DEMAND MANAGEMENT MODELS FOR TWO-ECHELON SUPPLY CHAINS

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In the majority of classical supply chain management and inventory theory literature, demand arises from exogenous sources upon which the firm has little or no control. In many practical contexts, however, enterprises have numerous tools to generate, shape and manipulate the demand they face. This phenomenon introduces a new dimension to supply chain planning problems; the interaction and coordination between the demand and supply side of the supply chain. In our study, we introduce and study demand management tools that are integrated into classical supply chain planning models.

We present a nonlinear, combinatorial optimization model to address planning decisions in both deterministic and stochastic supply chain settings, where a firm constructs a demand portfolio from a set of potential markets having price-sensitive demands. We first consider a pricing strategy that dictates a single price throughout all markets and provide an efficient algorithm for maximizing total profit. We also analyze the model under a market-specific pricing policy and describe its optimal solution. An extensive computational study characterizes the effects of key system parameters on the optimal value of expected profit, and provides some interesting insights on how a given market's characteristics can affect optimal pricing decisions in other markets.

We analyze the implications of order timing decisions in multi-retailer supply systems in a single period, newsvendor setting. Specifically, we investigate a supply chain with multiple retailers and a single supplier where one of the retailers is considered a preferred

or primary customer of the supplier. We compare the expected supplier and retailer profits under two order commitment strategies and specify conditions under which a particular commitment scheme benefits the supplier, the primary retailer, and the entire system. We compare our findings to a single-retailer system, and investigate the effects of capacitated supply. Observing that the outcome of the strategic interaction between the supplier and his primary customer is not in alignment with the supplier's preference, we propose and evaluate a number of demand management tools that the supplier can utilize in order to achieve his desired order commitment scheme. These tools include a capacity limit on the production quantity of the supplier, reallocation of the leftovers to the primary customer after demand realizations, and offering a discounted wholesale price. We also perform a comparative analysis of these tools and assess their effectiveness under various settings through a computational study.

CHAPTER 1
INTRODUCTION

Traditional supply chain management models have assumed that demand is an exogenous process, and it serves as an input to the supply chain. Given the demand characteristics, these models aim to align the supply chains such that demand is satisfied in the most efficient and effective way possible. The classical definition of supply chain management also emphasizes this point (Simchi-Levi(1999)):

> Supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that the merchandise is produced, and distributed at the right quantities, to the right locations, and at the right time, in order to minimize the systemwide costs while satisfying service level requirements.

In many practical contexts on the other hand, demand is not truly exogenous as suppliers are equipped with tools to influence and shape certain characteristics of demand. Such tools include, but are not limited to, pricing, promotions, discounts, product mix, shelf management and lead time. Demand management involves carefully selecting among these tools and working closely with customers so that the overall incoming demand for the enterprise and the supply chain will give rise to maximum values for the parties involved (Lee (2001)). This requires balancing the customer requirements with the firm's supply capabilities, and entails attempting to determine what and when the customers will purchase. The necessity of incorporating demand management into traditional supply chain management practices is becoming more evident among practitioners. For instance, the Council of Supply Chain Management Professionals provides the following definition:

> Supply Chain Management encompasses the planning and management of all activities in sourcing and procurement, conversion, and all logistics management activities. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third-party service providers, and customers. In essence, supply chain management integrates supply and demand management within and across companies.

Demand Management \ Traditional SCM	*No*	*Yes*
Yes	Mismatch of supply & demand, wrong demand signals, poor product/capacity plan	Optimal Results
No	Total Chaos	Costly promotion, inefficient sales deals, overheated supply chain, lost margins

Figure 1-1 Integrating demand and supply chain management (Lee (2001))

The failure to integrate demand management into supply chain management may result in serious inefficiencies for the supply chain members (see Figure 1-1). Lee (2001) discusses several industry practices that exemplify these inefficiencies. For instance, Volvo suffered from lack of proper communication and coordination between its supply chain planning and marketing groups in the mid-90's. In order to cut the excessive inventory of green cars, the marketing department offered deep discounts and aggressive deals to the distributors. However, the increased sales were interpreted by the supply chain group as the late success of the green car, and they produced even more, resulting in a larger inventory.

Failing to consider the operational implications of the demand management tools on the supply chain may also result in net losses for the company, although it may boost sales. Campbell Soup's chicken noodle soup experience is such an example (Clark and McKenney (1994)). Campbell promoted the product excessively around the winter season, when demand already peaked. With the further increased demand, the company had to prepare for the season in advance, which resulted in excessive storage in the spring. Production capacity was allocated to this product during the winter, which required other products to be manufactured in advance resulting in higher inventory and storage needs.

As a result, the increase in revenue due to increased sales was outweighed by the increase in operational costs.

In our study, we consider two structurally different problem settings, where we integrate classical supply chain management concepts with different demand management tools. In the first problem, we incorporate a market selection concept and pricing into an optimization model which generalizes well-known inventory models in the literature. The second model also applies to settings with multiple demand sources, but the demand management tools we offer differ. We consider order timing as the main instrument to control demand, and introduce capacity and inventory/pricing related measures to align the objectives of the supply chain members into a better managed demand.

The first part of our study deals with a supplier who faces price-sensitive demand streams from a number of sources and faces the problem of selecting the optimal subset of demand sources together with a selling price in each selected demand source. The demand streams may represent different customer orders, markets, downstream members of the supply chain, or segments of a single market, for example. Two different pricing strategies are evaluated in terms of the expected profit and market selection decisions. We introduce a general optimization model representing this problem, which generalizes well-known supply chain management problems under certain assumptions. After providing efficient solution algorithms, we investigate the effects of different system parameters on the pricing and market selection decisions of the firm under different inventory models. To our knowledge, this is the first model in the literature that incorporates pricing decision into a demand portfolio selection problem.

The second setting considers a stochastic supply chain model that consists of a supplier and multiple buyers. The buyers face stochastic demand and have a one time opportunity to order from the supplier. In this model, rather than focusing on market selection or pricing, we assume that the supplier accepts all retailer demands, and we investigate the effects of order timing decisions of the retailers. The order timing arrangement between the supplier and the retailers is another integrated form of demand and supply management as it not only affects the characteristics of the demand that the supplier faces but shapes the capacity restrictions that the retailers may encounter. In particular, we have observed examples in industry where the supplier is concerned about the reluctance of its major customers to commit to an order before the demand realization. We analyze the driving forces behind the commitment preference of the retailers and the preference of the supply chain members. We also identify the risks associated with different order timing decisions and how these risks affect the decision making procedures in the supply chain. As our findings support the conflicting preferences of the supply chain members in terms of order timing, we next focus on demand management tools that the supplier can utilize in order to have its main customers choose the supplier's desired commitment scheme.

CHAPTER 2
LITERATURE REVIEW

2.1 Pricing and Market Selection

The Economic Order Quantity (EOQ) and Newsvendor models are regarded as the building blocks of deterministic and stochastic inventory theory, respectively. Despite their restrictive assumptions, these models have been extensively utilized in practice due to their simple structures and robust performance. An important extension to such classical inventory control problems permits a price-dependent demand process.

The EOQ model, which was introduced by Harris (1913), considers an infinite horizon, continuous inventory model with a constant, continuous and deterministic demand rate. Whitin (1955) incorporates pricing into the classical EOQ model, which is later explicitly solved by Porteus (1985). Abad (1988) considers a similar problem with a more general demand function where the supplier offers an all-unit quantity discount. Abad (1996) studies perishable items and partial backordering in a similar setting. There are also studies that concentrate on similar models in multi-echelon settings. Weng (1995) considers a system consisting of a supplier and a group of homogeneous buyers, and analyzes the effectiveness of quantity discount in achieving channel coordination. Viswanathan and Wang (2003) model a single-retailer, single-supplier channel, where the retailer faces price-sensitive demand. They evaluate the effectiveness of quantity and volume discounts in terms of channel coordination. Lau and Lau (2003) investigate a joint-pricing model in the absence of setup costs, and consider different forms of demand curves. They conclude that even a small change in the appearance of the demand curve can cause a significant change in the optimal solution in a multi-echelon system. Ray et al. (2005) consider two pricing approaches (price as a decision variable and mark-up pricing) and concentrate on identifying managerial insights regarding the behavior of the optimal decisions.

The classical Newsvendor problem considers a single period model with stochastic demand. It dates back to Edgeworth (1888), who applied a variant to a financial setting.

The reader may refer to Khouja (1999) for a thorough review of the literature on this problem and its extensions. In Newsvendor models with pricing, Demand randomness is usually modelled either in an additive ($D(p) = q(p) + X$) or a multiplicative ($D(p) = q(p)X$) fashion, where $q(p)$ is a nonincreasing function of price and X is a random factor. Young (1978) proposes a demand model that handles both cases, and investigates the behavior of the optimal decision with respect to uncertainty. Petruzzi and Dada (1999) analyze both cases in detail and highlight the structural differences between these models.

Whitin (1955) appears to be the first study that links the newsvendor problem with pricing decisions. He assumes that expected demand is price-dependent, and provides necessary conditions for optimality using incremental analysis. Mills (1959) also assumes that demand is a random variable with an expected value that is decreasing in price, where randomness is modelled in an additive fashion with a constant variance. He shows that the optimal price is less than or equal to the optimal price under deterministic demand. Karlin and Carr (1962) consider multiplicative randomness, and show that the optimal price is greater than or equal to the optimal price under deterministic demand, which is the opposite of the additive demand case. Lau and Lau (1988) consider two different approaches to model demand randomness: (i) a simple homoscedastic regression model, $d(p) = a - bp + X$, and (ii) a demand distribution that is constructed using a combination of statistical data analysis and experts' subjective estimates. Polatoglu (1991) emphasizes the limitations of both the additive and multiplicative models, and formulates the problem with a general demand distribution to characterize the properties of the model, independent of the way randomness is handled. Instead of making specific assumptions about the demand distribution, Raz and Porteus (2006) assume that demand is a discrete random variable that depends on the price.

There are also studies in the literature that allow the firm to adjust price during the selling season, which leads to dynamic programming formulations. Khouja (2000) allows discounts on the initial price of the product in order to sell excess inventory remaining at the end of the period, where discounts are equally spaced and each discount has a fixed cost. In Petruzzi and Dada (2002), demand is assumed to be deterministic but unknown to the firm. Learning occurs as the firm observes the market's response to its decisions. The selling season is divided into multiple periods and the demand function is updated in each period. Monahan et al. (2004) analyze a multi-period problem, where demand in each period is modelled in a multiplicative way. They develop structural properties of the optimal pricing strategy and establish an efficient algorithm for computing optimal prices. The pricing models in the literature consist of far more than the studies that we have discussed. Bitran and Caldentey (2003) provide an excellent review of this area in a generic revenue management context. The reader may also refer to Chan et al. (2004) for a detailed review of the studies on the coordination of pricing and inventory decisions.

Note that all of these models consider a single market, i.e., the firm faces a single stream of demand. Eppen (1979) provides an early study considering a multi-location newsvendor problem where demands are normally distributed. Chen and Lin (1989) introduce general distributions for the demands and concave holding and shortage costs. Chang and Lin (1991) extend this work by considering transportation costs. Cherikh (2000) and Lin et al. (2001) employ a profit maximization perspective. Chen et al. (2001a) and (2001b) appear to be among the earliest studies that consider pricing and inventory control in a multilocation setting. They assume that demands occur at a constant deterministic rate that depends on the price and consider 'location specific pricing'. Federgruen and Heching (2002) consider a periodic review, stochastic model with multiple retailers where demand at a given retailer is price-sensitive. They consider a single-price strategy in a given period and develop an approximate model that is tractable. None of these models, however, consider market selection together with pricing, and they thus assume that the firm must satisfy all markets (or retailers).

In market selection problems, on the other hand, the firm has the flexibility to select the demands it will serve. Geunes et al. (2004) generalize the classical EOQ and EPQ models to address economic ordering decisions when a producer can choose whether to satisfy multiple markets. Geunes et al. (2006) consider order selection in a multi-period lot-sizing context with pricing decisions. They also incorporate limited capacity in a requirements planning model with order selection or pricing. Carr and Lovejoy (2000) introduce the 'inverse newsvendor problem' where the firm chooses an optimal demand portfolio with a fixed but uncertain capacity. The only study in the literature that considers market selection with demands dependent on endogenous variables is introduced by Taaffe et al. (2006). They present the 'selective newsvendor problem' (SNP), which addresses integrated market selection and ordering decisions where demand in each market is normally distributed and dependent on the marketing effort exerted. However, they only consider special functional forms of the relationship between the demand and marketing effort. These functional forms allow the market selection and marketing effort decisions to be separable. In our case, on the other hand, we allow general forms of revenue and cost functions, and market selection and pricing decisions are not separable. Furthermore, Taaffe et al. (2006) do not consider a setting where the endogenous variable is restricted to take the same value in all markets, which would be of little value in the marketing effort context.

Segments of the economics and marketing literature are also interested in similar market selection problems. However, their approach is quite different from ours. The analytical models presented in those studies usually employ game theoretic analysis to model market entry decisions of competing firms as discussed in Rhim et al. (2003). Moreover, they do not explicitly consider procurement quantity decisions and the operations-related costs that drive these decisions.

The fundamental difference between our model and existing market selection models is that we introduce a general optimization model and an efficient solution approach that not only incorporate endogenous pricing decisions but also apply to both deterministic and stochastic settings under certain assumptions. Hence, the firm must determine an optimal price (in each market or a single price for all markets), together with the market selection and inventory control policy parameter decisions. Based on the characteristics of the markets and the supplier's policies, the firm may choose to set a single price throughout all selected markets or an individual price for each selected market. To our knowledge, this is the first study that analyzes single-price and market-specific pricing schemes in a market selection context involving operations-related costs.

A stream of economics literature exists that deals with pricing decisions for a multi-market monopoly, which is referred to as 'third degree discrimination'. Third degree price discrimination can be broadly defined as 'charging different consumers (markets) different prices for the same good' (Armstrong (2007)), and has been studied since the 1920s (see Pigou (1920) and Robinson (1933) for the earliest discussions). This stream of literature also investigates the differences between third degree price discrimination, which corresponds to our 'market-specific pricing strategy', and 'uniform pricing', which corresponds to our 'single-price strategy', in terms of output and welfare implications (see e.g. Battalio and Ekelund (1972), Schmalansee (1981) and Varian (1985)). The reader may refer to Phlips (1988) and Armstrong (2007) for detailed reviews of studies on price discrimination. Our study differs from this stream in important ways. First of all, we consider general forms of revenue and cost functions, that in certain settings correspond to different supply chain management problems. This link has never been established in the economics literature. We also model market selection decisions explicitly. Although there are studies in the economics literature that recognize the fact that single-price strategy may exclude some of the markets (see e.g. Battalio and Ekelund (1972) and Layson (1994)), these studies consider a setting with only two markets and assume linear costs. Furthermore, the price variable dictates the market selection decisions in these studies. That is, a market is considered not served only if its demand at the optimal price level is zero. Our study, on the other hand, not only recognizes this phenomenon, but allows the firm to not serve a market that would have positive demand at the uniform price level offered in other markets. Furthermore, we provide an algorithm to solve this more general problem efficiently, which is not addressed in the economics literature.

2.2 Order Timing Tradeoffs

In the second part of our study, we focus on the order timing tradeoffs in a single-supplier, multiple-retailer supply chain (see Chapter 4), and analyze a number of demand management tools that the supplier can utilize in order to induce his preferred order timing scheme (see Chapter 5). In this section, we provide a review of the studies in the literature that examine similar settings.

Key elements governing supplier-buyer relations in a supply chain are the supply contracts that specify the conditions and parameters of the transactions within the supply chain. Lariviere (1999) and Tsay et al. (1999) provide excellent reviews of the literature on supply chain contracts. For a review of the literature on coordinating contracts, the reader may refer to Cachon (2003). The factors shaping these contracts, such as the length of the planning horizon and the timing and flexibility of the procurement decisions, have received considerable attention in the literature in recent years.

Several studies exist in the operations management literature that analyze the time frame of order commitment decisions between upstream and downstream members of the supply chain. To preserve consistency, we use 'retailer' and 'supplier' for the downstream and upstream members respectively. Iyer and Bergen (1997) investigate the effects of a 'quick response' model on the profits of the supply chain members, where the retailer is allowed to delay her order until having better information about demand. They investigate various mechanisms that provide incentives for the supplier to be involved. Contrary to their assumption that the supplier always provides the retailer with its order request, Ferguson (2003) models a 'strategic' supplier that gives little credibility to an order quantity without a firm commitment. He examines a retailer's choice of when to commit to an order quantity from its parts supplier. Both the supplier and the retailer have production lead-times and a signal about demand becomes available to the retailer after the seller's lead-time. The signal provides either no additional information or full information about the demand. Ferguson et al. (2005) extend Ferguson (2003) by defining the end-product demand as the sum of two random variables, which allows the demand information to vary continuously from non-informative to full information. Taylor (2006) considers a similar setting with the distinction that the demand is price-sensitive and the retailer exerts sales effort in the end-market. The supplier is assumed to be the dominant agent, and the timing decision depends on when the retailer exerts the sales effort. Cachon (2004) analyzes the commitment time frame within flexible supply contracts, and studies three different contracts in a newsvendor setting: with a push contract, the retailer prebooks inventory and bears all demand risk, whereas with a pull contract, she orders after the demand realization. The third contract type, advance purchase discount, provides the retailer two order opportunities; one before and one during the selling season.

The studies described above (except Cachon (2004)) allow the retailer to have a single, firm order; that is, once the retailer decides on the order quantity, she is not allowed to modify it and is not provided a second order opportunity. A significant number of studies exist, however, that grant the retailer some degree of flexibility such as quantity flexibility or buy-back arrangements, options, and second orders. These studies do not compare two different commitment schemes. Instead, they assume that the retailer places an early order, and analyze the effects of flexibility in order quantities. Next, we discuss a number of representative studies in the literature that fall into this category.

A quantity flexibility (QF) contract allows the retailer to modify its initial order quantity to some degree after acquiring better demand information. Tsay (1999) models a QF contract, where the supplier guarantees to deliver up to a certain percentage over the the initial forecast, and the retailer guarantees to purchase no less than a certain percentage below the forecast. The demand is modelled as the sum of two random variables, and the retailer updates her initial order after observing the first one. Wu (2005) incorporates bayesian updating into a QF contract. The contract terms between the supplier and the retailer are (i) the minimum order quantity of the retailer as a percentage of her initial forecast, (ii) the number of updates before the final order, (iii) and the transfer price. Durango-Cohen and Yano (2006) allow the supplier to differentiate between its commitment to the retailer and its actual production quantity. The supplier incurs a penalty for any shortfall of its commitment from the initial forecast of the retailer, and a penalty for any shortfall of its delivery from the realized demand or the commitment strategy. They analyze the supplier's decisions and show that the supplier commits either to the initial forecast or its production quantity. Huang et al. (2005) differ from these studies in that they only consider the retailer's problem, and the retailer can modify her initial order at the expense of a fixed cost in addition to a variable cost.

Another tool that provides additional flexibility in supply contracts is options, which can be defined as flexible commitments. With each option, the retailer pays the supplier an up-front option cost. At the final decision point, she may exercise any number of the options for an exercise cost. Cachon and Lariviere (2001) consider an options contract where the retailer with private demand information purchases firm commitments and options from the supplier to signal demand information. A final order is placed after demand is realized. In Brown and Lee (2003), on the other hand, the retailer makes her final decision before full information is available. Instead, after purchasing firm commitments and options, she observes a demand signal before her final decision. In this setting, they examine how quantity decisions are affected by the demand signal quality. Cheng et al. (2003) consider a similar model where they examine two different forms of the options: the *call* option, which the retailer can exercise to purchase additional units, and the *put* option, under which the retailer can send back inventory after observing demand. Wang and Tsao (2006) differ from Cheng et al. (2003) by modeling two directional options. That is, the retailer purchases a single type of option which can be exercised as either a *call* or a *put* option. Golovachkina and Bradley (2002) consider an options contract in the presence of a spot market and supplier capacity. The spot market price is uncertain and the retailer must satisfy her demand in full. In this setting, they consider the supplier's lead and characterize the optimal number of options that the retailer should purchase.

There are also a number of studies that model a two-mode production system for the supplier and/or two distinct order opportunities for the retailer. The first production mode is relatively cheap but requires a long lead time, whereas the second is expensive but enables quick response. Parlar and Weng (1997) consider such a setting, where the fixed and variable costs differ between the production modes. Perfect demand information becomes available before the second production run. They characterize the optimal solutions for the centralized and decentralized settings and show that the benefits of centralization may be substantial. Donohue (2000) considers a similar setting with the exception that demand information before the second production mode is not perfect, and there is no fixed production cost. She assumes that the supplier is the dominant firm and offers a wholesale price contract that also involves a buyback price. Choi et al. (2003) consider the retailer's problem, who has two order opportunities, and the unit price for the second order is initially uncertain. The second order is placed after demand information is updated by using a bayesian approach. The optimal ordering policy is characterized using a dynamic programming approach. Barnes-Schuster et al. (2002) also consider a setting with two production modes. However, they differ from the above studies in that they consider a two-period model with correlated demands. The retailer decides the firm commitments to be delivered at the beginning of first and second periods, and the quantity of options that can be exercised after observing the first period demand. They also consider standard holding and backorder costs. Milner and Rosenblatt (2002) also consider a two-period problem, but they only consider the retailer's problem. The retailer first places orders for two periods. After observing the first period demand, she is allowed to adjust her second order without any restriction by paying a per unit order adjustment penalty.

The studies discussed up to this point consider either single-period or two-period problems. There are also a large number of papers that analyze commitment schemes in multi-period settings. Most of these studies do not include the supplier as a strategic member and assume that the supplier must provide the retailer's order in full, and the order quantity of the retailer is restricted by a given form of contract. Bassok et al. (1997) analyze a multi-period, finite horizon problem where demand in each period is random and independent of the demands in other periods. Demands in different periods are not necessarily identically distributed. At the beginning of the horizon, the retailer makes a multi-period commitment. At the beginning of each period, she decides the purchase quantity for that period, and updates the commitments for the periods ahead, which should be within certain limits of the current commitments. Bassok and Anupindi (1997) consider a similar demand setting except that demands in different periods are identically distributed. In their case, the retailer specifies a total minimum quantity to be purchased over the planning horizon. This commitment quantity is assumed to be exogenous in their model. They derive the optimal ordering policy which is characterized by two order-up-to levels. Chen and Krass (2001) extend Bassok and Anupindi (1997) by allowing a non-stationary demand distribution and a different wholesale price for the orders that exceed the total minimum commitment. They show that the optimal policy can still be characterized by two order-up-to levels. Unlike the aforementioned studies, Cheung and Yuan (2003) consider an infinite horizon model where the retailer's commitment is to purchase at least a fixed amount in every period. The extra amount purchased is not subject to additional markup. They consider independent and identically distributed, discrete demands across the periods, and formulate a markov chain to analyze the behavior of the system. Serel et al. (2001) also consider an infinite horizon, periodic review inventory system. Unlike the others, they model a strategic supplier, who offers the retailer reserved capacity in return for a unit fee. The retailer also has an outside spot market option. For more information on supply contracts and quantity commitment schemes, the reader may refer to the excellent review by Anupindi and Bassok (1999).

The closest studies in the literature to ours that consider multiple retailer systems are Jin and Wu (2007), and Cvsa and Gilbert (2002). In Jin and Wu (2007), the supplier, rather than deciding on the entire production quantity in advance, announces the *excess capacity*, which is the amount of capacity the supplier provides in addition to the initial reservation of the retailer. They also extend their study to the multi-retailer setting where the retailers have equal power and reserve capacity. Cvsa and Gilbert (2002) consider a system with a single supplier and two identical retailers. The competition between the retailers is based on quantity and the price in the market is a linear function of the total order quantity. They analyze different models regarding the leadership structure between the retailers.

There is also a stream of literature that analyzes order timing from a marketing point of view. These studies usually consider a retailer's problem to generate better demand information by inducing early orders from the end customers via a price discount. Weng and Parlar (1999) consider a market of deterministic size, where each individual in the market has the same probability to make a purchase. In this setting, they analyze the optimal order quantity for the retailer and the optimal discount rate, assuming that the quantity of early orders generated by the discount is deterministic. Tang et al. (2004) extend Weng and Parlar (1999) in several ways. They consider a two-segment market, where some of the customers from the second segment can switch to the first segment when a discount is offered. The number of early sales is also stochastic. Furthermore, they include forecast updating, that is, the retailer can use the early orders in order to update her beliefs about the remaining demand. McCardle et al. (2004) extends Tang et al. (2004) by modeling the second customer segment strategically as well. That is, the second segment is now served by another distinct retailer. They analyze different scenarios depending on whether each retailer offers a discount or not, and identify the conditions under which both retailers offer an advance booking discount in the equilibrium solution.

Our models differ from the aforementioned studies in an important structural way. Unlike the multi-retailer models in the literature, one of the retailers in our model is the primary customer of the supplier. We aim to understand the relationship between the primary customer and the supplier under early and delayed commitment schemes. We provide the analytical tools and threshold values for the analysis of the value of different commitment schemes for multi-retailer systems. We also investigate the effects of the secondary retailer on the commitment preference of the primary retailer. This analysis leads to interesting insights on how suppliers can cope with low margin customers who demand that the supplier bears the demand risk, and on how retailers can benefit from sharing supplier output. Furthermore, we introduce a number of strategic tools for the supplier that can be utilized to induce his preferred order timing by the primary retailer.

CHAPTER 3
MARKET SELECTION DECISIONS FOR INVENTORY MODELS WITH PRICE-SENSITIVE DEMANDS

Standard approaches to classical inventory control problems typically assume that the firm has no effect on the revenue and demand parameters. Although this may be reasonable in perfectly competitive environments where firms are price-takers, in many contexts firms can manipulate demand to a certain degree using marketing tools such as pricing and advertising. In such contexts, a supplier of a good must often determine the good's price in addition to inventory policy parameters in order to respond to the implied demand. Another limitation of past standard models is that the firm effectively serves a single market. Although there are a number of studies in the literature (see Chapter 2 for a detailed discussion) that address multiple markets or locations, these also assume that the firm does not have the ability to choose whether or not to serve a market.

A considerable body of literature proposes extensions addressing the limitations of these models. The vast majority of these studies assume either exogenous demand and revenue, or a predetermined market portfolio. In this chapter, we introduce an optimization model that relaxes these assumptions. Our model applies to and generalizes related studies in the literature both in the deterministic (Economic Order Quantity [EOQ] model) and the stochastic (Newsvendor model) settings.

We consider a profit-maximizing firm offering a single product. A set of potential markets exists, and the firm must decide whether or not to serve each market (although we consider distinct markets, the setting also applies to a single-market problem with different customer classes). Revenue in each market is a function of the price offered. The firm must determine the markets it will serve, the price (or the prices in each market), and a procurement quantity that will be used to supply the selected markets. The resulting profit maximization problem is quite different from standard inventory control problems. In the traditional settings, the optimal inventory control policy parameter value(s) depends on a predefined demand rate (deterministic setting), or an exogenous probability distribution of demand (stochastic setting), over which the firm has little control. In the contexts we consider, however, the demand rate (or probability distribution) depends on the markets the firm selects and the selling price (or prices) offered in these markets. Moreover, since stock is pooled for all selected markets in the stochastic case, and fixed costs are shared in the deterministic case, the profitability of serving a market depends on the entire set of markets selected. As a result, in addition to containing nonlinear operations (EOQ/Newsvendor) and economics (pricing) elements, the model we develop also includes a substantial combinatorial optimization component.

In the absence of explicit constraints, a profit maximizing firm would naturally prefer to set different prices in different markets, assuming the market characteristics are different. However, the associated marketing and operational costs under market-specific prices may be higher than those in the single-price case. Even if such costs are not an issue, a firm might choose to set the same price in all markets as a marketing strategy in order to maintain a certain brand reputation and/or consistent customer experience. A number of additional reasons may also prevent a firm from applying a 'market-specific pricing' strategy. For instance, if the supplier uses regional distributors to supply different markets, in order to avoid conflicts with distributors and ensure equity, the supplier may apply a single price for all distributors (see, e.g., Balakrishnan et al. (2000)). Alternatively, regulations may exist that prevent charging different prices for the same good in different markets or to different customer classes (see e.g. Cabral (2000) for a uniform price imposed by an antitrust authority).

This work therefore studies an optimization problem that applies to both deterministic and stochastic inventory problems, and analyzes simultaneous market selection, pricing, and order quantity decisions for two potential cases: (i) the firm must offer the same selling price in all markets selected, and (ii) the firm has the flexibility to offer market-specific prices. At first glance, our study seems to be closely related to a branch of economics literature on multiproduct monopoly pricing and third degree price discrimination (see e.g.

Tirole (1988) for a brief description of these problems). As we discussed in more detail in Chapter 2, our model not only differs from this literature by modeling operational costs in inventory systems, but it incorporates explicit market selection decisions and provides an efficient algorithm for solving multi-market problems as well, which has not been done in the economics literature to our knowledge.

Under mild assumptions on the revenue and cost functions, we provide a polynomial-time solution for the single-price strategy and characterize the optimal solution for the market-specific pricing strategy. These models can be applied as benchmarks for making market selection, pricing, and procurement quantity decisions in stochastic environments with a short selling season, and deterministic environments with continuous and stationary demand. Using these models, we perform an extensive computational analysis to demonstrate the effects that different critical parameter settings have on the optimal value of (expected) profit. The results of this analysis provide some interesting and, in some cases, unexpected insights on how a market's characteristics can affect pricing decisions in other markets.

The remainder of this chapter is organized as follows: in Section 3.1, we introduce a general problem framework and key modeling assumptions. Section 3.2 proposes solution approaches for the 'single-price' and 'market-specific pricing' strategies. We provide an extensive computational study and present our main findings in Section 3.3. We conclude in Section 3.4 by summarizing our work.

3.1 Problem Description and Assumptions

In this section, we formally state our assumptions, and describe and formulate the model. Let n denote the number of potential markets available for a supplier to serve. The total (expected) revenue from market i is price dependent and is denoted by a continuous function, $R_i(p_i)$, where p_i is the price in market i. Let y denote the market selection vector, i.e., $y_i = 1$ if the firm decides to serve market i, and $y_i = 0$ otherwise. The total (expected) cost the supplier incurs when serving market i in isolation using a unit price p_i equals $S \times C_i(p_i)$, where S is a cost parameter that is context dependent, and $C_i(p_i)$ is a continuous, decreasing function of p_i. Moreover, let $\bar{p}$ denote the vector of n market prices, and the total cost incurred for serving all selected markets is represented by $C(\bar{p}, y)$. We will consider settings in which the individual market costs are not independent of one another when multiple markets are selected, that is, $C(\bar{p}, y) \neq \sum_{i=1}^{n} C_i(p_i) y_i$. Instead, we assume that the total cost function is of the form

$$C(\bar{p}, y) = S\sqrt{\sum_{i=1}^{n} [C_i(p_i)]^2 y_i}. \tag{3–1}$$

This cost structure encompasses both risk pooling in stochastic settings (such as the Newsvendor context discussed in Section 3.1.1) and economies-of-scale in procurement costs in deterministic settings (such as the EOQ context discussed in Section 3.1.2). The market selection problem with pricing (MSP) can then be constructed as follows:

$$\begin{aligned} \text{max} \qquad & G(\bar{p}, y) = \sum_{i=1}^{n} R_i(p_i) y_i - S\sqrt{\sum_{i=1}^{n} [C_i(p_i)]^2 y_i} \\ \text{subject to} \qquad & y_i \in \{0, 1\}, \qquad \forall i = 1 \ldots n, \\ & \bar{p} \in P. \end{aligned}$$

The definition of the set P depends on whether we consider the single-price case (in which case P consists of all vectors $\bar{p} \in \mathbb{R}^n$ such that $p_i = p$ for all $i = 1, \ldots, n$ and for some $p \in \mathbb{R}$) or the market-specific pricing case (in which case $P = \mathbb{R}^n$). We next discuss how the above formulation applies to different modeling environments.

3.1.1 Newsvendor Model with Market Selection and Pricing

(MSP) applies to a single-period, stochastic inventory problem under certain assumptions. Consider a set of potential markets where demand in market i is random and price-sensitive. In the vast majority of the literature (see Chapter 2), demand is typically defined as either $q(p) + X$ (additive demand model), or $q(p)X$ (multiplicative demand model), where $q(p)$ is a decreasing function of price, p, and X is a random variable independent of price. As X is assumed to be independent of the pricing decision, these models have important structural differences. When the random factor is additive, the standard deviation of demand is independent of the expected size of the market. As a result, the coefficient of variation of demand increases in price. In the multiplicative case, on the other hand, the coefficient of variation is constant, i.e., the standard deviation of demand decreases linearly in price. In our study, we let the distribution of the random factor depend on price. In particular, we model demand in market i as $D_i(p_i) = q_i(p_i) + X_i$ where X_is are independent random variables having pdf and cdf $f_i(x, p_i)$ and $F_i(x, p_i)$. Note that this demand model is quite similar to Young (1978), who formulates the demand as $\alpha(p)\epsilon + \beta(p)$ where $\alpha(p)$ and $\beta(p)$ are deterministic functions of p and ϵ is a random variable. In Appendix A.1, we demonstrate that the multiplicative and additive models can be equivalently represented by one another in our setting when the X_is are independent, normally distributed random variables. Hence, we restrict our analysis to the additive randomness case, noting that similar arguments and results are also valid for the multiplicative case. Next, we discuss the assumptions and their implications, under which (MSP) can handle the Newsvendor problem with market selection and pricing.

Assumption 3.1. *There is effectively a single pool of stock that serves all markets.*

Assumption 3.1 states that stock is allocated among selected markets after the uncertainty is resolved. This is quite reasonable when the markets are close to each other or when the firm offers the product on a ship-to-order basis. Inventory pooling naturally follows if individual markets represent different customer segments in a single market. Note that the problem would be trivial if inventory was not pooled, since each market would be considered separately in terms of inventory, pricing and selection decisions.

Assumption 3.2. *The random element in market i, X_i, is normally distributed with mean 0, and standard deviation $\sigma_i(p_i)$.*

Although the support of the normal distribution is the entire real line, and hence it may result in negative demand occurrences, it is widely employed in the literature.

Furthermore, we can limit the possibility of negative demand to negligible levels since the distribution of the random factor also depends on the price level. Moreover, if each market's demand is comprised of a large number of individual demands from different customers, then we would expect that the distribution of demand in each market can be closely approximated by a normal distribution as a result of the Central Limit Theorem (see *e.g.* Ross (2006), p. 79). Assuming $E[X_i] = 0$ is not restrictive since any nonzero mean value may be incorporated into the deterministic part of the demand, $q_i(p_i)$, without any effect on the model. The normality assumption enables us to model the aggregate demand explicitly since the sum of independent normal random variables is also normally distributed; that is, the aggregate demand is normally distributed with mean $D_y(\bar{p}) = \sum_{i=1}^{n} q_i(p_i)y_i$ and standard deviation $\sigma_y(\bar{p}) = \sqrt{\sum_{i=1}^{n} \sigma_i^2(p_i)y_i}$.

Assumption 3.3. *Shortages are either expedited or backlogged until the end of the selling season by placing an additional order that arrives at end of the selling season. In either case, the cost per shortage is independent of price.*

Note that in the backlogging case, Assumption 3.3 can be interpreted as having customers who are willing to wait until the end of the selling season to receive the product when the supplier faces a shortage.

Assumption 3.3, together with Assumption 3.2, results in separate inventory and pricing decisions as follows: recall that, by Assumption 3.2, the aggregate demand is normally distributed with mean $q_y(\bar{p}) = \sum_{i=1}^{n} q_i(p_i)y_i$ and standard deviation $\sigma_y(\bar{p}) = \sqrt{\sum_{i=1}^{n} \sigma_i^2(p_i)y_i}$. Hence, given the market selection and price vectors, the inventory decision is equivalent to a classical Newsvendor problem. Let $z = (Q - q_y(\bar{p}))/\sigma_y(\bar{p})$, where Q is the procurement quantity from an external supplier. Then, the expected shortages and leftovers are given by $\sigma_y(\bar{p})E[\epsilon - z]^+$ and $\sigma_y(\bar{p})E[z - \epsilon]^+$, respectively, where ϵ is a standard normal random variable. By Assumption 3.2, the expected sales are directly equal to the mean demand, i.e., $q_y(\bar{p})$. Hence, the expected total profit can be written as

$$G(\bar{p}, y, z) = \sum_{i=1}^{n} (p_i - c) q_i(p_i) y_i - K(z) \sqrt{\sum_{i=1}^{n} \sigma_i^2(p_i) y_i}, \tag{3–2}$$

where $K(z) = (c-v)E[z-\epsilon]^+ + (e-c)E[\epsilon - z]^+$, and c, e, v are per unit procurement cost, per unit shortage cost and per unit salvage value, respectively. Note that we have y_i in the square root term since $y_i^2 = y_i$ as $y_i \in \{0, 1\}$. Since per unit shortage cost is independent of price, the optimal value of z is independent of the prices set in the selected markets, and hence it is also independent of the market selection decisions; that is $z^* = s(\rho)$, where $s(\rho)$ is the standard normal variate associated with the fractile $\rho = (e-c)/(e-v)$. Therefore, the Newsvendor problem with market selection and pricing reduces to

$$\begin{aligned} \text{max} \quad & G(\bar{p}, y) = \sum_{i=1}^{n} (p_i - c) q_i(p_i) y_i - K^* \sqrt{\sum_{i=1}^{n} \sigma_i^2(p_i) y_i} \\ \text{subject to} \quad & y_i \in \{0, 1\}, \qquad \forall i = 1, \ldots, n, \\ & \bar{p} \in P, \end{aligned}$$

where $K^* = K(z^*)$. Note that this problem is a special case of (MSP), where $R_i(p_i) = (p_i - c) q_i(p_i)$, $S = K^*$, and $C_i(p_i) = \sigma_i(p_i)$.

3.1.2 EOQ Model with Market Selection and Pricing

Geunes et al. (2004) consider a standard EOQ problem with two exceptions. First, the producer can choose whether or not to satisfy each market's demand. Second, instead of minimizing average annual cost, they maximize average annual net profit. The resulting model (EOQMC) is

$$\begin{aligned} \text{max} \quad & G(y) = \sum_{i=1}^{n} r_i \lambda_i y_i - \sqrt{2Kh \sum_{i=1}^{n} \lambda_i y_i} \\ \text{subject to} \quad & y_i \in \{0, 1\}, \qquad \forall i = 1, \ldots, n, \end{aligned}$$

where r_i and λ_i denote the per unit net revenue and demand rate in market i, respectively, h denotes inventory holding cost rate, and K denotes the fixed setup/order cost.

