be relaxed in Chapter 4.

Both content providers offer their basic services at no cost to the end users.[1] In this model, I consider the revenue generation of the content provider as the average revenue generated (from all sources) per packet requested by the end consumer. Let $r_{\mathrm{Y}}$ and $r_{\mathrm{G}}$ denote the revenue rates of content provider Y and G, respectively, per packet for content. In other words, these two

[1] In order to draw a meaningful comparison between the current and the proposed environment, one needs to choose the same revenue model for the content providers under both environments, and I have therefore chosen the revenue model that mirrors current reality.

parameters denote the average rates at which the requests for content from the consumers provide revenues to the content providers from myriad types of advertisers that want to reach them. I assume that $r_G > r_Y$, which means that one content provider (G) is better than the other (Y) in getting the "right" consumers for its advertisers (and its other revenue sources) and therefore can charge higher advertising fees. This assumption does not affect the analysis and the generalized results (when $r_G < r_Y$ and $r_G = r_Y$) will be discussed later. Without net neutrality, the BSP provides preferential delivery service for content at a price $p$ which is the unit price for priority data packet transmission per packet. The technology to discriminate packets and streamline Internet traffic has been available at minimal fixed cost, and therefore the cost of implementing this mechanism of priority delivery of some content is assumed to be negligible. In response, the two content providers decide whether to pay for this preferential treatment. Then, the service decisions (whether to choose the preferential delivery service) for the two content providers can be represented by the indicator functions

$$I_Y = \begin{cases} 1, & \text{if Y pays} \\ 0, & \text{if Y does not pay} \end{cases} \text{ and } I_G = \begin{cases} 1, & \text{if G pays} \\ 0, & \text{if G does not pay} \end{cases}$$

Let $\lambda$ be the Poisson arrival rate of content requested by each consumer, and it is expressed in packets per unit of time. Content provider Y's decision problem is $\max_{I_Y} \left\{ r_Y \cdot \lambda \cdot x(I_Y, I_G) - I_Y \cdot p \cdot \lambda \cdot x(I_Y, I_G) \right\}$ and content provider G's decision problem is $\max_{I_G} \left\{ r_G \cdot \lambda \cdot \left[1 - x(I_Y, I_G)\right] - I_G \cdot p \cdot \lambda \cdot \left[1 - x(I_Y, I_G)\right] \right\}$. The demand for content providers $x(I_Y, I_G)$ and $1 - x(I_Y, I_G)$ depend on their service choices $I_Y$ and $I_G$. I will provide further analyses of the demand realization later.

A consumer's net utility from using the services of either Y or G depends on one's individual preferences, the "distance" of one's preferred provider (i.e., either Y or G) from one's ideal, and the cost of delay that is a result of the general congestion in the access network between the BSP and the consumer. Parameter $t$ measures the "fit cost" of the deviation from a consumer's ideal content in the Hotelling framework (Hotelling 1929). Following Mendelson (1985), I denote $V(\lambda)$ to be the gross value function of this content for each consumer, assumed to be twice differentiable and strictly concave. Furthermore, consumers get a congestion disutility due to waiting for packets: the delay cost parameter $d$ multiplied by the expected time in such a queuing system, $w$. The BSP charges a fixed Internet access fee $F$ per unit time to the consumers for Internet access. Both the fixed fee $F$ and the priority charge $p$ are expressed in the same unit of time. Therefore the utility function for an arbitrary consumer $\tilde{x} \in [0,1]$ is $U_{\mathrm{Y}}(\tilde{x}) = V(\lambda) - t\tilde{x} - F - d \cdot w$ if the content provider is Y and is $U_{\mathrm{G}}(\tilde{x}) = V(\lambda) - t(1-\tilde{x}) - F - d \cdot w$ if the content provider is G. In order to determine the delay $w$, I consider $\mu$ to be the capacity that the BSP provides to the consumers, expressed in packets per unit of time. This capacity constraint affects the service that the BSP renders to the consumers in a unique fashion. Specifically, one can think of the packets requested by the consumers as being serviced in an M/M/1 queuing system. I assume that customers are homogeneous in terms of having the same rate of requests for content, valuation of content, and sensitivity to delay. Currently under NN, the congestion delay is the same $w_{\mathrm{NN}} = \dfrac{1}{\mu - \lambda}$ for all consumers and does not figure into the consumers' decisions. However, under NNN, the congestion delay plays an important role. To understand the impact of abolishing net neutrality, consider a situation where the BSP starts charging content providers Y and G for preferential treatment of their packets and suppose

without loss of generality that only Y decides to pay for the service. As a result, any packet from Y that is received by the broadband service provider as a request from one of its customers now gets preferred treatment to the top of the queue (these packets still face the congestion from other similarly preferred packets from Y). I model the congestion in the network after Bandyopadhyay and Cheng (2006) and Mendelson (1985). Packets from G do not receive any preferential treatment and at any point in time are in fact queued after any packet from Y that might be requested at that point of time. Depending on the number of Y's packets in the channel, some consumers that previously preferred G might now find the congestion of G's packets causing enough disutility that they might now prefer Y's service. The delays of the content providers' packets are thus dependent on their service choices, denoted by $w(I_Y, I_G)$. Table 3-1 gives the delays of the four different outcomes under NNN for a two-class priority M/M/1 queue with service preemption. The $x$'s in the delay expressions are the corresponding marginal consumer who is indifferent between Y and G. Individual consumers then choose the content provider that yields the higher utility.

I assume that the consumer located at the two ends of the market is loyal to their corresponding content providers as the consumer at the end point receives content of perfect fit. That is, $V(\lambda) - \frac{d \cdot \mu}{(\mu - \lambda)^2} - F > V(\lambda) - t - \frac{d}{\mu - \lambda} - F$, an expression that can be simplified to $d < \frac{t(\mu - \lambda)^2}{\lambda}$. This assumption ensures the existence of a meaningful competition between the two content providers G and Y, one that is similar to a standard assumption in two-sided markets literature, e.g., Equation (8) of Armstrong (2006) and assumption A3 of Armstrong and Wright (2007). Armstrong (2006) notes that this assumption is the "necessary and sufficient condition for a market-sharing equilibrium to exist." (p. 674).

I assume that the Internet service provider captures all end consumers in $[0,1]$ under net neutrality. Further, the BSPs have stated that their intention is not to degrade the online experience for any current broadband subscriber even if net neutrality is abolished, and therefore we assume that the Internet service provider continues to serve all the current consumers when they start charging content providers for preferential delivery of their packets. In other words, we assume that the consumers' value function $V(\lambda)$ is sufficiently high that the utility for the indifferent consumer in any of the outcomes that follow is nonnegative.

I consider a monopolist BSP that delivers digital content from its local switching office to the end users. While the monopoly assumption is a simplification in some locales, unlike many other countries, the extent of competition in the local broadband services market is very limited in the United States, so much so that in many places, a single broadband service provider is often a de facto monopolist (Economides 2008; Hausman et al. 2001). The situation is aggravated by the high switching cost of long-term service contracts and incompatible broadband technologies between cable and phone companies. Further, many customers are not qualified for a digital subscriber's line (DSL) broadband service from phone companies, because they exceed the three miles distance limit from the phone company's nearest switching office, making the cable operators the de facto monopolistic broadband service provider in several local markets (Turner 2007). Thus, in addition to providing the benefit of making the analysis tractable, the assumption closely reflects the reality of local broadband services in the U.S. The BSP charges consumers a fixed Internet access fee $F$ per unit time. If it is allowed to charge the online content providers, the BSP would charge the content provider a price $p$, a per packet charge for the priority transmission of its packets. To keep the model tractable, I do not consider differential pricing that the BSP might employ (Shapiro and Varian 1998). Then the payoff function for the BSP is

$F$ under net neutrality and $F + I_Y \cdot p \cdot \lambda \cdot x + I_G \cdot p \cdot \lambda \cdot (1 - x)$ without net neutrality. A list of all notations is provided in Appendix A.

As shown in Figure 3-3, the timing of the game is as follows. The BSP announces the Internet access fee $F$ to consumers and the preferential delivery charge $p$ to content providers under NNN. Based on the announced fees, the two content providers decide simultaneously whether to pay the premium price for priority delivery of their content. After the BSP and the content providers make their respective decisions, consumers choose either content provider Y or content provider G. In the following sections, I use backward induction to deduce the subgame perfect Nash equilibria of the game with and without net neutrality regulation.

## Net Neutrality

In this section I analyze the model under net neutrality where the BSP decides on the optimal Internet access fee $F$, and consumers choose between content providers Y and G. The content providers do not have any decision to make here.

### Content Decisions for Consumers

Although individual consumers choose contents between Y and G independently, the consumers' decisions as a whole can be represented by the marginal consumer $x$ with all the consumers located in $[0, x]$ choosing Y and all the consumers located in $[x, 1]$ choosing G. The subscript NN denotes the case of net neutrality. Then $x_{NN}$ denotes the marginal consumer that is indifferent between content provider Y and content provider G under NN and can be specified by

$$V(\lambda) - tx_{NN} - \frac{d}{\mu - \lambda} - F_{NN} = V(\lambda) - t(1 - x_{NN}) - \frac{d}{\mu - \lambda} - F_{NN} \tag{3-1}$$

This leads to $x_{NN} = \frac{1}{2}$ implying equal market share for the two content providers. The payoff to content provider Y is $\Pi_{NN\_Y} = x_{NN}\lambda r_Y = \frac{1}{2}\lambda r_Y$ and the payoff to content provider G is $\Pi_{NN\_G} = (1 - x_{NN})\lambda r_G = \frac{1}{2}\lambda r_G$.

**Pricing Decisions for the BSP**

Under NN, Internet access fees collected from consumers are the only revenues for the BSP. Assume the BSP has negligible "running costs". Anticipating consumers' choices, the BSP solves the profit maximization problem as follows:

$$\begin{aligned} &\max_{F_{NN}} \Pi_{NN} = F_{NN} \\ &\text{s.t.} \quad U_{NN\_Y}(\tilde{x}) \geq 0, \quad 0 \leq \tilde{x} \leq x_{NN} \\ &\qquad U_{NN\_G}(\tilde{x}) \geq 0, \quad x_{NN} \leq \tilde{x} \leq 1 \end{aligned} \tag{3-2}$$

This leads to $\Pi_{NN} = F_{NN} = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$.

## No Net Neutrality

Next, I analyze the situation when the BSP is allowed to charge the content providers for preferential treatment of the latter's packets. Without net neutrality, the BSP charges the content provider $p$ per packet for priority over its competitor's packets, should its competitor choose to not pay. When both content providers pay the price $p$, both their packets receive equal treatment.

**Content Decisions for Consumers**

Given the BSP's choices of $F$, $p$ and content providers' choices $I_Y$ and $I_G$, consumers decide on their preferred content provider. Based on content providers' choices, there are essentially four possible outcomes: neither content provider pay ($I_Y = I_G = 0$); one content

provider pays and the other does not (which results in two different outcomes $I_Y = 1$, $I_G = 0$ and $I_Y = 0$, $I_G = 1$); and both content providers pay ($I_Y = I_G = 1$).

**Outcome 1:** Both content providers opt for not paying the priority price $p_1$ ($I_Y = I_G = 0$). The indifferent consumer $x_1$ is signaled by $V(\lambda) - tx_1 - \frac{d}{\mu - \lambda} - F_1 = V(\lambda) - t(1 - x_1) - \frac{d}{\mu - \lambda} - F_1$ which leads to $x_1 = \frac{1}{2}$.[2] Notice that this outcome amounts to the same result as in net neutrality (NN).

**Outcome 2:** Content provider Y pays $p_2$ while content provider G chooses not to pay ($I_Y = 1$, $I_G = 0$). In Outcome 2, content provider Y's packets are prioritized and therefore face congestion only to the extent of the traffic from Y, but content provider G's packets are not so that G's congestion is a function of the entire traffic. The marginal consumer $x_2$ who is indifferent between content provider Y and content provider G under NNN in Outcome 2 is specified by:

$$V(\lambda) - tx_2 - \frac{d}{\mu - x_2\lambda} - F_2 = V(\lambda) - t(1 - x_2) - \frac{d \cdot \mu}{(\mu - x_2\lambda)(\mu - \lambda)} - F_2 \tag{3-3}$$

This leads to $x_2 > \frac{1}{2}$ (see Appendix B for the proof[3]) meaning that content provider Y enjoys a larger market share $x_2$ at the price of $p_2$. Notice that G's traffic faces delay costs of an M/M/1 priority queue with preemption.

[2] To facilitate understanding, I have made the numerical subscripts (1 through 4) of the various variables correspond to the different Outcomes 1 through 4.

[3] All proofs are organized in Appendix B.

**Outcome 3:** This case is the opposite of Outcome 2. Content provider G decides to pay the preferential packet treatment price of $p_3$ per packet, while content provider Y chooses not to pay ($I_Y = 0$, $I_G = 1$). Carrying out a similar analysis, I denote the marginal consumer by $x_3$ who is indifferent between content provider Y and content provider G under NNN in Outcome 3 and is specified by:

$$V(\lambda) - tx_3 - \frac{d \cdot \mu}{\left[\mu - (1 - x_3)\lambda\right](\mu - \lambda)} - F_3 = V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda} - F_3 \tag{3-4}$$

It follows that $x_3 < \frac{1}{2}$, meaning that content provider G enjoys a larger market share $1 - x_3$ at the price of $p_3$.

**Outcome 4:** Both content providers pay the priority price $p_4$ to have their content delivered ($I_Y = I_G = 1$). Since both content providers' packets are treated the same, this leads to $x_4 = x_1 = x_{NN} = \frac{1}{2}$.

**Delivery Service Decisions for Content Providers**

Given certain values of $F$ and $p$, content providers Y and G decide whether to pay for the preferential delivery of data packets. Content provider Y's decision problem is represented by $\max_{I_Y}\left\{r_Y \cdot \lambda \cdot x(I_Y, I_G) - I_Y \cdot p \cdot \lambda \cdot x(I_Y, I_G)\right\}$, while content provider G's decision problem is represented by $\max_{I_G}\left\{r_G \cdot \lambda \cdot \left[1 - x(I_Y, I_G)\right] - I_G \cdot p \cdot \lambda \cdot \left[1 - x(I_Y, I_G)\right]\right\}$. Table 3-2 presents the payoff matrix for the content providers under the four outcomes.

**Outcome 1:** The revenues for content provider Y and G are $\Pi_{Y1} = \frac{1}{2}\lambda r_Y$ and $\Pi_{G1} = \frac{1}{2}\lambda r_G$, respectively. The incentive compatibility constraint for content provider Y is $\Pi_{Y1} - \Pi_{Y2} \geq 0$ and the incentive compatibility constraint for content provider G is $\Pi_{G1} - \Pi_{G3} \geq 0$.

**Outcome 2:** The revenue for Y is $\Pi_{Y2} = x_2 \lambda r_Y - x_2 \lambda p$ and the revenue for G is $\Pi_{G2} = (1 - x_2)\lambda r_G$. The incentive compatibility constraint for content provider Y is $\Pi_{Y2} - \Pi_{Y1} \geq 0$ and the incentive compatibility constraint for content provider G is $\Pi_{G2} - \Pi_{G4} \geq 0$. These two incentive compatibility constraints can be reduced to

$$p \leq \left( \frac{x_2 - 1/2}{x_2} \right) r_Y \tag{3-5}$$

and

$$p \geq \left( \frac{x_2 - 1/2}{1/2} \right) r_G \tag{3-6}$$

Compare the right hand side (RHS) of constraints (3-5) and (3-6) leads to $\frac{RHS_{(3\text{-}5)}}{RHS_{(3\text{-}6)}} = \frac{r_Y}{2 x_2 r_G} < 1$ since $x_2 > \frac{1}{2}$ and $r_G > r_Y$. Thus, there is no feasible $p$ such that content provider Y pays for the preferential delivery while content provider G does not. Therefore Outcome 2 cannot be an equilibrium. Note that this result is driven by the fact (or more correctly, the assumption) that $r_G > r_Y$. In other words, if $r_G > r_Y$, we can never have an outcome where content provider Y decides to pay and G does not. The assumption $r_G > r_Y$ does not affect the key results of the analyses. If, however, this assumption is reversed, then Outcome 3 instead of Outcome 2 cannot be an equilibrium.

**Outcome 3:** The revenues for Y and G are $\Pi_{Y3} = x_3 \lambda r_Y$ and $\Pi_{G3} = (1 - x_3)\lambda r_G - (1 - x_3)\lambda p$ respectively. The incentive compatibility constraint for content provider Y is $\Pi_{Y3} - \Pi_{Y4} \geq 0$ and the incentive compatibility constraint for content provider G is $\Pi_{G3} - \Pi_{G1} \geq 0$. These two incentive compatibility constraints can be reduced to

$$p \geq \left( \frac{1/2 - x_3}{1/2} \right) r_Y \tag{3-7}$$

and

$$p \leq \left( \frac{1/2 - x_3}{1 - x_3} \right) r_G \tag{3-8}$$

Comparing the RHS of (3-7) and (3-8) gives $\frac{RHS_{(3\text{-}7)}}{RHS_{(3\text{-}8)}} = \frac{2(1 - x_3) r_Y}{r_G}$. To analyze the magnitude of this ratio, we consider two possibilities in the relative values of $r_G$ and $r_Y$: Case I: $r_G < 2(1 - x_3) r_Y$; and Case II: $r_G \geq 2(1 - x_3) r_Y$. If Case I holds, there is no feasible $p$ such that content provider G pays for the preferential delivery and content provider Y does not, and therefore Outcome 3 cannot be an equilibrium. If Case II holds, Outcome 3 may be an equilibrium. I will further analyze the equilibrium results in next subsection (i.e., stage 1 of the game).

**Outcome 4:** In this case, content providers Y and G get the same revenues (i.e., $\frac{1}{2}\lambda r_Y$ and $\frac{1}{2}\lambda r_G$ for Y and G respectively) as those in Outcome 1. Both content providers, however, incur an extra expense of $\frac{1}{2}\lambda p$, which paid to the BSP. The incentive compatibility constraint for content provider Y is $\Pi_{Y4} - \Pi_{Y3} \geq 0$ and the incentive compatibility constraint for content

provider G is $\Pi_{G4} - \Pi_{G2} \geq 0$. We note that Outcome 4 is a classical prisoner's dilemma that has been described in other contexts (Brander and Spencer 1983; Roller and Tombak 1990) – both content providers know that they would be better off by not paying, but given the relative proximity of their per-consumer revenue streams, they end up paying the BSP.

**Pricing Decisions for the BSP**

Expecting the best responses of both content providers and consumers, the BSP analyzes the maximum profit that he can make under the various permutations of the choices of the content providers to pay him (recall that the content providers can decide either to pay or not pay the priority delivery charges) and the corresponding consumers' choices represented by the marginal consumer $x$.

**Outcome 1:** Both content providers opt to not pay the priority price $p_1$ ($I_Y = I_G = 0$). Similar to the NN case, the Internet access fees collected from consumers is the only revenue for the BSP. The BSP solves the following profit maximization problem:

$$\begin{aligned}
&\max_{F_1, p_1} \Pi_1 = F_1 \\
&\text{s.t.} \quad U_{NNN_Y}(\tilde{x}, I_Y, I_G) \geq 0, \quad 0 \leq \tilde{x} \leq x_1 \\
&\qquad U_{NNN_G}(\tilde{x}, I_Y, I_G) \geq 0, \quad x_1 \leq \tilde{x} \leq 1 \\
&\qquad \Pi_{Y1} - \Pi_{Y2} \geq 0 \\
&\qquad \Pi_{G1} - \Pi_{G3} \geq 0
\end{aligned} \tag{3-9}$$

The first two constraints are participation constraints for consumers of content provider Y and G respectively. The last two constraints are incentive compatibility constraints for the content providers. The consumers' participation constraints can be reduced to $p_1 \geq \left( \frac{x_2 - 1/2}{x_2} \right) r_Y$ and $p_1 \geq \left( \frac{1/2 - x_3}{1 - x_3} \right) r_G$. Since $x_2 - \frac{1}{2} = \frac{1}{2} - x_3$ (this is shown to be true in Outcome 3, which is

discussed later) and $r_G > r_Y$, $\left(\frac{1/2 - x_3}{1 - x_3}\right) r_G \geq \left(\frac{x_2 - 1/2}{x_2}\right) r_Y$, which implies that the BSP's optimal preferential delivery charge can be specified as $p_1 \geq \left(\frac{1/2 - x_3}{1 - x_3}\right) r_G$. Therefore the results of the profit maximization problem of the BSP in Outcome 1 are: the BSP makes a profit of $\Pi_1 = F_1 = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ and charges the content providers a fee $p_1$ such that $p_1 \geq \left(\frac{1/2 - x_3}{1 - x_3}\right) r_G$. The best response for both content provider Y and content provider G is to Not Pay the fee.

**Outcome 2:** Content provider Y pays $p_2$ while content provider G chooses not to pay ($I_Y = 1$, $I_G = 0$). In addition to the revenue from end users, the BSP also gains revenue from content provider Y for preferential delivery of Y's content. The broadband provider's profit maximization problem is thus:

$$\begin{aligned}
&\max_{F_2, p_2} \Pi_2 = F_2 + x_2 \lambda p_2 \\
&\text{s.t.} \quad U_{NNN_Y}(\tilde{x}, I_Y, I_G) \geq 0, \quad 0 \leq \tilde{x} \leq x_2 \\
&\qquad U_{NNN_G}(\tilde{x}, I_Y, I_G) \geq 0, \quad x_2 \leq \tilde{x} \leq 1 \\
&\qquad \Pi_{Y2} - \Pi_{Y1} \geq 0 \\
&\qquad \Pi_{G2} - \Pi_{G4} \geq 0
\end{aligned} \tag{3-10}$$

The first two constraints are the participation constraints of the consumers that prefer G and the consumers that prefer Y respectively. The last two constraints ensure that while content provider Y will pay, content provider G will not. As discussed in the second stage of the game, there is no feasible $p_2$ to induce Outcome 2.

**Outcome 3:** Content provider G pays $p_3$ while content provider Y chooses not to pay ($I_Y = 0$, $I_G = 1$). The BSP's profit maximization problem is given by:

$$
\begin{aligned}
&\max_{F_3, p_3} \Pi_3 = F_3 + (1 - x_3)\lambda p_3 \\
&\text{s.t.} \quad U_{NNN\_Y}(\tilde{x}, I_Y, I_G) \geq 0, \quad 0 \leq \tilde{x} \leq x_3 \\
&\qquad U_{NNN\_G}(\tilde{x}, I_Y, I_G) \geq 0, \quad x_3 \leq \tilde{x} \leq 1 \\
&\qquad \Pi_{Y3} - \Pi_{Y4} \geq 0 \\
&\qquad \Pi_{G3} - \Pi_{G1} \geq 0
\end{aligned}
\tag{3-11}
$$

The first two constraints are the consumers' participation constraints, while the last two ensure that this outcome actually holds – i.e., Y does not pay, but G does. Recall the two cases discussed in content providers' decisions. If Case I holds, i.e., $r_G < 2(1 - x_3) r_Y$, there is no feasible $p_3$ and Outcome 3 is not possible. If Case II holds, i.e., $r_G \geq 2(1 - x_3) r_Y$, the BSP's optimal choice of pricing strategy and its profits are given by the expressions

$$F_3 = V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda}, \quad p_3 = \left( \frac{1/2 - x_3}{1 - x_3} \right) r_G, \text{ and}$$

$$\Pi_3 = F_3 + (1 - x_3)\lambda p_3 = V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda} + \left( \frac{1}{2} - x_3 \right) \lambda r_G .$$

**Outcome 4:** Both content providers pay the priority price $p_4$ ($I_Y = I_G = 1$), so that neither content gets any relative advantage for delivery. The BSP now gains revenue from consumers as well as both content providers and therefore solves the following optimization problem:

$$
\begin{aligned}
&\max_{F_4, p_4} \Pi_4 = F_4 + \lambda p_4 \\
&\text{s.t.} \quad U_{NNN\_Y}(\tilde{x}, I_Y, I_G) \geq 0, \quad 0 \leq \tilde{x} \leq x_4 \\
&\qquad U_{NNN\_G}(\tilde{x}, I_Y, I_G) \geq 0, \quad x_4 \leq \tilde{x} \leq 1 \\
&\qquad \Pi_{Y4} - \Pi_{Y3} \geq 0 \\
&\qquad \Pi_{G4} - \Pi_{G2} \geq 0
\end{aligned}
\tag{3-12}
$$

It follows that the BSP's optimal pricing strategy is given by $F_4 = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ and $p_4 = (1 - 2x_3) r_Y$, while his profit is $\Pi_4 = F_4 + \lambda p_4 = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1 - 2x_3)\lambda r_Y$.

We note that $r_G \geq 2(1-x_3)r_Y$ (i.e., Case II) is a necessary condition for Outcome 3 to be an equilibrium. When this condition is not satisfied, i.e., $r_G < 2(1-x_3)r_Y$ (Case I), we have only two potential equilibria (Outcomes 1 and 4). Recall further that Outcome 2 is never an equilibrium as long as $r_G > r_Y$.

In order to determine his optimal pricing strategy $(F^*, p^*)$, the BSP compares the profits under the various outcomes and for a given set of parameter values, chooses its pricing strategy (which drives the equilibrium to one of the above-mentioned outcomes) to arrive at the highest profit. As the monopolist gatekeeper between the content providers and the customers, the BSP can essentially "drive" the direction of the equilibrium in such a way that it ensures the highest possible profits.

We can readily observe that $\Pi_4 > \Pi_1$ in both Case I and Case II. As a result, in Case I, the broadband provider will set the final $(F^*, p^*)$ to $F^* = F_4 = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$, $p^* = p_4 = (1-2x_3)r_Y$, and realize the profit $\Pi^* = \Pi_4 = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1-2x_3)\lambda r_Y$ since Outcomes 1 and 4 are the only two potential equilibria.

In Case II, the BSP needs to compare $\Pi_3$ with $\Pi_4$ in order to determine the outcome that gives the maximum profit, which leads to the following comparison:

$$\Pi_3 - \Pi_4 = \left[V(\lambda) - t(1-x_3) - \frac{d}{\mu - (1-x_3)\lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_G\right] - \left[V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1-2x_3)\lambda r_Y\right]$$

$= \left(\frac{1}{2} - x_3\right)\left[\lambda r_G - 2\lambda r_Y - t(1-2x_3)\right]$ after applying Equation (3-4) and some algebra.

We observe that if $r_G \geq 2r_Y + \frac{t}{\lambda}(1-2x_3)$, then $\Pi_3 \geq \Pi_4$. The broadband provider will then set the menu of prices $(F^*, p^*)$ to $F^* = F_3 = V(\lambda) - t(1-x_3) - \frac{d}{\mu-(1-x_3)\lambda}$, $p^* = p_3 = \left(\frac{1/2 - x_3}{1-x_3}\right) r_G$, and will realize the profit $\Pi^* = \Pi_3 = V(\lambda) - t(1-x_3) - \frac{d}{\mu-(1-x_3)\lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_G$.

Conversely, if $r_G < 2r_Y + \frac{t}{\lambda}(1-2x_3)$, then $\Pi_3 < \Pi_4$. The broadband provider will then set the menu of prices $(F^*, p^*)$ to $F^* = F_4 = V(\lambda) - \frac{t}{2} - \frac{d}{\mu-\lambda}$ and $p^* = p_4 = (1-2x_3)r_Y$ to attain a profit of $\Pi^* = \Pi_4 = V(\lambda) - \frac{t}{2} - \frac{d}{\mu-\lambda} + (1-2x_3)\lambda r_Y$.

Note that the condition that ensures the BSP to realize more profit in Outcome 3 than Outcome 4 also satisfies the condition for Outcome 3 to be feasible since $2r_Y + \frac{t}{\lambda}(1-2x_3) > 2(1-x_3)r_Y$. We thus simplify the combination of the profit comparison condition (between Outcomes 3 and 4) and the feasibility condition of Outcome 3 into the following two cases (as shown in Table 3-3).

**Case A:** Outcome 3 is not only feasible but also more profitable to the BSP than Outcome 4. Therefore, Outcome 3 is the equilibrium. The required condition for Case A is

$$r_G \geq 2r_Y + \frac{t}{\lambda}(1-2x_3) \tag{3-13}$$

**Case B:** Either Outcome 3 is not feasible, or when Outcome 3 is feasible, Outcome 4 is more profitable. Therefore, Outcome 4 is the equilibrium. The required condition for Case B is

$$r_G < 2r_Y + \frac{t}{\lambda}(1-2x_3) \tag{3-14}$$

Figure 3-4 summarizes these results graphically, by plotting $r_G$ against $r_Y$. The BSP's "choice" of the equilibrium in the game between the two content providers is dictated by the relative magnitudes of their revenue-generation capabilities. In Region A (corresponding to Case A), where $r_G$ is "significantly" greater than $r_Y$, the BSP chooses its pricing strategy in such a way as to drive the game to Outcome 3, where the content provider with higher profitability has the incentive to pay for priority delivery, while the content provider with lower profitability does not have the incentive to pay that fee. Conversely, in Region B (corresponding to Case B) where the content providers have more "comparable" revenue rates, the game will end up with Outcome 4, when both content providers pay the BSP the priority delivery fee.

The choice of making $r_G > r_Y$ is simply a matter of convenience of exposition, that the generalized results are symmetric on either side of the line $r_G = r_Y$ as indicated in Figure 3-5. Regions C and D can be interpreted analogously as Regions A and B.

**Winners and Losers – Comparison Between NN and NNN**

In this section, I consider the resulting surpluses for the various players. These results can then be used by the policymaker who has to decide whether to allow the BSP to charge for preferential service (i.e., opt for NNN), or continue to maintain the NN status quo. The policymaker can proceed to compare the equilibrium under NN and NNN, by evaluating the payoff for the BSP and content providers, consumer surplus, and social welfare under each regime.

From the BSP's point of view, NNN is preferred to NN since the profits in either Case A or Case B, $\Pi_3^*$ or $\Pi_4^*$ is higher than $\Pi_{NN}^*$. What is of interest to the policymaker is whether the other participants gain from this arrangement too and whether social welfare as a whole increases. The results of this analysis are summarized in Proposition 3-1.

**Proposition 3-1: (Winners and losers in the short run)**

The economic outcomes in the short-run under NN and NNN vary. Specifically, using NN as the benchmark,

- Social welfare would either increase or remain unchanged depending on parameter values as stated in conditions (3-13)-(3-14). Likewise, consumer surplus would increase or remain unchanged.
- Content providers are usually worse off under NNN except under Condition (3-13) when the content provider paying the priority delivery fee has the same surplus as under NN.
- The BSP is unambiguously better off.

Proof: See Appendix B.

These results are organized in Table 3-4. Appendix B contains the details of their derivations.

Clearly, the gains of abolishing net neutrality are not experienced equally. While the monopolist broadband service provider gains if no net neutrality were in place (in both Cases A and B of Table 3-4), the content providers are definitely worse off under this arrangement. Only content provider G's surplus is unchanged under Case A. It is interesting to note that G does not get to enjoy the increase in the number of consumers, since the extra rent is fully extracted by the Internet service provider. It is therefore no wonder why the content providers and the Internet service providers have been on the opposite sides of the net neutrality debate.

The fate of the end consumers is more nuanced. If the two content providers do not differ significantly (regions B or D in Figure 3-5) in terms of their revenue generation rates, the consumer surplus is unchanged. Consumers as a whole do stand to gain if one content provider is significantly better than the other in revenue generation (regions A and C in Figure 3-5). This increase in overall consumer surplus, however, is derived at the expense of the group of consumers whose content provider does not pay the priority charge, a result contrary to the

assertion of the BSPs that no consumer would be left worse off under the new arrangement (WSJ 2006).

Social welfare as a whole, in contrast, is at least as high under NNN as it is under NN, and is sometimes higher. Under NNN Case B, social welfare (like consumer surplus) does not change, but there is a transfer of wealth from the content providers to the Internet service provider. This transfer is made possible by the priority delivery charge that the Internet service provider extracts from both content providers, since the subscription fee to end users does not change ($F_4^* = F_1^*$). Under NNN Case A, the consumer surplus increases due for the most part to the lower subscription fees for all consumers ($F_3^* < F_1^*$). The winners under this arrangement are the consumers of content provider G (who are a majority) and the BSP, while the losers are the consumers of content provider Y and content provider Y itself. Content provider G's surplus from the additional consumers that have migrated from Y is fully siphoned away by the BSP.

**Capacity Expansion Decision – Does NN Hinder the Broadband Service Provider's Incentive to Expand Infrastructure Capacity?**

The other key question for the policymaker is the broadband service provider's motivation to expand capacity under NN. To discuss this question, I consider the long-run problem where the BSP can choose its capacity $\mu$.

Let $C(\mu)$ be the cost associated with capacity $\mu$. The long-run problem can be modeled as a three-stage game where the BSP chooses capacity $\mu$ and announces the Internet access fee $F$ to consumers and preferential delivery fee $p$ to content providers under NNN in the first stage. Based on the announced fees, in the second stage content providers choose whether to pay or not pay for preferential delivery, and in the third stage consumers choose between content provider Y and content provider G. In the long-run problem, we also need to consider the cost of

capacity expansion (this was not an issue in the preceding analysis, since in the short run the existing network capacity is fixed).

The objective is to determine the BSP's incentive to expand capacity and its optimal capacity decision under NN and NNN, and compare the two meaningfully in order to find the regime under which the incentive for the BSP to expand capacity is higher. Finally, the choice of regime (NN or NNN) is not under the control of the BSP, and is a choice that lies with the policymakers (who can calculate the aforementioned incentives).

I go through a process similar to that employed in analyzing the short-run problem to investigate the BSP's incentive to expand capacity and his optimal capacity choice.

**Decisions for Both Content Providers and Consumers in the Capacity Expansion Problem**

Given certain $F$ , $p$ , and $\mu$ , the analysis of the best responses for content providers and consumers do not differ from those in the short-term problem and is therefore not repeated here. The content providers' profits and consumers' utilities in the long-run problem will be multiplied by $\frac{1}{1-\delta}$ where $\delta$ is the discount factor. In other words, the content providers' long-run profits are $\pi_{\mathrm{Y}i} = \frac{1}{1-\delta}\Pi_{\mathrm{Y}i}$ for Y and $\pi_{\mathrm{G}i} = \frac{1}{1-\delta}\Pi_{\mathrm{G}i}$ for G, where $i = 1,2,3,4$ represents the four different outcomes. The long-run utility of an arbitrary consumer $\tilde{x} \in [0,1]$ is $u_{\mathrm{Y}}(\tilde{x}) = \frac{1}{1-\delta}U_{\mathrm{Y}}(\tilde{x})$ if content provider Y is chosen, and is $u_{\mathrm{G}}(\tilde{x}) = \frac{1}{1-\delta}U_{\mathrm{G}}(\tilde{x})$ if content provider G is chosen.

**Pricing and Capacity Expansion Decisions for the BSP in the Capacity Expansion Problem**

Expecting the best responses of content providers and consumers, I evaluate the optimal decision $(F^*, p^*, \mu^*)$ for the BSP.

**Under Net Neutrality or Outcome 1:** $x_1 = \frac{1}{2}$

The BSP's decision problem can be represented by Formulation (3-15).

$$\begin{aligned}
&\max_{F_1, p_1, \mu_1} \pi_1 = \frac{1}{1-\delta} F_1 - C(\mu_1) \\
&\text{s.t.} \quad u_{NNN_Y}(\tilde{x}, I_Y, I_G) \geq 0, \quad 0 \leq \tilde{x} \leq x_1 \\
&\qquad u_{NNN_G}(\tilde{x}, I_Y, I_G) \geq 0, \quad x_1 \leq \tilde{x} \leq 1 \\
&\qquad \pi_{Y1} - \pi_{Y2} \geq 0 \\
&\qquad \pi_{G1} - \pi_{G3} \geq 0
\end{aligned} \tag{3-15}$$

where $x_2$ and $x_3$ are as defined in Outcomes 2 and 3.

The long-run objective function represents the net cash flow for the BSP. We know that the first constraint is binding: i.e., $F_1^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu_1^* - \lambda}$. The optimal capacity $\mu_1^*$ can be derived by maximizing the long-term net cash flow $\pi_1^* = \frac{1}{1-\delta}\left[V(\lambda) - \frac{t}{2} - \frac{d}{\mu_1^* - \lambda}\right] - C(\mu_1^*)$. Equation (3-16) gives the first order condition of this optimization problem.

$$\frac{\partial \pi_1}{\partial \mu} = \frac{1}{1-\delta} \cdot \frac{d}{(\mu - \lambda)^2} - \frac{\partial C(\mu)}{\partial \mu} = 0 \tag{3-16}$$

The term $\frac{\partial \pi_1}{\partial \mu}$ is the marginal increase in profit of the BSP with respect to $\mu$, and as such we can interpret this term as the BSP's incentive to expand capacity.

**Outcome 2:** It can be proved analogously as discussed earlier in the short-run problem that given the nature of the parameter values, this outcome is not possible.

**Outcome 3:** $x_3 < \frac{1}{2}$, determined by $tx_3 + \frac{d\mu_3}{[\mu_3 - (1-x_3)\lambda](\mu_3 - \lambda)} = t(1-x_3) + \frac{d}{\mu_3 - (1-x_3)\lambda}$.

As before, $F_3^* = V(\lambda) - t(1-x_3) - \frac{d}{\mu_3^* - (1-x_3)\lambda}$, and $p_3^* = \left(\frac{1/2 - x_3}{1 - x_3}\right) r_G$.

The BSP will choose the optimal capacity $\mu_3^*$ to maximize the long-term net cash flows once again:

$$\pi_3^* = \frac{1}{1-\delta}\left[V(\lambda) - t(1-x_3) - \frac{d}{\mu_3^* - (1-x_3)\lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_G\right] - C(\mu_3^*)$$

$$= \frac{1}{1-\delta}\left[V(\lambda) - \frac{1}{2}t + \left(\frac{1}{2} - x_3\right)\left(\lambda r_G - 2t\frac{\mu}{\lambda} + t\right)\right] - C(\mu_3^*)$$

Then, $\mu_3^*$ can be characterized by the first order condition in Equation (3-17):

$$\frac{\partial \pi_3}{\partial \mu} = \frac{1}{1-\delta}\left[t\left(\frac{2\mu}{\lambda} - 1\right) - \lambda r_G\right]\frac{\partial x_3}{\partial \mu} - \frac{1}{1-\delta}\cdot\frac{2t}{\lambda}\left(\frac{1}{2} - x_3\right) - \frac{\partial C(\mu)}{\partial \mu} = 0 \tag{3-17}$$

**Outcome 4:** $x_4 = \frac{1}{2}$, $F_4^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu_4^* - \lambda}$ and $p_4^* = (1-2x_3)r_Y$.

Similarly, the corresponding net cash flows are given by

$\pi_4^* = \frac{1}{1-\delta}\left[V(\lambda) - \frac{t}{2} - \frac{d}{\mu_4^* - \lambda} + (1-2x_3)\lambda r_Y\right] - C(\mu_4^*)$ and $\mu_4^*$ is the solution of Equation (3-18).

$$\frac{\partial \pi_4}{\partial \mu} = \frac{1}{1-\delta}\cdot\frac{d}{(\mu-\lambda)^2} - \frac{2}{1-\delta}\cdot\frac{\partial x_3}{\partial \mu}\lambda r_Y - \frac{\partial C(\mu)}{\partial \mu} = 0 \tag{3-18}$$

Comparing the BSP's incentive to expand capacity under NNN (either $\frac{\partial \pi_3}{\partial \mu}$ or $\frac{\partial \pi_4}{\partial \mu}$) to that under NN, the BSP has more incentive to expand capacity under NN when

$$r_G < 2r_Y + \frac{t}{\lambda}(1-2x_3) \text{ or } r_G \geq \frac{2t}{\lambda}(1-2x_3) \tag{3-19}$$

and the BSP has more incentive to expand capacity under NNN when

$$2r_Y + \frac{t}{\lambda}(1-2x_3) \leq r_G < \frac{2t}{\lambda}(1-2x_3) \tag{3-20}$$

These conditions are graphed in Figure 3-6 with Condition (3-19) corresponding to the shaded

area and Condition (3-20) corresponding to the unshaded area. Proposition 3-2 summarizes the result concerning BSP's incentive to expand infrastructure capacity.

**Proposition 3-2: (The BSP's incentive to expand infrastructure capacity)**

Except for the region defined in Condition (3-20), the BSP has more incentive to expand capacity under net neutrality.

Proof: See Appendix B.

To understand the nature of the result in Figure 3-6, note that the capacity costs under NN and under NNN are identical given any same capacity level. Hence, we only need to focus on the revenue of the BSP. Further, under NN, the BSP's revenue $\frac{1}{1-\delta}\left[V(\lambda)-\frac{t}{2}-\frac{d}{\mu-\lambda}\right]$ is derived solely from the consumers while the BSP's revenue under NNN has two components: the contribution from the consumers and the contribution from the content providers. The revenue contribution from consumers increases in $\mu$ (as consumers enjoy reduced congestion), while the revenue contribution from the content providers decrease in $\mu$ (as the content providers have a reduced willingness to pay for priority delivery when congestion is reduced). Specifically, when both content providers pay (Regions B and D in Figure 3-6), the BSP's long-run revenue from consumers is $\frac{1}{1-\delta}\left[V(\lambda)-\frac{t}{2}-\frac{d}{\mu-\lambda}\right]$, which increases in $\mu$. The BSP's long-run revenue from the content providers is $\frac{1}{1-\delta}(1-2x_3)\lambda r_Y$, which decreases in $\mu$. Thus, by increasing the capacity $\mu$, the BSP platform gains from the consumers' side and loses from the content providers' side. Since the gains under NN and under NNN (Case B) are the same and the BSP incurs a loss from the content providers' side under NNN from expanding the infrastructure

capacity, the BSP will almost always have a higher incentive to increase capacity under net neutrality.

When only one content provider pays (Region A or C in Figure 3-6), and, the contribution from the consumers is $\frac{1}{1-\delta}\left[V(\lambda)-t(1-x_3)-\frac{d}{\mu-(1-x_3)\lambda}\right]$ which increases in $\mu$ and the contribution from the content providers is $\frac{1}{1-\delta}\left(\frac{1}{2}-x_3\right)\lambda r_G$ which decreases in $\mu$. The BSP gains from the consumers' side and loses from the content providers' side when increasing capacity $\mu$, a result similar to the case where both content providers pay. It is only in the small unshaded area in Figure 3-6 that under Condition (3-20) the gain outweighs the loss to give the BSP more incentive to expand the capacity under NNN.

From the policymaker's perspective, the questions of utmost interest concern the BSP's optimal capacity choices under NN and NNN, and whether the BSP's optimal capacity choices under NN and NNN are socially optimal. Propositions 3-3 and 3-4 provide useful guidance to the policymaker in addressing these questions.

**Proposition 3-3: (The BSP's optimal capacity choice)**

Except for the region defined in Condition (3-20), the optimal capacity choice under net neutrality (NN) is higher than under no net neutrality (NNN).

Proof: See Appendix B.

