THREE ESSAYS ON BUNDLING AND TWO-SIDED MARKETS

By

JIN JEON

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To my parents, wife, and two daughters

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TABLE OF CONTENTS

TABLE

LIST OF FIGURES

Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

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By

Jin Jeon

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Chair: Jonathan H. Hamilton
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This work addresses three issues regarding bundling and two-sided markets. It starts with a brief summary of the theories of bundling and of two-sided markets in Chapter 1.

Chapter 2 analyzes various aspects of bundling strategy by the monopolist of a primary good when it faces competition in the complementary good market. The main result is that the monopolist can use a bundling strategy in order to avoid commitment problem that arises in optimal pricing. Bundling increases the monopolist's profits without the rival's exit from the market. Bundling lowers social welfare in most cases, while it may increase consumers' surplus. One of the long-run effects of bundling is that it lowers both firms' incentives to invest in R&D.

Chapter 3 compares welfare implications of monopoly outcome and competitive outcome. Using a model of the credit card industry with various settings such as Bertrand and Hotelling competition with single-homing and multi-homing consumers as well as

proprietary and nonproprietary platforms, it is shown that introducing platform competition in two-side markets may lower social welfare compared to the case of monopoly platform. In most cases, monopoly pricing maximizes Marshallian social welfare since the monopolist in a two-sided market can properly internalize indirect network externalities by setting unbiased prices, while the competing platforms set biased prices in order to attract the single-homing side.

Chapter 4 analyzes the effects of distribution of consumers' expenditure volumes on the market outcomes using a model in which two card issuers compete à la Hotelling. The result shows that the effects of distribution of the expenditure volume are different for various cases of market coverage. For example, as the variance increases, issuers' profits decrease when the market is fully covered, while the profits increase when the market is locally monopolized. It is also shown that the neutrality of the interchange fee holds in the full-cover market under the no-surcharge-rule. Simulation results are provided to show other comparative statics that include the possibility of the positive relationship between the interchange fee and the cardholder fee.

Finally, Chapter 5 summarizes major findings with some policy implications.

CHAPTER 1
INTRODUCTION

This dissertation contains three essays on bundling and two-sided markets. These topics have recently drawn economists' attention due to the antitrust cases of Kodak and Microsoft, and movements in some countries to regulate the credit card industry.

In the Kodak case, independent service organizations (ISOs) alleged that Kodak had unlawfully tied the sale of service for its machines to the sale of parts, in violation of section 1 of the Sherman Act, and had attempted to monopolize the aftermarket in violation of section 2 of the Sherman Act.[1] In the Microsoft case, the United States government filed an antitrust lawsuit against Microsoft for illegally bundling Internet Explorer with Windows operating system.[2]

In the credit card industry, antitrust authorities around the world have questioned some business practices of the credit card networks, which include the collective determination of the interchange fee, the no-surcharge rule, and the honor-all-cards rule. As a result, card schemes in some countries such as Australia, United Kingdom, and South Korea have been required to lower their interchange fees or merchant fees.

To understand these antitrust cases, many economic models have been developed. In the following sections, brief summaries of the economic theories of bundling and of two-sided markets will be presented.

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[1] For more information about the Kodak case, see Klein (1993), Shapiro (1995), Borenstein, MacKie-Mason, and Netz (1995), and Blair and Herndon (1996).

[2] See Gilbert & Katz (2001), Whinston (2001), and Evans, Nichols and Schmalensee (2001, 2005) for further analysis of the Microsoft case.

1.1 Bundling

Economists' views regarding bundling or tying have shifted dramatically in recent decades.[3] The traditional view of tying can be represented by the leverage theory which postulates that a firm with monopoly power in one market could use the leverage to monopolize another market.

The Chicago School criticized the leverage theory, since such leveraging may not increase the profits of the monopolist. According to the single monopoly profit theorem supported by the Chicago School, the monopolist earns same profits regardless whether it ties if the tied good market is perfectly competitive. For example, suppose consumers' valuation of a combined product of A and B is \$10 and marginal cost of producing each good is \$1. Good A is supplied only by the monopolist, and good B is available in a competitive market at price equal to the marginal cost. Without bundling, the monopolist can charge \$9 for A—and \$1 for B—to make \$8 as unit profit per good A sold. If the monopolist sells A and B as a bundle, it can charge \$10 for the bundle and earn \$8 (\$10 − \$1 − \$1) per unit bundle. So the monopolist cannot increase profits by bundling in this case.

Economists led by the Chicago School proposed alternative explanations for bundling based on efficiency rationales. Probably the most common reason for bundling is it reduces the transaction costs such as consumers' searching costs and firms' packaging and shipping costs. Examples of this kind of bundling are abundant in the real

[3] Bundling is the practice of selling two goods together, while tying is the behavior of selling one good conditional on the purchase of another good. There is no difference between tying and bundling if the tied good is valueless without the tying good and two goods are consumed in fixed proportion. See Tirole (2005) and Nalebuff (2003) for the discussions of bundling and tying.

world: shoes are sold in pairs; personal computers (PCs) are sold as bundles of the CPU, a hard drive, a monitor, a keyboard and a mouse; cars are sold with tires and a car audio. In some sense, most products sold in the real world are bundled goods and services.

Another explanation for bundling in line with the efficiency rationale is price discrimination. That is, if consumers are heterogeneous in their valuations of products, bundling has a similar effect as price discrimination.[4] This advantage of bundling is apparent when consumers' valuations are negatively correlated. But bundling can be profitable even for nonnegative correlation of consumers' valuations (McAfee, McMillan, and Whinston, 1989). In fact, unless consumers' valuations are perfectly correlated, firms can increase profits by bundling.[5] Since price discrimination usually increases social welfare as well as firm's profits, bundling motivated by price discrimination increases efficiency of the economy.

The leverage theory of tying revived with the seminal work of Whinston (1990). He showed that the Chicago School arguments regarding tying can break down in certain circumstances which include 1) the monopolized product is not essential for all uses of the complementary good, and 2) scale economies are present in the complementary good. If there are uses of the complementary good that do not require the primary good, the monopolist of the primary good cannot capture all profits by selling the primary good only. So the first feature provides an incentive for the monopolist of the primary good to exclude rival producers of the complementary good. The second feature provides the monopolist with the ability to exclude rivals, since foreclosure of sales in the

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[4] See Adams and Yellen (1976) and Schmalensee (1984).

[5] Bakos and Brynjolfsson (1999) show the benefit of a very large scale bundling based on the Law of Large Numbers.

complementary market, combined with barriers to entry through scale economies, can keep rival producers of the complementary good out of the market.[6]

Bundling can also be used to preserve the monopolist's market power in the primary good market by preventing entry into the complementary market at the first stage (Carlton and Waldman, 2002a). This explains the possibility that Microsoft bundles Internet Explorer with Windows OS in order to preserve the monopoly position in the OS market, since Netscape's Navigator combined with Java technology could become a middleware on which other application programs can run regardless of the OS.

Choi and Stefanidis (2001) and Choi (2004) analyze the effects of tying on R&D incentives. The former shows that tying arrangement of an incumbent firm that produces two complementary goods and faces possible entries in both markets reduces entrants' R&D incentives since each entrant's success is dependent on the other's success. The latter analyzes R&D competition between the incumbent and the entrant, and shows that tying increases the incumbent's incentives to R&D since it can spread out the costs of R&D over a larger number of units, whereas the entrant's R&D incentives decrease.[7]

Chapter 2 presents a model of bundling that follows the basic ideas of the leverage theory. It shows that the monopolist of a primary good that faces competition in the aftermarkets can use the bundling strategy to increase profits to the detriment of the rival firm. Aftermarkets are markets for goods or services used together with durable equipment but purchased after the consumer has invested in the equipment. Examples include maintenance services and parts, application programs for operating systems, and

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[6] Nalebuff (2004) and Carlton and Waldman (2005a) also present models that show the entry deterrence effect in the tied good market.

[7] In chapter 2, I show that bundling reduces R&D incentives of the monopolist as well as of the rival.

software upgrades. One of the key elements of the aftermarket is that consumers buy the complementary goods after they have bought the primary good. For the monopolist of the primary good, the best way to maximize its profits is to commit to the second period complementary price. If this commitment is not possible or implementable, bundling can be used.

Unlike most of the previous models of the leverage theory, market foreclosure is not the goal of the bundling in this model. On the contrary, the existence of the rival firms is beneficial to the monopolist in some sense since it can capture some surplus generated by the rival firm's product.

1.2 Two-Sided Markets

Two-sided markets are defined as markets in which end-users of two distinctive sides obtain benefits from interacting with each other over a common platform.8 These markets are characterized by indirect network externalities, i.e., benefits of one side depend on the size of the other side.9 According to Rochet and Tirole (2005), a necessary condition for a market to be two-sided is that the Coase theorem does not apply to the transaction between the two sides. That is, any change in the price structure, holding constant the total level of prices faced by two parties, affects participation levels and the number of interactions on the platform since costs on one side cannot be completely passed through to the other side.

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[8] For general introductions to the two-sided market, see Roson (2005a), and Evans and Schmalensee (2005).

[9] In some cases such as media industries, indirect network externalities can be negative since the number of advertisers has a negative impact on readers, viewers, or listeners. See Reisinger (2004) for the analysis of two-sided markets with negative externalities.

Examples of the two-sided market are abundant in the real world. Shopping malls need to attract merchants as well as shoppers. Videogame consoles compete for game developers as well as gamers. Credit card schemes try to attract cardholders as well as merchants who accept the cards. Newspapers need to attract advertisers as well as readers.[10] Figure 1-1 shows the structure of the two-sided market in case of the credit card industry, both proprietary and nonproprietary schemes.

Although some features of two-sided markets have been recognized and studied for a long time,[11] it is only recently that a general theory of two-sided markets emerged.[12] The surge of interest in two-sided markets was partly triggered by a series of antitrust cases against the credit card industry in many industrialized countries including the United States, Europe and Australia. The literature on the credit card industry has found that the industry has special characteristics; hence conventional antitrust policies may not be applicable to the industry.[13]

Wright (2004b) summarizes fallacies that can arise from using conventional wisdom from one-sided markets in two-sided markets, which include: an efficient price structure should be set to reflect relative costs; a high price-cost margin indicates market power; a price below marginal cost indicates predation; an increase in competition necessarily results in a more efficient structure of prices; and an increase in competition necessarily results in a more balanced price structure.

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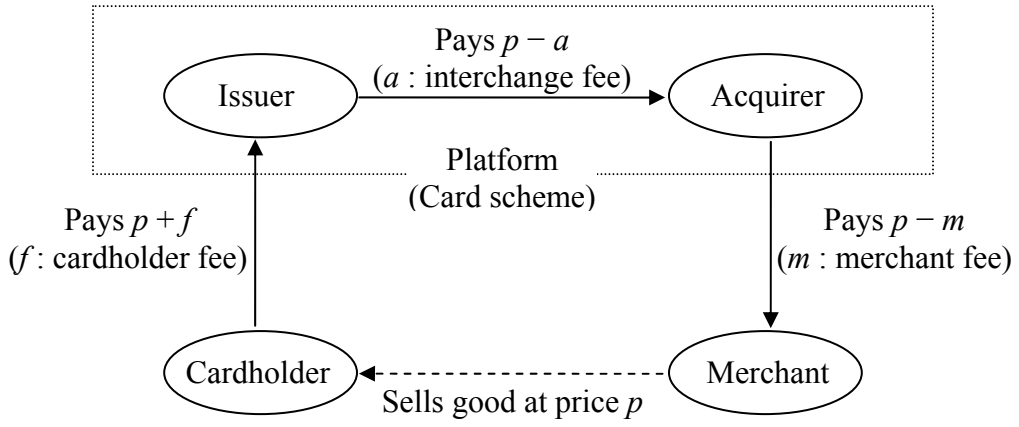
[10] See Rochet and Tirole (2003) for more examples of the two-sided market.

[11] For example, Baxter (1983) realized the two-sidedness of the credit card industry.

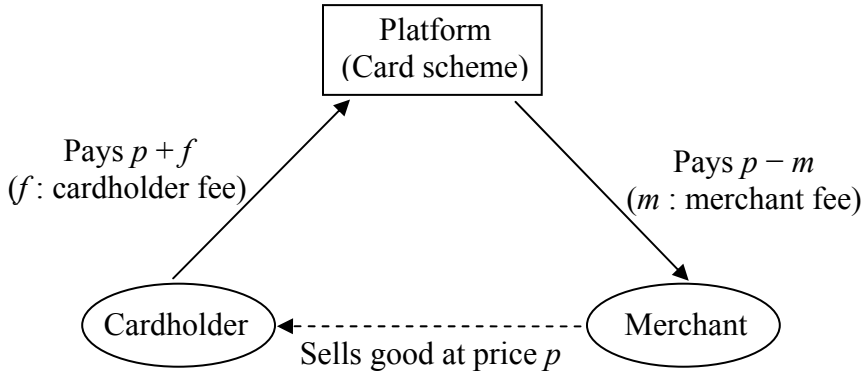
[12] The seminal papers include Armstrong (2005), Caillaud and Jullien (2003), and Rochet and Tirole (2003).

[13] The literature includes Gans and King (2003), Katz (2001), Rochet and Tirole (2002), Schmalensee (2002), Wright (2003a, 2003b, 2004a).

The theory of two-sided markets is related to the theories of network externalities and of multi-product pricing. While the literature on network externalities has found that in some industries there exist externalities that are not internalized by end-users, models are developed in the context of one-sided markets.[14] Theories of multi-product pricing stress the importance of price structures, but ignore externalities in the consumption of



(a) Nonproprietary card scheme



(b) Proprietary card scheme

Figure 1-1. Credit card schemes

[14] See Katz and Shapiro (1985, 1986), and Farrell and Saloner (1985, 1986).

different goods since the same consumer buys both goods. That is, the buyer of one product (say, razor) internalizes the benefits that he will derive from buying the other product (blades). The two-sided market theory starts from the observation that there exist some industries in which consumers on one side do not internalize the externalities they generate on the other side. The role of platforms in two-sided markets is to internalize these indirect externalities by charging appropriate prices to each side.

In order to get both sides on board and to balance demands of two sides, platforms in two-sided markets must carefully choose price structures as well as total price levels.[15] So it is possible that one side is charged below marginal cost of serving that side, which would be regarded as predatory pricing in a standard one-sided market. For this reason, many shopping malls offers free parking service to shoppers, and cardholders usually pay no service fees or even negative prices in the form of various rebates.

In a standard one-sided market, the price is determined by the marginal cost and the own price elasticity, as is characterized by Lerner's formula.[16] In two-sided markets, however, there are other factors that affect the price charged to each side. These are relative size of cross-group externalities and whether agents on each side single-home or multi-home.[17]

If one side exerts larger externalities on the other side than *vice versa*, then the platform will set a lower price for this side, *ceteris paribus*. In a media industry, for

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[15] In the credit card industry, non-proprietary card schemes choose interchange fees which affect the price structure of two sides.

[16] The standard Lerner's formula is $\frac{p-c}{p}=\frac{1}{\varepsilon}$ or $p=\frac{\varepsilon}{\varepsilon-1}c$, where p is the price, c is the marginal cost, and ε is the own price elasticity.

[17] An end-user is "single-homing" if she uses one platform, and "multi-homing" if she uses multiple platforms.

example, viewers pay below the marginal cost of serving while advertisers pay above the marginal cost since the externalities from viewers to advertisers are larger than those from advertisers to viewers.

When two or more platforms compete with each other, end-users may join a single platform or multiple platforms, depending on the benefits and costs of joining platforms. Theoretically, three possible cases emerge: (i) both sides single-home, (ii) one side single-homes while the other side multi-homes, and (iii) both sides multi-home.[18] If interacting with the other side is the main purpose of joining a platform, one can expect case (iii) is not common since end-users of one side need not join multiple platforms if all members of the other side multi-home.[19] For example, if every merchant accepts all kinds of credit cards, consumers need to carry only one card for transaction purposes. Case (i) is also not common since end-users of one side can increase interaction with the other side by joining multiple platforms. As long as the increased benefit exceeds the cost of joining additional platform, the end-users will multi-home.

On the contrary, one can find many examples of case (ii) in the real world. Advertisers place ads in several newspapers while readers usually subscribe to only one newspaper. Game developers make the same game for various videogame consoles while gamers each own a single console. Finally, merchants accept multiple cards while consumers use a single card.[20]

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[18] In most of the models on two-sided markets, single-homing and multi-homing of end-users are pre-determined for analytical tractability. For an analysis of endogenous multi-homing, see Roson (2005b).

[19] See also Gabszewicz and Wauthy (2004).

[20] According to an empirical study by Rysman (2006), most consumers put a great majority of their payment card purchases on a single network, even when they own multiple cards from different networks.

When end-users of one side single-home while those of the other side multi-home, the single-homing side becomes a “bottleneck” (Armstrong, 2005). Platforms compete for the single-homing side, so they will charge lower price to that side. As is shown in Chapter 3, platforms competing for the single-homing side may find themselves in a situation of the “Prisoner’s Dilemma”. That is, a lower price for the single-homing side combined with a higher price for the multi-homing side can decrease total transaction volume and/or total profits compared to the monopoly outcome. Further, competition in two-sided markets may lower social welfare since monopoly platforms can properly internalize the indirect externalities by charging unbiased prices, while competing platforms are likely to distort the price structure in favor of the single-homing side.

Chapter 3 presents a model of the credit card industry with various settings including single-homing vs. multi-homing cardholders, competition between identical card schemes (Bertrand competition) or differentiated schemes (Hotelling competition), and proprietary vs. non-proprietary card schemes. The main finding is that, unlike in a standard one-sided market, competition does not increase social welfare regardless of the model settings.

Chapter 4 tackles the assumption made by most models on the credit card industry that cardholders spend the same amounts with credit cards. By allowing heterogeneous expenditures among consumers, it shows the effects of a change in the variance of the expenditure on the equilibrium prices and profits. The results show that the effects are different depending on whether the market is fully covered, partially covered, or locally monopolized.

CHAPTER 2
BUNDLING AND COMMITMENT PROBLEM IN THE AFTERMARKET

2.1 Introduction

A monopolist of a primary good that faces competition in the aftermarket of the complementary goods often uses a bundling or tying strategy. Traditionally, bundling was viewed as a practice of transferring the monopoly power in the tying market to the tied market. This so-called "leverage theory" has been criticized by many economists associated with the Chicago School in that there exist other motives of bundling such as efficiency-enhancement and price discrimination. Further, they show that there are many circumstances in which firms cannot increase profits by leveraging monopoly power in one market to the other market, which is known as the single monopoly profit theorem.

Since the seminal work of Whinston (1990), the leverage theory revived as many models have been developed to show that a monopolist can use tying or bundling strategically in order to deter entry to the complementary market and/or primary market. The research was in part stimulated by the antitrust case against Microsoft filed in 1998, in which U.S government argued that Microsoft illegally bundles Internet Explorer with Windows operating system.[1] Most of the models in this line, however, have a commitment problem since the bundling decision or bundling price is not credible when the entrant actually enters or does not exit the market.

[1] For further analyses of the Microsoft case, see Gilbert & Katz (2001), Whinston (2001), and Evans, Nichols and Schmalensee (2001).

This paper stands in the tradition of the leverage theory and shows that the monopolist of a primary good can use a bundling strategy to increase profits as well as the market share in the complementary good market. Unlike the previous models, the monopolist's profits increase with bundling even if the rival does not exit the market. On the contrary, the existence of a rival firm is beneficial to the monopolist in some sense since the monopolist can capture some surplus generated by the rival firm's complementary good.

The model presented here is especially useful for the analysis of the Microsoft case. Many new features added to—i.e., bundled with—the Windows operating system (OS) had been independent application programs produced by other firms. For example, Netscape's Navigator was a dominant Internet browser before Microsoft developed Internet Explorer. Therefore, it is Microsoft, not Netscape, that entered the Internet browser market. Since Netscape's software development cost is already a sunk cost when Microsoft makes a bundling decision, the entry deterrent effect of bundling cannot be applied.

The main result is that the monopolist can use bundling to avoid the commitment problem[2] arising in the optimal pricing when consumers purchase the complementary good after they have bought the primary good. If the monopolist cannot commit to its optimal price for the complementary good at the first stage when consumers buy the primary good, then it may have to charge a lower price for the primary good and a higher price for the complementary good compared to its optimal set of prices since consumers

[2] This commitment problem is different from the one in the previous literature, in which the commitment problem arises since the bundling price is not credible if the would-be entrant actually enters the market.

rationally expect that the monopolist may raise its complementary good price after they have bought the primary goods. A double marginalization problem arises in this case since the monopolist has to charge the price that maximizes its second stage profits, while it also charges a monopoly price for the primary good at the first stage. Bundling makes it possible for the monopolist to avoid the double marginalization problem by implicitly charging a price equal to zero for the complementary good.

The model also shows that bundling generally lowers Marshallian social welfare except for the extreme case when the monopolist's bundled good is sufficiently superior to the rival's good. Social welfare decreases with bundling mainly because it lowers the rival's profits. Consumers' surplus generally increases with bundling. However, consumers' surplus also decreases when the rival's complementary good is sufficiently superior to the monopolist's.

The last result shows the effect of bundling on R&D investments. In contrast to the previous result of Choi (2004) that shows tying lowers the rival firm's incentive to invest in R&D while it increases the monopolist's incentive, I show that bundling lowers both firms' incentives to make R&D investments.

The literature on bundling or tying is divided into two groups − one finds the incentive to bundle from the efficiency-enhancing motives, and the other finds it from anticompetitive motives.[3] In the real world, examples of bundling motivated by efficiency reason are abundant. Shoe makers sell shoes as a pair, which reduces transaction costs such as consumers' searching costs and producers' costs of shipping and packaging. The personal computer is another example as it is a bundle of many parts such

[3] For a full review of the literature on bundling, see Carlton and Waldman (2005b).

as the CPU, a memory card, a hard drive, a keyboard, a mouse, and a monitor.[4] Carlton and Waldman (2002b) explain another efficiency motive for tying by showing that producers of a primary good may use tying in order to induce consumers to make efficient purchase decisions in the aftermarket when consumers can buy the complementary goods in variable proportions. If the primary good is supplied at a monopoly price while the complementary good is provided competitively, consumers purchase too much of the complementary good and too little of the primary good. Tying can reduce this inefficiency and increase profits.

Adams and Yellen (1976) provide a price discrimination motive for tying. Using some examples, they show that if consumers are heterogeneous in their valuations for the products, bundling has a similar effect as price discrimination. This advantage of bundling is apparent when consumers' valuations are negatively correlated. Schmalensee (1984) formalizes this theory assuming consumers' valuations follow a normal distribution. McAfee, McMillan, and Whinston (1989) show that bundling can be profitable even for nonnegative correlation of consumers' valuations. Bakos and Brynjolfsson (1999) show the benefit of a very large scale bundling of information goods based on Law of Large Numbers. Since price discrimination usually increases social welfare with an increase in total output, tying or bundling motivated by price discrimination can be welfare improving.

The anticompetitive motive of tying is reexamined by Whinston (1990). He recognizes that Chicago School's criticism of leverage theory only applies when the complementary good market is perfectly competitive and characterized by constant

[4] See Evans and Salinger (2005) for efficiency-enhancing motive of tying.

returns to scale, and the primary good is essential for use of the complementary good. He shows that in an oligopoly market with increasing returns to scale, tying of two independent goods can deter entry by reducing the entrant's profits below the entry cost. As was mentioned earlier, however, his model has a credibility problem since bundling is not profitable if entrance actually occurs.

Nalebuff (2004) also shows that bundling can be used to deter entry, but without a commitment problem since in his model the incumbent makes higher profits with bundling than independent sale even when the would-be entrant actually enters.[5] Carlton and Waldman (2002a) focus on the ability of tying to enhance a monopolist's market power in the primary market. Their model shows that by preventing entry into the complementary market at the first stage, tying can also stop the alternative producer from entering the primary market at the second stage.

Carlton and Waldman (2005a) shows that if the primary good is a durable good and upgrades for the complementary good are possible, the monopolist may use a tying strategy at the first stage in order to capture all the upgrade profits at the second stage. Especially when the rival's complementary good is superior to the monopolist's, the only way the monopolist sells second-period upgrades is to eliminate the rival's product in the first period by tying its own complementary good with its monopolized primary good. By showing that tying can be used strategically even when the primary good is essential for use of the complementary good, it provides another condition under which the Chicago School argument breaks down.

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[5] However, the optimal bundling price is higher when the entrant enters than the price that is used to threaten the entrant. So there exists a credibility problem with the price of the bundled good.

The model presented here also assumes the primary good is essential, but the primary good is not necessarily a durable good and constant returns to scale prevail. So it can be added to the conditions under which the Chicago School argument breaks down that bundling can be used strategically when consumers buy the primary good and the complementary good sequentially.

The rest of Chapter 2 is organized in the following way. Section 2.2 describes the basic setting of the model. Sections 2.3 to 2.5 show and compare the cases of independent sale, pricing with commitment, and bundling, respectively. Section 2.6 analyzes the welfare effect of bundling. Section 2.7 is devoted to the effect of bundling on R&D investments. The last section summarizes the results.

2.2 The Model

Suppose there are two goods and two firms in an industry. A primary good is produced solely by a monopolist, firm 1. The other good is a complementary good that is produced by both the monopolist and a rival, firm 2. The purchases of the primary good and the complementary good are made sequentially, i.e., consumers buy the complementary good after they have bought the primary good. Consumers buy at most one unit of each good,[6] and are divided into two groups. Both groups have same reservation value v_0 for the primary good. For the complementary good, however, one group has zero reservation value and the other group has positive reservation value v_i, where $i = 1, 2$ indicates the producer.[7] For modeling convenience, it is assumed that the

[6] So there is no variable proportion issue.

[7] Consumption of the complementary good may increase the reservation value of the primary good. It is assumed that v_i also includes this additional value.

marginal cost of producing each good is zero and there is no fixed cost for producing any good.[8]

The PC software industry fits in this model, in which Microsoft Windows OS is the monopolized primary good and other application programs are complementary goods. Microsoft also produces application programs that compete with others in the complementary good market. Sometimes Microsoft bundles application programs such as an Internet browser and a media player that could be sold separately into Windows OS. Consumers usually buy the Windows OS at the time they buy a PC, then buy application software later.

