# IN SEARCH OF A MATHEMATICS DISCOURSE MODEL: CONSTRUCTING MATHEMATICS KNOWLEDGE THROUGH ONLINE DISCUSSION FORUMS

By

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To my husband, Arturo Bird-Carmona, and our children, Maniel Bird-Ortiz and Nianti Bird-Ortiz. Their constant support and validation have been invaluable.

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TABLE OF CONTENTS

LIST OF TABLES

## LIST OF FIGURES

Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

IN SEARCH OF A MATHEMATICS DISCOURSE MODEL: CONSTRUCTING MATHEMATICS KNOWLEDGE THROUGH ONLINE DISCUSSION FORUMS

By

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The purpose of our study was to examine how participants of a public online mathematics discussion forum collaborated, negotiated, and generated new meaning and understanding through dialogue, intertextuality and polyvocality while constructing undergraduate mathematics knowledge. Our study was conducted under Kenneth Gergen's theoretical perspective of social constructionism and the methodology and methods proposed by J. P. Gee. Analysis of data included five months of threaded discussions divided into three periods of analysis which gave the researcher the opportunity to develop, review, and refine a preliminary mathematics discourse model.

Preliminary mathematics discourse models showed how participants engaged in dialogues that included specific activities and connections. Data also showed how participants used common language and mathematical symbols to communicate, the state of mind of the forum participants (social identity), the relationships they engaged in, and how they resolved their mathematical questions, problems, and inquiries.

# CHAPTER 1
# INTRODUCTION

> Only dialogue, which requires critical thinking, is also capable of generating critical thinking. Without dialogue there is no communication, and without communication there can be no true education.
>
> –Paulo Freire, *Pedagogy of the Oppressed* (1970, p. 81)

Teaching and learning is going through a continuous process of change that since the twentieth century can be thought of as a digital revolution (Roblyer, 2003). This revolution has influenced schools and colleges as well, including the teaching and learning of mathematics at all levels. However, this digital revolution is not a panacea in education. The impact on education of the use of computers, networks, and the Internet has been widely criticized (Oppenheimer, 1997, 2003; Bains, 1997; Postman, 1992, 1995; and Stoll, 2000). Nevertheless, it is research in theory and practice which can document the integration of new technologies into the educational environment, while helping to answer many of the questions posed by critics and teachers.

Changes in education are not new; they have occurred throughout history. In the 1800's, for example, teaching and learning incorporated the use of correspondence courses in addition to the existing traditional face-to-face education. Later, in the twentieth century, digital alternatives became available to society and rapidly made their way into the educational setting. *The World Wide Learn* (2005) indicated that digital environments in education continue to increase in every subject matter. The use of computers, networks, and the Internet has changed schools and colleges environment as well, allowing for more student-centered activities (Knowlton, 2000), different types of interaction (Moore, 1989; Gunawardena, Lowe, & Anderson, 1997), and different types of communication (Morrison & Guenther, 2000; Berge, 2000; Weiss, 2000; and Hacker & Niederhauser, 2000). Research by the Alliance for Higher Education Competitiveness has indicated that quality in online learning experiences is about teaching strategies and not technology (Abel, 2005). That is, software and hardware acquisition is not enough; "inservice computer training that focuses on classroom integration technology strategies" (Dawson, 1998, p.2), as well as research about formal and informal communities of learners, can make a difference in education.

New technological resources in communication and networking provide students and teachers with the opportunity to study, explore, collaborate, and investigate while joining together in virtual communities of learners. Students and teachers can communicate regularly via synchronous and asynchronous electronic tools that enable sending and receiving feedback in a timely fashion (Berge, 2000; Wheeler, 2002). Together, they become part of a community of learners (Lock, 2002) in which the distance from the educational institution to the students' location is no longer a limitation to acquiring and constructing new knowledge (Simonson, Smaldino, Albright, & Zvacek, 2003).

Virtual communities include people from different places and countries where "spatial and temporal boundaries are entirely symbolic" (Shumar & Renninger, 2002, p. 6). These communities are created and maintained through the use of synchronous and asynchronous communication tools (Lock, 2002) that help develop environments sustained by the interaction of their members. Synchronous communication tools are used at the same time from different places; examples of such include the use of chat rooms and videoconferences (Simonson, Smaldino, Albright, & Zvacek, 2003). Asynchronous communication occurs at different times and different places and includes tools such as email, discussion forums, and list-serves (Simonson, Smaldino, Albright, & Zvacek, 2003).

Gilbert and Moore (1998) and Wagner (1994, 1997) defined interaction as an interplay and exchange of ideas in which individuals and groups collaborate with each other (as cited in Roblyer & Ekhaml, 2000). Different types of interactions include learner-content, learner-instructor, learner-learner, (Moore, 1989), and learner-interface (Hillman, Willis, & Gunawardena, 1994). Researchers suggested that through interaction, it is possible to promote “critical thinking, higher-order thinking, distributed thinking, and constructive thinking (Berge & Muilenburg, 2001)” (as cited in Tu and Corry, 2003, p. 309) to provide an emotional, supportive space in which to share and find information (Stein & Glazer, 2003), to develop a sense of being self-directed while taking responsibility for learning (Lee & Gibson, 2003), and to interact with others while constructing knowledge beyond independent means (Vygotsky, 1978).

An asynchronous online communication tool, the discussion forum is a tool where people interested in the same topic or subject matter can interact with each other by asking questions, giving answers, and clarifying ideas. It is through this interaction that users develop a reciprocal communication system, one “that emphasize[s] learner developments in cognition, motivation, and social advancement” (Chou, 2004, p. 11). This process allows discussion forum participants to develop new skills and construct new knowledge (Hacker & Niederhauser, 2000). Gilbert and Moore (1998) also stated that social rapport and collaboration could lead to greater levels of interaction (as stated in Roblyer, 2000). In this way, a cyclical relationship takes place between collaboration and increased interaction and vice-versa.

Discussion forums can be classified as private or public digital spaces. The latter are not password protected or restricted for members of a specific organization or institution. Anyone with access to an Internet connection can log into a public discussion forum to post a message at any time and from any place. It is this flexibility that allows participants to contribute to and to be part of a larger community of learners.

An example of such a public environment is *The Math Forum @ Drexel*. This electronic space began as “a project to produce computer-generated videotapes” (Dawning, 1997, first paragraph) and later developed into the Geometry Forum (in 1992). This dynamic community continued to grow “under the leadership of Swarthmore College in Pennsylvania” (Grandgenett, 2001). Students, teachers, researches, parents, and math enthusiasts can benefit of the tools provided by this environment. It promotes interaction between these groups through the availability of chat rooms and discussion forums. It also houses bibliographies related to math education for teachers and researchers, as well as journal abstracts, articles, and other teaching and learning resources.

Research about the use of discussion forums in mathematics education is scarce and contradictory. Thus this study tries to clarify how knowledge construction takes place in such environments. To do so it examines writing in online asynchronous learning in combination with informal learning environments. That is, communities of practice where more knowledgeable others engage in collaborative practices with mathematics learners.

## Purpose

Our study connected research from four general areas of study: technology use in mathematics, writing in mathematics, communication in mathematics learning, and online communities of practice. Specifically, written communications in an online public discussion forum were examined to describe and analyze how mathematics knowledge was constructed.

The purpose of this study was to examine the types of online dialogues and discursive collaboration that took place in an online public discussion forum that facilitated the construction of mathematics knowledge, meaning making, and understanding. Construction of knowledge was defined from a social constructionist perspective (Chapter 3). Data analysis was based on Gee’s (1999, 2005) discourse analysis methods (Chapter 4).

## Significance of the Study

The use of discussion forums has been researched in terms of their relationship to distance learning, online courses, and blended courses (a combination of face-to-face learning and online courses) (Wegerif, 1998; Pérez-Prado and Thirunarayanan, 2002; Kanuka, Collet, & Caswell, 2002; Da Silva, 2003; Lee & Gibson, 2003; Woods & Ebersole, 2003; Knowlton, 2003; Schallert, et al., 2003-2004; Ferdig and Roehler, 2003-2004; Jetton, 2003-2004; Im & Lee, 2003-2004). However, most research studies focus on subject matter that is traditionally considered language-based.

Only a few studies have looked at the use of discussion forums in mathematics education. For example, Sener (1997) examined the use of asynchronous environments in a distance education science engineering degree that included mathematics courses. Sliva (2002) looked at an online discussion forum in a hybrid mathematics elementary methods course. Smith, Ferguson, and Caris (2003) studied mathematics instructors' experiences at the undergraduate college level in an online discussion-based environment. Lotze (2002) analyzed tutoring effectiveness. Bolin (2003) studied different communication tools and their impact in mathematics learning. Quinn (2005) looked at mathematics identity. Nevertheless, none of these studies examined the types of dialogue and discursive collaboration occurring in an online public discussion forum. Nor had they examined how participants co-constructed knowledge about high school mathematics, and first and second year undergraduate mathematics, or at how informal mathematics learning could contribute to knowledge construction. These studies did not analyze empirical accounts of discursive collaboration, where relationships between people assuming different roles are described (Satwicz & Stevens, 2008).

Sharma and Hannifin (2004) have also identified the need for further research about mathematics and technology in informal environments. They stated,

> Exploring the impact of different technologies and tools used by learners to develop their learning in informal environments can allow for more systematic and controlled introduction of informal technologies into formal educational settings. Equally important, such exploration may allow identification of appropriate combinations of structured scaffolding and less structured learning activities to support different types of learning. (p. 204)

In this way, this study extended the research base beyond language-based courses, beyond the point of view of the instructors, beyond formal learning, and beyond prescriptive research. Finally, this research could have implications on mathematics distance education, not only for undergraduate mathematics, but also for mathematics courses offered through virtual schools environments. As Sakshaug (2000) concluded, “Distance education is a growing area of instructional delivery. More schools are offering distance education courses than ever before. Mathematics educators must explore this venue of teaching in order to have input about what and how mathematics is taught” (p. 122). Nevertheless, few researches have been conducted in this area.

One way to contribute to these needs is to research a public digital space such as *The Math Forum @ Drexel* where a community of learners already exists and where mathematics knowledge is already being constructed through discussion forums (Renninger & Shumar, 2002). Such a study can inform communication and community building as well as informal learning in mathematics. In the *Math Forum @ Drexel,* participants use text to pose questions and answers, give feedback, recommend resources, and find solutions. As its participants interact with each other in a text-based environment, they become active learners, reflect about the problems posed, take time to reply, and collaborate with each other to develop new knowledge. In the *Math Forum @ Drexel,* users appear to be able to communicate effectively and to help each other develop new meanings and understanding. This study will examine how this happens.

## Research Question

This research will study the following question: How does online dialogue and discursive collaboration facilitate the co-construction of knowledge in high school and first and second year undergraduate mathematics via a discussion forum named *alt.math.undergrad* in the *Math Forum @ Drexel's* web site? This study analyzed digital archived data, a collection of primary sources organized as threaded discussion, available through the Web (Bolick, 2006). It examined how participants in the discussion forum constructed mathematics knowledge during the academic semester of August through December of 2004.

## Definition of Terms

- **Discussion forums:** asynchronous online communication tools where a group of people interested in the same topic or subject matter interact with each other by posting questions, answers, or both (Simonson, Smaldino, Albright, & Zvacek, 2003).

- **Public discussion forum:** discussion forums that are accessed freely through the Internet.

- **Community of learners:** group of people who share a common goal or objective and help each other to fulfill this goal (Lave & Wenger, 1991; Rogoff, 1994).

- **High school and first and second year undergraduate mathematics courses:** mathematics courses geared toward freshman and sophomore students, including General Math, College Algebra, Trigonometry, Elementary Statistics, Pre-Calculus, Calculus I, and Discrete Mathematics.

- **Voluntary participation:** self-initiated posting by help-seeking individuals or more knowledgeable others of a written contribution to the discussion forum that can include but is not limited to asking or answering a question, giving a hint, or adding comments.

- **Discursive collaboration:** when a group of people create meaning together through language to develop shared meaning (Satwicz & Stevens, 2008). According to Maye (2003), collaboration may take place in voluntary study groups and in consultation with professors, teaching assistants, and tutors. In this study, collaboration takes place in an online public discussion forum between voluntary participants.

- **Social constructionism of knowledge:** discourse that leads to meaning-making or generation of new understandings (Gergen, 1999). This research analyzes written discourse.

- **Informal learning environments:** learning spaces outside the classroom with no grades attached. According to Offer (2007), informal sources include peers, friends, and family members. However, in this research, sources may include more knowledgeable others that may or may not be known by help-seeking individuals.

## Delimitations

The following boundaries were established in order to conduct this research:

1. This study will analyze the postings to the public discussion forum "*alt.math.undergrad*" located in *The Math Forum @ Drexel* (http://www.mathforum.org).

2. "*alt.math.undergrad*" includes questions and answers from different mathematical topics, but only those threads related to high school and first and second year undergraduate mathematics as defined above will be examined.

3. The number of postings in each threaded discussion varies in terms of participation, from fewer than five to more than 100. In this research, threads with 10 to 25 posting were selected for analysis. Each thread is a discussion about a specific topic or question that can be subdivided into several conversations or stories. These conversations can have two or more postings each. A complete story can include six sections; these are setting, catalyst, crisis, evaluation, resolution, and coda (Gee, 2005).

4. The time period selected for study is one academic semester which took place from August to December of 2004.

5. Transcripts were developed by the researcher and reviewed by committee members.

# CHAPTER 2
# REVIEW OF LITERATURE

> Those who have studied the use of technology in mathematics teaching and learning have noted that technology mediates learning. That is, learning is different in presence of technology.
>
> –K. Heid, *Technology-Supported Mathematics Learning Environments* (2005, 67th NCTM Yearbook, p. 348)

Technology in education opened a new field in educational research studies which included analyzing and exploring the impact of audiovisual tools, instructional systems, vocational techniques, and computers on teaching and learning processes (Roblyer, 2003). However, research in educational technology is not about the tools or gadgets themselves (Clark, 1983; Kozma, 1991). Instead, it is about how both the old and the new can be used to develop meaning and understanding; it is about how these new tools and techniques can help students learn in a society that is always changing.

Learning environments are organized in different ways. One such distribution is presented by Rogoff (1994), where she summarized instructional approaches around three models: adult-run, child-run, and communities of learners. The first two organized learning activities from top-to-bottom and from bottom-up, respectively. In adult-run instruction, the teacher makes all the decisions, and the purpose of education is the transmission of knowledge. This type of education is based on behaviorist principles, one of its major proponents being B. F. Skinner (Lever-Duffy, McDonald, & Mizell, 2003). According to Rogoff (1994), the child-run model has children constructing knowledge on their own; adults only contribute to their learning by setting up learning environments. One of the proponents of this model was A. S. Neill, an English teacher who founded Summerhill, a school where children grew in liberty and learned as much as they wanted (Neill, 1963; Hemmings, 1975).

The third instructional model proposed and supported by Rogoff (1994), the communities of learners' model, is based on participatory theories, where learners collaborate with each other and where learning is the result of a process of interaction conducive to transformation. As Rogoff stated, this model is not a balance between adult-run and child-run instruction; it is a different type of model. In informal learning environments, more knowledgeable others support those in need of specific help. Roles are not static, however, and they will change at different times. According to Rogoff, Lave, and Wenger, however, informal learning environments can also promote the development of authentic communities of practice, where people with similar goals and interests interact and construct new knowledge (Rogoff & Lave, 1984; Lave, 1991, 1996; Lave & Wenger, 1991; Wenger, 2001; Rogoff, 1994). Even more, Resnick (1987) suggested there is a need to bring together formal learning (schooling based on individual performance) and informal learning (based on social interactions).

This dissertation investigated a community of learners made possible through the use of asynchronous communication, over the Internet, where informal interaction took place through written communication. In this environment, participants were located in different places and contributed at different times. The use of an asynchronous environment in mathematics learning was explored, specifically an online public discussion forum, where informal mathematics learning took place. This dissertation examined a text-based website, which allowed its users to construct mathematics knowledge through discourse while interacting, collaborating, and negotiating outside of the school setting. It focused on the generation of transformative written dialogue whose end result was the development of new mathematical understandings and knowledge. More specifically, this dissertation explored the use of an online public discussion forum as it related to high school and college first and second year undergraduate mathematics topics and the construction of knowledge developed through informal dialogues.

The next sections will present a review of related topics. These include (1) using technology in mathematics education, (2) writing in mathematics, (3) communication in mathematics learning, and (4) online communities of practice.

## Using Technology in Mathematics Education

The use of technological devices, gadgets, or machines in mathematics is as old as mathematics itself. From the use of pebbles to count to the use of digital technologies to visualize concepts and communicate ideas, technology has always played an important role in mathematics education (Dilson, 1968). The use of computers in mathematics dates from the second half of the twentieth century, becoming popular in schools in the 1970's and 1980's with mainframe computer systems and stand-alone personal computers used mostly for computer-assisted instruction (Roblyer, 2006). The advent of the Web at the end of the 1980's allowed its users in the following decade to share information and to communicate in synchronous and asynchronous environments in an easier way. These new Internet tools also permitted building virtual communities of learners throughout the world.

The following pages analyze research about the use of computers in mathematics teaching and learning. First, a series of articles by Crowe and Zand (2000a, 2000b, and 2001) and a master thesis by Morley (2007) researching the use of instructional technologies in mathematics are reviewed. As it will be shown in the following paragraphs, Crowe and Zand studied the use of computers from a comprehensive perspective and Morley developed a mathematics computer vision. Next, research by Samantha, Peressini, and Meymaris (2004) exploring effective learning environments in mathematics supported by technology are assessed. Lastly, Galindo's (2005) and Rose's (2001) studies showing how the Internet was used as a source of information and communication will be reviewed.

Crowe and Zand (2000a, 2000b, and 2001) researched and summarized computer use in mathematics by collecting data from British, American, and Australian universities. They developed a taxonomy of computer uses in mathematics that included three general categories: (1) the use of productivity tools (such as spreadsheets), (2) the use of general purpose mathematics software (such as *Mathlab*, *Maple*, and *Mathematica*), and (3) the use of information tools.

The taxonomy's second category was subdivided into two general subcategories: first, the use of didactic software packages, and second "doing" mathematics software. Under didactic software packages, Crowe and Zand included the use of multimedia and assessment. These were tools containing computer-assisted instruction software used to complement instruction. The second subcategory, "doing" mathematics software, included the use of programming languages, often associated with developing problem solving skills in mathematics (Papert, 1980). "Doing" mathematics software included the study of skills that allowed students to choose a particular procedure to solve a problem (Schoenfeld, 1989, as cited in Shield and Galbraith, 1998). This category also included geometric visualization and algebraic manipulation software, both generally used to help students conceptualize abstract ideas (Figure 2-1).

When analyzing information tools, Crowe and Zand (2000b) found that participants used three types of Internet resources: (1) "specific," including textual materials (books, digital libraries, tutorials, web pages), java modules (interactive demonstrations, investigative – dynamic- environments), and other media; (2) general, consisting of reference resources, journals, indices, and math web sites; and (3) dialogic communication tools, related to online support and online courses. These indicated that mathematics teaching and learning was possible through verbal communication (encouraging words and discourses), visual communication (graphics use and geometric representations), and the use of symbols (algebraic notation). This classification is similar to the one presented by Stonewater (2002), which he called the 'Rule of Three' for its graphic, numeric, and algebraic components.

Computer-written communication was reported as having more advantages than audio communication (Crowe & Zand, 2000b). According to these authors, the major advantage to computer-written communication is that it lets users have a permanent record of a dialogue, thus allowing them to revisit the same argument for different purposes as many times as needed. Written communication, they stated, could be saved in "a file, located, accessed, and edited in the same way as any other file" (p. 140). They also found that computer-written communication allowed for increased "speed of response" because students could receive prompt feedback. However, Crowe and Zand's investigation did not include the use of communication tools (synchronous or asynchronous) available through the web.

Crowe and Zand (2000b) concluded that "technology has a fundamental role to play, not only in teaching the present curriculum but also in shaping the curriculum of the future" (p. 146). This view was supported by Morley's (2007) mathematics technology vision. In her research, she concluded that "Using IT [instructional technology] in mathematics instruction can nurture positive attitudes toward mathematics while creating critical thinkers and lifelong learners" (p. 30). Morley contended that through IT, mathematical concepts could be studied through multiple representations and that IT could reduce the gap between mathematical concepts and real world data. As new technologies develop and old ones improve Crowe and Zand's conclusions seem to be disputed, still Morley's vision seems to stay the same.

Three examples of mathematics education research that concentrated on the use of technology to help students develop meaning in mathematics are those of Samantha Peressini, and Meymaris (2004), Galindo (2005), and Rose (2001). The first studied the use of calculators and computers in a learner-centered environment; the second two explored the use of the Internet as a source of information and collaboration.

According to Shamatha, Peressini, and Meymaris (2004), technology can be used to transform mathematics teaching and learning through the development of effective learning environments. These, they stated, included community-, learner-, knowledge-, and assessment-centered environments. They also agreed with Crowe and Zand's position regarding the use of technology in mathematics education when they concluded that "technology-supported activities can work meaningfully in a learning environment that research proves effective" (p. 378).

By taking a different angle, Galindo (2005) also explored the use of computers in mathematics education. He looked at how "the Internet is supporting and helping enhance the learning and teaching of mathematics" (p. 241). He listed different types of resources available on the Internet for teachers and students, including real-time data projects, existing data sets from different organizations, and collaborative projects. These resources, he stated, "can facilitate meaningful collaboration among individuals or groups to support mathematics learning" (Galindo, 2005, p. 251). He also added the possibilities of consulting with experts by submitting questions and engaging in conversations and collaborative interactions. Some collaborative efforts listed by Galindo included web sites such as *Conectando las Matemáticas a la Vida: Proyecto Internacional [Connecting Mathematics to Our Lives: International Project]* (http://www.orillas.org/math), *Class2Class* (http://www.mathforum.org/class2class), and *Global Schoolnet's Global Schoolhouse Organization* (http://www.globalschoolnet.org/GSH).

Rose (2001) went a step further, using the Internet as an information gathering resource in a college calculus course. Her goal was to increase classroom “dialogue, discourse, interaction, reflection, and writing about mathematics” (pp. 10-11) while students gathered information about how calculus was used in their particular majors. Students reported they were able to improve their skills in locating information not only about calculus but also about other mathematics topics. Rose also reported students were able to “speak of mathematics in a more positive nature and were able to find for themselves its connections to real-world situations . . . to their areas of study and/or personal lives” (Rose, 2001, p. ix).

Rose (2001), Galindo (2005), and Crowe and Zand (2000b), looked at the Internet as a source of information and collaborative effort. Galindo (2005) and Crowe and Zand (2000b) referenced *The Math Forum’s Ask Dr. Math* (http://mathforum.org/dr.math), a component of *The Math Forum* web site that concentrates on K-12 level problems. *Dr. Math* “is a question-and-answer service for mathematics students and their teachers” (Galindo, 2005, p. 254). None of these authors, however, looked at the *Discussion Forums* section of *The Math Forum @ Drexel* website, a place to “Read, post, browse, search, and subscribe to dozens of discussions, ranging in focus from AP courses and Investigations curricula to history, from policy and news to professional teaching associations, from Spanish puzzles to software, and more” (*The Math Forum @ Drexel*, 2006).

However, the Internet is also a place for formal mathematics learning. The increased number of courses and programs offered online has a direct impact on mathematics education. National reports have pointed to the increase of distance learning courses and programs, including those that include the teaching and learning of mathematics (Conference Board of Mathematical Sciences (Lutzer, 2000; The World Wide Learn, 2005). In its 2000 report, the

CBMS, an umbrella organization of professional mathematics organizations established in the 1960's, added a new section on distance learning and mathematics for the first time in its history. In addition, the World Wide Learn web site, a directory of higher education online programs, continuously adds new programs and offerings to its list of programs. In 2005, this web site listed more than 25 online programs related to mathematics offered in ten states around the United States. Associations such as the American Mathematical Association of Two-Year Colleges (AMATYC) and the Distance Learning Committee emphasized the importance of collaboration and communication in online education (AMATYC, 2005).

In summary, these authors examined the use of computers in mathematics education in different ways. They also looked at the Internet, including its tools and information repository as it related to mathematics learning. They set the groundwork for the integration of technology in mathematics education.

The next section will examine research about writing in mathematics. The introduction will examine the position taken by the National Council of Teachers of Mathematics. Then, different communication domains in mathematics and mathematics research about cognitive development and writing are examined. To end this section, research studies about writing in mathematics using digital and non-digital one-to-one and one-to-many environments are reviewed.

## Writing in Mathematics

Writing in mathematics is an outcome of the 1960's W*riting Across the Curriculum* movement (Clarke & Waywood, 1993, p. 235). As a result of it, the National Council of Teachers of Mathematics (NCTM), after a major revision of the mathematics curriculum during the 1980's, added communication standards to all grade levels that included talking, reading, and writing components. In their document, *Curriculum and Evaluation Standards for School Mathematics* (1989), they stated the importance of writing.

- **For grades K-4:** “Writing about mathematics, such as describing how a problem was solved, also helps students clarify their thinking and develop deeper understanding” (NCTM, 1989, p. 26).

- **For grades 5-8:** “Opportunities to explain, conjecture, and defend one’s ideas orally and in writing can stimulate deeper understandings of concepts and principles” (NCTM, 1989, p. 78).

- **For grades 9-12:** “All students need extensive experience listening to, reading about, writing about, speaking about, reflecting on, and demonstrating mathematical ideas” (NCTM, 1989, p. 140).

This was ratified by the NCTM in a following edition titled *Professional Standards for School Mathematics* (NCTM, 1991). It was the NCTM’s contention that writing in mathematics was connected to “deeper understanding.” However, as important as it may be, according to Quinn and Wilson (1997), writing activities are not used consistently in school mathematics (as cited by McIntosh & Draper, 2001).

This is also the case in college-level mathematics. Burton and Morgan (2000) reported that there has been an increase in the recognition of the importance of communication skills in mathematics by professional organizations and researchers but that “the training of mathematicians does not appear to include any systematic attention to the development of writing skills” (p. 448). They critiqued the teaching and learning of mathematics, as if its only purpose was “filling students’ head with facts and skills” (p. 450) and not initiating students into mathematical communities.

In terms of lifelong learning, Gross (1992) indicated that people needed to develop new kinds of learning, such as the ability to communicate “with colleagues around the world via computer bulletin boards” (p. 136), to learn with others, and to learn by teaching. He suggested using the “Invisible University” when referring to the World Wide Web. Gross argued that what we learn today will be obsolete in five years and that learning to communicate through different kinds of media is a necessity for the twenty-first century.

Most writing in mathematics, according to a study by Pearce and Davison (1988), is incidental and algorithmic. It mimics the teachers' presentations and includes "direct copying and notetaking (sic)" (p. 13). In the classroom culture, writing is used for "knowledge telling," which is, according to Bruer (1993), related to recitation or routine and mechanical activities. He affirmed, however, that the goal related to writing activities must be geared toward "knowledge transformation," which consists of authentic writing tasks and serving larger communicative purposes. Writing with a purpose is located in the latter type of writing, "knowledge transformation," which is a type of writing that includes going "back and forth between planning and text" (Bruer, 1993, p. 246). These authors agreed that writing must be more than just repeating what was said in the classroom.

The purpose of a written piece implies a cultural and methodological background that will shape its components (Richards, 1991). In this sense, "distinct domains of discourse" can be identified in mathematics. According to Richards, there are at least four types of domains: research, inquiry, journal, and school mathematics (Table 2-1). Richards looked at mathematics as a content area, and his taxonomy is general in scope. However, he sustained that two domains pertain to the teaching and learning of mathematics: the inquiry domain and school mathematics domain.

As was stated before by Pearce and Davison (1988), students tend to mimic the teacher's presentations, learning by repetition. Richards (1991) called this writing type the school mathematics discourse and described it as a sequence that included three steps: initiation, reply, and evaluation. The problem, he contended, was that it left very little or even no space for inquiry. Another way of looking at school mathematics was presented by Romberg (1992). He stated that most students are engaged in traditional settings of instruction instead of authentic instruction; they are led to store up information, “knowing what” instead of “knowing how” (p. 52).

Inquiry mathematics, where participants engage in dynamic discussions by asking questions, presenting conjectures, listening, reading, and problem solving, was favored by Richards (1991). He sustained that it is through reflexivity that participants begin to communicate with each other and through collaboration and negotiation that learners develop meaning together. These activities would help the learners construct their own knowledge (Richards, 1991); it is by being active in learning that students can construct their own knowledge (Romberg, 1992).

Developing meaning together by communicating, collaborating, and negotiating meaning is to Richards (1991) what Vygotsky (1978) referred to as the interpersonal and intrapersonal processes in learning. In order to develop meaning, there is a need to develop inquiry discourse, which includes authentic instruction, engaging in “knowing how,” interacting, collaborating and negotiating, all of which will be studied in this research.

Advocates of writing in mathematics argue that writing helps the learner develop deeper understanding (NCTM, 1989, 1991). Activities that engage the learner in writing include keeping a math journal during a specific period of time, expository writing, and using writing prompts. The objective of writing, according to Miller and England (1989), “is to focus the student’s thinking toward a better understanding of the subject matter” (p. 299). It should not be “to demonstrate writing ability;” instead, students should be encouraged to think, reflect, and record (Tichenor & Jewell, 2001).

The benefits of writing were stated by Cook (1995). He indicated that writing will (1) provide the opportunity to restructure new knowledge, (2) allow the student to review, reiterate, and deepen understanding, (3) encourage the student to clarify and consolidate new information, and (4) help the student to put in order his/her thoughts. Dusterhoff (1995) also added that writing helps the student explore, clarify, confirm, and extend his/her thinking and understanding (p. 48).

Cognitive skills, conducive to understanding, used when writing in mathematics include comparisons, analysis, and synthesis (Miller & England, 1989). By taking these skills one by one and comparing them to the cognitive domain of Bloom’s Taxonomy, it can be argued that most cognitive levels are covered. In order to make comparisons, basic knowledge needs to be included. To analyze a concept, the learner needs to break down a whole into its components and find relationships between them. To synthesize, the student will put together separate ideas, producing something new (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956).

Only two components of Bloom’s taxonomy are missing here: application and evaluation. However, Nahrgang and Peterson (1986) sustained that the use of journals in mathematics can help students develop intellectual skills such as synthesis, interpretation, translation, analysis, and evaluation. These skills are used in authentic problem solving and writing. Therefore, when learners write in authentic settings, they mostly use higher-level cognitive skills.

The following paragraphs present a series of research projects in which writing in mathematics is explored from different domain perspectives, as formulated by Richards (1991) above. These domains include expository writing or school mathematics domain, journal writing, inquiry and research about mathematics.

Writing with the intention to describe or explain a mathematical idea was categorized as expository writing by Shield and Galbraith (1998). In a study where expository writing was analyzed during a three month period of time, eighth graders were asked to write letters to an imaginary friend (Shield & Galbraith, 1998). Tasks included explaining “all about” a chosen procedure and explaining a mathematical idea to someone who had trouble with it (p. 37). In this study, Shield and Galbraith (1998) made no “aim to influence the development of student expository writing in mathematics . . . [although the teachers tried] to stimulate further elaboration through discussion” (p. 44). According to the authors, students wrote their letters using an algorithmic style. That is, a step-by-step process, similar to textbook presentations and teaching practices, or what Richards (1991) called school mathematics. Shield and Galbraith (1998) concluded that the argument which stated that writing in mathematics promoted deeper understanding was unsupported. They sustained that increasing students’ meaningfulness in mathematics writing would require the students to show higher levels of thinking about the ideas they present in writing.

What Shield and Galbraith (1998) did not consider in their conclusion, however, was that students were writing letters to imaginary friends who never answered back. There was no interaction, negotiation, or collaboration between the students and their imaginary friends and therefore no need to go beyond the minimum requirements of the task. Students had no reciprocity and letters were independent from one another. There was no implicit or explicit dialogue in writing letters that would not be answered. The task students were engaged in was not authentic, nor was it inquiry based. Therefore, in this particular research, the question still remained the same: can writing in mathematics promote the development of higher order skills?

In another research by Stonewater (2002), students were given an essay question for their second exam in a college calculus class. They were told to visit with the professor if they had questions about the exercise. This writing exercise was not new to the students, as they had had several writing assignments before and an essay question on their first exam. The purpose of this research was to identify criteria that discriminated successful from unsuccessful writers in mathematics. As a result, Stonewater (2002) created “The Mathematics Writer’s Checklist.”

The essay question analyzed in this research was very specific. It asked for definitions and explanations, as well as for the relationship between various concepts. In class discussions, topics were addressed from a conceptual rather than a procedural approach and problems were represented in three formats: algebraically, numerically, and graphically. According to Stonewater (2002), successful writers developed, elaborated, or clarified mathematical descriptions by using examples and mathematical notations and by being careful to address all the components of the exam question. In Richards (1991) domains, this exercise will also be classified as school mathematics.

Writing in mathematics can help students organize, clarify, and reflect on their own ideas (Burns, 2004). Using journal writing as a tool in mathematics, Nahrgang and Peterson (1986) and Shield and Galbraith (1998) invited students to reflect on their own learning. Journal writing was studied in secondary mathematics (grades 7-12) (Clarke & Waywood, 1993) and in college mathematics (Loud, 1999; Goss, 1998; DiBartolo, 2000). Researchers’ purposes were to help students see themselves as active learners while constructing mathematics knowledge through internal dialogue. Clarke and Waywood (1993) believed that through journal writing, students were able to engage in constructive personal dialogue. Loud (1999) found that students did better when “incorporating structured complex writing assignments into course requirements” (p. 95).

Goss (1998) found that by writing, students engaged in the construction of mathematics concepts and were able to organize and explain their ideas in a more precise and coherent way.

Three types of journal entries were identified in the data set collected by Clarke and Waywood (1993): recounting entries, summarizing entries, and dialogue entries. This last type of entry was more reflective, including (1) explanations about how students solved a problem and how new topics were related to old ones, (2) the identification and analysis of difficulties, and (3) questioning themselves and asking for help. In this research, interaction between teacher and students was limited. Still, Clarke and Waywood (1993) concluded that students actively constructed mathematics through writing. The categories they chose to divide students' entries presented a continuum of student's understanding of mathematics. Recounting was classified at the lowest level; that was where students described mathematics. Summarizing implied the capacity of grouping together and integrating mathematical concepts, and dialogue was at the highest level, where learners created and shaped mathematical knowledge.

Shield and Galbraith's (1998) research about expository writing had students write without interacting with others, but Clarke and Waywood's (1993) research about journal writing had students not only recounting and summarizing, but also engaging in personal dialogues. In the latter research, students had a chance to go back and reread what they had written before in their journals. This allowed students to move in a learning continuum that started with recounting and summarizing and ended with the engagement of personal dialogues; they were able to reflect on and transform their own knowledge. Through journal writing, Clarke and Waywood's students engaged in what Bruer (1993) called an "authentic writing task." Clarke and Waywood's (1993) research also supported Pearce and Davison's (1988) argument indicating that "writing can lead to a deeper understanding and improved mastery of a topic" (p. 6).

A study that engaged second graders and elementary education majors (undergraduate students) in a pen pal exercise during a 13 weeks period studied collaborative writing in mathematics (Tichenor & Jewell, 2001). College students emailed letters to the students through the college professor and classroom teacher. Letters included open-ended questions and sentence prompts for the kids. In return, second graders met individually with their teacher to discuss what they would write about and used the computer to answer questions, complete sentences, and ask math questions of their pen pals. According to Tichenor and Jewell (2001), writing also helped "preservice teachers . . . [gain] a better understanding of how children learn, think, feel, and write" (p. 304). The authors indicated that second graders "felt their math performance was better because of the keypal experience" (p. 305) and that teachers developed "a deeper understanding of teaching and learning" (p. 306). The Tichenor and Jewell (2001) study is an example of how writing in school mathematics can engage students in discourse, even if they are from different levels and from remote locations. It also exemplifies the use of technology as an aid in developing communication skills and a community of practice.

In Loud's (1999) research, students' attitudes toward mathematics positively changed after having written experiences in a college calculus course. DiBartolo's (2000) research also found similar outcomes when studying formal and informal writing in a college mathematics course and when evaluating "students' written responses to test questions in Set Theory, Combinatory, Probability, and Statistics" (p. 101). Furthermore, by writing about the importance of calculus and mathematics in their majors, college students were able to think, reflect, understand, find relevancy, and learn new mathematics (Rose, 2001).

In summary, according to the research cited above, writing had a positive impact on students' performance and attitudes toward mathematics. Writing in mathematics was researched using individual activities such as expository writing, essay writing, journal writing, research paper writing, and pen pal activities. In most cases, writing allowed students to reflect on their work and engage in the development of higher order skills.

## Communication in Mathematics Learning

Traditionally, communication in mathematics was initiated by the teacher with little input from the students. Nevertheless, changes in teaching and learning paradigms opened new windows of possibilities to teaching and learning mathematics. The use of computers and Internet communication tools has revolutionized the way people communicate and the way courses are imparted (from face-to-face to hybrid or full online courses). Synchronous and asynchronous communication tools are available in many schools, public libraries, and homes. The first happens at the same time, although not always from the same place. The second takes place at different times and usually from different locations.

Common types of digital asynchronous tools are email, discussion forums, blogs with comments, and wikis. Emails are one-to-one or one-to-many electronic communications that allow for personal communications. Discussion forums are mainly one-to-many communications that allow users “to engage in high-level discussion by framing and presenting ideas, formatting challenging questions for peers, and responding to those questions to clarify misconceptions” (Hacker & Niederhauser, 2000, p. 55).

### Using Discussion Forums

Regarding the use of discussion forum, most researchers point to the need to monitor and scaffold students’ participation (Kanuka, Collect, & Caswell, 2002; Tu & Corry, 2003; Wegerig, 1998); to give specific instructions and examples before the discussions start, including what to do and when to do it (discussion cycles, discussion duration, frequency of participation, depth of discussion) (Knowlton, 2003; Tu & Corry, 2003); to generate an evaluation rubric with specific criteria together with the students, as to increase collaboration and self-inclusiveness (Knowlton, 2003); to divide the class into small groups so that discussions can have more depth (10 to 15 students per group) (Tu & Corry, 2003); and to develop more structured activities at the beginning of its use, moving toward more abstract activities at the end (Wegerig, 1998).