**Proposition 3-4: (Whether the BSP's optimal capacity choice is socially optimal?)**

The BSP always invests at the socially optimal level under net neutrality. Abolishing net neutrality results in underinvestment in infrastructure capacity by the BSP when both content providers pay the priority delivery charge, and either underinvestment or overinvestment when only one content provider pays the priority charge.

Proof: See Appendix B.

A corollary of Propositions 3-3 and 3-4 is that while the BSP's optimal capacity choice might be higher under NNN than under NN in some specific instances, this higher capacity choice reduces social welfare.

The comparative statics for the various pricing variables and the surpluses of the various parties involved with respect to the capacity $\mu$ are summarized in Table 3-5.

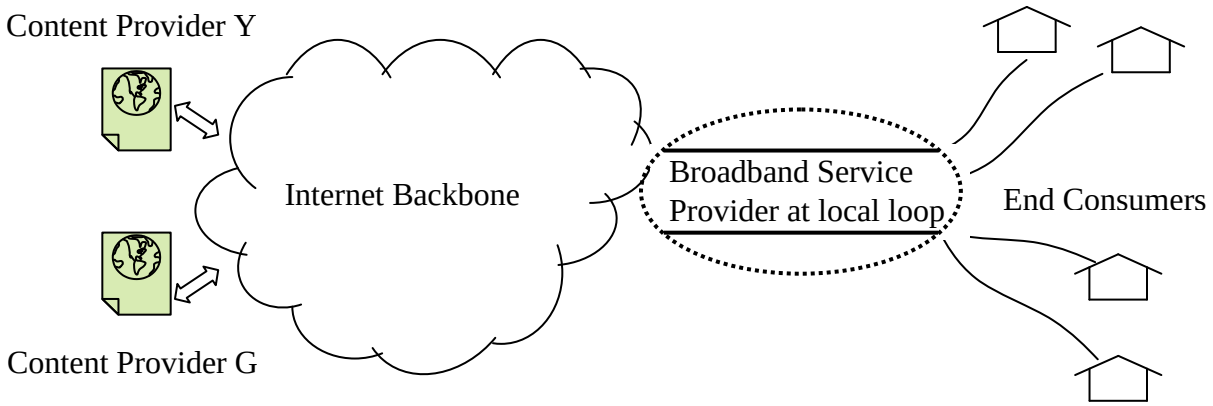



Figure 3-1. Schematic of the model


Figure 3-2. Content providers and their share of consumers with full market coverage

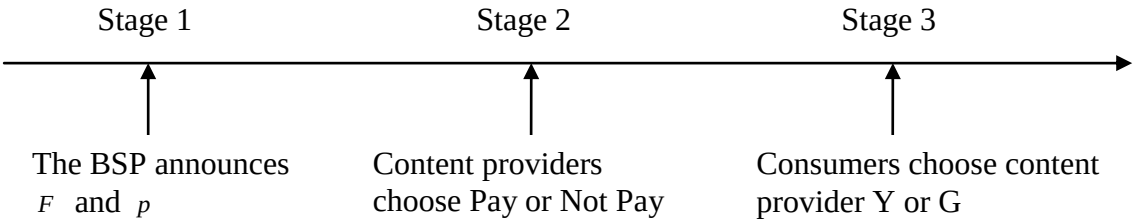



Figure 3-3. The sequence of events in the game

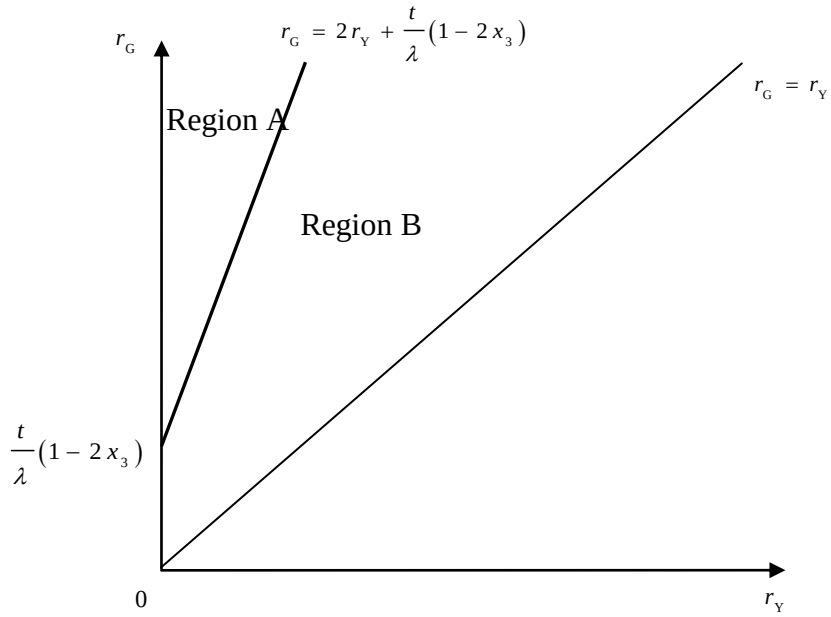



Figure 3-4. Graphical representation of the regions for arriving at different equilibria of the game when $r_G > r_Y$

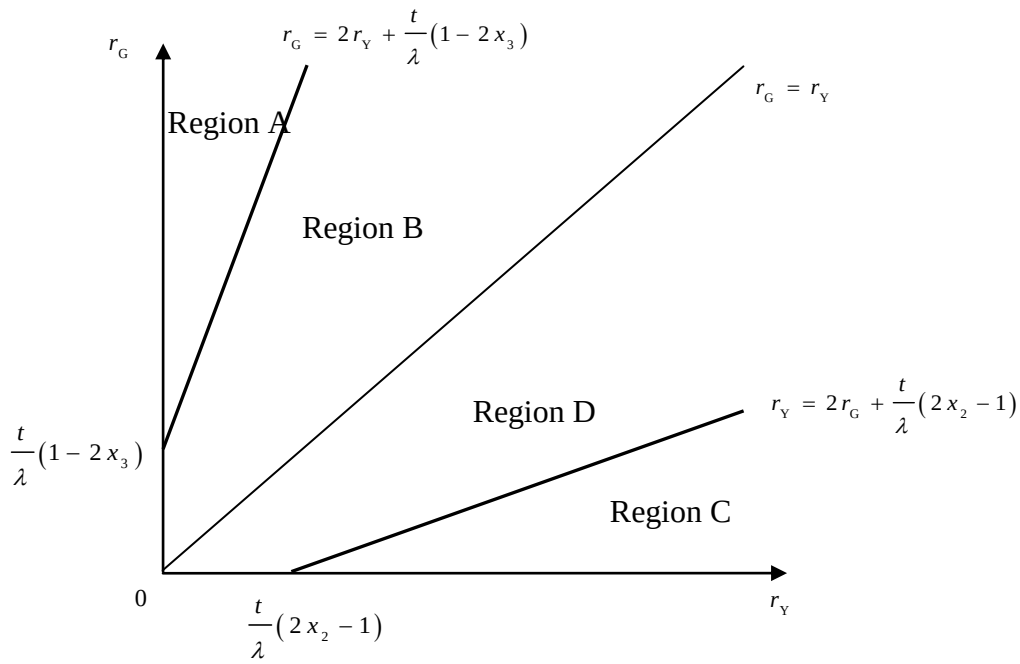



Figure 3-5. Generalized representation of the regions for arriving at different equilibria of the game

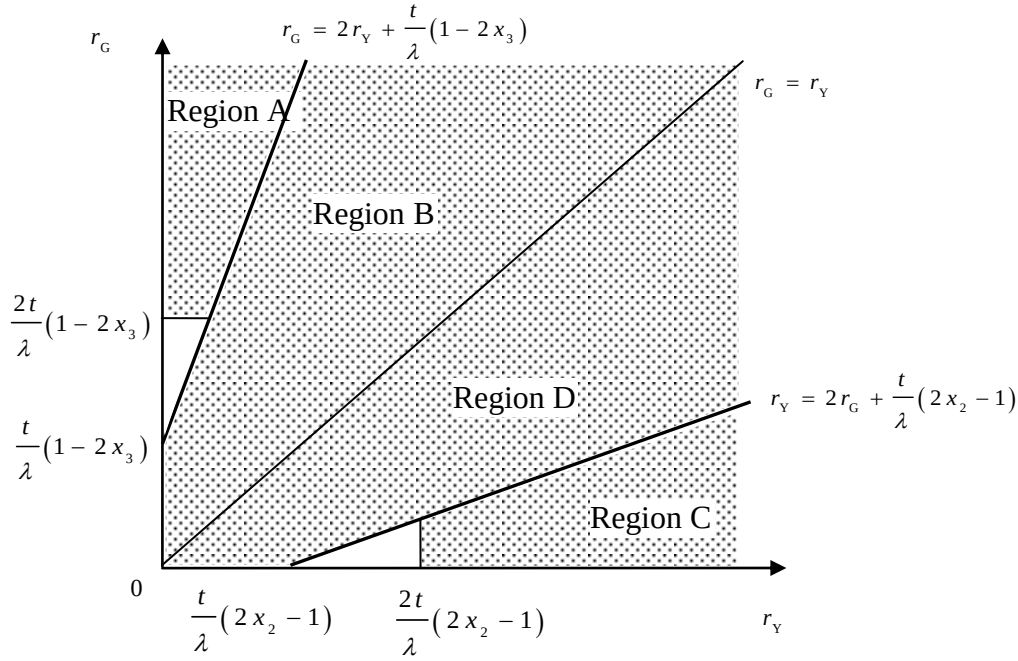



Figure 3-6. BSP's incentive to expand capacity. In the shaded region, the BSP has higher incentive to expand capacity and the optimal capacity level is higher under NN. In the white region, the BSP has lower incentive to expand capacity and the optimal capacity level is lower under NN.

Table 3-1. Delays under No Net Neutrality

| | G pays | G does not pay |
|---|---|---|
| Y pays | $w_{Y1} = \frac{1}{\mu-\lambda}$, $w_{G1} = \frac{1}{\mu-\lambda}$ | $w_{Y3} = \frac{\mu}{\left[\mu-(1-x_3)\lambda\right](\mu-\lambda)}$, $w_{G3} = \frac{1}{\mu-(1-x_3)\lambda}$ |
| Y does not pay | $w_{Y2} = \frac{1}{\mu-x_2\lambda}$, $w_{G2} = \frac{\mu}{(\mu-x_2\lambda)(\mu-\lambda)}$ | $w_{Y4} = \frac{1}{\mu-\lambda}$, $w_{G4} = \frac{1}{\mu-\lambda}$ |

Table 3-2. Content providers' payoffs

| | G does not pay | G pays |
|---|---|---|
| Y does not pay | $\Pi_{Y1} = \frac{1}{2}\lambda r_Y$ , $\Pi_{G1} = \frac{1}{2}\lambda r_G$ | $\Pi_{Y3} = x_3 \lambda r_Y$ , $\Pi_{G3} = (1 - x_3)\lambda r_G - (1 - x_3)\lambda p$ |
| Y pays | $\Pi_{Y2} = x_2 \lambda r_Y - x_2 \lambda p$ , $\Pi_{G2} = (1 - x_2)\lambda r_G$ | $\Pi_{Y4} = \frac{1}{2}\lambda r_Y - \frac{1}{2}\lambda p$ , $\Pi_{G4} = \frac{1}{2}\lambda r_G - \frac{1}{2}\lambda p$ |

Table 3-3. Summary of results of the game

| Case A: $r_G \geq 2r_Y + \frac{t}{\lambda}(1 - 2x_3)$ | Case B: $r_G < 2r_Y + \frac{t}{\lambda}(1 - 2x_3)$ |
|---|---|
| $F^* = F_3^* = V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda}$ <br> $p^* = p_3^* = \left(\frac{1/2 - x_3}{1 - x_3}\right) r_G$ <br> $\Pi^* = \Pi_3^* = V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_G$ | $F^* = F_4^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ <br> $p^* = p_4^* = (1 - 2x_3) r_Y$ <br> $\Pi^* = \Pi_4^* = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1 - 2x_3)\lambda r_Y$ |

Table 3-4. Comparison of various economic outcomes of interest under NN and NNN

| | NN (Benchmark) | NNN (Case A: Only G pays) | NNN (Case B: Both Y and G pay) |
|---|---|---|---|
| $F$ | $F_{NN} = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ | $F_{NNN\_A} = F_3 = V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda} < F_{NN}$ (Lower) | $F_{NNN\_B} = F_4 = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} = F_{NN}$ (Unchanged) |
| $p$ | N/A | $p_{NNN\_A} = p_3 = \left(\frac{1/2 - x_3}{1 - x_3}\right) r_G$ | $p_{NNN\_B} = p_4 = (1 - 2x_3) r_Y$ |
| BSP's Revenue | $\Pi_{NN\_BSP} = F_{NN} = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ | $\Pi_{NNN\_A\_BSP} = \Pi_3$ $= V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_G > \Pi_{NN}$ (Better off) | $\Pi_{NNN\_B\_BSP} = \Pi_4$ $= V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1 - 2x_3)\lambda r_Y > \Pi_{NN}$ (Better off) |
| Content Provider Y's Profit | $\pi_{NN\_Y} = \frac{1}{2}\lambda r_Y$ | $\pi_{NNN\_A\_Y} = x_3 \lambda r_Y < \pi_{NN\_Y}$ (Worse off) | $\pi_{NNN\_B\_Y} = x_3 \lambda r_Y < \pi_{NN\_Y}$ (Worse off) |
| Content Provider G's Profit | $\pi_{NN\_G} = \frac{1}{2}\lambda r_G$ | $\pi_{NNN\_A\_G} = \frac{1}{2}\lambda r_G = \pi_{NN\_G}$ (Unchanged) | $\pi_{NNN\_B\_G} = \frac{1}{2}\lambda \left[r_G - (1 - 2x_3) r_Y\right] < \pi_{NN\_G}$ (Worse off) |
| Consumer Surplus | $CS_{NN} = \frac{t}{4}$ | $CS_{NNN\_A} = t\left(x_3^2 - x_3 + \frac{1}{2}\right) > CS_{NN}$ (Better off) | $CS_{NNN\_B} = \frac{t}{4} = CS_{NN}$ (Unchanged) |
| Social Welfare | $SW_{NN}$ $= V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_Y + \frac{1}{2}\lambda r_G$ | $SW_{NNN\_A}$ $= V(\lambda) - t\left(\frac{1}{2} - x_3^2\right) - \frac{d}{\mu - (1 - x_3)\lambda} + x_3 \lambda r_Y + (1 - x_3)\lambda r_G$ $> SW_{NN}$ (Increased) | $SW_{NNN\_B}$ $= V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_Y + \frac{1}{2}\lambda r_G$ $= SW_{NN}$ (Unchanged) |

Note: The text in parenthesis shows how those specific economic outcomes change when moving from NN to NNN.

Table 3-5. Comparative statics with respect to capacity $\mu$

| | NN (Benchmark) | NNN (Case A: Only G pays) | NNN (Case B: Both Y and G pay) |
|---|---|---|---|
| $F$ | $F_{NN} = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ <br> $\boxed{+}$ | $F_{NNN\_A} = F_3 = V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda}$ <br> $\boxed{+}$ | $F_{NNN\_B} = F_4 = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ <br> $\boxed{+}$ |
| $p$ | N/A | $p_{NNN\_A} = p_3 = \left(\frac{1/2 - x_3}{1 - x_3}\right) r_G$ <br> $\boxed{-}$ | $p_{NNN\_B} = p_4 = (1 - 2x_3) r_Y$ <br> $\boxed{-}$ |
| BSP's Revenue | $\Pi_{NN\_BSP} = F_1 = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ <br> $\boxed{+}$ | $\Pi_{NNN\_A\_BSP} = \Pi_3$ <br> $= V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_G$ <br> $\boxed{?}$ | $\Pi_{NNN\_B\_BSP} = \Pi_4$ <br> $= V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1 - 2x_3)\lambda r_Y$ <br> $\boxed{?}$ |
| Content Provider Y's Profit | $\pi_{NN\_Y} = \frac{1}{2}\lambda r_Y$ <br> $\boxed{0}$ | $\pi_{NNN\_A\_Y} = x_3 \lambda r_Y$ <br> $\boxed{+}$ | $\pi_{NNN\_B\_Y} = x_3 \lambda r_Y$ <br> $\boxed{+}$ |
| Content Provider G's Profit | $\pi_{NN\_G} = \frac{1}{2}\lambda r_G$ <br> $\boxed{0}$ | $\pi_{NNN\_A\_G} = \frac{1}{2}\lambda r_G$ <br> $\boxed{0}$ | $\pi_{NNN\_B\_G} = \frac{1}{2}\lambda \left[ r_G - (1 - 2x_3) r_Y \right]$ <br> $\boxed{+}$ |
| Consumer Surplus | $CS_{NN} = \frac{t}{4}$ <br> $\boxed{0}$ | $CS_{NNN\_A} = t\left(x_3^2 - x_3 + \frac{1}{2}\right)$ <br> $\boxed{-}$ | $CS_{NNN\_B} = \frac{t}{4}$ <br> $\boxed{0}$ |
| Social Welfare | $SW_{NN} = V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_Y + \frac{1}{2}\lambda r_G$ <br> $\boxed{+}$ | $SW_{NNN\_A}$ <br> $= V(\lambda) - t\left(\frac{1}{2} - x_3^2\right) - \frac{d}{\mu - (1 - x_3)\lambda} + x_3 \lambda r_Y + (1 - x_3)\lambda r_G$ <br> $\boxed{?}$ | $SW_{NNN\_B}$ <br> $= V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_Y + \frac{1}{2}\lambda r_G$ <br> $\boxed{+}$ |

Legend: $\boxed{+}$ : increasing in $\mu$ ; $\boxed{-}$ : decreasing in $\mu$ ; $\boxed{0}$ : independent of $\mu$ ; $\boxed{?}$ : depends on parameter values.

# CHAPTER 4
# CONTENT PROVIDER DISCRIMINATION AND MARKET COVERAGE

In all extant literature on net neutrality, the focus has been either on the competition between content providers when the BSP institutes a fee for preferential delivery by assuming full market coverage (Chapter 3 in this dissertation) or on market coverage under the assumption of no competition between the content providers (Economides and Tag 2007; Hermalin and Katz 2007). The reason for these simplifications is analytical tractability: a two-sided market framework that models the externalities of limited bandwidth[1] along with intra-group and inter-group strategic effects between agents on both sides of the two-sided platform introduces significant analytical challenges by itself. For example, a standard assumption made in existing literature on two-sided markets is that of full market coverage in order to ensure analytical closure (Armstrong 2006).

In this chapter, I relax the requirement for full market coverage and analyze the effect of the proposed net neutrality legislation on broadband market coverage. The resulting analysis thus models the three fundamental aspects of the net neutrality debate: (1) the negative externalities associated with the constrained bandwidth between the BSP and the consumer; (2) the competition between content providers that makes it possible for the BSP to charge for preferential delivery;[2] and (3) the possibility that the content providers might subsidize broadband access for the consumers, which would result in enhanced broadband market coverage.

---

[1] A crucial aspect of the network neutrality debate is based on the fact that there is a constrained resource (the "pipe" between the BSP and the consumer) that is shared by packets of all the content providers, and the very existence of this constrained resource has negative externalities associated with it (to see the veracity of this claim, consider this – if this "pipe" were not a constrained resource, the BSPs would not have any credible mechanism to charge for preferential access).

[2] Modeling the competition between the content providers is essential – if there were no competition between the content providers, the BSPs could not have credibly instituted a charge for preferential delivery.

More importantly, addressing this modeling challenge enables us to answer three key research questions that are of interest to academics, practitioners and policymakers alike. First, how does net neutrality affect the BSP's market coverage? Second, how does the BSP's pricing affect consumer surplus and social welfare? And finally, can the abandonment of the net neutrality principle result in some providers being shut out of the market, thereby affecting "innovation at the edge" of the Internet? The last question addresses the concern of some net neutrality proponents who have argued that in the absence of net neutrality regulation, online competition and innovation will be reduced in the long run. Answers to all these questions have significant policy implications on the future of broadband access.

One novel aspect of this chapter is a methodological contribution (detailed in the "Computational Analyses" section), whereby I take advantage of the structural properties of the analytical model to suggest an innovative algorithm to tackle the model which is otherwise analytically intractable.

## The Model

Following the model proposed in Chapter 3, I consider a unit mass of consumers. As shown in Figure 4-1, consumers are uniformly located on $[0,1]$ with two content providers at the two ends of the line segment. For any $x \in [0,1]$, the consumer gets utility

$u_{\mathrm{Y}}(x) = V(\lambda) - t \cdot x - d \cdot w_{\mathrm{Y}} - F$ by subscribing to content provider Y, utility

$u_{\mathrm{G}}(x) = V(\lambda) - t(1-x) - d \cdot w_{\mathrm{G}} - F$ by subscribing to content provider G and utility $u_0(x) = 0$ by staying out of the market (i.e., by subscribing to neither). The consumer located at $x$ then compares the three options with their corresponding utilities of $u_{\mathrm{Y}}(x)$, $u_{\mathrm{G}}(x)$ and $u_0(x)$ and chooses the one with the highest utility.

The consumers' decisions can be characterized by their marginal consumers. As shown in Figure 4-1, $y$ and $1-z$ are marginal consumers for Y and G, respectively, with $y \le 1-z$. Then the market shares for content providers Y and G are $y$ and $z$ respectively. A special case of $y = 1-z$ corresponds to the case of full market coverage in which marginal consumers are indifferent between content provider Y and content provider G. When $y < 1-z$, the marginal consumers of each of the content providers are indifferent between subscribing to the preferred content provider and not subscribing to any Internet access service at all.

The timing of the game and other model setups are the same as the model presented in Chapter 3.

**Net Neutrality**

With net neutrality, the only revenue source for the BSP is the Internet access charge from consumers. Specifically, the BSP solves the following optimization problem:

$$
\begin{aligned}
\max_{F_1} \quad & \pi_1 = F_1 (y_1 + z_1) \\
\text{s.t.} \quad & V - ty_1 - \frac{d}{\mu - (y_1 + z_1)\lambda} - F_1 \ge 0 && (i) \\
& V - tz_1 - \frac{d}{\mu - (y_1 + z_1)\lambda} - F_1 \ge 0 && (ii) \\
& V - ty_1 - \frac{d}{\mu - (y_1 + z_1)\lambda} - F_1 \ge V - t(1 - y_1) - \frac{d}{\mu - (y_1 + z_1)\lambda} - F_1 && (iii) \\
& V - tz_1 - \frac{d}{\mu - (y_1 + z_1)\lambda} - F_1 \ge V - t(1 - z_1) - \frac{d}{\mu - (y_1 + z_1)\lambda} - F_1 && (iv)
\end{aligned}
\tag{4-1}
$$

Constraints $(i)$ and $(ii)$ in Formulation (4-1) are the participation constraints for the consumers and constraints $(iii)$ and $(iv)$ are the incentive compatibility constraints for the consumers. Solving the BSP's profit maximization problem yields Proposition 4-1.

**Proposition 4-1: (Market coverage under net neutrality)**

Under net neutrality, (1) when $V \geq V_{H1}$, where $V_{H1} \equiv t + \frac{d\mu}{(\mu - \lambda)^2}$, $y_1 = z_1 = 1/2$, i.e., the BSP covers the whole market; and (2) when $V < V_{H1}$, there exists a unique $0 < y_1 = z_1 < 1/2$, when the BSP covers only part of the market.

Proof: See Appendix B.

**No Net Neutrality**

Under NNN, four possible outcomes can take place: (1) The BSP elects not to charge either content provider for preferential delivery, which is the same outcome as would occur under a net neutrality regime (we call this Outcome 1); (2) the BSP decides to charge the content providers for preferential delivery of their packets, but only content provider Y decides to pay that fee (Outcome 2); (3) analogous to the previous outcome, but now only content provider G decides to pay (Outcome 3); and (4) given the BSP's menu of prices, both content providers decide to pay the preferential delivery fee. For each of these outcomes $i$, where $i = 1, 2, 3, 4$, we denote the fixed price that the BSP charges the consumers to be $F_i$, and a per-packet priority delivery fee $p_i$ (where $p_1 = 0$) to the content providers, and as a result gets a profit of $\pi_i$. The marginal consumer for content provider Y is denoted by $y_i$, while the marginal consumer for content provider G is denoted by $z_i$.

Each of these outcomes can arise in equilibrium, depending on the magnitudes of the exogenous parameters. The monopolist BSP can calculate its profit in each of the outcomes, and depending on the magnitudes of the exogenous parameters, any one of the outcomes can conceivably produce the highest profit for the BSP. In other words, the BSP, in its capacity as a

two-sided market platform, can effectively use its menu of prices $(F_i, p_i)$ to "guide" the equilibrium of the game to whichever outcome that would secure it the highest profit.

We next analyze the different outcomes, noting that **Outcome 1** (i.e., the outcome under the net neutrality regime) has already been analyzed in the previous section.

**Outcome 2:** (Y pays and G does not pay)

The BSP's problem is:

$$
\begin{aligned}
\max_{F_2, p_2} \quad & \pi_2 = F_2(y_2 + z_2) + y_2 \lambda p_2 \\
\text{s.t.} \quad & y_2 \left( V - ty_2 - \frac{d}{\mu - y_2\lambda} - F_2 \right) \geq 0 && (i) \\
& z_2 \left[ V - tz_2 - \frac{d\mu}{(\mu - y_2\lambda)(\mu - y_2\lambda - z_2\lambda)} - F_2 \right] \geq 0 && (ii) \\
& V - ty_2 - \frac{d}{\mu - y_2\lambda} - F_2 \geq V - t(1 - y_2) - \frac{d\mu}{(\mu - y_2\lambda)(\mu - y_2\lambda - z_2\lambda)} - F_2 && (iii) \\
& V - tz_2 - \frac{d\mu}{(\mu - y_2\lambda)(\mu - y_2\lambda - z_2\lambda)} - F_2 \geq V - t(1 - z_2) - \frac{d}{\mu - y_2\lambda} - F_2 && (iv) \\
& (y_2\lambda r_Y - y_2\lambda p_2) - y_1\lambda r_Y \geq 0 && (v) \\
& z_2\lambda r_G - (z_4\lambda r_G - z_4\lambda p_2) \geq 0 && (vi) \\
& F_2 \geq 0,\ p_2 \geq 0,\ 0 \leq y_2 \leq 1,\ 0 \leq z_2 \leq 1 && (vii)
\end{aligned}
\tag{4-2}
$$

In Formulation (4-2), constraints $(i)$ and $(ii)$ denote the consumers' participation constraints. Only when $y_2 > 0$, consumers' participation constraints for Y take effect. Similarly, only when $z_2 > 0$, consumers' participation constraints for G take effect. Notice when $y_2 = 0$ or $z_2 = 0$, constraint $(i)$ or $(ii)$ would be automatically satisfied – this type of a formulation is necessary in order to ensure that when Y or G have no market share, the BSP would not have to take into account their respective participation constraints. Constraints $(iii)$ and $(iv)$ represent the consumers' content choice constraints: the consumers of Y always prefer Y over G, while the

consumers of G prefer it over Y. The last two constraints are content providers' incentive compatibility constraints: that is, the content provider Y prefers this outcome (where it has a market share of $y_2$ by paying the priority delivery fee $p_2$ to the BSP) over Outcome 1 (and it compares its payoff in Outcome 2 to that of Outcome 1 since in that outcome G still does not pay), where it does not pay a fee and has a market share of $y_1$; and analogously, content provider G prefers this outcome to Outcome 4 (when it does pay the BSP, and so does Y).

Note that as a result of paying the priority delivery fee, packets from Y face an average delay only to the extent of the presence of other packets from Y, while the packets of G get "de-prioritized" as a result. In other words, we use a two-class priority queue with preemption to depict the waiting time for the consumers of these two content providers. For any consumer situated at $\tilde{x} \in [0,1]$, data packets from Y are transmitted with higher priority with a waiting time of $\frac{1}{\mu - \tilde{x}\lambda}$ while data packets from G are transmitted with lower priority with a waiting time of $\frac{\mu}{(\mu - \tilde{x}\lambda)(\mu - \lambda)}$.

**Lemma 4-1: (Feasibility Condition for Outcome 2):**

A necessary and sufficient condition for Outcome 2 to be feasible is

$$\left(1 - \frac{z_2}{z_4}\right) r_G \le \left(1 - \frac{y_1}{y_2}\right) r_Y .$$

Proof: See Appendix B.

**Lemma 4-2 (Optimization Condition for Outcome 2):**

When Outcome 2 is feasible, $p_2^* = \left(1 - \frac{y_1}{y_2}\right) r_Y$.

Proof: See Appendix B.

**Outcome 3:** (G pays and Y does not pay)

The BSP's profit maximization strategy now is:

$$\max_{F_3, p_3} \ \pi_3 = F_3 (y_3 + z_3) + z_3 \lambda p_3$$

$$\text{s.t.} \quad y_3 \left[ V - ty_3 - \frac{d\mu}{(\mu - z_3\lambda)(\mu - y_3\lambda - z_3\lambda)} - F_3 \right] \geq 0 \qquad (i)$$

$$z_3 \left( V - tz_3 - \frac{d}{\mu - z_3\lambda} - F_3 \right) \geq 0 \qquad (ii)$$

$$y_3 \lambda r_Y - (y_4 \lambda r_Y - y_4 \lambda p_3) \geq 0 \Rightarrow \ p_3 \geq \left( 1 - \frac{y_3}{y_4} \right) r_Y \qquad (iii)$$

$$(z_3 \lambda r_G - z_3 \lambda p_3) - z_1 \lambda r_G \geq 0 \Rightarrow \ p_3 \leq \left( 1 - \frac{z_1}{z_3} \right) r_G \qquad (iv)$$

$$V - ty_3 - \frac{d\mu}{(\mu - z_3\lambda)(\mu - y_3\lambda - z_3\lambda)} - F_3 \geq V - t(1 - y_3) - \frac{d}{\mu - z_3\lambda} - F_3 \qquad (v)$$

$$V - tz_3 - \frac{d}{\mu - z_3\lambda} - F_3 \geq V - t(1 - z_3) - \frac{d\mu}{(\mu - z_3\lambda)(\mu - y_3\lambda - z_3\lambda)} - F_3 \qquad (vi)$$

$$F_3 \geq 0, \ p_3 \geq 0, \ 0 \leq y_3 \leq 1, \ 0 \leq z_3 \leq 1 \qquad (vii) \qquad (4\text{-}3)$$

In Formulation (4-3), constraints $(i)$ and $(ii)$ denote the consumers' participation constraints, with the LHS of the two inequalities reflecting the fact that the BSP needs to consider a content provider's participation constraint only when its market share is positive, i.e., only when $y_3 > 0$, consumers' participation constraints for Y take effect; and similarly, only when $z_3 > 0$, consumers' participation constraints for G take effect. Notice when $y_3 = 0$ or $z_3 = 0$, constraint $(i)$ or $(ii)$ would be automatically satisfied. The constraints $(iii)$ and $(iv)$ represent the incentive compatibility constraints of the content providers (the relevant alternative outcome for Y is now Outcome 4, while that of G is Outcome 1); and $(v)$ and $(vi)$ represent the fact that the consumers of each of the content providers prefer it over its rival. Once again, the utility functions show the delay costs associated with a two-class priority queue.

**Proposition 4-2: (Outcome 3 dominates Outcome 2)**

Outcome 3 always yields at least as high a profit for the BSP as Outcome 2.

Proof: See Appendix B.

**Lemma 4-3 (Feasibility Condition for Outcome 3):**

A necessary and sufficient condition for Outcome 3 to be feasible is

$$\left(1 - \frac{y_3}{y_4}\right) r_Y \leq \left(1 - \frac{z_1}{z_3}\right) r_G \ .$$

Proof: Similar to the proof of Lemma 4-1.

**Lemma 4-4 (Optimization Condition for Outcome 3):**

When Outcome 3 is feasible, $p_3^* = \left(1 - \frac{z_1}{z_3}\right) r_G$ .

Proof: See Appendix B.

**Outcome 4:** (Both content providers decide to pay the BSP)

We note that under this outcome, neither content provider gets a differential advantage in terms of priority delivery of its packets: in other words, the only concession they extract by paying the fee is that its packets will not get de-prioritized with respect to its rival's packets. A logical question that arises is why would either content provider then decide to pay? The answer lies in the fact that this represents a Prisoner's dilemma for the content providers: neither of them would ideally have liked to pay the BSP (and would indeed be better off if both decided not to pay), but would be in a worse-off situation compared to the outcome where it decided not to pay but the other provider did.

The BSP's profit maximization problem under Outcome 4 is as follows:

$$
\begin{aligned}
\max_{F_4, p_4} \quad & \pi_4 = (F_4 + \lambda p_4)(y_4 + z_4) \\
\text{s.t.} \quad & V - ty_4 - \frac{d}{\mu - y_4\lambda - z_4\lambda} - F_4 \geq 0 && (i) \\
& V - tz_4 - \frac{d}{\mu - y_4\lambda - z_4\lambda} - F_4 \geq 0 && (ii) \\
& b_3\left[(y_4\lambda r_Y - y_4\lambda p_4) - y_3\lambda r_Y\right] \geq 0 && (iii) \\
& b_2\left[(z_4\lambda r_G - z_4\lambda p_4) - z_2\lambda r_G\right] \geq 0 && (iv) \\
& V - ty_4 - \frac{d}{\mu - y_4\lambda - z_4\lambda} - F_4 \geq V - t(1 - y_4) - \frac{d}{\mu - y_4\lambda - z_4\lambda} - F_4 \Rightarrow y_4 \leq 1/2 && (v) \\
& V - tz_4 - \frac{d}{\mu - y_4\lambda - z_4\lambda} - F_4 \geq V - t(1 - z_4) - \frac{d}{\mu - y_4\lambda - z_4\lambda} - F_4 \Rightarrow z_4 \leq 1/2 && (vi) \\
& F_4 \geq 0,\ p_4 \geq 0,\ 0 \leq y_4 \leq 1,\ 0 \leq z_4 \leq 1,\ b_2 \in \{0,1\},\ b_3 \in \{0,1\} && (vii)
\end{aligned}
\tag{4-4}
$$

In Formulation (4-4), constraints $(i)$ and $(ii)$ denote the consumers' participation constraints. The constraints $(iii)$ and $(iv)$ represent the incentive compatibility constraints of the content providers (the relevant alternative outcome for Y is now Outcome 3, while that of G is Outcome 2). Binary variables $b_2$ (or $b_3$) represents whether Outcome 2 (or Outcome 3) is feasible. These two variables are defined as:

$b_2 = \begin{cases} 1, & \text{if } \left(1 - \frac{z_2}{z_4}\right) r_G \leq \left(1 - \frac{y_1}{y_2}\right) r_Y \\ 0, & \text{otherwise} \end{cases}$ and $b_3 = \begin{cases} 1, & \text{if } \left(1 - \frac{y_3}{y_4}\right) r_Y \leq \left(1 - \frac{z_1}{z_3}\right) r_G \\ 0, & \text{otherwise} \end{cases}$, and ensure that the constraints $(iii)$ and $(iv)$ come into play only when the corresponding relevant outcome is feasible. Constraints $(v)$ and $(vi)$ represent the fact that the consumers of each of the content providers prefer it over its rival.

**Proposition 4-3: (Outcome 4 dominates Outcome 1)**

The BSP's profit is at least as high under Outcome 4 as under Outcome 1.

Proof: See Appendix B.

## Computational Analyses

### Solution Procedure

Based on the analytical model, we observe that Outcome 1 is independent of other outcomes. However, Outcome 2 depends on $y_1$ (in Outcome 1) and $z_4$ (in Outcome 4). Similarly, Outcome 3 depends on $z_1$ and $y_4$, and Outcome 4 depends on $y_3$ and $z_2$. Due to the interleaving structure of the four outcomes, it is impossible to derive analytical results from the model even with the help of technical computing tools such as Mathematica. However, there are several unique structures of the problem that allow us to analyze it and understand the nature of the equilibria. First, as stated earlier, Outcome 1 is independent. Second, Lemma 4-1 and 4-3 show that we can decompose the solution space into four components based on the feasibility conditions of Outcome 2 and Outcome 3. Third, Lemma 4-2 and 4-4 show that if we assume the feasibility of either Outcome 2 or Outcome 3, then their respective optimal solutions would depend only on Outcome 1. These insights lead us to develop Algorithm 1 that allows us to solve the problem as indicated below. Note that without employing this computational solution procedure, it would be impossible to gain the various important managerial and regulatory insights into this overly complicated and considerably generalized problem.

I believe that one of the main contributions of this chapter is this particular solution methodology. A two-sided platform, by its very nature, has different pricing options on the two sides it caters to. When the agents on the two sides of the platform can strategically interact among themselves in a manner that can affect their respective payoffs, the profit optimization

strategy for the platform with a particular pricing strategy (call it Option 1) will have to take into account other pricing strategies (say Option 2 or 3 and so on) that can plausibly be preferred by the agents: in other words, every possible outcome is intertwined with other outcomes. Such possibilities present unique challenges for the modelers, which is why they often resort to certain simplifying assumptions. I demonstrate that analyzing the structural properties of the model can often yield insights on the possible equilibria of the game, even if it is not possible to get closed-form solutions to the problem. This in turn can lead to the design of a suitable "divide-and-conquer" algorithm that effectively solves the problem.

**Algorithm 4-1:**

Step 1: Since Outcome 1 is independent, we solve Outcome 1 first and get $F_1^*$, $p_1^*$, $y_1^*$, and $z_1^*$.

Step 2: Although Outcome 2 is related to Outcome 4, the value of $y_4$ only determines the feasibility of Outcome 2. The optimal solution of Outcome 2 depends only on $y_1$. So we initially assume Outcome 2 to be feasible and solve for Outcome 2 based on the result of Outcome 1.

Step 3: Using similar reasoning, we then assume Outcome 3 to be feasible and solve for Outcome 3 based on the result of Outcome 1.

Step 4: Solve Outcome 4 in four different cases:

Case 1: (Both Outcome 2 and Outcome 3 are feasible)

Solve Outcome 4 with all the constraints

Add in the feasibility constraints for Outcome 2 and 3.

Case 2: (Only Outcome 2 is feasible)

Solve Outcome 4 in absence of the constraint involving Outcome 2

Add in the feasibility constraint for Outcome 2

Add in the infeasibility constraint for Outcome 3

Case 3: (Only Outcome 3 is feasible)

Solve Outcome 4 in absence of the constraint involving Outcome 3

Add in the feasibility constraint for Outcome 3

Add in the infeasibility constraint for Outcome 2

Case 4: (Neither Outcome 2 nor Outcome 3 is feasible)

Solve Outcome 4 in absence of the constraint involving Outcome 2 and 3

Add in the infeasibility constraints for Outcome 2 and 3

Step 5: Comparing all feasible $\pi_4(j)$, where $j = 1, 2, 3, 4$ denotes the above four cases, we can find the optimal $F_4^*$, $p_4^*$, $y_4^*$, and $z_4^*$.

Step 6: Compare $\pi_4^*$ to $\pi_3^*$ and determine the equilibrium (since $\pi_3^* \geq \pi_2^*$, from Proposition 4-2).

We note that even if Outcome 1 is dominated by all the other outcomes, we still need to solve for it, since the equilibrium output is used to find out the equilibria in outcomes 2 (Step 2) and 3 (Step 3). Similarly, even if Outcome 2 is dominated by outcomes 3 and 4, we still need to solve for its equilibrium in Step 2, since those results are used in Step 4.

To computationally explore the different outcomes and when each of them would dominate, we will need to effectively change the various exogenous parameter values so that we cover the entire parameter space. Note that while we can explore some of the parameters (specifically $\mu$, $\lambda$, $r_G$ and $r_Y$, as I will explain later) exhaustively for all possible values due to the nature of their impact on the solution, the other parameters can theoretically assume an infinite range of values. In the absence of a closed-form solution, what is necessary therefore is that for the latter set of parameters, we cover the entire range of values that can judiciously be considered within the limits of empirical reality. That process is explained in the next subsection.

For each of the combinations of the parameter values that I end up selecting, I then find the profit $\pi_3^*$ and $\pi_4^*$ in the manner outlined in the algorithm. This needs us to first select a reasonable "baseline" parameter values, and then perturb the different parameters around these baseline values, so that they cover all possible values that can be expected to be encountered in reality. Since every iteration with the parameter values involve solving several non-linear optimization problems, we would also have to specify a reasonable "step size" for incrementing the different parameter values, which ensures that while we effectively explore the parameter space, we do not end up having to perform an unmanageable number of optimization problems. As we detail in the discussion that follows, we ended up carrying out the solution procedure outlined in the algorithm for 10,890 sets of parameter values.

**The Baseline Model**

There are seven parameters in the analytical model. With so many parameters to be estimated for the computational analyses, it becomes a challenge to first set "reasonable" baseline values and then perturb those values sufficiently to explore the different possible scenarios and outcomes. In the baseline model, I set $V = 5$, $t = 2$, $d = 1$, $\mu = 1$, $\lambda = 0.5$, $r_G = 3.9$ and $r_Y = 0.6$, the rationale for which I explain next.

Note that not all the parameters in the model need to be changed independent of the other parameters. For example, with respect to the parameters $\mu$ and $\lambda$, what is important is not their absolute values but the utilization rate of the service queue $\frac{\mu}{\lambda}$, and hence I set the baseline utilization ratio at 0.5. We then change the utilization ratio (by changing $\lambda$ while keeping $\mu$ fixed at 1) from a low of 0.02 to a high of 0.98, increasing in step sizes of 0.12. Consider next the two parameters that estimate the revenue generation rates $r_G$ and $r_Y$. Once again, what concerns

us is the relative magnitude of these parameters rather than their absolute values.[3] To estimate the baseline values of the revenue generation rates $r_G$ and $r_Y$, I decided to look at the two companies that have come to symbolize the ad-supported revenue model in the online world that I model in this dissertation: Google and Yahoo! From the 10-Q documents of Google and Yahoo!, I obtained their 9-month revenue, and then converted these amounts into their per-day revenue. From Nielsen Online, I obtained their 1-month unique audience and analogously converted these numbers to a per-day unique audience figure for both the firms. Dividing the former per-day revenue amount by the per-day unique audience, I was able to designate the baselines values of $r_G$ and $r_Y$. Thus, when we perturb these values, we need to only change one of them – I chose that to be $r_Y$ – to find the effect of differing revenue generation rates. I then consider values of $r_Y$ to be as low as 0.1 and as high as 3.7 (i.e., similar to $r_G$ – going beyond 3.9 is unnecessary since in order to capture the dynamics of the game, one only need to consider relative magnitudes of the revenue generation rates).

The parameters that remain to be estimated are $v$, $t$ and $d$, and as stated earlier, they can theoretically vary within an infinite range. To find a practical range of values for these parameters, note that all of them go directly towards affecting the consumer's utility. Thus, one of these parameters can be kept fixed relative to the others, and here, I thought the unit fit cost $t$ would be the best choice, since one would be more interested in seeing how the different outcomes are affected by changing the other two.

[3] The BSP's pricing strategy for the content providers is centered on the criterion of whether it makes sense for it to charge both content providers of just content provider G. Thus, only the relative magnitudes of $r_G$ and $r_Y$ figure into the BSP's pricing strategy.

