ROTOR-BEARING SYSTEM DYNAMICS OF A HIGH SPEED MICRO END MILL SPINDLE

By

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Current micro-scale manufacturing technologies find limited application in a wide range of high strength engineering materials because of the difficulties encountered in creating complex three dimensional structures and features. Although milling is one of the most widely used processes for this type of manufacturing at the macro scale, it has yet to become an economically viable technology for micro-scale manufacturing. For optimal chip formation using very small diameter cutters, and to achieve economical material removal rates combined with good surface finish, high spindle speeds are needed. In addition, a low runout is desired to prevent premature tool breakage. However, the lack of suitable spindles capable of achieving rotational speeds in excess of 500,000 rpm coupled with sub-micrometer runout at the tool tip makes micro-scale milling commercially unviable.
This thesis demonstrates several means of analyzing the rotor dynamic behavior of a spindle in order to find critical speeds, unbalance response and linear stability margins. Experimental testing is performed to estimate bearing dynamic behavior at high speed.

The results of this study provide parameters for bearing stiffness and damping, bearing span and balancing limits to achieve sub-micrometer runout of tool tip for speeds up to 1 million rpm.
CHAPTER 1
INTRODUCTION

The technology development in the field of miniaturization has become a global phenomenon. Its impact is far and widespread across a broad application domain that encompasses many diverse fields and industries, such as telecommunications, portable consumer electronics, defense, and biomedical. The perfect example is the field of computers where modern computers which possess greater processing power and can fit under a desk or on a lap have replaced the bulky computers of the past such as the ENIAC (electronic numerical integrator and computer) which once filled large rooms. In recent times, more and more attention is being paid to the issues involved in the design, development, operation, and analysis of the equipment and processes of manufacturing micro components since the global trend toward the increased integration of miniaturized technology into society has gained enormous momentum. Currently, common techniques utilized in the fabrication of micro-components are based on the techniques developed for the silicon wafer processing industry. Unfortunately these processes are limited to production of simple planar geometries in a narrow range of material and are cost effective only in large volume [1]. Even though non traditional fabrication methods, such as focused ion beam machining, laser machining, and electrodischarge machining, are capable of producing high-precision micro-components, they have limited potential as mass production techniques due to the high initial cost, poor productivity, and limited material selection [2]. Micro milling has the potential to fabricate micro components and is capable of machining complex 3D shapes from wide variety of shapes and materials.
The objective of this research is to develop a micro-milling spindle which will rotate at over 500,000 rpm range with sub-micrometer runout, and thus become a commercially usable and cost effective manufacturing technology. Most machine tools such as lathes, milling machines, and all types of grinding machines, use a spindle or an axis of rotation for positioning work pieces or tools or machining parts and thus a large part of their accuracy can be attributed to the spindle. Consequently, the accuracy of the spindles used in their design directly influences the accuracy of the entire machine and thus can be considered as one of the most important components in the overall accuracy and operation of a machine tool.

Most commercial micro-tools have a 1/8th inch diameter shank (see Figure 1-1). This must be of utmost importance when designing the micro spindle. Another functional requirement is the ease of tool changing with minimal time and effort. The only viable way to meet the above design requirements while still obtaining satisfactory runout is to concentrate on designs incorporating the use of tool shank itself as the spindle shaft. To achieve desired performance, the following three functions must be satisfied:

1. **Bearing subsystem.** The bearing system must be so designed that it meets the following requirements. Firstly and fore mostly, it must be capable of supporting the tool shank without causing excessive runout. Also, it must support both radial and axial loads, and support rapid tool changes. Flexure Pivot Tilting Pad Bearings (FPTPB) are being studied as a potential bearing subsystem.

2. **Drive subsystem.** The tool drive system must be able to drive the tool at the required speed with enough torque and power to perform the desired machining operations. Also it should not introduce disturbance forces that cause excessive tool point runout. These requirements make air turbine drive as a potential drive subsystem. The system would incorporate the turbine blades directly into the tool shank of the micro-tool.

3. **Monitoring subsystem.** Theoretical and scientific understanding of micro-milling requires monitoring and recording of cutting forces. However, in the measurement bandwidth (that is 1,000,000 rpm with a 2-flute cutter and tooth passing frequency
of 33 KHz), the force measurement is extremely difficult because of the high
frequencies encountered even though the cutting forces are low.

The goals of this project are the following:

- To extend the capability to model and predict rotordynamics and bearing behavior at
  small sizes and high speeds.

- Development of a procedure for identification of dynamic stiffness and damping
  coefficients for the bearing.

Figure 1-1. Commercial micro-tool
CHAPTER 2
DEVELOPMENT OF EQUATION OF MOTION OF ROTOR-BEARING SYSTEM
AND PARAMETER IDENTIFICATION

It is of utmost importance in many companies that not only the operation should be
uninterrupted and reliable but also it should be carried out at high power and high speed.
Another vital requirement is the accurate prediction and control of the dynamic behavior
(unbalance response, critical speeds and instability). These factors were the motivations
for this research wherein rigid rotor analysis and finite element analysis was used to
investigate bearing coefficient parameters and the rotordynamics of micro spindle. Both
the rigid rotor analysis and finite element analysis have been performed simultaneously.
The tungsten carbide spindle has a first bending or flexure natural frequency of 2.2
million rpm for a bearing span of 1 inch. The spindle operating speeds are expected to be
about 500,000 rpm. Hence rigid rotor analysis can be justified. Finally, the experimental
setup was designed to find bearing parameters which were compared with analytical
results.

2.1 Rigid Rotor Analysis

In order to get generalized rotor dynamic models, the Jeffcott rotor is extended to a
four degree of freedom rigid rotor system as shown in the schematic diagram of Figure 2-1.
The four coordinates, which are the two geometric center translations (V, W) and the
two rotation angles (B, \( \Gamma \)) describe the rotor configuration relative to the fixed reference
(X, Y, Z). Bearing 1 and bearing 2 are located at an axial distance \( a_1 \) and \( a_2 \) from the
center of mass, respectively. Both these distances are defined as positive in the plus X direction. The rotor configuration is always defined so that $a_1$ is positive.

Figure 2-1. Rigid rotor schematic.

Where

- $(a, b, c)$ geometric center body reference
- $(\eta, \zeta)$ eccentricity components
- $(\phi)$ spin angle $= \Omega t$
- $(\Omega)$ constant spin frequency

\[
V_m(t) = V + (\eta \cos \phi - \zeta \sin \phi) \\
W_m(t) = W + (\zeta \cos \phi + \eta \sin \phi)
\] (2-1)

The angular rate of the rigid body is

\[
\omega_a = \Omega - \dot{\Gamma} \sin B \\
\omega_b = \dot{\Gamma} \cos B \sin \Omega t + \dot{B} \cos \Omega t \\
\omega_c = \dot{\Gamma} \cos B \cos \Omega t - \dot{B} \sin \Omega t
\] (2-2)
and the kinetic energy of the rigid body is

$$T = \frac{1}{2} m (\dot{V}_m^2 + \dot{W}_m^2) + \frac{1}{2} \left[ I_p \dot{\omega}_a^2 + I_d (\dot{\omega}_b^2 + \dot{\omega}_c^2) \right]$$  \hspace{1cm} (2-3)

By considering the variational work of the bearing forces, they are included in the equation of motion. The bearing force is a function of lateral shaft translations and velocities at the bearing location.

$$F_Y = F_Y(V, W, \dot{V}, \dot{W})$$

$$F_Z = F_Z(V, W, \dot{V}, \dot{W})$$ \hspace{1cm} (2-4)

Upon Taylor’s series expansion of eq. (2-4) about the origin, the force components in eq. (2-4) are approximated by their corresponding linear forms. At the ith typical bearing, forces are expressed in the following equation:

$$\begin{bmatrix} F_Y \\ F_Z \end{bmatrix} = \begin{bmatrix} k_{iYY} & k_{iYZ} \\ k_{iZY} & k_{iZZ} \end{bmatrix} \begin{bmatrix} V_i \\ W_i \end{bmatrix} - \begin{bmatrix} c_{iYY} & c_{iYZ} \\ c_{iZY} & c_{iZZ} \end{bmatrix} \begin{bmatrix} \dot{V}_i \\ \dot{W}_i \end{bmatrix}$$ \hspace{1cm} (2-5a)

or

$$\ddot{\mathbf{F}}_i = -k_i \ddot{\mathbf{r}}_i - c_i \dot{\mathbf{r}}_i$$ \hspace{1cm} (2-5b)

where

$$\ddot{\mathbf{r}}_i = \begin{bmatrix} 1 & 0 & 0 & -a_i \\ 0 & 1 & a_i & 0 \end{bmatrix} \ddot{\mathbf{q}} = A_i \ddot{\mathbf{q}}$$ \hspace{1cm} (2-6)

$$\ddot{\mathbf{q}}^T = [V \ W \ B \ \Gamma]$$ \hspace{1cm} (2-7)

The variational work done by bearing forces on the rotor is given by the following expression

$$\delta W_k = \sum_{i=1}^{2} \ddot{F}_i \delta \ddot{r}_i = \sum_{k=1}^{4} Q_k \delta q_k$$ \hspace{1cm} (2-8)

where $Q_k$ represents the generalized bearing forces.
Lagrange’s equations are of the following form:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial q_k} \right) - \frac{\partial T}{\partial q_k} = \dot{Q}_k
\]

\(k=1,2,3,4\) (2-9)

Using the above set of equations and inserting in eq (2-9) the following set of rigid rotor equations of motion has been obtained:

\[
M\ddot{q} + (C - \Omega G)\dot{q} + Kq = \ddot{Q}
\]

(2-10)

where

\[
M = \begin{bmatrix}
m & 0 & 0 & 0 \\
0 & m & 0 & 0 \\
0 & 0 & I_d & 0 \\
0 & 0 & 0 & I_d \\
\end{bmatrix}, \quad \quad C = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -l_p \\
0 & 0 & l_p & 0 \\
\end{bmatrix}
\]

\[
K = \sum_{i=1}^{2} A_i^T k_i A_i, \quad \quad C = \sum_{i=1}^{2} A_i^T c_i A_i
\]

\[
\ddot{Q} = m\Omega^2 \begin{bmatrix}
\eta \\
\zeta \\
0 \\
0
\end{bmatrix} \cos \Omega t + m\Omega^2 \begin{bmatrix}
-\zeta \\
\eta \\
0 \\
0
\end{bmatrix} \sin \Omega t
\]

(2-11)

2.2 Finite Element Analysis

Typically, it is not possible to obtain analytical solutions for problems involving complicated geometries, loadings and material properties. Based on the study and inspection of various approaches available for modeling, one of the most appropriate methods for modeling of high-speed micro spindle is the FEA, finite element method. It is also the only feasible type of computer simulation available for this purpose. The finite element method is generally a numerical method used for solving engineering and mathematical physics problems. The following steps are used in the FEA for dynamic response solution [3]:

...
• Form element stiffness matrix.
• Form element mass matrix.
• Assemble system stiffness matrix and incorporate constraints.
• Assemble system mass matrix and incorporate constraints.
• Solve eigenproblem and obtain a vector of frequencies and mode shapes.
• Form excitation vector in physical coordinates.

