THE IMPACT OF NONNORMALITY ON THE ASYMPTOTIC CONFIDENCE INTERVAL FOR AN EFFECT SIZE MEASURE IN MULTIPLE REGRESSION

By

LOU ANN MAZULA COOPER

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2007

ACKNOWLEDGMENTS

I would like to take this opportunity to thank the co-chairs of my dissertation committee, Dr. David Miller and Dr. James Algina. They are both exceptional teachers and without their guidance, support, and patience, this work would not have been possible. Dr. Miller, through his flexibility, approachable manner, and breadth of knowledge has been an invaluable resource throughout the course of my graduate studies. Dr. Algina is perhaps the most generous teacher I have ever known and his door was always open to me. His passion for research and his incredible work ethic had a profound influence on me.

I would also like to thank the other members of my committee for their help and encouragement. Special thanks go to Dr. Richard Davidson. I will always be grateful for the opportunity he provided to apply and expand my skills to experimental design, data analysis, and psychometric issues in medical education research. Thank you for giving me the most rewarding and stimulating job I have ever had. Dr. Walter Leite, although not there at the beginning, has provided a valuable sounding block and I look forward to our future collaborations.

I would also like to express my gratitude to my family for their love and encouragement during my mid-life career change. I know it has not always been easy to live with me. To my daughters, Abigail and Amanda, my hope is that I have provided a good example for you in pursuing my life-long love of learning. Finally, and most especially to Brian, my husband and best friend, whose love and unwavering belief in me made achieving this goal possible.

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Lou Ann Mazula Cooper

May 2007

Chair: M. David Miller
Co chair: James Algina
Major: Research and Evaluation Methodology

The increase in the squared multiple correlation coefficient, ΔR^2, associated with an individual predictor in a regression analysis is a measure commonly used to evaluate the importance of that variable in a multiple regression analysis. Previous research using multivariate normal data had shown that relatively large sample sizes are necessary for an acceptably accurate confidence interval for this regression effect size measure.

The coverage probability that an asymptotic confidence interval contained the population squared semipartial correlation, $\Delta\rho^2$, was investigated by simulating data from a range of nonnormal distributions such that (a) the predictors were nonnormal, (b) the error distribution was nonnormal, or (c) both predictors and errors were nonnormal. Additional factors manipulated included (a) the number of predictor variables, (b) the magnitude of the population squared multiple correlation coefficient in the original model, ρ_r^2, (c) the magnitude of the population squared semipartial correlation, $\Delta\rho^2$, and (d) sample size.

This study showed that when nonnormality is introduced, empirical coverage probability was always less than the nominal confidence level, often dramatically so. The degree of

nonnormality in the predictors was the most important factor influencing poor coverage probability. Although coverage probability increased as a function of sample size, when nonnormality in the predictors was substantial, the confidence interval is likely to be inaccurate no matter how large a sample size is used. With multivariate normal data, coverage probability improved as both ρ_r^2 and $\Delta\rho^2$ increased. When predictors are sampled from a nonnormal distribution, coverage probability tended to decrease as ρ_r^2 and $\Delta\rho^2$ increased and became even worse as the degree of nonnormality increased. It was further demonstrated that the asymptotic variance underestimates the sampling variance of ΔR^2. This produces standard errors that are too small and results in a confidence interval that is too narrow. Reliance on this confidence interval as a measure of the strength of the effect size will lead us to underestimate the importance of an individual predictor to the regression.

CHAPTER 1
INTRODUCTION

There is a growing consensus that the tradition of null hypothesis significance testing (NHST) has led to over-reliance on statistical significance in evaluating research results in the behavioral and social sciences. According to Cohen (1994), the biggest flaw in NHST is that it does not tell us what we want to know. A statistical test evaluates the probability of the sample results given the size of the sample assuming that the sample is drawn from a population where the null hypothesis is exactly true. In this framework, the outcome of a significance test is a dichotomous decision whether or not to reject the null hypothesis. As noted by Steiger and Fouladi (1997, p. 225), "this dichotomy is inherently dissatisfying to psychologists and educators, who frequently use the null hypothesis as a statement of no effect, and are more interested in knowing how big an effect is than whether it is (precisely) zero." Fundamentally, we are interested in determining how accurately the population effect has been estimated from the sample data and whether the observed effect size has practical significance. Statistical significance testing fails to provide the answers.

Within the behavioral and social sciences, methodological recommendations for reporting research results have increasingly emphasized the importance of reporting confidence intervals (Cumming & Finch, 2001; Smithson, 2001), effect sizes (Olejnik & Algina, 2002; Vacha-Hasse & Thompson, 2004), and confidence intervals for effect sizes (Cohen, 1990; Steiger & Fouladi, 1997; Thompson, 2002) to complement the results of hypothesis testing. Among the recommendations of the APA's Task Force on Statistical Inference (Wilkinson & Task Force on Statistical Inference, 1999) was a proposal to move away from routine reliance on NHST as a primary means of analyzing data to exploring, summarizing and analyzing data using visual representations, effect-size measures, and confidence intervals. The most recent edition of *The*

Publication Manual of the American Psychological Association (2001, p. 25-26) states, "For the reader to fully understand the importance of your findings, it is almost always necessary to include some index of effect size or strength of relationship in your Results section…The general principle to be followed, however, is to provide the reader not only with information about statistical significance but also with enough information to assess the magnitude of the observed effect or relationship." The *Manual* also states that failure to report an effect size is a "defect" (p. 5).

In 1996, Thompson recommended that American Educational Research Association (AERA) journals require that effect sizes be reported and interpreted in all studies. Ten years later the AERA Council recommends that statistical results should include an effect size measure as well as an indication of the uncertainty of that index of effect such as a confidence interval. The recently adopted Standards for Reporting on Empirical Social Science Research in AERA Publications (AERA, 2006) states that when quantitative methods are employed, "It is important to report the results of analyses that are critical for the interpretation of findings in ways that capture the magnitude as well as the significance of those results" (p. 37).

Editors of over 20 APA and other social science journals have published guidelines explicitly requiring authors to report effect sizes (Ellis, 2000; Harris, 2003; Heldref Foundation, 1997; Hresko, 2000; McLean & Kaufman, 2000; Royer, 2000; Snyder, 2000; Thompson, 1994; Vacha-Haase, Nilsson, Rentz, Lance, & Thompson, 2000) and the Editor of *Journal of Applied Psychology* requires an author to provide an explanation when an effect size is not reported (Murphy, 1997). Although this is evidence that editorial practices have evolved somewhat, effect size reporting is unlikely to become the norm until we move from recommendation and encouragement to requirement (Thompson, 1996; 1999).

Effect Sizes and Confidence Intervals

A confidence interval establishes a range of parameter values that are reasonably consistent with the data observed from a sample. Because a confidence interval gives a best point estimate of a parameter of interest and an interval about it reflecting an estimate of likely error, it contains all the information to be found in a significance test and more (Cohen, 1994). The likely range of the parameter values provides researchers with a better understanding of their data. If the parameter estimated has meaningful units, a confidence interval can be used to make statistical inferences that provide information in the same metric. According to Cumming and Finch (2001), there are four main reasons for promoting the use of confidence intervals: (a) they are readily interpretable, (b) are linked to familiar statistical tests, (c) can encourage replication and meta-analytic thinking, and (d) give information about precision.

The term effect size is broadly used to refer to any statistic that provides information that helps us judge the "practical significance" of the results of a study (Kirk, 1996). Cohen (1990) recommends that in addition to reporting an effect size, researchers should provide confidence intervals for effect sizes in order to gauge the possible range of values an effect size may assume. Absent a confidence interval, it is difficult to evaluate the accuracy of the effect size estimate. This, in turn, has implications for drawing meaningful conclusions.

Unfortunately, despite the increasing demand for researchers to do so, reporting effect sizes and confidence intervals has yet to become commonplace in educational and psychological journals. Vacha-Hasse, Nilsson, Rentz, Lance, and Thompson (2000) reviewed ten studies of effect size reporting in 23 journals, and found effect size(s) to be reported in roughly 10 to 50 percent of articles, notwithstanding the encouragement to do so from the fourth edition of the APA manual (1994). Empirical studies show that even when effect sizes are reported, interpretation is often given short shrift (Finch et al, 2002; Keselman et al., 1998).

It is likely that the emphasis on null hypothesis significance testing in graduate courses in statistics and research methodology has contributed to a general lack of knowledge concerning confidence intervals. Moreover, techniques for computing confidence intervals are often neglected in popular statistics textbooks and are not easily available in the statistical software that is routinely employed by applied researchers in the social sciences (Smithson, 2001). Even if these factors were not operating, researchers might be reluctant to report confidence intervals because as Steiger and Fouladi (1997, p. 228) observe, "interval estimates are sometimes embarrassing." Reporting confidence intervals can highlight the level of imprecision of statistical estimates and exposes the trivial nature of many published studies. Smithson (2001, p. 614) notes, "Almost any literature review or meta-analysis in psychology would give a very different impression from that conveyed by NHST if we routinely 'reconstructed' CIs for multiple R^2 and related GLM parameters."

Asymptotic Confidence Intervals for Correlations

A confidence interval establishes a range of hypothetical parameter values that cannot be ruled out given the observed sample data. The probability that the random interval includes, or covers, the true value of the parameter is the coverage probability of the interval. When the exact distribution of a statistic is known, the coverage is equal to the confidence level and the interval is said to be exact. A confidence interval is exact if it can be expected to contain a parameter's true value $100(1 - \alpha)\%$ of the time. Often exact intervals are not available or are difficult to calculate, and approximate intervals are used instead.

Confidence intervals are based on the sampling distribution of a statistic. Due to the central limit theorem, when sample size is sufficiently large, the sampling distribution of statistic will become more symmetric and eventually appear nearly normal, even when the population itself is not normally distributed. Methods based on asymptotic theory use approximations to the sampling variance of a statistic. If only the asymptotic distribution of the statistic is known, we can obtain an approximate confidence interval, which may or may not be reasonably accurate in finite samples. If the asymptotic confidence interval procedure is fully adequate, under repeated random sampling under identical conditions, a 95% confidence interval would contain the true population parameter 95% of the time. The accuracy of the approximation depends on whether there is a lack of bias and the degree to which the sampling distribution deviates from normality. If a statistic has no bias as an estimator of a parameter, its sampling distribution is centered at the true value of a parameter. An unbiased confidence interval is one where the probability of including any value other than the parameter's true value is less than or equal to $100(1 - \alpha)\%$. An interval is said to be conservative if the rate of coverage is greater than $100(1 - \alpha)\%$, the nominal confidence level. If the coverage probability is less than the nominal, the interval is said to be liberal. In general, conservative intervals are preferred over liberal ones (Smithson, 2003). Whenever a statistic based on asymptotic theory has poor finite sample properties, a confidence interval based on that statistic has poor coverage.

Multiple regression analysis is a common statistical application frequently used to predict a dependent variable (outcome) from two or more independent variables (predictors). The interpretation of results would be enhanced by the reporting of confidence intervals and effect sizes. The sample statistic, R^2, which estimates the proportion of variance in the dependent variable that is explained by the set of predictors, is commonly used to evaluate a multiple regression model. Published research studies frequently report R^2 values without any evidence of the precision with which they have been estimated. It is unfortunate that a confidence interval for the population parameter, ρ^2, is not computed by most popular statistical software packages.

Perhaps more significant, the topic is not even discussed in many applied or theoretical statistics texts.

In addition to the amount of variance explained by a given multiple regression model, researchers are often interested in evaluating the contribution that one variable makes to the regression, over and above a set of other explanatory variables. The increase in R^2, ΔR^2, when a variable (X_j) is added to a multiple regression model is a useful measure of the strength of the relationship between X_j and the dependent variable, Y, controlling for all other independent variables in the model. The change in R^2 that we observe by including each new X_j in the regression equation is the squared semipartial correlation corresponding to a given regression coefficient. Typically, whether X_j has made a statistically significant contribution to predicting Y is tested by conducting a t- or F-test on that regression coefficient. But, the squared semipartial correlation itself is a useful measure of effect size and as recommended by Cohen (1990) and Thompson (2002), we should calculate a confidence interval to evaluate the precision with which it has been estimated and the range of likely values.

Hedges and Olkin (1981) presented procedures for constructing a confidence interval for the squared semipartial correlation based on calculating the asymptotic covariance matrix for commonality components. Commonality analysis is a procedure by which the variance accounted for in the criterion is partitioned into two parts, the unique part and the common part. The unique part is attributable to the predictors individually. This is essentially the partial contribution of each predictor to the squared multiple correlation with the criterion. The second part is the common part, attributable to a combination of the predictors, which is the contribution to the multiple correlation with the criterion that all of the predictors in the combination share.

Thus, commonality analysis is a way to measure the importance of variables through the use of partial correlations.

Hedges and Olkin's results can be used to construct a confidence interval for ΔR^2. Olkin and Finn (1995) derived explicit expressions for asymptotic (large-sample) confidence intervals for functions of simple, partial, and multiple correlations. Since the focus of this study is on the squared semipartial correlation, the following discussion will be limited to Olkin and Finn's Model A (p. 157-159). Model A is the special case for use in determining whether an additional variable provides an improvement in predicting the criterion.

All of the procedures for comparing two sample correlation coefficients or two sample squared correlation coefficients described by Olkin and Finn have the same general form. Let r_A and r_B be the two sample correlations to be compared and ρ_A and ρ_B denote their corresponding population values. The large-sample distributional form for the difference in two correlations is

$$\left[(r_A - r_B) - (\rho_A - \rho_B)\right] \sim N\left(0, \sigma_\infty^2\right) \tag{1.1}$$

where

$$\sigma_\infty^2 = \operatorname{var}(r_A) + \operatorname{var}(r_B) - 2\operatorname{cov}(r_A, r_B) \tag{1.2}$$

is the asymptotic variance of the difference of the two correlation coefficients; σ_∞^2 is dependent on the population correlations (Olkin & Finn, 1995, p. 156).

When squared correlation coefficients are compared, the expressions in Equations 1.1 and 1.2 become

$$[(r_A^2 - r_B^2) - (\rho_A^2 - \rho_B^2)] \sim N(0, \sigma_\infty^2) \tag{1.3}$$

and

$$\sigma_\infty^2 = \operatorname{var}\left(r_A^2 - r_B^2\right) - \left(\rho_A^2 - \rho_B^2\right) - 2\operatorname{cov}\left(r_A^2, r_B^2\right). \tag{1.4}$$

Olkin and Finn present the general form for the large-sample variance of functions of correlations

$$\sigma^2_\infty f(r_{ij}, r_{ik}, r_{jk}) = \mathbf{a}\Phi\mathbf{a}' \quad (1.5)$$

specialized to a function of three correlations, r_{ij}, r_{ik}, and r_{jk} where $f(\ )$ is a function of the correlations, **Φ** is the sampling variance-covariance matrix of the correlations, and vector **a** contains a set of coefficients that depend on the function of the correlations to be evaluated. The variance of sample correlation r_{ij} is

$$\text{var}(r_{ij}) = (1-\rho^2_{ij})^2 / n \quad (1.6)$$

and the covariance of two correlations is

$$\begin{aligned}\text{cov}(r_{ij}, r_{ik}) = \frac{1}{2}\big[&\rho_{ij}\rho_{kl}(\rho^2_{ik} + \rho^2_{il} + \rho^2_{jk} + \rho^2_{jl}) + \rho_{ik}\rho_{jl} + \rho_{il}\rho_{jk} - \\ &(\rho_{ij}\rho_{ik}\rho_{il} + \rho_{ji}\rho_{jk}\rho_{jl} + \rho_{ki}\rho_{kj}\rho_{kl} + \rho_{li}\rho_{lj}\rho_{lk})\big] / n.\end{aligned} \quad (1.7)$$

When two correlations have one variable in common, Equation 1.7 simplifies to

$$\text{cov}(r_{ij}, r_{ik}) = \left[\tfrac{1}{2}(2\rho_{jk} - \rho_{ij}\rho_{ik})(1-\rho^2_{ij} - \rho^2_{ik} - \rho^2_{jk}) + \rho^3_{jk}\right] / n. \quad (1.8)$$

Large-sample estimates are obtained by replacing the population parameters with values computed from sample data. Using the delta method, it can be shown that if $f(r_{ij}, r_{ik}. r_{jk})$ is a function of the three correlations, then the vector **a** consists of the partial derivatives

$$\mathbf{a} = \left(\frac{\partial f}{\partial r_{12}}, \frac{\partial f}{\partial r_{13}}, \frac{\partial f}{\partial r_{23}}\right). \quad (1.9)$$

In the simplest case, suppose that two variables X_1 and X_2 are used to predict a third variable, X_0. In order to determine whether X_2, makes a significant contribution to the regression, we are interested in the difference, $R^2_{0(12)} - r^2_{01}$. Here, we use a capital "R" to signify a multiple correlation rather than a bivariate correlation, denoted by a lower case "r". The symbol $R^2_{0(12)}$

denotes the squared multiple correlation between X_0, X_1 and X_2, which is a function of the correlations among the variables r_{01}, r_{02}, and r_{12} given by

$$R^2_{0(12)} = \hat{\rho}^2_{0(12)} = \frac{r^2_{01} + r^2_{02} - 2r_{01}r_{02}r_{12}}{1 - r^2_{12}}. \tag{1.10}$$

The squared correlation between X_0 and X_1 is represented by r^2_{01}. Therefore, a confidence interval for $R^2_{0(12)} - r^2_{01}$ can be computed using Olkin and Finn's results for comparing two squared multiple correlation coefficients. In order to compare the population squared multiple correlations $\rho^2_{0(12)}$ and ρ^2_{01}, we use the estimates $R^2_{0(12)}$, r^2_{01}, and $\hat{\sigma}^2_{\infty}$, the estimated variance of the difference $R^2_{0(12)}$ - r^2_{01} where

$$\mathrm{var}(R^2_{0(12)} - r^2_{01}) = \mathbf{a\Phi a'}. \tag{1.11}$$

The upper triangular of the symmetric population correlation matrix is

$$\mathbf{P} = \begin{pmatrix} 1 & \rho_{01} & \rho_{02} \\ & 1 & \rho_{12} \\ & & 1 \end{pmatrix} \tag{1.12}$$

and the elements of the vector, **a**, are

$$a_1 = \frac{2\rho_{12}}{1-\rho^2_{12}}(\rho_{01}\rho_{12} - \rho_{02}), \tag{1.13}$$

$$a_2 = \frac{2}{1-\rho^2_{12}}(\rho_{02} - \rho_{01}\rho_{12}), \tag{1.14}$$

$$a_3 = \frac{2\rho}{(1-\rho^2_{12})^2}(\rho_{12}\rho^2_{01} + \rho_{12}\rho^2_{02} - \rho_{01}\rho_{02} - \rho_{01}\rho_{02}\rho^2_{12}). \tag{1.15}$$

The variance-covariance matrix for the sample correlations is

$$\begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ & \phi_{22} & \phi_{23} \\ & & \phi_{33} \end{pmatrix} = \begin{pmatrix} \mathrm{var}(r_{01}) & \mathrm{cov}(r_{01}, r_{02}) & \mathrm{cov}(r_{01}, r_{12}) \\ & \mathrm{var}(r_{02}) & \mathrm{cov}(r_{02}, r_{12}) \\ & & \mathrm{var}(r_{12}) \end{pmatrix}. \tag{1.16}$$

The sample correlation matrix, ***R***, estimates **P** and the sample values in ***R*** can be used to compute the elements of **a.**

Because the calculation of analytic derivatives becomes increasingly complicated as the number of variables increases, Olkin and Finn illustrated their method for a multiple regression model with no more than two predictors. Graf and Alf (1999) expanded Olkin and Finn's procedures to more general forms. Graf and Alf substituted numerical derivatives and offered two BASIC programs for calculating asymptotic confidence limits on the difference between two squared multiple correlations and the difference between two partial correlations. These programs, REDUX-AB, to compare two multiple correlations, and REDUX-CD, to compare two partial correlations, compute the **Φ** matrix, the partial derivatives in vector **a**, and a 95% confidence interval.

Alf and Graf (1999) present a further simplification that does not employ numerical derivatives, is less computationally demanding, and produces results equivalent to the method described by Olkin and Finn. All computations are based on sample estimates. The problem is approached by representing a multiple correlation as a zero-order correlation between the outcome variable and another single variable that is a weighted sum of the predictors. Alf and Graf defined

$$r_{AB} = \frac{r_{0B}}{r_{0A}} \tag{1.17}$$

where the subscripts A and B denote weighted sums of two sets of predictors and r_{AB} is the correlation between the two composite variables.

The confidence interval for the squared semipartial correlation coefficient is determined by the special case in which one set of predictors is a proper subset of the predictors in the other correlation. The two squared multiple correlations are computed using the same sample and the variables in the reduced model are a subset of the variables in the full model. Let ρ_r^2 and ρ_f^2 denote the population squared multiple correlation coefficients corresponding to R_r^2 and R_f^2. The subscript, *f*, refers to the "full" model with all predictors; the subscript, *r*, refers to the "reduced" model. The reduced model contains all predictors with the exception of the variable of interest. The asymptotic variance of R_f^2 is

$$Var(R_f^2) = \frac{4\rho_f^2\left(1-\rho_f^2\right)}{n}. \tag{1.18}$$

The asymptotic variance of R_r^2 is

$$Var(R_r^2) = \frac{4\rho_r^2\left(1-\rho_r^2\right)}{n}. \tag{1.19}$$

The asymptotic covariance between R_r^2 and R_f^2 is

$$Cov(R_f^2, R_r^2) = \frac{4\rho_f\rho_r\left[.5\left(2\rho_r/\rho_f - \rho_f\rho_r\right)\left(1-\rho_f^2-\rho_r^2-\rho_r^2/\rho_f^2\right)+\rho_r^3/\rho_f^3\right]}{n}. \tag{1.20}$$

For the squared semipartial correlation, let $\Delta R^2 = R_f^2 - R_r^2$. The asymptotic variance of ΔR^2 is

$$\sigma_\infty^2 = Var(R_f^2) + Var(R_r^2) - 2Cov(R_f^2, R_r^2). \tag{1.21}$$

An asymptotically correct 100(1 - α)% confidence interval for $\Delta\rho^2 = \rho_f^2 - \rho_r^2$ is

$$\Delta R^2 \pm z_{\alpha/2}\hat{\sigma}_\infty \tag{1.22}$$

where $z_{\alpha/2}$ is the (1 - α/2)th percentile of the standard normal distribution and $\hat{\sigma}_\infty$ is the estimate of σ_∞. In practice, the large-sample variance is estimated by substituting R_f^2 for ρ_f^2 and R_r^2 for ρ_r^2 in Equations 1.18, 1.19, and 1.20.

Equations 1.18 and 1.19 are problematic when the population squared multiple correlations are zero because the implication is that the sampling variance of R^2 is also zero (Stuart, Ord, &

Arnold, 1999). Similarly, Equation 1.20 implies that the sampling covariance is zero if either population multiple correlation coefficient, ρ_f or ρ_r, is zero. If it were known that both ρ_f^2 and ρ_r^2 were zero and these values were used to construct a confidence interval, we would incorrectly conclude that the width of the resulting interval is zero. This computational problem is unlikely to occur in practice since we substitute sample multiple correlation coefficients for their population values and it is doubtful that either R_f^2 or R_r^2 will ever be exactly zero.

The Alf and Graf formulas rely on asymptotic results. As such, they are only exactly correct for infinitely large samples. Thus, the accuracy of this approximation is heavily dependent on sample size. Alf and Graf (1999, p.74) concluded that "the correlation between two multiple correlations will be extremely high when the variables in one multiple correlation are a subset of the variables in another multiple correlation" and to ensure that coverage probability is equal to the nominal for the confidence interval on $\Delta\rho^2$, "moderately large to large" sample sizes are necessary.

In the absence of more specific recommendations on sample sizes, Algina and Moulder (2001) conducted a simulation study to evaluate the empirical probability that the interval in Equation 1.22 includes $\Delta\rho^2$ for 95% confidence interval. Algina and Moulder manipulated ρ_f^2, ρ_r^2, the number of predictors in the model (*k*), and the sample size (*n*). When the data are distributed multivariate normal, results indicate that when $\Delta\rho^2 > 0$, for sample sizes representative of those used in psychology (i.e., $n \leq 600$), coverage probabilities for a nominal 95% confidence interval were less than .95. This tends to be true even with relatively large sample sizes, i.e. between 600 and 1200.

When $\rho_f^2 - \rho_r^2 = 0$ all coverage probabilities were at least .999 for all sample sizes studied. That is, when ρ^2 does not increase when a predictor is added to a multiple regression model, the confidence interval is always too wide. Algina and Moulder (2001) posited two reasons for this defect in the confidence interval: (a) for all conditions in which $\rho_f^2 - \rho_r^2 = 0$ the asymptotic variance overestimated the sampling variance and (b) the distribution of $R_f^2 - R_r^2$ is positively skewed with a lower limit of 0. Because the confidence interval does not take this lower limit into account, even if the asymptotic variance was not overestimated, the lower limit would tend to be smaller than zero.

Algina and Moulder (2001) showed that coverage probability tends to increase as ρ_r^2 increases and as $\Delta\rho^2$ increases and tends to decrease as the number of predictors increases. Further, when the interval does not contain $\Delta\rho^2$, there is a tendency for the interval to be entirely below $\Delta\rho^2$. Algina and Moulder conclude that using the Alf and Graf method to compute a confidence interval with an inadequate sample size will underestimate the strength of the relationship between the predictor and the outcome variable.

The Impact of Nonnormality on Statistical Estimates

Every procedure used to make statistical inferences is based on a set of core assumptions. If the assumptions are met, the test will perform as theorized. However, the results may be misleading when the assumptions are violated. The most common method for estimating regression coefficients is ordinary least squares (OLS). Ordinary least squares yields unbiased, efficient, and normally distributed estimates when the following conditions are met: (a) No measurement error; (2) the mean of the residuals is zero; (3) the residuals have constant variance; (4) the residuals are not inter-correlated; and (5) the residuals are normally distributed.

In terms of power and accurate probability coverage, standard analysis of variance (ANOVA) and regression methods are affected by arbitrarily small departures from normality. As early as 1960, Tukey found that nonnormality could have a sizeable impact on power and measures of effect size could be misleading whenever means are being compared. By sampling from a contaminated normal distribution, Tukey showed that classical estimators are quite sensitive to distributions with heavy tails. The contaminated normal distribution is a mixture of two normal distributions, one of which has a large variance; the other distribution is standard normal. This results in a distribution with heavier tails than the Gaussian. Heavy-tailed distributions are characterized by unusually large or small values. Both heavy-tailed and skewed distributions are commonplace in applied work (Micceri, 1989). The presence of these characteristics in the data can "diminish the chances of detecting true associations among random variables and obtaining accurate confidence intervals for the parameters of interest" (Wilcox, 1998).

After reviewing over 400 large data sets from educational and psychological research, Micceri (1989) found the majority did not follow univariate normal distributions. Approximately two-thirds of ability measures and over 80% of the psychometric measures examined exhibited at least moderate asymmetry. For all data sets studied, 31% of the distributions showed skewness, γ_1, greater than .70 and 52% of psychometric measures demonstrated extreme to exponential asymmetry, $\gamma_1 > 2.00$. Psychometric measures also exhibited heavier tails than ability measures. Kurtosis estimates ranged from –1.70 to 37.37. To put this in some perspective, the kurtosis for the double exponential distribution is 3.0.

Breckler (1990) considered 72 articles in personality and social psychology journals and found that in analyses relying on the assumption of multivariate normality, only 19% of authors acknowledged this assumption and less than 10% considered whether it had been violated. Keselman and his colleagues (1998) reviewed articles in prominent educational and behavioral sciences research journals published during 1994 and 1995 and concluded (a) the majority of researchers conduct statistical analyses without considering the distributional assumptions of the tests they are using and therefore use analyses that are not robust; (b) researchers rarely reported effect sizes; and (c) researchers failed to perform power analyses in order to inform sample size decisions.

Statement of the Problem

Methods for constructing confidence intervals based on asymptotic theory, such as those proposed by Olkin and Finn and Alf and Graf, have the potential to be very attractive to applied researchers. In the case of the equations presented by Alf and Graf, a hand calculator can be used to compute a confidence interval using the appropriate estimates from the results of data analysis obtained using standard statistical analysis software. However, as Algina and Moulder demonstrated, even under the best case scenario, where data are drawn from a multivariate normal distribution, the coverage probability of the asymptotic confidence interval for $\Delta\rho^2$ is less than optimal, and when sample size is relatively small, e.g., < 200, would be considered unacceptable by most researchers. Since multivariate normal data is rare, the performance of Alf and Graf's procedure under "real world" conditions warrants further investigation.

Purpose of the Study

My dissertation will extend the work of Algina and Moulder (2001) and investigate the effect of the magnitude of population squared multiple correlation coefficients, ρ_r^2 and ρ_f^2, as well as the number of predictors, on the asymptotic confidence interval for $\Delta\rho^2$ under a range of nonnormal conditions. The study will investigate coverage probability when (a) the predictor variables are not distributed multivariate normal; (b) the residuals are not normal; and (c) both predictors and residuals are nonnormal. Empirical coverage probabilities will be compared to nominal coverage probabilities over a wide range of sample sizes. My research will address the following questions:

- How adequate is Alf and Graf's asymptotic confidence interval procedure for the squared semipartial correlation coefficient when used with sample sizes typically employed in research in education, psychology and the behavioral sciences under conditions of nonnormality?

- Is there a minimum sample size for which this method meets established standards for accuracy over a wide range of situations such that recommendations can be made for the use of this procedure in reporting the results of applied research?

CHAPTER 2
METHODS

In conducting a simulation study, especially when the goal is to inform the practice of researchers, it is important to ensure that the relevant factors are manipulated and that the levels of these factors reflect those routinely observed. To that end, six factors were manipulated in a factorial design using values typical of those observed in applied research: the number of predictors, the size of the squared multiple correlation in the reduced model, the size of the squared semipartial correlation, sample size, the distribution for the predictors, and the distribution for the error. These factors, and the levels of these factors, are detailed in Table 2-1.

Study Design

Number of predictors

Algina, Moulder, and Moser (2002) examined sample size requirements for accurate estimation of squared semipartial correlation coefficients and found a modest effect on the distribution of ΔR^2 due to the number of predictors included in the multiple regression model. Therefore, it follows that the sample size required for the confidence interval on $\Delta\rho^2$ to be robust, i.e. to have the coverage probability equal to the nominal confidence level, will likewise depend on the number of predictors. The number of predictors in the initial set of predictors ($k - 1$) ranged from 2 to 10 in increments of 2. This allowed investigation of the performance of the asymptotic confidence interval for a reasonable range of model sizes.

Squared multiple correlations

Algina, Moulder, and Moser also showed that the sampling distribution of ΔR^2 strongly depends on the population squared multiple correlations in both the full and reduced models, ρ_f^2 and ρ_r^2. Based on a survey of all APA journal articles published in 1992 reporting multiple regression results, Jaccard and Wan (1995) found the median squared multiple correlation in these studies to be .30. The 75th percentile for squared multiple correlations was approximately .50. Based on these results, the values for the squared multiple correlation coefficients for the predictors in the initial set (ρ_r^2) ranged from .00 to .60 in steps of .10 (7 levels of the factor). Cohen (1988) proposed, as a convention, that .02, .13, and .26 represent small, medium, and large effect sizes for squared semipartial correlations. By manipulating the squared multiple correlation coefficient for the entire set of predictors (ρ_f^2), such that it ranged from ρ_r^2 to ρ_r^2 + .30 in steps of .05, values for $\Delta\rho^2$ that ranged from .00 to .30 in steps of .05 were produced (7 levels of the factor). The values for $\Delta\rho^2$ are reasonably representative of likely effect sizes and the values selected for ρ_r^2 and ρ_f^2 cover a comprehensive range of population squared multiple correlations for multiple regression models from $\rho^2 = .00$ to $\rho^2 = .90$.

Sample size

Jaccard and Wan also reported typical sample sizes for studies using regression analysis. The median sample size was 175; a sample size of 400 was at the 75th percentile. However, Algina and Moulder found with multivariate normal data empirical estimates of the coverage probability were smaller than .95 even with a sample size as large as 1200. Since we expected empirical coverage probabilities to be worse for nonnormal data, larger sample sizes than are usually observed in psychological research were included. Sample size ranged from 100 to 1000 in steps of 100 and from 1000 to 2000 in steps of 250 (14 levels of the factor).

Distributions

The distributions chosen for study represent varying levels of nonnormality and were selected to: (a) allow examination of the effects of skewness and kurtosis; and (b) be representative of the types of univariate nonnormality commonly encountered in applied research in education and psychology. The method described in Hoaglin (1985) and Martinez and Iglewicz (1984) using the *g*-and-*h* distributions was used to generate data that is characterized by varying degrees of skewness (γ_1) and kurtosis (γ_2). A *g*-and-*h* distribution is generated by a single transformation of the standard normal distribution and allows for asymmetry and a variety of tail weights. In the case of the standard normal distribution, $g = h = 0$ and $\gamma_1 = \gamma_2 = 0$. When $g = 0$, a distribution is symmetric. Distributions with positive skew typically have $\gamma_1 > 0$ and in distributions with negative skew, $\gamma_1 < 0$. The tails of the distribution become heavier as *h* increases in value. Long-tailed distributions, such as the *t*-distribution, are characterized by $\gamma_2 > 0$. Short-tailed distributions, such as the uniform distribution, have $\gamma_2 < 0$.

The distributions selected for this study and their skewness and kurtosis are presented in Table 2-1. Distribution 1 is the multivariate normal case. Distribution 2 is symmetric and long-tailed and has the same skew and kurtosis as a *t*-distribution with 10 degrees of freedom. Distribution 3 is both asymmetric and leptokurtotic with the same skew and kurtosis as a χ^2 distribution with 10 degrees of freedom. Since distributions 2 and 3 have similar kurtosis, but differ with respect to asymmetry, this allowed us to evaluate the relative importance of skewness and kurtosis on the coverage probability of the confidence interval. Distribution 4 has the same skew and kurtosis as χ^2_4. Distribution 5 is extremely skewed with heavy tails and has skew and kurtosis equal to the exponential distribution. Nonnormality was manipulated in either (a) the predictors, (b) the residuals, or (c) in both the predictors and the residuals.

The error distribution is a univariate distribution. The empirical cumulative distribution functions for the four nonnormal distributions selected for this study, generated by sampling 1,000,000 random variates from each *g*-and-*h* distribution, are depicted in Figures 2-1 to 2-4. In addition, the deviation from normality is shown by including the normal curve with mean equal to μ_{gh} and standard deviation equal to σ_{gh} for each distribution. The population mean and standard deviation for each *g*-and-*h* distribution were calculated using the formulas given by Hoaglin (1985, p. 502-503).

In multiple regression, the predictors are multivariate. Multivariate normality, however, is a stronger assumption than univariate normality. Univariate normality of each of the variables is necessary, but not sufficient, and a nonnormal multivariate distribution can have normal marginals. Therefore, a preliminary step in evaluating multivariate normality is to study the reasonableness of assuming marginal normality for the observations on each of the variables (Gnanadesikan, 1997). In addition to graphical approaches, a common method for evaluating the normality of univariate observations is by means of skewness and kurtosis coefficients, $\sqrt{b_1}$ and b_2:

$$\sqrt{b_1} = \frac{\sqrt{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^3}{\left[\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2\right]^{3/2}} \tag{2.1}$$

and

$$b_2 = \frac{n\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^4}{\left[\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2\right]^{2}}. \tag{2.2}$$

These are sample estimates of the population skewness and kurtosis parameters $\sqrt{\beta_1}$ and β_2, respectively. When the population is normal, $\sqrt{\beta_1} = 0$ and $\beta_2 = 3$. If $\beta_2 < 3$, there is negative kurtosis; if $\beta_2 > 3$, there is positive kurtosis. Population skewness and kurtosis are also commonly described by γ_1 and γ_2 (Hoaglin, 1985) where

$$\gamma_1 = \sqrt{\beta_1} \tag{2.3}$$

and

$$\gamma_2 = \beta_2 - 3. \tag{2.4}$$

Mardia (1970) proposed indices for assessing multivariate normality that are generalizations of the univariate skewness and kurtosis measures $\sqrt{b_1}$ and b_2. Let $X_1,\ldots,X_n$ be a random sample from a population with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. The sample mean vector and covariance matrix are denoted by $\overline{X}$ and **S**, respectively. The skewness and kurtosis, $\beta_{1,k}$ and $\beta_{2,k}$, for a multivariate population, as defined by Mardia, are

$$\beta_{1,k} = E\left[\left(x_i - \mu\right)' \Sigma^{-1} \left(x_j - \mu\right)\right]^3 \tag{2.5}$$

and

$$\beta_{2,k} = E\left[\left(x_i - \mu\right)' \Sigma^{-1} \left(x_j - \mu\right)\right]^2. \tag{2.6}$$

According to Rencher (1995), since third order central moments for the multivariate normal distribution are zero, $\beta_{1,k} = 0$ when $\mathbf{X} \sim N(\boldsymbol{\mu},\boldsymbol{\Sigma})$. Furthermore, it can be shown that for multivariate normal **X**

$$\beta_{2,k} = k(k+2) \tag{2.7}$$

where k is equal to the number of variables. Sample estimates of $\beta_{1,k}$ and $\beta_{2,k}$ are given by

$$b_{1,k} = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left[(X_i - \overline{X})'S^{-1}\left(X_j - \overline{X}\right)\right]^3 \tag{2.8}$$

and

$$b_{2,k} = \frac{1}{n}\sum_{i}\left[(X_i - \bar{X})' S^{-1}(X_j - \bar{X})\right]^2. \tag{2.9}$$

Multivariate skewness and kurtosis were calculated by simulating 1,000,000 random variates sampled from each *g*-and-*h* distribution for each level of *k* under investigation and then applying equations 2.8 and 2.9 to obtain estimates of Mardia's multivariate measures, $b_{1,k}$ and $b_{2,k}$. The SAS program used to estimate these indices is included in Appendix A. Mardia's multivariate skewness estimates are presented in Table 2-2 and Table 2-3 presents Mardia's multivariate kurtosis estimates. Figures 2-5 and 2-6 are graphic presentations that compare the coefficients for the nonnormal distributions to the values expected under multivariate normality for the number of predictors under investigation in this study.

The design for the study is a 5 (data generating distribution for the predictors) × 5 (data generating distribution for the errors) × 7 (ρ_r^2) × 7 ($\Delta\rho^2$) × 5 (*k*) × 14 (*n*) fully crossed factorial. This resulted in a total of 85,750 unique conditions. Each combination of factors was replicated 10,000 times and for each replication, a 95% confidence interval was constructed using the Alf and Graf method.

Background and Theoretical Justification for the Simulation Method

The multiple regression model can be written as

$$Y_j = \beta_0 + \beta_1 X_{1j} + \beta_2 X_{2j} + \ldots + \beta_k X_{kj} + \varepsilon_j. \tag{2.10}$$

In the standardized multiple regression model, in the population with $k \geq 1$ predictors and one criterion, all variables are standardized to mean zero and unit variance so an intercept is not needed. This model is

$$Y_j = \beta_1 X_{1j} + \beta_2 X_{2j} + \ldots + \beta_k X_{kj} + \varepsilon_j = \sum_{i=1}^{k} \beta_i X_{ij} + \varepsilon_j \tag{2.11}$$

where β_i is the population standardized regression coefficient associated with the ith predictor; $e_{ij} \sim N(0,\sigma^2)$; $i = 1, \ldots, k$; $j = 1, \ldots, n$. Assuming that we are operating on the population and that the model is correct, predicted values are given by

$$\hat{Y}_j = \sum_{i=1}^{k} \beta_i X_{ij} \tag{2.12}$$

and the squared correlation between the observed (Y) and the predicted ($\hat{Y}$) values is denoted as $\rho^2_{Y\hat{Y}}$. In the sample, this is estimated by R^2. When the predictors are uncorrelated, the sum of the squared correlations is equal to the variation accounted for by all the predictors

$$\sum_{i=1}^{k} \rho^2_{YX_i} = \rho^2_{Y\hat{Y}}. \tag{2.13}$$

A simplifying transformation (Browne, 1975) holds that for any set of predictors that has a squared multiple correlation, ρ^2, with Y, it is always possible to transform the predictors so that (a) the transformed predictors are mutually uncorrelated, (b) have unit variance, and (c) the regression coefficients are equal to any set of values such that

$$\sum_{j=1}^{k} \beta_j^2 = \sigma_y^2 \rho^2. \tag{2.14}$$

The quantity $\Delta\rho^2$ is a function of the elements of the covariance matrix for the predictors and the criterion.

In order to illustrate the application of Browne's results to the current simulation, let $\mathbf{x}^\dagger$ denote the vector of standardized predictor variables, with $k \times k$ correlation matrix $\mathbf{P}$ and $k \times 1$ vector of correlation coefficients $\boldsymbol{\rho}$ between the predictors and the criterion variable, $\mathbf{y}$. The squared multiple correlation coefficient for all k variables is denoted by ρ_f^2 and for the first $k-1$ variables is denoted by ρ_r^2. We seek a transformation of the predictors to $\mathbf{x}$ such that the new variables are standardized and uncorrelated, and the regression coefficients relating **y** to the variables in $\mathbf{x}^{\dagger}$ are $\beta_i = 0$ for the first $k-2$ variables and $\beta_{k-1} = \sqrt{\rho_r^2}$ and $\beta_k = \sqrt{\rho_f^2 - \rho_r^2}$, for the last two variables, respectively.

