RELATIVE PRICES OF OPTIONS, FORWARD CONTRACTS, AND FUTURES CONTRACTS: THEORY AND EVIDENCE

BY

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To my parents

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LIST OF SYMBOLS

t ≡ Initiation date of the forward, futures and option contracts

s ≡ Maturity date of the forward, futures and option contracts, $s>t$

$B(T)$ ≡ Price, at time T, of a default-free discount bond which pays one dollar at s, $t<T<s$

$S(T)$ ≡ Price at time T of the asset on which the contracts are written

$f(T)$ ≡ Futures price at time T

$g(T)$ ≡ Forward price at time T

$g^{*}(T)$ ≡ Forward price inferred from option prices at time T

$c(T)$ ≡ Value of a European call option at time T

$C(T)$ ≡ Value of an American call option at time T

$p(T)$ ≡ Value of a European put option at time T

$P(T)$ ≡ Value of an American put option at time T

$k(T)$ ≡ Difference between the value of a European call and a European put option, i.e., $k(T)=c(T)-p(T)$

$K(T)$ ≡ Difference between the value of an American call and an American put option, i.e., $K(T)=C(T)-P(T)$

E ≡ Exercise price of calls and puts

$D(T)$ ≡ Future value of all dividends to be paid from T to s

$X(T)$ ≡ Difference between futures prices and forward prices, i.e., $X(T)\equiv f(T)-g(T)$

$Z(T)$ ≡ Difference between futures prices and inferred forward prices, i.e, $Z(T)\equiv f(T)-g^{*}(T)$

Abstract of Dissertation Presented to the Graduate School of the University of Florida in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

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Options, forward contracts and futures contracts are traded independently in the market. However, there are some common linkages among them. Some of these linkages are analyzed in this dissertation.

First, equilibrium forward prices are compared to observed futures prices to determine the impact of daily resettlement feature of futures contracts. Using the data on Major Market Index futures, significant differences between futures prices and forward prices are observed during 1985. The differences are too large to be explained by daily resettlement. In fact, the differences are large enough to allow for the existence of "quasi-arbitrage" opportunities between spot and futures markets. However, such opportunities seem to have declined

over time, presumably as a result of the actions of the arbitrageurs.

Second, a specific portfolio of options is compared to a forward contract. It has been shown that if the options are European, this portfolio is equivalent to a forward contract. It is argued in this study that this portfolio closely approximates a forward contract even when the options are American. The argument is even stronger for the case of index options. These options are used to infer forward prices which are then compared with observed futures prices. It is found that the difference between futures prices and <u>inferred</u> forward prices is substantially smaller than the difference between futures prices and <u>equilibrium</u> forward prices. This implies that during 1985, quasi-arbitrage opportunities in options and futures markets, if they existed at all, were less abundant than similar opportunities in spot and futures markets. Partial evidence regarding the existence of quasi-arbitrage opporunties within the options market is also uncovered.

As a by-product of the main analysis, a number of propositions regarding early exercise of American options are analyzed. It is found that dividends do not influence early exercises of index options in the manner suggested by theory for individual stock options.

CHAPTER I
INTRODUCTION

Financial markets have experienced a proliferation of securities in recent years. This propagation process is nowhere more evident than in the options and futures markets where a multitude of novel contracts have been introduced since the mid-1970s. Some of these contracts have gained widespread acceptance in the short span of time that they have been in existence. This is evident from the following table which provides an overview of the trading activity in some of the most popular financial futures and options.

Table 1
Number of Contracts Traded on June 2, 1986

Underlying Instrument	Options Volume	Futures Volume
1. NYSE Composite Index	7514	15400
2. Treasury Bonds	586	310718
3. Major Market Index	77591	7648
4. S&P 500 Index	6300	99929
5. S&P 100 Index	338823	NC
6. Value Line Index	9091	5316
7. Swiss Franc	14592	24516
8. W.German Mark	6924	26978

NC: no contracts available
(Source: The Wall Street Journal, June 3, and June 4, 1986)

Forward contracts do not find a place in this table because of lack of reported data about their trading activity. Nevertheless, they are an important part of the discussion that follows.

In recent years it has been common to find a number of assets on which all three types of contracts--options, futures and forwards--are traded simultaneously. It should not be surprising that some strong interrelationships exist among them stemming directly from the fact that they have the same underlying asset. Some of these interrelationships are anlayzed in this dissertation.

Objective of the Study

The objective of this study is twofold:

(1) To consolidate our knowledge regarding the interrelationships among the following three types of contracts

(a) Options

(b) Forward Contracts

(c) Futures Contracts

(2) To empirically test the interrelationships among these contracts.

Tasks of the Study

The following tasks are undertaken in order to achieve the objectives listed above.

The first issue addressed is the relationship between forward contracts and futures contracts. The major economic difference between the two contracts is that futures contracts are settled daily (i.e., are marked-to-market), whereas forward contracts are settled only at maturity. Several researchers have shown that when interest rates are non-stochastic, forward prices must be equal to futures prices. However, in the presence of stochastic interest rates, daily resettlement can cause futures prices to differ from forward prices.

A number of researchers have empirically investigated the effect of daily resettlement. A drawback of these studies is that they are unable to isolate the economic cause of the difference between futures prices and forward prices, namely, the daily resettlement feature, from the institutional causes such as higher liquidity and guaranteed performance of futures contracts. Moreover, forward contracts present a problem in testing, first, because there is a general lack of good quality data regarding their trading activity, and secondly, because the maturities of forward and futures contracts are difficult to match.

The approach used in this study to solve these problems is to compare futures prices observed in the market to the forward prices based on a simple, yet powerful, arbitrage model. In doing so the effect of daily resettlement can be analyzed by abstracting from the institutional reasons that may cause forward prices to differ from futures prices. A second advantage of this approach is that a large data set can be obtained so that more reliance can be placed on the statistical results. An empirical study is carried out to answer the following questions.

(1) Is there a difference between futures prices and forward prices for stock index contracts?

(2) Is the difference related to marking-to-market?

(3) Is the difference a function of

(a) Time to maturity of the contracts?

(b) Procedure for establishing the settlement price of futures contracts?

(4) Does the difference follow a systematic trend over time?

(5) Are there any arbitrage opportunities in the spot and futures markets related to the difference between futures prices and forward prices?

The detailed analysis and empirical results are presented in Chapter II.

The second task of this study is to analyze the relationship between a forward contract and a specific portfolio of American options called "The American Options Portfolio." It is easy to show that a portfolio of European options, called "The European Options Portfolio" is exactly equivalent to a forward contract on the underlying asset. However, when extended to American options the relationship between a similar options portfolio and a forward contract is less definite because of the possibility that American options may be exercised early.

To determine the importance of early exercise, it is necessary to empirically analyze relevant data regarding early exercises of American options. However, there have been no studies, to my knowledge, which have undertaken this task. I carry out an empirical study to ascertain the importance of early exercise. This study consists of determining the absolute magnitude of early exercises as well as testing some theoretical conjectures regarding early exercise of American options. The detailed analysis and relevant empirical results are presented in Chapter III.

The third task of the dissertation has its roots in the analysis done in Chapter III. It is argued that "The American Options Portfolio" closely approximates a forward contract. Therefore, the relationship between forward and futures contracts can be extended to cover "The American Options Portfolio" and futures contracts. This argument is used to infer forward prices from options and compare them to corresponding futures prices. It is expected that the results obtained in this section will be consistent with those reported in Chapter II. This task is described in detail in Chapter IV.

Chapter V summarizes and concludes the discussion by bringing together all the results that bind together options, forward contracts and futures contracts.

CHAPTER II
FORWARD CONTRACTS AND FUTURES CONTRACTS

Introduction

Forward contracts and futures contracts are quite similar in the sense that both involve buying or selling an asset at a future date for a fixed nominal price determined at the time the contract is written. However, there are some important economic and institutional differences between the two contracts. The economic difference is that futures contracts are settled daily whereas forward contracts are settled only at maturity. The institutional differences vary from contract to contract but, in general, forward contracts are less standardized, have poor secondary markets, and are not guaranteed by the exchange.

Forward Contract

A forward contract is an agreement to buy (or sell) the underlying asset at time s at a price called the forward price, g(t), determined at time t. The investment required at time t is zero and there are no payoffs from the contract until the maturity date s.

Futures Contract

A futures contract is an agreement to buy (or sell) the underlying asset at time s, at a futures price f(t) fixed at time t. Like a forward contract, the investment required for a futures contract at time t is zero.[1] However, unlike a forward contract, the futures contract is marked-to-market at the end of every day. The holder of a long position (the buyer) can withdraw profits at the end of the day, and, in case of a loss, must pay the difference to the seller. Since the profits from a futures contract are realized as they are earned, at maturity the price paid by the buyer to the seller is the spot price prevailing at that time.

By definition both forward price and futures price at maturity equal the spot price, i.e., $g(s)=f(s)=S(s)$.

Previous Research

The relationship between forward and futures contracts has been studied extensively. Black (1976) was one of the first researchers to distinguish between the two contracts by explicitly taking into account the daily resettlement feature

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[1]The margin required for forward and futures contracts is not considered an investment since the investor is assumed to be able to borrow at the riskfree rate. He can buy treasury securities with borrowed funds and deposit them as margin. As a result, the net cost of establishing a position is zero. Moreover, the treatment of margin is different for futures contract as compared to other assets. This fact is explained by Dusak (1973) as follows.

> Unlike other capital assets such as common stocks where the margin is transferred from buyer to seller, the margin on a futures contract is kept in escrow by the broker. Not only does the seller not receive the capital transfer from the buyer but he actually has to deposit an equivalent amount of his own funds in the broker's escrow account. . . . The margin . . . is . . . merely a good faith deposit to guarantee performance by the parties to the contract. (page 1391)

of futures contracts. Since then, other researchers have also focused their attention on this distinction and derived some important analytical results. Margrabe (1978) and Jarrow and Oldfield (1981) have derived a common result that if interest rates are non-stochastic, forward prices should be equal to futures prices. The intuition behind this result is quite simple, as explained below.

As a result of daily resettlement of futures contracts, the investor benefiting from the futures price movement on any given day receives the cash proceeds from the investor holding the opposite position and has the opportunity to invest those proceeds at the prevailing interest rate. The investor holding the opposite position must come up with the requisite cash, presumably by borrowing at the prevailing interest rate. For both investors the future interest rate is an important variable in determining the net benefit due to daily resettlement. Therefore, if there is no uncertainty regarding interest rate that will prevail at each point of time until maturity (i.e., if interest rates are non-stochastic), forward prices must be equal to futures prices. (See Jarrow and Oldfield (1981) for a lucid proof of this proposition).

Cox, Ingersoll and Ross (CIR) (1981) further investigate the effect of stochastic interest rates on the magnitude of the difference between forward prices and futures prices. CIR derive an arbitrage proof to show that the difference between the two prices depends upon the relationship between futures prices and short-term interest rates. If the two variables have positive covariance,[2] then forward prices must be lower than futures prices. The opposite is true if futures prices and short-term interest rates have negative covariance. The magnitude of the difference depends on the magnitude of the covariance between futures price and short-term interest rates and the time to maturity of the contracts.

The intution behind the CIR result is explained well by Klemkosky and Lasser (1985) as follows.