Note that (EOQMC) is a special case of (MSP) described earlier, where $R_i(p_i) = r_i\lambda_i$, $S = \sqrt{2K}$, and $C_i(p_i) = \sqrt{\lambda_i h}$. Moreover, (MSP) can easily incorporate a price-sensitive demand rate by setting $R_i(p_i) = (p_i - c_i)\lambda_i(p_i)$ and $C_i(p_i) = \sqrt{\lambda_i(p_i)h}$. Allowing different holding costs for markets can easily be handled by simply replacing h by h_i. As a result, (MSP) is capable of handling (EOQMC) described in Geunes et al. (2004), together with a number of its generalizations.

3.2 Solution Algorithms for (MSP)

In this section, we seek efficient solution algorithms for (MSP), which is a nonlinear, combinatorial optimization problem. We first consider (MSP) under a single-price strategy, i.e., the firm chooses to offer the product at the same price in all selected markets. This corresponds to the case in which the set P is limited to all vectors in $\mathbb{R}^n$ such that all n elements are identical, and we refer to this problem as (MSP-S). After introducing the algorithm for (MSP-S), we then consider the market-specific pricing strategy, which we refer to as (MSP-MS) from this point onward.

3.2.1 Market Selection with a Single Price – (MSP-S)

The market selection and pricing problem when the firm is required to set the same price in all markets selected can be formulated using a single price variable p as

$$\begin{array}{ll} \max & G(p, y) = \sum_{i=1}^{n} R_i(p)y_i - S\sqrt{\sum_{i=1}^{n}[C_i(p)]^2 y_i} \\ \text{subject to} & y_i \in \{0, 1\}, \qquad \forall i = 1, \ldots, n. \end{array}$$

Given a price, (MSP-S) reduces to a general version of the selective Newsvendor problem (SNP), which is discussed by Taaffe et al. (2006). Hence, for a fixed price, we can solve the market selection problem employing the *Decreasing Expected Revenue to*

Uncertainty Ratio Property introduced in Taaffe et al. (2006) (based on a result from Shen et al. (2003)), where the uncertainty in a market is characterized by its variance.

Property 3.1 (cf. Taaffe et al. (2006)). *After indexing markets in nonincreasing order of the ratio of expected net revenue to uncertainty, an optimal solution to [SNP] exists such that if we select market ℓ, we also select markets $1, 2, \ldots, \ell - 1$.*

Following Property 3.1 and sorting markets in nondecreasing order of the ratio of the (expected) revenue ($R_i(p)$) to the cost contribution ($[C_i(p)]^2$), an optimal solution to the market selection problem with a fixed price level can be found by selecting the best of n candidate solutions, where candidate solution ℓ selects markets 1 to ℓ (see Taaffe et al. (2006) for details). Note that the sorting mechanism works in favor of markets that have greater revenue and less cost contribution, which satisfies intuition.

For (MSP-S), however, price is a decision variable and the ordering of markets may differ at different price levels. To overcome this problem, the sorting scheme characterized above can be utilized to divide the feasible region in price into a set of contiguous, non-overlapping intervals, where the preference order of markets does not change within an interval. Hence, within each interval, we can utilize Property 1 with a slight modification to obtain an optimal set of markets for the price interval. In particular, for each candidate solution in each price interval, we need to maximize the objective function with respect to p with the constraint that p falls in the specified interval. The price intervals that enable this approach can be generated as follows.

Let P_{ij} denote the set of critical price levels where the preference ratios for markets i and j are equal, i.e., the threshold prices beyond which the order of these markets is reversed in the sorting scheme. For all (i, j) pairs such that $i < j$, P_{ij} is given by

$$P_{ij} = \left\{ c \leq p \leq \min(p_i^0, p_j^0) : \frac{R_i(p)}{[C_i(p)]^2} = \frac{R_j(p)}{[C_j(p)]^2} \right\}. \tag{3–3}$$

Recall that, in the specific applications that we considered in the previous section, demand in market i may be zero beyond a price level, p_i^0. In order to address this issue, we add these p_i^0 values to the critical price levels generated by (3–3). Assuming that the total number of critical price levels is finite, we reindex these critical price levels such that $c = p^0 < p^1 < p^2 < \cdots < p^m$, where $m = \sum_i \sum_{j>i} |P_{ij}| + n < \infty$. As we illustrate with some examples later, the sets P_{ij} often contain at most one element, which would lead to $m = O(n^2)$. The preference order of markets is the same within a price interval, $p \in (p^{k-1}, p^k)$. For two consecutive price intervals, (p^{k-1}, p^k) and (p^k, p^{k+1}), the ranking will be the same except that markets i and j will switch places in ordering sequence if $p^k \in P_{ij}$. Hence, we do not need to specifically rank order all markets for each price interval. Instead, we simply rank order them once, and determine which markets switch places at each price breakpoint. For each price interval indexed by $k = 1, \ldots, m$, we solve n maximization problems of the following form:

$$\max_{p \in (p^{k-1}, p^k)} \left\{ \sum_{i=1}^{\ell} R_i(p) - S\sqrt{\sum_{i=1}^{\ell} [C_i(p)]^2} \right\}. \tag{3–4}$$

After solving (3–4) for each price interval and for each $\ell = 1, 2, \ldots, n$ within each interval, the optimal solution is characterized by the solution to (3–4) that results in the highest optimal objective value. Note that, in any given interval, we may discard the markets at the end of the rank ordering with zero demand since they will not be selected anymore.

The running time of the above algorithm depends on the number of threshold price levels, m, and on how fast we can solve the maximization subproblem in the inner loop. Note that if each set P_{ij} contains at most one element and each market has a price where demand becomes zero, there exist $m = O(n^2 + n) = O(n^2)$ price intervals. Hence the running time of the algorithm becomes $O(Tn^3)$ where T denotes the time required to solve (3–4). In Appendix A.2, we show that the objective function (3–4) is concave if $R_i(p)$ is concave and $C_i(p)$ is convex for $p \leq p_i^0$ for all markets. In this case, we can utilize first order conditions to solve the subproblems efficiently.

Under the Newsvendor structure, with both the linear ($q_i(p) = a_i - b_i p$) and iso-elastic ($q_i(p) = \alpha_i p^{-\beta_i}$) demand models, each set P_{ij} contains at most one element when either the coefficient of variation or the standard deviation of demand is constant; that is, either $\sigma_i(p) = cv_i q_i(p)$ or $\sigma_i(p) = \sigma_i$, where cv_i denotes the coefficient of variation. In the linear demand case with a constant coefficient of variation, we have $P_{ij} = \{p_{ij}\}$ if $c \leq p_{ij} = (cv_j^2 a_j - cv_i^2 a_i)/(cv_j^2 b_j - cv_i^2 b_i) \leq \min(p_i^0, p_j^0)$. Otherwise, $P_{ij} = \emptyset$. With a constant standard deviation, we have $p_{ij} = (\sigma_j^2 a_i - \sigma_i^2 a_j)/(\sigma_j^2 b_i - \sigma_i^2 b_j)$ if $c \leq p_{ij} \leq \min(p_i^0, p_j^0)$, and $P_{ij} = \emptyset$ otherwise. In the iso-elastic demand case, these price levels are given by $p_{ij} = [(cv_j^2 \alpha_j)/(cv_i^2 \alpha_i)]^{1/(\beta_j - \beta_i)}$ for the constant coefficient of variation case, and $p_{ij} = [(\sigma_j^2 \alpha_i)/(\sigma_i^2 \alpha_j)]^{1/(\beta_i - \beta_j)}$ for the constant standard deviation case. Note that these results guarantee that $m = O(n^2 + n) = O(n^2)$ price intervals. Hence, the running time of the algorithm becomes $O(Tn^3)$ for all cases discussed above.

Under the EOQ structure, the ratio $R_i(p)/[C_i(p)]^2$ is equal to $(p - c_i)/h_i$ for a given price p, and thus P_{ij} again contains at most one element ($p_{ij} = (c_i h_j - c_j h_i)/(h_j - h_i)$), so that $m = O(n^2)$, and the running time of the algorithm is $O(Tn^3)$. When the procurement cost is the same for all markets (i.e., $c_i = c \; \forall i = 1, \ldots, n$), we have $P_{ij} = \emptyset$ for all (i, j) pairs, and for any price the rank order of the markets is the same, which is determined by the holding cost rates. Hence, the price intervals will be given only by $O(n)$ p_i^0 values, and the running time of the algorithm is then $O(Tn^2)$. When the holding cost rate is also the same for all markets, the ratio $R_i(p)/[C_i(p)]^2$ gives the same value at any price level for any market, which indicates that either all or none of the markets with a positive demand rate will be selected. Hence, the price intervals will again be formed only by p_i^0 values. However, in this case, there is only a single subproblem in each price interval, and the running time of the algorithm is $O(Tn + n \log n)$. The solution procedure described above can also be applied to the case where the cost contribution of each market is constant, i.e., $C_i(p) = C_i > 0$. We next consider the problem in contexts that permit market-specific pricing.

3.2.2 Market Selection with Market-Specific Prices – (MSP-MS)

In this section, we allow different prices in different markets. The notation remains the same except that p, which was the single price in the previous section, is replaced by the price vector $\bar{p}$ with components p_i. Despite the similarity to the single price case, the previous solution approach will not work for this problem since the concept of price intervals no longer exists, which complicates the problem significantly.

For (MSP-MS), we assume that the cost contribution of market i ($C_i(p_i)$) is a nonincreasing function of price and converges to zero when (expected) revenue term ($R_i(p_i)$) is zero, i.e., there exists a price level, p_i^0 such that $R_i(p_i^0) = 0$ and $C_i(p_i^0) = 0$. This assumption makes sense for the following reason: p_i^0 is usually characterized by the (expected) demand function in a market. That is, it is the price level beyond which there is no demand. When there is no demand, it is reasonable to assume that the cost of serving the market is also zero since the concept of serving a market without a demand is not meaningful. Hence, beyond a certain price level, the ideas of serving a market and the associated revenues and costs become irrelevant since there is no demand. We can thus eliminate the market selection variables, since market selection decisions can be inferred from the pricing decisions. In other words, $p_i = p_i^0$ is equivalent to $y_i = 0$, resulting in no revenue or cost associated with that market. (Note that although this assumption also makes sense for the original problem, i.e., market selection with single-price strategy, it does not help with the solution since the firm cannot set different prices in different markets.) Then, the problem reduces to

$$\begin{array}{ll} \max & \displaystyle\sum_{i=1}^{n} R_i(p_i) - S\sqrt{\sum_{i=1}^{n}[C_i(p_i)]^2} \\ \text{subject to} & 0 \le p_i \le p_i^0, \qquad \forall i = 1, \dots, n, \end{array}$$

which is a continuous optimization problem, and the characteristics of the optimal solution are highly dependent on the form of $R_i(p_i)$ and $C_i(p_i)$. If $R_i(p_i)$ is concave and $C_i(p_i)$ is convex for all $i = 1, \ldots, n$, the resulting formulation is a concave maximization problem as shown in Appendix A.3. This leads to Proposition 3.1.

Proposition 3.1. *If $R_i(p_i)$ is concave and $C_i(p_i)$ is convex, either all or none of the markets will be selected in the market-specific pricing case.*

Proof. Please see Appendix A.4. □

Proposition 3.1 implies that either the price in each market will equal p_i^0 or it will be strictly less than that; that is, either all demands are zero or all markets have strictly positive demand.

When the cost contribution of each market is independent of price (e.g., when the standard deviation of demand in the Newsvendor model is independent of price), Proposition 3.1 does not hold since such a case violates the assumption that $C_i(p_i)$ converges to zero when (expected) revenue term $R_i(p_i)$ is zero, i.e., there exists a price level, p_i^0 such that $R_i(p_i^0) = 0$ and $C_i(p_i^0) = 0$. However, the objective function of this problem is separable by markets and we can solve for the optimal price of each market individually. Let p_i^* denote the optimal price for market i. Since the market selection variable is zero for an unselected market, the prices in such markets can be set arbitrarily and we can reformulate the problem as

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} R_i(p_i^*) y_i - S \sqrt{\sum_{i=1}^{n} C_i^2 y_i} && (3\text{–}5) \\ \text{subject to} \quad & y_i \in \{0, 1\}, \quad \forall i = 1, \ldots, n. \end{aligned}$$

This problem has the same form as the selective Newsvendor problem (Taaffe et al. (2006)).

3.2.3 Application of the Algorithms: A Newsvendor Example

To illustrate the algorithms presented in the previous sections, we consider an example problem in a Newsvendor setting with three markets, $M1$, $M2$, and $M3$. Demand in each market is characterized by its expected value, $q_i(p_i)$, and coefficient of variation,

Table 3-1: Summary of the algorithm for the Newsvendor example.

					Markets Selected					
		Rank Order			$R1$		$R1$,$R2$		$R1$,$R2$,$R3$	
k	(p^{k-1}, p^k)	$R1$	$R2$	$R3$	p^*	$G(p^*)$	p^*	$G(p^*)$	p^*	$G(p^*)$
1	(40,77.5)	$M2$	$M3$	$M1$	77.50	2,323	77.50	6,536	77.50	7,329
2	(77.5, 88.75)	$M2$	$M1$	$M3$	88.75	2,652	85.20	3,197	88.75	8,250
3	(88.75,100)	$M1$	$M2$	$M3$	88.75	405	88.75	3,171	96.38	8,431
4	(100,150)	$M2$	$M3$	-	100.00	2,727	109.30	8,564	—	—
5	(150,200)	$M3$	-	-	150.00	5,227	—	—	—	—

cv_i. We have $q_1(p_1) = 100 - p_1$, $q_2(p_2) = 150 - p_2$, $q_3(p_3) = 200 - p_3$, and $cv_1 = 1/3$, $cv_2 = 1/7$, $cv_3 = 1/7$. Shortage and unit costs, and the salvage value are $e = 300$, $c = 40$, and $v = 20$, respectively. Then, we have $S = K(z^*) = 38.183$; see Section 3.1.1 for details. We next discuss the application of our algorithm for the (MSP-S) case ($p_1 = p_2 = p_3 = p$), and then consider the results for the (MSP-MS) case.

We first generate the critical price levels for each pair of markets in the (MSP-S) case. Note that this example considers a linear expected demand and a constant coefficient of variation for each market. Hence, we know that there is at most one solution to Equation (3–3) for each pair of markets. Solving Equation (3–3) for each pair, we get $P_{12} = \{88.75\}$, $P_{13} = \{77.5\}$, and $P_{23} = \varnothing$. Including the p_i^0 values, we reindex the critical price levels as follows: $p^0 = c = 40$, $p^1 = 77.5$, $p^2 = 88.75$, $p^3 = 100$, $p^4 = 150$, and $p^5 = 200$. We only rank order the markets at p^0, which corresponds to the interval (p^0, p^1). At subsequent critical price levels, either two markets switch or one of the markets is dropped since the expected revenue becomes zero. Table 3-1 summarizes the solution of the example.

Within each interval, we solve at most 3 subproblems. To illustrate, let us consider interval 2, i.e., $(77.5, 88.75)$. The rank order of markets is given as $(R1, R2, R3) = (M2, M1, M3)$. The first subproblem in this interval considers selecting the market that is ranked first, i.e., $M2$. The optimal price and the associated expected profit for this subproblem are 88.75 and 2,652, respectively. The second and third subproblems select $(R1, R2) = (M2, M1)$, and $(R1, R2, R3) = (M2, M1, M3)$, respectively. Having solved all subproblems, the optimal solution for this interval is $p^* = 88.75$ and the corresponding

Table 3-2: Optimal solutions for the Newsvendor example.

	p_1	p_2	p_3	$R_1(p_1)$	$R_2(p_2)$	$R_3(p_3)$	$S \times C(\bar{p}, y)$	$G(\bar{p}, y)$
(MSP-MS)	73.48	96.29	121.88	887.90	3023.33	6396.46	617.52	9690.16
(MSP-S)	-	109.30		-	2820.44	6285.56	542.25	8563.75

expected profit is 8,250. Note that this is a local optimal solution for the general problem. The global optimal is the largest of the local optimal solutions calculated for all intervals. In this example, it is $p^* = 109.3$, which corresponds to the optimal solution of interval 4. The associated expected profit is 8,564.

An important characteristic of the algorithm for the (MSP-S) problem is that it provides a set of additional (suboptimal) solutions for the entire feasible region in terms of the price variable, and it can easily be modified to capture additional constraints on the price level of the product. For instance, let's assume that the firm does not want to charge more than 90 for this particular example. Then, the optimal solution is found by considering the first three intervals only, where the third interval is modified to be $(88.75, 90)$. Another restriction that can be handled can be explained as follows: assume that the firm wants to serve specific markets. Then, we only need to consider the intervals and associated subproblems that select these markets. For instance, say $M1$ must be served in the above example. In such a case, only subproblem 3 of interval 1, subproblems 2 and 3 of interval 2, and all subproblems of interval 3 should be considered. The associated optimal solution is $p^* = 96.38$.

We now solve the same example for the (MSP-MS) case, allowing different prices in different markets. Recall that we can eliminate the market selection variables, and the resulting formulation is a concave maximization problem since $R_i(p_i) = (p_i - c)(a_i - b_i p_i)$ is concave and $C_i(p_i) = cv_i(a_i - b_i p_i)$ is convex. The optimal solution is provided in Table 3-2 along with the solution for the (MSP-S) case. In comparison to (MSP-S), (MSP-MS) not only selects the first market in addition to the others, but also generates more profit in markets 1 and 2 due to the flexibility to set different prices. As a result it provides 13.15% higher profits than (MSP-S).

3.3 Computational Analysis

This section discusses a set of computational tests on a two-market problem with both deterministic and stochastic demand models, which provides insight on market selection decisions and the value of market-specific versus single-price strategies. This analysis considers the impacts that relative market sizes and cost/demand parameters have on pricing and market selection decisions. For stochastic cases in which the model is repeatedly applied (as a sequence of single-period problems), we can gain some insight into how these decisions change as markets expand or contract. More generally, this analysis illustrates the benefits that the flexibility of market-specific pricing provides (relative to a single-price strategy) under a broad range of relative market sizes, price sensitivities and service levels. We focus on the two-market case in order to perform comparative statics to assess how different factors influence the attractiveness of a given market. Although we only consider two markets, one of the markets might correspond to a collection of existing markets, with the other corresponding to an individual market whose value we would like to characterize.

We consider two demand models that are widely used in the literature: linear and iso-elastic demand functions. The linear demand model is represented by a linear price-demand curve for each market $i \in \{1, 2\}$, i.e., $q_i(p) = a_i - b_i p$ where $a_i, b_i > 0$ and $p \leq a_i/b_i$, and the iso-elastic demand model is represented by $q_i(p) = \alpha_i p^{-\beta_i}$ where $\alpha_i > 0$ and $\beta_i > 1$. The parameters a_i and α_i denote the potential market size, and b_i and β_i denote the price sensitivity of demand. Since it possesses the basic model characteristics and enables closed-form solutions, we begin by providing analytical results for the deterministic single-period model. We then provide computational analysis for the Newsvendor and EOQ models and conclude that many of the basic observations about the system are the same as for the deterministic model. For these models, we also investigate the effects different parameters have on market selection decisions and the profitability of both models. We distinguish the structural differences between the linear and iso-elastic demand models under these settings.

3.3.1 Deterministic Single-Period Model

The deterministic single-period model is a special case of the Newsvendor model of Section 3.1.1 obtained by setting $\sigma_i(p_i) = 0$, $\forall i$. We start our analysis with the linear demand model, $q_i(p_i) = a_i - b_i p_i$. In order to analyze the effect of market sizes on market selection decisions, we treat a_1 as fixed and derive threshold values of a_2 that result in different qualitative decisions. Later, we perform the same analysis for price sensitivities, b_i. Recall that we only consider values such that $a_2 - bc > 0$. In the market-specific pricing model, both markets will be selected for all a_2 values since they have positive demands. The optimal prices are $\frac{a_1}{2b} + \frac{c}{2}$ and $\frac{a_2}{2b} + \frac{c}{2}$ respectively, resulting in a total profit of $\frac{(a_1-bc)^2+(a_2-bc)^2}{4b}$. In the single-price case, there are three possible decisions; either select both markets, select only market 1 (M1), or select only market 2 (M2). The optimal prices corresponding to these cases are $\frac{a_1+a_2}{4b} + \frac{c}{2}$, $\frac{a_1}{2b} + \frac{c}{2}$, and $\frac{a_2}{2b} + \frac{c}{2}$, respectively. The market selection decisions are made by comparing the profit levels of these cases and hence the optimal profit will be $\max\left\{\frac{(a_1+a_2-2bc)^2}{8b}, \frac{(a_1-bc)^2}{4b}, \frac{(a_2-bc)^2}{4b}\right\}$. Analyzing this expression, we can obtain threshold values of a_2 (which we denote by a_2' and a_2'') at which the market selection decisions change.

- If $a_2 < a_2' = (\sqrt{2}-1)(a_1 - bc) + bc$, only M1 is selected.
- If $a_2' < a_2 < a_2'' = \frac{a_1 - bc(2-\sqrt{2})}{\sqrt{2}-1}$, both markets are selected.
- If $a_2 > a_2''$, only M2 is selected.

Figures 3-1 and 3-2 illustrate the profits and the percentage difference in profits between the market-specific pricing (MSP-MS) and single-price (MSP-S) cases. Note that the profit in the (MSP-MS) case is strictly increasing in a_2. For the (MSP-S) case, it is constant up to a_2' since M2 is not selected if $a_2 < a_2'$, and the optimal price is constant on this interval for M1. When market sizes are equal, i.e., at $a_2 = a_1$, the profits are

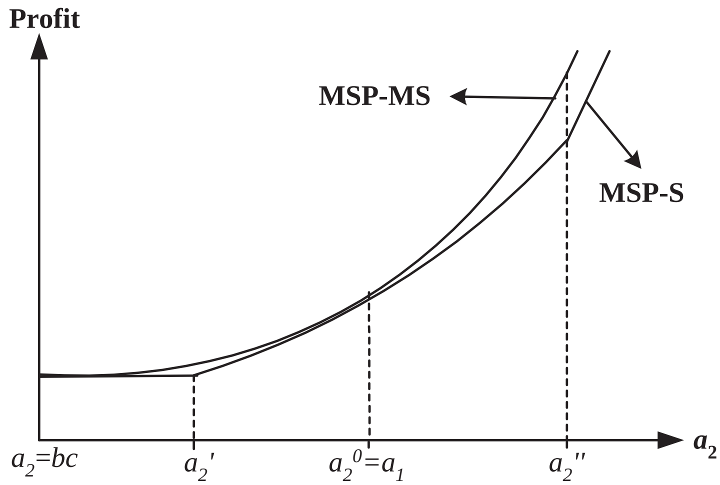

Figure 3-1 Deterministic demand analysis: profits as a function of a_2.

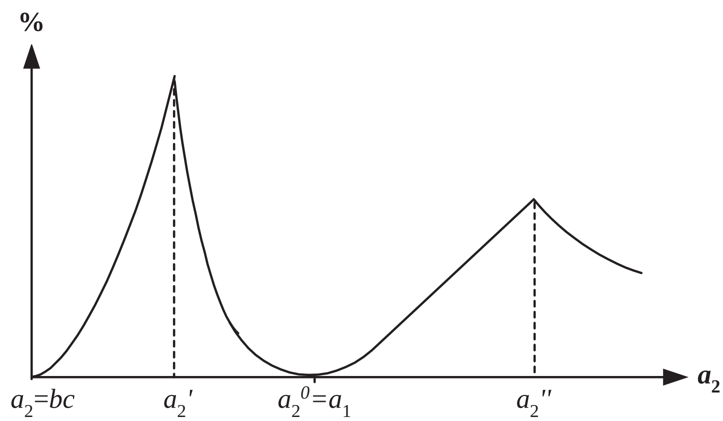

Figure 3-2 Deterministic demand analysis: percentage difference in profits.

also equal. When $a_2 > a_2''$, only M2 is selected in the (MSP-S) case and the difference in profits is constant thereafter, and equal to the profit generated in M1 in the (MSP-MS) case. This analysis illustrates the value that the flexibility of market-specific pricing provides as market sizes differ. It also illustrates the fact that a threshold value exists for a market's size at which point the market becomes an attractive market to supply under a single-price strategy. More interestingly, if some market (M1 in this case) maintains a constant size and another market (M2) grows, a threshold market size exists for the growing market at which point it becomes attractive to drop the market with a constant size (again, assuming a single-price strategy).

Similar arguments are also valid when we fix the potential market size for both markets to $a_1 = a_2 = a$, set $b_1 = b$ and vary b_2. In this case, for the single price case, there exist threshold values b_2' and b_2'' such that only M2 is selected when $b_2 \leq b_2'$, both markets are selected when $b_2' < b_2 \leq b_2''$, and only M1 is selected when $b_2 > b_2''$.

When the demand is iso-elastic, both markets will be selected regardless of the pricing scheme as both will have positive demands at any price level. Moreover, the optimal price of a single market is given by $\beta c/(\beta - 1)$, which is independent of the potential market size. Hence, when both markets have the same price sensitivity, $\beta_1 = \beta_2 = \beta$, the optimal price is given by $p = \beta c/(\beta - 1)$, and market-specific pricing and single pricing strategies coincide, even with different potential market sizes.

3.3.2 Newsvendor Model

In this section, we first highlight the similarities between the deterministic and the stochastic single-period models. Since we are unable to provide closed-form analytical results, we generate a test case that strongly resembles the analysis of the deterministic case. Let $q_1(p) = 500 - 3p$, $q_2(p) = a_2 - 3p$ and $\sigma_i(p) = q_i(p)/3$; also let $(c, v, e) = (100, 50, 300)$. Figures 3-3 and 3-4 depict the expected profits and the percentage difference in expected profits between the (MSP-MS) and (MSP-S) cases. Note that both figures are similar to those for the deterministic case. The smallest size at which M2 is selected is 412.72 for this example. The corresponding value in the deterministic case for the same parameters is 382.84, which is intuitively reasonable since uncertainty results in higher costs, and a greater expected demand is required to enter a market with uncertainty. Likewise, the threshold value a_2 value at which M1 is no longer selected is 711.15 in the stochastic case, whereas it is 782.84 when demand is deterministic. Note also that the highest percentage differences in profit also occur at these threshold values for both models.

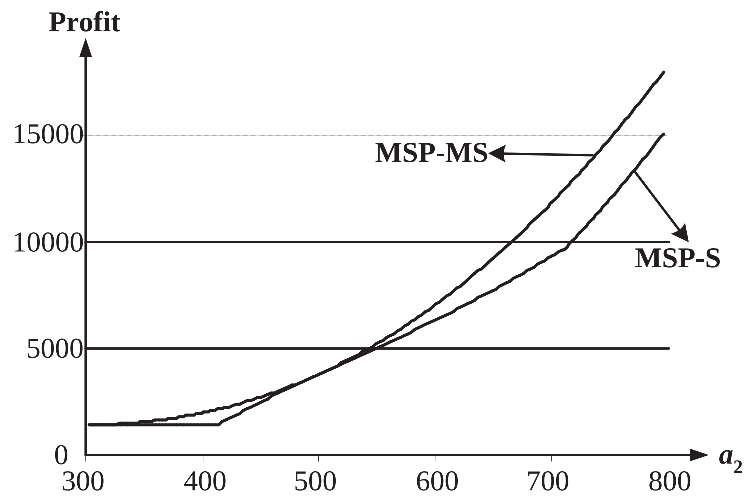

Figure 3-3 Stochastic demand analysis: profit as a function of a_2.

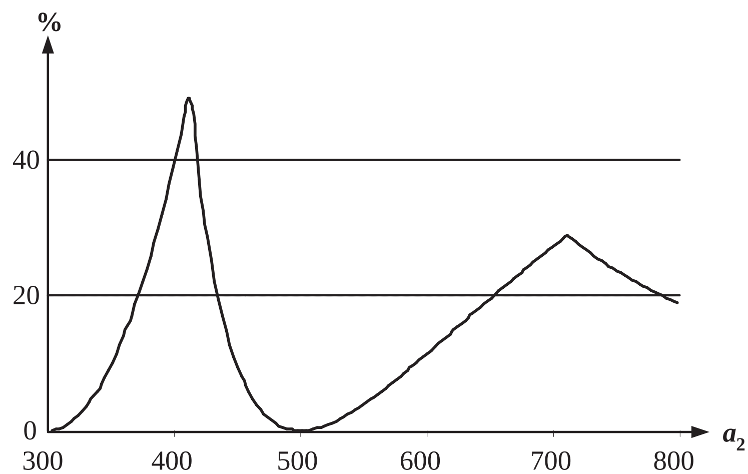

Figure 3-4 Stochastic demand analysis: percentage difference in profits.

We next perform a thorough computational analysis to study the market selection and pricing decisions in a Newsvendor setting, and to highlight the differences between the 'single pricing' and 'market-specific pricing' strategies.

3.3.2.1 Linear demand model

We start with the linear demand model and consider all combinations of the following parameter sets:

$(a_1, a_2) = \{(450, 450), (650, 450), (850, 450), (650, 650), (850, 650), (850, 850)\}$, $b_1 = \{2, 3, 4\}$, $b_2 = \{2, 3, 4\}$, $e = \{200, 300, 400\}$, $c = 100$, $v = 50$, $cv_1 = \{1/7, 1/5, 1/3\}$ and

Table 3-3: Market selection decisions for the Newsvendor model.

	a_1-a_2					
b_1-b_2	$450-450$	$650-450$	$850-450$	$650-650$	$850-650$	$850-850$
$2-2$	1/1	1/1	1/0	1/1	1/1	1/1
$2-3$	1/0	1/0	1/0	1/1	1/0	1/1
$2-4$	1/0	1/0	1/0	1/0	1/0	1/0
$3-2$	0/1	1/1	1/1	1/1	1/1	1/1
$3-3$	1/1	1/0	1/0	1/1	1/1	1/1
$3-4$	1/0	1/0	1/0	1/1	1/0	1/1
$4-2$	0/1	1/1	1/1	0/1	1/1	0/1
$4-3$	0/1	1/1	1/0	1/1	1/1	1/1
$4-4$	*/*	1/0	1/0	1/1	1/1	1/1

$cv_2 = \{1/7, 1/5, 1/3\}$, where cv_i denotes the coefficient of variation in market i. Note that although our models and solution approaches introduced in Sections 3.1 and 3.2 apply under general standard deviation functions, we assume $\sigma_i(p) = q_i(p)/cv_i$ to employ a single parameter for the uncertainty in the system, since our primary goal is to analyze the effects of such parameters on market selection decisions. Appendix A.5 contains a brief discussion of the implications of using different functional forms for the representation of $q_i(p)$ and $\sigma_i(p)$.

Table 3-3 provides the market selection decisions for (MSP-S) under different market sizes and price sensitivities where the entries are of the form (y_1^*/y_2^*). For all cases except when $a_1 = a_2 = 450$ and $b_1 = b_2 = 4$, the selection decisions are consistent across all remaining parameter settings, whereas for this particular case, the selection decisions are not consistent across all other parameter settings. In this case, however, the expected profit of the firm is so low that the market selection decisions are relatively unimportant. Hence, we do not consider this case for further analysis.

Observation 3.1. *Under (MSP-S), a market that is not selected at a low market size may become attractive when potential market size increases or price sensitivity decreases. More interestingly, increasing (decreasing) potential market size (price sensitivity) might cause the other market, which is attractive currently, to be dropped.*

Consider the case where $(a_1, a_2) = (450, 450)$ and $(b_1, b_2) = (3, 2)$, for instance. In this case, only M2 is selected. As b_2 increases, M2 becomes less attractive and is dropped when $b_2 = 4$. Although there is no corresponding change in its parameters, M1 becomes more attractive and is selected when $b_2 = 3$ or $b_2 = 4$. When the sensitivity of M1 is high relative to that of M2, M1 is not selected; however, when the same sensitivity is relatively small, it becomes the only profitable option.

Observation 3.2. *Under (MSP-S), an increase in the coefficient of variation of a selected market's demand results in an increase in the price for all markets.*

In this case, the firm's only tool to compensate for increased uncertainty in any market is the single price, and both markets are negatively affected by the resulting increase in price. Recall that one of the favorable aspects of single pricing is the fairness among markets that it provides. However, Observation 3.2 indicates that under (MSP-S), customers in a market are negatively affected by a change in another market, which calls into question the actual fairness of a single-price strategy.

Observation 3.3. *Under (MSP-S), a market that is currently selected may be dropped because of an increase in the coefficient of variation of the demand in any one of the markets.*

Note that the increasing price that results from a market's increase in uncertainty (see Observation 3.2) implies that the expected demands in both markets decrease. This decrease may cause a smaller market to be dropped because this provides the firm an opportunity to further increase price in larger markets and possibly generate greater profit. The parameter set given at the beginning of this section does not provide an example for the case where a market is dropped due to an increase in the coefficient of variation. In order to analyze this phenomenon more closely, we consider the following example. Let $(a_1, a_2) = (446, 410)$, $(b_1, b_2) = (3.2, 2.2)$, $e = 300$, $c = 100$ and $v = 50$. The selection and pricing decisions of (MSP-S) for this example are reported in Table 3-4. Note that the first market is dropped when the coefficient of variation of either market increases.

Table 3-4: Effects of uncertainty on market selection decisions.

cv_1	cv_2	p	y_1	y_2	$G(p,y)$	$(a_1 - b_1 p)y_1$	$(a_2 - b_2 p)y_2$
0.33	0.33	154.85	0	1	2185.27	0.00	69.34
0.33	0.20	150.18	0	1	2880.22	0.00	79.60
0.33	0.14	133.58	1	1	3283.08	18.54	116.12
0.20	0.33	154.85	0	1	2185.27	0.00	69.34
0.20	0.20	150.18	0	1	2880.22	0.00	79.60
0.20	0.14	132.30	1	1	3342.29	22.65	118.95
0.14	0.33	154.85	0	1	2185.27	0.00	69.34
0.14	0.20	132.48	1	1	2892.96	22.08	118.55
0.14	0.14	131.84	1	1	3363.65	24.11	119.95

Interestingly, M1 is more vulnerable to changes in cv_2 than it is to cv_1. For instance, when $cv_2 = 1/7$, M1 is not dropped even when cv_1 is as large as $1/3$. On the other hand, even if $cv_1 = 1/7$, it is dropped when $cv_2 = 1/3$, which can be explained as follows. Since the expected demand in M1 is far less than M2, an increase in cv_1 does not have a substantial effect on the aggregate standard deviation seen by the supplier. Hence, the supplier can still afford to select M1. However, an increase in cv_2 would result in considerably larger aggregate standard deviation. In this case, the firm may drop M1 to further increase the price and balance the increase in standard deviation.

Another parameter that affects selection and pricing decisions is the shortage cost. As shortage cost increases, the cost of the firm due to uncertainty increases. Hence, we expect that the firm would increase the price to decrease the aggregate variation. Similar to the results for an increase in coefficient of variation, increasing shortage cost also affects the selection decisions under (MSP-S); that is, an increase in shortage cost may force the firm to drop a currently selected market under (MSP-S). Consider the example above with $cv_1 = 1/7$ and $cv_2 = 1/5$. Both markets are selected when the shortage cost is \$300. When it increases to \$400, the supplier increases the price as an attempt to decrease the uncertainty, which causes the first market to be dropped.

We now compare the single-price strategy to the market-specific pricing strategy. Table 3-5 reports the average percentage difference in profits for different market sizes and price sensitivities. When attempting to interpret trends in these percentage differences,

Table 3-5: Average percentage differences in profits for the Newsvendor model.

	a_1-a_2					
b_1-b_2	$450-450$	$650-450$	$650-650$	$850-450$	$850-650$	$850-850$
$2-2$	0.19	11.12	0.06	15.42	4.01	0.03
$2-3$	25.67	7.59	14.37	3.60	19.71	9.87
$2-4$	1.99	0.61	15.40	0.29	7.33	23.99
$3-2$	25.67	0.26	14.37	4.98	1.20	9.87
$3-3$	1.42	20.88	0.22	8.01	7.10	0.09
$3-4$	10.66	1.65	17.24	0.65	16.29	8.20
$4-2$	1.99	19.78	15.40	0.31	15.80	23.99
$4-3$	10.66	4.90	17.24	17.48	0.23	8.20
$4-4$	20.62	5.14	0.86	1.40	15.57	0.24

we must keep in mind that the corresponding market selection decisions may change as we change parameter values (see Table 3-3). We can, however, draw certain conclusions based on these results, the first of which is fairly intuitive.

Observation 3.4. *The advantage of offering different prices in different markets becomes quite small when (a) the prices in the (MSP-MS) case are close to each other; (b) one of the markets generates much less profit than the other.*

In the first case (part (a)), (MSP-S) selects both markets and the optimal single price is close to the optimal market-specific prices for both markets. For instance, when $(a_1, a_2) = (650, 450)$, $(b_1, b_2) = (3, 2)$, $(cv_1, cv_2) = (1/7, 1/3)$ and $e = 200$, (MSP-MS) sets $(p_1, p_2) = (160.48, 170.09)$ and generates an expected profit of \$15,502. (MSP-S), in this case, selects both markets and sets $p = 164.31$ generating \$15,384. In the second case (part (b)), (MSP-S) selects the market with the higher profit only. Since the other market does not contribute to the profits significantly, it does not hurt (MSP-S) not to select it. For instance, when $(a_1, a_2) = (650, 450)$, $(b_1, b_2) = (3, 4)$, $(cv_1, cv_2) = (1/3, 1/3)$ and $e = 200$, (MSP-MS) sets $(p_1, p_2) = (167.34, 107.47)$. The mean demand and the profit margin in M2 are 20.1 and \$7.47 respectively, far less than M1. In this case, (MSP-S) selects only M1 and sets $p = 167.42$. The expected profits generated by (MSP-MS) and (MSP-S) are \$7,400 and \$7,275, respectively.

Observation 3.5. *Under (MSP-MS), an increase in the coefficient of variation of a market's (say M1) demand causes the price in that market to increase whereas the price in the other market (say M2) decreases.*

The first part of Observation 3.5 is obvious; since the standard deviation increases with the market demand, the firm increases the price to balance the increase in uncertainty. The fact that a market's price may decrease in response to an increase in another market's uncertainty, however, is surprising, and can be explained as follows. Because M2 becomes relatively less uncertain, the supplier decreases the price in this market to balance the decrease in total demand due to the higher price required in M1. Hence, we may conclude that the buyers in M2 face a lower price as a result of an increase in uncertainty in M1, although the characteristics of M2 are unchanged.

In summary, with a linear demand model, our computational results indicate that potential market size and price sensitivity are critical factors in driving market selection decisions, although coefficient of variation and shortage costs may also play a significant role in certain situations. (MSP-MS) always outperforms (MSP-S) as expected. Yet, the magnitude of the difference depends on the relative cost parameters, the similarities between markets in terms of resulting pricing decisions, and the coefficient of variation of the demands. We also observe that a market may be negatively affected (even dropped) because of the changes in the other market under (MSP-S), which makes the fairness assertion of (MSP-S) questionable.