2.3 Identifying Bearing Parameters By Experimental Method

The estimation of the dynamic bearing characteristics using theoretical methods usually results in an error in the prediction of the dynamic behavior of rotor-bearing systems. Reliable estimates of the bearing operating condition in actual test conditions are difficult to obtain and, therefore to reduce the discrepancy between the measurements and the prediction, physically meaningful and accurate parameter identification is required in actual test conditions. There are some similarities between various experimental methods for the dynamic characterization of rolling element bearings, fluid-film bearings and magnetic bearings. These methods require forces as input signals and displacement/velocities/accelerations of the dynamic system to be measured are usually the output signals, and input-output relationships are used to determine the unknown parameters of the system models. There are a lot of identification techniques of bearing parameters, which are based on methods used to excite the system [4], such as the following:

1. Methods using Incremental Static Load
2. Methods using Dynamic Load
3. Methods using an Excited Load
4. Method using Unbalance Mass
5. Methods using an Impact Hammer

6. Methods using Unknown Excitation

Appendix A summarizes the source material on the experimental dynamic parameter identification of bearings.

### 2.3.1 Methods Using Incremental Static Load

Mitchell et al. (1965-66) [5] performed experiments to incrementally load the bearing and measuring the change in position, and obtained the four stiffness coefficients of fluid-film bearings. They obtained the following simple relationships using the influence coefficient approach to

\[
\begin{align*}
k_{yy} &= \alpha_{zz} / \gamma \\
k_{yz} &= -\alpha_{yz} / \gamma \\
k_{zy} &= -\alpha_{zy} / \gamma \\
k_{zz} &= \alpha_{yy} / \gamma
\end{align*}
\]  \hspace{1cm} (2-12)

where

\[
\gamma = \alpha_{yy}\alpha_{zz} - \alpha_{yz}\alpha_{zy}
\]

\[
\begin{align*}
\alpha_{yy} &= y_1 / \Delta F_y \\
\alpha_{yz} &= z_1 / \Delta F_y \\
\alpha_{zy} &= y_2 / \Delta F_z \\
\alpha_{zz} &= z_2 / \Delta F_z
\end{align*}
\]  \hspace{1cm} (2-13)

Here \(y_1\) and \(z_1\) are displacements of the journal center from its static equilibrium position in vertical and horizontal directions respectively, on the application of a static incremental load \(\Delta F_y\) in the vertical direction; and \(y_2\) and \(z_2\) are displacements corresponding to a static incremental load \(\Delta F_z\) in the horizontal direction. This method can be applied to any type of bearing since the estimation of stiffness requires the establishment of a relationship between the force and the corresponding displacement.

### 2.3.2 Methods Using Dynamic Load

Dynamic load methods have been the most researched and widely used in the identification of dynamic bearing parameters in the last 45 years [4]. Their major
advantages are that they can be readily implemented on a real machine and the excitation can be applied either to the journal or to the bearing housing depending on practical constraints.

For the rigid rotor case, when the excitation is applied to the journal (Figure 2-2), the fluid-film dynamic equation can be written as

\[
\begin{bmatrix} m_{yy} & m_{yz} \\ m_{zy} & m_{zz} \end{bmatrix}\begin{bmatrix} \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} c_{yy} & c_{yz} \\ c_{zy} & c_{zz} \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} + \begin{bmatrix} k_{yy} & k_{yz} \\ k_{zy} & k_{zz} \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} f_y - m(\ddot{y} + \dot{y}_B) \\ f_z - m(\ddot{z} + \dot{z}_B) \end{bmatrix}
\]

(2-14)

where \( m \) is the mass of the journal, \( y \) and \( z \) represent the motion of the journal center from its equilibrium position relative to the bearing center, and \( y_B \) and \( z_B \) are the components of the absolute displacement of the bearing center in vertical and horizontal directions, respectively. In this case, the origin of the coordinate system is assumed to be at the static equilibrium position, so that gravity does not appear explicitly in the equation of motion. There will be one equation of this form for each of the bearings and the terms \( y_B, z_B \) represent the motion of the supporting structure. For the case of a rigid rotor with bearings on a rigid support, equation (2-14) can be expressed in the form

\[
M_B \ddot{\bar{q}} + C_B \dot{\bar{q}} + K_B \bar{q} = f - M_R \ddot{\bar{q}}
\]

(2-15)

The subscripts R and B refer to the rotor and bearings, respectively. On collecting the terms together, we get

\[
(M_B + M_R)\ddot{\bar{q}} + C_B \dot{\bar{q}} + K_B \bar{q} = f
\]

(2-16)

The overall system mass, damping and stiffness matrices can be formed by adding the separate contributions of the bearings and rotor in equation (2-16). This form was used by Arumugam et al. (1995) [6] to extract \( K_B \) and \( C_B \) in terms of the known and measurable quantities such as the rotor model, forcing and corresponding response. The
sinusoidal response of a rotor at speed $\Omega$ is studied using the modified form of this equation (2-16), and the response is of the form

$$q = Q e^{j\Omega t}$$

The governing equation of motion is given by

$$\begin{bmatrix} -M\Omega^2 + j\Omega C + K \end{bmatrix} Q = F_u = [Z(\Omega)]Q$$

(2-17)

where $[Z(\Omega)]$ is the dynamic stiffness matrix, $F_u$ is the unbalance force, and $\Omega$ is the rotational frequency of the rotor.

Figure 2-2. A non-floating bearing housing and a rotating journal.

2.3.3 Methods Using an Excitation Force

The application of a calibrated force to the journal can only rarely be applied in practical situations. Glienicke (1966–67) [7] adopted the technique of exciting the floating bearing bush (housing) sinusoidally in two mutually perpendicular directions (Figure 2-3) and measuring the amplitude and phase of the resulting motions in each case. The stiffness and damping coefficients were then calculated from the frequency-domain equations.
Morton (1971) [8] devised a measurement using the receptance coefficient method procedure for the estimation of the dynamic bearing characteristics. He excited the lightweight floating bearing bush by using very low forcing frequencies, \( \omega \) (10 and 15 Hz). Assuming the inertia force due the fluid film and bearing housing masses to be negligible, and for sinusoidal motion, equation (2-14) may be written as

\[
\begin{bmatrix}
z_{yy} & z_{yz} \\
z_{zy} & z_{zz}
\end{bmatrix}
\begin{bmatrix}
Y \\
Z
\end{bmatrix} = \begin{bmatrix} F_y \\ F_z \end{bmatrix}
\]

with

\[
z = k + j\omega c
\]

where \( Y \) and \( Z \) are complex displacements and \( F_y \) and \( F_z \) are complex forces in the vertical and horizontal directions, respectively. In equation (2-18) \( k \) represents the effective bearing stiffness coefficient, since while estimating the bearing dynamic stiffness, \( z \), the fluid-film added-mass and journal mass effects contribute to the real part of the dynamic stiffness and the effective stiffness is estimated.

Figure 2-3. A floating bearing housing and a fixed rotating shaft.
Someya (1976) [9], Hisa et al. (1980) [10] and Sakakida et al. (1992) [11] identified the dynamic coefficients of large-scale journal bearings by using simultaneous sinusoidal excitations on the bearing at two different frequencies and measuring the corresponding displacement responses. This is called the two-directional compound sinusoidal excitation method and all eight bearing dynamic coefficients can be obtained from a single test. When the journal is vibrating about the equilibrium position in a bearing, the dynamic component of the reaction force of the fluid film can be expressed by equation (2-18). If the excitation force and dynamic displacement are measured at two different excitation frequencies under the same static state of equilibrium and ignoring the fluid-film added-mass effects equation (2-18) can be solved for the eight unknown coefficients as

\[
\begin{bmatrix}
Y_1 & Z_1 & j\omega_1 Y_1 & j\omega_1 Z_1 \\
Y_2 & Z_2 & j\omega_2 Y_2 & j\omega_2 Z_2
\end{bmatrix}
\begin{bmatrix}
k_{yy} \\
k_{yz} \\
k_{zy} \\
k_{zz}
\end{bmatrix}
\begin{bmatrix}
F_{y1} - m_B \omega_1^2 Y_1 \\
F_{y2} - m_B \omega_2^2 Y_2 \\
c_{yy} \\
c_{yz}
\end{bmatrix}
= 0
\]

(2-19)

\[
\begin{bmatrix}
Y_1 & Z_1 & j\omega_1 Y_1 & j\omega_1 Z_1 \\
Y_2 & Z_2 & j\omega_2 Y_2 & j\omega_2 Z_2
\end{bmatrix}
\begin{bmatrix}
k_{zy} \\
k_{zz}
\end{bmatrix}
\begin{bmatrix}
F_{z1} - m_B \omega_1^2 Z_1 \\
F_{z2} - m_B \omega_2^2 Z_2 \\
c_{zy} \\
c_{zz}
\end{bmatrix}
= 0
\]

where \( \omega \) is the external excitation frequency and the subscripts 1 and 2 represent the measurements corresponding to two different excitation frequencies. Since equation (2-19) corresponds to eight real equations, the bearing dynamic coefficients can be obtained on substituting the measured values of the complex quantities \( F_y, F_z, Y, Z, Y_B \) and \( Z_B \).
2.3.4 Method Using Unbalance Mass

From a practical point of view, the simplest method of excitation is to use an unbalance force as this requires no sophisticated equipment for the excitation, and it is relatively easy to identify the rotational speed dependency of the bearing dynamic characteristics. However, the disadvantage is that information is limited to the synchronous response. Nevertheless, since this is the predominant requirement, the application of forces due to unbalance is extremely useful. Hagg and Sankey (1956, 1958) [12-13] were among the first to use the unbalance force only for experimentally measuring the oil-film elasticity and damping for the case of a full journal bearing. They used the experimental measurement technique of Stone and Underwood (1947) [14] in which they used the vibration diagram to measure the vibration amplitude and phase of the journal motion relative to the bearing housing. The direct stiffness and damping coefficients were only considered along the principal directions in their study (i.e., major and minor axes of the journal elliptical orbit).