> When the futures price falls, if there is a negative correlation between the futures price and short-term interest rates, the buyer of the contract must borrow for payment to the seller at a higher interest rate than existed when the contract was issued. When the futures price rises, the buyer will be able to invest the resettlement, but at a lower rate. The seller, on the other hand, will be able to invest when rates rise and must borrow when rates fall. (page 610)

CIR (1981) show that if forward prices and futures prices do not behave in this fashion, an arbitrage profit can be obtained by undertaking the following strategy: Buy a forward contract, sell B(j) futures contracts in each period j, liquidate them in the next period and invest the (possibly negative) proceeds into riskfree bonds. This arbitrage

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[2]Strictly speaking, it is the <u>local</u> covariance between percentage changes in futures prices and percentage changes in bond prices that should be either always negative or always positive for this proposition to be meaningful (Cox, Ingersoll, and Ross (1981), page 326). See French (1983) for a discussion of this assumption for empircial testing (page 330, footnote 21).

process prescribes the following relationship between futures prices and forward prices (CIR, Proposition 6, page 326).

$$f(t)-g(t) = PV_t \sum_{j=t}^{s-1} [f(j+1)-f(j)][B(j)/B(j+1)-1]/B(t) \qquad (1)$$

where PV is the present value operator.

In a continuous-time framework this equation reduces to the following equation.

$$g(t)-f(t) = PV_t[\int_t^s f(u)cov(f'(u),B'(u))du]/B(t) \qquad (2)$$

where cov(f'(u),B'(u)) is defined as the local covariance of the percentage change in the futures price, f'(u), and the percentage change in bond price, B'(u).

This result implies that if the local covariance between futures prices and bond prices is positive for every time from t to s, forward prices will be greater than futures prices. Conversely, for negative covariance futures prices will be greater than forward prices. Note that this equation does allow for the possibility that forward prices and futures prices may be equal even when interest rates are stochastic. This is possible if the local covariance between bond prices and futures prices is zero for each period until maturity.

A number of studies have empirically investigated the difference between futures prices and forward prices. Some of them have tested whether the observed differences are in line with the prediction of the CIR model.

Cornell and Reinganum (1981) find that there is no significant difference between forward prices and futures prices on foreign currencies. Since they find that the covariance between short-term interest rates and currency futures prices is negligible, their findings are consistent with the CIR model. Cornell and Reinganum also find that T-bills show greater difference between forward prices and futures prices even though the covariance between short-term interest rates and T-bill futures prices is negligible. This difference is apparently inconsistent with the CIR model. Cornell and Reinganum suggest that the inconsistency may be caused by factors other than marking-to-market. They offer tax treatment of T-bills and problems associated with shorting T-bills as primary candidates for explaining the discrepancy.

French (1983) compares forward prices and futures prices on two commodities--silver and copper--and finds significant differences between them. He finds some support for the CIR model in explaining the differences between the two prices.

Park and Chen (1985) find that there are no significant differences between forward and futures prices on foreign currencies, but such differences are significant for contracts based on physical commodities. They find strong support for the CIR model.

A New Approach to Comparing Forward and Futures Prices

The empirical studies described above have one or both of the following drawbacks.

(1) The data on forward contracts are not only difficult to obtain, they are often of poor quality too. This problem is evident in the studies by French (1983) and Park and Chen (1985).

French compares forward and futures prices which are observed in different countries and at different times and are denominated in different currencies. Park and Chen have problems in getting a large number of observations because forward contracts and futures contracts trade under different conventions. Forward contracts are issued with standard maturity periods, i.e., on every day, a one-month, a three-month, a six-month, and other such contracts are available. On the other hand, futures contracts are traded on the basis of standard maturity dates. Therefore, a three-month futures contract is available only on the day it is initiated or when a longer maturity contract has exactly three months left to maturity. For this reason it is difficult to obtain enough observations for which forward contracts and futures contracts have the same time to maturity.

(2) The second drawback of these studies is their inability to take into account qualitative factors such as higher

liquidity, greater degree of standardization, and guaranteed performance of futures contracts. These studies use forward prices observed in the market, compare them to futures prices and attribute the difference to marking-to-market. Since forward contracts differ from futures contracts along other qualitative dimensions too, it is not clear how the differences observed can be attributed solely to marking-to-market.

In order to isolate the marking-to-market effect forward prices, that are free from these extraneous factors, are needed. It is quite obvious that one cannot hope to observe such "perfect" forward prices in the market. However, they can be determined quite accurately by a simple, yet powerful, arbitrage model.

This well-known arbitrage model of forward prices, sometimes known as the cost-of-carry model, simply says that the forward price of an asset must equal its spot price plus the net costs associated with buying the asset today and holding it until maturity of the contract. If such is not the case then arbitrage will take place.

For a financial asset which provides no intermediate cash flows, the cost associated with holding the asset is simply the interest cost (the opportunity cost of money). Thus, the forward price for such an asset is given by the following model.

$$g(T) = \frac{S(T)}{B(T)} \tag{3}$$

where g is the forward price

S is the spot price of the asset

B is the price of a discount bond which pays \$1 at maturity, s.

If this price does not prevail, an arbitrage profit is available. For example, if $g(T)>S(T)/B(T)$, an investor can buy the asset in the spot market by borrowing the money at the riskfree rate,[3] and short a forward contract on the same asset. This strategy costs nothing and gives a positive payoff of $[g(T)-S(T)/B(T)]$ at maturity. If $g(T)<S(T)/B(T)$, then the strategy is reversed to make a riskless profit.

This model can be easily adjusted for assets that provide intermediate cash flows (e.g., dividends on common stock).

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[3] The assumption that investors can borrow and lend at the riskfree rate is not inordinately restrictive. The following quote from Cox and Rubinstein (1985) attests to that.

> The main reasons private borrowing rates exceed lending rates are transaction costs and differences in default risk. Transactions cost per dollar decline rapidly as the scale increases, so they are of secondary importance in a large operation. And if the arbitrage operation in which we are using these funds is indeed riskless, it should be possible to collateralize the loan so that the lender will bear no possibility of default. (page 40)

Moreover, this assumption does not require that all investors be able to borrow and lend at the riskfree rate. So long as there are a few previleged investors who can do so, the equilibrium forward prices will prevail.

Such cash inflows can be considered as negative carrying costs and the model can be rewritten as follows.

$$g(T) = \frac{S(T)}{B(T)} - D(T) \tag{4}$$

where D(T) is the known[4] future value of all dividends to be paid from T to s

This study proposes that in order to isolate the marking-to-market effect, forward prices, to be used for comparison with futures prices, be based upon the arbitarge model described above. This will eliminate the extraneous factors and also resolve the problem of mismatch of maturities of forward and futures contracts.

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[4]Most companies do not like their dividends to fluctuate dramatically. Hence, dividends on individual companies are, generally, quite predictable. The predictability of dividends is substantially higher for stock indices because fluctuations in the dividends of individual companies are smoothed out when they are aggregated in an index. Therefore, in my opinion, the assumption that dividends on a stock index are known, is reasonable.

Wu (1984) does not agree with this assumption. He argues that index futures prices are affected by uncertainty of dividends. He reports that when this uncertainty is taken into account the theoretical futures prices, given by the cost-of-carry model, decrease significantly. Since this model is used in this study to determine forward prices of stock indices, the forward prices used in this study would be upwardly biased according to Wu. A major result of this study is that futures prices inordinately exceed forward prices leading to quasi-arbitrage opportunities. This argument will become even stronger if Wu's contention is correct.

Selection of an Appropriate Asset

To get the most reliable results from the approach described above, one must be careful in the selection of contracts for empirical testing. Some of the criteria for selection are as follows.

(1) The underlying asset should be such that the cost of buying the asset and carrying it over a period can be determined fairly accurately. This criterion is essential for the equilibrium forward price, given by equation (4), to be measured accurately. On this criterion, all non-financial assets are ruled out since it is difficult to estimate their carrying costs precisely. For financial assets the carrying cost can reasonably be assumed to be the opportunity cost of money measured by the interest rate which is easily observable.

(2) The asset selected should have sufficient liquidity in futures trading so that a large sample can be obtained for reliable statistical testing. A number of financial assets satisfy this criterion as can be seen from Table 1. The most liquid futures contracts are T-Bond futures but they are not given to easy testing of the marking-to-market effect, primarily because of the delivery option associated with them. Stock index futures offer a good alternative since there is no delivery option associated with them.

(3) The choice of a particular index futures contract is dictated by the ease with which the index can be duplicated in the spot market. This criterion is essential for the arbitrage process, that determines the equilibrium forward price, to be successful. In these days of "program trading" it is quite easy to duplicate even a large index such as the S&P 500, but it is even easier to duplicate the Major Market Index (MMI) which consists of only 20 blue-chip stocks.

On the basis of these three criteria, the Major Market Index contracts are chosen for this study.

Data

The data for this part of the study are provided by the Chicago Board of Trade. Included in the data are daily observations on Major Market Index futures contracts for different maturities. The sample period is from January 2, 1985, to December 31, 1985. Usually three or four different maturities are available every day. The futures price used for analysis is the settlement price established at the end of trading every day. The closing values of the Major Market Index are also provided by the Chicago Board of Trade. A total of 945 observations are available for testing.

In order to determine the equilibrium forward price, dividends on the stocks comprising the MMI as well as prices of the discount bonds which mature on the same day as the futures contracts, are required. The dividends are obtained from Moody's Dividend Record and their dollar value is adjusted to the index.[5] The prices of the discount bonds are proxied by the prices of T-Bills which mature one day before the maturity of the futures contracts. The mismatch of one day in the maturity of T-bills and the futures contract is negligible and should not be of any consequence in statistical testing. The T-bill prices are calculated from the yields published daily in The Wall Street Journal.[6]

The maturities of the futures contracts in the sample range from one day to 186 days but maturities greater than 120 days are observed only infrequently.

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[5]Major Market Index (MMI) is an equally-weighted (or price-weighted) index designed to emulate the Dow-Jones Industrial Average. Changes in the index correspond to changes in the sum of the prices of one share each of the MMI's 20 stocks. The prices of 20 stocks are added and the sum is divided by a standard divisor. Periodically, this divisor is adjusted to reflect changes in the capitalization of the 20 companies. During 1985, the divisor value was changed twice--on May 20, 1985 and on December 31, 1985. The second change is of no significance to this study since none of the observations extend beyond the date of the change. The first change, made to reflect a stock dividend by Eastman Kodak, is relevant. On May 20, 1985, the value of the divisor was changed from 4.49699 to 4.41560. The adjustment to the dividends takes this fact into account. For example, a dividend of 65 cents by Procter and Gamble, on January 14, 1985, is divided by 4.49699, but an identical dividend by the same company on July 15, 1985, is divided by 4.41560.

[6]Usually, for the last two days before maturity of the futures contract, yield on a T-bill which expires one day before the maturity of the futures contract, is not available. In this case, the nearest T-bill, which in the sample used in this study is always a T-bill expiring on the thrusday after the maturity of the futures contract, is used. The bias in bond prices caused by this approximation is miniscule because the time to maturity is extremely small.

An interesting aspect of the data is related to the procedure for establishing the settlement price. The concept of settlement price is of special signficance for futures contracts and it is necessitated by the marking-to-market feature of these contracts.[7] The task of establishing the settlement price is easy if trading in the contracts is heavy and some trades take place near the end of the trading session. In such a case the average price of the trades in the last few seconds (usually 20-30 seconds) is used as the settlement price. If, however, the trading in the contracts is thin, the exchange establishes a settlement price which may or may not reflect the true closing price.