Consider now the expression for the consumer's utility under net neutrality, $V - ty_1 - \frac{d}{\mu - 2y_1\lambda} - F_1$. With $\mu$ set at the baseline value of 1, set the utilization ratio initially at 0.5. Assume now that Y has half the market share, i.e., $y_1 = 0.5$. Then, the denominator of the expression for delay is 0.5 (for a multiplicative effect of 2). Setting $t = 2$ and $d = 1$ (as we have under the baseline scenario) then makes the disutility of the delay in the baseline scenario to be twice as pernicious as the disutility of the fit cost, which we thought to be a reasonable baseline outcome (if one considers the two content providers to be Google and Yahoo!, such an assumption states that a consumer feels relatively more aggrieved with a delay in accessing their content than with the fact that she cannot access her "ideal" content provider). Finally, I felt that setting $V = 5$ made the gross valuation sufficiently larger than the delay for a baseline scenario, so that many consumers still have a large enough utility in accessing the services of the content providers. I then set a lower bound for $V$ to be 3 (at which point the net utility would be negligible or even negative), and the upper bound at 13 (at which point it dominates any other disutility), with a step size of 1. Keeping $t = 2$ throughout, I then modified $d$, in effect exploring the relative magnitude of the fit cost to the delay cost. If we keep all other parameter values at the baseline figures, keeping $d$ at a lower bound set at 0.5 gives the consumer a net utility (before accounting for access charges) of 3. That value goes down to 1 at the upper bound of $d$ at 1.5. The step size for the change in these $d$ values was kept at 0.1. Thus, the total number of exploration points for the entire parameter space was 10,890. The range of the parameter values and step sizes employed for each are summarized in Table 4-1.

As mentioned earlier, each run of Algorithm 4-1 involves solving several non-linear optimization problems. Since I had to run the algorithm for the 10,890 sets of parameter values, I

deployed the algorithm on subsets of the parameter space on three different desktop machines running Microsoft Excel 2007 with Premium Solver Version 8 on the Windows XP operating system (64-bit and 32-bit). The configurations of the three computers were: Intel Core 2 Quad processors with 8GB RAM; Intel Core 2 Duo processor with 4GB RAM; and Intel Core 2 Duo processor with 2GB RAM. The entire series of computations took a little more than a week to complete.

## Research Findings

The computational analysis produces several observations. These observations are important, since given the number of parameters in play, it is otherwise impossible to isolate the potential effect of each of them in the resultant equilibria. Even in those cases where I have managed to successfully explore the analytical solutions, the wide variety of parameters did not allow me to gauge the practical implications of the results. For example, I have proved analytically that the BSP will always prefer an NNN outcome (i.e., either Outcome 3 or 4) to the NN outcome (Outcome 1). However, this finding does not imply that the BSP would always opt for the NNN outcome in practice: for example, if the advantage gained from moving to the NNN regime is relatively small, and there are substantial administrative costs in implementing a pricing plan for the content providers (in terms of, say, new networking equipment like smart routers that identify the origin of the packets and then correctly place them in the two-tier queue without introducing any additional delay), then the BSP would probably not opt for the NNN outcomes. Further, it would be interesting to explore how the other constituents, i.e., the consumers and the content providers, fare under NNN – insights which we could not get using purely analytical techniques, but that are now available as a result of the computational analyses. For example, how would the fixed access fee compare under the two regimes? I discuss all such findings in the following sections.

**The Fate of the Consumers Under NNN**

**Fixed access fee:** I find that out of the total 10890 scenarios, $F^*_{NNN} < F^*_{NN}$ in 6287 (58%) scenarios and $F^*_{NNN} = F^*_{NN}$ in the remaining 4603 (42%) scenarios. In other words, the optimal fixed fee that the BSP would charge under NNN is never higher (and often lower) than the optimal fixed fee it would charge under NN. Further, when $F^*_{NNN}$ is smaller, the statistics of the computational analyses results indicate that it is on average nearly 21% lower than the corresponding $F^*_{NN}$.

**Profit contribution from consumers:** While the BSP gets a percentage of its revenue (profits) from the content providers under NNN, it would be interesting to find out whether the BSP gets less revenue as a whole from the consumers under NNN than under NN. The results of the computational analyses clearly indicate that that is true: in other words, $F^*_{NNN} \cdot \left( y^*_{NNN} + z^*_{NNN} \right) \leq F^*_{NN} \cdot \left( y^*_{NN} + z^*_{NN} \right)$ for all scenarios, and therefore the content providers clearly subsidize the consumers' access fees under NNN.

I also explore the average percentage contribution to the BSP's profits from the consumers under NNN (under NN, it is always 100%). Within the ambit of the results of the computational analyses, we observe that on average about 78% of the BSP's profits under NNN come from the consumers, while the rest 22% is generated from the content providers.

**The BSP's Incentives to Move to NNN**

**Percentage increase of the BSP's profits by moving from NN to NNN:** While we do know that the BSP prefers the NNN regime since it provides the flexibility to charge content providers, the surfeit of parameters do not allow us to determine the magnitude of this advantage analytically. Computational results reveal that on average, moving from NN to NNN increases the profit of the BSP by over a third – in other words, the BSP will usually have pronounced

incentive to move to the new regime. Thus, the results indicate that under NNN, the BSP lowers the fixed fee to expand the consumer base in order to extract more rent from one (Outcome 3) or both (Outcome 4) the content providers. Although BSP's profit from the consumers' side of the market is lower under NNN than that under NN, the overall profit is higher.

**Impact of different parameters on the BSP's profit:** I also examine the impact of the different parameters on the BSP's profit. Expectedly, the BSP's profit increases in consumers' valuation $v$ and content provider's revenue rate $r_{\mathrm{Y}}$ and decreases in consumers' unit delay cost. The consumers' content request rate $\lambda$ has two opposing effects – when $\lambda$ increases the BSP gains as a result of the usage-based charge on preferential delivery while the BSP stands to lose due to the increased congestion (and as a result the consumers can be charged less). I find the latter effect dominates the former and the BSP's profit decreases in $\lambda$.

**The Effect of NNN on Market Coverage**

One of the principal contributions of this chapter is to explore the effect of NNN on market coverage, even as I simultaneously modeled the competition between the content providers. In what follows, I denote market coverage (i.e., the total market covered) by $K_l$, $l \in \{\mathrm{NN}, \mathrm{NNN}\}$, and find that, under most cases, $K_{\mathrm{NNN}} \geq K_{\mathrm{NN}}$. In 10873 scenarios out of 10890 total scenarios, we find that market coverage under NNN is higher than NN. In the remaining 17 scenarios (representing about 0.16% of the total) where the market coverage is higher under NN, two common characteristics emerge: First, such a result always happens under Outcome 3, where only content provider G pays the BSP for priority delivery; and second, the ratio $\frac{r_{\mathrm{G}}}{r_{\mathrm{Y}}}$ is always very high (recall that I had assumed, without loss of generality, that $r_{\mathrm{G}} \geq r_{\mathrm{Y}}$).

**The percentage increase of market coverage under NNN:** Within the parameter space that I explored, I find that the mean market coverage increases by a little over 7% under NNN as compared to under NN.

**Impact of different parameters on market coverage:** I also examine the impact of various parameters on market coverage, and find that while, as one might expect, while $V$ and $r_Y$ have a slight positive impact on market coverage, the delay cost $d$ and the consumers' request arrival rate $\lambda$ have a negative effect on total market coverage.

**The Effect on Consumer Surplus**

From a policymaker's point of view, a very important aspect of allowing a NNN regime is to explore how the consumer surplus and social welfare change from those under NN. Specifically, if social welfare increases as a result of abandoning net neutrality – and more specifically, if the end consumers are better off – the idea for the proposed payment mechanisms would gain favor among policymakers. Conversely, if abandoning the principle of net neutrality results in helping just a few private agencies to extract more rent, the idea would find a much less sympathetic audience. The consumer surplus under NN is given by the following expression:

$$CS_{NN} = \left( V - \frac{d}{\mu - (y_1 + z_1)\lambda} - F_1 \right)(y_1 + z_1) - \frac{t}{2}\left(y_1^2 + z_1^2\right)$$

The consumer surplus under NNN is given by the following expression, depending on whether the final equilibrium is Outcome 3 or Outcome 4:

$$CS_{NNN} = \begin{cases} \left( V - \dfrac{d\mu}{(\mu - z_3\lambda)(\mu - y_3\lambda - z_3\lambda)} - F_3 \right) y_3 + \left( V - \dfrac{d}{\mu - z_3\lambda} - F_3 \right) z_3 - \dfrac{t}{2}\left(y_3^2 + z_3^2\right), \\ \qquad\qquad \text{if Outcome 3 is the equilibrium} \\ \left( V - \dfrac{d}{\mu - (y_4 + z_4)\lambda} - F_4 \right)(y_4 + z_4) - \dfrac{t}{2}\left(y_4^2 + z_4^2\right), \text{ if Outcome 4 is the equilibrium} \end{cases}$$

I find that it is always true that $CS_{NNN} \geq CS_{NN}$. Further, if Outcome 3 is the equilibrium, then it is strictly true that $CS_{NNN} > CS_{NN}$; if Outcome 4 is the equilibrium, I sometimes have $CS_{NNN} > CS_{NN}$ and in other cases $CS_{NNN} = CS_{NN}$.

The computational results suggest that the consumer surplus for G's consumers under NNN is higher than under NN in about 62% of the scenarios and equal to that under NN in the remaining 38% of the scenarios (but is never lower than that under NN). Consumer surplus for Y's consumers under NNN is lower in about 13% of the scenarios, higher in 49% of the scenarios, and equal to that under NN in the remaining scenarios. Thus the fate of the consumers is slightly nuanced – even though consumers as a whole are never worse off (and often better off), with some set of parameter values, the consumers of the less effective revenue-generating content provider end up being worse off under NNN as compared to NN. The results also indicate that the increase in consumer surplus ranges from 0% to 62% with a mean increase of about 10.4%.

**The Effect on Social Welfare**

Social welfare under NN is computed as

$$SW_{NN} = CS_{NN} + \pi^*_{NN} + (y_1 \lambda r_Y + z_1 \lambda r_G) \tag{4-5}$$

while the social welfare under NNN is computed as

$$SW_{NNN} = \begin{cases} CS_{NN} + \pi^*_3 + (y_3 \lambda r_Y + z_3 \lambda r_G - z_3 \lambda p_3), \text{ if outcome 3 is the equilibrium} \\ CS_{NN} + \pi^*_4 + (y_4 \lambda r_Y + z_4 \lambda r_G - y_4 \lambda p_4 - z_4 \lambda p_4), \text{ if outcome 4 is the equilibrium} \end{cases} \tag{4-6}$$

I find that it is always true that $SW_{NNN} \geq SW_{NN}$. If Outcome 3 is the equilibrium, then the strict inequality holds: $SW_{NNN} > SW_{NN}$; if Outcome 4 is the equilibrium, I find that sometimes $SW_{NNN} > SW_{NN}$ and at other time $SW_{NNN} = SW_{NN}$.

**Can NNN Reduce Competition among Content Providers? – The Issue of Innovation at the Edge**

Since implementing NNN can lead to an equilibrium that involves Outcome 3, when only G pays for preferential delivery, it is instructive to find out whether such an equilibrium can effectively force content provider Y out of the market (i.e., $y_3 = 0$). Note that such an outcome would not be possible in Outcome 4 – neither Y nor G would have any incentive to pay if it had no market share consequently. In 1358 out of the total 10890 (or in about an eighth of the scenarios), I find that to be indeed true. The common characteristics of such outcomes involve a low $r_Y$ (i.e., Y is less effective in generating revenue), and a high consumer request rate for packets $\lambda$ and delay cost ($d$). When $y_3 = 0$, we have a situation where abandoning NN reduces "innovation at the edge", as we end up driving out some content providers.

These results play into the hands of observers who have feared that not instituting net neutrality can reduce innovation in the Internet. For example, Vint Cerf, the renowned computer scientist who is commonly referred to as one of the "founding fathers of the Internet," has contended that such a payment structure would result in the Internet increasingly resembling today's mass media, where a few broadband service providers control what the customers effectively may have access to (Waldmeir 2006). Internet start-ups often have low revenue generation rates (in many cases they have to spend more than their revenue in order to generate consumer traffic), and resemble the content provider Y with a low $r_Y$. Established players (who might resemble content provider G) can preemptively pay the BSP and in some cases (when the packet traffic is relatively high and the consumers have a high disutility associated with delay) shut the new rival out of the market.

From a policymaker's perspective, the situation thus demands a studied response. Established content providers might see an opportunity in paying the priority delivery fees to the BSP and sacrifice profits in the short run in order to preemptively drive out possible competition from promising startups in future. While consumers might gain in the short run, with lower access fees (and therefore also support NNN), such a situation might lead to a less innovative Internet, and thereby lessen the experience for all consumers in the long run.

**The Possibility of Free Internet Access to Consumers**

Under NN, the BSP always charges a positive access fee $F$ since that represents its only revenue source. However, under NNN, it is possible with some sets of parameter values that the content providers effectively subsidize the consumers for Internet access, i.e., there might be situations where $F_{\mathrm{NNN}} = 0$. The computational results indicate the circumstances when this is possible. First, the consumer valuation for content $V$ has to be very low, while delay cost $d$ and packet arrival rate $\lambda$ are both high. I find that both Outcomes 3 and 4 may be the final equilibrium. When $r_{\mathrm{Y}}$ is relatively high compared to $r_{\mathrm{G}}$, Outcome 4 is equilibrium; when $r_{\mathrm{Y}}$ is relatively low, Outcome 3 is the equilibrium. The intuition for this type of a scenario is as follows: while consumers themselves might not value their Internet access very highly, they are nonetheless profitable to the content providers. Coupled with a high $d$ and $\lambda$, the consumers would easily drop out of the market if they are charged a positive $F$. This would not only deny the BSP from any revenue from the consumers, but would also stop the revenue source from the other side of the platform – since the content providers will pay the priority delivery fee only if there are consumers present on the other side of the platform. Therefore, the BSP decides to drop the access fee for the consumers, in order to retain them, and extract a part of the surplus that the content providers get from these consumers.

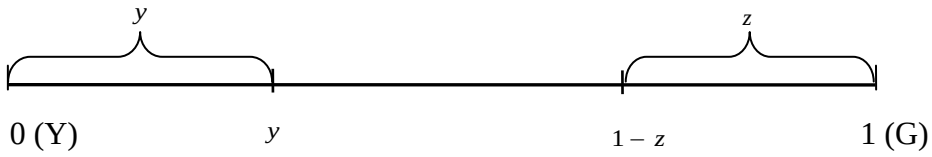



Figure 4-1. Consumers and the market shares of the two content providers

Table 4-1. Parameter values and step sizes

| | $V$ | $t$ | d | $\mu$ | $\lambda$ | $r_G$ | $r_Y$ |
|---|---|---|---|---|---|---|---|
| Baseline | 5 | 2 | 1 | 1 | 0.5 | 3.9 | 0.6 |
| Lower Bound | 3 | | 0.5 | | 0.02 | | 0.1 |
| Upper Bound | 13 | | 1.5 | | 0.98 | | 3.7 |
| Step Size | 1 | | 0.1 | | 0.12 | | 0.4 |
| Total Number | 11 | | 11 | | 9 | | 10 |

Note: The total number of iterations is 11*11*9*10 = 10,890.

# CHAPTER 5
# CONTENT PROVIDER DISCRIMINATION AND VERTICAL INTEGRATION

The literature on net neutrality thus far – especially those which look at the issue from an economic perspective – study the problem assuming that the BSP and the content providers are separate entities with conflicting objectives, with the two standing on opposing sides in this debate. However, recent developments indicate that the issue might not be that clearly delineated. Broadband service providers like Comcast and AT&T have struck deals with online content providers whereby the latter provide exclusive co-branded content through the former's "pipes". For example, Comcast and Yahoo! recently signed a multi-year agreement so that the latter can display its advertisements to the end consumers who subscribe to the former's broadband services (Shields 2007). Another example is that of AT&T and Yahoo! forming a strategic multi-year relationship whereby the two companies will offer a co-branded version of Yahoo!'s popular messaging platform, Yahoo! Messenger, to AT&T's broadband subscribers (BusinessWire 2006). As industry observers have noted, should net neutrality be abolished, it might motivate BSP to generate their own content (or equivalently, have a strategic relationship with a content provider) and then prioritize delivery of such content to the end consumers. If net neutrality is not enforced, and the BSP's in these examples are allowed to prioritize content from their strategic partner, a section of the consumers might switch from their erstwhile content provider to the BSP's strategic partner. A similar scenario can ensue with other classes of service like news, VoIP telephony, music streaming, etc.

These developments prompt a new set of questions for the policymaker:

- How does such vertical integration affect consumer surplus and social welfare? Specifically, under what conditions might the BSP want to vertically integrate with a content provider, even though the outcome might result in lower consumer surplus or social welfare?

- Since the vertically integrated firm can prioritize its own content without paying the priority delivery fees, how does such vertical integration affect the independent pure play content providers? Will such an arrangement amount to unfair competition?
- What are the possible different equilibrium outcomes when the BSP is vertically integrated with a content provider?
- Will the vertically integrated firm prefer no net neutrality over net neutrality?

In this chapter, I endogenize the BSP's vertical integration decision in the context of the net neutrality debate to answer these questions. In the model, I assume that there is a monopolist BSP who can form a strategic relationship with an online content provider to provide an online service that competes with that from a pure play competitor who only provides online content. The latter has to depend on the BSP's infrastructure for getting its content delivered to the end consumers.

## The Model

In this section I set up a game-theoretic model to analyze the impact of vertical integration between content providers and broadband service provider both in the presence and in the absence of net neutrality. The BSP has two vertical integration related decisions to make: (1) whether it would like to vertically integrate with one of the two content providers and (2) if so, which one. As the results for the case of no vertical integration are readily available from Chapter 3 in this dissertation, I thus focus the analyses in this chapter on the presupposition that the BSP has vertically integrated with one of the two content providers, and then compare and contrast these findings with those of Chapter 3 to address the BSP's vertical integration decision. Hence, there are three types of players in the game – the vertically integrated monopolist BSP, the two content providers (one of whom is vertically integrated with the BSP) and the end consumers. The BSP serves as an intermediary and transmits content from the content providers to end consumers. Since the BSP is vertically integrated, it has its own content (from its strategic

partner) that competes with the other independent pure play content provider (I call the independent content provider C) for the attention of the end consumers.

**Content Providers**

To model the competition between the content providers, I assume two content providers Y and G who offer their contents for free to the end consumers. (One of the two content providers is vertically integrated with the BSP.) I assume that the content from these two providers are horizontally differentiated in a Hotelling sense with the two of them located on the two ends of line segment $[0,1]$. As before, I assume that the content providers provide their content to the consumers for "free" and get compensated when the consumers interact with advertisers and other revenue generation mechanisms on the content providers' web sites. Let $r_Y$ and $r_G$ be the average revenue generated per packet requested by the end consumer, where $r_G > r_Y$.

I denote the revenue generation rate of the vertically integrated BSP by $r_{BSP}$ and the revenue generation rate of the independent content provider C, where $C \in \{Y, G\}$, by $r_C$. Thus, $r_{BSP} = r_G$ and $r_C = r_Y$ if the BSP vertically integrates with G, and $r_{BSP} = r_Y$ and $r_C = r_G$ if the BSP vertically integrates with Y. Therefore, depending on the BSP's choice, $r_{BSP}$ can be either greater than or less than $r_C$. The competition between content provider C and the BSP is driven by the fact that a larger consumer base will lead to greater advertising revenue.

**Vertically Integrated BSP**

Following the central tenets of the model in Chapter 3, I assume a monopolist BSP provides Internet access as well as its own content to the end consumers. The BSP can provide its content as the result of a vertical integration with a content provider – the exact mechanism

for this integration might be achieved through an outright merger between the two firms, or through a strategic alliance.[9] Figure 5-1 shows a schematic of the market structure of the model. The BSP charges the consumers a fixed fee $F$ for Internet access and potentially charges content provider C a per packet fee $p$ for preferential delivery of C's data packets if net neutrality is not enforced.

**Consumers**

Consumers request content from either the vertically integrated BSP or content provider C. Similar to the models in Chapters 3 and 4, for an arbitrary consumer $\tilde{x} \in [0,1]$, the fit cost associated with deviation from his ideal content is $t\tilde{x}$ if the consumer chooses the BSP's content and $t(1-\tilde{x})$ if the consumer chooses content from provider C. Then the utility function for the BSP's consumers under net neutrality is:

$$u_{NN\_BSP}(\tilde{x}) = V(\lambda) - t\tilde{x} - \frac{d}{\mu - \lambda} - F \tag{5-1}$$

The utility function for content provider C's consumers under net neutrality is:

$$u_{NN\_C}(\tilde{x}) = V(\lambda) - t(1-\tilde{x}) - \frac{d}{\mu - \lambda} - F \tag{5-2}$$

In the absence of net neutrality, I use a two-class priority queue to model the BSP's data transmission service. The detailed congestion cost and the corresponding utility functions for consumers under no net neutrality will be discussed in the section of no net neutrality.

I define two indicator functions as follows to represent whether the BSP would prioritize its own content and whether content provider C would pay for the preferential delivery.

[9] The details of the profit-sharing agreement between the two firms in a strategic alliance are not germane to this discussion, since the policy maker's main concern is the effect on social welfare.

$$I_{\text{BSP}} = \begin{cases} 1, & \text{The BSP prioritizes its own content} \\ 0, & \text{The BSP does not prioritize its own content} \end{cases} \tag{5-3}$$

$$I_{\text{C}} = \begin{cases} 1, & \text{Content provider C pays for the preferential delivery} \\ 0, & \text{Content provider C does not pay for the preferential delivery} \end{cases} \tag{5-4}$$

The timing of the four-stage game is as shown in Figure 5-2. In stage 1, the BSP decides (1) whether to integrate with a content provider and (2) if so, integrate with whom. In stage 2, the broadband provider announces $F$ and $p$. In stage 3, the BSP decides whether to give its own content preferential treatment and content provider C chooses to "Pay" or "Not Pay" for the preferential delivery. In stage 4, consumers choose content from either the BSP or content provider C. Given this particular timing, the strategy of the BSP is to calculate its maximum profit under different scenarios, and then, depending on the underlying parameter values (see Appendix A for a list of the various parameters and variables used in the text), choose whether to observe net neutrality, or – if the regulatory environment allows him to do so – choose to prioritize its own content and (for a fee) the content from provider C to the end consumers. Thus, the monopolist BSP can calculate what would be the profit under different scenarios and then choose that particular pricing strategy that will maximize its profit for a given set of parameter values. In the following two sections, I analyze these different scenarios by solving the BSP's profit maximization problems under different regulatory regimes (net neutrality and no net neutrality) and pricing arrangements. Note that there is only one pricing decision under net neutrality (the access fee that the BSP charges the consumers), while under no net neutrality, the BSP can charge both the consumers and content provider C (charging the latter for the service that prioritizes the delivery of its content to the end consumers).

## Net Neutrality

With net neutrality in place, the BSP is forbidden from providing the service of and charging for preferential delivery of data packets. Then the marginal consumer who is indifferent from the BSP and content provider C can be determined by $u_{NN\_BSP}(x_{NN}) = u_{NN\_C}(x_{NN})$, i.e.,

$$V(\lambda) - tx_{NN} - F_{NN} - \frac{d}{\mu - \lambda} = V(\lambda) - t(1 - x_{NN}) - F_{NN} - \frac{d}{\mu - \lambda} \tag{5-5}$$

which implies $x_{NN} = \frac{1}{2}$. Therefore the demand for the BSP and content provider C are both $\frac{1}{2}$. The resulting Internet access fee is $F_{NN} = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$. The BSP's revenue consists of both Internet access charge from consumers, i.e., $V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$, and advertisement revenues from advertisers, i.e., $\frac{1}{2}\lambda r_{BSP}$.

## No Net Neutrality

Without net neutrality, the BSP has the option to provide a preferential delivery for data packets from content provider C and the BSP itself. Depending on whether the BSP prioritizes its own content and whether C pays for the preferential delivery, there are four different outcomes as follows. Outcome 1: C does not pay for priority delivery of its own content, and content from neither provider is prioritized; Outcome 2: the BSP prioritizes its own content at the expense of that of C who does not pay the priority delivery fee; Outcome 3: the BSP prioritizes C's content for a fee so that the latter's content is prioritized at the expense of its own; and Outcome 4: C pays the BSP so that the BSP does not prioritize its own content over that of C (in other words, content from both providers receive the same priority). Note that all these four outcomes are under the "control" of the BSP – for example, if it is charging C for priority delivery, it can

create a contract that specifies whether it will in turn not prioritize its own content (i.e., Outcome 3), or whether the payment from C merely ensures that C's packets are not relatively "de-prioritized" with respect to its own (which is Outcome 4). We note further that a priori, none of the outcomes can be ruled out, since based on the values of the different parameters, any one of them might generate the highest profit for the BSP.

**Outcome 1:** The BSP does not prioritize its own content and content provider C does not pay for the preferential delivery ( $I_{BSP} = I_C = 0$ ).

Outcome 1 is essentially the equilibrium equivalent to net neutrality. The corresponding marginal consumer $x_1$ is determined by $u_{BSP1}(x_1) = u_{C1}(x_1)$, i.e.,

$$V(\lambda) - tx_1 - F_1 - \frac{d}{\mu - \lambda} = V(\lambda) - t(1 - x_1) - F_1 - \frac{d}{\mu - \lambda} \tag{5-6}$$

which implies $x_1 = \frac{1}{2}$. Under this scenario, the BSP's problem is:

$$\begin{aligned} &\max_{F_1, p_1} \ \pi_{BSP1} = F_1 + \frac{1}{2}\lambda r_{BSP} \\ &\text{s.t.} \quad u_{BSP1}(\tilde{x}) \geq 0, \qquad 0 \leq \tilde{x} \leq x_1 \qquad (i) \\ &\qquad u_{C1}(\tilde{x}) \geq 0, \qquad x_1 \leq \tilde{x} \leq 1 \qquad (ii) \\ &\qquad \pi_{BSP1} - \pi_{BSP2} \geq 0 \qquad (iii) \\ &\qquad \pi_{C1} - \pi_{C3} \geq 0 \qquad (iv) \end{aligned} \tag{5-7}$$

where constraints $(i)$ and $(ii)$ are consumers' participation constraints and constraints $(iii)$ and $(iv)$ are incentive compatibility constraints for the BSP and content provider C. Note that for its incentive compatibility considerations, the BSP compares its profit under Outcome 1 with that under Outcome 2 (where it prioritizes its own content over that of C). To consider the incentive compatibility constraint of content provider C, meanwhile, the BSP will need to compare C's

profit under Outcome 1 to that under Outcome 3 – when it prioritizes C's content at the expense of its own.

Some algebra shows that regardless of parameter values, the incentive compatibility constraint for the BSP can never be satisfied. In other words, if content provider C does not pay for the preferential delivery, the BSP is always better off prioritizing its own content (see Table 5-1 for a comparison of the BSP's profits under the different scenarios). Therefore Outcome 1 is not an equilibrium, unless the BSP is prohibited through regulation from differentially prioritizing its own content.

**Outcome 2:** Under Outcome 2, the BSP prioritizes its own content and content provider C does not pay for the preferential delivery ($I_{BSP} = 1$, $I_C = 0$).

In this case, data packets provided by the BSP and content provider C are transmitted with different priorities by the BSP. I use a two-class priority queue with preemption model to depict the waiting time. For a given consumer $\tilde{x}_2 \in [0,1]$, data packets from the BSP are transmitted with higher priority whose waiting time is $\frac{1}{\mu - \tilde{x}_2 \lambda}$ while data packets from content provider C are transmitted with lower priority whose waiting time is $\frac{\mu}{(\mu - \tilde{x}_2 \lambda)(\mu - \lambda)}$. Correspondingly the marginal consumer $x_2$ is determined by $u_{BSP2}(x_2) = u_{C2}(x_2)$, i.e.,

$$V(\lambda) - tx_2 - F_2 - \frac{d}{\mu - x_2 \lambda} = V(\lambda) - t(1 - x_2) - F_2 - \frac{d\mu}{(\mu - x_2 \lambda)(\mu - \lambda)} \tag{5-8}$$

which leads to a higher market share for the BSP ($x_2 > \frac{1}{2}$) than content provider C ($1 - x_2 < \frac{1}{2}$).

The BSP maximizes its profit by solving

$$
\begin{aligned}
\max_{F_2, p_2} \quad & \pi_{BSP2} = F_2 + x_2 \lambda r_{BSP} \\
\text{s.t.} \quad & u_{BSP2}(\tilde{x}) \geq 0, \qquad 0 \leq \tilde{x} \leq x_2 \qquad (i) \\
& u_{C2}(\tilde{x}) \geq 0, \qquad x_2 \leq \tilde{x} \leq 1 \qquad (ii) \\
& \pi_{BSP2} - \pi_{BSP1} \geq 0 \qquad (iii) \\
& \pi_{C2} - \pi_{C4} \geq 0 \qquad (iv)
\end{aligned}
\tag{5-9}
$$

where, just as in Formulation (5-7), constraints $(i)$ and $(ii)$ are consumers' participation constraints and constraints $(iii)$ and $(iv)$ are incentive compatibility constraints for the BSP and content provider C. Note that if Outcome 2 holds, then the BSP does not generate any revenue from priority delivery, since the only content that is prioritized is from its strategic partner. In this scenario, in order to ensure C's incentive compatibility constraint to hold true, the BSP will need to ensure that C's profit under this outcome when it does not pay the BSP (so that its content gets lower priority for delivery than the BSP's content) is at least as high as it is under Outcome 4, when C pays to ensure that the delivery of its content will not be relatively degraded with respect to the BSP's own content.

Outcome 2 can only arise in equilibrium if $r_{BSP} > r_C + \frac{t}{\lambda}(1 - 2x_3)$, i.e., the BSP will offer this pricing/prioritization policy only if it is the "sufficiently" more profitable content provider (since the online content provider's marginal cost of providing content can be approximated to zero, the relative profitability of the two content providers can be measured from their revenues). If however $r_{BSP} \leq r_C + \frac{t}{\lambda}(1 - 2x_3)$ [10], Outcome 2 is dominated by other scenarios.

[10] I assume that when the BSP is indifferent between two different outcomes (i.e., between Outcome 2 and Outcome 4, or between Outcome 3 and Outcome 4), it chooses Outcome 4.

**Outcome 3:** The BSP does not prioritize its own content and content provider C pays for the preferential delivery ($I_{BSP} = 0$, $I_C = 1$).

As opposed to Outcome 2, for a given consumer $\tilde{x}_3 \in [0,1]$ in Outcome 3 it is the data packets from content provider C that are transmitted with higher priority with a waiting time of $\frac{1}{\mu - (1-\tilde{x}_3)\lambda}$ while data packets from the BSP are transmitted with lower priority with waiting time of $\frac{\mu}{\left[\mu - (1-\tilde{x}_3)\lambda\right](\mu - \lambda)}$. Correspondingly the marginal consumer under Outcome 3, $x_3$ is determined by $u_{BSP3}(x_3) = u_{C3}(x_3)$, i.e.,

$$V(\lambda) - tx_3 - F_3 - \frac{d\mu}{\left[\mu - (1-x_3)\lambda\right](\mu - \lambda)} = V(\lambda) - t(1-x_3) - F_3 - \frac{d}{\mu - (1-x_3)\lambda} \tag{5-10}$$

which leads to a lower market share for the BSP ($x_3 < \frac{1}{2}$) than content provider C ($1 - x_3 > \frac{1}{2}$).The BSP maximize its profit by solving

$$\begin{aligned}
&\max_{F_3, p_3} \ \pi_{BSP3} = F_3 + x_3\lambda r_{ISP} + (1-x_3)\lambda p_3 \\
&\text{s.t.} \quad u_{BSP3}(\tilde{x}) \geq 0, \quad 0 \leq \tilde{x} \leq x_3 \\
&\qquad u_{C3}(\tilde{x}) \geq 0, \quad x_3 \leq \tilde{x} \leq 1 \\
&\qquad \pi_{BSP3} - \pi_{BSP4} \geq 0 \\
&\qquad \pi_{C3} - \pi_{C1} \geq 0
\end{aligned} \tag{5-11}$$

Now, in order to satisfy its own incentive compatibility constraints, the BSP has to make sure that its profits under Outcome 3, when C pays and the delivery of its own content is degraded with respect to that of C, is at least as high as under Outcome 4, when C pays and the content from both providers receive equal priority. Note that depending on the relative

magnitudes of the average per-consumer advertising revenue that is generated by the BSP and C, the BSP will sometimes willingly degrade delivery of its own content (Outcome 3) in order to extract the surplus from the advertising revenue that C generates from its bigger market share. The additional surplus extracted from C might be enough to more than compensate the loss of revenue from the loss of market share (compared to Outcome 4). In order to satisfy C's incentive compatibility constraints, the BSP has to ensure that by paying and getting priority delivery of its packets, C's profit is at least as high as under Outcome 1, when it does not pay and the delivery of its content does not receive priority (but is not degraded either).

Analogous to Outcome 2, Outcome 3 can only arise in equilibrium if $r_C > \left(\frac{1-x_3}{1/2-x_3}\right) r_{BSP} + \frac{t}{\lambda}(1-2x_3)$, i.e., the BSP will offer this pricing policy only if it is the "sufficiently" less profitable content provider. Conversely, if $r_C \leq \left(\frac{1-x_3}{1/2-x_3}\right) r_{BSP} + \frac{t}{\lambda}(1-2x_3)$, Outcome 3 is dominated by other scenarios. We also note that since the independent content provider is more effective in generating revenue, the BSP can credibly enforce this pricing/prioritization strategy where content provider C has reasons to believe that the BSP will degrade its own content's delivery, since by doing so, the market share for content provider C will increase and therefore the BSP can extract more surplus from content provider C through charges for priority service.

**Outcome 4:** The BSP prioritizes its own content and content provider C pays for the preferential delivery ($I_{BSP} = I_C = 1$) to ensure that its own packets do not get degraded with respect to those of the BSP as in Outcome 2.

In Outcome 4, data packets from both the BSP and content provider C get "preferential" delivery – in other words, delivery of neither content is degraded with respect to the other.

Therefore, packets from each provider face the same waiting time $\frac{1}{\mu - \lambda}$. So the marginal consumer $x_4$ is determined by $u_{BSP4}(x_4) = u_{C4}(x_4)$, i.e.,

$$V(\lambda) - tx_4 - F_4 - \frac{d}{\mu - \lambda} = V(\lambda) - t(1 - x_4) - F_4 - \frac{d}{\mu - \lambda} \tag{5-12}$$

which leads to $x_4 = \frac{1}{2}$. The BSP's profit maximization problem under Outcome 4 is:

$$\begin{aligned} &\max_{F_4, p_4} \ \Pi_{BSP4} = F_4 + \frac{1}{2}\lambda r_{BSP} + \frac{1}{2}\lambda p_4 \\ &\text{s.t.} \quad u_{BSP4}(\tilde{x}) \geq 0, \quad 0 \leq \tilde{x} \leq x_4 \\ &\qquad u_{C4}(\tilde{x}) \geq 0, \quad x_4 \leq \tilde{x} \leq 1 \\ &\qquad \pi_{BSP4} - \pi_{BSP3} \geq 0 \\ &\qquad \pi_{C4} - \pi_{C2} \geq 0 \end{aligned} \tag{5-13}$$

For the BSP, the scenario is the reverse of Outcome 3, and hence, to maintain incentive compatibility, the BSP has to ensure that its profit under Outcome 4 is at least as high as under Outcome 3. For content provider C, there remains an incentive to pay the BSP only if by doing so (and thereby not have the delivery of its content degraded with respect to those of the BSP as in Outcome 2), it can generate profits that are as high as under Outcome 2.

Proposition 5-1 summarizes the results of equilibrium of the game.

**Proposition 5-1: (Equilibrium of the game with a vertically integrated BSP)**

There are three possible equilibria of the game.

Under Case A: $r_C < r_{BSP} - \frac{t}{\lambda}(1 - 2x_3)$, Outcome 2 is the equilibrium.

Under Case B: $r_C > \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP} + \frac{t}{\lambda}(1 - 2x_3)$, Outcome 3 is the equilibrium.

Under Case C: $r_{BSP} - \frac{t}{\lambda}(1 - 2x_3) \le r_C \le \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP} + \frac{t}{\lambda}(1 - 2x_3)$, Outcome 4 is the equilibrium. Case C can be further divided into two sub-cases:

Under Case C1: $r_{BSP} - \frac{t}{\lambda}(1 - 2x_3) \le r_C \le \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP}$, $p_{41} = (1 - 2x_3) r_C$,

$$\pi_{41} = V(\lambda) - \frac{1}{2}t - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{BSP} + \left(\frac{1}{2} - x_3\right)\lambda r_C ;$$

Under Case C2: $\left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP} \le r_C \le \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP} + \frac{t}{\lambda}(1 - 2x_3)$, $p_{42} = r_{BSP}$,

$$\pi_{42} = V(\lambda) - \frac{1}{2}t - \frac{d}{\mu - \lambda} + \lambda r_{BSP} .$$

Proof: See Appendix B.

Figure 5-3 shows graphically the effect of the relative magnitudes of $r_C$ and $r_{BSP}$ on the final equilibrium. One can think of the graph being divided into two areas by the line $r_C = r_{BSP}$, with the bottom half corresponding to the case when the vertically integrated BSP is more profitable in generating advertising revenue than its pure play competitor, while the top half represents the opposite scenario. The figure can be divided into three main regions, which I denote as Case A, B and C respectively, and Case C can be further divided into two regions C1 and C2. The shaded region in the bottom half of the quadrant represents Case A, or Outcome 2. The other shaded region, which is in the top half of the quadrant represents Case B, or Outcome 3. The remaining area represents Case C, or Outcome 4.

With respect to the regions outlined in Figure 5-3, Outcome 4 is the equilibrium in the region in between that of Outcome 2 and Outcome 3. This is where the profitability of the BSP and the content provider is relatively “comparable”. Note that the fixed fee is higher in Outcome

4 than in Outcomes 2 and 3, and the BSP now gets relatively more of its profit from the consumers than from the content providers as compared to Outcomes 2 and 3.

As noted earlier, Case A or Case B occurs only when the relative revenue generation rates $r_{\mathrm{C}}$ and $r_{\mathrm{BSP}}$ are "sufficiently" different from one another (the exact magnitude of the difference necessary is given by the straight lines that demarcate the different regions). When the revenue generation rates are relatively "comparable", Outcome 4, or Case C, dominates. In this scenario, the BSP can credibly ask for compensation from its competitor content provider as an assurance for not prioritizing its (the BSP's) own content over that of content provider C. It is instructive to compare this outcome with that of Case B. In the latter case, the BSP, which is relatively much less profitable in generating revenue from its own content than its competitor, finds it to its advantage to prioritize its competitor's content relative to its own and then extract the surplus from the latter which has a higher market share, than to give its content the same priority as that of its competitor (which it does under Case C, when the BSP is relatively more profitable). This result is similar to Mandy's conclusion in the sabotage literature that a vertically integrated supplier might not want to "kill the goose that may have laid the golden egg" (Mandy 2000).

The two regions in Case C, C1 and C2, warrant a separate explanation. The level of the priority access fee $p$ is determined by two constraints: (1) it has to be low enough to attract content provider C to pay and (2) as opposed to Case B discussed above, $p$ has to be low enough so that the BSP does not have the incentive to degrade the delivery of its own content. Case C1 denotes the region where constraint 1 binds, while Case C2 denotes the region where constraint 2 binds.

I end this section with a discussion of the implications of Case A and B. Under Case A, the BSP prioritizes its own content at the expense of its competitor, who prefers not to pay for that

privilege. The BSP thus gets an advantage over its competitor without the requirement of a comparable rent (since the BSP doesn't have to pay for the priority service for its own content), and this is something that should definitely be of interest to a policymaker deliberating on this issue, since such an arrangement might be tantamount to unfair competition. Under Case B, we have an outcome which looks counter-intuitive at first sight: the BSP deliberately de-prioritizes its own content with respect to its competitor in exchange for a fee. As the discussion in the previous paragraph indicates, this outcome is plausible: the BSP gains more from the fee from its competitor (who has a higher market share) than it would have generated from the advertising revenue of its own content (and commanding the same market share) by delivering its own content with the same priority as that of its competitor.

**The BSP's Vertical Integration Decision**

Facing two content providers Y and G (with respective revenue rate $r_{\mathrm{Y}}$ and $r_{\mathrm{G}}$, where $r_{\mathrm{G}} > r_{\mathrm{Y}}$), the BSP needs to examine the area above the $r_{\mathrm{G}} = r_{\mathrm{Y}}$ line of Figure 5-4a in arriving its vertical integration decision. For an arbitrary point $T_0 = \{r_{\mathrm{Y}0}, r_{\mathrm{G}0}\}$ on the $\{r_{\mathrm{Y}}, r_{\mathrm{G}}\}$ parameter space in Figure 5-4a, the BSP's vertical integration choice is one of the following:

- "integrate with Y" (correspond to $T_1 = \{r_{\mathrm{Y}0}, r_{\mathrm{G}0}\}$ in Figure 5-4b)
- "integrate with G" (correspond to $T_2 = \{r_{\mathrm{G}0}, r_{\mathrm{Y}0}\}$ in Figure 5-4b)
- "integrate with neither"

Figure 5-4a shows the various regions which we need to analyze in order to exhaustively determine the BSP's decision. These are best explained using Table 5-1. The expressions in the cells of the table refer to the profit of the BSP under various conditions. The subscript VI (short for vertical integration) refers to the BSP's decision to integrate with one of the content providers. Further, the subscript $T_1$ refers to the BSP's decision to integrate with Y, and $T_2$ refers

to the BSP's decision to integrate with G. The subscript NVI (short for no vertical integration) refers to the BSP's decision to integrate with neither content provider. The numerical subscripts 2, 3, 41 and 42 refer to the four potential equilibria: 2 corresponds to Case A of Proposition 5-1 where Outcome 2 is the equilibrium, 3 corresponds to Case B of Proposition 5-1 where Outcome 3 is the equilibrium, 41 corresponds to Case C1 of Proposition 5-1 where Outcome 4 is the equilibrium and 42 corresponds to Case C2 of Proposition 5-1 where Outcome 4 is the equilibrium. Note that when there is no vertical integration, there are no equivalent cases that correspond to Case C1 and C2 of the vertical integration setting. However, to properly illustrate the region corresponding to Outcome 4 under no vertical integration, we need to include the straight line $r_G = 2r_Y + \frac{t}{\lambda}(1 - 2x_3)$ that divides regions 3 and 6. This line separates the two cases where Outcome 3 is the equilibrium and where Outcome 4 is the equilibrium (see Chapter 3 for details). Thus, region 1 represents the $\{r_Y, r_G\}$ space where Outcome 3 is the equilibrium if the BSP integrates either with content provider Y or neither of them, and Outcome 2 is the equilibrium if the BSP integrates with content provider G. The other regions can be interpreted similarly.

By comparing the profits of the BSP for the three different choices in each of these regions, I find that the BSP will always prefer to integrate with content provider G (the cells with stars in Table 5-1 represent the BSP's decision). The details of these profit comparisons are showed in Appendix B. These results however do not tell us whether the social planner would also prefer such an outcome. That discussion is provided later in the section of welfare comparison, where I consider the choices of the social planner and compare them with the BSP's choices.