Let the total number of consumers be normalized to one, and α be the portion of the consumers, group S, who have positive valuations for the complementary good. It is assumed that the consumers in S are distributed uniformly on the unit interval, in which the monopolist and firm 2 are located at 0 and 1, respectively.

The two complimentary goods are differentiated in a Hotelling fashion. A consumer located at x incurs an additional transportation cost tx when she buys the monopolist's complementary good, and $t(1 - x)$ when she buys firm 2's. So the gross utility of the complementary good for the consumer is $v_1 - tx$ when she buys from the monopolist, and $v_2 - t(1 - x)$ when she buys from firm 2. v_1 and v_2 are assumed to be greater than t in order to make sure that consumers in S cannot have a negative gross utility for any complementary good regardless of their positions. Further, in order to

[8] Unlike the models that explain tying as an entry deterrence device, the model in this paper assumes constant returns to scale.

make sure that all the consumers in S buy the complementary goods at equilibrium, it is assumed that[9]

$$v_1 + v_2 > 3t \tag{2-1}$$

The model presented here allows a difference between v_1 and v_2 in order to analyze bundling decision when the monopolist produces inferior—or superior—complementary good and the effect of bundling on R&D investments. But the difference is assumed to be less than t, i.e.,

$$| v_1 - v_2 | \leq t \tag{2-2}$$

since otherwise all consumers find one of the complementary goods superior to the other good.[10]

In the software industry, the primary good is the operating system (OS), and application programs like an Internet browser or a word processor are examples of complementary goods. The OS itself can be seen a collection of many functions and commands. Bakos and Brynjolfsson (1999) show that the reservation values among consumers of a large scale bundle converge to a single number, which justifies the assumption that consumers have the same valuation for the primary good. A single application program, however, is not as broadly used as an OS, so the valuation for the

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[9] The prices chosen by two firms could be too high so that some of the consumers in S may not want to buy the complementary good. The assumption $v_1+v_2 > 3t$ guarantees that every consumer in S buy a complementary good at equilibrium.

[10] This is also a sufficient condition for the existence of the various equilibria.

complementary good may vary among consumers. Furthermore, not all the application programs are produced for all consumers. Some of them are developed for a certain group of consumers such as business customers.

The game consists of two stages.[11] At the first stage consumers buy the primary good or bundled good at the price that the monopolist sets. The monopolist can set the price of its own complementary good with or without commitment, or sell both goods as a bundle. At the second stage, consumers buy one of the complementary goods, the prices of which are determined by the competition between the two firms.

Let p_0, p_1, and p_2 be the prices of the monopolist's primary good, the monopolist's complementary good, and firm 2's complementary good, respectively. Then the net utilities of the consumer located at x if she consumes the primary good only, the primary good with the monopolist's complementary good, and the primary good with firm 2's complementary good are, respectively,

$$u_0 = v_0 - p_0$$

$$u_1 = v_0 + v_1 - tx - p_0 - p_1$$

$$u_2 = v_0 + v_2 - t(1 - x) - p_0 - p_2$$

The consumer will buy only the primary good if

$$u_0 > u_1,\ u_0 > u_2, \text{ and } u_0 \geq 0$$

She will buy the primary good and the monopolist's complementary good if

[11] In section 2.7, an earlier stage will be added at which two firms make investment decisions that determine v_i's.

$u_1 \geq u_2$, $u_1 \geq u_0$, and $u_1 \geq 0$

She will buy the primary good and firm 2's complementary good if

$u_2 \geq u_1$, $u_2 \geq u_0$, and $u_2 \geq 0$

Lastly, she will buy nothing if

$u_0 < 0$, $u_1 < 0$, and $u_2 < 0$

2.3 Independent Sale without Commitment

In this section, it is assumed that the monopolist cannot commit to p_1 at the first stage. Without commitment, p_1 must be chosen to be optimal at the second stage. That is, in game-theoretic terms, the equilibrium price must be subgame perfect.

As in a standard sequential game, the equilibrium set of prices can be obtained by backward induction. Let x^* be the critical consumer who is indifferent between the monopolist's complementary good and firm 2's good. One can find this critical consumer by solving $v_1 - tx^* - p_1 = v_2 - t(1 - x^*) - p_2$, which gives

$$x^* = \frac{1}{2} + \frac{v_1 - v_2 + p_2 - p_1}{2t} \tag{2-3}$$

There are two cases to be considered: when the monopolist sells the primary good to all consumers, and when it sells its products to group S only. Consider first the case that the monopolist sells the primary good to all consumers. At the second stage, the monopolist will set p_1 to maximize $\alpha p_1 x^*$, while firm 2 will set p_2 to maximize $\alpha p_2(1 - x^*)$.

By solving each firm's maximization problem, one can obtain the following best response functions:

$$p_i = \frac{v_i - v_j + p_j + t}{2}, \quad i, j = 1, 2 \text{ and } i \neq j \tag{2-4}$$

from which one can obtain the following equilibrium prices for the case of the independent sale without commitment (IA case):

$$p_i^{IA} = \frac{v_i - v_j}{3} + t, \quad i, j = 1, 2 \text{ and } i \neq j$$

Plugging these into (2-3) gives the location of the critical consumer:

$$x^{IA} = \frac{1}{2} + \frac{v_1 - v_2}{6t}$$

At the first stage, the monopolist will set the price of the primary good equal to v_0 since consumers outside of group S will not buy the good for the price higher than v_0:

$$p_0^{IA} = v_0$$

One needs to check whether consumers actually buy the goods for this set of prices. This can be done by plugging the prices into the net utility of the critical consumer, i.e.,

$$u_1(x^{IA}) = v_0 + v_1 - tx^{IA} - p_0^{IA} - p_1^{IA} = \frac{v_1 + v_2 - 3t}{2} > 0$$

where the last inequality holds because of the assumption given in (2-1). As was noted in footnote 10, this assumption guarantees that all consumers in S buy both goods at equilibrium.

The profits of the firms at equilibrium are

$$\pi_1^{IA} = p_0^{IA} + \alpha p_1^{IA} x^{IA} = v_0 + \frac{\alpha(v_1 - v_2 + 3t)^2}{18t}$$

$$\pi_2^{IA} = \alpha p_2^{IA}(1 - x^{IA}) = \frac{\alpha(v_2 - v_1 + 3t)^2}{18t}$$

The monopolist may find it profitable to sell the primary goods exclusively to group S by charging the price higher than v_0. If a consumer located at x have bought the primary good at the first stage, the maximum prices she is willing to pay for the monopolist's and firm 2's complementary goods at the second stage are $v_1 - tx$ and $v_2 - t(1 - x)$, respectively, regardless how much she paid for the primary good at the first stage. Since the payment at stage one is a sunk cost to the consumer, she will buy a complementary good as long as the net utility from the complementary good is non-negative. This implies that when the monopolist sells the primary good to group S only without commitment to p_1 (IS case), the equilibrium prices and the location of the critical consumer at the second stage are exactly the same as in the IA case.[12] That is,

$$p_i^{IS} = \frac{v_i - v_j}{3} + t, \quad i, j = 1, 2 \text{ and } i \neq j$$

[12] There may exist multiple equilibria because of the coordination problem among consumers. For example, suppose consumers around at x^{IS} did not buy the primary good at stage 1. Then at stage 2, the two firms will charge higher prices than p_i^{IS}. At this price set, consumers who did not buy the base good will be satisfied with their decision.

$$x^{IS} = \frac{1}{2} + \frac{v_1 - v_2}{6t}$$

When consumers buy the primary good at stage 1, they rationally predict that the second period prices of the complementary goods are p_i^{IS}. So the monopolist will set the primary good price to make the critical consumer indifferent between buying the complementary good and not buying, which yields the following equilibrium price:

$$p_0^{IS} = v_0 + \frac{v_1 + v_2 - 3t}{2}$$

Note that the primary good price is higher than v_0 as is expected. By excluding the consumers who buy only the primary good, the monopolist can charge a higher price in order to capture some surplus that would otherwise be enjoyed by the consumers of the complementary goods.

The monopolist's profits may increase or decrease depending on the size of α, while firm 2's profits remain the same as in the IA case since the price and the quantity demanded in IS case are exactly the same as in the IA case:

$$\pi_1^{IS} = \alpha(p_0^{IS} + p_1^{IS} x^{IS}) = \alpha\left[v_0 + \frac{(v_1 - v_2)^2}{18t} + \frac{5v_1 + v_2 - 6t}{6}\right]$$

$$\pi_2^{IS} = \alpha p_2^{IS}(1 - x^{IS}) = \frac{\alpha(v_2 - v_1 + 3t)^2}{18t}$$

By comparing π_1^{IS} and π_1^{IA}, one can derive the condition in which the monopolist prefers the IS outcome to the IA outcome:

$$\alpha > \frac{2v_0}{2v_0 + (v_1 + v_2 - 3t)} \equiv \hat{\alpha}^{IS}$$

Note that $\hat{\alpha}^{IS}$ lies between 0 and 1 since $v_1 + v_2 - 3t > 0$ is assumed in (2-1).

2.4 Independent Sale with Commitment

The results of the previous section may not be optimal for the monopolist if it can choose both p_0 and p_1 simultaneously at the first stage and commit to p_1. To see this, suppose the monopolist can set both prices at the first stage with commitment. As in the previous section, one can distinguish two cases depending on the coverage of the primary good market. When the monopolist sells its primary good to all consumers with commitment to p_1 (CA case), the model shrinks to a simple game in which the monopolist set p_1 at the first stage and firm 2 set p_2 at the second stage since the primary good price should be set equal to v_0, i.e., $p_0^{CA} = v_0$. The equilibrium prices of the complementary goods can be derived using a standard Stackelberg leader-follower model.

The equilibrium can be found using backward induction. At the second stage, the critical consumer who is indifferent between the monopolist's complementary good and firm 2's good is determined by (2-3) with p_2 replaced by firm 2's best response function given by (2-4), i.e.,

$$x^* = \frac{3}{4} + \frac{v_1 - v_2 - p_1}{4t} \tag{2-5}$$

The monopolist will set p_1 to maximize $\alpha p_1 x^*$, which gives the following optimal price:

$$p_1^{CA} = \frac{v_1 - v_2 + 3t}{2}$$

The remaining equilibrium values can be obtained by plugging this into (2-4) and (2-5):

$$p_2^{CA} = \frac{v_2 - v_1 + 5t}{4}$$

$$x^{CA} = \frac{3}{8} + \frac{v_1 - v_2}{8t}$$

$$\pi_1^{CA} = v_0 + \frac{\alpha(v_1 - v_2 + 3t)^2}{16t}$$

$$\pi_2^{CA} = \frac{\alpha(v_2 - v_1 + 5t)^2}{32t}$$

The differences between the equilibrium prices of CA case and IA case are

$$p_1^{CA} - p_1^{IA} = \frac{v_1 - v_2 + 3t}{6} > 0$$

$$p_2^{CA} - p_2^{IA} = \frac{v_1 - v_2 + 3t}{12} > 0$$

The price differences are positive since the difference between v_1 and v_2 is assumed to be less than t. Since the monopolist's complementary good is a substitute for firm 2's good, p_1 and p_2 are strategic complements. If one firm can set its price first, it will set a higher price so that the rival also raises its own price compared to the simultaneous move game. With the increase in the prices, both firms enjoy higher profits as the following calculation shows:

$$\pi_1^{CA} - \pi_1^{IA} = \frac{\alpha(v_1 - v_2 + 3t)^2}{144t} > 0$$

$$\pi_2^{CA} - \pi_2^{IA} = \frac{\alpha(v_1 - v_2 + 3t)[27t - 7(v_1 - v_2)]}{288t} > 0$$

The profit of the monopolist must increase since it chooses a different price even if it could commit to p_1^{IA} at the first stage. Firm 2's profit also increases as both firms' prices of complementary goods increase while the price of the primary good remains the same.

When the monopolist covers only the consumers in group S with commitment to p_1 (CS case), the equilibrium can be found in a similar way as in the CA case. At the second stage, firm 2's best response function is the same as (2-4) and the critical consumer is also determined by (2-5). Since the monopolist will make the critical consumer indifferent between buying and not buying the complementary good, p_0 will be set to satisfy the following condition:

$$p_0 = v_0 + v_1 - p_1 - tx^* \tag{2-6}$$

Using (2-5) and (2-6), the monopolist's profits can be rewritten as a function of p_1 in the following way:

$$\pi_1 = \alpha(p_0 + p_1 x^*) = \alpha\left[v_0 + \frac{3v_1 + v_2 - 3t}{4} + \frac{p_1(v_1 - v_2 - p_1)}{4t}\right]$$

Maximizing this profit function w.r.t. p_1 yields the optimal price for the monopolist's complementary good, which is

$$p_1^{CS} = \frac{v_1 - v_2}{2}$$

Plugging this back to (2-4), (2-5) and (2-6), one can derive the remaining equilibrium values:

$$p_0^{CS} = v_0 + \frac{3v_1 + 5v_2 - 6t}{8}$$

$$p_2^{CS} = \frac{v_2 - v_1 + 2t}{4}$$

$$x^{CS} = \frac{3}{4} + \frac{v_1 - v_2}{8t}$$

$$\pi_1^{CS} = \alpha\left[v_0 + \frac{(v_1 - v_2)^2}{16t} + \frac{3v_1 + v_2 - 3t}{4}\right]$$

$$\pi_2^{CS} = \frac{\alpha(v_2 - v_1 + 2t)^2}{32t}$$

The differences between the equilibrium prices of CS case and IS case are as follows:

$$p_0^{CS} - p_0^{IS} = \frac{v_2 - v_1 + 6t}{8} > 0$$

$$p_1^{CS} - p_1^{IS} = \frac{v_1 - v_2 - 6t}{8} < 0$$

$$p_2^{CS} - p_2^{IS} = \frac{v_1 - v_2 - 6t}{12} < 0$$

When the monopolist can commit to its complementary good price, it charges a higher price for the primary good and a lower price for the complementary good. And the

rival firm also charges a lower price for its own complementary good. Since p_1 and p_2 are strategic complements, the monopolist can induce firm 2 to decrease p_2 by lowering p_1, which makes it possible for the monopolist to raise p_0 for higher profits. This would not be possible if the monopolist cannot commit to p_1 at the first stage since the monopolist has an incentive to raise the complementary good price at the second stage after consumers have bought the primary good.

The difference between the profits of CS case and IS case are as follows:

$$\pi_1^{CS} - \pi_1^{IS} = \frac{\alpha(v_2 - v_1 + 6t)^2}{144t} > 0$$

$$\pi_2^{CS} - \pi_2^{IS} = -\frac{\alpha[18t - 7(v_1 - v_2)][6t - (v_1 - v_2)]}{288t} < 0$$

The monopolist's profits increase when it can commit as in CA case. However, firm 2's profits decrease since the monopolist can capture some of the consumers' surplus generated by firm 2's complementary good by charging a higher price for the primary good.

Comparing π_1^{CS} and π_1^{CA}, one can derive the following condition for the monopolist to prefer the CS outcome to the CA outcome:

$$\alpha > \frac{16v_0}{16v_0 + (6v_1 + 10v_2 - 21t)} \equiv \hat{\alpha}^{CS}$$

$\hat{\alpha}^{CS}$ lies between 0 and 1 since $6v_1 + 10v_2 - 21t = 8(v_1 + v_2 - 3t) + 2(v_2 - v_1) + 3t > 0$ from the assumptions given in (2-1) and (2-2). The difference between $\hat{\alpha}^{IS}$ and $\hat{\alpha}^{CS}$ is

$$\hat{\alpha}^{IS} - \hat{\alpha}^{CS} = \frac{2v_0(2v_2 - 2v_1 + 3t)}{(2v_0 + v_1 + v_2 - 3t)(16v_0 + 6v_1 + 10v_2 - 21t)} > 0$$

The critical level of α with commitment is lower than with independent sale since the profit gain from commitment is higher in the CS case than in the CA case.[13] That is, the monopolist is willing to sell both goods to a smaller group of consumers when it can commit to the price of its own complementary good sold in the second period.

The problem that the monopolist earns lower profits when it cannot commit to the second period price of the complementary good is common in cases of durable goods with aftermarkets.[14] That is, rational consumers expect that the monopolist will set its second period price to maximize its second period profit regardless of its choice in the first period. The monopolist has an incentive to charge a higher p_1 after consumers in S have bought the primary good at the first stage, since the price consumers have paid for the primary goods is sunk cost at stage 2.[15] If the monopolist cannot commit to p_1^{CS}, therefore, some consumers in S would not buy the primary good at the first stage. So the monopolist would have to set a lower p_0 (p_0^{IS}) and a higher p_1 (p_1^{IS}) because of the hold-up problem.

One of the problems in relation to the pricing with commitment is that the optimal prices may not be implemented since p_1^{CS} is negative when $v_1 < v_2$.[16] The bundling

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[13] Note that $(\pi_1^{CS} - \pi_1^{IS}) - (\pi_1^{CA} - \pi_1^{IA}) = \dfrac{\alpha[2(v_2 - v_1) + 3t]}{16} > 0$

[14] See Blair and Herndon (1996)

[15] After consumers have bought the primary goods at stage 1, the monopolist has an incentive to charge p_1^{IS} which is higher than p_1^{CS}.

[16] If the marginal cost of producing the complementary good is positive, the optimal price

strategy that will be presented in the following section can resolve this problem as well as the commitment problem.

2.5 Bundling: An Alternative Pricing Strategy without Commitment

An alternative strategy for the monopolist when it cannot commit to the second period price or implement a negative price is bundling. That is, it sells both the primary good and its own complementary good for a single price. Note first that it is not optimal for the monopolist to sell the bundled good to all consumers since the bundled price must be equal to v_0 in that case. So the monopolist will sell the bundled good to group S only if it chooses the bundling strategy.

It is assumed that tying is reversible, i.e., a consumer who buys a bundled good may also buy another complementary good and consume it with the primary good.[17] Further, suppose consumers use only one complementary good, so the monopolist's bundled complementary good is valueless to the consumers who use firm 2's complementary good.[18]

At the second stage, a consumer who has bought the bundled good earlier may buy firm 2's good or not, depending on her location x. If she buys firm 2's complementary good, her net gain at stage 2 is $v_2 - t(1-x) - p_2$. If she does not buy, she can use the monopolist's complementary good included in the bundle without extra cost, and get net gain of $v_1 - tx$. So the critical consumer who is indifferent between buying firm 2's complementary good and using the bundled complementary good is

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can be positive even if $v_1 < v_2$.

[17] In the software industry, a consumer who uses Windows OS bundled with Internet Explorer may install another Internet browser.

[18] As long as there is no compatibility problem, consumers will use only one complementary good they prefer.

$$x^* = \frac{1}{2} + \frac{v_1 - v_2 + p_2}{2t} \tag{2-7}$$

Since the price paid for the bundled good is a sunk cost at the second stage, the critical consumer is determined by p_2 only. Firm 2 will choose p_2 to maximize $p_2(1 - x^*)$, which yields the following optimal price for firm 2:

$$p_2^{BS} = \frac{v_2 - v_1 + t}{2}$$

Plugging this into (2-7) gives the location of the critical consumer as follows:

$$x^{BS} = \frac{3}{4} + \frac{v_1 - v_2}{4t}$$

For this critical consumer to exist between 0 and 1, it is required that $-3t \leq v_1 - v_2 \leq t$. So the assumption of $|v_1 - v_2| \leq t$ given in (2-2) is also a sufficient condition for the existence of a bundling equilibrium without the exit of the rival firm. If $v_1 - v_2 \geq t$, then all consumers buy the bundled good only so the rival firm will exit the market. If $v_1 - v_2 \leq -3t$, on the other hand, all consumers buy both the bundled good and firm 2's complementary good.

At stage 1, the monopolist will set the bundled good price, p_b, that makes the critical consumer indifferent between buying and not buying:[19]

[19] At the second stage, the monopolist may have an incentive to unbundle the product and sell the primary good to the consumers outside of group S as long as the consumer's second stage valuation for the good is positive, i.e., higher than the marginal cost. Knowing this, some consumers in group S may want to wait until the second period,

$$p_b^{BS} = v_0 + \frac{3v_1 + v_2 - 3t}{4}$$

When consumers choose the monopolist's complementary good, the total price for the primary good and the complementary good decreases compared to the IS case since

$$p_b^{BS} - (p_0^{IS} + p_1^{IS}) = \frac{v_2 - v_1 - 3t}{12} < 0 \tag{2-8}$$

If consumers buy firm 2's complementary good as well as the monopolist's bundled good, the total price increases compared to the IS case since

$$(p_b^{BS} + p_2^{BS}) - (p_0^{IS} + p_2^{IS}) = \frac{v_1 - v_2 + 3t}{12} > 0 \tag{2-9}$$

Comparing (2-8) and (2-9) one can find that the total price decrease for the consumers of monopolist's complementary good is exactly the same as the total price increase for the consumers of firm 2's good. With the decrease of the total price, the number of consumers who choose to use the monopolist's complementary good increases compared to the IS case as the following shows:

$$x^{BS} - x^{IS} = \frac{v_1 - v_2 + 3t}{12t} > 0 \tag{2-10}$$

The profits of the firms are

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which will lower the monopolist's profits. To avoid this, the monopolist will try to commit to not unbundling. One way to commit is to make unbundling technologically difficult or impossible, as Microsoft combined Internet Explorer with Windows OS.

$$\pi_1^{BS} = \alpha p_b^{BS} = \alpha\left[v_0 + \frac{3v_1 + v_2 - 3t}{4}\right]$$

$$\pi_2^{BS} = \alpha p_2^{BS}(1 - x^{BS}) = \frac{\alpha(v_2 - v_1 + t)^2}{8t}$$

The following proposition shows that bundling increases monopolist's profits compared to the IS case.

Proposition 2-1 Suppose the monopolist sells its goods to consumers in *S* only. Then the monopolist's profit in the bundling equilibrium is strictly higher than under IS, but not higher than under CS.

Proof. The difference between profits with bundling and IS case is

$$\pi_1^{BS} - \pi_1^{IS} = -\frac{\alpha}{18t}(v_1 - v_2 + 3t)(v_1 - v_2 - \frac{3}{2}t) > 0$$

The inequality holds since $|v_1 - v_2| \leq t$.

On the other hand, the difference between profits with bundling and CS is

$$\pi_1^{BS} - \pi_1^{CS} = -\frac{\alpha(v_1 - v_2)^2}{16t} \leq 0$$

where the inequality holds steadily when $v_1 \neq v_2$. *Q.E.D.*

In most of the previous analysis of bundling based on the leverage therory, one of the main purposes of the bundling strategy is to foreclose the complementary good market. By lowering expected profits of the would-be entrants, bundling can be used to deter entry. The difference between the previous models and the current one is that bundling increases the profits of the monopolist even though the rival firm does not exit the market. On the contrary, the existence of the rival firm helps the monopolist in some sense since it creates demand for the monopolist's bundled good.

When compared to the CS case, bundling strategy generates the same profits for the monopolist if $v_1 = v_2$. Technically, bundling strategy is equivalent to setting $p_1 = p_b$ and $p_2 = 0$. When $v_1 = v_2$, the equilibrium commitment price for the monopolist's complementary good, p_1^{CS}, is zero, hence the monopolist's profits of bundling and CS cases are equal.[20] Since the optimal commitment price is either positive or negative if $v_1 \neq v_2$, the monopolist's bundling profits is less than the CS case.

By comparing bundling case with IA case, one can find the critical level of α above which the monopolist finds bundling is more profitable if commitment is not possible. The difference between the monopolist's profits is

$$\pi_1^{BS} - \pi_1^{IA} = \alpha\left[v_0 - \frac{(v_1 - v_2)^2}{18t} + \frac{5v_1 + 7v_2 - 15t}{12}\right] - v_0$$

[20] If the marginal cost (MC) of producing the complementary good is positive (c), the optimal commitment price for the good is c when $v_1 = v_2$ since the monopolist can avoid double marginalization problem by MC pricing for the downstream good. In this case, bundling cannot generate same profits as the CS case even when $v_1 = v_2$ since it implicitly charges zero price instead of the one equal to MC.

And the critical level of α at which the monopolist is indifferent between bundling and independent sale is

$$\hat{\alpha}^{BS} = \frac{36v_0 t}{36v_0 t + 3t(5v_1 + 7v_2 - 15t) - 2(v_1 - v_2)^2}$$

If α is higher than $\hat{\alpha}^{BS}$, the monopolist can make higher profit by bundling both goods together and selling it to group *S* only than by separately selling the primary good to all consumers. That is, bundling is profitable if the complementary good is widely used by the consumers of the primary good. In the software industry, Microsoft bundles Internet Explorer into Windows OS, while it sells MS Office as an independent product since Internet browser is a widely used product whereas the Office products are used by relatively small group of consumers.

Note that $\hat{\alpha}^{BS}$ lies between 0 and 1 since

$$\begin{aligned} 3t(5v_1 + 7v_2 - 15t) - 2(v_1 - v_2)^2 &> 3t(5v_1 + 7v_2 - 15t) - 2t^2 \\ &= 15t(v_1 + v_2 - 3t) + 2t(3v_2 - t) > 0 \end{aligned}$$

where the first and second inequalities hold because of the assumptions given in (2-2) and (2-1), respectively. The difference between $\hat{\alpha}^{IS}$ and $\hat{\alpha}^{BS}$ is

$$\hat{\alpha}^{IS} - \hat{\alpha}^{BS} = \frac{2v_0(v_1 - v_2 + 3t)[3t - 2(v_1 - v_2)]}{[3t(12v_0 + 5v_1 + 7v_2 - 15t) - 2(v_1 - v_2)^2](2v_0 + v_1 + v_2 - 3t)} > 0$$

The inequality holds because of the assumptions (2-1) and (2-2). Since the

monopolist can make much higher profits by bundling than IS case, it is willing to sell its goods to a smaller group of consumers than IS case if bundling is possible.