Research continued to develop connecting the use of discussion forums to blended learning environments. A sample of this was presented in the Special Issue: Computer-Mediated Communications, published in the *Journal of Research on Technology in Education* (Winter 2003-2004) edited by Suzanne Wade. Schallert, Reed, et al. (2003-2004) started this issue by considering the benefits of computer mediated discussions in students learning. This was a topic addressed from different perspectives by all the authors in this issue. Other topics included advantages, disadvantages, demographics, instructors' roles, and pedagogical implications when using discussion forums.

Shallert, Reed, et al. (2003-2004) stated that discussion forums offer the learner "the experience of thinking about an issue and commenting on it while reading others' comments … a valuable learning experience" (p. 111). Ferdig and Roehler (2003-2004) addressed a similar point when they stated, "multimedia environments generate higher level questions than students in classes without multimedia" (p. 119). Jetton (2003-2004) added that computer-mediated discussions facilitated learning by helping students: (1) make connections, (2) gain multiple perspectives, (3) develop problem solving skills, (4) add depth to their ideas, and (5) elicit instructors' input. Im and Lee (2003-2004) looked at student-to-student communication as a major tool to in developing a learning community.

Ferdig and Roehler (2003-2004) also presented five main advantages of using computer-mediated discussions. These are interactivity, active learning, teacher/student relationships, an increase in higher order thinking skills, and flexibility. The idea of interactivity was associated with collaboration, feedback, guidance, teamwork, and giving students a voice. Active learning was related to reflecting and making connections; promoting teacher/students relationships; and increasing flexibility, not only by allowing participation any time and from any place, but also by giving learners the time to think and structure a response, promoting reflexivity.

Fauske and Wade (2003-2004) and Im and Lee (2003-2004) evaluated the instructors' role in computer-mediated environment. Fauske and Wade (2003-2004) identified the following instructors' roles: monitoring discussion, "to develop netiquette collaboratively with students" (p. 147), determining "the appropriate level of structure needed and direction" (p. 147), modeling responses, and using alternative modes of communication when needed.

Nevertheless, none of these studies is directly related to mathematics education. They examine the use of discussion forums from different pedagogical perspectives unrelated to subject matter or content. This dissertation studied the development of threaded discussions in a specific discussion forum, a section of a community of practice, as it related to informal mathematics learning and inquiry learning. Although these discussions were initiated by a single person, all participants of the forum were able to read, reflect, and reply to the opening post (message). Still, some participants chose to stay in the background rather than actively participate.

**Discussion Forums Research in Mathematics Education**

Only a few studies were identified in which discussion forums were used in mathematics education. In the following paragraphs, researches concerning learning about teaching mathematics are divided into two general categories. The first category involves research in which discussion forums were used by pre-service and in-service teachers (Sliva, 2002; Smith, Ferguson, & Caris, 2003). The second category includes research in which discussion forums were used to learn how to do mathematics (Lotze, 2002; Bolin, 2003; and, Quinn, 2005). Only Lotze's (2002) research examined a semi-informal mathematics learning environment; that is, tutoring sessions that took place outside of the classroom and to which no grades were attached.

Sliva's (2002) research investigated how discussion forums were used to learn about mathematics. This study examined an online discussion forum in a hybrid mathematics elementary methods course with 20 pre-service elementary teacher candidates (students). Small group discussions explored five different topics related to mathematics education. These were "NCTM Standards, equity, technology in the mathematics classroom, brain research, and the Massachusetts Comprehensive Assessment System" (p. 84). In this research, students were using the discussion forum to learn about teaching mathematics and not to learn how to do mathematics.

By using the discussion forum, students began to develop community ties; they started thinking as researchers, became more reflective, and communicated openly in what they felt was a nonthreatening atmosphere (Sliva, 2002). This experience also allowed students to become more comfortable when using technology (Sliva, 2002). According to the author, the main implication of this study was the possibility of developing a community of learners that would provide support to future teachers, thus decreasing the sense of isolation many new teachers feel. Nevertheless, a limitation of this research was that it was conducted using a "password-protected asynchronous web-based discussion forum" (p. 81), and once students finished the semester, they would not have access to the system where previous discussions were located, thus minimizing the opportunities to continue collaborating and supporting one another. This pointed to the need of using public (or open) environments with free access over the Web where students could continue developing a mathematical community.

Secondly, research by Smith, Ferguson, and Caris (2003) studied how discussion forums were used to learn how to do mathematics from the instructor's perspective. The experiences online instructors had in an online, text-based environment were analyzed. Still, these authors did not analyze students' practice. Their goal was to investigate teaching and social issues, as well as differences between face-to-face and online teaching. They interviewed instructors from a wide spectrum of courses and then focused on mathematics instructors, following up with an extended sample of mathematics teachers.

Mathematics instructors in Smith, Ferguson, and Caris' (2003) research showed frustration when using text-based tools that limited their traditional teaching strategies, especially when they needed to communicate with formulas, symbols, and diagrams. They complained about "the need for greater precision in the use of [mathematics] language" (p. 41). It seems that instructors wanted to teach the way they were accustomed to on a chalkboard and expressed concern about the discussion forums' text-based format. This led Smith, Ferguson, and Caris (2003) to state, "The consensus . . . is that current Web-based distance learning environments do not adequately support mathematics" (p. 49). This study did not report about the teachers' technology skills and experience, which could have an impact on how they used technology to write mathematics in alternative ways.

The authors, Smith, Ferguson, and Caris (2003), suggested that there was a need to use new tools that would allow instructors to insert formulas and diagrams in an easier way. In addition, they stated, there was a need to consider the social impact that teaching online had on many of these instructors. New technologies now include the use of whiteboards (Glover, D., Miller, D., Averis, D., & Door, V, 2007), which can address some of the limitations presented by Smith, Ferguson and Caris (2003). In fact, Lotze's (2002) research participants used whiteboards, video, and audio to communicate. Comparisons are difficult to make, though, because Lotze's participants were students and tutors, and he compared online versus face-to-face college tutoring in mathematics and statistics in an effort to help determine the merits and drawbacks of new technologies.

In Lotze's (2002) research, students and tutors were paired and met a total of six times (three face-to-face sessions alternated with three online sessions). He reported that students with high levels of mathematics and technology anxiety where those with greater difficulties. This led Lotze (2002) to report that some students felt frustration, dissatisfaction, and technical problems during the online tutoring sessions. He also reported that some students needed more training to learn how to use and manipulate writing implements used with the whiteboard. Still, Lotze (2002) concluded (1) that the "medium was conducive to learning" (p. 125), even though it was not perfect, (2) that online tutoring took longer in time than face-to-face tutoring, (3) that interaction and communication was meaningful, and (4) that learning (knowledge construction) could take place in the online tutoring environment. Lotze (2002) ended his research hoping for a time when students could communicate with "cyber-tutors" at any time and from anywhere.

Bolin (2003) planned to use discussion forums in a hybrid college mathematics course so that all students could communicate and negotiate mathematical meanings with their classmates. Interview data, however, showed the researcher that the students lacked the confidence to participate in this environment, had no time or desire to participate, or did not know how to write the mathematical symbols that they thought were essential in writing mathematics. Instead, students preferred to use e-mail to communicate with the professor and to keep an e-journal about their meaning making processes, both more personal and individualized activities. Bolin (2003) recommended new research to examine the conversations that take place between professors and students and the “extensive, back-and-forth negotiation of meaning and understanding as they evolve over an extended period of time” (p. 108).

In Bolin’s (2003) research, students’ lack of confidence seemed to prevent them from becoming part of a community of learners through the discussion forum because it allowed their peers to see what their mathematical limitations (questions or misconceptions) were. Still, students did ask questions (e-mail), wrote about how to do mathematics (e-mail), and reflected about meaning making in a more personal environment (e-journal).

In another study, Quinn (2005) started to address some of Bolin’s (2003) questions when he studied online experiences and mathematical identity with undergraduate mathematics online students who volunteered to participate in his study. Participation in online communities was examined in relation to self-confidence, mathematics anxiety, self-concept, and gender. Quinn (2005) analyzed data from synchronous and asynchronous online communication tools and concluded that participating in online communication increased self-confidence, reduced anxiety, strengthened mathematics self-concept, and was not associated with gender differences.

In terms of mathematics identity, Quinn (2005) stated that volunteers had a relative anonymity when participating in the discussion forums that caused a relative comfort. He then recommended studying an environment where students could use avatars so that they would not have to expose themselves and their limitations to their classmates.

This supports Palloff and Pratt’s (1999) idea about the changes that teaching and learning on line have over its participants where teachers or students are no longer at the center of the learning process. Online learning can promote the development of communities of learners. As According to Palloff and Pratt (1999) stated “In the online classroom, it is the relationships and interactions among people through which knowledge is primarily generated” (p. 15). Moore

(1989) added that there is a need for continuous interaction between instructor and student, student and content, and student and student in distance learning environments. Increased interaction will minimize the sense of isolation students can feel when studying from remote locations (Moore, 1989). Increased interactions allow students to constitute a community of learners, one that allows participants to collaborate, negotiate, construct knowledge, and develop understanding (Palloff & Pratt, 1999; Li, 2004; Trentin, 2001; Dunlap, 2004).

The instructors' resistance in Smith, Ferguson, and Caris's (2003) research about using discussion forums reflected the changes new technologies brought to the way they taught. No longer was the teacher at the center of the lecture; their role had changed to one of a facilitator. A similar case can be seen in Bolin's (2003) study, when students seem to reject discussion forums because of lack of mathematics confidence. However, Quinn's (2005) research points in another direction, one that looks at the positive outcomes related to self-confidence, reduced anxiety, and strengthened mathematics self-concept when using communication tools while learning mathematics.

In the next section, communities of practice are furthered examined. Research by Lave (1991, 1996), Wenger (2001), Lave and Wenger (1991), Rogoff and Lave (1984), and others are discussed and related to this research project.

## Communities of Practice

Groups of people with common goals who are sharing ideas, making decisions together, and helping one another constitute a community of learners (Palloff & Pratt, 1999; Stepich & Ertmer, 2003; Trentin, 2001; Fielding, 1996). Together they interact, negotiate, and generate new meanings, taking "responsibility for determining what they need to know, and . . . [directing] their activities to effectively research, synthesize, and present their findings" (Dunlap, 2004, p. 41). In these environments, members may "have different interests, make diverse contributions, and hold varied viewpoints" (Lave & Wenger, 1991, p. 97). In a community of practice (CoP), there is a sense of "reciprocal caring" that goes beyond individualism (Fielding, 1996). Furthermore, communities of practice are characterized by "social issues of trust, reputation, space, and time" that help maintain knowledge ties (Nichini & Hung, 2002, p. 52).

New technologies have enabled the development of communities in which members no longer see each other face-to-face. As technology becomes transparent, unproblematic, and integrated into the community's activities (Lave & Wenger, 1991), different ways of interaction become possible (Moore, 1989; Hillman, Willis, & Gunawardena, 1994). Computer mediated-communication tools are used to communicate with others. These are synchronous and asynchronous communication tools that help establish ties between group members through active participation, even when members of the community are located at remote locations. Recently called "social software," they promote the development of supportive environments, scaffolding learning at different levels and allowing learners to try out ideas and challenge each other (Ferdig, 2007). Also, Web 2.0 tools such as podcasts, blogs, and wikis foster collaboration and sharing among its users (Boyd & Danielson, 2007).

Researchers relate communities of practice to informal learning, saying that they are rooted in everyday activities, and that they take place through demonstration, observation, and mimesis (Lave, 1996). Learning in communities of practice is mutual and reciprocal as opposed to formal learning or training that is directive (Trentin, 2001). The development of communities of practice is based on interaction, collaboration, scaffolding, and negotiation. The following sub-sections further explain these concepts and present an example of a community of practice.

**Interaction and Cooperation**

Interaction is defined as interplay, an exchange of ideas, and reciprocity of events among individuals (Gilbert & Moore, 1998; Wagner, 1994, 1997; Roblyer & Ekhaml, 2000). When interacting with others in face-to-face or online activities, learners might assume different roles. In the case of cooperative groups, learners are usually assigned to a specific role, each being responsible for a specific task or part of a problem (Hathorn & Ingram, 2002). Responsibility is divided, and collaboration might be implicit, but not a requirement.

In this cooperative group type of setting, interaction can be minimal, occurring at the beginning when tasks are divided among the members of a group. Once everyone knows what to do, communication among members of a group can become minimal. Learning, in this setting, is divided into chunks, and there is no need to share what has been learned. The emphasis is placed on the solution or outcome.

**Interaction, Negotiation, and Collaboration**

Nevertheless, in collaborative groups, interaction happens over time; it is “loose and voluntary” (Hathorn & Ingram, 2002, p. 37). Collaboration implies that learning is a shared responsibility where members take responsibility for one another (Hathorn & Ingram, 2002, p. 36). It is by articulating and elaborating their understanding and by sharing ideas and possible solutions to a problem that learners generate new knowledge (Dunlap, 2004).

In collaborative groups, negotiation is the source of learning (Sorensen & Munchú, 2004). By maximizing negotiation, interaction is enabled, and the generation of new learning is facilitated (Sorensen & Munchú, 2004). Collaborative groups act as “zones of proximal development” (Vygotsky, 1978) where any of its members can perform as a tutor or more “knowledgeable other” at different times. In this way, empowerment and ownership of meaning is encouraged (Sorensen & Munchú, 2004).

To negotiate is to interact with another (one or more people) and to reach an agreement. It implies continuous interaction, going back and forth, until an agreement is reached (Wenger, 2001). It is a process of interpretation and action, of making, remaking, thinking and rethinking, and of participation and comprehension (Wenger, 2001). It includes offering access to information, hearing or reading another person's perspectives, explaining why, inviting others to contribute, making others follow the rules, opening spaces for discussion, presenting a new argument or adding to an old one, sharing responsibilities, confronting others' positions and limits, and more (Wenger, 2001). Negotiation facilitates reflection and learning (Wenger, 2001). This is why learning is not the outcome of the individual mind but that of a participatory framework resulting from social practice (Lave & Wenger, 1991), that is, from interaction, negotiation, and collaboration.

**Communities of Practice in Mathematics**

There are many different kinds of communities of learners in mathematics. Some meet face-to-face every year, and some meet periodically over the Internet or the Web. Three examples of such communities are (1) the National Council of Mathematics that groups together mathematics teachers from K-12, mathematics teacher educators, and researchers, (2) the Mathematics American Association that represents mathematics professors, scientists, and investigators, and (3) the American Mathematical Association of Two-Year Colleges which embodies mathematics teachers and professors in small colleges and universities. These associations have the similar characteristic of meeting face-to-face every year in an annual conference and sharing their work in periodic publications and professional journals.

Other communities of mathematics learners meet online, over the Internet or the Web, or through news groups and email. They might not know each other personally, but they share ideas and collaborate with each other through electronic means. Two examples of such efforts include the *MathViaDistance* from Erie Community College in New York and the *Math Forum @ Drexel* in Pennsylvania. The former is an email list that communicates to its members by email only, and the latter is a more complex web site with several online components.

*The Math Forum @ Drexel* is self-defined as an online community of mathematicians, mathematics teachers and professors, mathematics learners, and enthusiasts. Participants use text-based asynchronous communication tools to pose questions, answer them, give feedback, recommend resources, and find solutions (Shumar & Renninger, 2002; Renninger & Shumar, 2002; Galindo, 2005; Crowe & Zand, 2000b). Mathematics learners interact with each other, become active learners, reflect about the problems posed by others, take time to reply, and contribute to knowledge building in community. On occasions they also watch without participating, following the turns of a discussion. Although there are limitations present in this environment, participants overcome them and continue working together toward the solution of problems.

*The Math Forum @ Drexel* environment is a public web site that can inspire communication and community building in mathematics. Members are voluntary participants working together in the generation of new knowledge, usually using pseudonyms. The research project presented here will examine one of its many discussion forums, over a period of five months, and how instances of dialogue (interaction, collaboration, and negotiation) contributed to knowledge construction in mathematics. More details about this community are included in the next chapter.

According to Shamatha, Peressini, and Meymaris (2004) learning through community-centered activities included “students [that] are encouraged and able to articulate their own ideas, challenge those of others, and negotiate deeper meaning along with other learners” (p. 263). They proposed that it is through learner-centered activities that “Students build new knowledge and understanding on what they already know and believe” (p. 364). These authors indicated that knowledge-centered activities will help students organize what they know, making connections that will later support planning and strategic thinking. It can be argued that by being part of a community of learners, students can partake in learner and knowledge centered activities.

## Summary

This chapter reviewed literature related to the use of technology in mathematics education, writing in mathematics, communication in mathematics, and specifically, the use of discussion forums in mathematics education, as well as the development of communities of practice. It examined the concepts related to communities of practice, such as interaction, collaboration, and negotiation. It also connected these topics and concepts with the research project presented in this dissertation.

The following two chapters look at qualitative research foundations (Chapter 3) and the methodology and methods (Chapter 4) used to develop this study. In the third chapter, you will also find the subjectivity statement, the research setting, and reflections on the Pilot Study. The methods used in this research are based on Gee's (1999, 2005) discourse analysis. These two chapters are then followed by the data analysis (Chapter 5, 6 & 7) and conclusion chapters.

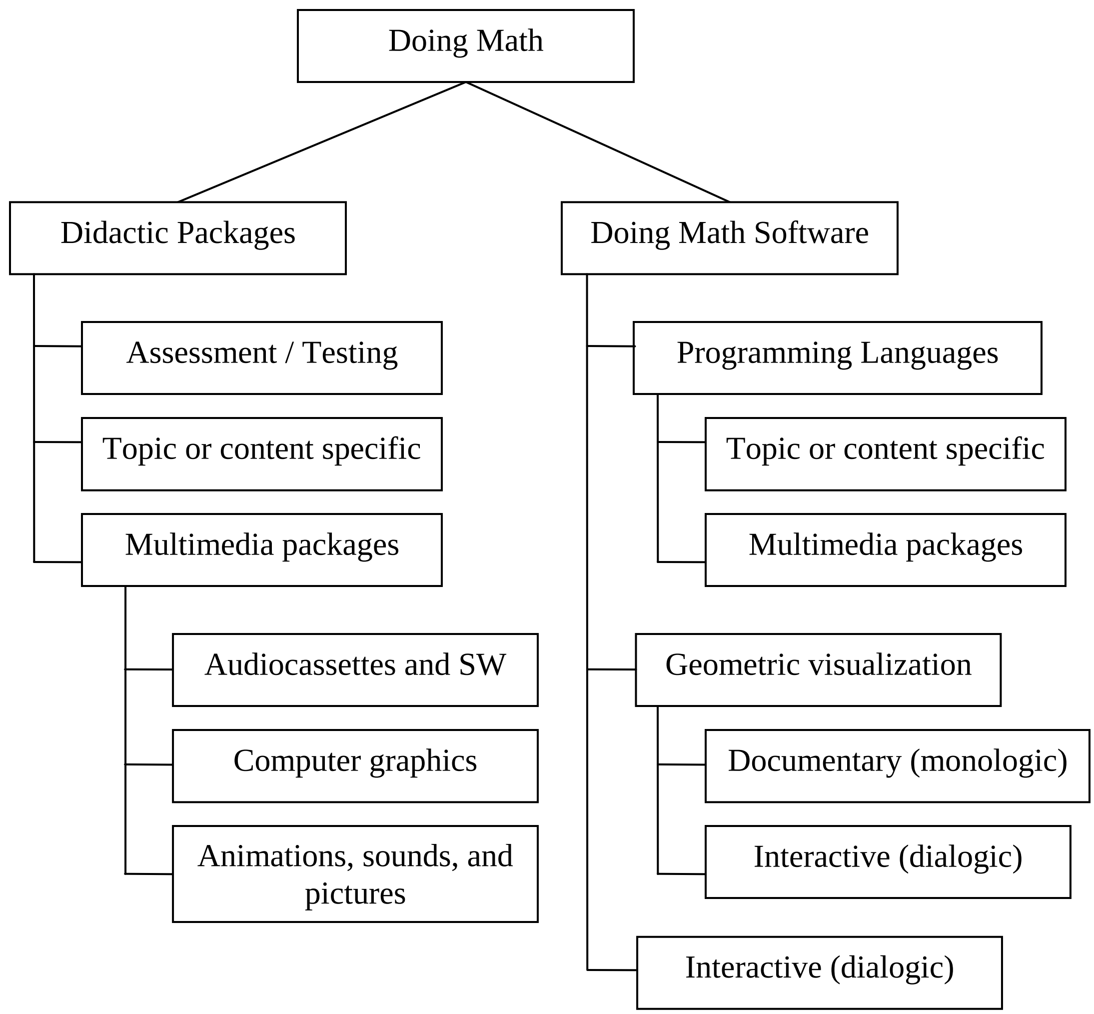


Figure 2-1. Software types used to construct knowledge in mathematics

Table 2-1. Domains of discourse in mathematics

| Domain of discourse | Communities using this type of discourse |
|---|---|
| Research math | Scientists and professional mathematicians |
| Inquiry math | Literate adults – includes mathematical discussions. Includes engaging in dynamic discussions by asking questions, presenting conjectures, listening, reading, and problem solving. Pertains to teaching and learning of mathematics. |
| Journal math | Used in publications and papers |
| School math | Used by teachers and students. Includes initiation, reply, and evaluation. Pertains to teaching and learning of mathematics |

# CHAPTER 3
# QUALITATIVE RESEARCH FOUNDATIONS

> . . . social analysis should help to generate vocabularies of understanding that can help us to create our future together. For the constructionist, the point of social analysis is not, then, to “get it right” about what is happening to us. Rather, such analysis should enable us to reflect and to create.
>
> –Kenneth J. Gergen, *An Invitation to Social Construction* (1999, p. 195)

For centuries, natural sciences were considered objective, a natural place for positivistic research. However, according to Burbules (2000), having “established a stronger set of common standards of practice, common vocabularies, and common techniques of inquiry” (p. 323) does not mean that physical and natural sciences’ knowledge does not get negotiated and constructed. Constructing knowledge in any discipline is a social process (Restivo, 1983). Accordingly, the study of the social construction of mathematics knowledge goes beyond a positivistic research approach.

The evolution of mathematics education studies was analyzed by Schoenfeld (1994) in the *Journal for Research in Mathematics Education*. In his article “A discourse on methods,” Schoenfeld showed how the focus of methodology in mathematics educational research changed from a positivist perspective “statistical in nature (mostly using hypothesis testing and regression analysis designs)” to a non-statistical, narrative, process-oriented methodology (p. 697). This dissertation continues this trend; it studied knowledge construction as it took place through transformative dialogue among groups of people in an online discussion forum using a qualitative approach.

The main purpose of this study was to examine the use of transformative dialogue to construct mathematics knowledge in a public online discussion forum. Transformative dialogue seeks a means to sustain the process of communication, acknowledges self expression, affirms the Other, coordinates actions that generate meaning together, affirms polyvocality, and promotes self-reflexivity – the questioning of one's own position. This research investigated written interactions during a period of one academic semester, archived at the *Math Forum @ Drexel* web site (http://www.mathforum.org) discussion groups' section, specifically the discussion group identified as *alt.math.undergrad*. The method used to analyze the data set was discourse analysis, as presented by Gee (1999, 2005). This method allowed the researcher to identify activities and connections within, between, and among the data set and to further develop a representation of how mathematics was constructed in an online asynchronous discussion forum, called a discourse model by Gee (1999, 2005). In general, this research analyzed a "socially constructed, complex, and ever changing" (Glesne, 1999, p. 5) electronic mathematics public environment.

A second purpose of this research was to contribute to the *sociology of mathematics*, a recent trend in mathematics' education research. This trend claims that "the reality of mathematics lies in discourse, so mathematics is as real – and only as real – as ordinary social life" (Restivo & Bauchspies, 2006, p. 198). Wittgenstein (1967, as cited in Restivo, 1983) emphasized the "art of *questioning*" in mathematics to establish a context from which a proposition could be "true." Lakatos (as cited in Restivo, 1983) looked at mathematics as a fallible discipline, one that is constructed through the development of conjectures, criticisms, and corrections and that is in continuous search for proofs and counterexamples. The sociology of mathematics moves away from the idea that mathematics is a static and rigid discipline and moves toward the idea of mathematics as a cultural expression. As such, mathematics "is constituted of mental-and-physical activities, culture, and history" (Restivo, 1983, p. 239).

Based on the purposes of this study, the research question for this study is as follows:

How does transformative dialogue and negotiation facilitate the social construction of mathematics knowledge in high school and first and second year undergraduate mathematics via an online discussion forum named *alt.math.undergrad* in the *Math Forum @ Drexel* web site?

This research studied an online asynchronous community and analyzed the corresponding digitally archived data during a specific period of time. It examined how participants in the discussion forum interacted and negotiated with each other to construct mathematics knowledge through the use of transformative dialogue. To accomplish these purposes, social constructionism was selected as the research's theoretical perspective, followed by a methodology and method that allowed the researcher to study language in context. This chapter will focus on the foundations of qualitative research and the analysis of the pilot study. It is divided into the next sections:

- Description of qualitative research fundamentals: research approach, epistemology, and theoretical perspective
- Subjectivity statement
- Description of the research setting
- Narrative with reflections about the pilot study

## Qualitative Research Fundamentals

The study presented here on transformative dialogue generated and negotiated in an online discussion forum searched for breadth and depth. It looked to understand how mathematics knowledge was constructed and how understanding of mathematics was developed through written interactions. It also acknowledged that there are multiple representations and multiple ways of learning mathematics. For this reason, a qualitative approach was selected to examine the data.

## Qualitative Research Approach

Qualitative research, a movement that began in the 1970's (Schwandt, 2000), searches for thick descriptions, those that cannot be found through statistical methods that are numerical in nature (Geertz, 1973). According to Patton (2002), "Qualitative methods facilitate [the] study of issues in depth and detail" (p. 14). These methods are associated with data that comes from words obtained from interviews, observations, and documents (Kvale, 1996; Jorgensen, 1989; Hill, 1993). Patton (2002) sustained that qualitative inquiry produces a great amount of information on a small number of persons or cases, thus reducing generalizations. It is through methodology rigor that the researcher analyzes data in order to represent findings (Anfara, Brown, & Mangione, 2002).

St. Pierre (2000) affirmed that qualitative researchers have the "responsibility . . . to keep educational research in play, increasingly unintelligible to itself, in order to produce different knowledge and produce knowledge differently as we work for social justice in the human sciences" (p. 27). Moreover, Erickson and Gutierrez (2002) stated that qualitative inquiry "can make valuable contributions to educational research, and that evidence-careful descriptive research falls within the range of methods in education that can be called *scientific*" (p. 21, emphasis in original). The controversies concerning research approach are many, especially now that the *No Child Left Behind Act* promotes quantitative research. However, as St. Pierre (2000) and Erickson and Gutierrez (2002) argued, qualitative research can also be scientific.

Crotty (1998) identified four elements of qualitative inquiry to justify a research project. These are epistemology, theoretical perspective, methodology, and method. In the next sections, the epistemology and theoretical perspective used in this research are discussed. Methodology and method will be elaborated in the next chapter.

## Epistemology: Constructionism

Epistemology is the theory of knowledge, of meaning making; it is that component of philosophy that studies how we learn. According to Crotty (1998), epistemology can be organized into three main divisions: objectivism, constructionism, and subjectivism. As in a continuum, at one end, objectivists believe "that meaning, and therefore meaningful reality, exists as such apart from the operation of any consciousness" (Crotty, 1998, p. 8); that knowledge can be objective, complete and unchanging (Burbules, 2000). At the other extreme, the subjectivist considers that meaning "is imposed on the object by the subject . . . [that] it is created out of nothing" (Crotty, 1998, p. 9). However, constructionism moves toward the center of this continuum. For Crotty (1998), "*all knowledge, and therefore all meaningful reality as such, is contingent upon human practices, being constructed in and out of interaction between human beings and their world, and developed and transmitted within an essentially social context*" (p. 42, emphasis in original). It is his contention that qualitative researchers tend to invoke a constructionist epistemology where meaning is created in relationship. This dissertation looks at knowledge from the constructionist standpoint, in which knowledge is constructed through interaction and not in the individual mind.

According to Crotty (1998), meaning making is the result of human beings being consciously engaged with the world. The process of constructing the world takes curiosity, imagination, and creativity (Crotty, 1998). Still, this does not mean falling to subjective, unfounded interpretations. Instead, "*[a] dialogue with the materials*" helps the researcher pay attention to the object of research (Crotty, 1998, p. 51, emphasis in original) while following a specific methodology or method rigor.

The idea of language and dialogue is at the center of the constructionism epistemology. Berger and Luckmann (1966), forerunners of social constructionism, stated that "sociology of knowledge presupposes sociology of language" (p. 185). Sampson (1993, as cited in Gergen, 2000) also indicated that meaning "is rooted in social process . . . sustained by conversations occurring between people" (p. 149). Therefore, it is through language and dialogue that people generate meaning together (Gergen & Gergen, 2004).

### Theoretical Perspective: Social Constructionism

Following the constructionism epistemology, this research is based on the social construction of knowledge theoretical perspective. Social constructionists are interested in the collective generation of meaning (Crotty, 1998) as it derives from collaboration and interaction and from reflexive questioning, dialogue, and negotiation (Gergen, 1999).

Gergen (2000) affirmed that meaning is neither the result of the individual mind (cognitive constructivism) nor that of the group (social constructivism), but that it is "a byproduct of language use within relationship" (p. 150). [For a discussion about the history, principles, similarities, and differences on cognitive constructivism and social constructivism, see Duffy and Cunningham (1996).] According to Gergen (2000), knowledge is not developed in a single mind or from a single individual, but rather from "language use within relationship" (p. 150). It is culture that molds knowledge, he says. That is why, for the social constructionist, "knowledge is a communal creation" (Gergen, 1994, p. 207).

Social constructionists are aware of the multiple causes that can produce specific outcomes. Thus, they cannot agree with the idea of causality proposed by positivistic researchers, as if the conditions of a specific experiment could be controlled to produce a specific human outcome. That is why for social constructionists, numbers and statistical analysis are not enough. They sustain numbers and statistics eliminate the voices of research participants, especially silencing those without sophisticated knowledge, therefore silencing those without power.

Furthermore, social constructionists argue in favor of a plurality of voices and disagree with the idea that there is “one true answer to any question” (Gergen, 1999, p. 92). As Gergen (1999) stated, “each construction has both potentials and limits, both scientifically and in terms of societal values” (p. 93). He also sustained that there is no need to abandon all voices to save only one. Thus, to say that there is one truth that can be generalized to the whole population is for the social constructionist “a form of cultural imperialism” (Gergen, 1999, p. 93). Imposing one view over another as if things were black or white would be imposing the voice of those in power.

For this reason, social constructionists step away from dichotomies and welcome plurality. They search for new ways of looking at things, ways that will contribute to the development of a “*generative theory*.” They also search for new possibilities that will include “*accounts of our world that challenge the taken-for-granted conventions of understanding, and simultaneously invite us into new worlds of meaning and action*” (Gergen, 1999, p. 116, emphasis in original).

The social constructionism theoretical perspective developed in different disciplines, such as therapy, organizational change, education, and scholarly expression (Gergen, 1999). However, since the focus of this research project is on the construction of mathematics knowledge in education, the following two sections will only present (1) how social constructionism is applied to education, including pedagogical alternatives, and (2) the social constructionist assumptions as they relate to this research project.

**Social Constructionism in Education: Alternative Pedagogies**

As previously stated, social constructionism is an outgrowth of communal relations (Gergen, 1999). Therefore, in education, social constructionists favor three pedagogical alternatives: reflexive deliberation, polyvocal pedagogy, and collaborative classrooms. In this research project, reflexive deliberation was used to generate the data set without the intervention of the researcher. Participants of the discussion forum were able to reflect before writing a message (post). Different solutions were introduced by the participants presenting different views and therefore resulting in collaborative polyvocality. While working together, the discussion forum participants engaged in negotiation and collaboration practices to generate new meanings.

These alternative pedagogies are explained below.

**Reflexive deliberation**

Reflexive deliberation is the outcome of communities of practice. Together, participants learn about the ways a community exists and how it thinks “about knowledge, language, discourse, and their relationships and distribution in society” (Gee, 2000, p. 522). In a community of practice, learners work together and interact with each other (Rogoff &Lave, 1984). They set goals together, negotiate appropriate means to reach them, and help each other throughout the process.

In this research project, the contributions that mathematics learners made to an online discussion forum were examined. Participants were part of a community where reflection was possible, even encouraged. The discussion forum allowed learners to present a problem in a post and to read postings by others. Its quality of asynchrony allowed participants to reflect before replying to others. Together, participants interacted and negotiated one or more answers to the posted questions.

**Polyvocal pedagogy**

Gergen (1999) stated that the Internet is an example of how people can generate new potential. This is possible when body and technology merge together, facilitating the development of polyvocality. According to Gergen (1999), the Internet can help “students . . . develop multiple voices, [different] forms of expression, or [different] ways of putting things” (p.

183). McCarty and Schwandt (2000) added that through polyvocal pedagogy, students could participate in a wide range of conversations and acquire different rhetorical skills. These in turn allow students to take persuasive positions by actively participating in different conversations.

Hatch (2002) also presented polyvocal methods as a means to find multiple perspectives. He stated that polyvocal analysis is interested in the existence of multiple truths; that is, in the multiple voices that are telling a story. In this research, this will translate into the multiple discourses written by the forum's participants. Through polyvocality, the researcher tries to identify the components of a story, which, according to Hatch (2002), is always partial, local, and historical. Therefore, the final purpose of polyvocal analysis is to capture the multiplicity of voices present in the scene and to tell as many stories as they generate. Gergen (2003) would agree with Hatch's (2002) view of polyvocal analysis.

In this research, polyvocality took place in different ways. After writing an original message (post) in a discussion forum, the participants of this community were able to present different answers to the same question or problem, developing a threaded discussion that branched in different directions. At the same time, they developed different ways of representing an answer (narrative, algebraic, graphical, or geometric), including different levels of abstraction. This, in turn, allowed the participants to look at mathematics from different perspectives.

**Collaboration**

Collaborative practices favor dialogue, consensus groups, and the generation of new ideas and opinions. It also opens the classroom to new experiences, permitting students to work within the community, establish Internet communications around the world, and even develop authentic projects (Gergen, 1999). The student is seen as an active participant in a community of learners (Lave & Wenger, 1991; Lave, 1991; Wenger, 2001).

This research analyzed the conversations in a mathematics online discussion forum available to those with access to a computer with an Internet connection. These conversations, generated asynchronously, made possible the discursive collaboration among its participants. Together, participants formed a community of mathematics learners. Learners from different backgrounds and nationalities worked together to find one or more solutions to a problem or question.

**Summary**

Reflexive deliberation, polyvocality, and collaboration are pedagogical alternatives in the social constructionism theoretical perspective, available in an online asynchronous discussion forum to those with access to a computer and an Internet connection. With this, the physical boundaries of a classroom disappear, and its walls are expanded to the world's cybernauts, where it is possible to become part of a larger community of learners.

In this study, the researcher examined how the Internet allowed high school and first and second year undergraduate mathematics learners to interact and collaborate, engaging in transformative dialogue. This research analyzed how participants in an online discussion forum constructed mathematics knowledge through reflexive deliberation, polyvocality, and discursive collaboration.

The social constructionism theoretical perspective is based on a set of assumptions. The next section will list these assumptions, as stated by Gergen (1994). Then an explanation of those that apply to this research follows.

**Social Constructionism Assumptions and Mathematics Knowledge**

Kenneth J. Gergen (1994) presented five major assumptions within the domain of social constructionism that broke away from the empiricist tradition of positivistic research. These are: (1) knowledge is socially constituted; (2) knowledge is embedded in historical developments; (3)

linguistic signals are related to experience and influenced by culture; (4) knowledge is influenced by personal values, ideologies, and visions; and (5) verification of theory is rendered suspect. In the following paragraphs, the first, third, and fifth assumptions are discussed as they relate to this research project.

**Knowledge is socially constituted**

Gergen (1994) sustained that social circumstances affect knowledge construction. He also maintained the importance of language in people's relationships. As stated before, knowledge is constructed in relationship with others in the collectivity, not in the individual mind but through social processes of communication.

The idea of socially constructing knowledge, that is, of socially constructing meaning in mathematics, is the main focus of this study. As Restivo and Bauchspies (2006) explained in "The will to mathematics: Minds, morals, and numbers,"

> Mathematical objects are things produced by, manufactured by, social beings through social means in social settings. There is no reason why an object such as a theorem should be treated any differently in this sense than a sculpture, a teapot, a painting, or a skyscraper. . . . Mathematicians work with notations, symbols, and rules; they have a general reservoir of resources, a toolkit, socially constructed around social interests and oriented to social goals. The objects they construct take their meaning from the history of their construction and usage, the ways they are used in the present, the consequences of their usage inside and outside of mathematics, and the network of ideas they are part of within math worlds and within larger societal worlds. (p. 210)

G. H. Hardy (1992), an English mathematician, also supported this idea when he wrote that "the function of the mathematician is to do something, to prove new theorems, to add to mathematics" (p. 61). Hardy talked about the importance of individually generating mathematical ideas and not about constructing mathematics in relationship. However, as documented in the foreword (by C. P. Snow) of Hardy's book and in Beckmann's (1971, p. 138) book, Hardy worked closely with Ramanujan, an Indian mathematician with little early formal education. Together, they produced “five papers of the highest class” (Hardy, 1992, p. 36). Together, Hardy and Ramanujan were able to construct new mathematics.

This research analyzed mathematical interactions and negotiation that took place in an online discussion forum. It studied how learners worked together, searching for one or more solutions to a problem, socially constructing mathematics meaning. It examined how transformative dialogue was used to construct mathematics knowledge.

**Linguistic signals, experience, and culture**

Berger and Luckmann (1966) stated that language has the “capacity to transcend the ‘here and now’ . . . [that it] bridges different zones within the reality of everyday life and integrates them into a meaningful whole” (p. 39). Be it face-to-face (first degree in Berger and Luckmann’s view) or written (second degree, according to Berger and Luckmann), “language is capable of becoming the objective repository of vast accumulations of meaning and experience, which it can then preserve in time and transmit to following generations” (Berger & Luckmann, 1966, p. 37). According to Phillips (2000), language is a human construct, and “different individuals may construct slightly different things with it, even when they use the same words” (p. 4); therefore words can be interpreted in different ways.

