PHENOMENOLOGY OF UNIVERSAL EXTRA DIMENSIONS

By

KYOUNGCHUL KONG

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by
Kyoungchul Kong
To my family
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KEY TO ABBREVIATIONS

CDMS: Cryogenic Dark Matter Search
CLIC: Compact Linear Collider
EW: Electroweak
EWSB: Electroweak Symmetry Breaking
ISR: Initial State Radiation
KK: Kaluza-Klein
LHC: Large Hadron Collider
LKP: Lightest KK particle
LSP: Lightest Supersymmetric particle
MSSM: Minimal Supersymmetric Standard Model
mSugra: Minimal Supergravity
MUED: Minimal Universal Extra Dimensions
SM: Standard Model
SPS: Snowmass Points and Slopes: Benchmarks for SUSY searches
SUSY: Supersymmetry
UED: Universal Extra Dimensions
WIMP: Weakly Interacting Massive Particle
WMAP: Wilkinson Microwave Anisotropy Probe
A major motivation for studying new physics beyond the Standard Model is the dark matter puzzle which finds no explanation within the Standard Model. Models with extra dimensions may naturally provide possible dark matter candidates if the theory is compactified at the TeV scale. In this dissertation, the phenomenology of Universal Extra Dimensions (UED), in which all the Standard Model fields propagate, is explored. We focus on models with one universal extra dimension, compactified on an $S_1/Z_2$ orbifold. We investigate the collider reaches for new particles and the cosmological implications of this model.

Models with Universal Extra Dimensions may provide excellent counter examples for typical supersymmetric theories with dark matter candidates. Therefore we contrast the experimental signatures of low energy supersymmetry and models with Universal Extra Dimensions and discuss various methods for their discriminations at colliders. We first study the discovery reach of the Tevatron and the LHC for level 2 Kaluza-Klein modes, which would indicate the presence of extra dimensions, since such particles are guaranteed by extra dimensions but not supersymmetry. We also investigate the possibility to differentiate the spins of the superpartners...
and KK modes by means of a dilepton mass method and the asymmetry method in the squark cascade decay to electroweak (EW) particles. We then study the processes of Kaluza-Klein muon pair production in universal extra dimensions in parallel to smuon pair production in supersymmetry at a linear collider. We find that the angular distributions of the final state muons, the energy spectrum of the radiative return photon and the total cross-section measurement are powerful discriminators between the two models. We also calculate the production rates of various Kaluza-Klein particles and discuss the associated signatures.

A prediction of the models with Universal Extra Dimensions with conserved KK-parity is the existence of dark matter. We calculate the relic density of the lightest Kaluza-Klein particle. We include coannihilation processes with all level one KK particles. In our computation we consider a most general KK particle spectrum, without any simplifying assumptions. We first calculate the Kaluza-Klein relic density in the minimal UED model, turning on coannihilations with all level one KK particles. We then go beyond the minimal model and discuss the size of the coannihilation effects separately for each class of level 1 KK particles. Our results provide the basis for consistent relic density computations in arbitrarily general models with Universal Extra Dimensions.

All these studies not only bring us to deeper understanding of new possibilities beyond the Standard Model but also provide strong phenomenological backgrounds and tools to identify the nature of new physics.
CHAPTER 1
INTRODUCTION

The Standard Model of particle physics is a theory which describes the strong, weak, and electromagnetic fundamental forces. This theory has been astonishingly successful in explaining much of the presently available experimental data. However, the Standard Model still leaves open a number of outstanding fundamental questions whose answers are expected to emerge in a more general theoretical framework.

One of the major motivations for pursuing new physics beyond the Standard Model is the dark matter problem which finds no explanation within the Standard Model. From the accumulated astrophysical data, we now know that ordinary matter comprises only about 4% ($\Omega_B$) of the Universe. The remaining 96% are divided between a mysterious form of matter called “dark matter” (22%, $\Omega_{CDM}$) and an even more perplexing entity called “dark energy” (74%, $\Omega_\Lambda$). From the inflationary big bang model,

\[ 1 = \Omega = \Omega_\Lambda + \Omega_{CDM} + \Omega_B , \tag{1-1} \]

is expected where $\Omega_B$ is the fractional energy density in baryons, $\Omega_{CDM}$ the fractional energy density in dark matter, and $\Omega_\Lambda$ the fractional energy density in dark energy. (The precise measured values are $\Omega_{CDM} = 0.22^{+0.01}_{-0.02}$, $\Omega_\Lambda = 0.74 \pm 0.02$, $\Omega_B = 0.044^{+0.002}_{-0.003}$ and $\Omega = 1.02 \pm 0.02$ [1].)

The microscopic nature of the dark matter is at present unknown. Perhaps the most attractive explanation is provided by the WIMP (weakly interacting massive particle) hypothesis: dark matter is assumed to consist of hypothetical stable particles with masses around the scale of electroweak symmetry breaking, whose
interactions with other elementary particles are of the strength and range similar to
the familiar weak interactions of the Standard Model. Such WIMPs naturally have
a relic abundance of the correct order of magnitude to account for the observed
dark matter, making them appealing from a theoretical point of view. The relic
density, $\Omega_{WIMP}$, of the WIMP dark matter is roughly estimated by

$$\Omega_{WIMP} \sim \left(\frac{1}{10^2\alpha}\right)^2 \left(\frac{m_{WIMP}}{1 \, \text{TeV}}\right)^2$$

$$\sim \mathcal{O}(1),$$

where $m_{WIMP}$ is the mass of the WIMP dark matter candidate and the magnitude
of electroweak interaction, $\alpha$ is expected to be of order 0.01. Therefore the relic
density of WIMP dark matter is expected to be of order 1 if the mass scale is
$\mathcal{O}(1) \, \text{TeV}$. The precise relic density including the correct coefficients in the above
equation needs to be calculated using the Boltzmann equation and the result
depends on the particular model. The above estimation tells us that the WIMP
hypothesis can naturally explain all or part of the dark matter. Moreover, many
extensions of the Standard Model contain particles which can be identified as
WIMP dark matter candidates. Examples include supersymmetric models, models
with Universal Extra Dimensions, little Higgs theories, etc.

An excellent candidate for such thermal WIMP arises in the R-parity conserv-
ing supersymmetric theories. New particles, called superpartners, predicted by the
supersymmetry are charged under this R-parity, while the Standard Model particles
are neutral under the symmetry. So the lightest supersymmetric particle (LSP) is
stable and can be a dark matter candidate. The supersymmetric models have other
side benefits:

1. R-parity also implies that superpartners interact only pairwise with SM
particles, which guarantees that the supersymmetric contributions to low
energy precision data only appear at the loop level and are small.
2. If the superpartners are indeed within the TeV range, the problematic quadratic divergences in the radiative corrections to the Higgs mass are absent, being canceled by loops with superpartners. The cancellations are enforced by the symmetry, and the Higgs mass is therefore naturally related to the mass scale of the superpartner.

3. The superpartners would modify the running of the gauge couplings at higher scales, and gauge coupling unification takes places with astonishing precision. Therefore supersymmetric extensions of the SM became the primary candidates for new physics at the TeV scale. Not surprisingly, therefore, the signatures of supersymmetry at the Tevatron and the LHC have been extensively discussed in the literature.

However, supersymmetry is not the only model which has WIMP candidates. Recent developments in string theory have spurred a revival of interest in the phenomenology of theories with extra spatial dimensions. Some or even all of the Standard Model particles could also propagate in the extra dimensions and it is suggested that a stable particle in the extra dimensional models may be able to account for the observed dark matter.

The immediate result from the hypothesis of extra dimensions is the existence of extra particle states. This can be understood easily in the following way. In 4 dimensions, we have the following energy-momentum relation,

\[ E^2 = p_{x_1}^2 + p_{x_2}^2 + p_{x_3}^2 + m^2, \quad (1-4) \]

where \( x_1, x_2, x_3 \) are the coordinates of the usual 3 dimensions, \( E \) is an energy of a particle and \( m \) is a mass of a particle. Suppose there was an extra dimension with a coordinate \( y \); then this relation becomes

\[ E^2 = p_{x_1}^2 + p_{x_2}^2 + p_{x_3}^2 + p_y^2 + m^2. \quad (1-5) \]
Figure 1–1: de Broglie’s particle-wave duality. As we go around the circle, we must fit an integer multiple of λ’s in its circumference.

Now recall the particle-wave duality,

\[ p_y = \frac{2\pi}{\lambda}. \] (1-6)

If the extra dimension is compact, e.g., a circle, then as we go around the circle, we must fit an integer multiple of λ’s in its circumference as shown in fig. 1–1.

Therefore periodicity implies a quantization of momentum along the extra dimension,

\[ \lambda = \frac{2\pi R}{n} \Rightarrow p_y = \frac{2\pi n}{2\pi R} = \frac{n}{R}. \] (1–7)

Substitute eqn. 1–7 into eqn. 1–5, then the energy-momentum relation becomes

\[ E^2 = p_{x_1}^2 + p_{x_2}^2 + p_{x_3}^2 + \frac{n^2}{R^2} + m^2 \equiv \tilde{p}^2 + M_n^2, \] (1–8)

where \( M_n = \sqrt{\frac{n^2}{R^2} + m^2} \) is the effective mass of the particle moving in the extra dimensions. This translates into a rich and exciting phenomenology at the LHC, since quantization of the particle momentum along the extra dimension necessarily implies the existence of whole tower of massive particles, called Kaluza-Klein (KK) modes or partners. The KK particles within each tower are nothing but heavier versions of their Standard Model counterpart. A discovery of a compact extra dimension at a collider can only be made through the discovery of the KK particles and measurement of their properties. In fig. 1–2, our 4 dimensional spacetime is one of the two branes, and the space between the two branes is usually referred to as “the bulk.” SM particles can either freely propagate into the bulk or remain on the brane. The mass spectrum of the KK partners even encodes information about
Figure 1–2: An illustration of bulk and brane. 4 dimensional spacetime is shown as a brane and the space between two brane is called a bulk, where extra dimensions exist

the space-time geometry: if the extra dimension is flat, the KK masses are roughly equally spaced [2, 3], and if the extra dimension is warped, the KK mass spectrum follows a non-trivial pattern [4, 5].

Now consider, for example, the most “democratic” scenario (which has become known as Universal Extra Dimensions) in which all Standard Model particles propagate in the bulk. Its simplest incarnation has a single extra dimension of size $R$, which is compactified on an $S_1/Z_2$ orbifold [6]. In fig. 1–3, we show $S_1/Z_2$ orbifold where the extra dimension is shown as a line in this geometry. Interestingly, the dark matter puzzle can be resolved in a compelling fashion in models with Universal Extra Dimensions. A peculiar feature of UED is the conservation of Kaluza-Klein number at tree level, which is a simple consequence of momentum conservation along the extra dimension. However, bulk and brane radiative effects break KK number down to a discrete conserved quantity, called KK-parity. The KK-parity adorns the UED scenario with many of the virtues typically associated with supersymmetry:
Figure 1–3: $S_1/Z_2$ orbifold. A half of the circle ($S_1$) is identified with the other half with a $Z_2$ symmetry. The geometry becomes a line with two fixed points. The line between two fixed points represents the bulk.

1. The lightest KK-partners (those at level 1) must always be pair-produced in collider experiments, which leads to relatively weak bounds from direct searches.

2. The KK-parity conservation implies that the contributions to various precisely measured low-energy observables only arise at the loop level and are small.

3. Finally the KK-parity guarantees that the lightest KK partner is stable, and thus can be a cold dark matter candidate.

As we will see in the next chapters, the phenomenology of this scenario clearly resembles that of supersymmetry. In this sense, many of the SUSY studies in the literature apply, and it is perhaps more important to find methods to distinguish between the two models. Recently, other models such as little Higgs theory with T-parity have been proposed as new physics beyond the Standard Model. Our studies can also apply in the case of little Higgs models since the first level of the UED model looks like the little Higgs particle spectrum.

Except for its abundance, no other properties of dark matter candidates are known at present. Therefore it is important to study the properties of new types of dark matter candidates in the extra dimensional models and compare them with those in supersymmetry. Then a number of questions can arise: What are the
properties of dark matter candidates in the extra dimensional models? How different are they from ones in supersymmetry? Can we see any evidences for the extra dimensions in dark matter or collider experiments? etc. In this dissertation, we want to answer at least some of these questions. Hence we investigate the collider phenomenology and astrophysical implications of Universal Extra Dimensions.

In chapter 2, we first show a simple example of a Lagrangian in extra dimensions and later introduce the complete model with Universal Extra Dimensions. We review the basic phenomenology of the UED model, contrasting it with a generic supersymmetric model as described above. The detailed properties of UED models are summarized in appendix A.

In chapter 3, we identify two basic discriminators between UED and SUSY, and proceed to consider each one in turn in the following sections. One of the characteristic features of extra dimensional models is the presence of a whole tower of Kaluza-Klein (KK) partners, labelled by their KK level \( n \). In contrast, \( N = 1 \) supersymmetry predicts a single superpartner for each SM particle. One might therefore hope to discover the higher KK modes of UED and thus prove the existence of extra dimensions. In section 3.1, we study the discovery reach for level 2 KK gauge boson particles and the resolving power of the LHC to see them as separate resonances. This study was done by our group for the first time [7–11]. The other fundamental difference between SUSY and UED is the spin of the new particles (superpartners or KK partners). Therefore in section 3.2, we investigate how well the two models can be distinguished at the LHC based on spin correlations in the cascade decays of the new particles. In particular, we use the asymmetry variable recently advertised by Barr [12], as well as dilepton mass distributions. Until recently there were no known methods for measuring the spins of new particles at the LHC but now the spin determination at the LHC has become a hot topic in collider physics [13–18]. In section 3.3, we contrast the
experimental signatures of low energy supersymmetry and the model of Universal Extra Dimensions, and this time at a linear collider, discuss various methods for their discrimination. This was also the first study to contrast SUSY and UED at a linear collider [10,19].

In chapter 4, we consider the astrophysical implications of the UED. We calculate the relic density of the KK dark matter and show new results on direct detection limits [20]. The first calculation of KK dark matter [21] was done in the past but under the assumption that all KK particles have the same masses. In addition, only a subset of the relevant coannihilation processes was included. Therefore in our new calculation [20], we include all possible coannihilation processes without assuming KK mass degeneracy. A similar calculation about one particular type of dark matter was done by a group at Princeton [22] independently and our results are in agreement. We then go beyond the minimal model and discuss the size of coannihilation effects separately for each class of level 1 KK particles. This calculation with different types of KK dark matter in nonminimal UED models was performed by our group only. The annihilation cross-sections for the dark matter calculation are listed in appendix B. In chapter 5, we conclude.
CHAPTER 2
UNIVERSAL EXTRA DIMENSIONS

2.1 Massive Scalar Field in Five Dimensions

The full Lagrangian of Universal Extra Dimensions is given in appendix A and here we consider a simple example to illustrate the physics of a theory with extra compact dimensions. As the simplest example of a Lagrangian in higher dimensions, we consider the action for a massive scalar field in 5 dimensions,

\[ S = \int d^4x dy \left[ \partial_M \Phi^*(x, y) \partial^M \Phi(x, y) - m^2 \Phi^*(x, y) \Phi(x, y) \right]. \] (2-1)

Here \( M, N = 0, 1, 2, 3, 5 \equiv \mu, 5, \) 5 dimensional metric is \( g_{MN} = (+ - - - -), \) \( \partial_M = (\partial_\mu, \partial_5), \) and \( y \) is the extra dimensional coordinate (5th component of a Lorentz index, \( M \)). In the case of a circular extra dimension \( (S_1) \), the 5 dimensional scalar field \( \Phi \) is expressed in terms of an exponential basis as follows:

\[ \Phi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi_n(x) e^{i n y R}, \] (2-2)

where \( R \) is the radius of the extra dimension. This exponential basis satisfies the following orthogonality relation between different modes,

\[ 2\pi R \delta_{n,m} = \int_0^{2\pi R} dy e^{i(n - m)y R}. \] (2-3)

Now we integrate out the extra dimensional coordinate \( y \) to get a 4 dimensional effective theory. Then the action becomes

\[ S = \sum_{n=-\infty}^{\infty} \int d^4x \left[ \partial_\mu \phi_n^*(x) \partial^\mu \phi_n(x) - \left( \frac{n^2}{R^2} + m^2 \right) \phi_n^*(x) \phi_n(x) \right], \] (2-4)
Here $\phi_n$ is a 4 dimensional scalar field of mass $m_n = \sqrt{\frac{n^2}{R^2} + m^2}$. We started with one massive scalar field in 5D and compactified this theory on $S_1$. As a result, we get an infinite number of scalar fields (called a KK-tower) with mass, $m_n = \sqrt{\frac{n^2}{R^2} + m^2}$, in 4 dimensions. $n = 0$ (the “zero” mode) corresponds to a regular massive scalar field in 4 dimensions with mass, $m$. For the nonzero modes (or KK modes), the mass comes mostly from the derivative with respect to the extra dimension ($\partial_5$). Notice that all KK modes have the same spin.

### 2.2 Universal Extra Dimensions

The models of Universal Extra Dimensions are similar to this example. In the simplest and most popular version, there is a single extra dimension of size $R$, compactified on an orbifold $(S_1/Z_2)$ instead of circle ($S_1$) [6]. The orbifold can introduce chiral fermions and project out unwanted 5th components of the gauge fields (see appendix A). More complicated 6-dimensional models have also been built [23–25]. The Standard Model is written in 5 dimensions as follows.

\[
\mathcal{L}_{\text{Gauge}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ \frac{-1}{4} B_{MN} B^{MN} - \frac{1}{4} W^a_{MN} W^{aMN} - \frac{1}{4} G^4_{MN} G^{AMN} \right\},
\]

\[
\mathcal{L}_{\text{GF}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ -\frac{1}{2\xi} (\partial^\mu B_\mu - \xi \partial_5 B_5)^2 - \frac{1}{2\xi} (\partial^\mu W^a_\mu - \xi \partial_5 W^a_5)^2 - \frac{1}{2\xi} (\partial^\mu G^A_\mu - \xi \partial_5 G^A_5)^2 \right\},
\]

\[
\mathcal{L}_{\text{Leptons}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{L}(x, y) \Gamma^M D_M L(x, y) + i \bar{E}(x, y) \Gamma^M D_M E(x, y) \right\},
\]

\[
\mathcal{L}_{\text{Quarks}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{Q}(x, y) \Gamma^M D_M Q(x, y) + i \bar{U}(x, y) \Gamma^M D_M U(x, y)
+ i \bar{D}(x, y) \Gamma^M D_M D(x, y) \right\},
\]

\[
\mathcal{L}_{\text{Yukawa}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ \lambda_u \bar{Q}(x, y) U(x, y) i \tau^2 H^*(x, y) + \lambda_d \bar{Q}(x, y) D(x, y) H(x, y)
+ \lambda_u \bar{L}(x, y) E(x, y) H(x, y) \right\},
\]

\[
\mathcal{L}_{\text{Higgs}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left[ (D_M H(x, y))^\dagger (D^M H(x, y)) + \mu^2 H^\dagger(x, y) H(x, y)
- \lambda \left( H^\dagger(x, y) H(x, y) \right)^2 \right],
\]
where covariant derivatives are defined in the appendix and each Standard Model field is expressed in terms of cos and sin modes on the orbifold,

\[
H(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ H(x) + \sqrt{2} \sum_{n=1}^{\infty} H_n(x) \cos\left(\frac{ny}{R}\right) \right\},
\]

\[
B_\mu(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ B_\mu^0(x) + \sqrt{2} \sum_{n=1}^{\infty} B_\mu^n(x) \cos\left(\frac{ny}{R}\right) \right\},
\]

\[
B_5(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} B_5^n(x) \sin\left(\frac{ny}{R}\right),
\]

\[
W_\mu(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ W_\mu^0(x) + \sqrt{2} \sum_{n=1}^{\infty} W_\mu^n(x) \cos\left(\frac{ny}{R}\right) \right\},
\]

\[
W_5(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} W_5^n(x) \sin\left(\frac{ny}{R}\right),
\]

\[
G_\mu(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ G_\mu^0(x) + \sqrt{2} \sum_{n=1}^{\infty} G_\mu^n(x) \cos\left(\frac{ny}{R}\right) \right\},
\]

\[
G_5(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} G_5^n(x) \sin\left(\frac{ny}{R}\right),
\]

\[
Q(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ q_L(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_L Q_L^n(x) \cos\left(\frac{ny}{R}\right) + P_R Q_R^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\},
\]

\[
U(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ u_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_R u_R^n(x) \cos\left(\frac{ny}{R}\right) + P_L u_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\},
\]

\[
D(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ d_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_R d_R^n(x) \cos\left(\frac{ny}{R}\right) + P_L d_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\},
\]

\[
L(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ L_0(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_L L_L^n(x) \cos\left(\frac{ny}{R}\right) + P_R L_R^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\},
\]

\[
E(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ e_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_R e_R^n(x) \cos\left(\frac{ny}{R}\right) + P_L e_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\},
\]

where \( H(x, y) \) is the 5D scalar field and \((B_\mu(x, y), B_5(x, y)), (W_\mu(x, y), W_5(x, y))\) and \((G_\mu(x, y), G_5(x, y))\) are the 5D gauge fields for \(U(1), SU(2)\) and \(SU(3)\) respectively. \(Q(x, y)\) and \(L(x, y)\) are the \(SU(2)\) fermion doublets while \(U(x, y), D(x, y)\) and \(E(x, y)\) are respectively the generic singlet fields for the up-type quark, the down-type quark and the lepton.
Figure 2–1: KK states after a compactification on the orbifold. Q (U) is a fermion state which is a doublet (singlet) under $SU(2)$ and $A$ is a gauge boson. L/R represent the chirality of each state. There is one corresponding KK fermion ($n \neq 0$) for each chirality of SM fermion ($n = 0$). Two KK states sharing an arrow make one Dirac fermion while a SM Dirac fermion needs one $SU(2)_W$-doublet and one $SU(2)_W$ singlet. $A_5$ is eaten by $A_\mu$ at each KK level after compactification and KK gauge bosons become massive while a SM gauge boson remains massless. These states are equally spaced since all KK states have the same mass $\frac{n}{R}$ before electroweak symmetry breaking. Crossed states do not exist on an orbifold compactification.

The 4 dimensional effective Lagrangian is obtained by integrating out the extra dimension using orthogonality relations between these trigonometric functions, which are given in eqns. (A-6). As a result of the compactification, we find the following properties of the 4 dimensional effective theory and list them below rather than showing the actual Lagrangians, which are are quite lengthy.

1. Each SM particle has an infinite number of KK partners. This is illustrated in fig. 2-1, where $n = 0$ corresponds to a SM particle and non-zero modes correspond to KK states. In fact, $\Lambda R$ is the number of KK levels below a cutoff scale, $\Lambda$, since this theory requires a cutoff at high energy.

2. KK particles have the same spin as SM particles. All KK particles at level $n$ have the same mass, $\frac{n}{R}$ before the Higgs gets a vacuum expectation value through the EWSB. The EWSB gives masses to Standard Model particles and changes KK masses to $\sqrt{m^2 + \frac{n^2}{R^2}}$ where $m$ is the mass term.
from EWSB. A peculiar feature of UED is that there are two KK Dirac fermions for each Dirac fermion in the SM. In fig. 2–1, L (R) represents a left (right) handed chirality of the SM fermion or KK fermion. In the SM, a fermion doublet (denoted by Q) is left handed and a fermion singlet (denoted by U) is right handed. Therefore there is no right handed fermion doublet and left handed fermion singlet in the SM. However, due to the orbifold boundary condition, there are two KK states with different chiralities for both Q and U. These two KK states make one Dirac spinor and therefore there are two Dirac fermions for each Dirac fermion in SM. In other words, the zero mode (SM particle) is either right handed or left handed but the KK mode (KK particle) comes in chiral pairs. This chiral structure is a natural consequence of the orbifold boundary conditions. The mass from EWSB appears as an off-diagonal entry in a fermion mass matrix (see eqn. B.96).

fig. 2–1 shows that the 5th components of 5 dimensional gauge bosons are eaten by KK gauge bosons and these KK gauge bosons become massive while the SM gauge bosons remain massless since there is no 5th component. The SM gauge boson can get a mass through EWSB.

3. **All vertices at tree level satisfy KK number conservation.** For each term in the Lagrangian, we have a $\Delta$ (see eqn. A-21) which is a linear combination of the Kronecker delta functions. Due to this structure, the

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1 Then there could be, in principle, a mixing between two KK Dirac fermions but the mixing angle is small since $R^{-1}$ is larger than fermion mass in the SM.

2 Similarly to the fermion case, there is a nonzero contribution to the diagonal part of the gauge boson mass matrix from EWSB (eqn. A-22) and therefore there is a mixing between KK partners of $U(1)$ hypercharge gauge boson ($B_n$) and KK partners of neutral $SU(2)_W$ gauge boson ($W^3_n$), as in the SM (see eqn. A-24). This mixing angle turns out to be small and we ignore it in our analysis.
Figure 2–2: KK number conservation and KK parity. KK parity is always conserved in all cases. (a) KK number is conserved and therefore this vertex exists at tree level. This coupling is the same as a SM coupling. (b) KK number is not conserved and it does not exist at tree level. It is generated at 1 loop. (c) KK number is conserved and it exists at tree level. This coupling does not involve any SM particle and its magnitude is less than SM coupling by $\sqrt{2}$. (d) Either KK number or KK parity are not conserved. It does not exist at any loop.

allowed vertices satisfy one of the following conditions,

$$|m \pm n \pm k| = 0,$$

$$|m \pm n \pm k \pm l| = 0.$$

This is the conservation of Kaluza-Klein number at tree level, which is a simple consequence of momentum conservation along the extra dimension. Therefore it is easy to see which vertices are allowed or which vertices are not. In fig. 2–2, (a) and (c) satisfy KK number conservation and those two vertices are allowed at tree level. (b) and (d) are not allowed at tree level.

4. **KK-parity is always conserved even at higher order.** Bulk and brane radiative effects [26–28] break KK number down to a discrete conserved quantity, the so called KK parity, $(-1)^n$, where $n$ is the KK level. KK parity ensures that the lightest KK partners (those at level one) are always pair-produced in collider experiments, just like in the $R$-parity conserving supersymmetry models. KK parity conservation also implies that the contributions to various low-energy observables [29–39] only arise at loop level and are small. As a result, the limits on the scale $R^{-1}$ of the extra
dimension from precision electroweak data are rather weak, constraining $R^{-1}$ to be larger than approximately 250 GeV [33]. Fig. 2–2(b) can be generated by 1 loop corrections with level 1 KK particles, however, KK-parity is not conserved in fig. 2–2(d), hence it can never be generated by higher order corrections.

5. **New vertices are basically the same as SM couplings (up to normalization).** Vertices which have both SM and KK particles are the same as the vertices in the SM if the KK particles are replaced by the corresponding SM particles. Vertices with KK particles only can differ by a factor such as $\sqrt{2}$ due to orthogonality relations (eqns. A-20) and normalization factors (eqn. 2–7). Of course, KK-parity must be always conserved in any case.

This UED framework has been a fruitful playground for addressing different puzzles of the Standard Model, such as electroweak symmetry breaking and vacuum stability [40–42], neutrino masses [43, 44], proton stability [45] or the number of generations [46].

To continue the study on the phenomenology of UED model, we need to know the mass spectrum. It depends on the interplay between the one-loop radiative corrections to the KK mass spectrum and the brane terms generated by unknown physics at high scales [28]. In fig. 2–3, the spectrum of the first KK level is shown at tree level (a) and one-loop (b), for $R^{-1} = 500$ GeV, $\Delta R = 20$, and assuming vanishing boundary terms at the cut-off scale $\Lambda$. Fig. 2–4 shows a qualitative sketch of the level 1 KK spectroscopy depicting the dominant (solid) and rare (dotted) transitions and the resulting decay product, based on the mass spectrum given in fig. 2–3. As indicated in fig. 2–3, in the minimal UED model (MUED) defined below, the LKP turns out to be the KK partner $\gamma_1$ (or the KK partner $B_1$ of hypercharge gauge boson since the Weinberg angle for KK states is small) of the photon [28] and its relic density is typically in the right ballpark:
in order to explain all of the dark matter, the $B_1$ mass should be in the range 500-600 GeV [20–22, 47–49]. Kaluza-Klein dark matter offers excellent prospects for direct [50–52] or indirect detection [50, 53–61]. Once the radiative corrections to the Kaluza-Klein masses are properly taken into account, the collider phenomenology of the minimal UED model exhibits striking similarities to supersymmetry [62, 63] and represents an interesting and well motivated counterexample which can “fake” supersymmetry signals at the LHC.

At hadron colliders, the dominant production mechanisms are KK gluon ($g_1$) or KK quark ($q_1$ or $Q_1$) productions. As shown in fig. 2–4, an $SU(2)_W$-singlet KK quark ($q_1$) dominantly decays into a jet and a KK photon ($\gamma_1$) while an $SU(2)_W$-doublet KK quark ($Q_1$) decays into level 1 EW gauge bosons ($Z_1$ or $W_1$). Level 1 gauge bosons decay into a KK lepton producing a SM lepton and later the KK lepton also produces a SM lepton. We can notice that this cascade decay looks like a typical SUSY cascade.

For the purposes of our study we have chosen to work with the minimal UED model considered in [62]. In UED the bulk interactions of the KK modes are fixed by the SM Lagrangian and contain no unknown parameters other than
the mass, $m_h$, of the SM Higgs boson. In contrast, the boundary interactions, which are localized on the orbifold fixed points, are in principle arbitrary, and their coefficients represent new free parameters in the theory. Since the boundary terms are renormalized by bulk interactions, they are scale dependent [26] and cannot be completely ignored since they will be generated by renormalization effects. Therefore, one needs an ansatz for their values at a particular scale. Like any higher dimensional Kaluza-Klein theory, the UED model should be treated only as an effective theory valid up to some high scale $\Lambda$, at which it matches to some more fundamental theory. The minimal UED model is then defined so that the coefficients of all boundary interactions vanish at this matching scale $\Lambda$, but are subsequently generated through RGE evolution to lower scales. The minimal UED model therefore has only two input parameters: the size of the extra dimension, $R$, and the cutoff scale, $\Lambda$. The number of KK levels present in the effective theory is simply $\Lambda R$ and may vary between a few and $\sim 40$, where the upper limit comes from the breakdown of perturbativity already below the scale $\Lambda$. Unless specified otherwise, for our numerical results below, we shall always choose the value of $\Lambda$.

Figure 2-4: Qualitative sketch of the level 1 KK spectroscopy depicting the dominant (solid) and rare (dotted) transitions and the resulting decay product. The figure is taken from Cheng et al. [62].
so that $\Delta R = 20$. Changing the value of $\Lambda$ will have very little impact on our results since the $\Lambda$ dependence of the KK mass spectrum is only logarithmic. For $R^{-1} > 500$ GeV, $\sin^2 \theta_n < 0.01$ where $\theta_n$ is the Weinberg angle for level $n$. Fig. 2-5 shows the discovery reach for MUEDs at the Tevatron (blue) and the LHC (red) in the $4\ell + E_T$ channel. A $5\sigma$ excess or the observation of 5 signal events is required, and lines show the required total integrated luminosity per experiment (in $fb^{-1}$) as a function of $R^{-1}$, for $\Delta R = 20$. In either case the two experiments are not combined. The figure is taken from Cheng et al. [62].

### 2.3 Comparison between UED and Supersymmetry

We are now in a position to compare in general terms the phenomenology of UED and supersymmetry at colliders. The discussion of Section 2.2 leads to the following generic features of UED:

1. For each particle of the Standard Model, UED models predict an infinite\(^3\) tower of new particles (Kaluza-Klein partners).

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\(^3\) Strictly speaking, the number of KK modes is $\Lambda R$, see Section 2.2.
2. The spins of the SM particles and their KK partners are the same.

3. The couplings of the SM particles and their KK partners are equal.

4. The generic collider signature of UED models with WIMP LKPs is missing energy.

Notice that the defining features 3 and 4 are common to both supersymmetry and UED and cannot be used to distinguish the two cases. We see that while \( R \)-parity conserving SUSY implies a missing energy signal, the reverse is not true: a missing energy signal would appear in any model with a dark matter candidate, and even in models which have nothing to do with the dark matter issue, but simply contain new neutral quasi-stable particles, e.g. gravitons [2, 64, 65]. Similarly, the equality of the couplings (feature No. 3) is a celebrated test of SUSY, but from the above comparison we see that it is only a necessary, but not a sufficient condition in proving supersymmetry. In addition, the measurement of superpartner couplings in order to test the SUSY relations is a very challenging task at a hadron collider. For one, the observed production rate in any given channel is only sensitive to the product of the cross-section times the branching fractions, and so any attempt to measure the couplings from a cross-section would have to make certain assumptions about the branching fractions. An additional complication arises from the fact that at hadron colliders all kinematically available states can be produced simultaneously, and the production of a particular species in an exclusive channel is rather difficult to isolate. The couplings could also in principle be measured from the branching fractions, but that also requires a measurement of the total width, which is impossible in our case, since the Breit-Wigner resonance cannot be reconstructed, due to the unknown momentum of the missing LSP (LKP).

We are therefore forced to concentrate on the first two identifying features as the only promising discriminating criteria. Let us begin with feature 1: the number of new particles. The KK particles at \( n = 1 \) are analogous to superpartners in
supersymmetry. However, the particles at the higher KK levels have no analogues in $N = 1$ supersymmetric models. Discovering the $n \geq 2$ levels of the KK tower would therefore indicate the presence of extra dimensions rather than SUSY. In this study we shall concentrate on the $n = 2$ level and in Section 3.1 we investigate the discovery opportunities at the LHC and the Tevatron (for linear collider studies of $n = 2$ KK gauge bosons, see [10,19,66,67]). Notice that the masses of the KK modes are given roughly by $m_n \sim n/R$, where $n$ is the KK level number, so that the particles at levels 3 and higher are rather heavy and their production is severely suppressed.

The second identifying feature – the spins of the new particles – also provides a tool for discrimination between SUSY and UED: the KK partners have identical spin quantum numbers as their SM counterparts, while the spins of the superpartners differ by $1/2$ unit. However, spin determinations are known to be difficult at the LHC (or at hadron colliders in general), where the parton-level center of mass energy $E_{CM}$ in each event is unknown. In addition, the momenta of the two dark matter candidates in the event are also unknown. This prevents the reconstruction of any rest frame angular decay distributions, or the directions of the two particles at the top of the decay chains. The variable $E_{CM}$ also rules out the possibility of a threshold scan, which is one of the main tools for determining particle spins at lepton colliders. We are therefore forced to look for new methods for spin determinations, or at least for finding spin correlations. Recently it has been suggested that a charge asymmetry in the lepton-jet invariant mass distributions from a particular cascade, can be used to discriminate SUSY from the case of pure phase space decays [12]. The possibility of discriminating SUSY and UED by this method will be the subject of Section 3.2 (see also [7–10] and [13]).

