REAL OPTIONS FRAMEWORK FOR ACQUISITION OF REAL ESTATE PROPERTIES WITH EXCESSIVE LAND

By

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To my husband, Lezhou Zhan, and my family,

Lau Leung, Sau-Pik Fung, Shing-Pen Leung, and Shing-Chiu Leung

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Our study touches a field that few researchers explore: the valuation model for acquisition of a property with excessive land that can be potentially converted into a new development.

Traditional valuation focuses mainly on the building improvement. With the drastic capitalization rate compression, however, it becomes critical to identify and explore any hidden value in an acquisition. One of such challenges is valuing a large partially vacant parcel that can be potentially converted into a new development.

Valuation of these parcels is not straightforward. Traditional discounted cash flow approach (DCF) cannot take into account the uncertainty and development flexibility. Alternative approaches are real options analysis (ROA) and decision tree analysis (DTA). However, the "twin asset" assumption required by the ROA methodology is often violated, especially for assets with private risk and rare events. The use of the same discount rate throughout valuation period in the DTA approach, regardless of changing risk characteristics upon the execution of decision making, allows for arbitrage opportunity.

Our proposed real estate with real options (RERO) model is a framework that combines DCF, ROA and DTA analyses to specifically value real estate acquisition with excessive infill

land. This methodology not only overcomes the shortcoming of current DCF method, but also is superior to the pure ROA or DTA analysis. Focusing on applicability in practice, this framework is developed intuitively with simple mathematics whenever possible. The study also explores a few unconventional real options cases, all of which could have been very complicated if modeled using the partial differential equations common in the academy, including (1) jump diffusion process that does not go back to normal diffusion, (2) risk drivers that do not follow the multiplicative stochastic movement, (3) private risk that has no market equivalent and hence violating the non-arbitrage option pricing assumption. All of these are implemented simply through binomial lattice with Monte Carlo simulation or DTA.

The RERO framework is applied to a real case in Atlanta. Valuation has two parts: (1) the improvement is modeled using a combined approach with Monte Carlo simulation, and (2) the incremental value using a separated decision approach with binomial lattice technique. The valuation result is very close to the actual closing price.

Three conclusions can be drawn from this study: (1) acquisition and development has different characteristics and deserve different kinds of attention; (2) consideration of managerial flexibility can change investment decisions; and (3) many unconventional real option valuation problems can be resolved by binomial lattice and Monte Carlo simulations.

The novelty of this study is the research subject: property acquisition with excessive land. From the methodology standing point, the RERO framework is developed with ease of applicability in mind. It bridges the gap between research and practice for real options applications in the real estate industry.

CHAPTER 1
INTRODUCTION

Background

Our study touches a field that very few academicians have explored: the valuation model for acquisition of a property with excessive land that can potentially be converted into a new development.

The three major schemes in real estate property investment are acquisition, development, and operation. Acquisition is the ownership transaction of land and improvement; development is the process of adding improvement to the land; and operation is the daily management of the property.

A majority of researchers focus on development, perhaps due to its high uncertainty. Acquisition, on the other hand, has been ignored to a certain extent considering its volume and size of transactions. Acquisition has been regarded as relatively low risk, since it is an investment on a touchable real property, which has historical operating track records, and numerous location attributes that last for decades and centuries.

In recent years, however, real estate capitalization rates (defined by dividing the acquisition cost by annual net operating income) have compressed dramatically, meaning real estate is far more expensive to acquire than ever before. It becomes critical to identify and explore any hidden value in an acquisition target in order to be competitive.

The proposed acquisition model has two parts: firstly, valuation of the income producing part of the property, mainly the improvement; secondly, the incremental value, mainly the excessive land that, depending on the circumstance of where the property is located, may have no value or substantial upside value.

The proposed real estate with real options (RERO) model is a framework that combines real options and decision tree analyses. This methodology not only overcomes the shortcoming of the current discounted cash flow method, but also is superior to the existing real options or decision tree analysis. Focusing on applicability in practice, this framework is developed intuitively using simple mathematics whenever possible. The improvement is modeled using a consolidated approach with Monte Carlo simulation, and the incremental value using a separated decision approach with binomial lattice technique.

Statement of Research Problem

The fundamental value of real estate is the income producing capability of the property, which depends on many factors such as the amount of rental income to collect, the operating and financing expenses, the level of risk of the cash flow, the appreciation or depreciation of property value, and the performance of alternative investment instruments in the financial market. Acquisition valuation is the projection of future earning capability of a property related to other alternative investments. Traditional valuation mainly focuses on the building improvement. With the drastic capitalization rate compression, however, it becomes critical to identify and explore any hidden value in an acquisition. One such challenge is valuing a large partially vacant parcel that can be potentially converted into a new development.

The attachment of excessive land to a property is not uncommon. Some developments were initially planned in phases, but the later phases were never implemented due to economic downturn or undesirable outcome of earlier phases. The land planned for later project phases thus remains vacant for a long time. Some early developments were planned on large parcels to insure sufficient space of surface parking. When the region becomes well developed and the economy turns to be more favorable, the vacant land becomes valuable for dense urban infill.

Valuation of these parcels, however, is not as straightforward as applying the traditional Discounted Cash Flow (DCF) approach, which discounts expected future cash flows at a certain discount rate to get the Net Present Value (NPV). In the case of infill land, without new development, all future cash flow will be 0; with certain assumptions of new development, it will generate a value. Intuitively, in a hot real estate market where demand for developable land is high, such as in the South Florida, those parcels are extremely valuable. But in a warm or cold real estate market, the best use of such parcels may remain undeveloped until the market matures. The uncertainty and development flexibility need to be taken into account. Whether or not the land would be developed, when, what type, and what size all matters during the property acquisition.

Alternative approaches are Real Options Analysis (ROA) and Decision Tree Analysis (DTA). The ROA approach has evolved from the financial option pricing theory to value real assets. Put simply, by acquiring a property, the owner has the right, but not the obligation, to develop the excessive land to its full use at a certain point of time in the future. Therefore, the value of a property with excess land should be higher than one without. The ROA methodology has been used to evaluate vacant land and to explain factors that affect development decisions. However, the ROA methodology requires one important assumption, that stochastic changes in the underlying value of the real asset to be developed are spanned by existing tradable assets or a dynamic portfolio of tradable assets, the price of which is perfectly correlated with the real asset (Pindyck, 1991). This so called twin asset is hard to find, especially for assets with private risk and rare events. Secondly, a lot of real options are compound options, which are options on options, not simply on a single asset, and consequently more complicated to solve by the pure option pricing methodology alone.

The DTA approach evolves from management science. It is a method to identify all alternative actions with respect to the possible random events in a hierarchical tree structure. The DTA approach is developed to handle interactions between random events and management decisions. However, a major limitation of the DTA method is its use of the same discount rate throughout the valuation period, regardless of changing risk characteristics upon the execution of decision making, and thus allows for arbitrage opportunity (Copeland and Antikarov, 2005).

Recent studies have turned to the combination of option pricing methodology, decision analysis, and game theory to solve real options problems. An ideal new approach should be able to address the unique characteristics of acquisition valuation with infill land, to handle the management flexibility, to take into account rare events such as new amenities driving up real estate value. It also needs to be intuitively simple for practical implementation.

Goal and Objectives

To overcome the above mentioned disadvantages of the current DCF, ROA, and DTA methodologies, this study has developed a framework, namely the Real Estate with Real Option (RERO) framework, as a combination of all three methods to specifically value real estate acquisition with excessive infill land. The objectives of this study are to:

- Develop a theoretical integrated framework to address real estate acquisition problems;
- Study factors affecting real estate acquisition and development, as well as their characteristics and statistical distributions;
- Test and validate the model by applying it to real cases.

Research Scope

The research subject is real estate acquisition, which includes the value of the structural improvement, and the incremental value represented by excess developable land. The definition of excess land is that in addition to the portion necessarily attached to the existing structural improvement; the excess portion that is large enough for new development and at the same time meets local regulation requirements. Development factors are outside of our scope. Potential users of the framework are real estate investors who need a tool to estimate the building value and the land value during property acquisition. The proposed valuation model addresses mainly the economic risk and uncertainty for acquisition and development.

Significance and Contributions

The novelty of our study is the research subject: property acquisition with excessive land. To our knowledge, this is a field that few researchers have addressed. From the methodology standing point, the RERO framework is developed with ease of applicability in mind. It bridges the gap between research and practice for real options applications in the real estate industry.

Organization of Dissertation

In Chapter 2 we review the characteristics of real estate acquisition, existing valuation approaches and their limitations, as well as what a new approach needs to achieve. In Chapter 3 we review the theory and technical details of the different approaches currently available, in preparation for developing the proposed framework. We introduce the RERO framework in Chapter 4, including valuation procedures, the combined and separated approaches, and some new techniques developed to specifically apply to the case studies followed. Chapter 5 and 6 are case studies of the combined approach and separated approach respectively. Collectively they illustrate how the RERO framework can be applied to a broad spectrum of scenarios in practice. In Chapter 7 we conclude the study and suggest future research directions.

CHAPTER 2
REVIEW OF REAL ESTATE VALUATION

This chapter discusses the current practice in acquisition valuation, alternative approaches and their limitations, followed by a review of real options in real estate. It also analyzes how the proposed RERO framework needs to resolve the practical problems unique to real estate acquisitions.

Current Practice

Distinguishing Acquisition and Development

Analogous to the financial market, the three major schemes in the real estate investment market are different and inter-related: acquisition, development, and operation. Acquisition is similar to a lumpy investment in a well established company with, in many cases, 100% ownership interest. Development is similar to the seeding of a start-up company and bringing it to Initial Public Offering. Operation is the income producing process in the daily management of the property.

This explains why research on development problems may not directly apply to acquisition valuation problems. A real estate investment firm may have a different agenda for the infill land than a real estate developer. The business of real estate development is to acquire and accumulate a considerable land bank, wait for appropriate timing and market demand to build new properties, and realize profit by selling the new properties to institutional investors. The business of commercial real estate investment, on the other hand, is to acquire existing properties, manage and improve the properties to receive the operating income from leasing. As an investment vehicle, commercial real estates tend to be traded more frequently than vacant land. As buildings get older and functionally obsolete, they usually change hands from passive institutional investors to active value-added investors for cosmetic and functional upgrade and tenant-mix adjustment. The developers, however, acquire land from different sources and wait more patiently in a real estate cycle before putting up new products to capture the maximal gain. Short holding periods and different business interest makes the infill land less valuable to an investor than the vacant land to a developer.

The major factors to consider during acquisition are quite different from those in the development and operation processes (Figure 2-1). During acquisition, the major factors are location, market condition, market rent, pricing of the building and the land. Development factors, such as impact fee and school zoning, are outside the scope. If the investor wins the bid, he goes through the due diligence and financing process before actually plans for development of the vacant land. Although our model consists of the building value and the land value assuming possible development, it is by no means to substitute for a detailed financial planning before the development breaks ground.

Typical Acquisition Valuation Process

A real estate investment company buys and manages properties to capture the cash flow from operation. Many of these companies specialize in one or a few product types, such as office, retail, industrial, or residential properties. To evaluate a property with infill land, the management needs to answer the following questions:

- What is the building worth?
- What is the market demand for space?
- What is the likelihood that the company, after acquiring the property, will put up new buildings?
- If the company does not intend to build new properties, what is the likelihood of the next buyer to put up new buildings?
- What type and size of development can add value to the land, and thus add value to the acquisition?

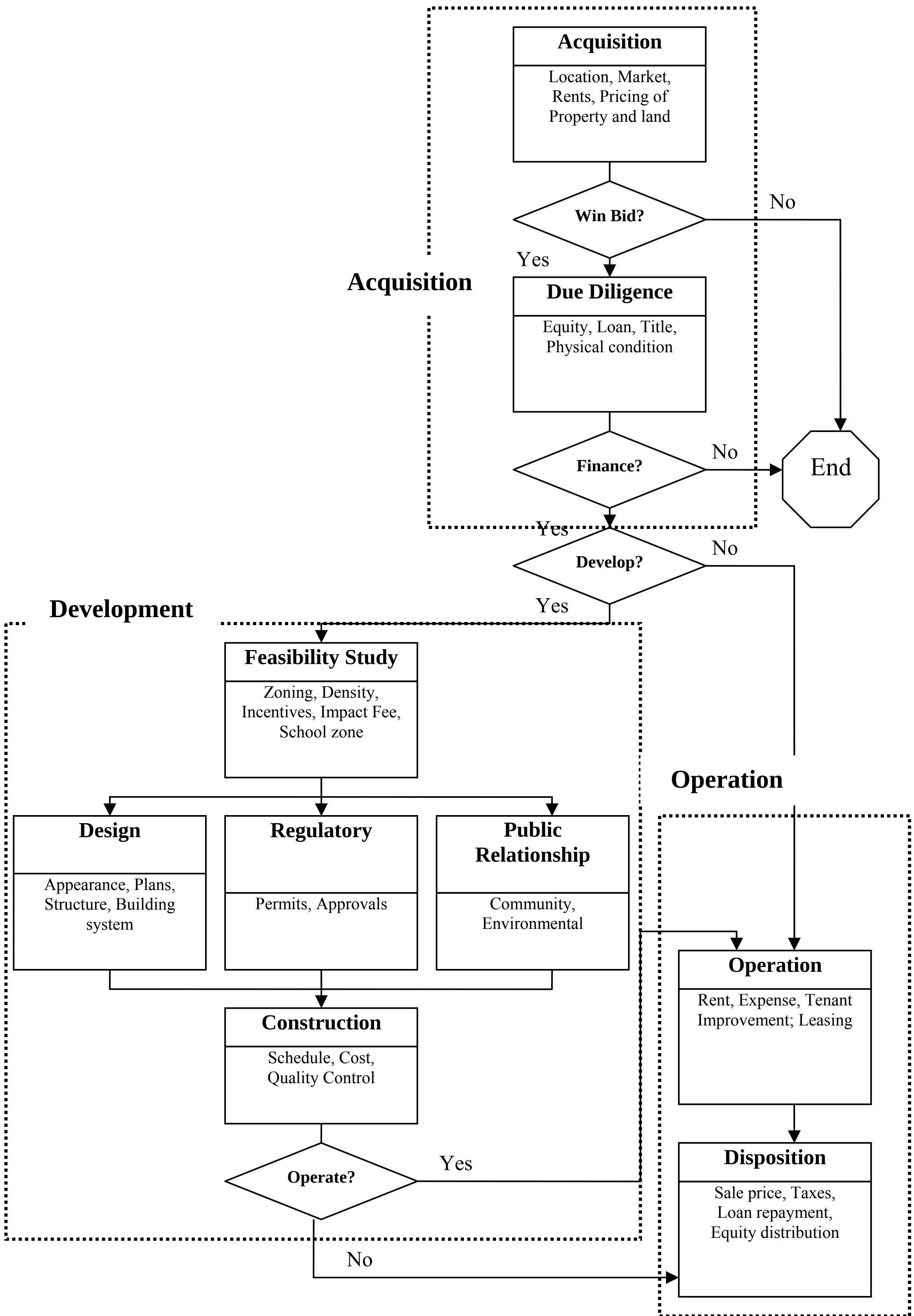

Figure 2-1. Real estate phases and major factors to consider.

The typical decision process followed in current practice to acquire a property (an office building for example) with infill land is shown in Figure 2-2. First, the building value and the land value are segregated. Building value is derived from the standard DCF projection. Depending on the investor's perspective towards the market, the land could have no value or some value. In a weak demand region, the land probably does not generate any additional income besides parking, thus it has little or no value to the investor. In a strong demand region the investor conducts further investigation on the suitable product type to develop. If the best product type to develop is one that the investor is familiar with, say an office tower, the investor will further evaluate the project and land worth through a development model. If the best product type is not one the investor is familiar with, say a residential condominium or an industrial building, the investor probably hesitates to get involved in the development alone.

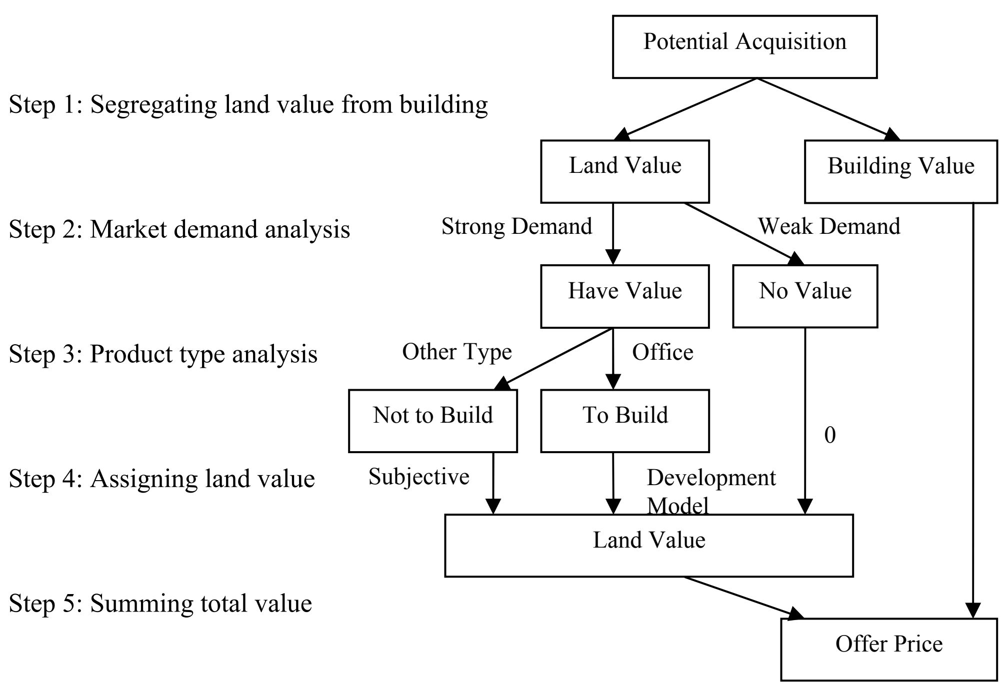

Figure 2-2. Current acquisition valuation process.

The investor might either find a development partner or consider selling off the land to such an interested party. In either case, for the acquisition purpose the investor will simply assign a subjective value to the land. The offer price consisting of the building and the land value is derived and submitted to the broker.

Current Real Option Approach and Limitations

In the ROA approach, by acquiring the property the investor not only receives all cash flows generated from leasing of the existing building, but also has the right, but not the obligation, to develop the vacant land to its full use at a certain point of time in the future. Therefore, the value of a property with infill land should be higher than one without.

However, the current ROA models are not without limitations. Firstly, valuation methods for vacant land may not be suitable for infill land due to their different characteristics in the following aspects: (1) the price of acquiring the land could be substantially lower; (2) the building type to be developed may be restricted by zoning regulation on current property; (3) the synergy effect could be substantial between the proposed building and the existing building; (4) The surface parking is an inseparable part of the existing property.

Secondly, a real estate investment firm has a different agenda for the infill land than a real estate developer. Short holding periods and different business interests make the infill land less valuable to an investor than to a developer.

Thirdly, the current theoretical models are on a higher level to address real estate as a whole, while investors need practical models to address individual cases. The current theoretical models are on an aggregate level to explain real estate value in general. They have rigid restrictions, and can only be applied to the simplest cases (Miller and Park, 2002). They also lack flexibility to change variables to model realistic assumptions for practical use. Real assets often possess unique location, physical and contractual characteristics, many of which are subjective and unquantifiable. Using the real option method alone may be insufficient.

Last, the existing "omnipotent" real options models are mathematically correct but too complicated to be used. Trigeorgis (2005) and others have advocated approximate methods to simplify the calculation for practical applications.

In summary, although the ROA approaches can overcome some of the drawbacks of DCF and provide better valuation for acquisition, the method itself is not fully developed to address the specific needs of acquisition valuation in practice.

Decision Tree Analysis and Limitations

Another available approach is the Decision Tree Analysis approach (DTA). DTA is a method to identify all alternative actions with respect to the possible random events in a hierarchical tree structure. It is developed to handle the interaction between random events and management decisions.

However, a major limitation of the DTA method is its use of the same discount rate throughout the valuation period, regardless of changing risk characteristics upon the execution of decision making, and thus allows for arbitrage opportunity (Copeland and Antikarov, 2005). This means using DTA alone is not sufficient for the acquisition with infill land problem.

Real Options in Real Estate

Applications of ROA in the real estate industry can be classified into the following categories: Vacant land for development, property redevelopment, and leasing (Ott, 2002). This section summarizes some theoretical models as well as empirical studies.

Theoretical Models

Titman (1985) developed a simple binominal tree model to explain why a piece of land could be more valuable remaining vacant today and when is optimal to develop. This seminal work is frequently cited in later papers, which all use Partial Differential Equations (PDE) and fall into two major categories by methodology: the optimal development timing problem, and the game theoretical problem. The optimal timing problem is represented by Clarke and Reed (1988, optimal timing and density for residential development), Capozza and Helsley (1990, conversion from agricultural to urban land use), Williams (1991, optimal timing and density to develop, optimal timing to abandon), and Geltner et al. (1996, two land use choice). The game theoretical problem is represented by Williams (1993, competition on simultaneous development), Grenadier (1996, competition on simultaneous or sequential development), and Childs et al. (2001, inefficient market with noisy effect on value). Figure 2-3 shows the genealogical relationship among these models. Table 2-1 itemizes the research subject, model variant, contributions and limitations of each study.

Besides land valuation, there are two types of real estate applications of the ROA that are closely related to our research: property redevelopment and operational research. Williams (1997), Childs at al. (1996), Cederborg and Ekeroth (2004) have researched on the redevelopment or renovation of real assets. They view existing buildings as assets that can be repetitively invested and improved, sometimes by changing functional attributes, e.g., switching from offices to apartments. Grenadier (1995, 2003), Adams, Booth and MacGregor (2001), Bellalah (2002), Grenadier and Wang (2005), Capozza and Sick (1991), among others have focused on options embedded in the commercial lease agreements, such as forward leases, escalation clauses, leases with options to renew or cancel, adjustable rate leases, purchase options, sale-leasebacks, ground leases, etc.

Acquisitions have not been thoroughly researched using the real options approach, though common in practice. As discussed earlier, acquisitions with excessive land differ from ground up development. They also differ from redevelopment, since they are not simple renovations of the existing buildings. They might include valuation of the leases as a source of cash flow for the potential development, but would require a much simpler valuation process on the leases. In summary, although acquisition valuation is close to the three subjects mentioned above, the approach is significantly different. A new approach needs to be able to address both the building value and the land value, if any, for potential development.

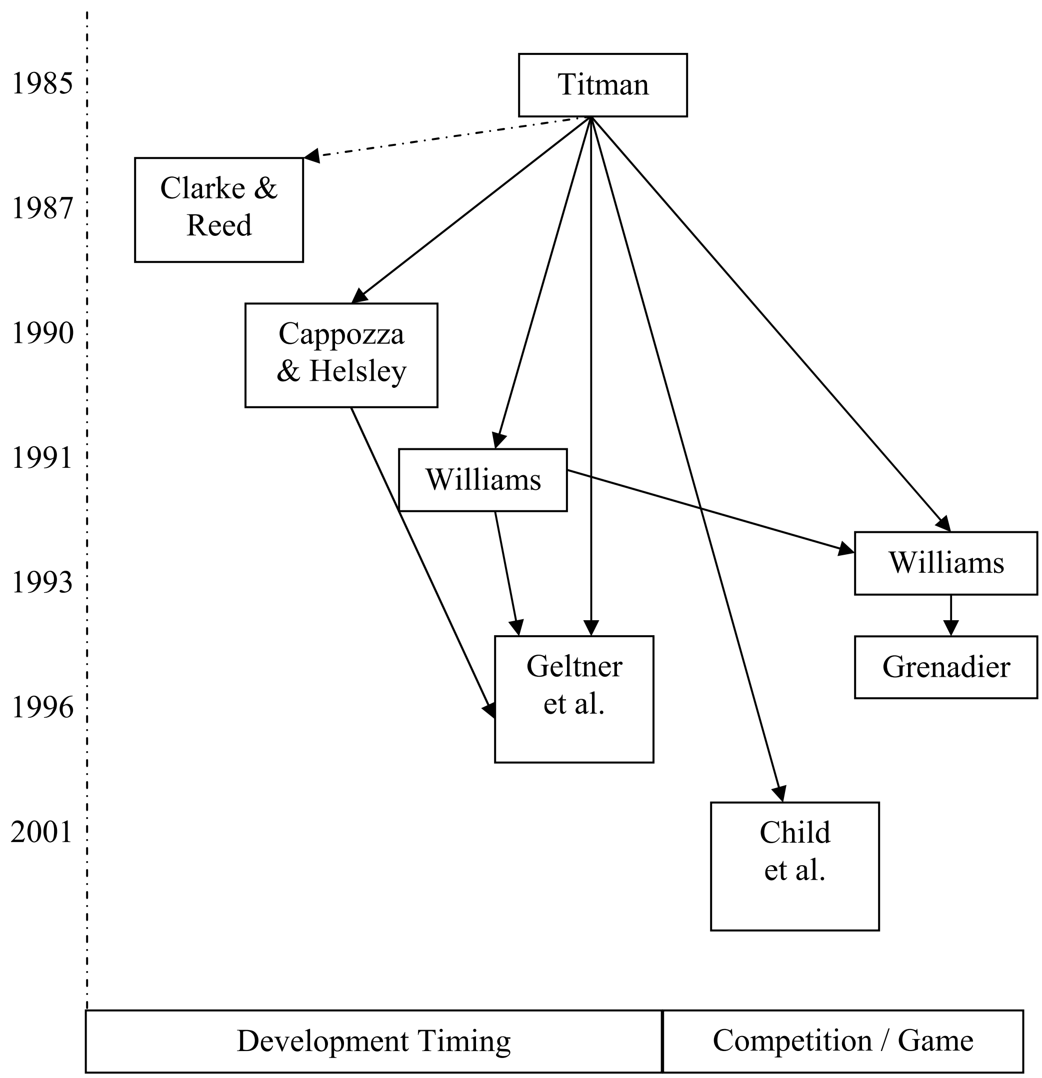

Figure 2-3. Real Options approaches for land valuation.

