# DOMAIN KNOWLEDGE, ATTITUDES, SELF-EFFICACY BELIEFS, AND ATTRIBUTIONS FOR ACHIEVEMENT WORKING TOGETHER IN THE COMMUNITY COLLEGE REMEDIAL MATHEMATICS CLASSROOM

By

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A THESIS PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
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To my hero, Kenneth Howard Murphy.

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Community colleges are faced with increasing numbers of students who are unprepared to complete and receive passing grades for college-level coursework. Mathematics is one subject area in which many students often require remediation. Researchers have discussed remedial mathematics in community colleges as it relates separately to mathematical domain knowledge. However, there are no studies which examine how the relationships between and among attitudes toward mathematics, self-efficacy beliefs, and attributions for achievement influence students' acquisition of mathematical domain knowledge. The findings of this study showed that attitudes toward mathematics, self-efficacy beliefs, and attributions for achievement influenced mathematical domain knowledge acquisition among four students.

# CHAPTER 1
# INTRODUCTION

## Introduction

Mathematics is a troublesome area of study for many students.   Many students either experienced difficulty with mathematics or simply have not successfully mastered a significant amount of mathematics.  From an educational standpoint, the former is preferable to the latter. Because mathematics is an exact science, students and teachers alike need not expect that the learning of mathematics will, or should, be without obstacles or a need for re-teaching.  Many factors affect the way that students relate to mathematics.  These factors include, but are not limited to, mathematical domain knowledge, attitudes toward mathematics, self-efficacy beliefs, and attributions for achievement in mathematics.  The researcher believes that knowledge affects attitude and efficacy beliefs, attitude and efficacy beliefs affect attribution, and vice-versa in both cases.  Through a description of students' previous experiences, this study explored situations in which attitude toward mathematics, mathematical efficacy beliefs, and attributions for achievement in mathematics influenced the acquisition of mathematical domain knowledge as it pertains to remedial algebra coursework.

## Statement of the Problem

Community colleges and many universities in America have adopted policies and mission statements that reflect the now commonly held belief that higher education should be made available to all who would make a serious attempt at such an endeavor.  This shift in focus has led to open-door policies and relaxed admission requirements, which have afforded opportunities for many to have access to education.  Community colleges, for example, are not critically selective in their admissions processes.  A student need not even possess a high school diploma because, in many cases, community colleges offer programs of study leading to G.E.D. or certificate of completion of high school courses. This has resulted in an influx of students who are not prepared for college-level coursework. Mathematics is one subject area in which many unprepared students require remediation. A lack of specific domain knowledge, negative attitudes toward mathematics, lack of self-efficacy, and external attributions for mathematics achievement often lead to a pattern of severely limited success in college-level mathematics courses.

## Purpose

As colleges and universities seek new ways to serve their students' needs, American higher education has witnessed the relocation of college preparatory courses, from highs schools or preparatory institutions, to the campuses of colleges themselves. Although universities sometimes offer these remedial courses, community colleges do so almost invariably. It has become common for students to attend one or more of these preparatory courses, usually not for credit, to ready themselves for the college-level material required for their major fields of study. As evidenced by proliferation of remedial courses, mathematics is one subject in which students typically lack adequate knowledge.

Since secondary schools in the United States presumably do not purposely furnish colleges with academically deficient students, one might wonder just how we have arrived at the point where we now stand. Despite how it has happened, community colleges, in particular, must find ways to help students overcome their academic deficiencies and move on to college-level coursework. The primary purpose of this study is to identify and describe relationships between and among attitudes about the usefulness of mathematics, self-efficacy beliefs, and attributions for achievement in mathematics and their influences on mathematical knowledge acquisition among community college students enrolled in a remedial algebra course.

# CHAPTER 2
# LITERATURE BACKGROUND

Research exists, albeit not very current, which addresses mathematical domain knowledge, attitudes and self-efficacy beliefs about mathematics, and attributions for achievement in mathematics separately. There exists some literature which explores relationships between these areas; for example, Xin Ma (1997), relates attitude toward mathematics to achievement in mathematics. Literature will be referenced below, particularly that which pertains to proposed or established relationships between mathematical domain knowledge, attitudes toward and self-efficacy beliefs about mathematics, and attributions for achievement in mathematics.

## Mathematical Domain Knowledge

Mathematical domain knowledge refers to knowledge that is specific to the domain of mathematics. For example, knowing that the quadratic formula is required to factor general equations of the form $Ax^2 + bx + c = 0$, requires mathematical domain knowledge. Mathematical domain knowledge consists of several types of knowing, including “knowing that”, “knowing how”, “knowing why”, and “knowing to”. According to Selden and Selden (1996), “knowing that” may simply refer to a student’s ability to reproduce facts they learned. “Knowing how” usually refers to knowing the process required to solve a problem. For example, a student who has observed the teacher solving quadratic equations using the quadratic formula ostensibly knows how to solve quadratic equations themselves. “By ‘knowing why’, (it is) meant having ‘various stories in one’s head’ about why a mathematical result is so” (Selden and Selden, 1996). It should be noted that knowing why and proving a thing are different; proofs require much more than just ‘stories in one’s head’. “Knowing to” refers to a student’s ability to act, and is arguably the most important type of knowing. This type of knowing allows learners to shift their attention from thinking to doing. According to Selden and Selden, “knowing to” is the type of knowing that most often translates to achievement; it is this type of knowing that teachers should try to teach most.