The difference between $\hat{\alpha}^{CS}$ and $\hat{\alpha}^{BS}$ is

$$\hat{\alpha}^{CS} - \hat{\alpha}^{BS} = \frac{4v_0[4(v_1 - v_2) + 3t][3t - 2(v_1 - v_2)]}{[3t(12v_0 + 5v_1 + 7v_2 - 15t) - 2(v_1 - v_2)^2](16v_0 + 6v_1 + 10v_2 - 21t)}$$

Using assumptions (2-1) and (2-2), one can find that $\hat{\alpha}^{CS}$ is higher than $\hat{\alpha}^{BS}$ except when $\frac{3}{4}t \le v_2 - v_1 < t$. Even though π^{CS} is not smaller than π^{BS}, the profit gain from selling group *S* only is higher in bundling case than CS case except $\frac{3}{4}t \le v_2 - v_1 < t$ as the following shows:

$$(\pi^{BS} - \pi^{IA}) - (\pi^{CS} - \pi^{CA}) = \frac{\alpha[4(v_1 - v_2) + 3t][3t - 2(v_1 - v_2)]}{144t}$$

This explains why the monopolist is willing to sell the goods to a smaller group of consumers than the commitment case.

2.6 Bundling and Social Welfare

Most previous analyses on bundling have ambiguous conclusions about the welfare effect of bundling. It has been said that bundling could increase or decrease welfare. In the model presented here, bundling decreases Marshallian social welfare except for an extreme case.

Marshallian social welfare consists of the monopolist profits, firm 2's profits, and consumers' surplus. When the monopolist bundles, its profits always increase compared

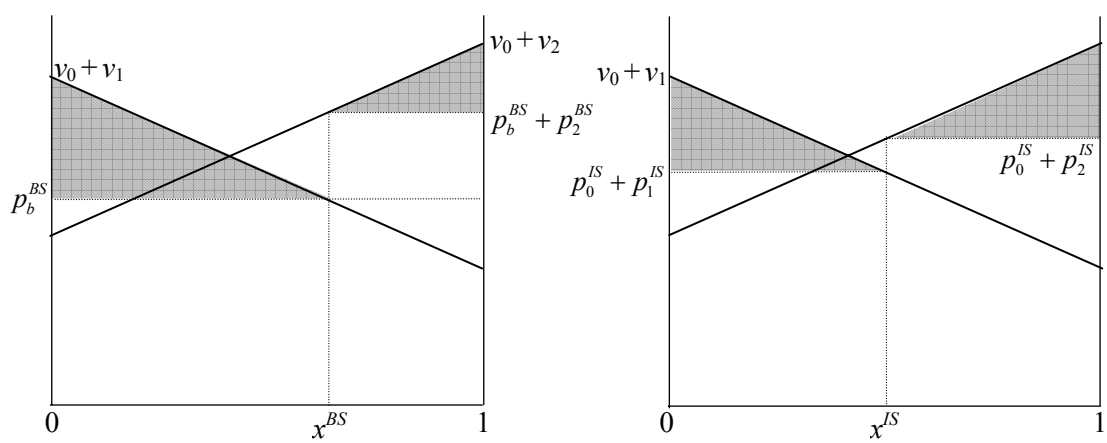
to the IS case. Firm 2's profits, on the other hand, decreases in bundling equilibrium since

$$\pi_2^{BS} - \pi_2^{IS} = \frac{\alpha(v_1 - v_2 + 3t)[5(v_1 - v_2) - 9t]}{72t} < 0$$

Consumers' surpluses with bundling and IS are

$$\begin{aligned} CS^{BS} &= \alpha\int_0^{x^{BS}} \left(v_0 + v_1 - tx - p_b^{BS}\right)dx + \alpha\int_{x^{BS}}^1 \left(v_0 + v_2 - t(1-x) - p_b^{BS} - p_2^{BS}\right)dx \\ &= \frac{\alpha[(v_1 - v_2)^2 + 2t(v_1 - v_2) + 5t^2]}{16t} \\ CS^{IS} &= \alpha\int_0^{x^{IS}} \left(v_0 + v_1 - tx - p_0^{IS} - p_1^{IS}\right)dx + \alpha\int_{x^{IS}}^1 \left(v_0 + v_2 - t(1-x) - p_0^{IS} - p_2^{IS}\right)dx \\ &= \frac{\alpha[(v_1 - v_2)^2 + 9t^2]}{36t} \end{aligned}$$

The shaded area of Figure 2-1 shows consumers' surplus of each case when $v_1 < v_2$. The difference between consumers' surplus with bundling and IS is



(a) Bundling (b) Independent sale (IS)

Figure 2-1. Consumers' surplus in bundling and IS cases when $v_1 < v_2$

$$CS^{BS} - CS^{IS} = \frac{\alpha(v_1 - v_2 + 3t)[5(v_1 - v_2) + 3t]}{144t}$$

which shows that consumers' surplus increases by the monopolist's decision to bundle unless $v_2 - v_1 > (3/5)t$. That is, unless firm 2's product is much superior to the monopolist's complementary good, consumers' surplus increases as the monopolist bundles. The consumers' surplus increases mainly because consumers who pay less in bundling case than in IS case outnumber consumers who pay more in bundling equilibrium. Unlike consumers' surplus, however, social welfare is more likely to decrease with bundling strategy by the monopolist, as the following proposition shows.

Proposition 2-2 Suppose the monopolist sells its goods to consumers in *S* only. Then Marshallian social welfare decreases with the monopolist's decision to bundle unless $\frac{3}{7}t < v_1 - v_2 < t$.

Proof. Marshallian social welfare is defined as the sum of consumers' surplus and profits of all firms. So social welfare with bundling is

$$W^{BS} = CS^{BS} + \pi_1^{BS} + \pi_2^{BS} = \alpha\left[v_0 + \frac{3(v_1 - v_2)^2}{16t} + \frac{10v_1 + 6v_2 - 5t}{16}\right]$$

And social welfare with IS is

$$W^{IS} = CS^{IS} + \pi_1^{IS} + \pi_2^{IS} = \alpha\left[v_0 + \frac{5(v_1 - v_2)^2}{36t} + \frac{2v_1 + 2v_2 - t}{4}\right]$$

The difference between them is

$$W^{BS} - W^{IS} = \frac{\alpha}{144t}(v_1 - v_2 + 3t)[7(v_1 - v_2) - 3t]$$

which is negative if $-3t < v_1 - v_2 < (3/7)t$, and positive otherwise. Since $|v_1 - v_2| < t$, the social welfare decreases except $(3/7)t < v_1 - v_2 < t$. *Q.E.D.*

The above proposition shows that unless the monopolist's complementary good is superior enough, the monopolist's bundling strategy lowers the social welfare. Especially, the social welfare always decreases when the monopolist bundles an inferior good or a good with the same quality as the rival's, i.e., $v_1 \le v_2$.

2.7 Bundling and R&D Incentives

One of the concerns about the bundling strategy by the monopolist of a primary good is that it may reduce R&D incentives in the complementary good industry. This section is devoted to the analysis of the effect of bundling on R&D incentives.

To analyze this, one needs to introduce an earlier stage at which two firms make decisions on the level of R&D investments to develop complementary goods. The whole game consists of three stages now. Let $R(v)$ be the minimum required investment level to develop a complementary good of value v. A simple form of the investment function is

$$R(v) = ev^2,\ e > 0$$

Using this, the firms' profit functions can be rewritten as follows:[21]

$$\hat{\pi}_1^{IS} = \alpha\left[v_0 + \frac{(v_1 - v_2)^2}{18t} + \frac{5v_1 + v_2 - 6t}{6}\right] - ev_1^2$$

$$\hat{\pi}_2^{IS} = \frac{\alpha(v_2 - v_1 + 3t)^2}{18t} - ev_2^2$$

$$\hat{\pi}_1^{BS} = \alpha\left[v_0 + \frac{3v_1 + v_2 - 3t}{4}\right] - ev_1^2$$

$$\hat{\pi}_2^{BS} = \frac{\alpha(v_2 - v_1 + t)^2}{8t} - ev_2^2$$

The following proposition shows that the monopolist's bundling strategy reduces not only the R&D incentive of the rival firm, but also its own incentive.

Proposition 2-3 Suppose the investment cost satisfies $e > \frac{3}{8t}$. Then the equilibrium values of v_i (i = 1, 2) are higher in the IS equilibrium than in the bundling equilibrium, i.e., $\hat{v}_2^{IS} > \hat{v}_2^{BS}$, and $\hat{v}_1^{IS} > \hat{v}_1^{BS}$. Further, firm 2's incentive decreases more than the monopolist's by bundling.

Proof. The first order conditions for profit maximization problems yield each firm's best response functions from which one can obtain the following equilibrium levels of v_i's for each equilibrium:

[21] In previous sections, it is assumed that the complementary goods already have been developed before the start of the game. The exclusion of the investment costs in profit function does not affect equilibrium since they are sunk costs.

$$\hat{v}_1^{IS} = \frac{\alpha(90et - 7\alpha)}{24e(9et - \alpha)}$$

$$\hat{v}_2^{IS} = \frac{\alpha(36et - 7\alpha)}{24e(9et - \alpha)}$$

$$\hat{v}_1^{BS} = \frac{3\alpha}{8e}$$

$$\hat{v}_2^{BS} = \frac{\alpha(8et - 3\alpha)}{8e(8et - \alpha)}$$

Since $0 \leq \alpha \leq 1$, the assumption $e > \frac{3}{8t}$ guarantees non-negative equilibrium values. Now the following comparisons prove the main argument:

$$\hat{v}_1^{IS} - \hat{v}_1^{BS} = \frac{\alpha(9et + 2\alpha)}{24e(9et - \alpha)} > 0$$

$$\hat{v}_2^{IS} - \hat{v}_2^{BS} = \frac{\alpha}{24e} + \frac{\alpha^2(10et - \alpha)}{8e(9et - \alpha)(8et - \alpha)} > 0$$

$$(\hat{v}_2^{IS} - \hat{v}_2^{BS}) - (\hat{v}_1^{IS} - \hat{v}_1^{BS}) = \frac{\alpha^2 t}{4(9et - \alpha)(8et - \alpha)} > 0 \qquad \textit{Q.E.D.}$$

Firm 2 has a lower incentive to invest in R&D because part of the rents from the investment will be transferred to the monopolist by bundling. The monopolist also has a lower incentive to invest because the bundling strategy reduces competitive pressure in the complementary good market.

2.8 Conclusion

It has been shown that the monopolist of a primary good has an incentive to bundle its own complementary good with the primary good if it cannot commit to the optimal set of prices when consumers buy the primary good and the complementary good

sequentially. Since the monopolist can increase its profits and the market share of its own complementary good by bundling, the model provides another case in which the Chicago School's single monopoly price theorem does not hold. While bundling lowers the rival firm's profits and Marshallian social welfare in general, it increases consumers' surplus except when the monopolist's complementary good is sufficiently inferior to the rival's good. Bundling also has a negative effect on R&D incentives of both firms.

Since bundling may increase consumers' surplus while it lowers social welfare, the implication for the antitrust policy is ambiguous. If antitrust authorities care more about consumers' surplus than rival firm's profits, this kind of bundling may be allowed. Even if total consumers' surplus increases, however, consumers who prefer the rival's complementary good can be worse off since they have to pay higher price for both the bundled good and the alternative complementary good. So bundling transfers surplus from one group to another group of consumers.

In addition to the problem of a redistribution of consumers' surplus, bundling also has a negative long-term effect on welfare since it reduces both firms' R&D incentives. This long-term effect of bundling on R&D investment may be more important than immediate effects on competitor's profit or consumers' surplus, especially for so-called high-tech industries that are characterized by high levels of R&D investments. For example, if a software company anticipates that development of a software program will induce the monopolist of the operating system to develop a competing product and bundle it with the OS, then the firm may have less incentive to invest or give up developing the software. This could be a new version of market foreclosure.

A related issue is that if the risk of R&D investments includes the possibility of the monopolist's developing and bundling of an alternative product, it can be said that bundling increases social costs of R&D investments. Furthermore, since the monopolist is more likely bundle a complementary good that has a broad customer base, bundling may induce R&D investments to be biased to the complementary goods that are for special group of consumers. A possible extension of the model lies in this direction.

Another extension could be to introduce competition in the primary good market, which is suitable for the Kodak case.[22] It has been pointed out that when the primary good market is competitive, the anticompetitive effect of bundling is limited. In the model presented here, firm 1 (the monopolist) could not set the bundling price so high if it faced competition in the primary good market. However, if the primary goods are also differentiated so that the producers of them have some (limited) monopoly powers, bundling may have anticompetitive effects. The result can be more complicated—but more realistic—if it is combined with the possibility of upgrade which is common in the software industry.

[22] See Klein (1993), Shapiro (1995), Borenstein, MacKie-Mason, and Netz (1995), and Blair and Herndon (1996).

CHAPTER 3
COMPETITION AND WELFARE IN THE TWO-SIDED MARKET: THE CASE OF CREDIT CARD INDUSTRY

3.1 Introduction

It is well known that a two-sided market—or more generally a multi-sided market—works differently from a conventional one-sided market. In order to get both sides on board and to balance the demands of both sides, a platform with two sides may have to subsidize one side (i.e., set the price of one side lower than the marginal cost of serving the side). In the credit card industry, cardholders usually pay no service fee or even a negative fee in various forms of rebate. In terms of the traditional one-sided market logic, this can be seen as a practice of predatory pricing. Several models of two-sided markets, however, show that the pricing rule of the two-sided market is different from the rule of the one-sided market, and a price below marginal cost may not be anti-competitive.[1]

Another feature of the two-sided market is that competition may not necessarily lower the price charged to the customers. In the credit card industry, competition between nonproprietary card schemes may raise the interchange fee, which in turn forces the acquirers to raise the merchant fee. The interchange fee is a fee that is paid by the acquirer to the issuer for each transaction made by the credit card. If the interchange fee decreases as a result of competition, the cardholder fee is forced to increase. For the

[1] Published papers include Baxter (1983), Rochet and Tirole (2002), Schmalensee (2002), and Wright (2003a, 2003b, 2004a).

proprietary card schemes that set the cardholder fees and the merchant fees directly, competition may lower one of the fees but not both fees.

The distinctive relationship between competition and prices raises a question about the welfare effect of competition in the two-sided market. Even if competition lowers the overall level of prices, it does not necessarily lead to a more efficient price structure. Previous models about competition in the two-sided markets focus mainly on the effect of competition on the price structure and derive ambiguous results on the welfare effects of competition. I present a model of the credit card industry in order to show the effects of competition on social welfare as well as on the price structure and level. The main result is that while the effects of competition on the price structure are different depending on the assumptions about whether consumers single-home or multi-home[2] and whether card schemes are identical (Bertrand competition) or differentiated (Hotelling competition), the effects of competition on social welfare do not vary regardless of different model settings. That is, competition does not improve the social welfare in the various models presented here.

The main reason for this result is that competition forces the platforms to set the price(s) in favor of one side that is a bottleneck part, while a monopoly platform can fully internalize the indirect network externalities that arise in the two-sided market.[3] In order to maximize the transaction volume (for nonproprietary schemes) or profits (for proprietary schemes), the monopolist first needs to make the total size of the network

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[2] If a cardholder (or merchant) chooses to use (or accept) only one card, she is said to single-home. If she uses multiple cards, she is said to multi-home.

[3] In a two-sided market, the benefit of one side depends on the size of the other side. This indirect network externality cannot be internalized by the end-users of the two-sided market. See Rochet and Tirole (2005).

externalities as large as possible. Competing card schemes, on the contrary, set biased prices since they share the market and try to attract single-homing consumers or merchants.

Since the first formal model by Baxter (1983), various models of two-sided markets have been developed. Many of them focus on the price structure of a monopolistic two-sided market.[4] It is in recent years that considerable attention has been paid to competition in two-sided markets. Rochet and Tirole (2003) study competition between differentiated platforms and show that if both buyer (consumer) and seller (merchant) demands are linear, then the price structures of a monopoly platform, competing proprietary platforms and competing (non-proprietary) associations are the same and Ramsey optimal. They measure the price structure and Ramsey optimality in terms of the price-elasticity ratio, so price levels and relative prices are not the same for different competitive environments. While they assume that consumers always hold both cards, the model presented here distinguishes cases with single-homing consumers and multi-homing consumers and uses Marshallian welfare measure which includes platforms' profits as well as consumers' and merchants' surpluses.

Guthrie and Wright (2005) present a model of competition between identical card schemes. They introduce the business stealing effect by allowing competing merchants and show that competition may or may not improve social welfare. I extend their model to the case of the competition between differentiated card schemes as well as the cases of proprietary card schemes, while removing the business stealing effect for simpler results.

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[4] The interchange fee is the main topic in these analyses of the credit card industry. See Rochet and Tirole (2002), Schmalensee (2002) and Wright (2003a, 2003b, 2004a) for the analyses of the credit card industry with monopoly card scheme.

Chakravorti and Roson (2004) also provide a model of competing card schemes and show that competition is always welfare enhancing for both consumers and merchants since the cardholder fee and the merchant fee in duopoly are always lower than in monopoly. To derive the results, they assume that consumers pay an annual fee while merchants pay a per-transaction fee and cardholder benefits are platform specific and independent of each other. In contrast to their model, this paper assumes both consumers and merchants pay per-transaction fees[5] and cardholder benefits are either identical or differentiated according to the Hotelling model, and concludes that competition does not improve Marshallian social welfare. Further, it shows competition may not always lower both the cardholder and merchant fees even for the proprietary scheme as well as non-proprietary scheme.

The rest of Chapter 3 proceeds as follows. Section 3.2 sets up the basic model of the non-proprietary card scheme. Section 3.3 and 3.4 show the effects of competition on the price structure and welfare for the cases of single-homing consumers and multi-homing consumers. Section 3.5 extends the model to the case of the proprietary card scheme and compares the results with those of the non-proprietary card scheme. The last section concludes with a discussion of some extensions and policy implications.

3.2 The Model: Nonproprietary Card Scheme

Suppose there are two payment card schemes, $i = 1, 2$, both of which are not-for-profit organizations of many member banks. A cardholder or consumer receives a per-transaction benefit b_{Bi} from using card i, which is assumed to be uniformly distributed between $(\underline{b}_B, \overline{b}_B)$. A merchant receives a per-transaction benefit, b_S, which is also

[5] The per-transaction fee paid by consumers can be negative in the various forms of rebates.

uniformly distributed between ($\underline{b}_S, \bar{b}_S$). It is assumed that merchants find no difference between two card schemes.

There are two types of member banks. Issuers provide service to consumers, while acquirers provide service to merchants. Following Guthrie and Wright (2006), both the issuer market and the acquirer market are assumed to be perfectly competitive. Card schemes set the interchange fees in order to maximize total transaction volumes.[6]

For modeling convenience, it is assumed that there is no fixed cost or fixed fee. Let c_I and c_A be per-transaction costs of a issuer and a acquirer, respectively. Then card scheme *i*'s per-transaction cardholder fee and merchant fee are, respectively,

$$f_i = c_I - a_i$$
$$m_i = c_A + a_i$$

where a_i is scheme *i*'s interchange fee. Note that the sum of the cardholder fee and the merchant fee is independent of the interchange fee since

$$f_i + m_i = c_I + c_A \equiv c$$

In order to rule out the possibility that no merchant accepts the card and all merchants accept the card, it is assumed that

[6] Rochet and Tirole (2003) assume constant profit margins for the issuers and the acquirers. Under this assumption, maximizing member banks' profits is same as maximizing total transaction volume, and the sum of the cardholder fee and the merchant fee is also independent of the interchange fee.

$$\bar{b}_B + \underline{b}_S < c < \bar{b}_B + \bar{b}_S \tag{3-1}$$

Both the numbers of consumers and merchants are normalized to one. Consumers have a unit demand for each good sold by a monopolistic merchant.[7] Merchants charge the same price to cash-paying consumers and card-paying consumers, i.e., the no-surcharge-rule applies.

The timing of the game proceeds as follows: i) at stage 1, the card schemes set the interchange fees, and the issuers and acquirers set the cardholder fees and merchant fees, respectively; ii) at stage 2, consumers choose which card to hold and use, and merchants choose which card to accept.

3.3 Competition between Identical Card Schemes: Bertrand Competition

In this section, two card schemes are assumed to be identical, i.e., $b_{B1} = b_{B2}$ ($\equiv b_B$). Consumers can hold one or both cards depending on the assumption of single-homing or multi-homing, while merchants are assumed to freely choose whether to accept one card, both cards, or none.

One of the key features of the two-sided market is that there exist indirect network externalities. As the number of members or activities increase on one side, the benefits to the members of the other side also increase. In the credit card industry, cardholders' benefits increase as the number of merchants that accept the card increases, while the merchants' benefits increase as the number of cardholders who use the card increases.

Some of the previous analyses of the credit card industry did not fully incorporate this network effect in their models by assuming homogeneous merchants, in which case

[7] Since merchants do not compete with each other, the business stealing effect does not exist in this model.

either all merchants or none accept the card.[8] So at any equilibrium where transactions occur, all merchants accept card and consumers do not need to worry about the size of the other side of the network. The model presented here takes into account this indirect network effect by assuming merchants are heterogeneous and the net utility of a consumer with b_B takes the following form:

$$U_{Bi} = (b_B - f_i)Q_{Si} = (b_B - c_I + a_i)Q_{Si}, \;\; i = 1, 2$$

where Q_{Si} is the number of merchants that accept card i.

For modeling convenience, it is assumed throughout this section that the issuer market is not fully covered at equilibrium, which requires

$$\bar{b}_B - \underline{b}_B > 2(\underline{b}_B + \bar{b}_S - c)$$

3.3.1 Single-Homing Consumers

If consumers are restricted to hold only one card, they will choose to hold card i if $U_{Bi} > U_{Bj}$ and $U_{Bi} \geq 0$. Note that the cardholding decision depends on the size of the other side as well as the price charged to the consumers. Even if $f_i > f_j$, a consumer may choose card i as long as the number of merchants that accept card i (Q_{Si}) is large enough compared to the number of merchants accepting card j (Q_{Sj}).

Merchants will accept card i as long as $b_S \geq m_i$ since accepting both cards is always a dominant strategy for an individual merchant when consumers single-home. So the

[8] See Rochet and Tirole (2002) and Guthrie and Wright (2006).

number of merchants that accept card i (quasi-demand function for acquiring service) is[9]

$$Q_{Si} = \frac{\bar{b}_S - m_i}{\bar{b}_S - \underline{b}_S} = \frac{\bar{b}_S - c_A - a_i}{\bar{b}_S - \underline{b}_S} \tag{3-2}$$

Using (3-2), the consumer's net utility can be rewritten as

$$U_{Bi} = \frac{(b_{Bi} - c_I + a_i)(\bar{b}_S - c_A - a_i)}{\bar{b}_S - \underline{b}_S}$$

Let b_B^* be the benefit of the critical consumer who is indifferent between card 1 and 2. One can obtain b_B^* by solving $U_{B1} = U_{B2}$, which is

$$b_B^* = \bar{b}_S + c_I - c_A - a_1 - a_2$$

A consumer with low b_B is more sensitive to the transaction fee, so she prefers the card with lower cardholder fee (i.e., higher interchange fee). On the other hand, a consumer with high b_B gets a larger surplus for each card transaction, so she prefers the card that is accepted by more merchants. Therefore, a consumer whose b_B is higher than b_B^* will choose a card with lower a_i, and a consumer whose b_B is lower than b_B^* will choose a card with higher a_i. If $a_i = a_j$, then consumers are indifferent between two cards, so they are assumed to randomize between card 1 and 2. This can be summarized by the following quasi-demand function of consumers:

[9] Schmalensee (2002) calls Q_{Si} and Q_{Bi} partial demands, and Rochet and Tirole (2003) call them quasi-demands since the actual demand is determined by the decisions of both sides in a two-sided market.

$$Q_{Bi} = \begin{cases} \dfrac{b_B^* - f_i}{\bar{b}_B - \underline{b}_B} = \dfrac{\bar{b}_S - c_A - a_j}{\bar{b}_B - \underline{b}_B} & \text{if } a_i > a_j \\ \dfrac{\bar{b}_B - b_B^*}{\bar{b}_B - \underline{b}_B} = \dfrac{\bar{b}_B - \bar{b}_S - c_I + c_A + a_i + a_j}{\bar{b}_B - \underline{b}_B} & \text{if } a_i < a_j \\ \dfrac{\bar{b}_B - c_I + a_i}{2(\bar{b}_B - \underline{b}_B)} & \text{if } a_i = a_j \end{cases} \tag{3-3}$$

At stage 1, the card schemes choose the interchange fees to maximize the transaction volume which is the product of Q_{Bi} and Q_{Si}. The following proposition shows the equilibrium interchange fee of the single-homing case of Bertrand competition.

Proposition 3-1 If two identical card schemes compete with each other and consumers single-home,

(i) the equilibrium interchange fee is

$$a^{bs} = \frac{1}{3}\left[2(\bar{b}_S - c_A) - (\bar{b}_B - c_I)\right]$$

(ii) a^{bs} maximizes total consumers' surplus

Proof. (i) Without loss of generality, suppose $a_1 > a_2$. Then scheme 2 will maximize the following objective function:

$$T_2(a_2; a_1) = Q_{B2}Q_{S2} = \frac{(\bar{b}_S - a_2 - c_A)(\bar{b}_B - \bar{b}_S - c_I + c_A + a_1 + a_2)}{(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)}$$

from which scheme 2's best response function can be obtained as follows:

$$R_2(a_1) = \frac{1}{2}\left(2\bar{b}_S - \bar{b}_B - a_1 + c_I - 2c_A\right)$$

Scheme 1's objective function is

$$T_1(a_1; a_2) = Q_{B1}Q_{S1} = \frac{(\bar{b}_S - a_1 - c_A)(\bar{b}_S - a_2 - c_A)}{(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)}$$

Since the function is a linear function of a_1 with negative coefficient, scheme 1 will set a_1 as low as possible, i.e., as close to a_2 as possible. So the best response function of scheme 1 is

$$R_1(a_2) = a_2$$

Solving $R_1(a_2)$ and $R_2(a_1)$ together, one can obtain the following Nash equilibrium:

$$a_1^* = a_2^* = \frac{1}{3}[2(\bar{b}_S - c_A) - (\bar{b}_B - c_I)] \equiv a^{bs}$$

The equilibrium transaction volume of scheme i when $a_1 = a_2 = a^{bs}$ is

$$T_i(a^{bs}; a^{bs}) = \frac{(\bar{b}_B + \bar{b}_S - c)^2}{9(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} \equiv T^{bs}$$

Since scheme 1's best response function seems to contradict the premise that $a_1 > a_2$, it is necessary to show that card schemes do not have an incentive to deviate from the equilibrium. To see this, suppose scheme 1 changes a_1 by Δa. Then the transaction volume of scheme 1 becomes

$$T_1(a^{bs} + \Delta a; a^{bs}) = \begin{cases} \dfrac{(\bar{b}_B + \bar{b}_S - c)(\bar{b}_B + \bar{b}_S - c - 3\Delta a)}{9(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} & \text{if } \Delta a > 0 \\ \dfrac{(\bar{b}_B + \bar{b}_S - c + 3\Delta a)(\bar{b}_B + \bar{b}_S - c - 3\Delta a)}{9(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} & \text{if } \Delta a < 0 \end{cases}$$

Both of them are less than T^{bs}, so there is no incentive for scheme 1 to deviate from a^{bs}.