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[7] A technical detail may be of interest to some readers. The daily resettlement feature also applies to contracts other than futures contracts, but it gets more prominence for futures contracts. Any position in any security that requires a margin, e.g., a short position in options, is generally marked-to-market. An option seller must deposit additional margin if the balance in the margin account is depleted as a result of the losses. By the same token, he can withdraw the excess balance in the margin account that may result because of profits on the position.

For most conventional options traded on the Chicago Board Options Exchange (CBOE), margin requirements are a function of the price of the underlying asset with an adjustment for the fact tht the option may be in- or out-of-the-money. For some recently introduced options (e.g., option on S&P 500 futures, traded on the Chicago Mercantile Exchange), margin is a function of the option price itself. For such options, the exchange establishes a settlement price for the option.

A hidden reason behind establishment of settlement price is to encourage "spread trading." If there is merit in this argument it is conceivable that CBOE may attempt to change its margin rules and start establishing settlement price even for conventional options.

The data used in this study explicitly distinguish between observations for which there is sufficient activity in the contracts at the end of the day, from those observations for which not enough trades take place at the close of the day.[8] One of the sub-tasks of the study is to find out if there is a difference between observations for which the settlement price reflects the true closing price and those for which the settlement price is established, somewhat artificially, by the exchange. The empirical results based on these data are presented in the next section.

Empirical Results

Direction and Magnitude of the Difference between Futures and Forward Prices

The first task is to examine the extent of the difference between futures prices and forward prices. Since equation (2) implies that the difference, if it exists, is likely to be related to the time to maturity of the contracts, the sample

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[8]The data obtained from the Chicago Board of trade come in a format that allows two closing prices to be published. These two prices describe the closing range. However, two prices are not available if trading at the close of the trading session is thin. In that case, only one closing price is published, and there is no way to determine the time at which the trade took place. Whenever two closing prices are published, it is reasonable to assume that the settlement price reflects the true closing price. Such observations are included in the first subset of observations. Whenever only one price is available, it is indeterminate whether the settlement price would also have been the closing price, if some trades had taken place near the end of the trading session. Such observations are included in the second subset of observations.

is divided into five categories on the basis of this criterion. The variable X is defined to denote the difference between futures prices and forward prices, i.e., $X \equiv f-g$. Some relevant statistics for X are presented in Table 2.

Table 2
Difference between Futures Prices and Forward Prices (in Cents)

Days to Maturity	0-14	15-29	30-59	60-89	≥ 90	Total
Number of observations	128	126	253	219	219	945
$\bar{X}$	12.1	20.5	47.1	81.0	160.1	72.9
t-statistic	3.45^{**}	4.19^{**}	10.5^{**}	15.4^{**}	24.9^{**}	24.6^{**}

**significant at 1% level

It is evident from this table that there is a significant difference between futures prices and forward prices. Futures prices are found to be in excess of fcrward prices by 73 cents on the average. The difference is highly significant as can be judged from the t-statstic of 24.64. As expected, the difference increases with an increase in time to maturity of the contracts. The average difference for contracts with less than 15 days to maturity is 12 cents but for contracts with more than 90 days to maturity, it is as high as 160 cents.

Test of Settlement Price Effect

Having observed significant difference between futures prices and forward prices, the next step is to check whether the difference is real, or due to the fact that some of the settlement prices used in testing may not reflect true closing prices. To carry out this test, the total sample is divided into two subsets.

The first set contains 405 observations for which there are at least two trades in the closing seconds and, therefore, it is almost certain that the settlement price is also the true closing price. This set of observations is loosely designated as the "Set of Liquid Contracts." The second set contains 540 observations for which there are either no trades or only one trade near the end of the trading session and, therefore, the settlement price set by the exchange may not reflect the true closing price. This set of observations is loosely designated as the "Set of Illiquid Contracts." The results obtained from the analysis are presented in Table 3.

Table 3
Difference between Futures Prices
and Forward Prices (in cents)

<table>
<tr><th>Days to Maturity</th><th>Liquid Contracts</th><th>Illiquid Contracts</th></tr>
<tr><td rowspan="3">0-14</td><td>N = 106</td><td>N = 22</td></tr>
<tr><td>$\bar{X}$ = 15.9</td><td>$\bar{X}$ = -6.2</td></tr>
<tr><td>t = 4.39**</td><td>t = -0.69</td></tr>
<tr><td></td><td colspan="2">t = 2.25*</td></tr>
<tr><td rowspan="3">15-29</td><td>N = 100</td><td>N = 26</td></tr>
<tr><td>$\bar{X}$ = 27.7</td><td>$\bar{X}$ = 7.0</td></tr>
<tr><td>t = 5.09**</td><td>t = 0.74**</td></tr>
<tr><td></td><td colspan="2">t = 3.17**</td></tr>
<tr><td rowspan="3">30-59</td><td>N = 117</td><td>N = 136</td></tr>
<tr><td>$\bar{X}$ = 63.2</td><td>$\bar{X}$ = 33.4</td></tr>
<tr><td>t = 8.87**</td><td>t = 6.21**</td></tr>
<tr><td></td><td colspan="2">t = 3.35**</td></tr>
<tr><td rowspan="3">60-89</td><td>N = 53</td><td>N = 166</td></tr>
<tr><td>$\bar{X}$ = 101.7</td><td>$\bar{X}$ = 74.4</td></tr>
<tr><td>t = 9.52**</td><td>t = 12.46**</td></tr>
<tr><td></td><td colspan="2">t = 2.23*</td></tr>
<tr><td rowspan="3">$\geq$90</td><td>N = 29</td><td>N = 190</td></tr>
<tr><td>$\bar{X}$ = 192.9</td><td>$\bar{X}$ = 155.2</td></tr>
<tr><td>t = 22.19**</td><td>t = 22.19**</td></tr>
<tr><td></td><td colspan="2">t = 2.34*</td></tr>
<tr><td rowspan="3">Total</td><td>N = 405</td><td>N = 540</td></tr>
<tr><td>$\bar{X}$ = 56.4</td><td>$\bar{X}$ = 85.3</td></tr>
<tr><td>t = 14.33**</td><td>t = 20.40**</td></tr>
<tr><td></td><td colspan="2">t = 5.03*</td></tr>
</table>

*significant 5% level
**significant at 1% level

Two types of t-statistics are presented in Table 3. First, there is one t-statistic each for liquid contracts and illiquid contracts corresponding to the null hypothesis that the mean value of X is zero for each of the two sets of observations. Second, there is a joint t-statistic which corresponds to the null hypothesis that the mean values of X for liquid contracts and illiquid contracts are equal.

At first glance there seems to be a sharp difference between the two sets of observations. The difference between futures prices and forward prices is consistently higher for liquid contracts as compared to illiquid contracts. Care should be taken in interpreting the average difference for the total sample reported in the last row. Even though illiquid contracts seem to have a higher mean difference, just the opposite is true. The deceptive result in the last row is a result of a sampling bias as illiquid contracts are usually the contracts with long maturities, for which it is natural to observe a greater difference between the two prices.

The preceding results are somewhat puzzling at first. It seems that the settlement price set by the exchange is usually a downward biased estimate of the true closing price. The term "settlement price effect" is used to refer to this apparently systematic discrepancy between liquid cotnracts and illiquid contracts. It seems odd that such a systematic difference should exist between futures prices set by the exchange and those observed in a liquid market. After a thorough analysis, a partial explanation of this phenomenon is uncovered. The apparent "settlement price effect" is partly due to the period to which the observations belong. It so happens that there were more illiquid contracts during the latter part of 1985. During the same period, the difference between futures prices and forward prices declined significantly. This fact is evident from the results presented in Table 4.

Table 4
Difference between Futures Prices and Forward Prices (in Cents)

Quarter	Liquid Contracts	Illiquid Contracts	Total
Quarter 1	N = 137 $\bar{X}$ = 91.5 t = 13.55**	N = 111 $\bar{X}$ = 157.4 t = 17.47**	N = 248 $\bar{X}$ = 120.9 t = 20.63**
Quarter 2	N = 119 $\bar{X}$ = 64.6 t = 9.47**	N = 130 $\bar{X}$ = 130.5 t = 18.97**	N = 249 $\bar{X}$ = 99.0 t = 18.78**
Quarter 3	N = 85 $\bar{X}$ = 39.5 t = 5.47**	N = 165 $\bar{X}$ = 72.8 t = 12.52**	N = 250 $\bar{X}$ = 61.5 t = 13.31**
Quarter 4	N = 64 $\bar{X}$ = -11.6 t = -1.84	N = 134 $\bar{X}$ = -3.1 t = -0.61	N = 198 $\bar{X}$ = -5.9 t = -1.46
Total	N = 405 $\bar{X}$ = 56.4 t = 14.33**	N = 540 $\bar{X}$ = 85.3 t = 20.40**	N = 945 $\bar{X}$ = 72.9 t = 24.64**

**significant at 1% level

The results presented in Table 4 indicate that the apparent "settlement price effect" implied by the results presented in Table 3 may be illusory. The real cause of the difference may, in fact, be the sampling feature that more liquid contracts and fewer illiquid contracts are observed during the time period when the differences between futures prices and forward prices are higher.

It is possible to isolate the "settlement price effect," if it does exist, by eliminating the time-of-the-year effect. However, an attempt to do so is thwarted by the small size of the data at hand.

Test of Daily Resettlement Effect

Having observed significant differences between forward and futures prices, the obvious question is "Are these differences in line with our expectations?" To answer this question, I go back to the CIR model which attmepts to predict the difference between the two prices. The CIR equation is reproduced below.

$$f(t)-g(t) = PV_t \sum_{j=t}^{s-1} [f(j+1)-f(j)][B(j)/B(j+1)-1]/B(t)$$

where PV is the present value operator.

The first noteworthy feature of the equation is the present value operator in front of the parentheses. The lack of a specific arithmetic expression for the present value operator indicates that the model is not set in an equilibrium pricing framework and, therefore, one does not know the discount rate which would adjust for the risk of the payoff given by the expression within the parentheses.

The second important feature of this equation is that it is formulated at time t in terms of future values of variables f (futures prices) and B (bond prices) which are unknown ex-ante (at time t).

These two characteristics, at first, seem to render testing of the model infeasible. However, the following discussion shows that one can still make useful comparisons between the predicted difference between forward and futures prices and that actually observed in the market.

As a first pass, assume that the correct discount rate, to be used for the payoff in question, is the riskfree rate. Also assume, for the moment, that the realized values of the two variables, f and B are exactly in line with ex-ante expectations. Under these two assumptions, the predicted value of the difference between futures and forward price is calculated using ex-post data in equation (1). This variable is denoted by $\dot{X}$. Table 5 presents some relevant statistics for $\dot{X}$.

Table 5
Predicted Difference between Futures Prices and Forward Prices (in Cents)

Days to Maturity	0-14	15-29	30-59	60-89	≥90	Total
Number of observations	116	125	252	218	219	930
Mean(X)	-0.03	-0.04	-0.13	-0.15	-0.42	-0.18
t-statistic	-1.87	-1.84	-5.71**	-4.18**	-8.65**	-10.80**

**significant at 1% level

A comparison of Table 2 and Table 5 reveals startling differences between predicted values of X and those actually observed in the market.[9] Not only is the <u>direction</u> of the difference between forward and futures prices exactly the opposite of that predicted by the model, the difference in <u>magnitude</u> also seems extremely large. For example, for contracts with less than 15 days to maturity, the CIR model predicts that forward prices should exceed futures prices by three-hundredths of one cent. Instead, it is found that futures prices on an average are higher than forward prices by 12.1 cents. For contracts with more than 90 days to maturity, futures prices exceed forward prices by 160 cents on the average whereas the average difference predicted by the CIR model is less than one-half of one cent.