The transformation can be constructed in two steps. It is well known that the variables in the vector $\tilde{\mathbf{x}} = \mathbf{A}\mathbf{x}^{\dagger}$, where **A** is a $k \times k$ matrix, will be uncorrelated dependent on an appropriate choice of **A.** For example, **A** can be selected as the inverse of the left Cholesky factor of $\mathbf{R}$ $\left(\text{i.e., } \mathbf{R} = \mathbf{A}^{-1}\mathbf{A}^{-\mathrm{T}}, \text{ where } \mathbf{A}^{-\mathrm{T}} \text{ indicates the inverse of } \mathbf{A}'\right)$. The vector of correlation coefficients between the transformed predictors and the criterion is $\mathbf{A\rho}$ and because the transformed variables are uncorrelated, $\tilde{\boldsymbol{\beta}} = \mathbf{A\rho}$ is the vector of regression coefficients relating the criterion variable to the variables in $\tilde{\mathbf{x}}$. Because the criterion is a standardized variable and $\tilde{\mathbf{x}} = \mathbf{Ax}$ is a nonsingular transformation, ρ_f^2 is unchanged by the transformation, and $\rho_f^2 = \tilde{\boldsymbol{\beta}}'\tilde{\boldsymbol{\beta}}$. We next seek a transformation $\mathbf{x} = \mathbf{T}'\tilde{\mathbf{x}}$, where $\mathbf{T}'$ is $k \times k$, such that the variables in $\mathbf{x}$ are standardized and uncorrelated and so that the regression coefficients for the variables in $\mathbf{x}$ are $\beta_i = 0$ for the first $k-2$ variables and $\beta_{k-1} = \sqrt{\rho_r^2}$ and $\beta_k = \sqrt{\rho_f^2 - \rho_r^2}$ for the last two variables, respectively. We see that $\boldsymbol{\beta}'\boldsymbol{\beta} = \rho_f^2$. Because the variables in $\tilde{\mathbf{x}}$ are standardized and uncorrelated, the matrix $\mathbf{T}'$ must be orthogonal so that the variables in $\mathbf{x}$ will be standardized and uncorrelated. With an orthogonal transformation, $\boldsymbol{\beta} = \mathbf{T}\tilde{\boldsymbol{\beta}}$. The matrix **T** can be constructed as follows (M. W. Browne, personal communication with J. Algina, 1999):

Let $\mathbf{u} = \tilde{\boldsymbol{\beta}} - \boldsymbol{\beta}$. Then, $\mathbf{T} = \mathbf{I} - 2\mathbf{u}\left(\mathbf{u}'\mathbf{u}\right)^{-1}\mathbf{u}'$ is an orthogonal matrix, and $\boldsymbol{\beta} = \mathbf{T}\tilde{\boldsymbol{\beta}}$. Because $\tilde{\boldsymbol{\beta}}'\tilde{\boldsymbol{\beta}} = \boldsymbol{\beta}'\boldsymbol{\beta}$, $\dfrac{2\mathbf{u}'\tilde{\boldsymbol{\beta}}}{\mathbf{u}'\mathbf{u}} = 1$, and $\boldsymbol{\beta} = \mathbf{T}\tilde{\boldsymbol{\beta}}$ it follows that $\mathbf{T}\tilde{\boldsymbol{\beta}} = \tilde{\boldsymbol{\beta}} - \dfrac{2\mathbf{u}'\tilde{\boldsymbol{\beta}}}{\mathbf{u}'\mathbf{u}}\left(\tilde{\boldsymbol{\beta}} - \boldsymbol{\beta}\right)$. Thus, if the variables in

$\mathbf{x}^{\dagger}$ are transformed to $\mathbf{x} = \mathbf{T}'\mathbf{A}\mathbf{x}^{\dagger}$, with $\mathbf{T}'$ and $\mathbf{A}$ defined as above, the transformed variables will be uncorrelated and standardized and the regression coefficients will be $\beta_{\iota} = 0$ for the first $k - 2$ variables, and $\beta_{k-1} = \sqrt{\rho_r^2}$ and $\beta_k = \sqrt{\rho_f^2 - \rho_r^2}$ for the last two variables, respectively. Because the variables are standardized and uncorrelated, the squared multiple correlation coefficient for the first $k - 1$ variables will be $\sum_{i-1}^{k-1} \beta_i = \rho_r^2$ and the squared multiple correlation coefficient for all k variables will be $\sum_{i-1}^{k} \beta_i^{\,2} = \rho_r^2 + \rho_f^2 - \rho_r^2 = \rho_f^2$.

The implication of Browne's result is that if the predictors are correlated, they can be transformed so that (a) the predictors are uncorrelated, (b) the predictive power of $k - 1$ of the predictors is channeled into one of the transformed predictors, (c) the predictive power of the remaining predictor is channeled into another of the transformed predictors, and (d) the remaining $k - 2$ predictors have no predictive power (Algina, Moulder, & Moser, 2002). Rather than simulating various covariance structures for the predictors, the application of Browne's results allows us to operate with uncorrelated predictors since it is always possible to transform these variables to correlated variables. This dramatically reduces the number of conditions in the simulation to a more manageable number. In addition, when the focus of the study is squared multiple correlation coefficients, there is no loss of generality if the means of the predictors and the criterion are rescaled to zero.

Therefore, in the simulation, (a) the independent variables are mutually uncorrelated with mean zero and variance one; (b) the criterion has mean zero and variance one; and (c) the regression coefficients are $\beta_1 = \rho_r$, $\beta^2 = \ldots = \beta_{k-1} = 0$, $\beta_k = \sqrt{\rho_f^2 - \rho_r^2}$. The squared multiple correlation is ρ_f^2 for variables X_1 to X_k and ρ_r^2 for variables X_1 to X_{k-1}. Given these conditions, the covariance between Y and X_1 is ρ_r, the covariances of Y with the remaining independent variables, X_2 to $X_{k\text{-}1}$ are all zero, the covariance between Y and X_k is $\sqrt{\rho_f^2 - \rho_r^2}$, and the covariance for any pair of X variables is zero.

Data Simulation

The data were simulated using the random-number generating function in SAS Version 9.13. Computations were performed using SAS Interactive Matrix Language (PROC IML). Data management and follow up analyses were also conducted using SAS. Normal random deviates were generated for the n x k data matrix of predictors, **X**, using the SAS RANNOR function. All nk scores were generated to be statistically independent. In order to generate data from a g-and-h distribution, standard unit normal variables, Z_{ij}, were transformed via the following equation

$$X_{ij} = \frac{\exp\left(gZ_{ij}\right) - 1}{g} \exp\left(\frac{hZ_{ij}^2}{2}\right) \tag{2.15}$$

when both g and h were nonzero. When g is zero, equation 2.15 is reduced to

$$X_{ij} = Z_{ij} \exp\left(\frac{hZ_{ij}^2}{2}\right). \tag{2.16}$$

The g-and-h distributed variables were then standardized by subtracting the population mean and dividing by the population standard deviation. If $g = 0$, $\mu_{gh} = 0$. When $g > 0$, the population mean is

$$\mu_{gh} = \frac{\exp\frac{g^2}{2(1-h)} - 1}{g\sqrt{1-h}} \tag{2.17}$$

and for $h \leq$ ½ the population standard deviation is

$$\sigma_{gh} = \sqrt{\frac{\left[\exp\frac{2g^2}{2(1-h)} - 2\exp\frac{g^2}{2(1-2h)} + 1\right]}{g^2\sqrt{1-2h}} - \frac{\left[\exp\frac{g^2}{2(1-h)} - 1\right]^2}{g^2(1-h)}}. \quad (2.18)$$

In a similar manner, an $n \times 1$ vector of standard normal random variables was generated. All n scores were generated to be statistically independent. The results of this vector were multiplied by $\sqrt{1-\rho_f^2}$. The result is a vector of residuals, **e**, with mean zero and variance equal to 1 - ρ_f^2. These steps ensured that the dependent variable, **y**, has mean of zero and variance equal to 1.0.

As detailed above, applying Browne's results, the $k \times 1$ vector of regression coefficients was constructed such that elements 1 to k –2 are zero and the next two elements are ρ_r and $\sqrt{\rho_f^2 - \rho_r^2}$, respectively. The sample covariance matrix, **S**, was calculated from the data according to the model $\mathbf{y} = \mathbf{X}\beta + \mathbf{e}$.

Let $\mathbf{R}_f$ be the correlation matrix for the full set of k predictor variables, $\mathbf{R}_{f+}$ be the k + 1 correlation matrix for all variables (including the criterion), $\mathbf{R}_r$ be the correlation matrix for the first k -1 predictors, and $\mathbf{R}_{r+}$ be the correlation matrix for the first k -1 predictors and the criterion variable. All four correlation matrices can then be calculated from **S**. The squared multiple correlation coefficients for the full and reduced models are given by

$$R_f^2 = 1 - \frac{\det\left(R_{f+}\right)}{\det\left(R_f\right)} \quad (2.19)$$

and

$$R_r^2 = 1 - \frac{\det\left(R_{r+}\right)}{\det\left(R_r\right)} \quad (2.20)$$

where det ( ) represents the determinant of the matrix (Mulaik, 1972). For each of the 10,000 replications of each distributional condition, the asymptotic confidence interval was calculated using the method described by Alf and Graf (1999).

Data Analysis

Coverage probability, the probability that a confidence interval contains the parameter for which the confidence interval was constructed, was used to evaluate the adequacy of the confidence intervals. Coverage probability was estimated as the proportion of the 10,000 replications in which the confidence interval contained the population squared semipartial correlation, $\Delta\rho^2$. In order to investigate bias, the probability that the confidence interval was wholly below $\Delta\rho^2$ and the probability the confidence interval was entirely above $\Delta\rho^2$ were also estimated.

To evaluate the conditions under which a hypothesis test is insensitive to assumption violations, Bradley (1978; 1980) proposed three criteria. Given the nominal Type I error rate, α, a test is robust if the empirical estimate of α falls within the interval $\alpha \pm \alpha/s$. A liberal criteria is established when s = 2 and the limits are given by $\alpha \pm .025 = [.025, .075]$. Using s = 5, the interval for a moderate criterion is [.04, .06]. To establish a strict criterion, s = 10 and the interval is [.045, .055]. If these recommendations are adapted and applied to criteria for a confidence interval with a nominal coverage probability of .95, the criterion intervals become (a) [.925, .975]; (b) [.94, .96]; and (c) [.945, .955].

Although there is no universally accepted standard by which procedures are considered robust or not, Lix and Keselman (1998) suggest that applied researchers should be comfortable working with a procedure that controls Type I error within the bounds established by Bradley's liberal criterion, as long as the procedure also limits the error rate across a wide range of assumption violations. Applying this recommendation to the procedure for constructing an asymptotic confidence interval means that in order to be controlled, the coverage probability should fall within the interval [.925, .975]. We used this interval for judging the adequacy of the confidence intervals. Because there are those who would consider this standard to be too lenient, confidence intervals were also evaluated according to the more stringent criterion level of .94 to .96.

Table 2-1. Study Design

Number of predictors, *k* (5 levels)

1. $k = 2$
2. $k = 4$
3. $k = 6$
4. $k = 8$
5. $k = 10$

Size of the squared multiple correlation coefficient for the reduced model (7 levels)

1. $\rho_r^2 = .00$
2. $\rho_r^2 = .10$
3. $\rho_r^2 = .20$
4. $\rho_r^2 = .30$
5. $\rho_r^2 = .40$
6. $\rho_r^2 = .50$
7. $\rho_r^2 = .60$

Size of the squared semipartial correlation coefficient (7 levels)

1. $\Delta\rho^2 = .00$
2. $\Delta\rho^2 = .05$
3. $\Delta\rho^2 = .10$
4. $\Delta\rho^2 = .15$
5. $\Delta\rho^2 = .20$
6. $\Delta\rho^2 = .25$
7. $\Delta\rho^2 = .30$

Sample size, *n* (14 levels)

1. $n = 100$
2. $n = 200$
3. $n = 300$
4. $n = 400$
5. $n = 500$
6. $n = 600$
7. $n = 700$
8. $n = 800$
9. $n = 900$
10. $n = 1000$
11. $n = 1250$
12. $n = 1500$
13. $n = 1750$
14. $n = 2000$

Table 2-1 Continued

Distribution for the predictor variables, **X** (5 levels)	
1. $g = 0, h = 0$	$\mu = 0, \ \sigma = 1, \gamma_1 = .00, \gamma_2 = .00$
2. $g = 0, h = .058$	$\mu = 0, \ \sigma = 1.097, \gamma_1 = .00, \gamma_2 = 1.00$
3. $g = .301, h = -.017$	$\mu = .150, \ \sigma = 1.041, \gamma_1 = .89, \gamma_2 = 1.20$
4. $g = .502, h = -.048$	$\mu = .249, \ \sigma = 1.108, \gamma_1 = 1.41, \gamma_2 = 3.00$
5. $g = .760, h = -.098$	$\mu = .378, \ \sigma = 1.252, \gamma_1 = 2.00, \gamma_2 = 6.00$
Distribution for the residuals, **e** (5 levels)	
1. $g = 0, h = 0$	$\mu = 0, \ \sigma = 1, \gamma_1 = .00, \gamma_2 = .00$
2. $g = 0, h = .058$	$\mu = 0, \ \sigma = 1.097, \gamma_1 = .00, \gamma_2 = 1.00$
3. $g = .301, h = -.017$	$\mu = .150, \ \sigma = 1.041, \gamma_1 = .89, \gamma_2 = 1.20$
4. $g = .502, h = -.048$	$\mu = .249, \ \sigma = 1.108, \gamma_1 = 1.41, \gamma_2 = 3.00$
5. $g = .760, h = -.098$	$\mu = .378, \ \sigma = 1.252, \gamma_1 = 2.00, \gamma_2 = 6.00$

Table 2-2. Mardia's Multivariate Skewness, $b_{1,k}$, for the Nonnormal Distributions.

	Distribution							
	g = 0 h = .058		g = .301 h = -.017		g = .502 h = -.048		g = .760 h = -.098	
k	$b_{1,k}$	Interval[1]	$b_{1,k}$	Interval[1]	$b_{1,k}$	Interval[1]	$b_{1,k}$	Interval[1]
2	.01	(-.03, .05)	1.55	(1.51, 1.59)	3.90	(3.81, 3.98)	7.87	(7.71, 8.03)
4	.02	(-.05, .08)	3.15	(3.09, 3.22)	7.65	(7.52, 7.78)	15.80	(15.57, 16.02)
6	.00	(-.09, .08)	4.71	(4.62, 4.80)	11.50	(11.33,11.66)	23.74	(23.47, 24.02)
8	-.01	(-.12, .10)	6.23	(6.11, 6.35)	15.47	(15.26,15.68)	31.64	(31.32, 31.96)
10	.01	(-.13, .14)	7.71	(7.57, 7.86)	19.43	(19.18,19.68)	39.61	(39.23, 39.98)

[1]This interval represents .025 and .975 percentiles of the 1,000,000 replications.

Table 2-3. Mardia's Multivariate Kurtosis, $b_{2,k}$, for the Nonnormal Distributions.

	Distribution							
	g = 0 h = .058		g = .301 h = -.017		g = .502 h = -.048		g = .760 h = -.098	
k	$b_{2,k}$	Interval[1]	$b_{2,k}$	Interval[1]	$b_{2,k}$	Interval[1]	$b_{2,k}$	Interval[1]
2	10.05	(10.03,10.08)	10.35	(10.32, 10.38)	13.84	(13.79,13.89)	19.95	(19.88, 20.02)
4	28.07	(28.02,28.11)	28.75	(28.70, 28.80)	35.69	(35.60,35.77)	48.01	(47.90, 48.13)
6	54.07	(54.00,53.13)	55.09	(55.02, 55.16)	65.50	(65.39,64.62)	84.08	(83.93, 84.23)
8	88.10	(88.01,88.19)	89.44	(89.34, 89.53)	103.41	(103.26,103.57)	128.05	(127.87, 128.24)
10	130.12	(130.01,130.23)	131.79	(131.67, 131.91)	149.21	(149.02,149.40)	180.03	(179.80, 180.25)

[1]This interval represents .025 and .975 percentiles of the 1,000,000 replications.

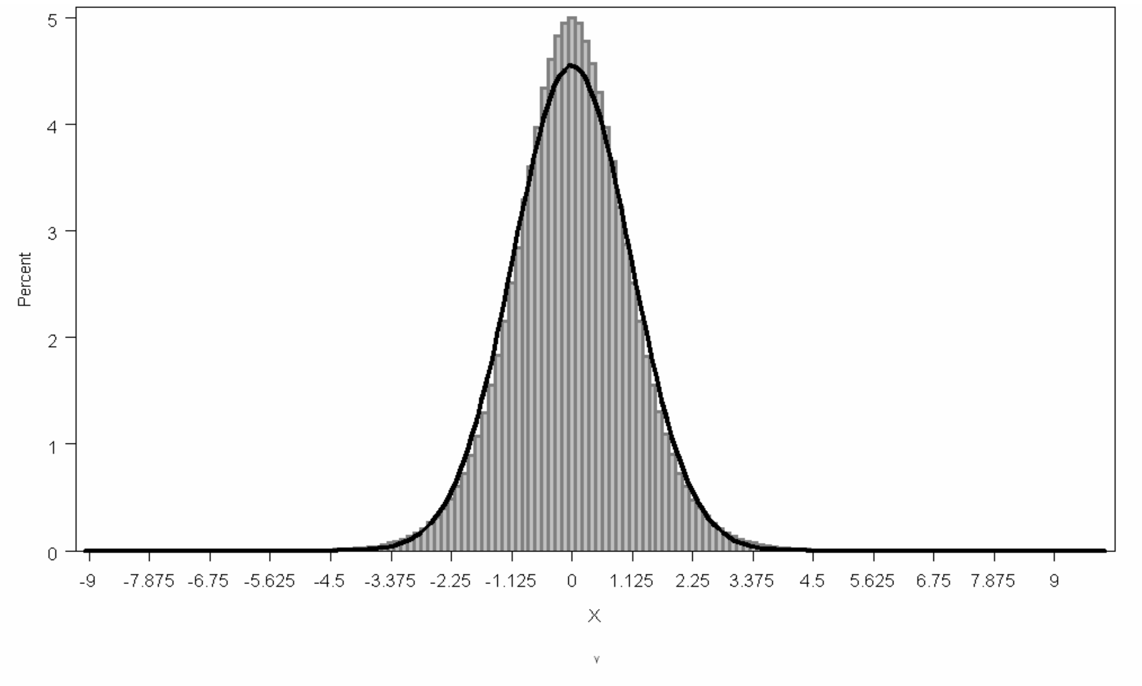


Figure 2-1. Plot of the empirical cumulative distribution function for a univariate nonnormal distribution where $g = 0$, $h = .058$ overlaid with a normal curve with $\mu_{gh} = 0$, $\sigma_{gh} = 1.097$.

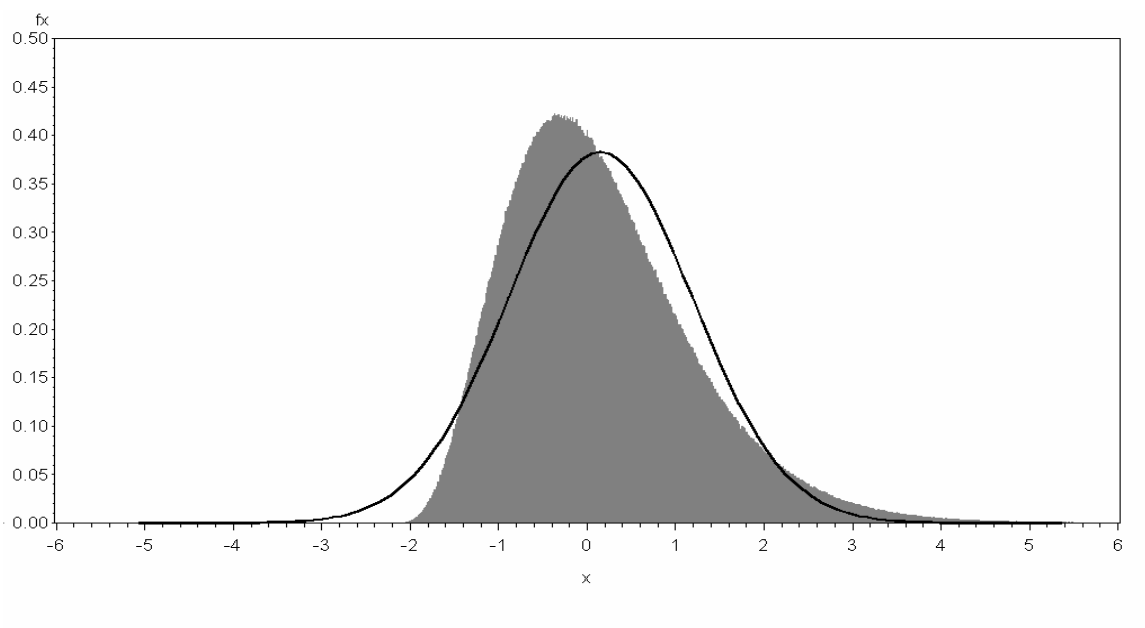


Figure 2-2. Plot of the empirical cumulative distribution function for a univariate nonnormal distribution where $g = .301$, $h = -.017$ overlaid with a normal curve with $\mu_{gh} = .150$, $\sigma_{gh} = 1.041$.

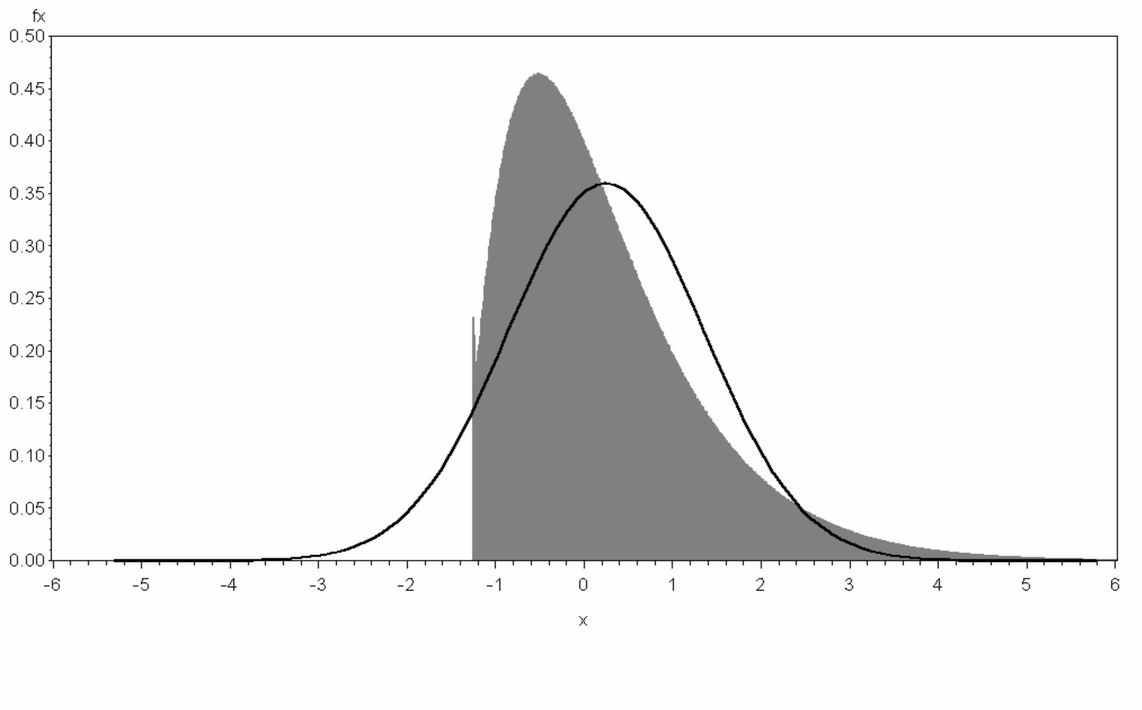


Figure 2-3. Plot of the empirical cumulative distribution function for a univariate nonnormal distribution where g = .502, h = -.048 overlaid with a normal curve with μ_{gh} = .249, σ_{gh} = 1.108.

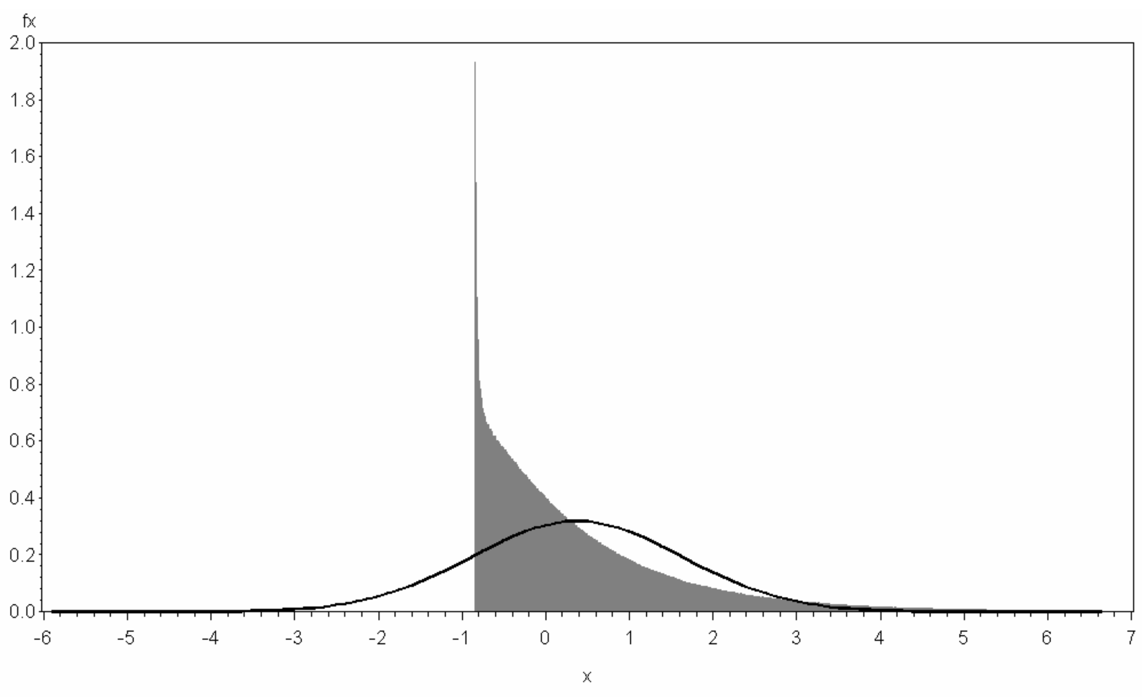


Figure 2-4. Plot of the empirical cumulative distribution function for a univariate nonnormal distribution where g = .760, h = -.098 overlaid with a normal curve with μ_{gh} = .378, σ_{gh} = 1.252

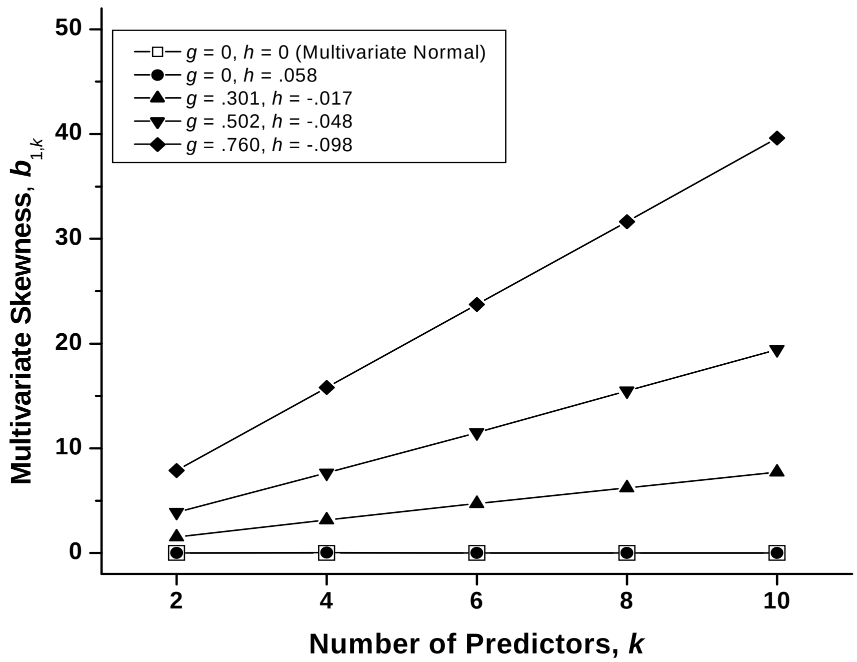


Figure 2-5. Comparison of Mardia's multivariate skewness for the multivariate normal distribution to that of the distributions investigated.

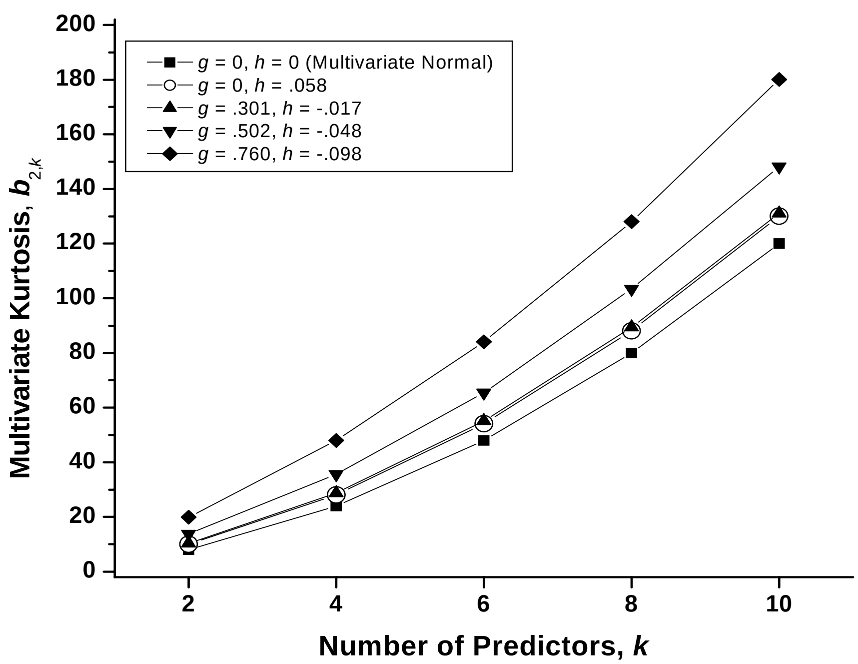


Figure 2-6. Mardia's multivariate kurtosis for the multivariate normal distribution and the nonnormal distributions investigated.

CHAPTER 3
RESULTS

Replication of Results for Multivariate Normal Data

Prior to conducting the study, data were simulated for the multivariate normal case in order to replicate key findings reported by Algina and Moulder (2001). Replication served two additional purposes. It verified that the simulation program was functioning properly and that reasonably close agreement was achieved between coverage probabilities estimated with 10,000 replications and coverage probabilities estimates reported by Algina and Moulder based on 50,000 replications. Results are compared for k = 2, 6, and 10 in Tables 3-1, 3-2, and 3-3. The shaded columns are the results from this simulation; the unshaded columns reproduce tabled results reported by Algina and Moulder (p. 638). In these tables, as well as subsequent tables reporting coverage probabilities, italics indicate that the estimated coverage probability falls within the interval from .925 to .975. Results in bold represent estimated coverage probabilities between .94 and .96.

As Olkin and Finn warned, and Algina and Moulder demonstrated, this procedure does not work at all when the population squared semipartial correlation is zero. Regardless of sample size, number of predictors, or the value of the population squared multiple correlation in the reduced model, the coverage probability when $\Delta\rho^2$ is zero is always too large, i.e., $\hat{p} \geq .999$. This is because if $\rho_f^2 = \rho_r^2$, $\sigma_\infty = 0$ even though the actual sampling variance of R^2 is not zero. Because of this defect in the asymptotic confidence interval, Alf and Graf recommended that researchers perform a hypothesis test of the significance of the corresponding regression coefficient and apply the asymptotic confidence interval procedure only when the null hypothesis is rejected. Given the coverage probability results when $\Delta\rho^2 = 0$, although coverage probabilities are reported in Tables 3-1 to 3-3, they are not included in the assessment of agreement that follows as doing so would tend to exaggerate the degree of correspondence between the two sets of estimates.

Comparing the coverage probability estimates generated by the two studies, for $\Delta\rho^2 > 0$, 79% were within ± .003 and 94% were within ± .005. Of the 504 comparisons, 73 (15%) showed no difference to 3 decimal places. When coverage probabilities differed, 208 (41%) estimates from the current study were greater and 223 (44%) were smaller than coverage probabilities reported by Algina and Moulder.

For $k = 2$, reported in Table 3-1, 90% of the estimates from the two simulations were within ± .003 and only 5 differences were greater than ± .005. For 15 of the 168 cases, estimated coverage probability would have been categorized differently with respect to Bradley's criteria for robustness, [.925,.975] or [.94,.96]. These discrepancies were evenly split with 8 estimates from Algina and Moulder's study falling in the more stringent interval, that is, closer to the nominal level, and 7 values of $\hat{p}$ estimated in this study satisfied the more stringent criterion.

Both sets of estimates when $k = 2$ showed that empirical coverage probability approached the nominal as sample size increased and as the magnitude of the squared semipartial correlation increased. The confidence interval was least accurate for the smallest sample size, $n = 175$, for all levels of ρ_r^2 when $\Delta\rho^2 = .05$. There was good coverage probability, i.e. at least .94, for $n \geq 425$ and $\Delta\rho^2 > .10$. Depending on the tolerance one has for the difference between coverage probability and the nominal confidence level, coverage probability could be considered marginally adequate, that is, at least .925, for all sample sizes and $\Delta\rho^2 > .10$.

The agreement between the two replications was somewhat worse as the number of predictors increased. As shown in Tables 3-2 and 3-3, for both $k = 6$ and $k = 10$, 127 (76%) comparisons were within ± .003. There were 8 (5%) differences greater than ± .005 with 6 predictors and 16 (9%) differences exceeded $\pm .005$ with 10 predictors. Although for $k = 6$ the large differences favored the results reported by Algina and Moulder (6 vs. 2), for $k = 10$ a large difference was just as likely to favor the estimates from the current simulation where "favoring" is defined as an estimated coverage probability that is closer in value to the nominal. In Algina and Moulder's data, there was also a tendency for the estimated coverage probability to meet the more stringent evaluation criterion when there was mismatch in categorization. For $k = 6$ and $\Delta\rho^2 > .10$, all coverage probabilities were greater than .925 for $n \geq 425$, and all but one were greater than .94 for $n = 600$. At $k = 10$, although all coverage probabilities met the liberal criterion at $n = 600$, there was no level of $\Delta\rho^2$ for which all were greater than .94. Overall, agreement between the two studies was quite good and therefore, the current study was conducted by simulating 10,000 replications of each condition.

Simulation Proper

In this simulation, 857,500,000 independent confidence intervals were calculated. Given there were 10,000 replications of each combination of **X**, **e**, n, k, ρ_r^2, and $\Delta\rho^2$, coverage probability was computed as the proportion of times the constructed confidence interval contained $\Delta\rho^2$, the population squared semipartial correlation. In this manner, 85,750 coverage probabilities were estimated.

Since the distribution from which predictors were sampled and the distribution for the residuals were both manipulated, this allowed us to examine four distinct situations that might be encountered when analyzing data using multiple regression: (a) normal **X**, normal **e**, (b) normal **X**, nonnormal **e**, (c) nonnormal **X**, normal **e**, and (d) nonnormal **X**, nonnormal **e**. Average empirical coverage probability estimates for these four scenarios, as a function of sample size, are depicted in Figure 3-1. Results for all values of k, ρ_r^2, and $\Delta\rho^2$, for selected sample sizes, are reported in Tables 3-4 to 3-7. Estimates for conditions where $\Delta\rho^2 = 0$ were omitted since all were either .999 or 1.000, rounded to three decimal places.

Table 3-4 presents results for normal predictors with normal errors. If we consider Bradley's liberal interval, .925 to .975, as evidence for robustness, for $k = 2$, 4, 6, 8, and 10, the percentages of nonrobust values at $n = 200$ were 9%, 12%, 14%, 38%, and 71%, respectively. At $n = 400$, the percentages of empirical values that were not robust decreased dramatically to 0%, 0%, 0%, 2%, and 2%. All estimated coverage probabilities were robust at $n \geq 600$. At the largest sample sizes reported, $n = 1500$ and $n = 2000$, all exceeded .94 and met the more stringent standard for robustness

When predictors were normal with nonnormal residuals, reported in Table 3-5, the percentage of nonrobust coverage probabilities increased. For $k = 2$, 4, 6, 8, and 10 and $n = 200$, the percentages of nonrobust values were 31%, 38%, 50%, 76%, and 100%, respectively. As expected, the number of nonrobust coverage probabilities decreased as n grew larger. This decrease was notable between $n = 200$ and $n = 400$ (7%, 7%, 5%, 14%, and 19%) and less so for $n = 600$ (5%, 2%, 2%, 5%, 7%) and $n = 800$ (2%, 2%, 2%, 5%, 5%). For $n \geq 1000$, all coverage probabilities were robust except when $\rho_r^2 = 0$ and $\Delta\rho^2 = .30$.

Table 3-6 shows coverage probability estimates when the predictors were nonnormal and the distribution of the residuals was normal. At $n = 200$, there were no robust empirical estimates at any level of k. For $n = 400$, the percentages of estimates outside Bradley's liberal interval were 64%, 62%, 76%, 95%, and 100% for $k = 2$, 4, 6, 8 and 10, respectively. For $n = 600$, the percentage of coverage probabilities that were nonrobust for these values of k were 50%, 55%, 60%, 64%, and 69%. For sample sizes greater than 600, improvement, as measured by a decrease in nonrobust values, was much more gradual. For $k = 2$, 4, 6, 8, and 10, and

n = 800, the percentages were 50%, 50%, 50%, 57%, and 60%; for n = 1000, 48%, 50%, 50%, 48%, and 55%; and for n = 1500, 45%, 45%, 50%, 48%, and 52%. At the largest sample size, n = 2000, at least 45% of empirical coverage probabilities at every level of k failed to meet even the liberal standard for robustness.

The coverage probabilities contained in Table 3-7 were estimated for the case where both predictors and errors were nonnormal. For $n \leq 400$, there were only 6 estimates greater than .925. Of these, 5 were observed for n = 400 and k = 2, and 1 at n = 400 and k = 4. For n = 600, the percentages of coverage probabilities that were nonrobust were 74%, 71%, 81%, 86%, and 88% for k = 2, 4, 6, 8, and 10, respectively. Similar to what was observed with nonnormal **X** and normal **e**, the improvement in coverage probabilities is minor for n > 600 such that when n = 2000, nonrobust estimates were 71% , 71% , 74% , 74% , and 76% for k = 2, 4, 6, 8, and 10, respectively.

For all four scenarios, coverage probability tended to decrease as more predictors were included in the model, particularly with smaller sample sizes. Coverage improved as sample size increased. Figure 3-1 suggests that nonnormality in the predictors was more detrimental to the adequacy of the confidence interval than was a nonnormal error distribution. A modest decline in coverage probability was observed between normal **X**, normal **e** and normal **X**, nonnormal **e**, but there was a considerable drop off in performance when **X** was nonnormal even when the errors were normally distributed.

In addition, coverage probability was examined by distributional condition. A distributional condition was defined by the combination of the distribution for the predictors and the distribution for the errors. There were 25 distributional conditions included in this study.

For clarity and ease of presentation, the *g*-and-*h* distributions from which data were generated will be referred to as: (a) pseudo-t_{10} for $g = 0$, $h = .058$; (b) pseudo-χ^2_{10} for $g = .301$, $h = -.017$; (c) pseudo-χ^2_4 for $g = .502$, $h = -.048$; and (d) pseudo-exponential for $g = .760$, $h = -.098$. The descriptive statistics reported in Table 3-8 were based on 2940 coverage probability estimates per distributional condition, excluding those cases where $\Delta\rho^2 = 0$.

Average coverage probability was closest to the nominal confidence level when both **X** and **e** were normally distributed. The average coverage probability was smallest for the most seriously nonnormal case, both **X** and **e** sampled from the pseudo-exponential distribution. Within each level of **X**, mean coverage probability decreased as the error distribution exhibited increasing nonnormality.

A similar pattern was observed for the median. In the extremes, the median for multivariate normal data was .944. In contrast, for the condition where both **X** and **e** were distributed pseudo-exponential, half the estimated coverage probabilities were less than .868. For all distributional conditions in which **X** was distributed pseudo-exponential, at each error distribution, at least 50% of the estimated coverage probabilities were below .90.

The variability in coverage probability increased with greater skewness and kurtosis in the data. When **X** was distributed pseudo-exponential, regardless of the distribution for **e**, the standard deviation was over three times that observed for the multivariate normal case. Although the maximum value did not differ a great deal as a function of distributional condition, the minimum was much lower and the range was wider for conditions with greater nonnormality.

Also included in Table 3-8 is an examination of the robustness of the confidence interval as a function of distributional condition at $n = 600$ and $n = 2000$. Applying the liberal criterion,

.925 to .975, all coverage probabilities were robust at n = 600 for multivariate normal data and when predictors were normally distributed and the distribution for the errors was either pseudo-t_{10} or pseudo-χ^2_{10}. There was no other distributional condition for which coverage probability was adequate for the entire range of values for k, ρ^2_r, and $\Delta\rho^2$, even for the largest sample size investigated, n = 2000. For the most extreme distributional condition simulated, both **X** and **e** drawn from a pseudo-exponential distribution, 100% of the coverage probabilities were nonrobust at n = 600. There was only slight improvement at n = 2000 where 90% of the estimates were not robust. Although it could be argued that data like this is unlikely to occur in practice, with an error distribution with severe nonnormality, i.e. pseudo-exponential, there was poor coverage even when the predictors were multivariate normal. At n = 2000, 25.2% of the estimates were not robust. Furthermore, when using multiple regression, applied researchers are much more likely to be concerned about the error distribution since violation of this assumption influences the power and accuracy of hypothesis tests. Researchers may not even investigate the multivariate skewness and kurtosis for the predictors. With a normal error distribution, the percentages of nonrobust estimates at n = 2000 for predictors distributed pseudo-t_{10}, pseudo-χ^2_{10}, pseudo-χ^2_4, and pseudo-exponential, were roughly 7%, 10%, 49%, and 96%, respectively. Although results are reported for only two sample sizes, for all distributional conditions and sample sizes investigated, when an estimated coverage probability was outside of either criterion interval, it was without exception, too small.