For the purposes of our study we have implemented the relevant features of the minimal UED model in the CompHEP event generator [68]. The minimal
Supersymmetric Standard Model (MSSM) is already available in CompHEP since version 41.10. We incorporated all $n = 1$ and $n = 2$ KK modes as new particles, with the proper interactions, widths, and one-loop corrected masses [28]. Similar to the SM case, the neutral gauge bosons at level 1, $Z_1$ and $\gamma_1$, are mixtures of the KK modes of the hypercharge gauge boson and the neutral $SU(2)_W$ gauge boson. However, as shown in [28], the radiatively corrected Weinberg angle at level 1 and higher is very small. For example, $\gamma_1$, which is the LKP in the minimal UED model, is mostly the KK mode of the hypercharge gauge boson. For simplicity, in the code we neglected neutral gauge boson mixing for $n \geq 1$. 
CHAPTER 3
COLLIDER PHENOMENOLOGY

In this chapter, we consider collider implications of Universal Extra Dimensions at the LHC and a future linear collider. Since the discovery of the first KK level is discussed in [62], we first focus on the discovery of level 2 KK particles at the LHC and the Tevatron. We then consider discrimination between supersymmetry and Universal Extra Dimensions with several different methods at the LHC and a linear collider.

3.1 Search for Level 2 KK Particles at the LHC

In this section we shall consider the prospects for discovery of level 2 Kaluza-Klein particles in UED. Our notation and conventions follow those of Ref. [62]. For example, $SU(2)_W$-doublet ($SU(2)_W$-singlet) KK fermions are denoted by capital (lowercase) letters. The KK level $n$ is denoted by a subscript. In fig. 3–1 we show the mass spectrum of the $n = 1$ and $n = 2$ KK levels in minimal UED, for $R^{-1} = 500$ GeV, $\Lambda R = 20$ and SM Higgs boson mass $m_h = 120$ GeV. We include the full one-loop corrections from Cheng et al. [28]. We have used RGE improved couplings to compute the radiative corrections to the KK masses (see appendix A.4). It is well known that in UED the KK modes modify the running of the coupling constants at higher scales. We extrapolate the gauge coupling constants to the scale of the $n = 1$ and $n = 2$ KK modes, using the appropriate $\beta$ functions dictated by the particle spectrum [69–71]. As a result the spectrum shown in fig. 3–1 differs slightly from the one in [28]. Most notably, the colored KK particles are somewhat lighter, due to a reduced value of the strong coupling constant, and overall the KK spectrum at each level is more degenerate.
3.1.1 Phenomenology of Level 2 Fermions

We begin our discussion with the $n = 2$ KK fermions. Since the KK mass spectrum is pretty degenerate, the production cross-sections at the LHC are mostly determined by the strength of the KK particle interactions with the proton constituents. As KK quarks carry color, we expect their production rates to be much higher than those of KK leptons. We shall therefore concentrate on the case of KK quarks only.

In principle, there are two mechanisms for producing $n = 2$ KK quarks at the LHC: through KK-number conserving interactions, or through KK-number violating (but KK-parity conserving) interactions. The KK number conserving QCD interactions allow production of KK quarks either in pairs or singly (in association with the $n = 2$ KK mode of a gauge boson). The corresponding production cross-sections are shown in fig. 3-2 (the cross-sections for producing $n = 1$ KK quarks have been calculated in [13, 72, 73]). In fig. 3-2a we show the cross-sections (in pb) for $n = 2$ KK-quark pair production, while in fig. 3-2b we show the results for $n = 2$ KK-quark/KK-gluon associated production and
Figure 3-2: Cross-sections of $n = 2$ KK particles at the LHC for (a) KK-quark pair production (b) KK-quark/KK-gluon associated production and KK-gluon pair production. The cross-sections have been summed over all quark flavors and also include charge-conjugated contributions such as $Q_2 \bar{q}_2$, $Q_2 q_2$, $g_2 Q_2$, etc.

for $n = 2$ KK-gluon pair production. We plot the results versus $R^{-1}$, and one should keep in mind that the masses of the $n = 2$ particles are roughly $2/R$. In calculating the cross-sections of fig. 3-2 we consider 5 partonic quark flavors in the proton along with the gluon. We sum over the final state quark flavors and include charge-conjugated contributions. We used CTEQ5L parton distributions [74] and choose the scale of the strong coupling constant $\alpha_s$ to be equal to the parton level center of mass energy. All calculations are done with CompHEP [68] with our implementation of the minimal UED model.

Several comments are in order. First, fig. 3-2 displays a severe kinematic suppression of the cross-sections at large KK masses. This is familiar from the case of SUSY, where the ultimate LHC reach for colored superpartners extends only to about 3 TeV. Notice the different mass dependence of the cross-sections for the three types of final states with $n = 2$ particles: quark-quark, quark-gluon, and gluon-gluon. This can be easily understood in terms of the structure functions of the quarks and gluon inside the proton. We also observe minor differences in the cross-sections for pair production of KK quarks with different $SU(2)_W$ quantum numbers. This is partially due to the different masses for $SU(2)_W$-doublet and
$SU(2)_W$-singlet quarks (see fig. 3–1), and the remaining difference is due to the contributions from diagrams with electroweak gauge bosons. Notice that since the cross-sections in fig. 3–2a are summed over charge conjugated final states, the mixed case of $Q_2 q_2$ contains twice as many quark-antiquark contributions (compare $Q_2 \bar{q}_2 + \bar{Q}_2 q_2$ to $q_2 \bar{q}_2$ or $Q_2 \bar{Q}_2$ alone).

If we compare the cross-sections for $n = 2$ KK quark production to the cross-sections for producing squarks of similar masses in SUSY, we realize that the production rates are higher in UED. This is due to several reasons. Consider, for example, s-channel processes. Well above threshold, the UED cross-sections are larger by a factor of 4 [19]. One factor of 2 is due to the fact that in UED the particle content at $n \geq 1$ is duplicated – for example, there are both left-handed and right-handed $SU(2)_W$-doublet KK fermions, while in SUSY there are only “left-handed” $SU(2)_W$-doublet squarks. Another factor of 2 comes from the different angular distribution for fermions, $1 + \cos^2 \theta$, versus scalars, $1 - \cos^2 \theta$. When integrated over all angles, this accounts for the second factor of 2 difference.

Furthermore, at the LHC new heavy particles are produced close to threshold, due to the steeply falling parton luminosities. In SUSY, the new particles (squarks) are scalars, and the threshold suppression of the cross-sections is $\sim \beta^3$, while in UED the KK-quarks are fermions, and the threshold suppression of the cross-section is only $\beta$. This distinct threshold behavior of the production cross-sections further enhances the difference between SUSY and UED. For example, we find that for $R^{-1} = 500$ GeV the pair production cross-section for charm KK-quarks is about 6 times larger than the cross-section for charm squarks. For processes involving first generation KK-quarks, where t-channel diagrams contribute significantly, the effect can be even bigger. For example, up KK-quark production and up squark production differ by about factor of 8.
Figure 3-3: Branching fractions of the level 2 “up” quarks versus $R^{-1}$ for (a) the $SU(2)_W$-doublet quark $U_2$ and (b) the $SU(2)_W$-singlet quark $u_2$.

In fig. 3–2 we have only considered production due to KK number conserving bulk interactions. The main advantage of those processes is that the corresponding couplings are unsuppressed. However, the disadvantage is that we need to produce two heavy particles, each of mass $\sim 2/R$, which leads to a kinematic suppression. In order to overcome this problem, one could in principle consider the single production of $n = 2$ KK quarks through KK number violating, but KK parity conserving interactions, for example

$$Q_2 \gamma^\mu T^a P_L Q_0 A^a_0 \mu ,$$  \hspace{1cm} \text{(3–1)}

where $A^a_\mu$ is a SM gauge field and $T^a$ is the corresponding group generator. However, (3–1) is forbidden by gauge invariance, and the lowest order coupling of an $n = 2$ KK quark to two SM particles has the form [28]

$$Q_2 \sigma^{\mu\nu} T^a P_L Q_0 F^a_0 \mu \nu .$$  \hspace{1cm} \text{(3–2)}

Such operators may in principle be present, as they may be generated at the scale $\Lambda$ by the unknown physics at higher scales. However, being higher dimensional, we expect them to be suppressed at least by $1/\Lambda$, hence in our subsequent analysis we shall neglect them. Having determined the production rates of level 2 KK quarks,
we now turn to the discussion of their experimental signatures. To this end we need to determine the possible decay modes of $Q_2$ and $q_2$. At each level $n$, the KK quarks are among the heaviest states in the KK spectrum and can decay promptly to lighter KK modes (this is true for the top KK modes [75, 76] as well). As can be seen from fig. 3–1, the KK gluon is always heavier than the KK quarks, so the two body decays of KK quarks to KK gluons are closed. Instead, $n = 2$ KK quarks will decay to the KK modes of the electroweak gauge bosons which are lighter. The branching fractions for $n = 2$ “up”-type KK quarks are shown in fig. 3–3. Fig. 3–3a (fig. 3–3b) is for the case of the $SU(2)_W$-doublet quark $U_2$ (the $SU(2)_W$-singlet quark $u_2$). The results for the “down”-type KK quarks are similar. We observe in fig. 3–3 that the branching fractions are almost independent of $R^{-1}$, unless one is close to threshold. This feature will persist for all branching ratios of KK particles which will be shown later.

Once we ignore the KK number violating coupling (3–2), only decays which conserve the total KK number $n$ are allowed. The case of the $SU(2)_W$-singlet quarks such as $u_2$ is simpler, since they only couple to the hypercharge gauge bosons. Recall that at $n \geq 1$ the hypercharge component is almost entirely contained in the $\gamma$ KK mode [28]. We therefore expect a singlet KK quark $q_2$ to decay to either $q_1 \gamma_1$ or $q_0 \gamma_2$, as seen in fig. 3–3b. The case of an $SU(2)_W$-doublet quark $Q_2$ is much more complicated, since $Q_2$ couples to the (KK modes of the) weak gauge bosons as well, and many more two-body final states are possible. Since the weak coupling is larger than the hypercharge coupling, the decays to $W$ and $Z$ KK modes dominate, with $BR(Q_2 \to Q_0 W_2)/BR(Q_2 \to Q_0 Z_2) = 2$ and $BR(Q_2 \to Q_1 W_1)/BR(Q_2 \to Q_1 Z_1) = 2$, as evidenced in fig. 3–3a. The branching fractions to the $\gamma$ KK modes are only on the order of a few percent. The threshold behavior seen in fig. 3–3a near $R^{-1} = 400$ GeV is due to the finite masses for the SM $W$ and $Z$ bosons, which enter the tree-level masses of $W_1^{\pm}$ and $Z_1$. Since
Figure 3-4: Branching fractions of the level 2 KK electrons versus $R^{-1}$. The same as fig. 3-3 but for the level 2 KK electrons: (a) the $SU(2)_W$-doublet $E_2$ and (b) the $SU(2)_W$-singlet $e_2$.

The mass splitting of the KK modes is due to the radiative corrections, which are proportional to $R^{-1}$, the channels with $W_1^\pm$ and $Z_1$ open up only for sufficiently large $R^{-1}$.

We are now in a position to discuss the experimental signatures of $n = 2$ KK quarks. The decays to level 2 gauge bosons will simply contribute to the inclusive production of $\gamma_2$, $Z_2$ and $W_2^\pm$, which will be discussed at length later in Section 3.1.2. On the other hand, the decays to two $n = 1$ KK modes will contribute to the inclusive production of $n = 1$ KK particles which was discussed in [62]. Naturally, the direct pair production of the lighter $n = 1$ KK modes has a much larger cross-section. Therefore, the indirect production of $n = 1$ KK modes from the decays of $n = 2$ particles can be easily swamped by the direct $n = 1$ signals and the SM backgrounds. For example, the experimental signature for an $n = 2$ KK quark decaying as $Q_2 \rightarrow Q_1 \gamma_1 \ (q_2 \rightarrow q_1 \gamma_1)$ is indistinguishable from a single $Q_1$ ($q_1$). This is because $\gamma_1$ does not interact within the detector, and there are at least two additional $\gamma_1$ particles in each event, so that we cannot determine how many $\gamma_1$ particles caused the measured amount of missing energy. The decays to $W_1$ and $Z_1$ may, however, lead to final states with up to four $n = 1$ particles,
each with a leptonic decay mode. The resulting multilepton signatures \( N\ell + \slashed{E}_T \) with \( N \geq 5 \) are therefore very clean and potentially observable. Distinguishing those events from direct \( n = 1 \) pair production would be an important step in establishing the presence of the \( n = 2 \) level of the quark KK tower. Unfortunately, the \( n = 2 \) sample is statistically very limited and this analysis appears very challenging. We postpone it for future work [77]. Much of the previous discussion applies directly to the level 2 KK leptons. Assuming the absence of the KK number violating coupling analogous to \((3-2)\), the branching fractions of the \( n = 2 \) KK electrons are shown in fig. 3–4. At each KK level, the KK modes of the weak gauge bosons are heavier than the KK leptons, therefore the only allowed decays are to \( \gamma_2 \) and \( \gamma_1 \). Just like KK quarks, KK leptons can be produced directly, through KK number conserving couplings, or indirectly, in \( W_2^\pm \) and \( Z_2 \) decays. In either case, the resulting cross-sections are too small to be of interest at the LHC.

### 3.1.2 Phenomenology of Level 2 Gauge Bosons

We now discuss the collider phenomenology of the \( n = 2 \) gauge bosons \( V_2 \). As we shall see, the KK gauge bosons offer the best prospects for detecting the \( n = 2 \) structure, since they have direct (but not tree level) couplings to SM particles, and can be discovered as resonances, e.g. in the dijet or dilepton channels. This is in contrast to the case of \( n = 2 \) KK fermions, which, under the assumptions of Sec. 3.1.1, do not have fully visible decay modes. Bump hunting will also help discriminate between \( n = 2 \) and \( n = 1 \) KK particles, since the latter are KK-parity odd, and necessarily decay to the invisible \( \gamma_1 \).

There are four \( n = 2 \) KK gauge bosons: the KK “photon” \( \gamma_2 \), the KK “Z-boson” \( Z_2 \), the KK “W-boson” \( W_2^\pm \), and the KK gluon \( g_2 \). Recall that the Weinberg angle at \( n = 2 \) is very small, so that \( \gamma_2 \) is mostly the KK mode of the hypercharge gauge boson and \( Z_2 \) is mostly the KK mode of the neutral W-boson of the SM. An important consequence of the extra dimensional nature of the model
Figure 3-5: Masses and widths of level 2 KK gauge bosons. (a) Masses of the four $n = 2$ KK gauge bosons as a function of $R^{-1}$. (b) Total widths of the $n = 2$ KK gauge bosons as a function of the corresponding mass. We also show the width of a generic $Z'$ whose couplings to the SM particles are the same as those of the $Z$-boson.

is that all four of the $n = 2$ KK gauge bosons are relatively degenerate, as shown in fig. 3-5a. The masses are all roughly equal to $2/R$. The mass splitting between the KK gauge bosons is almost entirely due to radiative corrections, which in the minimal UED model yield the mass hierarchy $m_{g_2} > m_{W_2} \sim m_{Z_2} > m_{\gamma_2}$. The KK gluon receives the largest corrections and is the heaviest particle in the KK spectrum at each level $n$. The $W_2^\pm$ and $Z_2$ particles are degenerate to a very high degree, due to $SU(2)_W$ symmetry. The KK number conserving interactions allow an $n = 2$ KK gauge boson $V_2$ to decay to two $n = 1$ particles, or to one $n = 2$ KK particle and one $n = 0$ (i.e., Standard Model) particle, provided that the decays are allowed by phase space. For example, the partial widths to fermion final states are given by

$$
\Gamma(V_2 \to f_2 \bar{f}_0) = \frac{c^2 g^2}{48 \pi m_{V_2}^3} \left( m_{V_2}^2 - m_{f_2}^2 - m_{f_0}^2 + \frac{m_{V_2}^4 - (m_{f_2} - m_{f_0})^2}{m_{V_2}^2} \right) \\
\times \sqrt{m_{V_2}^2 - (m_{f_2} - m_{f_0})^2} \left[ m_{V_2}^2 - (m_{f_2} + m_{f_0})^2 \right] \\
\approx \frac{c^2 g^2}{48 \pi m_{V_2}^3} \left( m_{V_2}^2 - m_{f_2}^2 \right)^2 \left( 1 + \frac{m_{V_2}^2 + m_{f_2}^2}{m_{V_2}^2} \right) \\
\approx \frac{c^2 g^2 m_{V_2}}{4\pi} \left( \frac{\delta m_{V_2}}{m_2} - \frac{\delta m_{f_2}}{m_2} \right)^2,
$$

\[3-3\]
\[ \Gamma(V_2 \to f_1 \bar{f}_1) = \frac{c^2 g^2}{24 \pi m_{V_2}^2} \left( m_{V_2}^2 - 4 m_{f_1}^2 \right)^{\frac{3}{2}} \]  
\[ \approx \frac{c^2 g^2 m_{V_2}}{6 \sqrt{2\pi}} \left( \frac{\hat{\delta} m_{V_2}}{m_2} - \frac{\hat{\delta} m_{f_1}}{m_1} \right)^{\frac{3}{2}} \left( \frac{m_2}{m_{V_2}} \right)^3 \]
\[ \approx \frac{c^2 g^2 m_{V_2}}{6 \sqrt{2\pi}} \left( \frac{\hat{\delta} m_{V_2}}{m_2} - \frac{\hat{\delta} m_{f_1}}{m_1} \right)^{\frac{3}{2}} \]  

where \( c \approx Y N_f^f / 2 \) for \( V_2 \approx \gamma_2 \), \( c \approx N_f^f / 2 \) for \( V_2 \approx Z_2 \), \( c = V_{CKM} N_f^f / \sqrt{2} \) for \( V_2 = W_2^\pm \) and \( c = 1 / \sqrt{2} \) for \( V_2 = g_2 \), with \( Y \) being the fermion hypercharge in the normalization \( Q = T_3 + Y / 2 \), \( V_{CKM} \) is the CKM mixing matrix, and \( N_f^f = 3 \) for \( f = q \) and \( N_f^f = 1 \) for \( f = \ell \). Here \( \hat{\delta} m \) stands for the total radiative correction to a KK mass \( m \), including both bulk and boundary contributions [28], \( m_2 \equiv 2 / R \), and \( g \) is the corresponding gauge coupling. The first lines in (3–3) and (3–4) give the exact result, while the last lines are the approximate formulas derived in [62] as leading order expansions in \( \hat{\delta} m / m \). The second line in (3–3) is an approximation neglecting the SM fermion mass \( m_{f_0} \). The second line in (3–4) is an alternative approximation which incorporates subleading but numerically important terms. In our code we have programmed the exact expressions and quote the approximations here only for completeness.

Note that the KK number conserving decays of the \( n = 2 \) KK gauge bosons are suppressed by phase space. This is evident from the approximate expressions in eqs. (3–3) and (3–4). The partial widths are proportional to the one-loop corrections, which open up the available phase space and allow the corresponding decay mode to take place. However, not all of the fermionic final states are available, for example, \( Z_2 \) and \( W_2^\pm \) have no hadronic decay modes to level 1 or 2, while \( \gamma_2 \) has no KK number conserving decay modes at all.

The \( n = 2 \) KK gauge bosons also have KK number violating couplings which can be generated either radiatively from bulk interactions, or directly at the scale
For example, the operator

$$\bar{f}_0 \gamma^\mu T^a P_L f_0 A_2^a$$

(3–5)

couples \(V_2\) directly to SM fermions \(f_0\), and leads to the following \(V_2\) partial width

$$\Gamma(V_2 \to f_0 \bar{f}_0) = \frac{c^2 g^2 m_{V_2}}{12\pi} \left( \frac{\bar{\delta}m_{V_2}}{m_2} - \frac{\bar{\delta}m_{f_2}}{m_2} \right)^2 \left( 1 - \frac{m_{f_0}^2}{m_{V_2}^2} \right) \sqrt{1 - 4 \frac{m_{f_0}^2}{m_{V_2}^2}} \quad (3–6)$$

where \(\bar{\delta}m\) stands for a mass correction due to boundary terms only [28]. In the second line we have neglected the SM fermion mass \(m_{f_0}\), recovering the result from Cheng et al. [62].

As we see from (3–6), the KK number violating decay is also suppressed, this time by a loop factor, and is proportional to the size of the radiative corrections to the corresponding KK masses. In spite of this suppression, the \(V_2 \to f_0 \bar{f}_0\) decays is most promising for experimental discovery. As long as the final state fermions can be reconstructed, the \(V_2\) particle can be looked for as a bump in the invariant mass distribution of its decay products. In this sense, the search is very similar to \(Z'\) searches, with one major difference. Since all partial widths (3–3–3–6) are suppressed, the total width of \(V_2\) is much smaller than the width of a typical \(Z'\). This is illustrated in fig. 3–5b, where we plot the widths of the KK particles \(\gamma_2\), \(W_{Z_2}^\pm\), \(Z_2\) and \(g_2\) in UED, as a function of the corresponding particle mass, and contrast to the width of a \(Z'\) with SM-like couplings. We see that the widths of the KK gauge bosons are extremely small. This has important ramifications for the experimental search, since the width of the resonance will then be determined by the experimental resolution, rather than the intrinsic particle width. In this sense the width must be included in the set of basic parameters of a \(Z'\) search [78].

Before we elaborate on the experimental signatures of the \(n = 2\) KK gauge bosons,
Figure 3–6: Cross-sections for single production of level 2 KK gauge bosons through the KK number violating couplings (3–5).

let us briefly discuss their production. There are three basic mechanisms:

1. **Single production through the KK number violating operator.**

The corresponding cross-sections are shown in fig. 3–6 as a function of $R^{-1}$. One might expect that these processes will be important, especially at large masses, since we need to make only a single heavy $n = 2$ particle, alleviating the kinematic suppression. If we compare the mass dependence of the Drell-Yan cross-sections in fig. 3–6 to the mass dependence of the $n = 2$ pair production cross-sections from fig. 3–2, indeed we see that the former drop less steeply with $R^{-1}$ and become dominant at large $R^{-1}$. On the other hand, the Drell-Yan processes of fig. 3–6 are mediated by a KK number violating operator (3–5) and the coupling of a $V_2$ to SM particles is radiatively suppressed. This is another crucial difference with the case of a generic $Z'$, whose couplings typically have the size of a normal gauge coupling and are unsuppressed [78].

Notice the roughly similar size of the four cross-sections shown in fig. 3–6. This is somewhat surprising, since the cross-sections scale as the corresponding gauge coupling squared, and one would have expected a wider spread in the values of the four cross-sections. This is due to a couple of things. First, for a
given $R^{-1}$, the masses of the four $n = 2$ KK gauge bosons are different, with $m_{g_2} > m_{W_2} \sim m_{Z_2} > m_{\gamma_2}$. Therefore, for a given $R^{-1}$, the heavier particles suffer a suppression. This explains to an extent why the cross-section for $\gamma_2$ is not the smallest of the four, and why the cross-section for $g_2$ is not as large as one would expect. There is, however, a second effect, which goes in the same direction. The coupling (3–5) is also proportional to the mass corrections of the corresponding particles:

$$\frac{\delta m_{V_2}}{m_{V_2}} = \frac{\delta m_{f_2}}{m_{f_2}}. \quad (3-7)$$

Since the QCD corrections are the largest, for $V_2 = \{\gamma_2, Z_2, W_2^{\pm}\}$, the second term dominates. However, for $V_2 = g_2$, the first term is actually larger, and there is a cancellation, which further reduces the direct KK gluon couplings to quarks.

2. Indirect production. The electroweak KK modes $\gamma_2$, $Z_2$ and $W_2^{\pm}$ can be produced in the decays of heavier $n = 2$ particles such as the KK quarks and/or KK gluon. This is well known from the case of SUSY, where the dominant production of electroweak superpartners is often indirect – from squark and gluino decay chains. The indirect production rates of $\gamma_2$, $Z_2$ and $W_2^{\pm}$ due to QCD processes can be readily estimated from figs. 3–2 and 3–3. Notice that $BR(Q_2 \to W_2^{\pm})$, $BR(Q_2 \to Z_2)$ and $BR(q_2 \to \gamma_2)$ are among the largest branching fractions of the $n = 2$ KK quarks, and we expect indirect production from QCD to be a significant source of electroweak $n = 2$ KK modes.

3. Direct pair production. The $n = 2$ KK modes can also be produced directly in pairs, through KK number conserving interactions. These processes, however, are kinematically suppressed, since we have to make two heavy particles in the final state. One would therefore expect that they will be the least relevant source of $n = 2$ KK gauge bosons. The only exception is KK gluon pair production which is important and is shown in fig. 3–2b. We see that it is comparable in size to KK quark pair production and $q_2 g_2 / Q_2 g_2$ associated production. We have
also calculated the pair production cross-sections for the electroweak $n = 2$ KK
gauge bosons and confirmed that they are very small, hence we shall neglect them
in our analysis below. In conclusion of this section, we discuss the experimental
signatures of $n = 2$ KK gauge bosons. To this end, we need to consider their
possible decay modes. Having previously discussed the different partial widths,
it is straightforward to compute the $V_2$ branching fractions. Those are shown
in fig. 3–7(a–d). Again we observe that the branching fractions are very weakly
sensitive to $R^{-1}$, just as the case of figs. 3–3 and 3–4. This can be understood as
follows. The partial widths (3–3) and (3–4) for the KK number conserving decays
are proportional to the available phase space, while the partial width (3–6) for the
KK number violating decay is proportional to the mass corrections (see eq. (3–7)).
Both the phase space and the mass corrections are proportional to $R^{-1}$, which then cancels out in the branching fraction.

Similarly to the case of $n = 2$ KK quarks discussed in Sec. 3.1.1, KK number conserving decays are not very distinctive, since they simply contribute to the inclusive $n = 1$ sample which is dominated by direct $n = 1$ production. The decays of $n = 1$ particles will then give relatively soft objects, and most of the energy will be lost in the LKP mass. In short, $n = 2$ signatures based on purely KK number conserving decays are not very promising experimentally — one has to pay a big price in the cross-section in order to produce the heavy $n = 2$ particles, but does not get the benefit of the large mass, since most of the energy is carried away by the invisible LKP. We therefore concentrate on the KK number violating channels, in which the $V_2$ decays are fully visible.

Fig. 3-7a shows the branching fractions of the KK gluon $g_2$. Since it is the heaviest particle at level 2, all of its decay modes are open, and have comparable branching fractions. The KK number conserving decays dominate, since the KK number violating coupling is slightly suppressed due to the cancellation in (3-7). In principle, $g_2$ can be looked for as a resonance in the dijet [79] or $t\bar{t}$ invariant mass spectrum, but one would expect large backgrounds from QCD and Drell-Yan. Notice that there is no indirect production of $g_2$, and its single production cross-section is not that much different from the cross-sections for $\gamma_2$, $Z_2$ and $W_2^\pm$ (see fig. 3-6). Therefore, the inclusive $g_2$ production is comparable to the inclusive $\gamma_2$ and $Z_2$ production, and then we anticipate that the searches for the $n = 2$ electroweak gauge bosons in leptonic channels will be more promising.

Figs. 3-7b and 3-7c give the branching fractions of $Z_2$ and $W_2^\pm$, correspondingly. We see that the decays to KK quarks have been closed due to the large QCD radiative corrections to the KK quark masses. Among the possible KK number conserving decays of $Z_2$ and $W_2^\pm$, only the leptonic modes survive, and they will
be contributing to the leptonic discovery signals of UED [62]. Recall that the KK number conserving decays are phase space suppressed, while the KK number violating decays are loop suppressed, and proportional to the mass corrections as in (3–7). The precise calculation shows that the dominant decay modes are $Z_2 \to q \bar{q}$ and $W_2^\pm \to q \bar{q}'$. This can be understood in terms of the large $\delta m_{q_t}$ correction appearing in (3–7). The resulting branching ratios are more than 50% and in principle allow for a $Z_2/W_2^\pm$ search in the dijet channel, just like the case of $g_2$. However, we shall concentrate on the leptonic decay modes, which have much smaller branching fractions, but are much cleaner experimentally.

Finally, fig. 3–7d shows the branching fractions of $\gamma_2$. This time all KK number conserving decays are closed, and $\gamma_2$ is forced to decay through the KK number violating interaction (3–5). Again, the jetty modes dominate, and the leptonic modes (summed over lepton flavors) have rather small branching fractions, on the order of 2%, which could be a potential problem for the search. In the following section we shall concentrate on the $Z_2 \to \ell^+\ell^-$ and $\gamma_2 \to \ell^+\ell^-$ signatures and analyze their discovery prospects in a $Z'$-like search [80,81].

### 3.1.3 Analysis of the LHC Reach for $Z_2$ and $\gamma_2$

We are now in a position to discuss the discovery reach of the $n = 2$ KK gauge bosons at the LHC and the Tevatron. We will consider the inclusive production of $Z_2$ and $\gamma_2$ and look for a dilepton resonance in both the $e^+e^-$ and $\mu^+\mu^-$ channels. An important parameter of the search is the width of the reconstructed resonance, which in turn determines the size of the invariant mass window selected by the cuts. Since the intrinsic width of the $Z_2$ and $\gamma_2$ resonances is so small (see fig. 3–5b), the mass window is entirely determined by the mass resolution in the dimuon and dielectron channels. For electrons, the resolution in CMS is approximately constant, on the order of $\Delta m_{ee}/m_{ee} \approx 1\%$ in the region of interest [82].
Figure 3–8: 5σ discovery reach for (a) $\gamma_2$ and (b) $Z_2$. We plot the total integrated luminosity $L$ (in fb$^{-1}$) required for a 5σ excess of signal over background in the dielectron (red, dotted) or dimuon (blue, dashed) channel, as a function of $R^{-1}$. In each plot, the upper set of lines labelled “DY” makes use of the single $V_2$ production of fig. 3-6 only, while the lower set of lines (labelled “All processes”) includes indirect $\gamma_2$ and $Z_2$ production from $n = 2$ KK quark decays. The red dotted line marked “FNAL” in the upper left corner of (a) reflects the expectations for a $\gamma_2 \rightarrow e^+e^-$ discovery at the Tevatron in Run II. The shaded area below $R^{-1} = 250$ GeV indicates the region disfavored by precision electroweak data [33].

the other hand, the dimuon mass resolution is energy dependent, and in preliminary studies based on a full simulation of the CMS detector has been parametrized as [83]

$$\frac{\Delta m_{\mu\mu}}{m_{\mu\mu}} = 0.0215 + 0.0128 \left( \frac{m_{\mu\mu}}{1 \, \text{TeV}} \right).$$

Therefore in our analysis we impose the following cuts

1. Lower cuts on the lepton transverse momenta $p_T(\ell) > 20$ GeV.
2. Central rapidity cut on the leptons $|\eta(\ell)| < 2.4$.
3. Dilepton invariant mass cut for electrons $m_{V_2} - 2\Delta m_{ee} < m_{ee} < m_{V_2} + 2\Delta m_{ee}$ and muons $m_{V_2} - 2\Delta m_{\mu\mu} < m_{\mu\mu} < m_{V_2} + 2\Delta m_{\mu\mu}$.

With these cuts the signal efficiency varies from 65% at $R^{-1} = 250$ GeV to 91% at $R^{-1} = 1$ TeV. The main SM background to our signal is Drell-Yan, which we have calculated with the PYTHIA event generator [84]. With the cuts listed above, we compute the discovery reach of the LHC and the Tevatron for the $\gamma_2$
and $Z_2$ resonances. Our results are shown in fig. 3–8. We plot the total integrated luminosity $L$ (in fb$^{-1}$) required for a 5$\sigma$ excess of signal over background in the dielectron (red, dotted) or dimuon (blue, dashed) channel, as a function of $R^{-1}$. In each panel in fig. 3–8, the upper set of lines labelled “DY” only utilizes the single $V_2$ production cross-sections from fig. 3–6. The lower set of lines (labelled “All processes”) includes in addition indirect $\gamma_2$ and $Z_2$ production from the decays of $n = 2$ KK quarks to $\gamma_2$ and $Z_2$ (we ignore secondary $\gamma_2$ production from $Q_2 \rightarrow Z_2 \rightarrow \ell_2 \rightarrow \gamma_2$). The shaded area below $R^{-1} = 250$ GeV indicates the region disfavored by precision electroweak data [33]. Using the same cuts also for the case of the Tevatron, we find the Tevatron reach in $\gamma_2 \rightarrow e^+e^-$ shown in fig. 3–8a and labelled “FNAL.” For the Tevatron we use electron energy resolution $\Delta E/E = 0.01 \oplus 0.16/\sqrt{E}$ [85]. The Tevatron reach in dimuons is worse due to the poorer resolution, while the reach for $Z_2$ is also worse since $m_{Z_2} > m_{\gamma_2}$ for a fixed $R^{-1}$.

Fig. 3–8 reveals that there are good prospects for discovering level 2 gauge boson resonances at the LHC. Already within one year of running at low luminosity ($L = 10$ fb$^{-1}$), the LHC will have sufficient statistics in order to probe the region up to $R^{-1} \sim 750$ GeV. Notice that in the minimal UED model, the “good dark matter” region, where the LKP relic density accounts for all of the dark matter component of the Universe, is at $R^{-1} \sim 500 - 600$ GeV [20–22]. This region is well within the discovery reach of the LHC for both $n = 1$ KK modes [62] and $n = 2$ KK gauge bosons (fig. 3–8). If the LKP accounts for only a fraction of the dark matter, the preferred range of $R^{-1}$ is even lower and the discovery at the LHC is easier.

From fig. 3–8 we also see that the ultimate reach of the LHC for both $\gamma_2$ and $Z_2$, after several years of running at high luminosity ($L \sim 300$ fb$^{-1}$), extends up to just beyond $R^{-1} = 1$ TeV. One should keep in mind that the actual KK masses
are at least twice as large: $m_{\gamma_2} \sim m_2 = 2/R$, so that the KK resonances can be discovered for masses up to 2 TeV. While the $n = 2$ KK gauge bosons are a salient feature of the UED scenario, any such resonance by itself is not a sufficient discriminator, since it resembles an ordinary $Z'$ gauge boson. If UED is discovered, one could then still make the argument that it is in fact some sort of non-minimal supersymmetric model with an additional gauge structure containing neutral gauge bosons. An important corroborating evidence in favor of UED would be the simultaneous discovery of several, rather degenerate, KK gauge boson resonances. While SUSY also can accommodate multiple $Z'$ gauge bosons, there would be no good motivation behind their mass degeneracy. A crucial question therefore arises: can we separately discover the $n = 2$ KK gauge bosons as individual resonances? For this purpose, one would need to see a double peak structure in the invariant mass distributions. Clearly, this is rather challenging in the dijet channel, due to the relatively poor jet energy resolution. We shall therefore consider only the dilepton channels, and investigate how well we can separate $\gamma_2$ from $Z_2$.