## The Welfare Effect of Net Neutrality

The different scenarios in the previous section, one under a net neutrality regime (or NN for short) and four under no net neutrality (NNN), list out exhaustively the various pricing/prioritization strategies of the BSP. Depending on the specific values of the parameters, the BSP can calculate the exact values of the optimum profit under each scenario, and decide on the best pricing strategy $F$ and $p$ correspondingly. We have already observed that given the choice, the BSP will never opt for abiding by the principles of net neutrality – in other words, unless enforced, net neutrality will never be a natural outcome of the game. It is imperative therefore from a regulatory perspective to find out whether it is in the best interests of the other players (the content providers and the consumers), as well as in terms of the total social welfare, to regulate enforcement of network neutrality. This section is devoted to that discussion – I discuss the impact of net neutrality (or its abolishment) on the payoffs to the three players and the overall welfare. Proposition 5-2 summarizes the findings.

**Proposition 5-2: (The welfare impact of net neutrality with a vertically integrated BSP)**

Under Case A where $r_C < r_{BSP} - \frac{t}{\lambda}(1 - 2x_3)$ and Case B where $r_C > \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP} + \frac{t}{\lambda}(1 - 2x_3)$, both consumer surplus and social welfare are higher under NNN than under NN.

Under Case C where $r_{BSP} - \frac{t}{\lambda}(1 - 2x_3) \le r_C \le \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP} + \frac{t}{\lambda}(1 - 2x_3)$, both consumer surplus and social welfare are the same under NN and NNN.

Proof: See Appendix B.

The expressions of consumer surplus and social welfare are provided in Table 5-2, which shows the comparison of the prices charged by the BSP, the profits of the BSP and that of the content provider C, the consumer surplus and the total social welfare under NN and in the three potential equilibria under NNN, which I denoted earlier as Cases A, B and C respectively. Case C is in turn divided into two subcases C1 and C2, as discussed in the previous section. Table 5-2 also shows the result of comparing the magnitude of these output variables under the different scenarios of NNN to that under NN, and in all the cases, we establish how the expressions compare regardless of input parameter values.

As can be seen from Table 5-2, the consumers' access fees are lower with NNN than with NN under Cases A and B and unchanged under Case C. The corresponding consumer surpluses are higher under Cases A and B as compared to NN, and unchanged under Case C. Similarly, the total social welfare with vertical integration increases with NNN under Cases A and B as compared to NN, while it is unchanged in Case C.

The vertically integrated BSP's profit is always higher under NNN, and therefore the BSP will always prefer NNN over NN.

The expressions in Table 5-2 establish that the content provider C has either the same profit (Case B) or is worse off under NNN. Interestingly, content provider C has the same profit in Case B that it has under NN, even though it has a higher market share than under NN – this is so because the BSP is able to extract the additional surplus from C completely through the priority access fee. In Case A, content provider C has a lower market share than under NN, and consequently a lower profit. In Case C, content provider C and the BSP have the same market share as under NN, but as a result of the priority access fee, content provider C's net surplus is lower than under NN.

**Welfare Comparison with Vertical Integration (VI) and No Vertical Integration (NVI)**

The issue of vertical integration in the online environment – whereby a BSP is both a provider and the delivery agent of the content – is especially significant from the perspective of the debate over net neutrality. Commentators of net neutrality have argued that the vertically integrated firm has an unfair competitive advantage over the pure play content provider, since the former can effectively act as a gatekeeper who can control the experience of the end consumers when they consume the content. Many consumers might find the delay of accessing particular websites having a disutility high enough to switch over to a competing provider, and since the vertically integrated firm can ask for compensation for not de-prioritizing the content of the rival content provider (or simply prioritize its own content without paying a fee), it can leave the pure play content provider at a competitive disadvantage. This insalubrious effect on the competition can be more palatable if it is found that the total social surplus as a result of the vertical integration is higher than what it would be if the BSP were not vertically integrated. In the latter scenario (i.e., with no vertical integration), the two competing content providers would have to pay the BSP in order to have their respective content delivered with priority to the end consumers. The various scenarios under this setting were analyzed in Chapter 3. We now proceed to compare the results in Chapter 3 with the findings in this chapter. Figure 5-5 visually summarizes the comparisons.

**Proposition 5-3: (The welfare effect of vertical integration)**

When $2r_{\mathrm{BSP}} + \frac{t}{\lambda}(1 - 2x_3) < r_{\mathrm{C}} < \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{\mathrm{BSP}} + \frac{t}{\lambda}(1 - 2x_3)$, social welfare is lower under VI than under NVI.

When $r_C + \frac{t}{\lambda}(1 - 2x_3) < r_{BSP} < 2r_C + \frac{t}{\lambda}(1 - 2x_3)$, social welfare is higher under VI than under NVI.

Otherwise, social welfare remains unchanged.

Proof: See Appendix B.

As can be seen from the graph in Figure 5-5, the crucial factors that determine the comparison of the social welfare with vertical integration (VI) as to when there is no vertical integration (NVI) are the relative magnitudes of the advertisement-generation capability of the two content providers (in other words, the ratio of $r_C$ to $r_{BSP}$), and whether the BSP has a strategic relationship with the more profitable content provider. In the graphs, the shaded regions represent the areas where the surpluses with VI and with NVI differ from one another. The term $\Delta SW$ signifies the difference between total social welfare under VI and under NVI. If the BSP has its strategic relationship with the less profitable content provider, the applicable region is the upper half of the quadrant that plots $r_C$ to $r_{BSP}$. In that situation, the surplus with VI is never greater than the surplus with NVI, and in fact, within the narrow strip of the shaded region, i.e., $2r_{BSP} + \frac{t}{\lambda}(1 - 2x_3) < r_C < \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP} + \frac{t}{\lambda}(1 - 2x_3)$, the surplus is lower. Conversely, if the BSP has its strategic relationship with the more profitable content provider[11], the total social welfare with VI is never lower than with NVI, and for the relatively thick shaded strip shown in Figure 5-5, the total surplus with vertical integration is actually higher than it would be with no vertical integration[12].

[11] The corresponding region in Figure 5-5 is the lower half of the quadrant where $r_C + \frac{t}{\lambda}(1 - 2x_3) < r_{BSP} < 2r_C + \frac{t}{\lambda}(1 - 2x_3)$.

[12] Simple algebra shows that lower strip is relatively thicker compared to the upper strip.

The reason for this can be understood as follows: In the region of the upper shaded strip, with no vertical integration, only content provider C pays and has its content prioritized at the expense of the content from the other content provider. However, with vertical integration, while content provider C is still the only paying content provider, in the shaded region, the content from the other content provider (now the BSP) receives equal priority. The corresponding loss of surplus of C is not fully compensated for by the gain of the BSP from its content, since the latter has a less effective revenue generation rate. The explanation for the lower shaded region is analogous: with no vertical integration, in this region, both content providers would have paid for equal priority of delivery, but with vertical integration, neither content providers pay; however with the content of the more effective content provider (now the BSP) being delivered with priority, the loss of C is more than compensated for by the gain of the BSP.

From the perspective of a policymaker, the results of this analysis are extremely significant. If net neutrality is not enforced, vertical integration can increase social welfare when the vertically integrated firm is also the more profitable content provider. However, the discussion might not always be this straightforward: one can certainly think of a scenario where the currently dominant content provider, envisaging future competition from a promising rival upstart (who right now lacks the financial prowess to generate more revenue from its own content), can preemptively merge with the BSP and put the rival startup at a disadvantage, and this might lead to a less socially beneficial outcome in the long run.

However, if the vertically integrated firm is the less profitable content provider (e.g., when the local BSP's search portal competes with the more established search engines), within a range of parameter values, social welfare can decrease (and will never increase), and therefore the policymaker will have reasons to subject such a service to more scrutiny.

Table 5-3 summarizes the preferences of the policymaker for vertical integration. As in Table 5-2, we need to consider the social welfare in all the seven regions. In regions 2 and 3 (corresponds to the upper shaded strip of Figure 5-5), the social planner prefers no vertical integration. In regions 4 and 6 (corresponds to the lower shaded strip of Figure 5-5), the social planner prefers vertical integration of the BSP and content provider G. In the other regions, the social planner has no preference between vertical integration and no vertical integration. The cells with stars in Table 5-3 represent the social planner's preferences. I summarize the comparison results in Proposition 5-4.

**Proposition 5-4: (The social planner's preferences for vertical integration compared to the BSP's choice)**

Comparing the BSP's vertical integration choice to the regulator's choice, we get the following:

When $2r_{BSP} + \frac{t}{\lambda}(1-2x_3) < r_C < \left(\frac{1-x_3}{1/2-x_3}\right) r_{BSP} + \frac{t}{\lambda}(1-2x_3)$, the BSP's vertical integration decision deviates from the social planner's preference. The social planner prefers no vertical integration while the BSP prefers to integrate with content provider G.

When $r_C + \frac{t}{\lambda}(1-2x_3) < r_{BSP} < 2r_C + \frac{t}{\lambda}(1-2x_3)$, the BSP's vertical integration decision is consistent with the social planner's preference. Both the social planner and the BSP prefer to integrate with content provider G.

Proof: See Appendix B.

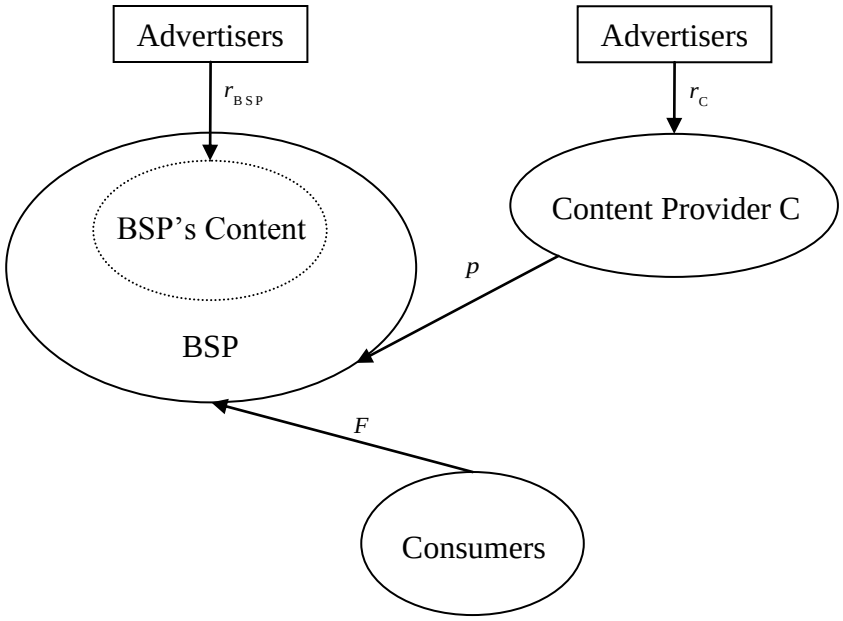



Figure 5-1. Market structure with vertical integration of content and broadband services

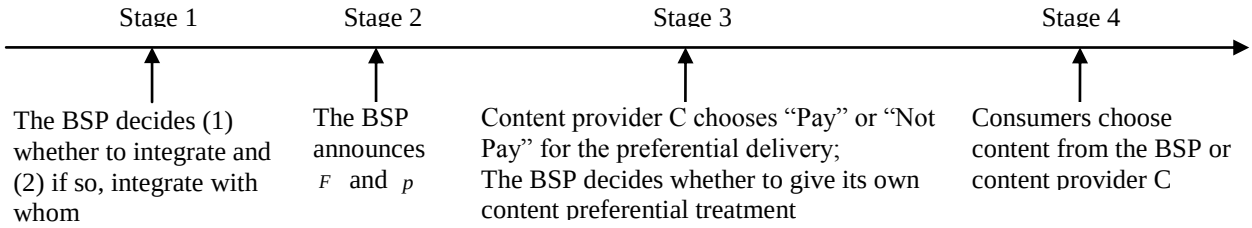



Figure 5-2. The sequence of events in the game with vertical integration

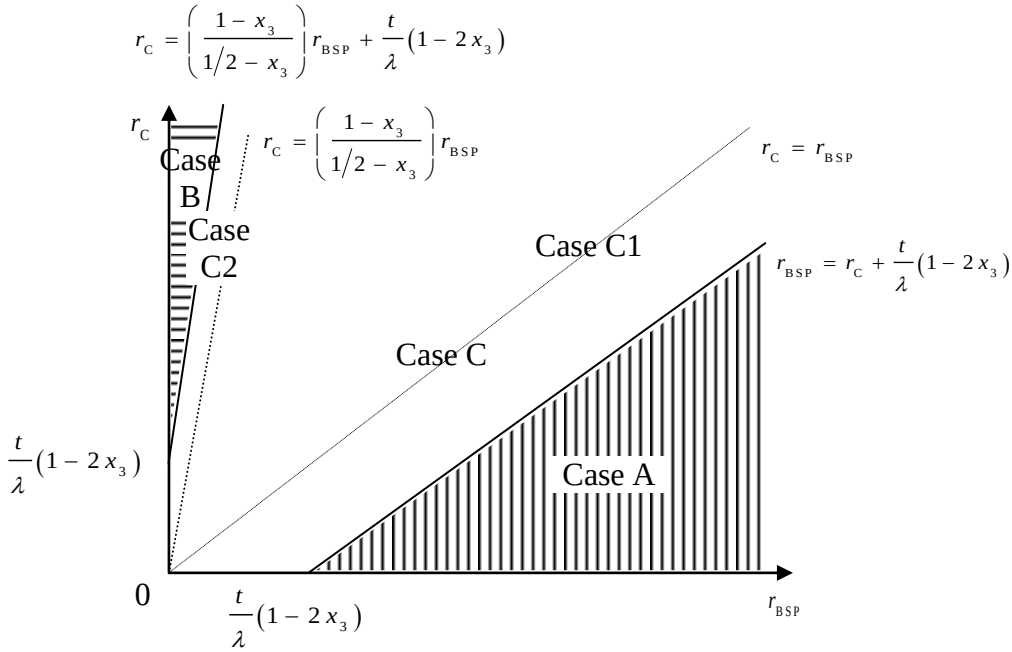
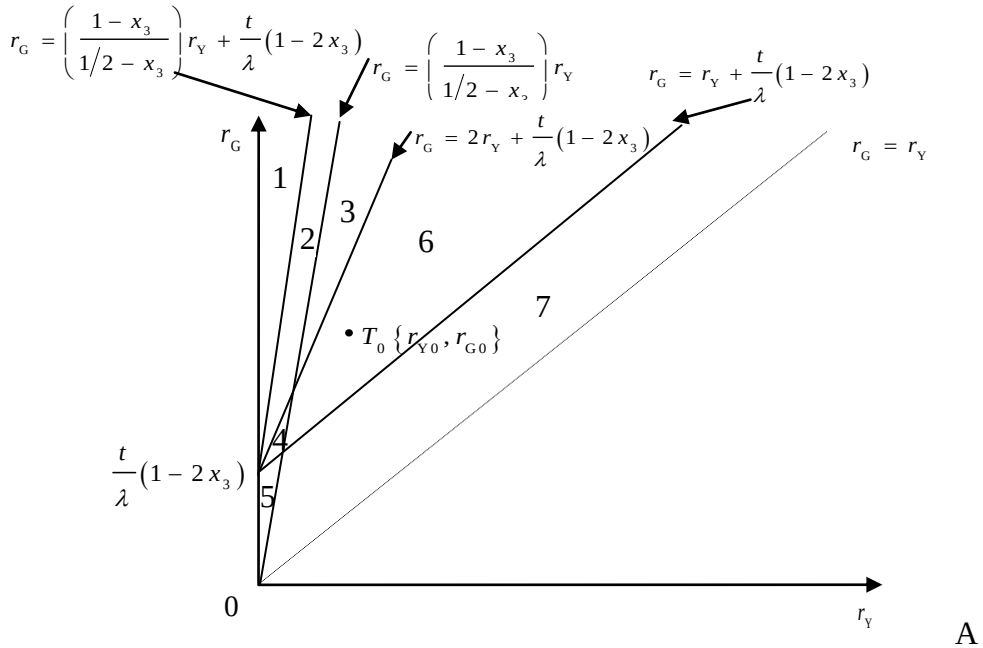
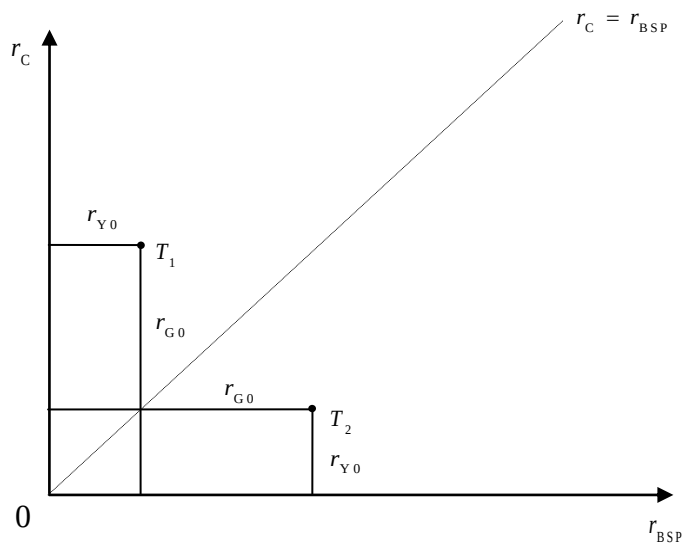



Figure 5-3. The equilibria with vertical integration




A




B

Figure 5-4. The BSP's vertical integration decision. A) The BSP's decision based on parameter space $\{r_Y, r_G\}$. B) The BSP's two options – integration with G or Y.

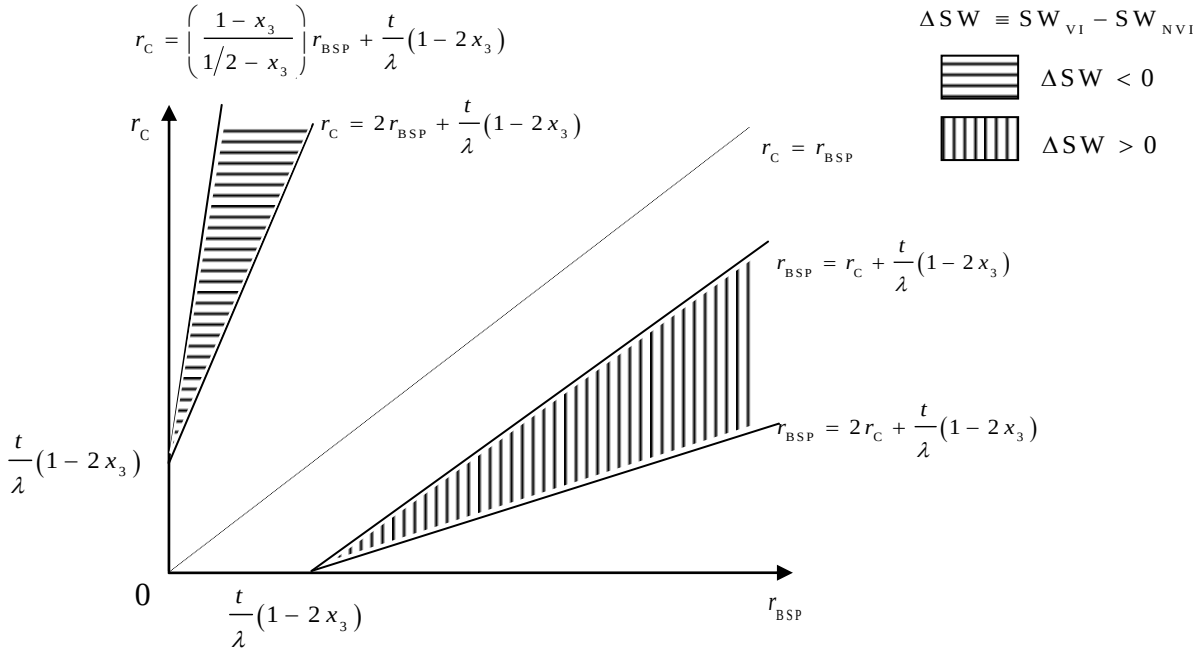



Figure 5-5. The effect of vertical integration on social welfare

Table 5-1. The BSP's vertical integration choices

| | integrate with Y | integrate with G | integrate with neither |
|---|---|---|---|
| | $T_1 = \{r_{Y0}, r_{G0}\}$ | $T_2 = \{r_{G0}, r_{Y0}\}$ | |
| Region 1 | $\pi_{VI3,T_1}$ | $\pi_{VI2,T_2}{}^*$ | $\pi_{NVI3}$ |
| Region 2 | $\pi_{VI42,T_1}$ | $\pi_{VI2,T_2}{}^*$ | $\pi_{NVI3}$ |
| Region 3 | $\pi_{VI41,T_1}$ | $\pi_{VI2,T_2}{}^*$ | $\pi_{NVI3}$ |
| Region 4 | $\pi_{VI42,T_1}$ | $\pi_{VI2,T_2}{}^*$ | $\pi_{NVI4}$ |
| Region 5 | $\pi_{VI42,T_1}$ | $\pi_{VI41,T_2}{}^*$ | $\pi_{NVI4}$ |
| Region 6 | $\pi_{VI41,T_1}$ | $\pi_{VI2,T_2}{}^*$ | $\pi_{NVI4}$ |
| Region 7 | $\pi_{VI41,T_1}$ | $\pi_{VI41,T_2}{}^*$ | $\pi_{NVI4}$ |

Table 5-2. Comparison of various economic outcomes of interest under NN and NNN

| | NN (Benchmark) | NNN (Case A: Outcome 2 – Only the BSP's content is prioritized) | NNN (Case B: Outcome 3 – Only C's content is prioritized) | NNN (Case C: Outcome 4 – Both the BSP and C's content are equally prioritized) | |
|---|---|---|---|---|---|
| | | | | Case C1 | CaseC2 |
| $F$ | $V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ | $V(\lambda) - tx_2 - \frac{d}{\mu - x_2\lambda}$ (Lower) | $V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda}$ (Lower) | $V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ (Unchanged) | $V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}$ (Unchanged) |
| $p$ | N/A | N/A | $\left(\frac{1/2 - x_3}{1 - x_3}\right) r_C$ | $(1 - 2x_3) r_C$ | $r_{BSP}$ |
| BSP's Profit | $V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{BSP}$ | $V(\lambda) - tx_2 - \frac{d}{\mu - x_2\lambda} + x_2\lambda r_{BSP}$ (Better off) | $V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_C + x_3\lambda r_{BSP}$ (Better off) | $V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{BSP} + \left(\frac{1}{2} - x_3\right)\lambda r_C$ (Better off) | $V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \lambda r_{BSP}$ (Better off) |
| Content Provider C's Profit | $\frac{1}{2}\lambda r_C$ | $(1 - x_2)\lambda r_C$ (Worse off) | $\frac{1}{2}\lambda r_C$ (Unchanged) | $x_3\lambda r_C$ (Worse off) | $\frac{1}{2}\lambda(r_C - r_{BSP})$ (Worse off) |
| Consumer Surplus | $\frac{t}{4}$ | $t\left(x_2^2 - x_2 + \frac{1}{2}\right)$ (Better off) | $t\left(x_3^2 - x_3 + \frac{1}{2}\right)$ (Better off) | $\frac{t}{4}$ (Unchanged) | $\frac{t}{4}$ (Unchanged) |
| Social Welfare | $V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{BSP} + \frac{1}{2}\lambda r_C$ | $V(\lambda) + t\left(x_2^2 - 2x_2 + \frac{1}{2}\right) - \frac{d}{\mu - x_2\lambda} + x_2\lambda r_{BSP} + (1 - x_2)\lambda r_C$ (Increased) | $V(\lambda) - t\left(\frac{1}{2} - x_3^2\right) - \frac{d}{\mu - (1 - x_3)\lambda} + x_3\lambda r_{BSP} + (1 - x_3)\lambda r_C$ (Increased) | $V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{BSP} + \frac{1}{2}\lambda r_C$ (Unchanged) | $V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{BSP} + \frac{1}{2}\lambda r_C$ (Unchanged) |

Note: The text in parenthesis shows how those economic outcomes change when moving from NN to NNN)

Table 5-3. The social planner's vertical integration preferences

| | integrate with Y | integrate with G | integrate with neither |
|---|---|---|---|
| | $T_1 = \{r_{Y0}, r_{G0}\}$ | $T_2 = \{r_{G0}, r_{Y0}\}$ | |
| Region 1 | $SW_{VI3,T_1}$ | $SW_{VI2,T_2}$ | $SW_{NVI3}$ |
| Region 2 | $SW_{VI42,T_1}$ | $SW_{VI2,T_2}$ | $SW_{NVI3}$ * |
| Region 3 | $SW_{VI41,T_1}$ | $SW_{VI2,T_2}$ | $SW_{NVI3}$ * |
| Region 4 | $SW_{VI42,T_1}$ | $SW_{VI2,T_2}$ * | $SW_{NVI4}$ |
| Region 5 | $SW_{VI42,T_1}$ | $SW_{VI41,T_2}$ | $SW_{NVI4}$ |
| Region 6 | $SW_{VI41,T_1}$ | $SW_{VI2,T_2}$ * | $SW_{NVI4}$ |
| Region 7 | $SW_{VI41,T_1}$ | $SW_{VI41,T_2}$ | $SW_{NVI4}$ |

# CHAPTER 6
# USER DISCRIMINATION

In 2007, it was independently verified that the broadband Internet service provider Comcast was slowing down network traffic within its servers that originated from the popular peer-to-peer (P2P) networks (McCullagh 2007). After initially denying any such behavior, Comcast defended its actions by claiming that the traffic from the P2P networks, which was dominated by just a small fraction of the total number of users, was slowing down the network traffic for the rest of the users. The United States Federal Communications Commission (FCC) later declared Comcast's actions to be illegal, thus providing further fuel to the "net neutrality" debate.

So far, the growing literature that has analyzed these economic issues of net neutrality has modeled that interested party as the content providers who are jockeying for a position in the consumers' minds. However, as the aforementioned Comcast example shows, the interested party might well be some of the consumers themselves who are willing to pay a fee to have their requested packets delivered with priority. In other words, a data packet traveling from its origin to its destination can be made "non-neutral" by the BSP at various stages of its journey – either at the "supply" side, whereby the BSP charges content providers for preferential delivery of their packets, or on the "demand" side, whereby the BSP charges the individual consumers a fee for the priority delivery of their requested content (or equivalently, de-prioritizes the requested content in the absence of the fee).

In this chapter, the focus is on the latter aspect of net neutrality, whereby I analyze the economic rationale for and against the proposals put forth by several broadband service providers who intend to differentiate between different classes of users. For example, the cable broadband service provider Time Warner Cable has recently started an experiment in certain

markets where they plan to charge Internet customers based on how much Web data they consume. The experiment started in a single market (Beaumont, TX) in the summer of 2008, and the company plans to introduce tiered pricing in several other markets in the near future. By charging a premium to the heaviest broadband users, much the same way cell phone providers collect fees from subscribers who exceed their allotted minutes, Time Warner would upend a longstanding uniform pricing strategy among (fixed-line) Internet service providers in the United States, whereby phone and cable companies have charged flat fees for unlimited access to the Web. AT&T has started a similar experiment with its own customers, also in Beaumont, TX.

As expected, such experiments have reignited the net neutrality debate. Proponents of net neutrality – consumer advocates and online content providers, for example – have opined that that a tiered Web-use pricing would limit customer choice and could stifle innovation by crimping demand for high-bandwidth services such as online video and music (Al-Chalabi 2008). However, cable and phone companies have countered by saying that they need the flexibility in setting prices for use of large, expensive, heavily used broadband networks, so as to effectively serve the majority of their customers and encourage greater efficiency in the way customers use capacity (Tweney 2008).

As consumers spend more time online, and also use the Internet to consume various types of data-intensive content (like music and video – a high-definition movie typically consumes around 8 GB of traffic), the decision to charge data consumption by volume can be expected to have profound implications in the way online content is consumed in future. In such scenarios, heavy users can expect to spend much more than what they currently spend on the erstwhile “all you can eat” plans. However, Time Warner has countered that most people are actually not downloading that much data. The company's trial in Beaumont, TX, lasted several months: of the

10,000 broadband customers enrolled – which represented about 25% of the company's total number of consumers in Beaumont – about 14% exceeded their cap and had to pay additional fees that averaged about $19 a month. Time Warner Cable also discovered that the top 25% of users consumed 100 times more data than the bottom 25% of users, suggesting an enormous gap in usage patterns.

Broadband service providers have often mentioned that as more and more people download TV shows and movies, particularly those in high-definition, the broadband network infrastructure faces enormous strain. Time Warner Cable has said its strategy is intended to alleviate some of that strain, with users self-regulating themselves under the new plan. But critics have expressed concerns that the pricing scheme will discourage broadband use and impede new online media businesses before they even have a chance to flourish.

The entire debate has raised a number of unanswered questions that are of interest to researchers and practitioners alike, not to mention the regulatory agencies. While legal scholars might debate whether such pricing plans (as those that Time Warner and AT&T are experimenting with) or prioritization strategies (as Comcast briefly attempted) are fair on the consumers, it is an entirely separate issue whether there are economic incentives for the BSPs to pursue such strategies that go against the net neutrality principle. In other words, facing a highly dynamic and differentiated data usage patterns from different classes of users, would a BSP gain (as compared to the status quo) by employing different pricing and/or packet prioritization strategies? In the first part of the analysis, I explore this issue.

While the BSP might prefer not to adhere to the principles of net neutrality under certain circumstances, such a move might be detrimental to the consumers or the society as a whole. Thus, from a social planner’s perspective, the issue is somewhat different: would the

abolishment of net neutrality on the "demand" side result in lower consumer surplus or social welfare? Depending on that answer, the social planner might wish to regulate on the issue.

In this chapter, I explore these issues and model them in an analytical framework and examine the economic impacts of user discrimination and net neutrality from the perspectives of both the BSP and the social planner. I characterize the dynamic and differentiated data demand of the end consumers by their valuations for data consumption and their usage patterns. Under the current scenario (which can be thought of as the net neutrality model with a uniform fixed fee pricing strategy), all users are charged the same fixed price for accessing broadband content. Both types of users face similar delays while accessing their desired content – the delay arises from the fact that the users' packets are serviced by the broadband service provider who has a fixed capacity. Facing this heterogeneous user data demand, broadband service providers have two potential instruments for user discrimination – price discrimination and traffic prioritization. If the BSP is allowed to differentially charge its users and/or prioritize their requested content, I explore six different strategies that it might employ:

- Broadband user traffic from different user types face the same delay, and all users are charged the same fixed fee (i.e., the status quo).
- Broadband user traffic from different user types face the same delay, and different types of users are charged different fixed fees.
- Broadband user traffic from different user types face the same delay, and different types of users are charged a two-part tariff.
- Broadband user traffic from different user types face different delays, and all users are charged the same fixed fee.
- Broadband user traffic from different user types face different delays, and different types of users are charged different fixed fees.
- Broadband user traffic from different user types face different delays, and different types of users are charged a two-part tariff.

The first three options (where all the broadband users face the same delay for their packets) cover different pricing strategies under net neutrality (or NN for short), while the last three options cover the different pricing strategies under no net neutrality (NNN). Another way to look at these options would be to think of the first three as representing strategies that the BSP can adopt if it is allowed to use only price discrimination, while the last three would represent strategies where the BSP is allowed to use both price and traffic prioritization as discriminating tools. These six different options help us model a broad swath of strategies that a BSP might employ under and in the absence of net neutrality. Depending on the characteristics of users' valuations for content and their usage patterns, different types of pricing and traffic prioritization regimes yield different profits for the BSP. However the optimal choice for the BSP might not coincide with that of a policymaker who intends to maximize the total social surplus. The results of the analysis should therefore be useful both for the broadband service providers as they mull over the introduction of the different pricing/prioritization strategies in an age where consumers increasingly get their information and entertainment online, and for policymakers who might wish to regulate the BSPs' practice of user discrimination in order to maximize social surplus.

## The Model

As before, I assume a monopolist BSP who provides Internet access to the end consumers. To model the demand for broadband Internet access service, I consider a unit mass of end consumers. As mentioned earlier, I assume that there are two types of users: a fraction $\alpha$ of H-type consumers and $1-\alpha$ fraction of L-type consumers. High-type users request more content (the requested rate of data packets by the two user types are given by $\lambda_H$ and $\lambda_L$ respectively, where $\lambda_H > \lambda_L$) and have higher valuation for that content ($v_H > v_L$) than the Low-type users. Considering the consumers' heterogeneous demand patterns, the BSP may charge a uniform

fixed fee ( $F$ ) per unit time to all consumers, different fixed fees ( $F_H$ and $F_L$ ) per unit time to different types of consumers, or a usage-based fee ( $p$ ) per packet to consumers for Internet access, a pricing strategy that has been already employed in some Scandinavian countries (Bandyopadhyay and Cheng 2006; Economist 2003). Since the consumers are serviced by the BSP which has a fixed network infrastructure capacity, the former encounter a disutility while they wait for the packets to arrive. The consumers' utility function thus takes the following form:

$$u_i = \begin{cases} V_i - d \cdot w_i - F, & \text{if the BSP charges a uniform fixed fee} \\ V_i - d \cdot w_i - F_i, & \text{if the BSP charges differential fixed fees} \\ V_i - d \cdot w_i - F - \lambda_i p, & \text{if the BSP charges a two-part tariff} \end{cases} \tag{6-1}$$

where $i = H$ or $L$ and $w_i$ is the delay for type $i$ consumers.

Consumers request data from various websites and the requested data packets are transmitted through the BSP's network. I assume consumers' request for data packets follows a Poisson process with arrival rate $\lambda_H$ and $\lambda_L$ for H-type and L-type consumers respectively. The gross valuations the two types of consumers receive are denoted by $V_H$ and $V_L$ . The delay for the consumers under net neutrality is:

$$w_i = \frac{1}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L}, \quad i = H \text{ or } L \tag{6-2}$$

In the absence of net neutrality, the BSP may prioritize data traffic based on user types. In this context, we note that the technology to discriminate packets and streamline Internet traffic has been available at minimal cost, and I therefore assume that there is no additional expense incurred by the BSP to implement a mechanism that enables preferential delivery of content. I use a two-class priority queue to model the BSP's data transmission service. If both H-type and L-type consumers receive the same priority for their traffic, then the congestion cost and the corresponding utility function would remain the same as in Equation (6-2). However, if H-type

consumers receive higher priority while L-type consumers receive lower priority, then the delay costs for the two types are given in (6-3).

$$w_H = \frac{1}{\mu - \alpha\lambda_H}, \; w_L = \frac{\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} \tag{6-3}$$

On the other hand, if L-type consumers receive higher priority while H-type consumers receive lower priority, then the delay for the two types are given in (6-4).

$$w_H = \frac{\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}, \; w_L = \frac{1}{\mu - (1-\alpha)\lambda_L} \tag{6-4}$$

In terms of pricing, the BSP may charge a uniform fixed fee to all consumers or charge different fixed fees to different types of consumers for Internet access. The potential regulation of net neutrality limits the BSP from selectively prioritizing the Internet traffic. In the absence of net neutrality, the BSP can also discriminate against different types of consumers through traffic prioritization. In the next two sections, I model these scenarios.

## Net Neutrality

In this section, I analyze three potential pricing structures for the BSP under net neutrality – uniform fixed fee, differential fixed fees and charging a two-part tariff.

### Option NN1: Uniform Fixed Fee Under Net Neutrality

Under net neutrality all consumers receive the same priority and therefore face the same congestion for data transmission. The most simple and common pricing mechanism for the BSP is to charge a uniform fixed fee for all consumers. The BSP's profit maximization problem is formulated in (6-5).

$$\max_{F_{NN1}} \pi_{NN1} = F_{NN1}$$

$$\text{s.t.} \quad V_H - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN1} \geq 0 \quad (i) \tag{6-5}$$

$$V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN1} \geq 0 \quad (ii)$$

Constraint $(i)$ is the participation constraint for H-type consumers and constraint $(ii)$ is the participation constraint for L-type consumers. Since $V_H > V_L$, the BSP will charge a fixed access fee that is high enough to just keep the L-type consumers to participate, i.e., $F^*_{NN1} = V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$, and the BSP then receives a corresponding profit of $\pi^*_{NN1} = F^*_{NN1} = V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$.

The corresponding consumer surplus, defined as the sum of the utility of all consumers, is given by

$$CS_{NN1} = \alpha\left(V_H - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN1}\right) + (1-\alpha)\left(V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN1}\right)$$

$= \alpha(V_H - V_L)$, and the social welfare, defined as the sum of both the BSP's profit and consumer surplus, is $SW_{NN1} = \pi^*_{NN1} + CS_{NN1} = \alpha V_H + (1-\alpha)V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$.

**Option NN2: Differential Fixed Fees Under Net Neutrality**

It is easy to see that this option reduces to the option NN1 above. This is because the BSP has just one service offering at its disposal, and therefore will not be able to differentiate between the two classes of users by using different prices (if the two user types are offered two different price points, the H-type users will always choose the lower price, as would the L-type users). The formal statement of the BSP's profit maximization problem is as follows:

$$\max_{F_{NN2\_H}, F_{NN2\_L}} \pi_{NN2} = \alpha F_{NN2\_H} + (1-\alpha) F_{NN2\_L}$$

$$\text{s.t.} \quad V_H - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN2\_H} \geq 0 \qquad (i)$$

$$V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN2\_L} \geq 0 \qquad (ii) \qquad (6\text{-}6)$$

$$V_H - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN2\_H} \geq V_H - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN2\_L} \qquad (iii)$$

$$V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN2\_L} \geq V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN2\_H} \qquad (iv)$$

Constraints $(i)$ and $(ii)$ are participation constraints for H-type and L-type consumers respectively. Constraints $(iii)$ and $(iv)$ are incentive compatibility constraints for H-type and L-type consumers respectively. Constraint $(iii)$ can be reduced to $F_{NN2\_H} \leq F_{NN2\_L}$ and Constraint $(iv)$ can be reduced to $F_{NN2\_H} \geq F_{NN2\_L}$. So $F_{NN2\_H} = F_{NN2\_L}$. As a result, under net neutrality Option NN2 can be reduced to Option NN1 with

$$\pi^*_{NN2} = F^*_{NN2\_H} = F^*_{NN2\_L} = V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}.$$

The corresponding consumer surplus will still be $CS_{NN2} = \alpha(V_H - V_L)$, and the social welfare will be given by $SW_{NN2} = \alpha V_H + (1-\alpha)V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$.

**Option NN3: Two-Part Tariff Under Net Neutrality**

Under this option, the BSP charges a two-part tariff for Internet access – a lump-sum fee $F$ and a per-unit charge $p$. Under net neutrality, the BSP cannot prioritize any user's requested content. The BSP's profit maximization problem is:

$$\max_{F_{NN3},\, p_{NN3}} \pi_{NN3} = F_{NN3} + \left[\alpha\lambda_H + (1-\alpha)\lambda_L\right] p_{NN3}$$

$$\text{s.t.} \quad V_H - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN3} - \lambda_H p_{NN3} \geq 0 \quad (i)$$

$$V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN3} - \lambda_L p_{NN3} \geq 0 \quad (ii) \tag{6-7}$$

Constraint $(i)$ is the participation constraint for H-type consumers and constraint $(ii)$ is the participation constraint for L-type consumers. By solving the BSP's problem (see Appendix B for derivation details), I find when the two types of consumers' valuations for data consumption are comparable (I denote this as Case NN3_1, with the exact criterion being $V_H \leq \frac{\lambda_H V_L}{\lambda_L} - \frac{d(\lambda_H - \lambda_L)}{\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$), the BSP will charge a positive lump-sum fee $F^*_{NN3\_1} = \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$ and a positive usage-based fee $p^*_{NN3\_1} = \frac{V_H - V_L}{\lambda_H - \lambda_L}$; however, if the two types of consumers differ significantly in their valuations for their requested content (or more precisely, if $V_H > \frac{\lambda_H V_L}{\lambda_L} - \frac{d(\lambda_H - \lambda_L)}{\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$, which I denote as Case NN3_2), the BSP will charge a zero lump-sum fee and rely only on usage-based fee: $F^*_{NN3\_2} = 0$ and $p^*_{NN3\_2} = \frac{1}{\lambda_L}\left[V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right]$. The corresponding consumer surpluses are $CS_{NN3\_1} = 0$ and $CS_{NN3\_2} = \alpha\left\{\left[\frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right]\left(\frac{\lambda_H - \lambda_L}{\lambda_L}\right) - \left(\frac{\lambda_H V_L - \lambda_L V_H}{\lambda_L}\right)\right\}$. The resulting social welfare is $SW_{NN3\_1} = SW_{NN3\_2} = \alpha V_H + (1-\alpha)V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$. Note that under Case NN3_1, the entire consumer surplus is extracted away completely by the BSP.

## No Net Neutrality

In this section, I consider the BSP's three pricing options (uniform fixed fee, differential fixed fees and two-part tariff) under NNN. In the absence of any net neutrality regulation, broadband service providers have one extra set of instruments to discriminate between end users: the BSP may assign different priorities to different types of traffic. Technically, BSPs first identify the data destination by inspecting data packets transmitted through the network. The BSPs then either charge the same access fee for all consumers and downgrade data transmission for heavy users (H-type consumers in the model), or charge a higher access fee and then assign a higher priority for the data packets requested by H-type consumers. Just as I analyzed the pricing strategies under NN, I now look into the three analogous pricing regimes under NNN.

### Option NNN1: Uniform Fixed Fee Under No Net Neutrality

When the BSP charge a uniform price to both types of the consumers, it has the incentive to assign a lower priority to data packets from H-type users because of their heavy use of the shared bandwidth. The BSP's decision problem can be formulated as:

$$\begin{aligned} &\max_{F_{NNN1}} \pi_{NNN1} = F_{NNN1} \\ &\text{s.t.} \quad V_H - \frac{d\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - F_{NNN1} \geq 0 \quad (i) \\ &\qquad V_L - \frac{d}{\mu - (1-\alpha)\lambda_L} - F_{NNN1} \geq 0 \quad (ii) \end{aligned} \tag{6-8}$$

Constraint $(i)$ is the participation constraint for H-type consumers (reflecting their higher wait times in the prioritized queue) and constraint $(ii)$ is the participation constraint for L-type consumers. Notice that I assume $V_H \geq \frac{d\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$ to ensure that this

scenario is feasible. Both constraints give upper bounds for the access fee $F_{NNN1}$. One can derive the equilibrium by comparing the two upper bounds.