The measured unbalance response whirl orbit gives the stiffness and damping coefficients. However, the results represent some form of effective rotor-bearing coefficients and not the true film coefficients as the cross-coupled coefficients are ignored. Duffin and Johnson (1966–67) [15] employed a similar approach to that of Hagg and Sankey to identify bearing dynamic coefficients of large journal bearings. They proposed an iterative procedure to calculate all eight coefficients. Four equations can be written relating the measured values of displacement amplitude and phase $Y$, $Z$, $\phi_y$ and $\phi_z$, together with the known value of the unbalance force, $F$, and four stiffness coefficients (obtained from static locus curve method; Mitchell et al., 1965–66) used to obtain the four unknown damping coefficients. This allows the solution of two sets of
simultaneous equations having two equations in each set. The results had a greater accuracy than the method (Glienecke, 1966–67) in which two sets of four simultaneous equations were used to obtain the stiffness and damping coefficients.

Murphy and Wagner (1991) [16] presented a method using a synchronously orbiting intentionally eccentric journal as the sole source of excitation for the extraction of stiffness and damping coefficients for hydrostatic bearings. The relative whirl orbits across the fluid film were made to be elliptic with asymmetric stiffness in the test bearing’s supporting structure. The study considered the bearing coefficients to be skew-symmetric and the elliptic nature was utilized in the data reduction process. Adams et al. (1992) [17] and Sawicki et al. (1997) [18] utilized experimentally measured responses corresponding to at least three discrete orbital frequencies, for a given operating condition to obtain twelve dynamic coefficients (stiffness, damping and added-mass) of hydrostatic and hybrid journal bearings, respectively. They assumed that the bearing dynamic coefficients are independent of frequency of excitation. The estimation equation was similar to equation (2-19) except the rotor mass was ignored and fluid film added-mass coefficients were considered. A confidence in the measurements was obtained by employing dual piezoelectric/strain gage load/displacement measuring systems. The difference between these two sets of dynamic force measurements was typically less than 2%. The test spindle (double-spool-shaft) had a provision for a circular orbit motion of adjustable magnitude with independent control over spin speed, orbit frequency and whirl direction. The least-squares linear regression fit to all frequency data points over the tested frequency range was used to obtain the bearing dynamic coefficients.
2.3.5 Methods Using an Impact Hammer

Until the early 1970s, the common method to obtain the dynamic characteristics of systems involved using sinusoidal excitation [4]. Downham and Woods (1971) [19] proposed a technique using a pendulum hammer to apply an impulsive force to a machine structure. Although they were interested in vibration monitoring rather than the determination of bearing coefficients, their work led to the idea that impulse testing could be capable of exciting all the modes of a linear system.

Nordmann (1975) [20] and Nordmann and Schöllhorn (1980) [21] identified the stiffness and damping coefficients of journal bearings by modal testing by means of the impact method wherein, a rigid rotor, running in journal bearings was excited by an impact hammer. Two independent impacts first in the vertical direction and then in the horizontal direction were applied to the rotor and the corresponding responses were measured. A transformation of input signals (forces) and output signals (displacements of the rotor) into the frequency domain was then carried out and the four complex FRFs were calculated. The bearing dynamic parameters were assumed to be independent of the frequency of excitation. The analytical FRFs, which depend on the bearing dynamic coefficients, were fitted to the measured FRFs. An iterative fitting process results in the stiffness and damping coefficients.

Zhang et al. (1992a) [22] fitted the measured FRFs to those calculated theoretically so as to obtain the eight bearing dynamic coefficients. They also quantitatively analyzed the influence of noise and measurement errors on the estimation in order to improve the accuracy of estimated bearing dynamic coefficients. They used a half-sinusoid impulse excitation and with a different level of noise added to the resulting response to test their algorithm and averaged the frequency responses to reduce the uncertainty due to noise in
the response. To reduce the effect of phase-measurement errors, they defined an error function using just the amplitude components of the FRFs. This non-linear objective function was then used to estimate the bearing parameters by an iterative procedure. It was also demonstrated by then that it was necessary to remove the unbalance response from the signal when an impact test was used, especially at higher speeds of operation, and they concluded this to be the reason for the scatter in the results by impact excitation, as compared to the discrete frequency harmonic excitation.

This method is time-consuming though since impact tests have to be conducted for each rotor speed at which bearing dynamic parameters are desired. In general, the amount of information that can be extracted from a single impulse test is limited as the governing equations for a bearing include coupling between the two perpendicular directions. Errors in the estimation will be greater for the case when bearing dynamic coefficients are functions of external excitation frequency as compared to the estimation from functions of rotor rotational frequency. Also impulse testing may lead to underestimation of input forces when applied to a rotating shaft as a result of the generation of friction-related tangential force components and, further, is prone to poor signal-to-noise ratios because of the high crest factor.

### 2.3.6 Methods Using Unknown Excitation

In industrial machinery, the application of a calibrated force is difficult to apply. Due to residual unbalance, misalignment, rubbing between the rotor and stator, aerodynamic forces, oil whirl, oil whip and instability, inherent forces are present in the system and these render the assessment of the forcing impossible. Adams and Rashidi (1985) [23] used the static loading method to measure bearing stiffness coefficients and determined orbital motion at an adjustable threshold speed to extract bearing damping
coefficients by inverting the associated eigenproblem. The approach stems from the physical requirement for an exact internal energy balance between positive and negative damping influences at an instability threshold. The approach was illustrated by simulation and does not require the measurement of dynamic forces.

Lee and Shih (1996) [24] found rotor parameters including bearing dynamic coefficients, shaft unbalance distribution and disk eccentricity in flexible rotors by presenting an estimation procedure based on the transfer matrix method. The relations between measured response data and the known system parameters were used to formulate the normal equations. The parameter estimation was then performed using the least squares method by assuming that the bearing dynamic coefficients were constant at close spin speeds.
CHAPTER 3

ROTOR DYNAMIC ANALYSIS OF MICRO SPINDLE

The aim of this project is to rotate a spindle supported by air bearings at up to 500,000 rpm, with sub-micrometer runout. The 1/8th inch diameter tool shank is used as a spindle shaft. As mentioned before, the only viable way to obtain satisfactory runout was to use the tool shank itself as the spindle shaft. An air turbine is used as a driving system for the spindle. Thus, the only viable way to assemble air turbine is to manufacture the turbine integral with the spindle, which is shown in Figure 3-1. Also from the practicality point of view the micro-spindle must accept a variety of tools with minimal time and effort required for tool change. Rotordynamics of high-speed flexible shafts is influenced by the complex interaction between the unbalance forces, bearing stiffness and damping, inertial properties of the rotor, gyroscopic stiffening effects, aerodynamic coupling, and speed-dependent system critical speeds. For stable high-speed operations, bearings must be designed with the appropriate stiffness and damping properties, selected on the basis of a detailed rotordynamic analysis of the rotor system. The two types of rotor dynamic analyses that are used for high-speed thin spindle are rigid rotor analysis and finite element analysis. The dynamic behavior of a spindle is analyzed in order to find critical speeds, unbalance response and linear stability margins by these methods.
3.1 Rigid Rotor Analysis

Rigid rotor analysis was initially used to get the rotor unbalance response. The air bearings were located on either side of the center of mass, as shown in Figure 3-2. In addition, center of mass is found by solid model ProE software.
The rigid rotor has 4 degrees of freedom (DOF) represented by two displacements \(V, W\) and two rotations \(B, \Gamma\) of the center of mass. The following equation, (derivation can be found in chapter 2), was used for a rigid rotor subjected to unbalance.

\[
M\ddot{q} + (C - \Omega G)\dot{q} + Kq = \bar{Q}
\]  

(2-1)

where \(M\), \(C\), \(G\), \(K\) are mass, damping, gyroscopic and stiffness matrices, respectively, \(\bar{Q}\) is the force vector. Expressions of these matrices can be found in chapter 2. \(\Omega\) is constant spin frequency.

In order to use equation (2-1) in the rotor orbit analysis, following procedure is applied. From chapter 2, it is known that displacement vector for 4 DOF is the following:

\[
\bar{q}^T = [V \ W \ B \ \Gamma]
\]

(3-1)

The shaft unbalance leads to harmonic synchronous excitation. Hence the displacement or response vector can be expressed as the following:

\[
\begin{align*}
\bar{q} &= \begin{bmatrix} V_c \\ W_c \\ B_c \\ \Gamma_c \end{bmatrix} \cos(\Omega t) + \begin{bmatrix} V_s \\ W_s \\ B_s \\ \Gamma_s \end{bmatrix} \sin(\Omega t) \\
&= \begin{bmatrix} \Omega \end{bmatrix} \\
\end{align*}
\]

(3-1)

As a result, first and second derivatives will have the following forms, respectively.

\[
\begin{align*}
\dot{q} &= -\Omega \begin{bmatrix} V_c \\ W_c \\ B_c \\ \Gamma_c \end{bmatrix} \sin(\Omega t) + \Omega \begin{bmatrix} V_s \\ W_s \\ B_s \\ \Gamma_s \end{bmatrix} \cos(\Omega t) \\
&= \begin{bmatrix} \Omega \end{bmatrix} \\
\end{align*}
\]

(3-2)

\[
\begin{align*}
\ddot{q} &= -\Omega^2 \begin{bmatrix} V_c \\ W_c \\ B_c \\ \Gamma_c \end{bmatrix} \cos(\Omega t) - \Omega^2 \begin{bmatrix} V_s \\ W_s \\ B_s \\ \Gamma_s \end{bmatrix} \sin(\Omega t) \\
&= \begin{bmatrix} \Omega \end{bmatrix} \end{align*}
\]