Table 2-1. Comparison of research subjects, model variants, contributions and limitations.

Author / title	Subject description	Model type & variant	Contribution / limitation
"Urban Land Prices under Uncertainty" (Titman, 1985)	Explain why land is more valuable remaining vacant for future development: increased uncertainty leads to a decrease in current development activity.	One time period binomial model assuming rents have two state values.	Seminal work of ROA in real estate. Simple. Two policy implications: (1) Government incentives to stimulate construction activities may actually lead to a decrease if the extent and duration of the activity is uncertain. (2) Initiation of height restrictions may lead to an increase in development activity due to reduced uncertainty regarding the optimal height of the area. One time period model. Assume only two states, and that construction costs are certain.
"A Stochastic Analysis of Land Development Timing and Property Valuation" (Clarke and Reed, 1987)	Examine the qualitative effects of the different types of uncertainty on the timing and structural density of land development on residential projects.	PDE to solve for optimal development timing and density assuming rents and development cost follows stochastic processes.	Limited to residential development. Two limited assumptions: (1) new construction is small so that rents and development costs are uninfluenced by the newly added construction. However, in reality development is lumpy and will affect market rents and vacancy rate. (2) Efficient market in which all agents have equal information about the future probability distributions of rentals and costs. However, in reality real estate leasing and sales information is not as transparent as that in the stocks market, but more predictable, at least in a short run.

Table 2-1. Continued.

Author / title	Subject description	Model type & variant	Contribution / limitation
"The Stochastic City" (Capozza and Helsley, 1990)	Examine the land value of conversion from agricultural to urban use based on spatial characteristic of real estate such as distance or commuting time to the CBD.	PDE model built on the traditional mono-centric urban theory to study spatial implication of land conversion value, assuming household income, rents and land prices follow stochastic processes.	Uncertainty (1) delays the conversion of land from agricultural to urban use, (2) imparts an option value to agricultural land, (3) causes land at the boundary to sell for more than its opportunity cost in other uses, and (4) reduces equilibrium city size. Does not explain very well land value in the emerging suburb economic centers.
"Real Estate Development as an Option" (Williams, 1991)	Optimal time to develop, optimal development density, and optimal time to abandon a project.	PDE model to solve for optimal timing of abandoning a project, in addition to optimal development timing and density, assuming carrying cost, rents and development cost follows GBM, also assuming carrying cost is significantly high so that during some circumstance it is better to abandon the project than bearing the cost.	Looks at the downside of a project: optimal time to abandon. This is a put option. Maximum feasible density is determined by zoning restrictions. Assumes perfectly competitive market and perpetual option.
"Insights on the Effect of Aland Use Choice" (Geltner et al. 1996)	Examine whether the multiple-use zoning add value to land by analyzing the land use choice between two different use types.	PDE to solve for optimal choice between two land use types, assuming development cost, value of first land use, value of second land use follow stochastic processes.	Land use type choice is a unique perspective in real estate. Assume construction unit cost is the same regardless of building type to be developed.

Table 2-1. Continued.

Author / title	Subject description	Model type and variant	Contribution/ limitation
"Equilibrium and Options on Real Assets" (Williams, 1993)	Examine industry equilibrium of optimal exercise policy under competition: the impact of competition erodes the value of the option to wait and leads to investment at very near zero net present value thresholds.	PDE to solve for perfect Nash equilibrium with finite elasticity of demand and finite development capacities in a less than perfectly competitive environment.	Among the first to consider the effect of competition. Exercising options to develop affects the aggregate supply of developed assets and market price, which preclude simultaneous exercise of the option among all developers.
"The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets" (Grenadier, 1996)	Explain why building booms in the face of declining demand and property values: fearing preemption by a competitor, developers proceed into a panic equilibrium in which all development occurs during a market downturn.	Three-stage model to explain real estate boom-and-bust cycle: valuation of land, construction lag, and "sticky vacancy" in operation	Extend the Williams model from symmetric and simultaneous equilibrium to either simultaneous or sequential development, and allows for preemptive equilibria. Powerful to explain boom-and-bust markets such as real estate. Assume individual firms are identical and have all information.
"Noise, Real Estate Markets, and Options on Real Assets: Theory" (Childs et al. 2001)	Optimal valuation of noisy real asset in an incomplete information game	PDE, assume optimal value include three terms: forward value estimate, historical value estimate, and the term that corrects for convexity effects due to incomplete information	Extend to include the price lagging effect in real estate, where estimate value is different from market value, i.e., in a less than perfect market.

Empirical Testing

A majority of the ROA empirical works in real estate has been in aggregate studies. Quigg (1993), Holland et al. (2000), Sivitanidou and Sivitanides (2000), Bulan et al. (2004) all use a large sample of real estate data to test the premium of land price over intrinsic value, whether irreversibility is an important factor for real estate investment, whether uncertainty delays construction, and whether competitions among developers decrease the option value of waiting.

As Bulan et al. (2004) point out, however, since real options models apply to individual investment projects and predict that trigger prices are non-linear, aggregate investment studies may obscure these relationships. Moreover, these empirical tests are limited to qualitative results, such as whether each variable in the ROA model has positive or negative effect on the overall option value. Few of the ROA empirical works has focused on individual case studies and its implication in practice.

The RERO Approaches

The RERO framework attempts to move beyond the realm of academic interest to be used quantitatively in practical problems of acquisition valuation, development decision making, and land policy analysis. The approach should be able to address the unique characteristics of acquisition valuation with infill land, to handle the management flexibility, to take into account rare events such as new amenities driving up real estate value. This calls for the combination of DCF, ROA and DTA methodologies. It also needs to be intuitively simple for practical implementation.

To achieve this goal, the problem is divided into two sub-problems: (1) valuation of the building structure and (2) valuation of the infill land. Valuation of the building structure represents a normal case of acquisition. On the other hand, valuation of the infill land represents the extra value stemmed from creative management, i.e., the ability to uncover the hidden value in real estate and realize it through active development.

Real estate valuation is an art and science. The RERO framework is not built on rigid reasoning and restricted assumptions to be precise, rather it is developed as a tool to solve a broad spectrum of practical real options problems. Specifically, it explores a few unconventional real option cases, including (1) jump diffusion process that does not go back to normal diffusion, (2) risk drivers that do not follow the multiplicative stochastic movement, (3) private risk that has no market equivalent and hence violating the non-arbitrage option pricing assumption. The mathematical models for these kinds of unconventional problems could be very complicated, if written in PDE equations. To facilitate practical implementation, the RERO framework applies the binomial lattice with Monte Carlo simulations and decision analysis method. The RERO framework is a simple yet powerful tool, intuitive to the practitioners, yet mathematically correct and precise.

Summary

This chapter compares the difference between real estate acquisition and development, reviews current practice of real estate acquisition valuation, discusses the three alternative valuation approaches, DCF, ROA, DTA and their limitations. Built on the strengths of these three approaches, the RERO framework needs to address practical problems of acquisition valuation, development decision making, and land policy analysis. The next few chapters explore modeling details of how this concept should be implemented.

CHAPTER 3
LITERATURE REVIEW

In Chapter 2 several different valuation methodologies were discussed conceptually: the Discounted Cash Flow approaches (DCF), the Real Option Analysis approaches (ROA), the Decision Tree Analysis approaches (DTA), and the proposed Real Estate with Real Option approaches (RERO). In this chapter the technical modeling details of the first three approaches, as well as the capital budgeting theory in finance will be discussed. The RERO approaches that built on the existing three will be discussed in Chapter 4.

Traditional Discounted Cash Flow Approaches

The Discounted Cash Flow (DCF) approaches include payback period, Internal Rate of Return (IRR), Net Present Value (NPV), and other forms such as Adjust Present Value. In this study DCF refers to the NPV method alone. The principle of the NPV method is to discount all projected free cash flow back to year 0, to get the net present value of the project (Equation 3-1). The NPV must be greater than 0, or the IRR must be greater than the company's hurdle rate, in order to justify the investment (Mun, 2002). If NPV is greater than 0, the project is regarded as optimal to be executed immediately.

$$NPV = \sum_{i=0}^{n} \frac{F_i}{(1+k)^i} \tag{3-1}$$

where
NPV is the net present value of the project at Year 0,
F_i is the projected free cash flow (including income, cost and terminal value) in year i,
k is the project discount rate.

The DCF method is suitable to evaluate projects that are well structured, with predictable future cash flows. For projects involve large uncertainty of timing, cost and cash flows, such as a real estate development, using the DCF approaches are difficult in the following three aspects (Miller and Park 2002; Feinstein and Lander 2002): firstly, selecting a fixed and appropriate discount rate; secondly, taking into account new information and changing the plan accordingly; thirdly, determining the optimal timing to carry out the project.

Capital Budgeting Theory

In the DCF approach and in all other approaches, one of the most influential factors is the discount rate to be used. To better understand discount rate, a brief discussion of the capital budgeting will follow.

Market Risk and Private Risk

Stocks are risky. For any individual stock, however, a large part of its risk can be eliminated by holding it in a large well-diversified portfolio. A portfolio consisting of all stocks is called a market portfolio. In reality, it can be approximated by a large amount of well-diversified stocks. The part of the risk of a stock that can be eliminated is called private risk, or diversifiable risk; while the part that cannot be eliminated is called market risk, or systematic risk (Brigham et al. 1999, p178). The Capital Asset Pricing Model (CAPM) indicates that the relevant riskiness of any individual stock is its contribution to the riskiness of a well-diversified portfolio, or the market risk portion only, which is measured by its β coefficient.

Capital Asset Pricing Model

If the market portfolio *m* is efficient, the required return $\bar{r}_s$ of any stock *i* is the risk-free interest rate *r* plus a risk premium, as shown in Equation 3-2.

$$\bar{r}_s = r + \beta_i(\bar{r}_m - r) \qquad (3\text{-}2)$$

Where
r is the risk-free return,
$\bar{r}_m$ is the expected market return,
$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$, where σ_{im} is the covariance between the stock and the market, and σ^2_m is the variance of the market portfolio.

β_i is an important variable to measure the risk characteristics of the stock i. If β_i is greater than 1, the stock is more volatile than the average stock market; and if β_i is less than 1, the stock is less volatile than the average stock market. The more volatile a stock is, the more risky it is, and consequently the higher the required return needs to be in order to justify the risk an investor takes.

Discount Rate

A firm's hurdle rate is usually its Weighted Average Cost of Capital (WACC). A large real estate investment firm is usually formed as a Real Estate Investment Trust (REIT), which does not pay income taxes, so long as 95% of its income from operation is distributed to the investors on an annual basis. The WACC k of a REIT is calculated by Equation 3-3.

$$k = r_s \frac{S}{V} + r_d \frac{D}{V} \tag{3-3}$$

where
r_s and r_d are the cost of equity and debt respectively,
S, D and V are the market values of equity, debt, and total asset respectively; $S + D = V$.

Equation 3-3 can also be used to value an investment project, as if every project was a separate mini company. However, it is difficult to determine the cost of equity and debt for a project, since the equity of a start-up project, for example, may not be publicly traded, and the risk characteristics of a project are quite different than that of the company as a whole.

The capital budgeting theory indicates that finding the right discount rate is extremely difficult, if not impossible. Since every company has different risk characteristics, the required discount rate is different from company to company. Also every project within the same company has different risk characteristics, and the correct discount rate required to value a project may not be the same as the company's WACC. This makes both the DCF and the DTA approached difficult to value infill land with development potential, although for an existing building with operating history the DCF and DTA approaches may work fine.

Option pricing theory, on the other hand, does not rely on the risk characteristics of a particular firm or project. Neither does it rely on the risk preference of an individual investor. It is discounted at the risk-free interest rate *r*. The reason is that "private risk is alleviated through portfolio diversification and market risk can be diminished through the option's replicating portfolio" (Miller 2002). For development project that involves a lot of uncertainty, this is a huge benefit over the traditional DCF method.

Option Pricing Theory

Definition and Type of Options

An option gives the holder the right but not the obligation to do something (Hull, 2006). In the financial market, there are two basic types of options: call options and put options. A call option gives the holder the right to buy the underlying asset by a certain date for a certain price. A put option gives the holder the right to sell the underlying asset by a certain date for a certain price (Figure 3-1). Based on exercise dates, options can be classified into two major types: American options can be exercised at any time up to the expiration date. European options can be exercised only on the expiration date. Most options are of the American type.

The value of a financial option is determined by the current price of the underlining asset S_0, the strike price at maturity date K, the risk-free interest rate r, maturity date T, return volatility of the underlining asset σ, and sometimes the dividends expected during the life of the option (Hull, 2006). Returns on options are asymmetric, i.e., options will only be exercised to the benefit of the holders. For example, if a holder of a call option can buy the stock 3 months later for $100 per share, and if the spot price at maturity becomes $120 per share, he will exercise this option, then sell the stock immediately, and earn $20 per share. However, if the spot price becomes \$83 per share at maturity, he can let the option expire without exercised, thus avoid losing \$17 per share. He only losses the premium initially paid for the option (Figure 3-2). His payoff is the difference between the spot price at maturity S_t and the exercise price K, or 0, whichever is greater (Equation 3-4).

$$Max(S_t - K, 0) \tag{3-4}$$

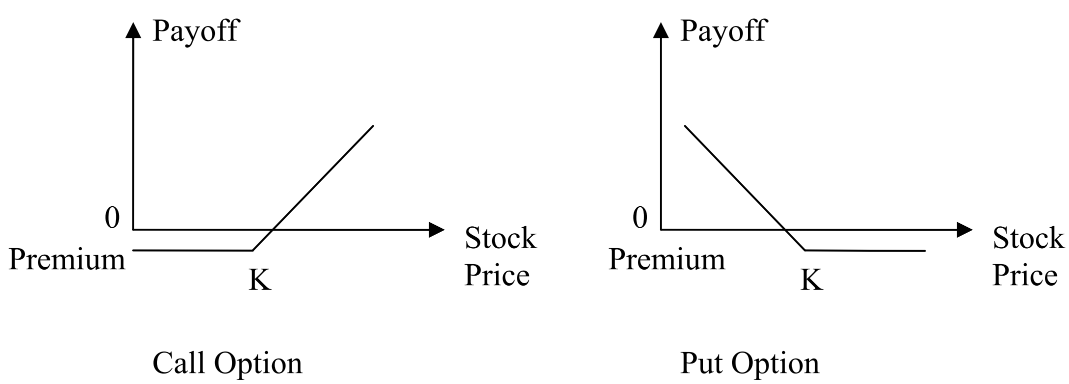

Figure 3-1. Payoff of call option and put option.

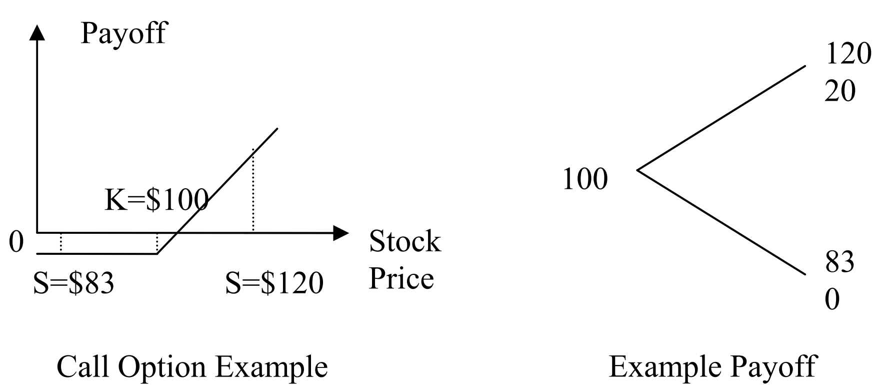

Figure 3-2. Call option payoff example.

Option pricing theory is to determine what premium, or option price, a holder should pay for such flexibility. The types of option pricing methodology include continuous- and discrete-time models (Miller and Park, 2002; Lander and Pinches, 1998). Continuous-time models include closed-form equations and stochastic partial differential equations. Discrete-time models are mostly lattice models and Monte Carlo simulation.

Black-Sholes Model and Stochastic Partial Differential Equations

The most famous closed-form equation is the Black-Scholes model, although it can only be used to price European options. The Black-Scholes (1973) pricing formula is developed under the following ideal assumptions: stock price change follows the Wiener process, distribution of return is lognormal, efficient market, constant short-term interest rate, no dividend payment, no transaction costs, and short selling is possible. A Wiener process, also called a Geometric Brownian Motion (GBM), is a random process with a mean change of 0 and a variance rate of 1. The values of *dz* for any two different short intervals of time *dt*, are independent (Equation 3-5).

$$dz = \varepsilon\sqrt{dt} \qquad (3\text{-}5)$$

Where ε has a standardized normal distribution $\phi(0,1)$, and $\phi(\mu,\sigma)$ denotes a probability distribution that is normally distributed with mean μ and standard deviation σ. A generalized Wiener process for a variable S can be defined by Equation 3-6.

$$dS = \mu S dt + \sigma S dz \qquad (3\text{-}6)$$

where
S is the underlying asset whose value change follows the Wiener process;
dS is the change of value S during an infinitesimal time interval dt.

Ito's Lemma (Hull, 2006, p273) is a theorem of stochastic calculus that shows second order differential terms of a Wiener Process can be considered to be deterministic when integrated over a non-zero time period. Since the stock price S follows the Wiener process, an option f (be it a call option or a put option) contingent on S follows the Ito's Lemma (Equation 3-7).

$$df = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial f}{\partial S}\sigma S dz \qquad (3\text{-}7)$$

The principle of option pricing methodology is to construct a riskless portfolio to prevent arbitrage. This portfolio *Π* is short one option and long $\partial f / \partial S$ shares of the underlying stock. When the stock price *S* changes, the $\partial f / \partial S$ shares must change accordingly. Later from Equation 3-10 we will see this portfolio is riskless because it does not involves *dz* over the time interval *dt*. The portfolio *Π* is written as Equation 3-8.

$$\Pi = -f + \frac{\partial f}{\partial S} S \tag{3-8}$$

During the time interval *dt*, the change in value of the portfolio is represented in Equation 3-9.

$$d\Pi = -df + \frac{\partial f}{\partial S} dS \tag{3-9}$$

Substitute *dS* from Equation 3-6 and *df* from Equation 3-7 into Equation 3-9,

$$d\Pi = \left(-\frac{\partial f}{\partial t} - \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt \tag{3-10}$$

To prevent arbitrage, the portfolio earns risk-free interest *r* during the time interval *dt*.

$$d\Pi = r\Pi dt \tag{3-11}$$

From Equation 3-8, Equation 3-10 and Equation 3-11, we have

$$\left(-\frac{\partial f}{\partial t} - \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt = r\left(-f + \frac{\partial f}{\partial S} S\right)dt$$

Which simplifies to

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S} S + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2 = rf \tag{3-12}$$

Equation (3-12) is the Black-Scholes partial differential equation. Subjected to the following boundary conditions:

$f = Max(S - K, 0)$, when $t = T$ in the case of a call option, and

$f = Max(K - S, 0)$, when $t = T$ in the case of a put option.

Integrating Equation 3-12, the Black-Scholes formula can be written as Equations 3-13 and 3-14 (Black and Scholes, 1973; Hull, 2006).

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{3-13}$$

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \tag{3-14}$$

where

$$d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \ ;$$

$$d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \ ;$$

c is the value of a European call option;
p is the value of a European put option;
S_0 is the current price of the underlying asset;
K is the strike price of the option at maturity;
r is the risk-free interest rate;
T is the time to maturity;
$N(\bullet)$ is the cumulative standard normal distribution function.

The Black-Scholes model can be divided into two parts: The first part, $S_0N(d_1)$, derives the expected benefit from acquiring a stock right now. This is found by multiplying stock price S_0 by the change in the call premium with respect to a change in the underlying stock price $N(d_1)$. The second part of the model, $Ke^{-rT}N(d_2)$, gives the present value of paying the exercise price on the expiration day. The fair market value of the call option is then calculated by taking the difference between these two parts.

The boundary condition of a call option is best depicted in Figure 3-3. The solid black line defines the call option value. The green line with square markers defines the maximum value of the option. For non-arbitrage, the option should never be worth more than the stock price S, otherwise an arbitrageur can easily make a risk-less profit by buying the stock and selling the call option. The blue line with triangle markers defines the minimum value of the option. The call option should be worth more than $Max(S_0 - Ke^{-rT}, 0)$, otherwise an arbitrageur can buy an option, short sell a share of stock, invest the surplus at risk-free interest rate and earn a profit. The possible option values fall in the region defined by the green line and the blue line and vary depending on the underlying stock volatility, option time to maturity, and risk-free interest rate.

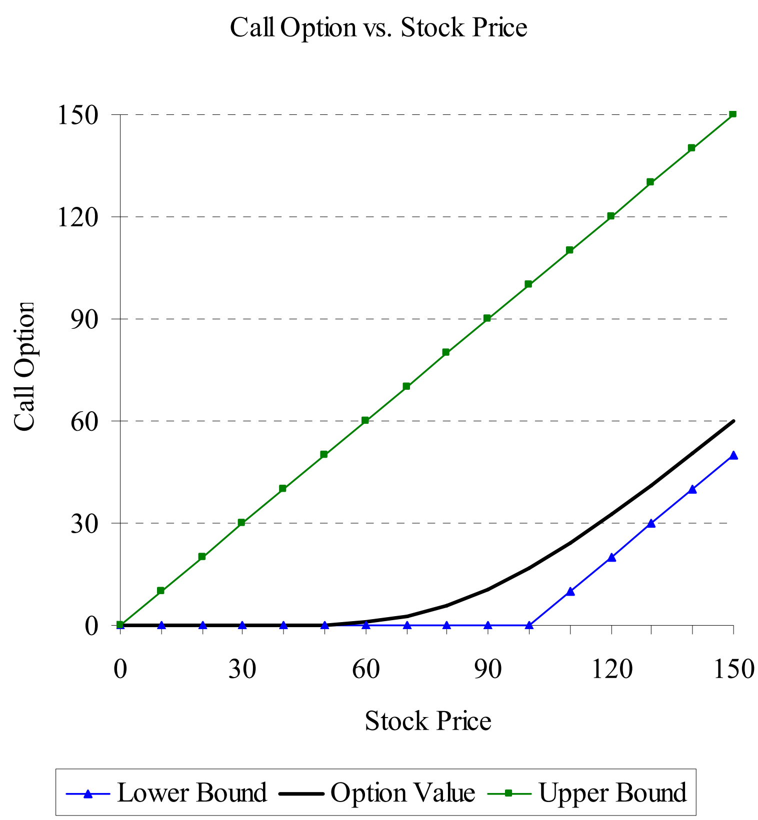

Figure 3-3. Call premium vs. security price.

Though the Black-Scholes pricing model has a lot of restrictions and can only value European options, there are a lot of stochastic partial differential equations with boundary conditions that relax some restrictions to a certain extent and can be used to value more specific questions. The benefits of these analytic continuous-time models are that they are flexible to model different circumstances, and mathematically accurate (Miller and Park, 2002). The drawback is that the modeling requires sophisticated mathematical knowledge, sometimes the solution does not exist, and even if it does, the process itself could become as complicated as a black-box for the practitioners to comprehend (Lander and Pinches, 1998).

In the case when analytical solutions to the stochastic differential equations do not exist, they must be solved numerically by using finite-difference methods, or Monte Carlo simulations (Miller and Park, 2002).

Lattices

Lattices are a type of discrete time model, which includes binomial tree, trinomial tree, quadranomial tree, and other multinomial models. Lattices are the approximation of the continuous models. The results of these two methods are very close when the time interval is infinitely small.

The most commonly used binomial lattice was developed by Cox et al. (1979), in which values of the underlying asset are assumed to follow a multiplicative binomial distribution. The model assumes the up and down parameters u and d, the volatility of the underlying asset σ, and risk-neutral probabilities p and $1 - p$ are constant (Figure 3-4).

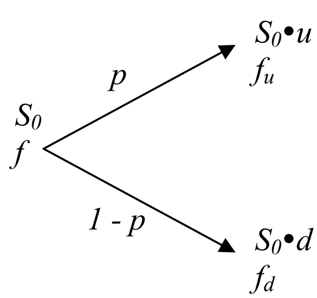

Figure 3-4. Stock and option price in a one-step binomial tree.

An option f (be it a call option or a put one) is valued by constructing a risk-less portfolio Π of a long position in δ shares of stock and a short position in 1 option (Equation 3-15).

$$\Pi = S_0\delta - f \tag{3-15}$$

In an up movement of the stock price, the value of the portfolio is

$$\Pi_u = S_0 u\delta - f_u$$

In a down movement of the stock price, the value of the portfolio is

$$\Pi_d = S_0 d\delta - f_d$$

The two are equal when

$$S_0 u\delta - f_u = S_0 d\delta - f_d$$

or when

$$\delta = \frac{f_u - f_d}{S_0(u-d)} \tag{3-16}$$

The portfolio is risk-less and must earn the risk-free interest rate *r*. The present value of the portfolio is represented by Equation 3-17.

$$\Pi = (S_0 u\delta - f_u)e^{-rT} \tag{3-17}$$

From Equation 3-15 and Equation 3-17, we have

$$S_0\delta - f = (S_0 u\delta - f_u)e^{-rT} \tag{3-18}$$

Substitute δ from Equation 3-16 into Equation 3-18,

$$f = e^{-rT}[pf_u + (1-p)f_d] \tag{3-19}$$

where

$$p = \frac{e^{rT} - d}{u - d}$$

$$u = e^{\sigma\sqrt{T}}$$

$$d = \frac{1}{u}$$

Equation 3-19 is a one-step binomial model, which can be generalized to two-step and multi-step models. Figure 3-5 shows a two-step binominal lattice. During each time step, the stock value either moves up to u or down to d of its previous value. Option value is derived by working backward from f_{uu} and f_{ud} to calculate f_u, from f_{ud} and f_{dd} to calculate f_d, then from f_u and f_d to calculate f (Equations 3-20, 3-21 and 3-22).