3.3.2.2 Iso-elastic demand model

We next consider the iso-elastic demand model and highlight its differences from the linear model. The expected demand in market i is given by $d_i(p_i) = \alpha_i p_i^{-\beta_i}$, and we consider all the combinations of the following parameter sets (where 1M = 1 million): $(\alpha_1, \alpha_2) = \{(1\text{M}, 1\text{M}), (2\text{M}, 1\text{M}), (2\text{M}, 2\text{M}), (3\text{M}, 1\text{M}), (3\text{M}, 2\text{M}), (3\text{M}, 3\text{M})\}$, $\beta_1 = \{1.4, 1.7, 2\}$, $\beta_2 = \{1.4, 1.7, 2\}$, $e = \{200, 300, 400\}$, $c = 100$, $v = 50$, $cv_1 = \{1/7, 1/5, 1/3\}$ and $cv_2 = \{1/7, 1/5, 1/3\}$.

Observation 3.6. *With the iso-elastic demand, both markets are selected in all instances regardless of the pricing strategy.*

This results because there is positive demand for any price level in each market. Hence, contrary to the linear demand model, the supplier does not require dropping one of the markets under a single pricing strategy in order to set the price high enough. Subsequent observations further compare the results of an iso-elastic demand model to a linear demand model.

Observation 3.7. *As in the linear demand model, under (MSP-S), an increase in the uncertainty of a market's demand results in an increase in the price.*

Observation 3.8. *The percentage difference of expected profits under the (MSP-MS) and (MSP-S) strategies are not as large as in the linear demand model.*

Observation 3.9. *As in the linear demand model, under (MSP-MS), an increase in the uncertainty of a market's demand causes the price in that market to increase whereas the price in the other market decreases.*

In summary, with an iso-elastic demand model, both markets are effectively selected under both pricing schemes. The difference in profits generated by different pricing schemes is not as significant as in the linear demand model. Hence, the supplier may employ the single pricing scheme without significant loss if the demands in the markets are iso-elastic. The pricing decisions are affected by a change in the cost/uncertainty parameters in the same way as the linear model.

Throughout our analysis of the Newsvendor model, we have assumed a constant coefficient of variation for both the linear and iso-elastic demand models, in order to understand the effects of uncertainty on market selection and pricing decisions. In order to examine the validity of our observations in the absence of this assumption, we performed computational tests with more general $\sigma_i(p)$ functions to see whether our findings apply to those cases. As we discuss in greater detail in Appendix A.5, under mild assumptions

Table 3-6: Market selection decisions for the EOQ model.

	a_1-a_2					
$b_1 - b_2$	$450-450$	$650-450$	$850-450$	$650-650$	$850-650$	$850-850$
$2-2$	1/1	1/1	1/0	1/1	1/1	1/1
$2-3$	1/*	1/0	1/0	1/1	1/0	1/1
$2-4$	1/0	1/0	1/0	1/0	1/0	1/0
$3-2$	*/1	1/1	1/1	1/1	1/1	1/1
$3-3$	1/1	1/*	1/0	1/1	1/1	1/1
$3-4$	1/0	1/0	1/0	1/1	1/0	1/1
$4-2$	0/1	1/1	1/1	0/1	1/1	0/1
$4-3$	0/1	1/1	1/*	1/1	1/1	1/1
$4-4$	0/0	1/0	1/0	1/1	1/1	1/1

(in particular, if $q_i(p)$ and $\sigma_i(p)$ approach zero at the same point or at the same rate) the observations we have discussed continue to hold.

3.3.3 Economic Order Quantity Model

We now focus on the deterministic, continuous, price-sensitive demand model, and highlight its similarities with the Newsvendor model in terms of market selection and pricing decisions.

3.3.3.1 Linear Demand Model

We consider the following parameter sets for the linear demand model: $(a_1, a_2) = \{(450, 450), (650, 450), (850, 450), (650, 650), (850, 650), (850, 850)\}$, $b_1 = \{2, 3, 4\}$, $b_2 = \{2, 3, 4\}$, $K = \{200, 300, 400\}$, $c = 100$, $h_1 = \{10, 20, 30\}$ and $h_2 = \{10, 20, 30\}$.

Table 3-6 provides the market selection decisions for (MSP-S) under different potential market sizes and price sensitivities.

The market selection decisions are almost identical to the Newsvendor model, which is quite intuitive since both models can be represented with almost the same mathematical model. Hence, our observations in Section 3.3.2.1 regarding the relation between market selection and potential market size and price sensitivity are also valid for the EOQ model (See Observation 3.1). The differences in the expected profits resulting from the (MSP-MS) and (MSP-S) strategies also follow a similar pattern to the Newsvendor model for the same reason (See Table 3-7 and Observation 3.4).

Table 3-7: Average percentage differences in profits for the EOQ model.

	$\mathbf{a_1 - a_2}$					
$b_1 - b_2$	$450 - 450$	$650 - 450$	$850 - 450$	$650 - 650$	$850 - 650$	$850 - 850$
$2 - 2$	0.01	8.72	14.65	0.00	3.42	0.00
$2 - 3$	23.46	6.83	3.25	11.78	19.11	8.67
$2 - 4$	0.92	0.33	0.18	14.75	6.98	23.69
$3 - 2$	23.46	0.14	3.41	11.78	1.10	8.67
$3 - 3$	0.12	18.05	6.99	0.01	5.29	0.00
$3 - 4$	4.65	0.81	0.36	11.41	15.05	6.45
$4 - 2$	0.92	14.92	0.29	14.75	13.40	23.69
$4 - 3$	4.65	1.63	14.59	11.41	0.04	6.45
$4 - 4$	n/a	2.21	0.71	0.04	9.36	0.01

We next focus on how fixed ordering cost and holding costs affect the market selection and pricing decisions.

Observation 3.10. *Under (MSP-MS), an increase in the holding cost of a market causes the price in that market to increase and whereas the price in the other market decreases.*

When holding cost increases, the supplier tends to have less inventory in that market. As the demand rate decreases in price and a lower demand rate results in lower order quantities and lower inventories, the price in that market is increased. For the other market, the relative holding cost is decreased, and as a result the price is decreased as well, to drive a larger demand rate.

Observation 3.11. *Under (MSP-S), an increase in the holding cost of a selected market results in an increased price.*

Observation 3.12. *A market that is currently selected may be dropped because of an increase in the holding cost of any one of the markets.*

Consider the instance with $a_1 = 650$, $a_2 = 450$, $b_1 = 3$, $b_2 = 3$, and $K = 300$. When $h_1 = h_2 = 10$, both markets are selected and the price is set at \$142.91, resulting in a demand rate of 221.27 and 21.27 in markets 1 and 2, respectively. As h_2 increases, it is no longer profitable for the supplier to serve market 2 since dropping it also enables to increase the price in market 1 to its individual optimal. As h_1 increases, the supplier increases the price to decrease the holding costs. Since the demand rate of market 2 is already low, the supplier no longer serves market 2 and sets the price in market 1 to its individual optimal.

Observation 3.13. *Under (MSP-S), an increase in the fixed ordering cost may cause the market with the lower demand to be dropped.*

With higher setup costs, the supplier would like to have fewer orders. Since decreasing the demand rate contributes to this goal, he may drop the market with the lower profit margin and increase the price so that the attractive market will have a lower demand rate with a higher profit margin. Consider the previous instance with $h_1 = h_2 = 10$. When $K = 200$ or $K = 300$, both markets are selected. However, when $K = 400$, the second market is dropped and the price is increased sharply to increase the profit margin of the first supply.

Note that these observations are also similar to the ones that we have for the coefficient of variation and the shortage cost in the Newsvendor model.

3.3.3.2 Iso-elastic Demand Model

For the iso-elastic demand case, we use the following parameter sets: $(\alpha_1, \alpha_2) = \{$(1M, 1M),(2M, 1M),(2M, 2M),(3M, 1M), (3M, 2M),(3M, 3M)$\}$, $\beta_1 = \{1.4, 1.7, 2\}$, $\beta_2 = \{1.4, 1.7, 2\}$, $K = \{200, 300, 400\}$, $c = 100$, $h_1 = \{10, 20, 30\}$ and $h_2 = \{10, 20, 30\}$.

As in the linear demand model cases, the computational analysis of the iso-elastic demand for the EOQ model reveal almost identical results to the Newsvendor model. Namely, all markets are selected at all instances regardless of the pricing strategy. The differences between the (MSP-MS) and (MSP-S) strategies with respect to the profits are not as large as the linear model.

Our analysis of the EOQ model with both linear and iso-elastic demand models reveals that the market selection decisions are almost identical to the Newsvendor model, which is quite intuitive since both models can be represented with almost the same mathematical model. Our observations in Section 3.3.2 regarding the relation between market selection and potential market size and price sensitivity, and the difference in expected profits have similar counterparts in the EOQ model. Moreover, the effects of holding and fixed ordering costs on market selection and pricing decisions are similar to the effects of uncertainty and shortage cost in the Newsvendor model, respectively.

3.4 Conclusion

This chapter considered the integration of two strategic demand management tools, pricing and market selection, through a nonlinear optimization model that involves binary variables together with continuous variables, and introduced an efficient solution algorithm for this problem. The formulation applies to a number of inventory problems involving simultaneous market/order selection, pricing and quantity decisions under certain assumptions. Two inventory models were explicitly discussed. The first one is a Newsvendor-type problem in multiple markets with pricing, where a supplier chooses the optimal set of markets to supply. We incorporated randomness in each market's demand in such a way that the standard deviation of the demand increases with the expected demand and showed that both multiplicative and additive randomness models can be handled in this manner. The second one is an EOQ type problem in multiple markets with pricing. We investigated two different pricing strategies for these models, where (i) the firm chooses to offer a single price in all selected markets, (MSP-S) and (ii) for each market, a different price is set, (MSP-MS).

The solution algorithm for the 'single-price' strategy employs the 'Decreasing Expected Revenue to Uncertainty' (DERU) ratio (Taaffe et al. (2006)) to determine relative market attractiveness for any given price. In particular, we generate a finite number of price intervals within which the rank order of the markets according to the DERU ratio does not change, and for each interval, we find the optimal price. This idea then leads to an efficient algorithm that solves the market selection problem with a single-price constraint. Under the 'market-specific pricing' strategy, we first observe that the market selection variables can be omitted. This yields a continuous nonlinear objective function in the price variables. Under mild conditions, we showed that the objective function is jointly concave, and hence the problem is efficiently solvable. We performed an extensive numerical analysis in order to further understand the dynamics of both strategies and observe their reactions to changing market conditions and cost parameters.

CHAPTER 4
ANALYSIS OF ORDER TIMING TRADEOFFS IN MULTI-RETAILER SUPPLY SYSTEMS

One of the key challenges in supply chain management is matching supply with demand, and supply chain members typically have conflicting objectives regarding this issue. While downstream members insist on responsive suppliers, upstream members are reluctant to allocate their supply before having information about customer orders. The CEO of Xilinx, a leading firm in electronics distribution, summarizes this problem as follows (Wilson (2004)):

> While it takes three to four months to print silicon, our customers want to promise to supply their customers in two to three weeks.

In order to cope with this problem, supply chain partners are working more closely and companies are now approaching their suppliers as ‘long-term’ partners. For instance, in the automobile sector, most of the manufacturers have launched programs to reduce the total number of direct suppliers and establish strategic partnerships with them (Bensaou (1999)). These long-term relationships can provide major benefits for both parties. While the upstream supplier provides the investment and technology for the downstream partner, the supplier in turn receives a more stable demand stream.

In this chapter, we investigate supplier-buyer relations in terms of the timing of order commitments and its implications for both the upstream and downstream tier in a single-period context. Specifically, we consider a two-echelon supply chain consisting of a single manufacturer (supplier) and multiple retailers (buyers), where buyers face stochastic demands. Since the manufacturing lead time is long compared to the selling season, the supplier must decide on the production quantity in advance of the selling season, and hence before demand uncertainty is resolved. An important question in supplier-buyer relations is the allocation of demand and supply risks among the parties. If buyers order after observing demand, the supplier is forced to decide on the production quantity under uncertainty and hence bears the ‘demand risk’. In this case, the buyers’ order quantities are limited by the supplier's production quantity, and they may face lost sales when the production quantity is less than the demand. We refer this phenomenon as the 'supply risk'. The buyers can avoid the supply risk by committing to an order quantity before the production run since the supplier can now produce and deliver the orders in full. However, the buyers bear the 'demand risk' in this setting.

Order timing decisions are particularly important in some industries, such as fashion apparel and consumer electronics. Cachon (2004) gives two examples from the sporting goods industry: Trek Inc. (a bicycle manufacturer) sells through independent retailers and bears all demand risk while O'Neill Inc. (apparels and accessories for water sports) accepts 'prebook' (early) and 'at-once' (in the selling season) orders from the retailers. 'Prebook' orders are placed in advance of the selling season and the retailers are guaranteed to receive these orders in full, but they assume all demand risk. O'Neill Inc. satisfies 'at-once' orders if inventory is available. Jin and Wu (2007), as part of a project with a major telecommunications component manufacturer in the U.S, investigate the role of capacity reservation contracts in high-tech industries. Gilbert and Ballou (1999) illustrate how advance information can be exploited to reduce operating costs, motivated by their work with a steel distributor.

In many industries, although suppliers accept orders from a large number of buyers, a few of these buyers are strategic partners with the supplier and they sometimes constitute the majority of the supplier's business. For instance, Shiroki Corporation, an automotive parts supplier, provides a wide variety of automotive parts to a large number of automobile manufacturers including Toyota, Honda, Nissan and Mitsubishi. However, its business with the Toyota Group constitutes more than half of Shiroki Corporation's sales. Another example is Quantum, a global leader in data storage solutions. Of all of its customers, Hewlett-Packard and Dell constituted around 40% of Quantum's revenues in 2004 and 2005. Jin and Wu (2007) discuss similar observations for the semiconductor industry. To reflect this phenomenon in our study, we designate one of the customers as the supplier's 'primary customer', and investigate the dynamics between the primary customer and the supplier in terms of the timing of the order commitment. The supplier works closely with the primary customer, and therefore has a strong relationship with and interest in approaching her with an order-timing scheme that may be mutually beneficial. Since our focus is on the relationship between the supplier and the primary customer, all remaining customers in our model are aggregated into a single party.

The suppliers and their customers usually have conflicting objectives with respect to order timing and commitment. Although the suppliers prefer a robust and predictable demand stream, we observe in many industries that they do not have firm commitments from their (possibly more powerful) customers that would achieve this outcome. Consider the situation faced by Mattel Inc., an American toy company (Kravetz (1999)):

> Mattel was hurt last year by inventory cutbacks at Toys "R" Us, and officials are eager to avoid a repeat of the 1998 Thanksgiving weekend. Mattel had expected to ship a lot of merchandise after the weekend, but retailers, wary of excess inventory, stopped ordering from Mattel. That led the company to report a $500 million sales shortfall in the last weeks of the year.... For the crucial holiday season of this year, Mattel sail it will require retailers to place their full orders before Thanksgiving. And, for the first time, the company will no longer take re-orders in December, Ms. Barad said. This will enable Mattel to tailor production more closely to demand and avoid building inventory for orders that don't come.

Although some suppliers can tailor the order timing of their retailers to their benefit, some may lack the channel power to do so. For instance, Quantum Corporation states in its 2005 annual report that more than half of its sales come from a few customers who have no minimum or long-term purchase commitments. Similarly, Jabil Circuit, which is the sole supplier to Quantum for certain products, notes in their 2005 annual report that their customers do not commit to firm production schedules in advance, which makes it difficult for Jabil to schedule production and maximize the utilization of the production facility. Hence, we consider a system in which the status quo involves delayed commitment for the primary retailer, i.e., the supplier produces in advance of the selling season, and the primary retailer places her order at the beginning of the selling season, when she has demand information. We develop models to analyze the expected profits of the supplier and its primary customer under two different settings. In the first one, the primary customer places an order after the realization of the demand. Although this helps her avoid the demand risk associated with demand uncertainty, she faces supply risk since the supplier, having produced in advance, rations the available quantity among the retailers according to a generalized uniform allocation rule (the details of this rule are discussed in subsequent sections), and this production quantity may not be sufficient to cover all demand. In the second case, the primary customer orders in advance of the selling season, before the supplier makes the production quantity decision, and has priority in inventory allocation. Hence, she eliminates the supply risk while taking on part of the demand risk. Following the terminology in Ferguson et al. (2005), we phrase the first case as the 'delayed commitment' model and the second case as the 'early commitment' model.

Note that the secondary retailer's order timing has not been specified. We assume that the secondary retailer is actually a collection of one or more retail customers who utilize the supplier as a secondary source of overflow supply post demand realization. That is, they might primarily be working with other suppliers, and contact our supplier only when their initial supply is insufficient. In this case, ordering after the demand realization is a natural result. Therefore, in our analysis, the secondary retailer's order timing is given and we focus on the relationship between the primary retailer and the supplier.

In this problem environment, we determine conditions under which the supplier (or the primary customer) prefers early (or delayed) commitment. We identify and model the tradeoff between demand and supply risks, and provide analytical results and insights with respect to this tradeoff. We derive conditions on system parameters that make an early (or delayed) commitment scheme pareto-optimal. We also compare a single-buyer system to our current setting and analyze the effect of the secondary retailer on the order timing preference of the primary retailer. We also examine the interaction between the retailers when both retailers has the flexibility to choose from the commitment schemes. Finally, we incorporate a production capacity for the supplier into our model and investigate its effects on the order timing preferences of the primary retailer and the supplier.

We observe that the supplier prefers the early commitment scheme under mild conditions. The preference of the primary retailer depends on the tradeoff between the supply and demand risks. Our findings indicate that she rarely prefers early commitment, which occurs when the supply risk is more critical. We also observe that both the supplier and the primary retailer benefit from the existence of another retailer (or collection of retailers) under delayed commitment in certain settings, which at first seems to be a counter-intuitive result because of the need to share the supplier's rationed output. However, our analysis reveals that in such cases, the increase in the supplier's production quantity due to the second retailer's demand provides the primary retailer with a larger reserved inventory than it would have in a single-retailer system. The supplier, in turn, gets to pool the uncertainty he faces while increasing expected sales. Hence, although an early commitment scheme is likely to be better for the supplier, he might consider the alternative of diversifying the retailer pool when his primary customer is reluctant to enter an early commitment agreement. More interestingly, the primary retailer, who prefers delayed commitment in the first place, may prefer to share the supplier's output with a non-competing retailer.

The rest of this chapter is organized as follows. In Section 4.1, we formally describe the problem environment, our assumptions, and the basics of the delayed and early commitment models. In Section 4.2, we analyze the expected profits of the supplier and primary manufacturer under both commitment schemes. Section 4.3 identifies the differences between a single-retailer system and a multi-retailer system. After numerically comparing the commitment schemes under various parameter settings in Section 4.4, we analyze the strategic interaction between the retailers in terms of order timing in Section

4.5. Section 4.6 investigates a capacitated setting and its implications regarding the order timing. We conclude this chapter in Section 4.7 by highlighting our findings.

4.1 Problem Description and Model Analysis

We consider a supply chain consisting of a single supplier (S) and two retailers (R_1 and R_2) in a single period setting. The retailers face independent, stochastic demands, modelled as continuous random variables X_1 and X_2 with probability density functions $f_1(\cdot)$ and $f_2(\cdot)$, respectively. $F_i(\cdot)$ denotes the cumulative distribution function of X_i. Because of the long production lead time, the supplier must make its production quantity decision, Q_S, in advance of the selling season, and thus before the uncertainty in demand is resolved. The order quantities of the retailers are denoted by Q_i. R_2 orders post demand realization. The order timing of R_1, which is the primary customer of the supplier, depends on the commitment scheme. We consider two different commitment structures; under early commitment, R_1 places an order before the production run of the supplier and hence before the realization of demand, whereas under delayed commitment, she orders after the realization of demand. The major differences between the early and delayed commitment schemes stem from the allocation of demand risk between the supplier and the retailers, the allocation of supply risk, and the allocation of inventory between the retailers. Under delayed commitment, the supplier bears all demand risk and does not differentiate between the retailers, allocating inventory according to a generalized uniform allocation rule when the production quantity is insufficient to satisfy the total demand. Note that even if there is a single retailer, supply risk still exists because the production quantity of the supplier creates a supply limitation for the retailer. In a multiple retailer system, this risk may increase or decrease depending on the demand parameters and the allocation of inventory among the retailers. Under early commitment, on the other hand, the primary retailer assumes her own demand risk, decreasing the uncertainty faced by the supplier. The supplier, in return, provides the primary retailer with priority in order fulfillment, which eliminates the supply risk for the primary retailer.

The selling price of the product and the transfer price between the supplier and retailer are exogenous and are denoted by r and w, respectively. Different unit revenues for the retailers can be handled with no additional effort. The unit production cost of the supplier is c. All information regarding demand, revenue and cost figures is symmetric, i.e., available to all parties. We do not include any explicit penalty costs associated with lost demand or any salvage value/cost associated with unsold inventory, and the retailers do not add any value to the product, although these extensions may be incorporated into the model easily. The profit function for the supplier under the delayed and early commitment schemes are represented by Π_S^D and Π_S^E, and for the primary retailer, they are Π_1^D and Π_1^E.

4.1.1 Delayed Commitment

When both retailers order after demand realization, the supplier employs a 'generalized uniform allocation rule' to allocate the production quantity between the retailers, and the quantity received by R_1 under this rule is given by Equation (4–1):

$$q_1 = \begin{cases} Q_1 & Q_1 + Q_2 \leq Q_S \\ \min\{Q_1, \rho Q_S\} & Q_1 + Q_2 > Q_S, Q_2 > (1-\rho)Q_S \\ \max\{\rho Q_S, Q_S - Q_2\} & Q_1 + Q_2 > Q_S, Q_1 > \rho Q_S \end{cases} \tag{4–1}$$

where ρ denotes the percentage of the supplier's production quantity reserved for R_1.

The essence of generalized uniform allocation can be explained as follows: The supplier reserves a certain fraction of the production quantity for each retailer; that is, each retailer is assured to have at least a fraction of the production no matter what the order size of the other retailer is. Furthermore, if one of the retailers orders less than her reserved amount, the remaining part of her share can be utilized to serve the other retailer when required. Proposition 4.1 characterizes the retailers' order quantities when the supplier allocates inventory according to a generalized uniform allocation mechanism.

Proposition 4.1. *In the presence of a generalized uniform allocation mechanism, the retailers' order quantities under delayed commitment match their respective demand realizations.*

Proof. We first prove the proposition for R_1. If $x_1 < \rho Q_S$ or $x_1 + Q_2 < Q_S$, R_1 will receive x_1 units if she orders x_1 units. Hence, she does not have any incentive to order more since ordering more results in having more inventory than demand. If $x_1 > \rho Q_S$ and $x_1 + Q_2 > Q_S$, ordering more than x_1 will not change her allocation (see Equation 4–1). Hence, given Q_S and Q_2, R_1 will order her demand realization. Similar arguments are also valid for R_2. That is, given Q_S and Q_1, we have $Q_2 = x_2$. Hence, at the equilibrium, both retailers order their true needs. □

The generalized uniform allocation rule prevents the retailers from manipulating their order quantities in order to get a greater share from the supplier since the allocation rule does not rely on the order quantities. Hence, the order quantities of the retailers match their demands under the delayed commitment scheme regardless of the production quantity of the supplier, i.e., $Q_1 = x_1$ and $Q_2 = x_2$. The reader may refer to Cachon and Lariviere (1999) for a detailed discussion of the allocation mechanisms.

Since the supplier does not differentiate between the retailers under the delayed commitment scheme, he faces an aggregate demand of $Z = X_1 + X_2$, and his profit is given by Equation (4–2).

$$\Pi_S^D(Q_S) = \begin{cases} w(x_1 + x_2) - cQ_S & x_1 + x_2 \leq Q_S \\ (w - c)Q_S & x_1 + x_2 > Q_S \end{cases} \tag{4–2}$$

Hence, the supplier's problem is $\max_{Q_S} \; E[\Pi_S^D(Q_S)]$. $E[\Pi_S^D(Q_S)]$ is characterized in Equation (4–3) where $h(z)$ and $H(z)$ denote the *pdf* and *cdf* of $Z = X_1 + X_2$ respectively.

$$E[\Pi_S^D(Q_S)] = (w - c)(\mu_1 + \mu_2) - \left[(w - c)\int_{Q_S}^{\infty}(z - Q_S)h(z)dz + c\int_{0}^{Q_S}(Q_S - z)h(z)dz\right] \tag{4–3}$$

This is the well-known newsvendor problem and the optimal solution is $Q_S^* = H^{-1}(\frac{w-c}{w})$. Note that the second term in Equation (4–3) quantifies the demand risk of the supplier since the supplier would generate a profit of $(w-c)(\mu_1+\mu_2)$ if the retailers' demands were deterministic.

We can now derive the expected profit function of the primary retailer, R_1. Recall that her order quantity is equal to the demand realization. Hence, her profit can be written as $(r-w)q_1$. Utilizing Equation (4–1), Equation (4–4) presents the expected profit of R_1:

$$\begin{aligned} E[\Pi_1^D] &= (r-w)\mu_1 - (r-w)\Bigg[\int\limits_0^{(1-\rho)Q_S}\int\limits_{Q_S-x_2}^{\infty}(x_1+x_2-Q_S)dF_1(x_1)dF_2(x_2) \\ &\quad + \int\limits_{\rho Q_S}^{\infty}(x_1-\rho Q_S)[1-F_2((1-\rho)Q_S)]dF_1(x_1)\Bigg] \end{aligned} \tag{4–4}$$

If the supplier's production quantity was infinite, the primary retailer's expected profit would be $(r-w)\mu_1$. Hence, we can deduce that the second term in Equation (4–4) quantifies the supply risk of R_1 under a delayed commitment scheme.

4.1.2 Early Commitment

R_1 receives her order in full under early commitment, i.e., $q_1 = Q_1$. Her expected profit, $(E[\Pi_1^E(Q_1)])$, is provided in Equation (4–5).

$$E[\Pi_1^E(Q_1)] = (r-w)\mu_1 - \left[(r-w)\int\limits_{Q_1}^{\infty}(x-Q_1)f_1(x)dx + w\int\limits_0^{Q_1}(Q_1-x)f_1(x)dx\right] \tag{4–5}$$

The optimal order quantity of R_1 is $Q_1^* = F_1^{-1}(\frac{r-w}{r})$. The second term in Equation (4–5) characterizes the decrease in R_1's expected profit due to the demand risk. However, the supply risk in the delayed commitment scheme (see Equation 4–4) diminishes under an early commitment scheme.

Since all information is symmetric, the supplier infers the order quantity of R_1 and produces exactly Q_1 units for R_1. The production quantity for R_2 must be determined under uncertainty. Letting $\hat{Q}_S$ denote the production quantity for the second retailer, we have $Q_S = Q_1 + \hat{Q}_S$. The supplier's expected profit function is presented in Equation (4–6).

$$E[\Pi_S^E(\hat{Q}_S)] = (w-c)(Q_1+\mu_2) - \left[(w-c) \int_{\hat{Q}_S}^{\infty} (x - \hat{Q}_S) f_2(x) dx + c \int_{0}^{\hat{Q}_S} (\hat{Q}_S - x) f_2(x) dx \right] \quad (4\text{–}6)$$

The supplier's expected profit is concave in $\hat{Q}_S$ and the optimal solution is characterized by the first order condition: $Q_S^* = Q_1 + F_2^{-1}(\frac{w-c}{w})$. The supplier still faces some demand risk. However, the degree of this risk is reduced because only R_2's demand is uncertain as opposed to both retailer's demand under delayed commitment. Hence, we can deduce that the demand risk of the supplier decreases under an early commitment scheme.

4.2 Comparison of the Strategies

We now compare the delayed and early commitment strategies in terms of production quantity and expected sales of the supplier, inventory allocation of the retailers, and expected profits of the supplier and primary retailer. Our aim is to derive analytical expressions that provide insights on the value of early commitment for the supplier and the primary retailer, the tradeoff between demand and supply risks, and observe whether settings exist where one of the commitment schemes is pareto-optimal. Finally, we compare a single retailer system to our current setting and analyze the effect of the secondary retailer on the order timing preference of the primary retailer and the supplier. For this purpose, we assume that R_i faces normally distributed demand with mean μ_i, and standard deviation σ_i. Superscripts D and E denote delayed and early commitment for R_1, respectively.

Recall that ρ, the fraction of the supplier's production quantity reserved for R_1, is the parameter that characterizes the generalized uniform allocation rule. From this section on, we assume that the supplier rations the production quantity among the retailers in proportion to the expected demands. Hence, $\rho = \mu_1/(\mu_1 + \mu_2)$. This is a special case of the generalized uniform allocation rule and can be regarded as a 'fair' allocation rule since it reserves larger capacity for the retailer with potentially larger demand. We refer to this rule as 'proportional allocation' for the rest of the manuscript.

4.2.1 The Supplier

When R_1 operates under a delayed commitment scheme, the total order quantity of the retailers, $Z = X_1 + X_2$, is normally distributed with mean $\mu_T = \mu_1 + \mu_2$ and standard deviation $\sigma_T = \sqrt{\sigma_1^2 + \sigma_2^2}$. The optimal production quantity of the supplier and his corresponding expected profit are given as

$$\begin{aligned} Q_S^D &= \mu_T + k_m \sigma_T \quad \text{and} \\ E[\Pi_S^D(Q_S^D)] &= (w-c)\mu_T - \sigma_T(wL(k_m) + ck_m), \end{aligned}$$

respectively. The quantity $L(z) = \int_z^\infty (u-z)\phi(u)du$ is the standard normal loss function, $k_m = \Phi^{-1}(\frac{w-c}{w})$, and $\phi(.)$ and $\Phi(.)$ denote the *pdf* and *cdf* of the standard normal distribution, respectively.

Under an early commitment scheme, the corresponding production quantity and the expected profit of the supplier are

$$\begin{aligned} Q_S^E &= \mu_T + k_1\sigma_1 + k_m\sigma_2 \quad \text{and} \\ E[\Pi_S^E(Q_S^E)] &= (w-c)(\mu_T + k_1\sigma_1) - \sigma_2(wL(k_m) + ck_m), \end{aligned}$$

respectively, where $k_1 = \Phi^{-1}(\frac{r-w}{r})$.

Let $\Delta[Q_S]$ denote the increase in the supplier's production quantity due to early commitment. Accordingly, let $\Delta[S_S]$ and $\Delta[\Pi_S]$ denote the increase in expected sales and expected profit, respectively. Then,

$$\begin{aligned} \Delta[Q_S] &= Q_S^E - Q_S^D = \sigma_1 k_1 - (\sigma_T - \sigma_2)k_m \\ \Delta[S_S] &= E[S_S^E] - E[S_S^D] = \sigma_1 k_1 + (\sigma_T - \sigma_2)L(k_m) \end{aligned}$$

There are a number of observations that we can generate from $\Delta[Q_S]$ and $\Delta[S_S]$. The supplier's expected sales increase as a result of the early commitment scheme if $k_1 \geq -\frac{(\sigma_T - \sigma_2)}{\sigma_1} L(k_m)$. Note that $k_1 \geq 0$ is equivalent to R_1's optimal service level being greater than 0.5, which one would usually expect. Observe that $\Delta[S_S] - \Delta[Q_S] = (\sigma_T - \sigma_2)(L(k_m) + k_m) > 0$, i.e., the increase in the supplier's expected sales as a result of early commitment is always greater than the increase in his production quantity. This also indicates that a decrease in the production quantity does not affect expected sales as greatly. Also note that expected sales may increase even if the total production quantity decreases. That is, the increase in certainty for the supplier due to R_1's early order may allow it to reduce the total production quantity while increasing expected sales when compared to the delayed commitment case. For instance, consider a case where $k_m > 0$ and $-\frac{(\sigma_T - \sigma_2)}{\sigma_1} L(k_m) < k_1 < 0$. Then, the production quantity and expected demand of the supplier decrease due to early commitment whereas his expected sales increase.

The increase in the supplier's expected profit due to early commitment is

$$\Delta[\Pi_S] = (w - c)\sigma_1 k_1 + (\sigma_T - \sigma_2)(wL(k_m) + ck_m) = (w - c)k_1\sigma_1 + (\sigma_T - \sigma_2)w\phi(k_m) \quad (4\text{–}7)$$

If $\Delta[\Pi_S]$ is greater than zero, the supplier benefits from the early commitment of R_1. Recall that if R_1 orders after demand realization, the burden of demand risk will be on the supplier. When R_1 commits early, the supplier does not fully bear the demand risk with respect to R_1's demand and hence faces a decreased level of uncertainty. Another factor that affects the profitability of the supplier is the order quantity of R_1. If Q_1^E is greater than the expected order quantity under a delayed commitment (which is equal to the expected demand, μ_1), the supplier certainly benefits from early commitment, since both factors work in his favor. Otherwise, the tradeoff between uncertainty and expected sales determines which commitment scheme is more profitable. Proposition 4.2 formally defines this relationship.

Proposition 4.2. *If $k_1 \geq -\frac{\sigma_T - \sigma_2}{\sigma_1} \frac{w}{w-c} \phi(k_m)$, the supplier benefits from the early commitment scheme. Otherwise, delayed commitment is more profitable.*

Proof. The proposition directly follows from Equation (4–7). □

The relation between the order quantities of R_1 under delayed and early commitment schemes is determined by k_1, i.e., if $k_1 > 0$, the order quantity under early commitment is greater, resulting in higher expected profit for the supplier, which is apparent from Proposition 4.2. When the order quantity of R_1 decreases with the early commitment, the difference between profits depends on the tradeoff we discussed earlier; early commitment may still provide higher profit when the decrease is moderate.

We now analyze the effects of uncertainty on the benefits of early commitment to the supplier, starting with the uncertainty in R_1's demand. Proposition 4.3 illustrates how the improvements in the supplier's expected profit from early commitment change with respect to the uncertainty in R_1's demand.

Proposition 4.3. *$\Delta[\Pi_S]$ is increasing in σ_1 if and only if $k_1 \geq -\frac{\sigma_1}{\sigma_T} \frac{w}{w-c} \phi(k_m)$.*

Proof. The proposition follows from the derivative of the difference with respect to σ_1: $\frac{d\Delta[\Pi_S]}{d\sigma_1} = (w-c)k_1 + \frac{\sigma_1}{\sigma_T} w\phi(k_m)$ □

The condition in Proposition 4.3 holds automatically when $k_1 \geq 0$, which is a mild assumption if we consider its implications on the service level of R_1. Hence, we can deduce, in general, that early commitment becomes more favorable to the supplier as the level of uncertainty in R_1's demand increases. Let k_1^{p1} and k_1^{p2} denote the threshold values for k_1 in Propositions 4.2 and 4.3, respectively. Then, we have $k_1^{p1} > k_1^{p2}$, which leads to Corollary 4.1.

Corollary 4.1.

If $k_1 > k_1^{p1}$, the supplier's profit improves under the early commitment scheme and this improvement increases in σ_1.

If $k_1^{p2} < k_1 < k_1^{p1}$, delayed commitment is more profitable for the supplier. However, increasing σ_1 works in the favor of early commitment scheme.
If $k_1 < k_1^{p2}$, early commitment is less profitable and $\Delta[\Pi_S]$ is decreasing in σ_1.

The effects of uncertainty in R_1's demand on the improvements resulting from early commitment are two-fold. Since early commitment decreases the demand risk of the supplier, increasing uncertainty increases the value of early commitment in terms of demand risk. Another effect of σ_1 is on the order quantity. If $k_1 > 0$, increasing uncertainty increases the order quantity of R_1 and hence the supplier's profit. However, when $k_1 < 0$, the order quantity decreases in σ_1, which is not favorable for early commitment from the supplier's perspective. Note that $\frac{d\Delta[\Pi_S]}{d\sigma_1} \geq 0$ when $k_1 \geq 0$; that is, the benefits from early commitment increase in σ_1 when $k_1 > 0$. This is reasonable since early commitment grants the supplier the opportunity to avoid the increasing uncertainty. Interestingly, early commitment is profitable for the supplier even when $k_1^{p1} \leq k_1 \leq 0$. In this case, increasing σ_1 decreases the supplier's profits under early commitment since the order quantity decreases. However, the benefits resulting from avoiding uncertainty outweigh this loss. If $k_1 < -\frac{\sigma_1 w\phi(k_m)}{\sigma_T(w-c)}$, on the other hand, the benefits from a deterministic order quantity no longer compensate for the loss from the decreased order quantity in σ_1, and the benefits from early commitment decrease. We next analyze the effects of uncertainty in R_2's demand.

Proposition 4.4. *$\Delta[\Pi_S]$ is decreasing in σ_2.*

Proof. $\frac{d\Delta[\Pi_S]}{d\sigma_2} = (\frac{\sigma_2}{\sigma_T} - 1)w\phi(k_m) < 0$ □

The only uncertainty involved in the early commitment scheme for the supplier is the demand uncertainty for R_2. In delayed commitment on the other hand, both demands are random and hence aggregate demand can be pooled to reduce the risk of uncertainty. Since this does not hold for the early commitment case, the supplier is more vulnerable to the uncertainty in R_2's demand, i.e., an increase in σ_2 results in a larger decrease in the supplier's profit under the early commitment scheme. Corollary 4.2 further analyzes the differences between delayed and early commitment schemes with respect to σ_2.

Corollary 4.2. *For the supplier:*

a) If $k_1 \geq 0$, early commitment always outperforms delayed commitment for any σ_2.

b) If $-\frac{w\phi(k_m)}{w-c} \leq k_1 < 0$, there exists a threshold value, σ_2^0 such that early commitment is better when $\sigma_2 < \sigma_2^0$ and delayed commitment performs better otherwise, where $\sigma_2^0 = \sigma_1 \frac{u^2-1}{2u}$, $u = \frac{(w-c)k_1}{w\phi(k_m)}$.

c) If $k_1 < -\frac{w\phi(k_m)}{w-c}$, delayed commitment always outperforms early commitment for any σ_2.

Proof. Part (a) directly follows from Proposition 4.2. For part c; $k_1 < -\frac{w\phi(k_m)}{w-c}$ implies $k_1 < -\frac{(\sigma_T-\sigma_2)w\phi(k_m)}{(w-c)\sigma_1}$ since $\frac{\sigma_T-\sigma_2}{\sigma_1} \leq 1$ for any σ_2. Part (c), hence, also follows from Proposition 4.2. For part (b); recall that $\Delta[\Pi_S] = (w-c)k_1\sigma_1 + (\sigma_T - \sigma_2)w\phi(k_m)$. When $\sigma_2 = 0$, $\Delta[\Pi_S]_{\sigma_2=0} = \sigma_1[(w-c)k_1 + w\phi(k_m)] > 0$. $\Delta[\Pi_S]$ decreases in σ_2 and becomes zero when $\sigma_2 = \sigma_2^0$. □

Corollary 4.2 provides an interval for k_1 where the degree of uncertainty in R_2's demand effects the commitment preference of the supplier. Specifically, if $k_1 \in (-\frac{w\phi(k_m)}{w-c}, 0)$, the supplier's choice of commitment scheme depends on σ_2 given fixed values of the other parameters. Otherwise, the supplier prefers one of the schemes over the other, regardless of the value of σ_2.