(ii) At symmetric equilibrium with common a, the consumers' demands for the card services are given by (3-3). So the total consumers' surplus is

$$\begin{aligned} TU_B^{bs}(f(a)) &= \sum_{i=1}^{2} Q_{Si} \int_f^{\bar{b}_B} Q_{Bi} df = \frac{(\bar{b}_B - f(a))^2 (\bar{b}_S - m(a))}{2(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} \\ &= \frac{(\bar{b}_B - c_I + a)^2 (\bar{b}_S - c_A - a)}{2(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} \end{aligned}$$

The optimal a that maximizes TU_B^{bs} is

$$a^* = \frac{1}{3}[2(\bar{b}_S - c_A) - (\bar{b}_B - c_I)]$$

which is same as a^{bs}. *Q.E.D.*

When consumers single-home, each card scheme has monopoly power over the merchants that want to sell their products to the consumers. This makes the card schemes try to attract as many consumers as possible by setting the interchange fee favorable to consumers. The resulting interchange fee chosen by the card schemes is one that maximizes total consumers' surplus.

An interchange fee higher than a^{bs} may attract more consumers due to the lower cardholder fee, but fewer merchants will accept the card due to the higher merchant fee. Therefore, a card scheme can increase the transaction volume by lowering its interchange fee, which attracts higher types of consumers who care more about the number of merchants that accept the card. On the other hand, an interchange fee lower than a^{bs} may attract more merchants, but fewer consumers will use the card. In this case, a card scheme can increase the transaction volume by raising its interchange fee.

In order to see how competition in the two-sided market affects the price structure, it is necessary to analyze the case in which the two card schemes are jointly owned by one entity. As the following proposition shows, it turns out that joint ownership or monopoly generates a lower interchange fee, which implies a higher cardholder fee and a lower merchant fee. In other words, competition between card schemes when consumers single-home raises the interchange fee.

Proposition 3-2 If two identical card schemes are jointly owned and consumers single-home,

(i) the symmetric equilibrium interchange fee is

$$a^{bj} = \frac{1}{2}\left[(\bar{b}_S - c_A) - (\bar{b}_B - c_I)\right] < a^{bs}$$

(ii) the joint entity may engage in price discrimination in which one scheme sets the interchange fee equal to a^{bj} and the other scheme sets the interchange fee at any level above a^{bj}, but the total transaction volume cannot increase by the price discrimination,

(iii) a^{bj} maximizes the social welfare, which is defined as the sum of the total consumers' surplus and the total merchants' surplus.

Proof. (i) Since the card schemes are identical, there is no difference between operating only one scheme and operating both schemes with same interchange fees. So suppose the joint entity operates only one scheme. Then the quasi-demand functions are

$$Q_B = \frac{\bar{b}_B - f}{\bar{b}_B - \underline{b}_B} = \frac{\bar{b}_B - c_I + a}{\bar{b}_B - \underline{b}_B}$$

$$Q_S = \frac{\bar{b}_S - m}{\bar{b}_S - \underline{b}_S} = \frac{\bar{b}_S - c_A - a}{\bar{b}_S - \underline{b}_S}$$

The joint entity will choose the optimal a in order to maximize the transaction volume $Q_B Q_S$. The optimal interchange fee obtained from the first-order condition is

$$a^* = \frac{1}{2}\left[(\bar{b}_S - c_A) - (\bar{b}_B - c_I)\right] \equiv a^{bj}$$

which is less than a^{bs} since

$$a^{bs} - a^{bj} = \frac{1}{6}\left(\bar{b}_S + \bar{b}_B - c\right) > 0$$

(ii) Without loss of generality, suppose $a_1 > a_2$. Then scheme 1 will attract low-type consumers and scheme 2 will attract high-type consumers. The quasi-demand functions are determined by (3-2) and (3-3). And the total transaction volume is

$$Q_{B1}Q_{S1} + Q_{B2}Q_{S2} = \frac{(\bar{b}_B - c_I + a_2)(\bar{b}_S - c_A - a_2)}{(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} \tag{3-4}$$

Note that (3-4) is independent of a_1, which implies a_1 can be set at any level above a_2. The optimal a_2 can be obtained from the first-order condition for maximizing (3-4):

$$a_2^* = \frac{1}{2}\left[(\bar{b}_S - c_A) - (\bar{b}_B - c_I)\right] \equiv a^{bj}$$

It is not difficult to check that the total transaction volume at equilibrium is also the same as in the symmetric equilibrium.

(iii) The sum of the total consumers' surplus and the total merchants' surplus is

$$\begin{aligned} TU^{bsj} &= TU_B^{bsj} + TU_S^{bsj} = Q_S \int_f^{\bar{b}_B} Q_B df + Q_B \int_m^{\bar{b}_S} Q_S dm \\ &= \frac{(\bar{b}_B - c_I + a)(\bar{b}_S - c_A - a)(\bar{b}_B + \bar{b}_S - c)}{2(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} \end{aligned}$$

The optimal a that maximizes TU^{bsj} is

$$a^{bw} = \frac{1}{2}\left[(\bar{b}_S - c_A) - (\bar{b}_B - c_I)\right]$$

which is same as a^{bj}. So a^{bj} maximizes social welfare. *Q.E.D.*

The most interesting result of the proposition is that the joint entity, which acts like a monopolist, chooses the socially optimal interchange fee. This is possible because both the issuing and acquiring sides are competitive even though the platform is monopolized, and the joint entity can internalize the indirect network externalities of both sides. Comparing propositions 3-1 and 3-2, one can find that competition between card schemes lowers social welfare as well as decreases total transaction volume. In a typical example of prisoner's dilemma in game theory, competing firms choose higher quantity and/or lower price, which is detrimental to themselves but beneficial to the society. But this example of the two-sided market shows that competitive outcome can be detrimental to the society as well as to themselves.

3.3.2 Multi-Homing Consumers

In this subsection, consumers are allowed to multi-home. Since there is no fixed fee or cost, individual consumer is always better off by holding both cards as long as $b_{Bi} > f_i$. So the number of consumers who hold card i is

$$Q_{Bi} = \frac{\bar{b}_B - f_i}{\bar{b}_B - \underline{b}_B} = \frac{\bar{b}_B - c_I + a_i}{\bar{b}_B - \underline{b}_B} \tag{3-5}$$

On the other hand, since merchants have monopoly power over the products they sell, they may strategically refuse to accept card i even if $b_S > m_i$.

If a merchant accepts card i only, it receives a surplus equal to

$$U_{Si} = (b_S - m_i)Q_{Bi} = \frac{(b_S - c_A - a_i)(\bar{b}_B - c_I + a_i)}{\bar{b}_B - \underline{b}_B} \tag{3-6}$$

If the merchant accept both cards, the surplus is

$$U_{Sb} = (b_S - m_1)Q_{b1} + (b_S - m_2)Q_{b2} = (b_S - c_A - a_1)Q_{b1} + (b_S - c_A - a_2)Q_{b2} \tag{3-7}$$

where Q_{bi} is the number of consumers who will use card i if the merchant accepts both cards.[10] When a consumer holding both cards buys from a merchant that accept both cards, the consumer will choose to use the card that gives a higher net benefit, i.e., she will use card i if $b_{Bi} - f_i > b_{Bj} - f_j$. And the consumer will randomize between card i and j if $b_{Bi} - f_i = b_{Bj} - f_j$.

If the two card schemes are identical ($b_{B1} = b_{B2}$), consumers will use the card that has a lower consumer fee if merchant accepts both cards, i.e.,

$$Q_{bi} = \begin{cases} Q_{Bi} & \text{if } a_i > a_j \ (f_i < f_j) \\ 0 & \text{if } a_i < a_j \ (f_i > f_j) \\ (1/2)Q_{Bi} & \text{if } a_i = a_j \ (f_i = f_j) \end{cases} \tag{3-8}$$

A merchant with b_S will accept card i only if $U_{Si} > U_{Sj}$ and $U_{Si} > U_{Sb}$. It will accept both cards if $U_{Sb} \geq U_{Si}$, $i = 1, 2$. To see the acceptance decision by a merchant, suppose

[10] Consumers' card-holding decision and card-using decision can be different since they can hold both cards but use only one card for each merchant.

$a_1 > a_2$ without loss of generality. Then the net surplus to the merchant if it accepts both cards is

$$U_{Sb} = (b_S - c_A - a_1)Q_{B1} + (b_S - c_A - a_2)\cdot 0 = U_{S1}$$

Merchants are indifferent between accepting card 1 only and accepting both cards since consumers will only use card 1 if merchants accept both cards. In other words, there is no gain from accepting both cards if consumers multi-home. So merchants' decision can be simplified to the choice between two cards. Let b_S^* be the critical merchant that is indifferent between accepting card 1 only and card 2 only, which can be obtained by setting $U_{S1} = U_{S2}$:

$$b_S^* = \bar{b}_B - c_I + c_A + a_1 + a_2$$

Merchants with low b_S will be sensitive to the merchant fee and prefer a card with low merchant fee (low interchange fee), while merchants with high b_S will prefer a card with low consumer fee (high interchange fee) since they care more about the number of consumers who use the card. Therefore, if $m_1 > m_2$ ($a_1 > a_2$), merchants with b_S smaller than b_S^* (and greater than m_2) will accept card 2 only and merchant with b_S higher than b_S^* will accept card 1.

If $a_1 = a_2$, all cardholders have both cards and it is indifferent for merchants whether they accept card 1, card 2 or both. For modeling simplicity, it is assumed that merchants will accept both cards if $a_1 = a_2$. The following summarizes the number of merchants that accept card i:

$$Q_{Si} = \begin{cases} \dfrac{\bar{b}_S - b_S^*}{\bar{b}_S - \underline{b}_S} = \dfrac{\bar{b}_S - \bar{b}_B + c_I - c_A - a_i - a_j}{\bar{b}_S - \underline{b}_S} & \text{if } a_i > a_j \\ \dfrac{b_S^* - m_i}{\bar{b}_S - \underline{b}_S} = \dfrac{\bar{b}_B - c_I + a_j}{\bar{b}_S - \underline{b}_S} & \text{if } a_i < a_j \\ \dfrac{\bar{b}_S - m_i}{\bar{b}_S - \underline{b}_S} = \dfrac{\bar{b}_S - c_A - a_i}{\bar{b}_S - \underline{b}_S} & \text{if } a_i = a_j \end{cases} \tag{3-9}$$

Proposition 3-3 If two identical card schemes compete with each other and consumers multi-home,

(i) the equilibrium interchange fee is

$$a^{bm} = \frac{1}{3}\left[(\bar{b}_S - c_A) - 2(\bar{b}_B - c_I)\right]$$

(ii) a^{bm} maximizes total merchants' surplus.

Proof. (i) Without loss of generality, suppose $a_1 > a_2$ ($m_1 > m_2$). Then scheme 1's best response function can be obtained by solving the optimization problem of the scheme, which is

$$R_1(a_2) = \frac{1}{2}\left(\bar{b}_S - 2\bar{b}_B + 2c_I - c_A - a_2\right)$$

Scheme 2's objective function is

$$T_2(a_2; a_1) = Q_{B2}Q_{S2} = \frac{(\bar{b}_B - c_I + a_1)(\bar{b}_B - c_I + a_2)}{(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)}$$

Since the function is linear in a_2 with positive coefficient, scheme 2 will set a_2 as high as possible, i.e., as close to a_1 as possible. So the best response function of scheme 2 is

$$R_2(a_1) = a_1$$

Solving $R_1(a_2)$ and $R_2(a_1)$ together, one can obtain the following Nash equilibrium:

$$a_1^* = a_2^* = \frac{1}{3}\left[(\bar{b}_S - c_A) - 2(\bar{b}_B - c_I)\right] \equiv a^{bm}$$

The equilibrium transaction volume of scheme i when $a_1 = a_2 = a^{bm}$ is

$$T_i(a^{bm}; a^{bm}) = \frac{(\bar{b}_B + \bar{b}_S - c)^2}{9(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} \equiv T^{bm}$$

As in Proposition 3-1, it is necessary to show that the card schemes do not have an incentive to deviate from a^{bm} in order to justify the equilibrium. To see this, suppose scheme 1 changes a_1 by Δa. Then the transaction volume of scheme 1 becomes

$$T_1(a^{bm} + \Delta a; a^{bm}) = \begin{cases} \dfrac{(\bar{b}_B + \bar{b}_S - c - 3\Delta a)(\bar{b}_B + \bar{b}_S - c + 3\Delta a)}{9(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} & \text{if } \Delta a > 0 \\ \dfrac{(\bar{b}_B + \bar{b}_S - c)(\bar{b}_B + \bar{b}_S - c + 3\Delta a)}{9(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} & \text{if } \Delta a < 0 \end{cases}$$

Both of them are less than T^{bm}, so there is no incentive for the scheme to deviate from a^{bm}.

(ii) At symmetric equilibrium with common a, the total merchants' surplus is

$$TU_S^{bm}(m(a)) = \sum_{i=1}^{2} Q_{Bi} \int_m^{\bar{b}_S} Q_{Si} dm = \frac{(\bar{b}_S - m(a))^2 (\bar{b}_B - f(a))}{2(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)}$$
$$= \frac{(\bar{b}_S - c_A - a)^2 (\bar{b}_B - c_I + a)}{2(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)}$$

The optimal interchange fee that maximizes TU_S^{bm} is

$$a^* = \frac{1}{3}\left[(\bar{b}_S - c_A) - 2(\bar{b}_B - c_I)\right]$$

which is equal to a^{bm}. So a^{bm} maximizes total merchants' surplus.

Q.E.D.

When consumers multi-home, the card schemes care more about merchants since they can strategically refuse to accept one card. By setting the interchange fee so as to maximize the merchants' surplus, the card schemes can attract as many merchants as possible. As in the single-homing case, an interchange fee higher or lower than a^{bm} is suboptimal and a card scheme can increase its transaction volume by changing the interchange fee closer to a^{bm}.

The interchange fee in the multi-homing case is lower than in the single-homing case since the fee is set in favor of the merchants. The following proposition shows that

the interchange fee is higher if the card schemes are jointly owned, which implies the interchange fee decreases as a result of competition between card schemes when consumers multi-home. It also shows that competition lowers social welfare as in the single-homing case.

Proposition 3-4 If two identical card schemes are jointly owned and consumers multi-home,

(i) the symmetric equilibrium interchange fee is

$$a^{bj} = \frac{1}{2}\left[(\bar{b}_S - c_A) - (\bar{b}_B - c_I)\right] > a^{bm}$$

(ii) the joint entity may engage in price discrimination in which one scheme sets the interchange fee equal to a^{bj} and the other scheme sets the interchange fee at any level below a^{bj}, but the total transaction volume cannot increase by the price discrimination,

(iii) a^{bj} maximizes social welfare.

Proof. (i) Regardless whether consumers single-home or multi-home, there is no difference for the joint entity between operating two card schemes with same interchange fee and operating only one scheme since the card schemes are identical. So the proof is the same as the first part of Proposition 3-2. And for multi-homing consumers, the monopolistic interchange fee is higher than the competitive interchange fee since

$$a^{bj} - a^{bm} = \frac{1}{6}\left(\bar{b}_S + \bar{b}_B - c\right) > 0$$

(ii) Without loss of generality, suppose $a_1 > a_2$. Then scheme 1 will attract low-type merchants and scheme 2 will attract high-type ones. Then the total transaction volume is

$$Q_{B1}Q_{S1} + Q_{B2}Q_{S2} = \frac{(\bar{b}_B - c_I + a_1)(\bar{b}_S - c_A - a_1)}{(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} \tag{3-10}$$

Note that (3-10) is independent of a_2, which implies a_2 can be set at any level below a_1. The optimal a_1 obtained from the first-order condition is

$$a_1^* = \frac{1}{2}\left[(\bar{b}_S - c_A) - (\bar{b}_B - c_I)\right]$$

which is equal to a^{bj}.

The total transaction volume at equilibrium is $\dfrac{(\bar{b}_B + \bar{b}_S - c)^2}{4(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)}$ which is the same as in the symmetric equilibrium.

(iii) The proof is the same as in part (iii) of Proposition 3-2.

Q.E.D.

The optimal interchange fee for the joint entity is the same as in the single-homing case since the card schemes do not compete for consumers or merchants. Unlike the single-homing case, however, the interchange fee decreases as a result of competition between the card schemes when consumers multi-home. Social welfare deteriorates since

competing card schemes set the interchange fee too low in order to attract more merchants.

Figure 3-1 shows the results of this section. As is clear in the figure, competitive equilibrium interchange fees maximize either consumers' surplus or merchants' surplus. Since monopoly interchange fee maximizes total surplus, competitive outcome is suboptimal in terms of social welfare.

3.4 Competition between Differentiated Card Schemes: Hotelling Competition

In this section, card schemes are assumed to be differentiated and compete à la Hotelling. As in a standard Hotelling model, suppose consumers are uniformly distributed between 0 and 1, and the card scheme 1 is located at 0 and scheme 2 is at 1. A consumer located at x receives a net benefit of $\bar{b}_B - tx\ (\equiv b_{B1})$ if she uses card 1, and $\bar{b}_B - t(1-x)\ (\equiv b_{B2})$ if she uses card 2. In order to comply with the assumption that consumers' benefits from card usage is uniformly distributed between $(\underline{b}_B, \bar{b}_B)$, the transportation cost t is assumed to be equal to $\bar{b}_B - \underline{b}_B$.

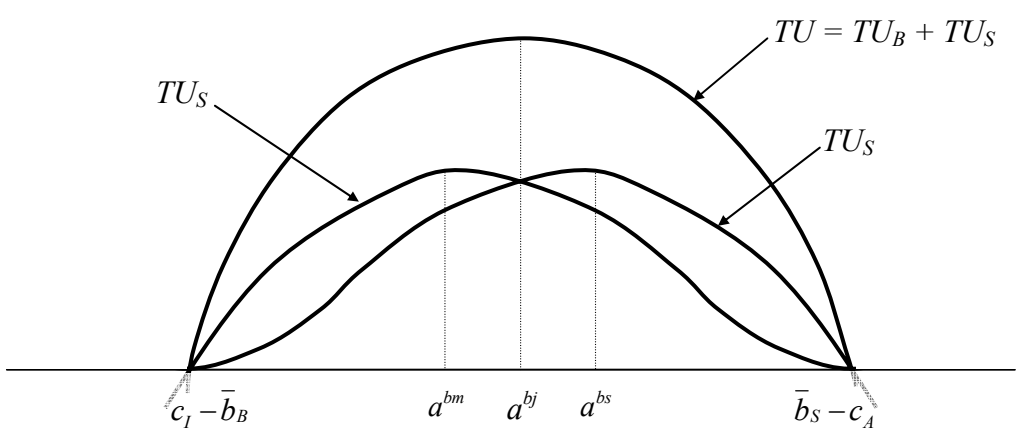

Figure 3-1. Welfare and interchange fees of Bertrand competition

The net utilities of a consumer located at x when she uses card 1 and 2 are

$$U_{B1} = \left(\bar{b}_B - tx - f_1\right)Q_{S1} = \left(\bar{b}_B(1-x) + \underline{b}_B x - c_I + a_1\right)Q_{S1}$$
$$U_{B2} = \left(\bar{b}_B - t(1-x) - f_2\right)Q_{S2} = \left(\bar{b}_B x + \underline{b}_B(1-x) - c_I + a_2\right)Q_{S2}$$

The critical consumer, x^*, who is indifferent between card 1 and 2 can be obtained by solving $U_{B1} = U_{B2}$:

$$x^* = \frac{(\bar{b}_B - c_I + a_1)Q_{S1} - (\underline{b}_B - c_I + a_2)Q_{S2}}{(\bar{b}_B - \underline{b}_B)(Q_{S1} - Q_{S2})} \tag{3-11}$$

If the issuer market is not fully covered, each card scheme has a full monopoly power over the consumers and the resulting equilibrium will be the same as in the monopoly case of the previous section. In order to obtain competitive outcomes, the issuer market is assumed to be fully covered at equilibrium. This requires the following assumption:[11]

$$\underline{b}_B + \bar{b}_S \geq c$$

Depending on whether consumers single-home or multi-home, and whether card schemes compete or collude, various equilibria can be derived. There may exist multiple equilibria including asymmetric ones. For expositional simplicity, however, only symmetric equilibria will be considered unless otherwise noted.

[11] For the issuer market to be fully covered, the net utility of the consumer located at x = ½ must be non-negative for the monopolistic interchange fee a^{bj}.

3.4.1 Single-Homing Consumers

When consumers are restricted to single-home, merchants will accept card i as long as $b_S \geq m_i$ as in the previous section. So Q_{Si} is determined by (3-2). Since the issuer market is fully covered, the number of consumers who choose card i is

$$Q_{B1} = x^* \quad \text{and} \quad Q_{B2} = 1 - x^*$$

where x^* is defined in (3-11).

The following proposition shows the symmetric equilibrium of the Hotelling competition when consumers single-home.

Proposition 3-5 If two differentiated card schemes compete à la Hotelling and consumers single-home,

(i) the symmetric equilibrium interchange fee is

$$a^{hs} = \begin{cases} c_I - \frac{1}{2}(\bar{b}_B + \underline{b}_B) & \text{if } \bar{b}_B - \underline{b}_B > 2(\underline{b}_B + \bar{b}_S - c) \\ \frac{1}{4}\left[2(\bar{b}_S - c_A) - 2(\bar{b}_B - c_I) - (\bar{b}_B - \underline{b}_B)\right] & \text{if } \bar{b}_B - \underline{b}_B \leq 2(\underline{b}_B + \bar{b}_S - c) \end{cases} \tag{3-12}$$

(ii) a^{hs} maximizes the weighted sum of total consumers' surplus and total merchants' surplus, $wTU_B^h + (1-w)TU_S^h$, where the weight for consumers' surplus is

$$w_1 = \frac{3(\bar{b}_B - \underline{b}_B) + 2(\underline{b}_B + \bar{b}_S - c)}{6(\bar{b}_B - \underline{b}_B) + 2(\underline{b}_B + \bar{b}_S - c)} \qquad \text{if } \bar{b}_B - \underline{b}_B \leq 2(\underline{b}_B + \bar{b}_S - c)$$

$$w_2 = \frac{2(\bar{b}_B - \underline{b}_B) + 4(\underline{b}_B + \bar{b}_S - c)}{3(\bar{b}_B - \underline{b}_B) + 8(\underline{b}_B + \bar{b}_S - c)} \qquad \text{if } \bar{b}_B - \underline{b}_B > 2(\underline{b}_B + \bar{b}_S - c)$$

Proof. (i) For a given a_2, card scheme 1 will set a_1 to maximize its transaction volume. The symmetric equilibrium interchange fee can be obtained from the first-order condition in which a_1 and a_2 are set to be equal to each other for symmetry:

$$a^* = \frac{1}{4}\left[2(\bar{b}_S - c_A) - 2(\bar{b}_B - c_I) - (\bar{b}_B - \underline{b}_B)\right]$$

For this fee to be an equilibrium, net benefit of the consumer at x = ½ must be nonnegative since the issuer market is assumed to be fully covered, which requires

$$\bar{b}_B - \frac{t}{2} - f(a^*) = \frac{1}{4}\left[2(\underline{b}_B + \bar{b}_S - c) - (\bar{b}_B - \underline{b}_B)\right] \geq 0$$

That is, a^* is an equilibrium interchange fee if $\bar{b}_B - \underline{b}_B \leq 2(\underline{b}_B + \bar{b}_S - c)$.

If $\bar{b}_B - \underline{b}_B > 2(\underline{b}_B + \bar{b}_S - c)$, the equilibrium interchange fee can be obtained by setting consumer's net benefit at x = ½ equal to zero:

$$a^{**} = c_I - \frac{1}{2}(\bar{b}_B + \underline{b}_B)$$

For a^{**} to be an equilibrium, it needs to be shown that the card schemes have no incentive to deviate from a^{**}. The transaction volume of card scheme 1 at a^{**} is

$$T_1(a^{**},a^{**}) = \frac{\bar{b}_B + \underline{b}_B + 2\bar{b}_S - 2c}{4(\bar{b}_S - \underline{b}_S)}$$

The right and left derivatives of scheme 1's profit at $a_1 = a^{**}$ are, respectively,

$$\lim_{\Delta a \to 0^+} \frac{T_1(a^{**}+\Delta a, a^{**}) - T_1(a^{**},a^{**})}{\Delta a} = \frac{2(\bar{b}_S + \underline{b}_B - c) - (\bar{b}_B - \underline{b}_B)}{4(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} < 0$$

$$\lim_{\Delta a \to 0^-} \frac{T_1(a^{**}+\Delta a, a^{**}) - T_1(a^{**},a^{**})}{\Delta a} = -\frac{(\underline{b}_B + \bar{b}_S - c)}{(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} < 0$$

So a^{**} is an equilibrium when $\bar{b}_B - \underline{b}_B > 2(\underline{b}_B + \bar{b}_S - c)$. Note that $a^* = a^{**}$ when $\bar{b}_B - \underline{b}_B = 2(\underline{b}_B + \bar{b}_S - c)$.