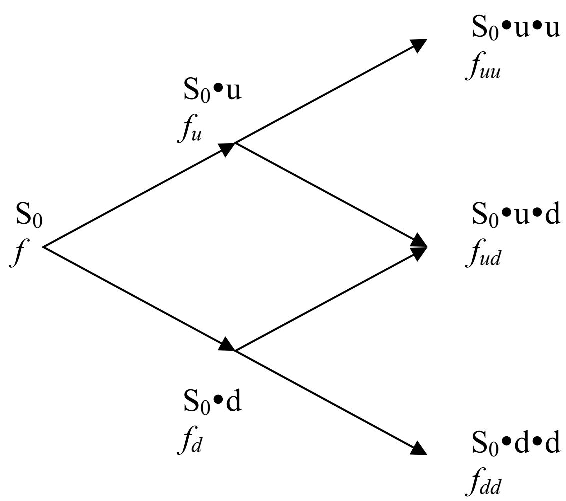

Figure 3-5. Stock and option prices in general two-step tree.

$$f_u = e^{-rt}[pf_{uu} + (1-p)f_{ud}] \tag{3-20}$$

$$f_d = e^{-rt}[pf_{ud} + (1-p)f_{dd}] \tag{3-21}$$

$$f = e^{-rt}[pf_u + (1-p)f_d] \tag{3-22}$$

Substituting from Equation 3-20 and Equation 3-21 into Equation 3-22, we get

$$f = e^{-2rt}[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd}] \tag{3-23}$$

where

$$p = \frac{e^{rt} - d}{u - d}$$
$$u = e^{\sigma\sqrt{t}}$$
$$d = \frac{1}{u}$$

In general, for a binomial lattice with n steps, the ith step ($0 \le i < n$) option value is calculated by Equation 3-24.

$$f_i = e^{-rt}[pf_{i+1,u} + (1-p)f_{i+1,d}] \tag{3-24}$$

Lattice, though still complicated, is more intuitive to the practitioners than continuous time models. It is especially useful to evaluate American options, since analytic solutions are almost non-existing in the continuous models. The drawback is that using lattice by itself is hard to model compound options. However, combined with DTA, lattice is capable to deal with a lot of complicated situations, even more flexible than PDEs in many circumstances.

Monte Carlo Simulation

Originally named after the casinos in Monte Carlo, Monaco, Monte Carlo simulation is about games of chance. It is now widely used to simulate stochastic processes by sampling large quantity of random outcomes for the processes (Figure 3-6). Because of the repetition of algorithms and the large number of calculations involved, Monte Carlo simulation is computationally complex, yet easy to model and understand.

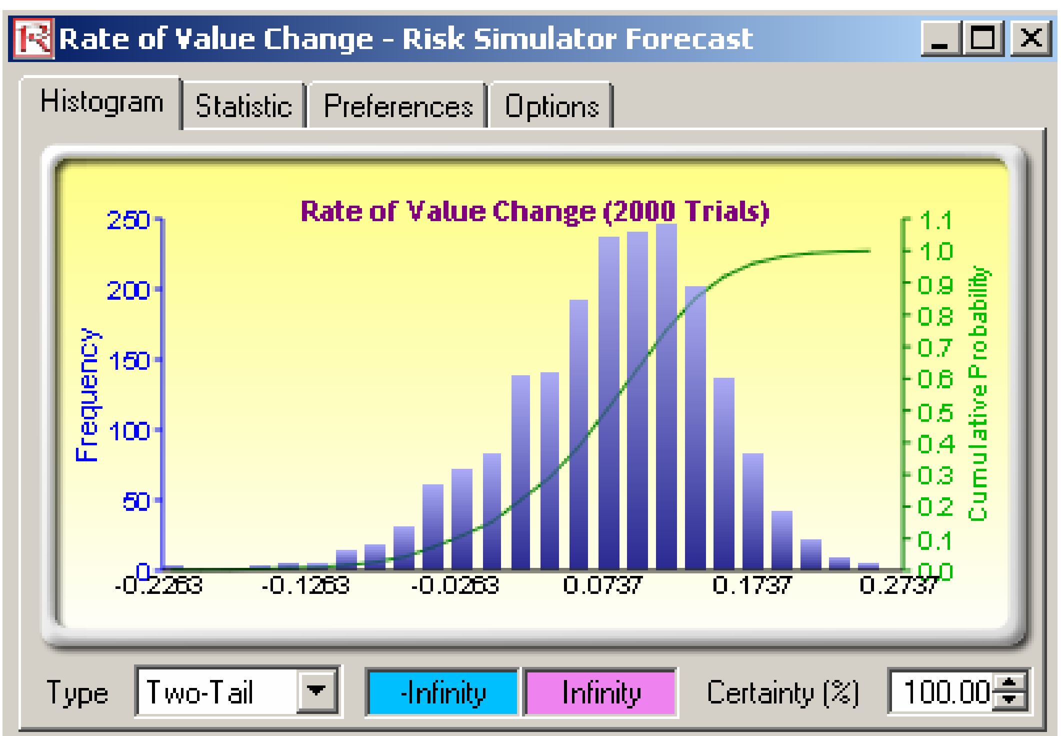

Figure 3-6. Monte Carlo simulation output.

In real options modeling, Monte Carlo simulation can be used where there are several underlying variables. The drawback is that it is difficult to work backward to determine option exercise strategy, since Monte Carlo simulation is forward looking. In the RERO model, it is used as an intermediate step to estimate volatility of the project stems from multiple risk drivers.

Real Options Analysis Approaches

First coined by Myers (1977), the ROA approaches are to apply financial option pricing theory and methodology to evaluate real assets (Miller and Park, 2002; Trigeorgis, 2005). In the financial market, a derivative is a security whose value changes depend on the value changes of some other underlying assets. In real asset valuation, the value of a project can be viewed as a derivative contingent upon input costs, output yield, time and uncertainty (Miller and Park, 2002), and therefore can be evaluated by applying the financial option pricing principles.

By using ROA, investment decisions are viewed as real options or combinations of real options, such as options to defer, expand, switch, contract, or abandon, as shown in Table 3-1 (Trigeorgis, 1996; Yao and Jaafari, 2003). Also included in the table are examples in the real estate and construction industry. Contrary to DCF method, in the ROA context greater volatility is not always worse, since losses are limited to the initial investment, or option premium, but the option holder can capture greater upswings if things turn out to be favorable. ROA is applied most commonly in the industries of natural resource, manufacturing, energy, research and development, start-up companies, and others (Lander and Pinches, 1998; Trigeorgis, 1996). Applications in the real estate and construction industries are still limited.

Although ROA borrows the option pricing theory, the distinguish characteristics of real assets demand different valuation assumptions and methodologies from direct applications of the option pricing theory without any modification. Table 3-2 lists the major differences between financial options and real options (Mun, 2002).

Table 3-1. Types of real options.

Options	Features	Examples
Defer	To postpone construction till optimal timing	Time to develop
Stage	To create a series of stages to allow for abandonment or expansion in later stages depending on outcomes of earlier stages	Phased development
Contract	To contract the project to a third party in order to mitigate risk or to speed up market domination	Franchise stores
Expand	To expand the project scale in favorable market conditions	Airport expansion
Abandon	To abandon the project and prevent severe lost in unfavorable market conditions	Bankruptcy of a project entity
Switch input/output	To change the output mix or input mix in response to changing market demand	Coal-fired vs. gas-fired power plants
Compound	Option on option, where the value of an earlier option can be affected by the value of later options. Most real world options are of this kind	Case study in Chapter 5 and 6

Table 3-2. Comparison between Financial Options and Real Options.

Characteristics	Financial options	Real options
Maturity	Short, usually in months	Long, usually in years
Underlying asset	Traded stocks, with comparables and pricing information	Not traded project free cash flow, proprietary in nature, with no explicit market comparables
Management manipulation	Value does not change due to individual management assumptions or actions	Value has to do with individual management assumptions and actions
Competition and market effect	Irrelevant to pricing	Direct drivers of value

One of the major differences between financial options and real options is how to handle private risk. The underlying assets of financial options are traded market assets, and market risk is the major source of risk among all financial options. Private risk can be treated simply as errors. The underlying assets of real options, however, are usually non-traded assets that do not have market equivalent. Private risks cannot be hedged. The other difference is the effect of management and competition. Financial options on the same underlying asset and the same maturity date are identical. They are widely held to be market efficient. A single transaction usually does not affect the pricing of financial options, neither does management or competition. Real options, on the other hand, are lumpy or one-of-the-kind in nature. Exercise of real options by management can have profound impact on the underlying asset value.

Consequently, there are a lot of debates in the academic world about how real options should be correctly priced. Borison (2005) classified existing real options approaches into 5 categories:

- The classic approach,
- The subjective approach,
- The Market Asset Disclaimer approach,
- The revised classic approach, and
- The integrated approach.

Borison also discussed the underlying assumptions of these approaches, the conditions that are appropriate for their applications, and the mechanics in applying them.

The classic approach assumes that the capital market is complete, and an identical twin asset or portfolio exits for every real asset under evaluation. It makes explicit use of no-arbitrage argument, and applies directly the Black-Shores formula.

The subjective approach also assumes that the capital market is complete. However, it relies on subjective judgment for input, as opposed to data from traded markets. This makes it an inconsistent approach, and limits to qualitative result.

The Market Asset Disclaimer (MAD) approach assumes that the capital market is not complete. It relies on the estimate value of the asset without flexibility as the “twin asset” for the purpose of calculating the option value of the flexibility. Data is drawn from traded markets when available, and subjective judgment when not. Proponents of this approach justified this step explicitly: the same, weaker assumptions that are used to justify the applications of DCF can be used to justify the applications of option pricing to flexible corporate investment (Copeland and Antikarov, 2001).

The revised classic approach assumes that the capital market is partially complete. It attempts to divide the world into black and white: For investments that have market equivalents, it applies the classic approach using market data; for investments that do not have market equivalents, it applies decision analysis using subjective judgment.

The integrated approach also assumes that the capital market is partially complete. However, it uses capital market data for market risk and subjective judgment for private risk in an integrated model.

The major difference among these approaches is how private risk is handled. The classic approach ignores private risk completely and treats real options exactly like financial options that all risks can be diversified away by constructing a hypothetical traded twin asset or portfolio. The subjective approach handles private risk by substituting market data by subjective assessment. The revised classic approach admits the limitations of direct applications of option pricing theory to real options analyses and classifies investments into those either dominated by market risk or by private risk. It applies the option pricing model only to investments dominated by market risk, and applies decision analysis to those dominated by private risk. Although it is a better approach than the previous two, the revised classic approach forces all investments into black or white, and implements two totally different approaches.

The MAD approach, on the other hand, admits the difficulty of handling private risk, thus does not rely on the existence of a traded replicating portfolio. Instead, it uses the project value itself without flexibility as the twin security, as if it were traded in the financial market. After all, the best correlation with the project is the project itself (Copeland and Antikarov, 2001). Trigeorgis (1996) also argued that the assumptions underlying the DCF approach are traded assets of comparable risk (same beta), and MAD assumptions are no stronger than those of DCF.

Contrary to Borison's understanding, Copeland and Antikarov (2005) clarified that the MAD approach does not blindly use all subjective assumptions. Similar to the integrated approach, MAD also uses traded market data whenever available, and uses subjective assumptions only when market estimates are impossible. The MAD approach and the integrated approach are considered to treat private risk in the same way, the difference remains only technical: MAD relies on simulations to evaluate project volatility, and attempts to combine all risks into one variable, whenever possible; while the integrated approach relies on utility functions, and models market risks and private risks explicitly and separately. Neither is superior to the other, and the selection of approaches depends on project characteristics on a case-by-case basis. For this reason, the proposed RERO approaches are built on the MAD and the integrated approaches.

Practical Real Options Model in Real Estate

Ghosh and Sirmans (1999) were among the first to address the applications of real options to the corporate real estate practitioners, by developing a look-up table for the options value, which is derived from an approximation of the Black-Scholes formula. They used the correspondence in Table 3-3 between financial and real options in order to apply the Black-Scholes formula directly to real options.

However, they did not explain whether the time value of money r is a risk-free rate or risk-adjusted discount rate, nor how the risk of project cash flows σ is determined.

Table 3-3. Correspondence between Financial and Real Options.

Variable	Financial options	Real options
S_0	Stock price	Present value of projects expected cash flows
K	Exercise/strike price	Cost of investment
T	Time to expiry	Length of time the decision can be deferred
r	Risk-free rate	Time value of money
σ	Standard deviation of stock returns	Risk of project cash flows

They also developed a three-step approach to calculate the option value:

Step 1: Calculate NPV_q from Equation 3-25.

$$NPV_q = \frac{S_0}{K/(1+r)^T} \tag{3-25}$$

Step 2: Calculate $\sigma\sqrt{T}$

Step 3: Read the value of the call option as a percentage of the value of the underlying asset from the table.

For example, if the stock price S is \$100, strike price K is \$100, time to expiry T is 1 year, time value of money r is 5%, standard deviation of annual return σ is 20%, then

$$NPV_q = S/[K/(1+r)^T] = 100/[100/(1.05)^1] = 1.05$$

$$\sigma\sqrt{T} = 0.20 \times 1 = 0.20$$

From the look-up table, C is 10.4% of the asset value, $C = 0.104 \times 100 = \$10.40$.

They did not specify how the look-up table is computed, but by comparing the Black-Scholes formula and their three-step approach, it is not difficult to find that they did some approximations in order to simplify the calculation.

From the Black-Scholes formula of Equation 3-13,

$$\frac{C}{S_0} = N(d_1) - \frac{Ke^{-rT}}{S_0} N(d_2) \tag{3-26}$$

where

$d_1 = \dfrac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$;

$d_2 = \dfrac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$.

$\dfrac{K}{(1+r)^T}$ is an approximation of Ke^{-rT}, and $\dfrac{S_0}{K/(1+r)^T}$ can substitute $\dfrac{S_0}{K}$, $(r + \sigma^2 / 2)T$ is ignored due to the low impact on the overall value. With the approximation and substituting Equation 3-25 into Equation 3-26, we have

$$\frac{C}{S_0} = \left(1 - \frac{1}{NPV_q}\right) N\left[\frac{\ln(NPV_q)}{\sigma\sqrt{T}}\right] \tag{3-27}$$

Equation 3-27 is the formula to develop the look-up table.

The Ghosh and Sirmans model falls into the subjective approach category of Borison's classification (Borison, 2005). As discussed in the previous section, this approach uses subjective assessment of variables without justification of its appropriateness. At a first glance, this approach is intuitive, especially for practitioners who are comfortable with NPV but unfamiliar with ROA. However, this direct application of the Black-Scholes model is not without its limitations. Firstly, it is restricted to European options, where timing of execution of the option is perfectly known in advance. Secondly, it assumes future cash flow is as deterministic as in the traditional NPV method, and allows for only one scenario analysis. It does not allow for stochastic and dynamic changes of the underlying variables, such as development cost and rental rate, does not solve for optimal development timing. Lastly, while there is a trade-off between simplicity and accuracy, the value derived from the look-up table has 10% variance from that calculated from the Black-Scholes model, which is deemed inaccurate in many circumstance. In summary, the model developed by Ghosh and Sirmans is a good attempt to build the understanding of management flexibility value of corporate real estate in practice, however, it lacks accuracy and depth of applicability in the real estate industry, which is what this study plans to overcome.

Decision Tree Analysis

First coined by Howard (1964, in Ng and Bjornsson, 2004), decision analysis is the discipline comprising the philosophy, theory, methodology, and practice necessary to address important decisions. Graphical representation of decision analysis problems commonly use influence diagrams and decision trees. DTA is a method to identify all alternative actions with respect to the possible random events in a hierarchical tree structure. It is developed to handle the interaction between random events and management decisions. Uncertainties are represented through probabilities and distributions. The attitude of a decision maker to risk is represented by utility functions.

Unlike the DCF approaches, there are no objectively correct DTA models. An appropriate model depends on the preferences and beliefs of the decision maker and hence is subjective. A decision analysis includes the following typical steps: first, defining the scope of the analysis; second, setting up a decision basis, including generating alternatives, collecting information, and estimating risk preference; third, constructing a decision tree with decision and uncertainty nodes; and forth, analyzing sensitivity of factors that have the largest effects (Ng and Bjornsson, 2004).

Decision analytic methods are used in a wide variety of fields, including business, environmental remediation, health care research and management, energy exploration, litigation and dispute resolution, etc.

DTA relies on subjective assessment of probabilities and distributions. This method alone cannot prevent arbitrage opportunity. However, the combination of ROA and DTA can eliminate the short-coming of both, and creates a much better approach.

Summary

In this chapter we reviews modeling details of the DCF, ROA, DTA approaches, as well as capital budgeting theory, ROA applications in real estate. Treatment of private risk differentiates these approaches from one another. In ROA methodologies alone, there are various approaches advocated and debated in the academic community. Due to the characteristics of real options, it is inappropriate and inaccurate to directly apply the option pricing formula without any modification. The correct real option methods must be able to handle private risk as well as market risk in a consistent way. Only the MAD and the integrated approaches are considered appropriate and are subject to further use.

CHAPTER 4
METHODOLOGY

The RERO framework consists of two approaches to value real estate acquisitions: the combined approach and the separated approach. This chapter introduces the key elements and steps of the RERO approaches. The next two chapters present case studies that implement the principles introduced in this chapter.

As mentioned in the previous chapter, the Market Asset Disclaimer (MAD) and the integrated approaches in ROA were adopted for this study.

RERO Modeling Procedures

The RERO framework adopts real options and decision analysis methodologies. It consists of a series of processes to solve a decision tree backward. The event tree starts by laying out all possible events and corresponding cash flows. Starting at the end of the analysis, we work backward through the tree at each decision node to calculate the payoff of all possible actions, using replicating portfolio or risk neutral discounting, choosing the optimal action that generates the highest payoff at each node. Eventually the possible cash flows generated by these future events and actions are folded back to a present value. The following 6 steps are critical in performing the RERO analysis (Figure 4-1):

- Problem framing;
- Approach selection;
- Risk drivers identification and estimation;
- Base case modeling;
- Option modeling; and
- Sensitivity analyses.

Problem Framing

For real estate acquisition, the first task is to review the case qualitatively, and to determine whether the asset itself is a sound investment. An investment that seems good by the numbers may not necessarily turn out to be a good investment in the end. Location, neighborhood development, economy growth, property visibility, accessibility, physical conditions, ownership and occupancy history, management capability, all these are unique characteristics of real estate that are non-quantifiable. Comprehensive local business knowledge and experience is needed to determine whether a piece of land is worth acquiring.

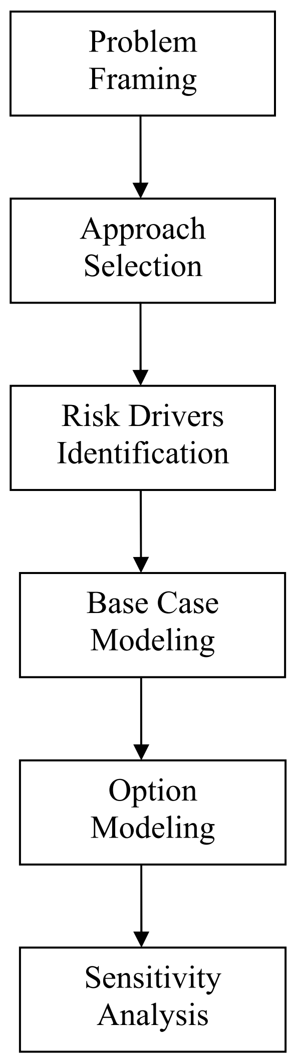

Figure 4-1. Critical steps in RERO analysis.

After this critical screening, if a property is good enough to go through the hassle of quantitative analysis, the problem is framed into a model and the story is told in a mathematical way. The goal becomes how much it is worth. Management flexibility and strategic options, if any, should be identified to determine which approach to use.

Approach Selection

DCF can solve most simple and conventional acquisition problems. It is only when a case has strategic options that cannot be valued by DCF should the RERO approaches be used. Depending on the characteristics of a project, the first step is to determine whether to use the combined approach or the separated approach. The differences between the two approaches are discussed in later sections.

Risk Drivers Identification and Estimation

The next step is to identify the risk drivers. Uncertainties of real estate acquisitions and development include rental income, operating costs, capital expenditure, discount rate, cap rate, development cost, etc. These variables flow through the model to affect the project value. Risk drivers are those key variables that have the most profound impact on project value change.

To estimate the volatility of each risk driver, objective methods such as time series forecast or regression analysis should be used, if historical or comparable data exists. Alternatively, subjective methods may be used, such as subjective guesses, growth rate assumptions, expert opinions, etc (Mun, 2002).

Base Case Modeling

The expected project value without flexibility is the base case for the subsequent option value analysis. The base case value acts as the "twin asset" that the real option approach is based on.

Option Modeling

From the problem framing step, some strategic options have been identified; from the approach selection step, the combined approach or the separated approach has been selected; from the risk driver identification and estimation approach, the key uncertainties have been identified and their volatilities quantified. Now in the option modeling step, a Monte Carlo simulation is run, an event tree is constructed, with managerial flexibilities incorporated in each node, option values are calculated, optimal decisions are made at each node, and the value are tracked from the end of the analysis back to the starting time of the analysis. This process may be run back and forth for several times to ensure all option values are calculated correctly and the corresponding rational decisions are made.

Sensitivity Analyses

Setting the project value with flexibility and/or option value as the dependent variables, each risk variable can be changed, and the trend of value changes in the dependent variables can be observed. This sensitivity analysis helps the user to see the whole picture and determine how each risk variable should be managed. It also helps in understanding how uncertainty could have otherwise altered decision making.

RERO Modeling Approaches

For different treatments of risk drivers, there are two types of RERO modeling approaches: the combined approach and the separated approach. The combined approach is used for valuation of an existing building with a historical operating track record. For uncertainties of infill land development, the separated approach is more suitable.

MAD has two key assumptions: firstly, the present value of the underlying risky asset without flexibility is the best estimate of the project value with flexibility. Secondly, properly anticipated cash flows fluctuate randomly. The second theorem allows the user to combine any number of uncertainties into a spreadsheet, and to produce an estimate of the project NPV conditional on the set of random variables drawn from their underlying distributions by using Monte Carlo simulation techniques (Copeland and Antikarov, 2001, p219). This is the theoretical foundation of the combined approach.

By using the combined approach, uncertainties are assumed to be able to be resolved continuously over time. This assumption generally holds for stabilized assets. However, many projects in real estate, such as infill land development, have major uncertainties that do not get resolved smoothly over time. Many rare events, e.g., permit approval, development activities in the neighborhood, a new mall, a new subway station, can significantly change the real estate value. For projects with any risk of such jumping effect, the actual event tree is asymmetric with changes in value occurring when a significant part of the uncertainty is resolved. The separated approach is used to isolate the risks with jump diffusion effect from those resolved continuously, and to model their interaction explicitly. In other words, the separated approach also assumes that the underlying project value without flexibility is the best estimate of the project value with flexibility, but it does not assume that the cash flows fluctuate randomly. Rather, it separates the risk drivers with jump effect from the others without, and models the jump effect explicitly.

The Combined Approach

The combined approach is most suitable for valuation with risks resolved continuously. This approach can be best applied to acquisition valuation of stabilized real estate assets. The process is to model the parameters of different uncertainties and to estimate their effect on the volatility of the project value using Monte Carlo simulation techniques. The effects of individual risk drivers are thus combined into the project volatility, which is used to generate a binomial event tree. Actions of managerial flexibility are added to solve for option value.

The following variables are typical in a property acquisition model: rental rate, occupancy rate, rentable square footage, expense recovery, operating expenses, capital expenditure, tenant improvement, leasing commission, going-out cap rate, discount rate, etc. Among these variables, the most influential ones are rental rate, stabilized occupancy rate, going-out cap rate, and discount rate. Rentable square footage is usually fixed; expense recovery and operating expenses vary but in a controllable small range related to the rental rate change; capital expenditure, tenant improvement, and leasing commission are tricky in reality, but could be assumed to be fixed on an annual basis for a high-end office building.

Rental rate and stabilized occupancy rate will be used as the two major variables in the case analyses. Rental rate is set by the market, and directly impacts the property value. For value-added type of investors, who intend to upgrade amenities and enhance occupancy, the stabilized occupancy rate is an important factor for revenue estimation. The discount rate, however, is subjective to each investor. In finance theory, the discount rate should reflect the level of risk of a project. In practice, however, for an individual investor, the discount rate is usually his weighted average cost of capital. Risk is mainly adjusted through the Cap rate rather than discount rate (Wheaton et al., 2001). The discount rate can therefore be regarded as fixed.

The change of rental rate depends on many factors, such as macro economics, employment growth, market occupancy rate, new construction pipeline, net absorption rate, etc. The change of rental rate is assumed to follow the multiplicative stochastic process. Historical data of rental rates will be examined in the next chapter.

Another factor that affects rental revenue is stabilized occupancy rate. For a building that is not fully leased, there might be upside potential to lease up the vacant space, depending on market demand. In a market with strong job growth, demand for office space is also strong. It is relatively easy to lease up the vacant space. Assuming that vacant space can be leased up, the incremental Net Operating Income (NOI) is substantial compared to the incremental revenue, since the incremental operating expense is minimal. In other words, whether a building is 50% occupied or 100% occupied, a majority of the operating expenses is fixed, the 50% lease-up can potentially triple the NOI. Note that a multi-tenant office building is seldom fully occupied, therefore stabilized occupancy rate usually is close to but never reaches 100%. A general vacancy factor is deducted from the fully leased revenue. The change of occupancy rate is assumed to follow the additive stochastic process. This process is similar to the multiplicative stochastic process with the only difference being that the up and down movements in the lattice are assumed to be additive rather than multiplicative (Copeland and Antikarov, 2001, p123).

The Separated Approach

The separated approach is more complicated than the combined approach and should be used only when needed. It is best used for projects with major private risks that do not get resolved continuously. The infill land valuation is an example in this study that can be better modeled using the separated approach.