4.2.2 Primary Retailer's Profit

The effects of an early commitment scheme on the expected profit of the primary retailer are also two-fold. Under a delayed commitment scheme, R_1 does not face any demand risk, but she bears a supply risk. With early commitment, R_1 has priority in inventory allocation, and she is guaranteed to receive her order in full, avoiding the supply risk. However, she now assumes the risk of her own demand. Below, we analyze the tradeoff between priority in inventory allocation and demand uncertainty from R_1's perspective and investigate the commitment preference of the primary retailer.

The expected profits of R_1 under delayed and early commitment schemes are given by Equations (4–4) and (4–5), respectively.

Proposition 4.5. *The expected profit of the primary retailer is greater with the delayed commitment scheme when* $k_1\sigma_1 \leq \rho k_m \sigma_T$.

Proof. Consider an allocation rule where retailer i gets $\min\{Q_i, \rho Q_S\} = \min\{x_i, \rho Q_S\}$. This rule is inferior to the proportional allocation rule since it does not utilize the capacity that is not used by a retailer to satisfy the demand of the other retailer. If we denote the expected profit of R_1 under this rule by $E[\Pi_1^{D'}]$, we can conclude that $E[\Pi_1^D] > E[\Pi_1^{D'}]$. Note that

$$E[\Pi_1^{D'}] = (r-w)\left[\mu_1 - \int_{\rho Q_S}^{\infty} (x-\rho Q_S) f_1(x)dx\right] = (r-w)\left[\mu_1 - \sigma_1 L\left(\frac{\rho Q_S - \mu_1}{\sigma_1}\right)\right]$$

Recall that the expected profit of R_1 under early commitment scheme is given by $E[\Pi_1^E] = (r-w)(\mu_1 - \sigma_1 L(k_1)) - w\sigma_1(L(k_1)+k_1)$. Thus,

$$\begin{aligned} E[\Pi_1^{D'}] - E[\Pi_1^E] &= (r-w)\sigma_1[L(k_1) - L(\frac{\rho k_m \sigma_T}{\sigma_1})] + w\sigma_1(L(k_1)+k_1) \\ &> (r-w)\sigma_1[L(k_1) - L(\frac{\rho k_m \sigma_T}{\sigma_1})] > 0. \end{aligned}$$

The first inequality follows since $L(k_1) + k_1 > 0$. The second inequality follows from the condition that $k_1\sigma_1 \leq \rho k_m \sigma_T$. $\square$

The reasoning behind the preceding proposition can be intuitively explained as follows: When $Q_1^E \leq \rho Q_S$, i.e., $k_1\sigma_1 \leq \rho k_m \sigma_T$, R_1 can already get Q_1^E units from the supplier with certainty even under the delayed commitment scheme. Hence, there is no need for her to pre-commit the same quantity that she can already receive, and bear the risks associated with the demand uncertainty. When $Q_1^E > \rho Q_S$, on the other hand, the preference of R_1 depends on the tradeoff between capacity allocation and demand uncertainty.

4.2.3 Total System Profits

In this section, we compare total system profits under the delayed and early commitment schemes. The expected total profit under a delayed commitment scheme is given by

$$E[\Pi_T^D] \;=\; r\left[\int_0^{Q_S} z h(z)dz + \int_{Q_S}^{\infty} Q_S h(z)dz\right] - cQ_S$$

where $h(\cdot)$ denotes the *pdf* of $Z = X_1 + X_2$. Recalling that the optimal production quantity of the supplier is $Q_S^* = \mu_T + k_m\sigma_T$ under a delayed commitment scheme, we can rewrite the expected total profit as

$$E[\Pi_T^D] \;=\; (r-c)\mu_T - \sigma_T(rL(k_m) + ck_m)$$

Similarly, the total expected profit for the early commitment scheme is

$$E[\Pi_T^E] = r\left[\int_0^{Q_1} x f_1(x)dx + \int_{Q_1}^{\infty} Q_1 f_1(x)dx\right] - cQ_1 + r\left[\int_0^{\hat{Q}_e} x f_2(x)dx + \int_{\hat{Q}_e}^{\infty} \hat{Q}_e f_2(x)dx\right] - c\hat{Q}_e$$

Noting that the optimal production quantity of the supplier is $Q_S^* = Q_1 + \hat{Q}_e$, where $Q_1 = \mu_1 + k_1\sigma_1$ and $\hat{Q}_e = \mu_2 + k_m\sigma_2$, the expected profit can be rewritten as

$$E[\Pi_T^E] \;=\; (r-c)\mu_T - \sigma_1(rL(k_1) + ck_1) - \sigma_2(rL(k_m) + ck_m)$$

Then the difference in total expected profits under early and delayed commitment schemes is given by

$$E[\Pi_T^E] - E[\Pi_T^D] \;=\; (\sigma_T - \sigma_2)(rL(k_m) + ck_m) - \sigma_1(rL(k_1) + ck_1) \qquad (4\text{–}8)$$

which leads to Proposition 4.6.

Proposition 4.6. *If $k_m \geq k_1$, total expected profits under a delayed commitment scheme are greater.*

Proof. Let $g(k) = rL(k) + ck$. Then $dg(k)/dk = -r(1 - \Phi(k)) + c$ and $d^2g(k)/dk^2 = r\phi(k) > 0$. Hence, the minimum of $g(k)$ occurs at k^* where $\Phi(k^*) = \frac{r-c}{r}$. Recall that $\Phi(k_m) = \frac{w-c}{w} < \Phi(k^*)$ and $\Phi(k_1) = \frac{r-w}{r} < \Phi(k^*)$. Since $\Phi(\cdot)$ is a monotone nondecreasing function and $g(\cdot)$ is convex, we can deduce that $k_1, k_m < k^*$. Hence, if $k_m > k_1$, then $g(k_m) < g(k_1)$. Since, $\sigma_T - \sigma_2 < \sigma_1$, $E[\Pi_T^E] - E[\Pi_T^L] < 0$. The proof is complete. □

Note that the condition in the proposition may also be written as $1 - c/w \geq 1 - w/r$, or $w/r \geq c/w$. Denoting m_w and m_r as the wholesale and retail markups, respectively, where $m_w = w/c$ and $m_r = r/w$, the condition $k_m \geq k_1$ is equivalent to $m_r \leq m_w$. This implies that whenever the retail markup is less than or equal to the wholesale markup, delayed commitment is preferred. This is because low retail markups discourage the retailer from a high early commitment order level, i.e., the retailer would in such cases order too little from a system perspective. Note also that above proposition provides a sufficient condition; that is, $k_m < k_1$ does not necessarily mean that total expected profits under an early commitment scheme are greater. This is mainly due to second retailer. The presence of R_2 results in a pooling effect under delayed commitment, which may compensate for the low production quantity of the supplier. In a single-supplier single-retailer system, on the other hand, $k_m < k_1$ implies early commitment since the supplier would not order a sufficient amount under delayed commitment.

From the system's perspective, early commitment is preferred if and only if $(\sigma_T - \sigma_2)(rL(k_m) + ck_m) \geq \sigma_1(rL(k_1) + ck_1)$, which can be rewritten as

$$\frac{g(k_m)}{g(k_1)} \geq \frac{\sigma_1}{\sigma_T - \sigma_2}, \tag{4–9}$$

where $g(k) = rL(k) + ck$. This inequality leads to Propositions 4.7 and 4.8.

Proposition 4.7. *For any k_1, there is a threshold k_m value, say $k'_m(k_1)$ such that for all $k_m \leq k'_m(k_1)$ early commitment is preferred from the system perspective.*

Proof. Note that $g(k)$ is convex. Let k^* be a global minimizer of $g(x)$. Suppose $k_1 > k_m$, which implies $g(k_1) < g(k_m)$, because $k_1, k_m < k^*$ and $g(k)$ is convex. As $g(k)$ is strictly decreasing in k for $k \leq k^*$, some value of k_m exists for any k, say $k'_m(k_1)$ such that for all $k_m \leq k'_m(k_1)$, condition (4–9) is satisfied. □

Proposition 4.8. *There exists a threshold σ_2 value, say σ'_2 such that for all $\sigma_2 \geq \sigma'_2$, delayed commitment maximizes total expected system profits.*

Proof. Note that $\frac{\sigma_1}{\sigma_T - \sigma_2}$ is increasing in σ_2 and $\lim_{\sigma_2 \to \infty} \frac{\sigma_1}{\sigma_T - \sigma_2} \to \infty$, which implies that condition (4–9) will not be satisfied for sufficiently large values of σ_2. Let σ'_2 denote the smallest σ_2 value that violates condition (4–9). Since $\frac{\sigma_1}{\sigma_T - \sigma_2}$ is increasing in σ_2, the condition will continue to be violated for any $\sigma_2 \geq \sigma'_2$ and the proposition follows. □

Proposition 4.7 implies that if the supplier's service level is sufficiently low, it is optimal for R_1 to order early in order to ensure a sufficient amount of sales for the system. We can also conclude from Proposition 4.8 that in terms of overall system performance, the supplier should not try to induce early commitment when the uncertainty in the secondary retailer's demand is above a certain level.

4.3 Comparison with the Single Retailer System

In this section, we briefly introduce the single-retailer model and compare it to our setting in order to generate insights on how another customer (or collection of customers) affects the commitment preferences of the supplier and the primary retailer.

Consider the system where the primary retailer of our original model is the only customer of the supplier. Under a delayed commitment scheme, the supplier produces $Q_S^D = \mu_1 + k_m\sigma_1$ units. Under early commitment, R_1 orders Q_1^E units before the production run, which also defines the supplier's production quantity. Hence, the supplier's expected profits under delayed and early commitment are given by $E[\Pi_S^D] = (w-c)\mu_1 - \sigma_1(wL(k_m) + ck_m)$, and $E[\Pi_S^E] = (w-c)(\mu_1 + k_1\sigma_1)$, respectively. The increase in the supplier's expected profit due to early commitment is given by

$$\Delta[\Pi_S] \quad = \quad (w-c)k_1\sigma_1 + \sigma_1 w\phi(k_m). \tag{4–10}$$

The expected profits of R_1 under delayed and early commitment schemes are $E[\Pi_1^D] = (r - w)(\mu_1 - \sigma_1 L(k_m))$ and $E[\Pi_1^E] = (r - w)\mu_1 - \sigma_1[rL(k_1) + wk_1]$, respectively. Likewise, the expected total profit of a single retailer system under delayed and early commitment schemes are characterized by $E[\Pi_T^D] = (r - c)\mu_1 - \sigma_1(rL(k_m) + ck_m)$ and $E[\Pi_T^E] = (r - c)\mu_1 - \sigma_1(rL(k_1) + ck_1)$, respectively. In a single retailer system, the increase in the expected total profit of the system is given by

$$\Delta[\Pi_T] \quad = \quad \sigma_1(rL(k_m) + ck_m - (rL(k_1) + ck_1)) \qquad (4\text{–}11)$$

Proposition 4.9 characterizes the preference of the commitment scheme from the system-wide profit perspective:

Proposition 4.9. *When the primary retailer is the only customer of the supplier, early commitment is preferred from the system's perspective if and only if* $k_1 > k_m$*.*

Proof. Early commitment will be preferred when $E[\Pi_T^E] > E[\Pi_T^D]$, i.e., when $g(k_1) < g(k_m)$, where $g(k) = rL(k) + ck$. As g(k) is decreasing in k (see the proof of Proposition 4.7), $g(k_1) < g(k_m)$ if and only if $k_1 > k_m$. □

Comparing Equation (4–10) with Equation (4–7), we observe that the increase in the supplier's expected profit due to early commitment is greater in a single-retailer system. That is, the supplier's valuation of the early commitment scheme decreases when there is another customer (or a collection of customers) who utilizes the supplier as an overflow supplier post demand realization.

We perform a similar analysis from the primary retailer's perspective. Proposition 4.10 provides conditions under which the primary retailer benefits from the existence of R_2 under delayed commitment.

Proposition 4.10. *Under a delayed commitment scheme,* R_1 *benefits from the existence of* R_2 *when (i)* $k_m \geq 0$ *and* $\sigma_1/\mu_1 \leq \sigma_T/\mu_T$ *or (ii)* $k_m < 0$ *and* $\sigma_1/\mu_1 > \sigma_T/\mu_T$*.*

Proof. The proof directly follows from the comparison of the inventory allocation of R_1 in both cases. The above conditions are sufficient since R_1's allocation in the multiple-retailer setting may exceed ρQ_S when the realization of R_2's demand is less than $(1-\rho)Q_S$. □

The existence of R_2 affects the commitment preference from the system's perspective, which is characterized by Proposition 4.11.

Proposition 4.11. *The existence of another customer ordering after her demand realization makes delayed commitment more attractive when compared to the single retailer system.*

Proof. The proof follows from Equation (4–8) and Equation (4–11), and the fact that $\sigma_1 > \sigma_T - \sigma_2$. □

Summing up what we have observed in this section, the existence of another customer makes delayed commitment more favorable from the system's perspective. The primary retailer (under certain settings, see Proposition 4.10) and the supplier value delayed commitment more when compared to the single-retailer system.

4.4 Computational Analysis

In this section, we perform a computational analysis to generate further insights on the preference of the primary retailer. As we have fully characterized the supplier's preferences analytically, this section mainly focuses on the primary retailer. We also focus on the differences of the two-retailer setting and the single-retailer system to illustrate the discussions on this issue in Section 4.3. For this purpose, we use the parameters presented in Table 4-1.

Table 4-1: Parameters.

c	w	r	μ_2	μ_1	σ_1/μ_1	σ_2/μ_2
5	10	40	80	80	1/7	1/7
		70	100	100	1/5	1/5
		100	120	120	1/3	1/3

Note that $k_1 > 0$ for our parameter set. Hence, we know that the supplier is always better off with the early commitment of R_1. Therefore, whenever R_1 benefits from the early commitment, early commitment is pareto-optimal for the supplier and R_1. Also note that this setting violates the sufficient condition, $\rho k_m \sigma_T > k_1 \sigma_1$ (see Proposition 4.5), under which we have shown analytically that R_1 prefers the delayed commitment scheme.

The parameter set in Table 4-1 corresponds to 243 test cases. The primary retailer prefers early commitment in 141 of these cases, which is equal to 58% (141 out of 243) of the time. Hence, we can conclude that although delayed commitment is frequently observed in industry, retailers can benefit from ordering early in certain contexts, even without the incentive of a discounted wholesale price. Furthermore, the aggregate profit of the supplier and the primary retailer is higher under an early commitment case in 89.3% (217 out of 243) of the cases, which implies that in some settings, the supplier generates high enough profits to compensate for the loss of the primary retailer.

When the retail markup is relatively low ($r = 40$), there is no case where R_1 benefits from an early commitment scheme. In this case, the unit revenue is not high enough for R_1 to justify the demand risk of early commitment. When $r = 100$, on the other hand, R_1 is better off with the early commitment scheme in all cases. This is intuitive since R_1 does not want to bear the high supply risk as unsatisfied demand due to insufficient supply would result in considerable loss of profit. When $r = 70$, R_1's preference depends on the demand parameters of both retailers. When her mean demand exceeds R_2's, she usually prefers early commitment. That is, when her demand is sufficiently large, R_1 prefers to secure inventory by committing early. When R_2's demand is larger than R_1's, the supply risk for R_1 is not severe because her inventory allocation is relatively large due to the large demand of R_2. Hence, in such cases, R_1 usually prefers delayed commitment.

Figure 4-1 depicts the average percentage increase in R_1's expected profit due to early commitment (compared to delayed commitment) as a function of unit revenue and her mean demand. R_1 values an early commitment scheme more as her expected demand increases for all retail markup levels considered. Similarly, Figure 4-2 depicts the average percentage increase in R_1's expected profit due to early commitment (compared to delayed

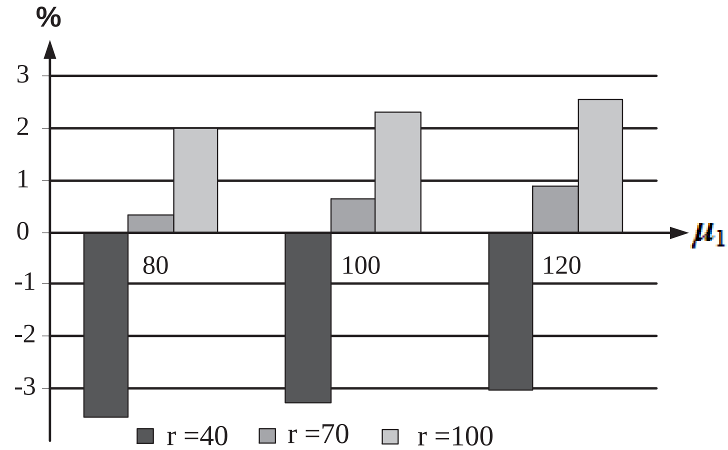

Figure 4-1 Percent increase in R_1's expected profit due to early commitment: effects of μ_1.

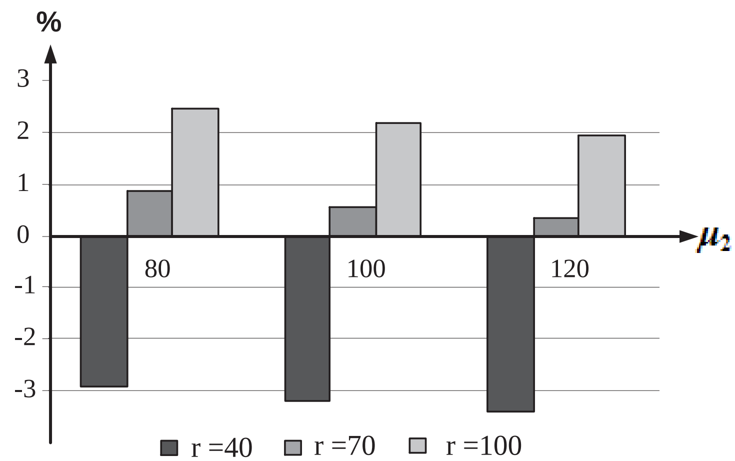

Figure 4-2 Percent increase in R_1's expected profit due to early commitment: effects of μ_2.

commitment) as a function of unit revenue and R_2's mean demand. R_1 values an early commitment scheme more as the expected demand of the secondary retailer decreases for all retail markup levels considered, since higher expected demand for R_2 increases the primary retailer's chance to receive sufficient inventory under a delayed commitment scheme.

We next analyze the effects of uncertainty on the expected profit of R_1 and her commitment preference. Figures 4-3 and 4-4 illustrate the relation between the average increase in R_1's expected profit due to early commitment and the coefficients of variation of demand for R_1 and R_2, respectively, for different values of unit revenue. Increasing

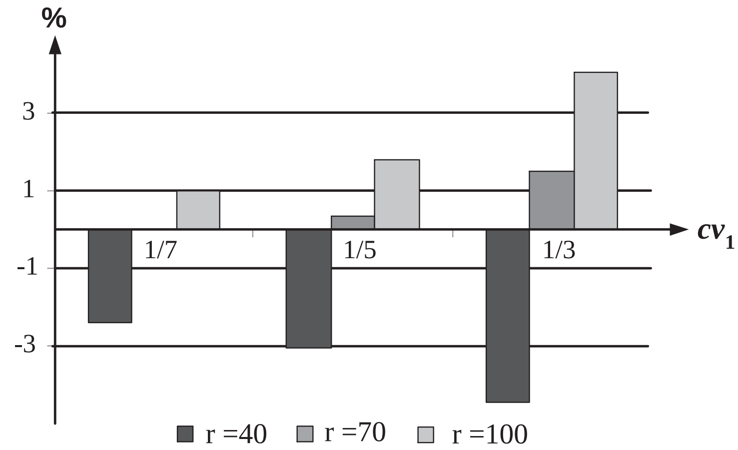

Figure 4-3 Percent increase in R_1's expected Profit due to early commitment: effects of cv_1.

uncertainty in R_1's demand (Figure 4-3) works in favor of the commitment scheme that provides more profits. When $r = 40$, R_1 prefers delayed commitment, which becomes more favorable as σ_1/μ_1 increases. When $r = 70$ or 100, early commitment is more favorable and the improvement in R_1's expected profit increases in σ_1/μ_1. This is due to the relative importance of supply and demand risks. When demand risk is more critical ($r = 40$), increasing uncertainty in demand amplifies this risk, and hence delayed commitment becomes more favorable. If R_1's primary concern is the supply risk, then increasing uncertainty makes it more crucial to commit early.

Increasing uncertainty in R_2's demand (Figure 4-4) decreases the percentage improvement in R_1's expected profit due to early commitment, which can be explained as follows: under delayed commitment, R_1 is reserved a certain amount of inventory no matter what R_2 orders. When uncertainty in R_2's demand increases, her order quantity becomes more uncertain. Higher order quantities will not hurt R_1 because of the reserved supply, whereas lower order quantities result in larger supply for her. Hence, increasing uncertainty of R_2's demand makes delayed commitment more favorable for R_1.

For the rest of this section, we focus on the single-retailer system and its comparison to the multi-retailer setting. We use the same parameter set (see Table 4-1) removing

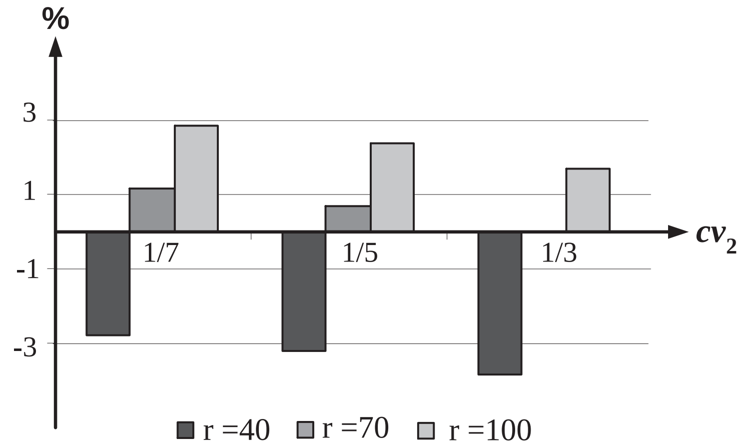

Figure 4-4 Percent increase in R_1's expected Profit due to early commitment: effects of cv_2.

the parameters related to the secondary retailer, which results in 27 test cases. Each of these cases is compatible with 9 cases in the multi-retailer setting that have the same parameters. For the comparison of the supplier's choice, we use the 'increase in expected profits due to early commitment' rather than the 'percentage increase' since the latter may be misleading in evaluating the supplier's profit (in the multi-retailer setting, the supplier serves the secondary retailer in both commitment schemes. Hence, using the percentage increase and comparing it with the single-retailer system underestimates the value of early commitment in a multi-retailer system).

In a single-retailer system, the supplier prefers early commitment in all the cases as in the multi-retailer setting. The average increase in his expected profit due to early commitment is \$203.5 when primary retailer is the only demand source whereas it is \$155.2 when there is another retailer (or collection of retailers). Furthermore, in each of the 27 cases considered, the increase in profits in a single-retailer system exceeds the maximum increase in the 9 corresponding cases in the multi-retailer setting. Hence, we conclude that the supplier values delayed commitment more in the presence of another retailer regardless of its demand parameters.

The primary retailer prefers early commitment in 67% (18 out of 27) of the cases. Recall that when $r = 70$, R_1's preference depends on the demand parameters in the multi-retailer setting. When she is the only customer, though, she always prefers early commitment when $r = 70$. The average percentage increase in her expected profit due to early commitment is 2.64% with a maximum of 7.84% and a minimum of -0.95% when she is the only demand source. However, the average is -0.12% when there is another retailer (or collection of retailers) with a maximum of 5.66% and a minimum of -6.13%. Furthermore, as in the supplier's case, in each of the 27 cases considered, the increase in her profits in a single-retailer system exceeds the maximum increase in the 9 corresponding cases in the multi-retailer setting. Hence, we conclude that the primary retailer values delayed commitment more in the presence of another retailer regardless of its demand parameters.

Next, we focus on the demand characteristics of the secondary retailer so as to see how they affect the valuation of the primary retailer. Recall that the percentage increase in R_1's expected profit due to an early commitment scheme decreases in μ_2 and σ_2 (see Figure 4-2 and Figure 4-4). That is, R_1 values the existence of another customer more when that customer's expected demand and coefficient of variation are higher, which can be explained as follows: note that $k_m = 0$ for our parameter set, which indicates that the primary retailer's reserved supply under a delayed commitment scheme is equal to μ_1 in both the single-retailer and multi-retailer systems. However, in the multi-retailer system, R_1 has the opportunity to use the excess supply that results when R_2's demand is less than her reserved supply. When μ_2 and/or σ_2 increase, the amount of possible excess supply increases.

Note that the parameters used in our computational test are biased in that they exclude instances where we could prove analytically through sufficient conditions that the primary retailer would select a delayed commitment scheme. Hence, the data set presented in Table 4-1 may lead to the overestimation of the instances where the primary retailer prefers an early commitment scheme. In order to have an unbiased analysis, we generated 1,000 random test instances with the following parameters: $\mu_i \sim U[0, 150]$, $cv_i \sim U[1/10, 1/3]$, $c = 5$; $w \sim U[c, 10c]$, $r \sim U[w, 10w]$ (this random data set will also be used in the subsequent sections). The primary retailer prefers early commitment in 105 of these instances. In all these instances, the supplier prefers early commitment too. There are 67 instances where both parties prefer a delayed commitment scheme. Hence, in only 17.2% of the test instances, the supplier's and the primary retailer's preferences match. The supplier prefers early commitment of the primary retailer in a total of 933 instances, i.e., 93.3% of the time. There are 828 instances where the primary retailer prefers delayed commitment whereas the supplier would like to have early commitment. Hence, we may deduce that although there are certain settings where the preferences of the primary retailer and the supplier are in alignment, the primary retailer prefers delayed commitment in the vast majority of the cases whereas the supplier prefers otherwise, which is consistent with industry practices.

4.5 Order Timing Interaction between the Retailers

Recall that the secondary retailer is modelled in our original setting as a collection of one or more retail customers who utilize the supplier as a secondary source of overflow supply post demand realization. Hence, R_2 naturally orders from the supplier after her demand realization. However, it is also worthwhile to analyze the setting where R_2 has the opportunity to select one of the commitment schemes as well. While evaluating the preferences of the retailers and the supplier, we assume that the retailers act simultaneously. That is, while evaluating the preference of one retailer, we do not assume an exogenous commitment scheme for the other retailer, which results in a strategic interaction between the retailers in terms of the order timing decision. This interaction can be modelled as a strategic game. The characteristics and components of a general strategic game are provided in Definition 4.1.

Table 4-2: Strategic interaction between the retailers.

	Early (E)	Delayed (D)
Early (E)	$(\Pi_1^E \vert R_2 = E, \Pi_2^E \vert R_1 = E)$	$(\Pi_1^E \vert R_2 = D, \Pi_2^D \vert R_1 = E)$
Delayed (D)	$(\Pi_1^D \vert R_2 = E, \Pi_2^E \vert R_1 = D)$	$(\Pi_1^D \vert R_2 = D, \Pi_2^D \vert R_1 = D)$

Definition 4.1 (cf. Osborne (2004)). *A strategic game consists of (i) a set of players, (ii) for each player, a set of actions, and (iii) for each player, preferences over the set of action profiles.*

In our model, the set of players corresponds to the retailers. For each retailer, the set of actions consists of selecting early commitment, and selecting delayed commitment. The preferences will be determined by the respective profit figures. Since the game has two players, it can be represented in a matrix format as depicted in Table 4-2, where rows correspond to the actions of R_1, and columns correspond to the actions of R_2. Each entry in the matrix consists of the expected profits of the retailers corresponding to the selected action pair. For instance, the top-right cell corresponds to R_1 and R_2 selecting early and delayed commitment schemes, respectively. The expected profits of R_1 and R_2 in this case are represented by $\Pi_1^E|R_2 = D$ and $\Pi_2^D|R_1 = E$, respectively.

An equilibrium of a strategic game can be characterized as follows:

Definition 4.2 (cf. Osborne (2004)). *A Nash equilibrium is an action profile a^* with the property that no player i can do better by choosing an action different from a_i^*, given that every other player j adheres to a_j^*.*

Definition 4.2 can be illustrated for our setting as follows: consider the action profile (E,D), that is, R_1 and R_2 select early and delayed commitment, respectively. This action profile is a Nash equilibrium if and only if R_1 has no incentive to select delayed commitment given R_2 selects delayed commitment (i.e., $\Pi_1^E|R_2 = D \geq \Pi_1^D|R_2 = D$), and R_2 has no incentive to select early commitment given R_1 selects early commitment (i.e., $\Pi_2^D|R_1 = E \geq \Pi_2^E|R_1 = E$).

Before further analyzing the strategic interaction between the retailers, the optimal production quantity of the supplier corresponding to every possible outcome of this interaction should be characterized. Note that the analysis in the previous sections is still valid with minor modifications. We already have the production quantity of the supplier when R_1 and R_2 select early and delayed, or delayed and delayed commitment schemes, respectively. His production quantities for other outcomes can also be generated similarly, which are presented in Equation (4–12):

$$Q_S = \begin{cases} H^{-1}(\frac{w-c}{w}) = \mu_T + k_m\sigma_T & (R_1, R_2) = (D, D) \\ F_1^{-1}(\frac{r-w}{e}) + F_2^{-1}(\frac{w-c}{w}) = \mu_T + k_1\sigma_1 + k_m\sigma_2 & (R_1, R_2) = (E, D) \\ F_1^{-1}(\frac{w-c}{w}) + F_2^{-1}(\frac{r-w}{r}) = \mu_T + k_m\sigma_1 + k_1\sigma_2 & (R_1, R_2) = (D, E) \\ F_1^{-1}(\frac{r-w}{r}) + F_2^{-1}(\frac{r-w}{r}) = \mu_T + k_1\sigma_1 + k_1\sigma_2 & (R_1, R_2) = (E, E) \end{cases} \quad (4–12)$$

The resulting expected profits of the supplier can also be inferred from the previous analysis, except the case $(R_1, R_2) = (E, E)$. In this case, both retailers order early, hence the supplier's profit is deterministic and given by $\Pi_S(E, E) = (w - c)(\mu_T + k_1\sigma_1 + k_1\sigma_2)$. Likewise, we can also compute the expected profits of the retailers as in the previous sections, except for the profit of a retailer who selects delayed commitment when the other retailer selects early commitment, which is given by $E[\Pi_i^D] = (r - w)[\mu_i - \sigma_i L(k_m)]$.

We next analyze the outcome of this setting using the random data set discussed in Section 4.4, considering only pure strategy Nash equilibria (see Osborne 2004 for the discussion of pure strategy and mixed strategy Nash equilibria). Note that the definition of Nash Equilibrium for the strategic game between the retailers (see Definition 4.2) does not guarantee a unique equilibrium. That is, more than one action profile can satisfy the requirements of a Nash Equilibrium. In those cases, we first use the equilibrium refinement technique called pareto-dominance (payoff dominance), which is defined below:

Definition 4.3. *(**cf. Harsanyi and Selten 1988**) Let a^* and $\bar{a}^*$ denote equilibrium points of a strategic game. a^* pareto-dominates $\bar{a}^*$ if we have $H_i(a^*) > H_i(\bar{a}^*)$ for every $i \in N$, where $H_i(x)$ denotes player i's payoff under action profile x, and N denotes the number of players.*

Table 4-3: Illustration of risk-dominance.

	U_2	V_2
U_1	(a_{11}, b_{11})	(a_{12}, b_{12})
V_1	(a_{21}, b_{21})	(a_{22}, b_{22})

When pareto-dominance does not provide a unique equilibrium, we utilize the concept of risk-dominance, which is described below:

Definition 4.4. ***(cf. Harsanyi and Selten 1988)*** *Let U and V be the equilibrium points of a two-player strategic game. Let u_i and v_i for $i = 1, 2$, be the deviation losses at U and V. Then, U risk-dominates V if $u_1u_2 > v_1v_2$, and V risk-dominates U if $u_1u_2 < v_1v_2$.*

To have a better understanding of the risk-dominance concept, consider the two-player strategic game presented in Table 4-3. Assume that $V = (V_1, V_2)$ and $U = (U_1, U_2)$ are two equilibrium points. Then, the deviation losses from equilibrium V are given by $v_1 = a_{22} - a_{12} > 0$ and $v_2 = b_{22} - b_{21} > 0$. Similarly, the deviational losses from equilibrium U are $u_1 = a_{11} - a_{21}$ and $u_2 = b_{11} - b_{12} > 0$. The deviational losses are greater than zero since U and V are equilibrium points.

In 1,000 test instances considered, there are 268 cases with multiple equilibria. In 232 of these 268 cases, (E,E) and (D,D) are the equilibria. Pareto-dominance eliminates the equilibrium (E,E) in all 232 cases. In the remaining 36 cases, the multiple equilibria are (E,D) and (D,E), and equilibrium refinement using pareto-optimality does not render a unique equilibrium. In these cases, risk-dominance is utilized to produce a unique equilibrium, which provides 15 of 36 cases with (E,D), and the remaining with (D,E). As a result, we have a unique equilibrium for each of the test instances. Both retailers prefer early (delayed) commitment 7.8% (87.8%) of the cases. R_1 and R_2 prefer early and delayed (delayed and early) commitment schemes 1.8% (2.6%) of the test instances, respectively. Delayed commitment is still the dominant strategy for the retailers. On the other hand, the supplier prefers the (D,D) outcome only in 53 cases. In the remaining 947 instances, the supplier prefers both retailers to commit early. However, only in 78 of such instances the retailers prefer to do so. Hence, we can deduce that the outcome of the strategic game between the retailers and the suppliers' preference coincide in 13.1% of the cases.

Note that throughout the analysis up to this point, we have assumed that both retailers' per unit revenues are equal. One may argue that this is the reason for having such a low number of equilibrium solutions where the retailers select different commitment schemes. To investigate this phenomenon, we generate an additional data set consisting of 1,000 random instances where $r_i \sim U[w, 10w]$, $i = 1, 2$. After eliminating multiple equilibria employing pareto-dominance and risk-dominance techniques successively, the results are as follows: both retailers prefer early (delayed) commitment 4.5% (86.8%) of the time. R_1 and R_2 prefer early and delayed (delayed and early) commitment schemes 4.4% (4.3%) of the test instances. Note that there is a significant increase in the number of equilibriums where retailers choose different commitment schemes. However, our major conclusion remains the same: delayed commitment scheme is still the dominant strategy for the retailers and the supplier prefers both retailers to commit early in the vast majority of the instances (87.8%).

4.6 Order Timing Decisions in a Capacitated Setting

Our analysis up to this point assumed that the supplier does not have any capacity limitations on production. In this section, we investigate order commitment preferences of the primary retailer and the supplier under capacitated supply, and investigate the effects of capacity on their preferences. In particular, we assume that the supplier has a deterministic production capacity, K. All other assumptions are still valid. This model will later serve as the building block for the analysis of strategic order timing when the supplier assumes the leader position (see Section 5.1.2).

4.6.1 The Supplier

When the primary retailer selects delayed commitment, the supplier's optimal unconstrained production quantity is $[Q_S^D]^U = H^{-1}(\frac{w-c}{w}) = \mu_T + k_m \sigma_T$. If this quantity is greater than the capacity, then the supplier will produce at the capacity level. That is, we have

$$Q_S^D \;=\; \begin{cases} K & K \le \mu_T + k_m\sigma_T \\ \mu_T + k_m\sigma_T & K > \mu_T + k_m\sigma_T \end{cases}. \tag{4–13}$$

Then the corresponding expected profit of the supplier is given by

$$E[\Pi_S^D] \;=\; \begin{cases} (w-c)\mu_T - \sigma_T\left(wL(\frac{K-\mu_T}{\sigma_T}) + c\frac{K-\mu_T}{\sigma_T}\right) & K \le \mu_T + k_m\sigma_T \\ (w-c)\mu_T - \sigma_T(wL(k_m) + ck_m) & K > \mu_T + k_m\sigma_T \end{cases}. \tag{4–14}$$

Similarly, when the primary retailer selects early commitment, the unconstrained production quantity is $[Q_S^E]^U \;=\; F_1^{-1}(\frac{r-w}{r}) + F_2^{-1}(\frac{w-c}{w}) \;=\; \mu_T + k_1\sigma_1 + k_m\sigma_2$. If this quantity is greater than the capacity, then the supplier will produce at the capacity level, That is, we have

$$Q_S^E \;=\; \begin{cases} K & K \le \mu_T + k_1\sigma_1 + k_m\sigma_2 \\ \mu_T + k_1\sigma_1 + k_m\sigma_2 & K > \mu_T + k_1\sigma_1 + k_m\sigma_2 \end{cases}. \tag{4–15}$$

The expected profit of the supplier under an early commitment scheme is then characterized by

$$E[\Pi_S^E] = \begin{cases} (w-c)(\mu_T + k_1\sigma_1) - \sigma_2(wL(\frac{K-\mu_T-k_1\sigma_1}{\sigma_2}) + c\frac{K-\mu_T-k_1\sigma_1}{\sigma_2}) & K \le \mu_T + k_1\sigma_1 + k_m\sigma_2 \\ (w-c)(\mu_T + k_1\sigma_1) - \sigma_2\,(wL(k_m) + ck_m) & K > \mu_T + k_1\sigma_1 + k_m\sigma_2 \end{cases} \tag{4–16}$$

Note that both $E[\Pi_S^D]$ and $E[\Pi_S^E]$ are piecewise functions. Hence, the increase in expected profit of the supplier due to early commitment, $\Delta[\Pi_S] \;=\; E[\Pi_S^E] \;-\; E[\Pi_S^D]$, should be examined separately in different intervals. Since these intervals depend on the relative magnitudes of $[Q_S^D]^U$ and $[Q_S^E]^U$, we will first consider the case where $[Q_S^D]^U \le [Q_S^E]^U$, i.e., $k_m\sigma_T \le k_1\sigma_1 + k_m\sigma_2$.