Our results are shown in fig. 3–9, where we show the invariant mass distribution in UED with $R^{-1} = 500$ GeV, for (a) the dimuon and (b) the dielectron channel at the LHC with $L = 100$ fb$^{-1}$. The SM background is shown with the (red) continuous underlying histogram.
channel at the LHC with $L = 100 \text{ fb}^{-1}$. We see that the diresonance structure is easier to detect in the dielectron channel, due to the better mass resolution. In dimuons, with $L = 100 \text{ fb}^{-1}$ the structure is also beginning to emerge. We should note that initially the two resonances will not be separately distinguishable, and each will in principle contribute to the discovery of a bump, although with a larger mass window. In our reach plots in fig. 3-8 we have conservatively chosen not to combine the two signals from $Z_2$ and $\gamma_2$, but show the reach for each one separately.

In this section we have discussed the differences and similarities in the hadron collider phenomenology of models with Universal Extra Dimensions and supersymmetry. We identified the higher level KK modes of UED and as a reliable discriminator between the two scenarios. We then proceeded to study the discovery reach for level 2 KK modes in UED at hadron colliders. We showed that the $n = 2$ KK gauge bosons offer the best prospects for detection, in particular the $\gamma_2$ and $Z_2$ resonances can be separately discovered at the LHC. Is this a proof of UED? Not quite – these resonances could still be interpreted as $Z'$ gauge bosons, but their close degeneracy is a smoking gun for UED. Furthermore, although we did not show any results to this effect, it is clear that the $W_2^{\pm}$ KK mode can also be looked for and discovered in its decay to SM leptons. One can then measure $m_{W_2}$ and show that it is very close to $m_{Z_2}$ and $m_{\gamma_2}$, which would further strengthen the case for UED.

Here we only concentrated on the minimal UED model, it should be kept in mind that there are many interesting possibilities for extending the analysis to a more general setup. For example, non-vanishing boundary terms at the scale $\Lambda$ can distort the minimal UED spectrum beyond recognition. A priori, in such a relaxed framework the UED-SUSY confusion can be “complete” in the context of a hadron collider and a preliminary study is under way to address this issue [14,15].
The UED collider phenomenology is also very different in the case of a “fat”
brane [86,87], charged LKPs [88] or KK graviton superwimps [89,90]. Notice that
Little Higgs models with $T$-parity [16,91–94] are very similar to UED, and can also
be confused with supersymmetry.

3.2 Spin Determination at the LHC

The fundamental difference between SUSY and UED is first the number of
new particles and second, the spins of new particles. The KK particles at $n = 1$
are analogous to superpartners in supersymmetry. However, the particles at the
higher KK levels have no analogues in $N = 1$ supersymmetric models. Discovering
the $n \geq 2$ levels of the KK tower would therefore indicate the presence of extra
dimensions rather than SUSY. However these KK particles can be too heavy to
be observed. Even if they can be observed at the LHC, they can be confused with
other new particles [10,11] such as $Z'$ or different types of resonances from extra
dimensions [25].

The second feature – the spins of the new particles – also provides a tool
for discrimination between SUSY and UED: the KK partners have identical spin
quantum numbers as their SM counterparts, while the spins of the superpartners
differ by $1/2$ unit. However, spin determinations are known to be difficult at the
LHC (or at hadron colliders in general), where the parton-level center of mass
energy $E_{CM}$ in each event is unknown. In addition, the momenta of the two dark
matter candidates in the event are also unknown. This prevents the reconstruction
of any rest frame angular decay distributions, or the directions of the two particles
at the top of the decay chains. The variable $E_{CM}$ also rules out the possibility
of a threshold scan, which is one of the main tools for determining particle spins
at lepton colliders. We are therefore forced to look for new methods for spin
determinations, or at least for finding spin correlations\(^1\). The purpose of this section is to investigate the prospects for establishing supersymmetry at the LHC by discriminating it from its look-alike scenario of Universal Extra Dimensions by measuring spins of new particles in two models\(^2\). As discussed before, the second fundamental distinction between UED and supersymmetry is reflected in the properties of the individual particles. Recently it has been suggested that a charge asymmetry in the lepton-jet invariant mass distributions from a particular cascade (see fig. 3–10), can be used to discriminate SUSY from the case of pure phase space decays [12] and is an indirect indication of the superparticle spins (A study of measuring sleptons spins at the LHC can be found in [17]). It is

\[^1\] Notice that in simple processes with two-body decays like slepton production \(e^+ e^- \rightarrow \tilde{\mu}^+ \tilde{\mu}^- \rightarrow \mu^+ \mu^- \tilde{\chi}_2^0 \tilde{\chi}_2^0\) the flat energy distribution of the observable final state particles (muons in this case) is often regarded as a smoking gun for the scalar nature of the intermediate particles (the smuons). Indeed, the smuons are spin zero particles and decay isotropically in their rest frame, which results in a flat distribution in the lab frame. However, the flat distribution is a necessary but not sufficient condition for a scalar particle, and UED provides a counterexample with the analogous process of KK muon production [19], where a flat distribution also appears, but as a result of equal contributions from left-handed and right-handed KK fermions.

\[^2\] The same idea can apply in the case of little Higgs models since the first level of the UED model looks like the new particles in little Higgs models [91–94].
therefore natural to ask whether this method can be extended to the case of SUSY versus UED discrimination. Following [12], we concentrate on the cascade decay $q \rightarrow q \tilde{\nu} \rightarrow q \ell^+ \ell^- \tilde{\nu}$ in SUSY and the analogous decay chain $Q_1 \rightarrow q Z_1 \rightarrow q \ell^+ \ell^- \tilde{\nu}$ in UED. Both of these processes are illustrated in fig. 3–10. Blue lines represent the decay chain in UED and red lines the decay chain in SUSY. Green lines are SM particles.

3.2.1 Dilepton Invariant Mass Distributions

First we will look for spin correlations between the two SM leptons in the final state. In supersymmetry, the slepton is a scalar particle and therefore there is no spin correlation between the two SM leptons. However in UED, the slepton is replaced by a KK lepton and is a fermion. We might therefore expect a different shape in the dilepton invariant mass distribution. To investigate this, we first choose a study point in UED (SPS1a in mSugra) with $R^{-1} = 500$ GeV taken from Cheng et al. [28, 62] and then we adjust the relevant MSSM parameters (UED parameters) until we get a matching spectrum. So the masses are exactly the same and they can not be used for discrimination and the only difference is the spin. In

Figure 3–11: Comparison of dilepton invariant mass distributions in the case of (a) UED mass spectrum with $R^{-1} = 500$ GeV (b) mass spectrum from SPS1a. In both cases, UED (SUSY) distributions are shown in blue (red). All distributions are normalized to $\mathcal{L} = 10$ fb$^{-1}$ and the error bars represent statistical uncertainty.
fig. 3–11, we show invariant mass distributions in UED and SUSY for two different types of mass spectrum. In fig. 3–11(a), all UED masses are adjusted to be the same as the SUSY masses in SPS1a ($m_0 = 100$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -100$, $\tan \beta = 10$ and $\mu = 0$) while in fig. 3–11(b) the SUSY masses are replaced by KK masses for $R^{-1} = 500$. In both cases, UED (SUSY) distributions are shown in blue (red). Squark/KK quark pair-production cross-sections are taken from Smillie et al. [13] and the relevant branching fractions are obtained from Cheng et al. [62] for UED and [95] for SUSY. All distributions are normalized to $\mathcal{L} = 10 \text{ fb}^{-1}$ and the error bars represent statistical uncertainty. In supersymmetry, the distribution is the same as the one in the case of pure phase space decay since the slepton has no spin. As we can notice, the two distributions are identical for both UED and SUSY mass spectrum even if the intermediate particles in UED and SUSY have different spins. The minor differences in the plot will completely disappear once the background, radiative corrections and detector simulation are included.

The invariant mass distributions for UED and SUSY/Phase space can be written as [13, 96]

\[
\text{Phase Space : } \frac{dN}{d\hat{m}} = 2\hat{m} \\
\text{SUSY : } \frac{dN}{d\hat{m}} = 2\hat{m} \\
\text{UED : } \frac{dN}{d\hat{m}} = \frac{4(y + 4z)}{(1 + 2z)(2 + y)} (\hat{m} + r\hat{m}^3)
\]

where the coefficient $r$ in the second term of the UED distribution is defined as

\[
r = \frac{(2 - y)(1 - 2z)}{y + 4z},
\]

$\hat{m} = \frac{m_{\ell\ell}}{m_{\ell\ell}^\text{max}}$ is the rescaled invariant mass, $y = \left(\frac{m_{\ell\ell}}{m_{\tilde{g}\tilde{g}}}\right)^2$ and $z = \left(\frac{m_{\tilde{g}\tilde{g}}}{m_{\ell\ell}}\right)^2$ are the ratios of the masses involved in the decay. $y$ and $z$ are less than 1 in the case of on-shell decay. From eqn. 3–8, there are two terms in UED. The first term is
Figure 3–12: A closer look into dilepton invariant mass distributions. (a) Contour dotted lines represent the size of the coefficient $r$ in eqn. 3–9. The minimal UED is a blue dot in the upper-right corner since $y$ and $z$ are almost 1 due to the mass degeneracy. The red dots represent several snowmass points: SPS1a, SPS1b, SPS5 and SPS3 from left to right. The green line represents gaugino unification so all SUSY benchmark points are close to this green line. (b) The dashed line represents the dilepton distribution in SUSY or pure phase space. The solid cyan (magenta) line represents the dilepton distribution in UED for $r = -0.3$ ($r = 0.7$).

linear in $\hat{m}$ like phase space and the second term is proportional to $\hat{m}^3$. So we see that whether or not the UED distribution is the same as the SUSY distribution depends on the size of the coefficient $r$ in the second term of the UED distribution. The UED distribution becomes exactly the same as the SUSY distribution if $r = 0.5$. Therefore we scan the $(y, z)$ parameter space, calculate the coefficient $r$ and show our result in fig. 3–12(a). In fig. 3–12(a), the contour dotted lines represent the size of the coefficient $r$ in eqn. 3–9. The minimal UED is blue dot in upper-right corner since $y$ and $z$ are almost 1 due to the degeneracy in the masses while red dots represent several snowmass points [97]: SPS1a, SPS1b, SPS5 and SPS3 from left to right. The green line represents gaugino unification so all SUSY benchmark points are close to this green line. As we see $r$ is small for both MUED and snowmass points and this is why we did not see any difference in the distributions from fig. 3–11. If the mass spectrum is either narrow (MUED mass spectrum) or generic mSugra type, the dilepton distributions are very similar.
and we can not tell any spin information from this distribution. However away from the mSugra model or MUED, we can easily find the regions where this coefficient $r$ is large and the spin correlation is big enough so that we can see a difference in shape. We show two points (denoted by ‘Good’ and ‘Better’) from fig.3–12(a) and show the corresponding dilepton distributions in fig.3–12(b). For the ‘Good’ point, the mass ratio is $m_{\tilde{\chi}_1^0} : m_{\tilde{\ell}} : m_{\tilde{\chi}_2^0} = 9 : 10 : 20$ and for the ‘Better’ point, $m_{\tilde{\chi}_1^0} : m_{\tilde{\ell}} : m_{\tilde{\chi}_2^0} = 1 : 2 : 4$. In fig.3–12(b), the dashed line represents dilepton distribution in SUSY or pure phase space and the solid cyan (magenta) line represents the dilepton distribution in UED for $r = -0.3$ ($r = 0.7$). Indeed for larger $r$, the distributions look different but background and detector simulation need to be included. Notice that in the mSugra model, the maximum of the coefficient $r$ is 0.4.

### 3.2.2 Lepton-Jet Invariant Mass - Charge Asymmetry

Now we look at spin correlations between $q$ and $\ell$ in fig. 3–10. In this case, there are several complications. First of all, we don’t know which lepton we need to choose. There are two leptons in the final state. One lepton, called ‘near’, comes from the decay of $\tilde{\chi}_2^0$ in SUSY or $Z_1$ in UED, while the other lepton, called ‘far’, comes from the decay of $\tilde{\ell}$ in SUSY or $\ell_1$ in UED. One can form the lepton-quark invariant mass distributions $m_{\ell q}$. The spin of the intermediate particle ($Z_1$ in UED or $\tilde{\chi}_2^0$ in SUSY) governs the shape of the distributions for the near lepton. However, in practice we cannot distinguish the near and far lepton, and one has to include the invariant mass combinations with both leptons (it is impossible to tell near and far leptons event by event but there can be an improvement on their selection [96]). Second, we do not measure charge of jets (or quarks). Therefore we do not know whether a particular jet (or quark) came from the decay of squark or anti-squark. This doubles the number of diagrams that we need to consider. These complications tend to wash out the spin correlations, but a residual effect remains,
which is due to the different number of quarks and anti-quarks in the proton,
which in turn leads to a difference in the production cross-sections for squarks and anti-squarks [12]. Most importantly, we do not know which jet is actually the correct jet in this cascade decay chain. We pair-produce two squarks (or KK quarks) particles and each of them produces one jet. Once ISR is included, there are many jets in the final state. For now, as in [13], we will assume that we know which jet is the correct one we need to choose. One never knows for sure which is the correct jet although there can be clever cuts to increase the probability that we picked the right one [96]. There are two possible invariant distributions in this case: 

\[ \frac{d\sigma}{dm}_{q^+} \text{ with a positively charge lepton and } \frac{d\sigma}{dm}_{q^-} \text{ with a negatively charged lepton.} \]

In principle, there are 8 diagrams that need to be included (a factor of 2 from quark/anti-quark combination, another factor of 2 from the two different leptons with different chiralities, a factor of 2 from the ambiguity between near and far leptons).

For this study, as in the dilepton case, we first start from a UED mass spectrum and adjust the MSSM parameters until we get a perfect match in the spectrum. In this case, \( Z_1 \) does not decay into right handed leptons. There are 4 contributions and they all contribute to both \( \frac{d\sigma}{dm}_{q^+} \) and \( \frac{d\sigma}{dm}_{q^-} \) distributions which are in fig. 3–13,

\[
\left( \frac{d\sigma}{dm} \right)_{q^+} = f_q \left( \frac{dP_2}{dm_n} + \frac{dP_1}{dm_f} \right) + f_{\bar{q}} \left( \frac{dP_1}{dm_n} + \frac{dP_2}{dm_f} \right) \\
\left( \frac{d\sigma}{dm} \right)_{q^-} = f_q \left( \frac{dP_1}{dm_n} + \frac{dP_2}{dm_f} \right) + f_{\bar{q}} \left( \frac{dP_2}{dm_n} + \frac{dP_1}{dm_f} \right),
\]

where \( P_1 \) (\( P_2 \)) represents distribution for a decay from a squark or KK quark (anti-squark or anti-KK quark) and \( f_q \) (\( f_{\bar{q}} \)) is the fraction of squarks or KK quarks (anti-squarks or anti-KK quarks) and by definition, \( f_q + f_{\bar{q}} = 1 \). This quantity \( f_q \) tells us how much squarks or KK quarks are produced compared to their anti-particles. For a UED mass spectrum and SPS1a, \( f_q \sim 0.7 \) [13]. These two
distributions in UED (SUSY) are shown in fig. 3–13(a) (fig. 3–13(b)) in different colors. The distributions are normalized to $\mathcal{L} = 10 fb^{-1}$ and the very sharp edge near $m_{q\ell} \sim 60$ GeV ($m_{q\ell} \sim 75$ GeV) is due to the near (far) lepton. However, once background and detector resolutions are included, the clear edges are smoothed out.

Now with these two distributions, a convenient quantity, ‘asymmetry’ [12] is defined below

$$A^{+-} = \frac{(\frac{d\sigma}{dm})_{q\ell^+} - (\frac{d\sigma}{dm})_{q\ell^-}}{(\frac{d\sigma}{dm})_{q\ell^+} + (\frac{d\sigma}{dm})_{q\ell^-}}.$$  \hspace{1cm} (3–11)

Notice that if $f_q = f_{\bar{q}} = 0.5$, $(\frac{d\sigma}{dm})_{q\ell^+} = (\frac{d\sigma}{dm})_{q\ell^-}$ and $A^{+-}$ becomes zero.

This is the case for pure phase space decay. So zero asymmetry means we don’t obtain any spin information from this decay chain, i.e., if we measure non-zero asymmetry, it means that the intermediate particle ($\tilde{\chi}_2^0$ or $Z_1$) has non-zero spin. So for this method to work, $f_q$ must be different from $f_{\bar{q}}$. So this method does not apply at $p\bar{p}$ collider such as the Tevatron since a $p\bar{p}$ collider produces the same amount of quarks and anti-quarks. The spin correlations are encoded in the charge asymmetry [12]. However, even in a $pp$ collider such as the LHC, whether or not we measure non-zero asymmetry depends on parameter space, e.g., in the focus point region, gluino production dominates and gluino produces equal amounts of squarks and anti-squarks. Therefore we expect $f_q \sim f_{\bar{q}} \sim 0.5$ and the asymmetry will be washed out.

Our comparison between $A^{+-}$ in the case of UED and SUSY for UED mass spectrum is shown in fig. 3–14(a). We see that although there is some minor difference in the shape of the asymmetry curves, overall the two cases appear to be very difficult to discriminate unambiguously, especially since the regions near the two ends of the plot, where the deviation is the largest, also happen to suffer from poorest statistics. Notice that we have not included detector effects
Figure 3–13: Jet-lepton invariant mass distributions. \( \frac{dN}{dm} q^+ \) (blue) and \( \frac{dN}{dm} q^+ \) (red) in the case of (a) UED and (b) SUSY for UED mass spectrum with \( R^{-1} = 500 \text{ GeV} \). \( q \) stands for both a quark and an antiquark, and \( N(q^+) \) (\( N(q^+) \)) is the number of entries with positively (negatively) charged lepton. The distributions are normalized to \( \mathcal{L} = 10 \text{fb}^{-1} \). A very sharp edge near \( m_{q^+} \approx 60 \text{ GeV} \) (\( m_{q^+} \approx 75 \text{ GeV} \)) is due to near (far) lepton. Once background and detector resolutions are included, the clear edges are smoothed out.

or backgrounds. Finally, and perhaps most importantly, this analysis ignores the combinatorial background from the other jets in the event, which could be misinterpreted as the starting point of the cascade depicted in fig. 3–10. Overall, fig. 3–14 shows that although the asymmetry (eqn. 3–11) does encode some spin correlations, distinguishing between the specific cases of UED and SUSY appears challenging.

Similarly in fig. 3–14(b), we show the asymmetry in UED and SUSY for a mass spectrum from the SPS1a point in the mSugra model. In this case, the mass spectrum is broad compared to the UED spectrum and \( \chi^0_2 \) in SUSY (\( Z_1 \) in UED) does not decay into left handed sleptons (\( SU(2)_W \) KK leptons). Unlike the narrow mass spectrum, in this study point with larger mass splittings, as expected in typical SUSY models, the asymmetry distributions appear to be more distinct than the case shown in fig. 3–14(a), which is a source of optimism. These results have been recently confirmed in [13]. It remains to be seen whether this
Figure 3–14: Asymmetries for UED and SUSY are shown in blue and red, respectively, in the case of (a) UED mass spectrum with $R^{-1} = 500$ GeV and (b) SPS1a mass spectrum. The horizontal dotted line represents pure phase space. The error bars represent statistical uncertainty with $L = 10 \text{ fb}^{-1}$.

Conclusion persists in a general setting, and once the combinatorial backgrounds are included [96]. Notice that comparing (a) and (b) in fig. 3–11, the signs of the two asymmetries have changed. The difference is the chirality of sleptons or KK leptons. In fig. 3–11(a) (fig. 3–11(a)), left handed sleptons or $SU(2)_W$ doublet KK leptons (right handed sleptons or $SU(2)_W$ singlet KK leptons) are onshell and the asymmetry starts out positive (negative) and ends negative (positive). By looking at the sign of the asymmetry, we can see which chirality was onshell.

What we did so far was, first we choose a study point in one model and fake parameters in other models until we see perfect match in the mass spectrum. However not all masses are observable and sometimes we get less constraints than the number of masses involved in the decay. So what we need to do is to match endpoints in the distributions instead of matching mass spectrum and ask whether there is any point in parameter space which is consistent with the experimental data. In other words, we have to ask which model fits the data better. We consider three kinematic endpoints: $m_{q\ell\ell}$, $m_{q\ell}$ and $m_{\ell\ell}$ (see fig. 3–10). In principle, we can find more kinematic endpoints such as a lower edge, here we are being conservative.
and take upper edges only [98–100]. In case of an onshell decay of $\chi_0^0$ and $\tilde{t}$, these three kinematic endpoints are written in terms of masses

\[
\begin{align*}
  m_{q\ell\ell} &= m_{\tilde{q}} \sqrt{(1 - x)(1 - yz)} \\
  m_{q\ell} &= m_{\tilde{q}} \sqrt{(1 - x)(1 - z)} \\
  m_{\ell\ell} &= m_{\tilde{q}} \sqrt{x(1 - y)(1 - z)}
\end{align*}
\] (3–12)

where $m_{\tilde{q}}$ is squark mass or KK quark mass and $x = \left(\frac{m_{\tilde{q}\ell}}{m_{\tilde{q}}}\right)^2$, $y = \left(\frac{m_{q}}{m_{\chi_0^0}}\right)^2$, and $z = \left(\frac{m_{q\ell}}{m_{\chi_0^0}}\right)^2$ are the ratios between masses in the cascade decay chain. By definition, $x$, $y$ and $z$ are less than 1.

We are now left with 2 free parameters: $f_q$ and $x$ and solve for $y$, $z$ and $m_{\tilde{q}}$ in terms of two free parameters. We minimize $\chi^2$,

\[
\chi^2 = \sum_{i=1}^{n} \frac{(x_i - \mu_i)^2}{\sigma_i^2},
\] (3–13)

between the two asymmetries in the $(x, f_q)$ parameter space to see whether we can fake a SUSY asymmetry in the UED model. $x_i$ is the theory prediction and $\mu_i$ is the experimental value with uncertainty $\sigma_i$. $\chi^2_{\text{dof}} = \chi^2/n$ is the ‘reduced’ $\chi^2$ or $\chi^2$ for $n$ degrees of freedom.

Our result is shown in fig. 3–15(a). We found the minimum $\chi^2$ is around 3 in the region where all KK masses are the same as the SUSY masses in the decay and $f_q$ is large. This means that $\chi^2$ is minimized when we have perfect match in mass spectrum. The red circle is the SPS1a point.

Now since we don’t have experimental data yet, we generated data samples from SPS1a assuming $10 fb^{-1}$ and constructed the asymmetries in SUSY and UED in fig. 3–15(b). We included 10% jet energy resolution. Red dots represent data points and the red line is the SUSY fit to the data points and the blue lines are the UED fits to the data points for two different $f_q$’s. For SUSY, $\chi^2$ is around 1 as we expect. We can get better $\chi^2$ for UED from 9.1 to 4.5 by increasing $f_q$. 
Figure 3–15: Asymmetries with relaxed conditions. (a) The contour lines show $\chi^2$ in the $(x, f_q)$ parameter space and the red dot represents the SPS1a point. $\chi^2$ is minimized when $f_q \approx 1$ and $x$ is the same as for SPS1a. (b) Red dots represent the data points with statistical error bars generated from SPS1a with $L = 10$ fb$^{-1}$ including 10% jet energy resolution. $\chi^2$-minimized UED (SUSY) fits to data are shown in blue (red). Since data was generated from SUSY, small $\chi^2$ in the SUSY fit is expected. $\chi^2$ in the UED fits is 9.1 (blue solid) and 4.5 (blue dotted) for $f_q = 0.7$ and $f_q = 1$, respectively.

It is still too big to fit the experimental data. So our conclusion for this study is that a particular point like SPS1a can not be faked through the entire parameter space of UED. However we need to check whether this conclusion will remain the same when we include the wrong jets which have nothing to do with this decay chain [96]. Notice that the clear edge at $m_{q\ell} \sim 300$ GeV in fig. 3–14(b) disappeared in fig. 3–15(b) after including jet energy resolution. From fig. 3–14, we see that SUSY has a larger asymmetry.

3.3 UED and SUSY at Linear Colliders

Universal Extra Dimensions and supersymmetry have rather similar experimental signatures at hadron colliders. The proper interpretation of an LHC discovery in either case may therefore require further data from a lepton collider. In this section we identify methods for discriminating between the two scenarios at the linear collider. We will consider 3 TeV Compact Linear Collider (CLIC). We study the processes of Kaluza-Klein muon pair production in universal extra
dimensions in parallel to smuon pair production in supersymmetry, accounting for the effects of detector resolution, beam-beam interactions and accelerator induced backgrounds. We find that the angular distributions of the final state muons, the energy spectrum of the radiative return photon and the total cross-section measurement are powerful discriminators between the two models. Accurate determination of the particle masses can be obtained both by a study of the momentum spectrum of the final state leptons and by a scan of the particle pair production thresholds. We also calculate the production rates of various Kaluza-Klein particles and discuss the associated signatures.

3.3.1 Event Simulation and Data Analysis

In order to study the discrimination of UED signals from supersymmetry, we have implemented the relevant features of the minimal UED model in the CompHEP event generator [68]. The MSSM is already available in CompHEP since version 41.10. All $n = 1$ KK modes are incorporated as new particles, with the proper interactions and one-loop corrected masses [28]. The widths can then be readily calculated with CompHEP on a case by case basis and added to the particle table. Similar to the SM case, the neutral gauge bosons at level 1, $Z_1$ and $\gamma_1$, are mixtures of the KK modes of the hypercharge gauge boson and the neutral $SU(2)_W$ gauge boson. However, it was shown in [62] that the radiatively corrected Weinberg angle at level 1 and higher is very small. For example, $\gamma_1$, which is the LKP in the minimal UED model, is mostly the KK mode of the hypercharge gauge boson. For simplicity, in the code we neglect neutral gauge boson mixing for $n \geq 1$.

In the next section we concentrate on the pair production of level 1 KK muons $e^+e^- \rightarrow \mu_1^+\mu_1^-$ and compare it to the analogous process of smuon pair production in supersymmetry: $e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-$. In UED there are two $n = 1$ KK muon Dirac fermions: an $SU(2)_W$ doublet $\mu_1^D$ and an $SU(2)_W$ singlet $\mu_1^S$, both of which contribute in eqn. (3–14) below (see also fig. 3–16). In complete
analogy, in supersymmetry, there are two smuon eigenstates, $\tilde{\mu}_L$ and $\tilde{\mu}_R$, both of which contribute in eqn. (3–15). The dominant diagrams in that case are shown in fig. 3–17. In principle, there are also diagrams mediated by $\gamma_n, Z_n$ for $n = 4, 6, \ldots$ but they are doubly suppressed - by the KK-number violating interaction at both vertices and the KK mass in the propagator - and here can be safely neglected. However, $\gamma_2$ and $Z_2$ exchange (fig. 3–16b) may lead to resonant production and significant enhancement of the cross-section, as well as interesting phenomenology as discussed below in Section 3.3.2.5. We have implemented the level 2 neutral gauge bosons $\gamma_2, Z_2$ with their widths, including both KK-number preserving and the KK-number violating decays as in Ref. [62]. We consider the final state consisting of two opposite sign muons and missing energy. It may arise either from KK muon production in UED

$$e^+e^- \rightarrow \mu_1^+\mu_1^- \rightarrow \mu^+\mu^-\gamma_1\gamma_1, \quad (3–14)$$
Table 3–1: Masses of the KK excitations for $R^{-1} = 500$ GeV and $\Lambda R = 20$ used in the analysis.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1^D$</td>
<td>515.0 GeV</td>
</tr>
<tr>
<td>$\mu_1^S$</td>
<td>505.4 GeV</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>500.9 GeV</td>
</tr>
</tbody>
</table>

with $\gamma_1$ being the LKP, or from smuon pair production in supersymmetry:

$$\mu^+ e^- \rightarrow \mu^+ \mu^- \rightarrow \mu^+ \mu^- \tilde{\chi}_1^0 \tilde{\chi}_1^0,$$

(3–15)

where $\tilde{\chi}_1^0$ is the lightest supersymmetric particle. We reconstruct the muon energy spectrum and the muon production polar angle, aiming at small background from SM processes with minimal biases due to detector effects and selection criteria.

The goal is to disentangle KK particle production (3–14) in UED from smuon pair production (3–15) in supersymmetry. We also determine the masses of the produced particles and test the model predictions for the production cross-sections in each case.

We first fix the UED parameters to $R^{-1} = 500$ GeV, $\Lambda R = 20$, leading to the spectrum given in Table 3–1. The ISR-corrected signal cross-section in UED for the selected final state $\mu^+ \mu^- \gamma_1 \gamma_1$ is 14.4 fb at $\sqrt{s} = 3$ TeV. Events have been generated with CompHEP and then reconstructed using a fast simulation based on parametrized response for a realistic detector at CLIC. In particular, the lepton identification efficiency, momentum resolution and polar angle coverage are of special relevance to this analysis. We assume that particle tracks will be reconstructed through a discrete central tracking system, consisting of concentric layers of Si detectors placed in a 4 T solenoidal field. This ensures a momentum resolution $\delta p/p^2 = 4.5 \times 10^{-5}$ GeV$^{-1}$. A forward tracking system should provide track reconstruction down to $\simeq 10^\circ$. We also account for initial state radiation (ISR) and for beamstrahlung effects on the center-of-mass energy. We assume that
muons are identified by their penetration in the instrumented iron return yoke of the central coil. A 4 T magnetic field sets an energy cutoff of \( \simeq 5 \) GeV for muon tagging.

The events from the CompHEP generation have been treated with the Pythia 6.210 parton shower [101] and reconstructed with a modified version of the SimDet 4.0 program [102]. Beamstrahlung has been added to the CompHEP generation. The luminosity spectrum, obtained by the GuineaPig beam simulation for the standard CLIC beam parameters at 3 TeV, has been parametrised using a modified Yokoya-Chen approximation [103,104]:

This analysis has backgrounds coming from SM \( \mu^+\mu^-\nu\bar{\nu} \) final states, which are mostly due to gauge boson pair production \( W^+W^- \rightarrow \mu^+\mu^-\nu_\mu\bar{\nu}_\mu, \ Z^0Z^0 \rightarrow \mu^+\mu^-\nu\bar{\nu} \) and from \( e^+e^- \rightarrow W^+W^-\nu_\mu\bar{\nu}_e, \ e^+e^- \rightarrow Z^0Z^0\nu_\nu\bar{\nu}_e \), followed by muonic decays. The background total cross-section is \( \simeq 20 \) fb at \( \sqrt{s} = 3 \) TeV. In addition to its competitive cross-section, this background has leptons produced preferentially at small polar angles, therefore biasing the angular distribution. In order to reduce this background, a suitable event selection has been applied. Events have been required to have two muons, missing energy in excess to 2.5 TeV, transverse energy below 150 GeV and event sphericity larger than 0.05. In order to reject the \( Z^0Z^0 \) background, events with di-lepton invariant mass compatible with \( M_{Z^0} \) have also been discarded. The underlying \( \gamma\gamma \) collisions also produces a potential background to this analysis in the form of \( \gamma\gamma \rightarrow \mu^+\mu^- \). This background has been simulated using the CLIC beam simulation and Pythia. Despite its large cross-section, it can be completely suppressed by a cut on the missing transverse energy \( E_T^{\text{missing}} > 50 \) GeV. Finally, in order to remove events with large beamstrahlung, the event sphericity had to be smaller than 0.35 and the acolinearity smaller than 0.8. These criteria provide a factor \( \simeq 30 \) background suppression, in the kinematical region of interest, while not significantly biasing the lepton momentum distribution.
Table 3–2: MSSM parameters for the SUSY study point used in the analysis. This choice of soft SUSY parameters in CompHEP leads to an exact match between the corresponding UED and SUSY mass spectra.

<table>
<thead>
<tr>
<th>MSSM Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>1000 GeV</td>
</tr>
<tr>
<td>$M_1$</td>
<td>502.65 GeV</td>
</tr>
<tr>
<td>$M_2$</td>
<td>1005.0 GeV</td>
</tr>
<tr>
<td>$M_{\mu_L}$</td>
<td>512.83 GeV</td>
</tr>
<tr>
<td>$M_{\mu_R}$</td>
<td>503.63 GeV</td>
</tr>
<tr>
<td>$\tan \beta$</td>
<td>10</td>
</tr>
</tbody>
</table>

3.3.2 Comparison of UED and Supersymmetry in $\mu^+\mu^-\not{E}_T$

In order to perform the comparison of UED and MSSM, we adjusted the MSSM parameters to get the two smuon masses $M_{\mu_L}$ and $M_{\mu_R}$ and the lightest neutralino mass $M_{\tilde{\chi}_1^0}$ matching exactly those of the two Kaluza-Klein muons $M_{\mu_1^0}$ and $M_{\mu_1'}$ and of the KK photon $M_{\gamma_1}$ for the chosen UED parameters. It must be stressed that such small mass splitting between the two muon partners is typically rather accidental in supersymmetric scenarios. The supersymmetric parameters used are given in Table 3–2. We then simulate both reactions (3–14) and (3–15) with CompHEP and pass the resulting events through the same simulation and reconstruction. The ISR-corrected signal cross-section in SUSY for the selected final state $\mu^+\mu^-\tilde{\chi}_1^0\tilde{\chi}_1^0$ is 2.76 fb at $\sqrt{s} = 3$ TeV, which is about 5 times smaller than in the UED case.

3.3.2.1 Angular Distributions and Spin Measurements

In the case of UED, the KK muons are fermions and their angular distribution is given by

$$\left( \frac{d\sigma}{d\cos \theta} \right)_{UED} \sim 1 + \frac{E_{\mu_1'}^2 - M_{\mu_1}^2}{E_{\mu_1}^2 + M_{\mu_1}^2} \cos^2 \theta.$$  \hspace{1cm} (3–16)

Assuming that at CLIC the KK production takes place well above threshold, the formula simplifies to:

$$\left( \frac{d\sigma}{d\cos \theta} \right)_{UED} \sim 1 + \cos^2 \theta.$$  \hspace{1cm} (3–17)
Figure 3-18: Differential cross-section $d\sigma/d\cos\theta_{\mu}$ for UED (blue, top) and supersymmetry (red, bottom) as a function of the muon scattering angle $\theta_{\mu}$. The figure on the left shows the ISR-corrected theoretical prediction. The two figures on the right in addition include the effects of event selection, beamstrahlung and detector resolution and acceptance. The left (right) panel is for the case of UED (supersymmetry). The data points are the combined signal and background events, while the yellow-shaded histogram is the signal only.