Specific domain knowledge in the area of mathematics is essential to the successful teaching and learning of mathematics. Unlike many other knowledge domains, mathematics requires linear thinking and a precise understanding of strategies that can be used to solve problems. That is not to say that teachers and students must always arrive at the correct answer to a mathematics problem in exactly the same way. In fact, employing several strategies is an asset to the mathematics problem solver. Given that teachers and mathematics students need to access the same canonical domain knowledge, we might expect that students enter college equipped with a certain amount of basic knowledge of mathematics. Basic knowledge of mathematics includes terminology along with knowledge of how to perform basic arithmetic and algebraic operations. This is not the case, in fact, “It can no longer be assumed that the beginning college algebra student understands the most basic concepts of mathematics” (Chang, 2000, p. 17). Community colleges are uniquely situated to confront this problem because teaching is the primary function of faculty at community colleges. That is, behavior management is not an issue as it may be in high school. In addition, community college faculty do not have research and service interests competing with teaching duties for their time as is the case in larger colleges and universities.

## Attitudes & Self-Efficacy Beliefs

An important factor bearing on the forethought phase of problem solving in mathematics is self-efficacy. Self-efficacy is one’s belief about their ability to be successful. Self-efficacious learners very often judge themselves able to solve problems a priori, and thus make choices about strategy and effort that often lead to successful problem solving. There is a related idea, that of self-concept, which is distinguishable from self-efficacy “in that self-efficacy is a context-specific assessment of competence to perform a specific task…" "Self-concept is not measured at that level of specificity and includes beliefs of self-worth associated with one's perceived competence." (Pajares & Miller, 1994) Learners may have a positive self-concept in general, but lack self-efficacy in a particular area. Also, a relationship exists between one's perceived ability to solve a specific mathematics problem and one's actual efficacy in solving that problem. As the former increases, so does the latter. That is not to say that self-efficacy alone can ensure success in mathematics problem solving, rather that when enhanced self-efficacy is combined with domain knowledge and proper use of strategies, a greater degree of success may be attained.

First, self-efficacy beliefs among learners of mathematics can be justifiably considered by instructors to be the single most important factor contributing to persistence in mathematical problem solving. It is not that self-efficacy is strictly more important than domain specific knowledge or achievement attribution strategies; rather self-efficacy occupies a position which is somewhat psychologically predominant. "Unless people believe that they can produce desired effects by their actions, they have little incentive to act." (Bandura, Barbaranelli, Caprara, & Pastorelli, 1996, p. 1206). Believing in their capabilities is what motivates mathematical learners to persist in the learning of mathematics. Without some degree of self-efficacy, students are likely to pursue mathematical learning only because it is required, and not for the purpose of mastery of the material. Mathematical self-efficacy beliefs are related to mathematical achievement in a number of ways. First, efficacy beliefs are formed and influenced by past attempts to achieve mathematical success. "The results…(indicate) that perceived self-efficacy is a significant contributor…including past performance" (Bandura & Locke, 2003, p. 90). Depending upon the level of success attained, students form beliefs about their future abilities to perform the same mathematical task or function. A string of unsuccessful attempts to perform mathematically could contribute to a negative set of mathematical self-efficacy beliefs. Once these negative beliefs exist, they are difficult to overcome. Often, students recall that an outstanding teacher in their educational experience was instrumental in their understanding of a certain subject. In the area of mathematics, it is probable that such teachers overcome negative self-efficacy beliefs that the students already possess, foster positive self-efficacy beliefs that the students do not possess, or both.

Second, according to Bandura (1996), efficacy beliefs are related to social beliefs of individuals. For example, a student may perceive that he or she is efficacious in mathematics based upon successes with math in their immediate family, or in their peer groups. "Parental aspirations and perceived efficacy build children's sense of efficacy and academic aspirations…" (Bandura, Barbaranelli, Caprara, & Pastorelli, 1996, p. 1213 ). When parents facilitate and encourage academic pursuits in their children, efficacy beliefs in those children are likely to be high. That is not to say that those same children are always the high achievers; rather that they are not hindered in their learning by the effects of low self-efficacy beliefs. Peer groups also can have an influence, positive or negative, on efficacy beliefs. "…Perceived academic efficacy affect(s) their academic achievement both independently and through the mediated effects of peer relations…" (Bandura et. al., 1996, p. 1211).

Third, efficacy beliefs have a reciprocal relationship with motivation. People generally avoid activities with which they do not feel comfortable. This is certainly true of college students, who tend to choose courses, lodging, and even friends based on comfort level and confidence. Mandatory mathematics requirements, however, force college students to complete a certain level of mathematics coursework. Since many students enroll in these mandatory courses with no other motivation than to satisfy general education requirements, it is important that instructors recognize the relationship between efficacy beliefs and motivation. According to Bandura and Locke (2003), “The evidence…is consistent in showing that efficacy beliefs contribute significantly to the level of motivation and performance.” (p. 87)

Another factor possibly contributing to the poor mathematical achievement of many college students is attitude toward mathematics. Negative attitudes toward mathematics are common among students today, and may be reinforced by parents and even teachers. That is not to say that most parents and teachers share a negative view of mathematics; rather that these groups evidently do not undertake to foster more positive perception of mathematics among students. Human activity has “determined that previous mathematics performance and perceived ability are both key elements for success in mathematics” (Kloosterman & Stage 1995, p. 296). Because rational consideration does not allow one to conclude that all students who are ill prepared for college mathematics courses are lazy and/or not very intelligent, attitudes and beliefs toward mathematics must play a substantial role in their poor performance. Certainly, there could be other factors at work as well, such as lack of exposure to mathematics and poor instruction. According to Mau (1991), “researchers have concluded that negative beliefs about learning and doing mathematics seem to be a key to many students’ inability to focus enough to ‘survive’ mathematics courses…” (p. 25).

## Attributions for Achievement

For the purposes of this study, the phrase ‘attributions for achievement’ will refer to what or to whom a person gives credit for mathematical achievement. Students might attribute success in the area of mathematics to various influences. A popular attribution for achievement is studying. Students associate studying well with academic success, not only in their own experience, but in general. This is a desirable attribution, but studying may be interpreted too broadly. Because of differences of opinion, it is difficult to measure some attributions made by students.  For example, what one student considers studying in depth, another might consider nothing more than reading over material.  Nevertheless, attributions are routinely made, and can serve in a general way to inform mathematics instructors as to the mindset of their students.