(3-3)
After substituting for \( M, C, G, K, Q \) into equation (2-1) using (3-1), (3-2), (3-3) and rearranging sine and cosine terms, and using harmonic balance, the following expression can be obtained:

\[
\begin{bmatrix}
 k_{yy}^1 + k_{yy}^2 - m\xi^2 & k_{yz}^1 + k_{yz}^2 & a_k^1 k_{yy}^1 + a_k^2 k_{yy}^2 & -(a_k^1 k_{yy}^1 + a_k^2 k_{yy}^2) \\
 k_{yz}^1 + k_{yz}^2 & k_{zz}^1 + k_{zz}^2 - m\Omega^2 & a_k^1 k_{yz}^1 + a_k^2 k_{yz}^2 & -(a_k^1 k_{yz}^1 + a_k^2 k_{yz}^2) \\
 a_k^1 k_{yy}^1 + a_k^2 k_{yy}^2 & a_k^1 k_{yz}^1 + a_k^2 k_{yz}^2 & 2k_{yy}^1 k_{yy}^2 + a_k^1 k_{zz}^2 - \Omega^2 d & -2k_{zz}^2 k_{yy}^2 - \Omega^2 d \\
 -\Omega(a_k^1 k_{yy}^1 + a_k^2 k_{yy}^2) & -\Omega(a_k^1 k_{yz}^1 + a_k^2 k_{yz}^2) & \Omega(a_k^1 c_{yy}^1 + a_k^2 c_{yy}^2) & \Omega(a_k^1 c_{yy}^1 + a_k^2 c_{yy}^2) \\
 -\Omega(a_k^2 c_{yy}^1 + a_k^2 c_{yy}^2) & -\Omega(a_k^2 c_{yy}^1 + a_k^2 c_{yy}^2) & \Omega(a_k^2 c_{yy}^1 + a_k^2 c_{yy}^2) & \Omega(a_k^2 c_{yy}^1 + a_k^2 c_{yy}^2) \\
 \Omega(a_k^1 c_{yy}^1 + a_k^2 c_{yy}^2) & \Omega(a_k^1 c_{yy}^1 + a_k^2 c_{yy}^2) & \Omega(a_k^1 c_{yy}^1 + a_k^2 c_{yy}^2) & -\Omega^2 p \\
 -\Omega(a_k^2 c_{yy}^1 + a_k^2 c_{yy}^2) & -\Omega(a_k^2 c_{yy}^1 + a_k^2 c_{yy}^2) & -\Omega(a_k^2 c_{yy}^1 + a_k^2 c_{yy}^2) & -\Omega(a_k^2 c_{yy}^1 + a_k^2 c_{yy}^2)
\end{bmatrix}
\]

\[
\begin{bmatrix}
 V_c \\
 W_c \\
 B_c \\
 \Gamma_c \\
 V_s \\
 W_s \\
 B_s \\
 \Gamma_s
\end{bmatrix}
= m\Omega^2
\]

\[
\begin{bmatrix}
 \eta \\
 \zeta \\
 0 \\
 0 \\
 -\zeta \\
 \eta \\
 0 \\
 0
\end{bmatrix}
\]

(3-4)
where $a_1$ and $a_2$ are the distance of the bearings from center of mass. $k_{yy}$, $k_{yz}$, $k_{zy}$, $k_{zz}$, and $c_{yy}$, $c_{yz}$, $c_{zy}$ and $c_{zz}$ are stiffness and damping coefficients of each bearing, respectively. $I_d$ and $I_p$ are the polar and diametric inertia, respectively.

The charts from ‘Rotor-Bearing Dynamics Design Technology’ [25], design handbook for fluid film type bearings were initially used to evaluate the damping and the stiffness coefficients. In order to use these charts, bearing length and diameter ratio was assumed to be two. Mass and gyroscopic matrices were found by hand calculation, which were later entered in the MathCAD program. The two types of forces on the system are the unbalance force and the cutting force. As the cutting force is much smaller than the unbalance force, it was neglected. An unbalance eccentricity of $e_u=0.000002$ inch was used initially to evaluate the unbalance force. The solution procedure was implemented in MathCAD to find the unbalance respond at the bearing locations.

The rotor orbits at the two bearing supports at 500,000 rpm for a rotor with a mass=$2.525\times10^{-5}$ lbf-sec^2/in, $I_p=4.187\times10^{-6}$ lbf-sec^2-in, $I_d=4.894\times10^{-8}$ lbf-sec^2-in and an unbalance eccentricity of 0.000002 inch are shown in Figure 3-3. The air bearing for this configuration has the following parameters, Table3-1.

Table 3-1. Bearing parameters at 500,000 rpm ($\Omega=52360$ rad/sec).

<table>
<thead>
<tr>
<th></th>
<th>$K_{yy1,2}=950.4$ lbf/in</th>
<th>$K_{yz1,2}=99.1$ lbf/in</th>
<th>$K_{zy1,2}=93.46$ lbf/in</th>
<th>$K_{zz1,2}=1050.4$ lbf/in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_{yy1,2}=0.021$ lbf-sec/in</td>
<td>$C_{yz1,2}=-0.004$ lbf-sec/in</td>
<td>$C_{zy1,2}=0.005$ lbf-sec/in</td>
<td>$C_{zz1,2}=0.106$ lbf-sec/in</td>
</tr>
</tbody>
</table>
Figure 3-3. Rotor orbits at two bearing supports at 500,000 rpm.
Even for the 1,000,000 rpm and same tool with same rigid rotor parameters, displacement of rotor in bearings is small (see figure 3-4).

Figure 3-4. Rotor orbits at the two bearing supports at 1,000,000 rpm.
One of the most critical places is also tool end, next to the air turbine. Thus, rotor orbits should be found at tool end. As it can be seen from Figure 3-5, runout in both directions is small at the tool end ($2 \times 10^{-6}$ inch).

![Figure 3-5. Rotor orbits at the tool end at 500,000 rpm.](image)

The radial clearance was chosen 0.0002 inch. The eccentricity ratio $\epsilon$ will be 0.01 ($\epsilon = e/c$) for the chosen unbalance.

Figure 3-6 shows that why figure 3-3 and figure 3-4 is same. It can be seen that after 500,000 rpm runout is nearly same and it is $2 \times 10^{-6}$ inch. In addition this figure shows that highest amplitude is between 100,000 rpm and 120,000 rpm. Later, it will be shown that the rigid body critical speeds are 105,042 rpm and 115,231 rpm.
Figure 3-6. Orbit amplitude at 1\textsuperscript{st}, 2\textsuperscript{nd} bearing and tool tip.

Linear stability analysis has been performed based on energy dissipated at bearings. System is stable, if total energy dissipated is negative. Based on this criteria the system is stable over the entire speed range. For 500,000 rpm, total work (energy dissipated) per cycle at each bearing is found \(-4.24253\times10^{-8}\) (see Appendix B).

Sample MathCAD program of rotor orbits can be found in Appendix B for a given spindle speed, 500,000rpm.

The bearing analysis code was coupled with the rotordynamics code in MATLAB, to efficiently explore the bearing/rotor design space. A direct interface to the bearing code used in XLTiltPadHGB was achieved by writing a MATLAB function. Input structures, viz: geometry, fluid, dynamic, operating, numerical were used to submit bearing parameters.

Thus the data plotting and visual inspection of bearing performance was facilitated by the MATLAB interface. This helped in the determination of factors that determine stiffness and damping. For example, stiffness as a function of the rotor eccentricity is
plotted in Figure 3-7. In this case, a constant gravity force was applied to the rotor and the bearing contained four pads.

The bearing stiffness and damping coefficients were passed from the bearing design code to the rotordynamic code. These coefficients are influenced by the bearing design parameters, and influence the rotordynamics. The rotor orbits within the bearings are calculated using the rotordynamic model and these orbits and their characteristics determine the rotor performance.

![Figure 3-7. Stiffness, Kyy versus rotor eccentricity.](image)

One of the most important aspects of research was to analyze rotor performance at critical speeds. Thus, rotor performance has been investigated at critical speeds. In order to find critical speeds, the following procedure has been adopted. It is known that eigenvalues can be found from equation with first order form. However equation of motion for rotor-bearing system is in second order. So equation (2-1) was modified in order to find eigenvalues.

\[
M \ddot{\mathbf{q}} + (C - \Omega \mathbf{Q}) \dot{\mathbf{q}} + K \mathbf{q} = \mathbf{F}
\]

(2-1)

\[
M^* = \begin{bmatrix}
0 & M \\
M & C
\end{bmatrix}

K^* = \begin{bmatrix}
-M & 0 \\
0 & K
\end{bmatrix}
\]
\[
\begin{align*}
\bar{x} &= \begin{bmatrix} \bar{q} \\ \bar{\theta} \end{bmatrix} \\
\bar{X} &= \begin{bmatrix} \bar{Q} \\ 0 \end{bmatrix}
\end{align*}
\quad (3-5)
\]

After entering equations (3-5) into (2-1), results in:

\[
M^* \bar{x} + K^* \bar{x} = \bar{X}
\quad (3-6)
\]

In order to find eigenvalues, right hand side of equation (3-6) is set to zero, and a harmonic solution \( \bar{x} = e^{j\omega t} \) and \( \bar{\theta} = j\omega e^{j\omega t} \) is assumed. After rearranging equation (3-6), the standard E.V.P (eigenvalue problem) of the term \( \Delta\bar{x} = \lambda\bar{x} \) can be set up, as shown below:

\[
(-jK^* - M^*)\bar{x}_0 = \lambda\bar{x}_0 \quad \text{where} \quad \lambda = \frac{1}{\omega}
\quad (3-7)
\]

In order to find all eight eigenvalues (four of which are complex conjugates of the other four) for a given spindle speed, a MathCAD program was generated to solve expression obtained above. The rotor whirl map was finally generated using these eigenvalues.

The whirl map for a rotor with a mass=2.525x10^{-4} lbf·sec^2/in, Ip=4.187x10^{-6} lbf·sec^2-in, Id=4.894x10^{-8} lbf·sec^2-in and an unbalance eccentricity of 0.000002 inch is shown in Figure 3-12; wherein the air bearing has the following parameters, Table 3-2.