As the supersymmetric muon partners are scalars, the corresponding angular distribution is

$$
\left( \frac{d\sigma}{d\cos\theta} \right)_{\text{SUSY}} \sim 1 - \cos^2 \theta.
$$

Distributions (3-17) and (3-18) are sufficiently distinct to discriminate the two cases. However, the polar angles $\theta$ of the original KK-muons and smuons are not directly observable and the production polar angles $\theta_{\mu}$ of the final state muons are measured instead. But as long as the mass differences $M_{\mu_1} - M_{\gamma_1}$ and $M_{\tilde{\mu}} - M_{\tilde{\chi}_1^0}$ respectively remain small, the muon directions are well correlated with those of their parents (see figure 3-18a). In fig. 3-18b we show the same comparison after detector simulation and including the SM background. The angular distributions are well distinguishable also when accounting for these effects. By performing a $\chi^2$ fit to the normalised polar angle distribution, the UED scenario considered here

\[ E^{-1} = 500 \text{ GeV} \]
\[ E_{\text{CM}} = 3 \text{ TeV} \]
Figure 3–19: The total cross-section $\sigma$ in pb as a function of the center-of-mass energy $\sqrt{s}$ near threshold for $e^+e^- \rightarrow \mu^+_1\mu^-_1 \rightarrow \mu^+\mu^-\gamma_1\gamma_1$. Left: the threshold onset with (line, blue) and without (dots) beamstrahlung effects. Right: a threshold scan at selected points. The green curve refers to the reference UED parameters while for the red (blue) curve the mass of $\mu^S_1$ ($\mu^D_1$) has been lowered by 2.5 GeV. The points indicate the expected statistical accuracy for the cross section determination at the points of maximum mass sensitivity. Effects of the CLIC luminosity spectrum are included.

could be distinguished from the MSSM, on the sole basis of the distribution shape, with $350 \text{ fb}^{-1}$ of data at $\sqrt{s} = 3 \text{ TeV}$.

3.3.2.2 Threshold Scans

At the $e^+e^-$ linear collider, the muon excitation masses can be accurately determined through an energy scan of the onset of the pair production threshold. This study not only determines the masses, but also confirms the particle nature. In fact the cross-sections for the UED processes rise at threshold $\propto \beta$ while in supersymmetry their threshold onset is $\propto \beta^3$, where $\beta$ is the particle velocity. Since the collision energy can be tuned at properly chosen values, the power rise of the cross-section can be tested and the masses of the particles involved measured. We have studied such threshold scan for the $e^+e^- \rightarrow \mu^+_1\mu^-_1 \rightarrow \mu^+\mu^-\gamma_1\gamma_1$ process at $\sqrt{s} = 1 \text{ TeV}$, for the same parameters as in Table 3–1. We account for the anticipated CLIC centre-of-mass energy spread induced both by the energy spread in the CLIC.
linac and by beam-beam effects during collisions. This been obtained from the detailed \textit{GuineaPig} beam simulation and parametrised using the modified Yokoya-Chen model [103, 105]. An optimal scan of a particle pair production threshold consists of just two energy points, sharing the total integrated luminosity in equal fractions and chosen at energies maximising the sensitivity to the particle widths and masses [106]. For the UED model scan we have taken three points, one for normalisation and two at the maxima of the mass sensitivity (see figure 3–19). Inclusion of beamstrahlung effects induces a shift of the positions of these maxima towards higher nominal $\sqrt{s}$ values [107]. From the estimated sensitivity $d\sigma/dM$ and the cross-section accuracy, the masses of the two UED muon excitations can be determined to $\pm0.11$ GeV and $\pm0.23$ GeV for the singlet and the doublet states respectively, with a total luminosity of 1 ab$^{-1}$ shared in three points, when the particle widths can be disregarded.

3.3.2.3 Production Cross-Section Determination

The same analysis can be used to determine the cross-section for the process $e^+e^- \rightarrow \mu^+\mu^- E_T$. The SM contribution can be determined independently, using anti-tag cuts, and subtracted. Since the cross-section for the UED process at 3 TeV is about five times larger compared to smuon production in supersymmetry, this measurement would reinforce the model identification obtained by the spin determination. This can be quantified by performing the same $\chi^2$ fit to the muon polar production angle discussed above, but now including also the total number of selected events. Since the cross-section depends on the mass of the pair produced particles, we include a systematic uncertainty on the prediction corresponding to a $\pm0.05$ % mass uncertainty, which is consistent with the results discussed below. At CLIC the absolute luminosity should be measurable to $\mathcal{O}(0.1 \%)$ and the average effective collision energy to $\mathcal{O}(0.01 \%)$.
Figure 3.20: The muon energy spectrum resulting from KK muon production (3–14) in UED (blue, top curve) and smuon production (3–15) in supersymmetry (red, bottom curve). The UED and SUSY parameters are chosen as in fig. 3–18. The plot on the left shows the ISR-corrected distribution, while that on the right includes in addition the effects of event selection, beamstrahlung and detector resolution and acceptance. The data points are the combined signal and background events, while the yellow-shaded histogram is the signal only.

3.3.2.4 Muon Energy Spectrum and Mass Measurements

The characteristic end-points of the muon energy spectrum are completely determined by the kinematics of the two-body decay and hence they don’t depend on the underlying framework (SUSY or UED) as long as the masses involved are tuned to be identical. We show the ISR-corrected expected distributions for the muon energy spectra at the generator level in fig. 3–20a, using the same parameters as in fig. 3–18. As expected, the shape of the $E_\mu$ distribution in the case of UED coincides with that for MSSM. The lower, $E_{min}$, and upper, $E_{max}$, endpoints of the muon energy spectrum are related to the masses of the particles involved in the decay according to the relation:

$$E_{max/min} = \frac{1}{2} M_\mu \left( 1 - \frac{M_{\tilde{\chi}_1^0}^2}{M_\mu^2} \right) \gamma (1 \pm \beta)$$  \hspace{1cm} (3–19)
where $M_{\tilde{\mu}}$ and $M_{\tilde{\chi}_1^0}$ are the smuon and LSP masses and $\gamma = 1/(1 - \beta^2)^{1/2}$ with $\beta = \sqrt{1 - M_{\tilde{\mu}}^2/E_{\text{beam}}^2}$ is the $\tilde{\mu}$ boost. In the case of the UED the formula is completely analogous with $M_{\mu_1}$ replacing $M_{\tilde{\mu}}$ and $M_{\gamma_1}$ replacing $M_{\tilde{\chi}_1^0}$.

Due to the splitting between the $\tilde{\mu}_L$ and $\tilde{\mu}_R$ masses in MSSM and that between the $\mu_1^D$ and $\mu_1^S$ masses in UED, in fig. 3-20a we see the superposition of two box distributions. The left, narrower distribution is due to $\mu_1^S$ pair production in UED ($\tilde{\mu}_R$ pair production in supersymmetry). The underlying, much wider box distribution is due to $\mu_1^D$ pair production in UED ($\tilde{\mu}_L$ pair production in supersymmetry). The upper edges are well defined, with smearing due to beamstrahlung and, but less importantly, to momentum resolution. The lower end of the spectrum has the overlap of the two contributions and with the underlying background.

Furthermore, since the splitting between the masses of the $\mu_1^D$, $\mu_1^S$ and that of $\gamma_1$ is small, the lower end of the momentum distribution can be as low as $O(1 \text{ GeV})$ where the lepton identification efficiency is cut-off by the solenoidal field bending the lepton before it reaches the electro-magnetic or the hadron calorimeter [109]. Nevertheless, there is sufficient information in this distribution to extract the mass of the $\gamma_1$ particle, using the prior information on the $\mu_1^D$ and $\mu_1^S$ masses, obtained by the threshold scan.

In fig. 3-20b we show the muon energy distribution after detector simulation. A one parameter fit gives an uncertainty on the $\gamma_1$ mass of $\pm 0.19$ (stat.) $\pm 0.21$ (syst) GeV, where the statistical uncertainty is given for 1 ab$^{-1}$ of data and the systematics reflects the effect of the uncertainty on the $\mu_1$ masses. The beamstrahlung introduces an additional systematics, which depends on the control of the details of the luminosity spectrum.

3.3.2.5 Photon Energy Spectrum and Radiative Return to the $Z_2$

With the $e^+e^-$ colliding at a fixed center-of-mass energy above the pair production threshold a significant fraction of the KK muon production will proceed
through radiative return. Since this is mediated by $s$-channel narrow resonances, a sharp peak in the photon energy spectrum appears whenever one of the mediating $s$-channel particles is on-shell. In case of supersymmetry, only $Z$ and $\gamma$ particles can mediate smuon pair production and neither of them can be close to being on-shell. On the contrary, an interesting feature of the UED scenario is that $\mu_1$ production can be mediated by $Z_n$ and $\gamma_n$ KK excitations (for $n$ even) as shown in fig. 3–16b. Among these additional contributions, the $Z_2$ and $\gamma_2$ exchange diagrams are the most important. Since the decay $Z_2 \rightarrow \mu_1 \mu_1$ is allowed by phase space, there will be a sharp peak in the photon spectrum, due to a radiative return to the $Z_2$. The photon peak is at

$$E_\gamma = \frac{1}{2} E_{CM} \left( 1 - \frac{M_{Z_2}^2}{E_{CM}^2} \right). \quad (3–20)$$

On the other hand, $M_{\gamma_2} < 2M_{\mu_1}$, so that the decay $\gamma_2 \rightarrow \mu_1 \mu_1$ is closed, and therefore there is no radiative return to $\gamma_2$. Notice that the level 2 Weinberg angle is very small [28] and therefore $Z_2$ is mostly $W_2^0$-like and couples predominantly to
\[ \mu_1^D \text{ and not } \mu_1^S. \] The photon energy spectrum in \( e^+e^- \rightarrow \mu_1^+\mu^-\gamma \) for \( R^{-1} = 1350 \) GeV, \( \Delta R = 20 \) and \( E_{CM} = 3 \) TeV is shown in fig. 3–21. On the left we show the ISR-corrected theoretical prediction from \texttt{CompHEP} while the result on the right in addition includes detector and beam effects. It is clear that the peak cannot be missed.

3.3.3 Prospects for Discovery and Discrimination in Other Final States

Previously in section 3.3.2 we considered the \( \mu^+\mu^-E_T \) final state resulting from the pair production of level 1 KK muons. However, this is not the only signal which could be expected in the case of UED. Due to the relative degeneracy of the KK particles at each level, the remaining \( n = 1 \) KK modes will be produced as well, and will yield observable signatures. In those cases, the discrimination techniques which we discussed earlier can still be applied, providing further evidence in favor of one model over the other. In this section we compute the cross-sections for some of the other main processes of interest, and discuss how they could be analyzed.

3.3.3.1 Kaluza-Klein Leptons

We first turn to the discussion of the other KK lepton flavors. The KK \( \tau \)-leptons, \( \tau_1^\pm \), are also produced in \( s \)-channel diagrams only, as in fig. 3–16, hence the \( \tau_1^+\tau_1^- \) production cross-sections are very similar to the \( \mu_1^+\mu^- \) case. The final state will be \( \tau^+\tau^-E_T \), and it can be observed in several modes, corresponding to the different options for the \( \tau \) decays. However, due to the lower statistics and the inferior jet energy resolution, none of the resulting channels can compete with the discriminating power of the \( \mu_1^+\mu^-E_T \) final state discussed in the previous section.

The case of KK electrons is more interesting, as it contains a new twist. The production of KK electrons can also proceed through the \( t \)-channel diagram shown in fig. 3–22c. As a result, the production cross-sections for KK electrons can be much higher than for KK muons. We illustrate this in fig. 3–23, where we
show separately the cross-sections for $SU(2)_W$ doublets (solid lines) and $SU(2)_W$ singlets (dotted lines), as a function of $R^{-1}$. (For the numerical results throughout section 3.3.3, we always fix $\Lambda R = 20$.) At low masses (i.e. low $R^{-1}$) the $e^+_1 e^-_1$ cross-sections can be up to two orders of magnitude larger, compared to the case of $\mu^+_1 \mu^-_1$. Another interesting feature is the resonant enhancement of the cross-section for $R^{-1} \sim 1450$ GeV, which is present in either case ($e$ or $\mu$) for the $SU(2)_W$ doublets (solid lines), but not the $SU(2)_W$ singlets (dotted lines). The feature is due to the on-shell production of the level 2 $Z_2$ KK gauge boson, which can then decay into a pair of level 1 KK leptons (see diagram (b) in figs. 3–16 and 3–22). Since the Weinberg angle at the higher ($n > 0$) KK levels is tiny [28], $Z_2$ is predominantly an $SU(2)_W$ gauge boson and hence does not couple to the $SU(2)_W$ singlet fermions, which explains the absence of a similar peak in the $e^+_1 S$ and $\mu^+_1 S$ cross-sections\(^3\). Because of the higher production rates, the $e^+_1 e^-_1 \slashed{E}_T$ event sample will be much larger and have better statistics than $\mu^+_1 \mu^-_1 \slashed{E}_T$. The $e^+_1 e^-_1 \slashed{E}_T$ final state has been recently advertised as a discriminator between UED and supersymmetry in [108]. However, the additional $t$-channel diagram (fig. 3–22c) has the effect of not only enhancing the overall cross-section, but also distorting

\(^3\) One might have expected a second peak closeby due to $\gamma_2$ resonant production, but in the minimal UED model the spectrum is such that the decays of $\gamma_2$ to level 1 fermions are all closed.
Figure 3–23: ISR-corrected production cross-sections of level 1 KK leptons ($e_1$ in red, $\mu_1$ in blue) at CLIC, as a function of $R^{-1}$. Solid (dotted) lines correspond to $SU(2)_W$ doublets (singlets).

the differential angular distributions discussed previously in Section 3.3.2.1, and creating a forward peak, which causes the cases of UED and supersymmetry to look very much alike. We show the resulting angular distributions of the final state electrons in fig. 3–24. For proper comparison, we follow the same procedure as before: we choose the UED spectrum for $R^{-1} = 500$ GeV, which yields KK electron masses as in Table 3–1. We then choose a supersymmetric spectrum with selectron mass parameters as in Table 3–2. This guarantees matching mass spectra in the two cases (UED and supersymmetry) so that any differences in the angular distributions should be attributed to the different spins. Unlike fig. 3–18, where the underlying shapes of the angular distributions were very distinctive (see eqs. (3–17) and (3–18)), the main effect in fig. 3–24 is the uniform enhancement of the forward scattering cross-section, which tends to wash out the spin correlations exhibited in fig. 3–18.

3.3.3.2 Kaluza-Klein Quarks

Level 1 KK quarks will be produced in s-channel via diagrams similar to those exhibited in fig. 3–16. The corresponding production cross-sections are shown in fig. 3–25, as a function of $R^{-1}$. We show separately the cases of the $SU(2)_W$
Figure 3-24: Differential cross-section \( \frac{d\sigma}{d\cos\theta_e} \) for UED and supersymmetry. The same as fig. 3-18 (left panel), but for KK electron production \( e^+e^- \rightarrow e_1^+e_1^- \), with \( \theta_e \) being the electron scattering angle.

doublets \( u_1^D \) and \( d_1^D \) and the \( SU(2)_W \) singlets \( u_1^S \) and \( d_1^S \). In the minimal UED model, the KK fermion doublets are somewhat heavier than the KK fermion singlets [28], so naturally, the production cross-sections for \( u_1^D \) and \( d_1^D \) cut off at a smaller value of \( R^{-1} \). Since singlet production is only mediated by \( U(1) \) hypercharge interactions, the singlet production cross-sections tend to be smaller.

We notice that \( u_1^S \bar{u}_1^S \) is larger by a factor of \( 2^2 \) compared to \( d_1^S \bar{d}_1^S \), in accordance with the usual quark hypercharge assignments.

The observable signals will be different in the case of \( SU(2)_W \) doublets and \( SU(2)_W \) singlets. The singlets, \( u_1^S \) and \( d_1^S \), decay directly to the LKP \( \gamma_1 \), and the corresponding signature will be 2 jets and missing energy. The jet angular distribution will again be indicative of the KK quark spin, and can be used to discriminate against (right-handed) squark production in supersymmetry, following the procedure outlined in section 3.3.2.1. The jet energy distribution will again exhibit endpoints, which will in principle allow for the mass measurements discussed in section 3.3.2.4. A threshold scan of the cross-section will provide further evidence of the particle spins (see section 3.3.2.2). The only major difference with respect to the \( \mu^+\mu^- E_T \) final state discussed in section 3.3.2, is the absence of the
monochromatic photon signal from section 3.3.2.5, since $Z_2$ is too light to decay to KK quarks. In spite of the many similarities to the dimuon final state considered in section 3.3.2, notice that jet angular and energy measurements are not as clean and therefore the lepton (muon or electron) final states would still provide the most convincing evidence for discrimination. The signatures of the $SU(2)_W$ doublet quarks are richer – both $u_1^D$ and $d_1^D$ predominantly decay to $Z_1$ and $W_1^\pm$ which in turn decay to leptons and the LKP [62]. The analogous process in supersymmetry would be left-handed squark production with subsequent decays to $\tilde{\chi}_2^0$ or $\tilde{\chi}_1^\pm$, which in turn decay to $\tilde{\ell}_L$ and $\tilde{\chi}_1^0$. In principle, the spin information will still be encoded in the angular distributions of the final state particles. However, the analysis is much more involved, due to the complexity of the signature, and possibly the additional missing energy from any neutrinos.

3.3.3.3 Kaluza-Klein Gauge Bosons

The ISR-corrected production cross-sections for level 1 electroweak\(^4\) KK gauge bosons ($W_1^\pm$, $Z_1$ and $\gamma_1$) at a 3 TeV $e^+e^-$ collider are shown in fig. 3–26, as

\(^4\) The level 1 KK gluon, of course, has no tree-level couplings to $e^+e^-$.  

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![Figure 3-25: ISR-corrected production cross-sections of level 1 KK quarks at CLIC, as a function of $R^{-1}$.](image-url)
a function of $R^{-1}$. The three relevant processes are $W_1^+W_1^-$, $Z_1Z_1$ and $Z_1\gamma_1$ ($\gamma_1\gamma_1$ is unobservable). In each case, the production can be mediated by a $t$-channel exchange of a level 1 KK lepton, while for $W_1^+W_1^-$ there are additional $s$-channel diagrams with $\gamma$, $Z$, $\gamma_2$ and $Z_2$. $Z_1$ and $W_1^\pm$ are almost degenerate [28], thus their cross-sections cut off at around the same point. The analogous processes in supersymmetry would be the pair production of gaugino-like charginos and neutralinos. The final states will always involve leptons and missing energy, since $W_1^\pm$ and $Z_1$ do not decay to KK quarks. In conclusion of this section, for completeness we also discuss the possibility of observing the higher level KK particles and in particular those at level 2. For small enough $R^{-1}$, level 2 KK modes are kinematically accessible at CLIC. Once produced, they will in general decay to level 1 particles and thus contribute to the inclusive production of level 1 KK modes. Uncovering the presence of the level 2 signal in that case seems challenging, but not impossible.

We choose to concentrate on the case of the level 2 KK gauge bosons ($V_2$), which are somewhat special in the sense that they can decay directly to SM fermions through KK number violating interactions. Thus they can be easily observed as dijet or dilepton resonances. In principle, there are two types of production mechanisms for level 2 gauge bosons. The first is single production $e^+e^- \rightarrow V_2$, which can only proceed through KK number violating (loop suppressed) couplings. The second mechanism is $e^+e^- \rightarrow V_2V_2$ pair production which is predominantly due to KK number conserving (tree-level) couplings. In fig. 3-27 we show the corresponding cross-sections for the case of the neutral level 2 gauge bosons, as a function of $R^{-1}$. For low values of $R^{-1}$, pair production dominates, but as the level 2 gauge boson masses increase and approach $E_{CM}$, single production becomes resonantly enhanced. Thus the first indication of the presence of the level 2 particles may come from pair production events, but once the mass of the dijet
Figure 3–26: ISR-corrected production cross-sections of level 1 KK gauge bosons at CLIC, as a function of $R^{-1}$.

or dilepton resonance is known, the collider energy can be tuned to enhance the cross-section and study the $V_2$ resonance properties in great detail. Supersymmetry and Universal Extra Dimensions are two appealing examples of new physics at the TeV scale, as they address some of the theoretical puzzles of the SM. They also provide a dark matter candidate which, for properly chosen theory parameters, is consistent with present cosmology data. Both theories predict a host of new particles, partners of the known SM particles. If either one is realized in nature, the LHC is expected to observe signals of these new particles. However, in order to clearly identify the nature of the new physics, one may need to contrast the UED and supersymmetric hypotheses at a multi-TeV $e^+e^-$ linear collider such as CLIC.\(^5\) We studied in detail the process of pair production of muon partners in the two theories, KK-muons and smuons respectively. We used the polar production angle to distinguish the nature of the particle partners, based on their spin. The same

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\(^5\) Similar studies can also be done at the ILC provided the level 1 KK particles are within its kinematic reach. Since precision data tends to indicate the bound $R^{-1} \geq 250$ GeV for the case of 1 extra dimension, one would need an ILC center-of-mass energy above 500 GeV in order to pair-produce the lowest lying KK states of the minimal UED model.
analysis could be applied for the case of other KK fermions, as discussed in section 3.3.3. We have also studied the accuracy of CLIC in determining the masses of the new particles involved both through the study of the energy distribution of final state muons and threshold scans. An accuracy of better than 0.1% can be obtained with 1 ab\(^{-1}\) of integrated luminosity. Once the masses of the partners are known, the measurement of the total cross-section serves as an additional cross-check on the hypothesized spin and couplings of the new particles. A peculiar feature of UED, which is not present in supersymmetry, is the sharp peak in the ISR photon energy spectrum due to a radiative return to the KK partner of the Z.

The clean final states and the control over the center-of-mass energy at the CLIC multi-TeV collider allows one to unambiguously identify the nature of the new physics signals which might be emerging at the LHC already by the end of this decade.
CHAPTER 4
COSMOLOGICAL IMPLICATIONS

4.1 Dark Matter Abundance

In this chapter, we now focus on the cosmological implications of Universal Extra Dimensions. We revisit the calculation of the relic density of the lightest Kaluza-Klein particle (LKP) in the model of Universal Extra Dimensions. The first and only comprehensive calculation of the UED relic density to date was performed in [21]. The authors considered two cases of LKP: the KK hypercharge gauge boson $B_1$ and the KK neutrino $\nu_1$. The case of $B_1$ LKP is naturally obtained in MUED, where the radiative corrections to $B_1$ are the smallest in size, since they are only due to hypercharge interactions. The authors of [21] also realized the importance of coannihilation processes and included in their analysis coannihilations with the $SU(2)_W$-singlet KK leptons, which in MUED are the lightest among the remaining $n = 1$ KK particles. It was therefore expected that their coannihilations will be most important. Subsequently, Refs. [48, 49] analyzed the resonant enhancement of the $n = 1$ (co)annihilation cross-sections due to $n = 2$ KK particles.

Our goal in this chapter will be to complete the LKP relic density calculation of Ref. [21]. We will attempt to improve in three different aspects:

- We will include coannihilation effects with all $n = 1$ KK particles. The motivation for such a tour de force is twofold. First, recall that the importance of coannihilations is mostly determined by the degeneracy of the corresponding particle with the dark matter candidate. In the minimal UED model, the KK mass splittings are due almost entirely to radiative corrections. In MUED, therefore, one might expect that, since the corrections to KK particles other
than the KK leptons are relatively large, their coannihilations can be safely neglected. However, the minimal UED model makes an ansatz [28] about the cut-off scale values of the so called boundary terms, which are not fixed by known SM physics, and are in principle arbitrary. In this sense, the UED scenario should be considered as a low energy effective theory with a multitude of parameters, just like the MSSM, and the MUED model should be treated as nothing more than a simple toy model with a limited number of parameters, just like the “minimal supergravity” version of supersymmetry, for example. If one makes a different assumption about the inputs at the cut-off scale, both the KK spectrum and its phenomenology can be modified significantly. In particular, one could then easily find regions of this more general parameter space where other coannihilation processes become active. On the other hand, even if we choose to restrict ourselves to MUED, there is still a good reason to consider the coannihilation processes which were omitted in the analysis of [21]. While it is true that those coannihilations are more Boltzmann suppressed, their cross-sections will be larger, since they are mediated by weak and/or strong interactions. Without an explicit calculation, it is impossible to estimate the size of the net effect, and whether it is indeed negligible compared to the purely hypercharge-mediated processes which have already been considered.

- We will keep the exact value of each KK mass in our formulas for all annihilation cross-sections. This will render our analysis self-consistent. All calculations of the LKP relic density available so far [21, 48, 49], have computed the annihilation cross-sections in the limit when all level 1 KK masses are the same. This approximation is somewhat contradictory in the sense that all KK masses at level one are taken to be degenerate with LKP, yet only a limited number of coannihilation processes were considered. In reality,
a completely degenerate spectrum would require the inclusion of all possible coannihilations. Conversely, if some coannihilation processes are being neglected, this is presumably because the masses of the corresponding KK particles are not degenerate with the LKP, and are Boltzmann suppressed. However, the masses of these particles may still enter the formulas for the relevant coannihilation cross-sections, and using approximate values for those masses would lead to a certain error in the final answer. Since we are keeping the exact mass dependence in the formulas, within our approach heavy particles naturally decouple, coannihilations are properly weighted, and all relevant coannihilation cross-sections behave properly. Notice that the assumption of exact mass degeneracy overestimates the corresponding cross-sections and therefore underestimates the relic density. This expectation will be confirmed in our numerical analysis in Section 4.3.

- We will try to improve the numerical accuracy of the analysis by taking into account some minor corrections which were neglected or approximated in [21]. For example, we will use a temperature-dependent $g_*$ (the total number of effectively massless degrees of freedom, given by eq. (4–6) below) and include subleading corrections (4–19) in the velocity expansion of the annihilation cross-sections.

The availability of the calculation of the remaining coannihilation processes is important also for the following reason. Coannihilations with $SU(2)_W$-singlet KK leptons were found to reduce the effective annihilation cross-section, and therefore increase the LKP relic density. This has the effect of lowering the range of cosmologically preferred values of the LKP mass, or equivalently, the scale of the extra dimension. However, one could expect that coannihilations with the other $n = 1$ KK particles would have the opposite effect, since they have stronger interactions compared to the $SU(2)_W$-singlet KK leptons and the $B_1$ LKP. As a
result, the preferred LKP mass range could be pushed back up. For both collider
and astroparticle searches for dark matter, a crucial question is whether there
is an upper limit on the WIMP mass which could guarantee discovery, and if
so, what is its precise numerical value. To this end, one needs to consider the
effect of all coannihilation processes which have the potential to enhance the LKP
annihilations. We will see that the lowering of the preferred LKP mass range in the
case of coannihilations with $SU(2)_W$-singlet KK leptons is more of an exception
rather than the rule, and the inclusion of all remaining processes is needed in order
to derive an absolute upper bound on the LKP mass.

4.2 The Basic Calculation of the Relic Density

4.2.1 The Standard Case

We first summarize the standard calculation for the relic abundance of a
particle species $\chi$ which was in thermal equilibrium in the early universe and
decoupled when it became nonrelativistic [21, 110, 111]. The relic abundance is
found by solving the Boltzmann equation for the evolution of the $\chi$ number density

$$\frac{dn}{dt} = -3Hn - \langle \sigma v \rangle (n^2 - n_{eq}^2) ,$$

(4.1)

where $H$ is the Hubble parameter, $v$ is the relative velocity between two $\chi$’s, $\langle \sigma v \rangle$ is
the thermally averaged total annihilation cross-section times relative velocity, and
$n_{eq}$ is the equilibrium number density. At high temperature ($T \gg m$), $n_{eq} \sim T^3$
(there are roughly as many $\chi$ particles as photons). At low temperature ($T \ll m$),
in the nonrelativistic approximation, $n_{eq}$ can be written as

$$n_{eq} = g \left( \frac{mT}{2\pi} \right)^{\frac{3}{2}} e^{-m/T} ,$$

(4.2)

where $m$ is the mass of the relic $\chi$, $T$ is the temperature and $g$ is the number
of internal degrees of freedom of $\chi$ such as spin, color and so on. We see from
eq (4.2) that the density $n_{eq}$ is Boltzmann-suppressed. At high temperature, $\chi$
particles are abundant and rapidly convert to lighter particles and vice versa. But shortly after the temperature \( T \) drops below \( m \), the number density decreases exponentially and the annihilation rate \( \Gamma = \langle \sigma v \rangle n \) drops below the expansion rate \( H \). At this point, \( \chi \)'s stop annihilating and escape out of the equilibrium and become thermal relics. \( \langle \sigma v \rangle \) is often approximated by the nonrelativistic expansion\(^1\)

\[
\langle \sigma v \rangle = a + b\langle v^2 \rangle + \mathcal{O}(\langle v^4 \rangle) \approx a + 6b/x + \mathcal{O}\left(\frac{1}{x^2}\right),
\]

where

\[
x = \frac{m}{T}.
\]

By solving the Boltzmann equation analytically with appropriate approximations \([21,110,111]\), the abundance of \( \chi \) is given by

\[
\Omega_\chi h^2 \approx \frac{1.04 \times 10^9}{M_{Pl}} \frac{x_F}{\sqrt{g_*(x_F)}} \frac{1}{a + 3b/x_F},
\]

where the Planck mass \( M_{Pl} = 1.22 \times 10^{19} \) GeV and \( g_* \) is the total number of effectively massless degrees of freedom,

\[
g_*(T) = \sum_{i=bosons} g_i + \frac{7}{8} \sum_{i=fermions} g_i.
\]

The freeze-out temperature, \( x_F \), is found iteratively from

\[
x_F = \ln \left( c(c + 2) \sqrt{\frac{45 g}{8 \pi^3}} m_{Pl}(a + 6b/x_F) \right),
\]

where the constant \( c \) is determined empirically by comparing to numerical solutions of the Boltzmann equation and here we take \( c = \frac{1}{2} \) as usual. The coefficient \( \frac{7}{8} \) in

\(1\) Note, however, that the method fails near \( s \)-channel resonances and thresholds for new final states \([112]\). In the interesting parameter region of UED, we are always sufficiently far from thresholds, while for the treatment of resonances, see \([48,49]\).
the right hand side of (4–6) accounts for the difference in Fermi and Bose statistics. Notice that \( g \) is a function of the temperature \( T \), as the thermal bath quickly gets depleted of the heavy species with masses larger than \( T \).

### 4.2.2 The Case with Coannihilations

When the relic particle \( \chi \) is nearly degenerate with other particles in the spectrum, its relic abundance is determined not only by its own self-annihilation cross-section, but also by annihilation processes involving the heavier particles. The previous calculation can be generalized to this “coannihilation” case in a straightforward way \([21, 111, 112]\). Assume that the particles \( \chi_i \) are labeled according to their masses, so that \( m_i < m_j \) when \( i < j \). The number densities \( n_i \) of the various species \( \chi_i \) obey a set of Boltzmann equations. It can be shown that under reasonable assumptions \([112]\), the ultimate relic density \( n \) of the lightest species \( \chi_1 \) (after all heavier particles \( \chi_i \) have decayed into it) obeys the following simple Boltzmann equation

\[
\frac{dn}{dt} = -3Hn - \langle \sigma_{eff} v \rangle (n^2 - n_{eq}^2),
\]

(4–8)

where

\[
\sigma_{eff}(x) = \sum_{ij} \sigma_{ij} \frac{g_i g_j}{g_{eff}^2} (1 + \Delta_i)^{3/2} (1 + \Delta_j)^{3/2} \exp(-x(\Delta_i + \Delta_j)),
\]

(4–9)

\[
g_{eff}(x) = \sum_{i=1}^{N} g_i (1 + \Delta_i)^{3/2} \exp(-x\Delta_i),
\]

(4–10)

\[
\Delta_i = \frac{m_i - m_1}{m_1}.
\]

(4–11)

Here \( \sigma_{ij} \equiv \sigma(\chi_i \chi_j \rightarrow SM) \), \( g_i \) is the number of internal degrees of freedom of particle \( \chi_i \) and \( n = \sum_{i=1}^{N} n_i \) is the density of \( \chi_1 \) we want to calculate. This Boltzmann equation can be solved in a similar way \([21, 112]\), resulting in

\[
\Omega_\chi h^2 \approx \frac{1.04 \times 10^9}{M_{Pl}} \frac{x_F}{\sqrt{g_*(x_F)}} \frac{1}{I_a + 3I_b/x_F},
\]

(4–12)
with

\[ I_a = x_F \int_{x_F}^{\infty} a_{\text{eff}}(x) x^{-2} dx, \quad (4-13) \]
\[ I_b = 2x_F^2 \int_{x_F}^{\infty} b_{\text{eff}}(x) x^{-3} dx. \quad (4-14) \]

The corresponding formula for \( x_F \) becomes

\[ x_F = \ln \left( c(c+2) \sqrt{\frac{45}{8} g_{\text{eff}}(x_F) m M_P (a_{\text{eff}}(x_F) + 6 b_{\text{eff}}(x_F)/x_F)} \right) / g_s(x_F) x_F. \quad (4-15) \]

Here \( a_{\text{eff}} \) and \( b_{\text{eff}} \) are the first two terms in the velocity expansion of \( \sigma_{\text{eff}} \)

\[ \sigma_{\text{eff}}(x) v = a_{\text{eff}}(x) + b_{\text{eff}}(x) v^2 + \mathcal{O}(v^4). \quad (4-16) \]

Comparing eqs. (4-9) and (4-16), one gets

\[ a_{\text{eff}}(x) = \sum_{ij}^{N} a_{ij} g_i g_j (1 + \Delta_i)^{3/2} (1 + \Delta_j)^{3/2} \exp(-x(\Delta_i + \Delta_j)) \right), \quad (4-17) \]
\[ b_{\text{eff}}(x) = \sum_{ij}^{N} b_{ij} g_i g_j (1 + \Delta_i)^{3/2} (1 + \Delta_j)^{3/2} \exp(-x(\Delta_i + \Delta_j)) \right). \quad (4-18) \]

where \( a_{ij} \) and \( b_{ij} \) are obtained from \( \sigma_{ij} v = a_{ij} + b_{ij} v^2 + \mathcal{O}(v^4) \).

Considering relativistic corrections [110, 113, 114] to the above treatment results in an additional subleading term which can be accounted for by the simple replacement

\[ b \rightarrow b - \frac{1}{4} a \quad (4-19) \]

in the above formulas, which will be explained in detail in next section.