When $[Q_S^D]^U \;\le\; [Q_S^E]^U$, we have three intervals to consider that are generated by the breakpoints of $E[\Pi_S^D]$ and $E[\Pi_S^E]$ in order to evaluate the increase in the supplier's expected profit due to early commitment: (i) $K \;\le\; \mu_T + k_m\sigma_T$, (ii) $\mu_T + k_m\sigma_T \;<\; K \;\le$

$\mu_T+k_1\sigma_1+k_m\sigma_2$, and (iii) $K > \mu_T+k_1\sigma_1+k_m\sigma_2$. In interval (i), the supplier's unconstrained production quantity under both commitment schemes exceed the capacity limit. Hence, he produces at the capacity limit. In interval (ii), he can achieve the unconstrained optimal solution under delayed commitment, but not under early commitment. In interval (iii), the solutions of the unconstrained problems are feasible under both commitment schemes. The detailed analysis of the supplier's profit in each of these intervals is presented in Appendix B.1.1, which leads to Proposition 4.12:

Proposition 4.12. *If* $[Q_S^D]^U \leq [Q_S^E]^U$*, i.e.,* $k_m \leq \frac{\sigma_1}{\sigma_T-\sigma_2}k_1$*, the supplier always benefits from an early commitment scheme. The difference gets smaller as the capacity constraint becomes tighter.*

Proof. $\Delta[\Pi_S] = 0$ when $K = 0$. The proposition then follows from Lemma B.1 and Lemma B.2. ☐

Whether the system is capacitated or not, the supplier benefits from an early commitment scheme if $[Q_S^D]^U \leq [Q_S^E]^U$. However, these benefits decrease as the capacity level decreases, which can be explained as follows: the inequality $[Q_S^D]^U \leq [Q_S^E]^U$ indicates that capacity is more crucial for the supplier under an early commitment scheme. Hence, the expected profit under early commitment is more vulnerable to the limitations on capacity. That is, the advantages of an early commitment scheme will gradually disappear as the capacity level decreases. Figure 4-5 illustrates $\Delta[\Pi_S]$ as a function of K for an example setting ($\mu_1 = \mu_2 = 100$, $\sigma_1 = \sigma_2 = 30$, $c = 10$, $w = 15$, $r = 100$). For this example, we have $[Q_S^D]^U = 181.73 < [Q_S^E]^U = 218.17$. Note that $\Delta[\Pi_S]$ is less than \$1 until $K > 90$ since the supplier utilizes the capacity at %100 almost with certainty under both commitment schemes when capacity is too low. From that point onward, the benefits of early commitment increases gradually.

We now analyze the setting where $[Q_S^D]^U > [Q_S^E]^U$. As for the case with $[Q_S^D]^U \leq [Q_S^E]^U$, we need to analyze the increase in expected profit of the supplier due to early commitment in three intervals: (i) $K \leq \mu_T + k_1\sigma_1 + k_m\sigma_2$, (ii) $\mu_T + k_1\sigma_1 + k_m\sigma_2 < K \leq$

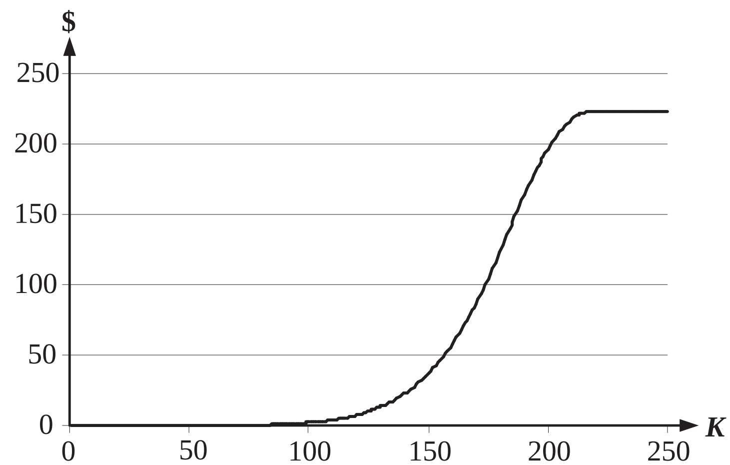

Figure 4-5 $\Delta[\Pi_S]$ as a function of K when $[Q_S^D]^U < [Q_S^E]^U$.

$\mu_T + k_m\sigma_T$, and (iii) $K > \mu_T + k_m\sigma_T$. The detailed analysis is presented in Appendix B.1.2, which leads to the characterization of $\Delta[\Pi_S]$ with respect to K:

Proposition 4.13. *When $[Q_S^D]^U > [Q_S^E]^U$, there exists a threshold capacity level, $K'_s = \mu_T + \frac{\sigma_T\sigma_1 k_1}{\sigma_T - \sigma_2}$, such that $\Delta[\Pi_S]$ is greater than zero and increasing in $(0, K'_s)$, decreasing in $(K'_s, \mu_T + k_m\sigma_T)$, and constant in $(\mu_T + k_m\sigma_T, \infty)$.*

Proof. The proposition follows directly from Lemma B.3 and the discussions thereafter in Appendix B.1.2. □

There is an important difference in the behavior of $\Delta[\Pi_S]$ when $[Q_S^D]^U > [Q_S^E]^U$, compared to the case $[Q_S^D]^U \leq [Q_S^E]^U$: it first increases, and then decreases in K, which can be explained as follows: when capacity is tight, the probability that an incremental increase in capacity will be utilized under an early commitment scheme is greater than the corresponding probability under delayed commitment scheme because of the early sales to the primary retailer. Hence, $\Delta[\Pi_S]$ increases in K. When capacity exceeds a critical value, it becomes more crucial under a delayed commitment scheme since the supplier will use capacity only for the secondary retailer's demand under an early commitment scheme once the primary retailer's order is satisfied, whereas under delayed commitment, there is still a possibility that both retailer will need capacity. Hence, $\Delta[\Pi_S]$ decreases in K. Note that

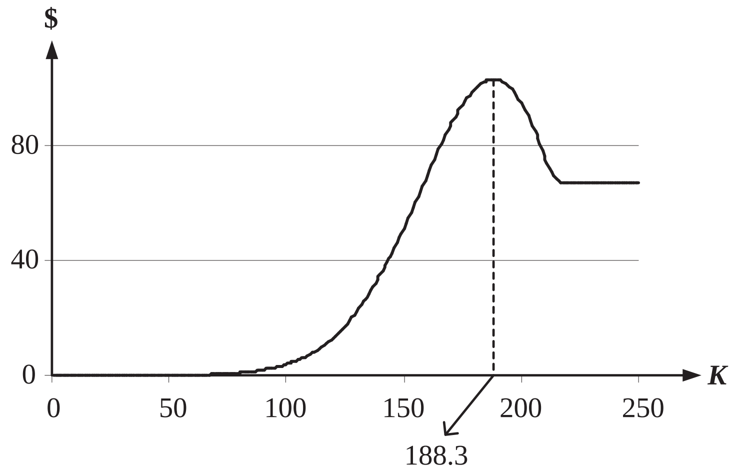

Figure 4-6 $\Delta[\Pi_S]$ as a function of K when $[Q_S^D]^U > [Q_S^E]^U$: sample data (i).

we cannot readily comment on the sign of $\Delta[\Pi_S]$ when $K \in (K_s', \mu_T + k_m\sigma_T)$. However, utilizing Proposition 4.2, we have the following corollary:

Corollary 4.3. *If* $k_1 \leq -\frac{\sigma_T-\sigma_2}{\sigma_1}\frac{w}{w-c}\phi(k_m)$*, that is, if the supplier prefers delayed commitment in an uncapacitated setting, there exists a capacity level,* K_s''*, such that* $\Delta[\Pi_S] > 0$ *for all* $K \in (0, K_s'')$*, and* $\Delta[\Pi_S] < 0$ *for all* $K \in (K_s'', \infty)$*. Otherwise, if the supplier prefers early commitment in an uncapacitated setting,* $\Delta[\Pi_S]$ *is greater than zero for all* K*.*

Figures 4-6 and 4-7 illustrate Proposition 4.13 and Corollary 4.3 for two example settings: (i) $\mu_1 = \mu_2 = 100$, $\sigma_1 = \sigma_2 = 30$, $c = 10$, $w = 30$, $r = 55$ (Figure 4-6), and (ii) $\mu_1 = \mu_2 = 100$, $\sigma_1 = \sigma_2 = 30$, $c = 10$, $w = 25$, $r = 40$ (Figure 4-7). In setting (i), the supplier always prefers delayed commitment. The threshold capacity level characterized in Proposition 4.13 is equal to 188.30. That is, when $K < 188.3$, $\Delta[\Pi_S]$ is increasing in K, and when $K > 188.3$, it is nonincreasing. In setting (ii), the supplier prefers delayed commitment if $K > K_s'' = 196.43$, and early commitment otherwise (see Corollary 4.3).

Having analyzed both cases, that is, $[Q_S^D]^U \leq [Q_S^D]^U$ and $[Q_S^D]^U > [Q_S^E]^U$, we may now combine our observations and comment on the supplier's choice of commitment scheme.

Proposition 4.14. *If the supplier benefits from an early commitment scheme in an uncapacitated system, he will benefit in a capacitated system too.*

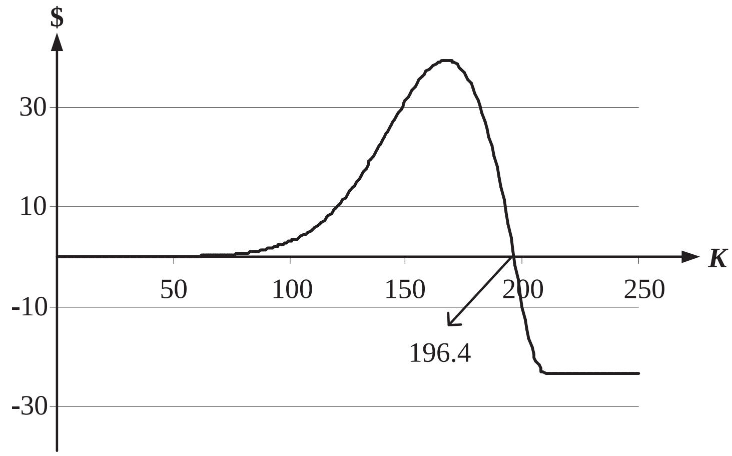

Figure 4-7 $\Delta[\Pi_S]$ as a function of K when $[Q_S^D]^U > [Q_S^E]^U$: sample data (ii).

Proof. The proposition follows from Proposition 4.12 for $[Q_S^D]^U \leq [Q_S^D]^U$, and Corollary 4.3 for $[Q_S^D]^U > [Q_S^D]^U$. □

Proposition 4.15. *If delayed commitment outperforms early commitment, there exists a threshold capacity level, K_s'', such that the supplier prefers early commitment when $K \leq K_s''$, and delayed commitment when $K > K_s''$.*

Proof. When $[Q_S^D]^U \leq [Q_S^D]^U$, the supplier prefers early commitment no matter what the capacity level is. Hence, we need to consider only the setting with $[Q_S^D]^U > [Q_S^D]^U$. Then, the proposition directly follows from Corollary 4.3. □

Contrary to the initial intuition, capacity does not necessarily favor early commitment from the supplier's perspective. That is, $\Delta[\Pi_S]$ of a capacitated system is not necessarily greater than that of an uncapacitated system. This is due to how a commitment scheme affects capacity utilization. Under an early commitment scheme, part of the capacity is utilized with certainty. However, the remaining capacity is more vulnerable. Under delayed commitment, a low realization of one retailer's demand can be compensated by a large realization of the other retailer's demand.

4.6.2 The Primary Retailer

If the supplier's capacity is greater than the order quantity of the primary retailer under an early commitment scheme, that is, $K \geq Q_1^E = \mu_1 + k_1\sigma_1$, the primary retailer's expected profit under an early commitment scheme is the same as in the uncapacitated system. On the other hand, under a delayed commitment scheme, R_1's expected profit is increasing in K, when $K \leq \mu_T + k_m\sigma_T$. Hence, we can deduce that $\Delta[\Pi_1]$ is decreasing in K for $K \in (\mu_1 + k_1\sigma_1, \mu_T + k_m\sigma_T)$. When $K \geq \mu_T + k_m\sigma_T$, the primary retailer's expected profits under delayed and early commitment match their uncapacitated counterparts. That is, $\Delta[\Pi_1]$ is constant. Unfortunately, we cannot comment on $\Delta[\Pi_1]$ when $K \leq \mu_1 + k_1\sigma_1$ since both profits are increasing in K, and the derivative of the difference does not provide useful insights. However, through extensive computational analysis, we have observed the following:

Conjecture 4.1. *If the primary retailer prefers early commitment in an uncapacitated model, she prefers early commitment in a capacitated setting too.*

Conjecture 4.2. *If the primary retailer prefers delayed commitment in an uncapacitated system, there exists a threshold capacity level, K_r, such that she prefers early commitment when $K \leq K_r$, and delayed commitment when $K > K_r$.*

4.6.3 System Perspective

We now compare total system profits under the delayed and early commitment schemes in the presence of a capacity limit. The expected total profit under delayed commitment is given by

$$E[\Pi_T^D] \;=\; r\left[\int_0^{Q_S^D} zh(z)dz + \int_{Q_S^D}^{\infty} Q_S^D h(z)dz\right] - cQ_S^D \tag{4–17}$$

where $h(\cdot)$ denotes the *pdf* of $Z = X_1 + X_2$, and Q_S^D is given by Equation (4–13). When retailer demands are normally distributed, Equation (4–17) reduces to

$$E[\Pi_T^D] = \begin{cases} (r-c)\mu_T - \sigma_T\left(rL\left(\frac{K-\mu_T}{\sigma_T}\right) + c\frac{K-\mu_T}{\sigma_T}\right) & K \le \mu_T + k_m\sigma_T \\ (r-c)\mu_T - \sigma_T(rL(k_m) + ck_m) & K > \mu_T + k_m\sigma_T \end{cases} \tag{4–18}$$

Similarly, the expected profit under early commitment is given by

$$E[\Pi_T^E] = \begin{cases} r\left[\mu_1 - \int\limits_K^\infty (x-K)f_1(x)dx\right] - cK & K \le Q_1^E \\ r\left[\mu_T - \int\limits_{Q_1^E}^\infty (x-Q_1^E)f_1(x)dx - \int\limits_{\hat{Q}_e}^\infty (x-\hat{Q}_e)f_2(x)dx\right] - cQ_S^E & K > Q_1^E \end{cases} \tag{4–19}$$

where $Q_1^E = F_1^{-1}(\frac{r-w}{r})$, Q_E^S is given by Equation (4–15), and $\hat{Q}_e = Q_S^E - Q_1^E$. When demands are normally distributed, Equation (4–19) reduces to

$$E[\Pi_T^E] = \begin{cases} (r-c)\mu_1 - \sigma_1\Psi\left(\frac{K-\mu_1}{\sigma 1}\right) & K \le \mu_1 + k_1\sigma_1 \\ (r-c)\mu_T - \sigma_1\Psi(k_1) - \sigma_2\Psi\left(\frac{K-\mu_T-\sigma_1 k_1}{\sigma_2}\right) & \mu_1 + k_1\sigma_1 < K \le \mu_T + \sigma_1 k_1 + \sigma_2 k_m \\ (r-c)\mu_T - \sigma_1\Psi(k_1) - \sigma_2\Psi(k_m) & K > \mu_T + \sigma_1 k_1 + \sigma_2 k_m \end{cases}$$

where $\Psi(x) = rL(x) + cx$.

From the system point of view, delayed commitment always performs better than early commitment whenever the supplier's production quantity is the same for both systems because of the inventory pooling under delayed commitment. This phenomenon can be explained intuitively as follows: under early commitment, a certain part of the inventory is allocated to the primary retailer independent of the demand realizations. Delayed commitment can also achieve this allocation. That is, the optimal solution under early commitment is a feasible allocation under delayed commitment. Hence, delayed commitment performs better than early commitment. The lemma below proves this assertion for any demand distribution.

Lemma 4.1. *Delayed commitment always outperforms early commitment from the system's perspective if production quantities are equal.*

Proof. See Appendix B.2.1. □

We next examine the increase in total expected profit due to early commitment as we did for the supplier. That is, we consider two cases separately: (i) $[Q_S^D]^U \leq [Q_S^E]^U$, and (ii) $[Q_S^D]^U > [Q_S^E]^U$.

When $[Q_S^D]^U \leq [Q_S^E]^U$, the breakpoints of $E[\Pi_T^E]$ and $E[\Pi_T^D]$ result in four intervals to consider in order to evaluate the increase in the supplier's expected profit due to early commitment: i) $K \leq \mu_1 + k_1\sigma_1$, (ii) $\mu_1 + k_1\sigma_1 < K \leq \mu_T + k_m\sigma_T$, (iii) $\mu_T + k_m\sigma_T < K \leq \mu_T + k_1\sigma_1 + k_m\sigma_2$, and (iv) $K > \mu_T + k_1\sigma_1 + k_m\sigma_2$. Each of these intervals is analyzed in detail in Appendix B.2.2, which reveals the following:

Proposition 4.16. *When $k_m \leq \frac{\sigma_1}{\sigma_T - \sigma_2} k_1$, (a) if delayed commitment is better for the uncapacitated system, it is better for the capacitated system too, (b) if early commitment is better for the uncapacitated system, there exists a threshold capacity level, K_T', such that delayed commitment is preferred when $K \in (0, K_T')$, and early commitment is preferred when $K \in (K_T', \infty)$.*

Proof. For part (a), from Lemma B.4, Lemma B.5, and Lemma B.6, we can conclude that $\Delta[\Pi_T] < 0$ for all K if $\Delta[\Pi_T] < 0$ for the uncapacitated system. For part (b), we know that $\Delta[\Pi_T] < 0$ when $K = \mu_T + k_m\sigma_T$ (Lemma 4.1), and it is nondecreasing thereafter (Lemma B.6). Hence, if $\Delta[\Pi_T] > 0$ in an uncapacitated setting, part (b) follows. □

Figures 4-8 and 4-9 illustrate the analysis in Appendix B.2.2 and Proposition 4.16 for two example settings: (i) $\mu_1 = \mu_2 = 100$, $\sigma_1 = \sigma_2 = 30$, $c = 10$, $w = 25$, $r = 100$ (Figure 4-8), and (ii) $\mu_1 = \mu_2 = 100$, $\sigma_1 = \sigma_2 = 30$, $c = 10$, $w = 15$ and $r = 100$ (Figure 4-9), respectively. For setting (i), the breakpoints of $\Delta[\Pi_T]$ are $Q_1^E = 120.23$, $[Q_S^D]^U = 210.75$, and $[Q_S^E]^U = 227.83$. $\Delta[\Pi_T]$ is decreasing in $(0, 120.23)$, increasing in $(120.23, 227.83)$, and constant thereafter, as characterized in Appendix B.2.2. Delayed commitment outperforms early commitment regardless of K as indicated in Proposition 4.16. For setting (ii), the corresponding breakpoints are $Q_1^E = 131.09$, $[Q_S^D]^U = 181.73$, and $[Q_S^E]^U = 218.17$.

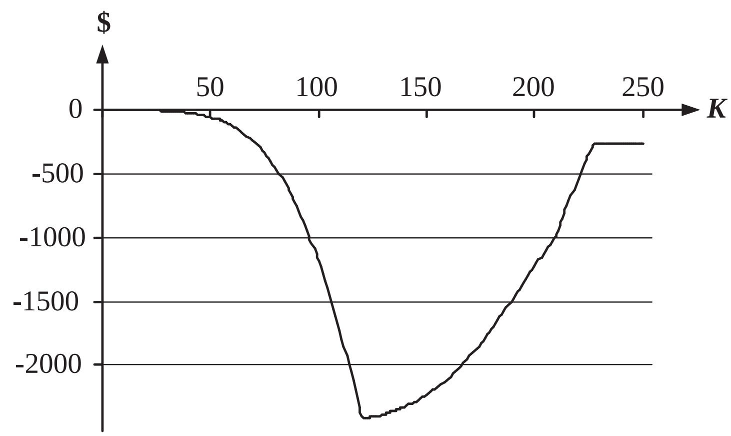

Figure 4-8 $\Delta[\Pi_T]$ as a function of K when $[Q_S^D]^U < [Q_S^E]^U$: sample data (i)

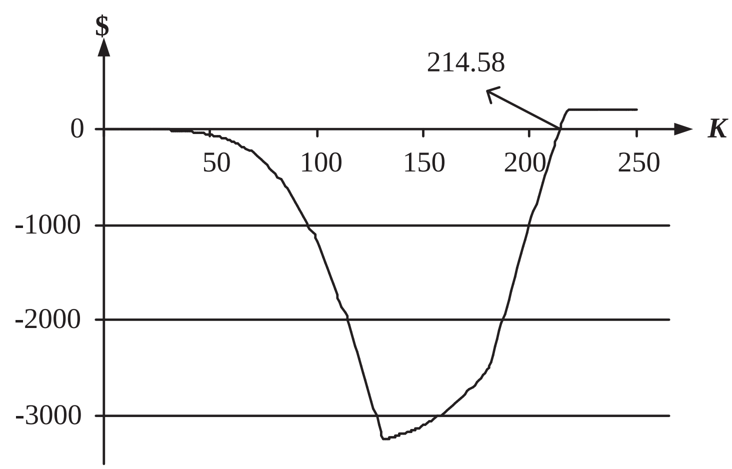

Figure 4-9 $\Delta[\Pi_T]$ as a function of K when $[Q_S^D]^U < [Q_S^E]^U$: sample data (ii)

Although the behavior of $\Delta[\Pi_T]$ with respect to K is similar to setting (i), there exists a threshold capacity level ($K_T' = 214.58$) in this setting such that $\Delta[\Pi_T] > 0$ if $K > 214.58$, and $\Delta[\Pi_T] < 0$ otherwise (see Proposition 4.16).

When $[Q_S^D]^U > [Q_S^E]^U$, we again have four intervals to consider in order to evaluate the increase in the total system profit due to early commitment: (i) $K \leq \mu_1 + k_1\sigma_1$, (ii) $\mu_1 + k_1\sigma_1 < K \leq \mu_T + k_1\sigma_1 + k_m\sigma_2$, (iii) $\mu_T + k_1\sigma_1 + k_m\sigma_2 < K \leq \mu_T + k_m\sigma_T$, and (iv) $K > \mu_T + k_m\sigma_T$. The detailed analysis of the expected total profit in each of these intervals is presented in Appendix B.2.3, which leads to Proposition 4.17:

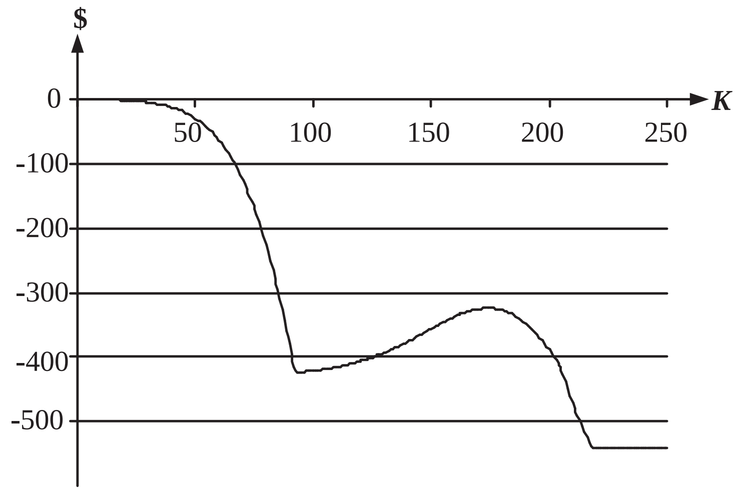

Figure 4-10 $\Delta[\Pi_T]$ as a function of K when $[Q_S^D]^U > [Q_S^E]^U$.

Proposition 4.17. *When* $[Q_S^D]^U > [Q_S^E]^U$*, (a)* $\Delta[\Pi_T]$ *is decreasing in* $K \in (0, \mu_1 + k_1\sigma_1)$*, increasing in* $K \in (\mu_1 + k_1\sigma_1, K_T'')$*, decreasing in* $K \in (K_T'', \mu_T + k_m\sigma_T)$*, and constant thereafter, and (b) delayed commitment always outperforms early commitment.*

Proof. Part (a) is the direct result of Lemmas B.4, B.7, and B.8. Part (b) follows from Lemma 4.1 and Lemma B.8. □

Figure 4-10 illustrates Proposition 4.17 for an example setting where $\mu_1 = \mu_2 = 100$, $\sigma_1 = \sigma_2 = 30$, $c = 10$, $w = 30$, and $r = 50$. In this setting, we have $Q_1^E = 92.4$, $[Q_S^E]^U = 205.32$, $[Q_S^D]^U$=218.27, and $K_T'' = 174.05$.

Now that we have analyzed both cases, that is, $[Q_S^D]^U \leq [Q_S^E]^U$ and $[Q_S^D]^U > [Q_S^E]^U$, we can evaluate the commitment schemes from the system's perspective in a capacitated setting.

Corollary 4.4. *If a delayed commitment scheme performs better in an uncapacitated setting, it also performs better in a capacitated setting.*

Proof. The corollary follows from Proposition 4.16 and Proposition 4.17. □

Corollary 4.5. *If an early commitment scheme is better in an uncapacitated setting, there exists a threshold capacity level, K'_T, such that expected total system profit is greater with a delayed (early) commitment scheme when $K \leq K'_T$ ($K > K'_T$).*

Proof. The corollary follows from Proposition 4.16 and Proposition 4.17. □

4.7 Concluding Remarks

This chapter analyzed supplier-buyer relations in terms of the timing of order commitments. We considered a multiple-retailer, single-supplier system where one of the retailers is designated as the primary customer of the supplier, and investigated two different commitment schemes; (i) the early commitment scheme where the primary retailer orders in advance of the selling season before observing the random demand and has priority in inventory allocation, and (ii) the delayed commitment scheme where she orders after demand realization along with a collection of customers that use the supplier as a source for their overflow demand, and the supplier rations inventory among the retailers according to a generalized uniform allocation rule. We derived analytical results characterizing the preferences of the primary retailer and the supplier, and the tradeoff between the supply and demand risks. We also conducted a computational analysis that supports and enhances our findings. Moreover, we considered an extension that allowed the secondary retailer to select her commitment scheme as well. Finally, we also investigated the capacitated setting and its implications with respect to the order timing preferences of the primary retailer and the supplier.

Our analysis indicates that the supplier prefers the early commitment scheme under mild conditions. The primary retailer's preference depends on the tradeoff between supply and demand risks. When her optimal service level is high, the supply risk is more critical, and she prefers the early commitment scheme. On the other hand, when her service level is lower, the demand risk is more critical, and she prefers the delayed commitment scheme. Having compared the multi-retailer setting to a single-retailer system where the primary retailer is the only customer of the supplier both analytically and numerically, we also observed that both the supplier and the primary retailer's valuations of delayed commitment increase when the supplier has another source of demand besides the primary retailer. Hence, we conclude that although an early commitment scheme is likely to provide higher profits for both parties in certain settings, the comparison of the commitment schemes is not as clear cut as in the single-retailer system.

The presence of a capacity constraint for the supplier may also affect the commitment preferences of both parties. If the supplier's choice is early commitment under in an uncapacitated system, the capacity constraint does not change his preference, which is also true for the primary retailer. On the other hand, if their choice in an uncapacitated setting is delayed commitment, it depends on the level of capacity in a capacitated system. If the capacity is less than a threshold value, both parties prefer early commitment. However, one should note that the respective threshold capacity levels for the primary retailer and the supplier are different.

CHAPTER 5
STRATEGIC TOOLS FOR DEMAND MANAGEMENT

In the previous chapter, we analyzed two commitment schemes and provided both analytical and numerical evidence that supports current industry practice, which can be summarized as follows: while the supplier side of the supply chain strives for early commitment in order to have a more efficient production plan, the retail side is reluctant to do so in certain settings due to the lack of sufficient incentives. Furthermore, the analysis of the capacitated system suggested that the primary retailer is more likely to select early commitment if the supplier's production quantity is limited. The characterization of the preferences of the supplier and the primary retailer depended on the comparison of their respective profit levels under delayed and early commitment schemes. For instance, while considering the preference of the primary retailer, we evaluated her expected profit under a delayed (early) commitment scheme assuming that the supplier also acts under a delayed (early) commitment scheme. That is, we have not yet analyzed the outcome of any strategic interaction between the supplier and the primary retailer, and the possible implications of the power structure in the supply chain.

In this chapter, we first characterize the outcome of strategic interaction between the supplier and the primary retailer in terms of order timing under the setting provided in the previous chapter, which we call 'strategic order timing'. We first investigate strategic order timing under the primary retailer's lead. This setting and its outcome are referred to as the 'original setting' and the 'original outcome', respectively and they serve as a benchmark for the demand management tools that the supplier can employ. We then analyze the results of strategic order timing under the supplier's lead and conclude that the supplier may use his production quantity, which serves as a capacity limit to the retailers, as a demand management tool in order to lead the system to his desired outcome, provided that he is the leader and his decision is observable by the primary retailer.

Another tool that the supplier can employ is to utilize the possibility of leftover stock after the realization of the secondary retailer's demand. Note that the design of the early commitment scheme in the previous chapter does not allow any flexibility, either to the supplier or to the primary retailer, in making use of such leftovers. Once the primary retailer submits her order quantity, neither the supplier nor the primary retailer has recourse in the case of a shortage in the primary retailer's market. Although the supplier already benefits from early commitment without recourse (under reasonable assumptions), the primary retailer may be reluctant to switch from the status quo unless provided further incentive. One such incentive that we analyze is to allocate leftovers at the supplier's site to the primary retailer provided that she faces stockouts. The supplier may have leftovers because he produces for the secondary retailer under uncertainty and the secondary retailer's demand may turn out to be less than the inventory built up for her. Hence, this arrangement may provide enough incentive for the primary retailer to select early commitment, as one side of the inventory risk that she faces will be partially removed. This demand management tool, where the supplier uses leftovers from the secondary retailer's demand to cover the shortage that the primary retailer faces, is called 'early commitment with recourse'.

The last instrument that we consider is the supplier's opportunity to charge a different wholesale price than the existing one. Our original model assumes that the wholesale price between the retailers and the supplier is fixed and does not change when the commitment scheme is changed. As we discussed before, the primary retailer opts to stick with delayed commitment in the majority of the cases we considered. The supplier, who benefits from early commitment, may offer a lower wholesale price in order to induce early commitment of the primary retailer. We will analyze two different models depending on the flexibility of the supplier: (i) the secondary retailer also benefits from the discounted wholesale price offered to the primary retailer that commits early, and (ii) the primary retailer is offered a discount on the wholesale price if she selects early commitment, and the wholesale price for the secondary retailer remains the same. Both arrangements may decrease the expected profit of the supplier under early commitment. Note that the supplier will also lose part of the profit from the secondary retailer by offering a discounted wholesale price in setting (i). Hence, when compared to (ii), the supplier is not willing to decrease the wholesale price as much in (i). That is, the supplier has higher flexibility if he can offer the discount only to the primary retailer. In both cases, if the benefits of an early commitment scheme are large enough, then the supplier may offer such a discount.

The remainder of this chapter is organized as follows: in Section 5.1, we characterize the strategic interaction between the supplier and the primary retailer and introduce the supplier's production capacity as a demand management tool. Section 5.2 presents the strategic use of leftovers under different power structures of the supply chain. After analyzing wholesale pricing in Section 5.3, we conclude this chapter by highlighting our main findings in Section 5.4.

5.1 Analysis of Strategic Order Timing

In this section, we aim to analyze the strategic interaction between the supplier and the primary retailer, which we call 'strategic order timing,' under different power structures. We first characterize the outcome of the resulting game under the primary retailer's lead. In this setting, first the primary retailer announces the commitment scheme, and then the supplier decides the production quantity. This framework serves as a benchmark for the demand management tools that the supplier can employ in order to induce his desired outcome. Then, we analyze the game under the supplier's lead. That is, we consider the setting where the supplier acts first by building up capacity or inventory, and the primary retailer selects a commitment scheme afterwards. The outcome of the resulting interaction depends on the information level of the primary retailer. We first assume that the capacity/inventory level built up by the supplier is observable by the primary retailer. This assumption is quite reasonable for our settings since we investigate the relationship between a supplier and a retailer that have been in business together and have a strong relationship. We then analyze the case where the supplier is still the leader, but capacity information is not available to the primary retailer when she makes her decision.

5.1.1 Primary Retailer's Lead

In this section, we analyze strategic order timing under the primary retailer's lead, which will be used as a benchmark for the demand management tools that we analyze in subsequent sections. Hence, the setting introduced in this section and its outcome will be referred to as the 'original setting' and the 'original outcome' in the rest of this chapter.

Under her lead, the primary retailer acts first by announcing a commitment scheme. The order of events for the 'strategic order timing' can be defined as follows:

1. The primary retailer selects the commitment scheme, and places an order if she selects early commitment.
2. The supplier decides the production quantity and carries out the production plan.
3. The primary retailer receives her order if she selected early commitment.
4. Demands are realized.
5. The supplier allocates inventory to the secondary retailer, and the primary retailer if she has selected delayed commitment.

Since the supplier and the primary retailer act sequentially, the resulting interaction can be represented by a decision tree as depicted in Figure 5-1, where the primary retailer acts first at the top level, and the supplier acts after that at the second level. The supplier's decision depends on the action of the primary retailer, i.e., the node he is at after the primary retailer's move. Note that the action set of the supplier in Figure 5-1 is limited to two for simplicity, whereas he actually has an infinite number of possible actions since we assume his production quantity is continuous.

The optimal solution of the decision tree can be characterized using backward induction. Being the leader, that is, having the first move, the primary retailer takes into

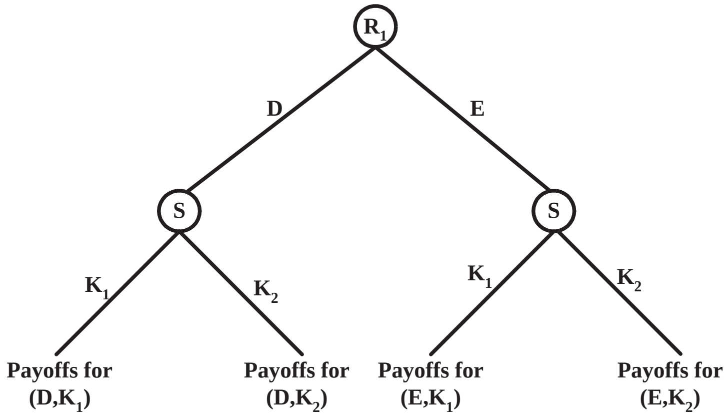

Figure 5-1 Sequential game under the primary retailer's lead.

account the optimal response of the supplier. Recall that we have already analyzed the optimal production quantity of the supplier given the selection of the primary retailer in Chapter 4. The supplier produces $Q_S^D = H^{-1}(\frac{w-c}{w}) = \mu_T + k_m\sigma_T$ if the primary retailer selects delayed commitment, and $Q_S^E = F_1^{-1}(\frac{r-w}{r}) + F_2^{-1}(\frac{w-c}{w}) = \mu_T + k_1\sigma_1 + k_m\sigma_2$ if she selects early commitment. Hence, in this setting, the primary retailer is able to dictate her choice of commitment scheme to the supply chain. That is, the primary retailer can determine what will happen, and picks her best outcome based on demand, cost and revenue parameters. Next, we revisit our computational study in Chapter 4 to illustrate the equilibrium solution for different parameter values.

In order to analyze the effects of different parameters on the outcome of the interaction between the supplier and the primary retailer, we start with the data set designed in Chapter 4, which is duplicated in Table 5-1. Of the 243 test cases, the primary retailer will select delayed commitment in 102 instances (42%). However, recall that the computational test was biased in that we excluded instances where we could prove analytically through sufficient conditions that the primary retailer would select a delayed commitment scheme. In order to have an unbiased analysis, we use the 1,000 random test instances that were generated in Chapter 4 with the following parameters: $\mu_i \sim U[0, 150]$, $cv_i \sim U[1/10, 1/3]$, $c = 5$, $w \sim U[c, 10c]$, $r \sim U[w, 10w]$. Tables C-1,

Table 5-1: Parameters.

c	w	r	μ_2	μ_1	σ_1/μ_1	σ_2/μ_2
5	10	40	80	80	1/7	1/7
		70	100	100	1/5	1/5
		100	120	120	1/3	1/3

C-2, and C-3 in Appendix C depict the distribution of these instances over different ranges of service levels, coefficient of variations, and mean demand levels, respectively. In 105 of these instances, the equilibrium is early commitment. That is, only 10.5% of the time does the primary retailer select early commitment. In each of these 105 instances, the supplier's preference is also early commitment. Hence, early commitment is pareto-optimal. However, the supplier prefers early commitment in 933 instances, i.e., 93.3% of the time. In the remaining 67 instances, he prefers a delayed commitment scheme, which is also the equilibrium solution. Hence, in only 17.2% of the test instances, the supplier's preference and the equilibrium outcome match. Therefore, we may deduce that the supplier has a significant incentive to work with the primary retailer in order to get her to select an early commitment scheme.

For the rest of this chapter, we analyze a number of demand management tools that the supplier can utilize to induce his desired outcome, and assess their effectiveness.

5.1.2 Supplier's Lead with Observable Capacity

The analysis of the original setting in Section 5.1.1 revealed that the original outcome of the strategic interaction between the supplier and the primary retailer is not in alignment with the supplier's preference in most cases, which is early commitment. Hence, the supplier has a significant motive to get the primary retailer to commit early. One such tool that the supplier can utilize is to act before the commitment decision of the primary retailer by building up capacity or inventory.

In this section, we argue that the supplier can influence the decision of the primary retailer if he can credibly share the production quantity information with the primary retailer, or if the primary retailer is able to observe the action taken by the supplier. Note that it will not make a difference whether the supplier builds up capacity first (with an associated unit cost) and produces after the commitment decision of the retailer, or he directly builds up inventory before the primary retailer's choice, since the supplier will never build up more capacity than the quantity he produces. Hence, the unit cost of capacity can be incorporated into the unit production cost if there is a capacity build-up process before the production. From this point on, we assume that the supplier first builds up capacity, and in this section this process is observable by the primary retailer. This assumption is reasonable for our settings since we investigate the relationship between a supplier and a retailer that have been in business together. We will also analyze the outcome of the game where the capacity information is not observable by the primary retailer in the next section.

The order of events in this setting can be summarized as follows:

1. The supplier builds up capacity and the primary retailer observes the capacity level.
2. The primary retailer selects a commitment scheme and places an order if she selects early commitment.
3. The supplier carries out production and delivers primary retailer's order if she selected early commitment.
4. Demands are realized.
5. The supplier allocates inventory to the secondary retailer, and the primary retailer if she has selected delayed commitment.

If both the supplier and primary retailer prefer early (delayed) commitment, the supplier's leadership will not yield a different outcome than the primary retailer's leadership. That is, the capacity level imposed by the supplier will match the uncapacitated solution, which is $Q_S^E = F_1^{-1}(\frac{r-w}{r})+F_2^{-1}(\frac{w-c}{w})$ ($Q_S^D = H^{-1}(\frac{w-c}{w})$). Furthermore, the supplier cannot get the primary retailer to select delayed commitment by imposing a capacity limit if her original choice is early commitment (see Section 4.6 for a detailed discussion of the capacitated setting). Hence, the supplier is going to use the leadership position effectively only if he and the primary retailer prefer early and delayed commitment schemes in the original setting, respectively.