Berger and Luckmann (1966) also shed light on how the social distribution of knowledge works. They sustained that “knowledge [is encountered] in everyday life as socially distributed, that is, as possessed differently by different individuals and types of individuals” (p. 46). They sustained that no one knows exactly the same as another. Moreover, Phillips (2000) cited Lorraine Code in saying that knowledge “is grounded in experiences and practices, in the efficacy of dialogic negotiation and of action” (p. 30).

In mathematics, meaning and experience are mainly represented through algebra and geometry. For many, algebra is a special language. According to Restivo and Bauchspies (2006), knowledge is constructed through the use of notations, rules, and theorems. As time goes on, more mathematical representations and applications are developed. In this research, discussion forum participants presented solutions in narrative, algebraic, and geometric forms through words, symbols, and diagrams. Participants interacted, negotiated, and collaborated with each other to find one or more answers to the posted questions, problems, and conundrums.

**Verification of theory through research is rendered suspect**

Gergen (1994) sustained that by testing a hypothesis, the researcher is already seeking for data that best serve their interests. He also affirmed that "any intelligible hypothesis can be 'verified' or 'falsified'" (p. 206). From the interpretive point of view, "the investigator attempts to document the rules of meaning within a specific context; the documentation . . . serves not as a validating device but as [a] rhetorical support" (Gergen, 1994, p. 206).

The investigator, following social constructionist principles, is not in search of specific data or results, nor of specific "truths" to accept or reject a hypothesis; instead, the researcher looks for meaning as it comes from the data itself, looking at the discourse of those that negotiate and collaborate while constructing meaning. In the social constructionism theoretical perspective, meaning is generated bottom-up instead of top-down. Gergen (1999) indicated that in "top-down" analysis, it is those in authority who set the rules. Indeed, transformative dialogue seeks for means of sustaining the process of communication, acknowledging self expression, affirming the Other, coordinating actions that generate meaning together, affirming polyvocality, and promoting self-reflexivity, the questioning of one's own position. It is the idea of togetherness that makes the difference; that is, working together, reflecting together, and coordinating actions together to generate new meanings.

In the research presented here, a discourse model was developed from within the data itself, analyzing the transformative dialogues occurred in the discussion forum. The model was then recursively verified twice throughout a period of five months. This research examined how participants generated meaning together and how interactive dialogue, negotiation, and discursive collaboration took place to construct mathematics meaning. To accomplish this, methodology rigor was performed through discourse analysis implemented with Gee’s (1999, 2005) discourse analysis methods. The next chapter will examine this methodology from a theoretical stance as well as its application in this research project. However, before presenting the methods used in this research and the application of Gee’s discourse analysis, three more sections are presented below. First, the researcher is introduced in the subjectivity statement; second, the research setting is described; and lastly, reflections on the pilot study are narrated.

## Subjectivity Statement[1]

Social constructionists cannot remain dispassionate about their work. They acknowledge that there is a reason that guides their work, motivating them to invest time in such a project. This researcher has taught junior high, high school, and undergraduate mathematics and is interested in researching the use of online communication tools in learning mathematics. By investigating the use of discussion forums in mathematics, the researcher was able to explore how students write mathematical ideas and search for meaning while interacting, negotiating, and collaborating with each other.

As a graduate student, this researcher studied the teaching of mathematics with technology and the importance of communication in e-learning environments, as well as the concept of quality in distance education from the students’ perspective (Ortiz-Rodríguez, Telg, Irani, Roberts, & Rhoades, 2005). Her interests also include researching online learning environments

[1] The subjectivity statement is written in third person, instead of first person, mainly because of cultural reasons. In the Puerto Rican culture we are taught not to talk about ourselves as we believe that this shows the character flow of lack of humility. The following paragraphs present a picture of who I am in third person.

and how they can help students develop communities of practice that promote meaning making and understanding.

Mathematics has always been this researcher’s favorite subject. In grade school, she was not a high achiever, although she strived to be one. She did not understood why the group of students, who were all supposedly at the same level of understanding, completed different sets of problems. By the time she reached the seventh grade, she had caught up with her peer group. By ninth grade, she started helping her classmates with their math homework. At that time, she admired her math teacher and her pedagogy. The teacher’s daily routine was simple and straightforward, following essentialist and behaviorist practices. Homework was checked first, then a new topic was presented, and finally, the teacher gave the students some practice and homework problems. The teacher’s elementary algebra class seemed so easy!

That time was when this researcher first became a tutor and felt empowered while helping her classmates understand and complete their homework. However, when she got to college, she was once again an underachiever in math, lacking the background knowledge needed to excel. Nevertheless, she worked hard to keep learning math and managed to complete a degree in Secondary Math Education. She then became a math teacher.

From 1961 to 1973, while she attended elementary and secondary school, there were no personal computers. Mainframe systems were a reality in a few school systems throughout the United States; but the public school system in Puerto Rico did not use computers, at least to her knowledge. The first personal computer widely available in K-12 settings was the Apple IIe, commercially marketed after 1977. During the 1970’s, some students used calculators, but she never had one due to a lack of financial resources. With a calculator, students were able to check the solutions to their homework problems, allowing them to look at patterns and to better develop concept knowledge.

The researcher taught math in high school (1977-1978) and in junior high school (1978-1979, 1991), as well as first and second year undergraduate college courses (1991-2002). What she loves the most is helping students feel that they can learn how to “do” mathematics, that they can be successful in mathematics. Finding out there are no secrets behind the numbers can help students understand the concepts at hand. This sense of empowerment in math is hard to accomplish because many students’ past experiences are grounded in behaviorist practices in which memorization had a major role. The daily routine that her old ninth grade teacher followed did not work for most students. That’s why she believes that learning mathematics has much to do with constructivist and constructionist practices, in which students can “do” mathematics in collaboration, developing concepts and understanding and applying them in everyday life (NCTM, 1989). Ideally, constructionism should be reached so that learning can focus on the “collective generation of meaning” (Crotty, 1998, p.58). Communication and collaboration are the building blocks for the development of communities of learners (Rogoff & Lave, 1984; Lave, 1991; Lave & Wenger, 1991; Lave, 1996; Wenger, 2001; Shamatha, Peressini, & Meymaris, 2004) and math communities are no exception.

## Research Setting

Our study examined interactions that took place in an online discussion forum located at a public web site and available to those interested in mathematics teaching and learning with access to the Internet. In the next sections, the web site is presented to the reader, including its history and services.

**The Math Forum @ Drexel Web Site**

Self-defined as “an online math education community center,” the *Math Forum*’s mission “is to provide resources, materials, activities, person-to-person interactions, and educational products and services that enrich and support teaching and learning in an increasingly technological world” (Drexel University, 1994-2006, paragraph 1). To fulfill their mission, the *Math Forum* has stated five main objectives. These are (1) to encourage communication throughout the mathematical community, (2) to offer model interactive projects, (3) to make math-related web resources more accessible, (4) to provide high-quality math and math education content, and (5) to spread news about new resources in the Internet. The *Math Forum* is a very dynamic community of mathematicians that continually adds more resources and services to its web site.

**History of the Math Forum @ Drexel**

In 1992, the *Geometry Forum* was founded by Eugene Klots, a mathematics professor from Swarthmore College and the developer of the software program *Geometer’s Sketchpad* (Kane, 2000). It was not until 1996 that the forum’s name changed to the *Math Forum*. During this year, Swarthmore College received a three million dollar grant from the National Science Foundation to further expand the site. A year later, in 1997, the *Math Forum* web site was helping close to 100,000 users a month (Downing, 1997). For many years, the media portrayed the *Math Forum* as a homework helper web site and a mathematics online community especially renowned for its “Problem of the Week” and “Ask Dr. Math” sections.

On April 6, 2000, *The Associated Press* reported that the *Math Forum* received “almost 1 [one] million visitors and 12 million hits per month” (“Popular math Web site is sold to WebCT,” 2000). It was in that same month, April of 2000, that the *Philadelphia Inquirer* reported that the *Math Forum* was sold to *WebCT*, the online software company (Woodall, 2000).This was mainly a financial arrangement, as its offices stayed at Swarthmore College, and the *Math Forum* continued to be managed by Eugene Klots (Research and Development) and Steve Weimar (Technology and Education). A year later, in 2001, Drexel University acquired the *Math Forum*.

In a special issue of the *Math Forum Internet News* (volume 6, number 36a), the President of Drexel University, Dr. Constantine Papadakis, informed the members of the mathematics community that the *Forum* had a new location and a new name, *The Math Forum @ Drexel.* He also pointed out that the web site would continue to offer the same services while introducing some new cutting edge features (*Math Forum Internet News*, 2001, paragraph 3). Since then, the web site changed its presence in the Internet and continued to expand.

**The Math Forum @ Drexel Services**

Services provided by the *Math Forum's* web site are divided into four main sections: (1) Main Areas, (2) Projects, (3) Features, and (4) Archives (*The Math Forum @ Drexel*, 1994-2004). *Main Areas* is subdivided into several sub-sections specially dedicated to students, teachers, parents and citizens, and researchers. The *Projects section* is geared toward the development of math skills, including a "Problem of the Week" area for each secondary math subject: Math Fundamentals, Pre-Algebra, Geometry, and Algebra. The *Math Forum Newsletter* (MFIN) is located under the *Features* section, as are the Mathematics Discussion Groups. Finally, the fourth section includes links to mailing lists, workshops, software, articles, book reviews, and more. (Table 3-1 for a complete list of the resources provided in the *Math Forum* web site).

The first issue of the *Math Forum Internet News* was published in October 7, 1996 (http://www.mathforum.org/electronic.newsletter/); since then, weekly issues were published. Eleven volumes, with added issues for special purposes, were published up through 2007. During the last 20 years, the newsletter kept the math community updated with new ideas, new developments, and the latest news in mathematics. The *Math Forum* also works in collaboration with professional organizations such as the *National Council of Teachers of Mathematics* (NCTM) and the *Mathematical American Association* (MAA).

The publication of the first *Math Forum Newsletter* issue in 1996 was an extension of a conversation already started, where references to math resources were made and geometry problems were posted through a mailing list. Email correspondence was accepted from community members and answered individually. Once the online newsletter was available through the site, it included a section called “Check out our Web Site” at the very end. In the first issue, only three resources were listed; the first was a link to “The Math Forum’s” home page, the second a link to the “Problem of the Week,” and the last to “Internet Resources (Steve’s Dump).”

It was not until September of 1998 that a link to the “Discussion Groups” was added in the forum’s newsletter. This was a special issue (3.39A) dedicated to describing “interesting conversations take[n] place during September of 1998 on Internet math discussion groups” (*The Math Forum News*, September 1998, first paragraph). This issue was also the beginning of a monthly series dedicated entirely to describe different discussion forums and conversations. Two references are made in the Discussion Groups series to the *alt.math.undergrad*, the first on July 28, 1999, and second on June 8, 2000. Both references include the description of the group, a link to its home page, and an example of the interactions that took place on that particular day. This information is similar to that given about any other discussion group. The Discussion Group’s series ended on August 2000.

**alt.math.undergrad Discussion Group**

Data for this research was located in the *Math Forum @ Drexel's* web site (http://www.mathforum.org) discussion section. The *alt.math.undergrad* discussion forum was chosen for analysis because it included high school and undergraduate first and second year mathematics discussions, courses the researcher has taught and tutored before. The *Math Forum @ Drexel* was selected since it is a well-established community of mathematics learners with more than two decades of existence and experience.

By November 8, 2006, there were a total of 68 active Discussion Groups and 13 inactive groups in this section. The active discussion groups were divided into ten categories. These are Courses, Curricula, Education, History, Math Topics, Online Projects, Policy and News, Professional Associations, sci.math, Software, and Inactive. The discussion group titled *alt.math.undergrad* belongs to the Math Topics category, and it is described as "an unmoderated newsgroup focused on undergraduate mathematics" (*Math Forum @ Drexel*, 2004). Other discussion groups in the Math Topics category include the following: *alt.algebra.help.independent, alt.math.recreational.independent, discretemath, geometry.college, geometry.pre-college*, *geometry.puzzles, geometry.research,* and *Snark*. Community members freely choose the group in which to participate, posting their messages from anywhere and anytime.

Archives in the *alt.math.undergrad* discussion forum include messages posted from July 7, 1996 to the present time. This is an active community that continues to receive messages up to the current date, expanding its archives day by day. As of November 8, 2006, the *alt.math.undergrad* included a total of 38,797 messages (posts), divided into 8,516 topics (threads). This research project will analyze the messages posted during the first academic semester of 2004 (August to December). More details about the data set as well as the selection criteria will be described in the methods chapter.

## Reflections about the Pilot Study

A preliminary study was conducted to examine how mathematics knowledge was socially constructed at the undergraduate level in the online public mathematics discussion forum called *alt.math.undergrad*. This initial phase allowed the researcher to investigate how data was organized, what subject matters were studied, and how discussion was generated. It also allowed the researcher to explore the activities and relationships taking place throughout the discussions.

The *alt.math.undergrad* discussion forum was chosen as the research focus because it includes high school and first and second year undergraduate mathematics topics. This discussion forum has free access through the *Math Forum @ Drexel's* web site (http://www.mathforum.org). The pilot study investigated threaded discussions (topics) and their corresponding postings (messages) occurring during a one-month period. Data was selected from October, 2004, because the pilot study started on that same month.

At first, a general analysis of all October, 2004, discussions showed that there were 167 threaded discussions (topics). From these, only three had more than 25 postings (messages), and twelve had between ten and twenty-five postings. A content analysis followed to identify the specific mathematics topics discussed in the latter group. In addition, the number of participants, frequency of participation, and time span of discussion was recorded. Further evaluation allowed the researcher to take general notes about the discussions, draw tree diagrams to identify the flow of interaction, and determine the number of stories in each threaded discussion. A selection from those threaded discussions covering topics offered in high school and first and second year undergraduate mathematics followed. This led to a total of five threaded discussions that met the data selection criteria (Table 3-2).

Next, mathematical stories were constructed using tree diagrams to show the flow of interactions present in each threaded discussion. The branches of the tree diagrams represented the way discussions were conducted, showing who replied to whom, which messages received one or more answers, and those that did not receive any reply. Each tree branch was identified as a story where participants interacted with each other by evaluating a problem, negotiating an answer, and generating new mathematical knowledge. Tree branches were collections of continuous postings that exemplified how interaction, negotiation, and discursive collaboration took place.

Once the stories were constructed, a decision was made to eliminate redundant intertextuality and personal identifications. Analysis focused on how participants developed mathematical knowledge; by eliminating redundant intertextuality, an understanding of thought processes in each story was facilitated. Since the focus of the study was on knowledge construction, the participants' identity was not relevant. This alone could be the object of future research.

At that point in the study, a question about methodology arose. Was content analysis enough? Was open coding and grounded theory the path to follow? Was phenomenology the correct methodology? What had to be the main focus when identifying how participants developed knowledge? Discourse analysis as stated by Gee (1999) was then chosen to further analyze the data set. Gee provided guide questions to analyze activities, which allowed the researcher to go beyond the mathematical content of each threaded discussion in search of negotiation and collaborative strategies. In Gee's discourse analysis, the *Activity building* looked at how participants use "cues or clues to assemble situated meanings about what activity or activities are going on" (Gee, 1999, p. 86). Gee focused on the specific actions that take place throughout discourse. To question the data, Gee (1999) formulated the following questions:

> "What is the larger or main activity (or set of activities) going on in the situation? What sub-activities compose this activity (or these activities)? What actions (down to the level of things like "requests for reason") compose these sub-activities and activities?" (p. 93)

Questioning the mathematical stories allowed the researcher to identify activities and specific actions conducted by the participants. However, at that point, each story was analyzed independently. Connections within, between, and among the data set were not considered in the pilot study.

As an experiment, the researcher then decided to go a step further into Gee's methodology and tried to identify the "parts of a story" present in each mathematical story (or tree branch). The researcher found that tree branches with five or more messages included all or most of the elements (body parts) of a story identified in Gee's discourse analysis methods. These include setting, catalyst, crisis, evaluation, resolution, and coda (Table 3-3).

Dividing the mathematical stories in this way – that is, using the body parts – allowed the researcher to identify types of activities that were conducted by the participants throughout the discussions. These activities were based on previous research about discussion forums. A report was then written based on these results (See Appendix A for the complete Pilot Study report).

**Lessons Learned through the Pilot Study**

Conducting the Pilot Study allowed the researcher to identify a methodology that fit the theoretical perspective as well as the data set. It helped explain how participants of a discussion forum constructed knowledge. The use of tree diagrams made it possible to represent the threaded discussions in a graphical format, to identify the flow of conversations, to identify the stories present in each threaded discussion, to breakdown the stories into body parts, and to complete discourse analysis – questioning the data and searching for breadth and depth.

The pilot study showed that users willingly posted questions, answers, suggestions, and references to Internet resources. Participants in the discussion forum worked together to find new meanings, presenting different ideas, some in algebraic form and others in graphical form through words. Polyvocality was present when participants presented similar ideas in different ways. As a community of practice, participants helped each other in different ways, sometimes identifying new resources and other times giving support to each other. Their voice helped the researcher build knowledge from the ground up through the analysis of transformative dialogue. Knowledge in the discussion forum was constructed through active participation.

**Criticisms**

As stated, the pilot study allowed the researcher to conduct a preliminary analysis; however, under-analysis occurred at different levels (Antaki, Billig, Edwards, & Potter, 2002; Burman, 2003). First, a summary-like report was presented, losing details and subtleties incorporated in the data. This was the result of spending more time organizing the data instead of analyzing utterances. Secondly, only isolated quotations were present in the report; even though a complete story was included as an example, very little analysis followed – that is, questioning the data was minimal. Mental constructs combined with previous research findings on discussion forums were used to find out these same constructs in the data, not allowing the data to speak for itself. The researcher moved toward general assumptions instead of going "back and forth between the general and the specific" (Antaki et al., 2002, section "9. Under-analysis through Spotting," paragraph 4). Finally, at the time the pilot study was conducted, sociology of mathematics was not yet identified in the research literature, limiting the analysis, findings, and conclusions.

However, after reviewing discourse analysis literature beyond Gee's methodology (Austin, 1962; Foucault, 1972; Parker, 2001; Hepburn & Potter, 2003; Van Dijk, 2003; Potter, 2003a, 2003b, 2004; Ainsworth & Hardy, 2004; Fairclough, 2004; McKenna, 2004; Rogers, 2004; Stevenson, 2004; Clarke, 2005), the researcher had a more global overview of this type of research methodology and methods, which, in turn, allowed the researcher to develop a more representative discourse model(s) of the data. Nevertheless, as Gergen (1999) stated, this model will not be final. Even Gee (2005) affirmed that "Discourse models, though they are theories (explanations), need not be complete, fully formed, or consistent" (p. 85). Moreover, as Foucault (1972) indicated in his book *The Archaeology of Knowledge*, "Discourse is the path from one contradiction to another: if it gives rise to those that can be seen, it is because it obeys that which it hides" (p. 151). The analysis of discourse is, for Foucault, a way to hide and reveal contradictions. However, contradictions were not identified in the pilot study, or at least they were not reported. The pilot study was just the first step of analysis, and a path had yet to be found.

In summary, in spite of the many limitations previously identified, it was through discourse analysis that the researcher was able to explore the transformative dialogue occurring in the *alt.math.undergrad* discussion forum. The researcher started to "explain why and how things happen as they do" (Gee, 2000, p. 196); that is, how mathematics knowledge was constructed in this digital environment. (A copy of the pilot study report is located in Appendix A: Pilot Study. Also see Appendix B: UF Institutional Review Board Letter, and Appendix C: Request for Copyright Permission.)

The following chapter presents discourse analysis as a methodology and a method. It begins with a revision of discourse analysis literature, and ends with Gee's discourse analysis methodology. This is then followed by the process of applying Gee's discourse analysis to study how discussion forums are used to construct mathematics knowledge in the particular discussion forum under study. Information about validity in qualitative research and Gee’s discourse analysis, and the limitations of the study will be analyzed at the end of the chapter.

Table 3-1. *Math Forum @ Drexel* web site resources by section.

| Section | Resources |
|---|---|
| Main Areas | Search for Math Resources |
| | Student Center |
| | Teachers' Place |
| | Parents & Citizens |
| | Research Division |
| | Math Resources by Subject |
| | Math Education |
| | Key Issues in Math |
| Projects | Ask Dr. Math |
| | Teacher2Teacher |
| | Math Fundamentals: Problem of the Week |
| | Pre-Algebra: Problem of the Week |
| | Geometry: Problem of the Week |
| | Algebra: Problem of the Week |
| | Active Problem Library |
| | Math Tools |
| Features | Dynamic Geometry Software |
| | Teacher Exchange |
| | Internet: Mathematics Library |
| | Math Forum Newsletter |
| | Mathematics Discussion Groups |
| | Math Awareness Month |
| | Math Forum Showcase |
| | What's New? |
| Archives | Articles & Book Reviews |
| | Geometry Newsgroups / Topics |
| | Internet Software: Mac & PC |
| | Learning & Math Discussions |
| | Mailing Lists & Newsgroups |
| | Math Software |
| | *Mathematics Teacher* Bibliographies |
| | Math Forum Workshops |

The *Math Forum* Quick *Reference*. (2006). Retrieved November 8, 2006, from http://www.mathforum.org/special.html.

Table 3-2. Data summary for October 2004: Threads with 10 to 25 postings

| Thread | Title & General Description | Evaluation |
|---|---|---|
| 1 | *Units, and algebraic integers*<br>Total Postings: 10<br>Frequency of participation:<br>• 1 post – 6 people<br>• 2 posts – 2 people<br>Time span: 2 days | Topic: Advanced Algebra<br>Storylines: 8<br>Notes:<br>• All storylines have two postings.<br>• The initiator of this thread presents an argument that is refuted by all others. They consider it flawed. |
| 2 | *A Question about Math Curriculum (Math Instructors and Professors Please Respond)*<br>Total Postings: 25<br>Frequency of participation:<br>• 1 post – 2 people<br>• 2, 3, 4, 6 posts – 1 person<br>• 8 posts – 1 person<br>Time span: 11 days | Topic: Undergraduate Math Curriculum<br>Story lines: 8<br>Notes:<br>• A student asks for feedback from math specialists to evaluate the curriculum changes in his program of study. |
| 3 | *Need help with some integral!*<br>Total Postings: 12<br>Frequency of participation:<br>• 1 post – 3 people<br>• 2 posts – 3 people<br>• 3 posts – 1 person<br>Time span: 3 days | Topic: Advanced Calculus<br>Storylines: 4<br>Notes:<br>• Topic not taught in first and second year undergraduate mathematics.<br>. |
| 4 | *2 sinA versus sin2A*<br>Total postings: 14<br>Frequency of participation:<br>• 1 post – 4 people<br>• 2 posts – 3 people<br>• 3 posts – 2 person<br>Time span: 4 days | Topic: Trigonometry<br>Storylines: 7<br>Notes:<br>• Topic is taught in first and second year undergraduate mathematics. |
| 5 | *Extrema / Diff*<br>Total postings: 14<br>Frequency of participation:<br>• 1 post – 5 people<br>• 2 posts – 2 people<br>• 4 & 8 posts – 1 person<br>Time span: 4 days | Topic: Calculus I<br>Storylines: 8<br>Notes:<br>• Topic is taught in first and second year undergraduate mathematics. |

Table 3-2. Continued

| Thread | Title & General Description | Evaluation |
|---|---|---|
| 6 | *Tan to Slope*<br>Total postings: 11<br>Frequency of participation:<br>• 1 & 2 posts – 2 people<br>• 5 posts – 1 people<br>Time span: 4 days | Topic: Trigonometry<br>Storylines: 3<br>Notes:<br>• Topic is taught in first and second year undergraduate mathematics. |
| 7 | *Norms!!!!* | Topic: Vectors<br>Notes:<br>• Topic not taught in first and second year undergraduate mathematics. |
| 8 | *Truth Tables Help* | Topic: Logic<br>Notes:<br>• Topic not included in study. |
| 9 | *Uniform Convergence* | Topic: Advanced Mathematics<br>Notes:<br>• Topic not taught in first and second year undergraduate mathematics. |
| 10 | *Statistics*<br>Total postings: 10<br>Frequency of participation:<br>• 1 & 3 posts – 2 people<br>• 3 posts – 1 people<br>Time span: 8 days | Topic: Central Tendency<br>Storylines: 4<br>Notes:<br>• Topic is taught in first and second year undergraduate mathematics. |
| 11 | *Need this explained*<br>Total postings: 10<br>Frequency of participation:<br>• 1 post – 2 people<br>• 2 posts – 4 people<br>Time span: 7 days | Topic: Logarithm<br>Storylines: 3<br>Notes:<br>• Topic is taught in first and second year undergraduate mathematics. |
| 12 | *Latest fuss, my apologies*<br>Total postings: 19<br>Frequency of participation:<br>• 1 post – 7 people<br>• 2 & 3 posts – 2 people<br>Time span: 4 days | Topic: Apologetic Argument<br>Notes:<br>• Not relevant for this study.<br>Decision: Will not be used in pilot study. |

Table 3-3. Parts of a story with higher-order structure

| Body parts | Description |
|---|---|
| Setting | Sets the scene in terms of time, space, and characters |
| Catalyst | Sets a problem |
| Crisis | Builds the problem to the point of requiring a resolution |
| Evaluation | Material that makes clear why the story is interesting and “tellable” |
| Resolution | Solves the problem |
| Coda | Closes the story |

Gee, J. P. (2005). *Discourse Analysis: Theory and method* (2nd ed., p. 131). New York: Routledge.

# CHAPTER 4
# METHODOLOGY AND METHODS

> To teach someone the meaning of [a] sentence . . . is to embed them in the conversational sea in which [the] sentence . . . swims.
>
> –James P. Gee, *An Introduction to Discourse Analysis: Theory and Method* (2005, 2nd ed., p. 46)

In this chapter, the methodology and methods used to study the construction of mathematics knowledge through the use of a particular discussion forum is presented to the reader. Based on Crotty (1998), methodology includes “the strategy, plan of action, process or design lying behind the choice and use of particular methods” (p. 3). He stated that the methods are the “techniques or procedures used to gather and analyse (sic) data related to some research question” (Crotty, 1998, p. 3). In general, methods are more specific than methodology.

However, in this research, the methodology and methods are intertwined in the same concept. They are both classified as discourse analysis: methodology in general terms, and more specifically, Gee’s (1999, 2005) discourse analysis processes as a method. The next sections explore discourse analysis as a methodology, Gee’s perspective of discourse analysis as a method, the application of Gee’s methods to this study, the validity, and the limitations.

## Discourse Analysis Methodology

Setting the basis for discourse analysis, Austin (1962), a forerunner, looked at the components of language. His analysis was that of utterances, that is, the analysis of sentences. Others looked at discourse analysis from different perspectives. For example, McKenna (2004) discussed the analysis of the “relationship between language and society” (p. 10), Joworski and Coupland (1999) talked about “the analysis of language in use” (p. 1, as cited in Clarke, 2005, p. 148), and still others used discourse analysis as a research tool (Fairclough, Graham, Lemke & Wodak, 2004). In general, researchers looked at discourse analysis as a way to analyze language *form and function* (Fairclough, Graham, Lemke, & Wodak, 2004; Gee, 2004, p. 19, emphasis in original; Austin, 1962).

However, most authors looked only at discourse as spoken language. That was the case of Austin (1962), who stated that "to say something is to do something" (p. 108). Years later, we found Foucault (1972) paraphrasing Austin when he wrote "to speak is to do something" (p. 209). Finally, McCarty and Schwandt (2000) sustained that in attempting to "talk," people "enter the world of discourse" (p. 55-56). These authors mostly looked at discourse as spoken language, but discourse is also found in text (Wodak, 1996; Mishler, 1990; Vygotsky, 1962).

Discourse analysis is applied across the social sciences, including education and learning processes (Potter, 2003a; Potter, 2003b; Rogers, Malancharuvil-Berkes, Moley, Hui, & Joseph, 2005; Rogers, 2004). It is also considered by many as a social action (Van Dijk, 2003; Wodak, 1996; McKenna, 2004; Potter, 2004). That is because, as Fairclough, Graham, Lemke, and Wodak's (2004) indicated, "different theoretical, academic and cultural traditions . . . push discourse . . . in different directions" (p. 4).

Two current classifications of discourse analysis, one by Joworski and Coupland (1999) and another by Fairclough, Graham, Lemke, and Wodak's (2004), show the focus discourse analysis takes in research. There are several similarities between both frameworks, which make possible their combination. In general, they can be listed as (1) the speech and conversational analysis or the study of individual text and talk, (2) the negotiation discourse in social relationships or the social agents and social change, and (3) the power/knowledge, ideology, and control discourse, or the analysis of social agents and social change, respectively.

Representing the negotiation discourse in social relationships' category, we have Fairclough's (2004) and Gee's (1999, 2005) discourse analysis methodology. Conversely, two authors who focused on power/knowledge discourse are Foucault (1972) and Van Dijk (2003); the former presented his views as the archaeology of knowledge and the latter as Critical Discourse Analysis.

In 1998, Gee and Green wrote, “Discourse analysis approaches have been developed to examine ways in which knowledge is socially constructed in classrooms and other educational settings” (p. 119). To study discourse, that is to study language-in-use, is, according to Gee (2004), “inherently political” and has implications on “status, solidarity, distribution of social goods, and power.” (p. 33). Rogers and others (2005) also stated that “Gee’s theory is inherently ‘critical’ in the sense of asserting that all discourses are social and thus ideological” (p. 370). In this way, Gee’s discourse analysis has similarities to that of Van Dijk’s (2003). As such, Gee’s discourse analysis also has a critical stance, is embedded in social constructionism practices, and maintains humanistic principles.

The critical component of discourse analysis “cannot be considered neutral, because it is caught up in political, social, racial, economic, religious, and cultural formations” (Rogers, Malancharuvil-Berkes, Moley, Hui, & Joseph, 2005, p. 369). It is in this sense that discourse analysis is critical. Following this view of discourse analysis, Gee (2004) differentiated between discourse analysis using lowercase and uppercase letters; that is, he differentiated between critical discourse analysis (cda - with lowercase) and Critical Discourse Analysis (CDA - with uppercase). One way to look at his representation is through a continuum. At one end, Gee (2004) presented critical discourse analysis as “anecdotal reflections on written or oral texts” (p. 20), and at the opposite end, he presented Critical Discourse Analysis as related to political proselytism (p. 20).

According to Gee (2004), the distinction between critical and non-critical discourse analysis is related to how social practices are studied; if social practices are treated "solely in terms of patterns of social interaction" (p. 32), then the study would be non-critical (cda). However, "critical approaches . . . go further and treat social practices not just in terms of social relationships . . . [but also] in terms of implications for things like status, solidarity, distribution of social goods, and power" (p. 33) (CDA). As an example, Gee (2005) presented how language is used as a gatekeeper in a job interview. The same happens with mathematics serving as the gatekeeper for white-collar professions in areas related to engineering and medicine (Restivo, 1983; Moses & Cobb, 2001).

Gee (2004) also wrote about how discourse analysis is applied to education. He suggested that it "needs to show how a distinctive community of practice is constituted out of specific social practices (across time and space) and how patterns of participation systematically change across time, both for individuals and the community of practice as a whole" (p. 39). He added that "learning is a type of social interaction in which knowledge is distributed across people and their tools and technologies" (Gee, 2004, p. 19). By moving across time and space and adding more data to the analysis, discourse analysis allows the researcher to identify different stories in a data set (polyvocality). However, data might not at all times be consistent, and finding discontinuities is always a possibility (Hatch, 2002; Foucault, 1972).

New data helps the researcher refine the previously identified discourse models (Gee, 1999, 2000). That is why Gee's (2005) discourse analysis allows a researcher "to think more deeply about the meanings" that people give to words "so as to make ourselves better, more human people and the world a better, more human place" (p. xii). Gee believes "*language has meaning only in and through social practices*, practices which often leave us morally complicit with harm and injustice unless we attempt to transform them" (p. 8, emphasis in original). Therefore, discourse analysis is an important "human task;" it is a way to better understand those around us.

In general, Gee's discourse analysis focuses on socio-cultural practices. Through his method, researchers can decompose text while searching for breath and depth, for the details that tell a story. Discourse analysis helps the researcher understand the socio-cultural practices happening in a specific community of practice. For this reason, Gee's (1999, 2005) method was chosen to analyze the community of mathematicians that constructed knowledge in the discussion forum *alt.math.undergrad* located at the *Math Forum @ Drexel*'s web site.

In this study, discourse analysis is defined as a search for meaning "situated in specific socio-cultural practices and experiences" (Gee, 2000, p. 195). This search for meaning came from the analysis of data in an asynchronous communication system, that is, the analysis of text generated without the intervention of the researcher. It was through the analysis of threaded discussions that the multiple discourses of those collectively constructing high school and first and second year undergraduate mathematics knowledge in an online discussion forum were identified. The analysis included threaded discussions from five months. The first month allowed the researcher to construct a preliminary discourse model. Data from subsequent months were used to revise and refine the previous discourse model, generating a new discourse model of how people constructed mathematics knowledge through the use of asynchronous threaded discussions after each period of analysis.

It was discourse analysis as stated by Gee (2005) that allowed the researcher to construct a set of models coming from the data itself, letting the online participants speak-out and introduce themselves to the researcher as they constructed mathematics knowledge and generated new meanings. This type of model was identified by Gee as a *cultural model* in 1999 and 2000 and later as a *discourse model* in 2005.

The next section will examine details about Gee's discourse analysis method, that is, Gee's discourse analysis processes.

**Gee's Discourse Analysis Method**

Gee (2005) stated that "All life for all of us is just a patchwork of thoughts, words, objects, events, actions, and interactions in Discourses" (p.7). When presenting his method, Gee started by making a distinction between "little d" and "big D" in their relationship to discourse. "Little d" is about "how language is used "on site" to enact activities and identities . . . [that is, about] language-in-use" (Gee, 2005, p. 7). It can be associated to the study of form and function in language. Yet, "big D" has to do with what accompanies language, that is "one's body, clothes, gestures, actions, interactions, symbols, tools, technologies (be they guns or graphs), values, attitudes, beliefs, and emotions . . ., and all at the 'right' places and times" (Gee, 2005, p. 7).

In this research, "little d" is used to analyze the form and function of language which makes possible the construction of high school and first and second year undergraduate mathematics, that is, the types of questions and inquiries posted and the replies that promoted or limited interaction. "big D" is used when analyzing specific actions and interactions made possible through the use of technology, specifically an on-line asynchronous communication tool, the public discussion forum *alt.math.undergrad* located at the *Math Forum @ Drexel* (http://www.mathforum.org).

Gee (2005) identified two types of discourse analysis: one that studied "general correlations between form (structure) and function (meaning) in language" (such as Austin's, 1962), and another that studied "specific interactions between language and context" (p. 54). It is this last type that Gee adopted to study discourse. Gee (1999, 2005) proposed a method that included the following general steps: (1) analyze raw transcriptions and prepare transformed and theorized transcriptions; (2) use building tasks (and the corresponding questions) and inquiry tools to identify themes; (3) write a preliminary Discourse model; (4) test with new data, as in a recursive analysis; and finally, (5) validate the Discourse model. This method is explored in the following paragraphs. And after, details of how Gee's method was applied in this research are explained.

**Working with Transcriptions**

The first step in Gee's methodology is to analyze raw transcriptions and prepare transformed and theorized transcriptions. Gee (2005) suggested looking "for patterns and links within and across utterances" (p. 188). In this way, the researcher starts making conjectures about the meaning of text from the start. When working with text, the researchers "have to 'say' the sentences of the text in their 'minds.' To do this, they must choose how to break them down into lines . . .Such choices are part of "imposing" a meaning (interpretation) on a text and different choices lead to different interpretations" (Gee, 2005, p. 126).

These lines will "reflect the information structure of a text" (Gee, 2005, p. 127). In turn, Gee called these sets of lines "stanzas," which are "devoted to a single topic, event, image, perspective, or theme" (Gee, 2005, p. 127). These are also identified by Gee (2005) as "microstructures" (p. 127). Subsequently, as stanzas accumulate into larger pieces of information, "macrostructures" are created (Gee, 2005, p. 128). Gee compared macrostructures to "stories," and as any other story in literature, they can be subdivided into different components. Gee (1999, 2005) suggested the following six body parts to a story: setting, catalyst, crisis, evaluation, resolution, and coda (Table 3-3).

Gee (1999, 2005) also noted that organizing text in this fashion allowed the researcher to check for patterns in people's speech or text and find basic themes. During the phase of analysis, the researcher would “shuttle (sic) back and forth between the actual lines [raw transcript/transformed transcript] and the idealized lines [theorized transcript]” (Gee, 2005, p. 129). Gee stated that “a line and stanza representation of a text . . . [can] simultaneously serve two functions. First, it represents . . . the patterns in terms of which the speaker has shaped her meanings . . . [and] second, it represents a picture of . . . [the] analysis, that is, of the meanings . . . [the researcher is] attributing to the text” (p. 136).

Groups of stanzas can be organized into stories and then divided into story body parts, connected blocks of information that can be used to discover structure in information, and which, in turn, can help the researcher “to look more deeply into the text and make new guesses about themes and meaning” (p. 136). Working with transcriptions is the first step in Gee’s analysis.

**Building Tasks**

The analysis will continue by questioning the data already converted into theorized transcriptions with stories divided into body parts. For this phase, Gee (1999, 2005) developed a set of building tasks, divided into seven “areas of reality.” These are significance, activities, identities, relationships, politics, connections, and sign systems and knowledge (Figure 4-1). Each one of the building tasks generates “a small part of the full picture” (Gee, 2005, p. 110) because they “are deeply inter-related” (p. 104), and “together [they] constitute a *system*” (p. 102, emphasis in original). Analysis is then produced by answering a set of questions related to each one of the building tasks. The reflection that accompanies this stage helps the researcher to “give meaning to . . . language” (p. 110) and to theorize with the data, creating a theorized transcript.

However, according to Gee (2005), “Actual analyses . . . usually develop in detail only a small part of the full picture” (p. 110); that is, not all building tasks are used in every analysis.

Two building tasks were selected to conduct this research; these are the activity and connections buildings. More details about this decision are included in the next section.