(ii) First, note that Q_{Bi} = ½ at symmetric equilibrium since the market is fully covered. The weighted sum of total consumers' surplus and merchants' surplus for scheme 1 is

$$\begin{aligned} wTU_B^h + (1-w)TU_S^h &= w\left(\int_0^{½} U_{B1}dx + \int_{½}^1 U_{B2}dx\right) + \frac{1-w}{2}\sum_{i=1}^{2}\int_m^{\bar{b}_S} Q_{Si}dm \\ &= \frac{(\bar{b}_S - c_A - a)\left[w(3\bar{b}_B + \underline{b}_B - 4c_I + 4a) + 2(1-w)(\bar{b}_S - c_A - a)\right]}{4(\bar{b}_S - \underline{b}_S)} \end{aligned} \tag{3-13}$$

The optimal interchange fee that maximizes this weighted surplus is

$$a_w^* = \frac{4(2w-1)(\bar{b}_S - c_A) - w(3\bar{b}_B + \underline{b}_B - 4c_I)}{4(3w-1)} \tag{3-14}$$

The size of the weight can be obtained by setting $a^{hs} = a_w^*$, which is

$$w_1 = \frac{3(\bar{b}_B - \underline{b}_B) + 2(\underline{b}_B + \bar{b}_S - c)}{6(\bar{b}_B - \underline{b}_B) + 2(\underline{b}_B + \bar{b}_S - c)} \qquad \text{if } \bar{b}_B - \underline{b}_B \le 2(\underline{b}_B + \bar{b}_S - c)$$

$$w_2 = \frac{2(\bar{b}_B - \underline{b}_B) + 4(\underline{b}_B + \bar{b}_S - c)}{3(\bar{b}_B - \underline{b}_B) + 8(\underline{b}_B + \bar{b}_S - c)} \qquad \text{if } \bar{b}_B - \underline{b}_B > 2(\underline{b}_B + \bar{b}_S - c)$$

Note that $w_1 = w_2 = 4/7$ if $\bar{b}_B - \underline{b}_B = 2(\underline{b}_B + \bar{b}_S - c)$. *Q.E.D.*

When the card schemes compete à la Hotelling, they have some monopoly power over the consumers. So unlike the Bertrand competition case, they do not need to set the interchange fee so high as to maximize total consumers' surplus. While the weight for consumers' surplus (w) in Bertrand competition is equal to 1, the weight in Hotelling competition ranges between 4/7 and 1. If $(\bar{b}_B - \underline{b}_B) = 2(\underline{b}_B + \bar{b}_S - c)$, the weight is equal to 4/7. It becomes close to one as $\bar{b}_B - \underline{b}_B$ approaches zero. Note that $\bar{b}_B - \underline{b}_B$ is equal to the transportation cost t. As in a standard Hotelling model, the monopoly power of a card scheme weakens as t becomes smaller. Therefore, the card scheme will set the interchange fee so as to maximize total consumers' surplus when the transportation cost becomes zero.

The following proposition shows the monopoly interchange fee in the Hotelling model also maximizes the social welfare as in the Bertrand model.

Proposition 3-6 If the two differentiated card schemes are jointly owned and consumers single-home,

(i) the joint entity will set the interchange fee equal to

$$a^{hj} = c_I - \frac{1}{2}(\bar{b}_B + \underline{b}_B)$$

(ii) a^{hj} maximizes the sum of the total consumers' surplus and the total merchants' surplus.

Proof. (i) I will prove this proposition in two cases: (a) when the joint entity sets the same interchange fees for scheme 1 and 2, and (b) when it sets two different fees (price discrimination).

(a) When the joint entity sets the same interchange fees for both schemes, the joint transaction volume is

$$T_M(a,a) = Q_{B1}Q_{S1} + Q_{B2}Q_{S2} = \frac{\bar{b}_S - c_A - a}{\bar{b}_S - \underline{b}_S}$$

where $Q_{B1} = Q_{B2} = ½$ since the issuer market is assumed to be fully covered.

Note that T_M is decreasing in a, which implies that the optimal a is the minimum possible level that keeps the issuer market covered. This fee can be obtained by setting the consumer's net benefit at $x = ½$ equal to zero, which is a^{hj}.

(b) Now suppose the joint entity tries a price discrimination by setting $a_1 = a^{hj} + \Delta a$ and $a_2 = a^{hj} - \Delta a$, $\Delta a > 0$. The joint transaction volume when it charges same fee, a^{hj}, is

$$T_M(a^{hj}, a^{hj}) = \frac{2(\bar{b}_S + \underline{b}_B - c) + (\bar{b}_B - \underline{b}_B)}{2(\bar{b}_S - \underline{b}_S)}$$

while the joint transaction volume of the price discrimination is

$$T_M(a^{hj} + \Delta a, a^{hj} - \Delta a) = \frac{2(\bar{b}_S + \underline{b}_B - c) + (\bar{b}_B - \underline{b}_B) - 2\Delta a}{2(\bar{b}_S - \underline{b}_S)}$$

It is not beneficial to engage in price discrimination since

$$T_M(a^{hj} + \Delta a, a^{hj} - \Delta a) - T_M(a^{hj}, a^{hj}) = -\frac{\Delta a}{\bar{b}_S - \underline{b}_S} < 0$$

(ii) Since $Q_{B1} = Q_{B2} = ½$ at full-cover market equilibrium, the sum of total consumers' surplus and total merchants' surplus is

$$\begin{aligned} TU_B^h + TU_S^h &= \int_0^{½} U_{B1} dx + \int_{½}^{1} U_{B2} dx + \sum_{i=1}^{2} \frac{1}{2} \int_m^{\bar{b}_S} Q_{Si} dm \\ &= \frac{(\bar{b}_S - c_A - a)(3\bar{b}_B + \underline{b}_B - 4c_I + 2\bar{b}_S - 2c_A + 2a)}{4(\bar{b}_S - \underline{b}_S)} \end{aligned}$$

The optimal a that maximizes social welfare is[12]

$$a_U^* = c_I - \frac{1}{4}\left(3\bar{b}_B + \underline{b}_B\right)$$

[12] The fee is equivalent to a_w^* in (3-14) when $w = ½$.

Note that the market is not fully covered at a_U^* since $a_U^* < a^{hj}$. In other words, a_U^* is not feasible. Therefore, a^{hj} maximizes the sum of the total consumers' surplus and the total merchants' surplus when the market is fully covered. *Q.E.D.*

Note that $a^{hs} = a^{hj}$ if $\bar{b}_B - \underline{b}_B \geq 2(\underline{b}_B + \bar{b}_S - c)$ and $a^{hs} > a^{hj}$ if $\bar{b}_B - \underline{b}_B < 2(\underline{b}_B + \bar{b}_S - c)$. As in the Bertrand competition case, competition does not lower the equilibrium interchange fee nor increase social welfare when the card schemes compete à la Hotelling and consumers single-home.

3.4.2 Multi-Homing Consumers

If consumers are allowed to multi-home, they will hold card *i* as long as $b_{Bi} > f_i$. So the number of consumers who hold card *i* is the same as (3-5). If the issuer market is fully covered and the merchants accept both cards, the critical consumer who is indifferent between card 1 and 2 is obtained by solving $\bar{b}_B - tx - f_1 = \bar{b}_B - t(1-x) - f_2$, which is

$$x^* = \frac{1}{2} + \frac{f_2 - f_1}{2t} = \frac{1}{2} + \frac{a_1 - a_2}{2(\bar{b}_B - \underline{b}_B)}$$

The number of consumers who use card *i* if merchants accept both cards is

$Q_{b1} = x^*$, and $Q_{b2} = 1 - x^*$

Lemma 3-1 If $a_i > a_j$, merchants accept either card *j* only or both cards, i.e., no merchant will accept card *i* only.

Proof. Without loss of generality, suppose $a_1 > a_2$. The critical merchant that is indifferent between accepting card 1 only and accepting card 2 only can be obtained by setting $U_{S1} = U_{S2}$, where U_{Si} is defined in (3.6):

$$b_S^* = \bar{b}_B - c_I + c_A + a_1 + a_2$$

Merchants with low b_S will be more sensitive to the merchant fee, while merchants with high b_S will care more about the number of consumers who use the card. So if $b_S > b_S^*$, the merchant prefers card 1 to card 2 and vice versa.

The critical merchant that is indifferent between accepting card *i* only and accepting both cards can be obtained by setting $U_{Sb} = U_{Si}$, where

$$U_{Sb} = (bs - m_1)Q_{b1} + (bs - m_2)Q_{b2}$$

Let b_{Si}^* be the critical merchant. That is,

$$b_{Si}^* = \frac{(a_j - a_i)(a_i + a_j + \bar{b}_B - \underline{b}_B) - 2a_i(a_j + \underline{b}_B + c_A - c_I) + 2c_A(c_I - \underline{b}_B)}{2(c_I - a_i - \underline{b}_B)}$$

If $b_S > b_{Si}^*$, accepting both cards is more profitable than accepting only card *i* since merchants with high b_S care more about the transaction volume. The difference between b_S^* and b_{Si}^* is

$$b_S^* - b_{Si}^* = \frac{a_1^2 + a_2^2 + (a_1 + a_2)(\bar{b}_B + \underline{b}_B - 2c_I) + 2(\bar{b}_B - c_I)(\underline{b}_B - c_I)}{2(c_I - a_i - \underline{b}_B)}$$

Note that the numerator is independent of i and the denominator is positive.[13] Since $a_1 > a_2$, $b_S^* > b_{S2}^* > b_{S1}^*$ if the numerator is positive, and $b_S^* < b_{S2}^* < b_{S1}^*$ if the numerator is negative. Note also that b_S^* is larger than m_i since

$$b_S^* - m_i = \bar{b}_B - f_j > 0, \ i \neq j,$$

and b_{S1}^* is smaller than m_1 since

$$b_{S1}^* - m_1 = -\frac{(a_1 - a_2)\left[(\bar{b}_B - \underline{b}_B) - (a_1 - a_2)\right]}{2(f_1 - \underline{b}_B)} < 0$$

which implies $b_S^* > b_{S2}^* > b_{S1}^*$. Note that the difference between two interchange fees, which is same as the difference between two cardholder fees, cannot exceed the difference between $\bar{b}_B$ and $\underline{b}_B$ since $\underline{b}_B < f_i < \bar{b}_B$.

As is shown in Figure 3-2, merchants will accept card 2 only if $b_S \in [m_2, b_{S2}^*)$, and accept both cards if $b_S \in [b_{S2}^*, \bar{b}_S]$.[14] *Q.E.D.*

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[13] $c_I - a_i - \underline{b}_B > 0$ since it is equal to $f_i - \underline{b}_B$ and the cardholder fee must be higher than $\underline{b}_B$.

[14] Since $b_{Si}^* - m_j = \dfrac{(a_j - a_i)\left[(\bar{b}_B + \underline{b}_B) - (f_1 + f_2)\right]}{2(f_i - \underline{b}_B)}$, $b_{S1}^* < m_2$ ($b_{S2}^* > m_1$) if and only if $\bar{b}_B + \underline{b}_B > f_1 + f_2$.

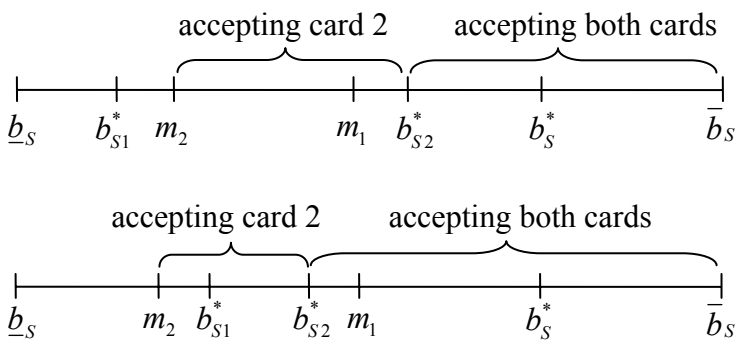

Figure 3-2. Merchants' acceptance decision when $a_1 > a_2$ ($m_1 > m_2$)

Based on Lemma 3-1, the number of merchants that accept card i is

$$Q_{Si} = \begin{cases} \dfrac{\overline{b}_S - b^*_{Sj}}{\overline{b}_S - \underline{b}_S} & \text{if } a_i > a_j \ (m_i > m_j) \\ \dfrac{\overline{b}_S - m_i}{\overline{b}_S - \underline{b}_S} & \text{if } a_i \le a_j \ (m_i \le m_j) \end{cases}$$

Let Q_{ai} be the number of merchants that accept card i only, and Q_{Sb} be the number of merchants that accept both cards. That is,

$$Q_{ai} = \begin{cases} Q_{Si} - Q_{Sj} & \text{if } a_i \le a_j \ (m_i \le m_j) \\ 0 & \text{if } a_i > a_j \ (m_i > m_j) \end{cases}$$

$$Q_{Sb} = Q_{Si} \qquad \text{where } a_i > a_j \ (m_i > m_j)$$

The following proposition summarizes the equilibrium interchange fee of the Hotelling competition with multi-homing consumers.

Proposition 3-7 If consumers can multi-home and card schemes compete à la Hotelling,

(i) the symmetric equilibrium interchange fee is

$$a^{hm} = \begin{cases} \frac{1}{2}\left(\bar{b}_S - c_A + c_I - \underline{b}_B - A\right) & \text{if } \bar{b}_B - \underline{b}_B \le 2(\underline{b}_B + \bar{b}_S - c) \\ c_I - \frac{1}{2}(\bar{b}_B + \underline{b}_B) & \text{if } \bar{b}_B - \underline{b}_B > 2(\underline{b}_B + \bar{b}_S - c) \end{cases}$$

where $A = \sqrt{2(\bar{b}_B - \underline{b}_B)^2 + (\underline{b}_B + \bar{b}_S - c)^2}$

(ii) $a^{hm} \le a^{hs}$, where the equality holds when $\bar{b}_B - \underline{b}_B \ge 2(\underline{b}_B + \bar{b}_S - c)$

(iii) a^{hm} maximizes the weighted sum of total consumers' surplus and total merchants' surplus, $wTU_B^{hs} + (1-w)TU_S^{hs}$, where the weight for the consumers' surplus is

$$w_1 = \frac{2(\underline{b}_B + \bar{b}_S - c) + 2A}{2(\underline{b}_B + \bar{b}_S - c) - 3(\bar{b}_B - \underline{b}_B) + 6A} \qquad \text{if } \bar{b}_B - \underline{b}_B < 2(\underline{b}_B + \bar{b}_S - c)$$

$$w_2 = \frac{2(\bar{b}_B - \underline{b}_B) + 4(\underline{b}_B + \bar{b}_S - c)}{3(\bar{b}_B - \underline{b}_B) + 8(\underline{b}_B + \bar{b}_S - c)} \qquad \text{if } \bar{b}_B - \underline{b}_B \ge 2(\underline{b}_B + \bar{b}_S - c)$$

Proof. (i) Without loss of generality, suppose $a_1 \ge a_2$ ($m_1 \ge m_2$). Then scheme 1 and 2's transaction volumes are, respectively,

$$T_1(a_1; a_2) = Q_{b1}Q_{Sb}$$
$$T_2(a_2; a_1) = Q_{B2}Q_{a2} + Q_{b2}Q_{Sb}$$

The symmetric equilibrium can be obtained by taking derivative of T_i w.r.t. a_i at $a_i = a_j$, which yields

$$a^* = \frac{1}{2}\left(\bar{b}_S - c_A + c_I - \underline{b}_B - \sqrt{2(\bar{b}_B - \underline{b}_B)^2 + (\underline{b}_B + \bar{b}_S - c)^2}\right)$$

At the symmetric equilibrium, all merchants accept both cards (i.e., $Q_{ai} = 0$) and $Q_{b1} = Q_{b2} = ½$. So the transaction volume of each card scheme is

$$T_i(a^*; a^*) = \frac{\underline{b}_B + \bar{b}_S - c + \sqrt{2(\bar{b}_B - \underline{b}_B)^2 + (\underline{b}_B + \bar{b}_S - c)^2}}{4(\bar{b}_S - \underline{b}_S)}$$

To see the card schemes do not have an incentive to deviate from a^*, suppose scheme 1 changes a_1 by Δa. Then the transaction volume of the scheme becomes

$$T_1(a^* + \Delta a; a^*) = \begin{cases} Q_{b1}Q_{Sb} & \text{if } \Delta a > 0 \\ Q_{B2}Q_{a2} + Q_{b2}Q_{Sb} & \text{if } \Delta a < 0 \end{cases}$$

The transaction volume does not increase by changing a since

$$T_1(a^* + \Delta a; a^*) - T_1(a^*; a^*)$$
$$= \begin{cases} -\dfrac{\Delta a^2(2(\bar{b}_B - \underline{b}_B) + \Delta a)}{2(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)(A - (\bar{b}_S + \underline{b}_B - c))} < 0 & \text{if } \Delta a > 0 \\ -\dfrac{\Delta a^2(3A - 2(\bar{b}_B - \underline{b}_B) - (\bar{b}_S + \underline{b}_B - c) - 3\Delta a)}{2(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)(A - (\bar{b}_S + \underline{b}_B - c) - 2\Delta a)} < 0 & \text{if } \Delta a < 0 \end{cases}$$

So the card schemes do not have an incentive to deviate from a^*.

For a^* to be an equilibrium, the issuer market must be fully covered at equilibrium. The net benefit of the consumer located at $x = ½$ is

$$\bar{b}_B - \frac{1}{2}t - (c_I - a^*) = \frac{1}{2}\left[(\bar{b}_B + \bar{b}_S - c) - \sqrt{2(\bar{b}_B - \underline{b}_B)^2 + (\underline{b}_B + \bar{b}_S - c)^2}\right]$$

This is nonnegative if and only if $\bar{b}_B - \underline{b}_B \le 2(\underline{b}_B + \bar{b}_S - c)$ since

$$(\bar{b}_B + \bar{b}_S - c)^2 - \left[2(\bar{b}_B - \underline{b}_B)^2 + (\underline{b}_B + \bar{b}_S - c)^2\right] = (\bar{b}_B - \underline{b}_B)\left[2(\underline{b}_B + \bar{b}_S - c) - (\bar{b}_B - \underline{b}_B)\right].$$

If $\bar{b}_B - \underline{b}_B > 2(\underline{b}_B + \bar{b}_S - c)$, as in the single-homing case, the equilibrium interchange fee can be obtained by setting consumer's net benefit at x = ½ equal to zero, which is

$$a^{**} = c_I - \frac{1}{2}(\bar{b}_B + \underline{b}_B)$$

Note that, as in the single-homing case, $a^* = a^{**}$ when $\bar{b}_B - \underline{b}_B = 2(\underline{b}_B + \bar{b}_S - c)$.

(ii) If $\bar{b}_B - \underline{b}_B < 2(\underline{b}_B + \bar{b}_S - c)$, $a^{hs} = ¼[2(\bar{b}_S - c_A) - 2(\bar{b}_B - c_I) - (\bar{b}_B - \underline{b}_B)]$ and the difference between the two equilibrium fees is

$$\begin{aligned} a^{hs} - a^{hm} &= \frac{1}{4}\left(2\sqrt{2(\bar{b}_B - \underline{b}_B)^2 + (\underline{b}_B + \bar{b}_S - l)^2} - 3(\bar{b}_B - \underline{b}_B)\right) \\ &> \frac{1}{4}\left(2\sqrt{2(\bar{b}_B - \underline{b}_B)^2 + [(1/2)(\bar{b}_B - \underline{b}_B)]^2} - 3(\bar{b}_B - \underline{b}_B)\right) = 0 \end{aligned}$$

If $\bar{b}_B - \underline{b}_B \ge 2(\underline{b}_B + \bar{b}_S - c)$, both a^{hs} and a^{hm} are equal to $c_I - ½(\bar{b}_B + \underline{b}_B)$.

(iii) The weighted sum of total consumers' surplus and total merchants' surplus is the same as (3-13), hence the optimal interchange fee maximizing the weighted surplus is

also the same as (3-14). The level of the weight can be obtained by setting $a^{hm} = a_w^*$, which is

$$w_1 = \frac{2(\underline{b}_B + \bar{b}_S - c) + 2A}{2(\underline{b}_B + \bar{b}_S - c) - 3(\bar{b}_B - \underline{b}_B) + 6A} \qquad \text{if } \bar{b}_B - \underline{b}_B < 2(\underline{b}_B + \bar{b}_S - c)$$

$$w_2 = \frac{2(\bar{b}_B - \underline{b}_B) + 4(\underline{b}_B + \bar{b}_S - c)}{3(\bar{b}_B - \underline{b}_B) + 8(\underline{b}_B + \bar{b}_S - c)} \qquad \text{if } \bar{b}_B - \underline{b}_B \geq 2(\underline{b}_B + \bar{b}_S - c)$$

Note that, as in the single-homing case, $w_1 = w_2 = 4/7$ if $\bar{b}_B - \underline{b}_B = 2(\underline{b}_B + \bar{b}_S - c)$.

Q.E.D.

When consumers multi-home, the equilibrium interchange fee is lower than that of the single-homing case. But unlike the Bertrand competition case in which card schemes set the interchange fee so as to maximize the merchants' surplus, the card schemes do not lower the fee enough. In the Bertrand competition with multi-homing consumers, merchants accept only one card if the merchant fees set by two card schemes are different. Therefore, a card scheme can maximize its transaction volume by attracting as many merchants as possible. In Hotelling competition, however, each card scheme has its own patronizing consumers since it provides differentiated service. This weakens merchant resistance, which forces many merchants to accept both cards.[15] Therefore, card schemes do not need to provide maximum surplus to the merchants.

If the card schemes are jointly owned, the result will be the same as in the single-homing case since the joint entity will split the issuer market so that each consumer holds

[15] See Rochet and Tirole (2002) for a discussion of merchant resistance.

only one card at equilibrium.

Figure 3-3 shows the relationship of various equilibrium interchange fees and welfare, which is drawn for the case of $\bar{b}_B - \underline{b}_B < 2(\underline{b}_B + \bar{b}_S - c)$.[16] The left side of a^{hj} is not feasible since the market cannot be fully covered. As is clear from the figure, competition not only increases the equilibrium interchange fee but also lowers social welfare. It also shows that allowing consumers to multi-home increases social welfare in the Hotelling competition case, although it lowers total consumers' surplus.

3.5 Proprietary System with Single-Homing Consumers

The analysis of the previous sections has been restricted to the competition between non-proprietary card schemes that set interchange fees and let the cardholder fees and

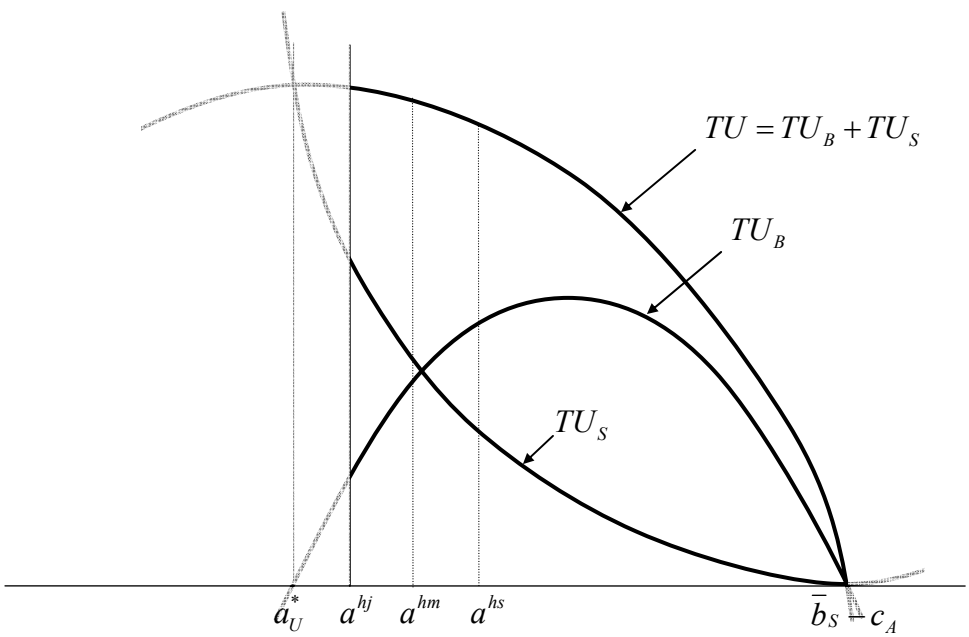

Figure 3-3 Welfare and interchange fees of Hotelling competition when $\bar{b}_B - \underline{b}_B < 2(\underline{b}_B + \bar{b}_S - c)$

[16] When $\bar{b}_B - \underline{b}_B \geq 2(\underline{b}_B + \bar{b}_S - c)$, $a^{hj} = a^{hm} = a^{hs}$.

merchant fees be determined by issuers and acquirers, respectively. Another type of credit card scheme, a proprietary scheme, serves as both issuer and acquirer. It sets the cardholder fee and merchant fee directly, so there is no need for an interchange fee.[17]

3.5.1 Competition between Identical Card Schemes

One of the features of the proprietary card scheme is that competition may not only alter the price structure but may also change the price level. In the previous sections, the sum of the cardholder fee and merchant fee does not change even after the introduction of competition between card schemes.[18] When a card scheme sets both the cardholder fee and the merchant fee, it may change one of the fees more than the other since the effects of competition on two sides are not equivalent.

To see how competition affects the equilibrium fees of the proprietary card scheme, the equilibrium of the monopoly case will be presented first. For the sake of simplicity, only the case of single-homing consumers will be considered.

When the monopoly proprietary card scheme sets f and m, the quasi-demand functions of consumers and merchants are

$$Q_B = \frac{\bar{b}_B - f}{\bar{b}_B - \underline{b}_B} \quad \text{and} \quad Q_S = \frac{\bar{b}_S - m}{\bar{b}_S - \underline{b}_S}$$

[17] In the United States, Discover and American Express are examples of this type of card scheme.