The types of attributions students make fall into one of two categories.  One is that of an external attribution, for example, “The test was too hard, so I failed it”.  The other is that of an internal attribution, for example, “I did not prepare properly for the exam, so I failed it.  The second type of attribution, internal attribution, “claims that the person was directly responsible for the event.” (Butterfield, 1996, p. 4).

Some examples of positive external attributions are:  I was just born with math intelligence; my brother is really good at math, I really like this teacher, my grade depended on it, the test was easy, etcetera.  There are, of course, also negative external attributions such as:  I was just not born with any math intelligence, the test was too hard, the teacher is terrible, etcetera.  These external attributions do not leave the student any recourse to perform better next time.  With this mindset, if the external factors remain the same, the student’s achievement level will also remain the same.  In the case of external attributions, the student places the responsibility for his/her performance on someone or something other than his/her own efforts.

Some examples of positive internal attributions are:  I studied hard for that test, I utilized the proper strategies, I recalled the algorithm for solving that problem, etcetera.  Here are a few examples of negative internal attributions:  I forgot to use the proper problem-solving strategy, I am not the kind of person who spends any time studying, I did not internalize the material in this unit, etcetera.  These attributions give room for improvement, may lead to increased self-efficacy, and as such are more desirable than external attributions for achievement.  According to Gredler (1992), attributions with an internal locus lead to feelings of confidence.  Internal attributions tend to be habit forming. For example, performing well on a mathematics exam after studying at a certain part of the library with another student might encourage the students to study together at the same place for the next exam. Over the course of the semester or year, it may become a habit for the students to meet at that same place to study for mathematics exams. So, the student may believe that their mathematical achievement is linked to studying with that other student (external), or at the location in the library (external), or the methods and strategies (internal) he or she used to study. Clearly, attributing mathematical success to the methods and strategies used for studying is the desirable attribution because the other two rely upon factors out of the student's control. In fact, attributing successes to external factors can "…undermine an existing habit" (Butterfield 1996, p. 6), such as might occur in our scenario above if all the attribution is given to the location and the study partner. The student may cease to employ the methods and strategies that caused the successes in the first place. Attributing successes to proper (mostly internal) factors reinforces good habits, thereby possibly increasing success in mathematics and mathematical self-efficacy beliefs

## CHAPTER 3
## FRAMEWORK AND DESIGN

This study utilizes description to isolate and magnify the evident relationships between and among attitudes toward mathematics, self-efficacy beliefs, and attributions for achievement. While the positive influences that these have on each other and on mathematical knowledge acquisition will be the focus, it is useful to understand the interplay in a practical way that includes the potential negative influences as well. The effects of domain knowledge on attitudes toward mathematics and self-efficacy beliefs can be positive, negative, or neutral. Increased mathematical knowledge may lead to better attitudes and self-efficacy beliefs; lack of such knowledge may lead to poor attitudes and self-efficacy beliefs, and it is possible that varying degrees of domain knowledge have no significant effect on attitudes toward mathematics and self-efficacy beliefs. It should be noted that effects of increased knowledge on attitudes and self-efficacy beliefs could be differential. That is, increased domain knowledge could improve attitudes without having a significant impact on self-efficacy. For the purposes of this study, a positive effect on attitudes and self-efficacy shall mean on either instead of on both, meaning that improvements in attitude are not assumed to be intrinsically linked to improvements in self-efficacy beliefs and vice-versa.

The effects of attitudes and self-efficacy beliefs on achievement attributions can be positive, negative, or neutral in the same way as described above. An example of a positive effect on achievement attributions might be that improved attitudes and self-efficacy beliefs led a student to attribute success on a mathematics assessment to studying or to strategies rather than to luck or to natural ability. By the arrows in the diagram, it is meant that self-efficacy beliefs and attitudes toward mathematics directly affect both domain knowledge and achievement attributions, while domain knowledge and attributions directly affect attitude/self-efficacy only.

This interpretation speaks to the importance of attitudes and self-efficacy beliefs in the area of mathematics. It would be difficult to over-emphasize the positive effects of improvement in attitudes toward mathematics and self-efficacy beliefs as they pertain to domain knowledge and achievement attributions.

## Participants

Four students at a community college in the southeastern United States participated in this study. Each of these students was, at the time of the study, enrolled in a remedial algebra course at the community college. The course, MAT0020 (Integrated Mathematics) is geared to begin with a brief review of arithmetic skills and move to basic algebra skills. The course culminates in a standardized state exit exam which requires mastery of algebra skills comparable to those required to pass Algebra II in a state’s high school. This level of mastery indicates that students possess the skills necessary to enroll in a college-level algebra course. The Common Placement Test (CPT) is a knowledge based exam which community colleges in the state use to place students. The focus, then, was the breadth and depth of domain knowledge up to and including algebraic thinking. Each student placed in MAT0020 demonstrates a reasonable level of arithmetic knowledge, but fails to demonstrate that same reasonable level of algebraic knowledge. Because each of the students in the study was enrolled in MAT0020, mathematical domain knowledge was uniformly less than desirable for college students. Participants shall hereafter be referred to as studentA, studentB, studentC, and studentD, which approximates their self-reported achievement levels (grade-wise) in MAT0020. These reports were confirmed via the course instructor with written, signed permission from the students. In each case, the student’s report of achievement level was a match with the grade reported by the MAT0020 instructor. StudentA and studentB occupy the high level of achievement range, studentC occupies the middle range, and studentD occupies the low range. StudentA is a caucasian female, 18-20 years of age, enrolled in her second semester of college. StudentB is a black male, 18-20 years of age, enrolled in his second semester of college. StudentC is a caucasian male, 18-20 years of age, enrolled in his second semester of college. StudentD is a hispanic male, 18-20 years of age, enrolled in his third semester of college. Each of the participants reported a somewhat weak background in mathematics, which stands to reason, as each of them is enrolled in a remedial algebra course. In order to be sure that achievement in terms of mastery and performance were uniform, the four participants were all chosen from the same section of MAT0020. Each of the participants took part in an impromptu end of class meeting attended by their instructor and the researcher; at this time they all verbally agreed to participate in the study. It was announced that, in addition to their grades and mathematical background information, a time commitment would be necessary for surveying and interviewing. There was an exchange of email addresses and phone numbers in preparation for beginning the study. The participants all voluntarily signed an informed consent document, and all spoke to the researcher about their experiences with mathematics.