We first consider the primary retailer's decision given the capacity imposed by the supplier, K. As we discussed in Section 4.6, if the primary retailer prefers early commitment in the original setting, she will continue to do so in the presence of limited capacity. On the other hand, if she prefers delayed commitment, we have the following:

Proposition 5.1. *If the primary retailer prefers delayed commitment in the original setting, there exists a threshold capacity level, K_r, such that the primary retailer is indifferent between a capacitated delayed commitment and early commitment.*

Proof. When $K \leq \mu_T + k_m\sigma_T$, R_1's expected profit under delayed commitment and its first derivative with respect to K are:

$$\begin{aligned} E[\Pi_1^D(K)] &= (r-w)\Bigg[\mu_1 - \int_0^{(1-\rho)K}\int_{K-x_2}^{\infty}(x_1+x_2-K)dF_1(x_1)dF_2(x_2) \\ &\quad - \int_{\rho K}^{\infty}(x_1-\rho K)[1-F_2((1-\rho)K)]dF_1(x_1)\Bigg] \end{aligned}$$

and

$$\frac{dE[\Pi_1^D(K)]}{dK} = (r-w)\Big(\int_0^{(1-\rho)K}[1-F_1(K-x_2)]f_2(x_2)dx_2 + \rho[1-F_1(\rho K)][1-F_2((1-\rho)K)]\Big)$$

Then, we have

$$0 < \left[\frac{dE[\Pi_1^D(K)]}{dK}\right]\Bigg|_{K\to 0^+} = (r-w)\rho \;<\; (r-w) = \left[\frac{dE[\Pi_1^E(K)]}{dK}\right]\Bigg|_{K\to 0^+} \qquad (5\text{–}1)$$

Inequality (5–1) indicates that there exists an interval $(0,\epsilon)$, $\epsilon > 0$, such that $E[\Pi_1^E(K)] > E[\Pi_1^D(K)]\ \forall\ K \in (0,\epsilon)$, which proves the proposition together with the condition that $[E[\Pi_1^D]]^U > [E[\Pi_1^E]]^U$, where $[E[\Pi_1^D]]^U$ and $[E[\Pi_1^E]]^U$ denote the primary retailer's expected profit under the original delayed and early commitment schemes, respectively. □

An immediate corollary of the above proposition is:

Corollary 5.1. *If the primary retailer prefers delayed commitment, there exists a capacity level that can be imposed by the supplier such that the primary retailer prefers early commitment.*

Corollary 5.1 does not necessarily guarantee that the supplier is going to set capacity equal to a level where the primary retailer selects early commitment. Whether the supplier should set capacity equal to this level depends on his expected profits under the original delayed commitment scheme and under a capacitated early commitment scheme where the capacity level is characterized by Proposition 5.1. Our approach to resolve this issue is as follows: we next show that if the supplier prefers early commitment in the original setting, there exists a threshold capacity level above which the supplier will generate more profit under an early commitment scheme compared to the original delayed commitment scheme. Then, by comparing this threshold with the one that makes the primary retailer indifferent, we can conclude whether the supplier should effectively assume the leadership position and build up capacity to have the primary retailer order early.

Lemma 5.1. *If the supplier prefers early commitment in the original setting, there exists a threshold capacity level, K_s, such that (i) $E[\Pi_S^E(K_S)] = [E[\Pi_S^D]]^U$, (ii) $E[\Pi_S^E(K)] > [E[\Pi_S^D]]^U \ \forall\ K > K_s$, and (iii) $E[\Pi_S^E(K)] < [E[\Pi_S^D]]U \ \forall\ K > K_s$.*

Proof. Let $\Delta[\Pi_S(K)]$ denote the difference in expected supplier profits of a capacitated early commitment and original delayed commitment, i.e., $\Delta[\Pi_S(K)] = E[\Pi_S^E(K)] - [E[\Pi_S^D]]^U$. When $K > \mu_T + k_1\sigma_1 + k_m\sigma_2$, the optimal unconstrained solution of the early commitment scheme is achievable. Since we assume the supplier prefers early commitment in the original setting, we have $\Delta[\Pi_S(K)] > 0$. When $K = 0$, $\Delta[\Pi_S(0)] = -[E[\Pi_S^D]]^U < 0$. Hence, part (i) follows. For $0 < K < \mu_T + k_1\sigma_1 + k_m\sigma_2$, we have

$$\Delta[\Pi_S(K)] \quad = \quad w\left[k_1\sigma_1 + \sigma_T L(k_m) - \sigma_2 L\left(\frac{K - \mu_T - k_1\sigma_1}{\sigma_2}\right)\right] + c(k_m\sigma_T + \mu_T - K)$$

and

$$\frac{d\Delta[\Pi_S(K)]}{dK} = -wL'\left(\frac{K-\mu_T-k_1\sigma_1}{\sigma_2}\right) - c = w\left[1-\Phi\left(\frac{K-\mu_T-k_1\sigma_1}{\sigma_2}\right)\right] - c > 0$$

The inequality follows since $\Phi\left(\frac{K-\mu_T-k_1\sigma_1}{\sigma_2}\right) < \Phi(k_m) = \frac{w-c}{c}$. $\square$

Proposition 5.2. *If the supplier and the primary retailer prefer early and delayed commitment schemes in the original setting, respectively, and $K_r > K_s$, then the supplier can increase his profits compared to the original setting, where the primary retailer is the leader, by assuming the leadership in the game and setting capacity equal to K_r.*

Proof. The proposition follows from Corollary 5.1 and Lemma 5.1. $\square$

Proposition 5.2 characterizes the outcome of the strategic interaction between the supplier and the primary retailer when the supplier is the leader. When both parties prefer the same commitment scheme, the supplier sets the capacity equal to his production quantity in the original setting. When the supplier and the primary retailer prefer delayed and early commitment schemes, respectively, the supplier cannot increase his profit by setting a different capacity level than the production quantity under primary retailer's lead. Hence, the outcome is again the same, independent of the leader. On the other hand, when the supplier and primary retailer prefer early and delayed commitment schemes, the outcome depends on the threshold capacity levels. If $K_r > K_s$, the supplier increases his profits by setting capacity to K_r and the primary retailer selects early commitment. Otherwise, the outcome is the same as the case where the primary retailer is the leader, which is the original delayed commitment (equivalently, $K = [Q_S^D]^U$). Table 5-2 summarizes the outcome of the game between the supplier and the primary retailer under the lead of both parties.

We next provide the results of our computational analysis to see how effective capacity is as a tool for the supplier to manipulate the primary retailer's demand, that is,

Table 5-2: Outcome of strategic order timing under supplier's and primary retailer's lead.

Supplier's Choice	R_1's Choice	R_1's Lead	Supplier's Lead
Early	Early	Early	Early , $K = [Q_S^E]^U$
Delayed	Delayed	Delayed	Delayed , $K = [Q_S^D]^U$
Early	Delayed	Delayed	Early , $K = K_r$ if $K_r > K_s$
			Delayed , $K = [Q_S^D]^U$ if $K_r < K_s$
Delayed	Early	Early	Early , $K = [Q_S^E]^U$

to have her commit early. We start with the data set presented in Table 5-1. Then, we move on to the random data set provided in Section 5.1.1.

Recall from Chapter 4 (Section 4.4) that the supplier prefers early commitment for all instances generated from the parameters in Table 5-1. Hence, whenever the primary retailer chooses to delay her order until after her demand, the supplier has an opportunity to use capacity in order to induce an early commitment scheme. There are 102 (out of 243) such instances, which includes all instances with $r = 40$ and 21 (out of 81) instances with $r = 70$. In all of these 102 instances, the supplier successfully uses capacity to induce early commitment. The maximum percentage gain for supplier is 16.65%, which occurs when $\mu_1 = 80$, $\mu_2 = 120$, $cv_1 = cv_2 = 1/3$, and $r = 70$. This is intuitively reasonable for the following reason: when compared to $r = 40$, the primary retailer is closer to early commitment with $r = 70$ (see our discussions in Chapter 4, Section 4.4). Hence, the supplier does not need to set the capacity too tight to get the primary retailer to commit early. Furthermore, the benefits of early commitment from the supplier's perspective will be larger when cv_1 is high since $k_1 > 0$. Similar arguments are also valid for the instance where the supplier has the minimum gain (2.70%), which occurs when $\mu_1 = 80$, $\mu_2 = 120$, $cv_1 = cv_2 = 1/7$, and $r = 40$. The average gain for the supplier from using capacity as a strategic tool is 7.41%.

In all 102 cases, the primary retailer gets her order quantity in full when she switches to early commitment due to the capacity limit. Furthermore, we know that she prefers delayed commitment in all these instances in the original setting. Hence, we can conclude that the primary retailer's expected profit decreases due to the limited capacity. The maximum (minimum) decrease is 5.21% (0.05%), which occurs when $\mu_1 = 120$, $\mu_2 = 120$, $cv_1 = cv_2 = 1/3$, and $r = 40$ ($\mu_1 = 80$, $\mu_2 = 120$, $cv_1 = cv_2 = 1/7$, and $r = 70$). Her average loss is 2.63%.

We now consider the effects of different demand parameters on the respective profits of the supplier and the primary retailer. We only consider the instances with $r = 40$ since they form a complete set for all demand parameters. The effects of the mean demands of both retailers are depicted in Figure 5-2 for the supplier, and Figure 5-3 for the primary retailer. When the primary retailer's demand increases, her order quantity under an early commitment scheme will also increase with certainty since $k_1 > 0$. Hence, the percentage increase in the supplier's profit also increases (see Figure 5-2). From the primary retailer's perspective, the benefits of delayed commitment decrease as μ_1 increases (see Chapter 4, Figure 4-1). Hence, being forced to commit early results in lower losses when μ_1 increases (see Figure 5-3). When μ_2 increases, the portion of the supplier's profit due to R_2 increases. Hence, we expect that the increase in profits due to the early commitment of R_1 will decrease. Furthermore, increasing μ_2 makes R_1 more reluctant to commit early (see Chapter 4, Figure 4-2), which forces the supplier to set a tighter capacity. Hence, the percentage increase in the supplier's profit decreases in μ_2 (see Figure 5-2). Due to the same considerations, the percentage loss of the primary retailer increases in μ_2 (see Figure 5-3).

The effects of coefficients of variation of the retailer demands (cv_1 and cv_2) are illustrated in Figure 5-4 for the supplier, and Figure 5-5 for the primary retailer. When cv_1 increases, the order quantity of the primary retailer under an early commitment scheme increases, which works in favor of capacitated early commitment. However, the retailer becomes more reluctant to commit early since the benefits of delayed commitment increase in cv_1 (see Chapter 4, Figure 4-3), resulting in a tighter capacity that decreases the supplier's profits due to R_2's demand. Figure 5-4 shows that the former dominates the latter, resulting in higher percentage increases in the supplier's profit as cv_1 increases. As

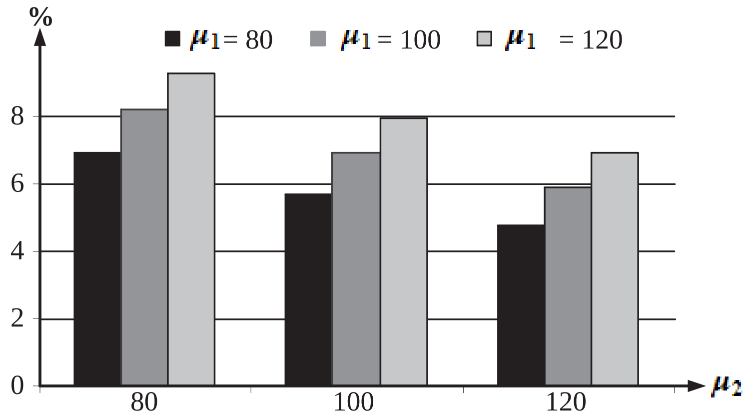

Figure 5-2 Percent increase in the supplier's expected profit: effects of μ_1 and μ_2.

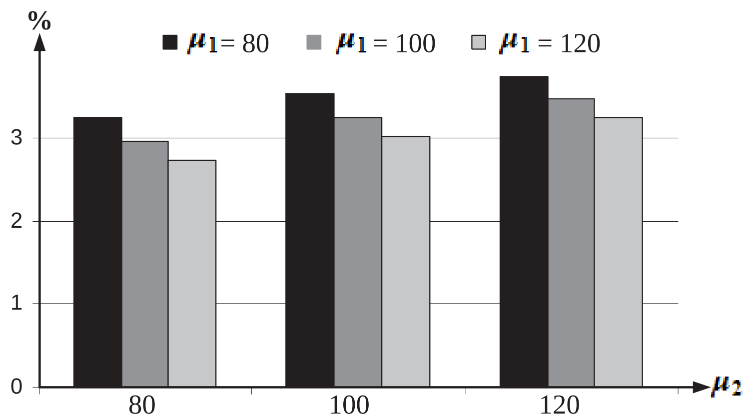

Figure 5-3 Percent decrease in R_1's expected profit: effects of μ_1 and μ_2.

the benefits of delayed commitment increase in cv_1, the percentage decrease in the primary retailer's profit increases in cv_1 (see Figure 5-5). The increase in the supplier's profit does not show a significant change in cv_2. However, the primary retailer's percentage loss increases in cv_2 (see Figure 5-5). This is mainly due to the increase in the benefits of a delayed commitment (see Chapter 4, Figure 4-4).

Recall that the supplier is able to use capacity as a strategic tool and increase his profits in all 102 instances in our data set in which the primary retailer selects delayed commitment originally. However, recall also that we have $k_1 > k_m$, i.e., the retail markup is greater than the wholesale markup, in all instances. We next consider the random data set discussed in Section 5.1.1 to see how effective capacity is when we do not assume any

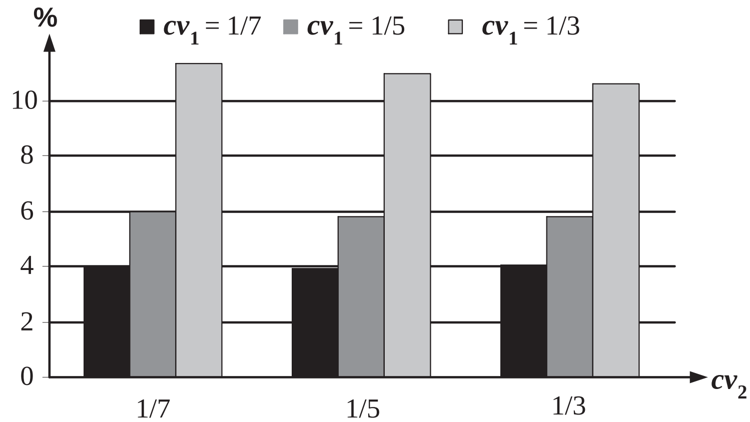

Figure 5-4 Percent increase in the supplier's expected profit: effects of cv_1 and cv_2.

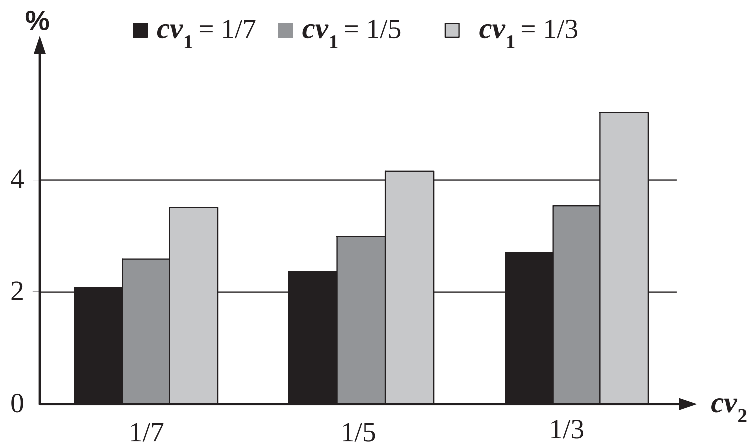

Figure 5-5 Percent decrease in R_1's expected profit: effects of cv_1 and cv_2.

order on k_1 and k_m. In 1000 instances examined, the supplier has the opportunity to use capacity in 828 instances. That is, the supplier and the primary retailer prefer early and delayed commitment in the original setting, respectively. In 505 of these instances (61%), the supplier benefits from inducing the primary retailer to commit early. The percentage increase in the supplier's profit can be as high as 63%, and the average is 6.68%. The average decrease in the primary retailer's profit is 4.55%, the maximum being 42%. Next, we analyze how these instances are distributed over demand and revenue parameters in terms of effectiveness, which we measure as the percentage of instances in which capacity can be used successfully to induce early commitment.

Table 5-3: Strategic use of capacity over k_1 and k_2.

	$k_m \leq -1$	$-1 < k_m \leq 0$	$0 < k_m \leq 1$	$k_m > 1$
$k_1 \leq -1$	$0 - (0)$	$0 - (0)$	$0 - (0)$	$0 - (0)$
$-1 < k_1 \leq 0$	$0 - (0)$	$0 - (3)$	$1 - (20)$	$0 - (4)$
$0 < k_1 \leq 1$	$0 - (0)$	$26 - (29)$	$114 - (228)$	$61 - (197)$
$k_1 > 1$	$0 - (0)$	$0 - (0)$	$169 - (183)$	$134 - (164)$

Table 5-3 depicts the number of instances that capacity can be successfully used as a strategic tool for different ranges of service levels for the primary retailer (k_1) and the supplier (k_m). Recalling that the unit cost was kept constant in all instances, k_m also implies the wholesale price. The numbers in parentheses are the numbers of instances where the supplier has a chance to induce early commitment. The strategic use of capacity is more successful when k_1 is larger, which is reasonable since the primary retailer will be more inclined to switch to early commitment when capacity is restricted. The opposite holds true for k_m: when the service level of the supplier is larger, the primary retailer is more likely to stick to delayed commitment, which would cause the supplier to reduce capacity significantly to induce early commitment. Hence, the success rate of using capacity as a strategic demand management tool decreases. Note that there is no case with $k_1 \leq -1$ or $k_m \leq -1$ where capacity can be used as a strategic tool. In the former case, the supplier prefers delayed commitment as well; hence he does not have an incentive to induce early commitment. In the latter case, the primary retailer already prefers early commitment; hence there is no need to use an extra measure to induce early commitment.

Table 5-4 depicts the number of instances that the supplier benefits from the use of capacity for different ranges of the coefficient of variations, cv_1 and cv_2. There is not an evident pattern in the number of instances as cv_1 and cv_2 changes. This is because the effects of uncertainty on the commitment scheme are closely related to the service levels of the supplier and the primary retailer. For instance, if $k_1 > 0$, the primary retailer's early order quantity increases as uncertainty increases, which may allow the supplier to induce early commitment more efficiently. On the other hand, if $k_1 < 0$, increasing uncertainty works in the opposite direction.

Table 5-4: Strategic use of capacity over cv_1 and cv_2.

	$0.1 \leq cv_2 \leq 0.178$	$0.178 < cv_2 \leq 0.256$	$0.256 < cv_2 \leq 0.333$
$0.1 \leq cv_1 \leq 0.178$	$62 - (97)$	$66 - (107)$	$53 - (107)$
$0.178 < cv_1 \leq 0.256$	$58 - (86)$	$54 - (82)$	$39 - (81)$
$0.256 < cv_1 \leq 0.333$	$56 - (92)$	$63 - (93)$	$54 - (83)$

We now investigate how mean demand levels alter the effectiveness of the use of capacity (see Table 5-5). The success rate of the strategic use of capacity increases (decreases) in μ_1 (μ_2), which is reasonable for the following reason: as μ_1 increases, the primary retailer's need to secure inventory increases, which makes it easier for the supplier to induce early commitment. On the other hand, a larger μ_2 increases the supplier's production quantity under a delayed commitment scheme, which benefits the primary retailer. Hence, the primary retailer will be more reluctant to commit early as μ_2 increases. As a result, the higher success rates occur when the primary retailer's expected demand is larger than the secondary retailer's. However, an exception occurs when the secondary retailer's mean demand is much lower than the primary retailer's, which can be explained as follows: in this case, the proportion of the inventory reserved for the primary retailer under a delayed commitment is so large that it outweighs her need to secure inventory in some instances.

5.1.3 Supplier's Lead with Unobservable Capacity

Our analysis of the interaction between the supplier and primary retailer with the supplier's lead assumed that the supplier's capacity level can be observed by the primary retailer. Now, we investigate the opposite setting, that is, the supplier cannot credibly share the capacity information. In this case, the primary retailer does not observe the capacity level of the supplier. Although the supplier can announce his capacity level, this information is not credible since the supplier may have an incentive to understate the capacity to induce early commitment. Hence, the primary retailer is bound to make her order timing decision without proper information about the capacity level of the supplier.

Table 5-5: Strategic use of capacity over μ_1 and μ_2.

	$0 < \mu_2 \leq 50$	$50 < \mu_2 \leq 100$	$100 < \mu_2 \leq 150$
$0 < \mu_1 \leq 50$	46 – (74)	30 – (87)	24 – (82)
$50 < \mu_1 \leq 100$	80 – (95)	62 – (91)	45 – (92)
$100 < \mu_1 \leq 150$	73 – (100)	83 – (108)	62 – (99)

There are a number of studies in the supply chain management literature that focus on information asymmetry between supply chain members. These studies deal with supply chains where one firm possesses superior information regarding its own costs (Corbett and de Groote (2000), Ha (2001), Corbett (2001)), operating characteristics (Gavirneni et al. (1999), Lim (2001)), and demand forecasts (Lau and Lau (2001), Lee et al. (2000), Li (2002)). When the firm with the superior information is the first party to act, the resulting interaction is characterized as a 'signaling game' (see Cachon and Netessine (2004)). Note that our setting in this section is quite similar to a 'signaling game'. However, there is an important distinction which prevents us from a similar analysis: in the literature, the information asymmetry between members is about an exogenous parameter, for which the firm without perfect information holds a prior probability distribution, and the firm with superior information 'signals' the information by choosing a specific action. In our setting, on the other hand, the information asymmetry is about the action chosen by the first party to act.

The resulting interaction between the supplier and the primary retailer is depicted in Figure 5-6 as an extensive game with imperfect information (see Osborne (2004) for a detailed discussion). For simplicity, it is assumed that the supplier has only two strategies to choose from, K_1 and K_2, whereas he actually has infinitely many. However, we will later on show that he actually has two candidate strategies for optimality. The other strategies will be dominated by these. The dashed line between the nodes corresponding to the primary retailer's decision indicates that the primary retailer does not know the supplier's capacity decision when she chooses the commitment scheme. In other words, she does not know what node she is at when she is to choose her strategy.

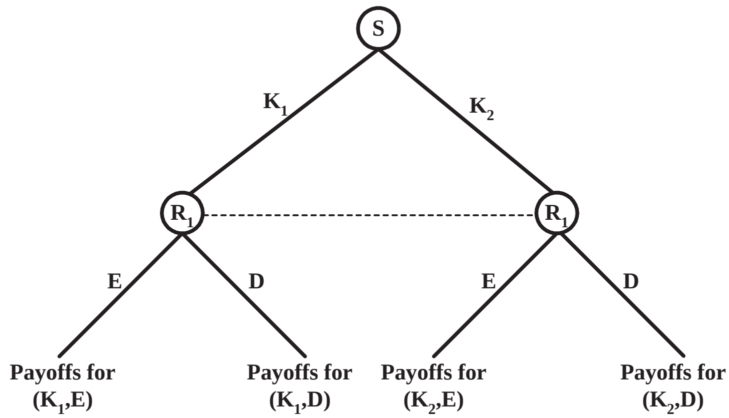

Figure 5-6: Sequential game between the supplier and the primary retailer with imperfect information.

Since each player moves only once, and no player, when moving, is informed of any other player's action, this game may alternatively be modelled as a strategic game where the supplier's action set contains infinitely many strategies (capacity level), and the primary retailer's action set consists of two strategies (early and delayed commitment). The reader may refer to Chapter 4, Section 4.5 for the discussion of strategic games.

Due to Definition 4.2 in Section 4.5, we can limit the action set of the supplier as follows: assume that the primary retailer selects delayed commitment. Then, the only viable action for the supplier is to set capacity equal to $[Q_S^D]^U$ since any other capacity level is dominated by $[Q_S^D]^U$. Similarly, if the primary retailer selects early commitment, the only action for the supplier is to set the capacity equal to $[Q_S^E]^U$. Hence, there are only two possible actions for the supplier that we may observe at equilibrium: $[Q_S^D]^U$ and $[Q_S^E]^U$. For expositional clarity, let $K_D = [Q_S^D]^U$ and $K_E = [Q_S^E]^U$. The resulting game is depicted in Table 5-6, where the supplier is the column player. Each entry in the matrix consists of the expected profits of the primary retailer and the supplier, corresponding to the selected action pair. For instance, the top-right cell corresponds to R_1 and the supplier selecting delayed commitment and K_E, respectively. The expected profits of R_1 and the supplier in this case are represented by $\Pi_1^D(K_E)$ and $\Pi_S^D(K_E)$, respectively.

Table 5-6: Strategic game between the supplier and the primary retailer

	K_D	K_E
Delayed (D)	$(\Pi_1^D(K_D), \Pi_S^D(K_D))$	$(\Pi_1^D(K_E), \Pi_S^D(K_E))$
Early (E)	$(\Pi_1^E(K_D), \Pi_S^E(K_D))$	$(\Pi_1^E(K_E), \Pi_S^E(K_E))$

The equilibrium concept of a strategic game (see Definition 4.2 in Section 4.5) can be illustrated for our setting as follows: consider the action profile (E, K_E), that is, R_1 and the supplier select early commitment and K_E, respectively. This action profile is a Nash equilibrium if and only if R_1 has no incentive to select delayed commitment given the supplier selects K_E (i.e., $\Pi_1^E(K_E) \geq \Pi_1^D(K_E)$), and the supplier has no incentive to select K_D given R_1 selects early commitment (i.e., $\Pi_S^E(K_E) \geq \Pi_S^E(K_D)$).

The supplier will select K_D (K_E) if the primary retailer selects delayed commitment (early commitment). Hence, only (D, K_D) and (E, K_E) are candidate equilibrium solutions. Proposition 5.3 characterizes the outcome of the game.

Proposition 5.3. *If the primary retailer prefers an early commitment scheme in the original setting, either (E, K_E) is the unique equilibrium or there is no pure strategy equilibrium. Otherwise, if the primary retailer prefers a delayed commitment scheme, (D, K_D) is an equilibrium solution.*

Proof. We start with the first part of the proposition, where the primary retailer prefers an early commitment scheme, that is, $\Pi_1^E(K_E) > \Pi_1^D(K_D)$. In this case, (D, K_D) cannot be an equilibrium since $\Pi_1^E(K_D) = \Pi_1^E(K_E) > \Pi_1^D(K_D)$. We now consider the other equilibrium candidate (E, K_E). If $K_E < K_D$, then $\Pi_1^E(K_E) > \Pi_1^D(K_D) > \Pi_1^D(K_E)$. Hence, (E, K_E) is an equilibrium. On the other hand, when $K_E > K_D$, we have $\Pi_1^D(K_E) > \Pi_1^D(K_D)$. Hence, we cannot argue whether $\Pi_1^E(K_E) \geq \Pi_1^D(K_E)$. In such a case, (E, K_E) may or may not be a equilibrium, concluding the proof of the first part of the proposition.

We now analyze the second part, where the primary retailer prefers a delayed commitment scheme, that is, $\Pi_1^D(K_D) > \Pi_1^E(K_E)$. Since $\Pi_1^D(K_D) > \Pi_1^E(K_E) = \Pi_1^E(K_D)$, (D, K_D) is an equilibrium. □

Table 5-7: Example (i) for the strategic game when R_1 prefers early commitment.

	$K_D = 186.9$	$K_E = 192.26$
Delayed	(8,126.93, 843.12)	(8,268.97, 831.49)
Early	(8,343.42, 859.75)	(8,343.42, 862.29)

To illustrate Proposition 5.3, we consider two example settings for the case where the primary retailer prefers early commitment, i.e., $\Pi_1^E(K_E) > \Pi_1^D(K_D)$, with the following parameters: (i) $\mu_1 = \mu_2 = 100$, $\sigma_1 = 5$, $\sigma_2 = 30$, $(c, w, r) = (10, 15, 100)$, and (ii) $\mu_1 = \mu_2 = 100$, $\sigma_1 = \sigma_2 = 30$, $(c, w, r) = (5, 10, 100)$. In example (i) (Table 5-7), (E, K_E) is the unique equilibrium, that is, neither the supplier nor the primary retailer has an incentive to deviate from this action profile, given the other player will adhere to its action in this profile. However, one should note that the primary retailer's preference of early commitment does not necessarily mean that she will continue to prefer early commitment given that the supplier selects K_E. Consider the strategic game in Table 5-8, which corresponds to example (ii). Although the primary retailer prefers an early commitment scheme, she will switch to delayed commitment if the supplier decides to produce K_E units. In this case, the game admits no equilibrium, i.e., we are not able to predict the outcome of the game.

Proposition 5.3 does not prove that (D, K_D) is a unique equilibrium when the primary retailer prefers delayed commitment, that is, it does not exclude (E, K_E) as an equilibrium. If $K_E > K_D$, then $\Pi_1^D(K_E) > \Pi_1^D(K_D) > \Pi_1^E(K_E)$. Hence, (E, K_E) is not an equilibrium. If $K_E < K_D$, then $\Pi_1^D(K_E) < \Pi_1^D(K_D)$. Hence, we cannot conclude whether $\Pi_1^E(K_E) \geq \Pi_1^D(K_E)$ or not, i.e., whether (E, K_E) may be an equilibrium or not. However, through an extensive computational analysis, we conjecture that (D, K_D) is the unique equilibrium when the primary retailer prefers a delayed commitment scheme.

As a result, the outcome of the interaction is quite similar to the outcome in the original setting, where the primary retailer is the leader. Whenever the primary retailer prefers delayed commitment, the outcome of the sequential game with the supplier's lead is delayed commitment if the capacity level/production quantity of the supplier cannot be

Table 5-8: Example (ii) for the strategic game when R_1 prefers early commitment.

	$K_D = 200$	$K_E = 238.45$
Delayed	(8,238.34, 830.74)	(8,810.43, 765.64)
Early	(8,473.51, 985.80)	(8,473.51, 1,072.55)

observed by the retailer. Hence, the supplier cannot induce the primary retailer to commit early. That is, the supplier has a chance to use capacity as a strategic tool to manipulate the primary retailer's demand only if he can credibly share the capacity information.

5.2 Early Commitment with Recourse

In the original early commitment model that we analyzed in Chapter 4, the primary retailer has a single order opportunity, which is utilized before the demand realizations. Due to the uncertainty involved in the demand process, the primary retailer usually opts to select delayed commitment and share a common pool of inventory with the secondary retailer after demand realizations in order to eliminate costs associated with demand uncertainty, rather than ordering early and facing the risk of uncertainty.

In this section, we analyze a more flexible version of the original early commitment model, 'early commitment with recourse', where the supplier offers the leftovers, if any, to the primary retailer after demands are realized. Assuming that the supplier produces a positive quantity for the secondary retailer, there is always a positive probability that the supplier will have leftovers under the original early commitment scheme because the production for the secondary retailer is delivered after her demand realization. If her demand turns out to be less than the production quantity, the supplier will have leftovers. Under early commitment with recourse, these leftovers will be allocated to the primary retailer if her demand realization exceeds her initial order. In this setting, we aim to analyze whether such an arrangement provides enough incentive for the primary retailer to choose early commitment over a delayed commitment scheme, and whether the supplier is willing to offer it.

The implications of such an arrangement seem trivial at first. Both the supplier and primary retailer appear to benefit; the supplier generates more profit from possible leftovers, and the primary retailer avoids possible lost sales. However, for the primary retailer to select early commitment with recourse over the status quo, this arrangement must be made before she makes her decision about order timing. That is, the supplier must inform her that he is willing to offer the possible leftovers to the primary retailer. In such a case, the retailer may decrease her order quantity compared to the original early commitment scheme in order to decrease her inventory risk related to unsold items. The supplier, anticipating such behavior, may increase her production quantity for the secondary retailer as there is now a greater chance that the primary retailer will face stockouts. That is, the recourse opportunity may serve as a second order for the primary retailer, and induce her to select early commitment. Moreover, it may also benefit the secondary retailer as she will have a larger inventory available. Conversely, the supplier may decrease the production quantity for the secondary retailer in order to increase the order quantity of the primary retailer. In such a case, the primary retailer is still better off when compared to the original early commitment scheme, whereas the recourse option hurts the secondary retailer since the inventory available for her is less than the original early commitment case. The dynamics of this arrangement are also closely related to the power structure of the supply chain, and hence the sequence of events. We first consider the setting where the primary retailer acts first by placing an early order, and the supplier follows with a production plan. Then, we analyze the system with the supplier's lead, where the supplier truthfully commits to the excess production quantity over the primary retailer's early order. Finally, we investigate the resulting model where the primary retailer cannot observe (or does not trust) the supplier's commitment to a production quantity for the selling season.

5.2.1 Primary retailer's Lead

When the primary retailer is the leader, she acts first by committing to an order quantity, Q_1. The supplier then responds by producing Q_S units in addition to Q_1, where Q_S is the production quantity for the selling season. Once demands are realized, Q_S

is allocated to the secondary retailer. Any leftover that may result from the secondary retailer's demand realization being less than Q_S is allocated to the primary retailer if the primary retailer's demand realization exceeds her initial order. We solve the resulting problem using backward induction.

We start by characterizing the optimal response of the supplier, given the order quantity of the primary retailer. The supplier's profit is given by

$$\Pi_S(Q_1, Q_S) = w \left\{ \begin{array}{ll} (Q_1 + x_2) & x_1 \leq Q_1, x_2 \leq Q_S \\ (Q_1 + Q_S) & x_2 > Q_S \\ (x_1 + x_2) & Q_1 < x_1 \leq Q_1 + Q_S - x_2, x_2 \leq Q_S \\ (Q_1 + Q_S) & x_1 > Q_1 + Q_S - x_2, x_2 \leq Q_S \end{array} \right\} - c(Q_1 + Q_S). \tag{5–2}$$

Taking expectation with respect to X_1 and X_2, we get

$$\begin{aligned} E[\Pi_S(Q_1, Q_S)] =& w \left[\int_0^{Q_S} \left((Q_1 + x_2)F_1(Q_1) + (Q_1 + Q_S)[1 - F_1(Q_1 + Q_S - x_2)] \right) dF_2(x_2) \right. \\ & \left. + \int_0^{Q_S} \left(\int_{Q_1}^{Q_1+Q_S-x_2} (x_1 + x_2) dF_1(x_1) \right) dF_2(x_2) + (Q_1 + Q_S)[1 - F_2(Q_S)] \right] \\ & -c(Q_1 + Q_S) \end{aligned} \tag{5–3}$$

The first and second derivatives of the supplier's expected profit with respect to Q_S are

$$\frac{\partial E[\Pi_S(Q_S, Q_1)]}{\partial Q_S} = w \left[\int_0^{Q_S} [1 - F_1(Q_1 + Q_S - x_2)] f_2(x_2) dx_2 + 1 - F_2(Q_S) \right] - c \tag{5–4}$$

and

$$\frac{\partial^2 E[\Pi_S(Q_S, Q_1)]}{\partial (Q_S)^2} = -w \left[f_2(Q_S)F_1(Q_1) + \int_0^{Q_S} f_1(Q_1 + Q_S - x_2) f_2(x_2) dx_2 \right] < 0,$$

respectively. Hence, $E[\Pi_S(Q_S|Q_1)]$ is concave in Q_S, and the optimal production quantity of the supplier is determined by the first order condition.

Proposition 5.4. *For any finite Q_1, the production quantity of the supplier for the selling season under early commitment with recourse (Q_S) is greater than the production quantity for R_2 under early commitment without recourse. That is, $Q_S^*|Q_1 > F_2^{-1}((w-c)/w)$.*

Proof.

$$\left.\frac{\partial E[\Pi_S(Q_S, Q_1)]}{\partial Q_S}\right|_{Q_S = F_2^{-1}(\frac{w-c}{w})} = w\left[\int_0^{Q_S} [1 - F_1(Q_1 + Q_S - x_2)] f_2(x_2) dx_2\right] > 0$$

The proposition follows since $E[\Pi_S(Q_S|Q_1)]$ is concave. □

Proposition 5.4 is intuitive for the following reason: in the original early commitment scheme, there is no leftover reallocation and the supplier produces $F_2((w-c)/w)$ units in addition to the order of the primary retailer, Q_1. Since the expected stockout for R_1 is greater than zero regardless of Q_1, the supplier produces more for the selling season in the presence of reallocation to account for this extra demand. Proposition 5.5 further characterizes the optimal response of the supplier, Q_S^*, to the early order of the primary retailer, Q_1.

Proposition 5.5. *Q_S^* is decreasing in Q_1. When $Q_1 = 0$, $Q_S^* = H^{-1}((w-c)/w)$, i.e., the production quantity under a delayed commitment scheme. When $Q_1 \to \infty$, $Q_S^* = F_2^{-1}((w-c)/w)$, i.e., the production quantity for R_2 under the original early commitment scheme.*

Proof. For a given Q_1, Q_S^* is characterized by setting Equation (5–4) to zero. That is, we have

$$\left.\frac{\partial E[\Pi_S(Q_S, Q_1)]}{\partial Q_S}\right|_{Q_S = Q_S^*} = 0. \tag{5–5}$$

Taking the implicit derivative of Equation (5–5) with respect to Q_1, we get

$$\frac{dQ_S^*}{dQ_1} = -\frac{\int\limits_0^{Q_S^*} f_1(Q_1 + Q_S^* - x_2) f_2(x_2) dx_2}{\int\limits_0^{Q_S^*} f_1(Q_1 + Q_S^* - x_2) f_2(x_2) dx_2 + f_2(Q_S^*) F_1(Q_1)} < 0,$$

which proves that Q_S^* is decreasing in Q_1. When $Q_1 = 0$, Equation (5–5) yields $(\int\limits_0^{Q_S^*}\int\limits_0^{Q_S^*-x_2} f_1(x_1)f_2(x_2)dx_1dx_2 = \frac{w-c}{w})$. Hence, we have $Q_S^* = H^{-1}(\frac{w-c}{w})$. When $Q_1 \to \infty$, Equation (5–5) yields $(F_2(Q_S^*) = \frac{w-c}{w})$. Hence, $Q_S^* = F_2^{-1}(\frac{w-c}{w})$. □

Since the expected stockout for the primary retailer is decreasing in Q_1, the supplier decreases Q_S^* as Q_1 increases. When $Q_1 = 0$, the system is equivalent to a delayed commitment scheme from the supplier's perspective, and he produces $H^{-1}(\frac{w-c}{w})$ units. However, from the primary retailer's perspective, there is a crucial difference from a delayed commitment scheme: the allocation mechanism used is no longer proportional since the supplier satisfies the secondary retailer's demand first.