**Inquiry Tools**

In addition to the building tasks, Gee (1999, 2005) uses a set of inquiry tools to help the researcher develop a Discourse model (Figure 4-2). The inquiry tools include the analysis of social languages, intertextuality, Conversations, Discourses, situated meaning, and Discourse models. These are described by Gee (1999, 2005) in the following way:

**Social languages** are “varieties of languages” that, in general, have two purposes: first, “to express different socially significant identities” and second, to “enact different socially meaningful activities” (Gee, 2005, p. 35); they are a representation of “what we learn and what we speak” (Gee, 2005, p. 37), the result of our cultural experiences and environment.

**Intertextuality** happens when “spoken or written text alludes to, quotes, or otherwise relates to, another [text],” it relates to “words that other people have said or written” (Gee. 2005, 21). Voithofer (2006) explained that texts “do not exist in a discursive vacuum,” that intertextuality interrelates “cultural, literary and historical factors that come together in a moment within a text” (p. 204). However, as will be shown below, Voithofer’s conception of intertextuality is closer to Gee’s notion of Conversations.

Expanding the notion of intertextuality, **Conversations** are related “to themes, debates, or motifs that have been the focus of much talk and writing in some social group with which we are familiar” (Gee, 2005, p. 21). When conversations include debates, people usually take sides. Conversations as an analytical tool will study “what ‘sides’ there are . . . and what sorts of people tend to be on each side” (Gee, 2005, p. 35). As Voithofer (2006) indicated, Conversations are related to cultural, literary, and historical factors.

**Discourses** study the “who” and the “what”: who are those who speak or write – their identities (their “*socially situated identity*”), and what are they doing (the activities they are completing; that is, the “socially situated *activity*”). Through discourses, the researcher can identify multiple entities (Gergen, 1999). Gee (2005) sustained that “Different social identities (different *whos (sic)*) may seriously conflict with one another” (p. 25, emphasis in original). Foucault (1972) identified this type of conflict as contradictions, irregularities in the use of words, “incompatible propositions” (p. 149).

In studying the “who” and the “what,” Gee (2005) listed the following Discourse characteristics: Discourses are always embedded in social institutions, have no clear/discrete boundaries, can be split into two or more, or can meld together. Discourses can change over time, emerge as new ones or die; be limitless. In addition, Discourses are defined in relationship with others; they are social practices and mental entities.

The next two inquiry tools, situated meaning and Discourse models, are closely interrelated, as were intertextuality and Conversations. According to Gee (2005), **situated meanings** are images and patterns, initially developed from a word and turned into a theory. Situated meanings are shared within the community in which people live; they are “rooted in the practices of the sociocultural group to which the learner belongs” (p. 60). To find meaning in context is to find the material setting where people are present; it is to find what they know and believe, “their social relationships . . . ethnic, gendered, and sexual identities, as well as cultural, historical, and institutional factors” (p. Gee, 2005, 57).

Gee (2005) agrees with Barsalou (1992), in considering situated meanings, “*mid-level patterns or generalizations*” (p. 66, emphasis in original), between two extremes, the general and the specific. However, situated meanings, like Discourses, are neither static nor definitions.

Instead, situated meanings are “flexible transformable patterns that come out of experience and, in turn, construct experience as meaningful in certain ways and not others” (Gee, 2005, p. 67). As an inquiry tool, “situated meaning” is a “thinking device” (Gee, 2005, 70).

The inquiry tools used in this research include social languages, intertextuality, discourses in terms of what people are doing and how they are interacting, and situated meanings - how they generate meaning together. These should help the researcher develop a discourse model explaining how mathematics knowledge was constructed through the use of the discussion forum *alt.math.undergrad* at *The Math Forum @ Drexel*.

**Discourse Models**

Initially called cultural models (Gee, 1999, 2000), discourse models (Gee, 2005) start by making assumptions and end with preliminary explanations of the world we live in. They help us make sense of things, understanding texts and the world; they “help us prepare for action in the world” (Gee, 2005, p. 75). Still, discourse models “need not be complete, fully formed, or consistent” (Gee, 2005, p. 85).

Moreover, discourse models are “shared by people belonging to specific social or cultural groups” (Gee, 2005, p. 95); “distributed across the different sorts of ‘expertise’ and viewpoints” (Gee, 2005, p. 95). They are what Foucault (1972) called discourse formations; that is, positivity’s. Gee (2005) also sustained that “Discourse models link to each other in complex ways to create bigger and bigger storylines” (p. 96), which in turn would approximate to Foucault’s (1972) conception of the archaeology of knowledge. The development and refinement of a discourse model will help validate the research findings.

**Reviewing the Preliminary Discourse Model**

Once the researcher is ready to construct a Discourse model, s/he has reached the third level of Gee’s discourse analysis. The initial *Discourse model statement* is then tested with more data in a recursive analysis (stage 4). As stated before, discourse models can change in different ways by adding, changing, refining, and deleting components until an explanation closer to the data set can be reached.

In this research, the preliminary discourse model is the result of Augusts' data analysis and interpretations. This model will then be revised and refined as more data is analyzed.

**To Summarize**

Gee's method offers the researcher the methodology rigor necessary to pay attention to the object of research. The process includes theorizing with data, questioning the data from different standpoints (building tasks), using inquiry tools to further analyze the data set, developing a discourse model (an explanation), reviewing, refining, and validating it with additional data. This process will help the researcher examine the transformative dialogue generated in an online asynchronous discussion forum, in which participants constructed new mathematical understandings about high school and first and second year undergraduate mathematics.

## Data Collection Procedures

As previously stated, this research analyzed asynchronous data archived in digital format from an online public discussion forum, "a leading center for mathematics and mathematics education on the Internet" (*The Math Forum @ Drexel*, 2004). Digital archived data "are free and accessible to all Internet users" (Bolick, 2006, p. 122) and take advantage of computers' characteristics. In this research, data was the product of voluntary participation in the *alt.math.undergrad* discussion forum. Data can also be classified as primary sources, since it was recorded while participants of the discussion forum interacted with each other without the intervention of others.

Data collection was limited by the time period under analysis, which included one academic semester, starting in August of 2004 and ending in December of 2004. During this period of time, the *alt.math.undergrad* discussion forum listed 761 threads that generated more than 3,800 postings. The selection of threads (topics) was narrowed down to select those with a specific number of postings and content areas offered in high school and first and second year undergraduate mathematics.

The number of postings (messages) in each threaded discussion (topic) varied in terms of participation, from less than five to more than 100. However, only threads composed of 10 to 25 messages each were chosen for analysis. The number of postings (messages) was important because the analysis incorporated identifying six body parts to a single mathematics story. This decision was based on the results of the pilot study that showed stories could be subdivided in their body parts for in-depth analysis. Threaded discussions are developed in a hierarchical style and include one or more stories, like branches of a tree. Therefore, having fewer than ten messages would make it difficult to find all of the components of a story (Table 3-3). A total of 37 threads met these criteria.

In addition, permission from the University of Florida's Institutional Review Board-02 was solicited, but reviewers indicated that no consent form was required from the participants since the archived data was previously collected (See Appendix B). Besides, no personal contacts with the users of the *Math Forum @ Drexel* discussion group were established. Only the postings with the indicated characteristics were chosen for analysis.

In summary, threaded discussions (topics) selected for analysis consisted of those with 10 to 25 postings (messages). They included topics related to high school and first and second year undergraduate mathematics. A time limit was established for analysis, and only those postings written from August to December of 2004 in the *alt.math.undergrad* discussion group of the *Math Forum @ Drexel* web site, located at a university in the east of the USA, were analyzed.

Participants who generated the data for this study were voluntary users of the discussion forum. Data was asynchronously generated, digitally archived, and accessible through the Internet. Although no consent forms were necessary (See Appendix C), the *Math Forum @ Drexel* was contacted and permission was granted to conduct this research.

**Data Analysis Process in this Dissertation Project**

Gee's (1999, 2005) discourse analysis was chosen to analyze the different storylines presented in the threaded data located at the *alt.math.undergrad* discussion forum. This type of analysis involved "asking questions about how language, at a given time and place, is used to construe the aspects of the situation network as realized at that time and place and how the aspects of the situation network simultaneously give meaning to that language (remember reflexivity)" (Gee, 2005, p. 110). Following up on Gee's statement, in this research, the time of analysis was one academic semester (August to December, 2004), the place was the *alt.math.undergrad* at the *Math Forum @ Drexel* web site, the setting was the online public discussion forum, the language was mathematics, and the simultaneous network was related to the different types of interaction, negotiation, and discursive collaboration practices taking place in each threaded discussion.

Threaded discussions are bounded by a beginning and an ending. As a result, the activity building task was selected as the main source of analysis. The connections building task helped establish relevancy within, between, and among threads. The inquiry tools used in this research (social languages, intertextuality, discourses, and situated meanings) supported the analysis of the data. This analysis permitted the identification of instances where coordinated actions conducted in collaboration with others generated meaning (Gergen, 1999).

This type of analysis also allowed the researcher to identify the sources of polyvocality present in the data. According to Gee (2005), human language supports "the performance of social activities and social identities . . . [as well as] . . . human affiliation within cultures, social groups, and institutions" (p. 1). Data in the *alt.math.undergrad* discussion forum was organized by topics (threaded discussions). The threads, composed of ten to twenty-five messages/postings, were further analyzed. Each topic (threaded discussion) was represented in a tree diagram to identify the flow of conversation and the interactions that took place between its participants. Each branch of the tree diagram represented a storyline. This also provided a means to identify the different parts of a story and to compare them with one another. In order to inform the research question, the content of each storyline and its corresponding body parts were then analyzed using the activity and connections building tasks questions designed for this research (for details go to the next section).

The use of inquiry tools and building tasks as presented by Gee (1999, 2005) allowed the researcher to develop a model of how mathematics knowledge was constructed and negotiated in an online discussion forum. This type of model was categorized by Gee in 2005 as a *discourse model* and previously, in 1999 and 2000, as a *cultural model*. Although both categorizations relate to the same concept, Gee, in 2005, decided to emphasize its discursive component. Still, the researcher prefers *cultural model* over *discourse model* because it not only takes into concern that knowledge is developed through the analysis of discourse, but it also emphasizes the context in which it was developed. The cultural component that evidences the idea of a mathematics community is emphasized in this research in accordance with Restivo's (1983) conception of sociology of mathematics.

## Questioning the Data

The specific questions used in this research were as follows:

A. From the activity building task:

1. What was the main activity going on in each threaded discussion? [Look at the main question in the first post/message, and the setting and catalyst body parts of the threaded discussion.]

2. What sub-activities compose this activity? [Look at the crisis, evaluation, and resolution body parts.]

3. What actions took place that composed the sub-activities of the activity? [Look at the crisis, evaluation, and resolution body parts.]

4. What types of solutions were presented to the participants of the threaded discussion? [Look at the resolution and coda body parts.]

B. From the connections building task: [Comparisons within the stories of a threaded discussion, and between and among threaded discussions]

1. What types of connections are made within a storyline, between storylines, and within a threaded discussion?

2. What types of connections are made with other threaded discussions and are there different threaded discussions studying the same question?

3. How is intertextuality used to create connections within, between, and among threaded discussions?

4. How do connections help to constitute ‘coherence’ in the discussion forum? [validity]

## Application of Gee’s Discourse Analysis Method

The procedure or method used in this research included the following steps; one through five identify how data was organized, and steps six through fifteen states the methods of analysis. These are as follows:

1. Identify the threads by month (August to December, 2004).

2. Choose the threads related to high school and first and second year undergraduate mathematics in each month with 10 to 25 postings.

3. Make a summary table with descriptive statistics of the threaded discussions related to this research by period of analysis, including content area, number of threads, number of postings, number of participants, and time span of discussion (from first to last posting).

4. Take each topic (threaded discussion) and draw a tree diagram showing the flow of conversation.

5. Convert each tree branch of each tree diagram into a mathematics story. In terms of Gee's discourse analysis, these are called macrostructures.

6. Analyze each story (macrostructure) to identify the story components; these are the "body parts" of a story, as stated in Gee (Table 3-3).

7. Divide mathematical stories into its components: (1) setting, (2) catalyst, (3) crisis, (4) evaluation, (5) resolution, and (6) coda. These components were used to answer the analysis building task questions.

8. Analyze the story lines (microstructure) to identify activities (or sequence of actions) taken place (building activities task from Gee's discourse analysis) – use of the questions listed above.

9. Look across the threaded discussion stories for themes; that is, types of activities the participants engaged in. These helped find patterns of communication, similarities, and differences and in turn, a discourse model.

10. Analyze the storylines to identify connections within and between stories in a thread, and between and among different threads.

11. Look across the threaded discussion stories for connecting themes taken place between the stories in each thread, and between and among the threads. These helped find patterns, similarities, and differences and in turn, a discourse model.

12. Develop a preliminary Discourse model/Cultural model.

13. Repeat steps six through twelve twice (one for data from September and October and another for data from November and December) and modify the Discourse model/Cultural model as needed.

14. Develop a cultural model, a theory that represented the data set.

15. Establish validity. (For more details on validity, see section below.). The following sources were used to review and refine the Discourse model:
    a. Reflexive notes written throughout the processes of organization and analysis
    b. Summaries developed throughout the organization and analysis of data
    **c.** Expert audits – dissertation committee

## Validity in Qualitative Research

Finding validity in qualitative research is about building credibility and trustworthiness (Mishler, 1990; Kvale, 1995; Lincoln & Denzin, 2000; Patton, 2002). For this reason, qualitative researchers open their work to the reader to be evaluated by them. They overtly present the trail that led them to the research interpretations. Still, from a postmodern view, no interpretation can be considered a final statement (Lincoln & Denzin, 2000) or a generalization or universal truth (Kvale, 1995).

To explore validity in qualitative research, arguments by Mishler (1990), Kvale (1995), and Patton (2002) are discussed below, including notes of how their ideas of validity were established in this research study. Then, a discussion will follow about how validity was explicitly accomplished in this research project.

Mishler (1990) presented validity "as a process through which a community of researchers evaluates the "trustworthiness" of a particular study" (p. 415). As such, validity is established in different ways, depending upon the type of study conducted by the researcher. In his article, Mishler presented three exemplary studies and showed how validity was established in each one. The first study was about "Life History Narratives and Identity Formation" by Mishler himself. In this study he demonstrated validity by making full transcripts and tapes available to other researchers. Methods were also used to link data, findings, and interpretation. The second study, "Narrativation in the Oral Style," illustrated validity by showing full texts and its "re-presentations," by theorizing with the transcripts, and by illustrating how interpretation was reached. In a third study, in which a "narrative strategy" was used, the author followed a sequence of steps, "a structural model for the analysis of . . . [a] passage" (p. 433), validating his work by making the process visible.

According to Mishler (1990), “validation of findings is embedded in cultural and linguistic practices” (p. 435); therefore, validation is provisional. Qualitative researchers need to show consistency in their work and thought processes so that their conclusions prove “trustworthiness.” In this research, data was available freely from the Internet, and a trail can be followed from the theorized transcripts to the data itself.

Another author who studied validity as it relates to social constructionism was Kvale (1995). He defined validity as a form to determine if a specified method can be used to investigate “what it is intended to investigate” (Kvale, 1995, “On validity and truth” section, first paragraph). However, for Kvale, that is not enough. He suggested the need to look at the researcher’s “ethical integrity” as well. For Kvale, that is a critical component used to establish the quality of scientific knowledge (Kvale, 1995, “Validity as quality of craftmanship” section, second paragraph).

Kvale’s conception of validity also included the process of recursively questioning, checking, and interpreting the research findings. Some of the strategies used in this process are “checking meaning of outliers, using extreme cases, following up surprises, looking for negative evidence, making if-then tests, ruling out spurious relations, replicating a finding, checking out rival explanations, and getting feedback from informants” (Kvale, 1995, “Validity as quality of craftsmanship” section, forth paragraph). He also stated that, in general, “to validate is to question” (“Validity of the validity question” section, second paragraph), to question everything. In this research, recursive questioning and reevaluation of a discourse model was accomplished throughout the process of analysis and interpretation across time.

Lastly, Patton’s (2002) conception of validity is related to three areas: (1) the use of rigorous methodologies; (2) the establishment of the researcher’s credibility; and (3) the presentation of philosophical beliefs. To determine the use of rigorous methodologies, Patton included Kvale (1995) questioning as well as triangulation, design, and applicability. According to Patton, triangulation can be accomplished in at least eight different ways. These are (1) using different data collection methods, (2) using different data sources, (3) having multiple analysts, (4) using multiple theories or perspectives to interpret data, (5) including participants' reviews of data and interpretations, (6) including the audience feedback, (7) adding personal reflections, and (8) using expert audits, such as doctoral committees and peer reviewers. A selection of these strategies is made to establish findings' credibility. In general, this selection is restricted by theoretical perspectives and methodologies. In this research, triangulation was possible through the use of reflexive deliberation made throughout the process of organization and analysis of data. It was also possible due to the support and guidance of the dissertation committee. Methodology rigor was accomplished by selecting epistemology, theoretical perspective, methodology, and methods that matched each other.

To determine the researcher's credibility, Patton (1995) stated that "a qualitative report should include some information about the researcher" (p. 566). This information would be related, in one way or another, to the research project. Additionally, background characteristics of the researcher and any personal information that might influence interpretations must be included. Peshkin (1988) called this type of personal report a "subjectivity statement." The researcher's credibility is also accomplished through intellectual rigor. Reviewing the data and rechecking interpretations over and over again to make sure they make sense increases the quality of analysis (Patton, 2002, p. 570). In this study, the researcher established credibility by exposing her background and personal characteristics to the reader in the subjectivity section. Intellectual rigor was possible by closely following Gee's discourse analysis methods.

Finally, validity is traditionally classified as internal or external. Internal validity refers to how findings correspond to “reality;” the closer the findings are to “reality,” the better. However, from a postmodern perspective, all findings and interpretations are mediated, viewed through the researchers’ eyes, through her/his beliefs, and through her/his background. Therefore, research findings and interpretations are limited by the implementation of methodology and the researcher her/himself. This is closely related to what Patton calls intellectual rigor, discussed above.

In qualitative research, external validity – that is, the generalization of interpretations or implications of a study – is limited to the context of the research. Therefore, many qualitative researchers will not talk about generalizations; instead, they will present preliminary findings based on the research data and its context. In Gee’s (1999, 2005) discourse analysis methods, the researcher presents a preliminary explanation in the form of a discourse model or cultural model. This research will present a discourse model of how mathematics knowledge was constructed through transformative dialogue in an online discussion forum. (chapters 5, 6, and 7).

Validation is a process that happens throughout the process of analysis. Gee (1999) identified four sources of validity in discourse analysis. These are convergence, agreement, coverage, and linguistic detail. **Convergence** is reached when “analysis offers *compatible* and *convincing* answers” to the research questions (p. 95, emphasis in original). **Agreement** is attained when “other sorts of research . . . tend to support our conclusions” (p. 95). **Coverage** is related to the applications that can be extrapolated from the data, or “being able to predict the sorts of things that might happen in related sorts of situations” (p. 95). **Linguistic details** are linked to the grammatical devices used in language. Still, validation in qualitative research is much more.

In this research, *linguistic details* validity was present when analyzing the transcripts, converting them into transformed transcripts (until storylines and body parts were identified), and then creating theorized transcripts. Examples of such are included in the data analysis chapters, evidencing case studies with raw data, corresponding transformed transcripts and theorized transcript (Mishler, 1990). This process included questioning the data in different ways and following Gee's methodology rigorously, which evidenced the researcher's ethical integrity and philosophical beliefs (Kvale, 1995; Patton, 2002). To triangulate data, the researcher used summaries, personal reflections, and expert audits (Patton, 2002). *Convergence* validity was then achieved when using theorized transcripts to answer the research question. Therefore, internal validity was accomplished through the use of *linguistic details* and *convergence* (Gee, 1999, 2005).

The research presented here analyzed a five-month period of interactions in the *alt.math.undergrad* discussion forum located at the *Math Forum @ Drexel* web site. By using Gee's (1999, 2005) methods and consistently following the set of steps presented above (intellectual rigor), a Discourse/Cultural Model was constructed. This model was first based on the findings of the first analyzed month and was then reviewed and refined with additional data of the following four months. Different storylines were analyzed to identify consistencies and contradictions, incorporating them to the initial Discourse/Cultural Model. The final Cultural Model allowed the researcher to find a preliminary answer to the research question: How does transformative dialogue and negotiation facilitate the social construction of mathematics knowledge in first and second year undergraduate mathematics via an online discussion forum named *alt.math.undergrad* in the *Math Forum @ Drexel* web site? Gee (1999, 2005) called this process *coverage validity*. Other researchers call it external validity.

## Limitations

Several limitations were present in this research project, some of which are identified in the following lines, although not in any particular order. First, this research used archived data, available through a public discussion forum on the Internet. Since the data was archived, no interviews were made, nor were interactions with the participants of the forum possible. Still, data included discursive collaborations, embedded in threaded discussions where mathematics knowledge was constructed. For this reason, the main limitation present in this research was not being able to clarify with the participants the messages (postings) they made in the threaded discussions. The use of triangulation by including participant's reviews of transformed and theorized transcripts and interpretations was not possible. However, to keep the interpretations as close as possible to the data, the researcher kept a journal with notes about the data for every month studied, wrote personal reflections throughout the processes of organizing and analyzing the postings/messages in each threaded discussion, and worked closely and in collaboration with expert auditors – the dissertation committee.

A second limitation related to the lack of interviews or personal communications with the participants is that of mainly focusing on the cognitive domain (how people develop mathematics knowledge) instead of the affective domain (what are their values, ideologies, or visions). This had a direct impact on the methodology, limiting the building tasks and inquiry tools used to analyze the data.

Other limitations were related to the theoretical perspective chosen for this research. By moving away from traditional ideas, the social constructionist theoretical perspective has generated many critiques mainly because for the social constructionist knowledge is constructed in relationship, in collaboration, in the community, and not in the individual mind. This has set the stage for criticisms about realism, experience, and other mental states such as relativism (Gergen, 1994).

One major concern of those who criticize social constructionism is their view of what is real, of "what is true, what really happened, what must be the case" (Gergen, 1994, p. 223). However, social constructionists value multiple views, multiple voices or discourses, and multiple perspectives. Social constructionism opens the door to transformative dialogue, negotiation, and increased possibilities of new understandings.

For the social constructionist, "social analysis should help to generate vocabularies of understanding that can help us create our future together" (Gergen, 1999, p. 195). Moreover, meaning is contextual, implying that meaning, understanding, and therefore knowledge are related to the context in which it is developed, to a particular time and space.

The example of the Earth being flat or round has been used to sustain one side of this controversy or the other (Phillips, 1997, section "Kenneth Gergen", paragraph 5; Gergen, 1994, p. 223). At one time in history, the world was thought to be flat, and that was true at that time and space, but later the study of the stars and new technological developments made it possible for Pythagoras (500 BC) and Aristotle (350 BC) to find new ways to describe the Earth's shape. A new conception of the Earth's shape was developed, and today it is believed that the Earth is "round." This example presents the importance of time and space and of context when making interpretations.

A second limitation to the theoretical perspective selected is related to the idea of experience and mental states. These are contested by social constructionists, mainly because they lend themselves "to an ideology of individualism – with all its invitations to alienation, narcissism, and exploitation," closing the door "to explore possibilities of alternative constructions" (Gergen, 1999, p. 227). To know what is real or objective, or even what is true or false, is not as important as the consequences they might have and the practical implications that can be made under a specific contention of truth (Gergen, 1999). However, for the constructionist, being settled in a position is never the end of the story – there can always be other ways of looking at things. In this sense, mathematics is a great example. The mathematical competencies that promoted new developments in the seventeenth century and the work of mathematicians to find "alternative constructions" for writing mathematics (notation development) would have not been possible if mathematicians had been content with previous findings or work. New possibilities and new ways of looking at mathematics generated new understandings and facilitated the construction of new mathematics. As was noted before, mathematics is a cultural expression (Restivo, 1983).

A third criticism of social constructionism has to do with the "incoherence of skepticism" (Gergen, 1999, p. 227). Some critics ask, "Isn't the social constructionist position itself a social construction?" (p. 228). The social constructionist would answer in the affirmative; and they will address the importance of reflexivity and new possibilities. (For more on this issue see the section titled "Social construction in education" at the beginning of Chapter 3.)

Again, there is no "final word" for the constructionist. Because of this, relativism is probably the main argument used against the social constructionism theoretical perspective. Critics may ask how moral and political deliberations can be made if there is no "final word." Social constructionists would answer with other questions such as: Whose moral or political stance must be taken? Is there something good for everyone, a universal truth? Why not accept that there are multiple and competing realities, a local morality, or local goods? For the social constructionist, no single voice could ever be the answered; instead, there should be conversation, dialogue, and negotiation; that is, meaning making in relationship.

In this research, transformative dialogue – interaction, discursive collaboration, and negotiation – was examined in order to develop a "discourse/cultural model" (Gee, 1999, 2005) of how mathematics knowledge was constructed in an online discussion forum. However, the researcher does not intend this model to be the final word about mathematics' knowledge construction in online environments. This model can be contested and reviewed by others, including mathematics sociologists, mathematics educators, and mathematics researchers. This model could also be the continuation of a conversation that addresses the use of technology in mathematics; it could even be an invitation for researchers to collaborate in finding new ways to teach and learn mathematics, new ways of meaning making that adds to the notion of *sociology of mathematics*. The possibilities are endless.

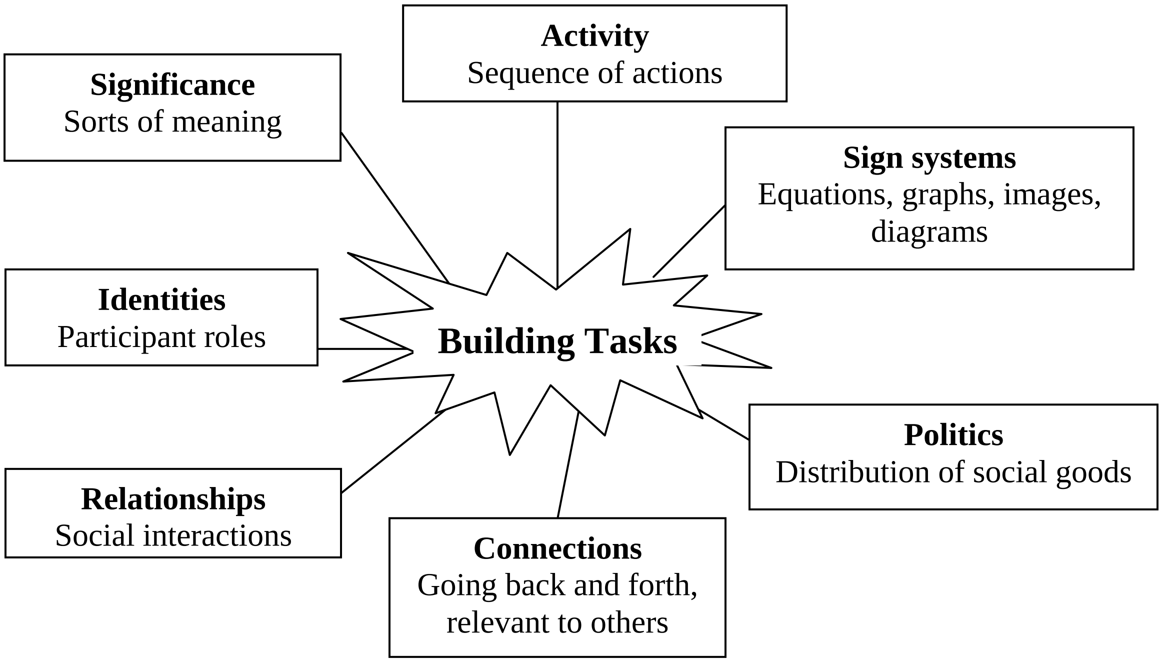


Figure 4-1. Components of an ideal discourse analysis

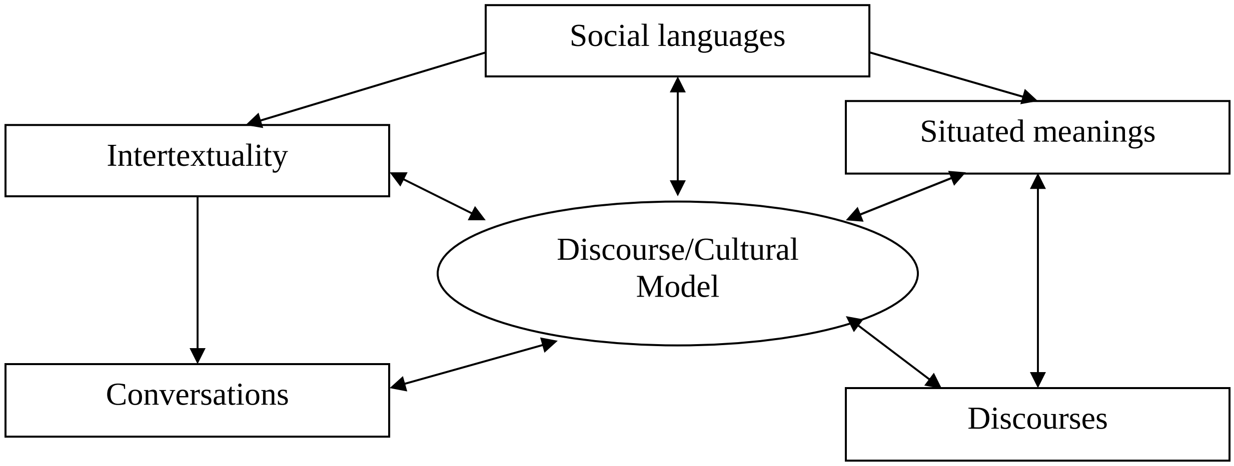


Figure 4-2. Inquiry tools used in Gee's discourse analysis

# CHAPTER 5
# CONSTRUCTING MATHEMATICS KNOWLEDGE THROUGH ONLINE COLLABORATION

## Preamble

This study analyzed five months of data from a discussion forum in the *Math Forum @ Drexel's* website. Discourse analysis, as stated by Gee (1999, 2005), was used to perform data analysis. Specifically, it followed the steps introduced in Chapter 4. In general, analysis was organized by period, thus providing the researcher the opportunity to develop a preliminary mathematics discourse model after the first period of analysis (August – Chapter 5) and then to review and refine the model throughout the following two periods (September and October – Chapter 6, and November and December – Chapter 7).

Each case of study was a threaded discussion, a set of related messages. These were analyzed to differentiate the stories present in each thread. Their structures were then converted into tree diagrams that showed several branches and sub-branches, which in turn identified different dialogues or stories within a discussion. These stories were rewritten into theorized transcripts and were further analyzed.

Each thread had a specific mathematics topic introduced as a question or problem (catalyst). These were included in the first message of the discussion (original post). On occasion, they also included a specific setting. The messages that followed examined the question or problem (evaluation and crisis components of a story) until a resolution was achieved. Some threads also included a coda adding additional information to the threads.

This chapter is the first of three data analysis chapters and includes the analysis of August's data. The organization of this chapter is similar to that of the next two data analysis chapters which include the analyses of the following months until December. The first three sections of each chapter explain the analysis of the main components of each story. The first includes the analysis of the setting and catalyst; the second contains the analysis of the evaluation, crisis and resolution; and the third provides the analysis of the coda. The researcher used the activity and connections' building tasks as the main source of analysis, using data questions listed in Chapter 4. However, data also allowed the researcher to explore other building tasks. Further details are included in the introduction of each chapter.

Finally, to end each chapter, a preliminary mathematics discourse model is presented to the reader. It shows how mathematics knowledge was negotiated and constructed throughout each period of study in the online discussion forum of the *Math Forum @ Drexel's* website. The first month, August, provided the basis for a general mathematics discourse model. The patterns that started to emerge at this point were reviewed and refined throughout the following months. Nevertheless, the final model can only be taken as a tentative way of constructing mathematics knowledge in a specific online discussion forum.

**Introduction to August's Data**

A total of four threaded discussions met the selection criteria in the month of August. As stated in Chapter 4, these were threaded discussions with 10 to 25 messages (posts) that included mathematics topics ranging from General Mathematics to Calculus. In the first period of analysis, the following mathematics topics were present: Pre-Calculus ("Exponents"), Calculus ("Radius of an Arc" and "Integrate!!"), and Discrete Mathematics ("Probabilities"). Table 5-1 shows the number of postings in each threaded discussion. They fluctuated between 10 and 17 messages and produced from one to 10 stories in a single threaded discussion.

Tree diagrams were constructed to follow the conversations that took place in the threaded discussions (Figures 5-1 and 5-2). These were sets of posts (messages), represented as rectangles, that followed an original communication (original post). Arrows coming out of a rectangle indicated that one or more replies were made to a message. Starting at the original post and following the arrows downward until there are no more arrows, a branch of the tree diagram was identified. Each tree diagram represented a threaded discussion and included one or more branches.

Branches were identified as Stories and were noted where they occurred in the diagrams. Examples of stories with two messages are found in Figure 5-1 (Stories 1, 9, and 10) and in Figure 5-2 (Story 3). Some stories included similar posts, and in the extreme cases only the last post was different. That is, they had more than one ending (different codas). A total of 20 stories were identified among the four threaded discussions selected for analysis in the August data.

Part I of this analysis describes data using the activities and connections' building tasks. Both building tasks provided the tools to identify main activities and sub-activities related to the construction of mathematics knowledge. These were organized into three main sections: "Problems, Questions, and Inquiries Introduction," with information about the setting, about how the authors of a problem or question of an original post guided the participants, and about how the authors positioned the questions; "Problem Evaluation and Solution Generation," concerning how inductive reasoning, algebra, and geometry were used throughout the evaluation, crisis, and resolution of the different stories; and "August Coda(s): Additional Information," suggesting how to use the forum to promote a better discourse among its participants.

Data from the first month also allowed the researcher to explore two more building tasks: sign systems and knowledge, and identities (Part II of the analysis). First, the sign systems and knowledge building task was used to analyze data that showed how authors used "common language and mathematics notations" as well as their comments regarding its importance. Second, the identities building task helped to examine the "Forum participants," thus allowing the researcher to describe those who interacted in the forum to construct mathematics knowledge.

## Part 1: Analysis of Activities and Connections

### Problems, Questions, and Inquiries Introduction

The setting and catalyst of each threaded discussion in the August data were included in the original post (first message of a thread). According to Gee (1999), the setting "sets the scene in terms of time, space, and characters," (p. 112) and the catalyst sets the problem.

### Setting: Use of time, space, and characters

Using the *Math Forum's* website, the Discussion Forum's space was controlled by the software itself. It provided participants two textboxes; the first was used to add a topic (which worked as a title) and the second to add a text message. This second textbox had no limitations on the number of words, symbols, lines of text, or equations used. It was in this second textbox that participants added questions or problems to further analyze them in collaboration with other participants. This site had no constraint of time, since it was available 24 hours, seven days a week.

Data showed that three out of four original posts did not include a specific problem setting; that is, they did not make reference to time, character, or space related to the math problem itself. The only thread that made reference to time and character was the "Probability" thread. Data follows:

I SETTING & CATALYST
*Stanza 1: Presenting the problem*
**Alfredo**
1 **Recently**,
2 **a prominent political figure** stated
3 that the probability of finding 2 persons
4 with the same Birthday date
5 in a group of 50
6 is almost 99%

7 . . . is it true?
8 how do i calculate that?

In terms of time, the author indicated that the problem was “recently" stated. Therefore, this problem was probably stated during the previous weeks, around the months of July or August. There were, however, no more indications of time. In this same message, a character was identified when indicating who proposed the problem, that is, “a prominent political figure.” However, no other characteristics of this person were added. Finally, there was no indication of where the author heard the problem.

The fact that only one out of the four threads selected for analysis included a setting and that it did not include many details pointed to the idea that stating a setting in a math problem in this environment was not as important for the participants as stating the problem itself. This was also a reflection of how mathematics is taught at most secondary schools and college. There is an emphasis on abstract ideas instead of concrete or real life problems. The following chapters will continue to examine how the setting was used to present mathematics problems and questions to confirm or reject this preliminary outcome.

**Catalyst: Analysis of problems and questions**

Most authors started the threaded discussions with the presentation of a question based on a specific problem. These were introduced in different ways. Participants included explanations and examples and paraphrased questions or problems. For example, in the thread “Radius of an Arc,” the question was written at the beginning of the message and was followed by an explanation that included a paraphrased question. The author then ended the message with a comparison to another problem. The following portion of data shows this interaction.

I CATALYST

*Stanza 1: The Problem*

**Sebastian**

1 Is it possible to find

2 the radius of an arc
3 given just two points
3a (coordinates)
4 of the arc?

*Stanza 2: Explanation*
5 In other words,
6 every arc
6a is part of a circle
6b with a certain radius.
7 If two coordinates
7a (x1, y1) and (x2, y2)
7b of the arc are given,
8 can the radius
9 be worked out
10 using those two coordinates?

*Stanza 3: Comparison*
11 I have been able to do this
12 given three points of the arc
13 but the application
14 I am working with
15 has only
16 two known points
16a of the arc.

In the first Stanza, Sebastian presented the main question of the problem to the forum participants and included specific conditions and an alternative way to identify the points in a graph (the coordinates). The second Stanza started with the connection clause “In other words” and was used to elaborate on the question presented in the first stanza.

As shown in the data sample above, Sebastian wanted to make sure that the question he posed was understood. For this reason, he decided to use different methods to state his problem. First, he presented the question (Lines 1-4), and then he included an explanation (Lines 5-7a). In this way, he stated the conditions under which the problem had to be evaluated. He also paraphrased the initial question (Lines 8-10) in an attempt to make clear what he initially meant. To end this original post, Sebastian added a comparison to a similar problem (Stanza 3), an explanation of when he was able to find the radius of a circle. He guided future participants in a specific direction so that the evaluation of his problem was as accurate as possible. Thus, the author tried to be very clear and specific about his question.

A counterexample was found in the "Integrate" thread, where the author asked for help by stating a direct question.

I CATALYST
*Stanza 1: Presenting the problem*
**María**
1 I need some help
2 to find step by step integral!!!!
3 sqrt(1+x^2)dx =??????

In this case, no details are offered to the participants; there is just a question with no examples or paraphrasing. No pattern seems to be used to present the mathematics question to the participants of the forum. The only commonality between this post and others was that like most of the original posts, there was a question.

During the first month of analysis, most participants (three out of four) used direct or indirect questions to introduce their problems or questions. For example, participants stated, "I need some help to find . . .," "Is it possible to find . . .?," "Can the . . . be worked out?," "Is it true?" "How can I calculate . .?" Through these indirect and direct questions, original posts' authors took the voice of those who seek for support from more knowledgeable others. Some included a tone of skepticism ("Is it true?"); some asked for assurance ("I have been able to. . ."); some overtly sought "step by step" procedures; and still others asked for analysis ("Is it possible to . . ."). In most cases, participants entered the Math Forum to find help, and they asked specific questions to locate more knowledgeable others who could provide that help.