Case NNN1_1: If $V_L - \frac{d}{\mu - (1-\alpha)\lambda_L} \le V_H - \frac{d\mu}{[\mu - (1-\alpha)\lambda_L][\mu - \alpha\lambda_H - (1-\alpha)\lambda_L]}$, i.e.,

$V_H - V_L \ge \frac{d[\alpha\lambda_H + (1-\alpha)\lambda_L]}{[\mu - (1-\alpha)\lambda_L][\mu - \alpha\lambda_H - (1-\alpha)\lambda_L]}$, then $F^*_{NNN1\_1} = V_L - \frac{d}{\mu - (1-\alpha)\lambda_L}$ and

$\pi^*_{NNN1\_1} = V_L - \frac{d}{\mu - (1-\alpha)\lambda_L}$. The corresponding consumer surplus is

$$CS_{NNN1\_1} = \alpha\left(V_H - \frac{d\mu}{[\mu - (1-\alpha)\lambda_L][\mu - \alpha\lambda_H - (1-\alpha)\lambda_L]} - F_{NNN1\_1}\right)$$
$$+ (1-\alpha)\left(V_L - \frac{d}{\mu - (1-\alpha)\lambda_L} - F_{NNN1\_1}\right)$$
$$= \alpha\left[V_H - V_L - \frac{d}{\mu - (1-\alpha)\lambda_L} \cdot \frac{\alpha\lambda_H + (1-\alpha)\lambda_L}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right].$$

The expression for social welfare is

$$SW_{NNN1\_1} = \pi^*_{NNN1\_1} + CS_{NNN1\_1}$$
$$= \alpha\left(V_H - \frac{d\mu}{[\mu - (1-\alpha)\lambda_L][\mu - \alpha\lambda_H - (1-\alpha)\lambda_L]}\right) + (1-\alpha)\left(V_L - \frac{d}{\mu - (1-\alpha)\lambda_L}\right).$$

Case NNN1_2: If $V_H - V_L < \frac{d[\alpha\lambda_H + (1-\alpha)\lambda_L]}{[\mu - (1-\alpha)\lambda_L][\mu - \alpha\lambda_H - (1-\alpha)\lambda_L]}$, then

$F^*_{NNN1\_2} = V_H - \frac{d\mu}{[\mu - (1-\alpha)\lambda_L][\mu - \alpha\lambda_H - (1-\alpha)\lambda_L]}$ and

$\pi^*_{NNN1\_2} = V_H - \frac{d\mu}{[\mu - (1-\alpha)\lambda_L][\mu - \alpha\lambda_H - (1-\alpha)\lambda_L]}$.

The corresponding consumer surplus is

$$CS_{NNN1\_2} = \alpha\left(V_H - \frac{d\mu}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]} - F_{NNN1\_2}\right)$$

$$+(1-\alpha)\left(V_L - \frac{d}{\mu-(1-\alpha)\lambda_L} - F_{NNN1\_2}\right)$$

$$= (1-\alpha)\left\{-V_H + V_L + \frac{d}{\mu-(1-\alpha)\lambda_L}\cdot\frac{\alpha\lambda_H+(1-\alpha)\lambda_L}{\mu-\alpha\lambda_H-(1-\alpha)\lambda_L}\right\},$$

and the corresponding social welfare is given by the following expression:

$$SW_{NNN1\_2} = \pi^*_{NNN1\_2} + CS_{NNN1\_2}$$

$$= \alpha\left(V_H - \frac{d\mu}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}\right) + (1-\alpha)\left(V_L - \frac{d}{\mu-(1-\alpha)\lambda_L}\right).$$

**Option NNN2: Differential Fixed Fees Under No Net Neutrality**

Under this option, the BSP charges a higher price for higher quality of the data transmission service through the Internet. Specifically, the BSP would offer the Internet access service with congestion cost $\frac{d}{\mu-\alpha\lambda_H}$ at a fixed price $F_{NNN2\_H}$ to H-type consumers and the Internet access service with congestion cost $\frac{d\mu}{(\mu-\alpha\lambda_H)\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}$ at a fixed price $F_{NNN2\_L}$ to L-type consumers. The BSP's profit maximization problem is then as follows:

$$\max_{F_{\text{NNN2\_H}},F_{\text{NNN2\_L}}} \pi_{\text{NNN2}} = \alpha F_{\text{NNN2\_H}} + (1-\alpha) F_{\text{NNN2\_L}}$$

$$\text{s.t.} \quad V_H - \frac{d}{\mu - \alpha\lambda_H} - F_{\text{NNN2\_H}} \geq 0 \qquad (i)$$

$$V_L - \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - F_{\text{NNN2\_L}} \geq 0 \qquad (ii)$$

$$V_H - \frac{d}{\mu - \alpha\lambda_H} - F_{\text{NNN2\_H}} \geq V_H - \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - F_{\text{NNN2\_L}} \qquad (iii)$$

$$V_L - \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - F_{\text{NNN2\_L}} \geq V_L - \frac{d}{\mu - \alpha\lambda_H} - F_{\text{NNN2\_H}} \qquad (iv)$$

(6-9)

Constraint $(i)$ is the participation constraint for H-type consumers and constraint $(ii)$ is the participation constraint for L-type consumers. Notice that I assume $V_L \geq \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$ to ensure the feasibility of this outcome. Constraints $(iii)$ and $(iv)$ are incentive compatibility constraints for H-type and L-type consumers respectively. Constraint $(iii)$ can be reduced to

$$F_{\text{NNN2\_H}} - F_{\text{NNN2\_L}} \leq \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - \frac{d}{\mu - \alpha\lambda_H}.$$

Constraint $(iv)$ can be reduced to

$$F_{\text{NNN2\_H}} - F_{\text{NNN2\_L}} \geq \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - \frac{d}{\mu - \alpha\lambda_H}.$$

Therefore, $F_{\text{NNN2\_H}} - F_{\text{NNN2\_L}} = \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - \frac{d}{\mu - \alpha\lambda_H}$.

From constraint $(i)$, we get $F_{\text{NNN2\_H}} \leq V_H - \frac{d}{\mu - \alpha\lambda_H}$.

From constraint $(ii)$, we get $F_{\mathrm{NNN2\_L}} \le V_L - \dfrac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$.

Since $\left\{V_H - \dfrac{d}{\mu - \alpha\lambda_H}\right\} - \left\{V_L - \dfrac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}\right\}$

$> \dfrac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - \dfrac{d}{\mu - \alpha\lambda_H}$, $F^*_{\mathrm{NNN2\_H}} = V_L - \dfrac{d}{\mu - \alpha\lambda_H}$,

$F^*_{\mathrm{NNN2\_L}} = V_L - \dfrac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$, and

$$\pi^*_{\mathrm{NNN2}} = \alpha F^*_{\mathrm{NNN2\_H}} + (1-\alpha) F^*_{\mathrm{NNN2\_L}} = V_L - \frac{d}{\mu - \alpha\lambda_H}\left[\frac{\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right].$$

The corresponding consumer surplus is

$$\begin{aligned} \mathrm{CS}_{\mathrm{NNN2}} &= \alpha\left(V_H - \frac{d}{\mu - \alpha\lambda_H} - F_{\mathrm{NNN2\_H}}\right) \\ &+ (1-\alpha)\left(V_L - \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - F_{\mathrm{NNN2\_L}}\right) = \alpha(V_H - V_L) \end{aligned}$$

, and the social welfare is given by

$$\begin{aligned} \mathrm{SW}_{\mathrm{NNN2}} &= \pi^*_{\mathrm{NNN2}} + \mathrm{CS}_{\mathrm{NNN2}} \\ &= \alpha\left(V_H - \frac{d}{\mu - \alpha\lambda_H}\right) + (1-\alpha)\left(V_L - \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}\right). \end{aligned}$$

**Option NNN3: Two-Part Tariff Under No Net Neutrality**

Under this scenario, the BSP charges the H-type consumers a two-part tariff to ensure a preferential delivery of their data packets, while the L-type consumers are charged only a lump-sum fee for their data delivery (which involves a higher delay). The BSP's decision problem can be formulated as:

$$\max_{F_{\mathrm{NNN3}},\, p_{\mathrm{NNN3}}} \quad \pi_{\mathrm{NNN3}} = F_{\mathrm{NNN3}} + \alpha \lambda_{\mathrm{H}}\, p_{\mathrm{NNN3}}$$

$$\text{s.t.} \quad V_H - \frac{d}{\mu - \alpha \lambda_H} - F_{\mathrm{NNN3}} - \lambda_{\mathrm{H}}\, p_{\mathrm{NNN3}} \geq 0 \qquad (i)$$

$$V_L - \frac{d\,\mu}{(\mu - \alpha \lambda_H)\left[\mu - \alpha \lambda_H - (1-\alpha)\lambda_L\right]} - F_{\mathrm{NNN3}} \geq 0 \qquad (ii)$$

$$V_H - \frac{d}{\mu - \alpha \lambda_H} - F_{\mathrm{NNN3}} - \lambda_{\mathrm{H}}\, p_{\mathrm{NNN3}} \geq V_H - \frac{d\,\mu}{(\mu - \alpha \lambda_H)\left[\mu - \alpha \lambda_H - (1-\alpha)\lambda_L\right]} - F_{\mathrm{NNN3}} \qquad (iii)$$

$$V_L - \frac{d\,\mu}{(\mu - \alpha \lambda_H)\left[\mu - \alpha \lambda_H - (1-\alpha)\lambda_L\right]} - F_{\mathrm{NNN3}} \geq V_L - \frac{d}{\mu - \alpha \lambda_H} - F_{\mathrm{NNN3}} - \lambda_{\mathrm{H}}\, p_{\mathrm{NNN3}} \qquad (iv)$$

(6-10)

Constraint $(i)$ is the participation constraint for H-type consumers and constraint $(ii)$ is the participation constraint for L-type consumers. Constraints $(iii)$ and $(iv)$ are incentive compatibility constraints for H-type and L-type consumers respectively. Constraint $(iii)$ can be reduced to

$$p_{\mathrm{NNN3}} \leq \frac{1}{\lambda_{\mathrm{H}}}\left\{\frac{d\,\mu}{(\mu - \alpha \lambda_H)\left[\mu - \alpha \lambda_H - (1-\alpha)\lambda_L\right]} - \frac{d}{\mu - \alpha \lambda_H}\right\}$$

Constraint $(iv)$ can be reduced to

$$p_{\mathrm{NNN3}} \geq \frac{1}{\lambda_{\mathrm{H}}}\left\{\frac{d\,\mu}{(\mu - \alpha \lambda_H)\left[\mu - \alpha \lambda_H - (1-\alpha)\lambda_L\right]} - \frac{d}{\mu - \alpha \lambda_H}\right\}.$$

So $p^*_{\mathrm{NNN3}} = \dfrac{1}{\lambda_{\mathrm{H}}}\left\{\dfrac{d\,\mu}{(\mu - \alpha \lambda_H)\left[\mu - \alpha \lambda_{\mathrm{H}} - (1-\alpha)\lambda_{\mathrm{L}}\right]} - \dfrac{d}{\mu - \alpha \lambda_H}\right\}.$

Substituting back to constraint $(i)$ gives

$$F_{\mathrm{NNN3}} \leq V_H - -\frac{d\,\mu}{(\mu - \alpha \lambda_H)\left[\mu - \alpha \lambda_H - (1-\alpha)\lambda_L\right]}.$$

Constraint $(ii)$ implies $F_{\mathrm{NNN3}} \leq V_L - \dfrac{d\,\mu}{(\mu - \alpha \lambda_H)\left[\mu - \alpha \lambda_H - (1-\alpha)\lambda_L\right]}.$

So $F^*_{\mathrm{NNN3}} = V_L - \dfrac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$ and

$$\pi^*_{\mathrm{NNN3}} = F^*_{\mathrm{NNN3}} + \alpha\lambda_{\mathrm{H}}\, p^*_{\mathrm{NNN3}} = V_L - \frac{d}{\mu - \alpha\lambda_H}\left[\frac{\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right].$$

The corresponding consumer surplus is

$$\mathrm{CS}_{\mathrm{NNN3}} = \alpha\left(V_H - \frac{d}{\mu - \alpha\lambda_H} - F_{\mathrm{NNN3}} - \lambda_{\mathrm{H}}\, p_{\mathrm{NNN3}}\right)$$
$$+ (1-\alpha)\left(V_L - \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - F_{\mathrm{NNN3}}\right) = \alpha(V_H - V_L).$$

Therefore the social welfare is $\mathrm{SW}_{\mathrm{NNN3}} = \pi^*_{\mathrm{NNN3}} + \mathrm{CS}_{\mathrm{NNN3}}$

$$= \alpha\left(V_H - \frac{d}{\mu - \alpha\lambda_H}\right) + (1-\alpha)\left(V_L - \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}\right).$$

Based on the BSP's pricing and traffic prioritization strategies, the BSP has six options and I summarize these six options in the Table 6-1.

In the previous two sections, I have analyzed the BSP's six options involving pricing and (under NNN) different priorities for different user classes. In the next section, I compare these different options, and thus explore the conditions under which the BSP might choose any one of them.

## The BSP's Choices

In this section I study the effects of pricing structure and traffic prioritization on the BSP's profit.

### The BSP's Preference for Pricing Structure Under Net Neutrality (Choice Among Three Options NN1, NN2, NN3)

Under net neutrality, the BSP is limited to just the pricing mechanisms to discriminate between the different user types. Comparing the BSP's three pricing options (NN1, NN2 and

NN3) under net neutrality yields $\pi^*_{NN3\_1} > \pi^*_{NN1} = \pi^*_{NN2}$ and $\pi^*_{NN3\_2} > \pi^*_{NN1} = \pi^*_{NN2}$. This result is summarized in Proposition 6-1.

**Proposition 6-1: (The BSP's preferred pricing structure under net neutrality)**

Under net neutrality, the BSP prefers a two-part tariff.

Proof: See Appendix B.

This result is intuitive since a two-part tariff provides two pricing instruments (fixed fee and usage-based fee) and therefore can extract more consumer surplus compared to the uniform fixed fee only for the BSP.

**The BSP's Preference for Pricing Structure Under No Net Neutrality (Choice Among Three Options NNN1, NNN2, NNN3)**

In the absence of net neutrality, the BSP may either charge the same price to both types of consumers and set a lower priority to data packets from H-type consumers (NNN1), or charge a higher price and set a higher priority to H-type consumers (NNN2), or charge H-type consumers a usage-based fee to get a higher priority (NNN3). The first option yields profit levels

$\pi^*_{NNN1\_1} = V_L - \dfrac{d}{\mu - (1-\alpha)\lambda_L}$ if $V_H - V_L \geq \dfrac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$ or

$\pi^*_{NNN1\_2} = V_H - \dfrac{d\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$ if

$V_H - V_L < \dfrac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$. The second and third option generates the same profit level for the BSP, i.e., $\pi^*_{NNN2} = \pi^*_{NNN3} = V_L - \dfrac{d\left[\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L\right]}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$. Comparing the three options, we find that if $V_H - V_L \geq \dfrac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$ then

$\pi^*_{\text{NNN1\_1}} > \pi^*_{\text{NNN2}} = \pi^*_{\text{NNN3}}$ i.e., when H-type consumers value their requested content more than the L-type consumers beyond a threshold, the BSP benefits from charging the same price to both types and assigning a lower priority to traffic from H-type consumers. If on the other hand $V_H - V_L < \frac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$, then $\pi^*_{\text{NNN1\_2}} < \pi^*_{\text{NNN2}} = \pi^*_{\text{NNN3}}$., i.e., when H-type consumers and L-type consumers have similar valuation for content, the BSP prefers to charge a higher price and in return offer preferential delivery to the data packets from the H-type consumers. This leads to Proposition 6-2.

**Proposition 6-2: (The BSP's preferred pricing structure under no net neutrality)**

(1) If $V_H - V_L \geq \frac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$, $\pi^*_{\text{NNN1}} = \pi^*_{\text{NNN1\_1}} > \pi^*_{\text{NNN2}} = \pi^*_{\text{NNN3}}$;

(2) If

$$\frac{d(1-\alpha)\left\{(\lambda_L - \alpha\lambda_H)\mu + \alpha\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]\right\}}{(\mu - \alpha\lambda_H)\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} \leq V_H - V_L < \frac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

, $\pi^*_{\text{NNN1}} = \pi^*_{\text{NNN1\_2}} > \pi^*_{\text{NNN2}} = \pi^*_{\text{NNN3}}$;

(3) If $V_H - V_L < \frac{d(1-\alpha)\left\{(\lambda_L - \alpha\lambda_H)\mu + \alpha\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]\right\}}{(\mu - \alpha\lambda_H)\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$,

$\pi^*_{\text{NNN1}} = \pi^*_{\text{NNN1\_2}} < \pi^*_{\text{NNN2}} = \pi^*_{\text{NNN3}}$.

Proof: See Appendix B.

**The BSP's Overall Preference for Pricing Structure (Choice Among the Six Options NN1, NN2, NN3, NNN1, NNN2, NNN3)**

In this subsection I address the question of what would be the equilibrium outcome if the BSP is given all six user discrimination options. Proposition 6-3 summarizes the comparison result of all six options.

**Proposition 6-3: (The BSP's overall preferred pricing structure)**

There are two potential preferred pricing structures for the BSP: NN3 or NNN1, depending on the parameter values.

Proof: see Appendix B.

I illustrate these results by adopting some real-life parameter values. AT&T has recently estimated that their top 5% of users (in terms of usage) account for about 40% of the total traffic, i.e., $\alpha = 0.05$ and $\frac{\alpha \lambda_H}{\alpha \lambda_H + (1-\alpha)\lambda_L} = 0.4$ (Tweney 2008). Using these parameter values, Figure 6-1 depicts the BSP's overall preferred pricing/prioritization strategy for such a traffic pattern in the $V_H - V_L$ space. The BSP prefers an NN3 outcome (two-part tariff with equal priority) in the shaded area and it prefers an NNN1 outcome (uniform fixed fee with low priority for heavy users) in the un-shaded area. The area marked by the bold lines represents the feasible parameter space. The different intercepts $C_0$, $C_1$, etc. on the two axes (the precise values of these intercepts have been defined in Appendix B) and the straight lines emanating from them represent the different regions (marked by numbers 1 through 7) in the parameter space within which we have to consider the optimal regime choice for the BSP. The BSP's preferred option is: NNN1_1 for region 1, NN3_2 for region 2, NN3_1 for region 3, NNN1_1 for region 4, NNN1_2 for region 5, NN3_1 for region 6, and NN3_1 for region 7.

A different traffic pattern would change the slope of the line that has the intercept of $-C_0$, but would not materially change the nature of the outcome – there would still be some regions where the BSP would opt for the NN3 outcome and the rest of the feasible region where the BSP would opt for the NNN1 outcome.

## The Social Planner's Preference for Pricing Structure

As outlined before, the choice of the social planner with regards to the pricing/prioritization regime might be at odds with that of the BSP, since social welfare is the sum of the BSP's profit and the consumers' surplus. Note that since the consumers' payments for the broadband services are effectively internal transfers as far as the calculation of the social welfare is concerned, the only measurable effect of the consumers on the social welfare comes from their valuation and the disutility that they attribute towards the congestion.

### The Social Planner's Preference for Pricing Structure Under Net Neutrality (NN1, NN2, NN3)

In this subsection, I examine the social planner's preference for different pricing structures under net neutrality by comparing the social welfare levels when the BSP adopts the three pricing structures. Proposition 6-4 summarizes the analysis.

**Proposition 6-4: (The Social planner's preferred pricing structure under net neutrality)**

When net neutrality is in place, social welfare is the same for one-level fixed fee, two-level fixed fee, and two-part tariff, i.e., $SW_{NN1} = SW_{NN2} = SW_{NN3}$. This is expected, since the effect of pricing is internalized, and there are no other effects to consider, as traffic prioritization is not allowed under net neutrality.

### The Social Planner's Preference for Pricing Structure Under No Net Neutrality (NNN1, NNN2, NNN3)

In this subsection, I examine the social planner's preference for different pricing structures under no net neutrality by comparing the social welfare levels when the BSP adopts the three pricing structures.

Propositions 6-5 and 6-6 summarize the results.

**Proposition 6-5: (The Social planner's preferred pricing structure under no net neutrality)**

Without net neutrality, the social planner always prefers the BSP charging a uniform fixed fee while downgrading heavy users, i.e., $SW_{NNN1} > SW_{NNN2} = SW_{NNN3}$.

Proof: See Appendix B.

**The Social Planner's Overall Preference for Pricing Structure (NN1, NN2, NN3, NNN1, NNN2, NNN3)**

**Proposition 6-6: (The Social planner's overall preferred pricing structure)**

$$SW_{NNN1} > SW_{NN1} = SW_{NN2} = SW_{NN3} > SW_{NNN2} = SW_{NNN3}$$

Proof: See Appendix B.

**Differences Between the BSP's and the Social Planner's Preferences**

Based on Propositions from 6-1 to 6-6, we can see the BSP has incentive to deviate its pricing choice from the social optimum. I summarize the differences in Proposition 6-7.

**Proposition 6-7: (The BSP's deviation from the social optimum)**

The BSP's preference differs from the social planner's preference under two scenarios:

(1) $V_H > V_L + \dfrac{d\lambda_H}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$ and

$V_L > \dfrac{d\left\{\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]\lambda_L + \mu(\lambda_H - \lambda_L)\right\}}{(\lambda_H - \lambda_L)\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$; and

(2) $V_H \le \dfrac{\lambda_H V_L}{\lambda_L} - \dfrac{d(\lambda_H - \lambda_L)}{\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$ and

$V_L < V_H < V_L + \dfrac{d\lambda_L}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$.

Proof: See Appendix B.

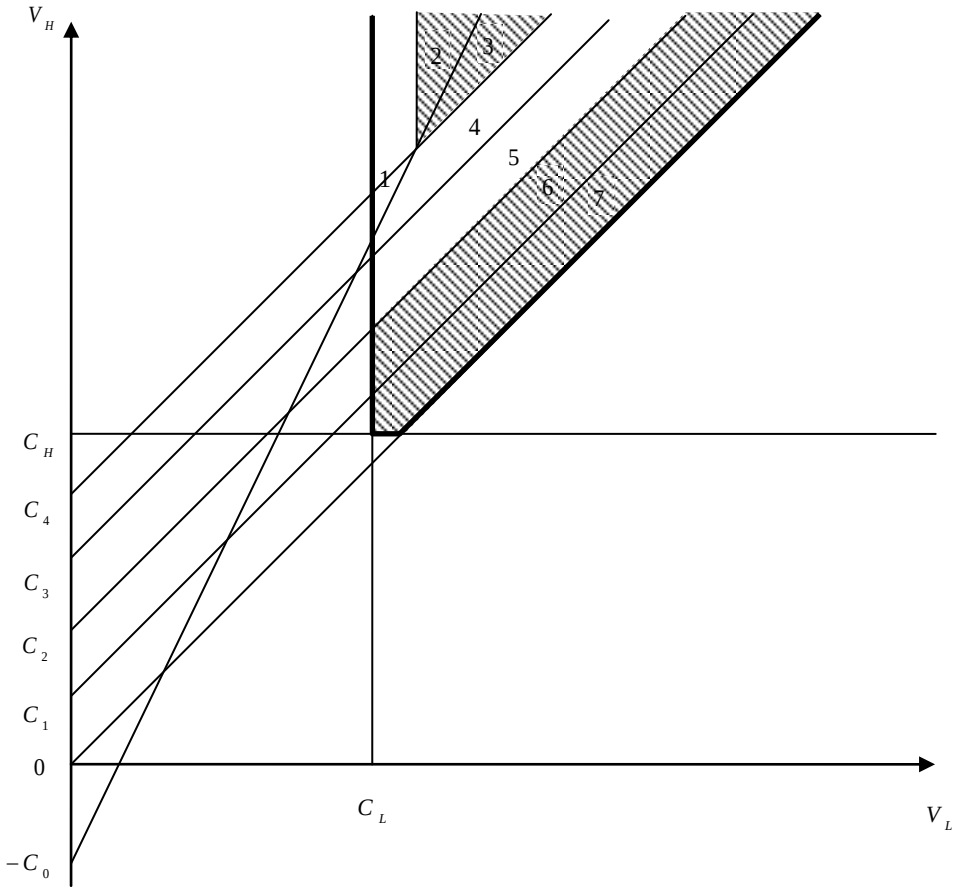



Figure 6-1. The BSP's overall preference for pricing structure and how it differs from the social planner's preference. Notes: (1) The area marked by the bold lines represents the feasible parameter space ($V_H \geq C_H$, and $V_L \geq C_L$, and $V_H > V_L$ where $C_H$ and $C_L$ are defined in the appendix). (2) The BSP's preferred option is: NNN1_1 for region 1, NN3_2 for region 2, NN3_1 for region 3, NNN1_1 for region 4, NNN1_2 for region 5, NN3_1 for region 6, and NN3_1 for region 7. (3) The shaded areas correspond to the regions where the BSP's preference is different from the social planner's preference.

Table 6-1. The BSP's six user discrimination options

| Options | | Results |
|---|---|---|
| Net Neutrality | NN1 | $\pi^*_{NN1} = F^*_{NN1} = V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$ |
| | | $CS_{NN1} = \alpha(V_H - V_L)$ |
| | | $SW_{NN1} = \alpha V_H + (1-\alpha)V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$ |
| | NN2 | $\pi^*_{NN2} = F^*_{NN2\_H} = F^*_{NN2\_L} = V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$ |
| | | $CS_{NN2} = \alpha(V_H - V_L)$ |
| | | $SW_{NN2} = \alpha V_H + (1-\alpha)V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$ |
| | NN3 | Case NN3_1: If $\dfrac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} \geq \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$, |
| | | $F^*_{NN3\_1} = \dfrac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$, $p^*_{NN3\_1} = \dfrac{V_H - V_L}{\lambda_H - \lambda_L}$ |
| | | $\pi^*_{NN3\_1} = \alpha V_H + (1-\alpha)V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$ |
| | | $CS_{NN3\_1} = 0$ |
| | | $SW_{NN3\_1} = \alpha V_H + (1-\alpha)V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$ |
| | | Case NN3_2: If $\dfrac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} < \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$, |
| | | $F^*_{NN3\_2} = 0$, $p^*_{NN3\_2} = \dfrac{1}{\lambda_L}\left[V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right]$ |
| | | $\pi^*_{NN3\_2} = \left[\dfrac{\alpha\lambda_H + (1-\alpha)\lambda_L}{\lambda_L}\right]\left[V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right]$ |
| | | $CS_{NN3\_2} == \alpha\left\{\left[\dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right]\left(\dfrac{\lambda_H - \lambda_L}{\lambda_L}\right) - \left(\dfrac{\lambda_H V_L - \lambda_L V_H}{\lambda_L}\right)\right\}$ |
| | | $SW_{NN3\_2} = \alpha V_H + (1-\alpha)V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$ |

Table 6-1. Continued

| Options | | Results |
|---|---|---|
| No Net Neutrality | NNN1 | Case NNN1_1: If $V_H - V_L \geq \dfrac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}$, |
| | | $\pi^*_{NNN1\_1} = F^*_{NNN1\_1} = V_L - \dfrac{d}{\mu-(1-\alpha)\lambda_L}$ |
| | | $CS_{NNN1\_1} = \alpha\left[V_H - V_L - \dfrac{d}{\mu-(1-\alpha)\lambda_L}\cdot\dfrac{\alpha\lambda_H+(1-\alpha)\lambda_L}{\mu-\alpha\lambda_H-(1-\alpha)\lambda_L}\right]$ |
| | | $SW_{NNN1\_1} = \alpha\left(V_H - \dfrac{d\mu}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}\right) + (1-\alpha)\left(V_L - \dfrac{d}{\mu-(1-\alpha)\lambda_L}\right)$ |
| | | Case NNN1_2: If $V_H - V_L < \dfrac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}$, |
| | | $\pi^*_{NNN1\_2} = F^*_{NNN1\_2} = V_H - \dfrac{d\mu}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}$ |
| | | $CS_{NNN1\_2} = (1-\alpha)\left\{-V_H + V_L + \dfrac{d}{\mu-(1-\alpha)\lambda_L}\cdot\dfrac{\alpha\lambda_H+(1-\alpha)\lambda_L}{\mu-\alpha\lambda_H-(1-\alpha)\lambda_L}\right\}$ |
| | | $SW_{NNN1\_2} = \alpha\left(V_H - \dfrac{d\mu}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}\right) + (1-\alpha)\left(V_L - \dfrac{d}{\mu-(1-\alpha)\lambda_L}\right)$ |
| | NNN2 | $F^*_{NNN2\_H} = V_L - \dfrac{d}{\mu-\alpha\lambda_H}$, $F^*_{NNN2\_L} = V_L - \dfrac{d\mu}{(\mu-\alpha\lambda_H)\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}$ |
| | | $\pi^*_{NNN2} = \alpha F^*_{NNN2\_H} + (1-\alpha)F^*_{NNN2\_L} = V_L - \dfrac{d}{\mu-\alpha\lambda_H}\left[\dfrac{\mu-\alpha^2\lambda_H-\alpha(1-\alpha)\lambda_L}{\mu-\alpha\lambda_H-(1-\alpha)\lambda_L}\right]$ |
| | | $CS_{NNN2} = \alpha(V_H - V_L)$ |
| | | $SW_{NNN2} = \alpha\left(V_H - \dfrac{d}{\mu-\alpha\lambda_H}\right) + (1-\alpha)\left(V_L - \dfrac{d\mu}{(\mu-\alpha\lambda_H)\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}\right)$ |
| | NNN3 | $F^*_{NNN3} = V_L - \dfrac{d\mu}{(\mu-\alpha\lambda_H)\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}$ |
| | | $p^*_{NNN3} = \dfrac{1}{\lambda_H}\left\{\dfrac{d\mu}{(\mu-\alpha\lambda_H)\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]} - \dfrac{d}{\mu-\alpha\lambda_H}\right\}$ |
| | | $\pi^*_{NNN3} = F^*_{NNN3} + \alpha\lambda_H p^*_{NNN3} = V_L - \dfrac{d}{\mu-\alpha\lambda_H}\left[\dfrac{\mu-\alpha^2\lambda_H-\alpha(1-\alpha)\lambda_L}{\mu-\alpha\lambda_H-(1-\alpha)\lambda_L}\right]$ |
| | | $CS_{NNN3} = \alpha(V_H - V_L)$ |
| | | $SW_{NNN3} = \alpha\left(V_H - \dfrac{d}{\mu-\alpha\lambda_H}\right) + (1-\alpha)\left(V_L - \dfrac{d\mu}{(\mu-\alpha\lambda_H)\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}\right)$ |

# CHAPTER 7
# CONCLUSIONS

In this chapter, I conclude my dissertation by summarizing major findings in previous chapters and discussing potential directions for future research.

## Summary of Major Findings

Chapter 3 in this dissertation aims to answer two fundamental issues of content provider discrimination: (1) the winners and losers of abandoning net neutrality; and (2) the BSP's incentive to expand their capacity without net neutrality. I find that if the principle of net neutrality is abolished, the BSP definitely stands to gain from the arrangement, as a result of extracting the preferential delivery charge from the content providers. The content providers are thus left worse off, mirroring the stances of the two sides in the debate. Depending on the parameter values in the framework, consumer surplus either does not change or is higher in the short-run, and in the latter case, while a majority of consumers are better off, a minority is left worse off with larger wait times to access their preferred content. Social welfare in the short-run increases when compared to the baseline net neutrality case when one content provider pays for preferential treatment, but remains unchanged when both content providers pay. The crucial parameter that determines the nature of the equilibrium is the relative magnitude of the revenue generation capabilities of the two content providers: if they differ significantly, the consumers of the less "effective" (i.e., in terms of revenue generation) content provider, who are a minority, are left worse off. The incentive for the broadband service provider to expand capacity under net neutrality is mostly higher than the incentive to expand when the principle of net neutrality is abolished. The exception to this outcome occurs when the BSP's profit accrues mostly from the consumers and only one content provider has the incentive to pay the priority delivery fees. Similarly, for most of the parameter space, the BSP's optimal capacity choice under net

neutrality (NN) is higher than that under the no net neutrality (NNN) regime. In fact, the experience in broadband markets around the world indicates that there might be some other forces in play that account for the infrastructure capacity expansions in other countries. In Japan, for example, fierce competition among broadband service providers has led to the introduction of download bandwidth speeds in excess of 100 Mbps as far back as in 2004 (Yang et al. 2004), with prices for the consumers significantly lower than that in the United States (Turner 2005). A final finding that should be of interest to policymakers is that under net neutrality, the BSP invests in broadband infrastructure to reach the socially optimal level, but when there is no net neutrality, the BSP either under- or over-invests in infrastructure.

In Chapter 4, I relax the full market coverage assumption and thereby simultaneously analyze the effect of abandoning net neutrality principles on the competition between content providers, the BSP's market coverage and access pricing for consumers. In the process, I design an algorithm that makes an important methodological contribution towards solving a problem that eluded analytical tractability but still had certain structural properties that enabled its deconstruction. From a theoretical perspective, I derive several important results. I find that if net neutrality regulation is not enforced, the BSP will always prefer to charge content providers for preferential delivery of their packets. Further, given the assumption of $r_G > r_Y$, Outcome 3 (G paying) is always preferable to the BSP than Outcome 2 (Y paying) (the result would have been reversed if one had assumed $r_Y > r_G$). Finally, I find that if the consumers' valuation for Internet access is above a certain threshold, the market will be fully covered. More crucially, from a practical perspective, there are several important implications of this chapter. The salubrious effects of NNN include higher market coverage in many cases, and an increased consumer surplus as a whole with certain sets of parameter values. Consumers subscribing to the paying

content provider often gain from the arrangement, but within a certain range of parameter values, consumers from the non-paying content provider are actually worse off. With content providers effectively subsidizing the consumers for accessing the Internet, access prices are never higher (and are often lower) with NNN. In fact, when consumers do not value their Internet access highly (and with some other conditions holding true), the BSP might even provide the Internet service for free. On the flip side, I find evidence that ushering NNN can reduce "innovations at the edge" of the Internet. Promising startups without the financial cushion of their established rivals can effectively be shut out of the market, as the BSP might find it profitable to institute a priority delivery pricing plan that is too high for them to pay, and in the presence of sufficient congestion in the network, subscribers might abandon that content provider altogether.

Chapter 5 analyzes the issue of vertical integration – if a BSP's service is vertically integrated with that of a content provider, will the vertically integrated firm have enough market power to negatively affect the social welfare? I find that the answer crucially depends on whether the BSP is integrated with the more or less effective content provider (effectiveness in this case being defined as the ability to generate advertising revenue from the consumer base). If the vertically integrated firm is relatively less effective in generating revenue from its content, I find that for a range of parameter values, the social welfare decreases as compared to a situation when there is no vertical integration. On the other hand, social welfare can actually increase if the BSP is vertically integrated with the more effective content provider. But in either case, the competing content provider is often left worse off (and is never better off). In order to decide on an effective policy that regulates vertical integration in an online environment, policymakers therefore will need to balance the priorities of increasing social welfare to that of ensuring effective competition. Allowing a BSP to form a strategic relationship with the more effective content

provider might increase social welfare in the short run. However, this decision might have the less desirable outcome of lowering the level of competition in that particular market segment in the long run, which in turn might lead to less desirable result for the society as a whole. This current research examines the various effects of vertical integration in the presence and absence of net neutrality on social welfare in the short-run, future research can look at the effects of vertical integration in the long run where the service capacity becomes a decision variable for the BSP, and explore whether the competing content providers are driven out of the market. Other possible areas of exploration include extending the analyses to the duopoly case of two competing BSPs, relaxing the constraint of market coverage, or allowing for different types of online content from different providers.

In contrast to the extant literature that has looked at the net neutrality problem from the "supply" perspective, in Chapter 6, I look at the issue from the "demand" perspective, and examine the economic impact of net neutrality on the BSP and the society as a whole, if the former is not allowed to either prioritize or degrade the delivery of content to one class of users. I consider scenarios with and without net neutrality and analyze the problem of the monopoly broadband service provider trying out different pricing and prioritization strategies. I find that the impact of net neutrality depends on both the characteristics of the Internet data consumption market and the BSP's pricing strategies. I find that with net neutrality in place, the BSP would prefer to charge a two-part tariff for Internet access, but without net neutrality, a BSP may choose to charge a uniform price and degrade heavy users or charge a higher price to high type users for preferential delivery of their data packets depending on the characteristics of users' valuations for content and their usage patterns. Interestingly, I find that without net neutrality in place, degrading the experience of the heavy users increases social welfare, a practice that was

recently banned by the FCC. The joint impact of both pricing and net neutrality under the framework of the proposed model has potentially very important policy implications. I identify conditions under which the BSP's user discrimination choices deviate from the social optimum. The last result helps illustrate the social planner's dilemma: even though it might decide not to enforce net neutrality, there will still be scenarios under which the BSP would opt for a net neutrality solution.

## Future Research

Some immediate areas of future research include consideration of a more sophisticated revenue model for content providers. It is important to note that in order to make a meaningful comparison between the two regimes, the content providers must follow the same revenue model under both NN and NNN, and the issue therefore is to choose a revenue model that accurately captures the incentives of the content providers. Thanks to content providers like Google or Yahoo!, the overwhelmingly popular revenue model for content providers in the online world is the advertisement-assisted model. In this framework, consumers get full access to all the content from the online providers. The reason for the popularity of this model is that free content brings about a large number of visitors, who in turn generate revenue by clicking on the advertisements. For several years, many content providers aimed for a hybrid model, a prominent example of which being the online edition of the New York Times, whereby some content, which was advertisement-assisted, was available for free, but other, ostensibly higher-quality content, was made available only to paying subscribers. The New York Times abandoned this revenue model in late 2007, after discovering that the free, advertisement-supported content that was viewed by a large number of users was more profitable than a limited audience of paying subscribers. In fact, Sydell (2007, audio broadcast) reports that "this might be the way that everything is going on the Web", and that the lone example of a mainstream content provider that relies on a

subscription model, the online version of the Wall Street Journal, is also expected to make most of its content available for free before the end of 2008 (Anderson 2008). While it has been argued that in the future a significant amount of content would have to be paid for, that right now that remains a conjecture, with a growing body of evidence that the advertisement-generated model is becoming more popular with time (Sydell 2007), even with many content providers that would have been expected to follow a subscriber revenue model. One example is Qtrax, a new business offering free and legal music downloads with 25 million songs in its inventory. Thus, under the current state of affairs, where the hybrid quality-differentiated model has been all but abandoned in favor of the purely advertisement-assisted model, advertisement-supported free content from the content providers seems to mirror the ground realities. In fact, the prominent technology journalist and author Chris Anderson has argued that this new revenue model based around "free" is here to stay (Anderson 2008), pointing out that today online content generates revenue from banner advertisements, affiliate revenues, rental of subscription lists, sale of aggregate information, licensing, live events, listing, paid inclusion, cost per install, getting users to create content for free, streaming audio and video advertising, and API fees, to name a few (Wilson 2008). Anderson (2008) argues that this is possible today, as the Web has made it possible to monetize two scarcities that are valuable, reputation and attention, and coupled with the fact that the marginal cost of the content is almost negligible, it has been possible for an increasing number of companies to generate more revenue from the free content (e.g., a free album by Radiohead) than they could have by charging for that content on traditional media (charging for that album on a compact disc).

In this dissertation, I do not consider the BSP's capacity (bandwidth) allocation issue. One interesting extension would be to study whether the BSP will find it optimal to partition the

capacity and whether such capacity partitioning will change the BSP's incentive to invest in expanding the infrastructure capacity. A major reason for the BSP to invest less under NNN than under NN is that capacity expansion reduces the attractiveness of priority delivery of packets for content providers. A partitioned capacity may enhance the content providers' willingness to pay for the priority charge under NNN, which in turn may make BSP's capacity expansion more desirable under NNN. Similarly, it is of interest to examine the implication of net neutrality on the ability of a content provider to provide premium services (e.g., real-time video or remote medical supervision) that require dedicated bandwidth.

# APPENDIX A
# LIST OF NOTATIONS

$x$ : The marginal consumer indifferent between content providers Y and G when the market is fully covered

$\tilde{x}$ : An arbitrary consumer on $[0,1]$

$r_{\mathrm{Y}}$ : Content provider Y's revenue rate per packet request for content

$r_{\mathrm{G}}$ : Content provider G's revenue rate per packet request for content

$p$ : Unit price per packet for data packet transmission

$I_{\mathrm{Y}}$ , $I_{\mathrm{G}}$ : Content providers' service choices

$\lambda$ : Poisson arrival rate of content requested from each consumer in packets per unit of time

$t$ : Fit cost for an end consumer away from the ideal content

$V(\lambda)$ : The gross value function of retrieving content for each consumer when consumers are considered homogeneous

$d$ : Consumers' delay parameter that converts the delay for consumers waiting for the content to arrive from the websites to the unit cost of delay per unit of time

$w$ : The expected time in the queuing system

$U$ : Consumers' utility function in the short-run problem

$u$ : Consumers' utility function

$F$ : A uniform fixed fee per unit of time charged by the broadband provider to the end consumers

$\mu$ : Capacity of the BSP in packets per unit of time

$\delta$ : The discount rate used in the long-run problem

$\Pi_{\mathrm{BSP}}$ : BSP's revenue in the short-run problem

$\pi_{\mathrm{BSP}}$ : BSP's profit

$\Pi_{\mathrm{Y}}$ , $\Pi_{\mathrm{G}}$ : Content providers' profit in the short-run problem

$\pi_{\mathrm{Y}}$ , $\pi_{\mathrm{G}}$ : Content providers' profit

$C(\mu)$ : BSP's cost for capacity $\mu$ in the long-run problem

$CS$ : Consumer surplus

$SW$ : Social welfare

$y$ , $1-z$ : Marginal consumers for content providers Y and G respectively when the full market coverage assumption is relaxed. So the corresponding market shares for content providers Y and G are $y$ and $z$ respectively.

$K$ : BSP's market coverage

$r_{BSP}$ : Revenue rate per data packet request for the vertically integrated BSP

$I_{BSP}$ : The vertically integrated BSP's service choices

$r_C$ : Revenue rate per data packet request for the independent content provider C in the vertical integration model

$I_C$ : Content provider C's service choices

$\pi_C$ : Content provider C's profit

$\alpha$ : Percentage of H-type consumers

$\lambda_H$ , $\lambda_L$ : Rate of content requested from H-type and L-type consumers in packets per unit of time

$V_H$ , $V_L$ : The gross value of retrieving content for H-type and L-type consumers respectively

$F_H$ , $F_L$ : Fixed fees charged to H-type and L-type consumers respectively

$w_H$ , $w_L$ : Consumers' delay cost (congestion cost) for H-type and L-type consumers respectively

$u_H$ , $u_L$ : The utility function for H-type and L-type consumers respectively

APPENDIX B
PROOFS OF PROPOSITIONS AND OTHER ANALYTICAL RESULTS

**Proof of $x_2 > \frac{1}{2}$**

Since $x_2$ is between 0 and 1 and $\mu - \lambda > 0$, Equation (3-3) can be rearranged as

$$2t\lambda(\mu-\lambda)x_2^2 - t(2\mu+\lambda)(\mu-\lambda)x_2 + t\mu(\mu-\lambda) + d\lambda = 0 .$$

Define $f(x_2) = 2t\lambda(\mu-\lambda)x_2^2 - t(2\mu+\lambda)(\mu-\lambda)x_2 + t\mu(\mu-\lambda) + d\lambda$. Then $x_2$ solves $f(x_2) = 0$.

Since $2t\lambda(\mu-\lambda) > 0$, $f(x_2)$ is a convex quadratic function. We also know that $f\left(\frac{1}{2}\right) = d\lambda > 0$

and $\left.\frac{\partial f}{\partial x_2}\right|_{x_2=\frac{1}{2}} = t(\mu-\lambda)\left[4\lambda\left(\frac{1}{2}\right) - 2\mu - \lambda\right] = -t(\mu-\lambda)(2\mu-\lambda) < 0$. Therefore $x_2 > \frac{1}{2}$.