## Instruments

The instruments utilized in this study may be viewed in the appendix and include the Student Survey and an interview guide designed by the researcher to reveal attributions for achievement in mathematics. The fifteen items in the Student Survey were separated for this purpose, with the first eleven indicating the student's attitude toward mathematics, and the remaining four indicating the student's level of self-efficacy with respect to mathematics. This Student Survey has been piloted in an ongoing study of college students enrolled in a remedial algebra course (MAT0020 or MAT0024). Except for cases of total disinterest, the Student Survey gives an indication of either positive or negative general attitude toward mathematics, and of either generally positive or generally negative mathematical self-efficacy beliefs. The personal interview was utilized to ascertain the level of mathematical domain knowledge possessed by each student. During the interviews, the students related grades, mastery level, and familiarity with various mathematical concepts. The interview items were designed to evoke responses which would indicate the achievement attributions of the student. That is, to what does the student attribute success (or failure) in the area of mathematics? For example, a student might attribute success on a mathematics assignment to having the natural ability to do well in math, or "I'm just smart" attribution. The fifteen item Student Survey has been piloted, in particular, on a group of over 100 remedial algebra students at the community college, excluding the four research participants of this study. This made for ease of use and interpretation, as the items had been edited over time to elicit responses aimed directly at the issues of attitudes toward mathematics and mathematical self-efficacy beliefs.

Within the structure of the five math-related statements used in the personal interview, tendencies toward one type of attribution or another were determined. Attributions for mathematical achievement were categorized according to their type. The two types of attributions were external attributions and internal attributions. The former indicates that the student attributes success or failure to some entity other than themselves, while the latter indicates that the student attributes success to studying, recalling examples, and generally working toward success in an incremental manner.

## Methods

The three indicators of knowledge, efficacy/attitude, and attribution were, respectively, personal interview/ course grades, Student Survey responses, and verbal responses to open-ended mathematics statements in the interview instrument (appendix). Within two weeks of the initial meeting, each student made contact with the researcher, and took time to respond to the fifteen item Student Survey (appendix). The responses to the Student Survey were analyzed to ascertain a general idea of the student's attitude toward mathematics and self-efficacy beliefs. At the time when the students responded to the Student Survey, a subsequent interview time was scheduled. The participant and researcher agreed to a forty-five minute window to explore the students' general mathematics background and to have them respond to five open-ended mathematics-related statements (appendix). Interviews were tape-recorded and conducted in a semi-private manner, meaning that while the student and researcher were in sight of many other students, their conversations could not be overheard. The interviews began with an informal chat about mathematics, more or less an icebreaker, followed by a request of the student to relate their general mathematics background. This information included memories of difficulty with mathematics, grades received in current or past mathematics courses, certain teachers that motivated the student (or not) to attempt to improve their mathematical situation, and general comments from the student about their feelings and thoughts about mathematics. After this informal beginning to the interview, each student was asked to formally respond to five mathematics-related statements. Students responded to each of the five items, and then were invited to make any closing statements they wished before concluding the interview. Without exception, the students spoke at length about their relationship to mathematics and how that relationship had developed over time.

# CHAPTER 4
# DATA ANALYSIS

Student D reported that his mathematics background was never good. He related that he has always received tutoring since early youth. He feels as though he will never need mathematics, and related that he becomes bored easily when doing mathematics. Student D expressed anxiety problems when dealing with mathematics, and stated that uncertainty is the norm for him when attempting to solve mathematics problems. He also stated that he has always had to play catch up in the area of mathematics, more often than not, having to work backwards from provided solutions to understand how to solve problems. Student D sees no reason to do mathematics without a calculator, and believes he performs satisfactorily when allowed to use a calculator to solve mathematics problems. In general, student D believes he possesses unacceptably little mathematical domain knowledge, and that what little he does possess is not very clear.

Although student D's responses to the Student Survey were mixed, his general attitude toward mathematics is negative. This is best evidenced by his agreement with the statements: "I often do not care whether I get math problems correct or incorrect", and "Mathematics is more something I have to do than something I choose to do." Student D's self-efficacy beliefs are high relative to his generally negative attitude toward mathematics. For example, even though he agreed with the statement: "I often think of other things while attempting to solve math problems", he also agreed with the statement: "Solving math problems is easy for me." It is also interesting to note that even after mentioning his mathematics anxiety problems, student D strongly disagreed with the statement: "Performing mathematical computations makes me anxious." In his interview, student D attributed success to eleventh hour studying, and included the possibility, though not for himself, that some people are born with mathematical ability.

Student C reported that his mathematics background was okay up until high school. He indicated that the difficulty arose when he began to become interested more in extracurricular activities than in school itself. Teachers play a large role in the learning process of student C**;** in fact, he claims to do little to no studying outside of class. He stated that in high school, his mathematics teachers did not care all that much, and that as a result, he did not care either. Student C related that he enjoyed no support structure, such as tutoring and the like, and still does not make use of such things even when they are available. Having already failed MAT0020 once, student C feels the need to perform this time, even though he is often bored with mathematics. He believes that, in general, he possesses a typical amount of mathematical domain knowledge, and that mathematics is not all that difficult.