Having the question at the beginning or end of a message did not significantly influence the number of replies in these threaded discussions. Original posts with questions at the beginning of the messages generated between eleven (11) and sixteen (16) replies. Those with questions at the end of the messages generated nine to fourteen (14) replies. However, not all original posts included a specific question. In the thread “Exponents,” the original post included a problem with a possible solution and an example.

I CATALYST

*Stanza 1: Presenting the problem*

**Frank**

1 Raising 3^x = 0 to 1/x
2 We get 3 = 0^(1/x)

*Stanza 2: Presenting a possible solution of the problem*

3 Using 3^x = y
4 taking the natural log
5 we have x ln 3 = ln y
6 or x = (ln y)/(ln 3),
7 a simple log function
8 not defined at y = 0

As shown above, there was no specific question in this thread; only a problem and a possible solution were stated. Frank’s mathematical statement was a statement that supported a specific idea instead of a question that looked for more information, help, or corroboration. In this post, there was no overt guidance of what the author wanted or needed. Instead, Frank showed how he started solving the problem by explaining how he had analyzed the problem. This post presented the opposite situation of that of Sebastian (above). Nevertheless, this did not discourage participants of the forum, and they engaged in an interchange of ideas on the topic that led to a discussion between two participants, a controversy interrupted by a more knowledgeable third party who ended the thread.

In summary, two major ideas can be gathered from these examples: first, most threads included a question in the original post, and second, the location of the question did not affect the number of replies. Additionally, in most threads (three out of four) the first message did not include a specific setting to the problem. Instead, authors chose to present the problems (catalysts) using different methods such as questions, paraphrased questions, examples, and other comments that guided the participants of the forum to answer the question presented to them. Only one thread included a partial setting that gave the time and character associated with the problem, but this information was not relevant to answering the question.

The questions in the catalyst section were included in different areas of the messages. Whether at the beginning or at the end of the post, the placement of the questions did not influence the number of replies received. A counterexample was found when the author of one post did not include an overt question (no direct or indirect question) but only included a problem with a possible solution. Although this did not discourage the users of the forum from engaging in the evaluation of the problem, only one story with two participants was generated from this original post, and it ended with a message from Tom, a third, more knowledgeable participant who clarified and thus ended the discussion. The author of this thread did not participate in the conversation; instead he stayed in the background until the problem was resolved by Tom.

**Problem Evaluation and Solution Generation**

After initiating a new thread by asking a question or presenting a problem, voluntary participants engaged in a dialogue that promoted the construction of mathematics knowledge. Participants worked together to build “the problem to the point of requiring a resolution” (crisis), including “material that [made] clear why the story [was] interesting and tellable (sic)” (evaluation) until they found one or more solutions (resolution) to the problem (Gee, 1999, p. 12).

The following pages include the analysis and interpretation of the participants’ interactions that exemplify how they collaborated to construct mathematics knowledge. First, algebra manipulations were used to solve mathematics problems. However, using algebra as the sole source of information to find the solution to problems was not always enough. The principles and definitions behind a problem pointed to specific algebra uses and manipulations. Participants also used geometric definitions and drawing explorations to develop mathematics knowledge. For example, in the thread “Radius of an Arc,” participants engaged in a series of posted contributions that included specific and general algebra solutions as well as geometric explorations. An example of algebraic misinterpretation was found in the thread “Exponents.”

In the thread “Radius of an Arc,” participants evaluated Sebastian’s question in different algebraic ways. For example, in Story 2, Ralph introduced inductive reasoning through algebraic manipulation. He tested different coordinates that satisfied the premises stated by another participant of the thread in a previous message.

*Stanza 15: Building a generic case through inductive reasoning: Case 1*

**Ralph**

104 How about: x^2 + (y - 4)^2 = 25<br>
105 Let's test it . . . for (3, 0):<br>
(3)^2 + (0 - 4)^2 = 25<br>
9 + 16 = 25<br>
25 = 25<br>
check!<br>
106 [test it for] (-3, 0):<br>
(-3)^2 + (0 - 4)^2 = 25<br>
9 + 16 = 25<br>
25 = 25<br>
check!

107 So a circle<br>
108 of radius 5 with center (0, 4)<br>
109 passes through (3, 0) and (-3, 0).

*Stanza 16: Case 2*

110 Or how about: x^2 + (y - 5)^2 = 34<br>
111 [test it for] (3, 0):<br>
(3)^2 + (0 - 5)^2 = 34<br>
9 + 25 = 34

34 = 34
check!
112 [test it for] (-3, 0):
(-3)^2 + (0 - 5)^2 = 34
9 + 25 = 34
34 = 34
check!
113 So a circle of radius sqrt(34)
114 with center (0, 5)
115 passes through (3, 0) and (-3, 0).

*Stanza 17: A generic case*
116 Or how about:
117 x^2 + (y - 6)^2 = 45
118 x^2 + (y - 7)^2 = 58
119 x^2 + (y - 10)^2 = 109
120 and even
121 x^2 + (y - k)^2 = k^2 + 9

In Stanzas 15 and 16, Ralph included detailed work, showing all the steps (algebraic manipulation) he made to test a possible solution. In Stanza 17, he presented three more equations (lines 117 to 119), but he did not include all the details, leaving it for the reader to complete the same way he previously did in Stanzas 15 and 16. In Line 121, Ralph presented a generic equation that could be used to find more specific cases.

In Story 4 of the same thread, Jack also built on Joe's coordinates to find a solution to the problem. He stated a similar argument to that of Ralph but from a more abstract position (See Stanza 15 below). Equations were stated, and possible values for the variables were given (Stanza 16 and 17), but details were left out for the reader to complete.

*Stanza 15: Equation: Abstract statement*
**Joe**
104 x^2 + (y-b)^2 = c^2
105 will be a circle
106 through both
107 (3,0) and (-3,0)
108 whenever 3^2 + b^2 = c^2,
109 which will be true
110 for infinitely many pairs

111 of values for b and c

*Stanza 16: Cases of study*

112 In particular,
113 for b = 0, c = 3 and
114 for b = 4, c = 5,
115 so right here we have
116 two circles of different radii.

*Stanza 17: Arriving to a generic solution*

117 In fact, for every real number b,
118 let c = sqrt(9 + b^2) and
119 you will get as many circles,
120 x^2 + (y-b)_^2 = c^2,
121 as there are real number values for b,
122 all passing through both
123 (3.0) and (-3,0).

RESOLUTION

*Stanza 18: Stating a solution*

124 So the number of circles
125 is uncountably (sic) infinite,
126 much "larger" than 1

Starting with a general argument (Stanza 15), Jack wrote an equation, an abstract statement (line 104) that was explained in the lines that followed. He based his argument on Joe's previous example (Stanza 4) of two possible coordinates that belong to a circle. Jack went on to show two particular cases where the equation held (Stanza 16, Lines 113 and 114), demonstrating that it was possible to find more than one circle passing through the same two coordinates. Then, in Stanza 17, Jack presented a general solution to the problem.

Ralph and Jack both used a similar perspective when analyzing this problem. They both used their knowledge of algebra to build a solution to the problem. Ralph started his presentation by showing the readers all of the details on how to solve the equations he posed. Jack, however, left these details to the reader. His presentation was more concise but also more abstract, leaving much of the work to the reader. The "Radius of an Arc" question was also analyzed from other mathematical perspectives in this thread. In the section “Geometry references and drawing explorations,” the geometric counterpart of this analysis will be presented to the reader. First, however, a look at algebraic misinterpretations will follow.

When users of math do not take into account the principles and definitions related to a specific problem, they could engage in algebraic misinterpretations. This is not only related to computation errors; it also includes the use of specific principles to analyze a problem and find a solution. That was the case presented in the thread titled “Exponents.”

Technical explanation: In pure mathematics, the equation 3^x = 0 has no solution. This equation reads three to the ‘x’ equals zero or three risen to the exponent ‘x’ equals zero. To find a solution to the equation 3^x = 0, you would need a value for ‘x’ that makes both sides of the equation equivalent, but that value does not exist. By definition 3^0 = 1, and as you substitute x for a positive integer, you will get a value greater than one, that is a positive integer such as 3^1 = 3, 3^2 = 9, and 3^3 = 27. If you use positive fractions to substitute for the exponent ‘x’, the equation will still give you a positive value. For example, 3^(1/2) = 1.73 (the square root of three) and 3^(1/3) = 1.44 (the cube root of three). Otherwise, if you consider negative fractions or negative integers such as -1, -2, -3, the solution to this equation will still be positive. Other examples can be 3^(-1) = 1/3 = .33 and 3^(-1/2) = 0.58. Thus, when evaluating 3^x, only positive solutions are possible; that is, the solutions are greater than 0. Therefore, 3^x is never equal to zero, and 3^x = 0 has no solution. Figure 5-3 shows a graph of 3^x = y (where ‘y’ is any possible solution). It shows how the curve of 3^x is above the x-axis, producing only positive solutions. A close examination of the graph shows that the curve approaches the x-axis in the negative side (to the left hand side on the horizontal axis), but never touches it.

This problem was the source of controversy developed by Mike and Jack, two of the four participants in this thread. The author of the thread, Frank, stayed in the background without making any other comments. The forth participant, Tom, also stayed silent until the end, at which time he presented the resolution. Throughout the evaluation and crisis of this thread, Mike and Jack engaged in a dialogue in which Jack tried to convince Mike of the possibilities of finding a solution to the equation 3^x=0. As shown above, in the technical explanation, such a solution was not possible. The controversy was not resolved until a third party interrupted the controversy. Mike and Jack started their interaction as follows:

*Stanza 3: Problem evaluation*

**Mike**

9 The exponential function 3^x
10 never takes (sic) the value zero.
11 You are starting from an “equation”
12 that can never be satisfied.

*Stanza 4: New possibilities*

**Jack**

13 True,
13a if we’re dealing
14 only with real numbers,
15 but in the extended real numbers
15a [-oo, +oo]
16 3^x = 0
16a when x = -oo

In Stanza 4, Jack tried to introduce a new mathematical interpretation to the problem. He explored the idea of the exponent as a negative infinitive (Line 16a), but Mike tried to maintain the definition of the equation based on mathematical principles. In the lines that followed, they pushed each other to try to interpret and reinterpret the equation beyond its mathematical meaning. Jack was trying to generate new meaning and Mike was keeping his ideas tied to mathematics fundamentals. Their voices took different positions, presenting their ideas in different ways.

This was an example of how algebra is restricted by mathematical definitions and principles. Although a math rule allows one to complete a step in algebra, taking that step does not necessarily make sense. There is more to algebraic manipulation than simply following the rules. In this particular case, the forum served as a means to explore possibilities that might have been dismissed at once in the classroom due to a lack of time for discussion. It also allowed the space for a more knowledgeable other to present the definition, origin, and use of exponents, not without first questioning the controversy generated by Mike and Jack about “extended numbers.” As Tom stated in the following stanza:

*Stanza 16: Evaluating the controversy*

**Tom**

96 What could be the motivation
97 of someone
98 to want to consider
98 Inf and -Inf as extended "numbers"?
99 That is not helpful

Afterwards, Tom went on to introduce the definition and history of exponents (Stanzas 17 & 18). He also made a reference to a professional article published in *The American Mathematical Monthly* by Knuth (1999). In this way, the forum expanded its frontiers to professional literature, which could serve as an invitation to go beyond the discussions and controversies taking place in the forum.

In summary, using algebraic manipulations to solve problems was used in the forum in different ways. These included three formats: one, giving all details of the work or step-by-step solutions; two, giving enough information to solve a problem but leaving out the details for the readers to complete; and third, giving abstract solutions to solve a problem to generalize a solution. The problems posted in this month also showed how algebraic manipulation was not always enough to solve a problem. It also showed how the use of mathematical definitions as well as the use of mathematical properties is important to reach mathematically supported solutions. Otherwise, confusion and misinterpretations can result.

In addition to algebra, geometry was also used to explore problems and find solutions. Geometry, the study of lines, surfaces, points, and curves, helped the participants of the forum to evaluate the problem posed by Sebastian. A rich set of possibilities was submitted and elaborated on throughout the messages of this thread by multiple participants. For example, John first used the concepts of "perpendicular bisector" and "three non-collinear points," Second, Joe wrote about the "endpoints of a diameter;" third, Jack introduced the concept of "the minimum possible radius," and finally Ben showed a way to explore this problem on the Cartesian graph. These ideas were spread throughout different stories of the same threaded discussion, thus connecting different ideas among the stories of this thread. Participants were collaborating with each other and helping Sebastian interpret the problem from different geometric angles. John presented the following resolution and evaluation:

*Stanza 4: Answering a question*

**John**

17 No
18 Its not possible to
19 uniquely determine
20 the center of the circle
21 given only two points
22 on the periphery

*Stanza 5: Perpendicular bisector*

23 Each point
24 on the **perpendicular bisector**
25 of the side connecting
26 the two points
26a (x1, y1) and (x2, y2)
27 will do.

*Stanza 6: Three non-collinear points and three perpendicular bisectors*

28 If you work with
29 **three non-collinear points**

30 the center of the circle
31 is the intersection
32 of the **three perpendicular bisectors**
33 of the sides connecting the points
33a (x1, y1) and (x2, y2),
33b (x1, y1) and (x3, y3),
33c and (x2, y2) and (x3, y3).

In Stanza 5, John gave a solution to the problem without an explanation. The reader needed to have an understanding of the geometry terms introduced in the message in order to understand it.

The same happened in Stanza 6 when John continued to debate about other ways to find the solution to the problem posed by Sebastian. This time John introduced two concepts in the same stanza. These were ‘three non-collinear points’ and ‘three perpendicular bisectors’ with their corresponding coordinates. To understand what John was saying, the readers could draw their interpretations on a piece of paper, use computer software to sketch out the propositions stated, or they could mentally draw the diagrams posed by John.

A second geometric analysis of the problem was posed by Joe when he used the concept of endpoints of a diameter to clarify when it was possible to find a single circle. This idea was previously posed by other members of the forum, when presenting the algebraic solution. According to Joe,

*Stanza 19: About endpoints*

**Joe**
116 It's true that
117 if
118 (-3,0) and (3.0)
119 are the **endpoints of a diameter**,
120 then
121 there is only one circle.

*Stanza 20: Infinite circles*

125 In fact
126 there are infinitely many circles

127 that contain the points
128 (-3,0) and (3,0).

*Stanza 21: Using an equation – back to algebra*
128 Any equation of the form
129 $x^2 + (y-k)^2 = 9 + k^2$
130 (for any real k)
131 is the equation of a circle
132 that contains those two points.

In Stanza 19, Joe used the logic construction “if-then” to introduce the concept of endpoints of a diameter. With this construction, he stated the conditions under which only one circle can pass through the coordinates (-3,0) and (3.0). As Joe stated, this can occur when the coordinates are the endpoints of a diameter. In Stanza 20, he further explained that there can be an infinite number of circles passing through these coordinates. He did not explain why or how this was possible in this story. Instead, he connected his geometric explanation with the algebraic solutions given before by other members of the forum.

Furthermore, Joe explained his last proposition in Story 2 when he stated that “any point on the y axis / could be a center” of the circle with coordinates (-3.0) and (3,0) (Lines 22 and 22a). He implicitly stated that the radius did not need to be three (3) at all times. Jack, in Story 10, also referred to this point when he stated the following:

*Stanza 4: Minimum possible radius*
**Jack**
18 One can find
19 a **minimum possible radius**,
19a half the distance
19b between the points
20 but no maximum

The idea that there was no maximum number for the radii suggested the infinite number of solutions in this problem. This last point was also explained by John in Story 3 when he suggested looking at the Cartesian graph and drawing some circles (See lines 145 to 149 below).

*Stanza 24: Ends of a diameter*

**John**

135 if you have the points
135a (-3,0) and (3,0),
136 those do not have to be
137 the **ends of the diameter**
138 of the circle
139 They could be points
140 near the top of
141 a really big circle
142 They could be
143 a little above or below
144 the diameter of a circle

*Stanza 25: Cartesian graph*

145 Look at a Cartesian graph
146 and draw some circles-
147 you'll see that you can draw
148 a LOT of circles
149 going through -3,3 on the x-axis.

In summary, by contributing to the analysis of the problem and working with different concepts and viewpoints, participants of the forum in this particular thread engaged in a distributed discussion giving pieces of the solution throughout their messages along different paths of the tree diagram. This showed how the messages in the threaded discussion, though divided into 10 different stories (Figure 5-1), were connected to one another. One reason that made this possible was that five of the seven participants, including the author of the original post, wrote from two to five messages each. Their dialogue represented an instance of intertextuality, where one idea was completed by the same person at another time, or even when somebody took the idea of another person and completed it. The asynchronous characteristic of the medium used in the forum allowed this to happen.

In general, participants constructed mathematics knowledge in four ways: (1) using algebraic manipulations, (2) clarifying algebraic misconceptions, (3) using geometric concepts, and (4) recommending the use of geometric constructions.

**August's Coda(s): Additional Information**

Codas were used to show how to address a question or idea to the forum participants. For example, in a follow-up message, the author of the "Radius of an arc" thread disagreed with another participant by insisting on the correctness of his position. He stated:

*Stanza 5: Insisting on his point*

**Sebastian**

23 I'm convinced
24 there is a way
. . .
27 You may have overlooked this
. . .
38 there is only ONE point
39a on the y axis
40 and NOT infinite points

Even though they included several misunderstandings, these statements were not challenged overtly in any of the immediate messages. Instead, participants gave examples and presented different ways to look at the problem from algebraic and geometric points of view; they also clarified misunderstandings. The significance of Sebastian's post was purposely overlooked by other participants of the forum who took a more constructive position and showed him how to evaluate the problem without overtly saying that he had a misunderstanding.

Sebastian's post generated four messages that led to even more replies. However, it was not until later, in Story 7, that Joe, in Post 13, addressed Sebastian's statements and gave him some alternative ways to present his ideas to the participants of the forum.

CODA from Story 7

*Stanza 19: Recommendations*

**Joe**

122 make statements like
123 "I don't see how
124 that can be true"
125 or
126 "But it seems to me
127 that 2 points are enough"
128 or

129 "Can you explain further
130 why there are
131 an infinite number of circles?
132 I can imagine
133 only the one with center at (0,0)
134 and radius 3."

*Stanza 20*: Apology

**Sebastian**

139 My sincere apologies
140 to everyone . . . :(

Joe's propositions evaluated the manner in which Sebastian presented his ideas to the forum. He showed Sebastian three alternative ways to follow-up on his initial question (See Lines 123-124, lines 126-127, and Lines 129-134). At that point, Sebastian was only able to apologize for his initial comments (See Stanza 20). His message had already been evaluated by other members of the forum who clarified his misconceptions with a variety of algebraic and geometric examples and arguments. Nevertheless, Joe's message was not unimportant; he gave others options and ideas about how to address the forum when asking follow-up questions in a way that would promote a better discourse between the participants of the forum.

## Part II: Analysis of Sign Systems, Knowledge, and Identities

### Symbols of Common Language and Mathematics Notation

In presenting their mathematics questions or problems, authors mostly used words, but on occasion they also used basic mathematics notation (symbols) or abbreviations. For example, in the thread "Radius of an arc," Sebastian identified two coordinates as (x1, y1) and (x2, y2). In mathematics, coordinates of a point use numbers as subscripts, but the discussion forum system did not provide the opportunity to format text in any way other than simple text (like that of a typewriter). Here Sebastian decided to write the number after the letter, which did not change its interpretation. This same notation was used by other participants of this thread to identify other geometric constructions.

In the thread about exponents, Frank also used the symbols available on the computer keyboard to present his problem. He stated the equation "3^x = 0", where x was the exponent. The notation he used included the caret (^), which was common in scientific and graphic calculators. These devices show the symbol on their screen but not on the keypad, where exponent keys, such as $x^2$, $x^y$ and $x^n$ are used. The computer keypad shows the caret symbol (^) on the number 6 key.

In the thread "Integrate," María presented a third example of how alternative ways to write mathematics symbols were used. She wrote the abbreviation of square root as *sqrt* in line 3, the same way it is used in some spreadsheet programs such as *Microsoft Excel*. She also spelled out the word "integral" without using abbreviations. There was no symbol available on the keyboard to substitute for the word integral, nor was there an abbreviation available in common software.

These examples show that transfer from using other technologies such as calculators, computer hardware (keypad), and computer software (*Microsoft Excel)* occurred when writing about mathematics problems. In an environment that only allowed for simple text, the writing limitations were not an obstacle for the forums' participants; these limitations were overcome by using other techniques borrowed from other technologies. Members of the forum used abbreviations, symbols available on the keypad, or even spelled out words to clearly communicate the problems and questions they were posting.

Although critics stated before that simple text environments were not conducive to mathematical language in online environments (Smith, Ferguson, & Caris, 2003), the participants of these threads found ways to introduce and evaluate mathematical terms. They transferred notation from calculators and computer software programs and spelled out complete words when no other symbols were available. In this way, the authors of the threads and those with whom they interacted were able to express themselves, to present mathematical questions or problems to the forum's participants, and to develop a set of possible solutions that allowed them to reach a resolution.

Mathematics notation is an important tool in writing mathematics. According to Tom, in the "Exponents" thread, the proper notation helps mathematicians in a number of ways. He made the following argument when addressing Mike and Jack and their controversy about the use of infinite numbers.

100 notational conventions are put in place
101 for reasons of efficiency
102 of thinking and writing
103 and an aid to instant
104 recognition / apprehension
105 of complicated expressions,
106 to simplify and clean up
107 our writing of math.

In lines 101 through 107, Tom explained why mathematical notation was important, giving three main reasons: (1) "efficiency of thinking and writing" (Lines 101 to 102), (2) "aid to instant recognition/apprehension of complicated expressions" (Lines103 to 105), and (3) simplification of mathematics writing (Lines 106 to 107). In this way, Tom reinforced the importance of using mathematical notations.

In summary, writing in mathematics is not restricted to words or text; it also includes the use of symbols. As Tom indicated, symbols are used to shorten written explanations or expositions. Symbols are also a part of the mathematics language used by all mathematicians. Even though their meanings can cause confusion to new users of math, the participants of the forum promoted their use and clarified misunderstandings when they arrived. They transferred their knowledge from other technologies such as calculators, computer software, and keyboards to express their mathematical ideas. In writing mathematics, participants of the forum generally used symbols, abbreviations, or spelled out words in their messages just as do authors of mathematics books.

**Identities of the Forum Participants**

Little information is known about the participants of the forum. They were voluntary participants who could be using pseudonyms. In this study, participants of the forum were not contacted at any time, and pseudonyms were used throughout the analysis to protect their online identities.

One reference was made about the type of participants of the forum. Tom, in the thread titled “Exponents” presented his concern about participants becoming confused when arguments were unclear. He identified three types of participants, as follows:

*Setting 15: About previous discussion*

. . .
69 novice or
70 casual students or
71 users of math

In this way, Tom took the position of a different type of participant, a more knowledgeable other who was concerned about the learner. Participants also included those who provided extra information, thus complementing and even supplementing the forum discourses. For example, Tom and Joe gave references to a journal article and a web page, respectively, that further explored the topic under consideration.

Therefore, there were at least two general roles engaged in the discourses of the forum: (1) on one side the novices, casual students, and users of math; and (2) on the other side, the more knowledgeable others, possibly mathematics teachers, instructors, or professors.

This concludes the analysis of data for the first period of analysis, the month of August. Putting together the findings stated above, a preliminary discourse model is presented to the reader in the next section.

## Moving toward a Discourse Model: Summary of August's Data Analysis

To develop the first discourse model the researcher decided to use a set of questions that could help her organize and summarize the data set. She used six basic questions: what, who, when, where, why, and how. The main focus of this research was answered with the how question which listed the activities and connections the participants engaged in (Figure 5-4).

The discourse model presented in this chapter is just a partial account of what the final proposed model will be. This model is based on data from four threads of the discussion forum. Patterns are not clear at this moment; they are just beginning to emerge. This preliminary discourse model only indicates how participants could construct knowledge in such an online environment. It will be revisited and reevaluated in the following months. This first model is a tool of inquiry for the following periods of analysis.

Novice, casual students, or users of mathematics, together with more knowledgeable others collaborated in evaluating and finding different kinds of solutions to mathematics problems in the areas of pre-calculus, calculus, and discrete mathematic, during the month of August. Participants used the following techniques to collaborate while constructing mathematics knowledge:

- Locating questions in different parts of the original message of a thread and accompanying such questions with examples, explanations, and paraphrased questions
- Presenting inductive algebraic reasoning and step-by-step procedures, presenting partial procedures so that others could complete the work, presenting generalizations and abstract statements, and connecting algebraic manipulations to mathematics definitions and principles
- Presenting different geometric concepts and constructions related to a single problem to explore different ways of analyzing and solving a problem.
- Connecting algebra and geometry to explore a single problem.
- Using common language as well as mathematics symbols and abbreviations to communicate.

- Complementing and supplementing evaluated problems by presenting references to documents outside the forum, such as a webpage and a professional journal article.

- Promoting the use of a more open and inviting language to ask for follow-up information by providing specific examples.

The next chapter presents the data analysis for the months of September and October. It concludes with a review of this Preliminary Mathematics Discourse Model

Table 5-1: General description of threaded discussions in August

| Title | Content Area | Number of Postings | Number of Stories | Number of Participants | Time Span in Days |
|---|---|---|---|---|---|
| Radius of an arc | Calculus | 17 | 10 | 7 | 4 |
| Exponents | Pre-calculus | 10 | 1 | 4 | 2 |
| Probability | Discrete Math | 14 | 4 | 9 | 2 |
| Integrate!!! | Calculus | 12 | 5 | 9 | 3[a] |

Note: a) The threaded discussion “Integrate!!” was developed in three different periods: one day in March, one day in July, and one day in August.

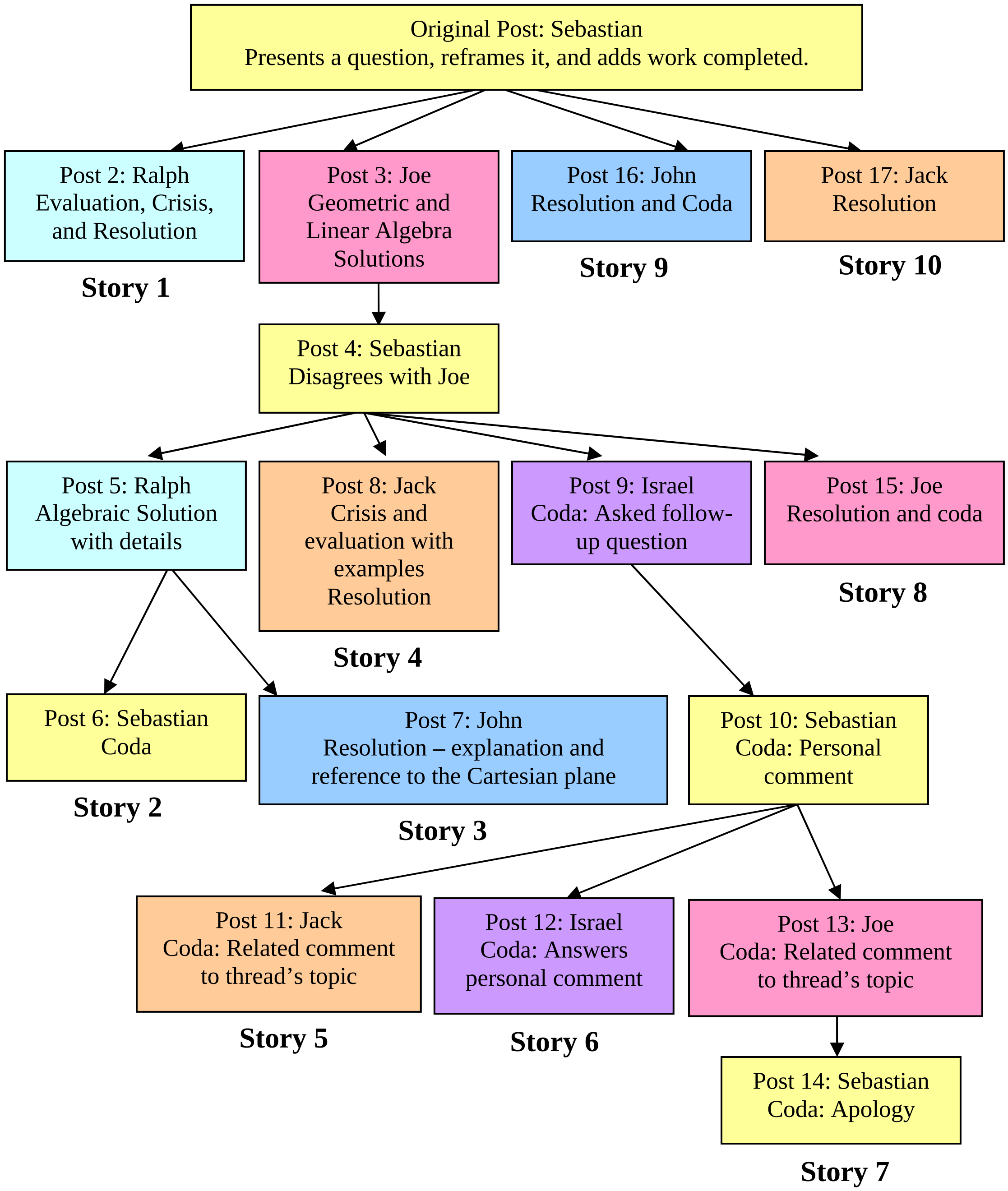


Figure 5-1. "Radius of an arc" tree. Each color represents a different participant. For example, the author of the original post participated five times in this thread (see yellow boxes) and was the one with the most participation. All other participants wrote from one to three messages each. This threaded discussion generated a total of 17 posts, organized into 10 stories, with 6 participants.

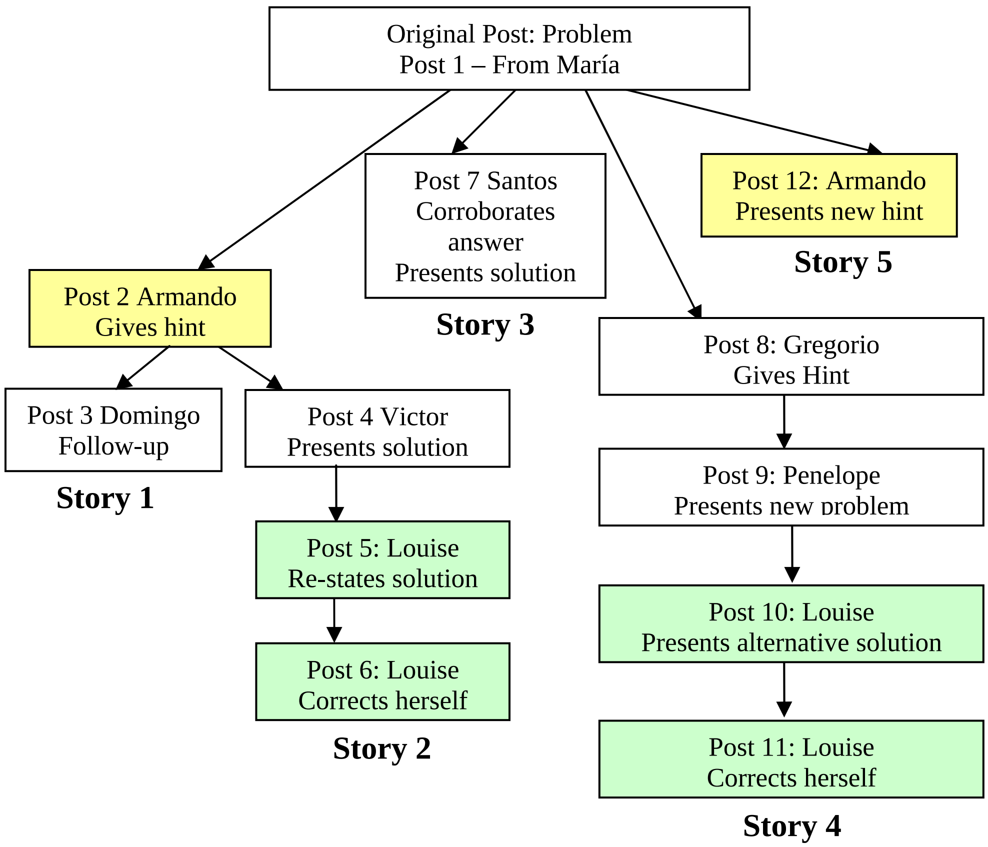


Figure 5-2. “Integrate!!!” tree. This threaded discussion has a total of 12 posts, organized into 5 stories with 8 participants. White color boxes show participants who only contributed once to this discussion.

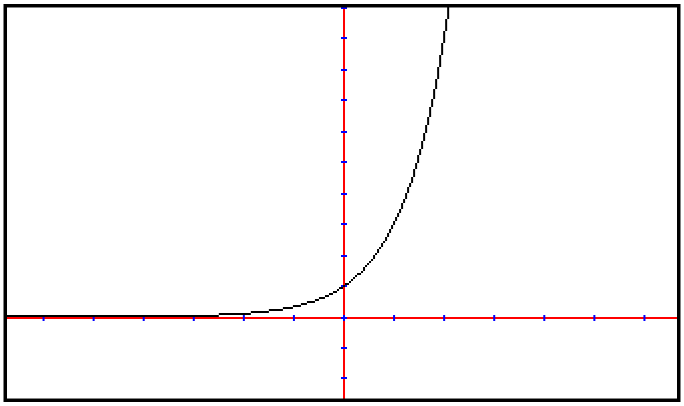

Figure 5-3. Graph of 3^x=y

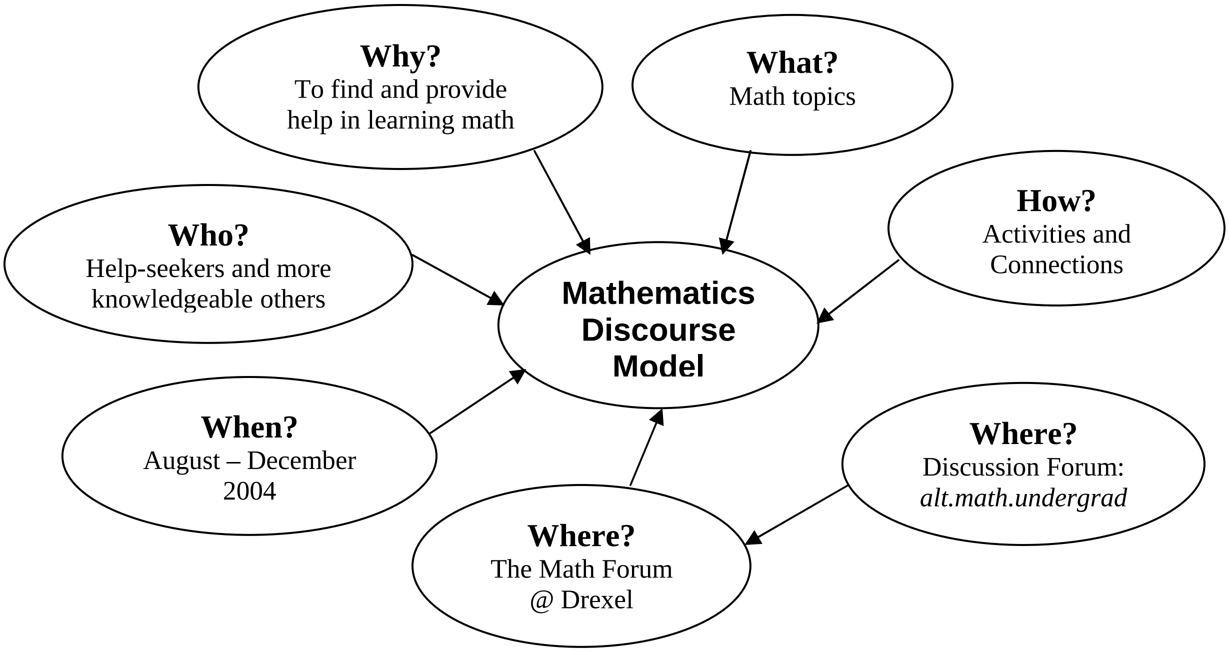


Figure 5-4. Writing the discourse model

# CHAPTER 6
# NEGOTIATING MATHEMATICS MEANING AND UNDERSTANDING

## Introduction to September's and October's Data

After stating a Preliminary Mathematics Discourse Model at the end of the last chapter, new data was analyzed to review the model. Following the same steps introduced before (in Chapter 4) and based on Gee's discourse model, this chapter presents the September and October data analysis. It comments, revises, and adds details to the previous analysis. At the end of this chapter, the Preliminary Mathematics Discourse Model is restated accordingly based on the analysis this new data showed the researcher. The next chapter will include the analysis of November's and December's data. In this way, the Preliminary Mathematics Discourse Model will be reanalyzed and refined to state a working Mathematics Discourse Model, one that came from the analysis of five months of threaded discussions in an online public forum located at the *Math Forum @ Drexel.*

The months of September and October provided the researcher with a total of thirteen new threaded discussions, seven from September and six from October (Table 6-1 for details). They each contained between ten (10) and twenty-five (25) messages and examined topics from General Mathematics through Calculus. Specifically, discussion topics were related to General Mathematics (two), Algebra (one), Statistics (one), Discrete Mathematics (two), Trigonometry (two), Pre-Calculus (one), and Calculus (four). As shown in Table 6-1, postings in each discussion included from ten (10) to twenty-three (23) messages, and included the participation of two to nine people. Data did not include a threaded discussion with only one story as it did in August.

The number of stories in each threaded discussion ranged from two (Figure 6-1) to nine (Figure 6-2). However, this did not mean the stories were all independent. In many occasions, stories were intertwined with one another as the following data analysis will show. An extreme case was represented in Figure 6-3 where a single conversation was divided into eight stories.

The next sections include the analysis of data using the activities and connections building tasks. Both provided the tools to identify how the opening post authors and how participants in general constructed and negotiated mathematics knowledge. As in Chapter 5, activities and sub-activities were organized by section. In comparison to Chapter 5, this chapter includes all of the same sections except for the Chapter 5 section titled “Common Language and Mathematics Notation.” Each section compares its contents with August’s data and incorporates new details to the previous analysis to include data from September and October.