The reader may wonder if the simplifying assumptions used to calculate the predicted values of X may have caused the discrepancy between observed and predicted values of X. The following arguments will show that these two assumptions are unlikely to explain the magnitude of the discrepancy.

First, consider the assumption that the riskfree interest rate can be used for discounting the payoff inside the present value operator in equation (1). We know that this

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[9]The total number of observations, 93[illegible]. reported in Table 5, is smaller than the total number of observations, 945, reported in Table 2. This happens because in calculating the value of predicted X using equation 2, the first observation for every contract with a different maturity is eliminated since it has no lagged value.

payoff is not riskfree and, therefore, should be discounted at a rate higher than the riskfree rate. But a higher discount rate will have no effect on the direction of the difference predicted by the CIR model. It would only affect the magnitude of the predicted difference, and considering the miniscule differences predicted by the model (e.g., one-half of one cent for contracts with greater than 90 days to maturity), this effect would also pale in comparison with the observed differences between the two prices. Hence, it seems that the assumption of using the riskfree rate as the discount rate has virtually no effect on the discrepancy between observed X and predicted X.

The second assumption is somewhat more crucial. It says that the futures prices and bond prices observed in the market are exactly in line with the ex-ante expectations of market participants. It is possible that this this assumption may introduce some bias, but there is no reason to believe that the bias will be systematic. So long as measurement errors caused by this assumption are random, statistical results presented in Table 5 should still be useful. More importantly, an even stronger statement, without utilizing this assumption, is made in the next section to show that the CIR model does not correctly predict the difference between futures and forward prices.

Quasi-Arbitrage Opportunities in the Futures Market

Since futures prices are found to be higher than forward prices by an amount greater than that predicted by the CIR model, and since the CIR model is based on a non-arbitrage condition, it would seem that a profit opportunity exists which can be exploited by taking opposite positions in spot market and futures market. However, the problem with this strategy is that it is not riskfree because futures prices and bond prices in future are unknown. An investor shorting a futures contract can lose on the futures position if futures prices go up, and this loss can be accentuated if interest rates also go up. At the same time, there will likely be a profit on the long spot position since spot prices usually move in tandem with futures prices. But the uncertainty regarding the future values of futures prices, spot prices, and bond prices implies that it is impossible to design a "perfect" (riskless) arbitrage strategy. Nevertheless, it is shown in this study that the market does offer opportunities to design trading strategies which require zero investment and still yield positive payoffs even under some of the most pessimistic scenarios. In order to distinguish such opportunities from "perfect" arbitrage opportunities, I use the term "quasi-arbitrage" opportunities. A "quasi-arbitrage" opportunity is one which requires zero investment but yields a positive payoff even under some of the most pessimistic scenarios.

Given the empirical result that futures prices seem excessively higher than forward prices, the correct strategy for the arbitrageur is to short a futures contract, buy the 20 stocks underlying the MMI using borrowed money and close out the position at maturity. Table 6 describes the payoff from this strategy.

Table 6
Payoff from the Quasi-Arbitrage Strategy

Time	t	t+1	. . .	s
Short one futures	0	$f(t)-f(t+1)$	. . .	$f(s-1)-f(s)$
Buy Spot Asset	$-S(t)$	0	. . .	$S(s)+D$
Borrow	$S(t)$	$f(t+1)-f(t)$	. . .	$-\frac{S(t)}{B(t)} - \sum_{j=t}^{s-2} [\frac{f(j+1)-f(j)}{B(j+1)}]$
	0	0	. . .	$g(s)-g(t)-\sum_{j=t}^{s-1} [\frac{f(j+1)-f(j)}{B(j+1)}]$

This strategy requires zero investment in every period until maturity. The loss on the futures position to the investor is $\sum_{j=t}^{s-1} [(f(j+1)-f(j))/B(j+1)]$ and the profit on the spot position is given by $g(s)-g(T)$. The strategy is profitable if

$$g(s)-g(T) > \sum_{j=t}^{s-1} [(f(j+1)-f(j))/B(j+1)] \tag{5}$$

Before testing inequality (5) in order to examine the possible existence of quasi-arbitrage opportunities, the definition of a pessimistic scenerio is needed. Instead of one, three different definitions based on varying degrees of pessimism are provided and the existence of quasi-arbitrage opportunities is examined under all three definitions. Since the arbitrageur shorts a futures contract, he stands to lose if afutures prices go up and also if interest rates go up.[10] With this in mind the following three defnitions of a pessimistic scenario are given.

(1) Futures prices and interest rates go up every day at a rate such that in 180 days they are 1.5 times their current levels.

(2) Futures prices and interest rates go up every day at a rate such that in 180 days they are twice their current levels.

(3) Futures prices and interest rates go up every day at a rate such that in 180 days they are 2.5 times their current levels.

Inequality (5) is tested for all three definitions using daily data on Major Market Index futures. Out of a total of

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[10]It can be easily shown that the other possibility where futures prices instead go down in every period is an obviously profitable one. To see this simply modify Table 6 for this new scenario. It would be seen that the strategy yields a non-negative payoff in each period, and the cash outflow at maturity is less than the sum of the cash inflows during intermediate periods.

945 observations a substantial proportion satisfy this inequality indicating that they could have been exploited by using the strategy outlined in Table 6. The number of such quasi-arbitrage opportunities and their average profit is given in Table 7.

Table 7
Number of Quasi-Arbitrage Opportunities in Spot and Futures Markets for MMI during 1985

	Definition 1	Definition 2	Definition 3
Number of Opportunities	430	301	249
Average Profit Per Index Unit (Cents)[11]	55.22	49.34	45.56

This analysis shows that there are opportunities for profit which are technically not riskfree but yield a profit even under some severely pessimistic scenerios. The next step is to check if these opportunities are profitable after transactions costs are taken into account.

[11]The face value of a 1985 MMI futures contract is calculated by multiplying the quoted price by 100. Thus, each contract consists of 100 index units. The profit given in Table 7 is cents per index unit. To get profit per contract shorted, these figures should be multiplied by 100. Incidentally, a similar contract with 250 index units (called 'maxi') has become popular recently and the volume in the 100 unit contract has declined sharply.

Transactions Costs and Quasi-Arbitrage

The first thing to note in this regard is that any arbitrage opportunities in the market are likely to be exploited by market professionals. These traders obviously have extremely low transaction costs since they pay no brokerage fee for trading on their own account. The major costs paid by them are mostly fixed in nature, e.g., the cost of obtaining a seat on the exchange. It is assumed here that these fixed cost are allocated to brokerage business for customers. The only relevant costs, then, are the marginal (or variable) costs of undertaking the arbitrage operation.

I know of only one truly variable cost for traders trading on their own account--the cost of clearing the trade through the clearing facility. For the type of arbitrage operation described above, clearing charges will have to be incurred in both futures and spot markets.

The clearing charge per MMI futures contract is 10 cents per side. This implies that per index unit, the cost is one-tenth of one cent. For spot assets, the 20 stocks that make up the MMI, clearing charges are levied by the National Securities Clearing Corporation. Two types of relevant costs are Trade Comparison Fees and Trade Clearance Fees.

Trade Comparison Fees represents the fees to enter trade data. Currently, for each side of each stock trade submitted, the fees is 3.3 cents per 100 shares, with a minimum fee of 6.6 cents and a maximum fee of $1.65.

Trade Clearance Fees represents fees for netting, issuance of instructions to receive or deliver and effecting book-entry deliveries. Currently, there are seven types of clearing fees out of which only the following two are applicable to a simple trade.

1. Receipts from CNS (Continuous Net Settlement) to satisfy a long valued position--45 cents per issue received.
2. Deliveries to CNS in the night processing cycle to cover a short valued position--45 cents per delivery.

Either one of these two types of fees is paid by a brokerage firm one time every day for each stock that it bought or sold during the day, whether for its customers or for its own account. For an arbitrage operation of the kind described in the previous section, the marginal Trade Clearance Fees is zero if the firm trades these stocks for its customers. Since MMI consists of 20 blue-chip stocks it is quite likely that there will a large number of customer trades in these stocks. Thus, it is reasonable to assume that Trade Clearance Fees of the kind mentioned above is zero. Nevertheless, the following calculations are made on the conservative assumption that customers do not trade in the stocks on the day that the arbitradeurs wants to undertake the arbitrage operation, and therefore, trade clearance fees is allocated to the arbitrage operation.

Assuming a hypothetical arbitrage operation of $1 million face value, some estimates are made for transactions costs.

It is worth mentioning that most arbitrage operations used in practice are of much bigger size, thus reducing the per unit cost even more.

July 1, 1985, is arbitrarily chosen as the day for which transactions costs are calculated. The average price of a share for 20 MMI companies on that day is $57.29375. These stocks are to be bought in equal proportion so that the total price is approximately $1 million. The number of shares thus calculated (rounded to nearest 100 to reduce transactions costs), is 900 per stock. The total investment is $1.03 million. The transactions costs are calculated as follows.

Trade Comparison Fees = (3.3 cents x 900/100) x 20
= $5.94
Trade Clearance Fees = (45 cents x 20)
= $9.00
Total Fees = $14.94

One-side fees of $14.94 is simply multiplied by 2 to get the two-way transactions costs of $29.88. To find out transactions costs per index unit, note that a face value of $1,031,287.50 implies 3940.12 index units (base on spot index value of 261.74 at the close of July 1, 1985). Thus, transactions costs per index unit are 2988/3940.12 = 0.76 cents. The round-trip clearing cost for the futures contract is 0.2 cents per index unit, thus giving the total round-trip transactions costs of 0.96 cents per index unit for the whole arbitrage operation.

Comparing these transactions costs with the discrepancy between observed and predicted differences, given in Table 7, it is easy to see that even after taking transactions costs into account, substantial pre-tax profit can still be made.

As regards taxes, there is no need to distinguish between ordinary gains and capital gains for professional traders. The only effect of taxes, then, is that they reduce, but do not eliminate, the profit from quasi-arbitrage.

It is worth noting that the strategy outlined in Table 6 involves buying the underlying stocks which is easier than shorting the stocks. Therefore, none of the objections concerning shorting of stocks apply to this strategy.

Out of curosity, I decide to check whether the transaction costs for ordinary individuals are large enough to eliminate their participation in quasi-arbitrage activities. To get some idea of transactions costs for such individuals, commission schedules for stocks and futures contracts are obtained from two discount brokers.[12]

For the futures contract, a quote of $27 per contract, per side for minimum volume and minimum frequency, is obtained. For larger volume and frequent trades, the cost declines to $12 per contract, per side. For a $1 million face value arbitrage program, the lower rate is applicable.

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[12]The commission schedules used in this study are not claimed to be representative of the market. Moreover, commission rates do vary frequently. Therefore, any results presented in this study should be interpreted accordingly.

Unfortunately, the quoted commission on futures contracts is for the newer and more popular MMI contract (called "Maxi") which contains 250 index units. The 1985 Major Market Index contracts consisted of only 100 units. The transactions costs for the old contracts are almost certainly lower than those specified above. By using these higher transactions costs, this study is being more coservative in its approach to finding quasi-arbitrage opportunities.