**Proof of Proposition 3-1 (The Results of the Short-Run Problem)**

Before we proceed to prove the results for the short-run problem, we prove the following important intermediate results which will be repeatedly used in later proofs.

It follows from Equation (3-4) that $tx_3 + \frac{d\cdot\mu}{[\mu-(1-x_3)\lambda](\mu-\lambda)} = t(1-x_3) + \frac{d}{\mu-(1-x_3)\lambda}$

$$\Rightarrow \frac{d\cdot\mu}{[\mu-(1-x_3)\lambda](\mu-\lambda)} - \frac{d}{\mu-(1-x_3)\lambda} = t(1-x_3) - tx_3 \tag{B-1}$$

$$\Rightarrow \left[\frac{d}{\mu-(1-x_3)\lambda}\right]\left(\frac{\mu}{\mu-\lambda} - 1\right) = t(1-2x_3)$$

$$\Rightarrow \left[\frac{d}{\mu-(1-x_3)\lambda}\right]\left(\frac{\lambda}{\mu-\lambda}\right) = 2t\left(\frac{1}{2} - x_3\right)$$

$$\Rightarrow \frac{d}{\mu-(1-x_3)\lambda} = 2t\left(\frac{1}{2} - x_3\right)\left(\frac{\mu}{\lambda} - 1\right) \tag{B-2}$$

Substituting (B-2) into (B-1), we get $\frac{d \cdot \mu}{\left[\mu - (1 - x_3)\lambda\right](\mu - \lambda)} - 2t\left(\frac{1}{2} - x_3\right)\left(\frac{\mu}{\lambda} - 1\right) = t(1 - x_3) - tx_3$

$$\Rightarrow \frac{d \cdot \mu}{\left[\mu - (1 - x_3)\lambda\right](\mu - \lambda)} = 2t\left(\frac{1}{2} - x_3\right) + 2t\left(\frac{1}{2} - x_3\right)\left(\frac{\mu}{\lambda} - 1\right)$$

$$\Rightarrow \left(\frac{d}{\mu - \lambda}\right)\left[\frac{\mu}{\mu - (1 - x_3)\lambda}\right] = 2t\left(\frac{1}{2} - x_3\right)\left(\frac{\mu}{\lambda}\right)$$

$$\Rightarrow \frac{d}{\mu - \lambda} = 2t\left(\frac{1}{2} - x_3\right)\left[\frac{\mu}{\lambda} - (1 - x_3)\right] \tag{B-3}$$

From Formula (B-2) and (B-3), we get

$$\begin{aligned} \frac{d}{\mu - \lambda} - \frac{d}{\mu - (1 - x_3)\lambda} &= 2t\left(\frac{1}{2} - x_3\right)\left[\frac{\mu}{\lambda} - (1 - x_3)\right] - 2t\left(\frac{1}{2} - x_3\right)\left(\frac{\mu}{\lambda} - 1\right) \\ &= 2t\left(\frac{1}{2} - x_3\right)\left[\frac{\mu}{\lambda} - (1 - x_3) - \frac{\mu}{\lambda} + 1\right] = 2tx_3\left(\frac{1}{2} - x_3\right) \end{aligned} \tag{B-4}$$

Consumer Surplus under NN

$$CS_{NN} = \int_0^{\frac{1}{2}}\left(V(\lambda) - tx - \frac{d}{\mu - \lambda} - F_{NN}\right)dx + \int_{\frac{1}{2}}^{1}\left(V(\lambda) - t(1 - x) - \frac{d}{\mu - \lambda} - F_{NN}\right)dx$$

$$= \int_0^{\frac{1}{2}}\left(V(\lambda) - tx - \frac{d}{\mu - \lambda} - \left(V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}\right)\right)dx + \int_{\frac{1}{2}}^{1}\left(V(\lambda) - t(1 - x) - \frac{d}{\mu - \lambda} - \left(V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}\right)\right)dx$$

$$= \int_0^{\frac{1}{2}}\left(\frac{t}{2} - tx\right)dx + \int_{\frac{1}{2}}^{1}\left(tx - \frac{t}{2}\right)dx = \left(\frac{t}{2}x - \frac{t}{2}x^2\right)\Bigg|_0^{\frac{1}{2}} + \left(\frac{t}{2}x^2 - \frac{t}{2}x\right)\Bigg|_{\frac{1}{2}}^{1} = \frac{t}{4}$$

Consumer surplus consists of two parts:

$$CS_{NN\_Y} = \int_0^{\frac{1}{2}}\left(V(\lambda) - tx - \frac{d}{\mu - \lambda} - F_{NN}\right)dx = \left(\frac{t}{2}x - \frac{t}{2}x^2\right)\Bigg|_0^{\frac{1}{2}} = \frac{t}{8}.$$

$$CS_{NN\_G} = \int_{\frac{1}{2}}^{1}\left( V(\lambda) - t(1-x) - \frac{d}{\mu-\lambda} - F_{NN} \right) dx = \left( \frac{t}{2}x^2 - \frac{t}{2}x \right)\Bigg|_{\frac{1}{2}}^{1} = \frac{t}{8}.$$

Social Welfare under NN

$$SW_{NN} = \Pi_{NN\_ISP} + \Pi_{NN\_Y} + \Pi_{NN\_G} + CS_{NN}$$

$$= \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu-\lambda} \right] + \left( \frac{1}{2}\lambda r_Y \right) + \left( \frac{1}{2}\lambda r_G \right) + \left( \frac{t}{4} \right)$$

$$= V(\lambda) - \frac{t}{4} - \frac{d}{\mu-\lambda} + \frac{1}{2}\lambda r_Y + \frac{1}{2}\lambda r_G$$

Internet Access Fee under NNN Case A

$$F_{NNN\_A} - F_{NN} = \left[ V(\lambda) - t(1-x_3) - \frac{d}{\mu-(1-x_3)\lambda} \right] - \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu-\lambda} \right]$$

$$= \frac{d}{\mu-\lambda} - \frac{d}{\mu-(1-x_3)\lambda} - t\left( \frac{1}{2} - x_3 \right)$$

Substituting (B-4) into the above, we get

$$= 2tx_3\left( \frac{1}{2} - x_3 \right) - t\left( \frac{1}{2} - x_3 \right)$$

$$= -2t\left( \frac{1}{2} - x_3 \right)^2 < 0$$

Therefore $F_{NNN\_A} < F_{NN}$.

BSP's Revenue under NNN Case A

$$\Pi_{NNN\_A\_ISP} - \Pi_{NN\_ISP} = \left[ V(\lambda) - t(1-x_3) - \frac{d}{\mu - (1-x_3)\lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_G \right] - \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} \right]$$

$$= \frac{d}{\mu - \lambda} - \frac{d}{\mu - (1-x_3)\lambda} - t\left(\frac{1}{2} - x_3\right) + \left(\frac{1}{2} - x_3\right)\lambda r_G$$

Substituting (B-4) into the above, we get

$$= 2tx_3\left(\frac{1}{2} - x_3\right) - t\left(\frac{1}{2} - x_3\right) + \left(\frac{1}{2} - x_3\right)\lambda r_G$$

$$= \left(\frac{1}{2} - x_3\right)\left[\lambda r_G - t(1 - 2x_3)\right]$$

Since we know under NNN Case A $r_G \geq 2r_Y + \frac{t}{\lambda}(1 - 2x_3)$, $\lambda r_G - t(1 - 2x_3) > 0$.

Therefore $\Pi_{NNN\_A\_ISP} > \Pi_{NN\_ISP}$.

Content Providers' Profit under NNN Case A

$$\Pi_{NNN\_A\_Y} = x_3 \lambda r_Y < \frac{1}{2}\lambda r_Y = \Pi_{NN\_Y}$$

$$\Pi_{NNN\_A\_G} = \frac{1}{2}\lambda r_G = \Pi_{NN\_G}$$

Consumer Surplus under NNN Case A

$$CS_{NNN\_A}$$

$$= \int_0^{x_3} \left( V(\lambda) - tx - \frac{d \cdot \mu}{\left[\mu - (1-x_3)\lambda\right](\mu - \lambda)} - F_3 \right) dx + \int_{x_3}^{1} \left( V(\lambda) - t(1-x) - \frac{d}{\mu - (1-x_3)\lambda} - F_3 \right) dx$$

Since $F_3 = V(\lambda) - tx_3 - \frac{d \cdot \mu}{\left[\mu - (1-x_3)\lambda\right](\mu - \lambda)} = V(\lambda) - t(1-x_3) - \frac{d}{\mu - (1-x_3)\lambda}$

$$= \int_0^{x_3} \left( V(\lambda) - tx - \frac{d \cdot \mu}{\left[\mu - (1 - x_3)\lambda\right](\mu - \lambda)} - \left( V(\lambda) - tx_3 - \frac{d \cdot \mu}{\left[\mu - (1 - x_3)\lambda\right](\mu - \lambda)} \right) \right) dx$$

$$+ \int_{x_3}^{1} \left( V(\lambda) - t(1 - x) - \frac{d}{\mu - (1 - x_3)\lambda} - \left( V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda} \right) \right) dx$$

$$= \int_0^{x_3} (tx_3 - tx)\, dx + \int_{x_3}^{1} (tx - tx_3)\, dx = t\left( x_3 x - \frac{1}{2} x^2 \right)\Bigg|_0^{x_3} + t\left( \frac{1}{2} x^2 - x_3 x \right)\Bigg|_{x_3}^{1}$$

$$= tx_3^2 - \frac{t}{2} x_3^2 + \frac{t}{2} - tx_3 - \frac{t}{2} x_3^2 + tx_3^2 = t\left( x_3^2 - x_3 + \frac{1}{2} \right)$$

$$= t\left[ \left( x_3 - \frac{1}{2} \right)^2 + \frac{1}{4} \right] > \frac{t}{4} = CS_{NN}$$

Therefore $CS_{NNN\_A} > CS_{NN}$, i.e., the consumer surplus is increased in NNN Case A compared to NN.

Consumer surplus consists of two parts:

$$CS_{NNN\_A\_Y} = \int_0^{x_3} \left( V(\lambda) - tx - \frac{d \cdot \mu}{\left[\mu - (1 - x_3)\lambda\right](\mu - \lambda)} - F_3 \right) dx$$

$$= t\left( x_3 x - \frac{1}{2} x^2 \right)\Bigg|_0^{x_3} = \frac{t}{2} x_3^2 < \frac{t}{8} = CS_{NN\_Y} \text{ since } 0 < x_3 < \frac{1}{2}.$$

$$CS_{NNN\_A\_G} = \int_{x_3}^{1} \left( V(\lambda) - t(1 - x) - \frac{d}{\mu - (1 - x_3)\lambda} - F_3 \right) dx = t\left( \frac{1}{2} x^2 - x_3 x \right)\Bigg|_{x_3}^{1}$$

$$= t\left( \frac{1}{2} x_3^2 - x_3 + \frac{1}{2} \right) > \frac{t}{8} = CS_{NN\_G} \text{ since } 0 < x_3 < \frac{1}{2}.$$

Social Welfare under NNN Case A

$$SW_{NNN\_A} = \Pi_{NNN\_A\_ISP} + \Pi_{NNN\_A\_Y} + \Pi_{NNN\_A\_G} + CS_{NNN\_A}$$

$$= \left[ V(\lambda) - t(1-x_3) - \frac{d}{\mu - (1-x_3)\lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_G \right] + (x_3 \lambda r_Y) + \left(\frac{1}{2}\lambda r_G\right) + \left[ t\left(x_3^2 - x_3 + \frac{1}{2}\right)\right]$$

$$= V(\lambda) - t\left(\frac{1}{2} - x_3^2\right) - \frac{d}{\mu - (1-x_3)\lambda} + x_3 \lambda r_Y + (1-x_3)\lambda r_G$$

$$SW_{NNN\_A} - SW_{NN}$$

$$= \left( V(\lambda) - t\left(\frac{1}{2} - x_3^2\right) - \frac{d}{\mu - (1-x_3)\lambda} + x_3 \lambda r_Y + (1-x_3)\lambda r_G \right) - \left( V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_Y + \frac{1}{2}\lambda r_G \right)$$

$$= -t\left(\frac{1}{4} - x_3^2\right) + \frac{d}{\mu - \lambda} - \frac{d}{\mu - (1-x_3)\lambda} - \left(\frac{1}{2} - x_3\right)\lambda r_Y + \left(\frac{1}{2} - x_3\right)\lambda r_G$$

$$= -t\left(\frac{1}{2} - x_3\right)\left(\frac{1}{2} + x_3\right) + 2tx_3\left(\frac{1}{2} - x_3\right) + \left(\frac{1}{2} - x_3\right)\lambda(r_G - r_Y) \text{ since Equation (B-4)}$$

$$= \left(\frac{1}{2} - x_3\right)\left[ -t\left(\frac{1}{2} + x_3\right) + 2tx_3 + \lambda(r_G - r_Y) \right] = \left(\frac{1}{2} - x_3\right)\left[ \lambda(r_G - r_Y) - t\left(\frac{1}{2} - x_3\right)\right]$$

$$> \left(\frac{1}{2} - x_3\right)\left[ \lambda\left(2r_Y + \frac{t}{\lambda}(1 - 2x_3) - r_Y\right) - t\left(\frac{1}{2} - x_3\right)\right]$$

$$= \left(\frac{1}{2} - x_3\right)\left[ \lambda r_Y + t\left(\frac{1}{2} - x_3\right)\right] > 0$$

Therefore $SW_{NNN\_A} > SW_{NN}$, i.e., social welfare is increased in NNN Case A compared to NN.

Internet Access Fee under NNN Case B

$$F_{NNN\_B} = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} = F_{NN}$$

BSP's Revenue under NNN Case B

$$\Pi_{NNN\_B\_ISP} = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1 - 2x_3)\lambda r_Y > V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} = \Pi_{NN\_ISP}$$

Content Providers' Profit under NNN Case B

$$\Pi_{NNN\_B\_Y} = x_3 \lambda r_Y < \frac{1}{2}\lambda r_Y = \Pi_{NN\_Y}$$

$$\Pi_{NNN\_B\_G} = \frac{1}{2}\lambda\left[r_G - (1 - 2x_3) r_Y\right] < \frac{1}{2}\lambda r_G = \Pi_{NN\_G}$$

Consumer Surplus under NNN Case B

$$CS_{NNN\_B} = \int_0^{\frac{1}{2}}\left(V(\lambda) - tx - \frac{d}{\mu - \lambda} - F_4\right)dx + \int_{\frac{1}{2}}^{1}\left(V(\lambda) - t(1 - x) - \frac{d}{\mu - \lambda} - F_4\right)dx$$

$$= \int_0^{\frac{1}{2}}\left(V(\lambda) - tx - \frac{d}{\mu - \lambda} - \left(V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}\right)\right)dx + \int_{\frac{1}{2}}^{1}\left(V(\lambda) - t(1 - x) - \frac{d}{\mu - \lambda} - \left(V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda}\right)\right)dx$$

$$= \int_0^{\frac{1}{2}}\left(\frac{t}{2} - tx\right)dx + \int_{\frac{1}{2}}^{1}\left(tx - \frac{t}{2}\right)dx = t\left(\frac{1}{2}x - \frac{1}{2}x^2\right)\Bigg|_0^{\frac{1}{2}} + t\left(\frac{1}{2}x^2 - \frac{1}{2}x\right)\Bigg|_{\frac{1}{2}}^{1} = \frac{t}{4} = CS_{NN}$$

Consumer surplus consists of two parts:

$$CS_{NNN\_B\_Y} = \int_0^{\frac{1}{2}}\left(V(\lambda) - tx - \frac{d}{\mu - \lambda} - F_4\right)dx = t\left(\frac{1}{2}x - \frac{1}{2}x^2\right)\Bigg|_0^{\frac{1}{2}} = \frac{t}{8} = CS_{NN\_Y}$$

$$CS_{NNN\_B\_G} = \int_{\frac{1}{2}}^{1}\left(V(\lambda) - t(1 - x) - \frac{d}{\mu - \lambda} - F_4\right)dx = t\left(\frac{1}{2}x^2 - \frac{1}{2}x\right)\Bigg|_{\frac{1}{2}}^{1} = \frac{t}{8} = CS_{NN\_G}$$

Therefore the consumer surplus remains unchanged.

Social Welfare under NNN Case B

$$SW_{NNN\_B} = \Pi_{NNN\_B\_ISP} + \Pi_{NNN\_B\_Y} + \Pi_{NNN\_B\_G} + CS_{NNN\_B}$$

$$= \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1 - 2x_3)\lambda r_Y \right] + (x_3 \lambda r_Y) + \left\{ \frac{1}{2}\lambda \left[ r_G - (1 - 2x_3) r_Y \right] \right\} + \left( \frac{t}{4} \right)$$

$$= V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_Y + \frac{1}{2}\lambda r_G = SW_{NN}$$

Therefore social welfare remains unchanged.

**Proof of a Necessary and Sufficient Condition for the Existence of a Unique** $x_3 \in (0,1)$

It follows from Equation (3-11) that $tx_3 + \dfrac{d \cdot \mu}{\left[\mu - (1 - x_3)\lambda\right](\mu - \lambda)} = t(1 - x_3) + \dfrac{d}{\mu - (1 - x_3)\lambda}$.

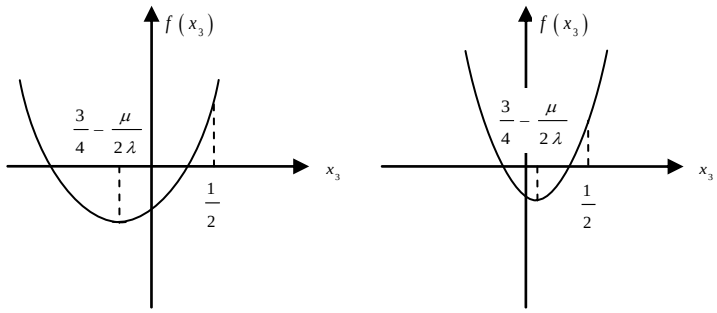
$$\Rightarrow t(2x_3 - 1) + \left( \frac{d}{\mu - (1 - x_3)\lambda} \right)\left( \frac{\lambda}{\mu - \lambda} \right) = 0$$

$$\Rightarrow t(2x_3 - 1)\left[\mu - (1 - x_3)\lambda\right](\mu - \lambda) + d\lambda = 0$$

$$\Rightarrow (2tx_3 - t)\left[\lambda(\mu - \lambda)x_3 + (\mu - \lambda)^2\right] + d\lambda = 0$$

$$\Rightarrow 2t\lambda(\mu - \lambda)x_3^2 + t(\mu - \lambda)(2\mu - 3\lambda)x_3 + d\lambda - t(\mu - \lambda)^2 = 0 \qquad \text{(B-5)}$$

We consider the function $f(x_3) = 2t\lambda(\mu - \lambda)x_3^2 + t(\mu - \lambda)(2\mu - 3\lambda)x_3 + d\lambda - t(\mu - \lambda)^2$. We know $f(0) = d\lambda - t(\mu - \lambda)^2$, $f(1) = d\lambda + t\mu(\mu - \lambda) > 0$, $f\left(\frac{1}{2}\right) = d\lambda > 0$, and $f(x_3)$ achieves the minimum at $x_3 = \frac{3}{4} - \frac{\mu}{2\lambda}$ with $f\left(\frac{3}{4} - \frac{\mu}{2\lambda}\right) = \dfrac{8d\lambda^2 - t(\mu - \lambda)(2\mu - \lambda)^2}{8\lambda}$.




There is a unique solution on $(0,1)$ if and only if $f(0) = d\lambda - t(\mu-\lambda)^2 < 0$ which is equivalent

to $d < \dfrac{t(\mu-\lambda)^2}{\lambda}$ (B-6)

Condition (B-6) is necessary and sufficient for the existence of a unique $x_3 \in (0,1)$. This

condition implies $V(\lambda) - \dfrac{d\cdot\mu}{(\mu-\lambda)^2} - F_3 > V(\lambda) - t - \dfrac{d}{\mu-\lambda} - F_3$ meaning the consumer located at

the two ends of the market is loyal to their corresponding content providers.

Under Condition (B-6), we know $\frac{3}{4} - \frac{\mu}{2\lambda} < x_3 < \frac{1}{2}$.

**Proof of $\frac{\partial x_3}{\partial \mu} > 0$**

Using implicit differentiation, $\frac{\partial x_3}{\partial \mu}$ can be derived from Equation (B-5) by solving

$$2t\lambda\left[x_3^2 + 2(\mu-\lambda)x_3\frac{\partial x_3}{\partial\mu}\right] + t\left[(4\mu-5\lambda)x_3 + (\mu-\lambda)(2\mu-3\lambda)\frac{\partial x_3}{\partial\mu}\right] - 2t(\mu-\lambda) = 0$$

$$\left[4\lambda(\mu-\lambda)x_3 + (\mu-\lambda)(2\mu-3\lambda)\right]\frac{\partial x_3}{\partial\mu} = 2(\mu-\lambda) - 2\lambda x_3^2 - (4\mu-5\lambda)x_3$$

$$(\mu-\lambda)(2\mu-3\lambda+4\lambda x_3)\frac{\partial x_3}{\partial\mu} = (1-2x_3)(2\mu-2\lambda+\lambda x_3)$$

Since we know $1-2x_3>0$, $2\mu-2\lambda+\lambda x_3>0$, $\mu-\lambda>0$, the sign of $\frac{\partial x_3}{\partial \mu}$ depends on $2\mu-3\lambda+4\lambda x_3$. Since we know $x_3>\frac{3}{4}-\frac{\mu}{2\lambda}$ from the proof of "a necessary and sufficient condition for the existence of a unique $x_3\in(0,1)$",

$$\frac{\partial x_3}{\partial \mu}=\frac{(1-2x_3)(2\mu-2\lambda+\lambda x_3)}{(\mu-\lambda)(2\mu-3\lambda+4\lambda x_3)}>0.$$

**Proof of Proposition 3-2 (The BSP's Incentive to Expand Capacity)**

In order to compare the BSP's incentive to expand capacity, we need to compare:

$$\frac{\partial \pi_{NN}}{\partial \mu}=\frac{1}{1-\delta}\cdot\frac{d}{(\mu-\lambda)^2}-\frac{\partial C(\mu)}{\partial \mu}$$

$$\frac{\partial \pi_{NNN\_A}}{\partial \mu}=\frac{1}{1-\delta}\cdot\left[t\left(\frac{2\mu}{\lambda}-1\right)-\lambda r_G\right]\frac{\partial x_3}{\partial \mu}-\frac{1}{1-\delta}\cdot\frac{2t}{\lambda}\left(\frac{1}{2}-x_3\right)-\frac{\partial C(\mu)}{\partial \mu}$$

$$\frac{\partial \pi_{NNN\_B}}{\partial \mu}=\frac{1}{1-\delta}\cdot\frac{d}{(\mu-\lambda)^2}-\frac{2}{1-\delta}\cdot\frac{\partial x_3}{\partial \mu}\lambda r_Y-\frac{\partial C(\mu)}{\partial \mu}$$

$$\frac{\partial \pi_{NN}}{\partial \mu}-\frac{\partial \pi_{NNN\_B}}{\partial \mu}=\left[\frac{1}{1-\delta}\cdot\frac{d}{(\mu-\lambda)^2}-\frac{\partial C(\mu)}{\partial \mu}\right]-\left[\frac{1}{1-\delta}\cdot\frac{d}{(\mu-\lambda)^2}-\frac{2}{1-\delta}\cdot\frac{\partial x_3}{\partial \mu}\lambda r_Y-\frac{\partial C(\mu)}{\partial \mu}\right]$$

$$=\frac{2}{1-\delta}\cdot\frac{\partial x_3}{\partial \mu}\lambda r_Y>0$$

$$\frac{\partial \pi_{NN}}{\partial \mu}-\frac{\partial \pi_{NNN\_A}}{\partial \mu}$$

$$=\left[\frac{1}{1-\delta}\cdot\frac{d}{(\mu-\lambda)^2}-\frac{\partial C(\mu)}{\partial \mu}\right]-\left\{\frac{1}{1-\delta}\cdot\left[t\left(\frac{2\mu}{\lambda}-1\right)-\lambda r_G\right]\frac{\partial x_3}{\partial \mu}-\frac{1}{1-\delta}\cdot\frac{2t}{\lambda}\left(\frac{1}{2}-x_3\right)-\frac{\partial C(\mu)}{\partial \mu}\right\}$$

$$= \frac{1}{1-\delta}\left\{\frac{d}{(\mu-\lambda)^2} - \left[t\left(\frac{2\mu}{\lambda}-1\right) - \lambda r_G\right]\frac{\partial x_3}{\partial \mu} + \frac{2t}{\lambda}\left(\frac{1}{2}-x_3\right)\right\}$$

Substituting $\frac{\partial x_3}{\partial \mu} = \frac{(1-2x_3)(2\mu-2\lambda+\lambda x_3)}{(\mu-\lambda)(2\mu-3\lambda+4\lambda x_3)}$ and $\frac{d}{\mu-\lambda} = 2t\left(\frac{1}{2}-x_3\right)\left[\frac{\mu}{\lambda}-(1-x_3)\right]$ into the above

gives $\frac{\partial \pi_{\mathrm{NN}}}{\partial \mu} - \frac{\partial \pi_{\mathrm{NNN\_A}}}{\partial \mu}$

$$= \frac{1}{1-\delta}\left\{\frac{1}{\mu-\lambda}2t\left(\frac{1}{2}-x_3\right)\left[\frac{\mu}{\lambda}-(1-x_3)\right] - \left[t\left(\frac{2\mu}{\lambda}-1\right)-\lambda r_G\right]\frac{(1-2x_3)(2\mu-2\lambda+\lambda x_3)}{(\mu-\lambda)(2\mu-3\lambda+4\lambda x_3)} + \frac{2t}{\lambda}\left(\frac{1}{2}-x_3\right)\right\}$$

$$= \frac{2}{1-\delta}\left(\frac{1}{2}-x_3\right)\left\{\frac{1}{\mu-\lambda}t\left[\frac{\mu}{\lambda}-(1-x_3)\right] - \left[t\left(\frac{2\mu}{\lambda}-1\right)-\lambda r_G\right]\frac{(2\mu-2\lambda+\lambda x_3)}{(\mu-\lambda)(2\mu-3\lambda+4\lambda x_3)} + \frac{t}{\lambda}\right\}$$

$$= \frac{2}{1-\delta}\left(\frac{1}{2}-x_3\right)\left\{\lambda r_G\frac{(2\mu-2\lambda+\lambda x_3)}{(\mu-\lambda)(2\mu-3\lambda+4\lambda x_3)} + \frac{t}{\lambda}\left[\frac{\mu-\lambda+\lambda x_3}{\mu-\lambda} - \frac{(2\mu-\lambda)(2\mu-2\lambda+\lambda x_3)}{(\mu-\lambda)(2\mu-3\lambda+4\lambda x_3)} + 1\right]\right\}$$

$$= \frac{2}{1-\delta}\left(\frac{1}{2}-x_3\right)\frac{(2\mu-2\lambda+\lambda x_3)}{(\mu-\lambda)(2\mu-3\lambda+4\lambda x_3)}\left[\lambda r_G - 2t(1-2x_3)\right]$$

Therefore $\frac{\partial \pi_{\mathrm{NN}}}{\partial \mu} - \frac{\partial \pi_{\mathrm{NNN\_A}}}{\partial \mu}\begin{cases} \geq 0, & \text{if } r_G \geq \frac{2t}{\lambda}(1-2x_3) \\ < 0, & \text{if } r_G < \frac{2t}{\lambda}(1-2x_3) \end{cases}$.

Recall that when $r_G \geq 2r_Y + \frac{t}{\lambda}(1-2x_3)$ Outcome 3 is the equilibrium and when

$r_G < 2r_Y + \frac{t}{\lambda}(1-2x_3)$, Outcome 4 is the equilibrium. We conclude that when $r_G < 2r_Y + \frac{t}{\lambda}(1-2x_3)$

or $r_G \geq \frac{2t}{\lambda}(1-2x_3)$ the BSP has more incentive to expand capacity under NN; when

$2r_Y + \frac{t}{\lambda}(1-2x_3) \leq r_G < \frac{2t}{\lambda}(1-2x_3)$ the BSP has more incentive to expand capacity under NNN.

**Proof of Proposition 3-3 (The BSP's Optimal Capacity Choice)**

Recall that the optimal capacity level for the BSP is determined by first order conditions

$\frac{\partial \pi_{NN}}{\partial \mu} = 0$ for NN, $\frac{\partial \pi_{NNN\_A}}{\partial \mu} = 0$ for NNN Case A, and $\frac{\partial \pi_{NNN\_B}}{\partial \mu} = 0$ for NNN Case B described in Equations (3-16), (3-17), and (3-18) respectively.

From the proof of Proposition 3-1, when $r_G < 2r_Y + \frac{t}{\lambda}(1-2x_3)$ or $r_G > \max\left\{2r_Y + \frac{t}{\lambda}(1-2x_3), \frac{2t}{\lambda}(1-2x_3)\right\}$, the BSP has more incentive to expand capacity under NN, i.e., $\frac{\partial \pi_{NN}}{\partial \mu} > \frac{\partial \pi_{NNN}}{\partial \mu}$. Let $\mu^*_{NN}$ and $\mu^*_{NNN}$ be the optimal capacity levels for NN and NNN respectively. Then $\left.\frac{\partial \pi_{NN}}{\partial \mu}\right|_{\mu = \mu^*_{NN}} = 0$ and $\left.\frac{\partial \pi_{NNN}}{\partial \mu}\right|_{\mu = \mu^*_{NNN}} = 0$. Since $\frac{\partial \pi_{NN}}{\partial \mu} > \frac{\partial \pi_{NNN}}{\partial \mu}$,

$$\left.\frac{\partial \pi_{NNN}}{\partial \mu}\right|_{\mu = \mu^*_{NNN}} = 0 = \left.\frac{\partial \pi_{NN}}{\partial \mu}\right|_{\mu = \mu^*_{NN}} > \left.\frac{\partial \pi_{NNN}}{\partial \mu}\right|_{\mu = \mu^*_{NN}}.$$

Second-order condition gives $\frac{\partial^2 \pi_{NN}}{\partial \mu^2} < 0$ and $\frac{\partial^2 \pi_{NNN}}{\partial \mu^2} < 0$. So $\frac{\partial \pi_{NN}}{\partial \mu}$ and $\frac{\partial \pi_{NNN}}{\partial \mu}$ decrease in $\mu$. Therefore $\mu^*_{NNN} < \mu^*_{NN}$.

Similarly, when $2r_Y + \frac{t}{\lambda}(1-2x_3) \le r_G \le \frac{2t}{\lambda}(1-2x_3)$, the BSP has more incentive to expand capacity under NNN, i.e., $\frac{\partial \pi_{NN}}{\partial \mu} \le \frac{\partial \pi_{NNN}}{\partial \mu}$. Let $\mu^*_{NN}$ and $\mu^*_{NNN}$ be the optimal capacity levels for

NN and NNN respectively. Then $\left.\frac{\partial \pi_{NN}}{\partial \mu}\right|_{\mu = \mu^*_{NN}} = 0$ and $\left.\frac{\partial \pi_{NNN}}{\partial \mu}\right|_{\mu = \mu^*_{NNN}} = 0$. Since

$\frac{\partial \pi_{NN}}{\partial \mu} \leq \frac{\partial \pi_{NNN}}{\partial \mu}$, $\left.\frac{\partial \pi_{NNN}}{\partial \mu}\right|_{\mu = \mu^*_{NNN}} = 0 = \left.\frac{\partial \pi_{NN}}{\partial \mu}\right|_{\mu = \mu^*_{NN}} \leq \left.\frac{\partial \pi_{NNN}}{\partial \mu}\right|_{\mu = \mu^*_{NN}}$. Second-order condition

gives $\frac{\partial^2 \pi_{NN}}{\partial \mu^2} < 0$ and $\frac{\partial^2 \pi_{NNN}}{\partial \mu^2} < 0$. So $\frac{\partial \pi_{NN}}{\partial \mu}$ and $\frac{\partial \pi_{NNN}}{\partial \mu}$ decrease in $\mu$. Therefore $\mu^*_{NNN} \geq \mu^*_{NN}$.

**Proof of Proposition 3-4 (Whether the BSP's Optimal Capacity Choice is Socially Optimal?)**

The long-run social welfare under net neutrality (NN) is

$$SW_{NN} = \frac{1}{1-\delta}\left[V(\lambda) - \frac{t}{4} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_Y + \frac{1}{2}\lambda r_G\right] - C(\mu).$$

The socially optimal capacity is derived by the first order condition $\frac{\partial SW_{NN}}{\partial \mu} = 0$. Since

$\frac{\partial \pi_{NN}}{\partial \mu} = \frac{\partial SW_{NN}}{\partial \mu}$, the BSP's optimal capacity decision that maximizes his profit coincides with that of the long-run social welfare. Namely, BSP always invests at the socially optimal level under net neutrality.

The long-run social welfare under Case A in the absence of net neutrality (NNN_A) is

$SW_{NNN\_A} = \frac{1}{1-\delta}\left[V(\lambda) - t\left(\frac{1}{2} - x_3^2\right) - \frac{d}{\mu - (1-x_3)\lambda} + x_3\lambda r_Y + (1-x_3)\lambda r_G\right] - C(\mu)$ which can be

rewritten as $SW_{NNN\_A} = \frac{1}{1-\delta}\left[V(\lambda) - t\left(\frac{1}{2} - x_3^2\right) - 2t\left(\frac{1}{2} - x_3\right)\left(\frac{\mu}{\lambda} - 1\right) + x_3\lambda r_Y + (1-x_3)\lambda r_G\right] - C(\mu)$.

After some algebra, one has $\frac{\partial SW_{NNN\_A}}{\partial \mu} - \frac{\partial \pi_{NNN\_A}}{\partial \mu} = \frac{1}{1-\delta}\left[\lambda r_Y - t(1-2x_3)\right]\frac{\partial x_3}{\partial \mu}$. We find that

$\frac{\partial SW_{NNN\_A}}{\partial \mu} > \frac{\partial \pi_{NNN\_A}}{\partial \mu}$ when $r_Y \geq \frac{t}{\lambda}(1-2x_3)$. That is, the BSP under-invests in capacity.

If, however, $r_Y < \frac{t}{\lambda}(1-2x_3)$, then $\frac{\partial SW_{NNN\_A}}{\partial \mu} < \frac{\partial \pi_{NNN\_A}}{\partial \mu}$, resulting in the BSP over-investing in capacity.

For the Case B in the absence of net neutrality (NNN_B), the long-run social welfare is

$SW_{NNN\_B} = \frac{1}{1-\delta}\left[V(\lambda) - \frac{t}{4} - \frac{d}{\mu-\lambda} + \frac{1}{2}\lambda r_Y + \frac{1}{2}\lambda r_G\right] - C(\mu)$. Since $\frac{\partial SW_{NNN\_B}}{\partial \mu} > \frac{\partial \pi_{NNN\_B}}{\partial \mu}$, the BSP always under-invests under Case B of no net neutrality.

**Proof of Proposition 4-1 (Market Coverage Under Net Neutrality)**

First notice that consumers' incentive compatibility constraints $(iii)$ and $(iv)$ in Formulation (4-1) can be simplified to $y_1 \leq 1/2$ and $z_1 \leq 1/2$.

Next we prove that "Consumers' participation constraints $(i)$ and $(ii)$ in Formulation (4-1) are binding."

Proof by contradiction:

Suppose $(F_1, y_1, z_1)$ is an optimal solution. There are three cases.

Case 1: $V - ty_1 - \frac{d}{\mu-(y_1+z_1)\lambda} - F_1 > 0$ and $V - tz_1 - \frac{d}{\mu-(y_1+z_1)\lambda} - F_1 > 0$.

Then substitute $F_1$ by $F_1 + \varepsilon$ would be a feasible solution which yields a higher objective value.

Case 2: $V - ty_1 - \frac{d}{\mu - (y_1 + z_1)\lambda} - F_1 > 0$ and $V - tz_1 - \frac{d}{\mu - (y_1 + z_1)\lambda} - F_1 = 0$.

Then $F_1 = V - tz_1 - \frac{d}{\mu - (y_1 + z_1)\lambda} < V - ty_1 - \frac{d}{\mu - (y_1 + z_1)\lambda}$. So $z_1 > y_1$. Then substitute $y_1$ by $y_1 + \varepsilon_y$ and $z_1$ by $z_1 - \varepsilon_z$, where $\varepsilon_y, \varepsilon_z > 0$ such that

$F_1 = V - tz_1 - \frac{d}{\mu - (y_1 + z_1)\lambda} = V - t(z_1 - \varepsilon_z) - \frac{d}{\mu - (y_1 + \varepsilon_y + z_1 - \varepsilon_z)\lambda}$ and

$V - t(y_1 + \varepsilon_y) - \frac{d}{\mu - (y_1 + \varepsilon_y + z_1 - \varepsilon_z)\lambda} - F_1 = t\left[(z_1 - y_1) - (\varepsilon_y + \varepsilon_z)\right] \geq 0$ would be a feasible solution. Since $F_1 = V - tz_1 - \frac{d}{\mu - (y_1 + z_1)\lambda} = V - t(z_1 - \varepsilon_z) - \frac{d}{\mu - (y_1 + \varepsilon_y + z_1 - \varepsilon_z)\lambda}$ implies

$\frac{d}{\mu - (y_1 + \varepsilon_y + z_1 - \varepsilon_z)\lambda} - \frac{d}{\mu - (y_1 + z_1)\lambda} = tz_1 - t(z_1 - \varepsilon_z) = t\varepsilon_z > 0$. So $\varepsilon_y - \varepsilon_z > 0$ and the objective value is higher, which is a contradiction.

Case 3: $V - ty_1 - \frac{d}{\mu - (y_1 + z_1)\lambda} - F_1 = 0$ and $V - tz_1 - \frac{d}{\mu - (y_1 + z_1)\lambda} - F_1 > 0$.

Then $F_1 = V - ty_1 - \frac{d}{\mu - (y_1 + z_1)\lambda} < V - tz_1 - \frac{d}{\mu - (y_1 + z_1)\lambda}$. So $y_1 > z_1$. Similar to Case 2, we get a contradiction.

Therefore the first two constraints are binding.

Since the first two constraints are binding, we get $y_1 = z_1$. Then the problem can be simplified as:

$$\max_{F_1} \ \pi_1 = 2F_1 y_1$$

s.t. $V - ty_1 - \frac{d}{\mu - 2y_1\lambda} - F_1 \geq 0$ and $0 \leq y_1 \leq 1/2$

Notice that $2y_1$ is the total market.

The participation constraint is always binding. So $F_1 = V - ty_1 - \frac{d}{\mu - 2y_1\lambda}$.

Then the problem becomes:

$$\max_{F_1} \pi_1 = 2\left(V - ty_1 - \frac{d}{\mu - 2y_1\lambda}\right)y_1$$

s.t. $0 \le y_1 \le 1/2$

FOC: $V - 2ty_1 - \frac{d\mu}{(\mu - 2y_1\lambda)^2} = 0$

Consider function $f_1(y_1) = V - 2ty_1 - \frac{d\mu}{(\mu - 2y_1\lambda)^2}$.

Take the first derivative, we know $f_1'(y_1) = -2t - \frac{4d\lambda\mu}{(\mu - 2y_1\lambda)^3} < 0$.

So $f_1(y_1)$ decreases in $y_1$ and therefore there is at most one solution to $f_1(y_1) = 0$ (the FOC condition) in $[0, 1/2]$.

Now check the boundary points:

When $y_1 = 0$, $f_1(y_1) = V - \frac{d}{\mu} > 0$.

When $y_1 = 1/2$, $f_1(y_1) = V - t - \frac{d\mu}{(\mu - \lambda)^2}$.

Define $V_{H1} = t + \frac{d\mu}{(\mu - \lambda)^2}$.

When $V < V_{H1}$, there exists an unique $0 < y_1^* < 1/2$ where $y_1^*$ solves the FOC condition $f_1(y_1) = 0$. When $V \geq V_{H1}$, $f_1(y_1) > 0$ in $[0,1/2]$ which means $\pi_1$ increases in $[0,1/2]$ and therefore $\pi_1$ achieves the highest value on $y_1^* = 1/2$.

**Proof of Lemma 4-1 (Feasibility Condition for Outcome 2)**

Proof of necessity: From content provider Y's incentive compatibility constraint, we get $p_2 \leq \left(1 - \frac{y_1}{y_2}\right) r_Y$. From content provider G's incentive compatibility constraint, we get $p_2 \geq \left(1 - \frac{z_2}{z_4}\right) r_G$. Therefore if Outcome 2 is feasible then $\left(1 - \frac{z_2}{z_4}\right) r_G \leq \left(1 - \frac{y_1}{y_2}\right) r_Y$.

Proof of sufficiency: If $\left(1 - \frac{z_2}{z_4}\right) r_G \leq \left(1 - \frac{y_1}{y_2}\right) r_Y$, then there exist a $p_2$ satisfying constraints both content providers' incentive compatibility constraints and so is a feasible $p_2$. We can easily show $F_2 = 0$ is always feasible. Therefore $\left(1 - \frac{z_2}{z_4}\right) r_G \leq \left(1 - \frac{y_1}{y_2}\right) r_Y$ is sufficient for Outcome 2 to be feasible.

**Proof of Lemma 4-2 (Optimization Condition for Outcome 2)**

Proof by contradiction. Suppose when Outcome 2 is feasible, $p_2^* < \left(1 - \frac{y_1}{y_2}\right) r_Y$. Then substitute $p_2^*$ by $p_2^* + \varepsilon$ would be a feasible solution which yields a higher objective value which is a contradiction.

**Proof Proposition 4-2 (Outcome 3 Dominates Outcome 2)**

By substituting $y_3 = z_2$ and $z_3 = y_2$ into this Formulation (4-3), we can see Formulation (4-3) has a greater feasible region than Formulation (4-2). Therefore Outcome 3 always yields at least as high a profit for the BSP as Outcome 2.

**Proof of Lemma 4-4 (Optimization Condition for Outcome 3)**

Proof by contradiction. Suppose when Outcome 3 is feasible, constraint (12) is not binding, i.e., $p_3^* < \left(1 - \frac{z_1}{z_3}\right) r_G$. Then substituting $p_3^*$ by $p_3^* + \varepsilon$ would be a feasible solution which yields a higher objective value.

**Proof of Proposition 4-3 (Outcome 4 Dominates Outcome 1)**

Compare the formulations of Outcome 1 and Outcome 4, we can see that Outcome 1 is equivalent to Outcome 4 plus the constraint of $p_4 = 0$. Therefore Outcome 4 always yields a profit for the BSP that is at least as high as that it gets under Outcome 1.