The second and third month of data also allowed the researcher to continue exploring the identity building task. A set of “I” statements taken from lines throughout the threads of September and October were used to identify “who” was seeking for help and what their state of mind was. Using these lines, the “socially situated identities” of the threads’ opening posts’ authors were explored and presented in the section titled “Identities of the Forum Participants.”

## Part I: Analysis of Activities and Connections

### Problems, Questions, and Inquiries Introduction

The first message (opening post) of each threaded discussion included the setting and catalyst of the stories. It was there where a problem was proposed, where one or more questions were asked, and where the author asked for help or corroboration. This post introduced the setting of the thread, which according to Gee (1999) “sets the scene in terms of time, space, and characters” (p. 112). Most authors also added additional information in the first post, including partial work done. The following sub-sections will analyze the opening posts and their components.

**Setting: Use of time, space, and character**

The setting during the second period of analysis was the same as that stated in the previous chapter. For most participants of the public online discussion forum (twelve (12) out of thirteen (13), 92%), the time, space, and character of the problem were not relevant. Instead, they used the opening post to introduce one or more questions or a problem.

As in Chapter 5, only one thread included a specific setting. Also, as in the previous chapter, this problem was related to probabilities. In this particular case, the author wanted to know how to calculate the probabilities of winning a legal case because he was going to court. The fact that the topic of this problem was the same as that of Chapter 5 points to a pattern in statistics, indicating that probabilities are most often tied to a specific setting. This was the only case out of 13 threaded discussions (approximately 8%) where a specific setting was described. Taken together with data from August, there are two out of 17 threads (11.8%) that present a setting, and both are related to probability problems. Data from next month will clarify this outcome.

**Catalyst: Analysis of problems and questions**

Topics in the threads were introduced in various ways. Participants stated a problem by introducing one or more questions; they demonstrated how they started their analysis by substantiating their questions with partial work done before posting; they looked for clarification, corroboration, and more. Catalysts were also used for different purposes, from trying to find the answer of a specific problem (to complete an assignment or answer a personal question), to trying to conceptualize a concept, or even to attempting to develop new mathematics.

As in data from the previous month, most threads added one or more questions, sometimes at the beginning of the thread and sometimes included at the end of the problem presentation or in between the problem explanation(s). If authors did not include sufficient details about their question, they were asked to guide the participants of the forum and indicate the specific help they needed. Participants wanted particulars of the work already done to address a specific question. They overtly stated that they needed more details, that is, more information about what the author's misconceptions were so that they could answer the opening post author's questions.

In the "*Basic Measures*" thread, for example, Leon presented a problem without any question. Here the author gave no guidance to the participants of the forum as to what he needed or what his misunderstanding was. He then received an immediate reply asking for more information. In this case, Leon was asked to show his work; he received a reply with the following question: "What have you done so far?" [*Basic Measures*: Story 1, Line 38]. Authors of most threads substantiated their questions with partial work already done. On some threads, this meant stating definitions of terms used or including the algebraic procedures used to start solving a problem. On other threads, it meant asking for corroboration or looking for understanding. This supports the idea that presenting a problem alone or a simple question at the end of a problem was not enough to guide the participants of the forum to negotiate meaning.

In the thread "Planarity of a Graph," Norman introduced a problem and then went on to state the definitions related to the specific problem. His was a Discrete Mathematics problem and algebra was not needed to solve the problem; he needed to write a proof. Norman did not ask for a solution or a step-by-step process to solve the problem; instead he asked for "tips." This thread is unique because it was the author himself who stated a possible solution at the end of the first story. A total of three participants (including the author of the original post) shared their ideas and collaborated to find a possible solution to the initial problem.

Problems related to calculus were supported using algebra. For example, in the thread “Trouble Finding Derivative” (Figure 6-2), the author demonstrated step-by-step procedures followed by little or no explanation of why he chose a particular method. His was an indirect question: first, he stated the problem, then he argued that “I’ve done this half-dozen ways, and none gets me the right answer” [Lines 4-5], and then he showed two algebraic attempts at solving the problem [Lines 7-15 and 16 to 20]. There was no specific question in the opening post of this thread, only the step-by-step procedure to substantiate his problem. The author was looking for clarification.

When solutions seemed incorrect, authors asked for corroboration. Such was the case of Jonathan in the thread “Tan to Slope,” in which he introduced a problem, showed his algebraic work, and indirectly asked for corroboration.

*Stanza 3: Looking for Corroboration*

**Jonathan continues**

13 And yet, while the slope is correct,
14 the 'b' is incorrect.
15 I don't see where
16 I could be going wrong
17 in such a short, short process.

He was confused because although he believed that he was following the right procedure, the answer he got as a result of his work was not the same as the one provided by a solution list. This thread generated 11 messages divided into three stories. In this case, the forum was used to confirm the results of the author’s work.

In another thread, John ended his opening post with the statement, “Any help / clearing this up / would be very much appreciated” [*Derivative*: Lines 29-31]. He was looking for-clarification and understanding. His opening message presented a problem regarding finding the first and second derivative of a logarithmic function. In his posting, he used algebra to show the work he had done, as well as written explanations (using words), as if he was thinking out loud. John also used the textbook to find out if his answer was correct but did not understand how the solution was found. His confusion was evident when he asked

> 23 . . . what I don't understand
> 24 Is how we got
> 25 The term 2 + ln t.
> 26 Where did the "2 + " come from?

Two more authors who looked for clarification were Josefina and Gabriel. Josefina was confused with a trigonometric notation. She stated, "I assume / 2 sin A is not the same as sin2A / Right?" [*2 sin A vs. sin* 2A: Lines 2-4]. Her question had to do with the concept of multiplication and the commutative law, as if in trigonometry the variable A was a specific number instead of an angle of which you had to find the sin function equivalence. This thread generated 14 messages divided into seven stories.

In Gabriel's case, he sought an explanation to further understand a logarithm rule (Figure 6-4). He wanted to know "why this is" [*Logs*: Line 5]; he wanted to conceptualize a rule that was given to him. This thread generated three stories and included a total of ten messages with a total of six participants. Three of these participants, John, Josefina, and Gabriel, used the forum to develop deep understanding of mathematics instead of surface learning. They were not satisfied with memorizing a classroom rule without knowing what it meant. For that reason, they looked for more details, information, and examples to clarify their misunderstandings with help from the *Math Forum @ Drexel* participants. They negotiated meaning and understanding with the more knowledgeable others present in the forum.

Deep understanding was not only related to homework problems. In Bernice's case, the goal was to find help in developing new mathematical equations. Hers was the case of "a high-school math addict" [*Consecutive Terms*: Line 75], as she identified herself, who was interested in finding general algebraic equations to solve consecutive terms that she was developing on her own.

In summary, the authors of the threaded discussions in the months of September and October looked for the following types of answers: corroboration, clarification, deep understanding, tips, and specific answers. Some original posts' authors were interested in completing homework questions or problems; others were looking for specific answers and understanding. Close to a third (four out of thirteen, 31%) of the authors in this period, overtly stated that they were trying to construct deep understanding of mathematical concepts.

**Problem Evaluation and Solution Generation**

Besides using algebraic manipulations, geometry references, and drawing explorations, as participants did throughout the August data, forum participants interacted with one another to clarify definitions and notations. Intertextuality was used to break down messages, to cite specific portions, to add, comment, and correct mathematics errors, and to answer questions. Most threaded discussions included citations from other posts or even from other stories. In this way, participants were able to analyze specific ideas presented by others while trying to minimize further misconceptions.

In "Graph Planarity," Norman ended the presentation of his problem with the question, "Any tips?" (Story 1, Line 39, Figure 6-1). To answer his question, two participants, Wilfred (in Story 1) and Sam (in Story 2), re-evaluated the original post and presented a theorem that could help Norman clarify the problem. However, the theorem still was not fully understood by Norman because of the notation "iff" (if and only if).  In mathematics, this notation is used to present a condition that needs to be satisfied. Norman was reminded of this by Wilfred and then again by Sam.

**Story 1**

*Stanza 11: Notation clarification*

**Wilfred continues**

86 Common mathematical usage
87 for 'if and only if' is 'iff'.

**Story 2**

**Sam**

*Stanza 7: Clarifying use of notation iff in Kuratowski's theorem*

**> From Norman**

**>**39 Any tips?
. . .
>> **From Wilfred**
>>63 Kuratowski's theorem:
>>64 G planar iff G has
>>65 no subgraph isomorphic
>>66 to K_5 or K_3,3,
>>67 the Kuratowski graphs.
. . .
**> From Norman**
>>>77 Yes,
>>>78 but what about graphs
>>>79 that are unplanar and
>>>80 do not have subgraphs
>>>81 isomorphic to K[5] and K[3,3].

53 They don't exist.
54 ('iff' = 'if and only if'.)

The use of intertextuality in this story allowed its participants to develop a conversation-like interaction. As shown above, the greater than sign (>) was used to identify the contribution of each participant. In Story 1, the notation “iff” is expanded to its mathematics meaning, “if and only if.” In Story 2, Sam used three portions of previous messages – note the number of greater than signs used – to indicate who said what.

Some misconceptions in mathematics occur because mathematics has its own language. In order to fully understand the mathematical theorems, corollaries, definitions, and propositions, one must understand what the notation and vocabulary means. Norman did not understand what “if and only if” meant. This limitation hindered him from fully understanding the topic of study.

Even though Wilfred and Sam tried to help Norman find a way to solve the problem, his limited understanding of the notation obstructed his learning. In this thread, no resolution was reached.

In the thread “Extrema,” Jonathan asked for help in differentiating the function f(x) = e^(-x)*sin x. Part of the discussion led to the analysis of e^(-x) and how to differentiate this factor. Five participants interacted and negotiated meaning until a solution to the problem was reached. From a total of eight stories in this thread, two (Story 5 and Story 6) evaluated Jonathan’s work in detail. Rogelio identified an error in Jonathan’s work and stated, “That’s incorrect” [Story 5 and 6, Line 18], and then corrected Jonathan’s work using intertextuality to explain why he thought there was an error. Jonathan followed up with another question, and two other participants, Morgan and Jake, answered by explaining the difference between positive and negative exponents when differentiating a function. In this thread, the use of intertextuality allowed the participants to introduce explanations and to reach a solution.

Besides misconception of mathematics principles and theorems (presented in Chapter 5), computation errors also generated confusion in learning mathematics. Participants corrected each other using intertextuality, and those who had made the errors accepted being corrected. For example, in “Trouble with Derivative” (Figure 6-2), Flores, one of the six participants of this thread, identified a computation error by Domingo (observe intertextuality in Lines 31 to 34) and presented the solution in the following way.

*Stanza 7: Making a correction*

**Flores**
**> From Domingo**
>31 So the function is the same as
>32 -8x ^ 9/2 - 7x ^ -7/2.
40 OK, so far.

>33 Now differentiate and get
>34 -16/9x ^ 2 + 49x ^ -9/2

35 I get -36x^(7/2) + 49/2 x^(-9/2)

IV RESOLUTION

*Stanza 8: Agrees with Flores observation*

**Jonathan**

36 So did I,

37 and it was the right answer.

. . .

*Stanza 8: Accepts correction*

**Domingo**

41 You're right, I stand corrected

Flores used intertextuality to evaluate Domingo's work. Flores re-stated the error (lines 33-34) and then added a line (35) with the correct solution. A third participant, Jonathan, agreed with Flores, confirming the correction, and then Domingo accepted the correction.

A second thread, "Logs" (Figure 6-4), included a correction concerning notation. This time it was an error in an explanation. Once again, intertextuality was used to present the error, and as before, the correction was presented in a single line. Although six participants negotiated meaning in this thread, only two participated in this interchange of ideas.

*Stanza 5: Correction of previous notation*

**Javier**

> **From Joe**

>6 a^t = b is the same equation as

>7 t = log_b(a), the base-b log of a.

15 ^^^^^^^^ log_a (b)

*Stanza 6: Accepts correction*

**Joe**

16 Omigosh. Thanks for correcting me.

When correcting errors, participants also corrected themselves. This was the case of Timothy who noticed an error after posting a message. In the thread titled "Integral," he added a post to correct himself. He stated

*Stanza 6: Correction*

**Timothy continues**

50 Whoops
51 -- book's answer is ok.
52 And I had a sign mistake
53 in my answer:
54 should be . . . + sqrt[x - x^2].
55 Otherwise the two answers,
56 while different in appearance,
57 are reconcilable.

In this thread, Timothy presented the correct answer and explained why his answer was incorrect. This did not happen in the previous examples where participants only included the lines with error(s) and a line with the correction.

Another source of misconceptions in mathematics comes from the use of tools and the limitations embedded in them. An example of such is presented in the “Zero Story.” In this thread, Bernice, the author, examined division by zero. She made a reference to a graphing program when stating the following:

**Bernice**:

20 Also
20 my graphing program
21 on my computer agrees:
22 y=2/x
23 if you find where x=0
24 the y will be at infinite.

Her argument was disputed by two other participants of the thread. First, Pepin conditioned his answer to the type of problem, one that included vertical asymptotes (lines that bound a graph, where a function has no meaning).

**Pepin continues**

52 Graphing programs are
53 not to be trusted if
54 there are vertical asymptotes.

And second, Mike supported Pepin’s points when he stated:

**Mike**
86 Well, no program
87 -- graphing or otherwise –
88 should be trusted "blindly".

Mike asked for careful consideration of any program used to solve a mathematics problem. Tools that help solve mathematics problems are not always one hundred percent accurate and can lead to confusion when their limitations are unknown. This was the case of Bernice. The forum participants then helped her clarify her misconceptions regarding division by zero. They used the formal mathematics definition of division as well as other examples and references to web articles.

In summary, participants of the forum evaluated problems and negotiated meaning by following up on questions, by adding comments, new information, and hints, by clarifying and defining concepts, mathematics terms, and notation, by presenting specific (step-by-step) and general examples, by introducing and explaining math rules and properties, by relating answers to other math content (Table 6-2), and by correcting mistakes. They also supported math teachers’ arguments, questioned book solutions, and analyzed graphing programs.

Interaction between participants was possible through the use of intertextuality, allowing authors of original posts and others to develop conversation-like interactions that promoted the development of new understandings and meanings as well as the clarification of different kinds of misconceptions. Corrections in the threaded discussions were accomplished in three different ways. First, participants corrected themselves; second, participants corrected another’s work; and third, participants confirmed other participants’ corrections. Participants who made mistakes accepted being corrected.

**September's and October's Coda(s): Additional Information**

Closing a story in a thread was like closing a chapter in a book. Participants of the discussion forum added different types of comments that could or could not be related to the topic of the thread itself. In the last chapter, we saw how participants used the coda to show other participants how to address a question or idea. In this period of analysis, data showed endings with generic statements that included one or more of the following: gratitude (62%), antagonistic or mocking remarks (15%), and different kinds of recommendations (15%). About a third of the endings were specific to the problems stated in the threads (31%). For more details see Table 6-3.

The following are examples of extreme codas presented in the forum, those that showed gratitude and those that presented antagonistic remarks. More than half (8 out of 13 threads, or 62%) of the endings included generic statements of gratitude in the form of thank you notes from the opening post authors. The following is from a simple statement in "Zero Story":

*Stanza 14: Gratitude*

**Bernice**

172 Thanks Mike
173 this [is] sorta (sic) exactly
174 what I was looking for. . .

Next is a statement that assessed the author's learning in "Probability":

*Stanza 21: Goodbye post*

**Gary**

174 To everyone:
175 Thanks for the education.
176 If nothing else, I've learned
177 to "correctly state the problem"

The following are statements that acknowledged new understanding in "Trouble finding a Derivative":

**V CODA**

*Stanza 15: Finds understanding and gratitude*

**Jonathan**

80 That makes it all
81 make /so/ much more sense.
82 I thank you.

Most authors of opening posts expressed their gratitude to the participants of the forum.

On a more negative side, 23% of the threads (2 of 13 threads) included antagonistic or mocking remarks. For example, in “Extrema” Jonathan (the author of the thread) and Jake (one of the additional eight participants) engaged in a controversy because Jonathan did not include details of his misconceptions when he posted the original message. Jake stated the following:

*Stanza 14: Continues reply to comment about laziness*

**Jake**

155 If you do work on the problems
156 as you say, that's comendable (sic).
157 But in that case, then
158 you would gain a ->lot<- more
159 from the group if,
160 instead of just posting your questions,
161 **you would tell us exactly what you tried,**
162 **how far you got,**
163 **and why you could not finish.**
. . .
170 If you have already done
171 some of the work yourself,
172 then let the people know so
173 they don't repeat it.
174 Have some consideration for ->our<- time,
175 just as you obviously hold yours
176 in such high regard.

In lines 161 through 163, he indicated exactly what he expected from an original post (see bold text above). Opening posts’ authors were to include specific ideas, indicating what they had done and where their misconceptions were. Jake’s position is again repeated in the “Consecutive Terms” thread in which he refuted another author’s post. This time he was arguing in favor of proper use of intertextuality.

*Stanza 7: The importance of intertextuality*

**Jake**

68 First:
69 If you don't quote
70 what you are replying to,
71 it is hard to figure out
72 what you are talking about.
73 If, on top of that,
74 your posting software
75 does not even know
76 how to make your messages
77 proper follow-ups,
78 it is nigh impossible
79 to know what you are talking about.
80 Learn some posting ettiquette (sic),
81 please.

Still, these two antagonistic cases are the exception when compared to all of the interactions in the forum. They both came from the same participant, indicating that most forum users found ways to help those who initiated a thread; they helped by example without making any negative comments. A friendlier atmosphere was prevalent in the forum.

## Part II: Analysis of Identities

As stated in the last chapter, only few references were made indicating whom the participants of the forum represented. Chapter 5 concluded that there were two types of participants: (1) the novice, casual student, and user of math, and (2) the more knowledgeable others, possibly mathematics teachers, instructors, or professors. This chapter's data confirms this finding.

For example, in "Consecutive Terms," the author of the thread referred to herself as "a high-school math addict" (Story 2, Line 75). Other threads also made references to students. For example, in "Distance Word Problem," Joe indicated that "students seem to think / that the unknown / must always be x" [Lines 22-24]. In another thread, "Tan to Slop," Jonathan indicated that he would email his professor [Lines 41-43], thus supporting the idea that he was a student.

In “Trouble Finding the Derivative,” Joe stated, “the rule I tell my students . . .” [Story 9, Line 26], covertly indicating that he was a teacher or professor. Furthermore, in the “Probability” thread, the author alluded to math “brainiacs” (sic) when referring to those who were able to help him [“Probability,” Story 3, Line 178]. Therefore, characters of the threads in September and October included students, teachers, and professors who voluntarily participated in the forum. This confirmed the statement presented in Chapter 5 regarding the identities of the forum’s participants.

Another feature that helped to discover the identity of the thread’s opening post authors was developed throughout the stories of the threads in this period of analysis. Their expressions of uncertainty while trying to learn, understand, and figure out mathematics illustrated the state of mind of the participants. In their search for meaning, authors wrote about their confusion and frustration. They used “I” statements throughout the different stories in the September and October data to show how they felt. For some, it not only meant having a problem; it was deeper than that. They were “trying to think,” and they were also “guessing” because they could not see where things went wrong. The following is a poem that presents these feelings of uncertainty. Numbers indicate in the thread where the “I” statements appear in accordance to Table 6-1.

**Voices calling out to be heard**

(2) I have this problem<br>
-- I agree<br>
-- I disagree

I want to make<br>
I would think

(3) I came up with . . . [and]<br>
I ‘m currently working on

(4) I am going to<br>
I have been

(5) I'm supposed to prove
I have to show
I stopped

I'm trying to think
I'm guessing
I'll have to describe

(7) I am getting confused
I assume

(8) I figure
I check it . . . [but]
I don't see why
I don't see where
I could be going wrong.

I'll repost
I answered
(9) I know that . . . [but]
(10) I need to find
I can figure it so far
I figure is right
I'd go on to say
I stop there

I should go from . . . to . . . [but]
I don't understand

(11) I'm having some trouble
I should note that

(12) I've done this
a half-dozen ways
I'll outline a couple
I must be messing up . . . [and]
I don't know what

Participants in the second period of analysis raised their voices beyond the text of their postings. When analyzing this data, the researcher was called upon to hear what they were saying. Nothing mattered until she took time to hear their voices calling out for help. The cognitive analysis had to be suspended, as it was impossible to continue without paying attention to their cries. Help-seekers wanted to learn mathematics, but they needed the support from others. It was as if the text jumped out of the page, calling for attention, making a statement about their state of mind. Once they were noticed, and once their cries were taken seriously, they set them aside and allowed the researcher continue with the cognitive analysis of this chapter.

The opening posts' authors looked for help in the forum because they wanted to succeed in mathematics. This space provided them with the support they needed, and they were able to interact with more knowledgeable others to negotiate meaning and understanding. As stated before, more than 60% of the authors openly stated they found a resolution to their problems. They all had the opportunity to engage in mathematical conversations. By themselves, authors did not know what to do; they were "confused," and they had to "guess" or even "stop." As they stated, they had a problem but were not able see "why" or "where" things went "wrong." It was together, working with more knowledgeable others that included math "brainiacs" (sic), teachers, or professors that they were able to succeed in most cases, as they overtly stated in the thank you notes they wrote.

Thus the analysis of the September's and October's data concludes. Throughout this analysis, original posts' authors have presented a problem or question; together with more knowledgeable others, they have evaluated and generated new meanings and understandings while sharing ideas and resources. The next section re-states the Preliminary Mathematics Discourse Model presented in Chapter 5 by including the analysis of the September's and October's data.

## Reviewing the Preliminary Mathematics Discourse Model

Putting together the findings stated above, a revision of the preliminary discourse model stated in Chapter 5 is presented below. This is still a model under construction; two more months of data must be evaluated to complete the analysis proposed in this dissertation. The reviewed preliminary model proposed in this chapter will be refined in the next chapter with data from November and December.

Novice, casual students, high-school math students, and users of mathematics, together with more knowledgeable others (math “brainiacs” (sic), teachers, and professors) collaborated and negotiated meaning and understanding. They evaluated mathematics problems, questions, and inquiries in the areas of general mathematics, algebra, statistics, discrete mathematics, trigonometry, pre-calculus, and calculus. Learners and tutors, during the months of August through October in a voluntary online public mathematics discussion forum that was available 24 hours a day and seven days a week, worked together and found different kinds of solutions as they generated and constructed new mathematics knowledge through the following activities:

- Presenting questions, problems, and inquiries located in different parts of the original message. Learners included examples, explanations, and paraphrased questions. They were looking for specific answers, corroboration, clarification, and understanding of mathematical concepts.

- Negotiating meaning by following up on questions and adding comments, new information, and hints. They clarified and defined mathematics concepts, terms, theorems, principles, and notations. They used algebraic manipulations to present inductive reasoning and step-by-step procedures as well as partial procedures so that others could complete the work; they also presented generalizations and abstract statements.

- Negotiating meaning by using geometry, presenting different geometric concepts and constructions. They explored different ways of analyzing and solving a problem.

- Adding references to famous mathematicians and different types of online sites and documents, including articles, tutorials, “webworks,” and other discussion groups.

- Using common language as well as mathematics symbols and abbreviations to communicate.

- Promoting the use of intertextuality to follow up on specific information, questions, and answers.

- Expressing sentiments of confusion and frustration when not succeeding in finding a solution or understanding a concept.

- Being grateful for the help received, assessing their own learning, and stating that they found understanding.

Table 6-1. General description of threaded discussions in September and October

| Title | Content Area | Number of Postings | Number of Stories | Number of Participants | Time Span in Days |
|---|---|---|---|---|---|
| Distance Word Problem (S) | General Mathematics | 13 | 6 | 8 | 8 |
| Zero Story (S) | General Mathematics | 15 | 6 | 6 | 10 |
| Consecutive Terms (S) | Algebra | 16 | 8 | 2 | 12 |
| Probability (S) | Statistics and Discrete Mathematics | 10 | 3 | 5 | 3 |
| Graph Planarity (S) | Discrete Mathematics | 10 | 2 | 3 | 4 |
| Basic Measures (O) | Statistics | 10 | 4 | 5 | 7 |
| 2sinA vs. sin2A (O) | Trigonometry | 14 | 7 | 8 | 4 |
| Tan to Slope (O) | Trigonometry | 11 | 3 | 5 | 3 |
| Logs (O) | Pre-Calculus | 10 | 3 | 6 | 4 |
| Derivative (S) | Calculus | 13 | 5 | 6 | 4 |
| Integral (S) | Calculus | 10 | 6 | 5 | 7 |
| Differentiate / Extrema (O) | Calculus | 21 | 8 | 9 | 2 |
| I Can't Believe (O) | Calculus | 23 | 9 | 6 | 12 |

Notes: S – September data. O – October data.

Table 6-2. Web resources referenced to in the threaded discussions of September and October data

| Thread Title | Web Resources |
|---|---|
| Tan to Slope | Webworks – online exercise system<br>Author emailed administrator about mistake. |
| Integral | Website – http://integrals.wolfram.com<br>Reference to support answer. |
| Word Problem | Discussion group in Google.com<br>Reference to other examples. |
| Logs | Web article – http://oakroadsystems.com/math/loglaws.htm#NewBase<br>Reference to support answer. |
| 2sinA vs. sin2A | Web Tutorial – "http://oakroadsystems.com/twt/spacial.htm#funcs30" changed to http://oakroadsystems.com/twt/double.htm#SineDouble<br>Recommendation: Online Trigonometry Tutorial. |
| Zero Story | Web articles – Interval arithmetic articles in Sun Microsystems website. Go to http://docs.sun.com/app/docs and search for the author William G. Walster. Reference to support argument. |

Table 6-3. Closing message topics of September and October data

| Topic | Percent | Threads |
|---|---|---|
| Gratitude | 62% | 6.2 Zero Story |
| | | 6.4 Probability |
| | | 6.7 2sinA versus sin2A |
| | | 6.9 Logs |
| | | 6.10 Derivative |
| | | 6.11 Integral |
| | | 6.12 Differentiate |
| | | 6.13 I Can't Believe |
| Antagonistic or Mockery Remarks | 23% | 6.1 Distance Word Problem |
| | | 6.3 Consecutive Terms |
| | | 6.12 Differentiate / Extrema |
| Recommendations | 15% | 6.2 Zero Story |
| | | 6.3 Consecutive Terms |
| Other | 31% | 6.4 Probability |
| | | 6.5 Graph Planarity |
| | | 6.6 Basic Measures |
| | | 6.8 Tan to Slope |

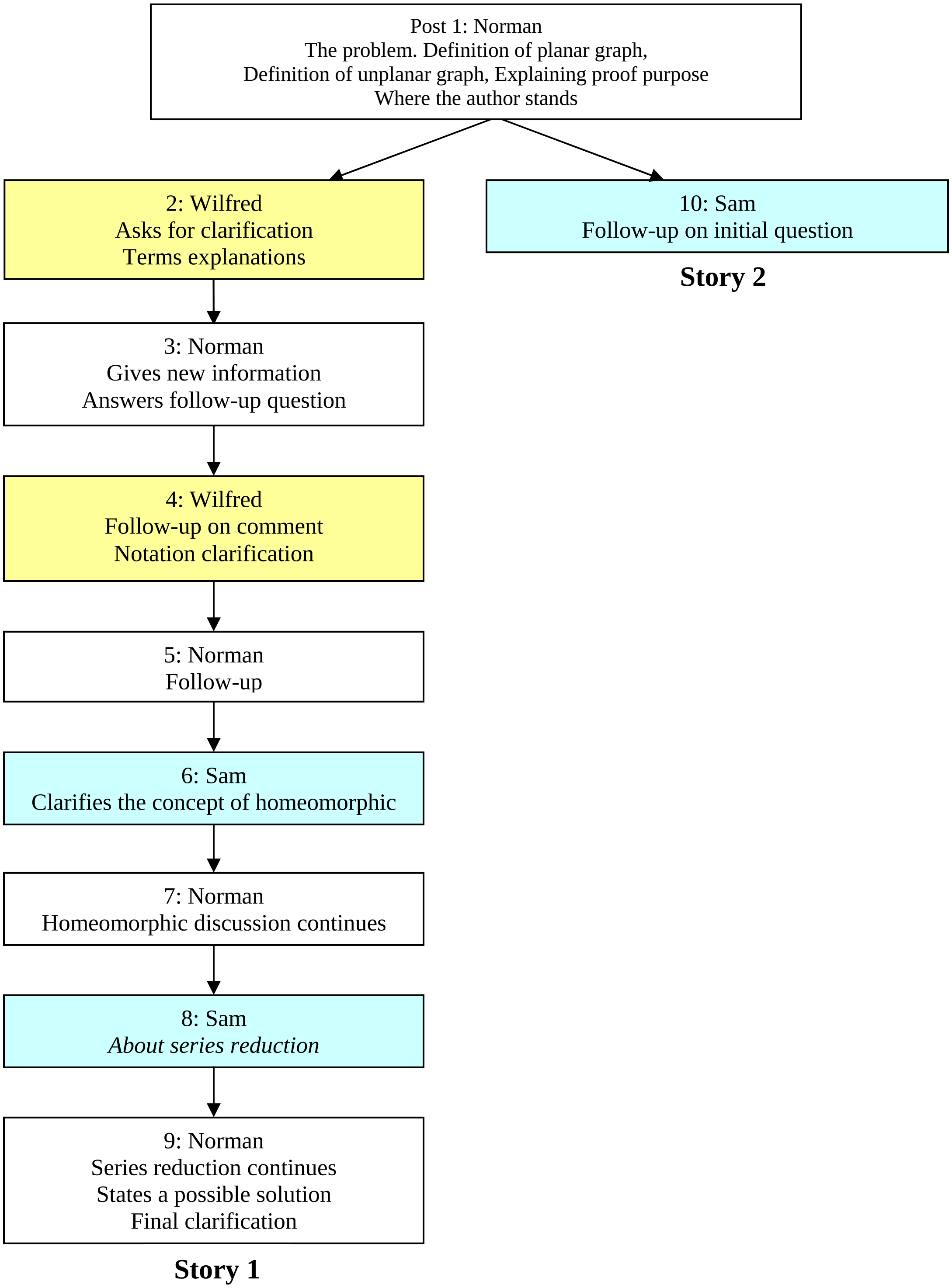


Figure 6-1. “Graph planarity” tree. This threaded discussion has a total of ten messages organized into two stories with three participants.

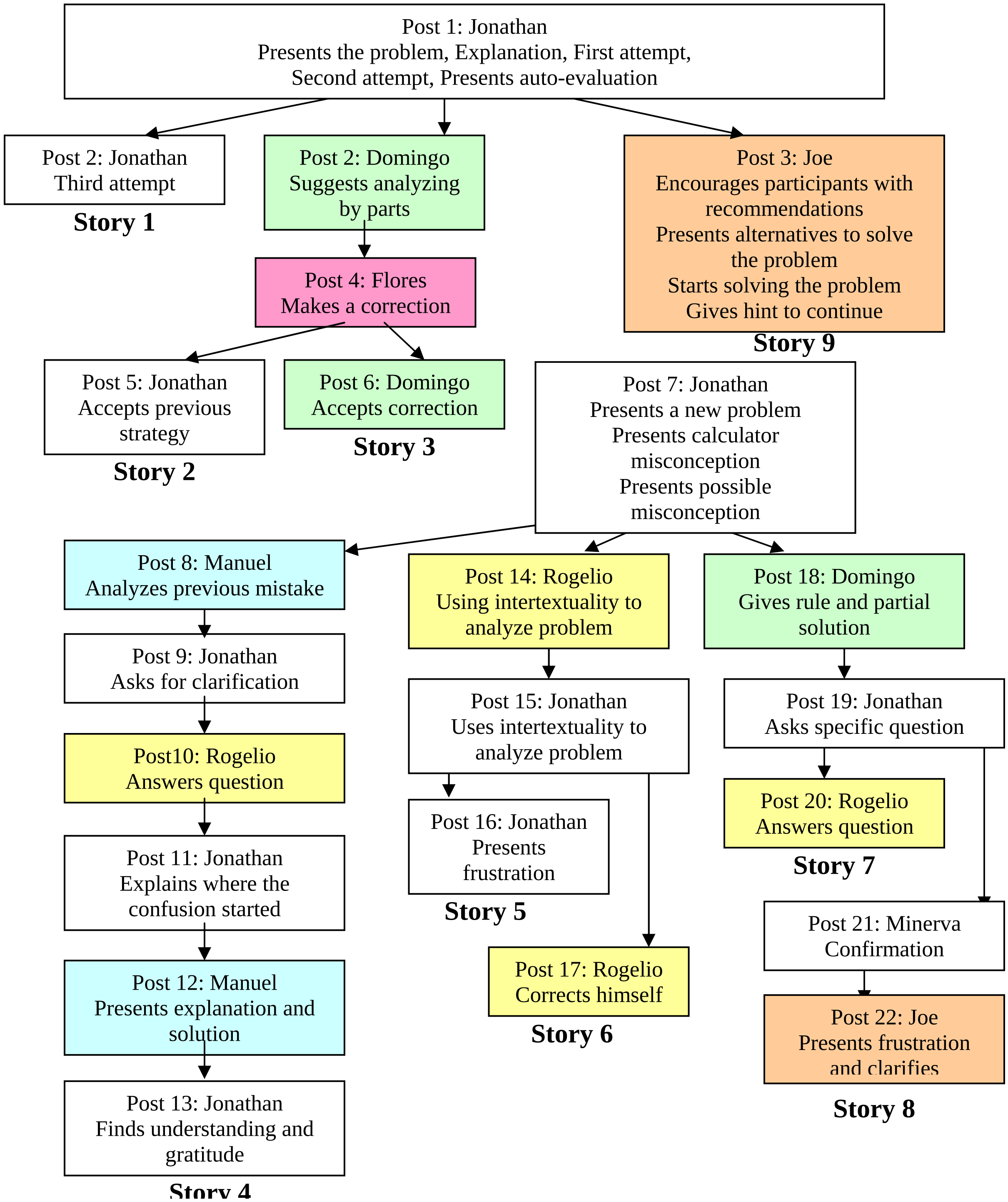


Figure 6-2. “Finding derivative” tree. This threaded discussion has a total of twenty-three messages organized into nine stories with six participants.

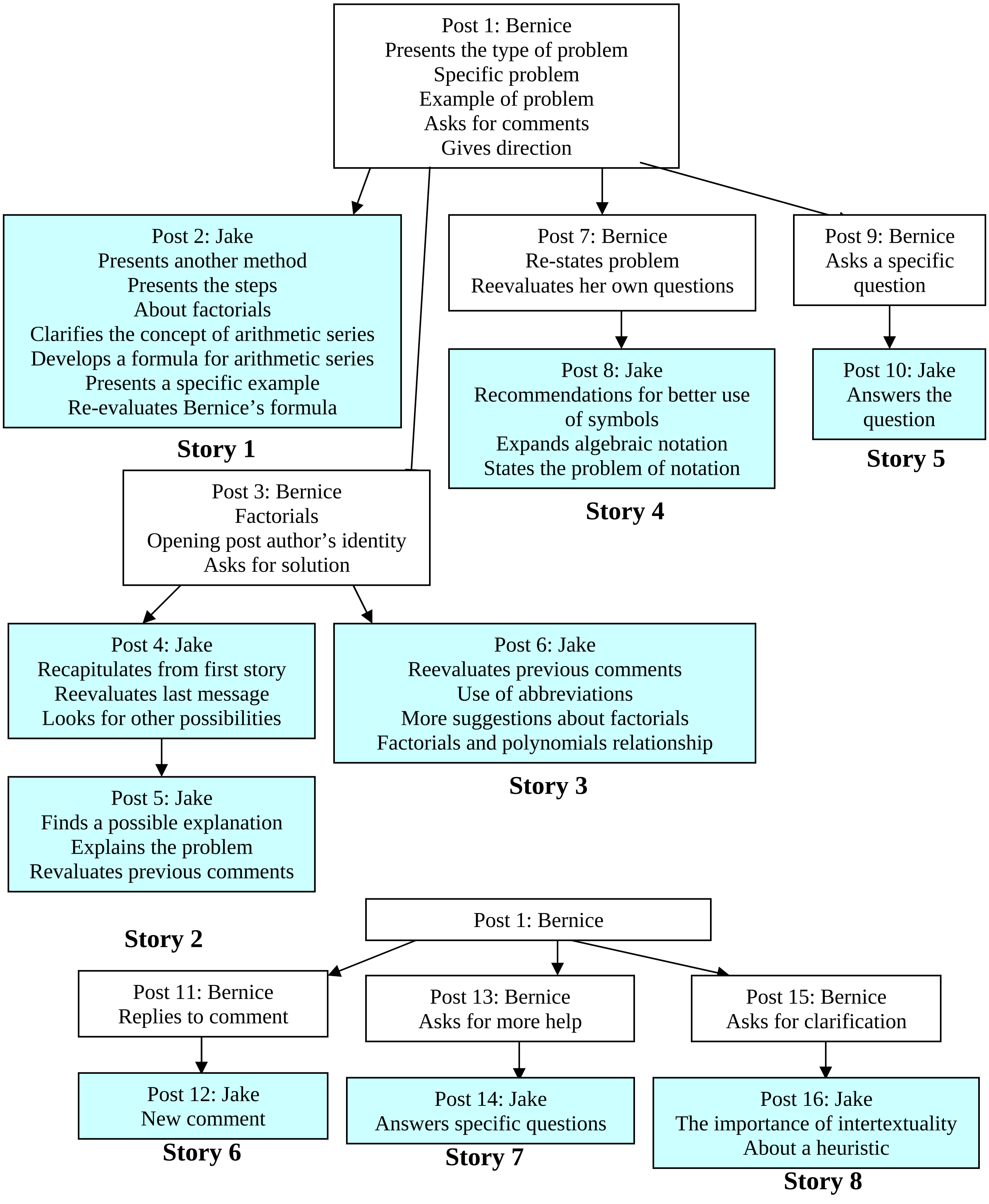


Figure 6-3. "Consecutive terms" tree. This threaded discussion has a total of 16 messages organized into eight stories with two participants.