Student C displays a generally positive attitude toward mathematics. He converses in an upbeat manner about the subject, and strongly disagreed with the statements: "Mathematics is not very useful in the real world." and "I often do not care whether I get math problems correct or incorrect." Student C responded in agreement to the statements: "Mathematics is more something I have to do than something I choose to do.", and "I often think of other things while attempting to solve math problems." This indicates that student C is not really interested in mathematics for reasons other than the general education requirement that he complete a certain amount of college level mathematics coursework. Student C's responses to the Student Survey items 12 through 15 indicated that he felt efficacious in the area of mathematics. Student C attributed success in mathematics to paying attention in class and to the influence of his instructor. He believes that seeing and hearing about a mathematics concept once is usually enough to master that concept, and that it is not necessary to repeatedly practice a skill for mastery. Student C does indicate that he believes he will have to study, eventually.

Student B indicated that his dealings with mathematics were good until high school. Similar to student C, student B related that the difficulty he experienced then was of his own making. He stated that this difficulty has made college very challenging, and that he has been forced to work much harder to make up for the lack of knowledge. Student B comes from a large family, and related that therein a support structure exists on which he regularly falls back. Having siblings who can help with school occasionally increases student B's will to succeed and enhances his self-efficacy beliefs. He believes that there are many people and support structures other than his family that students can take advantage of in the college setting; mathematics instructors and tutors were two that he mentioned. Student B possesses an acceptable amount of mathematical domain knowledge, but he does not seem to be comfortable utilizing the knowledge.

While student B's expressions would not likely give it away, he has a decidedly negative attitude toward mathematics. His agreement with items 3, 4, 7, and 8 of the Student Survey, which all address attitude toward mathematics, along with his disagreement with the statement: "I enjoy studying mathematics" confirms this negative attitude. Student B possesses low self-efficacy beliefs, as shown by his responses to items 12 through 15 of the Student Survey. He related that he experiences great difficulty in learning mathematics and that he believes he will never be good at doing mathematics. In spite of student B's negative attitude and low self-efficacy beliefs, he is performing well on exams in MAT0020. It is also noteworthy that he strongly agrees that mathematics is an important field of study. Student B attributes his success in MAT0020 to lots of behind the scenes work. In his interview he related that he spends a great deal of time mastering the concepts covered in the course. Student B represents a bit of an oddity because he is currently experiencing success with mathematics in spite of his low efficacy beliefs and his negative attitude toward mathematics.

Student A considers mathematics to be a very theoretical science, and she does not believe that her path in life will require her to make use of very much of the mathematics she is required to learn. She indicates that she has always memorized mathematical concepts, and does not usually see the big picture where mathematics is concerned. Student A believes that her middle school played a role in her struggle with mathematics, stating that she was placed in a lower level course because of overcrowding at the school. As a result, student A believes, she was ill-prepared for high school mathematics courses and subsequently for college mathematics courses as well. Although mathematics has never come easily to her, she does not experience great difficulty as long as she puts in the necessary study time. Student A describes her relationship to mathematics as a love/hate relationship.

Student A exhibits a positive attitude toward mathematics, as evidenced by her strong disagreement with items 4, 5, 6, and 8 of the Student Survey, all of which address attitude toward mathematics. She is the lone student in this study who disagrees at all with the statement: "Mathematics is more something I have to do than something I choose to do." In conversation, student A disparages the study of mathematics, but not because she finds it difficult. She speaks of a feeling of accomplishment when doing mathematics, but relates that washing her car might produce the same feeling. Her responses to items 12 through 15 of the Student Survey indicate that student A possesses high self-efficacy beliefs with respect to mathematics. She reports mathematics anxiety, but still believes she can perform mathematical operations successfully. The attributions for mathematical achievement given by student A included existing mathematical knowledge that she possesses, studying, and her current teacher's ability to teach effectively.

This study lends itself to descriptive analysis of the data acquired from the Student Surveys and the interviews. The connections and implications made will be more focused on why and how they occurred than on quantitative measures such as to what extent they occurred. The data have been summarized in the preceding pages; next they will be analyzed.

**Knowledge ↔ Attitudes & Efficacy**

Mathematical domain knowledge is sometimes difficult to measure because summative assessments, which are the usual measuring tool, sometimes do not give the best glimpse of domain content knowledge. This happens for a variety of reasons, including test anxiety, lack of feedback, and lack of motivation. For the participants in this study, mathematical domain knowledge is lacking in general, hence the remedial algebra course (MAT0020). With this in mind, performance in MAT0020, in the traditional grading scale sense, will be used to measure differences in the students' domain knowledge. Student A and student B are both performing near the high end of the grading scale, with student A slightly ahead. It will be assumed, then, that student A possesses slightly more mathematical domain knowledge than student B. Recall that student A exhibits a positive attitude toward mathematics, and that she believes that she is efficacious in the area of mathematics. Student B harbors a negative attitude toward mathematics, not outwardly, but in his candid responses to certain items of the Student Survey. He also possesses low self-efficacy beliefs with respect to mathematics. Since students A and B possess very similar amounts of domain knowledge, this factor may not have contributed to the large difference in self-efficacy beliefs and attitude. Rather, improvements in the attitude and efficacy beliefs of student B would serve to increase his domain knowledge. It is widely accepted that increased knowledge leads to increased efficacy, but perhaps the relationship is more reciprocal than commonly believed to be. Allowing domain knowledge, efficacy beliefs, attitudes, and attributions for achievement to function in a positive feedback cycle is certainly desirable, perhaps even necessary for some learners. An example of a positive feedback cycle would be a learning environment where equal emphasis was placed on each of the four factors mentioned above. In this way, the learning needs of the student could be met in a timely manner which could prevent undesired outcomes.

Student C is performing in the middle range of the grading scale for MAT0020. He possesses significantly less mathematical domain knowledge than student A and student B. Student C displays a positive attitude toward mathematics and believes he is efficacious in the area of mathematics. To date, his positive attitude and efficacy beliefs have not greatly enhanced his mathematical domain knowledge. Student C's upbeat personality and "can do" beliefs have likely prevented his lack of domain knowledge from having a negative impact on his attitudes and beliefs. This is good, but not enough to enhance his performance in MAT0020.