We next analyze the optimal order quantity for the primary retailer. Given $X_1 = x_1$ and $X_2 = x_2$, R_1's profit is given by

$$\Pi_1(Q_1, Q_S) = \begin{cases} rx_1 - wQ_1 & x_1 \leq Q_1 \\ (r-w)Q_1 & x_1 > Q_1, x_2 > Q_S \\ (r-w)x_1 & x_2 \leq Q_S, Q_1 \leq x_1 \leq Q_1 + Q_S - x_2 \\ (r-w)(Q_1+Q_S-x_2) & x_2 \leq Q_S, Q_1+Q_S-x_2 < x_1 \end{cases} . \quad (5\text{–}6)$$

Taking expectations with respect to X_1 and X_2, we get

$$\begin{aligned} E[\Pi_1(Q_1,Q_S)] &= \int_0^{Q_1}(rx_1 - wQ_1)dF_1(x_1) + (r-w)Q_1[1-F_1(Q_1)][1-F_2(Q_S)] \\ &+ (r-w)\int_0^{Q_S}\left(\int_{Q_1}^{Q_1+Q_S-x_2} x_1 dF_1(x_1)\right. \\ &+ \left.(Q_1+Q_S-x_2)[1-F_1(Q_1+Q_S-x_2)]\right)dF_2(x_2). \end{aligned} \quad (5\text{–}7)$$

When the primary retailer incorporates the optimal response of the supplier (Q_S^* as a function of Q_1, characterized by Equation 5–5), her expected profit becomes a function of only Q_1. Then, the first derivative of the primary retailer's expected profit is given by

$$\begin{aligned}\frac{dE[\Pi_1(Q_1)]}{dQ_1} &= -wF_1(Q_1) + (r-w)[(1-F_1(Q_1))(1-F_2(Q_S^*))] \\ &+ (r-w)\left[\int_0^{Q_S^*}(1+\frac{dQ_S^*}{dQ_1})(1-F_1(Q_1+Q_S^*-x_2))f_2(x_2)dx_2\right]. \qquad (5\text{–}8)\end{aligned}$$

Proposition 5.6. *Under early commitment with recourse, the primary retailer always utilizes the early order opportunity. The optimal order quantity is less than the order quantity under an early commitment scheme without recourse.*

Proof. Let Q_1^* denote the optimal order quantity of R_1, and recall that $Q_1^E = F_1^{-1}(\frac{r-w}{r})$ is the order quantity under an early commitment scheme without recourse.

$$\left.\frac{dE[\Pi_1(Q_1)]}{dQ_1}\right|_{Q_1=0} = (r-w)[1-F_2(Q_S^*) + \int_0^{Q_S^*}(1+\frac{dQ_S^*}{dQ_1})(1-F_1(Q_S^*-x_2))f_2(x_2)dx_2] > 0$$

Hence, $Q_1^* > 0$. Evaluating $\frac{dE[\Pi_1(Q_1)]}{dQ_1}$ for $Q_1 \geq Q_1^E$, we get

$$\begin{aligned}\frac{dE[\Pi_1(Q_1)]}{dQ_1} &< -w\frac{r-w}{r} + (r-w)\left[\frac{w}{r}(1-F_2(Q_S^*)) + \int_0^{Q_S^*}(1+\frac{dQ_S^*}{dQ_1})\frac{w}{r}f_2(x_2)dx_2\right] \qquad (5\text{–}9) \\ &< -w\frac{r-w}{r} + (r-w)\left[\frac{w}{r}(1-F_2(Q_S^*)) + \int_0^{Q_S^*}\frac{w}{r}f_2(x_2)dx_2\right] \qquad (5\text{–}10) \\ &= 0\end{aligned}$$

Inequality (5–9) follows since we have $F_1(Q_1) \geq \frac{r-w}{r}$ for all $Q_1 \geq Q_1^E$. Inequality (5–10) follows since $0 < (1+\frac{dQ_S^*}{dQ_1}) < 1$. □

It is intuitive that the primary retailer orders less than her order quantity under an early commitment scheme without recourse, since there is a possibility that there will be inventory left at the supplier after demand realizations.

We now focus on the outcome of the interaction between the primary retailer and the supplier. In order to investigate whether the primary retailer opts to select early commitment with recourse over delayed commitment, and whether the supplier should offer recourse to the primary retailer, we replicate our computational analysis, and start with the data set presented in Table 5-1. As we previously proved analytically, the order quantity of the primary retailer under early commitment with recourse is less than her order under the original early commitment scheme. As the coefficient of variation of R_2's demand (cv_2) increases, Q_1 decreases since the expected quantity of leftovers increase in cv_2, which enables R_1 to rely more on the leftovers. This also prompts the supplier to produce more in excess of Q_1, i.e., Q_S increases in cv_2. As r increases, the primary retailer increases Q_1 to secure more inventory, which leads the supplier to decrease Q_S. However, recall that we have proved analytically that Q_S under early commitment with recourse is greater than Q_S under early commitment without recourse.

In our computational tests, early commitment with recourse increases the primary retailer's profits by 1% on average when compared to the case without recourse. The increase gets more significant as r decreases. However, recall that the benefits of delayed commitment over early commitment also increase as r decreases. As a result, the supplier is not able to get the primary retailer to select early commitment with recourse when $r = 40$. Recall that the primary retailer selects delayed commitment in 102 of the 243 cases. These include all cases with $r = 40$; the remaining are with $r = 70$. When $r = 70$, the supplier can successfully employ the leftover-reallocation scheme to induce the primary retailer to order early whenever the primary retailer's original preference is delayed commitment, which corresponds to 21 cases. The average increase in the supplier's profit in these cases is 9.85%, whereas it is 0.5% for R_1. Hence, we can conclude that although the reallocation of leftovers is not powerful enough to induce the early commitment of the primary retailer in all cases, it provides a significant increase in the supplier's profit when it is possible to implement it. For this data set, when the original outcome is early commitment, the supplier is never better off with early commitment with recourse. It is also worthwhile to note that the secondary retailer will always benefit from the reallocation of the leftovers since it increases the production quantity available for her.

We also perform a similar analysis for 1000 randomly generated test cases presented in Section 5.1.1. Most of our findings above remain valid. In 7.7% of these test cases, the supplier offers early commitment with recourse and the primary retailer accepts it. In 25 of these cases, the original outcome was early commitment, which we did not observe in the previous data set. In these instances, the profit margin of the supplier is so low that he is better off offering leftovers to the primary retailer in order to reduce the cost of uncertainty due to the secondary retailer's demand. Since the primary retailer does not decrease her initial order significantly, the supplier is able to increase his profits. In the remaining 52 instances, the leftover reallocation introduced by the supplier successfully changes the order timing of the primary retailer. Hence, our initial observation that reallocation of the leftovers is not powerful enough to induce an early commitment outcome under the primary retailer's lead is still valid. However, again in alignment with our initial findings, it can increase the supplier's profit as much as 30% when it is successfully implemented.

5.2.2 Supplier's Lead with Credible Information Sharing

In this section, we consider early commitment with recourse where the supplier acts first by announcing the production quantity for the selling season truthfully, and the primary retailer responds by specifying an order quantity before the selling season. We utilize the backward induction method to characterize the optimal outcome of this arrangement. We then compare the expected profits of the primary retailer and the supplier in this setting to the original setting (see Section 5.1.1), and evaluate the effectiveness of the early commitment scheme with recourse under the supplier's lead.

We first solve the primary retailer's problem, given the production quantity of the supplier for the selling season, Q_S. The expected profit of the primary retailer is given by Equation (5–7). Given Q_S, the first and second derivatives of the primary retailer's expected profit function are given by Equations (5–11) and (5–12), respectively.

$$\frac{\partial E[\Pi_1(Q_1,Q_S)]}{\partial Q_1} = -wF_1(Q_1) + (r-w)\Bigg[(1-F_1(Q_1))(1-F_2(Q_S)) + \int_0^{Q_S} (1-F_1(Q_1+Q_S-x_2))f_2(x_2)dx_2\Bigg] \quad (5\text{–}11)$$

$$\frac{\partial^2 E[\Pi_1(Q_1,Q_S)]}{\partial Q_1^2} = -wf_1(Q_1) - (r-w)\Bigg[f_1(Q_1)(1-F_2(Q_S)) + \int_0^{Q_S} f_1(Q_1+Q_S-x_2)f_2(x_2)dx_2\Bigg] \quad (5\text{–}12)$$

Since $E[\Pi_1(Q_1,Q_S)]$ is concave ($\frac{\partial^2 E[\Pi_1(Q_1,Q_S)]}{\partial Q_1^2} < 0$), first order condition ($\frac{\partial E[\Pi_1(Q_1,Q_S)]}{\partial Q_1} = 0$) characterizes the optimal response of the primary retailer for any given Q_S.

Proposition 5.7. *For any finite $Q_S > 0$, the optimal order quantity of the primary retailer is less than her order under an early commitment scheme without recourse. That is, $Q_1^*|Q_S < F_1^{-1}((r-w)/r)$.*

Proof.

$$\left.\frac{\partial E[\Pi_1(Q_1,Q_S)]}{\partial Q_1}\right|_{Q_1=F_1^{-1}(\frac{r-w}{r})} < -w\frac{r-w}{r} + (r-w)\left[\frac{w}{r}(1-F_2(Q_S)) + \frac{w}{r}F_2(Q_S)\right] = 0 \quad (5\text{–}13)$$

The inequality in (5–13) holds since $F_1(Q_1^E) = (r-w)/r$, and $F_1(Q_1^E+Q_S-x_2) > (r-w)/r$ for all $x_2 \in (0, Q_S)$. The proposition follows since $E[\Pi_1(Q_1,Q_S)]$ is concave in Q_1. □

Considering Propositions 5.6 and 5.7 together, we can deduce that the primary retailer's order quantity is less than or equal to her order under early commitment without recourse regardless of the inventory available after demand realizations and the leadership of the supply chain. Proposition 5.8 further analyzes the optimal response of the primary retailer (Q_1^*) to the supplier's action, Q_S.

Proposition 5.8. *Q_1^* is decreasing in Q_S. When $Q_S = 0$, $Q_1^* = F_1^{-1}((r-w)/r)$, i.e., the order quantity under an early commitment scheme without recourse. When $Q_S \to \infty$, $Q_1^* = 0$, i.e., the primary retailer does not place an order.*

Proof. Q_1^* is characterized by setting Equation (5–11) to zero. That is, we have

$$\left.\frac{\partial E[\Pi_1(Q_1, Q_S)]}{\partial Q_1}\right|_{Q_1=Q_1^*} = 0. \tag{5–14}$$

Taking the implicit derivative of Equation (5–14) with respect to Q_S, we get

$$\frac{dQ_1^*}{dQ_S} = -\frac{\int_0^{Q_S} f_1(Q_1^* + Q_S - x_2) f_2(x_2) dx_2}{\int_0^{Q_S} f_1(Q_1^* + Q_S - x_2) f_2(x_2) dx_2 + w f_1(Q_1^*) + (r-w) f_1(Q_1^*)[1 - F_2(Q_S)]} < 0.$$

When $Q_S = 0$, Equation (5–14) yields $F_1(Q_1^*) = (r-w)/r$, i.e., $Q_1^* = F_1^{-1}((r-w)/r)$. When $Q_S \to \infty$, we have $-wF_1(Q_1^*) = 0$, i.e., $Q_1^* = 0$. □

When Q_S increases, the expected quantity of the leftovers that will be available to the primary retailer also increases. Because of the risk of uncertainty associated with the early order, the primary retailer decreases her order quantity, and relies more on the leftovers as Q_S increases. From Proposition 5.8, we can deduce that the supplier is able to induce the primary retailer to order any quantity in $(0, Q_1^E)$ by manipulating Q_S. In other words, the supplier can increase Q_1^* by decreasing Q_S, which costs the supplier the expected revenue that will be generated after the demand realizations. That is, the supplier faces the tradeoff between the revenue from R_1's order and the revenues from R_2's demand and possible reallocation to R_1. This tradeoff is captured by incorporating the optimal response of the primary retailer (Q_1^* as a function of Q_S) into the supplier's expected profit (Equation 5–3).

We now present our findings via computational analysis regarding the early commitment with recourse under supplier's lead, and investigate whether the supplier can use this alternative to induce the early commitment of the primary retailer. We start with the analysis of the numerical results corresponding to the data set presented in Table 5-1.

As in the setting with the primary retailer's lead, Q_1 decreases in cv_2. However, contrary to that setting, Q_S decreases in cv_2 under the supplier's lead since the supplier tries to prevent a further decrease in Q_1 by limiting the expected quantity of leftovers. The supplier's (primary retailer's) expected profit is greater under his (her) lead. Furthermore, Q_1 (Q_S) is greater under the supplier's (primary retailer's) lead.

Under the supplier's lead, the supplier is still unable to induce early commitment when $r = 40$, whereas he manages to do so when $r = 70$. Hence, one might conclude that early commitment with recourse does not provide enough incentive for the primary retailer to commit early, even under the supplier's lead. However, note that our analysis of this scheme does not take the primary retailer's choice of commitment into account. In other words, the supplier optimizes his expected profit under early commitment with recourse and the resulting outcome may not provide sufficient incentive for the primary retailer to switch from an existing delayed commitment scheme. In such cases, the supplier may induce early commitment by increasing the production quantity for the selling season (and hence the expected quantity of leftovers), which increases primary retailer's expected profits as well. If the supplier can increase Q_S to a certain level where the primary retailer is indifferent between the delayed commitment and early commitment with recourse, and if he can maintain a greater profit level than the delayed commitment case at the same time, then he can successfully induce the primary retailer to commit early by using the reallocation of leftovers strategically. For example, consider the following instance: $(c, w, r) = (5, 10, 40)$ and $(\mu_1, \sigma_1) = (\mu_2, \sigma_2) = (80, 80/3)$. In the original setting (no recourse, primary retailer is the leader), the primary retailer prefers delayed commitment, and the corresponding expected profits of the primary retailer and the supplier are 2,173.9 and 649.5, respectively. If the supplier offers early commitment with recourse by optimizing his own expected profits, the resulting order quantities are $Q_1 = 93.2$ and

Table 5-9: Strategic use of leftovers over k_1 and k_2.

	$k_m \leq -1$	$-1 < k_m \leq 0$	$0 < k_m \leq 1$	$k_m > 1$
$k_1 \leq -1$	0 – (1)	0 – (3)	0 – (5)	0 – (7)
$-1 < k_1 \leq 0$	1 – (2)	2 – (6)	14 – (42)	6 – (29)
$0 < k_1 \leq 1$	10 – (10)	34 – (41)	129 – (228)	118 – (197)
$k_1 > 1$	10 – (10)	0 – (0)	145 – (183)	111 – (164)

$Q_S = 78.0$. The corresponding expected profit figures are 2,107.7 and 776.3. Hence, the primary retailer rejects the early commitment with recourse and sticks with delayed commitment. However, if the supplier takes this into account and acts strategically, he sets $Q_S = 103.1$, which results in $Q_1 = 85.1$. The primary retailer's profit is the same as her profit under delayed commitment, and the supplier's profit is 733.4. Hence, the supplier increases his profits from 649.5, whereas the primary retailer is indifferent. With this strategic behavior, the supplier manages to get the primary retailer to select early commitment in 99 of the 102 instances where the original outcome is delayed commitment. When the early commitment scheme with recourse is implemented successfully, the average gain of the supplier is 9.1% with a maximum of 26.5%.

When we consider the 1000 random instances, we observe that the supplier has a chance to improve his profits in 928 cases by early commitment with recourse. In only 42 cases, the early commitment with recourse will replace the original outcome if the supplier insists on the optimal Q_S rather than implementing the one that provides the primary retailer with a larger expected profit. Of the remaining 886 cases, the supplier can manage to increase his profits in 539 instances (60.83%) via early commitment with recourse if he uses Q_S as a strategic tool. Next, we analyze how these instances are distributed over demand and revenue parameters in term of effectiveness, which we measure as the percentage of instances that early commitment with recourse can be utilized successfully to induce early commitment.

Table 5-9 depicts the number of instances that leftovers can be successfully used as a strategic tool for different ranges of service levels for the primary retailer (k_1) and the supplier (k_m). As for the strategic use of capacity, the strategic use of leftovers is more

Table 5-10: Strategic use of leftovers over cv_1 and cv_2.

	$0.1 \leq cv_2 \leq 0.178$	$0.178 < cv_2 \leq 0.256$	$0.256 < cv_2 \leq 0.333$
$0.1 \leq cv_1 \leq 0.178$	$76 - (112)$	$58 - (115)$	$40 - (118)$
$0.178 < cv_1 \leq 0.256$	$73 - (94)$	$60 - (99)$	$40 - (90)$
$0.256 < cv_1 \leq 0.333$	$88 - (101)$	$81 - (103)$	$65 - (96)$

(less) successful when k_1 (k_m) is larger for similar reasons. An important distinction from the strategic use of capacity is the success of the strategic use of leftovers when $k_m \leq -1$ and $k_1 \geq 0$. In these cases, the original outcome is early commitment. Recall that there is no case with $k_m \leq -1$ where capacity can be used as a strategic tool since the primary retailer already prefers early commitment. However, the supplier can still improve his profits with early commitment with recourse.

Table 5-10 depicts the number of instances that leftovers can be successfully used as a strategic tool for different ranges of the coefficients of variation, cv_1 and cv_2. The effectiveness of early commitment with recourse increases (decreases) in cv_1 (cv_2), which is not the case when capacity is used as a strategic tool. The reallocation of the leftovers under an early commitment scheme with recourse decreases the risk of uncertainty for the primary retailer. Hence, the supplier's success rate increases as uncertainty in the primary retailer's demand gets larger. On the other hand, an increase in the uncertainty of the secondary retailer's demand decreases the primary retailer's willingness to commit early, causing a lower success rate for the supplier.

We now investigate how mean demand levels affect the effectiveness of the use of leftovers (see Table 5-11). The success rate of the strategic use of leftovers increases (decreases) in μ_1 (μ_2), which is reasonable for the following reason: as μ_1 increases, the primary retailer's need to secure inventory increases which makes it easier for the supplier to induce early commitment. On the other hand, a larger μ_2 increases the supplier's production quantity under a delayed commitment scheme, which benefits the primary retailer. Hence, the primary retailer will be more reluctant to commit early as μ_2 increases.

Table 5-11: Strategic use of leftovers over μ_1 and μ_2.

	$0 < \mu_2 \leq 50$	$50 < \mu_2 \leq 100$	$100 < \mu_2 \leq 150$
$0 < \mu_1 \leq 50$	$59 - (83)$	$19 - (98)$	$14 - (99)$
$50 < \mu_1 \leq 100$	$94 - (105)$	$75 - (105)$	$44 - (103)$
$100 < \mu_1 \leq 150$	$109 - (112)$	$98 - (117)$	$69 - (106)$

Note that in 828 of the 928 instances where he may increase his profits, the supplier's original choice is not in alignment with the primary retailer (see Section 5.1.1). Hence, in 100 instances, the supplier has the opportunity to improve his profits, even if the original outcome was in his favor, which is an advantage of the early commitment with recourse over the strategic use of capacity.

Although early commitment with recourse may perform better than the strategic use of capacity in certain cases, one may argue that the credibility of the supplier's commitment to a production quantity is lower than the credibility of committing to a capacity level, as capacity may be observed by the primary retailer, whereas the supplier may change the production quantity under early commitment with recourse once the primary retailer places her order. Even though this argument may be correct in a single period context, the primary retailer can predict the supplier's action in the long term since the information about the quantity of leftovers and secondary retailer's demand is available to the primary retailer as well. Hence, the supplier is not going to manipulate the production quantity in a long-term relationship. We investigate the setting where the primary retailer does not rely on the supplier's commitment to a production quantity in the next section.

5.2.3 Suppliers Lead: Unreliable Supplier Case

When the primary retailer does not rely on the supplier's commitment to a production quantity for the selling season, the resulting structure of the strategic interaction between the parties is quite similar to the one analyzed in Section 5.1.3. That is, the sequential decision making under the supplier's lead with an unreliable quote from the supplier about the production quantity is equivalent to simultaneous decision making where both parties make their ordering decisions at the same time. The expected profits of the supplier and the primary retailer are given by Equations (5–3) and (5–7), respectively.

Proposition 5.9. *Early commitment with recourse under the supplier's lead and unreliable production quantity information admits a pure-strategy Nash equilibrium.*

Proof. Sufficient conditions under which *N-person, nonzero-sum* games admit a pure-strategy Nash equilibrium are provided in Basar and Oldser (1995). These conditions translate to our setting as follows:

- $E[\Pi_S(Q_1, Q_S)]$ and $E[\Pi_1(Q_1, Q_S)]$ are jointly continuous in (Q_1, Q_S), and strictly concave in Q_S and Q_1, respectively.
- $E[\Pi_S(Q_1, Q_S)] \to -\infty$ as $Q_S \to \infty$, and $E[\Pi_1(Q_1, Q_S)] \to -\infty$ as $Q_1 \to \infty$.

The proposition follows since these conditions hold. $\square$

When the number of strategies available to each party (in our model, Q_1 and Q_S for the primary retailer and the supplier, respectively) is a continuum, and the payoffs (in our model, expected profits) are continuous, a pure-strategy Nash equilibrium solution can be obtained as the common intersection points of the reaction curves of the parties. The reaction curve of a party is constructed by its optimal response set for every possible strategy set of the remaining parties, provided that the optimal response set is a singleton for a given strategy set of the remaining parties (see Basar and Oldser (1995) for a formal definition of the reaction curve). The reaction curves of the supplier and the primary retailer are characterized by Equations (5–15) and (5–16), respectively, since $E[\Pi_S(Q_1, Q_S)]$ and $E[\Pi_1(Q_1, Q_S)]$ are strictly concave in Q_S and Q_1, respectively.

$$RC_S(Q_1) = \left\{ Q_S : w\left[\int_0^{Q_S} [1 - F_1(Q_1 + Q_S - x_2)] f_2(x_2) dx_2 + 1 - F_2(Q_S) \right] - c = 0 \right\} \quad (5\text{–}15)$$

$$RC_1(Q_S) = \Bigg\{ Q_1 : -wF_1(Q_1) + (r-w)\Bigg[(1-F_1(Q_1))(1-F_2(Q_S)) \\ + \int_0^{Q_S}(1-F_1(Q_1+Q_S-x_2))f_2(x_2)dx_2\Bigg]\Bigg\} \tag{5–16}$$

We now analyze the reaction curves of the supplier ($RC_S(Q_1)$) and the primary retailer ($RC_1(Q_S)$) to further characterize the outcome of their interaction.

Proposition 5.10. *Early commitment with recourse under the supplier's lead and unreliable production quantity information admits a unique pure-strategy Nash equilibrium.*

Proof. Note that the reaction curve of the supplier can also be expressed as a function of Q_S (this is also true for the primary retailer). The derivatives of $RC_1(Q_S)$ and $RC_S(Q_S)$ with respect to Q_S are given by

$$\frac{dRC_1(Q_S)}{dQ_S} = -\frac{\int_0^{Q_S} f_1(Q_1+Q_S-x_2)f_2(x_2)dx_2}{\int_0^{Q_S} f_1(Q_1+Q_S-x_2)f_2(x_2)dx_2 + wf_1(Q_1) + (r-w)f_1(Q_1)[1-F_2(Q_S)]}$$

$$\frac{dRC_S(Q_S)}{dQ_S} = -\frac{\int_0^{Q_S} f_1(Q_1+Q_S-x_2)f_2(x_2)dx_2 + f_2(Q_S)F_1(Q_1)}{\int_0^{Q_S} f_1(Q_1+Q_S-x_2)f_2(x_2)dx_2}$$

The reaction curves cannot intersect more than once since $-1 < \frac{dRC_1(Q_S)}{dQ_S} < 0$, and $\frac{dRC_S(Q_S)}{dQ_S} < -1$. By Proposition 5.9, they intersect at least once. Hence, the equilibrium is unique. Figure 5-7 illustrates the reaction functions and the unique equilibrium. □

The preceding analysis characterizes the outcome of the interaction between the supplier and the primary retailer assuming that both parties agree to operate under an early commitment scheme with recourse. That is, we have not yet addressed whether the primary retailer is willing to switch to an early commitment scheme with recourse from the original setting or not. If the original outcome is early commitment, and the supplier

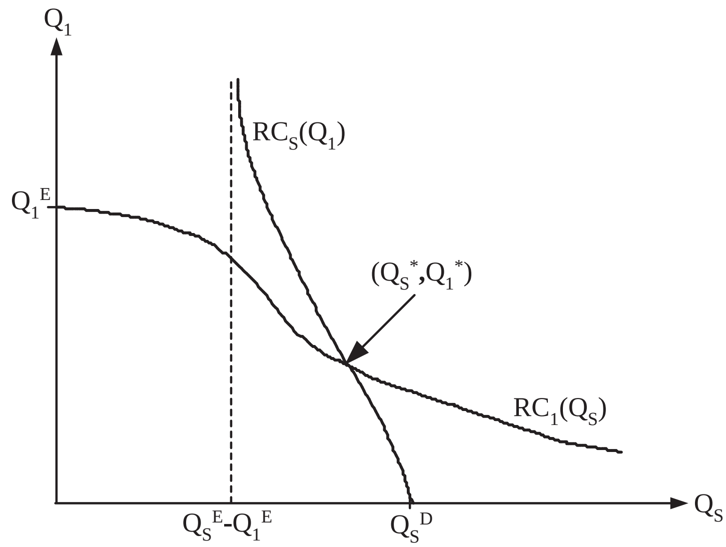

Figure 5-7 Response functions for the strategic game

offers the leftovers, the primary retailer will accept. However, if the primary retailer's original choice is delayed commitment, and the Nash equilibrium provides lower profits, the primary retailer will stick to delayed commitment. In such a case, one may argue that the supplier, being aware of the primary retailer's profit under a delayed commitment scheme, may try to use Q_S strategically to get to early commitment. That is, the supplier may increase Q_S up to a certain level such that the primary retailer is indifferent between early commitment with recourse and delayed commitment. We now elaborate on these issues, and argue that the strategic use of Q_S is not possible without credible information sharing.

Let's assume for now that there is no strategic use of Q_S, and let (Q_1^*, Q_S^*) be the unique equilibrium to the early commitment with recourse. If the primary retailer's expected profit under delayed commitment is larger compared to that of early commitment with recourse, the outcome of the interaction between the supplier and the primary retailer will be delayed commitment, where the supplier produces Q_S^D units and allocates to the retailers since neither of the parties has an incentive to deviate from this solution. Otherwise, the outcome is early commitment with recourse where the primary retailer orders Q_1^* units and the supplier produces Q_S^* for the selling season. Now, let's relax the assumption that there is no strategic use of Q_S to induce early commitment. Strategic use of Q_S is only meaningful when the primary retailer generates more profits in a delayed commitment scheme, whereas the supplier prefers early commitment with recourse. Assume that there exists an equilibrium solution (Q_1', Q_S') such that $Q_S' > Q_S^*$, and the primary retailer selects early commitment with recourse, that is, the supplier can use Q_S strategically. However, (Q_1', Q_S') cannot be on the reaction functions of both parties since there is a unique equilibrium to early commitment with recourse. That is, at least one of the parties has an incentive to deviate from (Q_1', Q_S'). Hence, we can conclude that when the supplier is not credible, the outcome is either delayed commitment or the unique Nash equilibrium of early commitment with recourse.

We now present our computational findings starting with the data set presented in Table 5-1. With respect to expected profits and order quantities, the early commitment model with recourse under the lead of an unreliable supplier is positioned between the model with the retailer's lead, and the one with a reliable supplier's lead, which is reasonable since this setting reduces to a strategic game where neither of the parties has the benefit of the leadership. In 21 of the 102 cases where delayed commitment is the original outcome, the supplier manages to induce early commitment with recourse, as under the retailer's lead. His expected profits increase by an average of 10.1%, which is slightly larger than the increase under the retailer's lead. Considering the random data set of 1000 instances, early commitment with recourse is implemented in only 45 instances. Early commitment without recourse was the original outcome in 30 of these instances. Hence, we can deduce that the supplier needs to concentrate on credible information sharing in order to use early commitment with recourse effectively.

5.3 Wholesale Pricing

Up to this point, we have assumed that the transfer price between the supplier and the retailers is exogenous and fixed. In this section, we examine the effectiveness of a discounted wholesale price from the supplier's perspective. That is, the supplier is allowed to offer a discounted wholesale price, provided that the primary retailer commits early. We consider two different settings depending on the flexibility of the pricing scheme. In both cases, the discounted price is valid only if the primary retailer accepts early commitment. Note that the supplier would not offer a discount if the primary retailer insisted on delayed commitment, since the supplier's profit under delayed commitment increases in the wholesale price. In the first setting, the secondary retailer benefits from the discounted wholesale price as well, whereas the discounted price is only offered to the primary retailer in the second case. It is intuitive that the supplier can use pricing more effectively in the second setting since the discounted wholesale price will only affect the profits due to the primary retailer. Next, we analyze both settings and evaluate their effectiveness in improving the supplier's profit.

5.3.1 Homogenous Wholesale Price

In this case, the supplier will offer a wholesale price discount to both retailers under an early commitment scheme provided that he can increase his expected profit level compared to the outcome of the original setting, which was characterized in Section 5.1.1. The supplier does not have any incentive to offer a discount under a delayed commitment scheme since his expected profit increases in the wholesale price in such a case. Therefore, we analyze the expected profit of the supplier under an early commitment scheme with a discounted wholesale price, and examine whether he can use wholesale price to improve his expected profit. Note that the primary retailer will not agree to commit early with the discounted price unless her expected profit is at least as large as her profit under the original setting. Note also that the primary retailer will certainly be better off with the discounted wholesale price if she already prefers early commitment under the original setting.

Let w_d denote the discounted wholesale price under an early commitment scheme. Then the supplier's expected profit function is given by

$$E[\Pi_S^E(w_d, Q_S, Q_1)] = (w_d - c)(Q_1 + \mu_2) - (w_d - c)\int_{Q_S}^{\infty}(x - Q_S)dF_2(x) - c\int_{0}^{Q_S}(Q_S - x)dF_2(x), \tag{5–17}$$

where Q_1 is the order quantity of the primary retailer, and Q_S is the production quantity of the supplier for the secondary retailer. Given w_d, we have

$$F_1(Q_1) = \frac{r - w_d}{r} \tag{5–18}$$

and

$$F_2(Q_S) = \frac{w_d - c}{w_d}. \tag{5–19}$$

From Equation (5–19), we get

$$w_d = \frac{c}{1 - F_2(Q_S)} \tag{5–20}$$

By substituting w_d given by Equation (5–20) into Equation (5–18), we get

$$[1 - F_1(Q_1)][1 - F_2(Q_S)] = \frac{c}{r}. \tag{5–21}$$

That is, Q_1 can also be expressed as a function of Q_S, which is characterized by Equation (5–21). Hence, the expected profit of the supplier under an early commitment scheme with a discounted wholesale price can be expressed as

$$E[\Pi_S^E(Q_S)] = c\left[\frac{F_2(Q_S)}{1 - F_2(Q_S)}\left(Q_1 + \mu_2 - \int_{Q_S}^{\infty}(x - Q_S)dF_2(x)\right) - \int_{0}^{Q_S}(Q_S - x)dF_2(x)\right] \tag{5–22}$$

where Q_1 is a function of Q_S, and $0 \leq Q_S \leq F_2^{-1}(\frac{w-c}{w})$.

If the original outcome of the interaction between the primary retailer and the supplier is early commitment, then maximizing $E[\Pi_S^E(Q_S)]$ is sufficient to solve the supplier's problem. Otherwise, the optimal expected profit level under an early commitment scheme may not be realizable since the primary retailer may refuse to switch to an early commitment scheme. In such cases, the supplier should also consider the possibility of offering a deeper discount since he may still improve his profits compared to the original agreement. Unfortunately, the analysis of Equation (5–22) does not provide useful insights about the preceding discussions. Hence, we next present our findings through computational analysis about the effectiveness of a homogenous wholesale price discount in increasing supplier's profits.

We first evaluate the data set presented in Table 5-1. Recall that there are 102 instances (out of 243) where the original outcome is delayed commitment, and the supplier always prefers early commitment. In 40 instances of these, the supplier is better off by offering a discounted wholesale price and inducing the primary retailer commit early. These include the 21 cases where the unit revenue for the retailers is 70. Recall that in the remaining instances where $r = 70$, the original outcome is already early commitment. When $r = 40$, the discounted wholesale price scheme works in just 19 of the 81 instances, which is reasonable since it would take a deeper discount to have the primary retailer commit early when her unit revenue is lower, which the supplier is reluctant to offer. Hence, offering a homogenous discount works better when r is larger. The average increase in the supplier's expected profit due to a discount in the wholesale price is 6.87%. It is 4.71% when $r = 40$ and 8.83% when $r = 70$. The average wholesale price is 9.28 when $r = 40$, and 9.87 when $r = 70$, which is in alignment with the above discussion (the original wholesale price is 10). The increase in the supplier's expected profit increases as μ_1 and/or cv_1 increase, which is reasonable since the early order quantity of the primary retailer increases in μ_1 and cv_1. The demand parameters of the secondary retailer affect the efficiency of the discounting scheme as well. The increase in the supplier's profit increases as μ_2 and/or cv_2 decrease. This is also intuitively reasonable for the following reason: higher mean demand and coefficient of variation for of the secondary retailer increase the availability of buffer stock for the primary retailer under a delayed commitment scheme. Hence, the primary retailer requires deeper discounts to commit early, which hurts the increase in the supplier's expected profits.

Out of 1000 random instances generated in Section 5.1.1, the supplier manages to induce early commitment of the primary retailer by offering a discounted wholesale price in 84 instances. The average increase in his expected profit in these instances is 8.57%. The maximum increase is over 28%. Hence, we can conclude that offering a homogenous discounted wholesale price can provide a significant increase in the supplier's profit although it cannot be utilized as frequently as the previous methods that we have discussed in Sections 5.1 and 5.2.

5.3.2 Different Wholesale Prices

In this section, we consider the setting where the supplier can charge the primary retailer a lower price than the secondary retailer if the primary retailer decides to commit early. In other words, the supplier will continue the original wholesale price for the delayed commitment, but he can decrease the wholesale price for the early commitment if it is going to increase his expected profit. Note that the secondary retailer does not have a direct effect on the pricing decision of the supplier since the discount will only be offered to the primary retailer. However, as in Section 5.3.1, the optimal wholesale price that maximizes the supplier's profit may be too high for the primary retailer to select early commitment. In such cases, the supplier may have to decrease the wholesale price for the early commitment to a critical level such that the primary retailer is indifferent between early and delayed commitment, and this critical wholesale price level is also affected by the demand parameters of the secondary retailer.

We first characterize the wholesale price that maximizes the supplier's expected profit given that the primary retailer commits early. Then, through a computational analysis, we explore how effective the discounting scheme is in terms of getting the primary retailer to commit early, and compare it with the homogenous wholesale price case that was discussed in Section 5.3.1.

Given the discounted wholesale price (w_d) by the supplier, the order quantity of the primary retailer under an early commitment scheme is given by

$$F_1(Q_1) = \frac{r - w_d}{r}. \tag{5–23}$$

Then, the supplier's expected profit is

$$E[\Pi_S^E(w_d)] = (w_d - c)Q_1, \tag{5–24}$$

where Q_1 is a function of w_d. Note that we exclude the supplier's profit due to the secondary retailer since it is not affected by the wholesale price offered to the primary retailer. Rewriting Equation (5–23) as $w_d = r[1 - F_1(Q_1)]$, and substituting into Equation (5–24), we get

$$E[\Pi_S^E(Q_1)] = (r[1 - F_1(Q_1)] - c)Q_1 \tag{5–25}$$

which is unimodal if the demand distribution of the primary retailer has the increasing generalized failure rate property (see Lariviere and Porteus (2001) for details).

We now present our findings on the effectiveness of the discounting scheme provided above and compare it with the homogenous wholesale price case, starting with the data set provided in Table 5-1. There are 82 out of 102 instances where the supplier manages to get the primary retailer to commit early by offering a discounted wholesale price. Recall that there were only 40 such cases when secondary retailer also benefits from the discount. The average increase in the supplier's profit is 6.95%, which at first seems counterintuitive since it is less than the average increase with the homogenous wholesale price case. However, if we only consider the 40 instances where the homogenous wholesale price was effectively used, the average increase in the supplier's profit is 11% when the discount is only offered to the primary retailer. Hence, it is superior to the homogenous wholesale price case because it not only works more frequently, but provides higher returns as well. For instance, when $r = 70$, both schemes manage to get the primary retailer commit early. However, the increase in the supplier's expected profit is 10.16% with different wholesale

Table 5-12: Strategic use of wholesale price over k_1 and k_2.

	$k_m \leq -1$	$-1 < k_m \leq 0$	$0 < k_m \leq 1$	$k_m > 1$
$k_1 \leq -1$	$0 - (0)$	$0 - (0)$	$0 - (0)$	$0 - (0)$
$-1 < k_1 \leq 0$	$0 - (0)$	$0 - (3)$	$0 - (20)$	$0 - (4)$
$0 < k_1 \leq 1$	$0 - (10)$	$19 - (29)$	$41 - (228)$	$2 - (197)$
$k_1 > 1$	$0 - (10)$	$0 - (0)$	$123 - (183)$	$47 - (164)$

Table 5-13: Strategic use of wholesale price over cv_1 and cv_2.

	$0.1 \leq cv_2 \leq 0.178$	$0.178 < cv_2 \leq 0.256$	$0.256 < cv_2 \leq 0.333$
$0.1 \leq cv_1 \leq 0.178$	$22 - (97)$	$21 - (107)$	$13 - (107)$
$0.178 < cv_1 \leq 0.256$	$31 - (86)$	$25 - (82)$	$12 - (81)$
$0.256 < cv_1 \leq 0.333$	$31 - (91)$	$42 - (93)$	$30 - (83)$

prices, while it was 8.83% with homogenous prices. The reaction of the increase in the supplier's expected profit to revenue and cost parameters is the same as the case with homogenous wholesale price setting. We next consider the random instances generated before to further investigate the use of different wholesale prices. Out of the 828 instances where the supplier is willing to induce the early commitment, offering a discount to the primary retailer works in 232 instances. The average increase in the supplier's profit is 5.56%, the maximum being almost 30%.

Table 5-12 depicts the number of instances that the wholesale price can be successfully used as a strategic tool for different ranges of service levels for the primary retailer (k_1) and the supplier (k_m). As for the strategic use of capacity and leftovers, the strategic use of the wholesale price is more (less) successful when k_1 (k_m) is larger for similar reasons.

