THE DYNAMICS OF NOMINAL EXCHANGE RATES

BY

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This dissertation is dedicated to my wife,

Wai-han Yu,

whose love and encouragement are my companions, always.

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This study describes the dynamic nature of nominal exchange rates, with special emphasis on their random walk structure. Although a lot of research favors the random walk model, economists are quite reluctant to accept it. To provide support for the random walk proposition, we apply unit root tests to four different currencies. Results of these tests confirm the existence of unit roots in the spot exchange rate, its expected value as well as the forward exchange rate. Our results show that we cannot reject the non-stationarity property of all these variables in the foreign exchange market.

Rational expectations and market efficiency hypotheses are also tested for the exchange rate market. Since the variables are shown to be non-stationary, the theory of co-

integration is introduced. This theory provides a different way to interpret the two hypotheses. In brief, it outlines a long run relationship between the variables being studied. By restricting the co-integrating factor to be unity, co-integration tests are applied to these hypotheses. The results show that the rational expectation hypothesis is accepted while the market efficiency hypothesis is rejected. Failure of the latter hypothesis is induced by the existence of risk premium.

An information matrix (IM) statistic is introduced to test for rational expectations. The test is applied to a subset of the parameter space to bypass problems raised by non-stationarity. The procedure is theoretically valid, though the Monte Carlo experiments are not encouraging. However, the results lead to important conclusions.

The IM test is a very general specification test. It is a portmanteau test for almost every sort of misspecification. As such it will have less power than tests designed for specific departures from the null hypothesis. Our results indicate that in the case of testing for unit roots, even specific tests like the Dickey-Fuller test have very low power. Thus it is not surprising that the IM test has still less power. Our results suggest that in testing for unit roots, general specification tests are not useful.

CHAPTER I
INTRODUCTION

General Background

Modelling with the incorporation of expectations has a long record in economic theory, both in the micro and macro economic areas. Early models of expectations, in general, postulated a 'model' outside the economic theory. The common practice in such models, because it simplified the analysis, was to use extrapolative or adaptive expectations. The former proposed the use of the first lag of the variable as the future prediction, while the latter was based on the idea of 'error correction through learning'. Unfortunately, both of them were built on ad hoc assumptions, casting serious doubts on their validity in practice.

The concept of rational expectations, first introduced by Muth (1961) suggested that theories of expectation formation should be consistent with the economic model being considered. Loosely speaking, an expectation is considered to be rational if the agent, in forming it, makes use of all the available information. This implies, in particular, that the forecast error cannot be reduced any further.

Surprisingly, a number of papers have found that the Muthian rational expectations hypothesis breaks down under empirical testing. Indeed, more evidence has emerged recently with the availability of survey data. Unlike the forecasts from the expectations models, survey forecasts are generated directly by the market participants. Evidence that the survey data reject the Muthian rationality hypothesis has been presented by Pesando (1975) and Mullineaux (1978) for the Livingston price expectations data, Frankel and Froot (1986, 1987) and Hsieh (1984) for the foreign exchange expectations data, Friedman (1980) and Froot (1989) for the interest rate expectations data, and so on. They all share the conclusion that survey expectations are irrational. In other words, these researchers find that the survey data fail the rational expectations test designed according to their interpretation of the Muthian hypothesis. These rational expectations tests, classified under the title 'weak', 'strong' and 'variance' test, will be discussed in detail in the next Chapter.

However, some of these 'proofs of irrationality' evidence is not strong enough to reject the hypothesis of rationality. This is true, in particular, when the variable under study is not stationary[1]. The idea that most economic

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[1]In the present and future context, non-stationarity and random walk process are considered equivalent. This is somewhat restrictive. However, it will simplify our work without affecting the main results.

variables are non-stationary came from the work of Nelson and Plosser (1982) who pointed out that unit roots are common in economic time series data. Dickey and Fuller (1981) as well as Nelson and Kang (1981, 1984) showed that conventional test statistics were inadequate in the presence of unit roots. Mankiw and Shapiro (1986) suggested that in the presence of unit roots, there was an over-rejection of the rational expectations hypothesis. All this work demonstrates that traditional rational expectations tests may not be applicable without checking in advance the dynamics of the variables and so the conclusions drawn are questionable. In fact, it can be shown that much of the evidence has to be re-interpreted if the non-stationarity of the variables is taken into account.

Purpose of the Study

Because of the inconclusive nature of the debate and the failure to pay attention to non-stationarity, the present dissertation examines the rational expectations hypothesis in detail, with special emphasis on the dynamics of the variables concerned. As it involves non-stationary series, the theory of co-integration will also be included. This concept is important because it describes the relationship between two non-stationary series, providing insight into the structure of the rational expectations hypothesis. It will be shown that the rational expectations

hypothesis is equivalent to requiring that the variable and its expectations be co-integrated with an integrating parameter one. In addition, the residuals from these two variables must be a white noise process. This means that tests for the existence of co-integration between variables need to be applied. Engle and Granger (1987) suggested seven test statistics for this purpose. They are: the Durbin and Watson (DW) test proposed by Sargan and Bhargava (1983); the Dickey and Fuller (DF) test and Augmented Dickey and Fuller (ADF) test suggested by Dickey and Fuller (1979, 1981); and the restricted and unrestricted vector autoregression (RVAR and UVAR) tests and their augmented counterparts. Most of these tests will be employed in this dissertation to check the validity of the rational expectations hypothesis. However, they will be applied only after suitable modification, for it is not co-integration but the hypothesis of rational expectations that we are interested in testing. In later chapters, these modified tests will be called restricted co-integrated tests. The theory will be applied to the foreign exchange market. This data set is chosen not only because of its popularity, so a lot of research is available for comparison, but also because of the easy availability of the data. Details about the data used in the analysis will be discussed in the next section.

In addition to all the tests mentioned above, a different statistic based on the Fisher information matrix will be introduced. The statistic was first developed by White (1982) to test for model misspecification. By applying the test to a subset of the parameter space, using the Lagrangian Multiplier principle, we find that the statistic can be used to test the rational expectations hypothesis. Moreover, it also avoids the problem associated with the non-stationarity of variables. However, the information matrix statistic will only converge to a chi square distribution asymptotically. This implies that the statistic works only when there is a sufficiently large number of observations. A series of Monte Carlo experiments with one hundred observations each will be employed to check the power of this statistic. Other test statistics, such as the DW, DF and ADF, will also be included in the experiments to serve as a reference. As it is believed that the nominal exchange rate is not stationary, the experiments concentrate on the ability to distinguish between a non-stationary series and a stationary one. This means that a random walk process will be taken as the null hypothesis of the experiments, while the alternatives are first order autoregressive series with parameters close to one.

Besides the rational expectations hypothesis, the above tests can also be applied to test the market efficiency hypothesis in the foreign exchange market. Most of the past

research concludes that the foreign exchange market is inefficient and the forward exchange rate is consequently not a good predictor of the future spot rate. Unfortunately, the exact cause of failure for the hypothesis has not yet been determined. Possible explanations include the violation of rational expectations or risk aversion of the agents. In most cases, the failure is attributed to the existence of a risk premium, rather than the irrationality of expectations. For example, both Hodrick and Srivastava (1986) and Park (1984) reach this conclusion. Others, however, take the opposite view: Frankel and Froot (1987) and Froot and Frankel (1989) are examples. Since the issue of rational expectations is addressed, it is natural to extend the work to include the market efficiency hypothesis. In fact, one major reason why there is no definite answer to the existence of an inefficient market is that the debate on the rational expectations hypothesis has never been settled. Basically, the two hypotheses are very closely related and it is difficult to separate them. For this reason, the present dissertation will also study the efficiency of the foreign exchange market. Once again, the restricted co-integration tests and the information matrix statistic will be employed. If nominal exchange rates are non-stationary, market efficiency requires the forward rate to be co-integrated with the spot rate with a factor one, and that

the residuals of the two variables be a white noise process. Details of this will be elaborated in later chapters.

To summarize, the purpose of the present study is to understand the dynamics of the nominal exchange rate, as well as to study two important hypotheses in the foreign exchange market. They are important for a proper understanding of foreign exchange market behavior, especially for the participants in the foreign exchange market.

Data Description

Three different kinds of data will be employed in the present study: (i) the survey data on future exchange rates, which are a proxy for the market expectations (ii) the actual exchange rates, both in spot and forward markets, and, (iii) the data used in the Monte Carlo experiments.

The Survey Data

The data are supplied by the Money Market Services (hereafter MMS) who collect survey data on four different currencies: the British pound (£), Deutsche mark (DM), Swiss franc (SF), and Japanese yen (¥), all denominated in US dollars per unit of the respective currency. The survey provides on a weekly basis one-week and one-month (30-day) ahead expectations of the value in dollars of these currencies. Participants of the survey include exchange rate dealers, banking and corporate economists, as well as

market economists. The data period is October 24, 1984 to May 19, 1989. We use the data to test the rationality of expectations and the efficiency of the exchange market. Because similar research has been conducted using monthly data, our emphasis will focus on the weekly survey. This provides us with the opportunity to examine the property of exchange rates in the very short run.

The Spot and Forward Exchange Rates

Spot and forward rates for each currency are needed when either the rational expectations or the efficient market hypothesis is tested. Both these rates can be obtained from the Wall Street Journal. Current spot and one-week from today exchange rates are used. These correspond to the date of the survey within the time range mentioned. However, the forward rate on a weekly basis is not available in the market. To obtain an equivalent series of the one-week forward exchange rate, as suggested by Hsieh (1984), the idea of covered interest parity will be used. The formula also uses the seven-day interest rates for the currencies concerned, which are published in the Financial Times. We will elaborate upon this procedure in a later chapter.

The Data for Monte Carlo Experiments

These data are constructed by the random generator in the Statistical Analysis System (SAS). The data are drawn from a standard normal distribution. Without loss of

generality, the variance of the generated series is assumed to be one. Based on these random series, several 'close to random walk' auto-regressive series with parameters equal to 0.99, 0.95 and 0.90 are created. In order to maintain the same random property for the initial value, the data series will be created as follows: First, by setting the initial value equal to zero, a series of one hundred and fifty observations is created. Second, after defining all the lagged variables, the first fifty observations will be deleted. This procedure has two advantages. First, problems associated with the randomness of the initial value are avoided. Second, there will be no loss of degrees of freedom when the lagged variables are created. The Dickey and Fuller (DF) test, Augmented Dickey and Fuller (ADF) test, Durbin and Watson (DW) test, Box and Pierce (Q) test, and the White Information Matrix (WIM) test will then be applied to this final series. All these statistics will be analyzed in full detail prior to the results of the experiments.

Layout of the Dissertation

Several important issues will be discussed in the next chapter. First, traditional tests for the rational expectations hypothesis will be presented to provide a clear picture of the development of this topic. Second, reasons for their inapplicability if the variable follows a random

walk process will be explored. A description of the theory of co-integration will follow immediately, acting as a background material for the set of rational expectations tests. These tests are based on the co-integration tests suggested by Engle and Granger (1987). Finally, the concept of co-integration, after a suitable modification, will be applied to test for the rational expectations hypothesis. The information matrix test will be discussed in Chapter III. The chapter discusses the basic theory associated with the White test, the procedure for calculating the statistic, the Monte Carlo experiments and its results. A review of the literature about the dynamics of exchange rates and the rational expectations hypothesis will start Chapter IV. As we claim that the exchange rates are non-stationary, evidence to support such a proposition is necessary before we proceed. For this reason, unit root tests will be applied to the exchange rate series. The empirical results of the tests will be listed. The market efficiency hypothesis and the debate surrounding it will be presented in Chapter V. In the same chapter we shall summarize results of previous studies of the same issues and describe the results of the restricted co-integration tests. The final chapter concludes the thesis.

CHAPTER II
TESTS FOR RATIONAL EXPECTATIONS: THEORY

Introduction

The Muthian rational expectations hypothesis states that the agent should make use of all available information in forming expectations. Since there is no specific test method associated with the hypothesis, researchers have tried to test its validity from various aspects, according to their interpretations of the hypothesis. In the next section, the important tests of rationality will be discussed. In case the variable under study is stationary, these tests will have the correct statistical properties. Unfortunately, most of the previous research did not take account of the dynamics of variables, raising serious doubts on their conclusion. It has been shown by Phillips (1986, 1987) that when the variable under study is non-stationary, the estimators and statistics generated by OLS do not have the distributions usually assumed. As most of the economic time series data are believed to have a unit root, the reliability of these traditional rationality tests becomes questionable. Reasons why these tests fail if the variable is non-stationary will also be discussed in the next

section. A set of tests of the rational expectations hypothesis, applicable to non-stationary series, will also be discussed.

In order to handle the dynamics of variables properly, when testing the rational expectations hypothesis, the theory of co-integration will be discussed. The theory is used to investigate the long run relationship among non-stationary variables. In the present context, these refer to the relationship between the spot and the forward rate or the spot rate and its expected value. In order to understand how co-integration theory fits into our context, a special section will be devoted to explaining this concept. This section will include the co-integration tests suggested by Engle and Granger (1987). Finally we discuss the modifications needed when the concept of co-integration is applied to the test for the rational expectations hypothesis. The full application will appear in Chapter IV.

Traditional Rational Expectations Tests

A general interpretation of 'exhausting all available information in forming expectations' is that there should be no systematic pattern in the forecast error. The forecast error defined as the difference between the variable y_t and its expected values y_t^e. More precisely, if the expectation is rational, the forecast error should be a white noise

process. Based on this, several tests for rational expectations were developed.