Figure 3-2 illustrates the relationship between coverage probability, distributional condition, and sample size. The best coverage probability, over the full range of sample sizes investigated, was observed for the condition in which both **X** and **e** were normal. However, at best, average coverage probability never reached the nominal confidence level, .95. There was a slight degradation in performance for conditions where **X** was normal and the nonnormal errors were distributed pseudo-t_{10} or pseudo-χ^2_{10}. Although it is a bit hard to discern because of the overlap for conditions where **X** was distributed pseudo-t_{10} and pseudo-χ^2_{10}, results were similar for normal **X** with **e** distributed pseudo-χ^2_4, and **X** sampled from pseudo-t_{10} with normal error. That is, coverage probability estimates were similar when the predictors were normal with markedly nonnormal errors and when predictors were sampled from a pseudo-t_{10} distribution with a normal error distribution. Similarly, the condition in which **X** was distributed pseudo-χ^2_4 with normal error exhibits coverage probability comparable to the conditions where predictors were moderately nonnormal, sampled from pseudo-t_{10} and pseudo-χ^2_{10}, with errors that were extremely skewed and kurtotic (pseudo-exponential). Thereafter, as the distribution for **X** became increasingly nonnormal, coverage probability decreased and was least adequate when the predictors were sampled from a pseudo-exponential distribution regardless of the distribution for the residuals. Within each condition for **X**, coverage probability decreased in the same systematic way as a function of the nonnormality in the error distribution such that coverage probability was best with normally distributed errors and worst when the errors were distributed pseudo-exponential.

Analysis of Variance and Mean Square Components

Given the sheer volume of data collected in this study, analysis of variance (ANOVA) was used to identify the experimental factors that were important in determining the estimated coverage probability, $\hat{p}$. Factorial ANOVA assumes that multiple factors contribute to the variance in the data. The total variance is partitioned into main effects corresponding to each factor, the interactions among them, and random error. The factors manipulated in the study were all treated as between-subjects effects in a fully-crossed ANOVA model that consisted of 6 main effects and 56 interactions. Since the procedure for calculating the confidence interval is clearly inappropriate when $\Delta\rho^2 = 0$, the 12,250 coverage probabilities calculated for this value were not included in this analysis. It was felt this provided a more accurate reflection of the data. ANOVA analyses and variance partitioning of coverage probabilities were therefore based on $N = 73{,}500$. The mean squares, F-statistics, and p-values associated with each effect in the full model were computed using the ANOVA procedure in SAS. These results are reported in Table 3-9. The combination of a large number of effects and a very large sample size ensured that there were many statistically significant effects, including higher-order interactions. In all, 34 of the 62 effects estimated were significant at $p < .0001$.

Because statistical significance is in large part a function of sample size, a statistically significant effect is not very informative when the sample size is very large. To better understand the relative impact of these effects on coverage probability, it was necessary to obtain a measure of influence to determine which effects were associated with a meaningful proportion of the variance. The term variance component is used in the context of analysis of variance with random effects and denotes the estimate of the amount of variance that can be attributed to each effect. In the current context, the levels of each factor were purposively selected. Because effects are fixed and not random, the more accurate term is mean square component. The ANOVA method for estimating mean square components equates mean squares to their expected values, ***EMS,*** and solves for the mean square components in those expectations. The estimated mean square component for each main effect and interaction was computed using the general formula

$$\theta^2_\alpha = \frac{MS(\alpha) - MS(\text{Residual})}{j}$$

where α is the effect of interest and j is the product of the number of levels for each factor not involved in α (Myers & Well, 2003). In this case, the residual mean square, .0000079, includes the mean square for the six-way interaction and the mean square for error. For example, the mean square component for **X** is given by

$$\theta_X^2 = \frac{7.5162 - .0000079}{(5)(5)(7)(6)(14)} = \frac{7.516192}{14700} = .0005113.$$

Since these are simultaneous linear equations with as many unknowns as there are equations, they have unique solutions and mean square components are estimated noniteratively. An unfortunate characteristic of ANOVA estimators is that they can yield negative estimates even though, by definition, they are nonnegative. Negative components were set equal to zero before calculating the proportion of variance that could be attributed to each effect. The components were then summed and the ratio of each mean square component to the sum was used as a measure of influence.

Effects significant at α = .0001 that accounted for at least .5% of the variance are reported in Table 3-9. The distribution for the predictors, **X**, was responsible for 44.51% of the total variance in coverage probability. The variance component associated with **X** was nearly four times greater than that of any other main effect. The main effects of $\Delta\rho^2$ and ρ_r^2 were comparatively less important factors in determining average coverage probability, accounting for 10.12% and 3.26% of the total variance, respectively. Effects of $\Delta\rho^2$ and ρ_r^2 were moderated by their interaction. This two-way interaction accounted for an additional 1.60% of the variance. The mean square component associated with sample size, n, accounted for 9.41% of the total variability in $\hat{p}$. The main effect of **e** accounted for only 3.38% of the variance indicating that the error distribution had a much smaller impact on the coverage probability of the confidence interval than did the distribution of the predictors. The number of predictor variables, k, had very little impact on $\hat{p}$, accounting for only .69% of the variability.

The critical importance of nonnormality in the predictors was further substantiated by the fact that interactions involving **X** explained an additional 22.24% of the variance in $\hat{p}$. The variance components for the two-way interactions between **X** and $\Delta\rho^2$ and **X** and ρ_r^2 were associated with 11.38% and 8.82% of the total variance, respectively. The three-way interaction of these factors, $\mathbf{X} \times \rho_r^2 \times \Delta\rho^2$, moderated the three main effects and the two-way interactions and accounted for an additional 2.04% of the total variance in coverage probability. The main effects of **X**, $\Delta\rho^2$, and ρ_r^2, and the interactions of these three factors explained 81.7% of the total variance in coverage probabilities.

The effect of **e** was also moderated, although to a lesser extent, by the two-way interactions between **e** and $\Delta\rho^2$ and **e** and ρ_r^2. These interaction effects were responsible for .70% and 1.28%, respectively. The three-way interaction, $\mathbf{e} \times \rho_r^2 \times \Delta\rho^2$, explained .54% of the total variance. The main effects of **e**, $\Delta\rho^2$, and ρ_r^2, and their interactions accounted for 5.85% of the variance in $\hat{p}$. This was further evidence that although a nonnormal error distribution had some effect on the coverage of the confidence interval it was not nearly as important as nonnormality in the predictors.

Sample size interacted with the number of predictors, k, and the size of the squared semipartial correlation coefficient, $\Delta\rho^2$. The $n \times k$ and $n \times \Delta\rho^2$ interaction effects each explained approximately 1% of the total variance. Important effects involving sample size were associated with 11.35% of the variability in coverage probability. Thus, it appears that sample size was also more important than nonnormality in the error distribution in determining the adequacy of the confidence interval. The effects reported in Table 3-9 accounted for an estimated 99.6% of the total variance in coverage probability. The following sections describe the important factors influencing coverage probability as identified by the mean square components analysis.

The Influence of Nonnormality on Coverage Probability

Nonnormal predictors

When coverage probability was averaged over all other factors, Table 3-10 shows the adequacy of the confidence interval, as measured by coverage probability, worsened as the distribution for the predictors became increasingly nonnormal. When **X** was distributed multivariate normal, average coverage probability was .935 (SD = .014). When the set of predictors was made up of variables sampled from a pseudo-t_{10} distribution, that is symmetric, but more peaked and heavier tailed than the normal distribution, average coverage probability dropped to .925 (SD = .015). A similar estimate of average coverage probability, $\hat{p}$ =.923 (SD = .015), was obtained when the explanatory variables were sampled from a population distributed as pseudo-χ^2_{10}. Because these distributions had similar values for both univariate and multivariate kurtosis, but differed with respect to skewness, this result seems to suggest, at least for moderate nonnormality, that skewness may be less important than kurtosis in determining the adequacy of the confidence interval procedure. When predictors were sampled from a pseudo-χ^2_4 population distribution, the average coverage probability was .906 (SD = .022). The average coverage probability when predictors were sampled from a distribution that has the same skewness and kurtosis as the exponential distribution was .877 (SD = .037).

The median was also related to the degree of nonnormality present and declined in a manner similar to the mean. In addition, the range of coverage probability values estimated in the simulation expanded as the degree of nonnormality became more extreme. Figure 3-3 presents boxplots that describe the distribution of coverage probability estimates as a function of the distribution for **X**. We see that all distributions for $\hat{p}$ are skewed to the right, but the distribution was flatter, more spread out, and longer-tailed as the degree of skewness and kurtosis in the distribution for the predictors increased.

Nonnormal error distribution

Table 3-11 shows descriptive statistics for the main effect of the distribution for error. The means, by error distribution, also declined as a function of the degree of nonnormality present. The range between the largest mean, .919 for normally distributed errors, and the smallest, .903 for errors distributed pseudo-exponential, was much smaller than observed in Table 3-10 for the main effect of the distribution for the predictors. There was also less variability in the median, ranging from .929 for normal errors to .911 for pseudo-exponential errors. The range of coverage probabilities and the standard deviations were essentially equal suggesting that there was little difference in the variability of coverage probability estimates as a function of the error distribution. The boxplots depicted in Figure 3-4 supported this contention.

The Impact of Squared Multiple Correlations on Coverage Probability

Figure 3-5 depicts the relationship between coverage probability and the magnitude of the population squared semipartial correlation. Averaged over all other factors, coverage probability tended to decrease as the size of the squared semipartial correlation increased. Figure 3-5 also shows that the effect of $\Delta\rho^2$ on coverage probability varied depending on the distribution for the predictors hence the significant interaction between **X** and $\Delta\rho^2$. Figure 3-5 and Table 3-12 show the relationship between $\Delta\rho^2$ and coverage probability within each distribution for **X**. Under normality there was actually a slight increase in $\hat{p}$ from $\Delta\rho^2 = .05$ to $\Delta\rho^2 = .10$, the smallest values investigated. This increase essentially leveled off thereafter. Stable coverage probability between $\Delta\rho^2 = .05$ and $\Delta\rho^2 = .10$ was observed for pseudo-t_{10} and pseudo-χ^2_{10} distributions. In both distributions, $\hat{p}$ showed a steady, but modest, decline for $\Delta\rho^2 > .10$. The decline in $\hat{p}$ when **X** was distributed pseudo-χ^2_4 was modest between $\Delta\rho^2 = .05$ ($\hat{p} = .924$) and $\Delta\rho^2 = .10$ ($\hat{p} = .919$). The rate of change was much steeper for $\Delta\rho^2 > .10$ such that $\hat{p}$ decreased to .886 at $\Delta\rho^2 = .30$. For **X** sampled from the pseudo-exponential distribution, coverage probability was essentially a linear function of $\Delta\rho^2$ that declined sharply over the range of $\Delta\rho^2$ from $\hat{p} = .915$ to $\hat{p} = .840$.

There was also a significant interaction, depicted in Figure 3-6, between **e** and $\Delta\rho^2$. However, as reported in Table 3-9, this effect while statistically significant, accounted for little of the variance in coverage probability. A comparison of Figure 3-6 with Figure 3-5 shows a similar pattern for the relationship between the error distribution and $\Delta\rho^2$ with less extreme variation in the rate at which coverage probability declined. When the error distribution was normal, pseudo-t_{10}, or pseudo-χ^2_{10}, $\hat{p}$ declined slightly between $\Delta\rho^2 = .05$ and $\Delta\rho^2 = .10$ with a steady, gradual decrease for $\Delta\rho^2 > .10$. The decline in coverage probability between $\Delta\rho^2 = .05$ and $\Delta\rho^2 = .60$ was more nearly linear, with a steeper slope, when the error distribution was sampled from either a pseudo-χ^2_4 or pseudo-exponential distribution. The decrease in coverage probability was most dramatic when errors were distributed pseudo-exponential. At $\Delta\rho^2 = .05$, $\hat{p} = .923$ and at $\Delta\rho^2 = .60$, coverage probability dropped to $\hat{p} = .883$. Coverage probabilities, as a function of **e** and $\Delta\rho^2$, are reported in Table 3-13.

Figure 3-7 depicts the relationship between ρ^2_r and coverage probability. Coverage probability stayed relatively constant between $\rho^2_r = .00$ and $\rho^2_r = .40$ and then decreased for

$\rho_r^2 = .50$ and $\rho_r^2 = .60$. The interaction between **X** and ρ_r^2 is also demonstrated in Figure 3-7. When the predictors were distributed multivariate normal, coverage probability was a linear function of ρ_r^2, gradually increasing from .928 at $\rho_r^2 = .00$ to .940 at $\rho_r^2 = .60$. For **X** distributed pseudo-t_{10} and pseudo-χ^2_{10}, there was a minor increase in coverage probability, roughly .92 to .95, between $\rho_r^2 = .00$ and $\rho_r^2 = .40$. Coverage probability was smaller for $\rho_r^2 \geq .50$. As the distribution for **X** demonstrated greater skewness and kurtosis, the coverage probability function tended to be more curvilinear. For **X** distributed pseudo-χ^2_4, coverage probability was relatively consistent between $\rho_r^2 = .00$ and $\rho_r^2 = .30$ and decreased steadily for $\rho_r^2 \geq .40$ to a minimum of .887 at $\rho_r^2 = .60$. When **X** was sampled from a pseudo-exponential distribution, coverage probability started out at $\hat{p} = .896$ at $\rho_r^2 = .00$ and decreased between $\rho_r^2 = .00$ and $\rho_r^2 = .30$ to .885. The decline in $\hat{p}$ was at a much faster rate thereafter such that when $\rho_r^2 = .60$, $\hat{p} = .837$. As Table 3-14 shows, the differences in coverage probabilities, as a function of the degree of nonnormality in **X,** had their smallest range of values at $\rho_r^2 = .00$ (.928 to .896) and the range was maximized at $\rho_r^2 = .60$ (.940 to .837).

While the behavior of $\hat{p}$ over the levels of $\Delta\rho^2$, as a function of the error distribution, was comparable to the relationship between $\Delta\rho^2$ and the distribution for the predictors, the $\mathbf{e} \times \rho_r^2$ interaction, presented in Figure 3-8, shows this was not the case for ρ_r^2. In contrast, the differences in $\hat{p}$, as a function of nonnormality in the error distribution, were greatest at $\rho_r^2 = .00$. Coverage probability, reported in Table 3-15, ranged from .927 for normal errors to .897 when errors were pseudo-exponential. By the time $\rho_r^2 = .60$, coverage probability had essentially converged and was approximately .90 regardless of the degree of nonnormality in the error distribution. Furthermore, for normal errors, maximum coverage probability, .927, occurred for $\rho_r^2 = .00$. For **e** distributed pseudo-t_{10} and pseudo-χ^2_{10}, the largest coverage probability, .922, occurred at $\rho_r^2 = .10$. For pseudo-χ^2_{10}, the largest average coverage probability, .915, was observed for $\rho_r^2 = .20$ and $\rho_r^2 = .30$. When the error distribution was sampled from a pseudo-exponential population distribution, the largest coverage probability, .907, was observed at $\rho_r^2 = .30$ and $\rho_r^2 = .40$. These results suggest that the **X** × ρ_r^2 and **e** × ρ_r^2 interactions might have a counterbalancing effect. However, the **e** × ρ_r^2 interaction, although statistically significant, explained a modest 1.3% of the total variance in coverage probability while the **X** × ρ_r^2 interaction accounted for 9.5% of the total variance.

The impact of the interaction between $\Delta\rho^2$ and ρ_r^2 on coverage probability is shown in Figure 3-9. Although there was a tendency for estimated coverage probability to be further from the nominal as ρ_r^2 increased, this was not the case for all values of $\Delta\rho^2$. When $\Delta\rho^2 = .05$, there was an increasing trend in coverage probability over the range of ρ_r^2 values. For $\Delta\rho^2 > .05$, coverage probability was relatively stable between $\rho_r^2 = .00$ and $\rho_r^2 =.30$, but then decreased substantially from $\rho_r^2 = .30$ to $\rho_r^2 = .60$.

However, the relationship of $\hat{p}$ to ρ_r^2 and $\Delta\rho^2$ varied depending on the distribution for **X**. Figure 3-10 shows the effect of the three-way interaction between **X**, ρ_r^2, and $\Delta\rho^2$ on coverage probability. To aid in the description and interpretation of effects, coverage probabilities, as a function of ρ_r^2 and $\Delta\rho^2$, for each level of **X** are reported in Tables 3-16 through 3-20. For the multivariate normal case, coverage probability tended to be worse when $\Delta\rho^2 = .05$ and for all levels of $\Delta\rho^2$ coverage probability increased as ρ_r^2 increased. The plots of coverage probability as a function of $\Delta\rho^2$ and ρ_r^2 for pseudo-t_{10} and pseudo-χ_{10}^2 look remarkably similar to each other and have the same pattern of results described for the two-way interaction of ρ_r^2 and $\Delta\rho^2$, albeit over a narrower range of values. Coverage probability increased over the levels of ρ_r^2 when $\Delta\rho^2 = .05$, but for $\Delta\rho^2 > .05$, coverage probability tended to increase from $\rho_r^2 = .00$, reached a maximum at $\rho_r^2 = .30$, and decreased thereafter. Although coverage probability was best for $\Delta\rho^2 = .05$ and $\rho_r^2 = .60$, for all other levels of $\Delta\rho^2$, coverage probability was lowest at $\rho_r^2 = .60$.

Coverage probability was consistent at approximately .925 over the full range for ρ_r^2 for $\Delta\rho^2 = .05$ when the predictors were distributed pseudo-χ_4^2. For $\Delta\rho^2 > .05$, coverage probability was stable between $\rho_r^2 = .00$ and $\rho_r^2 = .20$, but showed a decline between $\rho_r^2 = .30$ and $\rho_r^2 = .60$. The rate of decline was faster for larger values of $\Delta\rho^2$.

For **X** sampled from the pseudo-exponential distribution, coverage probability decreased as ρ_r^2 increased for all levels of $\Delta\rho^2$. The rate of decline varied according to the value of $\Delta\rho^2$ with steeper slopes associated with larger values of $\Delta\rho^2$. The drop in coverage probability was minor for $\Delta\rho^2 = .05$, where $\hat{p} = .917$ at $\rho_r^2 = .00$, falling to $\hat{p} = .907$ at $\rho_r^2 = .60$. However, when $\Delta\rho^2 = .30$, at $\rho_r^2 = .00$ coverage probability was .870 and decreased markedly to $\hat{p} = .777$ at $\rho_r^2 = .60$. Thus, when nonnormality in the predictors was extreme, the importance of the magnitude of the squared multiple correlations, $\Delta\rho^2$ and ρ_r^2, was critical for determining the adequacy of the confidence interval procedure. Although no condition, on average, demonstrated acceptable coverage over the entire range of factors manipulated in this study,

Figure 3-10 illustrates how inaccurate the asymptotic confidence interval can be under conditions that could occur in practice.

The Impact of Sample Size on Coverage Probability

As seen in Figures 3-1 and 3-2, regardless of the distribution for the predictors or the distribution for error, coverage probability increased rapidly between n = 100 and n = 400. The average coverage probability at n = 100 was .882 increasing to .912 at n = 400. The rate of increase, from .914 to .917, was considerably slower between n = 500 and n = 800. Furthermore, it appears that there was little to be gained by increasing the size of the sample beyond n = 1000 with respect to the adequacy of the confidence interval. Coverage probability is increasing so slowly between n = 1000 and n = 2000 (from .918 to .920) that it is likely that sample sizes well in excess of 2000 would be required to ensure the robustness of the confidence interval over a wide range of nonnormal conditions. Evidence to support this contention was evaluated by estimating coverage probabilities for **X** and **e** distributed pseudo-exponential; n = 5000; ρ_r^2 = .00, .30, and .60; and $\Delta\rho^2$ = .05, .10, .15, .20, .25, and .30. Results indicated that even with an extremely large sample size, when nonnormality is severe, coverage probability remained inadequate. Only 7 of 54 coverage probability estimates exceeded .925 and consequently, 87% were nonrobust. Six of the robust estimates were observed for ρ_r^2 = .00 or ρ_r^2 =.30 and $\Delta\rho^2$ = .05 for all three levels of k. The remaining robust estimate occurred for k = 10, ρ_r^2 = .00, and $\Delta\rho^2$ = .10.

Figure 3-11 shows that the effect of sample size was not the same at every level of $\Delta\rho^2$. The interaction between n and $\Delta\rho^2$ was due to the fact that the effect of $\Delta\rho^2$ was smaller when the sample size was smaller than the effect of $\Delta\rho^2$ when the sample size is larger. In addition, the average values for $\hat{p}$ are not is the same order as a function of $\Delta\rho^2$ for smaller sample sizes. For example, at $n = 100$, although coverage probability was clearly inadequate for all levels of $\Delta\rho^2$, it was worse for the smallest value, $\Delta\rho^2 = .05$, as well as the largest values, $\Delta\rho^2 = .25$ and $\Delta\rho^2 =.30$. Coverage probability improved noticeably for $\Delta\rho^2 = .05$ at $n = 200$ although it was still not as large as it was for $\Delta\rho^2 = .10$. By $n = 300$ coverage probability for $\Delta\rho^2 = .05$ and $\Delta\rho^2 = .10$ were equal. At $n \geq 400$, coverage probability was a function of $\Delta\rho^2$ growing worse as $\Delta\rho^2$ increased. Coverage probabilities, as a function of sample size and $\Delta\rho^2$, are presented in Table 3-21.

As shown in Figure 3-12, the rate of increase in coverage probability as a function of sample size depended on the number of predictors in the model. For the smaller sample sizes, most notably at $n = 100$, although average coverage probability was clearly inadequate, it was considerably worse when there were more predictors in the model. As the sample size increased, the difference between coverage probabilities as a function of the number of predictors became progressively smaller. Table 3-22 shows that at sample sizes greater than 1000, the difference in coverage probability was minimal and it appears that the number of predictors exerted very little influence on coverage probability.

Probability Above and Below the Confidence Interval

When the confidence interval did not contain the population squared semipartial correlation coefficient, the probability that the confidence interval was below $\Delta\rho^2$ and the probability that the confidence interval was above $\Delta\rho^2$ were also estimated. When $\Delta\rho^2 = 0$, average coverage probability was .9998. Only 18,754 of the 122,250,000 confidence intervals constructed did not contain the population parameter. There were only 4 instances in which the interval was wholly below $\Delta\rho^2$; 18,750 confidence intervals were wholly above $\Delta\rho^2$.

When the increase in the squared multiple correlation was zero the confidence interval was too conservative, but for all other values of $\Delta\rho^2$, the confidence intervals tended to be too liberal.

For the 73,500 conditions where $\Delta\rho^2 > .05$, the probability that the confidence interval was wholly below $\Delta\rho^2$ was twice the probability that the confidence interval was entirely above $\Delta\rho^2$ (.664 vs. .336). The confidence interval is biased in the sense that there is a systematic error that causes the estimated confidence limits to regularly miss the population parameter in the same direction. The tendency to underestimate $\Delta\rho^2$ occurs because the estimated asymptotic standard error declines as ΔR^2 declines. As a result, when $\Delta R^2 < \Delta\rho^2$ there is a tendency for the interval to be completely below $\Delta\rho^2$ (Algina & Moulder, 2001).

The Relationship between Estimated Asymptotic Variance, Empirical Sampling Variance of ΔR^2, and Coverage Probability

As previously noted, all coverage probabilities were at least .998 for $\Delta\rho^2 = 0$. This result indicates that when a predictor was added to a multiple regression model and there was no increase in ρ^2, the confidence interval was always too wide. As previously noted, there were two reasons for this shortcoming in the confidence interval. The distribution of ΔR^2 is skewed to the right and since the increase in R^2 cannot be less than zero it has a lower limit of zero. Because the confidence interval formula does not recognize this lower limit, when the population value was $\Delta\rho^2 = 0$, the confidence interval tended to have a lower limit less than zero.

The second basis for the problem, identified by Algina and Moulder, is that the asymptotic variance overestimates the sampling variance of ΔR^2. This was verified in the current study by calculating for each combination of $\mathbf{X}$, $\mathbf{e}$, n, k, ρ_r^2, and $\Delta\rho^2$ (a) the mean estimated asymptotic variance over the 10,000 replications and (b) the empirical sampling variance of ΔR^2. For all conditions where $\Delta\rho^2 = 0$, the ratio of the average value of (a) to (b), denoted as MEAV/VarΔR^2, ranged from 1.27 to 2.18 with a mean of 1.95 and a median of 1.96.

The ratio, MEAV/VarΔR^2, was also evaluated for $\Delta\rho^2 > 0$. ANOVA and mean square components analyses were conducted for MEAV/VarΔR^2 as the outcome variable. As was the case with coverage probability, due to the large sample size, only 24 of 62 effects failed to demonstrate significance at $p < .0001$. Effects significant at $\alpha = .0001$ that accounted for at least .5 % of the variance are reported in Table 3-23. These effects accounted for 97.8% of the variability in the variance ratio, MEAV/VarΔR^2. The distribution for the predictors explained 51.78% of the variance in the ratio. An additional 21.06% was attributable to the size of the squared semipartial correlation coefficient. Less important for accurate estimation of the variance were the main effects of **e** and ρ_r^2. These effects explained 6.26% and 2.67% of the total variance, respectively.

As observed for coverage probability, a substantial proportion of the variance, 89.5%, was accounted for by the main effects of **X**, $\Delta\rho^2$, and ρ_r^2, and the interaction of these effects: 6.81% was associated with the **X** × $\Delta\rho^2$ interaction, 6.45% was associated with the **X** × ρ_r^2 interaction, and the three-way interaction, **X** × ρ_r^2 × $\Delta\rho^2$, explained a modest .72%. The interaction between the distribution for the errors and ρ_r^2 accounted for an additional 2.02%. Although sample size plays a role in determining the coverage probability, it was not important in determining the ratio since the effect of n was included in calculating the variance.

Figure 3-13 illustrates how MEAV/VarΔR^2 varies as a function of the distribution for the predictors, ρ_r^2, and $\Delta\rho^2$. This figure corresponds to Figure 3-10, describing coverage probability as a function of the **X** × ρ_r^2 × $\Delta\rho^2$ interaction, and shows a similar pattern. For the multivariate normal case, variance ratios got further from 1.0 as $\Delta\rho^2$ increased for $\rho_r^2 = 0$. As ρ_r^2 increased, variance ratios improved for all values of $\Delta\rho^2$. This improvement was greater for larger values of $\Delta\rho^2$. By the time $\rho_r^2 = .60$, there was no difference in the MEAV/VarΔR^2 ratio as a function of ρ_r^2. The behavior of the variance ratio helps to explain the fact that for normal data coverage probability increases with both $\Delta\rho^2$ and ρ_r^2.

For **X** distributed pseudo-t_{10} and pseudo-χ_{10}^2, the pattern for MEAV/VarΔR^2 as a function of $\Delta\rho^2$ and ρ_r^2 was very similar. This was also observed for coverage probability. At all values of ρ_r^2, the variance ratio got smaller as $\Delta\rho^2$ increased. For $\Delta\rho^2 = .05$, the MEAV/VarΔR^2 ratio was consistent across the range for ρ_r^2. There was a slight curvilinear relationship in the $\Delta\rho^2 \times \rho_r^2$ plots for $\Delta\rho^2 > .15$ such that variance estimation improved slightly from $\rho_r^2 = .00$ to $\rho_r^2 = .30$ and then declined from $\rho_r^2 = .30$ to $\rho_r^2 = .60$. Therefore, variance estimates were best for all values of $\Delta\rho^2$ at $\rho_r^2 = .30$ and the most serious variance underestimation occurred when both ρ_r^2 and $\Delta\rho^2$ were largest.

When **X** was distributed pseudo-χ_4^2 or pseudo-exponential the difference between the variance ratio at the smallest value of $\Delta\rho^2$ and the largest was greater than for the previous distributions at $\rho_r^2 = .00$ and this difference became progressively larger as ρ_r^2 increased. For the most extreme degree of nonnormality, although MEAV/VarΔR^2 was never greater than .90, when $\Delta\rho^2$ represents a large effect size, the accuracy of the estimated variance was particularly poor over the range of ρ_r^2 values.

The scatterplot in Figure 3-14 is further evidence of a strong positive association between coverage probability and MEAV/VarΔR^2. The correlation between coverage probability and the variance ratio was $r = .91$. As the asymptotic variance more accurately estimated the actual sampling variance of ΔR^2, coverage probability approached the nominal confidence level. When coverage probabilities were poor, the estimated asymptotic variance could be less than half that of the empirical sampling variance of ΔR^2.

The strength of the relationship between coverage probability and MEAV/VarΔR^2 depends on the distribution for the predictors, as shown in Figures 3-15 to 3-19. For multivariate normal data, presented in Figure 3-15, the mean variance ratio was .946 (SD = .06). The median was .963 with a range from .666 to 1.050. Approximately 10% of the estimates were greater than 1.0 indicating that the asymptotic variance, albeit rarely, sometimes overestimated the empirical sampling variance. As the plot shows, however, a variance ratio near 1.0 was not a guarantee that the coverage probability will necessarily be close to .95 and coverage probability was as low as .85. Not surprisingly, the correlation between coverage probability and MEAV/VarΔR^2 was lower than that for the full data set, $r = .62$.

Although the correlation between coverage probability and MEAV/VarΔR^2 was similar to that for normal data, $r = .63$, when the predictors were sampled from the pseudo-t_{10} distribution, less than 1% of the variance ratios were above 1 (Figure 3-16). The mean variance ratio was .881 (SD=.065), the median was .886, and the range was .631 to 1.044.

The estimates from the pseudo-χ^2_{10} distribution again demonstrate close similarity to the pseudo-t_{10} distribution. Although the scatterplot in Figure 3-17 is somewhat less dispersed reflected in a slightly higher correlation, $r = .68$, the descriptive statistics show close agreement. The mean variance ratio was .870 (SD=.068), the median was .874, and the range was .626 to 1.044. Again, less than 1% of the ratio estimates were greater than 1.0.

As multivariate skewness and kurtosis increased, the correlation between coverage probability and MEAV/VarΔR^2 became much stronger. For the pseudo-χ^2_4 distribution r = .86. As Figure 3-18 demonstrates, the scatterplot was more compact and more spread out. The range of values was wider, 548 to 1.004, due to a lower minimum value. There was only 1 variance ratio greater than 1. The mean was .785 (SD = .100) and the median was .788.

Figure 3-19 shows the strongest relationship (r = .91) between coverage probability and MEAV/VarΔR^2 for the pseudo-exponential distribution. With skewness and kurtosis corresponding to the exponential distribution, the scatterplot was tightly concentrated and substantially more elongated. None of the variance ratios were greater than 1 and over 25% were less than .60. Variance ratios ranged from a low of .381 to a high of .972. The mean was .881 (SD=.132) and the median was .673.

In summary, for multivariate normal data, MEAV/VarΔR^2 was best when $\Delta\rho^2$ was small, but as ρ^2_r increased, variance was more accurately estimated and by the time ρ^2_r = .60, MEAV/VarΔR^2 was not dependent on $\Delta\rho^2$. This pattern of results did not hold when nonnormality was introduced in the predictors. For moderate nonnormality, MEAV/VarΔR^2 tended to be more dependent on the value of $\Delta\rho^2$ than on the magnitude of ρ^2_r. When nonnormality was more extreme, variance estimation became more inaccurate as both $\Delta\rho^2$ and ρ^2_r increased. Thus, when a variable was added to a multiple regression model that already explained a sizeable proportion of the variation in the outcome, for example, ρ^2_r = .60, the effect size associated with that variable was large, for example, $\Delta\rho^2$ = .30, and the data were not multivariate normal, using Alf and Graf's formula underestimated the variance. Furthermore, this study showed that when nonnormality was severe, the estimated asymptotic variance could be less than half that indicated by the sampling distribution of ΔR^2. In practice, this is likely to produce standard errors that are too small resulting in a confidence interval that is too narrow. Reliance on this confidence interval as a measure of the strength of the effect size will lead us to underestimate the importance of an individual predictor to the regression.

Table 3-1. Replication of Algina and Moulder's Results for Multivariate Data and Two Predictors.

		$\Delta\rho^2$													
n	ρ_r^2	0.00		0.05		0.10		0.15		0.20		0.25		0.30	
175	0.00	1.000	1.000	0.907	0.904	*0.925*	*0.925*	*0.931*	*0.930*	*0.936*	*0.937*	*0.938*	*0.939*	**0.940**	*0.934*
	0.10	1.000	1.000	0.911	0.910	*0.926*	*0.922*	*0.933*	*0.932*	*0.938*	*0.935*	**0.940**	*0.938*	*0.938*	*0.938*
	0.20	1.000	1.000	0.913	0.912	*0.930*	*0.929*	*0.935*	*0.933*	*0.938*	*0.938*	**0.942**	**0.940**	**0.940**	*0.939*
	0.30	1.000	1.000	0.919	0.919	*0.931*	*0.928*	*0.935*	**0.941**	*0.938*	*0.939*	*0.939*	**0.941**	**0.942**	*0.939*
	0.40	1.000	1.000	0.922	0.923	*0.934*	*0.934*	**0.940**	*0.939*	*0.939*	**0.941**	**0.942**	**0.940**	**0.941**	**0.944**
	0.50	1.000	1.000	0.923	*0.929*	*0.936*	*0.939*	*0.939*	*0.938*	**0.941**	**0.942**	**0.942**	*0.939*	**0.943**	**0.941**
	0.60	1.000	1.000	*0.931*	*0.928*	*0.939*	*0.937*	**0.941**	**0.943**	**0.941**	**0.940**	**0.942**	*0.939*	**0.943**	**0.940**
300	0.00	1.000	1.000	0.923	0.923	*0.937*	*0.932*	*0.938*	*0.936*	**0.943**	**0.943**	**0.943**	**0.945**	**0.944**	**0.944**
	0.10	1.000	1.000	*0.928*	*0.927*	*0.935*	*0.939*	**0.942**	**0.940**	**0.944**	**0.946**	**0.944**	**0.942**	**0.943**	**0.944**
	0.20	1.000	1.000	*0.929*	*0.931*	*0.938*	*0.936*	**0.940**	**0.946**	**0.942**	**0.941**	**0.943**	**0.946**	**0.946**	**0.946**
	0.30	1.000	1.000	*0.930*	*0.935*	**0.940**	*0.939*	**0.941**	**0.942**	**0.944**	**0.944**	**0.944**	**0.944**	**0.943**	**0.943**
	0.40	1.000	1.000	*0.934*	*0.929*	**0.941**	**0.943**	**0.944**	**0.942**	**0.945**	**0.947**	**0.946**	**0.946**	**0.946**	**0.946**
	0.50	1.000	1.000	*0.935*	*0.936*	**0.941**	**0.943**	**0.943**	**0.944**	**0.946**	**0.948**	**0.945**	**0.944**	**0.946**	**0.948**
	0.60	1.000	1.000	*0.938*	*0.937*	**0.943**	**0.942**	**0.944**	**0.943**	**0.944**	**0.942**	**0.945**	**0.945**	**0.945**	**0.945**
425	0.00	1.000	1.000	*0.931*	*0.933*	*0.938*	**0.942**	**0.942**	**0.943**	**0.944**	**0.948**	**0.945**	**0.944**	**0.947**	**0.947**
	0.10	1.000	1.000	*0.933*	*0.934*	**0.941**	**0.940**	**0.944**	**0.944**	**0.944**	**0.946**	**0.944**	**0.948**	**0.947**	**0.948**
	0.20	1.000	1.000	*0.933*	**0.940**	**0.942**	**0.942**	**0.945**	**0.946**	**0.945**	**0.944**	**0.947**	**0.942**	**0.946**	**0.947**
	0.30	1.000	1.000	*0.935*	*0.937*	**0.942**	**0.944**	**0.945**	**0.941**	**0.946**	**0.943**	**0.946**	**0.947**	**0.945**	**0.947**
	0.40	1.000	1.000	*0.936*	*0.935*	**0.943**	**0.941**	**0.945**	**0.946**	**0.944**	**0.949**	**0.946**	**0.947**	**0.949**	**0.948**
	0.50	1.000	1.000	*0.939*	*0.939*	**0.944**	**0.943**	**0.946**	**0.943**	**0.947**	**0.946**	**0.948**	**0.945**	**0.945**	**0.946**
	0.60	1.000	1.000	**0.941**	**0.941**	**0.944**	**0.947**	**0.946**	**0.945**	**0.945**	**0.947**	**0.947**	**0.947**	**0.946**	**0.945**
600	0.00	1.000	1.000	*0.935*	*0.935*	**0.943**	**0.941**	**0.945**	**0.944**	**0.945**	**0.946**	**0.947**	**0.944**	**0.946**	**0.950**
	0.10	1.000	1.000	*0.937*	*0.936*	**0.945**	**0.942**	**0.945**	**0.944**	**0.948**	**0.944**	**0.947**	**0.946**	**0.949**	**0.949**
	0.20	1.000	1.000	*0.939*	**0.941**	**0.945**	**0.947**	**0.946**	**0.942**	**0.946**	**0.944**	**0.948**	**0.948**	**0.948**	**0.944**
	0.30	1.000	1.000	*0.939*	*0.939*	**0.945**	**0.943**	**0.945**	**0.948**	**0.945**	**0.942**	**0.945**	**0.945**	**0.946**	**0.948**
	0.40	1.000	1.000	**0.940**	**0.942**	**0.945**	**0.945**	**0.947**	**0.946**	**0.949**	**0.948**	**0.948**	**0.951**	**0.948**	**0.948**
	0.50	1.000	1.000	**0.943**	**0.941**	**0.946**	**0.948**	**0.946**	**0.943**	**0.949**	**0.942**	**0.947**	**0.948**	**0.949**	**0.947**
	0.60	1.000	1.000	**0.943**	**0.942**	**0.946**	**0.945**	**0.946**	**0.945**	**0.949**	**0.948**	**0.948**	**0.948**	**0.947**	**0.945**

Note: Bold results are estimated coverage probabilities between .94 and .96; italicized results are estimated coverage probabilities between .925 and .975. Shaded columns are results from this study; unshaded columns are the results reported by Algina and Moulder (2001, p. 638-640).

Table 3-2. Replication of Algina and Moulder's Results for Multivariate Data and Six Predictors

		$\Delta\rho^2$													
n	ρ_r^2	0.00		0.05		0.10		0.15		0.20		0.25		0.30	
175	0.00	1.000	1.000	0.897	0.896	0.918	0.915	*0.927*	*0.925*	*0.930*	*0.928*	*0.933*	*0.937*	*0.935*	*0.936*
	0.10	1.000	1.000	0.903	0.906	0.920	0.920	*0.928*	0.923	*0.932*	*0.934*	*0.935*	*0.935*	*0.934*	*0.932*
	0.20	1.000	1.000	0.908	0.909	0.922	*0.926*	*0.926*	*0.931*	*0.931*	*0.934*	*0.934*	*0.928*	*0.934*	*0.935*
	0.30	1.000	1.000	0.909	0.914	0.922	*0.925*	*0.930*	*0.928*	*0.930*	*0.934*	*0.933*	*0.932*	*0.933*	*0.935*
	0.40	1.000	1.000	0.912	0.915	*0.925*	0.924	*0.930*	*0.932*	*0.932*	*0.925*	*0.932*	*0.932*	*0.934*	*0.933*
	0.50	1.000	1.000	0.918	0.918	*0.927*	*0.929*	*0.931*	*0.934*	*0.931*	*0.926*	*0.932*	*0.933*	*0.930*	*0.939*
	0.60	1.000	1.000	0.921	0.919	*0.929*	*0.933*	*0.929*	*0.934*	*0.930*	*0.929*	*0.932*	*0.931*	*0.931*	*0.927*
300	0.00	1.000	1.000	0.920	0.919	*0.932*	*0.932*	*0.935*	*0.936*	*0.938*	*0.937*	*0.939*	**0.940**	**0.942**	**0.941**
	0.10	1.000	1.000	0.923	0.918	*0.932*	*0.927*	*0.937*	*0.937*	*0.939*	*0.939*	*0.939*	**0.940**	**0.940**	**0.944**
	0.20	1.000	1.000	*0.925*	*0.926*	*0.933*	*0.935*	*0.938*	*0.939*	*0.939*	*0.939*	**0.940**	**0.941**	**0.940**	**0.942**
	0.30	1.000	1.000	*0.926*	0.924	*0.935*	*0.938*	*0.939*	*0.935*	**0.941**	**0.942**	**0.940**	*0.938*	**0.940**	**0.940**
	0.40	1.000	1.000	*0.927*	*0.931*	*0.936*	*0.931*	*0.936*	**0.940**	**0.941**	**0.942**	**0.941**	**0.940**	**0.940**	**0.942**
	0.50	1.000	1.000	*0.933*	*0.931*	*0.935*	*0.935*	*0.938*	*0.936*	*0.938*	*0.934*	**0.940**	**0.942**	*0.939*	*0.939*
	0.60	1.000	1.000	*0.933*	*0.933*	*0.938*	**0.943**	*0.938*	*0.934*	**0.940**	*0.939*	*0.939*	**0.940**	*0.939*	*0.937*
425	0.00	1.000	1.000	*0.927*	*0.925*	*0.938*	*0.939*	**0.940**	*0.937*	**0.941**	**0.945**	**0.943**	**0.941**	**0.944**	**0.944**
	0.10	1.000	1.000	*0.930*	*0.927*	*0.937*	*0.936*	**0.941**	*0.935*	**0.941**	**0.944**	**0.943**	**0.945**	**0.943**	**0.942**
	0.20	1.000	1.000	*0.931*	*0.932*	*0.939*	**0.942**	**0.940**	**0.942**	**0.943**	**0.941**	**0.944**	**0.944**	**0.943**	**0.945**
	0.30	1.000	1.000	*0.934*	*0.935*	*0.937*	**0.943**	**0.941**	**0.940**	**0.941**	**0.942**	**0.945**	*0.938*	**0.943**	**0.942**
	0.40	1.000	1.000	*0.935*	*0.937*	**0.941**	*0.937*	**0.942**	**0.944**	**0.943**	*0.939*	**0.941**	**0.944**	**0.943**	**0.946**
	0.50	1.000	1.000	*0.936*	*0.934*	**0.941**	**0.940**	**0.940**	*0.936*	**0.943**	**0.940**	**0.943**	**0.940**	**0.943**	**0.941**
	0.60	1.000	1.000	*0.936*	*0.939*	**0.942**	**0.944**	**0.942**	**0.943**	**0.941**	**0.944**	**0.941**	**0.944**	**0.941**	**0.944**
600	0.00	1.000	1.000	*0.933*	*0.934*	**0.941**	*0.938*	**0.944**	**0.941**	**0.944**	**0.944**	**0.945**	**0.949**	**0.946**	**0.947**
	0.10	1.000	1.000	*0.937*	*0.936*	**0.941**	**0.943**	**0.942**	**0.942**	**0.943**	**0.945**	**0.944**	**0.949**	**0.947**	**0.945**
	0.20	1.000	1.000	*0.937*	*0.935*	**0.942**	*0.936*	**0.941**	**0.943**	**0.943**	**0.947**	**0.944**	**0.946**	**0.945**	**0.946**
	0.30	1.000	1.000	*0.939*	**0.941**	**0.943**	*0.938*	**0.944**	**0.941**	**0.946**	**0.941**	**0.945**	**0.949**	**0.947**	**0.943**
	0.40	1.000	1.000	**0.940**	*0.939*	**0.942**	**0.941**	**0.946**	**0.942**	**0.945**	**0.941**	**0.945**	**0.946**	**0.946**	**0.941**
	0.50	1.000	1.000	**0.942**	*0.935*	**0.942**	**0.944**	**0.945**	**0.943**	**0.945**	**0.945**	**0.945**	**0.946**	**0.943**	**0.944**
	0.60	1.000	1.000	**0.941**	**0.942**	**0.942**	**0.942**	**0.945**	**0.945**	**0.945**	**0.946**	**0.943**	**0.942**	**0.944**	**0.946**

Note: Bold results are estimated coverage probabilities between .94 and .96; italicized results are estimated coverage probabilities between .925 and .975. Shaded columns are results from this study; unshaded columns are the results reported by Algina and Moulder (2001, p. 638-640).