Student D is performing near the low end of the grading scale. He possesses minimal mathematical domain knowledge and displays low self-efficacy beliefs and a negative attitude toward mathematics. When speaking with him, it is clear that the reciprocal relationship between domain knowledge and attitude/efficacy beliefs is working against student D. That is, his low efficacy beliefs and attitude actually serve to limit the amount of mathematical domain knowledge he possesses and his lack of domain knowledge perpetuates his negative attitudes and beliefs about mathematics. In particular, student D has a good self-concept, but fails to relate his troubles with mathematics to a specific context.

**Attitudes & Efficacy ↔ Attributions**

The relationship between self-efficacy beliefs/attitude and mathematical achievement attributions is clear. The potential positive effects of one on the other can be seen in any mathematics classroom.  For example, performing well on an assessment because of implementation of effective study habits leads to improved self-efficacy beliefs immediately and perhaps to an improved attitude toward mathematics ultimately.  Conversely, positive attitudes and efficacy beliefs about mathematics lead to proper attributions for achievement.  That is, a student who displays good attitudes and beliefs is much less likely to attribute success with mathematics to luck or natural ability as they are to attribute that success to hard work or good strategies.  Recall that attitudes and efficacy beliefs were measured with the Student Survey, and that attributions were documented during interviews.

Student D displays a negative attitude toward mathematics and possesses relatively low self-efficacy beliefs.  His attributions for achievement are less than desirable, indicating that his attitudes and beliefs about mathematics may be discouraging him from developing proper study habits and problem-solving strategies.  In addition, student D's attributions for achievement, which included being born with mathematical ability and cramming for exams, are not allowing for improvement of his attitudes and beliefs.  After all, being born with whatever mathematical abilities one possesses does not allow for increases in those abilities.  It is equally as disturbing to hear a student attribute failure on a mathematics assessment to lack of natural ability.  This type of attribution seems to doom the student to limited, if any, success in the area of mathematics.

Students **C**, **B**, and **A** all attributed successes in mathematics to desirable causes.  These included attending and paying attention in class, doing the assigned homework, lots of practice, and strategic studying.  In addition to these, student C reported that sometimes winging it, or hoping for the best, resulted in achievement gains.  In the case of student A, there seems to be a positive reciprocal relationship between her attributions and her attitudes and beliefs.  For student B, however, his desirable attributions do not seem to have had a positive influence on his attitudes and beliefs about mathematics. He attributes achievement in mathematics to hard work, etcetera, but does not believe that he is good at doing mathematics even though he puts in the work necessary. Student C possesses strong efficacy beliefs. Though his performance in MAT0020 is just average, he believes it would be much better if he so desired. In the interview, he indicated that he puts forth the necessary effort to pass the course, but no more effort than that. Ironically, it is probably this same self-assuredness which prompted student C to attribute success to winging it. This is an example of a positive influence having a negative effect. When this happens, there are more than likely overriding factors involved, such as student C's nearly invincible self-concept, which disallows him to entertain the thought of failure. In general, the students experienced a more dynamic relationship between attitudes/efficacy and attributions than between knowledge and attitudes/efficacy. In either case, attitudes and efficacy beliefs had the strongest influence over their counterparts.

# CHAPTER 5
# CONCLUSIONS

Because some degree of mathematical domain knowledge is necessary to perform any mathematical task, a rank ordering of domain knowledge, attitudes & beliefs, and attributions would place domain knowledge at the top. "Research on expertise indicates that domain-specific knowledge is the most important single component in effective learning, outstripping…other components" (Ericcson, 1996), (2004, p. 4). Attitudes & beliefs and attributions could be ranked in either order, depending upon the context. Interestingly, this ranking scheme limits the interplay between domain knowledge and other components because domain knowledge must exist, to a certain degree, a priori.

There were three primary conclusions reached as a result of this study. First, there is a reciprocal relationship between mathematical domain knowledge and mathematical self-efficacy. By understanding this relationship, instructors can provide instruction in a way that increases both efficacy and knowledge. Since efficacy beliefs are specific (e.g. I can solve quadratic equations easily), students can build efficacy beliefs slowly, as specific areas of mathematics are addressed. Conversely, mathematical self-efficacy beliefs can increase opportunities for students to gain mathematical domain knowledge. To use the earlier example, a student who believes he or she is efficacious in the area of solving quadratic equations would be more likely to excel in the area of applying solutions of quadratic equations to real-world problems. Students generally have an aversion to word problems, but efficacious beliefs with respect to the skills necessary to solve word problems can overcome this aversion. Repeated successes with solving word problems (i.e. displaying the knowledge), in this case applied quadratic equations, can lead to increased efficacy beliefs, thereby completing the reciprocal loop between efficacy beliefs and knowledge. Instructors can use their understanding of the reciprocal relationship between efficacy beliefs and domain knowledge.  However, instructors must also praise, support, and acknowledge student learning through a variety of feedback methods.