Weak and Strong Rational Expectations Tests

It is customary to start with a test of unbiasedness by estimating the regression equation,

$$y_t = \beta_0 + \beta_1 y_t^e + \epsilon_t$$

and testing the hypotheses $\beta_0 = 0$ and $\beta_1 = 1$, where y_t and y_t^e is defined as above. Frequently, this unbiasedness test is taken as a classic representation of the rational expectations test. Because the rational expectations hypothesis states that the variable should equal the sum of its expected value and a random white noise error. The most common test is to regress the forecast error on all the variables in the information set I_{t-1}. Rationality ensures that the parameters of this regression will be zero. The only problem is that the hypothesis rarely specifies what variables should be included in the information set. As there is no universal rule governing the choice of the variables, researchers must make their own choice, according to their interpretation of the theory or the availability of data. Clearly, y_{t-1} is in the information set I_{t-1}. Hence, the following equation is often estimated:

$$y_t - y_t^e = \alpha_0 + \alpha_1 y_{t-1} + \epsilon_t$$

and the hypotheses $\alpha_0 = 0$ and $\alpha_1 = 0$ are tested. If the null hypothesis is rejected, so is the rational expectations hypothesis. Since y^e_{t-1} is also in I_{t-1}, some tests are based on the equation:

$$y_t - y^e_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y^e_{t-1} + \epsilon_t$$

or,

$$y_t - y^e_t = \beta_0 + \beta_1 (y_{t-1} - y^e_{t-1}) + \epsilon_t$$

Rationality implies $\alpha_0 = 0$, $\alpha_1 = 0$ and $\alpha_2 = 0$ in the first equation, or $\beta_0 = 0$ and $\beta_1 = 0$ in the second. The former is an unrestricted test while the latter restricts the coefficients of y_{t-1} and y^e_{t-1} to be the same. In addition to these parameter tests, some tests focus on the behavior of the estimated error terms. Rational expectations implies that the error terms be serially un-correlated. Thus, when the forecast error exhibits a significant serial correlation, indicating that the information contained in previous forecast errors or related variables has not been fully utilized in performing future predictions, the rational expectations hypothesis is rejected.

Tests based on y_{t-1} or $(y_{t-1} - y^e_{t-1})$ or any combination of these variables are called 'weak' tests of the rational expectations hypothesis. The 'strong' tests, on the other

hand, require the forecast error to be un-correlated with any variable in the information set I_{t-1}. This includes any available economic variables other than y_t or y_t^e.

Variance Bounds Tests for Rationality

The main idea of Muthian rational expectations can be embodied in the following equation

$$y_t = y_t^e + \epsilon_t$$

where ϵ_t is the white noise error and is un-correlated with y_t^e. This implies that the covariance between the variables, $Cov(y_t^e, \epsilon_t)$, will be zero. For this reason,

$$Var(y_t) = Var(y_t^e) + Var(\epsilon_t)$$

and hence,

$$Var(y_t) > Var(y_t^e).$$

This is the basic idea of the variance test for rationality. If the reverse relationship is observed, the rational expectations hypothesis is rejected.

Another way to test for the rational expectations, as suggested by Lovell (1986), is to incorporate both the weak rationality test and the variance condition together. This

joint test will reject the rational expectations hypothesis, if either one of the two conditions fails.

Finally, a widely used procedure developed by Pesando (1975), Carlson (1977), Mullineaux (1978) and Friedman (1980) involves three steps:

1. Regress y_t on the variables in the information set I_{t-1}.
2. Regress y_t^e on the same set of variables in I_{t-1}.
3. Test the equality of the coefficients in the two regressions using Chow tests.

As long as the variables in I_{t-1} are not un-correlated with y_t and y_t^e, these procedures can be used to test whether the expectations are rational. Any significant difference in the coefficients in the two equations can be interpreted as evidence of irrationality.

Weaknesses Associated with These Rationality Tests

All of these tests mentioned are valid when the variables under study are stationary. However, available evidence shows that most of the economic time series data are not stationary, making such tests possible invalid if the usual critical values are used. Fortunately, in most of the cases, it is the critical region rather than the test procedure itself that makes the difference. This reduces the difficulties involved in revising the test. It has been

shown that when variables are non-stationary, the estimators of coefficients from the OLS regression do not follow the usual t- distribution. Thus the critical region is no longer around ±2.00 at 5% significance level. Instead, according to Dickey and Fuller (1979, 1981), the critical region should have an absolute value close to three. The exact critical value depends on the degrees of freedom and can be obtained from the Dickey and Fuller tables which are designed for regressions involving non-stationary series.

To be more specific, the following example can be used to clarify the problem. When the forecast error $(y_t - y_t^e)$ is regressed on the variable y_{t-1}, the assumption of a stationary white noise regression error becomes questionable if y_{t-1} is not stationary. Assume, for the moment, that the rational expectations hypothesis holds so that the forecast error is stationary. The regression error ϵ_t in the equation

$$y_t - y_t^e = \alpha_0 + \alpha_1 y_{t-1} + \epsilon_t$$

cannot be stationary if y_{t-1} is not. Because it is not meaningful to have a stationary series on the left-hand side of an equality sign but a non-stationary series on the right. This means that ϵ_t will violate the usual Gaussian assumption and the Student t- test for $\alpha_1 = 0$ is no longer

applicable. As mentioned above, the hypothesis has to be tested by using the Dickey and Fuller test.

Similar problems arise in the variance rationality tests. When both y_t and y_t^e are random walk process, their unconditional variances will tend to infinity as the number of observations becomes sufficiently large. In that case, it would be dangerous to draw any inference by comparing the two variances. A rejection of the hypothesis based on the fact that the variance of y_t is smaller than that of y_t^e is not convincing. The problem does not disappear even if the sample size is small. For example, assume that y_t and y_t^e have the following dynamic structures

$$y_t = y_{t-1} + \epsilon_t \quad \text{and} \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$
$$y_t^e = y_{t-1}^e + \eta_t \quad \text{and} \quad \eta_t \sim N(0, \sigma_\eta^2)$$

Then the variances of y_t and y_t^e, given that both y_0 and y_0^e equal zero, are $t\sigma_\epsilon^2$ and $t\sigma_\eta^2$ respectively. It is obvious that even if the latter variable has a larger variance it does not imply a rejection of the rational expectations hypothesis.

As a whole, the usefulness of the rational expectations tests mentioned in last section are doubtful when the variable under study is non-stationary. In order to analyze the problem suitably, the theory of co-integration, proposed by Engle and Granger (1987), will be introduced. To have a

better understanding of this theory, the following section will be devoted to explaining it.

Summary of the Theory of Co-integration

If x_t is a vector of economic variables with no deterministic component and has a stationary, invertible, ARMA representation after differencing d times, then x_t is said to be integrated of order d, denoted by $x_t \sim I(d)$. An important characteristic of such series is that if a and b are any constants with $b \neq 0$, then $a + bx_t$ is also an I(d) series.

This property forms the core of co-integration theory. If w_t and y_t are both I(d) processes, it is generally true that the linear combination of the two, say z_t, where

$$z_t = w_t - cy_t$$

will also be an I(d) process. However, it is also possible that z_t has a lower order, that is

$$z_t \sim I(d - k)$$

with $k > 0$ and hence $d - k < d$. This relationship reveals that there exists a constraint operating on the long-run components of the two series, w_t and y_t. A prominent example which is very useful in our analysis is when

$d = k = 1$. This happens when both w_t and y_t are I(1) series, in particular a random walk process, but a linear combination of the two variables z_t, is an I(0) stationary series.

In general, if both w_t and y_t are components of the vector x_t, then they are said to be 'co-integrated of order' (d, k), denoted by $x_t \sim CI(d, k)$, provided that both of the following conditions are satisfied. These are

1. all components of x_t are I(d);
2. there exists a vector $\alpha (\neq 0)$ so that $z_t = \alpha' x_t$ and $z_t \sim I(d - k)$. The vector α is called the co-integrating vector.

This is the fundamental idea of co-integration theory. By using it, tests for the rational expectations hypothesis when the variables are non-stationary can be formulated. Before deriving these tests, we will discuss the tests for co-integration proposed by Engle and Granger (1987).

Tests for Co-integration

Engle and Granger (1987) proposed seven different tests for co-integration between two I(1) series, say, w_t and y_t. The idea of these tests is to check, whether, after regressing w_t on y_t, any unit root exists in the estimated residual. Since w_t and y_t are I(1) series, they are not co-

integrated if the estimated residual series has a unit root. On the other hand, if the estimated residual is stationary, the null hypothesis of no co-integration would be rejected. In that case, w_t and y_t are co-integrated and the estimated coefficient will be the co-integrating vector.

Mathematically, a regression of the following form is estimated by the OLS estimation method,

$$w_t = \alpha_0 + \alpha_1 y_t + \mu_t$$

and the OLS residual $\hat{\mu}_t$ is obtained. All the tests for co-integration suggested by Engle and Granger (1987) are based on this estimated residual. These are as follows:

1. <u>Durbin Watson (DW) Test</u>

   The DW statistic is calculated based on the following equations:

$$\hat{\nu}_t = \hat{\mu}_t - \hat{\mu}_{t-1} ;$$

$$DW = \Sigma\hat{\nu}^2_{t-1} / \Sigma(\hat{\mu}_t - \hat{\mu}_m)^2 ,$$

   where

$$\hat{\mu}_m = \Sigma\hat{\mu}_t / T$$

   The statistic was first proposed by Sargan and Bhargava in 1983 to test for a unit root in the residual $\hat{\mu}_t$. If there is any unit root in the residual, the DW statistic should be close to zero, as the sum of squares of the difference between residuals should be small compared to the variance of the residual. Hence,

a large DW means a stationary residual and co-integration between w_t and y_t.

2. Dickey and Fuller (DF) Test

The DF statistic is obtained from the regression

$$\hat{\nu}_t = \alpha + \beta \hat{\mu}_{t-1} + \xi_t$$

and its value is equivalent to the Student statistic for testing the hypothesis of $\beta = 0$. The critical value for this statistic, however, is obtained from the Enger and Granger's table instead of the Student t-table or the Dickey and Fuller table.

3. Augmented Dickey and Fuller (ADF) Test

The process which obtains the ADF statistic is similar to the one that provides the DF statistic. One runs the regression,

$$\hat{\nu}_t = \alpha + \beta \hat{\mu}_{t-1} + \Sigma \delta_i \hat{\nu}_{t-i} + \xi_t$$

where i runs from 1 to any number p. In our context, p equals four. Similar to the DF statistic, the value of ADF statistics is equal to the Student t- statistic for testing $\beta = 0$. Once again the critical value for this test is not obtained from a t- table. Instead, it comes from a table developed by Engle and Granger.

4. Restricted VAR (RVAR) Test

This and the following tests use a different strategy. The test itself is similar to a two step estimator. It is based on the following two regressions:

$$w_t - w_{t-1} = \rho \hat{\mu}_t + \xi_{1t} \text{ ; and}$$

$$y_t - y_{t-1} = \lambda \hat{\mu}_t + \gamma(w_t - w_{t-1}) + \xi_{2t}$$

The RVAR test requires specification of the full system dynamics. For simplicity, and to be consistent with Engle and Granger (1987), a first order autoregression dynamics is assumed. The test statistic is the sum of squares of the Student t- statistics from testing $\rho = 0$ and $\gamma = 0$ in the two equations.

5. Augmented RVAR (ARVAR) Test

The ARVAR test is the same as RVAR except a higher order system of lagged $(w_t - w_{t-1})$ and $(y_t - y_{t-1})$ is postulated in the equations where the statistics are obtained. Again, the statistic is the sum of squares of the two Student t- statistics for testing $\rho = 0$ and $\gamma = 0$ in the two equations.

6. Unrestricted VAR (UVAR) Test

This test is based on vector autoregressions in the levels which are not restricted to satisfying the co-integration constraints. Two regressions are estimated:

$$w_t - w_{t-1} = \alpha_1 + \beta_1 w_{t-1} + \tau_1 y_{t-1} + \xi_{1t} \text{ ; and}$$

$$y_t - y_{t-1} = \alpha_2 + \beta_2 w_{t-1} + \tau_2 y_{t-1} + \delta(w_t - w_{t-1}) + \xi_{2t}$$

The statistic is equal to $2(F_1 + F_2)$, where F_1 is the F-statistic for testing both β_1 and τ_1 equal to zero in

the first equation, while F_2 is its counterpart in the second equation. Again, this test assumes a first order system.

7. Augmented UVAR (AUVAR) Test

This is an augmented or higher order version of the UVAR test. That is, higher order lagged values of $(w_t - w_{t-1})$ and $(y_t - y_{t-1})$ are included in the two equations in the UVAR test. The statistic is obtained by the same procedure as the UVAR test.

Relationship Between Co-integration and Rational Expectations

Co-integration theory proposes a different way to interpret the rational expectations hypothesis when the variables under study are I(1) processes. If both y_t and y_t^e are non-stationary, then the rational expectations hypothesis, which postulates the following relationship,

$$y_t = y_t^e + \epsilon_t$$

or,

$$y_t - y_t^e = \epsilon_t$$

is equivalent to requiring y_t and y_t^e to be co-integrated with a factor of unity, that is $\alpha = 1$. In addition, the

error term, ϵ_t, has to be a white noise process. This means that rational expectations is a stronger requirement because co-integration requires ϵ_t to be stationary, but rationality goes one step further by requiring ϵ_t to be a white noise error.