**Proof of Proposition 5-1 (Equilibrium of the Game with a Vertically Integrated BSP)**

Case A: $r_C < r_{BSP} - \frac{t}{\lambda}(1 - 2x_3)$

Outcome 1 and 3 are not equilibria. Considering Outcome 1 first, in the Formulation (5-7), the incentive compatibility constraint for BSP is $\pi_{BSP1} - \pi_{BSP2} \geq 0$, i.e., $-\lambda r_{BSP}\left(x_2 - \frac{1}{2}\right) \geq 0$. Since $x_2 > \frac{1}{2}$, this constraint can never be satisfied. Therefore Outcome 1 is not an equilibrium.

Now discussing Outcome 3, in the Formulation (5-11), $\pi_{BSP3} - \pi_{BSP4} \geq 0$ and $\pi_{C3} - \pi_{C1} \geq 0$ implies that $p \geq r_{BSP}$ and $p \leq \left( \frac{1/2 - x_3}{1 - x_3} \right) r_C$, respectively. But under Case A, $r_C < r_{BSP} - \frac{t}{\lambda}(1 - 2x_3)$, there is no feasible $p$ which satisfies both constraints.

Hence, we only consider Outcome 2 and 4, and note that BSP's profit in Outcome 4 is

$$\pi_{BSP4} = V(\lambda) - \frac{1}{2}t - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{BSP} + \left(\frac{1}{2} - x_3\right)\lambda r_C .$$

$$\pi_{BSP2} - \pi_{BSP4}$$

$$= \left[ V(\lambda) - (1 - x_3)t - \frac{d}{\mu - (1 - x_3)\lambda} + (1 - x_3)\lambda r_{BSP} \right] - \left[ V(\lambda) - \frac{1}{2}t - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{BSP} + \left(\frac{1}{2} - x_3\right)\lambda r_C \right]$$

$$= -\left(\frac{1}{2} - x_3\right)t + \left[ \frac{d}{\mu - \lambda} - \frac{d}{\mu - (1 - x_3)\lambda} \right] + \left(\frac{1}{2} - x_3\right)\lambda r_{BSP} - \left(\frac{1}{2} - x_3\right)\lambda r_C$$

$$= -\left(\frac{1}{2} - x_3\right)t - 2tx_3\left(\frac{1}{2} - x_3\right) + \left(\frac{1}{2} - x_3\right)\lambda r_{BSP} - \left(\frac{1}{2} - x_3\right)\lambda r_C$$

$$= -\left(\frac{1}{2} - x_3\right)(1 - 2x_3)t + \left(\frac{1}{2} - x_3\right)(r_{BSP} - r_C)\lambda$$

$$> -\left(\frac{1}{2} - x_3\right)(1 - 2x_3)t + \left(\frac{1}{2} - x_3\right)\lambda \left[ r_{BSP} - \left( r_{BSP} - \frac{t}{\lambda}(1 - 2x_3) \right) \right]$$ since under Case A

$r_C < r_{BSP} - \frac{t}{\lambda}(1 - 2x_3)$.

$$= -\left(\frac{1}{2} - x_3\right)(1 - 2x_3)t + \left(\frac{1}{2} - x_3\right)\lambda \left[ \frac{t}{\lambda}(1 - 2x_3) \right]$$

$$= -\left(\frac{1}{2} - x_3\right)(1 - 2x_3)t + \left(\frac{1}{2} - x_3\right)(1 - 2x_3)t = 0$$

Case B: $r_C > \left(\frac{1-x_3}{1/2-x_3}\right) r_{BSP} + \frac{t}{\lambda}(1-2x_3)$

Similar to Case A, Outcome 1 is not equilibrium. Hence we need only to consider Outcomes 2, 3 and 4, and note that now BSP's profit in Outcome 4 is $\pi_{BSP4} = V(\lambda) - \frac{1}{2}t - \frac{d}{\mu-\lambda} + \lambda r_{BSP}$.

$\pi_{BSP3} - \pi_{BSP2}$

$$= \left[V(\lambda) - t(1-x_3) - \frac{d}{\mu-(1-x_3)\lambda} + x_3\lambda r_{BSP} + \left(\frac{1}{2}-x_3\right)\lambda r_C\right] - \left[V(\lambda) - (1-x_3)t - \frac{d}{\mu-(1-x_3)\lambda} + (1-x_3)\lambda r_{BSP}\right]$$

$$= -(1-2x_3)\lambda r_{BSP} + \left(\frac{1}{2}-x_3\right)\lambda r_C = -2\left(\frac{1}{2}-x_3\right)\lambda r_{BSP} + \left(\frac{1}{2}-x_3\right)\lambda r_C = \left(\frac{1}{2}-x_3\right)\lambda\left(r_C - 2r_{BSP}\right)$$

$$> \left(\frac{1}{2}-x_3\right)\left[\left(\frac{1-x_3}{(1/2-x_3)} r_{BSP} + \frac{t}{\lambda}(1-2x_3)\right) - 2r_{BSP}\right]\lambda$$ since under Case B,

$$r_C > \left(\frac{1-x_3}{1/2-x_3}\right) r_{BSP} + \frac{t}{\lambda}(1-2x_3)$$

$$= \left(\frac{1}{2}-x_3\right)\left[\left(\frac{x_3}{1/2-x_3}\right) r_{BSP} + \frac{t}{\lambda}(1-2x_3)\right] > 0$$

$$\pi_{BSP3} - \pi_{BSP4} = \left[V(\lambda) - t(1-x_3) - \frac{d}{\mu-(1-x_3)\lambda} + x_3\lambda r_{BSP} + \left(\frac{1}{2}-x_3\right)\lambda r_C\right] - \left[V(\lambda) - \frac{1}{2}t - \frac{d}{\mu-\lambda} + \lambda r_{BSP}\right]$$

$$= -\left(\frac{1}{2}-x_3\right)t + \left[\frac{d}{\mu-\lambda} - \frac{d}{\mu-(1-x_3)\lambda}\right] - (1-x_3)\lambda r_{BSP} + \left(\frac{1}{2}-x_3\right)\lambda r_C$$

$$= -\left(\frac{1}{2}-x_3\right)t + 2tx_3\left(\frac{1}{2}-x_3\right) - (1-x_3)\lambda r_{BSP} + \left(\frac{1}{2}-x_3\right)\lambda r_C$$ since Equation (B-4)

$$= -t\left(\frac{1}{2}-x_3\right)(1-2x_3) - (1-x_3)\lambda r_{BSP} + \left(\frac{1}{2}-x_3\right)\lambda r_C$$

$$> -t\left(\frac{1}{2}-x_3\right)(1-2x_3)-(1-x_3)\lambda r_{BSP}+\left(\frac{1}{2}-x_3\right)\lambda\left[\left(\frac{1-x_3}{1/2-x_3}\right)r_{BSP}+\frac{t}{\lambda}(1-2x_3)\right]$$ since under Case

B, $r_C > \left(\frac{1-x_3}{1/2-x_3}\right)r_{BSP}+\frac{t}{\lambda}(1-2x_3)$

$$= -\left(\frac{1}{2}-x_3\right)(1-2x_3)t-(1-x_3)\lambda r_{BSP}+(1-x_3)\lambda r_{BSP}+\left(\frac{1}{2}-x_3\right)(1-2x_3)t = 0$$

Case C: $r_{BSP}-\frac{t}{\lambda}(1-2x_3) \le r_C \le \left(\frac{1-x_3}{1/2-x_3}\right)r_{BSP}+\frac{t}{\lambda}(1-2x_3)$ Firstly, I divide Case C into two subcases, Case C1: $r_{BSP}-\frac{t}{\lambda}(1-2x_3) < r_C < \left(\frac{1-x_3}{1/2-x_3}\right)r_{BSP}$, and Case C2:

$$\left(\frac{1-x_3}{1/2-x_3}\right)r_{BSP} < r_C < \left(\frac{1-x_3}{1/2-x_3}\right)r_{BSP}+\frac{t}{\lambda}(1-2x_3).$$

Under Case C1, similar to Case A, Outcome 1 is not equilibrium. In Formulation (5-13), we have $F_4 = V(\lambda)-\frac{1}{2}t-\frac{d}{\mu-\lambda}$ and $p_{41} = (1-2x_3)r_C$, which imply that the BSP's profit is

$$\pi_{41} = F_4+\frac{1}{2}\lambda r_{ISP}+\frac{1}{2}\lambda p_{41} = V(\lambda)-\frac{1}{2}t-\frac{d}{\mu-\lambda}+\frac{1}{2}\lambda r_{ISP}+\left(\frac{1}{2}-x_3\right)\lambda r_C.$$

Now, consider Outcomes 2, 3 and 4.

$$\pi_{BSP4}-\pi_{BSP2}$$

$$=\left[V(\lambda)-\frac{1}{2}t-\frac{d}{\mu-\lambda}+\frac{1}{2}\lambda r_{BSP}+\left(\frac{1}{2}-x_3\right)\lambda r_C\right]-\left[V(\lambda)-(1-x_3)t-\frac{d}{\mu-(1-x_3)\lambda}+(1-x_3)\lambda r_{BSP}\right]$$

$$=\left(\frac{1}{2}-x_3\right)t-\left[\frac{d}{\mu-\lambda}-\frac{d}{\mu-(1-x_3)\lambda}\right]-\left(\frac{1}{2}-x_3\right)\lambda r_{BSP}+\left(\frac{1}{2}-x_3\right)\lambda r_C$$

$$=\left(\frac{1}{2}-x_3\right)t-2tx_3\left(\frac{1}{2}-x_3\right)-\left(\frac{1}{2}-x_3\right)\lambda r_{BSP}+\left(\frac{1}{2}-x_3\right)\lambda r_C$$

$$= \left(\frac{1}{2} - x_3\right)(1 - 2x_3)\,t - \left(\frac{1}{2} - x_3\right)\lambda\,(r_{BSP} - r_C)$$

$$> \left(\frac{1}{2} - x_3\right)(1 - 2x_3)\,t - \left(\frac{1}{2} - x_3\right)\lambda\left[r_{BSP} - \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP}\right]$$ since under Case C1

$$r_{BSP} - \frac{t}{\lambda}(1 - 2x_3) < r_C < \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP}$$

$$= \left(\frac{1}{2} - x_3\right)(1 - 2x_3)\,t + \left(\frac{1}{2} - x_3\right)\left(\frac{1/2}{1/2 - x_3}\right) r_{BSP} > 0$$

$$\Pi_{BSP4} - \Pi_{BSP3}$$

$$= \left[V(\lambda) - \frac{1}{2}t - \frac{d}{\mu - \lambda} + \lambda r_{BSP}\right] - \left[V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda} + x_3\lambda r_{BSP} + \left(\frac{1}{2} - x_3\right)\lambda r_C\right]$$

$$= \left(\frac{1}{2} - x_3\right)t - \left[\frac{d}{\mu - \lambda} - \frac{d}{\mu - (1 - x_3)\lambda}\right] + (1 - x_3)\,\lambda r_{BSP} - \left(\frac{1}{2} - x_3\right)\lambda r_C$$

$$= \left(\frac{1}{2} - x_3\right)t - 2tx_3\left(\frac{1}{2} - x_3\right) + (1 - x_3)\,\lambda r_{BSP} - \left(\frac{1}{2} - x_3\right)\lambda r_C$$

$$= t\left(\frac{1}{2} - x_3\right)(1 - 2x_3) + (1 - x_3)\,\lambda r_{BSP} - \left(\frac{1}{2} - x_3\right)\lambda r_C$$

$$> t\left(\frac{1}{2} - x_3\right)(1 - 2x_3) + (1 - x_3)\,\lambda r_{BSP} - \left(\frac{1}{2} - x_3\right)\lambda\left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP}$$ since under Case C1,

$$r_{BSP} - \frac{t}{\lambda}(1 - 2x_3) < r_C < \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP}$$

$$= t\left(\frac{1}{2} - x_3\right)(1 - 2x_3) + (1 - x_3)\,\lambda r_{BSP} - (1 - x_3)\,\lambda r_{BSP}$$

$$= t\left(\frac{1}{2} - x_3\right)(1 - 2x_3) > 0$$

Under Case C2, similar to Case A, Outcome 1 is not equilibrium. In Formulation (5-13), we have

$F_4 = V(\lambda) - \frac{1}{2}t - \frac{d}{\mu - \lambda}$ and $p_{42} = r_{\mathrm{BSP}}$, which imply that the BSP's profit is

$$\pi_{42} = F_4 + \frac{1}{2}\lambda r_{\mathrm{BSP}} + \frac{1}{2}\lambda p_{42} = V(\lambda) - \frac{1}{2}t - \frac{d}{\mu - \lambda} + \lambda r_{\mathrm{BSP}}$$

Now, consider Outcomes 2, 3 and 4.

$$\pi_{\mathrm{BSP4}} - \pi_{\mathrm{BSP2}} = \left[V(\lambda) - \frac{1}{2}t - \frac{d}{\mu - \lambda} + \lambda r_{\mathrm{BSP}}\right] - \left[V(\lambda) - (1 - x_3)t - \frac{d}{\mu - (1 - x_3)\lambda} + (1 - x_3)\lambda r_{\mathrm{BSP}}\right]$$

$$= \left(\frac{1}{2} - x_3\right)t - \left[\frac{d}{\mu - \lambda} - \frac{d}{\mu - (1 - x_3)\lambda}\right] + x_3\lambda r_{\mathrm{BSP}}$$

$$= \left(\frac{1}{2} - x_3\right)t - 2tx_3\left(\frac{1}{2} - x_3\right) + x_3\lambda r_{\mathrm{BSP}}$$

$$= \left(\frac{1}{2} - x_3\right)(1 - 2x_3)t + x_3\lambda r_{\mathrm{BSP}} > 0$$

$$\pi_{\mathrm{BSP4}} - \pi_{\mathrm{BSP3}}$$

$$= \left[V(\lambda) - \frac{1}{2}t - \frac{d}{\mu - \lambda} + \lambda r_{\mathrm{BSP}}\right] - \left[V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda} + x_3\lambda r_{\mathrm{BSP}} + \left(\frac{1}{2} - x_3\right)\lambda r_{\mathrm{C}}\right]$$

$$= \left(\frac{1}{2} - x_3\right)t - \left[\frac{d}{\mu - \lambda} - \frac{d}{\mu - (1 - x_3)\lambda}\right] + (1 - x_3)\lambda r_{\mathrm{BSP}} - \left(\frac{1}{2} - x_3\right)\lambda r_{\mathrm{C}}$$

$$= \left(\frac{1}{2} - x_3\right)t - 2tx_3\left(\frac{1}{2} - x_3\right) + (1 - x_3)\lambda r_{\mathrm{BSP}} - \left(\frac{1}{2} - x_3\right)\lambda r_{\mathrm{C}}$$

$$= t\left(\frac{1}{2} - x_3\right)(1 - 2x_3) + (1 - x_3)\lambda r_{\mathrm{BSP}} - \left(\frac{1}{2} - x_3\right)\lambda r_{\mathrm{C}}$$

$$> t\left(\frac{1}{2}-x_3\right)(1-2x_3)+(1-x_3)\lambda r_{BSP}-\left(\frac{1}{2}-x_3\right)\lambda\left[\left(\frac{1-x_3}{1/2-x_3}\right)r_{BSP}+\frac{t}{\lambda}(1-2x_3)\right]$$ since under Case

C2, $\left(\frac{1-x_3}{1/2-x_3}\right)r_{BSP} < r_C < \left(\frac{1-x_3}{1/2-x_3}\right)r_{BSP}+\frac{t}{\lambda}(1-2x_3)$

$$= t\left(\frac{1}{2}-x_3\right)(1-2x_3)+(1-x_3)\lambda r_{BSP}-(1-x_3)\lambda r_{BSP}-t\left(\frac{1}{2}-x_3\right)(1-2x_3)=0$$

## Proof of the Results in Table 5-1

Case A: $r_C < r_{BSP}-\frac{t}{\lambda}(1-2x_3)$

Outcome 2 is the equilibrium. In the Formulation (5-9), the optimal solution is

$F_2 = V(\lambda)-tx_2-\frac{d}{\mu-x_2\lambda}$. Hence, the BSP's profit is

$$\pi_{BSP2} = F_2 + x_2\lambda r_{BSP} = V(\lambda)-tx_2-\frac{d}{\mu-x_2\lambda}+x_2\lambda r_{BSP}.$$

Content provider C's profit is $\pi_{C2} = (1-x_2)\lambda r_C$.

Consumer Surplus under Case A

$$CS_2 = \int_0^{x_2}\left[V(\lambda)-tx-\frac{d}{\mu-x_2\lambda}-F_2\right]dx+\int_{x_2}^{1}\left[V(\lambda)-t(1-x_2)-\frac{d\mu}{(\mu-x_2\lambda)(\mu-\lambda)}-F_2\right]dx$$

$$= \int_0^{x_2}\left\{V(\lambda)-tx-\frac{d}{\mu-x_2\lambda}-\left[V(\lambda)-tx_2-\frac{d}{\mu-x_2\lambda}\right]\right\}dx$$
$$+\int_{x_2}^{1}\left\{V(\lambda)-t(1-x)-\frac{d\mu}{(\mu-x_2\lambda)(\mu-\lambda)}-\left[V(\lambda)-t(1-x_2)-\frac{d\mu}{(\mu-x_2\lambda)(\mu-\lambda)}\right]\right\}dx$$

$$= \int_0^{x_2}(tx_2-tx)\,dx+\int_{x_2}^{1}(tx-tx_2)\,dx$$ since

$$F_2 = V(\lambda)-tx_2-\frac{d}{\mu-x_2\lambda} = V(\lambda)-t(1-x_2)-\frac{d\mu}{(\mu-x_2\lambda)(\mu-\lambda)}$$

$$= t\left( x_2 x - \frac{1}{2} x^2 \right)\Bigg|_0^{x_2} + t\left( \frac{1}{2} x^2 - x_2 x \right)\Bigg|_{x_2}^{1}$$

$$= t\left( x_2^2 - x_2 + \frac{1}{2} \right)$$

Social Welfare under Case A

$$SW_2 = \pi_{BSP2} + \pi_{C2} + CS_2$$

$$= \left[ V(\lambda) - tx_2 - \frac{d}{\mu - x_2\lambda} + x_2 \lambda r_{BSP} \right] + \left[ (1 - x_2)\lambda r_C \right] + t\left( x_2^2 - x_2 + \frac{1}{2} \right)$$

$$= V(\lambda) + t\left( x_2^2 - 2x_2 + \frac{1}{2} \right) - \frac{d}{\mu - x_2\lambda} + x_2 \lambda r_{BSP} + (1 - x_2)\lambda r_C$$

Case B: $r_C > \left( \frac{1 - x_3}{1/2 - x_3} \right) r_{BSP} + \frac{t}{\lambda}(1 - 2x_3)$

Outcome 3 is the equilibrium. In the Formulation (5-11), the optimal solution is

$F_3 = V(\lambda) - tx_3 - \frac{d\mu}{\left[\mu - (1 - x_3)\lambda\right](\mu - \lambda)}$ and $p_3 = \left( \frac{1/2 - x_3}{1 - x_3} \right) r_C$. Hence, the BSP's profit is

$$\pi_{BSP3} = F_3 + x_3 \lambda r_{BSP} + (1 - x_3)\lambda p_3 = V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda} + \left( \frac{1}{2} - x_3 \right)\lambda r_C + x_3 \lambda r_{BSP}.$$

Content provider C's profit is $\pi_{C3} = (1 - x_3)\lambda r_C - (1 - x_3)\lambda p_3 = \frac{1}{2}\lambda r_C$

Consumer Surplus under Case B

$$CS_3 = \int_0^{x_3} \left( V(\lambda) - tx - \frac{d\mu}{\left[\mu - (1 - x_3)\lambda\right](\mu - \lambda)} - F_3 \right) dx + \int_{x_3}^{1} \left[ V(\lambda) - t(1 - x) - \frac{d}{\mu - (1 - x_3)\lambda} - F_3 \right] dx$$

$$= \int_0^{x_3} \left\{ V(\lambda) - tx - \frac{d\mu}{\left[\mu - (1 - x_3)\lambda\right](\mu - \lambda)} - \left( V(\lambda) - tx_3 - \frac{d\mu}{\left[\mu - (1 - x_3)\lambda\right](\mu - \lambda)} \right) \right\} dx$$

$$+ \int_{x_3}^{1} \left\{ V(\lambda) - t(1 - x) - \frac{d}{\mu - (1 - x_3)\lambda} - \left[ V(\lambda) - t(1 - x_3) - \frac{d}{\mu - (1 - x_3)\lambda} \right] \right\} dx$$

$$= \int_0^{x_3} (tx_3 - tx)\,dx + \int_{x_3}^{1} (tx - tx_3)\,dx$$

since $F_3 = V(\lambda) - tx_3 - \dfrac{d\mu}{\left[\mu - (1-x_3)\lambda\right](\mu - \lambda)} = V(\lambda) - t(1-x_3) - \dfrac{d}{\mu - (1-x_3)\lambda}$

$$= t\left(x_3 x - \frac{1}{2}x^2\right)\Bigg|_0^{x_3} + t\left(\frac{1}{2}x^2 - x_3 x\right)\Bigg|_{x_3}^{1}$$

$$= t\left(x_3^2 - x_3 + \frac{1}{2}\right)$$

Social Welfare under Case B

$$SW_3 = \pi_{BSP3} + \pi_{C3} + CS_3$$

$$= \left[V(\lambda) - t(1-x_3) - \frac{d}{\mu - (1-x_3)\lambda} + x_3\lambda r_{BSP} + \left(\frac{1}{2} - x_3\right)\lambda r_C\right] + \frac{1}{2}\lambda r_C + t\left(x_3^2 - x_3 + \frac{1}{2}\right)$$

$$= V(\lambda) - t\left(\frac{1}{2} - x_3^2\right) - \frac{d}{\mu - (1-x_3)\lambda} + x_3\lambda r_{BSP} + (1-x_3)\lambda r_C$$

Case C1: $r_{BSP} - \dfrac{t}{\lambda}(1-2x_3) < r_C < \left(\dfrac{1-x_3}{1/2 - x_3}\right) r_{BSP}$

Outcome 4 (Case C1) is the equilibrium. In the Formulation (5-13), we have

$F_4 = V(\lambda) - \dfrac{1}{2}t - \dfrac{d}{\mu - \lambda}$ and $p_{41} = (1-2x_3)r_C$. Hence, the BSP's profit is

$$\pi_{41} = F_4 + \frac{1}{2}\lambda r_{BSP} + \frac{1}{2}\lambda p_{41} = V(\lambda) - \frac{1}{2}t - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{BSP} + \left(\frac{1}{2} - x_3\right)\lambda r_C.$$

Content provider C's profit is $\pi_{C41} = \dfrac{1}{2}\lambda r_C - \dfrac{1}{2}\lambda p_{41} = x_3\lambda r_C$.

Consumer Surplus under Case C1

$$CS_4 = \int_0^{1/2}\left[V(\lambda) - tx - \frac{d}{\mu-\lambda} - F_4\right]dx + \int_{1/2}^{1}\left[V(\lambda) - t(1-x) - \frac{d}{\mu-\lambda} - F_4\right]dx$$

Since $F_4 = V(\lambda) - \frac{1}{2}t - \frac{d}{\mu-\lambda}$

$$= \int_0^{1/2}\left\{V(\lambda) - tx - \frac{d}{\mu-\lambda} - \left[V(\lambda) - \frac{1}{2}t - \frac{d}{\mu-\lambda}\right]dx\right\}$$
$$+ \int_{1/2}^{1}\left\{V(\lambda) - t(1-x) - \frac{d}{\mu-\lambda} - \left[V(\lambda) - \frac{1}{2}t - \frac{d}{\mu-\lambda}\right]\right\}dx$$

$$= \int_0^{1/2} t\left(\frac{1}{2} - x\right)dx + \int_{1/2}^{1} t\left(x - \frac{1}{2}\right)dx = t\left(\frac{1}{2}x - \frac{1}{2}x^2\right)\Bigg|_0^{\frac{1}{2}} + t\left(\frac{1}{2}x^2 - \frac{1}{2}x\right)\Bigg|_{\frac{1}{2}}^{1} = \frac{t}{4}$$

Social Welfare under Case C1

$$SW_{41} = \pi_{BSP41} + \pi_{C41} + CS_4$$

$$= \left[V(\lambda) - \frac{1}{2}t - \frac{d}{\mu-\lambda} + \frac{1}{2}\lambda r_{BSP} + \left(\frac{1}{2} - x_3\right)\lambda r_C\right] + \left[(1-x_2)\lambda r_C\right] + \frac{t}{4}$$

$$= V(\lambda) - \frac{1}{4}t - \frac{d}{\mu-\lambda} + \frac{1}{2}\lambda r_{BSP} + \left(\frac{3}{2} - x_2 - x_3\right)\lambda r_C$$

$$= V(\lambda) - \frac{1}{4}t - \frac{d}{\mu-\lambda} + \frac{1}{2}\lambda r_{BSP} + \frac{1}{2}\lambda r_C \text{ since } x_2 - \frac{1}{2} = \frac{1}{2} - x_3$$

Case C2, $\left(\frac{1-x_3}{1/2 - x_3}\right)r_{BSP} < r_C < \left(\frac{1-x_3}{1/2 - x_3}\right)r_{BSP} + \frac{t}{\lambda}(1-2x_3)$

Outcome 4 (Case C2) is the equilibrium. In the Formulation (5-13), we have

$F_4 = V(\lambda) - \frac{1}{2}t - \frac{d}{\mu-\lambda}$ and $p_{42} = r_{BSP}$. Hence, the BSP's profit is $\pi_{42} = F_4 + \frac{1}{2}\lambda r_{BSP} + \frac{1}{2}\lambda p_{42}$

$$= V(\lambda) - \frac{1}{2}t - \frac{d}{\mu-\lambda} + \lambda r_{BSP}$$

Content provider C's profit is $\pi_{C42} = \frac{1}{2}\lambda r_C - \frac{1}{2}\lambda p_{42} = \frac{1}{2}\lambda\left(r_C - r_{BSP}\right)$.

Consumer Surplus under Case C2 is same as Case C1.

Social Welfare under Case C2

$$SW_{42} = \pi_{BSP42} + \pi_{C42} + CS_4 = \left[V(\lambda) - \frac{1}{2}t - \frac{d}{\mu - \lambda} + \lambda r_{BSP}\right] + \left[\frac{1}{2}\lambda\left(r_C - r_{BSP}\right)\right] + \frac{t}{4}$$

$$= V(\lambda) - \frac{1}{4}t - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_C + \frac{1}{2}\lambda r_{BSP}$$

**Proof of Proposition 5-2 (The Welfare Impact of Net Neutrality with a Vertically Integrated BSP)**

Case A, $r_C < r_{BSP} - \frac{t}{\lambda}(1 - 2x_3)$

$$SW_2 - SW_{NN}$$

$$= \left[V(\lambda) + t\left(x_2^2 - 2x_2 + \frac{1}{2}\right) - \frac{d}{\mu - x_2\lambda} + x_2\lambda r_{BSP} + (1 - x_2)\lambda r_C\right] - \left[V(\lambda) - \frac{1}{4}t - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{BSP} + \frac{1}{2}\lambda r_C\right]$$

$$= t\left(x_2^2 - 2x_2 + \frac{3}{4}\right) + \left(\frac{d}{\mu - \lambda} - \frac{d}{\mu - x_2\lambda}\right) + \left(x_2 - \frac{1}{2}\right)\lambda r_{BSP} - \left(x_2 - \frac{1}{2}\right)\lambda r_C$$

$$= t\left(x_2^2 - 2x_2 + \frac{3}{4}\right) + 2tx_3\left(\frac{1}{2} - x_3\right) + \left(x_2 - \frac{1}{2}\right)\lambda\left(r_{BSP} - r_C\right)$$

$$= t\left[\left(x_2^2 - 2x_2 + \frac{3}{4}\right) + 2(1 - x_2)\left(x_2 - \frac{1}{2}\right)\right] + \left(x_2 - \frac{1}{2}\right)\lambda\left(r_{BSP} - r_C\right) \text{ since } x_2 - \frac{1}{2} = \frac{1}{2} - x_3$$

$$= -t\left(x_2 - \frac{1}{2}\right)^2 + \left(x_2 - \frac{1}{2}\right)\lambda\left(r_{BSP} - r_C\right)$$

$$= \left(x_2 - \frac{1}{2}\right)\left[-t\left(x_2 - \frac{1}{2}\right) + \lambda\left(r_{BSP} - r_C\right)\right]$$

$$> \left(x_2 - \frac{1}{2}\right)\left\{-t\left(x_2 - \frac{1}{2}\right) + \lambda\left[r_C + \frac{t}{\lambda}(1 - 2x_3)\right] - \lambda r_C\right\} \text{ since } r_C < r_{BSP} - \frac{t}{\lambda}(1 - 2x_3)$$

$$= \left( x_2 - \frac{1}{2} \right) \left[ -t \left( x_2 - \frac{1}{2} \right) + \left( 1 - 2x_3 \right) t \right]$$

$$= \left( x_2 - \frac{1}{2} \right) \left\{ -t \left( x_2 - \frac{1}{2} \right) + t \left[ 1 - 2\left( 1 - x_2 \right) \right] \right\} \text{ since } x_2 - \frac{1}{2} = \frac{1}{2} - x_3$$

$$= \left( x_2 - \frac{1}{2} \right) \left[ -t \left( x_2 - \frac{1}{2} \right) + 2t \left( x_2 - \frac{1}{2} \right) \right]$$

$$= \left( x_2 - \frac{1}{2} \right)^2 > 0$$

$$CS_2 - CS_{NN} = t \left( x_2^2 - x_2 + \frac{1}{2} \right) - \frac{1}{4} t = t \left[ \left( x_2 - \frac{1}{2} \right)^2 + \frac{1}{4} \right] - \frac{1}{4} t = \left( x_2 - \frac{1}{2} \right)^2 t > 0$$

Case B, $r_C > \left( \frac{1 - x_3}{1/2 - x_3} \right) r_{BSP} + \frac{t}{\lambda} \left( 1 - 2x_3 \right)$

$SW_3 - SW_{NN}$

$$= \left[ V(\lambda) - t \left( \frac{1}{2} - x_3^2 \right) - \frac{d}{\mu - \left( 1 - x_3 \right) \lambda} + x_3 \lambda r_{BSP} + \left( 1 - x_3 \right) \lambda r_C \right] - \left[ V(\lambda) - \frac{1}{4} t - \frac{d}{\mu - \lambda} + \frac{1}{2} \lambda r_{BSP} + \frac{1}{2} \lambda r_C \right]$$

$$= -t \left( \frac{1}{4} - x_3^2 \right) + \left[ \frac{d}{\mu - \lambda} - \frac{d}{\mu - \left( 1 - x_3 \right) \lambda} \right] - \left( \frac{1}{2} - x_3 \right) \lambda r_{BSP} + \left( \frac{1}{2} - x_3 \right) \lambda r_C$$

$$= -t \left( \frac{1}{2} - x_3 \right) \left( \frac{1}{2} + x_3 \right) + 2tx_3 \left( \frac{1}{2} - x_3 \right) + \left( \frac{1}{2} - x_3 \right) \lambda \left( r_C - r_{BSP} \right)$$

$$= \left( \frac{1}{2} - x_3 \right) \left[ \lambda \left( r_C - r_{BSP} \right) - \left( \frac{1}{2} - x_3 \right) t \right]$$

$$> \left( \frac{1}{2} - x_3 \right) \left\{ \lambda \left[ \left( \frac{1 - x_3}{1/2 - x_3} \right) r_{BSP} + \frac{t}{\lambda} \left( 1 - 2x_3 \right) - r_{BSP} \right] - t \left( \frac{1}{2} - x_3 \right) \right\} \text{ since under Case B,}$$

$$r_C > \left( \frac{1 - x_3}{1/2 - x_3} \right) r_{BSP} + \frac{t}{\lambda} \left( 1 - 2x_3 \right)$$

$$= \left(\frac{1}{2} - x_3\right)\left[\lambda\left(\left(\frac{1/2}{1/2 - x_3}\right) r_{BSP} + \frac{t}{\lambda}(1 - 2x_3)\right) - \left(\frac{1}{2} - x_3\right) t\right]$$

$$= \left(\frac{1}{2} - x_3\right)\left\{\left[\left(\frac{1/2}{1/2 - x_3}\right)\lambda r_{BSP} + t(1 - 2x_3)\right] - t\left(\frac{1}{2} - x_3\right)\right\}$$

$$= \left(\frac{1}{2} - x_3\right)\left[\left(\frac{1/2}{1/2 - x_3}\right)\lambda r_{BSP} + t\left(\frac{1}{2} - x_3\right)\right] > 0$$

$$CS_3 - CS_{NN} = t\left(x_3^2 - x_3 + \frac{1}{2}\right) - \frac{1}{4}t = t\left[\left(\frac{1}{2} - x_3\right)^2 + \frac{1}{4}\right] - \frac{1}{4}t = \left(\frac{1}{2} - x_3\right)^2 t > 0$$

Case C, $r_{BSP} - \frac{t}{\lambda}(1 - 2x_3) \le r_C \le \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP} + \frac{t}{\lambda}(1 - 2x_3)$

Both Case C1 and C2 have the same social welfare, $SW_4 = V(\lambda) - \frac{1}{4}t - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda(r_C + r_{BSP})$,

and customer surplus, $CS_4 = \frac{t}{4}$, which remain unchanged compared to the benchmark case.

**Proof of Proposition 5-3 (The Welfare Effect of Vertical Integration)**

Case $(i)$ when $2r_{BSP} + \frac{t}{\lambda}(1 - 2x_3) < r_C < \left(\frac{1 - x_3}{1/2 - x_3}\right) r_{BSP} + \frac{t}{\lambda}(1 - 2x_3)$

$SW_{VI4} - SW_{NVI2}$

$$= \left[V(\lambda) - \frac{1}{4}t - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{BSP} + \frac{1}{2}\lambda r_C\right] - \left[V(\lambda) + t\left(x_2^2 - 2x_2 + \frac{1}{2}\right) - \frac{d}{\mu - x_2\lambda} + x_2\lambda r_{BSP} + (1 - x_2)\lambda r_C\right]$$

$$= t\left(\frac{1}{4} - x_3^2\right) - \left[\frac{d}{\mu - \lambda} - \frac{d}{\mu - (1 - x_3)\lambda}\right] + \left(\frac{1}{2} - x_3\right)\lambda r_{BSP} - \left(\frac{1}{2} - x_3\right)\lambda r_C$$

$$= t\left(\frac{1}{4} - x_3^2\right) - 2tx_3\left(\frac{1}{2} - x_3\right) + \left(\frac{1}{2} - x_3\right)\lambda r_{BSP} - \left(\frac{1}{2} - x_3\right)\lambda r_C$$

$$= t\left(\frac{1}{2} - x_3\right)\left(\frac{1}{2} + x_3\right) - 2tx_3\left(\frac{1}{2} - x_3\right) + \left(\frac{1}{2} - x_3\right)\lambda(r_{BSP} - r_C)$$

$$= t\left(\frac{1}{2} - x_3\right)\left[\left(\frac{1}{2} + x_3\right) - 2x_3\right] + \left(\frac{1}{2} - x_3\right)\lambda\left(r_{BSP} - r_C\right)$$

$$= t\left(\frac{1}{2} - x_3\right)^2 + \left(\frac{1}{2} - x_3\right)\lambda\left(r_{BSP} - r_C\right)$$

$$< t\left(\frac{1}{2} - x_3\right)^2 + \left(\frac{1}{2} - x_3\right)\lambda\left\{r_{BSP} - \left[2r_{BSP} + \frac{t}{\lambda}\left(1 - 2x_3\right)\right]\right\} \text{ since}$$

$$2r_{BSP} + \frac{t}{\lambda}\left(1 - 2x_3\right) < r_C < \left(\frac{1 - x_3}{1/2 - x_3}\right)r_{BSP} + \frac{t}{\lambda}\left(1 - 2x_3\right)$$

$$= t\left(\frac{1}{2} - x_3\right)^2 + \left(\frac{1}{2} - x_3\right)\left[-\lambda r_{BSP} - t\left(1 - 2x_3\right)\right]$$

$$= \left(\frac{1}{2} - x_3\right)\left[t\left(\frac{1}{2} - x_3\right) - \lambda r_{BSP} - 2t\left(\frac{1}{2} - x_3\right)\right]$$

$$= -\left(\frac{1}{2} - x_3\right)\left[\lambda r_{BSP} + t\left(\frac{1}{2} - x_3\right)\right] < 0$$

Case $(ii)$ when $r_C + \frac{t}{\lambda}\left(1 - 2x_3\right) < r_{BSP} < 2r_C + \frac{t}{\lambda}\left(1 - 2x_3\right)$

$SW_{VI4} - SW_{NVI3}$

$$= \left[V(\lambda) - \frac{1}{4}t - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{BSP} + \frac{1}{2}\lambda r_C\right] - \left[V(\lambda) - t\left(\frac{1}{2} - x_3^2\right) - \frac{d}{\mu - \left(1 - x_3\right)\lambda} + x_3\lambda r_{BSP} + \left(1 - x_3\right)\lambda r_C\right]$$

$$= t\left(\frac{1}{4} - x_3^2\right) - \left[\frac{d}{\mu - \lambda} - \frac{d}{\mu - \left(1 - x_3\right)\lambda}\right] + \left(\frac{1}{2} - x_3\right)\lambda r_{BSP} - \left(\frac{1}{2} - x_3\right)\lambda r_C$$

$$= t\left(\frac{1}{4} - x_3^2\right) - 2tx_3\left(\frac{1}{2} - x_3\right) + \left(\frac{1}{2} - x_3\right)\lambda r_{BSP} - \left(\frac{1}{2} - x_3\right)\lambda r_C$$

$$= t\left(\frac{1}{2} - x_3\right)\left(\frac{1}{2} + x_3\right) - 2tx_3\left(\frac{1}{2} - x_3\right) + \left(\frac{1}{2} - x_3\right)\lambda\left(r_{BSP} - r_C\right)$$

$$= t\left(\frac{1}{2} - x_3\right)\left[\left(\frac{1}{2} + x_3\right) - 2x_3\right] + \left(\frac{1}{2} - x_3\right)\lambda\left(r_{BSP} - r_C\right)$$

$$= t\left(\frac{1}{2} - x_3\right)^2 + \left(\frac{1}{2} - x_3\right)\lambda\left(r_{BSP} - r_C\right)$$

$$> t\left(\frac{1}{2} - x_3\right)^2 + \left(\frac{1}{2} - x_3\right)\lambda\left\{\left[r_C + \frac{t}{\lambda}\left(1 - 2x_3\right)\right] - r_C\right\} \text{ since } r_C + \frac{t}{\lambda}\left(1 - 2x_3\right) < r_{BSP} < 2r_C + \frac{t}{\lambda}\left(1 - 2x_3\right)$$

$$= t\left(\frac{1}{2} - x_3\right)^2 + 2t\left(\frac{1}{2} - x_3\right)^2 = 3t\left(\frac{1}{2} - x_3\right)^2 > 0$$

## Proof of Proposition 5-4 (The Social Planner's Preferences for Vertical Integration Compared to the BSP's Choice)

In Table 5-2,

$$\pi_{VI3,T_1} = V(\lambda) - t\left(1 - x_3\right) - \frac{d}{\mu - \left(1 - x_3\right)\lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_{G0} + x_3\lambda r_{Y0}$$

$$\pi_{VI41,T_1} = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{Y0} + \left(\frac{1}{2} - x_3\right)\lambda r_{G0}$$

$$\pi_{VI42,T_1} = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \lambda r_{Y0}$$

$$\pi_{VI2,T_2} = V(\lambda) - t\left(1 - x_3\right) - \frac{d}{\mu - \left(1 - x_3\right)\lambda} + \left(1 - x_3\right)\lambda r_{G0}$$

$$\pi_{VI41,T_2} = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{G0} + \left(\frac{1}{2} - x_3\right)\lambda r_{Y0}$$

$$\pi_{NVI3} = V(\lambda) - t\left(1 - x_3\right) - \frac{d}{\mu - \left(1 - x_3\right)\lambda} + \left(\frac{1}{2} - x_3\right)\lambda r_{G0}$$

$$\pi_{NVI4} = V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \left(1 - 2x_3\right)\lambda r_{Y0}$$

Region 1: $r_G > \left(\frac{1-x_3}{1/2-x_3}\right) r_Y + \frac{t}{\lambda}(1-2x_3)$

$$\pi_{VI2,T_2} - \pi_{VI3,T_1} = \left[V(\lambda) - t(1-x_3) - \frac{d}{\mu-(1-x_3)\lambda} + (1-x_3)\lambda r_{G0}\right]$$
$$-\left[V(\lambda) - t(1-x_3) - \frac{d}{\mu-(1-x_3)\lambda} + \left(\frac{1}{2}-x_3\right)\lambda r_{G0} + x_3\lambda r_{Y0}\right]$$

$$= \frac{1}{2}\lambda r_{G0} - x_3\lambda r_{Y0} > x_3\lambda r_{G0} - x_3\lambda r_{Y0} = x_3\lambda\left(r_{G0} - r_{Y0}\right) > 0$$

$$\pi_{VI3,T_1} - \pi_{NVI3} = \left[V(\lambda) - t(1-x_3) - \frac{d}{\mu-(1-x_3)\lambda} + \left(\frac{1}{2}-x_3\right)\lambda r_{G0} + x_3\lambda r_{Y0}\right]$$
$$-\left[V(\lambda) - t(1-x_3) - \frac{d}{\mu-(1-x_3)\lambda} + \left(\frac{1}{2}-x_3\right)\lambda r_{G0}\right]$$

$$= x_3\lambda r_{Y0} > 0$$

Region 2: $\max\left\{2r_Y + \frac{t}{\lambda}(1-2x_3), \left(\frac{1-x_3}{1/2-x_3}\right) r_Y\right\} < r_G < \left(\frac{1-x_3}{1/2-x_3}\right) r_Y + \frac{t}{\lambda}(1-2x_3)$

As we already know $\pi_{VI2,T_2} > \pi_{NVI3}$, we only need to compare $\pi_{VI2,T_2}$ to $\pi_{VI42,T_1}$.

$$\pi_{VI2,T_2} - \pi_{VI42,T_1} = \left[V(\lambda) - t(1-x_3) - \frac{d}{\mu-(1-x_3)\lambda} + (1-x_3)\lambda r_{G0}\right]$$
$$-\left[V(\lambda) - \frac{t}{2} - \frac{d}{\mu-\lambda} + \lambda r_{Y0}\right]$$

$$= -t\left(\frac{1}{2}-x_3\right) + \left[\frac{d}{\mu-\lambda} - \frac{d}{\mu-(1-x_3)\lambda}\right] + \left[(1-x_3)\lambda r_{G0} - \lambda r_{Y0}\right]$$

$$= -t\left(\frac{1}{2}-x_3\right) + 2tx_3\left(\frac{1}{2}-x_3\right) + \left[(1-x_3)\lambda r_{G0} - \lambda r_{Y0}\right]$$

$$= -t\left(\frac{1}{2}-x_3\right)(1-2x_3) + \left[(1-x_3)\lambda r_{G0} - \lambda r_{Y0}\right]$$

$$> -t\left(\frac{1}{2}-x_3\right)(1-2x_3)+\left\{(1-x_3)\lambda\left[2r_{Y0}+\frac{t}{\lambda}(1-2x_3)\right]-\lambda r_{G0}\right\}$$

$$= -t\left(\frac{1}{2}-x_3\right)(1-2x_3)+t(1-x_3)(1-2x_3)+(1-2x_3)\lambda r_{Y0}$$

$$= t(1-2x_3)\left(\frac{1}{2}\right)+(1-2x_3)\lambda r_{Y0} > 0$$

Region 3: $2r_Y+\frac{t}{\lambda}(1-2x_3) < r_G < \left(\frac{1-x_3}{1/2-x_3}\right)r_Y$

As we already know $\pi_{VI2,T_2} > \pi_{NVI3}$, we only need to compare $\pi_{VI2,T_2}$ to $\pi_{VI41,T_1}$.

$$\pi_{VI2,T_2}-\pi_{VI41,T_1}=\left[V(\lambda)-t(1-x_3)-\frac{d}{\mu-(1-x_3)\lambda}+(1-x_3)\lambda r_{G0}\right]$$
$$-\left[V(\lambda)-\frac{t}{2}-\frac{d}{\mu-\lambda}+\frac{1}{2}\lambda r_{Y0}+\left(\frac{1}{2}-x_3\right)\lambda r_{G0}\right]$$

$$= -t\left(\frac{1}{2}-x_3\right)+\left[\frac{d}{\mu-\lambda}-\frac{d}{\mu-(1-x_3)\lambda}\right]+\frac{1}{2}\lambda r_{G0}-\frac{1}{2}\lambda r_{Y0}$$

$$= -t\left(\frac{1}{2}-x_3\right)+2tx_3\left(\frac{1}{2}-x_3\right)+\frac{1}{2}\lambda(r_{G0}-r_{Y0})$$

$$= -t\left(\frac{1}{2}-x_3\right)(1-2x_3)+\frac{1}{2}\lambda(r_{G0}-r_{Y0})$$

$$> -t\left(\frac{1}{2}-x_3\right)(1-2x_3)+\frac{1}{2}\lambda\left[2r_{Y0}+\frac{t}{\lambda}(1-2x_3)-r_{Y0}\right]$$

$$= -t\left(\frac{1}{2}-x_3\right)(1-2x_3)+\frac{1}{2}t(1-2x_3)+\frac{1}{2}\lambda r_{Y0}$$

$$= tx_3(1-2x_3)+\frac{1}{2}\lambda r_{Y0} > 0$$

Region 4: $\max\left\{r_Y+\frac{t}{\lambda}(1-2x_3),\left(\frac{1-x_3}{1/2-x_3}\right)r_Y\right\} < r_G < 2r_Y+\frac{t}{\lambda}(1-2x_3)$

$$\pi_{\mathrm{VI2},T_2} - \pi_{\mathrm{NVI4}} = \left[ V(\lambda) - t(1-x_3) - \frac{d}{\mu - (1-x_3)\lambda} + (1-x_3)\lambda r_{\mathrm{G0}} \right]$$
$$- \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1-2x_3)\lambda r_{\mathrm{Y0}} \right]$$

$$= -t\left(\frac{1}{2} - x_3\right)(1-2x_3) + (1-x_3)\lambda r_{\mathrm{G0}} - (1-2x_3)\lambda r_{\mathrm{Y0}}$$

$$> -t\left(\frac{1}{2} - x_3\right)(1-2x_3) + (1-x_3)\lambda\left[r_{\mathrm{Y0}} + \frac{t}{\lambda}(1-2x_3)\right] - (1-2x_3)\lambda r_{\mathrm{Y0}}$$

$$= -t\left(\frac{1}{2} - x_3\right)(1-2x_3) + t(1-x_3)(1-2x_3) + \left[(1-x_3)\lambda r_{\mathrm{Y0}} - (1-2x_3)\lambda r_{\mathrm{Y0}}\right]$$

$$= \frac{1}{2}t(1-2x_3) + x_3\lambda r_{\mathrm{Y0}} > 0$$

$$\pi_{\mathrm{VI42},T_1} - \pi_{\mathrm{NVI4}} = \left[V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \lambda r_{\mathrm{Y0}}\right] - \left[V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1-2x_3)\lambda r_{\mathrm{Y0}}\right]$$

$$= 2x_3\lambda r_{\mathrm{Y0}} > 0$$

Now we move on to compare $\pi_{\mathrm{VI2},T_2}$ to $\pi_{\mathrm{VI42},T_1}$.