[18] This feature of the non-proprietary scheme requires an assumption of perfect competition among issuers and acquirers. If the perfect competition assumption is removed, competition may alter the price level as well as the price structure in the non-proprietary card scheme model.

and the profit of the scheme is[19]

$$\pi = (f + m - c)Q_B Q_S$$

From the first order condition for the profit maximization problem, one can obtain the following equilibrium cardholder fee and merchant fee:

$$\begin{aligned} f^M &= \frac{1}{3}\left(2\bar{b}_B - \bar{b}_S + c\right) \\ m^M &= \frac{1}{3}\left(2\bar{b}_S - \bar{b}_B + c\right) \end{aligned} \tag{3-15}$$

The following lemma shows that there does not exist a pure strategy equilibrium when two identical proprietary card schemes compete with each other.

Lemma 3-2 If two identical proprietary card schemes compete in a Bertrand fashion, no pure strategy equilibrium exists.

Proof. Note first that any set of prices that generates positive profit cannot be a symmetric equilibrium. If an equilibrium set of prices is (f, m) such that $f + m > c$, a card scheme can increase profit by lowering the cardholder fee marginally while keeping the merchant fee since the scheme can attract all consumers instead of sharing them with the other scheme.

Second, a set of prices which satisfies $f + m = c$ cannot be an equilibrium, either. To see this, let the equilibrium set of prices is (f, m) such that $f + m = c$. Without loss of

[19] The proprietary card scheme maximizes profits instead of card transaction volume.

generality, suppose scheme 2 lower the cardholder fee by d and raise the merchant fee by e, where $e > d > 0$. As in the Bertrand competition case of the previous section, consumers whose b_B is higher than b_B^* will choose card 1 while consumers with b_B lower than b_B^* will choose card 2, in which b_B^* is defined as

$$b_B^* = f - d + \frac{(\bar{b}_S - m)d}{e}$$

The quasi-demands of consumers and merchants for scheme 2's card service are

$$Q_{B2} = \frac{b_B^* - (f - d)}{\bar{b}_B - \underline{b}_B} = \frac{d(\bar{b}_S - m)}{e(\bar{b}_B - \underline{b}_B)}$$

$$Q_{S2} = \frac{\bar{b}_S - (m + e)}{\bar{b}_S - \underline{b}_S}$$

The profit of the scheme 2 is

$$\pi_2 = \frac{d(\bar{b}_S - m)(\bar{b}_S - m - e)(f + m - c + e - d)}{e(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} > 0$$

Since the scheme 2 can make positive profits by deviating from (f, m), it cannot be an equilibrium set of prices. *Q.E.D.*

The above lemma does not exclude the possibility of a mixed strategy equilibrium or asymmetric equilibrium. As the following proposition shows, however, competition

cannot improve social welfare since the monopolistic equilibrium set of prices maximizes social welfare.

Proposition 3-8 The equilibrium prices set by the monopolistic proprietary card scheme in the Bertrand model maximize Marshallian social welfare which is defined as the sum of cardholders' surplus, merchants' surplus and card schemes' profits.

Proof. Marshallian social welfare is defined as follows:

$$\begin{aligned} W &= TU_B + TU_S + \pi = \int_f^{\bar{b}_B} Q_B Q_S df + \int_m^{\bar{b}_S} Q_B Q_S dm + (f + m - c) Q_B Q_S \\ &= \frac{(\bar{b}_B - f)(\bar{b}_S - m)(\bar{b}_B + \bar{b}_S + f + m - 2c)}{2(\bar{b}_B - \underline{b}_B)(\bar{b}_S - \underline{b}_S)} \end{aligned} \tag{3-16}$$

The optimal prices that maximize welfare are

$$f^W = \frac{1}{3}\left(2\bar{b}_B - \bar{b}_S + c\right)$$

$$m^W = \frac{1}{3}\left(2\bar{b}_S - \bar{b}_B + c\right)$$

These are same as f^M and m^M, respectively. *Q.E.D.*

For comparison with other models, one may derive a set of Ramsey-optimal prices which is the solution of the following problem:

$$\underset{f,\,m}{Max}\ TU_B + TU_S \quad s.t.\ f + m = c$$

From the first-order condition of this maximization problem, the following Ramsey-optimal prices can be obtained:

$$f^R = \frac{1}{2}\left(\bar{b}_B - \bar{b}_S + c\right)$$

$$m^R = \frac{1}{2}\left(\bar{b}_S - \bar{b}_B + c\right)$$

The differences between two different optimal prices are same for both cardholder and merchant fees. That is,

$$f^W - f^R = m^W - m^R = \frac{1}{6}\left(\bar{b}_B + \bar{b}_S - c\right) > 0$$

Ramsey-optimal prices are lower than the prices that maximize Marshallian welfare since the former does not allow profits of the firms while the latter puts the same weight on profits as on customers' surplus. If social welfare is measured by the Ramsey standard, competition may increase the social welfare as long as competition lowers both cardholder and merchant fees.

It is also worth noting that the Ramsey-optimal fees of the proprietary scheme is equal to the consumer and merchant fees that are determined by the monopoly interchange fee of the nonproprietary scheme, i.e., $f^R = c_I - a^{bj}$ and $m^R = c_A + a^{bj}$, which confirms that a^{bj} maximizes both Marshallian and Ramsey social welfares.

3.5.2 Competition between Differentiated Card Schemes

When two proprietary card schemes are differentiated and compete à la Hotelling, the critical consumer, x^*, who is indifferent between card 1 and 2 is determined in the same way as (3-11) except that the card schemes set f_i and m_i instead of a_i:

$$x^* = \frac{(\bar{b}_B - f_1)(\bar{b}_S - m_1) + (f_2 - \underline{b}_B)(\bar{b}_S - m_2)}{(\bar{b}_B - \underline{b}_B)(2\bar{b}_S - m_1 - m_2)}$$

If the issuer market is not fully covered, each card scheme has a monopoly power over its own consumers, so the equilibrium set of prices will be same as f^M and m^M in (3-15).[20] In order to obtain a non-trivial result, suppose the issuer market is fully covered at equilibrium as in the previous section. This requires the following assumption:[21]

$$\bar{b}_B - \underline{b}_B \leq 2(\underline{b}_B + \bar{b}_S - c)$$

Using the first-order conditions, one can derive the best response functions of card schemes from which the following equilibrium prices can be obtained:

$$f^{phs} = \frac{1}{4}\left(5\bar{b}_B - 3\underline{b}_B - 2\bar{b}_S + 2c\right)$$

$$m^{phs} = \frac{1}{4}\left(2\bar{b}_S - \bar{b}_B - \underline{b}_B + 2c\right)$$

---

[20] Since merchants accept card i as long as $b_S \geq m_i$, the existence of competing card schemes does not affect the equilibrium merchant fee.

[21] For the issuer market to be fully covered, the net utility of the consumer located at x = ½ must be non-negative for the monopoly prices, f^M and m^M.

If two schemes collude and act like a monopolist, the joint entity will set the cardholder fee such that the critical consumer who is located at x = ½ is indifferent between using card and cash as well as between card 1 and 2. Since the transportation cost is assumed to be equal to $\bar{b}_B - \underline{b}_B$, the cardholder fee that will be set by the joint entity is

$$f^{phj} = \frac{1}{2}\left(\bar{b}_B + \underline{b}_B\right) \tag{3-17}$$

Given this cardholder fee, the joint profit can be rewritten as follows:

$$\pi = (f^{phj} + m - c)(Q_{B1}Q_{S1} + Q_{B2}Q_{S2}) = \frac{(\bar{b}_S - m)(\bar{b}_B + \underline{b}_B - 2c + 2m)}{2(\bar{b}_S - \underline{b}_S)}$$

The optimal merchant fee that maximizes this profit function is

$$m^{phj} = \frac{1}{4}\left(2\bar{b}_S - \bar{b}_B - \underline{b}_B + 2c\right)$$

Note that the merchant fee set by the joint entity is the same as the competitive merchant fee, i.e., $m^{phs} = m^{phj}$. This is because the issuer market is fully covered in both cases and the multi-homing merchants will accept any card as long as the merchant fee is less than b_S.

Proposition 3-9 When the two proprietary card schemes are differentiated in a Hotelling fashion, competition does not improve Marshallian social welfare.

Proof. If two card schemes charge same prices and the issuer market is fully covered, Marshallian social welfare is

$$\begin{aligned} W &= TU_B + TU_S + \pi_1 + \pi_2 \\ &= \int_0^{1/2} U_{B1} dx + \int^1 U_{B2} dx + \sum_{i=1}^{2} \frac{1}{2} \int_m^{\bar{b}_S} Q_{Si} dm + \sum_{i=1}^{2} (f + m - c) Q_{Bi} Q_{Si} \\ &= \frac{(3\bar{b}_B + \underline{b}_B - 4f)(\bar{b}_S - m)}{4(\bar{b}_S - \underline{b}_S)} + \frac{(\bar{b}_S - m)^2}{2(\bar{b}_S - \underline{b}_S)} + \frac{(f + m - c)(\bar{b}_S - m)}{(\bar{b}_S - \underline{b}_S)} \\ &= \frac{(\bar{b}_S - m)[3\bar{b}_B + \underline{b}_B + 2(\bar{b}_S - 2c + m)]}{4(\bar{b}_S - \underline{b}_S)} \end{aligned} \tag{3-18}$$

Note that social welfare is independent of *f*. That is, the cardholder fee has no effect on the welfare as long as the fee is low enough for the issuer market to be fully covered. An increase in the cardholder fee just transfers surplus from consumers to the card schemes.

Since the social welfare is only affected by the merchant fee and the equilibrium merchant fees of the competitive case and the monopoly case are equal to each other, competition does not improve the social welfare. *Q.E.D.*

The cardholder fee cannot affect social welfare since the issuer market is fully covered, i.e., the consumers' quasi-demand is fixed regardless of the cardholder fee. When the cardholder fee changes, it does not affect the demand of the issuer market, but

does affect consumers' surplus and card schemes' profits. Since an increase (decrease) in consumers' surplus is exactly offset by a decrease (increase) in the profits of the card schemes, the social welfare remains the same even though total customers' surplus, which is the sum of the consumers' surplus and the merchants' surplus, may increase due to the competition between the card schemes.

Note that the merchant fee that maximizes the social welfare represented by (3-18) is

$$m^{phw} = c - \frac{1}{4}\left(3\bar{b}_B + \underline{b}_B\right) \tag{3-19}$$

This merchant fee is not feasible since the card schemes' profits are negative at this fee. For the issuer market to be fully covered, the cardholder fee cannot exceed f^{phj} in (3-17). So the maximum possible profit margin when the card scheme charges f^{phj} and m^{phw} is

$$f^{phj} + m^{phw} - c = -\frac{1}{4}(\bar{b}_B - \underline{b}_B) < 0$$

When the first-best price is not feasible, one can think of the second-best price, or Ramsey price, which is the optimal price among the feasible prices. The Ramsey prices can be obtained by setting the cardholder fee equal to f^{phj} and the merchant fee equal to $c - f^{phj}$, i.e.,[22]

[22] These are the fees implied by the monopoly interchange fee a^{hj} in the nonproprietary Hotelling model, i.e., $f^{phr} = c_I - a^{hj}$ and $m^{phr} = c_A + a^{hj}$.

$$f^{phr} = \frac{1}{2}\left(\bar{b}_B + \underline{b}_B\right)$$

$$m^{phr} = c - \frac{1}{2}\left(\bar{b}_B + \underline{b}_B\right)$$

The merchant fee determined by the market—regardless whether it is monopoly or duopoly—is too high from the social point of view since the difference between the equilibrium merchant fee and Ramsey-optimal fee is

$$m^{phs} - m^{phr} = \frac{1}{4}\left((\bar{b}_B - \underline{b}_B) + 2(\bar{b}_S + \underline{b}_B - c)\right) > 0$$

3.6 Conclusion

This chapter shows the effects of competition in a two-sided market on the price structure and welfare using a formal model with various settings including single-homing vs. multi-homing consumers, Bertrand vs. Hotelling competition, and proprietary vs. non-proprietary card schemes. The effect of competition on the price structure depends on whether consumers single-home or multi-home since competing card schemes set lower prices for the single-homing side.

The most surprising result is that competition never improves social welfare regardless whether consumers single-home or multi-home, whether card schemes are identical or differentiated, or whether card schemes are proprietary or nonproprietary. In most cases, monopoly pricing maximizes Marshallian social welfare since the monopolist in a two-sided market can internalize indirect network externalities without bias to one side. The only exception is the case of the Hotelling model of the proprietary card scheme,

in which monopoly pricing dose not maximize the social welfare. But even in this case, competition does not improve Mashallian welfare.

The welfare effect of competition in the two-sided market may be different if the business stealing effect is introduced. Competing merchants may accept credit cards even if the merchant fees are higher than the direct benefit from the card service since accepting credit cards can attract card-using consumers. As is pointed by Rochet and Tirole (2002) and Guthrie and Wright (2005), the equilibrium interchange fees tend to be higher when there is a business stealing effect. Therefore, if the business stealing effect exists, monopoly pricing may not maximize the social welfare. And competition may improve social welfare if consumers multi-home and card schemes are nonproprietary and identical. If consumers single-home or card schemes compete à la Hotelling, however, competition may deteriorate social welfare since both competition and the business stealing effect tend to force the interchange fee upward.

Policy makers in many countries have investigated interchange fees and the rules set by the members of payment card systems, then moved to regulate card associations. Collective determination of the interchange fee and a lack of competition between card schemes are treated as main cause of the high interchange fee. Card schemes in some countries such as Australia, United Kingdom and South Korea have been required to lower their interchange fees or merchant fees. This chapter shows that high interchange fees or merchant fees may be a result of competition, not a result of the lack of competition. On the one hand, this implies that the interests of the regulatory authority and the card schemes can be aligned. That is, if lowering interchange fee or merchant fee increases merchant acceptance, both the social welfare and the card schemes' profits or

transaction volume can increase at the same time. On the other hand, it implies that two-sided markets should be regulated with discretion since, even though they may not be desirable, the outcomes of the market cannot be categorized as collusive or predatory actions − i.e., anticompetitive actions.

A possible extension of the model in this chapter, in addition to introducing the business stealing effect, lies in endogenizing the mechanism that determines single-homing or multi-homing of each side. One way to do it is, as most of other models do, introducing a fixed fee or a fixed cost of holding or accepting a card.

Although the model deals with the credit card industry, it can be easily extended to the other two-sided markets such as videogame consoles, shopping malls, telecommunications, and media industries.

CHAPTER 4
COMPETITION BETWEEN CARD ISSUERS WITH HETEROGENEOUS EXPENDITURE VOLUMES

4.1 Introduction

As credit and debit cards become an increasingly important part of the payment system, the card industry has drawn economists' attention. Theoretically, the credit card industry is analyzed as one of the typical examples of two-sided market. A two-sided (or multi-sided) market is a market in which two (or more) parties interact on a platform. The end-users enjoy indirect network externalities which increase as the size of the other side increases and cannot be internalized by themselves.[1] The platform enables the interaction by appropriately charging each side. The two largest credit card networks—MasterCard and Visa—use interchange fees to balance the demands of two sides. The interchange fee is a payment between the merchant's bank, known as the acquirer, and the consumer's bank, known as the issuer.

Another reason for the recent surge of interest in the credit card industry lies in government policies. Antitrust authorities around the world have questioned some business practices of the credit card networks. These include the collective determination of the interchange fee, the no-surcharge rule, and the honor-all-card rule.

[1] Rochet and Tirole (2004) point out that a necessary condition for a market to be two-sided is that the Coase theorem does not apply to the transaction between the two sides. For general introductions to the two-sided market, see Armstrong (2004), Roson (2005a), and Evans and Schmalensee (2005).

Because of its importance in theory and practice, the interchange fee has been the main topic in most of the literature on the credit card industry. To mitigate the complexity caused by two-sidedness of the market, however, these models make relatively simple assumptions on each side of the market. One of these simplifying assumptions is that cardholders have unit demand for all goods. The unit demand is a reasonable assumption in the analysis of many traditional markets—especially markets for durable goods such as houses and automobiles. But for the credit card industry, unit demand assumption is implausible since it implies cardholders' preferences are identical except for the credit card service.

The model presented here adopts a more plausible assumption that cardholders are heterogeneous in terms of the expenditure volume. Not only is this a more plausible assumption, it also makes possible a richer analysis on the competition on the issuer side of the market. The main finding is that the effects of a change in the variance of the expenditure volume on the equilibrium cardholder fees and profits are different for various cases of market coverage. As the variance of the expenditure volume increases, issuers' profits as well as the equilibrium cardholder fee decrease when the market is fully covered. When the market is locally monopolized, the profits increase while the cardholder fee remains the same as the variance increases. In case of the partial-cover market, the effect of an increase in the variance is mixed (i.e., the cardholder fee may increase or decrease as the variance increases).

The model also contains some new findings about the interchange fee. One of them is the neutrality of the interchange fee holds in the full-cover market even under the no-surcharge-rule. When the market is not fully covered, the neutrality does not hold since

there exist potential consumers that card issuers can attract. A simulation result also shows the possibility of the positive relationship between the interchange fee and the cardholder fee.

The first formal analysis of credit card industry in the context of two-sided market was provided by Baxter (1983). Although it is normative rather than positive, his model clearly shows that interchange fee is necessary to balance the demands of the two sides. It is only recently that more rigorous models were developed by economists as they started to pay attention to the two-sided market. Schmalensee (2002), Rochet and Tirole (2002), and Wright (2003a, 2003b, 2004) develop Baxter's idea in rigorous models with a single platform. Rochet and Tirole (2003) and Guthrie and Wright (2006) extend the models by allowing competition between platforms. The main focuses of these papers are how interchange fees are determined and how they are different from ordinary cartel price-fixing behavior. Although their models are more sophisticated than Baxter's, their treatment of each side—especially the issuer side—is relatively simple. For example, Schmalensee (2002) allows imperfect competition on both issuer and acquirer sides but the demands of each side are given, not derived.

Rochet and Tirole (2002) derive the demand for card service by endogenizing consumer behavior, but there is no difference between card-holding and card-using since they assume all consumers purchase the same amount of goods from each merchant. Further, by assuming identical merchants, their model cannot capture the trade-off between consumer demand and merchant demand caused by a change in the interchange fee. Wright (2002) allows heterogeneity among merchants, but also assumes unit demand by consumers.

Chapter 4 is organized as follows. The following section provides a basic model of the issuer market in the credit card industry and shows how the cardholder fee is determined in the various cases of market coverage. It also shows that the effects of a change in the variance of the expenditure volume are quite different for various cases of market coverage. Section 4.3 provides the determination of the interchange fee, also in different cases of market coverage. Section 4.4 provides other comparative statics and the results of the collusion between issuers. The last section summarizes the results and provides concluding remarks.

4.2 Equilibrium Cardholder Fee

4.2.1 The Model

Suppose there are two card issuers, $i = 1, 2$, associated with a single card scheme.[2] The issuers set cardholder fees, f_i, which can be negative, and the card scheme sets the interchange fee, a, which is a payment to card issuers from card acquirers. Merchants are not allowed to impose surcharges on consumers who pay with a card (i.e., the no-surcharge rule prevails).

Consumers or cardholders have the same valuation, b, for the card service but have different expenditure volumes, v, which are drawn with a positive density $g(v)$ over the interval $[\underline{v}, \bar{v}]$. It is assumed that each consumer spends the same amount at every merchant. That is, v can be interpreted as the purchasing amount from each merchant. So a consumer's total charging volume with the credit card is vQ_m, where Q_m is the number

[2] Visa and MasterCard are examples of this type of the credit card scheme. Competition between the card schemes is not an issue in this paper. Since many issuers issue both cards and most merchants accept both, they can be treated as one monopoly platform.

of merchants that accept the card. Consumers are also assumed to be distributed uniformly over the interval [0,1], where the issuers are located at two extremes.

The issuers compete à la Hotelling. A consumer located at x on the unit interval incurs a transportation cost of tx when she uses card 1 and $t(1-x)$ when using card 2.[3] The net utilities a consumer with v located at x receives from using card 1 and 2 are

$$\begin{aligned} u_1 &= v(b-f_1)Q_m - tx \\ u_2 &= v(b-f_2)Q_m - t(1-x) \end{aligned} \tag{4-1}$$

A consumer will choose to use card i if the following two conditions are satisfied:

$u_i \geq u_j$ (IR1), $i, j = 1,2, i \neq j$

$u_i \geq 0$ (IR2)

(IR1) requires the net utility from card i be at least as good as from card j, while (IR2) requires the net utility from card i be at least as good as from the other payment method, say cash. Note that the benefit of the card service, b, is measured relative to the benefit of using cash.

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[3] There are two types of transportation costs. One is the shipping cost which is proportional to the purchasing amount, and the other is the shopping cost which is a one-time cost and independent of the expenditure volume. In the credit card industry, both types of transaction costs exist. For example, average percentage rate (APR) for purchases is a shipping cost while APR for balance transfer is a shopping cost. For modeling simplicity, this paper assumes shopping costs only and no shipping costs. If one assumes shipping costs only, the expenditure volume plays no role in the model. To see this, suppose $u_1 = v(b - f_1 - tx)Q_m$ and $u_2 = v[b - f_2 - t(1-x)]Q_m$. Then the critical consumer who is indifferent between two cards is $x^* = \frac{1}{2} + \frac{f_2 - f_1}{2t}$, which is independent of v.

The critical consumers, x^*, who are indifferent between card 1 and 2 can be obtained by setting $u_1 = u_2$:[4]

$$x^*(f_1, f_2; v) = \frac{1}{2} + \frac{v(f_2 - f_1)Q_m}{2t} \tag{4-2}$$

Note that the locations of critical consumers varies as v changes if $f_1 \neq f_2$.

Consumers whose (IR2) condition is binding, x_i^*, are determined by setting $u_i = 0$. These consumers are indifferent between using card i and using cash:

$$\begin{aligned} x_1^*(f_1; v) &= \frac{v(b - f_1)Q_m}{t} \\ x_2^*(f_2; v) &= 1 - \frac{v(b - f_2)Q_m}{t} \end{aligned} \tag{4-3}$$

There may exist a consumer who is indifferent between using card 1 and 2, or not using any of the cards, i.e., both (IR1) and (IR2) conditions are binding. This consumer's expenditure volume, v^*, is determined by setting $x_1^* = x_2^*$:

$$v^*(f_1, f_2) = \frac{t}{(2b - f_1 - f_2)Q_m} \tag{4-4}$$

Depending on the size of v^* compared to $\bar{v}$ and $\underline{v}$, one can distinguish three regimes: full-cover market when $v^* \leq \underline{v}$, local monopoly when $v^* \geq \bar{v}$, and partial-cover market

[4] Consumers are assumed to use a single card or none at all, hence the possibility of multi-homing is excluded. In fact, there is no extra gain from multi-homing since the two issuers are in the same network.

when $\underline{v} < v^* < \bar{v}$. Figure 4-1 shows these three cases with the division of consumers in three parts. Consumers in area I and II use card 1 and 2, respectively. Consumers in area III choose not to use any of the cards.[5]

The consumers' demand for card i's service, q_i, is the sum of all consumers' expenditure volumes in area I or II. The demand functions of full-cover market are

$$\begin{aligned} q_1(f_1; f_2) &= \int_{\underline{v}}^{\bar{v}} v x^* g(v) dv \\ q_2(f_2; f_1) &= \int_{\underline{v}}^{\bar{v}} v(1 - x^*) g(v) dv \end{aligned} \tag{4-5}$$

In case of the local monopoly, the demand functions are

$$\begin{aligned} q_1(f_1) &= \int_{\underline{v}}^{\bar{v}} v x_1^* g(v) dv \\ q_2(f_2) &= \int_{\underline{v}}^{\bar{v}} v(1 - x_2^*) g(v) dv \end{aligned} \tag{4-6}$$

Last, the demand functions of partial-cover market are

$$\begin{aligned} q_1(f_1; f_2) &= \int_{\underline{v}}^{v^*} v x_1^* g(v) dv + \int_{v^*}^{\bar{v}} v x^* g(v) dv \\ q_2(f_2; f_1) &= \int_{\underline{v}}^{v^*} v(1 - x_2^*) g(v) dv + \int_{v^*}^{\bar{v}} v(1 - x^*) g(v) dv \end{aligned} \tag{4-7}$$

Issuer i's profit is as follows:

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[5] The split lines, $x_i^*(v)$, never cross the vertical axes since, at around $x = 0$ and $x = 1$, there always exist some consumers who will use one of the credit cards regardless of the transportation costs as long as $b > f_i$.