<table>
<thead>
<tr>
<th>Kyy1,2</th>
<th>Kyz1,2</th>
<th>Kzy1,2</th>
<th>Kzz1,2</th>
<th>[lbf/in]</th>
<th>( \Omega ) [rad/sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1069.2</td>
<td>99.1</td>
<td>-67.52</td>
<td>1069.2</td>
<td></td>
<td>62831</td>
</tr>
<tr>
<td>950.4</td>
<td>99.1</td>
<td>-93.46</td>
<td>1050.4</td>
<td></td>
<td>52000</td>
</tr>
<tr>
<td>831.6</td>
<td>105.04</td>
<td>-109.4</td>
<td>831.6</td>
<td></td>
<td>42000</td>
</tr>
<tr>
<td>751.9</td>
<td>115.92</td>
<td>-151.28</td>
<td>751.9</td>
<td></td>
<td>32000</td>
</tr>
<tr>
<td>700.8</td>
<td>118.8</td>
<td>-118.8</td>
<td>700.8</td>
<td></td>
<td>22000</td>
</tr>
<tr>
<td>623.4</td>
<td>178.2</td>
<td>-115.8</td>
<td>623.4</td>
<td></td>
<td>12000</td>
</tr>
<tr>
<td>594</td>
<td>190.08</td>
<td>-91.28</td>
<td>594</td>
<td></td>
<td>9500</td>
</tr>
<tr>
<td>159.2</td>
<td>137.22</td>
<td>-85.34</td>
<td>159.2</td>
<td></td>
<td>2000</td>
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</table>
Table 3-2. Continued

<table>
<thead>
<tr>
<th>Kyy1,2</th>
<th>Kyz1,2</th>
<th>Kzy1,2</th>
<th>Kzz1,2</th>
<th>[lbf/in]</th>
<th>Ω</th>
<th>[rad/sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.7</td>
<td>83.76</td>
<td>-53.76</td>
<td>36.7</td>
<td></td>
<td>500</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cyy1,2</th>
<th>Cyz1,2</th>
<th>Czy1,2</th>
<th>Czz1,2</th>
<th>[lbf-sec/in]</th>
<th>Ω</th>
<th>[rad/sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.015</td>
<td>-0.003</td>
<td>0.004</td>
<td>0.088</td>
<td></td>
<td>62831</td>
<td></td>
</tr>
<tr>
<td>0.021</td>
<td>-0.004</td>
<td>0.005</td>
<td>0.106</td>
<td></td>
<td>52000</td>
<td></td>
</tr>
<tr>
<td>0.033</td>
<td>-0.005</td>
<td>0.007</td>
<td>0.131</td>
<td></td>
<td>42000</td>
<td></td>
</tr>
<tr>
<td>0.053</td>
<td>-0.009</td>
<td>0.01</td>
<td>0.158</td>
<td></td>
<td>32000</td>
<td></td>
</tr>
<tr>
<td>0.086</td>
<td>-0.014</td>
<td>0.025</td>
<td>0.23</td>
<td></td>
<td>22000</td>
<td></td>
</tr>
<tr>
<td>0.18</td>
<td>-0.042</td>
<td>0.077</td>
<td>0.383</td>
<td></td>
<td>12000</td>
<td></td>
</tr>
<tr>
<td>0.287</td>
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<td>0.129</td>
<td>0.402</td>
<td></td>
<td>9500</td>
<td></td>
</tr>
<tr>
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<td>-0.011</td>
<td>0.018</td>
<td>0.322</td>
<td></td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>1.838</td>
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<td>0.017</td>
<td>0.147</td>
<td></td>
<td>500</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3-8. Whirl map.

It can be seen from graph (Figure 3-8), critical speeds are 11000 rad/sec (105,042 rpm) and 12067 rad/sec (115,231 rpm).

MathCAD program for different air bearing stiffness and damping values, found by using XLTiltPadHGB, can be found in Appendix C.
3.2 Finite Element Analysis

The rotordynamics analysis of the rotor-bearing system was performed using a special-purpose code. The model shown in Figure 3-9 has 10 finite elements, 11 nodes, with each node having 4 degrees of freedom. The rigid body and flexural natural frequencies and model shapes can be computed from this model. To use finite element method it is assumed stiffness coefficient fixed and no cross coupling stiffness. The model parameters are bearing stiffness = 1,000 lb/in (fixed), bearing span = 0.5 in, shaft dia = 0.125 in, shaft length = 0.925 in and the shaft material is tungsten carbide. In addition, $me_u = 10^{-6}$ oz-in is used in FE model. As a result four critical speeds are found for the following parameters.

![Figure 3-9. Finite element model.](image)

The rigid body critical speeds are 107,758, 115,298, while the flexural critical speeds are 2,042,349 and 4,955,776 rpm.

![Figure 3-10. Rigid body mode 1, 107758 rpm.](image)
Figure 3-11. Rotor analysis at 115298 rpm. A) Rigid body mode 2, B) Potential energy distribution.

Mode No. = 2, Critical Speed = 115298 rpm = 1921.84 Hz
Potential Energy Distribution (s/w=1)
Overall: Shaft(S) = 0.04%, Bearing(Brg) = 99.96%

A.

Figure 3-12. Rotor analysis at 2,042,349 rpm. A) Flexural mode 1, B) potential energy distribution.

Critical Speed = 2042349 rpm = 34039.16 Hz

A.
Figure 3-13. Flexural mode 2, 4,955,776 rpm.

Figure 3-14. Critical speed map.
The critical speed map in Figure 3-14 shows that the flexural modes are unaffected by bearing stiffness, while the two rigid body modes increase with bearing stiffness.

There are three critical points on the tool such as; two bearing positions and tool tip. As a result the following analysis are found for these positions.

Figure 3-15. 1st bearing. A) and B) Unbalance response, C) Amplitude and phase lag, D) Nyquist plot for displacement.
Figure 3-15. Continued
Figure 3-16. 2\textsuperscript{nd} bearing. A) and B) Unbalance response, C) Amplitude and phase lag, D) Nyquist plot for displacement.
Figure 3-16. Continued
Figure 3-17. Tool tip. A) Unbalance response, B) Amplitude and phase lag, C) Nyquist plot for displacement.
C.

Figure 3-17. Continued

A.

Figure 3-18. Shaft orbits. A), B), C), D), E), F) are shaft orbits as a function of speed.
Shaft Response - due to shaft 1 excitation
Rotor Speed = 100000 rpm, Response - FORWARD Precession
Max Orbit at stn 11, substn 1, with $a = 1.1509 \times 10^{-6}$, $b = 1.1509 \times 10^{-6}$

Figure 3-18. Continued
Shaft Response - due to shaft 1 excitation
Rotor Speed = 300000 rpm, Response - FORWARD Precession
Max Orbit at stn 11, substn 1, with $a = 5.0653E-006$, $b = 5.0653E-006$

Figure 3-18. Continued
Figure 3-18. Continued.

Figure 3-19. Stability Map (Note: Negative log decrements indicate instability).

3.3 Summary

Rigid body critical speeds (107,758 and 115,298 rpm) found by FEA are very close to the rotor analysis critical speeds (105,042 and 115,231 rpm) calculated by rigid rotor
analysis. The difference occurs, because in FEA analysis cross coupling stiffness terms and damping terms are eliminated and stiffness doesn’t change with rotor speed.

For rigid rotor analysis eccentricity ratio is calculated $\epsilon=0.01$, which can be questionable to get this value in practice, and it leads $m \times e_u = 1.22 \cdot 10^{-8}$ oz-in. So unbalance response analysis is performed again with $\epsilon=0.1$. Figure 3-20 shows that even with this eccentricity ratio sub micrometer runout has been obtained for a given stiffness and damping parameters.

![Orbit Amplitude](image)

Figure 3-20. Orbit amplitude at 1$^{st}$, 2$^{nd}$ bearing and tool tip, $\epsilon=0.1$.

Both rigid rotor analysis (see appendix B) and FEA (see figure 3-19) shows system is stable.
CHAPTER 4
EXPERIMENTAL IDENTIFICATION OF BEARING PARAMETERS

In order to ensure proper operation, the vibration phenomena, which the rotors supported by fluid film bearings are especially subject to, has to be properly predicted. Knowledge of dynamic coefficients, stiffness and damping of a bearing prior to its installation and operation can be highly influential in the operation costs of the final machine. Both an analytical and experimental approach can be employed to study the dynamic behavior of bearings. Numerical techniques and computer-based simulation are usually used to perform analytical studies. The current research has used XITiltPad program to identify bearing parameters.

The test set up aims at analyzing dynamic behavior of a tilted pad air bearing, which will be used in this research, and comparing the experimental results with analytical results. For the time being, experiments used a commercial air bearing as the tilted pad air bearing is in the process of being designed and for the initial studies; a flexure-supported ball bearing (see figure 4-1) is being used. Due to the difficulties found in exciting the rotor-bearing system and in the measurement of force and displacement data, experimental testing on fluid film bearings is known to be complex. The bearing parameters can be identified experimentally by six different methods as mentioned before. Since the research deals with a micro spindle supported by a small air bearing, the test setup will be also small making it extremely hard to use loading in order to excite the system. As a result, the viable way is using unbalance mass method.
4.1 Method of Measurement

The principle of operation of the test rig is rather simple: The bearing to be tested is placed on a chassis (see figure 4-2), which serves to support displacement probes and is mounted on dynamometer (load cell), used for measuring forces. A shaft is placed inside the bearing and one side of shaft is mounted to the high-speed spindle. Figure 4-3 shows a schematic of the test rig.

Figure 4-1. Flexure supported ball bearing: 1) Flexure, 2) Ball bearing.

Figure 4-2. A chassis.
The rotating part of the test bearing has been given an intentional eccentricity at the test bearing location. When the shaft rotates, the eccentricity generates an orbital pattern synchronous with shaft speed.

Since the shaft is driven with a synchronous harmonic load, the resulting shaft motion will, in general, be elliptic. Therefore, relative displacements describing ellipse as a function of time is in following form:

$$y(t) = a\cos(\omega t) + b\sin(\omega t)$$  \hspace{1cm} (4-1)

$$z(t) = g\cos(\omega t) + h\sin(\omega t)$$  \hspace{1cm} (4-2)

Following equations will be used in equation of motion:

$$\dot{y}(t) = -a\omega\sin(\omega t) + b\omega\cos(\omega t)$$  \hspace{1cm} (4-1a)

$$\dot{z}(t) = -g\omega\sin(\omega t) + h\omega\cos(\omega t)$$  \hspace{1cm} (4-2a)
The four coefficients \(a, b, g\) and \(h\) are termed Fourier coefficients, and \(\omega\) is the tester speed in radians per second. They are obtained from the synchronous components of complex frequency spectrums computed for the \(y\) and \(z\) displacements.

The air bearing will respond with reaction forces which will be read by dyno to the off centered rotor movement. The same procedure applied to the load data which load cells read and following equations are derived:

\[
F_y(t) = m(\cos \omega t) + n(\sin \omega t) \quad (4-3)
\]

\[
F_z(t) = p(\cos \omega t) + q(\sin \omega t) \quad (4-4)
\]

Equation of motion is the following:

\[
\begin{bmatrix}
F_y \\
F_z
\end{bmatrix} = \begin{bmatrix}
K_{yy} & K_{yz} \\
K_{zy} & K_{zz}
\end{bmatrix} \begin{bmatrix}
y \\
z
\end{bmatrix} + \begin{bmatrix}
C_{yy} & C_{yz} \\
C_{zy} & C_{zz}
\end{bmatrix} \begin{bmatrix}
y' \\
z'
\end{bmatrix} + \begin{bmatrix}
M_{yy} & M_{yz} \\
M_{zy} & M_{zz}
\end{bmatrix} \begin{bmatrix}
y'' \\
z''
\end{bmatrix} \quad (4-5)
\]

Inertia term can be neglected in equation, because shaft relative displacement is so small that inertia force will not contribute a lot for air film force. So new equation is the following:

\[
\begin{bmatrix}
F_y \\
F_z
\end{bmatrix} = \begin{bmatrix}
K_{yy} & K_{yz} \\
K_{zy} & K_{zz}
\end{bmatrix} \begin{bmatrix}
y \\
z
\end{bmatrix} + \begin{bmatrix}
C_{yy} & C_{yz} \\
C_{zy} & C_{zz}
\end{bmatrix} \begin{bmatrix}
y' \\
z'
\end{bmatrix} \quad (4-6)
\]

After substituting (4-1), (4-2), (4-1a), (4-2a), (4-3) and (4-4) into (4-6), the following equation is obtained:

\[
\begin{bmatrix}
b\omega & h\omega & a & g & 0 & 0 & 0 & 0 \\
-a\omega & -g\omega & b & h & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & b\omega & h\omega & a & g \\
0 & 0 & 0 & 0 & -a\omega & -g\omega & b & h
\end{bmatrix} \begin{bmatrix}
C_{yy} \\
C_{yz} \\
K_{yy} \\
K_{yz}
\end{bmatrix} = \begin{bmatrix}
m \\
n \\
p \\
q
\end{bmatrix} \quad (4-7)
\]
Since all these bearing parameters are functions of speed, it is necessary that the speeds span as wide a range as possible to give the best definition of the coefficients [16]. There is an example how to use equation (4-7) in Appendix D.