Second, the relationships among attitudes toward mathematics, mathematical self-efficacy beliefs, and attributions for achievement in mathematics are certainly more evident, and perhaps more important than the relationship between knowledge and efficacy beliefs.  By more important, the researcher means that the impact on learning mathematics is greater.  That is, attitudes toward mathematics, efficacy beliefs, and attributions for achievement can affect learning via their interaction with each other, whereas domain knowledge and efficacy beliefs primarily affect learning directly.  It is reasonable, then, to conclude that the dynamics of the relationships among attitudes toward mathematics, efficacy beliefs, and attributions for achievement are more important to increasing mathematical learning than is the dynamic between domain knowledge and efficacy beliefs.  Interaction with students and engaging students in the learning process is essential to the meaningful learning of mathematics.  Many students display a negative attitude toward mathematics, which is usually easy to detect.  This makes for an uphill battle in the mathematics classroom.  When students make comments such as "I'm just no good at math", it exhibits very clearly that the student harbors a negative attitude toward mathematics and that the student attributes his or her failures in mathematics to external factors.  These negative attitudes and these external attributions are undesirable as they lead to low self-efficacy beliefs among students.  The claim that these relationships are highly evident can be justified in almost any mathematics classroom on exam day.  Invariably, there will be some pre-test and post-test commentary by students which displays attitude, attribution, and efficacy beliefs.  This commentary can serve as the basis for classroom discussion and an opportunity to explore students' beliefs, learning needs, and/or misperceptions.  Clearly, simply overhearing negative comments on exam day and then mentioning it to the class will not suffice to accomplish this goal. Rather, some class time may be well spent in exploration of the origins of negative feelings about mathematics as well as exploration of reasons for poor performance on mathematics exams. It is difficult to predict the likely outcomes of such exploration. Changes in instruction, using activities that lead to success repeatedly and over time, may improve attitudes toward mathematics and promote internal attribution. In either case, the importance of efficacy beliefs, attitudes, and attributions is made clear in the sense that meaningful learning requires both responsibility for achievement level and acknowledgement of beliefs.

Third, attitudes toward mathematics and mathematical self-efficacy beliefs play an indispensable role in the learning of mathematics, even when the motivation for a student to engage in mathematical learning is completely external. The participants in this study were freshman community college students enrolled in a remedial algebra course (MAT0020), which usually indicates that mathematics is not a favored subject of study for a student. General education requirements mandate that students complete two college-level mathematics courses, which cover material significantly more involved than that of a remedial college algebra course. This requirement represents an external motivation for students to enroll in mathematics courses. Having solely external motivation to learn mathematics may not be the most desirable situation, but it does not preclude the meaningful learning of mathematics. Indeed, students who are externally motivated, by requirements, to study mathematics may become efficacious in the area of mathematics and come to hold positive attitudes toward mathematics. Other factors, such as mode of instruction and feedback also may have an effect on attitudes toward mathematics. This is the challenge for, and hopefully the goal of mathematics instructors. Attributions which allow for positive change in efficacy beliefs and attitudes, namely internal attributions, minimize external reasons for students to perform poorly in mathematics. In this way, students are prompted to take responsibility for their studies, and can enjoy successes in mathematics by giving themselves credit for those successes. This behavior enhances self-efficacy beliefs, and makes more sense to the student than attributing successes to luck of the draw or natural ability. In addition to improving mathematical performance, proper attributions which lead to increased efficacy beliefs and better attitudes can be brought to bear in other domains. Even the student who intends to complete exactly the required amount of mathematics and no more can benefit from making desirable attributions and building self-efficacy in mathematics in ways that enhance their ability to do well in other courses of study.

**Implications:** The conclusions reached in this study were of a general nature and should be discussed here and elsewhere to explore their applicability to particular mathematics environments. To that end, let us apply the findings to some specific situations, namely those surrounding students A, B, C, and D of this study.

Since student A is performing exceedingly well in MAT0020, and attributes part of that success to her instructor, it would be interesting to track her through her future mathematics coursework. To see that the reciprocal relationship between mathematical domain knowledge and mathematical self-efficacy beliefs can be utilized in the mathematics classroom in a way that increases both efficacy and knowledge, consider the situation of student B. Student B performs near the top of the grading scale for MAT0020, and possesses a good deal of mathematical domain knowledge. However, student B also possesses low self-efficacy beliefs and a negative attitude toward mathematics. His mathematical knowledge, which is substantial, could be utilized to foster better attitudes and efficacy beliefs about mathematics. Perhaps student B would benefit from journaling his beliefs and attitudes about mathematics. This could allow him to give himself credit for the good performance he enjoys in MAT0020. For example, having student B write down the reasons for each step when solving problems might prompt him to recognize that his persistence and strategies are the reasons he is successful with mathematics, and that he is actually fairly efficacious in the area of mathematics.

To see that the relationship among attitudes toward mathematics, mathematical self-efficacy beliefs, and attributions for achievement in mathematics is more evident, and perhaps more important than the relationship between knowledge and efficacy beliefs, consider student C. Student C, whose performance in MAT0020 is less than desirable, continues to display a positive attitude toward mathematics and exhibits high self-efficacy beliefs. His beliefs and attitudes influenced his internal attributions for achievement. This will allow student C to improve his performance, average though it is, because he realizes that success is mostly due to diligent studying and practice on his part. His attitudes, efficacy beliefs, and attributions being good could ultimately be more important for student C than his grade in MAT0020. Perhaps instruction could be geared to take advantage of student C’s willing attitude, assigning him formative assessments in addition to the existing summative assessments in hopes that his mathematical domain knowledge will be increased.

To see that attitudes toward mathematics and mathematical self-efficacy beliefs play an indispensable role in the learning of mathematics, even when the motivation for a student to engage in mathematical learning is completely external, consider that all of the participants in this study were forced to enroll in a remedial algebra course. Despite this, most of them are learning mathematics, and doing so in a way that minimizes the fact that their motivations are almost certainly external. Of the four participants, only student D, who is performing rather poorly in MAT0020, indicated that the course was simply a means to the end of satisfying general education requirements. In his case, it is hoped that the negative effects of low efficacy beliefs, poor attitude toward mathematics, and limited domain knowledge can be reversed through careful instruction and alternative assessment. Suggestions for future study or for advancement of this study include working with a larger population, following these participants through their college mathematics courses to provide longevity, and perhaps studying the relationship that exists directly between mathematical domain knowledge and attributions for achievement.