The following variables are typical in an infill land development model: rental rate, development cost, development timing, development scale, operating expenses, expense recovery, cap rate, discount rate, etc. Among these variables, the most uncertain ones are rental rate, development cost, and development timing. Development scale is regarded as a major economic factor, but not a major uncertainty in the context of our case study, due to approved permit of the development scale. Since the goal of most commercial developments is to maximize the investor’s wealth, developments are usually built to the largest size allowed by zoning and legal restrictions. Unless the development involves zoning changes, development scale is predictable, and thus is not modeled as a risk driver. As discussed in the combined approach, operating expenses and expense recovery are in a controllable range, and the discount rate for a particular project is fixed to a specific investor. Cap rate is assumed to be fixed in the integrated approach for simplicity.

Development costs include hard costs and soft costs, and can be subdivided into costs associated with land, structure, tenant improvement, leasing commission, legal, finance, taxes, insurance, marketing, etc. Hard costs are construction costs that include demolition, foundation, structure, mechanical and engineering systems, general conditions, bonds and insurance of construction, design and management fees, tenant improvement, etc. Soft costs are intangible costs that go to legal, survey, marketing, financing, taxes, leasing commissions, etc. Since every project is unique, development costs represent the major private risk that does not correlate with the traded financial market, and thus cannot be replicated by the so called traded twin asset.

Rental rate is discussed in the combined approach during normal circumstance. What needs to be pointed out in addition is the jump diffusion process. A jump diffusion process is defined as a type of stochastic process that has large discrete movements (jumps, or shocks), rather than small continuous movements (Amin, 1993). As Wheaton et al. (2001) noted: "In reaction to positive shocks, returns initially increase, but eventually diminish with the arrival of new supply. Similarly, negative shocks lead to building conversions, loss of stock and an eventual recovery of returns." One of the distinguishing characteristics of real estate, compared to traded securities, is its inelasticity, or slow reaction to shocks. The jump diffusion can be ignored in the acquisition of a nearly fully occupied property, since rental rates cannot be changed until lease expirations, which could be years from the emergence of the shock. But jump diffusion could be a major uncertainty in development, since all rental square footage is newly available. Developers can ask for higher rental rates in markets with rising demand.

Development timing is also important. Development timing is different from development duration. Given the size of a development project, the duration of construction is usually fixed, but when to start the project could have profound impact on the value, given the real estate cycle. One of the major disadvantages of DCF valuation is its inability to determine the optimal development timing. The RERO framework, on the other hand, can analyze all possible scenarios and indicate the best action at each point in time. It is extremely valuable for the investor to hold the option of when to start the development.

Another important factor is development scale, or the size of development. In the case study, the permit for around 1 million square feet of mix-used development has been approved. Consequently no assumption needs to be made for changing development scale. But in many cases, when rezoning is required in order to develop more density, development scale is an important factor and should be modeled in the decision tree as whether or not the rezoning requirement will be approved.

RERO Modeling Techniques

Rational for Using Binomial Lattices

Copeland and Antikarov (2001, p222) made the assumption that change in asset prices follow Geometric Brownian Motion, based on Samuelson's proof that "properly anticipated prices fluctuate randomly." In other words, change in asset value follows a random walk even if the risk drivers do not. This means multiple risk drivers, so long as they evolve continuously, can be combined and reduced to a single uncertainty, namely the expected underlying asset value change over time. This provides the rationale for using a binomial lattice to calculate real option value.

Monte Carlo Simulation

Monte Carlo simulation randomly generates values for uncertain variables to simulate a real-life model. In the combined approach, Monte Carlo simulation can be used as an intermediate step to estimate volatility of the project, the value of which is depended on multiple risk drivers. For this study Risk Simulator is used. Other simulation software available are Crystal Ball and @ Risk.

The steps followed in the combined approach are to:

1. Identify risk drivers;
2. Estimate the probability distribution of each risk driver using historical data or subjective estimates;
3. Build present value model;
4. Define input variables with the possible range of value and a probability distribution in an MS Excel spreadsheet equipped with Monte Carlo simulation tools;
5. Define correlations among the risk variables;
6. Define forecast variables., e.g., rate of return for the project;
7. Run the simulation a thousand times;
8. Read the outputs of the forecast variables and their volatility distributions; and
9. Use the outputs as input variables to build the event tree.

Replicating Portfolio

In most cases the project cash flows are discounted at the risk-adjusted rate to get to the project NPV. The risk-adjusted discount rate is higher than the risk-free discount rate, since it is adjusted up to accommodate higher risk of the project than that of the treasury bonds. In order to apply a binomial lattice that is developed based on risk-neutral probabilities and risk-free discount rates, risk-adjusted probabilities should be used together with risk-adjusted discount rates. To calculate the value of the option, the replicating portfolio method is used, but not the discounting method, since the risk characteristics of the project change over time depending on the decision made, and consequently the risk-adjusted discount rates also change over time (Copeland and Antikarov, 2001). The risk-adjusted up movement factor u and down movement factor d are the same as those in the risk-neutral binomial lattice (Equations 4-1 and 4-2).

$$u = e^{\sigma\sqrt{t}} \tag{4-1}$$

$$d = \frac{1}{u} \tag{4-2}$$

where σ is the project volatility, and t is the time in years of each step in the binomial tree.

The replicating portfolio formula can be derived by the same method as the option price is derived from binomial lattice. Construct a portfolio that consists of n shares of stock S and b amount of value in risk-free bonds. After a period of time t, the value of the portfolio can go up or down. Let the value be equal to the option value at that time.

$$nuS + be^{rt} = C_u \tag{4-3}$$

$$ndS + be^{rt} = C_d \tag{4-4}$$

From Equations 4-3 and 4-4, derive Equations 4-5 and 4-6.

$$n = \frac{C_u - C_d}{S(u-d)} \tag{4-5}$$

$$b = \frac{uC_d - dC_u}{e^{rt}(u-d)} \tag{4-6}$$

Consequently, the value of the option is calculated by Equation 4-7.

$$C = nS + b = \frac{C_u - C_d}{u-d} + \frac{uC_d - dC_u}{e^{rt}(u-d)} \tag{4-7}$$

Binomial Lattice with Dividend

Chapter 3 covers binomial lattice without dividend. In real estate, the net cash flows from operation are collected from the property and distributed to the investor, which is similar to the dividend distribution of a stock. The stock dividend is usually assumed to be distributed at a constant yield, since corporations plan and manage the distribution process. The net cash flows at the property level, on the other hand, are the actually amounts collected from the property, and hence vary from period to period. Denote δ_i to the dividend yield at Step i for $0 < i \leq n$, and using all other notions in Chapter 3, the asset value changes are depicted in Figure 4-2 for a two-period lattice.

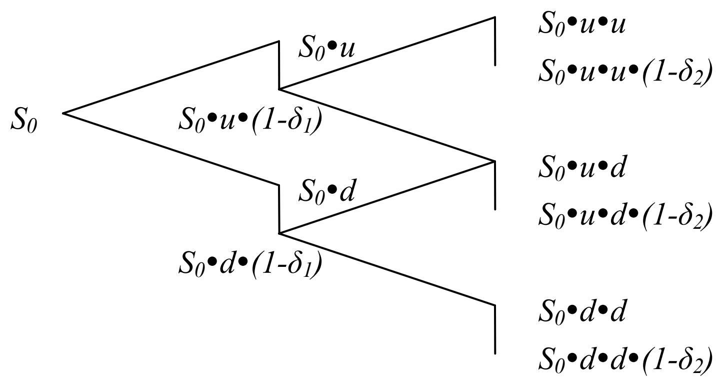

Figure 4-2. Two-step binomial lattice with different dividend yields.

At Step 2, the three possible values are calculated using Equations 4-8, 4-9, and 4-10.

$$C_{uu} = Max[S_0 uu(1-\delta_2) - K, 0] \tag{4-8}$$

$$C_{ud} = Max[S_0 ud(1-\delta_2) - K, 0] \tag{4-9}$$

$$C_{dd} = Max[S_0 dd(1-\delta_2) - K, 0] \tag{4-10}$$

To calculate the option value at Step 1, the dividend yield δ_2 needs to be added back to the option value, before discounting at the risk-free rate, which is shown in Equations 4-11 and 4-12.

$$C_u = \frac{pC_{uu} + (1-p)C_{ud}}{(1-\delta_2)e^{rt}} \tag{4-11}$$

$$C_d = \frac{pC_{ud} + (1-p)C_{dd}}{(1-\delta_2)e^{rt}} \tag{4-12}$$

The same method is followed to calculate the option value at Step 0, as shown in Equation 4-13.

$$C = \frac{pC_u + (1-p)C_d}{(1-\delta_1)e^{rt}} \tag{4-13}$$

In general, for a binomial lattice with *n* steps, the *i*th step ($0 \le i < n$) call option value with dividend is calculated by Equation 4-14.

$$C_i = \frac{pC_{i+1,u} + (1-p)C_{i+1,d}}{(1-\delta_{i+1})e^{rt}} \tag{4-14}$$

Binomial Lattice with Jump Process

Chapter 3 covers binomial lattice during normal circumstance that the underlying asset strictly follows the GBM movement. However, in reality, the asset movement could be a jump. For example, the zoning change from agricultural land to urban land, the establishment of new amenities in the neighborhood, the construction of new freeway exits, all can have a sudden and profound influence on the estate value in an area. These events seldom happen. But once occur, they will completely change the project payoff pattern. Hence, these jump diffusion effects cannot be priced using the binomial lattice developed by Cox et al. (1979). Amin (1993) developed a discrete time model to value options when the underlying process follows a jump diffusion process. Unlike the financial jump diffusion process that reverses back to normal value quickly, a jump diffusion process in real estate usually is irreversible, at lease not in a short period of time. That is, if a large scale development occurs that drives up the rental rate in a neighborhood, that rental rate is likely to remain at the same level for several years until a new event happens. In this study the Amin model was modified to accommodate this change. Based on the assumption that the jump risk is diversifiable, a one-period call option is priced in the Equation 4-15 (Figure 4-3).

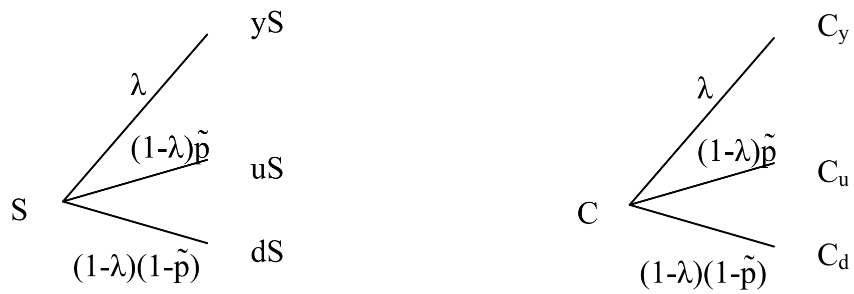

Figure 4-3. Binomial lattice with jump process.

$$C = e^{-rt}\{\lambda C_y + (1-\lambda)[\tilde{p}C_u + (1-\tilde{p})C_d]\} \tag{4-15}$$

where

λ is the probability of the jump event according to the Poisson distribution, and defined by $\lambda(x) = \dfrac{e^{-n}n^x}{x!}$ (where n is the expected number of successes, and x is the number of successes per unit);

y is the capital gain return on the underlying asset when the jump event occurs;

C_y is the option value at the time the jump event occurs;

$\tilde{p}$ is the adjusted probability of an up movement, and defined by

$$\tilde{p} = \frac{\dfrac{e^{rt} - \lambda y}{1-\lambda} - d}{u - d}.$$

Investment with Private Uncertainty

As discussed in Chapter 3, many investments include private and market uncertainties. Market uncertainty can be replicated with market participation and therefore diversifiable. Private uncertainty cannot. For example, the development project value depends on both the market uncertainty of rental rate and the private uncertainty of development cost.

The principle of pricing in such investment, if no correlation between the market risk and private risk exists, is to use risk-neutral probability for the market uncertainty and actual probability for the private uncertainty, both discounted at risk-free rate (Luenberger, 1998; Copeland and Antikarov, 2001; Smith and McCardle, 1999). Although formulas for pricing uncertainties with correlation exist, the no correlation assumption usually holds.

To implement this principle, there are two alternative methods: the quadranomial lattice and the decision analysis method.

The first method is to implement a quadranomial lattice. Figure 4-3 shows a one-step quadranomial lattice. If an option C is contingent upon the value of two underlying assets S_1 and S_2, assuming no correlation between S_1 and S_2, then the value of C is priced as Equation 4-16.

$$C = e^{-rt}(p_{11}C_{11} + p_{12}C_{12} + p_{21}C_{21} + p_{22}C_{22}) \tag{4-16}$$

where

$$
\begin{aligned}
p_{11} &= p_1 p_2 \\
p_{12} &= p_1(1 - p_2) \\
p_{21} &= (1 - p_1)p_2 \\
p_{22} &= (1 - p_1)(1 - p_2)
\end{aligned}
$$

p_i is the risk-neutral probability if S_i is market uncertainty, or the actual probability if S_i is private uncertainty. For each uncertainty, it can have more than two bifurcations. For example, if S_1 is a market risk with jump diffusion (three bifurcations), and S_2 is a private risk with three bifurcations, then C could be priced with nine nodes with corresponding probabilities and discount at the risk-free rate. In theory, an option can be contingent upon more than two separated assets, but in practice, the complexity of implementation will soon become intimidating. This study thus focuses on a few key risk drivers and combine them into two kinds of separated uncertainties: market uncertainty and private uncertainty.

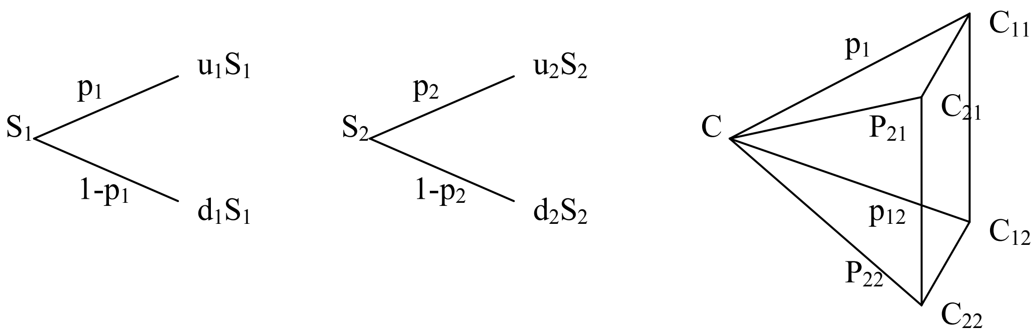

Figure 4-4. Quadranomial lattice.

Another way is to implement decision analysis methodology (Smith and Nau, 1995). For example, if the two underlying risks for a development are cost and rental rate, it can be modeled as shown in Figure 4-5. The expected value at each node is calculated and discounted at the risk-free rate. Equation 4-17 shows how the expected value $E(PV_0)$ can be calculated.

$$E(PV_0) = \sum_{j=1}^{m} p_j [E(PV_j)] \tag{4-17}$$

where

j is a scenario labeled from 1 to m, $1 \le j \le m$;

$E(PVj)$ is the expected present value of scenario j for all the years i, $1 \le i \le n$.

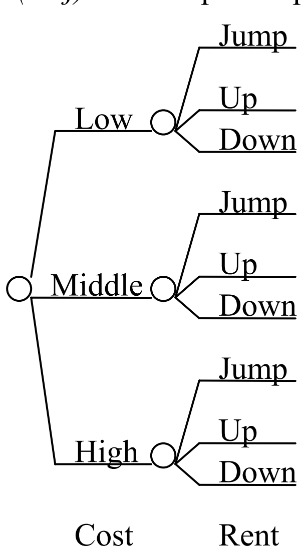

Figure 4-5. Decision analysis.

Summary

This chapter discusses the 6-steps RERO framework: problem framing; approach selection; risk drivers identification and estimation; base case modeling; option modeling; and sensitivity analysis. Two modeling approaches are introduced to deal with different risk characteristics: the combined approach for projects with risk drivers that get resolved continuously, and the separated approach for project either with risk drivers that follow the jump diffusion process or involving private risk. The modeling techniques that will be applied in the case studies are also introduced, including the rationale of using the binomial lattice, Monte Carlo simulation, replicating portfolio, binomial lattice with jump diffusion process, and investment with private risk.

CHAPTER 5
THE COMBINED APPROACH

Chapter 5 and 6 present case studies that implement the principles of RERO described in Chapter 4. The two chapters describe the valuation of two parts of one case: valuation of the building using the combined approach, and valuation of the infill land using the separated approach. Together, these two case studies demonstrate how the RERO framework can be applied to different scenarios in the real estate acquisition and development analysis.

Case Description

The case identified is 211 Perimeter in Atlanta. This property is located in the Central Perimeter submarket of Atlanta. Adjacent to the Perimeter Mall and a subway station, 211 Perimeter is located in one of the largest suburban office markets in Atlanta. The property has an office building of 226,000sf rentable area, and 13 acres total land. The current owner has got approvals for over 1 million square feet of mixed-use development on the 9.5 acres developable site, and has built a 6-storey parking garage with the intention to get as much value as the regulations allow from development of the excessive land (Figure 5-1). Furthermore, the property is strategically located within a larger neighborhood redevelopment planning of 38 acres and nearly 3 million square feet mixed-use development, although the timing of the neighborhood development is unknown.

The land obviously has some value, but development might not break ground immediately. The real estate market in Atlanta is a commodity market, which means developments are spread out with few restrictions. As 2005, the Central Perimeter office submarket was over built, with several old office buildings torn down for new residential developments. It would be interesting to know how current bidders should price the land in addition to the building.

Figure 5-1. 211 Perimeter site plan.

Building Valuation

In this chapter only the building is valuated using the combined approach with Monte Carlo simulation. The land valuation will be investigated in the next chapter using the separated approach. The following are the 6 steps used to perform the RERO valuation:

- Problem framing;
- Approach selection;
- Base case modeling;
- Risk drivers identification and estimation;
- Option modeling; and
- Sensitivity analyses.

Problem Framing

The property is located in a premium office market, with superior quality and tenant mix. Its strategic location within a larger neighborhood redevelopment plan makes real estate price appreciation in the future extremely promising, although the timing is still unknown. In short, the 211 Perimeter project is a sound investment that deserves further valuation.

After the preliminary qualitative analysis, this project appears acceptable for quantitative analyses. The 11-floor office building consists of 226,000sf rentable area. Current occupancy rate is 85%, with 15% upside potential to lease up the space. Major tenants collectively occupy 68% of the rentable square footage, which is deemed to be a sign of solid cash flow over the future.

One of the major decisions to make is about the chiller system upgrade. The existing chillers are still in working condition but are at their maximum capacity, and consume far more energy than new ones. Preliminary research shows that replacement of the existing chillers will cost $950,000, and will increase the net cash flow by 5% per year. If both rental rates and occupancy rates are good, replacement of the chillers can justify its cost, and add value to the property. Otherwise, the capital improvement may not break even, and keeping the existing chillers is more economical.

Approach Selection

The combined approach is selected since both the rental rate and occupancy rate are market driven, and can be combined into the Monte Carlo simulation.

Base case NPV calculation

The following variables are typical in the NPV valuation model: rental rate, occupancy rate, rentable square footage, expense recovery, operating expenses, capital expenditure, tenant improvement, leasing commission, going-out cap rate, discount rate. Table 5-1 shows the assumptions used in the base case NPV calculation. Figure 5-2 shows the cash flow output from Argus, a software package for real estate valuation.

Table 5-1. Major assumptions for Argus.

Average rental rate	$17/sf	Capital expenditure	$75,000
Occupancy rate	85%	Tenant improvement	$18/sf
Rentable sf	225,924 sf	Leasing commission	6.0%
Expense recovery	$0	Going-out Cap rate	7.0%
Operation expenses	$7.75/sf	Discount rate	9.0%

From the Argus cash flow output, modifications are made so that the model can be used for Monte Carlo simulation using Risk Simulator. Rental rate and occupancy rate have been identified as the two major risk variables that need to be simulated. Annual average rental rate and annual average occupancy rate are calculated from the Argus® output, which are used to derive annual net cash flow. Purchase price is assumed to be fixed, so that we can compare the project value with and without flexibility. Operating expenses and expense recoveries are controllable variables. Capital items, such as capital expenditure, tenant improvement, and leasing commission, are also controllable. Cap rate and discount rate are also assumed to be fixed.

Ignoring the option of chiller replacement, the project NPV has two components: (1) Total acquisition cost, including purchase price and closing cost; (2) Present value of annual net cash flow from operation and present value of net residual value (gross sale proceeds net out selling cost). These two parts are also called cost and benefit. The option of chiller replacement will be modeled later.

In real estate fundamental analysis, property value consists of residual value and net cash flow from operation. The residual value, or value when the project is sold, is the major part. It is determined by Net Operating Income (NOI) and Capitalization rate (Cap rate). NOI is the gross

Schedule Of Prospective Cash Flow
In Inflated Dollars for the Fiscal Year Beginning 10/1/2005

For the Years Ending	Year 1 Sep-2006	Year 2 Sep-2007	Year 3 Sep-2008	Year 4 Sep-2009
Potential Gross Revenue				
Base Rental Revenue	$3,865,560	$3,933,779	$4,055,362	$4,187,044
Absorption & Turnover Vacancy	(536,603)	(120,821)	(6,223)	(17,347)
Base Rent Abatements	(557,764)	(335,242)	(193,496)	
Scheduled Base Rental Revenue	2,771,193	3,477,716	3,855,643	4,169,697
Expense Reimbursement Revenue			26,417	52,555
Miscellaneous	2,400	2,472	2,546	2,623
Conference Room	6,000	6,180	6,365	6,556
Antenna Revenue	44,874	46,220	47,607	49,035
Total Potential Gross Revenue	2,824,467	3,532,588	3,938,578	4,280,466
General Vacancy			(137,519)	(149,114)
Effective Gross Revenue	2,824,467	3,532,588	3,801,059	4,131,352
Operating Expenses				
Cleaning	181,060	184,366	216,739	222,801
Repairs and Maint.	243,591	250,899	258,426	266,178
Utilities	318,091	326,111	355,126	365,464
Grounds	42,095	43,358	44,659	45,998
Security	128,504	132,359	136,330	140,420
Parking/Fitness Center	8,182	8,427	8,680	8,941
Management	116,706	106,141	133,037	144,597
Administrative	193,246	199,043	205,015	211,165
RE Taxes for Building	418,558	431,115	444,048	457,370
Insurance	62,129	64,277	66,233	68,197
Non-Recoverable	45,185	46,747	48,169	49,598
Total Operating Expenses	1,757,347	1,792,843	1,916,462	1,980,729
Net Operating Income	1,067,120	1,739,745	1,884,597	2,150,623
Leasing & Capital Costs				
Tenant Improvements	119,610	1,818,406	11,118	
Leasing Commissions	33,160	396,647	9,094	
Reserves	33,889	35,060	36,127	
8F Corridor & Common Area	75,000			
Total Leasing & Capital Costs	261,659	2,250,113	56,339	
Cash Flow Before Debt Service	805,461	(510,368)	1,828,258	

Figure 5-2. Base case NPV calculation.

income from all sources (rental, storage, tenant reimbursement, antenna lease, etc) minus all operating expenses (common area maintenance, management fee, security, landscaping, insurance, real estate taxes, etc). For this reason, NOI is also regarded as the net income of the property. This is different from what the investor actually gets, which is called the Net Cash Flow. Net cash flow is calculated by taking out capital items from NOI. These capital items, such as capital improvement, tenant improvement, and leasing commission, are one-time-off in nature. All these analyses are on an unleveraged before-tax basis, meaning debt financing and taxation are not considered.

Figure 5-3 shows the modified Argus® cash flow output for NPV calculation. For simulation simplicity, modifications of the Argus® output are made so that the net operating income and net cash flow are calculated by Equation 5-1 and Equation 5-2.

$$NOI = Q \cdot SF \cdot Occ + ER - OE \tag{5-1}$$

$$NCF = NOI - TI - LC - CapX \tag{5-2}$$

where

NOI is the net operating income;
Q is the average rental rate;
SF is the rentable square footage;
Occ is the actual occupancy rate;
ER is the expense recovery and other income;
OE is the operating expenses;
NCF is the net cash flow;
TI is the tenant improvement;
LC is the leasing commission;
CapX is the capital expenditure.

The residual value at sales is calculated by Equation 5-3.

$$V_n = \frac{NOI_{n+1}}{Cap} - SC \tag{5-3}$$

where

V_n is the net residual value at year n, and n is the holding period of the project;
Cap is the going-out Cap rate;
SC is the selling cost.

The total benefit of the project PV_j, which includes the present value of net cash flow NCF_i and residual value V_n, can be calculated by Equation 5-4.

$$PV_j = \sum_{i=j}^{n} \frac{NCF_i}{(1+k)^{i-j}} + \frac{V_n}{(1+k)^{n-j}} \tag{5-4}$$

where

PV_j is the project present value at Year j, and $0 \le j \le n$, where n is the holding period. When $j = 0$, it is the present value at time 0, or PV_0.
NCF_i is the net cash flow at Year i,
k is the discount rate of the project.

The NPV of the project is the present value of total cost PP_0 and total benefit PV_0 at time 0, as calculated by Equation 5-5.

$$NPV_0 = PV_0 - PP_0 \tag{5-5}$$

Risk Drivers Modeling

Among the variables, those that have the most profound impact on the project NPV changes are rental rate and stabilized occupancy rate, both are market driven. Rental rates differ lease-by-lease, but for simplicity we take the average rental rate over the entire building. Stabilized occupancy rate is subjective based on management's estimates In this case the 15% vacant space is assumed to be leased up within 2 years, after which a general vacancy factor of 3% is taken out.