4.2 Design Process

The test rig was designed and developed relying on an existing dynamometer, as seen in Figure 4-4. Detailed information about dynamometer is presented in Appendix E. In order to facilitate a better understanding of the setup, it was divided into four parts that is the dyno; a main chassis (see Figure 4-5), which serves to mount displacement probes in order to measure shaft displacement and to mount the test bearing; a shaft; and a spindle, which is an electric driven motor and rotates at 50000 rpm (max.).

Figure 4-4. Test Setup: 1) Base, 2) Dyno, 3) Chassis, 4) Test bearing, 5) Shaft, 6) Spindle
The shaft motion is determined by displacement probes, and results can be read on PC by using fiber optic sensors (RC20) connected to the PC. Dyno is used to measure the forces in x and y directions respectively, and the data is sent to the PC using a charge amplifier. A NSK spindle drive controller adjusts the spindle speed. The complete measurement system is illustrated in Figure 4-6.
CHAPTER 5
CONCLUSION AND RECOMMENDATIONS

The current research provides initial results for the development of a comprehensive rotordynamics analysis to describe high-speed micro spindle vibrations. Results demonstrate that the air bearing characteristics and spindle residual unbalance levels dictate the critical speed placement, unbalance response, and the ability to limit the tool tip runout to sub-micrometer levels.

Results demonstrate that the 1/8\textsuperscript{th} inch tungsten carbide micro spindle with the integrally machined air turbine at one end and the end mill cutter at the other end can indeed operate with sub-micrometer runout at the tool tip for spindle speeds up to 1 million rpm. Air bearing stiffness used is about 2000 lb/in. Spindle residual unbalance level assumed is $10^{-6}$ oz-in (me=. 000001 oz-in). Residual unbalance two to three times this level will also be adequate with higher bearing stiffness. Turbine engine shafts weighing 100 lbs or greater are routinely balanced to 0.1 oz-in. Considering that the end mill cutter weighs about 0.07 lb, the recommended unbalance limits can be achieved in practice.

Next step in this research is experimental analysis, which was undertaken to understand behavior of the air bearing. Since the air bearing is yet to be fabricated, a simple test rig has been built to test the spindle mounted on ball bearings with a flexure support. The x-y displacements at the bearings, as a function of speed, are being used to evaluate the support stiffness characteristics. Our goals are to demonstrate the feasibility of the proposed parameter estimation scheme to evaluate support stiffness.
# Appendix A

## Chronological List of Papers on the Experimental Dynamic Parameter Identification of Bearings

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<tr>
<th>References</th>
<th>Bearing type</th>
<th>Type of excitation</th>
<th>Vibration response measured</th>
<th>Identified dynamic parameters</th>
</tr>
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<tr>
<td>Hagg and Senkey (1956, 1958)</td>
<td>HDJ</td>
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<td>Displ. (frequency)</td>
<td>Direct damping and stiffness</td>
</tr>
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<td>Incremental static load</td>
<td>Displacement</td>
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<td>Displ. (frequency)</td>
<td>Damping and uncertainty</td>
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<tr>
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<td>Displacement</td>
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The following abbreviations are used in the table: AGS, annular gas seal; ANS, annular seal; ALS, annular liquid seal; BS, brush seals; DS, damper seals; ER, electrorheological fluid; FAB, foil air; FTB, foil thrust; GDS, gas damper seal; GJ, gas journal; GS, gas seal; HCS, honeycombed seal; HDJ, hydrodynamic journal; HSJ, hydrostatic journal; HYJ, hybrid journal; LGS, long gas seal; LS, long seal; MB, magnetic; MD, metal mess bearing damper; PLS, plain liquid seal; RB, recirculating ball; RE, rolling element; SPR, springs; SQF, squeeze film; TPJ, tilting pad journal; TR, tapered roller.

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APPENDIX B
GENERAL RIGID ROTOR SOLUTION 1
Input Rigid Rotor Parameters

\[ m := \frac{0.09748}{386} \text{ lbf-sec}^2/\text{in} \]

\[ l_p := \frac{0.00161\ell}{386} \text{ lbf-sec}^2/\text{in} \]

\[ l_d := \frac{1.889 \times 10^{-5}}{386} \text{ lbf-sec}^2/\text{in} \]

\[ \Omega := 52359.1 \text{ rad/sec} \]

\[ \begin{align*}
    a_1 & := 0.6 \text{ inch} \\
    a_2 & := -0.6 \\
    \epsilon & := \frac{\pi}{4} \\
    \eta & := \epsilon \cdot \cos(\epsilon \theta) \\
    \zeta & := \epsilon \cdot \sin(\epsilon \theta) \\
    \eta & = 1.41421 \times 10^{-6} \\
    \zeta & = 1.41421 \times 10^{-6} 
\end{align*} \]

\[ \begin{align*}
    K_{yy1} & := 2055.3 \text{ lbf/in} \\
    K_{yy2} & := 2055.3 \\
    C_{yy1} & := 0.014 \text{ lbf-sec/in} \\
    C_{yy2} & := 0.014 \\
    K_{yz1} & := 147.6 \text{ lbf-in} \\
    K_{yz2} & := 147.6 \\
    C_{yz1} & := -0.0071 \\
    C_{yz2} & := -0.0071 \\
    K_{zy1} & := -91.3 \text{ lbf-in} \\
    K_{zy2} & := -91.3 \\
    C_{zy1} & := 0.001 \text{ lbf-sec/in} \\
    C_{zy2} & := 0.001 \\
    K_{zz1} & := 1713.1 \text{ lbf-in} \\
    K_{zz2} & := 1713.1 \\
    C_{zz1} & := 0.03 \text{ lbf-sec/in} \\
    C_{zz2} & := 0.03 
\end{align*} \]
\[ \begin{bmatrix}
K_{yy_1} + K_{yy_2} & \cdots & a_1 K_{yy_1} + a_2 K_{yy_2} & \cdots & -\left( a_1 K_{yy_1} + a_2 K_{yy_2} \right) \\
K_{zy_1} + K_{zy_2} & \cdots & a_1 K_{zy_1} + a_2 K_{zy_2} & \cdots & -\left( a_1 K_{zy_1} + a_2 K_{zy_2} \right) \\
a_1 K_{zy_1} + a_2 K_{zy_2} & \cdots & a_1^2 K_{zy_1} + a_2^2 K_{zy_2} - (\Omega)^2 I d & \cdots & -\left( a_1^2 K_{zy_1} + a_2^2 K_{zy_2} - (\Omega)^2 I d \right) \\
\Omega \left( a_1 C_{yy_1} + a_2 C_{yy_2} \right) & \cdots & K_{yy_1} + K_{yy_2} - \left( \Omega \right)^2 m & \cdots & -\left( a_1 K_{yy_1} + a_2 K_{yy_2} \right) \\
\Omega \left( a_1 C_{zy_1} + a_2 C_{zy_2} \right) & \cdots & a_1 K_{zy_1} + a_2 K_{zy_2} & \cdots & -\left( a_1 K_{zy_1} + a_2 K_{zy_2} \right) \\
\Omega \left( a_1 C_{zy_1} + a_2 C_{zy_2} \right) & \cdots & a_1^2 C_{zy_1} + a_2^2 C_{zy_2} - (\Omega)^2 I p & \cdots & -\left( a_1^2 C_{zy_1} + a_2^2 C_{zy_2} - (\Omega)^2 I p \right) \\
\Omega \left( a_1 C_{yy_1} + a_2 C_{yy_2} \right) & \cdots & a_1^2 C_{yy_1} + a_2^2 C_{yy_2} + (\Omega)^2 I p & \cdots & -\left( a_1^2 C_{yy_1} + a_2^2 C_{yy_2} + (\Omega)^2 I p \right) \\
\end{bmatrix} \]

\[
\begin{bmatrix}
\eta \\
\zeta \\
0 \\
0 \\
-\zeta \\
0 \\
0 \\
\end{bmatrix} = m(\Omega)^2 
\]

(RHS unbalance forcing terms)
\[ A = \begin{pmatrix}
-67306.42171 & 25.2 & 0 & 0 & 3141.594 & -429.35118 & 0 & 0 \\
18.944 & -66987.02171 & 0 & 0 & 52.3599 & 3508.1133 & 0 & 0 \\
0 & 0 & 617.62206 & -6.81984 & 0 & 0 & 1196.94731 & 11458.76585 \\
0 & 0 & -9.072 & 675.11406 & 0 & 0 & -11323.04899 & 1262.92079 \\
-3141.594 & 429.35118 & 0 & 0 & -67306.42171 & 25.2 & 0 & 0 \\
-52.3599 & -3508.1133 & 0 & 0 & 18.944 & -66987.02171 & 0 & 0 \\
0 & 0 & -1196.94731 & -11458.76585 & 0 & 0 & 675.11406 & -6.81984 \\
0 & 0 & 11323.04899 & -1262.92079 & 0 & 0 & -9.072 & 560.13006
\end{pmatrix} \]

\[ q := A^{-1} \cdot \text{RHS} \]

**Bearing Orbits**

**Bearing #1**

\[ V1c := q_0 - a_1 \cdot q_3 \quad W1c := q_1 + a_1 \cdot q_2 \]

\[ V1s := q_4 - a_1 \cdot q_7 \quad W1s := q_5 + a_1 \cdot q_6 \]