However, the tests of co-integration from Engle and Granger (1987) cannot be applied directly to test for the rational expectations hypothesis as the latter hypothesis requires more than co-integration. Engle and Granger (1987) require only a stationary estimated residual, but not any restriction on the co-integrating factor or the randomness of the estimated error term. It can happen that y_t and y_t^e are co-integrated with a factor, say, 0.5 and that the error term follows an ARMA stationary process. Obviously, the y_t^e in this situation is not a rational expectation. To be more specific, co-integration is a necessary, but not a sufficient, condition for rational expectations when y_t and y_t^e follow random walks. Taken as a whole, an expectation is said to be rational, if the three following conditions have satisfied:

1. y_t and y_t^e must be co-integrated;
2. The co-integrating factor must be one;
3. The difference, $y_t - y_t^e$, must be a white noise process with no serial correlation.

In order to adjust the co-integration tests to fit the above requirements, a slight modification can be used to incorporate the first and second conditions above automatically. Instead of using the estimated residual, $\hat{\mu}_t$, a restricted residual μ_t, defined as the difference $y_t - y_t^e$, could be used. This will automatically restrict the co-integration factor to one. We call these restricted co-integrated tests. Hence, if μ_t is stationary, y_t and y_t^e are co-integrated with a factor one, provided that both of them are I(1) non-stationary series. The Box and Pierce (Q) test will be applied to μ_t to check for the existence of systematic patterns in the residuals. Any serial correlation that exists in μ_t will be captured by the Q statistics. If all these tests provide positive results, that is, y_t and y_t^e are co-integrated with a factor one, and μ_t is stationary and contains no systematic pattern, one can conclude that the expectations are rational.

The ideas outlined in this section will be applied to the foreign exchange market and these results will be presented in Chapter IV. Before this, however, another statistic for testing the rational expectations hypothesis will be introduced. The test is built on the Fisher information matrix. Its derivation, as well as the computations, are discussed in the next chapter.

CHAPTER III
THE INFORMATION MATRIX TEST FOR RATIONAL EXPECTATIONS

Introduction

Since the method of maximum likelihood was developed in the 1920s, it has become one of statistically most widely used tools for estimation and inference. A fundamental assumption of this method is that the stochastic law which governs the true phenomenon is known to the researcher. This means that the model proposed by the researcher must be correctly specified.

However, there is no reason to assume that the researcher has full knowledge about the data generating process. What will happen if the model is not correctly specified? Will there be any change in the properties of the likelihood function, the estimators, and their distributions? These questions were discussed by White (1982), who suggested a method to test for the misspecification of a model. The test is based on a well known property of the likelihood function, namely, that if the model is formulated correctly, the expected sum of the second derivative of the natural logarithm of a likelihood function (hereafter referred to as log-likelihood function)

and the square of its first derivative will be equal to zero. That is,

$$E_t[F_2\{\Theta\}] \quad + \quad E_t[F_1\{\Theta\}\cdot F_1\{\Theta\}'] \quad = \quad 0$$

where $F_2\{\Theta\}$ is the (pxp) matrix of the second derivatives of the log-likelihood function, that is, $\partial^2 \log f_t\{\Theta\}/\partial\Theta\cdot\partial\Theta'$; $F_1\{\Theta\}$ is the (px1) vector of the first derivatives of the log-likelihood function, denoted by $\partial \log f_t\{\Theta\}/\partial\Theta$; $E_t[\cdot]$ is the expectation operator, generally defined as $\int_\Theta[\cdot]f_t\{\Theta\}d\Theta$; $f_t\{\Theta\}$ is the likelihood function; and Θ is the (px1) vector containing all the p parameters in the system. The major contribution in White (1982) is the derivation of the asymptotic distribution of the components of this sum. Thus, the specification of any model can be tested by means of this distribution.

The next section derives this information matrix test. Some of the principal assumptions that lead to the final results are emphasized. Our major concern is to apply this test to examine the validity of the rational expectations and the market efficiency hypothesis in the exchange rate market. In particular, we want to apply the test when the variable under study is not a stationary series. As shown by Phillips (1986, 1987), whenever there exists a unit root in the series, the distribution of the test statistics can be very complicated and will not converge to any standard

distribution like the normal or χ^2, asymptotically. Phillips (1986) showed that if x_t has a dynamic process of the form

$$x_t = x_{t-1} + \epsilon_t$$

then as the number of observations T tends to infinity, we have the following results

$$T^{-3/2}\Sigma x_t \xrightarrow{a.s.} \sigma_\epsilon \int_0^1 W(t)dt \ ;$$

$$T^{-2}\Sigma x_t^2 \xrightarrow{a.s.} \sigma_\epsilon^2 \int_0^1 [W(t)]^2 dt \ ;$$

$$T^{-1}\Sigma x_{t-1}\epsilon_t \xrightarrow{a.s.} \{\tfrac{1}{2}\sigma_\epsilon^2[W(1)]^2 - \tfrac{1}{2}\Omega_{\epsilon 0}\} \ ;$$

$$T^{-1}\Sigma \epsilon_t^2 \xrightarrow{a.s.} \lim_{T\to\infty}(T^{-1}\Sigma E_t(\epsilon_t^2)) = \Omega_{\epsilon 0} \ ;$$

where σ_ϵ^2 is the variance of ϵ_t, W(t) is a Wiener process on C[0,1] and Σ is the sign of summation summing from 1 to T. Since the ordinary least squares estimators and the statistics are functions of the above sums, the asymptotic distribution of these random variables will not be simple normal distributions. However, it may be shown that if the information matrix test is applied to a subset of the parameter space, such problems can be avoided. The choice of parameters in the subset depends on the values of

parameters which are specified under the null hypothesis. Details of this argument and its application will be presented later in this chapter.

The final section gives the Monte Carlo experiment and its results. The experiment is essential as it compares the power of different statistics. Tests included in the experiment are the DF, ADF, DW, three Box and Pierce (Q) tests and the information matrix test. Since the test for co-integration is similar to the test for a unit root in a time series analysis, the experiment will concentrate on the ability of the tests to distinguish a random walk from a stationary AR(1) process that is close to being a non-stationary process.

The Information Matrix Test

The information matrix test is built on the assumption that if the model is correctly specified, the Fisher information matrix can be expressed either in the Hessian form, that is, $-E_t[F_2\{\theta\}]$, or in the outer product form, $E_t[F_1\{\theta\}\cdot F_1\{\theta\}']$. In other words, it is built on the fact that $E_t[F_2\{\theta\}] + E_t[F_1\{\theta\}\cdot F_1\{\theta\}'] = 0$ when the model's specification is correct. If this equality fails to hold, it follows that the model is not specified correctly. To derive the required statistics to test the hypothesis that this equality holds, let's assume that the log-likelihood function behaves properly, that is, the function is

continuous over the entire parameter space and differentiable up to, say, the third order. In addition, the existence of the first and second derivatives of the log-likelihood function, the non-singularity of Hessian and outer product matrix are also needed so that the statistics will converge, asymptotically, to a proper distribution. At this moment, no dynamics for the variables are specified and to simplify the derivation, all variables are assumed to be stationary.

For a p- parameter likelihood function, $f_t(\Theta)$, there are p^2 elements in the Hessian and the outer product matrix. Among them, only $\{p(p+1)\}/2$ are different. To compare these $\{p(p+1)\}/2$ distinct elements in the two matrices, White (1982) defined a statistics, d_{kt}, which was equal to the sum of the corresponding elements in the two matrices. Mathematically,

$$d_{kt}\{\Theta\} = \partial \log f_t\{\Theta\}/\partial\theta_i \cdot \partial \log f_t\{\Theta\}/\partial\theta_j + \partial^2 \log f_t\{\Theta\}/\partial\theta_i \partial\theta_j$$

where

$$k = 1, \ldots\ldots, p(p+1)/2 \;;$$

$$i,j = 1, \ldots\ldots, p \;;$$

$$t = 1, \ldots\ldots, T \;;$$

To simplify the notation, let $q = \{p(p+1)\}/2$. By means of the matrix notation, $d_{kt}\{\Theta\}$ $(k = 1,\ldots,q)$, for a particular observation t, can be grouped into a (qx1) vector such that

$$d_t\{\Theta\} = (F_{2t}\{\Theta\} + F_{1t}\{\Theta\}\cdot F_{1t}\{\Theta\}')^c$$

where c stands for the stacking of a (pxp) matrix. Notice that $d_t\{\Theta\}$ is only a (qx1) vector because only the distinct elements in the two matrices will be considered.

Assume the mean and partial derivatives of $d_t\{\Theta\}$ exist and define the mean as $D_T\{\Theta\}$ such that

$$D_T\{\Theta\} = E_t[d_t\{\Theta\}] \quad ;$$

and the (qxp) Jacobian matrix of $d_t\{\Theta\}$ as $\nabla D_T\{\Theta\}$ which is equal to

$$\nabla D_T\{\Theta\} = E_t[\{\partial d_t\{\Theta\}/\partial\theta_k\}] \quad ;$$

White (1982) argued that the random variable $D_T\{\Theta\}$ is normally distributed with zero mean and a constant variance $V\{\Theta\}$. That is,

$$D_T\{\Theta\} \sim N(0, V\{\Theta\}) \quad ;$$

where

$$V\{\Theta\} = E_t\{d_t\{\Theta\} - \nabla D_T\{\Theta\}\cdot\{E_t[F_{2t}\{\Theta\}]\}^{-1}\cdot E_t[F_{1t}\{\Theta\}]\}$$
$$\cdot\{d_t\{\Theta\} - \nabla D_T\{\Theta\}\cdot\{E_t[F_{t2}\{\Theta\}]\}^{-1}\cdot E_t[F_{1t}\{\Theta\}]\}'$$

T is the total numbers of observations. White (1982) proposed that the above test statistic can be calculated for any sample by substituting for Θ its maximum likelihood estimator, $\hat{\Theta}$, and replacing the expectation operator $E_t[.]$ by $1/T\cdot\Sigma[.]$, where the summation sign represents a sum from 1 to T. Under the null hypothesis of a correctly specified model, one can obtain the following sample statistics

1. $\sqrt{T}\cdot D_T\{\hat{\Theta}\} \overset{A}{\sim} N(0, V\{\Theta\})$;
2. $V\{\hat{\Theta}\} \xrightarrow{a.s.} V\{\Theta\}$;
3. $\vartheta_T = T\cdot D_T\{\hat{\Theta}\}'\cdot V^{-1}\{\hat{\Theta}\}\cdot D_T\{\hat{\Theta}\}$;

where ϑ_T is the information matrix test statistic, which follows a χ^2_q distribution asymptotically.

The calculation of the above statistic is tedious, especially when dealing with the variance $V\{\hat{\Theta}\}$ since it involves the third derivatives of the log-likelihood function. To simplify the work, Chesher (1983) and Lancaster (1984) defined a relatively easy formula to calculate ϑ_T. Define a Tx(q+p) matrix $Y\{\hat{\Theta}\}$ such that

$$Y\{\hat{\theta}\} = [d_t\{\hat{\theta}\}' \mid F_{1t}\{\hat{\theta}\}']$$

where t runs from 1 to T, $d_t\{\hat{\theta}\}'$ is a (1xq) row vector containing the sample values of the random variable $d_t\{\theta\}$ for each observation t, and $F_{1t}\{\hat{\theta}\}'$ is a (1xp) row vector containing the sample values of the first derivative of the log-likelihood function for the same observation. Hence $Y\{\hat{\theta}\}$ is a combination of the row vector $[d_t\{\hat{\theta}\}' \mid F_{1t}\{\hat{\theta}\}']$ from 1 to T. Both Chesher (1983) and Lancaster (1984) suggested that the information matrix statistic can be simplified into the form

$$\vartheta_T = \iota' \cdot Y\{\hat{\theta}\} \cdot [Y\{\hat{\theta}\}' \cdot Y\{\hat{\theta}\}]^{-1} \cdot Y\{\hat{\theta}\}' \cdot \iota$$

where ι is a (Tx1) vector such that $\iota'\iota = T$. The whole expression is equivalent to T times the R^2 statistic in the regression of ι on $Y\{\hat{\theta}\}$.

White (1982) emphasizes that, in many cases, it is inappropriate to conduct the test on all p elements in the system. There are several possible reasons for this: some of the elements can be identically zero; some may be a combination of others; and sometimes the degrees of freedom are too large if all the parameters are included. Another justification not mentioned by White (1982) is that in case there exists any non-stationary variable in the analysis, it

is very likely then that the components in $d_t\{\theta\}$ containing that variable are not stationary. It then turns out that $D_T\{\hat{\theta}\}$ will not converge to a normal distribution even when T tends to infinity. If that happens, one has to avoid the problem by considering a subset of the parameters that will converge to an appropriate distribution.