Table 3-3. Replication of Algina and Moulder's Results for Multivariate Data and Ten Predictors.

		$\Delta\rho^2$													
n	ρ_r^2	0.00		0.05		0.10		0.15		0.20		0.25		0.30	
175	0.00	1.000	1.000	0.890	0.893	0.910	0.913	0.917	0.919	0.921	0.920	0.921	0.920	*0.927*	*0.925*
	0.10	1.000	1.000	0.895	0.888	0.910	0.916	0.919	0.918	0.919	0.922	0.923	*0.926*	0.923	*0.926*
	0.20	1.000	1.000	0.897	0.898	0.913	0.911	0.919	0.917	0.921	0.920	0.923	*0.927*	0.921	0.921
	0.30	1.000	1.000	0.901	0.904	0.916	0.918	0.920	0.920	0.920	0.921	0.923	0.921	0.922	0.914
	0.40	1.000	1.000	0.904	0.907	0.918	0.914	0.920	0.924	0.921	0.922	0.919	0.917	0.918	0.916
	0.50	1.000	1.000	0.909	0.910	0.920	0.916	0.918	0.924	0.918	0.914	0.917	0.922	0.916	0.916
	0.60	1.000	1.000	0.911	0.912	0.919	0.917	0.919	0.922	0.915	0.916	0.911	0.912	0.910	0.904
300	0.00	1.000	1.000	0.914	0.922	*0.928*	*0.926*	*0.929*	*0.934*	*0.935*	*0.935*	*0.936*	*0.934*	*0.938*	*0.935*
	0.10	1.000	1.000	0.917	0.921	*0.926*	0.923	*0.933*	*0.934*	*0.934*	*0.936*	*0.935*	*0.937*	*0.936*	*0.935*
	0.20	1.000	0.999	0.919	0.919	*0.927*	0.924	*0.933*	*0.934*	*0.932*	*0.936*	*0.936*	*0.937*	*0.935*	*0.931*
	0.30	1.000	1.000	0.922	0.923	*0.931*	*0.931*	*0.932*	*0.934*	*0.934*	*0.936*	*0.934*	*0.936*	*0.934*	*0.930*
	0.40	1.000	1.000	0.924	0.919	*0.929*	*0.934*	*0.934*	*0.937*	*0.933*	*0.932*	*0.932*	*0.934*	*0.931*	*0.931*
	0.50	1.000	1.000	0.924	0.922	*0.930*	*0.931*	*0.933*	*0.938*	*0.932*	*0.934*	*0.931*	*0.933*	*0.930*	*0.930*
	0.60	1.000	1.000	*0.926*	*0.927*	*0.930*	0.924	*0.931*	*0.935*	*0.931*	*0.932*	*0.927*	*0.935*	*0.927*	*0.926*
425	0.00	1.000	1.000	0.923	0.924	*0.935*	*0.935*	*0.936*	**0.941**	*0.938*	*0.937*	*0.938*	**0.941**	**0.940**	**0.940**
	0.10	1.000	1.000	*0.927*	*0.926*	*0.934*	*0.931*	*0.938*	*0.934*	*0.939*	*0.935*	*0.938*	*0.938*	**0.940**	**0.940**
	0.20	1.000	1.000	*0.928*	*0.928*	*0.936*	*0.939*	*0.937*	*0.938*	**0.940**	*0.938*	*0.939*	*0.939*	*0.939*	*0.934*
	0.30	1.000	1.000	*0.930*	*0.934*	*0.936*	*0.934*	*0.936*	*0.934*	*0.939*	*0.939*	*0.939*	*0.939*	*0.938*	*0.936*
	0.40	1.000	1.000	*0.932*	*0.930*	*0.936*	*0.934*	*0.938*	*0.936*	*0.939*	*0.937*	**0.941**	*0.935*	*0.936*	*0.937*
	0.50	1.000	1.000	*0.930*	*0.931*	*0.936*	*0.939*	*0.936*	*0.939*	*0.938*	*0.939*	*0.939*	*0.934*	*0.935*	*0.935*
	0.60	1.000	1.000	*0.935*	*0.934*	*0.936*	*0.935*	*0.937*	*0.937*	*0.935*	*0.938*	*0.937*	*0.935*	*0.936*	*0.938*
600	0.00	1.000	1.000	*0.933*	*0.926*	*0.938*	*0.938*	**0.941**	**0.940**	**0.944**	*0.938*	**0.942**	**0.943**	**0.943**	**0.946**
	0.10	1.000	1.000	0.933	*0.939*	*0.937*	**0.941**	**0.941**	**0.940**	**0.941**	**0.941**	**0.945**	**0.944**	**0.943**	*0.939*
	0.20	1.000	1.000	*0.935*	*0.929*	*0.939*	*0.937*	**0.940**	*0.938*	**0.941**	**0.949**	**0.942**	**0.940**	**0.944**	**0.941**
	0.30	1.000	1.000	*0.935*	*0.930*	*0.939*	**0.941**	**0.941**	**0.940**	**0.941**	**0.942**	**0.941**	**0.942**	**0.943**	*0.937*
	0.40	1.000	1.000	*0.936*	*0.937*	**0.941**	**0.944**	**0.944**	**0.944**	**0.942**	**0.942**	**0.942**	**0.940**	**0.942**	*0.937*
	0.50	1.000	1.000	*0.934*	**0.940**	**0.942**	**0.942**	**0.941**	*0.939*	**0.942**	**0.941**	**0.940**	**0.942**	*0.939*	*0.938*
	0.60	1.000	1.000	**0.940**	*0.939*	**0.940**	**0.940**	**0.940**	*0.936*	*0.939*	**0.940**	*0.939*	**0.942**	**0.940**	*0.937*

Note: Bold results are estimated coverage probabilities between .94 and .96; italicized results are estimated coverage probabilities between .925 and .975. Shaded columns are results from this study; unshaded columns are the results reported by Algina and Moulder (2001, p. 638-640).

Table 3-4. Empirical Coverage Probabilities for Normal Predictors and Normal Errors.

			k				
n	ρ_r^2	$\Delta\rho^2$	2	4	6	8	10
200	0.00	0.05	0.912	0.911	0.901	0.896	0.898
		0.10	*0.926*	*0.925*	*0.926*	0.918	0.915
		0.15	*0.930*	*0.930*	*0.932*	*0.928*	0.916
		0.20	*0.934*	*0.936*	*0.934*	*0.928*	0.924
		0.25	**0.941**	*0.936*	*0.934*	*0.927*	*0.928*
		0.30	**0.943**	**0.940**	**0.940**	*0.932*	*0.931*
	0.10	0.05	0.912	0.907	0.904	0.904	0.906
		0.10	*0.932*	*0.932*	*0.925*	0.923	0.918
		0.15	*0.934*	*0.935*	*0.929*	*0.925*	0.922
		0.20	**0.942**	*0.935*	*0.938*	*0.925*	*0.927*
		0.25	**0.940**	*0.938*	*0.936*	*0.930*	0.923
		0.30	**0.942**	**0.940**	**0.942**	*0.937*	*0.930*
	0.20	0.05	0.916	0.917	0.916	0.916	0.902
		0.10	*0.929*	*0.930*	*0.928*	0.921	0.922
		0.15	**0.942**	*0.933*	*0.932*	0.924	*0.925*
		0.20	*0.938*	*0.938*	*0.938*	*0.928*	*0.928*
		0.25	**0.942**	*0.937*	*0.939*	*0.930*	*0.928*
		0.30	**0.940**	**0.942**	**0.942**	*0.930*	*0.925*
	0.30	0.05	*0.925*	0.913	0.915	0.911	0.906
		0.10	*0.935*	*0.932*	*0.927*	0.924	0.919
		0.15	*0.939*	*0.932*	*0.933*	*0.931*	*0.926*
		0.20	*0.938*	*0.936*	*0.931*	0.923	*0.926*
		0.25	**0.944**	**0.940**	*0.935*	*0.931*	*0.928*
		0.30	**0.943**	**0.940**	*0.937*	*0.932*	*0.927*
	0.40	0.05	0.924	0.919	0.919	0.917	0.909
		0.10	*0.935*	*0.936*	*0.928*	*0.927*	0.919
		0.15	**0.941**	*0.935*	*0.933*	*0.930*	0.920
		0.20	**0.942**	*0.938*	*0.938*	*0.928*	0.922
		0.25	**0.945**	*0.939*	*0.933*	*0.930*	0.924
		0.30	**0.949**	*0.939*	*0.933*	*0.931*	0.920
	0.50	0.05	*0.932*	*0.927*	*0.925*	0.913	0.914
		0.10	*0.934*	*0.935*	*0.931*	0.923	0.922
		0.15	**0.946**	**0.941**	*0.935*	*0.928*	0.923
		0.20	**0.942**	*0.938*	*0.930*	*0.928*	0.924
		0.25	**0.940**	**0.942**	*0.937*	*0.930*	0.918
		0.30	**0.941**	**0.942**	*0.935*	*0.927*	0.921
	0.60	0.05	*0.937*	*0.928*	0.922	0.923	0.918
		0.10	**0.941**	*0.939*	*0.931*	0.923	0.924
		0.15	**0.943**	*0.937*	*0.931*	*0.929*	0.923
		0.20	**0.940**	*0.938*	*0.935*	*0.927*	0.921
		0.25	**0.949**	*0.938*	*0.935*	*0.929*	0.916
		0.30	**0.944**	*0.939*	*0.934*	0.923	0.919

Note: Bold results are estimated coverage probabilities between .94 and .96; italicized results are estimated coverage probabilities between .925 and .975.

Table 3-4. Continued

n	ρ_r^2	$\Delta\rho^2$	k = 2	4	6	8	10
400	0.00	0.05	*0.931*	*0.927*	*0.929*	0.923	0.924
		0.10	*0.935*	**0.940**	*0.934*	*0.934*	*0.936*
		0.15	**0.942**	**0.941**	**0.940**	*0.937*	*0.939*
		0.20	**0.940**	**0.945**	**0.940**	**0.941**	*0.937*
		0.25	**0.945**	**0.943**	**0.942**	**0.947**	*0.937*
		0.30	**0.946**	**0.947**	**0.943**	**0.942**	*0.938*
	0.10	0.05	*0.929*	*0.932*	*0.928*	*0.928*	*0.927*
		0.10	**0.940**	**0.942**	*0.939*	*0.938*	*0.936*
		0.15	**0.943**	**0.942**	**0.941**	**0.940**	*0.937*
		0.20	**0.946**	**0.942**	*0.938*	*0.939*	*0.936*
		0.25	**0.946**	**0.942**	**0.941**	**0.942**	**0.941**
		0.30	**0.946**	**0.947**	**0.946**	**0.945**	*0.937*
	0.20	0.05	*0.931*	*0.929*	*0.929*	*0.930*	*0.929*
		0.10	*0.939*	**0.940**	**0.941**	*0.937*	*0.931*
		0.15	**0.944**	**0.942**	*0.936*	*0.936*	*0.936*
		0.20	**0.944**	**0.945**	**0.942**	**0.943**	*0.935*
		0.25	**0.947**	**0.947**	**0.942**	*0.939*	**0.941**
		0.30	**0.942**	**0.946**	**0.940**	**0.942**	*0.936*
	0.30	0.05	*0.938*	*0.933*	*0.934*	*0.935*	*0.929*
		0.10	**0.941**	**0.944**	*0.935*	*0.936*	*0.937*
		0.15	**0.942**	**0.940**	**0.941**	*0.938*	*0.939*
		0.20	**0.945**	**0.944**	**0.942**	**0.941**	*0.938*
		0.25	**0.946**	**0.944**	**0.943**	*0.935*	*0.938*
		0.30	**0.946**	**0.945**	**0.943**	**0.940**	*0.939*
	0.40	0.05	*0.933*	*0.935*	*0.934*	*0.930*	*0.930*
		0.10	**0.944**	**0.941**	*0.936*	*0.937*	*0.931*
		0.15	**0.950**	**0.940**	**0.941**	**0.941**	*0.934*
		0.20	**0.944**	**0.947**	**0.944**	*0.939*	*0.939*
		0.25	**0.948**	**0.944**	**0.940**	**0.944**	*0.933*
		0.30	**0.947**	**0.946**	**0.943**	**0.944**	*0.934*
	0.50	0.05	*0.935*	*0.935*	*0.936*	*0.932*	*0.931*
		0.10	**0.949**	**0.940**	**0.942**	*0.936*	*0.937*
		0.15	**0.946**	**0.942**	**0.945**	*0.937*	*0.939*
		0.20	**0.950**	**0.942**	*0.938*	**0.941**	*0.937*
		0.25	**0.946**	**0.946**	**0.942**	*0.938*	*0.939*
		0.30	**0.948**	**0.949**	**0.943**	*0.939*	*0.937*
	0.60	0.05	**0.943**	**0.940**	*0.938*	*0.935*	*0.932*
		0.10	**0.941**	**0.943**	*0.934*	*0.935*	*0.938*
		0.15	**0.943**	**0.944**	**0.943**	*0.938*	*0.935*
		0.20	**0.945**	**0.943**	**0.944**	*0.939*	*0.935*
		0.25	**0.943**	**0.943**	*0.938*	*0.932*	*0.935*
		0.30	**0.950**	**0.947**	**0.945**	*0.936*	*0.927*

Table 3-4. Continued

			k				
n	ρ_r^2	$\Delta\rho^2$	2	4	6	8	10
600	0.00	0.05	*0.931*	*0.935*	*0.937*	*0.927*	*0.929*
		0.10	**0.945**	**0.943**	**0.942**	**0.941**	*0.936*
		0.15	**0.947**	**0.946**	**0.945**	**0.942**	*0.937*
		0.20	**0.945**	**0.949**	**0.944**	**0.941**	**0.942**
		0.25	**0.942**	**0.950**	**0.948**	**0.944**	**0.944**
		0.30	**0.948**	**0.944**	**0.945**	**0.947**	**0.941**
	0.10	0.05	**0.943**	*0.933*	**0.940**	*0.935*	*0.932*
		0.10	*0.939*	**0.942**	**0.946**	**0.940**	**0.940**
		0.15	**0.943**	**0.945**	**0.942**	**0.942**	**0.941**
		0.20	**0.945**	**0.950**	**0.940**	**0.943**	**0.942**
		0.25	**0.948**	**0.947**	**0.947**	**0.944**	**0.940**
		0.30	**0.950**	**0.949**	**0.941**	**0.942**	**0.943**
	0.20	0.05	*0.938*	*0.936*	*0.939*	*0.938*	*0.936*
		0.10	**0.948**	**0.944**	**0.946**	**0.940**	**0.941**
		0.15	**0.949**	**0.946**	**0.945**	**0.940**	**0.945**
		0.20	**0.945**	**0.949**	**0.946**	**0.943**	**0.943**
		0.25	**0.946**	**0.947**	**0.944**	**0.941**	**0.944**
		0.30	**0.952**	**0.946**	**0.946**	**0.947**	**0.940**
	0.30	0.05	*0.939*	**0.940**	*0.933*	*0.938*	*0.936*
		0.10	**0.945**	**0.942**	**0.941**	**0.943**	*0.938*
		0.15	**0.943**	**0.951**	**0.945**	**0.943**	**0.943**
		0.20	**0.944**	**0.945**	**0.945**	**0.941**	*0.938*
		0.25	**0.949**	**0.945**	**0.941**	**0.947**	**0.941**
		0.30	**0.948**	**0.945**	**0.948**	**0.941**	*0.938*
	0.40	0.05	*0.937*	**0.943**	*0.936*	*0.934*	*0.939*
		0.10	**0.949**	**0.941**	**0.944**	**0.943**	**0.940**
		0.15	**0.949**	**0.945**	**0.945**	**0.944**	**0.944**
		0.20	**0.949**	**0.945**	**0.944**	**0.946**	*0.939*
		0.25	**0.949**	**0.949**	**0.948**	**0.944**	**0.943**
		0.30	**0.945**	**0.949**	**0.945**	**0.943**	**0.943**
	0.50	0.05	**0.941**	**0.941**	**0.940**	**0.943**	*0.936*
		0.10	**0.946**	**0.949**	**0.941**	**0.946**	**0.941**
		0.15	**0.944**	**0.941**	**0.945**	**0.942**	**0.948**
		0.20	**0.950**	**0.950**	**0.943**	**0.943**	**0.944**
		0.25	**0.946**	**0.946**	**0.941**	**0.946**	**0.941**
		0.30	**0.951**	**0.947**	**0.945**	**0.945**	*0.938*
	0.60	0.05	**0.940**	**0.945**	**0.944**	**0.942**	**0.944**
		0.10	**0.945**	**0.946**	**0.945**	*0.939*	**0.944**
		0.15	**0.948**	**0.945**	**0.946**	**0.943**	**0.942**
		0.20	**0.952**	**0.942**	**0.941**	**0.940**	*0.937*
		0.25	**0.950**	**0.947**	**0.945**	**0.945**	**0.944**
		0.30	**0.948**	**0.948**	**0.945**	**0.942**	**0.944**

Table 3-4. Continued

n	ρ_r^2	$\Delta\rho^2$	*k*				
			2	4	6	8	10
800	0.00	0.05	*0.939*	**0.940**	*0.939*	*0.937*	*0.936*
		0.10	**0.944**	**0.943**	**0.946**	**0.942**	**0.945**
		0.15	**0.947**	**0.941**	**0.946**	**0.945**	**0.945**
		0.20	**0.948**	**0.951**	**0.943**	**0.944**	**0.946**
		0.25	**0.948**	**0.949**	**0.948**	**0.944**	**0.946**
		0.30	**0.949**	**0.951**	**0.946**	**0.945**	**0.944**
	0.10	0.05	*0.938*	**0.942**	*0.939*	*0.938*	*0.939*
		0.10	**0.943**	**0.949**	**0.942**	**0.941**	*0.938*
		0.15	**0.945**	**0.942**	**0.946**	**0.947**	**0.943**
		0.20	**0.940**	**0.943**	**0.948**	**0.946**	**0.943**
		0.25	**0.942**	**0.945**	**0.946**	**0.947**	**0.946**
		0.30	**0.951**	**0.948**	**0.946**	**0.947**	**0.944**
	0.20	0.05	*0.938*	**0.940**	*0.938*	*0.935*	**0.940**
		0.10	**0.944**	**0.944**	**0.944**	**0.945**	*0.939*
		0.15	**0.950**	**0.944**	**0.941**	**0.945**	**0.946**
		0.20	**0.946**	**0.946**	**0.947**	**0.945**	**0.946**
		0.25	**0.949**	**0.947**	**0.944**	**0.948**	**0.946**
		0.30	**0.947**	**0.948**	**0.943**	**0.946**	**0.941**
	0.30	0.05	**0.944**	*0.938*	**0.942**	*0.939*	*0.939*
		0.10	**0.949**	**0.945**	**0.946**	**0.942**	**0.942**
		0.15	**0.947**	**0.949**	**0.945**	**0.948**	**0.943**
		0.20	**0.943**	**0.949**	**0.946**	**0.949**	**0.943**
		0.25	**0.947**	**0.948**	**0.944**	*0.939*	**0.944**
		0.30	**0.951**	**0.945**	**0.948**	**0.948**	**0.943**
	0.40	0.05	**0.946**	*0.938*	**0.941**	**0.944**	**0.942**
		0.10	**0.947**	**0.947**	**0.945**	**0.944**	**0.942**
		0.15	**0.946**	**0.948**	**0.944**	**0.943**	**0.941**
		0.20	**0.947**	**0.946**	**0.947**	**0.944**	**0.944**
		0.25	**0.948**	**0.947**	**0.950**	**0.943**	**0.946**
		0.30	**0.948**	**0.945**	**0.946**	**0.946**	**0.941**
	0.50	0.05	**0.947**	**0.940**	**0.951**	**0.942**	*0.939*
		0.10	**0.948**	**0.947**	**0.949**	**0.943**	**0.941**
		0.15	**0.950**	**0.946**	**0.949**	**0.944**	*0.939*
		0.20	**0.949**	**0.944**	**0.944**	**0.949**	**0.944**
		0.25	**0.951**	**0.946**	**0.946**	**0.947**	**0.943**
		0.30	**0.953**	**0.948**	**0.951**	**0.944**	**0.942**
	0.60	0.05	**0.945**	**0.944**	**0.945**	**0.943**	*0.937*
		0.10	**0.953**	**0.948**	**0.948**	**0.944**	**0.943**
		0.15	**0.948**	**0.946**	**0.943**	**0.941**	**0.942**
		0.20	**0.943**	**0.948**	**0.945**	**0.946**	**0.942**
		0.25	**0.947**	**0.948**	**0.945**	**0.943**	**0.944**
		0.30	**0.950**	**0.951**	**0.944**	**0.945**	**0.943**

Table 3-4. Continued

n	ρ_r^2	$\Delta\rho^2$	k				
			2	4	6	8	10
1000	0.00	0.05	**0.940**	**0.943**	*0.938*	*0.939*	*0.937*
		0.10	**0.947**	**0.946**	**0.942**	**0.944**	**0.943**
		0.15	**0.947**	**0.945**	**0.945**	**0.941**	**0.946**
		0.20	**0.947**	**0.947**	**0.944**	**0.948**	**0.941**
		0.25	**0.949**	**0.947**	**0.946**	**0.948**	**0.950**
		0.30	**0.953**	**0.950**	**0.949**	**0.945**	**0.948**
	0.10	0.05	**0.945**	**0.946**	**0.945**	**0.944**	**0.943**
		0.10	**0.950**	**0.944**	**0.945**	**0.946**	**0.942**
		0.15	**0.945**	**0.948**	**0.943**	**0.945**	**0.947**
		0.20	**0.950**	**0.951**	**0.946**	**0.944**	**0.944**
		0.25	**0.949**	**0.949**	**0.947**	**0.948**	**0.944**
		0.30	**0.949**	**0.951**	**0.946**	**0.945**	**0.950**
	0.20	0.05	**0.944**	**0.944**	**0.945**	**0.943**	**0.940**
		0.10	**0.949**	**0.941**	**0.946**	**0.944**	**0.944**
		0.15	**0.948**	**0.944**	**0.947**	**0.947**	**0.946**
		0.20	**0.947**	**0.946**	**0.943**	**0.945**	**0.945**
		0.25	**0.950**	**0.948**	**0.949**	**0.948**	**0.947**
		0.30	**0.947**	**0.947**	**0.949**	**0.946**	**0.945**
	0.30	0.05	**0.945**	*0.937*	**0.940**	**0.946**	**0.942**
		0.10	**0.946**	**0.949**	**0.943**	**0.943**	**0.944**
		0.15	**0.945**	**0.948**	**0.946**	**0.944**	**0.946**
		0.20	**0.948**	**0.948**	**0.945**	**0.948**	**0.945**
		0.25	**0.947**	**0.949**	**0.947**	**0.943**	**0.943**
		0.30	**0.951**	**0.948**	**0.947**	**0.945**	**0.950**
	0.40	0.05	**0.943**	**0.944**	**0.948**	**0.945**	**0.940**
		0.10	**0.944**	**0.945**	**0.948**	**0.946**	**0.946**
		0.15	**0.947**	**0.949**	**0.945**	**0.940**	**0.952**
		0.20	**0.948**	**0.947**	**0.944**	**0.947**	**0.949**
		0.25	**0.947**	**0.948**	**0.949**	**0.948**	**0.945**
		0.30	**0.951**	**0.946**	**0.950**	**0.949**	**0.947**
	0.50	0.05	**0.946**	**0.946**	**0.945**	**0.946**	**0.947**
		0.10	**0.950**	**0.943**	**0.948**	**0.942**	**0.943**
		0.15	**0.949**	**0.949**	**0.945**	**0.944**	**0.945**
		0.20	**0.951**	**0.948**	**0.948**	**0.943**	**0.946**
		0.25	**0.946**	**0.947**	**0.946**	**0.947**	**0.948**
		0.30	**0.948**	**0.945**	**0.951**	**0.947**	**0.941**
	0.60	0.05	**0.946**	**0.943**	**0.942**	**0.945**	**0.943**
		0.10	**0.951**	**0.945**	**0.946**	**0.945**	**0.946**
		0.15	**0.946**	**0.944**	**0.944**	**0.943**	**0.948**
		0.20	**0.951**	**0.950**	**0.947**	**0.945**	**0.942**
		0.25	**0.944**	**0.949**	**0.944**	**0.946**	**0.941**
		0.30	**0.947**	**0.951**	**0.949**	**0.944**	**0.943**

Table 3-4. Continued

n	ρ_r^2	$\Delta\rho^2$	k = 2	4	6	8	10
1500	0.00	0.05	**0.946**	**0.942**	**0.944**	**0.944**	**0.944**
		0.10	**0.949**	**0.947**	**0.949**	**0.943**	**0.945**
		0.15	**0.946**	**0.949**	**0.949**	**0.947**	**0.945**
		0.20	**0.949**	**0.944**	**0.949**	**0.945**	**0.947**
		0.25	**0.947**	**0.946**	**0.947**	**0.945**	**0.946**
		0.30	**0.950**	**0.948**	**0.949**	**0.945**	**0.944**
	0.10	0.05	**0.945**	**0.945**	**0.949**	**0.942**	**0.946**
		0.10	**0.945**	**0.944**	**0.948**	**0.945**	**0.947**
		0.15	**0.950**	**0.949**	**0.954**	**0.947**	**0.952**
		0.20	**0.945**	**0.951**	**0.953**	**0.946**	**0.945**
		0.25	**0.949**	**0.947**	**0.948**	**0.945**	**0.949**
		0.30	**0.949**	**0.948**	**0.947**	**0.949**	**0.944**
	0.20	0.05	**0.945**	**0.944**	**0.945**	**0.940**	**0.944**
		0.10	**0.951**	**0.943**	**0.945**	**0.945**	**0.948**
		0.15	**0.949**	**0.950**	**0.947**	**0.952**	**0.945**
		0.20	**0.948**	**0.949**	**0.950**	**0.948**	**0.948**
		0.25	**0.951**	**0.947**	**0.946**	**0.949**	**0.947**
		0.30	**0.947**	**0.946**	**0.948**	**0.952**	**0.952**
	0.30	0.05	**0.951**	**0.942**	**0.945**	**0.941**	**0.943**
		0.10	**0.950**	**0.950**	**0.947**	**0.947**	**0.945**
		0.15	**0.947**	**0.949**	**0.946**	**0.950**	**0.946**
		0.20	**0.950**	**0.947**	**0.950**	**0.949**	**0.948**
		0.25	**0.949**	**0.952**	**0.954**	**0.950**	**0.941**
		0.30	**0.953**	**0.952**	**0.949**	**0.949**	**0.944**
	0.40	0.05	**0.944**	**0.944**	**0.942**	**0.942**	**0.948**
		0.10	**0.949**	**0.949**	**0.947**	**0.948**	**0.948**
		0.15	**0.949**	**0.955**	**0.947**	**0.948**	**0.944**
		0.20	**0.947**	**0.950**	**0.947**	**0.946**	**0.945**
		0.25	**0.946**	**0.949**	**0.949**	**0.947**	**0.945**
		0.30	**0.947**	**0.952**	**0.947**	**0.947**	**0.949**
	0.50	0.05	**0.948**	**0.948**	**0.947**	**0.945**	**0.943**
		0.10	**0.949**	**0.951**	**0.950**	**0.950**	**0.942**
		0.15	**0.947**	**0.951**	**0.950**	**0.946**	**0.949**
		0.20	**0.951**	**0.951**	**0.947**	**0.948**	**0.945**
		0.25	**0.952**	**0.947**	**0.948**	**0.947**	**0.947**
		0.30	**0.947**	**0.952**	**0.945**	**0.950**	**0.949**
	0.60	0.05	**0.951**	**0.950**	**0.944**	**0.949**	**0.945**
		0.10	**0.948**	**0.948**	**0.948**	**0.947**	**0.947**
		0.15	**0.948**	**0.951**	**0.949**	**0.948**	**0.949**
		0.20	**0.953**	**0.952**	**0.944**	**0.946**	**0.948**
		0.25	**0.949**	**0.951**	**0.948**	**0.946**	**0.941**
		0.30	**0.950**	**0.951**	**0.952**	**0.949**	**0.947**

Table 3-4. Continued

n	ρ_r^2	$\Delta\rho^2$	k				
			2	4	6	8	10
2000	0.00	0.05	**0.946**	**0.945**	**0.945**	**0.943**	**0.940**
		0.10	**0.950**	**0.948**	**0.945**	**0.948**	**0.944**
		0.15	**0.946**	**0.944**	**0.948**	**0.952**	**0.950**
		0.20	**0.947**	**0.944**	**0.949**	**0.948**	**0.949**
		0.25	**0.945**	**0.945**	**0.949**	**0.948**	**0.945**
		0.30	**0.951**	**0.949**	**0.947**	**0.944**	**0.951**
	0.10	0.05	**0.949**	**0.948**	**0.943**	**0.946**	**0.946**
		0.10	**0.949**	**0.952**	**0.951**	**0.947**	**0.944**
		0.15	**0.947**	**0.945**	**0.945**	**0.947**	**0.948**
		0.20	**0.946**	**0.954**	**0.949**	**0.949**	**0.948**
		0.25	**0.945**	**0.950**	**0.948**	**0.946**	**0.948**
		0.30	**0.947**	**0.943**	**0.950**	**0.950**	**0.950**
	0.20	0.05	**0.951**	**0.946**	**0.944**	**0.947**	**0.944**
		0.10	**0.946**	**0.947**	**0.943**	**0.948**	**0.943**
		0.15	**0.949**	**0.952**	**0.950**	**0.951**	**0.949**
		0.20	**0.948**	**0.951**	**0.946**	**0.952**	**0.950**
		0.25	**0.948**	**0.950**	**0.947**	**0.944**	**0.949**
		0.30	**0.945**	**0.945**	**0.951**	**0.949**	**0.946**
	0.30	0.05	**0.947**	**0.948**	**0.945**	**0.950**	**0.944**
		0.10	**0.945**	**0.951**	**0.945**	**0.948**	**0.945**
		0.15	**0.946**	**0.947**	**0.950**	**0.947**	**0.948**
		0.20	**0.949**	**0.948**	**0.948**	**0.945**	**0.945**
		0.25	**0.945**	**0.951**	**0.950**	**0.947**	**0.946**
		0.30	**0.948**	**0.949**	**0.950**	**0.945**	**0.945**
	0.40	0.05	**0.948**	**0.951**	**0.948**	**0.947**	**0.948**
		0.10	**0.947**	**0.949**	**0.948**	**0.947**	**0.944**
		0.15	**0.946**	**0.951**	**0.950**	**0.951**	**0.949**
		0.20	**0.955**	**0.951**	**0.949**	**0.948**	**0.944**
		0.25	**0.949**	**0.947**	**0.948**	**0.950**	**0.948**
		0.30	**0.949**	**0.951**	**0.943**	**0.952**	**0.948**
	0.50	0.05	**0.949**	**0.944**	**0.946**	**0.946**	**0.948**
		0.10	**0.948**	**0.951**	**0.947**	**0.952**	**0.945**
		0.15	**0.949**	**0.946**	**0.950**	**0.947**	**0.947**
		0.20	**0.946**	**0.949**	**0.948**	**0.948**	**0.950**
		0.25	**0.948**	**0.953**	**0.951**	**0.950**	**0.947**
		0.30	**0.948**	**0.949**	**0.949**	**0.952**	**0.949**
	0.60	0.05	**0.948**	**0.950**	**0.943**	**0.944**	**0.945**
		0.10	**0.947**	**0.946**	**0.949**	**0.954**	**0.941**
		0.15	**0.949**	**0.949**	**0.950**	**0.948**	**0.946**
		0.20	**0.953**	**0.951**	**0.950**	**0.949**	**0.948**
		0.25	**0.946**	**0.951**	**0.947**	**0.951**	**0.944**
		0.30	**0.953**	**0.952**	**0.950**	**0.948**	**0.943**

Table 3-5. Empirical Coverage Probabilities for Normal Predictors and Nonnormal Errors.

			k				
n	ρ_r^2	$\Delta\rho^2$	2	4	6	8	10
200	0.00	0.05	0.910	0.906	0.902	0.901	0.898
		0.10	0.919	0.917	0.920	0.910	0.910
		0.15	0.921	0.920	0.919	0.916	0.909
		0.20	0.920	0.920	0.915	0.914	0.912
		0.25	0.916	0.917	0.914	0.913	0.907
		0.30	0.916	0.916	0.913	0.909	0.906
	0.10	0.05	0.913	0.908	0.906	0.902	0.901
		0.10	0.924	0.921	0.918	0.915	0.913
		0.15	*0.925*	*0.926*	0.921	0.918	0.915
		0.20	*0.925*	0.923	0.920	0.914	0.911
		0.25	0.923	0.924	0.918	0.918	0.911
		0.30	0.922	0.921	0.916	0.914	0.909
	0.20	0.05	0.915	0.912	0.907	0.909	0.904
		0.10	*0.927*	*0.926*	0.922	0.916	0.916
		0.15	*0.928*	0.924	0.922	0.922	0.920
		0.20	*0.931*	*0.927*	*0.925*	0.921	0.919
		0.25	*0.929*	*0.927*	0.924	0.919	0.913
		0.30	*0.927*	*0.925*	0.923	0.918	0.915
	0.30	0.05	0.920	0.916	0.914	0.909	0.906
		0.10	*0.930*	*0.927*	0.924	0.918	0.917
		0.15	*0.934*	*0.929*	*0.925*	0.923	0.918
		0.20	*0.932*	*0.929*	*0.925*	0.920	0.920
		0.25	*0.932*	*0.930*	*0.926*	0.923	0.919
		0.30	*0.934*	*0.930*	*0.928*	0.922	0.916
	0.40	0.05	0.924	0.919	0.914	0.913	0.910
		0.10	*0.932*	*0.930*	*0.926*	0.921	0.918
		0.15	*0.935*	*0.931*	*0.930*	*0.925*	0.919
		0.20	*0.935*	*0.932*	*0.927*	*0.925*	0.919
		0.25	*0.937*	*0.933*	*0.929*	*0.925*	0.917
		0.30	*0.938*	*0.935*	*0.931*	0.921	0.917
	0.50	0.05	*0.926*	0.924	0.920	0.915	0.913
		0.10	*0.935*	*0.931*	*0.928*	0.924	0.921
		0.15	*0.937*	*0.932*	*0.928*	*0.925*	0.920
		0.20	*0.938*	*0.935*	*0.933*	*0.928*	0.921
		0.25	**0.941**	*0.934*	*0.930*	0.924	0.917
		0.30	*0.939*	*0.936*	*0.931*	*0.927*	0.918
	0.60	0.05	*0.932*	*0.925*	*0.926*	0.920	0.914
		0.10	*0.938*	*0.937*	*0.930*	*0.925*	0.921
		0.15	**0.940**	*0.937*	*0.930*	*0.925*	0.920
		0.20	**0.942**	*0.938*	*0.930*	*0.925*	0.920
		0.25	**0.941**	*0.937*	*0.930*	*0.926*	0.917
		0.30	**0.943**	*0.937*	*0.930*	0.923	0.915

Note: Bold results are estimated coverage probabilities between .94 and .96; italicized results are estimated coverage probabilities between .925 and .975.