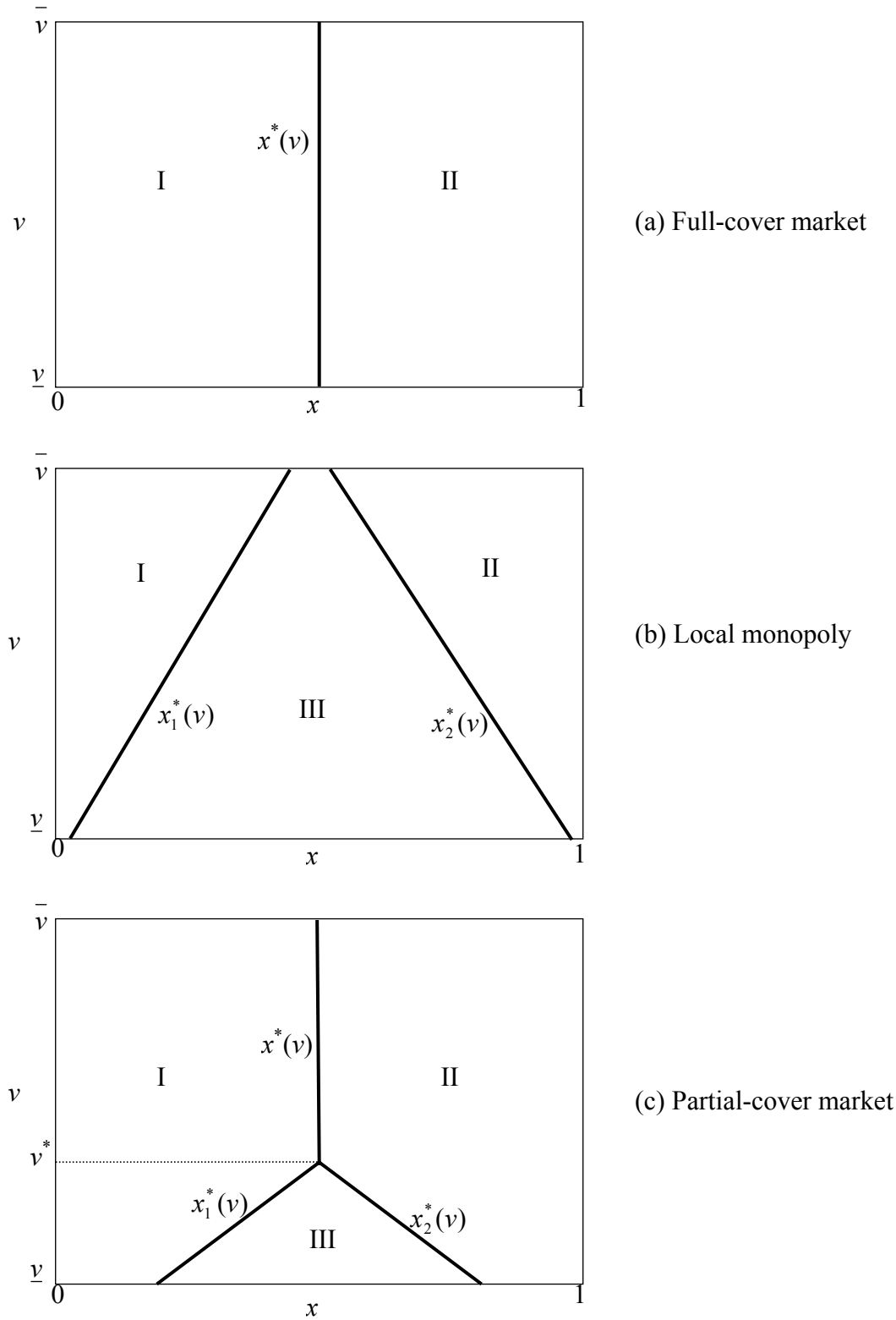

Figure 4-1. Division of consumers in three cases of market coverage

$$\pi_i = (f_i + a - c)Q_m q_i \tag{4-8}$$

where c is the marginal cost of the issuer, which is assumed to be same for both issuers.

The game proceeds as follows: at stage 1, the card scheme sets a; at stage 2, the competing card issuers set f_i; at stage 3, each consumer chooses whether to use card 1, 2 or not. The model can be solved by using backward induction. Since consumers' behavior at stage 3 has already been analyzed above, the next subsections will focus on the analysis of stage 2.

4.2.2 Full-Cover Market

In this subsection, the issuer market is assumed to be fully covered. This is possible if $\underline{v} \geq v^*$, or $t \leq 2\underline{v}(b - f^*)Q_m$, where f^* is the equilibrium cardholder fee of the symmetric issuers.

When the market is fully covered, the total demand for each issuer can be simplified as follows:

$$q_1(f_1; f_2) = \frac{1}{2}\int_{\underline{v}}^{\bar{v}} vg(v)dv + \frac{(f_2 - f_1)Q_m}{2t}\int_{\underline{v}}^{\bar{v}} v^2 g(v)dv = \frac{1}{2}E[v] + \frac{(f_2 - f_1)Q_m}{2t}E[v^2]$$

$$q_2(f_2; f_1) = \frac{1}{2}E[v] + \frac{(f_1 - f_2)Q_m}{2t}E[v^2] \tag{4-5'}$$

where $E[\cdot]$ is the mean of the variable in the bracket, i.e., $E[v] = \int_{\underline{v}}^{\bar{v}} vg(v)dv$ and $E[v^2] = \int_{\underline{v}}^{\bar{v}} v^2 g(v)dv$.

Issuer i's profit can be rewritten by plugging (4-5') into (4-8), which is

$$\pi_i = \frac{1}{2}(f_i + a - c)Q_m E[v] + \frac{(f_i + a - c)(f_j - f_i)Q_m^2}{2t} E[v^2]$$

Proposition 4-1 When the market is fully covered, the equilibrium cardholder fee and profits decrease as the variance of the expenditure volume increases while the mean of the volume remains constant.

Proof. At stage 2, each card issuer will maximize its profits by choosing optimal f_i. The first order condition for the profit maximization problem is

$$\frac{d\pi_i}{df_i} = \frac{Q_m}{2} E[v] + \frac{(f_j - 2f_i - a + c)Q_m^2}{2t} E[v^2] = 0 \tag{4-9}$$

Using (4-9), one can derive the issuer's best response function as follows:

$$f_i(f_j) = \frac{f_j - a + c}{2} + \frac{tE[v]}{2Q_m E[v^2]}$$

The symmetric Nash equilibrium of the model is

$$f^{FC} = c - a + \frac{tE[v]}{Q_m E[v^2]} \tag{4-10}$$

Note that, as in a standard Hotelling model, the equilibrium fee is the sum of the net marginal cost ($c - a$) and the profit margin $\frac{tE[v]}{Q_m E[v^2]}$.

The equilibrium profit is

$$\pi^{FC} = \frac{tE[v]^2}{2E[v^2]} \tag{4-11}$$

When the variance increases while the mean remains constant, $E[v^2]$ must increase since the variance of v is equal to $E[v^2] - E[v]^2$. So the equilibrium fee and profits decrease as the variance of v increases, holding the mean constant.

Q.E.D.

When a card issuer lowers its cardholder fee, the demand increases faster for the higher variance of v since $\frac{dq_i}{df_i} = -\frac{Q_m}{2t} E[v^2]$. But the other issuer will match the price decrease so the quantity sold will always be equal to $\frac{1}{2} E[v]$ at equilibrium, which implies that the demand becomes more elastic as the variance of v increases. As in a standard economic model, the equilibrium price decreases due to the increasing competition when the elasticity of demand increases. The profits decrease as a result of decreasing price without an increase in quantity sold. Note that the equilibrium fee can also be expressed in terms of elasticity (Lerner's formula):

$$f^{FC} = (c-a)\frac{\varepsilon^{FC}}{\varepsilon^{FC} - 1} \tag{4-12}$$

where $(c - a)$ is the net marginal cost, $\varepsilon^{FC} = -\frac{dq_i}{df_i}\frac{f}{q^{FC}}$ and $q^{FC} = q_i(f; f) = \frac{1}{2} E[v]$.

As is clear in (4-10) and (4-11), the equilibrium fee and profits increase when the mean of the expenditure volume increases while the variance remains the same. So it is not clear whether the fee and profits increase or decrease when both the mean and the variance of v increase. However, the following proposition shows that the equilibrium fee decreases when every cardholder increases expenditure volume at the same rate, so that both the mean and the variance increase.

Proposition 4-2 If every cardholder increases her expenditure volume at the same rate, the equilibrium cardholder fee decreases while the profits remain the same.

Proof. Suppose each cardholder's increased expenditure volume is $w = \beta v$, $\beta > 1$. Then the density of w, $h(w)$, is equal to $\frac{g(v)}{\beta}$, since w is distributed more widely. The means of w and w^2 are, respectively,

$$E[w] = \int wh(w)dw = \int \beta v g(v)dv = \beta E[v]$$
$$E[w^2] = \int w^2 h(w)dw = \int \beta^2 v^2 g(v)dv = \beta^2 E[v^2]$$

The equilibrium cardholder fee can be obtained using (4-10):

$$f_w^{FC} = c - a + \frac{tE[w]}{Q_m E[w^2]} = c - a + \frac{tE[v]}{Q_m \beta E[v]}$$

Since $\beta > 1$, the equilibrium fee decreases when every cardholder increases expenditure volume at the same rate.

Issuers' profits remain the same since

$$\pi_w^{FC} = \frac{tE[w]^2}{2E[w^2]} = \frac{t\beta^2 E[v]^2}{2\beta^2 E[v^2]} = \frac{tE[v]^2}{2E[v^2]} \equiv \pi^{FC}$$

Q.E.D.

This is a quite surprising result since the combined effects of increases in both mean and variance of the expenditure volume is to decrease the equilibrium cardholder fee. As the credit card industry grows, cardholders use more cards than other payment methods such as cash and checks. This increases the variance of the charging volume as well as the mean of the volume. So the increasing variance of the expenditure volume combined with an increase in the mean may be one of the reasons for the decrease in the cardholder fees over the history.

4.2.3 Local Monopoly

The local monopoly case arises when $v^* \geq \bar{v}$, or $t \geq 2\bar{v}(b - f^*)Q_m$. In this case, the demand for each issuer's service can be simplified as

$$q_i(f_i) = \int_{\underline{v}}^{\bar{v}} \frac{v^2(b - f_i)Q_m}{t} g(v)dv = \frac{(b - f_i)Q_m}{t} E[v^2] \qquad \text{(4-6')}$$

Using (4-6') and (4-8), issuer *i*'s profit can be rewritten as

$$\pi_i = \frac{(f_i + a - c)(b - f_i)Q_m^2}{t} E[v^2]$$

From the first-order condition for the profit maximization problem, one can derive the equilibrium cardholder fee as follows:

$$f^{LM} = \frac{b-a+c}{2} \tag{4-13}$$

As in the full-cover market case, demand increases faster for a given drop of the price as the variance of v increases since $\frac{dq_i}{df_i} = -\frac{Q_m}{t} E[v^2]$. Unlike in the full-cover market, however, the quantity demanded also increases when the variance of v increases since equilibrium quantity for each issuer is

$$q_i^{LM} = \frac{(b+a-c)Q_m}{2t} E[v^2]$$

As a result, the elasticity of demand is independent of the variance of v. Since the cardholder fee follows the Lerner's formula,[6] it is also independent of the variance of v.

The equilibrium profit of the local monopoly is

$$\pi^{LM} = \frac{(b+a-c)^2 Q_m^2}{4t} E[v^2] \tag{4-14}$$

Contrary to the full-cover market case, it is an increasing function of $E[v^2]$. This is because, as the variance of v increases, the quantity demanded also increases while the

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[6] $f^{LM} = (c-a)\frac{\varepsilon^{LM}}{\varepsilon^{LM}-1}$, where $\varepsilon^{LM} = -\frac{dq_i}{df_i}\frac{f}{q_i^{LM}}$.

price (cardholder fee) remains the same. The following proposition summarizes the above analysis.

Proposition 4-3 In case of a local monopoly, equilibrium profits increase while the equilibrium cardholder fee does not change when the variance of v increases, holding the mean constant.

4.2.4 Partial-Cover Market

The market is partially covered if $\underline{v} < v^* < \bar{v}$, or $2\underline{v}(b - f^*)Q_m < t < 2\bar{v}(b - f^*)Q_m$. When the market is partially covered, the demand for the card service is represented by (4-7), which can be rewritten as

$$\begin{aligned} q_1(f_1; f_2) &= \int_{\underline{v}}^{\bar{v}} v x^* g(v)dv - \int_{\underline{v}}^{v^*} v(x^* - x_1^*)g(v)dv \\ q_2(f_2; f_1) &= \int_{\underline{v}}^{\bar{v}} v(1 - x^*)g(v)dv - \int_{\underline{v}}^{v^*} v(x_2^* - x^*)g(v)dv \end{aligned} \tag{4-7'}$$

Plugging (4-2) and (4-3) into (4-7'), one can obtain

$$q_i(f_i; f_j) = \frac{1}{2}\int_{v^*}^{\bar{v}} v g(v)dv + \frac{(f_j - f_i)Q_m}{2t} E[v^2] + \frac{1}{2v^*}\int_{\underline{v}}^{v^*} v^2 g(v)dv \tag{4-7''}$$

The derivative of (4-7") with respect to f_i is

$$\frac{dq_i}{df_i} = -\frac{Q_m}{2t}\left(E[v^2] + \int_{\underline{v}}^{v^*} v^2 g(v)dv \right) \tag{4-15}$$

It is not clear from (4-15) that whether, as the variance of v increases, the demand changes faster for a given change in the price. Since $\int_{\underline{v}}^{v^*} v^2 g(v)dv$ also changes—it could increase or decrease—as the variance of v increases, $E[v^2]+\int_{\underline{v}}^{v^*} v^2 g(v)dv$ may be increasing or decreasing in the variance of v. Since $E[v^2]=\int_{\underline{v}}^{v^*} v^2 g(v)dv+\int_{v^*}^{\bar{v}} v^2 g(v)dv$, it is clear that

$$E[v^2]+\int_{\underline{v}}^{v^*} v^2 g(v)dv = 2\int_{\underline{v}}^{v^*} v^2 g(v)dv+\int_{v^*}^{\bar{v}} v^2 g(v)dv$$

Figure 4-2 shows why the expenditure volumes below v^* have a higher weight. When issuer 1 lowers f_1, consumers whose expenditure volumes are below v^* respond more sensitively than those with higher v. This is because issuer 1 competes with issuer 2 for higher type consumers, but not for lower type consumers. For notational convenience, define

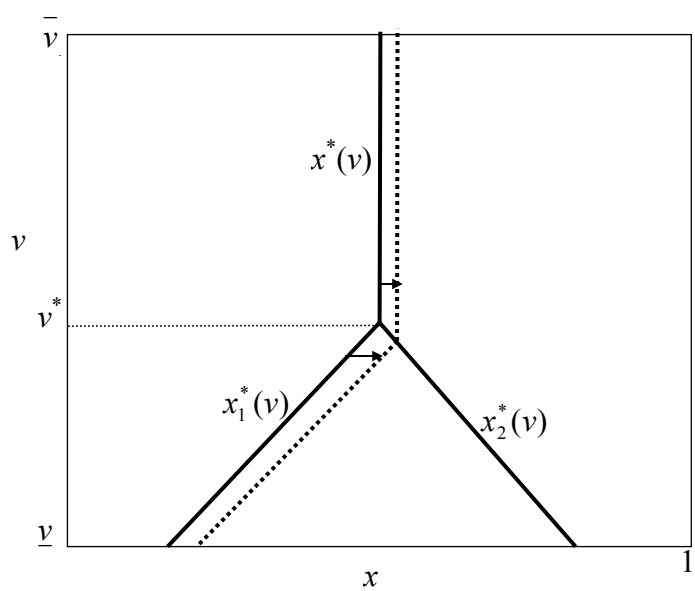

Figure 4-2. The effect of a price drop on demand

$$\hat{\sigma}^2 = E[v^2] + \int_{\underline{v}}^{v^*} v^2 g(v)dv$$

Then the derivative of the profit function given in (4-8) with respect to f_i is

$$\frac{d\pi_i}{df_i} = Q_m \left[q_i - \frac{(f_i + a - c)Q_m \hat{\sigma}^2}{2t} \right] \tag{4-16}$$

Lemma 4-1 The equilibrium cardholder fee is unique in the partial-cover market.

Proof. Using (4-16) and symmetry of the firms, the equilibrium fee, f^{PC}, is implicitly determined by

$$q_i(f^{PC}; f^{PC}) = \frac{(f^{PC} + a - c)Q_m \hat{\sigma}^2}{2t} \tag{4-17}$$

When $f = c - a$, the LHS of (4-17) is positive while the RHS of it is equal to zero. As f increases, the LHS of (4-17) decreases monotonically, while the RHS increases monotonically. So the equilibrium fee is uniquely determined by (4-17).

Q.E.D.

Using (4-7") and (4-17), one can obtain the equilibrium cardholder fee as a function of v^*:

$$f^{PC}(v^*) = c - a + \frac{t}{Q_m v^*}\left[\frac{v^*\int_{v^*}^{\bar{v}} vg(v)dv + \int_{\underline{v}}^{v^*} v^2 g(v)dv}{E[v^2] + \int_{\underline{v}}^{v^*} v^2 g(v)dv}\right] \tag{4-18}$$

Since v^* itself is a function of *f*, the equilibrium fee is still implicitly determined by (4-18). One of the benefits of expressing the equilibrium fee in terms of v^* is that it can be used to check the continuity of the equilibrium fee. To see this, suppose $v^* = \underline{v}$. Then (4-18) becomes equal to (4-10) so that $f^{PC} = f^{FC}$ at $v^* = \underline{v}$. When $v^* = \bar{v}$, (4-18) shrinks to

$$f^{PC}(\bar{v}) = c - a + \frac{t}{2Q_m \bar{v}} \tag{4-19}$$

Using the definition of v^*, one can show that (4-19) is equal to (4-13) so that $f^{PC} = f^{LM}$ at $v^* = \bar{v}$.

Propositions 4-1 and 4-3 have shown that, as the variance of the expenditure volume increases, the equilibrium fee decreases in the full-cover market or remains constant in the local monopoly case. From these results one may conjecture that in case of partial-cover market, the effect of a change in the variance of the expenditure volume should lie between the results of full-cover market and local monopoly cases. That is, the equilibrium fee may decrease or remain constant but never increase as the variance of the expenditure volume increases. But it turns out that the equilibrium fee may increase as well as decrease when the variance of *v* increases.[7]

[7] A simulation model is presented in the appendix that shows this result graphically.

Lemma 4-2 A sufficient condition for the equilibrium cardholder fee of the partial-cover market to increase when the variance of v increases, holding the mean constant, is

$$\frac{d\left(v^* \int_{v^*}^{\bar{v}} vg(v)dv\right)}{d\sigma^2} > 1$$

where $\sigma^2 = E[v^2] - E[v]^2$

Proof. A change in the variance of v affects the equilibrium fee in two ways. First, it affects the equilibrium fee directly through the changes in each term in the bracket of the right-hand side of (4-18). There also exists a second-order effect due to a change in v^*. However, the second-order effect is minor and cannot offset the first-order effect. So I will focus on the first-order effect only.

Note first that the bracket in (4-18) is less than 1 since

$$v^* \int_{v^*}^{\bar{v}} vg(v)dv = \int_{v^*}^{\bar{v}} v^* vg(v)dv < \int_{v^*}^{\bar{v}} v^2 g(v)dv < \int_{\underline{v}}^{\bar{v}} v^2 g(v)dv = E[v^2]$$

So the equilibrium cardholder fee will increase if the numerator increases more than the denominator when the variance increases. Since $\frac{dE[v^2]}{d\sigma^2} = 1$, the above condition is indeed a sufficient condition. *Q.E.D.*

Note that the equilibrium fee can be expressed in terms of the Lerner's formula as in the other two cases. That is, using (4-15) and (4-17), the equilibrium fee can be

rewritten as

$$f^{PC} = (c-a)\frac{\varepsilon^{PC}}{\varepsilon^{PC}-1}$$

where $\varepsilon^{PC} = -\frac{dq_i}{df_i}\frac{f}{q_i^{PC}}$

4.3 Equilibrium Interchange Fee

The analysis of the interchange fee is one of the main topics of the two-sided market literature focused on the credit card industry. Previous models on the credit card industry emphasize the balancing role of the interchange fee. A card scheme tries to optimize card transactions by achieving the right balance of cardholder demand and merchant acceptance. The interchange fee cannot be optimal if the demand of one side is too high while the demand of the other side is too low. By balancing the demands of both sides, the card scheme maximizes the aggregate profits of the member banks or total transaction volumes made by the card.

The model presented here follows the previous literature in that the card scheme sets the interchange fee in order to maximize combined profits. But the following analysis of the interchange fee is not complete due to the lack of in-depth analysis of the acquirer side. Most of the results here are obtained assuming the acquirer market is perfectly competitive, which simplifies the card scheme's objective as to maximize issuers' profits.

4.3.1 Full-Cover Market

As can be seen in (4-11), the equilibrium profit is independent of a and Q_m when the market is fully covered. This is because any gain from an increase in a or Q_m will be

competed away among the issuers. If the acquirer market is perfectly competitive as is often assumed in the credit card literature,[8] the card scheme will simply maximize the issuers' profits that are independent of the interchange fee. So the choice of the interchange fee is irrelevant to the overall profits as long as $Q_m(a) > 0$ and $f^*(a) < b$.

Even if the acquirer market is not perfectly competitive, the interchange fee may not affect the profits when the acquirer market is also fully covered. Suppose the acquirer market structure is similar to the issuer market. Then the resulting merchant fee and acquirers' profits will have similar structure as those of issuers so that the profits will be independent of the interchange fee.

When a change in the interchange fee does not alter any real variable in the economy, the interchange fee is neutral. Previous work finds that the neutrality of the interchange fee holds when both issuer and acquirer markets are perfectly competitive (Carlton and Frankel, 1995) or surcharge is possible (Rochet and Tirole, 2002).[9]

As will be clear in the following subsections, the neutrality of the interchange fee can hold in case of the full-cover market even if surcharge is not possible. When the market is not fully covered, lowering interchange fee can attract more consumers, which will affect issuers' profits as well as the quantity demanded. This can be summarized as follows.

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[8] Unlike the issuer market, there are few ways to differentiate in the acquiring market. Rochet and Tirole (2002) and Wright (2003) also assume perfect competition in the acquirer side. If acquirers are assumed to have market power, one would need to consider relative strength of issuers and acquirers to determine the interchange fee. See Schmalensee (2002).

[9] See Gans and King (2003) for a discussion of the neutrality of the interchange fee.

Proposition 4-4 Neutrality of the interchange fee holds under the no-surcharge-rule if both the issuer market and the acquirer market are fully covered.

4.3.2 Local Monopoly

When the market is locally monopolized, the card scheme will choose *a* to maximize industry profits represented by (4-14) if the acquirer side is perfectly competitive. Then the optimal interchange fee is implicitly determined by the following equation derived from the first-order condition:

$$a^{LM} = c - b - \frac{Q_m}{dQ_m / da} \tag{4-20}$$

or using the elasticity formula,

$$a^{LM} = (c - b)\frac{\varepsilon_m}{\varepsilon_m - 1} \tag{4-21}$$

where $\varepsilon_m = -\frac{dQ_m}{da}\frac{a}{Q_m}$

In a standard two-sided market model, the importance of an interchange fee lies in the role of balancing the demands of both sides. So it may look unconventional that (4-21) implies that the optimal interchange fee is related to the elasticity of the acquirer side only. But the fact is the elasticity of the issuer side is zero at the optimal level of the interchange fee. To see this, note first that the elasticity of the issuer side is

$$\varepsilon_a^{LM} = \frac{dq_i^{LM}}{da}\frac{a^{LM}}{q_i^{LM}} = \frac{a^{LM}}{b + a^{LM} - c} - \varepsilon_m \tag{4-22}$$

A simple calculation using (4-20) shows that (4-22) is equal to zero. This implies that the optimal interchange fee is set so as to maximize the quantity demanded in the issuer side. Rearranging (4-22), one can obtain the following expression:

$$a^{LM} = (c - b)\frac{\varepsilon_m + \varepsilon_a^{LM}}{\varepsilon_m + \varepsilon_a^{LM} - 1}$$

So the optimal interchange fee is indeed related to both sides.

4.3.3 Partial-Cover Market

In the partial-cover market, the equilibrium profits can be obtained by plugging (4-17) into the profit function (4-8), which is

$$\pi_i^* = \frac{(f^* + a - c)^2 Q_m^2}{2t}\left[E[v^2] + \int_{\underline{v}}^{v^*} v^2 g(v)dv\right] \tag{4-23}$$

If the acquirer side is perfectly competitive, the card scheme will choose optimal a to maximize π_i^*. A change in a affects profits in various ways. The derivative of the profit function w.r.t. the interchange fee can be decomposed as

$$\frac{d\pi_i^*}{da} = \frac{\partial \pi_i^*}{\partial a} + \frac{\partial Q_m}{\partial a}\frac{\partial \pi_i^*}{\partial Q_m} + \frac{\partial f^*}{\partial a}\frac{\partial \pi_i^*}{\partial f} \tag{4-24}$$

Since the issuer chooses the optimal f^* to maximize profit, the last term in (4-24) is zero at equilibrium. Using the envelope theorem, the first order condition can be simplified as

$$\frac{d\pi_i^*}{da} = \frac{(f^{PC} + a - c)Q_m}{2t}\left[(f^{PC} + a - c)[2\hat{\sigma}^2 - (v^*)^3 g(v^*)]\frac{dQ_m}{da} + 2\hat{\sigma}^2 Q_m\right] = 0 \qquad (4\text{-}25)$$

It follows from (4-25) that the optimal interchange fee is implicitly determined by

$$a^{PC} = c - f^{PC} - \frac{2\hat{\sigma}^2}{2\hat{\sigma}^2 - (v^*)^3 g(v^*)}\frac{Q_m}{dQ_m / da}$$

Unfortunately, no further analysis is possible unless one has more information about the distribution of v and the merchants' demand function for the card service.

4.4 Extension

4.4.1 Other Comparative Statics

The effects of a change in the variance of the expenditure volume on the equilibrium cardholder fees and profits have been analyzed in section 4.2. This subsection is devoted to the analysis of the other comparative statics.

Table 4-1 shows the main results of the comparative statics. One of the interesting results is the effect of t on cardholder fees and profits. Just like the variance of v, the effects of a change in t are opposite between full-cover market and local monopoly cases. In the full-cover market, "transportation cost" t works as in a standard Hotelling model. That is, a customer incurs a higher cost to switch to the other issuer as t increases. So the

Table 4-1 Comparative Statics

	$\frac{df^*}{d\sigma^2}$	$\frac{d\pi^*}{d\sigma^2}$	$\frac{df^*}{dt}$	$\frac{d\pi^*}{dt}$	$\frac{df^*}{dQ_m}$	$\frac{d\pi^*}{dQ_m}$	$\frac{df^*}{da}$
Full-cover market	−	−	+	+	−	0	±
Local monopoly	0	+	0	−	0	+	−
Partial-cover market	±	±	±	±	±	+	±

issuers can charge higher price and make higher profits.[10]

In the local monopoly case, however, customers either use a least-cost card or stop using the card. So when t increases, customers' cost of using the card increases while the cost of the alternative payment method—cash—remains the same. So the marginal customers will stop using the card for the given price, which causes a decrease in profits. The cardholder fee remains the same since, as t increases, the demand decreases proportionally for each level of v so that the elasticity of demand is independent of t.

In the partial-cover market, the effect of an increase in t is mixed. For customers above v^*, issuers can charge higher price when t increases since their switching cost increases just as in full-cover market. But for customers below v^*, issuers lose marginal customers due to the increase in the card-usage cost. Unlike local monopoly case, however, the demand does not decrease proportionally so it may be optimal for the issuers to lower the cardholder fee. The overall effects of a change in t on the cardholder fee and profits depend on the relative size of the two opposing effects.