**4.2.3 Thermal Average and Nonrelativistic Velocity Expansion**

The thermally averaged cross-section times relative velocity is defined as [110],

\[ \langle \sigma v_{\text{rel}} \rangle = \frac{\int d^3p_1 d^3p_2 e^{-E_1/T} e^{-E_2/T} \sigma v_{\text{rel}}}{\int d^3p_1 d^3p_2 e^{-E_1/T} e^{-E_2/T}}, \quad (4-20) \]
where \( v_{rel} \) is the relative velocity between two incoming particles. Since the relic particle decouples from the equilibrium when the particle is nonrelativistic, we can use nonrelativistic energy-momentum relation, \( p = mv + \mathcal{O}(v^2) \) and \( E = m + \frac{1}{2}mv^2 + \mathcal{O}(v^4) \). In the CM frame, above equation becomes

\[
\langle \sigma v_{rel} \rangle = \frac{1}{E_1 E_2 v_{rel}} \int \frac{d^3v_1 d^3v_2 e^{-E_1/T_1 - E_2/T_2}}{d^3v_1 d^3v_2 e^{-E_1/T_1 - E_2/T_2}} \sigma v_{rel} \\
= \frac{1}{E_1 E_2 v_{rel}} \int \frac{d^3V d^3v_{rel} e^{-\frac{m}{2}\left(V^2 + v_{rel}^2\right)}}{d^3V d^3v_{rel} e^{-\frac{m}{2}\left(V^2 + v_{rel}^2\right)}} \\
= \frac{1}{E_1 E_2 v_{rel}} \int \frac{dv_{rel} e^{-\frac{1}{4}x^2 v_{rel}^2} \sigma v_{rel}}{dv_{rel} e^{-\frac{1}{4}x^2 v_{rel}^2}},
\]

where \( \vec{V} = \vec{v}_1 + \vec{v}_2 \), \( \vec{v}_{rel} = \vec{v}_1 - \vec{v}_2 \) and \( E_1 + E_2 = 2m + \frac{1}{2}m^2\left(v_1^2 + v_2^2\right) = 2m + \frac{m}{4}(V^2 + v_{rel}^2) \) are used and \( x = \frac{m}{V} \). Now all we need to do is to expand \( \sigma v_{rel} \) in terms of \( v_{rel} \) and integrate over it. The cross-section is given by

\[
\sigma = \frac{1}{2E_1 E_2} \int (2\pi)^4 \delta^4 \left( p_1 + p_2 - \sum_j p_j \right) \prod_i \frac{d^3p_i}{(2\pi)^3 2p_i^0}. \tag{4-22}
\]

Now we define function \( w(s) \) using above equation,

\[
w(s) = \frac{1}{2E_1 E_2} \int (2\pi)^4 \delta^4 \left( p_1 + p_2 - \sum_j p_j \right) \prod_i \frac{d^3p_i}{(2\pi)^3 2p_i^0}. \tag{4-23}
\]

Now in CM frame we expand \( \frac{1}{E} w(s) \) in terms of \( s \) around \( 4m^2 \) with \( s = 4m^2 + m^2 v_{rel}^2 + \mathcal{O}(v_{rel}^4) \) and \( E = m + \frac{1}{2}mv_{CM}^2 + \mathcal{O}(v_{rel}^4) \) (\( 2V_{CM} = v_{rel} \)),

\[
\sigma v_{rel} = \frac{1}{E_1 E_2} w(s) = \frac{1}{E_2} w(s) = \frac{w(4m^2) + \frac{dv}{ds}(s - 4m^2) + \mathcal{O}(v_{rel}^4)}{\left(m + \frac{1}{2}mv_{CM}^2 + \mathcal{O}(v_{rel}^4)\right)^2} \\
= \frac{w_0 + w_0 \frac{v_{rel}^2}{4} + \mathcal{O}(v_{rel}^4)}{m^2 \left(1 + \frac{v_{rel}^2}{4} + \mathcal{O}(v_{rel}^4)\right)} \tag{4-25}
\]

\[
= \frac{1}{m^2} \left( w_0 + \left( w'_0 - w_0 \right) \frac{v_{rel}^2}{4} \right) + \mathcal{O}(v_{rel}^4),
\]
where

\[ w_0 = w(4m^2) \]
\[ w_0' = 4m^2 \left( \frac{dw}{ds} \right)_{s=4m^2}. \]  

(4–26)

Let us define two coefficients in the velocity expansion as \( a \) and \( b \),

\[ a = \frac{w_0}{m^2}, \]
\[ b = \frac{w_0' - w_0}{4m^2}. \]  

(4–27)

Therefore the thermally averaged cross-section is

\[ \langle \sigma v_{rel} \rangle = a + b(v_{rel}^2) + \mathcal{O}(v_{rel}^4) \]
\[ = a + 6bx + \mathcal{O}(x^2). \]  

(4–28)

However the full relativistic calculation gives us [110]

\[ \langle \sigma v_{rel} \rangle = \frac{w_0}{m^2} + \frac{3}{2m^2}(w_0' - 2w_0) + \mathcal{O}(x^2) \]
\[ = a + 6(b - \frac{1}{4}a)x + \mathcal{O}(x^2). \]  

(4–29)

So we expand \( \sigma v_{rel} \) in terms of relative velocity in the nonrelativistic limit to get two coefficients \( a \) and \( b \) and we substitute \( b \) by \( b - \frac{1}{4}a \) to recover relativistic correction [113, 114].

4.3 Relic Density in Minimal UED

For the purposes of our study we have implemented the relevant features of the minimal UED model in the CompHEP event generator [68]. We incorporated all \( n = 1 \) and \( n = 2 \) KK modes as new particles, with the proper interactions and one-loop corrected masses [28]. Similar to the SM case, the neutral gauge bosons at level 1, \( Z_1 \) and \( \gamma_1 \), are mixtures of the KK modes of the hypercharge gauge boson and the neutral \( SU(2)_W \) gauge boson. However, as shown in [28],
the radiatively corrected Weinberg angle at level 1 and higher is very small. For example, $\gamma_1$, which is the LKP in the minimal UED model, is mostly the KK mode of the hypercharge gauge boson. Therefore, for simplicity, in the code we neglected neutral gauge boson mixing for $n = 1$. We then use our UED implementation in CompHEP to derive analytic expressions for the (co)annihilation cross-sections between any pair of $n = 1$ KK particles. Our code has been subjected to numerous tests and cross-checks. For example, we reproduced all results from Servant et al. [21]. We have also used the same code for independent studies of the collider and astroparticle signatures of UED [10, 19, 61, 115] and thus have tested it from a different angle as well.

The mass spectrum of the $n = 1$ KK partners in minimal UED can be found, for example, in fig. 1 of [62]. In MUED the next-to-lightest KK particles are the singlet KK leptons and their fractional mass difference from the LKP is

$$\Delta_{\ell_R1} \equiv \frac{m_{\ell_{R1}} - m_{\gamma_1}}{m_{\gamma_1}} \sim 0.01 .$$

(4-30)

Notice that the Boltzmann suppression

$$e^{-\Delta_{\ell_R1} x_F} \sim e^{-0.01 \cdot 25} = e^{-0.25}$$

is not very effective and coannihilation processes with $\ell_{R1}$ are definitely important, hence they were considered in [21]. What about the other, heavier particles in the $n = 1$ KK spectrum in MUED? Since their mass splittings from the LKP

$$\Delta_i \equiv \frac{m_i - m_{\gamma_1}}{m_{\gamma_1}}$$

(4-31)

---

2 In this chapter we follow the notation of [21] where the two types of $n = 1$ Dirac fermions are distinguished by an index corresponding to the chirality of their zero mode partner. For example, $\ell_{R1}$ stands for an $SU(2)_W$-singlet Dirac fermion, which has in principle both a left-handed and a right-handed component.
are larger, their annihilations suffer from a larger Boltzmann suppression. However, the couplings of all $n = 1$ KK partners other than $\ell_{R1}$ are larger compared to those of $\gamma_1$ and $\ell_{R1}$. For example, $SU(2)_W$-doublet KK leptons $\ell_{L1}$ couple weakly, and the KK quarks $q_1$ and KK gluon $g_1$ have strong couplings. Therefore, their corresponding annihilation cross-sections are expected to be larger than the cross-section of the main $\gamma_1\gamma_1$ channel.

We see that for the other KK particles, there is a competition between the increased cross-sections and the larger Boltzmann suppression. An explicit calculation is therefore needed in order to evaluate the net effect of these two factors, and judge the importance of the coannihilation processes which have been neglected so far. One might expect that coannihilations with $SU(2)_W$-doublet KK leptons might be numerically significant, since their mass splitting in MUED is $\sim 3\%$ and the corresponding Boltzmann suppression factor is only $e^{-0.03-25} \sim e^{-0.75}$.

In our code we keep all KK masses different while we neglect all the masses of the Standard Model particles. As an illustration, let us show the $a$ and $b$ terms for $\gamma_1\gamma_1$ annihilation only. For fermion final states we find the $a$-term and $b$-term of $\sigma(\gamma_1\gamma_1 \rightarrow f \bar{f}) v$ as follows

\begin{align}
    a &= \sum_f \frac{32 \alpha_1^2 N_c m_\gamma^2}{9} \left( \frac{Y_{fL}^4}{(m_{\gamma_1}^2 + m_{fL1}^2)^2} + \frac{Y_{fR}^4}{(m_{\gamma_1}^2 + m_{fR1}^2)^2} \right) \quad (\text{4-32}) \\
    &\approx \sum_f \frac{8 \pi \alpha_1^2}{9 m_{\gamma_1}^2} N_c (Y_{fL}^4 + Y_{fR}^4) = \frac{8 \pi \alpha_1^2}{9 m_{\gamma_1}^2} \left( \frac{95}{18} \right), \quad (\text{4-33})
\end{align}

\begin{align}
    b &= - \sum_f \frac{4 \pi \alpha_1^2 N_c m_\gamma^2}{27} \left( Y_{fL}^4 \frac{11 m_{\gamma_1}^4 + 14 m_{fL1}^2 - 13 m_{fL1}^4}{(m_{\gamma_1}^2 + m_{fL1}^2)^4} \right) \\
    &\quad + Y_{fR}^4 \frac{11 m_{\gamma_1}^4 + 14 m_{fR1}^2 - 13 m_{fR1}^4}{(m_{\gamma_1}^2 + m_{fR1}^2)^4} \right) \quad (\text{4-34}) \\
    &\approx - \sum_f \frac{\pi \alpha_1^2}{9 m_{\gamma_1}^2} N_c (Y_{fL}^4 + Y_{fR}^4) = - \frac{\pi \alpha_1^2}{9 m_{\gamma_1}^2} \left( \frac{95}{18} \right), \quad (\text{4-35})
\end{align}
Figure 4–1: The $a$-term of the annihilation cross-section for (a) $\gamma_1 \gamma_1 \rightarrow e^+e^-$ and (b) $\gamma_1 \gamma_1 \rightarrow \phi\phi^*$, as a function of the mass of the $t$-channel particle(s). We fix the LKP mass at $m_{\gamma_1} = 500$ GeV and vary (a) the KK lepton mass $m_{e_R1} = m_{e_L1}$ or (b) the KK Higgs boson mass $m_{\phi}$. The blue solid lines are the exact results (4–32) and (4–36), while the red dotted lines correspond to the approximations (4–33) and (4–37).

where $g_1$ is the gauge coupling of the hypercharge $U(1)_Y$ gauge group, $\alpha_1 = \frac{g_1^2}{4\pi}$ and $N_c = 3$ for $f = q$ and $N_c = 1$ for $f = \ell$. $Y_f$ is the hypercharge of the fermion $f$.

For the Higgs boson final states we get

$$a = \sum_i \frac{2\alpha_i^2 Y_i^4}{9 m_{\gamma_1}^2} \left( \frac{11 m_{\gamma_1}^4 - 2 m_{\gamma_1}^2 m_{\phi_i}^2 + 3 m_{\phi_i}^4}{(m_{\gamma_1}^2 + m_{\phi_i}^2)^2} \right)$$ \hspace{1cm} (4–36)

$$\approx \sum_i \frac{2\alpha_i^2 Y_i^4}{3 m_{\gamma_1}^2} = \frac{4\alpha_1^2 Y_1^4}{3 m_{\gamma_1}^2}, \hspace{1cm} (4–37)$$

$$b = -\sum_i \frac{\pi \alpha_i^2 Y_i^4}{108 m_{\gamma_1}^2} \times \left( \frac{121 m_{\gamma_1}^8 + 140 m_{\gamma_1}^6 m_{\phi_i}^2 - 162 m_{\gamma_1}^4 m_{\phi_i}^4 + 60 m_{\gamma_1}^2 m_{\phi_i}^6 - 15 m_{\phi_i}^8}{(m_{\gamma_1}^2 + m_{\phi_i}^2)^4} \right)$$ \hspace{1cm} (4–38)

$$\approx -\sum_i \frac{\pi \alpha_1^2 Y_1^2}{12 m_{\gamma_1}^2} = -\frac{\pi \alpha_1^2 Y_1^2}{6 m_{\gamma_1}^2}. \hspace{1cm} (4–39)$$

In the limit where all KK masses are the same (the second line in each formula above), we recover the result of [21]. Notice the tremendous simplification which
arises as a result of the mass degeneracy assumption. In fig. 4–1 we show the $a$ terms of the annihilation cross-section for two processes: (a) $\gamma_1\gamma_1 \rightarrow e^+e^-$ and (b) $\gamma_1\gamma_1 \rightarrow \phi\phi^*$, as a function of the mass of the $t$-channel particle(s). We fix the LKP mass at $m_{\gamma_1} = 500$ GeV and vary (a) the KK lepton mass $m_{e_{R1}} = m_{e_{L1}}$ or (b) the KK Higgs boson mass $m_{\phi_1}$. The blue solid lines are the exact results (4–32) and (4–36), while the red dotted lines correspond to the approximations (4–33) and (4–37) in which the mass difference between the $t$-channel particles and the LKP has been neglected. We see that the approximations (4–33) and (4–37) can result in a relatively large error, whose size depends on the actual mass splitting of the KK particles. This is why in our code we keep all individual mass dependencies.

Another difference between our analysis and that of Ref. [21] is that here we shall use a temperature-dependent $g_*$ function as defined in (4–6). The relevant value of $g_*$ which enters the answer for the LKP relic density (4–12) is $g_*(T_F)$, where $T_F = m_{\gamma_1}/x_F$ is the freeze-out temperature. In fig. 4–2a we show a plot of $g_*(T_F)$ as a function of $R^{-1}$ in MUED, while in fig. 4–2b we show the corresponding values of $x_F$. In fig. 4–2a one can clearly see the jumps in $g_*$ when crossing the $b\bar{b}$, $W^+W^-$, $ZZ$ and $hh$ thresholds (from left to right). The $t\bar{t}$ threshold is further to the right, outside the plotted range. As we shall see below, cosmologically interesting values of $\Omega h^2$ are obtained for $R^{-1}$ below 1 TeV, where $g_*(T_F) = 86.25$, since we are below the $W^+W^-$ threshold. The analysis of Ref. [21] assumed a constant value of $g_* = 92$, which is only valid between the $W^+W^-$ and $ZZ$ thresholds. The expert reader has probably noticed from fig. 4–2b that the values of $x_F$ which we obtain in MUED are somewhat larger than the $x_F$ values one would have in typical SUSY models. This is due to the effect of coannihilations, which increase $g_{\text{eff}}$ (see fig. 4–5c below) and therefore $x_F$, in accordance with (4–15).

We are now in a position to discuss our main result in MUED. In fig. 4–3 we show the LKP relic density as a function of $R^{-1}$ in the minimal UED model. We show
the results from several analyses, each under different assumptions, in order to illustrate the effect of each assumption. We first show several calculations for the academic case of no coannihilations. The three solid lines in fig. 4–3 account only for the $\gamma_1\gamma_1$ process. The (red) line marked “a” recreates the analysis of Ref. [21], assuming a degenerate KK mass spectrum. The (blue) line marked “b” repeats the same analysis, but uses $T$-dependent $g_*$ according to (4–6) and includes the relativistic correction to the $b$-term (4–19). The (black) line marked “c” further relaxes the assumption of KK mass degeneracy, and uses the actual MUED mass spectrum.

Comparing lines “a” and “b,” we see that, as already anticipated from fig. 4–2a, accounting for the $T$ dependence in $g_*$ has the effect of lowering $g_*(x_F)$, $\sigma_{\text{eff}}(x_F)$, and correspondingly, increasing the prediction for $\Omega h^2$. This, in turns, lowers the preferred mass range for $\gamma_1$. Next, comparing lines “b” and “c,” we see that dropping the mass degeneracy assumption has a similar effect on $\sigma_{\text{eff}}(x_F)$ (see fig. 4–1), and further increases the calculated $\Omega h^2$. This can be easily understood from the $t$-channel mass dependence exhibited in (4–32) and (4–36). The $t$-channel
Figure 4-3: Relic density of the LKP as a function of $R^{-1}$ in the minimal UED model. The (red) line marked “a” is the result from considering $\gamma_1\gamma_1$ annihilation only, following the analysis of Ref. [21], assuming a degenerate KK mass spectrum. The (blue) line marked “b” repeats the same analysis, but uses $T$-dependent $g_*$ according to (4-6) and includes the relativistic correction to the $b$-term (4-19). The (black) line marked “c” relaxes the assumption of KK mass degeneracy, and uses the actual MUED mass spectrum. The dotted line is the result from the full calculation in MUED, including all coannihilation processes, with the proper choice of masses. The green horizontal band denotes the preferred Wilkinson Microwave Anisotropy Probe (WMAP) region for the relic density $0.094 < \Omega_{CDM} h^2 < 0.129$. The cyan vertical band delineates values of $R^{-1}$ disfavored by precision data.

masses appear in the denominator, and they are by definition larger than the LKP mass. Therefore, using their actual values can only decrease $\sigma_{eff}$ and increase $\Omega h^2$.

The dotted line in fig. 4-3 is the result from the full calculation in MUED, including all coannihilation processes, with the proper choice of masses. The green horizontal band denotes the preferred WMAP region for the relic density $0.094 < \Omega_{CDM} h^2 < 0.129$. The cyan vertical band delineates values of $R^{-1}$ disfavored by precision data [33]. We see that according to the full calculation, the cosmologically ideal mass range is $m_{\gamma_1} \sim 500 - 600$ GeV, when $\gamma_1$ accounts for all of the dark matter in the Universe. This range is somewhat lower than earlier studies have indicated, mostly due to the effects discussed above. Since the MUED model
will be our reference point for the investigations in Section 4.4, the dotted line from fig. 4-3 will be appearing in all subsequent plots in Section 4.4 below.

4.4 Relative Weight of Different Coannihilation Processes

As we already explained in 4.1, the assumptions behind the MUED model can be easily relaxed by allowing non-vanishing boundary terms at the scale $\Lambda$. This would modify the KK spectrum and correspondingly change our prediction for the KK relic density from the previous section [116–120]. Our code is able to handle such more general cases with ease, since we use as inputs the physical KK masses. In order to gain some insight into the cosmology of such non-minimal scenarios, we have studied the effects of varying the $n = 1$ KK masses one at a time. The change in any given KK mass will not only enhance or suppress the related coannihilation processes, but also impact any other cross-sections which happen to have a dependence on the mass parameter being varied. Thus the results in this section may allow one to judge the importance of each individual coannihilation process, and anticipate the answer for $\Omega h^2$ in non-minimal models.

We have classified the discussion in this section by particle types. Section 4.4.1 contains our results for the annihilation processes with KK leptons. Many of our results have already appeared in Ref. [21]. The new element here is the discussion of $\ell_{L1}$ coannihilations. The results presented in Sections 4.4.2 and 4.4.3 are completely new – there we investigate the coannihilation effects with strongly interacting KK modes and electroweak gauge bosons and/or Higgs bosons, respectively.

4.4.1 Coannihilations with KK Leptons

We begin with a discussion of $\gamma_1$ coannihilations with the $n = 1$ $SU(2)_W$-singlet leptons $\ell_{R1}$ and the $n = 1$ $SU(2)_W$-doublet leptons $\ell_{L1}$. One might expect that those processes will be important, since the KK leptons receive relatively small one-loop mass corrections. For example, in the minimal UED model $\Delta m_{\ell_{R1}} \sim 1\%$
and $\Delta_{\ell_{L1}} \sim 3\%$. It is natural to expect that this degeneracy might persist in non-minimal models as well.

Our approach is as follows. Since we keep separate values for the KK masses, when we start varying any one of them, we have to somehow fix the remainder of the KK mass spectrum. We choose to use MUED as our reference model, hence the masses which are not being varied, will be fixed according to their MUED values. We shall still show results for $\Omega h^2$ as a function of $R^{-1}$, but for various fixed values of the corresponding mass splitting $\Delta_i$ defined in eq. (4-31). We shall also always display the reference MUED model line, for which, of course, $\Delta_i$ takes its MUED value. Our first example is shown in fig. 4-4, where we illustrate the size of the coannihilation effects for (a) 1 generation or (b) 3 generations of degenerate singlet KK leptons $\ell_{R1}$. The lines show the LKP relic density as a function of $R^{-1}$, for several choices of the mass splitting $\Delta_{\ell_{R1}}$ between the LKP and the $SU(2)_W$-singlet KK fermions $\ell_{R1}$. The solid lines from top to bottom in both (a) and (b) correspond to $\Delta_{\ell_{R1}} = 0, 0.3, 0.1$. The dotted line is the nominal UED case from fig. 4-3.
both (a) and (b) correspond to $\Delta_{\ell_{R1}} = 0, 0.3, 0.1$, and the dotted line is the nominal UED case from fig. 4–3, for which $\Delta_{\ell_{R1}} = 0.01$. As expected, all lines follow the general trend of fig. 4–3. In accord with the observations of Ref. [21], we see that $\ell_{R1}$ coannihilations increase the prediction for $\Omega h^2$. Such a behavior may seem peculiar at first sight, since in supersymmetry one finds the opposite phenomenon — coannihilations with sleptons tend to reduce the SUSY WIMP relic density.

The difference between the two cases can be intuitively understood as follows. In SUSY, the cross-section for the main annihilation channel ($\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow f \bar{f}$) is helicity suppressed, but the coannihilation processes are not. Adding coannihilations therefore can only increase the effective cross-section (4–9) and correspondingly decrease $\Omega h^2$. In contrast, in UED the main annihilation channel ($\gamma_1 \gamma_1 \rightarrow f \bar{f}$) is already of normal strength. The effect of coannihilations can be easily guessed only if the additional processes have either much weaker or much stronger interactions. In the case of $\ell_{R1}$, however, the additional processes are of the same order (both $\gamma_1$ and $\ell_{R1}$ have hypercharge interactions only) and the sign of the coannihilation effect depends on the detailed balance of numerical factors, which will be illustrated in fig. 4–5 and discussed in more detail below.

The spread in the lines in fig. 4–4 is indicative of the importance of the coannihilations. Comparing fig. 4–4a and fig. 4–4b, we see that in the case of three generations, the effects are magnified correspondingly. A similar conclusion was reached in Ref. [21].

Notice the peculiar ordering of the lines corresponding to different $\Delta_{\ell_{R1}}$. With respect to variations of $\Delta_{\ell_{R1}}$, the maximum possible value of $\Omega h^2$ is obtained for $\Delta_{\ell_{R1}} \rightarrow 0$, where the effect of coannihilations is maximal. Then, as we increase the mass splitting between $\ell_{R1}$ and $\gamma_1$, at first $\Omega h^2$ decreases (see the sequence of $\Delta_{\ell_{R1}} = 0, \Delta_{\ell_{R1}} = 0.01$ and $\Delta_{\ell_{R1}} = 0.1$) but then starts increasing again and the $\Omega h^2$ values that we get for $\Delta_{\ell_{R1}} = 0.3$ are slightly larger than those for $\Delta_{\ell_{R1}} = 0.1$. This
Figure 4–5: Plots of various quantities entering the LKP relic density computation, as a function of the mass splitting $\Delta_{e_{R_1}}$ between the LKP and the $SU(2)_W$-singlet KK electron, for $R^{-1} = 500$ GeV in MUED. (a) Relic density, (b) $\alpha_{\text{eff}}(x_F)$, (c) $g_{\text{eff}}(x_F)$ and (d) $a_{\text{eff}}(x_F)g_{\text{eff}}^2(x_F)$.

behavior can be seen more clearly from fig. 4–5a, where we vary the mass of the $SU(2)_W$-singlet KK electron $e_{R_1}$ and plot $\Omega h^2$ versus $\Delta_{e_{R_1}}$ for a fixed $R^{-1} = 500$ GeV. The interesting behavior of $\Omega h^2$ exhibited in fig. 4–5a can be understood in terms of the $m_{e_{R_1}}$ dependence of the effective annihilation cross-section (4–9) which is dominated by its $a$-term (4–17). Both $\sigma_{\text{eff}}$ and $a_{\text{eff}}$ are functions of $x$, but for the purposes of our discussion here it is sufficient to concentrate on the fixed value $x = x_F$ which dominates the integrals (4–13) and (4–14). We plot $a_{\text{eff}}(x_F)$ as a function of $\Delta_{e_{R_1}}$ in fig. 4–5b. We see that $a_{\text{eff}}(x_F)$ exhibits exactly the
opposite dependence to $\Omega h^2$, and in particular, has an analogous local extremum at $\Delta_{eR_1} \sim 0.1$. Therefore, in order to understand qualitatively the behavior of $\Omega h^2$, we only need to concentrate on $a_{\text{eff}}(x_F)$.

Let us start with the large $\Delta_{eR_1}$ region in fig. 4–5b. The $SU(2)_W$-singlet KK electron $e_{R_1}$ is then too heavy to participate in any relevant coannihilation processes. The effective cross-section (4–9) then receives no contributions from processes with $e_{R1}$. Nevertheless, the mass of $e_{R1}$ enters $\sigma_{\text{eff}}$ through the cross-section for the process $\gamma_1 \gamma_1 \rightarrow e^+e^-$ (see eqs. (4–32) and (4–34)). Then as we lower $m_{e_{R1}}$, $\sigma(\gamma_1 \gamma_1 \rightarrow e^+e^-)$ is increased and this leads to a corresponding increase in $a_{\text{eff}}$ as seen in fig. 4–5b. This trend continues down to $\Delta_{eR_1} \sim 0.1$, where coannihilations with $e_{R1}$ start becoming relevant. This can be seen in fig. 4–5c, where we plot $g_{\text{eff}}(x_F)$ as a function of $\Delta_{eR_1}$. From its defining equation (4–10) we see that $g_{\text{eff}}(x)$ starts to deviate from a constant only when the exponential terms (which signal the turning on of coannihilations) become non-negligible. The exponential terms are all positive and increase $g_{\text{eff}}$. At the same time, there are new cross-section terms entering the sum for $\sigma_{\text{eff}}$, so we expect the numerator in (4–9) to increase as well. This is confirmed in fig. 4–5d, where we plot the numerator of (4–9) simply as $a_{\text{eff}}(x_F)g_{\text{eff}}^2(x_F)$. From figs. 4–5c and 4–5d we see that both the numerator and the denominator of (4–17) increase at low $\Delta_{eR_1}$, and so it is a priori unclear how their ratio will behave with $\Delta_{eR_1}$. In this particular case, $g_{\text{eff}}$ wins, and $a_{\text{eff}}(x_F)$ is effectively decreased as a result of turning on the coannihilations with $e_{R1}$. This feature was also observed in Ref. [21]. We are now in position to repeat the same analysis, but for the case of the $SU(2)_W$-doublet KK leptons $\ell_{L1}$. In fig. 4–6, in complete analogy to fig. 4–4, we illustrate the effects on the relic density from varying the $SU(2)_W$-doublet KK electron mass. From top to bottom, the solid lines show $\Omega h^2$ as a function of $R^{-1}$, for $\Delta_{eL_1} = 0.01, 0.001, 0$. The dotted line is again the MUED reference model. We see that the case of
Figure 4–6: The effects of varying the $SU(2)_W$-doublet KK electron mass. The same as fig. 4–4 but illustrating the effects of varying the $SU(2)_W$-doublet KK electron mass. From top to bottom, the solid lines show $\Omega h^2$ as a function of $R^{-1}$, for $\Delta_{\ell_L1} = 0.01, 0.001, 0$. The dotted line is the nominal UED case from fig. 4–3.

$SU(2)_W$-doublet KK leptons is different. Unlike $\ell_{R1}$, they have weak interactions, and the extra terms which they bring into the sum (4–9) are larger than the main annihilation channel. The increase in $g_{eff}$ is similar as before. As a result, this time the increase in the numerator of (4–9) wins, and the net effect is to increase the effective annihilation cross-section. This leads to a reduction in the predicted value for the relic density, as evidenced from fig. 4–6. Notice how the decrease in $\Omega h^2$ is monotonic with $\Delta_{\ell_L1}$.

Another difference between $\ell_{R1}$ and $\ell_{L1}$ coannihilations is revealed by comparing the case of 1 generation (panels (a) in figs. 4–4 and 4–6) and 3 generations (panels (b) in figs. 4–4 and 4–6). We see that for $SU(2)_W$ singlets, the coannihilations are more prominent for the case of 3 generations, while for $SU(2)_W$ doublets, it is the opposite. This is due to the different number of degrees of freedom contributed to $g_{eff}$ in each case, which shifts the delicate balance between the numerator and denominator of (4–9), as discussed above.
4.4.2 Coannihilations with KK Quarks and KK Gluons

We will now consider coannihilation effects with colored KK particles (KK quarks and KK gluons). Since they couple strongly, we expect on general grounds that the effective annihilation cross-sections will be enhanced, and the preferred range of the LKP mass will correspondingly be shifted higher. These expectations are confirmed by our explicit calculation whose results are shown in figs. 4–7 and 4–8. In fig. 4–7 we show the effects on the relic density from varying the masses of all three generations of (a) SU(2)$_W$-singlet KK quarks and (b) SU(2)$_W$-doublet KK quarks. The solid lines show $\Omega h^2$ as a function of $R^{-1}$, and are labeled by the corresponding value of $\Delta_{qR1}$ or $\Delta_{qL1}$ used. As before, the dotted line is the MUED reference model. Comparing the results in figs. 4–7a and 4–7b, we find that the coannihilations with $q_{R1}$ and $q_{L1}$ have very similar effects, as they are both dominated by the strong interactions, which are the same for $q_{R1}$ and $q_{L1}$. Fig. 4–8 shows the analogous result for the case of varying the KK gluon mass, where the labels now show the values of $\Delta_g$. There is a noticeable distortion of the lines around $R^{-1} \sim 2300$ GeV, which is due to the change in $g_*$ (see fig. 4–2a).
Figure 4-8: The effects of varying KK gluon mass. The same as fig. 4-7, but for the case of varying the KK gluon mass. The lines are labeled by the value of $\Delta g_1$. The dotted line is the nominal UED case from fig. 4-3.

From figs. 4-7 and 4-8 we see that in non-minimal UED models where the colored KK modes happen to exhibit some sort of degeneracy with the LKP, multi-TeV values for $m_{\chi_1}$ are in principle possible. From that point of view, unfortunately, there is no “no-lose” theorem for the LHC or ILC regarding a potential absolute upper bound on the LKP mass.

4.4.3 Coannihilations with Electroweak KK Bosons

We finally show our coannihilation results for the case of electroweak KK gauge bosons ($W^0_1$ and $W^\pm_1$) and KK Higgs bosons ($H^0_1$, $G^0_1$ and $G^\pm_1$). The results are displayed in figs. 4-9a and 4-9b, correspondingly. Due to the $SU(2)_W$ symmetry, all three $n = 1$ KK $W$-bosons are very degenerate, and we have assumed a common parameter $\Delta_{W_1}$ for all three. Similarly, the masses of the $n = 1$ KK Higgs bosons differ only by electroweak symmetry breaking effects, which we neglect throughout the calculation. We have therefore assumed a common parameter $\Delta_{H_1}$ for them as well. Since both the electroweak KK gauge bosons and the KK Higgs bosons have weak interactions, we expect the results to be similar to the case of
Figure 4–9: The effects of varying EW bosons. The same as fig. 4–8, but illustrating the effect of varying simultaneously the masses of all (a) $SU(2)$ KK gauge bosons and (b) KK Higgs bosons. In (a) the lines are labeled by the value of $\Delta W_1$, while in (b) the values of $\Delta H$ are (from top to bottom) $\Delta H = 0.05, 0.01, 0.001, 0$. The dotted line is the nominal UED case from fig. 4–3.

$SU(2)_W$-doublet leptons in the sense that coannihilations would lower the predictions for $\Omega h^2$. This is confirmed by fig. 4–9. We observe that the effects from the KK $W$-bosons are actually quite significant, and can push the preferred LKP mass as high as 1.4 TeV.

### 4.5 Other Dark Matter Candidates and Direct Detection

In previous sections we revisited the calculation of the LKP relic density in the scenario of Universal Extra Dimensions. We extended the analysis of Ref. [21] to include all coannihilation processes involving $n = 1$ KK partners. This allowed us to predict reliably the preferred mass range for the KK dark matter particle in the minimal UED model. We found that in order to account for all of the dark matter in the universe, the mass of $\gamma_1$ should be within $500 - 600$ GeV, which is somewhat lower than the range found in [21]. This is due to a combination of several factors. Among the effects which caused our prediction for $\Omega h^2$ to go up are the following: we used a lower value of $g_\ast$, we kept the individual KK masses in our formulas, and we accounted for the relativistic correction (4–19). On the other hand, as we saw
in Section 4.4, including the effect of coannihilations with KK particles other than $SU(2)_W$-singlet KK leptons, always has the effect of lowering the predicted $\Omega h^2$. Finally, the cosmologically preferred range for $\Omega h^2$ itself has shifted lower since the publication of [21].

The lower range of preferred values for $R^{-1}$ is good news for collider and astroparticle searches for KK dark matter. It should be kept in mind that it is quite plausible, and in fact very likely, that the dark matter is made up of not one but several different components, in which case the LKP could be even lighter. We should mention that several collider studies [10, 13, 19, 62] have already used an MUED benchmark point with $R^{-1} = 500$ GeV, a choice which we now see also happens to be relevant for cosmology.

In Section 4.4 we also investigated how each class of $n = 1$ KK partners impacts the KK relic density. We summarize the observed trends in fig. 4–10, where we fix $\Omega h^2 = 0.1$ and then show the required $R^{-1}$ for any given $\Delta_i$, for each class of KK particles. We show variations of the masses of one (red dotted) or three (red solid) generations of $SU(2)_W$-singlet KK leptons; three generations of $SU(2)_W$-doublet leptons (magenta); three generations of $SU(2)_W$-singlet quarks (blue) (the result for three generations of $SU(2)_W$-doublet quarks is almost identical); KK gluons (cyan) and electroweak KK gauge bosons (green). The circle on each line denotes the MUED values of $\Delta$ and $R^{-1}$. Fig. 4–10 summarizes our results from Section 4.4. It also provides a quick reference guide for the expected variations in the predicted value of $\Omega h^2$ as we move away from the minimal UED model. For example, it is clear that unlike the case of coannihilations with $\ell_{R1}$, which was considered in [21], coannihilations with all other KK particles will lower the prediction for $\Omega h^2$ and correspondingly increase the preferred range of $R^{-1}$. This is due to the larger couplings of those particles. Fig. 4–10 can also be used to
Figure 4–10: The change in the cosmologically preferred value for $R^{-1}$ as a result of varying the different KK masses away from their nominal MUED values. Along each line, the LKP relic density is $\Omega h^2 = 0.1$. To draw the lines, we first fix the MUED spectrum, and then vary the corresponding KK mass and plot the value of $R^{-1}$ which is required to give $\Omega h^2 = 0.1$. We show variations of the masses of one (red dotted) or three (red solid) generations of $SU(2)_W$-singlet KK leptons; three generations of $SU(2)_W$-doublet leptons (magenta); three generations of $SU(2)_W$-singlet quarks (blue) (the result for three generations of $SU(2)_W$-doublet quarks is almost identical); KK gluons (cyan) and electroweak KK gauge bosons (green). The circle on each line denotes the MUED values of $\Delta$ and $R^{-1}$.

quantitatively estimate the variations in the preferred value of $R^{-1}$ in non-minimal models.