Table 3-5. Continued

n	ρ_r^2	$\Delta\rho^2$	k = 2	4	6	8	10
400	0.00	0.05	*0.927*	*0.925*	*0.927*	0.922	0.922
		0.10	*0.930*	*0.929*	*0.931*	*0.928*	*0.927*
		0.15	*0.931*	*0.932*	*0.931*	0.923	0.924
		0.20	*0.927*	*0.929*	*0.925*	0.923	0.922
		0.25	0.924	0.921	0.922	0.921	0.921
		0.30	0.918	0.920	0.920	0.921	0.916
	0.10	0.05	*0.929*	*0.926*	*0.926*	*0.926*	0.920
		0.10	*0.933*	*0.932*	*0.930*	*0.928*	*0.927*
		0.15	*0.932*	*0.931*	*0.929*	*0.927*	*0.926*
		0.20	*0.930*	*0.930*	*0.929*	*0.928*	*0.925*
		0.25	*0.929*	*0.928*	*0.926*	*0.925*	*0.925*
		0.30	0.924	0.924	*0.925*	0.921	0.919
	0.20	0.05	*0.930*	*0.927*	*0.926*	*0.926*	*0.925*
		0.10	*0.934*	*0.934*	*0.933*	*0.930*	*0.929*
		0.15	*0.932*	*0.932*	*0.932*	*0.929*	*0.925*
		0.20	*0.933*	*0.933*	*0.930*	*0.930*	*0.927*
		0.25	*0.935*	*0.934*	*0.930*	*0.930*	*0.926*
		0.30	*0.932*	*0.931*	*0.930*	*0.926*	*0.925*
	0.30	0.05	*0.933*	*0.930*	*0.928*	*0.930*	0.924
		0.10	*0.936*	*0.938*	*0.933*	*0.932*	*0.932*
		0.15	*0.938*	*0.937*	*0.932*	*0.933*	*0.931*
		0.20	*0.937*	*0.937*	*0.933*	*0.932*	*0.930*
		0.25	*0.935*	*0.936*	*0.935*	*0.931*	*0.931*
		0.30	*0.938*	*0.936*	*0.934*	*0.931*	*0.930*
	0.40	0.05	*0.936*	*0.935*	*0.931*	*0.932*	*0.928*
		0.10	*0.937*	*0.938*	*0.938*	*0.934*	*0.934*
		0.15	*0.939*	*0.938*	*0.937*	*0.934*	*0.931*
		0.20	**0.940**	*0.939*	*0.937*	*0.934*	*0.932*
		0.25	**0.940**	*0.935*	*0.937*	*0.934*	*0.931*
		0.30	**0.943**	*0.937*	*0.938*	*0.933*	*0.930*
	0.50	0.05	*0.937*	*0.937*	*0.935*	*0.932*	*0.931*
		0.10	*0.939*	*0.939*	*0.937*	*0.937*	*0.933*
		0.15	**0.942**	**0.940**	*0.938*	*0.935*	*0.934*
		0.20	**0.945**	*0.939*	*0.936*	*0.937*	*0.933*
		0.25	**0.945**	**0.941**	*0.938*	*0.936*	*0.934*
		0.30	**0.944**	**0.941**	**0.940**	*0.939*	*0.931*
	0.60	0.05	*0.939*	*0.936*	*0.936*	*0.933*	*0.931*
		0.10	**0.944**	**0.943**	*0.938*	*0.937*	*0.934*
		0.15	**0.943**	**0.940**	**0.942**	*0.938*	*0.933*
		0.20	**0.946**	**0.943**	*0.939*	*0.935*	*0.934*
		0.25	**0.947**	**0.944**	**0.941**	*0.938*	*0.932*
		0.30	**0.944**	**0.944**	*0.939*	*0.938*	*0.932*

Table 3-5. Continued

n	ρ_r^2	$\Delta\rho^2$	k = 2	4	6	8	10
600	0.00	0.05	*0.932*	*0.933*	*0.931*	*0.932*	*0.927*
		0.10	*0.932*	*0.931*	*0.931*	*0.932*	*0.930*
		0.15	*0.933*	*0.930*	*0.931*	*0.930*	*0.928*
		0.20	*0.929*	*0.930*	*0.930*	*0.926*	*0.925*
		0.25	0.922	*0.926*	*0.926*	0.923	0.922
		0.30	0.924	0.922	0.921	0.918	0.922
	0.10	0.05	*0.935*	*0.935*	*0.932*	*0.930*	*0.930*
		0.10	*0.935*	*0.936*	*0.934*	*0.933*	*0.932*
		0.15	*0.937*	*0.932*	*0.934*	*0.931*	*0.930*
		0.20	*0.934*	*0.932*	*0.931*	*0.931*	*0.928*
		0.25	*0.929*	*0.930*	*0.927*	*0.928*	0.922
		0.30	*0.927*	*0.927*	*0.927*	*0.926*	*0.926*
	0.20	0.05	*0.936*	*0.935*	*0.933*	*0.932*	*0.933*
		0.10	*0.937*	*0.938*	*0.935*	*0.935*	*0.935*
		0.15	**0.940**	*0.935*	*0.934*	*0.934*	*0.932*
		0.20	*0.938*	*0.936*	*0.934*	*0.935*	*0.932*
		0.25	*0.935*	*0.931*	*0.931*	*0.932*	*0.928*
		0.30	*0.934*	*0.935*	*0.929*	*0.930*	*0.929*
	0.30	0.05	*0.938*	*0.936*	*0.936*	*0.934*	*0.935*
		0.10	**0.940**	*0.938*	*0.936*	*0.935*	*0.934*
		0.15	**0.940**	*0.937*	*0.938*	*0.935*	*0.932*
		0.20	*0.938*	*0.935*	*0.937*	*0.936*	*0.933*
		0.25	*0.939*	*0.937*	*0.937*	*0.936*	*0.932*
		0.30	*0.938*	*0.938*	*0.933*	*0.934*	*0.931*
	0.40	0.05	*0.937*	*0.938*	*0.936*	*0.935*	*0.935*
		0.10	**0.942**	**0.940**	**0.940**	*0.937*	*0.936*
		0.15	**0.941**	**0.940**	*0.939*	*0.938*	*0.935*
		0.20	**0.940**	*0.939*	*0.938*	*0.937*	*0.937*
		0.25	**0.941**	**0.941**	*0.939*	*0.939*	*0.934*
		0.30	**0.940**	**0.940**	*0.939*	*0.936*	*0.936*
	0.50	0.05	**0.941**	**0.941**	*0.938*	*0.937*	*0.936*
		0.10	**0.942**	**0.943**	*0.939*	*0.937*	*0.938*
		0.15	**0.945**	**0.941**	**0.941**	*0.938*	*0.939*
		0.20	**0.944**	**0.941**	**0.943**	*0.939*	*0.938*
		0.25	**0.943**	**0.944**	**0.942**	*0.939*	*0.937*
		0.30	**0.945**	**0.944**	*0.939*	**0.940**	*0.936*
	0.60	0.05	**0.944**	**0.941**	**0.940**	*0.939*	*0.937*
		0.10	**0.945**	**0.945**	**0.941**	**0.942**	*0.938*
		0.15	**0.943**	**0.942**	**0.941**	**0.942**	*0.936*
		0.20	**0.944**	**0.943**	**0.942**	**0.943**	*0.937*
		0.25	**0.947**	**0.943**	**0.942**	**0.941**	*0.938*
		0.30	**0.946**	**0.946**	**0.942**	*0.939*	*0.938*

Table 3-5. Continued

n	ρ_r^2	$\Delta\rho^2$	k = 2	4	6	8	10
800	0.00	0.05	*0.936*	*0.936*	*0.933*	*0.936*	*0.933*
		0.10	*0.937*	*0.933*	*0.934*	*0.933*	*0.931*
		0.15	*0.933*	*0.933*	*0.933*	*0.932*	*0.930*
		0.20	*0.932*	*0.929*	*0.928*	*0.928*	*0.926*
		0.25	*0.926*	*0.928*	*0.926*	0.924	0.924
		0.30	0.923	0.921	0.924	0.921	0.920
	0.10	0.05	*0.939*	*0.938*	*0.934*	*0.935*	*0.934*
		0.10	*0.938*	*0.934*	*0.938*	*0.933*	*0.934*
		0.15	*0.934*	*0.935*	*0.934*	*0.936*	*0.932*
		0.20	*0.933*	*0.935*	*0.931*	*0.930*	*0.930*
		0.25	*0.930*	*0.930*	*0.930*	*0.929*	*0.928*
		0.30	*0.929*	*0.926*	*0.928*	*0.925*	0.923
	0.20	0.05	*0.937*	*0.937*	*0.937*	*0.935*	*0.937*
		0.10	*0.939*	**0.940**	*0.939*	*0.938*	*0.934*
		0.15	*0.938*	*0.938*	*0.938*	*0.938*	*0.935*
		0.20	*0.937*	*0.935*	*0.937*	*0.934*	*0.933*
		0.25	*0.934*	*0.935*	*0.932*	*0.934*	*0.933*
		0.30	*0.933*	*0.932*	*0.933*	*0.931*	*0.930*
	0.30	0.05	**0.941**	*0.936*	*0.938*	*0.938*	*0.936*
		0.10	**0.942**	**0.943**	*0.939*	*0.935*	*0.939*
		0.15	**0.941**	**0.940**	*0.939*	*0.938*	*0.937*
		0.20	**0.941**	*0.938*	*0.939*	*0.937*	*0.934*
		0.25	*0.938*	*0.937*	*0.938*	*0.937*	*0.935*
		0.30	*0.938*	*0.937*	*0.938*	*0.936*	*0.933*
	0.40	0.05	**0.940**	**0.940**	**0.941**	*0.938*	*0.938*
		0.10	**0.943**	**0.941**	**0.941**	*0.939*	*0.937*
		0.15	**0.942**	**0.941**	**0.942**	**0.940**	*0.938*
		0.20	**0.942**	**0.941**	**0.940**	*0.937*	*0.939*
		0.25	**0.941**	**0.941**	*0.938*	*0.938*	*0.936*
		0.30	**0.943**	*0.938*	*0.938*	*0.939*	*0.937*
	0.50	0.05	**0.943**	**0.942**	**0.941**	**0.942**	*0.938*
		0.10	**0.943**	**0.940**	**0.942**	**0.942**	*0.939*
		0.15	**0.945**	**0.942**	**0.943**	**0.942**	**0.940**
		0.20	**0.944**	**0.943**	**0.942**	*0.938*	**0.940**
		0.25	**0.946**	**0.944**	**0.943**	*0.939*	**0.941**
		0.30	**0.945**	**0.943**	**0.944**	**0.942**	**0.941**
	0.60	0.05	**0.943**	**0.940**	**0.940**	**0.943**	**0.940**
		0.10	**0.946**	**0.944**	**0.945**	**0.942**	**0.942**
		0.15	**0.946**	**0.944**	**0.943**	**0.941**	**0.942**
		0.20	**0.945**	**0.945**	**0.943**	**0.943**	**0.941**
		0.25	**0.943**	**0.946**	**0.945**	**0.942**	**0.942**
		0.30	**0.948**	**0.944**	**0.944**	**0.941**	**0.941**

Table 3-5. Continued

n	ρ_r^2	$\Delta\rho^2$	*k* 2	4	6	8	10
1000	0.00	0.05	*0.938*	*0.939*	*0.936*	*0.935*	*0.935*
		0.10	*0.938*	*0.937*	*0.937*	*0.936*	*0.935*
		0.15	*0.936*	*0.932*	*0.932*	*0.934*	*0.937*
		0.20	*0.933*	*0.930*	*0.931*	*0.930*	*0.927*
		0.25	*0.927*	*0.930*	*0.927*	0.923	*0.927*
		0.30	0.924	0.924	0.922	0.922	0.923
	0.10	0.05	*0.938*	*0.938*	*0.937*	*0.938*	*0.937*
		0.10	*0.939*	*0.936*	*0.938*	*0.936*	*0.934*
		0.15	*0.936*	*0.939*	*0.937*	*0.935*	*0.935*
		0.20	*0.936*	*0.934*	*0.933*	*0.931*	*0.935*
		0.25	*0.929*	*0.930*	*0.931*	*0.928*	*0.929*
		0.30	*0.929*	*0.928*	*0.928*	*0.925*	*0.927*
	0.20	0.05	**0.940**	*0.939*	**0.940**	*0.938*	*0.937*
		0.10	*0.939*	**0.941**	*0.938*	*0.938*	*0.939*
		0.15	*0.939*	*0.939*	*0.936*	*0.938*	*0.936*
		0.20	*0.935*	*0.937*	*0.934*	*0.934*	*0.934*
		0.25	*0.936*	*0.934*	*0.933*	*0.934*	*0.933*
		0.30	*0.934*	*0.934*	*0.932*	*0.934*	*0.931*
	0.30	0.05	**0.940**	*0.939*	**0.940**	*0.939*	**0.940**
		0.10	**0.941**	**0.941**	**0.941**	**0.940**	*0.938*
		0.15	**0.942**	*0.939*	**0.941**	*0.939*	*0.936*
		0.20	**0.941**	*0.939*	*0.938*	*0.938*	*0.937*
		0.25	**0.941**	*0.937*	*0.936*	*0.938*	*0.936*
		0.30	*0.939*	*0.939*	*0.938*	*0.936*	*0.938*
	0.40	0.05	**0.943**	**0.940**	**0.941**	**0.940**	*0.938*
		0.10	**0.943**	**0.943**	**0.941**	**0.943**	**0.942**
		0.15	**0.943**	**0.941**	**0.943**	**0.940**	**0.940**
		0.20	**0.942**	**0.941**	*0.939*	*0.938*	*0.937*
		0.25	**0.941**	*0.939*	**0.940**	**0.940**	**0.941**
		0.30	**0.941**	**0.942**	*0.939*	**0.940**	**0.941**
	0.50	0.05	**0.943**	**0.944**	**0.943**	**0.940**	**0.940**
		0.10	**0.946**	**0.944**	**0.943**	**0.943**	**0.944**
		0.15	**0.942**	**0.945**	**0.942**	**0.941**	**0.942**
		0.20	**0.943**	**0.944**	**0.942**	**0.942**	**0.940**
		0.25	**0.943**	**0.943**	**0.943**	**0.941**	**0.940**
		0.30	**0.946**	**0.945**	**0.944**	**0.941**	**0.940**
	0.60	0.05	**0.945**	**0.944**	**0.944**	**0.941**	**0.941**
		0.10	**0.946**	**0.942**	**0.944**	**0.943**	**0.940**
		0.15	**0.945**	**0.945**	**0.945**	**0.941**	**0.942**
		0.20	**0.946**	**0.947**	**0.945**	**0.943**	**0.941**
		0.25	**0.947**	**0.947**	**0.944**	**0.943**	**0.941**
		0.30	**0.948**	**0.947**	**0.946**	**0.942**	**0.945**

Table 3-5. Continued

n	ρ_r^2	$\Delta\rho^2$	k = 2	4	6	8	10
1500	0.00	0.05	**0.941**	**0.940**	*0.939*	*0.939*	*0.938*
		0.10	*0.939*	*0.937*	**0.940**	*0.937*	*0.936*
		0.15	*0.937*	*0.935*	*0.935*	*0.934*	*0.935*
		0.20	*0.932*	*0.933*	*0.930*	*0.931*	*0.932*
		0.25	*0.929*	*0.927*	*0.929*	*0.926*	*0.928*
		0.30	0.924	*0.925*	0.921	0.924	0.922
	0.10	0.05	**0.943**	**0.941**	**0.940**	**0.940**	*0.938*
		0.10	**0.941**	*0.939*	**0.940**	**0.940**	*0.938*
		0.15	*0.937*	*0.939*	*0.939*	*0.935*	*0.936*
		0.20	*0.934*	*0.935*	*0.933*	*0.933*	*0.934*
		0.25	*0.933*	*0.931*	*0.932*	*0.931*	*0.930*
		0.30	*0.931*	*0.928*	*0.930*	*0.926*	*0.931*
	0.20	0.05	**0.944**	**0.942**	**0.944**	**0.941**	**0.942**
		0.10	**0.940**	**0.943**	**0.940**	**0.941**	*0.939*
		0.15	*0.939*	**0.940**	**0.940**	*0.937*	**0.941**
		0.20	*0.939*	*0.939*	*0.935*	*0.937*	*0.936*
		0.25	*0.933*	*0.935*	*0.937*	*0.934*	*0.935*
		0.30	*0.935*	*0.934*	*0.936*	*0.931*	*0.934*
	0.30	0.05	**0.942**	**0.942**	**0.943**	**0.941**	**0.944**
		0.10	**0.945**	**0.944**	**0.943**	**0.943**	**0.942**
		0.15	**0.943**	**0.940**	**0.941**	**0.942**	*0.939*
		0.20	*0.939*	**0.941**	**0.942**	*0.939*	*0.937*
		0.25	*0.939*	*0.939*	*0.938*	*0.937*	*0.936*
		0.30	**0.941**	**0.940**	*0.937*	*0.938*	*0.938*
	0.40	0.05	**0.945**	**0.944**	**0.943**	**0.943**	**0.942**
		0.10	**0.943**	**0.941**	**0.944**	**0.942**	**0.943**
		0.15	**0.944**	**0.941**	**0.942**	**0.940**	**0.943**
		0.20	**0.942**	**0.941**	**0.943**	**0.940**	**0.940**
		0.25	**0.942**	**0.943**	**0.942**	**0.941**	*0.939*
		0.30	**0.942**	**0.941**	**0.944**	**0.942**	**0.941**
	0.50	0.05	**0.944**	**0.946**	**0.945**	**0.943**	**0.945**
		0.10	**0.945**	**0.943**	**0.941**	**0.943**	**0.944**
		0.15	**0.944**	**0.944**	**0.945**	**0.943**	**0.942**
		0.20	**0.944**	**0.946**	**0.943**	**0.942**	**0.945**
		0.25	**0.945**	**0.945**	**0.943**	**0.946**	**0.941**
		0.30	**0.947**	**0.945**	**0.943**	**0.944**	**0.944**
	0.60	0.05	**0.947**	**0.947**	**0.945**	**0.947**	**0.946**
		0.10	**0.947**	**0.946**	**0.946**	**0.943**	**0.943**
		0.15	**0.944**	**0.946**	**0.945**	**0.944**	**0.945**
		0.20	**0.946**	**0.945**	**0.946**	**0.942**	**0.944**
		0.25	**0.948**	**0.944**	**0.947**	**0.947**	**0.944**
		0.30	**0.950**	**0.948**	**0.948**	**0.946**	**0.943**

Table 3-5. Continued

n	ρ_r^2	$\Delta\rho^2$	k = 2	4	6	8	10
2000	0.00	0.05	**0.944**	**0.944**	**0.940**	**0.940**	**0.941**
		0.10	**0.940**	*0.937*	**0.940**	*0.939*	*0.939*
		0.15	*0.936*	*0.938*	*0.935*	*0.936*	*0.935*
		0.20	*0.933*	*0.933*	*0.933*	*0.931*	*0.932*
		0.25	*0.928*	*0.930*	*0.928*	*0.931*	*0.926*
		0.30	0.924	*0.925*	0.923	0.921	0.924
	0.10	0.05	**0.941**	**0.941**	**0.942**	**0.941**	**0.941**
		0.10	**0.942**	**0.941**	**0.942**	**0.940**	**0.940**
		0.15	**0.940**	*0.935*	*0.938*	*0.938*	*0.937*
		0.20	*0.936*	*0.933*	*0.934*	*0.934*	*0.936*
		0.25	*0.929*	*0.931*	*0.932*	*0.930*	*0.930*
		0.30	*0.931*	*0.930*	*0.929*	*0.930*	*0.930*
	0.20	0.05	**0.943**	**0.944**	**0.943**	**0.942**	**0.943**
		0.10	**0.941**	**0.940**	**0.940**	**0.942**	**0.942**
		0.15	**0.942**	**0.940**	**0.940**	*0.939*	**0.940**
		0.20	**0.940**	*0.937*	*0.939*	*0.935*	*0.936*
		0.25	*0.935*	*0.936*	*0.936*	*0.935*	*0.935*
		0.30	*0.934*	*0.935*	*0.933*	*0.932*	*0.933*
	0.30	0.05	**0.943**	**0.944**	**0.943**	**0.944**	**0.943**
		0.10	**0.942**	**0.944**	**0.943**	**0.943**	**0.942**
		0.15	**0.942**	**0.940**	**0.940**	**0.940**	**0.940**
		0.20	**0.943**	**0.940**	**0.942**	**0.940**	*0.938*
		0.25	**0.941**	*0.938*	**0.941**	*0.938*	*0.936*
		0.30	*0.939*	*0.938*	*0.938*	*0.939*	*0.938*
	0.40	0.05	**0.947**	**0.947**	**0.946**	**0.945**	**0.945**
		0.10	**0.945**	**0.943**	**0.942**	**0.943**	**0.943**
		0.15	**0.942**	**0.944**	**0.943**	**0.944**	**0.944**
		0.20	**0.941**	**0.942**	**0.941**	**0.942**	**0.940**
		0.25	**0.943**	**0.944**	**0.942**	**0.942**	**0.941**
		0.30	**0.942**	**0.941**	**0.941**	**0.941**	**0.942**
	0.50	0.05	**0.945**	**0.948**	**0.946**	**0.947**	**0.944**
		0.10	**0.946**	**0.945**	**0.945**	**0.946**	**0.944**
		0.15	**0.944**	**0.945**	**0.946**	**0.943**	**0.942**
		0.20	**0.944**	**0.944**	**0.944**	**0.944**	**0.942**
		0.25	**0.944**	**0.944**	**0.943**	**0.943**	**0.942**
		0.30	**0.946**	**0.946**	**0.946**	**0.946**	**0.943**
	0.60	0.05	**0.946**	**0.945**	**0.947**	**0.945**	**0.945**
		0.10	**0.948**	**0.947**	**0.945**	**0.945**	**0.944**
		0.15	**0.946**	**0.949**	**0.948**	**0.943**	**0.946**
		0.20	**0.948**	**0.946**	**0.946**	**0.946**	**0.944**
		0.25	**0.948**	**0.949**	**0.947**	**0.946**	**0.946**
		0.30	**0.949**	**0.949**	**0.948**	**0.946**	**0.946**

Table 3-6. Empirical Coverage Probabilities for Nonnormal Predictors and Normal Errors.

			k				
n	ρ_r^2	$\Delta\rho^2$	2	4	6	8	10
200	0.00	0.05	0.904	0.904	0.898	0.898	0.892
		0.10	0.918	0.914	0.914	0.909	0.906
		0.15	0.919	0.918	0.916	0.911	0.909
		0.20	0.921	0.917	0.917	0.910	0.907
		0.25	0.919	0.917	0.913	0.911	0.908
		0.30	0.916	0.912	0.910	0.910	0.902
	0.10	0.05	0.910	0.908	0.903	0.897	0.898
		0.10	0.919	0.914	0.912	0.910	0.905
		0.15	0.921	0.918	0.915	0.911	0.906
		0.20	0.919	0.914	0.911	0.911	0.905
		0.25	0.918	0.915	0.913	0.906	0.902
		0.30	0.912	0.915	0.907	0.903	0.899
	0.20	0.05	0.910	0.911	0.907	0.902	0.899
		0.10	0.918	0.916	0.914	0.908	0.905
		0.15	0.920	0.917	0.913	0.908	0.905
		0.20	0.919	0.915	0.911	0.907	0.904
		0.25	0.917	0.910	0.909	0.903	0.900
		0.30	0.914	0.907	0.904	0.899	0.893
	0.30	0.05	0.914	0.910	0.909	0.903	0.901
		0.10	0.919	0.916	0.913	0.910	0.906
		0.15	0.918	0.915	0.911	0.907	0.903
		0.20	0.913	0.911	0.907	0.903	0.900
		0.25	0.906	0.904	0.903	0.901	0.893
		0.30	0.902	0.901	0.898	0.892	0.886
	0.40	0.05	0.918	0.913	0.909	0.905	0.902
		0.10	0.917	0.916	0.912	0.910	0.904
		0.15	0.914	0.911	0.909	0.903	0.897
		0.20	0.910	0.906	0.901	0.893	0.888
		0.25	0.901	0.897	0.894	0.890	0.885
		0.30	0.894	0.889	0.888	0.879	0.873
	0.50	0.05	0.916	0.913	0.909	0.906	0.901
		0.10	0.915	0.911	0.907	0.905	0.900
		0.15	0.907	0.906	0.900	0.898	0.890
		0.20	0.899	0.894	0.891	0.886	0.879
		0.25	0.890	0.887	0.881	0.876	0.871
		0.30	0.881	0.878	0.875	0.865	0.860
	0.60	0.05	0.917	0.913	0.910	0.905	0.900
		0.10	0.910	0.907	0.901	0.897	0.893
		0.15	0.896	0.896	0.891	0.880	0.879
		0.20	0.887	0.882	0.878	0.873	0.864
		0.25	0.871	0.870	0.863	0.858	0.853
		0.30	0.866	0.860	0.857	0.848	0.841

Note: Bold results are estimated coverage probabilities between .94 and .96; italicized results are estimated coverage probabilities between .925 and .975.

Table 3-6. Continued

n	ρ_r^2	$\Delta\rho^2$	k = 2	4	6	8	10
400	0.00	0.05	0.924	*0.925*	0.920	0.922	0.917
		0.10	*0.930*	*0.928*	*0.928*	*0.928*	0.922
		0.15	*0.926*	*0.929*	*0.926*	0.924	0.924
		0.20	*0.927*	*0.925*	*0.925*	0.924	0.920
		0.25	0.922	*0.925*	0.923	0.919	0.916
		0.30	0.919	0.922	0.917	0.914	0.913
	0.10	0.05	*0.927*	0.924	0.922	0.923	0.919
		0.10	*0.929*	*0.929*	*0.928*	*0.926*	0.923
		0.15	*0.927*	*0.927*	*0.926*	0.922	0.920
		0.20	0.924	*0.925*	0.922	0.923	0.917
		0.25	0.922	0.921	0.917	0.916	0.916
		0.30	0.916	0.919	0.917	0.911	0.909
	0.20	0.05	*0.926*	*0.928*	0.924	0.922	0.919
		0.10	*0.929*	*0.926*	*0.926*	0.924	0.922
		0.15	*0.926*	*0.927*	0.923	0.921	0.921
		0.20	*0.925*	0.922	0.919	0.919	0.916
		0.25	0.918	0.916	0.917	0.914	0.914
		0.30	0.915	0.914	0.912	0.912	0.908
	0.30	0.05	*0.928*	*0.929*	*0.926*	0.924	0.919
		0.10	*0.929*	*0.927*	*0.927*	0.924	0.921
		0.15	0.924	0.923	0.923	0.920	0.918
		0.20	0.917	0.918	0.916	0.912	0.911
		0.25	0.912	0.912	0.911	0.905	0.904
		0.30	0.906	0.904	0.904	0.902	0.899
	0.40	0.05	*0.929*	*0.926*	*0.925*	0.923	0.922
		0.10	0.924	0.924	0.924	0.920	0.919
		0.15	0.918	0.918	0.915	0.915	0.912
		0.20	0.914	0.914	0.909	0.911	0.904
		0.25	0.907	0.903	0.901	0.897	0.894
		0.30	0.898	0.897	0.896	0.895	0.891
	0.50	0.05	*0.926*	*0.926*	*0.925*	0.922	0.922
		0.10	0.923	0.920	0.920	0.918	0.914
		0.15	0.913	0.908	0.909	0.904	0.903
		0.20	0.901	0.903	0.901	0.898	0.892
		0.25	0.895	0.891	0.891	0.887	0.884
		0.30	0.887	0.886	0.882	0.878	0.873
	0.60	0.05	*0.926*	*0.925*	0.924	0.920	0.920
		0.10	0.915	0.916	0.910	0.909	0.908
		0.15	0.901	0.901	0.898	0.895	0.893
		0.20	0.891	0.887	0.888	0.881	0.876
		0.25	0.877	0.875	0.873	0.870	0.866
		0.30	0.870	0.866	0.861	0.859	0.856

Table 3-6. Continued

n	ρ_r^2	$\Delta\rho^2$	k				
			2	4	6	8	10
600	0.00	0.05	*0.931*	*0.929*	*0.930*	*0.928*	*0.927*
		0.10	*0.934*	*0.933*	*0.931*	*0.930*	*0.928*
		0.15	*0.934*	*0.932*	*0.928*	*0.929*	*0.928*
		0.20	*0.932*	*0.928*	*0.927*	*0.928*	*0.926*
		0.25	*0.927*	0.924	0.923	0.922	0.922
		0.30	0.922	0.922	0.922	0.919	0.918
	0.10	0.05	*0.934*	*0.933*	*0.928*	*0.928*	*0.928*
		0.10	*0.933*	*0.934*	*0.931*	*0.931*	*0.929*
		0.15	*0.931*	*0.929*	*0.928*	*0.928*	*0.926*
		0.20	*0.926*	*0.925*	*0.927*	0.923	0.923
		0.25	0.924	0.924	0.921	0.919	0.919
		0.30	0.918	0.921	0.916	0.916	0.917
	0.20	0.05	*0.932*	*0.932*	*0.929*	*0.930*	*0.929*
		0.10	*0.930*	*0.934*	*0.931*	*0.930*	*0.930*
		0.15	*0.930*	*0.930*	*0.925*	*0.925*	0.923
		0.20	*0.926*	*0.927*	0.923	0.920	0.919
		0.25	0.921	0.919	0.916	0.917	0.917
		0.30	0.917	0.915	0.914	0.911	0.912
	0.30	0.05	*0.932*	*0.930*	*0.929*	*0.930*	*0.929*
		0.10	*0.931*	*0.931*	*0.930*	*0.927*	*0.927*
		0.15	*0.927*	*0.927*	0.924	0.920	0.920
		0.20	0.919	0.916	0.917	0.917	0.912
		0.25	0.915	0.915	0.913	0.913	0.910
		0.30	0.909	0.905	0.906	0.908	0.904
	0.40	0.05	*0.931*	*0.931*	*0.931*	*0.929*	*0.929*
		0.10	*0.930*	*0.927*	*0.926*	0.923	0.922
		0.15	0.922	0.921	0.923	0.917	0.915
		0.20	0.914	0.914	0.911	0.911	0.907
		0.25	0.908	0.907	0.906	0.903	0.902
		0.30	0.898	0.902	0.895	0.894	0.892
	0.50	0.05	*0.931*	*0.933*	*0.930*	*0.928*	*0.926*
		0.10	*0.926*	0.921	0.919	0.918	0.919
		0.15	0.914	0.914	0.912	0.910	0.907
		0.20	0.905	0.904	0.904	0.901	0.897
		0.25	0.892	0.893	0.892	0.887	0.890
		0.30	0.886	0.885	0.884	0.880	0.878
	0.60	0.05	*0.930*	*0.925*	*0.929*	*0.927*	0.924
		0.10	0.918	0.917	0.915	0.913	0.910
		0.15	0.902	0.903	0.900	0.900	0.896
		0.20	0.887	0.889	0.889	0.885	0.881
		0.25	0.880	0.876	0.876	0.875	0.869
		0.30	0.868	0.867	0.863	0.863	0.863

Table 3-6. Continued

			k				
n	ρ_r^2	$\Delta\rho^2$	2	4	6	8	10
800	0.00	0.05	*0.937*	*0.935*	*0.931*	*0.934*	*0.932*
		0.10	*0.936*	*0.935*	*0.936*	*0.934*	*0.934*
		0.15	*0.933*	*0.932*	*0.934*	*0.930*	*0.931*
		0.20	*0.930*	*0.929*	*0.929*	*0.929*	*0.928*
		0.25	*0.925*	*0.926*	*0.925*	*0.925*	0.922
		0.30	0.924	0.922	0.922	0.923	0.922
	0.10	0.05	*0.938*	*0.933*	*0.934*	*0.935*	*0.932*
		0.10	*0.934*	*0.934*	*0.934*	*0.935*	*0.931*
		0.15	*0.931*	*0.931*	*0.932*	*0.928*	*0.931*
		0.20	*0.928*	*0.929*	*0.926*	*0.926*	0.924
		0.25	0.923	*0.926*	*0.926*	0.921	0.921
		0.30	0.920	0.920	0.921	0.918	0.918
	0.20	0.05	*0.935*	*0.937*	*0.933*	*0.935*	*0.932*
		0.10	*0.934*	*0.933*	*0.933*	*0.933*	*0.931*
		0.15	*0.932*	*0.928*	*0.929*	*0.928*	*0.927*
		0.20	*0.926*	0.924	*0.925*	0.924	0.922
		0.25	0.922	0.920	0.918	0.919	0.916
		0.30	0.918	0.914	0.915	0.912	0.914
	0.30	0.05	*0.936*	*0.937*	*0.933*	*0.931*	*0.931*
		0.10	*0.931*	*0.932*	*0.930*	*0.929*	*0.929*
		0.15	*0.927*	*0.927*	0.923	0.923	*0.927*
		0.20	0.920	0.919	0.921	0.917	0.917
		0.25	0.916	0.914	0.913	0.913	0.912
		0.30	0.909	0.908	0.907	0.907	0.906
	0.40	0.05	*0.936*	*0.934*	*0.935*	*0.935*	*0.934*
		0.10	*0.933*	*0.930*	*0.930*	*0.928*	*0.927*
		0.15	0.922	0.924	0.921	0.920	0.920
		0.20	0.916	0.913	0.910	0.913	0.910
		0.25	0.907	0.907	0.906	0.903	0.900
		0.30	0.902	0.899	0.899	0.897	0.895
	0.50	0.05	*0.933*	*0.934*	*0.931*	*0.932*	*0.928*
		0.10	*0.926*	*0.925*	*0.925*	0.923	0.922
		0.15	0.916	0.912	0.913	0.912	0.911
		0.20	0.907	0.906	0.908	0.902	0.898
		0.25	0.896	0.893	0.895	0.891	0.889
		0.30	0.885	0.885	0.884	0.883	0.881
	0.60	0.05	*0.932*	*0.929*	*0.931*	*0.927*	*0.928*
		0.10	0.918	0.917	0.918	0.915	0.914
		0.15	0.901	0.901	0.901	0.903	0.899
		0.20	0.893	0.889	0.891	0.887	0.887
		0.25	0.879	0.878	0.874	0.873	0.872
		0.30	0.869	0.864	0.866	0.866	0.861

Table 3-6. Continued

			k				
n	ρ_r^2	$\Delta\rho^2$	2	4	6	8	10
1000	0.00	0.05	*0.936*	*0.938*	*0.934*	*0.935*	*0.935*
		0.10	*0.938*	*0.938*	*0.936*	*0.935*	*0.934*
		0.15	*0.933*	*0.934*	*0.934*	*0.934*	*0.931*
		0.20	*0.931*	*0.931*	*0.929*	*0.929*	*0.927*
		0.25	*0.927*	*0.927*	*0.927*	*0.928*	0.924
		0.30	0.923	0.922	0.923	*0.925*	0.922
	0.10	0.05	*0.937*	*0.936*	*0.936*	*0.936*	*0.934*
		0.10	*0.936*	*0.936*	*0.935*	*0.934*	*0.932*
		0.15	*0.932*	*0.933*	*0.931*	*0.929*	*0.932*
		0.20	*0.930*	*0.930*	*0.927*	*0.928*	*0.926*
		0.25	*0.927*	0.924	*0.925*	*0.926*	0.923
		0.30	0.920	0.923	0.915	0.920	0.920
	0.20	0.05	**0.941**	*0.937*	*0.935*	*0.938*	*0.935*
		0.10	*0.935*	*0.935*	*0.933*	*0.933*	*0.932*
		0.15	*0.930*	*0.931*	*0.930*	*0.929*	*0.931*
		0.20	*0.927*	*0.928*	*0.925*	0.924	0.921
		0.25	0.922	0.921	0.920	0.920	0.919
		0.30	0.917	0.916	0.916	0.913	0.910
	0.30	0.05	*0.938*	*0.936*	*0.935*	*0.937*	*0.935*
		0.10	*0.933*	*0.936*	*0.934*	*0.932*	*0.929*
		0.15	*0.928*	*0.928*	*0.927*	*0.927*	*0.928*
		0.20	0.923	0.923	0.920	0.917	0.920
		0.25	0.917	0.915	0.915	0.914	0.914
		0.30	0.911	0.908	0.909	0.907	0.907
	0.40	0.05	*0.933*	*0.938*	*0.934*	*0.932*	*0.934*
		0.10	*0.932*	*0.929*	*0.928*	*0.929*	*0.928*
		0.15	0.924	0.922	0.920	0.922	0.922
		0.20	0.915	0.916	0.913	0.912	0.912
		0.25	0.907	0.908	0.906	0.904	0.902
		0.30	0.901	0.898	0.900	0.898	0.898
	0.50	0.05	*0.934*	*0.936*	*0.934*	*0.933*	*0.933*
		0.10	*0.928*	*0.927*	0.924	*0.926*	*0.925*
		0.15	0.914	0.914	0.914	0.914	0.915
		0.20	0.906	0.903	0.902	0.902	0.902
		0.25	0.897	0.894	0.893	0.897	0.894
		0.30	0.889	0.887	0.884	0.885	0.882
	0.60	0.05	*0.932*	*0.932*	*0.930*	*0.931*	*0.929*
		0.10	0.922	0.917	0.917	0.918	0.916
		0.15	0.904	0.906	0.902	0.902	0.900
		0.20	0.892	0.888	0.889	0.885	0.886
		0.25	0.880	0.880	0.876	0.874	0.872
		0.30	0.867	0.867	0.867	0.866	0.863

Table 3-6. Continued

n	ρ_r^2	$\Delta\rho^2$	k = 2	4	6	8	10
1500	0.00	0.05	**0.941**	*0.939*	*0.939*	**0.940**	**0.941**
		0.10	*0.939*	*0.938*	*0.938*	*0.938*	*0.934*
		0.15	*0.934*	*0.932*	*0.937*	*0.935*	*0.936*
		0.20	*0.932*	*0.931*	*0.929*	*0.931*	*0.931*
		0.25	*0.928*	*0.928*	*0.927*	*0.925*	*0.928*
		0.30	0.923	*0.925*	0.923	0.921	0.921
	0.10	0.05	**0.940**	**0.940**	*0.939*	*0.938*	*0.938*
		0.10	*0.936*	*0.937*	*0.939*	*0.938*	*0.934*
		0.15	*0.934*	*0.934*	*0.935*	*0.933*	*0.930*
		0.20	*0.930*	*0.928*	*0.930*	*0.930*	*0.929*
		0.25	*0.926*	*0.925*	*0.925*	0.924	0.924
		0.30	0.922	0.921	0.923	0.916	0.917
	0.20	0.05	**0.941**	**0.941**	*0.939*	*0.938*	*0.938*
		0.10	*0.937*	*0.936*	*0.934*	*0.935*	*0.935*
		0.15	*0.932*	*0.930*	*0.931*	*0.930*	*0.928*
		0.20	*0.928*	*0.927*	*0.926*	*0.925*	*0.925*
		0.25	0.922	0.922	0.921	0.919	0.920
		0.30	0.918	0.915	0.917	0.918	0.913
	0.30	0.05	*0.939*	*0.938*	*0.938*	*0.937*	**0.940**
		0.10	*0.933*	*0.933*	*0.935*	*0.930*	*0.932*
		0.15	*0.930*	*0.930*	*0.929*	*0.929*	*0.927*
		0.20	0.923	0.922	0.920	0.922	0.921
		0.25	0.915	0.918	0.914	0.915	0.913
		0.30	0.910	0.912	0.910	0.908	0.907
	0.40	0.05	*0.937*	*0.938*	*0.938*	*0.936*	*0.934*
		0.10	*0.932*	*0.932*	*0.932*	*0.931*	*0.929*
		0.15	*0.925*	0.923	0.923	*0.925*	0.919
		0.20	0.918	0.913	0.915	0.912	0.913
		0.25	0.909	0.908	0.905	0.909	0.908
		0.30	0.901	0.901	0.902	0.900	0.898
	0.50	0.05	*0.935*	*0.934*	*0.938*	*0.935*	*0.936*
		0.10	*0.927*	*0.925*	0.924	*0.925*	0.924
		0.15	0.919	0.918	0.914	0.916	0.911
		0.20	0.906	0.904	0.907	0.903	0.900
		0.25	0.896	0.897	0.896	0.898	0.893
		0.30	0.890	0.891	0.886	0.888	0.883
	0.60	0.05	*0.933*	*0.935*	*0.932*	*0.931*	*0.933*
		0.10	0.918	0.920	0.917	0.917	0.915
		0.15	0.906	0.903	0.903	0.905	0.903
		0.20	0.891	0.891	0.889	0.891	0.890
		0.25	0.880	0.875	0.878	0.878	0.878
		0.30	0.868	0.867	0.867	0.867	0.866

Table 3-6. Continued

n	ρ_r^2	$\Delta\rho^2$	k				
			2	4	6	8	10
2000	0.00	0.05	**0.941**	**0.941**	**0.940**	**0.941**	*0.939*
		0.10	*0.939*	*0.938*	*0.939*	*0.939*	*0.939*
		0.15	*0.937*	*0.935*	*0.936*	*0.935*	*0.935*
		0.20	*0.931*	*0.929*	*0.930*	*0.933*	*0.930*
		0.25	*0.926*	*0.927*	0.924	*0.927*	*0.928*
		0.30	*0.925*	*0.925*	*0.926*	0.924	0.922
	0.10	0.05	**0.941**	**0.942**	**0.941**	*0.939*	**0.941**
		0.10	**0.942**	*0.939*	*0.938*	*0.937*	*0.937*
		0.15	*0.936*	*0.934*	*0.934*	*0.936*	*0.933*
		0.20	*0.931*	*0.932*	*0.928*	*0.931*	*0.928*
		0.25	*0.926*	*0.926*	*0.927*	*0.925*	*0.926*
		0.30	0.922	0.920	0.922	0.920	0.924
	0.20	0.05	**0.943**	**0.942**	**0.940**	**0.940**	**0.941**
		0.10	*0.938*	*0.937*	*0.937*	*0.938*	*0.937*
		0.15	*0.933*	*0.931*	*0.931*	*0.932*	*0.930*
		0.20	*0.925*	*0.928*	*0.927*	*0.925*	*0.928*
		0.25	0.923	0.923	0.921	0.921	0.921
		0.30	0.917	0.919	0.917	0.916	0.917
	0.30	0.05	*0.939*	*0.938*	**0.942**	**0.941**	*0.938*
		0.10	*0.936*	*0.934*	*0.937*	*0.935*	*0.934*
		0.15	*0.927*	*0.928*	*0.926*	*0.928*	*0.928*
		0.20	0.921	0.920	0.923	0.921	0.924
		0.25	0.914	0.917	0.915	0.916	0.914
		0.30	0.910	0.907	0.911	0.912	0.908
	0.40	0.05	**0.941**	*0.939*	*0.939*	*0.937*	**0.941**
		0.10	*0.931*	*0.933*	*0.934*	*0.929*	*0.929*
		0.15	0.924	0.924	0.923	0.923	0.924
		0.20	0.915	0.915	0.916	0.916	0.913
		0.25	0.908	0.908	0.904	0.905	0.909
		0.30	0.904	0.899	0.897	0.899	0.898
	0.50	0.05	*0.939*	*0.937*	*0.938*	*0.935*	*0.935*
		0.10	*0.927*	*0.927*	*0.925*	*0.926*	*0.925*
		0.15	0.918	0.915	0.915	0.914	0.914
		0.20	0.907	0.906	0.905	0.907	0.905
		0.25	0.894	0.900	0.897	0.896	0.895
		0.30	0.889	0.889	0.888	0.885	0.881
	0.60	0.05	*0.934*	*0.933*	*0.933*	*0.934*	*0.930*
		0.10	0.918	0.920	0.917	0.918	0.916
		0.15	0.906	0.906	0.905	0.903	0.903
		0.20	0.892	0.890	0.892	0.888	0.890
		0.25	0.879	0.879	0.878	0.877	0.875
		0.30	0.872	0.870	0.869	0.868	0.866

Table 3-7. Empirical Coverage Probabilities for Predictors Nonnormal and Errors Nonnormal.

			k				
n	ρ_r^2	$\Delta\rho^2$	2	4	6	8	10
200	0.00	0.05	0.903	0.900	0.897	0.894	0.891
		0.10	0.911	0.908	0.905	0.902	0.899
		0.15	0.909	0.907	0.904	0.900	0.898
		0.20	0.904	0.901	0.898	0.895	0.893
		0.25	0.897	0.896	0.893	0.890	0.888
		0.30	0.891	0.890	0.888	0.885	0.881
	0.10	0.05	0.908	0.904	0.900	0.896	0.892
		0.10	0.914	0.911	0.909	0.903	0.901
		0.15	0.910	0.908	0.906	0.901	0.898
		0.20	0.906	0.904	0.901	0.898	0.892
		0.25	0.902	0.899	0.896	0.893	0.888
		0.30	0.894	0.893	0.891	0.886	0.882
	0.20	0.05	0.910	0.906	0.903	0.901	0.896
		0.10	0.915	0.911	0.909	0.906	0.901
		0.15	0.912	0.909	0.905	0.901	0.899
		0.20	0.906	0.904	0.901	0.897	0.893
		0.25	0.901	0.897	0.895	0.893	0.886
		0.30	0.896	0.893	0.891	0.887	0.881
	0.30	0.05	0.913	0.909	0.906	0.902	0.899
		0.10	0.916	0.913	0.908	0.905	0.901
		0.15	0.912	0.908	0.905	0.902	0.896
		0.20	0.906	0.902	0.900	0.896	0.891
		0.25	0.899	0.897	0.893	0.889	0.884
		0.30	0.894	0.892	0.888	0.883	0.878
	0.40	0.05	0.916	0.913	0.907	0.904	0.900
		0.10	0.915	0.913	0.908	0.904	0.901
		0.15	0.909	0.906	0.903	0.898	0.895
		0.20	0.903	0.899	0.894	0.891	0.885
		0.25	0.895	0.892	0.888	0.883	0.877
		0.30	0.888	0.886	0.883	0.875	0.868
	0.50	0.05	0.916	0.913	0.909	0.904	0.902
		0.10	0.912	0.909	0.906	0.901	0.897
		0.15	0.905	0.900	0.897	0.894	0.888
		0.20	0.896	0.893	0.887	0.882	0.876
		0.25	0.888	0.882	0.878	0.873	0.867
		0.30	0.879	0.875	0.871	0.865	0.857
	0.60	0.05	0.916	0.913	0.910	0.906	0.901
		0.10	0.908	0.905	0.902	0.895	0.891
		0.15	0.896	0.892	0.889	0.883	0.877
		0.20	0.885	0.881	0.875	0.870	0.863
		0.25	0.874	0.870	0.864	0.857	0.849
		0.30	0.863	0.860	0.856	0.847	0.838

Note: Bold results are estimated coverage probabilities between .94 and .96; italicized results are estimated coverage probabilities between .925 and .975.