Figure 5-4 shows the historical rental rates of the Central Perimeter Class A office market and the subject property in 15 years. The quarterly data is from CoStar®. The change of rental rate is assumed to follow GBM. This means the logarithm of the rental rate Qi is normally distributed; and the return (also called the change of rental rate) qi follows a random walk. Using Equation 5-6, a rental return analysis was performed and the scatter chart was plotted as shown in Figure 5-5, with market return variables on X-axis and corresponding subject property return variables on Y-axis. It shows negative correlation (-0.1445), which indicates that the rental

Rental Square Footage	225,924				
Base Rent psf		$17.11	$17.41	$17.95	$18.53
Rental Adjustment Multiple		16.9121	17.2087	17.7424	18.3157
Return Random Variable		1.0117	1.0117	1.0117	1.0117
Occupancy Rate		85.68%	89.15%	99.91%	99.72%
Occupancy Adjustment Variable		0.8542	0.8889	0.9965	0.9946
Occupancy Random Variable		0.0026	0.0026	0.0026	0.0026
	Year 0	Year 1	Year 2	Year 3	Year 4
For the Years Ending	Sep-2005	Sep-2006	Sep-2007	Sep-2008	Sep-2009
Potential Gross Revenue					
Base Rental Revenue		$3,865,560	$3,933,337	$4,055,336	$4,186,372
Vacancy		(553,548)	(426,767)	(3,650)	(11,722)
Abatements		(540,920)	(29,248)	(196,069)	(5,623)
Scheduled Base Rental Revenue		2,771,091	3,477,322	3,855,617	4,169,027
Other Reimbursement Revenue		53,274	54,872	82,935	110,769
Total Potential Gross Revenue		2,824,365	3,532,194	3,938,552	4,279,796
General Vacancy				(137,519)	(149,114)
Effective Gross Revenue		2,824,365	3,532,194	3,801,033	4,130,682
Operating Expenses					
Total Operating Expenses		1,757,347	1,792,843	1,916,462	1,980,729
Net Operating Income		1,067,018	1,739,351	1,884,571	2,149,953
Leasing & Capital Costs					
Total Leasing & Capital Costs		261,659	2,250,113	56,339	
Cash Flow Before Debt Service		805,359	(510,762)	1,828,232	
Net Sales Proceeds (Cap at 7%, 2% selling cost)				30,099,336	
Total Net Cash Flow		805,359	(510,762)	31,927,568	
Discount Rate	9%				
PV_0 of Cash Flow		738,862	(429,898)	24,653,941	
PV_1 of Cash Flow		805,359	(468,589)	26,872,795	
PV_0 of Cash Flow Static		738,862	(429,898)	24,653,941	
Forecasting Variable	**1.0900**				
Purchase and Closing Cost (3%)	(24,205,000)				
NPV of Project	**757,904**				

Figure 5-3. Spreadsheet model for Monte Carlo simulation.

rate change of the subject property, 211 Perimeter, has very weak, if not negligible, correlation with the market.

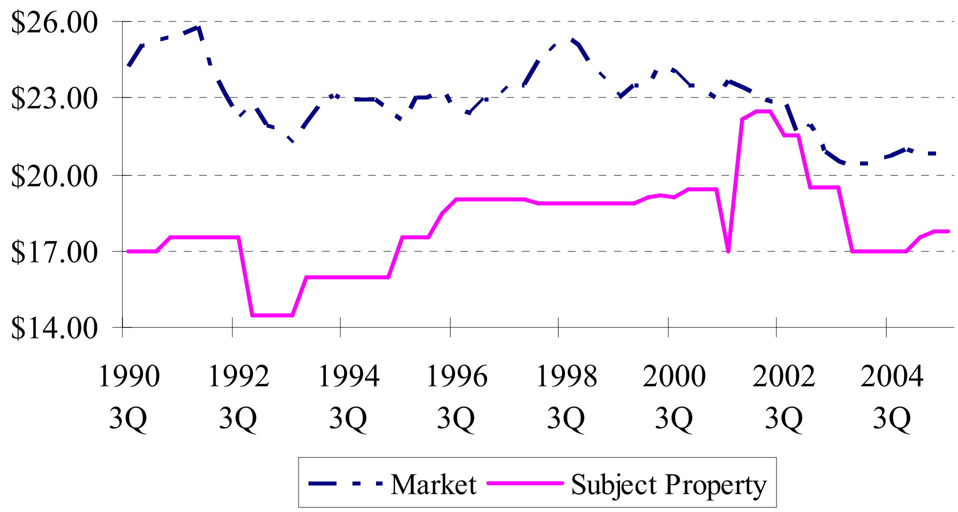

Figure 5-4. Historical market and subject property rental rates.

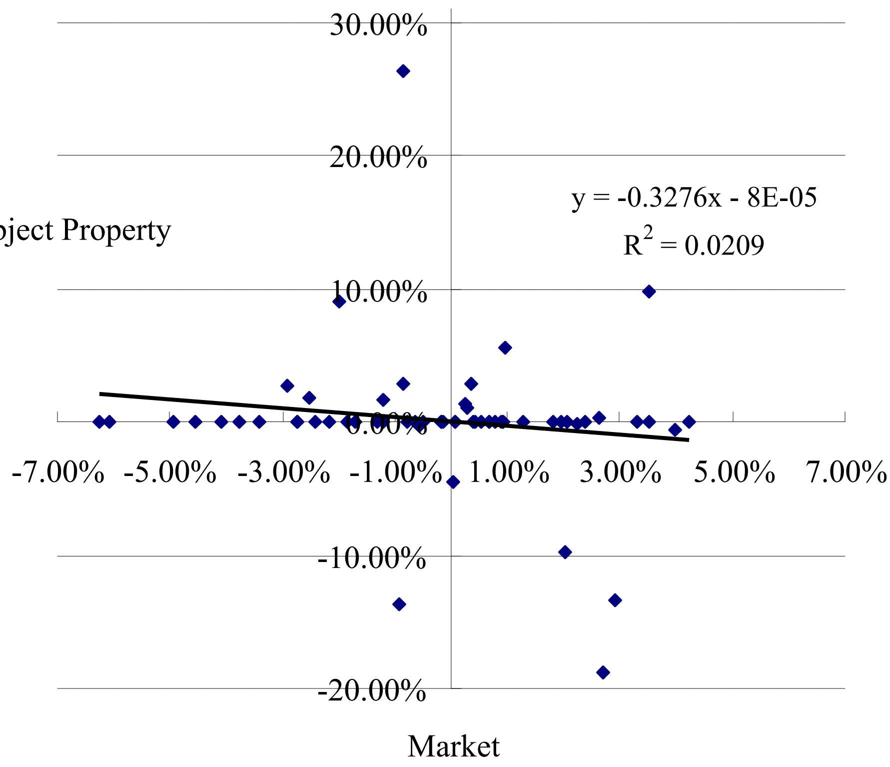

Figure 5-5. Returns correlation between market and subject property.

$$q_i = \ln(\frac{Q_i}{Q_{i-1}}) \tag{5-6}$$

The seemingly controversial result of weak or no correlation between the rental return of the subject property and that of the market can be explained as due to two reasons:

(1) Data reliability. CoStar started as a service portal mainly for commercial brokerage firms. In its early years data is derived from broker volunteer contributions. This would inevitably have led to data accuracy and timeliness issues. For example, from the first quarter of 1998 to the third quarter of 1999, the rental rates data of the subject property are missing, which are assumed to be $18.90/sf by the author for the purpose of data completeness.

(2) The inelastic nature of real estate market. Compared to the financial market, the real estate market is lumpy and the performance is somewhat predictable, at least for the near term. Commercial lease terms are usually 3 to 7 years for office leases, 5 to 20 years for anchor retail leases, and even 100 years for ground leases. In most cases, the rental payment is set and documented in the contract throughout the terms. Market rental rate changes can only slowly affect individual property ask rates, since the landlord can change rental rate only when a lease negotiation happens, usually before the lease expires. However, market rates can directly affect the rates for new construction, since all spaces are newly available.

Nevertheless, the data set from CoStar is the most comprehensive and consistent data available in the real estate industry. The characteristics of real estate require a different method than the one used to estimate stock volatility in the financial industry. Thus the correlation between the market and the subject property was ignored on purpose, and only the subject property rental rate data was used to estimate its volatility for acquisition.

Risk Simulator® is used to influence the distribution of the population from the available sample data. A lognormal distribution was chosen since the rental rates will never be negative. Due to the limitation of available data, the statistical significance of this distribution is low (P-Value of 0.1625). Nevertheless, this is the most reasonable fit for the data. By fitting the sample data into a lognormal distribution (Figure 5-6), the following variables are determined: μ is 0.0056 and σ is 0.0548. Annualizing the quarterly σ, and using Equation 5-7 and 5-8, mean of 18.0189 and standard deviation of 2.0218 for the return distribution are derived. To get the annual auto-correlation of the rental return, the quarterly return data is annualized by taking the average of the 4 quarters of each year, which turns out to be -0.0916. This auto-correlation of the samples is assumed to be the same as that of the population.

$$\overline{X} = e^{\mu + (\sigma^2/2)} \tag{5-7}$$

$$SD = \sqrt{e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}} \tag{5-8}$$

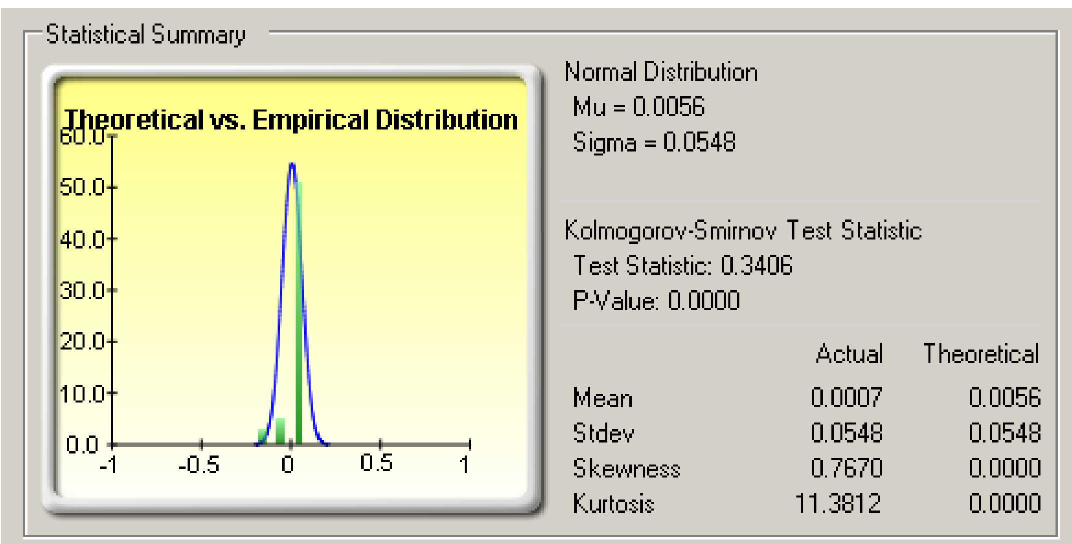

Figure 5-6. Normal distribution fit for historical returns on rental.

CoStar® also provides historical occupancy rates data for the market and subject property (Figure 5-7). Occupancy rate is assumed to follow the additive stochastic process. This means the change of occupancy rate o_i between any two quarters is simply the difference of the occupancy rate O_i and O_{i-1} (Equation 5-9). From the scatter plot of the change of occupancy data shown in Figure 5-8, it can be concluded that the occupancy rate of the property also has very weak correlation with the market (0.1263). Thus, this correlation is also ignored on purpose and only the historical occupancy rates of the subject property will be relied on for forecasting.

$$o_i = O_i - O_{i-1} \tag{5-9}$$

Using RiskSimulator®, the population μ and σ, the respective population mean and standard deviation of the normal distribution, are determined to be 0.0039 and 0.0471 respectively (Figure 5-9). Due to the limitation of available data, the statistical significance of this distribution is low (P-Value of 0.00004). However, this low P-value might be a limitation of the software itself, i.e., its estimation of data in a small range is inaccurate. Nevertheless, this is the most reasonable fit for the data. To preserve accuracy, it was decided to keep the sample mean as the population mean (0.0026), and annualize the sample standard deviation as the

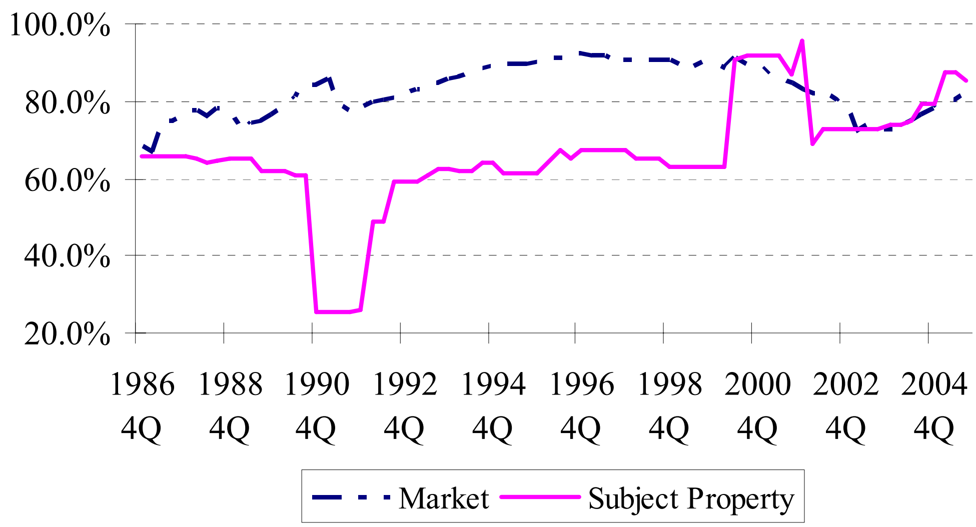

Figure 5-7. Historical market and subject property occupancy rates.

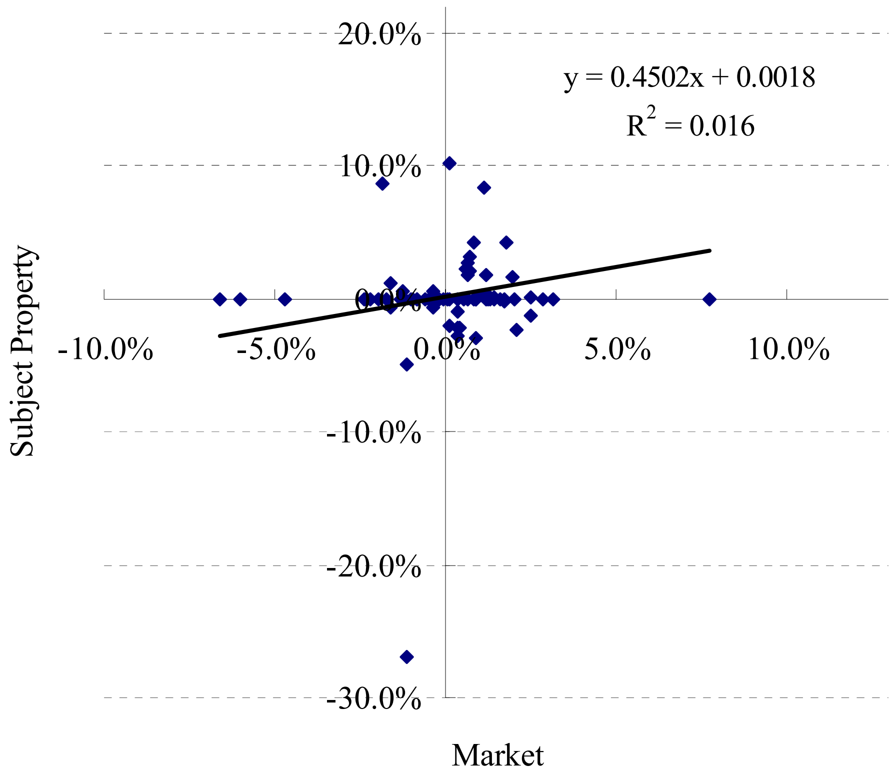

Figure 5-8. Occupancy changes correlation between the local real estate market and the subject property.

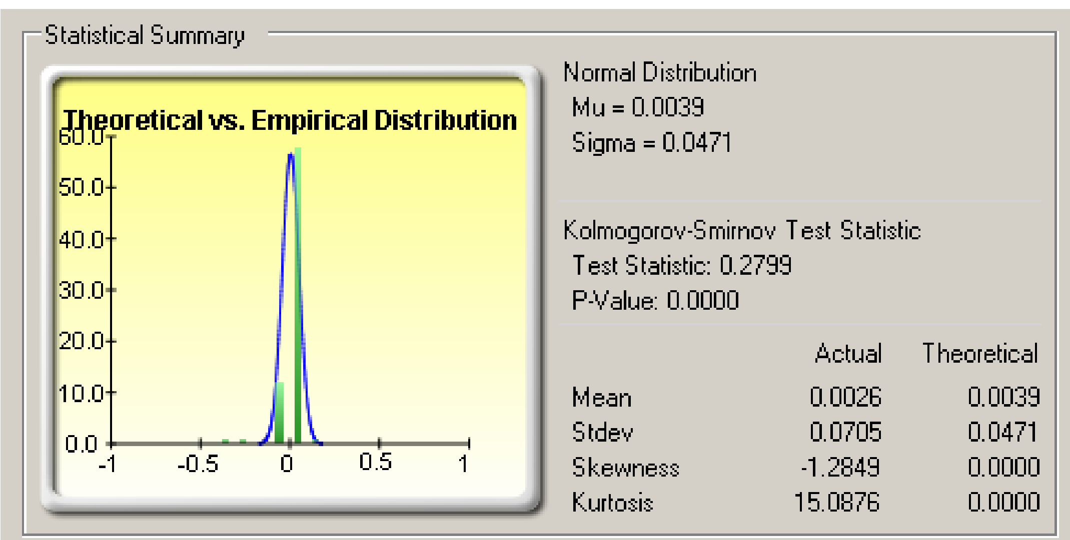

Figure 5-9. Normal distribution fit for historical occupancy rates.

population volatility (0.1409). To calculate the auto correlation, the change of occupancy rate data is annualized by taking the average of the 4 quarters of each year, which turns out to be 0.1185.

The correlation between rental return and change of occupancy rate is similar to the auto-correlation of the two, which comes out to be -0.1575.

For Monte Carlo simulation, the project volatility is the volatility of percentage changes in the value of the project from one time period to the next, defined by the forecasting variable z (Equation 5-10). This value is computed using the simulated present value of the project in Year 1 divided by the expected present value of the project in Year 0. In other words, PV_1 is dynamic, while PV_0 is static.

$$z = \frac{PV_1}{PV_0} \tag{5-10}$$

Option Modeling

In the previous step the rental rate, occupancy rate, their respective volatilities, auto-correlations, and the correlation between the two have been identified and quantified. With these variables, rental rates and occupancy rates for each year can be set as risk variables for the project value simulation. A total of 8 risk variables are defined and highlighted as shown in Figures 5-43, 5-10, and Table 5-2. The cash flows go through Equations 5-1 to 5-5 to generate annual net cash flow for the first 3 years. PV_0 and PV_1 are calculated based on the annual net cash flow. The forecasting variable z is defined in Equation 5-10. Setting PV_0 to be static and PV_1 to be dynamic, and running the simulation for a 1000 times, the simulation result of z is obtained as shown in Figure 5-11. Table 5-3 also shows the statistical summary of z, with a mean of 1.079 and standard a deviation of 0.3283.

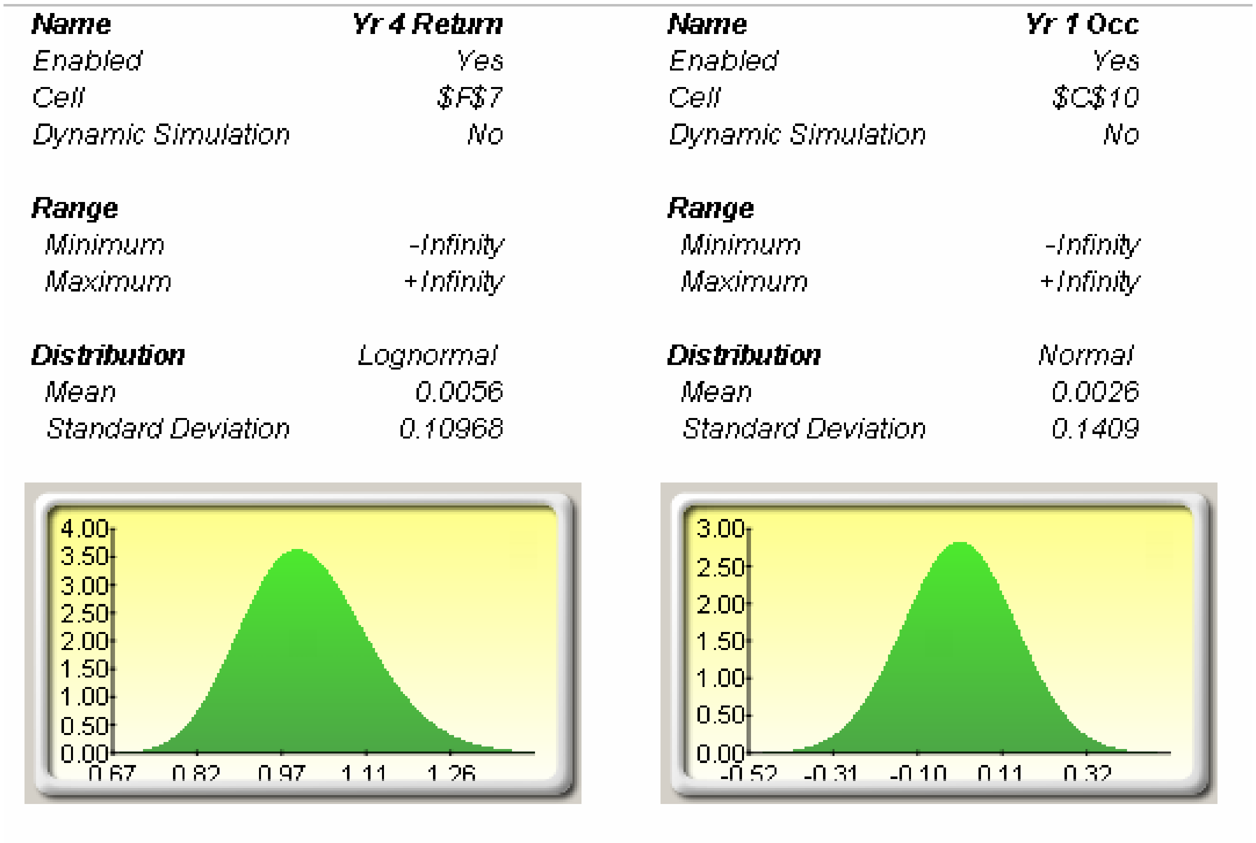

Figure 5-10. Snap shot of Monte Carlo simulation assumptions.

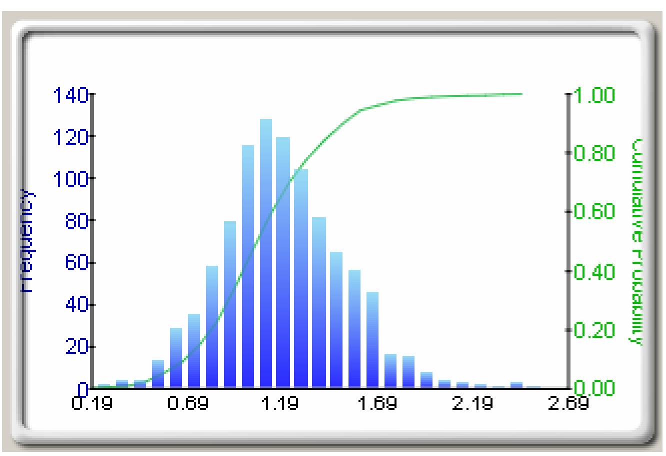

Figure 5-11. Monte Carlo Simulation Result of Forecasting Variable *z*.

The statistical distribution fit for Variable *z* is then performed. By plotting the 1000 *z* values from the simulation output, as shown in Figure 5-12, it is determined that they are normally distributed with P-Value of 0.8737. This result fits quite well with the theory

Table 5-2. Correlation between random variables.

	Yr 1 return	Yr 2 return	Yr 3 return	Yr 4 return	Yr 1 Occ	Yr 2 Occ	Yr 3 Occ	Yr 4 Occ
Yr 1 return	1.00							
Yr 2 return	-0.09	1.00						
Yr 3 return	0.00	-0.09	1.00					
Yr 4 return	0.00	0.00	-0.09	1.00				
Yr 1 occ	-0.16	0.00	0.00	0.00	1.00			
Yr 2 Occ	0.00	-0.16	0.00	0.00	0.12	1.00		
Yr 3 Occ	0.00	0.00	-0.16	0.00	0.00	0.12	1.00	
Yr 4 Occ	0.00	0.00	0.00	-0.16	0.00	0.00	0.12	1.00

Table 5-3. Statistical summary of Monte Carlo simulation result.

Description	Value
Number of data points	1000
Mean	1.0797
Median	1.0569
Standard deviation	0.3283
Variance	0.1078
Average deviation	0.2561
Maximum	2.4431
Minimum	0.0977
Range	2.3454
Skewness	0.3787
Kurtosis	0.7041
25% percentile	0.8722
75% percentile	1.2831
Error precision at 95%	0.0188

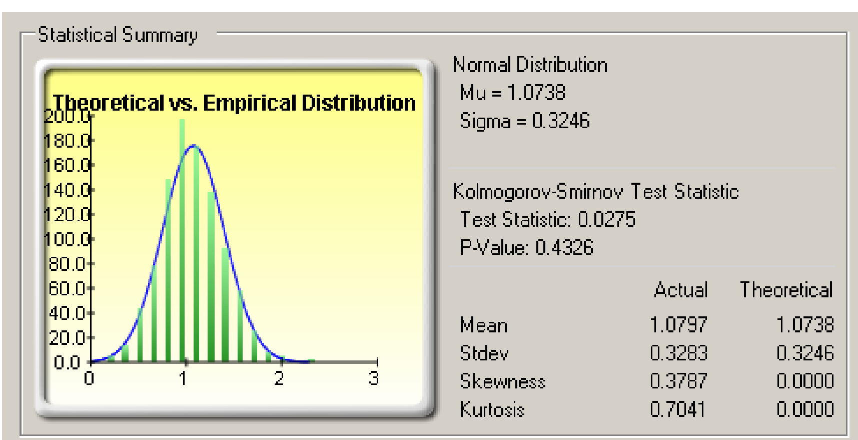

Figure 5-12. Normal distribution fit of forecasting variable *z*.

developed by Samuelson and adopted by Copeland and Antikarov (2001), as discussed in Chapter 4, that changes in correctly expected asset prices follow Geometric Brownian Motion.