The commission schedule on stocks is used to calculate the transaction costs for buying 900 shares of each of the 20 stocks. For all stocks, except A T & T, the commission is $112 plus 0.10 percent of the dollar amount. For A T & T, the commission is $61 plus 0.31 percent of the dollar amount. The total one-way commission thus calculated is $1250.12. The number of index units was calculated earlier to be 3940.12. Hence, per index unit cost for stocks is 31.73 cents per side. Adding to this the per side cost of 12 cents on the futures contract, the total one-way cost is 43.73 cents. The round-trip cost, therefore, is about 87 cents.

Comparing this cost with the average differences mentioned in Table 7, it can be seen that it is much more difficult for ordinary individuals to undertake the quasi-arbitrage operation. It is true that there will be some observations for which the discrepancy will be more than the transactions cost of 87 cents. But the profitability of such opportunities is, at best, substantially reduced and at worst, almost completely eliminated.

Time Trend in Quasi-Arbitrage Opportunities

Given that quasi-arbitrage opportunities existed in the market it seems reasonable to expect that they must have been utilized by discerning investors. It is also likely that actions of such investors would cause these opportunities to disappear over time. Since the quasi-arbitrage opportunities exist because of futures prices being inordinately higher than forward prices, the conjecture is that the difference between the two prices declined over time. To test this conjecture the sample is divided into four quarterly subsets. Table 8 presents the results of the test carried out to test this conjecture.

Table 8
Difference between futures prices and forward prices (in cents)

Quarter / Days to Mat.		Qtr.1	Qtr.2	Qtr.3	Qtr.4	Total
0-14	N =	33	29	33	33	128
	$\bar{X}$ =	26.9	16.2	10.1	-4.3	12.1
	t =	4.48**	2.35*	1.63	-0.55	3.45**
15-29	N =	30	32	32	32	126
	$\bar{X}$ =	62.7	23.0	3.7	-4.6	20.5
	t =	6.55**	2.79**	0.42	-0.56	4.19**
30-59	N =	64	64	65	60	253
	$\bar{X}$ =	105.7	61.1	39.9	-22.3	47.1
	t =	13.46**	9.41**	6.32**	-3.28**	10.5**
60-89	N =	54	61	62	42	219
	$\bar{X}$ =	120.8	118.4	60.3	6.0	81.0
	t =	9.82**	20.40**	8.81**	0.63	15.40**
≥90	N =	67	63	58	31	219
	$\bar{X}$ =	208.2	195.5	148.1	7.1	160.1
	t =	22.19**	23.33**	17.90**	0.57	24.98**
Total	N =	248	249	250	198	945
	$\bar{X}$ =	120.9	99.0	61.5	-5.9	72.9
	t =	20.63**	18.78**	13.21**	-1.46	24.64**

*significant at 5 % level
**significant at 1 % level

The results presented in Table 8 lend support to the conjecture that quasi-arbitrage opportunities in MMI contracts disappeared over time. This is reflected in the narrowing difference between futures and forward prices. The decline from the third to the fourth quarter is dramatic. One possible explanation for this behavior is that "program trading," which became extremely popular during 1985, caused arbitrage opportunities to decline sharply over time. The following quote from a recent news article by Zaslow (1986) supports this hypothesis.

> Arbitrage is one technique often employed by inter-market traders, who frequently swap baskets of large-capitalization stocks for offsetting stock-index futures to take advantage of price discrepencies. . . . Diminishing opportunities in the four-year old S&P 500 futures contract are driving some arbitragers to seek new trading frontiers. . . . With stock prices at their current lofty levels, <u>professional traders find the MMI a cheaper vehicle for arbitrage</u> (emphasis added). "To replicate the S&P index by buying underlying securities you need 40 to 50 stocks" . . . "You could replicate the entire MMI, all 20 stocks, and still get away cheaper."

Conclusion

Significant differences are found between futures and forward prices for MMI contracts during 1985. The differences are significantly higher than those predicted by the CIR model. This discrepancy between observed and predicted differences implies that quasi-arbitrage opportunities existed in MMI spot and futures markets during 1985. The empirical analysis confirms the existence of such quasi-arbitrage opportunities. However, it is also found that these opportunities have since disappeared, presumably because of the actions of the arbitrageurs.

CHAPTER III
OPTIONS AND FORWARD CONTRACTS--THE ISSUE OF EARLY EXERCISE

It has been shown by researchers (for example, Moriarty, Phillips, and Tosini (1981) and Cox and Rubinstein (1985)) that a specific portfolio of European options can be created such that its payoff is identical to the payoff from a forward contract on the underlying asset. However, the same is not quite true if instead the portfolio consists of American options because American options may be exercised before expiration. In this chapter the issue of early exercise and its effect on the relationship between "The American Options Portfolio" and a forward contract are discussed. The empirical analysis is done using daily data regarding actual exercises of MMI options. As a by-product of the main analysis, some well-known theoretical conjectures regarding early exercises of options are also tested.

Forward Contracts and The European Options Portfolio

At time t, "The European Options Portfolio"[13] is created by taking a long position in a European call option and a

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[13]Moriarty, Phillips, and Tosini (MPT), 1981, first described this portfolio as being equivalent to a futures contract under certain restrictive assumptions. Cox and Rubinstein (1985) realizing the nature of one of the restrictions imposed by MPT, correctly describe the portfolio as being equivalent to a forward contract.

short position in a European put option with the same maturity and identical exercise price. To finance this portfolio the investor borrows $k(t)$ so that the investment required at time t is zero (recall that $k(t) \equiv c(t)-p(t)$). The assumption here is that investors can borrow and lend at the riskfree rate.

The payoff from this portfolio, if held until maturity, is given in Table 9.

Table 9
Payoff from The European Options Portfolio

	Initial Cash Flow	Cash Flow at Expiration $S(s)>E$	$S(s)=E$	$S(s)<E$
Buy a call	$-c(t)$	$S(s)-E$	0	0
Sell a put	$+p(t)$	0	0	$S(s)-E$
Borrow	$k(t)$	$-\frac{k(t)}{B(t)}$	$-\frac{k(t)}{B(t)}$	$-\frac{k(t)}{B(t)}$
	0	$S(s)-E-\frac{k(t)}{B(t)}$	$-\frac{k(t)}{B(t)}$	$S(s)-E-\frac{k(t)}{B(t)}$

Intuitively, it is easy to see why this portfolio is like a forward contract on the underlying asset. If it is held until maturity, the holder of the portfolio ends up buying the underlying asset either by exercising the call option, if $(S(s)-E)>0$, or by being forced to buy by the put holder, if $(S(s)-E)<0$. In either case, he owns the underlying asset by paying the exercise price E and repaying the loan whose value at time s is $k(t)/B(t)$. Thus, "The European Options

Portfolio" is like a forward contract with the implied forward price, g^*, given by the following equation.

$$g^*(t) = E + \frac{k(t)}{B(t)} \tag{6}$$

Forward Contracts and The American Options Portfolio

If the portfolio of a long call and a short put is created with American options it is not sufficient to consider the payoff only at maturity because an American option gives its holder the right of early exercise. This causes a distortion in the possible equivalence of a forward contract and "The American Options Portfolio" because if either of the options comprising the portfolio is exercised before expiration, there is an intermediate payoff from the portfolio but no such payoff is forthcoming from the forward contract.

Note that once the equivalence is distorted due to early exercise it cannot, theoretically, be restored by simply buying or selling an identical option because the exercised option is exercised presumably because it is "optimal" for everyone to do so. Under this scenario the open interest in the option should go to zero.

It is quite obvious that the exact equivalence of a forward contract and "The European Options Portfolio" is of little practical interest since almost all traded options are American. The more interesting issue is that of the relationship between "The American Options Portfolio" and a forward contract. In order to make any meaningful statement about this relationship it is imperative that the issue of early exercise be analyzed in greater depth. This task is described in the next section.

Early Exercise of American Options

Previous Research

The theoretical implications of the possibility of early exercise have been discussed by a number of researchers. The most important contribution in this area is by Merton (1973) who developed a number of propositions regarding early exercise of options. Since then, other researchers (e.g., Cox and Rubinstein (1985), Geske and Shastri (1985), Evnine and Rudd (1985)) have elaborated on and supplemented his work.

The major results known regarding early exercise of options are summarized below. For ease of understanding, these propositions are presented assuming the option is written on a share of common stock.

(1) A call option on a stock that pays no dividend before the expiration of the option, should not be exercised before expiration. A call option on a stock that does pay some intermediate dividends may be exericsed early but the only times when it may be optimal to do so are when the stock is about to go ex-dividend.

(2) A put option on a stock which pays no dividend may be exercised early. Theoretically, it may be optimal to exercise a put at any time before expiration, but the

more likely points of time are immediately after the stock goes ex-dividend.

(3) If it is optimal to exercise a call then it is never optimal to leave unexercised an otherwise identical call that has either a lower exercise price or a shorter time to expiration.

(4) If it is optimal to exercise a put then it is never optimal to leave unexercised an otherwise identical put that has either a higher exercise price or a shorter time to expiration.

(5) It is not optimal to exercise an option if a better price can be obtained in the secondary market. By the same token, if it is optimal to exercise an option, its price in the secondary market should be exactly equal to its exercise value.

(6) Assuming that future dividends and interest rates are known, if the present value of all future dividends is less than the present value of the interest that can be earned on the exercise price, the call should never be exercised before expiration. Thus, higher dividends increase the probability of early exercise of call options.

(7) Assuming that future dividends and interest rates are known, if the present value of all future dividends is greater than the present value of interest that can be earned on the exercise price, a put option should not be

exercised early. This proposition implies that higher dividends deter early exercise of put options.

(8) From Propositions (3) and (4) it follows that an option with a longer time to expiration is less likely to be exercised than an otherwise identical option with a shorter time to expiration. Also, an option with smaller exercise value is less likely to be exercised than an otherwise identical option with higher exercise value.

(9) The last proposition has special relevance for index options. The dividend on an index is far more continuous than the dividend on an individual stock. Combining this knowledge with Proposition (7) and Proposition (7) one can see that dividends act as a weaker stimulant for early exercise of index call options than they do for indvidual stock options. Analogously, dividends act as a weaker deterrent to the early exercise of index put options than they do for individual stock options.

Even though the analytical results listed above have been known for quite some time, I know of no empirical research to validate these conjectures. The lack of data may partially explain the lack of research in this area. In order to carry out the second task outlined in the beginning, it is necessary to carry out an empirical study of early exercises of index options--specifically, MMI options.

Data

The data regarding early exercise of MMI options are provided by the Options Clearing Corporation (OCC). These data contain a listing of calls and puts exercised every day during 1985. The exercises are categorized according to exercise price and expiration month. These data are supplemented by other data regarding volume and open interest from The Wall Street Journal and Stock Option Guide respectively.

The data regarding open interest have two deficiencies which need to be pointed out. First, they are available for only 32 of the most popular MMI options contracts. This implies that a small number of contracts are left out. However, the contracts that are so missed are those for which open interest is low. The remaining sample is still of sufficient size to yield good statistical results. The second deficiency is that only weekly observations are available. This compels one to estimate daily open interest by interpolating between two adjacent weekly observations and making an adjustment for the number of options exercised. It is possible that this estimation may create some bias but it is not likely to be systematic. Moreover, a look at open interest data shows that they usually do not fluctuate dramatically over short periods and, therefore, the magnitude of the bias caused by interpolation is likely to be small.