Table 3-7. Continued

n	ρ_r^2	$\Delta\rho^2$	k				
			2	4	6	8	10
400	0.00	0.05	0.920	0.919	0.919	0.916	0.915
		0.10	0.922	0.921	0.920	0.917	0.915
		0.15	0.916	0.916	0.915	0.912	0.911
		0.20	0.910	0.907	0.907	0.906	0.905
		0.25	0.903	0.902	0.901	0.902	0.897
		0.30	0.896	0.895	0.894	0.893	0.890
	0.10	0.05	0.923	0.923	0.920	0.918	0.916
		0.10	0.923	0.922	0.922	0.919	0.917
		0.15	0.918	0.917	0.915	0.914	0.912
		0.20	0.912	0.910	0.909	0.907	0.906
		0.25	0.905	0.904	0.903	0.901	0.900
		0.30	0.901	0.899	0.897	0.895	0.893
	0.20	0.05	*0.926*	0.923	0.922	0.920	0.918
		0.10	0.924	0.920	0.922	0.919	0.917
		0.15	0.919	0.916	0.916	0.913	0.911
		0.20	0.911	0.910	0.909	0.907	0.906
		0.25	0.905	0.903	0.905	0.901	0.898
		0.30	0.900	0.898	0.897	0.896	0.894
	0.30	0.05	*0.926*	0.924	0.923	0.921	0.918
		0.10	0.923	0.922	0.920	0.918	0.916
		0.15	0.917	0.915	0.914	0.913	0.912
		0.20	0.910	0.907	0.907	0.905	0.902
		0.25	0.902	0.902	0.901	0.899	0.895
		0.30	0.899	0.896	0.893	0.893	0.891
	0.40	0.05	*0.926*	0.924	0.924	0.922	0.918
		0.10	0.923	0.919	0.920	0.917	0.916
		0.15	0.916	0.913	0.911	0.909	0.907
		0.20	0.907	0.905	0.902	0.900	0.898
		0.25	0.898	0.898	0.895	0.893	0.888
		0.30	0.891	0.889	0.889	0.885	0.883
	0.50	0.05	*0.927*	*0.926*	0.923	0.920	0.920
		0.10	0.921	0.918	0.918	0.913	0.912
		0.15	0.910	0.907	0.906	0.903	0.900
		0.20	0.900	0.897	0.894	0.893	0.890
		0.25	0.890	0.887	0.885	0.882	0.879
		0.30	0.880	0.880	0.878	0.875	0.872
	0.60	0.05	*0.925*	0.923	0.921	0.920	0.918
		0.10	0.913	0.912	0.910	0.907	0.906
		0.15	0.900	0.898	0.895	0.893	0.891
		0.20	0.887	0.886	0.883	0.881	0.875
		0.25	0.876	0.873	0.871	0.869	0.864
		0.30	0.864	0.864	0.862	0.859	0.853

Table 3-7. Continued

n	ρ_r^2	$\Delta\rho^2$	k = 2	4	6	8	10
600	0.00	0.05	*0.928*	*0.927*	*0.926*	0.924	0.923
		0.10	*0.926*	*0.925*	0.923	0.923	0.921
		0.15	0.919	0.918	0.918	0.918	0.916
		0.20	0.914	0.912	0.911	0.910	0.908
		0.25	0.904	0.905	0.903	0.903	0.903
		0.30	0.897	0.898	0.897	0.895	0.895
	0.10	0.05	*0.930*	*0.928*	*0.928*	*0.926*	*0.925*
		0.10	*0.927*	*0.926*	0.924	0.924	0.922
		0.15	0.921	0.918	0.918	0.917	0.917
		0.20	0.914	0.913	0.914	0.911	0.909
		0.25	0.908	0.905	0.904	0.904	0.902
		0.30	0.901	0.900	0.897	0.898	0.896
	0.20	0.05	*0.930*	*0.929*	*0.929*	*0.927*	*0.925*
		0.10	*0.927*	*0.925*	*0.925*	0.924	0.922
		0.15	0.921	0.919	0.919	0.917	0.915
		0.20	0.913	0.914	0.912	0.911	0.909
		0.25	0.906	0.907	0.903	0.903	0.903
		0.30	0.901	0.900	0.899	0.898	0.896
	0.30	0.05	*0.930*	*0.930*	*0.930*	*0.927*	*0.927*
		0.10	*0.926*	*0.926*	0.924	0.924	0.921
		0.15	0.919	0.919	0.918	0.915	0.914
		0.20	0.911	0.910	0.909	0.909	0.907
		0.25	0.905	0.904	0.903	0.901	0.899
		0.30	0.898	0.897	0.897	0.895	0.893
	0.40	0.05	*0.931*	*0.930*	*0.929*	*0.928*	*0.927*
		0.10	0.924	*0.925*	0.922	0.921	0.920
		0.15	0.916	0.915	0.913	0.913	0.910
		0.20	0.907	0.907	0.905	0.904	0.903
		0.25	0.900	0.898	0.897	0.896	0.895
		0.30	0.893	0.892	0.889	0.889	0.886
	0.50	0.05	*0.931*	*0.929*	*0.929*	*0.927*	*0.926*
		0.10	0.923	0.920	0.918	0.918	0.916
		0.15	0.910	0.909	0.908	0.907	0.903
		0.20	0.901	0.899	0.898	0.897	0.895
		0.25	0.889	0.891	0.888	0.887	0.884
		0.30	0.884	0.882	0.881	0.877	0.876
	0.60	0.05	*0.928*	*0.926*	*0.926*	*0.925*	0.923
		0.10	0.914	0.914	0.912	0.911	0.910
		0.15	0.901	0.900	0.898	0.896	0.895
		0.20	0.889	0.887	0.885	0.883	0.881
		0.25	0.876	0.876	0.874	0.872	0.869
		0.30	0.868	0.864	0.864	0.860	0.858

Table 3-7. Continued

			k				
n	ρ_r^2	$\Delta\rho^2$	2	4	6	8	10
800	0.00	0.05	*0.932*	*0.931*	*0.929*	*0.930*	*0.928*
		0.10	*0.928*	*0.926*	*0.926*	*0.926*	*0.925*
		0.15	0.922	0.919	0.920	0.919	0.917
		0.20	0.913	0.913	0.914	0.911	0.911
		0.25	0.906	0.905	0.906	0.905	0.904
		0.30	0.897	0.898	0.897	0.897	0.896
	0.10	0.05	*0.932*	*0.931*	*0.931*	*0.929*	*0.930*
		0.10	*0.928*	*0.927*	*0.927*	*0.927*	*0.925*
		0.15	0.921	0.920	0.922	0.920	0.919
		0.20	0.916	0.915	0.914	0.912	0.911
		0.25	0.907	0.908	0.905	0.905	0.905
		0.30	0.902	0.901	0.900	0.899	0.899
	0.20	0.05	*0.932*	*0.932*	*0.931*	*0.932*	*0.930*
		0.10	*0.929*	*0.928*	*0.927*	*0.927*	*0.926*
		0.15	0.923	0.921	0.920	0.918	0.918
		0.20	0.915	0.913	0.912	0.912	0.911
		0.25	0.907	0.908	0.907	0.906	0.905
		0.30	0.901	0.901	0.901	0.899	0.897
	0.30	0.05	*0.933*	*0.933*	*0.932*	*0.930*	*0.929*
		0.10	*0.928*	*0.927*	*0.927*	*0.926*	0.924
		0.15	0.921	0.919	0.918	0.918	0.916
		0.20	0.912	0.912	0.911	0.909	0.910
		0.25	0.905	0.904	0.904	0.903	0.902
		0.30	0.899	0.897	0.898	0.897	0.894
	0.40	0.05	*0.933*	*0.932*	*0.932*	*0.930*	*0.930*
		0.10	*0.925*	*0.926*	0.924	0.923	0.923
		0.15	0.916	0.918	0.914	0.915	0.914
		0.20	0.909	0.907	0.907	0.905	0.903
		0.25	0.900	0.899	0.898	0.897	0.895
		0.30	0.894	0.892	0.889	0.891	0.888
	0.50	0.05	*0.932*	*0.931*	*0.930*	*0.929*	*0.928*
		0.10	0.923	0.922	0.919	0.919	0.919
		0.15	0.911	0.911	0.910	0.907	0.907
		0.20	0.901	0.901	0.899	0.898	0.897
		0.25	0.891	0.890	0.890	0.888	0.887
		0.30	0.885	0.884	0.880	0.880	0.878
	0.60	0.05	*0.930*	*0.930*	*0.927*	*0.927*	*0.925*
		0.10	0.916	0.915	0.914	0.914	0.911
		0.15	0.901	0.901	0.899	0.898	0.896
		0.20	0.889	0.888	0.887	0.885	0.883
		0.25	0.878	0.876	0.875	0.873	0.871
		0.30	0.869	0.866	0.866	0.863	0.862

Table 3-7. Continued

			k				
n	ρ_r^2	$\Delta\rho^2$	2	4	6	8	10
800	0.00	0.05	*0.932*	*0.931*	*0.929*	*0.930*	*0.928*
		0.10	*0.928*	*0.926*	*0.926*	*0.926*	*0.925*
		0.15	0.922	0.919	0.920	0.919	0.917
		0.20	0.913	0.913	0.914	0.911	0.911
		0.25	0.906	0.905	0.906	0.905	0.904
		0.30	0.897	0.898	0.897	0.897	0.896
	0.10	0.05	*0.932*	*0.931*	*0.931*	*0.929*	*0.930*
		0.10	*0.928*	*0.927*	*0.927*	*0.927*	*0.925*
		0.15	0.921	0.920	0.922	0.920	0.919
		0.20	0.916	0.915	0.914	0.912	0.911
		0.25	0.907	0.908	0.905	0.905	0.905
		0.30	0.902	0.901	0.900	0.899	0.899
	0.20	0.05	*0.932*	*0.932*	*0.931*	*0.932*	*0.930*
		0.10	*0.929*	*0.928*	*0.927*	*0.927*	*0.926*
		0.15	0.923	0.921	0.920	0.918	0.918
		0.20	0.915	0.913	0.912	0.912	0.911
		0.25	0.907	0.908	0.907	0.906	0.905
		0.30	0.901	0.901	0.901	0.899	0.897
	0.30	0.05	*0.933*	*0.933*	*0.932*	*0.930*	*0.929*
		0.10	*0.928*	*0.927*	*0.927*	*0.926*	0.924
		0.15	0.921	0.919	0.918	0.918	0.916
		0.20	0.912	0.912	0.911	0.909	0.910
		0.25	0.905	0.904	0.904	0.903	0.902
		0.30	0.899	0.897	0.898	0.897	0.894
	0.40	0.05	*0.933*	*0.932*	*0.932*	*0.930*	*0.930*
		0.10	*0.925*	*0.926*	0.924	0.923	0.923
		0.15	0.916	0.918	0.914	0.915	0.914
		0.20	0.909	0.907	0.907	0.905	0.903
		0.25	0.900	0.899	0.898	0.897	0.895
		0.30	0.894	0.892	0.889	0.891	0.888
	0.50	0.05	*0.932*	*0.931*	*0.930*	*0.929*	*0.928*
		0.10	0.923	0.922	0.919	0.919	0.919
		0.15	0.911	0.911	0.910	0.907	0.907
		0.20	0.901	0.901	0.899	0.898	0.897
		0.25	0.891	0.890	0.890	0.888	0.887
		0.30	0.885	0.884	0.880	0.880	0.878
	0.60	0.05	*0.930*	*0.930*	*0.927*	*0.927*	*0.925*
		0.10	0.916	0.915	0.914	0.914	0.911
		0.15	0.901	0.901	0.899	0.898	0.896
		0.20	0.889	0.888	0.887	0.885	0.883
		0.25	0.878	0.876	0.875	0.873	0.871
		0.30	0.869	0.866	0.866	0.863	0.862

Table 3-7. Continued

n	ρ^2_r	$\Delta\rho^2$	k = 2	4	6	8	10
1000	0.00	0.05	*0.933*	*0.932*	*0.931*	*0.932*	*0.931*
		0.10	*0.929*	*0.930*	*0.927*	*0.927*	*0.926*
		0.15	0.922	0.922	0.921	0.921	0.919
		0.20	0.915	0.914	0.913	0.913	0.913
		0.25	0.905	0.906	0.906	0.905	0.904
		0.30	0.899	0.899	0.900	0.897	0.898
	0.10	0.05	*0.934*	*0.935*	*0.934*	*0.932*	*0.931*
		0.10	*0.929*	*0.930*	*0.929*	*0.928*	*0.926*
		0.15	0.923	0.922	0.921	0.921	0.920
		0.20	0.917	0.915	0.914	0.912	0.912
		0.25	0.908	0.908	0.907	0.906	0.906
		0.30	0.900	0.901	0.900	0.901	0.901
	0.20	0.05	*0.935*	*0.935*	*0.934*	*0.933*	*0.933*
		0.10	*0.929*	*0.929*	*0.927*	*0.928*	*0.926*
		0.15	0.922	0.922	0.921	0.920	0.920
		0.20	0.915	0.914	0.914	0.913	0.913
		0.25	0.908	0.907	0.906	0.906	0.906
		0.30	0.901	0.902	0.901	0.901	0.899
	0.30	0.05	*0.934*	*0.934*	*0.933*	*0.934*	*0.933*
		0.10	*0.929*	*0.928*	*0.928*	*0.926*	*0.926*
		0.15	0.922	0.921	0.919	0.921	0.918
		0.20	0.914	0.913	0.912	0.910	0.912
		0.25	0.906	0.905	0.905	0.905	0.903
		0.30	0.899	0.899	0.897	0.897	0.897
	0.40	0.05	*0.935*	*0.934*	*0.932*	*0.931*	*0.931*
		0.10	*0.927*	*0.927*	0.924	0.923	0.924
		0.15	0.917	0.916	0.915	0.915	0.915
		0.20	0.911	0.907	0.908	0.907	0.906
		0.25	0.900	0.901	0.900	0.899	0.896
		0.30	0.894	0.893	0.891	0.891	0.891
	0.50	0.05	*0.933*	*0.932*	*0.933*	*0.931*	*0.930*
		0.10	0.924	0.923	0.921	0.921	0.921
		0.15	0.911	0.911	0.910	0.910	0.909
		0.20	0.900	0.900	0.900	0.900	0.899
		0.25	0.893	0.890	0.889	0.889	0.888
		0.30	0.883	0.883	0.883	0.881	0.881
	0.60	0.05	*0.931*	*0.930*	*0.929*	*0.929*	*0.928*
		0.10	0.915	0.916	0.915	0.914	0.914
		0.15	0.901	0.902	0.900	0.900	0.896
		0.20	0.889	0.889	0.887	0.887	0.885
		0.25	0.879	0.877	0.875	0.874	0.873
		0.30	0.869	0.868	0.865	0.865	0.864

Table 3-7. Continued

			k				
n	ρ_r^2	$\Delta\rho^2$	2	4	6	8	10
1500	0.00	0.05	*0.937*	*0.935*	*0.935*	*0.935*	*0.935*
		0.10	*0.931*	*0.931*	*0.929*	*0.929*	*0.928*
		0.15	0.924	0.922	0.922	0.922	0.921
		0.20	0.915	0.914	0.916	0.915	0.914
		0.25	0.907	0.908	0.907	0.906	0.907
		0.30	0.900	0.898	0.898	0.898	0.898
	0.10	0.05	*0.938*	*0.935*	*0.936*	*0.936*	*0.934*
		0.10	*0.932*	*0.931*	*0.930*	*0.930*	*0.929*
		0.15	0.923	0.922	0.923	0.922	0.922
		0.20	0.915	0.916	0.916	0.915	0.914
		0.25	0.909	0.909	0.907	0.908	0.907
		0.30	0.902	0.902	0.901	0.901	0.899
	0.20	0.05	*0.938*	*0.937*	*0.936*	*0.935*	*0.936*
		0.10	*0.930*	*0.930*	*0.929*	*0.929*	*0.929*
		0.15	0.924	0.924	0.923	0.922	0.921
		0.20	0.916	0.915	0.915	0.915	0.912
		0.25	0.908	0.908	0.909	0.906	0.905
		0.30	0.903	0.902	0.901	0.901	0.900
	0.30	0.05	*0.937*	*0.936*	*0.935*	*0.935*	*0.934*
		0.10	*0.930*	*0.929*	*0.929*	*0.929*	*0.928*
		0.15	0.921	0.921	0.921	0.920	0.919
		0.20	0.912	0.914	0.913	0.911	0.913
		0.25	0.907	0.907	0.906	0.905	0.904
		0.30	0.901	0.899	0.900	0.899	0.898
	0.40	0.05	*0.937*	*0.936*	*0.937*	*0.935*	*0.935*
		0.10	*0.927*	*0.927*	*0.927*	*0.926*	*0.925*
		0.15	0.919	0.917	0.918	0.917	0.916
		0.20	0.912	0.909	0.908	0.908	0.907
		0.25	0.902	0.900	0.901	0.901	0.898
		0.30	0.896	0.893	0.893	0.891	0.892
	0.50	0.05	*0.934*	*0.934*	*0.934*	*0.934*	*0.933*
		0.10	0.924	0.924	0.923	0.923	0.923
		0.15	0.913	0.912	0.911	0.911	0.911
		0.20	0.903	0.902	0.902	0.900	0.899
		0.25	0.891	0.891	0.891	0.890	0.890
		0.30	0.884	0.885	0.883	0.881	0.880
	0.60	0.05	*0.931*	*0.931*	*0.930*	*0.930*	*0.931*
		0.10	0.918	0.917	0.917	0.915	0.914
		0.15	0.903	0.901	0.901	0.902	0.902
		0.20	0.889	0.888	0.888	0.888	0.885
		0.25	0.878	0.877	0.877	0.876	0.873
		0.30	0.867	0.866	0.866	0.866	0.864

Table 3-7. Continued

n	ρ_r^2	$\Delta\rho^2$	k = 2	4	6	8	10
2000	0.00	0.05	*0.938*	*0.938*	*0.937*	*0.938*	*0.936*
		0.10	*0.931*	*0.931*	*0.931*	*0.930*	*0.931*
		0.15	0.924	0.923	0.923	0.923	0.922
		0.20	0.916	0.915	0.915	0.914	0.915
		0.25	0.908	0.908	0.907	0.908	0.905
		0.30	0.900	0.899	0.898	0.900	0.898
	0.10	0.05	*0.939*	*0.938*	*0.937*	*0.937*	*0.936*
		0.10	*0.932*	*0.932*	*0.931*	*0.931*	*0.932*
		0.15	0.924	0.924	0.922	*0.925*	0.923
		0.20	0.917	0.917	0.915	0.916	0.916
		0.25	0.909	0.908	0.908	0.909	0.907
		0.30	0.903	0.901	0.902	0.900	0.902
	0.20	0.05	*0.939*	*0.939*	*0.938*	*0.937*	*0.936*
		0.10	*0.931*	*0.931*	*0.931*	*0.930*	*0.930*
		0.15	0.923	0.923	0.922	0.923	0.921
		0.20	0.916	0.915	0.914	0.915	0.916
		0.25	0.909	0.909	0.907	0.909	0.909
		0.30	0.903	0.902	0.902	0.903	0.902
	0.30	0.05	*0.936*	*0.938*	*0.937*	*0.936*	*0.936*
		0.10	*0.930*	*0.930*	*0.930*	*0.929*	*0.928*
		0.15	0.923	0.921	0.921	0.920	0.920
		0.20	0.913	0.913	0.913	0.914	0.912
		0.25	0.906	0.905	0.906	0.905	0.907
		0.30	0.901	0.900	0.899	0.899	0.898
	0.40	0.05	*0.937*	*0.937*	*0.937*	*0.936*	*0.937*
		0.10	*0.929*	*0.928*	*0.927*	*0.925*	*0.927*
		0.15	0.919	0.918	0.917	0.918	0.917
		0.20	0.909	0.911	0.908	0.909	0.908
		0.25	0.901	0.900	0.901	0.900	0.900
		0.30	0.895	0.894	0.894	0.893	0.894
	0.50	0.05	*0.936*	*0.936*	*0.934*	*0.934*	*0.934*
		0.10	*0.925*	*0.925*	0.924	0.923	0.923
		0.15	0.914	0.913	0.911	0.911	0.911
		0.20	0.900	0.902	0.902	0.901	0.900
		0.25	0.893	0.892	0.890	0.891	0.891
		0.30	0.883	0.885	0.885	0.884	0.883
	0.60	0.05	*0.933*	*0.933*	*0.931*	*0.931*	*0.931*
		0.10	0.920	0.918	0.916	0.915	0.916
		0.15	0.901	0.903	0.902	0.902	0.901
		0.20	0.890	0.890	0.889	0.887	0.887
		0.25	0.877	0.877	0.877	0.876	0.875
		0.30	0.868	0.868	0.868	0.868	0.866

Table 3-8. Descriptive Statistics for Coverage Probability by Distributional Condition.

Distributional Condition					Range		Percent Nonrobust	
Distribution for **X**	Distribution for **e**	$M_{\hat{p}}$	SD	Median	Minimum	Maximum	n = 600	n = 2000
Normal	Normal	0.940	0.0123	0.944	0.856	0.956	0.0	0.0
Normal	Pseudo - $t(10)$	0.938	0.0120	0.941	0.851	0.953	0.0	0.0
Normal	Pseudo - $\chi^2(10)$	0.937	0.0119	0.941	0.853	0.953	0.0	0.0
Normal	Pseudo - $\chi^2(4)$	0.933	0.0124	0.936	0.855	0.953	6.7	1.4
Normal	Pseudo - exponential	0.925	0.0155	0.928	0.856	0.951	34.3	25.2
Pseudo - $t(10)$	Normal	0.931	0.0131	0.934	0.855	0.950	11.0	7.1
Pseudo - $t(10)$	Pseudo - $t(10)$	0.928	0.0128	0.931	0.845	0.950	12.4	6.2
Pseudo - $t(10)$	Pseudo - $\chi^2(10)$	0.928	0.0127	0.930	0.842	0.949	15.2	7.6
Pseudo - $t(10)$	Pseudo - $\chi^2(4)$	0.923	0.0129	0.925	0.845	0.945	46.2	26.2
Pseudo - $t(10)$	Pseudo - exponential	0.915	0.0154	0.917	0.850	0.946	72.9	55.7
Pseudo - $\chi^2(10)$	Normal	0.929	0.0138	0.933	0.844	0.949	15.2	10.0
Pseudo - $\chi^2(10)$	Pseudo - $t(10)$	0.926	0.0132	0.929	0.841	0.948	21.4	12.4
Pseudo - $\chi^2(10)$	Pseudo - $\chi^2(10)$	0.926	0.0132	0.929	0.843	0.947	23.8	14.3
Pseudo - $\chi^2(10)$	Pseudo - $\chi^2(4)$	0.921	0.0133	0.923	0.838	0.948	52.9	33.8
Pseudo - $\chi^2(10)$	Pseudo - exponential	0.913	0.0156	0.915	0.844	0.944	74.3	61.0
Pseudo - $\chi^2(4)$	Normal	0.912	0.0213	0.917	0.796	0.946	62.4	49.1
Pseudo - $\chi^2(4)$	Pseudo - $t(10)$	0.909	0.0208	0.913	0.804	0.945	70.5	58.1
Pseudo - $\chi^2(4)$	Pseudo - $\chi^2(10)$	0.909	0.0209	0.913	0.800	0.945	71.9	63.3
Pseudo - $\chi^2(4)$	Pseudo - $\chi^2(4)$	0.904	0.0205	0.906	0.804	0.940	81.9	70.0
Pseudo - $\chi^2(4)$	Pseudo - exponential	0.896	0.0215	0.896	0.802	0.939	91.4	83.3
Pseudo - exponential	Normal	0.883	0.0373	0.891	0.733	0.940	81.4	96.2
Pseudo - exponential	Pseudo - $t(10)$	0.880	0.0367	0.887	0.729	0.939	99.5	86.2
Pseudo - exponential	Pseudo - $\chi^2(10)$	0.880	0.0369	0.886	0.741	0.938	95.7	85.7
Pseudo - exponential	Pseudo - $\chi^2(4)$	0.875	0.0363	0.879	0.745	0.936	99.0	87.1
Pseudo - exponential	Pseudo - exponential	0.867	0.0363	0.868	0.738	0.933	100.0	90.0

Table 3-9. Analysis of Variance, Estimated Mean Square Components, and Percentage of Total Variance in Coverage Probability Explained by the Study Variables.

Source of Variance	df	SS	Mean Square	F	p	Mean Square Component	Percentage of Total Variance
X	4	30.065	7.5162	951467.0	< .0001	.0005113	44.51
$X \times \Delta\rho^2$	20	6.408	.3204	40557.4	< .0001	.0001308	11.38
$\Delta\rho^2$	5	7.117	.1423	180191.0	< .0001	.0001162	10.12
n	13	7.378	.5675	71840.3	< .0001	.0001081	9.41
$X \times \rho_r^2$	24	5.107	.2128	26938.3	< .0001	.0001013	8.82
e	4	2.251	.5629	71251.3	< .0001	.0000383	3.33
ρ_r^2	6	2.357	.3929	49734.9	< .0001	.0000374	3.26
$X \times \rho_r^2 \times \Delta\rho^2$	120	.984	.0082	1038.4	< .0001	.0000234	2.04
$\rho_r^2 \times \Delta\rho^2$	30	.963	.0321	4062.8	< .0001	.0000183	1.60
$e \times \rho_r^2$	24	.738	.0308	3895.6	< .0001	.0000147	1.28
$n \times \Delta\rho^2$	65	.649	.0099	1263.4	< .0001	.0000114	.99
$n \times k$	52	.599	.0115	1459.0	< .0001	.0000110	.95
$e \times \Delta\rho^2$	20	.396	.0198	2505.4	< .0001	.0000081	.70
k	4	.465	.1162	14712.1	< .0001	.0000079	.69
$e \times \rho_r^2 \times \Delta\rho^2$	120	.259	.0022	273.5	< .0001	.0000062	.54

Table 3-10. Descriptive Statistics for Coverage Probability by Distribution for the Predictors.

				Range	
Distribution	M	SD	Median	Minimum	Maximum
Normal	0.935	0.014	0.939	0.851	0.956
Pseudo - $t(10)$	0.925	0.015	0.928	0.842	0.950
Pseudo - $\chi^2(10)$	0.923	0.015	0.926	0.838	0.949
Pseudo - $\chi^2(4)$	0.906	0.022	0.910	0.792	0.946
Pseudo - exponential	0.877	0.037	0.883	0.729	0.940

Table 3-11. Descriptive Statistics for Coverage Probability by Distribution for the Errors.

Distribution	M	SD	Median	Range Minimum	Range Maximum
Normal	0.919	0.0297	0.929	0.733	0.956
Pseudo - $t(10)$	0.916	0.0293	0.926	0.729	0.953
Pseudo - $\chi^2(10)$	0.916	0.0294	0.926	0.741	0.953
Pseudo - $\chi^2(4)$	0.911	0.0292	0.920	0.745	0.953
Pseudo - exponential	0.903	0.0302	0.911	0.738	0.951

Table 3-12. Coverage Probability by $\Delta\rho^2$ and the Distribution for the Predictors.

	Distribution for the Predictors (**X**)				
$\Delta\rho^2$	Normal	Pseudo- $t(10)$	Pseudo-$\chi^2(10)$	Pseudo-$\chi^2(4)$	Pseudo-exponential
0.05	0.932	0.929	0.929	0.924	0.915
0.10	0.936	0.930	0.929	0.919	0.901
0.15	0.936	0.927	0.926	0.911	0.885
0.20	0.935	0.924	0.922	0.902	0.868
0.25	0.935	0.921	0.918	0.894	0.853
0.30	0.934	0.917	0.915	0.886	0.840

Table 3-13. Coverage Probability by $\Delta\rho^2$ and the Distribution for the Errors.

	Distribution for the Errors (**e**)				
$\Delta\rho^2$	Normal	Pseudo- $t(10)$	Pseudo-$\chi^2(10)$	Pseudo-$\chi^2(4)$	Pseudo-exponential
0.05	0.928	0.927	0.927	0.925	0.923
0.10	0.927	0.925	0.925	0.922	0.917
0.15	0.922	0.920	0.920	0.915	0.907
0.20	0.917	0.914	0.914	0.908	0.899
0.25	0.912	0.908	0.908	0.901	0.890
0.30	0.907	0.903	0.902	0.895	0.883

Table 3-14. Coverage Probability by ρ_r^2 and the Distribution for the Predictors.

	Distribution for the Predictors (**X**)				
$\rho_r^{\ 2}$	Normal	Pseudo- $t\,(10)$	Pseudo-$\chi^2(10)$	Pseudo-$\chi^2(4)$	Pseudo-exponential
0.00	0.928	0.922	0.922	0.912	0.896
0.10	0.930	0.925	0.924	0.913	0.895
0.20	0.933	0.926	0.925	0.913	0.891
0.30	0.935	0.927	0.925	0.911	0.885
0.40	0.937	0.927	0.925	0.907	0.875
0.50	0.939	0.925	0.923	0.899	0.860
0.60	0.940	0.922	0.918	0.887	0.837

Table 3-15. Coverage Probability by ρ_r^2 and the Distribution for the Errors.

	Distribution for the Errors (**e**)				
$\rho_r^{\ 2}$	Normal	Pseudo- $t\,(10)$	Pseudo-$\chi^2(10)$	Pseudo-$\chi^2(4)$	Pseudo-exponential
0.00	0.927	0.922	0.921	0.913	0.897
0.10	0.926	0.922	0.922	0.914	0.902
0.20	0.925	0.921	0.921	0.915	0.905
0.30	0.922	0.920	0.919	0.915	0.907
0.40	0.918	0.916	0.916	0.913	0.907
0.50	0.912	0.910	0.910	0.908	0.905
0.60	0.902	0.902	0.901	0.900	0.899

Table 3-16. Coverage Probability by ρ_r^2 and $\Delta\rho^2$ for **X** Distributed Multivariate Normal.

	$\Delta\rho^2$					
$\rho_r^{\ 2}$	0.05	0.10	0.15	0.20	0.25	0.30
0.00	0.923	0.924	0.920	0.915	0.910	0.904
0.10	0.925	0.925	0.921	0.916	0.911	0.906
0.20	0.926	0.925	0.921	0.916	0.911	0.906
0.30	0.927	0.925	0.920	0.914	0.909	0.904
0.40	0.927	0.924	0.918	0.911	0.905	0.899
0.50	0.927	0.921	0.913	0.905	0.897	0.891
0.60	0.926	0.917	0.905	0.895	0.885	0.877

Table 3-17. Coverage Probability by ρ_r^2 and $\Delta\rho^2$ for **X** Distributed Pseudo-t_{10}(g = 0, h = .058).

	$\Delta\rho^2$					
$\rho_r^{\ 2}$	0.05	0.10	0.15	0.20	0.25	0.30
0.00	0.9248	0.9277	0.9259	0.9226	0.9190	0.9151
0.10	0.9266	0.9292	0.9274	0.9245	0.9213	0.9183
0.20	0.9282	0.9303	0.9282	0.9258	0.9232	0.9201
0.30	0.9293	0.9308	0.9289	0.9259	0.9233	0.9211
0.40	0.9311	0.9313	0.9286	0.9257	0.9226	0.9201
0.50	0.9317	0.9310	0.9276	0.9236	0.9201	0.9168
0.60	0.9324	0.9295	0.9241	0.9194	0.9145	0.9102

Table 3-18. Coverage Probability by ρ_r^2 and $\Delta\rho^2$ for **X** Distributed Pseudo-χ^2_{10} (g = .301, h = -.017).

	$\Delta\rho^2$					
$\rho_r^{\ 2}$	0.05	0.10	0.15	0.20	0.25	0.30
0.00	0.9247	0.9275	0.9252	0.9216	0.9181	0.9138
0.10	0.9265	0.9285	0.9265	0.9232	0.9202	0.9168
0.20	0.9280	0.9294	0.9273	0.9242	0.9211	0.9182
0.30	0.9291	0.9302	0.9277	0.9244	0.9216	0.9184
0.40	0.9303	0.9302	0.9272	0.9237	0.9199	0.9169
0.50	0.9312	0.9299	0.9253	0.9209	0.9162	0.9128
0.60	0.9313	0.9275	0.9213	0.9155	0.9098	0.9053

Table 3-19. Coverage Probability by ρ_r^2 and $\Delta\rho^2$ for **X** Distributed Pseudo-χ^2_4 (g = .502, h = -.048).

	$\Delta\rho^2$					
$\rho_r^{\ 2}$	0.05	0.10	0.15	0.20	0.25	0.30
0.00	0.9219	0.9219	0.9170	0.9109	0.9046	0.8980
0.10	0.9231	0.9224	0.9174	0.9113	0.9049	0.8987
0.20	0.9244	0.9224	0.9168	0.9104	0.9037	0.8976
0.30	0.9248	0.9217	0.9150	0.9075	0.9005	0.8939
0.40	0.9253	0.9198	0.9112	0.9026	0.8938	0.8862
0.50	0.9247	0.9158	0.9046	0.8935	0.8829	0.8737
0.60	0.9225	0.9089	0.8939	0.8788	0.8655	0.8542

Table 3-20. Coverage Probability by ρ_r^2 and $\Delta\rho^2$ for **X** Distributed Pseudo-exponential (g = .760, h = -.098).

	$\Delta\rho^2$					
$\rho_r^{\ 2}$	0.05	0.10	0.15	0.20	0.25	0.30
0.00	0.9169	0.9120	0.9026	0.8922	0.8811	0.8703
0.10	0.9179	0.9117	0.9013	0.8901	0.8795	0.8685
0.20	0.9181	0.9099	0.8981	0.8858	0.8737	0.8632
0.30	0.9172	0.9066	0.8925	0.8783	0.8644	0.8518
0.40	0.9157	0.9011	0.8836	0.8661	0.8496	0.8349
0.50	0.9125	0.8918	0.8689	0.8475	0.8278	0.8111
0.60	0.9063	0.8768	0.8469	0.8195	0.7959	0.7771

Table 3-21. Coverage Probability by Sample Size and $\Delta\rho^2$.

	$\Delta\rho^2$					
n	0.05	0.10	0.15	0.20	0.25	0.30
100	0.877	0.890	0.889	0.887	0.883	0.878
200	0.907	0.912	0.909	0.904	0.899	0.894
300	0.918	0.919	0.915	0.909	0.904	0.899
400	0.924	0.923	0.918	0.912	0.907	0.902
500	0.928	0.925	0.920	0.914	0.908	0.903
600	0.930	0.927	0.921	0.915	0.909	0.904
700	0.932	0.928	0.922	0.916	0.910	0.905
800	0.933	0.929	0.923	0.917	0.910	0.905
900	0.934	0.930	0.923	0.917	0.911	0.906
1000	0.935	0.930	0.924	0.917	0.911	0.906
1250	0.936	0.930	0.923	0.916	0.909	0.904
1500	0.937	0.931	0.923	0.916	0.910	0.904
1750	0.938	0.931	0.924	0.917	0.910	0.904
2000	0.938	0.931	0.924	0.917	0.910	0.905

Table 3-22. Coverage Probability by Sample Size and Number of Predictors.

n	k = 2	4	6	8	10
100	0.897	0.890	0.883	0.874	0.864
200	0.909	0.906	0.902	0.898	0.893
300	0.913	0.911	0.909	0.906	0.903
400	0.916	0.914	0.912	0.910	0.908
500	0.917	0.916	0.914	0.913	0.911
600	0.918	0.917	0.916	0.914	0.913
700	0.918	0.918	0.917	0.915	0.914
800	0.919	0.918	0.917	0.916	0.915
900	0.919	0.919	0.918	0.917	0.916
1000	0.920	0.919	0.918	0.918	0.917
1250	0.920	0.920	0.919	0.919	0.918
1500	0.921	0.920	0.920	0.919	0.918
1750	0.921	0.921	0.920	0.920	0.919
2000	0.921	0.921	0.920	0.920	0.920

Table 3-23. Analysis of Variance, Estimated Mean Square Components, and Percentage of Total Variance Explained in the Ratio of Mean Estimated Asymptotic Variance to the Empirical Sampling Variance of ΔR^2.

Source of Variance	df	SS	Mean Square	<u>F</u>	<u>p</u>	Mean Square Component	Percentage of Total Variance
X	4	640.90	160.22	1094917.0	< .0001	.0108996	51.78
$\Delta\rho^2$	5	271.56	54.31	371145.0	< .0001	.0044336	21.06
$X \times \Delta\rho^2$	20	70.21	3.51	23988.1	< .0001	.0014327	6.81
$X \times \rho_r^2$	24	68.45	2.85	19489.7	< .0001	.0013580	6.45
e	4	77.48	19.37	132371.0	< .0001	.0013177	6.26
ρ_r^2	6	35.40	5.90	40318.4	< .0001	.0005619	2.67
$e \times \rho_r^2$	24	21.42	0.89	6098.1	< .0001	.0004249	2.02
$X \times \rho_r^2 \times \Delta\rho^2$	120	6.42	0.05	365.8	< .0001	.0001525	0.72

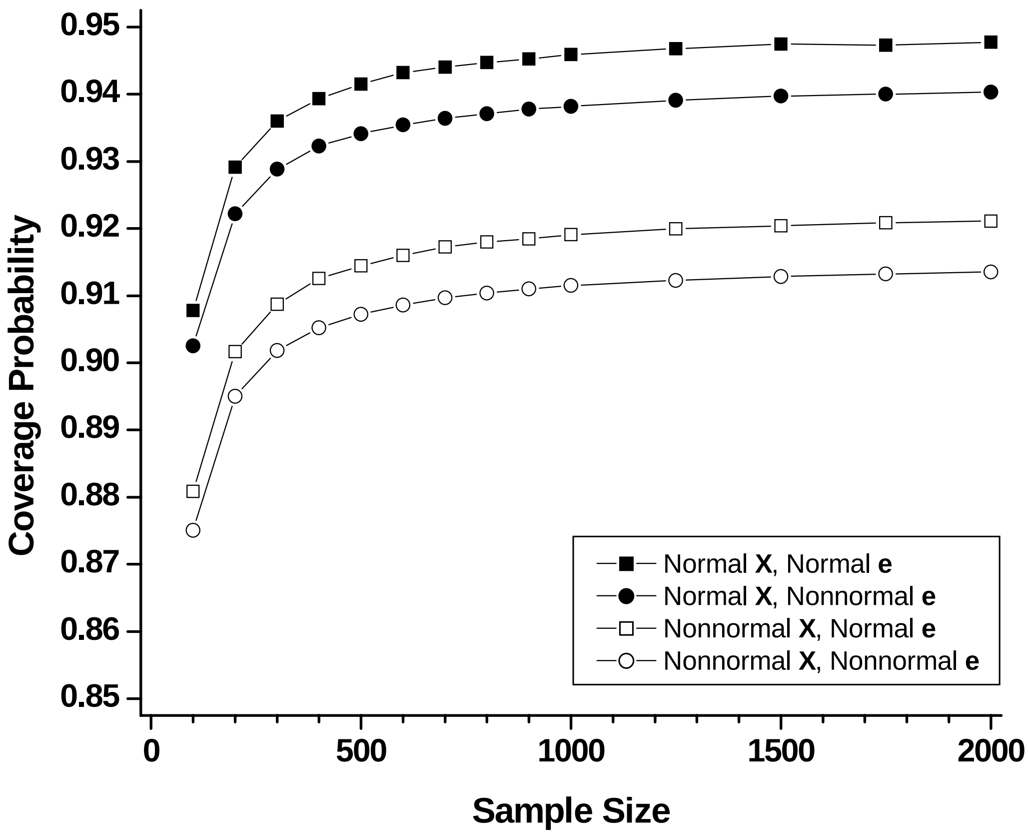


Figure 3-1. Mean estimated coverage probability by normality vs. nonnormality in the predictors, normality vs. nonnormality in the errors, and sample size.

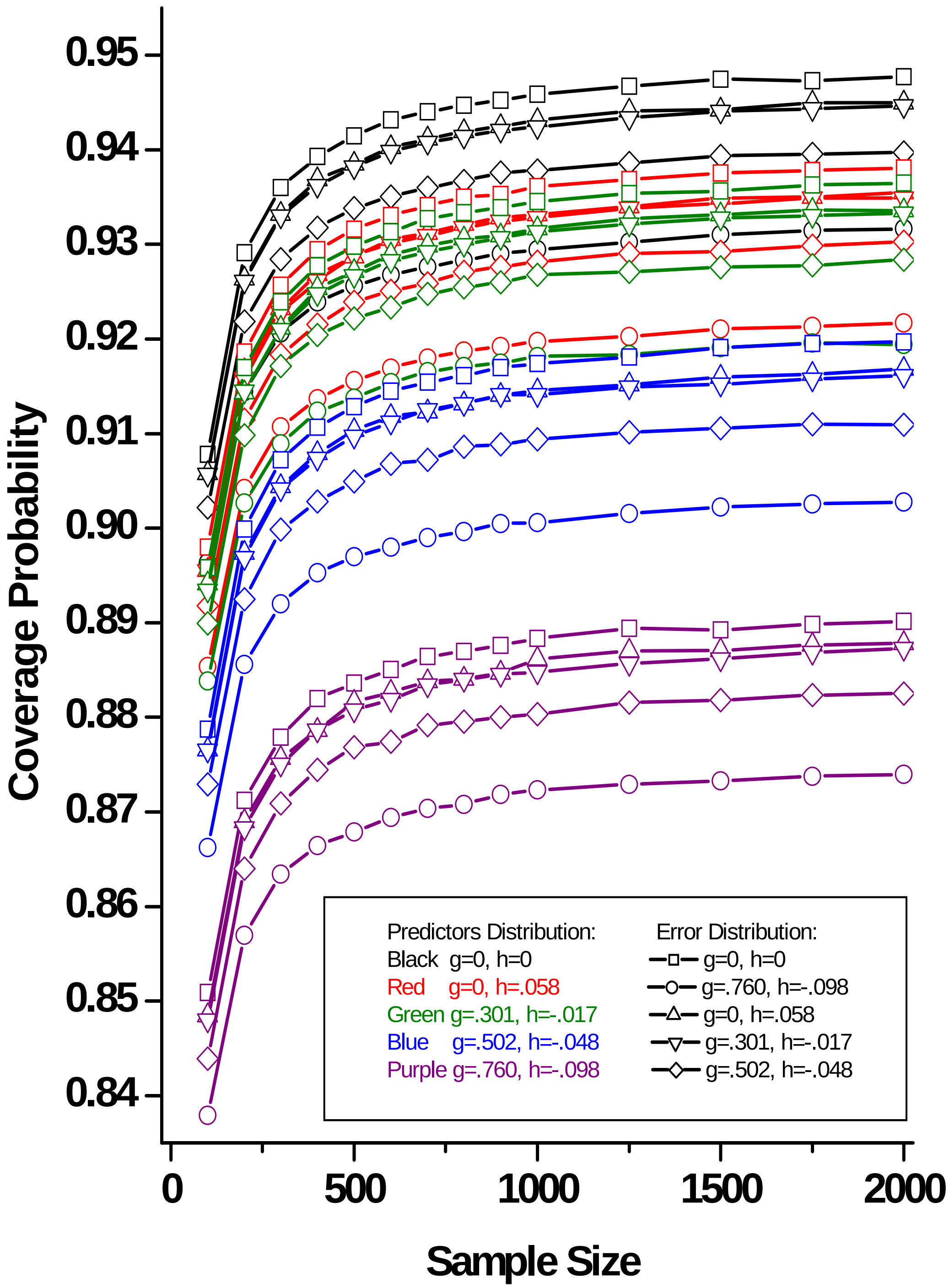


Figure 3-2. Empirical coverage probability as a function of distributional condition and sample size.