The Information Matrix Test and Rational Expectations

A basic formulation of a rational expectations model is rather simple, that is

$$y_t = \alpha + \beta y_t^e + \epsilon_t$$

with the hypothesis that $\alpha=0$, $\beta=1$ and ϵ_t is a white noise process with mean zero and constant variance σ^2. Although there is no special requirement on the distribution of the error term, it is customary to assume that it follows a normal distribution. This implies that the density function of the variable y_t, for a particular observation t, is

$$f_t(\alpha,\beta,\sigma^2) = 1/\sqrt{(2\pi\sigma^2)}\cdot\exp\{-\tfrac{1}{2}(y_t - \alpha - \beta y_t^e)^2/\sigma^2\}$$

and its log-likelihood function can be written as

$$\log f_t(\alpha,\beta,\sigma^2) = -\tfrac{1}{2}\log(2\pi) - \tfrac{1}{2}\log(\sigma^2) - \tfrac{1}{2}(y_t - \alpha - \beta y_t^e)^2/\sigma^2$$

Since there are three parameters (α, β and σ^2) in the model, there will be six distinct elements in the information matrix. The first order condition for any observation t, denoted by F_{1t}, is equal to

$$\begin{bmatrix} \partial \log f_t(\alpha,\beta,\sigma^2)/\partial\alpha \\ \partial \log f_t(\alpha,\beta,\sigma^2)/\partial\beta \\ \partial \log f_t(\alpha,\beta,\sigma^2)/\partial\sigma^2 \end{bmatrix}$$

and the second order condition for the same observation, denoted as F_{2t}, is

$$\begin{bmatrix} \partial^2 \log f_t/\partial\alpha\cdot\partial\alpha & \partial^2 \log f_t/\partial\alpha\cdot\partial\beta & \partial^2 \log f_t/\partial\alpha\cdot\partial\sigma^2 \\ \partial^2 \log f_t/\partial\beta\cdot\partial\alpha & \partial^2 \log f_t/\partial\beta\cdot\partial\beta & \partial^2 \log f_t/\partial\beta\cdot\partial\sigma^2 \\ \partial^2 \log f_t/\partial\sigma^2\cdot\partial\alpha & \partial^2 \log f_t/\partial\sigma^2\cdot\partial\beta & \partial^2 \log f_t/\partial\sigma^2\cdot\partial\sigma^2 \end{bmatrix}$$

With this information in hand, one can work out the vector $d_t\{\hat{\theta}\}$, the $D_T\{\hat{\theta}\}$ as well as the statistics ϑ_T, by substituting the maximum likelihood estimators of α, β and σ^2. However, when y_t, and most probably y_t^e as well, is non-stationary, elements in the $D_T\{\hat{\theta}\}$ matrix involving them will not converge to normal distributions as T tends to infinity. That is, as shown in previous section if y_t is not stationary, elements in F_{1t} or F_{2t} that contain Σy_t, Σy_t^2 or $\Sigma y_{t-1}\cdot\epsilon_t$ will be functions of the Wiener process after being

suitably scaled, preventing F_{1t} and F_{2t} from converging to the normal or χ^2 distributions. This implies that the statistic ϑ_T will no longer have a χ^2_q distribution.

Nevertheless, the problem can be avoided by considering a subset of the parameter space. Under the null hypothesis of rational expectations, α=0 and β=1. In this case, the whole formulation can be rewritten in a much simpler form

$$\epsilon_t = y_t - y_t^e$$

Even though both y_t and y_t^e are non-stationary, their difference, ϵ_t, has to be stationary if the null hypothesis is true. Under H_0, then, one can rewrite the log-likelihood function in the form

$$\log f_t(\sigma^2) = -\tfrac{1}{2}\log(2\pi) - \tfrac{1}{2}\log(\sigma^2) - \tfrac{1}{2}\epsilon_t^2/\sigma^2$$

Notice that only one parameter, σ^2, appears in the system. Hence the vector $d_t\{\Theta\}$ is reduced to a (1x1) scalar and the whole analysis can be simplified to a large extent.

Two different approaches may be adopted for this simplification. First, as White (1982) shows, it is legitimate to use a subset of parameters to perform the information matrix test. Based on the Lagrangian multiplier principle, one can always use the value of the parameters under the null hypothesis to work out the statistics. If

the null hypothesis describes the true values, this restricted statistic will have the same distribution as the unrestricted sample statistic. Thus, the parameters α and β can be replaced by 0 and 1 respectively, reducing $d_t\{\hat{\theta}\}$ to a (1x1) scaler instead of a (6x1) vector for each observation t. Also, the maximum likelihood estimator of σ^2 is replaced by its restricted counterpart under the Lagrangian multiplier principle, defined as

$$\dot{\sigma}^2 = \Sigma\epsilon_t^2/T$$

This, then, is the reformulation of the information matrix statistic for testing rational expectations when the variable under study is non-stationary. In brief, the idea is to reduce the number of parameters in the system by substituting parameter values under the null hypothesis. By this method, the problem associated with any non-stationary component in the system may be avoided. If the parameter values under the null hypothesis are the true values, the information matrix statistic will follow a χ^2 distribution.

Monte Carlo Experiments on the Information Matrix Test Statistic

In order to check the performance of the information matrix statistic, a Monte Carlo experiment was conducted to compare its effectiveness with other unit root tests. For

the sake of simplicity, only three of the seven tests suggested by Engle and Granger (1987) were included for comparison. They are the Dickey and Fuller (DF) test, Augmented Dickey and Fuller (ADF) test and the Durbin Watson (DW) test. Besides them, Box and Pierce (Q) statistics will be used to check the randomness of the restricted residual[1]. These Q statistics are included because, they can be calculated easily, and they show the properties of the residual. The idea is that if a variable follows a random walk process, the difference, $y_t - y_{t-1}$, will be a white noise process, and the Q statistic calculated will follow a χ^2 distribution.

The experiments worked out the distributions of the statistics under the null hypothesis and AR(1) alternatives with ρ equal to 0.99, 0.95 and 0.90. These alternatives were chosen because the purpose of the experiment was to distinguish between the null of a random walk from a series close to a random walk. The selection of alternatives is somewhat arbitrary. But they are common in most recent research. There are one thousand replications under the null and each of the alternative.

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[1]The simplest form of the Q statistic is defined as $T\Sigma\tau_i^2$ where i equals 1 to k and τ_i^2 is the ith sample autocorrelation in the residuals. If the model is correctly specified as a random walk process, then the Q statistic will have a χ^2 distribution with k degrees of freedom.

The experiments proceeded as follows: Using the random number generator in the SAS program, one thousand series of random numbers, with 150 observations each, denoted by ϵ_{it} (i=1,...,1000, t=1,...,150), were generated from a standard normal distribution. The initial condition y_{i0}, for all 'i', equalled zero. Then a thousand series of y_{it} was generated based on the following equation

$$y_{it} = \rho y_{i(t-1)} + \epsilon_{it} \quad i = 1,\ldots,1000 \text{ and } y_{i0} = 0$$

where ρ took the values of 1, 0.99, 0.95 and 0.90 respectively. When ρ equals one, y_{it} is a random walk process. By repeatedly running the tests on these series, the size of the statistics can be obtained. In some cases, the critical values obtained from the table may not have the correct size and hence some minor adjustment will be needed. For other values of ρ, y_{it} is a simple first order autoregressive AR(1) series. Since a fixed value of zero is assigned to be the initial value, only the last hundred observations were used for the experiment. The first fifty observations will were deleted.

The following tests were conducted: The Dickey and Fuller test (DF), augmented Dickey and Fuller test (ADF), Durbin and Watson test (DW), Box and Pierce test (Q), and the White information matrix test (WIM). When computing the information matrix test statistic, $Y\{\hat{\theta}\}$, $d_t\{\hat{\theta}\}$ and $F_{1t}\{\hat{\theta}\}$

were replaced by $Y\{\dot{\theta}\}$, $d_t\{\dot{\theta}\}$ and $F_{1t}\{\dot{\theta}\}$ respectively. The former were the maximum likelihood estimates of $Y\{\theta\}$, $d_t\{\theta\}$ and $F_{1t}\{\theta\}$, while the latter were their counterparts using the lagrangian multiplier estimators. The component of $d_t\{\theta\}$ and $F_{1t}\{\theta\}$ for a particular observation t, is defined as

$$d_t\{\theta\} = \tfrac{3}{4}\sigma^4 - 1\tfrac{1}{2}\epsilon_t^2/\sigma^6 + \tfrac{1}{4}\epsilon_t^4/\sigma^8 \quad ; \quad \text{and}$$

$$F_{1t}\{\theta\} = -\tfrac{1}{2}\sigma^2 + \tfrac{1}{2}\epsilon_t^2/\sigma^4 \quad ;$$

By substituting $\dot{\sigma}^2$ for σ^2 in these expressions, the value of $d_t\{\dot{\theta}\}$ and $F_{1t}\{\dot{\theta}\}$ can be obtained. Combining them together, the value for $Y_t\{\dot{\theta}\}$ as well as the information matrix statistic ϑ_T can be calculated easily.

Empirical Results of the Experiment

Results of the Monte Carlo experiment are summarized in the following tables. Table 3.1 states the critical values for various tests embodied in the experiment at the 10%, 5% and 1% significance levels. Figures in the table are the critical values obtained from tables associated with the tests. For example, the critical values for the DF test are obtained from the table in Fuller (1976), the critical values for the Q test are obtained from a χ^2 table and so

on. These critical values mean that if a sample statistic has a larger value, there will be 10%, 5% and 1% chance of incorrectly rejecting a correct null hypothesis.

TABLE 3.1

CRITICAL VALUES UNDER NULL HYPOTHESIS

	SIGNIFICANCE LEVEL		
	0.10	0.05	0.01
DF	-2.58(-2.514)	-2.89(-2.826)	-3.51(-3.338)
ADF	-2.58(-2.496)	-2.89(-2.775)	-3.51(-3.297)
DW	N.A. (0.206)	0.259(0.248)	0.376(0.351)
Q4	7.779(8.963)	9.488(11.74)	13.277(16.37)
Q8	13.362(15.11)	15.507(18.50)	20.090(26.48)
Q12	18.549(20.57)	21.026(24.58)	26.217(32.12)
WIM	2.076(3.693)	3.841(4.822)	6.635(6.764)

Note: These critical values are obtained from the tables associated with each test statistics. Values in parentheses are the corresponding values obtained from the null hypothesis. Very often, they are the size-adjusted critical values.

The critical values from published tables represent asymptotic values. We have only 100 observations. The figures in parentheses in table 3.1 represent the size-adjusted critical values obtained from the null hypothesis. These figures will also be used as a reference when counting the numbers of rejections under the different alternatives.

In some sense, these are better critical values because they represent the true size of the tests in the experiment.

Tables 3.2 to 3.4 present the results of the experiment when ρ takes the values of 0.99, 0.95 and 0.90 respectively. The figures in the tables represent the number of rejections in one thousand replications, based on the critical values from the tables. For each alternative, the numbers of rejections at 10%, 5% and 1% significance levels are recorded. Figures in parentheses are the corresponding numbers of rejections using the size-adjusted critical values.

Interpretation of the Empirical Results

The results from the experiment are not encouraging, especially for the information matrix statistic. As expected, the closer the value of ρ to one, the lower the ability of the tests to differentiate the alternatives from the null. When ρ equals 0.99, the power of all tests is no greater than 10% at 0.05 significance level. The situation gets worse when the 0.01 significance level is employed, where no test can pick up the alternative against the null hypothesis more than forty times out of one thousand. These confirm what the finding of many researchers that all the unit root tests have very low power in distinguishing an AR(1) stationary series with a high value of ρ from a random walk process.

Table 3.2

POWER OF THE TESTS (ρ=0.99)

ρ = 0.99	SIGNIFICANCE LEVEL		
	0.10	0.05	0.01
DF	0.107 (0.119)	0.060 (0.065)	0.012 (0.019)
ADF	0.091 (0.108)	0.052 (0.067)	0.008 (0.018)
DW	N.A. (0.133)	0.073 (0.080)	0.014 (0.017)
Q4	0.151 (0.100)	0.087 (0.051)	0.034 (0.009)
Q8	0.152 (0.103)	0.100 (0.049)	0.037 (0.010)
Q12	0.149 (0.103)	0.092 (0.048)	0.035 (0.009)
WIM	0.170 (0.100)	0.094 (0.055)	0.010 (0.010)

Note: The critical values are obtained from various tables corresponding to each statistics. Figures in parentheses are the corresponding numbers of rejections using the size-adjusted critical values obtained from Table 3.1.

Table 3.3

POWER OF TESTS (ρ=0.95)

ρ = 0.95	SIGNIFICANCE LEVEL		
	0.10	0.05	0.01
DF	0.237 (0.276)	0.129 (0.154)	0.029 (0.044)
ADF	0.182 (0.219)	0.097 (0.127)	0.023 (0.041)
DW	N.A. (0.335)	0.186 (0.205)	0.049 (0.067)
Q4	0.176 (0.127)	0.112 (0.068)	0.045 (0.016)
Q8	0.181 (0.132)	0.125 (0.068)	0.051 (0.015)
Q12	0.191 (0.136)	0.121 (0.072)	0.054 (0.016)
WIM	0.163 (0.098)	0.090 (0.051)	0.010 (0.010)

Note: See note under Table 3.2.

Table 3.4

POWER OF THE TESTS (ρ=0.90)

ρ = 0.90	SIGNIFICANCE LEVEL		
	0.10	0.05	0.01
DF	0.528 (0.563)	0.353 (0.393)	0.099 (0.151)
ADF	0.380 (0.431)	0.224 (0.273)	0.054 (0.099)
DW	N.A. (0.709)	0.504 (0.546)	0.170 (0.226)
Q4	0.299 (0.226)	0.203 (0.117)	0.085 (0.047)
Q8	0.316 (0.226)	0.215 (0.142)	0.112 (0.045)
Q12	0.309 (0.236)	0.223 (0.139)	0.113 (0.057)
WIM	0.154 (0.098)	0.091 (0.048)	0.070 (0.007)

Note: See note under Table 3.2.

Results improve significantly, when ρ decreases from 0.99 to 0.95 and even further to 0.90. The numbers of rejections for some of the tests increase up to 200 and 500 out of one thousand, depending on the level of significance used. Among them, the DW statistic has the best performance. Its power is the highest at the 5% significance level when compared to the other statistics.