[10] When t increases, the cost of using the card also increases. But the switching cost exceeds the usage cost.

Another interesting result is the effect of a change in the interchange fee on the equilibrium cardholder fee. Unlike conventional wisdom that cardholder fee decreases when the interchange fee increases, it turns out that the cardholder fee may increase as the interchange fee increases. The logic behind this is as follows. When the interchange fee increases, it raises the merchant fee so that the number of merchant that accept the card (Q_m) decreases, which in turn can cause an increase in the cardholder fee due to a decrease in the demand elasticity.[11]

The derivative of the full-cover market equilibrium cardholder fee w.r.t. a is

$$\frac{df^{FC}}{da} = -1 + \frac{tE[v]}{Q_m a E[v^2]} \varepsilon_m$$

The sign of this derivative depends on the relative size of each parameter. Although it is likely to be negative since the value of t should be small for the market to be fully covered, the sign could be positive especially when the elasticity of the acquirer market (ε_m) is extremely high.

4.4.2 Collusion

Since the equilibrium profits decrease in the full-cover market when the variance of v increases, the issuers have an incentive to exclude some low-volume consumers in order to reduce the variance. This is possible when they collude, and the following proposition shows that it is profitable to exclude some consumers.

[11] The negative relationship between f^{FC} and Q_m can be verified in (10).

Proposition 4-5 If two card issuers collude, the market cannot be fully covered at equilibrium.

Proof. It is enough to show that the issuers have an incentive to raise cooperatively the cardholder fee at $v^* = \underline{v}$.

When the two issuers collude, each firm's (common) demand function is

$$q^C(f;f) = \frac{1}{2}\int_{v^*}^{\bar{v}} vg(v)dv + \frac{1}{2v^*}\int_{\underline{v}}^{v^*} v^2 g(v)dv$$

The first-order condition for the joint profit maximization problem is

$$\frac{d\pi^C}{df} = Q_m\left[q^C - \frac{(f+a-c)Q_m}{t}\int_{\underline{v}}^{v^*} v^2 g(v)dv\right] = 0 \tag{4-26}$$

The equilibrium cardholder fee, f^C, is implicitly determined by the following condition:

$$q^C(f^C;f^C) = \frac{(f^C+a-c)Q_m}{t}\int_{\underline{v}}^{v^*} v^2 g(v)dv \tag{4-27}$$

When $f = b - \frac{t}{2\underline{v}Q_m}$, $v^* = \underline{v}$ by the definition of v^*. At this level of cardholder fee,

$$\left.\frac{d\pi^C}{df}\right|_{f=b-\frac{t}{2\underline{v}Q_m}} = \frac{Q_m}{2}E[v] > 0$$

since the second term inside the bracket in (4-26) is zero and $q^C = \frac{E[v]}{2}$. This implies that the optimal fee must be higher than $b - \frac{t}{2\underline{v}Q_m}$ so that v^* is greater than $\underline{v}$ at equilibrium. *Q.E.D.*

Comparing (4-27) with (4-17), it is clear that the collusive equilibrium fee is higher than the one without collusion in the partial-cover market, i.e., $f^C > f^{PC}$ since $E[v^2] > \int_{\underline{v}}^{v^*} v^2 g(v)dv$.

When the issuers collusively exclude low spending consumers, they can decrease the variance and increase the mean of the expenditure volume. Even though the total demand may decrease by this measure, the resulting profits increase.

Regulatory authorities in some countries are moving to regulate the credit card industry because of the allegedly too high interchange fees. If a policy measure lowering the interchange fee is accompanied by a higher cardholder fee, it also helps to reduce the variance of the charging volume among the credit card users.

4.5 Conclusion

Chapter 4 has proposed a framework for studying competition between card issuers when cardholders have heterogeneous expenditure volumes. What has been found is the effects of a change in variables on the competition vary depending on whether the market is fully covered, partially covered, or locally monopolized. When the market is locally monopolized, card issuers compete with other payment methods but not with each other. So any change that strengthen (weaken) the monopoly power will have a positive (negative) effect on profits. In case of the full-cover market, however, the only

competition issuers face is the one with each other. So any change that affects the competition between them has influence over issuers' profits. For example, suppose the transportation cost (t) increases. Then it decreases issuers' profits in case of local monopoly since it weakens the competitive power over the alternative payment methods. But in the full-cover market case, it increases the profits since it strengthens competitive power over the other issuer.

The results are mixed if the market is partially covered. The effects of a change in variables on the equilibrium price and profits are not constrained to the range between those of full-cover market and local monopoly. As the simulation results in the appendix show, however, the effects tend to become closer to those of full-cover market, the more the market is covered.

As the credit card industry grows, the market will become closer and closer to the full-cover market. If this happens with increasing variance of the expenditure volume, the overall profits of the industry may decrease even without competition in the card scheme level.

One of the policy implications is that regulating the interchange fee may have the same effect as reduced competition on the issuer side of the credit card industry if it induces a higher cardholder fee. Since the higher cardholder fee will exclude consumers with low expenditure volume from using the credit card even if the benefit from using the card (b) is the same as the high volume consumers', the policy may have an undesirable effect in terms of equality.

Although the model sheds new light on issuer side of the credit card industry by introducing heterogeneous expenditure volumes, a comparable analysis of the acquirer

side is missing. Since the card industry is categorized as a two-sided market, modeling of both sides is necessary in order to fully understand the working of the industry. Another possible extension of the model is to introduce shipping cost for the transportation cost of the model since many differentiated card benefits such as rebates in the form of frequent flyer miles are proportional to the charging volume. Last but not least, allowing platform competition will help understanding the difference between competition on the member bank level—issuers and acquirers—and platform competition.

CHAPTER 5
CONCLUDING REMARKS

This dissertation analyzes pricing strategies of multiproduct firms when they face competition in the complementary aftermarket (bundling), or there exist indirect network externalities that cannot be internalized by consumers of each good (two-sided markets).

Chapter 2 deals with a multiproduct firm that produces a monopolistic primary good and a competitive complementary good. If consumers buy the complementary good after they have bought the primary good, i.e., the complementary goods are sold in the aftermarket, the monopolist can make the highest profits by committing to the aftermarket price. But if credibility of commitment is an issue or the committed price is not feasible, the monopolist can sell them as a bundle and make higher profits than when it sells them independently.

It is also shown that bundling lowers social welfare in most cases while it increases consumers' surplus. So whether this kind of bundling should be allowed depends on the objective of policy makers. That is, bundling may be allowed if policy makers maximize consumers' surplus, whereas it should be regulated as an anticompetitive practice if Marshallian social welfare is the main concern. In the long-term point of view, however, bundling should be viewed with concern since it decreases both firms' incentives to invest in R&D.

Chapter 3 presents a model of a multiproduct firm (platform) that sells two products to two different types of end-users who interact with each other through the platform. Since the end-users cannot internalize indirect network externalities in this two-

sided market, the platform must choose the appropriate set of prices in order to get both sides on board and to maximize profits (for a proprietary platform) or output (for a non-proprietary platform). Using a case of the credit card industry, it is shown that competition may not improve social welfare or even have a negative effect on welfare since competing platforms set unbalanced prices in favor of the single-homing side. A lower price to the single-homing side is accompanied by a higher price to the multi-homing side. That is, in two-sided markets, higher prices may be a direct result of competition, not a sign of lack of competition. Besides, competing platforms choose a price structure that maximizes consumers' surplus if consumers single-home and merchants multi-home. So antitrust policy on two-sided market should be implemented with discretion.

Chapter 4 delves into the issuer side of the credit card industry by allowing heterogeneous expenditure volumes among consumers. The effects of a change in the variance of the expenditure volumes are mainly analyzed. The main finding is that the effects of a change in the variance on the equilibrium price and profits are different for various cases of market coverage. Especially, when all consumers' expenditures increase at the same rate so that both the mean and the variance increase, the equilibrium price decreases when the market is fully covered. This gives the issuers an incentive to cooperatively exclude consumers with low expenditure. This implies that any policy that increases cardholder fees may have a negative effect on consumers' welfare since it helps card issuers to reduce the variance of cardholders' charging volumes by excluding consumers with low expenditure.

Theories of two-sided markets are fairly new and are still in the middle of development, which includes the analysis of various strategies in two-sided markets, such as tying (Rochet and Tirole, 2006) and exclusive dealing (Armstrong and Wright, 2005). Despite rapid development in theory, empirical studies on two-sided markets are rare. As in the other areas of economic theory, more balanced empirical research is required to deepen our understanding of two-sided markets and to derive unbiased policy implications.

APPENDIX
SIMULATION ANALYSIS FOR CHAPTER 4

In this appendix, simulation results that show the behavior of the partial-cover market will be presented. Since it is difficult to solve the general model mathematically, a simulation model can be used to help understanding the partial-cover market.

In order to incorporate effects of a change in the variance of the expenditure volume, suppose the density function of v take the following form.

$$g(v) = \frac{2\left[y(\bar{v}-\underline{v})-1\right]\left[\left|2v-(\bar{v}+\underline{v})\right|-(\bar{v}-\underline{v})\right]}{(\bar{v}-\underline{v})^2} + y \tag{A-1}$$

Using this density function, the mean and the variance of v can be derived as

$$E[v] = \frac{\bar{v}+\underline{v}}{2}$$

$$E[v^2] = \frac{1}{24}\left[y(\bar{v}-\underline{v})^3 + 7(\bar{v}-\underline{v})^2 + 24\bar{v}\underline{v}\right]$$

$$Var[v] = \frac{1}{24}\left[y(\bar{v}-\underline{v})^3 + (\bar{v}-\underline{v})^2\right]$$

As can also be seen in Figure A-1, the variance of v increases when y increases, while the mean is independent of y. The derivative of the variance w.r.t y is

$$\frac{dVar[v]}{dy} = \frac{(\bar{v}-\underline{v})^3}{24} > 0$$

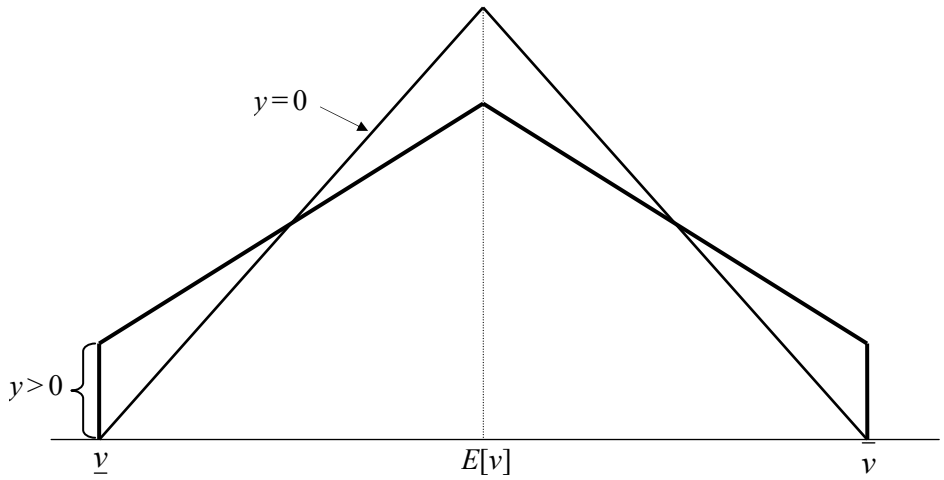

Figure A-1. The density function

The model can be simplified without loss of generality by setting $\underline{v} = 0$. To solve the model, note first that v^* increases as t increases in (4-4). As is shown in Table 4-1, f_i may increase or decrease when t increases. But even if f_i decreases, the resulting decrease in v^* is a second-order effect which cannot offset the first-order effect. This implies that there exists only one t for each v^* at equilibrium. Define

$$t_d = d(b - f)Q_m \bar{v}, \;\; 0 \le d \le 2 \tag{A-2}$$

where d is the parameter that correspond to v^*. Then t_d also corresponds to each value of v^*. Setting $t = t_d$ yields $v^* = \frac{d}{2}\bar{v}$. So $v^* = 0$ $(= \underline{v})$ if $d = 0$, and $v^* = \bar{v}$ if $d = 2$. Since the density function is symmetric, $v^* = E[v]$ if $d = 1$.

Now the equilibrium cardholder fee can be obtained as a function d (or v^*) using the first-order condition (4-16). Let $f^*(d)$ be the equilibrium cardholder fee obtained from this calculation. This equilibrium fee cannot be used directly to do comparative

statics because t cannot be held constant. To resolve this problem, one needs to apply implicit function theorem using (4-18). That is, define

$$H = c - a - f + \frac{t}{Q_m v^*}\left[\frac{v^*\int_{v^*}^{\bar{v}} vg(v)dv + \int_{\underline{v}}^{v^*} v^2 g(v)dv}{E[v^2] + \int_{\underline{v}}^{v^*} v^2 g(v)dv}\right]$$

Then the derivative of f w.r.t. a parameter x is

$$\frac{df}{dx} = -\frac{dH}{dx}\Big/\frac{dH}{df} \tag{A-3}$$

where x can be any parameter such as y, a, t, and Q_m. The final result is obtained by plugging t_d and $f^*(d)$ into (A-3).

The effect of an increase in the variance of v on the cardholder fee can be captured by df/dy. Figure A-2 shows that this derivative can be both negative and positive

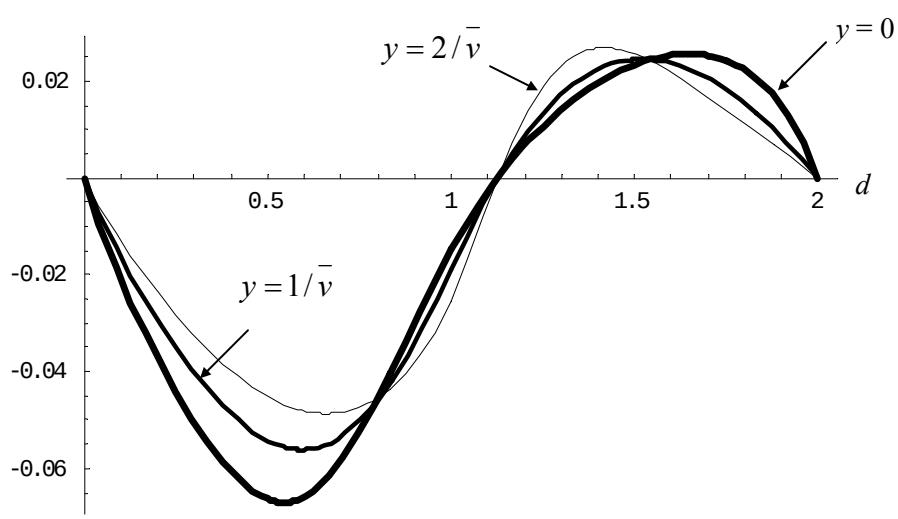

Figure A-2. Effects of an increase in the variance on the cardholder fee (df/dy)

depending on d (or v^*).[76] The three graphs are drawn for the cases of three different distributions of the expenditure volume. As is clear in the figure, the distribution of v does not affect the result qualitatively.

Another interesting result of the comparative statics is the effect of a change in the interchange fee on the cardholder fee (df/da). In order to derive df/da, an assumption regarding the acquirer market is necessary since Q_m also changes when a changes. To make the model tractable, assume a linear demand function in the acquirer market.

$$Q_m = r(k - a)$$

This linear demand function assumes that the acquirers transfer the whole interchange fee to the merchant fee, and the merchants accept the credit card as long as the merchant fee is lower than the per-transaction benefit they receive from the card service.

Figure A-3 shows the effects of the interchange fee on the cardholder fee. As can be seen in the figure, the cardholder fee may increase or decrease when the interchange fee increases depending on the parameter value. To obtain numerical results, arbitrary numbers are assigned to the parameters. The thick graph is drawn assuming $y = 0$, $b = 6$, $c = 2$, $k = 4$, and $a = 2$. And the thin graph is drawn assuming the same parameter values except $a = 1$. The higher the interchange fee is, the bigger is the elasticity of the acquirer market. And the issuers may raise the cardholder fee due to a lower Q_m despite of a higher a.

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[76] When drawing the graphs, it is assumed that $(a + b - c)\bar{v} = 1$ to get a numerical result.

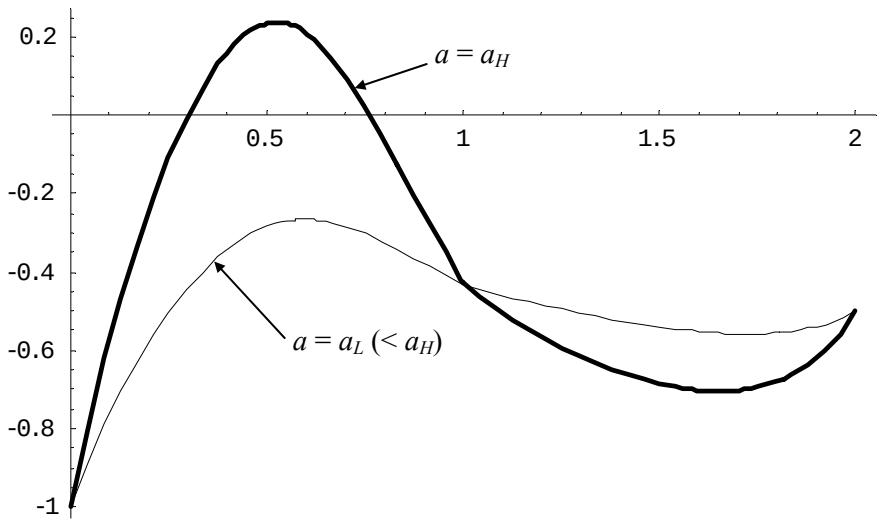

Figure A-3. Change in the interchange fee and the cardholder fee (*df*/*da*)

REFERENCES

Adams, William J. and Janet L. Yellen (1976) "Commodity Bundling and the Burden of Monopoly," *Quarterly Journal of Economics*, 90, pp. 475–498.

Armstrong, Mark (2005) "Competition in Two-Sided Markets," *Rand Journal of Economics*, forthcoming.

Armstrong, Mark and Julian Wright (2005) "Two-Sided Markets, Competitive Bottlenecks and Exclusive Contracts," mimeo.

Bakos, Yannis and Erik Brynjolfsson (1999) "Bundling Information Goods: Pricing, Profits and Efficiency," *Management Science*, 45, pp. 1613−1630.

Baxter, William F. (1983) "Bank Interchange of Transactional Paper: Legal and Economic Perspectives," *Journal of Law & Economics*, 26, pp. 541–588.

Blair, Roger D. and Jill B. Herndon (1996) "Restraints of Trade by Durable Good Producers," *Review of Industrial Organization*, 11, pp. 339−353.

Borenstein, Severin, Jeffrey K. MacKie-Mason, and Janet S. Netz (1995) "Antitrust Policy in Aftermarkets," *Antitrust Law Journal*, 63, pp. 455–482.

Caillaud, Bernard and Bruno Jullien (2003) "Chicken & Egg: Competition among Intermediation Service Providers," *Rand Journal of Economics*, 24, pp.309–328.

Carlton, Dennis W. and Michael Waldman (2002a) "The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries," *Rand Journal of Economics*, 33, pp. 194−220.

Carlton, Dennis W. and Michael Waldman (2002b) "Competition, Monopoly, and Aftermarket," mimeo.

Carlton, Dennis W. and Michael Waldman (2005a) "Tying, Upgrades, and Switching Costs in Durable-Goods Market," *NBER Working Paper*, 11407.

Carlton, Dennis W. and Michael Waldman (2005b) "Theories of Tying and Implications for Antitrust," mimeo.

Chakravorti, Sujit and Roberto Roson (2006) "Platform Competition in Two-Sided Markets: The Case of Payment Networks," *Review of Network Economics*, 5, pp. 118−142.

Choi, Jay P. (2004) "Tying and Innovation: A Dynamic Analysis of Tying Arrangements," *The Economic Journal*, 114, pp. 83−101.

Choi, Jay P. and Christodoulos Stefanadis (2001) "Tying, Investment, and the Dynamic Leverage Theory," *Rand Journal of Economics*, 32, pp. 52−71.

Evans, David (2003) "The Antitrust Economics of Multi-Sided Platform Markets," *Yale Journal on Regulation*, 20, pp. 325−381.

Evans, David and Richard Schmalensee (1999) *Paying with Plastic: The Digital Revolution in Buying and Borrowing*, MIT Press: Cambridge, MA.

Evans, David and Richard Schmalensee (2005) "The Industrial Organization of Markets with Two-Sided Platforms," NBER Working Paper, 11603.

Evans, David S. and Michael Salinger (2005) "Why Do Firms Bundle or Tie? Evidence from Competitive Markets and Implications for Tying Law," *Yale Journal of Regulation*, 22, pp. 37−89.

Evans, David S., Albert L. Nichols and Richard Schmalensee (2001) "An Analysis of the Government's Economic Case in U.S. v. Microsoft," *The Antitrust Bulletin*, 46, pp. 163−251.

Evans, David S., Albert L. Nichols and Richard Schmalensee (2005) "U.S. v. Microsoft: Did Consumers Win?," *NBER Working Paper*, 11727.

Farrell, Joseph and Garth Saloner (1985) "Standardization, Compatibility, and Innovation," *Rand Journal of Economics*, 16, pp. 70–83.

Farrell, Joseph and Garth Saloner (1986) "Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation," *American Economic Review*, 76, pp. 940–955.

Gabszewicz Jean J. and Xavier Y. Wauthy (2004) "Two-Sided Markets and Price Competition with Multi-homing," mimeo.

Gans, Joshua S. and Stephan P. King (2003) "The Neutrality of Interchange Fees in Payment Systems," *Topics in Economic Analysis & Policy*, 3, pp. 1–16.

Gilbert, Richard J. and Michael L. Katz (2001) "An Economist's Guide to U.S. v. Microsoft," *Journal of Economic Perspectives*, 15, pp. 25–44.

Guthrie, Graeme and Julian Wright (2006) "Competing Payment Schemes," *Journal of Industrial Economics*, forthcoming.

Katz, Michael L. (2001) *Reform of Credit Card Schemes in Australia II*, Reserve Bank of Australia.

Katz, Michael L. and Carl Shapiro (1985) "Network Externalities, Competition, and Compatibility," *American Economic Review*, 75, pp. 424–440.

Katz, Michael L. and Carl Shapiro (1986) "Technology Adoption in the Presence of Network Externalities," *Journal of Political Economy*, 94, pp. 822–841.

Klein, Benjamin (1993) "Market Power in Antitrust: Economic Analysis after Kodak," *Supreme Court Economic Review*, 3, pp. 43–92.

McAfee, R. Preston, John McMillan, and Michael D. Whinston (1989) "Multiproduct Monopoly, Commodity Bundling, and Correlation of Values," Quarterly Journal of Economics, 104, pp. 371–383.

Nalebuff, Barry (2003) *Bundling, Tying, and Portfolio Effects*, DTI Economics Paper No. 1.

Nalebuff, Barry (2004) "Bundling as an Entry Barrier," *Quarterly Journal of Economics*, 119, pp. 159−187.

Reisinger, Markus (2004) "Two-Sided Markets with Negative Externalities," mimeo.

Rochet, Jean-Charles and Jean Tirole (2002) "Cooperation among Competitors: Some Economics of Payment Card Associations," *Rand Journal of Economics*, 33, pp. 549–570.

Rochet, Jean-Charles and Jean Tirole (2003) "Platform Competition in Two-Sided Markets," *Journal of European Economic Association*, 1, pp. 990−1029.

Rochet, Jean-Charles and Jean Tirole (2005) "Two-Sided Markets: A Progress Report," *Rand Journal of Economics*, forthcoming.

Rochet, Jean-Charles and Jean Tirole (2006) "Tying in Two-Sided Markets and the Honor All Cards Rule," mimeo.

Roson, Roberto (2005a) "Two-Sided Markets: A Tentative Survey," *Review of Network Economics*, 4, pp. 142–160.

Roson, Roberto (2005b) "Platform Competition with Endogenous Multihoming," mimeo.

Rysman, Marc (2006) "An Empirical Analysis of Payment Card Usage," mimeo.

Schmalensee, Richard (1984) "Gaussian Demand and Commodity Bundling," *Journal of Business*, 57, pp. S211−S230.

Schmalensee, Richard (2002) "Payment Systems and Interchange Fees," *Journal of Industrial Economics*, 50, pp. 103–122.

Shapiro, Carl (1995) “Aftermarkets and Consumer Welfare: Making Sense of Kodak,” *Antitrust Law Journal*, 63, pp. 483−511.

Shapiro, Carl and David J. Teece (1994) “Systems Competition and Aftermarkets: an Economic Analysis of Kodak,” *Antitrust Bulletin*, 39, pp. 135−162.

Tirole, Jean (2005) “The Analysis of Tying Cases: A Primer,” *Competition Policy International*, 1, pp. 1−25.

Whinston, Michael D. (1990) “Tying, Foreclosure, and Exclusion,” *American Economic Review*, 80, pp. 837−859.

Whinston, Michael D. (2001) “Exclusivity and Tying in U.S. v. Microsoft: What We Know, and Don’t know,” *Journal of Economic Perspectives*, 15, pp. 63−80.

Wright, Julian (2003a) “Optimal Card Payment Systems”, *European Economic Review*, 47, pp. 587–612.

Wright, Julian (2003b) “Pricing in Debit and Credit Card Schemes,” *Economics Letters*, 80, pp. 305–309.

Wright, Julian (2004a). "The Determinants of Optimal Interchange Fees in Payment Systems." *Journal of Industrial Economics*, 52, pp. 1–26.

Wright, Julian (2004b) “One-Sided Logic in Two-Sided Markets,” *Review of Network Economics*, 3, pp. 44–64.

BIOGRAPHICAL SKETCH

Jin Jeon was born in Kunsan, Korea, in 1967. He received his B.A. and M.A. in economics from Seoul National University in Seoul, Korea. He joined the doctoral program at economics department of the University of Florida in 1999. He will receive his Ph.D. in economics in December 2006.