Open interest for each series of calls and puts for each day is estimated by using the following equation.

$$EOI(t) = EOI(t-1) + DAILY(w) - EX(t) \tag{7}$$

where $EOI(t)$ ≡ Estimated Open Interest for day t

$EOI(t-1)$ ≡ Estimated Open Interest for day t-1

$EX(t)$ ≡ Number of Options Exercised on day t

$DAILY(w)$ ≡ Daily factor defined as $(OI(W) - OI(w-1) + TOTEX(w))/5$

$OI(w)$ ≡ Actual Open interest at the end of the current week

$OI(w-1)$ ≡ Actual Open Interest at the end of the previous week

$TOTEX(w)$ ≡ Total Number of Options Exercised during the current week.

The last piece of information needed is the number of "Opening Purchases." This term refers to purchases which give the investor a new position where previously he had none. In contrast to "Opening Purchases," the term "Closing Purchases" refers to purchases which are entered into in order to close out an existing position. The following equation is used to determine the number of "Opening Purchases."

$$OP = (VOL + EX + EXP)/2 \tag{8}$$

where OP ≡ Number of Opening Purchases

VOL ≡ Total Volume (i.e., Total Number of Options Traded)

EX ≡ Number of Options Exercised

EXP ≡ Number of Options that Expired without being Exercised

The number of options that expired without being exercised is estimated by aggregating the open interest in options that are out-of-the-money at expiration. The implicit assumption here is that all options that are in-the-money are either exercised or closed-out in secondary market. This assumption is reasonable since every rational investor will likely exercise an in-the-money option at maturity.[14] Since at any time, data regarding only 32 contracts is available, some observations for variable EXP (Number of Options that Expired without being Exercised) in equation (8) are missed. However, it is not expected to introduce a serious bias since the number of contracts so missed is likely to be small. Moreover, the effect of a small error in estimating EXP is miniscule since the major driving force in equation (8) is Total Volume (VOL).

Empirical Results

The data are analyzed with the objective of determining the importance or non-importance of early exercise of MMI options. This is achieved by two different means. First, some summary statistics regarding the magnitude of early exercise are presented. Second, a test of some of the propositions

[14]Investors for whom transactions costs of exercising are not close to zero may decide to let an option expire without being exercised if transactions costs are more than the exercise value of the option.

regarding early exercise is conducted in order to understand the motivation behind early exercises by investors.

<u>Summary statistics</u>

During the year 1985 a total of 11.17 million MMI options contracts were traded. Of these, 6.44 million were call options and 4.73 million were put options. During the same period a total of 231,272 calls and 115,631 puts were actually exercised. The rest were either closed-out by taking an opposite position or allowed to expire worthless.

To measure the percentage of options that were actually exercised, one should not calculate the number exercised as a percentage of total number of options traded. In order to make a meaningful statement about the magnitude of the options exercised, the number of exercised options should be expressed as a percentage of total "Opening Purchases" for the following reason. An "Opening Purchase" can either be closed-out by taking an opposite position or the option can be exercised or the option can be allowed to expire if it is worthless. Therefore, to find out how many of the <u>originating</u> contracts are indeed exercised it is necessary to take the number of contracts exercised as a percentage of "Opening Purchases."

Using equation (8), it is found that there are a total of 3.70 million "Opening Purchases" for call options and 2.80 million "Opening Purchases" for put options, thus giving a total of 6.50 million "Opening Purchases" for MMI options during 1985.

Using these numbers in conjunction with the early exercise data it is found that 6.25 percent of MMI calls and 4.13 percent of MMI puts were actually exercised during 1985. However, for this study the important variable is not the percentage of options that were exercised, but the percentage of options that were exercised <u>early</u>. The analysis shows that out of a total of 231,272 calls that were exercised, 61.8 percent were exercised on maturity date itself. Out of a total of 115,631 puts exercised, 69.2 percent were exercised on maturity date. This implies that the number of calls and puts that were exercised <u>early</u> is 88,395 and 35,595 respectively. As a percentage of "Opening Purchases," it implies that a mere 2.39 percent of calls and 1.27 percent of puts were exercised early. For both types of options put together 1.91 percent of all opening contracts were exercised early. A summary of these statistics is provided in Table 10.

Table 10
Summary Statistics for
Major Market Index Options (1985)

	(1) Total Volume (millions)	(2) Opening Purchases (millions)	(3) Number Exer-cised	(4) (3) as % of (2)	(5) Early Exer-cises	(6) (5) as % of (3)	(7) (5) as % of (2)
Calls	6.44	3.70	231,272	6.25	88,395	38.2	2.39
Puts	4.73	2.80	115,631	4.13	35,595	30.8	1.27
Total	11.17	6.50	346,903	5.34	124,190	35.8	1.91

From these statistics it is clear that a very small percentage of options are acutally exercised. Not only is the proportion of options exercised small in absolute terms, it is even smaller as compared to marketwide statistics presented in Table 11.

Table 11
Comparison of Early Exercises
for MMI Options Vs. All Others (1985)

	Percent of Opening Purchases						
	MMI Options				All Options		
	Closing Sales	Early Exercises	Exercises at Maturity	Expired	Closing Sales	Total Exercises	Expired
Calls	74.01	2.39	3.86	19.74	58.4	12.5	31.2
Puts	68.89	1.27	2.86	26.98	63.6	13.0	27.3

Note: The figures for "All Options" do not add up to 100 percent for technical reasons. See CBOE brochure entitled Market Statistics, for details.

(Source: Chicago Board Options Exchange and Options Clearing Corporation)

The statstics given in Table 11 show that the magnitude of exercise for MMI options is substantially lower than the magnitude for all options put together. For all options on all exchanges, the percentage of "Opening Purchases" settled by exercising during 1985 is 12.3 percent for call options and 7.5 percent for put options. The comparable figures for MMI options are 6.25 percent and 4.13 percent respectively.

These results are not all that surprising considering the cash settlement feature of index options. Since exercising an index option does not involve physical transfer of the underlying stocks it eliminates investors who otherwise may have wanted to exercise the options in order to obtain the underlying stocks. For call options, there is an additional reason for fewer exercises. The dividend on an index is much more continuous than the dividend on an individual stock and, therefore, the non-exercise condition (given by Proposition 6) is satisfied more easily for call index options.

Even though summary statistics strongly support the view that early exercises are negligible, the data are further examined to test some of the early exercise propositions. Of particular interest is the role of dividends in the decision to exercise since most propositions have it as an important variable.

Test of dividend effect

The first task is to test the dividend effect on exercises as hypothesized by Proposition (1) and Proposition (2). This test should shed light on investors' motivations behind exercising options before expiration.

To carry out this test a distinction is made between "relevant days" and "other days." The term "relevant days" simply means the days for which there is a greater likelihood of early exercise than on other days, other things being equal. From Proposition (1), it can be seen that a "relevant day" for call options is the day just before the stock goes ex-dividend. From Proposition (2), a "relevant day" for put options is the ex-dividend day itself. If investors are indeed swayed by the dividend factor in making the decision to exercise, one should observe a greater percentage of exercises taking place, on an average, on "relevant days" than on "other days."

During 1985, dividends were paid by one or more Major Market Index companies on 59 different days. This implies that there were 59 "relevant days" for calls as well as for puts. For both types of options, the data are divided into two groups. The first group consists of obervations from "relevant days," and the second group consists of all other observations. These data are used to test the null hypothesis that the mean value of percentage of options outstanding exercised on any day is equal for both the groups. The results for this hypothesis are presented in Table 12.

Table 12
Test of Dividend Effect on Option Exercises

% of Calls outstanding exercised on relevant days	% of Calls outstanding exercised on other days	% of Puts outstanding exercised on relevant days	% of Puts outstanding exercised on other days
N = 781	N = 2541	N = 744	N = 2470
μ = 0.99	μ = 0.72	μ = 0.41	μ = 0 55
t = 1.04		t = 0.85	

The t-statistics presented in Table 12 show that the null hypothesis cannot be rejected even at 10 percent level of significance, i.e., it cannot be claimed with a high level of confidence that more exercises take place on "relevant days." Moreover, the results show that for put options the direction of the difference between the two groups is just the opposite of that suggested by theory. A smaller percentage of outstanding puts are exercised on "relevant days" than on "other days" but the difference is statistically insignificant. For call options the difference is in line with that predicted by Proposition (1). A larger percentage of outstanding calls are exercised on "relevant days" than on "other days," but the difference between the two groups is not statistically significant.

These results seem to imply that not only the magnitude of early exercises for index options is negligible, it is also not systematically related to dividends as predicted by Propositions (1) and (2). This does not, by any means, imply that Propositions (1) and (2) are incorrect. It simply means that for the special case of index options they do not hold as strongly as they are expected to, and probably do, for individual stock options.

Choice between closing-out in secondary market and exercising--Role of transactions costs

Propositions (3) and (4) alongwith some other propositions are tested in a regression analysis framework in the next section. Prcposition (5) which is related to the choice between exercising and closing out an option in the secondary market is interesting. It is analyzed in detail next.

The data are examined to see if the price that could have been realized in the secondary market, at the time the option was exercised, is greater than the exercise value (intrinsic value) of the option. In order for the test to be reliable, the time at which the price of the option is observed should coincide with the time the option is exercised. A special feature of index options ensures that the optimal time to make a decision regarding exercising the option is after 4:00 P.M. (Eastern Time) each day. The feature which causes such a phenomenon is cash settlement. When an index option is exercised, there is no delivery of underlying stocks. The holder of the option simply gets the difference between exercise price and the spot index value at the close of the day on which the exercise notice is tendered to the Options Clearing Corporation. It is, therefore, in the interest of the option holder to wait until the stock market has closed and the closing index value has been determined. Since the stock market closes at 4:00 P.M. (Eastern Time) the optimal time to make the exercise decision is after 4:00 P.M.

The next step is to check if there are any trades between 4:00 P.M. and 4:10 P.M. for the options that were exercised during 1985. Since this study focuses exclusively on <u>early</u> exercises, the data for 88,395 calls and 35,595 puts that were exercised early, are examined further. It is found that for 40,862 calls and 15,590 puts a better price in the secondary market was foregone by the investors.

At first glance, such behavior on the part of the investors seems to be irrational. However, there is one important variable that has not been considered yet. So far transactions costs have been ignored on the basis that for professional traders, they are minimal. However, when data on actual exercises is analyzed, it is inappropriate to make such an assumption since a number of these exercises are possibly undertaken by ordinary individuals, who are not immume from transactions costs.

The most common practice regarding transactions costs for exercising an <u>index</u> option is to charge the customer assuming the option had been closed out in the secondary market. For such cases there is no difference between transactions costs for exercising and those for closing out.[15] Therefore, the regular commission is irrelevant for comparing closing-out and exercising. However, there is another type of transaction cost that is still relevant. The bid-ask spread of the dealer is relevant when considering closing-out in the market. Cox and Rubinstein (1985) state that the one-way spread borne by an option investor is less than 1/16 of one dollar. With this in mind, I calculate the number of options for which the price available in the secondary market was higher than

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[15]All four brokerage houses (2 discount brokers and 2 full- service brokers) contacted, follow the practice of charging a customer for exercising an option by assuming that the option had been closed-out in secondary market. The possibility that some brokers may not follow this practice, cannot be denied.

exercise value to cover a bid-ask spread of $1/16. A second scenario envisaged is that of a higher bid-ask spread of $1/8. These calculations are presented in Table 13. The numbers in the second column entitled "Number Analyzable" refer to the number of options for which a trade between 4:00 P.M. and 4:10 P.M. could be traced. The term "Prem." refers to the premium over exercise value of the option.