The size-adjusted figures provide a better picture. The DW statistic dominates all the other tests in most of the situations. When the alternative is an AR(1) process with ρ equals to 0.90, the DW test rejects a random walk more than 70% of the time at the 10% significance level.

This is an encouraging result, for it means that when the alternative is not too 'close' to a random walk, the DW statistic can be used to test against ρ = 1.0 with high power. It would be too demanding, if we ask for a statistic that can differentiate ρ = 0.99 and ρ = 1.00 correctly with a high power.

The DF and ADF statistics improve gradually as ρ decreases. At the 10% significance level, DF can pick up more than half of the alternative when ρ equals to 0.90. In addition, the size-adjusted power of these two tests never drops below 10% when the same significance level is adopted. By comparison, the three Box and Pierce (Q) statistics improve by a smaller extent when the value of ρ decreases. They can only identify about 20% of the alternative if ρ equals 0.90 after adjusting for the size.

The results for the information matrix statistics are quite disappointing. Its power is never higher than 10% and, unlike the other statistics, its power does not improve as ρ decreases from 0.99 to 0.90. In fact, the number of rejections seems to decrease as ρ moves further away from one at all significance levels, both in the size unadjusted and adjusted cases. Though the decrease is comparatively small, it raises doubts about the usefulness of this statistic.

Several reasons may be offered to explain the poor performance of the information matrix statistics. First, as

White (1982) suggested, the statistic will converge to a χ^2 distribution 'asymptotically'. It is possible that a sample size of 100 is not large enough for the statistic to converge to its asymptotic distribution. Second, the choice of alternatives may be a hindrance for the statistic. It is likely that the test has a higher power in differentiating between an ARMA, or MA from a random walk than the case used in the present experiment. Finally, the most obvious answer is the non-stationarity property of the variable. However, the WIM depends only on ϵ_t, and ϵ_t is drawn from a stationary process irrespective of the null or alternative hypothesis. Thus, there is no point in blaming non-stationarity for the poor performance.

Summary

This chapter derives another statistic to test the rational expectations hypothesis. The intuitive idea is to substitute the parameter value given by the null hypothesis into the log-likelihood function, reducing it into a single parameter function that depends only on σ^2. Then the information matrix test is applied based on this function. If the expectation is rational, or in other words, the null hypothesis is true, the statistic should follow a χ_1^2 distribution. A Monte Carlo experiment is conducted to test the ability of this statistic, compared to other tests of co-integration, to differentiate an AR(1) process from a

random walk process. Although the information matrix statistic performs poorly, it seems that the other tests of co-integration, in particular the DW statistic, can serve the purpose moderately well. This provides some confidence in applying these tests to check the existence of a unit root, the rationality of expectations, and the efficiency of forward rate in the foreign exchange rate market in the next two chapters.

The Monte Carlo experiments, though apparently discouraging, lead to interesting conclusions. The information matrix test is a very general test. It is designed to test for all sorts of specification errors: non-normality, heteroskedasticity, lack of co-integration and so on. As such, it is expected to have less power than the DW, DF, ADF tests that are designed for specific alternatives, that is well-specified specification errors. Our Monte Carlo results show that in the case of testing for unit roots, even specific tests like the DW, DF, ADF tests have very low power. Thus, it is not surprising that the information matrix test has still less power. We are led to conclude that in testing for unit root, very general specification tests are not very useful. Our results cast doubt about the general applicability of the information matrix test.

CHAPTER IV
TESTS FOR RATIONAL EXPECTATIONS: EMPIRICAL RESULTS

Introduction

If exchange rates follow a random walk, the lagged value of the exchange rate is the best predictor for the future spot rate. Meese and Rogoff (1983) found that a random walk model performs as well as any structural model in the foreign exchange market. Others, like Somanath (1986), claim that the evidence is not conclusive. As a whole, there does not exist any definite answer to this question.

As mentioned in the previous chapters, whether exchange rates follow a random walk process greatly affects the reliability of the tests of the rational expectations hypothesis. If exchange rates are non-stationary, the legitimacy of many of the tests that have been used becomes questionable. It has been shown that if exchange rate is not stationary, usual test statistics do not follow the standard normal, Student t- or χ^2 distributions. For this reason, testing the existence of a unit root in exchange rates becomes an important prerequisite for any further analysis.

In order to have a better understanding of the controversy over exchange rates, the next section will be devoted to a review of some related research. Since so much work has been conducted before, this review can hardly be exhaustive. Emphasis will be on the random walk nature of exchange rates and tests for the rational expectations hypothesis in the exchange market. In particular, the 'evidence' in favor of rejecting the non-stationarity and irrationality of exchange rates will be reviewed. The purpose is to check whether there are strong enough reasons to turn down these propositions. Finally, in the third section, the empirical results of unit root and co-integration tests on weekly exchange rates are presented. These results suggest possible answers to the questions of whether nominal exchange rates follow a random walk and whether expectations are rational.

Debate on the Dynamics of Exchange Rates

Perhaps the earliest and most influential study on the dynamics of exchange rates is that by Meese and Rogoff (1983), who compare the time series and structural models of exchange rates on the basis of their out-of-sample forecasting accuracy. The models they included were flexible-price and sticky-price monetary models, as well as a sticky-price model that incorporates the current account. The out-of-sample accuracy is measured by mean error, mean

absolute error and root mean square error. Their empirical results show that all the structural models performed poorly. In contrast, a simple random walk time series model predicts exchange rates as well as any of the structural models.

Though Meese and Rogoff (1983) never concluded that nominal exchange rates follow a random walk process, they did identify the unpredictable nature of nominal exchange rates. Others who share a similar opinion are Mussa (1979) who showed that the spot exchange rate follows approximately a random walk, and Frenkel (1981) who showed that the spot exchange rate is highly volatile. More recently, Huang (1984) reported that, in general, random walk models perform better than other models in characterizing exchange rate behavior. Fratianni, Hur and Kang (1987) verified the robustness of the random walk hypothesis using time series of five major currencies; MacDonald and Torrance (1988) have confirmed the existence of a unit root in monthly exchange rate by direct testing. Hakkio and Rush (1989) also confirm that nominal exchange rates are non-stationary. All these show, either explicitly or implicitly, that nominal exchange rates are highly volatile and that it is very likely that they follow a random walk process.

The other side of the coin has also received some attention. Examples claiming that the nominal exchange rate is stationary can be easily found in any international

finance journal, particularly during the seventies. It was not until the mid eighties that some economists started challenging this view. There is still a substantial literature in support of this view. Park (1984), for example, rejected the random walk hypothesis in the foreign exchange market because of a systematic non-random component in the deviation of the current spot rate from the future spot rate. Somanath (1986), responded to Meese and Rogoff (1983) by considering a larger set of structural models. Utilizing both the out-of-sample and in-sample evidence, his results suggested that some structural models can dominate the random walk model in various sample periods. More importantly, he found that including lagged adjustment terms can contribute towards better performance in any models. Hakkio (1986) argued that the exchange rate is stationary but 'close' to a random walk, and he maintained that the low power of all unit root tests in distinguishing these two cases is responsible for the controversy. Frankel and Froot (1986) also rejected the non-stationarity hypothesis for exchange rates, though they concede that the process of exchange rate may be, once again, close to a random walk.

It is interesting to notice that even though many researchers rejected the non-stationarity of nominal exchange rate, they conceded that the rate is very 'close' to a random walk. Moreover, upon examination, some of the evidence appears weak. For example, Park (1984) rejected a

random walk because of the existence of a systematic component in exchange rates, even though all the coefficients of the lagged exchange rates are not significantly different from one when tested using the Dickey and Fuller test. In other words, it is also legitimate to accept the random walk hypothesis if a direct unit root test is used. Somanath (1986) found the random walk model could not dominate the structural models. However, his result also reveals that the ranking of the random walk model is very close to the top. Hakkio (1986) rejected the random walk hypothesis because it implied that the exchange rate has an unbounded unconditional variance. But he agreed that the evidence yields contradictory conclusions. He explained this by pointing to the low power of the unit root tests. Lastly, Frankel and Froot (1986) claimed that the exchange rate will become a non-interesting variable if it followed a random walk process. Like Park (1984), they claimed to discover a systematic relationship in expected depreciation. However, they did not conduct any direct unit root tests. It is probably the case that since unit root tests have low power, the power of an indirect test will be even worse. Others, for example, Hodrick and Srivastava (1984), simply ignored the existence of a unit root and did not conduct any test on this particular issue. We conclude that the evidence rejecting the random walk hypothesis in nominal exchange rate analysis is usually not

strong enough to give a definite answer. We need to conduct our own direct test.

Debate on the Rational Expectations Hypothesis

So far, the conclusions about the rational expectations hypothesis in the foreign exchange market seem less controversial. Most of the research, for instance, Dominguez (1986) and Frankel and Froot (1986) reject the rational expectations hypothesis. Hakkio and Rush (1989) reject it for one market. Lacking an independent set of expectations data, these papers have used the market forward exchange rate as a proxy for expectations. However, it may well be the case that the forward exchange rate contains more than expectations, specifically, a risk premium may also enter. Because this is a very important issue in international finance, we will pay more attention to it in the next chapter and so we postpone a discussion until then. In this chapter we concentrate on the tests using market survey data as a proxy for expectations.

The number of research papers using survey data has increased tremendously in the eighties, and their applications are spread over a wide range of areas in economics; for instance, in consumer behavior, price level forecasts, interest rate expectations and foreign exchange rate expectations. Among these areas, we confine ourselves to the foreign exchange market.

The earliest research that used survey data on foreign exchange rates was Frankel and Froot (1986). The survey data they used came from three different sources; the American Express Banking Corporation; the Economist Financial Report; and the Money Market Services, Inc. Using ordinary least squares estimation, they rejected the rational expectations hypothesis by finding an unconditional bias in the survey errors. Dominguez (1986), who also used forecast data in her work, regressed the actual spot depreciation on the corresponding forecast depreciation. The forecast data used in her paper were from the Money Market Services. Based on these data, she rejected the hypothesis of rationality in four foreign currency markets by finding that the estimated coefficient was significantly different from one. Ito (1988) also rejected the rational expectations hypothesis by using a set of cross-sectional survey data conducted by the Japan Center for International Finance. His results had two major conclusions. First, market expectations are rather heterogeneous, and second, many institutions are not expecting the future rationally.

These three papers constitute the leading research in applying survey data to test the rational expectations hypothesis. Their results are rather homogeneous in that the hypothesis is rejected when applied to the foreign exchange market. However, none of these papers takes account of the statistical consequences of the non-

stationarity of the exchange rate series. Non-stationarity has crucial effects when inference is drawn from the statistics involving that variable. For example, in Frankel and Froot (1986), the number of rejections of rationality decreases sharply if the critical value is obtained from a Dickey and Fuller table instead of the usual Student t-table. In fact, the number of rejections is far less than the number of acceptances if an absolute critical value of 3.00 is used (which is approximately the 95% level from the Dickey and Fuller table). Similar results appear in Ito (1988). None of the one-month and three-month coefficients in his table 4 is significant if the critical value is obtained from the Dickey and Fuller table. Dominguez (1986) has a stronger evidence to support her results. She rejects the rational expectations hypothesis in an overall sense. However, her results show that she can only reject the hypothesis using quarterly data, but not the monthly data. This means that one should be very cautious when interpreting the empirical results.

At the moment, the only paper that utilizes survey data and accepts the rational expectations hypothesis is Taylor (1989). Using individual rather than mean survey data, he cannot reject the null hypothesis of rational expectations because none of the coefficients in his regression is significant. The survey he used is a qualitative data survey, meaning that the data are recorded in a categorical

form. In fact, the participants respond to the survey only by answering whether the exchange rates may go up, down or stay the same twelve months hence. Taylor (1989) then formulated his research by quantifying the qualitative data using the Carlson-Parkin method. He formulated a subjective expectations distribution and used a scaling factor for each individual to obtain a set of aggregate point expectations. All his results are based on these aggregate estimates that at least some of the survey results could pass the tests for rationality. Although there may be some arguments about the way Taylor constructed his data, he demonstrated the robustness of his results by allowing the presence of random measurement errors.

In the next section, we will present our empirical results which use the restricted co-integration tests to test the validity of the rational expectations hypothesis in the foreign exchange market.

Empirical Results

Before we present the results of the restricted co-integration tests, it is necessary to know whether the exchange rate follows a random walk. For this purpose, three unit root tests, the DF, ADF and DW, and three Q statistics, namely Q(4), Q(8) and Q(12), are used. The first three are the standard unit root tests while the Q statistics are included to test the randomness of the

residuals. Although the results in the Monte Carlo experiments show the weakness of the information matrix statistic, we will include it as well. In order to have more confidence on the results of these tests, two different versions of these tests will be used: the unrestricted and restricted test statistics. The former uses the OLS estimated residuals, $\hat{\mu}_t$, from the equation

$$X_t = \alpha + \beta X_{t-1} + \mu_t$$

while the latter uses the restricted residuals defined as

$$\dot{\mu}_t = X_t - X_{t-1}$$

where X_t equals y_t, the nominal exchange rate, or y_t^e, the expected value of y_t. The logarithm of nominal exchange rates is used because it fits best most of the exchange rate models. Hsieh (1984) argued that the only justification to use the logarithmic form is if the exchange rate follows a log-normal distribution. We present results using both the level and the logarithmic form of exchange rate series.