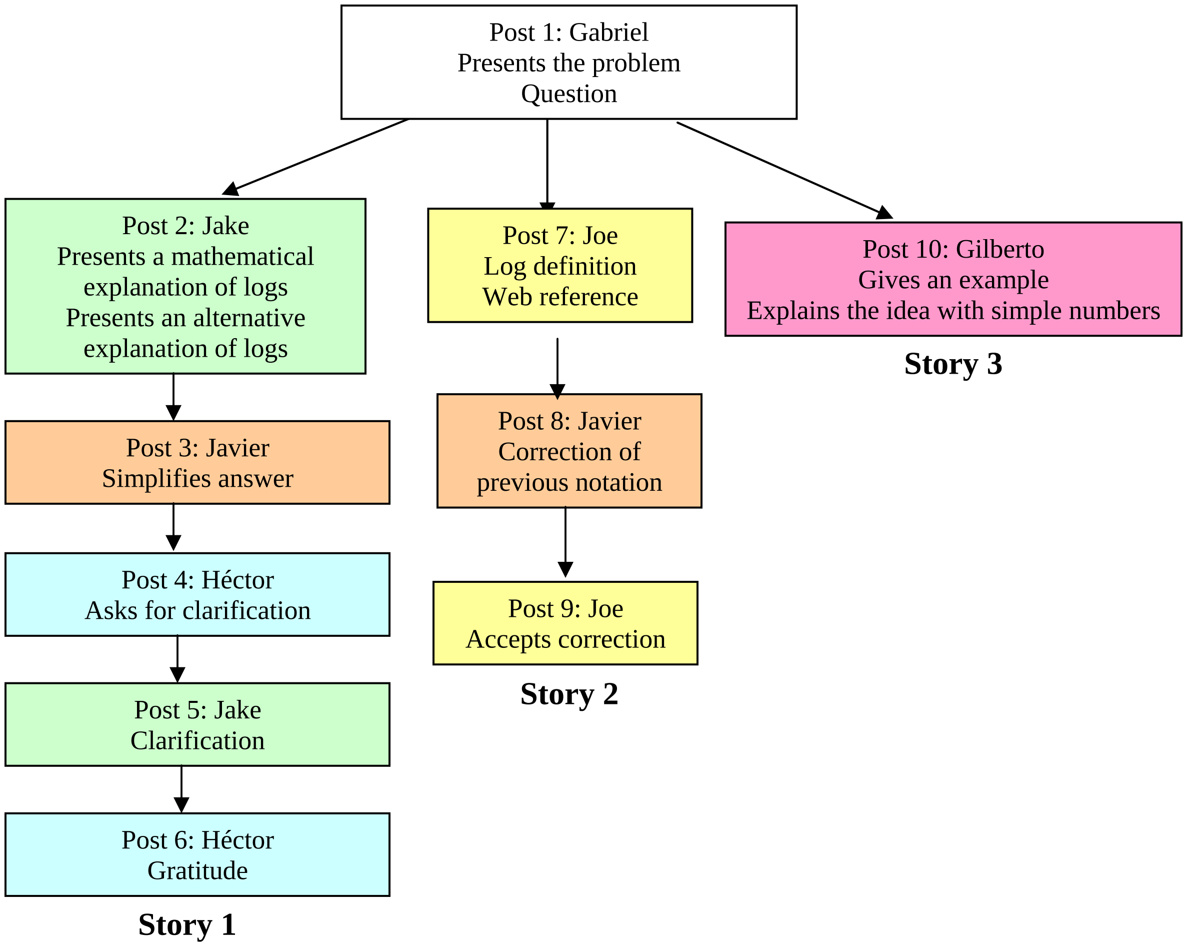


Figure 6-4. “Logs” tree. This threaded discussion has a total of ten messages organized into three stories with six participants.

# CHAPTER 7
# GENERATING NEW MEANING AND UNDERSTANDINGTHROUGH ONLINE POLYVOCAL COLLABORATION

## Introduction to November's and December's Data

The analysis presented in this chapter concludes a series of three data analysis chapters; as in Chapters five and six, it ends with a revision to the Preliminary Mathematics Discourse Model. This chapter refined previous outcomes with new data from November's and December's threaded discussions. Based on Gee's (1999, 2005) discourse analysis, the researcher focused on the analysis of the activities and connections building tasks as stated in the Methodology and Methods' chapter (Chapter 4).

Data from November and December provided the researcher with seventeen (17) new cases of study (Table 7-1 for details). Threads included ten (10) to twenty-five (25) messages generated from the interaction of three to eighteen (18) participants. As before, topics ranged from General Mathematics to Calculus, specifically including General Mathematics (one), Algebra (seven), Discrete Mathematics (four), Pre-Calculus (three), and Calculus (two). One of the Discrete Mathematics threads included a problem that could also be studied in Statistics; this discussion analyzed a problem about probabilities. The number of stories in each case of study ranged from two to sixteen (16), and as before, they were identified after creating tree diagrams with their corresponding branches. This period of study did not include a case with only one story as the August period did (Chapter 5), indicating that such structure was unique throughout the entire period of analysis. As data will show, connectivity during this period was accomplished among threaded discussions and discussion groups in the *Math Forum @ Drexel* as well as with other discussion groups and web resources over the Internet.

The organization of this chapter follows a similar structure to that of chapters five and six. Analysis is organized into two general parts: the first includes the analysis of the activities and connections building tasks, and the second looks at the identity building task. The first part of the chapter is sub-divided into three main sections corresponding to the body parts of a story (Gee, 1999, 2005). Activities and connections are included in each of these sections, titled as follows: "Problems, Questions, and Inquiries Introduction" with activities and connections related to the setting and problem or question of the thread; "Problem Evaluation and Solution Generation," including activities concerning how participants developed meaning and understanding; and "November and December's Coda(s): Additional Information" including suggestions of how to use the forum to promote better discourse among its participants (connections) and information about the use of gratitude messages to confirm that understanding had occurred (activities). The second part of this chapter includes analysis about the identity of the participants. This section confirms and expands the forum participants' identities presented in previous chapters.

## Part I: Analysis of Activities and Connections

### Problems, Questions, and Inquiries Introduction

As in previous chapters, the presentation of problems, questions, and inquiries were included primarily in the opening post (first message) of each thread. On few occasions, however, once the original problem was resolved, opening posts' authors or other members participating in the discussion added one ("Exponents versus multiplication notation" and "Number Patterns") or two additional questions ("Trig Identities"). These were mostly follow-up questions related to the original problem of the thread.

### Setting: Use of time, space, and character

In general, the setting of the discussions in this period of analysis remained the same as that in the previous chapters. Learners and more knowledgeable others used the spaces provided by the website to introduce topics and problems, questions, or inquiries related to mathematics problems studied at high school, college, or elsewhere. Again, only one thread included a problem about probabilities with a specific setting. Probabilities are used to establish the possibilities that a specific outcome will occur. These are usually stated in terms of percentages or decimals (numbers between 0 and 1). In school settings, probabilities are frequently studied by computing outcomes from a specific set of cards or color balls, given special characteristics of a whole. Nevertheless, the problem presented in the forum was an application problem, one that had to do with traffic lights. As it will be shown below, participants engaged in a series of interactions that explored the traffic lights' problem from different perspectives.

During the whole period of analysis, August through December, only three threads from a total of 34 threaded discussions included probability problems (8.8%). These were related to a specific setting. The opening post author of chapter five's problem wanted to confirm the probability of two persons in a group of 50 having the same birthday. In chapter 6, a threaded discussion author wanted to know the probability of winning a court case given a set of conditions. In chapter seven, the discussant wanted to know how to determine the probability of passing three traffic lights in a row without stopping. Data from this chapter confirmed the pattern proposed in last chapter about probability problems, that is, that probability questions in the forum were mostly tied to real life problems as opposed to classroom abstract problems.

Most problems in this environment did not include a setting. Opening posts' authors presented their questions or problems to the forum participants without presenting a setting, generally because they were abstract mathematics problems. In most cases, authors used the forum to find help from more knowledgeable others when they could not find a solution. They also used the forum to corroborate their mathematics solutions.

During this third period of analysis and in comparison with the previous two periods, the opening post messages showed one major change regarding setting. This occurred in the thread "Exponent versus multiplication notation," in which the author asked for corroboration without stating a specific problem. Sandra, the author, needed help to understand the solutions she had posted in another website. In the opening post, Sandra directed participants to a URL (web page address) where the problems she worked before were located, thus connecting this public website with a private website. In her post, she wrote the following:

*Stanza 1: Presenting URL*

**Sandra**

1 https://www.cotse.net/users/dns123/logit.htm

*Stanza 2: Presenting problem*

2 The link above
3 goes to 3 problems
4 that I worked.

This was the only time in the whole period of analysis (August through December) where a learner asked others to visit another website to check their work. In this way, Sandra's problem connected the *alt.math.undergrad* discussion group to another space on the Web. Even though this required going a step further to find out what the problems were and how she had solved them, the participants of the forum engaged in a discussion that generated 11 stories with 24 messages. This thread was one of three discussions with more than 20 messages and one of two with more than ten stories (Table 7-1). A total of nine participants, not including the opening post's author, engaged in an interaction that represented an instance of *polyvocal collaboration*, where participants contributed with different ideas to help Sandra understand why the answers she had were evaluated as wrong.

The coste.net website is a private space, so this group of participants needed a password to find Sandra's problem, one that she had not provided in her opening message. This means that they probably were a group of students working together in another environment, an online course or a face-to-face class. Still, there is no indication that they knew each other, but by being able to check Sandra’s answers, they indirectly stated that they were connected in some other way. Another possible explanation could be that once a participant had stated what the problem was, others were able to catch up and continue the interaction and negotiation of meaning.

During this period of analysis, the setting of the discussion forum was widened to include other web sites. This was a unique case, one of thirty-four (34) in the whole period of analysis. In previous threads, when making a connection to another website or web page, participants included them throughout the evaluation, crisis, or resolution portions of the story. Still, this new instance of connectivity showed that participants of this forum also used other web spaces to learn mathematics and that in this particular case, the *alt.math.undergrad* was used to corroborate the work done elsewhere. This confirmed the outcome presented in Chapter six that stated that users of the forum connected it to other digital math content (Table 7-2 for more details.). The forum was used as an additional learning resource.

**Catalyst: Analysis of problems and questions**

Problems and questions during this period of analysis were mostly stated in similar ways as before (chapters 5 & 6). Authors included problems with and without questions, explanations, examples, references to web pages, and possible solutions when looking for clarification, corroboration, and understanding. However, they did not include definitions or theorems in the opening post to further explain or guide the participants of the forum as they had done before.

Some participants used the first post to state their feelings. For example, they indicated “This is driving me crazy . . .,” and “I am going such a way round / that I feel I am missing / something very basic” (“Trig problem,” Line 9 and Lines 48-50, respectively), and “I am having a little trouble / understanding this trig book” (“Trig identities,” Lines 1-2). After overtly stating their feelings, they went on to state their specific mathematical misunderstandings using one or more of the following methods: presenting references to web readings already made (Table 7-2), showing algebraic work already done, making references to geometric concepts, adding algebra manipulations, stating trigonometric relationships, asking different types of questions, and quoting from a specific book. Using these methods, opening post authors guided participants to understand what their misconceptions where and what specific help they needed to overcome them.

When looking for answers, help, hints, or tips, most participants introduced the problem first and then asked for help at the end of the post. To accomplish this, they primarily used direct and indirect questions. However, there were some exceptions to this. For example, the author of one threaded discussion started his message by asking for help and then requested help again at the end of it, and another author used the first message of the discussion to ask a question that included the problem itself ("Three Digit Numbers"). In the first case, Antonio first stated the question: "How do you solve the question / plz help!!! thx!!!" ("Recursion," Lines 1-2). He then stated the problem he wanted others to consider and finally ended his opening post message by indirectly asking for specific help by saying, "plz list steps thx!!!" Two other cases also seemed different from the rest. The first one introduced a math problem without any personal question ("Close and Bounded Sequences"), it included only the problem statement without any work done or any type of question, and the second asked participants of the forum to provide new problems to be solved ("Looking for Problems").

Nevertheless, the pattern for most threads seemed to be to introduce the problem first and then ask for help. As before, most participants supported the presentation of a problem with partial work done, examples, explanations, readings already made, and even possible solutions to be evaluated by the participants of the forum.

When asking analysis questions in this period most authors of original posts used direct and indirect questions (16 out of 17, for 94%). Authors preferred two types of direct questions out of seven possible types (what, who, where, when, which, why, and how). These were "how" and "why" questions. In most cases authors used questions that started with or included the word "how." For example, authors proposed questions such as "**How** does . . . become . . .?" ("Derivative Question," Line 1) and "What I don't understand is, / **how** do we choose . . .?" ("Matrix Determinant," Lines 9-10). Examples of indirect questions used by opening post authors included "I don't know what to do next / to . . ." ("Function Value," Lines 7-8), "I can not figure out / the missing numbers / in these patterns . . ." ("Number Patterns," Lines 1-3), or even, "can someone please help me?" ("Trig Identities," Line 52). Only one thread included a math problem without a personal question from its author ("Close and Bounded Sequence"). In Table 7-3 questions from the original posts are included. They show how authors directed participants in their quest for understanding.

In this period of analysis, two new types of discussions were generated by the participants of the forum. The first one was asking for new problems and the second for someone to check the author's work posted in another web site. In the first case, Cirilo stated, "Has anyone got / a (not to hard) math problem? / If yes please post it" ("Looking for Problem," Lines 1-3). Instead of having a problem on hand for which he needed help in solving, the author in this particular post was the one looking for new problems to solve. In this way, the forum was also used as a source of mathematics problems. Even though there is a section in the *Math Forum @ Drexel's* website dedicated to "The problem of the week," where a new problem is posted every week, none of the participants alluded to it. Instead, they introduced different types of problems to satisfy Cirilo's inquiry.

In the second case, Sandra asked for corroboration, but what made hers a special case was that the problems were not stated in the opening post; instead she added a URL (a web page address which is no longer available) for others to go to, to check her answers. She overtly suggested this as follows:

**II CATALYST**

*Stanza 3: Looking for corroboration*

**Sandra continues**

8 Will someone please (sic)
9 check my answers?
10 I am told that
11 all 3 of my answers
12 are wrong, but
13 I was not told why
14 they were wrong.

Someone (it is not specified who) had told her the solutions she found were wrong. But this person did not add any explanation to the negative evaluation, and Sandra wanted to know why her answers "were wrong." She used the *Math Forum @ Drexel* to look for the corroboration and understanding she needed.

To conclude, authors of opening posts used the forum to write about their feelings and to introduce direct and indirect questions or problem statements in order to find help from more knowledgeable others. Most authors were looking for specific answers, corroboration, confirmation, and understanding. During this period of analysis, there were no authors looking to develop new mathematics formulas or ideas as they did in the second period of analysis (chapter six). Instead, one author wanted others to post new problems and another wanted others to visit a specific web page to review and explicate her misunderstandings. In this way, the *Math Forum @ Drexel* provided a space for math learners with access to the Internet to find more knowledgeable others to help them clarify mathematics misconceptions and develop new mathematics understandings and meanings. This space was not bounded by the site and had no access limitations. It is a public online website that provided free help to mathematics learners.

In general, authors were seeking information; details that could help them clarify misunderstandings, or hints to continue working with a math problem when they had to “stop” working on because they did not know what to do next or how to continue solving the problem. Most authors (16 out of 17) used direct and indirect questions to state their needs to participants. The types of questions asked did not discourage the participants of these threads; they collaborated with each other and negotiated answers and solutions providing different possibilities to solve the problems. The tree diagrams’ branches showed how participants interacted with one another, presented different ideas, negotiated meaning and understanding, and therefore constructed mathematics knowledge through instances of *polyvocal collaborations*.

**Problem Evaluation and Solution Generation**

The use of this distributed discussion board allowed participants to build problems up to the point of requiring different types of solutions. As stated in chapter five, participants were able to present algebra manipulations, discuss algebra misconceptions, relate algebra solutions to algebraic principles and definitions, state and use geometric definitions, relate algebra with geometry, and present written drawing explorations. The data from this last period of analysis (November’s and December’s data) also showed how participants engaged in three new activities: presenting simple images drawn with text to explicate a concept or rule, analyzing the difference between solving algebraic equations and trigonometric identities, and presenting different answers to a single problem, thus engaging in *polyvocal collaboration* (Gergen, 1999).

As in chapter6, participants used intertextuality (Gee, 2005) to break down messages, quoting portions of previous messages to answer specific questions or to clarify specific theoretical misconceptions and computation errors. They also used follow-up questions and comments, presented new information, gave hints and tips to guide the learners, explained notation problems, and presented rules and properties when finding the solution to a specific problem. In addition, this chapter looked at how participants of the forum used visual clues, how participants used encouraging remarks to build up learners' confidence, and how *polyvocal collaborations* were used by different respondents to present new ideas, thus adding different contributions and generating a set of options to help authors and others develop new meanings and understandings.

First, we will examine the use of visual clues, including the construction of text images and the use of upper case letters. Text images were used in three different threads: first, in the "Solving an Algebraic Inequality" thread, a number line was drawn to indicate intervals, second, in the "Matrix Determinant" stories, a Cartesian Plane was used to demonstrate an idea, and third, in the "Graph Planarity" thread, a simple drawing of a planar graph complemented information about its components (degree of its vertices) to show the difference between planar and non-planar graphs. The analysis of how these visual clues were used is presented below. Also analyzed below is the use of upper case letters to emphasize a misconception that needed clarification in the "Trig Identities" thread.

In the threaded discussion titled "Solving an Algebraic Inequality," Roberto presented a quadratic inequality and its solutions to the forum participants. He thought that he had found the solution but was not able to understand how his findings related to the book's answer and wondered if he had made a mistake.

4 2x^2 - 11x + 5 < 0
5 Well. . . I worked out the values
6 of ½ and 5 which are correct,
7 but would have thought that

the answer was ½ <x and 5<x.
After all I ended up
with an equation on the left
and the 'smaller' sign that side of x.
The answer is however ½ < x < 5.
I can't understand why!

His main problem was how to state the solution with the appropriate “greater than” or “less than” sign. He had stated the solution as two separate expressions in Line 8 (“½ < x and 5 < x”), which in one expression would be summarized as ½ < x > 5. The solution was very close and was stated as a single expression in Line 12 (“½ < x < 5”). However, Roberto could not understand why this was the solution.

Five participants engaged in this discussion. First, Wilfred evaluated his proposed solution and observed that if “½ < x” (x is greater than ½) and “x > 5” (x is greater than 5), then both expressions could be simplified as “x > ½” (x is greater than ½). When reading the expressions initially stated by Robert, one starts reading with the variable and ends with the number. So “½ < x” can be read as “x is greater than ½” and “5 < x” is read “x is greater than five.” As Wilfred explained, any number greater than 5 is already greater than ½; therefore, both expressions can be written as x > ½.

Second, Rosa asked Robert to try different values for x to corroborate his answer. She gave him two values (0 and -1) which both provided false statements. This message is not answered or further evaluated in the same story.

Then Denise evaluated Roberto’s solutions by going over his algebraic manipulations and presenting the number line image (drawn with keyboard characters) to further explicate how the solution had to be constructed:

**Denise**
>From Roberto
>27 so. . . x = 1/2 or x= 5

45 What you have so far is perfect
46 but you're not done yet.
47 Finding the solutions to
48 the corresponding equation,
49 as you did, is a good first step.

50 Now use those two values
51 (the zeros)
52 to divide the number line
53 into the intervals:

```
<-----------|--------------|------------->
           1/2             5
```

54 x < 1/2
54a or in interval notation (-oo,1/2)
55 1/2 < x < 5 . . . (1/2,5)
56 x > 5 . . . (5,oo)

In this way, Denise addressed Rosa’s misconception in providing possible values without directly stating that she had made an error. Instead of both (0 and -1) being to the left of the first intercept (x = ½), they should have been in different intervals, one to the left of ½ (for the first interval, x=0) and the second in between ½ and 5 (for the second interval, x = 1). In this way, the solutions would have different outcomes indicating what intervals corresponded to the solution. Denise also went on to explain how to use the number line to find the final solution to this problem. Her presentation provided visual clues to find the solution as well as its accompanying explanations.

**Denise continues**
57 To determine which interval(s)
58 comprise the solution set,
59 simply pick one number
60a (any number)
61 in each interval,

then test the expression
to see if it's positive or negative.
Since you already have it factored,
this should be quite easy
using what you know about
the product of two factors.
You are looking, of course,
for the interval(s) where
the expression is <0.

Two more collaborations were those of Kiko, who related the solution to functions, and Gary, who related the problem to quadratic equations and its graphical representations. It was not until Kiko expanded the idea of intervals, already presented in visual form and written expression by Denise, that Roberto was able to start understanding the problem solution. At first, Kiko's presentation was not understood by Roberto, who indicated "I can't see what / you're trying to show me" (Story 4, Lines 24-25) [negotiation meaning], to which Kiko responded, "Ok, here it goes . . ." (starting at Line 28 until Line 53). Later, Roberto overtly stated this point in the gratitude statement of the thread (see below). In this thread, participants used intertextuality to point to specific lines of text and to expand previous points. They also negotiated meaning by providing hints, asking for details, answering specific questions, and following up on ideas presented by other members of the group.

This exchange of ideas was an example of *polyvocal collaborations,* where different voices contributed different elements to the discussion, collaborating with one another to fulfill a common goal and opening new windows of understanding. Alone, Roberto was not able to understand the solution of the mathematical inequality problem, but together, they made the difference, allowing Roberto to find a way to understand how to state the solution to this type of problem and also clarifying Rosa's misconception.

This was not the only instance of *polyvocal collaboration* present in the whole period of analysis. Going back to chapter five, the thread “Radius of an Arc” also showed how six participants engaged in a series of contributions that included algebraic solutions (detailed and abstract), geometric concepts’ references, and written drawing explorations. No visual clues were used in that thread.

Through out the whole period of analysis (August to December), authors asked trigonometric questions related to knowledge construction and application of concepts. Most authors were looking for specific answers. None had made evaluation or synthesis questions. However, in this last period of analysis, the thread “Trig Identities” was used to analyze a quote from a trigonometry book, one that confused the author of the thread as to how identities should be solved. Her question compared how trigonometry identities and algebraic equations were solved. Cándida thought trig identities were solved the same way algebra equations are solved. She stated:

*Stanza 1: Presenting problem*
Post 1 - **Cándida**
1 I'm having a little trouble
2 understanding this trig book
. . .
*Stanza 3: Looking for understanding*
**Cándida continues**
. . .
34 maybe I've just had
35 that old algebra rule
36 pounded into my head
37 too many times:
38 ANYTHING YOU DO
39 TO ONE SIDE OF AN EQUATION
40 YOU MUST DO
41 TO THE OTHER SIDE!!!
42 I understand the last 2 lines:
43 1/2[sin^2 A + sin^2 A]
= sin^2 A
44 but where did 1-cos^2 A go to?

45 Maybe if someone just gives me
46 a step-by-step explanation
47 of what they're doing
48 I might get it,
49 or maybe the book left a major part
50 of the proof out.
51 Either way,
52 can someone please help me
53 with this (IMHO)
54 horrible explanations of trig proofs.

First, in Lines 38-41, Cándida emphasized her understanding about how to work with algebra equations using uppercase letters as a visual clue. In netiquette, this could mean someone is shouting or saying something really important. These lines were later quoted by four of the six participants who answered her post, demonstrating that they understood how important it was for her to analyze this point. Then, in Lines 45-46, she specifically stated that she needed more information, more details, and possibly intermediate steps that were missing from the textbook she was using and that were not obvious for her. In lines 52-54, she summarized her cry for help with an indirect question. This thread included 16 stories, of which half were about Cándida's problem. The second half worked with similar trigonometric identities.

To help Cándida clarify her misconceptions, a group of six forum participants gave her new ideas to consider. Wilfred presented arithmetic and algebra examples before he related his answer to trigonometry. In this way, he guided Cándida's thinking by starting with simple examples and moving toward more complex ideas. Jack and Petra talked about the use of equivalent expressions and substitution (respectively) when working with trigonometric equations instead of using algebraic manipulations that tried to isolate a variable to find its value. Dalila supported Jack's and Petra's ideas in her message and also presented a trig example that included a detailed, step-by-step procedure. Pablo explained why the equal sign symbol could be misleading. He provided algebraic examples and trigonometric identity's explanations comparing the procedures used to solve both types of problems.

In this thread, most participants did not overtly interact with one another; instead they presented their explanations in separate stories, some of which expanded previous points or even reanalyzed previous statements, thus connecting one story with another. Cándida did not ask follow-up questions or more details about their presentations. Instead, she stayed in the background without making any comments. Still, this thread represented an instance of *polyvocal collaborations* used in the forum to help Cándida clarify her misunderstanding. By herself, she felt lost; she needed a "step-by-step explanation of what they're doing" [in the book]. She needed help in understanding the trigonometric identities she was studying. In the *Forum,* she found not only one but six people who presented different examples at different levels of complexity, directly collaborating with her, indirectly collaborating with one another, and helping her to overcome her confusion. In this thread, negotiation was minimal. Still, the opening post author was able to find understanding (see next section).

During the months of November and December, participants of the forum engaged in similar activities as those described and analyzed in the previous data analysis chapters. One new activity was the construction of text images to complement their explanations and clarify misconceptions. A second activity was engaging in the analysis and evaluation of solving trigonometric identities, a discussion that led to the comparison of different types of mathematical procedures. In doing so, participants of the forum engaged in a third activity, the use of *polyvocal collaborations* that helped different authors and other participants gain new meaning and understanding.

## November’s and December’s Coda(s): Additional Information

Two general types of codas included in the third period of analysis were gratitude messages and netiquette discussions. First, authors of threads included gratitude statements in different ways and with different amounts of detail. These were addressed to individuals or groups of participants. Second, other participants pointed to the need to follow netiquette practices when engaging in discussions to make them easier to follow. Both types of codas were recurrent throughout the whole period of analysis (August to December), but what make them special in this period is the diversity of gratitude statements and the types of recommendations offered by the participants.

Gratitude messages included simple messages such as a “got it” and thank you in “*Trig problem*” (Story 7) directed to all the participants.

*Stanza 11: Auto-correction and gratitude*

Post 9 - **Robert**

170 Sorry, sorrry, sorrrry everybody
171 I got it!!!
172 Many thanks

They also wrote messages confirming that they had found what they needed by saying “this is exactly what I needed” in “Exponents versus Multiplication Notation” (Story 5). Authors addressed their gratitude statements to a single participant (in “Solving an Algebraic Inequality,” Story 5) and to the whole group of participants (in “Recursion,” Story 8). However, more elaborated gratitude messages were the exception. In “Trig identities” (Story 7), Cándida explained in detail how she was able to gain understanding.

*Stanza 5: Gratitude*

Post 9 - Cándida

55 DDDDDUUUUUHHHHH!!!
56 As soon as you all pointed out
57 "1-cos^2 A = sin^2 A"
58 as the reason why "1-cos^2 A"
59 disappeared

60 it just "clicked".
61 I realized exactly what you all
62 (and the book)
63 were trying [to] say about
64 manipulating part of the equation,
65 and how your (sic) not changing
66 the equation
67 your (sic) just substituting
68 proven parts to simplify it.
69 **I finally understand,**
70 **so I just want to say**
71 **THANK YOU.**
[lines 69-71: author's emphasis]

With this message, she concluded the stories related to her question. She posted the message in a separate thread so that all participants could see it. Six people collaborated in the development of Cándida's understanding, and she emphasized how "all" (Lines 56 and 61) helped her "finally understand" (Line 69) her problem. As was stated before, *polyvocal collaborations* were used in this thread to address the same problem from different perspectives or standpoints, going from lower level mathematics to upper level mathematics. As Cándida stated in her gratitude message, "it just 'clicked'. / I realized exactly what you all / (and the book) / were trying [to] say" (Lines 60 – 63). *Polyvocal collaborations* helped her generate new understanding.

The second type of coda, about netiquette statements, evaluated how posts could be distributed among multiple groups and the advantages of this. In the thread "Exponents versus Multiplication Notation," Denise stated the following benefits for authors (Lines 192-197) and their respondents (Lines 187-191):

*Stanza 17: Netiquette benefits: one*

**Denise**
187 This has multiple benefits.
188 For the others, it allows them to have
189 the entire conversation at their easy disposal
190 no matter which one of the addressed groups

191 they normally subscribe to.

*Stanza 18: Netiquette benefits: two*

**Denise**

192 For you, as the original poster,
193 you have the added benefit
194 of only having to follow one
195 of the addressed groups to see
196 each and every response
197 from *any* of the addressed groups.

The possibility of posting to several groups at the same time was not mentioned before. This established a connection among different discussion groups. In this way, the forum environment was expanded beyond the specific discussion forum and allowed participants to connect with a larger group of mathematics learners and more knowledgeable others. Not only were the authors of a thread able to connect with others among the threads in a period of study or within the stories of a thread, as it had been the case in the previous two periods of analysis, but according to Denise, participants of a specific discussion forum could also connect with participants of other forums.

Another member of the group, Manuel, also recommended using intertextuality to keep track of ideas by quoting “relevant portions of the original message” and writing “replies below the appropriate portions of the original message” (“Exponents versus Multiplication Notation,” Lines 108-119, Story 8). As he stated,

114 This makes it easier to keep track
115 of what has been said and
116 what you are referring to,
117 especially for those people who
118 aren't paying attention to every message
119 that goes by.

He also directed the participants to evaluate his own message as an example of what he was trying to say. In this way, netiquette was not only discussed by the participants but also referenced as an example. This was a way to contribute to the development of better discussions.

In general, coda statements were used by the participants of the *alt.math.undergrad* discussion forum to evaluate the medium used to communicate (a distributed asynchronous communication tool) and to try to help its users to better use this resource to learn mathematics. In addition, gratitude messages overtly confirmed that the forum participants were able to help each other generate new meanings and understandings.

## Part II: Analysis of Identities and Relationships

This chapter's data confirmed that the forum participants belonged to two general groups: on one side, the "newbie" or mathematics learners and on the other side, the more knowledgeable other that helped the first group construct mathematics knowledge while finding meaning and understanding. The first group included high school and college students. For example, in "Matrix Determinant," Alberto, the author of the thread, addressed himself to the forum by saying, "I'm in first year of high school" (Story 1, Line 1). On the other hand, Migdalia, in "Logs," indicted that she was a college student by stating that she had "embarked on a 3 year / bachelor's degree / in mathematics . . ." (Story 1, Lines 4-6). Nevertheless, most authors of questions, problems, or inquiries did not identify themselves as belonging to one or the other group; they just added a message to the forum, or started a threaded discussion that invited others to answer or comment.

As shown in the last two chapters, the more knowledgeable others group included teachers and professors. In "Trig Identities," Dalia also included instructors and book authors rather than simply kids who were learning new mathematics. This group of participants was also described by their character, that is, the way they contributed or interacted with others in the forum.

According to Sandra, in “Exponents versus Multiplication Notation,” there were “[t]hose who make it their life mission / to scorn others” (Story 4, Lines 137-138), flamers that make “the same suggestions / over and over each and every time / they see something not of their liking” (Story 4, Lines 131-134). There were also those who “have your [learners’] best interest in mind” (Story 4, Lines165). This last group might make an “occasional tactful suggestion” while trying to help the learner the most (Story 4, Lines 161-167) but will not add personal negative remarks.

Therefore, two general groups participated in the *alt.math.undergrad* discussion forum: the “newbie” or math learner and those who helped the learners. The first group was mostly composed of high school and college students; the second group acted as teachers, instructors, and professors. Both groups had special characteristics. The first group included learners that felt lost when trying to do mathematics by themselves; they were in need of others’ support, as it was shown in chapter six. The second group shared their knowledge of mathematics, presenting mathematics ideas in different ways and assisting others in their search for meaning and understanding through intertextuality and *polyvocal collaborations*.

Most interactions between learners and more knowledgeable others were conducted in a friendly manner. When evaluating others’ work, more knowledgeable others wrote comments such as, “what you have so far is perfect / but you’re not done yet” (“Solving an Algebraic Inequality,” Story 3, Lines 45-46), “You’re quite correct, / assuming that . . .” (“Traffic Light Probability,” Lines 32-33), “You are very much on the right track,” and “You just need / to tighten things up a bit. / Do you know any theorems / relating . . .” (“Graph Planarity,” Lines 72 and 85-88 respectively). Participants also answered specific questions and received follow-up replies from learners that, in turn, led to more interactions. For example,

**Solving an Algebraic Inequality**

**Story 4 of 6**

*Stanza 3: Asks for more details*

**Roberts**

22 Thanks.
23 I'm sure I am very dim,
24 but I can't see what
25 you're trying to show me.
26 Could you expand a bit, please
27 Thanks

**III RESOLUTION**

*Stanza 4: Provides more details*

**Kiko**

28 Hi. Ok, here it goes.
29 Remember that you are dealing
30 with an inequality.
31 That means that
32 the solutions are found
33 on intervals.
34 Also have in mind the rules
35 for multiplying signed numbers.

Nevertheless, there were also occasional antagonistic remarks. Some of these were presented in chapter six in presenting netiquette issues. In this period of analysis (November and December), one such interaction was related to the use of a calculator to compute logs. Wilfred started this type of interaction (in “Logs,” Story 3, Lines 13- 18) with the following remarks.

*Stanza 2: Derogatory remark*

**Wilfred**

13 Learn mathematics,
14 ditch the stupid calculator.
15 Have any of them ever
16 passed a math class?
17 Stick with them and
18 you'll be doing the same.

This message did not contribute to knowledge construction; instead, it insulted the author of the original post. Another message that started with a negative statement was that of Daniel.

*Stanza 4: Evaluates original post*

**Daniel**

22 It won't help the original poster,
23 though.
24 I have the 20'th edition
25 in front of me right now.
26 Common Logarithm Tables –
26a pp. 186-209
27 Natural Logarithm Tables –
27a pp. 210-217
28 Logarithms to another base –
28a no tables given

However, in this message, Daniel indirectly helped the author of the original post by including a reference to a specific book and those sections of it that could help her find the answer she needed. Participants of the forum expected learners to show that they had done something to try to solve the questions or problems they posted. As before, presenting a problem alone was not enough; participants wanted to know the details of the work already done by the opening post authors, and they needed more information to be able to answer specific questions. In general, negative remarks, when they occurred, were counteracted by more constructive statements or even dismissed by participants of the forum without saying much more.

Participants of the forum showed an emphasis toward constructive thinking. Negative remarks were the exception and when present, were counteracted by other more positive remarks or even ignored. A recommendation provided by Manuel in “Exponents versus Multiplication Notation” (Story 5) stated,

117 and therefore I would like to urge you
118 to work on how you word
119 your netiquette complaints to newbies.
120 If you don't think you have the time
121 to write out a polite response every time,
122 I suggest you build a boilerplate response
123 which refers them to a suitable webpage.

This entrance was related to netiquette complaints and the importance of making polite responses (Lines 117-119). It also included the recommendation of adding references to web pages (Lines 122-123) that could further help those in need to understand specific mathematics ideas and concepts. As stated before, the *alt.math.undergrad* discussion forum was an open environment because participants were not restricted to a particular group and because they were encouraged to use other resources available in the *Math Forum @ Drexel* as well as those in other websites, thereby connecting this online digital environment to other mathematics sites.

## Refining the Preliminary Mathematics Discourse Model

Data from November and December allowed the researcher to revise and refine the preliminary mathematics discourse model presented in the previous two chapters. As before, this is a model under construction, one that will need to be revised with new data in the future.

Novices, casual students, high-school and college math students, as well as users of mathematics, together with more knowledgeable others (math "brainiacs" (sic), teachers, instructors, and professors) collaborated and negotiated meaning and understanding to construct mathematics knowledge. Together they evaluated mathematics problems, questions, and inquiries in the areas of general mathematics, algebra, statistics, discrete mathematics, trigonometry, pre-calculus, and calculus. Working together during the months of August through December in a voluntary, online public mathematics discussion forum that was available 24 hours a day and seven days a week, learners and tutors generated different kinds of solutions and constructed new mathematics knowledge through the following activities:

- Presenting direct and indirect questions, problems, and inquiries located in different parts of the Original Message – learners included examples, explanations, and paraphrased questions. They were looking for specific answers, corroboration, clarification, and understanding of mathematical concepts.
- Negotiating meaning by following up on questions, adding comments, new information, hints, and tips. They clarified and defined mathematics concepts, terms, theorems,

- Negotiating meaning by using geometry: presenting different geometric concepts, written constructions, and visual clues such as text diagrams. They explored different ways of analyzing and solving a problem.

- Comparing procedures used to solve algebraic equations and trigonometric identities, thus developing higher order thinking skills such as analysis, synthesis, and evaluation.

- Referencing books, famous mathematicians, and different types of online sites and documents, including articles, tutorials, “webworks,” and other discussion groups, to add new information and complement ideas introduced in the forum.

- Using common language to communicate, as well as mathematics language including mathematics notation and abbreviations.

- Promoting the use of intertextuality to follow up on specific information, questions, and answers, thus encouraging participants to use netiquette strategies so that authors and respondents could follow entire conversations.

- Using polyvocal collaborations to present different ideas related to the same concept, including algebraic and geometric concepts as well as visual clues such as text diagrams to complement a discussion and upper case letters to emphasize a specific idea.

- Expressing sentiments of confusion and frustration when unable to find a solution or understand a concept and finding meaning and understanding to overcome such feelings through intertextuality and polyvocal collaborations.

- Being grateful for the help learners received, assessing their own learning, and stating that they found understanding.

This mathematics discourse model is the result of five months of data analysis from a specific mathematics discussion forum. Its validity will be examined in the next chapter, in which the summary, major findings, implications for action, and concluding remarks of this dissertation will be presented to the reader.

Table 7-1. General description of threaded discussions in November and December

| Title | Content Area | Number of Postings | Number of Stories | Number of Participants | Time Span in Days |
|---|---|---|---|---|---|
| Exponents Versus Multiplication Notation | General Mathematics | 24 | 11 | 10 | 5 |
| Number Patterns | Algebra | 14 | 9 | 12 | 5 |
| Three Digit Numbers | Algebra | 16 | 4 | 8 | 4 |
| Looking For Problems | Algebra | 16 | 4 | 6 | 7 |
| Solving an Algebraic Inequality | Algebra | 11 | 6 | 6 | 2 |
| Function Value | Algebra | 11 | 7 | 9 | 2 |
| Completing The Square | Algebra | 12 | 4 | 7 | 3 |
| Matrix Determinant | Algebra | 10 | 4 | 5 | 3 |
| Recursion | Discrete Math | 22 | 9 | 8 | 8 |
| Close and Bounded | Discrete Math | 10 | 5 | 5 | 3 |
| Traffic Light | Discrete Math / Statistics | 13 | 6 | 8 | 5 |
| Planarity of Graph | Discrete Math | 12 | 2 | 3 | 6 |
| Trig Identities | Pre-Calculus | 25 | 16 | 18 | 18 |
| Log | Pre-Calculus | 10 | 6 | 9 | 4 |
| Trig Problem | Pre-Calculus | 10 | 7 | 5 | 1 |
| Derivative Question | Calculus | 17 | 7 | 8 | 2 |
| Sine Curve | Calculus | 10 | 6 | 8 | 2 |

Notes: N – November data. D – December data.