From the Monte Carlo simulation, the mean μ and the volatility σ of forecasting variable z are calculated as 1.0797 and 0.3283 respectively. This means the expected average project return is 7.97% (1.0797 minus 1), and the volatility of the project is 30.4% (0.3283 divided by 1.0797).

Using the assumptions in Table 5-4, with 30.4% volatility, and $24,963,000 PV derived from the base case analysis, a value tree is constructed as shown in Figure 5-13. Net cash flows are modeled as dynamic dividend yield times PV in the base case (Refer to Chapter 4 for details of binomial lattice with dividend). For example, in Year 1, the PV can go up to $34,664,000 with an up factor of 1.3886, the post dividend cash flow is therefore $33,638,000 (after taking out 2.96% yield from the $34,664,000 before dividend cash flow).

Table 5-4. Event tree assumptions (Dollars in $1,000).

Assumptions		Intermediate computations	
PV of asset value	$24,963	Stepping time (dt)	1.0000
Implementation cost	$24,205	Up step size (up)	1.3886
Maturity (years)	3.00	Down step size (down)	0.7201
Risk-free discount rate (%)	5.00%		
Volatility (%)	32.83%		
Lattice steps	3		
Option type	Call		

NCF as percentage of PV

Year	1	2	3
NCF_i	$805	($511)	$1,828
PV_i	$27,210	$28,781	$31,928
Percentage	2.96%	-1.78%	5.73%

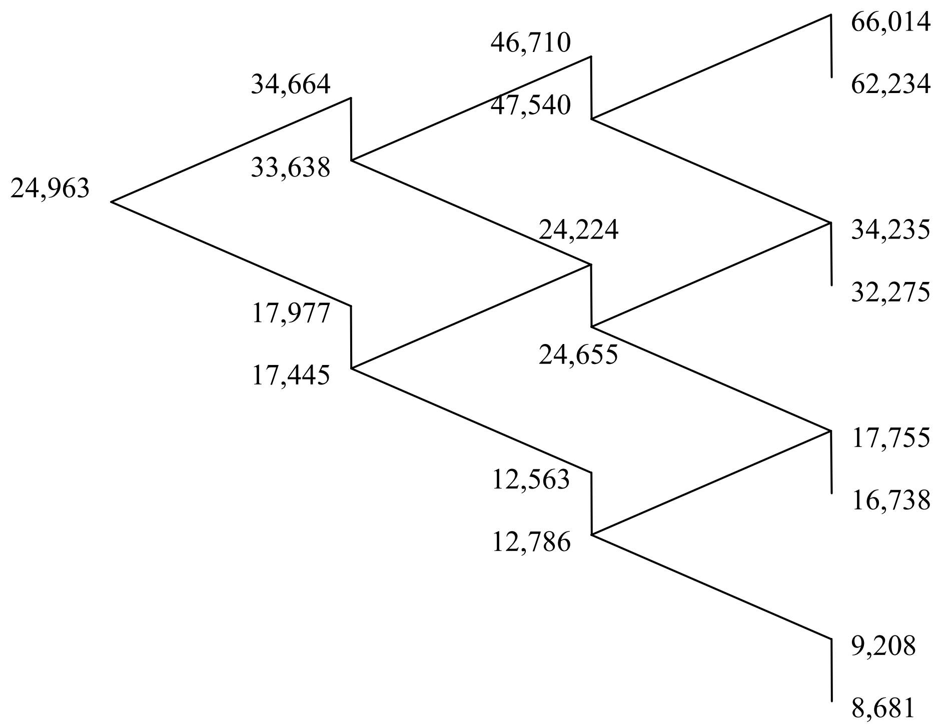

Figure 5-13. Event tree present value without flexibility (Number in $1,000).

With the event tree of PV without flexibility, the chiller replacement option can now be modeled. An event tree of PV with flexibility is constructed (Figure 5-14). At the end nodes, the decision is whether to keep the existing chillers or replace them with new ones. For example, the value of Node A` is calculated as follow.

Max (Replace, Keep) = Max (Present Value * 1.05 – Cost, Present Value)
= (62234 * 1.05 – 950, 62234)
= 64396 (Replace)

At the intermediate nodes, the decision is about whether to leave the option open or to execute it immediately. To calculate the value of leaving the option open, the replicating portfolio method developed in Chapter 4 must be used, but not the discounting method, since risk-adjusted probability and risk-adjusted discount rate are used to construct the spread sheet and event tree. Equation 4-7 is the replication portfolio formula to be applied.

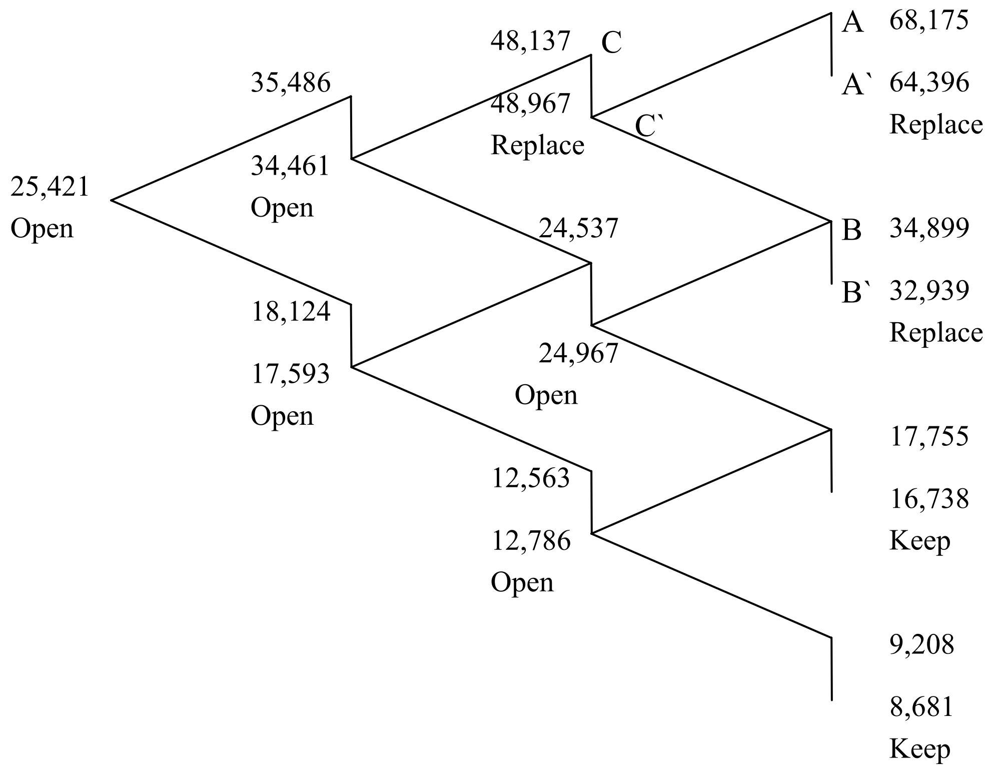

Figure 5-14. Present value with flexibility (Numbers in $1,000).

For example, the value of keeping the option open at Node C` is

$$C = \frac{68175 - 34899}{1.3886 - 0.7201} + \frac{1.3886 \times 34889 - 0.7201 \times 68175}{e^{0.05 \times 1}(1.3886 - 0.7201)} = 48877$$

Therefore, the value of node C` is

Max (Replace, Open) = Max (47540*1.05-950, 48877) = 48967 (Replace)

The decision is to replace the chiller system immediately. Using Equation 4-14 to add back the implied net cash flow of negative $830,000, the before dividend present value is $48,137,000. Working backward the value at each node can be similarly calculated and the optimal action can be selected to maximize the present value, and eventually the maximum present value can be derived at time 0. The present value increases from $24,963,000 (without flexibility) to $25,421,000 (with flexibility), or an increase by $458,000. The NPV of the project is now $1,216,000. In other words, the option to replace the chillers system creates $458,000 value. If the building could be purchased at $24,205,000, the NPV increases to $1,216,000.

Sensitivity Analyses

Sensitivity analyses are conducted using option value as dependent variable, and present value, replacement cost, discount rate and volatility as independent variables. Table 5-5 summarizes the effect of each independent variable as well as their combined effects on the option value.

Present value has positive effect on the option value (Figure 5-15). Replacement of the chiller system increases the annual net cash flow by 5%. And present value is positively related to net cash flow. Therefore, the higher the present value is, the higher the additional net cash flow would be when exercising the replacement option, and hence the higher the option value would be.

Table 5-5. Summary of variable effect on option value.

	Present value	Replacement cost	Discount rate	Volatility
Present value	Positive	Uncertain	Positive, most Sensitive when in-the-money	Positive, most Sensitive when at-the-money
Replacement cost		Negative	Uncertain, most Sensitive when at-the-money	Uncertain, most Sensitive when at-the-money
Discount rate			Positive	Positive, most Sensitive when at-the-money
Volatility				Positive

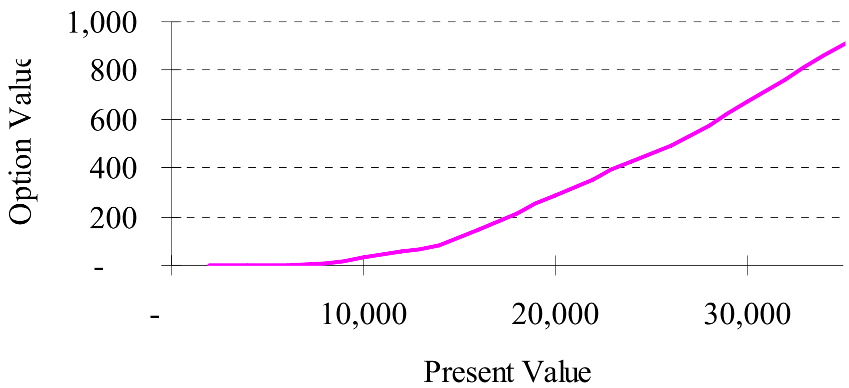

Figure 5-15. Option value in relation with present value.

As shown in Figure 5-16, the replacement cost has negative effect on the option value. The higher the replacement cost is, the less likely the replacement is breakeven, and hence the less likely the option would be exercised.

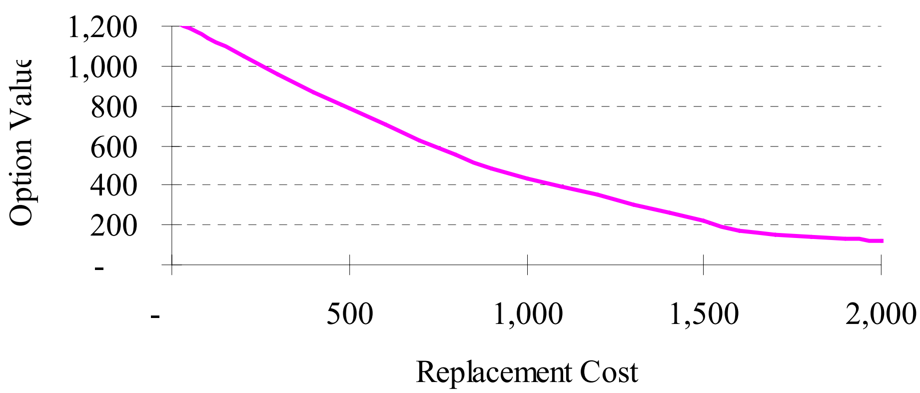

Figure 5-16. Option value in relation with replacement cost.

Volatility also has positive effect on the option value (Figure 5-17). The higher the volatility, the wider the present value spread becomes in later years, but the replacement option is only exercised in those scenarios with positive net cash flows. Therefore, the more uncertain the future cash flow is, the more valuable the option becomes.

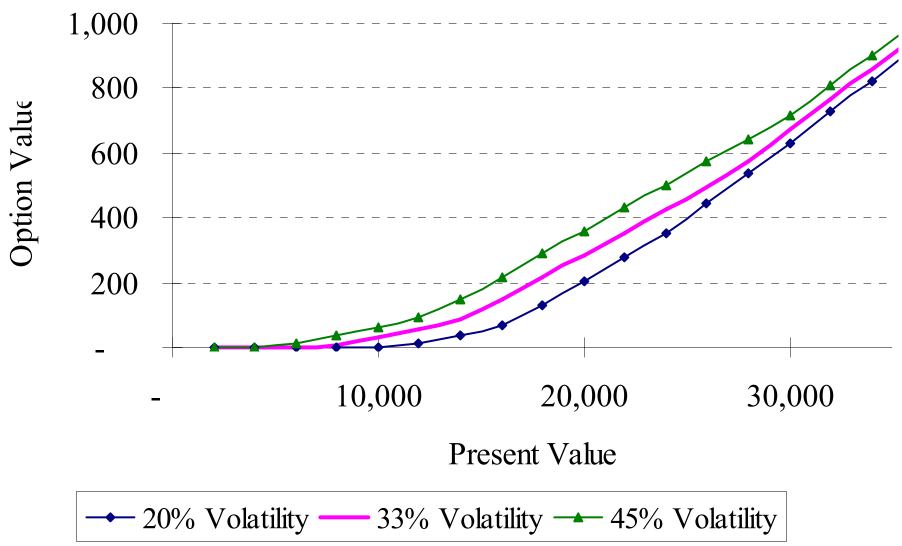

Figure 5-17. Option value in relation with present value and volatility.

Risk-free interest rate has positive effect on the option value. But the effect is not significant.

After examining the effect of each independent variable on the option value, combinations of each two independent variables can be looked at. The combination of present value and Risk-free interest rate has positive effect on the option value.

The two pairs of (1) present value and volatility (Figure 5-17), (2) volatility and risk-free rate (Figure 5-18) both exercise positive effect on option value, and are most sensitive when the option is at-the-money.

The three pairs of (1) replacement cost and volatility (Figure 5-19), (2) replacement cost and risk-free rate, (3) present value and replacement cost (Figure 5-20) all display uncertain effect on the option value. This conclusion is best illustrated in Figure 5-20. The 3-dimensional curve indicates that the higher the present value and the lower the replacement cost, the higher the option value. However, this effect is non-linear. With higher present value and higher replacement cost, the option value may be higher or lower, depending on whether the option value is in-the-money.

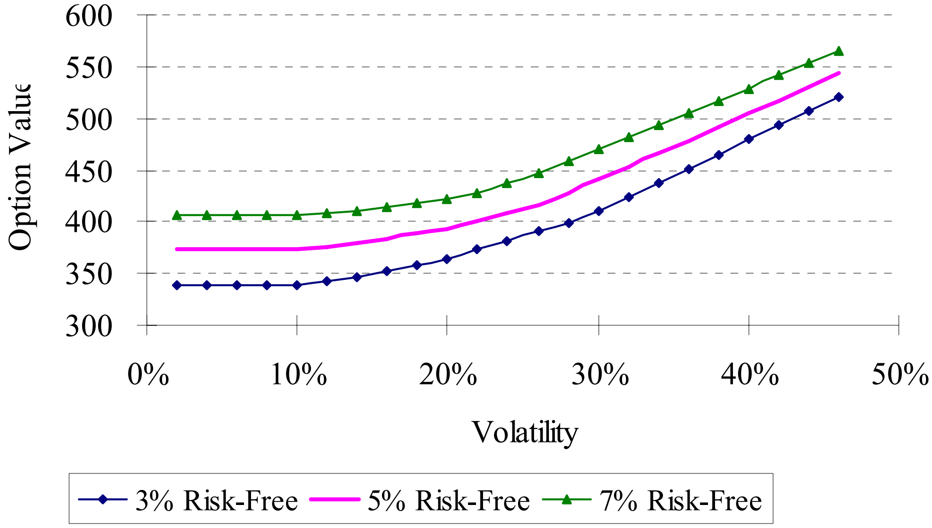

Figure 5-18. Option value in relation with volatility and discount rate.

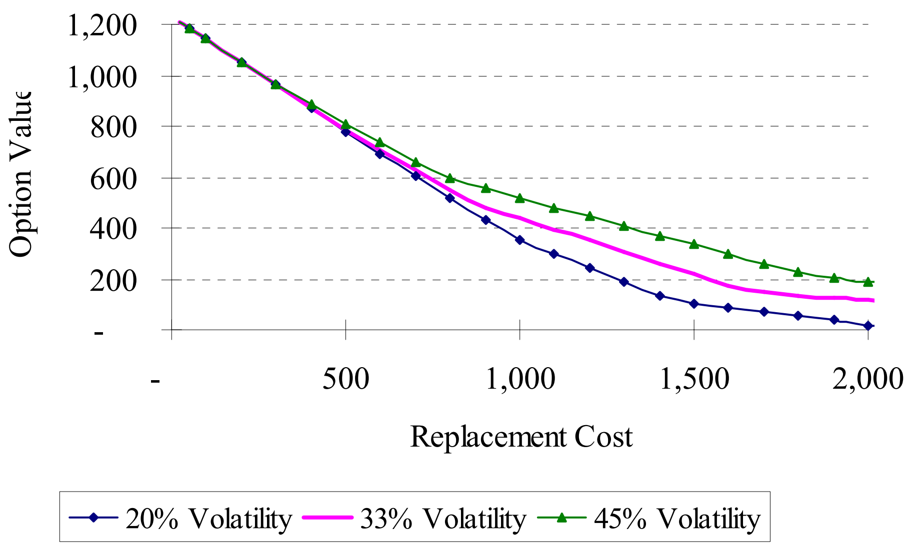

Figure 5-19. Option value in relation with replacement cost and volatility.

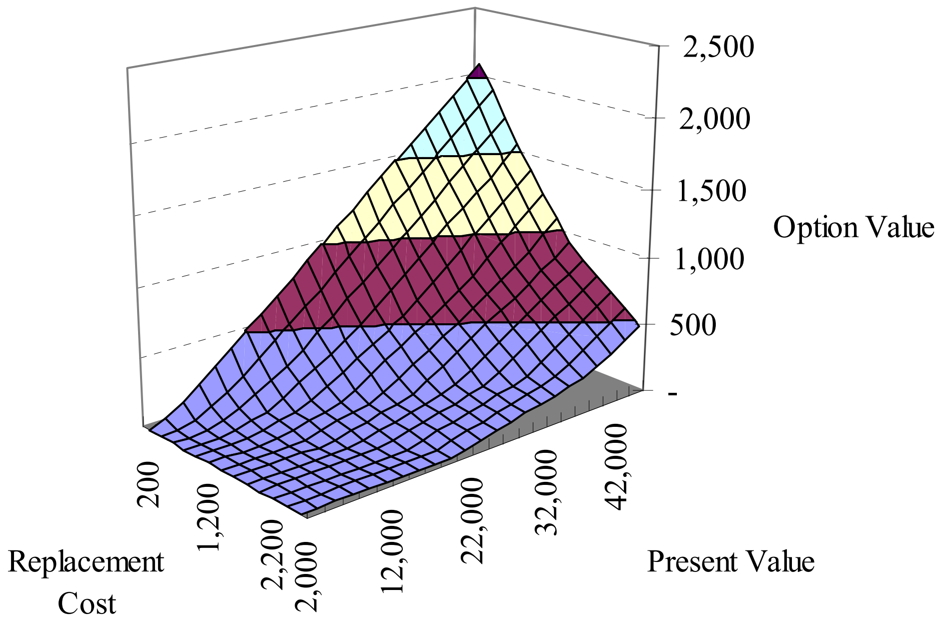

Figure 5-20. Option value in relation with present value and replacement cost.

Summary

This chapter applies the combined approach to determine the building value of the 211 Perimeter property in Atlanta. Rental rate and stabilized occupancy rate are identified as the two major risk drivers and their volatilities are estimated using historical data. The risk variables are combined in a spread sheet. Monte Carlo simulation is performed to estimate the project volatility. Event tree is constructed, in which the option to replace the chiller system is incorporated. The RERO approach indicates that the building is worth \$25,421,000, and the value of managerial flexibility is worth \$458,000.

CHAPTER 6
THE SEPARATED APPROACH

This chapter is the second part of the case study described in Chapter 5. In the previous chapter the RERO framework is applied to analyze the building structure and a managerial decision of chiller replacement. The combined approach with Monte Carlo simulation is used as the major methodology. This chapter, however, is about valuation of the infill land using the separated approach, with jump diffusion process and decision tree analysis techniques. Together, these two parts demonstrate how the RERO framework can be applied to different scenarios in the analysis of real estate acquisition and development.

Case Description

The previous chapter has full description of the case 211 Perimeter in Atlanta. This chapter only repeats the infill land portion. Besides the existing office building and the 6-story garage, the current owner has got approvals for over 1 million square feet of mixed-use development on the 9.5 acres developable site. Furthermore, the property is strategically located within a larger neighborhood redevelopment planning of 38 acres and nearly 3 million square feet mixed-use development, although the timing of neighborhood development is unknown.

The land obviously has some value, but development might not break ground immediately. The real estate market in Atlanta is a commodity market, which means, with little control of urban sprawl, developments are spread out easily as far as market demand exists. The Perimeter office submarket is currently overbuilt, with several old office buildings torn down for new residential developments. It would be interesting to know how current bidders should price the land in addition to the building.

Land Valuation

The value of the infill land (9.5 acres out of the 13 acres total) depends on the value and cost of the improvement should it be developed. The value of the improvement is determined by a function of its annual rental income and operating cost, just like the existing building. The cost of development includes hard costs and soft costs. Since every project is unique, development cost is assumed to be a private risk that does not correlate with the traded financial market.

Problem Framing

The addition of a 6-story garage has freed the infill land from its original function as surface parking. With the 1 million square feet mix-used development approval, the land can be sold for $4.75 million at anytime during the holding period. Its best value for the investor is being either developed or spin-off for $4.75 million.

Table 6-1 shows the development assumptions. Assume the land allows for 1 million square feet to be built, gross rent is $24.5/sf, stabilized occupancy rate is 85%, operating expense is $8.5/sf, required cap rate is 8%, risk-free interest rate is 5%. Expected development cost is $227.5/sf. Land carrying cost is assumed to be negligibly small compared to the development value. The land can be sold for $4.75 million at anytime. This can be viewed as the exercise price of a put option to the investor.

Table 6-1. Development assumptions.

Rentable sf	1,000,000	Site acres	9.50
Gross rent psf	$24.50	Land	$4.75
Occupancy rate	85.0%	Value	$154.06
Operating expenses psf	$8.50	Cost	$177.50
Net rent psf	$12.33		
Riskfree rate	5.0%	Cap rate	8.0%

In addition, management believes that the groundbreaking for the larger neighborhood redevelopment will have significant impact on the demand for new office space, and hence drive up rental rate of this development by 20%. This is a one-time event, but once the rental rate rises, it will remain at that level during the entire analysis period.

Approach Selection

The separated approach is selected because the impact when the rental rate jumps up by 20% is significant, and the chance is uncertain, depending on the timing of the neighborhood redevelopment. This is an example where one risk driver (the rental rate) does not get resolved smoothly, and must be modeled separately from the other risks.

Risk Drivers Identification and Estimation

The risk drivers are rental rates and development cost. Unlike the existing office building, the new building does not have a historical track record. For income, the building rental rate is assumed to have some premium over the average market rental rate. Changes in rental rate are assumed to follow the GBM movement, with a jump-diffusion process corresponding to the groundbreaking of the neighborhood project. Figure 6-1 shows the historical market average rental returns for Class A office properties in the Central Perimeter submarket. Using the Risk Simulator®, the quarterly lognormal returns are plotted into a normal fit as shown in Figure 6-2. Converted into annual data, the market rental rate volatility is 4.84%. As explained in Chapter 5, individual property is far more volatile than the market average. The management estimate doubles and becomes 9.68% per year for the infill land development project.

The current gross rental rate is $21/sf for the average Class A building in the Central Perimeter submarket. According to management experience, a $3.50/sf premium for a brand new building can be secured.

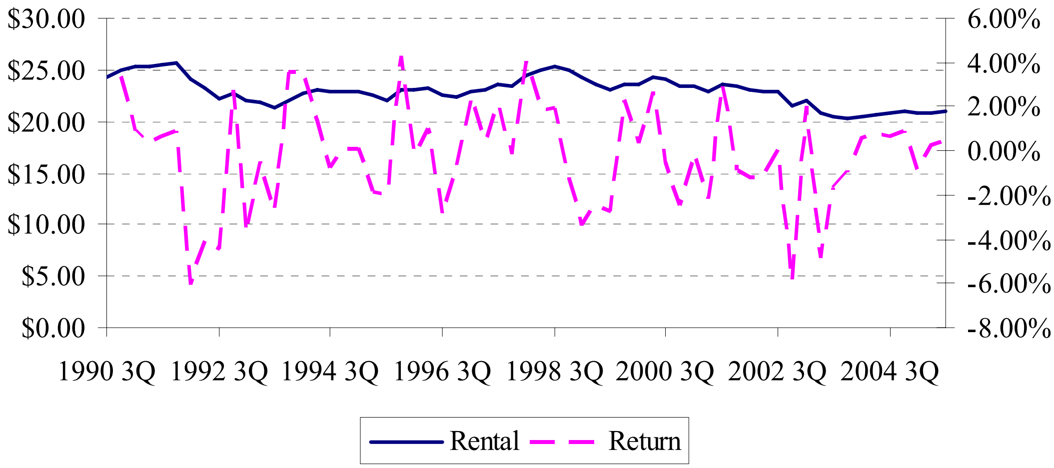

Figure 6-1. Historical market average rental rates and return volatility.