Results of the unit root tests are summarized in tables 4.1 to 4.4. Tables 4.1 and 4.2 present results of testing for unit roots in the spot exchange rate, y_t, while tables 4.3 and 4.4 present the corresponding results for the expected spot exchange rate series, y_t^e. In the tables the

top figures are the restricted test statistics while the figures in parentheses under them are their counterparts using the unrestricted residuals. Notice that none of the three conventional unit root tests in any of the tables has values large enough to reject the null hypothesis of non-stationarity. In fact, all of these statistics are so small that they are far below the critical values. This lends strong support to the proposition that spot exchange rates and their expected values follow a random walk process, no

TABLE 4.1

RESTRICTED (UNRESTRICTED) UNIT ROOT TESTS ON SPOT EXCHANGE RATES

Statistics	CURRENCIES £	DM	SF	¥
1. DW	0.0213 (0.0209)	0.0072 (0.0071)	0.0103 (0.0102)	0.00612 (0.00596)
2. DF	-1.5878 (0.0000)$^{+}$	-1.3136 (0.0000)$^{+}$	-1.2444 (0.0000)$^{+}$	-1.19573 (0.00000)$^{+}$
3. ADF	-1.6191 (-0.1706)	-1.3566 (-0.2046)	-1.3132 (-0.1812)	-1.27130 (-0.30353)
4. Q(4)	0.9270 (0.9107)	1.7647 (1.5900)	1.1378 (1.0867)	3.80696 (3.50184)
5. Q(8)	1.4233 (1.3806)	3.3393 (3.0365)	2.0689 (1.9470)	4.81714 (4.52426)
6. Q(12)	4.4297 (4.4291)	5.0061 (4.7664)	7.6403 (7.5538)	7.26992 (6.96995)
7. WIM	4.9917* (4.7080)*	3.6273 (3.1616)	8.5263* (8.0708)*	7.18138* (7.18862)*

$^{+}$ Value is too small to report.
* Significant at 5% level.
Note: Figures in parenthesis are values for each test using the unrestricted residuals.

TABLE 4.2

RESTRICTED (UNRESTRICTED) UNIT ROOT TESTS
ON THE LOGARITHM OF SPOT EXCHANGE RATES

Statistics	CURRENCIES £	DM	SF	¥
1. DW	0.0212 (0.0209)	0.0066 (0.0064)	0.0089 (0.0088)	0.00490 (0.00474)
2. DF	-1.6145 (-0.0000)$^{+}$	-1.3669 (0.0000)$^{+}$	-1.2670 (0.0000)$^{+}$	-1.28157 (-0.00000)$^{+}$
3. ADF	-1.6271 (-0.1054)	-1.4118 (-0.1498)	-1.3433 (-0.1590)	-1.35576 (-0.31336)
4. Q(4)	0.6654 (0.8134)	2.9799 (2.8956)	1.6990 (1.6647)	5.24761 (4.77394)
5. Q(8)	1.4652 (1.5987)	5.0971 (4.8932)	2.1408 (2.0419)	6.43465 (5.97671)
6. Q(12)	3.9153 (4.1114)	6.5092 (6.2864)	6.4263 (6.3407)	9.51398 (8.96821)
7. WIM	7.1709* (6.4564)*	5.7239* (5.5730)*	8.7964* (8.4112)*	7.15721* (6.98218)*

Note: See note under table 4.1.

matter whether they are measured in the level or logarithmic form. This may seem a bit odd since economists seldom encounter cases where both the variable and its logarithm are random walks. However, the Q statistics generally confirm the white noise properties of the restricted residuals. Almost all of the Q statistics are insignificant as shown in both tables 4.1 and 4.2. This shows that the increments of the spot exchange rate exhibit no serial correlation. Furthermore, these results are not changed

when the restricted residuals are replaced by their unrestricted counterparts. Values of all statistics in parentheses are close to their restricted counterparts. Combining these results together provides clear-cut evidence that the nominal spot exchange rate follows a random walk process.

As a whole, our results provide extremely strong evidence on the non-stationarity of spot exchange rates, both in the level and logarithmic form. In addition, our results show the survey expectations are also non-

TABLE 4.3

RESTRICTED (UNRESTRICTED) UNIT ROOT TESTS ON EXPECTED SPOT EXCHANGE RATES

Statistics	CURRENCIES £	DM	SF	¥
1. DW	0.0200 (0.0199)	0.0075 (0.0074)	0.0090 (0.0090)	0.00588 (0.00580)
2. DF	-1.2627 (0.0000)$^{+}$	-1.2318 (0.0000)$^{+}$	-0.5555 (-0.0000)$^{+}$	-1.25603 (0.00000)$^{+}$
3. ADF	-0.8557 (0.1839)	-0.4928 (0.5322)	-0.5309 (-0.1087)	-0.59586 (0.33404)
4. Q(4)	6.7013 (6.5578)	4.3505 (4.3528)	3.8313 (3.8438)	3.07689 (3.06462)
5. Q(8)	14.0664 (13.6559)	18.8592* (18.7621)*	14.3182 (14.2816)	6.31395 (6.28162)
6. Q(12)	16.0707 (15.6408)	22.0778* (21.9716)*	18.1758 (18.0243)	6.96748 (6.86930)
7. WIM	8.7954* (9.1795)*	0.7460 (0.5861)	0.0012 (0.0187)	6.46704* (5.91558)*

Note: See note under table 4.1.

TABLE 4.4

RESTRICTED (UNRESTRICTED) UNIT ROOT TESTS
ON THE LOGARITHM OF EXPECTED SPOT EXCHANGE RATES

Statistics	CURRENCIES £	DM	SF	¥
1. DW	0.0210 (0.0208)	0.0070 (0.0070)	0.0085 (0.0085)	0.00464 (0.00456)
2. DF	-1.2849 (-0.0000)$^{+}$	-1.0843 (-0.0000)$^{+}$	-0.4965 (0.0000)$^{+}$	-1.14455 (-0.00000)$^{+}$
3. ADF	-0.8695 (0.2551)	-0.3060 (0.6517)	-0.4357 (-0.0359)	-0.33612 (0.54323)
4. Q(4)	6.3661 (6.2494)	1.5237 (1.4805)	1.6571 (1.6106)	2.56690 (2.53612)
5. Q(8)	22.4215* (21.8374)*	23.6633* (23.4625)*	22.0866* (21.9455)*	6.66283 (6.60522)
6. Q(12)	28.0525* (27.2078)*	28.9086* (28.5699)*	29.9494* (29.6674)*	8.56825 (8.44601)
7. WIM	10.7279* (10.6235)*	4.9364* (4.0890)*	3.8557* (3.7502)	7.15269* (5.70230)*

Note: See note under table 4.1.

stationary, though the logarithm of the expectations may be less likely to be. The robustness of these results provides sufficient confidence for us to proceed further with the restricted co-integration tests.

Results of the restricted co-integration tests are presented in tables 4.5 and 4.6. Table 4.5 presents the results using the level of exchange rates and their expectations, while table 4.6 gives their counterparts in the logarithmic form. As in the previous tables, figures in

parentheses are results based on the unrestricted residuals. The first five statistics, DW, DF, ADF, RVAR and UVAR are used to test whether y_t is co-integrated with y_t^e, while the Q statistics are used to test the randomness of the residuals. Note that the null hypothesis for co-integration tests is that the variables are <u>not</u> co-integrated. This is equivalent to saying that a large value of the test

TABLE 4.5

RESTRICTED (UNRESTRICTED) CO-INTEGRATED TESTS IN LEVEL FORM

Statistics	CURRENCIES £	DM	SF	¥
1. DW	1.9397 (1.9467)	1.9269 (1.9609)	1.8420 (1.8467)	1.7498 (1.7698)
2. DF	-13.5792 (-13.6469)	-13.4387 (-13.5238)	-12.2783 (-12.2905)	-12.3170 (-12.3297)
3. ADF	-5.4159 (-5.4982)	-6.0112 (-6.1214)	-5.8968 (-5.9014)	-5.4539 (-5.4587)
4. RVAR	3182.89 (3079.99)	1714.22 (2157.62)	873.19 (994.49)	1777.18 (2021.59)
5. UVAR	3950.89 (3950.89)	2707.09 (2707.09)	1061.01 (1061.01)	2564.35 (2564.35)
6. Q(4)	4.0587 (2.7355)	7.0016 (6.2155)	6.9495 (6.4668)	4.8764 (4.3478)
7. Q(8)	9.6336 (8.0969)	19.8609* (18.5078)*	13.7977 (13.1403)	7.2079 (6.6025)
8. Q(12)	10.4481 (9.4921)	23.2457* (22.3792)*	18.8160 (18.3067)	7.8899 (6.9722)
9. WIM	8.5139* (8.1194)*	0.1552 (0.3613)	1.6293 (2.0117)	9.6953* (10.0522)*

Note: See note under table 4.1.

TABLE 4.6

RESTRICTED (UNRESTRICTED) CO-INTEGRATED TESTS
IN LOGARITHM FORM

Statistics	CURRENCIES £	DM	SF	¥
1. DW	1.8927 (1.9074)	1.9176 (1.9472)	1.8137 (1.8185)	1.7469 (1.7700)
2. DF	-13.1739 (-13.3397)	-13.2730 (-13.3853)	-12.3619 (-12.3867)	-12.3290 (-12.3596)
3. ADF	-5.4171 (-5.6019)	-6.2789 (-6.4363)	-6.0807 (-6.1025)	-5.6250 (-5.6424)
4. RVAR	2870.81 (2917.10)	1727.20 (1992.12)	958.67 (1061.80)	1633.25 (1744.20)
5. UVAR	3753.25 (3753.25)	2533.50 (2533.50)	1172.87 (1172.87)	2216.43 (2216.43)
6. Q(4)	4.6136 (2.3079)	4.7108 (3.9299)	4.6395 (4.0208)	3.9556 (3.3544)
7. Q(8)	15.8527* (13.0631)	21.7234* (20.4002)*	15.2921 (14.6575)	7.6146 (6.8609)
8. Q(12)	17.2410 (14.8813)	26.3354* (24.8650)*	23.1056* (22.3648)*	9.3474 (7.9749)
9. WIM	10.6859* (10.1548)*	1.9079 (1.5538)	2.7424 (3.0406)	5.3225* (5.5841)*

Note: See note under table 4.1.

statistic means a rejection of the null hypothesis and hence 'acceptance' of co-integration between the variables. To simplify the notation, the rejection of the co-integration tests is not indicated in the tables because the tests reject lack of co-integration. This means that nominal exchange rates and their expectations are co-integrated. In

addition, they are co-integrated with a factor one as the statistics are based on the restricted residuals. It should be noticed that figures in parentheses are nearly the same as their corresponding figures, showing no difference in using the restricted or unrestricted residuals. In fact, a careful examination of the auxiliary regressions, which give the unrestricted residuals, reveals that the coefficients of the regressors are very close to one, lending strong support to the hypothesis that the value of co-integrating factor is unity. This evidence strongly confirms the proposition that nominal exchange rates are co-integrated with their expectations with a factor of unity.

However, acceptance of the rational expectations hypothesis requires more than this. The randomness of the residuals is also an important factor. Absence of randomness in residuals means that there exists some way of predicting the future exchange rates, which violates the basic requirement of rational expectations. In this context, the Q statistic serves as an index to the randomness of the residuals since it is designed to capture any serial correlation between the residuals. In table 4.5, other than the Q(8) and Q(12) statistics for the Deutsche mark, which are significant at 5% but not at 1% level, all the Q statistics show no serial correlation in the restricted residuals. All the evidence in table 4.5 favors the conclusion of co-integration with unit factor between

the variables y_t and y_t^e, and a random white noise process for the residuals. In our terminology, these facts imply that the variable y_t is rationally expected by y_t^e and the rational expectations hypothesis in the foreign exchange market cannot be rejected.

Results in table 4.6 are less convincing as more Q statistics are significant at the 5% level. However, if 1% significance level is used, the only significant statistics will be Q(8) and Q(12) for Deutsche mark. As suggested earlier, there is some doubt about the non-stationarity of the exchange rate expectations in logarithmic form. Hence co-integration tests in this case cannot be considered conclusive. Nevertheless, most of the statistics in this table still favor the proposition that expectations are rational and so we have the same conclusion as in the previous paragraph.

Summary

The purpose of this chapter is to examine three important phenomena in the foreign exchange market. The first question is whether the nominal exchange rates y_t and their expectations y_t^e are random walk processes. The answer to this question is in tables 4.1 to 4.4. Almost all the evidence shows that these are non-stationary. The second question is whether y_t is co-integrated with y_t^e, with a co-integrating vector of unity. The answer to this question is

reported in tables 4.5 to 4.6. We see here that the two variables are co-integrated and the values of all the statistics testing co-integration are very close to those using unrestricted residuals. This implies that the restricted residuals successfully reflect the true value and using unity as a co-integrated factor is a correct choice. The last question is whether the residuals are stationary and random. This can be answered by again inspecting tables 4.5 and 4.6. Co-integration between y_t and y_t^e with a co-integrating vector of unity means their difference is a stationary process. Nearly all the Q statistics in the two tables suggest that no serial correlation exists. This is evidence that the residuals follow a white noise process. Combining all three answers together confirms the proposition that the rational expectations hypothesis is accepted in the foreign exchange market.