**Bearing #2**

\[ V2c := q_0 - a_2 \cdot q_3 \quad W2c := q_1 + a_2 \cdot q_2 \]

\[ V2s := q_4 - a_2 \cdot q_7 \quad W2s := q_5 + a_2 \cdot q_6 \]
PLOT BEARING ORBITS

\[ T := \frac{2 \pi}{\Omega} \quad T = 0.00012 \quad t := 0, \frac{T}{200}, T \]

Bearing #1
\[ V_1(t) := V_1c \cos(\Omega \cdot t) + V_1s \sin(\Omega \cdot t) \]
\[ W_1(t) := W_1c \cos(\Omega \cdot t) + W_1s \sin(\Omega \cdot t) \]

Bearing #2
\[ V_2(t) := V_2c \cos(\Omega \cdot t) + V_2s \sin(\Omega \cdot t) \]
\[ W_2(t) := W_2c \cos(\Omega \cdot t) + W_2s \sin(\Omega \cdot t) \]
Work done per cycle at the bearings

**Bearing #1**

\[
Kd_1 := \frac{Kyz_1 + Kzy_1}{2} \quad \text{Cm}_1 := \frac{Cyy_1 + Czz_1}{2}
\]

\[
WK_{1\text{Cir}} := 2\pi Kd_1 (V_{1c} W_{1s} - V_{1s} W_{1c})
\]

Work done by Circulation Force at Bearing #1

\[
WK_{1\text{Diss}} := -\pi \Omega \left[ Cyy_1 \left(V_{1c}^2 + W_{1s}^2 \right) + 2 Cm_1 (V_{1c} W_{1c} + V_{1s} W_{1s}) + Czz_1 \left(W_{1c}^2 + W_{1s}^2 \right) \right]
\]

Work done by dissipation forces

\[
WK_{1\text{Total}} := WK_{1\text{Cir}} + WK_{1\text{Diss}}
\]

Total work done at the bearing #1

**Bearing #2**

\[
Kd_2 := \frac{Kyz_2 + Kzy_2}{2} \quad \text{Cm}_2 := \frac{Cyy_2 + Czz_2}{2}
\]

\[
WK_{2\text{Cir}} := 2\pi Kd_2 (V_{2c} W_{2s} - V_{2s} W_{2c})
\]

Work done by Circulation Force at Bearing #2

\[
WK_{2\text{Diss}} := -\pi \Omega \left[ Cyy_2 \left(V_{2c}^2 + W_{2s}^2 \right) + 2 Cm_2 (V_{2c} W_{2c} + V_{2s} W_{2s}) + Czz_2 \left(W_{2c}^2 + W_{2s}^2 \right) \right]
\]

Work done by dissipation forces

\[
WK_{2\text{Total}} := WK_{2\text{Cir}} + WK_{2\text{Diss}}
\]

Total work done at the bearing #2
**Bearing #1**

WK1Cir = $4.13906 \times 10^{-11}$

WK1Diss = $-4.24666 \times 10^{-8}$

WK1Total = $-4.24253 \times 10^{-8}$

**Bearing #2**

WK2Cir = $4.13906 \times 10^{-11}$

WK2Diss = $-4.24666 \times 10^{-8}$

WK2Total = $-4.24253 \times 10^{-8}$

NOTE: Total work done at each bearing/cycle must be negative (subtracting energy from rotor), for stable operation.
Input Rigid Rotor Parameters

\[
\begin{align*}
\text{m} & := \frac{0.09748}{386} \text{ lbf-sec}^2/\text{in} \\
\text{lp} & := \frac{0.00161\epsilon}{386} \text{ lbf-sec}^2/\text{in} \\
\text{id} & := \frac{1.889 \times 10^{-5}}{386} \text{ lbf-sec}^2/\text{in} \\
\Omega & := 52359.5 \text{ rad/sec}
\end{align*}
\]

\[
\begin{align*}
a_1 & := 0.6 \text{ inch} \\
a_2 & := -0.6 \\
eu & := 0.00000 \text{ unbalance eccentricity, in} \\
eu\theta & := \frac{\pi}{4} \\
\eta & := \text{eu} \cdot \cos (\text{eu}\theta) \\
\zeta & := \text{eu} \cdot \sin (\text{eu}\theta) \\
\eta & = 1.41421 \times 10^{-6} \\
\zeta & = 1.41421 \times 10^{-6}
\end{align*}
\]

Bearing parameters found by using XLTiltPadHGB:

\[
\begin{align*}
\text{Kyy}_1 & := 2055.\epsilon & \text{Kyy}_2 & := 2055.\epsilon & \text{lb/in} & \text{Cyy}_1 & := 0.01\epsilon & \text{Cyy}_2 & := 0.014 \text{ lbf-sec/in} \\
\text{Kyz}_1 & := 147.\epsilon & \text{Kyz}_2 & := 147.\epsilon & \text{lb/in} & \text{Cyz}_1 & := -0.0071 & \text{Cyz}_2 & := -0.0071 \\
\text{Kzy}_1 & := -91.3\epsilon & \text{Kzy}_2 & := -91.3\epsilon & \text{lb/in} & \text{Czy}_1 & := 0.005 & \text{Czy}_2 & := 0.005 \\
\text{Kzz}_1 & := 1713.1\epsilon & \text{Kzz}_2 & := 1713.1\epsilon & \text{lb/in} & \text{Czz}_1 & := 0.03\epsilon & \text{Czz}_2 & := 0.03\epsilon
\end{align*}
\]
\[
\begin{bmatrix}
K_{yy1} & K_{yy2} & a_1K_{yz1} + a_2K_{yz2} & -a_1K_{yz1} + a_2K_{yz2} & \Omega (C_{yy1} + C_{yy2}) & \Omega (C_{yz1} + C_{yz2}) & \Omega (a_1C_{yz1} + a_2C_{yz2}) & -\Omega (a_1C_{yy1} + a_2C_{yy2}) \\
K_{zy1} & K_{zy2} & a_1K_{zz1} + a_2K_{zz2} & -a_1K_{zz1} + a_2K_{zz2} & \Omega (C_{zy1} + C_{zy2}) & \Omega (C_{zz1} + C_{zz2}) & \Omega (a_1C_{zz1} + a_2C_{zz2}) & -\Omega (a_1C_{zy1} + a_2C_{zy2}) \\
a_1K_{zy1} & a_1K_{zy2} & a_1^2K_{yz1} + a_1^2K_{yz2} & -a_1^2K_{yz1} + a_1^2K_{yz2} & \Omega (a_1C_{zy1} + a_2C_{zy2}) & \Omega (a_1C_{zz1} + a_2C_{zz2}) & \Omega (a_1^2C_{zy1} + a_1^2C_{zy2}) & -\Omega (a_1^2C_{zz1} + a_1^2C_{zz2}) \\
-\Omega (a_1C_{yy1} + a_2C_{yy2}) & -\Omega (a_1C_{zy1} + a_2C_{zy2}) & a_1^2K_{zz1} + a_1^2K_{zz2} & -a_1^2K_{zz1} + a_1^2K_{zz2} & \Omega (a_1C_{yy1} + a_2C_{yy2}) & -\Omega (a_1C_{zy1} + a_2C_{zy2}) & a_1^2K_{zz1} + a_1^2K_{zz2} & -\Omega (a_1C_{yy1} + a_2C_{yy2}) \\
-\Omega (a_1C_{zy1} + a_2C_{zy2}) & -\Omega (a_1C_{zz1} + a_2C_{zz2}) & a_1^2K_{zz1} + a_1^2C_{zz2} & -a_1^2K_{zz1} + a_1^2C_{zz2} & \Omega (a_1C_{yy1} + a_2C_{yy2}) & -\Omega (a_1C_{zy1} + a_2C_{zy2}) & a_1^2K_{zz1} + a_1^2C_{zz2} & -\Omega (a_1C_{yy1} + a_2C_{yy2}) \\
\Omega (a_1C_{yy1} + a_2C_{yy2}) & \Omega (a_1C_{zy1} + a_2C_{zy2}) & a_1^2C_{zz1} + a_2^2C_{zz2} & \Omega (a_1^2C_{zy1} + a_2^2C_{zy2}) & -\Omega (a_1K_{zy1} + a_2K_{zy2}) & -\Omega (a_1K_{zz1} + a_2K_{zz2}) & a_1^2K_{zy1} + a_2^2K_{zy2} & a_1^2K_{zz1} + a_2^2K_{yy2} \\
\end{bmatrix}
\]

\[
\text{RHS} := m \left( \Omega \right)^2 \cdot \begin{bmatrix}
\eta \\
\zeta \\
0 \\
0 \\
-\zeta \\
\eta \\
0 \\
0 
\end{bmatrix}
\]

(RHS unbalance forcing terms)

\[
\text{RHS} = \begin{bmatrix}
0.09791 \\
0.09791 \\
0 \\
-0.09791 \\
0.09791 \\
0 \\
0 \\
0 
\end{bmatrix}
\]
\[
A = \begin{pmatrix}
-65124.42171 & 295.2 & 0 & 0 & 1466.0772 & -743.51058 & 0 & 0 \\
-182.72 & -6508.76171 & 0 & 0 & 523.599 & 3665.193 & 0 & 0 \\
0 & 0 & 1222.46886 & 65.7792 & 0 & 0 & 923.62864 & 11289.11978 \\
0 & 0 & -106.272 & 1099.28766 & 0 & 0 & -11209.95161 & 1319.46948 \\
-1466.0772 & 743.51058 & 0 & 0 & -65124.42171 & 295.2 & 0 & 0 \\
-523.599 & -3665.193 & 0 & 0 & -182.72 & -6508.76171 & 0 & 0 \\
0 & 0 & -923.62864 & -11289.11978 & 0 & 0 & 1099.28766 & 65.7792 \\
0 & 0 & 11209.95161 & -1319.46948 & 0 & 0 & -106.272 & 1345.65006
\end{pmatrix}
\]

\( q := A^{-1} \cdot \text{RHS} \)

\[
q = \begin{pmatrix}
-1.46054 \times 10^{-6} \\
-1.5494 \times 10^{-6} \\
0 \\
0 \\
1.51235 \times 10^{-6} \\
-1.39413 \times 10^{-6} \\
0 \\
0
\end{pmatrix}
\]

**Bearing Orbits**

**Bearing #1**

\[
V_{1c} := q_0 - a_1 \cdot q_3 \\
V_{1s} := q_4 - a_1 \cdot q_7
\]

\[
W_{1c} := q_1 + a_1 \cdot q_2 \\
W_{1s} := q_5 + a_1 \cdot q_6
\]