On a final note, in the non-minimal UED model, other neutral KK particles such as $Z_1$ can also be dark matter candidates. On dimensional grounds, the relic density is inversely proportional to the square of the LKP mass,

$$\Omega h^2 \sim \frac{g_1^4}{m_{\gamma_1}^2},$$

$$\Omega h^2 \sim \frac{g_2^4}{m_{Z_1}^2}.$$  \hspace{1cm} (4–40) \hspace{1cm} (4–41)

Due to the larger coupling $g_2$ of the $SU(2)_W$ gauge interactions, we expect the upper bound on $m_{Z_1}$, consistent with WMAP, to be larger than the bound on $m_{\gamma_1}$ roughly by a factor of $g_2^2/g_1^2 \sim 3$. However, in the $Z_1$ LKP case, $SU(2)_W$ symmetry implies that the charged $W_1$ states are almost degenerate with $Z_1$, and therefore
Figure 4–11: The spin-independent direct detection limit from CDMS experiment. We show the relic density and spin-independent direct detection limit from CDMS experiment in the plane of mass splitting $\Delta q_1 = \Delta q_1 = \frac{m_{q_1} - m_{\gamma_1}}{m_{\gamma_1}}$ and LKP mass for (a) $\gamma_1$ LKP (b) $Z_1$ LKP. The red line accounts for all of the dark matter (100%) and the two red dotted lines show 10% and 1%, respectively. The blue (green) line shows the current CDMS limit with Ge-detector (Si-detector) and the three cyan lines represent projected SuperCDMS limits for each phase: A (25 kg), B (150 kg) and C (1 ton) respectively. In the case of $\gamma_1$ LKP, SuperCDMS rules out most of parameter space while there is little parameter space left in the case of $Z_1$ LKP. The yellow region in the case of $\gamma_1$ LKP shows parameter space that could be covered by the collider search in $4\ell + E_T$ channel at the LHC.

Coannihilations with $W_1^\pm$ will be very important and will need to be considered.

The analysis of the cases of $Z_1$ and $H_1$ LKP and their direct and indirect detection prospects is currently in progress [121]. In fig. 4–11, we show the relic density and spin-independent direct detection limit [50] from CDMS experiment [122,123] in the plane of mass splitting, $\Delta q_1 = \Delta q_1 = \frac{m_{q_1} - m_{\gamma_1}}{m_{\gamma_1}}$ and LKP mass for (a) $\gamma_1$ LKP (b) $Z_1$ LKP. The red line accounts for all of the dark matter (100%) and the two red dotted lines show 10% and 1%, respectively. The blue (green) line shows the current CDMS limit with Ge-detector (Si-detector) and the three cyan lines represent projected SuperCDMS limits for each phase: A (25 kg), B (150 kg) and C (1 ton) respectively. In the case of $\gamma_1$ LKP, SuperCDMS rules out most of parameter space while there is little parameter space left in the case of $Z_1$ LKP.
The yellow region in the case of $\gamma_1$ LKP shows parameter space that could be covered by the collider search in $4\ell + E_T$ channel at the LHC [62].

The results presented here are also relevant for the case of KK graviton superwimps [89,124,125], whose relic density is still determined by the freeze-out of the next-to-lightest KK particle.

In conclusion, dark matter candidates from theories with extra dimensions should be considered on an equal footing with more conventional candidates such as SUSY dark matter or axions. The framework of Universal Extra Dimensions provides a useful playground for gaining some experience about the signals one could expect from extra dimensional dark matter. If extra dimensions have indeed something to do with the dark matter problem, the explicit realization of that idea may look quite differently (see for example [88,126,127]), especially if one wants to resolve the radion stabilization problem [128,129]. Nevertheless, we believe that the methods and insight we developed in this chapter will prove useful in more general contexts.
CHAPTER 5
CONCLUSIONS

A major motivation for studying new physics beyond the Standard Model is the dark matter puzzle which finds no explanation within the Standard Model. The models with Universal Extra Dimensions naturally provide possible candidates if the theory is compactified at the TeV scale and KK-parity is conserved. The models are within the reach of current or future collider and astroparticle experiments.

For the collider studies, we have written a code to simulate new physics signals in UED and it is already being used by experimentalists. For the dark matter study, we wrote a code which calculates the relic abundance of the KK dark matter. In this code, we do not assume any specific dark matter candidate and therefore it can be used for any other neutral dark matter candidates within the model. Importantly, any coannihilation processes can be included since our code was written with a general mass spectrum of the UED model.

In this dissertation, we concentrated on the collider phenomenology and astrophysical implications of Universal Extra Dimensions. In chapter 3, we studied the discovery reach of the Tevatron and the LHC for level 2 Kaluza-Klein modes, which would indicate the presence of extra dimensions. We found that with 100 fb$^{-1}$ of data the LHC will be able to discover the $\gamma_2$ and $Z_2$ KK modes as separate resonances if their masses are below 2 TeV. We also investigated the possibility to differentiate the spins of the superpartners and KK modes by means of the dilepton mass method and the asymmetry method in a squark cascade decay. However this method may not be generalized for all points in parameter space. At a $e^+e^-$ linear collider, we found that in the $\mu^+\mu^- E_T$ channel the angular distributions of the
final state muons, the energy spectrum of the radiative return photon and the total cross-section measurement are powerful discriminators between the two models.

In chapter 4, we revisited the calculation of the relic density of the lightest Kaluza-Klein particle. We included coannihilation processes with all level one KK particles. In our computation we considered a most general KK particle spectrum, without any simplifying assumptions. In particular, we did not assume a completely degenerate KK spectrum and instead retain the dependence on each individual KK mass. We found that in order to account for all of the dark matter in the universe, the mass of $\gamma_1$ should be within $500 - 600$ GeV, which is somewhat lower than the range found in previous calculation. In nonminimal UED models, any neutral KK particle, in principle, could be a dark matter candidate and for these candidates more work need to be done. We have also discussed current limits from direct detections.

To conclude, models with Universal Extra Dimensions are interesting proposals for physics beyond the standard model. The attractive merit of analogy to supersymmetry particularly makes this model more interesting. Studies of their phenomenology show that the idea of Universal Extra Dimensions is viable at present. Our studies on their phenomenology will provide tools to understand the nature of new physics beyond the Standard Model.
APPENDIX A
STANDARD MODEL IN 5D

A.1 Lagrangian of the Standard Model in 5D

Here we consider the minimal model of universal extra dimensions. The model is defined in five dimensions with one extra dimension compactified. In 5D there are two problems. First we cannot write the usual Dirac mass term because a chiral representation of a fermion does not exist in 5D. Second we have the fifth component (scalar in 4D) of the 5D vector field after the compactification and therefore we end up with too many zero mode scalar particles in 4D since there are three gauge groups in the SM. To introduce chiral fermions and project out unwanted scalars, a $Z_2$ symmetry is imposed on the $S_1$. There are two fixed points in this geometry $S_1/Z_2$, called the orbifold (see fig. 1–3). We can see that this geometry is still invariant under the exchange of two fixed points ($Z_2$). This symmetry is called KK parity. We impose the following special boundary conditions for fermions and vector fields at the fixed points. We want a field $\phi$ to be either even or odd under the transformation $P_5: y \rightarrow -y$ then at $y = 0, \pi R$

$$\partial_5 \phi^+ = 0 \quad \text{for even fields},$$

$$\phi^- = 0 \quad \text{for odd fields},$$

(A-1)

where $\partial_5$ is the derivative with respect to the extra dimensional coordinate. These are Neumann and Dirichlet boundary conditions respectively at the fixed points. The associated KK expansions are

$$\phi^+(x, y) = \frac{1}{\sqrt{2\pi R}} \phi_0^+ + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_n^+(x) \cos \frac{ny}{R},$$

(A-2)

$$\phi^-(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_n^+(x) \sin \frac{ny}{R},$$

(A-3)
where $x$ is the 4-dimensional spacetime coordinate $x^\mu = (t, x, y, z)$ and $y$ is the extra dimensional coordinate. $R$ is the size of the extra dimension and $n$ represents the KK-level. $n = 0$ is the SM mode. The orbifold compactification forces the first four components to be even under $\mathcal{P}_5$ while the fifth component is odd:

$$\begin{align*}
\partial_5 A^\mu &= 0 \\
A^5 &= 0
\end{align*}$$

at the fixed points. (A-4)

Hence the KK expansion of a vector field is

$$A_\mu(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ A^\mu_0(x) + \sqrt{2} \sum_{n=1}^{\infty} A^n_\mu(x) \cos\left(\frac{ny}{R}\right) \right\}, \quad (A-5)$$

$$A_5(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} A^n_5(x) \sin\left(\frac{ny}{R}\right). \quad (A-6)$$

Imposing

$$\begin{align*}
\partial_5 \psi^+_R &= 0 \\
\psi^+_L &= 0
\end{align*}$$

at $y = 0$, $\pi R$ or

$$\begin{align*}
\partial_5 \psi^+_L &= 0 \\
\psi^+_R &= 0
\end{align*}$$

at $y = 0$, $\pi R$, (A-7)

the respective KK mode expansions are

$$\psi^+(x, y) = \frac{1}{\sqrt{2\pi R}} \psi^0_R(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty}\left( \psi^n_R(x) \cos\frac{ny}{R} + \psi^n_L(x) \sin\frac{ny}{R} \right), \quad (A-8)$$

$$\psi^-(x, y) = \frac{1}{\sqrt{2\pi R}} \psi^0_L(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty}\left( \psi^n_L(x) \cos\frac{ny}{R} + \psi^n_R(x) \sin\frac{ny}{R} \right). \quad (A-9)$$

So the zero mode is either right handed or left handed. However KK modes come in chiral pairs. This chiral structure is a natural consequence of the orbifold
boundary conditions. The 5 dimensional SM fields are defined as follows.

\[
H(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ H(x) + \sqrt{2} \sum_{n=1}^{\infty} H_n(x) \cos\left(\frac{ny}{R}\right) \right\},
\]

\[
B_\mu(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ B_\mu^0(x) + \sqrt{2} \sum_{n=1}^{\infty} B_\mu^n(x) \cos\left(\frac{ny}{R}\right) \right\},
\]

\[
B_5(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} B_5^n(x) \sin\left(\frac{ny}{R}\right),
\]

\[
W_\mu(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ W_\mu^0(x) + \sqrt{2} \sum_{n=1}^{\infty} W_\mu^n(x) \cos\left(\frac{ny}{R}\right) \right\},
\]

\[
W_5(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} W_5^n(x) \sin\left(\frac{ny}{R}\right),
\]

\[
G_\mu(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ G_\mu^0(x) + \sqrt{2} \sum_{n=1}^{\infty} G_\mu^n(x) \cos\left(\frac{ny}{R}\right) \right\},
\]

\[
G_5(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} G_5^n(x) \sin\left(\frac{ny}{R}\right),
\]

\[
Q(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ q_L(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_LQ_L^n(x) \cos\left(\frac{ny}{R}\right) + P_RQ_R^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\},
\]

\[
U(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ u_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_Ru_R^n(x) \cos\left(\frac{ny}{R}\right) + P_Lu_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\},
\]

\[
D(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ d_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_Rd_R^n(x) \cos\left(\frac{ny}{R}\right) + P_Ld_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\},
\]

\[
L(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ L_0(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_LL_0^n(x) \cos\left(\frac{ny}{R}\right) + P_RL_R^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\},
\]

\[
E(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ e_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_Re_R^n(x) \cos\left(\frac{ny}{R}\right) + P_Le_L^n(x) \sin\left(\frac{ny}{R}\right) \right] \right\},
\]

where \(H(x, y)\) is the 5D scalar field and \((B_\mu(x, y), B_5(x, y)), (W_\mu(x, y), W_5(x, y))\) and \((G_\mu(x, y), G_5(x, y))\) are the 5D gauge fields for \(U(1), SU(2)\) and \(SU(3)\) respectively. \(Q(x, y)\) and \(L(x, y)\) are the \(SU(2)\) fermion doublets while \(U(x, y), D(x, y)\) and \(E(x, y)\) are respectively the generic singlet fields for the up-type quark,
the down-type quark and the lepton. The $SU(2)$ and $SU(3)$ gauge fields are

$$W_M \equiv W_M^a \frac{\tau^a}{2},$$

$$G_M \equiv G_M^A \frac{\lambda^A}{2},$$

where $\tau^a$’s are the usual Pauli’s matrices and $\lambda^A$’s are the usual Gell-Mann matrices. $P_{L,R} = \frac{1 \pm \gamma^5}{2}$ and $M = \mu, 5$ and $\mu = 0, 1, 2, 3$. Now we write the 5 dimensional Lagrangian invariant under $SU(3) \times SU(2) \times U(1)$ and compactify over the orbifold to get the 4 dimensional effective Lagrangian. First let us set up the conventions for the ingredients that go into the Lagrangian. The gamma matrices in 5D

$$\Gamma^M = (\gamma^\mu, i\gamma^5),$$

satisfy the Dirac-Clifford algebra

$$\{\Gamma^M, \Gamma^N\} = 2g^{MN},$$

where $g^{MN}$ is the 5D metric

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix},$$

and $g^{\mu\nu} = (+ - - -)$ is the usual 4D metric. The gauge couplings are denoted by

$$g_i = \frac{g_i^{(5)}}{\sqrt{\pi R}},$$

where $i = 1, 2, 3$ stands for $U(1)$, $SU(2)$, and $SU(3)$, $g_i^{(5)}$’s are the 5-dimensional gauge couplings and $g_i$’s are the 4-dimensional gauge couplings. The covariant
The five dimensional Standard Model can be written as,

\[ L_{\text{Gauge}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ -\frac{1}{4} B_{MN} B^{MN} - \frac{1}{4} W_{MN} W^{MN} - \frac{1}{4} G_{MN}^{A} G^{AMN} \right\} , \]

\[ L_{\text{GF}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ -\frac{1}{2\xi} (\partial^\mu B_\mu - \xi \partial_5 B_5)^2 - \frac{1}{2\xi} (\partial^\mu W_\mu^a - \xi \partial_5 W_5^a)^2 \right\} , \]

\[ L_{\text{Leptons}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{L}(x, y) \Gamma^M D_M L(x, y) + i \bar{E}(x, y) \Gamma^M D_M E(x, y) \right\} , \]

\[ L_{\text{Quarks}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ i \bar{Q}(x, y) \Gamma^M D_M Q(x, y) + i \bar{U}(x, y) \Gamma^M D_M U(x, y) + i \bar{D}(x, y) \Gamma^M D_M D(x, y) \right\} , \]

\[ L_{\text{Yukawa}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left\{ \lambda_u \bar{Q}(x, y) U(x, y) i \tau^2 H^*(x, y) + \lambda_d \bar{Q}(x, y) D(x, y) H(x, y) \right. \]

\[ + \lambda_e L(x, y) E(x, y) H(x, y) \right\} , \]

\[ L_{\text{Higgs}} = \frac{1}{2} \int_{-\pi R}^{\pi R} dy \left[ (D_M H(x, y))^{\dagger} (D^M H(x, y)) + \mu^2 H^\dagger(x, y) H(x, y) \right. \]

\[ - \lambda (H^\dagger(x, y) H(x, y))^2 \right] . \]

We express the 5 dimensional fields in terms of the trigonometric functions due to the orbifold structure. So the orthogonality relations are very important when we
compactify. The following are the relations we need

\[
\begin{align*}
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \cos\left(\frac{my}{R}\right) \cos\left(\frac{ny}{R}\right) &= \frac{\pi R}{2} \delta_{m,n}, \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) &= \frac{\pi R}{2} \delta_{m,n}, \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \cos\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) \cos\left(\frac{ly}{R}\right) &= \frac{\pi R}{4} \Delta_{mnl}^1, \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \cos\left(\frac{ny}{R}\right) \cos\left(\frac{ly}{R}\right) &= \frac{\pi R}{8} \Delta_{mnlk}^2, \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) \sin\left(\frac{ly}{R}\right) &= \frac{\pi R}{8} \Delta_{mnlk}^3, \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) \cos\left(\frac{ly}{R}\right) \cos\left(\frac{ky}{R}\right) &= \frac{\pi R}{4} \Delta_{mnlk}^4, \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \cos\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) &= 0, \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) \sin\left(\frac{ly}{R}\right) &= 0, \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \cos\left(\frac{ny}{R}\right) \cos\left(\frac{ly}{R}\right) &= 0, \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \cos\left(\frac{ny}{R}\right) \cos\left(\frac{ly}{R}\right) \cos\left(\frac{ky}{R}\right) &= 0, \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) \sin\left(\frac{ly}{R}\right) \cos\left(\frac{ky}{R}\right) &= 0, \\
\frac{1}{2} \int_{-\pi R}^{\pi R} dy \sin\left(\frac{my}{R}\right) \sin\left(\frac{ny}{R}\right) \cos\left(\frac{ly}{R}\right) \cos\left(\frac{ky}{R}\right) &= 0,
\end{align*}
\]

(8-19)

and \(\Delta\)'s are defined below.

\[
\begin{align*}
\Delta_{mnl}^1 &= \delta_{l,m+n} + \delta_{n,l+m} + \delta_{m,l+n}, \\
\Delta_{mnlk}^2 &= \delta_{k,l+m+n} + \delta_{l,m+n+k} + \delta_{m,n+k+l} + \delta_{n,k+l+m} + \delta_{k+m,l+n} + \delta_{k+n,l+m}, \\
\Delta_{mnlk}^3 &= -\delta_{k,l+m+n} - \delta_{l,m+n+k} - \delta_{m,n+k+l} - \delta_{n,k+l+m} + \delta_{k+l,m+n} + \delta_{k+n,l+m}, \\
\Delta_{mnl}^4 &= -\delta_{l,m+n} + \delta_{n,l+m} + \delta_{m,l+n}, \\
\Delta_{mnlk}^5 &= -\delta_{k,l+m+n} - \delta_{l,m+n+k} + \delta_{m,n+k+l} + \delta_{n,k+l+m} - \delta_{k+l,m+n} + \delta_{k+m,l+n} + \delta_{k+n,l+m}.
\end{align*}
\]

\(\Delta\)s are defined below.
After integrating out the extra dimensional coordinate, we find the following properties of the 4 dimensional effective theory.

- Each SM particle has an infinite number of KK partners
- The KK particles have the same spin as SM particles
- New vertices are the same as the SM couplings (up to normalization)
- All vertices satisfy KK number conservation (see fig. 2-2)

For each term in the Lagrangian we have a $\Delta$ which is a linear combination of the Kronecker delta functions. Due to this structure, the allowed vertices satisfy one of the following conditions,

$$|m \pm n \pm k| = 0 ,$$

$$|m \pm n \pm k \pm l| = 0 .$$

Therefore it is easy to see which vertices are allowed or not. In addition, the coupling is the same as the one in the SM (up to normalization factor $\sqrt{2}$).

However the KK number conservation is broken to the discrete symmetry, called KK-parity, due to the radiative corrections. The characteristic feature of this theory is that there exist maximum and minimum bounds on the size of the extra dimension $R$. The minimum bound comes from the electroweak precision data measurement which tells us $R^{-1} \geq 250 \text{ GeV}$ [6, 33]. And the maximum bound is related to the cosmology. According to cosmological observations, only 23% is the matter type and the rest is some sort of energy. So if this theory suggests dark matter, the relic density for this particle should not be greater that 23% ($\Omega h^2 \leq 0.13$). This gives us an upper bound on $R^{-1} \leq \text{a few TeV}$ [20-22].

### A.2 The Kaluza-Klein Fermions and Gauge bosons

Now let us summarize the fermion content of this theory. As we can see from the table A-1, there are two KK particles corresponding to one SM particles. The
Table A-1: Fermion content of the Standard Model and corresponding Kaluza-Klein fermions

<table>
<thead>
<tr>
<th>SU(2) Symmetry</th>
<th>SM mode</th>
<th>KK mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quark doublet</td>
<td>$q_L(x) = \begin{pmatrix} U_L(x) \ D_L(x) \end{pmatrix}$</td>
<td>$Q_L^n(x) = \begin{pmatrix} U_L^n(x) \ D_L^n(x) \end{pmatrix}$, $Q_R^n(x) = \begin{pmatrix} U_R^n(x) \ D_R^n(x) \end{pmatrix}$</td>
</tr>
<tr>
<td>Lepton doublet</td>
<td>$L_0(x) = \begin{pmatrix} \nu_L(x) \ E_L(x) \end{pmatrix}$</td>
<td>$L_L^n(x) = \begin{pmatrix} \nu_L^n(x) \ E_L^n(x) \end{pmatrix}$, $L_R^n(x) = \begin{pmatrix} \nu_R^n(x) \ E_R^n(x) \end{pmatrix}$</td>
</tr>
<tr>
<td>Quark Singlet</td>
<td>$u_R(x)$</td>
<td>$u_R^n(x), u_L^n(x)$</td>
</tr>
<tr>
<td>Quark Singlet</td>
<td>$d_R(x)$</td>
<td>$d_R^n(x), d_L^n(x)$</td>
</tr>
<tr>
<td>Lepton Singlet</td>
<td>$e_R(x)$</td>
<td>$e_R^n(x), e_L^n(x)$</td>
</tr>
</tbody>
</table>

Table A-2 summarizes the quantum numbers for the mass eigenstates when we ignore the mass term from the Yukawa coupling.

<table>
<thead>
<tr>
<th>KK Fermions</th>
<th>$I_3$</th>
<th>$Y$</th>
<th>$Q = I_3 + \frac{1}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quark Doublet</td>
<td>$U_n = U_L^n(x) + U_R^n(x)$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>$D_n = D_L^n(x) + D_R^n(x)$</td>
<td>$-\frac{2}{3}$</td>
<td>$-\frac{2}{3}$</td>
</tr>
<tr>
<td>Quark Singlet</td>
<td>$\nu_n = \nu_L^n(x) + \nu_R^n(x)$</td>
<td>$\frac{1}{2}$</td>
<td>$-1$</td>
</tr>
<tr>
<td></td>
<td>$E_n = E_L^n(x) + E_R^n(x)$</td>
<td>$-\frac{1}{2}$</td>
<td>$-1$</td>
</tr>
<tr>
<td>Lepton Singlet</td>
<td>$\nu_n = \nu_L^n(x) + \nu_R^n(x)$</td>
<td>$\frac{1}{2}$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

Table A-3: Fermions and gauge bosons in the Standard Model

<table>
<thead>
<tr>
<th>SM Fermions</th>
<th>SM Gauge Bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u = U_L(x) + u_R(x)$</td>
<td>$W^\pm = \frac{W^3 \pm B^e}{\sqrt{2}}$</td>
</tr>
<tr>
<td>$d = D_L(x) + d_R(x)$</td>
<td>$Z_\mu = -\sin \theta_W B_\mu + \cos \theta_W W^3_\mu$</td>
</tr>
<tr>
<td>$e = E_L(x) + e_R(x)$</td>
<td>$A_\mu = \cos \theta_W B_\mu + \sin \theta_W W^3_\mu$</td>
</tr>
<tr>
<td>$\nu_L = \nu_L(x)$</td>
<td></td>
</tr>
</tbody>
</table>

The particles in the Standard Model are summarized in the table A-3. Note that in UED, the Dirac fermions $F^n_n(x) = F_L^n(x) + F_R^n(x)$, as shown above, are constructed out of $F_L^n(x)$ and $F_R^n(x)$ which have the same $SU(2) \times U(1)$ quantum numbers. Contrast this with a SM Dirac fermion “$f$” which is constructed out of
\( f_L(x) \) and \( f_R(x) \) which have different \( SU(2) \times U(1) \) quantum numbers. This difference shows up in the processes involving gauge-bosons couplings with fermions. The mass terms for the KK singlets appear with the wrong sign in the fermionic Lagrangian and this affect shows up as a wrong sign in Yukawa interaction through the redifinition of fermion field (see appendix B.6 for detail).

The mass eigenstates of KK photons and \( Z \) can be obtained by diagonalizing the following mass matrix in \( W_3^n \) and \( B_n \) basis

\[
\begin{pmatrix}
\frac{n^2}{R^2} + \delta m_{B_n}^2 + \frac{1}{4}g_1^2v^2 & \frac{1}{4}g_1g_2v^2 \\
\frac{1}{4}g_1g_2v^2 & \frac{n^2}{R^2} + \delta m_{W_3^n}^2 + \frac{1}{4}g_2^2v^2
\end{pmatrix}.
\] (A-22)

The Lagrangian for the EW gauge bosons can be obtained using the following definitions

\[
W_{n\mu}^\pm = \frac{W_{n\mu}^1 \mp iW_{n\mu}^2}{\sqrt{2}},
\]

\[
Z_n^\mu = - \sin \theta_n B_\mu^n + \cos \theta_n W_3^n W_{n\mu}^3 \approx W_{n\mu}^3,
\] (A-23)

\[
A_{n\mu} = \cos \theta_n B_\mu^n + \sin \theta_n W_3^n W_{n\mu}^3 \approx B_\mu^n.
\]

The neutral gauge boson eigenstates become approximately pure \( B_n \) and \( W_3^n \) since the Weinberg angles for KK states are small as shown in A-1. Therefore the tree level mass of the nth KK mode is

\[
m_{n}^2 = \frac{n^2}{R^2} + m^2.
\] (A-24)

### A.3 The Decay Widths of KK Particles

With the Feynman rules, we calculate the decay widths for each particle at the first KK level and each gauge boson at the second KK level. For level 1 KK
Figure A–1: Dependence of the “Weinberg” angle $\theta_n$ for the first few KK levels $(n = 1, 2, \cdots, 5)$ on $R^{-1}$ for fixed $\Lambda R = 20$. The figure is taken from Cheng et al. [28].

particles, the decay widths are given below. For level 1 fermion,

$$
\Gamma(f_1 \to f_0 V_1) = \frac{c^2 g^2 m_{f_1}^3}{32 \pi m_{V_1}^2} \left( m_{f_1}^2 + m_{f_0}^2 - m_{V_1}^2 + \frac{(m_{f_1}^2 - m_{f_0}^2 - m_{V_1}^2)(m_{f_1}^2 - m_{f_0}^2 + m_{V_1}^2)}{m_{V_1}^2} \right) \times \sqrt{\left( m_{f_1}^2 - (m_{f_0} - m_{V_1})^2 \right) \left( m_{f_1}^2 - (m_{f_0} + m_{V_1})^2 \right)}, \quad (A-25)
$$

$$
\approx \frac{c^2 g^2 m_{f_1}^3}{32 \pi m_{V_1}^2} \left( 1 - \frac{m_{V_1}^2}{m_{f_1}^2} \right)^2 \left( 1 + 2 \frac{m_{V_1}^2}{m_{f_1}^2} \right). \quad (A-26)
$$

For level 1 gauge boson,

$$
\Gamma(V_1 \to f_0 f_0) = \frac{c^2 g^2 m_{V_1}^3}{48 \pi m_{V_1}^2} \left( m_{V_1}^2 - m_{f_1}^2 - m_{f_0}^2 + \frac{m_{V_1}^4 - (m_{f_1}^2 - m_{f_0}^2)^2}{m_{V_1}^2} \right) \times \sqrt{\left( m_{V_1}^2 - (m_{f_1} - m_{f_0})^2 \right) \left( m_{V_1}^2 - (m_{f_1} + m_{f_0})^2 \right)}, \quad (A-27)
$$

$$
\approx \frac{c^2 g^2 m_{f_1}^4}{48 \pi m_{V_1}^2} \left( 1 - \frac{m_{V_1}^2}{m_{f_1}^2} \right)^2 \left( 2 + \frac{m_{V_1}^2}{m_{f_1}^2} \right). \quad (A-28)
$$

Now we give the decay widths for level 2 gauge bosons. For two SM fermions in the final state,

$$
\Gamma(V_2 \to f_0 f_0) = \frac{c^2 g^2 m_{V_2}}{12 \pi} \left( \frac{\delta m_{V_2}}{m_2} - \frac{\delta m_{f_2}}{m_2} \right)^2 \left( 1 - \frac{m_{f_0}^2}{m_{V_2}^2} \right) \sqrt{\left( 1 - 4 \frac{m_{f_0}^2}{m_{V_2}^2} \right)}, \quad (A-29)
$$

$$
\approx \frac{c^2 g^2 m_{V_2}}{12 \pi} \left( \frac{\delta m_{V_2}}{m_2} - \frac{\delta m_{f_2}}{m_2} \right)^2. \quad (A-30)
$$
For one level 2 KK fermion and one SM fermion, 

\[ \Gamma(V_2 \to f_2 f_0) = \frac{c^2 g^2}{48\pi m_{V_2}^3} \left( m_{V_2}^2 - m_{f_2}^2 - m_{f_0}^2 + \frac{m_{f_2}^4 - (m_{f_2}^2 - m_{f_0}^2)^2}{m_{V_2}^2} \right) \times \sqrt{(m_{V_2}^2 - (m_{f_2} - m_{f_0})^2) (m_{V_2}^2 - (m_{f_2} + m_{f_0})^2)}, \] (A-31)

\[ \approx \frac{c^2 g^2}{48\pi m_{V_2}^3} (m_{V_2}^2 - m_{f_2}^2)^2 \left( 1 + \frac{m_{V_2}^2 + m_{f_2}^2}{m_{V_2}^2} \right), \] (A-32)

\[ \approx \frac{c^2 g^2 m_{V_2}}{4\pi} \left( \frac{\delta m_{V_2}}{m_2} - \frac{\delta m_{f_2}}{m_2} \right)^2. \] (A-33)

For two level 1 KK fermions, 

\[ \Gamma(V_2 \to f_1 f_1) = \frac{c^2 g^2}{24\pi m_{V_2}^2} (m_{V_2}^2 - 4m_{f_1}^2)^{\frac{3}{2}}, \] (A-34)

\[ \approx \frac{c^2 g^2 m_{V_2}}{6\sqrt{2}\pi} \left( \frac{\delta m_{V_2}}{m_2} - \frac{\delta m_{f_1}}{m_1} \right)^{\frac{3}{2}} \left( \frac{m_{V_2}}{m_1} \right)^3, \] (A-35)

\[ \approx \frac{c^2 g^2 m_{V_2}}{6\sqrt{2}\pi} \left( \frac{\delta m_{V_2}}{m_2} - \frac{\delta m_{f_1}}{m_1} \right)^{\frac{3}{2}}. \] (A-36)

For each decay widths corresponding to level 2 gauge bosons, \( m_2^2 = \left( \frac{m}{R} \right)^2 + \delta m_2^2 \) was used in the last approximation. For the decay of level 2 KK fermion into one SM fermion and one level 2 gauge boson, 

\[ \Gamma(f_2 \to V_2 f_0) = \frac{g^2 c^2}{32\pi m_{f_2}^2} \sqrt{(m_{f_2}^2 - (m_{f_0} - m_{V_2})^2) (m_{f_2}^2 - (m_{f_0} + m_{V_2})^2)} \times \left( m_{f_2}^2 + m_{f_0}^2 - m_{V_2}^2 + \frac{(m_{f_2}^2 - m_{f_0}^2 - m_{V_2}^2)(m_{f_2}^2 - m_{f_0}^2 + m_{V_2}^2)}{m_{V_2}^2} \right). \] (A-37)

For two level 1 KK particles in the final state, 

\[ \Gamma(f_2 \to V_1 f_1) = \frac{g^2 c^2}{32\pi m_{f_2}^2} \sqrt{(m_{f_2}^2 - (m_{f_1} - m_{V_1})^2) (m_{f_2}^2 - (m_{f_1} + m_{V_1})^2)} \times \left( m_{f_2}^2 + m_{f_1}^2 - m_{V_1}^2 - 6m_{f_2} m_{f_1} + \frac{(m_{f_2}^2 - m_{f_1}^2 - m_{V_1}^2)(m_{f_2}^2 - m_{f_1}^2 + m_{V_1}^2)}{m_{V_1}^2} \right). \] (A-38)
Figure A–2: Running coupling constants in SM (a) and UED (b)

For one SM gauge boson and one level 2 KK fermion,

\[ \Gamma(f_2 \to W_0 f'_2) = \frac{g^2 V^2_{CKM}}{32 \pi m^2_{f_2}} \sqrt{\left( m^2_{f_2} - (m_{f_2} - m_{V_0})^2 \right) \left( m^2_{f_2} - (m_{f_2} + m_{V_0})^2 \right) } \]  
\[ \times \left( m^2_{f_2} + m^2_{f_2} - m^2_{V_0} - 6 m_{f_2} m_{f_2} + \frac{(m^2_{f_2} - m^2_{f_2} - m^2_{V_0})(m^2_{f_2} - m^2_{f_2} + m^2_{V_0})}{m^2_{V_2}} \right), \]  

(A-39)

where \( c = \frac{\gamma}{2} \) for \( B_n \), \( c = \frac{1}{2} \) for \( Z_n \), \( c = \frac{1}{\sqrt{2}} \) for \( G_n \) and \( c = \frac{V_{CKM}}{\sqrt{2}} \) for \( W_n \). \( \delta m \) represents the total mass correction \((\delta m + \bar{\delta} m)\) and \( \bar{\delta} m \) only contains the mass corrections due to the boundary terms. (though typically \( \delta m \approx \bar{\delta} m \). We can generalize those formulas as follows,

\[ \Gamma(f_1 \to f_0 V_1) \to \Gamma(f_2 \to f_0 V_2) \to \Gamma(f_n \to f_0 V_n) \]
\[ \Gamma(V_1 \to f_1 f_0) \to \Gamma(V_2 \to f_2 f_0) \to \Gamma(V_n \to f_n f_0) \]  
\[ \Gamma(f_2 \to f_1 V_1) \to \Gamma(f_{2n} \to f_n V_n) . \]  

(A-40)

A.4 Running Coupling Constants in Extra Dimensions

The couplings in the Standard Model,

\[ \alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z}, \]  

(A-41)
satisfy the following simple differential equation

\[ \frac{d\alpha_i^{-1}}{dt} = -\frac{1}{2\pi b_i} , \quad \text{(A-42)} \]

where \( M_Z \) is the mass of Z and \( \alpha_i = \frac{g_i^2}{4\pi} \), \( t = \ln \mu \) and \( i = 1, 2, 3 \), corresponding to the gauge group \( U(1) \times SU(2) \times SU(3) \) \( (\alpha_1 = \frac{5}{3}\alpha_Y \) and \( b_1 = \frac{3}{2}b_Y) \). For any gauge group, the coefficients are given by

\[ b = -\frac{11}{3} C_{\text{adj}} + \frac{2}{3} \sum_f C_f + \frac{1}{6} \sum_h C_h , \quad \text{(A-43)} \]

where \( C_{\text{adj}} \) is the Dynkin index of the adjoint representation of the gauge group, \( C_f \) is the Dynkin index of the representation of the left-handed Weyl fermions and \( C_h \) is that of the representation of the real Higgs field. \( C_{\text{adj}} = n \) for \( SU(n)(n > 2) \) and \( C_f = \frac{1}{2} \) for the fundamental representation of \( SU(2) \) and \( C_f = \frac{5}{2}Y_f^2 \) for \( U(1) \).