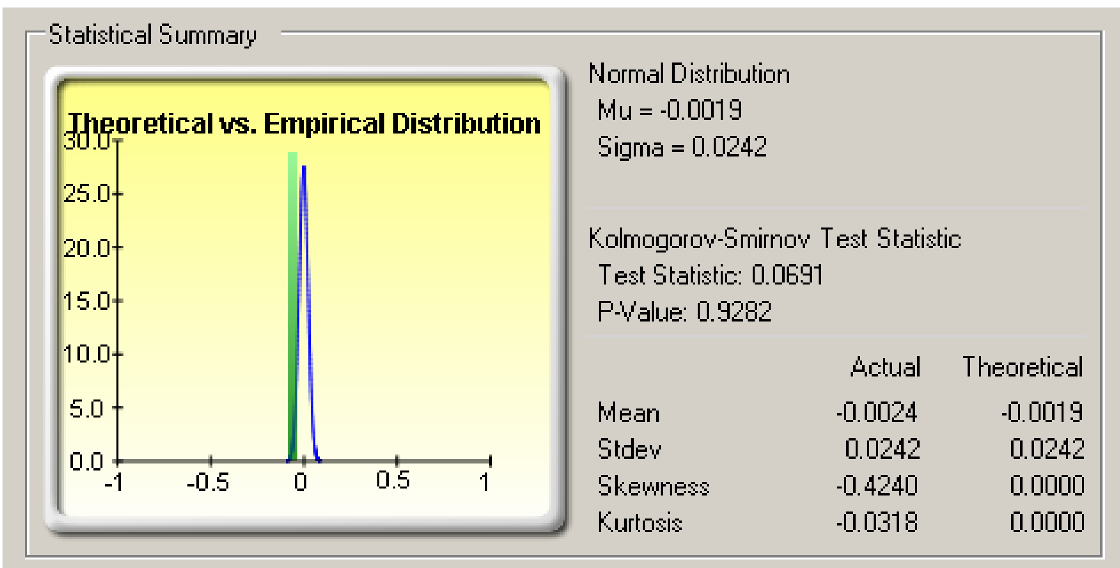

Figure 6-2. Normal distribution fit for historical market rental returns.

Rental rate changes are assumed to follow the GBM movement. A Poisson distribution jump-diffusion process corresponds to the groundbreaking of the neighborhood residential project, with 10% annual probability. The option value is calculated using Equation 4-15 developed in Chapter 4, where λ is 10% and y is 1.2 (1 plus 20%).

Figure 6-3 shows how to get rental rate changes from one period to the next period. At Year 0, gross rental rate is \$24.50/sf. It could have three values in the next year: \$29.40/sf (1.2

times \$24.50/sf) if the neighborhood development breaks ground, \$26.99/sf (up movement) or \$22.24/sf (down movement) if the neighborhood development does not break ground, with probabilities of 0.10, 0.5895, and 0.3105 respectively. In year 2, it could have five values. If the neighborhood development breaks ground in Year 1, the rental rate \$29.40/sf will follow the GBM movement with possible value of \$32.39/sf or \$26.69/sf, with probabilities of 0.7402 and 0.2598 respectively. If no development breaks ground in Year 1, the rental rates of \$26.99/sf and \$22.24/sf each follows the GBM with jump diffusion process and has three values, which combine into 5 possible values. In Year 3 the rental rates follow the same process and can have seven values. Notice, however, the probabilities to get to these values are different with and without the jump diffusion process.

Sigma	9.68%	u	1.1016	λ	10.00%
t	1	d	0.9077	y	1.20
r_f	5.00%	p	0.7402	$\acute{p}$	0.6550
		1-p	0.2598	$(1-\lambda)\acute{p}$	0.5895
				$(1-\lambda)(1-\acute{p})$	0.3105

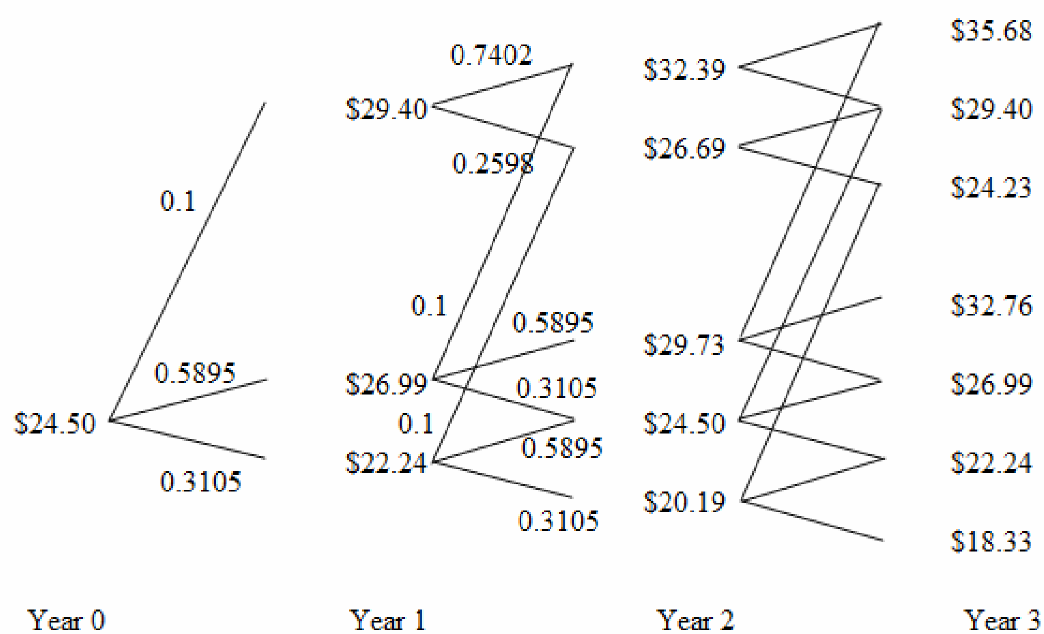

Figure 6-3. Gross rental rate movement and probabilities.

Taking out revenue lost from the 15% vacant space, $8.5/sf operating expense, and capping the net cash flow at 8% Cap rate, we can get the corresponding per square foot building value contingent upon the gross rental rate, stabilized occupancy, operating expenses, cap rate, and the likelihood of the neighborhood residential development (Figure 6-4).

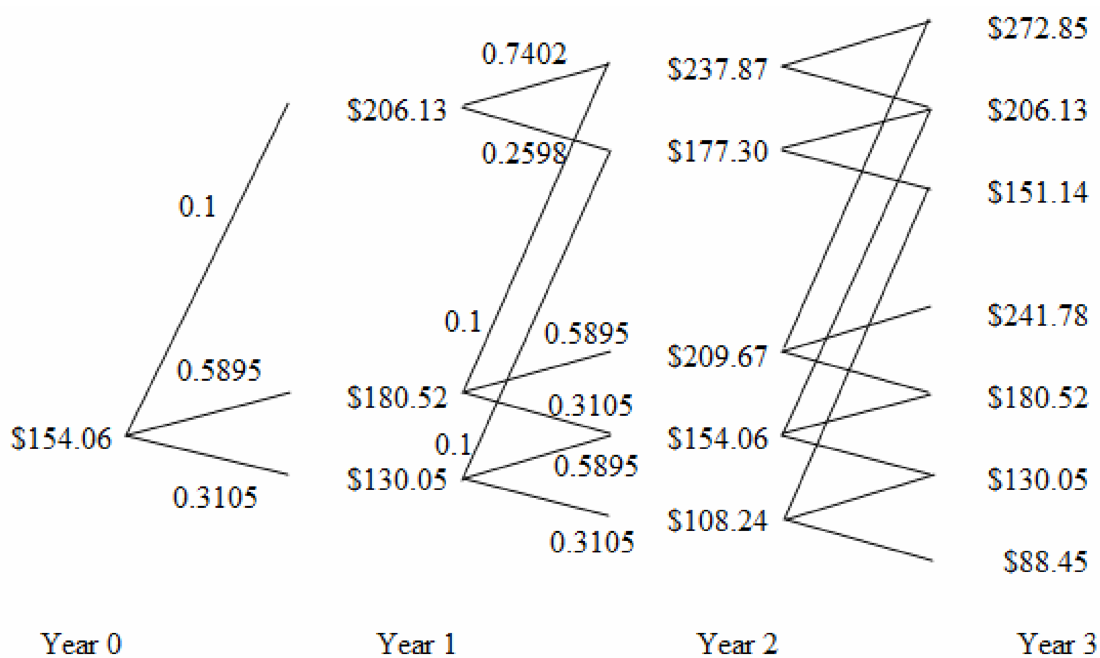

Figure 6-4. Building value movement and probabilities.

There is no direct comparable data on development cost. Development cost includes hard and soft costs. For hard cost, the RS Means Building Cost Data manual (RS Means, 1998-2006) can be used. The cost per square foot data for high-rise office buildings from Year 1998 to Year 2006 is shown in Figure 6-5. The historical data shows an upward trend, at a pace generally consistent with the inflation rate from inflatiodata.com (Figure 6-6). RS Means compiles market average data nation wide, which does not reflect the volatility of local markets. More over, there are no data about the soft cost. Each project is unique in some soft cost items, such as land acquisition cost, permit application cost, unexpected cost, etc. The best estimate would be from experienced managers. The development cost is assumed not to change with the financial market. It is a private risk that depends on the geological condition of the site, material and labor condition of the local market, etc. Management has estimated that with 50% probability the development cost would be \$175/sf, with 20% to be \$150/sf, and with 30% to be \$200/sf, or

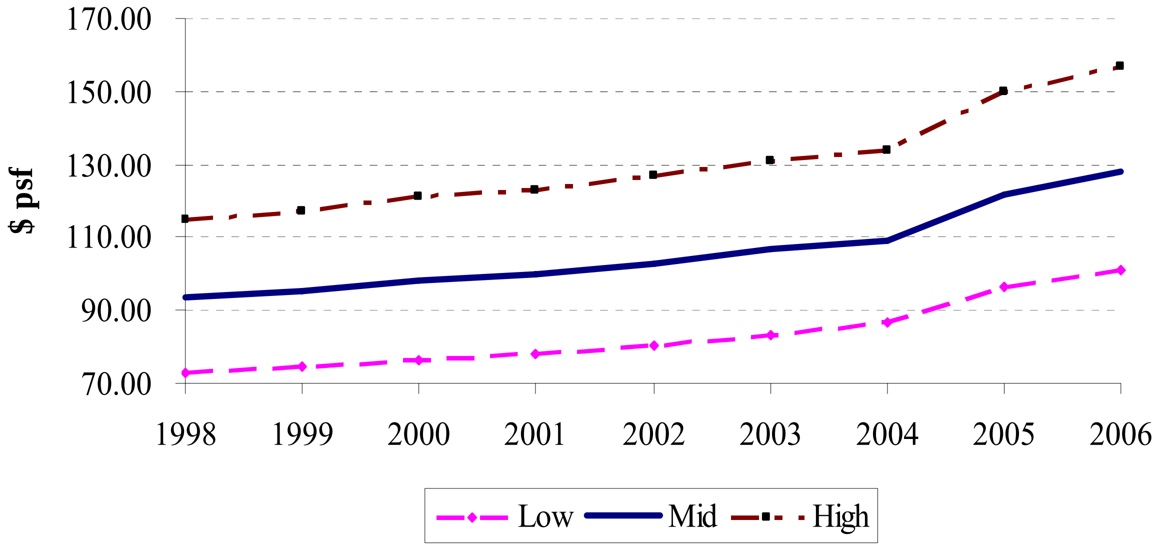

Figure 6-5. Historical construction cost for high-rise office building.

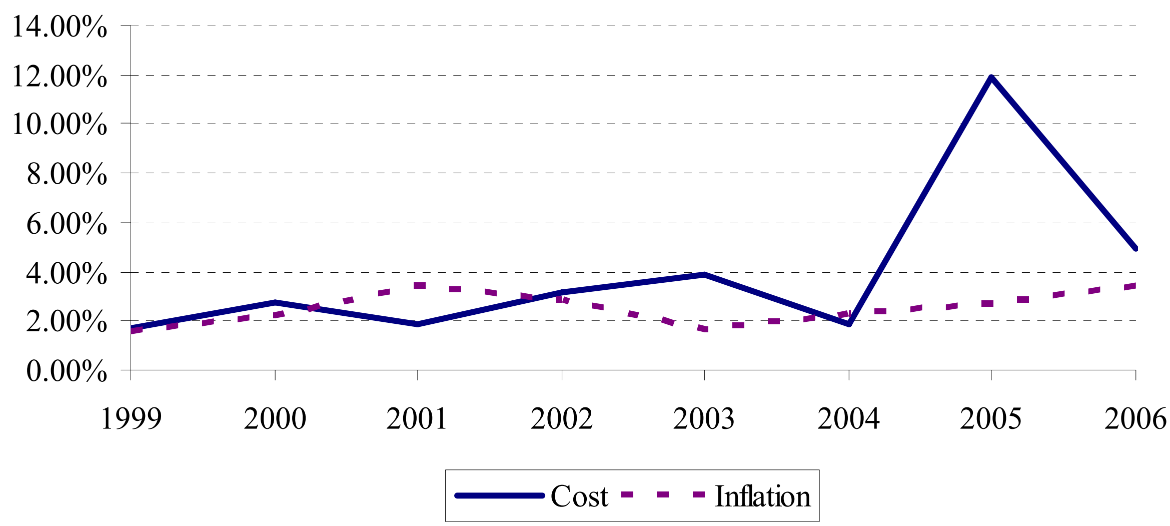

Figure 6-6. Construction cost change rate and inflation rate.

expected cost of $177.50/sf (Figure 6-7). Cost increases by 3% annually, consistent with the average inflation rate over the past 7 years. For simplicity, the buildable square footage is assumed to be the same as the rentable square footage.

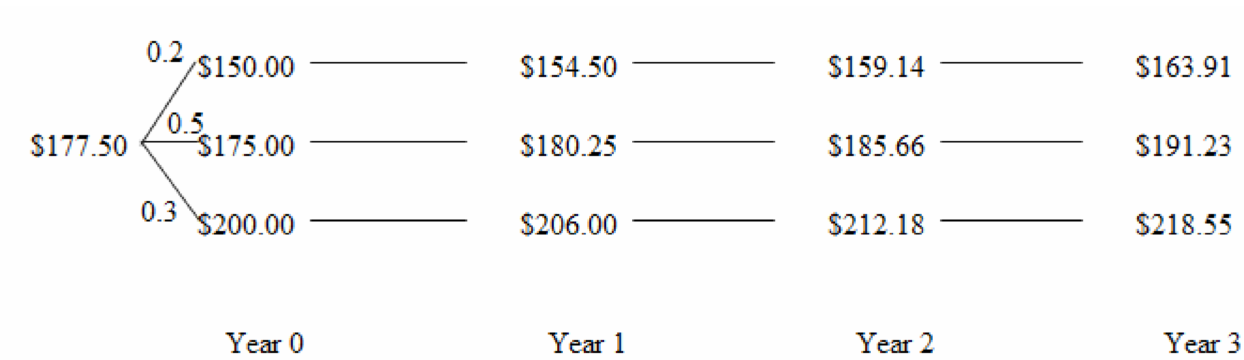

Figure 6-7. Development cost assumptions.

Base Case Modeling

The expected PV without any flexibility is calculated as shown in Figure 6-8. It is better represented in matrices. Each table in Figure 6-9 is a matrix of possible PVs for a given year. Starting from Year 3, the possible outcomes of building values are listed in the first row, and the possible outcomes of development costs are listed in the first column. The values inside the rectangle are all possible combinations of costs and values. The same applies to the values for Year 2, Year 1 and Year 0. In Year 0, the expected value is calculated as the sum of the three values times the respective probabilities of their development cost.

Option Modeling

There are three possible kinds of decisions at each node: (1) to develop the land, (2) to keep the land as-is, and (3) to sell it for $4.75 million. Figure 6-10 depicts the decisions and payoffs corresponding to the matrices in Figure 6-11. In this lattice, the notation below the value represents the optimal decision to be made: D for developing the land; K for keeping the option alive; and S for selling the land.

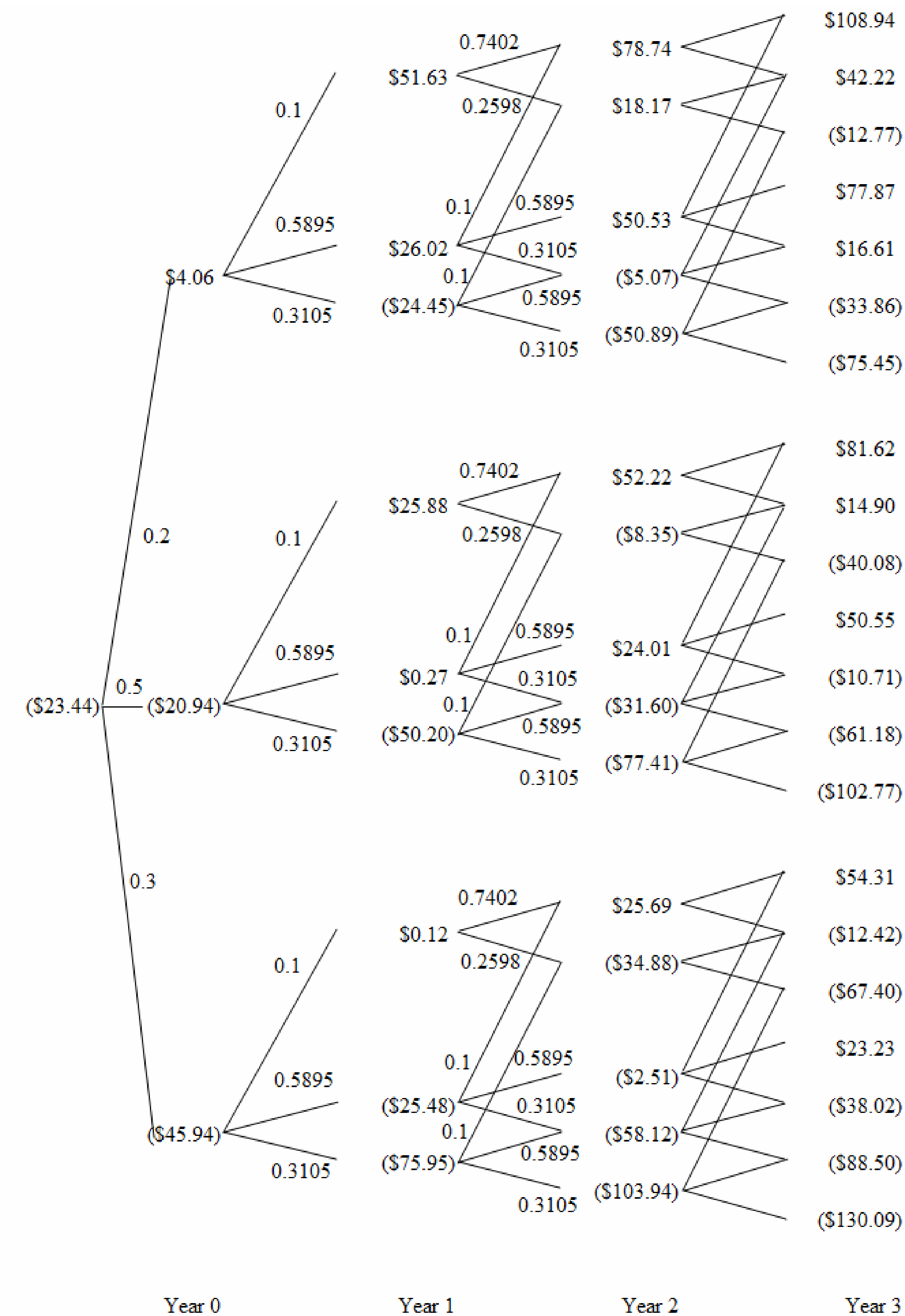

Figure 6-8. Payoff and probabilities without flexibility (Dollars in $1,000,000).

T=3

		Building Value						
		272.85	206.13	151.14	241.78	180.52	130.05	88.45
Development	163.91	108.94	42.22	(12.77)	77.87	16.61	(33.86)	(75.45)
Cost	191.23	81.62	14.90	(40.08)	50.55	(10.71)	(61.18)	(102.77)
	218.55	54.31	(12.42)	(67.40)	23.23	(38.02)	(88.50)	(130.09)

T=2

		Building Value				
		237.87	177.30	209.67	154.06	108.24
Development	159.14	78.74	18.17	50.53	(5.07)	(50.89)
Cost	185.66	52.22	(8.35)	24.01	(31.60)	(77.41)
	212.18	25.69	(34.88)	(2.51)	(58.12)	(103.94)

T=1

		Building Value		
		206.13	180.52	130.05
Development	154.50	51.63	26.02	(24.45)
Cost	180.25	25.88	0.27	(50.20)
	206.00	0.12	(25.48)	(75.95)

T=0

		Building Value	
		154.06	
Development	150.00	4.06	
Cost	175.00	(20.94)	→ (23.44)
	200.00	(45.94)	

Figure 6-9. Payoff matrices for project values without flexibility (Numbers in $1,000,000).

In Year 3, the decision will be either to develop the land or to sell it for $4.75 million, whichever generates the higher payoff. For example, the PV of Node A is calculated as follows:

Max (Develop, Sell) = Max (Building Value – Cost, Salvage Value)
= (206.13 – 163.61, 4.75)
= 42.22 (Develop)

Working backward, in Year 2, the payoff is the greatest of the three: (1) the payoff of developing the land, which is the building value minus development cost; (2) the payoff of keeping the option open, i.e., the corresponding payoff in Year 3 discounted at risk-free interest rate using the binomial or jump diffusion probabilities calculated in Table 6-2; (3) the payoff of

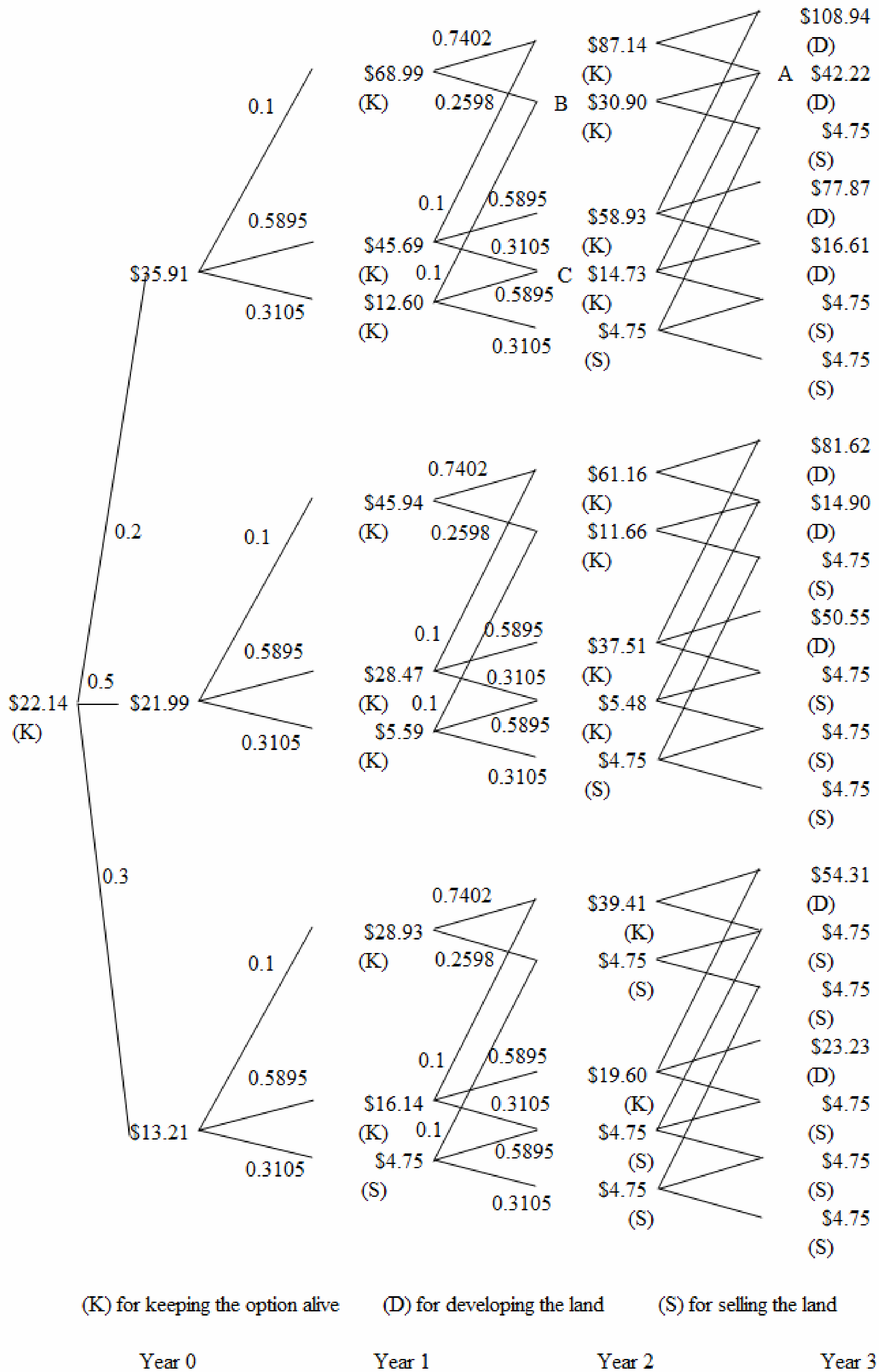

Figure 6-10. Decision payoff and probabilities with flexibility (Dollars in $1,000,000).

T=3

Building Value

		272.85	206.13	151.14	241.78	180.52	130.05	88.45
Development	163.91	108.94	42.22	4.75	77.87	16.61	4.75	4.75
Cost	191.23	81.62	14.90	4.75	50.55	4.75	4.75	4.75
	218.55	54.31	4.75	4.75	23.23	4.75	4.75	4.75

T=2

Building Value

		237.87	177.30	209.67	154.06	108.24
Development	159.14	87.14	30.90	58.93	14.73	4.75
Cost	185.66	61.16	11.66	37.51	5.48	4.75
	212.18	39.41	4.75	19.60	4.75	4.75

T=1

Building Value

		206.13	180.52	130.05
Development	154.50	68.99	45.69	12.60
Cost	180.25	45.94	28.47	5.59
	206.00	28.93	16.14	4.75

T=0

Building Value

		154.06	
Development	150.00	35.91	
Cost	175.00	21.99	→ 22.14
	200.00	13.21	

Figure 6-11. Payoff matrices of project value with flexibility (Numbers in $1,000,000).

Table 6-2. Probabilities of jump diffusion and binomial processes.

Jump diffusion			No jump	
Jump λ	Up $(1-\lambda)\tilde{p}$	Down $(1-\lambda)(1-\tilde{p})$	Up p	Down $1-p$
0.1000	0.5895	0.3105	0.7402	0.2598

the put option, which is to sell the land for $4.75 million. For the normal stochastic process, the payoff of keeping the option open at Node B, for example, is calculated using Equation 3-19 as follows:

$$C = e^{-rt}[pC_u + (1-p)C_d] = e^{-0.05\times 1}[0.7402\times 42.22 + 0.2598\times 4.75] = 30.90$$

Consequently, the PV of Node B is calculated as follow.