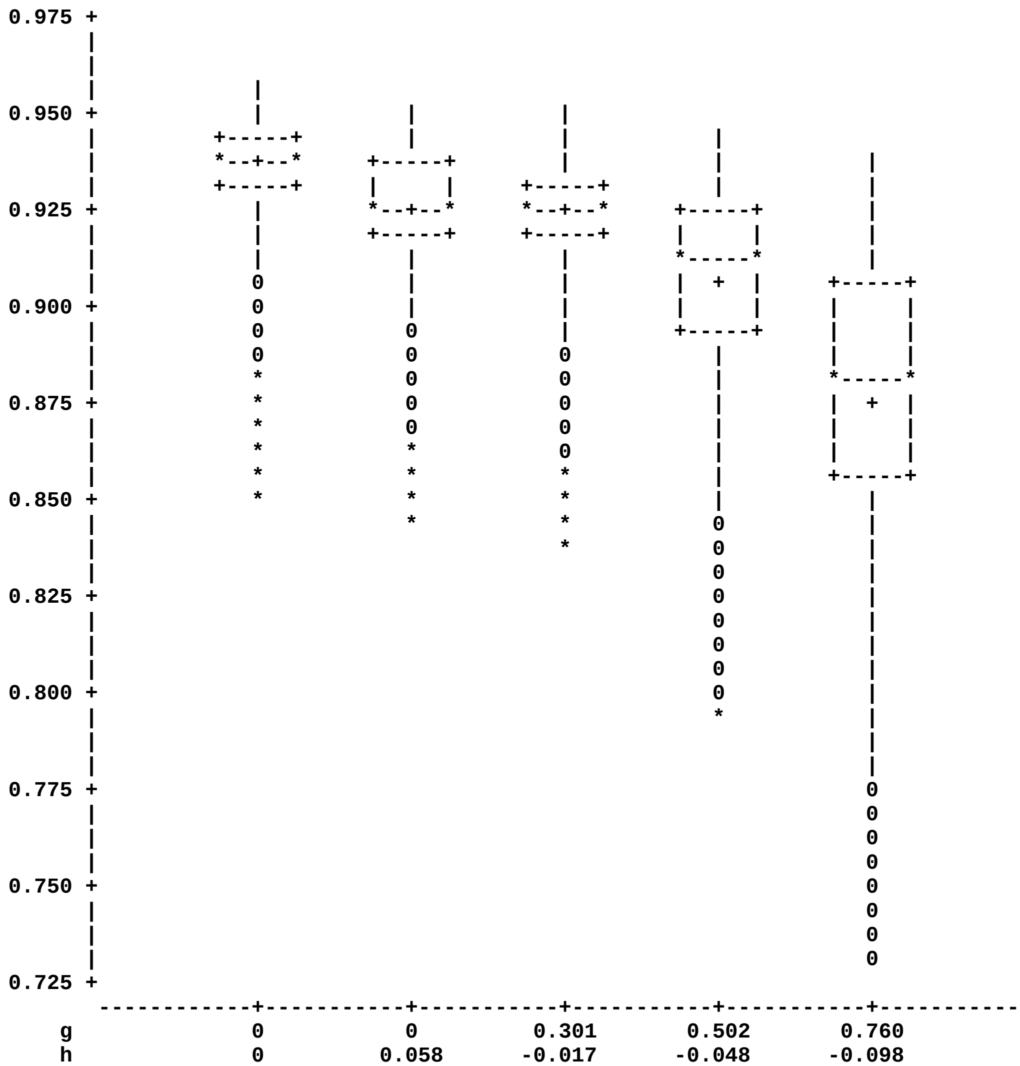


Figure 3-3. Box plots of the distributions of coverage probability estimates by distribution for the predictors (n_i = 14,700).

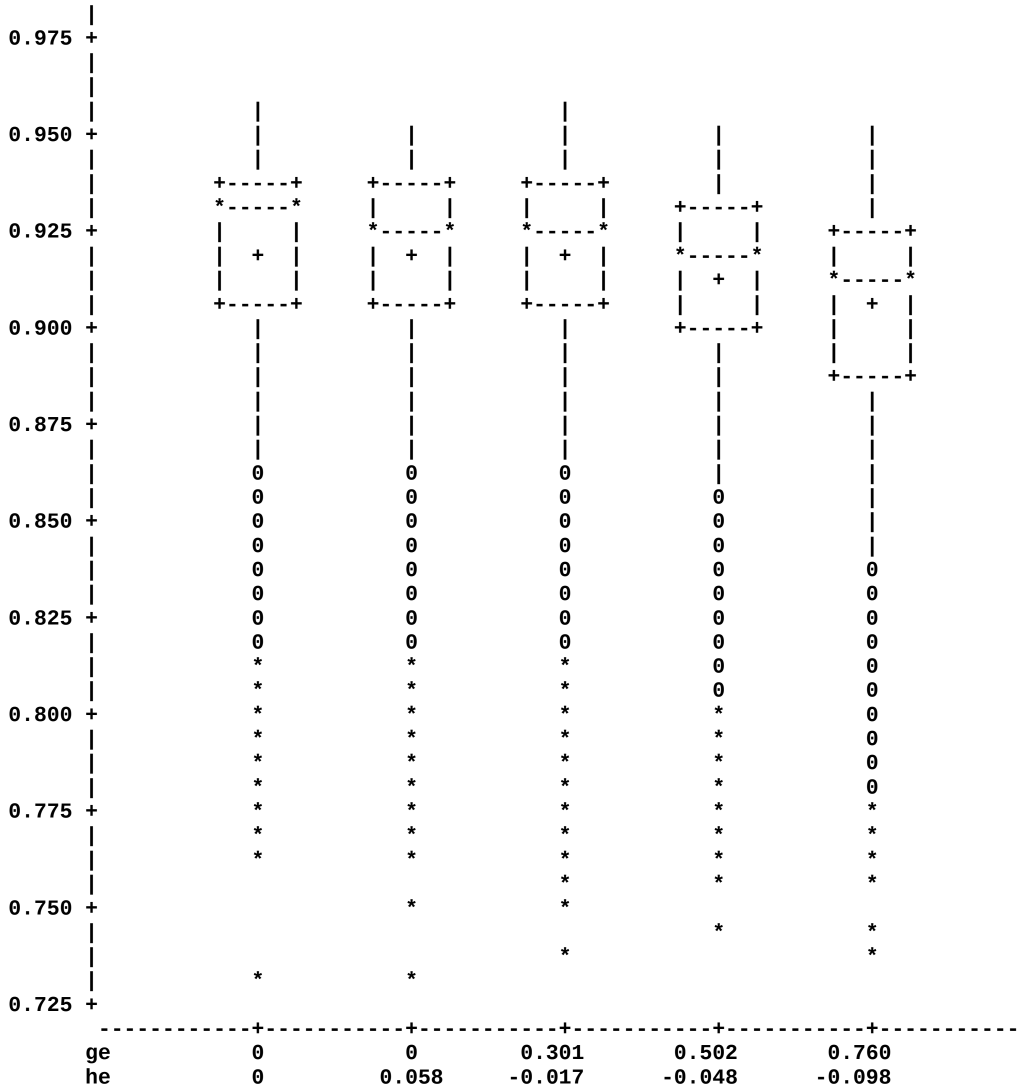


Figure 3-4. Box plots of the distributions of coverage probability estimates by distribution for the errors (n_i = 14,700).

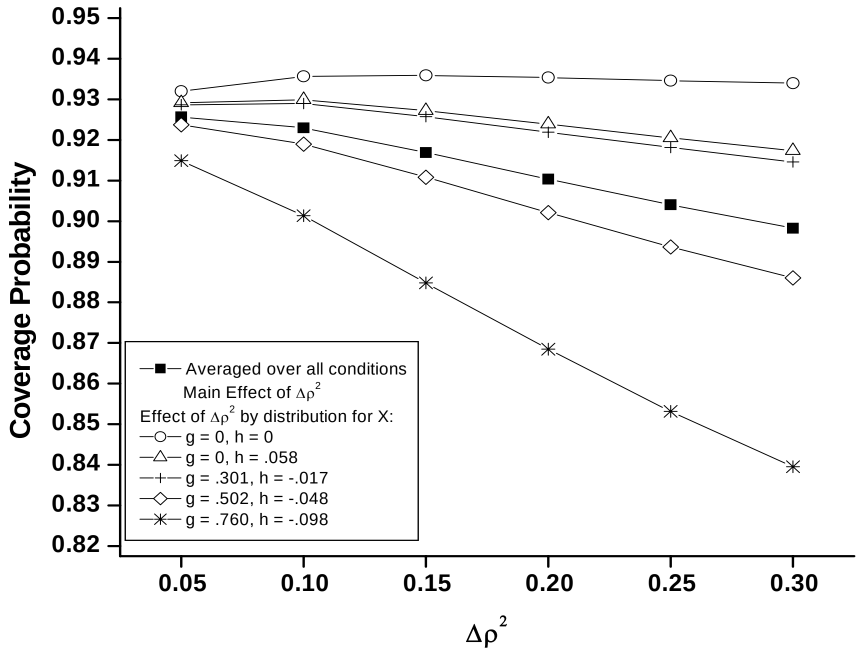


Figure 3-5. Main effect of the squared semipartial correlation coefficient , $\Delta\rho^2$, and the effect of the interaction of $\Delta\rho^2$ and **X** on coverage probability for $\Delta\rho^2 > 0$.

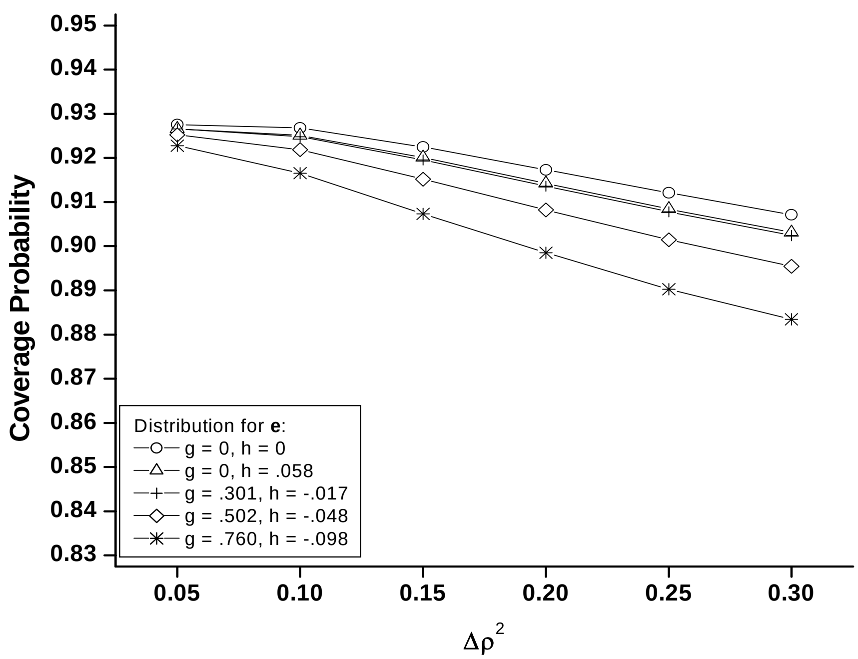


Figure 3-6. Main effect of the squared semipartial correlation coefficient , $\Delta\rho^2$, and the effect of the interaction of $\Delta\rho^2$ and **e** on coverage probability for $\Delta\rho^2 > 0$.

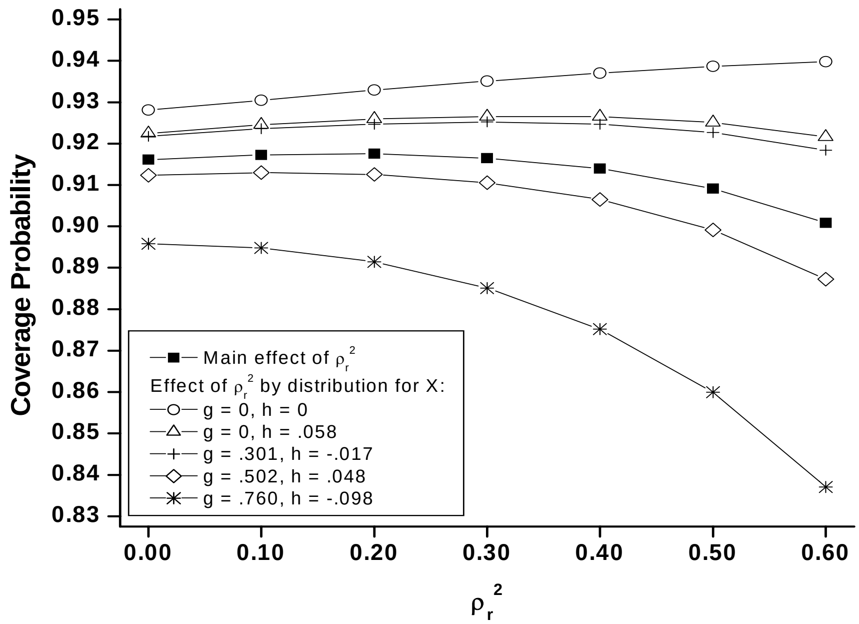


Figure 3-7. Effect of the interaction between the size of the squared multiple correlation in the reduced model, ρ_r^2, and the distribution for the predictors, **X**, on coverage probability.

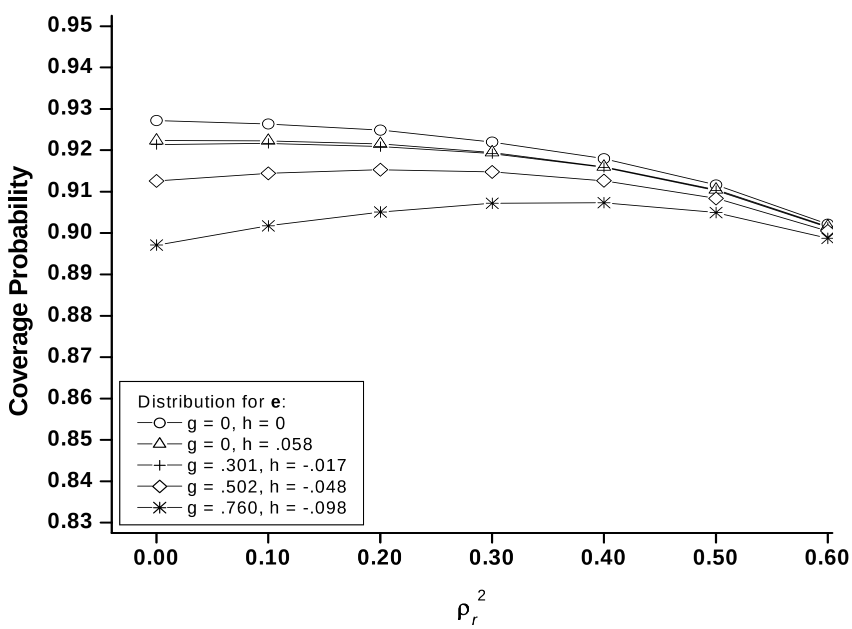


Figure 3-8. Interaction between the size of the squared multiple correlation in the reduced model, ρ_r^2, and the distribution for the errors, **e**, and its relationship to coverage probability. ρ_r^2

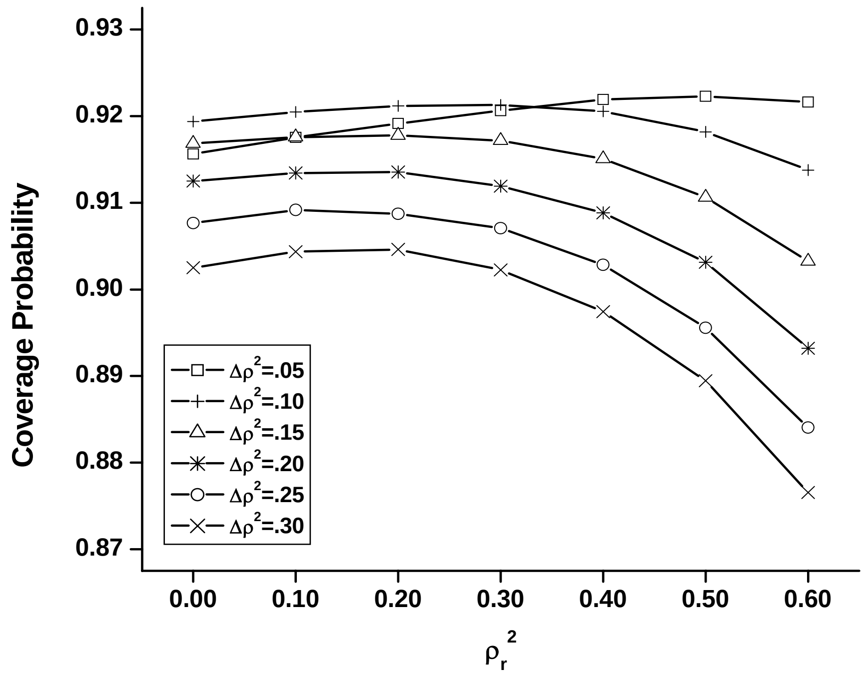


Figure 3-9. Effect of the $\rho_r^2 \times \Delta\rho^2$ interaction on coverage probability for $\Delta\rho^2 > 0$.

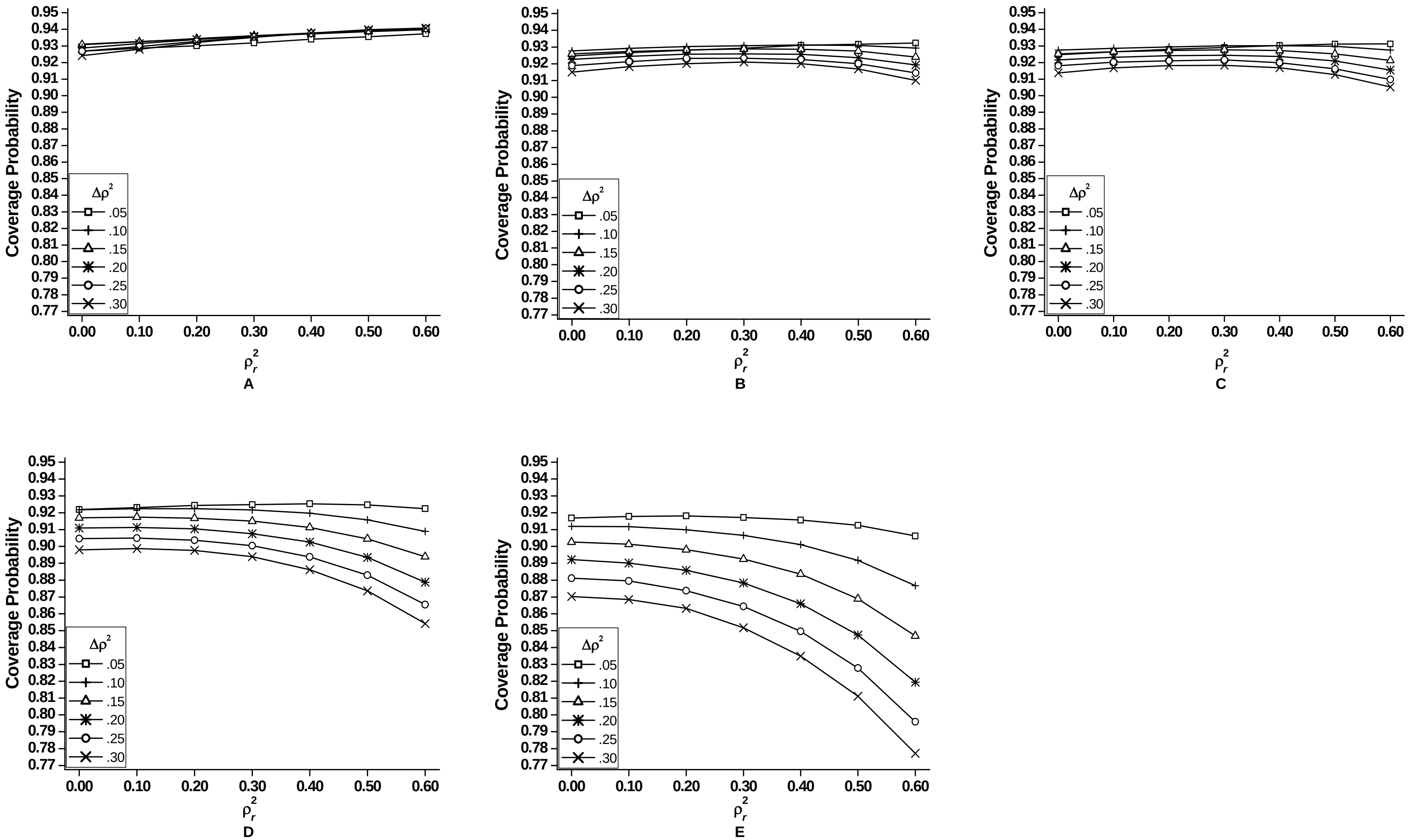


Figure 3-10. Effect of the **X** $\times\ \rho_r^2 \times \Delta\rho^2$ interaction on coverage probability for $\Delta\rho^2 > 0$. A) **X** sampled from a multivariate normal population. B) **X** sampled from a pseudo-t(10) distribution (g = 0, h = .058). C) **X** sampled from a pseudo-χ^2(10) distribution (g = .301, h = -.017). D) **X** sampled from a pseudo-χ^2(10) distribution (g = .502, h = -.048). E) **X** sampled from a pseudo-exponential distribution (g = .760, h = -.098).

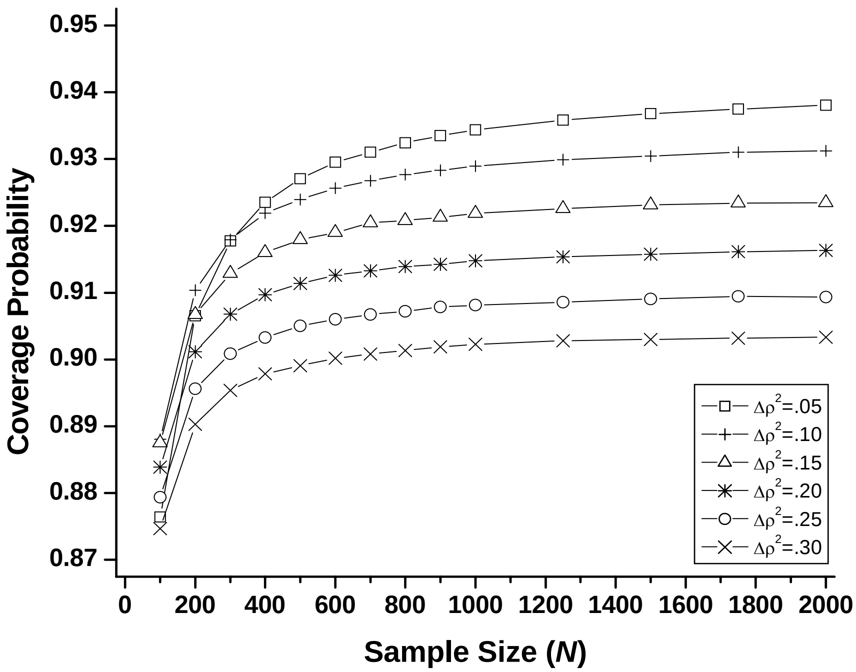


Figure 3-11. Interaction between sample size, *n*, and the population squared semipartial correlation, $\Delta\rho^2$, and the impact on coverage probability for $\Delta\rho^2 > 0$.

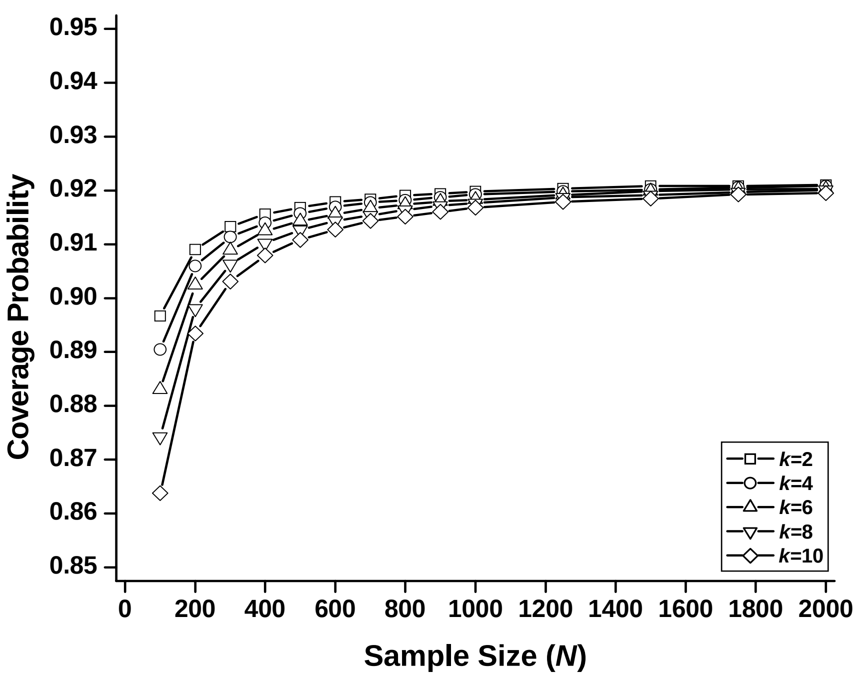


Figure 3-12. Effect of the interaction between sample size, *n* , and number of predictors, *k,* on coverage probability.

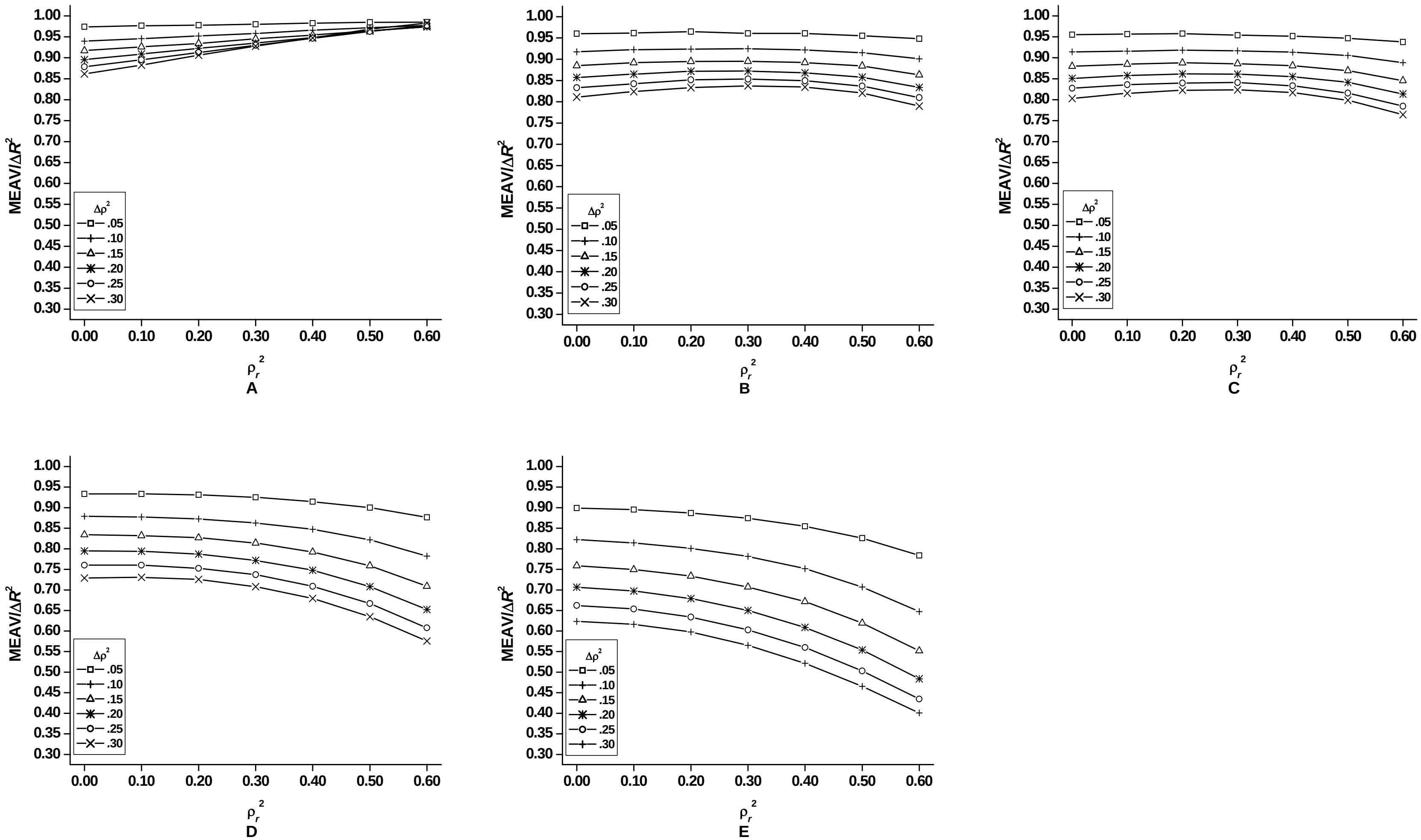


Figure 3-13. Ratio of mean estimated asymptotic variance to the variance in ΔR^2 (MEAV/Var ΔR^2) as a function of the distribution for the predictors, $\Delta\rho^2$, and ρ_r^2 .A) **X** sampled from a multivariate normal population. B) **X** sampled from a pseudo-*t*(10) distribution (g = 0, h = .058). C) **X** sampled from a pseudo-χ^2(10) distribution (g = .301, h = -.017). D) **X** sampled from a pseudo-χ^2(10) distribution (g = .502, h = -.048). E) **X** sampled from a pseudo-exponential distribution (g = .760, h = -.098).

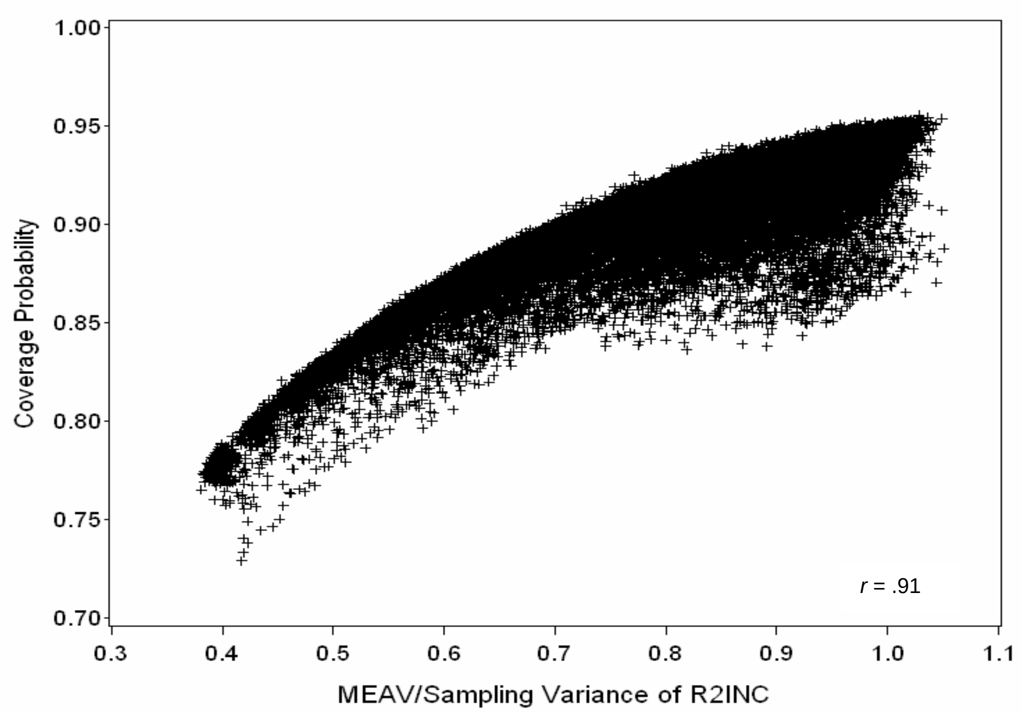


Figure 3-14. Relationship between coverage probability and the ratio of mean estimated asymptotic variance to the empirical sampling variance of ΔR^2 for $\Delta\rho^2 > 0$

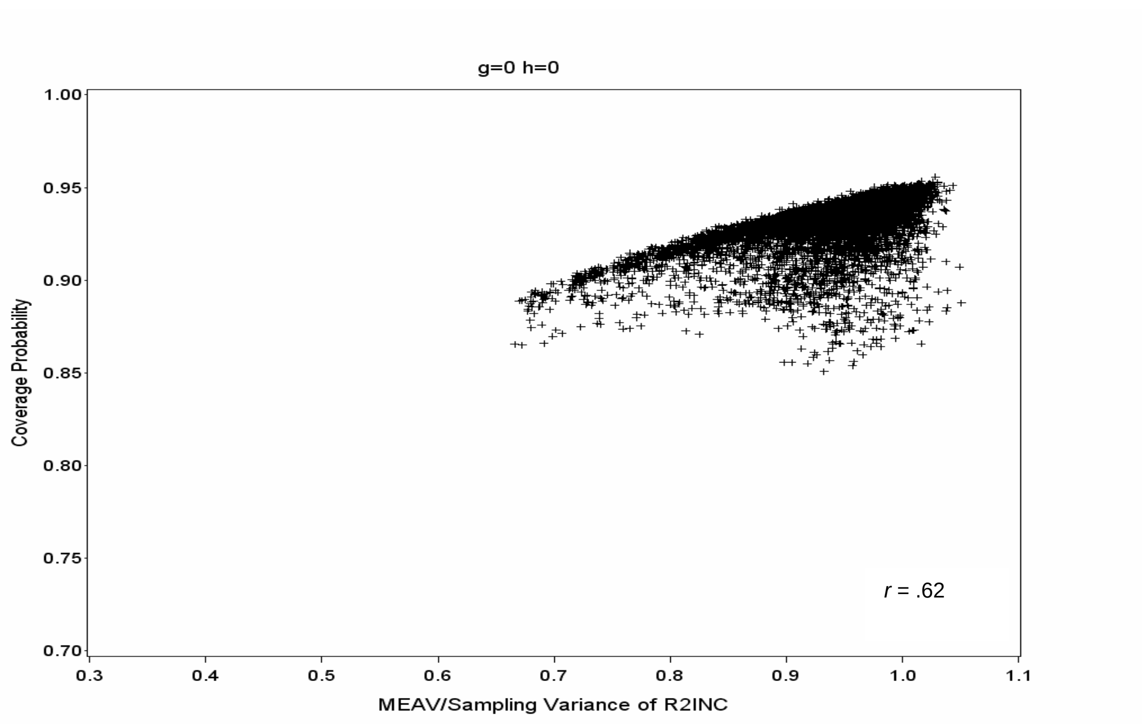


Figure 3-15. Relationship between coverage probability and the ratio of mean estimated asymptotic variance to the empirical sampling variance of ΔR^2 for $\Delta\rho^2 > 0$ for multivariate normal data ($g = 0$, $h = 0$).

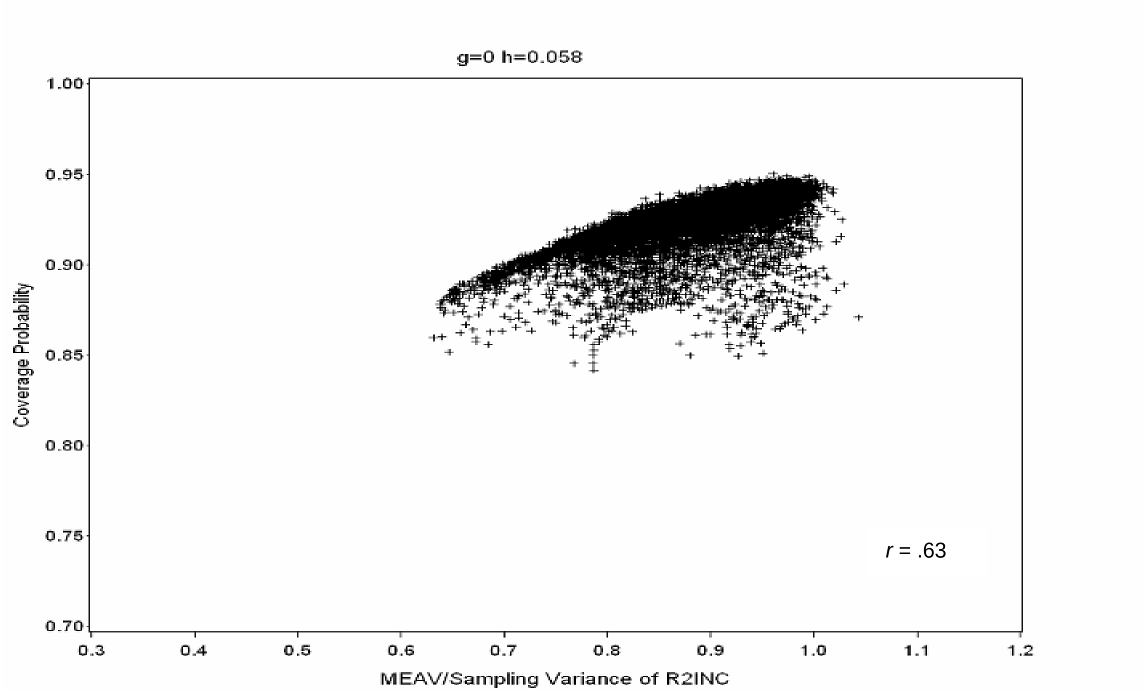


Figure 3-16. Relationship between coverage probability and the ratio of mean estimated asymptotic variance to the empirical sampling variance of ΔR^2 for $\Delta\rho^2 > 0$ and **X** distributed pseudo-t_{10} (g = 0, h = .058).

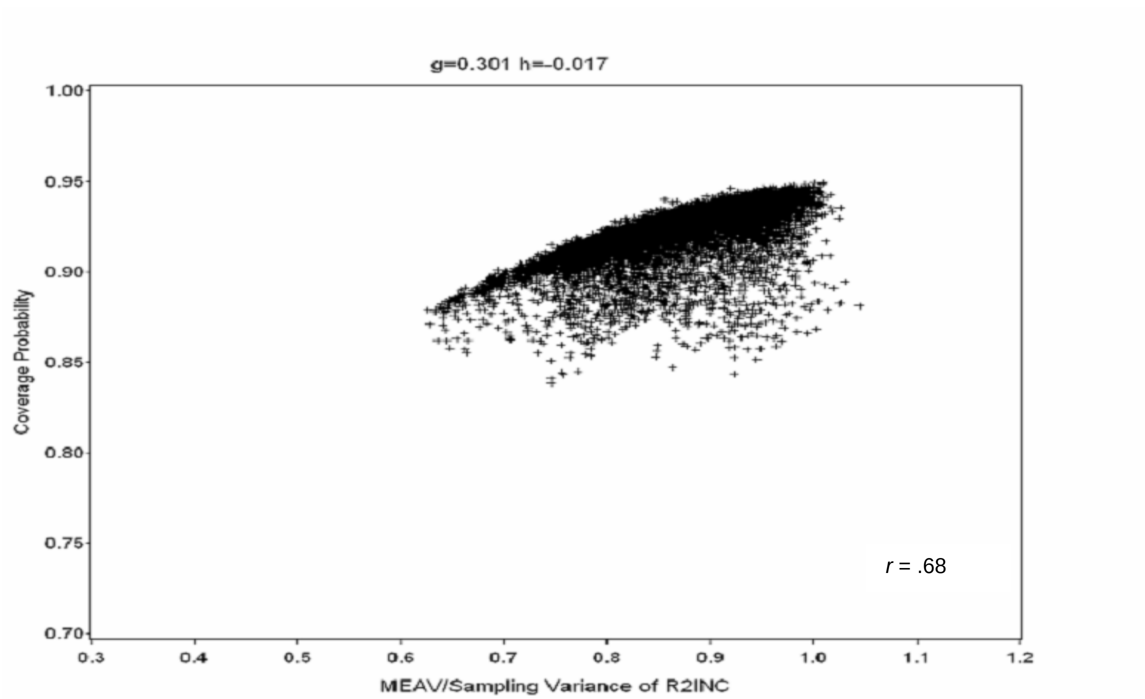


Figure 3-17. Relationship between coverage probability and the ratio of mean estimated asymptotic variance to the empirical sampling variance of ΔR^2 for $\Delta\rho^2 > 0$ and **X** distributed pseudo-χ^2_{10} (g = .502, h = -.048).

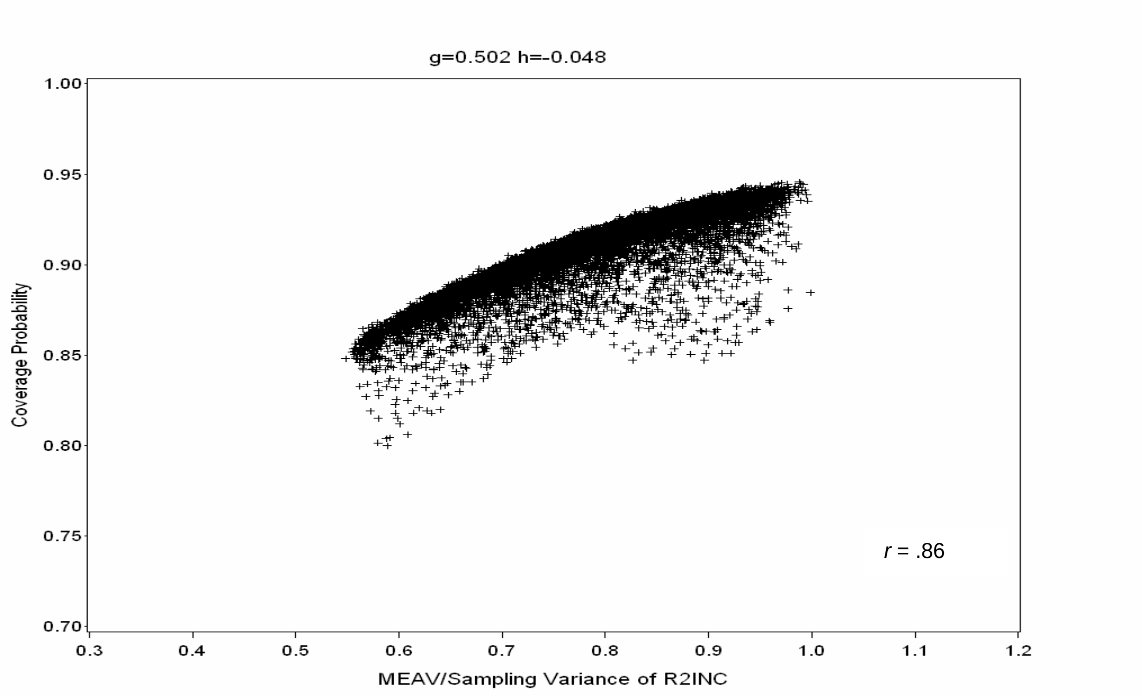


Figure 3-18. Relationship between coverage probability and the ratio of mean estimated asymptotic variance to the empirical sampling variance of ΔR^2 for $\Delta\rho^2 > 0$ and **X** distributed pseudo-χ^2_4 (g = .502, h = -.048).