Table 13
Choice between Exercising and Closing-Out

	(1) # of Early Exercises	(2) Number Analyzable	(3) # for which Prem. >0	(4) (3) as % of (2)	(5) # for which Prem. >1/16	(6) (5) as % of (2)	(7) # for which Prem. >1/8	(8) (7) as % of (2)
Calls	88395	64949	40862	62.9	16502	25.4	13506	20.8
Puts	35595	29070	14768	50.8	7562	26.0	3843	13.2

From the results presented above it can be seen that when bid-ask spread is taken into account, the number of options that apparently should not have been exercised declines sharply. Nevertheless, there are still a few option exercises that cannot be explained even by a bid-ask spread of $1/8.

An interesting phenomenon is discovered for some exercises. There are instances where options are being exercised at maturity even though their exercise value is extremely low, and apparently not enough to cover the transaction costs. Two cases stand out in this regard. On March 16, 1985, a total of 2,678 March 250 calls were exercised at a time when their exercise value was a mere eight cents. Similarly, on May 18, 1985, a total of 10,827 May 255 puts were exercised when their exercise value was eight cents. Since the transaction costs of exercising are usually greater than eight cents (see Chapter IV for details regarding transactions costs) it seems surprising that these options should have been exercised. Three possible explanations exist.

(1) These exercises were undertaken by brokers themselves and, therefore, transactions costs were limited to the nominal sum charged by the OCC.
(2) It is possible that some brokerage houses do not follow the practice of charging their customers for exercises as if they had been closed-out.
(3) These exercises are irrational.

The first two explanations seem more likely, but it is impossible to prove one way or the other since no information is available as to who indeed exercised these options.

Test of other early exercise propositions

Propositions (6), (7), and (8) are tested next in a regression analysis framework. Since Proposition (8) is based on Propositions (3) and (4), the following test also applies to the latter two. These propositions offer the following conjectures.

(1) The greater the exercise value (intrinsic value) of an option, greater the likelihood of its early exercise.

(2) The greater the present value of future dividends minus the present value of interest on exercise price, greater the likelihood of early exercises for call options. For put options, the opposite holds true.

(3) The greater the time to maturity of an option, lower the likelihood of its early exercise.

Based on these three conjectures the following regression model is formulated.

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$$

where $Y \equiv$ Percentage of Outstanding Options Exercised (i.e., $Y \equiv EX(t)/EOI(t)*100$)

$X_1 \equiv$ Exercise Value (Intrinsic Value) of Option (for calls: $X_1 \equiv S-E$, for puts: $X_1 \equiv E-S$)

$X_2 \equiv$ Difference between PV(Dividend) and PV(Interest) (for calls: $X_2 \equiv$ PV(Dividend)-PV(Interest on E) for puts: $X_2 \equiv$ PV(Interest on E)-PV(Dividend)

$X_3 \equiv$ Time to Maturity of the Contract

On the basis of the analytical results listed earlier, a priori, the coefficients are expected to have the following signs.

$$\beta_1 > 0,\ \beta_2 > 0,\ \beta_3 < 0$$

The results from this regression model are presented in Table 14.

Table 14
Results from the Regression
$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$

Y	N	Intercept	X1	X2	X3	R^2
Calls	3322	2.004	0.099	0.326	-0.023	5.5 %
		(11.06)**	(9.99)**	(1.61)	(3.72)**	
Puts	3214	1.849	0.095	0.024	-0.021	6.1 %
		(13.10)**	(12.55)**	(0.16)	(5.16)**	
Total	6536	1.753	0.098	0.158	-0.025	5.8 %
		(17.22)**	(16.29)**	(2.50)*	(9.597)**	

*significant at 5 % level
**significant at 1 % level

All three coefficients have their expected signs. The difference, however, is that "Intrinsic Value" and "Time to Maturity" are good explanatory variables, but the "Difference between Present Value of Dividends and Present Value of Interest on Exercise Price" is not as significant in explaining the magnitude of early exercise. Once again, a relative lack of dividend effect for index options is demonstrated.

Conclusion

In this chapter a number of results regarding early exercises of index options were presented. It was shown that a small number of options are exercised early, and especially for index options, a negligible number are exercised early. Given this information it seems plausible that even "The American Options Portfolio" can closely approximate a forward contract. The next chapter uses this conjecture further for exploring the relationship between options and futures contracts.

CHAPTER IV
FUTURES CONTRACTS AND THE AMERICAN OPTIONS PORTFOLIO

Introduction

The year 1985 will be remembered in financial circles as the year when words like "arbitrage" and "arbitrageurs" became commonly known. Much of the credit for this goes to "program trading" which was increasingly used by arbitrageurs to lock in nearly riskfree profits by taking advantage of the discrepancy in prices between different markets. The Major Market Index contracts were not immune from this phenomenon as this quote from a news article by Zaslow (1986) confirms.

> A growing number of big institutional traders are using a Chicago Board of Trade futures contract and an American Stock Exchange option on the Major Market Index . . . As <u>traders buy or sell underlying MMI stock or options and offset their positions in futures</u>, they are creating some of the most unpredictable price moves yet in the booming stock and stock-index markets. (emphasis added) (April 7, 1986, page 40)

The analysis presented in the preceding chapters provides the tools to explore the relationship between MMI options and MMI futures. From the analysis of early exercises of MMI options presented in Chapter III, it seems that their effect is minimal, and that "The American Options Portfolio" can be used to approximate a forward contract. In that case, the relationship between forward and futures contracts described and tested in Chapter II should also hold true for "The American Options Portfolio" and a futures contract.

Recall that equation (6) provided the definition of inferred forward prices using European options. This equation is now rewritten for American options.

$$g^{*} = E + \frac{K(T)}{B(T)} \tag{9}$$

Forward prices inferred using this equation are compared to futures prices to determine the direction and magnitude of the differences between them. Futher, the possibility of existence of arbitrage opportunities is examined.

Data

To study the relationship between "The American Options Portfolio" and a futures contract, daily observations for all MMI calls and puts traded during 1985 are obtained from Daily Market Publications. A major strength of the data is that they are trade-by-trade with time of trade stamped alongside the price of the option. This feature of the data enables me to minimize the non-synchronicity problem so common in studies using options data.

The non-synchronicity problem arises when the prices of two contracts being compared are observed at different times. The following process is used to minimize the effect of this problem. It is known that trading in the three markets, i.e., the stock market, the options market, and the futures market, stops within a span of 15 minutes. The market for spot asset is the New York Stock Exchange and it stops trading at 4:00 P.M. (Eastern Time). The MMI options, traded on the American Stock Exchange, stop trading at 4:10 P.M. (Eastern Time) and the futures contracts, traded on the Chicago Board of Trade, stop trading at 4:15 P.M. (Eastern Time).

A data set containing the last trade of each call and put for every day in 1985 is created, provided a trade took place between 4:00 P.M. and 4:10 P.M. This means that the maximum discrepancy between the time the option prices and the futures prices are observed, is 15 minutes. Even though, in my opinion, this is within acceptable limits, I double-check by creating two subsets of data--one consisting of only those options for which a trade took place exactly at 4:10 P.M., and another consisting of all other options. The results of this exercise are presented in the next section.

Minimizing the non-synchronicity problem has one drawback in that the deep in- and out-of-the-money and longer maturity options are represented only infrequently in the sample.

Empirical Results[16]

Direction and Magnitude of the Difference between Futures Prices and Inferred Forward Prices

Similar to Chapter II, the first task is to examine the direction and the magnitude of the difference between futures

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[16]The empirical results presented in this section should be interpreted carefully. An unexpected phenomenon, found in later research, seems to weaken these results. See footnote 17 for more details regarding this phenomenon.

prices and <u>inferred</u> forward prices. The data are divided into four categories on the basis of time to maturity. The following table contains some statistics regarding the difference between futures prices and inferred forward prices. The difference $(f-g^*)$ is denoted by variable Z.

Table 15
Difference between Futures Prices and Inferred Forward Prices (in cents)

Days to Maturity	0-14	15-29	30-44	$\geq$45	Total
Number of observations	340	350	254	153	1097
$\bar{Z}$	7.9	11.9	21.2	24.2	14.5
t-statistic	5.55**	6.29**	8.77**	6.40**	13.37**

Comparing the results presented in Table 15 to those in Table 2, a few similarities as well as several important differences are obvious. The two sets of results are similar in that futures prices are found to be greater than forward prices as well as inferred forward prices. In both cases, the difference between the two prices increases with time to maturity. Even though the direction of the results in both tables is identical, the difference in magnitude is unmistakeable. The difference between futures prices and inferred forward prices is substantially smaller than the difference between futures prices and forward prices. To examine the cause of this discrepancy the first possibility examined is the effect of non-synchronicity.

Test of Non-Synchronicity Effect

To examine whether a significant difference is caused by the small degree of non-synchronicity left in the sample, the sample is divided into two sets. The first set contains only those option trades which took place at 4:10 P.M. Since the futures market closes at 4:15 P.M., the non-synchronicity for such trades is 5 minutes. The second set contains the last trade for each call and put series, provided the trade took place beteen 4:00 P.M. and 4:09 P.M. The non-synchronicity for this set ranges from 6 minutes to 15 minutes. Table 16 presents the results of a test of the difference in the value of $\bar{Z}$ for the two groups.

Table 16
Difference between futures prices and inferred forward prices (in cents)

Days to Maturity	4:10 P.M. Trades		4:00-4:09 P.M. Trades
0-14	N = 186 $\bar{X}$ = 8.1 t = 4.57**		N = 154 $\bar{X}$ = 7.7 t = 3.32**
		t = 0.13	
15-29	N = 188 $\bar{X}$ = 11.5 t = 4.57**		N = 162 $\bar{X}$ = 12.4 t = 4.31**
		t = 0.24	
30-44	N = 85 $\bar{X}$ = 19.3 t = 4.79**		N = 169 $\bar{X}$ = 22.1 t = 7.34**
		t = 0.56	
≥45	N = 32 $\bar{X}$ = 13.9 t = 1.61		N = 121 $\bar{X}$ = 26.9 t = 6.43**
		t = 1.36	

*significant at 5% level
**significant at 1% level

The results presented in the Table 16 indicate that the two groups are are very similar. It is not surprising that observations from 4:00 P.M. trades are, on an average, similar to those observed at 4:10 P.M. because an important determinant of option prices, the spot index, is fixed at

4:00 P.M. when the stock market stops trading. Since the two groups are so similar, the following discussion does not distinguish between the two sets.

Time Trend in the Difference between Futures Prices and Inferred Forward Prices

Recall that in Chapter II it was argued that the differences between futures and forward prices are large enough to allow for the existence of quasi-arbitrage opportunities. If a similar argument is be applied to the difference between futures prices and inferred forward prices, it would seem that quasi-arbitrage opportunities, if they exist, may not be as abundant in options and futures markets as they are in futures and spot markets because the differences between futures and inferred forward prices are relatively smaller.

Another argument advanced in Chapter II was that there is a decline in the difference between forward and futures price over time, presumably as a result of the actions of the arbirageurs. A similar reasoning can be applied to inferred forward prices and futures prices. It is expected that the difference between futures prices and inferred forward prices will decline over time but the decline may not be as sharp or consistent since the difference is relatively small to begin with. To test this conjecture the sample is divided into four quarters and the trend in $\bar{Z}$ is observed over time.