CHAPTER V
TESTS FOR THE MARKET EFFICIENCY HYPOTHESIS: THEORY AND EMPIRICAL RESULTS

Introduction

It is impossible to study the rational expectations hypothesis in the foreign exchange market without referring to the market efficiency hypothesis. The two are so related that they are two sides of the same coin. Before the availability of a reliable set of survey data, economists took the forward rate as a proxy for the market expectation of the future value of exchange rate. For example, Dornbusch (1976) assumed that the forward rate is an unbiased predictor of the future spot rate, while Cornell (1977) claimed that the forward rate can be used as a proxy for the market expectations. This is natural because, the forward rate was the only available set of data relating to the future spot rate and, it was generated from the market. What economists had in mind was that if the foreign exchange market was efficient, the forward bias, defined as the difference between the forward rate and the corresponding spot rate at its date of maturity, should be unpredictable. This is because every profit opportunity in the market would be closed by the invisible hand and hence the forward rate

would be the same as the market's expectation. This basic logic contributed to the core of the market efficiency hypothesis: if the market is efficient, the difference between forward exchange rate and future spot rate is a random error.

However, a lot of recent empirical evidence showed that the forward exchange rate is a biased predictor of the future spot rate. Baillie, Lippens and McMahon (1983) rejected the hypothesis for six currencies they considered. Hansen and Hodrick (1983) found evidence to reject this hypothesis from the 1920s and the 1970s. Hsieh (1984) claims that his results provided the strongest rejection of the hypothesis ever seen. This evidence is so strong that from the early eighties, there exists only a little argument about the failure of the market efficiency hypothesis in the foreign exchange market. The only disagreement revolves around what causes the failure of the hypothesis.

Different researchers came up with different arguments to explain this failure. After much contention, the debate seems to have resolved down to two possible reasons: either the failure of the rational expectations hypothesis or the existence of a risk premium, or both. Although the number of possible choices has been reduced tremendously, the debate does not seem to have ended. For example, Hansen and Hodrick (1983) found that risk premiums are important in at least two of the five currencies they studied. Fama

(1984) concluded that most of the variation in forward rates is variation in the risk premium and the premium is negatively correlated with the expected future spot rate components of the forward rates. Park (1984) also provided evidence in favor of a risk premium and claimed that it accounts for 10-20% of the total variance in future spot rates. However, Frankel and Froot (1987) claim that the forward bias cannot be attributed to a risk premium. Froot and Frankel (1989), once again, found no sign of risk premium in the bias of forward exchange rate.

In the present chapter, the problem of efficiency in the exchange rate market is discussed. Since the results in chapter four support the rational expectations hypothesis, it is likely that the failure of standard market efficiency tests is due to the risk premium. Using survey expectations data, any forward bias can be decomposed into portions attributable to the risk premium and expectational errors. This decomposition allows us to determine whether a risk premium is the real cause of the failure of the market efficiency hypothesis. The next section will be devoted to explaining how the decomposition helps in testing market efficiency.

Similar to previous chapters, stationarity of the forward rate is emphasized because whether the forward rate has a unit root will affect the reliability of the results from the previous research. Once again, the tests mentioned

in chapter two will be applied to the forward rate. The restricted co-integration tests will be used to test whether the forward rate is co-integrated with the future spot rate with a unit factor and white noise residuals. Previous research documenting the failure of market efficiency suggests that these two variables will not be co-integrated, or at least not co-integrated in the way described.

Nearly all previous work in this area uses one-month or three-month data since forward rates are available only on this basis. The present research differs from this by using data on a weekly basis. This is quite distinctive because the weekly forward rate is not regularly reported. Even so, it can be generated by means of the covered interest parity. We assume that this will be a good proxy because of the profit seeking behavior in the foreign exchange market, where all possible revenue opportunities will be driven away. Details about the covered interest parity and this generated forward rate will be included in the third section. In the final section empirical results of the restricted co-integrated tests will be presented. This will serve as evidence for the failure of the market efficiency hypothesis and the existence of risk premium as a possible answer to its failure.

The Market Efficiency Hypothesis

Let the market expectations of the future spot rate be y_t^e and the rationally expected spot rate conditional on the information set I_{t-1} be $E_{t-1}[y_t/I_{t-1}]$. One way to interpret the rational expectations hypothesis is that y_t^e equals $E_t[y_t/I_{t-1}]$. That is

$$y_t^e = E_t[y_t/I_{t-1}]$$

and a testable form of the hypothesis derived from this relationship is

$$\upsilon_t = y_t - y_t^e$$

The theory states that $E_t[y_t/I_{t-1}]$ differs from y_t only by a random error. Since the results from the last chapter suggest that both y_t and y_t^e are non-stationary, the hypothesis in fact requires the two variables to be co-integrated with a unit factor and υ_t to follow a white noise process.

Another hypothesis called 'no risk premium in the forward rate' states that the forward rate should equal the market expectations, that is, $y_t^f = y_t^e$. Combining these two hypotheses together forms the central core of the market efficiency hypothesis:

$$y_t^f = E_t[y_t/I_{t-1}]$$

Again, a testable implication of this hypothesis can be written in the following form

$$\mu_t = y_t - y_t^f$$

Market efficiency implies that μ_t has zero mean and is uncorrelated with any information in I_{t-1}. The analysis of this is similar to testing the rational expectations hypothesis in chapter four. If y_t^f is also a non-stationary series, the market efficiency hypothesis is equivalent to saying that y_t and y_t^f are co-integrated with a unit factor and the restricted residuals μ_t form a white noise error. Any violation of the above conditions is evidence rejecting the efficiency hypothesis.

Before the availability of a reliable set of survey data, there was no way to separate the rational expectations and the 'no risk premium' hypothesis. The only testable form was a joint hypothesis that involved both y_t and y_t^f, which were the only available data set. This is one possible reason why there is no consensus on the causes of the failure of the hypothesis. In fact, there is no way to identify the risk premium in the analysis and hence no way to prove its existence. It is not until recently that the availability of survey expectations made it possible to

decompose the two hypotheses. Since y_t^e is accessible from the survey, the rational expectations hypothesis can be tested directly. This has already been conducted in chapter four and the results showed no sign of irrationality. Then the 'no risk premium in the forward rate' can also be tested by considering the difference between y_t^f and y_t^e, that is

$$y_t^f - y_t^e$$

By allowing the existence of a random error, the relationship becomes

$$\eta_t = y_t^f - y_t^e$$

The 'no risk premium in the forward rate' requires η_t to follow a white noise process. In particular, if both the variables in the equation are non-stationary, the concept of co-integration can be applied. Any violation of these conditions will become evidence in favor of the existence of a risk premium.

Before the presentation of empirical results, it is necessary to explain how the weekly forward exchange rate is generated. The theory of covered interest parity is employed. Details of this will be discussed in the next section.

Covered Interest Parity

The idea of covered interest parity is very simple. It says that by means of the forward exchange market the return from investing one dollar will be the same whether the dollar is invested in the domestic or the foreign market. Let y_t stand for the spot exchange rate in units of US dollars per unit of foreign currency, and let y_t^e and y_t^f represent a similar exchange rate obtained from the survey and forward market respectively. If an investor deposits one dollar in the US market, his returns on this investment after one period will be $(1 + i_{US})$ dollars, where i_{US} is the interest rate in the United States. However, if he deposits the dollar in a foreign country and covers it through the forward market, his returns will be $y_t^f(1 + i^*)/y_t$ after one period, where i^* is the interest rate in the foreign market. These returns must be equal, that is,

$$(1 + i_{US}) = y_t^f(1 + i^*)/y_t$$

or in terms of y_t^f

$$y_t^f = y_t(1 + i_{US})/(1 + i^*)$$

This provides a formula to calculate the forward exchange rate whenever it is not available. Values of the variables on the right-hand side are easily available in the Wall

Street Journal or the Financial Times. The only problem in this parity condition would be a large transaction cost that prevents the equality of the two investments. However, Hsieh (1984), who used the same method to obtain a series of weekly forward rates, pointed out that the cost of transactions are not large enough to alter any conclusions. Additionally, any existence of a risk premium causes no trouble to the parity as the values of all variables are known in the current period, so that the gain or loss can be known exactly at this moment. This means that there are sufficient reasons to believe the robustness of using the generated y_t^f as a forward rate in later analysis.

Empirical Results

The empirical results are presented in tables 5.1 to 5.6. The first two tables present the statistics of unit root tests on the generated forward rate. Table 5.1 shows the results from testing the hypothesis in the level form while table 5.2 gives the counterparts in the logarithmic form. Using the same terminology as in chapter four, the numbers in the tables are the restricted test statistics and numbers in parentheses are the corresponding unrestricted values. The first three statistics, that is DW, DF and ADF, test the existence of a unit root. None of the statistics in tables 5.1 and 5.2 are significant at 5% level, whether in the restricted or unrestricted form. These results lend

a strong support to the proposition that the forward rate is a non-stationary process. It would be rather surprising if both the spot rate and its expectations are non-stationary but the forward rate is a stationary process. Together with the unit root tests in chapter four, our results strongly suggest that all variables in the foreign exchange market, at least, on a weekly basis, are non-stationary. The Q statistics in tables 5.1 and 5.2 are used to test for serial correlation between the residuals of forward rate and its

TABLE 5.1

RESTRICTED (UNRESTRICTED) UNIT ROOT TESTS ON THE GENERATED FORWARD EXCHANGE RATES

Statistics	CURRENCIES £	DM	SF	¥
1. DW	0.0185 (0.0183)	0.0070 (0.0069)	0.0404 (0.0397)	0.00538 (0.00526)
2. DF	-1.4873 (-0.0000)$^{+}$	-1.5142 (0.0000)$^{+}$	-2.0160 (-0.0000)$^{+}$	-1.26423 (0.00000)$^{+}$
3. ADF	-1.5872 (-0.3796)	-1.5500 (-0.2513)	-1.3495 (2.4327)	-1.36515 (-0.29068)
4. Q(4)	6.7880 (6.7990)	1.7665 (1.5048)	57.1981* (55.2307)*	1.57655 (1.35989)
5. Q(8)	7.5078 (7.5266)	4.4429 (4.0733)	57.5441* (56.1332)*	1.77425 (1.58596)
6. Q(12)	12.3287 (12.2915)	8.4421 (8.2983)	80.0100* (80.0250)*	5.79349 (5.60798)
7. WIM	8.9325* (7.6494)*	4.6552 (4.6310)	3.7118* (5.5148)*	8.18977* (7.33910)*

$^{+}$ Value is too small to report.
* Significant at 5% level.
Note: Figures in parenthesis are values for each test using the unrestricted residuals.

TABLE 5.2

RESTRICTED (UNRESTRICTED) UNIT ROOT TESTS
ON THE LOGARITHM OF GENERATED FORWARD EXCHANGE RATES

Statistics	CURRENCIES £	DM	SF	¥
1. DW	0.0185 (0.0183)	0.0063 (0.0062)	0.0446 (0.0437)	0.00430 (0.00417)
2. DF	-1.5000 (-0.0000)$^{+}$	-1.5670 (0.0000)$^{+}$	-2.1387 (0.0000)$^{+}$	-1.37455 (-0.00000)$^{+}$
3. ADF	-1.5825 (-0.3519)	-1.6091 (-0.1928)	-1.3553 (3.2265)	-1.48566 (-0.32229)
4. Q(4)	5.8413 (5.8548)	2.4359 (2.3241)	69.7092* (65.1210)*	2.16717 (1.78748)
5. Q(8)	6.7272 (6.7769)	6.3839 (6.1910)	71.1988* (66.6002)*	2.30786 (1.94912)
6. Q(12)	10.7412 (10.7935)	8.5794 (8.4622)	88.2966* (87.4711)*	7.89605 (7.39278)
7. WIM	9.6574* (9.7281)*	2.4697 (2.1208)	12.7123* (11.3970)*	12.83453* (11.16364)*

Note: See note under table 5.1.

lagged value. Except for the Swiss Franc, all the other currencies show no sign of serial correlation in the residuals at the 5% significance level. In particular, figures for the restricted and unrestricted statistics are very close to each other, showing that there is no difference in using the restricted or unrestricted residuals in performing these tests. The equivalence of these two residuals gives further confidence about the results of the unit root tests.

The highly significant Q statistics for the Swiss Franc show that this currency is quite different from the others as its residuals exhibit high serial correlation. After a careful examination, these high serial correlations are found to be the consequence of a highly volatile 7-day Swiss interest rate. The sample variances of the interest rate for all five countries, namely, the United States, Great Britain, West Germany, Switzerland and Japan, are 1.2037, 2.8569, 0.7936, 10.7336 and 1.5833 respectively. Notice that the interest rate for Switzerland is at least three and a half times to thirteen times more volatile than in the other countries. Since the interest rate is an important component in covered interest parity, its high volatility has significant effects on the generated forward rate. For this reason, it would be dangerous to draw any conclusion, either positively or negatively, from the results of the Swiss Franc. Nevertheless, the high significance of all the unit root statistics, together with the strong evidence from other currencies, is sufficient to support the proposition that the forward exchange rate is non-stationary.