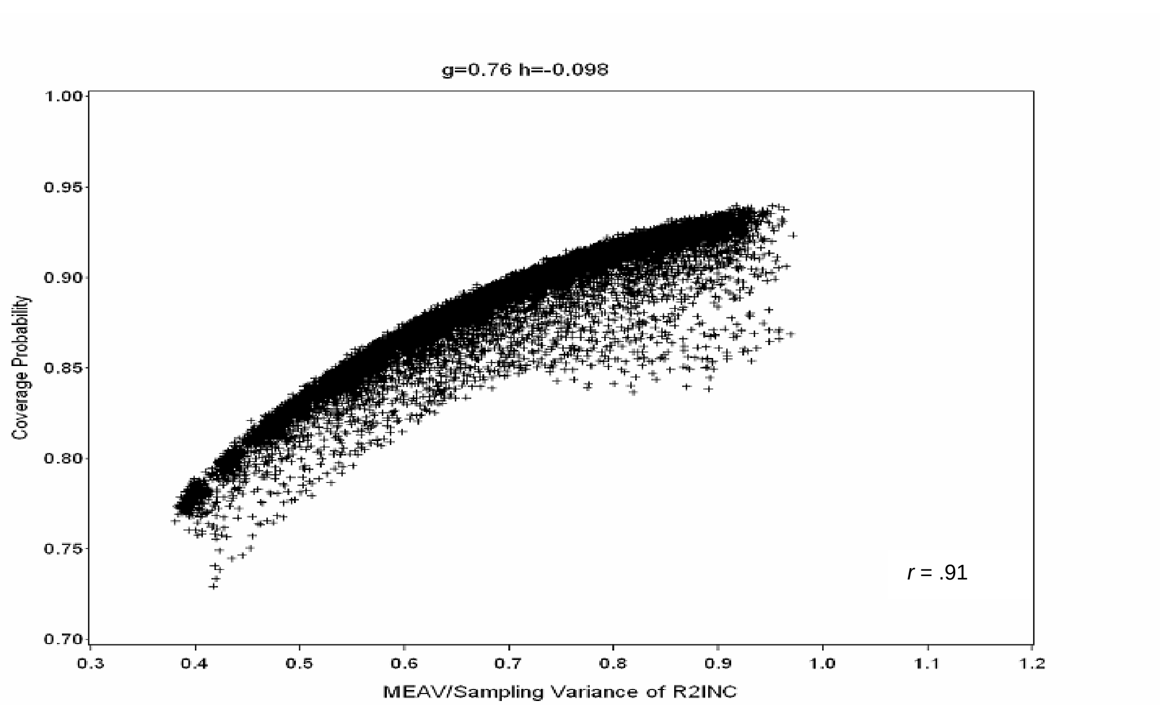


Figure 3-19. Relationship between coverage probability and the ratio of mean estimated asymptotic variance to the empirical sampling variance of ΔR^2 for $\Delta\rho^2 > 0$ and **X** distributed pseudo-exponential (g =.760, h = -.098)..

CHAPTER 4
DISCUSSION

Given the current emphasis on reporting effect sizes and confidence intervals, Alf and Graf's approach to constructing confidence intervals for the squared semipartial correlation is appealing in its simplicity. Confidence limits can be computed using only a hand calculator and the computer output from a multiple regression analysis. Results of this study would caution against widespread use because the procedure demonstrated poor control of coverage probability. Even when the distributional assumption of multivariate normality holds for the data, the asymptotic confidence interval procedure is biased and with sample sizes typically used in psychology, the coverage probability for a nominal 95% confidence interval will tend to be less than .95. As shown in this study, when nonnormality is introduced, depending on the degree, coverage probability can be dramatically less than .95 even with samples as large as 2000. This is especially true when the predictors included in a multiple regression model do not follow a multivariate normal distribution. When the predictors are nonnormal, the Alf and Graf procedure produces a confidence interval that tends to be too liberal. With extreme nonnormality, the interval will be much too narrow and the contribution of a single variable to the regression will be minimized. Since multivariate normality is rarely observed in practice, the poor performance of this procedure for use as a measure of effect size accuracy is particularly disappointing. It appears that accuracy is sacrificed for the sake of computational facility.

One of the goals of this study was to determine if we could identify a minimum, "fail safe", sample size for which the confidence interval offers adequate coverage probability over a wide range of distributional conditions and regression model characteristics. Despite the sizeable amount of data simulated and analyzed, we must conclude that a sample size well in excess of 2000 would be necessary to demonstrate the robustness of this procedure against nonnormality even if we were willing to adopt a more liberal standard, i.e. > .925. Increasing the sample size to a level that is atypical for research in the behavioral and social sciences (e.g., ≥ 1000) in an attempt to ensure adequate coverage probability when the distributional assumption is violated will not yield the desired result. In the case of extreme nonnormality in the predictors, the extremely slow incremental improvement in coverage probability, as a function of sample size, suggests that the procedure is likely to be inaccurate no matter how large a sample is used.

For both normal and nonnormal data, coverage probability is also dependent on the population effect sizes for both the squared semipartial correlation coefficient and the squared multiple correlation coefficient for the model to which the variable of interest has been added; however, the pattern of empirical coverage probabilities over the range of factors manipulated is completely different. In order to illustrate this, Figure 4-1 presents three hypothetical situations: (a) a variable with a small population effect size is added to a model that explains none of the variance in the criterion, $\Delta\rho^2 = .05,\ \rho_r^2 = .00;$ (b) a variable with a medium population effect size is added to a model in which the effect size associated with the population squared multiple correlation coefficient is already relatively large, $\Delta\rho^2 = .15,\ \rho_r^2 = .30;$ and (c) a variable with a large population effect size is added to a model for which the effect size associated with the population squared multiple correlation coefficient is very large. For each of these situations, average coverage probability for each nonnormal distribution is compared to the case where **X** is multivariate normal as a function of sample size. Coverage probabilities are averaged over the distribution for **e** and the number of predictors, *k*. Although estimates are based on only 25 cases, there is a clear trend. Not only is coverage probability worse for nonnormal data, but the behavior of the confidence interval shows a pattern that is the direct opposite of that observed for normal predictors regardless of the degree of nonnormality. For normal predictors, coverage probability improves as both ρ_r^2 and $\Delta\rho^2$ become larger. This suggests that coverage probability is closer to the nominal confidence level for larger effect sizes than it is when the effect size associated with an added variable is small. The reverse is true when there is nonnormality in the predictors. Here, the confidence interval is more accurate for smaller effect sizes and coverage probability decreases as the effect size gets larger. Table 4-1 shows coverage probability is clearly unacceptable for a large effect under the most nonnormal conditions investigated. Due to this inadequacy, this procedure for estimating confidence intervals falls far short as a reliable measure of importance in reporting research results and its use defeats the purpose of those interested in reforming statistical practice.

Limitations

It is obviously not possible to investigate every imaginable type of nonnormality that might occur in applied research situations. This study has investigated a reasonably broad range of nonnormal distributions without claiming to have exhausted the possibilities. Some might argue that the method of simulating nonnormality is not representative of real world data. For example, it is unlikely that all of the variables in a multiple regression analysis are samples from a population that is distributed with the same skewness and kurtosis as the exponential distribution and this is where estimated coverage probability deviates markedly from the nominal. However, the premise in simulation studies investigating robustness is that if a procedure performs well under extreme conditions, it can be expected to work under most conditions likely to be encountered by researchers. Therefore, adequate coverage probability across a wide range of possible conditions should be demonstrated in order to recommend a statistical procedure.

In this study, coverage probability characteristics were very similar when the data were distributed either pseudo-t_{10} or pseudo-χ^2_{10}. Both univariate and multivariate kurtosis measures are nearly equal for the two distributions, but pseudo-t_{10} is symmetrical while pseudo-χ^2_{10} is moderately skewed. Although this would suggest that skewness has less influence on coverage probability than kurtosis, the evidence is based on only one comparison. In this study, the distribution for the predictors, in a sense, serves as a proxy for the degree of multivariate nonnormality and the distribution for the errors is a reflection of the additive influence of univariate nonnormality. In the ANOVA and variance components analyses reported, these variables were treated as categorical, as were the levels of n, k, $\Delta\rho^2$ and ρ^2_r. Since multivariate skewness and kurtosis can be quantified as continuous variables using Mardia's measures of multivariate skewness, $b_{1,k}$, and multivariate kurtosis, $b_{2,k}$, and univariate skewness can be measured by γ_1 and γ_2, the impact of skewness and kurtosis on coverage probability could be modeled using multiple regression.

The size of the squared semipartial correlation may have examined too wide a range and the values studied were somewhat coarse. The large values studied, i.e. .25 and .30, where the procedure tends to perform especially poorly, correspond to a large effect size and may be relatively rare in practice. At the other extreme, Algina and Moulder reported results for $\Delta\rho^2 < .05$ that were similar to the coverage probabilities observed for $\Delta\rho^2 = .05$ with multivariate normal data. Since small to medium effect sizes are much more common than large effect sizes in research in the social and behavioral sciences, values for $\Delta\rho^2$ less than .05 and in smaller steps between .05 and .15 would have given a more complete picture. Nonetheless, this would not have changed the findings or conclusions of this study.

Further Research

The asymptotic method of constructing confidence intervals assumes symmetry by referring to a single normal distribution with two-equal sized tails sliced off. Our concern in using an approximation based on asymptotic theory is whether the distribution of the difference in squared correlations approaches normality for a given sample size. This approach assumes that the sampling distribution retains the same shape regardless of the value of the population parameter. However, confidence intervals are not always symmetrical around sample statistics and their sampling distributions do not always have the same shape for different values. If the distribution that generates the data is not symmetric, using the asymptotic variance derived under the assumption of symmetry typically underestimates the actual asymptotic variance.

When one set of predictors is a subset of the other, as is true for the case where, $R_f^2 - R_r^2 = \Delta R^2$, R_r^2 will never be larger than R_f^2 and as a result, the sampling distribution of ΔR^2 is truncated at zero. Therefore, if the difference between R_f^2 and R_r^2 is not significant, the approximation will be inappropriate regardless of sample size (Graf & Alf, 1999). Research directed at understanding the distribution of ΔR^2, with the goal of developing more appropriate methods of approximation, is needed.

The poor coverage of the asymptotic confidence interval described in this study suggests that developing an alternative method for constructing confidence intervals for $\Delta\rho^2$ that has adequate probability coverage for practical sample sizes is an important goal. The effectiveness of asymptotically distribution-free and nonparametric methods employing the bootstrap for use in constructing a confidence interval for $\Delta\rho^2$ should be explored. It may be possible to obtain much more accurate confidence intervals than were demonstrated in this study, with smaller, more realistic sample sizes, using bootstrap methods. These studies will be demanding given the computational demands of simulations employing resampling techniques, the wide variety of nonnormal distributions that could be studied, and the number of potential combinations that could be generated for the predictors and the residuals.

In addition, when one set of variables is not a subset of the other, there are no small sample procedures to test the significance of the difference in correlations. Thus, when we are interested in whether a set of predictors performs equally well in two populations, i.e. the difference between two independent samples, $R_a^2 - R_b^2$, asymptotic confidence interval methods are the only procedures available (Graf & Alf, 1999, p. 119). For the comparison of squared multiple correlations from independent samples, Algina and Keselman (1999) found that asymptotic confidence intervals were inaccurate when the population multiple correlations were zero or very small and in some conditions, were inaccurate when sample sizes and coefficients were unequal. The procedure worked reasonably well with equal sample sizes as small as n = 40 and equal population multiple correlations that were sufficiently different from zero. When multiple correlation coefficients and sample sizes were unequal, the sample sizes required for control of coverage probabilities ranged from 40 to 960 for the smaller sample. These results, however, were obtained by simulating multivariate normal data. Since the present study showed unequivocally that asymptotic confidence interval procedures are inaccurate with nonnormal data when the two multiple correlation coefficients compared are estimated from the same sample, an important next step is to evaluate coverage probability for the comparison of multiple correlation coefficients from independent samples under conditions of nonnormality. This will be a huge undertaking since the range of nonnormal distributions is considerable. For independent samples, this problem is compounded by the necessity of manipulating not only the degree of nonnormality in the predictor and error distributions, but simulating a broad range of conditions for two populations instead of one. The number of possible combinations is extraordinary. Further complexity is introduced in that the manipulation of sample size must include conditions in which the two samples are equal and unequal.

Conclusion

The purpose of this dissertation was to evaluate the accuracy of confidence intervals around an effect size measure in multiple regression analysis (ΔR^2), based on an asymptotic approach to the problem as outlined by Hedges and Olkin (1981), Olkin and Finn (1995), and Graf and Alf (1999) when the distributional assumption of multivariate normality does not hold. Algina and Moulder (2001) found, and this study confirms, that even with multivariate normal data asymptotic confidence intervals were generally inaccurate except when sample sizes were large, i.e. $n \geq 600$. There are many considerations that influence sample size decisions, including power and accuracy. With the current emphasis on reporting effect sizes, it is recommended that researchers plan studies with sufficient sample size so that effect sizes are estimated with adequate accuracy and hypotheses are tested with sufficient power. A researcher interested in setting confidence intervals for $\Delta\rho^2$ to estimate the importance of individual variables to the regression will find it challenging to design a study with a large enough sample to ensure accuracy even in the unlikely event that multivariate normal data is anticipated.

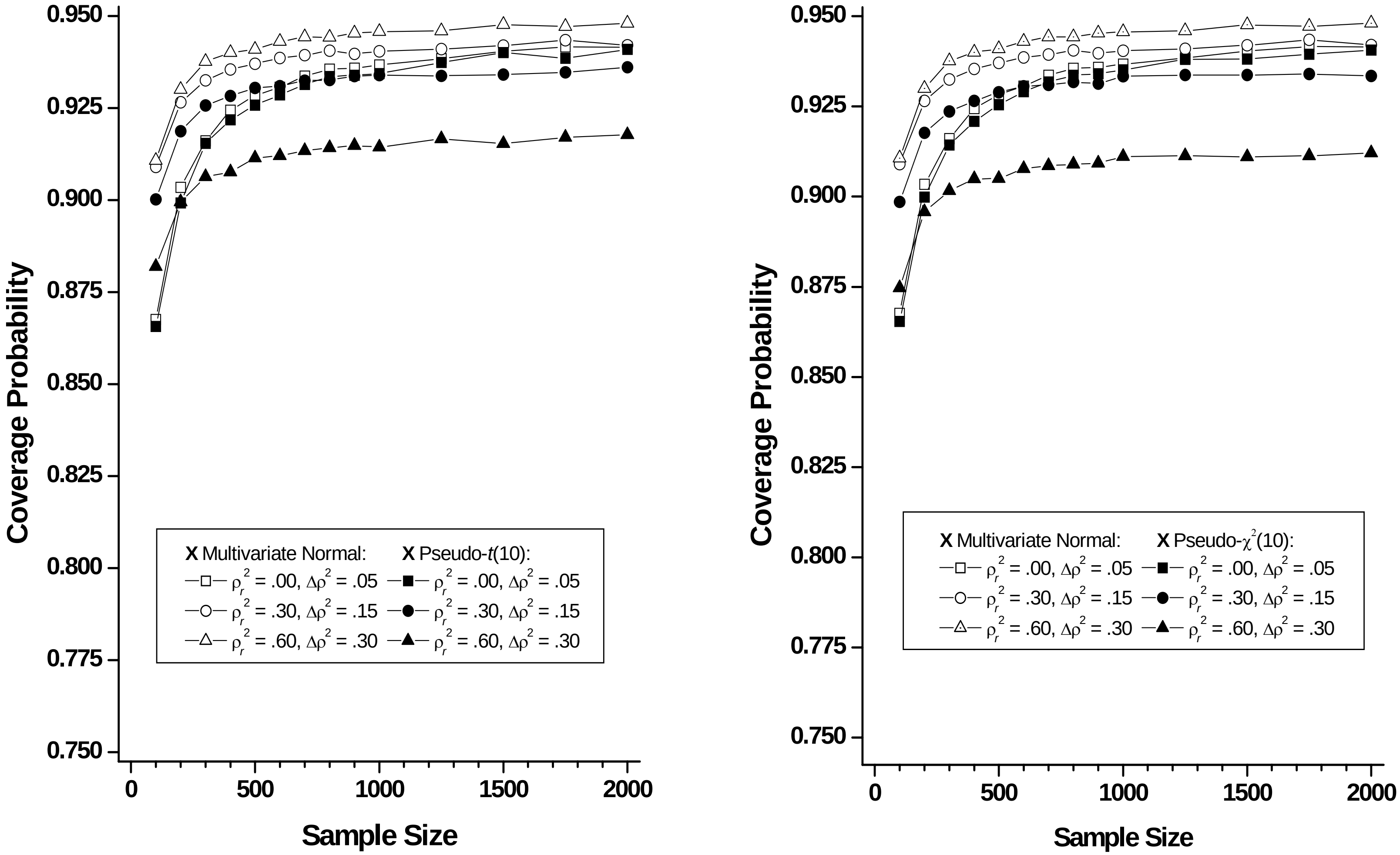
It might be tempting for researchers to continue to use the asymptotic method for constructing confidence intervals in situations where there is the expectation that the data is approximately multivariate normal. This should be discouraged. Although it is highly recommended that one should always carefully inspect one’s data, there are very few tests for examining multivariate normality. Graphical methods are not reliable for evaluating the degree of nonnormality present in multivariate data. The graphical test, similar to the *Q-Q* plot discussed for the univariate case, is a plot of the ordered squared Mahalanobis distance against the χ^2 distribution with k degrees of freedom. The analytical tests simply assess the multivariate measures of skewness and kurtosis and the distribution of these test statistics is not known. In addition, indices of skew and kurtosis can be deceptive because these estimates are likely to have very large standard errors unless the sample size is very large (Algina, Keselman, & Penfield, 2005).

Our conclusion is that use of Alf and Graf's method for constructing confidence intervals for the squared semipartial correlation coefficient should be abandoned. Rather, we should turn our efforts toward the search for a confidence interval that has good coverage probability for sample sizes typical of research in the behavioral and social sciences, over a wide range of distributions and values for ρ_r^2, ρ_f^2, and k.

Table 4-1. Coverage Probability as a Function of n, Selected Values for ρ_r^2 and $\Delta\rho^2$, and Distribution for the Predictors.

Predictor Distribution	ρ_r^2	$\Delta\rho^2$	Sample Size (n) 100	200	300	400	500	600	700	800	900	1000	1250	1500	1750	2000
Normal	.00	.05	.864	.904	.916	*.927*	*.932*	*.932*	*.937*	*.938*	*.938*	*.939*	**.940**	**.944**	**.945**	**.944**
	.30	.15	.913	*.932*	*.937*	**.940**	**.941**	**.945**	**.944**	**.946**	**.944**	**.945**	**.948**	**.948**	**.949**	**.948**
	.60	.30	.915	*.932*	*.937*	**.941**	*.939*	**.945**	**.945**	**.947**	**.946**	**.947**	**.947**	**.950**	**.948**	**.949**
Pseudo-*t* (10)	.00	.05	.866	.899	.915	.922	*.926*	*.929*	*.931*	*.934*	*.934*	*.934*	*.937*	**.940**	*.939*	**.941**
	.30	.15	.900	.919	*.926*	*.928*	*.931*	*.931*	*.932*	*.933*	*.934*	*.934*	*.934*	*.934*	*.935*	*.936*
	.60	.30	.882	.900	.906	.908	.911	.912	.913	.914	.915	.914	.917	.915	.917	.918
Pseudo-χ^2(10)	.00	.05	.865	.900	.914	.921	*.926*	*.929*	*.932*	*.934*	*.934*	*.935*	*.938*	*.938*	**.940**	**.941**
	.30	.15	.899	.918	.924	*.927*	*.929*	*.931*	*.931*	*.932*	*.931*	*.933*	*.934*	*.934*	*.934*	*.933*
	.60	.30	.875	.896	.902	.905	.905	.908	.909	.909	.909	.911	.911	.911	.911	.912
Pseudo-χ^2(4)	.00	.05	.862	.897	.911	.919	.923	*.926*	*.928*	*.930*	*.934*	*.933*	*.934*	*.936*	*.937*	*.938*
	.30	.15	.889	.904	.912	.915	.916	.917	.919	.919	.920	.921	.921	.921	.920	.920
	.60	.30	.829	.845	.851	.853	.855	.857	.858	.858	.859	.858	.858	.858	.860	.860
Pseudo-exponential	.00	.05	.854	.893	.907	.913	.918	.921	.924	*.925*	*.927*	*.928*	*.930*	*.931*	*.932*	*.933*
	.30	.15	.862	.882	.888	.893	.893	.894	.896	.896	.896	.899	.899	.899	.900	.900
	.60	.30	.761	773	.776	.777	.778	.777	.779	.779	.779	.780	.781	.778	.780	.782

Note: Bold results are estimated coverage probabilities between .94 and .96; italicized results are estimated coverage probabilities between .925 and .975.



A B

Figure 4-1. Coverage probability as a function of sample size and several combinations of ρ_r^2 and $\Delta\rho^2$ for predictors sampled from a normal distribution and (A) pseudo-t_{10}; (B) pseudo- χ^2_{10}; (C) pseudo-χ^2_4; and (D) pseudo-exponential distributions.

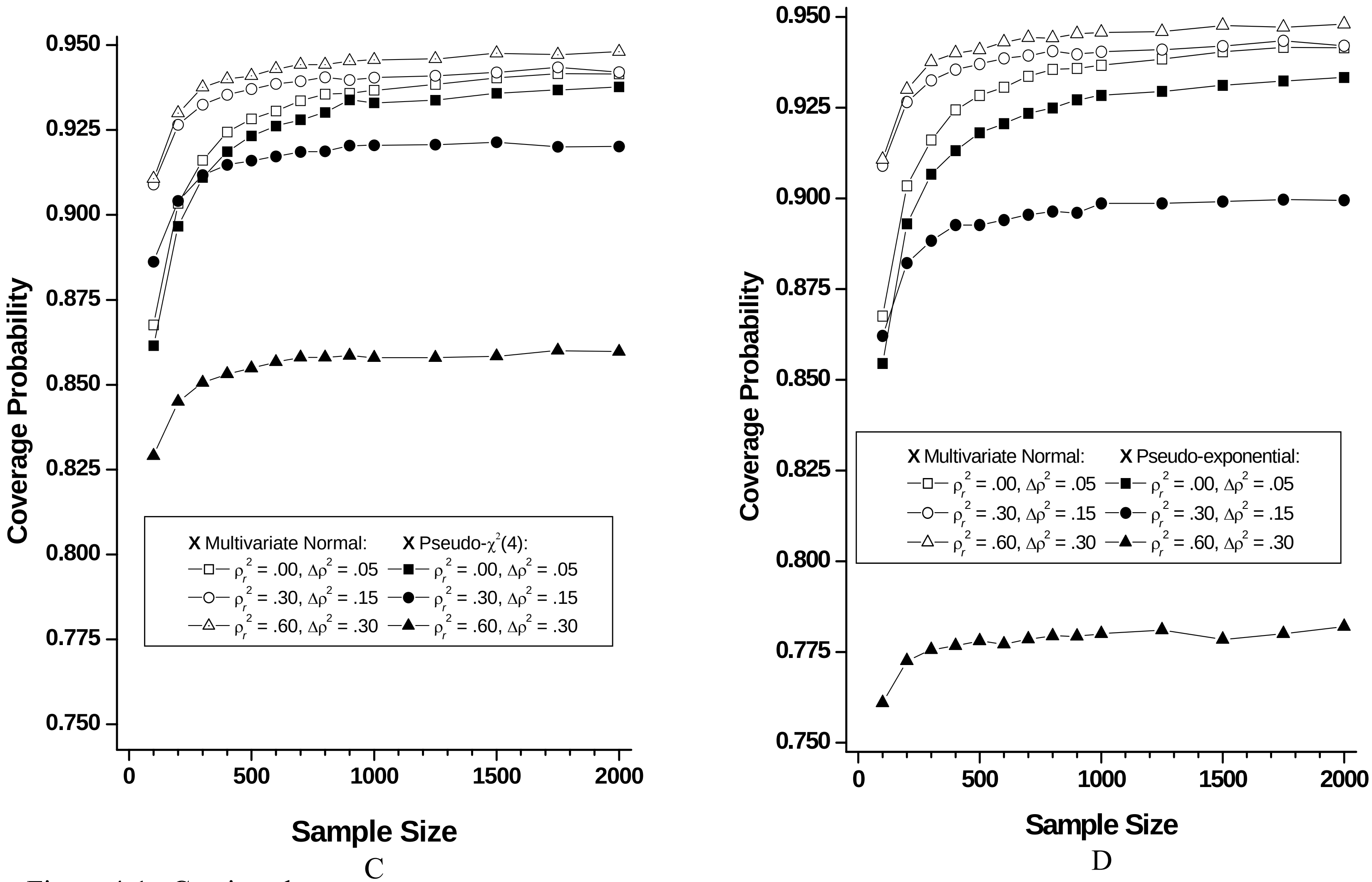


Figure 4-1. Continued

APPENDIX A
PROGRAM FOR COMPUTING MARDIA'S MULTIVARIATE MEASURES OF SKEWNESS AND KURTOSIS IN SAS

```
/*This program was used to compute values for Mardia's multivariate indices
of skewness and kurtosis for the four nonnormal distributions investigated in
this study.*/

data;
g=.00;
h=.058;
*g=.301;
*h=-.017;
*g=.502;
*h=-.048;
*g=.760;
*h=-.098;

if g^=0 then do;
  popmu=(exp(g**2/(2*(1-h)))-1)/(g*(sqrt(1-h)));
   num1=exp(2*g**2/(1-2*h));
   num2=2*exp(g**2/(2*(1-2*h)));
   den=g**2*(sqrt(1-2*h));
   popvar=(-1)*popmu**2+
       (num1-num2+1)/den;;
end;

if g=0 then do;
   popmu=0;
   popvar=1/((1-2*h)**(3/2));
end;

do i =1 to 1000000;
tempx1=rannor(0);
tempy1=rannor(0);
tempx2=rannor(0);
tempy2=rannor(0);
tempx3=rannor(0);
tempy3=rannor(0);
tempx4=rannor(0);
tempy4=rannor(0);
tempx5=rannor(0);
tempy5=rannor(0);
tempx6=rannor(0);
tempy6=rannor(0);
tempx7=rannor(0);
tempy7=rannor(0);
tempx8=rannor(0);
tempy8=rannor(0);
tempx9=rannor(0);
tempy9=rannor(0);
tempx10=rannor(0);
tempy10=rannor(0);
```

```
if g=0 then do;
zx1=(tempx1*exp(h*tempx1**2/2)-popmu)/sqrt(popvar);
zy1=(tempy1*exp(h*tempy1**2/2)-popmu)/sqrt(popvar);
zx2=(tempx2*exp(h*tempx2**2/2)-popmu)/sqrt(popvar);
zy2=(tempy2*exp(h*tempy2**2/2)-popmu)/sqrt(popvar);
zx3=(tempx3*exp(h*tempx3**2/2)-popmu)/sqrt(popvar);
zy3=(tempy3*exp(h*tempy3**2/2)-popmu)/sqrt(popvar);
zx4=(tempx4*exp(h*tempx4**2/2)-popmu)/sqrt(popvar);
zy4=(tempy4*exp(h*tempy4**2/2)-popmu)/sqrt(popvar);
zx5=(tempx5*exp(h*tempx5**2/2)-popmu)/sqrt(popvar);
zy5=(tempy5*exp(h*tempy5**2/2)-popmu)/sqrt(popvar);
zx6=(tempx6*exp(h*tempx6**2/2)-popmu)/sqrt(popvar);
zy6=(tempy6*exp(h*tempy6**2/2)-popmu)/sqrt(popvar);
zx7=(tempx7*exp(h*tempx7**2/2)-popmu)/sqrt(popvar);
zy7=(tempy7*exp(h*tempy7**2/2)-popmu)/sqrt(popvar);
zx8=(tempx8*exp(h*tempx8**2/2)-popmu)/sqrt(popvar);
zy8=(tempy8*exp(h*tempy8**2/2)-popmu)/sqrt(popvar);
zx9=(tempx9*exp(h*tempx9**2/2)-popmu)/sqrt(popvar);
zy9=(tempy9*exp(h*tempy9**2/2)-popmu)/sqrt(popvar);
zx10=(tempx10*exp(h*tempx10**2/2)-popmu)/sqrt(popvar);
zy10=(tempy10*exp(h*tempy10**2/2)-popmu)/sqrt(popvar);
end;

if g^=0 then do;
zx1=((1/g)*(exp(g*tempx1)-1)*exp(h*tempx1**2/2)-popmu)/sqrt(popvar);
zy1=((1/g)*(exp(g*tempy1)-1)*exp(h*tempy1**2/2)-popmu)/sqrt(popvar);
zx2=((1/g)*(exp(g*tempx2)-1)*exp(h*tempx2**2/2)-popmu)/sqrt(popvar);
zy2=((1/g)*(exp(g*tempy2)-1)*exp(h*tempy2**2/2)-popmu)/sqrt(popvar);
zx3=((1/g)*(exp(g*tempx3)-1)*exp(h*tempx3**2/2)-popmu)/sqrt(popvar);
zy3=((1/g)*(exp(g*tempy3)-1)*exp(h*tempy3**2/2)-popmu)/sqrt(popvar);
zx4=((1/g)*(exp(g*tempx4)-1)*exp(h*tempx4**2/2)-popmu)/sqrt(popvar);
zy4=((1/g)*(exp(g*tempy4)-1)*exp(h*tempy4**2/2)-popmu)/sqrt(popvar);
zx5=((1/g)*(exp(g*tempx5)-1)*exp(h*tempx5**2/2)-popmu)/sqrt(popvar);
zy5=((1/g)*(exp(g*tempy5)-1)*exp(h*tempy5**2/2)-popmu)/sqrt(popvar);
zx6=((1/g)*(exp(g*tempx6)-1)*exp(h*tempx6**2/2)-popmu)/sqrt(popvar);
zy6=((1/g)*(exp(g*tempy6)-1)*exp(h*tempy6**2/2)-popmu)/sqrt(popvar);
zx7=((1/g)*(exp(g*tempx7)-1)*exp(h*tempx7**2/2)-popmu)/sqrt(popvar);
zy7=((1/g)*(exp(g*tempy7)-1)*exp(h*tempy7**2/2)-popmu)/sqrt(popvar);
zx8=((1/g)*(exp(g*tempx8)-1)*exp(h*tempx8**2/2)-popmu)/sqrt(popvar);
zy8=((1/g)*(exp(g*tempy8)-1)*exp(h*tempy8**2/2)-popmu)/sqrt(popvar);
zx9=((1/g)*(exp(g*tempx9)-1)*exp(h*tempx9**2/2)-popmu)/sqrt(popvar);
zy9=((1/g)*(exp(g*tempy9)-1)*exp(h*tempy9**2/2)-popmu)/sqrt(popvar);
zx10=((1/g)*(exp(g*tempx10)-1)*exp(h*tempx10**2/2)-popmu)/sqrt(popvar);
zy10=((1/g)*(exp(g*tempy10)-1)*exp(h*tempy10**2/2)-popmu)/sqrt(popvar);
end;

b12=(zx1*zy1+zx2*zy2)**3;
b22=(zx1**2+zx2**2)**2;
b14=(zx1*zy1+zx2*zy2+zx3*zy3+zx4*zy4)**3;
b24=(zx1**2+zx2**2+zx3**2+zx4**2)**2;
b16=(zx1*zy1+zx2*zy2+zx3*zy3+zx4*zy4+zx5*zy5+zx6*zy6)**3;
b26=(zx1**2+zx2**2+zx3**2+zx4**2+zx5**2+zx6**2)**2;
b18=(zx1*zy1+zx2*zy2+zx3*zy3+zx4*zy4+zx5*zy5+zx6*zy6+zx7*zy7+zx8*zy8)**3;
b28=(zx1**2+zx2**2+zx3**2+zx4**2+zx5**2+zx6**2+zx7**2+zx8**2)**2;
b110=(zx1*zy1+zx2*zy2+zx3*zy3+zx4*zy4+zx5*zy5+zx6*zy6+zx7*zy7
      +zx8*zy8+zx9*zy9+zx10*zy10)**3;
```

```
b210=(zx1**2+zx2**2+zx3**2+zx4**2+zx5**2+zx6**2+zx7**2+zx8**2
        +zx9**2+zx10**2)**2;
uniskew=(zx1*zx2)**3;
unikurt=zx1**4;
output;
end;
keep b12 b22 b14 b24 b16 b26 b18 b28 b110 b210 uniskew unikurt;
proc means;
var b12 b22 b14 b24 b16 b26 b18 b28 b110 b210 uniskew unikurt;
run;
quit;
```

APPENDIX B
DATA SIMULATION SAS PROGRAM

```
/*This program was used to simulate data for the conditions under study,
construct a 95% confidence interval for the squared semipartial correlation
according to the formulas presented by Alf and Graf, and compute the coverage
probability.*/

proc iml;
filename output 'C:\Dissertation\nx1nne4.dat';
start1=0;
reps = 10000;
n=nrow(x);
p=ncol(x);

do i=1 to 5;                          /*Predictor distributions*/
   if i=1 then do;                    /*Normal distribution*/
      g=0;
      h=0;
   end;
   if i=2 then do;                    /*t-distribution with 10 df*/
      g = 0;
      h = .058;
   end;
   if i=3 then do;                    /*Chi-sq with 10 df*/
      g=.301;
      h=-.017;
   end;
   if i=4 then do;                    /*Chi-sq with 4 df*/
      g=.502;
      h=-.048;
   end;
   if i=5 then do;                    /*Exponential distribution*/
      g =.760;
      h=-.098;
   end;

do j=1 to 5;                          /*Distributions for errors*/
   if j=1 then do;                    /*Normal distribution*/
      ge=0;
      he=0;
   end;
   if j=2 then do;                    /*t-distribution with 10 df*/
      ge = 0;
      he = .058;
   end;
   if j=3 then do;                    /*Chi-sq with 10 df*/
      ge=.301;
      he=-.017;
   end;
   if j=4 then do;                    /*Chi-sq with 4 df*/
      ge=.502;
      he=-.048;
   end;
   if j=5 then do;                    /*Exponential*/
      ge =.760;
```

```
      he=-.098;
   end;

   do n = 100 to 1000 by 100;                         /*Sample size*/
      do k = 2 to 10 by 2;                            /*Predictors*/
         do rho2r = .00 to .60 by .10;                /*R-sq for reduced
                                                        model*/
            do rho2f = rho2r to (rho2r + .30) by .05; /*R-sq for full model*/
              do rep = 1 to reps;                     /*Replications*/
              start1=start1+1;
              counter1=0;
              counter2=0;
              counter3=0;
              rhor = root(rho2r);
              rhof = root(rho2f);
              rho2inc = rho2f-rho2r;

/*Calculate vector of regression coefficients transform to uncorrelated
  Predictors*/

                  B=j(k,1,0);
                  B[k-1,1]=rhor;
                  B[k,1]=rho2inc##.5;

/*Generates a matrix of predictor variables with mean=0 and variance = 1*/

                  z=rannor(repeat(0,n,k));
                  if g^=0 then do;  /*uses g and h generator to transform x*/
                  x=((exp(g#z)-j(n,p,1))/g)#exp(h#z##2/2);
                  popmu=(exp(g##2/(2#(1-h)))-1)/(g#(sqrt(1-h)));
                  num1=exp(2#g##2/(1-2#h));
                  num2=2#exp(g##2/(2#(1-2#h)));
                  den=g##2#(sqrt(1-2#h));
                  popvar=(-1)#popmu##2+(num1-num2+1)/den;
                  xx=(x-popmu)/popvar##.5;
                  end;
                    if g=0 then do;
                    popmu=0;
                    popvar=1/((1-2#h)##(3/2));
                    x=z#exp(h#z##2/2);
                    xx=(x-popmu)/popvar##.5;
                    end;

/*Generate a vector of errors with mean = 0 and variance = 1-rho2f*/

                        temp=j(n,1,0);
                        ze = rannor(temp);
                        if ge^=0 then do;
                        e = ((exp(ge#ze)-j(n,1,1))/ge)#exp(he#ze##2/2);
                              popmue =(exp(ge##2/(2#(1-he)))-1)/(ge#(sqrt
                              (1-he)));
                        num1e=exp(2#ge##2/(1-2#he));
                        num2e=2#exp(ge##2/(2#(1-2#he)));
                        dene=ge##2#(sqrt(1-2#he));
                        popvare=(-1)#popmue##2+(num1e-num2e+1)/dene;
                        ee = ((e-popmue)/popvare##.5)#((1-rho2f)##.5);
                        end;
```

```
                          if ge=0 then do;
                          popmue=0;
                          popvare=1/((1-2#he)##(3/2));
                          e=ze#exp(he#ze##2/2);
                          ee = ((e-popmue)/popvare##.5)#((1-rho2f)##.5);
                          end;

/*Calculate y according to the model*/

                          y = (xx*B) + ee;
                          YX=y||xx;  /*YX is the matrix with all variables*/
                          sum=YX[+,];

/*Calculate covariance and correlation matrices based on YX*/

                          YXPYX=YX`*YX-SUM`*SUM/N;
                          S=DIAG(1/SQRT(VECDIAG(YXPYX)));
                          R=S*YXPYX*S;
                          R2 = R[2:p+1,2:p+1];

/*Calculate the remaining R matrices*/

                          R3 = R[1:k,1:k];
                          R4 = R[k:p,2:k];
                          R2F=1-((det(R))/(det(R2)));
                          R2R=1-((det(R3))/(det(R4)));
                          R2INC=R2F-R2R;
                          RR = R2R##.5;
                          RF = R2F##.5;
                          RFR = RR/RF;
                          if RF = 0 then do;
                          print RF;
                          RFR=0;
                          end;

/*Alf and Graf variance formula for case 2 where one set of predictors is a
subset of another*/
                           V=(4#R2F#(1-R2F)##2/n) + (4#R2R#(1-R2R)##2/n)
                           (8#RF#RR#(.5#(2#RFR-(RF#RR))*(1-R2F-R2R-(RFR##2))
                           + (RFR##3))/n);
                           if V < 0 then do;
                           print R2F R2R;
                           V = 0;
                           end;
                              LCL=R2INC-(1.96*(V##.5));
                              UCL=R2INC+(1.96*(V##.5));
                              if LCL<=rho2inc & UCL>=rho2inc then
                              counter1=counter1+1;
                              if LCL>rho2inc then counter2=counter2+1;
                              if UCL<rho2inc then counter3=counter3+1;
file output;
put n 4.0 +2 k 2.0 +2 rho2r 4.2 +2 rho2f 4.2 +2 rho2inc 4.2 +2
R2R 6.4 +2 R2F 6.4 +2 R2INC 6.4 +2 LCL 8.6 +2 UCL 8.6 +2 counter1 5.0 +2
counter2 5.0 +2 counter3;

              end; *for reps;
            end;  *end for rho2f;
```

```
          end; *end for rho2r;
        end;  *end for k;
      end; *end for n;
    end; *for k;
  end; *for i;
closefile output;
quit;
```

/*Calculate coverage probability*/

```
data;
infile 'C:\Dissertation\nx1nne4.dat';
input n 4.0 +2 k 2.0 +2 rho2r 4.2 +2 rho2f 4.2 + 2 rho2inc 4.2 +2
      R2R 6.4 +2 R2F 6.4 +2 R2INC 6.4 + 2 LCL 8.6 +2 UCL 8.6 +2 counter1 5.0
      +2 counter2 5.0 +2 counter3;
proc sort;
by k n rho2r rho2inc;
proc means noprint n mean std maxdec=4;
by k n rho2r rho2inc;
var counter1 counter2 counter3;
output mean=covprob below above out = res14;
proc print noobs data = res14;
var k n rho2r rho2inc covprob below above;
run;
```

/*Calculate mean estimated asymptotic variance and sampling variance of ΔR^2*/

```
data varratio;
infile 'c:\Dissertation\nx1nne4.dat';
input n k rho2r rho2f rho2inc R2R R2F R2INC;
RF=sqrt(R2F);
RR=sqrt(R2R);
RFR=RR/RF;
if RF = 0 then RFR = 0;
EAV=(4*R2F*(1-R2F)**2/N) + (4*R2R*(1-R2R)**2/N) -
(8*RF*RR*(.5*(2*RFR-(RF*RR))*(1-R2F-R2R-(RFR**2)) + (RFR**3))/N);
proc means noprint n mean var maxdec=4;
by n k rho2r rho2inc;
var R2INC EAV;
output mean=MR2INC MEAV var=VR2INC VEAV out=res14;
run;

data varratio2;
set res14;
ratio = MEAV/VR2INC;
proc print noobs data=varratio2;
var n k rho2r rho2inc MR2INC VR2INC MEAV RATIO;
run;
```

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BIOGRAPHICAL SKETCH

Lou Ann Mazula Cooper, the daughter of William and Priscilla Mazula, was born in Cedar Falls, Iowa. She graduated from Parkersburg High School and majored in psychology at the University of Iowa where she earned a Bachelor of Science degree with honors in 1977. While a student at the University of Iowa, she met and married Brian Cooper. When her husband accepted a postdoctoral fellowship at the University of Florida in 1979, she moved to Gainesville, Florida. She worked for the Florida Department of Labor, coordinating youth and employment and training programs. She was later employed at Santa Fe Community College where she held several positions: work evaluator, career counselor, and adjunct faculty. In 1992, she earned a Master of Education degree in educational psychology from the University of Florida in 1992. She developed and wrote successful grant proposals to fund educational opportunity programs for local high school students. She was the director of SFCC's Upward Bound Program from 1995 to 2000. In 2000, she began her doctoral studies in research and evaluation methods in the Department of Educational Psychology at the University of Florida. She earned a Master of Arts in Education in 2004 and will graduate with the Ph.D. degree in May 2007.