Applying this formula for one Higgs doublet and \( n_f \) chiral families, we find

\[ \begin{align*}
    b_1 &= + \frac{4}{3} n_f + \frac{1}{10} , \\
    b_2 &= - \frac{22}{3} + \frac{4}{3} n_f + \frac{1}{6} , \\
    b_3 &= -11 + \frac{4}{3} n_f .
\end{align*} \quad \text{(A-44)} \]

In the Standard Model, with \( n_f = 3 \), the numerical values of these coefficients are

\[ (b_1, b_2, b_3) = \left( \frac{41}{10}, -\frac{19}{6}, -7 \right) . \quad \text{(A-45)} \]

The couplings in the Universal Extra Dimensions

\[ \alpha_i^{-1} = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z} + \frac{\tilde{b}_i}{2\pi} \ln \frac{\mu}{\mu_0} - \frac{\tilde{b}_i X_{\delta}}{2\pi \delta} \left[ \left( \frac{\mu}{\mu_0} \right)^\delta - 1 \right] , \quad \text{(A-46)} \]

satisfy

\[ \frac{d\alpha_i^{-1}}{dt} = -\frac{b_i - \tilde{b}_i}{2\pi} - \frac{\tilde{b}_i X_{\delta}}{2\pi} \left( \frac{\mu}{\mu_0} \right)^\delta , \quad \text{(A-47)} \]
where $\delta$ is the number of extra dimensions and $X_\delta = \frac{2\pi^{\delta/2}}{\Gamma(\delta/2)}$. The new beta function coefficients $\tilde{b}_i$ correspond to the contributions of the Kaluza-Klein states at each massive KK excitation level. In the minimal universal extra dimension,

$$(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = \left(\frac{41}{10}, -\frac{17}{6}, -\frac{13}{2}\right),$$

(A-48)

where we used

$$b = -\frac{11}{3}C_{adj} + \frac{2}{3}\sum_f C_f + \frac{1}{6}\sum_h C_h + \frac{1}{6}C_{adj}.$$  

(A-49)
APPENDIX B
ANNIHILATION CROSS-SECTIONS

In this section, we summarize the annihilation cross sections of any pair of $n = 1$ KK particles into SM fields in the limit of no electroweak symmetry breaking, as in [21]. In order to render the formulas manageable, in this Appendix we list our results in the limit of equal KK masses. However, in our numerical calculation, we kept different masses for all KK particles, which often leads to enormously complicated analytical expressions. We also assume all SM particles to be massless, since we are working in the limit where we neglect electroweak symmetry breaking (EWSB) effects of order $vR$, where $v$ is the Higgs vacuum expectation value in the SM. All cross-sections are calculated at tree level. All vertices satisfy KK-number conservation and KK-parity since KK-number violating interactions are only induced at the loop level [28]. Some of the cross-sections have already appeared in [21] and we find perfect agreement with those results.

We define a few constants below which are commonly used in our formulas for the cross-sections.

\[
g_1 = \frac{e}{c_w}, \quad (B.1)
\]
\[
g_2 = \frac{e}{s_w}, \quad (B.2)
\]
\[
g_Z = \frac{e}{2s_w c_w}, \quad (B.3)
\]
\[
\beta = \sqrt{1 - \frac{4m^2}{s}}, \quad (B.4)
\]
\[
L = \log \left( \frac{1 - \beta}{1 + \beta} \right) = -2 \tanh^{-1} \beta. \quad (B.5)
\]

Here $g_1$, $g_2$ and $g_3$ are the gauge couplings of $U(1)_Y$, $SU(2)_W$ and $SU(3)$. $\beta$ is the velocity of the incoming KK particle in the annihilation process. Notice that $L$
Table B–1: A guide to the formulas in the Appendix. Each box in the table corresponds to a particular type of an initial state. The entry points to the section in the Appendix where the corresponding annihilation cross-sections can be found. Here “gauge bosons” include EW KK gauge bosons ($W^+_1$, $Z_1$ and $\gamma_1$) and the KK gluon ($g_1$). “Higgses” stands for the KK Higgs ($H_1$) and KK Goldstone bosons ($G_1^{\pm}$ and $G_1^0$). Leptons contain both $SU(2)_W$-singlet KK leptons ($\ell_{R1}$) and $SU(2)_W$-doublet KK leptons ($\ell_{L1}$ and $\nu_{L1}$). Quarks include both $SU(2)_W$-doublet KK quarks ($q_{R1}$) and $SU(2)_W$-singlet KK quarks ($q_{L1}$).

<table>
<thead>
<tr>
<th></th>
<th>Gauge bosons</th>
<th>Leptons</th>
<th>Quarks</th>
<th>Higgses</th>
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</thead>
<tbody>
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<td>Gauge bosons</td>
<td>B.2</td>
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<tr>
<td>Leptons</td>
<td>B.3</td>
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</tr>
<tr>
<td>Quarks</td>
<td>B.3</td>
<td>B.5</td>
<td>B.4</td>
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<td>Higgses</td>
<td>B.7</td>
<td>B.8</td>
<td>B.8</td>
<td>B.6</td>
</tr>
</tbody>
</table>

is negative since $0 < \beta < 1$. $m$ is the KK mass which for the purposes of this appendix is the same for all KK particles, $e$ is the electric charge and $c_w$ and $s_w$ are the cosine and sine of the Weinberg angle in the SM. Table B–1 provides a quick reference guide for the different process types.

**B.1 Leptons**

Coannihilations with $SU(2)_W$-singlet KK leptons $\ell_{R1}$ are important since they are expected to be the next-to-lightest KK particles in the minimal UED model [21, 28]. For fermion final states with $f \neq \ell$, the cross-section is

$$\sigma(\ell_{R1}^+\ell_{R1}^- \rightarrow f\bar{f}) = \frac{N_c g_1^4 Y_{\ell_{R1}} \left(Y_{\ell_{L1}}^2 + Y_{\ell_{R1}}^2\right)(s + 2m^2)}{24\pi\beta s^2} ,$$  
(B.6)

where $N_c$ is 3 for quarks and 1 for leptons. For cases with the same lepton flavor in the initial and final state, we have

$$\sigma(\ell_{R1}^+\ell_{R1}^- \rightarrow \ell^+\ell^-) = \frac{g_1^4 Y_{\ell_{R1}}^4 \left(5\beta s + 2(2s + 3m^2)L\right)}{32\pi\beta^2 s^2}$$

$$+ \frac{g_1^4 Y_{\ell_{R1}}^4 \left(\beta(4s + 9m^2) + 8m^2L\right)}{64\pi^2 m^2 \beta^2 s}$$

$$+ \frac{g_1^4 Y_{\ell_{R1}}^2 \left(Y_{\ell_{R1}}^2 + Y_{\ell_{L1}}^2\right)(s + 2m^2)}{24\pi\beta s^2} ,$$  
(B.7)

$$\sigma(\ell_{R1}^\pm\ell_{R1}^\mp \rightarrow \ell^\pm\ell^\mp) = \frac{g_1^4 Y_{\ell}^4 \left(-m^2(4s - 5m^2)L - \beta s(2s - m^2)\right)}{32\pi\beta^2 s^2 m^2} ,$$  
(B.8)
\[
\sigma(\ell_{R1}^+ \ell_{R1}^\pm \rightarrow \ell'^\pm \ell'^\mp) = \frac{g_1^4 Y_\ell^4 (4s - 3m^2)}{64\pi \beta s m^2}, \tag{B.9}
\]
\[
\sigma(\ell_{R1}^\pm \ell_{R1}^\mp \rightarrow \ell^\pm \ell'^\pm) = \frac{g_1^4 Y_\ell^4 (\beta (4s + 9m^2) + 8m^2 L)}{64\pi \beta^2 s m^2}, \tag{B.10}
\]

where \(\ell\) and \(\ell'\) are the leptons from different families. For the remaining final states we get
\[
\sigma(\ell_{R1}^+ \ell_{R1}^- \rightarrow \phi \phi^*) = \frac{g_1^2 Y_\ell^2 Y_\phi^2 (s + 2m^2)}{48\pi \beta^2 s^2}, \tag{B.11}
\]
\[
\sigma(\ell_{R1}^+ \ell_{R1}^- \rightarrow B_0 B_0) = \frac{g_1^4 Y_\ell^4 (2(s^2 + 4m^2 s - 8m^4) - \beta s (s + 4m^2))}{8\pi \beta^2 s^3}. \tag{B.12}
\]

Our results, (B.6-B.12), exactly agree with (C.1) - (C.8) from Servant et al. [21].

The cross-sections among left handed fermions are somewhat complicated since they involve \(SU(2)_W\) gauge bosons as well as \(U(1)_Y\) gauge bosons. For KK neutrinos we find
\[
\sigma(\nu_{\ell 1} \bar{\nu}_{\ell 1} \rightarrow f \bar{f}) = \frac{N_c g_Z^2 (g_k^2 + \bar{g}_K^2) (s + 2m^2)}{24\pi \beta s^2}, \tag{B.13}
\]
\[
\sigma(\nu_{\ell 1} \bar{\nu}_{\ell 1} \rightarrow \phi \phi^*) = \frac{g_5^2 g_Z^2 (s + 2m^2)}{48\pi \beta s^2}, \tag{B.14}
\]
\[
\sigma(\nu_{\ell 1} \bar{\nu}_{\ell 1} \rightarrow ZZ) = \frac{g_5^4 (8m^4 - s^2 - 4m^2 s L - \beta s (s + 4m^2))}{8\pi \beta^2 s^3}, \tag{B.15}
\]
\[
\sigma(\nu_{\ell 1} \bar{\nu}_{\ell 1} \rightarrow W^+ W^-) = -\frac{5g_5^2 (s + 2m^2)}{96\pi \beta s^2} + \frac{g_5^2 (\beta s - 2m^2 L)}{32\pi \beta^2 s^2} - \frac{g_5^4 (\beta (s + 4m^2) + (s + 2m^2)L)}{32\pi \beta^2 s^2}, \tag{B.16}
\]
\[
\sigma(\nu_{\ell 1} \nu_{\ell 1} \rightarrow \nu_{\ell} \nu_{\ell}) = \frac{g_5^2 (\beta s (2s - m^2) - m^2 (4s - 5m^2) L)}{32\pi \beta^2 s^2 m^2}, \tag{B.17}
\]
\[
\sigma(\nu_{\ell 1} \nu_{\ell 1} \rightarrow \nu_{\ell} \nu_{\ell}) = \frac{g_5^2 (4s - 3m^2)}{64\pi \beta s m^2}, \tag{B.18}
\]
\[
\sigma(\nu_{\ell 1} \bar{\nu}_{\ell 1} \rightarrow \nu_{\ell} \bar{\nu}_{\ell}) = \frac{g_5^4 (\beta (4s + 9m^2) + 8m^2 L)}{64\pi \beta^2 s m^2}, \tag{B.19}
\]
\[
\sigma(\nu_{\ell 1} \bar{\nu}_{\ell 1} \rightarrow \ell^- \ell'^+) = \frac{g_5^4 (\beta (4s + 9m^2) + 8m^2 L)}{256\pi \beta^2 s m^2}. \tag{B.20}
\]
for at least one charged KK lepton in the initial state we get

\[ \sigma(\nu_{l1}\bar{\nu}_{l1} \to \ell^+\ell^-) = \frac{gz_\gamma g_L^2 g_L (5\beta s + 2(2s + 3m^2)L)}{32\pi\beta^2 s^2} \]

\[ + \frac{g^2_2 (g^2_L + g^2_R)(s + 2m^2)}{24\pi\beta s^2} + \frac{g^4_1 (\beta(4s + 9m^2) + 8m^2L)}{64\pi m^2\beta^2 s^2}. \]  

(B.21)

Here \( \tilde{g}_{L(R)} = \frac{e}{s_w c_w} (T^3 - Q_f s^2_w) \), \( \hat{g}_L = g_Z \) for neutrinos and \( \hat{g}_L = g_Z / \sqrt{2} \) for charged leptons. \( g_\phi = \frac{e}{s_w c_w} (T^3 - Q_f s^2_w) \) with \( Q_\phi = 1 \) for the upper entry in the Higgs doublet and \( Q_\phi = 0 \) for the lower entry. Since we ignore EWSB, all gauge bosons have transverse polarizations only. \( \phi \) represents either a charged Higgs boson (\( \phi_u \), isospin \( \frac{1}{2} \)) or a neutral Higgs boson (\( \phi_d \), isospin \( -\frac{1}{2} \)).

The previous results allow us to immediately obtain

\[ \sigma(\nu_{l1}\bar{\nu}_{l1} \to \phi\phi^*) = \sigma(\ell^+_{L1}\ell^-_{L1} \to \phi\phi^*), \]

\[ \sigma(\nu_{l1}\bar{\nu}_{l1} \to ZZ) = \sigma(\ell^+_{L1}\ell^-_{L1} \to ZZ + Z\gamma + \gamma\gamma), \]

\[ \sigma(\nu_{l1}\bar{\nu}_{l1} \to W^+W^-) = \sigma(\ell^+_{L1}\ell^-_{L1} \to W^+W^-), \]

\[ \sigma(\nu_{l1}\nu_{l1} \to \nu_{l1}\nu_{l1}^* \to \ell^+\ell^+ \)\]

\[ = \sigma(\nu_{l1}\ell^+_{L1} \to \nu_{l1}\ell^+), \]

\[ = \sigma(\ell^+_{L1}\ell^+_{L1} \to \ell^+\ell^+), \]

\[ \sigma(\nu_{l1}\bar{\nu}_{l1} \to \phi\phi^*) = \sigma(\ell^+_{L1}\ell^-_{L1} \to \ell^+\ell^+). \]  

(B.22)

For at least one charged KK lepton in the initial state we get

\[ \sigma(\ell^+_{L1}\ell^-_{L1} \to f \bar{f} \) \text{ or } \ell^+_{R}\ell^- \) = \]

\[ \frac{N_c g^4(s + 2m^2)}{24\pi\beta s^2}, \]  

(B.23)

\[ \sigma(\ell^+_{L1}\ell^-_{L1} \to \nu_{l1}\bar{\nu}_{l1} \) \text{ or } \ell^+_{L}\ell^- \) = \]

\[ \frac{g_2^2 g^2 (5\beta s + 2(2s + 3m^2)L)}{32\pi\beta^2 s^2} \]

\[ + \frac{g^4_1 (\beta(4s + 9m^2) + 8m^2L)}{64\pi m^2\beta^2 s^2} + \frac{g^4_1 (s + 2m^2)}{24\pi\beta s^2}. \]  

(B.24)

\[ \sigma(\ell^+_{L1}\ell^-_{L1} \to \phi^*_u\phi_d) = \frac{g^4_2 (s + 2m^2)}{192\pi \beta s^2}, \]  

(B.25)
\[
\sigma(\ell_{L1} \bar{\nu}_{L1} \to W^- B_0) = \frac{g_1^2 g_2^2 ((8m^4 - s^2 - 4m^2 s)L - \beta(s + 4m^2))}{32\pi^2 s^3} , \tag{B.27}
\]
\[
\sigma(\ell_{L1} \bar{\nu}_{L1} \to W^- W_3^0) = -\frac{5g_1^2 (s + 2m^2)}{48\pi s^2 \beta} + \frac{g_2^4 (\beta s - 2m^2 L)}{32\pi s^2 \beta^2} - \frac{g_2^4 (\beta(s + 4m^2) + (s + 2m^2)L)}{64\pi s^2 \beta} + \frac{g_1^2 m^2 L}{16\pi s^2} , \tag{B.28}
\]
\[
\sigma(\ell_{L1} \bar{\nu}_{L1} \to \ell^- \nu_\ell) = \frac{g_1 g_2^3 (5\beta s + 2(2s + 3m^2)L)}{64\pi \beta^2 s^2 (2s_w^2 - 1)} + \frac{g_1 g_2^2 (\beta(4s + 9m^2) + 8m^2 L)}{64\pi m^2 \beta^2 s^2 (2s_w^2 - 1)^2} + \frac{g_1^3 (s + 2m^2)}{96\pi \beta s^2} , \tag{B.29}
\]
\[
\sigma(\ell_{L1} \bar{\nu}_{L1} \to \ell^- \nu_\ell) = \frac{g_1 g_2^3 (-2m^2(4s - 5m^2)L + m^2 \beta s)}{64\pi \beta^2 s^2 m^2 (2s_w^2 - 1)} + \frac{\beta s(4s - 3m^2)}{64\pi \beta^2 s^2 m^2} \left( \frac{g_1 g_2^3}{(2s_w^2 - 1)^2} + \frac{g_1^3}{4} \right) , \tag{B.30}
\]
\[
\sigma(\nu_{L1} \ell_{L1} \to \nu_\ell \ell^-) = \frac{g_1^3 (4s - 3m^2)}{256\pi m^2 s \beta} , \tag{B.31}
\]
\[
\sigma(\nu_{L1} \ell_{L1} \to \nu_\ell \ell^-) = \frac{g_1^2 g_2^2 (4s - 3m^2)}{64\pi m^2 \beta (2s_w^2 - 1)^2} , \tag{B.32}
\]
\[
\sigma(\bar{\nu}_{L1} \ell_{L1} \to \bar{\nu}_\ell \ell^-) = \frac{g_1^2 g_2^2 (\beta(4s + 9m^2) + 8m^2 L)}{64\pi m^2 \beta^2 (2s_w^2 - 1)^2} , \tag{B.33}
\]
\[
\sigma(\ell_{L1} \bar{\nu}_{L1} \to \nu_\ell \bar{\nu}_\ell) = \frac{g_1^4 (\beta (9m^2 + 4s) + 8m^2 L)}{256\pi m^2 s \beta^2} , \tag{B.34}
\]
where $g^2 = g_1^2 Y_1 Y_1 + g_2^2 T^3_1 T^3_1$. The above cross-sections, (B.13-B.34) are consistent with (B.48) - (B.62) and (B.71) - (B.74) in [21]. For one $SU(2)_W$-singlet KK lepton and one $SU(2)_W$-doublet KK lepton we get
\[
\sigma(\ell_{R1} \ell_{L1} \to \ell \ell) = \frac{g_1^4 Y_{e_l}^2 Y_{e_R}^2}{64\pi m^2 s \beta^2} \left( 8m^2 L + \beta(9m^2 + 4s) \right) , \tag{B.35}
\]
\[
\sigma(\ell_{R1} \bar{\ell}_{L1} \to \ell \bar{\ell}) = \frac{g_1^4 Y_{e_l}^2 Y_{e_R}^2}{64\pi m^2 s \beta} \left( 4s - 3m^2 \right) . \tag{B.36}
\]
These two formulas have the same structure as $\sigma(\ell_{R1}^\pm \ell_{R1}^\pm \to \ell_{R1}^\pm \ell_{R1}^\pm)$ and $\sigma(\ell_{R1}^\mp \ell_{R1}^\mp \to \ell_{R1}^\mp \ell_{R1}^\mp)$. 
B.2 Gauge Bosons

The self-annihilation cross-sections of $\gamma_1$ are

$$\sigma(\gamma_1 \gamma_1 \rightarrow f \bar{f}) = \frac{N_c g_1^4 (Y_{fL} + Y_{fR})}{72 \pi s^2 \beta^2} \left(-5s(2m^2 + s)L - 7s\beta \right), \quad (B.37)$$

$$\sigma(\gamma_1 \gamma_1 \rightarrow \phi \phi^*) = \frac{g_1^4 Y_\phi^4}{12 \pi s \beta}. \quad (B.38)$$

These two cross-sections are identical to (A.44) and (A.47) in [21]. For $Z_1$ self-annihilation into fermions and Higgs bosons,

$$\sigma(Z_1 Z_1 \rightarrow f \bar{f}) = \frac{N_c g_2^4}{1152 \pi s^2 \beta^2} \left(-5(2m^2 + s)L - 7s\beta \right), \quad (B.39)$$

$$\sigma(Z_1 Z_1 \rightarrow \phi \phi^*) = \frac{g_2^4}{192 \pi s \beta}. \quad (B.40)$$

The cross-section for the above two processes are obtained from $\sigma(\gamma_1 \gamma_1 \rightarrow f \bar{f})$ and $\sigma(\gamma_1 \gamma_1 \rightarrow \phi \phi^*)$ by replacing $g_1 Y$ with $g_2 / 2$, which corresponds to the $Z$ couplings to SM fermions and Higgs bosons. For the coannihilations of $SU(2)_W$ KK bosons into SM gauge bosons, we get

$$\sigma(Z_1 Z_1 \rightarrow W^+ W^-) = \frac{g_2^4}{18 \pi m^2 s^3 \beta^2} \left(12m^2(s - 2m^2)L + s\beta(12m^4 + 3sm^2 + 4s^2) \right), \quad (B.41)$$

$$\sigma(W_1^+ W_1^+ \rightarrow W^+ W^+) = \frac{g_2^4}{36 \pi m^2 s^3 \beta^2} \left(12m^2(s - 2m^2)L + s\beta(12m^4 + 3sm^2 + 4s^2) \right), \quad (B.42)$$

$$\sigma(W_1^+ W_1^- \rightarrow \gamma \gamma) = \frac{e^4}{36 \pi m^2 s^3 \beta^2} \left(12m^4(s - 2m^2)L + s\beta(12m^4 + 3sm^2 + 4s^2) \right), \quad (B.43)$$

$$\sigma(W_1^+ W_1^- \rightarrow \gamma Z) = \frac{g_2^2 e^2 c_w^2}{18 \pi m^2 s^3 \beta^2} \left(12m^4(s - 2m^2)L + s\beta(12m^4 + 3sm^2 + 4s^2) \right), \quad (B.44)$$

$$\sigma(W_1^+ W_1^- \rightarrow ZZ) = \frac{g_2^4 c_w^2}{36 \pi m^2 s^3 \beta^2} \left(12m^4(s - 2m^2)L + s\beta(12m^4 + 3sm^2 + 4s^2) \right). \quad (B.45)$$

We see that the above five cross-sections contain similar expressions up to overall factors due to the gauge structure of $SU(2)_W \times U(1)_Y$. For $W_1^\pm$ annihilation into
other final states, we have

$$\sigma(W_1^+W_1^- \rightarrow f\bar{f}) = \frac{-N_c g_2^2}{576\pi s^2\beta^2} ((12m^2 + 5s)L + 2\beta(4m^2 + 5s)) \ , \quad (B.46)$$

$$\sigma(W_1^+W_1^- \rightarrow W^+W^-) = \frac{g_2^4}{18\pi m^2 s^2\beta^2} (2m^2(3m^2 + 2s)L + \beta(11m^4 + 5sm^2 + 2s^2)) \ , \quad (B.47)$$

$$\sigma(W_1^+W_1^- \rightarrow \phi\phi^*) = \frac{g_2^2(s - m^2)}{144\pi s^2\beta^2} \ . \quad (B.48)$$

These three cross-sections are different since they involve s-channel Z diagrams.

For \(\gamma_1 Z_1\) and \(\gamma_1 W_1^-\) into fermions we can recycle \(\sigma(\gamma_1 \gamma_1 \rightarrow f\bar{f})\) and obtain

$$\sigma(\gamma_1 Z_1 \rightarrow f\bar{f}) = \frac{-N_c g_1^2 g_2^2 Y_f^2}{288\pi s^2\beta^2} (5(2m^2 + s)L + 7s\beta) \ , \quad (B.49)$$

$$\sigma(\gamma_1 W_1^- \rightarrow f\bar{f}^0) = \frac{-N_c g_1^2 g_2^2 Y_f^2}{144\pi s^2\beta^2} (5(2m^2 + s)L + 7s\beta) \ . \quad (B.50)$$

For the annihilation of two different KK gauge bosons into Higgs bosons we have

$$\sigma(\gamma_1 Z_1 \rightarrow \phi\phi^*) = \frac{g_1^2 g_2^2}{192\pi s\beta} \ , \quad (B.51)$$

$$\sigma(\gamma_1 W_1^- \rightarrow \phi_d\phi_u^*) = \frac{g_1^2 g_2^2}{96\pi s\beta} \ , \quad (B.52)$$

$$\sigma(Z_1 W_1^- \rightarrow \phi_d\phi_u^*) = \frac{g_4^2\beta}{288\pi s} \ , \quad (B.53)$$

which can be obtained from \(\sigma(\gamma_1 \gamma_1 \rightarrow \phi\phi^*)\). The cross-section for \(Z_1 W_1^-\) into fermions

$$\sigma(Z_1 W_1^- \rightarrow f\bar{f}' \bar{f}) = \frac{-N_c g_2^4}{576\pi s^2\beta^2} ((14m^2 + 5s)L + \beta(16m^2 + 13s)) \ , \quad (B.54)$$

has a different structure compared to other fermion final states due to an s-channel W diagram. The cross-sections for \(Z_1 W_1^-\) into gauge boson final states

$$\sigma(Z_1 W_1^- \rightarrow ZW^-) = \frac{g_2^4 c_w^2}{18\pi m^2 s^2\beta^2} (2m^2(3m^2 + 2s)L + \beta(11m^4 + 5sm^2 + 2s^2)) \ , \quad (B.55)$$
\[ \sigma(Z_1 W_1^- \rightarrow \gamma W^-) = \frac{e^2 g_2^2}{18 \pi m^2 s^2 \beta^2} (2m^2(3m^2 + 2s)L + \beta(11m^4 + 5sm^2 + 2s^2)) , \]  
(B.56)

can be obtained from \( \sigma(W_1^+ W_1^- \rightarrow W^+ W^-) \). For KK gluons we get
\[ \sigma(g_1 g_1 \rightarrow gg) = \frac{g_3^4}{64 \pi m^2 s^3 \beta^2} (8m^2(s^2 + 3sm^2 - 3m^4)L + s\beta(34m^2 + 13sm^2 + 8s^2)) , \]  
(B.57)
\[ \sigma(g_1 g_1 \rightarrow q\bar{q}) = \frac{-g_4^4}{3456 \pi s^2 \beta^2} (2(20s + 49m^2)L + \beta(72m^2 + 83s)) , \]  
(B.58)
for which there are no analogous processes. The cross-sections associated with one gluon and one electroweak gauge bosons in the initial state
\[ \sigma(g_1 \gamma_1 \rightarrow q\bar{q}) = \frac{g_3^2 g_5^2 (Y_{qL}^2 + Y_{qR}^2)}{144 \pi s^2 \beta^2} (-5(2m^2 + s)L - 7s\beta) , \]  
(B.59)
\[ \sigma(g_1 Z_1 \rightarrow q\bar{q}) = \frac{g_2^2 g_3^2}{576 \pi s^2 \beta^2} (-5(2m^2 + s)L - 7s\beta) , \]  
(B.60)
\[ \sigma(g_1 W_1^- \rightarrow q\bar{q}') = \frac{g_2^2 g_3^2}{288 \pi s^2 \beta^2} (-5(2m^2 + s)L - 7s\beta) , \]  
(B.61)
are obtained from \( \sigma(\gamma_1 \gamma_1 \rightarrow f\bar{f}), \sigma(\gamma_1 Z_1 \rightarrow f\bar{f}) \) and \( \sigma(\gamma_1 W_1^- \rightarrow f\bar{f'}) \) by simple coupling replacements and accounting for the additional color factors.

### B.3 Fermions and Gauge Bosons

Note that in UED, the Dirac KK fermions are constructed out of two Weyl fermions with the same \( SU(2)_W \times U(1)_Y \) quantum numbers while a Dirac fermion in the Standard Model is made up of two Weyl fermions of different \( SU(2)_W \times U(1)_Y \) quantum numbers. Therefore the couplings of KK fermions to zero mode gauge bosons are vector-like. This difference shows up in processes involving gauge-boson couplings with fermions. For the vertices which involve \( n = 1 \) gauge bosons, we need one KK fermion and one SM fermion in order to conserve KK number. In this case, there is always a projection operator associated with the subscript \( (L/R) \) of the KK fermion. The annihilation cross-sections with \( SU(2)_W \)-singlet KK fermions
and $SU(2)_W$ KK gauge bosons ($Z_1$ and $W_1^\pm$) are zero:

\[
\sigma(f_{R1}Z_1 \to SM) = 0 ,
\]
\[
\sigma(f_{R1}W_1^\pm \to SM) = 0 .
\] (B.62)

The cross-section for $SU(2)_W$-singlet leptons and $\gamma_1$ is

\[
\sigma(\gamma_1\ell_R^\pm \to B_0\ell^\pm) = \frac{Y_\ell^4 g_1^4 (-6L - \beta)}{96\pi s\beta^2} .
\] (B.63)

For a KK quark and $\gamma_1$ we have

\[
\sigma(q_1\gamma_1 \to qq) = \frac{g_1^2 g_3^2 Y_\ell^2}{72\pi s\beta^2} (-6L - \beta) .
\] (B.64)

This cross-section is basically the same as $\sigma(\gamma_1\ell_R^\pm \to B_0\ell^\pm)$, up to a group factor. In $\sigma(q_1\gamma_1 \to qq)$, the vertex associated with the SM gluon $g$ contains a Gell-Mann matrix $t^a_{ij}$. In the squared matrix element, we then get [130]

\[
\sum_{a=1}^{8} \frac{1}{3} \sum_{i,j=1}^{3} t^a_{ij} t^a_{ji} = \frac{1}{3} \times \frac{4}{3} \times 3 = \frac{4}{3} .
\] (B.65)

Similarly, for the $SU(2)_W$-doublet quarks with $Z_1$, we get

\[
\sigma(q_L Z_1 \to qq) = \frac{g_2^2 g_3^2}{288\pi s\beta^2} (-6L - \beta) ,
\] (B.66)

by replacing the $g_1$ coupling in $\sigma(q_1\gamma_1 \to qq)$ with $g_2/2$. For $W_1^\pm$, one should use the coupling $g_2/\sqrt{2}$ instead:

\[
\sigma(q_L W_1 \to qq') = \frac{g_2^2 g_3^2}{144\pi s\beta^2} (-6L - \beta) .
\] (B.67)

For the $SU(2)_W$-doublet KK leptons and $\gamma_1$ or $Z_1$, we get

\[
\sigma(\ell_L\gamma_1 \to \ell_L\gamma/Z) = \frac{g_1^4}{1536\pi s\beta^2 s_w^2} (-6L - \beta) ,
\] (B.68)
\[
\sigma(\ell_L Z_1 \to \ell_L\gamma/Z) = \frac{g_2^4}{1536\pi s\beta^2 c_w^2} (-6L - \beta) ,
\] (B.69)
\[ \sigma(\ell_{L1} \gamma_1 \rightarrow \nu_{\ell} W) = \frac{g_1^2 g_2^2}{68\pi s \beta^2} (-6L - \beta). \]  

(B.70)

The last 7 cross-sections have a similar structure since they all have \( s \)- and \( t \)-channel diagrams only. For the cross-sections associated with \( SU(2)_W \)-doublet KK leptons and electroweak KK gauge bosons into other final states, we have

\[ \sigma(\ell_{L1} Z_1 \rightarrow \nu_{\ell} W) = \frac{g_2^4}{68\pi m^2 s \beta^2} \left( 26m^2 L + \beta (23m^2 + 32s) \right), \]  

(B.71)

\[ \sigma(\ell_{L1} W_1 \rightarrow \nu_{\ell'} (\gamma + Z)) = \frac{g_2^4}{68\pi m^2 s \beta^2} \left( m^2 (32c_w^2 - 6) L \right. \]  
\[ \left. + \beta m(24c_w^2 - 1) + 32s \beta c_w^2 \right), \]  

(B.72)

\[ \sigma(\ell_{L1} W_1^\pm \rightarrow \ell_L W^\pm) = \frac{g_2^4}{192\pi m^2 s \beta^2} \left( -3m^2 L + 4s \beta \right), \]  

(B.73)

\[ \sigma(\ell_{L1} W_1^\pm \rightarrow \ell_L W^\pm) = \frac{g_2^4}{384\pi m^2 s \beta^2} \left( 16m^2 L + \beta (11m^2 + 8s) \right), \]  

(B.74)

\[ \sigma(\ell_{L1} W_1^- \rightarrow \ell_L W^-) = \sigma(\nu_{\ell_1} W_1^+ \rightarrow \ell_L W^+), \]  

(B.75)

\[ \sigma(\ell_{L1} W_1^- \rightarrow \ell_L W^-) = \sigma(\nu_{\ell_1} W_1^- \rightarrow \ell_L W^-). \]  

(B.76)

For KK gluon - KK quark annihilation, we obtain

\[ \sigma(g_1 q_1 \rightarrow g q) = \frac{g_3^4}{846\pi m^2 s \beta^2} \left( 24m^2 L + \beta (25m^2 + 36s) \right). \]  

(B.77)

**B.4 Quarks**

The annihilation cross-section of two KK quarks into SM quarks of different flavor

\[ \sigma(q_1 \bar{q}_1 \rightarrow q \bar{q}') = \frac{g_3^4 (s + 2m^2)}{72\pi s \beta^2} \]  

(B.78)

can be obtained from \( \sigma(\ell_{R1}^+ \ell_{R1}^- \rightarrow f \bar{f}) \) by multiplying with the following group factor [130]

\[ \frac{1}{3} tr \left( t^a t^b \right) tr \left( t^b t^c \right) = \frac{1}{9} C(r)^2 \delta^{ab} \delta^{ab} = \frac{1}{9} \cdot \frac{1}{4} \cdot 8 = \frac{2}{9}. \]  

(B.79)
Here \( C(r) = 1/2 \) is the quadratic Casimir operator for the fundamental representation of \( SU(3) \). KK quark annihilation into same flavor SM quarks is given by

\[
\sigma(q_1 q_1 \rightarrow qq) = \frac{g_3^4}{432\pi m^2 s^2 \beta^2} (2m^2(4s - 5m^2)L + (6s - 5m^2)s\beta) .
\]  

(B.80)

In this process, there are three terms in the squared matrix element since we have both \( t \)- and \( u \)-channel diagrams. This process also has an analogy with \( \sigma(\ell_R^{+} \ell_R^{-} \rightarrow \ell^+ \ell^-) \). However, each term gets a different group factor. The squares of the \( t \)- and \( u \)-channel diagrams get the same factor of \( 2/9 \) but for the cross term we obtain [130]

\[
\left(\frac{1}{3}\right)^2 tr (t^a t^b t^a t^b) = \frac{1}{9} \left(C_2(2) - \frac{1}{2} C_2(G)\right) tr (t^a t^a) \\
= \frac{1}{9} \left(\frac{4}{3} - \frac{3}{2}\right) \frac{4}{3} = -\frac{2}{9} = -\frac{2}{27} .
\]  

(B.81)

For \( q_1 \bar{q}_1 \) annihilation into same flavor SM quarks we get

\[
\sigma(q_1 \bar{q}_1 \rightarrow q\bar{q}) = \frac{g_3^4}{864\pi m^2 s^2 \beta^2} \left(4m^2(4s - 3m^2)L + \beta(32m^4 + 33sm^2 + 12s^2)\right) ,
\]  