As shown as Figure 5-4b, note that the intersection point of $r_{\mathrm{G}} = r_{\mathrm{Y}} + \frac{t}{\lambda}(1-2x_3)$ and

$r_{\mathrm{G}} = \left(\frac{1-x_3}{1/2 - x_3}\right) r_{\mathrm{Y}}$ is $\left\{\frac{4t}{\lambda}\left(\frac{1}{2} - x_3\right)^2, \frac{4t}{\lambda}\left(\frac{1}{2} - x_3\right)(1-x_3)\right\}$, which means that in region 4 if

$r_{\mathrm{Y}} < \frac{4t}{\lambda}\left(\frac{1}{2} - x_3\right)^2$, $r_{\mathrm{G}} > r_{\mathrm{Y}} + \frac{t}{\lambda}(1-2x_3)$; otherwise, $r_{\mathrm{G}} > \left(\frac{1-x_3}{1/2 - x_3}\right) r_{\mathrm{Y}}$. Now show that

$\pi_{\mathrm{VI2},T_2} - \pi_{\mathrm{VI42},T_1} > 0$.

If $r_{\mathrm{Y}} < \frac{4t}{\lambda}\left(\frac{1}{2} - x_3\right)^2$, then

$$\pi_{\mathrm{VI2},T_2} - \pi_{\mathrm{VI42},T_1} = \left[ V(\lambda) - t(1-x_3) - \frac{d}{\mu - (1-x_3)\lambda} + (1-x_3)\lambda r_{\mathrm{G}0} \right] - \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \lambda r_{\mathrm{Y}0} \right]$$

$$= -t\left(\frac{1}{2} - x_3\right)(1-2x_3) + \left[(1-x_3)\lambda r_{\mathrm{G}0} - \lambda r_{\mathrm{Y}0}\right]$$

$$> -t\left(\frac{1}{2} - x_3\right)(1-2x_3) + \left[(1-x_3)\lambda\left(r_{\mathrm{Y}0} + \frac{t}{\lambda}(1-2x_3)\right) - \lambda r_{\mathrm{Y}0}\right] \text{ since } r_{\mathrm{G}} > r_{\mathrm{Y}} + \frac{t}{\lambda}(1-2x_3)$$

$$= \frac{t}{2}(1-2x_3) - \lambda x_3 r_{\mathrm{Y}0}$$

$$> \frac{t}{2}(1-2x_3) - \lambda x_3\left[\frac{4t}{\lambda}\left(\frac{1}{2} - x_3\right)^2\right] \text{ since } r_{\mathrm{Y}} < \frac{4t}{\lambda}\left(\frac{1}{2} - x_3\right)^2$$

$$= t\left(\frac{1}{2} - x_3\right) - 4tx_3\left(\frac{1}{2} - x_3\right)^2 = t\left(\frac{1}{2} - x_3\right)\left(1 - 2x_3 + 4x_3^2\right) = t\left(\frac{1}{2} - x_3\right)\left[(1-x_3)^2 + 3x_3^2\right] > 0$$

If $r_{\mathrm{Y}} > \frac{4t}{\lambda}\left(\frac{1}{2} - x_3\right)^2$, then

$$\pi_{\mathrm{VI2},T_2} - \pi_{\mathrm{VI42},T_1} = \left[ V(\lambda) - t(1-x_3) - \frac{d}{\mu - (1-x_3)\lambda} + (1-x_3)\lambda r_{\mathrm{G}0} \right] - \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \lambda r_{\mathrm{Y}0} \right]$$

$$= -t\left(\frac{1}{2} - x_3\right)(1-2x_3) + \left[(1-x_3)\lambda r_{\mathrm{G}0} - \lambda r_{\mathrm{Y}0}\right]$$

$$> -t\left(\frac{1}{2} - x_3\right)(1-2x_3) + \left[(1-x_3)\lambda\left(\frac{1-x_3}{1/2 - x_3}\right) r_{\mathrm{Y}0} - \lambda r_{\mathrm{Y}0}\right] \text{ since } r_{\mathrm{G}} > \left(\frac{1-x_3}{1/2 - x_3}\right) r_{\mathrm{Y}}$$

$$= -t\left(\frac{1}{2} - x_3\right)(1-2x_3) + \lambda\left(\frac{1/2 - x_3 + x_3^2}{1/2 - x_3}\right) r_{\mathrm{Y}0}$$

$$> -t\left(\frac{1}{2} - x_3\right)(1-2x_3) + \lambda\left(\frac{1/2 - x_3 + x_3^2}{1/2 - x_3}\right)\left[\frac{4t}{\lambda}\left(\frac{1}{2} - x_3\right)^2\right] \text{ since } r_{\mathrm{Y}} > \frac{4t}{\lambda}\left(\frac{1}{2} - x_3\right)^2$$

$$= -t\left(\frac{1}{2} - x_3\right)(1-2x_3) + 4t\left(\frac{1}{2} - x_3 + x_3^2\right)\left(\frac{1}{2} - x_3\right)$$

$$= t\left(\frac{1}{2} - x_3\right)\left(1 - 2x_3 + 4x_3^2\right) = t\left(\frac{1}{2} - x_3\right)\left[\left(1 - x_3\right)^2 + 3x_3^2\right] > 0$$

Region 5: $\left(\frac{1 - x_3}{1/2 - x_3}\right) r_Y < r_G < r_Y + \frac{t}{\lambda}\left(1 - 2x_3\right)$

As we already know $\pi_{VI42,T_1} > \pi_{NVI4}$, we only need to compare $\pi_{VI41,T_2}$ to $\pi_{VI42,T_1}$.

$$\pi_{VI41,T_2} - \pi_{VI42,T_1} = \left[V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{G0} + \left(\frac{1}{2} - x_3\right)\lambda r_{Y0}\right] - \left[V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \lambda r_{Y0}\right]$$

$$= \frac{1}{2}\lambda r_{G0} - \left(\frac{1}{2} + x_3\right)\lambda r_{Y0} > \left(\frac{1 - x_3}{1 - 2x_3}\right)\lambda r_{Y0} - \left(\frac{1}{2} + x_3\right)\lambda r_{Y0} = \left[\frac{\left(1 - 2x_3\right)^2 + 2x_3}{2\left(1 - 2x_3\right)}\right]\lambda r_{Y0} > 0$$

Region 6: $r_Y + \frac{t}{\lambda}\left(1 - 2x_3\right) < r_G < \min\left\{2r_Y + \frac{t}{\lambda}\left(1 - 2x_3\right), \left(\frac{1 - x_3}{1/2 - x_3}\right) r_Y\right\}$

As we already know $\pi_{VI2,T_2} > \pi_{NVI4}$ when $r_G > r_Y + \frac{t}{\lambda}\left(1 - 2x_3\right)$, we only need to compare $\pi_{VI2,T_2}$ to $\pi_{VI41,T_1}$.

$$\pi_{VI2,T_2} - \pi_{VI41,T_1}$$

$$= \left[V(\lambda) - t\left(1 - x_3\right) - \frac{d}{\mu - \left(1 - x_3\right)\lambda} + \left(1 - x_3\right)\lambda r_{G0}\right] - \left[V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{Y0} + \left(\frac{1}{2} - x_3\right)\lambda r_{G0}\right]$$

$$= -t\left(\frac{1}{2} - x_3\right)\left(1 - 2x_3\right) + \frac{1}{2}\lambda\left(r_{G0} - r_{Y0}\right)$$

$$> -t\left(\frac{1}{2} - x_3\right)\left(1 - 2x_3\right) + \frac{1}{2}\lambda\left[r_{Y0} + \frac{t}{\lambda}\left(1 - 2x_3\right) - r_{Y0}\right]$$

$$= tx_3\left(1 - 2x_3\right) > 0$$

Region 7: $r_Y < r_G < \min\left\{r_Y + \frac{t}{\lambda}\left(1 - 2x_3\right), \left(\frac{1 - x_3}{1/2 - x_3}\right) r_Y\right\}$

$$\pi_{\mathrm{VI41},T_2} - \pi_{\mathrm{VI41},T_1}$$

$$= \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{\mathrm{G0}} + \left(\frac{1}{2} - x_3\right)\lambda r_{\mathrm{Y0}} \right] - \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{\mathrm{Y0}} + \left(\frac{1}{2} - x_3\right)\lambda r_{\mathrm{G0}} \right]$$

$$= \frac{1}{2}\lambda\left(r_{\mathrm{G0}} - r_{\mathrm{Y0}}\right) - \left(\frac{1}{2} - x_3\right)\lambda\left(r_{\mathrm{G0}} - r_{\mathrm{Y0}}\right) = x_3\lambda\left(r_{\mathrm{G0}} - r_{\mathrm{Y0}}\right) > 0$$

$$\pi_{\mathrm{VI41},T_2} - \pi_{\mathrm{NVI4}} = \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + \frac{1}{2}\lambda r_{\mathrm{G0}} + \left(\frac{1}{2} - x_3\right)\lambda r_{\mathrm{Y0}} \right] - \left[ V(\lambda) - \frac{t}{2} - \frac{d}{\mu - \lambda} + (1 - 2x_3)\lambda r_{\mathrm{Y0}} \right]$$

$$= \frac{1}{2}\lambda r_{\mathrm{G0}} - \left(\frac{1}{2} - x_3\right)\lambda r_{\mathrm{Y0}} > \left(\frac{1}{2} - x_3\right)\lambda r_{\mathrm{G0}} - \left(\frac{1}{2} - x_3\right)\lambda r_{\mathrm{Y0}} = \left(\frac{1}{2} - x_3\right)\lambda\left(r_{\mathrm{G0}} - r_{\mathrm{Y0}}\right) > 0$$

## Solution of Formulation (6-7) – Option NN3

In order to simplify the proofs in Chapter 6, we introduce the following notations.

Define $C_H \equiv \dfrac{d\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$

$$C_L \equiv \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

$$C_0 \equiv \frac{d(\lambda_H - \lambda_L)}{\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

$$C_1 \equiv \frac{(1-\alpha)d\left\{(\lambda_L - \alpha\lambda_H)\mu + \alpha\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]\right\}}{(\mu - \alpha\lambda_H)\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

$$C_2 \equiv \frac{d\lambda_L}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

$$C_3 \equiv \frac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

$$C_4 \equiv \frac{d\lambda_H}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

We know that $C_2 < C_3 < C_4 < C_H$ since $\mu > \lambda_H > \alpha\lambda_H + (1-\alpha)\lambda_L > \lambda_L$.

Next we show $C_1 < C_2$ as below.

$$C_2 - C_1 = \frac{d\lambda_L}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - \frac{(1-\alpha)d\left\{(\lambda_L - \alpha\lambda_H)\mu + \alpha\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]\right\}}{(\mu - \alpha\lambda_H)\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

$$\geq \frac{(1-\alpha)d\lambda_L}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - \frac{(1-\alpha)d\left\{(\lambda_L - \alpha\lambda_H)\mu + \alpha\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]\right\}}{(\mu - \alpha\lambda_H)\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

$$= \frac{\alpha(1-\alpha)d(\lambda_H - \lambda_L)}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} > 0$$

The intersection between $V_H = \frac{\lambda_H V_L}{\lambda_L} - \frac{d(\lambda_H - \lambda_L)}{\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} = \frac{\lambda_H V_L}{\lambda_L} - C_0$ and

$V_H = V_L + \frac{d\lambda_H}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} = V_L + C_4$ is

$$V_L = \frac{d\left\{\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]\lambda_L + \mu(\lambda_H - \lambda_L)\right\}}{(\lambda_H - \lambda_L)\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}.$$

The intersection between $V_H = \frac{\lambda_H V_L}{\lambda_L} - \frac{d(\lambda_H - \lambda_L)}{\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} = \frac{\lambda_H V_L}{\lambda_L} - C_0$ and

$V_H = \frac{d\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} = C_H$ is

$$V_L = \frac{d\left[\mu\lambda_H - (1-\alpha)(\lambda_H - \lambda_L)\lambda_L\right]}{\lambda_H\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} < C_H$$

since $\frac{d\left[\mu\lambda_H - (1-\alpha)(\lambda_H - \lambda_L)\lambda_L\right]}{\lambda_H\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - C_H$

$$= \frac{d\left[\mu\lambda_H - (1-\alpha)(\lambda_H - \lambda_L)\lambda_L\right]}{\lambda_H\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - \frac{d\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

$$= \frac{-(1-\alpha)\, d\, (\lambda_H - \lambda_L)\, \lambda_L}{\lambda_H \left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} < 0$$

Now I move on to solve Formulation **(6-7)** – Option NN3

In Formulation (6-7), from $(i)$, we get $F_{\mathrm{NN3}} \le V_H - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - \lambda_H p_{\mathrm{NN3}}$. From $(ii)$, we get $F_{\mathrm{NN3}} \le V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - \lambda_L p_{\mathrm{NN3}}$. We consider two cases:

**Case 1:** $p_{\mathrm{NN3}} \ge \dfrac{V_H - V_L}{\lambda_H - \lambda_L}$. Then

$V_H - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - \lambda_H p_{\mathrm{NN3}} \le V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - \lambda_L p_{\mathrm{NN3}}$. So constraint $(i)$ is binding, i.e., $F_{\mathrm{NN3}} = V_H - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - \lambda_H p_{\mathrm{NN3}}$. Substituting into the objective function gives $V_H - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - (1-\alpha)(\lambda_H - \lambda_L)\, p_{\mathrm{NN3}}$. The optimal solution is

$F^*_{\mathrm{NN3}} = -\dfrac{\lambda_L V_H}{\lambda_H - \lambda_L} + \dfrac{\lambda_H V_L}{\lambda_H - \lambda_L} - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$, $p^*_{\mathrm{NN3}} = \dfrac{V_H - V_L}{\lambda_H - \lambda_L}$, and

$$\pi^*_{\mathrm{NN3}} = \alpha V_H + (1-\alpha) V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}.$$

**Case 2:** $p_{\mathrm{NN3}} \le \dfrac{V_H - V_L}{\lambda_H - \lambda_L}$. Then

$V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - \lambda_L p_{\mathrm{NN3}} \le V_H - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - \lambda_H p_{\mathrm{NN3}}$. So constraint $(ii)$ is

binding, i.e., $F_{NN3} = V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - \lambda_L p_{NN3}$. Substituting into the objective function

gives $V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} + \alpha(\lambda_H - \lambda_L) p_{NN3}$.

**Case 21:** $\dfrac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} \geq \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$. The optimal solution is

$F^*_{NN3} = \dfrac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$, $p^*_{NN3} = \dfrac{V_H - V_L}{\lambda_H - \lambda_L}$, and

$\pi^*_{NN3} = \alpha V_H + (1-\alpha)V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$.

**Case 22:** $\dfrac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} \leq \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$. The optimal solution is $F^*_{NN3} = 0$,

$$p^*_{NN3} = \frac{1}{\lambda_L}\left[V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right]$$

(Since $\dfrac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} \leq \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$,

$$p^*_{NN3} = \frac{1}{\lambda_L}\left[V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right] \leq \frac{1}{\lambda_L}\left(V_L - \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L}\right) = \frac{V_H - V_L}{\lambda_H - \lambda_L}.)$$

$$\pi^*_{NN3} = \left[\frac{\alpha\lambda_H + (1-\alpha)\lambda_L}{\lambda_L}\right]\left[V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right].$$

The above cases can be summarized as:

**Case NN3_1:** If $\dfrac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} \geq \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$, i.e.,

$V_H \leq \dfrac{\lambda_H V_L}{\lambda_L} - \dfrac{d(\lambda_H - \lambda_L)}{\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} = \dfrac{\lambda_H V_L}{\lambda_L} - C_0$, $F^*_{NN3\_1} = \dfrac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$,

$p^*_{NN3\_1} = \frac{V_H - V_L}{\lambda_H - \lambda_L}$, $\pi^*_{NN3\_1} = \alpha V_H + (1-\alpha)V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$. The corresponding consumer surplus is

$$CS_{NN3\_1} = \alpha\left(V_H - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN3\_1} - \lambda_H p_{NN3\_1}\right) + (1-\alpha)\left(V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN3\_1} - \lambda_L p_{NN3\_1}\right) = 0.$$

Therefore the social welfare is

$$SW_{NN3\_1} = \pi^*_{NN3\_1} + CS_{NN3\_1} = \alpha V_H + (1-\alpha)V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}.$$

**Case NN3_2:** If $\frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} < \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$, i.e.,

$$V_H > \frac{\lambda_H V_L}{\lambda_L} - \frac{d(\lambda_H - \lambda_L)}{\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} = \frac{\lambda_H V_L}{\lambda_L} - C_0, \; F^*_{NN3\_2} = 0,$$

$$p^*_{NN3\_2} = \frac{1}{\lambda_L}\left[V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right], \; \pi^*_{NN3\_2} = \left[\frac{\alpha\lambda_H + (1-\alpha)\lambda_L}{\lambda_L}\right]\left[V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right].$$

The corresponding consumer surplus is

$$CS_{NN3\_2} = \alpha\left(V_H - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN3\_2} - \lambda_H p_{NN3\_2}\right) + (1-\alpha)\left(V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - F_{NN3\_2} - \lambda_L p_{NN3\_2}\right)$$

$$= \alpha V_H + (1-\alpha)V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - \left[\frac{\alpha\lambda_H + (1-\alpha)\lambda_L}{\lambda_L}\right]\left[V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right].$$

$$= \alpha\left\{\left[\frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right]\left(\frac{\lambda_H - \lambda_L}{\lambda_L}\right) - \left(\frac{\lambda_H V_L - \lambda_L V_H}{\lambda_L}\right)\right\}$$

Therefore the social welfare is

$$SW_{NN3\_2} = \pi^*_{NN3\_2} + CS_{NN3\_2} = \alpha V_H + (1-\alpha)V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}.$$

**Proof of Proposition 6-1 (The BSP's Preferred Pricing Structure Under Net Neutrality)**

Now we move on proving Proposition 6-1. Comparing the BSP's three options under net neutrality, we know $\pi^*_{NN1} = V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} = \pi^*_{NN2}$. Then we compare $\pi^*_{NN1}$ and $\pi^*_{NN2}$ to $\pi^*_{NN3}$.

If $\frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} \geq \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$, i.e., $V_H \leq \frac{\lambda_H V_L}{\lambda_L} - \frac{d(\lambda_H - \lambda_L)}{\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} = \frac{\lambda_H V_L}{\lambda_L} - C_0$,

$$\pi^*_{NN3} = \pi^*_{NN3\_1} = \alpha V_H + (1-\alpha)V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}.$$

Since $\alpha V_H + (1-\alpha)V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} > V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$, $\pi^*_{NN3\_1} > \pi^*_{NN1} = \pi^*_{NN2}$.

If $\frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} < \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$, i.e., $V_H > \frac{\lambda_H V_L}{\lambda_L} - \frac{d(\lambda_H - \lambda_L)}{\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} = \frac{\lambda_H V_L}{\lambda_L} - C_0$,

$$\pi^*_{NN3} = \pi^*_{NN3\_2} = \left[\frac{\alpha\lambda_H + (1-\alpha)\lambda_L}{\lambda_L}\right]\left[V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right]$$

$$> V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} = \pi^*_{NN1} = \pi^*_{NN2} \text{ since } \frac{\alpha\lambda_H + (1-\alpha)\lambda_L}{\lambda_L} > 1.$$

**Proof of Proposition 6-2 (The BSP's Preferred Pricing Structure Under No Net Neutrality)**

Comparing the BSP's three options under no net neutrality, we know

$$\pi^*_{\text{NNN2}} = \pi^*_{\text{NNN3}} = V_L - \frac{d}{\mu - \alpha\lambda_H}\left[\frac{\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right].$$

Then we compare $\pi^*_{\text{NNN2}}$ and $\pi^*_{\text{NNN3}}$ to $\pi^*_{\text{NNN1}}$.

If $V_H - V_L \geq \dfrac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$, i.e.,

$$V_H \geq V_L + \frac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} = V_L + C_3, \text{ then}$$

$$\pi^*_{\text{NNN2}} - \pi^*_{\text{NNN1\_1}} = V_L - \frac{d}{\mu - \alpha\lambda_H}\left[\frac{\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right] - V_L + \frac{d}{\mu - (1-\alpha)\lambda_L}$$

$$= -\frac{d}{\mu - \alpha\lambda_H}\left[\frac{\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right] + \frac{d}{\mu - (1-\alpha)\lambda_L}$$

$$= \frac{-\alpha d(1-\alpha)(\lambda_H - \lambda_L)\left[\mu - (1-\alpha)\lambda_L\right] - \alpha d\lambda_H(\mu - \alpha\lambda_H)}{(\mu - \alpha\lambda_H)\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} < 0.$$

So $\pi^*_{\text{NNN1\_1}} > \pi^*_{\text{NNN2}} = \pi^*_{\text{NNN3}}$.

If $V_H - V_L < \dfrac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$, i.e.,

$$V_H < V_L + \frac{d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} = V_L + C_3, \text{ then}$$

$$\pi^*_{\text{NNN2}} - \pi^*_{\text{NNN1\_2}} = V_L - \frac{d\left[\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L\right]}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - V_H + \frac{d\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

$$= -(V_H - V_L) + \frac{d\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - \frac{d\left[\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L\right]}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

$$= -(V_H - V_L) + \frac{(1-\alpha)\, d\left\{(\lambda_L - \alpha\lambda_H)\,\mu + \alpha\lambda_L\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]\right\}}{(\mu - \alpha\lambda_H)\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} = -(V_H - V_L) + C_1 .$$

Since $C_1 < C_3$, we get if $V_H < V_L + C_1$, $\pi^*_{\mathrm{NNN1\_2}} < \pi^*_{\mathrm{NNN2}} = \pi^*_{\mathrm{NNN3}}$; If $V_L + C_1 \le V_H < V_L + C_3$,

$\pi^*_{\mathrm{NNN1\_2}} > \pi^*_{\mathrm{NNN2}} = \pi^*_{\mathrm{NNN3}}$.

Summarizing the results, we have: if $V_H \ge V_L + C_3$, $\pi^*_{\mathrm{NNN1}} = \pi^*_{\mathrm{NNN1\_1}} > \pi^*_{\mathrm{NNN2}} = \pi^*_{\mathrm{NNN3}}$; if

$V_L + C_1 \le V_H < V_L + C_3$, $\pi^*_{\mathrm{NNN1}} = \pi^*_{\mathrm{NNN1\_2}} > \pi^*_{\mathrm{NNN2}} = \pi^*_{\mathrm{NNN3}}$; if $V_H < V_L + C_1$,

$\pi^*_{\mathrm{NNN1}} = \pi^*_{\mathrm{NNN1\_2}} < \pi^*_{\mathrm{NNN2}} = \pi^*_{\mathrm{NNN3}}$.

**Proof of Proposition 6-3 (The BSP's Overall Preferred Pricing Structure)**

From the results of Proposition 6-1 and Proposition 6-2, we know: under net neutrality,

If $V_H \le \dfrac{\lambda_H V_L}{\lambda_L} - C_0$, $\pi^*_{\mathrm{NN}} = \pi^*_{\mathrm{NN3\_1}} = \alpha V_H + (1-\alpha)V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$.

If $V_H > \dfrac{\lambda_H V_L}{\lambda_L} - C_0$, $\pi^*_{\mathrm{NN}} = \pi^*_{\mathrm{NN3\_2}} = \left[\dfrac{\alpha\lambda_H + (1-\alpha)\lambda_L}{\lambda_L}\right]\left[V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right]$.

Under no net neutrality, If $V_H \ge V_L + C_3$, then $\pi^*_{\mathrm{NNN}} = \pi^*_{\mathrm{NNN1\_1}} = V_L - \dfrac{d}{\mu - (1-\alpha)\lambda_L}$;

If $V_L + C_1 \le V_H < V_L + C_3$, $\pi^*_{\mathrm{NNN}} = \pi^*_{\mathrm{NNN1\_2}} = V_H - \dfrac{d\,\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$;

If $V_H < V_L + C_1$, $\pi^*_{\mathrm{NNN}} = \pi^*_{\mathrm{NNN2}} = \pi^*_{\mathrm{NNN3}} = V_L - \dfrac{d\left[\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L\right]}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$.

Considering the BSP's overall preference, there are six cases:

**Case 1:** If $V_H \le \dfrac{\lambda_H V_L}{\lambda_L} - C_0$ and $V_H \ge V_L + C_3$, then

$$\pi^*_{NNN} - \pi^*_{NN} = \pi^*_{NNN1\_1} - \pi^*_{NN3\_1} = \left[ V_L - \frac{d}{\mu - (1-\alpha)\lambda_L} \right] - \left[ \alpha V_H + (1-\alpha) V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} \right]$$

$$= -\alpha (V_H - V_L) + \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} - \frac{d}{\mu - (1-\alpha)\lambda_L}$$

$$= -\alpha (V_H - V_L) + \frac{\alpha d \lambda_H}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

If $V_L + C_4 = V_L + \dfrac{d\lambda_H}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} \le V_H \le \dfrac{\lambda_H V_L}{\lambda_L} - C_0$, then $\pi^*_{NNN1\_1} \le \pi^*_{NN3\_1}$.

If $V_L + C_3 \le V_H < V_L + C_4$ and $V_H \le \dfrac{\lambda_H V_L}{\lambda_L} - C_0$, then $\pi^*_{NNN1\_1} > \pi^*_{NN3\_1}$.

**Case 2:** If $V_H \le \dfrac{\lambda_H V_L}{\lambda_L} - C_0$ and $V_L + C_1 \le V_H < V_L + C_3$, then $\pi^*_{NNN} - \pi^*_{NN} = \pi^*_{NNN1\_2} - \pi^*_{NN3\_1}$

$$= \left[ V_H - \frac{d\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} \right] - \left[ \alpha V_H + (1-\alpha) V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} \right]$$

$$= (1-\alpha)(V_H - V_L) - \frac{d\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} + \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$$

$$= (1-\alpha)(V_H - V_L) - \frac{(1-\alpha) d \lambda_L}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

If $V_L + C_2 = V_L + \dfrac{d\lambda_L}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} \le V_H < V_L + C_3$ and $V_H \le \dfrac{\lambda_H V_L}{\lambda_L} - C_0$, then

$\pi^*_{NNN1\_2} \ge \pi^*_{NN3\_1}$.

If $V_L + C_1 \le V_H < V_L + C_2$ and $V_H \le \dfrac{\lambda_H V_L}{\lambda_L} - C_0$, then $\pi^*_{NNN1\_2} < \pi^*_{NN3\_1}$.

**Case 3:** If $V_H \le \dfrac{\lambda_H V_L}{\lambda_L} - C_0$ and $V_H < V_L + C_1$, then $\pi^*_{NNN} - \pi^*_{NN} = \pi^*_{NNN2} - \pi^*_{NN3\_1}$

$$= \left[ V_L - \frac{d\left[\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L\right]}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} \right] - \left[ \alpha V_H + (1-\alpha) V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} \right]$$

$$= -\alpha\left(V_H - V_L\right) - \frac{d\left[\mu - \alpha^2\lambda_H - \alpha\left(1-\alpha\right)\lambda_L\right]}{\left(\mu - \alpha\lambda_H\right)\left[\mu - \alpha\lambda_H - \left(1-\alpha\right)\lambda_L\right]} + \frac{d}{\mu - \alpha\lambda_H - \left(1-\alpha\right)\lambda_L}$$

$$= -\alpha\left(V_H - V_L\right) - \frac{\alpha\left(1-\alpha\right)d\left(\lambda_H - \lambda_L\right)}{\left(\mu - \alpha\lambda_H\right)\left[\mu - \alpha\lambda_H - \left(1-\alpha\right)\lambda_L\right]} < 0 .$$

So $\pi^*_{\mathrm{NN3\_1}} > \pi^*_{\mathrm{NNN2}}$.

**Case 4:** If $V_H > \frac{\lambda_H V_L}{\lambda_L} - C_0$ and $V_H \geq V_L + C_3$, then $\pi^*_{\mathrm{NNN}} - \pi^*_{\mathrm{NN}} = \pi^*_{\mathrm{NNN1\_1}} - \pi^*_{\mathrm{NN3\_2}}$

$$= \left[V_L - \frac{d}{\mu - \left(1-\alpha\right)\lambda_L}\right] - \left[\frac{\alpha\lambda_H + \left(1-\alpha\right)\lambda_L}{\lambda_L}\right]\left[V_L - \frac{d}{\mu - \alpha\lambda_H - \left(1-\alpha\right)\lambda_L}\right]$$

$$= \frac{-\alpha\left(\lambda_H - \lambda_L\right)V_L}{\lambda_L} + \frac{\alpha d\left\{\left[\alpha\lambda_H + \left(1-\alpha\right)\lambda_L\right]\lambda_L + \mu\left(\lambda_H - \lambda_L\right)\right\}}{\lambda_L\left[\mu - \left(1-\alpha\right)\lambda_L\right]\left[\mu - \alpha\lambda_H - \left(1-\alpha\right)\lambda_L\right]}$$

If $V_H > \frac{\lambda_H V_L}{\lambda_L} - C_0$, $V_H \geq V_L + C_3$, and $V_L \geq \frac{d\left\{\left[\alpha\lambda_H + \left(1-\alpha\right)\lambda_L\right]\lambda_L + \mu\left(\lambda_H - \lambda_L\right)\right\}}{\left(\lambda_H - \lambda_L\right)\left[\mu - \left(1-\alpha\right)\lambda_L\right]\left[\mu - \alpha\lambda_H - \left(1-\alpha\right)\lambda_L\right]}$, then

$\pi^*_{\mathrm{NNN1\_1}} \leq \pi^*_{\mathrm{NN3\_2}}$. Notice $V_L = \frac{d\left\{\left[\alpha\lambda_H + \left(1-\alpha\right)\lambda_L\right]\lambda_L + \mu\left(\lambda_H - \lambda_L\right)\right\}}{\left(\lambda_H - \lambda_L\right)\left[\mu - \left(1-\alpha\right)\lambda_L\right]\left[\mu - \alpha\lambda_H - \left(1-\alpha\right)\lambda_L\right]}$ is also where the

line $V_H = \frac{\lambda_H V_L}{\lambda_L} - \frac{d\left(\lambda_H - \lambda_L\right)}{\lambda_L\left[\mu - \alpha\lambda_H - \left(1-\alpha\right)\lambda_L\right]} = \frac{\lambda_H V_L}{\lambda_L} - C_0$ intersects the line

$$V_H = V_L + \frac{d\lambda_H}{\left[\mu - \left(1-\alpha\right)\lambda_L\right]\left[\mu - \alpha\lambda_H - \left(1-\alpha\right)\lambda_L\right]} = V_L + C_4 .$$

If $V_H > \frac{\lambda_H V_L}{\lambda_L} - C_0$, $V_H \geq V_L + C_3$, and $V_L < \frac{d\left\{\left[\alpha\lambda_H + \left(1-\alpha\right)\lambda_L\right]\lambda_L + \mu\left(\lambda_H - \lambda_L\right)\right\}}{\left(\lambda_H - \lambda_L\right)\left[\mu - \left(1-\alpha\right)\lambda_L\right]\left[\mu - \alpha\lambda_H - \left(1-\alpha\right)\lambda_L\right]}$, then

$\pi^*_{\mathrm{NNN1\_1}} > \pi^*_{\mathrm{NN3\_2}}$.

**Case 5:** If $V_H > \frac{\lambda_H V_L}{\lambda_L} - C_0$ and $V_L + C_1 \le V_H < V_L + C_3$, then $\pi^*_{NNN} - \pi^*_{NN} = \pi^*_{NNN1\_2} - \pi^*_{NN3\_2}$

$$= \left[ V_H - \frac{d\,\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} \right] - \left[ \frac{\alpha\lambda_H + (1-\alpha)\lambda_L}{\lambda_L} \right] \left[ V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} \right]$$

$$= \frac{\lambda_L V_H - \left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]V_L}{\lambda_L} + \frac{\alpha\, d\,\mu\,(\lambda_H - \lambda_L) - (1-\alpha)\, d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]\lambda_L}{\lambda_L\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$$

If $\lambda_L V_H - \left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]V_L \ge \frac{\alpha\, d\,\mu\,(\lambda_H - \lambda_L) - (1-\alpha)\, d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]\lambda_L}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$, i.e.,

$V_H \ge \frac{\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]V_L}{\lambda_L} + \frac{\alpha\, d\,\mu\,(\lambda_H - \lambda_L) - (1-\alpha)\, d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]\lambda_L}{\lambda_L\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$, then

$\pi^*_{NNN1\_2} \ge \pi^*_{NN3\_2}$.

If $\lambda_L V_H - \left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]V_L < \frac{\alpha\, d\,\mu\,(\lambda_H - \lambda_L) - (1-\alpha)\, d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]\lambda_L}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$, i.e.,

$V_H < \frac{\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]V_L}{\lambda_L} + \frac{\alpha\, d\,\mu\,(\lambda_H - \lambda_L) - (1-\alpha)\, d\left[\alpha\lambda_H + (1-\alpha)\lambda_L\right]\lambda_L}{\lambda_L\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}$, then

$\pi^*_{NNN1\_2} < \pi^*_{NN3\_2}$.

**Case 6:** If $V_H > \frac{\lambda_H V_L}{\lambda_L} - C_0$ and $V_H < V_L + C_1$, then $\pi^*_{NNN} - \pi^*_{NN} = \pi^*_{NNN2} - \pi^*_{NN3\_2}$

$$= \left[ V_L - \frac{d\left[\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L\right]}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} \right] - \left[ \frac{\alpha\lambda_H + (1-\alpha)\lambda_L}{\lambda_L} \right] \left[ V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} \right]$$

$$< \left[ V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} \right] - \left[ \frac{\alpha\lambda_H + (1-\alpha)\lambda_L}{\lambda_L} \right] \left[ V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} \right]$$

$$< \left[ V_L - \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L} \right] \left[ 1 - \frac{\alpha\lambda_H + (1-\alpha)\lambda_L}{\lambda_L} \right] < 0$$

So $\pi^*_{NN3\_2} > \pi^*_{NNN2}$.

**Proof of Proposition 6-5 (The Social Planner's Preferred Pricing Structure Under No Net Neutrality)**

Comparing social welfare under NNN1, NNN2, and NNN3 gives

$$SW_{NNN2} = SW_{NNN3} = \alpha\left(V_H - \frac{d}{\mu - \alpha\lambda_H}\right) + (1-\alpha)\left(V_L - \frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}\right).$$

Since $SW_{NNN1\_1} = SW_{NNN1\_2}$

$$= \alpha\left(V_H - \frac{d\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}\right) + (1-\alpha)\left(V_L - \frac{d}{\mu - (1-\alpha)\lambda_L}\right),$$

$$SW_{NNN2} - SW_{NNN1\_1} = \alpha\left(\frac{d\mu}{\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - \frac{d}{\mu - \alpha\lambda_H}\right)$$

$$-(1-\alpha)\left(\frac{d\mu}{(\mu - \alpha\lambda_H)\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]} - \frac{d}{\mu - (1-\alpha)\lambda_L}\right)$$

$$= \frac{d}{\mu - (1-\alpha)\lambda_L}\left[\frac{\mu - \alpha(1-\alpha)\lambda_H - (1-\alpha)^2\lambda_L}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right] - \frac{d}{\mu - \alpha\lambda_H}\left[\frac{\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\right]$$

$$= \frac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}\left[\frac{\mu - \alpha(1-\alpha)\lambda_H - (1-\alpha)^2\lambda_L}{\mu - (1-\alpha)\lambda_L} - \frac{\mu - \alpha^2\lambda_H - \alpha(1-\alpha)\lambda_L}{\mu - \alpha\lambda_H}\right]$$

$$= \frac{-\alpha(1-\alpha)d(\lambda_H - \lambda_L)\left[2\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]}{\left[\mu - \alpha\lambda_H - (1-\alpha)\lambda_L\right]\left[\mu - (1-\alpha)\lambda_L\right]\left[\mu - \alpha\lambda_H\right]} < 0$$

Therefore $SW_{NNN1} > SW_{NNN2} = SW_{NNN3}$.

**Proof of Proposition 6-6 (The Social Planner's Overall Preferred Pricing Structure)**

Recall $SW_{NN1} = SW_{NN2} = SW_{NN3} = \alpha V_H + (1-\alpha)V_L - \dfrac{d}{\mu - \alpha\lambda_H - (1-\alpha)\lambda_L}$ and

$$SW_{NNN1} = \alpha\left(V_H - \frac{d\mu}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}\right) + (1-\alpha)\left(V_L - \frac{d}{\mu-(1-\alpha)\lambda_L}\right)$$

$$> SW_{NNN2} = SW_{NNN3} = \alpha\left(V_H - \frac{d}{\mu-\alpha\lambda_H}\right) + (1-\alpha)\left(V_L - \frac{d\mu}{(\mu-\alpha\lambda_H)\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}\right)$$

Since $SW_{NN1} - SW_{NNN2} = \frac{\alpha d}{\mu-\alpha\lambda_H} + \frac{(1-\alpha)d\mu}{(\mu-\alpha\lambda_H)\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]} - \frac{d}{\mu-\alpha\lambda_H-(1-\alpha)\lambda_L}$

$$= \frac{\alpha d}{\mu-\alpha\lambda_H} - \frac{\alpha d(\mu-\lambda_H)}{(\mu-\alpha\lambda_H)\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]} = \frac{\alpha(1-\alpha)d(\lambda_H-\lambda_L)}{(\mu-\alpha\lambda_H)\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]} > 0,$$

$$SW_{NN1} - SW_{NNN1} = \frac{\alpha d\mu}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]} + \frac{(1-\alpha)d}{\mu-(1-\alpha)\lambda_L} - \frac{d}{\mu-\alpha\lambda_H-(1-\alpha)\lambda_L}$$

$$= \frac{(1-\alpha)d}{\mu-(1-\alpha)\lambda_L} - \frac{(1-\alpha)d(\mu-\lambda_L)}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]} = \frac{-\alpha(1-\alpha)d(\lambda_H-\lambda_L)}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]} < 0,$$

$$SW_{NNN1} > SW_{NN1} = SW_{NN2} = SW_{NN3} > SW_{NNN2} = SW_{NNN3}.$$

**Proof of Proposition 6-7 (The BSP's Deviation from the Social Optimum)**

From Proposition 6-6, we know that the social planner always prefers NNN1. From Proposition 6-3, we know that under the following two scenarios: (1)

$V_H > V_L + \frac{d\lambda_H}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}$ and

$V_L > \frac{d\left\{\left[\alpha\lambda_H+(1-\alpha)\lambda_L\right]\lambda_L+\mu(\lambda_H-\lambda_L)\right\}}{(\lambda_H-\lambda_L)\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}$; (2) $V_H \le \frac{\lambda_H V_L}{\lambda_L} - \frac{d(\lambda_H-\lambda_L)}{\lambda_L\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}$

and $V_L < V_H < V_L + \frac{d\lambda_L}{\left[\mu-(1-\alpha)\lambda_L\right]\left[\mu-\alpha\lambda_H-(1-\alpha)\lambda_L\right]}$, the BSP prefers NN3 and therefore the BSP would deviate from the social planner's preferred option.

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