After determining the existence of a unit root in the forward rate, the concept of co-integration can be applied. The first set of variables to be tested for co-integration is the forward rate and the corresponding spot rate. This is equivalent to testing the market efficiency hypothesis. The results are listed in tables 5.3 and 5.4, with the usual

format that the first table gives the results from using the level of the variables while the second table gives similar results for the logarithmic form. The figures in the tables are statistics obtained from the restricted tests and the figures in parenthesis under them are the unrestricted statistics. As in the tables in chapter four, the first

TABLE 5.3

RESTRICTED (UNRESTRICTED) CO-INTEGRATED TESTS IN LEVEL FORM

Statistics	CURRENCIES £	DM	SF	¥
1. DW	1.3134 (1.3396)	1.2015 (1.4326)	1.8148 (1.9185)	0.7992 (1.1353)
2. DF	-10.8832 (-11.2301)	-9.9509 (-11.4912)	-14.0278 (-14.9760)	-7.4727 (-9.6047)
3. ADF	-4.1219 (-4.4106)	-3.5910 (-4.6170)	-3.8791 (-4.6160)	-2.0349# (-3.2875)#
4. RVAR	103.84 (484.58)	58.67 (842.55)	74.63 (309.91)	52.09 (343.48)
5. UVAR	756.96 (756.96)	1270.84 (1270.84)	352.20 (352.20)	524.56 (524.56)
6. Q(4)	18.8437* (14.1628)*	16.6717* (12.2973)*	37.9994* (31.8220)*	22.7927* (20.3231)*
7. Q(8)	20.1405* (15.3443)	20.2191* (14.9319)	38.9374* (32.8547)*	26.4670* (23.1337)*
8. Q(12)	23.9973* (19.7549)	23.3382* (17.6883)	56.8005* (48.5617)*	31.7049* (26.7408)*
9. WIM	7.5910* (1.8002)	23.0573* (2.1769)	32.8941* (4.4429)*	0.0108 (2.3056)

Note: See note under table 5.1.
Not significant at 5% level.

TABLE 5.4

RESTRICTED (UNRESTRICTED) CO-INTEGRATED TESTS
IN LOGARITHM FORM

Statistics	CURRENCIES £	DM	SF	¥
1. DW	1.2184 (1.3225)	1.4664 (1.4869)	2.0539 (2.0179)	0.9756 (1.1182)
2. DF	-10.2712 (-11.2469)	-11.6631 (-11.8975)	-16.1632 (-15.9829)	-8.5103 (-9.5327)
3. ADF	-3.8574 (-4.5481)	-4.7883 (-5.0074)	-4.8632 (-4.8124)	-2.6105 (-3.3489)
4. RVAR	108.50 (525.83)	64.45 (961.18)	87.57 (316.32)	60.80 (345.63)
5. UVAR	807.76 (807.76)	1456.77 (1456.77)	356.41 (356.41)	538.17 (538.17)
6. Q(4)	21.0980* (10.5440)*	11.5887* (10.3323)*	30.8954* (30.3248)*	21.3194* (19.2380)*
7. Q(8)	22.7571* (11.8103)	15.1395 (13.5730)	32.9769* (32.2937)*	24.9134* (22.1948)*
8. Q(12)	25.7405* (16.1365)	17.6372 (15.9874)	48.7884* (48.1922)*	30.2574* (26.3840)*
9. WIM	0.2539 (7.5707)*	70.3592* (3.0687)	69.8353* (5.4836)*	13.4684* (2.1176)

Note: See note under table 5.1.

five statistics, DW, DF, ADF, RVAR and UVAR, are tests for co-integration under the null hypothesis that no co-integration exists between the two variables. Hence a large value of the statistics will represent a rejection of the null hypothesis or, in other words, an acceptance of co-integration.

Almost all the co-integration statistics in these two tables are large enough to reject the null hypothesis of no co-integration at the 5% significance level. The only exception is the restricted ADF statistics for the Japanese Yen, both in the level and the logarithmic form. However, both of them become significant if the unrestricted statistics are considered instead. In fact, the results in the tables outside and inside the parentheses are rather different. This difference becomes extreme in the case of the RVAR statistics, where the values in parentheses are at least four times higher. The Q statistics also have high significant values, showing serial correlations as being important in the restricted residuals.

These results contain two important points. The forward exchange rate and the corresponding spot rate may be co-integrated, because nearly all the statistics are significant in rejecting the null hypothesis. But it is very likely that they are not co-integrated with a factor one because of the great difference between the restricted and unrestricted statistics. Additional evidence comes from the restricted Q statistics, which have a higher rejection rate than the unrestricted case. This means that constraining the co-integration factor to unity introduces serial correlation in the residuals. This suggests that the forward and spot rate are not co-integrated with a factor one, and their difference is not a random white noise error.

This is equivalent to saying that, as most of the recent research did, the market efficiency hypothesis is rejected.

We can now decompose the forward bias into an expectation error and a risk premium. Using the following formula, the forward bias can be rewritten as

$$y_t - y_t^f = (y_t - y_t^e) - (y_t^e - y_t^f)$$

Since the rational expectations hypothesis is accepted based on the results from chapter four, the difference between y_t and y_t^e would be a white noise error, say ϵ_t. In other words, the equation becomes

$$y_t - y_t^f = \epsilon_t - (y_t^e - y_t^f)$$

Hence, any bias that exists on the left hand side must come from the difference $(y_t^e - y_t^f)$. A test of this hypothesis, denoted as the 'no risk premium' hypothesis, can be written as

$$y_t^e - y_t^f = \upsilon_t$$

If there is no risk premium in the forward exchange market, υ_t must be a white noise error. Any violation of this formulation, either in the coefficient of the forward rate or in the randomness of the error, is evidence of risk

premium and is support for the presumption that risk premium is the major cause for the rejection of the market efficiency hypothesis.

Results of this 'no risk premium' tests are presented in tables 5.5 and 5.6, testing the level and the logarithmic form of the variables. In general, we cannot

TABLE 5.5

RESTRICTED (UNRESTRICTED) 'NO RISK PREMIUM' TESTS IN LEVEL FORM

Statistics	CURRENCIES £	DM	SF	¥
1. DW	0.4104 (0.4172)	0.3816 (0.5486)	1.7573 (1.8407)	0.2545# (0.4748)
2. DF	-5.1249 (-5.1623)	-4.7463 (-5.9825)	-14.1153 (-14.7760)	-3.6879 (-5.2680)
3. ADF	-3.2658 (-3.2931)#	-2.1398 (-2.8676)#	-3.8458 (-4.6563)	-0.8313 (-1.7349)#
4. RVAR	6.3667# (42.1648)	3.3106# (54.7821)	57.3573 (249.9081)	4.4415# (48.4715)
5. UVAR	43.4438 (43.4438)	56.1800 (56.1800)	262.9344 (262.9344)	50.5202 (50.5202)
6. Q(4)	48.2803* (46.2234)*	45.8696* (40.2844)*	44.9390* (44.6178)*	29.1464* (32.9468)*
7. Q(8)	59.9273* (58.1069)*	53.7556* (49.7513)*	45.9077* (45.7182)*	43.0062* (47.6340)*
8. Q(12)	62.6342* (60.5989)*	60.4681* (56.4360)*	78.3148* (77.7783)*	45.5293* (50.3223)*
9. WIM	200.8925* (2.3042)	196.7702* (2.3100)	4.9741* (11.9574)*	17.3664* (0.0044)

Note: See note under table 5.1.
Not significant at 5% level.

TABLE 5.6

RESTRICTED (UNRESTRICTED) 'NO RISK PREMIUM' TESTS IN LOGARITHM FORM

Statistics	CURRENCIES £	DM	SF	¥
1. DW	0.3866 (0.4478)	0.6309 (0.6308)	1.9752 (1.9461)	0.4177 (0.5015)
2. DF	-4.9713 (-5.4873)	-6.3374 (-6.3369)	-15.5185 (-15.2613)	-4.9326 (-5.4782)
3. ADF	-3.1339# (-3.4987)#	-2.8355# (-2.8341)#	-4.8044 (-4.7135)	-1.5026# (-2.0082)#
4. RVAR	6.7686# (45.9360)	3.4698# (60.6551)	65.5005 (265.6032)	6.6391# (60.5095)
5. UVAR	47.1672 (47.1672)	61.6078 (61.6078)	279.2266 (279.2266)	62.8484 (62.8484)
6. Q(4)	55.0510* (42.0222)*	37.5930* (37.4967)*	48.0486* (48.5537)*	26.9205* (28.1553)*
7. Q(8)	71.2517* (59.5246)*	49.6387* (49.6676)*	55.0649* (55.4194)*	37.9044* (38.8121)*
8. Q(12)	75.5385* (63.1332)*	55.7035* (55.7357)*	83.8934* (85.0640)*	39.7021* (40.5636)*
9. WIM	190.2147* (8.6399)*	209.3365* (1.0970)	55.8230* (17.9048)*	87.2403* (0.3741)

Note: See note under table 5.1.
Not significant at 5% level.

reject the null hypothesis of no co-integration based on the ADF and RVAR statistics. In particular, the values of the RVAR statistics change greatly under the restricted and unrestricted case, showing either that the two variables are not co-integrated or that they are co-integrated with a factor other than one. In any case, the 'no risk premium'

hypothesis is definitely rejected. This idea is further confirmed by the Q statistics in both tables. None of them has a value less than 25, showing that a high serial correlation exists between the residuals. This conclusion is not affected by using the unrestricted instead of the restricted residuals, or by taking the variable in the logarithmic form. This provides strong evidence that it is the existence of a risk premium that causes the failure of the market efficiency hypothesis.

Summary

This chapter tested the stationarity of the forward exchange rate and the validity of the market efficiency hypothesis in the foreign exchange market. From chapter four, since we found that both the spot rate and its expectations are non-stationary, it is very likely that the forward rate follows a similar process. Tables 5.1 and 5.2 provide support for this. Another result obtained from chapter four is the acceptance of the rational expectations hypothesis. Because of this, any inefficiency in the exchange market would be the consequence of a risk premium. Tables 5.3 and 5.4 provide evidence for the failure of the market efficiency hypothesis. The last two tables show indirectly the existence of risk premium.

CHAPTER VI
SUMMARY AND CONCLUSION

Several important issues relating to the dynamics of nominal exchange rates are studied in the previous chapters. These include the stationarity of the nominal rates, the validity of the rational expectations hypothesis and the market efficiency hypothesis. Results in chapter four and five confirm clearly that the nominal spot rate, its expected future value and the nominal forward rate are all non-stationary. This is at variance with the results of previous research. This could be partly attributed to the fact that this study uses weekly data, whereas the others use monthly or quarterly data. It has been shown that the distribution of the test statistics will be different from the traditional normal distributions when the variables under consideration are non-stationary. By using the theory of co-integration, the rational expectations hypothesis is reformulated. The hypothesis requires the nominal spot rate and its expected value to be co-integrated with a unit factor, and requires the residuals to follow a white noise process. Based on this result, tests for co-integration are applied to the nominal exchange rate and its expected value.

Four currencies, including the British pound, Deutsche mark, Swiss Franc and Japanese Yen, against US dollar are employed to test rationality and the results are presented in chapter four. All the test results are conclusive. First, all the co-integration statistics reject the null hypothesis of no co-integration, thus suggesting that the nominal spot rate and its expected value are perhaps co-integrated. Second, all the three Q statistics show no sign of serial correlation in the residuals, confirming that the residual series is white noise. Finally, values of all the restricted test statistics are extremely close to their unrestricted counterparts, suggesting that there is no difference in using either one of them. In other words, restricting the co-integration factor to be unity is legitimate. Combining all this evidence helps us to conclude that the rational expectations hypothesis is accepted in the foreign exchange market.

By contrast, the market efficiency hypothesis is rejected when the same procedure is applied to the forward rate and its corresponding future spot rates. Though the two series may still be co-integrated, the residual is not a white noise process. This conclusion is based on the high Q statistics values in tables 5.3 and 5.4. Moreover, the large difference between the restricted and unrestricted test statistics, at least for some of the tests, can be considered as supportive of the fact that restricting the

co-integration factor to be one is invalid. Putting this evidence together suggests that even though there may be a long run relationship between the forward rate and the spot rate, they are not related in a way described by the market efficiency hypothesis.

Earlier studies could not determine satisfactorily the causes of the failure of the market efficiency hypothesis because the forward bias could not be decomposed. With the availability of survey data on expectations, the forward bias can be decomposed into two components, expectations error and risk premium. As the acceptance of the rational expectations hypothesis is evidence in favor of a random expectations error, the only cause of the failure of the market efficiency hypothesis would appear to be the existence of risk premium. This proposition is tested in chapter five and the results are listed under the 'no risk premium' tables. High Q statistics as well as large difference in values between the restricted and the unrestricted tests reject the null hypothesis of no risk premium in the market.

Our empirical results are based entirely on the weekly data which has seldom been used in previous research. This is especially true with the studies of the market efficiency hypothesis as the forward rate is not available on a weekly basis. The difference in the time span used may be one major reason why we get results contradicting earlier work.

In fact, we can observe higher absolute values of the DF and ADF test statistics when the restricted co-integration tests are applied to monthly survey data. It seems fair to say that economic agents can predict accurately within a short time span, but not so accurately when the time span is extended, say, to a month.

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Peter Chi-Wah Liu was born in Hong Kong September 1, 1957. After completing the first year of a three-year diploma program in the Department of Mechanical Engineering in the Hong Kong Polytechnic, he shifted to the Chinese University of Hong Kong in 1978. There he began his undergraduate study in economics for the next four years. In 1982, with a second upper division honors bachelor degree, he continued to pursue his master's degree in the same university.

He joined the Lingnan College of Hong Kong in 1984 as an assistant lecturer. The two-year teaching experience provided him an incentive to pursue the degree of Doctor of Philosophy. In 1986, he quit his job and joined the Department of Economics at the University of Florida as a Ph.D. student. There he received the Rafael Lusky Prize in Economics as the most outstanding first-year graduate student and the Robert Lanzillotti Prize in Economics for the best second-year paper. His expected date of graduation is in December 1989. After that, he will start his job in the Department of Economics at the University of Miami as an assistant professor.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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