(B.82)

which can also be obtained using the analogy to \( \sigma(\ell_R^{+} \ell_R^{-} \rightarrow \ell^+ \ell^-) \) and taking into account group factors. For the final state with gluons we have

\[
\sigma(q_1 \bar{q}_1 \rightarrow gg) = \frac{-g_3^4}{54\pi s^3 \beta^2} \left(4(m^4 + 4sm^2 + s^2)L + s\beta(31m^2 + 7s)\right) .
\]  

(B.83)

For different quark flavors in the initial state, we have

\[
\sigma(q_1 q_1' \rightarrow qq') = \frac{g_3^4(4s - 3m^2)}{288\pi m^2 s\beta} ,
\]  

(B.84)

\[
\sigma(q_1 \bar{q}_1' \rightarrow q\bar{q}') = \frac{g_3^4}{288\pi m^2 s\beta^2} \left(8m^2L + \beta(9m^2 + 4s)\right) .
\]  

(B.85)

The above two cross-sections can be obtained from \( \sigma(\ell_R^{+} \ell_R^{-} \rightarrow \ell^+ \ell^-) \) and \( \sigma(\ell_R^{+} \ell_R^{-} \rightarrow \ell^+ \ell^-) \), correspondingly, by multiplying with the group factor \( 2/9 \). The
remaining cross-sections are

\[
\sigma(q_1 q_1' \rightarrow q q') = \sigma(u_{R1} \bar{d}_{R1} \rightarrow u \bar{d}) ,
\]

\[
= \sigma(u_{R1} d_{L1} \rightarrow u d) ,
\]

\[
= \sigma(u_{R1} u_{L1} \rightarrow u u) ,
\]

(B.86)

and

\[
\sigma(q_1 q_1' \rightarrow q q') = \sigma(u_{R1} d_{R1} \rightarrow u d) ,
\]

\[
= \sigma(u_{R1} \bar{d}_{L1} \rightarrow u \bar{d}) ,
\]

\[
= \sigma(u_{R1} \bar{u}_{L1} \rightarrow u \bar{u}) .
\]

(B.87)

\[
\sigma(q_1 q_1' \rightarrow q q') = \sigma(u_{R1} u_{R1} \rightarrow u u) .
\]

\[
= \sigma(u_{R1} \bar{d}_{L1} \rightarrow u \bar{d}) ,
\]

\[
= \sigma(u_{R1} \bar{u}_{L1} \rightarrow u \bar{u}) .
\]

(B.88)

\[
\sigma(q_1 q_1' \rightarrow q q') = \sigma(u_{R1} d_{L1} \rightarrow u d) ,
\]

\[
= \sigma(u_{R1} d_{L1} \rightarrow u d) ,
\]

\[
= \sigma(u_{R1} \bar{u}_{L1} \rightarrow u \bar{u}) .
\]

(B.89)

\[
\sigma(q_1 q_1' \rightarrow q q') = \sigma(u_{R1} u_{L1} \rightarrow u u) .
\]

\[
= \sigma(u_{R1} \bar{d}_{L1} \rightarrow u \bar{d}) ,
\]

\[
= \sigma(u_{R1} \bar{u}_{L1} \rightarrow u \bar{u}) .
\]

(B.90)

\[
\sigma(q_1 q_1' \rightarrow q q') = \sigma(u_{R1} d_{L1} \rightarrow u d) ,
\]

\[
= \sigma(u_{R1} \bar{d}_{L1} \rightarrow u \bar{d}) ,
\]

\[
= \sigma(u_{R1} \bar{u}_{L1} \rightarrow u \bar{u}) .
\]

(B.91)

\[
\sigma(q_1 q_1' \rightarrow q q') = \sigma(u_{R1} u_{L1} \rightarrow u u) .
\]

\[
= \sigma(u_{R1} \bar{d}_{L1} \rightarrow u \bar{d}) ,
\]

\[
= \sigma(u_{R1} \bar{u}_{L1} \rightarrow u \bar{u}) .
\]

(B.92)

\[
\sigma(q_1 q_1' \rightarrow q q') = \sigma(u_{R1} d_{L1} \rightarrow u d) ,
\]

\[
= \sigma(u_{R1} \bar{d}_{L1} \rightarrow u \bar{d}) ,
\]

\[
= \sigma(u_{R1} \bar{u}_{L1} \rightarrow u \bar{u}) .
\]

(B.93)

These three cross-sections can be obtained from \(\sigma(\ell_{R1}^{\pm} \ell_{R1}^{\mp} \rightarrow \ell^{\pm} \ell^{\mp})\). For one KK lepton and one KK anti-quark in the initial state, we have

\[
\sigma(\nu_{L1} \bar{u}_{L1} \rightarrow \nu u) = \frac{(4g_1^2 Y_{\nu_{L1}} Y_{u_{L1}} - g_2^2)^2 (4s - 3m^2)}{1024 \pi m^2 s \beta} \),
\]

(B.88)

\[
\sigma(\nu_{L1} \bar{u}_{L1} \rightarrow \nu u) = \frac{(4g_1^2 Y_{\nu_{L1}} Y_{u_{L1}} + g_2^2)^2 (4s - 3m^2)}{1024 \pi m^2 s \beta} \),
\]

(B.89)

\[
\sigma(\ell_{L1} \bar{u}_{L1} \rightarrow \nu d) = \frac{g_2^4 (4s - 3m^2)}{256 \pi m^2 s \beta} .
\]

(B.90)

These three cross-sections can be obtained from \(\sigma(\ell_{R1}^{\pm} \ell_{R1}^{\mp} \rightarrow \ell^{\pm} \ell^{\mp})\). For one KK lepton and one KK anti-quark in the initial state, we have

\[
\sigma(\nu_{L1} \bar{u}_{L1} \rightarrow \nu u) = \frac{(4g_1^2 Y_{\nu_{L1}} Y_{u_{L1}} - g_2^2)^2 (8m^2 L + \beta(9m^2 + 4s))}{256 \pi m^2 s \beta^2} \),
\]

(B.91)

\[
\sigma(\ell_{L1} \bar{u}_{L1} \rightarrow \ell u) = \frac{(4g_1^2 Y_{\ell_{L1}} Y_{u_{L1}} - g_2^2)^2 (8m^2 L + \beta(9m^2 + 4s))}{1024 \pi m^2 s \beta^2} \),
\]

(B.92)

\[
\sigma(\nu_{L1} \bar{u}_{L1} \rightarrow \nu u) = \frac{(4g_1^2 Y_{\nu_{L1}} Y_{u_{L1}} + g_2^2)^2 (8m^2 L + \beta(9m^2 + 4s))}{256 \pi m^2 s \beta^2} \),
\]

(B.93)

which can be obtained from \(\sigma(\ell_{R1}^{\pm} \ell_{R1}^{\mp} \rightarrow \ell^{\pm} \ell^{\mp})\). If one of the particles in the initial state is an \(SU(2)_W\)-singlet fermion, only \(\gamma_1\) can mediate the process and the
cross-sections can be obtained from our previous results:
\[
\sigma(\ell R_1 u L_1 \rightarrow \ell u) = \sigma(\ell L_1 u R_1 \rightarrow \ell u) = \sigma(\ell R_1 \bar{u}_R \rightarrow \ell \bar{u}) = \sigma(\ell R_1 \ell L_1 \rightarrow \ell \ell), \tag{B.94}
\]
\[
\sigma(\ell R_1 \bar{u}_L \rightarrow \ell \bar{u}) = \sigma(\ell L_1 \bar{u}_R \rightarrow \ell \bar{u}) = \sigma(\ell R_1 u R_1 \rightarrow \ell u) = \sigma(\ell R_1 \bar{e}_L \rightarrow \ell \bar{u}), \tag{B.95}
\]

### B.6 Higgs Bosons

The mass terms for the KK $SU(2)_W$-singlets appear with the wrong sign in the fermion Lagrangian. For example, the mass term for the KK quarks leads to the following structure for the mass matrix at tree level
\[
\begin{pmatrix}
\bar{u}_{L_n}(x), \bar{u}_{R_n}(x)
\end{pmatrix}
\begin{pmatrix}
\frac{n}{R} & m \\
m & -\frac{n}{R}
\end{pmatrix}
\begin{pmatrix}
u_{L_n}(x) \\
u_{R_n}(x)
\end{pmatrix}. \tag{B.96}
\]

The corresponding mass eigenstates $u'_{L_n}$ and $u'_{R_n}$ have mass
\[
M_n = \sqrt{\left(\frac{n}{R}\right)^2 + m^2}. \tag{B.97}
\]

These mass eigenstates receive different radiative corrections which lift the degeneracy [28]. The interaction eigenstates are related to the mass eigenstates by
\[
\begin{pmatrix}
u_{L_n} \\
u_{R_n}
\end{pmatrix}
= \begin{pmatrix}
\cos \alpha & \gamma_5 \sin \alpha \\
\sin \alpha & -\gamma_5 \cos \alpha
\end{pmatrix}
\begin{pmatrix}
u'_{L_n} \\
u'_{R_n}
\end{pmatrix}, \tag{B.98}
\]
where $\alpha$ is the mixing angle between $SU(2)_W$-singlet and $SU(2)_W$-doublet fermions defined by
\[
\tan 2\alpha = \frac{m}{n/R}. \tag{B.99}
\]

This mixing is very small except for the top quark. However, even with $\alpha \approx 0$ the effect of the rotation (B.98) is present in the Yukawa couplings through the
redefinition $u_R \rightarrow -\gamma^5 u_R$. It does not affect the gauge-fermion couplings. We use the following notation for KK Higgs bosons,

$$
\begin{pmatrix}
G_1^+ \\
\frac{H_1 + iG_1}{\sqrt{2}}
\end{pmatrix}.
$$

We keep only the top-Yukawa coupling and we also keep the Higgs self-coupling assuming $m_h = 120$ GeV. Below we list the cross-sections associated with two KK Higgs bosons in the initial state.

$$
\sigma(H_1 H_1 \rightarrow G^+ G^-) = \frac{1}{64\pi m^2 s^2 s_w^2} \left( 8e^2 m^2 \lambda^2 s_w^2 L + \beta \{(2s + m^2)e^4 + 4\lambda m^2 s_w^2 e^2 + 4\lambda^2 m^2 s_w^4 \} \right),
$$

$$
\sigma(H_1 H_1 \rightarrow H H) = \frac{9\lambda^2}{32\pi s \beta},
$$

$$
\sigma(H_1 H_1 \rightarrow GG) = \frac{1}{128\pi m^2 s^2 \beta^2 s_w^4 c_w^4} \left( 8e^2 m^2 \lambda s_w^2 c_w^2 L + \beta \{(2s + m^2)e^4 + 4\lambda m^2 s_w^2 c_w^2 + 4\lambda^2 m^2 s_w^4 c_w^4 \} \right),
$$

$$
\sigma(H_1 H_1 \rightarrow ZZ) = \frac{g_3^4}{16\pi s^3 \beta^2 c_w^4} \left( s \beta (s + 4m^2) + 4m^2 (s - 2m^2) L \right),
$$

$$
\sigma(H_1 H_1 \rightarrow W^+ W^-) = \frac{g_2^4}{32\pi s^3 \beta^2 c_w^4} \left( s \beta (s + 4m^2) + 4m^2 (s - 2m^2) L \right),
$$

$$
\sigma(H_1 H_1 \rightarrow t\bar{t}) = \frac{3y_t^4}{16\pi s^2 \beta^2} \left( -(s + 2m^2) L - 2s \beta \right),
$$

$$
\sigma(G_1^+ G_1^+ \rightarrow G^+ G^+) = \frac{1}{128\pi m^2 s^2 \beta^2 s_w^4 c_w^4} \left( -16e^2 m^2 \lambda s_w^2 c_w^2 L \right.
+ \left. \beta \{(2s + m^2)e^4 - 8\lambda m^2 e^2 s_w^2 c_w^2 + 16\lambda^2 m^2 s_w^4 c_w^4 \} \right),
$$

$$
\sigma(G_1^+ G_1^+ \rightarrow t\bar{t}) = \frac{1}{1152\pi s^2 \beta^2 s_w^4 c_w^4} \left( 72s_w^4 c_w^2 y_t^2 (-3s_w^2 y_t^2 - 4m^2 e^2) L - 432s \beta s_w^4 c_w^4 y_t^4 
+ s \beta^3 (20s_w^4 - 12s_w^2 + 9) e^4 - 144s \beta s_w^4 c_w^2 y_t^2 e^2 \right),
$$
\[ \sigma(G_1^+G_1^- \rightarrow b \bar{b}) = \frac{1}{1152 \pi s \beta^2 c^2 \epsilon^4 c_w} (72 s_w^2 c_w y_t^2 (-3 s_w^2 c_w y_t^2 + e^2 m^2 (4 s_w^2 - 3)) L - 432 s \beta^4 c_w y_t^4 + s \beta^4 (20 s_w^2 - 24 s_w^2 + 9) e^4 \]
\[ - 36 s \beta^2 y_t^2 e^2 (4 s_w^2 - 7 s_w^2 + 3) \] ,

\[ \sigma(G_1^+G_1^- \rightarrow G^+G^-) = \frac{-1}{192 \pi m^2 s^2 \beta^2 c_w^4} (6 m^2 e^2 s (e^2 + 4 \lambda s_w^4 c_w^4) L - 48 \lambda m^2 s^2 c_w^4 + \beta \{ e^2 (m^4 - 7 s^2 - 3 s^2) - 12 \lambda m^2 s^2 c_w^4 \}) \] ,

\[ \sigma(G_1^+G_1^- \rightarrow GH) = \frac{1}{128 \pi m^2 s \beta^2 c_w^4} (8 s_w^2 m^2 e^2 \lambda L + \beta \{ (2 s + m^2) e^4 + 4 \lambda s_w^2 m^2 e^2 + 4 \lambda^2 m^2 c_w^4 \}) \] ,

\[ \sigma(G_1^+G_1^- \rightarrow GH) = \frac{g_2^4}{48 \pi s^2 \beta^2} (24 m^2 c_w^2 s L - \beta \{ 4(1 - 2 s_w^2)^2 m^4 + s (92 s_w^4 - 140 s_w^2 + 47) - 24 s^2 c_w^4 \}) \] ,

\[ \sigma(G_1^+G_1^- \rightarrow ZZ) = \frac{g_2^4 (1 - 2 s_w^2)^4}{4 \pi s^3 \beta^2} (4 m^2 (s - 2 m^2) L + s \beta (s + 4 m^2)) \] ,

\[ \sigma(G_1^+G_1^- \rightarrow \gamma Z) = \frac{e^2 g_2^4 (1 - 2 s_w^2)^2}{2 \pi s^3 \beta^2} (4 m^2 (s - 2 m^2) L + s \beta (s + 4 m^2)) \] ,

\[ \sigma(G_1^+G_1^- \rightarrow \gamma \gamma) = \frac{e^4}{4 \pi s^3 \beta^2} (4 m^2 (s - 2 m^2) L + s \beta (s + 4 m^2)) \] ,

\[ \sigma(G_1^+G_1^- \rightarrow W^+W^-) = \frac{g_1^4}{24 \pi s^2 \beta^2} (6 m^2 L + \beta (s + 11 m^2)) \] ,

\[ \sigma(G_1^+G_1^- \rightarrow f \bar{f}) = \frac{N_c g_2^4 \beta}{24 \pi s} (g^2_L + g^2_R) \] ,

where \( f \) represents leptons and first two generations of quarks and

\[ \hat{g}_{L(R)}^2 = -e^2 Q_f - 2 g_2^2 (1 - 2 s_w^2) (T_f^3 - Q_f s_w^2) . \]
A number of cross-sections can be simply related:

\[
\begin{align*}
\sigma(G_1 G_1 \rightarrow G^+ G^-) &= \sigma(H_1 H_1 \rightarrow G^+ G^-), \\
\sigma(G_1 G_1 \rightarrow GG) &= \sigma(H_1 H_1 \rightarrow HH), \\
\sigma(G_1 G_1 \rightarrow HH) &= \sigma(H_1 H_1 \rightarrow GG), \\
\sigma(G_1 G_1 \rightarrow ZZ) &= \sigma(H_1 H_1 \rightarrow ZZ), \\
\sigma(G_1 G_1 \rightarrow W^+ W^-) &= \sigma(H_1 H_1 \rightarrow W^+ W^-), \\
\sigma(G_1 G_1 \rightarrow t\bar{t}) &= \sigma(H_1 H_1 \rightarrow t\bar{t}), \\
\sigma(G_1^+ G_1^- \rightarrow GG) &= \sigma(G_1^+ G_1^- \rightarrow HH), \\
\sigma(G_1^+ G_1^- \rightarrow HH) &= \frac{1}{2} \sigma(H_1 H_1 \rightarrow G^+ G^-).
\end{align*}
\] (B.118)

The rest of the cross-sections are

\[
\begin{align*}
\sigma(H_1 G_1 \rightarrow HG) &= \frac{1}{192\pi m^2 s^2 \beta^2 s_w^4 c_w^4} (6m^2 e^2 s(e^2 - 2\lambda s_w^2 c_w^2)L \\
&\quad - \beta ((m^4 - 7s^2 - 3s^2)e^4 \\
&\quad + 6\lambda m^2 s s_w^2 c_w^2 e^4 - 12\lambda^2 m^2 s s_w^4 c_w^4)),
\end{align*}
\] (B.119)

\[
\begin{align*}
\sigma(H_1 G_1 \rightarrow G^+ G^-) &= \frac{g_4^2}{48\pi m^2 s^2 \beta^2} (24m^2 c_w^2 s L + \beta (-4(1 - 2s_w^2)^2 m^4 \\
&\quad + s m^2 (-92s_w^4 + 140s_w^2 - 47) + 24s^2 c_w^4)),
\end{align*}
\] (B.120)

\[
\begin{align*}
\sigma(H_1 G_1 \rightarrow W^+ W^-) &= \frac{g_4^2}{96\pi s^3 \beta^2} (12m^2 (2m^2 + s)L + s \beta (32m^2 + s)),
\end{align*}
\] (B.121)

\[
\begin{align*}
\sigma(H_1 G_1 \rightarrow t\bar{t}) &= \frac{1}{288\pi s^2 \beta^2 s_w^2 c_w^2 s_w^2} (-54y_t^2 (e^2 m^2 + c_w^2 s_w^2 (s - 2m^2) y_t^2)L \\
&\quad + \beta (-54s y_t^2 c_w^2 s_w^2 - 27e^2 y_t^2 s + e^2 g_2^2 s \beta^2 (32s_w^4 - 24s_w^2 + 9)),
\end{align*}
\] (B.122)

\[
\begin{align*}
\sigma(H_1 G_1 \rightarrow f\bar{f}) &= \frac{N_c g_Z^2}{24\pi s} (g_L^2 + g_R^2),
\end{align*}
\] (B.123)
\[ \sigma(H_1 G_1^+ \rightarrow HG^+) = \frac{1}{192\pi m^2 s^2 \beta^2 s_w^4} \left( 6m^2 e^2 s(e^2 - 2\lambda s_w^2)L - 12\lambda^2 m^2 s s_w^4 \right) \]

\[ - \beta \{(m^4 - 7s m^2 - 3s^2)e^4 + 6\lambda m^2 s s_w^4 e^4 \} , \quad (B.124) \]

\[ \sigma(H_1 G_1^+ \rightarrow GG^+) = \frac{g_Z^4}{48\pi m^2 s^2 \beta^2} \left( 12m^2 (1 - 2s_w^2)(2 - 3s^2)L \right) \]

\[ - \beta \{(4m^4 + 47s m^2 - 24s^2)c_w^4 \} \]

\[ - 24(m^2 - s) s s_w^2 c_w^2 - 3s(m^2 + 4s)s_w^4 \} , \quad (B.125) \]

\[ \sigma(H_1 G_1^+ \rightarrow ZW^+) = \frac{g_Z^4}{6\pi s^3 \beta^2} \left( 6m^2 L \{(1 - 2s_w^2)(4m^2 - 2s^2_w + s) + s \} \right) \]

\[ + s\beta \{4s_w^4(s + 11m^2) + (1 - 2s_w^2)(s + 32m^2) \} , \quad (B.126) \]

\[ \sigma(H_1 G_1^+ \rightarrow \gamma W^+) = \frac{e^2 g_Z^2}{24\pi s^2 \beta^2} \left( 6m^2 L + \beta(11m^2 + s) \right) , \quad (B.127) \]

\[ \sigma(H_1 G_1^+ \rightarrow t \bar{b}) = \frac{1}{64\pi s^2 \beta^2 s_w^4} \left( -6s_w^2(2e^2 m^2 + s s_w y_t^2)y_t^2 L \right) \]

\[ + \beta \{s^2 e^4 - 6s s_w^4 y_t^2 e^2 - 12s s_w^4 y_t^4 \} , \quad (B.128) \]

\[ \sigma(H_1 G_1^+ \rightarrow f \bar{f}^\prime) = \frac{N_c g_Z^4 \beta}{192\pi s} , \quad (B.129) \]

\[ \sigma(G_1 G_1^+ \rightarrow GG^+) = \sigma(H_1 G_1^+ \rightarrow HG^+) , \]

\[ \sigma(G_1 G_1^+ \rightarrow HG^+) = \sigma(H_1 G_1^+ \rightarrow GG^+) , \]

\[ \sigma(G_1 G_1^+ \rightarrow ZW^+) = \sigma(H_1 G_1^+ \rightarrow ZW^+) , \quad (B.130) \]

\[ \sigma(G_1 G_1^+ \rightarrow \gamma W^+) = \sigma(H_1 G_1^+ \rightarrow \gamma W^+) , \]

\[ \sigma(G_1 G_1^+ \rightarrow f \bar{f}^\prime) = \sigma(H_1 G_1^+ \rightarrow f \bar{f}^\prime) . \]

**B.7 Higgs Bosons and Gauge Bosons**

The cross-sections involving one KK Higgs boson and one KK gauge boson are given below. They are rather simple compared to the cross-sections from
\[
\sigma(H_1g_1 \to t\bar{t}) = \frac{g_2^2y_1^2}{48\pi m^2s^2_{\beta} \beta} (2m^2(s - m^2)L + (2s - 5m^2)s\beta), \quad (B.131)
\]

\[
\sigma(H_1Z_1 \to ZH) = \frac{g_2^4}{96\pi c_w^2s^\beta} (L + 4\beta), \quad (B.132)
\]

\[
\sigma(H_1Z_1 \to W^-G^+) = \frac{g_2^4}{96\pi m^2s^2_{\beta} \beta} (4m^2L + s\beta(4s + m^2)), \quad (B.133)
\]

\[
\sigma(H_1Z_1 \to t\bar{t}) = \frac{g_2^2y_1^2}{64\pi m^2s^2_{\beta} \beta} (2m^2L + (4s - 11m^2)s\beta), \quad (B.134)
\]

\[
\sigma(G_1^+Z_1 \to ZG^+) = \frac{g_2^4(1 - 2s_w^2)^2}{96\pi c_w^2s^\beta} (L + 4\beta), \quad (B.135)
\]

\[
\sigma(G_1^+Z_1 \to \gamma G^+) = \frac{e^2g_2^2}{24\pi s^2_{\beta}} (L + 4\beta), \quad (B.136)
\]

\[
\sigma(G_1^+Z_1 \to t\bar{b}) = \frac{g_2^2y_1^2}{64\pi m^2s^2_{\beta} \beta} (2m^2L + (4s - 11m^2)s\beta), \quad (B.137)
\]

\[
\sigma(H_1\gamma_1 \to ZH) = \frac{g_1^2g_2^2}{96\pi c_w^2s^\beta} (L + 4\beta), \quad (B.138)
\]

\[
\sigma(G_1\gamma_1 \to W^-G^+) = \frac{g_2^4}{96\pi s^2_{\beta} \beta} (L + 4\beta), \quad (B.139)
\]

\[
\sigma(H_1\gamma_1 \to t\bar{t}) = \frac{g_2^2y_1^2}{576\pi m^2c_w^2s^2_{\beta} \beta} (-2m^2(7s + 8m^2)L + (4s - 43m^2)s\beta), \quad (B.140)
\]

\[
\sigma(G_1^+\gamma_1 \to ZG^+) = \frac{g_1^2g_2^2(1 - 2s_w^2)^2}{96\pi c_w^2s^\beta} (L + 4\beta), \quad (B.141)
\]

\[
\sigma(G_1^+\gamma_1 \to \gamma G^+) = \frac{e^2g_1^2}{24\pi s^2_{\beta}} (L + 4\beta), \quad (B.142)
\]

\[
\sigma(G_1^+W_1^+ \to G^+W^+) = \frac{g_2^4}{96\pi m^2s^2_{\beta} \beta} (12m^2L + \beta(6m^2 + 5s)), \quad (B.143)
\]

\[
\sigma(H_1W_1^+ \to G^+Z) = \frac{g_2^4}{96\pi m^2s^2_{\beta} c_w^2 \beta} (m^2(2 - s_w^2 - 2s_w^4)L \\
- \beta\{m^2(4s_w^4 - s_w^2 + 1) + (3s_w^4 - 7s_w^2 + 4)\}) \quad (B.144)
\]

\[
\sigma(H_1W_1^+ \to G^+\gamma) = \frac{g_1^2e^2}{96\pi m^2s^2_{\beta} \beta} (-2m^2L + \beta(4m^2 + 3s)), \quad (B.145)
\]

\[
\sigma(H_1W_1^+ \to HW^+) = \frac{g_2^4}{96\pi s^2_{\beta} \beta} (L + 4\beta), \quad (B.146)
\]
\[ \sigma(G_1^-W_1^+ \rightarrow W^+G^-) = \frac{g_2^4}{96\pi m^2 s^2 \beta^2} \left( -2m^2 L + \beta(4m^2 + 3s) \right), \quad (B.147) \]

\[ \sigma(G_1^-W_1^+ \rightarrow ZG) = \frac{g_2^4}{96\pi m^2 s^2 \beta^2 C_w^2} \left[ m^2 (12s_w^4 - 15s_w^2 + 4)L \\
+ \beta\{(6s_w^4 - 3s_w^2 + 1)m^2 + s(5s_w^4 - 9s_w^2 + 4)\} \right], \quad (B.148) \]

\[ \sigma(G_1^-W_1^+ \rightarrow \gamma G) = \frac{g_2^4 e^2}{96\pi m^2 s^2 \beta^2} \left( 12m^2 L + \beta(6m^2 + 5s) \right), \quad (B.149) \]

\[ \sigma(G_1^-W_1^+ \rightarrow t\bar{t}) = \frac{g_2^3}{64\pi m^2 s^2 \beta^2} \left( -4m^2 y_t^2 L + \beta\{e^2m^2 + 2(4s - 11m)y_t^2\} \right), \quad (B.150) \]

where \( y_t \) is the top quark Yukawa coupling. The cross-sections listed below are obtained from our previous calculations. For one KK Higgs boson and one KK gluon we get

\[ \sigma(H_1 g_1 \rightarrow t\bar{t}) = \sigma(G_1^-g_1 \rightarrow t\bar{t}), \]

\[ = \sigma(G_1^+ g_1 \rightarrow t\bar{b}). \quad (B.151) \]

For one KK Higgs boson and one \( Z_1 \) we have

\[ \sigma(H_1 Z_1 \rightarrow ZH) = \sigma(G_1^-Z_1 \rightarrow ZH), \]

\[ \sigma(H_1 Z_1 \rightarrow W^-G^+) = \sigma(G_1^-Z_1 \rightarrow W^-G^+), \]

\[ = \sigma(G_1^+ Z_1 \rightarrow W^+G), \]

\[ = \sigma(G_1^+ Z_1 \rightarrow W^+H), \]

\[ = \sigma(W_1^+ H_1 \rightarrow W^+G), \]

\[ = \sigma(W_1^+ H_1 \rightarrow W^+H), \quad (B.152) \]

\[ \sigma(H_1 Z_1 \rightarrow t\bar{t}) = \sigma(G_1^-Z_1 \rightarrow t\bar{t}), \]

\[ \sigma(G_1^+ Z_1 \rightarrow t\bar{b}) = \sigma(H_1 W_1^+ \rightarrow t\bar{b}), \]

\[ = \sigma(G_1 W_1^+ \rightarrow t\bar{b}). \]
For one KK Higgs boson and one $\gamma_1$ we obtain

$$
\sigma(H_1 \gamma_1 \rightarrow ZH) = \sigma(G_1 \gamma_1 \rightarrow ZG) ,
\sigma(G_1 \gamma_1 \rightarrow W^- G^+) = \sigma(H_1 \gamma_1 \rightarrow W^- G^+) ,
= \sigma(G_1^+ \gamma_1 \rightarrow W^+ G) ,
= \sigma(G_1^+ \gamma_1 \rightarrow W^+ H) ,
\sigma(H_1 \gamma_1 \rightarrow t\bar{t}) = \sigma(G_1 \gamma_1 \rightarrow t\bar{t}) ,
= \sigma(G_1^+ \gamma_1 \rightarrow t\bar{b}) .
$$

(B.153)

For one KK Higgs boson and one $W_1^\pm$, we get

$$
\sigma(H_1 W_1^+ \rightarrow G^+ Z) = \sigma(G_1 W_1^+ \rightarrow G^+ Z) ,
\sigma(H_1 W_1^+ \rightarrow G^+ \gamma) = \sigma(G_1 W_1^+ \rightarrow G^+ \gamma) ,
\sigma(H_1 W_1^+ \rightarrow H W^+) = \sigma(G_1 W_1^+ \rightarrow GW^+) ,
\sigma(G_1^- W_1^+ \rightarrow ZG) = \sigma(G_1^- W_1^+ \rightarrow ZH) ,
\sigma(G_1^- W_1^+ \rightarrow \gamma G) = \sigma(G_1^- W_1^+ \rightarrow \gamma H) .
$$

(B.154)

B.8 Higgs Bosons and Fermions

For the cross-sections between one KK Higgs boson and one KK SU(2)$_W$-singlet fermion, we have

$$
\sigma(H_1 f_{R1} \rightarrow f G) = \frac{g_1^4 f^2}{32 \pi m^2 s \beta^2} (m^2 L + s \beta) ,
\sigma(H_1 t_{R1} \rightarrow gt) = -\frac{g_3^2 y_t^2}{48 \pi s \beta^2} (2L + 3 \beta) ,
\sigma(G_1^+ t_{R1} \rightarrow t G^+) = \frac{1}{288 \pi m^2 s \beta^2 c_w^2} \left(12c_w^2 e^2 m^2 y_t^2 + m^2 L \{+12c_w^2 (m^2 - s) y_t^2 e^2 \\
+ 9c_w^4 (m^2 - s) y_t^2 + 4s e^4 \} + s \beta \{4s e^4 - 9c_w^4 m^2 y_t^4 \} \right) .
$$

(B.155)
\[ \sigma(H_1 f_{R1} \rightarrow fG) = \sigma(G_1 f_{R1} \rightarrow fH) , \]
\[ = \sigma(G_1^+ f_{R1} \rightarrow fG^+) , \]
\[ = \sigma(G_1^- f_{R1} \rightarrow fG^-) , \]
\[ \sigma(H_1 t_{R1} \rightarrow gt) = \sigma(H_1 t_{L1} \rightarrow gt) , \]
\[ = \sigma(G_1 t_{R1} \rightarrow gt) , \]
\[ = \sigma(G_1^- t_{R1} \rightarrow gb) , \]
\[ = \frac{1}{2} \sigma(G_1^+ b_{L1} \rightarrow gt) . \]

For the cross-sections between one KK Higgs boson and one KK \textit{SU}(2)_W-doublt fermion, we get

\[
\sigma(H_1 f_{L1} \rightarrow Gf) = \frac{e^2(T_f^3 c_w g_2 - 2g_1 s_w Y_f)^2}{128\pi m^2 s_w^2 c_w^2 s^2 \beta^2} (m^2 L + s\beta) , \tag{B.159}
\]
\[
\sigma(H_1 f_1^+ \rightarrow f^- G^+) = \frac{g_2^2}{64\pi m^2 \beta} (m^2 L + s\beta) , \tag{B.160}
\]
\[
\sigma(G_1^+ t_{L1} \rightarrow tG^+) = \frac{e^2(c_w g_2 - 2g_1 s_w Y_f)^2}{128\pi m^2 s_w^2 c_w^2 s^2 \beta^2} (m^2 L + s\beta) + \frac{y_t^4}{32\pi s^2 \beta^2} (m^2 L + s\beta) , \tag{B.161}
\]
\[
\sigma(G_1^+ t_{L1} \rightarrow tW^+) = -\frac{g_2 y_t^2 L}{32\pi s \beta^2} , \tag{B.162}
\]
\[
\sigma(G_1^- b_{L1} \rightarrow bG^-) = \frac{e^2(c_w g_2 - 2g_1 s_w Y_f)^2}{128\pi m^2 s_w^2 c_w^2 s^2 \beta^2} (m^2 L + s\beta)
+ \frac{1}{32\pi c_w^2 s_w^2 s^2 \beta^2} \left( y_t^2 L \{ s e^2 (c_w^2 - 2s_w^2 Y_b) 
+ c_w^2 s_w^2 (s - m^2) y_t^2 - s_w^2 c_w^2 s^2 \beta^2 y_t^2 \} \right) . \tag{B.163}
\]
where $T_f^3$ denotes the fermion isospin.

$$\sigma(H_1 b_{L1} \rightarrow G f) = \frac{1}{2} \sigma(G^+_1 t_{L1} \rightarrow t W^+) ,$$

$$\sigma(H_1 f_{L1} \rightarrow G f) = \sigma(G_1 f_{L1} \rightarrow H f) ,$$

$$= \sigma(G^+_1 f_{L1} \rightarrow G^+ f) ,$$

$$\sigma(H_1 f^+_1 \rightarrow f^- G^+) = \sigma(H_1 f^-_1 \rightarrow f^+ G^-) ,$$

$$= \sigma(G_1 f^+_1 \rightarrow f^- G^+) ,$$

$$= \sigma(G^+_1 f^-_1 \rightarrow f^+ G^-) ,$$

$$= \sigma(G^+_1 f^-_1 \rightarrow f^+ G^-) ,$$

$$= \sigma(G^-_1 f^+_1 \rightarrow f^- G) ,$$

where $f$ stands for any lepton or quark, except $t_{L1}$ and $b_{L1}$, and $f^+ (f^-)$ denotes isospin $+1/2$ (isospin $-1/2$) fermions.
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BIOGRAPHICAL SKETCH

Born in 1974 in Pusan, South Korea, I received a B.S. degree in 1997 and an M.S. degree in 1999 from Pusan National University. I received a Ph.D. from the University of Florida in 2006.

I have received Korean Graduate Student Research Award (supported by University of Florida alumni and the New York Times) in 2005, Outstanding International Student Award in 2005, Presidential Recognition in 2004 in recognition of outstanding achievements and contribution to the University of Florida, Physics Department Teaching Assistant of the Year in 2001 and 5 certificates of achievement for outstanding academic accomplishments every year from 2001 to 2006.

I started working with Prof. Konstantin Matchev in 2002 and am interested in particle and astroparticle phenomenology, and especially in signatures of new physics beyond the Standard Model. My research so far has concentrated on the following areas: collider phenomenology of new physics at the LHC and ILC, phenomenology of dark matter and development of event generators and implementation of new models.