Table 17
Difference between Futures Prices and Inferred Forward Prices (in Cents)

Days to Maturity		Qtr.1	Qtr.2	Qtr.3	Qtr.4	Total
	N =	86	70	82	102	340
0-14	$\bar{Z}$ =	7.4	15.3	10.4	1.3	7.9
	t =	3.37**	5.47**	4.13**	0.40	5.55**
	N =	83	81	81	105	350
15-29	$\bar{Z}$ =	14.1	17.8	16.9	1.7	11.9
	t =	2.94**	4.35**	5.34**	0.59	6.29**
	N =	53	47	66	88	254
30-44	$\bar{Z}$ =	23.2	12.4	33.2	15.6	21.2
	t =	3.28**	3.17**	6.82**	4.68**	8.77**
	N =	14	30	55	54	153
$\geq$45	$\bar{Z}$ =	8.8	45.8	22.4	17.9	24.2
	t =	0.67	5.44**	4.04**	2.69**	6.40**
	N =	236	228	284	349	1097
Total	$\bar{Z}$ =	13.4	19.6	19.9	7.6	14.5
	t =	5.18**	8.66**	9.96**	4.03**	13.37**

**significant at 1 % level

As expected the decline in the difference between futures prices and inferred forward price is not as noticeable as the decline in the difference between futures prices and forward prices as reported in Table 8. This is consistent with the conjecture that quasi-arbitrage opportunities in options and futures market, if they existed at all during 1985, were probably not very large to begin with and, therefore, the difference between futures price and inferred price does not decline as dramatically.

Quasi-Arbitrage Opportunities in Options Markets

An interesting aspect of equation (9) is that g^*, the inferred forward price, can be observed from any exercise price so long as the corresponding call and put prices can be observed. It can be easily shown that if two different exercise prices yield two different g^* for European options, arbitrage profit can be made as described below.

Without any loss of generality, assume that $E_1<E_2$. Forward prices inferred from these options are denoted by g_1^* and g_2^* respectively. An arbitrage strategy is described in Table 18 for the case where $g_1^*>g_2^*$. In case $g_1^*<g_2^*$, this strategy can simply be reversed.

Table 18
Arbitrage Strategy Using European Options

	Initial Cash Flow	At Maturity		
		$S(s)>E_2>E_1$	$E_2>S(s)>E_1$	$E_2>E_1>S(s)$
Buy Call 2	$-c_2$	$S(s)-E_2$	-	-
Sell Put 2	p_2	-	$S(s)-E_2$	$S(s)-E_2$
Buy Put 1	$-p_1$	-	-	$E_1-S(s)$
Sell Call 1	c_1	$E_1-S(s)$	$E_1-S(s)$	-
Lend	$-B(E_2-E_1)$	E_2-E_1	E_2-E_1	E_2-E_1
	$\varepsilon>0$	0	0	0

This strategy gives a positive payoff at the beginning and a zero payoff at maturity date. In an efficient market, such opportunities should be eliminated instantly. Thus, for European options, a non-arbitrage condition is that options with two different exercise prices should yield the same g^*.

In chapter III it was argued that the early exercise feature of American options on stock indicies is sparingly utilized. Hence, it seems that even for American options, a change in the exercise price should have a negligible effect on the inferred forward price.

To test this conjecture the data are divided into two subsets. Every day for which inferred forward prices based on two or more different exercise prices are available, is selected for the test. Prices of options with the lowest exercise price are denoted by C_{MIN} and P_{MIN} and those with highest exercise price are denoted by C_{MAX} and P_{MAX}. Based on these two sets of observations, two different values of g^* are calculated using the following equations.

$$g^*_{MIN} = E_{MIN} + \frac{C_{MIN} - P_{MIN}}{B} \tag{10}$$

$$g^*_{MAX} = E_{MAX} + \frac{C_{MAX} - P_{MAX}}{B} \tag{11}$$

To test whether the mean value of g^*_{MIN} is equal to the mean value of g^*_{MAX}, a simple t-test is performed. A new variable, W, is defined to denote the difference between g^*_{MIN} and g^*_{MAX}. The relevant statistics for W are presented in Table 19.

Table 19

Difference between g^*_{MIN} and g^*_{MAX} (in Cents)

Days to Maturity	0-14	15-29	30-44	≥45	Total
Number of observations	116	114	82	38	350
$\bar{W}$	14.4	21.7	21.2	16.9	18.6
t-statistic	5.49**	4.97**	4.63**	1.53	8.07**

**significant at 1 % level

The results presented in Table 19 are somewhat surprising. The variable g^*_{MIN} is higher than g^*_{MAX} for all maturities.

This implies that as exercise price decreases, the inferred forward price, g^*, also increases.[17]

If early exercise is not an issue then an arbitrage as shown in Table 18 should be feasible. Since it was argued in the previous chapter that early exercise is minimal, but not zero, a "perfect" arbitrage strategy seems infeasible. A quasi-arbitrage strategy may be to adopt the strategy outlined in Table 18 and take positions with different brokers. It has been shown empirically that early exercise probability itself is small. Even if that small probability were to become a reality, it is unlikely that it would happen for investor's positions with all brokers. It is, however, difficult to quantify the net effect of this strategy given the subjective factors involved.

Transactions Costs and Quasi-Arbitrage

A casual check of transactions costs for floor traders shows that they are minimal. The only kind of transactions costs that these traders pay are clearing charges to the OCC and any per contarct charges levied by the options exchange. At present, the OCC charges 7.5 cents per contract per side for clearing the trade. Since the strategy outlined in Table

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[17]This unexpected phenomenon reduces the credibility of the results presented in the previous section. It was shown that futures prices, f, exceeded inferred forward prices, g*. Now that we find that g^* varies systematically with E, it is not clear whether the difference between f and g* is real or caused by the exercise prices that happened to be selected for the results presented in Table 15 through Table 19.

18 involves taking a position in four different options, the total one-way transaction costs are 30 cents for one contract in each option. For each index unit it implies a total one-way transactions cost of 0.3 cents. As regards the charges by the options exchange, traders must pay the exchange 6 cents per contract per side for trading on their own account. This implies an additional charge of 24 cents per side charge for trading in four options simultaneously. The addition to the per index unit cost is a negligible 0.24 cents.

From this discussion it is obvious that the transactions costs for floor traders are negligible. Therefore, such costs will not be a deterrent to exploitation of quasi-arbitrage opportunities, if such opportunities exist.

I also check to see if transactions costs for ordinary individuals are low enough for them to consider jumping into the fray to exploit the quasi-arbitrage opportunities. Ignoring early exercise, this strategy is feasible for ordinary individuals if the transactions costs are less than the differences reported in Table 19. The transactions costs are obtained from two discount brokers and two full-service brokers. Since similar results are obtained from different transactions costs data, only one of them is reported here.

Dollar Amount	Commission
$ 0 - 2,000	$18 + 1.8 % of dollar amount
$ 2,001 - $11,000	$38 + 0.8 % of dollar amount
$ 11,001 and over	$98 + 0.25 % of dollar amount

These commission rates are applied to the data on options to see if the arbitrage strategy described in Table 18 is feasible for ordinary individuals. Three different scenerios are envisaged on the basis that the arbitrage strategy is undertaken by taking a position in 10 contracts, 100 contracts, and 1,000 contracts. Table 20 presents the summary of one-way transactions costs for all three scenerios.[18]

Table 20
One-Way Transactions Costs Per Index Unit for Ordinary Individuals using the Strategy Outlined in Table 18 (in cents)

Days to Maturity / No. of Contracts	0-14	15-29	30-44	≥ 44	Total
10	21.2	25.7	28.4	29.9	25.3
100	6.0	7.5	8.4	8.8	7.4
1000	3.3	4.3	5.0	5.4	4.2

Comparing the transaction costs given in Table 20 to the magnitude of difference between g^*_{MIN} and g^*_{MAX} given in Table

[18]Only one-way transactions costs are presented in this table since they are sufficient to make the point here. The round-trip costs can be approximated by noting that out of the four options in which a position is taken, two will expire worthless. For the other two, closing trades will have to be made and transactions costs will have to be incurred. A rough estimate of round-trip transaction costs, therefore, is to multiply the numbers presented in Table 20 by 1.5.

19, it is easy to see that it is much more difficult for ordinary individuals to undertake the operation, unless they are willing to take on large positions. The transactions costs for ordinary individuals are not negligible as they are for floor traders.

Conclusion

It was shown in this chapter that for the sample used, futures prices exceed forward prices inferred from option prices. The difference between the two prices is a function of time to maturity, but not a function of time of trade for observations taken between 4:00 P.M. and 4:10 P.M. A weakness of the results is that they are sensitive to the exercise prices chosen for inferring forward prices. Such sensitivity weakens the results presented in Table 15 through Table 18. However, it proves to be the basis of the discovery of potential quasi-arbitrage opportunities in the options market.

CHAPTER V
SUMMARY AND CONCLUSIONS

This dissertation examines the interrelationships among options, forward contracts and futures contracts. The first issue addressed is the relationship between forward contracts and futures contracts. The controversy about the difference between forward prices and futures prices is addressed from a different perspective using stock index futures. It is shown that during 1985 significant differences existed between equilibrium forward prices and observed futures prices on Major Market Index. The differences were too large to be explained by the CIR model which takes into account the daily resettlement feature of futures contracts. The magnitude of the differences is large enough to allow for the existence of quasi-arbitrage opportunities between spot and futures markets. Even after the transactions costs are taken into account, profitable opportunities exist for professional traders. For ordinary investors, transactions costs are apparently high enough to perclude their participation in arbitarge operations. Further, it is found that during 1985 such opportunities declined over time presumably because of the actions of the arbitrageurs.

The second issue addressed is the relationship between a forward contract and a specific portfolio of options. It is well-known that the two are equivalent if the portfolio consists of European options. It is argued in this study that even when the portfolio consists of American options, it may closely approximate a forward contract. The essence of this argument is that a miniscule proportion of options are exercised early thus reducing the distinction between European and American options. The argument is especially strong for the special case of index options which experience a much smaller incidence of early exercise. It is also shown that for index options, some of the theoretical conjectures regarding early exercises are not very useful in explaining even the small incidence of early exercise. In particular, dividends do not seem to influence early exercise of index options in the manner suggested by theory for individual stock options.

The third issue addressed is the relationship between American options and futures contracts. It is argued that since "The American Options Portfolio" closely approximates a forward contract, it should be related to a futures contract in the same way as a forward contract. In other words, forward prices inferred from option prices should be related to futures just like the eqlibrium forward prices would be. A test of this conjecture cofirms that the direction of the difference between inferred forward prices and futures prices is the same as the difference between equilibrium forward prices and futures prices, but their magnitudes differ significantly. The difference between futures prices and inferred forward prices are much smaller indicating that fewer, if any, quasi-arbitrage opportunities existed between options and futures markets as compared to such opportunities in spot and futures markets. In doing this analysis, it is discovered that a different kind of quasi-arbitrage opportunity may have existed within the options market. This discovery is a result of the observation that option prices based on different exercise prices yield different inferred forward prices. Lower exercise prices yield systematically higher forward prices. If early exercise is not a major problem, as argued in chapter III, then a quasi-arbitrage opportunity seems to exist. However, the gain from this opportunity is not quantifiable.

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BIOGRAPHICAL SKETCH

Gautam Dhingra was born on December 11, 1961, in New Delhi, India. He earned a Bachelor of Commerce (Honors) degree from University of Delhi in 1980 and was awarded the MBA degree in 1982 by the same institution. In 1983, he entered the University of Florida and was awarded a doctoral degree in Finance in August 1986. Upon completion of the degree he joined Hewitt Associates, Lincolnshire, Illinois, as an Investment Analyst.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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