Max (Develop, Keep, Sell)
= Max (Building Value – Cost, Payoff of Keeping Option Open, Salvage Value)
= (177.30 – 159.14, 30.90, 4.75)
= 30.90 (Keep)

For the jump diffusion, the payoff of keeping the option open at Node C, for example, is calculated using Equation 4-15 as follows:

$$C = e^{-rt}\{\lambda C_y + (1-\lambda)[\tilde{p}C_u + (1-\tilde{p})C_d]\}$$
$$= e^{-0.05\times 1}\{0.1\times 42.22 + 0.5895\times 16.61 + 0.3105\times 4.75\} = 14.73$$

Consequently, the PV of Node C is calculated as follows:

Max (Develop, Keep, Sell)
= Max (Building Value – Cost, Payoff of Keeping Option Open, Salvage Value)
= (154.06 – 159.14, 14.73, 4.75)
= 14.73 (Keep)

Working backward to Year 0, the PV of the project is expected PV of each cost scenario times its corresponding probability. The PV of Node D is calculated using Equation 4-17 as follows:

$$E(PV_0) = \sum_{j=1}^{m} p_j[E(PV_j)] = 0.2\times 35.91 + 0.5\times 21.99 + 0.3\times 13.21 = 22.14$$

The PV of the project increases from negative \$23.44 million without flexibility to positive \$22.14 million with the development and sell-off flexibility. The option value is $\$22.14 - (-\$23.44) = \$45.57$ million.

Sensitivity Analyses

Sensitivity analyses are conducted using gross rental rates, occupancy rates, volatility, Cap rates, and development cost as independent variables, and on two dependent variables: project value and option value. Project value is the PV of the project with the flexibility of deferred development, spin-off the land, and immediate development. Option value is the difference between PV with flexibility and PV without flexibility. Since the PV without flexibility also changes with variables, the project value and option value analyses have quite different results and implications.

As shown in Figure 6-12 the rental rate has a positive effect on project value. Rental rate is directly linked to revenue. The higher the rental rate is, the higher the income the project will generate, and hence the higher the project value is. However, as shown in Figure 6-13 it has a negative effect on option value. This is because the higher the rental rate is, the more likely the project will be developed immediately, hence the option to wait or abandon the development by selling off the land is less worthy. In other words, higher rental rate not only increases the project value with flexibility, it also increases the value without flexibility at even higher pace. These two values cancel out each other, resulting in minimal option value.

The combination of rental rate and occupancy rate has the same result: positive effect on the project value (Figure 6-12), and negative effect on the option value (Figure 6-13). Note that the option value is sensitive to stabilized occupancy rate when the option is at-the-money.

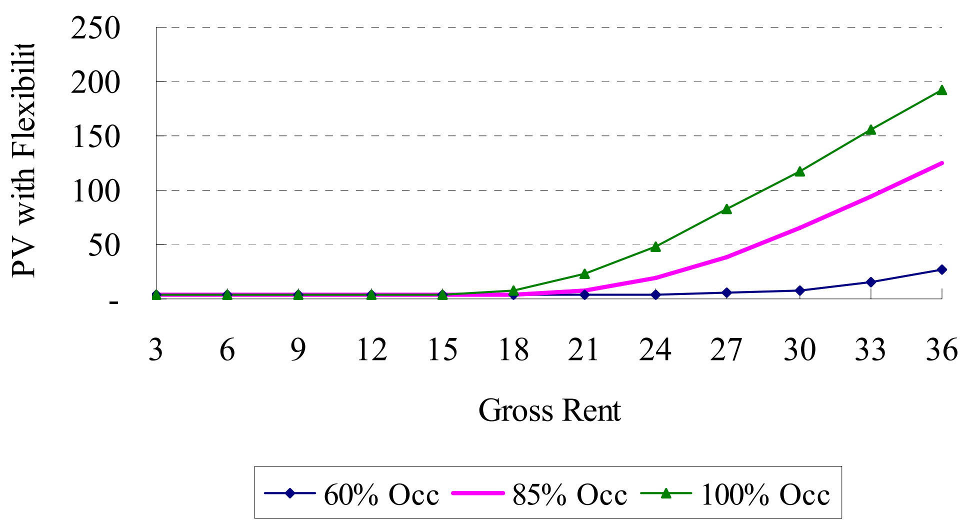

Figure 6-12. Present value in relation with rental rate and occupancy rate.

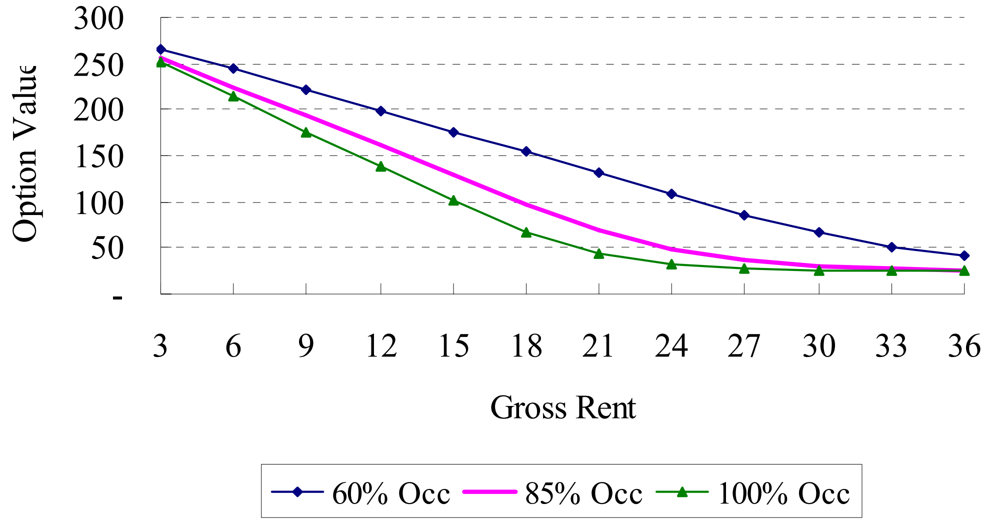

Figure 6-13. Option value in relation with rental rate and occupancy rate.

Just opposite to the effect of rental rate, as shown in Figure 6-14, development cost has a negative effect on project value, but positive effect on option value (Figure 6-15), for the same reason as explained above.

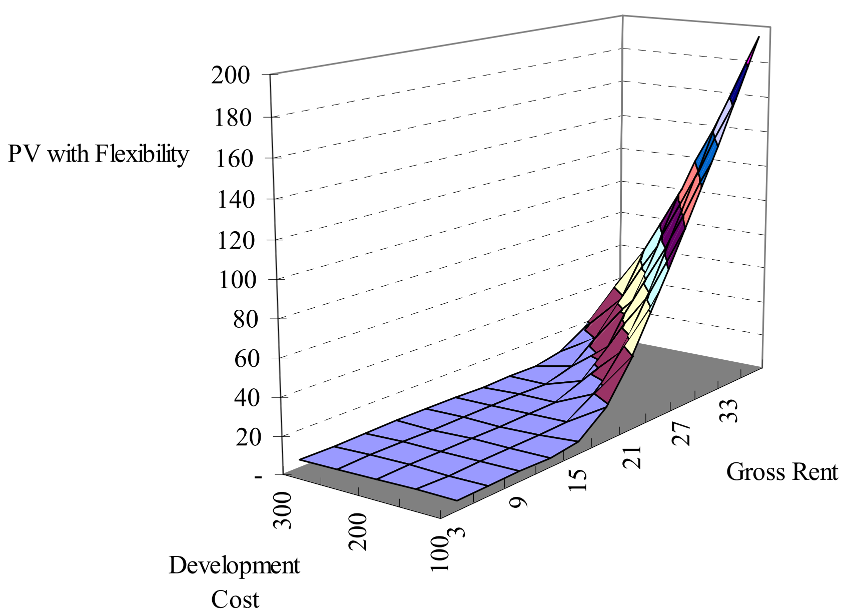

Figure 6-14. Present value in relation with rental rate and development cost.

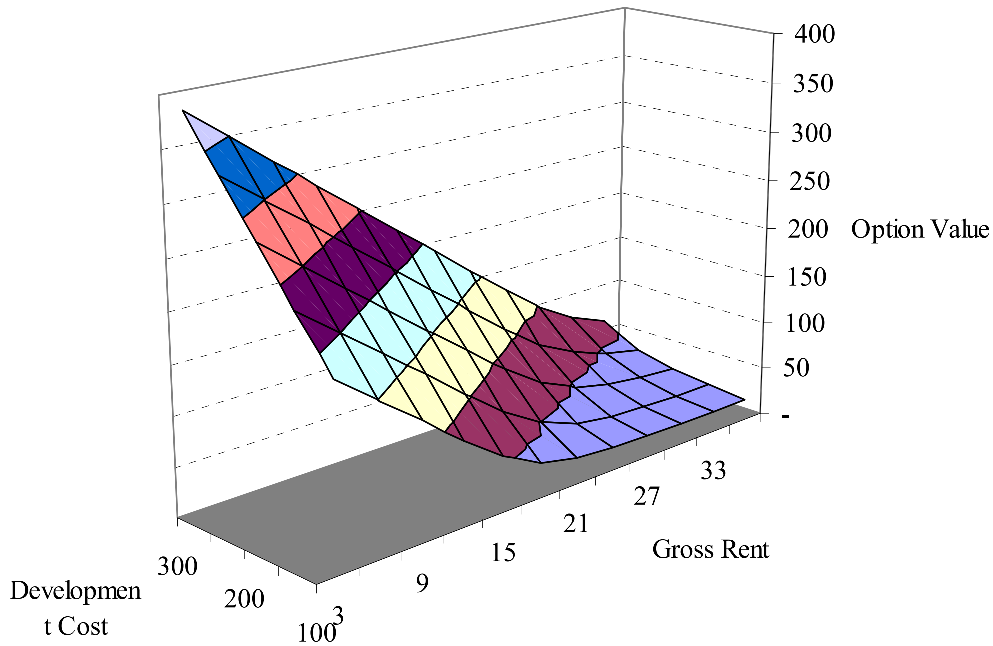

Figure 6-15. Option value in relation with rental rate and development cost.

As shown in Figure 6-16, cap rate has negative effect on project value. This is because cap rate is inversely related to property value. (Property value is determined by dividing net operating income by cap rate.) However, the effect of cap rate on option value is more profound. Figure 6-17 shows that at normal rental rate range (\$11/sf to \$31/sf), cap rate has a positive impact on the option value; however, in the low rental rate range (\$0/sf to \$11/sf), its impact is the opposite. Figure 6-18 illustrates how the combination of rental rate and cap rate results in different option value. Unlike most situations where a variable has monotonic impact on the option value, the shape of cap rate on option value is convex. For example, at \$20/sf gross rent, the option value at 2% cap rate is \$71 million, at 4% cap rate the option value drops to \$47 million, and at 8% cap rate the option value comes back to \$77 million.

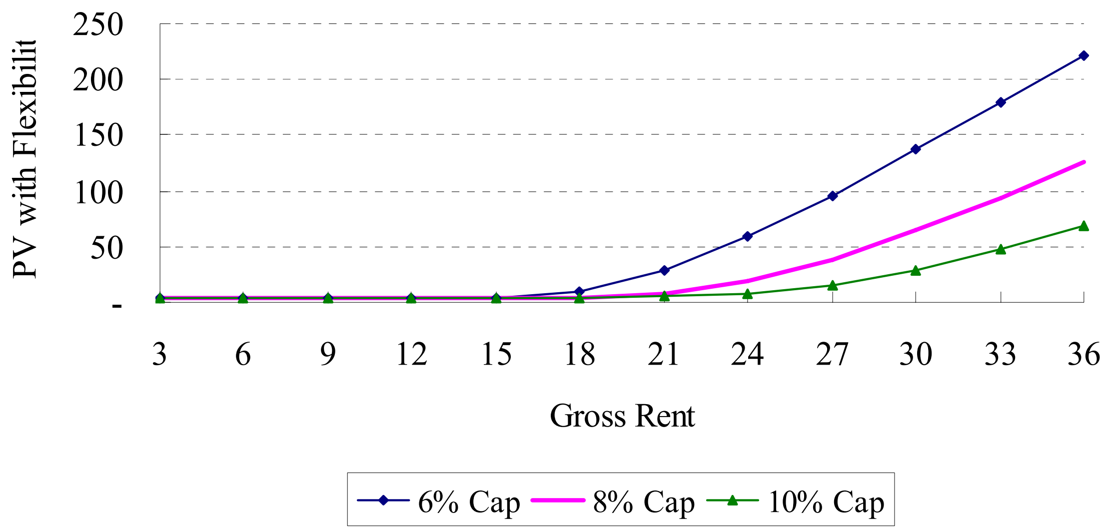

Figure 6-16. Present value in relation with rental rate and Cap rate.

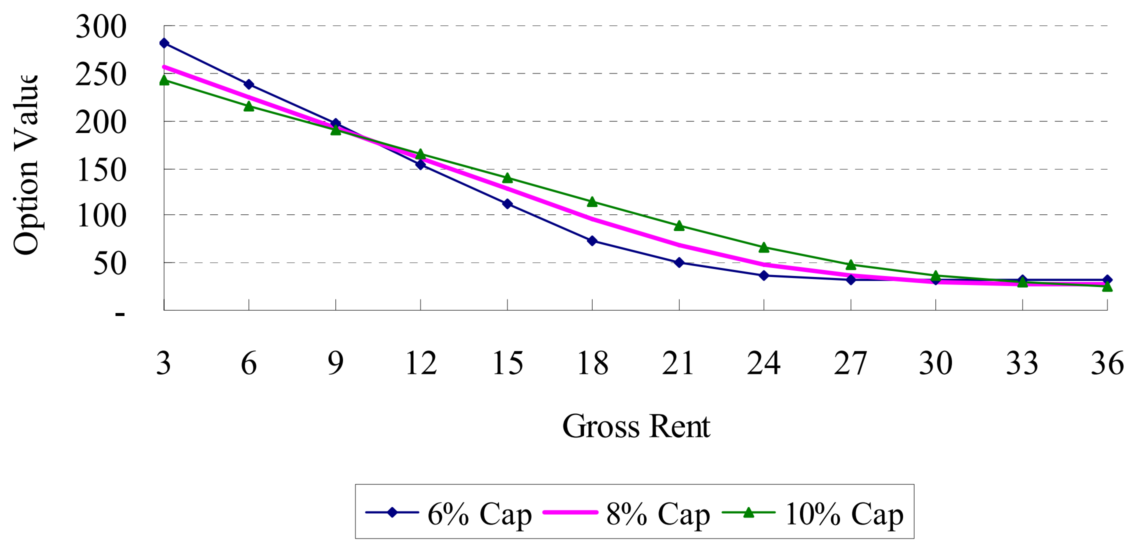

Figure 6-17. Option value in relation with rental rate and Cap rate.

As shown in Figures 6-19 and 6-20 volatility has positive impact on both project value and option value. This finding is consistent with many observations in real options research (Titman, 1985; Williams, 1991; Quigg, 1993) that greater volatility increases option value, which is also the reason why the real options methodology should be applied to projects with high uncertainty.

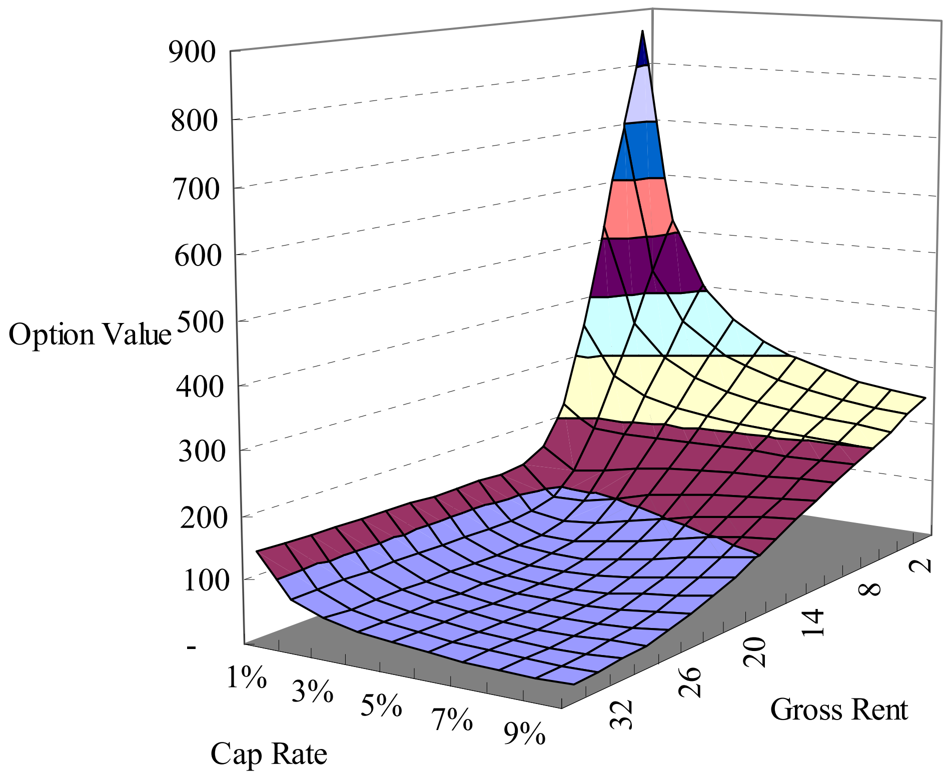

Figure 6-18. Option value in relation with rental rate and Cap rate in 3D.

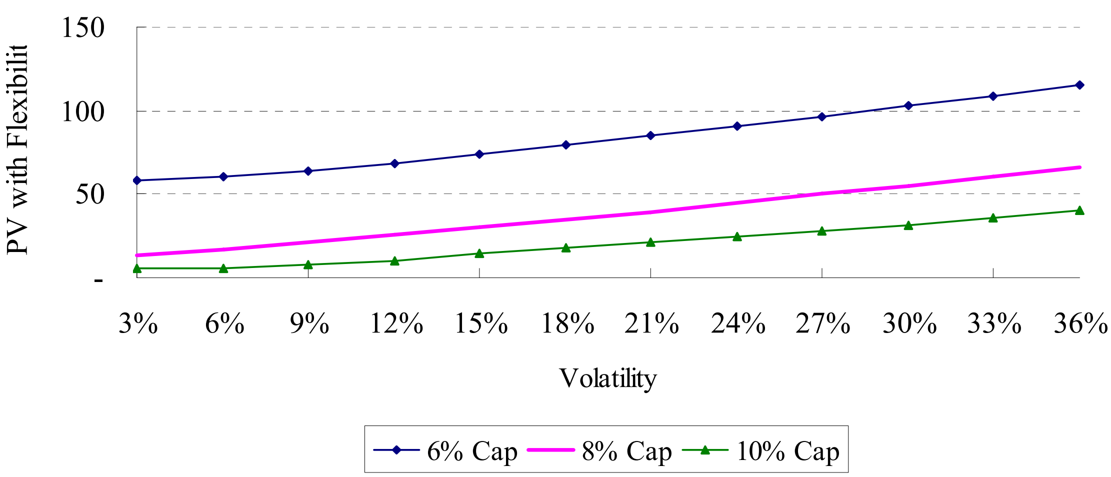

Figure 6-19. Present value in relation with volatility and Cap rate.

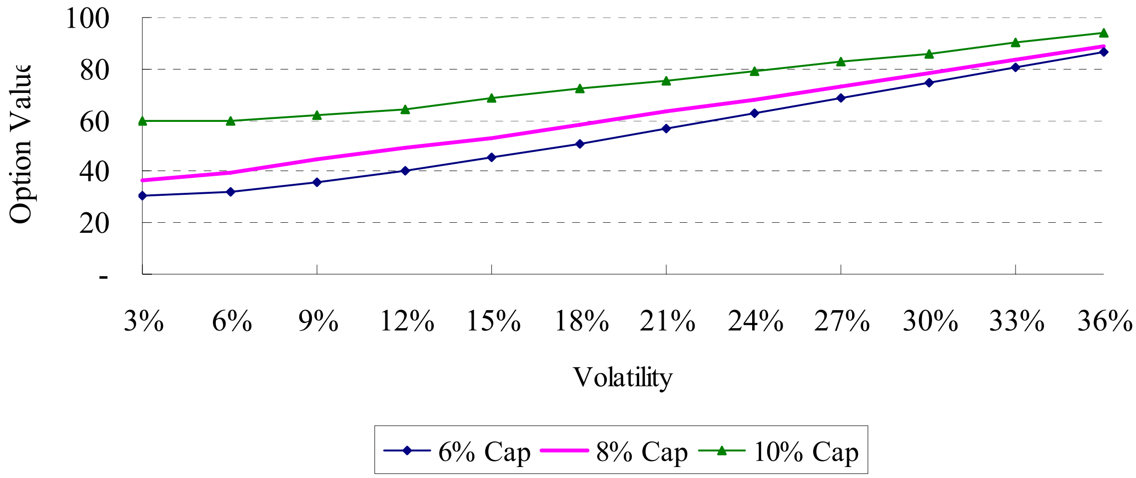

Figure 6-20. Option value in relation with volatility and Cap rate.

Summary

This chapter applies the separated approach to value the infill land of the 211 Perimeter property in Atlanta. Rental rate and development cost are identified as the two major risk drivers. Rental rate is assumed to have jump diffusion effect due to the uncertainty of the larger neighborhood redevelopment project. Development cost is assumed to be a private risk with no corresponding traded twin asset and it is estimated subjectively based on management's experience. DTA methodology is applied and an event tree is constructed, in which three options are incorporated: the option to develop immediately, the option to delay development, and the option to sell the land. The RERO approach indicates that the land is worth $22,140,000 and the value of managerial flexibility is worth $45,570,000.

In Chapter 5, the building is estimated to be worth $25 million; in this chapter, the land is estimated to be worth $22 million, totaling $47 million. This is very close to reality, because the property was actually sold for $43.5 million in 2005.

CHAPTER 7
CONCLUSIONS AND RECOMMENDATIONS

Conclusions

Three main conclusions are drawn from this research: (1) acquisition and development has different characteristics and deserve different kinds of attention; (2) consideration of managerial flexibility can change investment decisions; and (3) many unconventional real option valuation problems can be realized by using binomial lattice and Monte Carlo simulations.

Acquisition and development have different characteristics and thus deserve different kinds of valuation. The option value of acquisition is usually on a much lower scale than that of development, but by no means is it less significant. In the case studies, the option in the existing building is replacement of the chiller system. Its value is $496,000, or 52% of the replacement cost of $950,000. On the other hand, the option on the infill land is development timing and abandonment. The option value is as high as $45.65 million, but only 26% of the development cost of $177.5 million. Due to the scale of the valuations, it is better to have the option in the building and the options in the land valued separately. But the impact of management flexibility on acquisition and operation is as significant as, if not more than, that on development.

The consideration of operating flexibility in acquisition is important. It adds competitive value to the bid for a property. In the case studies, the building is worth $25 million, and the land is worth $22 million, totaling $47 million. In other words, the infill land is worth almost as much as the building. This is very close to reality, because the property was actually sold for $43.5 million. Note that the present value of the development project without any flexibility is negative $23 million. With negative NPV, the project will not break ground. This means if management does not incorporate the flexibilities into the land valuation, the development is deemed worthless, and so is the land.

The RERO framework explores a few unconventional real option cases, including (1) jump diffusion process that does not go back to normal diffusion, (2) risk drivers that do not follow the multiplicative stochastic movement, (3) private risk that has no market equivalent and hence violating the no-arbitrage option pricing assumption. All of these can be implemented through a binomial lattice with Monte Carlo simulations or the DTA approach. The RERO framework is a simple yet powerful tool, intuitive to the practitioners, yet mathematically correct and precise.

Recommendations for Future Research

There are at least three directions that future research can go in: model perfection, game theory and phase investment. Model perfection is to improve the preciseness of outcome from the RERO models. Lattice is a discrete-time method for option pricing. The smaller the time step, the closer the result will be to that calculated by continuous-time methods. At the same time, the development cost is assumed to have three values in our case study: the optimistic value, the most likely value, and the pessimistic value. More branches can be added to produce a more precise result. By dividing the lattice into more time steps, and breaking the development cost into more branches, a more precise result will be generated.

A significant factor not considered in this study is competition. Without the consideration of competition, in most cases it is optimal to defer exercising an option until the end of the holding period. However, competition erodes the value of waiting, affects the value of option as competitors enter or exit the market place and changes the market dynamics (Williams 1993; Myerson, 1991). Should game theory be incorporated into the RERO framework, we predict the option value would be slightly lower, and hence even closer to the closing price.

The other direction is stage investment and phased investment. Real estate development is a lengthy process, and it usually takes 2 to 3 years, if not longer. During this period, a lot of uncertainties can change the managerial strategies. Stage investment refers to dividing a real estate development project into different stages: planning, design, construction, sales, etc. This process can be valued similar to pharmaceutical research and development. Phased investment refers to dividing a large real estate development project into different phases, for example, Phase I retail corridor, Phase II residential condominium, Phase III office and hotel towers, etc. Decisions at later phases are contingent upon the outcome of earlier phases. However, there can be timing overlaps between two phases. While this problem is best solved by decision tree analysis, the combination of real options and decision analysis could be beneficial.

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BIOGRAPHICAL SKETCH

Nga-Na Leung earned her PhD degree in building construction from the University of Florida, Gainesville, FL. While earning this degree, she worked as an acquisition analyst for Parmenter Realty Partners in Miami, FL later for Acadia Realty Trust in White Plains, NY, and now for Antares Investment Partners in Greenwich, CT.

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Nga-Na worked as an assistant project manager in the Environetics Design Group in Shanghai, China prior to coming to the US. At UF, she was supported by the Alumni Fellowship, the highest merit-based award for graduate students. After graduation Nga-Na will continue her career in commercial real estate investment, including acquisitions, development, and management.