Table 5-13 depicts the number of instances that the wholesale price can be successfully used as a strategic tool for different ranges of the coefficients of variation, cv_1 and cv_2. There is not an evident pattern in the number of instances as cv_1 and cv_2 vary because of similar reasons discussed in the use of capacity as a strategic rule.

We now investigate how mean demand levels affect the effectiveness of the use of wholesale price (see Table 5-14). The success rate of the strategic use of wholesale pricing increases (decreases) in μ_1 (μ_2) for similar reasons discussed in the analysis of the use of capacity and leftovers.

Table 5-14: Strategic use of wholesale price over μ_1 and μ_2.

	$0 < \mu_2 \leq 50$	$50 < \mu_2 \leq 100$	$100 < \mu_2 \leq 150$
$0 < \mu_1 \leq 50$	$21 - (74)$	$11 - (87)$	$6 - (82)$
$50 < \mu_1 \leq 100$	$39 - (95)$	$24 - (91)$	$17 - (92)$
$100 < \mu_1 \leq 150$	$47 - (100)$	$44 - (108)$	$23 - (99)$

Note that the wholesale price is not as powerful as the capacity and leftovers in terms of the effectiveness in inducing early commitment. However, this observation alone is not sufficient to conclude that it is inferior. There may be instances where it provides larger increases in the supplier's expected profit.

5.4 Comparative Analysis and Concluding Remarks

In this section, we summarize our findings about the strategic tools that the supplier can utilize to manipulate the primary retailer's order timing by comparing them with respect to both effectiveness and the increase in the supplier's expected profit. The original setting analyzed in Section 5.1.1 serves as a benchmark to assess the success of these tools. We consider the use of (i) capacity under the supplier's lead with credible information, (ii) early commitment with recourse (leftover reallocation) under the supplier's lead with credible information, and (iii) different wholesale prices. The random data set characterized in Section 5.1.1 forms the basis for our observations.

Since the use of capacity is only effective when the preferences of the supplier and the primary retailer are not in alignment, we evaluate the effectiveness of the demand management tools in these 828 instances in order to perform a fair comparison. However, it is worthwhile to note that early commitment with recourse works in 40 instances where the preference of the supplier and the primary retailer is the same in the original setting, which can be considered as an advantage of this mechanism over the others.

Recall that the strategic use of capacity works in 505 of 828 instances where the supplier and the primary retailer have conflicting preferences. Early commitment with recourse and different wholesale prices are successful in 541 and 232 instances, respectively. There are 613 (74%) instances where at least one of the tools is successful. That is, the supplier can improve his expected profits in 74% of the instances by employing one of the techniques we discussed in this chapter.

There is no case where offering different wholesale prices succeeded alone. Furthermore, there are only three cases where a discounted wholesale price performed the best among the alternatives we considered. In other words, for almost all the instances where a discounted wholesale price works, either the use of capacity or the leftover reallocation works better from the supplier's perspective, which may lead to the conclusion that offering a discounted wholesale price is inferior to the other methods. However, one should also note that both of the other methods require credible information sharing between the supplier and the primary retailer in order to be effective, which may not be the case all the time. In such cases, a discounted wholesale price may increase the supplier's profit significantly. Furthermore, it does not decrease the expected profit of the primary retailer whereas imposing a capacity limit might, which is another advantage over the use of capacity as a strategic tool. Keeping these issues in mind, we next compare the use of capacity and leftovers.

There are 433 cases where both methods are implementable, and in 324 of these, the use of capacity as a strategic tool increases the supplier's profit more when compared to early commitment with recourse. In 108 (72) instances, early commitment with recourse (strategic use of capacity) is successful whereas strategic use of capacity (early commitment with recourse) is not. Hence, we may conclude that neither of the methods provides a significant advantage over the other in terms of the supplier's expected profit. While the strategic use of capacity provides more improvement when we consider the cases where both methods work, early commitment with recourse is implementable in more instances. This is mainly due to the flexibility of the early commitment scheme with recourse. Since the primary retailer has more flexibility in this regime, the possibility that she will select it is higher when compared to the strategic use of capacity. On the other hand, this flexibility causes the supplier not to increase his profits as much as the model with the strategic use of capacity.

CHAPTER 6
CONCLUSION

In our study, we considered the integration of demand management tools with classical supply chain management concepts in two distinct modelling environments. In the first model, we introduce a general optimization model that not only applies to a number of well-known inventory planning models but incorporates demand management tools such as pricing and market selection decisions into these models as well. In the second model, we analyze the order timing preferences of different supply chain levels, and propose demand management tools for the upstream level. These tools include operational strategies such as the allocation of inventory or restricting available capacity, and marketing tools such as wholesale pricing.

Although there is a well-established stream of literature that incorporates pricing into inventory models, and an emerging stream that considers flexibility in demand/market selection from an operations management perspective, the former assumes the supply chain must respond to every market's/customer's needs, whereas the latter considers fixed revenues. In the first part of our study, we present a nonlinear, combinatorial optimization model that considers pricing together with market/order/demand selection. This model not only generalizes well-known inventory models under simple assumptions, but overlaps and advances the stream of economics literature that deals with third-degree price discrimination as it allows two different pricing schemes; (i) a single price that applies all selected markets, and (ii) a specific price for each selected market. We provide solutions algorithms and characterize the optimal solutions for general classes of revenue and cost functions. Furthermore, we analyze the effects of certain market and cost characteristics on the pricing and market selection decisions, and the difference between the pricing schemes.

The market/order selection concept is a quite new idea in the operations management literature. Our current study deliberates on geographically dispersed markets; hence the demands in these markets are assumed to be independent. One possible extension is to investigate the correlated demand case in conjunction with pricing decisions. Another possible avenue for further research related with market/order selection is the determination of a portfolio for substitutable products. It is a common practice in many industries to offer practically the same or very similar products at the same time in order to satisfy different customer segments. An extension of the market selection model can be utilized to model this setting to determine the optimal set of products to offer, their prices and corresponding advertising efforts. The studies in the literature assume a centralized structure for the market selection problem. However, this problem can also be employed in decentralized settings where a manufacturer sells its product in different markets through a set of retailers. In that case, the manufacturer should also consider the retailers willingness to enter the business, which depends on the contract parameters between the manufacturer and the retailers. It should also be noted that the demand substitutability in such a setting brings retailer competition into the picture, which is also worth further consideration.

In the second part of our study, we focus on the order timing decisions between different levels of the supply chain. Investigating the tradeoffs between different commitment schemes, we observe that current industry practice, where the downstream members of the supply chain are reluctant to place early orders while the upstream members prefer early orders, is supported by our analytical findings. We then introduce three different mechanisms that the upstream members can utilize to induce their preference: (i) capacity, (ii) allocation of inventory, and (iii) pricing. We observe that although capacity and allocation of inventory are more effective than pricing, they require credible information sharing between the supply chain members. In the absence of information sharing, using wholesale pricing as a strategic tool to affect the order timing decision's of downstream member's may result in significant improvement in the profit levels of the upstream suppliers.

In our study, there is a single order opportunity for both retailers with no flexibility to change. An immediate extension would consider more flexible supply contracts that allow the primary retailer to adjust her order quantity under the early commitment strategy. Tools that enable such flexibility include quantity flexibility, options, second orders and buy-back arrangements. Another avenue for further exploration is competition between retailers. In our models, the only relation between the retailers is the inventory allocation policy under delayed commitment, which they cannot affect. Hence, there is no interaction between the retailers. Future work may extend this setting by introducing demand and/or capacity competition between the retailers. Another possible research direction is to introduce a certain degree of flexibility for the production process of the supplier such as a second, possibly more expensive, production run for the supplier closer to (or during) the selling season.

APPENDIX A
APPENDIX FOR CHAPTER 3

A.1 Equivalence of Additive and Multiplicative Randomness

Let the demand in each market be $D_i = q_i(p)X_i$ where X_i is normally distributed with mean 1 and standard deviation σ_i. Let X_y denote the aggregate demand, i.e. $X_y = \sum_{i=1}^n D_i y_i$. Note that X_y is also normally distributed with mean $\sum_{i=1}^n q_i(p)y_i$ and standard deviation $\sqrt{\sum_{i=1}^n [q_i(p)]^2\sigma_i^2}$. Denoting the pdf of X_y by $f_y(x,p)$, the expected profit of the firm is

$$G(p,Q,y) = (p-c)\sum_{i=1}^n q_i(p)y_i - (c-v)\int_{-\infty}^{Q}(Q-x)f_y(x,p)dx - (e-c)\int_{Q}^{\infty}(x-Q)f_y(x,p)dx.$$

Optimizing over Q given y and p and substituting optimal order quantity in the expected profit, the problem reduces to

$$\begin{array}{ll} \max & (p-c)\displaystyle\sum_{i=1}^n q_i(p)y_i - K\sqrt{\sum_{i=1}^n [q_i(p)]^2\sigma_i^2 y_i} \\ \text{subject to} & y_i \in \{0,1\} \qquad \forall i = 1,\ldots,n. \end{array}$$

The only difference between the multiplicative and additive randomness is the standard deviation of the random variable X_y. If we let the standard deviation of additive randomness in each market, $\sigma_i(p) = q_i(p)\sigma_i$, where σ_i is the standard deviation of X_i in the multiplicative case, then both models have the same structure. Hence, we can conclude that multiplicative randomness is a special case of the additive model and can be handled in exactly the same way. Similar arguments are also valid when the firm is able to offer market-specific prices.

A.2 Proof of Concavity: (MSP-S)

The objective function of each subproblem is given by $G(p) = \sum_{i=1}^{\ell} R_i(p) - S\sqrt{\sum_{i=1}^{\ell}[C_i(p)]^2}$. Since each $R_i(p)$ is assumed to be concave, the first part of $G(p)$ is concave.

Below, we show that $C(p) = \sqrt{\sum_{i=1}^{\ell}[C_i(p)]^2}$ is convex. Hence, we can conclude that $G(p)$ is concave.

The second derivative of $C(p)$ with respect to p is given by

$$\begin{aligned}\frac{d^2C(p)}{dp^2} &= \left[\sum_{i=1}^{\ell}[C_i(p)]^2\right]^{-\frac{3}{2}} \left[-\left(\sum_{i=1}^{\ell} C_i(p)C_i'(p)\right)^2\right. \\ &+ \left.\left(\sum_{i=1}^{\ell}[C_i(p)]^2\right)\left(\sum_{i=1}^{\ell}([C_i'(p)]^2 + C_i(p)C_i''(p))\right)\right]\end{aligned}$$

where $C_i'(p)$ and $C_i''(p)$ denote the first and second derivatives of $C_i(p)$, respectively. Let $T = \left[\sum_{i=1}^{\ell}[C_i(p)]^2\right]^{-\frac{3}{2}}$. Also denote $C_i(p)$, $C_i'(p)$, and $C_i''(p)$ by A_i, B_i and C_i. Then,

$$\begin{aligned}\frac{d^2C(p)}{dp^2} &= T\left[\left(\sum_{i=1}^{\ell} A_i^2\right)\left(\sum_{i=1}^{\ell}\left[B_i^2 + A_iC_i\right]\right) - \left(\sum_{i=1}^{\ell} A_iB_i\right)^2\right] \\ &> T\left[\left(\sum_{i=1}^{\ell} A_i^2\right)\left(\sum_{i=1}^{\ell} B_i^2\right) - \left(\sum_{i=1}^{\ell} A_iB_i\right)^2\right] \\ &= T\left[\sum_{i=1}^{\ell}\sum_{j\neq i} A_i^2B_j^2 - 2\sum_{i=1}^{\ell}\sum_{j>i} A_iB_iA_jB_j\right] \\ &= T\left[\sum_{i=1}^{\ell}\sum_{j>i}(A_iB_j - A_jB_i)^2\right] > 0\end{aligned}$$

A.3 Proof of Concavity: (MSP-MS)

The objective function of the problem is $\sum_{i=1}^{n} R_i(p_i) - S\sqrt{\sum_{i=1}^{n}[C_i(p_i)]^2}$. The first part is concave since each $R_i(p_i)$ is assumed to be concave. We need to show that $C(p) = \sqrt{\sum_{i=1}^{n}[C_i(p_i)]^2}$ is convex in p. Let $\overrightarrow{C}(p) = [C_1(p_1), \cdots, C_n(p_n)]^T$. Then $C(p)$ is the ℓ_2 norm of $\overrightarrow{C}(p)$, i.e., $C(p) = ||\overrightarrow{C}(p)||$. For any $\lambda \in (0,1)$, the following holds for any p_1 and p_2:

$$\begin{aligned}C(\lambda p_1 + (1-\lambda)p_2) &= ||\overrightarrow{C}(\lambda p_1 + (1-\lambda)p_2)|| \leq ||\lambda\overrightarrow{C}(p_1) + (1-\lambda)\overrightarrow{C}(p_2)|| \\ &\leq ||\lambda\overrightarrow{C}(p_1)|| + ||(1-\lambda)\overrightarrow{C}(p_2)|| = \lambda C(p_1) + (1-\lambda)C(p_2)\end{aligned}$$

The first inequality holds since $C_i(p_i)$'s are all convex. The result directly follows.

A.4 Proof of Proposition 3.1

We show that if any market is selected in the optimal solution, then market i should also be selected, and if market j is not selected in the optimal solution, then market i will not be selected either for any i.

Since the expected profit function $G(p)$ is jointly concave in p, we can utilize first-order conditions to characterize the optimal solution. The first derivative with respect to p_i is given by

$$\frac{\partial G(p)}{\partial p_i} = R_i'(p_i) - S\frac{C_i(p_i)C_i'(p_i)}{\sqrt{\sum_{j=1}^{n}[C_i(p_i)]^2}} \tag{A–1}$$

Let p_i^0 denote the minimum price at which $R_i(p_i)$ and hence $C_i(p_i)$ are equal to zero. At p_i^0, $\left(\sum_{j=1}^{n}[C_j(p_j)]^2\right)^{-1/2}$ is well defined since at least one market is selected and the first derivative evaluated at p_i^0 is

$$\left.\frac{\partial G(p)}{\partial p_i}\right|_{p_i=p_i^0} = R_i'(p_i^0) < 0 \tag{A–2}$$

indicating that the optimal p_i should be less than p_i^0, i.e., market i should be selected.

Assume market j is not selected in the optimal solution. We next characterize the decision regarding market i. If market i is to be selected, then due to the first part of the proof, market j should also be selected, which violates the assumption that market j is not selected. Hence, the optimal decision for market i should be 'not selecting'.

A.5 Different Standard Deviation Functions for the Newsvendor Model

Recall that in Section 3.3, for the newsvendor model, we only considered $\sigma_i(p) = cv_i \times q_i(p)$ and varied the value of cv_i to examine the effects of uncertainty. We now extend our computations to include general $\sigma_i(p)$ functions. Figures A-1 and A-2 illustrate the nonlinear functions used in the linear and iso-elastic demand cases, respectively. In both cases, the $\sigma(p)$ functions are generated such that the coefficient of variation is

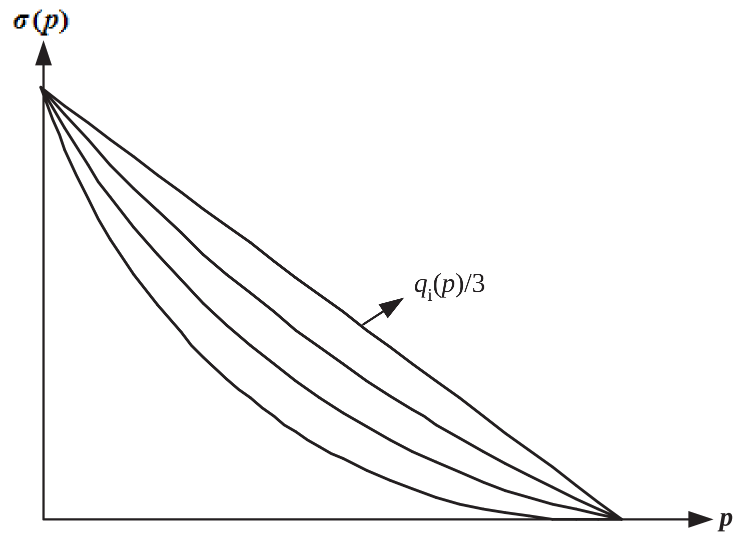

Figure A-1 Standard deviation function for linear demand.

less than 1/3. In the linear case, the standard deviation function is given by $\sigma(p) = s - \sqrt{R^2 - (p-t)^2}$. The parameters s, t and R are generated so that $\sigma(a/b) = 0$, and $q(p)/3 > \sigma(p) > 0$ for all $p < a/b$. They also define the level of uncertainty in the market. In the iso-elastic demand case, the standard deviation function is given by $\sigma(p) = \bar{\alpha}p^{-\bar{\beta}}$. The parameters $\bar{\alpha}$ and $\bar{\beta}$ are computed so that $q(p)/3 > \sigma(p) > 0$. For both cases, we generate $\sigma(p)$ functions for each set of demand parameters considered in Section 3.3. Replicating our computational study with these standard deviation functions, we observed that all of our observations are still valid for both the linear and the iso-elastic demand cases.

For the linear demand we also consider a linear standard deviation function. Note that the constant coefficient case corresponds to the linear standard deviation function as depicted in Figure A-3. We now consider a linear function that approaches to zero before the demand does. In such a case, the demand becomes deterministic when price exceeds a certain threshold. We generate different levels of uncertainty by changing this threshold price level (See Figure A-4).

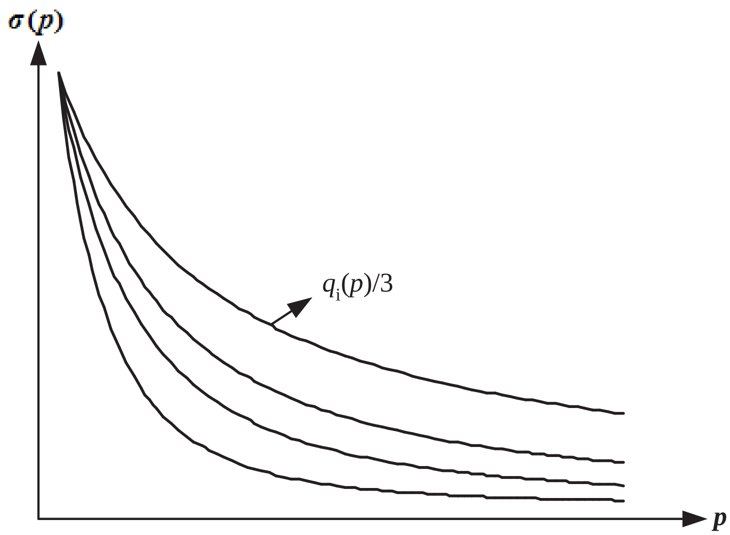

Figure A-2 Standard deviation function for iso-elastic demand.

Replicating our computational analysis with this form of the standard deviation function, we found that our observations regarding the market selection decisions still hold. However, our observations about the effects of uncertainty on the pricing decisions do not generalize to this case. This is because we have a standard deviation function that allows deterministic demand when price exceeds a certain threshold value. In some instances, the supplier sets the price such that the resulting demand in the market(s) is deterministic. When uncertainty increases, that is, when the threshold price level increases, the supplier may also increase the price to remain in the (smaller) region where demand is effectively deterministic. Further increasing uncertainty increases the threshold price at which demand is effectively deterministic, and may force the supplier to decrease the price in order to increase demand at the expense of incurring some degree of uncertainty. Hence, the reaction of optimal prices to changes in uncertainty is not necessarily consistent in this special case.

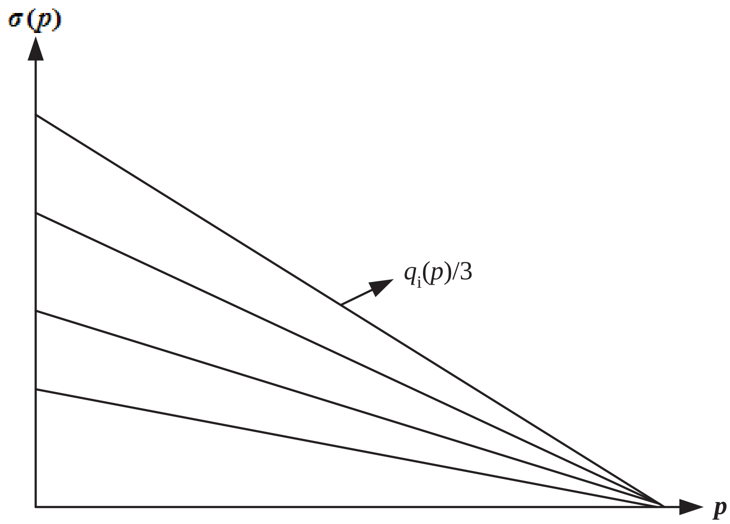

Figure A-3 Linear standard deviation function for linear demand: I.

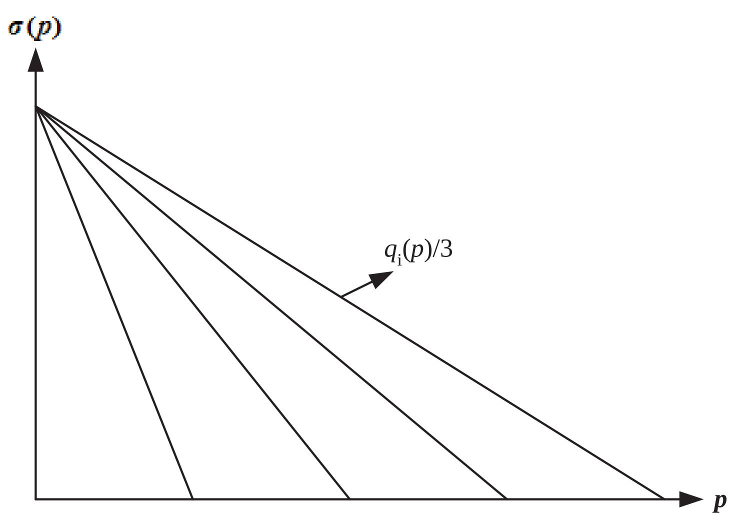

Figure A-4 Linear standard deviation function for linear demand: II

APPENDIX B
APPENDIX FOR CHAPTER 4

B.1 Evaluation of the Supplier's Profit

B.1.1 $[\mathbf{Q_S^D}]^{\mathbf{U}} \le [\mathbf{Q_S^E}]^{\mathbf{U}}$: $\mathbf{k_m}\sigma_{\mathbf{T}} \le \mathbf{k_1}\sigma_{\mathbf{1}} + \mathbf{k_m}\sigma_{\mathbf{2}}$

Interval (i): $K \le \mu_T + k_m\sigma_T$

When $K \le \mu_T + k_m\sigma_T$, the supplier's unconstrained optimal production quantities under early and delayed commitment schemes exceed the capacity limit. That is, the supplier will be producing at the capacity level in both schemes. Hence, the expected profit of the supplier under delayed and early commitment schemes is given by the first parts of Equation (4–14) and Equation (4–16), respectively. Then, we have

$$\Delta[\Pi_S] = w\left[\sigma_1 k_1 + \sigma_T L\left(\frac{K-\mu_T}{\sigma_T}\right) - \sigma_2 L\left(\frac{K-\mu_T-k_1\sigma_1}{\sigma_2}\right)\right], \tag{B–1}$$

and

$$\frac{d\Delta[\Pi_S]}{dK} = w\left[\Phi\left(\frac{K-\mu_T}{\sigma_T}\right) - \Phi\left(\frac{K-\mu_T-k_1\sigma_1}{\sigma_2}\right)\right], \tag{B–2}$$

which leads to the following lemma:

Lemma B.1. *The benefits to the supplier of an early commitment scheme increase in K within interval (i). Moreover, early commitment always outperforms delayed commitment.*

Proof. $\frac{d\Delta[\Pi_S]}{dK} \ge 0$ if and only if $\frac{K-\mu_T}{\sigma_T} \ge \frac{K-\mu_T-k_1\sigma_1}{\sigma_2}$. The condition can be rewritten as $K \le \mu_T + \frac{\sigma_T\sigma_1 k_1}{\sigma_T-\sigma_2} = K_s'$. However, note that $K_s' > \mu_T + k_m\sigma_T$ since we are currently focusing on the setting where $k_m\sigma_T \le k_1\sigma_1 + k_m\sigma_2$. That is, K_s' is outside the interval we are considering, which is (i). Hence, $\Delta[\Pi_S]$ increases in K in interval (i). The lemma follows since $\Delta[\Pi_S] = 0$ when $K = 0$. □

Note that both $E[\Pi_S^E]$ and $E[\Pi_S^D]$ decrease as the capacity becomes more restricted. The decrease in $E[\Pi_S^E]$ is larger than the decrease in $E[\Pi_S^D]$. This can intuitively explained as follows: the advantage of an early commitment scheme over a delayed commitment scheme from the supplier's perspective is the guaranteed sales to the primary retailer, which ensures that part of the capacity will definitely be utilized. However, as capacity decreases, the possibility that aggregate retailer demands will exceed capacity under a delayed commitment scheme increases. That is, delayed commitment also provides a high capacity utilization. Hence, the benefits of early commitment are less significant when the capacity of the supplier is tight.

Interval (ii): $\mu_T + k_m\sigma_T < K \leq \mu_T + k_1\sigma_1 + k_m\sigma_2$

In this interval, the supplier's optimal unconstrained production level under a delayed commitment scheme is achievable. Hence, his expected profit under delayed commitment is constant and equal to the uncapacitated case. On the other hand, $E[\Pi_S^E]$ is increasing since K is less than $\mu_T + k_1\sigma_1 + k_m\sigma_2$, the unconstrained optimal production quantity, which leads to the following lemma.

Lemma B.2. $\Delta[\Pi_S]$ *is increasing in* K *when* $K \in (\mu_T + k_m\sigma_T, \mu_T + k_1\sigma_1 + k_m\sigma_2)$. *Hence, early commitment always outperforms delayed commitment in terms of the supplier's profit.*

Proof. The first part follows from the discussions preceding the lemma. The second part then follows from Lemma B.1. □

Interval (iii): $K > \mu_T + k_1\sigma_1 + k_m\sigma_2$

This interval corresponds to the uncapacitated case. Since $\Delta[\Pi_S]$ is greater than zero and increasing in K in interval (ii) (see Lemma B.2), we can conclude that $\Delta[\Pi_S]$ is greater than zero in this interval as well. Moreover, $\Delta[\Pi_S]$ is constant since uncapacitated optimal solutions are achievable.

B.1.2 $[\mathbf{Q_S^D}]^{\mathbf{U}} > [\mathbf{Q_S^E}]^{\mathbf{U}}$: $\mathbf{k_m\sigma_T > k_1\sigma_1 + k_m\sigma_2}$

Interval (i): $K \leq \mu_T + k_1\sigma_1 + k_m\sigma_2$

In this interval, the supplier's unconstrained optimal production quantities under both delayed and early commitment exceed the capacity. Hence, as in the previous section, $\Delta[\Pi_S]$ and its derivative are given by Equation (B–1) and Equation (B–2), respectively.

Lemma B.3. *There exists a threshold capacity level, $K_s' \in (0, \mu_T + k_1\sigma_1 + k_m\sigma_2)$, such that when $K \in (0, K_s')$, $\Delta[\Pi_S]$ is greater than zero and is increasing in K, and when $K \in (K_s', \mu_T + k_1\sigma_1 + k_m\sigma_2)$, it is decreasing in K.*

Proof. $\frac{d\Delta[\Pi_S]}{dK} \geq 0$ if and only if $\frac{K-\mu_T}{\sigma_T} \geq \frac{K-\mu_T-k_1\sigma_1}{\sigma_2}$. The condition can be rewritten as $K \leq \mu_T + \frac{\sigma_T\sigma_1 k_1}{\sigma_T-\sigma_2} = K_s'$. $K_s' < \mu_T + k_1\sigma_1 + k_m\sigma_2$ since we are considering the setting where $[Q_S^D]^U > [Q_S^E]^U$. Hence, K_s' divides interval (i) into 2 parts: when $K \leq K_s'$, $\Delta[\Pi_S]$ is increasing in K. It is also greater than zero in this interval since $\Delta[\Pi_S] = 0$ when $K = 0$. When $K_s' < K \leq \mu_T + k_1\sigma_1 + k_m\sigma_2$, it is decreasing in K. □

Interval (ii): $\mu_T + k_1\sigma_1 + k_m\sigma_2 < K \leq \mu_T + k_m\sigma_T$

In this interval, the expected profit of the supplier under an early commitment scheme is constant as the unconstrained optimal solution is achievable. On the other hand, $E[\Pi_S^D]$ is increasing in K. Hence, we can conclude that in interval (ii), $\Delta[\Pi_S]$ is decreasing in K.

Interval (iii): $K > \mu_T + k_m\sigma_T$

Uncapacitated solutions are achievable. Hence, $\Delta[\Pi_S]$ is constant in this interval.

B.2 Evaluation of the Total System Profit

B.2.1 Proof of Lemma 4.1

Since production quantities and unit revenues are equal, it is sufficient to show that total sales quantity under delayed commitment is larger. Below, we show that given any realization of X_1. The result will then hold when we take expectation over X_1. Given $X_1 = x_1$, let S_D and S_E denote the sales quantities under delayed and early commitment schemes, respectively, and let $\Delta[S] = S_D - S_E$. Furthermore, let K denote the production quantity. Since the proportion of the retailers' demands satisfied does not make a difference from the system's perspective, we assume that primary retailer's demand has priority. First, consider any realization of X_1 such that $X_1 = x_1 \leq Q_1^E$.

$$
\begin{aligned}
\Delta[S] &= x_1 + \mu_2 - \int\limits_{K-x_1}^{\infty} (x_1 + x_2 - K) f_2(x_2) dx_2 - x_1 - \mu_2 + \int\limits_{K-Q_1^E}^{\infty} (Q_1^E + x_2 - K) f_2(x_2) dx_2 \\
&= \int\limits_{K-Q_1^E}^{\infty} (Q_1^E + x_2 - K) f_2(x_2) dx_2 - \int\limits_{K-x_1}^{\infty} (x_1 + x_2 - K) f_2(x_2) dx_2 \\
&> \int\limits_{K-Q_1^E}^{\infty} (Q_1^E + x_2 - K) f_2(x_2) dx_2 - \int\limits_{K-x_1}^{\infty} (Q_1^E + x_2 - K) f_2(x_2) dx_2 \\
&= \int\limits_{K-x_1}^{K-Q_1^E} (Q_1^E + x_2 - K) f_2(x_2) dx_2 > 0
\end{aligned}
$$

Now, consider $Q_1^E < X_1 = x_1 \leq K$.

$$
\begin{aligned}
\Delta[S] &= x_1 + \mu_2 - \int\limits_{K-x_1}^{\infty} (x_1 + x_2 - K) f_2(x_2) dx_2 - Q_1^E - \mu_2 + \int\limits_{K-Q_1^E}^{\infty} (Q_1^E + x_2 - K) f_2(x_2) dx_2 \\
&= (x_1 - Q_1^E) + \int\limits_{K-Q_1^E}^{\infty} (Q_1^E + x_2 - K) f_2(x_2) dx_2 - \int\limits_{K-x_1}^{\infty} (x_1 + x_2 - K) f_2(x_2) dx_2 \\
&> (x_1 - Q_1^E) + \int\limits_{K-x_1}^{\infty} (Q_1^E + x_2 - K) f_2(x_2) dx_2 - \int\limits_{K-x_1}^{\infty} (x_1 + x_2 - K) f_2(x_2) dx_2 \\
&= (x_1 - Q_1^E) - \int\limits_{K-x_1}^{\infty} (x_1 - Q_1^E) f_2(x_2) dx_2 \\
&= (x_1 - Q_1^E) F_2(K - x_1) > 0
\end{aligned}
$$

When $X_1 = x_1 > K$, the expected sales under delayed commitment is equal to K, which is obviously greater than the expected sales under early commitment. Hence, the proof is complete.

B.2.2 $[\mathbf{Q_S^D}]^{\mathbf{U}} \leq [\mathbf{Q_S^E}]^{\mathbf{U}}$: $\mathbf{k_m}\sigma_{\mathbf{T}} \leq \mathbf{k_1}\sigma_{\mathbf{1}} + \mathbf{k_m}\sigma_{\mathbf{2}}$

Interval (i): $K \leq \mu_1 + k_1\sigma_1$

When $K \leq \mu_1 + k_1\sigma_1$, the supplier will produce at the capacity level under both commitment schemes. Due to Lemma 4.1, we can conclude that delayed commitment always outperforms early commitment from the system's point of view. Lemma B.4 characterizes how the difference between profits changes with respect to K in this interval.

Lemma B.4. *The benefits of delayed commitment become more significant as K increases in $(0, \mu_1 + k_1\sigma_1)$.*

Proof. The increase in expected total profit due to early commitment and its derivative are given by

$$\Delta[\Pi_T] = \sigma_T \Psi\left(\frac{K-\mu_T}{\sigma_T}\right) - \sigma_1 \Psi\left(\frac{K-\mu_1}{\sigma_1}\right) - (r-c)\mu_2$$

and

$$\begin{aligned}\frac{d\Delta[\Pi_T]}{dK} &= r\left[\Phi\left(\frac{K-\mu_T}{\sigma_T}\right) - \Phi\left(\frac{K-\mu_1}{\sigma_1}\right)\right] \\ &< 0\end{aligned}$$

respectively. The inequality follows since $\frac{K-\mu_T}{\sigma_T} < \frac{K-\mu_1}{\sigma_1}$. Since $\Delta[\Pi_T] = 0$ when $K = 0$, the lemma follows. □

<u>Interval (ii): $\mu_1 + k_1\sigma_1 < K \leq \mu_T + k_m\sigma_T$</u>

In this interval, the supplier produces at the capacity level as in interval (i). Under an early commitment scheme, primary retailer receives $\mu_1 + k_1\sigma_1$ units and the remaining inventory, $K - \mu_T - k_1\sigma_1$, is allocated to the secondary retailer. Note that Lemma 4.1 is still valid in this interval. That is, delayed commitment always outperforms early commitment. The following lemma illustrates the change in the difference between expected profits with respect to K.

Lemma B.5. *Within interval (ii), the benefits of delayed commitment decrease in K.*

Proof. The increase in expected total profit due to early commitment and its derivative are given by

$$\Delta[\Pi_T] = \sigma_T \Psi\left(\frac{K-\mu_T}{\sigma_T}\right) - \sigma_1 \Psi(k_1) - \sigma_2 \Psi\left(\frac{K-\mu_T-k_1\sigma_1}{\sigma_2}\right)$$

and

$$\frac{d\Delta[\Pi_T]}{dK} = r\left[\Phi\left(\frac{K-\mu_T}{\sigma_T}\right) - \Phi\left(\frac{K-\mu_T-k_1\sigma_1}{\sigma_2}\right)\right]$$

respectively. Referring to the proof of Lemma B.1, we deduce that $\Delta[\Pi_S]$ is increasing in K. □

Interval (iii): $\mu_T + k_m\sigma_T < K \leq \mu_T + k_1\sigma_1 + k_m\sigma_2$

In this interval, the supplier's unconstrained optimal production quantity under delayed commitment is achievable. Hence, total expected profit under delayed commitment is constant. On the other hand, $E[\Pi_T^E]$ is increasing since $K \leq \mu_T + k_1\sigma_1 + k_m\sigma_2 < Q_T^*$, where Q_T^* denotes the optimal production quantity under early commitment from the system's perspective (the second inequality follows from the proof of Proposition 4.6. Hence, we have the following:

Lemma B.6. $\Delta[\Pi_T]$ *is increasing in interval (iii).*

Interval (iv): $K > \mu_T + k_1\sigma_1 + k_m\sigma_2$

This interval corresponds to the uncapacitated setting for both commitment schemes. Hence, $\Delta[\Pi_T]$ is constant.

B.2.3 $[\mathbf{Q_S^D}]^{\mathbf{U}} > [\mathbf{Q_S^E}]^{\mathbf{U}}$: $\mathbf{k_m\sigma_T > k_1\sigma_1 + k_m\sigma_2}$

Interval (i): $K \leq \mu_1 + k_1\sigma_1$

For this interval, our analysis in Section B.2.2 is still valid. That is, delayed commitment always outperforms early commitment (see Lemma 4.1), and the benefits of delayed commitment increase in K (Lemma B.4).

Interval (ii): $\mu_1 + k_1\sigma_1 < K \leq \mu_T + k_1\sigma_1 + k_m\sigma_2$

The supplier's production quantity is equal to the capacity for both commitment schemes. Hence, due to Lemma 4.1, delayed commitment always outperforms early commitment.

The change in the difference between expected profits with respect to K is characterized in the following lemma.

Lemma B.7. *There exists a threshold capacity level, K_T'', such that the benefits of delayed commitment decrease in K when $K \in (\mu_1 + k_1\sigma_1, K_T'')$, and increase in K when $K \in (K_T'', \mu_T + k_1\sigma_1 + k_m\sigma_2)$.*

Proof. See the proof of Lemma B.3. □

Interval (iii): $\mu_T + k_1\sigma_1 + k_m\sigma_2 < K \leq \mu_T + k_m\sigma_T$

In this interval, the optimal production quantity of the supplier under an early commitment scheme is achievable. Hence, expected total profit under early commitment is constant. On the other hand, expected total profit under delayed commitment is increasing. Hence, we have the following:

Lemma B.8. $\Delta[\Pi_T]$ *is decreasing in interval (iii).*

Interval (iv): $K > \mu_T + k_m\sigma_T$

This interval corresponds to the uncapacitated setting. Hence, $\Delta[\Pi_T]$ is constant.

APPENDIX C
APPENDIX FOR CHAPTER 5: DISTRIBUTION OF RANDOM INSTANCES

Table C-1: Distribution of instances over k_1 and k_m.

	$k_m \leq -1$	$-1 < k_m \leq 0$	$0 < k_m \leq 1$	$k_m > 1$
$k_1 \leq -1$	1	3	5	7
$-1 < k_1 \leq 0$	2	6	42	29
$0 < k_1 \leq 1$	10	54	229	197
$k_1 > 1$	10	38	203	164

Table C-2: Distribution of instances over cv_1 and cv_2.

	$0.1 \leq cv_2 \leq 0.178$	$0.178 < cv_2 \leq 0.256$	$0.256 < cv_2 \leq 0.333$
$0.1 \leq cv_1 \leq 0.178$	117	122	127
$0.178 < cv_1 \leq 0.256$	104	107	98
$0.256 < cv_1 \leq 0.333$	110	114	101

Table C-3: Distribution of instances over μ_1 and μ_2.

	$0 < \mu_2 \leq 50$	$50 < \mu_2 \leq 100$	$100 < \mu_2 \leq 150$
$0 < \mu_1 \leq 50$	89	102	102
$50 < \mu_1 \leq 100$	116	116	113
$100 < \mu_1 \leq 150$	119	130	113

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BIOGRAPHICAL SKETCH

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