## APPENDIX A
## INTERVIEW QUESTIONS

Interview questions designed to reveal attributions for successes and failures in mathematics among remedial college algebra students:

1. If I have performed well on a mathematics exam, it is because…

2. Lack of ability in the area of mathematics can cause…

3. Trying hard to solve mathematics problems will result in…

4. Factors which are out of my control where mathematics is concerned are…

5. Mathematicians most likely have success with mathematics because…

APPENDIX B
STUDENT SURVEY

1 Mathematics is an important field of study.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
2 I enjoy studying mathematics.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
3 A calculator is necessary when doing mathematics.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
4 Many mathematical concepts are too difficult to understand.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
5 Mathematics is not very useful in the real world.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
6 I often do not care whether I get math problems correct or incorrect.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
7 Mathematics is more something I have to do than something I choose to do.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
8 Mathematics is best left to mathematicians.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
9 I often think of other things while attempting to solve math problems.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
10 Knowing why mathematics works is as important as knowing how it works.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
11 Mathematics is a powerful tool.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
12 Performing mathematical computations makes me anxious.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
13 I experience great difficulty in learning mathematics.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
14 Solving math problems is easy for me.
( )strongly agree ( )agree ( )disagree ( )strongly disagree
15 I believe I will never be good at doing mathematics.
( )strongly agree ( )agree ( )disagree ( )strongly disagree

LIST OF REFERENCES

Ball, D. L., Hill, H. C., & Rowan, B. (2004). *Effects of teachers' mathematical knowledge for teaching on student achievement.* Presented at the 2004 annual meeting of the American Educational Research Association, San Diego, CA.

Bandura, A., Barbaranelli, C., Caprara, Gian V., & Pastorelli, C., (1996). Multi-faceted impact of self-efficacy beliefs on academic functioning. *Child Development* **67**, 1206-1222.

Bandura, A., & Locke, E. A., (2003). Negative self-efficacy and goal effects revisited. *Journal of Applied Psychology*, **88** # 1, 87-99.

Bandy, I. G. (1985). Ready or not, high school graduates are going to college. *NASSP Bulletin*, 87-89.

Brooks, D., & Schraw, G., website, (2004). Improving college teaching using an interactive, compensatory model of learning. Retrieved April 13, 2007, from http://dwbrr.unl.edu/iTech/TEAC859/Read/ICML.html

Butterfield, S. B., website, (1996). Attribution theory: the problem with external attributions. Retrieved April 13, 2007, from http://www.as.wvu.edu/~sbb/comm221/chapters/attrib.htm

Chang, P. T.(2002). The ostentatious world of algebra. University of Alaska Anchorage.

Ericsson, K. A., & Smith, J. (1991). Toward a general theory of expertise: Prospects and limits. Cambridge, England: Cambridge University Press.

Ericsson, K. A., Krampe, R. T., & Tesch-Römer, C. (1993). The Role of Deliberate Practice in the Acquisition of Expert Performance. Psychological Review, 100(3), 363-406.

Gredler, M.E. (1992). Learning and instruction: Theory into practice. (2nd ed.). New York:Macmillan

Hall, M. & Ponton, M., (2002). A comparative analysis of mathematics self-efficacy of developmental and non-developmental freshman mathematics students. Louisiana/Mississippi section of the *Mathematics Association of America*.(MAA) 2-3.

Kishor, N., & Ma, X., (1997). Assessing the relationship between attitude toward mathematics and achievement in mathematics: a meta-analysis. *Journal for Research in Mathematics Education*, **28** #1, 26-47.

Kloosterman, P. & Stage, F. K. (1995).

Gender, beliefs, and achievement in remedial college-level mathematics.*The Journal of Higher Education*, **66** #3, 294-311.

Ma, X., (1997). Reciprocal relationships between attitude toward mathematics and achievement in mathematics. *The Journal of Educational Research,* **90.**

Marsh, H. W., Miller, D., Pajares, F., Roche & Lawrence, A.,(1997). Item-specific efficacy judgments in mathematical problem solving: the downside of standing too close to trees in a forest. *Contemporary Educational Psychology* **22,** 363-377.

Mau, S. (1991). *Belifs of college-level-remedial mathematics students.* Paper presented at the annual AERA meeting in Chicago, April 1991; 25.

Pajares, F. & Miller, D., (1994). *Role of self-efficacy and self-concept beliefs in mathematical problem solving: a path analysis..*

Selden, A.& J., (1996). Of what does mathematical knowledge consist? *MAA Online*

Sorensen, C. T. & Hoyt, J. E. (1999). Promoting academic standards?: The link between remedial education in college and student preparation in high school. *Department of Institutional Research and Management Studies*, 1-2,5,7.

Zimmerman, B.J., & Paulsen, A.S. (1995). Self-monitoring during collegiate studying: an invaluable tool for academic self-regulation. *New diections in college teaching and learning: Understanding self-regulated learning,* **63**, 13-27.

Zimmerman, B.J., & Kitsantas, A. (1997). Developmental phases in self-regulation: Shifting from process to outcome goals. *Journal of Educational Psychology*, **89**, 29-36.

Zimmerman, B.J. (1998). Developing Self-Fulfilling Cycles of Academic Regulation: An Analysis of Exemplary Instructional Models. In D.H. Schunk & B.J. Zimmerman (eds.), *Self-regulated learning: from teaching to self-reflective practice* (pp.1-19). New York: Guilford Press.

## BIOGRAPHICAL SKETCH

Scott Murphy was born in Gainesville, Florida, in the late sixties. As a native son of the area, he realized that it does not get much better than this. Having never entertained the idea of going elsewhere to college, Scott obtained a Bachelor of Science in mathematics in the early nineties and a Master of Arts in Education, specializing in mathematics, in August 2007. Scott is a mathematics instructor at Santa Fe Community College in Gainesville and has nearly ten years of service to SFCC. While Scott's real passion in life is his family, which includes his wife Jessica and ten-year-old son Jordyn, mathematics runs a close second. Scott intends to remain in Gainesville, where his extended family also resides and where Gator sports events are held. Scott is dedicated to helping students who have trouble with mathematics overcome those issues and also enjoys teaching those who are motivated mathematicians.