**Bearing #2**

\[
V_{2c} := q_0 - a_2 \cdot q_3 \\
V_{2s} := q_4 - a_2 \cdot q_7
\]

\[
W_{2c} := q_1 + a_2 \cdot q_2 \\
W_{2s} := q_5 + a_2 \cdot q_6
\]
PLOT BEARING ORBITS

\[ T := \frac{2 \cdot \pi}{\Omega} \quad T = 0.00012 \]

\[ t := 0, T \frac{0.00012}{200} \]

**Bearing #1**

\[ V_1(t) := V_{1c} \cos(\Omega \cdot t) + V_{1s} \sin(\Omega \cdot t) \]

\[ W_1(t) := W_{1c} \cos(\Omega \cdot t) + W_{1s} \sin(\Omega \cdot t) \]

**Bearing #2**

\[ V_2(t) := V_{2c} \cos(\Omega \cdot t) + V_{2s} \sin(\Omega \cdot t) \]

\[ W_2(t) := W_{2c} \cos(\Omega \cdot t) + W_{2s} \sin(\Omega \cdot t) \]
Work done per cycle at the bearings

Bearing #1

\[ K_{d1} := \frac{K_{yz1} + K_{zy1}}{2}, \quad C_{m1} := \frac{C_{yy1} + C_{zz1}}{2} \]

\[ W_{K1Cir} := 2\pi K_{d1} V_{1c} W_{1s} - V_{1s} W_{1c} \]  
Work done by Circulation Force at Bearing #1

\[ W_{K1Diss} := -\pi \Omega \left[ C_{yy1} V_{1c}^2 + W_{1s}^2 \right] + 2 C_{m1} (V_{1c} W_{1c} + V_{1s} W_{1s}) + C_{zz1} (W_{1c}^2 + W_{1s}^2) \]  
Work done by dissipation forces

\[ W_{K1Total} := W_{K1Cir} + W_{K1Diss} \]  
Total work done at the bearing #1

Bearing #2

\[ K_{d2} := \frac{K_{yz2} + K_{zy2}}{2}, \quad C_{m2} := \frac{C_{yy2} + C_{zz2}}{2} \]

\[ W_{K2Cir} := 2\pi K_{d2} V_{2c} W_{2s} - V_{2s} W_{2c} \]  
Work done by Circulation Force at Bearing #2

\[ W_{K2Diss} := -\pi \Omega \left[ C_{yy2} V_{2c}^2 + W_{2s}^2 \right] + 2 C_{m2} (V_{2c} W_{2c} + V_{2s} W_{2s}) + C_{zz2} (W_{2c}^2 + W_{2s}^2) \]  
Work done by dissipation forces

\[ W_{K2Total} := W_{K2Cir} + W_{K2Diss} \]  
Total work done at the bearing #2
**Bearing #1**

- WK1Cir = $7.73768 	imes 10^{-10}$
- WK1Diss = $-3.56451 	imes 10^{-8}$
- WK1Total = $-3.48713 	imes 10^{-8}$

**Bearing #2**

- WK2Cir = $7.73768 	imes 10^{-10}$
- WK2Diss = $-3.56451 	imes 10^{-8}$
- WK2Total = $-3.48713 	imes 10^{-8}$

NOTE: Total work done at each bearing/cycle must be negative (subtracting energy from rotor), for stable operation.
APPENDIX D
EXAMPLE OF FINDING BEARING PARAMETERS

Let's choose two different speeds, and different displacement components for each speed.

\[ \Omega := 2 \cdot 10^4 \, \text{rad/sec} \]
\[ a := 0.02 \quad b := 0.01 \quad g := 0.02 \quad h := 0.02 \]
\[ \Omega_1 := 2.1 \cdot 10^4 \]
\[ a_1 := 0.03 \quad b_1 := 0.01 \quad g_1 := 0.02 \quad h_1 := 0.03 \]
\[ \theta := 0, 2 \cdot \frac{\pi}{100} \ldots 2 \cdot \pi \quad \text{where } \theta = \Omega t \]

\[ y(\theta) := a \cdot \cos(\theta) + b \cdot \sin(\theta) \]
\[ z(\theta) := g \cdot \cos(\theta) + h \cdot \sin(\theta) \]

\[ y_1(\theta) := a_1 \cdot \cos(\theta) + b_1 \cdot \sin(\theta) \]
\[ z_1(\theta) := g_1 \cdot \cos(\theta) + h_1 \cdot \sin(\theta) \]

Let

\[ \text{K}_{yy} := 100 \, \text{lbf/in} \quad \text{K}_{zy} := -50 \, \text{lbf/in} \]
\[ \text{C}_{yy} := 15 \quad \text{C}_{zy} := -4 \, \text{lbf-sec/in} \]
\[ \text{K}_{yz} := 50 \, \text{lbf/in} \quad \text{K}_{zz} := 90 \, \text{lbf/in} \]
\[ \text{C}_{yz} := 4 \quad \text{C}_{zz} := 15 \, \text{lbf-sec/in} \]

So by assuming all variables, forces can be found by using equation of motion (4-6), where

\[ F_y(\theta) = m \cdot \cos(\theta) + n \cdot \sin(\theta) \] and \[ F_z(\theta) = p \cdot \cos(\theta) + q \cdot \sin(\theta). \]
Now by using force and displacement components, lets find stiffness and damping coefficients.

\[
\begin{align*}
\mathbf{P} & := \begin{bmatrix} b \cdot \Omega & h \cdot \Omega & a & g & 0 & 0 & 0 & 0 \\
-a \cdot \Omega & -g \cdot \Omega & b & h & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & b \cdot \Omega & h \cdot \Omega & a & g \\
0 & 0 & 0 & -a \cdot \Omega & -g \cdot \Omega & b & h \\
b_1 \cdot \Omega & h_1 \cdot \Omega & a_1 & g_1 & 0 & 0 & 0 & 0 \\
-a_1 \cdot \Omega & -g_1 \cdot \Omega & b_1 & h_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & b_1 \cdot \Omega & h_1 \cdot \Omega & a_1 & g_1 \\
0 & 0 & 0 & -a_1 \cdot \Omega & -g_1 \cdot \Omega & b_1 & h_1 \\
\end{bmatrix}, \\
\mathbf{T} & := \begin{bmatrix} m \\
n \\
p \\
q \\
\end{bmatrix}
\end{align*}
\]

For each speed, there are set of four equations, expressed above. Thus for two different speeds, following expression is obtained:

\[
\begin{align*}
\mathbf{P}_1 & := \begin{bmatrix} b \cdot \Omega & h \cdot \Omega & a & g & 0 & 0 & 0 & 0 \\
-a \cdot \Omega & -g \cdot \Omega & b & h & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & b \cdot \Omega & h \cdot \Omega & a & g \\
0 & 0 & 0 & -a \cdot \Omega & -g \cdot \Omega & b & h \\
b_1 \cdot \Omega & h_1 \cdot \Omega & a_1 & g_1 & 0 & 0 & 0 & 0 \\
-a_1 \cdot \Omega & -g_1 \cdot \Omega & b_1 & h_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & b_1 \cdot \Omega & h_1 \cdot \Omega & a_1 & g_1 \\
0 & 0 & 0 & -a_1 \cdot \Omega & -g_1 \cdot \Omega & b_1 & h_1 \\
\end{bmatrix}, \\
\mathbf{T}_1 & := \begin{bmatrix} m \\
n \\
p \\
q \\
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\text{Ans} := \mathbf{P}_1^{-1} \cdot \mathbf{T}_1 \\
\text{Ans} = \begin{bmatrix} 15 \\
4 \\
1000 \\
500 \\
-4 \\
15 \\
-500 \\
900 \\
\end{bmatrix}
\end{align*}
\]

As it can be seen, stiffness and damping coefficients are the same values as assumed.
Let's assume force data were read in error, because of noise. Forces in $y$ direction and forces in $z$ direction are read in ten and four percent error, respectively.

The result shows that the stiffness and damping coefficients are changed by the same percentage as forces. So it means noise in force affects bearing parameters at the same percentage as in force itself.

\[
m := m \cdot 1.1 \quad n := n \cdot 1.1 \quad p := p \cdot 1.0^4 \quad q := q \cdot 1.0^4 \\
m_1 := m_1 \cdot 1.1 \quad n_1 := n_1 \cdot 1.1 \quad p_1 := p_1 \cdot 1.0^4 \quad q_1 := q_1 \cdot 1.0^4
\]

\[
P_1 := \begin{pmatrix}
  b \cdot \Omega & h \cdot \Omega & a & g & 0 & 0 & 0 & 0 \\
  -a \cdot \Omega & -g \cdot \Omega & b & h & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & b \cdot \Omega & h \cdot \Omega & a & g \\
  0 & 0 & 0 & 0 & -a \cdot \Omega & -g \cdot \Omega & b & h \\
  b_1 \cdot \Omega_1 & h_1 \cdot \Omega_1 & a_1 & g_1 & 0 & 0 & 0 & 0 \\
  -a_1 \cdot \Omega_1 & -g_1 \cdot \Omega_1 & b_1 & h_1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & b_1 \cdot \Omega_1 & h_1 \cdot \Omega_1 & a_1 & g_1 \\
  0 & 0 & 0 & 0 & -a_1 \cdot \Omega_1 & -g_1 \cdot \Omega_1 & b_1 & h_1
\end{pmatrix}
\]

\[
T_1 := \begin{pmatrix}
m \\
n \\
p \\
q \\
m_1 \\
n_1 \\
p_1 \\
q_1
\end{pmatrix}
\]

\[
\text{Ans} := P_1^{-1} \cdot T_1
\]

\[
\begin{pmatrix}
  16.5 \\
  4.4 \\
  1.1 \times 10^3 \\
  550 \\
  -4.16 \\
  15.6 \\
  -520 \\
  936
\end{pmatrix}
\]

The result shows that the stiffness and damping coefficients are changed by the same percentage as forces. So it means noise in force affects bearing parameters at the same percentage as in force itself.
## APPENDIX E
### KISTLER DYNANOMETER

**Force – FMD**

![Kistler Dynanometer Image](image)

### 3-Komponenten-Dynanometer Fx, Fy, Fz

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* 1 N (Newton) = 1 kg · m · s² = 0,1020 lbf, lbf = 0,2248... lbf, 1 inch = 25,4 mm, 1 kg = 2,2046... lbf, 1 Nm = 0,73756... lbf ft
LIST OF REFERENCES


BIOGRAPHICAL SKETCH

The author of the thesis was born on July 2, 1981, in Azerbaijan. He grew up in Azerbaijan. In 1998 he moved to Turkey to attend Middle east Technical University, where he received his Bachelor of Science in Mechanical Engineering. In 2004 he traveled to United States of America for the pursuit of a master’s degree in mechanical engineering at University of Florida. He is planning to complete the degree of Master of Science in Mechanical Engineering in August 2006.