Table 7-2. Connecting the discussion forum with other math resources (from November's and December's Data)

| Thread Tile | Resources |
|---|---|
| Exponents versus multiplication notation | (1) Private web space – the author of the thread asked participants to check her work posted here. https://www.cotse.net/users/dns123/logit.htm (2) Reference to web article about exponents – http://oakradsystems.com/gen1/unice.htm#upside" changed to http://oakroadsystems.com/math/expolaws.htm |
| Number patterns | (1) Reference to support reply – http://www.research.att.com/~njas/sequences/index.html The On-line Encyclopedia of Integer Sequences, located at AT&T Research Labs Website |
| Matrix determinant | (1) Reference to complement message about vectors and triangles – http://www.scienceoxygen.com/mathnote/vector206.html (2) Reference to web article – http://mathforum.org/library/drmath/view/55063.html |
| Logs | (1) Reference to web article about logarithms – http://oakroadsystems.com/math/loglaws.htm#NewBase (2) CRC book – *CRC Standard Mathematical Tables and Formulae (31st ed., 2002)* by Daniel Zwillinger |
| Trig Problem | (1) Reference to web article about trigonometry – http://oakroadsystems.com/twt/solving.htm#Cases |

Table 7-3. Use of direct and indirect questions in the opening posts of the threads

| Title | Direct questions | Indirect question |
|---|---|---|
| Exponents Versus Multiplication Notation | | After introducing a web page address and setting: "Will someone please check my answers?" |
| Number Patterns | | "I can not figure out the missing numbers in these pattern (sic) . . ." And then presents two number patterns. |
| Three Digit Numbers | Only the math question: "**How** many 3-digit numbers consist only of odd digits?" | |
| Looking For Problems | | Asking for problems: "has any1 got a (not too hard) math problem?" |
| Solving an Algebraic Inequality | | After presenting problem, personal solution, and book's solution: "I can't understand **why**! Help" |
| Function Value | | After presenting the math problem and initial steps: "Now I don't know **what** to do next to find . . ." |
| Completing The Square | | After presenting the math problem and initial steps: "And if that is ok, then **what**? . . . Help" |
| Matrix Determinant | First: After presenting the problem: "What I don't understand is **how** do we choose . . .?"<br>Second: After comparing with other problems: ". . . **why** does this same formula hold true even when . . .? | |
| Recursion | | **Before** presenting the problem: "How do you solve this problem plz help!!! thx!!!"<br>**After** presenting the problem: "plz list steps thx!!!" |
| Close and Bounded Sequence | (No direct questions, only the math problem.) | (No indirect questions, only the math problem.) |
| Traffic Light | | After presenting the problem and solution: "that's a low number, so I'm not too sure . . ." |

Table 7-3. Continued

| Title | Direct questions | Indirect question |
|---|---|---|
| Planarity of Graph | | After presenting the problem and several attempts already made: “I can’t think of any other strategy at the moment for this. Any tips?” |
| Trig Identities | | After stating the math problem: “can someone please help me with this … horrible explanations of . . .” |
| Log | | After stating general calculator problem: “Hints anyone??” |
| Trig Problem | | After presenting the problem and several attempts: “Can you help, please?” |
| Derivative Question | Only math questions: “**How** does . . . become . . .? **How** do you handle derivatives for . . .?” | |
| Sine Curve | After presenting a math problem statement: “**How** do I define . . .?” | After the direct question: “Any help would be great.” |

# CHAPTER 8
# CONCLUSIONS

The purpose of this study was to examine the types of online dialogues and discursive collaborations that took place in an online public discussion forum that facilitated the construction of knowledge, meaning making, and understanding in mathematics. Based on social constructionism conceptualizations (Gergen, 1994, 1999; Gergen & Gergen, 2003), in which learning is the result of interaction, negotiation, discursive collaboration, reflexive questioning, and dialogue (Chapter 3), and using discourse analysis methods as proposed by Gee (1999, 2005) to analyze the data set (Chapter 4), the researcher was able to develop, revise, and review a preliminary mathematics discourse model (chapters 5, 6, and 7) that showed the types of activities and connections in which people engaged while constructing mathematics knowledge. In general, this study examined how participation in a discussion forum helped mathematics learners construct new mathematics knowledge, meaning, and understanding while engaging in generative dialogue, negotiation, and discursive collaborations.

In this research, discussion forums were defined as online asynchronous communication tools where a group of people interested in the same topic or subject matter interacted with each other by posting questions, answers, or both. This type of communication was organized as threaded discussions with a beginning and an end. Discussions included interactions that connected participants within, between, and among threads and even between different discussion forums and websites. The interactions in which participants of the forum engaged are discussed below.

## Summary of Major Findings

By examining written discursive collaborations, the researcher was able to study how participants of a discussion forum engaged in different types of social interactions and negotiation practices. She focused on the activities and connections in which participants engaged, although data also provided the elements needed to look at the identity of the participants and the sign-systems they used to communicate. Major findings include the following:

- All participation of the forum seemed to be voluntary and assumed one of two roles, mathematics help-seekers and more knowledgeable others.
- Participants used different types of sign systems to communicate, including algebraic and geometric concepts, principles, symbols, ideas, and visual clues.
- Participants engaged in different types of activities related to mathematics cognitive development, including know-how and inquiry-based learning.
- Participants provided affective and emotional support while interacting and negotiating mathematics meaning and understanding.
- Interactions and negotiation practices connected the *alt.math.undergrad* forum threads within, between, and among themselves.
- Interactions and negotiation practices connected the *alt.math.undergrad* forum threads to other resources outside the *Math Forum @ Drexel.*

### Identity of Participants

Forum users seemed to be voluntary contributors with an identity or pseudonym they themselves had chosen when subscribing to the public website. Study participants assumed one of two roles, mathematics help-seekers (learners of mathematics) or more knowledgeable others.

These were not fixed roles, and participants were able to assume one or both roles at any time when evaluating a problem or question. Their contributions to the forum included asking follow-up questions as well as answering questions or making comments related to others postings.

From a total of 111 participants, twenty-four (24) participants, or 22%, were help-seekers, initiating one or more threads. Only four help-seekers introduced more than one question. They represented 17% of the help-seekers and only 3.6% of all participants. The ratio between help-seekers and more knowledgeable others was 1 to 4. Together, as voluntary participants of the forum, they interacted, negotiated, and collaborated with one another using different strategies and helped each other to construct and generate new mathematics meanings and understandings. With the collaboration of more knowledgeable others, help seekers were able to strengthen their mathematics self concept to the point of becoming empowered to address their professors and others with their own questions and findings.

**Sign Systems or Alternative Ways to Communicate**

Because participation seemed to be voluntary, the levels of technology anxiety appeared low. This allowed users to find alternative ways to communicate and write mathematics symbols, notations, and ideas. For example, some participants transferred knowledge from other technological devices, such as calculators, computer hardware, and computer software. When participants did not know the mathematics symbols they needed to use, they made up new symbols or abbreviations. More knowledgeable others then informed them about the symbols or abbreviations commonly used in the forum, as if they were teaching them how to communicate in this medium. This result refuted Smith, Ferguson, and Caris (2003), who concluded that "Web-based distance learning environments do not adequately support mathematics" (p. 49).

Participants also interpreted problems from different mathematical points of view. They used algebraic, geometric, narrative forms of communication, as well as visual clues to explicate ideas, give examples, and present different types of solutions to a single problem. This allowed the participants of the forum to engage in dialogues at their own level of understanding, to choose one message over another, and to engage in further discursive collaborations.

**Activities Related to Mathematics Cognitive Development**

Throughout the discussions developed in the forum, users engaged in all levels of knowledge construction, going from low-level knowledge skills (knowledge, comprehension, and application) to high-level knowledge skills (analysis, synthesis, and evaluation) (Bloom, et al., 1956). Participants resembled a heterogeneous group of mathematicians participating and contributing with mathematical ideas at different levels of understanding. This provided a rich environment where help-seekers were able to find and correspond with others at similar level of understanding and to have a preview of more sophisticated or advanced mathematics knowledge.

Participants of the forum were able to develop new meanings and understanding based on the questions help-seekers presented to the forum. The use of intertextuality allowed all participants to focus upon and to analyze specific ideas, as well as to clarify misconceptions. Participants also engaged in negotiation practices, providing hints, tips, new ideas, theorems, definitions, and visual clues to help mathematics learners develop meaning and understanding. More knowledgeable others followed up on questions and clarified ideas, thus developing in-depth discussions. They presented solutions in different levels of understanding (cognitive levels) going from low-level mathematics to advanced mathematics. They also included different types of answers, engaging in polyvocal collaborations.

Discussions in the forum allowed students to store up information and to develop new meanings through “knowing how” activities. This contradicted Romberg’s (1992) findings, which analyzed school mathematics and found that in most cases, mathematics education allowed students to store up information by “knowing what” instead of “knowing how.” He found that students had few opportunities to engage in authentic learning or inquiry-based learning. This discussion forum provided an authentic learning space where participants engaged in know-how activities and in in-depth analysis of mathematics concepts and ideas.

In this forum, participants negotiated meaning by asking and answering follow-up questions, by writing solutions in different levels of difficulty, and by writing various types of solutions to a single problem, engaging in polyvocal collaboration. Participants also added tips or hints, comments, new information, and references to digital and non-digital resources. They clarified misconceptions, corrected one another and even themselves when mistakes were made. The forum allowed participants to engage in “knowing how” activities that promoted the construction of mathematics knowledge through writing activities.

While negotiating meaning, participants of the forum – help-seekers as well as more knowledgeable others – also engaged in inquiry-based learning (Richards, 1991). They did not use the forum to do research or journal-type writing. However, the forum allowed space for reflexive learning in collaboration with others. Authors of questions or problems were expected to explicate why they needed help and to state what work they had done already to try to find the solution to their conundrums. These annotations helped others to focus on the particular misconceptions or misinterpretations of the author. Most participants were able to find a solution to the problems they posted through collaborative practices.

Mathematics learners in this research were able to interact with others in an authentic, inquiry-based environment. This study refutes Shield and Galbraith’s (1998) research outcomes related to writing for deeper understanding. In Shield and Galbraith’s research, students wrote questions to imaginary friends, thus eliminating all possibilities of authentic interactions. Nevertheless, in the discussion forum, help-seekers posted questions and expected to receive different types of answers. While interacting with other participants, they followed-up with tips, hints, new ideas, or comments. This dynamic environment allowed learners to be active participants, negotiating meaning and collaborating with one another, even if they did not know each other personally.

In this research, participants resembled those of Stonewater (2002), as they were able to develop, elaborate, and clarify mathematics ideas by using algebraic and geometric examples, mathematics notation, visual clues, and narrative explanations. As Restivo (1983) stated, mathematics showed signs of cultural expressions.

**Activities Related to Affective and Emotional Support**

The forum was a learning environment, a community of practice where participants not only engaged in mathematics knowledge building but also helped develop the affective skills of its participants. Help-seekers and more knowledgeable others engaged in negotiation and collaborative practices that included receiving, responding, valuing, organizing, questioning, discriminating, and justifying their arguments (Krathwohl, Bloom, & Masia, 1964). These were exemplified by the following activities: (1) receiving new ideas and comments, (2) responding to questions and follow-up questions, (3) valuing arguments presented by both learners and more knowledgeable others, (4) valuing algebraic answers and notations, arithmetic problems and solutions, and geometric propositions, (5) organizing ideas, (6) questioning statements and propositions made by others, (7) discriminating between different answers posted by others at different levels of difficulty and making corrections when needed, and (8) justifying arguments with definitions, prepositions, theorems, and other sources that included documents available on the Web, other websites, journal articles, and books.

Some opening post authors expressed feelings of uncertainty, confusion, despair, and frustration when trying to find a solution or solve a problem. They were help-seeking individuals who used the forum as a tutoring center because they knew this was a place where more knowledgeable others were available and willing to help. Signing in to the forum was voluntary, and participants were able to use pseudonyms if they desired.

When help-seekers manifested feelings of frustration, more knowledgeable others responded in ways that allowed the learners to move forward and find solutions to their problems or questions. The statements of despair stated by some at the beginning or in the middle of a threaded discussion (See poem “Voices calling out to be heard” in Chapter 6) changed into notes of gratitude at the end of the threads.  For example, frustration and despair was overtly stated with phrases such as “I have this problem,” “I am getting confused,” “I don’t understand,” “I must be messing up,” and “I stopped.” These later changed into statements of relief such as “this [is] sorta (sic) exactly what I was looking for,” “it just clicked,” “I got it!!!,” or “this is exactly what I needed.”

On several occasions, a single thread was used to ask and answer more than one question. Learners felt free to ask more questions related to the initial one after their first question or problem was resolved. The support received by learners was then translated into statements of gratitude which they openly manifested at the end of the threaded discussions. Working together in the forum empowered some participants to communicate with professors and instructors to make corrections or recommendations. As Quinn (2005) stated, using communication tools enabled students to build self-confidence in their capacity for learning mathematics.

The forum not only allowed its participants to develop knowledge and affective skills, it also provided the emotional support needed by learners in their quest for meaning making and understanding. This finding supports Stein and Glazer’s (2003) research conclusions. It also supports Ferdig’s (2007) statements about how the use of social software promotes the development of supportive environments, scaffolding learning at different levels while participants try out different ideas and challenge each other.

## Connections Within, Between, and Among the *alt.math.undergrad* Forum Threads

Within threads, participants engaged in polyvocal contributions, presenting partial solutions in algebraic, geometric, and narrative forms. They also used visual clues such as text diagrams to further explicate a concept and upper case letters to emphasize a specific question. In this way participants were able to evaluate a problem or question from different perspectives, and help-seekers were able to choose to follow up with those they understood the best. They also benefited from these different types of presentations by gaining a global view of how mathematics ideas can be used from different perspectives and levels of difficulty.

Between and among threads, participants engaged in Conversations (Gee, 1999, 2005) that promoted the integrity of the forum as a whole. They engaged in Conversations about the importance of netiquette practices, about the importance of intertextuality – quoting sections of a previous posting – when answering or following up on a question, and about the use of mathematics notation and abbreviations.

Participants developed a sense of community and worked toward maintaining the integrity of the interactions that took place in the forum. Netiquette was a Conversation (Gee, 1999, 2005) that developed throughout the whole period of analysis. More knowledgeable others provided hints on how to introduce problems and questions. They emphasized the importance of supporting one's initial statements with work already done and with specifics about the questions they were presenting to the forum so that others could best know how to help. More knowledgeable others also clarified mathematics notations and made recommendations about how to write mathematics symbols in a text-based environment.

As in any other community of practice, antagonistic remarks were also present throughout the participants' dialogues; however, they were discouraged or avoided by most participants. Instead, participants gave tips about how to use intertextuality to pose their answers or replies, suggested better ways to present questions, clarified mathematics notation, and elaborated on how netiquette could benefit both the learner and the more knowledgeable other when negotiating meaning.

### Connections to Other Resources outside the *Math Forum @ Drexel*

While negotiating meaning and collaborating with one another, participants made references to specific books, famous mathematicians who had worked with a specific topic, and different types of online sites and documents that had more information on the question being discussed. These included online and paper journal articles, web tutorials, "webworks," and other discussion groups outside the *Math Forum @ Drexel*. In this way, more knowledgeable others complemented the capabilities of the forum, moving toward other mathematics resources available elsewhere.

### In Summary

By using Gee's discourse analysis methods, the researcher was able to develop a series of three discourse models, allowing her to review and revise outcomes from the first set of data with more data. This model was the result of a series of answers to the questions initially proposed by Gee (1999, 2005) and reinterpreted by the researcher. Building task questions used in this research initially included the activities and connections building tasks. Still data allowed the researcher to explore the identities of the participants and sign systems used to construct mathematics knowledge. Questions were used to analyze the data set and were simplified by the researcher to construct a mathematics discourse model (See Figure 5-4). The components of the model included answers to the following questions:

- Who? Identities: help-seekers and more knowledgeable others.
- Why? To find and provide help in learning mathematics.
- What? Mathematics topics in high-school and first- and second-year college level.
- Where? At the *Math Forum @ Drexel*, specifically the *alt.math.undergrad* discussion forum.
- When? August through December, 2004.
- How? Activities, connections, and sign systems participants engaged in.

The mathematics discourse model explained how participants engaged in written discursive collaborations and negotiation practices. It showed how online intertextuality allowed participants of the forum to construct new knowledge and understandings, and it showed how they received affective and emotional support from other participants of the forum throughout this process. It also showed how participants became part of a community of mathematics' learners that protected their space from disruption, maintaining its integrity.

Social languages were related to mathematics construction of knowledge at all times, and a Conversation (Gee, 1999, 2005) of how to use the forum to promote learning tied in the different threads throughout the whole period of analysis. Other interruptions were dismissed, even ignored, when they occurred. At the end, help-seekers were grateful for more knowledgeable others and for the cognitive, affective, and emotional support they had received. Together, they engaged in a mathematics Discourse (Gee, 1999, 2005).

The main focus of this research was to find out how participants of the forum constructed mathematics knowledge. For this reason, it closely analyzed and reported the activities and connections in which participants engaged. Still, this research did not overtly report on the significance of the interactions, nor did it closely examine the relationships and politics embedded in the discussions. These can be the object of further research.

## Implications and Recommendations

The following implications and recommendations can be extrapolated from the research findings:

**Implication 1:** Participation in a heterogeneous community of practice can have a positive impact on mathematics learning. It can allow help-seekers to engage in authentic learning experiences with more knowledgeable others, to engage in know-how activities and inquiry-based learning, to engage in different types of mathematical discussions at different levels of difficulty and abstraction, and to engage in reflexive deliberations and discursive collaborations when analyzing contributions made to the community (Lave & Wenger, 1991; Palloff & Pratt, 1999; Renninger, & Shumar, 2002; Rogoff, 1994; Rogoff & Lave, 1984; Wenger, 2001).

**Implication 2:** Co-construction of knowledge can allow help-seekers and more knowledgeable others to negotiate meaning and understanding (Gergen, 1994, 1999; Gergen & Gergen 2003), allowing them to clarify misconceptions and to construct new knowledge. It can also allow participants to engage in polyvocal collaborations when participants introduce different ideas and different solutions from different standpoints to a single problem. This can also allow participants to have a larger view of the problem at hand and to see how it can be applied to different settings.

**Implication 3:** Writing in mathematics can help students develop higher order cognitive skills. This research supports Miller and England's (1989) argument about writing in mathematics. They contended that writing is conducive to understanding and to the development of cognitive skills. In this study, participants had the opportunity to write in an authentic environment. This allowed learners to engage in inquiry-based learning activities.

**Implication 4:** Participation in a heterogeneous community of practice can empower help-seekers to communicate with those in authority positions, such as professors and administrators. This can help learners assume a proactive attitude toward learning. According to Quinn (2005), participating in such environments can also reduce mathematics anxiety while increasing mathematics self-concept.

**Implication 5:** Participation in an online heterogeneous community of practice can provide the support needed by students working on distance education programs or virtual school. Students can feel isolated when learning in distance education programs, especially if asked to work by themselves (Moore, 1989; Weis, Knowlton, &Speck, 2000; and Simonson, Smaldino, Albright, & Zvacek, 2003). However, online heterogeneous communities of practice can provide the space participants need to find other learners at similar and at different knowledge levels, providing them the opportunity to work in the zone of proximal development, both with less and with more knowledgeable others. As help-seekers, they can find more knowledgeable others creating a zone of proximal development that can enable the first move beyond their independent means in their quest for learning (Vygotsky, 1978).

## Recommendations for Practice

This study supports the importance of communities of practice among learners. It recommends organizing heterogeneous informal learning spaces for students in distance learning programs, virtual schools, and community centers. These environments can allow help-seekers and more knowledgeable others to meet over extended periods of time. Participants that start as help-seekers can move toward positions of more knowledgeable others over a period of time. For example, in a virtual school, this would mean that students from different mathematics courses can use pseudonyms or avatars to participate as both help-seekers and more knowledgeable others. Students can be taught how to ask questions and how to provide partial solutions at the initial stage of engagement.

**Recommendations for Research**

- A study that analyzes the relationships and power structures developed in informal learning environments.
- A study that examines how students develop mathematics knowledge across time in an informal learning environment, analyzing the transformation of a help-seeker (probably a first-year student) into a more knowledgeable other (senior student). This study can also explore (1) how the use of pseudonyms, avatars, or the use of students' real names impact their participation, including what types of questions they ask, and what types of problems they post, and (2) how students' mathematics self-concept and self confidence changes through time (Quinn, 2005).
- A longitudinal study that examines the notion of empowerment among participants of online heterogeneous communities of practice and its influence on students' pursuits of higher goals. An example of such is Moses and Cobb's (2001) *Algebra Project*, a face-to-face college preparatory program for minority students.
- A study that analyzes the relationships among participants of online heterogeneous community and the influence of those relationships on retention rates in distance learning programs or virtual schools. The increased number of online courses and programs has an impact on mathematics education as well (Lutzer, 2000; The World Wide Learn, 2005). There is a need to continue exploring how these courses are offered, what are the teachers' and students' needs, and what are the exemplary practices conducted in such environments.

- Future research should also analyze how the availability and ease of use of new technologies, such as whiteboards, real video, real audio, and Web 2.0 tools, can promote the development of formal and informal learning communities. As technology becomes more transparent and easily available, replication of previous research with these technologies is also needed (Lotze, 2002; Bolin, 2003).

## Conclusions

This study analyzed written discursive collaborations in a mathematics discussion forum. It was based on social constructionism conceptualizations (Gergen, 1994, 1999; Gergen & Gergen 2003), and data was analyzed using Gee's (1999, 2005) discourse analysis methods. Participants assumed two major roles, help-seekers and more knowledgeable others. They participated in an active community of practice where informal learning took place. Help-seekers engaged in collaborative practices while interacting and negotiating with more knowledgeable others to clarify ideas and misconceptions. The diversity of answers presented in the forum allowed its users to engage in polyvocal collaborations that resulted in the construction of new mathematical knowledge. Participants negotiated meaning together by using intertextuality to quote portions of text and to focus on specific ideas. This supported Lave and Wenger's (1991) and Fielding's (1996) arguments about how participants of a community of practice introduce different views and care for each other.

The discussion forum studied here was a learning environment, a community of practice where free mathematics tutoring was offered to help-seekers. Learners interacted with more knowledgeable others to negotiate meaning and understanding, working with different kinds of problems in different mathematical areas and at different levels of difficulty. The diversity of problems posted to the forum did not limit the participation of its users. If authors lacked confidence and expressed feelings of frustration, a large group of more knowledgeable others was ready to state different ways to solve a problem, thus engaging in a sea of collaborations by providing hints, tips, definitions, explanations, visual clues, and different ways to help solve a problem or find the answer to a question. Interactions between and among its participants included the development of cognitive and affective skills as well as the emotional support needed to succeed in mathematics. Help-seekers felt free to express themselves: they felt part of a nurturing community where more knowledgeable others were ready to support them at all levels.

# APPENDIX A
# PILOT STUDY

## Using Public Discussion Forums to Construct Mathematics Knowledge

As access to the Internet increases, more people of all ages are able to enter a dynamic and interactive world full of new and innovative resources. The Internet provides its users with a set of communication tools that can be used to construct knowledge. These tools have been classified as asynchronous (different time, different place) and synchronous (same time, different place). Examples of asynchronous tools include e-mail, listserv, discussion forums, and blogs. Whiteboards, video and audio conferences, chat rooms, and instant messaging are examples of synchronous tools.

Construction of knowledge is no longer limited to the tools and resources available through personal means; having access to the Internet allows the users to have access to an unlimited number of resources that go beyond the students' physical space and surroundings, thus expanding their horizons all over the world and enabling them to participate in different types of communities of practice.

In this research, discussion forums were the vehicle that allowed students to interact with other content, students, teachers, and more knowledgeable others (mentors). The use of discussion forums made possible the clarification of ideas. The student was no longer left alone to construct knowledge; instead, students became partners in learning, participating in a community that helped them build knowledge and negotiate meaning together. The zone of proximal development became evident, and students were able to learn beyond their independent means (Vygotsky, 1978) by actively participating in a community of learners and constructing new knowledge.

Discussion forums are asynchronous on-line communication tools where "reciprocal communication" (Chou, 2004) is used for interaction between people interested in a specific topic or subject matter, thereby building a community of learners. Members are located in different parts of the world and can log in at different times to contribute answers or new questions. It is this flexibility that allows a participant to contribute to a larger community of learners.

The construction of knowledge in this study is defined from a social constructionist theoretical perspective, where the learner in collaboration with others actively participates "in generating meaning or understanding" (Ornstein & Hunkins, 2004, p. 117). Jonassen, et al. (1995) identified four attributes of constructivism: context, construction, collaboration, and conversation. All of these are closely related to building online communities of learners where applications to real life can be found (context), where reflection is possible (construction), where working with peers take place (collaboration), and where planning and making meaning of content happens as more students engage in learning interactions (conversation) (Chou, 2004). These are also components of social constructionism pedagogical alternatives, in which learners engage in reflexive deliberations, polyvocal pedagogies, and collaborative interactions (Gergen, 1999; Gee,1999).

## Review of Literature

Tu and Corry (2003) stated, "The goal of online discussion is to promote constructive thinking and maximize interactions between and among instructors, students, contents, and interface" (p. 303). It was Michael Moore who in 1989 identified three basic types of interaction: student-teacher, student-content, and student-student interaction. In 1994, Hillman, Willis, and Gunawardena added a forth type of interaction: interface-student interaction. The possibilities of this last type of interaction can be twofold: at the initial stages of human computer interaction, it can be viewed as a barrier, but as experience builds up, students and instructors are able to use technology to its full capabilities (Kanuka, Collet, & Caswell, 2002) until technology is no longer an issue and turns out to be almost invisible to the user.

Other research by Pérez-Prado and Thirunarayanan (2002) found that “interacting with peers fortified the learning process and made it more enjoyable” (p. 197). Stein and Glazer (2003) concluded that when communities of learners are built in distance education, they help students emotionally, providing a supportive space where they can share and find information. Lee and Gibson (2003) researched the mechanisms students use to be self-directed in an online course and concluded that students can develop the sense of being self-directed if the environment encourages dialogue, provides flexibility, and allows them to take responsibility for their own learning.

Wegerif (1998) studied the social dimensions related to asynchronous learning environments. He identified a set of factors that can influence community building in online classes, including “course design, the role of the moderators, the interaction styles of course participants, and features of the technological medium used” (p. 48). Even though the development of technology has made great advances and access to technology has increased, these factors are still the object of research.

Knowlton (2003) answered some of the questions posed by Wegerif (1998) in a research conducted to present a system to evaluate students’ contributions to a discussion forum in an on-line course, in which he included formative and summative activities. He emphasized the importance of working in a structured environment so that students could develop the skills of critical thinking and critical writing, as well as the skills related to providing constructivist feedback. In his research, he provided students with self-evaluation questions to use before their own postings and with examples of critical writing, rubrics, and generative questions to evaluate their peers. In this way, Knowlton (2003) connected "writing-across-the-curriculum" to asynchronous discussion (p. 39).

Kanuka, Collet, and Caswell (2002) studied the impact asynchronous, text-based Internet communication technology has on instruction when integrated into distance courses. Although working from the instructors' perspectives, they also addressed the issue of structured environments, finding that undergraduate students needed more structure and dialogue than graduate students. Structured environments can help students reduce the transactional distance and the feeling of isolation that sometimes is associated with online courses. The authors also found that the degree of flexibility is dependent on the degree of control students have and that there is a need to "model effective teaching" by moderating discussions and "contributing special knowledge and insights, weaving together various discussion threads and course components, and maintaining group harmony" (p. 164, cited from Rohfeld and Hiemstra, 1995, p. 91 by Kanuka, Collet, & Caswell, 2002).

Tu and Corry (2003) proposed a set of dimensions, a group of properties that try to define good discussion forum practices. These are: (1) discussion cycle – Wednesday to Tuesday, (2) discussion duration – two weeks, (3) class size – ten to fifteen students, (3) depth of threads – controlled by the instructor, (5) discussion frequency – too much or too little activity should be avoided, (6) learner-learner interaction – time is crucial to maintain communication, (7) moderation – to provide effective guidance, (8) number of postings per students – two to four, (9) instructions for discussions – should be provided prior the beginning of the discussions, and (10) evaluation measures for quality – should be provided overtly (for example, to use a rubric for formative evaluation).

Tu and Corry's (2003) research is consistent with previous research. For example, they agree with Knowlton who addressed the importance of giving students examples and self-evaluation guides in order to make better postings. They also agree with how Wegerif (1998) and Stein and Glazer (2003) viewed the moderator's role. Nevertheless, Tu and Corry's (2003) position about the number of students per group is challenged by Chou (2004), who "observed that there was more equal participation in the discussion in three-member small groups than in large groups" (p. 16).

Most of the available research studies investigate the use of discussion forums as they relate to distance learning, on-line courses, or mixed courses – face-to-face and on-line courses. Little has been done with public discussion forums and how they help students construct knowledge in informal learning environments. This research identified the activities that took place during the construction of knowledge in a public discussion forum and described the types of interaction occurring in such an environment where voluntary interchange of knowledge was conducted.

## Methodology

This study investigated how mathematics' public discussion forums are used to socially construct knowledge while developing communities of learners. Gee's (1999) discourse analysis method allowed the researcher to look at two case studies from the activity building perspective. Two threads were analyzed using a combination of (1) Gunawardena, Lowe, and Anderson (1997) interaction analysis model and (2) Lee and Gibson's (2003) interpretation of self-direction.

Gunawardena, Lowe, and Anderson's assessment model was based on content analysis, dividing the construction of knowledge into five phases: (1) sharing and comparing information; (2) discovering and exploring dissonance or inconsistency among ideas, concepts, or statements;

(3) negotiating meaning, or co-construction of knowledge; (4) testing and modifying the proposed synthesis or co-construction, and (5) agreeing statements or applications of newly-constructed meaning (p. 419).

Lee and Gibson (2003) analyzed the indicators of self-direction from three different dimensions: control (interdependence, proficiency, and resources), critical reflection, and responsibility. They also looked at the content of the messages and classified them as cognitive, meta-cognitive, social, organizational, and technical.

**Participants**

Data was available from *The Math Forum* discussion forums' web site (http://www.mathforum.org), "a leading center for mathematics and mathematics education on the Internet" located physically in Philadelphia, PA (The Math Forum @ Drexel, 2004). The discussion forum is a public environment where mathematicians, students, teachers, and math enthusiasts meet at any time and from anywhere. Contributions are received from the United States, the United Kingdom, the Netherlands, Australia, Micronesia, and other countries. They come from different types of domains, including private networks (net), commercial (com), non-profit organizations (org), and education (edu) – especially universities (such as University of Maryland, University of California (Berkley & Stanford), Oklahoma State University, University of Georgia, and others). Other participants' addresses cannot be easily classified as to their location and type of domain.

Questions or comments are posted in this discussion forum, initiating a discussion that during the month evaluated reached over 140 messages in a single thread. *The Math Forum* "mission is to provide resources, materials, activities, person-to-person interactions, and educational products and services that enrich and support teaching and learning in an increasingly technological world" (The Math Forum @ Drexel, 2004). The public math discussion forum has no official moderator, users post their messages at any time, interaction occurs regularly, modeling is realized by the contributions of the users themselves, and participants are not identified as undergraduate or graduate, students or instructors, or in any other way, unless they had volunteered such information. The users of the forum maintain the integrity of its contents.

In this research, pseudonyms are used to ensure participants anonymity in case they had included personal information. An IRB was submitted, but this research was exempt from getting the approval of the participants since data is archived, no personal identifiers are used, and no personal contact with participants would take place.

**Researcher**

The researcher is an undergraduate first and second year mathematic professor interested in the use of communication tools that allow students to write about mathematics and to search for meaning while interacting with more knowledgeable others such as students, mentors, and teachers. As a graduate student, she has researched the teaching of mathematics with technology, including the possibilities available through the Internet. She has also researched the importance of communication in the e-learning environments and its impact on quality education.

In terms of theory, she believes in the importance of Vygotsky's zone of proximal development: the impact that a mentor can have academically and psychologically over the mentee related to the construction of knowledge and the sense of empowerment and self-actualization that can be developed. She is also interested in the importance of social construction of knowledge where communities of learners share, explore, and develop new understanding.

**Procedure**

After identifying the source of data for this research, *The Math Forum* and a specific discussion forum, “alt.math.undergrad”, the researcher observed that the discussions were organized by month. She selected one month for evaluation and analyzed the threads with more than ten postings. In this way, she expected to find a more comprehensive overview of the interaction that takes place in the discussion forum. The topics under consideration would be first and second year mathematics.

Taking Gee’s (1999) activity building task questions as the basis for analysis, the next step was to answer the questions related to the set of activities and sub-activities that were taking place in the specific threads and to answer the questions related to the actions generated by the postings. The activities were determined through the use of Gunawardena, Lowe, and Anderson’s (1997) interaction analysis model and Lee and Gibson’s (2003) interpretation of self-direction.

In this research, there was no need to develop transcript data since it was already posted in the discussion forum. A limitation to this process, however, is that there was no opportunity to clarify the information presented by the participants; nor was there the opportunity to help empower them through the acquisition of new knowledge and the clarification of misunderstandings. Empowerment and knowledge building was accomplished in most cases through the discussion generated by the participants of the threads. The analysis conducted in this research was equivalent to the analysis of historical archives because there was no personal contact with the participants and because the threads have a beginning and an ending date.

## Results

Two case studies are presented in this paper: (1) The case of the integral, and (2) The trigonometric identity. Each of these cases represents a single thread in the discussion forum with more than ten postings, seven to eleven (11) participants, and a discussion that lasted between three and five days.

**Case of the Integral**

The main activity on in this thread was finding the solution to the integral $\int dx/[(x^2)*\ln(x)]$. Ralph presented this problem and received two immediate responses, including hints on how to solve the problem. The following sub-actions took place: requests for clarification, provision of explanations, presentation of new hints, corrections to misunderstandings, confirmation of information, and presentation of the solution. These were then summarized by one participant.

In this case, generation of knowledge took place during a period of three days. Seven people interacted with each other, sharing ideas, asking more questions, and developing understanding of the problem until a solution was presented. Four people posted more than once in an online conversation that had order, empowering its participants through the generation of new knowledge and the clarification of ideas.

**Trigonometric Identity Case**

In this case, Stacy was looking for clarification. She had solved a problem, but the textbook that she was using indicated her solution was wrong. Her post asked the question “Is this right?” and users confirmed her findings by answering in the affirmative and by trying to find out why she was confused. The actions that took place in this thread included answering the specific question post by Stacy (with a simple Right!), posting new examples, connecting with previous knowledge, relating the question to other topics and mathematical rules, giving references to tools available in the web, asking new questions related to trigonometry, and presenting new ways of looking at the problem by moving from the algebraic version to the geometric version of the problem.

Both positive and negative social interventions took place in this thread. The negative intervention occurred when Susan came in and stated, “From reading his posts, nothing mathematical is being thought at all”. This comment was made in response to Tyler, who in a previous posting related the concepts to different mathematical rules. The positive intervention occurred when Tyler thanked Israel for making a reference to his work with a simple “<<blush>> Thank you.” In this way, physical expressions were made possible and his previous intervention is rendered as important. No other social interventions happened in this thread.

The generation of knowledge in this case is more complex than that of the previous case. This discussion lasted four days and had more postings. At the end, Stacy not only got the answer to her first question but also to two other questions she had. In this discussion, algebra was related to geometry, a visual example was given, and an explanation of why sin 60° = square root of (3) /2 was presented using an equilateral triangle, a bisector and the definition of sin.

## Discussion

The review of literature showed that discussion forums were mostly associated with courses in which students analyzed literary work. Little was done in the area of mathematics. Although a need for writing in mathematics was established by the National Council of Mathematics Teachers (1989), most research in mathematics was related to writing journals and writing new problems.

This research investigated how mathematics’ public discussion forums were used to socially construct knowledge while developing communities of learners. Participants in this forum included students, teachers, professors, and math enthusiasts. Together they helped each other clarify their questions and construct new knowledge. In the two examples presented here, discussions about a particular math problem occurred in an ordered fashion. Most interventions were content related, very few were social.

The social construction of knowledge took place when people from different places and in different countries interacted with one another. The zone of proximal development was then increased beyond the physical surroundings and resources available to the mentee. Mentors, beyond the physical range of the mentee, were able to help them clarify and find solutions to the problems they posted to the forum. As the use of the Internet increases, the possibilities for building new communities of learners are made possible.

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## BIOGRAPHICAL SKETCH

Madeline Ortiz-Rodríguez graduated from the University of Puerto Rico, Río Piedras Campus, in 1977 with a bachelor’s degree in secondary mathematics education. In 1987, she graduated with a master’s degree in educational research and evaluation from the same college. In 1990 she completed a second master’s degree in secondary mathematics education from the University of Iowa, Iowa City, IA. She has worked as a mathematics teacher at junior high, high school, and college levels. She has also taught computer science courses and educational technology courses at the college level, directed the Center for Instructional Design (1994-1998), and served as the chairperson of the Science and Technology Department (1999-2001) at the Inter American University of Puerto Rico, Fajardo Campus.

Madeline started her doctoral program in curriculum and instruction with emphasis on Educational Technology in August 2002 at the University of Florida, Gainesville, FL. Throughout her doctoral studies she had the opportunity to participate in an internship program, visiting and presenting her research at the Institute of Education in London, UK, as well as working and presenting her research at the University of Barcelona in Spain. She also had the opportunity to publish in collaboration with her professors in the *Quarterly Review of Distance Education* (2005). Her research interests include integrating technology in teaching and learning practices, the study of social software, mathematics education, and distance education.

At present, Madeline continues to teach at the Inter American University of Puerto Rico, Fajardo Campus.